RESEARCH STUDY ON HEALTHCARE MANAGEMENT OPERATIONS AND MARKETING INTERFACE

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Research Study on Healthcare Management Operations and Marketing Interface

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CERTIFICATE OF ORIGINALITY

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Abstract

This thesis studies three topics from the interface of Healthcare Operations Management (HOM) and marketing. Based on the major issues of the healthcare market: healthcare resources delivery, policymaking, and the interaction among multiple entities (such as the hospitals, drug manufacturers, retailers, and patients/consumers), this thesis conducts three detailed studies.

The first study investigates the operation of the inventory sharing mechanism between two independent hospitals considering patients' behavior. When the stockout of rescue medical items happens in the hospital, compared with placing an expensive emergent replenishment order with the dealer, requesting inventory sharing from another hospital with enough stocks could save time and cost. We first identify the inventory decisions without hospitals' inventory sharing action and then derive hospitals' sharing decisions and inventory decisions under the sharing scenario. Through numerical experiments, we find that hospitals benefit from the inventory sharing option rather than the emergent replenishment policy. Furthermore, we investigate the effects of patients' behavior (patient's emergent request rate), the hospital's safety inventory level, and other cost parameters on inventory decisions. Under the sharing policy, when hospital j's emergent request rate or the safety inventory level increases, then hospital i's optimal initial inventory level increases, while the increase of hospital j's initial inventory level causes the decrease of hospital i's optimal inventory level. This study provides more practical suggestions for hospitals' inventory sharing operations.

The second study explores the interaction between retailers' sharing action and return action for unused items in the consignment contract. Hospitals purchase medical supplies from the dealer on consignment contracts. Dealer provides

a return policy for unused inventory but charges a return fee. Two hospitals could share inventory to reduce the amount of return to the dealer. Motivated by this consignment contract policy for the medical supply chain, we develop a framework (a common dealer and two independent retailers) that considers retailers' sharing action and return problems. We aim at developing a coordinating mechanism to manage the retailers' sharing and return action. The dealer-dominated sharing and retailer-dominated sharing are compared from the perspective of sharing performance and expected profits. We analyze the condition that the dealer is better off from retailers' sharing when the dealer has the power to encourage retailers' sharing. We further explore the dealer's trading preference for an individual retailer or cooperative retailers when the dealer has no power to encourage retailers' sharing. Numerical experiments are conducted to examine the sensitivity of retailers' sharing decisions, retailers' profits, and dealer's profit to the return price.

In the third study, we develop a three-echelon model to study the interaction among the upstream manufacturer, the downstream pharmacy benefit manager (PBM), and the consumers in the pharmaceutical supply chain. The PBM provides a drug at the wholesale price to the insurance company and benefits as an intermediary between the drug manufacturer and the insurance company. To increase the influence on distribution channels, the drug manufacturer considers the vertical integration with the PBM based on the direct retail strategy. First, we obtain the equilibrium solution of drug quality decision and pricing decision when there exist the direct retail channel and vertical integration channel simultaneously. Second, from the manufacturer's perspective, we consider the quality differentiation level between the vertical integration channel and the direct retail channel. Then, we divide the integration cases into low-quality and high-quality vertical integration cases. Furthermore, we examine the effect of quality differentiation level on consumer performance (co-payment and demand for the drug) and the joint profit of the PBM and manufacturer. We find that consumers do not always choose the drug with high-level quality from the vertical integration channel. The co-payment level also increases with the increasing quality differentiation level. Additionally, the joint profit of the manufacturer and PBM does not always decrease with the increasing manufacturing cost. As the quality differentiation level increases, PBM sets a higher retail price to offset the increased manufacturing cost. This study provides some managerial suggestions for pharmaceutical implications through numerical experiments.

Publications Arising from the Thesis

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Chapter 1

Introduction

1.1 Background and Motivation

Healthcare Operations Management (HOM), as an emerging research direction of operations management, has attracted more attention from academia and industry. Especially with the outbreak of Covid-19, more operational challenges arise and need timely solutions. After the 21st century, the research on HOM focuses on the operational issues that examine the interaction among multiple entities (e.g. hospitals, drug manufacturers, retailers, insurance companies, and patients/consumers) in the healthcare ecosystem, which is referred to as HOM 2.0, while HOM 1.0 analyzes and improves the operation of a single entity (hospital or other medical organizations) (Dai and Tayur 2020). In the recent HOM literature review, Dai and Tayur (2020) provide a taxonomy of the previous research directions of healthcare operations. There are four major components of the healthcare ecosystem: delivery, financing, policymaking, and innovation. Similarly, in a literature review of empirical research on healthcare operations, Kc et al. (2020) divide the burgeoning field of HOM into three research parts: the allocation of healthcare resources, the delivery system designing, and the innovative technologies & models designing. As a whole, there are three divisions in emerging healthcare operations management: (1) the delivery of healthcare resources in the HOM; (2) the policymaking in the HOM; (3) the interaction among multiple entities in the HOM.

The first issue of HOM is the allocation and delivery of healthcare resources, which is to ensure that patients obtain treatment or care from the provider efficiently. Much academic research pays attention to the mainstream directions, such as the patients' admission for walk-in patients (Barz and Rajaram 2015, Meng et al. 2015, Samiedaluie et al. 2017) and emergency department patients (Carmen et al. 2018, Guo et al. 2016, Niyirora and Zhuang 2017), operating room scheduling (Cardoen et al. 2010, Eun et al. 2019, Guido and Conforti 2017), and ambulance dispatching (Erkut et al. 2008, Knight et al. 2012, Enayati et al. 2018). These studies have provided managerial insights and practical guidelines for improving the delivery efficiency of healthcare services. However, there still exists an imbalance of healthcare resources. Especially when some urgent situations occur (e.g., traffic accidents, infectious disease outbreaks), the demand for medical items increases highly, and some hospitals face stockout while some hospitals have enough stock. Therefore, in our first study, we develop the inventory sharing policy between two hospitals based on the expensive emergency replenishment policy. We aim to explore the feasibility of the sharing policy and its effects on the hospital's original operations.

The second issue is the policymaking of healthcare operations; many operational policies are updated or proposed to decrease the healthcare operations cost. In the United States, almost 1/3 of the medical expenditures occur in hospitals (Martin et al. 2019). Therefore, many studies focus on the cost-saving operational policies in hospital operations management (Keskinocak and Savva 2020). For instance, Robinson and Chen (2010) consider the random open-access scheduling for patients who make appointments in the early morning. They show that open-access scheduling outperforms traditional appointments scheduling. Saghafian et al. (2014) show that considering patient complexity to the triage system could increase patients' safety and hospital triaging efficiency. Some studies focus on the operational policies in the interaction among multiple entities in the healthcare supply chain. Zhao et al. (2012) explore the effect of the fee-for-service (FFS) contract in pharmacy distribution and show that the FFS contract could increase the overall supply chain benefit more than the forward buying contract. Zhou et al. (2011) consider the replenishment issue of perishable medical inventory (platelet) and propose the expedited replenishment policy based on the regular replenishment policy. Similarly, we study the inventory problem of the long-leading-time disposable medical items, such as intraocular lenses and orthopedic implants. These items need a long leading time and expensive stock cost, so hospitals usually purchase them by the consignment contract from the retailer. Therefore, in our second study, we extend the model in our first study and build a coordinating mechanism to explore the effect of the inventory sharing policy on the return decision for unused items in consignment contracts.

The third issue is about the interaction among multiple healthcare entities, especially the interaction with the market interface. The healthcare entities mainly contain the providers (drug/medical items manufacturers, hospitals, pharmacies), the payers (insurance companies, employers), and patients. Patients purchase the medical plan from the insurance company by the employer or by themselves. However, in the recent five years, patients afford more for buying medical services as the insurer pays 90% of the costs to about 70% (Barkholz 2017). Facing the challenge, a growing number of scholars have focused on the issue. Guo et al. (2019) explore the differences between two reimbursement schemes, the fee-for-service payment, and the bundled payment, and analyze the effects of the payment schemes on patients' revisit rate, patients waiting time, and social welfare. Dai et al. (2017) consider patient insurance coverage and explore the effect of insurance structure on the service utility in the healthcare market. Kouvelis et al. (2015) study the competition among multiple pharmacy benefit managers (PBMs) and investigate how the PBMs decide the drug resale prices and formulary tiers in consumers' medical plans. In contrast, our third study focuses on the vertical integration between the manufacturer and PBM. It examines the effect of the upstream integration on downstream consumers' performance (co-payment level and demand for the drug).

1.2 Research Design

Based on the above background of HOM, the thesis is motivated by the following structures: Chapter 2 relates to the first study: (1) inventory sharing policy between two hospitals. Chapter 3 describes the second study: (2) coordinating retailer's return policy with inventory sharing in the consignment contract. Chapter 4 relates to the third study: (3) the vertical integration between the manufacturer and the PBM in the pharmaceutical supply chain. Finally, chapter 5 summarizes the three studies and future research directions.

The series of research questions and research design are shown from the thesis:

- Chapter 2.

  (1) When the hospital anticipates a stockout on some disposable medical items, the best response is to request inventory sharing or conduct the emergency replenishment?

  (2) At the beginning of a period, how does the hospital decide the regular replenishment quantity based on the emergency replenishment option and inventory sharing option?

  (3) How do the cost features and patient's behavior affect the hospital's replenishment decision and the optimal order-up-to level?

  To solve the above research questions, we construct a mathematical model to explore the inventory sharing between two hospitals. Before considering the sharing policy, we first model the regular replenishment process with the emergency replenishment option. Then with the inventory sharing option, we divide the replenishment process into six cases according to the hospital's demand and inventory level. We derive the optimal replenishment strategies of two hospitals. Additionally, we explore the effects of the patient's emergency request rate and the hospital's safety stock level on the optimal inventory decision. We find that the sharing policy is more profitable than the emergency replenishment policy for the cooperative hospitals when the sharing condition is satisfied. Under the sharing mechanism, the optimal inventory of one hospital increases as the partner hospital's emergent request rate increases, the partner hospital's safety inventory level increases, and the partner hospital's order-up-to level decreases.

- Chapter 3.

  (1) Considering retailers' sharing action downstream, how does the dealer decide the consignment price (wholesale price) and the return price for unsold items?

  (2) Dividing retailers' sharing cases into the dealer-dominated and retailer-dominated cases according to the entity that decides the sharing price, how do the dealer and retailers benefit from the sharing policy?

  (3) What are the impacts of the dealer's return decision on the retailer's sharing action, retailers' profits, and dealer's profit?

We build a two-echelon model based on the above research questions, that considers a common upstream dealer and two downstream retailers. The retailer places an order from the dealer according to the demand and returns unused items to the dealer with the return price. We aim at exploring the interaction among dealer's pricing decision, return decision, and retailers' sharing decisions in the consignment contract. According to the market power, we consider the retailers' sharing into the dealer and retailer-dominated case. We find that the beneficial effect of the retailers' sharing action depends on who dominates the sharing price. Under the retailer-dominated case, the dealer is worse off as retailers choose to form a cooperative alliance with a lower sharing price. Simultaneously, the dealer may increase the return price for unsold items to incentivize retailers' sharing, which stunts retailers' sharing action, as retailers are worse off under the sharing case. Counter-intuitively, when the dealer decreases the return price, the retailers with excess inventory are incentivized to share as the consignment price increases. Consequently, retailers in the alliance are better off from sharing action.

- Chapter 4.

  (1) Suppose that the drug quality is endogenous; how do the drug manufacturer and the PBM make the pricing decision and quality decision vertically integrated?

  (2) Considering the manufacturer's profit and the overall profit of the pharmaceutical supply chain, what are the conditions that encourage or impede the drug manufacturer from vertical integration with the PBM?

  (3) Suppose that the manufacturer applies the direct retail channel and vertical integration channel simultaneously; how would an integrated drug manufacturer differentiate in terms of drug quality between two distribution channels. How would the drug quality differentiation decision affect the joint profit of the manufacturer and the PBM?

Considering the above research questions, we construct a three-echelon model containing the upstream drug manufacturer, the download PBM, and consumers. We first model the interaction among the entities and derive the equilibrium solutions without vertical integration between the manufacturer and PBM. Then we explore the conditions that encourage the manufacturer to choose the vertical integration channel with the PBM. Above all, we focus on how the manufacturer differentiates the drug quality level between two distribution channels (direct retail channel and vertical integration channel). We also investigate the effects of drug manufacturing cost and quality differentiation level on consumers' behavior as well as the profit of the manufacturer and the PBM. From this study, we summarize some conclusions that provide suggestions for practice. First and foremost, although the manufacturer differentiates the drug quality level between two distribution channels, consumers do not always choose the higher-quality drug as the quality differentiation level increases. Second, the manufacturing cost does not always hurt the joint profit of the manufacturer and PBM. Instead, it affects the joint profit by affecting the manufacturer's differentiation decision. When the differentiation level is lower (≤ 1), the joint profit first decreases and then increases with the increasing manufacturing cost, while at a higher differentiation level (> 1), the joint profit increases with the increasing manufacturing cost.

Chapter 2

Inventory Sharing Strategy for Disposable Medical Items between Two Hospitals

2.1 Introduction

Hospital inventory management is a challenge as patients' demand for medical items is difficult to predict. Natural disasters, massive traffic accidents, and infectious disease outbreaks bring a steep increase in emergent patients and difficulty in hospital order-related decision-making. Hospitals always face stockout on medical inventory, especially for disposable medical items such as sanitary materials, disposable medical gloves, disposable infusion/blood transfusion, medical textiles, surgical instruments, disposable catheters, vascular surgical instruments, anesthesia instruments, obstetric instruments, oxygen masks, and other necessities (Lapierre and Ruiz 2007). Additionally, some small hospitals cannot provide enough beds for patients when urgent situations occur. For instance, in Hong Kong, the bed utilization of some public hospitals reaches 120% during flu outbreak season such that patients transfer between hospitals further delays the medical treatment of patients. In such situations, the demand for medical items at a hospital increases suddenly, and stockout occurs as the regular safety stock cannot handle the increasing demand (Saedi et al. 2016). Different from industrial products, stockout of critical medical supplies may result in a life-threatening situation for patients thus hospitals try to ensure various medical supplies are adequate, even in unanticipated situations (Little and Coughlan 2008, Chen et al. 2013). However, the inventory holding cost is high for hospitals. Wang et al. (2015) mention that inventory cost is a significant component of all expenses in a hospital. For the sake of profit, hospitals always choose the expensive emergent replenishment to address the stockout problem. Emergent replenishment can guarantee immediate supply, but hospitals need to pay higher prices and additional delivery charges. Therefore, there are some management issues regarding the medical inventory system. First, because of high inventory holding cost and limited warehouse capacity, hospitals cannot stock a large volume of inventory. Hospitals can only adopt the expensive emergent replenishment policy to satisfy the sudden demand. It is not an economical inventory policy as it incurs higher purchase prices and expensive delivery charges. Second, when one hospital faces stockout on some medical items, another nearby hospital may be holding idle inventory. Therefore, it is critical to explore more effective inventory policies to mitigate stockout risk with a lower operating cost for hospitals (Nicholson et al. 2004, Katsaliaki and Brailsford 2007, Royston 2016).

We construct a sharing mechanism for disposable medical inventories between two hospitals to solve the above medical inventory management problems. Although inventory sharing has received some attention in traditional supply chain management, it is rare in hospitals. When a huge traffic accident happens or an infectious disease outbreak, the local hospitals may anticipate medical inventory stockout. Simultaneously, hospitals in other regions may keep enough inventory. In this case, inventory sharing is feasible when one hospital has excess demand and another has excess inventory. Sharing inventory policy could save time for patients than transferring patients to another hospital.

Furthermore, in contrast to customer demands, patients' demands cannot be backlogged, which means patients' behavior has some effects on hospitals' decisions. For example, for unsatisfied customers, the retailer may compensate the customer in the future. However, for unsatisfied patients, the hospital will make replenishment decisions or request possible sharing to fulfill patients' demands according to patients' actions.

This chapter builds an inventory sharing model between two decentralized hospitals, where they make independent inventory decisions in each period. Demand realization happens at the beginning of a period, and then these two hospitals place a regular replenishment order for the next period according to demand forecast information. Hospitals fulfill the demand by a regular order placed in the last period together with the inventory carried forward from the last period. If stockout is anticipated to occur, hospitals take corresponding actions according to the patients' behavior. In the hospital, elective patients make appointments in advance; hospitals can forecast their demands. On the other hand, emergent patients arrive at the hospital randomly. Some emergent patients choose to "stay-in" the hospital and wait for treatment, and some patients leave the hospital.

This chapter focuses on emergent patients and defines the percentage of emergent patients as the emergent request rate. It is a common phenomenon that less-urgent patients give up waiting and turn to another hospital, especially in flu season when medical inventory stockout happens frequently. To satisfy the requests of "stay-in" patients, hospitals make emergent orders to the dealer or request inventory sharing with the partner hospital according to the transaction cost incurred. If emergent replenishment is preferred, the regular inventory decisions of the two hospitals are independent and not influenced by each other. On the other hand, if inventory sharing is chosen, then the partner hospital decides the sharing quantity according to multiple factors, such as its demand and the safety inventory level. Hospitals benefit from the inventory sharing mechanism intuitively. If inventory sharing is adopted, for the hospital facing stockout, it saves the emergent procurement cost; for the hospital with excessive inventory, it saves the inventory holding cost by sharing inventories with the partner hospital. Simultaneously, the medical supplies utilization rate is enhanced from the perspective of social welfare. This chapter further explores how the hospitals decide the order quantities with the sharing option and whether there is an interaction in their decisions considering patients' behavior.

We propose three research problems for the inventory sharing of a single item between two hospitals.

(1) What are the conditions that hospitals benefit from inventory sharing than emergent replenishment?

(2) At the beginning of one period, what are the optimal regular replenishment quantity and the optimal inventory (order-up-to level) for the next period's consumption?

(3) What are the effects of the cost features of hospitals and patients' behavior on hospitals' inventory decisions?

Similar with our research question, Park et al. (2016) study the multi-period inventory sharing problem in the spot and forward market. A firm with excessive demand either purchases from the spot/forward market or sends a sharing request to other firms. The firm with excess inventory can sell to the spot/forward market or accept another firm's sharing request. The authors investigate the equilibrium strategies of two firms and develop a structured transshipment pricing scheme to increase the value of inventory sharing. Their work and ours both assume that the demand cannot be backlogged and needs to be satisfied immediately (no delay) by the spot market ("emergent replenishment" in our study). The main difference lies in their focus on the sharing price structure, and we assume that sharing price equals the regular replenishment price. Additionally, we consider the effects of patients' behavior and the safety inventory level of the hospital on the sharing decision. We also investigate the interaction on inventory decisions between two sharing hospitals.

The rest of the chapter is structured as follows. Section 2.2 reviews related literature on inventory sharing & inventory transshipment and shows our main research contributions. Section 2.3 introduces the background and model setting. Next, sections 2.4 and 2.5 analyze the emergent replenishment/emergent sharing decision as well as the optimal inventory policy under the no-sharing case and the sharing case, respectively. Then, we discuss the preliminary results by several numerical experiments in Section 2.6 and summarize conclusions in Section 2.7.

2.2 Literature Review

There is very little literature on inventory sharing in the hospital setting. Therefore, we review literature mainly in the three relevant categories: (1) inventory sharing literature in the setting of industry or enterprise; (2) inventory transshipment, which is similar to the concept of inventory sharing in the operations perspective; and (3) research on healthcare materials and logistics management. The corresponding literature is summarized in more detail below.

The first stream of literature is relevant to inventory sharing, which includes several common considerations: high-priority demand or low-priority demand, single period or multiperiod, and sharing between two parties or among multiple parties. Zhao et al. (2005) consider the inventory sharing problem in a decentralized dealer network where each dealer faces high and low-priority demand. They focus on two research issues. The first issue is, when one dealer accepts the sharing request of another dealer, which dealer should place a replenishment order to their common manufacturer. The second problem is determining the replenished inventory quantity for each dealer after the sharing action. It is found that there exists a pure-strategy equilibrium under the full-inventory-sharing game and fixed-portion-sharing game. The unsatisfied sharing request can be backlogged in their work, and the rejected sharing request is supposed to be made up later. Unlike their model setting, we assume that unsatisfied demand cannot be backlogged. Once the sharing request is rejected, the hospital needs to place an emergent replenishment order from the dealer, which fits the hospital setting. Based on the two-dealer sharing problem, Yan and Zhao (2015) construct a multi-dealer ($n > 2$) inventory sharing mechanism, where n dealers make the replenishment decisions independently but share inventory cooperatively. In the case of asymmetric demand information, the authors analyze the effects of complete information sharing and no information sharing on the coordination mechanism, respectively. This study provides managerial insights on coordinating a multi-dealer inventory sharing mechanism when considering asymmetric information. Yan and Zhao (2011) also consider the effects of asymmetric information on inventory sharing in a decentralized system. To increase the information truth in inventory sharing between retailers, a coordination mechanism is proposed between the manufacturer and retailers to maximize profit. Park et al. (2016) consider the multi-period inventory sharing problem in the spot and forward market. They develop the optimal equilibrium strategies for two firms and construct a structured transshipment pricing mechanism to make more benefits from inventory sharing.

The second category of literature concerns the inventory transshipment problem. Inventory transshipment implies transferring inventory from one location to another when a retailer has excess demand for a certain inventory item, and another retailer has excess inventory of the same item. Due to the long procurement lead time and the difficulty of predicting demand in some industrial operations, transshipment becomes a routine activity to better match supply and demand. Transshipment usually occurs when demands are observed/realized and before they are satisfied (Robinson 1990). In general, transshipment is investigated under the centralized distribution system and the decentralized distribution system. Under the centralized system, a centralized decision-maker (supplier/distributor) makes replenishment and transshipment decisions among retailers to maximize the total profits of all retailers in the system. While in a decentralized system, each retailer makes replenishment and transshipment decisions independently, aiming at maximizing their independent profit. Earlier studies focus on the inventory transshipment problem under a centralized setting. Tagaras (1989) proposes a two-location centralized system with an optimal ordering and transshipment policy. Robinson (1990) considers the multi-period inventory problem with transshipment among multiple locations. Based on previous research outputs, Dong and Rudi (2004) explore which party benefits more from transshipment under a centralized distribution system when a common manufacturer serves multiple retailers. Then, the effect of transshipment on retailers and manufacturer are compared in two cases: the manufacturer is a wholesale price setter or a wholesale price taker. Liao et al. (2014) tackle a similar inventory transshipment problem in an industrial setting. They compare the options of lateral transshipment and emergent order when stockout occurs. From their research outputs, lateral transshipment between two retailers saves more cost and time than emergent replenishment. The authors also propose optimal inventory response policies for different scenarios under a single-period setting. Besides, the effects of customer behavior on inventory decisions are considered, such as customer request rate and rate of switching to another store. The major difference between their work and ours lies in they investigating the inventory transshipment between two centralized retailers, while we explore the sharing problem between two decentralized hospitals. There are some studies about decentralized inventory transshipment. Anupindi et al. (2001), Granot and Sošić (2003), and Slikker et al. (2005) use the cooperative game theory to study the transshipment problem in a decentralized distribution system and aim at obtaining the Nash equilibrium on inventory decisions.

Besides, Rudi et al. (2001) consider two independent inventory locations and examine the effects of intrafirm transshipment and interfirm transshipment on the optimal inventory decision. In this paper, intrafirm transshipment is the inventory transshipment under a centralized system while interfirm transshipment is equivalent to transshipment under a decentralized system. After comparing these two cases, it is found that when the single retailer maximizes the independent profit, the joint profit of a centralized system cannot be realized. Zhao et al. (2006) study two issues about emergent transshipment among multiple independent dealers under a decentralized system. When one dealer faces stockout, the first issue is when the dealer should send a transshipment request to another dealer with excess inventory (requesting decision), and the second issue is when the dealer with excess inventory should fill the request (filling demand decision). It is found that a threshold rationing policy can determine the transshipment decision. Hu et al. (2007) generalize the study of (Rudi et al. 2001). They prove that a transshipment price does not always exist by a counterexample and explore the sufficient and necessary conditions under which the transshipment price does exist. Hanany et al. (2010) develop a transshipment coordination mechanism in which a third party coordinates the inventory transshipment among multiple independent retailers. Yao et al. (2016) consider the pre-season initial stocking decision and the in-season inventory transshipment decision simultaneously between two locations. They identify the optimal initial inventory decision and optimal transshipment policy by dynamic programming. Li et al. (2022) explore the transshipment scheme of perishable products in offline groceries. They differentiate and separate inventory by applying the Last-In-First-Out rule in the two-outlet transshipment.

We also review literature about hospitals' materials and logistics management. Volland et al. (2017) provide a detailed review to summarize the previous research on hospitals' logistics problems. Wieser (2011) focus on the healthcare logistics optimization issue. They also suggest considering the service level of patients in operational practice, such as the service quality, traceability, and the information system. Kritchanchai and MacCarthy (2017) investigate the application of vendor-managed inventory (VMI) in pharmaceutical transportation for hospitals, while Kritchanchai et al. (2018) study the performance of healthcare logistics from the following perspectives: purchasing and supply policy, warehousing, inventory management, transportation and distribution, and information technologies. Scholars also propose incorporating the behavior of healthcare personnel when improving the healthcare inventory transportation efficiency. For instance, Stefanini et al. (2020) consider the behaviors of medical staff and health managers when developing resource planning strategies for lung cancer patients. Additionally, Adida et al. (2011) explore the hospital stockpiling policy for disaster prevention, and a proactive inventory transshipment policy before the disaster happens. In contrast to their work, our study analyzes the emergency inventory sharing policy for the stockout in the hospital, which is a reactive inventory transshipment policy after the stockout happens. Another difference lies in Adida et al. (2011) focus on stockpile decision-making and neglect of the hospital's safety stock. Our study explores the sharing decision-making when considering the effect of the hospital's safety stock level. Most importantly, we also investigate the impacts of patients' behavior (emergent request rate for treatment) on a hospital's inventory decisions.

Our research is different from the previous studies in the following ways: (1) the demand for disposable medical inventory cannot be backlogged in a hospital setting; (2) it is in a decentralized system, where two hospitals operate independently; (3) it proposes the sharing mechanism when an emergent replenishment option is available; (4) it investigates the effects of patient's behavior and the safety inventory level of the hospital on sharing decisions. We summarize the twofold contribution of this study based on the differences with the above-related literature review. First, from the perspective of the model background, we investigate the inventory sharing mechanism between two independent hospitals considering the hospital's regular and emergent replenishment policy. Hospitals benefit from inventory sharing compared with the expensive emergent replenishment policy under some conditions. This study provides some managerial suggestions for the hospital's operations practice. Second, from the perspective of the mathematical model setting, we consider the specific characteristics of patients' behavior in the inventory sharing model. We capture the patient's emergent request rate when the hospital faces inventory stockout and explore the effects of the emergent request rate on hospitals' order decisions. Therefore, this study also enriches the literature on inventory sharing, especially in the healthcare setting.

2.3 Model Setup

We consider a sharing mechanism of a single item in two cooperative hospitals i and j $(i, j = 1, 2, i \neq j)$ in a time period. Each hospital faces independent stochastic demand D_i and D_j, with probability density function g_i, g_j and distribution function G_i, G_j. In a hospital, the demand for medical items should be satisfied within the same period and cannot be backlogged to the next period. Daily demand comes from appointment patients and emergent patients. The demand for emergent patients is difficult to forecast, and therefore stockout happens occasionally. In the following sections, we assume that hospital i faces stockout and hospital j has adequate inventory in a period; the opposite case is symmetric.

Hospitals apply a periodic review policy for most medical items and make replenishment decisions at the beginning of each period. Since the lead time of regular replenishment is one period (i.e., the order is placed in the last period and received at the beginning of this period), hospital i can only use the inventory carried forward and the amount ordered in the last period to fulfill the demand in this period. If stockout happens, the hospital chooses either the emergent replenishment to the dealer or inventory sharing from the partner hospital to consider the related cost. The hospital generally pays a higher price and transportation cost for emergent replenishment. If another hospital accepts the inventory sharing request, the hospital with excess demand needs to bear the transshipment cost. Considering the specialty of patient demands and medical inventory, hospitals are risk-averse compared with commercial organizations. Hence, we consider the safety inventory level k_j, which is a fraction of the order-up-to level x_j. The partner hospital j will reject any inventory sharing request when its on-hand inventory level is lower than its safety inventory level. In general, two cooperative hospitals are willing to share when one hospital faces stockout and the partner hospital has enough inventory above the safety stock level.

In addition, we consider patients' behavior in our model. When stockout happens in a hospital, some patients may decide to leave the hospital, while others stay and wait for emergent treatment until the replenishment inventory arrives. We use the emergent request rate w_i to denote the rate of patients that stay in the hospital $w_i \in [0, 1]$ (i.e., $(D_i - x_i)w_i$). Patients choose to wait in the hospital i when stockout happens. We assume that each hospital has a unique w_i. For instance, for a large and famous hospital, the corresponding w_i is high; patients are more willing to stay in the hospital and wait for service. In contrast, for a hospital in a central location where many other hospitals are located nearby, the corresponding w_i is low because patients may choose another hospital.

The sequence of events in each period is illustrated as follows (taking hospital i as an example). At the beginning of a period, the hospital receives the regular replenishment order that was placed in the last period with price p_r^l and observes the order-up-to level x_i. Meanwhile, the hospital realizes demand D_i, the regular replenishment price p_r and emergent replenishment price p_e. We define $\mathcal{D} := (D_i, D_j)$ and $\mathcal{X} := (x_i, x_j)$ to represent the demand and initial inventory level for hospitals i and j respectively. If stockout occurs in hospital i, hospital i will either place an emergent replenishment order e_i to the dealer or request sharing inventory from partner hospital j according to the operating costs. For the emergent replenishment, the hospital needs to pay the emergent procurement cost p_e and transportation cost τ_e, respectively. Under the inventory sharing policy, the partner hospital j shares excess inventory with hospital i at a regular replenishment price p_r, but the sharing amount s_j is determined by the request level of hospital i, the internal demand of hospital j, and the safety inventory k_j. The hospital i also pays τ_s as transshipment cost in the sharing process. Once the demand is satisfied, the remaining inventory is carried forward to the next period with the holding cost of h. Note that hospital j can save relative holding cost in this period by sharing its inventory. We assume that hospital i will return the sharing inventory to hospital j in the next period and ignore the unpunctual return issue in this study. Besides, we assume that $p_e > p_r$, $\tau_e > \tau_r$, and $\tau_s > \tau_r$ to prevent a hospital from always choosing to request sharing instead of making regular replenishment orders to the dealer. We set $\tau_s > \tau_e$ in the numerical experiments to prove that inventory sharing may still occur even if the sharing transportation cost is higher than the emergent transportation cost. The savings from the emergent purchase price and inventory holding cost outweighs the additional transportation cost.

We define the following notations applied in our model in Table 2.1:

In the remaining sections of this chapter, we first analyze the emergent replen-

Table 2.1: Notations in the model.

Notations	Descriptions
D_i	The demand for hospital i
p_r	Purchase price of regular replenishment in current period
p_r^l	Purchase price of regular replenishment in last period
p_e	Purchase price of emergent replenishment in current period
τ_r	Transportation cost of regular replenishment
τ_e	Transportation cost of emergent replenishment
τ_s	Transaction cost of inventory sharing
h	Inventory holding cost
k_i	The safety inventory level of hospital i
w_i	The emergent request rate of patients in hospital i
r_i	The regular replenishment order for hospital i in current period
e_i	The order amount of emergent replenishment for hospital i
s_i	Hospital i shares s_i units to hospital j
x_i	The order-up-to level of hospital i at the beginning of current period

ishment policy without the inventory sharing action, and we use the superscript "e" to denote the case. Then, we suggest a sharing policy that combines sharing action and emergent replenishment policy, represented by superscript "s". Under these two policies, we aim at obtaining an optimal sharing/emergent replenishment order amount, optimal regular replenishment amount in the current period, and the optimal order-up-to level at the beginning of the next period. We also investigate the effects of the hospital's safety stock level and patient emergent request rate on the hospital's choice of inventory policy. Furthermore, we explore the response function of one hospital to the partner hospital on the optimal inventory order-up-to level in the sharing mechanism.

2.4 Benchmark Case

Before considering the sharing mechanism, we first analyze the benchmark case without inventory sharing. In this case, each hospital makes inventory decisions independently to minimize the expected cost of the next period. When stockout is anticipated to occur, the emergent replenishment policy is executed. $O_i^e(x_i)$ denotes the total expected cost in a period.

$$O_i^e(x_i) = E\left\{\int_0^{x_i} O_i^{e1}(x_i)g_i(D_i)dD_i + \int_{x_i}^{\infty} O_i^{e2}(x_i)g_i(D_i)dD_i\right\} \tag{2.1}$$

where $O_i^{e1}(x_i)$ and $O_i^{e2}(x_i)$ represent the expected cost when stockout happens and without stockout happens, respectively. The logic of the solution is that in a period, hospital i receives the regular replenishment quantity, observes the inventory level, and aims at minimizing the expected cost of this period. If stockout happens, unsatisfied demand is fulfilled by the emergent replenishment order. Therefore, we explore the order-up-to level x_i which arrives at the beginning of this period that minimizes $O_i^e(x_i)$ for hospital i. More model setting details are presented in Appendix A.

Proposition 2.1. *Under the no-sharing case, for hospital i, given D_i and x_i, e_i increases as w_i increases.*

In the hospital, the emergent demand cannot be backlogged to the next period and should be satisfied in the current period. Therefore, when the stockout is anticipated in hospital i, if w_i is high (which means more emergent patients choose to stay and wait for medical supply), then the hospital needs to place a larger emergent order. We provide the proofs of the following propositions in Appendix A.

Proposition 2.2. *Under the no sharing case, the expected cost of hospital i, $O_i^e(x_i)$, increases as w_i increases. In a period, there exists a unique x_i^* that minimizes $O_i^e(x_i)$ when $w_i \geq \frac{p_r+\tau_r-h}{p_e+\tau_e}$.*

Under the emergent replenishment policy, the patient's emergent request rate increases the total operating cost of the hospital. If stockout happens, a higher percentage of patients are willing to wait for emergent medical supply; then the hospital needs to pay more to satisfy the demand. When the hospital decides the regular replenishment quantity, a unique optimal order-up-to level x_i^* can be determined to minimize the cost of hospital i only when the emergent request rate of patients $w_i \geq \frac{p_r+\tau_r-h}{p_e+\tau_e}$.

2.5 Inventory Policies with Sharing

This section proposes the inventory sharing mechanism between two hospitals. Two hospitals form a cooperative alliance in which they make inventory decisions independently and share inventory cooperatively. We assume that the two cooperative hospitals are willing to share when the sharing action is beneficial to the alliance.

2.5.1 Safety Complete Sharing Policy

Taking the case where hospital i faces stockout and hospital j has sufficient inventory in a period as an example, when the demand of hospital i exceeds the order-up-to level ($D_i \geq x_i$), hospital i prefers to send a sharing request to the partner hospital j if sharing saves costs rather than place an emergent replenishment order ($p_e + \tau_e \geq p_r + \tau_s$). If hospital j accepts the request and shares excess inventory with hospital i, the inventory holding cost h is reduced in the current period—especially when the medical items have special storage requirements, the holding cost could be very high. However, hospital j keeps k_j inventory as the safety stock and makes a sharing decision rationally. The sharing condition is wider when two hospitals are centralized, ($p_e + \tau_e + h - p_r - \tau_s \geq 0$). Two corollaries are derived based on the sharing condition.

Corollary 2.1. *Under the sharing mechanism, the positive cost reduction of two hospitals increases in τ_e, h and decreases in τ_s.*

Compared to the emergent replenishment policy, the cost reduction of the sharing policy mainly comprises the savings in transportation cost and inventory holding cost. Corollary 2.1 shows that the positive cost reduction of sharing action increases as the emergent replenishment transportation cost and holding cost increase. Conversely, it decreases as the sharing transshipment cost increases.

Corollary 2.2. *The sharing mechanism has positive benefits only if $p_e + \tau_e + h - p_r - \tau_s \geq 0$.*

We have $p_e - p_r + h \geq 0$, which means that there exists the case: when $\tau_e \leq \tau_s$, the transportation cost to share the inventory from hospital j to hospital i is more expensive than the emergent transportation cost from dealer to hospital i, the inventory sharing is more economical for the hospital. Therefore, transportation cost is not always considered a top priority when stockout happens in a hospital.

Based on the complete pooling policy (Tagaras 1989), we propose the safety complete sharing policy as a sharing rule for two cooperative hospitals in Proposition 2.3.

Proposition 2.3. *When stockout is anticipated in hospital i in a period, for $j \neq i$:*

i. If $(x_j - D_j)(1 - k_j) \geq (D_i - x_i)w_i$, then $s_j = (D_i - x_i)w_i$.

ii. If $(x_j - D_j)(1 - k_j) < (D_i - x_i)w_i$, then $s_j = (x_j - D_j)(1 - k_j)$.

We further consider the effect of safety stock level and patient's emergent request rate on the hospital's sharing decision. Only when hospital j has surplus inventory after satisfying its internal demand does it have the capacity to share. The "complete" in our proposition means that when the hospital has excess inventory, it can share completely. Under scenario (i), when the surplus inventory of hospital j is enough to cover the emergent request amount of hospital i, the sharing amount is equal to the requested amount, and the emergent replenishment is not needed. Under scenario (ii), hospital j can satisfy the partial amount of hospital i's sharing request; then hospital i still needs to place an emergent replenishment order to meet the remaining demand.

2.5.2 Six Scenarios under the Sharing Case

Under the sharing mechanism, if the sharing condition is satisfied, the hospital that faces stockout will send a sharing request to the partner hospital that has surplus inventory. We first obtain the optimal sharing/emergent replenishment quantity, then explore the optimal regular replenishment quantity for the next period. In our model, the expected total cost of the two cooperative hospitals is set as the objective function. At the beginning of a period, an order-up-to level x_i is decided to minimize the total cost when x_j is given. The expected total cost of the hospital alliance is denoted as $O^s(x_i, x_j)$. Following the cost structure under the emergent replenishment policy, we analyze the sharing policy between two cooperative hospitals under the following six scenarios.

Scenario 1: $x_i \geq D_i$**,** $x_j \geq D_j$

In this scenario, both hospitals i and j have surplus inventory in a period; thus no sharing or emergent replenishment occurs, and both hospitals conduct inventory planning independently. We denote the expected total cost of the hospital alliance as $O^{s1}(x_i, x_j)$ in this period:

$$\begin{aligned} O^{s1}(x_i, x_j) = {} & (p_r^l + \tau_r)x_i + (p_r^l + \tau_r)x_j + (h - p_r - \tau_r)(x_i - D_i) \\ & + (h - p_r - \tau_r)(x_j - D_j) \end{aligned} \tag{2.2}$$

Scenario 2: $x_i < D_i$**,** $x_j \geq D_j$**,** $(x_j - D_j)(1 - k_j) \geq (D_i - x_i)w_i$

In this period, hospital j has surplus inventory and hospital i has excess demand. The hospital i prefers inventory sharing instead of emergent replenishment to fulfill the demand of emergent patients. Meanwhile, the sharing quantity of j is enough to cover the requesting quantity of i, and thus the optimal sharing amount is $s_j = (D_i - x_i)w_i$. The emergent replenishment amount is $e_i = e_j = 0$. Notice that if sharing occurs, the immediate cost structure changes since hospital i has to pay a sharing cost for transportation, denoted as $\tau_s(D_i - x_i)w_i$. Hospital j saves the corresponding holding cost $h(D_i - x_i)w_i$. The expected total cost of hospital i and j is denoted as $O^{s2}(x_i, x_j)$:

$$\begin{aligned} O^{s2}(x_i, x_j) = {} & (p_r^l + \tau_r)x_i + (p_r^l + \tau_r)x_j + \tau_s(D_i - x_i)w_i \\ & + (h - p_r - \tau_r)[x_j - D_j - (D_i - x_i)w_i] \end{aligned} \tag{2.3}$$

Scenario 3: $x_i < D_i$, $x_j \geq D_j$, $(x_j - D_j)(1 - k_j) < (D_i - x_i)w_i$

In scenario 3, hospital j keeps the safety stock and does not have enough inventory to satisfy the full amount requested by hospital i. Therefore, the optimal sharing amount is $s_j = (x_j - D_j)(1 - k_j)$, and hospital i still places an emergent replenishment to the dealer for the remaining demand, that is, $e_i = (D_i - x_i)w_i - (x_j - D_j)(1 - k_j)$. The corresponding expected total cost is:

$$\begin{aligned} O^{s3}(x_i, x_j) = {} & (p_r^l + \tau_r)x_i + (p_r^l + \tau_r)x_j + \tau_s(x_j - D_j)(1 - k_j) \\ & + (p_e + \tau_e)[(D_i - x_i)w_i - (x_j - D_j)(1 - k_j)] \\ & + (h - p_r - \tau_r)(x_j - D_j)k_j \end{aligned} \tag{2.4}$$

Scenario 4: $x_i \geq D_i$, $x_j < D_j$, $(x_i - D_i)(1 - k_i) \geq (D_j - x_j)w_j$

Scenario 4 is symmetric to scenario 2, and the expected total cost can be represented as:

$$\begin{aligned} O^{s4}(x_i, x_j) = {} & (p_r^l + \tau_r)x_i + (p_r^l + \tau_r)x_j + \tau_s(D_j - x_j)w_j \\ & + (h - p_r - \tau_r)[x_i - D_i - (D_j - x_j)w_j] \end{aligned} \tag{2.5}$$

Scenario 5: $x_i \geq D_i$, $x_j < D_j$, $(x_i - D_i)(1 - k_i) < (D_j - x_j)w_j$

Scenario 5 is a symmetric example of scenario 3, and the expected total cost is:

$$\begin{aligned} O^{s5}(x_i, x_j) = {} & (p_r^l + \tau_r)x_i + (p_r^l + \tau_r)x_j + \tau_s(x_i - D_i)(1 - k_i) \\ & + (p_e + \tau_e)[(D_j - x_j)w_j - (x_i - D_i)(1 - k_i)] \\ & + (h - p_r - \tau_r)(x_i - D_i)k_i \end{aligned} \tag{2.6}$$

Scenario 6: $x_i < D_i$, $x_j < D_j$

In scenario 6, inventory sharing does not occur since two cooperative hospitals are facing stockout, and emergent replenishment is the only option. The emergent replenishment quantities of hospital i and j are $e_i = (D_i - x_i)w_i$ and $e_j = (D_j - x_j)w_j$, respectively. Under this case, the expected total cost can be denoted as:

$$
\begin{aligned}
O^{s6}(x_i, x_j) = (p_r^l + \tau_r)x_i + (p_r^l + \tau_r)x_j + (p_e + \tau_e)(D_i - x_i)w_i \\
+ (p_e + \tau_e)(D_j - x_j)w_j
\end{aligned} \tag{2.7}
$$

Combining the above six scenarios, the expected total cost of two hospitals under the inventory sharing mechanism is derived by the following equation:

$$
\begin{aligned}
O^s(x_i, x_j) = & \int_0^{x_i} \int_0^{x_j} O^{s1}(x_i, x_j) g_i(D_i) g_j(D_j) dD_j dD_i \\
& + \int_0^{x_i} \int_{x_j}^{\frac{(x_i - D_i)(1-k_i)}{w_j} + x_j} O^{s2}(x_i, x_j) g_i(D_i) g_j(D_j) dD_j dD_i \\
& + \int_0^{x_i} \int_{\frac{(x_i - D_i)(1-k_i)}{w_j} + x_j}^{\infty} O^{s3}(x_i, x_j) g_i(D_i) g_j(D_j) dD_j dD_i \\
& + \int_0^{x_j} \int_{x_i}^{\frac{(x_j - D_j)(1-k_j)}{w_i} + x_i} O^{s4}(x_i, x_j) g_i(D_i) g_j(D_j) dD_j dD_i \\
& + \int_0^{x_j} \int_{\frac{(x_j - D_j)(1-k_j)}{w_i} + x_i}^{\infty} O^{s5}(x_i, x_j) g_i(D_i) g_j(D_j) dD_j dD_i \\
& + \int_{x_i}^{\infty} \int_{x_j}^{\infty} O^{s6}(x_i, x_j) g_i(D_i) g_j(D_j) dD_j dD_i
\end{aligned} \tag{2.8}
$$

The logic of the sharing policy is summarized as follows: inventory sharing only occurs when one hospital has excess demand and another hospital has excess inventory. For the hospital that receives a sharing request, if the surplus inventory is still more than the requested amount after maintaining sufficient safety stock, then it is optimal to fulfill the shortage of the partner hospital to minimize the expected total cost for two hospitals. In contrast, if the surplus inventory is not sufficient to fulfill the whole shortage amount, it is optimal to share as much as possible and make up the remaining demand through the emergent replenishment policy.

We obtain more structural properties in the following propositions.

Proposition 2.4. *Under the sharing case, given $\mathcal{D}$ and $\mathcal{X}$, the optimal s_j increases as w_i increases and decreases as k_j increases.*

For the hospital with excess demand, inventory sharing between the two cooperative hospitals is preferred if it saves more cost than emergent replenishment.

Suppose that the hospital i faces stockout and the partner hospital j has excess inventory, a higher w_i increases the sharing amount s_j when hospital j can cover all the shortage of hospital i. However, when hospital j can only fulfill the partial unsatisfied demand of hospital i, the final sharing amount depends on the safety stock level of hospital j. If hospital j keeps more safety stock, then the available sharing quantity is less. After obtaining the cost structure of the expected total cost of two cooperative hospitals under the inventory sharing mechanism, we identify the effects of the patients' behavior (emergent request rate) and the hospital's safety stock level on the expected total cost. The detailed observation is presented in Proposition 2.5.

Proposition 2.5. *Under the sharing case, for two cooperative hospitals, when $h \leq \tau_s + p_r + \tau_r$, the optimal expected total cost increases as w_i and w_j increase. When $\tau_s \leq p_e + \tau_e + h - p_r - \tau_r$, the optimal expected total cost increases as k_i and k_j increase.*

With the mathematical proof, we find that only when $h \leq \tau_s + p_r + \tau_r$, does a higher w_i or w_j increase the total cost of two hospitals. In general, for the alliance of two hospitals i and j, a higher w_i or w_j leads to a higher sharing amount/emergent replenishment amount and a higher total cost. However, if h is very high ($h > \tau_s + p_r + \tau_r$), more holding cost is saved through the sharing mechanism and can offset some cost caused by increasing w_i or w_j, and the expected total cost may not increase. Similarly, only when $\tau_s \leq p_e + \tau_e + h - p_r - \tau_r$ is satisfied, does a higher k_j make hospital j share less inventory with hospital i, then hospital i spends more on emergent replenishment. Total cost increases as k_j increases under the above condition. If τ_s is very high, $\tau_s > p_e + \tau_e + h - p_r - \tau_r$, inventory sharing is not preferred for hospital i when stockout happens. Under the case, k_j does not effect on sharing action and the expected total cost; a higher total cost of the alliance results from the emergent replenishment order placed by hospital i.

Proposition 2.6. *Under the sharing case, for two cooperative hospitals i and j, given x_j, there exists a unique pair of order-up-to levels (x_i, x_j) such that $x_i^*(x_j)$*

minimizes $O^s(x_i, x_j)$. *When* $\tau_s \leq p_e + \tau_e + h - p_r - \tau_r$, $x_i^*(x_j)$ *has the following properties:*

i. $x_i^*(x_j)$ *increases as* w_j *increases.*

ii. $x_i^*(x_j)$ *increases as* k_j *increases.*

iii. $x_i^*(x_j)$ *decreases as* x_j *increases.*

Without hospitals' sharing action, the inventory decisions in hospital j do not affect the inventory decisions of hospital i. However, in the sharing mechanism, the two hospitals form an alliance and aim at preventing stockout with a minimal total cost. Therefore, the emergent request rate of patients in hospital j, the safety stock level of hospital j and the order-up-to quantity of hospital j affect the decision process of hospital i. Under the condition that $\tau_s \leq p_e + \tau_e + h - p_r - \tau_r$, inventory sharing is preferred when stockout occurs in hospital i. If hospital i realizes that hospital j may face an increasing w_j (which means hospital j needs more inventory to satisfy its internal demand), then hospital i will increase its inventory level $x_i^*(x_j)$ in case of a stockout.

On the other hand, if hospital i is informed that the safety stock level k_j in hospital j increases (which means hospital j wants to keep more safety stock), the possible sharing amount to hospital i will decrease. Thus, hospital i can only increase $x_i^*(x_j)$ in advance in case of expensive emergent replenishment. Besides, if inventory sharing is preferred, when hospital j increases the inventory level x_j, hospital i will decrease $x_i^*(x_j)$ to avoid overstock.

2.6 Results and Discussion

In this part, we compare two inventory policies (emergent replenishment policy and sharing policy) with various parameter settings (replenishment price, transportation cost, inventory handling cost, patient's emergent request rate, and safety stock levels of the hospitals) using numerical experiments. For simplicity, we denote the emergency replenishment policy as the no-sharing policy.

In our model setting, the decision process has Markov property, which means the decision in a period only depends on the decision in the last period, but is independent of any other previous periods. Then our numerical experiments consist of four parts. The first part investigates the effects of emergency replenishment price and transportation cost on hospitals' decisions and the expected costs. The second part explores the optimal inventory decisions of hospitals under the two inventory policies. Then, in the third part, we compare the expected total cost of two hospitals when they are independent (without sharing policy) and in an alliance (under the sharing policy), and explore the cost reduction of the inventory sharing option. In the fourth part, we investigate how hospital i determines the inventory order-up-to level according to the order-up-to level of hospital j.

The numerical experiments assume that two hospitals face the stochastic demand, which follows a normal distribution with $\mu = 100$ and standard deviation $\delta = 50$. Other parameters are: the regular replenishment price for $p_r = 40$ for all periods, emergent replenishment price $p_e = 50$, per-unit transportation cost for regular replenishment $\tau_r = 5$, per-unit transportation cost for emergent replenishment $\tau_e = 10$, per-unit transportation cost for sharing $\tau_s = 12$, emergent request rate of hospital i $w_i = 0.8$, and safety stock rate of hospital i $k_i = 0.1$. Without inventory sharing, hospital i and hospital j are independent. The optimal inventory level depends on the demand forecast and the emergent request rate for each hospital. A higher emergent request rate leads to higher emergent replenishment quantity and higher cost. Therefore, the impacts of w_i on hospital i or w_j on hospital j are not included in this experiment. Instead, we focus on the effects of the emergent request rate of hospital j and the safety stock level of hospital j on the order-up-to level of hospital i under the sharing case. In the result presentations, we denote the no-sharing case as "N" and the sharing case as "S".

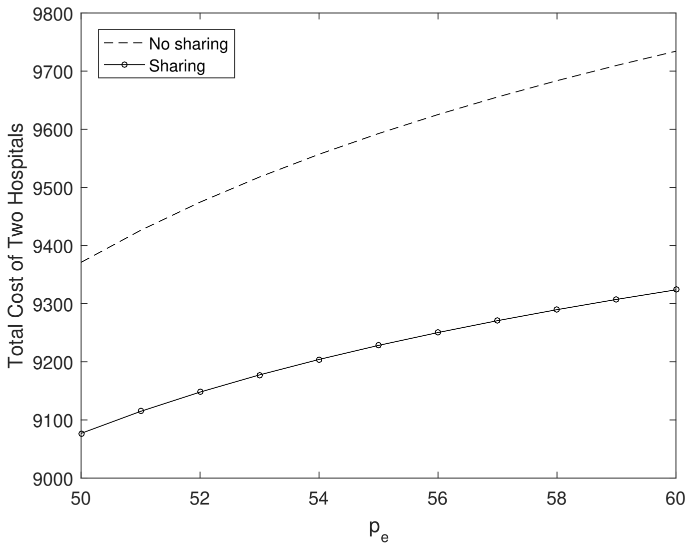
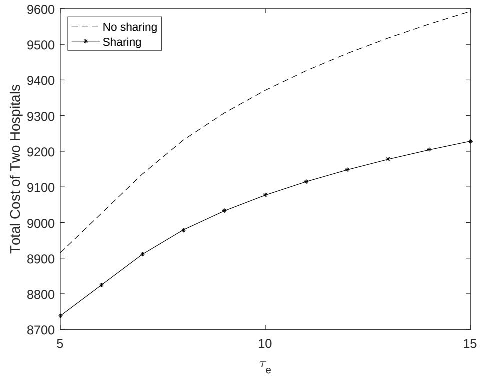


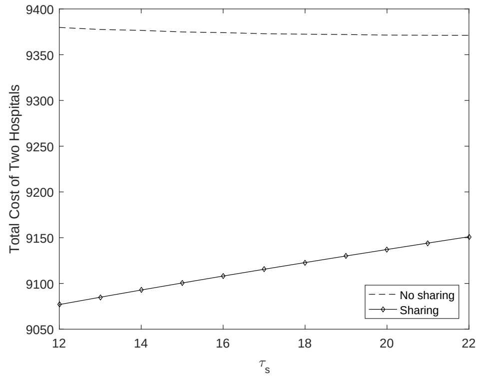
Figure 2.1: The impacts of emergent replenishment price on hospitals' total costs.

2.6.1 The Impacts of Emergent Replenishment Price and Transportation Cost

In our study, hospitals' inventory decisions, sharing decisions, and expected total cost are affected by many factors, including internal factors (replenishment price, transportation cost, inventory handling cost, safety inventory level) and external factors (demand of patients and patient's behavior). We first explore the impacts of replenishment price and transportation cost using numerical experiments. Figure 2.1 shows that a higher emergent replenishment price increases hospitals' total cost and induces the inventory sharing action. Figure 2.2a,b identify the effects of emergent replenishment transportation cost and sharing transportation cost on hospitals' ordering decisions, respectively. Under the sharing case, a higher emergent replenishment transportation cost and a higher emergent replenishment price increase the hospitals' cost reduction compared to the no-sharing case. However, a higher sharing transportation cost decreases the hospital's cost reduction compared to the no-sharing case. These results verify Corollary 2.1 well.



(a) The impacts of emergent replenishment transportation cost.

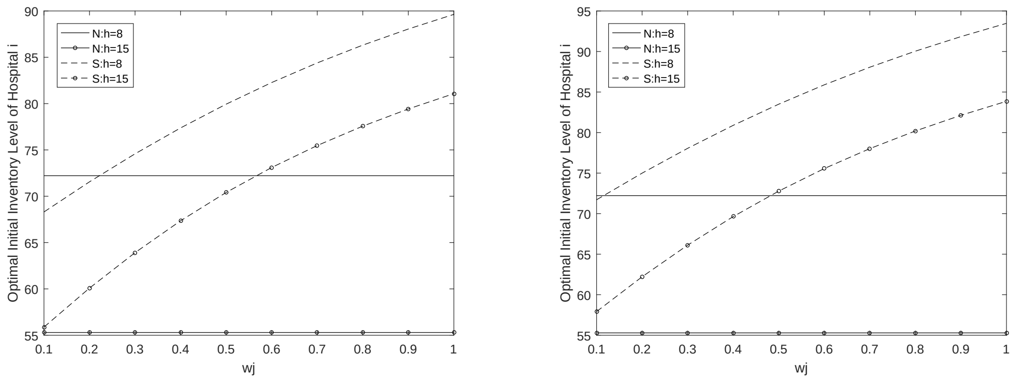


(b) The impacts of sharing transportation cost.

Figure 2.2: The impacts of transportation cost on hospitals' total costs.

2.6.2 The Optimal Order-up-to Levels of Hospitals

This section explores the optimal order-up-to level of a hospital with and without the sharing option, respectively. Figure 2.3a shows that: (a) Under the no-sharing case, the optimal inventory level of hospital i is independent of the emergent request rate of hospital j. (b) Under the sharing case, the optimal order-up-to level of hospital i increases as the emergent request rate w_j increases, regardless of the holding cost. It means that when more patients request emergent service in hospital j, then hospital j cannot share its inventory with hospital i. Hospital i could only increase the order-up-to level to reduce stockout risk. (c) Higher inventory holding cost ($h = 15$) decreases the optimal inventory order-up-to level under two cases. The explanation is intuitive—the hospital will reduce the order quantity for medical items with higher holding cost (e.g., some items needing cryopreservation) to minimize total inventory cost, especially when demand is uncertain. Figure 2.3b shows similar results to Figure 2.3a; by comparing the two figures, it can be found that under the sharing policy, when the safety stock level of hospital j is higher ($k_j = 0.5$), the corresponding order-up-to level of hospital i is slightly higher than that of the lower k_j case ($k_j = 0.1$). The main reason is that a higher safety stock level will decrease the sharing amount; hospital i that anticipates a stockout may not be satisfied by the sharing action and needs to improve the order-up-to level in advance.

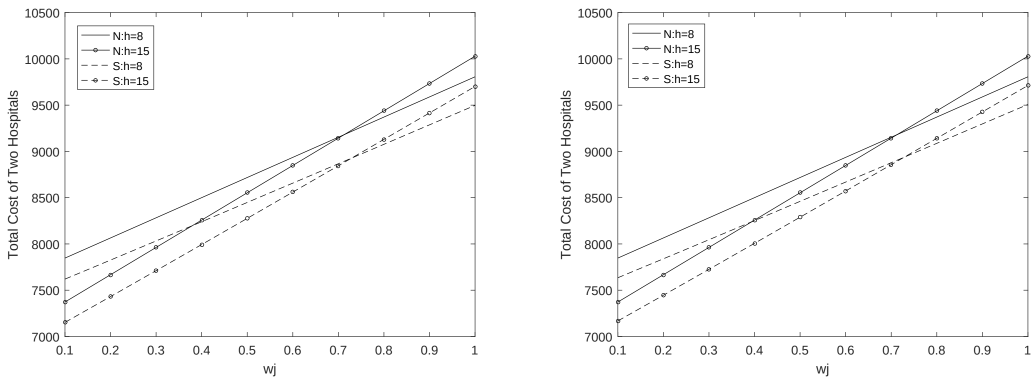


(a) $k_j = 0.1$ (b) $k_j = 0.5$

Figure 2.3: Optimal order-up-to level of hospital i.

2.6.3 The Optimal Expected Total Cost of Hospitals

In the third part of the numerical experiments, we compute the expected total cost of hospitals under the two inventory policies. Under the no-sharing case, we consider the two hospitals separately and compute the sum of the expected cost. The hospital i determines its optimal inventory level and the expected total cost by assuming w_i is constant at 0.8. The cost of hospital j increases with the increasing w_j from 0.1 to 1. Under the sharing case, we take the two hospitals as an alliance and compute the expected total cost when w_j increases from 0.1 to 1. Figure 2.4 illustrates the following results: (a) When the per-unit holding cost is the same, the sharing policy saves more costs than the no-sharing policy, regardless of w_j or k_j. The results indicate that if the sharing condition is satisfied, inventory sharing is more economical than the emergent replenishment policy. (b) In both no-sharing and sharing cases, the optimal total cost increases as w_i or w_j increases (we only show the effects of w_j here, as w_i has a symmetric effect). When w_j is small enough (≤ 0.7), we find that under the same policy, if h is higher, then the total cost is lower since a higher holding cost encourages the hospital to order less (result from Figure 2.3). However, when w_j is large enough (> 0.7), a higher h leads to a higher total cost. Compared to the lower h case, hospital i provides less s_i to hospital j and leads to more e_j under the higher h case. Therefore, under the same policy, if the emergent request rate is low, reducing the inventory level is economical for the medical items with higher holding cost. By contrast, if the emergent request rate is high, increasing the inventory level saves more costs.



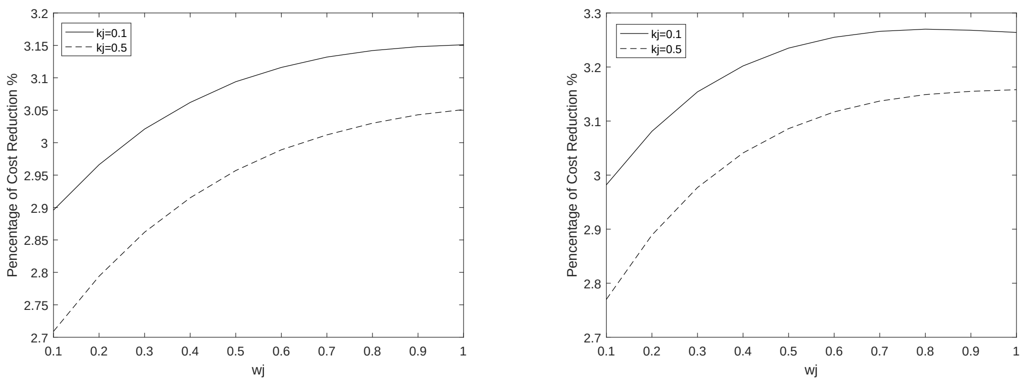
(a) $k_j = 0.1$ (b) $k_j = 0.5$

Figure 2.4: Optimal expected total costs of the two hospitals.

Besides, under the sharing policy, the expected total cost increases as k_i or k_j increases (we only show the effects of k_j, as k_i is having a symmetric impact). To present the slight difference between Figure 2.4a and b, we compare the expected percentage of cost reduction under the lower k_j case ($k_j = 0.1$) and higher k_j case ($k_j = 0.5$). Figure 2.5 shows that hospitals in the sharing mechanism save more costs than in the no-sharing mechanism when k_j is lower. Since if the sharing action is preferred, a higher safety stock level will decrease the sharing amount, and hospitals will satisfy the remaining demand by the emergent replenishment, leading to a higher total cost.

2.6.4 The Response Inventory Decisions of Hospitals

We investigate how the hospital responds to another hospital's inventory decision under the sharing mechanism in this section. Figure 2.6 presents the response inventory level of hospital i to hospital j with different values of w_j. Under the sharing case, if inventory sharing is economical, one hospital will decrease the inventory level if the partner hospital increases its inventory order-up-to level. This avoids the overstock problem at the hospital. Furthermore, when the emer-



(a) $h = 8$ (b) $h = 15$

Figure 2.5: Percentage of cost reduction for different k_j.

gent request rate of hospital j is higher, although hospital i has an incentive to decrease its order-up-to level as hospital j increases its inventory level, hospital i still orders more than that in the smaller w_j case.

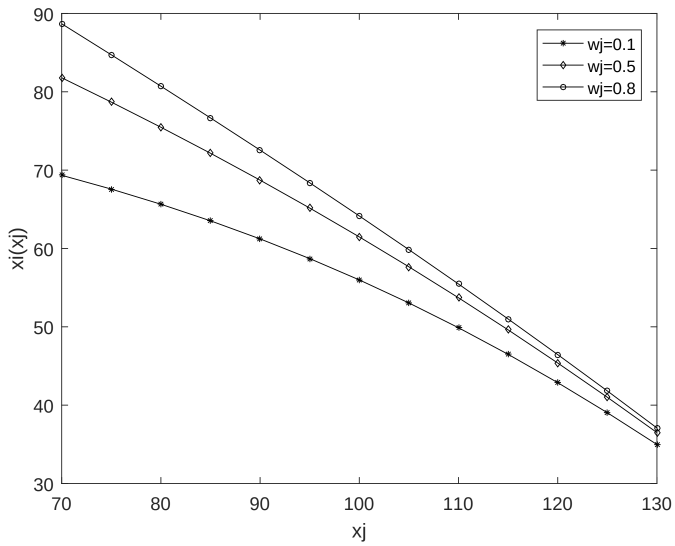


Figure 2.6: The response order-up-to level of hospital i to hospital j.

2.7 Conclusions

This chapter constructs an inventory sharing model for two hospitals to address the inventory stockout problem. We first derive the emergent replenishment policy and optimal inventory order-up-to level for each hospital when we do not consider inventory sharing. Then we propose an inventory sharing policy and obtain the optimal inventory order-up-to level when the two hospitals are cooperative. Furthermore, we explore the impacts of the patient's emergent request rate and the safety stock level of the hospital, as well as other cost parameters on the optimal inventory decisions. It is found that when the sharing condition is satisfied, the inventory sharing policy is more economical than the emergent replenishment policy. Under the sharing case, the expected total cost of two hospitals increases when one hospital's emergent request rate increases and the safety inventory level increases. The optimal inventory order-up-to level of one hospital increases when the partner hospital's emergent request rate increases, the partner hospital's safety inventory level increases, or the partner hospital's order-up-to level decreases.

Furthermore, this chapter provides the following contribution: (1) It solves the stockout problem of medical items in hospitals; we propose the inventory sharing mechanism and derive optimal inventory policies for two cooperative hospitals. (2) Under the sharing policy, we investigate the impacts of patients' behavior (the emergent request rate), the hospital's safety inventory level, and other cost parameters on the inventory decisions. On one hand, from the perspective of research novelty, this study enriches the previous literature review about inventory sharing in the healthcare setting. On the other hand, the results from this study could provide some managerial insights into hospitals' operational practice. (3) The one-period model setting can be extended to a multi-period setting when the scenarios have the following features: demand updates in every period and the inventory decision in a period only depends on the decisions of the previous period. Therefore, this study also has good applicability.

This study also has some limitations which can be considered in future work. First, although our mathematical model describes hospitals' ordering process and inventory sharing process completely, it still neglects some complex characteristics in actual healthcare operations, such as medical manager behaviors (Stefanini et al. 2020), the service level of patients (Wieser 2011), information asymmetry between the dealer and hospitals, and delivery reliability in transportation (Kritchanchai and MacCarthy 2017). By considering these specific characteristics in the mathematical model, the research would provide more managerial insights for practice. Second, this study is in a two-hospital sharing setting, and we will consider an n-hospitals inventory sharing problem in future research. Furthermore, in the current study, our sharing mechanism assumes that the borrowing hospital (requests sharing) would return the amount shared to the lending hospital (accepts a sharing request) in the next period. In further research, we will also consider the impacts of the unpunctual return problem on the sharing mechanism.

Chapter 3

Coordinating Inventory Sharing with Retailer's Return in the Consignment Contracts

3.1 Introduction

Although hospitals or clinics apply various purchasing or inventory policies to manage disposable medical items, the shortage or waste of expired medical inventory exists as the difficulty of predicting patient's demand. Healthcare organizations afford the high costs to manage such medical inventory. For instance, a small private clinic needs to dispose of an average of 750$ worth of drugs and disposable medical items per month in Hong Kong. Large hospitals adopt consignment contracts to manage drugs and disposable medical inventories to reduce such inventory management costs. Consignment is a business model in that a dealer places products at the retailer's warehouse but receives the payment until these items are sold. In general, long-lead-time disposable medical items have strong consignment potential, such as intraocular lenses and orthopedic implants. For example, intraocular lenses have multiple models and sizes for fitting different patients and are widely used to treat cataracts. If hospitals purchase and stock all sizes of intraocular lenses in the warehouse, the inventory cost would be very high. Therefore, the hospital prefers to allow the dealer to consign these items to the hospital and pays the used quantity at the end of a consignment cycle. In addition, these inventories have a long manufacturing lead time, and cannot be replenished in a short time. Therefore, hospitals benefit from the consignment inventory policy. First, under the consignment policy, hospitals could hold flexible inventory, especially for emergent surgical items. As a result, patients face less risk of treatment delay since consignment inventory guarantees immediate supply for urgent demand. Second, the hospital's cash flow can be significantly improved since the used medical products can be charged to patients before the hospital pays the dealer (Ballard 1991).

Although the consignment contract reduces the waste of disposable inventory and mitigates the risk of overstock for hospitals, there are still some issues with the consignment contracts. The first issue is the increasing return of unused medical items. Considering the quick update of medical item categories, high inventory cost, and demand uncertainty, hospitals would rather return unsold/unused (we use these interchangeably in the following sections) items to the dealer than keep them in their warehouses. Dealer provides a return policy for hospitals, under which hospitals are allowed to return unsold items at the end of a selling season. Consequently, the dealer needs to dispose of the returned medical items. Unlike traditional commodities (such as toys or clothes), returned medical items have low salvage values and are quickly expired, and cannot even be traded in the secondary market. Therefore, the dealer wants to reduce the hospital's return with a more reliable return policy acceptable to the hospitals. For instance, "Cardinal Health", an integrated healthcare services and products company, provides medical products to hospitals by consignment contracts. It charges a hospital 10% of the invoice price as a restocking fee when the hospital returns unsold products in a saleable condition and charges the hospital 25% of the invoice price as a restocking fee for returns not in a saleable condition. Although the hospital can return the unsold products to the medical dealer at the end of a selling season, the dealer still charges some fees for restocking according to the condition of the returned products. Such a consignment contract with return issues also exists in industrial practices. For instance, some big retailers (e.g. Walmart, Target, Meijer) sell seasonal items (e.g. Christmas decorations or toys) by the consignment contract (Lee and Wai 2005). These retailers will return unsold items to the vendor at the end of the selling season (Hu et al. 2014). In addition, in the book trading industry, the wholesalers and retailers can return unsold books to the publisher at a full price minus the shipping fee and handling fee (Rungtusanatham et al. 2007). Therefore, the return policy in consignment contracts is worth more academic and practical concerns.

The second issue is about resource utilization in the consignment contracts. Traditional consignment focuses on a one-to-one contract, one dealer to one hospital. In practice, a large medical company serves two or more hospitals. Hospital's inventory information is shared with the common dealer but not shared among hospitals. Therefore, for the same medical item, one hospital might return many unsold medical items while another hospital might face a stockout of the same medical item. In the case, the inventory sharing policy provides a better solution to increase the overall resource utilization rate. Shao et al. (2011) mention that inventory sharing between retailers or dealers has been widely applied in industry and has drawn academic attention. In recent years, hospitals and other healthcare organizations have also considered inventory sharing as a new inventory management approach to reduce the stockout of medical supplies. For example, more than 50% of US hospitals join in the multi-hospital system. This type of hospital consolidation encourages medical resource sharing (i.e. medical items, vaccines, and blood) (Cutler et al. 2011, Burns et al. 2015). Zepeda et al. (2016) find that the multi-hospital system is beneficial to reduce the medical supply chain risk. Although the application of revenue-sharing contracts in the consignment contract has concentrated in academia (Bart 2021, Heydari 2021), there is still no analytical work on the consignment contracts with retailers' inventory sharing, especially considering retailers' return problem.

Motivated by the above-discussed problems of consignment contracts, we aim at developing a sharing and return framework for two retailers and one common dealer, exploring the interactive effects among retailers' sharing decision, return decision, and dealer's pricing decision, and providing insights for a dealer to choose a non-cooperative retailer or cooperative retailers, for retailers to be individual or cooperative.

This chapter tackles the following three research problems:

(1) Considering retailers' sharing action, how does the dealer decide the wholesale and design the return policy for retailers?

(2) How do the dealer and retailers benefit from the sharing policy in a dealer-dominated case and a retailer-dominated case, respectively?

(3) How does the dealer's return policy affect the retailer's sharing action, retailers' profits, and the dealer's profit?

Our study provides a twofold contribution to production research. First is the novel model design. We construct a framework that includes a common dealer and two retailers, in which we consider the retailers' inventory sharing action and return action. To our knowledge, we are the first to investigate the interactive effects of inventory sharing decisions, return decisions, and pricing decisions in consignment contracts. The second is the practical application. Our research outputs provide some managerial insights for production practice in real life. For instance, the powerful dealer benefits from retailers' sharing by reducing the refund products. In contrast, the weak dealer is better off by transacting with the individual retailer rather than cooperative retailers (such as retailers of chain stores). Furthermore, increasing the return price that retailers need to pay for refund products cannot encourage retailers' inventory sharing; retailers prefer sharing excess inventory with a decreasing return price counter-intuitively.

The remaining sections of this chapter are arranged as follows: Section 3.2 reviews the relevant literature and identifies the main differences between the previous studies and ours. In Section 3.3, we develop a two-echelon model between the dealer and two retailers, which captures the feature of sharing and return in consignment contracts. Section 3.4 analyzes the dealer's pricing & return policy and retailer's ordering decision without the sharing option. Section 3.5 compares the performance of the dealer and retailers under two different cases: the dealer controls retailers' inventory sharing, and retailers control their sharing. This section also identifies the effects of return price on sharing decisions and the expected profits. Section 3.6 summarizes the main findings of this study. Finally, all proofs and results of supplemental numerical experiments are presented in Appendix B.

3.2 Literature Review

As our research explores the effects of the return policy on consignment inventory sharing, we mainly review the following three streams of literature: inventory return problem, consignment contracts, and inventory sharing policy. Considering the generality of our model, we review literature that covers one or two of the above features in the industrial setting. Additionally, as our research is motivated by the hospital return problem in the consignment contract, we also review the literature on hospital or healthcare inventory management.

The first category of literature tackles the pricing and order problem in the supply chain with the consideration of return policy. There are two categories of return problems in the traditional supply chain, channel return and customer return (Hu et al. 2014). Channel return policies are offered by suppliers/dealers to retailers for returning unsold products, while retailers offer return policies to customers for not-fitting tastes or expectations. Gümüş et al. (2013) also denote the channel return as intrachannel return and the customer return as extrachannel return, respectively. Yao et al. (2008) explore the effects of price sensitivity on the return policy. When the price sensitivity is low, the return policy can better coordinate the channel profits than the wholesale-price-only contract. Chen and Bell (2009) construct a return function to measure the customer return quantity, depending on the retail price and selling quantity. Chen and Bell (2011) consider both channel return and customer return. They suggest two categories of buyback prices for unsold return and customer return respectively. A buyback policy is an approach in which the manufacturer buys unsold products with a buyback price from the retailer. They also find that retailers need to make a joint decision on price and order quantity when facing price-sensitive stochastic demand. Xiao et al. (2010) develop a coordination mechanism between a manufacturer and a retailer when considering the customer return. They examine the effects of full refund policy, partial refund policy, and no-refund policy on profitability. Chen (2011) explores whether the retailer sharing customer return information to the manufacturer will affect the retailer's order and manufacturer's pricing decisions. Liu et al. (2014) investigate the effect of the buyback policy on supply chain coordination. They find that the buyback policy can coordinate the manufacturer and retailer when the return quantity is decided exogenously. However, when the return quantity is endogenous, the coordination is not induced.

The secondary category of literature concerns the return issue in the consignment contracts. Regarding the consignment contracts, scholars have done a wide range of studies, such as the research on channel performance of consignment contracts, the comparison of the Vendor Management Inventory (VMI) and the Retailer Management Inventory (RMI), the pricing & ordering decision in consignment, and consignment contract design. Under the consignment contracts, Wang et al. (2004) examine the channel performance and the individual performance of firms in a centralized system and decentralized system respectively. Ru and Wang (2010) compare the performance between Vendor Management Consignment Inventory mechanism (VMCI) and the Retailer Management Inventory mechanism (RMCI). It is found that the former is always better than the latter for both suppliers and retailers. Some scholars focus on the specific features of consignment contract design. For instance, Yang et al. (2019) explore the manufacturer-retailer matching policy considering revenue sharing and slotting fees in the consignment agreement. Sen et al. (2021) investigate the effects of warehouse space constraints of both the consignor and consignee on supply chain performance. They also consider the deterioration effect of the product in the consignment contract design. However, the literature on return policy in consignment contracts is scarce. In the consignment contracts, the supplier/dealer offers a free return policy for unsold products to the retailer to capture a larger market share, which puts the supplier/dealer into a low-power position. To reduce profit loss caused by free return, the supplier/dealer charges retailers additional fees for restocking or handling unsold products. Extended the model setup of Ru and Wang (2010), Hu et al. (2014) consider the salvage value of returns, incorporate the customer return and channel return in consignment contracts, and show that VMCI mechanism is always more beneficial than the RMCI mechanism regardless of the return policy.

The third stream of literature focuses on the effect of the return policy on inventory decisions when inventory sharing/transshipment exists. Shao et al. (2011) construct a sharing inventory mode with two retailers and a common dealer and identify the difference between dealer-control sharing and retailer-control sharing. However, they do not incorporate the product return issue. Our study focuses on the downstream sharing policy when a consignment contract's return option is available. Furthermore, we identify the effects of the return policy on sharing decisions. Dan et al. (2016) model the preventative manufacturer-dominated inventory transshipment policy in retailers. Under this setting, they investigate the effects of return constraints on pricing and ordering decisions. Although their work is similar to our study, there are still many differences. The first difference is that they explore the return issue in a regular wholesale-price contract while we focus on the consignment contracts. The second difference is the type of transshipment; they apply the preventative inventory transshipment (before stockout) while the sharing policy in our model is reactive (when stockout happens). The last difference lies in the return policy design. In their work, return price is a parameter, which reflects the refund price that the manufacturer refunds to the retailer for unsold products, but in our model setting, return price is a decision variable and denotes the additional restocking fee that the dealer charges the retailer for returning unsold inventory.

Besides, we review some research on hospital inventory management. Saha and Ray (2019) provide a detailed review that classifies the modeling methodologies and solution methods for healthcare inventory management. Based on their work, we find that the difference in hospital inventory management approaches lies in the replenishment policies. Two traditional replenishment policies have been widely used in the industrial setting: periodic review policy (the inventory is replenished at the beginning of the replenishment period) and continuous review policy (the inventory is replenished when the warehouse is empty) (Rosales 2015). Bijvank and Vis (2012) investigate the application of two types of periodic review policy in hospital inventory replenishment. The first model maximizes the hospital service level with a capacity constraint while the second model minimizes the hospital capacity with a service level constraint. Saedi et al. (2016) apply a continuous review policy to mitigate the drug shortage when considering the uncertain demand in the hospital. However, Rosales (2014) and Rosales (2015) find that the hybrid inventory management policy, which combines the low-cost periodic-review replenishment policy and the high-cost continuous-review replenishment policy, is more profitable for hospitals. Furthermore, Wang et al. (2015) propose an innovative, dynamic replenishment policy where the buffer size is adjusted according to the dynamic demand.

After conducting an exhaustive analysis of related literature, we summarize the major differences between the representative literature and our research in Table 3.1. In short, the main contribution of our study to production research literature is twofold. First, our study makes up the research gap on the consignment contracts considering inventory sharing and return policy. We develop a two-echelon (upstream dealer and downstream retailers) model with downstream inventory sharing as well as downstream return to the upstream dealer. This framework in the consignment contracts has not been considered in previous research. Second, we identify the complex interaction among upstream dealer's pricing decisions, return decisions, and downstream retailers' sharing decisions. The findings in this study could provide some theoretical support for future research on consignment contract design with the interactive effect of inventory sharing and return policy.

Table 3.1: The differences between several representative literature and ours.

	Centralized	Decentralized	Channel return	Customer return	Consignment contract	Wholesale price contract	Sharing/ transshipment
Yao et al. (2008)		$\diamond$	$\diamond$			$\diamond$	
Xiao et al. (2010)	$\diamond$	$\diamond$	$\diamond$	$\diamond$		$\diamond$	
Chen (2011)		$\diamond$	$\diamond$	$\diamond$		$\diamond$	
Hu et al. (2014)	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$		
Dan et al. (2016)		$\diamond$	$\diamond$	$\diamond$		$\diamond$	$\diamond$
Wu et al. (2016)	$\diamond$	$\diamond$	$\diamond$	$\diamond$	$\diamond$		
Shao et al. (2011)	$\diamond$	$\diamond$				$\diamond$	$\diamond$
ours	$\diamond$	$\diamond$	$\diamond$		$\diamond$		$\diamond$

3.3 Model Description

We consider a single-period model consisting of one common dealer and two retailers. The dealer (he) sells a disposable item to two independent retailers (she) by consignment contract. The dealer decides the unit consignment price to two retailers and the unit return price (restocking price, we use them interchangeably in the following sections) for unsold returns. Before the demands are realized, two retailers decide their order quantity simultaneously, and the dealer delivers the orders to the retailers' consignment warehouses at the beginning of a consignment cycle. Considering the long lead time of the product, we suppose that no replenishment is provided during the selling season. The dealer's unit transaction cost (handling cost) is c for processing the retailer's order. Retailers can fulfill the demand of customers by consignment inventory. The stochastic demand of retailer i is denoted by D_i, where $i = 1, 2$ represents the retailer, F_i and f_i denote the distribution function and density function of D_i, respectively.

When a consignment cycle begins, the dealer determines a consignment price w per unit item, and a return price r charged to retailers for unsold returns at the end of the cycle. The retailer charges the customer p per unit item, and the market determines the retail price. We assume that the sharing transportation cost is 0. The agreed sharing price w_s is predetermined by their common dealer or set by retailers (Shao et al. 2011). Besides, the sharing amount from retailer i to retailer j is denoted by T_i, which equals the minimum of the excess inventory of retailer i and the excess demand of retailer j ($T_i = min\{(Q_i - D_i)^+, (D_j - Q_j)^+\}$). At the end of the consignment cycle, the retailer pays the dealer the net used amount and returns the unused inventory with r. For the dealer, the salvage value per unit return is s. We assume that $s + r < w$ to prevent the arbitrage opportunity. $c > s$ is also a critical assumption that allows the dealer to charge some return fee to retailers in consignment contracts.

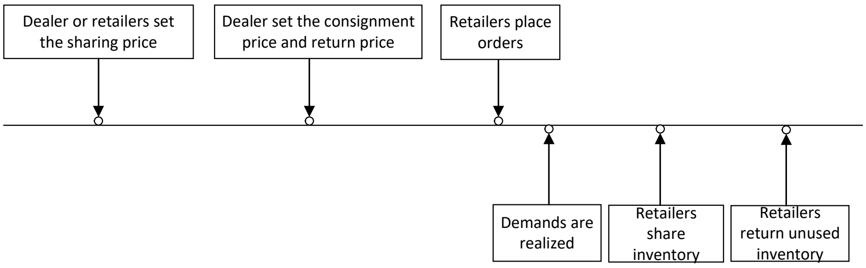


Figure 3.1: The timeline of events

Based on the above framework, the sequence of events under the sharing mechanism is described as follows (shown in Figure 3.1):

Step 1. The powerful dealer sets the sharing price or retailers set a sharing price w_s.

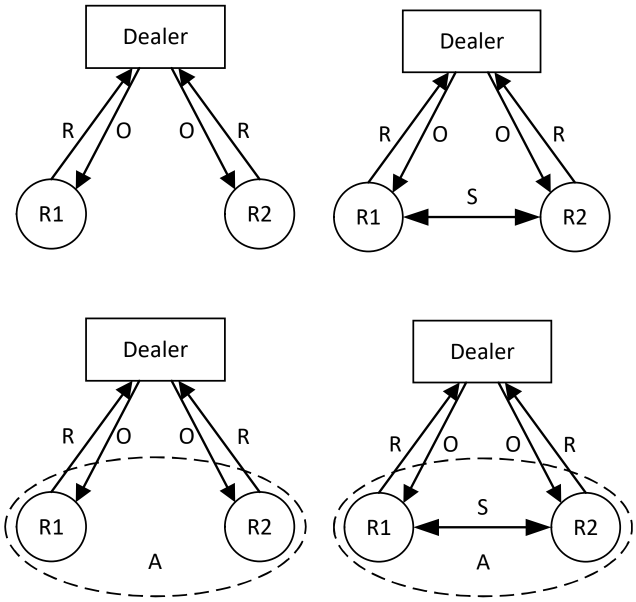
Step 2. The dealer decides the consignment price w charged to the retailer and the return price r for unsold return at the end of the consignment cycle.

Step 3. Retailers decide the order quantity Q_i simultaneously and receive the order at the beginning of the consignment cycle. We assume no emergent or additional replenishment during the consignment cycle for such a long-lead-time item.

Step 4. After the demand is realized, the retailer charges customers the market price p for each unit item.

Step 5. When one retailer anticipates stockout and another retailer has excess inventory, they share inventory with the given sharing price w_s.

Step 6. At the end of the consignment cycle, two retailers pay the dealer the net used amount and refund unused inventory to the dealer with a per unit return price r.



(a) No sharing case (b) Sharing case

Figure 3.2: The operation of the supply chain without retailers' sharing & with retailers' sharing.

The sequence of events without retailers' sharing can ignore step 1 and step 5. As Figure 3.2 shows, "O" denotes the retailer places an order from the dealer, "R" denotes the retailer returns the unused inventory to the dealer, "S" represents the inventory sharing action between retailers, and "A" means that retailers form a cooperative alliance. Therefore, we first consider the benchmark case without retailers' sharing action and explore the difference between individual-retailers-sharing and cooperative-retailers-sharing (Figure 3.2a). Then further investigate the effects of retailers' sharing action on dealer's profit and retailers' profits, respectively. Furthermore, we analyze the performance of the dealer and retailers when they control the sharing price, respectively.

3.4 No Sharing Case

When the inventory sharing option is not considered, the decision sequence of the events is as follows: the dealer decides consignment price w and the return price r charged to the retailer, respectively. Then the retailer decides the order quantity Q_i for the whole consignment cycle. We aim at obtaining the Nash equilibrium by backward induction.

3.4.1 The Ordering Policy of Retailer

Considering the ordering decision of the retailer i, the expected profit is denoted by $\Pi_N^{h_i}(Q_i)$,

$$\Pi_N^{h_i} = E\left\{(p-w)\min(D_i, Q_i) - r(Q_i - D_i)^+\right\} \tag{3.1}$$

The profit of retailer i equals the net profit of sold quantity minus return fee of unsold refunds. Defining $\Lambda(Q_i) = \int_A^{Q_i}(Q_i - D_i)f_i(D_i)dD_i$ and $\Theta(Q_i) = \int_{Q_i}^B(D_i - Q_i)f_i(D_i)dD_i$, we obtain the unique Q_i (denoted by Q_N^i) that maximizes the expected profit of retailer i when w, r and p are given.

$$Q_N^i = F_i^{-1}(\frac{p-w}{p-w+r}) \tag{3.2}$$

3.4.2 The Pricing Policy of Dealer

The dealer's expected profit is denoted by Π_N^d,

$$\Pi_N^d = E\left\{\sum_{i=1}^{2} w\min(D_i, Q_i) + (r+s)(Q_i - D_i)^+ - cQ_i\right\} \tag{3.3}$$

Considering the centralized system as a benchmark, we obtain the total expected profit of the dealer and two retailers, which is denoted by Π_N^c,

$$\Pi_N^c = E\left\{\sum_{i=1}^{2} p\min(D_i, Q_i) + s(Q_i - D_i)^+ - cQ_i\right\} \tag{3.4}$$

The first-order condition satisfies $\frac{\partial \Pi_N^c}{\partial Q_i} = 0$. We obtain the expression of Q_N^{ci}, which maximizes the total expected profit of the centralized system.

$$Q_N^{ci} = F_i^{-1}(\frac{p-c}{p-s}) \tag{3.5}$$

To motivate the retailer to order Q_N^{ci} under the decentralized system, the dealer offers retailers specific pricing and return policies.

Proposition 3.1. *When the dealer and retailers are coordinated without an inventory sharing option, the dealer provides a return policy (w, r) for retailers, in* *which, the range of return price is denoted as* (r_N^l, r_N^h)*, where*

$$r_N^l = 0 \tag{3.6}$$

$$r_N^h = c - s \tag{3.7}$$

$$w = p - \frac{(p-c)r}{c-s} \tag{3.8}$$

Proposition 3.1 indicates that $w(r)$ is a decreasing function of r according to $c - s > 0$. Without retailers' sharing option, the dealer offers a return policy $r \in (r_N^l, r_N^h)$ that ensures both the dealer and retailers earn positive profits. If the dealer charges a higher return price r to retailers ($r > r_N^h$), the dealer earns a negative profit since the consignment price is lower than c. If the dealer charges a lower return price ($r < r_N^l$), retailers will make a negative profit because the consignment price provided by the dealer is higher than the retail price charged to customers.

3.5 Sharing Case

In this section, we construct a framework consisting of two retailers, i and j, and their common dealer. We first analyze the ordering decision of retailers, then explore how the dealer decides consignment price and return price with the consideration of retailers' sharing action. Finally, we investigate the setting of sharing price under two cases: the dealer controls retailers' sharing and retailers control their sharing.

3.5.1 The Ordering Policy of Retailer

We first consider how the retailers make order decisions with the sharing option by the backwards induction. The decision process is similar to the general Newsvendor model when the sharing amount between two retailers is decided. Let $\Pi_S^{h_i}$ denote the total expected profit of retailer i,

$$\Pi_S^{h_i} = E\left\{(p-w)\min(D_i, Q_i) + (w_s - w)T_i + (p - w_s)T_j - r(Q_i - D_i - T_i + T_j)^+\right\} \tag{3.9}$$

Taking the first derivative with Q_i, we obtain the unique Q_i (denoted as Q_S^i) that maximizes retailer i's expected profit.

$$F_i(Q_S^i) = 1 + \frac{w_s - w}{p - w}\frac{\partial T_i}{\partial Q_i} + \frac{p - w_s}{p - w}\frac{\partial T_j}{\partial Q_i} - \frac{r}{p - w}F_i(Q_i - T_i + T_j) \tag{3.10}$$

3.5.2 The Pricing Policy of Dealer

Under the sharing mechanism, inventory sharing between retailers reduces the retailer's return quantity to the dealer. Dealer's expected profit is also affected by retailers' sharing. Therefore, the dealer needs to consider the possible effects of sharing action when he determines the consignment price and return price. Let $\Pi_N^d(w, r)$ denote the total expected profit of the dealer,

$$\Pi_S^d = E\left\{\sum_{i=1, j\neq i}^{2} w(\min(D_i, Q_i) + T_i) + (r + s)(Q_i - D_i - T_i + T_j)^+ - cQ_i\right\} \tag{3.11}$$

Before tackling the optimal pricing decision of the dealer, we also consider the centralized system of the dealer and retailers as a benchmark case. By adding (3.9) and (3.11), we obtain the total expected profit of the centralized system as follows,

$$\Pi_S^c = E\left\{\sum_{i=1, j\neq i}^{2} \{p\min(D_i, Q_i) + w_sT_i + (p - w_s)T_j + s(Q_i - D_i - T_i + T_j)^+ - cQ_i\right\} \tag{3.12}$$

Solving the first condition $\frac{\partial \Pi_S^c}{\partial Q_i} = 0$, we find that there exists a unique Q_S^{ci} that maximizes the total expected profit of the system. The corresponding Q_S^{ci} satisfies the below condition:

$$F_i(Q_S^{ci}) = \frac{p - c}{p} + \frac{w_s}{p}\frac{\partial T_i}{\partial Q_i} + \frac{p - w_s}{p}\frac{\partial T_j}{\partial Q_i} + \frac{s}{p}F_i(Q_i - T_i + T_j) \tag{3.13}$$

for any i.

To encourage the coordination among the dealer and two retailers in the decentralized system, the dealer proposes a specific return and pricing policy (w, r) with the consideration of retailers' sharing.

Proposition 3.2. *When the dealer and retailers are coordinated with the inventory sharing option, the dealer provides a return policy (w, r) for retailers, in which the range of return price is denoted as (r_S^l, r_S^h), where*

$$r_S^l = 0 \tag{3.14}$$

$$r_S^h = c - s \tag{3.15}$$

$$w = \frac{p[c - (r+s)F_i(Q_i - T_i + T_j)]}{c - sF_i(Q_i - T_i + T_j) - (p - w_s)(\frac{\partial T_j}{\partial Q_i} - \frac{\partial T_i}{\partial Q_i})} \tag{3.16}$$

Considering retailers' sharing action, Proposition 3.2 indicates that the dealer offers such a (w, r) policy that ensures the dealer and retailers make positive profits, where $r \in (r_S^l, r_S^h)$. In addition, the range of r is independent of sharing parameters. The consignment price w is affected by r and w_s, it decreases as return price r increases and increases as sharing price w_s increases.

3.5.3 Who Dominates the Sharing Price

In this section, we analyze the effect of sharing parameters on retailers' sharing performance and profits. The sharing action has effects on the ordering decision of retailers. First, for retailer i, if she decreases the order quantity from Q_N^i, she will send a sharing request to retailer j, pay the sharing price w_s, and charge patients the market price p. Therefore, the retailer i can obtain a margin $p - w_s$. Second, if retailer i increases the order quantity, she will share more units to retailer j by charging w_s, rather than returning to the dealer and paying the return fee r.

Lemma 3.1. *w_s increases, then Q_S^i increases.*

We identify the impact of sharing price w_s on the retailer's ordering quantity under the sharing case. When consignment price and return price are given, if the sharing price w_s increases, the retailer with excess inventory is willing to increase the order quantity such that she can share out. Simultaneously, as w_s increases, the retailer orders more and keeps enough inventory to fulfill demand since the margin of sharing request $(p - w_s)$ decreases.

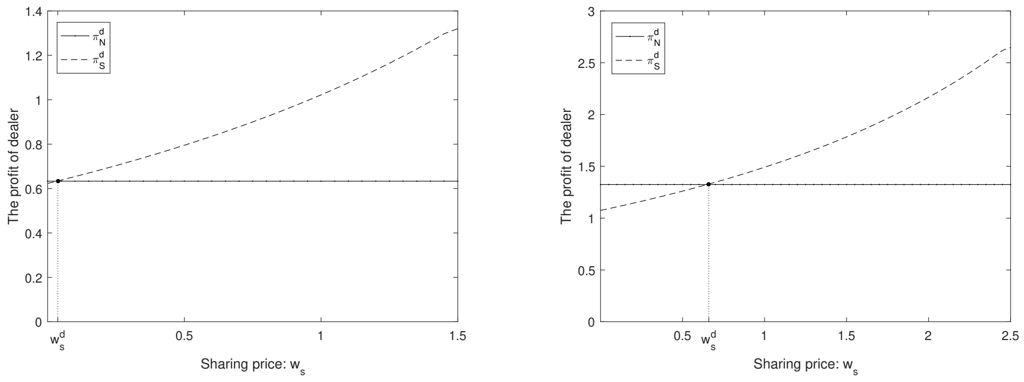
Proposition 3.3.

i. *For retailer i, when the consignment price and return price are given, $Q_N^i > Q_S^i$ at $w_s = 0$; $Q_N^i < Q_S^i$ at $w_s = p$.*

ii. *For the dealer, when he sets the optimal consignment price and return price, his profit increases as the sharing price w_s increases. The dealer makes a lower profit under the inventory sharing mechanism at $w_s = 0$ and a higher profit at $w_s = p$.*

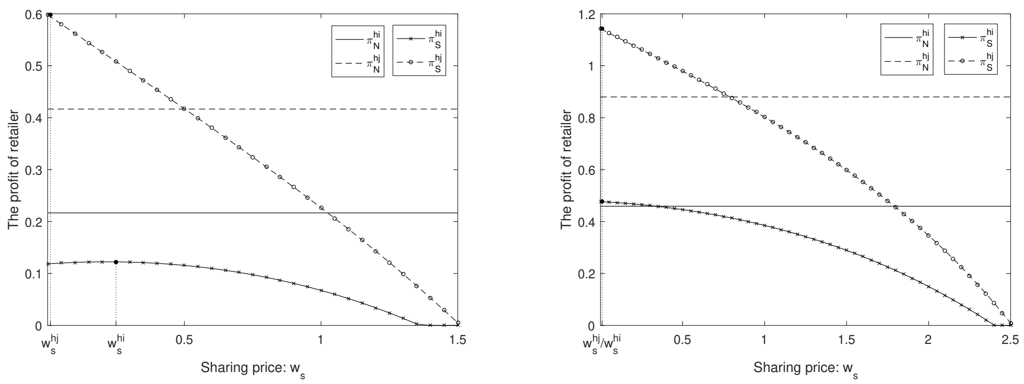
Proposition 3.3 illustrates that the sharing price w_s would incentivize or stunt the sharing action. Although the dealer determines the optimal consignment price and the return price that benefit both of dealer and the retailer, the dealer still prefers a higher sharing price under the sharing mechanism. For retailers, the sharing price determines their inventory choices: share excess inventory with another retailer or return it to the dealer. Therefore, the determination of sharing price is critical.

In the following section, we consider two possible cases in the transaction process: first, the dealer dominates the inventory sharing in the supply chain. Under the case, the dealer offers a sharing price that benefits himself and can be accepted by retailers. Second, retailers dominate inventory sharing and set a profitable sharing price for themselves. We design numerical experiments to analyze the effect of w_s on dealer's profit, retailers' profit, and the profit of retailers' alliance with and without sharing action. Let $r = 0.1$, $c = 0.5$, $s = 0.3$, and $D_i \sim U(0, 1), i = 1, 2$. We compare the results by setting $p = 1.5$ and $p = 2.5$ sequentially. Furthermore, we show the above results with the demand submits to the normal distribution in Appendix B.2. In the experiments, we use Π_N^{hi}, Π_N^{hj}, Π_N^h, and Π_N^d to denote the profit of retailer i, the profit of retailer j, the profit of retailers-alliance, and dealer's profit without inventory sharing, while Π_S^{hi}, Π_S^{hj}, Π_S^h, and Π_S^d denote the profit that under sharing case, respectively.



(a) $p = 1.5$ (b) $p = 2.5$

Figure 3.3: Dealer's profit.



(a) $p = 1.5$ (b) $p = 2.5$

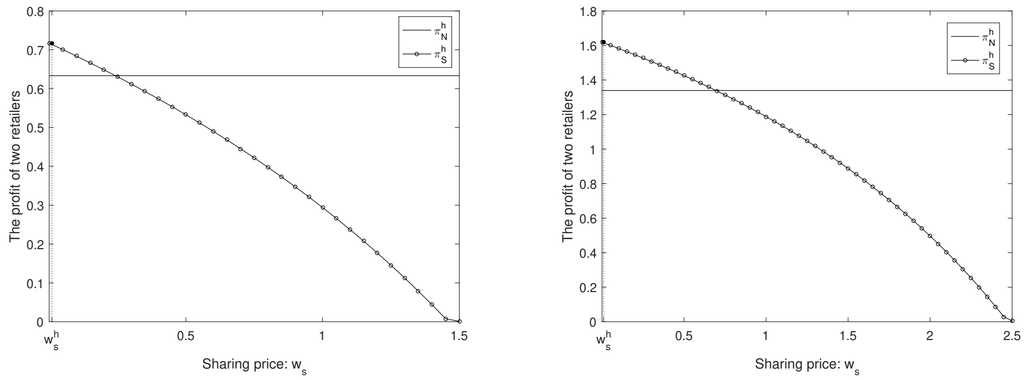
Figure 3.4: Retailers' profits when they are individual.

(1)The Dealer Dominates the Sharing Price

Proposition 3.4. *When the dealer dominates the sharing price, he is better off with retailers' sharing and makes more profits as the sharing price increases.*

Observation 3.1. *When the dealer dominates retailers' sharing action, retailers may be worse off with sharing.*

If the dealer dominates retailers' sharing action, that means the dealer can set a sharing price for retailers and has the power to induce retailers to share. Figure 3.3 shows that the dealer is better off with retailers' sharing if $w_s > w_s^d$. Therefore, as Proposition 3.4 indicates, the dealer always prefers setting a higher w_s. In addition, we observe that retailers may be worse off because of sharing.



(a) $p = 1.5$ (b) $p = 2.5$

Figure 3.5: Retailers' profits when they form an alliance.

Although the profit of retailer i suffers an increasing trend before decreases, it is still lower than the profit without sharing (Figure 3.4a). Retailer j with excess demand benefits from sharing when w_s is lower (around < 0.5). Therefore, when the dealer dominates sharing price, he prefers increasing w_s to p, while retailers are worse off under the case.

(2)Retailers Dominate the Sharing Price

Proposition 3.5. *When retailers dominate their sharing action, they set the sharing price $w_s^{\hat{h}}$, the dealer is better off with retailers' sharing action if $w_s^{\hat{h}} > w_s^d$, worse off if $w_s^{\hat{h}} < w_s^d$, and indifferent if $w_s^{\hat{h}} = w_s^d$.*

Observation 3.2. *When retailers dominate their sharing action, if $w_s^{\hat{h}} > w_s^h$, the dealer prefers consigning with individual retailers; otherwise, individual retailers and cooperative retailers-alliance are indifferent to the dealer.*

When retailers dominate the sharing action, they decide the sharing price without the dealer's interference. Additionally, two retailers decide to be independent or cooperative. If they prefer to be independent, they maximize their individual profit, respectively. w_s^{hi} and w_s^{hj} denote retailer's preferred sharing prices, under which retailer i and j are most profitable respectively. Under the independent case, two retailers will decide a sharing price $w_s^{\hat{h}}$ in the range of (w_s^{hj},w_s^{hi}). Dealer's profit is affected by retailers' sharing decisions. Figure 3.3a shows that the dealer benefits from retailers' sharing when $w_s^{\hat{h}} > w_s^d$ and hurts by sharing when $w_s^{\hat{h}} < w_s^d$. But in the case of Figure 3.3b, the dealer is always better off from sharing because the retailer will set $w_s^{\hat{h}} = 0$ and $w_h^d > 0$.

Then we focus on the dealer's preference for retailers' cooperation. When two retailers decide to cooperate, they aim to maximize their total profit but still make the order decision independently. w_s^h is denoted as the sharing price that the cooperative retailers determine. From Figure 3.5, retailers set the sharing price $w_s^h = 0$ and obtain the highest total profit. Therefore, the dealer makes more profits if retailers are individual than cooperative ($w_s^{\hat{h}} > w_s^h$). We also observe that the dealer is indifferent to retailers' cooperation when $w_s^{\hat{h}} = w_s^h$.

3.5.4 The Impacts of the Return Policy on Sharing Performance

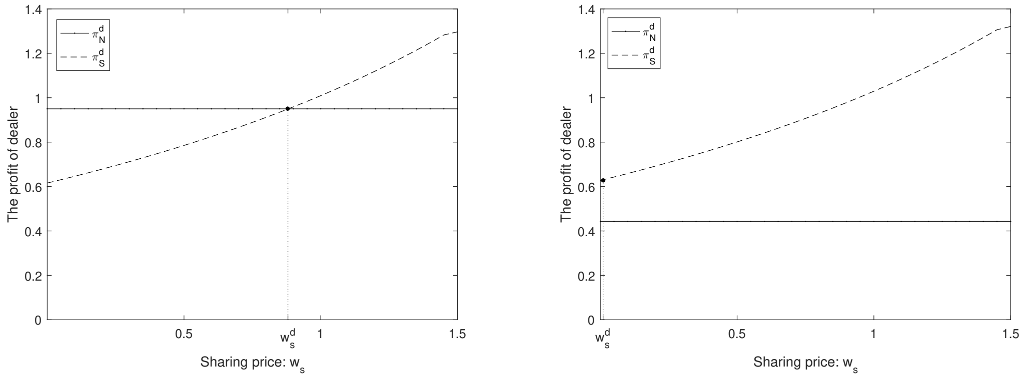
The decision of sharing price depends on the consignment price, the return price for unused refunds, the given market price, and the uncertain demand. Among them, return price is critical because it affects the consignment price. In this section, we aim at exploring the effects of return price on retailers' sharing performance and profits as well as the dealer's profit. Let $p = 1.5$, $c = 0.5$, $s = 0.3$, $D_i \sim U(0,1), i = 1,2$. We obtain the dealer's profit, retailer's profit, and the profit of retailers' alliance with and without sharing action when $r = 0.05$ and $r = 0.13$ sequentially. Additionally, we show the above results with the demand submits to the normal distribution in Appendix B.2.

Proposition 3.6.

i. When the dealer increases return price r,

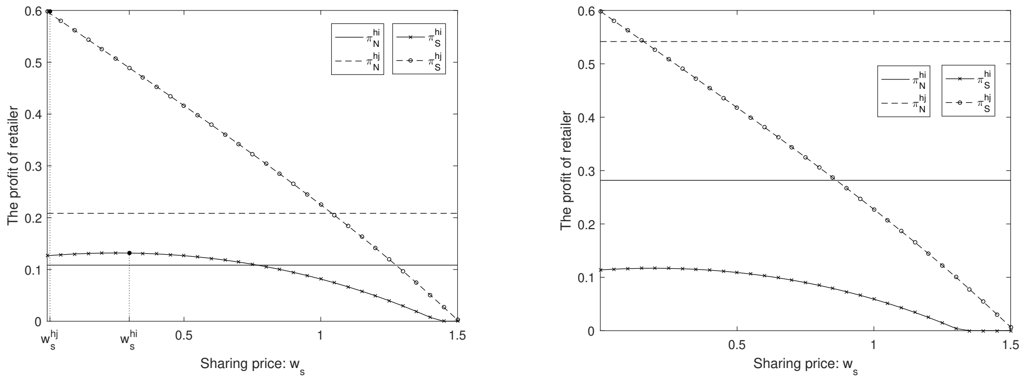
- *if the dealer dominates sharing action, the dealer is always better off with retailers' sharing.*
- *if retailers dominate sharing action, they reject the sharing action. Dealer is always worse off without retailers' sharing.*

ii. When the dealer decreases return price r,



(a) $r = 0.05$ (b) $r = 0.13$

Figure 3.6: Dealer's profit.

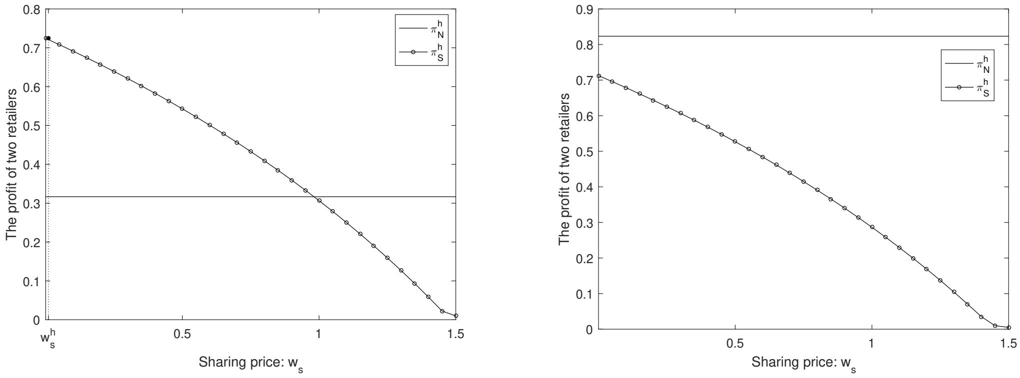


(a) $r = 0.05$ (b) $r = 0.13$

Figure 3.7: Retailers' profits when they are individual.

- *if the dealer dominates sharing action, he is better off with retailers' sharing when $w_s > w_s^d$.*
- *if retailers dominate sharing action, they set the sharing price w_s^h. The dealer is better off with retailers' sharing action if $w_s^{\hat{h}} > w_s^d$, worse off if $w_s^{\hat{h}} < w_s^d$, and indifferent if $w_s^{\hat{h}} = w_s^d$. In addition, the dealer prefers that retailers are individual.*

Proposition 3.6 shows that the return price significantly affects retailers' sharing decisions, retailers' profits, and dealer's profit. When the dealer increases the return price ($r = 0.13$) in the numerical experiments, we consider the sharing performance under two cases. First, if the dealer dominates sharing action, that means retailers submit to the dealer's decision. Figure 3.6b shows that the dealer



(a) $r = 0.05$ (b) $r = 0.13$

Figure 3.8: Retailers' profits when they form an alliance.

always benefits from retailers' sharing action, and he prefers a high sharing price ($0 < w_s \leq p$). However, in this case, sharing is not profitable for retailers. Second, if retailers dominate the sharing action, retailers reject sharing, since there is no motivation to share (Figure 3.7b and 3.8b). Retailers are almost worse off with sharing action (except the retailer j benefits from sharing when w_s is in a small range). Under this case, the dealer is always worse off without retailers' sharing because the increase of r leads to the decrease of consignment price w. Therefore, increasing the return price is not always available for the dealer to encourage retailers' sharing, especially when retailers dominate their sharing action.

When the dealer decreases the return price ($r = 0.05$), if the dealer dominates retailers' sharing, the dealer is better off with sharing when $w_s > w_s^d$ (Figure 3.6a). If retailers dominate their sharing action, they decide a sharing price between w_s^{hi} and w_s^{hj}, where w_s^{hi} and w_s^{hj} represents that retailer i and j make the highest profit respectively. For retailer i with excess inventory, the decrease of r leads to an increase of w. Consequently, the profit of retailer i decreases $(p - w)$. Therefore, as the w_s increases, retailer i begins to benefit from sharing out. Simultaneously, as r decreases, the profit of retailer j decreases than that of a higher r case, so retailer j is better off sharing in a wider w_s range (Figure 3.7a). Considering the aim of profit maximization, it is possible that retailers form an alliance and set the sharing price $w_s^h = 0$, and then reallocate the profit. In this study, we do not consider the details of profit reallocation. When retailers dominate sharing action, the dealer prefers that retailers are individual rather than a cooperative alliance.

3.6 Conclusions

In this chapter, we construct a framework that contains a common dealer and two retailers considering the retailer's sharing action and return action to the dealer. Retailers place orders to the dealer by a consignment contract, which means they only pay for used items at the end of the consignment cycle. Retailers can choose to return the unused items to the dealer by paying a return fee or share with another retailer after demand realization. We aim at coordinating retailers' sharing with the retailer's return action under two cases: dealer-dominated sharing case and retailer-dominated sharing case. We further explore the effect of the retailer's return policy on sharing performance and profits. Some managerial insights are concluded as follows:

(1) Retailers share excess inventory and reduce the return amount to the dealer. It seems that retailers and the dealer benefit from the sharing action especially when the salvage value of refunds is very low. However, the benefit effect depends on who controls the sharing action. If the dealer has complete power in the consignment contract (e.g. the dealer dominates the market), he sets a higher sharing price, and retailers are worse off with sharing; retailers prefer to return excess inventory to the dealer rather than share with each other. On the other hand, when retailers have complete power (e.g. multiple dealers provide the same products), retailers prefer to set a lower sharing price and form a cooperative alliance to share excess inventory. In this case, the dealer is worse off with retailers' sharing most time.

(2) Dealer can choose to trade with individual retailers or cooperative retailer alliance. However, when the dealer cannot dominate retailers' sharing, he

prefers trading with the individual retailer such that he makes more profit under the sharing mechanism because individual retailers set a higher sharing price than cooperative retailers.

(3) The return price significantly affects the retailer's sharing action, retailers' profit, and the dealer's profit. When the dealer cannot dominate sharing price, he may set a higher return price to encourage retailers' sharing. However, it does not work; a higher return price increases the consignment price, which causes retailers worse off under sharing mechanism. Therefore, retailers will reject inventory sharing action and the dealer is worse off without sharing. On the contrary, a decreasing return price will encourage retailers' sharing because retailers begin to be better off with sharing when the consignment price is higher.

We also propose two new research directions based on this study. First, considering the interaction between OM and marketing in this topic, for example, modeling the market share of retailers and the dealer in the decision process, may provide some marketing managerial insights into the dealer's pricing decision. Second, when the dealer decides to trade with individual retailers or cooperative retailers, he trusts the retailers as it is supposed that the information is complete. However, information asymmetry will induce the dealer to consider the possibility of the information truth. As a result, the dealer may adjust his pricing and return policy for retailers. Therefore, we will extend the work by considering information asymmetry in future research.

Chapter 4

Quality Design, Drug Pricing and Vertical Integration in the Healthcare Market

4.1 Introduction

The research study presented in this chapter is motivated by the popular phenomenon in the US healthcare market, the vertical integration between the pharmaceutical manufacturer and the distributors. For instance, in the 1990s, the vertical integration activities between the drug manufacturer and the Pharmacy Benefit Manager (PBM) showed an increasing trend (Simonet 2007b). In 1993, the drug manufacturing company Merck spent US$ 6.6 billion to vertical integrate with Medco, which was one of the top PBM companies in the US (Lehnhausen 2017). In the pharmaceutical supply chain, the PBM acts as an intermediary between the pharmaceutical manufacturer and the medical insurance company. The PBM has the bargaining power to wholesale drugs from the pharmaceutical manufacturer and then provide formularies for insurance companies (Simonet 2007a). Formularies contain lists of drugs that are covered by a medical insurance benefits plan. According to the formularies list with drug prices, the insurance company designs tiered medical insurance plans for consumers. Each tiered plan covers a specific drug list and corresponding co-payment level that consumers need to pay. From the perspective of the pharmaceutical market, the PBM manages consumers' drug selection of formularies while the drug manufacturer cannot control the downstream distribution of drugs. By vertical integrating with an independent PBM, the drug manufacturer can impede the drugs of rival manufacturing companies on the formularies of PBM (Simonet 2007b).

On the other hand, the drug manufacturer has little influence on the distribution channels of their products, through the integration with the PBM, the drug manufacturer can obtain information about physicians' prescription preferences and consumers' preferences for drugs on prices and qualities (Kanavos and Vandoros 2010, Chen and Maskus 2005). Therefore, forward vertical integration is supposed to help pharmaceutical manufacturing companies to increase their market shares and adjust their pricing and quality decision to meet consumer's demand. PBM also benefits from vertical integration with the drug manufacturer. In 1994, Caremark, as an independent PBM, negotiated with four pharmaceutical manufacturers: Rhone Poulenc-Rorer, Bristol Myers Squibb, Eli Lilly, and Pfizer, and allowed them to access Caremark's formularies. In addition, through the integration with multiple upstream drug manufacturers, Caremark received rebates or price discounts from 85% of the drugs of the formulary (Simonet 2007b). Therefore, PBMs could increase their market power and profits by vertical integration with drug manufacturers.

The upstream vertical integration in the pharmaceutical supply chain also has significant effects on consumer choice and consumer surplus, which is why the pharmaceutical vertical integration in the healthcare market has attracted more academic and industrial concerns. Cuesta et al. (2019) find patients (consumers) from integrated insurers pay 23% less than consumers from non-integrated insurers for prescriptions annually, which is consistent with the conclusion of Lehnhausen (2017). In addition, the integrated drug manufacturer sells at a lower price than the non-integrated manufacturer, which leads to a lower charge to consumers. However, there are also some oppositions from the industrial perspective. The Insurance Commission[4.1] and American Medical Association[4.2] do not recommend vertical integration because they think that the integration in the healthcare market is anti-competitive and not in consumers' interest.

Furthermore, empirical proof shows that removing vertical integration from the healthcare market would increase consumer surplus by $4.3 billion annually (Diebel 2018). Therefore, the vertical integration in the pharmaceutical market has drawn numerous public controversies. Three major concerns are shown below: first, both the manufacturer and the PBM may benefit from the vertical integration, such as the increasing market share and profit. Second, consumers may purchase drugs distributed from the integrated manufacturer at a lower price. Simultaneously, consumers have fewer choices in drug categories since the PBM prefers selecting drugs from its integrated manufacturer. Third, except the purchase price differentiation brought by the vertical integration, the drug manufacturer's quality decision also has been affected. Furthermore, the endogenous drug quality also affects the drug manufacturer's integration decision with the PBM.

There are no such studies on the effects of vertical integration between the drug manufacturer and the PBM in the pharmaceutical supply chain. Therefore, our study constructs a three-echelon framework including the manufacturer, the PBM, and consumers. We aim at investigating the effects of vertical integration between the drug manufacturer and the PBM on their decisions and profits, exploring the effects of endogenous drug quality on the integration decision as well as on the profits of the manufacturer and the PBM, and providing practical managerial insights for drug manufacturer's pricing & quality designing, and PBM's formulary designing. We propose to address the following research questions to fulfill the current research gaps in the related field.

[4.1] Dave Jones, an Insurance Commissioner from the State of California, proposes the opposition to the integration in the healthcare market in a statement to the United States Department of Justice. http://www.insurance.ca.gov/0400-news/0100-press-releases/2018/upload/nr085LtrJonestoUSAGSessionsreCVS-AetnaMerger.pdf.

[4.2] American Medical Association requests that the Utah Insurance Commissioner oppose the vertical integration in the healthcare market. https://www.ama-assn.org/delivering-care/patient-support-advocacy/cvs-aetna-merger.

- First, when the drug quality is endogenous, how do the drug manufacturer and the PBM make the pricing decision when they are vertically integrated?

- Second, what conditions encourage or impede the drug manufacturer from vertical integration with the PBM?

- Third, how would an integrated drug manufacturer differentiate in terms of drug quality between two distribution channels and how would the drug quality differentiation decision affect the joint profit of the manufacturer and the PBM?

We build a three-echelon model based on the above research questions, including the upstream drug manufacturer, the download PBM, and consumers. In the first stage, we suppose that there is no vertical integration between the manufacturer and PBM. The PBM distributes a drug with the retail price after the manufacturer sets the drug quality level and wholesale price. Consumers choose the drug according to their preferences or doctor's prescription. We obtain the equilibrium solutions about PBM's ordering decision, manufacturer's quality decision, and pricing decision by backward induction analysis. In the second stage, we suppose that the drug manufacturer applies the mixed distribution strategy to retail the drug, vertical integration with a PBM and direct retail with another PBM. The manufacturer differentiates the drug quality between two channels. Considering the vertical integration with the manufacturer, the PBM aims at maximizing their joint profit. We explore the equilibrium when considering two distribution channels and the condition that the manufacturer benefits from the vertical integration channel. In the third stage, we focus on the decision of quality differentiation level between two channels and investigate the effects of quality differentiation on consumers' behavior. We find some counter-intuitive results verified by numerical experiments. First, after the join-in of vertical integration in distribution channels, as the quality differentiation level increases, consumers do not always choose the drug with a higher-level quality. Second, we analyze the effects of manufacturing cost on the manufacturer's quality differentiation level and the joint profit. We find that the joint profit does not always decrease as the manufacturing cost increases; it also depends on the quality differentiation level. Given a lower differentiation level (≤ 1), the joint profit first decreases and then increases as the manufacturing cost increases. At the same time, given a larger differentiation level (> 1), the joint profit increases as the increasing retail price covers the increasing manufacturing cost.

The remaining sections of this chapter are shown as follows: In Section 4.2, we review the relevant literature (vertical integration in distribution channels; quality decisions in the manufacturing industry; interaction between the insurance company and the PBM) and highlight the main differences between the relevant studies and ours. Section 4.3 develops a three-echelon framework that includes the drug manufacturer, the PBM, and consumers and models the vertical integration between the drug manufacturer and the PBM mathematically. Section 4.4 analyzes the PBM's ordering decision, manufacturer's pricing, and quality design under the direct retail strategy. Section 4.5 explores the overall game equilibrium of quality decisions and pricing decisions in two distribution channels. Section 4.6 identifies the effects of quality differentiation level and quality cost. Finally, section 4.7 summarizes the major conclusions of this chapter. All proofs of the propositions and corollaries are shown in Appendix C.

4.2 Literature Review

This study relates three streams of previous studies. First is the vertical integration in distribution channels. The second is quality decision designing. The third is the interaction among the pharmaceutical supply chain, including medical insurance plan design and consumer co-payment level decision.

This research is related to the literature on channel integration, especially the vertical integration in the distribution channels. McGuire and Staelin (1983) show that when the product substitutability is high, the manufacturer prefers to choose a decentralized distribution system rather than an integrated channel. Moorthy (1988a) then investigates the necessary conditions that the manufacturer favors vertical integration rather than decentralization. Vertical integration is divided into forward integration and backward integration according to the integrating direction. Arya and Mittendorf (2013) find that the forward integration between the manufacturer and the retailer increases both the self-investment and the cross-investment in the product demand. Lin et al. (2014) examine three strategies (no vertical integration, forward integration, and backward integration) for the manufacturer, and show that when the backward integration is always profitable, the forward integration is not beneficial for the manufacturer. Li and Chen. (2018) explore the effect of backward integration strategy on price competition, quality differentiation, and supply chain structure. Li and Chen. (2020) compare the manufacturer's choice for the forward integration or backward integration considering the feature of product quality (exogenous or endogenous).

Another stream of the literature studies quality decisions. Some studies focus on quality decisions in a vertical competition setting. For instance, Moorthy (1988b) examines the quality positioning strategies for two competing manufacturers. Economides (1999) finds that the quality and market share of a product are lower when there lacks a vertical integration in distribution. Then Choudhary et al. (2005) study the personalized pricing and quality decision in a duopoly framework, which is vertically differentiated. Chambers (2006) studies how the variable cost functions affect the manufacturer's pricing and quality decision. Anand et al. (2011) characterize the consumer intensity of service and show that consumer intensity affects the quality decision for the service provider. Dumrongsiri et al. (2008) explore the equilibrium of decisions between direct and retail channels and find that as the retailer's service quality increases, the manufacturer's profit will increase in dual distribution channels. Shi et al. (2013) explore the impacts of centralized or decentralized distribution channels on quality decisions considering consumer heterogeneity. Ha et al. (2016) focus on the impacts of the manufacturer's encroachment decisions on quality decisions and provide insights for the encroaching manufacturer about how to differentiate in product qualities.

Furthermore, this study relates to the literature on the interaction between the insurance company and the PBM. Some scholars focus on insurance plan design and drug competition. For instance, Cui et al. (2016) study the effect of formulary structure (numbers of drugs, patient's co-payment) on price competition and drug market shares. Dai et al. (2017) consider patients' insurance coverage and investigate the impact of insurance structure on the consumer utility in the healthcare market. Mehta et al. (2017) find that inefficiency exists in consumers' choice of different medical types. For example, most consumers prefer expensive curative care types rather than the secondary or primary care type though the cost increases. Mehta et al. (2017) apply the practical data set to optimize consumers' insurance plan decisions annually and the consumption decisions periodically.

Additionally, some studies explore the function of the PBM in the pharmaceutical supply chain, especially the impacts of PBM's decisions. The PBM designs a medical plan for consumers, including a drug formulary list with the specific retail price and co-payment level charged to consumers. Kouvelis et al. (2015) show that the Nash equilibrium of pricing decision exists under PBMs' competition. They also study the effect of PBMs' merger on the pharmaceutical supply chain. Then Kouvelis et al. (2018) explore PBM's optimal pricing decision and co-payment decision under the integration between the drug manufacturer and the PBM and analyze the impacts of vertical integration on the market share and profit of integrated PBM. The major distinction between this study and ours is that Kouvelis et al. (2018) considers exogenous drug quality while we consider the endogenous drug quality decision in the model setting. In addition, we consider the quality differentiation level and investigate how it affects the performance of the manufacturer, the PBM, and consumers.

Our research differs from the aforementioned studies in the following aspects. First, compared with the traditional supply chain, we build a model in the pharmaceutical supply chain with more specific considerations in this setting. For example, we consider the consumer's co-payment level and the employer's afforded level in the prescription. Then we model the vertical integration between the manufacturer and the PBM based on the above setting and analyze the corresponding impacts of vertical integration. Second, different from the previous research on the interaction among the drug manufacturer, the PBM, the insurer, and consumers, we focus on the effect of vertical integration on drug quality differentiation decision in two distribution channels (the direct retail channel and the vertical integration channel). Therefore, on the one hand, this study fulfills the research gap in academia about the interaction among the upstream drug manufacturer, the downstream PBM cooperated with the insurance company and consumers. On the other hand, this study provides some managerial suggestions for the decision-maker to design the production or distribution strategy.

4.3 Model Setup

We consider that the upstream pharmaceutical manufacturer (he) provides an essential drug for consumers (patients), which is covered in the formulary list in the consumer's medical plan. Under the single distribution strategy, the drug is distributed by the downstream PBM (she) with a retail price p, then according to the medical plan, the consumer only pays the co-payment t and the employer affords the remaining part when the consumer buys the drug. Now, the manufacturer could choose to be vertically integrated with the PBM to increase the profit. Under the mixed distribution strategy, the manufacturer has two distribution channels, direct retail with the PBM and vertically integrated with another PBM. For instance, the manufacturer distributes some volumes of the drug to a PBM by a direct retail channel. Under the channel, the manufacturer decides the wholesale price to the PBM, and then the PBM decides the retail price of consumers. Simultaneously, the manufacturer vertically integrates with another PBM and decides the retail price to consumers together. Under the integration channel, the manufacturer and this PBM maximize their joint profit. The manufacturer also provides differentiated quality under two distribution channels, we denote the quality differentiation level as θ. When $\theta = 1$, that means the manufacturer provides the drug with uniform quality under two channels. Otherwise, the manufacturer provides the same drug with different levels of quality under distribution channels.

We model the consumer's utility for using the drug provided by the PBM as $u = \alpha q - \beta t$, where $\alpha > 0$ and $\beta > 0$ capture the consumer's sensitivity to the drug quality and co-payment level, respectively. We assume that consumers have heterogeneous preferences for drug price and quality, α is distributed on $[0, 1]$ uniformly. Therefore, consumers buy the drug with the preference $\alpha \geq \beta t/q$. Then, the corresponding demand is $d = 1 - \beta t/q$. Following the previous studies (Choudhary et al. 2005, Ha et al. 2016), we assume that $\lambda > 0$ measures the manufacturer's sensitivity to the drug quality, and the unit manufacturing cost is $c = \lambda q^2$. Our study considers that the drug manufacturer could choose the drug quality level flexibly according to the manufacturing plan. For instance, there are two categories of Ibuprofen Sustained-release Capsule for pain treatment in the market, which is different in the recommended dosage. Consumers choose one category according to their preferences. Therefore, we consider the endogenous quality in Section 4.4 and 4.5 in our model. This is the major difference between Kouvelis et al. (2018) and ours; they assume that the drug quality is exogenously given and cannot be changed. Additionally, we assume that the drug manufacturer faces the fixed cost k to vertically integrate with the PBM, including the time cost and transportation cost for contracting ($k \geq 0$).

The timeline of events is shown as follows: without the vertical integration between the manufacturer and the PBM, (i) the drug manufacturer decides the quality level q and the wholesale price w of the drug in the formulary list. (ii) the PBM decides the retail price p of the drug charged to consumers. (iii) the insurer that cooperated with the employer decides the co-payment level t for the drug in consumers' prescriptions. Considering the vertical integration, no wholesale price is needed between the integrated manufacturer and PBM. They aim at maximizing the joint profit. In addition, we explore the impacts of the drug quality differentiation level θ when considering the exogenous drug quality. The manufacturer decides θ at the time that he chooses the type of vertical integration channel (high-quality VI or low-quality VI).

We define the notations that are applied in our model in Table 4.1.

Table 4.1: Notations.

Notations	**Descriptions**
α	Consumer's sensitivity to the drug quality.
β	Consumer's sensitivity to the drug co-payment level.
λ	Manufacturer's sensitivity to the drug quality.
θ	Drug's quality differentiation level.
u	Consumer's utility for using the drug.
c	Drug's manufacturing cost with quality q.
k	Manufacturer's cost for the vertical integration with the PBM.
w^*	The wholesale price of the drug.
q	The quality of the drug.
p	The drug's retail price for a prescription.
d	Consumer's demand for the drug.
t	The co-payment level of the drug afforded by the consumer.
e	The remaining part afforded by the employer after consumer's co-payment.

* From the notation w to e, we use the superscript 0 to denote the notation (e.g. w^0) when not considering the vertical integration, while the superscript N and V (e.g. w^N, w^V) denote the direct retail channel and vertical integration channel under the mixed distribution strategy, respectively.

4.4 No Vertical Integration

This section explores the interaction in the pharmaceutical supply chain only considering the direct retail channel between the drug manufacturer and the PBM. This case is called the single distribution strategy of the manufacturer. We apply the backward induction to obtain the equilibrium of decisions. The sequence of events is shown as follows: first, according to the PBM's and the employer's decisions, the drug manufacturer sets the wholesale price w^0 and corresponding quality level q^0 for the drug. Second, given the wholesale price w^0 and drug quality q^0, the PBM decides the drug's retail price p^0. Third, the insurer decides the co-payment level t^0 of the drug in the consumer's prescription.

4.4.1 PBM's Decisions

Under the direct retail strategy, the PBM maximizes her profit Π_p^0 given the upstream manufacturer's wholesale price w^0 and quality level q^0.

$$\max_{d^0} \Pi_p^0 = (\frac{q^0(1-d^0)}{\beta} + e^0 - w^0)d^0 \tag{4.1}$$

4.4.2 Manufacturer's Decisions

With the derivation of the order quantity $d^0 = (\beta e^0 - \beta w^0 + q^0)/2q^0$, the manufacturer maximizes his profit Π_m^0 by setting wholesale price w^0.

$$\max_{w^0,q^0} \Pi_m^0 = (w^0 - \lambda {q^0}^2)(\frac{\beta e^0 - \beta w^0 + q^0}{2q^0}) \tag{4.2}$$

The expression of the wholesale price is $w^0 = \frac{\lambda {q^0}^2}{2} + \frac{e^0}{2} + \frac{q^0}{2\beta}$. By the backward induction, the PBM could obtain the optimal p^0 to maximize its profit. Then the insurer cooperated with the employer will set an optimal e^0 by minimizing the cost for the consumer's prescription $\Pi_e^0 = e^0 d^0$. Finally, the optimal retail price and ordering quantity can be derived as a function of quality q^0, where $p^0 = \frac{5\lambda {q^0}^2}{8} + \frac{3q^0}{8\beta}$, $d^0 = \frac{1-\beta\lambda q^0}{8}$. The manufacturer sets the optimal wholesale price as $w^0 = \frac{1}{6\beta^2\lambda}$, $q^0 = \frac{1}{3\beta\lambda}$.

4.5 Vertical Integration with Endogenous Quality

This section investigates the equilibrium when two distribution channels exist simultaneously. The manufacturer applies the mixed distribution strategy and distributes the drugs with two qualities by two different channels: direct retail the drug with quality q^N to one PBM, and distribute the drug with quality q^V to another vertically integrated PBM. The quality-differentiated drugs from the above two channels enter the market with the retail price of p^N and p^V, respectively.

4.5.1 Consumer's Decision

Consumers choose the drug according to their preferences or doctor's prescription. They only pay the co-payment level of the drug price and the remaining part is afforded through their medical plans. The co-payment price of the drug from the direct retail channel and the vertical integration channel is t^N and t^V, respectively. Observing the differentiated drug price, consumers choose a drug based on their expected valuation of price and quality. We assume that the consumer's utility by purchasing the drug from two channels is $u^N = \alpha q^N - \beta t^N$ and $u^V = \alpha q^V - \beta t^V$, respectively. Then consumers with the valuation $\alpha \geq \frac{\beta(t^V - t^N)}{q^V - q^N}$ would purchase the drug that comes from the vertical integration channel, and the consumers with the valuation $\frac{\beta t^N}{q^N} \leq \alpha < \frac{\beta(t^V - t^N)}{q^V - q^N}$ would purchase the drug that comes from the direct retail channel.

4.5.2 PBM's Decisions

Considering the vertical integration with the upstream manufacturer, the downstream PBM aims at maximizing the profit Π_p^V, which includes the joint profit with the manufacturer under the vertical integration channel and the profit by the direct retail channel. We define the order quantity d^V, retail price p^V, and quality level q^V under the vertical integration channel; and d^N, p^N, and q^N under the direct retail channel, respectively.

$$\max_{p^V, p^N, q^V} \Pi_p^V = (p^V - \lambda {q^V}^2)d^V + (p^N - w^N)d^N - k \tag{4.3}$$

Where the first term is the joint profit under the vertical integration channel, the second term is the profit under the direct retail channel, and the last term is the vertical integration cost. The PBMs first decide the retail price of the drug (p^N, p^V) under two distribution channels, and then decide the drug quality q^V distributed under the vertical integration channel.

4.5.3 Manufacturer's Decisions

With the given retail price p^V and p^N, the manufacturer only decides the wholesale price w^N and quality level q^N, and then maximizes the profit Π_m^N for the direct retail channel.

$$\max_{w^N, q^N} \Pi_m^N = (w^N - \lambda {q^N}^2) d^N \tag{4.4}$$

Additionally, the insurer (cooperated with consumers' employer) minimizes the cost $\Pi_e^N = e^N d^N$ and $\Pi_e^V = e^V d^V$ by setting e^N and e^V to afford the remaining fee after the consumer's co-payment.

4.5.4 The Equilibrium Solutions

According to the above profit function, we derive the equilibrium decisions of the manufacturer and PBM under the mixed distribution strategy. All proofs are attached in the Appendix C.

Proposition 4.1. *There exist the following optimal decisions:*

i Without the vertical integration between the manufacturer and the PBM:

$$\begin{cases} w^{0*} = \frac{3\lambda {q^0}^2}{4} + \frac{q^0}{4\beta}, \\ p^{0*} = \frac{5\lambda {q^0}^2}{8} + \frac{3q^0}{8\beta}, \end{cases} \tag{4.5}$$

ii When the vertical integration between the manufacturer and the PBM exists:

$$\begin{cases} w^{N*} = \frac{q^N(17\beta\lambda {q^N}^2 - 27\beta\lambda q^N q^V - 2\beta\lambda {q^V}^2 + 2q^N - 2q^V)}{4\beta(5q^N - 8q^V)}, \\ p^{N*} = \frac{q^N(13\beta\lambda {q^N}^2 - 21\beta\lambda q^N q^V + 2\beta\lambda {q^V}^2 + 8q^N - 14q^V)}{4\beta(5q^N - 8q^V)}, \\ e^{N*} = \frac{q^N(3\beta\lambda {q^N}^2 - 5\beta\lambda q^N q^V + 2\beta\lambda {q^V}^2 - 2q^N + 2q^V)}{\beta(5q^N - 8q^V)}, \end{cases} \tag{4.6}$$

$$\begin{cases} p^{V*} = \frac{q^V(\beta\lambda {q^N}^2 + 8\beta\lambda q^N q^V - 12\beta\lambda {q^V}^2 + 1q^N - 4q^V)}{2\beta(5q^N - 8q^V)}, \\ e^{V*} = \frac{q^V(\beta\lambda {q^N}^2 + 3\beta\lambda q^N q^V - 4\beta\lambda {q^V}^2 - 4q^N + 4q^V)}{\beta(5q^N - 8q^V)}, \end{cases} \tag{4.7}$$

Proposition 4.2. *The manufacturer benefits from the vertical integration channel, if and only if $k < \frac{0.0404}{\beta^2\lambda}$; otherwise, the manufacturer will not apply the mixed distribution strategy.*

Proposition 4.3. *The overall pharmaceutical supply chain benefits from the vertical integration channel, if and only if* $k < \frac{0.0477}{\beta^2\lambda}$*; otherwise, the manufacturer will not apply the mixed distribution strategy.*

With the equilibrium results from Table 4.2, we could verify the Proposition 4.2 and 4.3. With the consideration of vertical integration, the manufacturer has two distribution strategies according to the profit performance. The first strategy is a single strategy, which is directly distributing drugs to one PBM. The second strategy is a mixed strategy, which is to distribute the drugs with two qualities by two channels, direct retail the drug with quality q^N to one PBM, and distribute the drug with quality q^V to another vertically integrated PBM. From the equilibrium results of Table 4.2, we learn that when the condition $(\Pi_p^V - \Pi_p^0) - \Pi_m^0 \geq 0$ is satisfied, the manufacturer benefits from the vertical integration channel. That is when the integration cost $k < \frac{0.0404}{\beta^2\lambda}$, the manufacturer will apply the mixed distribution strategy. On the other hand, from the perspective of the overall profits of the pharmaceutical supply chain Π_t, it consists of the manufacturer's profit and PBM's profit. We derive the condition $(\Pi_p^V + \Pi_m^N + \Pi_p^N) - (\Pi_p^0 + \Pi_m^0) \geq 0$, that is $k < \frac{0.0477}{\beta^2\lambda}$, under the condition, the overall supply chain benefits from the mixed distribution strategy by considering the vertical integration channel simultaneously. Furthermore, Proposition 4.2 and 4.3 state that the manufacturer has more incentives to vertically integrated with the PBM considering overall supply chain profits rather than its own profit.

4.6 Vertical Integration with Quality Differentiation Level

The pharmaceutical manufacturer provides a drug with two dosages. The quality level is distinguished by θ, then the manufacturer distributes the drug with quality q to the PBM by the direct retail channel and the drug with quality θq by the vertical integration channel respectively. To explore how the manufacturer differentiates the drug quality under two channels, we consider the drug quality is exogenous in the following model setting. Two cases are designed to analyze the

Table 4.2: The equilibrium results under Vertical Integration vs without Vertical Integration.

	single strategy	**mixed strategy**	
	direct retail channel	direct retail channel	VI channel
q	$\frac{0.3333}{\beta\lambda}$	$\frac{1.1429}{\beta\lambda}$	$\frac{0.7143}{\beta\lambda}$
w	$\frac{0.1666}{\beta^2\lambda}$	$\frac{1.1838}{\beta^2\lambda}$	-
p	$\frac{0.1944}{\beta^2\lambda}$	$\frac{1.3470}{\beta^2\lambda}$	$\frac{0.7653}{\beta^2\lambda}$
d	0.0833	0.1786	0.0714
Π_m	$\frac{0.0046}{\beta^2\lambda}$	$\frac{-0.0219}{\beta^2\lambda}$	-
Π_p	$\frac{0.0023}{\beta^2\lambda}$	$\frac{0.0292}{\beta^2\lambda}$	$\frac{0.0474}{\beta^2\lambda} - k$
Π_t	$\frac{0.0069}{\beta^2\lambda}$	$\frac{0.0547}{\beta^2\lambda} - k$	

impacts of the quality differentiation level θ on the manufacturer's integration decision. One case is $0 < \theta \leq 1$, the drug that is distributed under the vertical integration channel is lower than the drug distributed under the direct retail channel, therefore, we define it as *low-quality Vertical Integration*; another case is $\theta \geq 1$, that is the drug that distributed under the vertical integration channel is higher than the drug distributed under the direct retail channel, defined as *high-quality Vertical Integration.* The manufacturer makes decisions on quality differentiation level θ and chooses low-quality or high-quality vertical integration channels simultaneously.

4.6.1 *Low-Quality Vertical Integration* Case

Consumer's Decision

Consumers still choose the drug according to their preferences for the drug utility. Under the *low-quality VI* case, the consumers with the valuation $\frac{\beta t^V}{\theta q} \leq \alpha < \frac{\beta(t^V - t^N)}{\theta q - q}$ would buy the drug from the vertical integration channel. The consumers with the valuation $\alpha \geq \frac{\beta(t^V - t^N)}{\theta q - q}$ would purchase the drug from the direct retail channel.

PBM's Decisions

$$\max_{p^V, p^N} \pi_p^V = (p^V - \lambda {q^V}^2) d^V + (p^N - w^N) d^N - k \tag{4.8}$$

We use $m^N = p^N - w^N$ to denote the profit margin of the PBM by the direct retailing unit drug. Given $t^V = p^V - e^V$, and $t^N = w^N + m^N - e^N$, substituting d^N with $1 - \frac{\beta(t^V - t^N)}{\theta q - q}$, d^V with $\frac{\beta(t^V - t^N)}{\theta q - q} - \frac{\beta t^V}{\theta q}$ in Equation 4.8, we obtain π_p^V as below:

$$\max_{p^V, p^N} \pi_p^V = (p^V - \lambda q^{V2})(\frac{\beta(t^V - t^N)}{\theta q - q} - \frac{\beta t^V}{\theta q}) + m^N(1 - \frac{\beta(t^V - t^N)}{\theta q - q}) - k \tag{4.9}$$

The profit π_p^V contains three parts, where the first term is the joint profit under the vertical integration channel, the second term is the profit under the direct retail channel, and the last term is the vertical integration cost. The PBMs decide the retail price of the drug (p^N, p^V) under two distribution channels.

Manufacturer's Decisions

$$\max_{w^N, \theta} \pi_m^N = (w^N - \lambda q^{N2}) d^N \tag{4.10}$$

By substituting d^N with $1 - \frac{\beta(t^V - t^N)}{\theta q - q}$ in Equation 4.10, we rewrite π_m^N as below:

$$\max_{w^N, \theta} \pi_m^N = (w^N - \lambda q^{N2})(1 - \frac{\beta(p^V - e^V - w^N - m^N + e^N)}{\theta q - q}) \tag{4.11}$$

In this stage, to maximize the profit under direct retail channel π_m^N, the manufacturer first decides the wholesale price for the drug that is distributed under the direct retail channel and then decides the quality differentiation level θ by substituting w^N, p^N, p^V into the objective function.

4.6.2 *High-Quality Vertical Integration* Case

Consumer's Decision

Similarly, under the *high-quality VI* case, consumers with the valuation $\alpha \geq \frac{\beta(t^V - t^N)}{\theta q - q}$ would buy the drug from the vertical integration channel, and the consumers with the valuation $\frac{\beta t^N}{\theta q} \leq \alpha < \frac{\beta(t^V - t^N)}{\theta q - q}$ would purchase the drug from the direct retail channel.

PBM's Decisions

Similar to the *low-quality VI* case. Given $t^V = p^V - e^V$, and $t^N = w^N + m^N - e^N$, substituting d^N with $\frac{\beta(t^V - t^N)}{\theta q - q} - \frac{\beta t^N}{q}$, d^V with $1 - \frac{\beta(t^V - t^N)}{\theta q - q}$ in Equation 4.8, we obtain π_p^V as below:

$$\max_{p^V, p^N} \pi_p^V = (p^V - \lambda {q^V}^2)(1 - \frac{\beta(t^V - t^N)}{\theta q - q}) + m^N(\frac{\beta(t^V - t^N)}{\theta q - q} - \frac{\beta t^N}{q}) - k \tag{4.12}$$

Manufacturer's Decisions

Then, by substituting d^N with $\frac{\beta(t^V - t^N)}{\theta q - q} - \frac{\beta t^N}{q}$ in Equation 4.10, we obtain π_m^N as below:

$$\max_{w^N, \theta} \pi_m^N = (w^N - \lambda {q^N}^2)(1 - \frac{\beta(p^V - e^V - w^N - m^N + e^N)}{\theta q - q}) \tag{4.13}$$

Following similar steps for obtaining the game equilibrium in the low-quality VI case, we derive the above objective functions of PBM and manufacturer under the high-quality VI case. All proofs are attached in the Appendix C.

4.6.3 The Effects of the Quality Differentiation Level Decision

In this section, we aim at exploring the impacts of the endogenous quality differentiation level θ by assuming that the drug quality is given. This section covers two research issues; how the manufacturer chooses the vertical integration strategy with the PBM and how the quality differentiation level affects the consumer's co-payment level and drug preference when the drug quality q and vertical integration cost k are given.

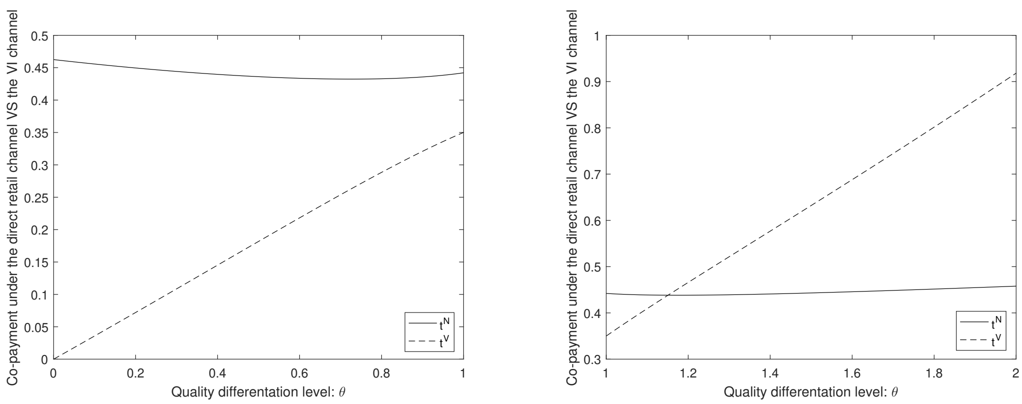
Proposition 4.4. *Given q, when the vertical integration happens in the manufacturer and the PBM, there exists:*

i When the manufacturer chooses the low-quality VI strategy, if $0 < \beta\lambda q \leq \frac{7-4\sqrt{2}}{17}$ or $\frac{7+4\sqrt{2}}{17} < \beta\lambda q < \frac{3}{4}$, then $\theta(q) = \frac{\beta\lambda q + 1 + \sqrt{17\beta^2\lambda^2 q^2 - 14\beta\lambda q + 1}}{2\beta\lambda q}$, otherwise, $\theta(q) = 1$.

ii When the manufacturer chooses the high-quality VI strategy, if $\beta\lambda q > 2$*, then* $\theta(q) = \frac{3\beta\lambda q-2}{2\beta\lambda q}$*, otherwise,* $\theta(q) = 1$*.*

Proposition 2.4 illustrates how the drug manufacturer decides the vertical integration strategy with the PBM in drug distribution (low-quality VI or high-quality VI). The decision process depends on the drug quality-related parameters, including the drug quality q, the consumer's sensitivity to the drug quality β, and the manufacturing cost λ. We find that if the manufacturer decides to distribute a low-quality level drug by integration channel, then he sets the quality differentiation level as $\theta(q) = \frac{\beta\lambda q+1+\sqrt{17\beta^2\lambda^2q^2-14\beta\lambda q+1}}{2\beta\lambda q}$ with the condition of $0 < \beta\lambda q \leq \frac{7-4\sqrt{2}}{17}$ or $\frac{7+4\sqrt{2}}{17} < \beta\lambda q < \frac{3}{4}$. Otherwise, the manufacturer will keep uniform quality in two distribution channels. Additionally, if the manufacturer distributes a high-quality level drug by the integration channel with the PBM, then the quality differentiation level is $\theta(q) = \frac{3\beta\lambda q-2}{2\beta\lambda q}$ with $\beta\lambda q > 2$, otherwise, the manufacturer will not differentiate the drug quality in two distribution channels.

Observation 4.1. *As the quality differentiation level* θ *increases, the consumer's co-payment level for the drug from the VI channel increases. When* $\theta \leq 1$*, the consumer's co-payment for the drug from the VI channel is lower than that from the direct retail channel; when* $\theta > 1$*, the consumer's co-payment for the drug from the VI channel is larger than that from the direct retail channel gradually.*



(a) $\theta \leq 1$

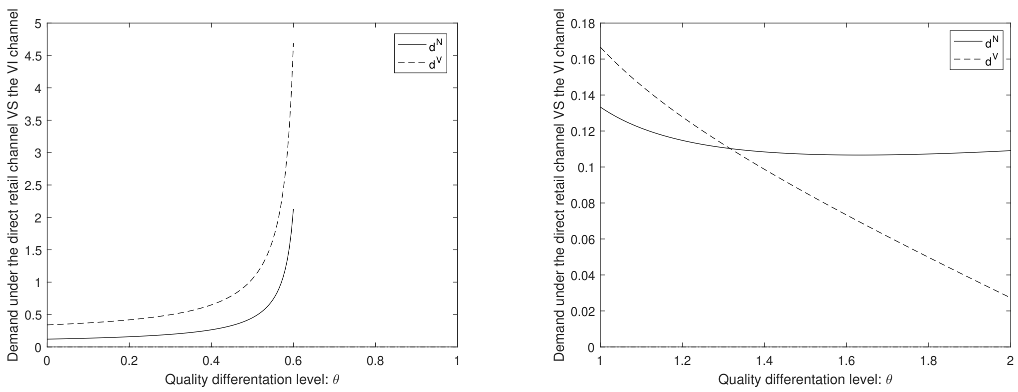
(b) $\theta > 1$

Figure 4.1: The consumer's co-payment with the quality differentiation level decision.

Observation 4.2. *The quality differentiation level has effects on the consumer's demand and co-payment level under the VI channel and the direct retail channel.*

- *When the quality differentiation level $\theta \leq 1$, the consumer's demand for the drug increases as θ increases; the co-payment level of the VI channel is lower than that of the direct retail channel.*

- *When the quality differentiation level $\theta > 1$, the consumer's demand for the drug decreases as θ increases; the co-payment level of the VI channel is lower firstly, and then higher than that of the direct retail channel as θ increases.*



(a) $\theta \leq 1$ (b) $\theta > 1$

Figure 4.2: The drug demand with the quality differentiation level decision.

We conduct numerical experiments and obtain results by Figure 4.1, which shows the effect of quality differentiation level on consumers' co-payment level for the drug in prescription. From the results of Observation 4.2, we conclude that the manufacturer's quality differentiation decision on the direct retail channel and VI channel has some impacts on downstream consumers. Figure 4.2a shows that when the manufacturer chooses a low-quality VI strategy with the PBM, the drug quality under the VI channel is lower than that under the direct retail channel. As a result, more consumers prefer to buy the drug from the VI channel rather than from the direct retail channel, which means the demand for the lower-quality drug is higher than that for the higher-quality drug. Both consumers' demand for the drug under the VI and the direct retail channel increase as the quality differentiation level θ increases. However, Figure 4.2b shows that when the manufacturer chooses a high-quality VI strategy with the PBM, the consumer's demand for the same drug under the VI channel is first higher than that under the direct retail channel. Then, as θ increases (> 1.35), the demand under the VI channel is lower than that under the direct retail channel. Additionally, both consumers' demand under the VI channel and the direct retail channel decrease as θ increases. Summarizing the results from two groups of figures, we find that for the manufacturer, a higher-level drug quality will not always cater to the consumer's demand with the increasing quality differentiation level, because the consumer's co-payment level increases simultaneously. As a result, consumers will sacrifice their quality requirements for price advantages to some extent.

4.6.4 The Effects of the Quality Cost

Following the findings in Section 4.6.3, we explore how the manufacturing cost of the drug affects the manufacturer's quality differentiation level as well as the joint profit of the manufacturer and the PBM.

Corollary 4.1. *The quality cost has effects on the quality differentiation level and the joint profit of the PBM and the drug manufacturer.*

i As the quality cost λ increases, the quality differentiation level θ first keeps uniform on 1, experiences a decreasing trend then increases and larger than 1.

ii When the quality differentiation level $\theta \leq 1$, the joint profit of the PBM and drug manufacturer first decreases and then increases as the quality cost λ increases. When $\theta > 1$, the joint profit of the PBM and drug manufacturer increases as the quality cost λ increases.

Corollary 4.1 summarizes the effect of drug manufacturing cost in vertical integration. By conducting some numerical experiments, Figure 4.3 shows how

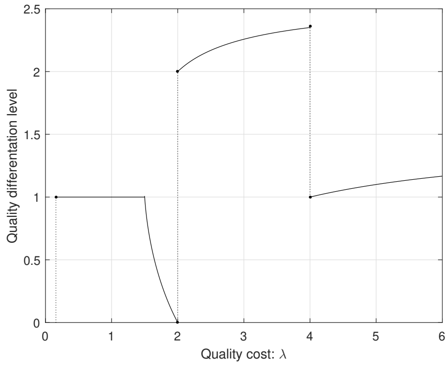


Figure 4.3: Manufacturer's quality differentiation level with the quality cost.

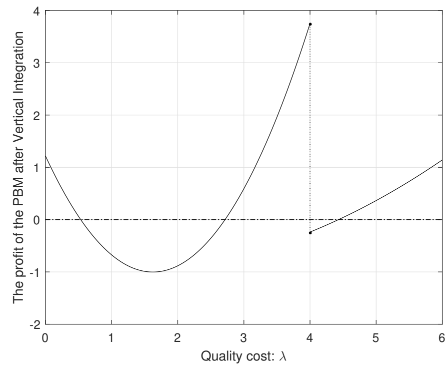


Figure 4.4: The joint profit of PBM and manufacturer with the quality cost.

the manufacturer decides the quality differentiation level θ as quality cost λ increases, while Figure 4.4 illustrates the sequential effects on the joint profit of the integrated PBM and manufacturer as λ increases. The results show that the increasing drug manufacturing cost λ will not always hurt the joint profit under the vertical integration channel. The quality differentiation level θ also has critical effects in the process. As Figure 4.4 shows, under the low-quality VI case, Π_p^V first decreases and then increases (at around $\lambda > 1.8$) as λ increases, while under the high-quality VI case, Π_p^V increases as λ increases. Simultaneously, we find that manufacturer's decision on the drug quality differentiation level θ is also affected by λ. Intuitively, as the drug manufacturing cost λ increases, the manufacturer will decrease θ to reduce the overall manufacturing cost. However, Figure 4.3 shows that under the low-quality VI case, the upstream manufacturer keeps uniform θ between two distribution channels when $\lambda < 1.5$, then θ decreases (tends to 0) firstly and increases with the increasing λ. By contrast, under the high-quality VI case, θ increases with the increasing λ. Additionally, the increasing joint profit Π_p^V reflects that the integrated PBM sets a higher retail price to offer a higher-quality level drug for the high-segment consumers.

4.7 Conclusions

This chapter develops a three-echelon framework (manufacturer-PBM-consumers), which focuses on the vertical integration between the upstream manufacturer and the downstream PBM in the pharmaceutical supply chain. We first explore how vertical integration affects the drug quality designing, pricing as well as profit of the manufacturer and PBM. Then, we investigate the overall game equilibrium considering the vertical integration channel and derive the conditions that encourage the manufacturer to apply vertical integration with the PBM. Additionally, suppose that both the direct retail channel and the vertical integration channel exist, we examine two cases according to the quality differentiation level, the low-quality VI case ($\theta \leq 1$) and the high-quality case ($\theta > 1$). Under the cases, we analyze the impacts of quality differentiation level on consumers' behavior (drug demand and co-payment level) and the impacts of drug manufacturing cost on the quality differentiation level as well as profit level. Finally, we summarize the following conclusions:

(1) Given the consumer's sensitivity to co-payment level and the manufacturer's sensitivity to quality, the manufacturer decides whether to vertically integrate with the downstream PBM depending on the integration cost k. Considering the overall profit of the pharmaceutical supply chain, the range of integration costs that the manufacturer can accept is wider than the range when only considering the manufacturer's profit.

(2) The choice of vertical integration channel (high-quality VI or low-quality VI) and the quality differentiation level between the direct retail channel and vertical integration channel depend on the quality-related parameters. Under the high-quality VI case, consumers first prefer the higher-quality level drug, then begin to choose lower-quality level drug as the quality differentiation level increases.

(3) The manufacturing cost has effects on the joint profit of the manufacturer and PBM by influencing the quality differentiation levels between the direct retail channel and vertical integration channel. Counter-intuitively, under the high-quality VI case, the increasing manufacturing cost improves the joint profit of the PBM and manufacturer.

In future research work, we will consider two related directions based on this study. First, we would combine the market interface with operations in the model. For instance, to consider the effect of vertical integration on the market share of the drug manufacturer and the PBM. The second direction is to explore the impacts of vertical integration in distribution channels on manufacturers' drug Research Development (RD) strategies. For example, the integration strategy affects the consumer's co-payment level and quality requirement for the drug. Therefore, the manufacturer will change decisions on drug production, including developing new drugs or suspension of some drugs.

Chapter 5

Summary and Future Work

Based on the emerging issues on HOM, this thesis explores three topics about healthcare operations and marketing interface: the operation of the inventory sharing policy between two hospitals; the coordination and interaction of retailers' return policy and inventory sharing policy in the consignment contract; the effects of the vertical integration between the manufacturer and PBM on their decisions and consumer's performance. In the following paragraphs, we summarize three studies' major contributions and future research directions.

In the first study, we model the operation of the inventory sharing process between two independent hospitals and find that under some special conditions, the inventory sharing policy is more profitable than the emergent replenishment policy for hospitals. Specifically, we investigate the effects of patients' behavior (patient's emergent request rate) on the hospital's ordering and sharing decisions. Under hospitals' sharing policy, the patient's emergent request rate to a hospital will increase the partner hospital's optimal inventory level while a hospital's initial inventory level will decrease the partner hospital's optimal inventory level. However, there are still some limitations of the first study. In future work, we could extend the study on inventory sharing from a two- hospital setting into a n-hospitals setting based on the previous research results.

The second study focuses on the interaction between the retailers' return action and inventory sharing action. The critical innovation of this study lies in the model design. We build a framework that includes a common dealer and two retailers in the consignment contract, in which the return of unused items to the dealer is allowed. Retailers choose to share inventory according to the related costs. We divide the sharing cases into the dealer-dominated case and retailers-dominated case. We explore the effects of the dealer's return policy on retailers' sharing decisions and profit performance. From the study results, we find that when the dealer cannot control retailers' sharing action, he prefers trading with the individual retailer to be better off. Counter-intuitively, retailers will be encouraged to share inventory when the dealer decreases the return price. This study still has some possible extensions in future work. For instance, to consider more marketing characteristics based on the operational model, such as the manufacturer's market share. By modeling multiple categories of drugs, we could investigate the effects of retailers' inventory sharing on the manufacturer's market share with the retailer's return action in the consignment contract.

In the third study, we construct a three-echelon model with the drug manufacturer, the PBM, and consumers and explore the vertical integration between the manufacturer and PBM. We suppose that the drug quality is endogenous in the first stage, and then the manufacturer differentiates the drug quality under two distribution channels (direct retail channel and vertical integration channel) in the second stage. We analyze the effects of vertical integration on the manufacturer's quality decision and profit performance. From the results, we find that the manufacturer's quality differentiation level decision on the direct retail channel and vertical integration channel affects consumers' behavior (drug demand and co-payment level). Consumers do not always choose the high-quality drug as the quality differentiation level increases. Under the high-quality vertical integration channel, the manufacturer and PBM benefit from the increased manufacturing cost as their profit is influenced simultaneously by the quality differentiation level. In this study, we provide some managerial insights for the drug manufacturer from the consumer welfare perspective. In future work, we aim to focus on the impacts of vertical integration on the manufacturer's drug Research Development (RD) decisions. For example, we will investigate how does the vertical integration between the manufacturer and the PBM affect the new drug development and outdated drug suspension.

In addition, based on the mainstream research directions of HOM, the online healthcare platform operation and digital health operation are also worth more academic attention (Keskinocak and Savva 2020). Such as the substitution of the online diagnosis for the offline diagnosis and the effects of online reviews of physicians on patients' choice (Xu et al. 2021).

Appendix A

Proofs for Chapter 2

Proof of Proposition 2.1 We first introduce more details in the benchmark case. In scenario 1: $x_i \geq D_i$, we denote the total expected cost of hospital i as $O_i^{e1}(x_i)$, which consists of two components. The first component is the cost of initial inventory (order-up-to level) x_i, which represents the cost of the initial inventory, denoted by $(p_r^l + \tau_r)x_i$. The second component is the cost that happens in the current period, denoted by $(p_r + \tau_r)n_i + h(x_i - D_i)$. In addition, as the inventory level at the beginning of the next period equals $x_i - D_i + r_i$, hence, the cost of $(p_r + \tau_r)(x_i - D_i)$ is counted repeatedly for the cost of the initial inventory in the next period, so we cut it in this period. Therefore, we obtain:

$$O_i^{e1}(x_i) = (p_r^l + \tau_r)x_i + (h - p_r - \tau_r)(x_i - D_i) \tag{A.1}$$

Similarly, the expected cost of hospital i in scenario 2 ($D_i > x_i$) is derived.

$$O_i^{e2}(x_i) = (p_r^l + \tau_r)x_i + (p_e + \tau_e)(D_i - x_i)w_i \tag{A.2}$$

where $e_i = (D_i - x_i)w_i$ represents the emergent replenishment order amount. Then $O_i^e(x_i)$ is obtained in Section2.4. □

Proof of Proposition 2.2 Given x_i,

$$\frac{\partial O_i^e(x_i)}{\partial w_i} = \int_{x_i}^{\infty} (p_e + \tau_e)(D_i - x_i)g_i(D_i)d(D_i) > 0. \tag{A.3}$$

Thus, $O_i^e(x_i)$ increases as w_i increases.

$$\begin{aligned}\frac{\partial O_i^e(x_i)}{\partial x_i} &= \int_0^{x_i} (h + p_r^l - p_r) g_i(D_i) d(D_i) \\ &+ \int_{x_i}^{\infty} [p_r^l + \tau_r - (p_e + \tau_e) w_i] g_i(D_i) d(D_i)\end{aligned} \tag{A.4}$$

$$\frac{\partial^2 O_i^e(x_i)}{\partial x_i^2} = [h - p_r - \tau_r + (p_e + \tau_e) w_i] g_i(x_i) > 0 \tag{A.5}$$

if $h - p_r - \tau_r + (p_e + \tau_e) w_i > 0$. Therefore, we notice that the above equation is positive, and $O_i^e(x_i)$ is convex in x_i when $w_i > \frac{p_r + \tau_r - h}{p_e + \tau_e}$. □

Proof of Proposition 2.4 We consider the cases where inventory sharing occurs:

in scenario 2, $x_i < D_i$, $x_j \geq D_j$, $(x_j - D_j)(1 - k_j) \geq (D_i - x_i) w_i$,

$$\frac{\partial s_j}{\partial w_i} = D_i - x_i \geq 0, \frac{\partial s_j}{\partial k_j} = 0; \tag{A.6}$$

in scenario 3, $x_i < D_i$, $x_j \geq D_j$, $(x_j - D_j)(1 - k_j) < (D_i - x_i) w_i$,

$$\frac{\partial s_j}{\partial w_i} = 0, \frac{\partial s_j}{\partial k_j} = D_i - x_i < 0. \tag{A.7}$$

In scenario 4 and scenario 5, taking the first derivative of s_i with respect to w_j, k_i, we can draw similar conclusions. □

Proof of Proposition 2.5 Given (x_i, x_j),

$$\begin{aligned}\frac{\partial O^s(x_i, x_j)}{\partial w_j} &= \int_0^{x_i} \int_{x_j}^{\frac{(x_i - D_i)(1-k_i)}{w_j} + x_j} (\tau_s + p_r + \tau_r - h)(D_j - x_j) \\ &\times g_i(D_i) g_j(D_j) dD_j dD_i \\ &+ \int_0^{x_i} \int_{\frac{(x_i - D_i)(1-k_i)}{w_j} + x_j}^{\infty} (p_e + \tau_e)(D_j - x_j) g_i(D_i) g_j(D_j) dD_j dD_i \\ &+ \int_{x_i}^{\infty} \int_{x_j}^{\infty} (p_e + \tau_e)(D_j - x_j) g_i(D_i) g_j(D_j) dD_j dD_i > 0.\end{aligned} \tag{A.8}$$

Thus, if $(\tau_s + p_r + \tau_r - h) \geq 0$, the monotonicity maintains, $O^s(x_i, x_j)$ increases as w_j increases.

$$\begin{aligned}\frac{\partial O^s(x_i, x_j)}{\partial k_j} &= \int_0^{x_j} \int_{\frac{(x_j - D_j)(1-k_j)}{w_i} + x_i}^{\infty} (h - \tau_s + p_e + \tau_e - p_r - \tau_r) \\ &\times (x_j - D_j) g_i(D_i) g_j(D_j) dD_i dD_j > 0\end{aligned} \tag{A.9}$$

If $(h-\tau_s+p_e+\tau_e-p_r-\tau_r)\geq 0$, the monotonicity maintains, and $O^s(x_i,x_j)$ increases as k_j increases. The same result can be obtained by differentiating with respect to w_i, k_i. □

Proof of Proposition 2.6

$$
\begin{aligned}
\frac{\partial O^s(x_i,x_j)}{\partial x_i} &= \int_0^{x_i}\int_0^{x_j}(h+p_r^l-p_r)g_i(D_i)g_j(D_j)dD_jdD_i \\
&+\int_0^{x_i}\int_{x_j}^{\frac{(x_i-D_i)(1-k_i)}{w_j}+x_j}(h+p_r^l-p_r)g_i(D_i)g_j(D_j)dD_jdD_i \\
&+\int_0^{x_i}\int_{\frac{(x_i-D_i)(1-k_i)}{w_j}+x_j}^{\infty}[(\tau_s-p_e-\tau_e)(1-k_i)+(h-p_r-\tau_r)k_i \\
&+p_r^l+\tau_r]\times g_i(D_i)g_j(D_j)dD_jdD_i \\
&+\int_0^{x_j}\int_{x_i}^{\frac{(x_j-D_j)(1-k_j)}{w_i}+x_i}[(h-\tau_s-p_r-\tau_r)w_i+p_r^l+\tau_r] \\
&\times g_i(D_i)g_j(D_j)dD_jdD_i \\
&+\int_0^{x_j}\int_{\frac{(x_j-D_j)(1-k_j)}{w_i}+x_i}^{\infty}[p_r^l+\tau_r-(p_e+\tau_e)w_i]g_i(D_i)g_j(D_j)dD_jdD_i \\
&+\int_{x_i}^{\infty}\int_{x_j}^{\infty}[p_r^l+\tau_r-(p_e+\tau_e)w_i]g_i(D_i)g_j(D_j)dD_jdD_i
\end{aligned}
\tag{A.10}
$$

$$
\begin{aligned}
\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i^2} &= \int_0^{x_j}[(h-p_r-\tau_r)(1-w_i)+\tau_s w_i]g_i(x_i)g_j(D_j)dD_j \\
&+\frac{(1-k_i)^2}{w_j}\int_0^{x_i}(h-\tau_s+p_e+\tau_e-p_r-\tau_r) \\
&\times g_j(\frac{(x_i-D_i)(1-k_i)}{w_j}+x_j)g_i(D_i)dD_i \\
&+w_i\int_0^{x_j}(h-\tau_s+p_e+\tau_e-p_r-\tau_r) \\
&\times g_i(\frac{(x_j-D_j)(1-k_j)}{w_i}+x_i)g_j(D_j)dD_j \\
&+\int_{x_j}^{\infty}[(p_e+\tau_e)(w_i+k_i-1)+\tau_s(1-k_i)+(h-p_r-\tau_r)k_i] \\
&\times g_i(x_i)g_j(D_j)dD_j
\end{aligned}
\tag{A.11}
$$

We note that if the above items are positive, then $O^s(x_i,x_j)$ is convex in x_i. When $O^s(x_i,x_j)$ is unimodal, we have:

$$\begin{aligned}\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i \partial x_j} &= (1-k_i)\int_0^{x_i}(h-\tau_s+p_e+\tau_e-p_r-\tau_r) \\ &\times g_j(\frac{(x_i-D_i)(1-k_i)}{w_j}+x_j)g_i(D_i)dD_i \\ &+(1-k_i)\int_0^{x_j}(h-\tau_s+p_e+\tau_e-p_r-\tau_r) \\ &\times g_i(\frac{(x_j-D_j)(1-k_j)}{w_i}+x_i)g_j(D_j)dD_j > 0\end{aligned} \tag{A.12}$$

$$\frac{\partial x_i(x_j)}{\partial x_j} = -\left.\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i \partial x_j}\middle/\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i^2}\right. \tag{A.13}$$

since $\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i^2} > \frac{\partial^2 O^s(x_i,x_j)}{\partial x_i \partial x_j} > 0$ and $\left|-\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i \partial x_j}\middle/\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i^2}\right| < 1$.

Therefore, if $\tau_s \le p_e+\tau_e+h-p_r-\tau_r$ is satisfied, $x_i(x_j)$ is unique when x_j is given (the proof is according to the proof of Proposition 1 in (Rudi et al. 2001)).

$$\begin{aligned}\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i \partial w_j} &= -\frac{(x_i-D_i)(1-k_i)^2}{w_j^2}\int_0^{x_i}(h-\tau_s+p_e+\tau_e-p_r-\tau_r) \\ &\times g_j(\frac{(x_i-D_i)(1-k_i)}{w_j}+x_j)g_i(D_i)dD_i\end{aligned} \tag{A.14}$$

$$\begin{aligned}\frac{\partial^2 O^s(x_i,x_j)}{\partial x_i \partial k_j} &= -(x_j-D_j)\int_0^{x_j}(h-\tau_s+p_e+\tau_e-p_r-\tau_r) \\ &\times g_i(\frac{(x_j-D_j)(1-k_j)}{w_i}+x_i)g_j(D_j)dD_j\end{aligned} \tag{A.15}$$

We obtain that there exists a unique $x_i^*(x_j)$, which decreases as x_j increases. When $x_i < D_i$, $x_i^*(x_j)$ increases as w_j increases if $\tau_s \le p_e+\tau_e+h-p_r-\tau_r$. When $x_j < D_j$, $x_i^*(x_j)$ increases as k_j increases if $\tau_s \le p_e+\tau_e+h-p_r-\tau_r$. □

Appendix B

Proofs and Supplemental numerical experiments for Chapter 3

B.1 Proofs

Proof of Proposition 3.1 Under a decentralized system, the condition $Q_N^i = Q_N^{ci}$ is satisfied when the dealer and retailers are coordinated. Therefore, we obtain $w(r)$ in Proposition 2.1. Plugging (3.8) into (3.3) and taking the first derivative with r, we obtain that for $i = 1, 2$,

$$\frac{\partial \Pi_N^d}{\partial r} = -\frac{p-c}{c-s}(Q_i - \Lambda(Q_i)) + \Lambda(Q_i) < 0 \tag{B.1}$$

It shows that the Π_N^d is a decreasing function of r; hence, $\Pi_N^{h_i}$ is an increasing function of r as Π_N^c is independent of r. Let $\Pi_N^{h_i} = 0$ and $\Pi_d = 0$, we obtain r_l and r_h respectively.

We consider the dealer firstly, and take the first derivative with Q_i for (3.3):

$$\frac{\partial \Pi_N^d}{\partial Q_i} = w(1 - F_i(Q_i)) + (r+s)F_i(Q_i) - c \tag{B.2}$$

which represents the expected marginal profit of the dealer for selling a product, and it should be positive. Therefore, we obtain $w(r)(1 - F_i(Q_i)) + (r+s)F_i(Q_i) - c > 0$. Replacing w with $w(r)$ in (3.3):

$$\Pi_N^d = E\left\{\sum_{i=1, j\neq i}^{2} w(r)(Q_i - \Lambda(Q_i)) + (r+s)\Lambda(Q_i) - cQ_i\right\} \tag{B.3}$$

when $\Pi_N^d = 0$, $r = r_N^h = c - s$.

Then considering the retailer, we also obtain $w(r) = p - r\frac{F_i(Q_i)}{1-F_i(Q_i)}$ by solving the first-order condition:

$$\frac{\partial \Pi_N^{h_i}}{\partial Q_i} = (p_i - w)(1 - F_i(Q_i)) - rF_i(Q_i) = 0 \tag{B.4}$$

then replacing w with $w(r)$ in (3.1):

$$\Pi_N^{h_i} = E[r\frac{F_i(Q_i)}{1 - F_i(Q_i)}(Q_i - \Lambda(Q_i)) - r\Lambda(Q_i)] \tag{B.5}$$

when $\Pi_N^{h_i} = 0$, $r = r_N^l = 0$ and $r_N^h > r_N^l$ is obvious with the assumption $c > s$. $\square$

Proof of Proposition 3.2 We obtain the $w(r)$ by coordinating the dealer and two retailers in decentralized system. Then plugging (3.16) into (3.11) and taking the first derivative with r,

$$\frac{\partial \Pi_S^d}{\partial r} = \frac{(Q_i - \Lambda(Q_i) + T_i)p\lambda}{c - s\lambda - (p - w_s)(\beta - \alpha)} + \Lambda(Q_i - T_i + T_j) < 0 \tag{B.6}$$

where $\alpha = \frac{\partial T_i}{\partial Q_i}$, $\beta = \frac{\partial T_j}{\partial Q_i}$, and $\lambda = F_i(Q_i - T_i + T_j)$. We can observe that Π_S^d is a decreasing function of r under coordination case. $\Pi_S^{h_i}$ increases as r increases since Π_S^c is independent of r. Then let $\Pi_S^{h_i} = 0$ and $\Pi_S^d = 0$, we obtain the r_S^l and r_S^h respectively.

Considering the dealer first and taking the first derivative with Q_i for (3.11),

$$\begin{aligned}\frac{\partial \Pi_S^d}{\partial Q_i} &= \sum_i^2 [w(1 - F_i(Q_i) + \frac{\partial T_i}{\partial Q_i}) + (r + s)F_i(Q_i - T_i + T_j) - c] \\ &\geq [(w - (r + s))(1 - F_i(Q_i - T_i + T_j)) + w\frac{\partial T_i}{\partial Q_i} + (r + s - c)] > 0\end{aligned} \tag{B.7}$$

when $r = r_S^h = c - s$, Π_S^d has the minimum value. When considering the retailer, we obtain

$$w(r) = p - \frac{(p - w_s)(\frac{\partial T_i}{\partial Q_i} - \frac{\partial T_j}{\partial Q_i}) - rF_i(Q_i - T_i + T_j)}{1 - F_i(Q_i) + \frac{\partial T_i}{\partial Q_i}}$$

by solving the first-order condition:

$$\frac{\partial \Pi_S^{h_i}}{\partial Q_i} = (p-w)(1-F_i(Q_i))+(w_s-w)\frac{\partial T_i}{\partial Q_i}+(p-w_s)\frac{\partial T_j}{\partial Q_i}-rF_i(Q_i-T_i+T_j) \tag{B.8}$$

then replacing w with $w(r)$ in (3.9):

$$\begin{aligned}\Pi_S^{h_i}(w(r)) = E[(p-w(r))(Q_i-\Lambda(Q_i)) + (w_s-w(r))T_i + (p-w_s)T_j \\ - r\Lambda(Q_i-T_i+T_j) - 0]\end{aligned} \tag{B.9}$$

for each retailer i. When $\Pi_S^{h_i}(w(r)) = 0$,

$$\begin{aligned} r &= r_S^l \\ &= \frac{(p-w_s)[\frac{\partial T_i}{\partial Q_i}(Q_i-\Lambda(Q_i)+T_j) - \frac{\partial T_j}{\partial Q_i}(Q_i-\Lambda(Q_i)+T_i) - (T_i-T_j)(1-F_i(Q_i))]}{\Lambda(Q_i-T_i+T_j)(1-F_i(Q_i) + \frac{\partial T_i}{\partial Q_i} - F_i(Q_i-T_i+T_j)(Q_i-\Lambda(Q_i)+T_i)}\end{aligned}$$

when w_s increases to p, and r decreases to 0, then $\Pi_S^{h_i} = 0$. Therefore, under the retailers' sharing option, the dealer still provides a return policy $r_S^l = 0$ and $r_S^h = c - s$ to make both the dealer and retailers profitable. □

Proof of Proposition 3.3 Under no sharing case, the ordering quantity of retailer i satisfies $F_i(Q_N^i) = \frac{p-w}{p-w+r}$. Under sharing case, when the consignment price w and return price r are determined by the dealer, we consider the ordering quantity of the retailer under two extreme cases: $w_s = 0$ and $w_s = p$. When $w_s = 0$, $F_i(Q_S^i) = 1 + \frac{p}{p-w}\frac{\partial T_j}{\partial Q_S^i} - \frac{r}{p-w}F_i(Q_S^i - T_i + T_j)$, we obtain that $F_i(Q_S^i) \leq \frac{p-w}{p-w+r} + \frac{p}{p-w+r}\frac{\partial T_j}{\partial Q_S^i}$. Because $\frac{\partial T_j}{\partial Q_S^i} < 0$ and the monotonicity of $F_i(\cdot)$, $Q_N^i > Q_S^i$ at $w_s = 0$. Therefore, when the dealer sets the optimal consignment price and return price under no sharing case, he will make a higher profit than under sharing case at $w_s = 0$.

When $w_s = p$, $F_i(Q_S^i) = 1 + \frac{\partial T_i}{\partial Q_S^i} - \frac{r}{p-w}F_i(Q_S^i - T_i + T_j)$, we observe that $F_i(Q_S^i) \geq \frac{p-w}{p-w+r} + \frac{\partial T_i}{\partial Q_S^i}\frac{p-w}{p-w+r}$. Because $\frac{\partial T_i}{\partial Q_S^i} > 0$ and the monotonicity of $F_i(\cdot)$, $Q_N^i < Q_S^i$ at $w_s = p$. The dealer makes a higher profit under sharing case when $w_s = p$. □

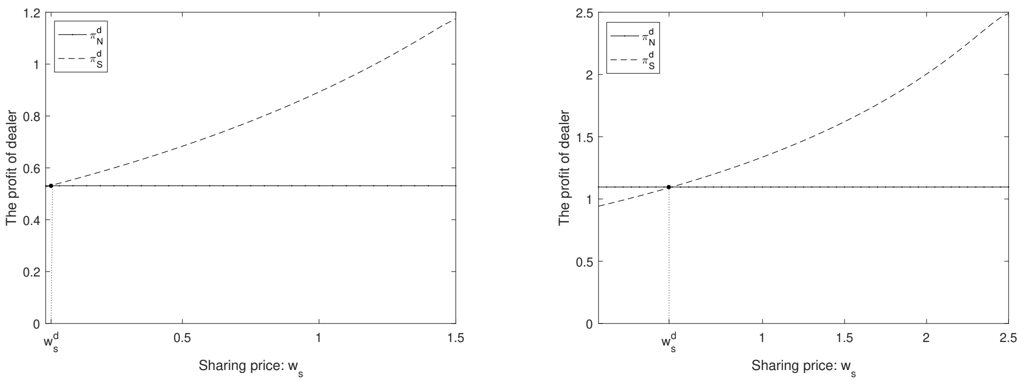
Proof of Proposition 3.4 According to the monotonicity of the dealer's profit, Proposition 3.4 can be obtained. □

Proof of Proposition 3.5 The proof of Proposition 3.5 is similar to Proposition 3.4. □

Proof of Proposition 3.6 According to Observation 3.1, Observation 3.2 and Proposition 3.5, Proposition 3.6 can be obtained. □

B.2 Supplemental Numerical Experiments

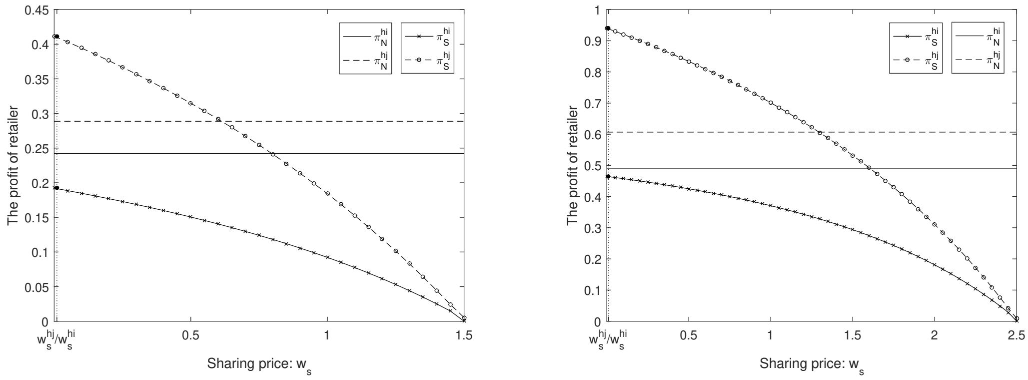
To examine the robustness of the conclusions about the effects of sharing price, we conduct another group of numerical experiments where the demand submits to the normal distribution. Let $r = 0.1$, $c = 0.5$, $s = 0.3$, $D_i \sim N(0.5, 0.08), i = 1, 2$. We obtain the dealer's profit, the retailer's profit and the profit of retailers' alliance with and without sharing action when $p = 1.5$ and $p = 2.5$ sequentially. The results also validate the conclusions between dealer-dominated sharing case and retailers-dominated sharing case.



(a) $p = 1.5$ (b) $p = 2.5$

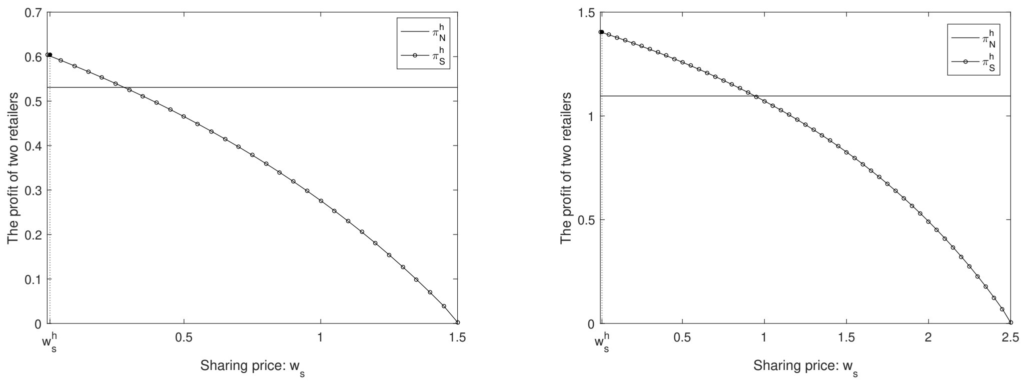
Figure B.1: Dealer's profit.

To examine the robustness of the conclusions about the effects of return price, we conduct another group of numerical experiments where the demand submits to the normal distribution. Let $p = 1.5$, $c = 0.5$, $s = 0.3$, $D_i \sim N(0.5, 0.08), i = 1, 2$. We obtain the dealer's profit, the retailer's profit and the profit of retailers' alliance with and without sharing action when $r = 0.05$ and $r = 0.13$ sequentially. The results also validate the conclusions in Proposition 3.6.



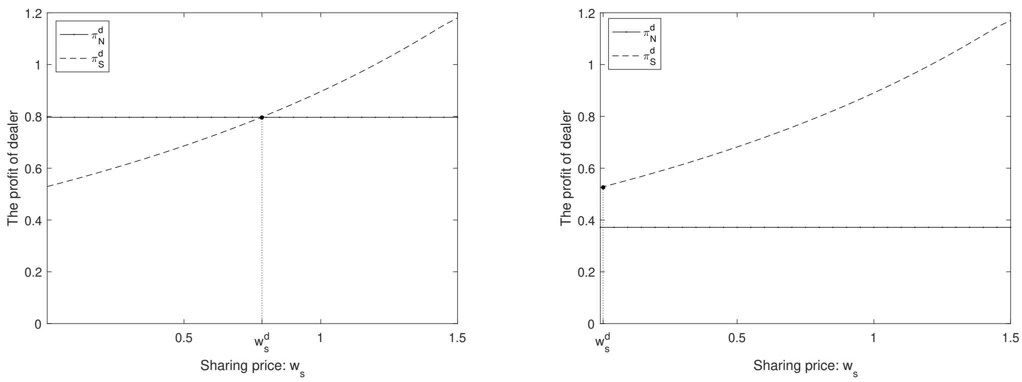
(a) $p = 1.5$ (b) $p = 2.5$

Figure B.2: Retailers' profits when they are individual.



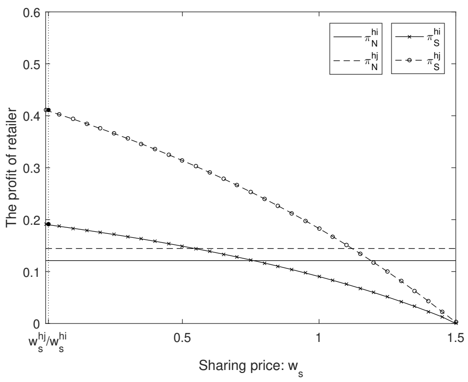
(a) $p = 1.5$ (b) $p = 2.5$

Figure B.3: Retailers' profits when they form an alliance.

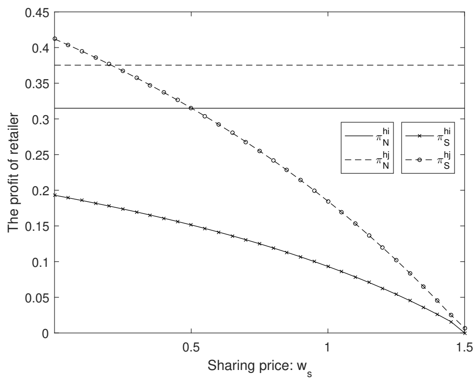


(a) $r = 0.05$ (b) $r = 0.13$

Figure B.4: Dealer's profit.

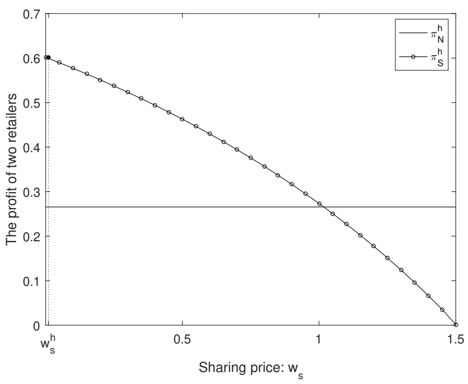


(a) $r = 0.05$

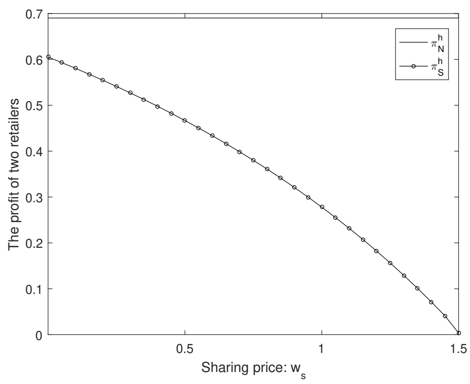


(b) $r = 0.13$

Figure B.5: Retailers' profits when they are individual.



(a) $r = 0.05$



(b) $r = 0.13$

Figure B.6: Retailers' profits when they form an alliance.

Appendix C

Proofs for Chapter 4

Proof of Proposition 4.1 For deriving the optimal decisions in Proposition 4.1, we first consider the case without the vertical integration between the drug manufacturer and the PBM, that is under the direct retail channel (Section 4.4).

We take the partial derivatives of Π_m^0 (Equation 4.2) with w^0:

$$\frac{\partial \Pi_m^0}{\partial w^0} = \frac{\beta e^0 - \beta w^0 + q^0}{2q^0} - \frac{\beta(w^0 - \lambda {q^0}^2)}{2q^0} \tag{C.1}$$

By solving $\frac{\partial \Pi_m^0}{\partial w^0} = 0$, we obtain:

$$w^0 = \frac{\lambda {q^0}^2}{2} + \frac{e^0}{2} + \frac{q^0}{2\beta} \tag{C.2}$$

Then substituting w^0 into Π_p^0 and taking the partial derivatives with p^0, by solving $\frac{\partial \Pi_p^0}{\partial p^0} = 0$, we obtain:

$$p^0 = \frac{\lambda {q^0}^2}{4} + \frac{3e^0}{4} + \frac{3q^0}{4\beta} \tag{C.3}$$

Similarly, we have $e^0 = \frac{\lambda {q^0}^2}{2} - \frac{q^0}{2\beta}$. Substituting e^0 into w^0 and p^0, we derive the optimal wholesale price and retail price of the drug without the vertical integration between the drug manufacturer and the PBM: $w^{0*} = \frac{3\lambda {q^0}^2}{4} + \frac{q^0}{4\beta}$ and $p^{0*} = \frac{5\lambda {q^0}^2}{8} + \frac{3q^0}{8\beta}$.

Second, we consider the case under vertical integration, where the manufacturer can choose two distribution channels with the PBM, the direct retail channel and the vertical integration channel. We suppose that both of the channels are applied by the manufacturer and the drug quality under the VI channel is larger than the quality under the direct retail channel $q^V > q^N$. We use $m^N = p^N - w^N$ to denote the profit margin of the PBM by direct retail channel.

Step 1, given p^V, e^N, e^V, $t^V = p^V - e^V$, and $t^N = w^N + m^N - e^N$, substituting d^N with $\frac{\beta(t^V - t^N)}{q^V - q^N} - \frac{\beta t^N}{q^N}$, d^V with $1 - \frac{\beta(t^V - t^N)}{q^V - q^N}$, we take the partial derivatives of Π_m^N (Equation 4.4) with w^N; by setting $\frac{\partial \Pi_m^N}{\partial w^N} = 0$, we obtain the expression of w^N with related p^V, e^N, e^V.

$$w^N = \frac{q^V(c^N + e^N - m^N) + q^N(p^V - e^V)}{2q^V} \tag{C.4}$$

Simultaneously, $\frac{\partial^2 \Pi_m^N}{\partial {w^N}^2} = -\frac{2\beta}{q^V - q^N} - \frac{2\beta}{q^N} < 0$ verifies the w^N is a unique optimal solution.

Step 2, putting the w^N into d^N and d^V, taking partial derivatives of Equation 4.3 with p^V, we obtain:

$$\frac{\partial \Pi_p^V}{\partial p^V} = \frac{\beta q^V(c^N - e^N + 2m^N - q^N/\beta) + (\beta q^N - 2\beta q^V)(2p^V - c^V - e^V - q^V/\beta)}{2q^V(q^V - q^N)} \tag{C.5}$$

By solving $\frac{\partial \Pi_p^V}{\partial p^V} = 0$ with $\frac{\partial^2 \Pi_p^V}{\partial {p^V}^2} = \frac{\beta(q^N - 2q^V)}{q^V(q^V - q^N)} < 0$, we derive the optimal pair of (m^N, p^V) as

$$\begin{cases} m^N = \frac{q^N}{2\beta} - \frac{c^N}{2} + \frac{e^N}{2} \\ p^V = \frac{q^V}{2\beta} + \frac{c^V}{2} + \frac{e^V}{2} \end{cases} \tag{C.6}$$

Then substituting the m^N and p^V in the demand d^N and d^V.

Step 3, considering $c^N = \lambda {q^N}^2$, $c^V = \lambda {q^V}^2$, simultaneously taking the partial derivatives of Π_e^N and Π_e^V with e^N and e^V respectively, with

$$\begin{cases} \frac{\partial^2 \Pi_e^N}{\partial {e^N}^2} = \frac{\beta q^V}{2q^N(q^V - q^N)} > 0 \\ \frac{\partial^2 \Pi_e^V}{\partial {e^V}^2} = \frac{\beta(2q^V - q^N)}{2q^V(q^V - q^N)} > 0 \end{cases} \tag{C.7}$$

we derive the unique optimal pair of (e^N, e^V), then, substituting the e^N, e^V in the w^N, m^N and p^V, we obtain the optimal solutions in Proposition 4.1. □

Proof of Proposition 4.2 & Proposition 4.3 We obtain the equilibrium results (Table 4.2) by considering the single distribution strategy (only the direct retail channel) and the mixed distribution strategy (the direct retail channel and the vertical integration channel). Take the optimal expression in Proposition 4.1 into the Equation 4.3 and 4.4; we obtain the new expression of manufacturer's profit without VI channel Π_m^N and joint profit of manufacturer and the PBM with VI channel Π_p^V as follows:

$$\begin{aligned}\Pi_p^V = {} & [\frac{q^V(\beta {q^N}^2\lambda - 2\beta q^N q^V\lambda + 4\beta {q^V}^2\lambda + q^N - 4q^V)}{2\beta(5q^N - 8q^V)}] \\ & \cdot [\frac{\beta {q^N}^2\lambda + 2\beta q^N q^V\lambda - 8\beta {q^V}^2\lambda - 4q^N + 8q^V}{-4(5q^N - 8q^V)}] \\ & + [\frac{q^N(2\beta {q^N}^2\lambda - 3\beta q^N q^V\lambda - 2\beta {q^V}^2\lambda - 3q^N + 6q^V)}{-2\beta(5q^N - 8q^V)}] \\ & \cdot [\frac{q^V(3\beta q^N\lambda - 2\beta q^V\lambda - 2)}{4(5q^N - 8q^V)}] - k\end{aligned} \tag{C.8}$$

$$\Pi_m^N = [\frac{q^N(3\beta {q^N}^2\lambda - 5\beta q^N q^V\lambda + 2\beta {q^V}^2\lambda - 2q^N + 2q^V)}{-4\beta(5q^N - 8q^V)}][\frac{q^V(3\beta q^N\lambda - 2\beta q^V\lambda - 2)}{4(5q^N - 8q^V)}] \tag{C.9}$$

By taking the partial derivatives of Π_m^N with q^N and Π_p^V with q^V, we obtain the optimal q^N, q^V with the first-order condition: $\frac{\partial \Pi_m^N}{\partial q^N} = 0$ and $\frac{\partial \Pi_p^V}{\partial q^V} = 0$. Then we have $q^N = \frac{1.1429}{\beta\lambda}, q^V = \frac{0.7143}{\beta\lambda}$. Simultaneously, we verify the second-order condition for the optimal q^N, q^V (setting $\beta = 1, \lambda = 0.5$):

$$(\frac{\partial^2 \Pi_m^N}{\partial {q^N}^2})(\frac{\partial^2 \Pi_p^V}{\partial {q^V}^2}) - (\frac{\partial^2 \Pi_m^N}{\partial q^N \partial q^V})(\frac{\partial^2 \Pi_p^V}{\partial q^V \partial q^N}) = 2.4118e + 16 > 0 \tag{C.10}$$

□

Proof of Proposition 4.4 Considering the drug quality differentiation level θ, we first analyze the *low-quality VI* case, where $\theta \leq 1$. Similar to the steps in Section 4.5, we obtain the $w^N(q)$ by $\partial \frac{\Pi_m^V}{w^N} = 0$.

$$w^N(q) = \frac{q(\beta\lambda\theta^3 q - 2\beta\lambda\theta^2 q + 17\beta\lambda\theta q - 28\beta\lambda q - \theta^2 + 5\theta - 4)}{4\beta(5\theta - 8)} \tag{C.11}$$

then taking the partial derivatives of Π_e^N and Π_e^V with e^N and e^V respectively, with

$$\begin{cases} \frac{\partial^2 \Pi_e^N}{\partial {e^N}^2} = \frac{\beta(4-3\theta)}{4q(\theta-2)(\theta-1)} > 0 \\ \frac{\partial^2 \Pi_e^V}{\partial {e^V}^2} = \frac{\beta(\theta-2)}{2\theta q(\theta-1)} > 0 \end{cases} \tag{C.12}$$

we derive the optimal pair of $(e^N(q), e^V(q))$:

$$\begin{cases} e^N(q) = \frac{q(\beta\lambda\theta^3 q - 2\beta\lambda\theta^2 q - 3\beta\lambda\theta q + 4\beta\lambda q - \theta^2 + 5\theta - 4)}{-\beta(5\theta-8)} \\ e^V(q) = \frac{\theta q(3\beta\lambda\theta^2 q - 4\beta\lambda\theta q + \beta\lambda q - 3\theta + 3)}{\beta(5\theta-8)} \end{cases} \tag{C.13}$$

we also obtain the $d^N(q)$ by substituting the $w^N(q)$ as follows:

$$d^N(q) = \frac{\beta\lambda\theta^2 q - \beta\lambda\theta q - 4\beta\lambda q - \theta + 4}{-4(5\theta-8)} \tag{C.14}$$

plugging the $w^N(q)$, $d^N(q)$ into $\Pi_m^N(\theta, q)$, then the manufacturer maximizes:

$$\begin{aligned} \Pi_m^N(\theta, q) = {} & [\frac{q(\beta\lambda\theta^3 q - 2\beta\lambda\theta^2 q - 3\beta\lambda\theta q + 4\beta\lambda q - \theta^2 + 5\theta - 4)}{4\beta(5\theta-8)}] \\ & \cdot [\frac{\beta\lambda\theta^2 q - \beta\lambda\theta q - 4\beta\lambda q - \theta + 4}{-4(5\theta-8)}] \end{aligned} \tag{C.15}$$

Given q, by partial deriving $\Pi_m^N(\theta, q)$ with θ, we could obtain $\theta_1 = \frac{\beta\lambda q+1+\sqrt{17\beta^2\lambda^2q^2-14\beta\lambda q+1}}{2\beta\lambda q}$, $\theta_2 = \frac{\beta\lambda q+1-\sqrt{17\beta^2\lambda^2q^2-14\beta\lambda q+1}}{2\beta\lambda q}$. Consider the constraints of $\theta \leq 1$ and $17\beta^2\lambda^2q^2 - 14\beta\lambda q + 1 \geq 0$, we conclude that if $0 < \beta\lambda q \leq \frac{7-4\sqrt{2}}{17}$ or $\frac{7+4\sqrt{2}}{17} < \beta\lambda q < \frac{3}{4}$, then $\theta(q) = \frac{\beta\lambda q+1+\sqrt{17\beta^2\lambda^2q^2-14\beta\lambda q+1}}{2\beta\lambda q}$; otherwise, the manufacturer sets $\theta(q) = 1$.

Second, we analyze the *high-quality VI* case, where $\theta > 1$.

$$w^N(q) = \frac{q(2\beta\lambda\theta^2 q + 27\beta\lambda\theta q - 17\beta\lambda q + 2\theta - 2)}{4\beta(-5+8\theta)} \tag{C.16}$$

also taking the partial derivatives of Π_e^N and Π_e^V with e^N and e^V respectively, with

$$\begin{cases} \frac{\partial^2 \Pi_e^N}{\partial {e^N}^2} = \frac{\beta\theta(4\theta-3)}{4q(2\theta-1)(\theta-1)} > 0 \\ \frac{\partial^2 \Pi_e^V}{\partial {e^V}^2} = \frac{\beta(2\theta-1)}{2\theta q(\theta-1)} > 0 \end{cases} \tag{C.17}$$

we obtain the optimal pair of $(e^N(q), e^V(q))$ as follows:

$$\begin{cases} e^N(q) = -\frac{q(2\beta\lambda\theta^2 q - 5\beta\lambda\theta q + 3\beta\lambda q + 2\theta - 2)}{\beta(8\theta - 5)} \\ e^V(q) = \frac{\theta q(4\beta\lambda\theta^2 q - 3\beta\lambda\theta q - \beta\lambda q - 4\theta + 4)}{\beta(8\theta - 5)} \end{cases} \tag{C.18}$$

by substituting the $w^N(q)$ into the $d^N(q)$, we obtain:

$$d^N(q) = \frac{\theta(2\beta\lambda\theta q - 3\beta\lambda + 2)}{4(8\theta - 5)} \tag{C.19}$$

then the manufacturer maximizes:

$$\Pi_m^N(\theta, q) = [\frac{q(\beta\lambda\theta^2 q - 5\beta\lambda\theta q + 3\beta\lambda q + 2\theta - 2)}{4\beta(-5 + 8\theta)}][\frac{\theta(2\beta\lambda\theta q - 3\beta\lambda q + 2)}{4(-5 + 8\theta)}] \tag{C.20}$$

Similarly, given q, by partial deriving $\Pi_m^N(\theta, q)$ with θ, we obtain $\theta = \frac{3\beta\lambda q - 2}{2\beta\lambda q}$. Then considering the constraint of $\theta > 1$, we conclude that if $\beta\lambda q > 2$, then $\theta(q) = \frac{3\beta\lambda q - 2}{2\beta\lambda q}$. □

Proof of Proposition 4.1 We conduct the numerical experiments to analyze the effects of drug quality cost λ on the quality differentiation level θ as well as the joint profit of the manufacturer and the PBM, by setting $\beta = 1$, $k = 1$, $q = 1/2$. The results in Figures 4.3 and 4.4 verify Corollary 4.1.

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