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Co-ordination Models of a Single-vendor Multi-buyer Supply Chain

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CERTIFICATE OF ORIGINALITY

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Abstract

Supply chain management has become a critical issue in current business environments. Much research has emphasized the co-ordination that reduces the total system cost in a supply chain network. In the last three decades, various integrated inventory co-ordinated models have been established (Sarmah et al. (2006), Khouja and Goyal (2008)). Chan and Kingsman (2005, 2006, 2007) developed a synchronized cycles model that allows each buyer to choose its ordering cycle, where the length of the cycle is a submultiple of the vendor's production cycle. In order to further minimize the total cost, under the synchronized cycle the vendor may schedule the time of the delivery within an ordering cycle, and this delivery time may be different from buyer to buyer. It has been shown, by many numerical experiments, that the synchronized cycles model can significantly reduce the total system cost and make a significant reduction in the vendor's cost compared to the independent policy and the common replenishment cycle (e.g. Banerjee and Burton (1994)). However, the cost to all the buyers is significantly increased.

This research analyses what mechanisms are needed from the vendor to motivate the buyers to change their policies so as to allow the saving from coordination to be achieved. The first mechanism proposed by the research is quantity discounts. Three models of quantity discounts are proposed. The second mechanism proposed by this research is a trade credit policy, in which the supplier will offer the retailer a delay period, that is, *the trade credit period*, in paying for the amount of purchasing cost. Such credit policies may be applied as an alternative to quantity discounts to motivate buyers to participate in the supply chain co-ordination. The final mechanism proposed by this research is a cost sharing policy in which a portion of the buyer's holding cost is borne by the vendor. While the vendor benefits from the co-ordination by synchronized cycles, the mechanisms proposed by this research can guarantee that a buyer's total relevant cost of coordination will not be increased when compared with independent optimization. Hence, both the vendor and the buyers are motivated to co-ordiante in the supply chain.

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Chapter 1

Introduction and Literature Review

1.1 Introduction

Supply chain management has become a critical issue in current business environments. An effective supply chain policy can reduce the average holding inventory level and the total expected cost. From classical inventory theory, the economic ordering quantity (EOQ) concept has been widely applied. Under EOQ, a buyer determines the optimal ordering size that minimizes its total cost. In a two-level supply chain, under individual optimal policies, the buyer orders at the EOQ and the vendor uses the information from the buyer to determine its own optimal production schedule, i.e. economic production quantity (EPQ). However, such a policy may not be optimal for the whole supply chain system. Such an individual policy is known as the independent optimization. Many researchers, beginning in the 1970's, started to explore modes of co-ordination that perform better than independent optimization in terms of total system cost. In the last three decades, various integrated inventory co-ordinated models have been developed.

Chan and Kingsman (2005, 2007) developed a synchronized cycle model that allows each buyer to choose its ordering cycle, while the length of the cycle should be kept as a factor of the vendor's production cycle. In their paper, the synchronized cycles model out-performs the independent policy and the common cycle policy in a number of numerical examples with different ranges of demands and numbers of buyers. In addition, the paper illustrated that the common cycle policy can only outperform the independent policy in some limited cases. It is shown in Chan and Kingsman (2005, 2007) that the synchronized cycles model can be used to plan the ordering intervals in a one-vendor many-buyer supply chain so as to reduce significantly the system costs compared to each partner operating completely independently. However, the mechanism of how to motivate the buyers to participate in the coordination was not considered.

1.2 Outline of the Thesis

Chapter 1 introduces the background and evolution of supply chain management. A literature survey on the supply chain and various kinds of co-ordination models are also presented.

Three supply chain models, the independent policy model, the synchronized cycles model, and the synchronized cycles model with quantity discounts are introduced in the Chapter 2. Assumptions and notations used in the thesis are also presented in this chapter. Chapter 2 also states the conditions that a quantity discount model has to satisfy.

Chapter 3, 4 and 5 present three quantity discount models respectively. All three models are capable in obtaining a more coordinated results, all parties in the coordination can benefit when compared with his independent policy. A reasonable and necessary discount is offered by the vendor to motivate the buyers to change their policies so as to allow the savings from coordination to be achieved and each buyer is better off.

Chapter 6 investigates how the credit policies can help the synchronized cycles model to achieve the coordination. Credit policies may be applied as an alternative to quantity discounts. This chapter develops an algorithm to minimize the total relevant cost of the coordinated system and also an equitable profit-sharing scheme which depends on the minimum of the system surplus with different delay payments. No matter what is the situation regarding the vendor's or buyer's capital cost structure, the synchronized cycles model with delay periods can reduce both the vendor's and each buyer's cost when compared with his independent cost. The trade credit policy is a good mechanism to make the allocation of the system surplus between vendor and buyers in the coordination, particularly when the majority of buyers' capital cost are larger than vendor's capital cost.

Chapter 7 proposes a coordination model by synchronizing ordering and production cycles with cost sharing which is based on the buyers' inventory holding costs. In the numerical experiments, while ensuring that all the buyers are not worse off, the vendor still can have a substantial saving by synchronizing the ordering cycles of the buyers, and more importantly the total system cost is also reduced. Most significant in this cost sharing policy is that the vendor does not need to know information of the buyers' cost structures.

Chapter 8 summarizes the thesis and suggests possible future research opportunities arising from the results of the thesis. This chapter also investigates how the vendor's cost information, e.g. different values of set-up cost and ordering and shipping cost, would affect the performance of the coordination models with quantity discounts.

1.3 Literature Review

1.3.1 Introduction

The classical inventory theory, the economic ordering quantity (EOQ) policy has been widely studied. Goyal (1977) pointed out that in a typical industrial purchasing situation, a buyer's ordering cost per order is usually smaller than the supplier's setup cost per production run. As a consequence, the adoption of the buyer's optimal ordering policy places the vendor at a cost disadvantage. By the same token, the adoption of the supplier's independently derived optimal production and supply policy is disadvantageous from the buyers' perspectives. Many researchers, beginning in 1970's, started to study models of co-ordination which perform better than the individual independent policy in terms of the total system cost. Effective coordination plays an important role in the successful operation of modern manufacturing and distribution systems. To achieve effective coordination between the supplier and the buyers is both a current managerial concern and an important research issue.

Coordination models have been categorized as follows:

(i)The models that maximize vendor's/supplier's net yearly profit by giving some incentives to buyers (quantity discounts). These models are classified as vendor's/supplier's perspective coordination models.

(ii)The models that minimize the total system cost with respect to coordinated lot size or order quantity. These models improve the system savings and need some mechanisms (quantity discount, cost sharing) to allocate the surplus between vendor and buyers. These models are classified as joint buyer and seller/manufacturer perspective coordination models or joint economic lot sizing problem (JELSP).

(iii)A manufacturer and multiple buyers coordination models, a special case of (i) or (ii), which are based on one of the objective functions of the first two categories of models to achieve channel coordination.

(iv)The models with trade credit period based on one of the objective functions of the first two categories. The trade credit period is another mechanism to achieve the channel coordination.

1.3.2 Vendor's/Supplier's Perspective Co-ordination Models

The problem of quantity discount and the efficiency of buyer-seller transactions were studied in 50's. The traditional quantity discount problem assumes that the discount schedule already exists and the vendor has the full information of buyer's cost structure. With the assumption that vendor has all information about the buyer's cost construction, many researchers have studied various co-ordination models with the objective function to maximize vendor's profit. The vendor uses a quantity discount to entice the buyer to change his order from the independent policy computed without the price-discount. The pricing scheme is profitable as long as the total discount offered to the buyers is less than vendor's cost saving.

Monahan (1984) assumed the price of the item as a decision variable. Monahan suggested a policy for a vendor to entice his major customers to increase their order quantity from EOQ, i.e. Q^*, by offering a price discount. Monahan considered this model under a lot-for-lot policy. He showed that the factor K by which the buyer should increase the order quantity (i.e., optimal value of K) is independent of the opportunity cost of holding inventory for both the buyer and the vendor. One important issue here is that when the buyer is exactly compensated for the increase in cost due to a larger order size, the buyer will be indifferent towards increasing his order quantity. The price discount schedule suggested by Monahan's model is equivalent to an all unit discount schedule with only one price break. The ultimate schedule of the model is one which maximizes the supplier's resultant economic gain, with no added cost to the buyer. However, Monahan recognized that this discount plan "earmarks nearly all economic benefits for the vendor" and stated that "other more equitable benefit sharing arrangements could be considered".

Monahan's work obviously opens a significant research direction on lot sizing problems with quantity discount. However, a number of researchers criticized the reasonability of Monahan's following assumptions:

(1) The supplier's production frequency is the same as the buyer's ordering frequency (lot-for-lot policy).
(2) Changes in the buyer's order quantities (or order frequencies) do not affect the supplier's inventory cost.

Lee and Rosenblatt (1986) further generalized Monahan's model by pointing out two deficiencies in his exposition: (i) an additional constraint on the minimum acceptable profit margin for the vendor: since no constraints are imposed on the amount of price discount offered in the model**,** it is possible that the model could generate a scenario in which the amount of price discount given by the vendor exceeds the selling price of the item; (ii) it may not be advisable for the vendor to use lot-for-lot policy when the set up cost of manufacturing is high. They formulated their model by assuming that the cost for the vendor to process the buyer's order is negligible, compared to the vendor's set up cost. However, when this is not the case, then the processing cost of the buyer's orders should be included. They allowed the vendor to purchase an integer multiple (k) of the buyer's order quantity (KQ), where (k) is a positive integer, and maximize the vendor's yearly net profit subject to the constraint on the discount amount offered to the buyer. The authors also develop an efficient algorithm to determine the values of k and K. They use the same price discount d_k as in Monahan's model.

Joglekar (1988) pointed out that Monahan's (1984) assumption (1) above is likely to be applicable only when: (i) the buyer is willing to accept a long lead time in getting his orders filled, (ii) the buyer's order frequency is periodic and is known to the vendor in such a way that he manages to schedule the production of the buyer's order quantity, (iii) the vendor has substantial unused capacity which can be used to schedule the production of the buyer's order without disturbing the schedules of any other items already in the master production schedule, and (iv) the vendor's inventory carrying costs are so high and his manufacturing set-up costs are so low that the use of a production lot size greater than the buyer's order size is economically inferior to the policy of lot-for-plot production. Joglekar extended Monahan's model, using the optimal production lot size policy and pointed that it is superior to the optimal price discount policy particularly when the setup cost of the manufacturer is substantially larger than the ordering cost of the buyer. He also pointed that it is possible and reasonable for the vendor to use both the optimal production lot size policy and optimal price discount schedules.

While the previous models determined the optimal order size and the optimal price discount maximizing the supplier's profit function, Drezner and Wesolowsky (1989) focused on the optimal price break quantity and the discount price associated with the optimal price break quantity when the supplier deals with a single buyer. The authors also provided a model finding the optimal price break quantity and discount price for the supplier offering a single quantity discount schedule to multiple buyers. Through a search procedure, the model found the combination of the optimal price break quantity and the optimal discount price maximizing the supplier's aggregate profit function. However, the search procedure became exhaustive as the number of buyers increases.

Weng and Wong (1993) developed general all-unit quantity discount models for a single buyer (with constant demand or price-sensitive demand) or multiple buyers. They considered the supplier's profitability with a quantity discount policy. Their analysis provided methods for simultaneously determining the optimal decision policies.

1.3.3 Joint Economic Lot Sizing Problem

With the assumption that the vendor has all information about the buyer's cost structure, some researchers have used quantity discounts as a coordination mechanism to maximize the joint profit of the system or minimize the total channel cost. This part of the literature review focuses on deterministic JELSP. This problem evolved from a simple model with infinite production rate and lot-for-lot assumption (Goyal, 1977), to a general model with a finite production rate and a shipment policy that is not restricted in any way (Hill, 1999).

One of the early works related to JELSP was due to Goyal (1977). He suggested a joint economic lot size model where the objective is to minimize the total relevant costs for both the vendor and the buyer. This model is suitable when a collaborative arrangement between the buyer and the vendor is enforced by some contractual agreement. This situation is not uncommon in organizations which have implemented JIT purchasing. Goyal assumed an infinite replenishment rate for the vendor (i.e., the vendor does not manufacture the items himself but in turn buys it from his vendor) and ignored the effect of a finite production rate in computing his inventory carrying costs. Moreover, he assumed that the inventory holding costs are independent of the price of the item (i.e., the price of the item was assumed fixed). Goyal (1977) suggested a solution to the problem under the assumption of having a lot-for-lot policy for the shipments from the vendor to the buyer. In this policy, each production lot is sent to the buyer as a single shipment. This implies that the entire production lot should be ready before shipment.

Banerjee (1986) relaxed the infinite production rate assumption of Goyal (1977) while retaining his lot-for-lot policy. This study is the one that coined the term JELP. In his model, with a suitable saving allocation scheme, both the vendor and the buyer could be better off.

Goyal (1988) contributed to the efforts of generalizing the problem by relaxing the lot-for-lot policy. He assumed that the production lot is shipped in an integer multiple, denoted by n, of the buyer’s order size, but only after the entire lot has been produced. This provides a lower or equal joint total relevant cost as compared to Banerjee's (1986) model.

An essential assumption in all previous work is that before implementing these models, the vendor must have knowledge of buyer’s annual demand, and the holding and ordering costs governing the buyer’s inventory policy. While buyer’s annual demand can be inferred from past ordering behavior of the buyer, it is difficult to estimate a buyers’ holding and ordering costs unless the buyer is willing to reveal the true values of his cost parameters. There probably is no best method to estimate buyers’ costs accurately.

Lu (1995) considered another circumstance in which the objective function is to minimize the vendor’s total annual cost subject to the maximum cost that the buyer may be prepared to incur. He relaxed the assumption of Goyal (1988) about completing a batch before starting shipments and explored a model which allowed shipments to take place during production. This new model will be suitable when the vendor has an advantage over the buyer in the purchasing negotiation.

He found the optimal solution for the one-vendor one-buyer case and presented a heuristic approach for the one-vendor multi-buyer case.

Weng (1995) pointed out that there are two streams of research on the roles of quantity discounts in channel coordination in previous research. The first considers operating cost as a function of order quantities but treats demand as constants, while the second considers the demand as a decreasing function of the selling price but treats operating costs as constants. Weng presented a general model by considering both channel coordination and operating cost minimization in one single work. This work focused on the control mechanism provided by quantity discounts in channel coordination. With the assumption that the buyer will receive a fixed fraction of the incremental profit, the author has shown that a quantity discount for the buyer along with the franchise fee paid to the supplier is sufficient to induce the buyer to make joint profit maximization. Furthermore, this work showed that quantity discounts alone are not sufficient to guarantee joint profit maximization.

Goyal (1995) used the same numerical examples as Lu (1995) to show that a different shipment policy could give a better solution. This alternative policy involved successive shipments within a production batch increasing by a factor equal to the production rate divided by the demand rate.

The production batch is sent to the buyer in n shipments and the *ith* shipment size to the buyer within a production batch is determined by $q_1(P/D)^{i-1}$, where q_1 is the first shipment size, P/D is vendor's production rate divided by buyer's demand rate. This policy was based on a much earlier idea set out by Goyal (1977) to solve a very similar problem in a slightly different setting. The author formulated the problem and gave the optimal expression for the first shipment size as a function of the number of shipments. Although this involved unequal shipment sizes, there is some intuitive appeal in a policy where all the available stock is shipped out when another shipment is needed and also in providing the buyer with smaller quantities ‘to be getting on with’ during the early stages of a production run. This form of policy can result in a lower joint total cost than the equal shipment size policy.

Hill (1997) further generalized the model of Goyal (1995) by taking the geometric growth factor as a decision variable, of which the equal shipment size policy and Goyal’s (1995) policy represent special cases. He suggested that the *ith* shipment size should be determined by $q_1\lambda^{i-1}$, where $1 \le \lambda \le P/D$. This method is based on exhaustive search for both the growth factor and the number of shipments in certain ranges. It is not surprising that this more general class of policies gives rise to a lower joint total cost solution than do either of the special cases, but this is at the expense of production solutions that are less likely to be of practical interest.

Another simple geometric-then-equal policy that produces good results was suggested in Goyal and Nebebe (2000). They proposed a policy which calls for a small shipment followed by a series of larger and equal-sized shipments. The ratio of the first shipment size to the size of the remaining equal shipments is set to be the production rate divided by the demand rate as was done by Goyal (1995). This policy ensures a quick delivery of the first shipment to the buyer and avoids excessive inventory levels of higher order shipments at the buyer’s end. This method is easy to implement and conceptually simpler than the methods suggested by Goyal (1995) and Hill (1997). This policy tries to exploit the benefit of both the equal size and the geometric policies.

Goyal (2000) suggested a method to improve the solutions obtained by the method given in Hill (1997). In his method, based on first shipment size, the following shipment sizes are increased by the ratio of production rate to demand rate as long as it is feasible to do so. For the remaining shipments, the rest of the production run is equally distributed. The resulting improvement was demonstrated with a small number of experiments. It was unclear whether the improvement was in general significant, or for what kind of problems it is so.

Unlike all the above-mentioned researchers finding the optimal solution from a given structure of policy, Hill (1999) derived the structure of the globally optimal policy of shipments. He showed that the structure of the optimal policy includes shipments increasing in size according to a geometric series followed by equal-sized shipments. He also suggested an exact iterative algorithm for solving the problem. However, Hill's model (1999) was built on the base of assumption that the buyer's inventory holding cost per unit time is always bigger than the vendor's. Although he listed some reasons that such an assumption is reasonable, they are not very intuitively obvious.

Most previous work has been based on the assumption that unit stockholding costs increase as stock moves down the supply chain, but the opposite may sometimes hold. Hill and Omar (2006) revisited JELSP by relaxing an assumption regarding holding costs, which were allowed to decrease down the supply chain. They also pointed an interested research into consignment stock is the splitting of the holding cost into a financial component and a physical component. It is fairly natural to relate the financial component, the cost of capital tied up in stock, to average stock and to relate the physical storage component to the maximum inventory held, reflecting the need to allocate or rent storage space for a production.

Zhou and Wang (2007) developed a more general production-inventory model for a single-vendor-buyer integrated system. This model neither requires the buyer's unit holding cost greater than the vendor's, nor assumes the structure of shipment policy. This model also extends to the situation with shortages permitted. They showed that when the vendor's unit holding cost is greater than the buyer's, the optimal shipment policy consists only of unequal-sized shipments with all successive shipment sizes increasing by a fixed factor equal to the ratio of the production rate to the demand rate. A significant insight observed is that it is more beneficial for the integrated system to make the vendor's holding cost higher than the buyer's, than to make the vendor's holding cost lower than the buyer's if shortages are not permitted to occur otherwise it just the opposite.

Traditional ELS models do not consider inventory capacity constraints, which are embedded in most inventory systems. Deng and Yano (2006) studied an economic lot sizing (ELS) problem with both upper and lower inventory bounds. Bounded ELS models address inventory control problems with time-varying inventory capacity and safety stock constraints. An O(n^2) algorithm is found by using net cumulative demand (NCD) to measure the amount of replenishment requested to fulfill the cumulative demand till the end of the planning horizon. An O(n) algorithm is found for the special case, the bounded ELS problem with non-increasing marginal production cost.

A recent review focusing on the coordination mechanisms between vendor and buyer through quantity discount schemes is presented in Sarmah et al. (2006).

1.3.4 A Manufacturer and Multiple Buyers Coordination Models

Another major research direction considered the case of one vendor multi-buyer supply chains.

Lal and Staelin (1984) addressed the problem of why and how a seller should develop a discount pricing structure even if such a pricing structure does not alter ultimate demand. Their model initially studied one vendor and one group of homogeneous buyers. This was extended to heterogeneous group of buyers, variable ordering and shipping costs and situations where the seller faces numerous groups of buyers, each having different ordering policies.

Kim and Hwang (1988) considered the multiple customers case in determining the discount rate and price break point. The mathematical analysis in the paper is based on the assumption that a seller suppliers a single item to multiple customers and a major benefit for the supplier is directly related to the number of set-ups (without incurring any additional cost to the retailers) when an incremental discount system is implemented. They also assumed that each customer follows an EOQ model when he determines the order size.

Drezner and Wesolowsky (1989) also studied the multiple buyers case. They gave a method for solving the problem when one seller offers a single quantity discount schedule to many buyers. The discount is of the “all quantity” type.

Joglekar and Tharthare (1990) refined the JELP model by relaxing the lot-for-lot assumption, and separated the vendor’s setup cost into two parts, the first is the standard manufacturing setup cost per production run, and the second is vendor’s ordering and handling cost from a buyer. They presented an alternative approach to minimizing the total inventory carrying and ordering costs for both with one vendor many identical buyers and many non-identical buyers: the individually responsible and rational decision (IRRD) approach. An algebraic proof of IRRD’s superiority over JELS is offered in the more general and realistic case of a vendor dealing with n non-identical purchasers with reasonably predictable annual demand but uncertain order quantities and timings.

Banerjee and Burton (1994) developed an integrated production/inventory model: a common cycle co-ordination system in their paper for a single vendor and multiple buyers under deterministic conditions. They have shown that in multiple buyers’ cases, classical economic lot size model may not be able to truly reflect the exact scenario due to discrete vendor inventory depletion. In particular, a stockout may occur, even under deterministic situations, in the absence of an adequate reorder point policy. In their model, they have considered the common replenishment cycle to all buyers, and the supplier's manufacturing cycle time is an integer multiple of it. The results showed that such coordination was more desirable than independent optimization from a system point of view.

Bylka (1999) relaxed the assumption of Banerjee and Burton's (1994) model that vendor demand rate was approximately constant. He considered the demand as a periodic sequence and each buyer used his own replenishment policy. In his study, an optimal vendor production schedule is determined.

When a supplier purchases and delivers an item to several buyers it could use a common replenishment epochs (CRE) strategy and provide a discount to the participants. Viswanathan and Piplani (2001) first studied this model and suggested that the supplier offer a discount that is the maximum of the discount required by all buyers to participate. The demand faced by the buyers occurs at a constant rate. Before replenishment co-ordination, replenishment orders from the buyers to the vendor could be placed at any arbitrary point in time. Each buyer places order for replenishment at equidistant intervals (as per the lot size formula) such that their own inventory costs minimized. The orders from different buyers, however, need not occur at the same epochs. Under the proposed CRE strategy, the vendor specifies common replenishment epochs and requires all the buyers to replenish only at those epochs. Having to place replenishment orders only at specific points in time will obviously increase the cost for the buyer(s). Therefore, in order to entice the buyers to accept this strategy, the vendor offers a price discount. They would agree to replenish at the same epochs suggested by the vendor only if the price discount offered helped them to compensate for any increase in inventory costs, and possibly provide additional savings. The minimum discount required by each buyer is calculated and a price discount equal to the maximum of the minimum discounts for all the buyers is offered to the buyers to adopt the CRE scheme.

Mishra (2004) suggested a modified CRE strategy, where the vendor offers only a “selective” discount. The “selective” discount is the maximum of the minimum discount required by a subset of all the buyers. This may result in some of the buyers dropping out of the scheme (and ordering as per the pre-CRE scenario) as their required minimum discount may be higher than the discount being offered. Mishra showed that this strategy results in reduced costs for the vendor in many scenarios.

A more recent study that assumes the multi-buyer case is Yau and Chiou (2004). They considered an integrated supply chain model in which one vendor supplies items for the demand of multiple buyers. The objective of this model is to minimize the vendor’s total annual cost subject to the maximum cost that the buyer may be prepared to incur. They explored the optimality structure of this integrated model and assert that the optimal cost curve is piece-wise convex. The authors have developed a very efficient search algorithm (its run time is extremely short) to solve the optimal cost curve which turned out to be piece-wise convex.

A new methodology to obtain the joint economic lot size in the case where multiple buyers are demanding one type of item from a single vendor was presented in Siajadi et al. (2006). The production is organized in such a way that the first shipment for each buyer is done in a sequence. Following this sequence, the first delivery starts from the first buyer followed by the second, the third and so on. The duration from one delivery to the next is fixed for each buyer, with equal cycle times for buyers and the vendor. `

Deng and Yano (2006) generalized the single-product, discrete-time model to consider capacity constraints. The manufacturer faces a known demand curve in each period, which is assumed to be continuously differentiable and strictly decreasing with respect to price. The demand curves may vary from period to period. They investigated the problem of joint production and pricing decisions under capacity constraints in a discrete-time framework with setup costs. They characterize properties of the optimal solution, considering cases with constant and time-varying capacity, and with and without speculative motive for holding inventory. They showed that, counter to intuition, optimal prices may increase as the capacity increases, even when capacity is constant over the horizon. They also showed that increases in capacity do not always exhibit diminishing marginal returns. The results also suggested that firms with seasonal demand and tight capacity constraints should be more aggressive in setting prices to manage their demands than what is typically done in practice. They finally discussed how a decision maker can use the procedure as an aid in solving multi-product versions of the problem.

Chu and Leon (2007) considered the problem of coordinating a single-vendor multi-buyer inventory system when there are privacy restrictions in the information required to solve the problem. The objective function and cost parameters of each facility are regarded as private information that no other facilities in the system have access to. Moreover, each facility is responsible to specify its own replenishment policy. The objective is to minimize the total average setup/ordering and inventory related cost. Solution methodologies under private and global information are developed to find two types of nested power-of-two stationary policies. The first policy assumes all the buyers must replenish simultaneously. The second policy is a more general case where the common replenishment assumption is relaxed. A simple form of information exchange is uncovered that allows the solution methodologies for private and global information to yield the same results. The experimental results suggest that the performance of the proposed heuristics is comparable to or better than an existing method.

Chan and Kingsman (2005, 2007) proposed a coordinated single-vendor multi-buyer supply chain model by synchronizing delivery and production cycles. The synchronization is achieved by scheduling the actual delivery days of the buyers and coordinating them with the vendor’s production cycle whilst allowing the buyers to choose their own lot sizes and order cycles. A mathematical model was developed and analyzed. The results of the numerical examples show that the synchronized-cycles policy works better than an independent optimization and restricts buyers to adopt a common order cycle. They also pointed out that it is interesting in examining what price and quantity discounts are needed from the vendor to motivate the buyers to change their policies to allow the savings from coordination to be achieved.

1.3.5 Trade Credit Policy

The lot-sizing problem is based on two conflicting cost functions, which are the order/set-up and the inventory holding costs. The earliest lot-sizing problem is the economic order quantity (EOQ) model. Ever since its introduction, it has been subjected to several extensions. The EOQ model under permissible delay in payment is among these extensions found in the literature.

The traditional economic order quantity (EOQ) model assumes that the retailer's capitals are unrestricting and must be paid for the items as soon as the items were received. However, this may not be true. In practice, the supplier will offer the retailer a delay period, that is, *the trade credit period*, in paying for the amount of purchasing cost. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged by the vendor if the payment is not settled by the end of the trade credit period. In a real world, the supplier often makes use of this policy to promote his commodities. Suppliers often resort to the practice of offering extended payment privileges to a retailer which is quite prevalent in some industries today. Such credit policies may be applied as an alternative to price discounts to induce larger orders, because such policies are not thought to provoke competitors to reduce their prices and thus introduce lasting price reductions, or because such policies are traditional in the firm's industry. From the practical point of view, a few pharmaceutical companies and agricultural machinery manufacturers offer a larger credit period for a larger amount of purchase rather than giving some discount on unit price.

From the buyer's perspective, Goyal (1985) is the first person who developed the EOQ model under conditions of a permissible delay in payments.

Jamal et al. (1997, 2000) considered the lot-sizing problem for a deteriorating item under conditions of fixed demand and permissible delay in payment. Wang and Shinn (1997) considered the pricing and the lot-sizing problem under conditions of price sensitive demand and permissible delay in payment. They show that when the end demand is price sensitive, the lot size for the buyer is not invariant to the length of the credit period.

Goyal (1985) is frequently cited when the inventory systems under conditions of permissible delay in payments are discussed. An implicit assumption of Goyal is that the items are obtained from an outside supplier and the entire lot size is delivered at the same time, that means that the replenishment rate is infinite. When the replenishment rate is much larger than the demand rate, this assumption is probably satisfactory as an approximation. However, if the rate of replenishment is comparable to the rate of demand, Goyal's analysis needs to be modified to reflect this situation. Chung and Huang (2003) extended Goyal's model to the case that all items are replenished at a finite rate.

Abad and Jaggi (2003) adopted similar assumption of a fixed length trade credit period, or in other words, seller specified. They have formulated a model of seller-buyer relationship when end demand is price sensitive, which views both the unit price the seller charges and the length of the credit period as decision variables, since they both influence the end demand for the product. However, in Abad and Jaggi (2003) it is a decision variable in the seller's model. In practice the credit period is set by the seller. The primary purpose for the seller in offering trade credit to the buyer is the stimulation of the end demand for the product. Offering trade credit will be economic for the seller if the additional profit generated by the increased sales is sufficient to compensate for the opportunity cost incurred. On the other side, the buyer can take advantage of a credit period and reduce his costs and increase his profit. The trade credit can be beneficial to both the seller and the buyer in certain circumstances.

Chung and Liao (2004) studied the problem of determining the EOQ for exponentially deteriorating items under the conditions of permissible delay in payments. This delay in payments depends on the quantity ordered. That is, when the order quantity is less than that which the delay in payments is permitted, the payment for the product must be made immediately, otherwise, the fixed trade credit period is permitted.

In all these articles described above, the EOQ is invariant to the length of the permissible delay period, which was assumed to be of a fixed length.

There have been not many published work that investigated coordinating orders in a two-level supply chain with delay in payments. Contrary to earlier works that investigate the EOQ model under the assumption of permissible delay in payments, Jaber and Osman (2006) assumed that the length of the permissible delay period is a supply chain decision variable. That is, both the seller and the buyer have to agree on such a period so that their individual costs are either less, or invariant.

Chapter 2

Quantity Discount Models

2.1 Introduction

Chan and Kingsman (2007) considered the situation of one vendor and multiple buyers under the deterministic demand. They also considered the vendor as a manufacturer and let the buyer choose his ordering cycle which must be an integer factor of vendor's manufacturing cycle. From the viewpoint of the system cost, the results showed that their synchronized model works better than the independent policy. However the division of the surplus between parties of the system has not yet been discussed. This research analyses what quantity discounts are needed from the vendor to motivate the buyers to change their policies so as to allow the saving from coordination to be achieved.

In the majority of models of quantity discount**,** one of the limitations is that holding cost of the buyer is considered to be independent of purchase price. In this research, the buyer's holding cost h_i is expressed as a percentage of the capital, so buyer's holding cost is considered to be dependent on purchasing price. By using the quantity discount, this will make the buyers' holding cost more complicated in the model, and make it more difficult to find the suitable discounted price $p_d(i)$ for each buyer. However it is quite necessary to express the buyer's holding cost as a percentage of the capital, particularly in the problem of quantity discounts. The discounts in the purchase price can reduce the buyer's holding cost to some extent, and so will reduce the system cost when compared with the synchronized model without quantity discount.

Most of the previous models are developed where the supplier offers an all unit quantity discount with a single price break point. In Chapter 3, 4 and 5 three models of quantity discounts are proposed in the research to ascertain when each buyer can be better off with the new ordering cycle decided by the vendor. The discounted price depends on buyer's new ordering cycle $k_i T$ and his optimal EOQ T_i^*, and this makes the discounts different for each buyer. Unlike the all-unit quantity discount, where the total discounts of the buyer only depend on the buyer's ordering quantity, the total discounts in this research depend rather on the buyer's contribution in the co-ordination system. That is, if a buyer's new ordering cycle $k_i T$ in co-ordination is longer than his optimal EOQ T_i^*, the vendor will give the buyer a reasonable and necessary discount. A necessary discount means that with this discount the buyer will not increase his cost when compared with his independent policy. A reasonable discount depends on a buyer's contribution in the co-ordiantion system, which are different among buyers.

Two objective functions are also proposed in this research. The first objective function is to minimize vendor's cost, and the second objective function is to minimize the total system cost.

2.2 Independent policy

We assume that each of the n buyers faces a deterministic demand at rate d_i per unit time, incurs an ordering cost A_i each time it places an order and incurs an inventory holding cost h_i (expressed as a percentage of the capital per unit time). If the buyers and the supplier operate independently then each buyer will order a quantity Q_i at time intervals of T_i units apart, which are determined only on the basis of the costs and demands of the ith buyer. The total costs per unit time for the ith buyer can thus be expressed as:

$$TCB\_ind(i) = \frac{A_i}{T_i} + \frac{h_i d_i T_i p_s}{2} + d_i p_s \qquad (2.2.1)$$

where p_s is the selling price of the item, and $Q_i = d_i T_i$. This is the simple standard EOQ model so that the costs per unit time are minimized when

$$T_i^* = \sqrt{\frac{2A_i}{h_i d_i p_s}} \text{ and } TCB_i^* = \sqrt{2A_i h_i d_i p_s} + d_i p_s \qquad (2.2.2)$$

The vendor is faced with orders from each of the n buyers based on demand rates of $d_1, d_2, ..., d_n$ per unit time. Thus the vendor has to satisfy a demand that occurs at an average rate of $D = \sum_{i=1}^{n} d_i$ per unit time. The vendor produces new items at a rate P per unit time. We assume that the vendor incurs a set-up cost S_v for each production run and incurs a holding cost of h per unit held per unit time. If the vendor operates independently of the buyers and aims to satisfy the average demand rate D per unit time, then we have the simple EBQ model where the vendor starts a production run every T_v units of time and produces a total lot size of Q_v, where $Q_v = DT_v$. The costs per unit time for the vendor are given by

$$TCV\_ind = \frac{S_v}{T_v} + \frac{hDT_v}{2}(1-\frac{D}{P}) + \sum_{i=1}^{n}\frac{C_i}{T_i} + p_m D \qquad (2.2.3)$$

where the p_m is the manufacturing cost per unit for the vendor, C_i is ordering and shipping cost for the vendor. The third term in Eq. (2.2.3) covers the order processing and fixed shipment costs in supplying the order quantities $Q_i = d_i T_i$ to each of the buyers. These depend only on the T_i, which are determined by the buyers and outside the control of the vendor, so they do not affect the determination of T_v and Q_v for the vendor. The fourth term of Eq.(2.2.3) is the total manufacturing cost for the vendor.

The above is the standard inventory theory, but details can be found in Banerjee and Burton (1994).

The above model for the vendor assumes that the vendor actually occur as aggregated orders $Q_1, Q_2, ..., Q_n$. Therefore, the above model for the vendor cannot guarantee that there will never be any stock outs, i.e. failures to meet the buyers' demands on time. The maximum demand that can occur at any time is when all buyers require a delivery order at the same time. So to ensure that all demands are satisfied on time, then the vendor should not have less than $Q_1 + Q_2 + ... + Q_n$ in stock at the time the vendor starts a new production run. This will be the case for instantaneous delivery. This quantity $Q_1 + Q_2 + ... + Q_n$ becomes the re-order level. Thus an extra term $h\sum_{i=1}^{n} Q_i$ needs to be added to Eq. (2.3.3) to give the true costs per unit time for the vendor if the vendor is to have zero stock outs. If the vendor starts a production run with a stock level less than $Q_1 + Q_2 + ... + Q_n$ then stock outs are may occur. Hence,

$$TCV\_ind = \frac{S_v}{T_v} + \frac{hDT_v}{2}(1 - \frac{D}{P}) + \sum_{i=1}^{n} \frac{C_i}{T_i} + p_m D + h\sum_{i=1}^{n} Q_i \qquad (2.2.4)$$

Optimization of vendor’s total cost (Eq. (2.2.4)) yields the simple EBQ model where the costs of the vendor per unit time are minimized by

$$T_v^* = \sqrt{\frac{2S_v}{hD(1 - \frac{D}{P})}} \qquad (2.2.5)$$

2.3 The Synchronized Cycles Model

2.3.1 Introduction

Clearly, in the previous situation where the vendor and the buyers are operating independently, the vendor needs to carry a large stock of items to satisfy all demands on time, or the buyers will have to suffer stockouts and late deliveries. Co-ordinating the timing of deliveries of the buyers with the production policy of the vendor may enable the stock needed in the system to avoid stockouts to be reduced. Banerjee and Burton (1994) and others proposed that the buyers all adopt a common order cycle of placing orders every T time units apart. In order to meet these scheduled demands the vendor will have to use a production cycle that is some integer multiple of T, say NT.

However, forcing all buyers to use the same common cycle time T will be costly for both the small buyers, forced to carry higher stocks than they would wish, and the larger buyers, forced to place more orders than they would wish. It would be more economical to have small cycle times for the low demand buyers and large cycle times for the high demand buyers. Chan and Kingsman (2007) proposed that this can be achieved by having some basic cycle time, T, and insisting that each buyer use an integer multiple of that basic cycle time, say k_iT for the *ith* buyer. Let the vendor production cycle time be denoted by NT, where N is also an integer. The idea is closely akin to the Extended Basic Period approach for the Economic Lot Scheduling Problem introduced by Haessler (1979).

For simplicity, Chan and Kingsman (2007) assumed that delivery to the buyers is instantaneous, or more exactly that buyers' orders are received and deducted from the vendor's inventory at regular interval T apart. The result of the co-ordination will be a set of demands $D_1, D_2, ..., D_N$ over the NT periods of the vendor production cycle, where each demand is some subset of the buyers' order quantities. To determine the vendor's stock holding cost, the model first needs to consider how to meet these demands $D_1, D_2, ..., D_N$. If two buyers order every two periods, they could be allocated both to periods 1,3,5,… or to periods 2,4,6,… or allocate one buyer to periods 1,3,5,… and the other buyer to 2,4,6,…

2.3.2 The synchronized cycles model of the coordinated system

In addition to the Chan and Kingsman (2007) synchronized cycles model, this research also consider the purchasing costs of buyers and the manufacturing cost for the vendor. Furthermore the models in this research assume that each buyer places its orders as early as possible in each of its order cycles. Then the total relevant costs of the vendor and buyers are:

Buyers' ordering cost= $\sum_{i=1}^{n} \frac{A_i}{k_i T}$

Buyers' holding cost= $\frac{1}{2} \sum_{i=1}^{n} h_i d_i k_i T p_s$

Buyers' purchasing cost= $p_s D = \sum_{i=1}^{n} p_s d_i$

Vendor's setup cost= $\frac{S_v}{NT}$

Vendor's shipping cost= $\sum_{i=1}^{n} \frac{C_i}{k_i T}$

Vendor's holding cost= $[\frac{hD}{2} - \frac{hD^2}{2P}]NT + \sum_{i=1}^{n} d_i h(\frac{D}{P} - 0.5)k_i T$

Vendor's manufacturing cost= $p_m D$

The total relevant cost of the vendor (TCV_cor) and the total relevant cost of the system (*TCS_cor*) in the co-ordinated model without quantity discount can be written as

$$TCV\_cor = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} + p_m D$$

$$\begin{aligned} TCS\_cor = &\{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i}{k_i T} \\ &+ d_i[\frac{hD}{P} - 0.5(h - h_i p_s)]k_i T\} + (p_s + p_m)D \end{aligned} \quad (2.3.2)$$

Some numerical experiments have been carried out to investigate the performance of the synchronized cycles model to see how allowing each buyer to have its own individual cycle rather than following a common cycle performs. Three examples are used for the purpose of illustration, and the data are shown in Appendix 1. By using the algorithm in Chan and Kingsman (2007), the results of the examples are found as follows：

(In Table 2.3.2.1, Table 2.3.2.2 and Table 2.3.2.3), the first three columns are independent costs of buyers (*TCBS_ind*), vendor (*TCV_ind)* and system (*TCS_ind*), and the last three columns are coordination cost in the synchronized cycles model for buyers (*TCBS_cor)*, vendor (*TCV_cor)* and system (*TCS_cor)*. Results show that regardless numbers of buyers, the synchronized cycles model works well in the coordination. It can significantly reduce the total system cost when compared to the independent policy. However, the cost to buyer is significantly increased when compared with his independent policy.

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor
0.1	123.95	115.62	239.58	128.54	97.65	226.19
0.2	123.95	114.97	238.92	128.33	98.50	226.83
0.3	123.95	114.27	238.23	126.37	100.87	227.25
0.4	123.95	113.53	237.48	126.37	101.06	227.43
0.5	123.95	112.71	236.67	126.37	101.24	227.61
0.6	123.95	111.81	235.77	125.37	102.23	227.60
0.7	123.95	110.79	234.75	125.37	101.94	227.31
0.8	123.95	109.58	233.54	125.37	101.65	227.02
0.9	123.95	108.01	231.96	125.01	101.14	226.15

Table 2.3.2.1 Example 1 Synchronized Model when $p_m = \$1.5$, $p_s = \$2$

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor
0.1	2378.20	1802.90	4181.20	2422.70	1678.80	4101.50
0.2	2378.20	1796.20	4174.40	2422.80	1673.20	4096.00
0.3	2378.20	1788.90	4167.10	2421.70	1667.50	4089.20
0.4	2378.20	1781.20	4159.40	2421.70	1660.70	4082.40
0.5	2378.20	1772.70	4150.90	2421.70	1653.90	4075.60
0.6	2378.20	1763.40	4141.60	2422.30	1645.00	4067.20
0.7	2378.20	1752.80	4131.00	2422.20	1635.40	4057.60
0.8	2378.20	1740.20	4118.40	2422.20	1623.70	4045.90

Table 2.3.2.2 Example 2 Synchronized Model when $p_m = \$1.5$, $p_s = \$2$

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor
0.1	6703.7	4936.7	11640	6830.5	4680.5	11511
0.2	6703.7	4921.7	11625	6830.5	4668.7	11499
0.3	6703.7	4905.7	11609	6830.2	4654.5	11485
0.4	6703.7	4888.6	11592	6830.2	4638.1	11468
0.5	6703.7	4869.9	11574	6830.2	4621.8	11452
0.6	6703.7	4849.3	11553	6830.3	4602.7	11433
0.7	6703.7	4825.9	11530	6829.2	4582.7	11412
0.8	6703.7	4798.2	11502	6828.7	4556.8	11385
0.9	6703.7	4762	11466	6830.1	4520.9	11351

Table 2.3.2.3 Example 3 Synchronized Model when $p_m = \$1.5$, $p_s = \$2$

2.4 Quantity Discount Model

2.4.1 Introduction

The results in the previous section show that co-ordination by synchronized cycles leads to savings in the system when compared with the independent policy. For a win-win situation, both the parties of a supply chain should benefit in the co-ordination exercise. However, in the synchronized cycles model, the cost of each buyer is increased while there is a significant reduction in the vendor's and system's cost. This appears to be a general result that applies in all analyses of co-ordinated ordering, inventory and production planning models. The vendor is motivated to seek to co-ordinate decisions in the whole supply chain but the buyers are not. Hence, the interest is in examining what price and quantity discount are needed from the vendor to motivate the buyers to change their policies to allow the savings from co-ordination to be achieved.

2.4.2 The Costs of the Co-ordinated System with Quantity Discount

Buyers' ordering cost= $\sum_{i=1}^{n}\frac{A_i}{k_iT}$

Buyers' holding cost= $\frac{1}{2}\sum_{i=1}^{n}h_i d_i k_i T p_d(i)$

Buyers' purchasing cost= $\sum_{i=1}^{n}p_d(i)d_i = p_s D - \sum_{i=1}^{n}(p_s - p_d(i))d_i$

where $p_s D$ is buyers' purchasing cost without quantity discount, and $\sum_{i=1}^{n}(p_s - p_d(i))d_i$ is buyers' total received discounts, i.e. buyers' additional gains (savings) from quantity discount. Note that $p_d(i)$ is the discounted price for the *ith* buyer.

Vendor's setup cost= $\frac{S_v}{NT}$

Vendor's shipping cost= $\sum_{i=1}^{n}\frac{C_i}{k_iT}$

Vendor's holding cost= $[\frac{hD}{2}-\frac{hD^2}{2P}]NT + \sum_{i=1}^{n}d_i h(\frac{D}{P}-0.5)k_i T$

Vendor's manufacturing cost= $p_m D$

Vendor's additional loss caused by quantity discount $= \sum_{i=1}^{n}(p_s - p_d(i))d_i$ (i.e. loss in the revenue which equals the buyers' additional savings in purchasing cost)

Therefore, the total buyers' cost (*TCBS_qd*), total vendor's cost (*TCV_qd*), and total system cost (*TCS_qd*) with quantity discount are given by:

$$TCBS\_qd = \sum_{i=1}^{n}\{\frac{A_i}{K_i T} + 0.5 d_i h_i k_i T p_d(i) + d_i p_d(i)\} \tag{2.4.2.1}$$

$$\begin{aligned} TCV\_qd = &\{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} \\ &+ p_m D + \sum_{i=1}^{n}(p_s - p_d(i))d_i \end{aligned} \tag{2.4.2.2}$$

$$\begin{aligned} TCS\_qd = &\{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i}{k_i T} \\ &+ d_i[\frac{hD}{P} - 0.5(h - h_i p_d(i))]k_i T)\} + (p_s + p_m)D \end{aligned} \tag{2.4.2.3}$$

Now, we need to find the suitable discounted prices $p_d(i)$ which satisfy the followings:

1) A reduction in vendor's cost when compared with his cost in the independent policies.

2) A reduction in each buyer's cost when compared with his cost in the independent policies.

If the discounted price $p_d(i)$ can make a reduction in both the vendor's and buyer's costs, then obviously they can reduce the system cost when compared with the independent policy. This is a win-win situation, since both parties of the supply chain can participate in the division of the surplus.

Another important problem is whether the division of the system surplus is reasonable or equitable between the system parties. It is deemed that not only a reasonable division between the vendor and buyers is necessary, but also a reasonable division among buyers.

2.4.3 The Discounted Price from the Buyer's Viewpoint

The vendor offers a discounted price $p_d(i)$ to each buyer to entice him to adopt the co-ordinated model (i.e. buyer will change his ordering cycle from his optimal T_i^* to $k_i T$ in the co-ordinated model), this discounted price $p_d(i)$ will be accepted by each buyer only if it can make the buyer's cost in the co-ordinated model less than or equal to his cost in the independent policy. The discounted price should also be between zero and the original selling price. Therefore, the discounted price from each buyer's viewpoint should satisfy the following conditions:

qd_condition(1) **:** $TCB\_qd(i) \le TCB_i^*$

qd_condition(2) **:** $0 \le p_d(i) \le p_s$

qd_condition(3) **:** $p_d(i) = p_s$ when $k_i T = T_i^*$ (2.4.3.1) where TCB_i^* is buyer's optimal cost in the independent policy, see Eq. (2.2.2), and $TCB\_qd(i)$ is buyer's coordination cost with quantity discount.

If the buyer's co-ordinated ordering cycle k_iT is just his *EOQ* optimal T_i^*, the vendor does not need to offer him a discount to entice him to adopt the co-ordinated model, since the buyer is simply adopting his independent policy. So, the *qd_condition(3)* seems quite reasonable.

Question: The traditional model often uses an all unit quantity discount with a single price break point. Is it really suitable for a single-vendor and multi-buyer situation?

The all unit quantity discount is the pre-determined discount which depends on buyer's quantity of demand. As mentioned before, in the multi-buyer situation, the division of the system surplus should be reasonable among buyers. The buyers which have the same quantity of demand will have the same received discounts in an all unit quantity discount policy. However, a buyer will have an increased holding cost which depends on the buyer's ordering cycle k_iT in the co-ordination. Some buyers may have a large increased holding cost caused by a longer k_iT, while some buyers may have a little increase only. So, a buyer who makes a larger contribution in the co-ordination (since his ordering cycle k_iT is much longer than his *EOQ* T_i^*) has relatively less cost saving when compared with other buyers who have the same demands. It seems that the all unit quantity discount cannot give a reasonable division of the co-ordination surplus among buyers.

This research develops a more reasonable quantity discounts policy which depends on a buyer's ordering cycle k_iT in the co-ordination, and his independent optimal $EOQ\,T_i^*$. It is designed so that the more contribution the buyer makes in the co-ordination (distance between k_iT and T_i^*), the more discounts the buyer could receive from the vendor. So, the buyer will get no discount when $k_iT = T_i^*$.

The total costs of each buyer in the co-ordinated model with the price discount is

$$TCB\_qd(i) = \frac{A_i}{k_iT} + \frac{1}{2}h_i d_i k_i T p_d(i) + d_i p_d(i) \qquad (2.4.3.2)$$

The optimal total costs of each buyer in the independent policy without quantity discount is

$$TCB_i^* = \sqrt{2A_i h_i d_i p_s} + d_i p_s \qquad (2.4.3.3)$$

In chapter 3, 4 and 5, we propose three models with discounted prices $p_d(i)$ which satisfy the *qd_condition(1)* (i.e. Eq. (2.4.3.2) $\le$ (2.4.3.3)) , *qd_condition(2)* (i.e. $0 \le p_d(i) \le p_s$), and *qd_condition(3)* (i.e. $p_d(i) = p_s$ when $k_iT = T_i^*$).

2.4.4 Two Objective Functions

This research proposes two objective functions to be used by the quantity discount models proposed in chapter 3 , 4 and 5. The first objective function is to minimize the vendor's total relevant cost, the second is to minimize the total costs (i.e. vendor and buyers) of the system.

2.4.4.1 Minimizing Vendor's Cost

The total discount offered is the vendor's loss of revenue led by quantity discount. So, this part of loss should be added to the vendor's cost in the model with quantity discount. The model of minimizing the vendor's total relevant cost, including his setup cost, shipping and ordering cost, holding cost, manufacturing cost and his loss led by the quantity discount is given as:

$$\text{Min. } TRC_v = setup + shipping + holding + manufacturing + \sum_{i=1}^{n} discount(i) d_i$$

$$\begin{aligned} &= \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} \\ &+ p_m D + \sum_{i=1}^{n} discount(i) d_i \end{aligned} \qquad (2.4.4.1)$$

where *discount(i)* is buyer *i*'s discount offered by the vendor.

2.4.4.2 Minimizing Total System Cost

The total relevant cost of the system in the co-ordinated model with quantity discount can be written as

$$\begin{aligned} TCS\_qd &= \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i}{k_i T} + d_i[\frac{hD}{P} - 0.5(h - h_i p_d(i))]k_i T)\} \\ &+ (p_s + p_m)D \end{aligned} \qquad (2.4.4.2)$$

Compared with the total system cost Eq. (2.3.2) in the synchronized cycles model without quantity discount, the only difference is that the holding cost of the buyers' is calculated from $p_d(i)$ instead of p_s. The $p_d(i)$ also depends on k_iT which is smaller than p_s, so there will be at least a saving of buyers' holding cost when compared with the synchronized cycles model without quantity discount.

2.5 Algorithm of Quantity Discount Models

No matter the total relevant cost of the vendor when minimizing vendor's cost, or the total system cost in the synchronized model, the expression of the cost consists of two parts.

The first part is $\frac{S_v}{NT} + (\frac{hD}{2} - \frac{hD^2}{2P})NT$ and it is fixed when the value of N is given.

The second part depends on the value of buyer's ordering cycle k_iT, and also depends on N since k_i should be a factor of N.

Full algorithm for the model with quantity discount

Step 1: Set $N = low\_N$ *and* $T = 1$.

Step 2: Use Sub-algorithm to find the optimal k_i^* *values for fixed* N *and* T *in the second part of the total cost.*

Step 3: If N *<365, then set* $N = N + 1$ *and go back to step 2.*

Step 4: Take the N *which gives the least total relevant cost of both the first and second part.*

Sub-algorithm to find optimal K_i for fixed N and T

Step 1: Find all factors of N .

Step 2: Let K_i be all the possible factors of N. Find the best factor for k_i which can minimize the "ith" independent cost in the second part of the total cost.

The above algorithm can also apply to other quantity discount models discussed in Chapter 3, 4, and 5. In each model, N starts with the value low_N which depends on the lower limit value of the constraint in that model.

In Model 1, the constraint is $k_i T \geq 0.5T^*_i$, so $low\_N = \max(0.5T_i^*)$, for i=1 to n.

In Model 2, the constraint is $k_i T \geq T^*_i$, so $low\_N = \max(T_i^*)$, for i=1 to n.

In Model 3, the constraint is $T_i^* \leq k_i T \leq T^*_i + \frac{2}{h_i}$, the lower limit is also T_i^*,

so $low\_N = \max(T_i^*)$, for i=1 to n.

Chapter 3

Quantity Discount Model 1

3.1 Discounted Price

In section 2.4.3, we stated the three conditions which the discounted price has to satisfy from the viewpoint of each buyer. In this chapter, we present the first model of quantity discount and the results show that the discounted price can help vendor to achieve the co-ordination and not increasing buyer's cost when compared with his independent cost.

The discounted price $p_d(i)$ given to each buyer should be less than the original selling price p_s (by *qd_condition (2)*). This means that each buyer should always have a saving in the purchasing cost in the co-ordinated model with the quantity discount when compared with the independent policy. We ignore the part of the purchasing cost in both Eq. (2.4.3.2) and (2.4.3.3), and try to find the discounted price $p_d(i)$ to make the remaining terms be equal in Eq. (2.4.3.2) and (2.4.3.3), i.e.

$$\frac{A_i}{k_i T} + \frac{1}{2} h_i d_i k_i T p_d(i) = \sqrt{2 A_i h_i d_i p_s} \tag{3.1.1}$$

This gives that the discounted price for each buyer is

$$p_d(i) = p_s - p_s(1-\frac{T_i^*}{k_iT})^2 \quad \text{and} \quad discount(i) = p_s(1-\frac{T_i^*}{k_iT})^2 \qquad (3.1.2)$$

The proof of the Eq.(3.1.2) is shown below:

$$\frac{A_i}{k_iT} + \frac{1}{2}h_i d_i k_i T p_d(i) = \sqrt{2A_i h_i d_i p_s}$$

$$\Rightarrow \quad p_d(i) = 2(\sqrt{2A_i h_i d_i p_s} - \frac{A_i}{k_iT})/(h_i d_i k_i T)$$

$$= \frac{2(T_i^* h_i d_i p_s - \frac{A_i}{k_iT})}{h_i d_i k_i T}$$

$$= \frac{2T_i^* p_s}{k_iT} - \frac{2A_i}{h_i d_i (k_iT)^2}$$

$$= p_s(\frac{2T_i^*}{k_iT} - (\frac{T_i^*}{k_iT})^2)$$

$$= p_s - p_s(1-\frac{T_i^*}{k_iT})^2$$

with $T_i^* = \sqrt{\frac{2A_i}{h_i d_i p_s}}$.

The discounted price in Eq.(3.1.2) is obviously less than p_s and it will be larger than zero if $k_iT \geq 0.5T_i^*$. So, by using this discounted price (Eq. (3.1.2)) in the co-ordinated model with the constraint $k_iT \geq 0.5T_i^*$, it ascertains that each buyer can reduce his cost when compared with the independent policy. That is, each buyer has a saving in the purchasing cost with the quantity discount, while the sum of his ordering cost and holding cost is just the same as that of his independent policy. Moreover, the part of the discount $discount(i) = p_s(1-\frac{T_i^*}{k_iT})^2$ will be zero when $k_iT = T_i^*$. Each buyer will have a total cost savings when compared with the independent policy as:

$$p_s(1-\frac{T_i^*}{k_iT})^2 d_i\,. \qquad (3.1.3)$$

This is the benefit each buyer can get from the co-ordinated model. It seems quite reasonable and equitable, since each buyer will get more benefit if his co-ordinated ordering cycle k_iT is farther away from T_i^*.

3.2 Results of Objective Function 1: Minimizing Vendor's Cost

3.2.1 Results

The discounted price in first quantity discounts model is:

$$p_d(i) = p_s - p_s(1-\frac{T_i^*}{k_iT})^2 \text{ where } k_iT \geq 0.5T_i^*$$

The total discounts which vendor offers to buyers is :

$$\sum_{i=1}^{n} discount(i)d_i = \sum_{i=1}^{n} p_s(1-\frac{T_i^*}{k_iT})^2 d_i$$

The model of minimizing vendor's cost is given as:

$$\text{Min. } TRC_v = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} + p_m D + \sum_{i=1}^{n} p_s (1 - \frac{T_i^*}{k_i T})^2 d_i \quad (3.2.1)$$

with $p_d(i) = p_s - p_s(1 - \frac{T_i^*}{k_i T})^2$,

subject to: $k_i T \geq 0.5 T_i^*$

The summarized results of Example 1 (Table 3.2.1.1) show that the total system cost by quantity discount TCS_qd is always smaller than that in the independent case when D/P increases from 0.1 to 0.9, and both the vendor and each buyer reduce their cost when compared with their independent cost.

The summarized results of Example 2 and 3 (Table 3.2.1.2, Table 3.2.1.3) show that the vendor offers too much of a discount to each buyer than he could afford. In order to control the discounts offered to each buyer, the factor $q, (0 < q \leq 1)$ is added to the part of the discount in the discounted price $p_d(i)$. This research proposes two methods: 1) "q factor" where q is a constant for all buyers, and 2) "q_i factor" where q_i is a variable for each buyer.

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_ind	TCV_ind	TCS_ind
0.10	123.36	108.75	232.11	123.95	115.62	239.58
0.20	123.36	108.55	231.91	123.95	114.97	238.92
0.30	123.36	108.35	231.71	123.95	114.27	238.23
0.40	123.36	108.15	231.51	123.95	113.53	237.48
0.50	123.36	107.95	231.31	123.95	112.71	236.67
0.60	123.36	107.75	231.11	123.95	111.81	235.77
0.70	123.36	107.04	230.39	123.95	110.79	234.75
0.80	123.36	106.14	229.50	123.95	109.58	233.54
0.90	123.36	105.07	228.43	123.95	108.01	231.96

Table 3.2.1.1 Example 1 Model 1 when $p_m = \$1.5$, $p_s = \$2$

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_ind	TCV_ind	TCS_ind
0.10	2267.60	1828.70	4096.30	2378.20	1802.90	4181.20
0.20	2267.60	1821.70	4089.30	2378.20	1796.20	4174.40
0.30	2267.60	1814.80	4082.30	2378.20	1788.90	4167.10
0.40	2267.60	1807.80	4075.30	2378.20	1781.20	4159.40
0.50	2267.60	1800.80	4068.40	2378.20	1772.70	4150.90
0.60	2267.60	1793.90	4061.40	2378.20	1763.40	4141.60
0.70	2262.90	1785.00	4048.00	2378.20	1752.80	4131.00
0.80	2267.60	1770.60	4038.10	2378.20	1740.20	4118.40
0.90	2267.60	1755.30	4022.80	2378.20	1723.80	4102.00

Table 3.2.1.2 Example 2 Model 1 when $p_m = \$1.5$, $p_s = \$2$

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_ind	TCV_ind	TCS_ind
0.10	6394.50	5107.10	11502.00	6703.70	49367.00	11640.00
0.20	6394.50	5095.00	11489.00	6703.70	49217.00	11625.00
0.30	6394.50	5080.00	11475.00	6703.70	49057.00	11609.00
0.40	6394.50	5063.30	11458.00	6703.70	48886.00	11592.00
0.50	6394.50	5046.60	11441.00	6703.70	48699.00	11574.00
0.60	6393.40	5027.30	11421.00	6703.70	48493.00	11553.00
0.70	6394.50	5006.00	11401.00	6703.70	48259.00	11530.00
0.80	6394.90	4979.40	11374.00	6703.70	47982.00	11502.00
0.90	6393.60	4944.60	11338.00	6703.70	47620.00	11466.00

Table 3.2.1.3 Example 3 Model 1 when $p_m = \$1.5$, $p_s = \$2$

3.2.2 Modified Model with "q Factor"

The q is added to the part of the discount, so Eq. (3.2.1) becomes as:

$$\begin{aligned}\text{Min. } TRC_V &= \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i}{k_iT} + d_i h(\frac{D}{P} - 0.5)k_iT\} \\ &\quad + p_m D + \sum_{i=1}^{n} qp_s(1 - \frac{T_i^*}{k_iT})^2 d_i\end{aligned} \tag{3.2.2}$$

with $p_d(i) = p_s - qp_s(1 - \frac{T_i^*}{k_iT})^2$,

subject to: $0 \le q \le 1$ and $k_iT \ge 0.5T_i^*$

Starting with q=1 (see Appendix 2 "Example 2 Model 1 "q_factor" Minimizing Vendor's cost" for details), the results show that the vendor's cost is larger than his independent cost, while the buyers' cost is much smaller than his independent cost.

When q decreases from 1 to 0.1, the total cost of all buyers *TCBS_qd* is increased while the vendor's cost *TCV_qd* is decreased. The optimal value of q is approximately 0.7 in example 2, the point at which each buyer and the vendor reduces his cost when compared with the independent case. The results of q=0.7 are shown as below:

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	Inc_V	Inc_S
0.1	2307	1795.3	4102.4	2.99	0.42	1.88
0.2	2307	1788.4	4095.4	2.99	0.43	1.89
0.3	2307	1781.4	4088.4	2.99	0.42	1.89
0.4	2307	1774.4	4081.5	2.99	0.38	1.87
0.5	2307	1767.5	4074.5	2.99	0.3	1.84
0.6	2307	1760.5	4067.5	2.99	0.16	1.79
0.7	2304.4	1750.4	4054.9	3.1	0.13	1.84
0.8	2307	1737.2	4044.3	2.99	0.17	1.8
0.9	2307	1721.9	4029	2.99	0.11	1.78

Table3.2.2.1 Example2 Modified Model 1 when q=0.7

In Table 3.2.2.1, inc_BS, inc_V and inc_S are the percentage cost savings for the buyers, vendor and system respectively, detailed calculation can be found on page 60.

However in example 3, it is impossible to find the constant value q for all the buyers to let both vendor's and buyers' cost be reduced. See Appendix 3 "Example 3 Model 1 "*q*_factor" Minimizing Vendor's Cost" for details.

3.2.3 Modified Model with "$q(i)$ Factor"

In Model 1 when vendor's cost is minimized, both examples 2 and 3 have the problem that the vendor's cost is larger than his independent cost. Although the "q factor" method proposed can control the discounts of the buyer, it cannot guarantee that both the vendor and the buyer can be benefited by the coordination, and it may be difficult to find the constant value q for all the buyers. This section presents the "q_i" factor such that q_i is different for each buyer.

In the Chan and Kingsman (2007) synchronized model without quantity discount, the co-ordinated system has the cost savings defined as $\pi_{cor} = TCS\_ind - TCS\_cor$, the vendor has the cost savings defined as $\pi_{v\_cor} = TCV\_ind - TCV\_cor$, and the buyers has the cost saving defined as $\pi_{BS\_cor} = TCBS\_ind - TCBS\_cor$. In their model, the vendor and the system have a significant reduction of the cost while buyers have their costs increased when compared with the independent policy. This shows that $\pi_{v\_cor} > \pi_{cor}$, $\pi_{BS\_cor} < 0$, and $\pi_{v\_cor} = \pi_{cor} + (-\pi_{BS\_cor\_})$.

The last term in Eq. (3.2.2) is vendor's additional loss in revenue caused by the quantity discount, denoted the "discounted part", then the "remaining part" in Eq.(3.2.2) is

$$\{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} + p_m D \qquad (3.2.3.1)$$

which is actually the expression of the vendor's cost in the Chan and Kingsman (2007) synchronized model without quantity discount Eq(2.3.2). The "remaining part" will just be the value TCV_cor in tables 2.3.2.1, 2.3.2.2 and 2.3.2.3 by substituting N and k_iT by the optimal values determined in their models.

In this research, the cost saving of the vendor with the discounted price

$p_d(i) = p_s - q_i p_s (1 - \frac{T_i^*}{k_iT})^2$ is given by:

$$\pi_{v\_qd} = TCV\_ind - TCV\_qd$$

$$= TCV\_ind - \text{"}remaining\ \ part\text{"} - \text{"}discounted\ \ \ part\text{"} \qquad (3.2.3.2)$$

The term q_i in the discounted price is the factor to control the discount. (i)On one hand, it controls the discount such that vendor's cost saving π_{v_qd} is positive. (ii)On the other hand, it controls the discount such that buyer's discount is more than his increased cost.

By using the optimal solution in Chan and Kingsman's model, the suitable q_i for each buyer is found to make sure that π_{v_qd} in Eq. (3.2.3.2) is positive. Let it be $\pi_{v\_qd}{}^*$. It is obvious that these q_i values can make sure that the optimal value in Eq. (3.2.3.2) $\pi_{v\_qd}{}^{**}$ is larger than or equal to $\pi_{v\_qd}{}^*$, which should also be positive.

When substituting the optimal solution of Chan and Kingsman (2007) synchronized cycles model without quantity discount into Eq. (3.2.3.2), then the vendor's cost saving with the discounted price in Model 1 will be:

$$\pi_{v\_qd} = TCV\_ind - TCV\_cor - "discounted \quad part"$$
$$= \pi_{v\_cor} - "discounted \quad part"$$

The $"discounted \quad part"$, $\sum_{i=1}^{n} discount(i)d_i$, in Eq. (3.2.3.2) should make the value of π_{v_qd} be positive. This indicates that:

$$\sum_{i=1}^{n} discount(i)d_i = \sum_{i=1}^{n} q_i p_s (1 - \frac{T_i^*}{k_{cor}(i)T})^2 d_i \qquad (3.2.3.3)$$
$$= (\pi_{v\_cor})\beta \qquad \text{with } 0 \le \beta \le 1$$
$$= (TCV\_ind - TCV\_cor)\beta$$

Note that $k_{cor}(i)$ is the optimal k_i values in the co-ordination model without quantity discount.

Let β in Eq(3.2.3.3) just be 1 and Eq. (3.2.3.3) will become:

$$\sum_{i=1}^{n} discount(i)d_i = \sum_{i=1}^{n} q_i p_s (1 - \frac{T_i^*}{k_{cor}(i)T})^2 d_i = \pi_{v\_cor} = \pi_{cor} + (-\pi_{BS\_cor\_}) \qquad (3.2.3.4)$$

This is the sum of the independent buyers' discounts, and the value of q_i for each buyer is proposed as follows:

$$q_i p_s (1-\frac{T_i^*}{k_{cor}(i)T})^2 d_i = \pi_{cor}\frac{d_i}{D} + (-\pi_{B\_cor}(i)), \tag{3.2.3.5}$$

or $$q_i p_s (1-\frac{T_i^*}{k_{cor}(i)T})^2 d_i = \pi_{cor}\frac{(1-\frac{T_i^*}{k_{cor}(i)T})^2}{\sum_{i=1}^{n}(1-\frac{T_i^*}{k_{cor}(i)T})^2} + (-\pi_{B\_cor}(i)) \tag{3.2.3.6}$$

The RHS of Eq. (3.2.3.5) and Eq. (3.2.3.6) both consist of two parts. The second part $-\pi_{B\_cor}(i)$ is the increased cost of buyer i in the co-ordinated model without quantity discount when compared with his independent cost. The first part is the proposed buyer i's share of the system savings in the co-ordinated model without quantity discount. The share in Eq. (3.2.3.5) depends on buyer's ordering quantity, while the share in Eq. (3.2.3.6) depends on buyer's contribution (the distance between $k_{cor}(i)$ and T_i^*) in the co-ordinated model without quantity discount.

Let $X_i = (1-\frac{T_i^*}{k_{cor}(i)T})^2$. We can find the q_i value for each buyer such that:

$$q_i = \min\{\frac{\pi_{cor}}{p_s X_i D} + \frac{(-\pi_{B\_cor}(i))}{p_s X_i d_i},\ 1\} = \{q_i^1 + q_i^2,\ 1\} \tag{3.2.3.7}$$

$$q_i = \min\{\frac{\pi_{cor}}{p_s d_i \sum_{i=1}^{n} X_i} + \frac{(-\pi_{B\_cor}(i))}{p_s d_i X_i},\ 1\} = \{q_i^1 + q_i^2,\ 1\} \tag{3.2.3.8}$$

The q_i^2 is the same in both of the two equations, and q_i^1 depends on the buyer *i*'s proportion of the system savings in the co-ordinated model without quantity discount.

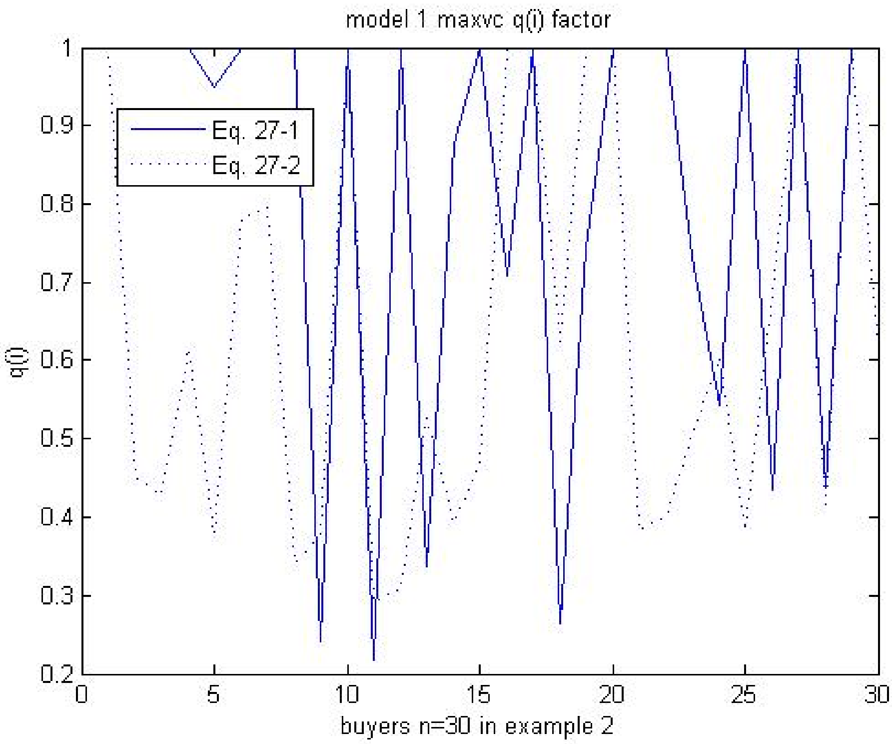

Figure 3.1 Example 2 Model 1 *q(i)* Values

From Figure 3.1 above, it is shown that the q_i in Eq. (3.2.3.7) has a larger amplitude than that in Eq. (3.2.3.8). The value q_i's (actually q_i^1) in Eq. (3.2.3.7) are very small for the 9th, 11th, 13th, 18th, 24th, 26th, and 28th buyer, since these buyers have large X_i values, which correspond to the Figure 3.2 below.

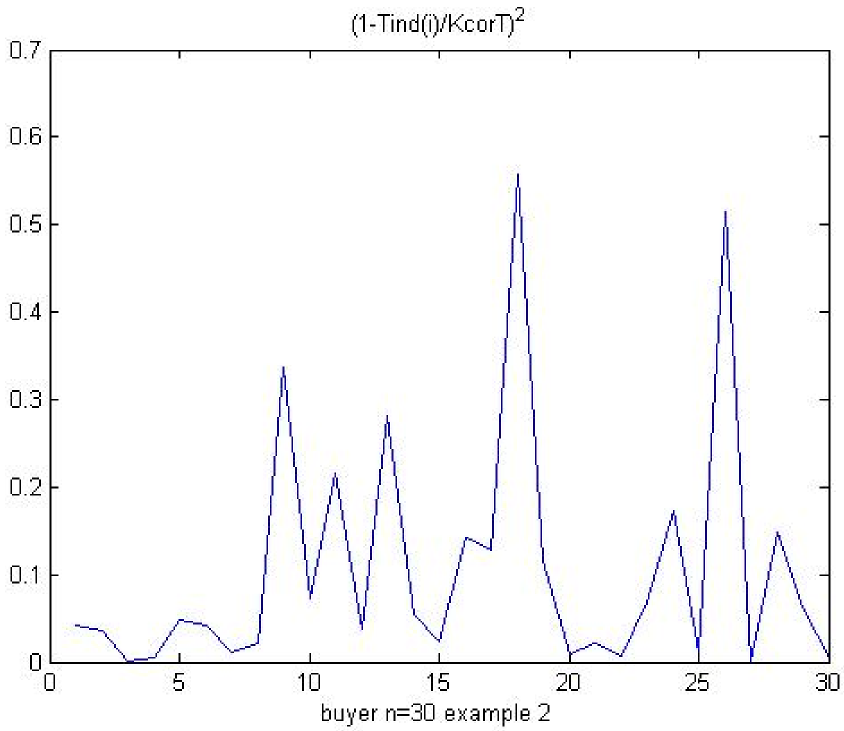

Figure 3.2 Example 2 Model 2 X_i in "q_i _ *factor*"

The larger X_i means that buyer i makes a larger contribution in the co-ordinated model without quantity discount, however he gets a smaller q_i value by Eq. (3.2.3.7) (e.g. 9th, 11th, 13th, 18th, 24th, 26th, and 28th buyers). It seems that these offers of discounts among buyers are not fair. Actually, the result of example 2 showed that Eq. (3.2.3.5) has a problem for the 18th buyer (buyer's cost increases when compared with the independent policy), since his value of q_i is small.

This seems that Eq. (3.2.3.6) in which the offers of discounts depend on each buyer's contribution works better. In this research, we use the q_i factor from Eq. (3.2.3.8) and it works well in all of the three examples.

$$q_i = \min\{(\frac{\pi_{cor}}{p_s d_i \sum_{i=1}^{n} X_i} + (-\frac{\pi_{B\_cor}(i)}{p_s d_i X_i})),1\} \tag{3.2.3.9}$$

where: $X_i = (1 - \frac{T_i^*}{k_{cor}(i)T})^2$

$$\pi_{cor} = TCS\_ind - TCS\_cor$$

$$-\pi_{B\_cor}(i) = (TCB\_cor(i) - TCB\_ind(i)) > 0$$

Question: Is there any saving left for the vendor?

By using the q_i in Eq (3.2.3.9), the value of $\pi_{v\_qd}{}^*$ is just zero. There is no need to worry about the value of $\pi_{v\_qd}{}^{**}$ which is the vendor's cost saving because of two reasons:

(i) Usually the quantity discount model will have more system saving π_{qd} than the co-ordinated model without quantity discount π_{cor}.

(ii) The buyer's final cost saving is just his received discounts in model 1 when $q = 1$. But by using the factor q_i ($q_i \le 1$) to control the discount, the buyer's discounted price increases, his holding cost which also depends on the discounted price will also increase, and therefore the buyer's cost is larger than that with $q=1$ in Eq. (3.2.3.8). Finally, the vendor has a larger share of the cost saving in the system.

The summarized results of this Modified Model 1 for the three examples are shown in Table 3.2.3.1, Table 3.2.3.2 and Table 3.2.3.3. See Appendix 4 for details of N, k_i, $p_d(i)$ and q_i.

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	Inc_V	inc_S
0.1	121.674	108.3844	230.0584	1.84	6.26	3.97
0.2	121.827	108.0361	229.8631	1.72	6.03	3.79
0.3	122.2398	107.9245	230.1644	1.38	5.56	3.38
0.4	122.3728	107.522	229.8948	1.27	5.29	3.19
0.5	122.515	107.1105	229.6256	1.16	4.97	2.97
0.6	123.2581	107.0256	230.2837	0.56	4.28	2.33
0.7	123.3161	106.4571	229.7732	0.51	3.91	2.12
0.8	123.0823	105.7654	228.8476	0.7	3.48	2.01
0.9	122.6499	104.6979	227.3478	1.05	3.06	1.99

Table 3.2.3.1 Example 1 Modified Model 1 with "$q(i)$ factor"

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	Inc_V	inc_S
0.1	2324.3	1776.8	4101.1	2.27	1.45	1.91
0.2	2326.7	1769.7	4096.3	2.17	1.47	1.87
0.3	2324	1765	4089	2.28	1.34	1.88
0.4	2324.5	1757.6	4082.1	2.26	1.32	1.86
0.5	2325.4	1749.9	4075.3	2.22	1.28	1.82
0.6	2325.6	1740.9	4066.5	2.21	1.27	1.81
0.7	2326.2	1730.5	4056.7	2.19	1.27	1.8
0.8	2328.3	1717.3	4045.6	2.1	1.32	1.77
0.9	2327	1703.1	4030.1	2.15	1.2	1.75

Table 3.2.3.2 Example 2 Modified Model 1 with "$q(i)$ factor"

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	Inc_V	inc_S
0.1	6605.1	4894.1	11499	1.47	0.86	1.21
0.2	6607	4880.5	11488	1.44	0.84	1.19
0.3	6608.3	4864.5	11473	1.42	0.84	1.18
0.4	6608.8	4847.5	11456	1.42	0.84	1.17
0.5	6610.1	4829.8	11440	1.4	0.82	1.16
0.6	6606.6	4812.5	11419	1.45	0.76	1.16
0.7	6609.6	4789.7	11399	1.4	0.75	1.13
0.8	6610.6	4762.6	11373	1.39	0.74	1.12
0.9	6610.5	4726.9	11337	1.39	0.74	1.12

Table 3.2.3.3 Example 3 Modified Model 1 with "$q(i)$ factor"

By definition, $inc\_BS = \dfrac{TCBS\_ind - TCBS\_qd}{TCBS\_ind} \times 100\%$,

$$inc\_V = \frac{TCV\_ind - TCV\_qd}{TCV\_ind} \times 100\%$$

$$inc\_S = \frac{TCS\_ind - TCS\_qd}{TCS\_ind} \times 100\%$$

They are the percentage cost savings for the buyers, vendor and system respectively.

These percentages are all positive numbers in Example 1, 2 and 3 when D/P increases from 0.1 to 0.9. This means that in these three examples, both the vendor and each buyer have a cost savings when compared with his independent cost.

3.3 Results of Objective Function 2: Minimizing the Total System Cost

3.3.1 Results

The total system cost with discounted price in Model 1 is given by:

$$\begin{aligned} TCS\_qd = &\{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i}{k_i T} \\ &+ d_i[\frac{hD}{P} - 0.5(h - h_i p_d(i))]k_i T\} + (P_s + P_m)D \end{aligned} \tag{3.3.1.1}$$

where $p_d(i) = p_s - p_s(1 - \dfrac{T_i^*}{k_i T})^2$,

subject to $k_i T \geq 0.5 T_i^*$

where $p_d(i)$ is the discounted price for the ith buyer. This model makes sure that the sum of buyer's ordering cost and holding cost is the same as their sum in the independent case. So the problem can also be written as:

$$\text{Min. } TCS\_qd = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} + fixed\_part$$

$$\text{where } fixed\_part = \sum_{i=1}^{n}\sqrt{2A_i h_i d_i p_s} + (p_s + p_m)D \qquad (3.3.1.2)$$

subject to: $k_i T \geq 0.5T_i^*$.

We do not need to consider the $fixed\_part$ in Eq. (3.3.1.2), and the remaining parts are just the expression of the vendor's cost in the co-ordinated model without quantity discount. In order to minimizing the total system costs, the determination of N and k_i only depends on the vendor's cost structure, and the vendor can decide these values without the information of the buyers' cost structure. However, the results of the three examples (Table 3.3.1.1) show that the vendor's cost is much worse than those in the independent policy when D/P is increasing from 0.1 to 0.9. The percentage savings of vendor's cost when compared with the independent policy are very large negative values, some are near minus one (this means that the vendor's cost in the co-ordination is nearly twice his independent cost).

D/P	Example1	Example2	Example3
0.1	-62.17	-79.76	-91.69
0.2	-58.16	-79.91	-91.71
0.3	-55.46	-80.26	-91.91
0.4	-53.42	-80.78	-92.18
0.5	-52.13	-81.34	-92.67
0.6	-51.1	-73.93	-82.79
0.7	-39.53	-69.51	-76.81
0.8	-33.85	-66.32	-73.82
0.9	-28.62	-64.22	-70.75

Table 3.3.1.1 Percentages Saving of Vendor's Cost （*inc_V%*）

Without considering the part of discounts, using Eq. (3.3.1.2), the optimal solutions of N and k_i are determined by the vendor's optimal policy which gives the best cost for the vendor. However, these optimal values of k_iT determined only from the vendor's viewpoint may be quite longer than the buyer's T_i^*, so the part of the discounts for each buyer will be very large. That is why the vendor's cost increases considerably in the previous table. It seems that it is necessary to use a "q factor" or "q_i factor" to control the discounts offered by the vendor.

3.3.2 Modified Model with "q Factor"

$$TCS\_qd = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i}{k_i T} + d_i[\frac{hD}{P} - 0.5(h - h_i p_d(i))]k_i T\} + (p_s + p_m)D \tag{3.3.2.1}$$

where $p_d(i) = p_s - q \cdot p_s(1 - \frac{T_i^*}{k_i T})^2$,

subject to: $0 \le q \le 1$

$\Rightarrow$

$$TCS\_qd = TCS\_cor - \sum_{i=1}^{n} 0.5 d_i h_i q p_s (1 - \frac{T_i^*}{k_i T})^2 k_i T$$

$$= TCS\_cor - \sum_{i=1}^{n} 0.5 d_i h_i q p_s (k_i T + \frac{T_i^{*2}}{k_i T} - 2T_i^*)$$

$$= \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i - 0.5 d_i h_i q p_s T_i^{*2}}{k_i T} + d_i[h(\frac{D}{P} - 0.5) + 0.5 h_i p_s (1-q)]k_i T\} + (p_s + p_m)D + \sum_{i=1}^{n} d_i h_i q p_s T_i^*$$

where $T_i^* = \sqrt{\frac{2A_i}{h_i d_i p_s}}$

subject to: $0 \le q \le 1$

$\Rightarrow$

$$TCS\_qd = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i(1-q)}{k_i T} + d_i[h(\frac{D}{P} - 0.5) + 0.5 h_i p_s (1-q)]k_i T\} + fixed\_part$$

where $fixed\_part = (p_s + p_m)D + \sum_{i=1}^{n} d_i h_i q p_s T_i^*$ (3.3.2.2) subject to: $0 \le q \le 1$

Results:

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S
0.1	111.7264	112.8959	224.6224	9.86	2.36	6.24
0.2	111.9998	113.3595	225.3594	9.64	1.40	5.68
0.3	112.3112	113.7591	226.0703	9.39	0.45	5.10
0.4	115.7145	110.9179	226.6324	6.65	2.30	4.57
0.5	115.7145	111.1129	226.8274	6.65	1.42	4.16
0.6	115.7145	111.3079	227.0224	6.65	0.45	3.71
0.7	117.2197	109.6187	226.8383	5.43	1.06	3.37
0.8	117.2197	109.3037	226.5233	5.43	0.26	3.00
0.9	118.5128	107.2666	225.7794	4.39	0.68	2.66

Table3.3.2.1 Example 1 Modified Model 1 when q=0.3

Table 3.3.2.1 shows that when q=0.3, the co-ordination works well in Example 1. However, similar to Example 3 in section 3.2.1, it is difficult to find the proper q as a constant for all the buyers in Examples 2 and 3.

3.3.3 Modified Model with "$q(i)$ Factor"

$$\text{Min. } TSC\_qd = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i + A_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T + 0.5 d_i h_i k_i T p_d(i)\} + fixed\_part \tag{3.3.3.1}$$

where $fixed\_part = (p_s + p_m)D$,

$$p_d(i) = p_s - q_i p_s (1 - \frac{T_i^*}{k_i T})^2$$

subject to: $0 \le q_i \le 1$ and $k_i T \ge 0.5 T_i^*$

As the results suggested, the vendor needs more control of the discounts. So, we add *num* which is a positive number larger than 1 into the previous "q_i factor", such that:

$$q_i p_s (1 - \frac{T_i^*}{k_{cor}(i)T})^2 d_i = \pi_{cor} \frac{p_s (1 - \frac{T_i^*}{k_{cor}(i)T})^2}{num \cdot \sum_{i=1}^{n} p_s (1 - \frac{T_i^*}{k_{cor}(i)T})^2} + (-\pi_{B\_cor}(i)) \qquad (3.3.3.2)$$

So, q_i becomes:

$$q_i = \min\{\frac{\pi_{cor}}{num \cdot d_i p_s \sum_{i=1}^{n} X_i} + (-\frac{\pi_{B\_cor}(i)}{d_i p_s X_i}),\ 1\} \qquad (3.3.3.3)$$

where: $X_i = (1 - \frac{T_i^*}{k_{cor}(i)T})^2$,

$num \geq 1$,

$\pi_{cor} = TCS\_ind - TCS\_cor$, and

$-\pi_{B\_cor}(i) = TCB\_cor(i) - TCB\_ind(i) > 0$

Note that the *num* has to be larger than or equal to one, to ensures that the total discounts is not larger than the total surplus of co-ordination. Furthermore, the larger the *num* is, the fewer discounts the vendor will give to the buyer. In our later examples *num* is set equal to 3 as the illustration. It is possible to set num equal to other positive numbers larger than 1, it depends on how much the vendor is willing to share the system savings with the buyers.

The summarized results of minimizing total system cost of this modified model with "*q(i)* factor" are shown in Tables 3.3.3.1, 3.3.3.2, & 3.3.3.3, in which *inc_qd_cor* is the percentage system cost saving with quantity discount mechanism, when compared with the initial synchronized model without the quantity discount.

See Appendix 5 for details of N, k_i, $p_d(i)$ and q_i.

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S	inc_qd_cor
0.1	118.4309	106.7847	225.2155	4.45	7.64	5.99	0.43
0.2	118.9894	106.972	225.9615	4.00	6.96	5.42	0.38
0.3	119.4068	107.227	226.6338	3.67	6.17	4.87	0.27
0.4	119.9163	106.946	226.8623	3.26	5.8	4.47	0.25
0.5	120.3008	106.7841	227.0849	2.95	5.26	4.05	0.23
0.6	119.7412	107.4905	227.2317	3.40	3.87	3.62	0.16
0.7	120.8296	106.2092	227.0387	2.52	4.14	3.28	0.12
0.8	121.2161	105.5348	226.7508	2.21	3.69	2.91	0.12
0.9	121.4298	104.5114	225.9412	2.04	3.24	2.59	0.09

Table 3.3.3.1 Example 1 Modified Model 1 with "$q(i)$ factor" $p_s = 2,$ and $p_m = 1.5$

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S	inc_qd_cor
0.1	2305.3	1764.7	4070	3.07	2.12	2.66	0.77
0.2	2307.3	1757.8	4065.1	2.98	2.14	2.62	0.75
0.3	2304.6	1752.8	4057.4	3.1	2.02	2.63	0.78
0.4	2305.3	1745.7	4051	3.07	1.99	2.6	0.77
0.5	2306.2	1738.2	4044.4	3.03	1.95	2.57	0.77
0.6	2308.7	1728.3	4036.9	2.92	1.99	2.53	0.75
0.7	2310.3	1717.4	4027.7	2.86	2.01	2.5	0.74
0.8	2311.8	1704.9	4016.7	2.79	2.03	2.47	0.72
0.9	2308.5	1692.6	4001	2.93	1.81	2.46	0.74

Table 3.3.3.2 Example 2 Modified Model 1 with "$q(i)$ factor" $p_s = 2,$ and $p_m = 1.5$

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S	inc_qd_cor
0.1	6564.6	4885.4	11450	2.08	1.04	1.64	0.53
0.2	6575.4	4864.3	11440	1.91	1.17	1.6	0.52
0.3	6573.6	4851.3	11425	1.94	1.11	1.59	0.52
0.4	6574.1	4835	11409	1.93	1.1	1.58	0.52
0.5	6577.4	4816	11393	1.88	1.11	1.56	0.51
0.6	6574	4800.3	11374	1.94	1.01	1.55	0.51
0.7	6574.4	4779.1	11354	1.93	0.97	1.53	0.51
0.8	6577	4750.9	11328	1.89	0.99	1.51	0.51
0.9	6576.9	4717.4	11294	1.89	0.94	1.5	0.5

Table 3.3.3.3 Example 3 Modified Model 1 with "$q(i)$ factor" $p_s = 2,$ and $p_m = 1.5$

The modified Model 1 "$q(i)$ factor" works well in Examples 1, 2 and 3 (Table 3.3.3.1, Table 3.3.3.2 & Table 3.3.3.3), where each buyer and vendor have their costs reduced. In Example 1 (Table 3.3.3.1), buyers have a percentage cost savings ranging from 2.04% to 4.45%, and the vendor has a percentage cost savings ranging from 3.24% to 7.64%. In Example 2 (Table 3.3.3.2), buyers have a percentage cost savings ranging from 2.79% to 3.07%, and the vendor has a percentage cost savings ranging from 1.81% to 2.12%. In Example 3 (Table 3.3.3.3), buyers have a percentage cost savings ranging from 1.88% to 2.08%, and the vendor has a percentage cost savings ranging from 0.94% to 1.17%. So the buyer is willing to accept the new ordering cycle, and the vendor also can get the benefit from the model. Moreover, the total system cost with quantity discount in the synchronized model is better than that without the quantity discount. The percentage system cost savings when compared with the initial synchronized model without the quantity discount (*inc_qd_cor*) are all positive numbers in the three examples.

3.4 Conclusions

The formula of the discounted price $p_d(i) = p_s - p_s(1 - \frac{T_i^*}{k_i T})^2$ does not need any buyer's cost information, it only needs the buyer's EOQ ordering cycle, T_i^*, which should already be known by the vendor. Co-ordinated models in the literature usually assume that both the parties share all the cost information. However, in practice, the members of a supply chain may not be interested to disclose all the information.

In minimizing vendor's cost in Model 1, the vendor can calculate his cost and compare with his independent cost without the information of the buyers. Although the vendor cannot calculate the buyer's cost, he can be sure that each buyer will adopt the co-ordinated model since each buyer can reduce his cost when compared with his independent cost.

In some cases, if the result is not good, i.e. the vendor's cost is larger than his independent cost (e.g. Examples 2 & 3), this means that the vendor offers more discounts than he should. So, the vendor needs to control the discount. There are two proposed methods in this research. The first one is the "q factor" method, and the advantage of this is that the vendor does not need to know the buyer's information. The disadvantage is that it can be difficult to find the constant q for all the buyers to control the discounts (this happens in Example 3). The second method to control the discount is the "q_i factor" method, and the advantage of this is that this factor is different for each buyer and it depends on the buyer's own situation and his needed discounts, so it has a more powerful control than the "*q*_factor". However the q_i calculation needs the solution of the Chan and Kingsman's (2005, 2007) synchronized cycles model without quantity discount.

From the viewpoint of system savings, minimizing total system cost in Model 1 is better than minimizing vendor's cost. Table 3.4.1 shows the percentage system savings when compared with the independent system cost under the model of "q_i factor" by using two different objective functions: minimizing total system cost (*Minsc*) or minimizing total vendor's relevant cost (*Minvc*).

Minsc			*Minvc*		
Example1	Example2	Example3	Example1	Example2	Example3
5.99	2.66	1.64	3.97	1.91	1.21
5.42	2.62	1.6	3.79	1.87	1.19
4.87	2.63	1.59	3.38	1.88	1.18
4.47	2.6	1.58	3.19	1.86	1.17
4.05	2.57	1.56	2.97	1.82	1.16
3.62	2.53	1.55	2.33	1.81	1.16
3.28	2.5	1.53	2.12	1.80	1.13
2.91	2.47	1.51	2.01	1.77	1.12
2.59	2.46	1.50	1.99	1.75	1.12

Table 3.4.1 Modified Model 1 with "$q(i)$ factor" Percentage System Saving (*inc_S*)

Chapter 4

Quantity Discount Model 2

4.1 Discounted Price

In traditional quantity discount models, the vendor usually offers the quantity discounts to the buyer to entice him to make a larger ordering quantity cycle in the coordinated model. With the assumption that the ordering cycle for each buyer in the coordinated model k_iT is larger than T_i^*, then the buyer's ordering cost $\frac{A_i}{k_iT}$ will be less than $\frac{A_i}{T_i^*}$. Hence, we ignore the part of the ordering cost in both Eq. (2.4.3.2) and (2.4.3.3), and try to find the discounted price $p_d(i)$ to make the remaining terms be equal in Eq. (2.4.3.2) and (2.4.3.3), i.e.

$$0.5h_i d_i k_i T p_d(i) + d_i p_d(i) = 0.5h_i d_i T_i^* p_s + d_i p_s. \tag{4.1.1}$$

This gives the discounted price for each buyer

$$p_d(i) = p_s \frac{0.5h_i T_i^* + 1}{0.5h_i k_i T + 1} \tag{4.1.2}$$

and $$discount(i) = p_s \frac{0.5h_i (k_i T - T_i^*)}{0.5h_i k_i T + 1} \quad . \tag{4.1.3}$$

With the constraint of $k_i T > T_i^*$, the $p_d(i)$ in Eq. (4.1.2) is obviously less than p_s and larger than zero. More-over, the $discount(i) = p_s \dfrac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1}$ will be zero when $k_i T = T_i^*$.

4.2 Results of Objective Function 1: Minimizing Vendor's Cost

The discounted price in model 2 is:

$$p_d(i) = p_s \frac{0.5h_i T_i^* + 1}{0.5h_i k_i T + 1} = p_s\left(1 - \frac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1}\right) \quad \text{and} \quad k_i T \geq T_i^* \tag{4.2.1}$$

The part of the discounts the vendor offers to each buyer will be:

$$discount(i) = p_s \frac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1} \tag{4.2.2}$$

The model of minimizing the vendor's cost is :

$$\begin{aligned} \text{Min. } TRC_v = & \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} \\ & + p_m D + \sum_{i=1}^{n} p_s \frac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1} d_i \end{aligned} \tag{4.2.3}$$

where $p_d(i) = p_s \dfrac{0.5h_i T_i^* + 1}{0.5h_i k_i T + 1}$

$$= p_s\left(1 - \frac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1}\right),$$

and $discount(i) = p_s \dfrac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1}$

subject to: $k_i T \geq T_i^*$

Model 2 works well in Examples 1 and 2, and each buyer and vendor has a cost less than the cost in the independent case. In Table 4.2.1 Example 1, buyers have a percentage cost savings ranging from 0.94% to 1.97%, and the vendor has a percentage cost savings ranging from 4.26% to 10%. In Table 4.2.2 Example 2, buyers have a percentage cost savings ranging from 2.16% to 2.29%, and the vendor has a percentage cost savings ranging from 2.08% to 2.41%. So the buyer is willing to accept the new ordering cycle, and the vendor can get the benefit from the model. Moreover, the total system cost is even less than the co-ordinated model without quantity discount.

However, the results in Table 4.2.3 Example 3 show that the vendor has a cost which is a little more than that in his independent policy (increases from 0.03% to 0.3% when D/P increases from 0.1 to 0.9) while both the buyer's cost and the system cost are much less than those of the independent policy (reduce about 2.8% and 1.6% respectively). The system cost with quantity discount is also less than that of the co-ordinated model without the quantity discount.

See Appendix 6 for details of N, k_i and $p_d(i)$ for the three examples.

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_cor	TCV_cor	TCS_cor	TCBS_ind	TCV_ind	TCS_ind	inc_BS	inc_V	inc_S
0.1	121.5098	104.0556	225.5654	128.5358	97.6525	226.1883	123.9529	115.6222	239.5751	1.97	10.00	5.85
0.2	121.5811	104.6928	226.2739	128.3262	98.5033	226.8295	123.9529	114.9689	238.9218	1.91	8.94	5.29
0.3	121.9765	104.9599	226.9364	126.3706	100.8746	227.2451	123.9529	114.2732	238.2261	1.59	8.15	4.74
0.4	122.0534	105.134	227.1874	126.3706	101.0566	227.4271	123.9529	113.5259	237.4788	1.53	7.39	4.33
0.5	122.2622	105.2595	227.5217	126.3706	101.2386	227.6091	123.9529	112.7132	236.6661	1.36	6.61	3.86
0.6	122.5674	105.07	227.6373	125.3672	102.2343	227.6015	123.9529	111.8143	235.7672	1.12	6.03	3.45
0.7	122.4348	104.8025	227.2373	125.3672	101.9418	227.309	123.9529	110.794	234.7469	1.22	5.41	3.20
0.8	122.7145	104.3178	227.0323	125.3672	101.6493	227.0165	123.9529	109.5837	233.5366	1.00	4.81	2.79
0.9	122.7931	103.4013	226.1944	125.0103	101.1423	226.1527	123.9529	108.0064	231.9593	0.94	4.26	2.49

Table 4.2.1 Example 1 Model 2 Percentage Saving Data

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_cor	TCV_cor	TCS_cor	TCBS_ind	TCV_ind	TCS_ind	inc_BS	inc_V	inc_S
0.1	2324.1	1759.5	4083.6	2422.7	1678.8	4101.5	2378.2	1802.9	4181.2	2.27	2.41	2.33
0.2	2323.7	1753.8	4077.6	2422.8	1673.2	4096	2378.2	1796.2	4174.4	2.29	2.36	2.32
0.3	2323.7	1746.9	4070.7	2421.7	1667.5	4089.2	2378.2	1788.9	4167.1	2.29	2.35	2.31
0.4	2323.7	1740	4068	2421.7	1660.7	4082.4	2378.2	1781.2	4159.4	2.29	2.31	2.30
0.5	2324.6	1733.1	4057.8	2421.7	1653.9	4075.6	2378.2	1772.7	4150.9	2.25	2.23	2.24
0.6	2326.7	1724.7	4051.4	2422.3	1645	4067.2	2378.2	1763.4	4141.6	2.17	2.19	2.18
0.7	2326.2	1715.3	4041.5	2422.2	1635.4	4057.6	2378.2	1752.8	4131	2.19	2.14	2.17
0.8	2325.8	1703.1	4029	2422.2	1623.7	4045.9	2378.2	1740.2	4118.4	2.20	2.13	2.17
0.9	2326.9	1687.9	4014.8	2421.6	1609.1	4030.7	2378.2	1723.8	4102	2.16	2.08	2.13

Table 4.2.2 Example 2 Model 2 Percentage Saving Data

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_cor	TCV_cor	TCS_cor	TCBS_ind	TCV_ind	TCS_ind	inc_BS	inc_V	inc_S
0.1	6513.9	4938.1	11452	6830.5	4680.5	11511	6703.7	4936.7	11640	2.83	-0.03	1.62
0.2	6513.9	4926.4	11440	6830.5	4668.7	11499	6703.7	4921.7	11625	2.83	-0.10	1.59
0.3	6514.7	4911.5	11426	6830.2	4654.5	11485	6703.7	4905.7	11609	2.82	-0.12	1.58
0.4	6514.7	4895.1	11410	6830.2	4638.1	11468	6703.7	4888.6	11592	2.82	-0.13	1.57
0.5	6514.7	4878.8	11394	6830.2	4621.8	11452	6703.7	4869.9	11574	2.82	-0.18	1.56
0.6	6516.8	4858.9	11376	6830.3	4602.7	11433	6703.7	4849.3	11553	2.79	-0.20	1.53
0.7	6516.8	4837.9	11355	6829.2	4582.7	11412	6703.7	4825.9	11530	2.79	-0.25	1.52
0.8	6516	4812	11328	6828.7	4556.8	11385	6703.7	4798.2	11502	2.80	-0.29	1.51
0.9	6518.2	4776.1	11294	6830.1	4520.9	11351	6703.7	4762	11466	2.77	-0.30	1.49

Table 4.2.3 Example 3 Model 2 Percentage Saving Data

4.3 Results of Objective Function 2: Minimizing the Total System Cost

$$\text{Min. } TCS\_qd = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i}{k_i T} + d_i[\frac{hD}{P} - 0.5(h - h_i p_d(i))]k_i T\} + (p_s + p_m)D \tag{4.3.1}$$

where $p_d(i) = p_s \frac{0.5h_i T_i^* + 1}{0.5h_i k_i T + 1} = p_s(1 - \frac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1}),$

subject to: $k_i T \geq T_i^*$

$\Rightarrow$

$$\text{Min. } TCS\_qd = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i + A_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} + \sum_{i=1}^{n} p_s \frac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1} d_i + fixed\_part \tag{4.3.2}$$

where $fixed\_part = \sum_{i=1}^{n} 0.5h_i d_i T_i^* p_s + (p_s + p_m)D$

where $p_d(i) = p_s \frac{0.5h_i T_i^* + 1}{0.5h_i k_i T + 1} = p_s(1 - \frac{0.5h_i(k_i T - T_i^*)}{0.5h_i k_i T + 1}),$

subject to: $k_i T \geq T_i^*$

Model 2 works well in Examples 1 and 2 in minimizing total system costs, and each buyers and vendor have their costs reduced. In Example 1 (Table 4.3.1), buyers have a percentage cost savings ranging from 1.22% to 2.05%, and the vendor has a percentage cost savings ranging from 4.11% to 9.96%. In Example 2 (Table 4.3.2), buyers have a percentage cost savings ranging from 4.22% to 4.35%, and the vendor has a percentage cost savings ranging from 0.22% to 0.66%. So the buyer is willing to accept the new ordering cycle, and the vendor can get the benefit from the model. Moreover, the total system cost with quantity discount in the synchronized model is better than that without the quantity discount.

For Example 3, it can be seen from Table 4.3.3 that the vendor's cost is worse than his independent cost. The results are not surprising as this situation also exists in the model of minimizing vendor's cost.

See Appendix 7 for details of N, k_i and $p_d(i)$ for the three examples.

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_cor	TCV_cor	TCS_cor	TCBS_ind	TCV_ind	TCS_ind	inc_V	inc_S	inc_S
0.1	121.414	104.1015	225.5155	128.5358	97.6525	226.1883	123.9529	115.6222	239.5751	2.05	9.96	5.87
0.2	121.4445	104.7597	226.2042	128.3262	98.5033	226.8295	123.9529	114.9689	238.9218	2.02	8.88	5.32
0.3	121.8385	105.0295	226.868	126.3706	100.8746	227.2451	123.9529	114.2732	238.2261	1.71	8.09	4.77
0.4	121.8385	105.218	227.0565	126.3706	101.0566	227.4271	123.9529	113.5259	237.4788	1.71	7.32	4.39
0.5	121.8385	105.4065	227.245	126.3706	101.2386	227.6091	123.9529	112.7132	236.6661	1.71	6.48	3.98
0.6	122.2457	105.1636	227.4093	125.3672	102.2343	227.6015	123.9529	111.8143	235.7672	1.38	5.95	3.54
0.7	122.2457	104.8711	227.1168	125.3672	101.9418	227.309	123.9529	110.794	234.7469	1.38	5.35	3.25
0.8	122.0836	104.7375	226.8211	125.3672	101.6493	227.0165	123.9529	109.5837	233.5366	1.51	4.42	2.88
0.9	122.4348	103.5704	226.0052	125.0103	101.1423	226.1527	123.9529	108.0064	231.9593	1.22	4.11	2.57

Table 4.3.1 Example 1 Model 2 Percentage Saving Data

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_cor	TCV_cor	TCS_cor	TCBS_ind	TCV_ind	TCS_ind	inc_BS	inc_V	inc_S
0.1	2274.9	1791.3	4066.2	2422.7	1678.8	4101.5	2378.2	1802.9	4181.2	4.35	0.64	2.75
0.2	2277	1784.3	4061.2	2422.8	1673.2	4096	2378.2	1796.2	4174.4	4.26	0.66	2.71
0.3	2277.5	1777.2	4054.6	2421.7	1667.5	4089.2	2378.2	1788.9	4167.1	4.24	0.66	2.7
0.4	2277.9	1770.1	4048	2421.7	1660.7	4082.4	2378.2	1781.2	4159.4	4.22	0.62	2.68
0.5	2275.1	1766	4041.1	2421.7	1653.9	4075.6	2378.2	1772.7	4150.9	4.34	0.38	2.65
0.6	2276.7	1756.2	4032.9	2422.3	1645	4067.2	2378.2	1763.4	4141.6	4.27	0.41	2.62
0.7	2276	1747.3	4023.3	2422.2	1635.4	4057.6	2378.2	1752.8	4131	4.3	0.31	2.61
0.8	2277.9	1734.1	4012	2422.2	1623.7	4045.9	2378.2	1740.2	4118.4	4.22	0.35	2.58
0.9	2276.9	1719.9	3996.8	2421.6	1609.1	4030.7	2378.2	1723.8	4102	4.26	0.22	2.56

Table 4.3.2 Example 2 Model 2 Percentage Saving Data

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S
0.1	6479.8	4956.7	11437	3.34	-0.41	1.75
0.2	6479.8	4945.2	11425	3.34	-0.48	1.72
0.3	6481.1	4929.4	11411	3.32	-0.48	1.71
0.4	6481.1	4913.3	11394	3.32	-0.51	1.71
0.5	6481.1	4897.1	11378	3.32	-0.56	1.69
0.6	6480.9	4878.5	11359	3.32	-0.6	1.68
0.7	6482.1	4856.5	11339	3.31	-0.63	1.66
0.8	6482.6	4830	11313	3.3	-0.66	1.65
0.9	6481.5	4796.3	11278	3.31	-0.72	1.64

Table 4.3.3 Example 3 Model 2 Percentage Saving Data

4.4 Conclusions

Chan and Kingsman (2005, 2007) developed a synchronized cycles model that allows each buyer to choose its ordering cycle, while the length of the cycle should be kept as a factor of the vendor's production cycle. They showed that the co-ordination synchronized cycles model works well. It has been shown, by many numerical experiments, that the synchronized cycles model can significantly reduce the total system cost and make a significant cost reduction compared to the independent policy and the common replenishment cycle (e.g. Banerjee and Burton (1994)). However, the cost to all the buyers is significantly increased. Hence, the mechanism to attract buyers to join the co-ordination is deemed necessary.

With the assumption that the ordering cycle for each buyer in the coordinated model $k_i T$ is larger than T_i^*, this chapter presents another quantity discounts model, to achieve the system co-ordination and make an equitable division of the system surplus among system members.

The buyer's holding cost h_i is expressed as a percentage of the capital, so buyer's holding cost is considered to be dependent on purchasing price.

Two objective functions for each quantity discount model are also proposed in this research. The first objective function is to minimize vendor's cost, and the second objective function is to minimize the total system cost.

The quantity discount model can ascertain the benefit of the buyer and it is not pre-determined (e.g. all unit discount) but depends on the buyer's ordering cycle in the co-ordination model. So, the manufacturer's savings can be passed by the discount to each buyer and the system cost can further be reduced by the reduction of buyers' holding cost. However, in some cases, the quantity discounts models may have a solution in which the vendor may have his cost increased in the co-ordination when compared with his independent cost. This is due to the reason that the discounts offered are too large.

Model 2 ascertains that each buyer can reduce his cost when compared with his independent cost. The vendor can also get benefits by using Model 2 in Examples 1 and 2, but there exists the problem that the vendor's cost is a little worse than his independent cost in Example 3. Comparing the two objective functions, minimizing total system cost (*Minsc*) is better than minimizing vendor's cost (*Minvc*) in the first two examples.

Minsc		*Minvc*	
Example1	Example2	Example1	Example2
5.87	2.75	5.85	2.33
5.32	2.71	5.29	2.32
4.77	2.7	4.74	2.31
4.39	2.68	4.33	2.3
3.98	2.65	3.86	2.24
3.54	2.62	3.45	2.18
3.25	2.61	3.2	2.17
2.88	2.58	2.79	2.17
2.57	2.56	2.49	2.13

Table 4.4.1 Examples1 and 2 System Percentage Saving Data (*inc_S*) in Model 2

Chapter 5

Quantity Discount Model 3

5.1 Discounted Price

This model is similar to the "break-even price discount" in Monahan's model (1984). We firstly modify Eq. (2.4.3.2) by overstating the buyer's inventory holding cost. This is achieved by using p_s instead of $p_d(i)$ for its calculation (since $p_d(i) \le p_s$) .

$$TCB\_qd(i) = \frac{A_i}{k_i T} + \frac{1}{2} h_i d_i k_i T p_s + d_i p_d(i) \qquad (5.1.1)$$

As Monahan (1984) pointed, "This discount, as far as the buyer is concerned, must be sufficient to at least compensate him for his additional inventory expenses." We then try to find the discounted price $p_d(i)$ to make Eq. (5.1.1) and (2.4.3.3) equal, i.e.

$$\frac{A_i}{k_i T} + \frac{1}{2} h_i d_i k_i T p_s + d_i p_d(i) = \sqrt{2 A_i h_i d_i p_s} + d_i p_s \qquad (5.1.2)$$

The RHS of Eq. (5.1.2) also equals to $\frac{A_i}{T_i^*} + \frac{1}{2} h_i d_i T_i^* p_s + d_i p_s$, and this gives that the discounted price for each buyer is

$$p_d(i) = p_s[1 - \frac{h_i}{2}(k_i T - T_i^*)] + \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})$$

and $$discount(i) = p_s \frac{h_i}{2}(k_i T - T_i^*) - \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T}) \qquad (5.1.3)$$

It can be proved that the $p_d(i)$ in Eq. (5.1.3) is less than p_s if $k_i T > T_i^*$.

Proof:

The quantity discounts in Model 2 is:

$$p_d(i) = p_s[1 - \frac{h_i}{2}(k_i T - T_i^*)] + \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})$$

$$p_d(i) \le p_s \Rightarrow \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T}) \le \frac{h_i p_s}{2}(k_i T - T_i^*)$$

$$\Rightarrow \frac{A_i}{d_i}(\frac{k_i T - T_i^*}{T_i^* k_i T}) \le \frac{h_i p_s}{2}(k_i T - T_i^*)$$

$$\Rightarrow \frac{2A_i}{d_i h_i T_i^* p_s} \le k_i T \qquad \text{where } \frac{2A_i}{d_i h_i p_s} = T_i^{*2}$$

$$\Rightarrow T_i^* \le k_i T$$

And $p_d(i)$ in Eq. (5.1.3) is larger than zero if $k_i T \le T_i^* + \frac{2}{h_i}$.

Proof

$$p_d(i) = p_s[1 - \frac{h_i}{2}(k_i T - T_i^*)] + \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})$$

If $T_i^* \le k_i T$, then the second part in the $p_d(i)$ is larger than zero. We want to make sure that $p_d(i)$ is positive, so we can simply make sure that the first part is positive.

$$p_s[1 - \frac{h_i}{2}(k_i T - T_i^*)] \ge 0 \quad \Rightarrow \quad k_i T \le T_i^* + \frac{2}{h_i}$$

So, with the constraint of $T_i^* \le k_i T \le T_i^* + \frac{2}{h_i}$, substitute the $p_d(i)$ in Eq. (5.1.3) into Eq. (2.4.3.2), (or replacing back p_s by $p_d(i)$ in Eq (5.1.1)), then we have Eq.(2.4.3.2)<Eq.(2.4.3.3). Moreover, the

$discount(i) = p_s \frac{h_i}{2}(k_i T - T_i^*) - \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})$ will be zero when $k_i T = T_i^*$.

5.2 Results of Objective Function 1: Minimizing Vendor's Cost

The discounted price of Model 3 is:

$$p_d(i) = p_s - [p_s \frac{h_i}{2}(k_i T - T_i^*) - \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})] \tag{5.2.1}$$

subject to: $T_i^* \le k_i T \le T_i^* + \frac{2}{h_i}$

Hence, the discount for each buyer is:

$$discount(i) = p_s \frac{h_i}{2}(k_i T - T_i^*) - \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T}) \tag{5.2.2}$$

The model of minimizing vendor's cost is :

$$\begin{aligned} \text{Min. } TRC_v = &\{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n}\{\frac{C_i}{k_i T} + d_i h(\frac{D}{P} - 0.5)k_i T\} \\ &+ p_m D + \sum_{i=1}^{n} p_s \frac{h_i(k_i T - T_i^*)}{2} d_i - \sum_{i=1}^{n} A_i(\frac{1}{T_i^*} - \frac{1}{k_i T}) \end{aligned} \tag{5.2.3}$$

where $p_d(i) = p_s - [p_s \frac{h_i}{2}(k_i T - T_i^*) - \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})]$,

$$discount(i) = p_s \frac{h_i}{2}(k_i T - T_i^*) - \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})$$

subject to: $T_i^* \le k_i T \le T_i^* + \frac{2}{h_i}$,

From Table 5.2.1, Table 5.2.2, and Table 5.2.3 , it can be seen that minimizing vendor's cost in Model 3 can work better than the independent policy. Both parties in the system, the vendor and each buyer, can reduce his cost when compared with his independent cost. Moreover, the system cost is also better than that of the co-ordinated model without quantity discount.

See Appendix 8 for details of N, k_i and $p_d(i)$ for the three examples.

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_cor	TCV_cor	TCS_cor	TCBS_ind	TCV_ind	TCS_ind	inc_BS	inc_V	inc_S
0.1	123.503	102.2354	225.7385	128.5358	97.6525	226.1883	123.9529	115.6222	239.5751	0.36	11.58	5.78
0.2	123.533	102.8766	226.4095	128.3262	98.5033	226.8295	123.9529	114.9689	238.9218	0.34	10.52	5.24
0.3	123.7305	103.2922	227.0227	126.3706	100.8746	227.2451	123.9529	114.2732	238.2261	0.18	9.61	4.7
0.4	123.7305	103.4742	227.2047	126.3706	101.0566	227.4271	123.9529	113.5259	237.4788	0.18	8.85	4.33
0.5	123.7305	103.6562	227.3867	126.3706	101.2386	227.6091	123.9529	112.7132	236.6661	0.18	8.04	3.92
0.6	123.8586	103.6486	227.5073	125.3672	102.2343	227.6015	123.9529	111.8143	235.7672	0.08	7.3	3.5
0.7	123.8586	103.3561	227.2148	125.3672	101.9418	227.309	123.9529	110.794	234.7469	0.08	6.71	3.21
0.8	123.8586	103.0636	226.9223	125.3672	101.6493	227.0165	123.9529	109.5837	233.5366	0.08	5.95	2.83
0.9	123.8876	102.1998	226.0873	125.0103	101.1423	226.1527	123.9529	108.0064	231.9593	0.05	5.38	2.53

Table 5.2.1 Example1 Model 3 Percentage Saving Data

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_cor	TCV_cor	TCS_cor	TCBS_ind	TCV_ind	TCS_ind	inc_BS	inc_V	inc_S
0.1	2364.3	1723.5	4087.8	2422.7	1678.8	4101.5	2378.2	1802.9	4181.2	0.59	4.4	2.23
0.2	2365.4	1718.0	4083.4	2422.8	1673.2	4096.0	2378.2	1796.2	4174.4	0.54	4.35	2.18
0.3	2365.5	1711.2	4076.7	2421.7	1667.5	4089.2	2378.2	1788.9	4167.1	0.54	4.34	2.17
0.4	2365.5	1704.5	4069.9	2421.7	1660.7	4082.4	2378.2	1781.2	4159.4	0.54	4.31	2.15
0.5	2365.5	1697.7	4063.2	2421.7	1653.9	4075.6	2378.2	1772.7	4150.9	0.54	4.23	2.11
0.6	2364.3	1689.3	4053.7	2422.3	1645	4067.2	2378.2	1763.4	4141.6	0.58	4.2	2.12
0.7	2364.3	1679.8	4044.1	2422.2	1635.4	4057.6	2378.2	1752.8	4131.0	0.58	4.16	2.1
0.8	2365.5	1668.0	4033.5	2422.2	1623.7	4045.9	2378.2	1740.2	4118.4	0.54	4.15	2.06
0.9	2365.5	1652.9	4018.3	2421.6	1609.1	4030.7	2378.2	1723.8	4102.0	0.54	4.11	2.04

Table 5.2.2 Example2 Model 3 Percentage Saving Data

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_cor	TCV_cor	TCS_cor	TCBS_ind	TCV_ind	TCS_ind	inc_BS	inc_V	inc_S
0.1	6665.8	4812.4	11478	6830.5	4680.5	11511	6703.7	4936.7	11640	0.57	2.52	1.39
0.2	6665.8	4800.8	11467	6830.5	4668.7	11499	6703.7	4921.7	11625	0.57	2.46	1.37
0.3	6665.0	4786.3	11451	6830.2	4654.5	11485	6703.7	4905.7	11609	0.58	2.43	1.36
0.4	6665.0	4770.0	11435	6830.2	4638.1	11468	6703.7	4888.6	11592	0.58	2.43	1.36
0.5	6665.0	4753.8	11419	6830.2	4621.8	11452	6703.7	4869.9	11574	0.58	2.39	1.34
0.6	6664.9	4734.6	11400	6830.3	4602.7	11433	6703.7	4849.3	11553	0.58	2.37	1.33
0.7	6664.9	4713.7	11379	6829.2	4582.7	11412	6703.7	4825.9	11530	0.58	2.32	1.31
0.8	6665.7	4687.5	11353	6828.7	4556.8	11385	6703.7	4798.2	11502	0.57	2.31	1.29
0.9	6664.3	4652.5	11317	6830.1	4520.9	11351	6703.7	4762.0	11466	0.59	2.30	1.30

Table 5.2.3 Example3 Model 3 Percentage Saving Data

5.3 Results of Objective Function 2: Minimizing the Total System Cost

$$\text{Min. } TCS\_qd = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \{\sum_{i=1}^{n}(\frac{C_i + A_i}{k_i T} + d_i[\frac{hD}{P} - 0.5(h - h_i p_d(i))]k_i T\} + (p_s + p_m)D \qquad (5.3.1)$$

where $p_d(i) = p_s - [p_s \frac{h_i}{2}(k_i T - T_i^*) - \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})]$

subject to: $T_i^* \le k_i T \le T_i^* + \frac{2}{h_i}$

Results:

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S	TCBS_ind	TCV_ind	TCS_ind
0.1	123.41	102.29	225.70	0.44	11.53	5.79	123.95	115.62	239.58
0.2	123.47	102.91	226.38	0.39	10.49	5.25	123.95	114.97	238.92
0.3	123.70	103.31	227.01	0.20	9.60	4.71	123.95	114.27	238.23
0.4	123.70	103.50	227.20	0.20	8.83	4.33	123.95	113.53	237.48
0.5	123.73	103.66	227.39	0.18	8.04	3.92	123.95	112.71	236.67
0.6	123.86	103.65	227.51	0.08	7.30	3.50	123.95	111.81	235.77
0.7	123.86	103.36	227.21	0.08	6.71	3.21	123.95	110.79	234.75
0.8	123.86	103.06	226.92	0.08	5.95	2.83	123.95	109.58	233.54
0.9	123.89	102.20	226.09	0.05	5.38	2.53	123.95	108.01	231.96

Table 5.3.1 Example1 Model 3 Percentage Saving Data

The results of Example 1 (Table 5.3.1) show that the vendor's cost in the co-ordinated model with quantity discount is still better than his cost in the independent policy. All the percentage cost saving for the vendor, buyers and the system are positive numbers. Hence, each system member can get benefit in the co-ordination model.

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S	TCBS_ind	TCV_ind	TCS_ind
0.1	1273.9	2636.2	3910.1	46.43	-46.22	6.48	2378.2	1802.9	4181.2
0.2	1273.9	2633.5	3907.4	46.43	-46.62	6.4	2378.2	1796.2	4174.4
0.3	1273.9	2630.7	3904.7	46.43	-47.06	6.3	2378.2	1788.9	4167.1
0.4	1316.6	2585.2	3901.8	44.64	-45.14	6.19	2378.2	1781.2	4159.4
0.5	1233.4	2663.4	3896.8	48.14	-50.24	6.12	2378.2	1772.7	4150.9
0.6	1233.4	2651.3	3884.7	48.14	-50.36	6.2	2378.2	1763.4	4141.6
0.7	1346.7	2524	3870.7	43.37	-44	6.3	2378.2	1752.8	4131
0.8	1346.7	2508.2	3854.9	43.37	-44.14	6.4	2378.2	1740.2	4118.4
0.9	1190.5	2641.4	3831.9	49.94	-53.23	6.58	2378.2	1723.8	4102

Table 5.3.2 Example2 Model 3 Percentage Saving Data

There exists the situation in Example 2 (Table 5.3.2) that the total vendor's cost in the co-ordinated model with quantity discount, *TCV_qd*, is larger than that in the independent policy, *TCV_ind*. However, the buyers' cost has a large reduction ranging from 43.37% to 49.94%, and the system cost also has a reduction of about 6% when compared with the independent policy. Such results may be due to the reason that some buyers (5^{th}, 14^{th}, 18^{th}, 21^{th}, 25^{th}, 28^{th}) get a very large discount from the vendor since his ordering cycle $k_i T$ is much longer than T_i^* (Figure 5.1).

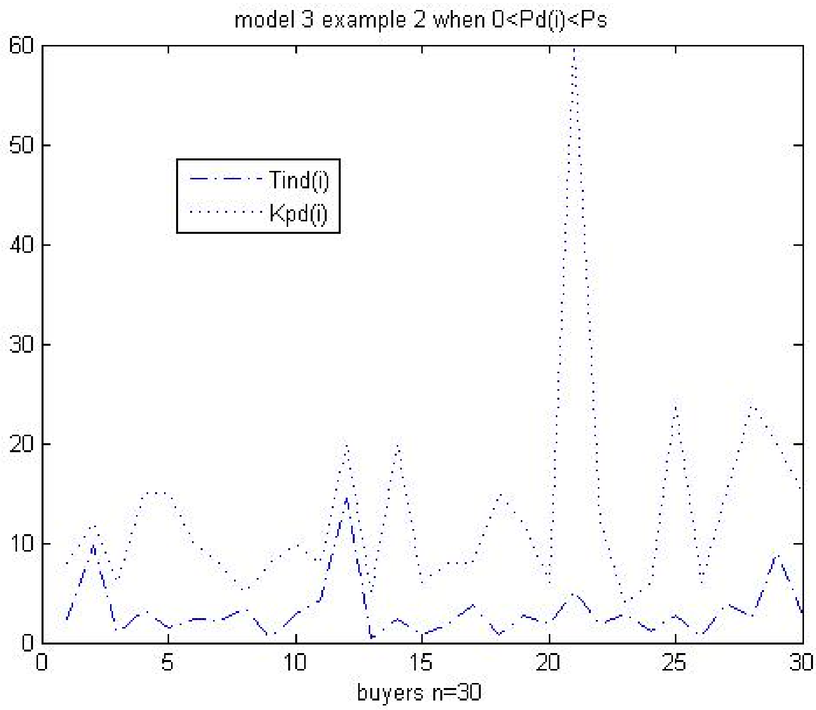

Figure 5.1 Example 2 Model 3 Comparison of Buyers' Ordering Cycle

From the vendor's viewpoint, it is not advantageous to offer this kind of discount to the buyer even if it reduces the system cost.

In Example 3 (Table 5.3.3), the total vendor's cost in the co-ordinated model with discount is also larger than that in the independent policy.

D/P	inc_BS	inc_V	inc_S
0.1	36.85	-39.46	4.49
0.2	36.85	-39.44	4.55
0.3	36.85	-39.44	4.61
0.4	36.84	-39.47	4.66
0.5	36.84	-39.55	4.7
0.6	36.84	-39.68	4.72
0.7	36.84	-39.9	4.72
0.8	44.29	-50.6	4.71
0.9	40.83	-46.01	4.76

Table 5.3.3 Example 3 Model 3 Percentage Saving Data

Hence, we should have a constraint of the discounts that the vendor could offer. On one hand, the discounted price $p_d(i)$ should be less than p_s, this derives that $k_i T \geq T_i^*$, on the other hand, $p_d(i)$ should be larger than the vendor's manufacturing cost per unit p_m, i.e.

$$p_d(i) = p_s[1 - \frac{h_i}{2}(k_i T - T_i^*)] + \frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T}) \geq p_m \tag{5.3.2}$$

As $\frac{A_i}{d_i}(\frac{1}{T_i^*} - \frac{1}{k_i T})$ must be positive since $k_i T \geq T_i^*$, the equation can be simplified such that

$$p_s[1 - \frac{h_i}{2}(k_i T - T_i^*)] \geq p_m$$

Hence,

$$k_i T \leq T_i^* + \frac{2(1 - \frac{p_m}{p_s})}{h_i} \tag{5.3.3}$$

Therefore, the modified constraint is :

$$T_i^* \le k_i T \le T_i^* + \frac{2(1-\frac{p_m}{p_s})}{h_i} \tag{5.3.4}$$

By using this new constraint in Examples 1-3, the results are shown in Tables 5.3.4, 5.3.5 & 5.3.6.

D/P	TCBS_qd	TCV_qd	TCS_qd	TCBS_ind	TCV_ind	TCS_ind
0.1	123.407	102.2916	225.6986	123.9529	115.6222	239.5751
0.2	123.4721	102.9107	226.3828	123.9529	114.9689	238.9218
0.3	123.7015	103.3083	227.0098	123.9529	114.2732	238.2261
0.4	123.7015	103.4968	227.1983	123.9529	113.5259	237.4788
0.5	123.7305	103.6562	227.3867	123.9529	112.7132	236.6661
0.6	123.8586	103.6486	227.5073	123.9529	111.8143	235.7672
0.7	123.8586	103.3561	227.2148	123.9529	110.794	234.7469
0.8	123.8586	103.0636	226.9223	123.9529	109.5837	233.5366
0.9	123.8876	102.1998	226.0873	123.9529	108.0064	231.9593

Table 5.3.4 Example 1 Model 3 with Constraint

The results of Example 1 (Table 5.3.4) are the same as those of Example 1 with the initial constraint $T_i^* \le k_i T \le T_i^* + \frac{2}{h_i}$ (or $0 \le p_d(i) \le p_s$).

The results of Example 2 and 3 (Table 5.3.5, and Table 5.3.6) with constraint $T_i^* \le k_i T \le T_i^* + \frac{2(1-\frac{p_m}{p_s})}{h_i}$ show that Model 3 can work better than the independent policy. All the percentage cost savings for the vendor, buyers and system are positive numbers. The vendor and each buyer can reduce his cost when compared with his independent cost. Moreover, the system cost in Model 3 (*TCS_qd*) is also less than that of the co-ordinated model without quantity discount (*TCS_cor*).

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S	TCBS_ind	TCV_ind	TCS_ind	TCS_cor
0.1	2363.2	1729.6	4092.8	0.63	4.07	2.11	2378.2	1802.9	4181.2	4101.5
0.2	2361.5	1724.7	4086.2	0.7	3.98	2.11	2378.2	1796.2	4174.4	4096
0.3	2361.5	1717.9	4079.5	0.7	3.97	2.1	2378.2	1788.9	4167.1	4089.2
0.4	2361.6	1711.2	4072.7	0.7	3.93	2.08	2378.2	1781.2	4159.4	4082.4
0.5	2361.6	1704.4	4066	0.7	3.85	2.05	2378.2	1772.7	4150.9	4075.6
0.6	2363.2	1695.6	4058.8	0.63	3.84	2	2378.2	1763.4	4141.6	4067.2
0.7	2363.1	1686.1	4049.3	0.63	3.8	1.98	2378.2	1752.8	4131	4057.6
0.8	2361.5	1674.8	4036.3	0.7	3.76	1.99	2378.2	1740.2	4118.4	4045.9
0.9	2361.6	1659.6	4021.2	0.7	3.72	1.97	2378.2	1723.8	4102	4030.7

Table 5.3.5 Example 2 Model 3 with Constraint

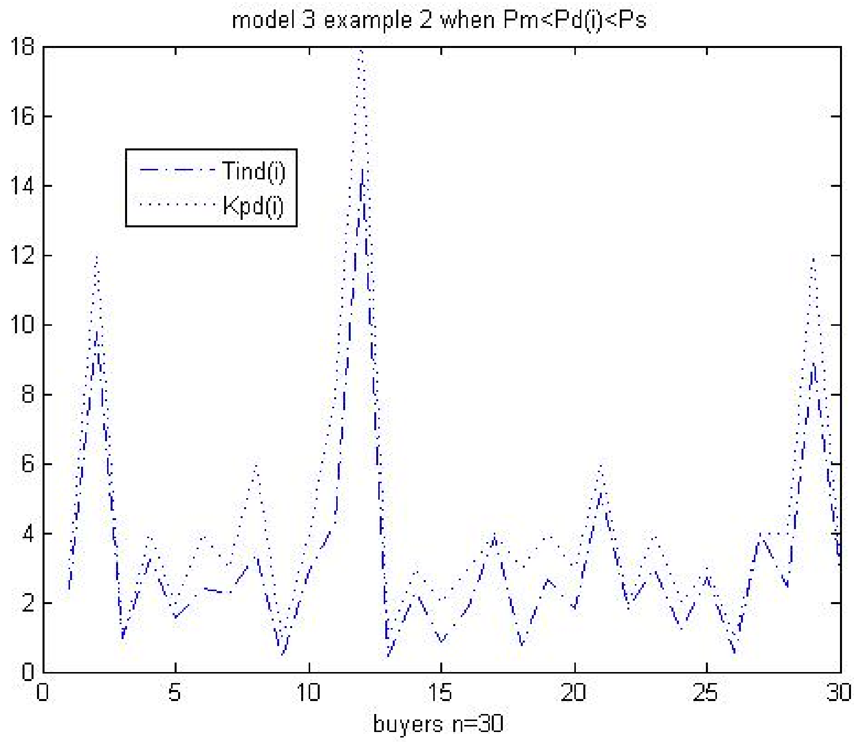

Figure 5.2 Example 2 Model 3 Comparison of Buyers'

Ordering Cycle with Constraint

Figure 5.1 shows that some buyers (5^{th}, 14^{th}, 18^{th}, 21^{th}, 25^{th}, 28^{th}) of Example 2 get a very large discount from the vendor since their ordering cycles k_iT in Model 3 are much longer than their T_i^* in the independent policy. It can be seen in Figure 5.2 that the ordering cycle k_iT of buyers in Model 3 is close to their optimal independent T_i^* when constraint (5.3.4) is introduced. For example, the k_iT of the 21^{st} buyer in Figure 5.1 without the constraint is about 60, but is very much reduced to 6 in Figure 5.2. Hence Model 3 with the constraint (5.3.4) can help to control the discounts in Model 3. This is also explained by Figure 5.4, that the optimal discounted price with the constraint Pm is much reduced when compared with the optimal discounted price without the constraint.

Similar results for Example 3 can also be found in Figure 5.3 and Figure 5.5.

D/P	TCBS_qd	TCV_qd	TCS_qd	inc_BS	inc_V	inc_S	TCBS_ind	TCV_ind	TCS_ind	TCS_cor
0.1	6661	4817.4	11478	0.64	2.42	1.39	6703.7	4936.7	11640	11511
0.2	6661	4805.8	11467	0.64	2.35	1.36	6703.7	4921.7	11625	11499
0.3	6659.5	4792.7	11452	0.66	2.3	1.35	6703.7	4905.7	11609	11485
0.4	6659.5	4776.5	11436	0.66	2.29	1.35	6703.7	4888.6	11592	11468
0.5	6659.5	4760.3	11420	0.66	2.25	1.33	6703.7	4869.9	11574	11452
0.6	6660.6	4740.3	11401	0.64	2.25	1.32	6703.7	4849.3	11553	11433
0.7	6660.6	4719.5	11380	0.64	2.21	1.3	6703.7	4825.9	11530	11412
0.8	6660.5	4693.3	11354	0.64	2.18	1.29	6703.7	4798.2	11502	11385
0.9	6659.8	4659.3	11319	0.66	2.16	1.28	6703.7	4762	11466	11351

Table 5.3.6 Example 3 Model 3 with Constraint

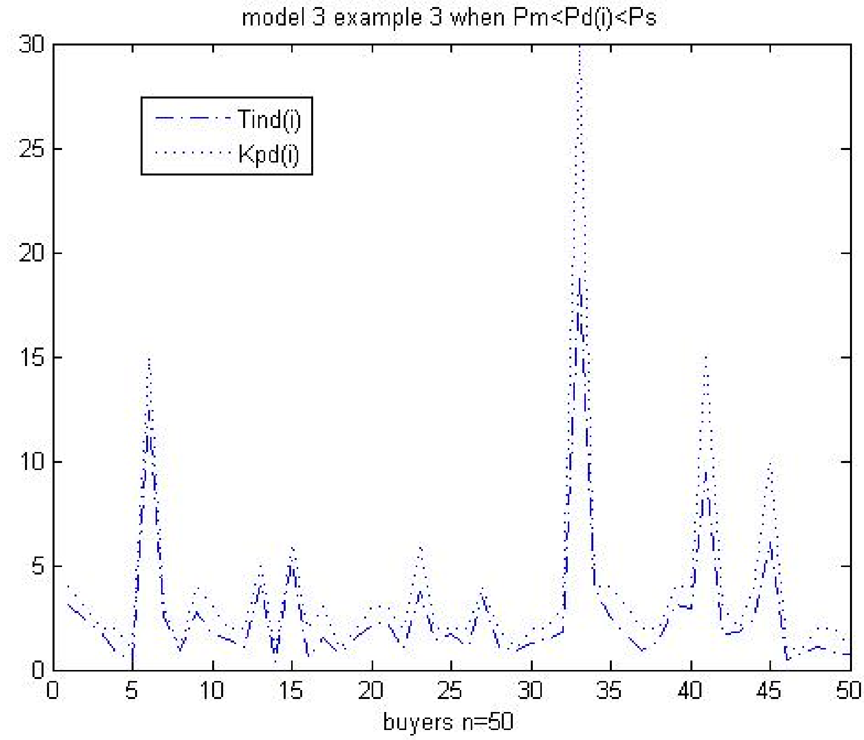

Figure 5.3 Example 3 Model 3 Comparison of Buyers'

Ordering Cycle with Constraint

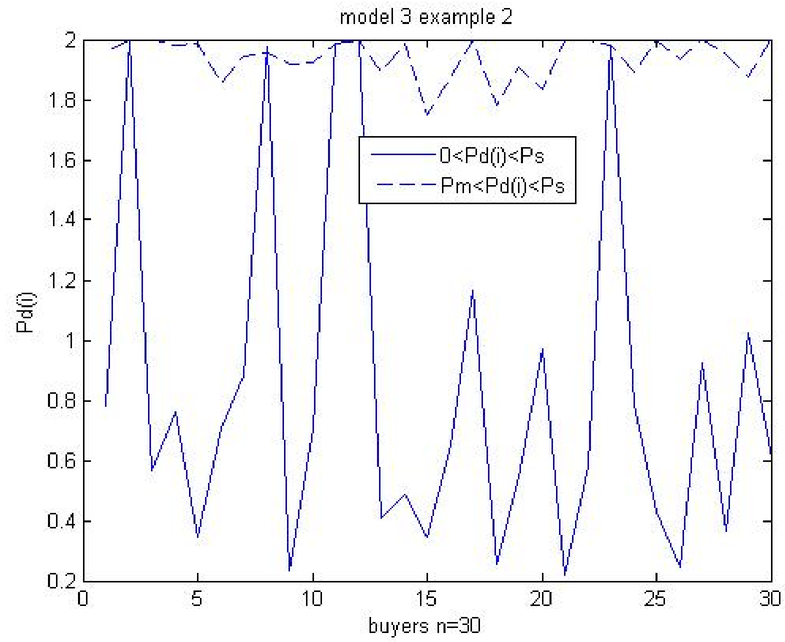

Figure 5.4 Example 2 Model 3 Comparison of the Discounted Price

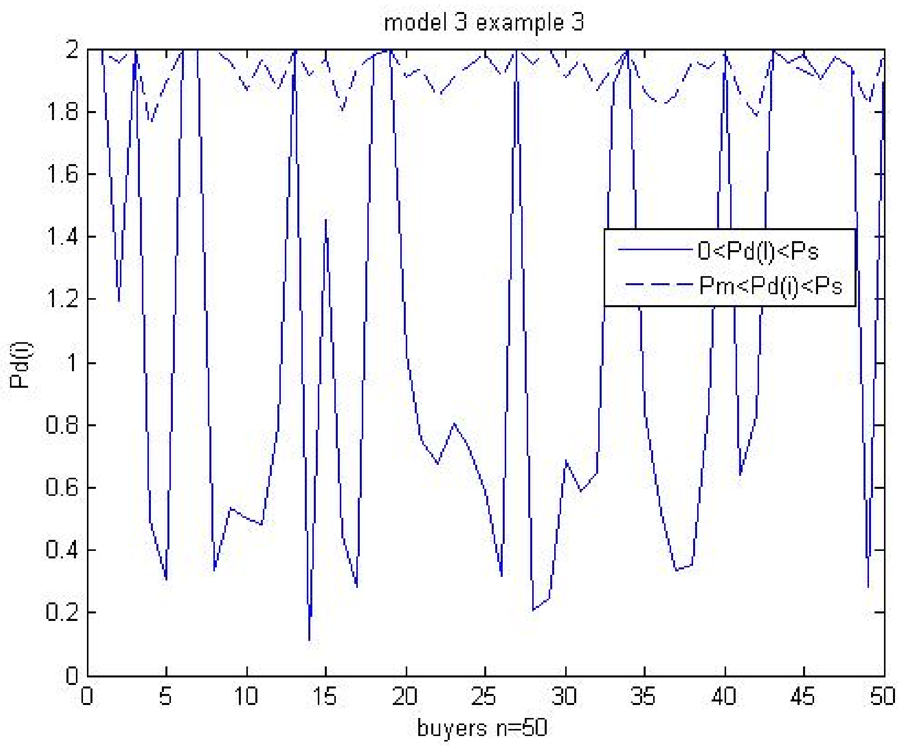

Figure 5.5 Example 3 Model 3 Comparison of the Discounted Price

5.4 Conclusions

Model 3 is similar to the "break-even price discount" in Monahan's model(1984), but is applied to the multi-buyer synchronized cycles model. Two objective functions are considered in Model 3, one is to minimize the vendor's cost, and the other is to minimize total system's cost. Table 5.4.1 shows that under both of the two objective functions, Model 3 has a positive system cost saving in the coordination with quantity discount when compared with the independent policy cost.

Model 3 can ascertain that each buyer can reduce his cost when compared with his independent cost. In minimizing vendor's cost, all the three examples show that both the vendor and the buyer get a cost reduction. However, a modified constraint has to be introduced to the model when the total system cost is minimized.

In Tables 5.4.2, 5.4.3 and 5.4.4 where the objective function is to minimize the total system cost, the lower boundaries of the constraint of p_d is 0, 0.85 and p_m respectively in the three tables. When the lower boundary of the constraint of p_d ($0 \le p_d(i) \le p_s$) increases from 0 to p_m, the buyers' percentages saving *inc_BS* is decreased, the vendor's percentage saving *inc_V* is increased, and the system's percentage saving *inc_S* is decreased. Example 1 is an exception since the buyers' holding cost h_i's are very small values when compared with examples 2 and 3.

So, setting the lower boundary of the constraint of p_d can help the vendor to control the discounts. Certainly, a discounted price should not be less than the vendor's manufacturing cost p_m.

Result of system percentage saving in Model 3

D/P	Minsc			Minvc		
	Ex.1	Ex.2	Ex.3	Ex.1	Ex.2	Ex.3
0.1	5.79	2.11	1.39	5.78	2.23	1.39
0.2	5.25	2.11	1.36	5.24	2.18	1.37
0.3	4.71	2.1	1.35	4.7	2.17	1.36
0.4	4.33	2.08	1.35	4.33	2.15	1.36
0.5	3.92	2.05	1.33	3.92	2.11	1.34
0.6	3.5	2	1.32	3.5	2.12	1.33
0.7	3.21	1.98	1.3	3.21	2.1	1.31
0.8	2.83	1.99	1.29	2.83	2.06	1.29
0.9	2.53	1.97	1.28	2.53	2.04	1.3

Table 5.4.1 Model 3 Percentage of System Saving (*inc_S*) under Two Objective Functions

D/P	Ex.1 inc_BS	Ex.1 inc_V	Ex.2 inc_BS	Ex.2 inc_V	Ex.3 inc_BS	Ex.3 inc_V
0.1	0.44	11.53	46.43	-46.22	36.85	-39.46
0.2	0.39	10.49	46.43	-46.62	-39.46	4.49
0.3	0.2	9.6	46.43	-47.06	-39.44	4.55
0.4	0.2	8.83	44.64	-45.14	-39.44	4.61
0.5	0.18	8.04	48.14	-50.24	-39.47	4.66
0.6	0.08	7.3	48.14	-50.36	-39.55	4.7
0.7	0.08	6.71	43.37	-44	-39.68	4.72
0.8	0.08	5.95	43.37	-44.14	-39.9	4.72
0.9	0.05	5.38	49.94	-53.23	-50.6	4.71

Table 5.4.2 Model 3 System Saving Data of Minimizing System’s Cost $0<Pd(i)<Ps$

D/P	Ex.1 inc_BS	Ex.1 inc_V	Ex.2 inc_BS	Ex.2 inc_V	Ex.3 inc_BS	Ex.3 inc_V
0.1	0.44	11.53	2.64	2.29	2.05	0.72
0.2	0.39	10.49	2.71	2.16	2.05	0.68
0.3	0.2	9.6	2.71	2.13	1.62	1.22
0.4	0.2	8.83	2.71	2.08	1.62	1.2
0.5	0.18	8.04	2.71	1.99	2.24	0.31
0.6	0.08	7.3	2.8	1.81	2.24	0.31
0.7	0.08	6.71	2.8	1.77	2.24	0.25
0.8	0.08	5.95	2.97	1.57	1.68	0.98
0.9	0.05	5.38	2.97	1.47	2	0.55

Table 5.4.3 Model 3 System Saving Data of Minimizing System’s Cost $0.85<Pd(i)<Ps$

D/P	Ex.1 inc_BS	Ex.1 inc_V	Ex.2 inc_BS	Ex.2 inc_V	Ex.3 inc_BS	Ex.3 inc_V
0.1	0.44	11.53	0.63	4.07	0.64	2.42
0.2	0.39	10.49	0.7	3.98	0.64	2.35
0.3	0.2	9.6	0.7	3.97	0.66	2.3
0.4	0.2	8.83	0.7	3.93	0.66	2.29
0.5	0.18	8.04	0.7	3.85	0.66	2.25
0.6	0.08	7.3	0.63	3.84	0.64	2.25
0.7	0.08	6.71	0.63	3.8	0.64	2.21
0.8	0.08	5.95	0.7	3.76	0.64	2.18
0.9	0.05	5.38	0.7	3.72	0.66	2.16

Table 5.4.4 Model 3 System Saving Data of Minimizing System’s Cost $Pm<Pd(i)<Ps$

Chapter 6

Trade Credit Policy

6.1. Introduction

In the real world, a supplier often makes use of a trade credit policy to promote his commodities. Suppliers often resort to the practice of offering extended payment privileges to a retailer which is quite prevalent in some industries today. Such credit policies may be applied as an alternative to price discounts to induce larger orders, because such policies are not thought to provoke competitors to reduce their prices and thus introduce lasting price reductions, or because such policies are traditional in the firm's industry.

The objective of this chapter is to propose a co-ordinated single-vendor multi-buyer supply chain model by synchronizing ordering and production cycles with a trade credit policy. Firstly, we will develop a model considering the vendor as a manufacturer producing an item to supply multiple heterogeneous buyers, which also incorporates a trade credit policy that can guarantee that every buyer will at most has the same total inventory costs as in independent optimization. As no additional cost will be incurred, the buyers will be motivated to participate in the proposed co-ordination. Secondly, we will develop an algorithm to minimize the total relevant cost of the co-ordinated system. Finally, we also develop an equitable profit-sharing scheme that makes a fair situation to all the parties in the supply chain.

6.2 The Independent Policy and the Synchronized Cycles Model

Notations and assumptions can be seen in Section 2.2 and 2.3 of the Quantity Discount Models chapter. The differences are: firstly, we do not consider the purchasing and manufacturing costs in trade credit policy; secondly, the buyer's inventory holding cost is independent of the capital cost (i.e. purchasing cost).

The total cost per unit time incurred by the i^{th} buyer can be expressed as:

$$TC_i^{IND}\left(Q_i\right) = \frac{A_i d_i}{Q_i} + \frac{h_i Q_i}{2} \qquad (6.2.1)$$

By applying the basic economic order quantity (EOQ) model, the optimal ordering quantity is

$$Q_i = \sqrt{\frac{2A_i d_i}{h_i}} \qquad (6.2.2)$$

and $T_i^{IND} = \frac{Q_i}{d_i} = \sqrt{\frac{2A_i}{d_i h_i}}$ units of time.

Hence total cost per unit time incurred by the i^{th} buyer can then be expressed as:

$$TC_i^{IND} = \sqrt{2A_i h_i d_i} \qquad (6.2.3)$$

The total cost per unit time incurred by the vendor can be expressed as:

$$TC_v^{IND}(Q_v) = \frac{S_v D}{Q_v} + \frac{h_v Q_v}{2}\left(1 - \frac{D}{P}\right) + \sum_{i=1}^{n} \frac{C_i d_i}{Q_i} + h_v \sum_{i=1}^{n} Q_i \qquad (6.2.4)$$

By the standard inventory model, the optimal economic batch production is

$$Q_v = \sqrt{\frac{2S_vD}{h_v\left(1-\frac{D}{P}\right)}} \tag{6.2.5}$$

and the total system cost for the independent policy model is

$$TC^{IND}(Q_1, Q_2, \ldots, Q_v) = \sqrt{2S_vh_vD\left(1-\frac{D}{P}\right)} + \sum_{i=1}^{n}\left(\frac{C_id_i}{Q_i} + \sqrt{2A_id_ih_i} + h_vQ_i\right). \tag{6.2.6}$$

The total relevant cost of the whole system in the Synchronized Cycles Model is given by:

$$\begin{aligned} TC^{SCA}(k_i, N, T) = &\left\{\frac{S_v}{NT} + \left[\frac{h_vD}{2} - \frac{h_vD^2}{2P}\right]NT\right\} + \\ &\left\{\sum_{i=1}^{n}\left(\frac{C_i + A_i}{k_iT} + d_i\left[\frac{h_v}{P}D - \frac{1}{2}(h_v - h_i)\right]k_iT\right)\right\} \end{aligned} \tag{6.2.7}$$

6.3 The Trade Credit Policy

As discussed in Chapter 2, it is shown in Chan and Kingsman (2005, 2007) that the synchronized cycles model can be used to plan the ordering intervals in a one-vendor many-buyer supply chain so as to reduce significantly the system costs compared to each partner operating completely independently. However, the total relevant cost of the i^{th} buyer of the co-ordinated system is always higher than that of independent optimization. Hence, a buyer would not be motivated to participate in the co-ordination. In order to motivate the buyer, we propose a trade credit policy as an incentive mechanism in the synchronized cycles model.

Throughout this Chapter, we assume that the holding cost for both the vendor (h_v) and the buyers (h_i) are divided into two components, one associated with the opportunity cost of capital (h_v^c and h_i^c), and the other associated with the storage cost per unit of item (h_v^s and h_i^s) per unit time. If the vendor allows a credit period of M_i to buyer *i*, the vendor would need to bear an additional capital opportunity cost of $\sum_{i=1}^{n} h_v^c d_i M_i$; while the buyer gains a capital opportunity benefit during this credit period and thus the cost borne by the buyer can be discounted by an amount of $\sum_{i=1}^{n} h_i^c d_i M_i$ from his original total cost.

Hence, the total buyer cost and total vendor cost under the trade credit policy is:

Buyer Cost: $\dfrac{A_i}{k_i T} + \dfrac{1}{2} h_i d_i k_i T - h_i^c d_i M_i$ (6.3.1)

Vendor Cost: $\dfrac{S_v}{NT} + \left(\dfrac{h_v D}{2} - \dfrac{h_v D^2}{2P} \right) NT + \sum_{i=1}^{n} d_i h_v \left(\dfrac{D}{P} - 0.5 \right) k_i T + \sum_{i=1}^{n} \dfrac{C_i}{k_i T} + \sum_{i=1}^{n} h_v^c d_i M_i$

(6.3.2)

The total relevant cost of the system with trade credit period M_i is therefore:

$$\left\{ \frac{S_v}{NT} + \left(\frac{h_v D}{2} - \frac{h_v D^2}{2P} \right) NT + \sum_{i=1}^{n} \frac{A_i + C_i}{k_i T} + \sum_{i=1}^{n} d_i h_v \left(\frac{D}{P} - 0.5 \right) k_i T \right\} + \sum_{i=1}^{n} \frac{1}{2} d_i h_i k_i T + \sum_{i=1}^{n} \left(h_v^c - h_i^c \right) d_i M_i \qquad (6.3.3)$$

To make the co-ordination possible, the vendor should determine the value of M_i such that the buyers are not worse off when compared to the independent policy, i.e.

$$\frac{A_i}{k_i T} + \frac{1}{2} h_i d_i k_i T - h_i^c d_i M_i \le \sqrt{2 A_i h_i d_i}$$

$$\Rightarrow M_i \ge \frac{\frac{A_i}{k_i T} + \frac{1}{2} h_i d_i k_i T - \sqrt{2 A_i h_i d_i}}{h_i^c d_i} \qquad (6.3.4)$$

In the vendor's perspective, it is more preferable that the trade credit period is as short as possible (i.e. M_i equals the right-hand side of Eq (6.3.4)); while in the buyer's perspective, M_i should satisfy Eq (6.3.4) and of course, the longer the trade credit period the more the buyer can be benefited. If we impose the minimum value of M_i ($M_{\min}(i)$) in Eq (6.3.4), the cost borne by the buyers is the same as that of the independent policy and this may not be sufficient to entice all the buyers to participate in the synchronized cycles co-ordination. Therefore, it is necessary for the vendor to set a "fair" length of the trade credit period so that both parties are willing to co-ordinate.

To cope with the above, we need to determine the longest period $M_{\max}(i)$ the vendor is still better off under the synchronized cycles model when compared with the independent policy.

The value of $M_{\max}(i)$ can be determined by setting $TC_V^M = TC_V^{IND}$, that is:

$$\sum_{i=1}^{n} d_i M_{\max}(i) h_v^c = \left[\sqrt{2S_v h_v D(1-\frac{D}{P})} + \sum_{i=1}^{n} \frac{C_i}{T_i^*} + h_v \sum_{i=1}^{n} Q_i \right] - \left[\frac{S_v}{NT} + (\frac{h_v D}{2} - \frac{h_v D^2}{2P}) NT + \sum_{i=1}^{n} \frac{C_i}{k_i T} + \sum_{i=1}^{n} d_i h_v (\frac{D}{P} - 0.5) k_i T \right] \qquad (6.3.5)$$

The right hand side of Eq. (6.3.5) is the difference of the vendor's independent cost and his cost in the co-ordination, which should equal the vendor's extra capital cost during the buyers' maximum delay periods, and the total system surplus goes to buyers' side.

The co-ordination is feasible for all M_i satisfying the constraint $M_{min}(i) \le M_i \le M_{max}(i)$. However, the vendor would choose $M_i = M_{min}(i)$ to optimize his own profit. Similarly, the buyers would prefer $M_i = M_{max}(i)$.

6.4 An Equitable Profit-Sharing Scheme

To obtain a fair co-ordination, an equitable sharing scheme is developed to determine M_i such that both the vendor and all the buyers should have a certain amount of cost savings when compared to their costs under the independent policy.

Initially, we apply the algorithm developed in Chan and Kingsman (2007) to determine the solutions of the synchronized cycles model. Then, we seek an appropriate value of M_i which leads to an equitable division of the system surplus between the buyers and the vendor.

The total costs per unit time for the vendor in the independent policy can be expressed as

$$TC_V^{IND} = \sqrt{2S_v h_v D\left(1-\frac{D}{P}\right)} + \sum_{i=1}^{n}\frac{C_i}{T_i^*} \qquad (6.4.1\ )$$

The total costs per unit time for the vendor in the coordination model with credit period can be expressed as

$$TC_V^M = \left\{\frac{S_v}{NT} + \left(\frac{h_v D}{2} - \frac{h_v D^2}{2P}\right)NT + \sum_{i=1}^{n}\frac{C_i}{k_i T} + \sum_{i=1}^{n} d_i h_v\left(\frac{D}{P}-0.5\right)k_i T\right\} + \sum_{i=1}^{n} h_v^c d_i M_i$$

(6.4.2)

The terms inside the curly bracket is the total cost borne by the vendor under the synchronized cycles model.

We can then obtain the value of $M_{\max}(i)$ by setting $TC_V^M = TC_V^{IND}$,

$$\sum_{i=1}^{n} d_i M_{\max}(i) h_v^c = \left[\sqrt{2S_v h_v D\left(1-\frac{D}{P}\right)} + \sum_{i=1}^{n}\frac{C_i}{T_i^*}\right] - \left[\frac{S_v}{NT} + \left(\frac{h_v D}{2} - \frac{h_v D^2}{2P}\right)NT + \sum_{i=1}^{n}\frac{C_i}{k_i T} + \sum_{i=1}^{n} d_i h_v\left(\frac{D}{P}-0.5\right)k_i T\right]$$

(6.4.3)

The total system cost with trade credit is :

$$TC^M = TC_V^M + \sum TC_i^M = TC^{SCA} + \sum_{i=1}^{n}(h_v^c - h_i^c) d_i M_i \qquad (\ 6.4.4)$$

The total surplus of the system's savings with trade credit (S^M) is:

$$S^M(M_i) = S^{SCA} + \sum_{i=1}^{n}(h_i^c - h_v^c)d_i M_i \qquad (6.4.5)$$

where S^{SCA} is the surplus obtained by co-ordination by the synchronized cycles model.

Based upon (6.4.5), we can conclude that

- if $h_i^c > h_v^c$, then the trade credit M_i increases the system surplus when compared with the synchronized cycles model without trade credit.
- if $h_i^c < h_v^c$, then the trade credit M_i decreases the system surplus when compared with the synchronized cycles model without trade credit.
- if $h_i^c = h_v^c$, then the trade credit M_i has no influence to the total system cost, it is only a mechanism to re-distribute the surplus between buyers and the vendor.

6.4.1 Determine the Minimum and Maximum Trade Credit Periods

If the vendor offers a trade credit for $M_{\min}(i)$ periods for the i^{th} buyer, then the total surplus $M_S^* = S^M(M_{\min}(i))$ is absorbed by the vendor and each buyer has the same cost as under the independent policy, i.e.

$$TC_V^{IND} - \left(TC_V^{SCA} + \sum_{i=1}^{n} h_v^c d_i M_{\min}(i) \right) = M_S^* \qquad \text{(Vendor)}$$

$$\sum TC_i^{IND} - \left(\sum TC_i^{SCA} - \sum_{i=1}^{n} h_i^c d_i M_{\min}(i) \right) = 0 \qquad \text{(Buyers)}$$

On the other hand, if the vendor offers a trade credit for $M_{\max}(i)$ periods for the i^{th} buyer, then the total surplus $M_S^{**} = S^M(M_{\max}(i))$ is absorbed by the buyers and the vendor has the same cost as under the independent policy, i.e.

$$TC_V^{IND} - \left(TC_V^{SCA} + \sum_{i=1}^{n} h_v^c d_i M_{\max}(i) \right) = 0 \qquad \text{(Vendor)}$$

$$\sum TC_i^{IND} - \left(\sum TC_i^{SCA} - \sum_{i=1}^{n} h_i^c d_i M_{\max}(i) \right) = M_S^{**} \qquad \text{(Buyers)}$$

and this gives

$$M_S^* = \sum_{i=1}^{n} h_v^c d_i M_{\max}(i) - \sum_{i=1}^{n} h_v^c d_i M_{\min}(i) = h_v^c \cdot \beta \qquad (\ 6.4.1.1\ )$$

$$M_S^{**} = \sum_{i=1}^{n} h_i^c d_i M_{\max}(i) - \sum_{i=1}^{n} h_i^c d_i M_{\min}(i) \geq \min\left\{ h_i^c \right\} \cdot \beta \qquad (\ 6.4.1.2)$$

where $\beta = \sum_{i=1}^{n} d_i \left(M_{\max}(i) - M_{\min}(i) \right)$.

To obtain a feasible coordination with trade credit policy, we must have

$$S_{\min}^M = \min\left\{ \min\left\{ h_i^c \right\}, h_v^c \right\} \cdot \beta \,. \qquad (\ 6.4.1.3\ )$$

where $S_{\min}^M$ is the minimum of system surplus in the co-ordination with $M_{\min}(i) \leq M(i) \leq M_{\max}(i)$.

6.4.2 Algorithm for Determining the "Equitable" Trade Credit Period

An extra credit period ΔM_i should be added to the buyer's $M_{\min}(i)$ to ascertain that the system surplus is divided equitably between the buyers and the vendor. ΔM_i can be determined by

$$\sum_{i=1}^{n}\left(d_i h_i^c \Delta M_i\right) = \frac{S_{\min}^M}{\alpha}, \qquad (6.4.2)$$

where $\alpha \geq 1$ is negotiated among the vendor and the buyers, and the credit period given to the i^{th} buyers becomes $M_i = M_{\min}(i) + \Delta M_i$.

From the viewpoint of the vendor, he is sure of getting benefit in the coordination model if $\alpha \geq 1$. The $S_{\min}^M$ is the minimum system surplus with all possible delay periods in the coordination model that the vendor and buyer can share among them.

ΔM_i is the extra delay period offered to each buyer which will lead to the buyer's cost savings in the coordination model when compared with his independent policy. If we want that each buyer and the vendor have same percentage improvement in the coordination, that is an equitable division of system surplus between members, so ΔM_i can be calculated and be different value among buyers.

For simplicity, we assume $\Delta M_i = \Delta M$ for all buyers and $\alpha = 2$ (The vendor gets half of the surplus and the rest are shared among the buyers) and the algorithm can be stated as:

Step1: Calculate $M_{\min}(i)$ by (6.3.4).

Step2: Calculate $\sum_{i=1}^{n} d_i M_{\max}(i) h_v^c$ by (6.4.3).

Step3: Calculate M_S^* by (6.4.1.1).

Step 4: Calculate $\beta = \dfrac{M_S^*}{h_v^c}$.

Step 5: Calculate $S_{\min}^M$ by (6.4.1.3).

Step 6: Calculate $\Delta M = \dfrac{S_{\min}^M / \alpha}{\sum_{i=1}^{n} d_i h_i^c}$.

Step 7: The trade credit period for the i^{th} buyer is then equal to $M_i = M_{\min}(i) + \Delta M$

6.5 Results and Discussions

Some numerical experiments have been carried out to illustrate the performance of the synchronized cycles model with trade credit policy. These results are compared with the performance of the synchronized cycles model without trade credit policy (Chan and Kingsman, 2007) as well as that of buyers and supplier operating independently. Three examples (Chan and Kingsman, 2007) are used in our experiments, and some additional data about capital cost are also assumed. The data are shown in Appendix 1. For each example, we also consider three different cases of vendor's and buyers' capital cost structure, that is:

(i) $\min\{h_i^c\} = h_v^c$, i.e. buyers' minimum capital cost equals vendor's capital cost;

(ii) $\min\{h_i^c\} < h_v^c$, i.e. buyers' minimum capital cost is less than vendor's capital cost;

(iii) $\min\{h_i^c\} > h_v^c$, i.e. buyers' minimum capital cost is larger than vendor's capital cost.

See Appendix 9 for details of N, k_i and M_i of the three examples.

Example 1 is that used by Banerjee and Burton (1994) common order cycle method, multiple buyers data were randomly generated for 30 buyers and 50 buyers in Examples 2 and 3, respectively. This enables us to see if the results for the many buyers case differ from those with only a few buyers. We also include a full range of different values of D/P from 0.1, 0.2, …, up to 0.9 for comparison in our experiments, since the vendor's inventory cost as well as the total relevant cost depend directly on the ratio D/P in the synchronized cycles model.

D/P	Case 1			Case 2			Case 3			Case 4								
	TC_{BS}^{TCR}	TC_{V}^{TCR}	TC^{TCR}	TC_{BS}^{TCR}	TC_{V}^{TCR}	TC^{TCR}	TC_{BS}^{TCR}	TC_{V}^{TCR}	TC^{TCR}	TC_{BS}^{TCR}	TC_{V}^{TCR}	TC^{TCR}	TC_{BS}^{IND}	TC_{V}^{IND}	TC^{IND}	TC_{BS}^{SYN}	TC_{V}^{SYN}	TC^{SYN}
0.1	0.05	15.47	15.52	0.68	17.13	17.81	-0.21	14.76	14.56	1.26	21.93	23.19	7.95	28.62	36.58	12.54	10.65	23.19
0.2	0.76	15.99	16.74	1.35	17.53	18.88	0.50	15.32	15.83	1.91	21.92	23.83	7.95	27.97	35.92	12.33	11.50	23.83
0.3	1.91	17.22	19.13	2.54	18.71	21.25	1.73	16.71	18.45	2.46	21.78	24.25	7.95	27.27	35.23	10.37	13.87	24.25
0.4	2.38	17.24	19.62	3.00	18.70	21.71	2.20	16.76	18.96	2.93	21.50	24.43	7.95	26.53	34.48	10.37	14.06	24.43
0.5	2.87	17.26	20.13	3.50	18.67	22.18	2.70	16.79	19.48	3.42	21.18	24.61	7.95	25.71	33.67	10.37	14.24	24.61
0.6	3.52	17.44	20.97	3.78	18.22	22.01	3.42	17.13	20.55	3.87	20.73	24.60	7.95	24.81	32.77	9.37	15.23	24.60
0.7	3.89	17.03	20.91	4.15	17.78	21.92	3.79	16.72	20.51	4.23	20.08	24.31	7.95	23.79	31.75	9.37	14.94	24.31
0.8	4.35	16.58	20.93	4.61	17.29	21.90	4.25	16.29	20.54	4.69	19.32	24.02	7.95	22.58	30.54	9.37	14.65	24.02
0.9	4.79	15.74	20.54	4.98	16.31	21.29	4.71	15.51	20.22	5.05	18.10	23.15	7.95	21.01	28.96	9.01	14.14	23.15

Table 6.5.1: Results of Example 1

Case 1: $\min\{h_i^c\} = h_v^c$, $h_v^c = 0.002$, $h_i^c = [0.002\ \ 0.004\ \ 0.003\ \ 0.008\ \ 0.01]$;

Case 2: $\min\{h_i^c\} < h_v^c$, $h_v^c = 0.002$, $h_i^c = [0.001\ \ 0.002\ \ 0.003\ \ 0.0018\ \ 0.01]$;

Case 3: $\min\{h_i^c\} > h_v^c$, $h_v^c = 0.002$, $h_i^c = [0.003\ \ 0.004\ \ 0.006\ \ 0.008\ \ 0.01]$;

Case 4 $h_v^c = 0.002$, $h_i^c = 0.002\ \ \forall i$.

The first three columns under the 4 different cases are the buyer's, vendor's and system cost in the co-ordination with trade credit policy; the following three columns are, respectively, the independent costs; the final three columns are, respectively, the co-ordination costs by the synchronized cycles method without trade credit policy.

For Example 1, it can be seen from Table 6.5.1 that the performance of the synchronized cycles model with trade credit policy outperforms the independent policy over the whole range of D/P. Each party in the co-ordination can get benefits by the trade credit policy when compared with his independent cost. The improvement of the total system relevant cost in the synchronized cycles model with trade credit policy (TC^{M}) over the independent policy (TC^{IND}) ranges from 27.09% to 57.56%, 26.5% to 51.3%, 30.17% to 60.2%, for the three cases respectively. In the independent policy, both the vendor's cost and the total relevant cost decrease as D/P increases. As D/P increases, the total buyers' cost with trade credit policy (TC_{BS}^{M}) increases.

However, both the total relevant cost of the vendor and the system cost with trade credit policy increase as D/P increases from 0.1 to 0.6 (or 0.4), they then decrease as D/P increases further. The ratio of D/P also has significant influence on the results of independent optimization, buyer's and vendor's costs decrease as D/P increases.

A common point of the three cases given by Example 1 is that a majority of the buyers' capital cost is larger than vendor's capital cost, especially for cases 1 and 3. From Equation (6.4.5), we can find that the difference of the new system surplus with trade credit policy and the initial synchronized cycles model ($S^M(M_i) - S^{SCA}$) depends on the difference of buyers' and vendor's capital cost. The results here show that the system cost with trade credit policy (TC^M) will decrease when compared with the initial synchronized cycles model (TC^M) when a majority of buyers' capital cost is less than vendor's capital cost. From Equation (6.4.5), it is not surprising that the trade credit policy does not lead to a change of system surplus when each buyer's capital cost is the same as vendor's capital cost, that is $(TC^M = TC^{SCA})$, which can be seen in the columns of Table 6.5.1 Case 4.

If $h_i^c = h_v^c$, then the trade credit policy has no influence on the total system cost. The policy is only a mechanism to re-distribute the system surplus among buyers and the vendor. The first example shows that the trade credit period M_i is a good mechanism to make the allocation of the system surplus between the vendor and buyers, particularly when a majority of buyers' capital cost is larger than vendor's capital cost. There is some extra system surplus by used the trade credit policy in the co-ordination.

For the results of Example 2, Table 6.5.2 shows that the performance of the synchronized cycles model with trade credit policy performs better than the independent policy over the whole range of D/P. The improvement ranges from 9.29% to 8.73%, 6.67% to 6.17%, 14.74% to 14.08% , for the three cases respectively. Each party in the co-ordination can benefit by the trade credit policy when compared with his independent cost. As D/P increases, the total buyers' cost with trade credit policy (TC_{BS}^{M}) increases. However, both the total relevant cost for the vendor (TC_{V}^{M}) and the system cost with trade credit policy (TC_{S}^{M}) decrease as D/P increases.

D/P	Case 1			Case 2			Case 3								
	TC_{BS}^{TCR}	TC_{V}^{TCR}	TC^{TCR}	TC_{BS}^{TCR}	TC_{V}^{TCR}	TC^{TCR}	TC_{BS}^{TCR}	TC_{V}^{TCR}	TC^{TCR}	TC_{BS}^{IND}	TC_{V}^{IND}	TC^{IND}	TC_{BS}^{SYN}	TC_{V}^{SYN}	TC^{SYN}
0.1	467.54	404.85	872.39	484.71	412.86	897.57	455.52	364.49	820.01	512.21	449.52	961.73	560.27	312.12	872.39
0.2	469.02	397.26	866.28	485.83	405.27	891.10	457.01	357.64	814.65	512.21	440.45	952.65	560.27	306.01	866.28
0.3	469.92	388.49	858.42	486.06	395.91	881.97	458.80	350.67	809.46	512.21	430.78	942.99	556.70	301.71	858.42
0.4	470.50	378.69	849.20	486.49	386.11	872.60	459.38	341.15	800.53	512.21	420.40	932.61	556.70	292.49	849.20
0.5	471.54	368.44	839.98	487.27	375.85	863.12	460.42	331.42	791.83	512.21	409.11	921.32	556.70	283.27	839.98
0.6	472.17	356.58	828.76	487.62	363.83	851.45	461.30	320.25	781.55	512.21	396.62	908.83	555.71	273.05	828.76
0.7	473.20	343.43	816.63	488.39	350.68	839.07	462.32	307.61	769.93	512.21	382.44	894.65	555.71	260.92	816.63
0.8	474.23	327.65	801.87	489.19	334.93	824.11	463.31	292.27	755.58	512.21	365.63	877.83	555.90	245.97	801.87
0.9	474.84	306.34	781.18	489.59	313.55	803.15	464.02	271.42	735.44	512.21	343.71	855.92	555.49	225.69	781.18

Table 6.5.2: Results of Example 2

Case 1: $h_i^c = h_v^c$, $h_v^c = 0.002$, $h_i^c = 0.002 \quad \forall i$;

Case 2: $h_i^c < h_v^c$, $h_v^c = 0.002$, $h_i^c = 0.0015 \quad \forall i$;

Case 3: $h_i^c > h_v^c$, $h_v^c = 0.002$, $h_i^c = 0.004 \quad \forall i$.

Co-ordination can achieve the system surplus, but usually only the vendor can benefit by the co-ordination, buyers will not. The synchronized cycles model proposed by Chan and Kingsman (2007) is significantly better than the common order cycle method of Banerjee and Burton (1994) for problems of a moderately large size (50 buyers). The results of Example 3 (Table 6.5.3) show that the performance of the synchronized cycles model with trade credit policy also performs better than the independent policy by 18.06% to 18.64% over the whole range of D/P values.

D/P	TC_{BS}^{TCR}	TC_{V}^{TCR}	TC^{TCR}	TC_{BS}^{IND}	TC_{V}^{IND}	TC^{IND}	TC_{BS}^{SYN}	TC_{V}^{SYN}	TC^{SYN}
0.1	1128.00	595.90	1723.90	1255.70	850.66	2106.40	1382.50	594.48	1977.00
0.2	1129.60	584.16	1713.80	1255.70	835.68	2091.40	1382.50	582.75	1965.30
0.3	1130.40	569.81	1700.20	1255.70	819.72	2075.40	1382.20	568.47	1950.70
0.4	1130.80	553.45	1684.30	1255.70	802.58	2058.30	1382.20	552.11	1934.40
0.5	1132.00	537.09	1669.10	1255.70	783.94	2039.70	1382.20	535.76	1918.00
0.6	1132.70	517.97	1650.70	1255.70	763.33	2019.00	1382.30	516.67	1898.90
0.7	1134.40	497.99	1632.40	1255.70	739.93	1995.60	1381.20	496.74	1877.90
0.8	1135.30	472.03	1607.40	1255.70	712.17	1967.90	1380.70	470.78	1851.40
0.9	1135.50	436.11	1571.60	1255.70	675.99	1931.70	1382.10	434.87	1817.00

Table 6.5.3: Results of Example3 ($\min\{h_i^c\} > h_v^c$, $h_v^c = 0.001$, $\min\{h_i^c\} = 0.002$)

6.6 Conclusions

As discussed in Chapter 2, it is shown in Chan and Kingsman (2005, 2007) that the synchronized cycles model can be used to plan the ordering intervals in a one-vendor many-buyer supply chain so as to reduce significantly the system costs compared to each partner operating completely independently. However, the total relevant cost of the ith buyer of the co-ordinated system is always higher than that of independent optimization. Hence, a buyer would not be motivated to participate in the co-ordination.

Trade credit policy may be applied as an alternative to price discounts to induce buyers to make larger orders and adopt the coordination model. The objective of this chapter is to propose a co-ordinated single-vendor multi-buyer supply chain model by synchronizing ordering and production cycles with a trade credit policy. In this chapter, we develop an equitable profit-sharing scheme that makes a fair situation to all the parties in the supply chain.

We assume that the holding cost for both the vendor (h_v) and the buyers (h_i) are divided into two components, one associated with the opportunity cost of capital (h_v^c and h_i^c), and the other associated with the storage cost per unit of item (h_v^s and h_i^s) per unit time. We determine the minimum and maximum delay period from both the vendor's and buyer's perspective, and find the expression of the system surplus with these two extreme delay periods. An algorithm is developed to find the suitable delay period given to buyers.

The results of the numerical experiments show that trade credit policy works well in the synchronized cycles model. Irrespective of the capital cost structure of the vendor and buyers, the synchronized cycles model with delayed period M_i can reduce the vendor's and each buyer's cost when compared with his independent cost. The above results also echo the conclusion mentioned before:

- If a majority of buyers' capital cost are larger than the vendor's capital cost, then the trade credit period M_i increases the system surplus when compared with the co-ordinated model without trade credit.
- If a majority of buyers' capital cost are less than the vendor's capital cost, then the trade credit period M_i decreases the system surplus when compared with the co-ordinated model without trade credit.
- If each buyer's capital cost is the same as the vendor's capital cost, the trade credit policy has no influence on the total system cost, it is only a mechanism to re-distribute the surplus between buyers and the vendor.

In conclusion, trade credit policy is a good mechanism to allocate the system surplus between the vendor and the buyers in the co-ordination, particularly when the majority of buyers' capital costs are larger than the vendor's capital cost.

Chapter 7

Delay Payment Methods Based on Cost Sharing

7.1 The Cost Sharing Scheme on Buyer's Total Cost (TCB_i^{IND})

It is shown in Chan and Kingsman (2005, 2007) that the total relevant cost of the i^{th} buyer of the co-ordinated system is always higher than that of independent optimization. Hence, a buyer would not be motivated to participate in the co-ordination. In order to motivate the buyer, here we propose a cost-sharing scheme in which the vendor pays a proportion, r_i, of the buyer's holding cost such that the buyer's total relevant cost in the co-ordinated system is the same as that of independent optimization. Under the cost sharing co-ordinated system the vendor will need to pay an additional amount of

$$\frac{1}{2} r_i d_i h_i k_i T \ , \qquad (7.1.1)$$

where $r_i = \left(1 - \frac{T_i}{k_i T}\right)^2$ for each buyer.

Proof:

Let B_i^{IND} and B_i^{SCA} be the cost of buyer i under the independent policy and the synchronized cycles model respectively. That is,

$$B_i^{IND} = \sqrt{2 A_i h_i d_i} = h_i d_i T_i \text{ and}$$

$$B_i^{SCA} = \frac{A_i}{k_i T} + \frac{1}{2} h_i d_i k_i T = \frac{1}{k_i T}\left(\frac{h_i d_i T_i^2}{2}\right) + \frac{1}{2} h_i d_i k_i T$$

After a little mathematical manipulation we can obtain

$$B_i^{SCA} = B_i^{IND} \frac{1}{k_i T}\left(\frac{T_i}{2}\right) + B_i^{IND} \frac{k_i T}{2T_i} = \frac{B_i^{IND}}{2}\left(\frac{T_i}{k_i T} + \frac{k_i T}{T_i}\right).$$

The difference between B_i^{SCA} and B_i^{IND} can then be expressed as

$$B_i^{SCA} - B_i^{IND} = \frac{1}{2}\left(1 - \frac{T_i}{k_i T}\right)^2 h_i d_i k_i T$$

and we obtain the additional amount the vendor is required to pay for each buyer under the cost sharing co-ordinated model given by Eq. (7.1.1).

7.2 Cost Sharing by Credit Policy and or Price Discount

The proposed cost-sharing scheme in Section 7.1 requires the vendor to pay a proportion of each buyer's holding costs. However, the buyers may not wish to reveal the actual values of their holding costs in real life. Hence, this research further proposes two mechanisms (under two scenarios) with which the vendor can pay (or compensate) the proportion of buyers' holding costs suggested in Section 7.1.

7.2.1 Delay Payment Method

In this method, we assume that both the vendor's and buyers' holding costs consist solely of the capital opportunity cost. As capital opportunity cost is a time value of money, the buyers' holding costs can be compensated by a delay payment period. It is therefore proposed in this research that the vendor offers each buyer *i* a delay payment period M_i such that:

$$M_i = \frac{1}{2}(1 - \frac{T_i^*}{k_i T})^2 \cdot k_i T \qquad (7.2.1.1)$$

It can be seen from Eq. (7.2.1.1) that the delay payment period is a proportion, $\frac{1}{2}(1 - \frac{T_i^*}{k_i T})^2$, of the ordering cycle $k_i T$ of buyer *i*. This means that the purchasing cost (i.e. capital) of buyer *i* is not tied up for a proportion of his ordering cycle, and hence the proportion of his inventory holding cost is saved. Note that this proportion is the same as the proportion r_i proposed in Section 7.1.

With the delay payment periods, the vendor is required to bear an extra holding cost:

$$\sum_{i=1}^{n} hd_i M_i = 0.5 \sum_{i=1}^{n} hd_i (1 - \frac{T_i^*}{k_i T})^2 k_i T$$

$$= 0.5 \sum_{i=1}^{n} hd_i (k_i T + \frac{T_i^{*2}}{k_i T} - 2T_i^*) \qquad (7.2.1.2)$$

Hence, the vendor's total relevant cost with delay payment policy is :

$$TCV = \{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\}$$

$$+\sum_{i=1}^{n}\{\frac{C_i + 0.5hd_iT_i^{*2}}{k_iT} + d_i\frac{h}{P}Dk_iT\} - fixedpart \qquad (7.2.1.3)$$

where $fixedpart = \sum_{i=1}^{n} hd_iT_i^*$

As the total relevant cost of each buyer in the co-ordinated system is kept the same as that of independent optimization, i.e. a constant, our objective is now to find the nonnegative values for N and k_i's that minimize the above vendor's total relevant cost in Eq. (7.2.1.3).

The vendor's total relevant cost in Eq. (7.2.1.3) is a function of NT and k_iT. Thus we can arbitrarily set T = 1. However, the k_i are functions of N or vice versa, since k_i have to be factors of N. For fixed N and T, it can be shown that vendor's total relevant cost is a convex function of k_i, and we need to search for the minimum integer factor point of a given N. For this kind of synchronized cycles optimization, the solution method has been developed by Chan and Kingsman (2005, 2007).

The results of Example 1, 2 and 3 are shown in Table 7.2.1.1, Table 7.2.1.2, and Table 7.2.1.3 below.

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor	N_cor
0.1	7.9529	28.6222	36.5751	7.9529	12.9429	20.8958	56
0.2	7.9529	27.9689	35.9218	7.9529	13.7299	21.6828	53
0.3	7.9529	27.2732	35.2261	7.9529	14.4852	22.4381	51
0.4	7.9529	26.5259	34.4788	7.9529	15.192	23.1449	60
0.5	7.9529	25.7132	33.6661	7.9529	15.4843	23.4372	62
0.6	7.9529	24.8143	32.7672	7.9529	15.6858	23.6387	62
0.7	7.9529	23.794	31.7469	7.9529	15.6311	23.584	84
0.8	7.9529	22.5837	30.5366	7.9529	15.3226	23.2755	96
0.9	7.9529	21.0064	28.9593	7.9529	14.6111	22.564	120

Table 7.2.1.1 Example 1

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor	N_cor
0.1	512.2091	403.4493	915.6583	512.2091	132.8642	645.0733	52
0.2	512.2091	396.6565	908.8656	512.2091	134.6151	646.8241	54
0.3	512.2091	389.4242	901.6333	512.2091	133.9271	646.1361	60
0.4	512.2091	381.6541	893.8631	512.2091	131.8054	644.0145	60
0.5	512.2091	373.2045	885.4136	512.2091	128.2957	640.5047	72
0.6	512.2091	363.8584	876.0674	512.2091	123.4409	635.65	72
0.7	512.2091	353.25	865.4591	512.2091	116.9006	629.1097	90
0.8	512.2091	340.6665	852.8756	512.2091	108.1982	620.4072	108
0.9	512.2091	324.2675	836.4765	512.2091	95.3304	607.5395	144

Table 7.2.1.2 Example 2

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor	N_cor
0.1	1255.7	850.6612	2106.4	1255.7	293.481	1549.2	40
0.2	1255.7	835.677	2091.4	1255.7	295.4639	1551.2	42
0.3	1255.7	819.7233	2075.4	1255.7	292.7006	1548.4	42
0.4	1255.7	802.583	2058.3	1255.7	286.3023	1542	48
0.5	1255.7	783.944	2039.7	1255.7	278.0975	1533.8	48
0.6	1255.7	763.3273	2019	1255.7	266.3699	1522.1	60
0.7	1255.7	739.9263	1995.6	1255.7	251.6122	1507.3	72
0.8	1255.7	712.1683	1967.9	1255.7	231.2978	1487	80
0.9	1255.7	675.9935	1931.7	1255.7	202.1314	1457.9	120

Table 7.2.1.3 Example 3

From the results of all the three examples, the total buyer's cost remains the same as in the independent policy, while the vendor's cost and total system cost have substantial savings by synchronizing the ordering cycles of the buyers. The savings of the vendor ranges from 30.44% to 54.78% compared to the independent policy for Example 1. For Examples 2 and 3, the savings ranges from 65.46% to 70.6% and 64.29% to 70.1% respectively. For the total system cost, the savings ranges from 22.08% to 42.87%, 27.26% to 29.55%, and 24.44% to 26.45% for Example 1, 2, and 3 respectively. Details of the optimal values of k_i and M_i are shown in Appendix 10.

The buyer's costs, as expected, are the same as that of independent optimization. Comparing the relative savings percentages of the vendor and the total system cost, the vendor's saving percentage is far greater than the total system saving percentage. Thus, there is sufficient motivation for the vendor to entice the buyers to join the co-ordinated system by bearing some of the buyers' holding cost. The vendor's saving percentage first decreases then increases as the D/P ratio increases. There is no concrete relationship between the D/P ratio and the saving percentage.

7.2.2 Delay Payment Period and Price Discounts Method

Both the buyers' and vendor's unit holding costs consisting of two components: $h_i = h_i^c + h_i^s$ and $h = h_v^c + h_v^s$

where: h_i^c = buyer's capital opportunity cost, h_i^s = buyer's unit storage cost
h_v^c = vendor's capital opportunity cost , h_v^s = vendor's unit storage cost

To compensate the buyer's increased cost in the synchronized model, the vendor needs to share a proportion of the holding costs of each buyer (see Eq.(7.1)). A delay payment period alone can only compensate the buyer's capital opportunity cost, hence we propose in this section that the vendor would offer each buyer both a delay payment and a price discount. While the delay payment period M_i is to compensate buyer *i*'s capital opportunity cost, the discount P_{di} is to compensate buyer *i*'s storage cost.

$$M_i = 0.5 r_i^* k_i T = 0.5(1 - \frac{T_i^*}{k_i T})^2 \cdot k_i T \qquad (7.2.2.1)$$

$$P_{di} = 0.5 r_i^* k_i T h_i^s = 0.5(1 - \frac{T_i^*}{k_i T})^2 \cdot k_i T \cdot h_i^s \qquad (7.2.2.2)$$

where $r_i^* = (1 - \frac{T_i^*}{k_i T})^2$ in Section 7.1.

The sum of Eq. (7.2.2.1) and Eq. (7.2.2.2) is:

$$M_i d_i h_i^c + P_{di} d_i = 0.5 r_i k_i T d_i (h_i^c + h_i^s) = 0.5 r_i d_i h_i k_i T \qquad (7.2.2.3)$$

It can be seen from the RHS of Eq.(7.1.1) that this is exactly the same as the cost that buyer *i* needs to be compensated. Therefore, the total relevant cost of each buyer in the co-ordinated system is kept as the same as that of independent optimization.

With the buyers' delay payment periods and total offered discounts, the vendor's extra costs become:

$$\sum_{i=1}^{n} h_v^c d_i M_i + \sum_{i=1}^{n} P_{di} d_i = 0.5 \sum_{i=1}^{n} h_v^c d_i (1 - \frac{T_i^*}{k_i T})^2 k_i T + 0.5 \sum_{i=1}^{n} h_i^s d_i (1 - \frac{T_i^*}{k_i T})^2 k_i T$$

$$= 0.5 \sum_{i=1}^{n} (h_v^c + h_i^s) d_i (k_i T + \frac{T_i^{*2}}{k_i T} - 2T_i^*) \qquad (7.2.2.4)$$

So, the vendor's total relevant cost is :

$$\begin{aligned} TCV = &\{\frac{S_v}{NT} + [\frac{hD}{2} - \frac{hD^2}{2P}]NT\} + \sum_{i=1}^{n} \{\frac{C_i + 0.5(h_v^c + h_i^s) d_i T_i^{*2}}{k_i T} \\ &+ d_i (\frac{h}{P} D - 0.5h + 0.5(h_v^c + h_i^s)) k_i T - (h_v^c + h_i^s) d_i T_i^*\} \end{aligned} \qquad (7.2.2.5)$$

Our objective is now to find the nonnegative values for the N and k_i's that minimize the above vendor's total relevant cost.

Let $TCV = TCV^* - fixedpart$,

where $fixedpart = \sum_{i=1}^{n} (h_v^c + h_i^s) d_i T_i^*$

$$\text{and } TCV^{*}=\{\frac{S_v}{NT}+[\frac{hD}{2}-\frac{hD^2}{2P}]NT\}+\sum_{i=1}^{n}\{\frac{C_i+0.5(h_v^c+h_i^s)d_iT_i^{*2}}{K_iT}$$
$$+d_i(\frac{h}{P}D-0.5h+0.5(h_v^c+h_i^s))K_iT\} \quad (7.2.2.6)$$

Applying a similar algorithm as that of Chan and Kingsman (2005,2007), the results of Example 1 , 2 and 3 are shown in Table 7.2.2.1, Table 7.2.2.2, and Table 7.2.2.3 below.

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor	N_cor
0.1	7.9529	28.6222	36.5751	7.9529	15.3316	23.2845	44
0.2	7.9529	27.9689	35.9218	7.9529	15.9633	23.9162	43
0.3	7.9529	27.2732	35.2261	7.9529	16.3717	24.3246	56
0.4	7.9529	26.5259	34.4788	7.9529	16.5525	24.5054	54
0.5	7.9529	25.7132	33.6661	7.9529	16.728	24.6809	54
0.6	7.9529	24.8143	32.7672	7.9529	16.668	24.6209	78
0.7	7.9529	23.794	31.7469	7.9529	16.3755	24.3284	78
0.8	7.9529	22.5837	30.5366	7.9529	16.0651	24.018	96
0.9	7.9529	21.0064	28.9593	7.9529	15.2077	23.1606	144

Table 7.2.2.1 Example 1 $h_v^c=0.002,\ h_i^c=[0.001\ 0.002\ 0.003\ 0.0018\ 0.001]$

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor	N_cor
0.1	512.2091	403.4493	915.6583	512.2091	323.7898	835.9989	48
0.2	512.2091	396.6565	908.8656	512.2091	318.2423	830.4513	60
0.3	512.2091	389.4242	901.6333	512.2091	311.4715	823.6806	60
0.4	512.2091	381.6541	893.8631	512.2091	304.6796	816.8886	60
0.5	512.2091	373.2045	885.4136	512.2091	297.8876	810.0967	60
0.6	512.2091	363.8584	876.0674	512.2091	289.5316	801.7407	72
0.7	512.2091	353.25	865.4591	512.2091	279.9312	792.1402	84
0.8	512.2091	340.6665	852.8756	512.2091	268.1517	780.3608	120
0.9	512.2091	324.2675	836.4765	512.2091	252.995	765.204	180

Table 7.2.2.2 Example 2 $h_v^c=0.001,\ h_i^c=0.001\ \forall i$

D/P	TCBS_ind	TCV_ind	TCS_ind	TCBS_cor	TCV_cor	TCS_cor	N_cor
0.1	1255.7	850.6612	2106.4	1255.7	647.548	1903.3	36
0.2	1255.7	835.677	2091.4	1255.7	636.2682	1892	36
0.3	1255.7	819.7233	2075.4	1255.7	621.8548	1877.6	48
0.4	1255.7	802.583	2058.3	1255.7	605.8072	1861.5	48
0.5	1255.7	783.944	2039.7	1255.7	589.6854	1845.4	48
0.6	1255.7	763.3273	2019	1255.7	570.7863	1826.5	60
0.7	1255.7	739.9263	1995.6	1255.7	550.0576	1805.8	60
0.8	1255.7	712.1683	1967.9	1255.7	524.0797	1779.8	84
0.9	1255.7	675.9935	1931.7	1255.7	489.3466	1745.1	120

Table 7.2.2.3 Example 3 $h_v^c = 0.001,\ h_i^c = 0.5h_i\ \forall i$

From the results of all the three examples, the total buyers' cost remains the same as in the independent policy, while the vendor's cost and total system cost have substantial savings by synchronizing the ordering cycles of the buyers. Comparing with results under Section 7.2.1, the vendor's and system's cost are both increased. This is natural since the vendor not only offers a delay payment period but also a price discount. The savings of the vendor ranges from 29.5% to 53.48% compared to the independent policy for Example 1. For Examples 2 and 3, the saving ranges from 19.74% to 21.98% and 23.88% to 27.61% respectively. For the total system cost, the saving ranges from 21.4% to 41.85%, 8.47% to 8.7%, and 9.51% to 9.64% for Example 1, 2, and 3 respectively. Details of the optimal values of k_i and M_i are shown in Appendix 11.

7.3 Conclusions

It is shown in Chan and Kingsman (2005, 2007) that the total relevant cost of the i^{th} buyer of the co-ordinated system is always higher than that of independent optimization. Hence, a buyer would not be motivated to participate in the co-ordination. In order to motivate the buyer, here we propose a cost-sharing scheme in which the vendor pays a proportion of the buyer's holding cost. However, the buyers may not wish to reveal the actual values of their holding costs in real life. Hence, this research further proposes two mechanisms (under two scenarios) with which the vendor can pay (or compensate) the proportion of buyers' holding costs. The purpose of the cost sharing policy is to ensure the total costs borne by all the buyers are the same as in the independent model.

In the first mechanism, we assume that both the vendor's and buyers' holding costs consist solely of capital opportunity cost, that the buyers' holding costs can be compensated by a delay payment period.

In the second mechanism, we assume that the vendor's and buyers' holding costs consist both of capital opportunity cost and storage cost. While the delay payment period is to compensate the capital opportunity cost, an extra discount is offered by the vendor to compensate the buyer's storage cost.

In the numerical experiments, while keeping all the buyers not to be worse off when compared with his independent policy, the vendor still has a substantial savings by synchronizing the ordering cycles of the buyers with two mechanisms, and more importantly the total system cost is also reduced. The vendor's saving percentage is far greater than the total system saving percentage. Thus, there is sufficient motivation for the vendor to entice the buyers to join the co-ordinated system by bearing some of the buyers' holding cost. The co-ordination scheme is very effective, providing a win-win-win situation.

The synchronized cycles model is very effective in a one-vendor many-buyer supply chain system, but some policies (i.e. quantity discount and credit period) can be incorporated into the model for better co-ordination and can add more reality to our business environment.

Co-ordinated models in the literature usually assume that both the parties share all the cost information. However, in practice, the members of a supply chain may not be interested to disclose all the information. Most significantly, the mechanisms proposed in this chapter do not require the information of the buyers' cost structure. The only information that needs to be known when calculating Eq. (7.2.1.1), (7.2.2.1) and (7.2.2.2) is T_i^*, the buyer's optimal ordering cycle in the independent policy, which is considered to be known by the vendor by historical demand. In the first mechanism, the proportion of buyers' holding cost is compensated by the vendor offering a delay payment period as buyers' holding cost is considered consisting solely of the capital opportunity cost. In the second mechanism, it is further considered that the buyers' holding cost consists of both capital opportunity cost and storage cost. The capital opportunity cost, which is usually unknown to the vendor, can be compensated by again a delay period. For the storage cost (usually the rent of the warehouse) of the buyers, which can be easily known from the market, it is proposed to be compensated by a price discount.

Chapter 8

Conclusion

8.1 Introduction

Chan and Kingsman (2005, 2007) developed a synchronized cycles model that allows each buyer to choose its ordering cycle, while the length of the cycle should be kept as a factor of the vendor's production cycle. They showed that the co-ordination synchronized cycles model works well. It has been shown, by many numerical experiments, that the synchronized cycles model can significantly reduce the total system cost and make a significant cost reduction compared to the independent policy and the common replenishment cycle (e.g. Banerjee and Burton (1994)). However, the cost to all the buyers is significantly increased. Hence, mechanisms to attract buyers to join the co-ordination is deemed necessary.

This thesis presents three types of mechanisms (quantity discounts models, trade credit policy and delay payment method based on cost sharing of holding costs) to help to achieve the system co-ordination and make an equitable division of the system surplus among system members.

8.2 Quantity Discount Models

In the majority of models of quantity discounts, one of the limitations is that the holding cost of the buyer is considered to be independent of purchase price. In this thesis, the buyer's holding cost h_i is expressed as a percentage of the capital, so buyer's holding cost is considered to be dependent on purchasing price.

Another major limitation of most of the models in the literature is the assumption that a supply chain partner has complete information (including cost, demand, lead time, etc.) about the other partners. In practice, it is difficult for the vendor to know the cost structure of the buyers. Coordination under limited information sharing is an important issue of concern to be studied. The Quantity Discount Model 1 proposed in Chapter 3 can solve this problem in some extents, with the assumption that the buyer's demand is deterministic.

Many models in the literature also failed to specify how the incremental savings to the manufacturer can be passed to the buyer. Some authors have mentioned about equal splitting of the surplus, whereas some have suggested splitting the surplus according to their investment. In this thesis, the quantity discounts can ascertain the benefit of the buyer and it is not pre-determined (e.g. all unit discount) but depends on the buyer's ordering cycle in the co-ordination model. So, the manufacturer's savings can be passed by the discount to each buyer and the system cost will also be reduced by the reduction of buyers' holding cost. However, in some cases, the quantity discounts models may have a solution in which the vendor may have his cost increased in the co-ordination when compared with his independent cost. This is due to the reason that the discounts offered are too large. Chapter 3 further proposes the modified quantity discounts model with "q" or "q_i" factors to solve this problem.

8.3 Trade Credit Policy

Trade credit policies may be applied as an alternative to quantity discounts to motivate buyers to participate in supply chain co-ordination. Chapter 6 presents a trade credit policy to be applied in the synchronized cycles model. An algorithm to minimize the total relevant cost of the co-ordinated system is developed and an equitable profit-sharing scheme is proposed to make a fair distribution of system savings. The results of the mechanism of implementing a trade credit period show that it works well in the synchronized cycles model. Unlike quantity discounts models, regardless of the vendor's or buyers' capital cost structure, the synchronized cycles model with trade credit policy can reduce both vendor's and each buyer's cost when compared with his independent cost. Hence, the trade credit policy is a good mechanism to perform equitable sharing of the system savings between vendor and buyers in the co-ordination. However, the policy requires the vendor to have information of the buyers' cost structure.

8.4 Delay Payment Method Based on Cost Sharing of Holding Cost

Similar to the trade credit policy, Chapter 7 proposes a delay payment method, which is actually based on cost sharing of buyer's holding cost. In this method, a portion $r_i = \left(1 - \frac{T_i}{k_i T}\right)^2$ of the buyer's holding cost is borne by the vendor. The purpose of the cost sharing is to ensure the total cost of each buyer in the co-ordination system is the same as that in the independent model. The vendor can offer this portion of holding cost by two mechanisms, a delay payment method alone or a delay payment together with a price discount. The two mechanisms, on one hand keep all the buyers not to be worse off, and on the other hand provide the vendor with a substantial saving obtained by synchronizing the ordering cycles of the buyers. Most significantly, this delay payment method based on cost sharing can further reduce the total system cost and does not require the vendor to know the information of the buyers' cost structure. The only information that the vendor needs to know is the buyers' optimal ordering cycles in the independent policy, which are considered to be known before.

Hence, the delay payment method based on cost sharing of holding cost takes the advantage of the trade credit policy which is a practical mechanism in the synchronized cycles model to achieve the system saving without increasing the buyers' cost. However, the vendor may still need the information of the buyers' cost structure if he wants to share the system saving equitably among members.

8.5 Future Research

There are interesting avenues of further research emanating from this research. A general result that applies in all analyses of co-ordinated vendor-buyer models is that, when compared with independent optimization, a coordinated model makes a significant reduction in the vendor's cost but the cost to all the buyers is significantly increased. Hence, the interest is in examining what mechanisms are needed from the vendor to motivate the buyers to change their policies to allow the savings from co-ordination to be achieved. It is shown in Chan and Kingsman (2005, 2007) that their synchronized cycles model can be used to plan the ordering intervals in a one-vendor many-buyer supply chain so as to reduce significantly the system costs compared to each partner operating completely independently. However, the mechanism of how to motivate the buyers to participate in the co-ordination was not considered. This research has proposed a number of mechanisms for the synchronized cycles model. The mechanisms not only can motivate buyers to participate in the co-ordination, but can also achieve equitable sharing of system savings between vendor and buyers. However, in order to achieve equitable sharing of system savings resulting from the co-ordination, the proposed mechanisms still require the vendor to have the cost information of the buyers.

For another direction of future research, we may consider some other important cost elements in supply chain models, e.g. transportation cost. Different models of shipping freight costs are typically classified as either full truckload (FTL) transportation or less than truckload (LTL) transportation. There is a fixed cost C_T per load up to a given capacity Q_T in FTL transportation. For small quantities, a cost per unit item s is the most efficient method for transportation in LTL. In the real world, LTL is a complex problem since the LTL cost structure has several breakpoints with discounts on the cost per unit when the quantity is increasing. Co-ordiantion may lead to a saving of transportation cost in the system. Some mechanisms like price discounts, trade credit, cost sharing could also be considered in the new synchronized cycles model including transportation cost so as to achieve the sharing of system savings between buyers and vendor.

Furthermore, revenue is a very important element in a supplier and retailer system. Under a revenue-sharing contract, a retailer pays a supplier a wholesale price for each unit purchased, plus a percentage of the revenue the retailer generate. Such contracts have become more prevalent in the videocassette rental industry relative to the more conventional wholesale price contract. Given a single supplier and retailer, it coordinates the supply chain and arbitrarily divides the resulting profits for any reasonable revenue function that depends on the retailer's purchase quantity and price. The supplier sells at a wholesale price below marginal cost, but her participation in the retailer's revenue more than offsets the loss on sales.

With the growth of new products and markets, the supply chain system supporting both manufactured products and services have become intertwined directly or indirectly with the logistics operation for other competing products as well. While e-business is simplifying the communications between suppliers and customers, many suppliers are still finding it challenging to provide timely deliveries of goods and services because of geographical distance and resource limitations. Thus, strategic planning and scheduling of logistics operations are also an important research issue.

Appendix

Appendix 1 - Dataset

Example 1 (5 Buyers; $S_v = 250$; $h = 0.005$)

Buyer i	*di*	*Ci*	*Ai*	*hi*
1	8	40	20	0.008
2	15	40	15	0.009
3	10	40	6	0.010
4	5	40	10	0.010
5	20	40	18	0.007

Data for Example #1

Example 2 (30 Buyers; $S_v = 3{,}000$; $h = 0.0028$)

Buyer i	*di*	*Ci*	*Ai*	*hi*	*Buyer i*	*di*	*Ci*	*Ai*	*hi*
1	16	16	23	0.167	16	25	24	28	0.434
2	37	16	35	0.207	17	27	29	32	0.620
3	11	6	3	0.449	18	5	8	15	0.575
4	17	23	24	0.511	19	32	11	36	0.446
5	47	25	17	0.326	20	19	15	7	0.570
6	29	19	27	0.226	21	33	7	24	0.525
7	29	30	4	0.763	22	38	12	6	0.064
8	37	22	31	0.444	23	13	12	33	0.690
9	13	16	30	0.260	24	47	23	12	0.016
10	39	10	25	0.517	25	25	27	37	0.787
11	30	5	18	0.334	26	45	11	4	0.142
12	34	19	33	0.178	27	46	4	12	0.216
13	32	14	39	0.368	28	16	20	25	0.243
14	4	2	32	0.615	29	34	10	18	0.350
15	20	9	37	0.135	30	27	6	35	0.496

Data for Example #2

Example 3 (50 Buyers; S_v = 5,000; h = 0.0028)

Buyer i	*di*	*Ci*	*Ai*	*hi*	*Buyer i*	*di*	*Ci*	*Ai*	*hi*
1	26	10	8	0.147	26	21	18	17	0.081
2	6	5	19	0.203	27	1	13	26	0.977
3	49	27	7	0.548	28	31	22	14	0.003
4	3	14	26	0.874	29	48	6	5	0.152
5	11	24	22	0.324	30	48	4	22	0.873
6	15	17	24	0.522	31	4	26	28	0.119
7	26	12	21	0.160	32	13	11	8	0.500
8	48	17	20	0.998	33	7	9	11	0.271
9	33	10	1	0.183	34	42	11	6	0.960
10	24	16	4	0.471	35	14	4	10	0.383
11	18	21	11	0.964	36	20	16	39	0.155
12	20	24	27	0.876	37	16	20	22	0.677
13	16	15	22	0.791	38	24	10	14	0.196
14	30	8	17	0.977	39	37	17	26	0.576
15	32	12	2	0.735	40	45	29	20	0.110
16	1	9	10	0.887	41	2	26	4	0.076
17	7	11	5	0.094	42	37	7	16	0.282
18	2	11	1	0.484	43	16	2	29	0.446
19	20	13	5	0.857	44	34	26	23	0.047
20	31	17	18	0.170	45	47	25	20	0.987
21	29	26	11	0.708	46	15	14	17	0.358
22	24	24	19	0.361	47	33	15	10	0.817
23	7	7	12	0.952	48	31	1	18	0.112
24	21	14	22	0.277	49	35	20	27	0.659
25	22	22	15	0.438	50	20	2	5	0.606

Data for Example #3

Appendix 2 - Example 2 Model 1 "*q*_factor" Minimizing Vendor's Cost

D/P	TCBS_ind	TCV_ind	TCS_ind
0.1	2.3782	1.8029	4.1812
0.2	2.3782	1.7962	4.1744
0.3	2.3782	1.7889	4.1671
0.4	2.3782	1.7812	4.1594
0.5	2.3782	1.7727	4.1509
0.6	2.3782	1.7634	4.1416
0.7	2.3782	1.7528	4.131
0.8	2.3782	1.7402	4.1184
0.9	2.3782	1.7238	4.102

q=1 D/P	(ok) TCBS_qd	(bad) TCV_qd	TCS_qd	q=0.9 D/P	(ok) TCBS_qd	(bad) TCV_qd	TCS_qd
0.1	2267.6	1828.7	4096.3	0.1	2280.2	1817.6	4097.8
0.2	2267.6	1821.7	4089.3	0.2	2280.2	1810.6	4090.9
0.3	2267.6	1814.8	4082.3	0.3	2280.2	1803.7	4083.9
0.4	2267.6	1807.8	4075.3	0.4	2280.2	1796.7	4076.9
0.5	2267.6	1800.8	4068.4	0.5	2280.2	1789.7	4069.9
0.6	2267.6	1793.9	4061.4	0.6	2280.2	1782.8	4063
0.7	2267.6	1785	4048	0.7	2276.8	1773.5	4050.3
0.8	2267.6	1770.6	4038.1	0.8	2280.2	1759.5	4039.7
0.9	2267.6	1755.3	4022.8	0.9	2280.2	1744.2	4024.4

q=0.8 D/P	(ok) TCBS_qd	(bad) TCV_qd	TCS_qd	q=0.7 D/P	(ok) TCBS_qd	(good) TCV_qd	TCS_qd
0.1	2293.6	1806.5	4100.1	0.1	2307	1795.3	4102.4
0.2	2293.6	1799.5	4093.1	0.2	2307	1788.4	4095.4
0.3	2293.6	1792.5	4086.2	0.3	2307	1781.4	4088.4
0.4	2293.6	1785.6	4079.2	0.4	2307	1774.4	4081.5
0.5	2293.6	1778.6	4072.2	0.5	2307	1767.5	4074.5
0.6	2293.6	1771.6	4065.2	0.6	2307	1760.5	4067.5
0.7	2290.6	1762	4052.6	0.7	2304.4	1750.4	4054.9
0.8	2293.6	1748.4	4042	0.8	2307	1737.2	4044.3
0.9	2293.6	1733.1	4026.7	0.9	2307	1721.9	4029

q=0.6 D/P	(not ok) TCBS_qd	(good) TCV_qd	TCS_qd	q=0.5 D/P	(not ok) TCBS_qd	(good) TCV_qd	TCS_qd
0.1	2315.6	1783.2	4098.8	0.1	2317.9	1767.9	4085.7
0.2	2317.9	1776.6	4094.4	0.2	2319.7	1761.6	4081.4
0.3	2317.9	1769.6	4087.5	0.3	2319.7	1754.7	4074.4
0.4	2317.9	1762.7	4080.5	0.4	2320.4	1747.8	4068.2
0.5	2317.9	1755.7	4073.6	0.5	2320.4	1740.8	4061.3
0.6	2315.7	1748.3	4064.1	0.6	2317.7	1733.2	4050.9
0.7	2315.7	1738.3	4054	0.7	2317.7	1723.2	4040.9
0.8	2318.1	1725.5	4043.6	0.8	2319.4	1710.7	4030.1
0.9	2317.9	1710.2	4028.1	0.9	2319.9	1695.4	4015.3

q=0.4 D/P	(not ok) TCBS_qd	(good) TCV_qd	TCS_qd	q=0.3 D/P	(not ok) TCBS_qd	(good) TCV_qd	TCS_qd
0.1	2747.8	1749.1	4496.9	0.1	3181.3	1723.4	4904.8
0.2	2860.9	1743.2	4604.1	0.2	3181.3	1717.8	4899.2
0.3	2860.9	1737	4598	0.3	3175.2	1712.2	4887.4
0.4	2352.3	1730.5	4082.8	0.4	3044.1	1706.6	4750.7
0.5	2352.3	1723.6	4075.9	0.5	3044.1	1700.5	4744.6
0.6	2347.8	1715.9	4063.7	0.6	2590.8	1693.1	4283.9
0.7	2347.8	1705.9	4053.7	0.7	2487.7	1683.3	4171.1
0.8	2348.8	1693.6	4042.5	0.8	2467.9	1671.5	4139.4
0.9	2350.6	1678.4	4029	0.9	2462.1	1656.4	4118.5

Appendix 3 - Example 3 Model 1 "*q*_factor" Minimizing Vendor's Cost

D/P	TCBS_ind	TCV_ind	TCS_ind
0.1	6.7037	4.9367	11.64
0.2	6.7037	4.9217	11.625
0.3	6.7037	4.9057	11.609
0.4	6.7037	4.8886	11.592
0.5	6.7037	4.8699	11.574
0.6	6.7037	4.8493	11.553
0.7	6.7037	4.8259	11.53
0.8	6.7037	4.7982	11.502
0.9	6.7037	4.762	11.466

q=0.9	ok	bad		q=0.8	ok	bad	
D/P	TCBS_qd	TCV_qd	TCS_qd	D/P	TCBS_qd	TCV_qd	TCS_qd
0.1	6.4217	5.0759	11.498	0.1	6.457	5.0437	11.501
0.2	6.4217	5.0638	11.485	0.2	6.457	5.0316	11.489
0.3	6.4217	5.0488	11.471	0.3	6.4576	5.0167	11.474
0.4	6.4217	5.0321	11.454	0.4	6.4576	5	11.458
0.5	6.4217	5.0155	11.437	0.5	6.4576	4.9833	11.441
0.6	6.4208	4.9961	11.417	0.6	6.4574	4.9639	11.421
0.7	6.4208	4.9748	11.396	0.7	6.4574	4.9427	11.4
0.8	6.422	4.9484	11.37	0.8	6.4575	4.9162	11.374
0.9	6.421	4.9135	11.334	0.9	6.4573	4.8813	11.339

q=0.7	ok	bad		q=0.6	not ok	bad	
D/P	TCBS_qd	TCV_qd	TCS_qd	D/P	TCBS_qd	TCV_qd	TCS_qd
0.1	6.4944	5.0114	11.506	0.1	6.5286	4.9785	11.507
0.2	6.4944	4.9993	11.494	0.2	6.5286	4.9664	11.495
0.3	6.4948	4.9844	11.479	0.3	6.529	4.9516	11.481
0.4	6.4956	4.9677	11.463	0.4	6.529	4.9349	11.464
0.5	6.4956	4.9511	11.447	0.5	6.529	4.9183	11.447
0.6	6.494	4.9315	11.426	0.6	6.5298	4.8988	11.429
0.7	6.494	4.9103	11.404	0.7	6.5298	4.8775	11.407
0.8	6.4955	4.8839	11.379	0.8	6.5285	4.8512	11.38
0.9	6.4943	4.849	11.343	0.9	6.53	4.8163	11.346

q=0.5 D/P	not ok TCBS_qd	bad TCV_qd	TCS_qd	q=0.4 D/P	not ok TCBS_qd	good TCV_qd	TCS_qd
0.1	6.5709	4.944	11.515	0.1	6.6477	4.9021	11.55
0.2	6.5709	4.932	11.503	0.2	6.6477	4.8903	11.538
0.3	6.5719	4.9173	11.489	0.3	6.6481	4.8757	11.524
0.4	6.5696	4.9007	11.47	0.4	6.6481	4.8592	11.507
0.5	6.5696	4.8841	11.454	0.5	6.6493	4.8428	11.492
0.6	6.5735	4.8647	11.438	0.6	6.6084	4.8237	11.432
0.7	6.5735	4.8435	11.417	0.7	6.6087	4.8024	11.411
0.8	6.5698	4.817	11.387	0.8	6.6023	4.7759	11.378
0.9	6.5698	4.7823	11.352	0.9	6.601	4.7416	11.343

Appendix 4 - Example 1 Model 1 "*q(i)*_factor" Minimizing Vendor's Cost

D/P	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
N	54.00	54.00	60.00	60.00	60.00	72.00	72.00	108.00	144.00
K(i)									
Buyer1	27.00	27.00	30.00	30.00	30.00	24.00	24.00	27.00	24.00
Buyer2	18.00	18.00	20.00	20.00	20.00	18.00	18.00	18.00	18.00
Buyer3	18.00	18.00	15.00	15.00	15.00	12.00	12.00	12.00	16.00
Buyer4	27.00	27.00	20.00	20.00	20.00	24.00	24.00	27.00	24.00
Buyer5	18.00	18.00	20.00	20.00	20.00	18.00	18.00	18.00	18.00

Pd(i)									
Buyer1	1.99	1.99	1.97	1.97	1.97	2.00	2.00	1.99	2.00
Buyer2	1.98	1.98	1.96	1.96	1.97	1.98	1.98	1.98	1.98
Buyer3	1.86	1.87	1.94	1.94	1.95	1.99	1.99	1.99	1.92
Buyer4	1.89	1.90	2.00	2.00	2.00	1.95	1.95	1.90	1.96
Buyer5	1.99	1.99	1.98	1.98	1.98	1.99	1.99	2.00	1.99

q(i)									
Buyer1	0.52	0.49	0.57	0.53	0.49	0.57	0.53	0.47	0.50
Buyer2	0.33	0.31	0.31	0.29	0.27	0.32	0.30	0.27	0.28
Buyer3	0.46	0.43	0.44	0.41	0.37	0.46	0.43	0.38	0.41
Buyer4	0.80	0.75	0.88	0.81	0.75	0.89	0.82	0.73	0.78
Buyer5	0.25	0.24	0.23	0.22	0.20	0.24	0.23	0.20	0.21

Appendix 4 - Example 2 Model 1 “*q(i)*_factor” Minimizing Vendor’s Cost

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	48	60	60	60	60	72	84	120	180
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	10	10	10	10	9	12	10	10
Buyer 3	1	1	1	1	1	1	1	1	1
Buyer 4	3	3	3	3	3	3	3	3	3
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	3	3	3	3	3	3	3	3	3
Buyer 7	2	2	2	2	2	2	2	2	2
Buyer 8	4	4	4	4	4	4	4	4	4
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	4	4	4	4	4	4	4	4	4
Buyer 11	4	5	5	5	5	4	4	5	5
Buyer 12	16	15	15	15	15	18	14	15	15
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	3	3	3	3	3	3	3	3	3
Buyer 15	1	1	1	1	1	1	1	1	1
Buyer 16	3	3	3	3	3	3	3	3	3
Buyer 17	6	5	5	5	5	6	6	5	5
Buyer 18	1	1	1	1	1	1	1	1	1
Buyer 19	3	3	3	3	3	3	3	3	3
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	6	6	6	6	6	6	6	6	6
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	3	3	3	3	3	3	3	3	3
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	1	1	1	1	1	1	1	1	1
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	3	3	3	3	3	3	3	3	3
Buyer 29	12	12	12	12	12	12	12	12	12
Buyer 30	3	3	3	3	3	3	3	3	3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
discounted Price Pd(i)									
Buyer 1	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92
Buyer 2	1.97	2.00	2.00	2.00	2.00	1.99	1.97	2.00	2.00
Buyer 3	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 4	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 5	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 6	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.94	1.93
Buyer 7	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 8	1.98	1.98	1.98	1.98	1.98	1.99	1.99	1.98	1.98
Buyer 9	1.74	1.75	1.73	1.74	1.74	1.75	1.75	1.75	1.75
Buyer 10	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 11	2.00	1.99	1.99	1.99	1.99	2.00	2.00	1.99	1.99
Buyer 12	1.99	2.00	2.00	2.00	2.00	1.98	2.00	2.00	2.00
Buyer 13	1.70	1.70	1.69	1.69	1.70	1.70	1.71	1.71	1.70
Buyer 14	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 15	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 16	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71
Buyer 17	1.75	1.90	1.90	1.90	1.90	1.75	1.75	1.90	1.90
Buyer 18	1.93	1.93	1.92	1.92	1.92	1.93	1.93	1.93	1.93
Buyer 19	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 20	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 21	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 22	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 23	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 24	1.79	1.79	1.78	1.78	1.79	1.79	1.79	1.79	1.79
Buyer 25	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 26	1.74	1.74	1.73	1.73	1.74	1.74	1.74	1.75	1.74
Buyer 27	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 28	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 29	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 30	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99

D/P q(i)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 2	0.46	0.45	0.48	0.48	0.47	0.45	0.45	0.44	0.45
Buyer 3	0.44	0.43	0.45	0.45	0.44	0.43	0.42	0.42	0.43
Buyer 4	0.63	0.62	0.65	0.64	0.63	0.62	0.61	0.60	0.62
Buyer 5	0.39	0.38	0.40	0.40	0.39	0.38	0.38	0.37	0.38
Buyer 6	0.80	0.78	0.82	0.81	0.80	0.78	0.78	0.77	0.78
Buyer 7	0.81	0.80	0.84	0.83	0.82	0.80	0.79	0.78	0.80
Buyer 8	0.35	0.37	0.38	0.38	0.37	0.34	0.34	0.35	0.36
Buyer 9	0.38	0.38	0.40	0.39	0.39	0.38	0.37	0.37	0.38
Buyer 10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 11	0.30	0.30	0.30	0.30	0.30	0.29	0.29	0.28	0.28
Buyer 12	0.33	0.31	0.33	0.33	0.32	0.31	0.31	0.30	0.31
Buyer 13	0.54	0.53	0.55	0.55	0.54	0.53	0.52	0.51	0.53
Buyer 14	0.40	0.39	0.41	0.41	0.40	0.39	0.39	0.38	0.39
Buyer 15	0.48	0.47	0.49	0.49	0.48	0.47	0.47	0.46	0.47
Buyer 16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 18	0.64	0.63	0.66	0.65	0.64	0.62	0.62	0.61	0.62
Buyer 19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 20	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	1.00
Buyer 21	0.39	0.39	0.41	0.40	0.40	0.38	0.38	0.37	0.38
Buyer 22	0.41	0.40	0.42	0.42	0.41	0.40	0.40	0.39	0.40
Buyer 23	0.52	0.51	0.53	0.53	0.52	0.51	0.50	0.49	0.51
Buyer 24	0.61	0.60	0.63	0.62	0.62	0.60	0.60	0.59	0.60
Buyer 25	0.39	0.38	0.40	0.40	0.39	0.38	0.38	0.37	0.38
Buyer 26	0.70	0.69	0.72	0.71	0.70	0.69	0.68	0.68	0.69
Buyer 27	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 28	0.42	0.42	0.44	0.43	0.43	0.42	0.41	0.41	0.42
Buyer 29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 30	0.62	0.61	0.64	0.64	0.63	0.61	0.61	0.60	0.61

Appendix 4 - Example 3 Model 1 "*q(i)*_factor" Minimizing Vendor's Cost

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	36	36	48	48	48	60	72	84	120
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	3	3	3	3	3	3	3	3	3
Buyer 3	2	2	2	2	2	2	2	2	2
Buyer 4	1	1	1	1	1	1	1	1	1
Buyer 5	1	1	1	1	1	1	1	1	1
Buyer 6	12	12	12	12	12	12	12	12	12
Buyer 7	3	3	3	3	3	3	3	3	3
Buyer 8	1	1	1	1	1	1	1	1	1
Buyer 9	3	3	3	3	3	3	3	3	3
Buyer 10	2	2	2	2	2	2	2	2	2
Buyer 11	2	2	2	2	2	2	2	2	2
Buyer 12	2	2	2	2	2	2	2	2	2
Buyer 13	4	4	4	4	4	4	4	4	4
Buyer 14	1	1	1	1	1	1	1	1	1
Buyer 15	9	9	8	8	8	10	8	7	8
Buyer 16	1	1	1	1	1	1	1	1	1
Buyer 17	2	2	2	2	2	2	2	2	2
Buyer 18	1	1	1	1	1	1	1	1	1
Buyer 19	2	2	2	2	2	2	2	2	2
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	4	4	4	4	4	4	4	4	4
Buyer 22	1	1	1	1	1	1	1	1	1
Buyer 23	4	4	4	4	4	4	4	4	4
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	2	2	2	2	2	2	2	2	2

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	36	36	48	48	48	60	72	84	120
Buyer 26	1	1	1	1	1	1	1	1	1
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	2	2	2	2	2	2	2	2	2
Buyer 29	1	1	1	1	1	1	1	1	1
Buyer 30	2	2	2	2	2	2	2	2	2
Buyer 31	2	2	2	2	2	2	2	2	2
Buyer 32	2	2	2	2	2	2	2	2	2
Buyer 33	18	18	24	24	24	20	24	21	24
Buyer 34	4	4	4	4	4	4	4	4	4
Buyer 35	3	3	3	3	3	3	3	3	3
Buyer 36	2	2	2	2	2	2	2	2	2
Buyer 37	2	2	2	2	2	2	2	2	2
Buyer 38	2	2	2	2	2	2	2	2	2
Buyer 39	4	4	4	4	4	5	4	4	5
Buyer 40	3	3	3	3	3	3	3	3	3
Buyer 41	12	12	12	12	12	10	12	12	10
Buyer 42	2	2	2	2	2	2	2	2	2
Buyer 43	2	2	2	2	2	2	2	2	2
Buyer 44	3	3	3	3	3	3	3	3	3
Buyer 45	6	6	6	6	6	6	6	6	6
Buyer 46	1	1	1	1	1	1	1	1	1
Buyer 47	1	1	1	1	1	1	1	1	1
Buyer 48	1	1	1	1	1	1	1	1	1
Buyer 49	1	1	1	1	1	1	1	1	1
Buyer 50	1	1	1	1	1	1	1	1	1

Table A4.1 The solutions of $k(i)$ of each buyer for Example 3 Model 1

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	Buyer 26	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 2	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	Buyer 27	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 3	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	Buyer 28	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
Buyer 4	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	Buyer 29	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 5	1.81	1.81	1.81	1.81	1.82	1.81	1.82	1.82	1.82	Buyer 30	1.86	1.86	1.87	1.87	1.87	1.86	1.87	1.87	1.87
Buyer 6	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	Buyer 31	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 7	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	Buyer 32	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 8	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	Buyer 33	2.00	2.00	1.91	1.91	1.91	1.99	1.91	1.98	1.91
Buyer 9	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	Buyer 34	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 10	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	Buyer 35	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 11	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	Buyer 36	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 12	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.81	Buyer 37	1.74	1.74	1.74	1.75	1.75	1.74	1.75	1.75	1.75
Buyer 13	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	Buyer 38	1.94	1.94	1.94	1.94	1.95	1.94	1.94	1.95	1.95
Buyer 14	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	Buyer 39	1.90	1.90	1.90	1.90	1.90	1.71	1.90	1.90	1.71
Buyer 15	1.66	1.66	1.77	1.77	1.77	1.56	1.77	1.88	1.77	Buyer 40	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 16	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.86	Buyer 41	1.91	1.91	1.91	1.91	1.91	1.99	1.91	1.91	1.99
Buyer 17	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	Buyer 42	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 18	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	Buyer 43	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 19	1.98	1.98	1.98	1.98	1.99	1.98	1.99	1.99	1.99	Buyer 44	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 20	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	Buyer 45	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 21	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	Buyer 46	1.78	1.78	1.78	1.79	1.79	1.78	1.79	1.79	1.79
Buyer 22	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	Buyer 47	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.96
Buyer 23	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	Buyer 48	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 24	1.90	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	Buyer 49	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 25	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	Buyer 50	1.97	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98

Table A4.2 The solutions of $p_d(i)$ of each buyer for Example 3 Model 1

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	0.23	0.22	0.22	0.22	0.22	0.22	0.19	0.19	0.19
Buyer 2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 3	0.22	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.20
Buyer 4	0.54	0.54	0.54	0.53	0.53	0.54	0.53	0.53	0.53
Buyer 5	0.30	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.28
Buyer 6	0.24	0.23	0.22	0.22	0.22	0.23	0.22	0.22	0.21
Buyer 7	0.22	0.22	0.20	0.20	0.20	0.20	0.20	0.20	0.19
Buyer 8	0.47	0.46	0.46	0.45	0.45	0.46	0.45	0.45	0.44
Buyer 9	0.61	0.60	0.59	0.59	0.58	0.60	0.59	0.58	0.58
Buyer 10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 11	0.51	0.51	0.50	0.50	0.50	0.50	0.50	0.49	0.49
Buyer 12	0.46	0.46	0.45	0.45	0.45	0.46	0.45	0.45	0.45
Buyer 13	0.14	0.13	0.13	0.13	0.13	0.14	0.13	0.13	0.13
Buyer 14	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.22	0.22
Buyer 15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 16	0.43	0.43	0.42	0.42	0.42	0.43	0.42	0.42	0.41
Buyer 17	0.26	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.24
Buyer 18	0.51	0.50	0.49	0.49	0.48	0.50	0.49	0.48	0.48
Buyer 19	0.13	0.13	0.12	0.12	0.12	0.13	0.12	0.12	0.12
Buyer 20	0.90	0.89	0.88	0.88	0.87	0.89	0.87	0.87	0.86
Buyer 21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 22	0.25	0.25	0.25	0.25	0.24	0.25	0.25	0.24	0.24
Buyer 23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 24	0.62	0.61	0.61	0.60	0.60	0.61	0.60	0.60	0.59
Buyer 25	0.53	0.52	0.52	0.52	0.51	0.52	0.52	0.51	0.51

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 26	0.34	0.34	0.33	0.33	0.33	0.34	0.33	0.33	0.33
Buyer 27	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.16
Buyer 28	0.26	0.26	0.26	0.26	0.25	0.26	0.25	0.25	0.25
Buyer 29	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.21
Buyer 30	0.52	0.51	0.51	0.51	0.51	0.51	0.51	0.50	0.50
Buyer 31	0.35	0.35	0.34	0.34	0.34	0.34	0.34	0.34	0.34
Buyer 32	0.89	0.88	0.87	0.87	0.86	0.88	0.86	0.86	0.85
Buyer 33	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 34	0.21	0.21	0.20	0.20	0.20	0.21	0.20	0.20	0.20
Buyer 35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 36	0.60	0.60	0.59	0.59	0.58	0.59	0.59	0.58	0.58
Buyer 37	0.46	0.45	0.45	0.45	0.44	0.45	0.45	0.44	0.44
Buyer 38	0.28	0.28	0.28	0.28	0.27	0.28	0.27	0.27	0.27
Buyer 39	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 40	0.23	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
Buyer 41	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 42	0.82	0.81	0.80	0.80	0.79	0.81	0.80	0.79	0.79
Buyer 43	0.31	0.31	0.31	0.31	0.30	0.31	0.30	0.30	0.30
Buyer 44	0.38	0.38	0.37	0.37	0.37	0.37	0.37	0.37	0.36
Buyer 45	0.78	0.77	0.73	0.73	0.72	0.69	0.72	0.69	0.71
Buyer 46	0.42	0.42	0.41	0.41	0.41	0.41	0.41	0.41	0.40
Buyer 47	0.31	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Buyer 48	0.31	0.31	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Buyer 49	0.68	0.67	0.66	0.66	0.65	0.66	0.65	0.65	0.64
Buyer 50	0.19	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18

Table A4.3 The solutions of $q(i)$ of each buyer for Example 3 Model 1

Appendix 5 - Example 1 Model 1 “*q(i)*_factor” Minimizing System’s Cost

D/P N	0.1 49	0.2 47	0.3 60	0.4 58	0.5 58	0.6 58	0.7 84	0.8 84	0.9 120
K(i)									
Buyer 1	49	47	60	58	58	58	42	42	40
Buyer 2	49	47	30	29	29	29	28	28	24
Buyer 3	49	47	30	29	29	29	28	28	30
Buyer 4	49	47	60	58	58	58	42	42	40
Buyer 5	49	47	30	29	29	29	28	28	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Discounted Price Pd(i)									
Buyer 1	1.89	1.90	1.82	1.84	1.85	1.84	1.93	1.93	1.94
Buyer 2	1.83	1.84	1.93	1.93	1.94	1.93	1.94	1.94	1.96
Buyer 3	1.73	1.75	1.84	1.86	1.87	1.85	1.86	1.87	1.86
Buyer 4	1.76	1.79	1.66	1.69	1.71	1.69	1.81	1.83	1.84
Buyer 5	1.88	1.89	1.95	1.96	1.96	1.96	1.96	1.96	1.98

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
q(i)									
Buyer 1	0.23	0.22	0.27	0.25	0.24	0.24	0.23	0.21	0.22
Buyer 2	0.18	0.17	0.14	0.14	0.13	0.15	0.14	0.13	0.13
Buyer 3	0.23	0.22	0.19	0.18	0.17	0.20	0.19	0.17	0.18
Buyer 4	0.34	0.32	0.39	0.36	0.34	0.36	0.34	0.31	0.32
Buyer 5	0.14	0.13	0.11	0.11	0.10	0.11	0.11	0.10	0.10

Appendix 5 - Example 2 Model 1 "*q(i)*_factor" Minimizing System's Cost

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	48	60	60	60	72	72	84	120	144
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	15	12	12	12	12	12	12	12
Buyer 3	1	1	1	1	1	1	1	1	1
Buyer 4	4	4	4	4	4	4	4	4	4
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	4	4	4	4	4	4	4	4	4
Buyer 7	3	3	3	3	3	3	3	3	3
Buyer 8	6	5	5	5	6	6	6	5	6
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	12	10	10	10	9	9	7	8	9
Buyer 11	8	10	10	10	8	8	7	8	8
Buyer 12	24	20	20	20	18	18	21	20	18
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	3	3	3	3	3	3	3	3	3
Buyer 15	2	2	2	2	2	2	2	2	2
Buyer 16	6	6	6	6	6	6	6	6	6
Buyer 17	48	60	60	60	72	72	84	60	72
Buyer 18	3	3	3	3	3	3	3	3	3
Buyer 19	6	5	5	5	6	6	6	5	6
Buyer 20	3	3	3	3	3	3	3	3	3
Buyer 21	6	6	6	6	6	6	7	6	6
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	4	4	4	4	4	4	4	4	4
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	4	4	4	4	4	4	4	4	4
Buyer 29	48	60	60	60	72	72	84	120	72
Buyer 30	3	3	3	3	3	3	3	3	3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Discounted Price Pd(i)									
Buyer 1	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94
Buyer 2	1.99	1.95	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 3	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 4	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 5	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 6	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
Buyer 7	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94
Buyer 8	1.94	1.96	1.96	1.96	1.93	1.94	1.94	1.96	1.93
Buyer 9	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86
Buyer 10	0.95	1.09	1.06	1.07	1.16	1.18	1.39	1.28	1.18
Buyer 11	1.95	1.92	1.92	1.92	1.95	1.95	1.97	1.95	1.95
Buyer 12	1.96	1.98	1.98	1.98	1.99	1.99	1.98	1.98	1.99
Buyer 13	1.83	1.83	1.82	1.82	1.83	1.83	1.83	1.83	1.83
Buyer 14	1.97	1.98	1.97	1.97	1.97	1.98	1.98	1.98	1.98
Buyer 15	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81
Buyer 16	1.25	1.25	1.23	1.23	1.24	1.26	1.26	1.27	1.26
Buyer 17	0.31	0.25	0.25	0.25	0.21	0.21	0.18	0.25	0.21
Buyer 18	1.62	1.62	1.61	1.61	1.62	1.62	1.63	1.63	1.62
Buyer 19	1.61	1.73	1.72	1.72	1.61	1.61	1.62	1.73	1.61
Buyer 20	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.83	1.82
Buyer 21	1.99	1.99	1.99	1.99	1.99	1.99	1.97	1.99	1.99
Buyer 22	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 23	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 24	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86
Buyer 25	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 26	1.52	1.52	1.51	1.51	1.51	1.52	1.52	1.52	1.52
Buyer 27	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 28	1.92	1.93	1.92	1.92	1.92	1.93	1.93	1.93	1.93
Buyer 29	0.68	0.55	0.55	0.55	0.47	0.47	0.41	0.29	0.47
Buyer 30	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
q(i)									
Buyer 1	0.77	0.76	0.79	0.78	0.78	0.76	0.76	0.75	0.76
Buyer 2	0.20	0.19	0.20	0.20	0.20	0.19	0.19	0.19	0.19
Buyer 3	0.26	0.26	0.26	0.26	0.26	0.26	0.25	0.25	0.26
Buyer 4	0.34	0.34	0.35	0.35	0.35	0.34	0.34	0.33	0.34
Buyer 5	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22
Buyer 6	0.49	0.48	0.50	0.49	0.49	0.48	0.48	0.48	0.48
Buyer 7	0.45	0.44	0.46	0.46	0.45	0.44	0.44	0.44	0.44
Buyer 8	0.17	0.19	0.20	0.20	0.19	0.17	0.17	0.19	0.19
Buyer 9	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.20	0.21
Buyer 10	0.92	0.91	0.93	0.93	0.92	0.90	0.90	0.89	0.90
Buyer 11	0.12	0.13	0.12	0.12	0.12	0.12	0.11	0.12	0.11
Buyer 12	0.14	0.13	0.13	0.13	0.13	0.12	0.13	0.12	0.12
Buyer 13	0.31	0.31	0.31	0.31	0.31	0.30	0.30	0.30	0.30
Buyer 14	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.22	0.23
Buyer 15	0.28	0.28	0.29	0.29	0.29	0.28	0.28	0.28	0.28
Buyer 16	0.79	0.79	0.81	0.81	0.80	0.78	0.78	0.77	0.78
Buyer 17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 18	0.34	0.34	0.35	0.35	0.34	0.34	0.34	0.33	0.34
Buyer 19	0.62	0.62	0.63	0.63	0.62	0.61	0.61	0.61	0.61
Buyer 20	0.58	0.57	0.59	0.58	0.58	0.57	0.57	0.56	0.57
Buyer 21	0.20	0.20	0.21	0.20	0.20	0.20	0.20	0.20	0.20
Buyer 22	0.25	0.24	0.25	0.25	0.25	0.24	0.24	0.24	0.24
Buyer 23	0.27	0.27	0.28	0.28	0.28	0.27	0.27	0.27	0.27
Buyer 24	0.41	0.41	0.42	0.42	0.41	0.41	0.41	0.41	0.41
Buyer 25	0.21	0.21	0.22	0.22	0.21	0.21	0.21	0.21	0.21
Buyer 26	0.47	0.47	0.48	0.47	0.47	0.47	0.47	0.46	0.47
Buyer 27	0.71	0.70	0.73	0.73	0.72	0.70	0.69	0.68	0.70
Buyer 28	0.25	0.25	0.26	0.26	0.26	0.25	0.25	0.25	0.25
Buyer 29	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 30	0.35	0.34	0.35	0.35	0.35	0.34	0.34	0.34	0.34

Appendix 5 - Example 3 Model 1 "*q(i)*_factor" Minimizing System's Cost

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	36	40	48	48	48	60	72	84	120	N	36	36	48	48	48	60	72	84	120
Buyer 1	4	4	4	4	4	4	4	4	4	Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 2	36	20	12	12	8	12	9	7	8	Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 3	2	2	2	2	2	2	2	2	2	Buyer 28	2	2	2	2	2	2	2	2	2
Buyer 4	2	2	2	2	2	2	2	2	2	Buyer 29	1	1	1	1	1	1	1	1	1
Buyer 5	1	1	1	1	1	1	1	1	1	Buyer 30	2	2	2	2	2	2	2	2	2
Buyer 6	36	20	16	16	16	15	12	14	12	Buyer 31	2	2	2	2	2	2	2	2	2
Buyer 7	4	4	4	4	4	3	3	3	3	Buyer 32	4	4	4	4	4	4	4	4	4
Buyer 8	1	1	1	1	1	1	1	1	1	Buyer 33	36	40	48	48	48	60	72	84	120
Buyer 9	4	4	4	4	4	4	4	4	4	Buyer 34	4	4	4	4	4	4	4	4	4
Buyer 10	4	4	4	4	4	4	4	4	4	Buyer 35	9	8	8	8	8	6	8	7	8
Buyer 11	2	2	2	2	2	2	2	2	2	Buyer 36	3	2	2	2	2	2	2	2	2
Buyer 12	2	2	2	2	2	2	2	2	2	Buyer 37	2	2	2	2	2	2	2	2	2
Buyer 13	4	5	4	4	4	5	4	4	5	Buyer 38	2	2	2	2	2	2	2	2	2
Buyer 14	1	1	1	1	1	1	1	1	1	Buyer 39	36	40	48	48	48	60	72	84	60
Buyer 15	36	40	48	48	48	60	72	84	60	Buyer 40	4	4	4	4	4	4	4	4	4
Buyer 16	2	2	2	2	2	2	2	2	2	Buyer 41	36	40	48	48	48	60	72	84	60
Buyer 17	3	2	3	3	3	3	3	3	3	Buyer 42	3	2	3	3	3	3	3	3	3
Buyer 18	1	1	1	1	1	1	1	1	1	Buyer 43	2	2	2	2	2	2	2	2	2
Buyer 19	2	2	2	2	2	2	2	2	2	Buyer 44	4	4	4	4	4	4	4	4	4
Buyer 20	3	2	3	3	3	3	3	3	3	Buyer 45	9	8	8	8	8	10	8	7	8
Buyer 21	36	40	48	48	48	60	72	84	60	Buyer 46	1	1	1	1	1	1	1	1	1
Buyer 22	1	1	1	1	1	1	1	1	1	Buyer 47	1	1	1	1	1	1	1	1	1
Buyer 23	9	8	8	8	8	6	8	7	8	Buyer 48	2	2	2	2	2	2	2	2	2
Buyer 24	2	2	2	2	2	2	2	2	2	Buyer 49	2	2	2	2	2	2	2	2	2
Buyer 25	2	2	2	2	2	2	2	2	2	Buyer 50	1	1	1	1	1	1	1	1	1

Table A5.1 The solutions of $k(i)$ of each buyer for Example 3 Model 1

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 2	0.26	0.47	0.74	0.74	1.04	0.74	0.95	1.16	1.05
Buyer 3	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 4	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71	1.71
Buyer 5	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 6	1.92	1.97	1.99	1.99	1.99	1.99	2.00	2.00	2.00
Buyer 7	1.97	1.97	1.98	1.98	1.98	2.00	2.00	2.00	2.00
Buyer 8	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 9	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94
Buyer 10	1.60	1.60	1.60	1.60	1.61	1.60	1.60	1.61	1.61
Buyer 11	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 12	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 13	2.00	2.00	2.00	2.00	2.00	1.99	2.00	2.00	2.00
Buyer 14	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
Buyer 15	0.54	0.49	0.42	0.42	0.42	0.34	0.28	0.24	0.34
Buyer 16	1.72	1.72	1.73	1.73	1.73	1.73	1.73	1.73	1.73
Buyer 17	1.92	1.98	1.92	1.92	1.92	1.92	1.92	1.92	1.92
Buyer 18	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 19	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 20	1.91	1.99	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 21	0.24	0.22	0.18	0.18	0.18	0.15	0.12	0.10	0.15
Buyer 22	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 23	1.61	1.68	1.69	1.69	1.69	1.84	1.69	1.76	1.69
Buyer 24	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93
Buyer 25	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 26	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
Buyer 27	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 28	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93
Buyer 29	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 30	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
Buyer 31	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 32	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.67
Buyer 33	1.55	1.44	1.26	1.26	1.26	1.06	0.91	0.80	0.58
Buyer 34	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 35	1.18	1.27	1.27	1.27	1.28	1.48	1.28	1.37	1.28
Buyer 36	1.83	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 37	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.82
Buyer 38	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 39	0.33	0.30	0.25	0.25	0.25	0.20	0.17	0.15	0.20
Buyer 40	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 41	0.91	0.83	0.71	0.71	0.71	0.58	0.49	0.42	0.58
Buyer 42	1.80	1.98	1.80	1.80	1.80	1.80	1.80	1.80	1.80
Buyer 43	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 44	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94
Buyer 45	1.92	1.96	1.96	1.96	1.96	1.90	1.96	1.99	1.96
Buyer 46	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.87
Buyer 47	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 48	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92
Buyer 49	1.73	1.73	1.73	1.73	1.74	1.73	1.74	1.74	1.74
Buyer 50	1.98	1.98	1.98	1.99	1.99	1.98	1.99	1.99	1.99

Table A5.2 The solutions of $p_d(i)$ of each buyer for Example 3 Model 1

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9		D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	0.14	0.14	0.14	0.14	0.13	0.14	0.11	0.11	0.11		Buyer 26	0.28	0.28	0.27	0.27	0.27	0.28	0.27	0.27	0.27
Buyer 2	1.00	0.99	0.98	0.98	0.97	0.99	0.97	0.97	0.96		Buyer 27	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
Buyer 3	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11		Buyer 28	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
Buyer 4	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.40		Buyer 29	0.16	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Buyer 5	0.21	0.21	0.21	0.21	0.20	0.21	0.21	0.20	0.20		Buyer 30	0.41	0.41	0.40	0.40	0.40	0.40	0.40	0.40	0.40
Buyer 6	0.09	0.09	0.09	0.08	0.08	0.09	0.08	0.08	0.08		Buyer 31	0.27	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
Buyer 7	0.10	0.10	0.09	0.09	0.09	0.09	0.09	0.09	0.09		Buyer 32	0.58	0.58	0.58	0.57	0.57	0.58	0.57	0.57	0.57
Buyer 8	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28		Buyer 33	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 9	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.34		Buyer 34	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
Buyer 10	0.64	0.63	0.63	0.63	0.62	0.63	0.62	0.62	0.62		Buyer 35	0.79	0.78	0.78	0.78	0.77	0.78	0.77	0.77	0.77
Buyer 11	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.32	0.32		Buyer 36	0.40	0.39	0.39	0.39	0.39	0.39	0.39	0.39	0.39
Buyer 12	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35		Buyer 37	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.32
Buyer 13	0.07	0.07	0.07	0.07	0.07	0.08	0.07	0.07	0.07		Buyer 38	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
Buyer 14	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16		Buyer 39	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		Buyer 40	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
Buyer 16	0.28	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27		Buyer 41	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Buyer 17	0.17	0.17	0.16	0.16	0.16	0.17	0.16	0.16	0.16		Buyer 42	0.53	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.51
Buyer 18	0.27	0.27	0.27	0.27	0.26	0.27	0.26	0.26	0.26		Buyer 43	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
Buyer 19	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06		Buyer 44	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.19
Buyer 20	0.57	0.56	0.56	0.56	0.55	0.56	0.56	0.55	0.55		Buyer 45	0.41	0.40	0.37	0.37	0.37	0.33	0.37	0.35	0.37
Buyer 21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00		Buyer 46	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
Buyer 22	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19		Buyer 47	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
Buyer 23	0.57	0.57	0.56	0.56	0.55	0.56	0.56	0.55	0.55		Buyer 48	0.21	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
Buyer 24	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.44	0.44		Buyer 49	0.41	0.40	0.40	0.40	0.40	0.40	0.40	0.40	0.39
Buyer 25	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36		Buyer 50	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11

Table A5.3 The solutions of $q(i)$ of each buyer for Example 3 Model 1

Appendix 6 - Example 1 Model 2 Minimizing Vendor's Cost

D/P N	0.1 45	0.2 43	0.3 54	0.4 52	0.5 56	0.6 66	0.7 72	0.8 84	0.9 120
K(i)									
Buyer 1	45	43	54	52	28	33	36	28	30
Buyer 2	45	43	27	26	28	22	24	21	20
Buyer 3	45	43	27	26	28	33	24	28	24
Buyer 4	45	43	54	52	56	33	36	42	40
Buyer 5	45	43	27	26	28	22	24	21	20

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Discounted Price Pd(i)									
Buyer 1	1.93	1.93	1.90	1.90	1.99	1.97	1.96	1.99	1.98
Buyer 2	1.88	1.88	1.95	1.95	1.94	1.97	1.96	1.97	1.98
Buyer 3	1.85	1.86	1.92	1.93	1.92	1.90	1.94	1.92	1.94
Buyer 4	1.89	1.90	1.85	1.86	1.84	1.94	1.93	1.90	1.91
Buyer 5	1.91	1.91	1.96	1.97	1.96	1.98	1.97	1.98	1.99

Appendix 6 - Example 2 Model 2 Minimizing Vendor's Cost

D/P N	0.1 48	0.2 60	0.3 60	0.4 60	0.5 72	0.6 72	0.7 96	0.8 120	0.9 144
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	10	10	10	12	12	12	10	12
Buyer 3	1	1	1	1	1	1	1	1	1
Buyer 4	4	4	4	4	4	4	4	4	4
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	3	3	3	3	3	3	3	3	3
Buyer 7	3	3	3	3	3	3	3	3	3
Buyer 8	4	4	4	4	4	4	4	4	4
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	3	3	3	3	3	3	3	3	3
Buyer 11	6	6	6	6	6	6	6	6	6
Buyer 12	16	15	15	15	18	18	16	15	16
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	3	3	3	3	3	3	3	3	3
Buyer 15	1	1	1	1	1	1	1	1	1
Buyer 16	4	4	4	4	4	4	4	4	4
Buyer 17	4	5	5	5	4	4	4	5	4
Buyer 18	3	3	3	3	3	3	3	3	3
Buyer 19	4	4	4	4	4	3	3	3	3
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	6	6	6	6	6	6	6	6	6
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	3	3	3	3	3	3	3	3	3
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	3	3	3	3	3	3	3	3	3
Buyer 29	12	10	10	10	9	9	12	10	9
Buyer 30	3	3	3	3	3	3	3	3	3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Discounted Price Pd(i)									
Buyer 1	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 2	1.98	2.00	2.00	2.00	1.98	1.98	1.98	2.00	1.98
Buyer 3	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 4	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92
Buyer 5	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 6	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
Buyer 7	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 8	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 9	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 10	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 11	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 12	1.99	2.00	2.00	2.00	1.99	1.99	1.99	2.00	1.99
Buyer 13	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83
Buyer 14	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94
Buyer 15	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 16	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61
Buyer 17	1.97	1.77	1.77	1.77	1.97	1.97	1.97	1.77	1.97
Buyer 18	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
Buyer 19	1.81	1.81	1.81	1.81	1.81	1.94	1.94	1.94	1.94
Buyer 20	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 21	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 22	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 23	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 24	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
Buyer 25	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 26	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
Buyer 27	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 28	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 29	1.75	1.91	1.91	1.91	2.00	2.00	1.75	1.91	2.00
Buyer 30	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97

Appendix 6 - Example 3 Model 2 Minimizing Vendor's Cost

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
Buyer 1	4	4	4	4	4	4	4	4	4
Buyer 2	3	3	3	3	3	3	3	3	3
Buyer 3	2	2	2	2	2	2	2	2	2
Buyer 4	2	2	2	2	2	2	2	2	2
Buyer 5	1	1	1	1	1	1	1	1	1
Buyer 6	18	18	16	16	16	15	15	14	15
Buyer 7	3	3	3	3	3	3	3	3	3
Buyer 8	1	1	1	1	1	1	1	1	1
Buyer 9	3	3	3	3	3	3	3	3	3
Buyer 10	2	2	2	2	2	2	2	2	2
Buyer 11	2	2	2	2	2	2	2	2	2
Buyer 12	2	2	2	2	2	2	2	2	2
Buyer 13	6	6	6	6	6	5	5	6	5
Buyer 14	1	1	1	1	1	1	1	1	1
Buyer 15	9	9	8	8	8	10	10	7	8
Buyer 16	2	2	2	2	2	2	2	2	2
Buyer 17	2	2	2	2	2	2	2	2	2
Buyer 18	1	1	1	1	1	1	1	1	1
Buyer 19	2	2	2	2	2	2	2	2	2
Buyer 20	3	3	3	3	3	3	3	3	3
Buyer 21	6	6	6	6	6	5	5	6	5
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	6	6	6	6	6	5	5	6	5
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	2	2	2	2	2	2	2	2	2

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	2	2	2	2	2	2	2	2	2
Buyer 29	1	1	1	1	1	1	1	1	1
Buyer 30	2	2	2	2	2	2	2	2	2
Buyer 31	2	2	2	2	2	2	2	2	2
Buyer 32	2	2	2	2	2	2	2	2	2
Buyer 33	36	36	48	48	48	30	30	28	30
Buyer 34	4	4	4	4	4	4	4	4	4
Buyer 35	4	4	4	4	4	4	4	4	4
Buyer 36	2	2	2	2	2	2	2	2	2
Buyer 37	2	2	2	2	2	2	2	2	2
Buyer 38	2	2	2	2	2	2	2	2	2
Buyer 39	6	6	6	6	6	5	5	6	5
Buyer 40	3	3	3	3	3	3	3	3	3
Buyer 41	12	12	12	12	12	10	10	12	10
Buyer 42	2	2	2	2	2	2	2	2	2
Buyer 43	2	2	2	2	2	2	2	2	2
Buyer 44	3	3	3	3	3	3	3	3	3
Buyer 45	9	9	8	8	8	10	10	7	8
Buyer 46	1	1	1	1	1	1	1	1	1
Buyer 47	1	1	1	1	1	1	1	1	1
Buyer 48	2	2	2	2	2	2	2	2	2
Buyer 49	2	2	2	2	2	2	2	2	2
Buyer 50	1	1	1	1	1	1	1	1	1

Table A6.1 The solutions of $k(i)$ of each buyer for Example 3 Model 2

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 2	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 3	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 4	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70
Buyer 5	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
Buyer 6	1.99	1.99	1.99	1.99	1.99	1.99	1.99	2.00	1.99
Buyer 7	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 8	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 9	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 10	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 11	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
Buyer 12	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79
Buyer 13	1.97	1.97	1.97	1.97	1.97	1.98	1.98	1.97	1.98
Buyer 14	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 15	1.49	1.49	1.60	1.60	1.60	1.40	1.40	1.73	1.60
Buyer 16	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77
Buyer 17	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 18	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93
Buyer 19	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 20	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79
Buyer 21	1.40	1.40	1.40	1.40	1.40	1.53	1.53	1.40	1.53
Buyer 22	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
Buyer 23	1.82	1.82	1.82	1.82	1.82	1.90	1.90	1.82	1.90
Buyer 24	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 25	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 26	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83
Buyer 27	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 28	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 29	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 30	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81
Buyer 31	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
Buyer 32	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 33	1.81	1.81	1.70	1.70	1.70	1.87	1.87	1.89	1.87
Buyer 34	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 35	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
Buyer 36	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 37	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78
Buyer 38	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 39	1.55	1.55	1.55	1.55	1.55	1.68	1.68	1.55	1.68
Buyer 40	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 41	1.87	1.87	1.87	1.87	1.87	1.97	1.97	1.87	1.97
Buyer 42	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92
Buyer 43	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 44	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 45	1.88	1.88	1.92	1.92	1.92	1.85	1.85	1.96	1.92
Buyer 46	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
Buyer 47	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 48	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88
Buyer 49	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
Buyer 50	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96

Table A6.2 The solutions of $p_d(i)$ of each buyer for Example 3 Model 2

Appendix 7 - Example 1 Model 2 Minimizing System's Cost

D/P N	0.1 48	0.2 47	0.3 58	0.4 58	0.5 58	0.6 78	0.7 78	0.8 84	0.9 144
K(i)									
Buyer 1	48	47	58	58	58	39	39	42	36
Buyer 2	48	47	29	29	29	26	26	28	24
Buyer 3	48	47	29	29	29	26	26	28	24
Buyer 4	48	47	58	58	58	39	39	42	36
Buyer 5	48	47	29	29	29	26	26	28	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Discounted Price Pd(i)									
Buyer 1	1.92	1.92	1.88	1.88	1.88	1.95	1.95	1.94	1.96
Buyer 2	1.87	1.87	1.94	1.94	1.94	1.95	1.95	1.94	1.96
Buyer 3	1.83	1.84	1.92	1.92	1.92	1.93	1.93	1.92	1.94
Buyer 4	1.88	1.88	1.83	1.83	1.83	1.91	1.91	1.90	1.93
Buyer 5	1.90	1.90	1.96	1.96	1.96	1.97	1.97	1.96	1.97

Appendix 7 - Example 2 Model 2 Minimizing System's Cost

D/P N	0.1 48	0.2 60	0.3 60	0.4 60	0.5 72	0.6 72	0.7 84	0.8 120	0.9 144
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	15	12	12	12	12	12	12	12
Buyer 3	2	2	2	2	2	2	2	2	2
Buyer 4	4	4	4	4	4	4	4	4	4
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	4	4	4	4	4	4	4	4	4
Buyer 7	3	3	3	3	3	3	3	3	3
Buyer 8	6	5	5	5	6	6	6	5	6
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	6	6	6	6	6	6	6	6	6
Buyer 11	8	10	10	6	8	8	7	8	8
Buyer 12	24	20	20	20	18	18	21	20	18
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	4	4	4	4	4	4	4	4	4
Buyer 15	2	2	2	2	2	2	2	2	2
Buyer 16	6	5	5	5	6	6	6	5	6
Buyer 17	16	15	15	15	18	18	14	15	16
Buyer 18	3	3	3	3	3	3	3	3	3
Buyer 19	6	5	5	5	6	6	6	5	6
Buyer 20	3	3	3	3	3	3	3	3	3
Buyer 21	8	6	6	6	8	6	7	6	6
Buyer 22	3	3	3	3	3	3	3	3	3
Buyer 23	4	4	4	4	4	4	4	4	4
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	5	5	5	4	4	4	5	4
Buyer 28	4	4	4	4	4	4	4	4	4
Buyer 29	24	30	30	30	24	24	28	24	24
Buyer 30	4	4	4	4	4	4	4	4	4

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Discounted Price Pd(i)									
Buyer 1	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 2	1.98	1.95	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 3	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74	1.74
Buyer 4	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92
Buyer 5	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 6	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75	1.75
Buyer 7	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 8	1.90	1.94	1.94	1.94	1.90	1.90	1.90	1.94	1.90
Buyer 9	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 10	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55	1.55
Buyer 11	1.97	1.95	1.95	1.99	1.97	1.97	1.98	1.97	1.97
Buyer 12	1.97	1.98	1.98	1.98	1.99	1.99	1.98	1.98	1.99
Buyer 13	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83
Buyer 14	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86
Buyer 15	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69	1.69
Buyer 16	1.36	1.48	1.48	1.48	1.36	1.36	1.36	1.48	1.36
Buyer 17	0.85	0.89	0.89	0.89	0.78	0.78	0.94	0.89	0.85
Buyer 18	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
Buyer 19	1.58	1.69	1.69	1.69	1.58	1.58	1.58	1.69	1.58
Buyer 20	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73
Buyer 21	1.91	1.97	1.97	1.97	1.91	1.97	1.94	1.97	1.97
Buyer 22	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 23	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93
Buyer 24	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
Buyer 25	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 26	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
Buyer 27	2.00	1.90	1.90	1.90	2.00	2.00	2.00	1.90	2.00
Buyer 28	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
Buyer 29	1.18	1.01	1.01	1.01	1.18	1.18	1.06	1.18	1.18
Buyer 30	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87

Appendix 7 - Example 3 Model 2 Minimizing System's Cost

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	36	36	48	48	48	60	60	84	120
Buyer 1	4	4	4	4	4	4	4	4	4
Buyer 2	4	4	4	4	4	4	4	4	4
Buyer 3	2	2	2	2	2	2	2	2	2
Buyer 4	2	2	2	2	2	2	2	2	2
Buyer 5	1	1	1	1	1	1	1	1	1
Buyer 6	18	18	16	16	16	15	15	14	15
Buyer 7	4	4	4	4	4	4	3	3	3
Buyer 8	1	1	1	1	1	1	1	1	1
Buyer 9	4	4	4	4	4	5	5	4	5
Buyer 10	3	3	3	3	3	3	3	3	3
Buyer 11	2	2	2	2	2	2	2	2	2
Buyer 12	2	2	2	2	2	2	2	2	2
Buyer 13	6	6	6	6	6	5	5	6	5
Buyer 14	1	1	1	1	1	1	1	1	1
Buyer 15	36	36	48	48	48	60	30	42	40
Buyer 16	2	2	2	2	2	2	2	2	2
Buyer 17	3	3	3	3	3	3	3	3	3
Buyer 18	1	1	1	1	1	1	1	1	1
Buyer 19	2	2	2	2	2	2	2	2	2
Buyer 20	3	3	3	3	3	3	3	3	3
Buyer 21	9	9	8	8	8	6	6	7	8
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	9	9	8	8	8	6	6	7	8
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	2	2	2	2	2	2	2	2	2

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	36	36	48	48	48	60	60	84	120
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	2	2	2	2	2	2	2	2	2
Buyer 29	1	1	1	1	1	1	1	1	1
Buyer 30	2	2	2	2	2	2	2	2	2
Buyer 31	2	2	2	2	2	2	2	2	2
Buyer 32	4	4	4	4	4	4	4	4	4
Buyer 33	36	36	48	48	48	60	30	42	40
Buyer 34	4	4	4	4	4	4	4	4	4
Buyer 35	6	6	6	6	6	6	6	6	6
Buyer 36	3	3	3	3	3	3	3	3	3
Buyer 37	2	2	2	2	2	2	2	2	2
Buyer 38	2	2	2	2	2	2	2	2	2
Buyer 39	9	9	8	8	8	10	10	12	10
Buyer 40	4	4	4	4	4	4	4	4	4
Buyer 41	18	18	16	16	16	15	15	14	15
Buyer 42	3	3	3	3	3	3	3	3	3
Buyer 43	2	2	2	2	2	2	2	2	2
Buyer 44	4	4	4	4	4	4	4	4	4
Buyer 45	9	9	8	8	8	10	10	7	8
Buyer 46	1	1	1	1	1	1	1	1	1
Buyer 47	1	1	1	1	1	1	1	1	1
Buyer 48	2	2	2	2	2	2	2	2	2
Buyer 49	2	2	2	2	2	2	2	2	2
Buyer 50	1	1	1	1	1	1	1	1	1

Table A7.1 The solutions of $k(i)$ of each buyer for Example 3 Model 2

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 2	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66	1.66
Buyer 3	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 4	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70	1.70
Buyer 5	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
Buyer 6	1.99	1.99	1.99	1.99	1.99	1.99	1.99	2.00	1.99
Buyer 7	1.97	1.97	1.97	1.97	1.97	1.97	1.99	1.99	1.99
Buyer 8	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 9	1.89	1.89	1.89	1.89	1.89	1.80	1.80	1.89	1.80
Buyer 10	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77
Buyer 11	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
Buyer 12	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79
Buyer 13	1.97	1.97	1.97	1.97	1.97	1.98	1.98	1.97	1.98
Buyer 14	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 15	0.52	0.52	0.41	0.41	0.41	0.33	0.61	0.46	0.48
Buyer 16	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77
Buyer 17	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
Buyer 18	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93
Buyer 19	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 20	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79	1.79
Buyer 21	1.14	1.14	1.21	1.21	1.21	1.40	1.40	1.30	1.21
Buyer 22	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
Buyer 23	1.62	1.62	1.68	1.68	1.68	1.82	1.82	1.75	1.68
Buyer 24	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 25	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93	1.93

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 26	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83	1.83
Buyer 27	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 28	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 29	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 30	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81	1.81
Buyer 31	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
Buyer 32	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61	1.61
Buyer 33	1.81	1.81	1.70	1.70	1.70	1.59	1.87	1.75	1.77
Buyer 34	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 35	1.51	1.51	1.51	1.51	1.51	1.51	1.51	1.51	1.51
Buyer 36	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72	1.72
Buyer 37	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78
Buyer 38	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 39	1.25	1.25	1.34	1.34	1.34	1.18	1.18	1.05	1.18
Buyer 40	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 41	1.63	1.63	1.71	1.71	1.71	1.75	1.75	1.79	1.75
Buyer 42	1.68	1.68	1.68	1.68	1.68	1.68	1.68	1.68	1.68
Buyer 43	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 44	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
Buyer 45	1.88	1.88	1.92	1.92	1.92	1.85	1.85	1.96	1.92
Buyer 46	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
Buyer 47	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 48	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88
Buyer 49	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
Buyer 50	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96

Table A7.2 The solutions of $p_d(i)$ of each buyer for Example 3 Model 2

Appendix 8 - Example 1 Model 3 Minimizing Vendor's Cost

D/P N	0.1 45	0.2 44	0.3 56	0.4 56	0.5 56	0.6 78	0.7 78	0.8 78	0.9 144
K(i)									
Buyer 1	45	44	56	56	56	39	39	39	36
Buyer 2	45	44	28	28	28	26	26	26	24
Buyer 3	45	44	28	28	28	26	26	26	24
Buyer 4	45	44	56	56	56	39	39	39	36
Buyer 5	45	44	28	28	28	26	26	26	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Discounted Price Pd(i)									
Buyer 1	1.96	1.97	1.93	1.93	1.93	1.98	1.98	1.98	1.99
Buyer 2	1.91	1.91	1.97	1.97	1.97	1.98	1.98	1.98	1.98
Buyer 3	1.87	1.88	1.95	1.95	1.95	1.96	1.96	1.96	1.96
Buyer 4	1.93	1.93	1.88	1.88	1.88	1.95	1.95	1.95	1.96
Buyer 5	1.93	1.94	1.98	1.98	1.98	1.99	1.99	1.99	1.99

Appendix 8 - Example 2 Model 3 Minimizing Vendor's Cost

D/P N	0.1 48	0.2 60	0.3 60	0.4 60	0.5 60	0.6 72	0.7 84	0.8 120	0.9 180
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	12	12	12	12	12	12	12	12
Buyer 3	1	1	1	1	1	1	1	1	1
Buyer 4	4	4	4	4	4	4	4	4	4
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	3	3	3	3	3	3	3	3	3
Buyer 7	3	3	3	3	3	3	3	3	3
Buyer 8	4	5	5	5	5	4	4	5	5
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	4	4	4	4	4	4	4	4	4
Buyer 11	8	10	6	6	6	8	7	8	6
Buyer 12	24	20	20	20	20	18	21	20	18
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	3	3	3	3	3	3	3	3	3
Buyer 15	1	1	1	1	1	1	1	1	1
Buyer 16	3	3	3	3	3	3	3	3	3
Buyer 17	6	5	5	5	5	6	6	5	5
Buyer 18	3	3	3	3	3	3	3	3	3
Buyer 19	4	4	4	4	4	4	4	4	4
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	6	6	6	6	6	6	6	6	6
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	4	4	4	4	4	4	4	4	4
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	4	4	4	4	4	4	4	4	4
Buyer 29	12	12	12	12	12	12	12	12	12
Buyer 30	3	3	3	3	3	3	3	3	3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Discounted Price Pd(i)									
Buyer 1	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 2	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 3	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 4	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 5	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 6	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 7	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 8	2.00	1.98	1.98	1.98	1.98	2.00	2.00	1.98	1.98
Buyer 9	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92
Buyer 10	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92	1.92
Buyer 11	1.99	1.97	2.00	2.00	2.00	1.99	1.99	1.99	2.00
Buyer 12	1.99	1.99	1.99	1.99	1.99	2.00	1.99	1.99	2.00
Buyer 13	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
Buyer 14	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 15	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 16	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88
Buyer 17	1.70	1.90	1.90	1.90	1.90	1.70	1.70	1.90	1.90
Buyer 18	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78	1.78
Buyer 19	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 20	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 21	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 22	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 23	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 24	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
Buyer 25	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 26	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63	1.63
Buyer 27	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 28	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 29	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88	1.88
Buyer 30	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00

Appendix 8 - Example 3 Model 3 Minimizing Vendor's Cost

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
Buyer 1	4	4	4	4	4	4	4	4	4
Buyer 2	3	3	3	3	3	3	3	3	3
Buyer 3	2	2	2	2	2	2	2	2	2
Buyer 4	2	2	2	2	2	2	2	2	2
Buyer 5	1	1	1	1	1	1	1	1	1
Buyer 6	18	18	16	16	16	15	15	14	15
Buyer 7	4	4	3	3	3	3	3	3	3
Buyer 8	1	1	1	1	1	1	1	1	1
Buyer 9	4	4	4	4	4	4	4	4	4
Buyer 10	3	3	3	3	3	3	3	3	3
Buyer 11	2	2	2	2	2	2	2	2	2
Buyer 12	2	2	2	2	2	2	2	2	2
Buyer 13	6	6	6	6	6	5	5	6	5
Buyer 14	1	1	1	1	1	1	1	1	1
Buyer 15	6	6	8	8	8	6	6	7	8
Buyer 16	2	2	2	2	2	2	2	2	2
Buyer 17	3	3	3	3	3	3	3	3	3
Buyer 18	1	1	1	1	1	1	1	1	1
Buyer 19	2	2	2	2	2	2	2	2	2
Buyer 20	3	3	3	3	3	3	3	3	3
Buyer 21	4	4	4	4	4	4	4	4	4
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	6	6	6	6	6	6	6	6	6
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	2	2	2	2	2	2	2	2	2

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	2	2	2	2	2	2	2	2	2
Buyer 29	1	1	1	1	1	1	1	1	1
Buyer 30	2	2	2	2	2	2	2	2	2
Buyer 31	2	2	2	2	2	2	2	2	2
Buyer 32	3	3	3	3	3	3	3	3	3
Buyer 33	36	36	48	48	48	30	30	42	30
Buyer 34	4	4	4	4	4	4	4	4	4
Buyer 35	4	4	4	4	4	4	4	4	4
Buyer 36	2	2	2	2	2	2	2	2	2
Buyer 37	2	2	2	2	2	2	2	2	2
Buyer 38	2	2	2	2	2	2	2	2	2
Buyer 39	4	4	4	4	4	5	5	4	5
Buyer 40	4	4	4	4	4	4	4	4	4
Buyer 41	12	12	12	12	12	12	12	12	12
Buyer 42	2	2	2	2	2	2	2	2	2
Buyer 43	2	2	2	2	2	2	2	2	2
Buyer 44	3	3	3	3	3	3	3	3	3
Buyer 45	9	9	8	8	8	10	10	7	8
Buyer 46	1	1	1	1	1	1	1	1	1
Buyer 47	1	1	1	1	1	1	1	1	1
Buyer 48	2	2	2	2	2	2	2	2	2
Buyer 49	2	2	2	2	2	2	2	2	2
Buyer 50	1	1	1	1	1	1	1	1	1

Table A8.1 The solutions of $k(i)$ of each buyer for Example 3 Model 3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 2	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 3	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 4	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76	1.76
Buyer 5	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
Buyer 6	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 7	1.99	1.99	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 8	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 9	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 10	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 11	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 12	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 13	1.99	1.99	1.99	1.99	1.99	2.00	2.00	1.99	2.00
Buyer 14	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 15	1.97	1.97	1.67	1.67	1.67	1.97	1.97	1.85	1.67
Buyer 16	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
Buyer 17	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94
Buyer 18	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 19	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 20	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 21	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77	1.77
Buyer 22	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
Buyer 23	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 24	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94
Buyer 25	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 26	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 27	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 28	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95	1.95
Buyer 29	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 30	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91	1.91
Buyer 31	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 32	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 33	1.89	1.89	1.75	1.75	1.75	1.94	1.94	1.82	1.94
Buyer 34	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 35	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87	1.87
Buyer 36	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 37	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85	1.85
Buyer 38	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 39	1.94	1.94	1.94	1.94	1.94	1.79	1.79	1.94	1.79
Buyer 40	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 41	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96	1.96
Buyer 42	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98	1.98
Buyer 43	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Buyer 44	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99
Buyer 45	1.96	1.96	1.98	1.98	1.98	1.93	1.93	1.99	1.98
Buyer 46	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90	1.90
Buyer 47	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97	1.97
Buyer 48	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94	1.94
Buyer 49	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82	1.82
Buyer 50	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99	1.99

Table A8.2 The solutions of $p_d(i)$ of each buyer for Example 3 Model 3

Appendix 9 - Example 1 Case1 Credit policy

D/P N	0.1 45	0.2 44	0.3 56	0.4 56	0.5 56	0.6 78	0.7 78	0.8 78	0.9 144
K(i)									
Buyer 1	45	44	56	56	56	39	39	39	36
Buyer 2	45	44	28	28	28	26	26	26	24
Buyer 3	45	44	28	28	28	26	26	26	24
Buyer 4	45	44	56	56	56	39	39	39	36
Buyer 5	45	44	28	28	28	26	26	26	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	40.6	37.2	51.8	50.4	49.0	22.9	21.8	20.5	15.9
Buyer 2	45.5	42.4	24.3	23.0	21.6	18.1	17.1	15.7	13.0
Buyer 3	65.8	62.2	34.8	33.4	32.0	27.3	26.3	24.9	21.0
Buyer 4	31.5	29.0	31.9	30.6	29.1	18.6	17.5	16.2	13.6
Buyer 5	29.4	27.0	19.2	17.9	16.5	14.1	13.1	11.8	10.1

Appendix 9 - Example 1 Case2 Credit policy

D/P N	0.1 45	0.2 44	0.3 56	0.4 56	0.5 56	0.6 78	0.7 78	0.8 78	0.9 144
K(i)									
Buyer 1	45	44	56	56	56	39	39	39	36
Buyer 2	45	44	28	28	28	26	26	26	24
Buyer 3	45	44	28	28	28	26	26	26	24
Buyer 4	45	44	56	56	56	39	39	39	36
Buyer 5	45	44	28	28	28	26	26	26	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	61.8	56.6	88.2	86.5	84.7	35.2	33.8	32.2	24.2
Buyer 2	71.5	67.1	33.3	31.6	29.8	25.7	24.4	22.7	18.5
Buyer 3	69.2	65.2	36.8	35.2	33.4	29.6	28.2	26.6	22.6
Buyer 4	64.8	60.2	83.8	82.2	80.4	40.8	39.4	37.8	30.5
Buyer 5	32.8	30.1	21.3	19.7	17.9	16.4	15.1	13.4	11.7

Appendix 9 - Example 1 Case3 Credit policy

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	45	44	56	56	56	78	78	78	144
K(i)									
Buyer 1	45	44	56	56	56	39	39	39	36
Buyer 2	45	44	28	28	28	26	26	26	24
Buyer 3	45	44	28	28	28	26	26	26	24
Buyer 4	45	44	56	56	56	39	39	39	36
Buyer 5	45	44	28	28	28	26	26	26	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	33.1	30.3	39.1	37.9	36.6	18.5	17.5	16.4	12.9
Buyer 2	43.9	41.0	23.1	21.9	20.6	17.1	16.2	15.0	12.3
Buyer 3	42.7	40.1	24.8	23.6	22.3	19.0	18.1	16.9	14.3
Buyer 4	29.9	27.6	30.7	29.4	28.2	17.6	16.6	15.4	12.9
Buyer 5	27.8	25.6	18.0	16.8	15.5	13.1	12.2	11.0	9.4

Appendix 9 - Example 1 Case4 Credit policy

D/P N	0.1 45	0.2 44	0.3 56	0.4 56	0.5 56	0.6 78	0.7 78	0.8 78	0.9 144
K(i)									
Buyer 1	45	44	56	56	56	39	39	39	36
Buyer 2	45	44	28	28	28	26	26	26	24
Buyer 3	45	44	28	28	28	26	26	26	24
Buyer 4	45	44	56	56	56	39	39	39	36
Buyer 5	45	44	28	28	28	26	26	26	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	75.5	68.5	81.7	77.6	73.4	45.2	42.1	38.2	31.8
Buyer 2	103.0	95.4	61.1	57.1	52.8	45.8	42.7	38.8	32.8
Buyer 3	122.1	114.2	73.3	69.3	65.0	57.0	53.8	49.9	42.8
Buyer 4	92.4	84.8	105.2	101.2	96.9	58.3	55.2	51.2	42.8
Buyer 5	90.3	83.2	56.3	52.3	48.0	41.9	38.7	34.8	29.7

Appendix 9 - Example 2 Case1 Credit policy

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	12	12	12	12	12	12	12	12
Buyer 3	1	1	1	1	1	1	1	1	1
Buyer 4	3	3	3	3	3	3	3	3	3
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	3	3	3	3	3	3	3	3	3
Buyer 7	2	2	2	2	2	2	2	2	2
Buyer 8	6	6	4	4	4	5	5	4	5
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	4	4	4	4	4	4	4	4	4
Buyer 11	9	9	8	8	8	6	6	7	6
Buyer 12	36	36	24	24	24	20	20	14	15
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	3	3	3	3	3	3	3	3	3
Buyer 15	1	1	1	1	1	1	1	1	1
Buyer 16	3	3	3	3	3	3	3	3	3
Buyer 17	6	6	6	6	6	5	5	6	5
Buyer 18	3	3	3	3	3	3	3	3	3
Buyer 19	4	4	4	4	4	4	4	4	4
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	6	6	6	6	6	6	6	6	6
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	4	4	4	4	4	4	4	4	4
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	4	4	4	4	4	4	4	4	4
Buyer 29	12	12	12	12	12	12	12	12	12
Buyer 30	3	3	3	3	3	3	3	3	3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	43.1	42.3	41.8	41.5	40.9	40.6	40.0	39.5	39.2
Buyer 2	26.1	25.3	24.9	24.5	24.0	23.7	23.1	22.6	22.2
Buyer 3	24.2	23.4	23.0	22.6	22.1	21.8	21.2	20.7	20.3
Buyer 4	25.1	24.3	23.8	23.5	22.9	22.6	22.0	21.5	21.1
Buyer 5	30.7	29.9	29.4	29.1	28.5	28.2	27.7	27.1	26.8
Buyer 6	38.0	37.2	36.7	36.4	35.8	35.5	34.9	34.4	34.1
Buyer 7	27.2	26.4	25.9	25.6	25.0	24.7	24.1	23.6	23.3
Buyer 8	47.4	46.6	24.5	24.2	23.6	32.0	31.5	22.2	30.6
Buyer 9	65.3	64.5	64.0	63.7	63.1	62.8	62.2	61.7	61.3
Buyer 10	61.5	60.8	60.3	60.0	59.4	59.1	58.5	58.0	57.6
Buyer 11	34.5	33.7	30.0	29.7	29.2	23.6	23.0	24.9	22.1
Buyer 12	46.4	45.6	29.2	28.9	28.3	24.1	23.5	20.4	20.1
Buyer 13	78.6	77.8	77.3	77.0	76.4	76.1	75.5	75.0	74.7
Buyer 14	31.8	31.0	30.5	30.2	29.7	29.3	28.8	28.2	27.9
Buyer 15	28.4	27.6	27.1	26.8	26.2	25.9	25.3	24.8	24.4
Buyer 16	85.4	84.6	84.1	83.8	83.2	82.9	82.3	81.8	81.5
Buyer 17	172.7	171.9	171.5	171.2	170.6	72.2	71.6	169.2	70.7
Buyer 18	132.0	131.2	130.8	130.4	129.9	129.5	129.0	128.4	128.1
Buyer 19	70.2	69.4	68.9	68.6	68.0	67.7	67.1	66.6	66.3
Buyer 20	26.9	26.1	25.6	25.3	24.7	24.4	23.8	23.3	23.0
Buyer 21	26.3	25.5	25.0	24.7	24.1	23.8	23.2	22.7	22.3
Buyer 22	25.1	24.3	23.8	23.5	23.0	22.6	22.1	21.5	21.2
Buyer 23	34.4	33.6	33.2	32.9	32.3	32.0	31.4	30.9	30.5
Buyer 24	78.0	77.3	76.8	76.5	75.9	75.6	75.0	74.5	74.1
Buyer 25	25.3	24.5	24.0	23.7	23.1	22.8	22.2	21.7	21.4
Buyer 26	207.2	206.4	205.9	205.6	205.0	204.7	204.1	203.6	203.3
Buyer 27	23.9	23.1	22.7	22.4	21.8	21.5	20.9	20.4	20.0
Buyer 28	49.0	48.3	47.8	47.5	46.9	46.6	46.0	45.5	45.1
Buyer 29	84.9	84.1	83.6	83.3	82.7	82.4	81.9	81.3	81.0
Buyer 30	25.2	24.4	23.9	23.6	23.0	22.7	22.2	21.6	21.3

Appendix 9 - Example 2 Case2 Credit policy

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	12	12	12	12	12	12	12	12
Buyer 3	1	1	1	1	1	1	1	1	1
Buyer 4	3	3	3	3	3	3	3	3	3
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	3	3	3	3	3	3	3	3	3
Buyer 7	2	2	2	2	2	2	2	2	2
Buyer 8	6	6	4	4	4	5	5	4	5
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	4	4	4	4	4	4	4	4	4
Buyer 11	9	9	8	8	8	6	6	7	6
Buyer 12	36	36	24	24	24	20	20	14	15
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	3	3	3	3	3	3	3	3	3
Buyer 15	1	1	1	1	1	1	1	1	1
Buyer 16	3	3	3	3	3	3	3	3	3
Buyer 17	6	6	6	6	6	5	5	6	5
Buyer 18	3	3	3	3	3	3	3	3	3
Buyer 19	4	4	4	4	4	4	4	4	4
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	6	6	6	6	6	6	6	6	6
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	4	4	4	4	4	4	4	4	4
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	4	4	4	4	4	4	4	4	4
Buyer 29	12	12	12	12	12	12	12	12	12
Buyer 30	3	3	3	3	3	3	3	3	3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	51.7	50.7	50.4	50.0	49.3	48.9	48.2	47.5	47.1
Buyer 2	29.1	28.1	27.8	27.4	26.7	26.4	25.6	24.9	24.5
Buyer 3	26.6	25.5	25.3	24.9	24.2	23.8	23.1	22.3	21.9
Buyer 4	27.7	26.6	26.4	26.0	25.3	24.9	24.2	23.4	23.0
Buyer 5	35.2	34.1	33.9	33.5	32.8	32.4	31.7	30.9	30.5
Buyer 6	44.9	43.8	43.6	43.2	42.5	42.1	41.4	40.6	40.2
Buyer 7	30.5	29.4	29.2	28.8	28.1	27.7	27.0	26.2	25.9
Buyer 8	57.5	56.4	27.3	26.9	26.2	37.6	36.8	24.4	35.7
Buyer 9	81.3	80.2	80.0	79.6	78.9	78.5	77.8	77.0	76.6
Buyer 10	76.3	75.3	75.1	74.6	73.9	73.6	72.8	72.1	71.7
Buyer 11	40.2	39.2	34.7	34.3	33.6	26.2	25.5	27.9	24.3
Buyer 12	56.1	55.0	33.7	33.2	32.5	26.9	26.2	22.0	21.6
Buyer 13	99.0	98.0	97.7	97.3	96.6	96.3	95.5	94.8	94.4
Buyer 14	36.7	35.6	35.4	35.0	34.3	33.9	33.2	32.4	32.0
Buyer 15	32.1	31.0	30.8	30.4	29.7	29.3	28.6	27.8	27.4
Buyer 16	108.1	107.1	106.8	106.4	105.7	105.3	104.6	103.9	103.5
Buyer 17	224.6	223.5	223.3	222.9	222.2	91.1	90.3	220.3	89.2
Buyer 18	170.3	169.3	169.0	168.6	167.9	167.5	166.8	166.1	165.7
Buyer 19	87.8	86.8	86.5	86.1	85.4	85.1	84.3	83.6	83.2
Buyer 20	30.1	29.0	28.8	28.4	27.7	27.3	26.6	25.8	25.4
Buyer 21	29.3	28.2	28.0	27.6	26.9	26.5	25.8	25.0	24.6
Buyer 22	27.8	26.7	26.5	26.1	25.3	25.0	24.3	23.5	23.1
Buyer 23	40.2	39.1	38.9	38.5	37.8	37.4	36.7	35.9	35.6
Buyer 24	98.3	97.3	97.1	96.6	95.9	95.6	94.8	94.1	93.7
Buyer 25	28.0	26.9	26.7	26.3	25.5	25.2	24.5	23.7	23.3
Buyer 26	270.5	269.4	269.2	268.8	268.1	267.7	267.0	266.2	265.8
Buyer 27	26.2	25.1	24.9	24.5	23.8	23.4	22.7	21.9	21.6
Buyer 28	59.7	58.6	58.4	58.0	57.2	56.9	56.2	55.4	55.0
Buyer 29	107.5	106.4	106.2	105.8	105.0	104.7	104.0	103.2	102.8
Buyer 30	27.9	26.8	26.6	26.2	25.4	25.1	24.4	23.6	23.2

Appendix 9 - Example 2 Case3 Credit policy

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	12	12	12	12	12	12	12	12
Buyer 3	1	1	1	1	1	1	1	1	1
Buyer 4	3	3	3	3	3	3	3	3	3
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	3	3	3	3	3	3	3	3	3
Buyer 7	2	2	2	2	2	2	2	2	2
Buyer 8	6	6	4	4	4	5	5	4	5
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	4	4	4	4	4	4	4	4	4
Buyer 11	9	9	8	8	8	6	6	7	6
Buyer 12	36	36	24	24	24	20	20	14	15
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	3	3	3	3	3	3	3	3	3
Buyer 15	1	1	1	1	1	1	1	1	1
Buyer 16	3	3	3	3	3	3	3	3	3
Buyer 17	6	6	6	6	6	5	5	6	5
Buyer 18	3	3	3	3	3	3	3	3	3
Buyer 19	4	4	4	4	4	4	4	4	4
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	6	6	6	6	6	6	6	6	6
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	4	4	4	4	4	4	4	4	4
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	4	4	4	4	4	4	4	4	4
Buyer 29	12	12	12	12	12	12	12	12	12
Buyer 30	3	3	3	3	3	3	3	3	3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	24.8	24.4	23.9	23.7	23.4	23.2	22.9	22.7	22.5
Buyer 2	16.3	15.9	15.4	15.3	15.0	14.7	14.5	14.2	14.0
Buyer 3	15.3	14.9	14.5	14.3	14.0	13.8	13.5	13.3	13.1
Buyer 4	15.8	15.4	14.9	14.7	14.4	14.2	13.9	13.7	13.5
Buyer 5	18.6	18.2	17.7	17.5	17.3	17.0	16.7	16.5	16.3
Buyer 6	22.2	21.8	21.3	21.2	20.9	20.7	20.4	20.1	19.9
Buyer 7	16.8	16.4	15.9	15.8	15.5	15.3	15.0	14.7	14.5
Buyer 8	26.9	26.5	15.2	15.1	14.8	18.9	18.7	14.0	18.2
Buyer 9	35.8	35.5	35.0	34.8	34.5	34.3	34.0	33.8	33.6
Buyer 10	34.0	33.6	33.1	33.0	32.7	32.4	32.2	31.9	31.7
Buyer 11	20.5	20.1	18.0	17.8	17.6	14.7	14.4	15.4	14.0
Buyer 12	26.4	26.0	17.6	17.4	17.2	15.0	14.7	13.1	12.9
Buyer 13	42.5	42.1	41.6	41.5	41.2	41.0	40.7	40.4	40.2
Buyer 14	19.1	18.7	18.2	18.1	17.8	17.6	17.3	17.0	16.8
Buyer 15	17.4	17.0	16.5	16.4	16.1	15.9	15.6	15.3	15.1
Buyer 16	45.9	45.5	45.0	44.9	44.6	44.4	44.1	43.8	43.6
Buyer 17	89.6	89.2	88.7	88.6	88.3	39.0	38.7	87.5	38.3
Buyer 18	69.2	68.8	68.4	68.2	67.9	67.7	67.4	67.2	67.0
Buyer 19	38.3	37.9	37.4	37.3	37.0	36.8	36.5	36.2	36.0
Buyer 20	16.7	16.3	15.8	15.6	15.3	15.1	14.8	14.6	14.4
Buyer 21	16.3	16.0	15.5	15.3	15.0	14.8	14.5	14.3	14.1
Buyer 22	15.8	15.4	14.9	14.7	14.5	14.2	14.0	13.7	13.5
Buyer 23	20.4	20.0	19.6	19.4	19.1	18.9	18.6	18.4	18.2
Buyer 24	42.2	41.8	41.4	41.2	40.9	40.7	40.4	40.2	40.0
Buyer 25	15.9	15.5	15.0	14.8	14.5	14.3	14.0	13.8	13.6
Buyer 26	106.8	106.4	105.9	105.8	105.5	105.3	105.0	104.7	104.5
Buyer 27	15.2	14.8	14.3	14.2	13.9	13.6	13.4	13.1	12.9
Buyer 28	27.7	27.3	26.9	26.7	26.4	26.2	25.9	25.7	25.5
Buyer 29	45.7	45.3	44.8	44.6	44.4	44.1	43.8	43.6	43.4
Buyer 30	15.8	15.4	14.9	14.8	14.5	14.3	14.0	13.7	13.5

Appendix 9 - Example 3 Credit policy

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 72	0.8 84	0.9 120
Buyer 1	4	4	4	4	4	4	3	3	3
Buyer 2	3	3	3	3	3	3	3	3	3
Buyer 3	2	2	2	2	2	2	2	2	2
Buyer 4	2	2	2	2	2	2	2	2	2
Buyer 5	1	1	1	1	1	1	1	1	1
Buyer 6	18	18	16	16	16	15	12	14	12
Buyer 7	4	4	3	3	3	3	3	3	3
Buyer 8	1	1	1	1	1	1	1	1	1
Buyer 9	4	4	4	4	4	4	4	4	4
Buyer 10	3	3	3	3	3	3	3	3	3
Buyer 11	2	2	2	2	2	2	2	2	2
Buyer 12	2	2	2	2	2	2	2	2	2
Buyer 13	4	4	4	4	4	5	4	4	4
Buyer 14	1	1	1	1	1	1	1	1	1
Buyer 15	6	6	8	8	8	6	8	7	8
Buyer 16	2	2	2	2	2	2	2	2	2
Buyer 17	3	3	3	3	3	3	3	3	3
Buyer 18	1	1	1	1	1	1	1	1	1
Buyer 19	2	2	2	2	2	2	2	2	2
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	4	4	4	4	4	4	4	4	4
Buyer 22	1	1	1	1	1	1	1	1	1
Buyer 23	6	6	6	6	6	6	6	6	6
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	2	2	2	2	2	2	2	2	2

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 72	0.8 84	0.9 120
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	2	2	2	2	2	2	2	2	2
Buyer 29	1	1	1	1	1	1	1	1	1
Buyer 30	2	2	2	2	2	2	2	2	2
Buyer 31	2	2	2	2	2	2	2	2	2
Buyer 32	3	3	3	3	3	3	3	3	3
Buyer 33	36	36	48	48	48	30	36	42	30
Buyer 34	4	4	4	4	4	4	4	4	4
Buyer 35	4	4	4	4	4	4	4	4	4
Buyer 36	2	2	2	2	2	2	2	2	2
Buyer 37	2	2	2	2	2	2	2	2	2
Buyer 38	2	2	2	2	2	2	2	2	2
Buyer 39	4	4	4	4	4	5	4	4	5
Buyer 40	4	4	4	4	4	4	4	4	4
Buyer 41	12	12	12	12	12	12	12	12	12
Buyer 42	2	2	2	2	2	2	2	2	2
Buyer 43	2	2	2	2	2	2	2	2	2
Buyer 44	3	3	3	3	3	3	3	3	3
Buyer 45	9	9	8	8	8	6	8	7	8
Buyer 46	1	1	1	1	1	1	1	1	1
Buyer 47	1	1	1	1	1	1	1	1	1
Buyer 48	2	2	2	2	2	2	2	2	2
Buyer 49	2	2	2	2	2	2	2	2	2
Buyer 50	1	1	1	1	1	1	1	1	1

Table A9.1 The solutions of $k(i)$ of each buyer for Example 3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	0.44	0.44	0.44	0.44	0.43	0.43	0.24	0.24	0.24
Buyer 2	0.37	0.37	0.36	0.36	0.36	0.36	0.36	0.36	0.36
Buyer 3	0.26	0.26	0.26	0.26	0.26	0.25	0.25	0.25	0.25
Buyer 4	0.96	0.96	0.96	0.96	0.96	0.95	0.95	0.95	0.95
Buyer 5	0.57	0.56	0.56	0.56	0.56	0.56	0.55	0.55	0.55
Buyer 6	2.01	2.00	1.06	1.06	1.06	0.70	0.25	0.42	0.24
Buyer 7	0.76	0.76	0.31	0.31	0.30	0.30	0.30	0.30	0.30
Buyer 8	0.25	0.25	0.25	0.25	0.24	0.24	0.24	0.24	0.24
Buyer 9	0.61	0.60	0.60	0.60	0.60	0.60	0.59	0.59	0.59
Buyer 10	0.77	0.77	0.76	0.76	0.76	0.76	0.76	0.75	0.75
Buyer 11	0.39	0.39	0.39	0.39	0.38	0.38	0.38	0.38	0.38
Buyer 12	0.68	0.68	0.68	0.68	0.67	0.67	0.67	0.67	0.67
Buyer 13	0.25	0.24	0.24	0.24	0.24	0.41	0.24	0.23	0.23
Buyer 14	0.58	0.57	0.57	0.57	0.57	0.57	0.57	0.56	0.56
Buyer 15	0.33	0.33	1.16	1.16	1.16	0.32	1.15	0.65	1.15
Buyer 16	1.25	1.25	1.25	1.25	1.24	1.24	1.24	1.24	1.24
Buyer 17	0.98	0.98	0.97	0.97	0.97	0.97	0.97	0.97	0.97
Buyer 18	0.31	0.31	0.31	0.31	0.31	0.31	0.30	0.30	0.30
Buyer 19	0.37	0.36	0.36	0.36	0.36	0.36	0.36	0.35	0.35
Buyer 20	0.26	0.26	0.26	0.25	0.25	0.25	0.25	0.25	0.25
Buyer 21	1.03	1.02	1.02	1.02	1.02	1.02	1.01	1.01	1.01
Buyer 22	0.25	0.24	0.24	0.24	0.24	0.24	0.23	0.23	0.23
Buyer 23	1.08	1.08	1.08	1.08	1.07	1.07	1.07	1.07	1.07
Buyer 24	0.40	0.40	0.40	0.40	0.39	0.39	0.39	0.39	0.39
Buyer 25	0.30	0.30	0.29	0.29	0.29	0.29	0.29	0.29	0.28

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 26	0.62	0.62	0.62	0.61	0.61	0.61	0.61	0.61	0.61
Buyer 27	0.31	0.31	0.30	0.30	0.30	0.30	0.30	0.29	0.29
Buyer 28	0.69	0.68	0.68	0.68	0.68	0.68	0.67	0.67	0.67
Buyer 29	0.25	0.25	0.24	0.24	0.24	0.24	0.24	0.23	0.23
Buyer 30	0.51	0.51	0.51	0.51	0.50	0.50	0.50	0.50	0.50
Buyer 31	0.40	0.39	0.39	0.39	0.39	0.39	0.38	0.38	0.38
Buyer 32	0.70	0.70	0.70	0.70	0.69	0.69	0.69	0.69	0.69
Buyer 33	8.37	8.37	17.9	17.9	17.9	4.35	8.36	12.9	4.34
Buyer 34	0.25	0.24	0.24	0.24	0.24	0.24	0.24	0.23	0.23
Buyer 35	0.79	0.79	0.78	0.78	0.78	0.78	0.78	0.77	0.77
Buyer 36	0.32	0.31	0.31	0.31	0.31	0.31	0.31	0.30	0.30
Buyer 37	0.82	0.81	0.81	0.81	0.81	0.81	0.80	0.80	0.80
Buyer 38	0.45	0.44	0.44	0.44	0.44	0.44	0.43	0.43	0.43
Buyer 39	0.45	0.44	0.44	0.44	0.44	0.95	0.43	0.43	0.95
Buyer 40	0.52	0.52	0.52	0.51	0.51	0.51	0.51	0.51	0.51
Buyer 41	0.79	0.79	0.79	0.78	0.78	0.78	0.78	0.78	0.78
Buyer 42	0.29	0.29	0.29	0.29	0.29	0.28	0.28	0.28	0.28
Buyer 43	0.27	0.27	0.27	0.27	0.27	0.27	0.26	0.26	0.26
Buyer 44	0.36	0.35	0.35	0.35	0.35	0.35	0.34	0.34	0.34
Buyer 45	1.16	1.16	0.68	0.68	0.67	0.24	0.67	0.34	0.67
Buyer 46	0.51	0.50	0.50	0.50	0.50	0.50	0.50	0.49	0.49
Buyer 47	0.32	0.32	0.32	0.32	0.31	0.31	0.31	0.31	0.31
Buyer 48	0.64	0.63	0.63	0.63	0.63	0.63	0.62	0.62	0.62
Buyer 49	0.91	0.91	0.91	0.91	0.90	0.90	0.90	0.90	0.90
Buyer 50	0.31	0.31	0.31	0.31	0.31	0.30	0.30	0.30	0.30

Table A9.2 The solutions of $M(i)$ of each buyer for Example 3

Appendix 10 - Example 1 Cost Sharing by Delay Payment

D/P N	0.1 56	0.2 53	0.3 51	0.4 60	0.5 62	0.6 62	0.7 84	0.8 96	0.9 120
K(i)									
Buyer 1	56	53	51	60	62	62	42	48	40
Buyer 2	56	53	51	30	31	31	28	32	24
Buyer 3	56	53	51	60	31	31	42	32	30
Buyer 4	56	53	51	60	62	62	42	48	40
Buyer 5	56	53	51	30	31	31	28	24	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	8.6	7.4	6.6	10.2	11.0	11.0	3.4	5.5	2.8
Buyer 2	15.1	13.7	12.8	3.8	4.2	4.2	3.1	4.6	1.7
Buyer 3	18.1	16.7	15.7	20.0	6.5	6.5	11.5	6.9	6.0
Buyer 4	11.6	10.3	9.4	13.3	14.2	14.2	5.8	8.2	5.0
Buyer 5	14.3	12.9	12.0	3.3	3.6	3.6	2.6	1.3	1.3

Appendix 10 - Example 2 Cost Sharing by Delay Payment

D/P N	0.1 52	0.2 54	0.3 60	0.4 60	0.5 72	0.6 72	0.7 90	0.8 108	0.9 144
K(i)									
Buyer 1	26	18	15	15	12	12	10	9	9
Buyer 2	52	27	30	30	24	24	18	18	18
Buyer 3	26	18	15	12	12	12	10	9	9
Buyer 4	26	18	15	12	12	12	10	9	9
Buyer 5	26	18	20	15	12	12	10	12	9
Buyer 6	52	54	30	30	24	24	30	27	24
Buyer 7	26	18	15	15	12	12	10	9	9
Buyer 8	52	27	30	20	18	18	15	18	16
Buyer 9	13	9	10	6	6	6	6	6	6
Buyer 10	52	54	60	30	36	36	30	27	24
Buyer 11	26	27	20	20	18	12	15	12	12
Buyer 12	52	54	30	30	24	24	18	18	18
Buyer 13	26	27	20	15	18	12	15	12	12
Buyer 14	52	27	20	20	18	18	15	12	12
Buyer 15	52	27	20	20	18	18	15	12	12
Buyer 16	52	54	60	60	36	36	30	36	36
Buyer 17	52	54	60	60	72	36	45	36	36
Buyer 18	52	27	30	30	24	24	18	18	18
Buyer 19	52	54	60	30	36	36	30	27	24
Buyer 20	52	27	30	20	24	18	18	18	16
Buyer 21	52	27	30	20	18	18	18	18	16
Buyer 22	26	18	15	15	12	12	10	12	9
Buyer 23	52	27	20	20	18	18	15	12	12
Buyer 24	52	27	30	20	18	18	18	18	16
Buyer 25	13	9	10	6	6	6	5	6	4
Buyer 26	52	27	30	30	24	24	18	18	18
Buyer 27	26	18	12	10	9	9	9	9	8
Buyer 28	52	27	30	20	24	18	18	18	16
Buyer 29	52	54	60	60	72	72	45	54	48
Buyer 30	26	27	20	20	18	12	15	12	12

D/P M(i)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	10.7	6.8	5.3	5.3	3.8	3.8	2.9	2.4	2.4
Buyer 2	17.2	5.5	6.8	6.8	4.2	4.2	1.9	1.9	1.9
Buyer 3	12.1	8.1	6.6	5.1	5.1	5.1	4.1	3.6	3.6
Buyer 4	10.0	6.1	4.6	3.2	3.2	3.2	2.3	1.9	1.9
Buyer 5	11.5	7.5	8.5	6.0	4.5	4.5	3.6	4.5	3.1
Buyer 6	23.7	24.7	12.7	12.7	9.7	9.7	12.7	11.2	9.7
Buyer 7	10.9	6.9	5.4	5.4	4.0	4.0	3.0	2.6	2.6
Buyer 8	22.7	10.3	11.8	6.9	5.9	5.9	4.5	5.9	5.0
Buyer 9	6.1	4.1	4.6	2.6	2.6	2.6	2.6	2.6	2.6
Buyer 10	23.2	24.2	27.2	12.2	15.2	15.2	12.2	10.7	9.3
Buyer 11	9.1	9.6	6.2	6.2	5.2	2.5	3.8	2.5	2.5
Buyer 12	13.5	14.4	4.0	4.0	1.9	1.9	0.3	0.3	0.3
Buyer 13	12.5	13.0	9.5	7.0	8.5	5.5	7.0	5.5	5.5
Buyer 14	23.8	11.3	7.8	7.8	6.8	6.8	5.4	3.9	3.9
Buyer 15	25.2	12.7	9.2	9.2	8.2	8.2	6.7	5.2	5.2
Buyer 16	24.2	25.2	28.2	28.2	16.2	16.2	13.2	16.2	16.2
Buyer 17	22.3	23.3	26.3	26.3	32.2	14.3	18.8	14.3	14.3
Buyer 18	25.2	12.8	14.3	14.3	11.3	11.3	8.3	8.3	8.3
Buyer 19	23.4	24.4	27.4	12.5	15.5	15.5	12.5	11.0	9.5
Buyer 20	24.2	11.7	13.2	8.3	10.3	7.3	7.3	7.3	6.3
Buyer 21	21.1	8.9	10.3	5.5	4.6	4.6	4.6	4.6	3.7
Buyer 22	11.2	7.3	5.8	5.8	4.3	4.3	3.3	4.3	2.9
Buyer 23	23.1	10.7	7.3	7.3	6.3	6.3	4.8	3.4	3.4
Buyer 24	24.8	12.4	13.9	8.9	7.9	7.9	7.9	7.9	6.9
Buyer 25	4.1	2.2	2.7	0.9	0.9	0.9	0.5	0.9	0.2
Buyer 26	25.4	12.9	14.4	14.4	11.4	11.4	8.4	8.4	8.4
Buyer 27	9.3	5.5	2.7	1.8	1.4	1.4	1.4	1.4	1.0
Buyer 28	23.6	11.2	12.6	7.7	9.7	6.7	6.7	6.7	5.7
Buyer 29	17.8	18.8	21.7	21.7	27.6	27.6	14.4	18.8	15.9
Buyer 30	10.4	10.9	7.4	7.4	6.4	3.6	5.0	3.6	3.6

Appendix 10 - Example 3 Cost Sharing by Delay Payment

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	40	42	42	48	48	60	72	80	120
Buyer 1	20	14	14	12	8	10	8	8	8
Buyer 2	40	42	42	24	24	20	24	20	20
Buyer 3	20	14	14	8	8	6	8	8	6
Buyer 4	40	42	21	24	16	20	18	16	15
Buyer 5	20	21	14	12	12	10	9	8	8
Buyer 6	40	21	21	16	16	15	12	16	12
Buyer 7	20	14	14	12	8	10	8	8	8
Buyer 8	20	14	14	12	12	10	9	8	8
Buyer 9	40	42	42	24	24	20	18	20	20
Buyer 10	40	42	42	24	24	20	24	20	20
Buyer 11	40	21	21	16	16	15	12	16	12
Buyer 12	40	21	21	16	16	15	12	16	12
Buyer 13	20	14	14	8	8	10	8	8	6
Buyer 14	40	21	21	16	12	12	12	10	10
Buyer 15	40	42	42	48	48	60	72	80	60
Buyer 16	40	42	21	24	16	20	18	16	15
Buyer 17	40	21	21	16	16	15	12	10	12
Buyer 18	20	14	7	8	8	6	6	5	5
Buyer 19	10	7	7	6	6	5	6	5	4
Buyer 20	40	21	21	24	16	15	18	16	15
Buyer 21	40	42	42	48	48	60	36	40	40
Buyer 22	40	21	14	16	12	12	12	10	10
Buyer 23	40	42	42	48	48	30	36	40	30
Buyer 24	40	21	14	16	12	12	12	10	10
Buyer 25	20	14	14	12	12	10	9	8	8

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	40	42	42	48	48	60	72	80	120
Buyer 26	20	21	14	12	12	10	9	10	8
Buyer 27	20	14	7	8	6	6	6	5	5
Buyer 28	40	21	21	16	16	15	12	10	12
Buyer 29	20	14	14	12	8	10	8	8	8
Buyer 30	20	21	14	16	12	12	12	10	10
Buyer 31	20	21	14	12	12	10	9	8	8
Buyer 32	40	42	42	24	24	30	24	20	20
Buyer 33	40	42	42	48	48	60	72	80	60
Buyer 34	20	14	7	8	8	6	6	5	6
Buyer 35	40	42	42	48	48	30	36	40	30
Buyer 36	40	42	21	24	24	20	18	16	15
Buyer 37	40	42	21	24	16	20	18	16	15
Buyer 38	20	14	14	12	8	10	8	8	6
Buyer 39	40	42	42	48	48	60	36	40	40
Buyer 40	40	21	21	16	12	12	12	10	10
Buyer 41	40	42	42	48	48	60	36	40	40
Buyer 42	40	42	21	24	24	20	18	16	15
Buyer 43	20	14	7	8	8	6	6	5	6
Buyer 44	40	21	21	24	16	15	18	16	12
Buyer 45	40	42	42	24	24	20	18	20	20
Buyer 46	10	7	7	6	6	5	4	4	4
Buyer 47	20	14	14	12	12	10	9	8	8
Buyer 48	20	14	14	12	8	10	8	8	8
Buyer 49	40	42	21	24	24	20	18	16	15
Buyer 50	5	6	3	3	3	3	2	2	2

Table A10.1 The solutions of $k(i)$ of each buyer for Example 3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	7.1	4.2	4.2	3.3	1.5	2.4	1.5	1.5	1.5
Buyer 2	17.7	18.7	18.7	9.7	9.7	7.8	9.7	7.8	7.8
Buyer 3	8.3	5.3	5.3	2.4	2.4	1.5	2.4	2.4	1.5
Buyer 4	19.2	20.2	9.7	11.2	7.2	9.2	8.2	7.2	6.7
Buyer 5	9.6	10.1	6.6	5.6	5.6	4.6	4.1	3.6	3.6
Buyer 6	9.5	1.8	1.8	0.4	0.4	0.2	0.0	0.4	0.0
Buyer 7	7.6	4.7	4.7	3.7	1.8	2.8	1.8	1.8	1.8
Buyer 8	9.1	6.1	6.1	5.1	5.1	4.1	3.6	3.1	3.1
Buyer 9	17.3	18.3	18.3	9.4	9.4	7.4	6.4	7.4	7.4
Buyer 10	18.3	19.3	19.3	10.3	10.3	8.3	10.3	8.3	8.3
Buyer 11	18.6	9.1	9.1	6.6	6.6	6.1	4.6	6.6	4.6
Buyer 12	18.9	9.5	9.5	7.0	7.0	6.5	5.0	7.0	5.0
Buyer 13	6.3	3.5	3.5	1.0	1.0	1.8	1.0	1.0	0.3
Buyer 14	19.6	10.1	10.1	7.6	5.6	5.6	5.6	4.6	4.6
Buyer 15	15.1	16.0	16.0	19.0	19.0	24.9	30.9	34.9	24.9
Buyer 16	19.4	20.4	9.9	11.4	7.4	9.4	8.4	7.4	6.9
Buyer 17	18.5	9.0	9.0	6.6	6.6	6.1	4.6	3.6	4.6
Buyer 18	9.3	6.3	2.8	3.3	3.3	2.3	2.3	1.8	1.8
Buyer 19	3.6	2.2	2.2	1.7	1.7	1.2	1.7	1.2	0.8
Buyer 20	17.9	8.4	8.4	9.9	6.0	5.5	7.0	6.0	5.5
Buyer 21	17.8	18.8	18.8	21.8	21.8	27.8	15.8	17.8	17.8
Buyer 22	19.0	9.5	6.0	7.0	5.0	5.0	5.0	4.0	4.0
Buyer 23	16.4	17.4	17.4	20.4	20.4	11.5	14.4	16.4	11.5
Buyer 24	18.6	9.1	5.6	6.6	4.6	4.6	4.6	3.7	3.7
Buyer 25	8.4	5.4	5.4	4.4	4.4	3.5	3.0	2.5	2.5

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 26	8.9	9.4	5.9	4.9	4.9	3.9	3.4	3.9	2.9
Buyer 27	6.8	3.9	0.9	1.3	0.5	0.5	0.5	0.2	0.2
Buyer 28	19.0	9.5	9.5	7.0	7.0	6.5	5.0	4.0	5.0
Buyer 29	9.1	6.1	6.1	5.1	3.1	4.1	3.1	3.1	3.1
Buyer 30	8.8	9.3	5.8	6.8	4.8	4.8	4.8	3.8	3.8
Buyer 31	8.6	9.1	5.6	4.6	4.6	3.7	3.2	2.7	2.7
Buyer 32	18.2	19.2	19.2	10.2	10.2	13.2	10.2	8.3	8.3
Buyer 33	5.6	6.4	6.4	8.8	8.8	14.1	19.6	23.3	14.1
Buyer 34	6.5	3.6	0.7	1.0	1.0	0.4	0.4	0.1	0.4
Buyer 35	17.6	18.5	18.5	21.5	21.5	12.6	15.6	17.6	12.6
Buyer 36	18.4	19.4	8.9	10.4	10.4	8.4	7.5	6.5	6.0
Buyer 37	19.1	20.1	9.6	11.1	7.1	9.1	8.1	7.1	6.6
Buyer 38	8.7	5.7	5.7	4.7	2.8	3.7	2.8	2.8	1.8
Buyer 39	17.0	18.0	18.0	21.0	21.0	27.0	15.0	17.0	17.0
Buyer 40	17.2	7.8	7.8	5.3	3.4	3.4	3.4	2.5	2.5
Buyer 41	11.7	12.6	12.6	15.5	15.5	21.3	9.8	11.7	11.7
Buyer 42	18.3	19.3	8.9	10.4	10.4	8.4	7.4	6.4	5.9
Buyer 43	8.3	5.4	2.0	2.4	2.4	1.5	1.5	1.0	1.5
Buyer 44	17.6	8.2	8.2	9.7	5.8	5.3	6.7	5.8	3.8
Buyer 45	14.3	15.3	15.3	6.7	6.7	4.8	3.9	4.8	4.8
Buyer 46	4.5	3.0	3.0	2.5	2.5	2.0	1.5	1.5	1.5
Buyer 47	9.3	6.3	6.3	5.3	5.3	4.3	3.8	3.3	3.3
Buyer 48	8.9	5.9	5.9	4.9	3.0	3.9	3.0	3.0	3.0
Buyer 49	19.2	20.2	9.7	11.2	11.2	9.2	8.2	7.2	6.7
Buyer 50	1.8	2.3	0.9	0.9	0.9	0.9	0.4	0.4	0.4

Table A10.2 The solutions of $M(i)$ of each buyer for Example 3

Appendix 11 - Example 1 Cost Sharing by Delay Payment and Discount

D/P N	0.1 44	0.2 43	0.3 56	0.4 54	0.5 54	0.6 78	0.7 78	0.8 96	0.9 144
K(i)									
Buyer 1	44	43	56	54	54	39	39	32	36
Buyer 2	44	43	28	27	27	26	26	24	24
Buyer 3	44	43	28	27	27	26	26	24	24
Buyer 4	44	43	56	54	54	39	39	32	36
Buyer 5	44	43	28	27	27	26	26	24	24

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	4.1	3.8	8.6	7.8	7.8	2.5	2.5	0.8	1.7
Buyer 2	9.6	9.2	3.1	2.7	2.7	2.4	2.4	1.7	1.7
Buyer 3	12.4	11.9	5.2	4.8	4.8	4.4	4.4	3.5	3.5
Buyer 4	6.5	6.2	11.6	10.7	10.7	4.6	4.6	2.3	3.6
Buyer 5	8.9	8.5	2.6	2.2	2.2	1.9	1.9	1.3	1.3

Appendix 11 - Example 2 Cost Sharing by Delay Payment and Discount

D/P N	0.1 48	0.2 60	0.3 60	0.4 60	0.5 60	0.6 72	0.7 84	0.8 120	0.9 180
K(i)									
Buyer 1	3	3	3	3	3	3	3	3	3
Buyer 2	12	12	12	12	12	12	12	12	12
Buyer 3	1	1	1	1	1	1	1	1	1
Buyer 4	3	3	3	3	3	3	3	3	3
Buyer 5	2	2	2	2	2	2	2	2	2
Buyer 6	3	3	3	3	3	3	3	3	3
Buyer 7	2	2	2	2	2	2	2	2	2
Buyer 8	4	5	5	5	5	4	4	5	5
Buyer 9	1	1	1	1	1	1	1	1	1
Buyer 10	4	4	4	4	4	4	4	4	4
Buyer 11	8	10	6	6	6	8	7	8	6
Buyer 12	24	20	20	20	20	18	21	20	18
Buyer 13	1	1	1	1	1	1	1	1	1
Buyer 14	3	3	3	3	3	3	3	3	3
Buyer 15	1	1	1	1	1	1	1	1	1
Buyer 16	3	3	3	3	3	3	3	3	3
Buyer 17	6	5	5	5	5	6	6	5	5
Buyer 18	3	3	3	3	3	3	3	3	3
Buyer 19	4	4	4	4	4	4	4	4	4
Buyer 20	2	2	2	2	2	2	2	2	2
Buyer 21	6	6	6	6	6	6	6	6	6
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	4	4	4	4	4	4	4	4	4
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	3	3	3	3	3	3	3	3	3
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	4	4	4	4	4	4	4	4	4
Buyer 29	12	12	12	12	12	12	12	12	12
Buyer 30	3	3	3	3	3	3	3	3	3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
M(i)									
Buyer 1	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Buyer 2	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
Buyer 3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Buyer 4	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Buyer 5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Buyer 6	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Buyer 7	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Buyer 8	0.04	0.25	0.25	0.25	0.25	0.04	0.04	0.25	0.25
Buyer 9	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
Buyer 10	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
Buyer 11	0.87	1.64	0.25	0.25	0.25	0.87	0.53	0.87	0.25
Buyer 12	1.87	0.75	0.75	0.75	0.75	0.34	1.00	0.75	0.34
Buyer 13	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
Buyer 14	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Buyer 15	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Buyer 16	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
Buyer 17	0.38	0.13	0.13	0.13	0.13	0.38	0.38	0.13	0.13
Buyer 18	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84	0.84
Buyer 19	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
Buyer 20	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Buyer 21	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
Buyer 22	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Buyer 23	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
Buyer 24	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
Buyer 25	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Buyer 26	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51
Buyer 27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Buyer 28	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Buyer 29	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
Buyer 30	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Appendix 11 - Example 3 Cost Sharing by Delay Payment and Discount

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
Buyer 1	4	4	4	4	4	4	4	4	4
Buyer 2	3	3	3	3	3	3	3	3	3
Buyer 3	3	3	3	3	3	3	3	3	3
Buyer 4	2	2	2	2	2	2	2	2	2
Buyer 5	1	1	1	1	1	1	1	1	1
Buyer 6	36	36	24	16	16	15	15	14	12
Buyer 7	4	4	4	4	4	4	4	4	4
Buyer 8	1	1	1	1	1	1	1	1	1
Buyer 9	4	4	4	4	4	5	5	4	5
Buyer 10	3	3	3	3	3	3	3	3	3
Buyer 11	2	2	2	2	2	2	2	2	2
Buyer 12	2	2	2	2	2	2	2	2	2
Buyer 13	6	6	6	6	6	5	5	4	5
Buyer 14	2	2	2	2	2	2	2	2	2
Buyer 15	9	9	8	8	8	10	10	7	8
Buyer 16	2	2	2	2	2	2	2	2	2
Buyer 17	3	3	3	3	3	3	3	3	3
Buyer 18	1	1	1	1	1	1	1	1	1
Buyer 19	2	2	2	2	2	2	2	2	2
Buyer 20	3	3	3	3	3	3	3	3	3
Buyer 21	6	6	6	6	6	5	5	6	5
Buyer 22	2	2	2	2	2	2	2	2	2
Buyer 23	6	6	8	8	8	6	6	7	8
Buyer 24	2	2	2	2	2	2	2	2	2
Buyer 25	2	2	2	2	2	2	2	2	2

D/P N	0.1 36	0.2 36	0.3 48	0.4 48	0.5 48	0.6 60	0.7 60	0.8 84	0.9 120
Buyer 26	2	2	2	2	2	2	2	2	2
Buyer 27	4	4	4	4	4	4	4	4	4
Buyer 28	3	3	3	3	3	3	3	3	3
Buyer 29	1	1	1	1	1	1	1	1	1
Buyer 30	2	2	2	2	2	2	2	2	2
Buyer 31	2	2	2	2	2	2	2	2	2
Buyer 32	3	3	3	3	3	3	3	3	3
Buyer 33	36	36	48	48	48	60	60	42	40
Buyer 34	4	4	4	4	4	4	4	4	4
Buyer 35	6	6	6	6	6	5	5	6	5
Buyer 36	3	3	3	3	3	3	3	3	3
Buyer 37	2	2	2	2	2	2	2	2	2
Buyer 38	2	2	2	2	2	2	2	2	2
Buyer 39	6	6	6	6	6	6	6	6	6
Buyer 40	4	4	4	4	4	4	4	4	4
Buyer 41	12	12	12	12	12	12	12	14	12
Buyer 42	3	3	3	3	3	3	3	3	3
Buyer 43	2	2	2	2	2	2	2	2	2
Buyer 44	4	4	4	4	4	4	4	4	4
Buyer 45	9	9	8	8	8	10	10	7	8
Buyer 46	1	1	1	1	1	1	1	1	1
Buyer 47	1	1	1	1	1	1	1	1	1
Buyer 48	2	2	2	2	2	2	2	2	2
Buyer 49	2	2	2	2	2	2	2	2	2
Buyer 50	1	1	1	1	1	1	1	1	1

Table A11.1 The solutions of $k(i)$ of each buyer for Example 3

D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	D/P	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Buyer 1	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	Buyer 26	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
Buyer 2	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	Buyer 27	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
Buyer 3	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	Buyer 28	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63	0.63
Buyer 4	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	0.36	Buyer 29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Buyer 5	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	Buyer 30	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
Buyer 6	7.75	7.75	2.82	0.41	0.41	0.23	0.23	0.09	0.01	Buyer 31	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Buyer 7	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	Buyer 32	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23	0.23
Buyer 8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	Buyer 33	4.06	4.06	8.82	8.82	8.82	14.1	14.1	6.35	5.57
Buyer 9	0.18	0.18	0.18	0.18	0.18	0.48	0.48	0.18	0.48	Buyer 34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Buyer 10	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	Buyer 35	1.01	1.01	1.01	1.01	1.01	0.61	0.61	1.01	0.61
Buyer 11	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	Buyer 36	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32
Buyer 12	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	0.22	Buyer 37	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28	0.28
Buyer 13	0.31	0.31	0.31	0.31	0.31	0.09	0.09	0.00	0.09	Buyer 38	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
Buyer 14	0.62	0.62	0.62	0.62	0.62	0.62	0.62	0.62	0.62	Buyer 39	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
Buyer 15	0.77	0.77	0.46	0.46	0.46	1.11	1.11	0.21	0.46	Buyer 40	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
Buyer 16	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	Buyer 41	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.74	0.27
Buyer 17	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	Buyer 42	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29	0.29
Buyer 18	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	Buyer 43	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Buyer 19	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	Buyer 44	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31	0.31
Buyer 20	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	Buyer 45	0.46	0.46	0.22	0.22	0.22	0.75	0.75	0.05	0.22
Buyer 21	1.18	1.18	1.18	1.18	1.18	0.76	0.76	1.18	0.76	Buyer 46	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
Buyer 22	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	Buyer 47	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Buyer 23	0.42	0.42	1.12	1.12	1.12	0.42	0.42	0.75	1.12	Buyer 48	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
Buyer 24	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	Buyer 49	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33
Buyer 25	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	Buyer 50	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03

Table A11.2 The solutions of $M(i)$ of each buyer for Example 3

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