The Hong Kong Polytechnic University

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MODELLING ACTIVITY AND TRAVEL CHOICE BEHAVIOUR: A NETWORK EQUILIBRIUM APPROACH

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CERTIFICATE OF ORIGINALITY

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ABSTRACT

Travel demands are derived from the desire of individuals to participate in various activities such as home, work, shopping, etc. Individuals' travel choice and activity choice, together with the interdependence of activity and travel scheduling, should be comprehensively investigated by means of transport modelling. Network equilibrium models with taking account of congestion effects, which provide valuable insights into understanding individuals' activity and travel choice behaviour, are widely used for long-term transport planning.

In densely populated urban areas such as Hong Kong, multi-modal trips have been increasing in magnitude in recent years. This situation is similar in many fast-growing cities in Asia. Hence finding equilibrium results in congested multi-modal transport/transit networks is an important issue in long-term transport planning. The research presented in this thesis is novel in its aim to address the activity and travel choice behaviour simultaneously with consideration of congestion effects such as the crowding at activity locations and within transit vehicles in multi-modal transport/transit networks.

In the literature, most existing network equilibrium models for travel behaviour analysis adopt a trip-based approach. In the trip-based approach, trips are adopted as the basic unit of analysis, and trip chains made by individuals are considered as separable and independent entities.

The first contribution of this thesis is that a trip-based network equilibrium model is

proposed to model travel choice behaviour in congested multi-modal transport networks under demand uncertainty. In the proposed trip-based model, crowding discomfort in transit vehicles, boarding congestion effect, and congestion impact of road traffic are explicitly modelled. The stochastic bus frequency derived from unstable road travel time is particularly investigated. The impacts of demand uncertainty on passenger flows and travel times are effectively captured. Individuals' route and mode choice behaviour under travel time uncertainty are intensively explored.

The trip-based network equilibrium model, however, ignores the underlying motivation of trip making, and cannot reflect the linkages between activities and travels. To understand the limitation and shortcoming of the trip-based approach, the above trip-based proposed model is extended to an activity-based approach. The activity-based approach enables an integrated investigation into the activity-travel scheduling mechanism, i.e. what activities to be conducted, in what sequence, when and for how long, when each trip starts, which transport mode/route is to be used, and how the activities and travels interrelate in congested multi-modal transport/transit networks. As previous studies have shown that crowding discomfort has an important effect on individuals' choice of transit service for long-term planning, the in-vehicle crowding discomfort is considered in the proposed activity-based model particularly for congested transit networks in Asia.

The second contribution of this thesis is the proposal of an activity-based network equilibrium model to solve the daily activity-travel pattern (DATP) scheduling problem. The resultant DATP choice reflects individuals' various activity choices (e.g.

activity sequence, start time and duration), travel choices (e.g. departure time, transfer, and route/mode), and the relationship between activity and travel choice behaviour in congested multi-modal transit networks.

As a pioneering endeavour, the proposed activity-based network equilibrium model extends existing theories by developing an integrated framework which incorporates the flexible activity sequence and duration, the stochastic effects of activity utility, together with the route/mode choice under network congestion. A novel activity-time-space multi-modal super-network platform is constructed to explicitly address the relationship between activity choices and travel choices in time and space coordinates in a congested multi-modal transit network. By using the proposed super-network platform, the time-dependent DATP scheduling problem can be converted into a static traffic assignment problem.

A number of empirical studies have investigated the recurrent effects of adverse weather on individuals' DATP choice and such effects are obviously greater in cities which suffer frequent rainy periods. The long-term transit planning for areas with high average annual rainfall should be considerably different from the planning for areas with less rainfall. Thus, clearly, particularly in areas such as Hong Kong, rain effects should be considered when modelling individuals' activity and travel choices. An activity-based network equilibrium model for scheduling DATPs in multi-modal transit networks under adverse weather conditions (with different rainfall intensities) is proposed, which is the third contribution of this thesis. The interdependency of individuals' activity/travel choices and weather conditions are intensively investigated in congested multi-modal transit networks. As vehicle capacity and frequency of different transit modes are influenced by adverse weather conditions, in-vehicle crowding discomfort taking account of adverse weather impacts is specifically considered in the proposed model. The effects of adverse weather on different transit modes and different activities are also explicitly investigated.

It should be noted in the above three network equilibrium models that the activity and travel choice behaviour of individual is assumed to be independent, so called one-individual level. As travel surveys indicate, joint participation in activities and travels represent a substantial portion of individuals' DATPs. Most existing studies on activity-based network equilibrium models, however, are confined to the one-individual level. Less attention has been given to the interdependence between individuals' joint activities and travels. Obviously, there is a need to investigate the effects of the joint activity-travel pattern (JATP) choice for long-term transit planning.

The fourth contribution of this thesis is the development of an activity-based network equilibrium model which can solve two-individual JATP scheduling problem in congested multi-modal transit networks. The proposed JATP scheduling model extends existing theories by developing a unified framework to capture both independent and joint activity/travel choices in congested multi-modal transit networks. To capture the effect of activity location capacity in long-term transit planning, the proposed model is extended to incorporate the crowding discomfort at activity location. A measure of JATP utility is proposed to incorporate the joint travel benefit. Individuals' preference towards joint travel is explicitly examined by the proposed model, and the impacts of joint travel benefit on individuals' independent and joint activity/travel choices are intensively explored.

Network equilibrium models are capable of predicting traffic flow patterns subject to network congestion phenomena. In this thesis, four network equilibrium models are proposed to comprehensively investigate individuals' activity and travel choice behaviour in congested multi-modal transport/transit networks. In the proposed models, different congestion effects are considered such as in-vehicle crowding discomfort, road traffic congestion, and crowding at activity location. The proposed models offer the flexibility and feasibility to comprehensively consider different congestion effects in multi-modal transport/transit networks for future extensions. The ultimate aim of the proposed network equilibrium models is to make valuable contributions to the new avenue of research on activity and travel choice behaviour for design of multi-modal transport networks and evaluation of alternative transport systems with consideration of their congestion effects.

PUBLICATIONS ARISING FROM THIS THESIS

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NOTATIONS

The following notations are used throughout the thesis unless otherwise specified.

Abbreviations

ATS	Activity-Time-Space
CV	Coefficient of Variation
DATP	Daily Activity-Travel Pattern
FIFO	First-In-First-Out
JATP	Joint Activity-Travel Pattern
JATS	Joint-Activity-Time-Space
MSA	Method of Successive Average
M-U	Mean-Utility
M-V	Mean-Variance
OD	Origin-Destination
RSUE	Reliability-based Stochastic User Equilibrium
RUE	Reliability-based User Equilibrium
SAM	State-Augmented Multi-modal
SD	Standard Deviation
UE	User Equilibrium

Network Representation

A	the set of links in SAM network, $A = A_t \cup A_d$;
	or the set of links in ATS-SAM network, $A = A_a \cup A_t \cup A_d$;
	or the set of links in JATS network, $A = A_a \cup A_t \cup A_d \cup A_w \cup A_m$

A_{a}	the set of activity links in ATS-SAM or JATS network; $A_{\mathrm{a}}=\{a_{\mathrm{a}}\}$
$A_{\mathrm{a}}^{\mathrm{indep}}$	the set of independent activity links in JATS network
$A_{\mathrm{a}}^{\mathrm{joint}}$	the set of joint activity links in JATS network
A_{t}	the set of transfer links in SAM or ATS-SAM or JATS network; $A_{\mathrm{t}}=\{a_{\mathrm{t}}\}$
A_{d}	the set of direct in-vehicle links in SAM or ATS-SAM or JATS network
A_{w}	the set of waiting links in JATS network; $A_{\mathrm{w}}=\{a_{\mathrm{w}}\}$
A_{m}	the set of meeting links in JATS network; $A_{\mathrm{m}}=\{a_{\mathrm{m}}\}$
B	the set of transport modes; $B=\{b\}$
G	the SAM or ATS-SAM or JATS network; $G=(N,A)$
I_{a}	the set of activity locations; $I_{\mathrm{a}}=\{i_{\mathrm{a}}\}$
M	conventional multi-modal transport network; $M=\left(U,V\right)$
M_b	sub-network of mode b; $M_b=(U_b,V_b)$
N	the set of nodes in SAM or ATS-SAM or JATS network
P	the set of DATPs or JATPs in ATS-SAM or JATS network
P^{od}	the set of routes between OD pair od; $P^{od}=\{p^{od}\}$
P^{ou}	the set of routes from origin node o to node u; $P^{ou}=\{p^{ou}\}$
S	the set of probable transfer states
SE	the scan eligible set in route searching process
U	the set of physical nodes; $U=\left\{i\right\}$

U_b	the set of nodes associated with the sub-network M_b; $U_b \subseteq U$
V	the set of physical links; $V = \{v\}$
V_b	the set of physical links associated with the sub-network M_b; $V_b \subseteq V$
a_{sn}^{ij}	direct in-vehicle link from location i to location j with transfer state s as its n th transfer in the trip in SAM network
a_{d}	direct in-vehicle link in ATS-SAM or JATS network
b	transport mode; $b \in B$; b_1 (subway), b_2 (auto), b_3 (bus)
d	destination node
i	physical location of node
ind	individual(s) indicator
k	time of day
l	alight or aboard indicator
n	number of prior transfers
o	origin node
p	a route /DATP /JATP
s	transfer state; $s \in S$
$\eta(s)$	associated transport mode of transfer state s; $\eta(s) \in B$
$\xi(s)$	the set of probable transfers from state s; $\xi(s) \subseteq S$
π	the optimal route/DATP/JATP

Variables

c_p	travel time budget of route p

cf_p	commonality factor of JATP p
$DISU_v(k)$	stochastic dis-utility of physical link v at time interval k
$DISU_{a_\mathrm{d}}$	stochastic dis-utility of in-vehicle link a_d in ATS-SAM network
$disu_v(k)$	mean of dis-utility of physical link v at time interval k
$disu_v(k,wc)$	the dis-utility of physical link v at time interval k under weather category wc
$disu_{a_\mathrm{d}}$	mean of in-vehicle link dis-utility in ATS-SAM network
$disu_{a_\mathrm{t}}$	dis-utility of transfer link a_t in ATS-SAM network
$disu_{a_\mathrm{w}}$	dis-utility of waiting link a_w in JATS network
$disu^p_{travel}$	overall travel dis-utility of JATP p
$disu^p_{joint}$	individuals' joint travel dis-utility in JATP p
F_p	stochastic passenger flow on route p
f_p	mean of passenger flow on route p in SAM network, or passenger flow on DATP/JATP p in ATS-SAM/JATS network
F^{ij}_{sn}	stochastic passenger flow on direct in-vehicle link a^{ij}_{sn} in SAM network
f^{ij}_{sn}	mean of passenger flow on in-vehicle link a^{ij}_{sn} in SAM network
F_{a_t}	stochastic passenger flow on transfer link a_t
f_{a_t}	mean of passenger flow on transfer link a_t
f_{a_a}	passenger flow on activity link a_a
F_v	stochastic passenger flow on physical link v

$F_v(k)$	stochastic passenger flow on the physical link v at time interval k
f_v	mean of passenger flow on physical link v
$f_v(k)$	mean of passenger flow on physical link v at time interval k
$F_{v_{b_2}}$	stochastic passenger flow of mode b_2 on road link v
$f_{v_{b_2}}$	mean passenger flow of mode b_2 on road link v
F_{a_t}	stochastic passenger volume at transfer link a_t
$\overline{F_{bi}}$	prior passenger volume already in mode b prior to picking up passengers at location i
F_{a_d}	stochastic passenger flow on in-vehicle link a_d in ATS-SAM network
f_{a_d}	mean of passenger flow on in-vehicle link a_d in ATS-SAM network
f_1	mean of $(F_{a_t}+\overline{F_{bi}})$
G_b	stochastic frequency of the transport mode b
L^p_{joint}	"length" of joint travel in JATP p
L^p_{total}	overall "lengths" of individuals' total travel in JATP p
Q^{od}	stochastic travel demand between OD pair od
q^{od}	mean of travel demand between OD pair od
T_v	stochastic travel time of physical link v
t_v	mean travel time of physical link v
$t_v(k,wc)$	travel time of physical link v at time interval k under weather

category wc

T_{sn}^{ij}	stochastic travel time of in-vehicle link a_{sn}^{ij}
t_{sn}^{ij}	mean of travel time on in-vehicle link a_{sn}^{ij}
$T_{a_{\mathrm{t}}}$	stochastic waiting time of transfer link a_{t}
$t_{a_{\mathrm{t}}}$	mean waiting time of transfer link a_{t}
T_p	stochastic travel time of route p
t_p	mean travel time of route p
$U_{a_{\mathrm{a}}}$	stochastic utility of activity link a_{a}
$\bar{u}_{a_{\mathrm{a}}}(k)$	marginal utility of performing activity link a_{a} at time k
$u_{a_{\mathrm{a}}}$	utility of activity link a_{a}
$u_{a_{\mathrm{a}}}^{ind}$	utility of individual(s) ind performing activity link a_{a}
$u_{activity}^{p}$	overall activity utility of JATP p
U_p	stochastic utility of DATP p in ATS-SAM network
u_p	mean utility of DATP p in ATS-SAM network
X_v	stochastic total traffic volume on road link v
x_v	mean of total traffic volume on road link v
ε	random term in travel demand
σ_q^{od}	SD of travel demand between OD pair od
$\sigma_v(k)$	SD of dis-utility of physical link v at time interval k
$\sigma_{a_{\mathrm{d}}}$	SD of in-vehicle link dis-utility in ATS-SAM network
σ_f^p	SD of passenger flow on route p

σ_{fsn}^{ij}	SD of passenger flow on in-vehicle link a_{sn}^{ij} in SAM network
$\sigma_f^{a_t}$	SD of passenger flow on transfer link a_t
σ_f^v	SD of passenger flow on physical link v
$\sigma_f^v(k)$	SD of passenger flow on physical link at time interval k
$\sigma_f^{v_{b_2}}$	SD of passenger flow of mode b_2 on road link v
$\sigma_f^{a_d}$	SD of passenger flow on in-vehicle link a_d in ATS-SAM network
σ_1	SD of $(F_{a_t} + \overline{F_{bi}})$
σ_t^v	SD of travel time of physical link v
σ_{tsn}^{ij}	SD of travel time on in-vehicle link a_{sn}^{ij}
σ_t^p	SD of travel time of route p
$\sigma_t^{a_t}$	SD of waiting time on transfer link a_t
σ_{a_a}	SD of utility of activity link a_a
σ_p	SD of utility of DATP p in ATS-SAM network
σ_{x_v}	SD of total traffic volume on road link v
u_p	mean utility of DATP p in ATS-SAM network
φ_p	dis-utility of route p in SAM network, or budget utility of DATP p in ATS-SAM network, or JATP utility in JATS network

Parameters

cv_{a_a}	model parameter relevant to the activity type of a_a

cv^{od}	CV of travel demand between OD pair od
e_{b_2}, e_{b_3}	passenger car equivalents for mode b_2 (auto) and b_3 (bus)
e_0	average vehicle occupancy parameter representing the number of passengers per auto
g_b	frequency of the transport mode b
$g_b(wc)$	frequency of mode b under weather category wc
h_b	vehicle capacity of the transport mode b
$h_b(wc)$	vehicle capacity of mode b under weather category wc
K	total number of time intervals
$p'_{wc}(k)$	posterior probability of occurrence of wc given the weather forecast for time interval k to $k+1$
pen_b	mode-specified transfer penalty
q	total population
s_0	bus fleet size
$s_{u_{a_{\mathrm{a}}}}(wc)$	scale function of activity utility under weather category wc
$s_{t_v}(wc)$	scale function of physical link travel time under weather category wc
$t_{a_{\mathrm{d}}}^0$	travel time of in-vehicle link a_{d}
t_v^0	free flow travel time of physical link v
$u_{a_{\mathrm{a}}}^{\max}, \alpha_{a_{\mathrm{a}}}, \beta_{a_{\mathrm{a}}}, \gamma_{a_{\mathrm{a}}}$	activity-specific parameters in marginal utility function
vot	value of time
wc	weather category

w_{ind}	individual ind 's weight parameter
α	probability of on-time arrival in SAM network, or probability of gaining budget utility of DATP in ATS-SAM network
$\Phi^{-1}(\alpha)$	inverse of standard normal cumulative distribution function at the probability of α
β_1, k_1	parameters in the travel time functions of subway link and bus link
β_2, γ_2	parameters in the travel time function of bus link
β_b, θ_b	parameters in physical link dis-utility function in activity-based models
$\beta'_{a_a}, \theta'_{a_a}$	parameters in activity utility considering crowding effect at activity location
β_{cf}	commonality factor parameter
γ_1, k_2	parameters in the travel time functions of road link
μ, λ	parameters in the transfer waiting time function
ω_1	parameter in route dis-utility function
$\delta(p,a)$	incidence relationship between route and link
$\delta(a_{sn}^{ij}, v)$	incidence relationship between in-vehicle link and physical link in SAM network
$\delta(a_{\mathrm{d}}, v)$	incidence relationship between in-vehicle link and physical link in ATS-SAM or JATS network
κ	a restricted number of total transfers
κ_v	capacity of the road link v

$\kappa_{a_{\mathrm{a}}}$	capacity of the activity location
χ	interaction parameter between individuals
$\rho_{a_{\mathrm{d}}}$	fare of in-vehicle link a_{d} in ATS-SAM or JATS network
$\rho_{a_{sn}^{ij}}$	fare of in-vehicle link a_{sn}^{ij} in SAM network
ρ_p	fare of route p
π_{wc}	rainfall intensity of weather category wc

1 Introduction

1.1 Statement of the problem

Hong Kong, with a population of 7.1 million and a land area of only 1104 square kilometres, is one of the most densely populated cities in the world. Over 11 million personal trips are made for various activities, every day and are increasing. This increase in travel demand has put great pressure on the existing transport networks. Network equilibrium models can provide a comprehensive understanding of individuals' activity and travel choice behaviour and present a more accurate interpretation of traffic conditions in congested transportation networks. Such models are widely used for long-term transport planning.

In most conventional transportation studies, the four-step method (i.e. trip generation, trip distribution, modal split, and traffic assignment) is widely adopted in travel behaviour modelling. In the literature, the four-step method is classified as the trip-based approach in which the trip is the basic unit of analysis. Trip chains made by an individual are considered as separable and independent entities. In congested transport networks, travel times vary greatly from day to day due to network uncertainties such as random demand fluctuations. Great strides have been made in network equilibrium models for investigating individuals' travel behaviour in road networks under demand uncertainty using the trip-based approach (Chen and Ji, 2005; Chen *et al*., 2011, 2012; Shao *et al*., 2006a, 2006b). In recent years, multi-modal trips in Hong Kong have been increasing in magnitude. However, less effort has been made in modelling travel choice behaviour for multi-modal transport networks under uncertainty (Szeto *et al*., 2013). In multi-modal transport networks, various transport modes differ in their travel time reliability. Therefore, for fast developing cities in Asia, the development of network equilibrium models for modelling travel choice behaviour in congested multi-modal transport networks under demand uncertainty is an important issue.

As travel demands are derived from the desire of individuals to participate in various activities, individuals' activity choice significantly influences travel demand and travel choice. An understanding of the interactions between individuals' activity and travel choice behaviour plays an important role in long-term transport planning. The linkage between activities and travels, the temporal and spatial constraints, and the interdependence of activity and travel scheduling should be comprehensively investigated by network equilibrium models. Unlike the trip-based approach, the activity-based approach covers another class of models. This class provides a better understanding of the interaction between activity choice and travel choice. Hence, extending the above mentioned trip-based network equilibrium model under uncertainty to the activity-based approach could better enable a comprehensive modelling of individuals' daily activity-travel patterns (DATPs) under uncertainty.

It is of note that the current development of activity-based models lacks a rigid and comprehensive modelling framework. Most existing activity-based network equilibrium models cover only a few choice dimensions. The specification of choice dimensions is based either on the available travel survey data or on a relatively ad hoc method. The research presented in this thesis appears to be the first devoted exclusively to the development of unified network equilibrium frameworks for modelling individuals' activity and travel choice behaviour in congested multi-modal transport/transit networks.

In general, adverse weather has a significant influence on individuals' activity and travel choice behaviour and such influence is obviously greater in those cities which suffer frequent rainy periods. Given the above, it would be advantages for the impacts of weather conditions to be taken into account in long-term transport planning. Hence an activity-based network equilibrium model for scheduling DATPs in multi-modal transit networks under adverse weather could provide solution.

Many activity-based travel behaviour models are based on individual decision making in which individuals' joint decisions are not explicitly considered. With the rapid development of information and telecommunication technology, joint activity/travel constitutes an ever-increasing share of an individual's DATP (Ronald *et al*., 2012; Vovsha *et al*., 2003). This emphasises the importance of modelling joint activity and travel choices for long-term transport planning and policy analysis. This research, thus respond to the above implied developing travel need by proposing an activity-based network equilibrium model for solving joint activity-travel pattern (JATP) scheduling problem in multi-modal transit network.

In summary, this research is devoted to developing network equilibrium models for the purpose of long-term strategic planning, in which individuals' activity and travel choice behaviour in congested multi-modal transport/transit networks can be comprehensively investigated.

1.2 Objectives of the research

The aim of this research is to develop network equilibrium models for modelling individuals' activity and travel choice behaviour in multi-modal transport/transit networks. The following objectives are designed to achieve this aim.

Objective 1: to develop a trip-based network equilibrium model for traffic assignment in multi-modal transport networks under demand uncertainty.

Objective 2: to develop an activity-based network equilibrium model for solving the DATP scheduling problem in multi-modal transit networks under activity uncertainty.

Objective 3: to develop an activity-based network equilibrium model for solving the DATP scheduling problem in multi-modal transit networks under adverse weather conditions.

Objective 4: to develop an activity-based network equilibrium model for solving the JATP scheduling problem in multi-modal transit networks.

The inter-relationships of the four research objectives are depicted in Figure 1.1. A trip-based network equilibrium model is developed for traffic assignment in multi-modal transport network under demand uncertainty (Objective 1). The proposed trip-based model (Objective 1) is extended to an activity-based model for solving DATP scheduling problem in multi-modal transit networks under activity uncertainty (Objective 2). To incorporate the effects of adverse weather, another activity-based model is proposed to take into account adverse weather conditions (Objective 3). To extend the one-individual activity-travel choice problem to two-individual joint choice problem, an activity-based network equilibrium model is developed for JATP scheduling (Objective 4) on the basis of one-individual DATP scheduling models (Objectives 2 and 3).

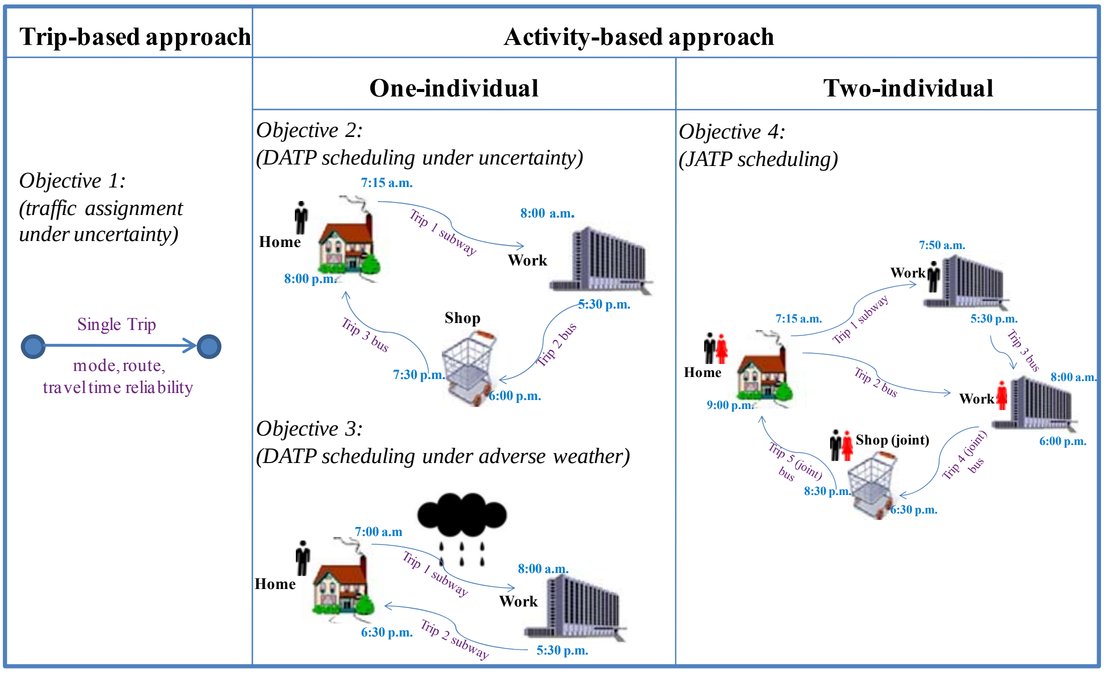


Figure 1.1 The inter-relationships of research objectives

1.3 Structure of the thesis

This thesis is composed of seven chapters. The relationships between these seven chapters are illustrated in Figure 1.2. A brief introduction of this research is given in Chapter 1. Chapter 2 reviews relevant literature on trip-based and activity-based network equilibrium models. The core of this research consists of four Chapters. Chapter 3 focuses on a trip-based network equilibrium model. Chapters 4, 5 and 6 discuss activity-based network equilibrium models.

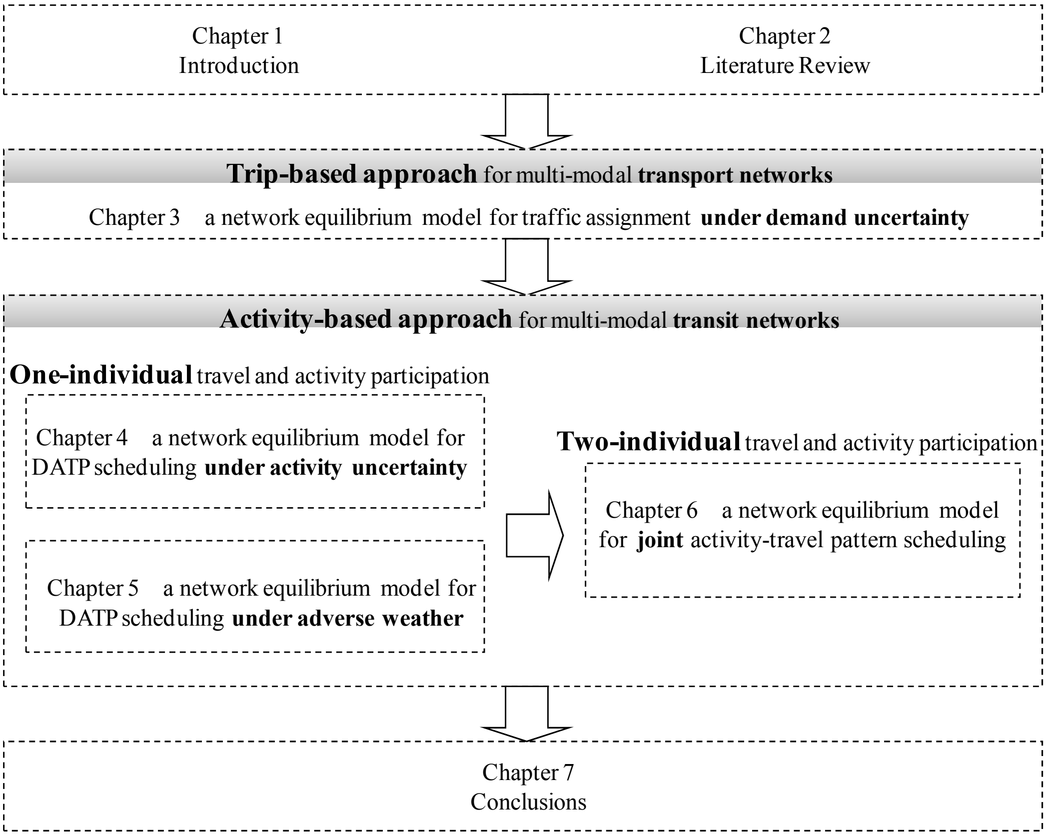


Figure 1.2 Structure of the thesis

Chapter 3 presents a trip-based reliability-based user equilibrium (RUE) model for traffic assignment in congested multi-modal transport networks under demand uncertainty. The distributions of stochastic passenger flows and travel times are studied. The stochastic bus frequency due to the unstable road travel time is explicitly considered. Probable transfers and non-linear fare structures are effectively modelled by adapting the state-augmented multi-modal (SAM) transport network (Lo *et al.*, 2003). By the proposed RUE model, individuals' route and mode choice behaviour are intensively explored.

In Chapter 4, the trip-based RUE model for traffic assignment proposed in Chapter 3 is extended to an activity-based RUE model for solving the DATP scheduling problem.

A novel activity-time-space state-augmented multi-modal (ATS-SAM) super-network platform is proposed to simultaneously address the activity choices and travel choices in multi-modal transit networks. In order to capture the stochastic characteristics of different activities, activity utilities are assumed in Chapter 4 to be time-dependent and stochastic in relation to the activity types. Individuals' activity and travel choices are simultaneously investigated by the extended RUE model with consideration of in-vehicle crowding effect and activity uncertainty.

In Chapter 5, the activity-based network equilibrium model proposed in Chapter 4 is extended to consider the effects of adverse weather conditions (with different rainfall intensities). A user equilibrium (UE) model is proposed for solving the DATP scheduling problem in congested multi-modal transit networks under adverse weather. The effects of adverse weather on different transit modes and different activities are explicitly modelled. The interdependence of individuals' DATP choice and weather conditions are comprehensively investigated.

The activity-based network equilibrium models proposed in Chapters 4 and 5 both address the modelling of one-individual activity and travel participation. Chapter 6 considers two-individual's activity and travel participation. A UE model is proposed for scheduling two-individual JATPs in multi-modal transit networks. By proposing a novel joint-activity-time-space (JATS) super-network platform, individuals' activity and travel choices, including both independent ones and joint ones, can be comprehensively investigated. A measure of JATP utility is proposed to incorporate joint travel benefit. The effects of joint travel length are modelled in joint travel benefit.

Research findings are summarized in the last Chapter of this thesis, Chapter 7, followed by suggestions for further study.

2 Literature Review

The long-term planning of urban congested multi-modal transport/transit networks relies on the use of network equilibrium models for predicting the way in which individuals choose routes/modes and activities. Over the past decades, network equilibrium models have thus received much attention. This research, as was indicated in Chapter 1, is believed to be the first attempt to comprehensively investigate individuals' activity and travel choice behaviour in congested multi-modal transport/transit networks using network equilibrium models.

In the literature, few attempts have been reported regarding the development of network equilibrium models for comprehensively investigating individuals' travel and activity choice behaviour in congested multi-modal transport/transit networks. However, much valuable information can be derived from previous related studies. In this regard, a literature review has been conducted to identify ideas relevant to the objectives of this research.

This Chapter is structured as follows. Section 2.1 focuses on the trip-based travel analysis approach. Both the concept of travel time reliability is introduced, and reliability-based network equilibrium models are reviewed in Section 2.1. In Section 2.2, the differences between trip-based and activity-based approaches are discussed. Section 2.3 focuses on the activity-based approach, giving a brief review of related activity-based studies. A summary of key points concludes the Chapter.

2.1 Trip-based network equilibrium models with travel time uncertainty

In densely populated areas, travel times in congested transport networks generally vary from day to day due to network uncertainties such as random demand fluctuations. Many empirical studies have found that travel time uncertainty has significant impacts on individuals' route and mode choice behaviour (Abdel-Aty *et al.*, 1995; Lam and Small, 2001; Brownstone *et al.*, 2003; De Palma and Picard, 2005).

In congested networks with travel time uncertainty, individuals consider travel time uncertainty as a risk for their travels. To reduce the risk of late arrival, individuals may prefer to secure the probability that a trip can be successfully fulfilled within a given travel time threshold. In the literature, such probability is referred as travel time reliability (Asakura and Kashiwadani, 1991). Travel time reliability provides a quantitative measure of stochastic travel time. In densely populated areas, for many reasons, travel time reliability has become a matter of increasing.

It has been identified in many studies that travel time reliability is one of the key factors affecting individuals' choice behaviour (Jackson and Jucker, 1982; Abdel-Aty *et al.*, 1995). Jackson and Jucker (1982) proved that the variance of route travel time , in particular, influenced travel choices. Abdel-Aty *et al.* (1995) found that travel time reliability was the most or second most crucial factor concerning route choices made. Therefore, it is of advantage for travel time reliability to be explicitly considered when modelling individuals' choice behaviour, particularly in multi-modal transport networks with uncertainty.

To quantitatively model travel time reliability, a widely used concept: travel time budget has been proposed by Lo *et al*. (2006). The travel time budget can be expressed as the summation of the mean route travel time and a safety margin of route travel time. The concept of travel time budget is adopted in this research (Chapter 3), and as such is also extended to the concept of daily activity-travel pattern (DATP) budget utility (Chapter 4).

Individuals' route and mode choice behaviour can be investigated by the analysis and further investigations of network equilibriums. A classification of network equilibrium models is given in Table 2.1. In conventional network equilibrium models for deterministic transport networks, the expected travel time is the only criterion influencing individuals' route choice decisions (Wardrop, 1952; De Cea and Fernandez, 1993; Aashtiani, 1979). As indicated above, in recent years, travel time reliability is an increasing concern for the travelling public, in particular for travellers such as air passengers, for whom expectations of on-time arrival is high. Thus, several studies recently, have been devoted to considering travel time reliability in traffic assignment models.

Lo *et al*. (2006) extended the well-known user equilibrium (UE) model (Wardrop, 1952) to a reliability-based user equilibrium (RUE) model by using the concept of travel time budget. The travel time budget is defined as the summation of the mean route travel time and a safety margin of route travel time. The safety margin is the extra time added to journey time expectation by individuals aiming to ensure the achievement of the probability of on-time arrival. Under the RUE principle, individuals choose the route with minimum travel time budget instead of expected travel time in the UE model.

Table 2.1 Classification of trip-based network equilibrium models

	Deterministic	**Stochastic (Considering travel time reliability)**
Road Network	Wardrop (1952) …	Shao *et al.* (2006a, 2006b, 2008) Siu and Lo (2008)
Transit Network	De Cea and Fernandez (1993) …	Yang and Lam (2006) Zhang *et al.* (2010) Szeto *et al.* (2013)
Multi-modal Transport Network	Aashtiani (1979) …	Sumalee *et al.* (2011) **This research**

Following the RUE framework, much attention has been given to travel behaviour modelling for either road or transit networks. In road networks, Shao *et al.* (2006a) proposed a RUE model to investigate the effects of demand uncertainty. Siu and Lo (2008) developed a RUE model which considered both demand and supply uncertainties. Zhou and Chen (2008) compared three RUE models under demand uncertainty. Reliability-based stochastic user equilibrium (RSUE) models were further developed to take account of individuals' perception errors (Shao *et al.*, 2006b; Shao *et al.*, 2008).

In transit networks, Yang and Lam (2006) presented a RSUE model in congested transit networks with unreliable transit services. Zhang *et al.* (2010) developed a schedule-based RSUE model to investigate travel choice behaviour, in terms of departure time and route choices, in transit networks with demand and supply uncertainties.

In multi-modal transport networks, Sumalee *et al*. (2011) proposed a RSUE model in multi-modal transport networks with adverse weather conditions under the common-line framework. The research presented in this thesis proposes a RUE model using state-augmented multi-modal transport network under uncertainty. The interactions between public transit networks and road networks are explicitly considered. Problems of unrealistic transfers and non-linear fare structures are also tackled in this research.

2.2 From trip-based approach to activity-based approach

In most of the conventional transportation studies, the trip-based approach is adopted with the aim of analyzing travel choice behaviour. In the trip-based approach, the basic unit of analysis is the trip. Trip chains made by an individual are considered as separate and independent entities. Latterly, however, a growing awareness of the limitations of the conventional trip-based approach in analyzing travel choice behaviour has developed largely because the underlying motivation of trip making is ignored. The trip-based approach does not reflect the linkages between activities and trips, the temporal and spatial constraints, and the interdependence of activity and travel scheduling.

Travel demands are derived from individuals' needs/desires to participate in economic and social activities such as work, eating and shopping. Compared to the trip-based approach, another approach the activity-based approach has surfaced in the late 1980s for the analysis of travel choice behaviour and predicting travel demand (Kitamura, 1988; Jones, 1990). In this approach, the trip is no longer considered the basic unit of analysis. The various activities conducted by individuals assumed the position of high focus (McNally, 2000). Goodwin (1983) defined the activity-based approach as "the consideration of revealed travel patterns in the context of the structure of activities, of the individual or household, with a framework emphasizing the importance of time and space constraints".

The activity-based approach covers another class travel choice behaviour analysis models. By using the activity-based approach, it was felt that temporal and spatial constraints, scheduling of activities, participation time and location of each activity, and the linkage among activities and trips can be better investigated. Therefore, there is an increasing awareness that the activity-based approach provides a better understanding of individuals' choice behaviour than does the trip-based approach. Travel behaviour analysis through the activity-based approach is more complex than that through the trip-based approach.

2.3 Activity-based approach

In this section, a review of activity-based network equilibrium models is first given in Section 2.3.1. The activity utility concept is discussed in Section 2.3.2. Section 2.3.3 introduces the impacts of adverse weather on activity and travel choice. Section 2.3.4 reviews the joint activity/travel choice problem in the literature.

2.3.1 Activity-based network equilibrium models

Activity-based network equilibrium models comprehensively reflect travel choices, interdependence of different trips, and the scheduling of activities in temporal and spatial dimensions. A substantial body of literature has been developed on activity-based network equilibrium models.

Some studies have been developed with given travel times but ignoring congestion effects. Recker (1995) formulated a household activity pattern model which optimized the activity sequence and trip sequence in the context of pre-determined activity-travel patterns with given and fixed travel times.

Some models were proposed with the aim of considering the effect of network congestion on simultaneous activity and travel choices. Lam and Yin (2001) presented an activity-based time-dependent traffic assignment model. Lam and Huang (2003) and Huang and Lam (2005) extended Lam and Yin's model to dynamic stochastic user equilibrium models for investigating both the activity and travel choices.

Some studies have been developed based on pre-determined activity sequence. For example, Zhang *et al*. (2005) studied work duration determination problem together with the relationship between work duration and trip departure time choices in the queuing network throughout the whole day. Li *et al*. (2010) proposed an activity-based transit assignment model for solving the transit scheduling problems in multi-modal transit networks.

To comprehensively study individuals' activity and travel choices, some relevant models have recently been developed for congested road networks. Ramadurai and Ukkusuri (2010) proposed a single unified dynamic framework to model jointly the activity location, time of participation, activity duration, and route choices. Ouyang *et al*. (2011) proposed an activity-based traffic assignment model for solving the DATP scheduling problem.

However, these models were all developed for modelling DATP scheduling problems in congested road networks. Little effort has been found as regards solving the DATP choice problem in multi-modal transit networks. In metropolitan areas such as Hong Kong, over 90% of daily travel is made using a variety of public transit modes. In view of this, the research presented in this thesis is devoted to modelling individuals' activity choices and route/mode choices simultaneously in multi-modal transit networks with consideration of congestion effect and activity uncertainty.

In most existing activity-based traffic assignment models, there is a need to enumerate all activity-travel patterns (Lam and Huang, 2002; Huang and Lam, 2005; Zhang *et al.*, 2005). Such enumeration is time consuming for complicated multi-modal transit networks. In this research, this difficulty is tackled by incorporating an activity-travel pattern searching algorithm into the proposed activity-based traffic assignment model.

2.3.2 Activity utility

Individuals gain utility from the activities they conduct. The concept of activity utility is used in the utility maximization framework (Adler and Ben Akiva, 1979; Kitamura, 1984). The amount of utility gained is determined by the characteristics of the activity, the activity duration, and the degree of individual's need.

In the literature, activity utility functions associate selected activity characteristics. The numerical values of these activity characteristics are assumed to relate to particular utility levels. It is assumed in the literature that activity utility is a function of the activity duration and the activity characteristics.

It is believed that various activity participations have different preferred times. Activity participation usually starts with a warming up phase in which the marginal activity utility increases. After reaching a maximum point, the marginal utility decreases. In this research, the activity utility is determined by a bell-shaped marginal utility function proposed by Joh *et al*. (2002) and Ettema and Timmermans (2003):

$$\bar{u}_{a_{\mathrm{a}}}(k) = \frac{\gamma_{a_{\mathrm{a}}} \beta_{a_{\mathrm{a}}} u_{a_{\mathrm{a}}}^{\max}}{\exp[\beta_{a_{\mathrm{a}}}(k-\alpha_{a_{\mathrm{a}}})]\left\{1+\exp[-\beta_{a_{\mathrm{a}}}(k-\alpha_{a_{\mathrm{a}}})]\right\}^{\gamma_{a_{\mathrm{a}}}+1}}, \tag{2.1}$$

where k is the time of day; $u_{a_{\mathrm{a}}}^{\max}$ is the maximum accumulated utility of activity a_{a}, and $\alpha_{a_{\mathrm{a}}}$, $\beta_{a_{\mathrm{a}}}$, $\gamma_{a_{\mathrm{a}}}$ are the activity-specific parameters to be estimated. These parameters can be estimated on the basis of survey data (Ettema and Timmermans, 2003; Ashiru *et al*., 2004).

Many related studies have adopted this type of function for modelling the marginal utility of activity (Ashiru *et al*., 2004; Zhang *et al*., 2005; Li *et al*., 2010). This function does not consider the needs of individuals. In further studies, need-based utility functions (Arentze and Timmermans, 2009) can also be incorporated in the models proposed in this research.

Individuals also receive dis-utility from the travels between activities. Supernak (1992) adopted the total utility of activity-travel pattern in a typical utility maximization context. The total utility obtained from an activity-travel pattern is the summation of the utility gained from activities and the dis-utility resulting from travels. Individuals' activity-travel pattern choices are decided by underlying activity utilities and travel dis-utilities. Individuals select the activity-travel pattern with the largest total utility.

Individuals' activity-travel pattern choices are influenced by the temporal profiles of activity utilities.

In previous related studies, the activity utility is considered as the summation of a systematic component which is a deterministic representative value of utility and a random component which represents the variation in individuals' perceptions (Kitamura, 1984; Lam and Yin, 2001). In other words, the uncertainty of activity utility in previous studies lies in individuals' various perceptions. Broadly, the activity utility may consist of the following attributes: (a) the activity time window; (b) the degree of need for the activity; (c) the degree of satisfaction gained from the process; (d) the financial gain or loss. In reality, these attributes vary from day to day. Thus, the utility profile of each activity should not be a single curve but rather an area which indicates a probability distribution.

In people's daily life, various activities should be conducted to meet people's different needs. Some activities conducted are compulsory, such as work, while the need for others is more flexible such as shopping (Kitamura, 1984). Thus, in previous studies, activities are classified into two categories: compulsory (or mandatory) ones such as work and home, and non-compulsory (or discretionary) ones such as shopping and eating. This research appears to be the first attempt to capture the stochastic characteristics of different activities in modelling individuals' activity and travel choice behaviour.

2.3.3 Impacts of adverse weather on activity and travel choice

From Section 2.3.1, it can be seen that several network equilibrium models, providing valuable insights into understanding individuals' activity-travel scheduling behaviour, have been proposed for long-term transport planning. These models aim at and succeed in more comprehensively studying individuals' activity and travel choices.

None of the above models, however, has explicitly incorporated the weather/climate effects on activity-travel pattern scheduling, although a number of empirical studies have investigated the recurrent effects of adverse weather on individuals' activity choice and travel behaviour. Some studies have reported individuals' mode and departure time changes as affected by weather conditions (Khattak and De Palma, 1997; Guo *et al*., 2007), and some have indicated activity behaviour changes (Smith, 1993; Khattak and De Palma, 1997; Cools *et al*., 2010). Rainfall has the most frequent and significant adverse weather effect on individuals' activity and travel choices in tropical and subtropical areas such as Hong Kong and Singapore. Rainfall has been found to significantly affect individuals' activity and travel choice behaviour such as activity duration and travel mode choice. Long-term transit planning for areas with high average annual rainfall is considerably different from that for areas with less rainfall. Thus, particularly in tropical and subtropical areas such as Hong Kong and Singapore, rain effects should be considered when modelling individuals' activity and travel choices.

In order to incorporate rain effects in travel behaviour modelling, Lam *et al*. (2008) proposed a network equilibrium model for road networks giving specific consideration to the effect of rain on road capacity and link travel time. Sumalee *et al*. (2011) extended this work to model multi-modal transport networks under adverse weather conditions. The above two models are both trip-based transport models, hence, the specific trip making motivation, plus the interdependence of activities and travels are not considered. Cools *et al*. (2010) found that individuals' travel behaviour under adverse weather conditions was highly dependent on trip purpose (i.e. activities to be conducted). Thus, a network equilibrium model is proposed in this research to comprehensively model individuals' activity and travel choice behaviour under adverse weather conditions.

2.3.4 Joint activity/travel choice problem

Many activity-based travel behaviour models are based on individual decision making but joint decisions are not explicitly considered. In reality, however, individuals undertake both independent and joint activities/travels as essential parts of their DATPs. For example, household members meet at subway stations after work, then travel jointly such as to have dinner in a shopping mall. With the rapid development of information and telecommunication technology, as mentioned in the Introduction of this thesis, such joint activity constitutes an ever-increasing share of an individual's DATP (Ronald *et al*., 2012). Travel surveys indicate that joint travel has now become a significant portion of travel within regions (Vovsha *et al*., 2003). This form of behaviour emphasises the need and importance of the explicit analysis and modelling joint activity-travel pattern (JATP) scheduling problem for long-term transport planning and policy analysis.

Currently, a number of transportation studies have investigated the joint activity and travel choice problem with consideration of inter-personal dependencies. Table 2.2 gives a classification of previous studies on this subject.

Table 2.2 Classification of joint activity-travel choice studies

	Joint activity-travel choices
Simulation models	Miller and Roorda, 2003; Arentze and Timmermans, 2009; Dubernet and Axhausen, 2013.
Econometric models	Globe and McNally, 1997; Gliebe and Koppelman, 2002; Zhang *et al*., 2009.
Network equilibrium models	**This research**.

The complex nature of inter-personal dependencies results in many studies using the simulation technique. For example, Miller and Roorda (2003) proposed a micro-simulation model to generate DATPs for all individuals in a household on the basis of a conventional trip diary survey. Arentze and Timmermans (2009) developed a need-based model of activity generation for a multi-day planning period taking account of household members' interactions. Dubernet and Axhausen (2013) included joint travels in a multi-agent micro-simulation.

Apart from simulation models, a number of econometric models have also been proposed with the aim of exploring the intra-household behavioural interactions in relation to activity-travel choice behaviour, using structural equation modelling or the random utility approach. For example, the study of out-of-home activities and travel durations by Globe and McNally (1997), a time allocation model for two-individual households that accounts for joint activity participation by Gliebe and Koppelman (2002), and the work of Zhang *et al*. (2009) in which different household utility functions are introduced to represent household members' joint decision making interactions.

Compared to the development of activity-based simulation models and econometric models, fewer studies have been devoted to developing activity-based mathematical analytical models such as network equilibrium models. Activity-based network equilibrium models can provide a comprehensive understanding of individuals' activity and travel choice behaviour, and present more accurate traffic conditions in a transportation network. Most existing studies on activity-based network equilibrium , however, are on the basis of one individual level and ignore individuals' joint activity-travel choices (Lam and Yin, 2001; Lam and Huang, 2002, 2003; Huang and Lam, 2005; Zhang *et al*., 2005; Li *et al*., 2010; Ramadurai and Ukkusuri, 2010, 2011; Ouyang *et al*., 2011; Fu and Lam, 2014; Fu *et al*., 2014b). As joint activities/travels represent a substantial portion of individuals' DATPs, it is of serious interest to develop network equilibrium models which can comprehensively consider both individuals' independent and joint activity/travel choice behaviour.

2.4 Summary

The long-term transport planning relies on the use of network equilibrium models for comprehensively predicting individuals' activity and travel choice behaviour and present more accurate traffic conditions in a transportation network. As a result, great strides have been made in the development of these models.

The trip-based approach and the activity-based approach are two main approaches to analyzing individuals' activity and travel choice behaviour. In the trip-based approach, trip is adopted as the basic unit of travel analysis, while the activity-based approach focuses on activities conducted by individuals.

In the trip-based approach, travel time reliability induced by network uncertainty has been explicitly considered in transportation planning and modelling. In the review of reliability-based studies, it was found that most studies are developed for road networks. As multi-modal trips have increased in magnitude in recent years, there is a practical need to develop a comprehensive reliability-based traffic assignment model for multi-modal transport networks with uncertainty. Problems of unrealistic transfers and non-linear fare structures should be tackled, and individuals' route and mode choices should be intensively explored.

Travel demands are derived from people's desires to participate in various activities. Thus, an understanding of the interactions between individuals' activities and travel choice behaviour plays an important role for long-term transport planning. By reviewing the activity-based approach to travel analysis, it is found that little effort has been made to develop network equilibrium models which can comprehensively model individuals' activity-travel choice behaviour in multi-modal transport networks. It is also demonstrated that the uncertainty of activity utility and adverse weather may affect individuals' DATP choices. In addition, individuals' joint activity/travel choice behaviour should be modelled in activity-travel pattern scheduling. Thus, various network equilibrium models for DATP/JATP scheduling are needed.

Based on the previous related work, a trip-based network equilibrium model is proposed in Chapter 3, and an activity-based model is proposed in Chapter 4 as an extension of this trip-based model. On the basis of the activity-based model proposed in Chapter 4, another activity-based model considering effects of adverse weather is developed and presented in Chapter 5. Finally, a network equilibrium model for two-individual JATP scheduling is proposed and described in Chapter 7.

3 A Network Equilibrium Model for Traffic Assignment under Demand Uncertainty

In densely populated and congested urban areas, travel times in congested multi-modal transport networks are generally varied and stochastic in practice. Stochastic travel times are likely to be the result of day-to-day demand fluctuations and in all probability affect individuals' route and mode choice behaviour based on their expectations of on-time arrival. Reliability-based user equilibrium (RUE) models can provide a comprehensive understanding of travel behaviour in multi-modal transport networks under uncertainty. In this Chapter, a trip-based RUE model is proposed for modelling individuals' travel choice behaviour in congested multi-modal transport networks under demand uncertainty.

The work presented in this Chapter contributes the literature in three aspects. Firstly, to capture the effects of demand uncertainty, passenger flows and generalized travel times of different transport modes are all formulated as random variables. Secondly, the congestion effect of road traffic and the crowding discomfort in vehicle are explicitly modelled. The stochastic bus frequency derived from unstable road travel time is considered. Thirdly, a stochastic state-augmented multi-modal (SAM) transportation network is adapted to model explicitly both the probable transfers and non-linear fare structures, particularly in Asian cities like Hong Kong. Using the proposed RUE model, individuals' route and mode choice behaviour in congested multi-modal transport networks can be intensively explored.

This Chapter is structured as follows. Section 3.1 gives the background and the motivation of the study presented in this Chapter. The model assumptions are given in Section 3.2. The distributions of passenger flow and travel time are derived and described in Section 3.3. The RUE model formulation and solution algorithm are presented in Section 3.4. A numerical example illustrating the proposed model is provided in Section 3.5. A summary of this Chapter is given in Section 3.6.

3.1 Background

In metropolitan areas, travel times in multi-modal transport networks generally vary from day to day as a result of random demand fluctuations and supply degradations. Many empirical studies have found that travel time uncertainty has significant impacts on individuals' route and mode choice behaviour in congested transport networks (Abdel-Aty *et al*., 1995; Lam and Small, 2001; Brownstone *et al*., 2003; De Palma and Picard, 2005). These empirical studies revealed that individuals do indeed consider travel time uncertainty a travel risk. To ensure the trip is timely achieved to enable fulfilment of the purpose of the trip, individuals may have more concerns on the probability that a trip can be successfully fulfilled within a given travel time, referred to travel time reliability in the literature. Travel time reliability should therefore be explicitly considered when modelling the mode and route choice behaviour particularly in multi-modal transport networks with uncertainty.

In view of the above, Lo *et al*. (2006) extended the well-known user equilibrium (UE) model (Wardrop, 1952) to RUE model by using a concept of travel time budget. The travel time budget is the summation of mean route travel time and a safety margin of route travel time. The latter is an extra time added by an individual to achieve his/her desired probability of on-time arrival. Under the RUE principle, individuals choose the optimal route with minimum travel time budget instead of expected travel time in the conventional UE model. Following the RUE framework, as elaborated in Section 2.1 of Chapter 2, much attention has been given to travel behaviour modelling for either road or transit networks. However, less effort has been found in modelling individuals' mode and route choice behaviour in congested multi-modal transport networks under uncertainty.

In reality, there is a practical need for providing a reliability-based network equilibrium model in congested multi-modal transport networks for two reasons. Firstly, an interaction between public transit networks and road networks is evident especially during rush hours. Regular bus service frequency may be disrupted by traffic congestion on associated road networks. Using either transit networks or road networks in modelling cannot demonstrate the interactions between public transit and road traffic. Thus in this circumstance a multi-modal network model is practically required. Secondly, multi-modal trips have increased in magnitude in recent years. Individuals may take trips by autos, by public transit, or park-and-ride for their daily travels. Therefore, an exploration of individuals' route and mode choice behaviour in congested multi-modal transport networks, and with the inclusion of travel time reliability is of value in transit-orientated cities like Hong Kong.

Hence, this Chapter presents a RUE model for investigating individuals' travel choice behaviour in congested multi-modal transport networks. To capture the effects of demand uncertainty, passenger flows and generalized travel times of different transport modes are formulated as random variables. Stochastic bus frequency derived from the variability of road travel time is explicitly considered, and the derivations of mean and standard deviation (SD) of link and route travel times are provided. Note that unrealistic transfers are avoided and the difficulty of non-linear fare structures is tackled by using a state-augmented multi-modal (SAM) transport network proposed by Lo *et al*. (2003).

3.2 Model assumptions

To facilitate the presentation of the essential ideas without loss of generality, the following assumptions are made in this Chapter.

A3.1: Origin-destination (OD) demands are assumed to follow independent normal distributions similar to those made in previous studies (Asakura and Kashiwadani, 1991; Waller *et al*., 2001; Chen *et al*., 2003).

A3.2: Route flows are assumed to be mutually independent and follow the same type of statistical distribution as OD demand distribution. The coefficient of variation (CV) of route flow is assumed to be equal to that of OD demand distribution as the works of Shao *et al.* (2006a, 2006b). Justification of this assumption with empirical data is advisable in further study.

A3.3: The multi-modal network model investigated in this study falls within the category of static model for long term planning at the strategic level. Therefore, it is assumed that all individuals in the multi-modal transport network would have perfect knowledge towards traffic condition based on past experience.

A3.4: All individuals can get on the buses or subways, i.e. there is no vehicle capacity constraint. It is because all the demand will be catered by the supply for long-term planning purpose.

A3.5: Link and route travel times are assumed to be mutually independent and follow normal distributions (Shao *et al*., 2006a; Shao *et al*., 2006b).

3.3 Formulation of passenger flow and travel time distributions

3.3.1 Network representation

In this Chapter, the SAM network proposed by Lo *et al*. (2003) is adapted to avoid unrealistic transfers and to represent the non-linear fare structures involved in multi-modal transport networks.

Consider a multi-modal transport network $M=(U,V)$, where $U=\{i\}$ and $V=\{v\}$ are respectively the set of physical nodes and the set of physical links. The multi-modal network M can be divided into w sub-networks $M_b=(U_b,V_b)$, $b\in B$, $U_b\subseteq U$, $V_b\subseteq V$, where $b\in B$ is a specified transport mode, and U_b and V_b, respectively are the set of nodes and the set of links associated with the sub-network M_b. In this Chapter, three transport modes: subway, auto, and bus, respectively denoted by b_1, b_2, b_3 are considered. All sub-networks are combined and represented by a strongly connected graph $G=(N,A)$ through a state-augmentation approach (Bertsekas, 1995), where N is a set of nodes and A is a set of links. The resulting network G is termed the SAM network.

In the SAM network, each node is described as (i,s,n,l), where i indicates the physical location of the node, s is the transfer state which is used to model probable transfers, n is the number of modal transfers that have been made by an individual, and l is the alight or aboard indicator. n is used as a constraint on the maximum number of transfers. The value of l equals to 1 (0), indicating that an individual is at the beginning (end) of a direct in-vehicle link. Each transfer state $s \in S$ is specifically, associated with a transport modal usage $\eta(s) \in B$ and a set of probable transfers $\xi(s) \subseteq S$. If individuals are at state s, the indication is that these individuals are using mode $\eta(s)$ and that they can only transfer to any state in $\xi(s)$.

Links are divided into two categories in the SAM network, i.e. $A = A_{\mathrm{t}} \cup A_{\mathrm{d}}$, where A_{t} denotes a set of transfer links between modes and A_{d} denotes a set of direct in-vehicle links, and the latter is made up of consecutive physical links. Each transfer link $a_{\mathrm{t}} \in A_{\mathrm{t}}$ is constructed according to the probable transfer states. Each in-vehicle link $a_{sn}^{ij} \in A_{\mathrm{d}}$ represents a direct in-vehicle movement from location i to location j with transfer state s as its n^{th} transfer in the trip.

In many transit-orientated cities like Hong Kong, public transit fares, such as those for buses and subways are not proportional to travel distance. The route fares are non-additive, and thus cannot be simply determined by summing the fares of relevant physical links along that route. In this study, direct in-vehicle links in the SAM networks are represented as a set of consecutive physical links; so that the fares of direct in-vehicle links are additive.

Let nodes o and d be the OD nodes， and $P^{od} = \{p\}$ be the set of routes linking OD pair od . Let ρ_p be the fare of route $p \in P^{od}$. It can be expressed as the summation of fares of direct in-vehicle links:

$$\rho_p = \sum_{a_{sn}^{ij} \in A_d} \rho_{a_{sn}^{ij}} \delta(p, a_{sn}^{ij}), \tag{3.1}$$

where $\rho_{a_{sn}^{ij}}$ denotes the fare of travelling on in-vehicle link a_{sn}^{ij} . $\delta\left(p, a_{sn}^{ij}\right)$ is the incidence relationship between in-vehicle link and route; $\delta\left(p, a_{sn}^{ij}\right)$ equals 1 if in-vehicle link a_{sn}^{ij} is used in route p , 0 otherwise. In this way, non-linear fare structures can be taken into account as the travel fares can be represented on a node to node basis.

3.3.2 Passenger flow distribution

For notation consistency, the variables with capital letters used throughout this Chapter represent random variables and variables with lower-case letters represent deterministic variables. Following the model assumption A3.1, the travel demand between OD pair od (denoted as Q^{od}) is a random variable following a normal distribution,

$$Q^{od} = q^{od} + \varepsilon, \tag{3.2}$$

where q^{od} is the mean demand, $E\left[Q^{od}\right] = q^{od}$; ε is the random term, $E[\varepsilon] = 0$. Let σ_q^{od} be the SD of the OD demand:

$$\sigma_q^{od} = \sqrt{Var\left[Q^{od}\right]} = \sqrt{Var[\varepsilon]}. \tag{3.3}$$

The CV of the travel demand between OD pair *od* (denoted as cv^{od}) can be expressed as

$$cv^{od} = \frac{\sigma_q^{od}}{q^{od}}. \tag{3.4}$$

Denote the passenger flow along a route $p \in P^{od}$ as F_p . Following the model assumption A3.2, the flow conservation can then be expressed by following equations:

$$Q^{od} = \sum_{p \in P^{od}} F_p, \tag{3.5}$$

$$q^{od} = \sum_{p \in P^{od}} f_p, \tag{3.6}$$

$$\sigma_f^p = \sqrt{Var\left[F_p\right]} = f_p cv^{od} \quad \forall p \in P^{od}, \tag{3.7}$$

where f_p and σ_f^p , respectively, are the mean and the SD of passenger flow along route p .

Denote the passenger flow on direct in-vehicle link a_{sn}^{ij} as F_{sn}^{ij}. It can be expressed by the summation of passenger flows on all routes using this in-vehicle link:

$$F_{sn}^{ij} = \sum_{p \in P^{od}} \delta(p, a_{sn}^{ij}) F_p \quad \forall a_{sn}^{ij} \in A_{\mathrm{d}}, \tag{3.8}$$

$$f_{sn}^{ij} = \sum_{p \in P^{od}} \delta(p,a_{sn}^{ij}) f_p \quad \forall a_{sn}^{ij} \in A_{\mathrm{d}}, \tag{3.9}$$

$$\sigma_{fsn}^{ij} = \sqrt{Var[F_{sn}^{ij}]} = \sqrt{\sum_{p \in P^{od}} Var[F_p]\delta(p,a_{sn}^{ij})} \quad \forall a_{sn}^{ij} \in A_{\mathrm{d}}, \tag{3.10}$$

where f_{sn}^{ij} and σ_{fsn}^{ij}, respectively, are the mean and the SD of passenger flow on link a_{sn}^{ij}; $\delta(p,a_{sn}^{ij})$ is the incidence relationship between in-vehicle link and route; $\delta(p,a_{sn}^{ij})$ equals 1 if in-vehicle link a_{sn}^{ij} is on route p, 0 otherwise.

The passenger flow on each transfer link $a_{\mathrm{t}} \in A_{\mathrm{t}}$ is denoted as $F_{a_{\mathrm{t}}}$. It can be calculated by summing the passenger flows of all routes using this transfer link as

$$F_{a_{\mathrm{t}}} = \sum_{p \in P^{od}} \delta(p,a_{\mathrm{t}}) F_p \quad \forall a_{\mathrm{t}} \in A_{\mathrm{t}}, \tag{3.11}$$

$$f_{a_{\mathrm{t}}} = \sum_{p \in P^{od}} \delta(p,a_{\mathrm{t}}) f_p \quad \forall a_{\mathrm{t}} \in A_{\mathrm{t}}, \tag{3.12}$$

$$\sigma_f^{a_{\mathrm{t}}} = \sqrt{Var[F_{a_{\mathrm{t}}}]} = \sqrt{\sum_{p \in P^{od}} Var[F_p]\delta(p,a_{\mathrm{t}})} \quad \forall a_{\mathrm{t}} \in A_{\mathrm{t}}, \tag{3.13}$$

where $f_{a_{\mathrm{t}}}$ and $\sigma_f^{a_{\mathrm{t}}}$ are the mean and the SD of passenger flow on link a_{t}, respectively; $\delta(p,a_{\mathrm{t}})$ is the incidence relationship between transfer link and route; $\delta(p,a_{\mathrm{t}})$ equals 1 if transfer link a_{t} is on route p, 0 otherwise.

Let F_v be the passenger flow of mode b on physical link $v \in V_b$. It can be expressed as the summation of passenger flows on all direct in-vehicle links consisting of this physical link:

$$F_v = \sum_{a_{sn}^{ij} \in A_{\mathrm{d}}} \delta(a_{sn}^{ij},v) F_{sn}^{ij} = \sum_{a_{sn}^{ij} \in A_{\mathrm{d}}} \sum_{p \in P^{od}} \delta(a_{sn}^{ij},v)\delta(p,a_{sn}^{ij}) F_p \quad \forall v \in V_b, \eta(s) = b, \tag{3.14}$$

$$f_v = \sum_{a_{sn}^{ij} \in A_d} \delta(a_{sn}^{ij}, v) f_{sn}^{ij} = \sum_{a_{sn}^{ij} \in A_d} \sum_{p \in P^{od}} \delta(a_{sn}^{ij}, v) \delta(p, a_{sn}^{ij}) f_p \quad \forall v \in V_b, \eta(s) = b, \quad (3.15)$$

$$\sigma_f^v = \sqrt{Var[F_v]} = \sqrt{\sum_{a_{sn}^{ij} \in A_d} Var[F_{sn}^{ij}] \delta(a_{sn}^{ij}, v)} = \sqrt{\sum_{a_{sn}^{ij} \in A_d} \sum_{p \in P^{od}} \delta(a_{sn}^{ij}, v) \delta(p, a_{sn}^{ij}) Var[F_p]} \quad \forall v \in V_b, \eta(s) = b, \quad (3.16)$$

where f_v and σ_f^v are the mean and the SD of passenger flow on link v, respectively; $\delta(a_{sn}^{ij}, v)$ is the incidence relationship between in-vehicle link and physical link; $\delta(a_{sn}^{ij}, v)$ equals 1 if physical link v is in the in-vehicle link a_{sn}^{ij}, 0 otherwise.

According to the model assumptions A3.1 and A3.2, passenger flows of links and routes all follow normal distributions:

$$F_{sn}^{ij} \sim N\left(f_{sn}^{ij}, \left(\sigma_{fsn}^{ij} \right)^2 \right) \quad \forall a_{sn}^{ij} \in A_d, \quad (3.17)$$

$$F_{a_t} \sim N\left(f_{a_t}, \left(\sigma_f^{a_t} \right)^2 \right) \quad \forall a_t \in A_t, \quad (3.18)$$

$$F_v \sim N\left(f_v, \left(\sigma_f^v \right)^2 \right) \quad \forall v \in V, \quad (3.19)$$

$$F_p \sim N\left(f_p, \left(\sigma_f^p \right)^2 \right) \quad \forall p \in P^{od}. \quad (3.20)$$

3.3.3 Link travel time distribution

In this section, travel time distributions for physical links, in-vehicle links and transfer links are derived.

3.3.3.1 Physical links

The concept of generalized travel time is adopted in this Chapter, to model crowding discomfort in vehicles and congestion in road traffic. The generalized travel time of the physical link v is assumed to be increasing with respect to link flow F_v:

$$T_v = t_v\left(F_v\right), \tag{3.21}$$

$$t_v = E\left[T_v\right] = \int_{-\infty}^{+\infty} t_v(x)\varphi_v(x)dx \quad \forall v \in V, \tag{3.22}$$

$$(\sigma_t^v)^2 = \int_{-\infty}^{+\infty} (t_v(x))^2 \varphi_v(x)dx - \left(\int_{-\infty}^{+\infty} t_v(x)\varphi_v(x)dx\right)^2 \quad \forall v \in V, \tag{3.23}$$

where T_v is the generalized physical link travel time; $t_v(.)$ is the physical link travel time function; $\varphi_v(.)$ is the probability density function of link flow; t_v and σ_t^v, respectively, are the mean and the SD of travel time on physical link v.

Subway (mode b_1)

The generalized travel time T_v considering in-vehicle crowding discomfort (Spiess, 1983; Nielsen, 2000) on physical link v for mode b_1 can be expressed as

$$T_v = T_v\left(t_v^0, F_v, h_{b_1}, g_{b_1}\right) = t_v^0\left(1 + \beta_1\left(\frac{F_v}{h_{b_1} g_{b_1}}\right)^{k_1}\right) \quad \forall v \in V_{b_1}, \tag{3.24}$$

where h_{b_1} is the subway vehicle capacity (passengers per vehicle), and g_{b_1} is the subway deterministic frequency (vehicles per hour); t_v^0 is the free flow travel time of link v; β_1 and k_1 are model parameters. This equation is similar to the BPR formula adopted for road traffic. It is used to quantify the increasing discomfort when the number of passengers increases. Parameter β_1 is the weighting factor of the in-vehicle crowding dis-comfort. With a high value of k_1 in Equation (3.24), the crowding effect increases considerably (Spiess, 1983). Normally, $\beta_1 > 0$ and $k_1 > 1$.

As link flows follow normal distributions, the probability density function of link flow distributions can be expressed as

$$\varphi_v(x) = \frac{1}{\sqrt{2\pi}\sigma_f^v} \exp\left(-\frac{\left(x - f_v\right)^2}{2\left(\sigma_f^v\right)^2} \right) \quad \forall v \in V. \tag{3.25}$$

Substituting Equations (3.24) and (3.25) into Equations (3.22) and (3.23), the mean and the SD of physical link travel time in mode b_1 can be re-written as

$$t_v = t_v^0 + t_v^0 \frac{\beta_1}{\left(h_{b_1} g_{b_1}\right)^{k_1}} \sum_{i=0,i=even}^{k_1} \binom{k_1}{i} \left(\sigma_f^v\right)^i \left(f_v\right)^{k_1-i} (i-1)!! \quad \forall v \in V_{b_1}, \tag{3.26}$$

$$\sigma_t^v = \sqrt{\left(t_v^0 \frac{\beta_1}{\left(h_{b_1} g_{b_1}\right)^{k_1}} \right)^2 \left(\begin{array}{l} \sum\limits_{i=0,i=even}^{2k_1} \binom{2k_1}{i} \left(\sigma_f^v\right)^i \left(f_v\right)^{2k_1-i} (i-1)!! \\ -\left(\sum\limits_{i=0,i=even}^{k_1} \binom{k_1}{i} \left(\sigma_f^v\right)^i \left(f_v\right)^{k_1-i} (i-1)!! \right)^2 \end{array} \right)} \quad \forall v \in V_{b_1}. \tag{3.27}$$

The detailed derivations of Equations (3.26) and (3.27) can be found in Shao *et al.* (2006b).

Auto (mode b_2)

The link travel time considering congestion in road traffic (mode b_2) can be modelled by the most widely used Bureau of Public Roads (BPR) function (Sheffi, 1985) as

$$T_v = T_v\left(t_v^0, X_v, \kappa_v\right) = t_v^0 \left(1 + \gamma_1 \left(\frac{X_v}{\kappa_v} \right)^{k_2} \right) \quad \forall v \in V_{b_2}, \tag{3.28}$$

where X_v is the total traffic volume on road link v; κ_v is the capacity of the road link, and γ_1 and k_2 are model parameters. It should be noted that X_v consists of two components, that is, traffic volume of mode b_2 (auto) and traffic volume of mode b_3 (bus), because they both belong to road traffic. Therefore,

$$X_v = \frac{F_{v_{b_2}}}{e_0} e_{b_2} + G_{b_3} e_{b_3} \quad \forall v \in V \setminus V_{b_1}, \tag{3.29}$$

where G_{b_3} is the bus frequency; $F_{v_{b_2}}$ is the passenger flow of mode b_2 on road link v; e_{b_2} and e_{b_3} are passenger car equivalents for mode b_2 and b_3; e_0 is the average vehicle occupancy parameter representing the number of passengers per auto. The first term in Equation (3.29) represents the traffic volume of auto, and the second term represents the traffic volume of bus.

In the congested multi-modal transport network, the bus frequency may not be fixed because of the variability of road travel time. Therefore, in this Chapter bus frequency is considered to be a random variable for modelling the interactions between bus usage and auto usage. For simplicity, the bus fleet size s_0 is assumed to be fixed, and G_{b_3} is determined by s_0 and the cycle time of bus route. It is also assumed that cycle time can be represented by $2T_p$, where T_p is the one-way travel time of the bus route that contains the physical link v, and $T_p \sim N\left(t_p, \left(\sigma_t^p\right)^2\right)$. Thus, the stochastic bus frequency G_{b_3} can be calculated as

$$G_{b_3} = \frac{s_0}{2T_p}. \tag{3.30}$$

The mean and the variance of the second term in Equation (3.29) can be obtained according to Li *et al*. (2009):

$$E\left[G_{b_3}e_{b_3}\right]=E\left[\frac{s_0e_{b_3}}{2T_p}\right]=\frac{s_0e_{b_3}}{2t_p}\left(1+\frac{\left(\sigma_t^p\right)^2}{t_p^{\;2}}\right), \tag{3.31}$$

$$Var\left[G_{b_3}e_{b_3}\right]=Var\left[\frac{s_0e_{b_3}}{2T_p}\right]=\frac{s_0^{\;2}e_{b_3}^{\;2}\left(\sigma_t^p\right)^2}{4t_p^{\;4}}. \tag{3.32}$$

As mentioned above, $F_{v_{b_2}} \sim N\left(f_{v_{b_2}},\left(\sigma_f^{v_{b_2}}\right)^2\right)$ where $f_{v_{b_2}}$ is the mean passenger flow of mode b_2 on road link v and $\sigma_f^{v_{b_2}}$ is the SD of passenger flow of mode b_2 on road link v. Assuming that the traffic volume of bus and auto on the road are mutually independent, the mean and the SD of X_v (denoted as x_v and σ_{x_v}, respectively) can be expressed as follows:

$$x_v=\frac{e_{b_2}}{e_0}f_{v_{b_2}}+\frac{s_0e_{b_3}}{2t_p}\left(1+\frac{\left(\sigma_t^p\right)^2}{t_p^{\;2}}\right) \quad \forall v\in V\setminus V_{b_1}, \tag{3.33}$$

$$\sigma_{x_v}=\sqrt{\frac{e_{b_2}^2}{e_0^2}\left(\sigma_f^{v_{b_2}}\right)^2+\frac{s_0^{\;2}e_{b_3}^{\;2}\left(\sigma_t^p\right)^2}{4t_p^{\;4}}} \quad \forall v\in V\setminus V_{b_1}. \tag{3.34}$$

The mean and the SD of physical link travel time in mode b_2 can then, be expressed as follows:

$$t_v=t_v^0+t_v^0\frac{\gamma_1}{\left(\kappa_v\right)^{k_2}}\sum_{i=0,i=even}^{k_2}\binom{k_2}{i}\left(\sigma_{x_v}\right)^i\left(x_v\right)^{k_2-i}\left(i-1\right)!! \quad \forall v\in V_{b_2}, \tag{3.35}$$

$$\sigma_t^v = \sqrt{\left(t_v^0 \frac{\gamma_1}{(\kappa_v)^{k_2}}\right)^2 \left(\begin{array}{l} \sum_{i=0,i=even}^{2k_2} \binom{2k_2}{i} \left(\sigma_{x_v}\right)^i \left(x_v\right)^{2k_2-i} (i-1)!! \\ -\left(\sum_{i=0,i=even}^{k_2} \binom{k_2}{i} \left(\sigma_{x_v}\right)^i \left(x_v\right)^{k_2-i} (i-1)!! \right)^2 \end{array} \right)} \quad \forall v \in V_{b_2}. \quad (3.36)$$

Bus (mode b_3)

The generalized travel time on physical link v for mode b_3 can be expressed as

$$T_v = T_v\left(t_v^0, F_v, h_{b_3}, G_{b_3}, X_v, \kappa_v\right) = t_v^0 \left(1 + \beta_2 \left(\frac{F_v}{h_{b_3} G_{b_3}}\right)^{k_1} + \gamma_2 \left(\frac{X_v}{\kappa_v}\right)^{k_2}\right) \quad \forall v \in V_{b_3}, \quad (3.37)$$

where h_{b_3} denotes the bus capacity; G_{b_3} is the stochastic bus frequency (referring to Equation (3.30)), and β_2 and γ_2 are model parameters;. The last two terms in Equation (3.37) represent the crowding discomfort in vehicle and the congestion in road traffic.

The mean and the SD of physical link travel time in mode b_3 are as follows:

$$\begin{aligned} t_v = t_v^0 + \left(\frac{2^{k_1} \beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1} \left(s_0\right)^{k_1}}\right) \sum_{i=0,i=even}^{k_1} \binom{k_1}{i} \left(\sigma_f^v\right)^i f_v^{k_1-i} (i-1)!! \sum_{j=0,j=even}^{k_1} \binom{k_1}{j} \left(\sigma_t^p\right)^j t_p^{k_1-j} (j-1)!! \\ + \frac{\gamma_2 t_v^0}{(\kappa_v)^{k_2}} \sum_{i=0,i=even}^{k_2} \binom{k_2}{i} \left(\sigma_{x_v}\right)^i x_v^{k_2-i} (i-1)!! \qquad \forall v \in V_{b_3}, \end{aligned} \quad (3.38)$$

$$\sigma_t^v = \sqrt{\begin{array}{l} \left(\dfrac{2^{k_1}\beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1}\left(s_0\right)^{k_1}}\right)^2 \sum_{i=0,i=even}^{2k_1}\binom{2k_1}{i}\left(\sigma_f^v\right)^i f_v^{\,2k_1-i}(i-1)!! \sum_{j=0,j=even}^{2k_1}\binom{2k_1}{j}\left(\sigma_t^p\right)^j t_p^{\,2k_1-j}(j-1)!! \\ -\left[\left(\dfrac{2^{k_1}\beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1}\left(s_0\right)^{k_1}}\right)\sum_{i=0,i=even}^{k_1}\binom{k_1}{i}\left(\sigma_f^v\right)^i f_v^{\,k_1-i}(i-1)!! \sum_{j=0,j=even}^{k_1}\binom{k_1}{j}\left(\sigma_t^p\right)^j t_p^{\,k_1-j}(j-1)!!\right]^2 \\ +\dfrac{\left(\gamma_2\right)^2\left(t_v^0\right)^2}{\left(\kappa_v\right)^{2k_2}}\left[\sum_{i=0,i=even}^{2k_2}\binom{2k_2}{i}\left(\sigma_{x_v}\right)^i x_v^{\,2k_2-i}(i-1)!! - \left(\sum_{i=0,i=even}^{k_2}\binom{k_2}{i}\left(\sigma_{x_v}\right)^i x_v^{\,k_2-i}(i-1)!!\right)^2\right] \end{array}}$$

$$\forall v \in V_{b_3}. \tag{3.39}$$

The detailed manipulations on deducing Equations (3.38) and (3.39) are given in Appendix A.

3.3.3.2 In-vehicle links

The travel time of direct in-vehicle link (denoted as T_{sn}^{ij}) can be expressed as the summation of relevant physical link travel times (denoted as T_v):

$$T_{sn}^{ij} = \sum_{v\in V_b} T_v \delta\left(a_{sn}^{ij}, v\right) \quad \forall a_{sn}^{ij} \in A_{\mathrm{d}},\ \eta(s) = b. \tag{3.40}$$

As the mean and the standard deviation (SD) of travel time for each physical link $v \in V_b$ (denoted as t_v and σ_t^v respectively) are given in this Chapter, following the model assumption A3.5, the mean and the SD of in-vehicle link travel time (denoted as t_{sn}^{ij} and σ_{tsn}^{ij} respectively) then, can be expressed as

$$t_{sn}^{ij} = \sum_{v\in V_b} t_v \delta\left(a_{sn}^{ij}, v\right) \quad \forall a_{sn}^{ij} \in A_{\mathrm{d}},\ \eta(s) = b, \tag{3.41}$$

$$\sigma_{tsn}^{ij} = \sqrt{\sum_{v\in V_b}\left(\sigma_t^v\right)^2 \delta\left(a_{sn}^{ij}, v\right)} \quad \forall a_{sn}^{ij} \in A_{\mathrm{d}},\ \eta(s) = b, \tag{3.42}$$

where t_v and σ_t^v respectively denotes the mean and the SD of travel time for each physical link $v \in V_b$. $\delta(a_{sn}^{ij}, v)$ is the incidence relationship between in-vehicle link and physical link; $\delta(a_{sn}^{ij}, v)$ equals 1 if physical link v is in the in-vehicle link a_{sn}^{ij}, 0 otherwise.

3.3.3.3 Transfer links

There is no transfer waiting time if individuals transfer to mode b_2 (auto). When individuals transfer to b_1 (subway) or b_3 (bus), the waiting time for transfer link a_t (denoted as T_{a_t}) can be expressed as the work by Lo *et al*. (2003):

$$T_{a_t} = \frac{\lambda}{G_b} + \frac{1}{G_b}\left(\frac{F_{a_t} + \overline{F_{bi}}}{G_b h_b}\right)^{\mu} \quad \forall a_t \in A_t, b \in \{b_1, b_3\}, \qquad (3.43)$$

where F_{a_t} is the passenger volumn on this transfer link; $\overline{F_{bi}}$ is the prior passenger volume already in mode b prior to picking up passengers at location i; G_b and h_b, respectively, are the frequency and vehicle capacity of the boarding transport mode b; μ and λ are model parameters. The first term in Equation (3.43) expresses the waiting time for the next arriving vehicle, and the second term is related to the boarding congestion effect.

Specifically, the prior passenger volume can be obtained by summing the passenger flows on relevant direct in-vehicle links which do not start from location i (Lo *et al*., 2003):

$$\overline{F_{bi}} = \sum_{a_{sn}^{yz} \in A_d, \forall y \neq i, \eta(s)=b} F_{sn}^{yz} \delta\left(a_{sn}^{yz}, v^{i,i+1}\right) = \sum_{a_{sn}^{yz} \in A_d, \forall y \neq i, \eta(s)=b} \sum_{p \in P^{od}} F_p \delta\left(a_{sn}^{yz}, v^{i,i+1}\right) \delta\left(p, a_{sn}^{yz}\right), \qquad (3.44)$$

where $v^{i,i+1}$ is the physical link from i to its next station $i+1$ in mode b_1 or b_3; F_{sn}^{yz}

is the passenger flow on in-vehicle link a_{sn}^{yz} from y to z that does not start from i. For simplicity, F_{a_t} and $\overline{F_{bi}}$ are assumed to be independent from each other, then the mean and the SD of $F_{a_t}+\overline{F_{bi}}$ (denoted as f_1 and σ_1) can be expressed as follows:

$$f_1=\sum_{p\in P^{od}} f_p\delta(p,a_t)+\sum_{a_{sn}^{yz}\in A_d,\forall y\neq i,\eta(s)=b}\ \sum_{p\in P^{od}} f_p\delta\left(a_{sn}^{yz},v^{i,i+1}\right)\delta\left(p,a_{sn}^{yz}\right), \tag{3.45}$$

$$\sigma_1=\sqrt{\sum_{p\in P^{od}}\left(\sigma_f^p\right)^2\delta(p,a_t)+\sum_{a_{sn}^{yz}\in A_d,\forall y\neq j,\eta(s)=b}\ \sum_{p\in P^{od}}\left(\sigma_f^p\right)^2\delta\left(a_{sn}^{yz},v^{i,i+1}\right)\delta\left(p,a_{sn}^{yz}\right)}. \tag{3.46}$$

It is assumed that $F_{a_t}+\overline{F_{bi}}$ follows a normal distribution: $\left(F_{a_t}+\overline{F_{bi}}\right)\sim N(f_1,\sigma_1^2)$.

As indicated earlier, the subway frequency $G_b=g_{b_1}$ is a constant. Thus, the mean and the SD of waiting time for each transfer link to b_1 (subway) can be expressed as

$$t_{a_t}=\frac{\lambda}{g_{b_1}}+\frac{1}{\left(g_{b_1}\right)^{\mu+1}\left(h_{b_1}\right)^{\mu}}\sum_{i=0,i=even}^{\mu}\binom{\mu}{i}(\sigma_1)^i(f_1)^{\mu-i}(i-1)!!\ \ \forall a_t\in A_t, \tag{3.47}$$

$$\sigma_t^{a_t}=\sqrt{\left(\frac{1}{\left(g_{b_1}\right)^{\mu+1}\left(h_{b_1}\right)^{\mu}}\right)^2\left(\begin{array}{l}\sum_{i=0,i=even}^{2\mu}\binom{2\mu}{i}(\sigma_1)^i(f_1)^{2\mu-i}(i-1)!!\\-\left(\sum_{i=0,i=even}^{\mu}\binom{\mu}{i}(\sigma_1)^i(f_1)^{\mu-i}(i-1)!!\right)^2\end{array}\right)}\ \forall a_t\in A_t. \tag{3.48}$$

However, when individuals transfer to b_3 (bus), G_b (i.e. G_{b_3}) is stochastic as formulated in Section 3.3.3.1. Thus the mean and the SD of waiting time for each transfer link to b_3 (bus) can be expressed as

$$t_{a_t}=\frac{2\lambda t_p}{s_0}+\left(\frac{2^{\mu+1}}{s_0^{\mu+1}h_{b_3}^{\ \mu}}\right)\sum_{i=0,i=even}^{\mu}\binom{\mu}{i}(\sigma_1)^i f_1^{\mu-i}(i-1)!!\sum_{j=0,j=even}^{\mu+1}\binom{\mu+1}{j}\left(\sigma_t^p\right)^j t_p^{\ \mu+1-j}(j-1)!!, \tag{3.49}$$

$$\sigma_t^{a_t} = \sqrt{\begin{aligned} &\frac{4\lambda^2}{s_0^{\ 2}}\left(\sigma_t^p\right)^2 + \left(\frac{2^{2\mu+2}}{s_0^{\ 2\mu+2} h_{b_3}^{\ 2\mu}}\right) \sum_{i=0, i=even}^{2\mu} \binom{2\mu}{i} \sigma_1^i f_1^{2\mu-i} (i-1)!! \sum_{j=0, j=even}^{2\mu+2} \binom{2\mu+2}{j} \left(\sigma_t^p\right)^j t_p^{\ 2\mu+2-j} (j-1)!! \\ &-\left[\left(\frac{2^{\mu+1}}{s_0^{\ \mu+1} h_{b_3}^{\ \mu}}\right) \sum_{i=0, i=even}^{\mu} \binom{\mu}{i} \sigma_1^i f_1^{\mu-i} (i-1)!! \sum_{j=0, j=even}^{\mu+1} \binom{\mu+1}{j} \left(\sigma_t^p\right)^j t_p^{\ \mu+1-j} (j-1)!!\right]^2 \\ &+2\left(\frac{2^{\mu+2}\lambda}{s_0^{\ \mu+2} h_{b_3}^{\ \mu}}\right) \sum_{i=0, i=even}^{\mu} \binom{\mu}{i} \sigma_1^i f_1^{\mu-i} (i-1)!! \sum_{j=0, j=even}^{\mu+2} \binom{\mu+2}{j} \left(\sigma_t^p\right)^j t_p^{\ \mu+2-j} (j-1)!! \\ &-2\left(\frac{2^{\mu+2}\lambda t_p}{s_0^{\ \mu+2} h_{b_3}^{\ \mu}}\right) \sum_{i=0, i=even}^{\mu} \binom{\mu}{i} \sigma_1^i f_1^{\mu-i} (i-1)!! \sum_{j=0, j=even}^{\mu+1} \binom{\mu+1}{j} \left(\sigma_t^p\right)^j t_p^{\ \mu+1-j} (j-1)!! \end{aligned}} . \tag{3.50}$$

The detailed manipulations used in deducing Equations (3.49) and (3.50) can be found in Appendix B.

3.3.4 Route travel time distribution

The route travel time (denoted as T_p) can be obtained by summing the travel times on direct in-vehicle links and waiting times on transfer links:

$$T_p = \sum_{a_{sn}^{ij} \in A_d} T_{sn}^{ij} \delta\left(p, a_{sn}^{ij}\right) + \sum_{a_t \in A_t} T_{a_t} \delta\left(p, a_t\right) \quad \forall p \in P^{od}. \tag{3.51}$$

Following the model assumption A3.5, the mean and the SD of route travel time (denoted as t_p and σ_t^p respectively) can be expressed as

$$t_p = \sum_{a_{sn}^{ij} \in A_d} t_{sn}^{ij} \delta(p, a_{sn}^{ij}) + \sum_{a_t \in A_t} t_{a_t} \delta(p, a_t) \quad \forall p \in P^{od}, \tag{3.52}$$

$$\sigma_t^p = \sqrt{\sum_{a_{sn}^{ij} \in A_d} \left(\sigma_{tsn}^{ij}\right)^2 \delta\left(p, a_{sn}^{ij}\right) + \sum_{a_t \in A_t} \left(\sigma_t^{a_t}\right)^2 \delta\left(p, a_t\right)} \quad \forall p \in P^{od}, \tag{3.53}$$

and the route travel time follows a normal distribution: $T_p \sim N\left(t_p, \left(\sigma_t^p\right)^2\right)$.

3.4 Model formulation and solution algorithm

3.4.1 Model formulation

In network with travel time uncertainty, individuals tend to assign extra time to their prospective journey, so as to ensure a high probability of on-time arrival. The concept of travel time budget proposed by Lo *et al*. (2006) is adopted in this study. The travel time budget in this study is defined as the summation of the mean route travel time and a safety margin of route travel time.

Let α be the probability of arriving at a destination within the travel time budget, and c_p be the travel time budget for a given reliability threshold α. The value of α expresses individuals' attitude regarding on-time arrival. A larger α indicates a higher expectation on on-time arrival. This value of α can be pre-determined according to individuals' socio-economic characteristics and trip purpose (Chen *et al*., 2011).

Following the model assumption A3.5, route travel time follows a normal distribution: $T_p \sim N\left(t_p,\left(\sigma_t^p\right)^2\right)$. The travel time budget then, can be expressed as

$$c_p = t_p + \Phi^{-1}\left(\alpha\right)\sigma_t^p \quad \forall p \in P^{od}, \tag{3.54}$$

where $\Phi^{-1}\left(\alpha\right)$ is the inverse of standard normal cumulative distribution function at the probability of α. If $\alpha = 0.5$, $\Phi^{-1}\left(\alpha\right) = 0$ and the safety margin is equal to zero. It implies that individuals do not take travel time uncertainty into account and are only concerned about the expected travel time. The RUE results should be close to that of the UE model when $\alpha = 0.5$. However, it should be noted that in the RUE model when $\alpha = 0.5$, the mean travel times are still related to the SD of traffic flow. As such, the RUE model is not exactly equivalent to the conventional UE model when $\alpha = 0.5$.

In multi-modal transport networks, route fare is also an important individuals' route choice criterion. Hence, in this study, route fare is incorporated into route dis-utility by converting route fare into a time unit using a value of time (HK$ per minute, denoted as vot). The route dis-utility function can then be represented as

$$\varphi_p = -\omega_1 \left(c_p + \frac{\rho_p}{\text{vot}} \right) \quad \forall p \in P^{od}, \tag{3.55}$$

where ω_1 is a model parameter.

Because this study falls in the category of static model for long-term planning at the strategic level, it is postulated that all individuals in multi-modal transport networks would have a RUE route choice pattern: for each OD pair, the dis-utilities of all used routes are smallest and equal, and all unused routes have larger dis-utilities. Denote $\pi \in P^{od}$ as the most reliable route which has the smallest route dis-utility. The RUE condition can be formally expressed as

$$f_p \left(\varphi_\pi - \varphi_p \right) = 0 \quad \forall p \in P^{od}, \tag{3.56}$$

$$\varphi_\pi - \varphi_p \geq 0 \quad \forall p \in P^{od}. \tag{3.57}$$

The aforementioned RUE problem can be further expressed as the following gap function formulation:

$$\min \; GAP = \sum_{p \in P^{od}} f_p \left(\varphi_\pi - \varphi_p \right), \tag{3.58}$$

$$\varphi_{\pi} - \varphi_{p} \geq 0, \tag{3.59}$$

$$f_{p} \geq 0. \tag{3.60}$$

The gap function can be referred to as the overall gap capturing the complementary slackness conditions of the RUE model. According to Facchinei and Pang (2003), it can be proved that at least one solution of the RUE problem exists. In general, the uniqueness of the solution cannot be guaranteed due to the complex form of the route dis-utility function. However, in a special case, when the OD demand is deterministic, the RUE model becomes a UE model. In this case, the RUE model solution is unique (Sheffi, 1985; Shao *et al.*, 2006).

3.4.2 Solution algorithm

Most traditional solution algorithms cannot be used to solve the proposed RUE model, because it is difficult to determine the descent direction for solving the problem concerned. The widely used method of successive average (MSA) is a heuristic method with a forced convergence property. Therefore, proposed in this Chapter, is a solution algorithm based on MSA solving the aforementioned RUE problem. The detailed steps for the solution algorithm are presented as follows.

Step 0: Transform the traditional multi-modal transport network to a SAM network. List all the feasible routes in the SAM network according to the pre-defined probable transfer states.

Step 1: Calculate free-flow route travel times $\left\{t_{p}\right\}$. Set $\{\sigma_{t}^{p}\} = \left\{0\right\}$ .Then get free-flow route dis-utilities $\{\varphi_{p}\}$ on the basis of $\left\{t_{p}\right\}$, $\{\sigma_{t}^{p}\}$ and fares. Perform all-or-nothing assignment on the basis of $\{\varphi_p\}$ to obtain route flows and link flows $\mathbf{f}^1 = \{f_v, \forall v \in V\}$. Set $\mathrm{n}=1$.

Step 2: Get the SDs of link flow $\{\sigma_f^v\}$ and the SDs of route travel time $\{\sigma_t^p\}$. Use $\mathbf{f}^{\mathrm{n}}$, $\{\sigma_f^v\}$, $\left\{t_p\right\}$, $\{\sigma_t^p\}$ to update link travel times. Then get new $\left\{t_p\right\}$ and $\{\sigma_t^p\}$. After that, get new route dis-utilities $\{\varphi_p\}$.

Step 3: Perform all-or-nothing assignment on the basis of route dis-utilities $\{\varphi_p\}$, yielding auxiliary link flows $\overline{\mathbf{f}}^{\mathrm{n}}$.

Step 4: Calculate new link flows with an MSA scheme $\mathbf{f}^{\mathrm{n+1}} = \mathbf{f}^{\mathrm{n}} + (1/\mathrm{n})(\overline{\mathbf{f}}^{\mathrm{n}} - \mathbf{f}^{\mathrm{n}})$.

Step 5: For an acceptable convergence level τ, if $\max\limits_{v}\left|\mathbf{f}^{\mathrm{n+1}} - \mathbf{f}^{\mathrm{n}}\right| \le \tau$, stop; otherwise, set n=n+1, go back to Step 2.

Theorem 3.1. When the algorithm terminates, the RUE solution can be obtained.

Proof

In this algorithm aimed at solving the user equilibrium problem, the convergence test is based on the maximum change in link flow between successive iterations (Sheffi, 1985). Other criteria can also be used. According to Step 5, when the solution algorithm terminates, the maximum change in link flow is sufficiently small, so that the RUE condition is achieved approximately, i.e. $f_p(\varphi_\pi - \varphi_p) \approx 0$. Summing up the RUE condition for all paths yields that GAP $\approx$ 0. As GAP $\geq$ 0, the minimum value of the objective function is approximately obtained at the termination.

3.5 Numerical example

To demonstrate the aforementioned RUE formulation, adopted in this Chapter is the network presented by Lo *et al*. (2003). As shown in Figure 3.1, the network consists of 9 nodes and 16 physical links. There were three transport modes (auto, subway, bus) in this example and only one OD pair was considered. The origin was set as Node 1, and the destination was set as Node 9. Modal transfers follow the probable transfer states defined in Lo *et al*. (2003). The resulting feasible routes are listed in Table 3.1.

In this numerical example, the model parameters were set as follows: $cv^{od}=0.3$, $\beta_1=0.01$, $k_1=2$, $\gamma_1=0.3$, $k_2=2$, $e_0=1.2$, $e_{b_2}=1$, $e_{b_3}=3$, $\beta_2=0.007$, $\gamma_2=0.01$, $\lambda=0.5$, $\mu=2$, $\omega_1=0.123$, $\kappa=2$, and vot=1.37 HK$/min. The link capacity of each road link was 800 vehicles per hour. Free-flow travel time of each road link was set as 20 minutes and travel time of each subway link was set as 19 minutes. The fare of travelling by auto was nine units per link (fuel cost), whereas the non-linear fares of subway and bus are shown in Tables 3.2 and Table 3.3 respectively. The capacity of subway was 3200 passengers per vehicle, and the frequency was 8 vehicles per hour. The capacity of bus was 180 passengers per vehicle, and the fleet size was 20 vehicles.

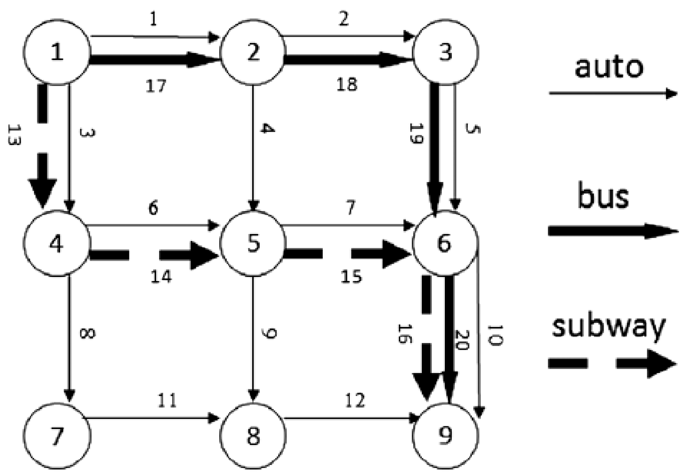


Figure 3.1 The multi-modal transport network

Table 3.1 Feasible routes according to probable transfer states

Route number	Modal usage (1-subway; 2-auto; 3-bus)	Transfer segment (node to node)
1	1	1→ 9
2	3	1→ 9
3	2	1→ 9
4	3-1	1→ 6→ 9
5	2-1	1→ 6→ 9
6	1-3	1→ 6→ 9
7	2-1	1→ 5→ 9
8	2-3	1→ 6→ 9
9	2-3	1→ 3→ 9
10	2-3	1→ 2→ 9
11	2-1	1→ 4→ 9
12	2-3-1	1→ 3→ 6→ 9
13	2-1-3	1→ 5→ 6→ 9
14	2-3-1	1→ 2→ 6→ 9
15	2-1-3	1→ 4→ 6→ 9

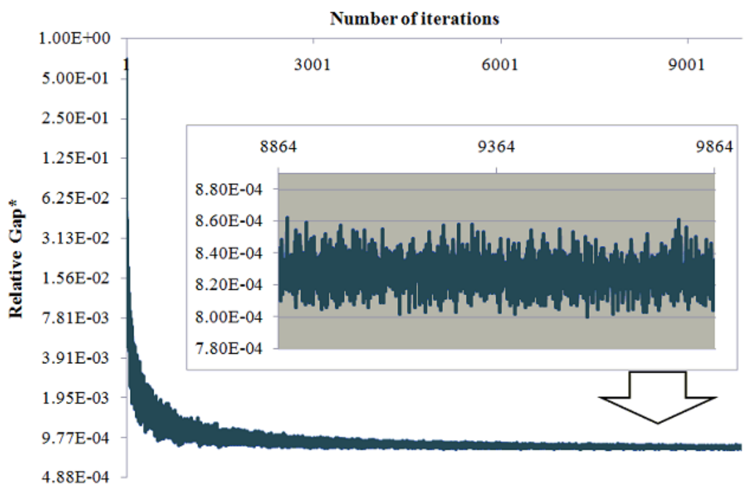
Table 3.2 Non-linear fares of the subway

From node	**To node**			
	4	**5**	**6**	**9**
1	10	25	40	40
4		10	25	40
5			10	25
6				10

Table 3.3 Non-linear fares of the bus

From node	To node			
	2	**3**	**6**	**9**
1	8	8	8	8
2		8	8	8
3			4	4
6				4

The convergence characteristics of the proposed RUE solution algorithm are illustrated in Figure 3.2. It can be seen that the RUE condition at the relative gap of 10^{-4} has been achieved after 9864 iterations (when $q^{od} = 3000$, $\tau = 0.1$). This result indicates that the proposed MSA solution algorithm can solve the RUE problem for this example network with an acceptable accuracy level.



*Relative Gap= $\left| GAP / \sum_{p \in P^{od}} f_p \varphi_p \right|$

Figure 3.2 Convergence characteristics of the solution algorithm

Individuals' travel behaviour in congested multi-modal transport networks, in terms of mode and route choices, were investigated under different levels of OD demand and on-time arrival probability. Figure 3.3 shows the variation of modal split when the mean of the OD demand (q^{od}) increases from 3000 passengers per hour to 30000 passengers per hour under different probabilities of on-time arrival (α). Figure 3.3(a)-(c), respectively, depicts the percentages of individuals using different transport modes (subway, bus and auto).

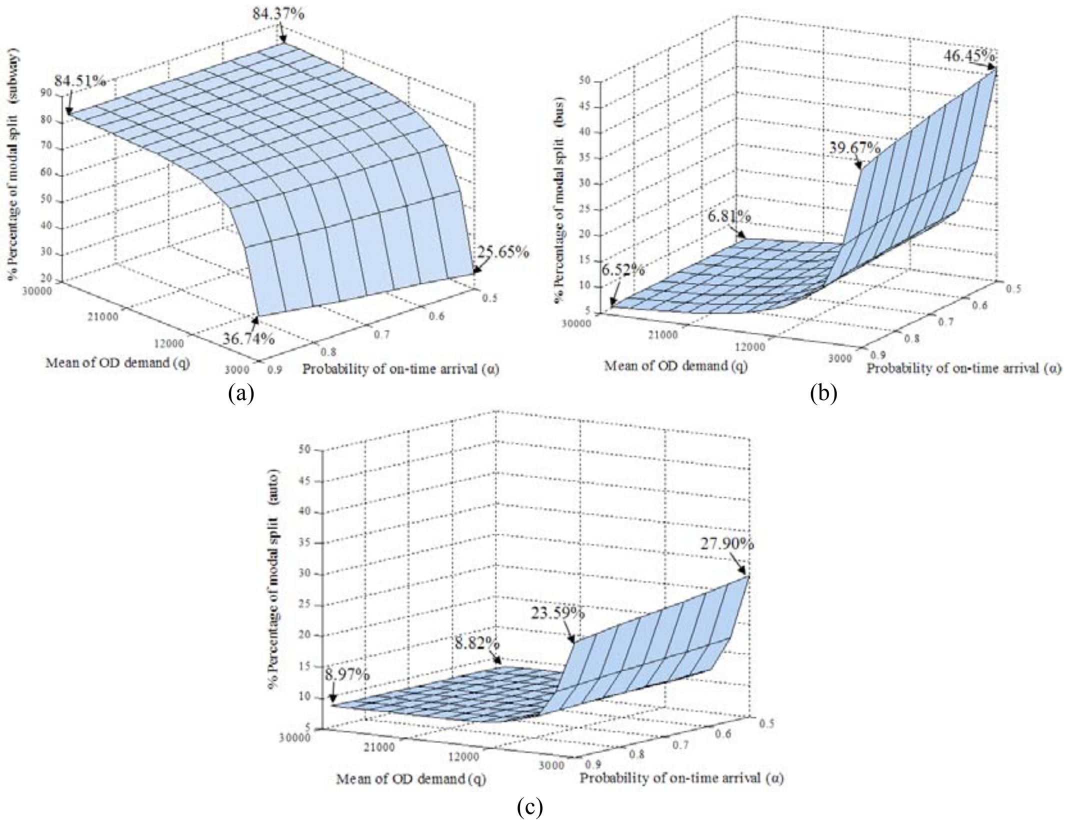


Figure 3.3 Modal splits under different levels of OD demand and on-time arrival probabilities: (a) subway, (b) bus, and (c) auto

It can be seen from Figure 3.3 that as the mean of the OD demand increases from 3000 to 30000, the percentage of modal share on subway increases dramatically. For example, with a 90% probability of on-time arrival (α =0.9), the percentage of subway users increases from 36.74% to 84.51%. In contrast, the percentages of modal share on bus and auto both decrease (from 39.67% to 6.52% and from 23.59% to 8.97%, respectively). This may be due to that large OD demand results in severe traffic congestion on the road. In view of this, individuals tend to choose the more reliable subway mode that has fixed frequency and no congestion interactions with bus and auto.

From the above possibilities posed, it was found that OD demand level influences individuals' route and mode choice behaviour. In addition, the on-time arrival probability also has a significant effect on individuals' route and mode choice behaviour.

For a certain level of OD demand, with the increase of α, the number of people travelling by subway rises, whereas the numbers of people travelling by auto and bus decrease. For example, when $q^{od} = 3000$, with α=50%, 25.65% of individuals use the subway, and this percentage increases to 36.74% when α reaches 0.9. For bus and auto usage, however, the percentages decrease from 46.45% to 39.67% and from 27.90% to 23.59%, respectively. This may be due to that under demand variation, the generalized subway travel time has a smaller variation than that of road traffic.

However, when q^{od} becomes larger, this phenomenon is less prominent. For instance, when $q^{od} = 30000$, with the increase of α, variations of modal split are all within 1% (84.37% to 84.51% for subway, 6.81% to 6.52% for bus, and 8.82% to 8.97% for auto). For auto, there is even a slight increase (8.82% to 8.97%) as compared with the downtrend in a smaller demand. This shows that when the OD demand is very large, most individuals will not change their mode choices to improve the probability of on-time arrival. This may be due to the fact that the large OD demand considerably increases subway crowding discomfort. In this situation, none of the available transport modes are reliable; and therefore, some people prefer to use autos to avoid the discomfort of transit vehicle travels. Thus, in the congested road-based transport networks in metropolitan areas, the subway is normally more reliable than other transport modes if an individual has a higher expectation of on-time arrival, but when the network becomes extremely overcrowded, subways will no longer be attractive as a result of the considerable crowding discomfort on trains.

Additionally, individuals' attitudes toward modal transfers under different probabilities of on-time arrival have been examined. The results are illustrated in Figure 3.4. As discussed in the modal formulation, each modal transfer has a transfer waiting time and thus brings a penalty into the route utility. Transfer waiting times are generally varied and stochastic because of the demand uncertainty, thus modal transfers may bring a degree of uncertainty to individuals' on-time arrivals. As α increases, individuals tend to choose the routes without any modal transfer. For example, when α increases from 0.5 to 0.9 ($q^{od}=3000$), the number of individuals using single transport mode increases from 2494 to 2596, whereas the number of individuals who use modal transfers decreases from 506 to 404.

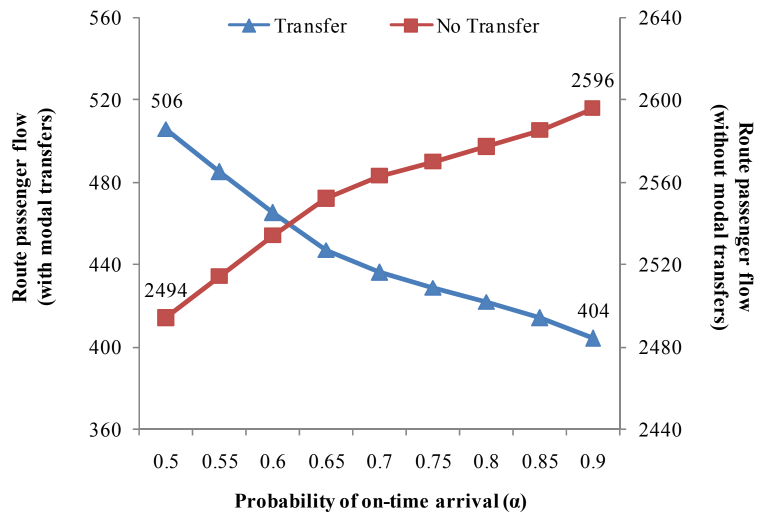


Figure 3.4 Individuals' attitudes toward modal transfer under different on-time arrival probabilities

3.6 Summary

This Chapter proposes a trip-based network equilibrium model in congested multi-modal transport networks (including auto, bus and subway modes) with demand variations. In the proposed trip-based model, crowding discomfort in transit vehicles, boarding congestion effect, and congestion impact of road traffic are explicitly modelled. To capture the effects of demand uncertainty, passenger flows and generalized travel times of different transport modes are formulated as random variables. In this Chapter, the stochastic bus frequency derived from unstable travel time of bus route is explicitly considered. This differs from the conventional approach in which bus frequency is independent of road travel time. Thus, the proposed network equilibrium model is more realistic for congested urban areas such as that in Hong Kong. In addition, the probable transfers and non-linear fare structures, involved in the multi-modal transport networks, are explicitly modelled in this

Chapter by using the SAM network.

Individuals' route and mode choice behaviour under stochastic multi-modal networks are also incorporated in the proposed network equilibrium model. To capture individuals' route and mode choice behaviour, the travel time budget, defined as the summation of the mean and the safety margin of generalized route travel time, is adopted in this new model. On the basis of this travel choice criterion, a RUE condition is then proposed. The RUE problem is solved by a solution algorithm using the MSA. The proposed network equilibrium model and the solution algorithm are tested using a hypothetical network. The results of the numerical example indicate that with a high expectation of on-time arrival, individuals tend to use the subway mode and prefer not to change mode during their travel.

In this Chapter, most random quantities are assumed to be normally distributed to better facilitate the presentation of those ideas which are essential. However, further study of the non-linear relationship among these quantities is advisable. Other types of probability distribution can also be adopted in further studies such as Lognormal distribution (Zhao and Kockelman, 2002), Poisson distribution (Clark and Watling, 2005), and truncated normal distribution. If different distributions are adopted, further investigations on the properties of these distributions should be carried out and various analyses may be needed. Statistical simulation can also be used to estimate the random flow distributions in the network.

As travel demands are derived from the desire of individuals to participate in various activities, an understanding of the interaction between individuals' activity and travel choice behaviour plays an important role in long-term transport planning. Therefore, the trip-based model in this Chapter is extended to an activity-based network equilibrium model in Chapter 4 to model the activity-travel pattern scheduling problem in multi-modal transit networks with uncertainty in different types of activity.

4 A Network Equilibrium Model for DATP Scheduling under Activity Uncertainty

Travel demands are derived from the desire of people to participate in various economic and social activities such as those associated with work, eating and shopping. Thus, an understanding of the interaction between individuals' activities and travel choice behaviour plays an important role for long-term transportation planning. A growing awareness in transportation research is that the activity-based approach can provide a better understanding of individuals' choice behaviour than that resulting from the trip-based approach. The activity-based network equilibrium models offer a comprehensive way to reflect travel choices, interdependency of trips, and scheduling of activities in temporal and spatial dimensions. In this Chapter, the trip-based network equilibrium model proposed in Chapter 3 is extended to an activity-based reliability-based user equilibrium (RUE) model for solving the daily activity-travel pattern (DATP) scheduling problems in congested multi-modal transit networks under activity uncertainty. Previous studies have indicated that crowding discomfort has a significant impact on individuals' choice of transit service for long-term planning. Thus, the in-vehicle crowding discomfort is explicitly considered in the proposed activity-based model particularly for congested transit networks in Asia.

In the proposed RUE model, the activity-travel choice problem is converted into a static traffic assignment problem by constructing a new super-network platform. With the use of the new super-network platform, individuals' activity and travel choices, such as time and space coordination, activity sequence and duration, and route/mode choices, can be simultaneously investigated. In this Chapter, in order to capture the stochastic characteristics of different activities, the activity utilities are assumed to be time-dependent and stochastic in relation to activity types (compulsory or non-compulsory in nature). To take account of the uncertainty of activity utility for modelling the DATP scheduling problem in congested multi-modal transit network, a concept of DATP budget utility is proposed in this Chapter.

This Chapter is structured as follows. Section 4.1 gives the background and the motivation of the study presented in this Chapter. A new super-network platform is introduced and model assumptions are given in Section 4.2. The problem statement and model formulation are presented in Section 4.3 for modelling the DATP scheduling problem in congested multi-modal transit network with activity uncertainty. The solution algorithm for solving the proposed RUE model is given in Section 4.4. A numerical example is provided in Section 4.5 for illustration of the proposed RUE model and solution algorithm. The key findings of this Chapter are summarized in Section 4.6.

4.1 Background

In past decades, increasing attention has been given to an activity-based approach in studying travel choice behaviour (Hirsh *et al*., 1986; Recker, 1995; Yamamoto and Kitamura, 1999; Yamamoto *et al*., 2000; Pendyala *et al*., 2002; Ruiz and Roorda, 2011; Chow and Recker, 2012; Zhang and Timmermans, 2012; Hannes *et al*., 2012).

The super-network representation has been adapted to model the activity-travel scheduling problem. For congested road networks, Ramadurai and Ukkusuri (2010) proposed a unified dynamic framework referred as activity-travel networks to model activity location, activity starting time, activity duration, and route choice simultaneously. Ouyang *et al*. (2011) studied the DATP scheduling problem in congested road network by constructing an expanded time-space network. For multi-modal transport networks, Liao (2011) proposed a multi-state super-network and made all link costs time-dependent when modelling the activity-travel scheduling problem. His proposed multi-state super-network, however, has conceivable difficulty in eliminating some unrealistic transfers and also cannot tackle the non-linear fare structures of public transit systems such as that in Hong Kong. Activity sequence and activity duration need to be pre-determined in his model.

In view of the above, proposed in this Chapter is a new super-network platform which integrates both the activity-time-space (ATS) network (Ouyang *et al*., 2011) and the state-augmented multimodal (SAM) network (Lo *et al*., 2003) to explicitly model multi-modal trips and individuals' activity choices. With this new super-network platform, the transfers and non-linear fare structures in multi-modal transit networks can be explicitly modelled and simultaneously address the relationship between activity choices and travel choices. Each route from origin to destination in the new super-network platform represents a specific DATP.

Individuals' equilibrium choices can be obtained by applying traffic assignment algorithms to the new super-network. As reviewed in Chapter 2, most studies were developed for modelling DATP scheduling problems in congested road networks. Little effort to solve the comprehensive DATP choice problem in congested multi-modal transit networks has been observed. In metropolitan areas such as Hong Kong, over 90% of daily travel appears to be made using various public transit modes. Hence a need is likely to exist for a simultaneous modelling of individuals' activity choices and route/mode choices in multi-modal transit networks.

The concept of activity utility is widely used in the activity-based approach (Adler and Ben-Akiva, 1979; Kitamura, 1984). In the previous related studies, the perceived activity utility is considered as the summation of a systematic component which is a deterministic representative value of utility and a random component which represents the variation in individuals' perceptions (Kitamura, 1984; Lam and Yin, 2001). That is, the uncertainty of activity utility, mentioned in previous studies, lies in variation of individuals' perception.

Activity utility may broadly, consist of the following attributes: (a) activity time window; (b) degree of need for the activity; (c) degree of satisfaction from the process; (d) money gain or loss. In reality, these attributes vary from day to day. For example, the utility of working from 8:00 to 10:00 a.m. on Monday may be much higher than working at that time on Tuesday, if such as an important meeting took place on Monday. Thus, on this basis, the systematic component in activity utility should be stochastic, and the activity utility has a day-to-day variation. The utility profile of an activity should not be a single curve but rather an area which indicates a probability distribution. However, little information regarding modelling the day-to-day variation of activity utility is evident, despite the need to take into account the uncertainty of activity utility as suggested above. This uncertainty may, in practice, have a significant influence on individuals' DATP choices.

Described in this Chapter is a pioneering endeavour devoted to capturing the stochastic characteristics of different activities. Activity utility is assumed to be time-dependent and stochastic in relation to different activity types (compulsory and non-compulsory in nature), and the travel dis-utility is also stochastic. A concept of DATP budget utility is proposed in the modelling of the uncertainties in activity utility and travel dis-utility.

In light of the above, the trip-based RUE model proposed in Chapter 3 for traffic assignment is extended to an activity-based model for scheduling DATPs in congested multi-modal transit networks under uncertainty and is described in this Chapter. The time and space coordination, activity location, activity sequence and duration, and the relationship between activity and route/mode choices, can be simultaneously investigated by solving the RUE problem on the new super-network platform. Hence existing theories are extended by the development of a comprehensive framework which incorporates the flexible activity sequence and duration, the stochastic effects of activity utility, route and mode choices, together with in-vehicle crowding effects.

4.2 Network representation and model assumptions

In this section, a new super-network platform is introduced in Section 4.2.1. Model assumptions are given in Section 4.2.2. In Section 4.2.3, the utilities/dis-utilities of different links on the new super-network platform are discussed.

4.2.1 A new super-network platform

A super-network platform named the ATS-SAM super-network is constructed and described in this section. The ATS-SAM super-network is an integration and expansion of the ATS network and the SAM transport network. The ATS network is an expanded network in which activity links are introduced into the conventional time-space network. As a result, both individuals' activity and travel choices can be captured in this super-network. The SAM network can be used to eliminate the unrealistic transfers and to model the non-linear fare structures in multi-modal transport networks. By constructing the ATS-SAM super-network, the merits of these two networks are achieved simultaneously as a consequence.

The SAM network introduced in Chapter 3 is augmented to ATS-SAM super-network by incorporating time-space coordinates and activity links. The study horizon is divided into K equally spaced time intervals (Lam and Yin, 2001; Lam and Huang, 2002; Huang and Lam, 2005; Zhang *et al.*, 2005; Li *et al.*, 2010). Let $k = 1, 2, ..., K, K+1$ be the start time of a node or link. The framework of ATS-SAM super-network is given as below (refer to Figure 4.1 as an example of the ATS-SAM super-network consisting of three transport modes, i.e. subway, bus, and auto).

Nodes: Each node in the SAM network is augmented into $K+1$ nodes in ATS-SAM super-network. Each node is described as $((i, s, n, l), k)$, where i is the physical location of the node for a particular activity, and s is the transfer state used to model probable transfers. n is the number of transfers that has been made by an individual, and l is the alight or aboard indicator. k is the start time of the node. The value of l is equal to 1 (0) indicating that the individual is at the beginning (end) of an in-vehicle link. Specifically, each transfer state $s \in S$ associates with the use of a particular transport mode $\eta(s) \in B$ and a set of probable transfers $\xi(s) \subseteq S$. If individuals are at state s, the indication is that these individuals are using mode $\eta(s)$ and can only transfer to any state in $\xi(s)$. Modal transfers in this Chapter follow the probable transfer states defined by Lo *et al*. (2003).

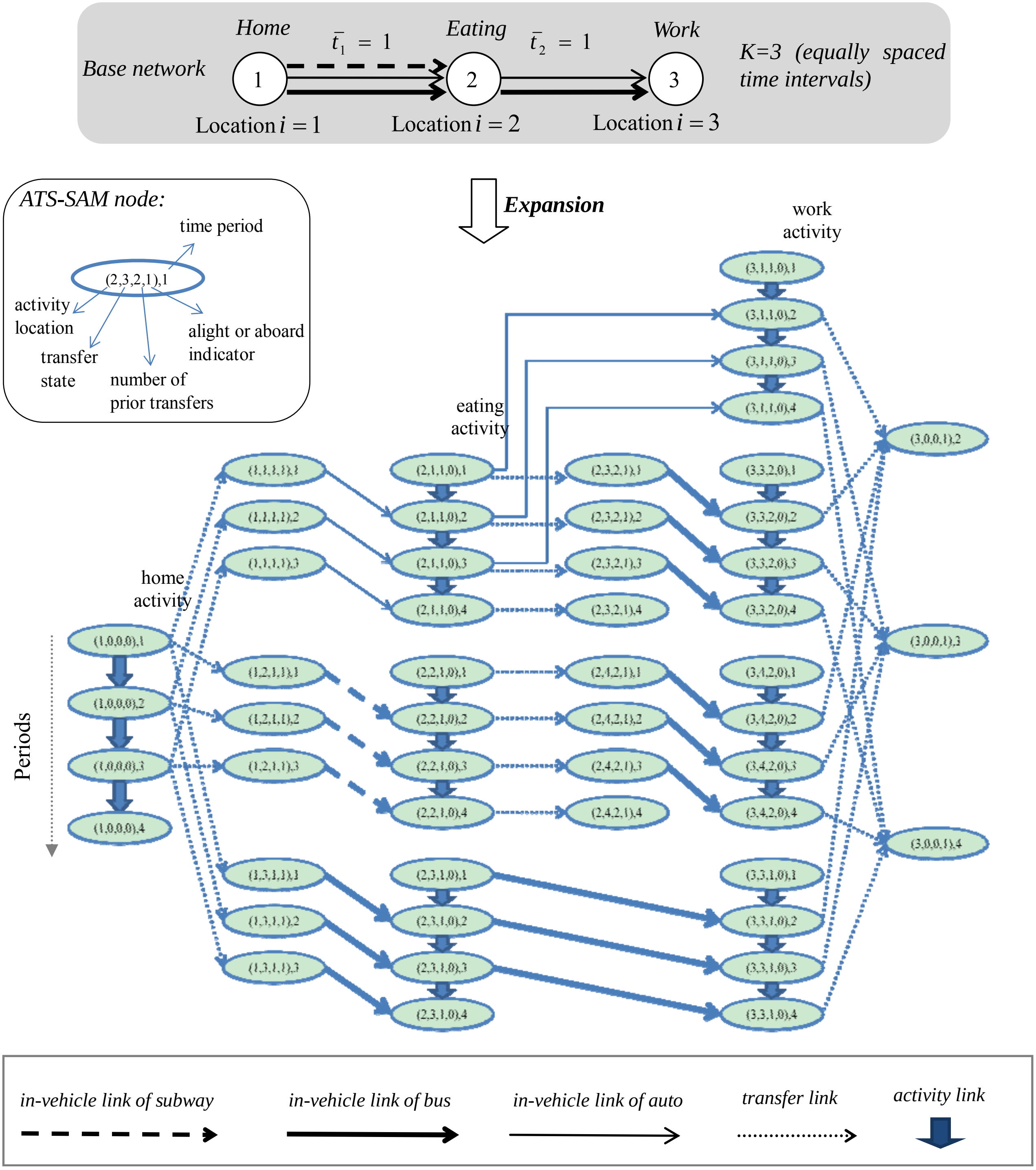


Figure 4.1 An illustrative example of ATS-SAM super-network

Links: Links in the ATS-SAM super-network are classified into three categories, i.e.

$A = A_t \cup A_d \cup A_a$, where A_t is the set of transfer links between modes, and A_d is the set of direct in-vehicle links made up of physical links. A_a is the set of activity links. Each transfer link $a_t \in A_t$ is constructed according to the probable transfer states defined by Lo *et al*. (2003). The duration of a transfer link is assumed to be zero in this Chapter but transfer dis-utility is considered. Each in-vehicle link $a_d \in A_d$ represents a direct in-vehicle movement. It should be noted that a direct in-vehicle link may consist of more than one consecutive physical link. In this way, non-linear fares can be directly represented on a node to node basis. A_a is constructed between the augmented nodes at the same location to indicate that a particular activity is conducted. Each $a_a \in A_a$ is characterized by the activity location, the activity type, activity start time, and activity duration. The activity time window is not required as the activity utility by time of day is adopted in this Chapter (Lam and Yin, 2001). The process of route searching in the ATS-SAM super-network can lead to realistic and more generalized results regarding the times to perform activities during the study period.

A rule-based algorithm is proposed to generate the ATS-SAM super-network. With this rule-based algorithm, the conventional multi-modal transit network can easily be automatically transformed into the ATS-SAM super-network. In the novel super-network, each route from origin to destination represents a feasible DATP. The detailed steps of the proposed ATS-SAM super-network expansion algorithm are presented below.

Input: a multi-modal transit network M, activity locations ($i_a \in I_a$), transfer states

$s \in S$, probable transfers $\xi(s) \subseteq S$ for each $s \in S$, maximum number of transfers κ, and number of time intervals K.

Output: the ATS-SAM super-network.

Step 1. Node augmentation.

For each node $i \in U$, expand the node into into ATS-SAM node:

$\left((i,0,0,l),k\right)$, $l = 0,1,$ $k = 1,2,...,K,K+1$; and $\left((i,s,n,l),k\right)$, $s = 1,2,3,4,5,$ $n = 1,2,...,\kappa,$ $l = 0,1,$ $k = 1,2,...,K,K+1$. Denote the ATS-SAM node set as N.

Step 2. Construction of ATS-SAM activity links.

Scan all nodes in set N. Construct activity links $a_{\mathrm{a}} \in A_{\mathrm{a}}$ between $\left((i_{\mathrm{a}},s,n,0),k\right)$ and $\left((i_{\mathrm{a}},s,n,0),k+1\right)$.

Step 3. Construction of ATS-SAM transfer links.

Scan all nodes in set N. Construct transfer links $a_{\mathrm{t}} \in A_{\mathrm{t}}$ between $\left((i,s,n,0),k\right)$ and $\left((i,\xi(s),n+1,1),k\right)$.

Step 4. Construction of ATS-SAM direct in-vehicle links.

Find all in-vehicle links in network M on the basis of physical travel links. Obtain in-vehicle link travel times $t_{a_{\mathrm{d}}}$.

For each $i \in U$, find all $i' \in U$ which are connected to i by in-vehicle links. Record the mode b and the travel time $t^0_{a_{\mathrm{d}}}$ of each in-vehicle link.

For each i', construct ATS-SAM in-vehicle links between $\left((i,s,n,1),k\right)$ and $\left((i',s,n,0),k+t^0_{a_{\mathrm{d}}}\right)$, $\eta(s) = b$.

Step 5. Simplification of the super-network.

Delete the augmented nodes which are not two-way connected except for the origin node and the destination node. Delete the redundant links.

4.2.2 Model assumptions

In order to facilitate the essential ideas without loss of generality, the following assumptions are made in this Chapter.

A4.1: The DATP is considered in a fixed study horizon, divided into K equally spaced time intervals (Lam and Yin, 2001; Huang and Lam, 2005; Zhang *et al*., 2005; Ouyang *et al*., 2011).

A4.2: The proposed model falls within the static model category for long-term planning at the strategic level. Therefore, it is assumed that individuals have perfect knowledge of traffic conditions throughout the whole network (Ouyang *et al*., 2011).

A4.3: The utility maximization approach is employed to formulate the individuals' DATP choices (Lam and Huang, 2002; Zhang *et al*., 2005; Li *et al*., 2010). Activity utility only depends on the start time of the activity and its duration. The activity utility is determined by a bell-shaped marginal utility function proposed by Joh *et al*. (2002) and Ettema and Timmermans (2003). Many related studies have adopted this type of function for modelling the marginal utility of activity (Ashiru *et al*., 2004; Zhang *et al*., 2005; Li *et al*., 2010). This function does not consider the needs of individuals. In further studies, the need-based utility functions (Arentze and Timmermans, 2009) can also be incorporated in the model proposed in this Chapter.

A4.4: Transit vehicles are assumed to fully follow a run schedule which is given and fixed (Tong and Wong, 1998; Tong *et al*., 2001; Li *et al*., 2010). Link travel times are deterministic. No vehicle capacity constraint exists. In-vehicle crowding discomfort is modelled (Spiess, 1983; Nielsen, 2000; Lo *et al*., 2003; Sumalee *et al*., 2011).

A4.5: In this Chapter, only one behaviourally homogeneous group is considered for facilitation of the presentation of the essential ideas (Lam and Yin, 2001; Huang and Lam, 2005; Ouyang *et al*., 2011). Multiple groups can be considered as an extension of the model proposed in this Chapter (Chen *et al*., 2011). Activity interdependency of household members is also not considered in this Chapter but is investigated in Chapter 6 of this thesis.

A4.6: Link utilities/dis-utilities and the DATP utility are assumed to follow normal distributions.

The time period of the proposed model is from 06:00 to 24:00 and is divided into 108 intervals each with ten minutes. Four types of activities are investigated in this Chapter; namely, home, work, dinner, and shopping activities. The activity sequence and duration are not fixed (Ouyang *et al*., 2011). Home and work are considered as compulsory activities, while dinner and shopping are non-compulsory activities.

4.2.3 Link utility/dis-utility in ATS-SAM super-network

Following the model assumption A4.6, the utility of activity link a_{a} (denoted as $U_{a_{\mathrm{a}}}$) is stochastic and assumed to follow the normal distribution. The mean utility of performing activity link a_{a} from start time k for one time interval is expressed as:

$$u_{a_{\mathrm{a}}} = \int_{k}^{k+1} \bar{u}_{a_{\mathrm{a}}}(\omega) d\omega, \tag{4.1}$$

where $\bar{u}_{a_{\mathrm{a}}}(k)$ denotes the marginal utility of performing activity link a_{a} at time k. The standard deviation (SD) of activity utility is expressed as a function of the $u_{a_{\mathrm{a}}}$ in this Chapter:

$$\sigma_{a_{\mathrm{a}}} = cv_{a_{\mathrm{a}}} \cdot u_{a_{\mathrm{a}}}, \tag{4.2}$$

where cv_{a_a} is a model parameter relevant to the activity type (compulsory or non-compulsory) of a_a.

The dis-utility of physical link v with start time interval k (denoted as $DISU_v(k)$) is modelled to represent in-vehicle crowding discomfort (Spiess 1983; Nielsen 2000; Lo *et al*. 2003):

$$DISU_v(k) = -\text{vot} \cdot t_v^0 \left(1 + \beta_b \left(\frac{F_v(k)}{h_b \cdot g_b} \right)^{\theta_b} \right), \quad v \in V_b \tag{4.3}$$

where t_v^0 is the travel time of physical link v; h_b is the vehicle capacity of mode b; g_b denotes the frequency of mode b; vot is the value of time; β_b and θ_b are model parameters relevant to mode b. $F_v(k)$ is the stochastic passenger flow on the physical link v at time interval k.

Travel demands are derived from the need of individuals to participate in various activities, thus in view of the uncertainty of activity utility, the demands for different activities are stochastic. Under demand uncertainty, the passenger flows are also stochastic. In this Chapter, F_{a_d} is assumed to follow normal distribution. Denote the mean and the SD of F_{a_d} as f_{a_d} and $\sigma_f^{a_d}$, respectively. It is assumed in this Chapter that the coefficient of variation (CV) of link flow is equal to the CV of the activity (which can be conducted at the end of the in-vehicle link in the ATS-SAM super-network) utility, so the SD of F_{a_d} can be expressed as

$$\sigma_f^{a_d} = cv_{a_a} \cdot f_{a_d}, \tag{4.4}$$

where f_{a_d} denotes the mean of F_{a_d}. a_a is the activity which is at the end of the in-vehicle link a_d in the ATS-SAM super-network. If no activity can be conducted at the end of the in-vehicle link, $\sigma_f^{a_d}$ is assumed to be zero. The mean of F_{a_d} (denoted as f_{a_d}) is expressed as

$$f_{a_d} = \sum_{p \in P} f_p \delta(p, a_d), \tag{4.5}$$

where $\delta(p, a_d)$ is equal to 1 if in-vehicle link a_d is used in DATP p; 0 otherwise.

The mean of $F_v(k)$ can be expressed as

$$f_v(k) = \sum_{a_d \in A_d} f_{a_d}(k) \delta(a_d, v), \tag{4.6}$$

where $\delta(a_d, v)$ is equal to 1 if physical link v is in direct in-vehicle link a_d; 0 otherwise. The SD of $F_v(k)$ can be expressed as

$$\sigma_f^v(k) = \sqrt{\sum_{a_d \in A_d} (\sigma_f^{a_d})^2 \delta(a_d, v)}. \tag{4.7}$$

By assuming that $F_v(k)$ follows normal distribution, the mean and the SD of $DISU_v(k)$ (denoted as $disu_v(k)$ and $\sigma_v(k)$, respectively) can be obtained as discussed in Chapter 3 (Shao *et al.*, 2006; Fu *et al.*, 2014a). Assuming physical link dis-utilities are mutually independent, the mean of in-vehicle link dis-utility $DISU_{a_d}$ can be obtained by the summation of related physical links' mean dis-utilities and transit fare:

$$disu_{a_d} = \sum_{v \in V_b} disu_v(k) \delta(a_d, v) \text{-} \rho_{a_d}, \tag{4.8}$$

where ρ_{a_d} is the transit fare with respect to the direct in-vehicle link a_d. In this way, non-linear fares can be directly represented by node-to-node basis. The SD of in-vehicle link dis-utility can be expressed as

$$\sigma_{a_\mathrm{d}} = \sqrt{\sum \sigma_v(k)^2 \delta(a_\mathrm{d}, v)}. \tag{4.9}$$

As regards transfer links by mode, the link dis-utility can be expressed as

$$disu_{a_\mathrm{t}} = -\mathrm{vot} \cdot \frac{1}{2g_b} - pen_b, \qquad a_\mathrm{t} \in A_\mathrm{t} \tag{4.10}$$

where g_b is the frequency of the transit line to which individuals transfer on the transfer link concerned, and pen_b is the mode-specified transfer penalty.

Let P be the route set in the ATS-SAM super-network (i.e. DATP set). The daily utility gain, i.e. the utility of DATP $p \in P$ (denoted as U_p), can be obtained by summing the dis-utilities of in-vehicle links, dis-utilities of transfer links, and utilities of activity links:

$$U_p = \sum_{a_\mathrm{d} \in A_\mathrm{d}} DISU_{a_\mathrm{d}} \delta(p, a_\mathrm{d}) + \sum_{a_\mathrm{t} \in A_\mathrm{t}} disu_{a_\mathrm{t}} \delta(p, a_\mathrm{t}) + \sum_{a_\mathrm{a} \in A_\mathrm{a}} U_{a_\mathrm{a}} \delta(p, a_\mathrm{a}), \tag{4.11}$$

where $\delta(p,a)$ is the incidence relationship between DATP and link; $\delta(p,a)$ equals 1 indicates that this link is used in the DATP, 0 otherwise.

Link utilities/dis-utilities are assumed to be mutually independent and follow normal distributions in this Chapter. Therefore, the mean and the SD of the DATP utility can be respectively expressed as

$$u_p = \sum_{a_\mathrm{d} \in A_\mathrm{d}} disu_{a_\mathrm{d}} \delta(p, a_\mathrm{d}) + \sum_{a_\mathrm{t} \in A_\mathrm{t}} disu_{a_\mathrm{t}} \delta(p, a_\mathrm{t}) + \sum_{a_\mathrm{a} \in A_\mathrm{a}} u_{a_\mathrm{a}} \delta(p, a_\mathrm{a}), \tag{4.12}$$

$$\sigma_p = \sqrt{\sum_{a_\mathrm{d} \in A_\mathrm{d}} \sigma_{a_\mathrm{d}}^2 \delta(p, a_\mathrm{d}) + \sum_{a_\mathrm{a} \in A_\mathrm{a}} \sigma_{a_\mathrm{a}}^2 \delta(p, a_\mathrm{a})}. \tag{4.13}$$

Following model assumption A4.6, the DATP utility follows a normal distribution:

$U_p \sim N(u_p, \sigma_p^{\ 2})$.

4.3 Definitions and problem statement

In this section, a definition of DATP budget utility is first discussed in Section 4.3.1. Using this definition, the model formulation is proposed in Section 4.3.2.

4.3.1 Definition of DATP budget utility

Individuals conduct activities to meet a variety of needs in their daily life. Activity participation returns utility as a reward for a meeting of needs. Individuals also receive dis-utility from the travel between activities. Supernak (1992) adopted the total utility of activity-travel pattern in a typical utility maximization context. The total utility obtained from an activity-travel pattern is the summation of the utility gained from activities and the dis-utility resulting from travels. Individuals' activity-travel pattern choices are decided by the underlying activity utilities and travel dis-utilities. Individuals select the activity-travel pattern with the maximum total utility.

As a pioneering endeavour, this Chapter is devoted to capturing the stochastic characteristics of different activities. Activity utility is assumed in this Chapter to be time-dependent and stochastic in relation to different activity types. The primary activities in people's daily life can be divided into two major groups based on activity types, i.e. compulsory activities such as work and home, and non-compulsory activities such as eating at restaurants, and shopping in malls. In general, the utility of non-compulsory activity has a much larger day-to-day variation than does the utility of compulsory activity. The first reason is the fact that the time window for non-compulsory activity apparently has greater variation than that for compulsory activity.

The second reason is that the need and the satisfaction of performing non-compulsory activity are not as stable as the compulsory one. For example, people need to go back home every day, but they may not go for shopping so frequently. The third reason is that people may obtain or lose reward in utility or money in a more constant or consistent manner from compulsory activities than from non-compulsory activities. For example, people's reward in terms of salary for work activity is often the same for every day, but how much they spend on shopping each day is much more discretionary. Therefore, modelling the uncertainty of activity utility and investigating how it would affect individuals' DATP choices are significant in multi-modal transit networks with uncertainty.

Under the uncertainty of activity utility, individuals tend to assign an extra dis-utility as a safety margin to ensure a higher probability of gaining a certain level of DATP utility. In view of this, a concept of DATP budget utility is proposed in this Chapter.

In this Chapter, the DATP budget utility is defined as

[DATP Budget Utility] = [Expected DATP Utility] + [DATP Utility Margin].

Mathematically, the DATP budget utility associated with DATP p, φ_p, can be expressed as

$$\varphi_p = u_p + \lambda'\sigma_p, \tag{4.14}$$

where λ' is a negative parameter. It is related to the requirement on ensuring a certain utility gain. For individuals who want to ensure a higher probability of utility gain, they regard the DATPs as having a relatively small φ_p, or equivalently, a small value of λ'. Formally, λ' can be related mathematically to the probability that the individuals can gain the DATP budget utility, written as

$$P\left\{U_p \geq \varphi_p = u_p + \lambda' \sigma_p\right\} = \alpha, \tag{4.15}$$

where α is the probability of gaining DATP budget utility. Re-arranging terms in Equation (4.15), we obtain

$$P\left\{\frac{U_p - u_p}{\sigma_p} \geq \lambda'\right\} = \alpha. \tag{4.16}$$

The following equation can then, be obtained.

$$P\left\{\frac{U_p - u_p}{\sigma_p} < \lambda'\right\} = 1 - \alpha. \tag{4.17}$$

Let $\Phi(\cdot)$ be the standard normal cumulative distribution function. Equation (4.17) can be re-written as

$$\Phi(\lambda') = 1 - \alpha. \tag{4.18}$$

As $1 - \Phi(\lambda') = \Phi(-\lambda'),$ Equation (4.18) can be transformed as

$$\Phi(-\lambda') = \alpha. \tag{4.19}$$

Take the inverse of Equation (4.19), we obtain $\lambda' = -\Phi^{-1}(\alpha).$

Therefore, the DATP budget utility, defined as the summation of the expected DATP utility and a negative safety margin of DATP utility, can be expressed as

$$\varphi_p = u_p - \Phi^{-1}(\alpha)\sigma_p. \tag{4.20}$$

The value of α represents individuals' risk attitude toward utility gain. A larger α indicates a higher expectation of daily utility gain (i.e. a higher probability of gaining DATP budget utility), and results in a larger negative safety margin and a smaller φ_p. The negative safety margin represents a risk in utility gain. This margin is assigned by individuals as an extra dis-utility to ensure the expected probability of gaining the budget utility.

In this Chapter, it is assumed that the SD of activity utility σ_{a_a} is linear correlated with the mean of activity utility u_{a_a} (as shown in Equation (4.2)). For compulsory activities, the cv_{a_a} is smaller than that of non-compulsory activities. When u_{a_a} of two activities are equal, individuals choose the activity with a smaller cv_{a_a}, since a smaller cv_{a_a} leads to a smaller σ_{a_a} and a larger budget utility.

Figure 4.2 gives a simple example to illustrate the budget utility concept proposed in this Chapter. Two types of activities (work and shopping) are depicted in Figure 4.2. For the time period 17:00-17:01, when $\alpha = 50\%$, the $\Phi^{-1}(\alpha)\cdot\sigma_{a_a}$ of these two activities are zero and the budget utilities of these two activities are equal ($\varphi = 6$). However, when $\alpha = 90\%$, the $\Phi^{-1}(\alpha)\cdot\sigma_{a_a}$ of shopping activity is larger than that of work activity. As work activity is a compulsory activity and shopping activity is a non-compulsory activity, work should be more reliable than shopping in utility gain. Comparing the budget utility of these two activities ($\varphi = 4.5$ for work and $\varphi = 3.5$ for shopping), people tend to choose work rather than shopping. It is shown in this illustration that the uncertainty of activity utility affects people's activity choices.

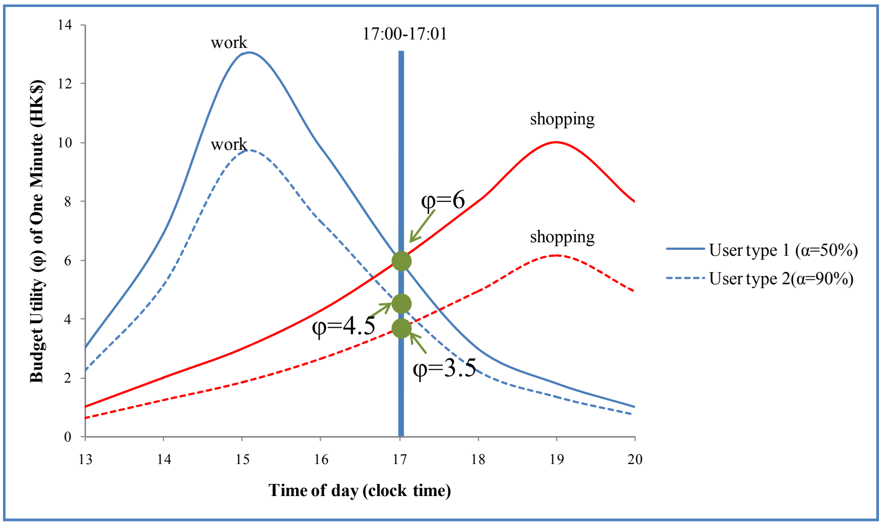


Figure 4.2 An illustration of budget utility variation for different types of activities

4.3.2 Model formulation

With the use of the proposed ATS-SAM super-network, individuals' activity choices (i.e. activity locations, sequence and durations) and travel choices (i.e. route and mode choices, transfers) are explicitly represented by different links in the proposed super-network platform. Activities with different durations or different start times are constructed as different activity links. In this way, the time-dependent activity utility, in this Chapter, can be modelled in terms of static values. The relationships between activity and travel choices are reflected by the ATS-SAM super-network topology. Each route from origin to destination in the ATS-SAM super-network represents a feasible DATP. Therefore, by using the ATS-SAM super-network, the proposed time-dependent traffic assignment model is equivalent to a static reliability-based user equilibrium (RUE) model.

The proposed RUE model falls into the category of static model in nature for long-term planning at the strategic level, thus it is postulated that all individuals would have a RUE activity-travel choice pattern: for each day, the budget utilities of all used

DATPs are largest and equal, and all unused DATPs have smaller budget utilities. Denote π as the reliable optimal route (i.e. the reliable optimal DATP) with the largest budget utility in the ATS-SAM super-network. The RUE condition can be formally expressed as

$$f_p(\varphi_\pi - \varphi_p) = 0, \tag{4.21}$$

$$q = \sum_{p \in P} f_p, \tag{4.22}$$

$$\varphi_\pi - \varphi_p \geq 0, \tag{4.23}$$

$$f_p \geq 0, \tag{4.24}$$

where f_p denotes the passenger flow on DATP p, and q denotes the total population in the study network.

The previously mentioned RUE problem can be further expressed as the following gap function formulation:

$$\min\ GAP = \sum_{p \in P^{od}} f_p(\varphi_\pi - \varphi_p). \tag{4.25}$$

The gap function refers to the overall gap capturing the complementary slackness conditions of the proposed RUE model.

The above RUE condition can also be formulated as a variational inequality (VI) problem:

Find $f_p^* \in \Omega$ such that

$$\sum_{p \in P^{od}} \varphi_p^*(f_p^* - f_p) \geq 0, \quad \forall f_p \in \Omega \tag{4.26}$$

where Ω denotes the set of feasible route flows, and f_p^* is the equilibrium route flow.

According to Facchinei and Pang (2003), it can be proved that at least one solution of the VI problem exists. In general, the uniqueness of the solution cannot be guaranteed in the RUE model, because the monotone property of VI problem cannot be guaranteed due to the complex form of DATP budget utility. However, in a special case, when the link utility/dis-utility is deterministic, the RUE model becomes a UE model. In this case, the solution of the RUE model is unique if the route set is given and fixed (Sheffi, 1985; Shao *et al*., 2006).

4.4 Solution algorithm

First, an effective route searching algorithm is developed in Section 4.4.1. The algorithm is capable of finding the optimal DATP (i.e. the optimal route in ATS-SAM super-network) with largest budget utility. The proposed route searching algorithm is then, further incorporated into a network equilibrium solution algorithm in Section 4.4.2 to search for a reliable optimal DATP at each iteration. Such network equilibrium algorithm can solve the RUE problem without the requirement of route enumeration as the route choice set is not pre-determined.

4.4.1 Solution algorithm for searching the reliable optimal DATP

Individuals schedule their activities and trips to maximize their DATP budget utility. This is equivalent to finding the route with maximum budget utility from origin to destination in the ATS-SAM super-network. Therefore, the DATP searching problem can be converted into a reliable shortest route problem by using the ATS-SAM super-network. The reliable optimal DATP searching problem can be formulated as a bi-criterion problem with respect to two independent decision variables, i.e. the mean of DATP's utility and the variance of DATP's utility. It is unlikely to find a single optimal pattern for each day because of the conflicting criteria in the bi-criterion shortest route problem, but a set of non-dominated routes can be obtained in the ATS-SAM super-network. The definition of non-dominated routes is that, it is not possible to find another route with a better value in one criterion without worsening the other criterion. The mean-variance (M-V) dominant condition can be defined as follows:

Definition 4.1 (M-V dominant condition) Given two routes $p_i \neq p_j \in P$, p_i M-V dominates p_j, if p_i and p_j satisfy either

(i) $u_{p_i} \geq u_{p_j}$ and $\sigma_{p_i} < \sigma_{p_j}$, or

(ii) $u_{p_i} > u_{p_j}$ and $\sigma_{p_i} \leq \sigma_{p_j}$.

As the number of non-dominated routes grows exponentially with the network size, it is computationally intractable to identify all non-dominated routes under the M-V dominant condition. Thus, in this Chapter, a stronger mean-budget (M-B) dominant condition is adopted to reduce the number of non-dominated routes by using the DATP mean utility and the DATP budget utility as two criteria.

Definition 4.2 (M-B dominant condition) Given a confidence level α and two routes $p_i \neq p_j \in P$, p_i M-B dominates p_j, if p_i and p_j satisfy $u_{p_i} \geq u_{p_j}$ and $\varphi_{p_i} > \varphi_{p_j}$.

In this Chapter, a label-selection label-correcting method (Guerriero and Musmanno, 2001; Chen *et al.*, 2011) is adopted in the development of an efficient solution algorithm for reliable optimal DATP generation in multi-modal transit networks with uncertainty in activity utility of different activity types.

Let P^{ou} be a set of non-dominated routes maintained at each node u from origin o, and non-dominated routes from origin to all nodes are maintained in a scan eligible set, denoted as SE. For each iteration, one non-dominated route p_i^{ou} is selected from SE in first-in-first-out (FIFO) order for route extension. A temporary route is constructed by extending the selected route p_i^{ou} to its successor link, denoted as p_i^{ov}. The dominant relationship between the newly generated route p_i^{ov} and the set of non-dominated routes P^{ov} at node v is determined, based on the M-B dominant condition (Definition 4.2). If p_i^{ov} is a non-dominated route at node v, it is then inserted into P^{ov} and SE. As the newly generated route p_i^{ov} may also dominate some routes in P^{ov}, these dominated routes should be eliminated from P^{ov} and SE. The proposed algorithm continues the route searching process until SE becomes empty. At the last step of this algorithm, the reliable optimal DATP can be determined by choosing the route with the maximum budget utility.

The detailed steps of the proposed DATP searching algorithm are listed as follows.

Inputs: an origin node o, a maximum number of allowable transfers κ, a probability of gaining DATP budget utility α.

Returns: the reliable optimal DATP in the ATS-SAM super-network.

Step 1. Initialization:

Create a route p_i^{oo} from o to itself, and set $u_{i^{oo}} = 0, \left(\sigma_{i^{oo}}\right)^2 = 0, t_{i^{oo}} = 0$. Add p_i^{oo} into label-vector P^{oo} and the list of candidate labels SE.

Step 2. Label selection:

Take label $p_i^{ou} \in P^{ou}$ at node u from SE in FIFO order. If $SE = \phi$, then go to Step 4; otherwise go to Step 3.

Step 3. Route extension:

For every outgoing link a of chosen node u (v denotes a successor node of node u). Generate a new label $p_i^{ov} \in P^{ov}$. Set $u_{p_i^{ov}} := u_{p_i^{ou}} + u_a$, $\left(\sigma_{p_i^{ov}}\right)^2 := \left(\sigma_{p_i^{ou}}\right)^2 + \left(\sigma_a\right)^2$, $\varphi_{p_i^{ov}} := u_{p_i^{ov}} - \Phi^{-1}(\alpha)\sigma_{p_i^{ov}}$. If p_i^{ov} is a non-dominated route under M-B dominant condition, insert p_i^{ov} into P^{ov} and SE, and remove all routes M-B dominated by p_i^{ov} from P^{ov} and SE.

Go back to Step 2.

Step 4. Determine the reliable optimal DATP. Stop.

4.4.2 Solution algorithm for solving the RUE problem

Most conventional solution algorithms cannot be used to solve the proposed RUE model, as it is difficult to determine the descent direction for solving the DATP scheduling problem in multi-modal transit networks. The widely used method of successive average (MSA) is a heuristic method with a forced convergence property. Thus, a solution algorithm based on MSA is proposed for solving the RUE problem (Fu *et al.*, 2014a). The DATP searching algorithm proposed in Section 4.4.1 is incorporated in this solution algorithm to search, at each iteration, for a reliable optimal DATP. The solution algorithm for solving the proposed RUE model is outlined as follows.

Step 0. Transform the traditional multi-modal transit network to the ATS-SAM super-network.

Step 1. Initialization. Let $n=0$. Call the DATP searching algorithm proposed in Section 4.4.1 to find the reliable optimal route in the ATS-SAM super-network (i.e. DATP) with the largest budget utility. Perform an all-or-nothing assignment.

Step 2. Update link dis-utilities.

Step 3. Call the DATP searching algorithm proposed in Section 4.4.1 to find the optimal route with the largest budget utility. Perform an all-or-nothing assignment and yield auxiliary link flows in the ATS-SAM super-network.

Step 4. Update the link flows using an MSA process.

Step 5. If the stopping criterion is satisfied, then stop. Otherwise let $n=n+1$ and go back to Step 2.

4.5 Numerical example

The purposes of the numerical example are to illustrate: (a) the application of the proposed model and solution algorithm; (b) how the uncertainty of activity utility affects individuals' DATP choices; (c) the effects of traffic congestion on individuals' activity choices; (d) individuals' mode choice behaviour under different population levels.

It is believed that various activity participations have different preferred times. Activity participation usually starts with a warming up phase in which the marginal activity utility increases. After reaching a maximum point, the marginal utility decreases. In this Chapter, the following marginal utility function proposed by Ettema and Timmermans (2003) is adopted.

$$\bar{u}_{a_a}(k)=\frac{\gamma_{a_a}\beta_{a_a}u_{a_a}^{\max}}{\exp[\beta_{a_a}(k-\alpha_{a_a})]\left\{1+\exp[-\beta_{a_a}(k-\alpha_{a_a})]\right\}^{\gamma_{a_a}+1}}, \tag{4.27}$$

where k is the time of day; $u_{a_a}^{\max}$ is the maximum accumulated utility of activity a_a, and α_{a_a}, β_{a_a}, γ_{a_a} are the activity-specific parameters to be estimated. These parameters can be estimated on the basis of survey data (Ettema and Timmermans, 2003; Ashiru *et al.*, 2004). Table 4.1 shows the given parameters in the marginal utility function for the numerical example in this Chapter.

Table 4.1 Given parameters in the marginal utility function

	Work in the morning	Work in the afternoon	Home	Shopping	Eating (dinner)
$u_{a_a}^{\max}$ (HK$)	720	600	1440	1080	600
α_{a_a}	600	900	680	1180	1080
β_{a_a}	0.021	0.021	0.0048	0.018	0.05
γ_{a_a}	0.8	1.2	1.8	1	1

The total study period was from 06:00 to 24:00 (18 hours per day) and was equally divided into 108 intervals (i.e. 10 minutes per interval). Figure 4.3 depicts a simple multi-modal transit network. One subway line and two bus lines served in the network. There are four nodes and seven physical links. The four nodes represent four study zones: home area (H), restaurant area (R), shop area (S), and work place (W). Four activities (i.e. home, dinner, shopping and work) can be conducted at the four nodes respectively.

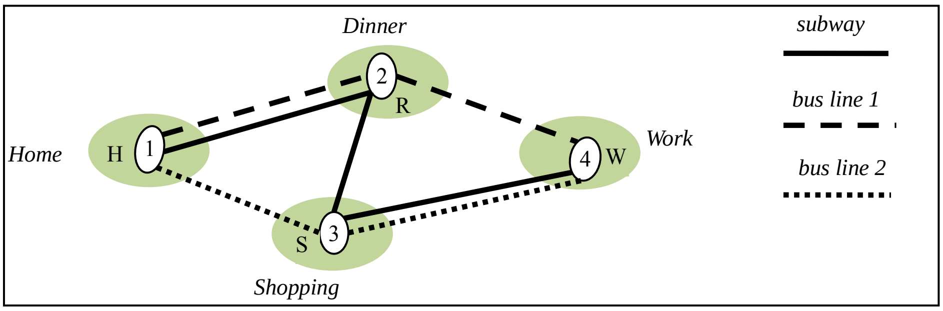


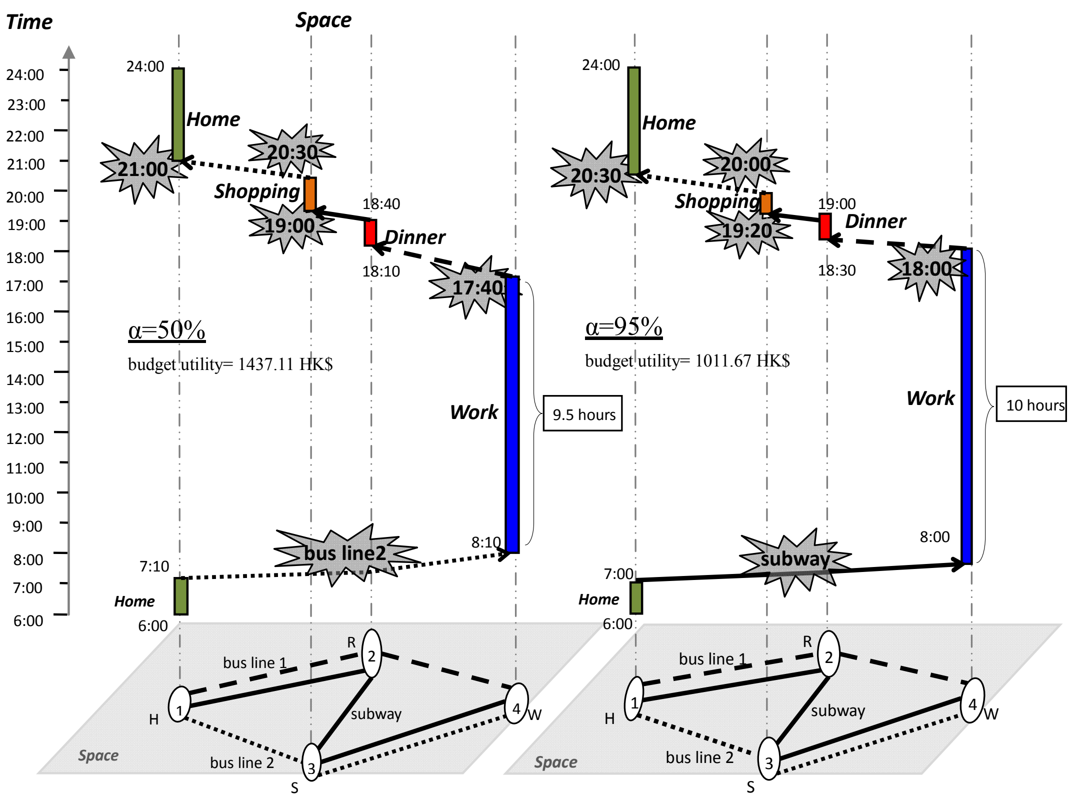
Figure 4.3 The multi-modal transit network

The value of time was HK$ 60.00/hour. The travel time of each bus link was 30 minutes and the travel time of each subway link was 20 minutes. The non-linear subway fare was set as: using one physical subway link costing HK$ 8.00, using two links costing HK$ 15.00, and three links costing HK$ 20.00. The bus fare for each bus line was HK$ 5.00 per physical link. Note that US$ 1.00 is approximately equal to HK$ 7.80. The cv_{a_a} for compulsory activity was 0.1, and for non-compulsory activity was 0.9.

Figure 4.4 shows reliable optimal DATPs generated by the proposed DATP searching algorithm under different expectations of daily utility gain (i.e. different probabilities of gaining DATP budget utility). The activity sequence of the DATPs in Figure 4.4 is home-work-dinner-shopping-home. Activity start/end time, activity duration, activity location can be traced. The travel time of each trip, route choices and mode choices can also be found. Figure 4.4(a) illustrates the reliable optimal DATP under $\alpha = 50\%$ and Figure 4.4(b) depicts the reliable optimal DATP when $\alpha = 95\%$. A comparison of these two results indicates that when individuals improve their expectations of daily utility gain (from $\alpha = 50\%$ to $\alpha = 95\%$), the DATP budget utility decreases (from HK$ 1437.11 to HK$ 1011.67). This is due to the budget utility being the summation of mean utility and a negative safety margin (as Equation (4.20) shows). A larger α results in a larger negative safety margin, so the DATP budget utility decreases with increase of α.

To ensure a high probability of gaining DATP budget utility, people tend to conduct compulsory activities. It can be seen in Figure 4.4 that when α increases from 50% to 95%, people extend the work activity duration for another half hour (from 9.5 hours to

10 hours). They also return home earlier (changing from 21:00 to 20:30). In contrast, shopping time is reduced by 50 minutes (when $\alpha = 50\%$, from 19:00 to 20:30; when $\alpha = 95\%$, from 19:20 to 20:00). For individuals' mode choice, it is noted from Figure 4.4 that individuals tend to use subways as α increases. For example, when individuals depart from home to work in the morning, they choose bus line 2 when $\alpha = 50\%$, but when α increases to 95% they choose the subway, because the subway is more reliable and has a smaller variation in dis-utility gain than that of the bus. If individuals have a high expectation of daily utility gain (i.e. a high probability of gaining DATP budget utility), they tend to use the subway rather than the bus in the multi-modal transit networks.



(a) **(b)**

Figure 4.4 Results of reliable optimal daily activity-travel patterns under different expectations of daily utility gain

To explore how traffic congestion affects individuals' activity choice behaviour, another example was explored with the use of the proposed DATP searching algorithm. Table 4.2 shows the example of reliable optimal DATPs under three scenarios with different link travel times when $\alpha = 95\%$. When link travel times for all links by mode in the network are halved, it was found that individuals leave home quite late (7:30) in the morning. They work for a long period (10 hours) to gain more utility, and perform non-compulsory activities after work (1.33 hours for dinner and 1 hour for shopping). However, if traffic congestions occur in the network and all link travel times are doubled, individuals have to leave home quite early (6:10) in the morning. To ensure that they can arrive home as early as possible, they do not perform non-compulsory activities (0 hour for dinner and shopping).

Table 4.2 Activity duration for scenarios with different link travel times

activity duration / link travel time	Departure time from home	Work duration	Dinner duration	Shopping duration	Arrival time at home
link travel time*0.5	**7:30**	**10h**	**1.33h**	**1h**	20:30
link travel time*1	7:00	10h	0.5h	0.67h	20:30
link travel time*2	**6:10**	9.5h	**0h**	**0h**	19:40

Individuals' mode choice behaviour can be investigated by the proposed RUE model. Figure 4.5 depicts the modal split for scenarios with different population levels in the study network. It can be seen in Figure 4.5 that with population increases, individuals tend to use the subway rather than bus. When the population is only 500, less congestion occurs on the network. The result is that the percentages of people choosing subway, bus line 1, and bus line 2 are almost equal. However, when the congestion increases (e.g. when population reaches 4000), the percentage of those travelling by subway (53.20%) is much higher than that of those travelling by bus line 1 (22.71%) and bus line 2 (24.09%). This is due to the fact that, in the study network, the subway has a larger capacity than the bus. People choose the subway for their travel to avoid bus travel in-vehicle crowding. In addition, as shown in Figure 4.4, the subway is more reliable and has a smaller variation in dis-utility gain than that of the bus. If individuals have a high expectation of daily utility gain, subways rather than buses are used in multi-modal transit networks.

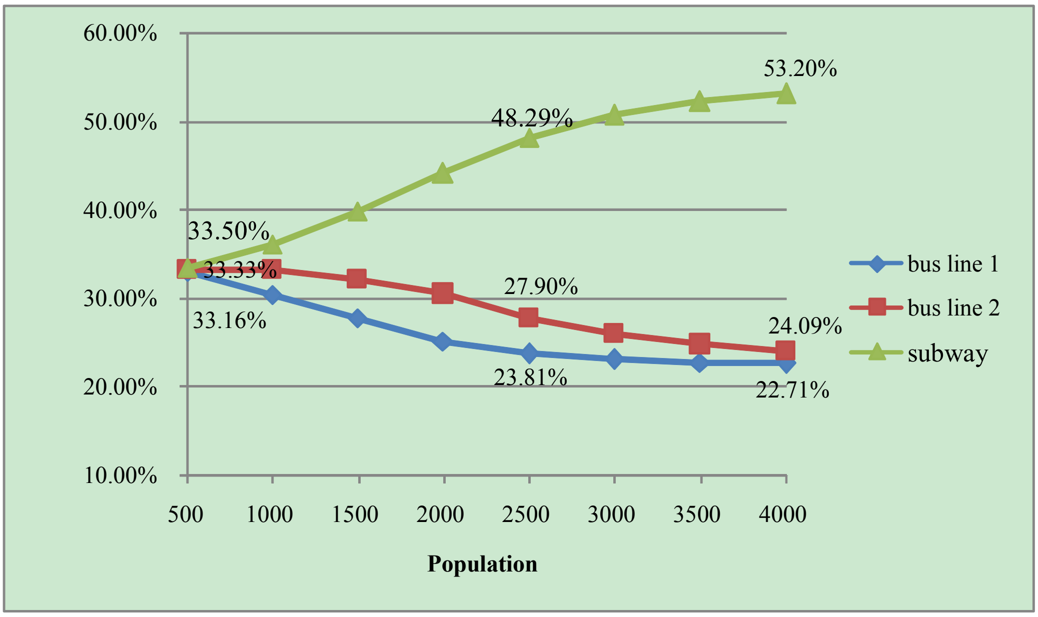


Figure 4.5 Modal split for scenarios with different population levels in the study network

It can be seen from Figure 4.5 that a significant difference between two bus lines exists when the population equals 2500, whereas there is no great difference when the population equals 500 or 4000. This is because in the study network, individuals' travel choices are influenced by travel dis-utilities of different transit lines. When the network is not congested (i.e. population equals 500), the dis-utilities of different transit lines are all quite small. Thus, the percentages of people choosing different lines are almost equal. As the population increases, the dis-utilities of the two bus lines start to diverge. Thus, in Figure 4.5, when the population equals 2500, there is a significant difference between two bus lines. However, when the network becomes extremely congested, compared to the large difference between the bus dis-utility and the subway dis-utility, there is no significant difference in choice preference between the two bus lines. Thus, it appears that individuals' preference regarding the two bus lines is small when population equals 4000.

Individuals' activity choice behaviour can also be investigated under different expectations of daily utility gain by the use of the proposed RUE model. Figure 4.6 shows the variation of average duration of compulsory and non-compulsory activities when the probability of gaining DATP budget utility (α) increases from 50% to 90% under different CV values. It is observed from Figure 4.6 that the average duration of compulsory activities increases with α, while the duration of non-compulsory activities decreases. For example, when the CV of compulsory activity utility equals 0.1 and the CV of non-compulsory equals 0.9, the duration of compulsory activities increases from 13.39 hour/individual to 13.68 hour/individual as α increase from 50% to 90%. The duration of non-compulsory activities, however, decreases from 2.12 hour/individual to 1.81 hour/individual, because non-compulsory activities bring larger variation in utility gain. In view of this, people tend to perform non-compulsory activities for a shorter period.

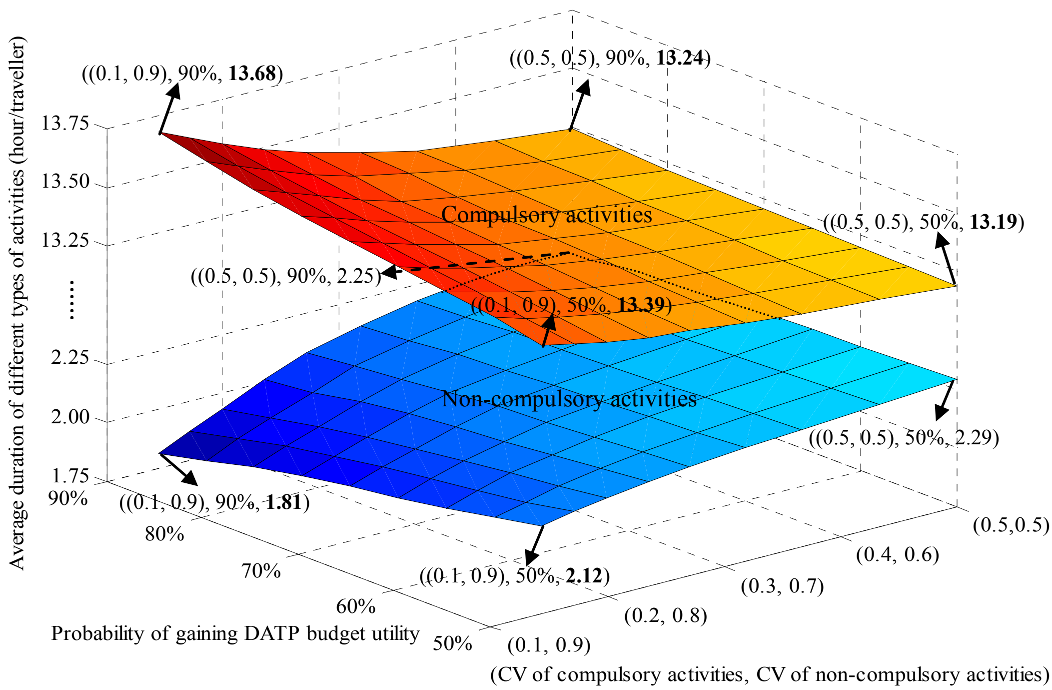


Figure 4.6 Average durations of compulsory and non-compulsory activities under different expectations of daily utility gain and different CV values

The difference between the CV of compulsory activity utility and the CV of non-compulsory activity utility also affects the durations of these two types of activities. When CVs of compulsory and non-compulsory activity utilities are the same (both equal to 0.5), it can be seen in Figure 4.6 that the activity duration variations are small (from 13.19 hour/individual to 13.24 hour/individual for compulsory activities as α increases). However, when the CV of compulsory activity utility is much smaller than the CV of non-compulsory activity utility, the activity durations change more significantly as α increases. A larger CV indicates a larger variation of activity utility. Facing the uncertainty of activity utility, people tend to conduct compulsory activities which have smaller utility variation to ensure a higher probability of gaining DATP budget utility.

4.6 Summary

This Chapter presents an activity-based network equilibrium model for scheduling DATPs in congested multi-modal transit networks under uncertainty. A new super-network platform (i.e. the ATS-SAM super-network) is proposed to explicitly model the transfers and non-linear fare structures in multi-modal transit networks, and also address the activity choices and travel choices simultaneously. It is shown that the DATP scheduling problem can be converted into a static traffic assignment problem on the proposed ATS-SAM super-network.

In this Chapter, the crowding discomfort in transit vehicles is modelled for long-term strategic planning. The uncertainty of activity utility and the resultant uncertainty of travel dis-utility are explicitly considered. A concept of DATP budget utility is used to model the uncertainties of activity utility and travel dis-utility. The effects of these uncertainties on passengers' DATPs can be assessed with the use of the proposed RUE model. An efficient solution algorithm without prior enumeration of DATPs is developed for solving the DATP scheduling problem. The numerical results show that the proposed RUE model can be used to investigate the passengers' DATPs in congested multi-modal transit networks. People's attitudes towards compulsory and non-compulsory activities vary and would affect their DATP choices under different expectations on daily utility gain. The results indicate that with a high expectation on daily utility gain, individuals tend to use subway and tend to conduct compulsory activities for a longer period.

This Chapter extends existing theories by developing a comprehensive super-network framework which incorporates the congestion effect in transit vehicles, stochastic effects of activity utility, flexible activity sequence and duration, and the route and mode choices. Further study is required for model calibration and validation with empirical data in a case study with realistic size of network in practice.

An extension of this activity-based model is proposed and described in next Chapter to consider adverse weather conditions. A network equilibrium model for DATP scheduling under adverse weather is presented in Chapter 5.

5 A Network Equilibrium Model for DATP Scheduling under Adverse Weather

In general, adverse weather has significant influence on individuals' activity/travel choice behaviour and such influence is obviously greater in cities which suffer frequent rainy periods. Thus, the impacts of weather conditions should be taken into account in long-term transit service planning. In this Chapter, an activity-based network equilibrium model is developed for scheduling daily activity-travel patterns (DATPs) under adverse weather conditions (with different rainfall intensities). The interdependency of individuals' activity/travel choices and weather conditions are comprehensively investigated in congested multi-modal transit networks.

In the proposed activity-based network equilibrium model, the DATP choice problem under adverse weather conditions is converted into an equivalent static transit assignment problem by using the ATS-SAM super-network presented in Chapter 4. As vehicle capacity and frequency of different transit modes are influenced by adverse weather conditions, in-vehicle crowding discomfort taking account of adverse weather impacts is specifically considered in the proposed model. The effects of adverse weather on different transit modes and different activities are also explicitly modelled.

The outline of this Chapter is as follows. Assumptions and notations are firstly given in Section 5.1. The problem statement is elaborated in Section 5.2. In Section 5.3, a DATP choice network equilibrium model is formulated as a variational inequality (VI) over the super-network platform, and an efficient solution algorithm is also given.

Section 5.4 gives numerical examples illustrating the proposed model and algorithm. Finally, a summary of this Chapter is outlined in Section 5.5.

5.1 Background

As discussed in Chapter 4, several network equilibrium models, which provide valuable insights into understanding individuals' activity-travel scheduling behaviour, have been proposed for long-term transport planning over the past decades (Lam and Yin, 2001; Lam and Huang, 2002, 2003; Huang and Lam, 2005; Zhang *et al*., 2005; Li *et al*., 2010; Ramadurai and Ukkusuri, 2010; Ouyang *et al*., 2011). None of these models, however, has incorporated the weather/climate effects on activity-travel pattern scheduling explicitly.

A number of empirical studies have investigated the recurrent effects of adverse weather on individuals' activity choice and travel behaviour. Some studies have reported travellers' mode and departure time changes as affected by weather conditions (Khattak and De Palma, 1997; Guo *et al*., 2007), and some have indicated activity behaviour changes (Smith, 1993; Khattak and De Palma, 1997; Cools *et al*., 2010). Rainfalls have the most frequent and significant adverse weather effects on individuals' activity and travel choices in tropical and subtropical areas such as Hong Kong and Singapore. Based on data from the World Weather Information Services (http://www.worldweather.org/), Hong Kong has the highest average annual rainfall (2383 mm) of all the major Pacific Rim Cities, with Singapore achieving the second highest (2150 mm). The average annual number of rainy days in Hong Kong is as high as 104. Rainfall significantly affects individuals' activity and travel choices such as activity duration and travel mode choice. The long-term transit planning for areas with high average annual rainfall is considerably different from the planning for areas with less rainfall. Thus, clearly, particularly in areas such as Hong Kong and Singapore, rain effects should be considered when modelling individuals' activity and travel choices.

In order to incorporate rain effects in travel behaviour modelling, Lam *et al*. (2008) proposed a network equilibrium model for road networks with specific consideration of rain effects on road capacity and link travel time, and Sumalee *et al*. (2011) extended this work to model multi-modal transport networks under adverse weather conditions. The above two models are both trip-based transport models, so the trip making motivation, and the interdependency of activities and trips are not considered. Cools *et al*. (2010) found that individuals' travel choice behaviour under adverse weather conditions is highly dependent on trip purpose (i.e. activities). It is, thus, of serious interest to comprehensively model and investigate individuals' activity and travel choice behaviour under adverse weather conditions.

In many Asian cities such as Hong Kong and Singapore, most daily travel is made using various public transit modes (over 90% and over 55%, respectively). Hence, as a pioneering endeavour, a network equilibrium model for scheduling DATPs under adverse weather conditions (with different rainfall intensities) in congested multi-modal transit networks is proposed and described in this Chapter. By using the ATS-SAM super-network presented in Chapter 4, the DATP choice problem is converted into an equivalent static transit assignment problem. The time and space coordination, activity location, activity sequence and duration, and the relationship between activity and route/mode choices, can be simultaneously investigated by solving the user equilibrium (UE) problem on the super-network platform. The study presented in this Chapter extends existing theories by developing a comprehensive framework which incorporates flexible activity sequence and duration, route and mode choices, together with effects of adverse weather conditions.

5.2 Problem statement

5.2.1 Model assumptions and network representation

In order to facilitate the essential ideas without loss of generality, besides adopting the assumptions A4.1 - 4.5 made in Chapter 4, the following assumptions are made in this Chapter.

A5.1: Individuals can acquire weather forecast information for each time interval over the whole day (Lam *et al*., 2008; Sumalee *et al*., 2011).

A5.2: The subway is weather-proof. Bus frequency and capacity are assumed to vary with weather condition (Sumalee *et al*., 2011).

A5.3: The weather conditions for all zones in the study area are identical (Lam *et al*., 2008; Sumalee *et al*., 2011).

A5.4: The in-vehicle travel time for the bus mode is given exogenously by a scaled function dependent on rainfall intensity. The in-vehicle travel times by modes in the road network under different rainfall intensities, however, can be modelled explicitly by the activity-based traffic assignment model proposed by Ouyang *et al*. (2011).

A5.5: In reality, weather conditions dynamically change. Individual's activity and travel choices are likewise dynamic. In this Chapter, it is assumed that the dynamic situations are not considered in the proposed static model. Individuals are assumed not to be re-optimizing their activities and travels during the study time period (Lam *et al*., 2008). This assumption is made because the proposed model falls within the static model category for long-term planning at the strategic level. Individuals have perfect knowledge of traffic and weather conditions throughout the whole network and the whole time period concerned.

In accordance with the model presented in Chapter 4, four types of activities are investigated in this Chapter; namely, home, work, dinner, and shopping activities. The time period of the proposed model is from 06:00 to 24:00 and is divided into 108 intervals each with ten minutes. The activity sequence and durations are not fixed (Ouyang *et al*., 2011; Fu and Lam, 2014). Home and work are considered as compulsory activities, while dinner and shopping are non-compulsory activities (Fu and Lam, 2014).

The super-network platform proposed in Chapter 4, i.e. the ATS-SAM super-network, is adopted in this Chapter to simultaneously consider time and space coordination, activity sequence and duration, and the relationship between activity and route/mode choices. With this rule-based algorithm proposed in Chapter 4, conventional multi-modal transit networks can easily be automatically transformed into ATS-SAM super-networks. It should be noted that, in this Chapter, in-vehicle links are constructed based on the weather conditions during each time interval, because different weather conditions result in different in-vehicle travel times. Figure 5.1 is an example of the ATS-SAM super-network considering weather conditions consisting of two transit modes, i.e. subway, bus. Three activities (i.e. home, work, and dinner) are considered in this example. In this small example, the study time horizon is divided into three equally spaced time intervals. Two weather categories (i.e. rain and no-rain) are considered. Travel time for each link under no-rain conditions is one interval. Travel time for bus link under rain condition is two intervals. Weather forecast indicates the rain starts from the second time interval. The probable transfer states in Figure 5.1 follow that used by Lo *et al*. (2003). It can be seen from Figure 5.1 that different links are constructed according to the forecast of weather conditions by time of day. Individuals' activity and travel choices under different weather conditions can be explicitly depicted by the ATS-SAM super-network.

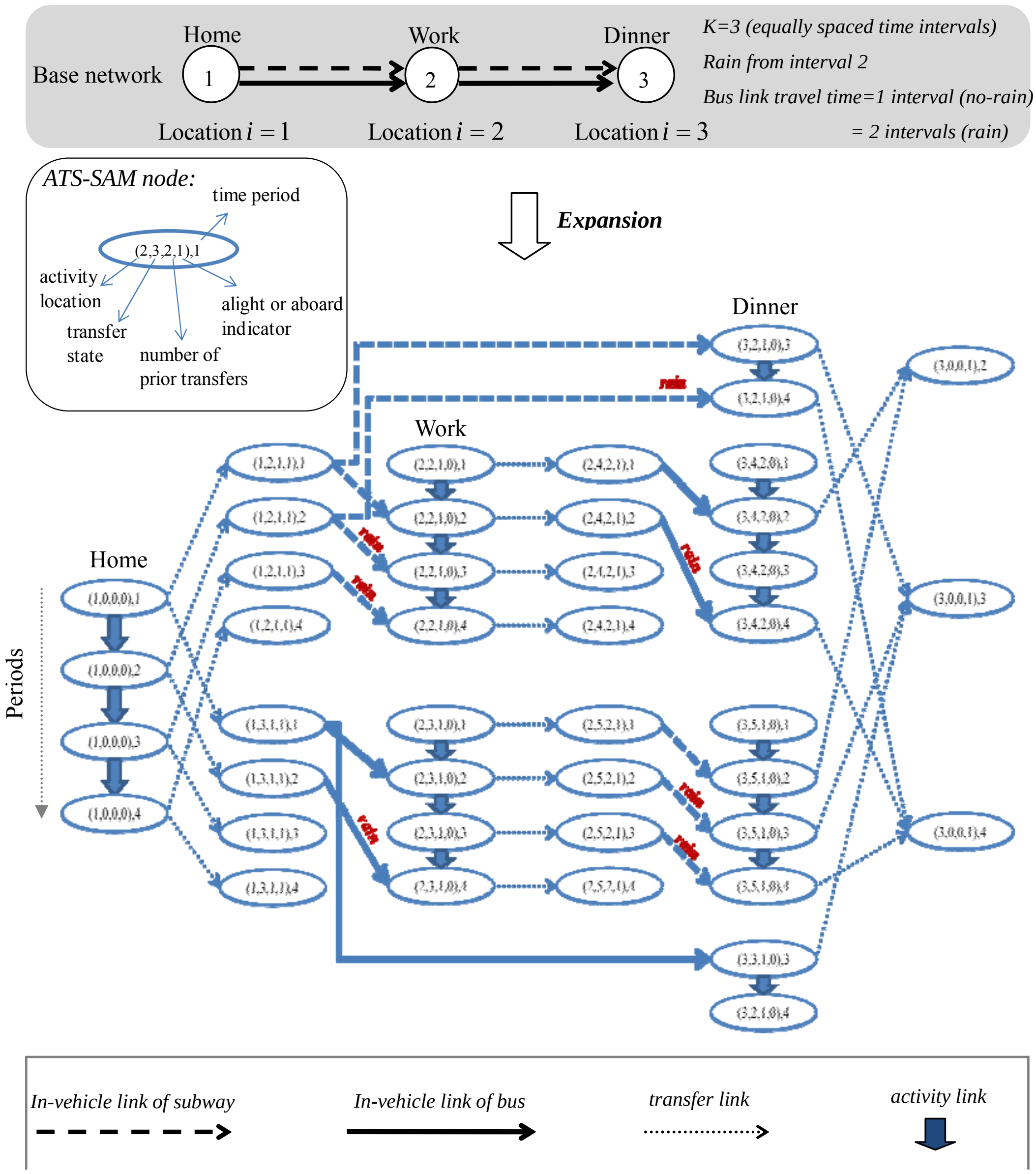


Figure 5.1 An example of the ATS-SAM super-network considering weather conditions

5.2.2 Effects of weather forecast information

Five weather categories (denoted as wc) with different average rainfall intensity levels for each time period (denoted as π_{wc}) are adopted (Lam *et al*., 2008). With a 10-min interval in the study period, wc_1 indicates no rain or light rain with average rainfall intensity $\pi_{wc1} = 5mm/h = 0.3mm/interval$; wc_2 indicates normal rain with $\pi_{wc2} = 20mm/h = 3.3mm/interval$; wc_3 indicates amber rainstorm with $\pi_{wc3} = 30mm/h = 5mm/interval$; wc_4 indicates red rainstorm with $\pi_{wc4} = 50mm/h = 8.3mm/interval$; wc_5 indicates black rainstorm with $\pi_{wc5} = 70mm/h = 11.7mm/interval$. It is assumed that weather forecast provides the chance of each weather category for each time interval in the study period. Each possible weather category is forecast with the probability of its occurrence $\breve{p}_{wc}(k)$. $\breve{p}_{wc}(k)$ is the prior probability of the weather category wc for time k to time $k+1$. For example, with a 10-min interval in the study period, $\breve{p}_{wc_4}(9:00) = 40\%$ means that, on the basis of weather forecast, there is a 40% chance of a red rainstorm in the period 9:00-9:10.

However, the weather forecast may not be accurate. Thus, based on past experiences, individuals may perceive a posterior probability for each weather category. To ensure a more precise investigation of weather effects, the approach proposed by Lam *et al*. (2008) is adopted in this Chapter. Bayes' Theorem is used to combine prior weather forecast accuracy beliefs and the current weather forecast information. Let $\hat{p}_{wc/\breve{p}}$ be the conditional probability of $\breve{p}(k)$ (a vector of $\breve{p}_{wc}(k)$) given weather category wc occurs. The posterior probability of occurrence of wc given the weather forecast $\breve{p}(k)$ for time k to $k+1$ (denoted as $p'_{wc}(k)$) can then, be obtained on the basis of

Bayes' Theorem (refer to Equation (32) in Lam *et al*., 2008).

5.2.3 Link utility/dis-utility in ATS-SAM super-network under adverse weather conditions

Many empirical studies reveal that adverse weather conditions have significant impacts on individuals' travel and activity decisions (Khattak and De Palma, 1997; Cools *et al*., 2010). Under adverse weather conditions, individuals have less desire to take part in out-door activities or non-compulsory activities such as eating at restaurants and shopping in malls (Cools *et al*., 2010). Activity utility may broadly consist of the following attributes: (a) an activity time window; (b) a degree of need for the activity; (c) a degree of satisfaction from the process; and (d) money gain or loss. Thus, it can be said that the utilities of some activities are in fact influenced by weather conditions due to the variation of need and satisfaction. The higher the rain fall intensity the lower the activity utility.

The previously proposed activity utility functions were mainly concerned with activity participation time and activity type. These functions may not be applicable directly in the case of adverse weather with various rainfall intensities. To capture the rain effects on individuals' activity choices, a modified activity utility function is proposed. The utility of performing activity link a_{a} from start time k for one interval under weather category wc is expressed by

$$u_{a_{\mathrm{a}}}(wc) = s_{u_{a_{\mathrm{a}}}}(wc) \cdot \int_{k}^{k+1} \overline{u}_{a_{\mathrm{a}}}(\omega) d\omega, \tag{5.1}$$

where $\overline{u}_{a_{\mathrm{a}}}(k)$ denotes the marginal utility of performing activity link a_{a}; k is the start time of activity link a_{a}; $s_{u_{a_{\mathrm{a}}}}(wc)$ is the scale function of activity utility under weather category wc . $s_{u_{a_a}}(wc) \leq 1$ is a decreasing function with respect to wc , implying that activity utility decreases with the rainfall intensity. In this Chapter, two activity types (i.e. compulsory/obligatory and non-compulsory/discretionary in nature) are considered. For compulsory activities such as home and work, $s_{u_{a_a}}(wc)$ equals 1 for all wc , and for non-compulsory activities, $s_{u_{a_a}}(wc)$ is less than 1. It can be seen that the higher the rainfall intensity the lower the utility of non-compulsory activity, and that the utility of compulsory activity is not influenced by weather conditions. Thus, under adverse weather conditions individuals may reduce or cancel their non-compulsory activities. This property is in accordance with the contentions expressed in the empirical study by Cools *et al*. (2010).

The resultant activity utility for time k to $k+1$ from different possible weather categories (denoted as u_{a_a}) can be expressed as

$$u_{a_a} = \sum_{wc_1}^{wc_5} p'_{wc}(k) u_{a_a}(wc), \tag{5.2}$$

where $p'_{wc}(k)$ denotes the posterior probability of weather category wc from time k to $k+1$. u_{a_a} is the mixture activity utility considering all weather categories.

In this Chapter, the activity marginal utility function used in Chapter 4 is adopted. Activity utility in this Chapter is a function of activity time and activity duration regardless of the passenger flow at activity location. In the next Chapter, the crowding effects at activity locations will be considered by incorporating passenger flow into the activity utility function.

To capture the rain effects, the physical link travel time at time interval k under weather category wc (denoted as $t_v(k, wc)$) can be expressed as

$$t_v(k, wc) = t_v^0 \cdot s_{t_v}(wc), \tag{5.3}$$

where t_v^0 is the travel time of physical link v under no-rain weather condition, and $s_{t_v}(wc) \geq 1$ is the scale function of physical link travel time under weather category wc (Lam *et al*., 2008). $s_{t_v}(wc)$ represents the effects of adverse weather conditions on physical link travel times. For the subway mode, $s_{t_v}(wc)$ equals 1.

The physical link dis-utility is modelled with consideration of rainfall intensity. For congested physical links, the dis-utility of physical link v for mode b at time interval k under weather category wc (denoted as $disu_v(k, wc)$) is expressed to represent in-vehicle crowding discomfort (Sumalee *et al*., 2011):

$$disu_v(k, wc) = -\text{vot} \cdot t_v(k, wc) \cdot \left(1 + \beta_b \left(\frac{f_v(k)}{h_b(wc) \cdot g_b(wc)}\right)^{\theta_b}\right), \tag{5.4}$$

where $h_b(wc)$ is the vehicle capacity of mode b under weather category wc. As regards the bus mode, $h_b(wc)$ decreases with wc because umbrellas occupy a degree of space. $g_b(wc)$ denotes the frequency of mode b under weather category wc. As regards the bus mode, $g_b(wc)$ decreases with wc due to the increased road travel time (Sumalee *et al*., 2011). $f_v(k)$ denotes the passenger flow on the physical link at time interval k; vot is the value of time; β_b and θ_b are model parameters relevant to mode b; wc is the weather category at time interval k.

The resultant dis-utility of physical link v at time interval k from different possible weather categories (denoted as $disu_v(k)$) can be expressed as

$$disu_v(k) = \sum_{wc_1}^{wc_5} p'_{wc}(k) disu_v(k, wc). \tag{5.5}$$

The dis-utility of in-vehicle link a_d can be obtained by the summation of related physical link dis-utilities and transit fare:

$$disu_{a_\mathrm{d}} = \sum_{v \in V_b} disu_v(k) \delta\left(a_\mathrm{d}, v\right) - \rho_{a_\mathrm{d}}, \tag{5.6}$$

where ρ_{a_d} is the transit fare with respect to the direct in-vehicle link a_d.

As regards transfer links by mode, the transfer link dis-utility under weather category wc can be expressed as

$$disu_{a_\mathrm{t}}(wc) = -\mathrm{vot} \cdot \frac{1}{2 g_b(wc)} - pen_b, \tag{5.7}$$

where $g_b(wc)$ is the frequency of the mode to which individuals transfer on the transfer link concerned under weather category wc, and pen_b is the mode-specified transfer penalty. The resultant dis-utility of transfer link at time interval k from different possible weather categories (denoted as $disu_{a_\mathrm{t}}$) can be expressed as

$$disu_{a_\mathrm{t}} = \sum_{wc_1}^{wc_5} p'_{wc}(k) \cdot disu_{a_\mathrm{t}}(wc). \tag{5.8}$$

Let P be the set of routes in the ATS-SAM super-network (i.e. DATP set). The daily utility gain, i.e. the utility of DATP $p \in P$ (denoted as u_p), can be obtained by summing dis-utilities of in-vehicle links, dis-utilities of transfer links, and utilities of activity links:

$$u_p = \sum_{a_\mathrm{d} \in A_\mathrm{d}} disu_{a_\mathrm{d}} \cdot \delta(p, a_\mathrm{d}) + \sum_{a_\mathrm{t} \in A_\mathrm{t}} disu_{a_\mathrm{t}} \cdot \delta(p, a_\mathrm{t}) + \sum_{a_\mathrm{a} \in A_\mathrm{a}} u_{a_\mathrm{a}} \cdot \delta(p, a_\mathrm{a}), \tag{5.9}$$

where $\delta(p,a)$ is the incidence relationship between DATP and link; $\delta(p,a)$ equals 1 indicating that this link is used in the DATP, 0 otherwise.

5.3 Model formulation and solution algorithm

With the use of the proposed ATS-SAM super-network, individuals' activity choices (i.e. activity locations, sequence and durations) and travel choices (i.e. route, mode, transfers, and departure time) under different weather conditions are explicitly represented by different links in the proposed super-network platform. Activities with different start times are constructed as different activity links. The time-dependent relationships between activity and travel choices can be modelled by the ATS-SAM super-network topology. Each route from origin to destination in the ATS-SAM super-network represents a feasible DATP. Therefore, the proposed time-dependent DATP scheduling problem is equivalent to a static multi-modal transit assignment model on the ATS-SAM super-network.

The proposed model falls into the category of static UE model in nature for long-term transit planning at the strategic level. As we postulate, individuals would select DATPs to maximize the daily utility and settle into a long-term equilibrium. Thus, although the activity utility may be specific to each individual in reality, it is postulated in this Chapter that all individuals would have a UE activity-travel choice pattern: for each day, the utilities of all used DATPs are the largest and equal, and all unused DATPs have smaller utilities. Denote π as the optimal route (i.e. the optimal DATP) with the largest utility in the ATS-SAM super-network. u_π denotes the utility of route π. The UE condition can be formally expressed as

$$f_p(u_\pi - u_p) = 0, \tag{5.10}$$

$$q = \sum_{p \in P} f_p, \tag{5.11}$$

$$u_\pi - u_p \geq 0, \tag{5.12}$$

$$f_p \geq 0, \tag{5.13}$$

where f_p denotes the passenger flow on DATP p and q denotes the total population in the study network.

The above UE condition can be formulated as a VI: Find $f_p^* \in \Omega$ such that

$$\sum_{p \in P} u_p^* (f_p^* - f_p) \geq 0, \quad \forall f_p \in \Omega \tag{5.14}$$

where Ω denotes the set of feasible DATP flow solutions; f_p^* and u_p^* denote the equilibrium DATP flow and equilibrium DATP utility, respectively. In this Chapter, the DATP utility is continuous and strictly monotone with respect to the DATP flow, and the feasible set Ω is compact and convex. Facchinei and Pang (2003) indicate that it can be proved that the solution of this VI problem exists and the uniqueness of the solution can be guaranteed. In this Chapter, the VI problem is solved by the widely used method of successive average (MSA).

The solution algorithm for solving the DATP scheduling problem is outlined as follows.

Step 0. Calculate in-vehicle link travel time on the basis of the weather condition for each time interval. Transform the traditional multi-modal transit network to the ATS-SAM super-network by using the rule-based super-network expansion algorithm.

Step 1. Initialization. Let $n = 0$. Call the shortest path faster algorithm (SPFA) (Duan, 1994) to find the optimal route in the ATS-SAM super-network (i.e. DATP) with the largest utility. Perform an all-or-nothing assignment. Obtain the route and link flows $\mathbf{f}^n$ in the ATS-SAM super-network.

Step 2. Update in-vehicle link dis-utilities.

Step 3. Call the SPFA algorithm to find the optimal route with the largest utility. Perform an all-or-nothing assignment and yield auxiliary link flows in the ATS-SAM super-network.

Step 4. Obtain updated link flows $\mathbf{f}^{n+1}$ using an MSA process.

Step 5. For an acceptable convergence level τ, if $\max_a \left|\mathbf{f}^{n+1} - \mathbf{f}^n\right| \leq \tau$, then stop. Otherwise let $n = n+1$ and go back to Step 2.

5.4 Numerical example

The purposes of the numerical example are to illustrate: (a) application of the proposed model and solution algorithm; (b) how the adverse weather affects individuals' activity choices; (c) individuals' mode choice behaviour under various weather conditions; (d) the effects of adverse weather on individuals' departure time choices; (e) the impacts of adverse weather on the overall performance of the multi-modal transit networks such as the daily average travel time per individual.

In this numerical example, the total study period was from 06:00 to 24:00 (18 h per day) and was equally divided into 108 intervals (i.e. 10 min per interval). The weather forecast information for each time interval in the study period was given.

Figure 5.2 depicts a multi-modal transit network based on a study area in Singapore with various bus and subway lines. Two subway (i.e. MRT in Singapore) lines and three bus lines serve this study network. Four activities (i.e. home, work, shopping and dinner) are considered. The three circles shown in Figure 5.2 represent three study zones located in Singapore: (1) home area (H), (2) work area (W), and (3) shopping/dinner area (S&D). The home area, Clementi, is a major residential area. The work area, Tanjong Pagar, is one zone of the Central Business District. The shopping/dinner area, Harbour Front, is a recreational area with a large shopping mall. In this example, after deleting the nodes which are not two-way connected (except for origin and destination), the numbers of nodes and links in the super-network are 9811 and 25,441 respectively.

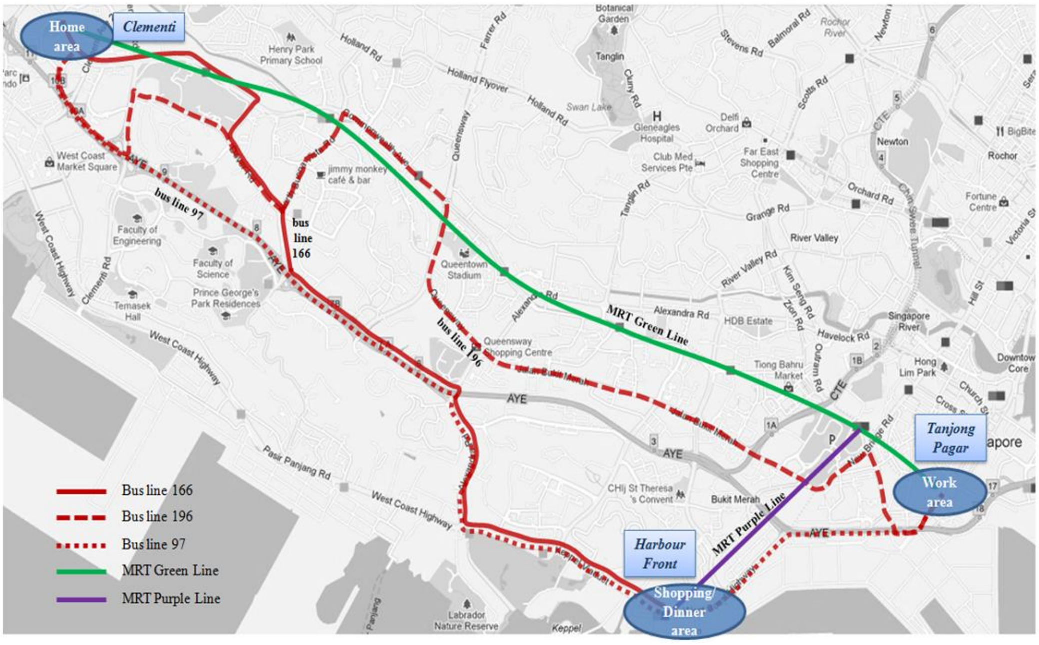


Figure 5.2 The multi-modal transit network in study area

The data relating to transit lines were obtained from the website of the Land Transport Authority of Singapore. Table 5.1 shows the given parameters in the marginal utility function for the numerical examples. The scale function for the utility of non-compulsory activities is set as $s_{u_{a_a}}(wc)=\exp(-0.6\cdot\pi_{wc})$. The scale function for bus travel time is set as $s_{t_v}(wc)=\exp(0.12\cdot\pi_{wc})$ (Lam *et al.*, 2008; Sumalee *et al.*, 2011). The vot is S\$ 60.00/h. Note that US\$ 1.00 is approximately equal to S\$1.30.

$\beta_b = 0.1$, $\theta_b = 2$, $pen_b = 0.5$, $\tau = 0.1$.

Table 5.1 Given parameters in the marginal utility function

	Work (6:00-12:00)	Work (12:00-24:00)	Home (6:00-12:00)	Home (12:00-24:00)	Shopping	Dinner
$u_{a_a}^{\max}$ (S\$)	1440	1440	1000	1000	1080	1440
α_{a_a}	600	900	360	1440	1180	1080
β_{a_a}	0.021	0.021	0.0048	0.0048	0.018	0.05
γ_{a_a}	0.8	0.8	1.8	1.8	1	1

The traffic assignment model proposed in this Chapter falls within the category of static UE model for strategic policy planning. Several weather forecast scenarios are obtained based on samples of multiple days' weather forecasts. The proposed model will be solved for each scenario to determine average effects of weather on the study area for long-term planning purpose. The five weather category scenarios used by Sumalee *et al*. (2011), shown in Figure 5.3, were adopted in this Chapter. According to this figure, it is found that from scenario S1 to S5, the weather conditions become increasingly adverse. This being the case, S1 represents good weather, and S5 represents severe weather in the following discussion. Note that in the numerical example, the five scenarios are applied at the morning peak (assumed 7 a.m. to 9 a.m.) and evening peak (assumed 5 p.m. to 8 p.m.) periods in the travel and activity choice investigation, as the weather conditions during the other time periods (i.e. work and home time) have little impact on individuals' travel and activity choices. For the other time periods, a good weather (i.e. S1) is applied. Under scenario S1, bus frequency and capacity are 9 veh/h and 110 passenger/veh. Under scenarios S2-S5, bus frequency and capacity are assumed to reduce to 6 veh/h and 100 passenger/veh, respectively (Sumalee *et al*., 2011). Subway frequency and capacity for all scenarios are 12 veh/h and 400 passenger/veh.

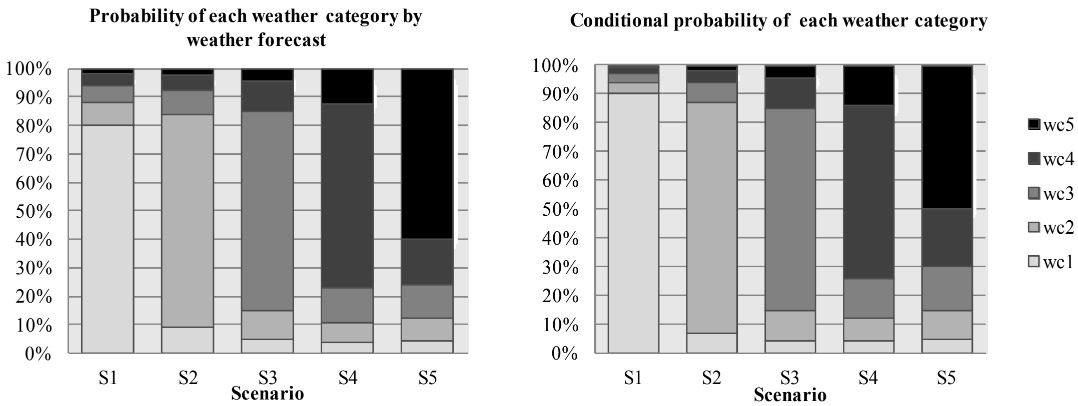
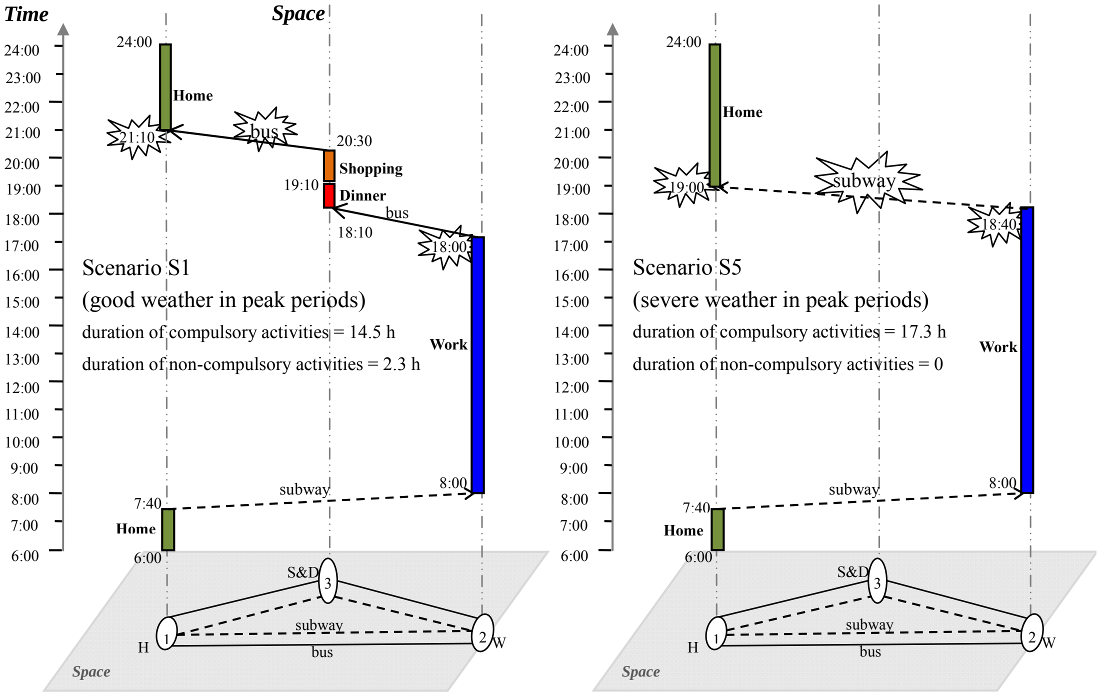


Figure 5.3 Scenarios for different weather forecast information

Figure 5.4 shows two optimal DATPs under two weather scenarios (applying S1 and S5 in peak periods). The two DATPs are route searching results under free-flow condition. It can be seen that using the proposed super-network, activity choice, activity start/end time, activity duration, and activity location can be traced. Travel time of each trip, route choice and mode choice can also be found. Figure 5.4(a) illustrates the DATP under scenario S1 (i.e. good weather) and Figure 5.4(b) depicts the DATP under scenario S5 (i.e. severe weather). A comparison of these two DATPs indicates that under adverse weather conditions, individuals tend to carry out their compulsory activities and use the subway. It can be seen from Figure 5.4 that as the rainfall intensity increases, the duration of compulsory activities (i.e. home and work) is extended by about 3 h (from 14.5 to 17.3). In contrast, non-compulsory activities (i.e. shopping and dinner) are cancelled. Individuals leave work later (changing from 18:00 to 18:40), and return home earlier (changing from 21:10 to 19:00) under severe weather conditions so as to obtain maximum daily utility.



(a) **(b)**

Figure 5.4 Results of daily activity-travel patterns under different weather scenarios

Individuals' overall activity choice behaviour can be effectively investigated under scenarios for different weather conditions by using the proposed model. Table 5.2 shows average duration variation of different activities under different weather scenarios (applying S1-S5 in peak periods). It is clear from Table 5.2 that the average duration of compulsory activities (i.e. home and work) increases with rainfall intensity, while the duration of non-compulsory activities (i.e. dinner and shopping) decreases. As the rainfall intensity increases, the average home activity and work activity durations show respective increases from 4.82 h/individual to 6.20 h/individual and from 10.03 h/individual to 10.65 h/individual. In contrast, the dinner duration and shopping duration, show respective decreases from 0.95 h/individual to 0.10 h/individual and from 1.23 h/individual to 0.24 h/individual. It is due to that adverse weather significantly affects the utility of non-compulsory activities, while compulsory-activities have to be performed regardless of weather condition. It can also be seen from the study network that as most people tend to cancel non-compulsory activities under severe weather conditions, the daily average travel time decreases from 0.97 h/individual under S1 to 0.81 h/individual under S5.

Table 5.2 Average durations of activities and travel under different weather scenarios

		Scenario				
		S1 (good weather)	**S2**	**S3**	**S4**	**S5** (severe weather)
Activity	**Home**	**4.82 h (26.8%)**	5.54 h (30.8%)	5.93 h (32.9%)	6.09 h (33.8%)	**6.20 h (34.4%)**
	Work	**10.03 h (55.7%)**	10.54 h (58.5%)	10.65 h (59.2%)	10.65 h (59.2%)	**10.65 h (59.2%)**
	Dinner	**0.95 h (5.3%)**	0.40 h (2.2%)	0.19 h (1.1%)	0.12 h (0.7%)	**0.10 h (0.6%)**
	Shopping	**1.23 h (6.8%)**	0.64 h (3.6%)	0.40 h (2.2%)	0.32 h (1.8%)	**0.24 h (1.3%)**
Travel		**0.97 h (5.4%)**	0.88 h (4.9%)	0.83 h (4.6%)	0.82 h (4.5%)	**0.81 h (4.5%)**
Total time		18 h (100%)	18 h (100%)	18 h (100%)	18 h (100%)	18 h (100%)

Under adverse weather conditions, individuals who choose the bus mode for travel may change their departure time in the morning to accommodate the increased road travel time. Table 5.3 shows the average departure time and average travel time per trip for bus riders. It is seen that under severe weather conditions, earlier morning departure times are chosen by bus riders, and the average bus trip travel time increases. For instance, under scenario S1 (i.e. good weather), bus riders depart to work at 8:37 a.m., and the average per-trip travel time over the whole day is 26.3 min. However, under scenario S5 (i.e. severe weather), they should depart quite early, i.e. 6:30 a.m. in the morning, and that the average per-trip travel time increases to 33.0 min. This is due to the influence of adverse weather conditions on bus frequency, bus capacity, and road travel time.

Table 5.3 Average departure time and travel time per trip for bus riders

Scenario	Average departure time in the morning	Average travel time per trip (minutes)
S1	**8:37 a.m.**	**26.3**
S2	7:18 a.m.	26.8
S3	6:43 a.m.	29.8
S4	6:30 a.m.	32.2
S5	**6:30 a.m.**	**33.0**

Individuals' travel mode choice behaviour can also be examined by the proposed model. Figure 5.5 depicts the variation of modal split with different population levels under different weather scenarios (applying S1-S5 in peak periods). The test fully considers the increased road travel time, reduced bus frequency and capacity under severe weather. It can be found from Figure 5.5 that a drastic demand shift from bus to subway exists under severe weather conditions. For instance, with a population of 5000, the modal share for subways is 18% under weather scenario S1 (i.e. good weather). However, under S5 (i.e. severe weather), the modal share for subways increases to 42% as the subway mode is weather-proof. From Figure 5.5, population level effects on individuals' mode choice can also be found. It is seen that, with a large population, individuals tend to use subways rather than buses under any weather scenario. For example, under scenario S1, most individuals choose buses for their travel (i.e. 82%) when the population is only 5000. However, when the population is 50,000, the bus modal share decreases to 50%. In the study network, this figure can be explained by the fact that, the subway has a larger capacity than that of bus. When the population is large, individuals tend to choose the subway to avoid bus in-vehicle crowding.

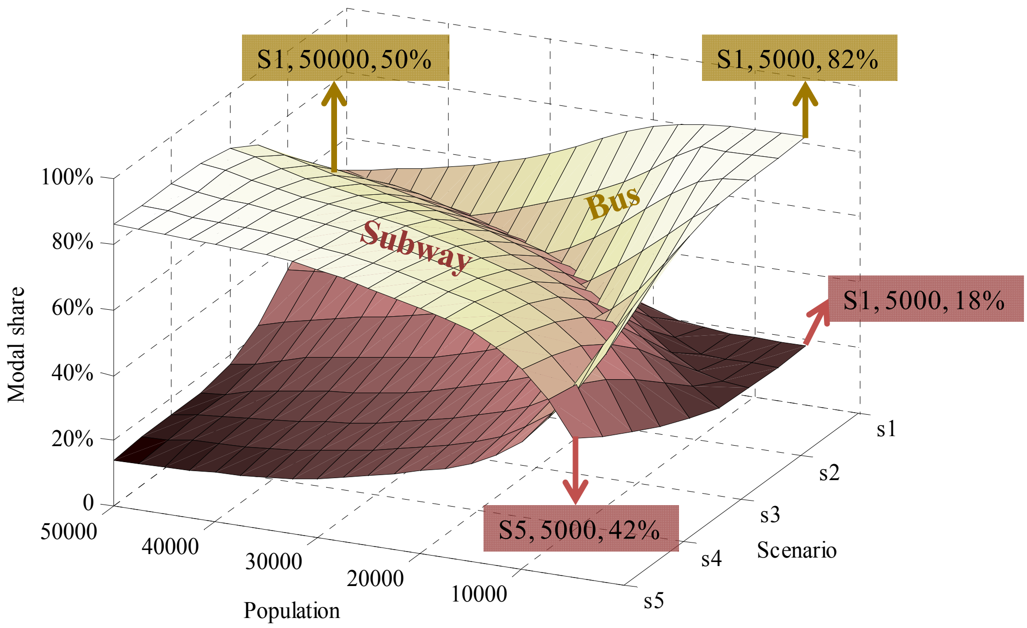


Figure 5.5 Modal shares under different weather scenarios and different population levels

5.5 Summary

This Chapter presents an activity-based network equilibrium model for scheduling DATPs in multi-modal transit networks under adverse weather conditions with different rainfall intensities. The proposed model is designed for long-term planning of congested multi-modal transit network in cities with frequent rainy periods (e.g. Singapore and Hong Kong). In the proposed model, weather forecast information was incorporated for solving the individuals' DATP scheduling problem. In-vehicle crowding discomfort taking account of adverse weather impacts is specifically considered. This model explicitly considers the effects of adverse weather on the performances of different transit modes, and the effects on the utilities of the various activities (i.e. compulsory or non-compulsory). The proposed model extends existing studies by developing a comprehensive framework which incorporates flexible activity sequences and durations, route and mode choices, and also adverse weather effects.

The ATS-SAM super-network introduced in Chapter 4 is adopted. Not only can this network explicitly model the transfers and non-linear fare structures in multi-modal transit networks but also simultaneously addresses the activity choices and travel choices in time-space coordinates under conditions of different rainfall intensities. The ATS-SAM super-network is constructed based on link travel times for different times of day according to the provided weather forecast information. Individuals schedule their DATPs based on the trade-off between the utility gained from activity participation and the dis-utility of the travel required.

An efficient solution algorithm without prior DATP enumeration is developed for solving the equivalent static transit assignment problem on the ATS-SAM super-network. The proposed model and solution algorithm are tested with a real multi-modal transit network in Singapore. The numerical results show that the proposed model can be used to investigate individuals' DATPs and overall average effects on multi-modal transit networks under adverse weather conditions. The numerical results highlight the key role of weather-proof systems (i.e. subways) as the main transit mode under severe weather condition. In addition, individuals' attitudes towards compulsory and non-compulsory activities vary and their DATP choices change according to weather conditions. It is shown that the carrying out of compulsory activities and the use of subways will be underestimated if weather effects are not explicitly considered for long-term transit planning. On the basis of the proposed model, further work is required for model calibration and validation with empirical data. Rainfall intensity data, mobile phone data and activity-travel diaries data will be collected.

In Chapters 4 and 5, network equilibrium models are proposed focusing on one-individual independent travel and activity participation. In the next Chapter 6, the proposed network equilibrium model is extended to consider two-individual joint travel and activity participation.

6 A Network Equilibrium Model for Joint Activity-Travel Pattern Scheduling

Over the past decades, many activity-based travel behaviour models have been proposed based on individuals' independent decision making. Individuals' joint decisions, however, are not explicitly considered. In reality, both independent and joint activities/travels form individual's normal daily activity-travel patterns (DATPs). Travel surveys have indicated that joint travel/activity constitutes an important part in individuals' DATPs. On this basis, explicit modelling of joint activity and travel choices is a natural and necessary component in long-term transport planning and policy analysis.

To ensure relevance and accuracy in this respect, a comprehensive dual investigation of individuals' independent and joint activity choices (e.g. activity start time and duration, activity sequence) and travel choices (e.g. departure time, route and mode) is necessary. In this Chapter, an activity-based network equilibrium model is proposed for scheduling two-individual joint activity-travel patterns (JATPs) in congested multi-modal transit networks. The interdependency of individuals' independent and joint activity/travel choices is comprehensively investigated in the proposed model.

In this proposed network equilibrium model for JATP scheduling, a measure of JATP utility is proposed to model the joint travel benefit. By constructing a joint-activity-time-space (JATS) multi-modal super-network platform, the time-dependent JATP scheduling problem is converted into an equivalent static user equilibrium (UE) model. Flexible activity start time and duration are incorporated in the proposed model. Both in vehicle and at activity location crowding discomforts are considered. An efficient solution algorithm without prior JATP enumeration is proposed to solve the JATP scheduling problem on the JATS super-network.

The outline of this Chapter is as follows. Essential background knowledge is presented in Section 6.1. A novel JATS super-network platform and problem statement are amplified in Section 6.2. A network equilibrium model for JATP scheduling is formulated as a variational inequality (VI) over the JATS super-network platform and given in Section 6.3. The solution algorithm is given in Section 6.4. Section 6.5 gives numerical examples to illustrate the proposed model, and finally, a summary of key points of the Chapter is presented in Section 6.6.

6.1 Background

As is shown in Chapters 4 and 5, travel demands are derived from the desire of people to participate in various financially and socially stimulated activities such as work, eating and shopping. Over the past decades, to perceive the underlying motivation of trip making, increased attention has been given to the activity-based approach in travel behaviour modelling (Kitamura, 1988; Axhausen and Gärling, 1992; Recker, 1995; Yamamoto *et al.*, 2000; Chow and Recker, 2012; Zhang and Timmermans, 2012). It is widely recognized that the activity-based approach can reflect temporal and spatial constraints, household influence, interdependencies of trips, scheduling of activities, and also the linkage between activities and trips.

Many activity-based travel behaviour models are based on individual decision making but joint decisions are not explicitly considered. In reality, however, both independent and joint activities/travels form essential parts of individuals' JATPs. For example, household members meet at subway stations after work, then travel jointly to have dinner in a shopping mall. With the rapid development of information and telecommunication technology, such joint activity constitutes an ever-increasing share of an individual's daily activity-travel pattern (Ronald *et al*., 2012). Travel surveys indicate that joint travel has now become a significant portion of travel within regions (Vovsha *et al*., 2003). From such findings, the importance of explicit analysis and modelling of joint activity-travel choices for long-term transport planning and policy analysis is clear.

Currently, a number of activity-based simulation models and econometric models have investigated the joint activity and travel choice problem with consideration of inter-personal dependencies (Globe and McNally, 1997; Gliebe and Koppelman, 2002; Miller and Roorda, 2003; Arentze and Timmermans, 2009; Zhang *et al*., 2009; Dubernet and Axhausen, 2013). Fewer studies have been devoted to developing activity-based mathematical analytical models such as network equilibrium models. Activity-based network equilibrium models can provide a comprehensive understanding of individuals' activity and travel choice behaviour, and present more accurate traffic conditions in a congested transportation network. Most existing studies on activity-based network equilibrium, however, are on the basis of one individual level and also ignorance of individuals' joint activity-travel choices (Lam and Yin, 2001; Lam and Huang, 2002, 2003; Huang and Lam, 2005; Zhang *et al*., 2005; Li *et al*., 2010; Ramadurai and Ukkusuri, 2010, 2011; Ouyang *et al*., 2011; Fu and Lam, 2014). As joint participation in activities and travels represent a substantial portion of individuals' DATPs, it is of important interest to develop network equilibrium models which can comprehensively consider individuals' independent and joint activity/travel choices in congested multi-modal transit networks.

In many Asian cities such as Hong Kong, most daily travel is conducted using various public transit modes (over 90% in Hong Kong) rather than privately owned cars. Joint travels using public transit may benefit individuals by satisfying a need for communal activity or by offering pleasurable travel experience. The consideration of joint activity-travel choices in long-term transport planning is an important research area, as yet largely unexplored. Hence, a network equilibrium model for scheduling joint activity-travel patterns (JATPs) in multi-modal transit networks is proposed in this Chapter. Individuals' preference for joint travel and various activity/travel choices made by individuals are explicitly explored by means of the proposed model.

The problem of coupling constraints, which is a major challenge in JATP modelling, is solved in this Chapter by extending the activity-time-space super-network proposed in Chapter 4 to a novel joint-activity-time-space (JATS) multi-modal super-network. Using the JATS super-network platform, both the independent activity/travel choice and joint choice can be modelled simultaneously. The relationship between activity choices and travel choices can be effectively captured by solving the user equilibrium (UE) problem on the JATS super-network platform. Existing theories are extended in this Chapter by developing a comprehensive framework to capture independent and joint activity/travel choices in multi-modal transit networks. A network equilibrium model with consideration of joint travel benefit is explicitly proposed. The ultimate aim of the proposed model is to assess the effects of alternative transport policies and to be used for future long-term strategic planning.

6.2 Problem statement and network representation

6.2.1 Joint activity-travel pattern (JATP)

In this Chapter, a JATP concept is proposed to model the activity and travel choices of a two-individual household within the study time period. Figure 6.1 shows an example of a two-individual (i.e. individual A and individual B) JATP from 6:00 to 24:00. The two individuals' all independent and joint activity/travel choices (e.g. time and space coordination, activity sequence and location, activity start time and duration, route and mode choices) throughout the whole time period are depicted as a JATP. It can be seen that the activity sequence of this JATP is home-work-shopping-home. The activity start/end time, activity duration, activity location can be traced. Several trips are conducted between different activities. The travel time of each trip, route choice and mode choice can also be found. Note that both independent and joint activities/travels are included in the JATP. In example Figure 6.1, the two individuals shop together after independent work activities, then jointly travel home. The activity-travel choice problem of a two-individual household is termed the JATP scheduling problem. This problem is solved in this Chapter.

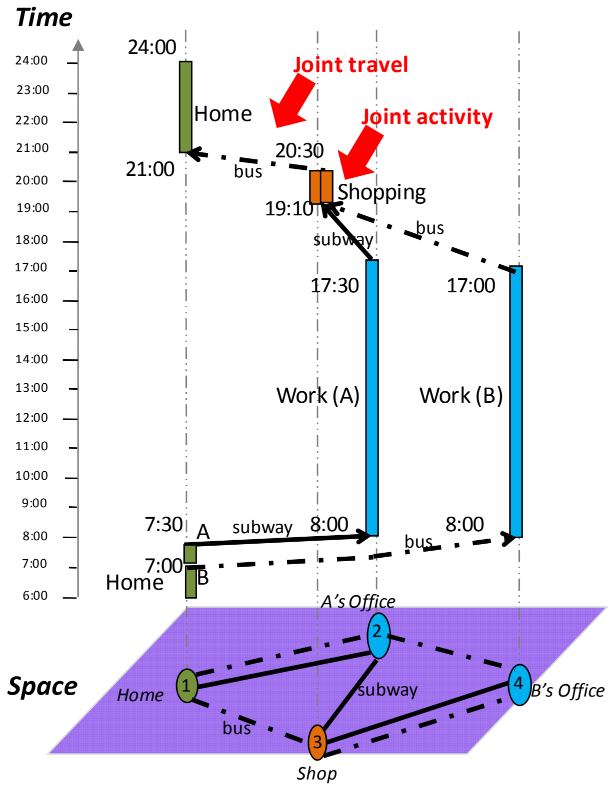


Figure 6.1 An illustration of a two-individual JATP

6.2.2 Model assumptions

In order to facilitate the essential ideas without loss of generality, besides adopting the assumptions A4.1 - 4.4 in Chapter 4, the following assumptions are made in this Chapter.

A6.1: A commonality factor is proposed to capture the impact of joint travel length on individuals' JATP utility. This is inspired by the commonality factor used in the C-logit model (Cascetta *et al*., 1996; Zhou *et al*., 2010).

A6.2: In urban areas such as those in Hong Kong, most individuals remain in the work place during the noon period. In this Chapter, the JATP scheduling problem is divided into two time periods (i.e. morning period before 12:00 noon and afternoon period after 12:00 noon). Individuals are assumed to start the morning period with joint home activity and end that period with independent work activities. In the afternoon period, individuals are assumed to start with independent work activities and end that afternoon period with joint home activity.

A6.3: It is assumed that the total population in the study network consists of multiple two-individual households. The two individuals in a household jointly make activity-travel decisions, and the joint decision-making process seeks to maximize the utility of the entire household (Zhang *et al*., 2005).

Three types of activities are investigated and described in this Chapter: work, shopping, and home activities. Work is considered as an independent activity, while shopping and home activities can be conducted independently or jointly. In accordance with that described in Chapters 4 and 5, home and work are considered as compulsory activities, while shopping is a non-compulsory activity (Fu and Lam, 2014). The activity choices, including activity sequence, activity location, activity start time and duration are not fixed.

6.2.3 A joint-activity-time-space (JATS) super-network platform

The synchronization problem (i.e. the temporal and spatial co-ordination among individuals) poses a challenge in joint activity-travel modelling. To produce consistent activity/travel choices, some studies concerned with synchronization of joint activity/travel participation (Dubernet and Axhausen, 2013; Fang *et al*., 2011; Liao *et al*., 2013). Liao *et al*. (2013) developed joint multi-state super-networks to address the independent and joint mode/route choices of two interacting household members. The above study is the first attempt to extend individual multi-state super-networks to joint multi-state super-networks. The synchronization of mode/route choice, where and when to meet or depart can be represented by the proposed super-network of Liao *et al*. (2013). Liao *et al*.'s multi-state super-network, however, has difficulty in tackling the non-linear fare structures of public transit systems, such as the system in Hong Kong. In Liao *et al*.'s model, activity duration has to be pre-determined, and link cost is independent of the influence of crowding effect. Furthermore, the joint travel benefit has not been considered in their model. Therefore, as presented in this Chapter, a novel super-network platform is proposed to incorporate independent and joint activity/travel choices, non-linear fare structures, flexible activity start time and duration, and the crowding effects in the multi-modal transit network.

The joint-activity-time-space (JATS) super-network expansion approach proposed in this Chapter is based on the ATS-SAM super-network presented in Chapter 4. The objective is to represent individuals' independent and joint activity choices and travel choices over a multi-modal transit network. In this approach, the ATS-SAM super-network delivered in Chapter 4 is further developed by incorporating joint activity and travel links. The framework of the JATS super-network is given below.

Nodes: Each node in the JATS super-network is described as JATS node $(ind,(i,l),k)$, where ind is the individual(s) indicator. The value of ind is equal to 1 (2) indicating individual A (B) is at the node, and the value of ind is equal to 12 indicating both A and B are at the node. The JATS nodes with $ind = 12$ are called joint nodes, while the ones with $ind = 1$ or 2 are independent nodes. The definitions of i, l, and k are in accordance with those in Chapter 4.

Links: Links in the JATS super-network are classified into five categories, i.e.

$A = A_a \cup A_t \cup A_d \cup A_w \cup A_m$.

- A_a is the set of activity links as seen in Chapter 4. A_a is constructed between the augmented nodes with the same individual(s) and at the same location to indicate that a particular activity is conducted for one interval. Each $a_a \in A_a$ is characterised by individual(s), activity location, activity type, and activity start time. $A_a = A_a^{\text{indep}} \cup A_a^{\text{joint}}$, where A_a^{indep} denotes the set of independent activity links and A_a^{joint} denotes the set of joint activity links.
- A_t is the set of transfer links as seen in Chapter 4.
- A_d is the set of direct in-vehicle links in accordance with that in Chapter 4.
- A_w is the set of waiting links. Each $a_w \in A_w$ is constructed between the independent nodes at the same location to indicate an individual waiting for the other individual for one time interval.
- A_m is the set of meeting links. Each $a_m \in A_m$ is constructed between an independent node and a joint node at the same location to represent individuals meeting each other at the node. The duration of a meeting link is assumed to be zero (Liao *et al*. 2013).

Figure 6.2 shows an example of the JATS super-network consisting of two transit modes (i.e. subway and bus) and two activities (i.e. work and shopping). The two individuals (A and B) work at different places and shop together after work. The study horizon is divided into four equally spaced time intervals. The travel time of each physical link is one interval.

6.2.4 JATS super-network expansion algorithm

A rule-based algorithm is proposed to generate the JATS super-network for two-individual household JATP scheduling. With this rule-based algorithm, the conventional multi-modal transit network can be automatically transformed into the JATS super-network.

Figure 6.2 is an example of the network expansion result for a time period after work. Each joint route from the two origins (i.e. one origin for one individual) to the destination (i.e. the same destination for the two individuals) in the JATS super-network represents a feasible JATP in the afternoon period. The JATS super-network expansion algorithm is extended from the network expansion algorithm proposed in Chapter 4 and is presented as follows:

Input: a multi-modal transit network M, two origin locations for individual A and B (i_A and i_B), one destination location (i_{AB}), activity locations ($i_{\mathrm{a}} \in I_{\mathrm{a}}$), and number of time intervals K.

Output: the JATS super-network.

Step 1. Node augmentation.

For each node $i \in U$, expand the node into JATS node $\left(ind,(i,l),k\right)$, $ind = 1,2,12$, $l = 0,1$, $k = 1,2,...,K,K+1$. Denote the JATS node set as N.

Step 2. Construction of JATS activity links.

Scan all nodes in set N. Construct JATS activity links $a_{\mathrm{a}} \in A_{\mathrm{a}}$ between $\left(ind,(i_{\mathrm{a}},0),k\right)$ and $\left(ind,(i_{\mathrm{a}},0),k+1\right)$.

Step 3. Construction of JATS transfer links.

Scan all nodes in set N. Construct JATS transfer links $a_{\mathrm{t}} \in A_{\mathrm{t}}$ between

$\left(ind,(i_{a_\mathrm{a}},0),k\right)$ and $\left(ind,(i_{a_\mathrm{a}},1),k\right)$.

Step 4. Construction of JATS direct in-vehicle links.

Find all in-vehicle links in network M on the basis of physical travel links.

For each $i \in U$, find all $i' \in U$ which are connected to i by in-vehicle links. Record the mode b and the travel time $t^0_{a_\mathrm{d}}$ of each in-vehicle link.

For each i', construct JATS direct in-vehicle links between $\left(ind,(i,1),k\right)$ and $\left(ind,(i',0),k+t^0_{a_\mathrm{d}}\right)$.

Step 5. Construction of JATS waiting links.

Scan all nodes in set N. Construct JATS waiting links $a_\mathrm{w} \in A_\mathrm{w}$ between $\left(ind,(i,0),k\right)$ and $\left(ind,(i,0),k+1\right)$.

Step 6. Construction of JATS meeting links.

Scan all nodes in set N. Construct JATS meeting links $a_\mathrm{m} \in A_\mathrm{m}$ between $\left(1,(i,0),k\right)$ and $\left(12,(i,0),k\right)$, and between $\left(2,(i,0),k\right)$ and $\left(12,(i,0),k\right)$.

Step 7. Simplification of the super-network.

Delete the augmented nodes which are not two-way connected except for the origin nodes (i.e. $\left(1,(i_A,0),1\right)$ and $\left(2,(i_B,0),1\right)$) and the destination node (i.e. $\left(12,(i_{AB},0),k+1\right)$). Delete the redundant links.

Following the model assumption A6.2, in the JATS super-network for the morning period, each joint route from the origin (at home) to two destinations (at work places) represents a feasible JATP. The network expansion steps are similar and not listed here. With the use of the JATS super-network, individuals' activity choices (i.e. activity locations, sequence and durations) and travel choices (i.e. route and mode choices, transfers), including both independent ones and joint ones, are explicitly represented by different links in the proposed super-network platform. The relationships between activity and travel choices are reflected by the JATS super-network topology. The non-linear fare structures in multi-modal transit networks can be explicitly modelled. Flexible activity start time and duration are incorporated. Each joint route in the JATS super-network platform represents a feasible JATP.

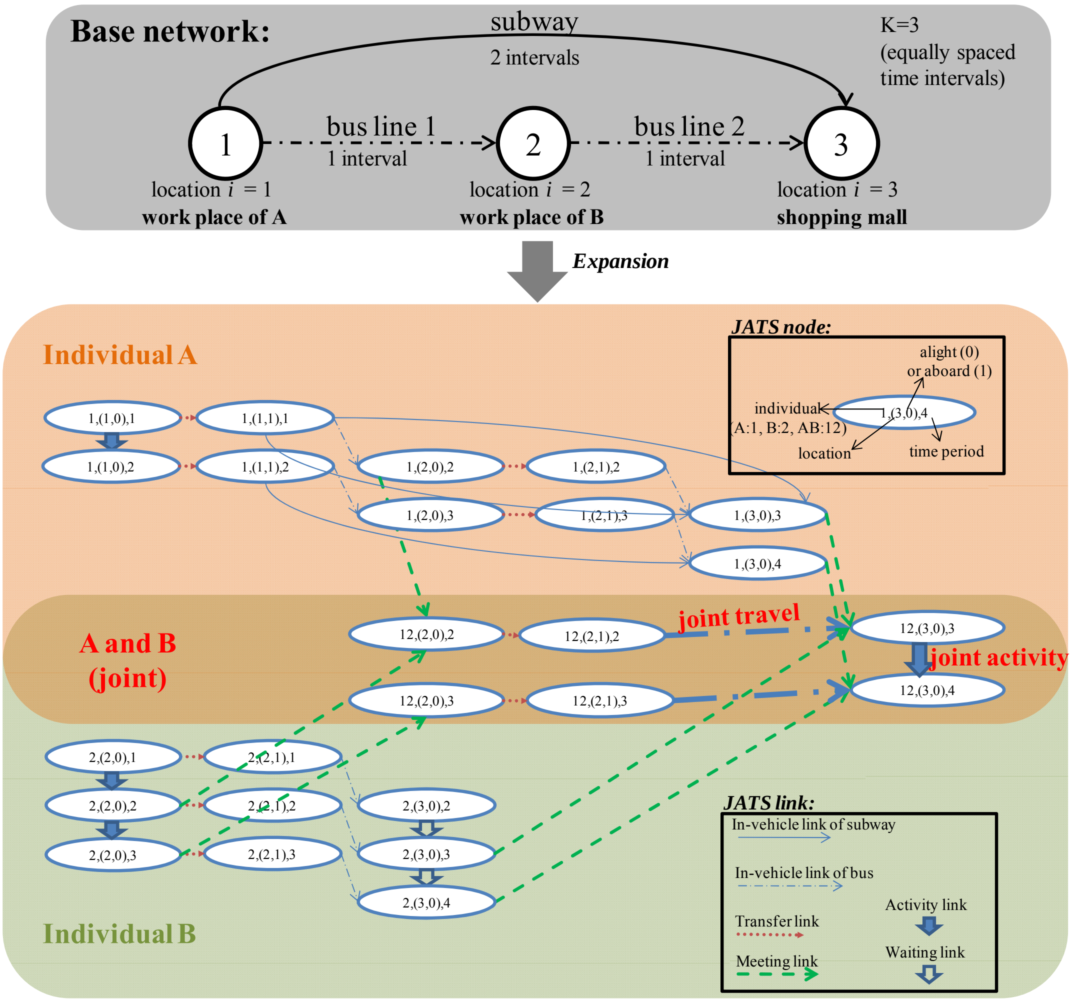


Figure 6.2 An illustration of the JATS super-network

6.2.5 Link utility/dis-utility in JATS super-network

In this Chapter, marginal activity utility is specified for different individuals, and the crowding discomfort at activity location is considered. The utility of individual *ind* performing independent activity link a_a is expressed by

$$u_{a_a}^{ind} = \left(1 + \beta'_{a_a}\left(\frac{f_{a_a}}{\kappa_{a_a}}\right)^{\theta'_{a_a}}\right) \cdot \int_k^{k+1} \overline{u}_{a_a}^{ind}(\omega)d\omega, \qquad a_a \in A_a^{\text{indep}},\ ind = 1,2 \qquad (6.1)$$

where k is the time interval of activity link a_a; $\overline{u}_{a_a}^{ind}(k)$ denotes the marginal activity utility of individual ind; f_{a_a} is the passenger flow on the activity link at time interval k; κ_{a_a} is the capacity of the activity location; β'_{a_a} and θ'_{a_a} are model parameters relevant to activity type. β'_{a_a} is equal to 0 for compulsory activities (e.g. home and work), as the utility of compulsory activities is not affected by the crowding at the activity locations.

As regards joint activity utility, a group utility function proposed by Zhang *et al*. (2002) is adopted to represent the preference for performing joint activities with consideration of intra-household interactions. The utility of joint activity link a_a is expressed by

$$u_{a_a}^{12} = w_1 \cdot u_{a_a}^1 + w_2 \cdot u_{a_a}^2 + \chi \cdot u_{a_a}^1 \cdot u_{a_a}^2, \qquad a_a \in A_a^{\text{joint}} \qquad (6.2)$$

The joint activity utility is the summation of weighted individuals' utility and a weighted interaction effect. $u_{a_a}^1$ and $u_{a_a}^2$ are independent utilities of two individuals which can be obtained from Equation (6.1). w_{ind} is individual ind's weight parameter. $w_1 \geq 0$, $w_2 \geq 0$, and $w_1 + w_2 = 1$. w_{ind} ($ind = 1,2$) can be interpreted as a measure of the individual ind's power in household's decision making. The parameter χ moderates the effect of joint activity benefit and reflects household members' concern for joint activity. A detailed interpretation of this function and other types of household utility function can be found in Zhang *et al*. (2002, 2009).

The dis-utility of physical link v with start time interval k (denoted as $disu_v(k)$) is expressed to represent in-vehicle crowding discomfort (Spiess 1983; Nielsen 2000; Lo *et al*. 2003):

$$disu_v(k) = -\text{vot} \cdot t_v^0 \left(1 + \beta_b \left(\frac{f_v(k)}{h_b \cdot g_b}\right)^{\theta_b}\right), \quad v \in V_b \tag{6.3}$$

where t_v^0 is the free-flow travel time of physical link v; h_b is the vehicle capacity of mode b; g_b denotes the frequency of mode b; vot is the value of time; β_b and θ_b are model parameters relevant to mode b. $f_v(k)$ is the passenger flow on the physical link v at time interval k, which can be expressed as the summation of passenger flows on all in-vehicle links consisting of this physical link:

$$f_v(k) = \sum_{a_\text{d} \in A_\text{d}} f_{a_\text{d}} \cdot \delta(a_\text{d}, v), \tag{6.4}$$

where $\delta(a_\text{d}, v)$ is equal to 1 if physical link v is in direct in-vehicle link a_d; 0 otherwise.

The in-vehicle link dis-utility can be obtained by the summation of related physical link dis-utilities and transit fare:

$$disu_{a_\text{d}} = \sum_{v \in V} disu_v \cdot \delta(a_\text{d}, v) \text{-} \rho_{a_\text{d}}, \tag{6.5}$$

where ρ_{a_d} is the transit fare with respect to the direct in-vehicle link a_d. In this way, non-linear fares can be directly represented by node-to-node basis.

As regards transfer links by mode, the link dis-utility is in accordance with that in Chapters 4 and 5. In the JATS super-network, each waiting link indicates waiting for one time interval, thus, the dis-utility of waiting link (denoted as $disu_{a_{\mathrm{w}}}$) can be expressed as $disu_{a_{\mathrm{w}}} = -\mathrm{vot}$. The dis-utility of meeting link $a_{\mathrm{m}} \in A_{\mathrm{m}}$ is assumed to be zero (Liao *et al*., 2013).

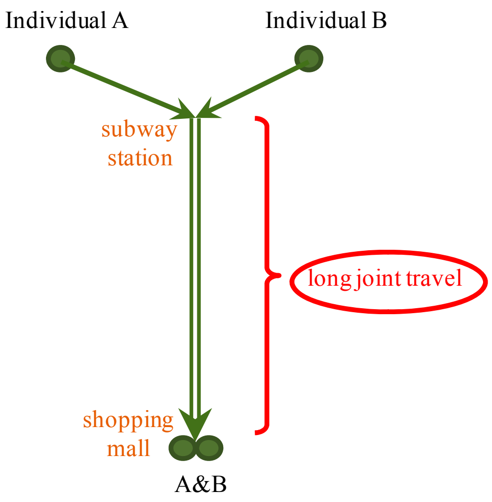
6.3 The JATP scheduling model

6.3.1 Impact of joint travel length

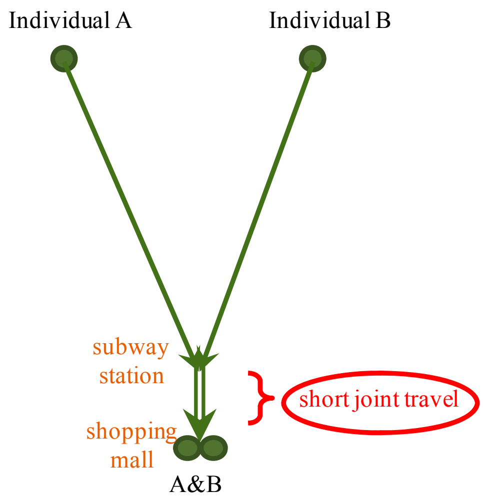
Individuals are known to gain benefits from joint travel and joint activity. To comprehensively model individuals' independent and joint activity/travel choice behaviour, individuals' preference for joint activity/travel should be investigated. However, the investigation of joint travel preference is still largely unexplored in the literature.

Activity-travel surveys indicate individuals are willing to travel further and pursue activities for longer durations when the activity-travel is being pursued jointly with family or friends. According to the findings of Srinivasan and Bhat (2008), joint episodes are often of long durations. In this Chapter, the benefits gained from joint activity is modelled by incorporating interaction parameter in joint activity utility function (refer to Equation (6.2)). The benefits from joint travel is modelled by considering the joint travel length. In general, individuals make JATP choices based on the consideration of different joint travel lengths (refer to Figure 6.3). For example, individuals can meet at the nearest subway station, and jointly take a lengthy journey to the shopping mall (as is shown in Figure 6.3(a)). They can also meet at a subway station near the shopping mall, and take a short joint journey (as Figure 6.3(b) shows).

Although a short joint bus/train journey to the shopping mall (i.e. Figure 6.3(b)) is a possibility, in reality, it is not likely. Individuals are likely to conduct such travel independently. Faced with waiting either at the subway station or a shopping mall, prior to meeting, a traveller is more likely to avoid the joint train journey preferring to meet at the shopping mall destination. Thus, the length of joint travel should be explicitly considered in modelling individuals' JATP choice behaviour.



(a) Long joint travel



(b) Short joint travel

Figure 6.3 Comparison of two JATPs with different joint travel lengths

A JATP scheduling model with explicit consideration of joint travel benefit is described in this Chapter. A measure of JATP utility is proposed. The impact of joint travel length is modelled in the JATP utility by using a commonality factor (Cascetta *et al.*, 1996; Zhou *et al.*, 2010).

The commonality factor was first proposed by Cascetta *et al.* (1996) in the C-logit model. In the C-logit model, the commonality factor is added into the route utility function to account for the similarities between overlapping routes. Utilities of overlapping routes are modified by incorporating this factor in the C-logit model. As joint travel brings individuals benefit and increases individuals' utility, in this Chapter, the concept of commonality factor is adopted to enable the consideration the similarity of individuals' routes (i.e. the proportion of joint travel in total travel). A measure of JATP utility with commonality factor is proposed in this Chapter.

The JATP scheduling model can be interpreted as an implicit JATP availability choice model, where the JATP utility with commonality factor can be considered as a normalised measure of the availability of a JATP as an alternative for a generic traveller. The commonality factor is calculated based on the joint travel proportion in total travel. The JATP utility increases with the proportion of joint travel.

Let $P=\{p\}$ be the joint route set in the JATS super-network (i.e. JATP set). The proposed network equilibrium model for JATP scheduling keeps the mathematical structure of conventional UE model, but with a modified route utility. In this Chapter, a measure of JATP utility is proposed to represent the household utility gain from all independent and joint choices. The JATP utility (denoted as φ_p), is defined as the sum of the overall activity utility of JATP p (denoted as $u_{activity}^{p}$) and overall travel dis-utility of JATP p (denoted as $disu_{travel}^{p}$) times a commonality factor (denoted as cf_p):

$$\varphi_p = u_{activity}^{p} + disu_{travel}^{p} \cdot cf_p. \tag{6.6}$$

By considering the act of waiting for another individual as an activity, the overall activity utility of the JATP p (i.e. $u_{activity}^{p}$) can be expressed by summing the weighted utilities of activity links and dis-utilities of waiting links:

$$u^p_{activity} = \sum_{a_a \in A_a} w_{ind} \cdot u^{ind}_{a_a} \cdot \delta(p, a_a) + \sum_{a_w \in A_w} w_{ind} \cdot disu_{a_w} \cdot \delta(p, a_w). \qquad (6.7)$$

The overall travel dis-utility of the JATP p (i.e. $disu^p_{travel}$) can be obtained by summing the weighted dis-utilities of in-vehicle links and transfer links:

$$disu^p_{travel} = \sum_{a_d \in A_d} w_{ind} \cdot disu_{a_d} \cdot \delta(p, a_d) + \sum_{a_t \in A_t} w_{ind} \cdot disu_{a_t} \cdot \delta(p, a_t). \qquad (6.8)$$

where $\delta(p,a)$ is the incidence relationship between JATP and link; $\delta(p,a)$ is equal to 1 indicates that this link is used in the JATP, 0 otherwise. w_{ind} is the individual weight parameter concerning the link is conducted by which individual(s).

The commonality factor cf_p of JATP p is directly proportional to the joint travel degree of the individuals' overall travel. The role played by cf_p is as follows: a JATP with a large proportion of joint travel has a smaller cf_p, thus a larger JATP utility with respect to a JATP with a small proportion of joint travel.

The commonality factor can be specified in different functional forms, resulting in different JATP utility. One possible way to specify the commonality factor is:

$$cf_p = 1 / e^{\beta_{cf} \cdot \frac{L^p_{joint}}{L^p_{total}}}, \quad p \in P \qquad (6.9)$$

where L^p_{joint} is the "length" of joint travel; L^p_{total} gives the overall "lengths" of individuals' total travel in JATP p; $\frac{L^p_{joint}}{L^p_{total}}$ indicates the proportion of joint travel in total travel.

β_{cf} is the commonality factor parameter. It is greater than or equal to 0. If β_{cf} is equal to 0, the commonality factor is equal to 1. This indicates that the joint travel benefit is not considered and the proposed JATP scheduling model collapses to become the conventional activity-travel pattern scheduling model.

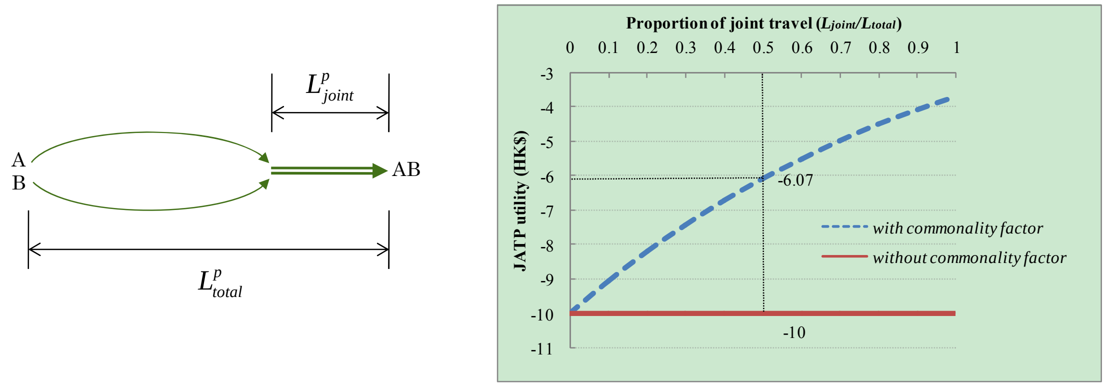
It can be shown that the above specification of the commonality factor cf_p has the following properties:

i) If JATP p does not include any joint travel (i.e. all travels in the JATP are independent), L^p_{joint} is equal to 0 and cf_p is equal to 1. Thus, the JATP utility is equal to the simple summation of activity utility and travel dis-utility. The indication is that there is no benefit from joint travel.

ii) If individuals take only joint travel in the JATP (i.e. no independent travel), $L^p_{joint} = L^p_{total}$ and cf_p is equal to $1/e^{\beta_{cf}}$. As $\beta_{cf} \geq 0$, $0 < 1/e^{\beta_{cf}} \leq 1$, the JATP utility is increased, which means individuals obtain benefit from joint travel.

iii) It is not difficult to see that $L^p_{total} \geq L^p_{joint}$, thus, $0 \leq \frac{L^p_{joint}}{L^p_{total}} \leq 1$ and $1/e^{\beta_{cf}} \leq cf_p \leq 1$.

The effect of the commonality factor in the JATP utility is exemplified in a simple JATP shown by Figure 6.4(a). Assuming β_{cf} is equal to 1 and the total travel dis-utility ($disu^p_{travel}$) is equal to HK$ -10, it can be found from Figure 6.4(b) that, φ_p is equal to HK$ -10 regardless of joint travel length if the commonality factor is not incorporated. However, if joint travel benefit is considered by using the proposed commonality factor cf_p, φ_p will increase with the proportion of joint travel in total travel. For example, φ_p increases to HK\$ -6.07 if the proportion of joint travel (i.e. $\frac{L^p_{joint}}{L^p_{total}}$) is equal to 0.5. It can be seen from Figure 6.4 that when $\frac{L^p_{joint}}{L^p_{total}}$ increases from 0 to 1, the JATP utility varies from HK\$ -10 to HK\$ -3.7.



(a) a small JATP **(b) JATP utility**

Figure 6.4 Effect of commonality factor on JATP utility

In Equation (6.9), the "length" can either be a flow-independent attribute (e.g. free-flow travel time) or flow-dependent attribute (e.g. travel dis-utility). The former case is clearly a special case of the latter when the congestion effect is negligible. The selection of appropriate attributes in the commonality factor depends on the specifics of the intended scenarios. For example, for individuals who have better knowledge of the network conditions such as commuters equipped with traveller information, it would be more appropriate to choose a flow-dependent commonality factor. On the other hand, a flow-independent commonality factor would be more suitable for modelling route choice behaviour of individuals who have little information about the network conditions. As regards the flow-independent case, the JATP utility can be expressed as

$$\varphi_p = u^p_{activity} + disu^p_{travel} / e^{\beta_{cf} \cdot \frac{t^p_{joint}}{t^p_{travel}}}, \tag{6.10}$$

where t_{joint}^{p} denotes the individuals' joint travel time (including joint in-vehicle travel time and joint transfer time) during the whole JATP, while t_{travel}^{p} denotes the individuals' total travel time (including joint in-vehicle travel time and transfer time, and independent weighted in-vehicle travel time and transfer time) during the whole JATP. As regards the flow-dependent case, the JATP utility can be expressed as

$$\varphi_p = u_{activity}^{p} + disu_{travel}^{p} / e^{\beta_{cf} \cdot \frac{disu_{joint}^{p}}{disu_{travel}^{p}}}, \tag{6.11}$$

where $disu_{joint}^{p}$ denotes the individuals' joint travel dis-utility. In this Chapter, the proposed model, in nature, falls into the category of a static UE model for long-term transit planning at the strategic level, and individuals are assumed to have perfect knowledge of traffic conditions throughout the whole network. Thus, the flow-dependent case as Equation (6.11) is adopted in this Chapter.

6.3.2 Model formulation

With the use of the proposed JATS super-network, both individuals' independent and joint activity/travel choices are explicitly represented by different links in the proposed JATS super-network platform. The time-dependent relationships between activity and travel choices can be modelled by the JATS super-network topology. Each joint route from origin to destination in the JATS super-network represents a feasible JATP. Therefore, the proposed time-dependent JATP scheduling problem is equivalent to a static multi-modal transport assignment model on the JATS super-network.

The proposed model falls into the category of static transport network equilibrium model in nature for long-term planning at the strategic level. It is thus postulated that all households would have a UE activity-travel choice pattern: for each day, the utilities of all used joint JATPs are the largest and equal, and all unused JATPs have smaller utilities. Denote π as the optimal route (i.e. the optimal JATP) with the largest utility in the JATS super-network. The UE condition can be formally expressed as

$$f_p(\varphi_\pi - \varphi_p) = 0, \tag{6.12}$$

$$q = \sum_{p \in P} f_p, \tag{6.13}$$

$$\varphi_\pi - \varphi_p \geq 0, \tag{6.14}$$

$$f_p \geq 0, \tag{6.15}$$

where f_p denotes the household flow on JATP p, and q denotes the total household number in the study network.

The previously mentioned UE problem can be further expressed as the following gap function formulation:

$$\min \; GAP = \sum_{p \in P} f_p(\varphi_\pi - \varphi_p). \tag{6.16}$$

The gap function refers to the overall gap capturing the complementary slackness conditions of the proposed UE model. The gap function is non-negative, $GAP \geq 0$.

The above UE condition can also be formulated as a variational inequality (VI):

Find $f_p^* \in \Omega$ such that

$$\sum_{p \in P^{od}} \varphi_p^*(f_p^* - f_p) \geq 0, \quad \forall f_p \in \Omega \tag{6.17}$$

where Ω is the feasible set of JATP flows defined by (6.13) and (6.15).

Theorem 6.1. The solution of the VI problem (6.17) is equivalent to the UE condition (6.12) - (6.15).

Proof

For the proof, readers are referred to Smith (1979) on the route VI formulation for the static traffic assignment problem.

Theorem 6.2. At least one solution of the VI problem (6.17) exists.

Proof

According to Facchinei and Pang (2003), the proof can be completed by the following two properties: (a) The JATP utility is continuous; (b) the feasible set Ω is compact and convex.

In general, the uniqueness of the solution depends on the monotone property of VI formulation. According to the definition of the JATP utility and commonality factor, the uniqueness of the solution cannot be guaranteed due to the non-additive form of JATP utility and the non-separable flow-dependent commonality factor.

6.4 Solution algorithm

In this section, a route searching algorithm to determine the optimal JATP is first presented in Section 6.4.1. Based on this algorithm, a solution algorithm for solving the UE model is proposed and given in Section 6.4.2.

6.4.1 Solution algorithm for searching the optimal JATP

Household members schedule their independent and joint activities and trips to maximize their JATP utility. Such actions are the equivalent of finding the route with maximum JATP utility from origin to destination in the JATS super-network. Therefore, the JATP searching problem can be converted into a shortest route problem by using the JATS super-network. It can be seen from Equation (6.11) that the JATP utility cannot be calculated by simple summation of the link (dis-)utilities. This non-additive property indicates that a sub-route between any pair of nodes on the shortest route may not be the shortest route itself. Therefore, conventional single-criterion shortest route algorithms such as the Dijkstra's algorithm and the Bellman-Ford algorithm cannot be adapted for finding the optimal JATP. The JATP searching problem can be formulated as a multi-criterion problem with respect to three decision variables, i.e. total activity utility in JATP p (i.e. $u^{p}_{activity}$), total travel dis-utility (i.e. $disu^{p}_{travel}$), and joint travel dis-utility (i.e. $disu^{p}_{joint}$). It is unlikely that a single optimal pattern can be found because of the conflicting criteria in the multi-criterion shortest route problem, but a set of non-dominated routes can be obtained in the JATS super-network. The definition of non-dominated routes is that, it is not possible to find another route with a better value in one criterion without worsening another criterion. The JATP dominant condition can be defined as follows:

Definition 6.1 (JATP dominant condition)**.** Given two JATPs $p_i \neq p_j \in P$, p_i dominates p_j, if p_i and p_j satisfy

(i) $u^{p_i}_{activity} > u^{p_j}_{activity}$ and $disu^{p_i}_{travel} \geq disu^{p_j}_{travel}$ and $disu^{p_i}_{joint} \leq disu^{p_j}_{joint}$, or

(ii) $u^{p_i}_{activity} \geq u^{p_j}_{activity}$ and $disu^{p_i}_{travel} > disu^{p_j}_{travel}$ and $disu^{p_i}_{joint} \leq disu^{p_j}_{joint}$, or

(iii) $u^{p_i}_{activity} \geq u^{p_j}_{activity}$ and $disu^{p_i}_{travel} \geq disu^{p_j}_{travel}$ and $disu^{p_i}_{joint} < disu^{p_j}_{joint}$.

A label-selection label-correcting method (Guerriero and Musmanno, 2001; Chen *et al.*, 2011) is adopted in the development of an efficient solution algorithm for finding the optimal JATP in multi-modal transit networks. The solution algorithm is an extension of the DATP searching algorithm proposed in Chapter 4. Following the model assumption A6.2, in this Chapter, the JATP scheduling problem is divided into two time periods (i.e. morning period before 12:00 noon and afternoon period after 12:00 noon). In the morning period, two individuals start journeys from the same node in the JATS super-network, and end at different destinations. The morning JATP search is from the origin to the two destinations. Regarding the afternoon period, the destination of the two individuals in the JATS super-network is arrival at the same node, thus the proposed afternoon JATP searching algorithm looks for the optimal JATP backwards, that is from the destination point to the origin points (i.e. consider the JATP destination node d as the route searching origin, and the two JATP origin nodes y_A and y_B as the route search destinations). The JATP searching algorithm for the afternoon period is as below. The algorithm for the morning period is similar and not shown here.

Let $P^{dy_Ay_B}$ be a set of non-dominated routes maintained at nodes y_A and y_B, and the non-dominated routes from destination d to all node pairs are maintained in a scan eligible set, denoted as SE. At each iteration, one non-dominated route $p_i^{dy_Ay_B}$ is selected from SE in a first-in-first-out (FIFO) order for route extension. A temporary route is constructed by extending the selected route $p_i^{dy_Ay_B}$ to its successor link whose end node is y_A or y_B (y_A for example here, and the temporary route is denoted as $p_j^{dy_A'y_B}$). The dominant relationship between the newly generated route $p_j^{dy_A'y_B}$ and the set of non-dominated routes $P^{dy_A'y_B}$ at nodes y_A' and y_B is determined based on JATP dominant condition (Definition 1). If $p_j^{dy_A'y_B}$ is a non-dominated route at nodes y_A' and y_B, it is then inserted into $P^{dy_A'y_B}$ and SE. As the newly generated route $p_j^{dy_A'y_B}$ may also dominate some routes in $P^{dy_A'y_B}$, these dominated routes should be eliminated from $P^{dy_A'y_B}$ and SE. The proposed algorithm continues the route searching process until SE becomes empty. At the last step of this algorithm, the optimal JATP can be determined by choosing the route with the largest JATP utility.

The detailed steps of the proposed algorithm for finding the optimal joint route in JATS super-network are listed as follows.

Inputs: destination node d

Returns: the optimal joint route in the JATS super-network (i.e. the optimal JATP)

Step 1. Initialization:

Create a route p_i^{ddd} from node d to itself, and set $u_{activity}^{p_i^{ddd}} = 0, disu_{travel}^{p_i^{ddd}} = 0, disu_{joint}^{p_i^{ddd}} = 0$.

Add p_i^{ddd} into label-vector P^{ddd} and the list of candidate labels SE.

Step 2. Label selection:

Take label $p_i^{dy_Ay_B} \in P^{dy_Ay_B}$ from SE in FIFO order. If $SE = \phi$, then go to Step 4; otherwise go to Step 3.

Step 3. Route extension:

If $y_A = y_B$ (denoted as y for uniformity), go to Step 3.1.; otherwise go to Step 3.2.

Step 3.1. For every link a (with start node x) whose end node is y: If link a is a meeting link, go to Step 3.1.1; If link a is an activity/waiting link, go to Step 3.1.2; If link a is an in-vehicle/transfer link, go to Step 3.1.3.

Step 3.1.1. Find the corresponding meeting link a' (with start node x') of the other individual. Generate a new label $p_j^{dxx'} \in P^{dxx'}$. Set $u_{activity}^{p_j^{dxx'}} = u_{activity}^{p_i^{dyy}}$, $disu_{travel}^{p_j^{dxx'}} = disu_{travel}^{p_i^{dyy}}$, and $disu_{joint}^{p_j^{dxx'}} = disu_{joint}^{p_i^{dyy}}$.

Step 3.1.2. Generate a new label $p_j^{dxx} \in P^{dxx}$. Set $u_{activity}^{p_j^{dxx}} = u_{activity}^{p_i^{dyy}} + u_a$, $disu_{travel}^{p_j^{dxx}} = disu_{travel}^{p_i^{dyy}}$, and $disu_{joint}^{p_j^{dxx}} = disu_{joint}^{p_i^{dyy}}$.

Step 3.1.3. Generate a new label $p_j^{dxx} \in P^{dxx}$. Set $u_{activity}^{p_j^{dxx}} = u_{activity}^{p_i^{dyy}}$, $disu_{travel}^{p_j^{dxx}} = disu_{travel}^{p_i^{dyy}} + u_a$, and $disu_{joint}^{p_j^{dxx}} = disu_{joint}^{p_i^{dyy}} + u_a$.

Step 3.2. For every link a (with start node x) the end node of which is y_A or y_B (denoted as y for uniformity), if link a is an activity/waiting link, go to Step 3.2.1; If link a is an in-vehicle/transfer link, go to Step 3.2.2.

Step 3.2.1. Generate a new label $p_j^{dxy} \in P^{dxy}$. Set $u_{activity}^{p_j^{dxy}} = u_{activity}^{p_i^{dyy}} + u_a$, $disu_{travel}^{p_j^{dxy}} = disu_{travel}^{p_i^{dyy}}$, and $disu_{joint}^{p_j^{dxy}} = disu_{joint}^{p_i^{dyy}}$.

Step 3.2.2. Generate a new label $p_j^{dxy} \in P^{dxy}$. Set $u_{activity}^{p_j^{dxy}} = u_{activity}^{p_i^{dyy}}$, $disu_{travel}^{p_j^{dxy}} = disu_{travel}^{p_i^{dyy}} + u_a$, and $disu_{joint}^{p_j^{dxy}} = disu_{joint}^{p_i^{dyy}}$.

If the new label $p_j^{dxx'}$ (or p_j^{dxx} or p_j^{dxy}) is a non-dominated route under the JATP dominant condition, then insert the new label into $P^{dxx'}$ (or P^{dxx} or P^{dxy}) and SE, and remove all routes dominated by the new label from $P^{dxx'}$ (or P^{dxx} or P^{dxy}) and SE.

Go back to Step 2.

Step 4. Determine the optimal JATP with the largest JATP utility. Stop.

6.4.2 Solution algorithm for solving the UE problem

In this section, a path-based solution algorithm is proposed for solving the UE problem by using the JATP searching algorithm proposed in Section 6.4.1. The path set (i.e. JATP set) is generated by the column generation technique using the JATP searching algorithm on a need basis. This avoids the burden of enumerating a pre-defined set of JATPs. Most conventional solution algorithms cannot be used to solve the proposed UE model as it is difficult to determine the descent direction for solving the JATP scheduling problem in multi-modal transit network. The widely used method of successive average (MSA) is a heuristic method with a forced convergence property. Thus, a solution algorithm based on MSA is proposed for solving the JATP scheduling problem (Fu *et al.*, 2014a).

The UE solution accuracy level is measured by the relative gap $RGAP$ as

$$RGAP = GAP / \sum_{p \in P} f_p \varphi_p . \tag{6.18}$$

The GAP in Equation (6.18) refers to Equation (6.16). The smaller $RGAP$ value indicates better approximation of the UE solution.

The solution algorithm for solving the JATP scheduling problem is outlined as follows.

Step 1. Transform the traditional multi-modal transit network into the JATS super network by using the JATS super-network expansion algorithm.

Step 2. Initialization. Let $n = 0$. Call the JATP searching algorithm proposed in Section 6.4.1 to find the optimal path $\pi \in P$ in the JATS super-network (i.e. JATP) with the largest JATP utility. Assign all individuals on π . Update link flows and link (dis-)utilities.

Step 3. Column generation. Call the JATP searching algorithm proposed in Section 6.4.1 to find the optimal JATP $\pi \in P$. If φ_π is larger than any φ_p in P , add π into P .

Step 4. Flow update. Perform an all-or-nothing assignment based on JATP utilities, and yield auxiliary JATP flows. Obtain updated JATP flows using an MSA process. Update link flows and link (dis-)utilities.

Step 5. Convergence test. For an acceptable convergence level τ , if $RGAP \le \tau$, stop. Otherwise let $n = n+1$ and go back to Step 3.

6.5 Numerical examples

The point of the numerical examples is to illustrate: (a) application of the proposed model and solution algorithm; (b) the effects of joint travel benefit on individuals' activity and travel choices.

In this Chapter, the activity marginal utility function used in Chapters 4 and 5 is adopted. Table 6.1 shows the given parameters in the marginal utility function for the numerical examples in this Chapter. The value of time was HK$ 60.00/hour. Other parameters were set as $w_1 = 0.5$, $w_2 = 0.5$, $\chi = 0.01$, $\beta'_{a_a} = 0.001$, $\theta'_{a_a} = 2$, $\beta_b = 0.1$, $\theta_b = 2$.

Table 6.1 Given parameters in the marginal utility function

	Work (A) (morning)	Work (B) (morning)	Work (A) (afternoon)	Work (B) (morning)	Home (morning)	Home (afternoon)	Shopping
$u_{a_a}^{\max}$ (HK$)	1800	1700	1800	1700	1000	2500	800
α_{a_a}	600	600	900	900	360	1320	1140
β_{a_a}	0.021	0.021	0.021	0.021	0.0048	0.0048	0.018

γ_{a_a}	0.8	0.8	0.8	0.8	1.8	1.8	1

6.5.1 A small network

The study time period for the small example was from 5:00 p.m. to 7:00 p.m. and was equally divided into 12 intervals (i.e. 10 minutes per interval). Figure 6.5 depicts a simple multi-modal transit network. One subway line and two bus lines served in the network. Included are three nodes and three physical links. The three nodes represent three study zones: work place of individual A, work place of individual B, shop area. Three activities (i.e. work (A), work (B), shopping) can be performed at the respective three nodes. Link L2 is an overlapping link on which individuals can conduct joint travel.

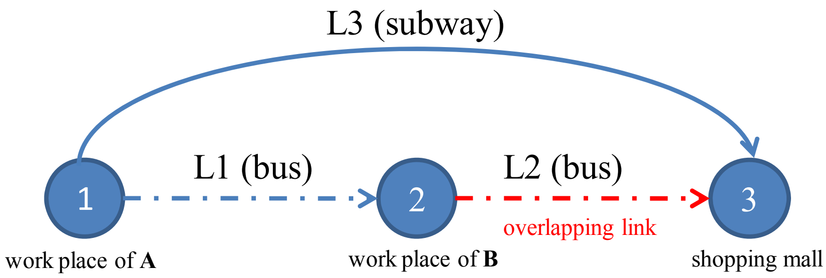
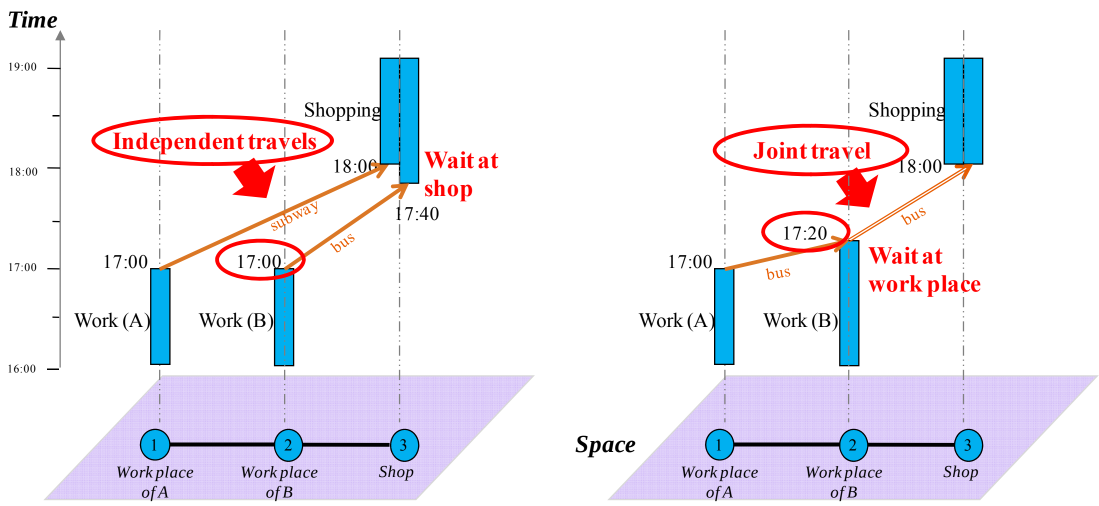


Figure 6.5 A small multi-modal transit network

The travel time of bus link L1 was 20 minutes, and the travel time of bus link L2 was 40 min. The travel time of using subway from node 1 to node 3 was the same as the time using bus. The bus fare was HK$ 2.00 per physical link. The subway fare was HK$ 12.00. The total household number in this small network was 2000.

Figure 6.6 illustrates the representative JATP which most households choose under the UE condition. It is seen that using the proposed novel super-network, individuals' independent and joint activity choices can be traced. Such choices include activity start/end time, independent and joint activity duration. Individuals' independent and joint travel choices can also be found, such as departure time, route/mode choice, and meeting time/location. Figure 6.6(a) depicts the resultant JATP without considering joint travel benefit (i.e. $\beta_{cf} = 0$) and Figure 6.6(b) illustrates the resultant JATP with consideration of joint travel benefit (i.e. $\beta_{cf} = 1$). A comparison of these two JATPs indicates that without considering joint travel benefit explicitly, individuals tend to depart earlier after work (i.e. 17:00), and meet at shopping location after independent travels. It can be seen from Figure 6.6(b) that if joint travel benefit is considered, individual B's work time is extended by about 20 min (i.e. the departure time changes from 17:00 to 17:20). Individual B waits individual A at the work place, and then they travel jointly to shop, hence obtaining maximum JATP utility. Figure 6.6 illustrates that individuals' preference towards joint travel can be effectively captured by the proposed JATP scheduling model. Individuals' travel and activity choices, including departure time, route choice, activity start time, and activity duration, are affected by JATP utility.



(a) conventional models
(**without considering** joint travel benefit)

(b) proposed JATP scheduling model
(**considering** joint travel benefit)

Figure 6.6 Comparison of JATP choice with and without considering joint travel benefit

The effects of in-vehicle travel time on travel choice are investigated by the proposed model as shown in Figure 6.7. In the small network, two individuals can conduct joint travel at overlapping link L2. The model results are compared by changing the travel time of the overlapping link. It can be seen from the figure that with the JATP scheduling model, the increase of link travel time has resulted in an increase in the number of people choosing joint travel. For example, if L2 link travel time is 20 min, about 52.17% people choose joint travel. If the travel time is 70 min, the percentage of joint travel increases to 63.70%. This is due to people can obtain much benefit from joint travel if the travel time is long.

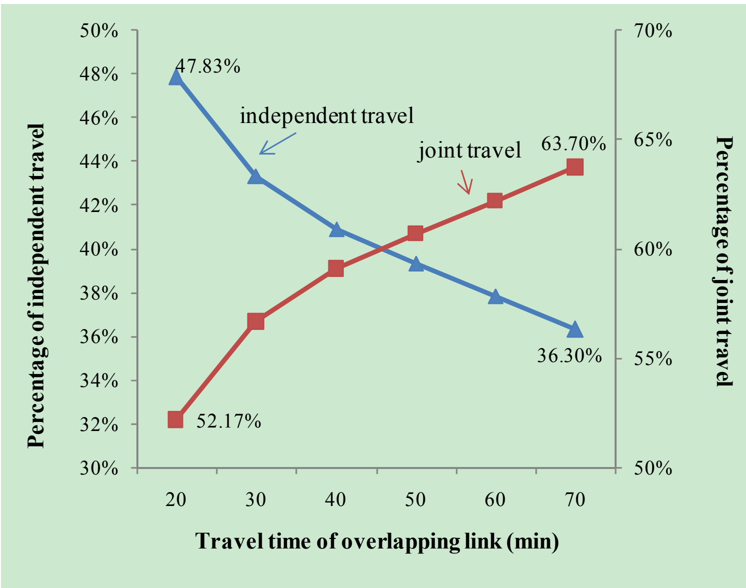


Figure 6.7 Effects of in-vehicle travel time on travel choice behaviour

Table 6.2 shows the proportion of people choosing joint travel and average joint travel time and equilibrium JATP utility from tests with different commonality factor parameters (β_{cf}). From the table, the higher the commonality factor parameter, the larger the joint travel proportion and thus larger average joint travel time per person. $\beta_{cf} = 0$ indicates that joint travel benefit is not considered explicitly. With the increase of β_{cf}, the joint travel proportion increases from 2.27% to 99.00%, and the average joint travel time per person is from 0.9 min to 39.9 min. The equilibrium JATP utility is as large as HK$ 229.33 under $\beta_{cf} = 3$ compared to HK$168.46 under $\beta_{cf} = 0$. This illustrates the benefits gained from joint travel decisions.

Table 6.2 Joint travel choices under different commonality factors

	Proportion of joint travel	Average joint travel time per person	Equilibrium JATP utility
$\beta_{cf} = 0$	**2.27%**	**0.9 min**	**HK$ 168.46**
$\beta_{cf} = 1$	59.09%	23.6 min	HK$ 194.05
$\beta_{cf} = 2$	83.33%	33.3 min	HK$ 214.18
$\beta_{cf} = 3$	**99.00%**	**39.9 min**	**HK$ 229.33**

6.5.2 The Sioux-Falls network

The proposed model and algorithm was also tested using the Sioux-Falls network, shown in Figure 6.8. The study period was from 6:00 a.m. to 9:00 p.m. Two assignments were conducted, one for the morning period and the other for the afternoon period. To reduce the size of super-network, in this example, it was assumed that individuals stayed at the work places from 10:00 a.m. until 5:00 p.m. The time interval was 10 min. Transit lines in the network were created based on some transit lines in Szeto and Jiang (2014). Bus line number 10 in Szeto and Jiang (2014) was considered as a subway line in this Chapter. The headway of each transit line was two time intervals. The in-vehicle travel time of each physical link in the network was one time interval. The non-linear fares were set as: using less than or equal to 4 physical links costs HK$ 5.00; using more than 4 physical links costs HK$ 8.00. The total household number in the network was 8000.

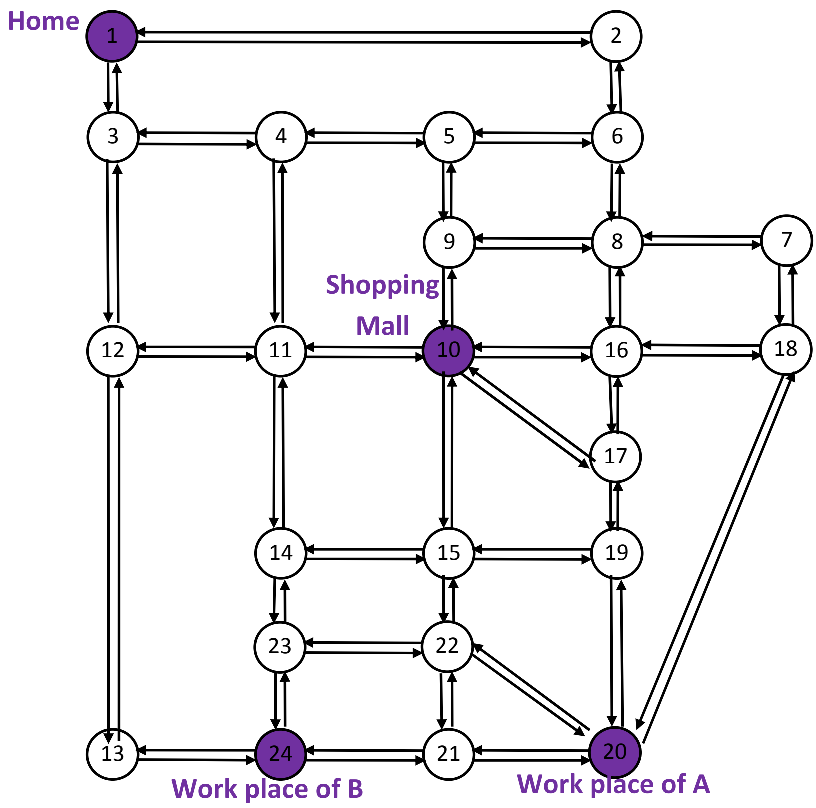


Figure 6.8 Sioux-Falls network

The convergence characteristics of the proposed UE solution algorithm are illustrated in Figure 6.9. It can be seen that the UE condition at the relative gap (as shown in Equation (6.18)) of 0.01 has been achieved within 100 iterations. This result indicates that the proposed MSA solution algorithm can solve the UE problem for this typical network with an acceptable accuracy level.

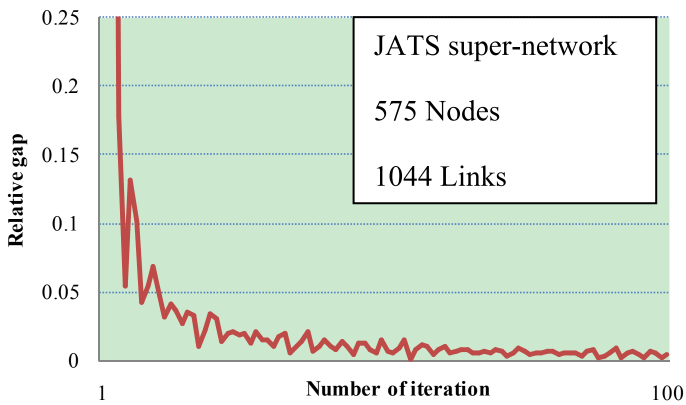
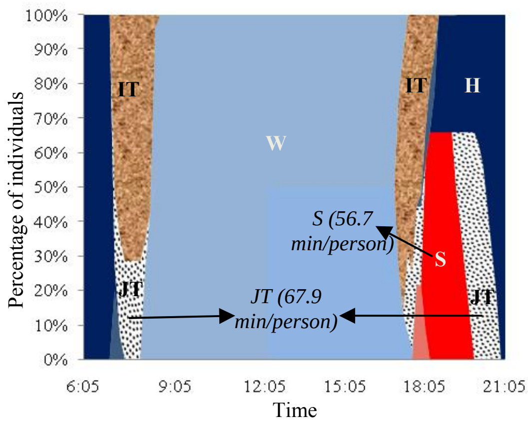


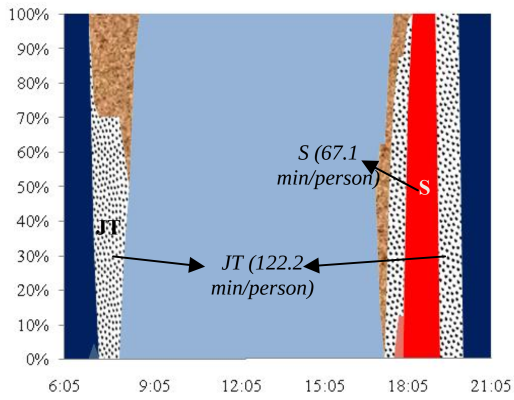
Figure 6.9 Convergence result for the Sioux-Falls network

Figure 6.10 presents the temporal population distribution for different activities (home, work, and shopping) and travels (independent travel and joint travel) under four scenarios with different link travel times. Scenarios 1 and 3 are model results without considering joint travel benefit explicitly ($\beta_{cf} = 0$). Scenarios 2 and 4 are results with joint travel benefit ($\beta_{cf} = 1$). By comparing the four scenarios, it was found that under Scenario 2 and 4 (i.e. considering joint travel benefit), individuals tend to conduct joint travels (JT) to work in the morning and after work in the afternoon. The average daily joint travel time is 67.9 min per person under Scenario 1, compared to 122.2 min per person in Scenario 2.

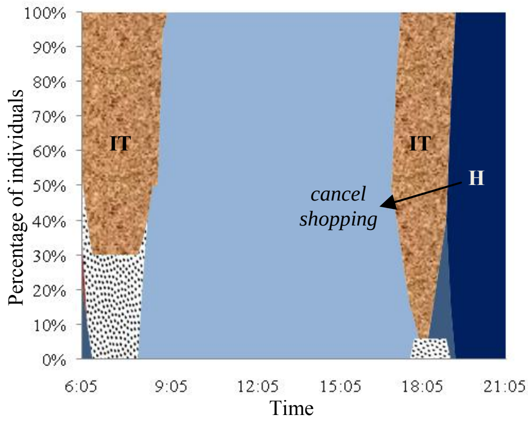
Comparing Scenario 2 to Scenario 1, more individuals conduct the shopping activity (S) jointly after work. However, it is noted that if traffic congestion on the network reaches the point of doubling all link travel times (Scenarios 3 and 4), individuals will leave home earlier in the morning, and in addition not conduct non-compulsory shopping to ensure they can return home as early as possible. Thus, previous activity-travel scheduling models which do not explicitly consider joint travel benefit, may underestimate individuals' joint travel choices. Individuals' departure times, activity start times and durations may also be biased.



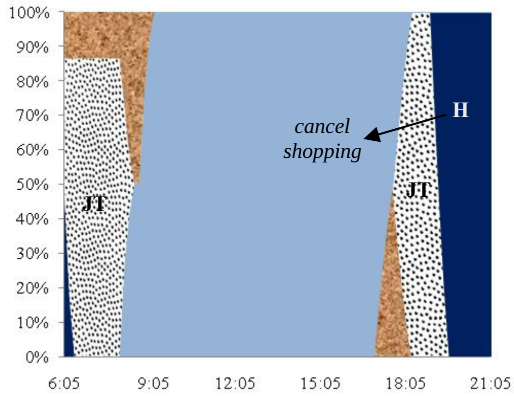
Scenario 1: Link travel time***1**
without joint travel benefit



Scenario 2: Link travel time***1**
with joint travel benefit



Scenario 3: Link travel time***2**
without joint travel benefit



Scenario 4: Link travel time***2**
with joint travel benefit

W (work) **S** (shopping) **H** (home) **JT** (joint travel) **IT** (independent travel)

Figure 6.10 Temporal population distributions under different link travel times with and without joint travel benefit

6.6 Summary

The network equilibrium models proposed in Chapters 3, 4 and 5 are based on individual decision making without considering individuals' joint decisions. This Chapter presents an activity-based network equilibrium model for scheduling two-individual JATPs in congested multi-modal transit networks. The novel super-network platform introduced in Chapter 4 is extended to a JATS super-network for simultaneously addressing individuals' independent and joint activities/travels. It is shown that the JATP scheduling problem can be converted into a static traffic assignment problem on the proposed JATS super-network. A solution algorithm without prior enumeration of JATPs is proposed for solving the JATP scheduling problem on the JATS super-network. Numerical results show that both individuals' independent and joint activity/travel choices can be simultaneously investigated by the proposed model. Included are such as activity start time, activity duration, departure time of each person, independent and joint travel routes/modes.

The benefit from joint travel is explicitly modelled in this Chapter by incorporating a commonality factor in JATP utility. It was found that the joint travel benefit significantly influences individuals' activity/travel choices. If joint travel benefit is not considered, the investigation of individuals' activity and travel choices (e.g. departure time, activity duration, joint/independent travel duration) would be biased. It is also found that the in-vehicle travel time also affects individuals' JATP choice. People tend to conduct joint travel when the in-vehicle travel time is long. Further study is required for calibration of utility functions and the commonality factor with empirical data.

7 Conclusions

In this Chapter, the key findings obtained from this research are summarized in Section 7.1. Section 7.2 gives detailed research findings together with conclusions drawn. Recommendations for further study are given in Section 7.3.

7.1 Summary of research findings

In long-term strategic transport planning, network equilibrium models provide a promising avenue for a comprehensive understanding of individuals' activity choices (e.g. activity sequence, activity start time, activity duration) and travel choices (e.g. departure time, route, mode) in congested multi-modal transport/transit networks. As a pioneering endeavour, this research has aimed to develop network equilibrium models to investigate simultaneously individuals' activity and travel choice behaviour for long-term transport planning. In the proposed network equilibrium models, network congestion effects are explicitly considered such as crowding discomfort in vehicle, congestion effect of road traffic, and crowding at activity location.

The research presented in this thesis contributes to current literature related to the topic concerned, in that a more comprehensive understanding of individuals' activity and travel choice behaviour in congested multi-modal transport/transit networks is given. This research has generated several key findings and in particular, the work makes contributions to the advancement of knowledge in network equilibrium models for integrated modelling of the activity and travel choice behaviour in a consistent manner. The four research objectives outlined in Section 1.2 of Chapter 1 have been addressed and achieved in this thesis.

The first contribution of this research relates to Objective 1. A trip-based network equilibrium model is proposed to assess individuals' travel choice behaviour in congested multi-modal transport networks under demand uncertainty (Chapter 3). It has been found that individuals' route/mode choice and transfer behaviour are significantly affected by travel time uncertainty. Based on the knowledge that individuals' activities form the underlying reasons for trip making, a second finding relates to Objective 2. It has been found that individuals' activity and travel choice behaviour can be investigated simultaneously by extending the trip-based network equilibrium to the activity-based approach (Chapter 4). The proposed activity-based model for daily activity-travel pattern (DATP) scheduling problem can capture the interaction between activity and travel choices with taking account of crowding discomfort in transit vehicles for long-term transport planning.

A third contribution relates to the Objective 3 that, adverse weather conditions can be successfully considered in the activity-based network equilibrium model proposed in Chapter 5. Impacts of adverse weather on the performance of different transit modes and the effects on the utility of various activities can be explicitly modelled. It is shown that individuals' activity and travel choices might be biased without taking into account adverse weather conditions. The above one-individual network equilibrium is extended to a two-individual case for the achievement of Objective 4, which becomes the fourth contribution of this research. A network equilibrium model is proposed in Chapter 6 to solve the joint activity-travel pattern (JATP) scheduling problem. By the JATP scheduling model, individuals' independent and joint activity/travel choices can be simultaneously studied. The crowding effect at activity location is explicitly modelled. It is demonstrated that the estimation of individuals' activity and travel choice behaviour is biased if the joint travel benefit is not taken into account.

7.2 Detailed research findings

In Chapter 3 of this thesis, individuals' travel choice behaviour in multi-modal transport networks under demand uncertainty is investigated by a trip-based network equilibrium model. To capture travel choice behaviour under travel time uncertainty, a travel time budget is adapted and a reliability-based user equilibrium model is proposed. Passenger flows and generalized travel times of different transport modes are formulated as random variables to capture the impacts of demand uncertainty. The resultant network equilibrium model can take into account the congestion effect of each mode together with their inter-modal interactions. The probable transfers and the non-linear fare structures, involved in the multi-modal transport networks, are also explicitly modelled.

A finding revealed by a numerical example indicates that expectation of on-time arrival is high, individuals tend to use the subway (e.g. the Mass Transit Railway in Hong Kong) and prefer not to change mode during their journey. It is also demonstrated that individuals' travel choice behaviour is significantly affected by network congestion. Under severe traffic congestion, individuals tend to choose the more reliable subway mode which has an exclusive right-of-way without congestion interactions with other modes.

To understand the motivation of trip making and the subsequent linkage between activity and travel, the trip-based network equilibrium model proposed in Chapter 3 is extended in Chapter 4, to an activity-based model for solving the DATP scheduling problem in congested multi-modal transit networks. The stochastic effects of activity utility and travel dis-utility can be captured by proposing a concept of DATP budget utility. It is shown that the DATP scheduling problem can be converted into a static traffic assignment problem by proposing an activity-time-space super-network platform. The novel super-network platform contributes new knowledge on simultaneous investigation of individuals' various activity and travel choices, such as time and space coordination, activity sequence, activity start time, activity duration, and route/mode choice.

The contribution of the activity-based model proposed in Chapter 4 is revealed by numerical examples. It is demonstrated that the proposed activity-based model is able to investigate individuals' DATP choice in multi-modal transit networks with taking account of in-vehicle crowding effect. One major finding is that, the uncertainty of activity utility has a significant influence on individuals' activity and travel choices. Individuals' attitudes toward compulsory and non-compulsory activities vary with their expectations of daily utility gain. Individuals would tend to use the subway mode when the in-vehicle crowding discomfort on the bus mode is increasing due to larger population in the study area with higher travel demand. This finding is consistent with the modal split result in Chapter 3. With a high expectation of daily utility gain, individuals conduct compulsory activities for a longer period (more than 13 hours) within a day rather than non-compulsory activities. If severe traffic congestion occurs in the network, individuals leave home early (i.e. 6:10 a.m.) in the morning and may not perform non-compulsory activities after work.

Based on the activity-based network equilibrium model in Chapter 4, a further extension for assessing the weather effects is presented in Chapter 5. An activity-based model is proposed for scheduling DATPs under adverse weather conditions with different rainfall intensities. It is found that weather forecast information can be incorporated for solving the DATP scheduling problem. The proposed model contributes to long-term planning of multi-modal transit networks in cities with frequent rainy periods over the year. The impacts of adverse weather on the performance of different transit modes and the effects on the utility of various activities can be explicitly considered.

A major finding revealed by numerical examples is that the model proposed in Chapter 5 can be used to investigate individuals' DATPs and the weather effects on multi-modal transit networks. The numerical results highlight the key role of weather-proof systems (i.e. subways) as the main transit mode under severe weather condition. The modal share for subways is as high as 42% under adverse weather condition compared to 18% under good weather environment. In addition, individuals' attitudes towards compulsory and non-compulsory activities are varied and their DATP choices change according to the weather conditions. It is shown in the numerical example that both the carrying out of compulsory activities and the travel by subways may be underestimated if weather effects are not explicitly considered for long-term transit planning. From a good weather scenario to a severe weather scenario, the average duration of compulsory activities increases by about 2 hours and most individuals would cancel their non-compulsory activities.

The network equilibrium models proposed in Chapters 3, 4 and 5 are based on individual decision making. Note that individuals' joint decisions are not explicitly considered. In Chapter 6, the above activity-based models for one-individual level are extended to consider the crowding at activity location for incorporating the impacts of activity location capacity for long-term strategic planning. The final contribution of this research is to propose another activity-based network equilibrium model for scheduling two-individual JATPs in multi-modal transit networks. It is demonstrated that the interdependence of individuals' independent and joint activity/travel choices can be comprehensively investigated. In the proposed model for the JATP scheduling, the joint travel benefit is successfully modelled by incorporating a commonality factor in the JATP utility. By extending the activity-time-space super-network proposed in Chapter 4 to a joint-activity-time-space super-network platform, the time-dependent JATP choice problem can be converted into an equivalent static network equilibrium problem.

Numerical examples are used to show the practicability and performance of the proposed model for JATP scheduling. The results illustrate that the individuals' independent and joint activity/travel choices can be simultaneously investigated by the proposed model. It is shown that if the joint travel benefit is not considered, individuals' daily joint travel time and joint shopping duration would be underestimated. The departure time of each trip and the start time of each activity are also likely to be biased. Another finding is the revelation of the role of in-vehicle travel time as a significant affecting factor in individuals' joint travel choice. Long in-vehicle travel time leads to a large percentage of individuals choosing joint travel.

7.3 Recommendations for further study

This research covers a wide horizon in the area of modelling activity and travel choice behaviour using a network equilibrium approach, however, several related important issues and interesting questions merit further study. Some directions for further study are outlined below.

- The proposed network equilibrium models (Chapters 3, 4, 5, and 6) in this research would benefit from the following important extension. Model calibration and validation with empirical data should be conducted. The activity-based network equilibrium models given in Chapters 4, 5, and 6, would especially benefit from the attempt to empirically measure the utility functions of different activities.

- As the model assumptions adopted in this research may cause some potential biases, some of them merit being relaxed in further study. For example, in order to facilitate the presentation of the essential ideas, vehicle capacity constraints are not considered in this research. In some Asian cities, not all individuals are able to get on the first arrival transit vehicle during peak periods. Capacity constraints do exist and may result in crowding effects on transit systems. Thus, the incorporation of capacity constraints is an important issue for further research.

- By using the proposed novel super-network platforms in Chapters 4, 5, and 6, activities in different time periods, route and mode choice can be automatically captured. However, the construction of the novel complicated super-network platforms presents difficulties in using this approach. When constructing the novel super-network platforms, activity links for each time period should be included, with travel links built with different start times for each mode, and all probable transfers defined. The super-network construction results in a huge complicated network to represent a multi-modal transport network which, initially, was one of a relatively small size. Thus, the reduction of the size of the super-network and the development of efficient solution algorithms are worthy of further study. Activity time windows can be pre-determined and fixed, and uneven time periods can be specified for different activities so as to reduce the network size. Efficient solution algorithms should also be examined for solving the network equilibrium models in real-size transport networks.

- The proposed activity-based models for multi-modal transit networks in Chapters 4, 5, and 6 should be extended to multi-modal transport networks with inclusion of road networks. In order to investigate the effects of road congestion of the DATP/JATP scheduling problem, further work is required for consideration of road congestion and travel time variation in advanced activity-based network equilibrium models.

- This thesis contributes to the literature on the simultaneous modelling of individuals' activity and travel choice behaviour. The new approach can be used to assess the impacts of land use and network changes not just on travel choices only, but also on activity choices in which activity sequence and duration can be changed. The proposed models could be extended to enable evaluation and assessment of various transport planning policies and infrastructure projects on activity choice behaviour. Based on the research presented in this thesis, it can be acknowledged that activity-based bi-level programming models could be developed for government's policy optimization. Various optimization models such as sustainable land use and transport optimization problems could be considered as the upper level. The network equilibrium models presented in this thesis could be adapted as the lower level to model individuals' activity and travel choice behaviour when the land use and transport plan is given.

APPENDIX A

The detailed manipulations on deducing Equations (3.38) and (3.39) are given in this appendix.

Substituting Equation (3.30) in Equation (3.37) gives the following equation.

$$T_v = t_v^0 + \beta_2 t_v^0 \left(\frac{2F_v T_p}{h_{b_3} s_0} \right)^{k_1} + \gamma_2 t_v^0 \left(\frac{X_v}{\kappa_v} \right)^{k_2} = A_1 + A_2 + A_3.$$

As discussed in Chapter 3, $T_p \sim N\left(t_p, \left(\sigma_t^p\right)^2\right)$, $F_v \sim N\left(f_v, \left(\sigma_f^v\right)^2\right)$, $X_v \sim N\left(x_v, \left(\sigma_{x_v}\right)^2\right)$. Then, following equations can be obtained.

$$E\left[A_2\right] = \left(\frac{2^{k_1} \beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1} \left(s_0\right)^{k_1}} \right) \sum_{i=0, i=even}^{k_1} \binom{k_1}{i} \left(\sigma_f^v\right)^i f_v^{\,k_1-i} (i-1)!! \sum_{j=0, j=even}^{k_1} \binom{k_1}{j} \left(\sigma_t^p\right)^j t_p^{\,k_1-j} (j-1)!!,$$

$$E\left[A_3\right] = \frac{\gamma_2 t_v^0}{\left(\kappa_v\right)^{k_2}} \sum_{i=0, i=even}^{k_2} \binom{k_2}{i} \left(\sigma_{x_v}\right)^i x_v^{\,k_2-i} (i-1)!!,$$

$$\begin{aligned} Var[A_2] = & \left(\frac{2^{k_1} \beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1} \left(s_0\right)^{k_1}} \right)^2 \sum_{i=0, i=even}^{2k_1} \binom{2k_1}{i} \left(\sigma_f^v\right)^i f_v^{\,2k_1-i} (i-1)!! \sum_{j=0, j=even}^{2k_1} \binom{2k_1}{j} \left(\sigma_t^p\right)^j t_p^{\,2k_1-j} (j-1)!! \\ & - \left[\left(\frac{2^{k_1} \beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1} \left(s_0\right)^{k_1}} \right) \sum_{i=0, i=even}^{k_1} \binom{k_1}{i} \left(\sigma_f^v\right)^i f_v^{\,k_1-i} (i-1)!! \sum_{j=0, j=even}^{k_1} \binom{k_1}{j} \left(\sigma_t^p\right)^j t_p^{\,k_1-j} (j-1)!! \right]^2, \end{aligned}$$

$$Var[A_3] = \frac{\left(\gamma_2\right)^2 \left(t_v^0\right)^2}{\left(\kappa_v\right)^{2k_2}} \left[\sum_{i=0, i=even}^{2k_2} \binom{2k_2}{i} \left(\sigma_{x_v}\right)^i x_v^{\,2k_2-i} (i-1)!! - \left(\sum_{i=0, i=even}^{k_2} \binom{k_2}{i} \left(\sigma_{x_v}\right)^i x_v^{\,k_2-i} (i-1)!! \right)^2 \right].$$

Particularly, the manipulations of $E\left[A_2\right]$ and $Var[A_2]$ are similar to the detailed manipulations of $E\left[I_2\right]$ and $Var[I_2]$ in Appendix B.

For simplicity, it is assumed that A_2 and A_3 are mutually independent. Then, Equations (3.38) and (3.39) can be obtained as follows.

$$
\begin{aligned}
t_v &= E[A_1]+E[A_2]+E[A_3] \\
&= t_v^0 + \left(\frac{2^{k_1}\beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1}\left(s_0\right)^{k_1}} \right) \sum_{i=0,i=even}^{k_1} \binom{k_1}{i} \left(\sigma_f^v\right)^i f_v^{\,k_1-i}(i-1)!! \sum_{j=0,j=even}^{k_1} \binom{k_1}{j} \left(\sigma_t^p\right)^j t_p^{\,k_1-j}(j-1)!! \\
&+ \frac{\gamma_2 t_v^0}{\left(\kappa_v\right)^{k_2}} \sum_{i=0,i=even}^{k_2} \binom{k_2}{i} \left(\sigma_{x_v}\right)^i x_v^{\,k_2-i}(i-1)!!,
\end{aligned}
$$

$$
\sigma_t^v = \sqrt{Var[A_2]+Var[A_3]}
$$

$$
= \sqrt{
\begin{aligned}
&\left(\frac{2^{k_1}\beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1}\left(s_0\right)^{k_1}} \right)^2 \sum_{i=0,i=even}^{2k_1} \binom{2k_1}{i} \left(\sigma_f^v\right)^i f_v^{\,2k_1-i}(i-1)!! \sum_{j=0,j=even}^{2k_1} \binom{2k_1}{j} \left(\sigma_t^p\right)^j t_p^{\,2k_1-j}(j-1)!! \\
&- \left[\left(\frac{2^{k_1}\beta_2 t_v^0}{\left(h_{b_3}\right)^{k_1}\left(s_0\right)^{k_1}} \right) \sum_{i=0,i=even}^{k_1} \binom{k_1}{i} \left(\sigma_f^v\right)^i f_v^{\,k_1-i}(i-1)!! \sum_{j=0,j=even}^{k_1} \binom{k_1}{j} \left(\sigma_t^p\right)^j t_p^{\,k_1-j}(j-1)!! \right]^2 \\
&+ \frac{\left(\gamma_2\right)^2\left(t_v^0\right)^2}{\left(\kappa_v\right)^{2k_2}} \left[\sum_{i=0,i=even}^{2k_2} \binom{2k_2}{i} \left(\sigma_{x_v}\right)^i x_v^{\,2k_2-i}(i-1)!! - \left(\sum_{i=0,i=even}^{k_2} \binom{k_2}{i} \left(\sigma_{x_v}\right)^i x_v^{\,k_2-i}(i-1)!! \right)^2 \right]
\end{aligned}
} .
$$

APPENDIX B

The detailed manipulations on deducing Equations (3.49) and (3.50) are given as follows.

Substituting Equation (3.30) in Equation (3.43) gives the following equation.

$$T_{a_t} = \frac{2\lambda T_p}{s_0} + \left(\frac{2^{\mu+1}}{s_0^{\mu+1} h_{b_3}^{\ \mu}} \right) \left(F_{a_t} + \overline{F_{bi}} \right)^{\mu} T_p^{\ \mu+1} = I_1 + I_2.$$

As discussed in Chapter 3, $T_p \sim N\left(t_p, \left(\sigma_t^p\right)^2\right)$, $\left(F_{a_t} + \overline{F_{bi}}\right) \sim N(f_1, \sigma_1^2)$. For simplicity, we transfer some notations in the following equations in this appendix, that is, $t = t_p$, $\sigma_2 = \sigma_t^p$, $F = F_{a_t} + \overline{F_{bi}}$, and $f = f_1$. Then,

$$E[I_1] = \frac{2\lambda t}{s_0},$$

$$Var[I_1] = \frac{4\lambda^2}{s_0^{\ 2}} \sigma_2^{\ 2},$$

$$E[I_2] = \left(\frac{2^{\mu+1}}{s_0^{\mu+1} h_{b_3}^{\ \mu}} \right) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x^{\mu} y^{\mu+1} \cdot \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x-f)^2}{2\sigma_1^{\ 2}} \right) \cdot \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(y-t)^2}{2\sigma_2^{\ 2}} \right) dxdy.$$

Substituting $z_1 = \dfrac{x-f}{\sigma_1}, z_2 = \dfrac{y-t}{\sigma_2}$ in the above equations gives

$$E[I_2] = \left(\frac{2^{\mu}}{\pi s_0^{\ \mu+1} h_{b_3}^{\ \mu}} \right) \int_{-\infty}^{+\infty} \left[(z_2\sigma_2 + t)^{\mu+1} \exp\left(-\frac{z_2^{\ 2}}{2} \right) \int_{-\infty}^{+\infty} (z_1\sigma_1 + f)^{\mu} \exp\left(-\frac{z_1^{\ 2}}{2} \right) dz_1 \right] dz_2$$

$$= \left(\frac{2^{\mu}}{\pi s_0^{\ \mu+1} h_{b_3}^{\ \mu}} \right) \int_{-\infty}^{+\infty} \left[\sum_{j=0}^{\mu+1} \binom{\mu+1}{j} (z_2\sigma_2)^j t^{\mu+1-j} \exp\left(-\frac{z_2^{\ 2}}{2} \right) \int_{-\infty}^{+\infty} \sum_{i=0}^{\mu} \binom{\mu}{i} (z_1\sigma_1)^i f^{\mu-i} \exp\left(-\frac{z_1^{\ 2}}{2} \right) dz_1 \right] dz_2$$

$$= \left(\frac{2^{\mu}}{\pi s_0^{\ \mu+1} h_{b_3}^{\ \mu}} \right) \int_{-\infty}^{+\infty} \left[\sum_{j=0}^{\mu+1} \binom{\mu+1}{j} (z_2\sigma_2)^j t^{\mu+1-j} \exp\left(-\frac{z_2^{\ 2}}{2} \right) \sum_{i=0}^{\mu} \binom{\mu}{i} (\sigma_1)^i f^{\mu-i} \int_{-\infty}^{+\infty} z_1^{\ i} \exp\left(-\frac{z_1^{\ 2}}{2} \right) dz_1 \right] dz_2$$

According to the MathWorld website (Weisstein, 2005), it follows that

$$\int_{-\infty}^{+\infty} z_1^{\,i} \exp\left(-\frac{z_1^{\,2}}{2}\right) dz_1 = \begin{cases} 0 & \text{if i is odd.} \\ \sqrt{2\pi}(i-1)!! & \text{if i is even.} \end{cases}$$

Therefore,

$$E[I_2] = \left(\frac{2^{\mu}}{\pi s_0^{\,\mu+1} h_{b_3}^{\,\mu}}\right) \int_{-\infty}^{+\infty} \left[\sum_{j=0}^{\mu+1} \binom{\mu+1}{j} (z_2\sigma_2)^j t^{\mu+1-j} \exp\left(-\frac{z_2^{\,2}}{2}\right) \sum_{i=0,i=even}^{\mu} \sqrt{2\pi} \binom{\mu}{i} (\sigma_1)^i f^{\mu-i} (i-1)!! \right] dz_2$$

$$= \left(\frac{2^{\mu}}{\pi s_0^{\,\mu+1} h_{b_3}^{\,\mu}}\right) \sum_{i=0,i=even}^{\mu} \sqrt{2\pi} \binom{\mu}{i} (\sigma_1)^i f^{\mu-i} (i-1)!! \left[\sum_{j=0}^{\mu+1} \binom{\mu+1}{j} (\sigma_2)^j t^{\mu+1-j} \int_{-\infty}^{+\infty} z_2^{\,j} \exp\left(-\frac{z_2^{\,2}}{2}\right) dz_2 \right]$$

$$= \left(\frac{2^{\mu+1}}{s_0^{\,\mu+1} h_{b_3}^{\,\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} (\sigma_1)^i f^{\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+1} \binom{\mu+1}{j} (\sigma_2)^j t^{\mu+1-j} (j-1)!!,$$

With similar manipulations, the following equations can be obtained.

$$E\left[I_2^{\,2}\right] = \left(\frac{2^{2\mu+2}}{s_0^{\,2\mu+2} h_{b_3}^{\,2\mu}}\right) \sum_{i=0,i=even}^{2\mu} \binom{2\mu}{i} \sigma_1^{\,i} f^{2\mu-i} (i-1)!! \sum_{j=0,j=even}^{2\mu+2} \binom{2\mu+2}{j} \sigma_2^{\,j} t^{2\mu+2-j} (j-1)!!,$$

$$Var[I_2] = E\left[I_2^{\,2}\right] - \left(E\left[I_2\right]\right)^2$$

$$= \left(\frac{2^{2\mu+2}}{s_0^{\,2\mu+2} h_{b_3}^{\,2\mu}}\right) \sum_{i=0,i=even}^{2\mu} \binom{2\mu}{i} \sigma_1^{\,i} f^{2\mu-i} (i-1)!! \sum_{j=0,j=even}^{2\mu+2} \binom{2\mu+2}{j} \sigma_2^{\,j} t^{2\mu+2-j} (j-1)!!$$

$$- \left[\left(\frac{2^{\mu+1}}{s_0^{\,\mu+1} h_{b_3}^{\,\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} (\sigma_1)^i f^{\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+1} \binom{\mu+1}{j} (\sigma_2)^j t^{\mu+1-j} (j-1)!! \right]^2,$$

$$E\left[I_1 I_2\right] = \left(\frac{2^{\mu+2}\lambda}{s_0^{\,\mu+2} h_{b_3}^{\,\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} \sigma_1^{\,i} f^{\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+2} \binom{\mu+2}{j} \sigma_2^{\,j} t^{\mu+2-j} (j-1)!!,$$

$$Cov(I_1, I_2) = E\left[I_1 I_2\right] - E\left[I_1\right] E\left[I_2\right]$$

$$= \left(\frac{2^{\mu+2}\lambda}{s_0^{\,\mu+2} h_{b_3}^{\,\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} \sigma_1^{\,i} f^{\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+2} \binom{\mu+2}{j} \sigma_2^{\,j} t^{\mu+2-j} (j-1)!!$$

$$- \left(\frac{2^{\mu+2}\lambda t}{s_0^{\,\mu+2} h_{b_3}^{\,\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} (\sigma_1)^i f^{\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+1} \binom{\mu+1}{j} (\sigma_2)^j t^{\mu+1-j} (j-1)!!.$$

Therefore, Equations (3.47) and (3.48) can be obtained by the following equations.

$$t_{a_t} = E\left[T_{a_t}\right] = E\left[I_1\right] + E\left[I_2\right]$$

$$= \frac{2\lambda t_p}{s_0} + \left(\frac{2^{\mu+1}}{s_0^{\,\mu+1} h_{b_3}^{\,\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} (\sigma_1)^i f_1^{\,\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+1} \binom{\mu+1}{j} \left(\sigma_t^p\right)^j t_p^{\,\mu+1-j} (j-1)!!,$$

$$\sigma_t^{a_t} = \sqrt{Var[I_1] + Var[I_2] + 2Cov(I_1, I_2)}$$

$$= \sqrt{\begin{aligned} &\frac{4\lambda^2}{s_0^{\,2}}\left(\sigma_t^p\right)^2 + \left(\frac{2^{2\mu+2}}{s_0^{\,2\mu+2} h_{b_3}^{\;2\mu}}\right) \sum_{i=0,i=even}^{2\mu} \binom{2\mu}{i} \sigma_1^i f_1^{2\mu-i} (i-1)!! \sum_{j=0,j=even}^{2\mu+2} \binom{2\mu+2}{j} \left(\sigma_t^p\right)^j t_p^{\,2\mu+2-j} (j-1)!! \\ &- \left[\left(\frac{2^{\mu+1}}{s_0^{\,\mu+1} h_{b_3}^{\;\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} \sigma_1^i f_1^{\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+1} \binom{\mu+1}{j} \left(\sigma_t^p\right)^j t_p^{\,\mu+1-j} (j-1)!!\right]^2 \\ &+2\left(\frac{2^{\mu+2}\lambda}{s_0^{\,\mu+2} h_{b_3}^{\;\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} \sigma_1^i f_1^{\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+2} \binom{\mu+2}{j} \left(\sigma_t^p\right)^j t_p^{\,\mu+2-j} (j-1)!! \\ &-2\left(\frac{2^{\mu+2}\lambda t_p}{s_0^{\,\mu+2} h_{b_3}^{\;\mu}}\right) \sum_{i=0,i=even}^{\mu} \binom{\mu}{i} \sigma_1^i f_1^{\mu-i} (i-1)!! \sum_{j=0,j=even}^{\mu+1} \binom{\mu+1}{j} \left(\sigma_t^p\right)^j t_p^{\,\mu+1-j} (j-1)!! \end{aligned}}\,.$$

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