QUANTITATIVE ANALYSIS

IN SUPPLY CHAIN OUTSOURCING

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Quantitative Analysis in Supply Chain Outsourcing

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CERTIFICATE OF ORIGINALITY

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Abstract

In the manufacturing industry, the production output is constrained due to the limited capital flow. It is a fatal weakness to the manufacturer as well as to component suppliers. When component suppliers incur budget constraints, the manufacturer can share a proportion of each supplier's production cost to relieve her from such restraint. We intend to study the manufacturer's optimal solution of the cost share amounts and analyze how such sharing will affect the suppliers' performances. Since it involves multiple suppliers, we will further explore how different decision sequences will affect the equilibrium. On the other hand, when two competing manufacturers (or firms) incur capacity constraints in the demand market, they have to make the decisions on the capacity investment. Therefore, we propose two types of model in this thesis to study the above problems.

In the first model, we consider one manufacturer who sources multiple complementary components, each from one independent supplier who has the budget constraint. Facing with the uncertain demand, the manufacturer decides on the cost share amounts given to his suppliers. We analyze three different decision sequences in this model: a) simple sequential decisions in wholesale prices, b) simultaneous decisions in wholesale prices and c) hybrid sequential decision in alternating cost share amounts and wholesale prices.

Compare first two sequences, we find that the manufacturer strictly prefers the simultaneous setting. On the contrary, the supplier who makes the decision first in the simple sequential setting gets worse in the simultaneous decision sequence. We then study the manufacturer's optimal solutions in the hybrid sequential decision setting. Different from the simple sequential setting, the equilibrium is asymmetric in hybrid

setting. Furthermore, the supplier who decides first also charges a higher wholesale price than other suppliers which underlines our exploration that early action boosts the supplier's bargaining power no matter whether the manufacturer shares the cost or not.

In the second model, we consider a setting of two firms serving one market whose demand is price sensitive and uncertain. We characterize the equilibrium capacity and production decisions by the two firms. The result shows that the firm whose process efficiency is more prone to improving as capacity expands will invest in more capacity and achieve a more efficient process given that production is not overly labor and material intensive.

To the best of our knowledge, this is the initial research to study cost sharing in a multiple-suppliers model with three decision sequences, especially with the new hybrid decision setting. While the cost sharing is always beneficial to the manufacturer, we surprisingly find that the supplier who charges the highest wholesale price is averse it even if her cost burden is loosed after cost sharing. We also show the conditions under which cost sharing benefits the suppliers. Furthermore, the result also illustrates that different decision sequences have different impacts on the equilibrium and the resulting channel performance. In the second model, we find out the optimal solution under the circumstance where two firms make the competing decisions on the capacity investment.

PUBLICATIONS ARISING FROM THE THESIS

Hong Yan, Li Jiang and **Yanmin Jiang**. "The Effect of Cost Sharing in a Decentralized Supply Chain", Working Paper, Presented in MSOM 2012

Li Jiang and **Yanmin Jiang**. "Capacity Investment and Process Efficiency at Flexible Firms", Working Paper

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Chapter 1 Introduction

1.1 Background of Research

In the manufacturing industry, especially in the automobile trade, there are two major approaches for the manufacturer to procure components: producing himself or purchasing from the component suppliers. No matter which approach the manufacturer chooses, it inevitably encounters with the budget or capacity constraint due to the limited resource. When outsourcing components from suppliers who have such constraints, the manufacturer usually pushes his suppliers to reduce their costs as low as possible. Thus, the manufacturer can capture a volume advantage in the final market. However, it is hard for component suppliers to cut the cost to the ideal extent while still offering a relative low wholesale price to the manufacturer. Once these component suppliers go to bankrupt due to heavy cost burdens, it could be a disaster to all players in the supply chain, including the manufacturers who regularly order components from them. Although the manufacturer can seek for other suppliers, it is both time and cost consuming. Moreover, it also takes a longer time for the manufacturer to establish a stable relationship with new suppliers. In this case, we incline to the approach that the manufacturer may finance his important suppliers rather than harshly pushing them. One example is that when Ford Motor Company's (FM's) supplier, Visteon, was threatened to bankruptcy in 2005, FM paid between \$1.6 billion and \$1.8 billion to help Visteon in restructuring (White 2005). Another example shows that, when the component supplier Collins & Aikman—about 90%

of vehicles made in North America have at least one component produced by this supplier—considered filing to bankruptcy in 2005, the Big Three auto manufacturers had seldom choice but to sustain the supplier by costing 532 million dollars (Barkholz et al. 2007). The similar attitude of the above manufacturers demonstrates the necessity to finance the suppliers. The third example shows the benefits for the manufacturer to help his suppliers. In 2005, for example, General Motors Corporation's (GM's) annual output decreased by 1 million vehicles due to Delphi's bankruptcy (Delphi is GM's largest part supplier, and correspondingly, GM is Delphi's largest customer, accounting for half of Delphi's sales). In the meanwhile, Toyota—GM's main competitor in automobile market—promoted the production capacity of his motor-vehicle assembly plant in Canada by 50%. In that year, stable supply of the components helped Toyota to win a much higher market share than GM.

Motivated by these cases, we derive the first model in this dissertation in which the manufacturer faces a price-sensitive and uncertain demand. The manufacturer orders N complementary components from N independent suppliers, respectively. Without loss of generality, we assume that only one unit of each N components is needed by one unit of final product. Each supplier has an upper limit on the capital flow, which further restricts the supplier's production capacity. That is to say, the supplier could only produce a limited quantity of product with the constraint of capital flow. It is straightforward that the operation of production cannot sustain without sufficient cashes. We also assume that the manufacturer can only share part of suppliers' costs, namely "deductable cost". Chapter 3 and 4 respectively consider two canonical decision sequences, sequential decision sequence and simultaneous decision sequence. In the sequential decision setting, the manufacturer first decides on the cost share amounts given to the suppliers. Then the suppliers sequentially decide the wholesale price. Finally, the manufacturer orders the production quantity. To distinguish it from the new sequential sequence showed below, we name this canonical sequence as *simple sequential sequence*. In the simultaneous setting, the suppliers simultaneously decide their own wholesale prices after the manufacturer deciding the cost share amounts. To better characterize players' behaviors in reality, we further consider a new decision sequence besides the above two typical scenarios. The new setting contains a decision sequence of alternating cost share amount and wholesale price. That is, the manufacturer only decides on the cost share amount to one supplier at the beginning of the scenario, and then the corresponding supplier chooses the wholesale price. After N rounds of such competing decisions, the manufacturer orders the quantity. All the decisions are made before the realization of the demand uncertainty. We term this sequence as *hybrid sequential sequence*. Each supplier faces a direct competition from a potential supplier who produces the same component. The intensity of the competition is measured by the cost difference between two suppliers (Jiang and Wang 2007).

In the second model, when the manufacturer himself incurs costs to build capacity and produce them, it may relate to the manufacturing flexibility that has long aroused the interests in the academic community. One of the flexibility endowments that have received much attention is volume flexibility that confers one firm with the capability to produce below capacity when the realized market demand is low. The furniture industry provides a canonical example that is equipped with such a capability. The entire industry is highly market oriented and under the strong influence of economy to display a cyclical pattern. This makes volume flexibility a must-have capability. To focus on the stratgic implications of flexibility capability, the vast majority of existing literature assumes away production costs, and neglect the mutual influences between capacity and production cost. Therefore, we set up the second model in this dissertation to investigate the competitive capacity and production decisions by volume-flexible firms. We incorporate the impacts of capacity on the production cost through process efficiency in a setting of two firms serving a price sensitive and uncertain market. In addition to the cost to invest in capacity, each firm incurs a production cost that consists of two components. One is the *input* cost that is in direct proportion to the level of production, including material and direct labor costs. The other is the *efficiency cost* which can be attributed to the organization and management of the production process. We model process efficiency by the curvature of efficiency cost, referred to as *efficiency factor*, and use an *efficiency index* to capture the sensitivity of process efficiency with respect to capacity expansion.

1.2 Aims and Objectives

Our models intended to study the following research questions:

1. In what cases should the manufacturer share his component suppliers' production costs and how much should he share?

2. How cost-sharing decisions affect suppliers' profits? Is it beneficial to all suppliers?

3. How different decision sequences affect the effect of cost sharing on the manufacturer and suppliers?

4. Does production cost sharing improve the channel profit?

5. How two firms with volume flexibility competitively decide their capacity investments in a price-sensitive market?

Our work makes four major contributions to the existent literatures. Firstly, we explore a new decision sequence with alternating cost sharing decision (by the manufacturer) and wholesale price decision (by suppliers). This hybrid sequence is more approximately to the reality compared to the simple sequential sequence or the simultaneous decision sequence. Secondly, we reveal the effect of cost sharing in the production process with one manufacturer and multiple component suppliers. Thirdly, we study how the decision sequence may affect the cost sharing decision. Last but not the least, we examine how the volume-flexible firms compete in the capacity which has impacts on the production cost through process efficiency.

The remainder of the dissertation is organized as follow: Chapter 2 introduces the related literatures we have referred in this dissertation. Chapter 3 describes the simple sequential wholesale price decision sequence in a Stackelberg game. We adopt backward induction to solve this problem and then compare the channel performance with no cost sharing setting. Chapter 4 studies the simultaneous decision sequence and compares the results with that in Chapter 3. We analyze the effect of different decision sequences on different players in the cost sharing setting. Chapter 5 reveals the equilibrium of cost sharing amounts in the hybrid decision sequence. We then analyze the effect of cost sharing on each player's profit under such sequence. Chapter 6 explores the manufacturing flexibility and capacity investment by setting up the second model. It then follows by Chapter 7 as the conclusions and future work remarks.

Chapter 2 Literature Review

2.1 Literatures on Cost Share Problem

Since Pasternack (1985) first launched the notion of supply chain contract, many researchers have designed a variety of contract mechanisms to coordinate the supply chain, such as quantity-discount contract, buy-back contract, and option contract etc. Kreps (1990) considers a monopoly setting in which the manufacturer, as the Stackelberg leader, decides the wholesale price by maximizing his own profit. The retailer, as a follower, could only choose "accept" or "reject" this wholesale price. If the expected revenue of "accept" is larger than its opportunity cost, the retailer will accept this contract, otherwise, reject. Spengler (1950), however, illustrates that double marginalization leads to the supply chain inefficiency when price-only contract is adopted. In this case, the retail price in a decentralized system is higher than that in a centralized system, and thus decreasing the market demand. Our research still assumes the price-only contract because it is widely used in business practice due to its feasibility. An extensive review in contracting and supply chain coordination can be found in Lariviere (1999).

Petrruzi and Dada (1999) assumes an uncertain and price sensitive demand based on which we build up our demand function. We also refer to Wang et al. (2004) and Jiang and Wang (2010) for characterizing an assembly system who faces a random and price-elasticity market demand. Wang et al. (2004) demonstrates the equilibrium in the de-centralized supply chain in which the manufacturer contracts with the supplier by revenue sharing scheme. Jiang and Wang (2010) first introduce the suppliers' direct competition in the decentralized supply chain. Their results show that the manufacturer benefits from suppliers' direct competition, which is according with our preliminary results in an alternative supplier competition model. They also point out that the manufacturer prefers mergers of suppliers, especially the merger of those suppliers who face less direct competition. Lariviere and Porteus (2001) analyze the pricing decision in a single-manufacturer and single-retailer newsvendor model. They find out that there exists a unique solution in this monopoly pricing setting if the stochastic demand function suffices IGFR condition. Lariviere (2006) further discusses the properties of IGFR condition. We apply this condition in our models.

Literatures on supply disruption and supplier bankruptcy also give us some insights in supply chain outsourcing. Bollapragada et al. (2004) studies the inventory and supply arrangements under supply uncertainty. Based on this paper, Tomlin & Wang (2005) and Tomlin (2006) make extensions in different dimensions. Tomlin & Wang (2005) studies the manufacturer's choice of parts supplier in an unstable supply chain. This model contains two suppliers with different costs. The result shows that if the manufacturer is risk-neutral, the optimal policy is to order parts from both suppliers. If not, it is better to order parts from one of them. Tomlin (2006) considers a model which contains one manufacturer and two different suppliers (one is reliable and another is not). The result indicates that given a production period, the manufacturer is prone to order products from the reliable supplier only if the frequency of supply disruption is high. However, if the duration of supply disruption is short and the manufacturer is risk-averse, it is better for the manufacturer to order products from both suppliers. Hendricks and Singhal (2003), Hendricks and Singhal (2005a) and Hendricks and Singhal (2005b) study how the supply disruption affects the manufacturer's performance. Sheffi (2001), Chopra and Sodhi (2004) and Sheffi (2005) analyze the causes of supply disruption via case study. Tang (2006) gives a recent review of supply disruption management. Swinney and Netessine (2009) considers a two-period contracting game between a single buyer and two identical suppliers. The buyer faces deterministic demand while two suppliers have uncertain production costs. The result shows that the long-term contract, which allows the buyer to offer a higher wholesale price to the supplier in the first period, can reduce the supplier's possibility of default in the second period. Babich et al. (2007) studies the effects of supplier disruption risk in a channel where one retailer deals with two competing risky suppliers. The results show that the retailer prefers the supplier with highly correlated default events while the suppliers and the channel prefer the negatively correlated defaults. Babich (2007) characterizes a stochastic process of supplier's bankruptcy and studies the manufacturer's financial arrangement to the supplier. This paper gives the manufacturer's optimal joint ordering and subsidy policy.

Some papers focus on the capacity investment in supply chain outsourcing. Taylor and Plambeck (2007) studies the effect of two alternative simple contracts on the performance of capacity investment. The results show that price and quantity contract is preferred when the production cost is low and either the capacity cost is low or the discount factor is high. Otherwise, price-only contract is chosen. Li and Debo (2009) compares the performance of capacity investment between second sourcing and sole sourcing and points out that the results depend on the demand distribution and capacity cost.

Several economics researchers study cost sharing from the viewpoint of policy-making. Mas-Colel and Silvestre (1989) reveals the cost share equilibrium for the benefit approach to the allocation of the public good costs. Adams and Rask (1968) explores the reason why cost-share lease is less preferred than output-share lease in less developed country although the former is more social efficiency. In the operations research, Bhaskran and Krishnan (2009) studies the cost and work sharing between two collaborating firms in the new product development. In this paper, development cost sharing is a kind of investment sharing. The result shows that both cost and work sharing are particularly relevant for products with no preexisting revenues. Moreover, cost sharing plays an important role in environments where new-to-world product projects have significant timing uncertainty.

2.2 Literatures on Flexibility

The second model relates to the stream of literature on flexibility. Gerwin (1993) classifies manufacturing flexibility into six categories, and outlines a procedure for altering the type and amount of flexibility over time. The analytical models in OM tap two flexibility types, product flexibility and volume flexibility. Fine and Freund (1990) model a firm producing n products. The firm first installs capacities for N dedicated resources and a flexible resource that produce all products. After the actual demand curve is known, produces under capacity constraints. They find that value of product flexibility depends on cost difference between dedicated and flexible technologies. Van Mieghem (1998) develops a similar model and finds flexibility is beneficial when demands are perfectly correlated if product margins are different. Other works on similar subjects include Harrison and Van Mieghem (1998), Netessine et al. (2002), and Tomlin and

Wang (2005). Jordan and Graves (1995) examine total flexibility versus partial flexibility under the concept of chains, where each chain consists of product-plant links with more links corresponding to higher flexibility. They find that adding limited flexibility can achieve nearly all the benefits of total flexibility. Graves and Tomlin (2003) extend to a multi-echelon setting. These papers center on a monopoly setting.

On strategic selections of product flexibility, Roller and Tombak (1990, 1993) model two firms competing on technology. A firm with flexible technology enters two markets, while a firm with dedicated technology enters one market. They show that firms end up with a Prisoner's dilemma-like situation: while each can enter one market and earn a monopoly profit, both choose to enter two markets with flexible technology under the threat that the rival may be flexible. The retail prices in these papers are exogenous. Chod and Rudi (2005) endogenizes price decisions, and find that the capacity and profit of a firm with a flexible resource for two products increase with demand uncertainty. Goyal and Netessine (2007) investigate flexibility selection by two firms serving two markets with price sensitive and uncertain demand. Anand and Girotra (2008) endogenizes the supply chain configuration for firms to choose between early differentiation and late differentiation, which is essentially the strategic decision on product flexibility.

Research on volume flexibility is relatively sparse. Several models in Van Meighem and Dada (1999) on the effects of different sequences of product and price decisions have the flavor of volume flexibility. They focus on a monopoly setting with limited analysis of duopoly models. Anupindi and Jiang (2008) provides a detailed analysis of volume flexibility in both monopoly and duopoly models with price sensitive and uncertain demand. While not very closely related to the model in this paper, we deem it worthwhile to refer to one stream of Economics literature that defines flexibility as efficient operation scale. Once committed to a capacity, a flexible firm can produce above it, but at a much higher marginal cost. Higher flexibility corresponds to lower marginal cost increase. Vives (1986) uses this notion in an N-firm oligopoly where firms first commit to a capacity and then produce. The result shows that capacity is a good commitment variable when technology is inflexible. The equilibrium is close to the Cournot outcome. However, capacity is not a good commitment variable when technology is flexible and equilibrium is close to Bertrand outcome with low profit margin. Boyer and Moreaux (1997) considers a discrete scenario when flexibility allows firms to produce at identical marginal costs for all output levels, but inflexibility makes production over pre-committed capacity exorbitantly costly. They show that flexibility choices and market equilibrium depend on market volatility and market size.

In the extant literature, the study of flexibility focuses on the value of certain capability and firms' strategic flexibility selections. More often than not, the conclusion is to install flexibility when the added cost is not too high. Production cost is usually assumed away for tractability. We shift the focus to firms' capacity and production efficiency on a given flexibility platform, where capacity enforces output limit and affects the production cost through its effect on the production efficiency. We incorporate non-zero production costs and allow them to differ across firms, and explicitly define process efficiency. The notion was initiated by Stigler (1939), and later formalized by Marschak and Nelson (1962). Mills (1984) proposes a functional form for production cost; and shows if there is a continuum of production costs for varying flexibility levels, firms will use a more flexible cost structure when demand is more volatile. We modify his proposed production cost function in our model. Mills (1986) extends his earlier work to a setting where the firms have a finite set of flexibility options. All the past papers assume a perfectly competitive market with price-taking firms, but neglect responsive capabilities and capacity limits at the firms. We analyze price-setting firms with volume flexibility, and impose capacity constraints in an imperfectly competitive market.

Chapter 3 Simple Sequential Decision in Wholesale Prices

We consider one manufacturer who sources complementary components from N suppliers. Each supplier incurs a production cost of $(k_i + c_i)$. k_i presents the cost of supplier i's core production steps which is mastered only by the supplier and thus cannot be outsourced. c_i is the cost of the normal steps which could be shared by the manufacturer. We then term c_i as supplier i's *deductable cost*. The production cost at the manufacturer is c_M. Each supplier also has a capital constraint B_i. Thus, it derives supplier i's capacity constraint as $B_i/(k_i + c_i)$.

The market demand is a random factor ε with CDF $F(\cdot)$ and PDF $f(\cdot)$. The probability distribution has support on $[A, B]$ with $B > A \geqslant 0$. For tractability, we consider an example where ε follows a uniform distribution on $[0, H]$. Then we have $f(x) = 1/H, F(x) = x/H$. The market price p is exogenous.

We then model the implication of cost sharing decision made by the manufacturer who may relieve suppliers' from capacity constraints. To each supplier, the manufacturer can decide to share supplier i's cost at the amount of Δ_i, where $0 \leqslant \Delta_i \leqslant c_i$. Especially, when the cost share amount is equal to c_i, we say that supplier i is fully-shared.

Denote $c = c_M + \sum_{i=1}^{n} c_i$ as the total unit production cost without cost sharing. After cost sharing, the total production cost turns to $\bar{c} = c + \sum_{i=1}^{n}(\beta_i \Delta_i)$, where $\beta_i > 0$ indicates that the manufacturer costs more in producing the normal steps than the supplier does. Since the shared normal steps are attributed to the suppliers instead the manufacturer, this assumption makes sense in that the manufacturer who is not so familiar with the normal steps costs more in production process than the supplier does.

In the sequential model, the manufacturer first decides cost share amounts to N suppliers respectively. Then the suppliers sequentially decide their own wholesale prices. Finally, the manufacturer decides the ordering quantity.

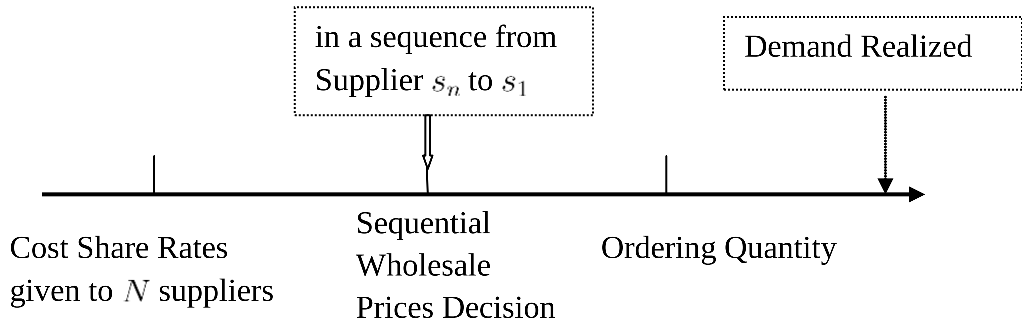


Figure 3.1 Decision Sequence in Simple Sequential Setting

3.2 Stackelberg Game in the Simple Sequential Setting

3.2.1 Manufacturer's Decision on Production Quantity

For given his own decisions on cost share amounts $\Delta_i, 0 \leqslant \Delta_i \leqslant 1, (i = 1, ..., n)$ and suppliers' decisions on the wholesale price w_i, the manufacturer chooses the ordering quantity q to maximize his profit. We have

$$\begin{aligned} \Pi_M &= pE[min(D,q)] - (\bar{c} + \sum_{i=1}^{n} m_i)q \\ &= (p - \bar{c} - \sum_{i=1}^{n} m_i)q - \frac{pq^2}{2H}, \end{aligned} \tag{3.1}$$

where $m_i = w_i - (k_i + c_i - \Delta_i)$ represents supplier i's marginal profit.

Intuitively, the optimal quantity q_d^* is given by

$$q^*(m_i, \Delta_i) = H - \frac{H}{p}(\bar{c} + \sum_{i=1}^{n} m_i) \tag{3.2}$$

3.2.2 Suppliers' Problem

By given the manufacturer's decisions on cost sharing amounts Δ_i and knowing his response to the wholesale prices, the suppliers sequentially decide their own wholesale prices. Without loss of generality, we assume that supplier s_n decides first and then supplier s_{n-1} does, until supplier s_1. Denote $G_j(\Delta_j) = B_j/(k_j + c_j - \Delta_j),\ j = 1, ..., n$ as supplier j's capacity after cost sharing, named *attainable capacity*. Especially, when $\Delta_j = 0$, $G_j(0)$ indicates supplier j's *initial capacity*. Then we have supplier s_i's expect profit as follows,

$$\begin{aligned} &\Pi_{s_1} = m_{s_1} q \\ &s.t. \quad q \leqslant G_{s_1} \end{aligned} \tag{3.3}$$

Solving this problem, we have

$$m^*_{s_1}(m_{-s_1}, \Delta) = \begin{cases} \frac{1}{2}(p - \bar{c} - \sum_{t=2}^{n} m_{s_t}) & \bar{c} + \sum_{t=2}^{n} m_{s_t} \geqslant p - \frac{2p}{H} G_{s_1} \\ (p - \bar{c} - \sum_{t=2}^{n} m_{s_t}) - \frac{p}{H} G_{s_1} & c < \bar{c} + \sum_{t=2}^{n} m_{s_t} < p - \frac{2p}{H} G_{s_1} \end{cases} \tag{3.4}$$

where $\Delta = (\Delta_1, \Delta_2, ..., \Delta_n)$ and $m_{-s_i} = (m_{s_{i+1}}, m_{s_{i+2}} .., m_{n-1}, m_{s_n})$. To facilitate the denotation, we simplify $G_j(\Delta_j)$ to G_j when analyzing suppliers' best responses of wholesale prices. Since $\bar{c} > c$ and $\sum_{t=2}^{n} m_{s_t} \geqslant 0$, we must have $\bar{c} + \sum_{t=2}^{n} m_{s_t} > c$.

Denote $G_{r_i}(\Delta_{r_i}) = min\left(G_{s_1}(\Delta_{s_1}), G_{s_2}(\Delta_{s_2}), ..., G_{s_i}(\Delta_{s_i})\right),\ i = 1, ..., n.$

$G_{r_i}(\Delta)$ indicates the minimum *attainable capacity* among supplier s_1 to s_i. Particularly, if two or more suppliers reach the minimum, we can arbitrarily choose one as supplier r_i. Furthermore, when $\Delta = 0$ and $i = n$, $G_{r_n}(0)$ indicates the minimum capacity among N

suppliers before cost sharing, and the corresponding supplier is denoted as *supplier* I. When $\Delta = C = (c_1, c_2, ..., c_n)$, $G_{r_n}(C)$ indicates the minimum capacity among N suppliers whose attainable costs are totally shared by the manufacturer. Moreover, the corresponding supplier is denoted as *supplier* F. That is, supplier I indicates the supplier who has the minimum initial capacity and supplier F indicates the one who has the minimum maximum capacity. Both of them are independent of the decision sequence. Similarly, if two or more suppliers reaches $G_{r_n}(0)$ or $G_{r_n}(C)$, we can arbitrarily choose one as supplier I or F.

Then we obtain supplier s_i's best response function as below,

$$m_{s_i}^*(m_{-s_i}, \Delta) = \begin{cases} \frac{1}{2}(p - \bar{c} - \sum_{t=i+1}^{n} m_{s_t}) & \bar{c} + \sum_{t=i+1}^{n} m_{s_t} \geqslant p - \frac{2^i p}{H} G_{r_i} \\ (p - \bar{c} - \sum_{t=i+1}^{n} m_{s_t}) - \frac{2^{i-1} p}{H} G_{r_i} & otherwise \end{cases} \tag{3.5}$$

Proof:

It follows (3.4) that,

$$\Pi_{s_2} = m_{s_2} q = \begin{cases} m_{s_2}(p - \bar{c} - \sum_{t=2}^{n} m_{s_t}) \frac{H}{2p} & m_{s_2} \geqslant p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - \frac{2p}{H} G_{s_1} \\ m_{s_2} G_{s_1} & otherwise \end{cases},$$

$$s.t. \begin{cases} m_{s_2} \geqslant p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - \frac{2p}{H} G_{s_2} & m_{s_2} \geqslant p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - \frac{2p}{H} G_{s_1} \\ G_{s_1} \leqslant G_{s_2} & otherwise \end{cases}.$$

where $m_{s_2} G_{s_1}$ is linearly increase in m_{s_2}. The function $m_{s_2}(p - \bar{c} - \sum_{t=2}^{n} m_{s_t}) H/2p$, which is unimodal in m_{s_2}, reaches maximum at $\hat{m}_{s_2} = (p - \bar{c} - \sum_{t=3}^{n} m_{s_t})/2$.

When $G_{s_1} \leqslant G_{s_2}$,

$$\Pi_{s_2} = \begin{cases} m_{s_2}(p - \bar{c} - \sum_{t=2}^{n} m_{s_t})\frac{H}{2p} & m_{s_2} \geqslant p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - \frac{2p}{H}G_{s_1} \\ m_{s_2}G_{s_1} & p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - \frac{2p}{H}G_{s_2} \leqslant m_{s_2} < p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - \frac{2p}{H}G_{s_1}. \end{cases}$$

Thus, suppliers_2's best response is $m_{s_2} = max(\hat{m}_{s_2}, p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - 2pG_{s_1}/H)$.

When $G_{s_1} > G_{s_2}$, we have,

$$\Pi_{s_2} = m_{s_2}(p - \bar{c} - \sum_{t=2}^{n} m_{s_t})\frac{H}{2p}$$
$$s.t. \quad m_{s_2} \geqslant p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - \frac{2p}{H}G_{s_2}$$

Thus, supplier s_2's best response is $m_{s_2} = max(\hat{m}_{s_2}, p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - 2pG_{s_2}/H)$. Combine the above cases, we obtain supplier s_2's best response as follows,

$$m^*_{s_2}(m_{-s_2}, \mathit{\Delta}) = max\left(\hat{m}_{s_2}, p - \bar{c} - \sum_{t=3}^{n} m_{s_t} - \frac{2p}{H}G_{r_2}\right),$$

where $G_{r_2} = min(G_{s_1}, G_{s_2})$.

From the above result, we conjecture that, for supplier s_i, the best response to m_{-s_i} and $\mathit{\Delta}$ is $m^*_{s_i}(m_{-s_i}, \mathit{\Delta}) = max(\hat{m}_{s_i}, p - \bar{c} - \sum_{t=i}^{n} m_{s_t} - 2^{i-1}pG_{r_i}/H)$. The mathematical induction is adopted to prove this conjecture.

For $i = k$, we assume that the above best response is established. It can be rewritten as

$$m^*_{s_k}(m_{-s_k}, \mathit{\Delta}) = \begin{cases} \frac{p - \bar{c} - \sum_{t=k+1}^{n} m_{s_t}}{2} & \bar{c} + \sum_{t=k+1}^{n} m_{s_t} \geqslant p - \frac{2^i p}{H}G_{r_k} \\ p - \bar{c} - \sum_{t=k+1}^{n} m_{s_t} - \frac{2^{k-1}p}{H}G_{r_k} & otherwise \end{cases}.$$

Thus, for $i = k + 1$, supplier s_{k+1}'s objective function is given by

$$\Pi_{s_{k+1}} = \begin{cases} m_{s_{k+1}}(p - \bar{c} - \sum_{t=k+1}^{n} m_{s_t})\dfrac{H}{2^k p} & m_{s_{k+1}} \geqslant p - \bar{c} - \sum_{t=k+2}^{n} m_{s_t} - \dfrac{2p^k}{H} G_{r_{k+1}} \\ m_{s_{k+1}} G_{r_k} & m_{s_{k+1}} \leqslant p - \bar{c} - \sum_{t=k+2}^{n} m_{s_t} - \dfrac{2p^k}{H} G_{r_k} \end{cases},$$

where $m_{s_{k+1}} G_{r_k}$ is linearly increase in $m_{s_{k+1}}$. Since $m_{s_{k+1}}(p - \bar{c} - \sum_{t=k+1}^{n} m_{s_t})H/(2^k p)$ is unimodal in $m_{s_{k+1}}$, it reaches maximum at $\hat{m}_{s_{k+1}} = (p - \bar{c} - \sum_{t=k+2}^{n} m_{s_t})/2$. $\hat{m}_{s_{k+1}} = \frac{1}{2}\left(p - \bar{c} - \sum_{t=k+2}^{n} m_{s_t}\right)$. Similar to supplier s_2's problem, we have

$$m^*_{s_{k+1}}(m_{-s_{k+1}}, \Delta) = max\left(\hat{m}_{s_{k+1}}, p - \bar{c} - \sum_{t=k+2}^{n} m_{s_t} - \frac{2^k p}{H} G_{r_{k+1}}\right).$$

Thus, the result of (3.5) is proved.

3.3.3 Cost Sharing Decision

Knowing that suppliers choose wholesale prices according to (3.5), the manufacturer decides on cost share amounts Δ to maximize his own profit,

$$\Pi_M(\Delta) = \begin{cases} \dfrac{H[p - \bar{c}(\Delta)]^2}{2^{2n+1} p} & \bar{c}(\Delta) \geqslant p - \dfrac{2^n p}{H} G_{r_n}(\Delta) \\ \dfrac{p}{2H}[G_{r_n}(\Delta)]^2 & otherwise \end{cases} \tag{3.6}$$

Recalling the definition of $\bar{c}(\Delta)$ and $G_{r_n}(\Delta)$, we can find that both of them increase in Δ. Thus, the optimal solution is unique.

Theorem 3.1 *When suppliers sequentially decide the wholesale prices, the manufacturer has a unique optimal solution for the cost share amounts at the beginning point, which is given by*

$$\Delta^*_{s_i} = \begin{cases} min\left(\bar{\Delta}^+, k_{s_i} + c_{s_i} - \dfrac{B_{s_i} k_F}{B_F}\right) & s_i = I \\ \left[k_{s_i} + c_{s_i} - \dfrac{B_{s_i}(k_I + c_I - \Delta^*_I)}{B_I}\right]^+ & s_i = s_1, s_2, ..., s_n \ and \ s_i \neq I \end{cases} \tag{3.7}$$

where $\bar{\Delta}$ satisfies

$$L(\bar{\Delta}) = p - c - \sum_{i=1}^{n} \left\{ \beta_{s_i} \left[k_{s_i} + c_{s_i} - \frac{B_{s_i}(k_I + c_I - \bar{\Delta})}{B_I} \right]^+ \right\} - \frac{2^n p}{H} G_I(\bar{\Delta}) = 0.$$

Proof:

We know from (3.7) that, $\Pi_M(\Delta)$ increases in Δ when $\bar{c}(\Delta) \geqslant p - 2^n p G_{r_n}(\Delta)/H$ and decreases in it when $\bar{c}(\Delta) < p - 2^n p G_{r_n}(\Delta)/H$. Rewrite $\bar{c}(\Delta) = p - 2^n p G_{r_n}(\Delta)/H$ as $p - c - \sum_{i=1}^{n}\{\beta_{s_i}[k_{s_i} + c_{s_i} - \frac{B_{s_i}(k_I + c_I - \hat{\Delta}_I)}{B_I}]^+\} - 2^n p G_I(\hat{\Delta}_I)/H = 0$, where I indicates the supplier who has the smallest initial capacity. So, the apex of $\Pi_M(\Delta)$ locates at $\Delta_I^* = \hat{\Delta}_I$. For optimality, the manufacturer will keep all the shared suppliers with the same capacity, i.e., $\Delta_j^* = [k_j + c_j - B_j(k_i + c_i - \Delta_i^*)/B_i]^+$ $(i, j = 1, 2, ..., n)$. Therefore, Δ_I^* is supported in the interval of $[0, k_I + c_I - B_I k_F / B_F]$, where F indicates the supplier indicates the supplier who has the smallest fully-shared capacity. Therefore, the optimal cost share rate to supplier I is given by $\Delta_I^* = min(max(0, \hat{\Delta}_I), k_{s_i} + c_{s_i} - B_{s_i} k_F / B_F)$. Correspondingly, the optimal cost share rates to other suppliers are $\Delta_{s_i}^* = [k_{s_i} + c_{s_i} - B_{s_i}(k_I + c_I - \Delta_I^*)/B_I]^+$.

Thus it is proved.

The manufacturer's optimal cost share rates to the suppliers in (3.7) are irrelevant to the suppliers' decision sequence, but affected by their capacity constraints.

Substituting (3.7) into the manufacturer's best response function on the ordering quantity, we have

$$q^* = \begin{cases} \dfrac{H[p - \bar{c}(\Delta_I^*)]}{2^n p} & \bar{c}(\Delta_I^*) \geqslant p - \dfrac{2^n p}{H} G_I(\Delta_I^*) \\ G_I(\Delta_I^*) & otherwise \end{cases} \tag{3.8}$$

We obtain from the above equilibrium that, the threshold for cost sharing is $c = p - 2^n pG_I(0)/H$. When $c \leqslant p - 2^n pG_I(0)/H$, the manufacturer does not need to share supplier's cost as he can extract enough capacity from each supplier. When $c > p - 2^n pG_I(0)/H$, the manufacturer will first share cost with the supplier who has the severest initial capacity constraint, i.e., supplier I, and then together with the supplier whose initial capacity limit is the second severest. Analogously, the manufacturer may share i $(i \leqslant n)$ suppliers' costs until the market demand is satisfied or supplier F is fully shared.

For easier reference, we sort *N* suppliers by their initial capacity in the ascending order, i.e., $G_1(0) \leqslant G_2(0) \leqslant ... \leqslant G_n(0)$. Supplier 1, who has the smallest initial capacity in the inequality, is equivalent to supplier I as we denote before. Since the size of initial capacity is independent of the decision sequence, the denotation of supplier 1 to n is also independent of that of supplier s_1 to s_n which is sorted by the decision sequence of wholesale price. That is, although the set of supplier i $(i = 1, 2, ..., n)$ equals to the set of supplier s_i $(i = 1, 2, ..., n)$, supplier i can be different from supplier s_i.

To specify the manufacturer's cost share decision, we take a two-suppliers setting as example. According to the above definition, supplier s_2 decides her wholesale price ahead of supplier s_1 while supplier 1's initial capacity is smaller than supplier 2's. If supplier 2 is identical to supplier s_2, it indicates that the supplier whose capacity is much tighter decides the wholesale price first. But if she is identical to supplier s_1, it indicates that the supplier whose capacity is much tighter decides the wholesale price later.

As theorem 3.1 illustrated, the decision order of the suppliers' wholesale prices does not affect the manufacturer's cost sharing decision. No matter whether the supplier decides the wholesale price earlier or later, she receives the same cost share rate depending on the suppliers' capacity constraints and market price. Thus, the pattern of the optimal solution $(\Delta^*_{s_1}, \Delta^*_{s_2})$ is symmetric about the straight line $G_{s_1}(0) = G_{s_2}(0)$. So we only need to show the half of the shape by choosing supplier 1 and supplier 2 as the coordinator (See Figure 3.1 in below).

Denote

$L_1(\Delta_1) = p - c - \beta_1\Delta_1 - 2wG_1(\Delta_1)$,

$L_2(\Delta_1) = p - c - \beta_1\Delta_1 - \beta_2[k_2 + c_2 - B_2(k_1 + c_1 - \Delta_1)/B_1] - 2wG_1(\Delta_1)$,

where $w = 2p/H$.

Thus, we can specify the optimal cost share rates as follows,

$$(\Delta_1^*, \Delta_2^*) = \begin{cases} (c_1, 0) & In\ Section\ I \\ (\bar{\Delta}_1, 0) & In\ Section\ II \\ (0,0) & In\ Section\ III \\ \left(\bar{\Delta}_2, k_2 + c_2 - \frac{B_2}{B_1}(k_1 + c_1 - \bar{\Delta}_2)\right) & In\ Section\ IV \\ \left(k_1 + c_1 - \frac{B_1}{B_2}k_2, c_2\right) & In\ Section\ V \\ \left(c_1, k_2 + c_2 - \frac{B_2}{B_1}k_1\right) & In\ Section\ VI \end{cases} \tag{3.9}$$

where

$\bar{\Delta}_1 = k_1 + c_1 - 4wB_1/[p - c - \sigma_3 + \sqrt{(p - c - \sigma_3)^2 + 8w\beta_1 B_1}]$, $\sigma_3 = \beta_1(k_1 + c_1)$,

$\bar{\Delta}_1 = k_1 + c_1 - 4wB_1/[p - c - \sigma_2 + \sqrt{(p - c - \sigma_2)^2 + 8wh}]$, $h = \beta_1 B_1 + \beta_2 B_2$, and

$\sigma_2 = \beta_1(k_1 + c_1) + \beta_2(k_2 + c_2)$.

Rewrite $L_1(\bar{\Delta}_1)$, we have

$$\begin{aligned} L_1(\bar{\Delta}) &= p - c - \beta_1\Delta_1 - 2wG_1(\Delta_1) \\ &= p - c - \sigma_3 + \frac{\beta_1 B_1}{G_1(\bar{\Delta}_1)} - 2wG_1(\bar{\Delta}_1) \end{aligned}$$

where $\sigma_3 = \beta_1(k_1 + c_1)$. Let $L_1(\bar{\Delta}_1) = 0$ so that $G_1(\bar{\Delta}_1)$ is given by

$$G_1(\bar{\Delta}_1) = \frac{B_1}{k_1 + c_1 - \bar{\Delta}_1} = \frac{p - c - \sigma_3 \pm \sqrt{(p - c - \sigma_3)^2 + 8w\beta_1 B_1}}{4w}$$

Since $G_1(\bar{\Delta}_1) > 0$, the negative root is excluded. Thus, we obtain $\bar{\Delta}_1$.

Similarly, let $L_2(\bar{\Delta}_2) = 0$, we have

$$p - c - \sigma_2 - 2wG_1(\bar{\Delta}_2) + \frac{h}{G_1(\bar{\Delta}_2)} = 0$$
$$G_1(\bar{\Delta}_2) = \frac{p - c - \sigma_2 + \sqrt{(p - c - \sigma_2)^2 + 8wh}}{4w}$$

where $\sigma_2 = \beta_1(k_1 + c_1) + \beta_2(k_2 + c_2)$ and $h = \beta_1 B_1 + \beta_2 B_2$. The negative root has been excluded since $G_1(\bar{\Delta}_2) > 0$. Thus, we obtain $\bar{\Delta}_2$.

Area I to IV are shown in Figure 3.1.

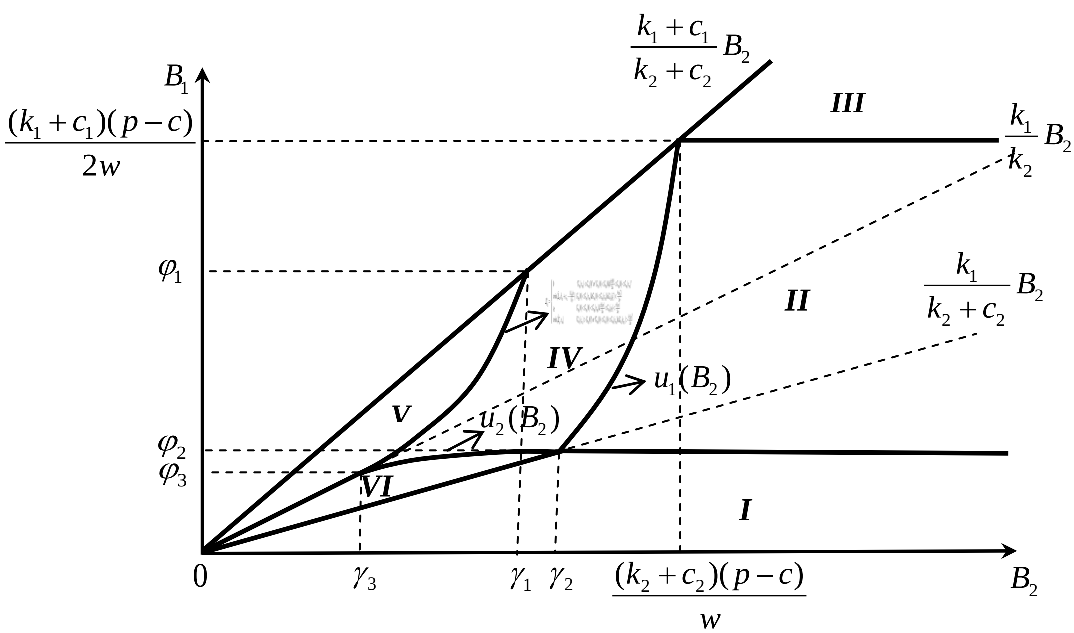


Figure 3.2 Cost Sharing Equilibrium in Simple Sequential Setting

In Figure 3.1,

$$
\begin{aligned}
&\sigma_1 = \beta_1 c_1 + \beta_2 c_2, \\
&\sigma_2 = \beta_1 (k_1 + c_1) + \beta_2 (k_2 + c_2), \\
&\varphi_1 = \frac{k_2 (k_1 + c_1)}{2w(k_2 + c_2)} \left[p - c - \frac{c_2 \sigma_2}{k_2 + c_2} \right], \\
&\varphi_2 = \frac{k_1}{2w} (p - c - \beta_1 c_1), \\
&\varphi_3 = \frac{k_1}{2w} (p - c - \sigma_1), \\
&\gamma_1 = \frac{k_2}{2w} \left[p - c - \frac{c_2 \sigma_2}{k_2 + c_2} \right], \\
&\gamma_2 = \frac{k_2 + c_2}{2w} (p - c - \beta_1 c_1), \\
&\gamma_3 = \frac{k_2}{2w} (p - c - \sigma_1),
\end{aligned}
$$

$B_1 = u_1(B_2)$ does not have an explicit expression, but satisfies $L_1 \left(k_1 + c_1 - \frac{B_1 (k_2 + c_2)}{B_2} \right) = 0$,

$B_1 = u_2(B_2)$ satisfies $L_2(c_1) = 0$, and $B_1 = u_3(B_2)$ satisfies $L_2 \left(k_1 + c_1 - \frac{B_1 k_2}{B_2} \right) = 0$.

In Lemma 3.1, we characterize the properties of the boundary conditions.

Lemma 3.1 *a)* $u_1(B_2)$*,* $u_2(B_2)$ *and* $u_3(B_2)$ *are increasing in* B_2*.*

b) The equilibrium $(\Delta_1^*, \Delta_2^*) = (k_1 + c_1 - B_1 k_2 / B_2, c_2)$ *exists i.f.f.* $k_1 c_2 \leqslant k_2 c_1$.

Proof:

Denote

$$
\begin{aligned}
z &= L_1 \left[k_1 + c_1 - \frac{B_1 (k_2 + c_2)}{B_2} \right] \\
&= p - c - \beta_1 \left[k_1 + c_1 - \frac{B_1 (k_2 + c_2)}{B_2} \right] - \frac{2w B_2}{k_2 + c_2}
\end{aligned}
$$

z increases in B_1 and decreases in B_2. By the Implicit Function Theorem, we have $du_1(B_2)/dB_2 = -(\partial z / \partial B_2)/(\partial z / \partial B_1) > 0$. Thus, $u_1(B_2)$ increases in B_2.

Similarly, we have $u_2(B_2)$ and $u_3(B_2)$ increases in B_2.

Suppose $k_1c_2 > k_2c_1$, so that $(k_2+c_2)/k_2 < (k_1+c_1)/k_1$. Since $(k_1+c_1)/B_1 \geqslant (k_2+c_2)/B_2$, we have $G_1(c_1) \leqslant G_2(c_2)$. In this case, the manufacturer cannot extract profit via fully sharing supplier 2's deductable cost since the ordering quantity is finally constrained by supplier 1's capacity. Therefore, the optimal Δ_2 is less than c_2.

3.3 Profit Comparison with No-Cost-Share Sequential Setting

In the equilibrium of cost sharing model, the optimal ordering quantity is given by,

$$q_s^* = \begin{cases} \dfrac{p-\bar{c}(\Delta_1^*)}{2w} & \bar{c}(\Delta_1^*) \geqslant p - 2wG_1(\Delta_1^*) \\ G_1(\Delta_1^*) & otherwise \end{cases} \tag{3.10}$$

Therefore, supplier s_2's profit is given by,

$$\Pi_{s_2}^* = \begin{cases} \dfrac{[p-\bar{c}(\Delta_1^*)]^2}{4w} & \bar{c}(\Delta_1^*) \geqslant p - 2wG_1(\Delta_1^*) \\ [p-\bar{c}(\Delta_1^*) - wG_1(\Delta_1^*)]G_1(\Delta_1^*) & otherwise \end{cases} \tag{3.11}$$

and supplier s_1's profit is given by,

$$\Pi_{s_1}^* = \begin{cases} \dfrac{[p-\bar{c}(\Delta_1^*)]^2}{8w} & \bar{c}(\Delta_1^*) \geqslant p - 2wG_1(\Delta_1^*) \\ \dfrac{w}{2}[G_1(\Delta_1^*)]^2 & otherwise \end{cases} \tag{3.12}$$

and we obtain the manufacturer's profit as follows,

$$\Pi_M^* = \begin{cases} \dfrac{[p-\bar{c}(\Delta_1^*)]^2}{16w} & \bar{c}(\Delta_1^*) \geqslant p - 2wG_1(\Delta_1^*) \\ \dfrac{w}{4}[G_1(\Delta_1^*)]^2 & otherwise \end{cases} \tag{3.13}$$

Proposition 3.1 *In the setting where suppliers sequentially decide on the wholesale prices, the decision sequence does not affect the manufacturer's decision and profit, but only suppliers'. The supplier who decides first can charge a higher wholesale price than other suppliers can if the manufacturer shares any supplier's total deductable cost.*

By given the simple sequential decision setting, the manufacturer's optimal cost-sharing policy and profit are irrelevant to the suppliers' decision order on the wholesale prices. However, the suppliers prefer to make the decision early so that they can earn more profits. Thus, the cost sharing equilibrium in this setting is symmetric.

3.3.1 Profit Comparison of the Manufacturer

Denote Π_m^* as the manufacturer's optimal profit without cost sharing. We have,

$$\Pi_M^* - \Pi_m^* = \begin{cases} 0 & c \geqslant p - 2wG_1(0) \\ \dfrac{w}{4}\left\{[G_1(\Delta_1^*)]^2 - [G_1(0)]^2\right\} > 0 & otherwise \end{cases} \tag{3.14}$$

Thus, cost sharing can benefit the manufacturer if the market demand is not fully satisfied.

3.3.2 Profit Comparison of Supplier s_1

Denote $\Pi_{n_i}^*$ as supplier s_i's optimal profit without cost sharing. Then, supplier s_1's is given by

$$\Pi_{s_1}^* - \Pi_{n_1}^* = \begin{cases} 0 & c \geqslant p - 2wG_1(0) \\ \dfrac{w}{2}\left\{[G_1(\Delta_1^*)]^2 - [G_1(0)]^2\right\} > 0 & otherwise \end{cases} \tag{3.15}$$

Similar to the manufacturer, supplier s_1 always benefits from cost sharing when the production quantity cannot satisfy the market demand under suppliers' initial capacity constraints. Furthermore, in an N-suppliers model, all the suppliers can benefit from cost sharing except the one who first decides the wholesale price.

3.3.1 Profit Comparison of the Manufacturer

Denote Π_m^* as the manufacturer's optimal profit without cost sharing. We have,

$$\Pi_M^* - \Pi_m^* = \begin{cases} 0 & c \geqslant p - 2wG_1(0) \\ \dfrac{w}{4}\left\{[G_1(\Delta_1^*)]^2 - [G_1(0)]^2\right\} > 0 & otherwise \end{cases} \tag{3.16}$$

Thus, cost sharing can benefit the manufacturer if the market demand is not fully satisfied.

3.3.2 Profit Comparison of Supplier s_1

Denote $\Pi_{n_i}^*$ as supplier s_i's optimal profit without cost sharing. Then, supplier s_1's is given by

$$\Pi_{s_1}^* - \Pi_{n_1}^* = \begin{cases} 0 & c \geqslant p - 2wG_1(0) \\ \dfrac{w}{2}\left\{[G_1(\Delta_1^*)]^2 - [G_1(0)]^2\right\} > 0 & otherwise \end{cases} \tag{3.17}$$

Similar to the manufacturer, supplier s_1 always benefits from cost sharing when the production quantity cannot satisfy the market demand under suppliers' initial capacity constraints. Furthermore, in an N-suppliers model, all the suppliers can benefit from cost sharing except the one who first decides the wholesale price.

3.3.3 Profit Comparison of Supplier s_2

Supplier s_2's optimal profit without cost sharing is given by

$$\Pi_{n_2}^* = \begin{cases} \dfrac{(p-c)^2}{4w} & c \geqslant p - 2wG_1(0) \\ [p - c - wG_1(0)]G_1(0) & otherwise \end{cases} \tag{3.18}$$

Denote

$$\tilde{\xi}_1 = min\left(\frac{k_1 + c_1}{c_1}\sigma_1, \frac{k_1^2 + (k_1 + k_2)^2}{c_1^2}\beta_1\right),$$

$$\hat{\xi}_1 = max\left(\frac{k_1 + c_1}{c_1}\sigma_1, \frac{k_1^2 + (k_1 + k_2)^2}{c_1^2}\beta_1\right),$$

$$\tilde{\xi}_2 = min\left(\sigma_2, \frac{k_1^2 + (k_1 + k_2)^2}{c_1^2}\sigma_1\right),$$

$$\hat{\xi}_2 = max\left(\sigma_2, \frac{k_1^2 + (k_1 + k_2)^2}{c_1^2}\sigma_1\right),$$

$$\tau_1 = min\left(\hat{\xi}_1, \sigma_2\right),$$

$$\tau_2 = max\left(\bar{\xi}_2, \frac{k_1 + c_1}{c_1}\sigma_2\right).$$

According to (3.11) and (3.18), we compare supplier s_2's profit as below,

Lemma 3.2 *Supplier's profit comparison between cost sharing and non-cost sharing is given by*

(I) When $c < p < c + \beta_1(k_1 + c_1)$, there must be $\Pi_{s_2}^* < \Pi_{n_2}^*$.

(II) When $c + \beta_1(k_1 + c_1) \leqslant p < c + \tilde{\xi}_1$, $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is in the shading of Figure 3.3.

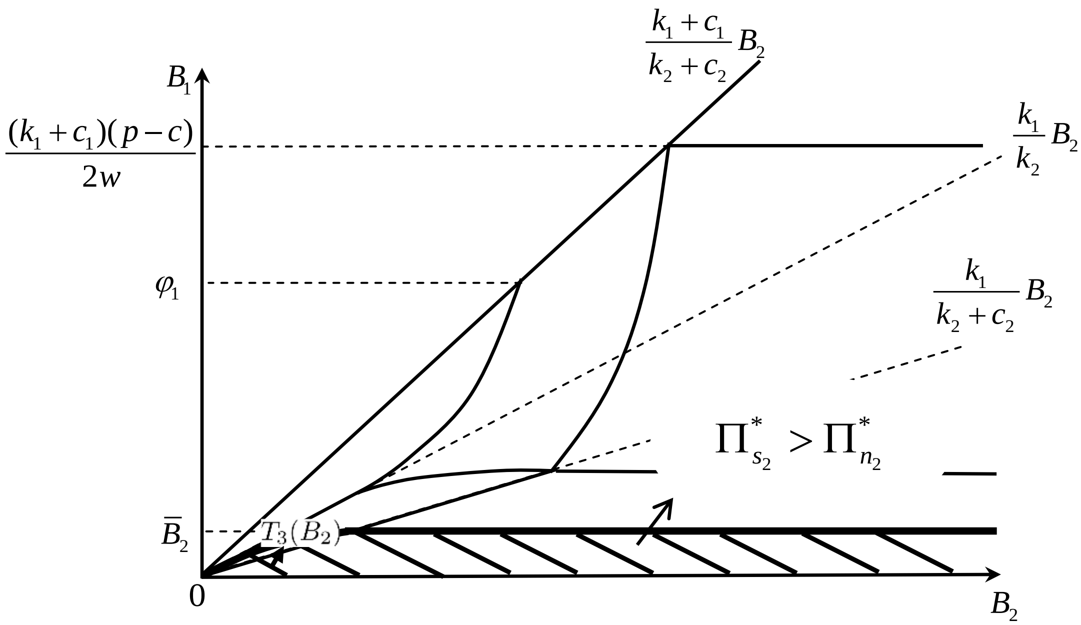


Figure 3.3 Supplier s_2's Profit Comparison in (II)

(III) When $c+\tilde{\xi}_1 \leqslant p < c+\tau_1$, the shading shape for $\Pi^*_{s_2} \geqslant \Pi^*_{n_2}$ is affected by β_i, k_i and c_i:

(a) When $0 < \beta_1/\beta_2 < \tilde{\theta}$ or when $\tilde{\theta} \leqslant \beta_1/\beta_2 < \hat{\theta}$ and $k_2/c_2 \geqslant (2k_1+c_1)(k_1+c_1)/c_1^2$, $\Pi^*_{s_2} \geqslant \Pi^*_{n_2}$ is in the shadow area of Figure 3.4.

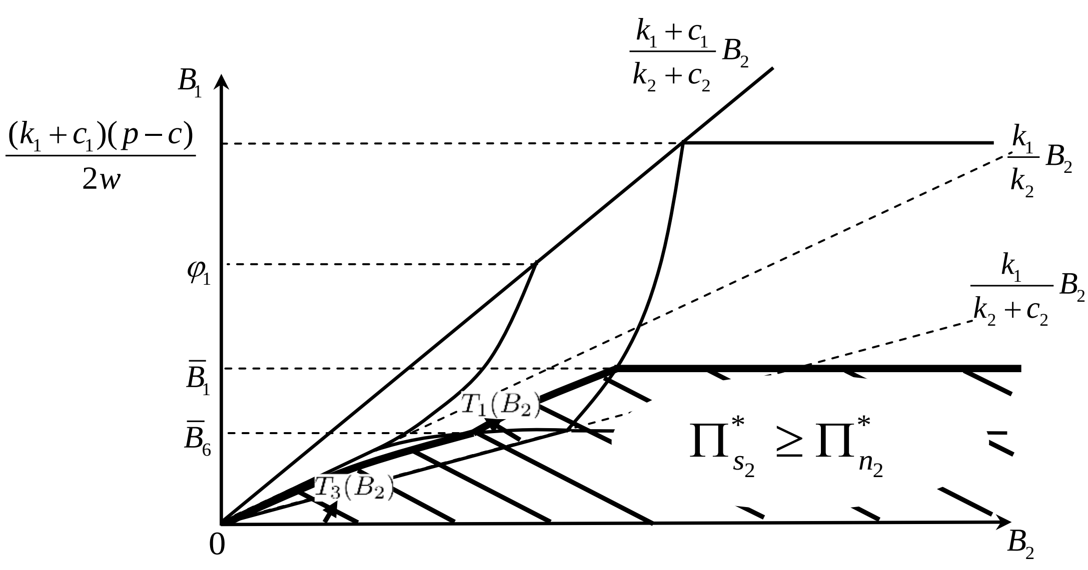


Figure 3.4 Supplier s_2's Profit Comparison in (IIIa)

(b) When $\tilde{\theta} < \beta_1/\beta_2 < \hat{\theta}$ and $k_2/c_2 < (2k_1 + c_1)(k_1 + c_1)/c_1^2$, or when $\beta_1/\beta_2 \geqslant \hat{\theta}$, $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is in the shadow area of Figure 3.5.

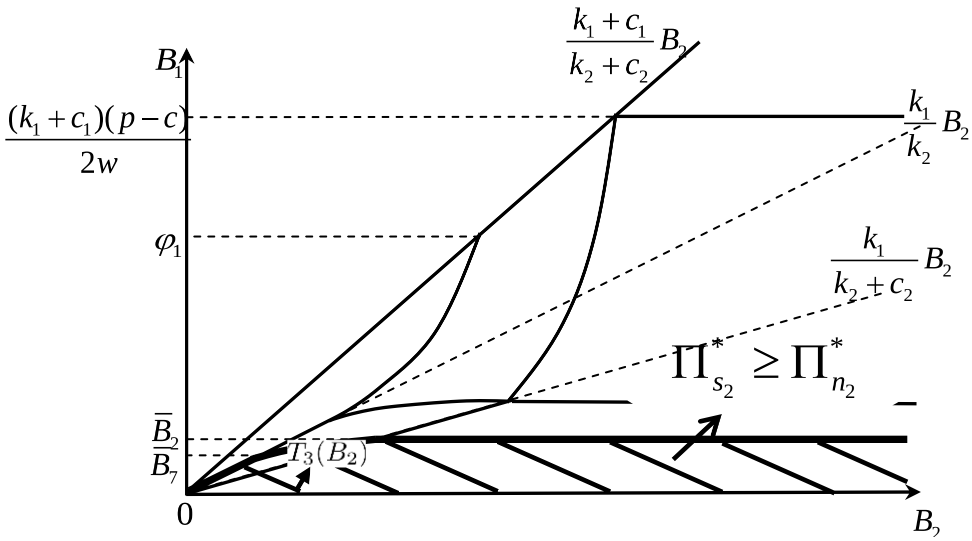


Figure 3.5 Supplier s_2's Profit Comparison in (IIIb)

(IV) When $c + \tau_1 \leqslant p < c + \tau_2$, the shading shape for $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is affected by β_i, k_i and c_i:

(a) When $0 < \beta_1/\beta_2 < [c_1(k_2 + c_2)]/[k_1(2k_1 + c_1)]$, $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is in the shadow area of Figure 3.6.

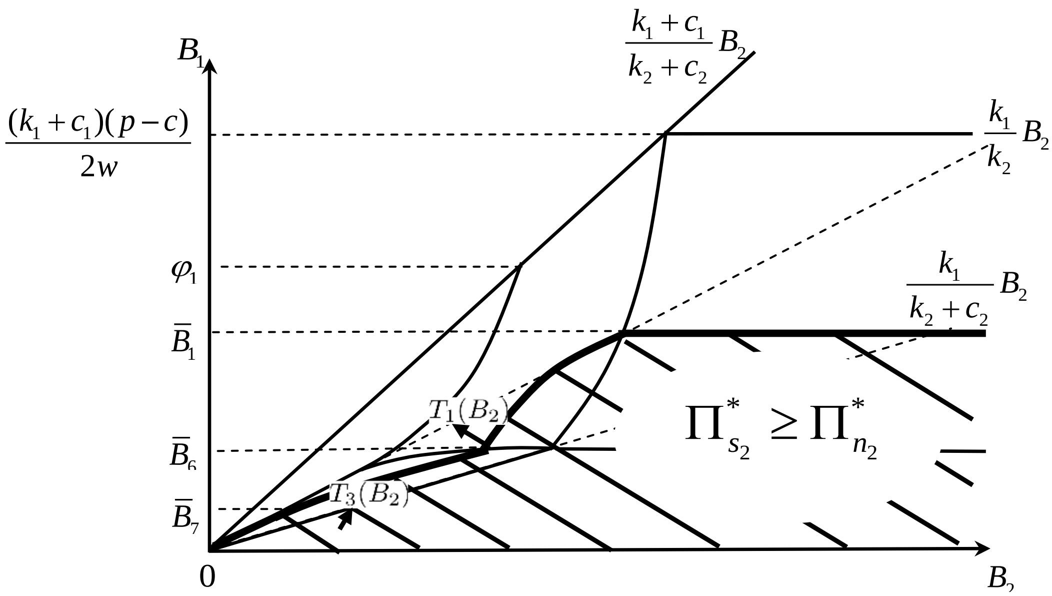


Figure 3.6 Supplier s_2's Profit Comparison in (IVa)

(b) When $\beta_1/\beta_2 \geqslant [c_1(k_2+c_2)]/[k_1(2k_1+c_1)]$, $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is in the shadow area of Figure 3.7.

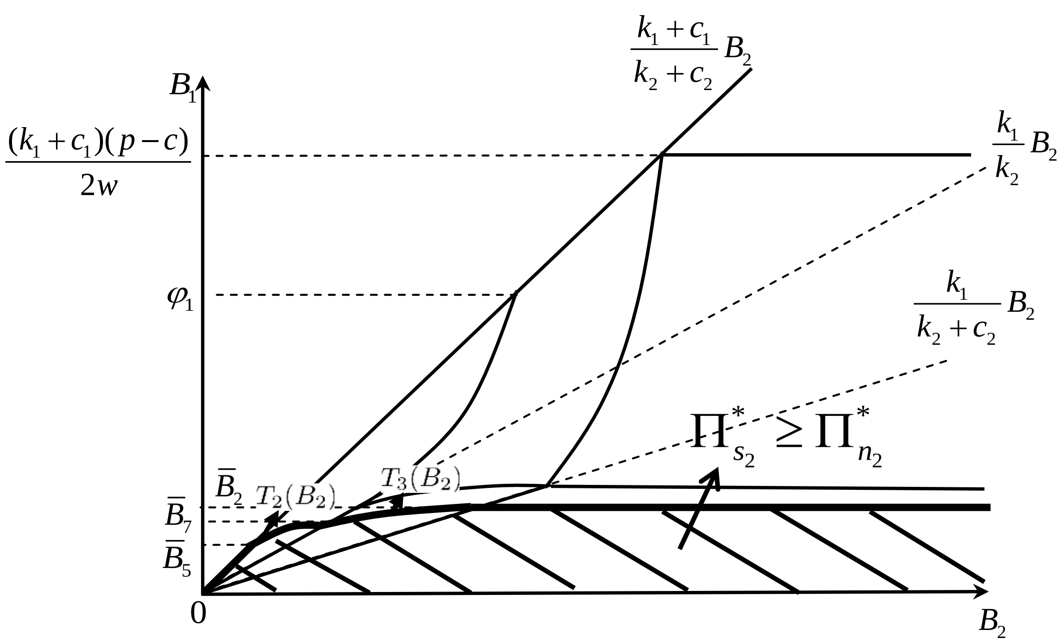


Figure 3.7 Supplier s_2's Profit Comparison in (IVb)

(V) When $c+\tau_2 \leqslant p < c+\hat{\xi}_2$, the shading shape for $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is affected by β_i, k_i and c_i:

(a) When $0 < \beta_1/\beta_2 < \tilde{\theta}$ or when $\tilde{\theta} \leqslant \beta_1/\beta_2 < \hat{\theta}$ and $k_2/c_2 < [(2k_1+c_1)(k_1+c_1)]/c_1^2$, $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is in the shadow area of Figure 3.8.

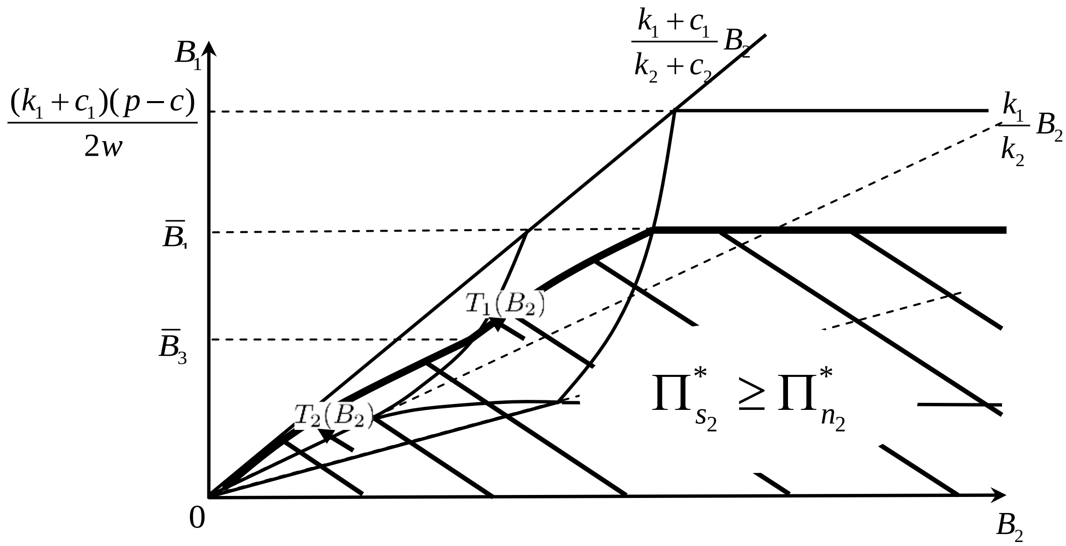


Figure 3.8 Supplier s_2's Profit Comparison in (Va)

(b) When $\tilde{\theta} \leqslant \beta_1/\beta_2 < \hat{\theta}$ and $k_2/c_2 \geqslant [(2k_1+c_1)(k_1+c_1)]/c_1^2$ or when $\beta_1/\beta_2 \geqslant \hat{\theta}$, $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is in the shadow area of Figure 3.9.

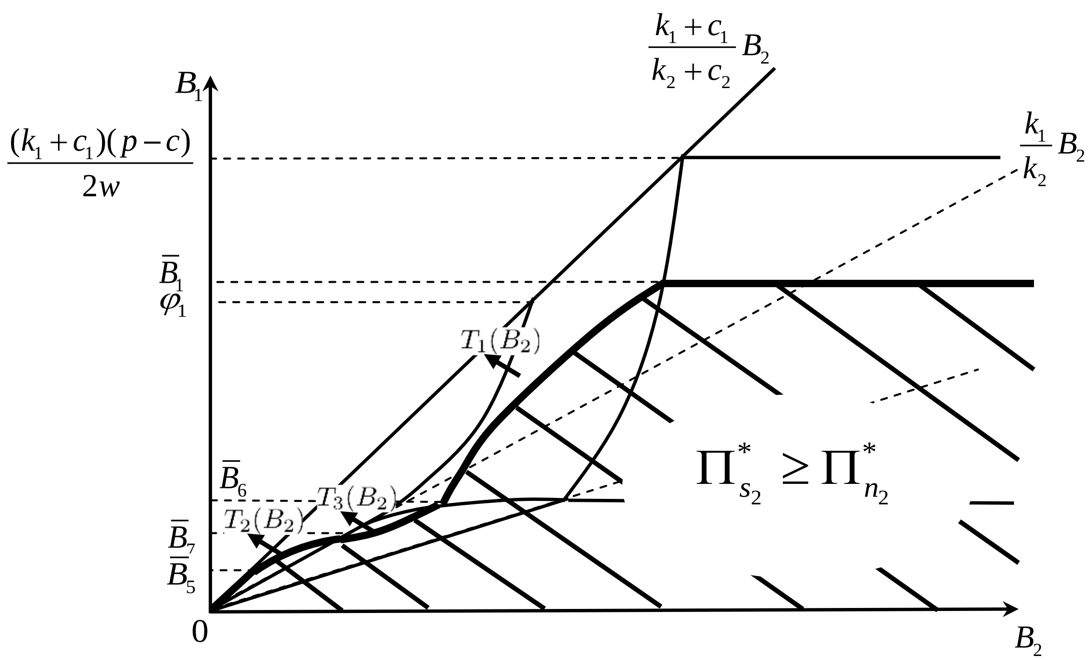


Figure 3.9 Supplier s_2's Profit Comparison in (Vb)

(VI) When $c+\hat{\xi}_2 \leqslant p < c+[k_2^2+(k_2^2+c_2^2)^2]\sigma_2/[c_2(k_2+c_2)]$, the shading of Figure 3.10 indicates $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$.

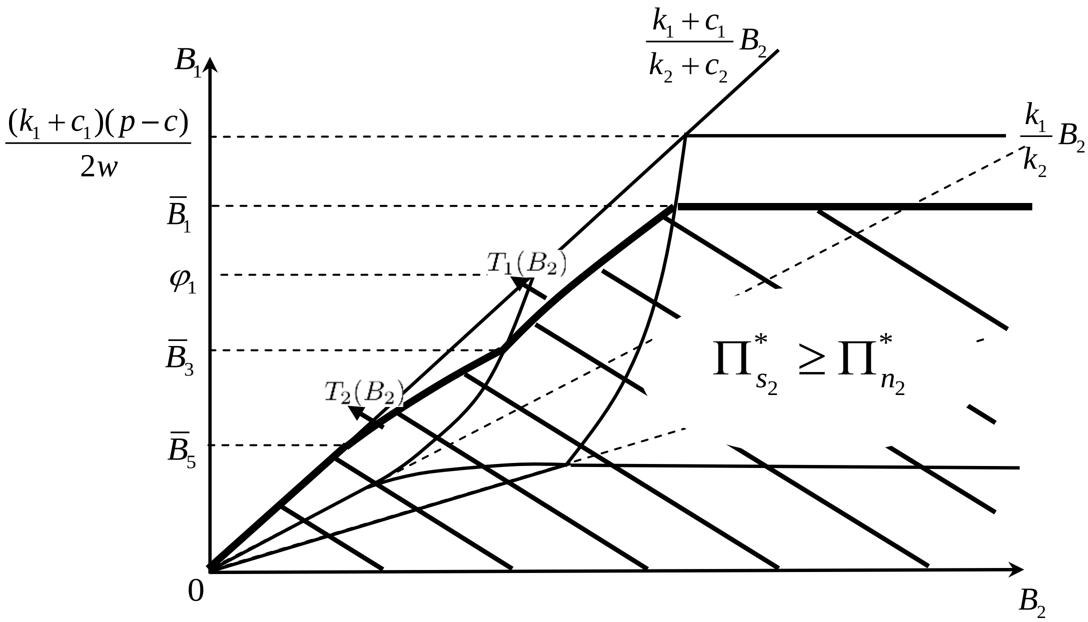


Figure 3.10 Supplier s_2's Profit Comparison in (VI)

(VII) When $p \geqslant c+[k_2^2+(k_2^2+c_2^2)^2]\sigma_2/[c_2(k_2+c_2)]$, $\Pi_{s_2}^* \geqslant \Pi_{n_2}^*$ is in the shadow area of Figure 3.11.

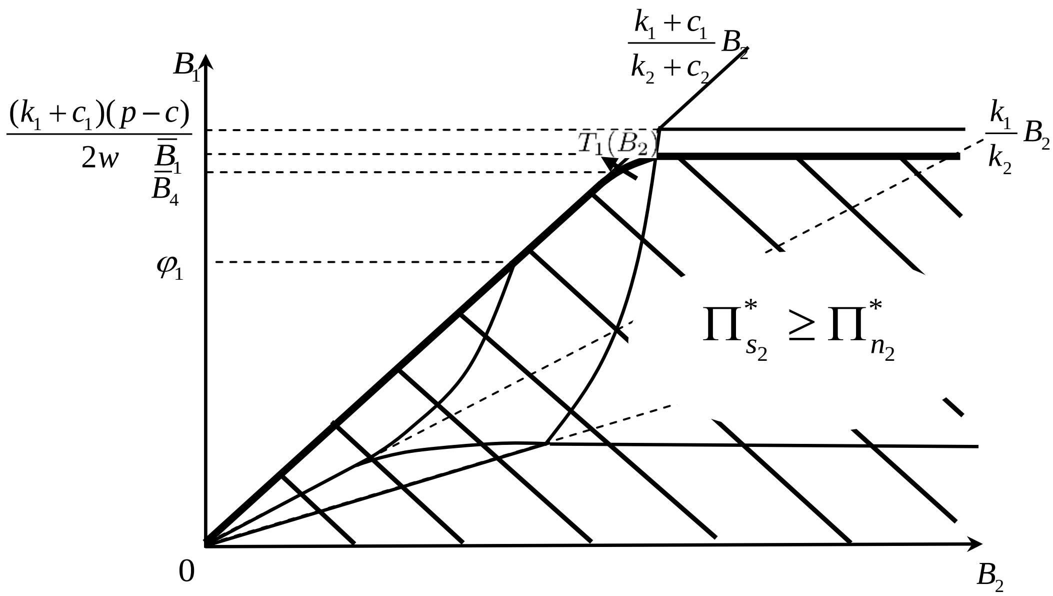


Figure 3.11 Supplier s_2's Profit Comparison in (VII)

Proof:

From Figure 3.3 to 3.11, we denote

$$\bar{B}_1(B_2) = \frac{k_1+c_1}{4w}\left[3(p-c) - 2\beta_1(k_1+c_1) - \sqrt{[p-c+2\beta_1(k_1+c_1)]^2 - 8\beta_1^2(k_1+c_1)^2}\right],$$

$$\bar{B}_2(B_2) = \frac{k_1(k_1+c_1)(p-c-\beta_1 k_1 - \beta_1 c_1)}{(2k_1+c_1)w},$$

$\bar{B}_3(B_2)$ does not has an explicit express, but satisfies

$$\frac{8w\bar{B}_3}{k_1+c_1}\left(p-c-\frac{w\bar{B}_3}{k_1+c_1}\right) =$$
$$(p-c-\beta_1 k_1 - \sigma_1)^2 + (p-c-\beta_1 k_1 - \sigma_1)\sqrt{(p-c-\beta_1 k_1-\sigma_1)^2 + 8w\beta_1\bar{B}_3} + 4w\beta_1\bar{B}_3,$$

$\bar{B}_4(B_2)$ does not has an explicit express, but satisfies

$$\left[p-c-\sigma_2 + \sqrt{(p-c-\sigma_2)^2 + \frac{8w\bar{B}_4}{k_1+c_1}\sigma_2}\right]^2 = \frac{16w\bar{B}_4}{k_1+c_1}\left(p-c-\frac{w\bar{B}_4}{k_1+c_1}\right),$$

$$\bar{B}_5 = \frac{k_2(k_1+c_1)}{(2k_2+c_2)w}(p-c-\sigma_2),$$

$$\bar{B}_6 = \frac{p-c}{w}\frac{k_1^2(k_1+c_1)}{k_1^2+(k_1+c_1)^2},$$

$$\bar{B}_7 = \frac{k_1(k_1+c_1)}{(2k_1+c_1)w}\left[p-c-\frac{k_1+c_1}{c_1}\sigma_1\right].$$

Proof:

According to Figure 3.1, we prove the positivity of $(\Pi_{s_2}^* - \Pi_{n_2}^*)$ in six different equilibrium areas.

CASE 1: $(\Delta_1^*, \Delta_2^*) = (c_1, 0)$

Supplier s_2's profit without cost share is $\Pi_{n_2}^* = B_1[p-c-wB_1/(k_1+c_1)]/(k_1+c_1)$. And her profit with cost share is $\Pi_{s_2}^* = B_1[p-c-\beta_1 c_1 - wB_1/k_1]/k_1$ in this case.

Let $\Pi_{s_2}^* > \Pi_{n_2}^*$, we have $B_1 < \bar{B}_2 = [k_1(k_1+c_1)(p-c-\sigma_3)]/[(2k_1+c_1)w]$. Denote $\varphi_2 = k_1(p-c-\beta_1 c_1)/w$. The positivity of $(\Pi_{s_2}^* - \Pi_{n_2}^*)$ is given by

$$\Pi_{s_2}^* - \Pi_{n_2}^* \begin{cases} > 0 & 0 \leqslant B_1 < \bar{B}_2 \\ \leqslant 0 & \bar{B}_2 \leqslant B_1 < \varphi_2^+ \end{cases}$$

where $c + \sigma_3 \leqslant p < c + \beta_1[k_1^2 + (k_1+c_1)^2]/c_1$.

CASE 2: $(\Delta_1^*, \Delta_2^*) = (\bar{\Delta}_1, 0)$

Since B_1 is supported in $[\varphi_2^+, (p-c)(k_1+c_1)/2w]$, $\Pi_{n_2}^*$ is concavely increasing in B_1. According to (3.9), supplier s_2's profit with cost share is given by

$$\Pi_{s_2}^* = \frac{\left[p-c-\sigma_3+\sqrt{(p-c-\sigma_3)^2+8w\beta_1 B_1}\right]^2}{16w}$$

Where $\sigma_3 = \beta_1(k_1+c_1)$.

Denote $v = p-c-\sigma_3$. Let $\Pi_{n_2}^* = \Pi_{s_2}^*$, we have

$$\frac{[v+\sqrt{v^2+8w\beta_1 B_1}]^2}{16w} = \left[p-c-\frac{wB_1}{k_1+c_1}\right]\frac{B_1}{k_1+c_1}$$
$$v\sqrt{v^2+8w\beta_1 B_1} = \left[p-c-\frac{wB_1}{k_1+c_1}\right]\frac{8wB_1}{k_1+c_1} - v^2 - 4w\beta_1 B_1$$

Solve the above equation and exclude the false roots, we obtain three roots $B_{1_1} = \frac{(p-c)(k_1+c_1)}{2w}$.

Take $\Pi_{s_2}^*$'s first-order derivative with respect to B_1, we have

$$\begin{aligned}\frac{d\Pi_{s_2}^*}{dB_1} &= \frac{\beta_1\left[p-c-\beta_1(k_1+c_1)+\sqrt{[p-c-\beta_1(k_1+c_1)]^2+8w\beta_1 B_1}\right]}{2\sqrt{[p-c-\beta_1(k_1+c_1)]^2+8w\beta_1 B_1}} \\ &= \frac{\beta_1[p-c-\beta_1(k_1+c_1)]}{2\sqrt{[p-c-\beta_1(k_1+c_1)]^2+8w\beta_1 B_1}} + \frac{\beta_1}{2}\end{aligned} \tag{3.19}$$

Since $8w\beta_1 B_1 > 0$, $\frac{d\Pi_{s_2}^*}{dB_1} > 0$. $\frac{d\Pi_{s_2}^*}{dB_1}$ increases in B_1 when $c < p < c+\sigma_3$ and decreases in it when $p \geqslant c+\sigma_3$.

Knowing that $\Pi_{s_2}^* = \Pi_{n_2}^*$ when $B_1 = \frac{(p-c)(k_1+c_1)}{2w}$, we have

i) When $c < p < c+\beta_1 c_1$, $\Pi_{s_2}^*|_{B_1=0} = \Pi_{n_2}^*|_{B_1=0} = 0$ and $\varphi_2 < 0$. So $\Pi_{s_2}^*$ and $\Pi_{n_2}^*$ have no intersection in the interval of $B_1 \in (0, \frac{(p-c)(k_1+c_1)}{2w})$. Since $\Pi_{s_2}^*$ is convexly increasing and $\Pi_{n_2}^*$ is concavely increasing in the referring interval, we have $\Pi_{s_2}^* < \Pi_{n_2}^*$.

ii) When $c+\beta_1 c_1 \leqslant p < c + \frac{k_1^2+(k_1+c_1)^2}{c_1}\beta_1$,

$\Pi_{s_2}^*|_{B_1=0} = \frac{p-c-\sigma_3+|p-c-\sigma_3|}{16w} \geqslant 0 = \Pi_{n_2}^*|_{B_1=0}$ and $\varphi_2 \geqslant 0$. Suppose $\Pi_{s_2}^*$ and $\Pi_{n_2}^*$ intersect at $\bar{B}_1 \in (\varphi_2, \frac{(p-c)(k_1+c_1)}{2w})$. Since $\Pi_{s_2}^*|_{B_1=\varphi_2} < \Pi_{n_2}^*|_{B_1=\varphi_2}$, it infers $\Pi_{s_2}^* < \Pi_{n_2}^*$ when $B_1 < \bar{B}_1$ and $\Pi_{s_2}^* > \Pi_{n_2}^*$ when $B_1 > \bar{B}_1$. It contradict with the fact that $\Pi_{s_2}^*|_{B_1=0} \geqslant \Pi_{n_2}^*|_{B_1=0}$. So we have $\Pi_{s_2}^* < \Pi_{n_2}^*$.

iii) When p$\geqslant c+\frac{k_1^2+(k_1+c_1)^2}{c_1}\beta_1$, $\Pi_{s_2}^*|_{B_1=0} > \Pi_{n_2}^*|_{B_1=0}$ and $\varphi_2 \geqslant 0$. It can be inferred that $\Pi_{s_2}^*$ and $\Pi_{n_2}^*$ have one and only one intersection point if $B_1 < \frac{(p-c)(k_1+c_1)}{2w}$. Since $\Pi_{s_2}^*|_{\varphi_2} > \Pi_{n_2}^*|_{\varphi_2}$, the intersection point $\bar{B}_1$ is in the interval of $(\varphi_2, \frac{(p-c)(k_1+c_1)}{2w})$. As we obtained above, the possible $\bar{B}_1$ is

$\frac{k_1+c_1}{4w}\left[3(p-c)-2\beta_1(k_1+c_1)\pm\sqrt{(p-c)^2+4\beta_1(p-c)(k_1+c_1)-4\beta_1^2(k_1+c_1)^2}\right]$.

Since $\bar{B}_1 < \frac{(p-c)(k_1+c_1)}{2w}$ and p$\geqslant c+\frac{k_1^2+(k_1+c_1)^2}{c_1}\beta_1$, we exclude the false root and obtain $\bar{B}_1$ as follows,

$$\frac{k_1+c_1}{4w}\left[3(p-c)-2\beta_1(k_1+c_1)-\sqrt{(p-c)^2+4\beta_1(p-c)(k_1+c_1)-4\beta_1^2(k_1+c_1)^2}\right].$$

Therefore, the positivity of $(\Pi_{s_2}^* - \Pi_{n_2}^*)$ in case 2 is given by

$$\Pi_{s_2}^* - \Pi_{n_2}^* \begin{cases} > 0 & \varphi_2^+ \leqslant B_1 < \bar{B}_1 \\ \leqslant 0 & \bar{B}_1 \leqslant B_1 < \frac{(p-c)(k_1+c_1)}{2w} \end{cases}$$

where $\bar{B}_1$ is supported in $(\varphi_2^+, \frac{(p-c)(k_1+c_1)}{2w})$ i.f.f. $p > c+\frac{k_1^2+(k_1+c_1)^2}{c_1}\beta_1$.

CASE 3: $(\varDelta_1^*, \varDelta_2^*) = (0,0)$. In this case, the manufacturer will not share any supplier's cost, so $\Pi_{s_2}^* = \Pi_{n_2}^*$.

CASE 4: $(\varDelta_1^*, \varDelta_2^*) = (\bar{\varDelta}_2, k_2+c_2-\frac{B_2}{B_1}(k_1+c_1-\bar{\varDelta}_2))$

According to (3.9), supplier s_2's profit with cost sharing is given by

$$\begin{aligned}\Pi_{s_2}^* &= w\left[G_1(\bar{\varDelta}_2)\right]^2 \\ &= \frac{\left(p-c-\sigma_2+\sqrt{(p-c-\sigma_2)^2+8w(\beta_1 B_1+\beta_2 B_2)}\right)^2}{16w}\end{aligned}$$

Since $\Pi_{s_2}^*$ increases in B_2 and $\Pi_{n_2}^*$ is irrelevant to B_2, $(\Pi_{s_2}^* - \Pi_{n_2}^*)$ is increasing in B_2. Denote $B_2 \in (g_1(B_1), g_2(B_1))$, where $g_1(B_1)$ and $g_2(B_1)$ are increasing function. There exist $\tilde{B}_2 \in (g_1, g_2)$ that satisfies $\Pi_{s_2}^* - \Pi_{n_2}^* = 0$ if $\Pi_{s_2}^* - \Pi_{n_2}^* < 0$ when $B_2 = g_1(B_1)$.

Check the positivity of $(\Pi_{s_2}^* - \Pi_{n_2}^*)$:

a) when $B_2 = \frac{k_2+c_2}{k_1+c_1} B_1$, supplier s_2's profit with cost share is given by

$$\begin{aligned}\Pi_{s_2}^* &= w\left[G_1(\bar{\Delta}_2)\right]^2 \\ &= \frac{\left(p - c - \sigma_2 + \sqrt{(p-c-\sigma_2)^2 + \frac{8w\sigma_2}{k_1+c_1}B_1}\right)^2}{16w}\end{aligned}$$

First-order derivative of $\Pi_{s_2}^*$ with respect to B_1 is

$$\frac{d\Pi_{s_2}^*}{dB_1} = \frac{\beta_1}{2}\left[\frac{p-c-\sigma_2}{\sqrt{(p-c-\sigma_2)^2 + \frac{8w\sigma_2}{k_1+c_1}B_1}} + 1\right]$$

$B_1 > 0$, so that $\frac{d\Pi_{s_2}^*}{dB_1} > 0$, $\frac{d^2\Pi_{s_2}^*}{dB_1^2} > 0$ when $c < p < c + \sigma_2$ and $\frac{d^2\Pi_{s_2}^*}{dB_1^2} \leqslant 0$ when $p \geqslant c + \sigma_2$. $\Pi_{n_2}^*$ is concavely increasing in $B_1 \leqslant \frac{(p-c)(k_1+c_1)}{2w}$.

Denote $\varphi_1 = \frac{k_2(k_1+c_1)}{w(k_2+c_2)}\left[p - c - \frac{c_2\sigma_2}{k_2+c_2}\right]$. Substituting $B_1 = \varphi_1$ into $(\Pi_{s_2}^* - \Pi_{n_2}^*)$, we have

$$\begin{aligned}\Pi_{s_2}^* - \Pi_{n_2}^* &= w\left[\frac{(k_2+c_2)\varphi_1}{k_2(k_1+c_1)}\right]^2 - \left(p - c - \frac{w\varphi_1}{k_1+c_1}\right)\frac{\varphi_1}{k_1+c_1} \\ &= \frac{\left(p - c - \frac{c_2\sigma_2}{k_2+c_2}\right)^2}{4w} - \frac{k_2}{2w(k_2+c_2)}\left(p - c - \frac{c_2\sigma_2}{k_2+c_2}\right)\left[p - c - \frac{k_2}{2(k_2+c_2)}\left(p - c - \frac{c_2\sigma_2}{k_2+c_2}\right)\right]\end{aligned}$$

Let $\Pi_{s_2}^* - \Pi_{n_2}^* > 0$, we have $p > c + \frac{(k_2+c_2)^2 + k_2^2}{c_2(k_2+c_2)}\sigma_2$.

$\Pi_{s_2}^*$ is increasing in B_1 and $\Pi_{n_2}^*$ is concavely increasing in B_1 where $B_1 \in [0, \frac{(p-c)(k_1+c_1)}{2w}]$. It infers that $\Pi_{s_2}^* < \Pi_{n_2}^*$ in a neighborhood $(\frac{(p-c)(k_1+c_1)}{2w} - \varepsilon, \frac{(p-c)(k_1+c_1)}{2w})$, where $\varepsilon > 0$.

i) When $c < p < c + \frac{c_2\sigma_2}{k_2+c_2}$, $\varphi_1 < 0$ and $\Pi^*_{s_2}|_{B_1=0} = \Pi^*_{n_2}|_{B_1=0} = 0$. Suppose $\Pi^*_{s_2} > \Pi^*_{n_2}$ in $B_1 \in (x_1, x_2)$ where $x_1, x_2 \in [0, \frac{(p-c)(k_1+c_1)}{2w} - \varepsilon]$. Since $\Pi^*_{s_2} = \Pi^*_{n_2}$ at $B_1 = 0$ and $\Pi^*_{s_2} < \Pi^*_{n_2}$ in $B_1 \in (\frac{(p-c)(k_1+c_1)}{2w} - \varepsilon, \frac{(p-c)(k_1+c_1)}{2w})$, there must be $\Pi^*_{s_2} = \Pi^*_{n_2}$ at $B_1 = x_2 \in (0, \frac{(p-c)(k_1+c_1)}{2w})$, i.e., $\Pi^*_{s_2}$ and $\Pi^*_{n_2}$ intersects at $B_1 = 0, x_2, \frac{(p-c)(k_1+c_1)}{2w}$. It contradicts with the fact that $\Pi^*_{s_2}$ and $\Pi^*_{n_2}$ at most have two intersections. So we have $\Pi^*_{s_2} \leqslant \Pi^*_{n_2}$ when $B_1 \in [0, \frac{(p-c)(k_1+c_1)}{2w}]$.

ii) When $c + \frac{c_2\sigma_2}{k_2+c_2} \leqslant p < c + \frac{(k_2+c_2)^2+k_2^2}{c_2(k_2+c_2)}\sigma_2$, $\varphi_1 \geqslant 0$. Since $\Pi^*_{s_2} < \Pi^*_{n_2}$ at $B_1 = \varphi_1$, we similarly have $\Pi^*_{s_2} \leqslant \Pi^*_{n_2}$ when $B_1 \in [\varphi_1, \frac{(p-c)(k_1+c_1)}{2w}]$.

iii) When $p \geqslant c + \frac{(k_2+c_2)^2+k_2^2}{c_2(k_2+c_2)}\sigma_2$, $\Pi^*_{s_2} \geqslant \Pi^*_{n_2}$ at $B_1 = \varphi_1 > 0$. Since $\Pi^*_{s_2} < \Pi^*_{n_2}$ in $B_1 \in (\frac{(p-c)(k_1+c_1)}{2w} - \varepsilon, \frac{(p-c)(k_1+c_1)}{2w})$, $\Pi^*_{s_2}$ and $\Pi^*_{n_2}$ must intersect in the interval of $B_1 \in [\varphi_1, \frac{(p-c)(k_1+c_1)}{2w})$.

The positivity of $(\Pi^*_{s_2} - \Pi^*_{n_2})$ at $B_2 = \frac{k_2+c_2}{k_1+c_1}B_1$ is

$$\Pi^*_{s_2} - \Pi^*_{n_2} \begin{cases} > 0 & \varphi_1 \leqslant B_1 < \bar{B}_4 \\ \leqslant 0 & \bar{B}_4 \leqslant B_1 < \dfrac{(p-c)(k_1+c_1)}{2w} \end{cases}$$

where $\bar{B}_4$ does not have a explicit expression but satisfies

$$w\left[\frac{p-c-\sigma_2+\sqrt{(p-c-\sigma_2)^2+\frac{8w\bar{B}_4\sigma_2}{k_1+c_1}}}{4w}\right]^2 = \left(p-c-\frac{w\bar{B}_4}{k_1+c_1}\right)\frac{\bar{B}_4}{k_1+c_1}.$$

And $\bar{B}_4 \in [\varphi_1, \frac{(p-c)(k_1+c_1)}{2w})$ i.f.f. $p \geqslant c + \frac{(k_2+c_2)^2+k_2^2}{c_2(k_2+c_2)}\sigma_2$.

b) When $B_2 = u_3^{-1}(B_2)$, $G_1(\bar{\Delta}_2) = \frac{B_2}{k_2}$. It follows, after some algebra, that,

$$\Pi^*_{s_2} = \left(\frac{p-c-\sigma_2+\beta_2k_2+\sqrt{(p-c-\sigma_2+\beta_2k_2)^2+8w\beta_1B_1}}{4w}\right)^2$$

Similarly, we have

$$\Pi_{s_2}^* - \Pi_{n_2}^* \begin{cases} > 0 & \varphi_3^+ \leqslant B_1 < \bar{B}_3 \\ \leqslant 0 & \bar{B}_3 \leqslant B_1 < \varphi_1^+ \end{cases}$$

where $\varphi_3 = \frac{k_1}{2w}(p - c - \sigma_1)$. Denote $\sigma_4 = p - c - \beta_1(k_1 + c_1) - \beta_2 c_2$. $\bar{B}_3$ does not have an explicit expression but satisfies

$$\frac{(p-c-\sigma_4)^2 + 4w\beta_1\bar{B}_3 + (p-c-\sigma_4)\sqrt{(p-c-\sigma_4)^2 + 8w\beta_1\bar{B}_3}}{8w} = \left(p - c - \frac{w\bar{B}_3}{k_1 + c_1}\right)\frac{\bar{B}_3}{k_1 + c_1}$$

And $\bar{B}_3 \in [\varphi_3, \varphi_1)$ i.f.f. $c + \frac{[k_1^2 + (k_1+c_1)^2]\sigma_1}{c_1^2} \leqslant p < c + \frac{(k_2+c_2)^2 + k_2^2}{c_2(k_2+c_2)}\sigma_2$.

Then we have to prove $\frac{[k_1^2 + (k_1+c_1)^2]\sigma_1}{c_1^2} \leqslant \frac{(k_2+c_2)^2 + k_2^2}{c_2(k_2+c_2)}\sigma_2$. Suppose this inequality holds, it follows that

$$\left\{\frac{k_1^2 + (k_1+c_1)^2}{c_1} - \frac{[k_2^2 + (k_2+c_2)^2](k_1+c_1)}{(k_2+c_2)c_2}\right\}\beta_1 \leqslant \left\{\frac{k_2^2 + (k_2+c_2)^2}{c_2} - \frac{[k_1^2 + (k_1+c_1)^2]c_2}{c_1^2}\right\}\beta_2$$
$$\frac{(k_1c_2 - k_2c_1)\left[2k_1(k_2+c_2) + (2k_2+c_2)c_1\right]}{k_2+c_2}\beta_1 \leqslant \frac{(k_2c_1 - k_1c_2)\left[(k_2+c_2)c_1 + (k_1+c_1)c_2 + k_2c_1 + k_1c_2\right]}{c_1}\beta_2$$

where the items in the square brackets are positive. According to Proposition 1(b), $k_1c_2 \leqslant k_2c_1$. So the left item is non-positive while the right one is non-negative. Hence, the assumption is proved.

c) When $B_2 = u_2^{-1}(B_2)$, $G_1(\bar{\Delta}_2) = \frac{B_1}{k_1}$. B_1 is supported in $[\varphi_3, \varphi_2)$. Similar to the above case, the comparison of supplier s_2's profit is given by

$$\Pi_{s_2}^* - \Pi_{n_2}^* \begin{cases} > 0 & \bar{B}_6 \leqslant B_1 < \varphi_2^+ \\ \leqslant 0 & \varphi_3^+ \leqslant B_1 < \bar{B}_6 \end{cases}$$

where $\bar{B}_6 = \frac{p-c}{w}\frac{k_1^2(k_1+c_1)}{k_1^2 + (k_1+c_1)^2}$ and it is supported in $[\varphi_3, \varphi_2)$ when $c + \frac{[k_1^2 + (k_1+c_1)^2]\beta_1}{c_1} \leqslant p < c + \frac{[k_1^2 + (k_1+c_1)^2]\sigma_1}{c_1^2}$.

Denote $T_1(B_2) = arg\tilde{B}_2(B_1)$ The existence of $T_1(B_2)$ is as follows: 1) When $c \leqslant p < c + \frac{[k_1^2+(k_1+c_1)^2]\beta_1}{c_1}$, $T_1(B_2)$ does not exist, so that $\Pi_{s_2}^* < \Pi_{n_2}^*$; 2) When $c + \frac{[k_1^2+(k_1+c_1)^2]\beta_1}{c_1} \leqslant p < c + \frac{[k_1^2+(k_1+c_1)^2]\sigma_1}{c_1^2}$, $T_1(B_2)$ insects with $u_1(B_2)$ and $u_2(B_2)$; 3) When $c + \frac{[k_1^2+(k_1+c_1)^2]\sigma_1}{c_1^2} \leqslant p < c + \frac{(k_2+c_2)^2+k_2^2}{c_2(k_2+c_2)}\sigma_2$, $T_1(B_2)$ insects with $u_1(B_2)$ and $u_3(B_2)$; 4) $p \geqslant c + \frac{(k_2+c_2)^2+k_2^2}{c_2(k_2+c_2)}\sigma_2$, $T_1(B_2)$ insects with $u_1(B_2)$ and $\frac{k_1+c_1}{k_2+c_2}B_2$. When $T_1(B_2)$ exists, $\Pi_{s_2}^* < \Pi_{n_2}^*$ when $B_1 > T_1(B_2)$ and $\Pi_{s_2}^* < \Pi_{n_2}^*$ when $B_1 < T_1(B_2)$.

CASE 5: $(\Delta_1^*, \Delta_2^*) = (k_1 + c_1 - \frac{B_1 k_2}{B_2}, c_2)$

Supplier s_2's profit with cost share is $\Pi_{s_2}^* = [p - c - \frac{wB_2}{k_2}]\frac{B_2}{k_2}$. Similar to the above case, there may exist $B_1 = T_2(B_2)$ which satisfies $\Pi_{s_2}^* = \Pi_{n_2}^*$. Check the positivity of $(\Pi_{s_2}^* - \Pi_{n_2}^*)$, we have

a) When $B_2 = \frac{k_2 B_1}{k_1}$, $B_1 \in [0, \varphi_3^+)$.

$$\Pi_{s_2}^* - \Pi_{n_2}^* \begin{cases} > 0 & \bar{0} \leqslant B_1 < \bar{B}_7 \\ \leqslant 0 & \bar{B}_7 \leqslant B_1 < \varphi_3^+ \end{cases}$$

where $\bar{B}_7 = \frac{k_1(k_1+c_1)}{(2k_1+c_1)w}(p - c - \frac{k_1+c_1}{c_1}\sigma_1)$ and is supported in $[0, \varphi_3^+)$ i.f.f. $c + \frac{(k_1+c_1)\sigma_1}{c_1} \leqslant p < c + \frac{[k_1^2+(k_1+c_1)^2]\sigma_1}{c_1^2}$.

b) When $B_2 = \frac{k_2+c_2}{k_1+c_1}B_1$, $B_1 \in [0, \varphi_1^+)$.

$$\Pi_{s_2}^* - \Pi_{n_2}^* \begin{cases} > 0 & \bar{0} \leqslant B_1 < \bar{B}_5 \\ \leqslant 0 & \bar{B}_5 \leqslant B_1 < \varphi_1^+ \end{cases}$$

where $\bar{B}_5 = \frac{k_2(k_1+c_1)}{(2k_2+c_2)w}[p - c - \sigma_2]$ and is supported in $[0, \varphi_1^+)$ i.f.f. $c + \sigma_2 \leqslant p < c + \frac{(k_2+c_2)^2+k_2^2}{c_2(k_2+c_2)}\sigma_2$.

CASE 6: $(\Delta_1^*, \Delta_2^*) = (c_1, k_2 + c_2 - \frac{B_2 k_1}{B_1})$

Supplier s_2's profit with cost share is $\Pi_{s_2}^* = [p - c - \frac{wB_1}{k_1}]\frac{B_1}{k_1}$. Intuitively, there may exist $B_1 = T_3(B_2)$ which satisfies $\Pi_{s_2}^* = \Pi_{n_2}^*$. And we have $\Pi_{s_2}^* < \Pi_{n_2}^*$ when $B_1 > T_3(B_2)$ and $\Pi_{s_2}^* < \Pi_{n_2}^*$ when $B_1 < T_3(B_2)$ if $T_3(B_2)$ exists. Denote $\tilde{\xi}_1 = min\left(\frac{k_1+c_1}{c_1}\sigma_1, \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1\right)$ and $\hat{\xi}_1 = max\left(\frac{k_1+c_1}{c_1}\sigma_1, \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1\right)$. After some algebra, we have $\frac{k_1+c_1}{c_1}\sigma_1 > \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1$ when $\frac{\beta_1}{\beta_2} < \frac{(k_1+c_1)c_2}{k_1(2k_1+c_1)}$.

According to the above analysis on the positivity of $(\Pi_{s_2}^* - \Pi_{n_2}^*)$, we have: 1) When $c < p < c + \sigma_3$, $T_3(B_2)$ does not exist and $\Pi_{s_2}^* < \Pi_{n_2}^*$; 2) When $c + \sigma_3 \leqslant p < c + \tilde{\xi}_1$, $T_2(B_2)$ intersects with $\frac{k_2+c_2}{k_1}B_1$; 3) When $c + \tilde{\xi}_1 \leqslant p < c + \hat{\xi}_1$, $T_2(B_2)$ intersects with $u_2(B_2)$ if $0 < \frac{\beta_1}{\beta_2} < \frac{(k_1+c_1)c_2}{k_1(2k_1+c_1)}$ or intersects with $\frac{k_1}{k_2+c_2}B_2$ and $\frac{k_1}{k_2}B_2$ if $\frac{\beta_1}{\beta_2} \geqslant \frac{(k_1+c_1)c_2}{k_1(2k_1+c_1)}$; 4) When $c + \hat{\xi}_1 \leqslant p < c + \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\sigma_1$, $T_2(B_2)$ intersects with $u_2(B_2)$ and $k_1 B_2/k_2$; 5) When $p \geqslant c + \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\sigma_1$, $T_2(B_2)$ does not exist and $\Pi_{s_2}^* > \Pi_{n_2}^*$.

To combine the above six cases, we have to clarify the relationship of some conditions: When $\frac{\beta_1}{\beta_2} < \frac{c_1(k_2+c_2)}{k_1(2k_1+c_1)}$, we have $\Delta_{s_2}^* = \begin{cases} 0 & \text{[illegible]} \\ \min(\hat{\Delta}_2, k_{s_2} + c_{s_2} - \frac{B_{s_2}k_{s_1}}{B_{s_1}}) & \text{[illegible]} \\ \beta_1 & \text{[illegible]} \\ \min(\hat{\Delta}_2^+, c_{n_2}) & \text{[illegible]} \end{cases}$; When $\frac{\beta_1}{\beta_2} < \frac{(k_1+c_1)c_2}{k_1(2k_1+c_1)}$, we have $\frac{k_1+c_1}{c_1}\sigma_1 > \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1$; When $\frac{\beta_1}{\beta_2} < \frac{[k_2 c_1^2 - 2k_1 c_2(k_1+c_1)]^+}{k_1 c_1(2k_1+c_1)}$, we have $\frac{k_1^2+(k_1+k_2)^2}{c_1^2}\sigma_1 < \sigma_2$. Since $k_1 c_2 \leqslant k_2 c_1$ when the equilibrium $(\Delta_1^*, \Delta_2^*) = (k_1 + c_1 - \frac{B_1 k_2}{B_2}, c_2)$ exists, we must have $\frac{k_1+c_1}{c_1}\sigma_1 < \sigma_2$ and $\frac{k_1+c_1}{c_1}\sigma_1 > \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1$ if $\sigma_2 > \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1$. Compare $\frac{(k_1+c_1)c_2}{k_1(2k_1+c_1)}$ and $\frac{k_2 c_1^2 - 2k_1 c_2(k_1+c_1)}{k_1 c_1(2k_1+c_1)}$, we have

$$\frac{(k_1+c_1)c_2}{k_1(2k_1+c_1)} - \frac{k_2 c_1^2 - 2k_1 c_2(k_1+c_1)}{k_1 c_1(2k_1+c_1)} = \frac{1}{k_1(2k_1+c_1)}\left[\frac{(2k_1+c_1)(k_1+c_1)}{c_1^2} - \frac{k_2}{c_2}\right].$$

Denote $\tilde{\theta} = min(\frac{(k_1+c_1)c_2}{k_1(2k_1+c_1)}, \frac{[k_2c_1^2-2k_1c_2(k_1+c_1)]^+}{k_1c_1(2k_1+c_1)})$,

and $\hat{\theta} = max(\frac{(k_1+c_1)c_2}{k_1(2k_1+c_1)}, \frac{[k_2c_1^2-2k_1c_2(k_1+c_1)]^+}{k_1c_1(2k_1+c_1)})$, we have

a) When $0 < \frac{\beta_1}{\beta_2} < \tilde{\theta}$, $\frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1 < \frac{k_1+c_1}{c_1}\sigma_1 < \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\sigma_1 < \sigma_2$.

b) When $\tilde{\theta} < \frac{\beta_1}{\beta_2} < \hat{\theta}$,

$\frac{k_1+c_1}{c_1}\sigma_1 < \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1 < \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\sigma_1 < \sigma_2$ if $\frac{k_1}{c_1} < \frac{k_2}{c_2} < \frac{(2k_1+c_1)(k_1+c_1)}{c_1^2}$; and $\frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1 < \frac{k_1+c_1}{c_1}\sigma_1 < \sigma_2 < \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\sigma_1$ if $\frac{k_2}{c_2} > \frac{(2k_1+c_1)(k_1+c_1)}{c_1^2}$.

c) When $\hat{\theta} < \frac{\beta_1}{\beta_2} < \frac{c_1(k_2+c_2)}{k_1(2k_1+c_1)}$, $\frac{k_1+c_1}{c_1}\sigma_1 < \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1 < \sigma_2 < \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\sigma_1$.

d) When $\frac{\beta_1}{\beta_2} > \frac{c_1(k_2+c_2)}{k_1(2k_1+c_1)}$, $\frac{k_1+c_1}{c_1}\sigma_1 < \sigma_2 < \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\beta_1 < \frac{k_1^2+(k_1+k_2)^2}{c_1^2}\sigma_1$.

Thus, we prove the movement from Figure 3.2 to Figure 3.10.

From the above comparisons, we can find that the supplier who first decides the wholesale price may not always benefit from cost sharing. When the market price p is lower than τ_1, supplier s_2's profit is always hurt by cost sharing. And with the increase of p, cost sharing benefits supplier s_2 when B_1 is low and B_2 is high, but still hurt it when B_1 is high enough. Beside it, cost sharing's effect on supplier s_2 is also affected by the ratio of β_1 and β_2. Other suppliers, however, can always earn extra profits from cost sharing.

We know from (3.10) that, the optimal cost share amounts are not affected by N suppliers' decision sequence by given the simple sequential setting. However, such sequence has influences on the suppliers' wholesale prices, and further affects their profits. Similar to the results without cost sharing, the earlier the supplier makes the decision, the more profits she can obtain from early movement. The manufacturer, thus, obtains the least. It indicates that cost-sharing policy could affect the players' profits, but not the profit shares. When the market price is low, the increased profits derived from cost sharing are allocated to all the players in the same proportion as the profit share in the no-cost-sharing setting. When the market is high, the first-moving supplier incurs profit cutting due to cost sharing. The profits of the manufacturer and the other supplier, however, still increase in the same proportion as that in the no-cost-sharing case.

In a N-suppliers case, the manufacturer' optimal profit is given as follows,

$$\Pi_M^* = \begin{cases} \dfrac{[p-\bar{c}(\varDelta_{r_n}^*)]^2}{2^{2n}w} & \bar{c}(\varDelta_{r_n}^*) \geqslant p - 2wG_{r_n}(\varDelta_{r_n}^*) \\ \dfrac{w}{4}[G_{r_n}(\varDelta_{r_n}^*)]^2 & otherwise \end{cases}$$

Supplier s_n's optimal profit is

$$\Pi_{s_n}^* = \begin{cases} \dfrac{[p-\bar{c}(\varDelta_{r_n}^*)]^2}{2^{n}w} & \bar{c}(\varDelta_{r_n}^*) \geqslant p - 2wG_{r_n}(\varDelta_{r_n}^*) \\ \left[p-\bar{c}(\varDelta_{r_n}^*) - wG_{r_n}(\varDelta_{r_n}^*)\right] G_{r_n}\varDelta_{r_n}^*) & otherwise \end{cases}$$

and other suppliers' optimal profits are given as follows,

$$\Pi_{s_i}^* = \begin{cases} \dfrac{[p-\bar{c}(\varDelta_{r_n}^*)]^2}{2^{2n-i}w} & \bar{c}(\varDelta_{r_n}^*) \geqslant p - 2wG_{r_n}(\varDelta_{r_n}^*) \\ \dfrac{w}{2^{i-2}}[G_{r_n}(\varDelta_{r_n}^*)]^2 & otherwise \end{cases}.$$

When the number of suppliers increases, suppliers' profits keep in a similar function with that in the duel-suppliers case. Therefore, all the results in the duel-suppliers case can be extended to the N-suppliers case.

Chapter 4 Hybrid Sequential Decision in Alternating Cost Sharing and Wholesale Price

4.1 Model Description

In this setting, the manufacturer sequentially decides N cost share amounts to N suppliers. Similar to the previous model, the decision sequence follows s_n, s_{n-1},…, until s_1. However, in each sub-game, the manufacturer first decides the cost share amount Δ_i $(i = s_1, s_2, ..., s_n)$, and then supplier i decides his wholesale price [illegible]. Finally, the manufacturer decides the ordering quantity. For tractability, we initially consider a two-supplier model in which supplier s_2 moves first and supplier s_1 follows. This sequence is more accurate to the reality when the manufacturer sources components from short-term or new suppliers. Since the manufacturer has to make business with suppliers in turns, it is more likely that the decisions of the cost share amounts are also made in sequence.

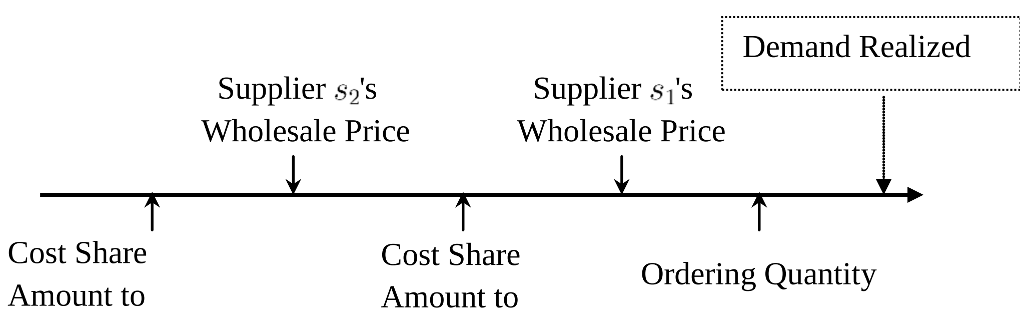


Figure 4.1 Decision Sequence in Hybrid Sequential Setting

Similar to the previous chapter, we adopt backward induction to solve the problem and obtain the manufacturer's best response function of ordering quantity as follows,

$$q^*(m_i, \Delta_i) = H - \frac{2}{w}(\bar{c} + \sum_{i=1}^{n} m_i)$$

In the next section, we solve two sub-games between the manufacturer and two suppliers, respectively. Different from the simple sequential setting, the supplier does not know the cost share amounts of the suppliers who decide the wholesale prices later than she does. Therefore, she could only maximize her profit based on these suppliers' best responses.

4.2 Stackelberg Game in the Hybrid Sequential Setting

4.2.1 Sub-game between manufacturer and supplier s_1

By given the manufacturer's decision on Δ_{s_2} and supplier s_2's wholesale price w_{s_2}, the manufacturer chooses Δ_{s_1} and then supplier s_1 decides on w_{s_1}.

Supplier s_1's Problem

Knowing the manufacturer's response to suppliers' wholesale prices, supplier s_1 decides on his wholesale price to maximize his profit,

$$\begin{aligned} &\Pi_{s_1} = m_{s_1}[H - \tfrac{2}{w}(\bar{c} + \textstyle\sum_{i=1}^{2} m_{s_i})] \\ &s.t. \quad H - \tfrac{2}{w}(\bar{c} + \textstyle\sum_{i=1}^{2} m_{s_i}) \leqslant G_{s_1} \end{aligned} \tag{4.1}$$

Solving this problem, we have

$$G_I = G_{rn}(O) \tag{4.2}$$

In this setting, the manufacturer cannot decide cost share amounts for all suppliers simultaneously at the beginning of the game. Before deciding to share supplier s_1's cost, the manufacturer has already offered supplier s_2 a cost share amount and has known her wholesale price w_{s_2}.

Knowing his own response to the given wholesale prices and supplier s_1's best response to the given cost share amount, the manufacturer chooses Δ_{s_1} to maximize his profit,

$$\Pi_M = \begin{cases} \frac{w}{4}[G_{s_1}(\Delta_{s_1})]^2 & 0 \leq G_{s_1}(\Delta_{s_1}) < \frac{(p-\bar{c}-m_{s_2})}{w} \\ \frac{(p-\bar{c}-m_{s_2})^2}{4w} & G_{s_1}(\Delta_{s_1}) \geqslant \frac{(p-\bar{c}-m_{s_2})}{w} \end{cases} \tag{4.3}$$

After some algebra, we obtain the manufacturer's best response to Δ_{s_1} as follow

$$\Delta^*_{s_1}(m_{s_2}, \Delta_{s_2}) = \begin{cases} c_{s_1} & 0 \leqslant m_{s_2} < \vartheta_1 - \beta_{s_1} c_{s_1} - wG_{s_1}(c_{s_1}) \\ \tilde{\Delta}_{s_1} & \vartheta_1 - \beta_{s_1} c_{s_1} - wG_{s_1}(c_{s_1}) \leqslant m_{s_2} < \vartheta_1 - wG_{s_1}(0) \\ 0 & m_{s_2} \geqslant \vartheta_1 - wG_{s_1}(0) \end{cases} \tag{4.4}$$

where $\vartheta_1 = p - c - \beta_{s_2}\Delta_{s_2}$, $\widetilde{\Delta}_{s_1} = k_{s_1} + c_{s_1} - \frac{\sqrt{(\vartheta_2 - m_{s_2})^2 + 4w\beta_{s_1}B_{s_1}} - \vartheta_2 + m_{s_2}}{2\beta_{s_1}}$ and $\vartheta_2 = \vartheta_1 - \beta_{s_1}(k_{s_1} + c_{s_1})$.

Denote $\mathcal{L}_1(\Delta_{s_1}) = p - \bar{c} - m_{s_2} - wG_{s_1}(\Delta_{s_1})$. Since both $\bar{c}$ and $G_{s_1}(\Delta_{s_1})$ are monotonously increasing in the interval of $\Delta_{s_1} \in [0, c_{s_1}]$, $\mathcal{L}_1(\Delta_{s_1})$ decreases in Δ_{s_1}. Π_M increases in Δ_{s_1} when $\mathcal{L}_1(\Delta_{s_1}) > 0$ and decreases in Δ_{s_1} when $\mathcal{L}_1(\Delta_{s_1}) \leqslant 0$. As a consequence, when $m_{s_2} \geqslant \vartheta_1 - wG_{s_1}(0)$, Π_M is a decreasing function in Δ_{s_1}. So the best response is $\Delta_{s_1} = 0$.

When $\vartheta_1 - \beta_{s_1} c_{s_1} - wG_{s_1}(c_{s_1}) \leqslant m_{s_2} < \vartheta_1 - wG_{s_1}(0)$, Π_M is a unimodal function and its peak is at $\Delta_{s_1} = \tilde{\Delta}_{s_1}$, where $\tilde{\Delta}_{s_1}$ satisfies $\mathcal{L}_1(\tilde{\Delta}_{s_1}) = 0$. Denote $t = p - c - \beta_{s_2}\Delta_{s_2} - m_{s_2}$, which is irrelevant to $\tilde{\Delta}_{s_1}$. Rewrite $\mathcal{L}_1(\tilde{\Delta}_{s_1}) = 0$ as

$t-\beta_{s_1}\tilde{\Delta}_{s_1}-\frac{wB_{s_1}}{k_{s_1}+c_{s_1}-\tilde{\Delta}_{s_1}}=0$. Since $\mathcal{L}_1(\Delta_{s_1})$ monotonously decreases in $\Delta_{s_1}\in[0,c_{s_1}]$, $\tilde{\Delta}_{s_1}$ is unique. After solving the formula, we obtain

$$\tilde{\Delta}_{s_1}=\frac{t+\beta_{s_1}(k_{s_1}+c_{s_1})\pm\sqrt{[t-\beta_{s_1}(k_{s_1}+c_{s_1})]^2+4w\beta_{s_1}B_{s_1}}}{2\beta_{s_1}}.$$

Rewrite it as $\widetilde{\Delta}_{s_1}=k_{s_1}+c_{s_1}+\frac{t-\beta_{s_1}(k_{s_1}+c_{s_1})\pm\sqrt{[t-\beta_{s_1}(k_{s_1}+c_{s_2})]^2+4w\beta_1 B_1}}{2\beta_{s_1}}$. Since $4w\beta_{s_1}B_{s_1}>0$, we obtain $\sqrt{[t-\beta_{s_1}(k_{s_1}+c_{s_2})]^2+4w\beta_1 B_1}>t-\beta_{s_1}(k_{s_1}+c_{s_1})$. Thus we exclude the root which is larger than c_{s_1} and obtain $\tilde{\Delta}_{s_1}=\frac{t+\beta_{s_1}(k_{s_1}+c_{s_1})-\sqrt{[t-\beta_{s_1}(k_{s_1}+c_{s_1})]^2+4w\beta_{s_1}B_{s_1}}}{2\beta_{s_1}}$.

When $0\leqslant m_{s_2}<\vartheta_1-\beta_{s_1}c_{s_1}-wG_{s_1}(c_{s_1})$, Π_M is an increasing function and its best response is $\Delta_{s_1}=c_{s_1}$. Thus it is proved.

4.2.2 Sub-game between manufacturer and supplier s_2

In this sub-game, supplier s_2 decides on the wholesale price w_{s_2} after the manufacturer provides the cost share amount Δ_{s_2}.

Supplier s_2's Problem

Supplier s_2's objective function is given by,

$$\Pi_{s_2}=\begin{cases} m_{s_2}G_{s_1}(c_{s_1}) & 0\leqslant m_{s_2}<\vartheta_1-\beta_{s_1}c_{s_1}-wG_{s_1}(c_{s_1}) \\ m_{s_2}\frac{\vartheta_2-m_{s_2}+\sqrt{(\vartheta_2-m_{s_2})^2+4w\beta_{s_1}B_{s_1}}}{2w} & \vartheta_1-\beta_{s_1}c_{s_1}-wG_{s_1}(c_{s_1})\leqslant m_{s_2}<\vartheta_1-wG_{s_1}(0) \\ \frac{m_{s_2}\vartheta_1-m_{s_2}^2}{w} & m_{s_2}\geqslant\vartheta_1-wG_{s_1}(0) \end{cases} \tag{4.5}$$

and it subjects to

$$s.t.\begin{cases} G_{s_1}(c_{s_1})\leqslant G_{s_2}(\Delta_{s_2}) & 0\leqslant m_{s_2}<\vartheta_1-\beta_{s_1}c_{s_1}-wG_{s_1}(c_{s_1}) \\ G_{s_1}(\widetilde{\Delta}_{s_1})\leqslant G_{s_2}(\Delta_{s_2}) & \vartheta_1-\beta_{s_1}c_{s_1}-wG_{s_1}(c_{s_1})\leqslant m_{s_2}<\vartheta_1-wG_{s_1}(0) \\ p-c-\beta_{s_2}\Delta_{s_2}-m_{s_2}\leqslant wG_{s_2}(\Delta_{s_2}) & m_{s_2}\geqslant\vartheta_1-wG_{s_1}(0) \end{cases}$$

By solving this problem, we obtain supplier s_2's best response to the wholesale price as below,

$$m_{s_2}^*(\Delta_{s2}) = \begin{cases} \vartheta_1 - wG_{s_2}(\Delta_{s_2}) & 0 \leqslant G_{s_2}(\Delta_{s_2}) \leqslant \mu_3 < \frac{\vartheta_2}{2w} \\ \vartheta_2 + \frac{\beta_{s_1} B_{s_1}}{\mu_3} - w\mu_3 & G_{s_1}(0) \leqslant \mu_3 < \frac{\vartheta_2}{2w} \\ \frac{\vartheta_2}{2} + \frac{2w\beta_{s_1} B_{s_1}}{\vartheta_2} & G_{s_1}(0) \leqslant \frac{\vartheta_2}{2w} \leqslant \mu_3 \\ \vartheta_1 - w\mu_1 & \frac{\vartheta_2}{2w} \leqslant \mu_1 < \frac{\vartheta_1}{2w} \\ \frac{\vartheta_1}{2} & \mu_1 > \frac{\vartheta_1}{2w} \end{cases} \tag{4.6}$$

where $\mu_1 = min\,(G_{s_2}(\Delta_{s_2}), G_{s_1}(0))$, $\mu_2 = max\,(G_{s_2}(\Delta_{s_2}), G_{s_1}(0))$, and $\mu_3 = min\,(\mu_2, G_{s_1}(c_{s_1}))$.

Π_{s_2} is a continuous piecewise function. When $0 \leqslant m_{s_2} < \vartheta_1 - \beta_{s_1} c_{s_1} - wG_{s_1}(c_{s_1})$, Π_{s_2} is linearly increasing in m_{s_2} if and only if $G_{s_2}(\Delta_{s_2}) \geqslant G_{s_1}(c_{s_1})$.

When $\vartheta_1 - \beta_{s_1} c_{s_1} - wG_{s_1}(c_{s_1}) \leqslant m_{s_2} < \vartheta_1 - wG_{s_1}(0)$, Π_{s_2} exists if and only if $G_{s_1}(\widetilde{\Delta}_{s_1}) \leqslant G_{s_2}(\Delta_{s_2})$. Denote $\alpha = \vartheta_2 - m_{s_2}$, so we have $d\alpha/dm_{s_2} = -1$. Rewrite the constraint $G_{s_1}(\widetilde{\Delta}_{s_1}) \leqslant G_{s_2}(\Delta_{s_2})$ as $\sqrt{\alpha^2 + 4w\beta_{s_1} B_{s_1}} \leqslant 2wG_{s_2}(\Delta_{s_2}) - \alpha$. If $2wG_{s_2}(\Delta_{s_2}) - \alpha < 0$, this inequality is not established. Thus, Π_{s_2} does not exist. If $2wG_{s_2}(\Delta_{s_2}) - \alpha \geqslant 0$, it can be rewritten as $\alpha^2 + 4w\beta_{s_1} B_{s_1} \leqslant [2wG_{s_2}(\Delta_{s_2}) - \alpha]^2$. Substitute $m_{s_2} = \vartheta_2 - \alpha$ to it, we obtain $m_{s_2} \geqslant \bar{m}_{s_2} = \vartheta_2 + \frac{\beta_{s_1} B_{s_1}}{G_{s_2}(\Delta_{s_2})} - wG_{s_2}(\Delta_{s_2})$.

In this case, m_{s_2} is supported in the interval of $[\vartheta_1 - \beta_{s_1} c_{s_1} - wG_{s_1}(c_{s_1}), \vartheta_1 - wG_{s_1}(0))$ only.

Rewrite it as $\left[\vartheta_2 + \frac{\beta_{s_1} B_{s_1}}{G_{s_1}(c_{s_1})} - wG_{s_1}(c_{s_1}), \vartheta_2 + \frac{\beta_{s_1} B_{s_1}}{G_{s_1}(0)} - wG_{s_1}(0)\right)$.

If $0 \leqslant G_{s_2}(\Delta_{s_2}) < G_{s_1}(0)$, we have $m_{s_2} < \vartheta_2 + \frac{\beta_{s_1} B_{s_1}}{G_{s_1}(0)} - wG_{s_1}(0) < \vartheta_2 + \frac{\beta_{s_1} B_{s_1}}{G_{s_2}(\Delta_{s_2})} - wG_{s_2}(\Delta_{s_2})$. Thus Π_{s_2} does not exist; If $G_{s_1}(0) \leqslant G_{s_2}(\Delta_{s_2}) < G_{s_1}(c_{s_1})$, Π_{s_2} exists only when $\bar{m}_{s_2} \leqslant m_{s_2} < \vartheta_1 - wG_{s_1}(0)$; If $G_{s_2}(\Delta_{s_2}) \geqslant G_{s_1}(c_{s_1})$, Π_{s_2} exists if and only if $m_{s_2} \geqslant \bar{m}_{s_2}$.

Since $\mathcal{L}_1(0) > 0$, it infers $\alpha > wG_{s_1}(0) > 0$. Then, we take Π_{s_2}'s first-order derivative at m_{s_2} as follows,

$$\begin{aligned}\frac{d\Pi_{s_2}}{dm_{s_2}} &= \frac{\alpha + \sqrt{\alpha^2 + 4w\beta_{s_1}B_{s_1}}}{2w} - (\vartheta_2 - \alpha)\frac{\alpha + \sqrt{\alpha^2 + 4w\beta_{s_1}B_{s_1}}}{2w\sqrt{\alpha^2 + 4w\beta_{s_1}B_{s_1}}} \\ &= \frac{\alpha + \sqrt{\alpha^2 + 4w\beta_{s_1}B_{s_1}}}{2w}\left[1 - \frac{\vartheta_2 - \alpha}{\sqrt{\alpha^2 + 4w\beta_{s_1}B_{s_1}}}\right]\end{aligned}$$

Let $d\Pi_{s_2}/dm_{s_2} = 0$, we have $m_{s_2} = \vartheta_2/2 + (2w\beta_{s_1}B_{s_1})/\vartheta_2$. Since both m_{s_2} and α are positive, $1 - \frac{\vartheta_2 - \alpha}{\sqrt{\alpha^2 + 4w\beta_{s_1}B_{s_1}}}$ increases in α and decreases in m_{s_2}. Thus, $d\Pi_{s_2}/dm_{s_2}$ is positive when $m_{s_2} < m^*_{s_2}$ and is negative when $m_{s_2} \geqslant m^*_{s_2}$. Therefore, Π_{s_2} reaches maximum at $m^*_{s_2} = \vartheta_2/2 + 2w\beta_{s_1}B_{s_1}/\vartheta_2$.

Compare $\bar{m}_{s_2}$ with $m^*_{s_2}$, we have

$$\begin{aligned}\bar{m}_{s_2} - m^*_{s_2} &= \vartheta_2 + \frac{\beta_{s_1}B_{s_1}}{G_{s_2}(\Delta_{s_2})} - wG_{s_2}(\Delta_{s_2}) - \frac{\vartheta_2}{2} - \frac{2w\beta_{s_1}B_{s_1}}{\vartheta_2} \\ &= -\frac{w}{G_{s_2}(\Delta_{s_2})}\left[G_{s_2}(\Delta_{s_2}) - \frac{\vartheta_2}{2w}\right]\left[G_{s_2}(\Delta_{s_2}) + \frac{2\beta_{s_1}B_{s_1}}{\vartheta_2}\right]\end{aligned}$$

Since $\vartheta_2 > 0$, it infers that $\bar{m}_{s_2} > m^*_{s_2}$ when $0 < G_{s_2}(\Delta_{s_2}) < \frac{\vartheta_2}{2w}$, and $\bar{m}_{s_2} \leqslant m^*_{s_2}$ when $G_{s_2}(\Delta_{s_2}) \geqslant \frac{\vartheta_2}{2w}$. As a consequence, when $0 < G_{s_2}(\Delta_{s_2}) < \frac{\vartheta_2}{2w}$, Π_{s_2} monotonously decreases in $m_{s_2} \in [\bar{m}_{s_2}, +\infty)$; when $G_{s_2}(\Delta_{s_2}) \geqslant \frac{\vartheta_2}{2w}$, Π_{s_2} is unimodal in the supported interval.

When $m_{s_2} \geqslant \vartheta_1 - wG_{s_1}(0)$, Π_{s_2} is subject to $m_{s_2} \geqslant \vartheta_1 - wG_{s_2}(\Delta_{s_2})$. Therefore, Π_{s_2} exists if and only if $m_{s_2} \geqslant \vartheta_1 - w\mu_1$, $\mu_1 = min\left(G_{s_2}(\Delta_{s_2}), G_{s_1}(0)\right)$. Take Π_{s_2}'s first-order derivative at m_{s_2}, we have $d\Pi_{s_2}/dm_{s_2} = (\vartheta_1 - 2m_{s_2})/w$. Intuitively, the unique peak of Π_{s_2} is at $m_{s_2}^* = \vartheta_1/2$.

Combine the above three pieces of the function, we obtain supplier s_2's best response to the manufacturer's cost share decision as follows:

a) When $0 \leqslant G_{s_2}(\Delta_{s_2}) < G_{s_1}(0)$, Π_{s_2} is a unimodal function which is subjected to $m_{s_2} \geqslant \vartheta_1 - wG_{s_2}(\Delta_{s_2})$. In this case, supplier s_2's best response is as follows,

$$m_{s_2}^* = \begin{cases} \vartheta_1 - wG_{s_2}(\Delta_{s_2}) & 0 \leqslant G_{s_2}(\Delta_{s_2}) < \frac{\vartheta_1}{2w} \\ \frac{\vartheta_1}{2} & G_{s_2}(\Delta_{s_2}) \geqslant \frac{\vartheta_1}{2w} \end{cases}$$

b) When $G_{s_1}(0) \leqslant G_{s_2}(\Delta_{s_2}) < min\left(G_{s_1}(c_{s_1}), \frac{\vartheta_2}{2w}\right)$, Π_{s_2} decreases in m_{s_2}. Since $m_{s_2} \geqslant \vartheta_2 + \frac{\beta_{s_2} B_{s_1}}{G_{s_2}(\Delta_{s_2})} - wG_{s_2}(\Delta_{s_2})$, the best response is $m_{s_2}^* = \vartheta_2 + \frac{\beta_{s_2} B_{s_1}}{G_{s_2}(\Delta_{s_2})} - wG_{s_2}(\Delta_{s_2})$.

c) When $G_{s_1}(0) \leqslant \frac{\theta_2}{2w} < min\left(G_{s_2}(\Delta_{s_2}),\ G_{s_1}(c_{s_1})\right)$, Π_{s_2} is a unimodal function so that the best response reaches its apex $m_{s_2}^* = \frac{\vartheta_2}{2} + \frac{2w\beta_{s_1} B_{s_1}}{\vartheta_2}$.

d) When $\frac{\vartheta_2}{2w} \leqslant G_{s_1}(0) \leqslant G_{s_2}(\Delta_{s_2})$, Π_{s_2} is a unimodal function in the interval of $m_{s_2} \geqslant \vartheta_1 - wG_{s_1}(0)$. Thus, supplier s_2's best response is given by

$$m_{s_2}^* = \begin{cases} \vartheta_1 - wG_{s_1}(0) & \frac{\vartheta_2}{2w} \leqslant G_{s_2}(\Delta_{s_2}) < \frac{\vartheta_1}{2w} \\ \frac{\vartheta_1}{2} & G_{s_2}(\Delta_{s_2}) \geqslant \frac{\vartheta_1}{2w} \end{cases}$$

e) When $G_{s_1}(c_{s_1}) < min\left(\frac{\vartheta_2}{2w},\ G_{s_2}(\Delta_{s_2})\right)$, Π_{s_2} decreases in m_{s_2} , where $m_{s_2} \geqslant \vartheta_2 + \frac{\beta_{s_1} B_{s_1}}{G_{s_1}(c_{s_1})} - wG_{s_1}(c_{s_1})$.

So the best response is $m_{s_2}^* = \vartheta_2 + \frac{\beta_{s_1} B_{s_1}}{G_{s_1}(c_{s_1})} - wG_{s_1}(c_{s_1})$.

Thus, it is proved.

Manufacturer's Decision on Δ_{s_2}

Based on the suppliers' best response of m_{s_i}, the manufacturer chooses Δ_{s_2} to maximize his profit,

$$\Pi_M(\Delta_{s2}) = \begin{cases} \frac{w}{4}[G_{s_2}(\Delta_{s_2})]^2 & 0 \leqslant G_{s_2}(\Delta_{s_2}) \leqslant \mu_3 < \frac{\vartheta_2}{2w} \\ \frac{w}{4}\mu_3^2 & G_{s_1}(0) \leqslant \mu_3 < \frac{\vartheta_2}{2w} \\ \frac{\vartheta_2^2}{16w} & G_{s_1}(0) \leqslant \frac{\vartheta_2}{2w} \leqslant \mu_3 \\ \frac{w}{4}\mu_1^2 & \frac{\vartheta_2}{2w} \leqslant \mu_1 < \frac{\vartheta_1}{2w} \\ \frac{\vartheta_1^2}{16w} & \mu_1 > \frac{\vartheta_1}{2w} \end{cases} \tag{4.7}$$

By solving this problem, we obtain the optimal cost share amount that the manufacturer may offer to supplier s_2.

$$\Delta_{s_2}^* = \begin{cases} 0 & 'G_{s_1}(c_{s_1}) < G_{s_2}(0)' \; or \; 'G_{s_1}(0) < G_{s_2}(0) \& \frac{\vartheta_2(0)}{2w} < G_{s_2}(0) < G_{s_1}(c_{s_1})' \\ min(\tilde{\Delta}_2, k_{s_2} + c_{s_2} - \frac{B_{s_2}k_F}{B_F}) & G_{s_2}(0) < G_{s_1}(c_{s_1}) \& G_{s_1}(0) < G_{s_2}(c_{s_2}) \& G_{s_2}(\rho_1^+) < \frac{\vartheta_2(\rho_1^+)}{2w} \\ \rho_1 & G_{s_2}(0) < G_{s_1}(0) < G_{s_2}(c_{s_2}) \& \frac{\vartheta_2(\rho_1)}{2w} < G_{s_2}(\rho_1) < \frac{\vartheta_1(\rho_1)}{2w} \\ min(\hat{\Delta}_2^+, c_{s_2}) & 'G_{s_1}(c_{s_1}) > G_{s_2}(0)' \; or \; 'G_{s_2}(0) < G_{s_1}(0) < G_{s_2}(c_{s_2}) \& G_{s_2}(c_{\rho_1}) > \frac{\vartheta_1(\rho_1)'}{2w} \end{cases} \tag{4.8}$$

$G_I = min\{G_{s_1}(0), G_{s_2}(0), ..., G_{s_n}(0)\}$, and $G_F = min\{G_{s_1}(c_{s_1}), G_{s_2}(c_{s_2}), ..., G_{s_n}(c_{s_n})\}$.

where $\rho_1 = k_{s_2} + c_{s_2} - \frac{B_{s_2}(k_{s_1}+c_{s_1})}{B_{s_1}}$. $\widetilde{\Delta}_2$ does not have an explicit expression but satisfies $G_{s_2}(\widetilde{\Delta}_2) = \frac{\vartheta_2(\widetilde{\Delta}_2)}{2w}$. Similarly, $\hat{\Delta}_2$ does not have an explicit expression but satisfies $G_{s_2}(\hat{\Delta}_2) = \frac{\vartheta_1(\hat{\Delta}_2)}{2w}$.

Proof:

According to (4.8), the optimal Δ_{s_2} is affected by the relationship among $G_{s_1}(0)$, $G_{s_1}(c_{s_1})$, $G_{s_2}(0)$ and $G_{s_2}(c_{s_2})$.

Case 1: $G_{s_1}(c_{s_1}) < G_{s_2}(0)$

For $\forall \; \Delta_{s_2} \in [0, \; c_{s_1}]$, $G_{s_2}(\Delta_{s_2}) > G_{s_1}(c_{s_1})$. Thus, we have $\mu_3 = G_{s_1}(c_{s_1})$. Both ϑ_1 and ϑ_2 are positive and linearly decreasing in Δ_{s_2}based on suppliers' best responses. As a result, both ϑ_1^2 and ϑ_2^2 decrease in Δ_{s_2}, which follows that $\Pi_M(\Delta_{s_2})$ is non-increasing in Δ_{s_2}. In this case, the manufacturer will not share any cost with supplier s_2.

Case 2: $G_{s_1}(0) < G_{s_2}(0) \leqslant G_{s_1}(c_{s_1})$

When $k_{s_1}c_{s_2} > k_{s_2}c_{s_1}$, we must have $G_{s_1}(c_{s_1}) < G_{s_2}(c_{s_2})$ in this case. But when $k_{s_1}c_{s_2} \leqslant k_{s_2}c_{s_1}$, we have to explore further:

a) $G_{s_1}(0) < G_{s_2}(0) \leqslant G_{s_1}(c_{s_1}) \leqslant G_{s_2}(c_{s_2})$

Denote $\mathcal{L}_2(\Delta_{s_2}) = \vartheta_2(\Delta_{s_2}) - 2wG_{s_2}(\Delta_{s_2})$. Intuitively, $\mathcal{L}_2$ decreases in Δ_{s_2}. Denote $\rho_2 = k_{s_2} + c_{s_2} - \frac{B_{s_2}k_{s_1}}{B_{s_1}}$. The optimal $\Delta^*_{s_2} = \rho_2$ according to (4.9) when $\mathcal{L}_2(\rho_2) \geqslant 0$. When $\mathcal{L}_2(\rho_2) < 0$, $\Pi_M(\Delta_{s_2})$ reaches its maximum at $\Delta^*_{s_2} = \tilde{\Delta}_{s_2}$.

b) $G_{s_1}(0) < G_{s_2}(0) \leqslant G_{s_2}(c_{s_2}) \leqslant G_{s_1}(c_{s_1})$

Since $G_{s_1}(0) < G_{s_2}(\Delta_{s_2}) < G_{s_1}(c_{s_1})$ for $\forall\ \Delta_{s_2} \in [0,\ c_{s_1}]$, $\mu_3 = G_{s_2}(\Delta_{s_2})$. When $\tilde{\Delta}_{s_2} \leqslant 0$, it follows that $B_{s_2} \geqslant \frac{k_{s_2}+c_{s_2}}{2w}[p - c - \beta_{s_1}(k_{s_1} + c_{s_1})]$. In this case, Π_M is non-increasing in Δ_{s_2} so that the manufacturer will not share supplier s_2's cost; When $\tilde{\Delta}_{s_2} \geqslant c_{s_2}$, Π_M increases in Δ_{s_2}. The manufacturer will share supplier s_2's cost; When $0 \leqslant \tilde{\Delta}_{s_2} < c_{s_2}$, Π_M reaches its maximum at $\Delta^*_{s_2} = \tilde{\Delta}_{s_2}$.

Case 3: $G_{s_2}(0) \leqslant G_{s_1}(0) < G_{s_2}(c_{s_2})$

Denote $\rho_1 = k_{s_2} + c_{s_2} - \frac{B_{s_2}(k_{s_1}+c_{s_1})}{B_{s_1}}$ and $\mathcal{L}_3(\Delta_{s_2}) = \vartheta_1(\Delta_{s_2}) - 2wG_{s_2}(\Delta_{s_2})$. If $k_{s_1}c_{s_2} > k_{s_2}c_{s_1}$, $\mathcal{L}_3$ decreases in Δ_{s_2}.

a) $G_{s_2}(0) \leqslant G_{s_1}(0) < G_{s_1}(c_{s_1}) < G_{s_2}(c_{s_2})$

When $\tilde{\Delta}_{s_2} < \rho_1 < \rho_2$, it infers that $\mathcal{L}_2(\rho_2) < \mathcal{L}_2(\rho_1) < 0$. Thus, Π_M is concavely increasing in $\Delta_{s_2} \in [0,\ \tilde{\Delta}_{s_2})$. When $\rho_1 \leqslant \hat{\Delta}_{s_2}$, Π_M continuously increases in $\Delta_{s_2} \in [\ \tilde{\Delta}_{s_2}, \rho_1)$ till $\frac{w}{4}[G_{s_1}(0)]^2$. When $\Delta_{s_2} \geqslant \rho_1$, Π_M is an non-increasing function. When $\rho_1 > \hat{\Delta}_{s_2}$, Π_M is a unimodal function which increases in $\Delta_{s_2} \in [\ \tilde{\Delta}_{s_2}, \hat{\Delta}_{s_2})$ and decreases in $\varDelta_{s_2} \in [\hat{\varDelta}_{s_2}, +\infty)$. Combine four situations, Π_M is a unimodal function and the optimal solution is $\varDelta_{s_2}^* = min(\rho_1, \hat{\varDelta}_{s_2})$.

When $\rho_1 < \tilde{\varDelta}_{s_2} < \rho_2$, Π_M increases in $\varDelta_{s_2} \in [0, \tilde{\varDelta}_{s_2})$ and is non-increasing afterwards. Thus, $\varDelta_{s_2}^* = \tilde{\varDelta}_{s_2}$.

When $\rho_1 < \rho_2 < \tilde{\varDelta}_{s_2}$, it follows that $G_{s_1}(c_{s_2}) < \frac{\vartheta_2(\rho_1)}{2w}$. So Π_M only increases when $0 \leqslant \varDelta_{s_2} < \rho_2$. Therefore, $\varDelta_{s_2}^* = \rho_2$.

b) $G_{s_2}(0) \leqslant G_{s_1}(0) < G_{s_2}(c_{s_2}) < G_{s_1}(c_{s_1})$

Similar to the above case, we obtain $G_{s_1}(c_{s_1}) < \frac{\vartheta_2(\rho_2)}{2w}$ when $\tilde{\varDelta}_{s_2} < \rho_1$.In this case, Π_M is increasing in $\varDelta_{s_2}$ only when $\varDelta_{s_2} < min(\rho_1, \hat{\varDelta}_{s_2})$. So the optimal solution $\varDelta_{s_2}^* = min(\rho_1, \hat{\varDelta}_{s_2})$. When $\tilde{\varDelta}_{s_2} \geqslant \rho_1$, Π_M reaches its maximum in the interval of $\frac{\vartheta_2}{2w} < \mu_1 < \frac{\vartheta_1}{2w}$. In this case, $\varDelta_{s_2}^* = min(\rho_2, \tilde{\varDelta}_{s_2})$.

Case 4: $\frac{4w^4}{(k_1+c_1)^4} B_1^3 - [] B_1^3 -$

In this case, Π_M increases in $0 \leqslant \varDelta_{s_2} < \hat{\varDelta}_{s_2}$ and decreases in $\varDelta_{s_2} \geqslant \hat{\varDelta}_{s_2}$. Since $\varDelta_{s_2} \in [0, c_{s_2}]$, the optimal $\varDelta_{s_2}^* = max\left(min(\tilde{\varDelta}_{s_2}, c_{s_2}), 0\right)$.

Thus, it is proved.

4.2.3 The Equilibrium in Hybrid Sequential Setting

Supplier s_2's Decision

We obtain the optimal wholesale price for supplier s_2 by substituting the manufacturer's decision of $\varDelta_{s_2}^*$ into the best response function.

$$m_{s_2}^* = \begin{cases} p - c - \beta_{s_1} c_{s_1} - wG_{s_1}(c_{s_1}) & G_{s_1}(c_{s_1}) < \frac{\theta_2(0)}{2w} \ \& \ G_{s_1}(c_{s_1}) < G_{s_2}(0) \\ \frac{\theta_2(0)}{2} + \frac{2w\beta_{s_1} B_{s_1}}{\theta_2(0)} & G_{s_1}(0) < \frac{\theta_2(0)}{2w} < min\left(G_{s_2}(0), G_{s_1}(c_{s_1})\right) \\ p - c - wG_{s_1}(0) & G_{s_1}(0) \leqslant G_{s_2}(0) \ \& \ \frac{\theta_2(0)}{2w} \leqslant G_{s_1}(0) \leqslant \frac{\theta_1(0)}{2w} \\ \frac{p-c}{2} & min\left(G_{s_1}(0), G_{s_2}(0)\right) > \frac{\theta_1(0)}{2w} \\ \theta_2(\rho_2) + \frac{\beta_{s_1} B_{s_1}}{G_{s_1}(c_{s_1})} - wG_{s_1}(c_{s_1}) & G_{s_2}(0) \leqslant G_{s_1}(c_{s_1}) < G_{s_2}(c_{s_2}) \ \& \leqslant B_{s_1} < V_2(B_{s_2}) \\ \frac{\theta_2(\tilde{\Delta}_2)}{2} + \frac{2w\beta_{s_1} B_{s_1}}{\theta_{s_1}(\tilde{\Delta}_2)} & V_2(B_{s_2}) \leqslant B_{s_1} < V_1(B_{s_2}) \ \& \ G_{s_2}(c_{s_2}) > \frac{\theta_2(c_{s_2})}{2w} \ \& \ G_{s_2}(0) < \frac{\theta_2(0)}{2w} \\ \theta_2(c_{s_2}) + \frac{\beta_{s_1} B_{s_1}}{G_{s_2}(c_{s_2})} - wG_{s_2}(c_{s_2}) & G_{s_1}(0) \leqslant G_{s_2}(c_{s_2}) < \frac{\theta_2(c_{s_2})}{2w} \ \& \ G_{s_1}(c_{s_1}) > G_{s_2}(0) \\ \theta_1(\rho_1) - wG_{s_1}(0) & G_{s_2}(0) \leqslant G_{s_1}(0) < G_{s_2}(c_{s_1}) \ \& \ V_1(B_{s_2}) \leqslant B_{s_1} < V_2(B_{s_1}) \\ \frac{\theta_1(\hat{\Delta}_2)}{2} & B_{s_1} \geqslant V_3(B_{s_2}) \ \& \ G_{s_2}(c_{s_2}) > \frac{\theta_1(c_{s_2})}{2w} \ \& \ G_{s_2}(0) \leqslant \frac{\theta_1(0)}{2w} \\ \theta_1(c_{s_2}) - wG_{s_2}(c_{s_2}) & G_{s_1}(0) \geqslant G_{s_2}(c_{s_2}) \ \& \ 0 \leqslant G_{s_2}(c_{s_2}) < \frac{\theta_1(c_{s_2})}{2w} \end{cases} \quad (4.10)$$

where $\rho_2 = k_{s_2} + c_{s_2} - \frac{B_{s_2} k_{s_1}}{B_{s_1}}$. $B_{s_1} = V_1(B_{s_2})$ does not have an explicit expression but satisfies $\vartheta_2(\rho_1) - 2wG_{s_2}(\rho_1) = 0$.

Similarly, $B_{s_1} = V_2(B_{s_2})$ satisfies $\vartheta_2(\rho_2) - 2wG_{s_2}(\rho_2) = 0$ $B_{s_1} = V_3(B_{s_2})$ satisfies $\vartheta_1(\rho_1) - 2wG_{s_2}(\rho_1) = 0$.

Lemma 4.1 shows the properties of $V_i(B_{s_2})$.

Lemma 4.1 *Given $\delta_i \geqslant 0$,*

a) $V_1(B_{s_2})$ increases in B_{s_2} when $\frac{k_{s_2}}{k_{s_1}+c_{s_1}}\delta_3 \leqslant B_{s_2} \leqslant \frac{k_{s_2}+c_{s_2}}{k_{s_1}+c_{s_1}}\delta_4$.

b) $V_2(B_{s_2})$ concavely increases in $B_{s_2} \in \left[\frac{k_{s_2}+c_{s_2}}{k_{s_1}}\delta_1, \frac{k_{s_2}+c_{s_2}}{k_{s_1}}\delta_2\right]$. Moreover, $V_2(B_{s_2})$is smaller than $\frac{k_{s_1}}{c_{s_1}} B_{s_2}$.

c) $V_3(B_{s_2})$ is a convexly increasing function in the interval of $\frac{k_{s_2}}{k_{s_1}+c_{s_1}}\delta_5 \leqslant B_{s_2} \leqslant \frac{k_{s_2}+c_{s_2}}{k_{s_1}+c_{s_1}}\delta_6$.

where $\delta_1 = \frac{k_{s_1}(p-c-\bar{\sigma}_2)}{2w}$, $\delta_2 = \frac{k_{s_1}(p-c-\bar{\sigma}_1)}{2w}$, $\delta_3 = \frac{(k_{s_1}+c_{s_1})(p-c-\bar{\sigma}_2)}{2w}$, $\delta_4 = \frac{(k_{s_1}+c_{s_1})(p-c-\bar{\sigma}_1)}{2w}$, $\delta_5 = \frac{(k_{s_1}+c_{s_1})(p-c-\beta_{s_2}c_{s_2})}{2w}$,$\delta_6 = \frac{(k_{s_1}+c_{s_1})(p-c)}{2w}$, $\bar{\sigma}_1 = \beta_{s_1}(k_{s_1} + c_{s_1})$, $\bar{\sigma}_2 = \beta_{s_1}(k_{s_1} + c_{s_1}) + \beta_{s_2} c_{s_2}$.

Proof:

Rewrite $\mathcal{L}_2(\rho_2)=0$ in the form of $B_{s_1}=V_2(B_{s_2})$. Intuitively, $V_2(B_{s_2})$ is an increasing function. Substitute $B_{s_1}=\delta_1$ and $B_{s_2}=\frac{k_{s_2}}{k_{s_1}}\delta_1$ into $\mathcal{L}_2(\rho_2)$, we have $\mathcal{L}_2(\rho_2)=\frac{\delta_1}{k_{s_1}}-\frac{p-c-\beta_{s_1}(k_{s_1}+c_{s_1})-\beta_{s_2}c_{s_2}}{2w}=0$. It implies that $V_2(B_{s_2})$ intersects with the line $B_{s_1}=\frac{k_{s_1}}{c_{s_1}}B_{s_2}$ in the point of $(B_{s_2},B_{s_1})=(\frac{k_{s_2}}{k_{s_1}}\delta_1,\delta_1)$.

Take $V_2(B_{s_2})$'s first-order derivative with respect to B_{s_2}, we have $dV_2/dB_{s_2}=\beta_{s_2}k_{s_1}/(\frac{B_{s_2}}{B_{s_1}}\beta_{s_2}k_{s_1}+\frac{2wB_{s_1}}{k_{s_1}})>0$. Take $V_2(B_{s_2})$'s second-order derivative to B_{s_2}, we have

$$\frac{d^2V_2}{dB_{s_2}^2}=-\frac{\beta_{s_2}k_{s_1}}{\left(\frac{B_{s_2}\beta_{s_2}k_{s_1}}{B_{s_1}}+\frac{2wB_{s_1}}{k_{s_1}}\right)^2}\left[\beta_{s_2}k_{s_1}\cdot\frac{B_{s_1}+B_{s_2}V_2'(B_{s_2})}{B_{s_1}^2}+\frac{2w}{k_{s_1}}\right]$$

Since $V_2'(B_{s_2})=dV_2/dB_{s_2}>0$, it infers that $d^2V_2/dB_{s_2}^2<0$.

Substitute $B_{s_1}=\delta_1, B_{s_2}=\frac{k_{s_2}}{k_{s_1}}\delta_1$ into dV_2/dB_{s_2}, we have

$$\frac{dV_2(B_{s_1})}{dB_{s_2}}\Big|_{B_{s_2}=\frac{k_{s_2}}{c_{s_2}}\delta_1}=\frac{\beta_{s_2}k_{s_1}}{k_{s_2}\beta_{s_2}+2w\delta_1/k_{s_1}}$$

Since $\delta_1>0$, it follows that $dV_2(B_{s_1})/dB_{s_2}<k_{s_1}/k_{s_2}$ when $B_{s_2}=k_{s_2}\delta_1/c_{s_2}$. As it proved above, $dV_2(B_{s_1})/dB_{s_2}$ is positive and decreasing in $B_{s_2}>k_{s_2}\delta_1/c_{s_2}$. Therefore, we can infer that $V_2(B_{s_2})\leqslant\frac{k_{s_1}}{k_{s_2}}B_{s_2}$ in $B_{s_2}\in[\frac{k_{s_2}}{k_{s_1}}\delta_1,\frac{k_{s_2}+c_{s_2}}{k_{s_1}}\delta_2]$. Thus, it is proved.

By solving $\mathcal{L}_3(\rho_1)=0$, we obtain

$$B_{s_2}=V_3(B_{s_1})=\frac{2w}{\beta_{s_2}(k_{s_1}+c_{s_1})^2}B_{s_1}^2-\frac{p-c-\beta_{s_2}(k_{s_2}+c_{s_2})}{\beta_{s_2}(k_{s_1}+c_{s_1})}B_{s_1}.$$

Since $\frac{d^2V_3(B_{s_1})}{dB_{s_1}^2}>0$, $V_3(B_{s_1})$ is a convex function. According to the proof in (24), ρ_1 increases in B_{s_1} and decreases in B_{s_2}. Since ϑ_1 decreases in Δ_{s_2}, it implies that $\vartheta_1(\rho_1)$ is decreasing in B_{s_1} and increasing in B_{s_2}. As $G_{s_1}(0)$ increases in B_{s_1}, it infers that $L_3(\rho_1)$

increases in B_{s_2} and decreases in B_{s_1}. Thus, $V3(B_{s_1})$ is an increasing function in the supported interval.

Manufacturer's Decision on $\Delta^*_{s_2}$

By given $(\Delta^*_{s_2}, m^*_{s_2})$, the manufacturer's optimal cost share amount offered to supplier s_1 could be solved. Combining $\Delta^*_{s_2}$ with $\Delta^*_{s_1}$, we acquire Theorem 4.1 as below.

Theorem 4.1 *When the cost share amounts and the wholesale prices are decided in an alternated sequence, the optimal cost share amount to each supplier is unique.*

$$(\Delta^*_{s_1}, \Delta^*_{s_2}) =$$
$$\begin{cases} \left(min\left(c_{s_1}, (k_{s_1} + c_{s_1} - \frac{2wB_{s_1}}{p-c-\bar{\sigma}_1})^+\right), 0\right); & 'G_{s_1}(c_{s_1}) < G_{s_2}(0)' \ or \ 'G_{s_1}(0) < G_{s_2}(0) \& \frac{\vartheta_2(0)}{2w} < G_{s_2}(0) < G_{s_1}(c_{s_1})' \\ \left(k_{s_1} + c_{s_1} - \frac{B_{s_1}(k_{s_2}+c_{s_2}-\Delta^*_{s_2})}{B_{s_2}}, min(\tilde{\Delta}_2, k_{s_2} + c_{s_2} - \frac{B_{s_2}k_F}{B_F})\right); & G_{s_2}(0) < G_{s_1}(c_{s_1}) \& G_{s_1}(0) < G_{s_2}(c_{s_2}) \& G_{s_2}(\rho_1^+) < \frac{\vartheta_2(\rho_1^+)}{2w} \\ (0, \rho_1); & G_{s_2}(0) < G_{s_1}(0) < G_{s_2}(c_{s_2}) \& \frac{\vartheta_2(\rho_1)}{2w} < G_{s_2}(\rho_1) < \frac{\vartheta_1(\rho_1)}{2w} \\ \left(0, min(\hat{\Delta}_2^+, c_{s_2})\right); & 'G_{s_1}(c_{s_1}) > G_{s_2}(0)' \ or \ 'G_{s_2}(0) < G_{s_1}(0) < G_{s_2}(c_{s_2}) \& G_{s_2}(\rho_1) > \frac{\vartheta_1(\rho_1)}{2w}' \end{cases}$$

The above theorem illustrates that the manufacturer's optimal cost share amounts given to the suppliers are affected by suppliers' capacities.

Figures below give us a more intuitive view of the optimal policy. Different from the equilibrium in the last chapter, this solution shape is asymmetric due to the influence of the decision sequence.

Denote $r_{s_i} = \frac{c_{s_i}}{k_{s_i}+c_{s_i}}$ as supplier s_i's deductable cost amount, which indicates the proportion of the deductable cost in supplier s_i's cost. The relationship between r_{s_1} and r_{s_2} may also affect the equilibrium.

Case 1 $r_{s_1} \geqslant r_{s_2}$

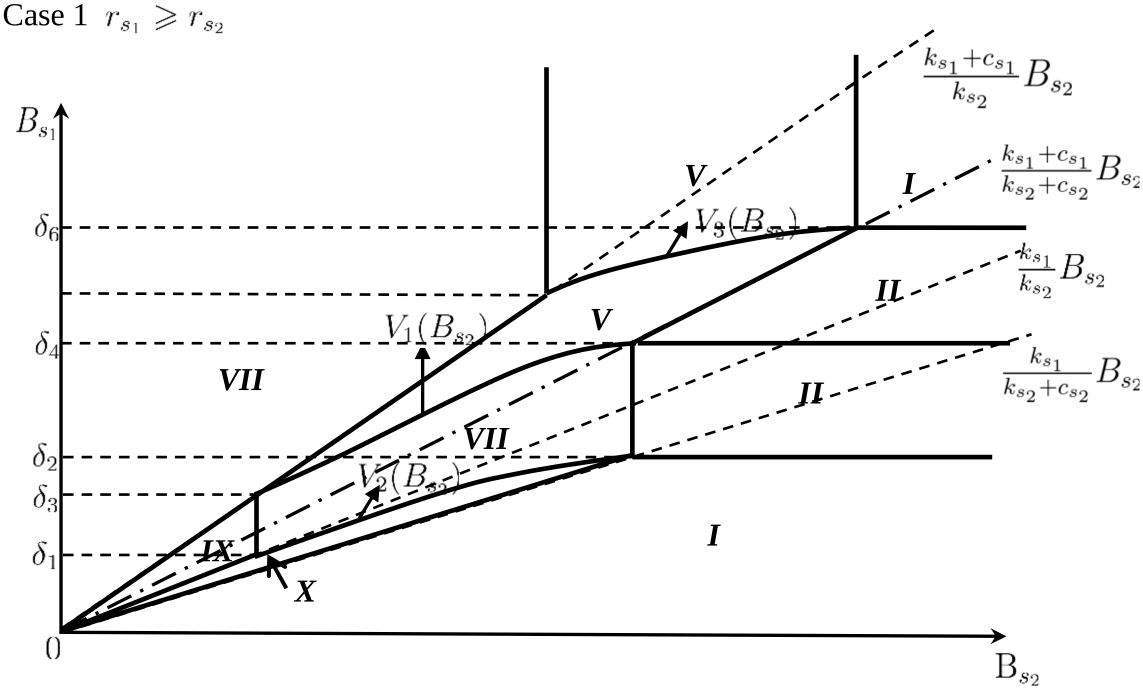


Figure 4.2 Equilibrium in Hybrid Setting 1

Case 2 $r_{s_1} < r_{s_2}$

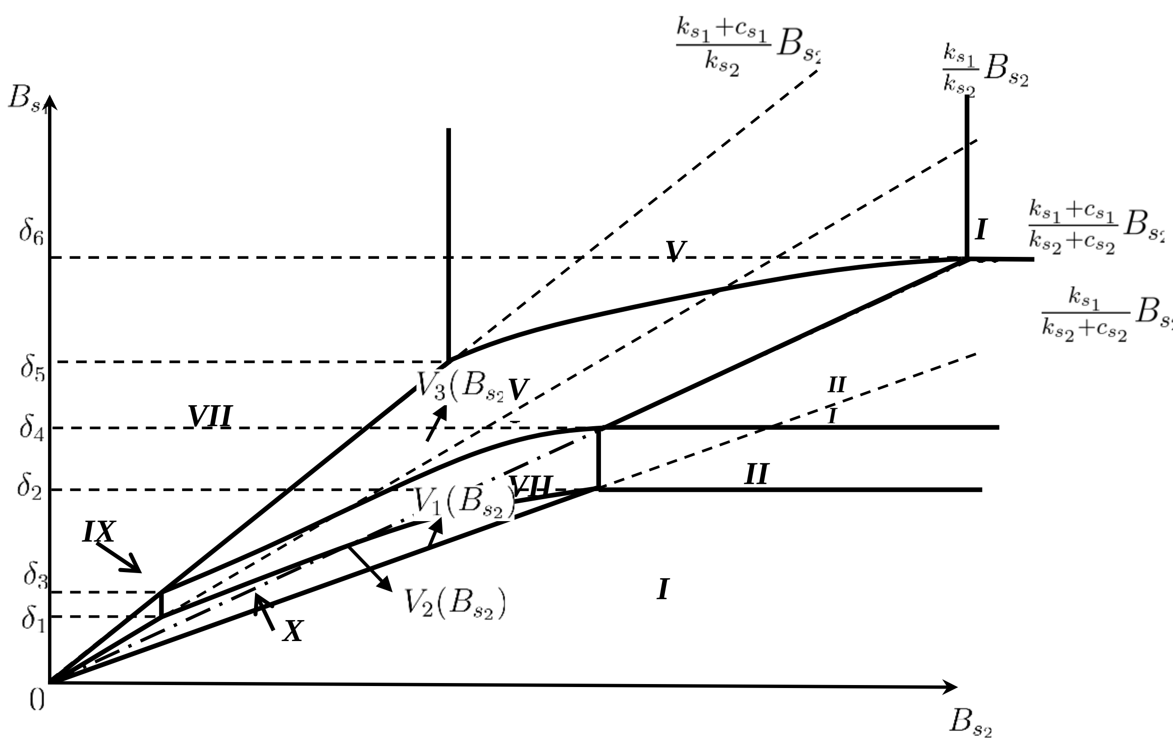


Figure 4.3 Equilibrium in Hybrid Setting 2

When r_{s_1} becomes smaller than r_{s_2}, the shape of the equilibrium rotates with the origin as the center counter-clockwise. When $B_{s_1} < \delta_1$, the manufacturer will always share the total deductable cost with the supplier who has a smaller $G_{s_i}(c_{s_i})$ to extract more profit. However, when $B_{s_1} > \delta_6$, the manufacturer cannot earn any extra profit by cost sharing. δ_i vary with different sales price and suppliers' costs. Since $p > c$, we have $\delta_6 > 0$. Intuitively, $\delta_6 = max(\delta_i)$ and $\delta_1 = min(\delta_i)$. For δ_i $(i = 1, ..., 5)$, however, they are not always positive and even not in a fixed order from small to large. That is, the above figure only demonstrates one particular case of δ_i. Since δ_1 has the same positivity as δ_3, lemma 4.2 illustrates the properties of δ_2 to δ_5.

Lemma 4.2 *For given β_{s_i}, k_{s_i} and c_{s_i}, the positivity and the size order of $\delta_i(i = 2, ..., 5)$ depends on p:*

Case 1: $\bar{\sigma}_1 \geqslant \beta_{s_2}c_{s_2}$

a) $\delta_i < 0$ when $0 \leqslant p < c + \beta_{s_2}c_{s_2}$;

b) $\delta_5 \geqslant 0$ when $c + \beta_{s_2}c_{s_2} \leqslant p < c + \bar{\sigma}_1$;

c) $\delta_5 \geqslant \delta_4 > \delta_2 > 0$ when $c + \bar{\sigma}_2 \leqslant p < c + \bar{\sigma}_3$;

d) $\delta_5 \geqslant \delta_4 > \delta_2 > \delta_{>}0$ when $c + \bar{\sigma}_3 \leqslant p < c + \bar{\sigma}_4$;

e) $\delta_5 \geqslant \delta_4 > \delta_3 > \delta_2 > 0$ when $p \geqslant c + \bar{\sigma}_4$.

Case 2: $\bar{\sigma}_1 < \beta_{s_2}c_{s_2}$

a) $\delta_i < 0$ when $0 \leqslant p < c + \bar{\sigma}_1$;

b) $\sigma_4 > \sigma_2 \geqslant 0$ when $\Delta^*_{s_1} = \begin{cases} 0 & 'G_{s_1}(c_{s_1}) < G_{s_2}(0)' \text{ or } 'G_{s_1}(0) < G_{s_2}(0) \& \frac{\phi_2(0)}{2w} < G_{s_2}(0) < G_{s_1}(c_{s_1})' \\ min(\bar{\Delta}_2, k_{s_1} + c_{s_1} - \frac{B_{s_1}k_{s_1}}{B_{s}}) & G_{s_2}(0) < G_{s_1}(c_{s_1}) \& G_{s_1}(0) < G_{s_2}(c_{s_2}) \& G_{s_2}(\rho_1^+) < \frac{\phi_2(\rho_1^+)}{2w} \\ \rho_1 & G_{s_2}(0) < G_{s_1}(0) < G_{s_2}(c_{s_2}) \& \frac{\phi_2(\rho_1)}{2w} < G_{s_2}(\rho_1) < \frac{\phi_1(\rho_1)}{2w} \\ min(\bar{\Delta}_2^+, c_{s_1}) & 'G_{s_1}(c_{s_1}) > G_{s_2}(0)' \text{ or } 'G_{s_2}(0) < G_{s_1}(0) < G_{s_2}(c_{s_2}) \& G_{s_2}(c_{s_1}) > \frac{\phi_1(\rho_1)}{2w}' \end{cases}$;

c) $\delta_4 > \delta_2 \geqslant \delta_5 \geqslant 0$ when $c + \beta_{s_2}c_{s_2} \leqslant p < c + min(\bar{\sigma}_3, \bar{\sigma}_5)$;

d) *When* $c + min(\bar{\sigma}_3, \bar{\sigma}_5) \leqslant p < c + max(\bar{\sigma}_3, \bar{\sigma}_5)$, $\delta_4 > \delta_2 > \delta_5 > \delta_3 > 0$ *if* $0 < \bar{\sigma}_1 < \frac{k_{s_1}}{k_{s_1}+c_{s_1}}\beta_{s_2}c_{s_2}$*; and* $\delta_4 > \delta_2 > \delta_5 > 0$ *if* $\frac{k_{s_1}}{k_{s_1}+c_{s_1}}\beta_{s_2}c_{s_2} < \bar{\sigma}_1 < \beta_{s_2}c_{s_2}$*;*

e) $\delta_4 > \delta_5 > \delta_2 > \delta_3 \geqslant 0$ *when* $c + max(\bar{\sigma}_3, \bar{\sigma}_5) \leqslant p < c + \bar{\sigma}_4$*;*

f) $\delta_4 > \delta_5 > \delta_3 \geqslant \delta_2 > 0$ *when* $p \geqslant c + \bar{\sigma}_4$.

where $\bar{\sigma}_3 = \beta_{s_2}c_{s_2} + \beta_{s_1}(k_{s_1} + c_{s_1})$, $\bar{\sigma}_4 = \beta_{s_1}(k_{s_1} + c_{s_1}) + \beta_{s_2}\frac{(k_{s_1}+c_{s_1})c_{s_2}}{c_{s_1}}$, $\bar{\sigma}_5 = \frac{(k_{s_1}+c_{s_1})c_{s_2}}{c_{s_1}}\beta_{s_2} - \frac{k_{s_1}(k_{s_1}+c_{s_1})}{c_{s_1}}\beta_{s_1}$.

Proof:

Intuitively, we must have $\delta_4 > \delta_2$ and $\delta_5 > \delta_3$. When $\beta_{s_1}(k_{s_1} + c_{s_1}) \geqslant \beta_{s_2}c_{s_2}$, it infers that $\delta_5 \geqslant \delta_4 > \delta_2$. After some algebra, we have $\delta_5 \geqslant \delta_4 > \delta_2 > \delta_3$ when $p < c + \beta_{s_1}(k_{s_1} + c_{s_1}) + \frac{(k_{s_1}+c_{s_1})c_{s_2}}{c_{s_1}}\beta_{s_2}$.

When $\beta_{s_1}(k_{s_1} + c_{s_1}) < \beta_{s_2}c_{s_2}$, we have $\delta_4 > \delta_5 > \delta_3$. Furthermore, we have $\delta_2 > \delta_5$ when $p < c + \left[\frac{(k_{s_1}+c_{s_1})c_{s_2}}{c_{s_1}}\beta_{s_2} - \frac{k_{s_1}(k_{s_1}+c_{s_1})}{c_{s_1}}\beta_{s_1}\right]^+$. Since $\beta_{s_1}(k_{s_1} + c_{s_1}) < \beta_{s_2}c_{s_2}$, the item in the square brackets is positive. If this inequality holds, we have $\delta_4 > \delta_2 > \delta_5 > \delta_3$. Otherwise, we know from the above proof that, $\delta_4 > \delta_5 > \delta_2 > \delta_3$ when $p < c + \beta_{s_1}(k_{s_1} + c_{s_1}) + \frac{(k_{s_1}+c_{s_1})c_{s_2}}{c_{s_1}}\beta_{s_2}$. Thus, Lemma 4.2 is proved.

The threshold for cost sharing is $p > min(\beta_{s_2}c_{s_2}, \bar{\sigma}_1)$. When the price is low than the threshold, the profit extracted from cost sharing cannot make up the additional cost that the manufacturer bears.

Proposition 4.1 *When the manufacturer sequentially decides cost share amounts to N suppliers,* *a) the equilibrium still keeps the balance* $G_{s_1}(\Delta^*_{s_1}) = G_{s_2}(\Delta^*_{s_2})$ *if both suppliers are shared by the manufacturer.*

b) the rank of δ_i *changes with sales price, which may further affect the shape of the equilibrium.*

c) the equilibrium is mutually affected by suppliers' capacity constraints and the decision sequence. When the supplier who has a limited initial capacity moves later, the manufacturer may not share her cost as he does in the simple sequential setting.

When $\delta_4 < B_{s_1} < \delta_6$ and $G_{s_1}(0) \leqslant G_{s_2}(0)$, supplier s_2 charges a high wholesale price since she can produce sufficient products without cost sharing. As a consequence, the manufacturer is not able to share supplier s_1's cost in the second stage even if he can extract more quantity to satisfy market demand via this approach. It implies that early negotiation boosts the bargaining power of the supplier who has abundant budget.

Supplier s_1's Decision

Substituting the optimal cost sharing amounts into supplier s_1's best response function, we obtain the optimal m_{s_1} as follows,

$$m^*_{s_1} = \begin{cases} \frac{w}{2}G_{s_1}(c_{s_1}) & G_{s_1}(c_{s_1}) < \frac{\theta_2(0)}{2w} \ \& \ 0 \leqslant B_{s_1} < V_2(B_{s_2}) \\ \frac{p-c-\bar{\sigma}_1}{4} & G_{s_1}(0) < \frac{\theta_2(0)}{2w} < min\left(G_{s_2}(0), G_{s_1}(c_{s_1})\right) \\ \frac{w}{2}G_{s_1}(0) & '\frac{\theta_2(0)}{2w} < G_{s_1}(0) \leqslant min\left(\frac{\theta_1(0)}{2w}, G_{s_2}(0)\right)' \ or \ 'G_{s_2}(0) \leqslant G_{s_1}(0) < G_{s_2}(c_{s_1}) \ \& \ V_1(B_{s_2}) \leqslant B_{s_1} < V_2(B_{s_1})' \\ \frac{p-c}{4} & min\left(G_{s_1}(0), G_{s_2}(0)\right) > \frac{\theta_1(0)}{2w} \\ \frac{w}{2}G_{s_2}(\tilde{\Delta}_2) & V_2(B_{s_2}) \leqslant B_{s_1} < V_1(B_{s_2}) \ \& \ G_{s_2}(c_{s_2}) > \frac{\theta_2(c_{s_2})}{2w} \ \& \ G_{s_2}(0) < \frac{\theta_2(0)}{2w} \\ \frac{w}{2}G_{s_2}(c_{s_2}) & '0 \leqslant G_{s_2}(c_{s_2}) < min\left(G_{s_1}(0), \frac{\theta_1(c_{s_2})}{2w}\right)' \ or \ 'G_{s_1}(0) \leqslant G_{s_2}(c_{s_2}) < min\left(G_{s_1}(0), \frac{\theta_2(c_{s_2})}{2w}\right)' \\ \frac{w}{2}G_{s_2}(\hat{\Delta}_2) & B_{s_1} \geqslant V_3(B_{s_2}) \ \& \ G_{s_2}(c_{s_2}) > \frac{\theta_1(c_{s_2})}{2w} \ \& \ G_{s_2}(0) \leqslant \frac{\theta_1(0)}{2w} \end{cases}$$

4.3 Channel Performance

Since optimal cost sharing amounts are affected by the decision sequence, we further study whether it may reallocate the profit among the suppliers and the manufacturer.

Manufacturer's Profit

Substituting the optimal cost share amounts and suppliers' wholesale price decisions into the manufacturer's objective function, we obtain his optimal profit as below,

$$\Pi_M^* = \begin{cases} \frac{w}{4}[G_{s_1}(c_{s_1})]^2 & G_{s_1}(c_{s_1}) < \frac{\theta_2(0)}{2w} \;\&\; 0 \leqslant B_{s_1} < V_2(B_{s_2}) \\ \frac{(p-c-\bar{\sigma}_1)^2}{16w} & G_{s_1}(0) < \frac{\theta_2(0)}{2w} < min\,(G_{s_2}(0), G_{s_1}(c_{s_1})) \\ \frac{w}{4}[G_{s_1}(0)]^2 & '\frac{\theta_2(0)}{2w} < G_{s_1}(0) \leqslant min\left(\frac{\theta_1(0)}{2w}, G_{s_2}(0)\right)' \; or \; 'G_{s_2}(0) \leqslant G_{s_1}(0) < G_{s_2}(c_{s_1}) \;\&\; V_1(B_{s_2}) \leqslant B_{s_1} < V_2(B_{s_1})' \\ \frac{(p-c)^2}{16w} & min\,(G_{s_1}(0), G_{s_2}(0)) > \frac{\theta_1(0)}{2w} \\ \frac{w}{4}[G_{s_2}(\tilde{\Delta}_2)]^2 & V_2(B_{s_2}) \leqslant B_{s_1} < V_1(B_{s_2}) \;\&\; G_{s_2}(c_{s_2}) > \frac{\theta_2(c_{s_2})}{2w}) \;\&\; G_{s_2}(0) < \frac{\theta_2(0)}{2w} \\ \frac{w}{4}[G_{s_2}(c_{s_2})]^2 & '0 \leqslant G_{s_2}(c_{s_2}) < min\left(G_{s_1}(0), \frac{\theta_1(c_{s_2})}{2w}\right)' \; or \; 'G_{s_1}(0) \leqslant G_{s_2}(c_{s_2}) < min\left(G_{s_1}(0), \frac{\theta_2(c_{s_2})}{2w}\right)' \\ \frac{w}{4}[G_{s_2}(\hat{\Delta}_2)]^2 & B_{s_1} \geqslant V_3(B_{s_2}) \;\&\; G_{s_2}(c_{s_2}) > \frac{\theta_1(c_{s_2})}{2w} \;\&\; G_{s_2}(0) \leqslant \frac{\theta_1(0)}{2w} \end{cases} \tag{4.11}$$

Supplier s_2's Profit

Substitute (4.11) into supplier s_2's best response, we have

$$\Pi_{s_2}^* = \begin{cases} [p-c-\beta_{s_1}c_{s_1} - wG_{s_1}(c_{s_1})]\,G_{s_1}(c_{s_1}) & G_{s_1}(c_{s_1}) < \frac{\theta_2(0)}{2w} \;\&\; G_{s_1}(c_{s_1}) < G_{s_2}(0) \\ \frac{[\theta_2(0)]^2}{4w} + \beta_{s_1}B_{s_1} & G_{s_1}(0) < \frac{\theta_2(0)}{2w} < min\,(G_{s_2}(0), G_{s_1}(c_{s_1})) \\ [p-c-wG_{s_1}(0)]\,G_{s_1}(0) & G_{s_1}(0) \leqslant G_{s_2}(0) \;\&\; \frac{\theta_2(0)}{2w} \leqslant G_{s_1}(0) \leqslant \frac{\theta_1(0)}{2w} \\ \frac{(p-c)^2}{4w} & min\,(G_{s_1}(0), G_{s_2}(0)) > \frac{\theta_1(0)}{2w} \\ [p-c-\beta_{s_1}c_{s_1} - \beta_{s_2}\rho_2 - wG_{s_1}(c_{s_1})]\,G_{s_1}(c_{s_1}) & G_{s_2}(0) \leqslant G_{s_1}(c_{s_1}) < G_{s_2}(c_{s_2}) \;\& \leqslant B_{s_1} < V_2(B_{s_2}) \\ \frac{[\theta_2(\tilde{\Delta}_2)]^2}{4w} + \beta_{s_1}B_{s_1} & V_2(B_{s_2}) \leqslant B_{s_1} < V_1(B_{s_2}) \;\&\; G_{s_2}(c_{s_2}) > \frac{\theta_2(c_{s_2})}{2w}) \;\&\; G_{s_2}(0) < \frac{\theta_2(0)}{2w} \\ \theta_2(c_{s_2}) \cdot G_{s_2}(c_{s_2}) + \beta_{s_1}B_{s_1} - w[G_{s_2}(c_{s_2})]^2 & G_{s_1}(0) \leqslant G_{s_2}(c_{s_2}) < \frac{\theta_2(c_{s_2})}{2w} \;\&\; G_{s_1}(c_{s_1}) > G_{s_2}(0) \\ \theta_1(\rho_1) \cdot G_{s_2}(0) - w[G_{s_1}(0)]^2 & G_{s_2}(0) \leqslant G_{s_1}(0) < G_{s_2}(c_{s_1}) \;\&\; V_1(B_{s_2}) \leqslant B_{s_1} < V_2(B_{s_1}) \\ \frac{[\theta_1(\hat{\Delta}_2)]^2}{4w} & B_{s_1} \geqslant V_3(B_{s_2}) \;\&\; G_{s_2}(c_{s_2}) > \frac{\theta_1(c_{s_2})}{2w} \;\&\; G_{s_2}(0) \leqslant \frac{\theta_1(0)}{2w} \\ [p-c-\beta_{s_2}c_{s_2} - wG_{s_2}(c_{s_2})]\,G_{s_2}(c_{s_2}) & G_{s_1}(0) \geqslant G_{s_2}(c_{s_2}) \;\&\; 0 \leqslant G_{s_2}(c_{s_2}) < \frac{\theta_1(c_{s_2})}{2w} \end{cases} \tag{4.12}$$

Supplier s_1's Profit

Similarly, we obtain supplier s_1's optimal profit as below,

$$\Pi_{s_1}^* = \begin{cases} \frac{w}{2}[G_{s_1}(c_{s_1})]^2 & G_{s_1}(c_{s_1}) < \frac{\theta_2(0)}{2w} \ \& \ 0 \leqslant B_{s_1} < V_2(B_{s_2}) \\ \frac{[p-c-\bar{\sigma}_1]^2}{8w} & G_{s_1}(0) < \frac{\theta_2(0)}{2w} < min\left(G_{s_2}(0), G_{s_1}(c_{s_1})\right) \\ \frac{w}{2}[G_{s_1}(0)]^2 & '\frac{\theta_2(0)}{2w} < G_{s_1}(0) \leqslant min\left(\frac{\theta_1(0)}{2w}, G_{s_2}(0)\right)' \ or \ 'G_{s_2}(0) \leqslant G_{s_1}(0) < G_{s_2}(c_{s_1}) \ \& \ V_1(B_{s_2}) \leqslant B_{s_1} < V_2(B_{s_1})' \\ \frac{(p-c)^2}{8w} & min\left(G_{s_1}(0), G_{s_2}(0)\right) > \frac{\theta_1(0)}{2w} \\ \frac{w}{2}[G_{s_2}(\tilde{\Delta}_2)]^2 & V_2(B_{s_2}) \leqslant B_{s_1} < V_1(B_{s_2}) \ \& \ G_{s_2}(c_{s_2}) > \frac{\theta_2(c_{s_2})}{2w}) \ \& \ G_{s_2}(0) < \frac{\theta_2(0)}{2w} \\ \frac{w}{2}[G_{s_2}(c_{s_2})]^2 & '0 \leqslant G_{s_2}(c_{s_2}) < min\left(G_{s_1}(0), \frac{\theta_1(c_{s_2})}{2w}\right)' \ or \ 'G_{s_1}(0) \leqslant G_{s_2}(c_{s_2}) < min\left(G_{s_1}(0), \frac{\theta_2(c_{s_2})}{2w}\right)' \\ \frac{w}{2}[G_{s_2}(\hat{\Delta}_2)]^2 & B_{s_1} \geqslant V_3(B_{s_2}) \ \& \ G_{s_2}(c_{s_2}) > \frac{\theta_1(c_{s_2})}{2w} \ \& \ G_{s_2}(0) \leqslant \frac{\theta_1(0)}{2w} \end{cases} \tag{4.13}$$

Different from the previous chapter, the cost sharing decision is more limited by the suppliers' capacity constraints in the hybrid sequential setting. The players' profits have changes in this setting due to the mutual effects caused by alternating decisions between the cost share amount and the wholesale price. However, the players' profit share is still keeps in this case.

The manufacturer's cost sharing decision is affected by the correlation between the suppliers' decision sequence and capacity constraint order. Thus the cost sharing equilibrium is asymmetric. It is quite different from the simple sequential setting.

Extending the model into multiple-suppliers case, we obtain the manufacturer's optimal profit as follows,

$$\Pi_M^* = \begin{cases} \dfrac{\theta^2}{2^{2n}w} & G_{s_n}(\Delta_{s_n}^*) \geqslant \dfrac{\theta}{2^{n-1}w} \\ \dfrac{w}{4}[G_{s_1}(0)]^2 & G_{s_1}(0) = G_I \& \dfrac{p-c-\beta_{s_1}(k_{s_1}+c_{s_1})}{w^{n-1}} \leqslant G_{s_1}(0) < \dfrac{p-c}{2w} \\ \dfrac{w}{4}[G_{s_n}(\Delta_{s_n}^*)]^2 & otherwise \end{cases}.$$

Supplier s_n's optimal profit is given by

$$\Pi_{s_n}^* = \begin{cases} \dfrac{\theta^2}{2^n w} + X_{s_{i-1}}\beta_{s_{i-1}}B_{s_{i-1}} & G_{s_n}(\Delta_{s_n}^*) \geqslant \dfrac{\theta}{2^{n-1}w} \\ [p-c-2^{n-2}wG_{s_1}(0)]G_{s_1}(0) & G_{s_1}(0) = G_I \& \dfrac{p-c-\beta_{s_1}(k_{s_1}+c_{s_1})}{w^{n-1}} \leqslant G_{s_1}(0) < \dfrac{p-c}{2w} \\ [p-c-2^{n-2}G_{s_n}(\Delta_{s_n}^*)]G_{s_n}(\Delta_{s_n}^*) & otherwise \end{cases}$$

where $\theta = p - c - \sum_{i \in S, i \neq s_n} \beta_{s_i}(k_{s_i} + c_{s_i}) - \beta_{s_n}\Delta_{s_n}^*$ and

$$X_{s_j} = \begin{cases} 1 & \Delta_{s_j}^* > 0 \\ 0 & \Delta_{s_j}^* = 0 \end{cases}.$$

Other suppliers' optimal profits are given by

$$\Pi_{s_i}^* = \begin{cases} \dfrac{\theta^2}{2^{2n-i}w} + X_{s_{i-2}}\beta_{s_{i-1}}B_{s_{i-1}} & G_{s_n}(\Delta_{s_n}^*) \geqslant \dfrac{\theta}{2^{n-1}w} \\ 2^{i-2}w[G_{s_1}(0)]^2 & G_{s_1}(0) = G_I \& \dfrac{p-c-\beta_{s_1}(k_{s_1}+c_{s_1})}{w^{n-1}} \leqslant G_{s_1}(0) < \dfrac{p-c}{2w} \\ 2^{i-2}w[G_{s_n}(\Delta_{s_n}^*)]^2 & otherwises \end{cases}.$$

The above results indicate that each supplier's capital flow directly affects the profit of the supplier who makes the decision next to her when the manufacturer does not share any supplier's total attainable cost. For two neighboring suppliers, the supplier who moves first can obtain an extra marginal profit if the manufacturer shares the cost of the later-moving supplier. Moreover, the extract profit increases when the later-moving supplier is not prone to cost sharing.

Chapter 5 Simultaneous Decision Setting in Both Cost Sharing and Wholesale Price

Simultaneous setting is different from sequential model in that N suppliers simultaneously decide wholesale prices after the manufacturer decides cost share amounts. Without loss of generality, we denote the supplier in this chapter by supplier i instead of s_i. As it assumes in the previous chapter, suppliers' initial capacities keep the inequality of $G_1(0) \leqslant G_2(0) \leqslant ... \leqslant G_n(0)$.

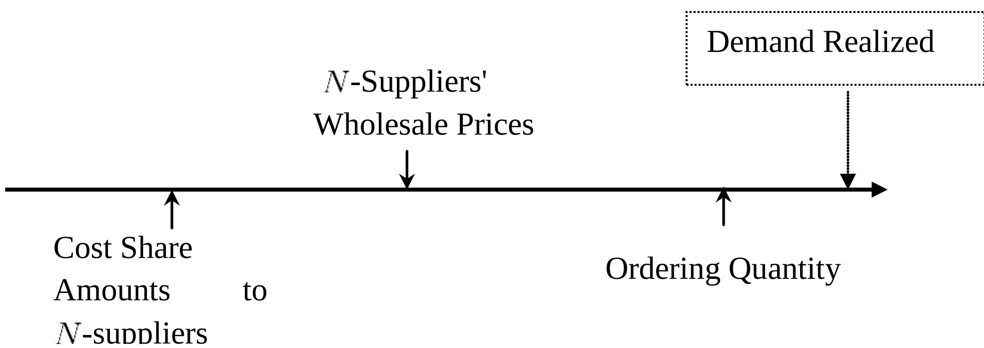


Figure 5.1 Decision Sequence in Simultaneous Setting

5.1 Suppliers' Problem

Given the manufacturer's decisions on cost share amounts offered to the suppliers, N suppliers simultaneously decide their own wholesale prices. By knowing other suppliers' best response functions on wholesale prices, supplier i maximizes her own profit as follows,

$$
\begin{aligned}
\Pi_i &= m_i q \\
&= m_i \left[H - \frac{2}{w}(\bar{c} + \sum_{j=1}^{n} m_i) \right] \\
s.t. \quad & m_i \geqslant p - \bar{c} - \frac{w}{2} G_i(\Delta_i) - M_{-i}
\end{aligned} \tag{5.1}
$$

where $M_{-i} = \sum_{j=1, j\neq i}^{n} m_j$.

Refer to the previous chapter, we denote $G_{r_n}(\Delta_{r_n})$ as the minimum attainable capacity among N suppliers. Correspondingly, the subscription r_n indicates the supplier who has the minimum attainable capacity. Thus, we obtain supplier i's best response function by solving ,

For $G_{r_n}(\Delta_{r_n}) \geqslant \frac{2(p-\bar{c})}{(n+1)w}$,

$$
m_i^*(\Delta) = \frac{p - \bar{c}(\Delta)}{n+1} \tag{5.2}
$$

For $G_{r_n}(\Delta_{r_n}) < \frac{2(p-\bar{c})}{(n+1)w}$,

$$
m_i^*(\Delta) = \begin{cases} p - \bar{c}(\Delta_{r_n}) - \dfrac{nw}{2} G_{r_n}(\Delta_{r_n}) & i = r_n \\ \dfrac{w}{2} G_{r_n}(\Delta_{r_n}) & i = 1, 2, .., n; i \neq r_n \end{cases} \tag{5.3}
$$

Denote $\mathcal{G}_{r_i}(\Delta_{r_i}) = min\left(G_1(\Delta_1), G_2(\Delta_2), ..., G_i(\Delta_i)\right),\ i = 1, ..., n$. $\mathcal{G}_{r_i}(\Delta_{r_i})$ indicates the minimum attainable capacity among supplier 1 to supplier i. Since supplier i can be different from supplier s_i, $\mathcal{G}_{r_i}(\Delta_{r_i})$ is not equivalent to $G_{r_i}(\Delta_{r_i})$. Only when $i = n$, we have $\mathcal{G}_{r_n}(\Delta_{r_n}) = G_{r_n}(\Delta_{r_n})$.

Proof:

We first discuss the case that involves three suppliers. Take Π_i's first-order derivative at m_i,

$$\frac{d\Pi_i}{dm_i} = H - \frac{2}{w}(\bar{c} + m_i + M_{-i}) - \frac{2}{w}m_i$$
$$= \frac{2}{w}(p - \bar{c} - M_{-i} - 2m_i)$$

Let $d\Pi_i/dm_i = 0$, we obtain $m_i = (p - \bar{c} - M_{-i})/2$. The profit function is unimodal.

When $0 \leqslant M_{-i} < p - \bar{c} - \frac{w}{2}\mathcal{G}_i(\mathit{\Delta}_i)$, $m_i^*(M_{-i}) = \frac{1}{2}(p - \bar{c} - M_{-i})$. When $M_{-i} \geqslant p - \bar{c} - \frac{w}{2}\mathcal{G}_i(\mathit{\Delta}_i)$, $m_i^*(-M_{-i}) = max\left(\frac{p-\bar{c}-M_{-i}}{2}, p - \bar{c} - M_{-i} - \frac{w}{2}\mathcal{G}_i(\mathit{\Delta}_i)\right)$.

Combine these two cases, we have

$$m_i^*(M_{-i}) = \begin{cases} p - \bar{c} - \frac{w}{2}G_i(\mathit{\Delta}_i) - M_{-i} & 0 \leqslant M_{-i} < p - \bar{c} - wG_i(\mathit{\Delta}_i) \\ \frac{1}{2}(p - \bar{c} - M_{-i}) & M_{-i} \geqslant p - \bar{c} - wG_i(\mathit{\Delta}_i) \end{cases} \tag{5.4}$$

Intuitively, $m_i^*(M_{-i})$ is linear and decreasing in M_{-i}. Particularly, $m_i^*(M_i) = p - \bar{c} - \frac{w}{2}\mathcal{G}_i(\mathit{\Delta}_i) - M_{-i}$ are parallel to each other.

Substitute $m_2 = \frac{1}{2}(p - \bar{c} - M_{-2})$ into $m_1^*(M_{-1})$, it follows that

$$m_1(m_3) = \begin{cases} \frac{p-\bar{c}-m_3}{3} & \mathcal{G}_i(\mathit{\Delta}_i) > \frac{2(p-\bar{c}-m_3)}{3w} \\ p - \bar{c} - m_3 - w\mathcal{G}_1(\mathit{\Delta}_1) & \mathcal{G}_1(\mathit{\Delta}_1) < min(\mathcal{G}_2(\mathit{\Delta}_2), \frac{2(p-\bar{c}-m_3)}{3w}) \end{cases} \tag{5.5}$$

Substitute $m_2 = p - \bar{c} - M_{-2} - \frac{w}{2}G_2(\mathit{\Delta}_2)$ into $m_1^*(M_{-1})$, it infers that

$$M_{-3}(m_3) = \begin{cases} \frac{2(p-\bar{c}-m_3)}{3} & \mathcal{G}_{r_2}(\mathit{\Delta}_{r_2}) \geqslant \frac{2(p-\bar{c}-m_3)}{3w} \\ p - \bar{c} - m_3 - \frac{w}{2}\mathcal{G}_{r_2}(\mathit{\Delta}_{r_2}) & 0 \leqslant \mathcal{G}_{r_2} < \frac{2(p-\bar{c}-m_3)}{3w} \end{cases}$$

Substitute the above function into $m_3^*(M_{-3})$, we obtain suppliers' best response function as follows,

Case 1 $\frac{2(p-\bar{c}-m_3)}{3} < p - \bar{c} - wG_3(\mathit{\Delta}_3)$

Supplier 3's best response function is given by,

$$m_3 = p - \bar{c} - \frac{w}{2}G_3(\mathit{\Delta}_3) - \frac{2(p - \bar{c} - m_3)}{3}$$
$$m_3^*(\mathit{\Delta}) = p - \bar{c} - \frac{3w}{2}G_3(\mathit{\Delta}_3)$$

Since it must satisfy $\mathcal{G}_{r_2}(\Delta_{r_2}) \geqslant \frac{2(p-\bar{c}-m_3)}{3w}$, we have $G_3(\Delta_3) = G_{r_3}(\Delta_{r_3}) < (p-\bar{c})/2w$. Consequently, we have $m_1^*(\Delta) = m_2^*(\Delta) = \frac{w}{2}G_3(\Delta_3)$.

Case 2 $\frac{2(p-\bar{c}-m_3)}{3} \geqslant p - \bar{c} - wG_3(\Delta_3)$

We obtain supplier 3's best response function as below,

$$m_3 = \frac{p-\bar{c}}{6} + \frac{m_3}{3}$$
$$m_3^*(\Delta) = \frac{p-\bar{c}}{4}$$

In this case, it must satisfy the constraint that $\mathcal{G}_{r_2}(\Delta_{r_2}) > 2(p-\bar{c}-\frac{p-\bar{c}}{4})/3w = (p-\bar{c})/2w$. After some algebras, we $m_1^*(\Delta_1) = m_2^*(\Delta_2) = \frac{p-\bar{c}}{4}$.

Case 3 $p - \bar{c} - m_3 - \frac{w}{2}\mathcal{G}_{r_2}(\Delta_{r_2}) \geqslant p - \bar{c} - wG_3(\Delta_3)$

In this case, supplier 3's best response function is given by

$$m_3 = \frac{1}{2}\left(m_3 + \frac{w}{2}\mathcal{G}_{r_2}(\Delta_{r_2})\right)$$
$$m_3^*(\Delta) = \frac{w}{2}\mathcal{G}_{r_2}(\Delta_{r_2})$$

Substitute it into the above constraint, we have $\mathcal{G}_{r_2}(\Delta_{r_2}) = G_{r_3}(\Delta_{r_3})$. Since $G_{r_3}(\Delta_{r_3}) < 2(p-\bar{c}-m_3)/3w$, it infers that $G_{r_3}(\Delta_{r_3}) < (p-\bar{c})/2w$. So we have

$$(m_1^*(\Delta), m_2^*(\Delta)) = \begin{cases} \left(p-\bar{c}-\frac{3w}{2}G_{r_3}(\Delta_{r_3}), \frac{w}{2}G_{r_3}(\Delta_{r_3})\right) & G_1(\Delta_1) \leqslant G_2(\Delta_2) \\ \left(\frac{w}{2}G_{r_3}(\Delta_{r_3}), p-\bar{c}-\frac{3w}{2}G_{r_3}(\Delta_{r_3})\right) & G_1(\Delta_1) > G_2(\Delta_2) \end{cases}$$

Case 4 $p - \bar{c} - m_3 - \frac{w}{2}\mathcal{G}_{r_2}(\Delta_{r_2}) < p - \bar{c} - wG_3(\Delta_3)$

In this case, supplier 3's best response function must satisfy $G_3(\Delta_3) = \mathcal{G}_{r_2}(\Delta_{r_2})$. Obviously, it has no solution.

Combine the above four cases, we obtain suppliers' best response functions in a three-suppliers setting. Extending the result to the N-suppliers model, we conjecture suppliers' response function as follow,

a) When $G_{r_n}(\Delta_{r_n}) \geqslant \frac{2(p-\bar{c})}{(n+1)w}$,

$$m_i^*(\Delta) = \frac{p - \bar{c}(\Delta)}{n+1} \tag{5.6}$$

b) When $G_{r_n}(\Delta_{r_n}) < \frac{2(p-\bar{c})}{(n+1)w}$

$$m_i^*(\Delta) = \begin{cases} p - \bar{c}(\Delta) - \frac{nw}{2} G_{r_n}(\Delta_{r_n}) & i = r_n \\ \frac{w}{2} G_{r_n}(\Delta_{r_n}) & i = 1, 2, .., n; i \neq r_n \end{cases} \tag{5.7}$$

Then, we adopt mathematical induction to prove it.

For $i = 2, 3$, we have proved that the above best response functions are established. Assuming that it still stand for $i = k - 1$, we consider the case when $i = k$.

For supplier j, $j = 1, 2, ..., k-1$, we have $m_j(m_k) = (p - \bar{c} - m_k)/k$ when $\mathcal{G}_{r_{k-1}} \geqslant \frac{2(p-\bar{c}-m_k)}{kw}$. When $0 < \mathcal{G}_{r_{k-1}} < \frac{2(p-\bar{c}-m_k)}{kw}$, we have $m_{-r_{k-1}}(m_k) = \frac{w}{2} \mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}})$ and $m_{r_{k-1}}(m_k) = p - \bar{c} - \frac{(k-1)w}{2} \mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}}) - m_k$.

Similar to (5.7), we solve supplier k's profit function as follows,

$$m_k^*(M_{-k}) = \begin{cases} p - \bar{c} - \frac{w}{2} G_k(\Delta_k) - M_{-k} & 0 \leqslant M_{-k} < p - \bar{c} - w G_k(\Delta_k) \\ \frac{1}{2}(p - \bar{c} - M_{-k}) & M_{-k} \geqslant p - \bar{c} - w G_k(\Delta_k) \end{cases} \tag{5.8}$$

Substitute $m_j(m_k), j = 1, ..., k-1$ into (5.8), we obtain that

Case 1 $m_j(M_{-j}) = (p - \bar{c} - M_{-j})/2,\ j = 1, 2, ..., k$

Supplier k's best response is

$$m_k = \frac{1}{2} \left[p - \bar{c} - \frac{k-1}{k}(p - \bar{c} - m_k) \right]$$
$$m_k^*(\Delta) = \frac{p - \bar{c}}{k+1}$$

Since $M_{-k} \geqslant p-\bar{c}-wG_k(\Delta_k)$ in this case, we have

$$\frac{(k-1)(p-\bar{c})}{k+1} > p-\bar{c}-wG_k(\Delta_k)$$
$$G_k(\Delta_k) > \frac{2(p-\bar{c})}{w(k+1)}$$

Thus, supplier j's best response is $m_j^*(\Delta)=\frac{p-\bar{c}}{k+1}$.

Case 2 $\quad M_{-k} \geqslant p-\bar{c}-wG_k(\Delta_k)$ & $0<\mathcal{G}_{r_{k-1}}<\frac{2(p-\bar{c}-m_k)}{kw}$

Substitute $M_{-k}=p-\bar{c}-\frac{w}{2}\mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}})-m_k$ into $m_k^*(M_{-k})$, we obtain supplier k's best response function as follows,

$$m_k=\frac{1}{2}\left[m_k+\frac{w}{2}\mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}})\right]$$
$$m_k^*(\Delta)=\frac{w}{2}\mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}})$$

Since $M_{-k} \geqslant p-\bar{c}-wG_k(\Delta_k)$, we have $G_{r_k}=\mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}})$. Furthermore, it must satisfy that

$$\mathcal{G}_{r_{k-1}}<\frac{2\left[p-\bar{c}-\frac{w}{2}\}\mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}}\right]}{kw}$$
$$G_{r_k}(\Delta_{r_k})<\frac{2(p-\bar{c})}{(k+1)w}$$

We further have

$$m_{r_k}^*(\Delta)=p-\bar{c}-\frac{kw}{2}G_{r_k}(\Delta_{r_k}),\ m_{-r_k}^*(\Delta)=\frac{w}{2}G_{r_k}(\Delta_{r_k}).$$

Case 3 $\quad 0 \leqslant M_{-k}<p-\bar{c}-wG_k(\Delta_k)$ & $\mathcal{G}_{r_{k-1}} \geqslant \frac{2(p-\bar{c}-m_k)}{kw}$

Supplier k's best response function is

$$m_k=p-\bar{c}-\frac{w}{2}G_k(\Delta_k)-\frac{k-1}{k}(p-\bar{c}-m_k)$$
$$m_k^*(\Delta)=p-\bar{c}-\frac{kw}{2}G_k(\Delta_k)$$

where $\mathcal{G}_{r_{k-1}} \geqslant G_{r_k}$ and $G_k(\Delta_k) \leqslant \frac{2(p-\bar{c})}{(k+1)w}$.

After some algebras, we have $m_j^*(\Delta) = m_{r_k}^*(\Delta) = \frac{w}{2}G_k(\Delta_k)$.

Case 4 $0 \leqslant M_{-k} < p - \bar{c} - wG_k(\Delta_k)$ and $0 < \mathcal{G}_{r_{k-1}} < \frac{2(p-\bar{c}-m_k)}{kw}$

We have

$$m_k = -\frac{w}{2}G_k(\Delta_k) + \frac{w}{2}\mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}}) + m_k$$
$$G_k(\Delta_k) = \mathcal{G}_{r_{k-1}}(\Delta_{r_{k-1}})$$

Intuitively, there is no solution in this case.

Combine the above cases, we can conclude that (5.7) and (5.8) are still established for $i = k$. Thus, it is proved.

Particularly, when two or more suppliers reach $G_{r_n}(\Delta) < \frac{2(p-\bar{c})}{(n+1)w}$, only one of them can charge $p - \bar{c}(\Delta_{r_n}) - \frac{nw}{2}G_{r_n}(\Delta_{r_n})$.

5.2 Manufacturer's Problem on Cost Sharing Decision

By knowing suppliers' best response on the wholesale prices, the manufacturer choose $\Delta_i,\ i = 1, 2, ..., n$ to maximize his own profit,

$$\Pi_M = \begin{cases} \dfrac{2(p-\bar{c})^2}{(2n+1)(n+1)w} & G_{r_n}(\Delta_{r_n}) \geqslant \dfrac{2(p-\bar{c})}{(n+1)w} \\ \dfrac{w}{4}G_{r_n}(\Delta_{r_n}) & 0 < G_{r_n}(\Delta_{r_n}) < \dfrac{2(p-\bar{c})}{(n+1)w} \end{cases} \tag{5.9}$$

Solving this problem, we obtain the equilibrium of Δ_i^* as below.

Pf

When $G_{r_n}(\Delta_{r_n}) \geqslant \frac{2(p-\bar{c})}{(n+1)w}$

We have $q^*(\Delta) = H - \frac{2}{w}(\frac{np}{n+1} + \frac{\bar{c}}{n+1}) = \frac{2(p-\bar{c})}{(n+1)w}$. Thus we obtain the manufacturer's profit function as below

$$
\begin{aligned}
\Pi_M(\Delta) &= \left[p - \bar{c} - \frac{n(p-\bar{c})}{n+1}\frac{p-\bar{c}}{2(n+1)}\right]\frac{2(p-\bar{c})}{(n+1)w} \\
&= \frac{2(p-\bar{c})^2}{(2n+1)(n+1)w}
\end{aligned}
$$

When $0 < G_{r_n}(\Delta_{r_n}) < \frac{2(p-\bar{c})}{(n+1)w}$,

After some algebra, we have $q^*(\Delta) = H - \frac{2}{w}[p - \frac{w}{2}G_{r_n}(\Delta_{r_n})] = G_{r_n}(\Delta_{r_n})$. In this case, the manufacturer's profit function is given by

$$
\begin{aligned}
\Pi_M(\Delta) &= \frac{w}{2}G_{r_n}(\Delta_{r_n}) \cdot G_{r_n}(\Delta_{r_n}) - \frac{w}{4}[G_{r_n}(\Delta_{r_n})]^2 \\
&= \frac{w}{4}[G_{r_n}(\Delta_{r_n})]^2
\end{aligned}
$$

Thus it is proved.

Theorem 5.1 *When N suppliers simultaneously decide on wholesale prices, there exist a unique solution of Δ_i $(i = 1, 2, ..., n)$ that the manufacturer will offer to suppliers.*

$$
\Delta_i{}^* = \begin{cases} min(\hat{\Delta}^+, k_i + c_i - \dfrac{B_i}{G_F(c_F)}) & i = 1 \\ \left[k_i + c_i - \dfrac{B_i(k_i + c_i - \Delta_1^*)}{B_1}\right]^+ & i = 2, 3, ..., n \end{cases} \tag{5.10}
$$

where $G_F(c_F) = min(G_1(c_1), G_2(c_2), ..., G_n(c_n))$. $\hat{\Delta}$ does not have an explicit expression but satisfies

$$
\mathcal{L}(\hat{\Delta}) = p - c - \textstyle\sum_{i=1}^{n}\{\beta_i[k_i + c_i - \frac{B_i(k_1+c_1-\hat{\Delta})}{B_I}]^+\} - \frac{(n+1)w}{2}G_1(\hat{\Delta}) = 0.
$$

Proof:

is a unimodal function in Δ_{r_n}.

Denote $\mathcal{L}(\Delta_{r_n}) = p - \bar{c} - \frac{(n+1)w}{2}G_{r_n}(\Delta_{r_n})$. Since $\mathcal{L}(\Delta_{r_n})$ decreases in Δ_{r_n}, there exist a unique $\hat{\Delta}$ which satisfy $\mathcal{L}(\hat{\Delta}) = 0$. We conclude that, $\Delta_{r_n}^* = min(\hat{\Delta}^+, G_{r_n}(c_{r_n}))$, $\Delta_{-r_n}^* = \left[k_{-r_n} + c_{-r_n} - \frac{B_{-r_n}}{G_{r_n}(\Delta_{r_n})}\right]^+$. Thus it is proved.

The above theorem shows that the solution structure is similar to that in the simple sequential setting in chapter 3. For given N suppliers, however, we can infer that $\bar{\Delta} < \hat{\Delta}$ since $2^n > n+1, \forall\ n = 2, 3, ..., k$. It implies that, the simultaneous setting can release more potential market demand than the first sequential setting. Thus, the manufacturer may share more suppliers' costs to extract capacities.

5.3 Profit Comparison

In this chapter, we compare the channel performance between simultaneously setting and first sequential setting which involves two suppliers.

5.4.1 Comparison of Manufacturer's Profits

For $n = 2$, substitute (5.10) into the manufacturer's best response on the ordering quantity,

$$q_d^* = \begin{cases} \dfrac{2[p - \bar{c}(\Delta_1^*)]}{3w} & \bar{c}(\Delta_1^*) \geqslant p - 2wG_1(\Delta_1^*) \\ G_1(\Delta_1^*) & \bar{c} < \bar{c}(\Delta_1^*) < p - 2wG_1(\Delta_1^*) \end{cases} \tag{5.11}$$

Then we obtain the manufacturer's optimal profit as follows,

$$\Pi_M = \begin{cases} \dfrac{2\left[p - \bar{c}(\Delta_1^*)\right]^2}{9w} & \bar{c}(\Delta_1^*) \geqslant p - \dfrac{3w}{2}G_1(\Delta_1^*) \\ \dfrac{w}{\left[G_1(\Delta_1^*)\right]^2} & 0 \leqslant \bar{c}(\Delta_1^*) < p - \dfrac{3w}{2}G_1(\Delta_1^*) \end{cases} \tag{5.12}$$

According to (3.13), we compare the manufacturer's profit in different decision sequence as follow,

Lemma 5.1 a) $\Delta_s^* \leqslant \Delta_d^*$;

b) $(\Pi_M^s)^* < (\Pi_M^d)^*$.

Refer to Figure 5.1, we have $(\Delta_1^d)^* = (\Delta_1^s)^*$ in area I, III, V and VI. In these cases, it is intuitive that $(\Pi_M^s)^* = \frac{w}{4}[G_{r_2}(c_{r_n})]^2 < w[G_{r_2}(c_{r_2})]^2 = (\Pi_M^d)^*$ or $(\Pi_M^s)^* = \frac{(p-c)^2}{16w} < \frac{2(p-c)^2}{9w} = (\Pi_M^d)^*$.

Proof:

According to the optimal function of [illegible] and $\hat{\Delta}$, it can be infer that there must be $\bar{\Delta} < \hat{\Delta}$ if the manufacturer shares the same number of suppliers in both settings. In this case, $(\Pi_M^s)^* = \frac{w}{4}[G_1(\bar{\Delta})]^2 < w[G(\hat{\Delta})]^2 = (\Pi_M^d)^*$, where the superscript d indicates the simultaneous setting while s indicates the sequential setting.

When $(\Pi_M^s)^* = \frac{(p-c)^2}{16w}$ and $(\Pi_M^d)^* = \frac{2[p-\bar{c}(\hat{\Delta})]^2}{9w} = w[G_1(\hat{\Delta})]^2$, we know from (3.11) that $G_1(0) \geqslant \frac{p-c}{2w}$. It follows that $\frac{(p-c)^2}{4w} \leqslant w[G_1(0)]^2 < w[G_1(\hat{\Delta})]^2$, so we have $(\Pi_M^s)^* < (\Pi_M^d)^*$.

In conclusion, we have $(\Pi_M^s)^* < (\Pi_M^d)^*$. Thus it is proved.

When suppliers decide the wholesale price simultaneously, the manufacturer is willing to share more suppliers' costs than that in the sequential decision setting, and thus extracting more profit from the increased capacity. In the equilibrium, simultaneous setting can always benefit the manufacturer even if he provides the same cost share amounts as that in the sequential setting. That is, the manufacturer strictly prefers to simultaneous setting.

Figure 5.2 shows the condition under which the manufacturer would provide higher cost share amounts in the simultaneous setting. W.L.O.G, we assume that $G_1(c_1) \leqslant G_2(c_2)$.

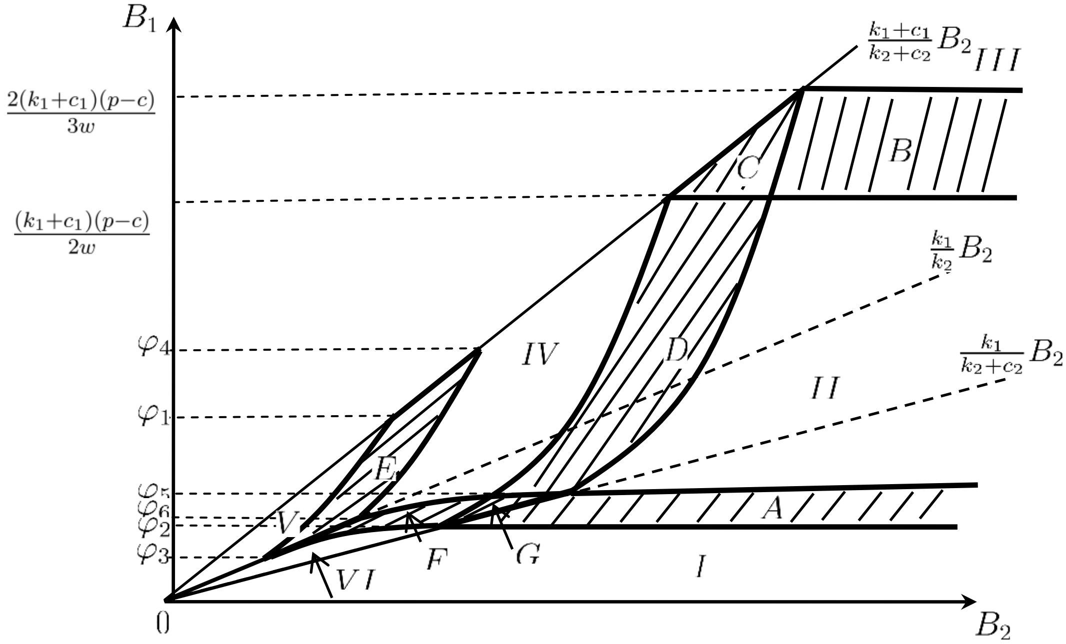


Figure 5.2 Comparison of the manufacturer's Profit

where

$$\varphi_4 = \frac{2k_2(k_1+c_1)}{3w(k_2+c_2)}\left[p-c-\frac{c_2\sigma_2}{k_2+c_2}\right]$$
$$\varphi_5 = \frac{2k_1}{3w}(p-c-\beta_1 c_1)$$
$$\varphi_6 = \frac{2k_1}{3w}(p-c-\sigma_1)$$

Expect for the area I, IV and V, the manufacturer offers higher cost share amounts to suppliers than that in the sequential setting.

5.4.2 Comparison of Suppliers' Profits

In the simultaneous setting, we denote d_1 as the supplier who charges a high wholesale price when the market demands more than the capacity. Thus, we obtain supplier d_1's profit as below,

$$\Pi_{d_1}^* = \begin{cases} \dfrac{2\left[p-\bar{c}(\varDelta_1^*)\right]^2}{9w} & \bar{c}(\varDelta_1^*) \geqslant p - \dfrac{3w}{2}G_1(\varDelta_1^*) \\ \left[p-\bar{c}(\varDelta_1^*) - wG_1(\varDelta_1^*)\right]G_1(\varDelta_1^*) & c < \bar{c}(\varDelta_1^*) < p - \dfrac{3w}{2}G_1(\varDelta_1^*) \end{cases} \tag{5.13}$$

Correspondingly, the other supplier (denoted as d_2)'s optimal profit is given by

$$\Pi_{d_2}^* = \begin{cases} \dfrac{2\left[p-\bar{c}(\varDelta_1^*)\right]^2}{9w} & \bar{c}(\varDelta_1^*) \geqslant p - \dfrac{3w}{2}G_1(\varDelta_1^*) \\ \dfrac{w}{2}\left[G_1(\varDelta_1^*)\right]^2 & c < \bar{c}(\varDelta_1^*) < p - \dfrac{3w}{2}G_1(\varDelta_1^*) \end{cases} \tag{5.14}$$

Intuitively, we can infer that $\Pi_{d_1}^* \geqslant \Pi_{d_2}^*$.

Referring to the previous chapter, the supplier's profit is affected by the decision sequence in the sequential setting. Consequently, we have to discuss the suppliers' profit into two cases:

Lemma 5.2 *a) When supplier d_1 moves first in the sequential model, there must be $\Pi_{d_1}^* \leqslant \Pi_{s_2}^*$, $\Pi_{d_2}^* \geqslant \Pi_{s_1}^*$. The equalities hold when and only when $q_d^* = q_s^* = G_{r_2}(c_{r_2})$.*

b) When supplier d_1 moves later, there must be $\Pi_{d_1}^* > \Pi_{s_1}^*$, $\Pi_{d_2}^* < \Pi_{s_2}^*$.

Proof:

Case 1 supplier d_1 moves first in the sequential setting

Supplier d_1 is equivalent to supplier s_2 in this case. Referring to Figure 13, we compare $\Pi_{d_1}^*$ and $\Pi_{s_2}^*$ as follows,

In section I (see Figure 3.2), $\Pi_{d_1}^* = \Pi_{s_1}^* = [p - \bar{c}(c_1) - wG_1(c_1)]G_1(c_1)$.

In section A, $\Pi_{d_1}^* = [p - c - \beta_1 c_1 - w\frac{B_1}{k_1}]\frac{B_1}{k_1}$, $\Pi_{s_2}^* = \frac{(p-c-\beta_1\bar{\varDelta}_1)^2}{4w}$. As it is proved in Lemma 1.2, $\Pi_{d_1}^*$ decreases in $B_1 \in (\varphi_2, \varphi_5)$ and $\Pi_{s_2}^*$ increases in B_1 when $c < p < c + \sigma_3$. Since $\Pi_{d_1}^*|_{B_1=\varphi_2} = \Pi_{s_1}^*|_{B_1=\varphi_2} = \frac{(p-c-\beta_1 c_1)^2}{4w}$, it infer that $\Pi_{d_1}^* < \Pi_{s_2}^*$.

In section II and IV, $\Pi^*_{d_1} = \frac{2[p-\bar{c}(\hat{\Delta})]^2}{9w} = \frac{w}{2}G_1(\hat{\Delta})$, $\Pi^*_{s_2} = \frac{[p-\bar{c}(\hat{\Delta})]^2}{4w} = wG_1(\bar{\Delta})$. Since $\hat{\Delta} > \bar{\Delta}$, and $\bar{c}$ increases in $\bar{c}(\Delta)$, we have $\Pi^*_{d_1} < \Pi^*_{s_2}$. Similarly, we have $\Pi^*_{d_1} < \Pi^*_{s_2}$ in area C and D.

In section B, $\Pi^*_{d_1} = \frac{2[p-\bar{c}(\hat{\Delta})]^2}{9w}$, $\Pi^*_{s_2} = \frac{(p-c)^2}{4w}$. Thus, we have $\Pi^*_{d_1} < \Pi^*_{s_2}$.

In section III, $\Pi^*_{d_1} = \frac{2(p-c)^2}{9w} < \frac{(p-c)^2}{4w} = \Pi^*_{s_2}$.

In section V and VI, $\Pi^*_{d_1} = \Pi^*_{s_2} = [p - \bar{c}(c_{r_2}) - wG_{r_2}(c_{r_2})]\, G_{r_2}(c_{r_2})$.

In section E,

Take $\Pi^*_{d_1}$'s first-order partial derivative at B_1, it follows that $\partial\Pi^*_{d_1}/\partial B_1 = \beta_1 > 0$. $\Pi^*_{s_2}$'s first-order partial derivative at B_1 is given by

$$\partial\Pi^*_{s_2}/\partial B_1 = \frac{\beta_1}{2}\left[\frac{p-c-\sigma_2}{\sqrt{(p-c-\sigma_2)^2 + 8w(\beta_1 B_1 + \beta_2 B_2)}} + 1\right]$$

It follows that

$$\partial(\Pi^*_{d_1} - \Pi^*_{s_2})/\partial B_1 = \frac{\beta_1}{2}\left[1 - \frac{p-c-\sigma_2}{\sqrt{(p-c-\sigma_2)^2 + 8w(\beta_1 B_1 + \beta_2 B_2)}}\right]$$

Since $\beta_1 B_1 + \beta_2 B_2 > 0$, we have $\partial(\Pi^*_{d_1} - \Pi^*_{s_2})/\partial B_1 > 0$. It implies that $\Pi^*_{d_1} - \Pi^*_{s_2}$ increases in B_1 for given B_2.

Take $\Pi^*_{d_1}$'s first-order partial derivative at B_2 as follows,

$$\partial\Pi^*_{d1}/\partial B_2 = \frac{1}{k_2}\left[p - \beta_1(k_1 + c_1) - \beta_2 c_2\right] - \frac{2wB_2}{k_2^2}$$

Furthermore, we take $\Pi^*_{d_1}$'s second-order partial derivative at B_2 and obtain $\partial(\Pi^*_{d_1})^2/\partial^2 B_2 = -\frac{2w}{k_2^2} < 0$. It infers that $\Pi^*_{d_1}$ is concave in B_2, and the apex locates at $\hat{B}_2 = \frac{k_2}{2w}\left[p - \beta_2(k_1 + c_1) - \beta_2 c_2\right]$. So we have $\partial\Pi^*_{d1}/\partial B_2 < 0$.

Take $\Pi_{s_2}^*$'s first-order partial derivative at B_2, we obtain

$$\partial\Pi_{s_2}^*/\partial B_2 = \frac{\beta_1}{2}\left[\frac{p-c-\sigma_2}{\sqrt{(p-c-\sigma_2)^2+8w(\beta_1 B_1+\beta_2 B_2)}}+1\right]$$

Obviously, $\Pi_{d_1}^* - \Pi_{s_2}^*$ decreases in B_2 by given B_1.

Thus, $\Pi_{s_2}^* \geqslant \Pi_{d_1}^*$ when $B_2 = \frac{k_2}{k_1}B_1$, $B_2 = \frac{k_2+c_2}{k_1+c_1}B_1$ and $B_2 = arg\ u_6(B_2)$. We can further infer that $\Pi_{s_2}^* \geqslant \Pi_{d_1}^*$ in this case.

In area F, $\Pi_{d_1}^* = [p-\beta_1 c_1 - \beta_2(k_2+c_2-\frac{B_2}{G_1(c_1)}) - wG_1(c_1)]G_1(c_1)$, $\Pi_{s_2}^* = \frac{[p-\bar{c}(\bar{\Delta}_2)]^2}{4w}$. In area G, $\Pi_{d_1}^* = [p-\beta_1 c_1 - \beta_2(k_2+c_2-\frac{B_2}{G_1(c_1)}) - wG_1(c_1)]G_1(c_1)$, $\Pi_{s_2}^* = \frac{[p-\bar{c}(\bar{\Delta}_1)]^2}{4w}$. Similar to area E, we obtain that $\Pi_{s_2}^* \geqslant \Pi_{d_1}^*$ in these two areas. Thus, we prove that $\Pi_{s_2}^* \geqslant \Pi_{d_1}^*$.

Similar to the comparison of the manufacturer's profit, we can obtain that $\Pi_{d_2}^* \geqslant \Pi_{s_1}^*$.

Case 2 supplier d_2 moves first in the sequential setting

Since $\Pi_{d_1}^* \geqslant \Pi_{d_2}^*$, it is intuitive that $\Pi_{s_2}^* > \Pi_{d_1}^* > \Pi_{d_2}^*$ and $\Pi_{d_1}^* > \Pi_{d_2}^* > \Pi_{s_1}^*$. The equality does not hold in this case.

Thus, it is proved.

Lemma 5.2 illustrates that the sequential setting benefits the supplier who makes the decision first, no matter whether she has the heaviest cost burden or not. Other suppliers, however, would prefer simultaneous setting. Although the first-moving supplier may incur cost cutting due to cost sharing in the sequential setting, she still prefers this setting because early movement allows her to charge a higher wholesale price than others. This result indicates that decision sequence plays a more important role in players' profits than the cost-sharing policy does.

Chapter 6 Capacity Investment In a Flexible Platform

6.1 Introduction

Fierce competition and volatile consumer demand have made flexible capability indispensible to any manufacturer in today's market, for it quick response to customer orders, or quick changeovers between products. The push to make processes flexible has spread throughout the entire manufacturing industry, and even permeated chemical and paper industries where the rule of game has long been that plants with the longest production runs are the most competitive. Firms across the industries have collectively invested billions of dollars in machines and computer integrated systems to establish platforms for their desired flexibility capabilities. On the other hand, manufacturers have been under the directives to reduce costs. As globalization has made further compression of materials and labor costs quite infeasible, they are forced to seek cost reduction by enhancing process efficiency to lower marginal production costs. In many circumstances, process efficiency is tied to the scale of capacity at one firm. More often than not, a more efficient production process, with a lower production cost, is associated with a higher capacity, attributed to the experience the firm has accumulated in arranging the resources to manage production activities. On a given flexibility platform, the capacity level imposes a physical limit on the scale of the production one firm can engage and, in the meantime, to a large extent influences the way at which the production process is managed. Since both the capacity efficiency and process efficiency are intertwined, it is imperative for the operation managers to understand their interplay to better align their initiatives. In this chapter, we will explicitly build the connection between capacity and process efficiency.

We consider a setting of two firms each with volume flexibility, referring to the capability to produce below capacity. The market demand is price sensitive and uncertain. We define an explicit production cost function form to capture these features, and use an efficiency factor to relate the process efficiency to the scale of the capacity. We will explore the competitive capacity investments by the two firms, with implications on process efficiency and profit performance, by use of the decision sequence illustrated in Figure 6.1. Two firms first simultaneously invest in capacities (which determine their process efficiencies) before actual demand curve reveals. After the actual demand curve is known, firms simultaneously produce under capacity limit and sell products in the market. Recently, there is a stream of literature on manufacturing flexibility, mostly in a monopoly setting. The relevant study in a competitive setting is sparse, and, for tractability, a symmetric setting is assumed, and production cost is often assumed away to focus on the strategic effects of flexibility on the individual firms and the system. We incorporate production cost and introduce an efficiency factor to relate it to the capacity at an individual firm. By making the efficiency factors differ across the firms, we allow asymmetry in the operational decision makings.

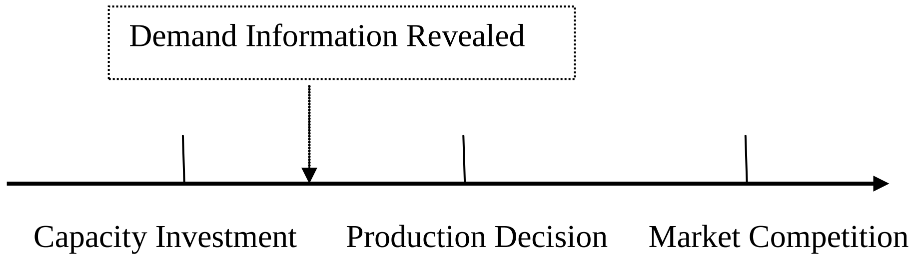


Figure 6.1 Decision Sequence in Capacity Investment Model

6.2 The Basic Model

We consider a setting of two firms that serve one market. The market demand is price sensitive and uncertain, and let the inverse demand function be:

$$P(q, \mathrm{A}) = (\mathrm{A} - q)^+ \tag{6.1}$$

where q is the quantity and A the random variable that models the market size. We assume that A has a non-negative support with mean $\mu > 0$, and follows a general distribution function with PDF $f(\cdot)$ and CDF $F(\cdot)$. α represents a specific realization of A.

Firms have installed volume-flexible facility capacity that allows them to produce with hold under capacity limit after the actual demand curve is unveiled (see van Meighem and Dada 1999). We assume that, given its capacity K_i, firm i's production function takes the following form (see, for instance, Mills 1984):

$$c_i(q \mid K_i) = \beta q + \frac{q^2}{2\gamma_i(K_i)} \tag{6.2}$$

where q is output, βq is the *proportional* cost influenced by factors like raw materials and direct labor that are in direct proportions to the outputs. Moreover, $\frac{q^2}{2\gamma_i(K_i)}$ is attributed to *process efficiency*. The convex functional form, as given in (6.2), displays diseconomy of scale.

Hossain et al. (2004) applied an econometric analysis to measure the production processes in 21 industries, and conclude that more than 40 percent of the industries experience decreasing returns to inputs. $\gamma_i(K_i)$ determines the curvature of the production function: a large value of it implies *more efficient* process with a flatter curve. We refer to $\gamma_i(K_i)$ as the *efficiency factor*, and let it be influenced by the capacity level K_i. In general, when a firm builds a higher capacity level, it is expected to have a higher production efficiency, with a larger efficiency factor. When $\gamma_i = 0$, its production cost is so large as to prevent firm i from generating output; we call such a firm an inefficient firm. As $\gamma_i \to \infty$, the marginal production cost is β, and we call such a firm a *fully efficient* firm.

To facilitate expressions of the decisions and performance measures, we define

$$L(x) \equiv \int_x^\infty (\alpha - x) dF(\alpha), \qquad \text{for } x \in [0, +\infty), \tag{6.3}$$

and let $x(c)$ be the unique solution to $L(x) = c$, for $0 < c \le \mu$. $L(x)$ can be understood as the expected clearing price with x units in the market. $x(c)$ is its inverse function, or the amount of units in market at which the expected market clearing price equals c. Based on these definitions, we further define, for $x(c_0) \ge \beta$,

$$K_0 \equiv \frac{x(c_0)-\beta}{2}, \text{ and } \Pi_0 \equiv \int_{\beta}^{x(c_0)} \frac{(\alpha-\beta)^2}{4} dF(\alpha) + \int_{x(c_0)}^{\infty} \frac{(x(c_0)-\beta)^2}{4} dF(\alpha) \qquad (6.4)$$

We next analyze capacity decisions of firms in a duopoly. We add subscript d on the operational performance measures for given capacity levels and hence efficiency factors at two firms, and subscript D on the quantities of interest in the equilibrium. Subscripts i and j, for $i, j = 1,2$ and $i \neq j$, are added to identify individual firms. The decision framework is as given in Figure 1. Two firms first simultaneously make capacity investments. After the actual demand curve reveals, they simultaneously decide respective outputs under their capacity limits to devote into the market. Then sales are made and profits accrue to the firms.

6.3 Two Firms' Equilibrium Outputs by Given Capacity

Given process efficiencies at two firms (that may well result in different production costs), $\underline{\gamma} = (\gamma_1, \gamma_2)$, where we simply write γ_i instead of $\gamma_i(K_i)$, they each produce with holdback under capacity limit when the realized market size is α, by solving the following problem:

$$\begin{aligned} &Max \quad \Pi_{d,i}(q_i \mid \alpha, \underline{K}) = q_i(\alpha - q_i - q_j)^+ - c_i(q_i \mid K_i) && (6.5) \\ &s.t. \qquad 0 \le q_i \le K_i \end{aligned}$$

We first study the case where firms produce without capacity limits. The equilibrium, as shown in Lemma A, offers insights in the impacts of the efficiencies on the firms' outputs.

Lemma 6.1 *Suppose two firms have efficiency factors $\underline{\gamma} = (\gamma_1, \gamma_2)$. When the realized market size is α, the unconstrained production quantity is*

$$q_i^e(\alpha \mid \underline{\gamma}) = (\alpha - \beta)^+ T_i(\underline{\gamma})$$

where $T_i(\underline{\gamma}) \equiv \dfrac{\gamma_i(1+\gamma_j)}{1+2\gamma_i+2\gamma_j+3\gamma_i\gamma_j}$, *for* $i, j = 1,2$ *and* $i \neq j$.

Suppose the two firms have efficiency factors $\underline{\gamma} = (\gamma_1, \gamma_2)$.

Given that firm j produces $q_j \geq 0$, the profit of firm i by producing q_i is:

$\pi_{d,i}(q_i \mid q_j, \alpha, \underline{\gamma}) = q_i \cdot (\alpha - q_i - q_j) - c_i(q_i \mid K_i)$, for $i, j = 1,2$ and $i \neq j$,

where $c_i(q_i) = \beta q_i + q_i^2 / 2\gamma_i$.

The first- and second-order derivatives of $\pi_{d,i}(q_i \mid q_j, \alpha, \underline{\gamma})$ with respect to q_i are, respectively,

$$\pi_{d,i}^{(1)}(q_i \mid q_j, \alpha, \underline{\gamma}) = \alpha - \beta - q_j - \frac{1+2\gamma_i}{\gamma_i} q_i, \text{ and } \pi_{d,i}^{(2)}(q_i \mid q_j, \alpha, \underline{\gamma}) = -\frac{1+2\gamma_i}{\gamma_i} q_i < 0.$$

When $\alpha \leq \beta$, $\pi_{d,i}^{(1)}(q_i \mid q_j, \alpha, \underline{\gamma}) \leq 0$ and hence $q_i(q_j) = 0$ for $q_j \geq 0$.

When $\alpha > \beta$, the best response (BR) of firm i is $q_i(q_j \mid \alpha, \underline{\gamma}) = \dfrac{\gamma_i}{1+2\gamma_i}(\alpha - \beta - q_j)^+$.

The unconstrained equilibrium is obtained by solving the BR functions simultaneously as:

$q_i^e(\alpha \mid \underline{\gamma}) = (\alpha - \beta) T_i(\underline{\gamma})$, where $T_i(\underline{\gamma}) \equiv \dfrac{\gamma_i(1+\gamma_j)}{1+2\gamma_i+2\gamma_j+3\gamma_i\gamma_j}$, for $i, j = 1,2$ and $i \neq j$.

Thus, it its proved.

Consider that capacity K_i enforces an upper limit on firm i's output. For a volatile market, a higher capacity gives the firm more room in output control, especially in large markets. We let $q_i(\alpha\,|\,\underline{K})$ be the equilibrium output of firm i in market α, and $\Pi_{d,i}(\alpha\,|\,\underline{K})$ its ex-post profit, for $i=1,2$:

$$\Pi_{d,i}(\alpha\,|\,\underline{K}) = q_i(\alpha\,|\,\underline{K})\cdot(\alpha - q_i(\alpha\,|\,\underline{K}) - q_j(\alpha\,|\,\underline{K}))^+ - c_i(q_i(\alpha\,|\,\underline{K})).$$

Knowing firm j has a capacity of K_j, firm i selects its capacity by solving the following problem:

$$K_i(K_j) \equiv ArgMax\left\{\int \Pi_{d,i}(\alpha\,|\,\underline{K})dF(\alpha) - c_0 K_i : K_i \geq 0\right\} \qquad (6.6)$$

We make the following definitions for $\alpha > \beta$, $i, j = 1,2$ and $i \neq j$,

D1. $\quad \bar{q}_i(\alpha\,|\,\underline{\gamma}) = Min\{q_j : b_i(q_j\,|\,\alpha,\underline{\gamma}) = 0\} = \alpha - \beta$.

D3. $\quad K_i(\alpha\,|\,\underline{\gamma}) \equiv b_i(0\,|\,\alpha,\underline{\gamma}) = \dfrac{\gamma_i}{1+2\gamma_i}(\alpha - \beta)$.

D4. $\quad \underline{q}_i(x\,|\,\alpha,\underline{\gamma})$ satisfies $b_i(\underline{q}_i(x\,|\,\alpha,\underline{\gamma})\,|\,\alpha,\underline{\gamma}) = x$. So $\underline{q}_i(x\,|\,\alpha,\underline{\gamma}) = \alpha - \beta - \dfrac{1+2\gamma_i}{\gamma_i}x$.

Based on definitions D1-D3, the following properties are straightforward.

P1. $\quad \bar{q}_i(\alpha\,|\,\underline{\gamma})$, $K_i(\alpha\,|\,\underline{\gamma})$, $\underline{q}_i(x\,|\,\alpha,\underline{\gamma})$, and $b_i(q_j\,|\,\alpha,\underline{\gamma})$ are unique.

P3. $\quad \bar{q}_i(\alpha\,|\,\underline{\gamma})$, $K_i(\alpha\,|\,\underline{\gamma})$, $\underline{q}_i(x\,|\,\alpha,\underline{\gamma})$ increase in α.

P4. $\quad \bar{q}_i(\alpha) > K_j(\alpha) > q_j^e(\alpha)$, for $i, j = 1,2$ and $i \neq j$.

To further facilitate expressions, we define four terms

$$A(\underline{K}) \equiv \frac{K_1}{T_1(\underline{\gamma})},\; B(\underline{K}) \equiv \frac{K_2}{T_2(\underline{\gamma})},\; C(\underline{K}) \equiv K_1 + \frac{1+2\gamma_2}{\gamma_2}K_2, \text{ and } D(\underline{K}) \equiv K_2 + \frac{1+2\gamma_1}{\gamma_1}K_1.$$

Lemma 6.2

a) If $K_1 \geq \dfrac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)}K_2$, *then* $A(\underline{K}) \geq D(\underline{K}) \geq C(\underline{K}) \geq B(\underline{K})$.

b) If $K_1 < \frac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)}K_2$*, then* $A(\underline{K}) < D(\underline{K}) < C(\underline{K}) < B(\underline{K})$.

To prove this lemma, we have

$$A(\underline{K}) - D(\underline{K}) = \frac{K_1}{T_1(\underline{\gamma})} - \left(K_2 + \frac{1+2\gamma_1}{\gamma_1}K_1\right)$$

$$= \frac{\gamma_1(1+\gamma_2)+\gamma_2(1+\gamma_1)+(1+\gamma_1)(1+\gamma_2)}{\gamma_1(1+\gamma_2)}K_1 - \left(K_2 + \frac{1+2\gamma_1}{\gamma_1}K_1\right) = \frac{\gamma_2(1+\gamma_1)}{\gamma_1(1+\gamma_2)}K_1 - K_2.$$

So $A(\underline{K}) \geq D(\underline{K})$ iff $K_1 \geq \frac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)}K_2$.

$$D(\underline{K}) - C(\underline{K}) = \left(K_2 + \frac{1+2\gamma_1}{\gamma_1}K_1\right) - \left(K_1 + \frac{1+2\gamma_2}{\gamma_2}K_2\right) = \frac{1+\gamma_1}{\gamma_1}K_1 - \frac{1+\gamma_2}{\gamma_2}K_2.$$

So $D(\underline{K}) \geq C(\underline{K})$ iff $K_1 \geq \frac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)}K_2$.

$$C(\underline{K}) - B(\underline{K}) = \left(K_1 + \frac{1+2\gamma_2}{\gamma_2}K_2\right) - \frac{K_2}{T_2(\underline{\gamma})}$$

$$= \left(K_1 + \frac{1+2\gamma_2}{\gamma_2}K_2\right) - \frac{\gamma_1(1+\gamma_2)+\gamma_2(1+\gamma_1)+(1+\gamma_1)(1+\gamma_2)}{\gamma_2(1+\gamma_1)}K_2 = K_1 - \frac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)}K_2$$

So $C(\underline{K}) \geq B(\underline{K})$ iff $K_1 \geq \frac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)}K_2$. Thus, it is proved.

Then we further have Lemma 6.4.

Lemma 6.3 *Suppose firm i installs* K_i. *In market* α, *its BR function* $q_i(q_j \mid \alpha, \underline{K})$ *if firm* j *outputs* q_j, *for* $i, j = 1,2, i \neq j$ *is:*

If $0 \leq \alpha \leq \beta$, *then* $q_i(q_j \mid \alpha, \underline{K}) = 0$.

If $\alpha > \beta$, *then*

a) $K_i \geq K_i(\alpha \mid \underline{\gamma})$ *:* $q_i(q_j \mid \alpha, \underline{K}) = \frac{\gamma_i}{1+2\gamma_i}(\alpha - \beta - q_j)^+$

b) $0 \leq K_i \leq K_i(\alpha \mid \underline{\gamma}) : q_i(q_j \mid \alpha, \underline{K}) = \begin{cases} K_i, & 0 \leq q_j \leq \underline{q}_i(K_i \mid \alpha) \\ b_i(q_j \mid \alpha, \underline{\gamma}), & \underline{q}_i(K_i \mid \alpha) < q_j \leq \overline{q}_i(\alpha \mid \gamma) \\ 0, & q_j > \overline{q}_i(\alpha \mid \gamma) \end{cases}$,

where $b_i(q_j \mid \alpha, \underline{\gamma}) = \frac{\gamma_i}{1+2\gamma_i}(\alpha - \beta - q_j)$, $K_i(\alpha \mid \underline{\gamma}) = \frac{\gamma_i}{1+2\gamma_i}(\alpha - \beta)$,

and $\underline{q}_i(K_i \mid \alpha) = \alpha - \beta - \frac{1+2\gamma_i}{\gamma_i} K_i$.

Proof:

Firm i faces the following problem in market α:

$$Max\ \pi_{d,i}(q_i \mid q_j, \alpha, \underline{\gamma}) = q_i \cdot (\alpha - q_i - q_j)^+ - c_P(q_i \mid \gamma_i)$$

$$s.t. \qquad 0 \leq q_i \leq K_i$$

The first- and second-order derivatives of $\pi_{d,i}$ with respect to q_i are, respectively,

$$\pi_{d,i}^{(1)}(q_i \mid q_j, \alpha, \underline{\gamma}) = \alpha - \beta - q_j - \frac{1+2\gamma_i}{\gamma_i} q_i, \text{ and } \pi_{d,i}^{(2)}(q_i \mid q_j, \alpha, \underline{\gamma}) = -\frac{1+2\gamma_i}{\gamma_i} < 0.$$

$\pi_{d,i}$ is strictly concave in q_i. Now that $q_i \in [0, K_i]$, we evaluate $\pi_{d,i}^{(1)}(q_i \mid q_j, \alpha, \underline{\gamma})$ at the two boundaries and first evaluate its derivative at $q_i = 0$: $\pi_{d,i}^{(1)}(0 \mid q_j, \alpha, \underline{\gamma}) = \alpha - \beta - q_j$.

Case 1: $0 \leq \alpha \leq \beta$: $\pi_{d,i}^{(1)}(0 \mid q_j, \alpha, \underline{\gamma}) \leq 0$ for $q_j \geq 0$, so that $\pi_{d,i}$ decreases and attains optimum at $q_i(q_j \mid \alpha, \underline{K}, \underline{\gamma}) = 0$. This is Part (*a*).

Case 2: $\alpha > \beta$ the value of $\pi_{d,i}^{(1)}(0 \mid q_j, \alpha, \underline{\gamma})$ is influenced by q_j.

If $q_j \ge \alpha - \beta$, then $\pi^{(1)}_{d,i}(0 \mid q_j, \alpha, \underline{\gamma}) \le 0$. So $\pi_{d,i}$ decreases and $q_i(q_j \mid \alpha, \underline{K}, \underline{\gamma}) = 0$, regardless of K_i.

If $0 \le q_j < \alpha - \beta$, then $\pi^{(1)}_{d,i}(0 \mid q_j, \alpha, \underline{\gamma}) > 0$, and we check if the capacity is binding.

$$\pi^{(1)}_{d,i}(K_i \mid q_j, \alpha, \underline{\gamma}) = \alpha - \beta - q_j - \frac{1+2\gamma_i}{\gamma_i} K_i.$$

where $\pi^{(1)}_{d,i}(K_i \mid q_j, \alpha, \underline{\gamma})$ decreases in K_i and q_j. $\pi^{(1)}_{d,i}(K_i \mid 0, \alpha, \underline{\gamma}) = \alpha - \beta - \frac{1+2\gamma_i}{\gamma_i} K_i$; and it strictly decreases in K_i.

$\pi^{(1)}_{d,i}(K_i(\alpha) \mid 0, \alpha, \underline{\gamma}) = 0$ by definition D2, so that $\pi^{(1)}_{d,i}(K_i \mid 0, \alpha, \underline{\gamma}) \ge 0$ if $0 \le K_i \le K_i(\alpha)$ and $\pi^{(1)}_{d,i}(K_i \mid 0, \alpha, \underline{\gamma}) < 0$ otherwise. We consider these two situations sepaamountly.

Case 3.1 $K_i > K_i(\alpha)$. $\pi^{(1)}_{d,i}(K_i \mid 0, \alpha, \underline{\gamma}) < 0$.

Since $\pi^{(1)}_{d,i}(K_i \mid q_j, \alpha, \underline{\gamma})$ decreases in q_j, $\pi^{(1)}_{d,i}(K_i \mid q_j, \alpha, \underline{\gamma}) < 0$ and capacity is unbinding. By concavity, $q_i(q_j \mid \alpha, \underline{K}) = b_i(q_j \mid \alpha, \underline{\gamma})$.

Case 3.2 $0 \le K_i \le K_i(\alpha)$. $\pi^{(1)}_{d,i}(K_i \mid 0, \alpha, \underline{\gamma}) \ge 0$.

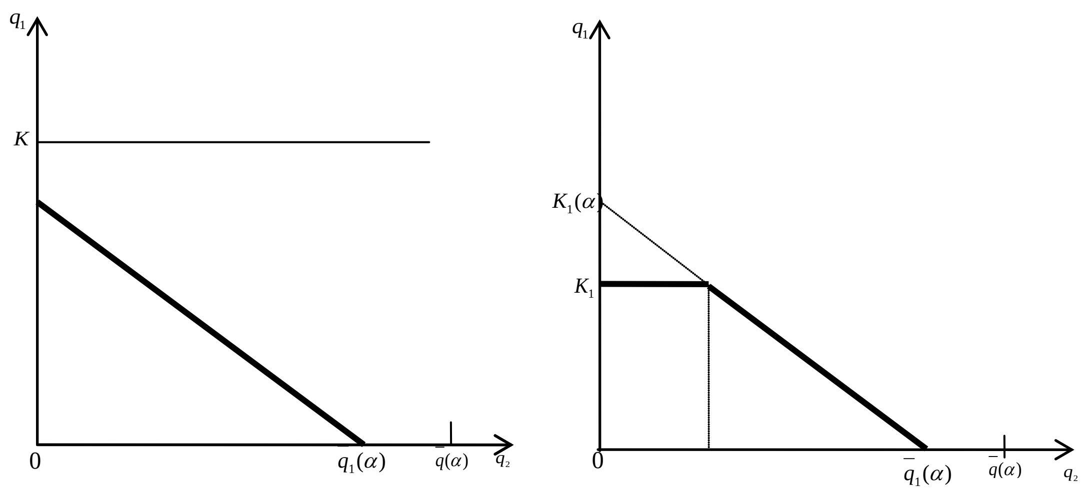
By definition of $\underline{q}_i(x \mid \alpha, \underline{\gamma})$ in D3, $\pi^{(1)}_{d,i}(K_i \mid q_j, \alpha, \underline{\gamma}) > 0$ and $\pi_{d,i}$ is an increasing function for $0 \le q_j < \underline{q}_i(K_i \mid \alpha, \underline{\gamma})$, whereas $\pi^{(1)}_{d,i}(K_i \mid q_j, \alpha, \underline{\gamma}) \le 0$ and $\pi_{d,i}$ is concave otherwise.

$$\pi_{d,i}^{(1)}(K_i \mid \bar{q}_i(\alpha \mid \underline{\gamma}), \alpha, \underline{\gamma}) = \alpha - \beta - \bar{q}_i(\alpha \mid \underline{\gamma}) - \frac{1+2\gamma_i}{\gamma_i} K_i \le 0 \quad , \quad \text{so}$$

$\underline{q}_i(K_i \mid \alpha, \underline{\gamma}) \le \bar{q}_i(\alpha \mid \underline{\gamma})$, and hence $q_i(q_j \mid \alpha, \underline{K}, \underline{\gamma}) = b_i(q_j \mid \alpha, \underline{\gamma})$ if $\underline{q}_i(K_i \mid \alpha, \underline{\gamma}) < q_j \le \bar{q}_i(\alpha \mid \underline{\gamma})$; $q_i(q_j \mid \alpha, \underline{K}, \underline{\gamma}) = K_i$ if $0 \le q_j < \underline{q}_i(K_i \mid \alpha, \underline{\gamma})$.

Combining the results in the two cases, the equilibrium in Lemma 6.3 holds.

Figure 6.2 shows firm i's best response in the two cases.



$a)$ $K_1 > K_1(\alpha)$ $b)$ $0 \le K_1 \le K_1(\alpha)$

Figure 6.2 Best Response of Firm i in Market $\alpha > \beta$

By results in Lemma 6.3., for $0 \le \alpha \le \beta$, it is dominant for each firm not to produce. For $\alpha > \beta$, the capacities affect the shapes. After detailed scenario analysis, we directly show the equilibrium in Lemma 6.4 and illustrate it in Figure 6.3.

Lemma 6.4: *Suppose that two firms have capacities* $\underline{K} = (K_1, K_2)$ *with efficiency factors*
$\underline{q}_1(K_1, \alpha)$
$\underline{\gamma} = (\gamma_1, \gamma_2)$, *equilibrium quantities when* $\alpha \ge \beta$ *are:*

a) $(q_1^e(\alpha|\underline{\gamma}), q_2^e(\alpha|\underline{\gamma}))$ *if* $K_i > q_i^e(\alpha|\underline{\gamma})$ *for* $i=1,2$, *or equivalently* $\alpha < \beta + A(\underline{K})$ *and* $\alpha < \beta + B(\underline{K})$.

b) $(K_1, b_2(K_1|\alpha,\underline{\gamma}))$ *if* $0 \le K_1 \le q_1^e(\alpha|\underline{\gamma})$ *and* $K_2 > b_2(K_1|\alpha)$, *or equivalently* $\alpha \ge \beta + A(\underline{K})$ *and* $\alpha < \beta + C(\underline{K})$.

c) $(b_1(K_2|\alpha,\underline{\gamma}), K_2)$ *if* $0 \le K_2 \le q_2^e(\alpha|\underline{\gamma})$ *and* $K_1 > b_1(K_2|\alpha,\underline{\gamma})$, *or equivalently* $\alpha \ge \beta + B(\underline{K})$ *and* $\alpha < \beta + D(\underline{K})$.

d) (K_1, K_2) *if* $0 \le K_1 \le q_1^e(\alpha|\underline{\gamma})$ *and* $0 \le K_2 \le b_2(K_1|\alpha,\underline{\gamma})$; *or* $q_1^e(\alpha|\underline{\gamma}) < K_1 \le K_1(\alpha|\underline{\gamma})$ *and* $0 \le K_2 \le \underline{q}_1(K_1|\alpha,\underline{\gamma})$, *or equivalently* $\alpha \ge \beta + A(\underline{K})$ *and* $\alpha \ge \beta + C(\underline{K})$, *or* $\beta + \frac{1+2\gamma_1}{\gamma_1}K_1 < \alpha < \beta + A(\underline{K})$ *and* $\alpha \ge \beta + D(\underline{K})$.

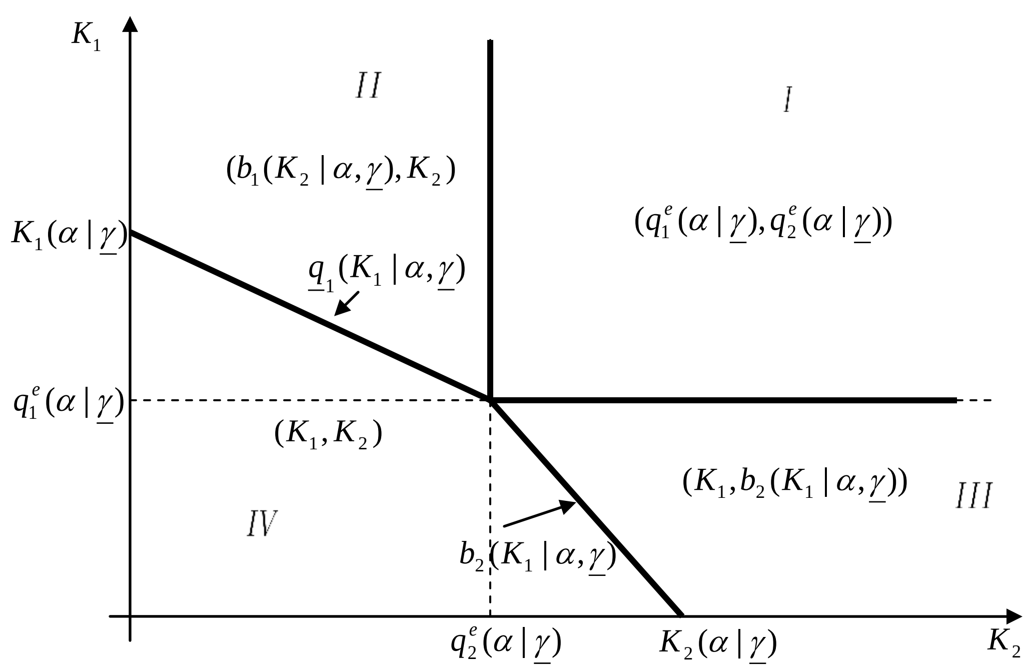


Figure 6.3 Sub-game Equilibrium Production under Capacity Constraint

Figure 6.3 partitions the space of (K_1, K_2) when the market size $\alpha > \beta$, and marks the sub-game equilibrium production under capacity constraints in each area. In area I, the capacity at neither firm caps production, although the capacity does affect the efficiency factor and hence the production cost at each firm. In area IV, the capacities at two firms are sufficiently low to cap their productions. In other circumstances (area II and III), the firm with less capacity will have its production capped, while the other firm will make production decision accordingly without capacity restriction.

6.4 Equilibrium Capacities

We next investigate the firms' capacity investments. Consider the capacity decision of firm 1, given capacity K_2 at firm 3.

Suppose that firm 1 chooses K_1 such that $\gamma_2(1+\gamma_1)K_1 \geq \gamma_1(1+\gamma_2)K_2$. Then by Lemma 6.4, the equilibrium in market α is:

a) $0 \leq \alpha \leq \beta$

It is dominant strategy for each firm not to produce. So $\Pi_1(K_1 \mid K_2, \alpha) = 0$.

b) $\beta < \alpha \leq \beta + B(\underline{K})$: $\left(q_1(\alpha \mid \underline{K}), q_2(\alpha \mid \underline{K})\right) = \left(q_1^e(\alpha \mid \underline{\gamma}), q_2^e(\alpha \mid \underline{\gamma})\right)$

$$= (\alpha-\beta)T_1(\underline{\gamma})\left[\alpha - (\alpha-\beta)(T_1+T_2)\right] - \left(\beta(\alpha-\beta)T_1 + \frac{(\alpha-\beta)^2}{2\gamma_1}T_1^2\right)$$

$$= (\alpha-\beta)^2\left[T_1 - T_1(T_1+T_2) - \frac{T_1^2}{2\gamma_1}\right]$$

$$= \frac{\gamma_1(1+2\gamma_1)(1+\gamma_2)^2}{2(1+2\gamma_1+2\gamma_2+3\gamma_1\gamma_2)^2}(\alpha-\beta)^2.$$

c) $\beta + B(\underline{K}) < \alpha \leq \beta + D(\underline{K})$: $\left(q_1(\alpha \mid \underline{K}), q_2(\alpha \mid \underline{K})\right) = \left(b_1(K_2 \mid \alpha), K_2\right)$

$$\Pi_1(K_1 \mid K_2, \alpha, \underline{\gamma}) = b_1(K_2 \mid \alpha, \underline{\gamma})\left[\alpha - \left(b_1(K_2 \mid \alpha, \underline{\gamma}) + K_2\right)\right] - \left(\beta b_1(K_2 \mid \alpha, \underline{\gamma}) + \frac{b_1^2\left(K_2 \mid \alpha, \underline{\gamma}\right)}{2\gamma_1}\right)$$

$$= \frac{\gamma_1}{2(1+2\gamma_2)}(\alpha - \beta - K_2)^2 .$$

d) $\beta + D(\underline{K}) < \alpha$: $\left(q_1(\alpha \mid \underline{K}), q_2(\alpha \mid \underline{K})\right) = \left(K_1, K_2\right)$

$$\Pi_1(K_1 \mid K_2, \alpha) = K_1\left[\alpha - \left(K_1 + K_2\right)\right] - \left(\beta K_1 + \frac{K_1^2}{2\gamma_1}\right) = K_1\left(\alpha - \beta - K_2\right) - \frac{1+2\gamma_1}{2\gamma_1} K_1^2 .$$

$\Pi_1(K_1 \mid K_2, \alpha)$ is continuous in α .

The expected profit of firm 1 for $K_1 \in \left(\frac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)} K_2, +\infty\right)$ is:

$$\Pi_1(K_1 \mid K_2) = -c_0 K_1 + \int_{\beta}^{\beta+B(\underline{K})} \frac{\gamma_1(1+2\gamma_1)(1+\gamma_2)^2}{2\left(1+2\gamma_1+2\gamma_2+3\gamma_1\gamma_2\right)^2}\left(\alpha - \beta\right)^2 dF(\alpha) +$$

$$\int_{\beta+B(\underline{K})}^{\beta+D(\underline{K})} \frac{\gamma_1}{2\left(1+2\gamma_1\right)}\left(\alpha - \beta - K_2\right)^2 dF(\alpha) + \int_{\beta+D(\underline{K})}^{\infty} K_1\left(\alpha - \beta - K_2 - \frac{1+2\gamma_1}{2\gamma_1} K_1\right)^2 dF(\alpha) .$$

Suppose that firm 1 chooses a capacity K_1 such that $0 \le K_1 < \frac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)} K_2$. Then by Lemma 6.4, the equilibrium quantity in market α is:

a) $0 \le \alpha \le \beta$

It is dominant strategy for it not to produce. So $\Pi_1(K_1 \mid K_2, \alpha) = 0$.

b) $\beta < \alpha \le \beta + A(\underline{K})$: $\left(q_1(\alpha \mid \underline{K}), q_2(\alpha \mid \underline{K})\right) = \left(q_1^e(\alpha \mid \underline{\gamma}), q_2^e(\alpha \mid \underline{\gamma})\right)$

$$\Pi_1(K_1 \mid K_2, \alpha) = q_1^e \cdot (\alpha - q_1^e - q_2^e) - \left(\beta q_1^e + \frac{\left(q_1^e\right)^2}{2\gamma_1}\right)$$

$$= \frac{\gamma_1(1+2\gamma_1)(1+\gamma_2)^2}{2\left(1+2\gamma_1+2\gamma_2+3\gamma_1\gamma_2\right)^2}(\alpha - \beta)^2 .$$

c) $\beta + A(\underline{K}) < \alpha \le \beta + C(\underline{K})$: $\left(q_1(\alpha \mid \underline{K}), q_2(\alpha \mid \underline{K})\right) = \left(K_1, b_2(K_1 \mid \alpha, \underline{\gamma})\right)$

$$\Pi_1(K_1 \mid K_2, \alpha, \underline{\gamma}) = K_1\left[\alpha - \left(K_1 + b_2(K_1 \mid \alpha, \underline{\gamma})\right)\right] - \left(\beta K_1 + \frac{K_1^2}{2\gamma_1}\right)$$

$$= K_1\left[\alpha - \left(K_1 + \frac{\gamma_2}{1+2\gamma_2}(\alpha - \beta - K_1)\right)\right] - \left(\beta K_1 + \frac{K_1^2}{2\gamma_1}\right)$$

$$= K_1\left[(\alpha - \beta)\frac{1+\gamma_2}{1+2\gamma_2} - \frac{1+2\gamma_1+2\gamma_2+2\gamma_1\gamma_2}{2\gamma_1(1+2\gamma_2)}K_1\right].$$

d) $\beta + C(\underline{K}) < \alpha$: $\left(q_1(\alpha \mid \underline{K}), q_2(\alpha \mid \underline{K})\right) = \left(K_1, K_2\right)$

$$\Pi_1(K_1 \mid K_2, \alpha) = K_1\left[\alpha - \left(K_1 + K_2\right)\right] - \left(\beta K_1 + \frac{K_1^2}{2\gamma_1}\right) = K_1\left(\alpha - \beta - K_2\right) - \frac{1+2\gamma_1}{2\gamma_1}K_1^2.$$

The expected profit of firm 1 for $K_1 \in \left(0, \frac{\gamma_1(1+\gamma_2)}{\gamma_2(1+\gamma_1)}K_2\right)$ can be written as

$$\Pi_1(K_1 \mid K_2) = -c_0 K_1 + \int_{\beta}^{\beta + A(\underline{K})} \frac{\gamma_1(1+2\gamma_1)(1+\gamma_2)^2}{2\left(1+2\gamma_1+2\gamma_2+3\gamma_1\gamma_2\right)^2}\left(\alpha - \beta\right)^2 dF(\alpha) +$$
$$\int_{\beta + A(\underline{K})}^{\beta + C(\underline{K})} K_1\left[(\alpha - \beta)\frac{1+\gamma_2}{1+2\gamma_2} - \frac{1+2\gamma_1+2\gamma_2+2\gamma_1\gamma_2}{2\gamma_1(1+2\gamma_2)}K_1\right] dF(\alpha) + \int_{\beta + C(\underline{K})}^{\infty} K_1\left(\alpha - \beta - K_2 - \frac{1+2\gamma_1}{2\gamma_1}K_1\right)^2 dF(\alpha).$$

The profit function of firm 2 for given capacity at firm 1 follows by symmetry. Theorem 6.1 establishes the existence of Nash equilibrium in pure strategy, and characterizes the capacities at the two firms.

Theorem 6.1 *If $\mu > c_0$, then there exist pure- strategy Nash equilibrium capacities for the firms, (K_1^e, K_2^e), which satisfy that*

$$K_1^e \frac{1 + r_1^e}{r_1^e} = K_2^e \frac{1 + r_2^e}{r_2^e} \tag{6.7}$$

and

$$\frac{(1+\gamma_2^e)(1+2\gamma_1^e+2\gamma_2^e+5\gamma_1^e\gamma_2^e)}{2(1+2\gamma_1^e+2\gamma_2^e+3\gamma_1^e\gamma_2^e)^3}\cdot\frac{t_1\gamma_1^e}{K_1^e}S(G(K^e))+\frac{t_1K_1^e}{2\gamma_1^e}\bar{F}(G(K^e))+L(G(K^e))=c_0 \tag{6.8}$$

where $S(x)\triangleq\int_\beta^x(\alpha-\beta)^2dF(\alpha)$, $L(x)\triangleq\int_x^\infty(\alpha-x)dF(\alpha)$, $G(K^e)=\beta+\frac{K_1^e}{T_1(\gamma_1^e,\gamma_2^e)}$, and $\gamma_i^e=\gamma_i(K_i^e)$.

Proof:

For given K_2, firm1's second-order derivative follows that,

Case1: when $K_1\geq\frac{r_1(1+r_2)}{r_2(1+r_1)}K_2$

$$\begin{aligned}
\frac{d\prod_1(K_1|K_2)}{dK_1}&=\frac{(D(K)-K_2)^2}{2}f(B+D(K))\\
&+\frac{dr_1}{dK_1}\left[\frac{(1+r_2)^2(1+2r_1+2r_2+5r_1r_2)}{2(1+2r_1+2r_2+3r_1r_2)^3}\int_\beta^{\beta+B(K)}(\alpha-\beta)^2dF(\alpha)\right]\\
&+\frac{dr_1}{dK_1}\left[\frac{r_1(1+r_2)^3(1+2r_1)}{2r_2^3(1+r_1)^4}K_2^3\cdot f(\beta+B(K))\right]\\
&+\frac{dr_1}{dK_1}\left[\frac{1}{2(1+2r_1)^2}\int_{\beta+B(K)}^{\beta+D(K)}(\alpha-\beta-K_2)^2dF(\alpha)-\frac{K_1(D(K)-K_2)^2}{2r_1(1+2r_1)}f(\beta+D(K))\right]\\
&-\frac{dr_1}{dK_1}\left[\frac{r_1(1+r_2)K_2(B(K)-K_2)^2}{2r_2(1+2r_1)(1+r_1)^2}f(\beta+B(K))\right]\\
&+\left[\frac{K_1^2(1+2r_1)^2}{2r_1^2}-\frac{1+2r_1}{r_1}K_1(D(K)-K_2)\right]f(\beta+D(K))\\
&+\int_{\beta+D(K)}^\infty(\alpha-\beta-K_2-\frac{1+2r_1}{r_1}K_1)dF(\alpha)\\
&+\frac{dr_1}{dK_1}\left\{\left[\frac{K_1^2}{r_1^2}(D(K)-K_2)-\frac{1+2r_1}{2r_1^3}K_1^3\right]f(\beta+D(K))+\frac{K_1^2}{2r_1^2}\int_{\beta+D(K)}^\infty dF(\alpha)\right\}-c_0\\
&=\int_{\beta+D(K)}^\infty(\alpha-\beta-K_2-\frac{1+2r_1}{r_1}K_1)dF(\alpha)\\
&+\frac{dr_1}{dK_1}\left[\frac{(1+r_2)^2(1+2r_1+2r_2+5r_1r_2)}{2(1+2r_1+2r_2+3r_1r_2)^3}\int_\beta^{\beta+B(K)}(\alpha-\beta)^2dF(\alpha)\right]\\
&+\frac{dr_1}{dK_1}\left[\frac{1}{2(1+2r_1)^2}\int_{\beta+B(K)}^{\beta+D(K)}(\alpha-\beta-K_2)^2dF(\alpha)+\frac{K_1^2}{2r_1^2}\int_{\beta+D(K)}^\infty dF(\alpha)\right]-c_0
\end{aligned}$$

Then, we further have

$$\frac{d^2\prod_1(K_1|K_2)}{dK_1}=-t_1(1-t_1)K^{t_1-2}\left[L(r)\int_{\beta}^{\beta+B(K)}(\alpha-\beta)^2\,dF(\alpha)\right]$$
$$-t_1(1-t_1)K^{t_1-2}\left[\frac{1}{2(1+2r_1)^2}\int_{\beta+B(K)}^{\beta+D(K)}(\alpha-\beta-K_2)^2\,dF(\alpha)+\frac{K_1^2}{2r_1^2}\int_{\beta+D(K)}^{\infty}dF(\alpha)\right]$$
$$-\frac{1+2r_1-2t_1}{r_1}\int_{\beta+D(K)}^{\infty}dF(\alpha)$$
$$+t_1^2K_1^{2t_1-2}\left[\frac{dL(r)}{dr_1}\int_{\beta}^{\beta+B(K)}(\alpha-\beta)^2\,dF(\alpha)-\frac{2}{(1+2r_1)^3}\int_{\beta+B(K)}^{\beta+D(K)}(\alpha-\beta-K_2)^2\,dF(\alpha)\right]$$
$$+t_1^2K_1^{2t_1-2}\left[\frac{r_1(1+r_2)^3}{r_2^2(1+r_1)^4(1+2r_1+2r_2+3r_1r_2)}K_2^3 f(\beta+B(K))-\frac{K_1^2}{r_1^3}\int_{\beta+D(K)}^{\infty}dF(\alpha)\right]$$

where $L(r)=\dfrac{(1+r_2)^2(1+2r_1+2r_2+5r_1r_2)}{2(1+2r_1+2r_2+3r_1r_2)^3}$,

$$\frac{dL(r)}{dr_1}=\frac{(1+r_2)^2\left[2r_2(1+2r_1+2r_2+3r_1r_2)-(2+3r_2)(1+4r_1+4r_2+12r_1r_2)\right]}{2(1+2r_1+2r_2+3r_1r_2)^4}<0.$$

Denote
$$M_1=-\frac{(t_1-1)^2+2r_1}{r_1}\int_{\beta+D(K)}^{\infty}dF(\alpha)$$
$$-t_1^2K_1^{2t_1-2}\frac{2}{(1+2r_1)^3}\int_{\beta+B(K)}^{\beta+D(K)}(\alpha-\beta-K_2)^2\,dF(\alpha)$$
$$+t_1^2K_1^{2t_1-2}\frac{r_1(1+r_2)^3}{r_2^2(1+r_1)^4(1+2r_1+2r_2+3r_1r_2)}K_2^3 f(\beta+B(K)).$$

Since $(\alpha-\beta-K_2)^2$ is increasing in $\alpha, \forall\alpha\in\left(\beta+B(K),\ \beta+D(K)\right)$, we have

$$M_1<-Q_1\int_{\beta+D(K)}^{\infty}dF(\alpha)-Q_2\int_{\beta+B(K)}^{\beta+D(K)}dF(\alpha)+Q_3K_2 f(\beta+B(K)).$$

where

$$Q_1=\frac{(t_1-1)^2+2r_1}{r_1};\ Q_2=\frac{K_2^{2-2t_2}}{K_1^{2-2t_1}}\cdot\frac{2t_1^2}{1+2r_1}\left(\frac{1+r_2}{1+r_1}\right)^2;\ Q_3=\frac{K_2^{2-2t_2}}{K_1^{2-2t_1}}\cdot\frac{t_1^2r_1\left(1+r_2\right)^3}{\left(1+r_1\right)^4(1+2r_1+2r_2+3r_1r_2)}.$$

Note that $Q_1>Q_2$, therefore

$$M_1 < -(Q_1 - Q_2)\int_{\beta+D(K)}^{\infty} dF(\alpha) - Q_2\int_{\beta+B(K)}^{\infty} dF(\alpha) + Q_3K_2 f(\beta+B(K))$$

$$= -(Q_1 - Q_2)\int_{\beta+D(K)}^{\infty} dF(\alpha) - Q_2\overline{F}(\beta+B(K))\left[1 - \frac{r_1(1+r_2)(1+2r_1)}{2(1+r_1)^2(1+2r_1+2r_2+3r_1r_2)}K_2\frac{f(\beta+B(K))}{\overline{F}(\beta+B(K))}\right]$$

$$< -(Q_1 - Q_2)\int_{\beta+D(K)}^{\infty} dF(\alpha) - Q_2\overline{F}(\beta+B(K))\left[1 - (\beta+B(K))\frac{f(\beta+B(K))}{\overline{F}(\beta+B(K))}\right]$$

Assume that $F(\alpha)$ is IGFR in the support of $[a,b)$. Therefore, $(\beta+B(K))\frac{f(\beta+B(K))}{\overline{F}(\beta+B(K))} \le 1$. Note that $\frac{d^2\prod_1(K_1|K_2)}{dK_1^{\ 2}} < M_1$, we then have $\frac{d^2\prod_1(K_1|K_2)}{dK_1^{\ 2}} < M_1 < 0$. So it guarantees the quasi-concavity of $\prod_1(K_1|K_2)$ when $K_1 > \frac{r_1(1+r_2)}{r_2(1+r_1)}K_2$.

Case2: $K_1 < \frac{r_1(1+r_2)}{r_2(1+r_1)}K_2$

$$
\begin{aligned}
\frac{d\prod_1(K_1|K_2)}{dK_1} &= \frac{K_1^2(1+2r_1)}{2r_1T_1}f(\beta+A(K)) + \int_{\beta+C(K)}^{\infty}(\alpha-\beta-K_2)dF(\alpha) \\
&+\left(\frac{K_1^2}{2r_1}-\frac{1+r_2}{r_2}K_1K_2\right)f(\beta+C(K)) - \int_{\beta+C(K)}^{\infty}\frac{1+2r_1}{r_1}K_1dF(\alpha) \\
&+\frac{dr_1}{dK_1}\left[L(r)\int_{\beta}^{\beta+A(K)}(\alpha-\beta)^2dF(\alpha)\right] \\
&-\frac{dr_1}{dK_1}\left[\frac{(1+2r_1)(1+2r_2)}{2r_1^3(1+r_2)}K_1^3f(\beta+A(K))\right] \\
&+\frac{dr_1}{dK_1}\int_{\beta+C(K)}^{\infty}\frac{K_1^2}{2r_1^2}dF(\alpha) \\
&+\int_{\beta+A(K)}^{\beta+C(K)}\frac{(\alpha-\beta)r_1(1+r_2)-K_1(1+2r_1+2r_2+2r_1r_2)}{r_1(1+2r_2)}dF(\alpha) \\
&+K_1\left[\frac{(1+r_2)K_2}{r_2}-\frac{K_1}{2r_1}\right]f(\beta+C(K)) - \frac{K_1^2(1+2r_1)}{r_1T_1}f(\beta+A(K)) \\
&+\frac{dr_1}{dK_1}\left[\frac{K_1^3(1+2r_1)(1+2r_2)}{2r_1^3(1+r_2)}f(\beta+A(K)) + \int_{\beta+A(K)}^{\beta+C(K)}\frac{K_1^2}{2r_1^2}dF(\alpha)\right] - c_0 \\
&=\frac{dr_1}{dK_1}\left[L(r)\int_{\beta}^{\beta+A(K)}(\alpha-\beta)^2dF(\alpha) + \int_{\beta+A(K)}^{\infty}\frac{K_1^2}{2r_1^2}dF(\alpha)\right] \\
&+\frac{1}{1+2r_2}\int_{\beta+A(K)}^{\beta+C(K)}\left[(1+r_2)(\alpha-\beta) - \frac{K_1(1+2r_1+2r_2+2r_1r_2)}{r_1}\right]dF(\alpha) \\
&+\int_{\beta+C(K)}^{\infty}\left(\alpha-\beta-K_2-\frac{1+2r_1}{r_1}K_1\right)dF(\alpha) - c_0
\end{aligned}
$$

After some algebra, it follows,

$$
\begin{aligned}
\frac{d^2\prod_1(K_1|K_2)}{dK_1^2} &= \left[t_1(t_1-1)K_1^{t_1-2}L(r) + t_1K_1^{t_1-1}\frac{dL(r)}{dr_1}\frac{dr_1}{dK_1}\int_{\beta}^{\beta+A(K)}(\alpha-\beta)^2dF(\alpha)\right] \\
&-\left[\frac{1-t_1}{r_1}\left(\frac{1+2r_1+2r_2+2r_1r_2}{1+2r_1}-\frac{t_1}{2}\right)+\frac{t_1(2+2r_2)}{1+2r_1}\right]\int_{\beta+A(K)}^{\beta+C(K)}dF(\alpha) \\
&-\frac{dA(K)}{dK_1}f(\beta+A(K))\left(\frac{r_1r_2K_1^{1-t_1}}{1+2r_2}-\frac{t_1r_2K_1}{1+2r_1+2r_2+3r_1r_2}\right) \\
&-\frac{(t_1-1)(t_1-2)+4r_1}{2r_1}\int_{\beta+C(K)}^{\infty}dF(\alpha) + \frac{r_2K_1}{1+2r_2}f(\beta+C(K))
\end{aligned}
$$

Let

$$M_2 = -\frac{(t_1-1)(t_1-2)+4r_1}{2r_1}\int_{\beta+C(K)}^{\infty} dF(\alpha) + \frac{r_2K_1}{1+2r_2} f(\beta+C(K))$$
$$= -\frac{(t_1-1)(t_1-2)+4r_1}{2r_1}\bar{F}(\beta+C(K))\left[1-\frac{r_2K_1}{1+2r_2}\cdot\frac{2r_1}{(t_1-1)(t_1-2)+4r_1}\cdot\frac{f(\beta+C(K))}{\bar{F}(\beta+C(K))}\right]$$

Note that the item $(t_1-1)(t_1-2)+4r_1 = \left(t_1-\frac{3}{2}\right)^2 - \frac{1}{4} + 4r_1$ is decreasing in $t_1, \forall t_1 \in [0,1]$.

Thus, we have

$$\frac{r_2K_1}{1+2r_2}\cdot\frac{2r_1}{(t_1-1)(t_1-2)+4r_1}\cdot\frac{f(\beta+C(K))}{\bar{F}(\beta+C(K))} \le \frac{r_2}{2(1+2r_2)}K_1\frac{f(\beta+C(K))}{\bar{F}(\beta+C(K))} < (\beta+C(K))\frac{f(\beta+C(K))}{\bar{F}(\beta+C(K))}$$

When $F(\alpha)$ is IGFR in the support of $[a,b)$, $(\beta+C(K))\frac{f(\beta+C(K))}{\bar{F}(\beta+C(K))} \le 1$. It follows that $\frac{d^2\prod_1(K_1|K_2)}{dK_1^{\,2}} < M_2 < 0$. $\prod_1(K_1|K_2)$ is quasi-concave when $K_1 < \frac{r_1(1+r_2)}{r_2(1+r_1)}K_2$.

Case 1 and Case 2 shows that $\prod_1(K_1|K_2)$ is quasi-concave in K_1 and its maximizer is determined by solving $\frac{d\prod_1(K_1|K_2)}{dK_1} = 0$. By given K_2, firm 1 reaches optimality at the point of $K_1 = K_2\cdot\frac{1+r_2}{r_2}\cdot\frac{r_1}{1+r_1}$ if c_0 satisfies (6.8). On the other hand, by given K_1, Firm 2's objective function is also quasi-concave in K_2 in a symmetric setting, which implies that its maximizer is determined by solving $\frac{d\prod_2(K_2|K_1)}{dK_2} = 0$. When c_0 satisfies (6.8), it is intuitive that firm 2's maximizer is $K_2 = K_1\cdot\frac{1+r_1}{r_1}\cdot\frac{r_2}{1+r_2}$. That is to say, by given the other firm's strategy K_{3-i}, $K_i \frac{1+r_i}{r_i} = K_{3-i} \frac{1+r_{3-i}}{r_{3-i}}$ is always the optimal strategy for firm i, $i=1,2$. We thus complete the proof of Theorem 6.1.

As shown in (6.7), in the equilibrium, two firms strive to match their respective capacity Ki adjusted by the marginal efficiency cost attained at the capacity level $\frac{K_i}{\gamma_i(K_i)} = K_i^{1-t_i}$. The magnitude of the adjustment for the firm with a lower efficiency index is larger than that for the firm with a higher efficiency index when the capacity level is high, but smaller when the capacity level is low.

The equilibrium profit of firm $i,\ i=1,2$, is given by

$$\Pi_i^e = \frac{\gamma_i^e(1+\gamma_{3-i}^e)(1+2\gamma_i^e-t_i)(1+2\gamma_i^e+2\gamma_{3-i}^e+3\gamma_1^e\gamma_2^e)+t_i[\gamma_i^e+2(r_i^e)^2+3(r_i^e)^2 r_{3-i}^e]}{2(1+2\gamma_1^e+2\gamma_2^e+3\gamma_1^e\gamma_2^e)^3} S(G(K^e)) + \frac{1+2\gamma_i^e-t_i}{2\gamma_i^e}(K_i^e)^2\bar{F}(G(K^e))$$

where $S(\cdot)$, $G(K^e)$, and γ_i^e are as defined in Theorem 6.1.

6.5 Capacity Comparison

We next compare the capacities at the two firms, where we assume, without loss of generality, that firm 1 has a larger efficiency index than firm 3.

Proposition 6.1 *Assume without loss of generality that* $t_1 \geqslant t_2$.

Define $\beta_1 \triangleq inf\{\beta \geqslant 0 : L(\beta+4) \leqslant c_0\}$,

and $\beta_2 \triangleq sup\{\geqslant 0 : \frac{5S(\beta+4)}{128} + \frac{\bar{F}(\beta+4)}{2} + L(\beta+4) \geqslant c_0\}$, *where* $S(\cdot)$ *and* $L(\cdot)$ *are as defined in Theorem 6.1. Then* $0 \leqslant \beta_1 \leqslant \beta_2$, *and:*

a) When $\beta \geqslant \beta_2$,$K_1^e \leqslant K_2^e$;

b) When $0 \leqslant \beta \leqslant \beta_1$, $K_1^e > K_2^e$;

c) Otherwise, there exist $\beta(t_1,t_2) \in [\beta_1,\beta_2]$ such that $K1^e \leqslant K_2^e$ for $\beta \in [\beta(t_1,t_2),\beta_2]$, and $K_1^e > K_2^e$ for $\beta \in [\beta(t_1,t_2),\beta_2]$.

Moreover, β_1 and β_2 first-order increase, and given its mean, second-order decrease with $F(\cdot)$. Given t_1 and *t_2, $\beta(t_1,t_2)$ first-order increases, and, given its mean, second-order decreases with $F(\cdot)$.*

Proof:

Denote

$$c(K_1^e,K_2^e) = \left.\frac{d\prod_1(K_1|K_2)}{dK_1}\right|_{K_1^e=\frac{r_1^e(1+r_2^e)}{r_2^e(1+r_1^e)}K_2^e}$$

$$= \frac{(1+r_2^e)^2(1+2r_1^e+2r_2^e+5r_1^er_2^e)}{2(1+2r_1^e+2r_2^e+3r_1^er_2^e)^3}\cdot\frac{t_1r_1^e}{K_1^e}S\left(G(K^e)\right)+\frac{t_1r_1^e}{2K_1^e}\bar{F}\left(G(K^e)\right)+L\left(G(K^e)\right).$$

As Theorem 6.1 shows, $K_1^e = K_2^e$ only when $c_0 = c(1,1)$, for all $K_i > 0, i = 1,2$. Since firm i's objective function is quasi-concave in K_i, we have $K_1^e < K_2^e$ when $K_1^e < 1$ and $K_1^e > K_2^e$ when $K_1^e > 1$. Derive $c(1,1)$ to β, we have

$$\frac{dc}{d\beta} = \frac{5t_1}{128}\left\{(\beta+4)^2 f(\beta+4) - 2\beta^2 f(\beta) - 2\beta\left[(\beta+4)f(\beta+4) - \beta f(\beta)\right]\right\}$$
$$+\frac{5t_1}{128}\left[2\beta\int_\beta^{\beta+4}dF(\alpha)+\beta^2 f(\beta+4) - 2\int_\beta^{\beta+4}\alpha dF(\alpha)\right] - \int_{\beta+4}^{\infty}dF(\alpha) - \frac{t_1}{2}f(\beta+4)$$
$$= -\frac{5t_1}{64}\int_\beta^{\beta+4}(\alpha-\beta)dF(\alpha) - \bar{F}(\beta+4)\left[1-\frac{t_1 f(\beta+4)}{8\bar{F}(\beta+4)}\right].$$

Since $F(\alpha)$ is IGFR, $\frac{dc}{d\beta} \leq 0$, i.e., c decreases in β, $\forall \beta \geq 0$. Intuitively, $F(\alpha)$ linearly increases in $t_1, \forall t_1 \in [0,1]$, i.e., $c(K_1^e,K_1^e)\big|_{t_1=0} \leq c(K_1^e,K_1^e) \leq c(K_1^e,K_1^e)\big|_{t_1=1}$ by given β. When $0 \leq \beta < \beta_1$, it guarantees $c_0(K_1^e,K_2^e) < c(1,1)\big|_{t_1=0}$, so $K_1^e > K_2^e$; when

$\beta \geq \beta_2$, $c_0(K_1^e, K_2^e) \geq c(1,1)\big|_{t_1=1}$, therefore, $K_1^e \leq K_2^e$; when $\beta_1 \leq \beta < \beta_2$, $c(K_1^e, K_1^e)$ increases in t_1 , so there must exists $\underline{\beta}(t_1, t_2) \in \left[\beta_1, \beta_2\right)$ such that $K_1^e > K_2^e$ for $\beta \in \left[\beta_1, \underline{\beta}(t_1, t_2)\right)$ and $K_1^e \leq K_2^e$ for $\beta \in \left[\underline{\beta}(t_1, t_2), \beta_2\right)$. β_1, β_2 are defined in Proposition 6.1.

Proposition 6.1 identifies two threshold marginal input costs, β_1 and β_2 with $\beta_1 \leqslant \beta_2$, which are not affected by process efficiencies. Part 1) shows that, when the marginal input cost is sufficiently large, i.e., $\beta \geqslant \beta_2$, the firm with a larger efficiency index will invest in less capacity than the firm with a smaller efficiency index. That is, when production is highly labor and material intensive, the firm whose process efficiency is more prone to improving as capacity expands, which we call the *more efficiency prone* firm, will invest in less capacity. Part 2) shows that the reverse is true when marginal input cost is sufficiently low, i.e., $0 \leqslant \beta \leqslant \beta_1$, or when process efficiency is more essential a cost determinant. When it is below the threshold level $\beta(t_1, t_2)$ for the marginal input cost in Part 3), the more efficiency prone firm will invest in more capacity and achieve a more efficient process, whereas a high labor and material content in the production process will deter firm from investing in the capacity to develop efficiency. Moreover, β_1, β_2, and $\beta(t_1, t_2)$ are first-order increasing but second-order decreasing with market demand. $\beta(t_1, t_2)$ increases in t_1 but decreases in t_2. This implies that the more efficiency prone firm is more likely to invest in more capacity and attain a more efficient process than the less efficiency prone firm, as the market expands or becomes more volatile; an expanding and more volatile market would thus favor the firms to scale up capacity investment.

It is quite involved, if possible, to analytically investigate the equilibrium profits of the two firms, and we resort to numerical studies.

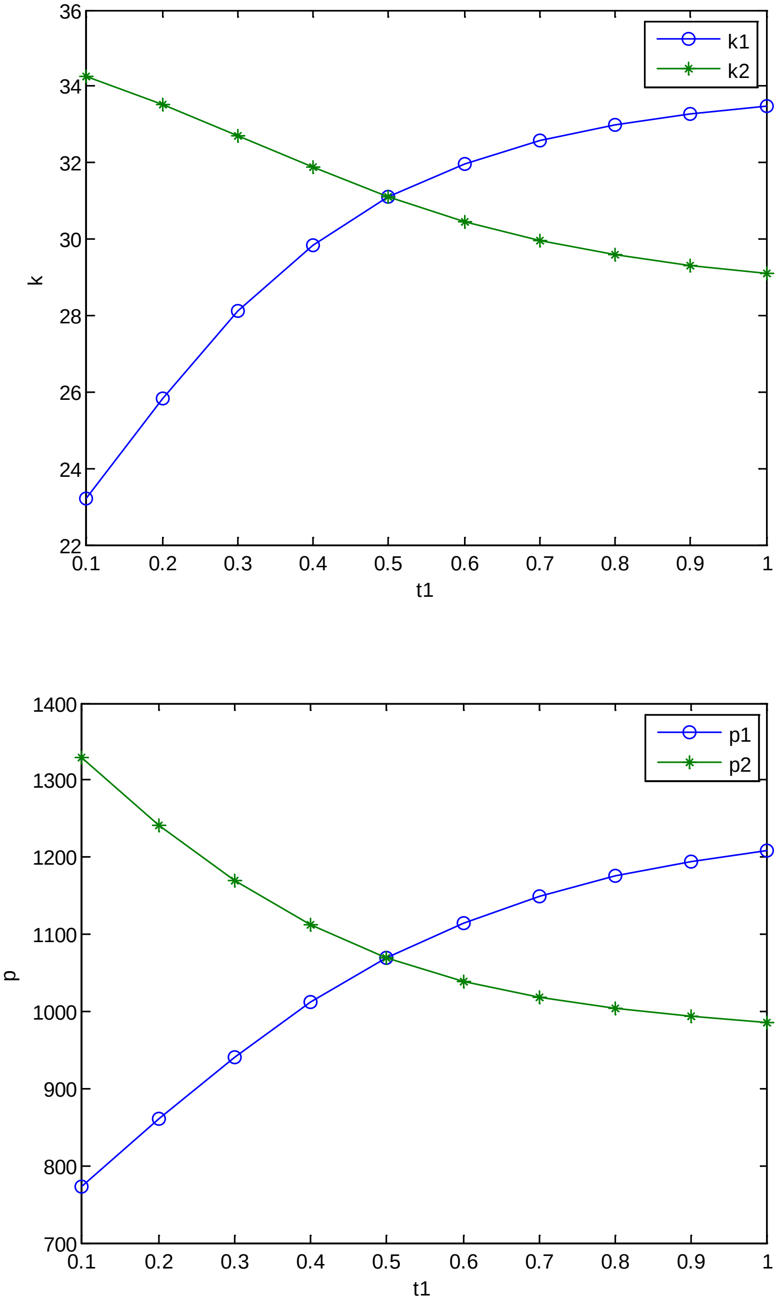


Figure 6.4 Effects of Efficiency Index on Capacities and Profits

Figure 3 illustrates the typical patterns for capacities and profits at two firms, where we fix the efficiency index at firm 2 but vary that at firm 1, by keeping the marginal input cost at a moderate level. Observe that the capacity and profit of firm 1 increase with its own efficiency index but decrease with that at firm 2. Hence, as the production process at one firm becomes more efficiency prone, this particular firm will boost capacity to be more efficient in production and reap in a higher profit, which will however force its competitor to de-invest in capacity and suffer a profit reduction. Between two firms, it is the more efficiency prone firm who will invest in more capacity and earn a higher profit.

Chapter 7 Conclusions

This dissertation deals with two models. In the first model, we consider one manufacturer who orders components from multiple suppliers with budget constraints. Facing with an uncertain demand market, the manufacturer decides on the cost share amounts given the suppliers to relieve them from cost burden. Three decision sequences are considered in this model. In the first sequence, the manufacturer initially decides N suppliers' cost share amounts, and then the suppliers sequentially decide their own wholesale price. In the second sequence, N suppliers simultaneously decide the wholesale price after the manufacturer makes the decision of cost share amounts. In the third sequence, both the manufacturer and N suppliers sequentially decide on the cost share amount and wholesale price, in an alternate sequence.

We obtain all equilibriums and channel profits under the above sequences. By comparing results among different models, we find that the supplier who moves first in the sequential setting, no matter which type of sequential sequence it is, can always charges the highest price among the suppliers. It is the same privilege as that without cost sharing. However, the first decision-maker is the only player who will be hurt by cost sharing. When the sales price is low, the first decision-maker is strictly reluctant to cost sharing even if the manufacturer relieves her product cost. When the sales price increases, the first decision-maker can benefit from cost sharing in some circumstances. Nevertheless, she still incurs profit reduction if other suppliers have enough budgets. The manufacturer and other suppliers, however, always prefer cost sharing.

Similar to the no cost sharing case, the manufacturer strictly prefer simultaneous decision sequence than the sequential setting as he may reap in more profits by sharing the same costs from the suppliers. The supplier who decides first, in the contrast, prefer the sequential decision sequence than the simultaneous one even if her profit is hurt by cost sharing in the previous decision sequence. Different from the sequential setting that the first decision-maker charges the highest wholesale price, the supplier who has the heaviest cost burden charges the highest wholesale price in the simultaneous setting. The decision sequence reallocates the profit share between the first decision-maker and the deepest cost-bounded supplier. If the first decision-maker has the heaviest capital constraints, she still prefers sequential setting. It indicates that the decision order has deep impact on the supplier's preference in the decision sequence regardless of the capital constraint.

The comparison of three settings in Chapter 3, 4 and 5 indicates that cost-sharing policy obviously affects each player's profit, but very weakly on the profit share. In most cases, the profit share among the players keeps as they are without cost sharing. Even in the sequential cases where the first decision-making supplier incurs profit cutting, she still earns more profits than other players do.

In the simple sequential setting, the correlation between the suppliers' decision sequence and capacity constraint order do not affect the manufacturer's cost sharing decision. Thus, the cost sharing equilibrium is symmetric to all the players. However, in the hybrid sequential setting, the cost sharing equilibrium is asymmetric. Moreover, each supplier's profit depends on the capital flow of the supplier who makes the decision next to her. When the latter has inefficient capital, the previous supplier may earn extra profits.

In the second model, we consider a setting of two firms serving a price-sensitive and uncertain market demand. Each firm is endowed with volume flexibility, which refers to the capability to produce below capacity when mark demand is low. Firms incur costs to invest in capacities and produce. We contribute to the literatures on flexibility with an equilibrium analysis for the competitive capacity and production decisions of firms in an asymmetric duopoly, incorporating the production cost at each firm and allowing it to be affected by capacity through process efficiency. Our results show that the firm whose process efficiency is more prone to improving with capacity expansion will invest in more capacity and achieve a more efficient process only when the production is not too labor and material intensive. Moreover, an expanding and more volatile market together with a stronger learning effect on efficiency from capacity expansion will favor the firm to scale up capacity. There are several plausible avenues to extend the work in this paper. Firstly, we will consider other functional forms for the market demand to examine the robustness and generality of the existing findings. In this paper, we only consider a linearly additive demand function. Secondly, we may expand the setting to an oligopoly that includes more than two firms, where more interactions, vertical and horizontal, are likely to be incubated among the participants.

Further research will focus on the extension of the cost-sharing model into an asymmetric setting. We will assume suppliers' costs as random factors and then study how the equilibrium moves and how the decision sequence affects the equilibrium in such case. Moreover, we are going to discuss the impact of asymmetric information on the manufacturer and suppliers’ performance. We will also explore the similar extensions in the hybrid sequential decision sequence in the future.

Reference

1. Adams, D. W., N. Rask. 1968. Economics of cost-share leases in less-developed countries. *Journal of Agricultural Economics*. 50(4): 935-942.
2. Anand, K., K. Girotra. 2007. The strategic perils of delayed differentiation. *Management Science*. 53(5): 697-713.
3. Anupindi, R., L. Jiang. 2008. Capacity investment under postponement stamountgies, market competition, and demand uncertainty. *Management Science*. 54 (11): 1876-1890.
4. Babich, V. 2007. Dealing with supplier bankruptcies: costs and benefits of financial subsidies. *Working paper*. Industrial and Operations Engineering. University of Michigan.
5. Babich, V., A. N. Burnetas, P. H. Ritchken. 2007. Competition and Diversification Effects in Supply Chains with Supplier Default risk. *Manufacturing & Service Operations Management*. 9(2) 123-146.
6. Barkholz, D., R. Sherefkin. 2007. C&A debacle will cost automakers $665 million. *Automotive News*. 81(6248): 1-47.
7. Barlow, R.E., F. Proschan. 1965. *Mathematical theory of reliability*. John Wiley &Sons. New York.
8. Bhaskaran, S. R., V. Krishnan. 2009. Effort, revenue, and cost sharing mechanisms for collaborative new production development. *Management Science*. 55(7): 1152-1169.

9. Bollapragada, R., U. S. Rao, J. Zhang. 2005. Managing inventory and supply performance in assembly systems with random supply capacity and demand. *Management Science*. 50(12).

10. Boyer, M., M. Moreaux. 1997. Capacity commitment versus flexibility. *Journal of Economics & Management Strategy*. 6(2): 347-376.

11. Chod, J., N. Rudi .2005. Resource flexibility with responsive pricing. *Operations Research*. 53(3): 532-548.

12. Chopra, S., M. Sodhi. 2005. Managing risk to avoid supply-chain breakdown. *MIT Sloan Management Review*. 46(1): 53-61.

13. Eliashberg, J., R. Steinberg. 1991. Competitive stamountgies for two firms with asymmetric production cost structures. *Management Science.* 37(11): 1452-1474.

14. Li, C., L. G. Debo. 2009. Second sourcing vs. sole sourcing with capacity investment and asymmetric information. *Manufacturing & Service Operations Management*. 11(3): 448-470.

15. Gerwin, D. 1994. Manufacturing flexibility: a strategic perspective. *Management Science*. Vol. 39: 395-410.

16. Goyal, M., S. Netessine. 2007. Strategic technology choice and capacity investment under demand uncertainty. *Management Science*. 55 (2): 192-207.

17. Goyal, M., S. Netessine. 2011. Volume flexibility with multiple products and the trade-off with product flexibility. Forthcoming. *Manufacturing & Service OperationsManagement.*

18. Graves, S., B. Tomlin. 2004. Process flexibility in supply chains. *Management Science*. 49 (7): 907-919.

19. Hendricks, K.B., V. R. Singhal. 2004. The effect of supply chain glitches on shareholder wealth. *Journal of Management*. 21: 501-523.

20. Hendricks, K.B., V. R. Singhal. 2005a. Association between supply chain glitches and operating performance. *Management Science*. 51(5): 695-711.

21. Hendricks, K.B., V.R. Singhal. 2005b. An empirical analysis of the effect of supply chain disruptions on long-turn stock price performance and equity risk of the firm. *Production and Operations Management*. 14(1): 35-53.

22. Hiebert, L. 1989. Cost Flexibility and Price Dispersion. *The Journal of Industrial Economics*. 38 (1): 103-109.

23. Hossain, M., M. Bhatti, M. Ali. 2005. An econometric analysis of major manufacturing industries. *Managerial Auditing Journal*. 19 (6): 790-795.

24. Jiang, L., Y. Wang. 2010. Supplier competition in decentralized assembly systems with price-sensitive and uncertain demand. *Management Science*. 12(1) 93-101.

25. Jones, R., J. Ostroy. 1985. Flexibility and uncertainty. *Review of Economic Studies*. 13-33.

26. Jordan, W.C., S. Graves. 1995. Principles on the benefits of manufacturing process flexibility. *Management Science*. 41 (4): 577-595.

27. Kreps, D. 1990. A course in microeconomic theory. Princeton University Press, Princeton, NJ.

28. Lariviere, M. A. 1999. Supply chain contracting and coordination with stochastic demand [A]. In: Tayur S, Magazine M and Ganeshan R. Quantitative models of supply chain management. *Boston, MA: Kluwer Academic Publishers*. 233-268.

29. Lariviere, M. A., E. L. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing and Service Operations Management*. 3(4) 293-305.
30. Lariviere, M. A. 2006. A note on probability distributions with increasing generalized failure amounts. *Operations Research*. 54(3): 602-605.
31. Marschak, T., R. Nelson. 1963. Flexibility, uncertainty and economic theory. *Metroeconomica*. 14 (2): 42-60.
32. Mas-Colell, A., M. Whinston, J. Green. 1995. *Microeconomic Theory*, Oxford Press.
33. Mas-Colell, A., J. Silvestre. 1989. Cost share equilibria: a Lindahlian approach. Journal of *Microeconomic Theory*. 47: 239-256.
34. Milgrom, P., J. Roberts. 1990. The economics of modern manufacturing technology, strategy, and organization. *American Economics Review*. 80 (3): 511-528.
35. Mills, D. 1985. Demand fluctuations and endogenous firm flexibility. *The Journal of Industrial Economics*. 33 (1): 55-71.
36. Mills, D. 1986. flexibility and firm diversity with demand fluctuations. *International Journal of Industrial Organization*. 4: 302-315.
37. Pasternack, B. A. 1985. Optimal pricing and return policies for perishable commodities. *Marketing Science.* 4(2): 166-176.
38. Petruzzi, N., M. Dada. 1999. Pricing and the newsvendor problem: A review with extensions. *Operations Research*. 47(2) 183-195.

39. Roller, L., M. Tombak. 1990. Strategic choice of flexible production technologies and welfare implications. *The Journal of Industrial Economics*. 38(4): 417-431.

40. Roller, L., M. Tombak. 1994. Competition and investment in flexible technologies. *Management Science*. 39(3): 107-115.

41. Sheffi, Y. 2001. Supply chain management under the threat of international terrorism. *International Journal of Logistics Management.* 12(2): 1-11.

42. Sheffi, Y. 2005. The resilient enterprise: overcoming vulnerability for competitive advantage. *MIT Press*. Boston, MA.

43. Spengler, J. 1950. Vertical integration and antitrust policy. *Political Economy*. 58(4): 347-353.

44. Stigler, G. 1939. Production and distribution in the short run. *Journal of Political Economy*. 47: 304-327.

45. Swinney, R., S. Netessine. 2009. Long-term contracts under the threat of supplier default. *Manufacturing & Service Operations Management*. 11(1): 1-19.

46. Taylor, T. A., E. L. Plambeck. Simple relational contracts to motivate capacity investment: price only vs. price and quantity. *Manufacturing & Service Operations Management*. 9(1): 94-114.

47. Tang, C. S. 2006. Perspectives in supply chain risk management. *International Journal of Production Economics*. 103(2): 451-488.

48. Tomlin, B. 2006. On the value of mitigation and contingency stamountgies for managing supply chain disruption risks. *Management Science*. 52(5): 639-657.

49. Tomlin, B., Y. Wang. 2005. On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management*. 7(1): 37-57.

50. Upton, D. 1995. What really makes factories flexible. *Harvard Business Review*. 74-85.

51. Van Mieghem, J. A. 1998. Investment stamountgies for flexible resources. *Management Science*. 44: 1071-1078.

52. Van Mieghem, J. A., M. Dada. 1999. Price versus production postponement: capacity and competition. *Management Science*. 45 (12): 1631-1649.

53. Vives, X. 1986. Commitment, flexibility and market outcomes. *International Journal of Industrial Organization*. 4: 217-229.

54. Wang, Y., L. Jiang, Z. Shen. 2005. Channel performance under consignment contract with revenue sharing. *Management Science*. 50(1): 34-47.

55. White, J. 2005. Ford to pay up to $1.8b on Visteon. Wall Street Journal. May 26.