Development of Production and Inventory Control Policies in Imperfect Production Systems

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2016

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A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

December 2015

CERTIFICATE OF ORIGINALITY

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Abstract

The production and inventory control problems in manufacturing systems have attracted a large number of researchers in the past few decades. Holding inventory, such as raw material, tools and assemblies, not only ties up capital but also generates an associated carrying cost and even has the potential for inventory depreciation. Ideally, the inventory level should be kept as low as possible. However, in reality, manufacturing companies face uncertainties from the market demand, production processes and supply. Advanced planning of production activities to hedge against these uncertainties is necessary and important for manufacturing companies. What is a reasonable amount of inventory to hold and how to manage the corresponding production planning and control have raised challenges to decision makers. Hence the aim of production and inventory control, in general, is to minimize the overall cost, including the inventory holding/backlogging cost and production cost, by determining the optimal control variables, such as production quantity, safety stock level and production speed.

Although the problem of production and inventory control has been studied intensively, research gaps still exist and more practical factors should be taken into consideration. For example, stationary demand is always taken as an assumption in previous research works. However, in real cases, non-stationary demand can be commonly observed. Corresponding optimal control policies should be investigated. In addition, a large number of previous research studies normally assumed that the production system is perfect and the inventory is non-perishable, and the rest either only looked at problems with deteriorating production processes or perishable

inventories. An integrated study considering both of the factors needs to be carried out.

Hence this research aims to address the issues and has filled the research gaps mentioned above in the field of production and inventory control. There are five main deliverables provided in this research:

First of all, non-stationary demand and forecasting have first been introduced into hedging point based production and inventory control policy. Two forecast corrected control policies have been proposed and optimized through an integrated simulation, design of experiment and response surface methods. Secondly, the impact of forecasting inaccuracy on hedging point based control policies has been investigated which can provide aid in the decision making process of choosing control policy. Thirdly, product and process deterioration have been jointly examined in two economic production quantity models together with backorder and rework. In addition, both piece-wise linear increasing and stochastic production process deterioration have been modeled. Optimality conditions for cost functions were obtained and numerical examples were employed to illustrate the performance of the proposed models. The optimum combination of production and backorder quantity in each production run were determined and sensitivity analysis showed the influence of different parameters on inventory behavior.

Publications

Journal Papers:

1. N. Li, Felix T.S. Chan, S.H. Chung, and Allen H. Tai, (2015). An EPQ Model for Deteriorating Production System and Items with Rework, *Mathematical Problems in Engineering*, Vol.2015. doi:10.1155/2015/957970

2. N. Li, Felix T.S. Chan, S.H. Chung, and Allen H. Tai. A Stochastic Production-Inventory Model in a Two-State Production System with Inventory Deterioration, Rework Process and Backordering, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*. (In press)

3. N. Li, Felix T.S. Chan, S.H. Chung. Development of Production-Inventory Control Policy with Non-Stationary Demand, Machine Failures and Demand Forecasting, *European Journal of Industrial Engineering*.(Under review with major revision)

4. Felix T.S. Chan, N. Li, S.H. Chung, M. Saadat. Management of Sustainable Manufacturing Systems-A Review on Mathematical Problems, *International Journal of Production Research*. (Under review with minor revision)

Conference Papers:

5. N. Li, Felix T.S. Chan, S.H. Chung, Allen H. Tai, M. Saadat and Z.X. Wang, (2015). A Stochastic Production-Inventory Model in a Two-state Production

System with Deteriorating Product, Rework Process and Backordering, *2015 Global Engineering & Applied Science Conference*, Tokyo, Japan, Dec 2-4.

6. N. Li, Felix T.S. Chan, S.H. Chung, B. Niu, (2015). The Impact of Non-stationary Demand and Forecasting on a Failure-prone Manufacturing System, *The 2015 International Conference on Industrial Engineering and Operations Management*, Dubai, March 3-5.

7. N. Li, Felix T.S. Chan, S.H. Chung, (2014). Development of Failure-Prone Manufacturing System Control with Inventory Inaccuracy and Stochastic Demand, *The 18th International Symposium on Inventories*, Budapest, Hungary, August 18-22.

Acknowledgements

First of all, I wishes to express my gratitude to my supervisor, Professor Felix T.S. Chan, for his suggestions and guidance during the process of research, the publication of this work and personal development. And also thanks him for his patience and encouragement to me and when i made mistakes and when I found it difficult to move on. I further wishes to thank Dr. S.H. Chung for his precious help and advice provided to my research work, project and papers in the past three years. I have learned a lot from him.

Secondly, I want to say thank you to Dr. Andrew Ip who was my confirmation examiner and have provided me precious guidance and suggestion on the direction of my research. I also would like to thank the support from ISE departement, especially Prof. Yue, Ms. Cammy Chiu, Ms. Iris Ko.

Finally, I would like to thank my parents and my wife Christina who are the most important people in my life. Taking up a 3-year PhD study is never a easy decision, not to mention the sacrifice and compromise they made. Without their encouragement and trust, i wouldn't be able to achieve this far, not only as a researcher but also as a person.

Table of Contents

List of Figures

List of Tables

List of Abbreviations

HPP	Hedging Point Policy
EPQ	Economic Production Quantity
EOQ	Economic Order Quantity
ELSP	Economic Lot Scheduling Problem
EMQ	Economic Manufacturing Quantity
ELQ	Economic Lot Quantity
SMA	Simple Moving Average
SES	Simple Exponential Smoothing
ARMA	Autoregressive Moving Average
ARIMA	Autoregressive Integarated Moving Average
FMS	Flexible Manufacturing Systems
PHP	Prioritized Hedging Point
MPC	Model Predictive Control
IMC	Internal Model Control
HWS	Holt-Winter Seasonal
MAE	Mean Absolute Error

MAPE	Mean Absolute Percentage Error
MSE	Mean Squared Error
WIP	Work In Progress
FCFS	First Come First Served
CCD	Central Composite Designs
DOE	Design of Experiments
GA	Genetic Algorithms
JIT	Just In Time
ANOVA	Analysis of Variance
PCR	Percentage Cost Reduction

List of Notations

Notations in Chapter 3

i	The index of batch
j	The index of order
$x_i(t)$	WIP level in batch i
$y(t)$	Inventory level
$u(t)$	Production rate
u_{max}	Maximum production rate
$S(t)$	Status of machine at time t
L_i	Lot size of i_{th} batch
$\gamma_{\alpha\beta}$	Transition time from α to β
$\gamma_{\beta\alpha}$	Transition time from β to α
$TB(t)$	Total batch number up to time t
d_j	The demand size of j_{th} order
$\lambda(t)$	Arrival rate of customer orders at time t
d_j	The demand size of j_{th} order

Chapter 1 Introduction

First of all, Chapter 1.1 describes the relevant background information for this research. Then the problems identified from the literatures are presented in Chapter 1.2. Chapter 1.3 presents the research scope and objectives and the contribution of this research is showed in Chapter 1.4. Lastly the structure of this thesis is elaborated in Chapter 1.5.

1.1 Research Background

As a part in the whole supply chain, the production and inventory control has fundamental and crucial impact on the performance of the whole supply chain. Inventory, which could be as small as a bolt and as large as a mechanical excavator, help manufacturers hedge against the potential risks from production capacity, price and demand. Having an extremely low inventory level puts the company at the risk of losing incoming orders and customers (Waters, 2003) . However, it doesn't mean that holding as much inventory as possible is a wise choice for manufacturers, as too much inventory causes high inventory cost and holds excessive capital which reduces their flexibility. Finding a balance between production/inventory and demand is always a challenge for industries (Goldsby and Martichenko, 2005)

In addition, in recent decades, increasing market competitiveness, quick market changes and high requirements on the customization of products have raised new challenges to manufacturing companies. How to control the relevant cost generated from production, inventory management, as well as satisfy customers demand has become more and more important in a fast changing market environment (Fredendall and Hill, 2000). More and more uncertainties, such as inventory inaccuracy, stochastic demand, production quality and the failure of machines, appear and further challenge the management capability in terms of scheduling and planning. (Boyer and Verma, 2009)

As a result, the production and inventory control problems in manufacturing systems have received a great deal of attention in the past 4 decades. The problem, in general, is to minimize overall cost, including the inventory holding/backlogging cost, production cost and transportation cost, by determining the optimal control variables, such as reordering policy, production speed and quantity.

Both in academia and the industrial world, there is a large number of existing systematic methodologies focusing on solving the problem and providing managerial insights to manufacturing companies. Among all the relevant literature, two streams of research are especially popular and have been intensively studied: Hedging Point Policy(HPP) based production and inventory control in failure-prone manufacturing systems and the lot sizing problems.

Hedging Point Policy (HPP)is a result of a theoretical control approach to the production and inventory control problem. HPP was first proposed by Kimemia and Gershwin (1981) as a optimal feedback control structure in Flexible Manufacturing Systems (FMS). In 1986, complete proof and demonstration of the optimum hedging point for a single machine and a single product system was provided through the work of Akella and Kumar (1986). In their research, the problem was formulated as a continuous-time model with jump Markov disturbances. They showed that a certain critical number in the inventory level forms the optimal solution. A formula was also determined to calculate the optimal inventory level. Hedging point policy is a specifically useful in the failure prone system.

While lot sizing problems tries to solve production problems concerned with the suitable quantity of products to be produced in each production cycle with the objective of minimizing the overall production and inventory cost and the consideration of different factors such as setup cost, demand and replenishment. They can be further categorized into four different problems: Economic Order Quantity (EOQ) problem, Economic Production Quantity(EPQ) problem, Economic Lot Sizing problem and Economic Lot Scheduling problem. Among the four categories, EPQ is especially popular. Hence in this research, we mainly focus on EPQ problems and its models.

1.2 Problem Statement

However, despite research work on the two problems mentioned above, especially in the field of failure prone manufacturing systems, there are still several research gaps that exist in the current literature.

For hedging point based production and inventory control problem:

- The majority of the studies assumed that the demand process is stationary. In terms of stationary demand, there are two types of assumptions. First of all, the demand is viewed as a constant throughout the whole period. Secondly the demand is modeled with a stochastic process, such as the Bernoulli Process, Poisson Process or Compound Poisson Process, in which the mean and variance usually are kept constant. For instance, in research on production and inventory control problems (Hajji et al., 2009; Sethi and Zhang, 1999) formulated models with constant demand rate only, and a stationary control policy is generated correspondingly.

The demand process in actual industry, on the contrary, normally follows a non-stationary process. The reasons behind this phenomenon can be summarized as intensive market competition, high frequency of new products, short life cycles and seasonality (Neale and Willems, 2009). All those factors have led to the fact that a stationary demand process is not adequate in reflecting the real situation in supply chain management.

- The impact of forecasting on HPP based control policy with non-stationary demand has not been investigated. There are some studies discussing the relationship between forecasting and production/inventory control. For example, Fildes and Beard (1992) looked at the application of forecasting on production and inventory control and how the accuracy can be improved in all different forecasting methods. Similarly, a forecasting-production-inventory system was analyzed by (Toktay and Wein, 2001), using the Martingale model of forecast evolution to provide updates for the forecast of correct stock policy. However, in their studies, only stationary demand process is considered and a reliable manufacturing system is normally modeled in which machine isn't subject to failure. So integrated research considering non-stationary demand, forecasting and hedging point based production and inventory control is of great interest.

For EPQ problem, the main research gaps can be summarized as:

- The majority of papers have only considered either inventory deterioration or unreliable production systems. For instance, Pal et al. (Pal et al., 2013) considered only production process deterioration is considered. However both aspects are important in production and inventory control and should be taken into consideration.

  In terms of product deterioration, perishable goods such as food, fruit, drink and other products like electronic devices and metal processing are good examples and their quality is heavily influenced by the storage conditions and storage time after production. If the quality does not meet the standard or requirement anymore, either extra cost is needed to recover the quality or the product has to be disposed of (Raafat, 1991).

  The reliability of production systems, on the other hand, addresses the quality of production processes, such as defective rate, machine breakdown and production speed. Defects can be generated from processing mistakes, setup mistakes, adjustment mistakes and tooling mistakes (Hinckley, 1997). Although management techniques, such as quality control and the improvement of manufacturing technology, have successfully reduced the defect rate to a relatively low level, it is still a problem for companies, especially when the complexity of production is high. The mobile phone industry can be used as a good example of having extra cost caused by defects (Worstall, 2013).

  Similar to the defective rate, the decrease of production and machine breakdowns are also obstacles to achieving a high productivity in a manufacturing system (Iravani and Duenyas, 2002). Maintenance is an effective method that prevents the serious machine breakdown and restores production quality. However, for deterioration within each production cycle, maintenance cannot solve the problem.

1.3 Research Scope and Objectives

In order to solve the problems mentioned in the previous section and bridge the existing gaps between academic and industrial world in area of production and inventory control, this research mainly focuses on developing production and inventory control policy in imperfect manufacturing systems together with consideration of practical uncertainties. To be more specific, the major research objectives can be summarized as follows:

- Investigate the modified hedging point policy and optimization methods to take non-stationary demand and forecast into consideration.

- Investigate the practical factors in economic production quantity problem such as imperfect production, process deterioration, product deterioration, backorder and rework.

- Research into the optimization methodologies for the both the hedging point policy problem and economic production quantity problem.

1.4 Structure of the Thesis

The rest of the thesis is arranged as follows:

Chapter 2 presents a thorough review on the literature related to the production and inventory control problem. The research on hedging point based control problem and EPQ problem are introduced and discussed. The development of these two problems is outlined and, in the end, the research gaps are summarized to help readers better understand the problem itself and the work done in this research.

Chapter 3 describes the modified hedging point policies and corresponding methodologies for production and inventory control problems with consideration of forecasting and non-stationary demand. Numerical experiments have been conducted to validate the performance of the proposed model and methodologies.

Chapter 4 first shows the modified model for the EPQ problem with both inventory and piece-wise linear production process deterioration. Then, an analytical approach was used to solve the problem. Lastly numerical experiments and sensitivity analysis are illustrated to show the performance of the model.

Similar to Chapter 4, Chapter 5 starts the chapter with an introduction of the mathematical formulation of the proposed modified model. While in the model, stochastic production process deterioration is considered instead of a piece-wise linear one.

Chapter 6 summarizes all the models and methodologies proposed in this research. Discussion of the results and their applications are also presented.

Chapter 7 lists the conclusions made based on the results generated from Chapters 3 to 5. The existing limitations of the current research and highlights the potential research directions to be considered in the future are points out in the end.

Chapter 2 Literature Reviews

2.1 Failure-Prone Production and Inventory Control

This chapter presents a thorough review of the existing literature in the field of production and inventory control. The review is further divided into four parts. Firstly, the relevant works on failure-prone production and inventory control are discussed. Then the economical lot sizing problem is looked into. Lastly, the existing research on studying the relationship among forecasting, stochastic, non-stationary demand and production and inventory control is examined. The research gaps are identified and elaborated at the end.

2.1.1 Traditional hedging point policy

The optimality of the Hedging Point Policy in a failure-prone manufacturing system has been proven by Akella and Kumar (1986) with discounted cost, as mentioned earlier. They showed that under the assumption of constant demand, and for a single machine single product manufacturing system, there is a key parameter, the so-called hedging point H, that exists in the optimal policy. $u(t)$ and $x(t)$ are the production rate and inventory level at time t, respectively. d represents the demand rate and u_max is the maximum production rate available. The traditional optimal control policy is generalized as shown in Equation 1:

$$I_1(t_1) = \begin{cases} 0 & if \quad x(t) > H \\ d & if \quad x(t) = H \\ u_{max} & if \quad x(t) < H \end{cases} \tag{2.1}$$

This control policy shows that the production rate adjusts its value according to the inventory level. For example, when the inventory level $x(t)$ is lower than H, the maximum production rate will be adopted. In their work, the failure of the machine is modeled by a two-state Markov chain. Similarly, Bielecki and Kumar(1988) obtained the long term minimum cost for the single-product, single-machine control problem with an unreliable machine, linear holding cost and backlog cost. Using the result of hedging point policy, they also proved that zero-inventory policy is possible, even with the existence of uncertainties in the systems, under certain conditions.

2.1.2 Generalization of the problem

Later, studies focusing on the more general problem were carried out. Boukas and Haurie (1990) firstly studied the case with age dependent machine failure rate, while Hu et al (1994) looked at the hedging point policy with consideration of production-rate-dependent failure rate. The results indicated that a linear failure rate function is the necessary and sufficient condition to get an optimal hedging point policy. Sharifnia (1988) generalized the work of Bielecki and Kumar (1988) and assumed a system with multiple states. A similar approach can also be found in the study done by Liberopoulos and Hu (1995).

The extended work for a multi-product, single machine hedging point policy was carried out by Sethi and Zhang (1999) in which the hedging points for different products were determined separately. Sethi's research showed that the structure of the original hedging point policy was also valid for multi-product problems. Meanwhile, a modified policy called prioritized hedging point(PHP) policy was proposed by Perkins and Srikant (2001) to deal with multi-product systems. Differing from the traditional approach, in PHP policy, weights are given to various part types. If the inventory levels of all part types are below their respective hedging points, the one with the highest priority or weight is produced at the maximum production speed, while the others are kept around their hedging points. A sequence of two part-type problems was used to replace the multi-part-type problem, and explicit expressions of the buffer level probability density function and the corresponding optimal hedging point were obtained. In 2001, Shu and Perkins(2001) further extended the PHP policy by taking quadratic buffer costs into account, and the optimal priority sequence was also determined in their work.

Wang and Yu (2012) investigated the effect of inaccurate inventory observation in a single machine, single product production control policy scenario. A modified control policy with consideration of the observation error in the hedging point was adopted. The research showed that the modified policy provides better robustness than the traditional hedging point policy. However, when the ratio between the failure time and functional time is large enough, the traditional hedging point policy performs better. Production quality decision making is also introduced into the study of production control policy, and one with multiple part-types, multi-machine was obtained by the same group of researchers (Chan and Wang, 2014). Another work from Hajji et al (2012) assumed that manufacturing systems produce both non-confirming and confirming product, subject to probability. The engineering specification requirement was used as a control variable which significantly influenced the rate of conforming of the products. Differing with another paper done by Hajji et al. (2012), this work adopted the traditional structure of hedging point policy directly, without any numerical approximation.

The integration with manufacturing network has been studied by two separate research groups. Chan et al. (2008) proposed a two level hedging point control policy for networked manufacturing systems in which extra production resources can be used with extra cost. Multiple-product type manufacturing systems were modeled and for each product there were both a higher and lower hedging point, which is the major difference between their research and the previous work on the same problem. Meanwhile, Sajadi et al. (2011) also modelled the production control problem in a manufacturing network. In this model, the production process consisted of several steps and each machine created only one step in the whole production process, and the production speed of different machines was influenced by the machine in the previous step. An approximated control policy was utilized and the corresponding hedging point for each machine was generated through a combined method of simulation and statistical analysis.

Chen (2004) provided the characteristics of HPP and the corresponding switching curves for both reliable and unreliable systems with random demands. Chen also proved that whether there was a finite or infinite planning horizon, the proposed hedging point policy is optimal.

2.1.3 Integration with other problems

The 21st century has witnessed a trend of combining production and inventory control with more uncertainties and other problems, for instance, maintenance, replenishment and transportation.

Gharbi and Kenne (2000) integrated maintenance activities into the problem of production and inventory control. An age-dependent hedging point policy was proposed and optimal hedging point, machine switching age and maintenance age parameter were determined when the minimum inventory cost was achieved. Rivera-Gomez et al. (2013) also considered maintenance activities in a deteriorating manufacturing system. The study aimed at minimizing the total cost by obtaining the production planning and overhaul schedule while inventory holding, shortage, overhaul cost were taken into consideration. As a result, a machine deterioration-dependent hedging point policy was structured. Berthaut et al. (2011) looked into the integration of block replacement and HPP problem in failure-prone manufacturing cells. A modified block replacement/hedging point policy was adopted and proved to be better than the traditional HPP or block replacement policy.

Joint replenishment and production control were considered in the work of Hajji et al. (2009). A two-stage supply chain was modelled in which both the suppliers and manufacturers were unreliable. The optimum production control policy and supply policy were approximated simultaneously using numerical resolution methods. Bouslah et al. (2012) studied the production and inventory control problem in an unreliable batch production system with constant demand. Lot size was considered together with the hedging point. By using the numerical resolution approach, their research first approximated the structure of the control policy on a small scale. The results showed that the new control policy still followed the pattern of the traditional control policy as in equation 1. Then simulation, experimental design and response surface method were combined again to determine the optimal combination of hedging point and batch size with higher accuracy than numerical resolution approach. Other similar research work also include the integration with setup(Assid et al., 2015b), lockout/tagout (Charlot et al., 2007), transportation delay (Mourani et al., 2008), subcontracting planning (Assid et al., 2015a) and adjustable capacity(Gharbi et al., 2011).

From all the research work mentioned above, a conclusion can be made that the complexity of the production and inventory control problem is always increasing dramatically with more and more practical factors considered. In addition, the integration among different problems also makes the model complicated; for instance, the integration between hedging point based control and forecasting required forecast corrected hedging point policy which make the mathematical analysis hard to conduct.

In the tradition production/inventory control problem, analytical analysis methods, as shown, need lengthy proof and demonstration to solve. For integrated problems, pure analytical analysis is not capable of solving the problem in a reasonable time anymore (Sajadi et al., 2011). Hence, methods such as numerical approximation, simulation and some statistical techniques are then introduced. In a model that considered both production and preventive maintenance rates control (Gharbi and Kenne, 2000), simulation, experimental design and response surface methodology are applied. This method was also utilized in most of the studies mentioned earlier in this section.

Some researchers have employed other methods such as principles and theories from the field of control engineeringto tackle this problem. Perea et al. (2000) proposed dynamic modelling of the system using a fluid analogy. Model Predictive Control (MPC) was used to produce control policies. Similar study was carried out earlier by Schwartz and Rivera (2010) who also made use of classic control theory in production and inventory control problems. As an extension of the previous work, both Internal Model Control (IMC) method and MPC were applied and it enhanced the performance of the proposed method.

2.2 Lot Sizing Problems

The lot sizing problem is a critical element in production planning processes. It raises the question about how many products need to be ordered or produced in one production cycle in order to minimize the inventory, production, setup cost and satisfy the demand at the same time. To be more specific, large lot sizes lead to extremely high inventory cost. However, small lot sizes produce excessive setup cost, and when uncertainties are taken into consideration, how to determine the right lot size becomes more and more complex. It is especially applicable to industries with a single production process and in the metal processing industries. (Karimi et al., 2003)

The general lot sizing problem can be further described by four different models: Economic Order Quantity (EOQ), Economic Production Quantity(EPQ), Economic Lot Sizing Problem and Economic Lot Scheduling Problem. EOQ and EPQ are the fundamental problems in lot sizing. The only difference between EOQ and EPQ is that in EPQ, companies manufacture the product by themselves and the inventory is continuously and gradually built up along with the production process for one order. While in the EOQ model,orders are received only after the whole lot is completed. A sudden increase in the inventory level can be observed. It is common in the situation in which the production is outsourced to other companies. We believe that's the reason why Rogers (1958) used Economic Lot Quantity (ELQ) to describe the general problem of EPQ and EOQ.

If more than one product needs to be produced consecutively on a single manufacturing facility, EOQ and EPQ models are no longer applicable to this specific situation. The Economic Lot Scheduling Problem was developed and extended from ELQ, as a result, to handle the economic quantity problem together with production scheduling. (Eilon, 1957; Rogers, 1958)

While the Economic Lot Sizing problem refers to the case in which demand varies in different periods, Wagner and Whitin (1958) first proposed a dynamic lot-size model to solve the problem. In their model, the demand over a certain period was assumed to be known and the number of planning periods was finite. Dynamic programming was used to formulate the problem and an algorithm was developed to obtain the optimal solutions. However, due to its assumption on unlimited capacity and its high analysis complexity, researchers have proposed other methods to find the near optimal solution. Brahimi et al. (2006) provided a comprehensive review of the literature addressing lot sizing problems. In their survey, the lot sizing problem is categorized into incapacitated and capacitated cases.

2.2.1 Economic order quantity problem

Harris (1990) pioneered research on EOQ problems and a EOQ formula was proposed. In general, the EOQ model aims to find a trade off between inventory holding cost and ordering cost (Zipkin, 2000). Later on, in order to make the model more practical, factors such as payment delay, freight discount costs, nonlinear holding costs and deteriorating inventory were considered and integrated with traditional EOQ approach.

Goyal (1985) stated that in the traditional EOQ model, it was assumed that payment for an order must be made at the moment the order is received. However it's not true in industries where a fixed period of time is allowed for payment delay and no interest is charged if payment is settled on time. Hence the author modified the EOQ model to tackle this specific problem. Chung (1998) studied the same problem as Goyal but provided an alternative way to obtain the optimal solution. Both Shah (1993) and Aggarwal and Jaggi (1995) extended Goyal's work by taking deteriorating/decaying inventory into account. By proving the convexness of the cost function, Chu et al(1998) improved the derivation process used in Aggarwal and Jaggi's model to find the optimal solution. Weibull distributed inventory deterioration was studied by Covert and Philip(1973), and a model with time-dependent deteriorating rate was developed by Manna and Chaudhuri (2006) as an extension of previous work. In their research, not only the deteriorating rate was time-dependent but also the production rate was proportional to demand rate.

Imperfect production processes were also examined in Cheng's work(1991). Instead of using constant unit production cost, a demand-dependent unit production cost was assumed. A geometric program method was used to formulate the problem and a closed-form solution was determined. For more literature on the EOQ model with an imperfect product, please refer to the review paper of Khan et al. (2011).

Jamal(1997) further looked at a model with allowable shortage. Various facets of payment delay were discussed. Similarly, Eroglu and Ozdemir (2007) combined the EOQ model with backorder, but the production process was assumed to be imperfect and a portion of the products were defective. The good-quality products and defective products were sold at different prices. Both linear and fixed backlog costs were considered by Sphicas (2006) and Cardenas-Barron (2011). Sphicas employed an analyzed process without calculus, while Cardenas-Barron proposed a simple solution method based on analytic geometry and algebra, and the effectiveness was proven. Pentico and Drake (2009) studied the case with partial backordering as a supplement of the basic model, and one with full-backordering. The fuzzy version of the EOQ problem was taken into consideration by Kazemi et al. (2010) and all the parameters and decision variables were fuzzified and represented with two types of fuzzy numbers.

Schrady (1967) pioneered research on the Economic Order Quantity (EOQ) problem with repairable items. A deterministic inventory model was developed with the assumption of constant demand and repair rate. And the optimum order quantity was determined when the setup cost and inventory holding cost were minimized.

A large number of studies have been carried out using Schradys model. Richter (1994) extended the original EOQ repair model in waste disposal, which was represented as a product disposal rate. In this model, the formula for optimal lot size and cost function were first derived as a function of disposal rate. Then the optimal disposal rate was determined when the cost was minimized. The study also showed that the disposal price had a significant impact on the optimal disposal rate. For instance, the optimal disposal rate was a convex function of small disposal price. Richter further proposed two models considering variable setup numbers and variable collection time intervals (Richter, 1996a,b). The results indicated that the behavior of collection intervals, setup numbers and cost varied with different values of the disposal rate. Konstantaras and Skouri (2010) relaxed the assumption that no backlog was allowed in the previous research, following the study of Richter, but extending the model to the case allowing inventory backlog.

The disposal rate was also utilized by Teunter (2001) for a similar EOQ model with a recoverable item. Various holding costs were applied for both manufactured and recovered products and a more generalized EOQ formula was obtained. However in the studies above, infinite production rates were mostly assumed to determine the lot size formulae. Teunter (2004) looked at cases with finite and infinite production rates, and provided a more gereral lot sizing formula for the two cases. Widyadana and Wee (2010) revisited the model proposed by Teunter (2004) and introduced an alternative method using algebraic approaches to solve the problem. Feng and Viswanathan (2011) also proposed a new heuristic for lot sizing problem with product recovery. A more general set of (P, R) policy was utilized in which the lots for manufacturing and remanufacturing were interleaved, while in traditional (P, R) policies, all the lots for manufacturing are arranged together. The results showed that the new heuristics have better performance than those used in Teunter (2004).

2.2.2 Economic lot sizing problem

Due to the complexity of Wagner and Whitin's method, Silver and Meal (1973) developed a heuristic algorithm to solve the dynamic lot sizing problem. Both studies had the same assumption that the production capacity is unlimited. The integration with deteriorating inventory can be found in the works of Papachristos and Skouri (2000) and Hsu (2000). Zangwill (1969) looked at the backlog allowed economic lot sizing problem and a network approach was utilized for formulation. Both parallel and series facilities were taken into consideration. The case with stockout instead of backlog was studied by Sandbothe and Thomson (1990) in which unsatisfied demands were lost directly rather than backordered.

The dynamic lot sizing in manufacturing and remanufacturing system has also been researched. Richter and Sombrutzki (2000) modified the classical Wagner/Whitin model with consideration of remanufacturing. They addressed lot sizing in planing the manufacturing and remanufacturing activities, for a number of periods, with dynamic but deterministic demand. The zero inventory property was proven in their model. However, their model was based on the assumption of a large quantity of low cost used products. Golany et al. (2001) tackled the same problem and provided a more general model and solution. Teunter et al. (2006) also studied the problem and it was first formulated in mixed integer linear programming, and then solved by using a dynamic programming algorithm and some heuristics.

The capacitated dynamic lot sizing problem in closed-loop supply chain was studied by Pan et al. (2009), in which the capacities for remanufacturing, manufacturing and disposal were assumed to be limited. Similar to the work done by Teunter et al.

(2006), a dynamic programming algorithm was developed. Capacitated lot sizing was also researched by Li et al.(2007) and Zhang et al.(2012). The first study adopted a heuristic GA to analyze the capacitated lot sizing. The periods that needed setups were firstly determined by the GA stage, then a dynamic programing approach was used to compute the optimal lot size for both types of products. Zhang et al. (2012) introduced a lagrangian relaxation based method to tackle the same problem and proved that the proposed method outperformed the other methods in terms of the solution quality. Fazle et al. (2014) investigated the effect of different demand patterns and return patterns on dynamic lot sizing with product recovery and remanufacturing. A dynamic programming heuristic algorithm was developed and the results showed the solution ability.

2.2.3 Economic production quantity problem

In the academic world, the EPQ problem has attracted a large number of researchers over the past 80 years. It was first proposed by Taft (1918) to determine the quantity of products to be manufactured each time, in order to balance the holding/backlogging cost and fixed set-up cost. It provided a simple but useful way to calculate the right amount of production and help with the decision making process in the production process when a single item is considered (Dobson, 1987). However, a number of assumptions make the model unrealistic. For instance, perfect production process, product quality and constant demand are the common assumptions made in traditional EPQ models. To make the EPQ model more practical, some researchers have combined the EPQ with various uncertainties:

Researchers have pointed out that in real industry production processes quality is affected by many factors, such as production speed, setup cost and production quantity. Hence, the assumption that the production is perfect in the classical EPQ model is no longer valid. Rosenblatt and Lee (1986) first studied the Economic Production Quantity model with imperfect production processes. In their model, a production process shifts from a good condition to a bad condition in a random time. The results showed that the production run time was shorter than the one in the classical model. They also extended the model by taking the setup up cost dependent deterioration rate into consideration.

Khouja and Mehrez (1994) proposed a modified EPQ model where the production rate was taken as a control variable and the deterioration rate was dependent on the production rate. Hayek and Salameh (2001) further considered the rework of defective products on the same machine, which was not included in previous papers. In the model, the rework starts right after normal production and backordering is also allowed. Similarly, the work done by Salameh and Jaber (2000) showed that the value of EPQ increases along with the rise of the imperfect quality rate. Chiu (2003) extended the work done in (2001) and included an integrated random defective rate into the problem. Also, in Chiu's model, the repair process was assumed to be imperfect and any scrapped products are disposed of. A fuzzified EPQ model was also proposed by Shekarian et al. (2014a) in which the defective rate and demand rate were represented by fuzzy numbers and later extended with consideration of backorders and rework(Shekarian et al., 2014b). Taleizadeh et al. (2013) investigated the impact of repair failure on EPQ. In their model, a percentage of defective products were turned into scrapped products after repair and a disposal cost for the scrap product was taken into consideration in the overall cost. Recently another similar model was also proposed in (Cárdenas-Barrón, 2009; Jamal et al., 1997).

The effect of machine breakdown in EPQ has also been examined. Groenvelt et al. (1992) developed two control policies to deal with machine breakdown together with corrective maintenance, while Chiu and Chang (2014) studied the EPQ problem together with stochastic breakdowns and an imperfect rework process. Taleizadeh et al. (2014) examined a multi-product and single machine EPQ model in which process interruption was allowed.

Lin and Kroll (2006) stated that the majority of previous studies either considered only machine breakdowns or process deterioration. So in their model, both linear and exponential production process deterioration were examined together with machine breakdowns. However, backordering was not allowed in their model. Preventive maintenance and repair warranties were taken into consideration in the work done by Pal et al. (2013). Similar to the work of Rosenblatt and Lee (1986), the production process switches from an in-control and an out-of-control state after a random point in their model. Wee et al. (2014) looked at a case where screening and production were not synchronized and a feasible optima policy was developed for different scenarios. Other research studies with an imperfect production system were given in (Sarkar et al., 2014).

For product deterioration, Goyal and Giri (2001) have summarized the causes for deterioration, such as damage, dryness, vaporization etc. The earliest study on deteriorating products can be traced back to the 1960s, where Hadley and Whitin (1963) first developed an inventory model with the product having an obsolescence date. Later on, several production lot sizing inventory models considering deteriorating products were proposed. Mak (1982) looked at an exponentially decaying case and backlog was also allowed. Partial backlog was introduced into the model with product deterioration by Wee (1993). Pricing policy was also combined with the traditional EPQ model by several groups of researchers. Among them, the work done by Goyal and Gunasekaran (1995) and Wee and Law (1999) are representative of this type of model. Teunter and Flapper (2003) integrated both rework and product deterioration, and examined a single-stage single product production system. In their model, the time and cost for rework had a linear relationship with time and the switch between production and rework needed a fixed time and cost. They aimed to maximize the expected profit per time by determining the optimal production quantity. Inderfurth et al. (2005) generalized the model proposed in the previous paper and reduced the restriction in the production and rework capacity. Hence all the demand can be satisfied and backlog was not allowed in their model. Compared with the other models mentioned above, it is assumed that there is a constant defective rate rather than a stochastic one.

Golhar and Sarker (1992) examined the EPQ model in a Just-In-Time (JIT) delivery system. Under the principle of JIT, the delivery system requires small shipment sizes which helps reduce the inventory cost and set-up cost. Their research proved that the total cost had a negative linear relationship with shipment size. Later, Banerjee and Kim(1995) also studied EPQ in a JIT environment, compared with the work of Golhar and Sarker (1992), they introduced the multiple delivery lot into the problem.In 1993, Wee (1993) first proposed an EPQ model with partial backlog and product deterioration, and later Wee and Law (1999) integrated the problem with the consideration of pricing policy, in a finite planning horizon. Goyal and Gunasekaran (1995) also combined the pricing problem with the EPQ problem on a deteriorating product. In addition, advertisement frequency was also considered in their model.

Backlog was also discussed in the following research work, such as (Cárdenas-Barrón, 2009; Cárdenas-Barrón and Cardenas-Barron, 2001; Chiu, 2003; Eroglu and Ozdemir, 2007; Pentico and Drake, 2009) which examined the EPQ problem with full/planned/partial backlog. Other similar works include Abad (1996) and Sarkar (2014).

All the previous mentioned research studies assumed a continuous issuing policy to meet the demand of customers. Disontinuous issuing or delivery policy was taken into consideration by Chiu et al (2011). Discontinuous issuing policy, as stated in their work, is more practical and common in industry. In their paper, the delivery of products was divided into n installments with a fixed time interval and quantity. Numerical methods were utilized to obtain the optimal solution. Later,they further looked at the discontinuous issuing EPQ problem with partial rework (Chiu et al., 2012). However in these two papers, the number of shipments was assumed to be known. Cardenas-Barron et al (2013) jointly considered the multi delivery EPQ problem with both lot size and number of delivery to minimize the inventory cost.

The learning effect was integrated into the EPQ model by Jaber et al. (2008). In their study, a fixed percentage of products manufactured was assumed to be defects. Due to the learning effect, the percentage of defective products decreases along with the increase of total production cycles. The learning curve was generated and validated by industrial data. Konstantaras et al. (2010) took the quality of remanufactured product into consideration in the EPQ problem. They studied the situation where a certain percentage of remanufactured products were sold as refurbished items at reduced prices due to the secondary quality. Inspection and sorting were introduced and the corresponding setup time and cost were considered in the inventory model.For review papers specifically on EPQ, the following works by Sarmah et.al (2006) and Yano and Lee (1995) are relevant.

2.2.4 Economic lot scheduling problem

As mentioned above, the EPQ model is specific for the manufacturing model with a single item. It is also called a single product lot scheduling method. However, if there are more than one type of product in a manufacturing system, the EPQ model cannot sort out a cyclical scheduling and production quantities for all the product types at the same time (Glock, 2012; Zhang et al., 2013). The Economic Lot Scheduling Problem (ELSP), however, can successfully lead to a solution.

Jack Rogers (1958) was the first researcher to propose a computational approach to combine the Economic Manufacturing Quantity (EPQ) with production scheduling, aimed at minimizing the sum of the setup cost and inventory cost. Bomberger's (1966) work established the traditional ELSP approach with known setup cost, setup time, holding cost, production and constant demand rate. It was assumed that one production facility could only proceed with one product at a time. Hsu (1983) later proved that the ELSP problem is NP-hard after testing the feasibility of different production schedules. Other research works also point out the infeasibility of using the analytical approach to solve the ELSP (Gallego and Shaw, 1997; Yao, 2001).

In general, there are three policies commonly used in ELSP: the basic period approach, the time varying approach and the common cycle approach. The traditional ELSP model in Bambergers work (1966) first adopted the basic period approach in which cycle times vary with different products. In addition, the cycle time for a certain product must be multiple times of the basic period. Moon et al.

(2002) also employed the basic period policy together with imperfect production processes and consideration of the set-up times. In the common cycle approach, the cycle time for each every product is defined with same value. It enables the use of analytical approaches to solve the ELSP (Torabi et al., 2005). However, a performance of this approach was not as good as the other two due to its strict requirement on cycle time. Lastly, a time varying lot size allows for flexible lot sizes or cycle times for all the products. The cycle time is decided by parameters such as inventory level or demand.

A dynamic version was proposed by Dobson (1987) and an optimum solution was generated. Because of the complexity, heuristic and meta-heuristic methods such as Genetic Algorithms (GA) (Jenabi et al., 2007) were used to find the near-optimal solutions. Other researchers have also developed heuristic algorithms to solve the time-varying problem. Please refer to (Leachman and Gascon, 1988; Raza and Akgunduz, 2008; Zipkin, 1991).

Recently more and more researchers shifted their focus to stochastic economic lot scheduling problem. The introduction of uncertainties in demand, setup time and production rate makes the problem more complex and realistic compared with the deterministic ELSP problem. Winands et al. (2011) summarized the main differences between the two problems, in a review paper. First of all, a flexible production scheduling plan is preferred than to a rigid one. Secondly, inventory control plays a more important role in the problem. Especially in the case with uncertain demand or random machine failures, inventory should hedge against the potential changes.

Similar to the deterministic ELSP problem, there are three types of production policies related to the sequence of production and the cycle time. Dynamic production sequence (Karmarkar and Yoo, 1994; Sox and Muckstadt, 1997), fixed production sequence/dynamic cycle length (Bourland and Yano, 1994; Bradley and Conway, 2003) and fixed production sequence/fixed cycle length (Bruin, 2007). With respect to the lot sizing policy, two types of lot sizing policy, global and local lot sizing policy, were defined in this problem. In local lot sizing policy, the decision about lot sizing is made independently according to a given inventory control policy, such as (s,S), and the sequence of production is dependent on priority rules. The two decision processes are separated and conducted individually (Fransoo et al., 1995; Grasman et al., 2008). On the other hand, global lot sizing policy takes the state of machine and inventory level of all the products into consideration (Jin and Loulou, 1995).

The ELSR problem with an imperfect production system addresses a realistic problem in actual manufacturing systems where the production quality is not perfect. Hence, the rework of imperfect or defective products should also be taken into consideration. Roy et al. (2009) used fuzzy numbers instead of crisp values to represent the defective rate. A genetic algorithm (GA) was used to obtain the optimal value of the total number of cycles, production time and, lastly, the total profit. Taleizadeh et al. (2013) in addition, considered factors such as service level and budget limit in ELSR with an imperfect production process. Pasandideh et al. (2013) employed two separate meta-heuristic algorithms to solve the problem. Both the inventory cost and also the warehouse space utilization were minimized.

Tang and Teunter (2006) were the first group of researchers that examined ELSP with remanufacturing. In their case, both the production quantity and the sequence of different products must be determined at the same time with consideration of remanufacturing in a single production line. A common cycle time policy was adopted and mixed integer linear programming was used to formulate the solution. It was also assumed that each manufacturing lot was followed by a remanufacturing lot. An exact solution was obtained as a result. Further, the work considering product substitution together with remanufacturing and lot sizing was conducted by Pineyro and Viera (2010) and Ahiska and Kurtul (2014) respectively.

2.3 Forecasting with Inventory Control

2.3.1 Stationary and non-stationary demand

The past 20 years have witnessed revolutionary changes in different perspectives such as customers' purchasing behaviour, manufacturing technology, global markets and supply chain management (Autry et al., 2012). Especially, the fast evolving technology and the ever competitive market significantly reduce the life cycle of a product. All of these facts make companies begin to face non-stationary demand processes. According to Neale et al. (2009), the frequent introduction of new products shortens the life cycle of a product. The stages of launch, stabilization and drop in the product life cycle make the demand change with time. An example from Hewlett Packard also supports the idea that demand cannot be stationary in the current market. Ozkaya (2008) also mentions that non-stationary demand is suitable, specifically for a high-tech industry, in which short life cycles are often observed.

Study on non-stationary demand models is getting more important. However, compared with the stationary demand model, works with non-stationary demand models are relatively fewer. This phenomenon can be attributed to the complexity in applying non-stationary approaches in both academic and practical situations. A similar idea was stated by Silver (2008) that in inventory management non-stationary demand is not easy for routine use. As stated in the introduction chapter, the majority of the works done in the area of failure-prone production and inventory control assume a stationary demand process.

Nonetheless, there are some works that have already studied the impacts of non-stationary demand and the corresponding control policy in areas such as inventory control and production control. Tunc et al. (2011) studied the cost of using stationary inventory policies on the condition of non-stationary demand processes, and implied that stationary policies are preferred due to their simplicity in real cases. This research raises an important question as to what is the general cost of adopting a stationary policy under the condition of non-stationary demand. The results show that using a stationary inventory policy under a non-stationary demand process is expensive, especially when the demand variation is high. However, in Tuncs work (2011), the non-stationary demand was assumed to be known for each single period. However in reality, demand information is forecasted based on historical data and inaccuracy exists. The impact of inaccuracy or forecast errors on the non-stationary demand is unknown. So, a further study simultaneously considering advanced demand information and non-stationary control is necessary.

2.3.2 Forecasting and non-stationary demand

In order to gain information on non-stationary demand, forecasting is usually utilized in supply chain management. Based on existing information, such as historical data and market trends, forecasting provides an estimation of the product quantity that will be consumed in a certain period of time (Chase, 2013). Generally speaking, forecasting can be divided into two categories: Qualitative and Quantiative forecasting.

Qualitative forecasting, for instance, the Delphi method and market research, are usually used for medium to long term forecasting. The results are mainly based on the judgment and experience of decision makers (Witt and Witt, 1992). Compared with quantitative methods, qualitative forecasting can take more non-numerical factors into consideration (Makridakis et al., 2008). Time series methods are commonly used for both stationary and non-stationary demand forecasting. They are mainly based on the available historical data. Among the time series forecasting methods, Simple Moving average (SMA), Single Exponential Smoothing (SES), Holt-Winter Seasonal (HWS) method and Autoregressive Integrated Moving Average (ARIMA) are popular in both practical applications and academic research (Bermúdez, 2013; Box et al., 2013). It is suggested in the research of Acar and Gardner (2012) that the choice of forecasting methods significantly affects supply chain performance. The improvement of forecasting accuracy can generate more benefits than the appropriate inventory control policy. SMA, Holts additive trend and damped additive trend methods are compared with respect to the total supply chain cost.

Strijbosch and Moors (2005) compared the performances of Single Exponential Smoothing, Simple Moving Average and ARIMA against a (R,S) inventory policy. Fill rate was selected instead of inventory cost to evaluate the performance in the case of non-stationary demand. These three methods were also adopted in the research of Warren and Chang (2010). Similar to the work by Strijbosch and Moors (2005), the impact of the forecast on inventory policy performance was examined and compared. However, the previous work focused more on finding the optimal forecasting parameters while the later optimized the inventory policy. Please refer to the following paper for more examples and details about the application of forecasting methods (Syntetos et al., 2009).

However forecasting cannot produce 100% accurate results. Even for a given demand, different forecasting methods will produce various accuracies. This issue was not considered in the work of Tunc et al.(2011) on which the demand was assumed to be known to the system. The cost related to inaccuracy was neglected. The accuracy of forecasting can be measured by parameters such as the Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Mean Squared Error (MSE) (Hosoda and Disney, 2006; Robert Fildes, 1992)

In our research, specifically WMA, ES, HWS and ARIMA are chosen to forecast the demand. The optimal parameters for each method are firstly determined, which minimizes the prediction errors based on a given demand set. The forecast errors are calculated based on the value of MAPE in order to evaluate the accuracy of the forecasting methods.

The relationship between forecasting accuracy and inventory control has been carried out on the condition of non-stationary demand by a number of researchers.

A paper from Strijbosch et al. (2011) looked at the interaction between forecasting and stock control in a periodic order-up-to level inventory system. Non-stationary demand was used and the fill rate of a manufacturing system under different forecasting methods was evaluated as a performance indicator. Ali et al. (2012) discussed the impact of forecasting errors on inventory parameters. Similarly, Babai et al. (2013) proposed to investigate the relationship between forecasting accuracy and inventory performance, in terms of inventory holding and cost. In addition, an order-up-to-level policy and information sharing mechanism were applied in their supply chain model which extended the research of Ali et al. (2012). In their models, inventory policies such as (s, S) and (R, S) were examined against non-stationary demand. Other similar research can also be found in (Syntetos et al., 2010; Warren Liao and Chang, 2010). Our work, on the other hand, looks at the impact of non-stationary demand on the production and inventory control problem.

2.4 Discussion

The literature reviewed above revealed the following existing research gaps:

1. According to literature review, although the hedging point based control problem has been heavily studied, such as in the work of Gharbi and Kenne (2000) the integration between lot sizing and hedging point policy was researched. A similar model was proposed by Bouslah et.al (2012) where stationary demand was assumed. The impracticality of the stationary demand process have been discussed previously, the models with stationary demand cannot be applied to industry directly. Hence in this thesis, to fill in the gaps between academic and industries, non-stationary demand is introduced and

considered into the traditional hedging point policy. And In order to tackle the non-stationary demand, forecasting methods were used.

2. The impact of forecasting methods and its respective accuracies have been investigated by many researchers. However in the problem of hedging point based control, this issue hasn't been investigated.

3. The extant literature shows that the majority of papers have only considered either rework and an imperfect system or product deterioration. A few integrated studies can be found. In the first paper by Teunter and Flapper (2003), rework and production was assumed to use the same machine and the defective rate followed a given probability distribution. The production quantity needed to be determined to maximize the profit obtained. In another paper by Inderfurth et al. (Inderfurth et al., 2005), the problem is generalized by assuming the demand for good items was limited and the demand would be entirely satisfied. A closed form results were obtained by utilizing a constant and deterministic defective and deteriorating rate. Tai (2013) added inspection errors into the model with deteriorating items and imperfect production. In addition, Tai also investigated the effect of selling imperfect products to customers. However, in all of the three studies mentioned above, the defective rate was regarded as stationary and would not change with time. In addition, backlog was not considered in the first two models.

4. Stochastic deterioration has not been considered in the integration problem of imperfect systems or the product deterioration.Rosenblatt and Lee (1986) proposed a similar model in which the machine changed from an in-control

condition to an out-of-control condition, after a stochastic period of time. However their model only considered a deteriorating production process.

In order to fill the research gaps, three different models were formulated and experiments were conducted in my project. Model 1 was designed to solve the first two research gaps while models 2 and 3 were employed to handle the third and fourth research gaps respectively.

The research objectives can then be summarized as follows:

- To develop a two-level control policy for the production inventory control problem with a Markov modulated Poisson demand process using integrated simulation and experimental design method

- To develop a forecast-corrected control policy for the production inventory control problem using the time series forecasting method and the integrated simulation and experimental design method.

- To develop an EPQ model with inventory a deterioration and deterministic deteriorating production process in which the defective rate increases after each certain period of time.

- To develop an EPQ model when the production process is subjected to random deterioration along with the increase of defective rate.

Detailed methodologies used for each model and problem are introduced in the following chapters.

Chapter 3 Development of Forecast Corrected Two-level Hedging Point Policies

3.1 Notations and Assumptions

This section lists all the notations and assumptions used throughout this research.

Notations

i	The index of batch
j	The index of order
$x_i(t)$	WIP level in batch i
$y(t)$	Inventory level
$u(t)$	Production rate
u_{max}	Maximum production rate
$S(t)$	Status of machine at time t
L_i	Lot size of i_{th} batch
$\gamma_{\alpha\beta}$	Transition time from α to β
$\gamma_{\beta\alpha}$	Transition time from β to α
$TB(t)$	Total batch number up to time t
d_j	The demand size of j_{th} order
$\lambda(t)$	Arrival rate of customer orders at time t
d_j	The demand size of j_{th} order

$df(t)$	Forecast demand at time t
μ	Mean of demand size
σ	Standard deviation of demand size
$N(t)$	The number of arrived orders up to t
$D(t)$	Cumulative demand from time 0 to t
θ_i	Finishing time of $i^(th)$ batch
ζ_i	Starting time of $i^(th)$ batch
H	Hedging point
C_h	Holding cost ($/unit/time)
C_b	Backlogging cost ($/unit/time)
C_s	Fixed setup cost ($/cycle)
AC_c	Average cost from corrected policy
AC_o	Average cost from original policy
PCR	Percentage cost reduction
α	Status that machine in good condition
β	Status that machine is in failure
θ	Time interval of in-control state
δ	Deteriorating rate of product
μ	Demand rate (unit/time)
p	Mean time for high demand
q	Mean time for low demand
λ_l	Order arrival rate of low demand period
λ_h	Order arrival rate of high demand period
H_l	Low hedging point
H_h	High hedging point
L_l	Low batch size

L_h	High batch size
L_m	Medium batch size

Assumptions

1. During the production of one batch, the lot level is simply the integral of the production rate with respect to time. The status of the machine is modeled by a two-state Markov Process. The average transition time from the functional state to failure state α and the average transition time from failure to the functional state γ are described by two individual Poisson processes, with different mean values.
2. Unmet demand will be backlogged and the backlog will be satisfied first.
3. The production speed within one batch is assumed to be constant.
4. The replenishment of raw material is assumed to be instantaneous and infinite. The production speed in one batch is viewed as constant.

3.2 Problem Formulation

In this section, the inventory model is first introduced with respect to the mathematical formulation. Then second part presents the two modified hedging point policies and the mechanism of the control policies is elaborated in detail. Lastly, the proposed simulation, design of experiments and response surface method are explained.

3.2.1 Inventory model

A single machine, single product, failure-prone manufacturing system subject to non-stationary demand is modeled in this chapter. In batch i, products are manufactured from time ζ_i with the production rate $u(t)$ through which the Work-In-Process (WIP) batch level $y(t)$ increases accordingly. WIP products are stored beside the machine until the number of products for one batch is satisfied. The WIP inventory behavior is described in Equ.(3.1). At time θ_i, the finished batch is sent to the inventory warehouse. It results in a sudden increase in inventory level $x(t)$ by the specific batch size L_i which is shown in Equ.(3.2). Customers Orders arrive at the manufacturing system and are served based the principle of First-Come-First-Served (FCFS). Corresponding numbers of products are delivered to the customers immediately when the order arrives. In Equ.(3.3), the behavior of the inventory level between two batches is illustrated, where d_j represents the demand of order j and the total amount of demand aggregated from the batch starting time ζ_i to time t equals to $\sum_1^{N(t-\zeta_i)} d_j$.

$$y(t) = \int_0^t u(\tau)d\tau \quad t \in (\zeta_i, \theta_i) \quad where \quad y(0) = 0 \quad and \quad u(\tau) \in (0, u_{max}) \tag{3.1}$$

$$x(\zeta_{i+1}) = x(\theta_i) + L_i \quad where \quad x(0) = x_0 \tag{3.2}$$

$$x(t) = x(\zeta_i) - \sum_1^{N(t-\zeta_i)} d_j \quad t \in (\zeta_i, \theta_i) \tag{3.3}$$

Inventory surplus or backlog in each period generates inventory holding cost and backlog cost at the rate of C_h and C_b. For each batch produced, a fixed setup cost C_s is taken into consideration. The aggregated setup cost is represented as $C_s * TB(t)$ where $TB(t)$ is the number of batches produced up to time t. Following the method used in Sethi and Zhangs work (1999), the corresponding dynamic programming cost function of the model is defined:

$$J(x,u,\alpha) = \lim_{T\to\infty} \frac{1}{T} E \int_0^T \big(C_h * x^+(t) + C_b * x^-(t) + C_s * TB(t)\big) dt \tag{3.4}$$

where $x^+(t) = max(0, x(t))$ *and* $x^-(t) = max(0, -x(t))$.

The overall objective of this model is to minimize the expected long run average cost $J(x,u,\alpha)$ by finding the optimal control policy. However solving the cost function is complex analytically, even for a simple model with constant demand, not to mention the batch production (Bouslah et al., 2012; Sajadi et al., 2011). In this model, batch production and non-stationary demand are considered which significantly increases the complexity of the analytical method. Hence, we do not adopt the analytical approach, but instead, a modified Hedging point policy is adopted. The structure of the hedging point policy proven by (Akella and Kumar, 1986; Gharbi and Kenne, 2000) is used. This practice is also utilized in (Sajadi et al., 2011). Under the condition of non-stationary demand and batch production in this chapter, it can provide an approximation of the optimal solution when the analytical analysis is hard to be implemented.

3.2.2 Case 1: HPP with qualitative forecasting

In order to model the non-stationary demand, we firstly use a compound Poisson process demand to model the intermittent arrival of orders which is the result of expert judgment. The size of demand d_i follows a normal distribution and the arrival rate is represented as $\lambda(t)$. Then, a two-state Markov chain is combined with the compound Poisson process to model the mean time in the high demand period and low demand periods with their respective values of p and q. The stochastic demand is formulated as a compound Poisson demand in which the arrivals of orders follow a Poisson distribution and the demand size in each order follows a normal distribution. To be specific, the cumulated demand $D(t)$ can be expressed as:

$$D(t) = \sum_{1}^{N(t)} d_j \tag{3.5}$$

$N(t)$ is the number of order arrived up to time t and follows a Poisson process with an arrival rate of λ. At the same time d_j follows a normal distribution with mean μ and standard deviation σ. This modelling method has been widely used in the academic literature because of its simplicity and practicality (Shang, 2012). According to the theory of the compound Poisson process, the expected value for $D(t)$ is calculated as follows:

$$E[D(t)] = E[N(t)]E[d] = t\lambda(t)E[d_j] = t\mu\lambda(t) \tag{3.6}$$

To cope with the proposed non-stationary demand, a modified HPP is developed accordingly. In terms of production speed, as we can see, when the inventory level $x(t)$ is larger than hedging point H, the production rate will be reduced to zero. When $x(t)$ is smaller than H but larger than the difference between H and L_i, the production rate will be set to the value of the forecast demand $df(t+1)$ in the next period, which equals $\lambda t\mu$. Lastly, if the inventory level is smaller than $H - L_i$, then the maximum production rate will be applied to the production system. For hedging point H and lot size L_i, when $\lambda(t) = \lambda_l$, the set of (H_l, L_h) is applied where relatively larger hedging point and smaller lot size is implemented. A combination of (H_h, L_l) is used instead on the condition of low demand. It is inspired by the theory of Rossi and Lodding (2012) in which they state that a small lot size helps minimize the amplification of the demand fluctuation. As explained above, the following equation illustrates the control policy developed.

$$
P(t) = \begin{cases} u(t) = \begin{cases} 0 & when \quad x(t) \geq H \\ df(t+1) & when \quad H - L_i \leq x(t) \leq H \\ u_{max} & when \quad x(t) < H - L_i \end{cases} \\ (H, L_i) = \begin{cases} (H_l, L_h) & if \quad \lambda(t) = \lambda_l \\ (H_h, L_l) & if \quad \lambda(t) = \lambda_h \end{cases} \end{cases} \tag{3.7}
$$

3.2.3 Case 2: HPP with quantitative forecasting

In this case, the time series forecasting method is used to forecast the non-stationary demand. In order to take the forecast demand into consideration, a forecast-corrected hedging policy is proposed. Similar to the previous control policy, the forecast-corrected control policy also consists of three parts, but the demand forecast is added into the control policy. The hedging point is corrected according to the amount of demand in the next time period $t+1$.

This work classifies the demand data into three levels: high, medium and low. The classification is decided according to the historical data. It means that if one specific forecast demand is higher than one third of the recorded demand data in the past, then this demand data is viewed as high demand and a small lot size L_s is used for production. Similarly, a forecast demand that is lower than one third but higher than two thirds of the historical data is classified as medium demand. Correspondingly, a medium lot size L_m will be applied. In the chapter, all the optimal values of the hedging point and high/medium/low lot sizes are determined simultaneously. The forecast-corrected control policy can be constructed as shown below.

$$u(t) = \begin{cases} 0 & when \quad x(t) \geq H \quad + \quad df(t+1) \\ d & when \quad H \quad + \quad df(t+1) \quad - \quad L_i \leq \quad x(t) < H \quad + \quad df(t+1) \\ u_{max} & when \quad x(t) \leq H \quad + \quad df(t+1) \quad -L_i \end{cases} \tag{3.8}$$

3.2.4 Proposed methodology

The whole process mainly consists of 4 stages: initialization stage, design of experiment, the simulation and response surface methodology stage, as shown in Figure 1. In initialization stage, the proposed hedging point policy and forecasted demand are generated. Especially, in Case 2, they are firstly forecasted using the proposed time series methods. The data are then stored separately and later are employed as the input to the simulation process. For instance, for 3-period WME and SES, the following equations are applied to generate the forecast:

$$n_(t+1) = \alpha m_t + \beta m_(t-1) + \gamma m_(t-2) \tag{3.9}$$

$$n_(t+1) = \alpha m_t + (1-\alpha) n_t \tag{3.10}$$

Where n_t is the forecast for period t and m_t is the actual demand for period t, the optimal values of α,β and γ are decided by minimizing the MAPE for the whole forecasting period and $\alpha + \beta + \gamma = 1$. In terms of ARIMA and HWS, the forecasting is implemented by using statistical software *R*. The design of experiments stage provides a formal planned experimentation design to change the value of different parameters in simulation model. To be more specific, in this research, the Box-

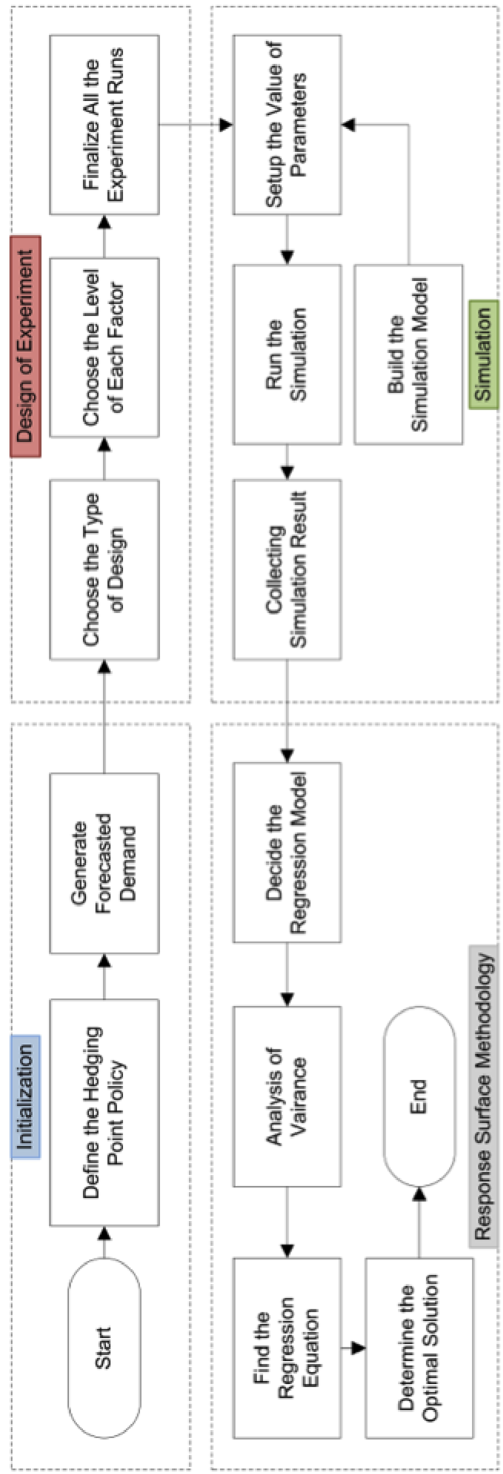

Figure 3.1: The flow of simulation, experimental design and response surface methodology

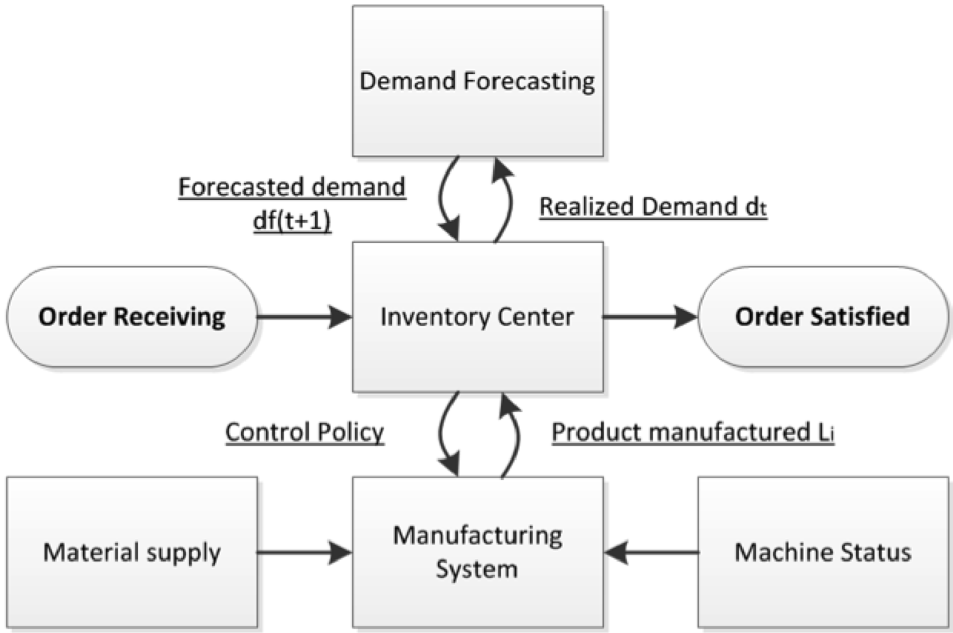

Figure 3.2: The flow chart of simulation model

Wilson Central Composite Designs (CCD) method is employed which is commonly used in DOE (Myers et al., 2009). The simulation model is based on the proposed manufacturing system. The discrete event simulation software ARENA is used to execute the simulation process.

In the simulation stage, first of all, demand for the next period is forecast based on the information of the historical demand. Orders are received and the inventory is reduced accordingly, to satisfy the demand. Control policy is adjusted based on the information of demand forecasting and current inventory level. Then, the manufacturing system produces products according to the defined control policy. At the end of the simulation, the value of the average inventory cost in the whole simulation time is calculated and recorded. The simulation process is illustrated in Figure 3.2. Lastly, response surface methodology helps determine the relationship between control factors and response. After finishing all the simulation experiments, the values of the simulated average inventory cost under different control policies are recorded. Response surface methodology uses the values of the inventory cost as the response and the optimal control policy can be obtained as a result. In this research, a second-degree regression model is used to find the relationship between the cost and the control variables, as shown in Equ.(3.11). x_i represents the control variable, while β_i is the coefficient to be estimated from simulation results. n is the number of control variables in the design and in this paper it equals to 4 in both cases.

$$C = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum\sum \beta_{ij} x_i x_j + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \varepsilon \tag{3.11}$$

3.3 Numerical Experiments

This section mainly looks at the numerical study process. First of all, Case 1 is examined using the Markov modulated compound Poisson process. Then, the second case with historical demand data and forecast-correct hedging point policy is investigated. The impact of the forecasting methods and accuracy is also provided.

3.3.1 Case 1: two-level control policy

In this scenario, the modified two-level hedging point policy is adopted to control the production-inventory system. There are 4 control variables in total which are H_l, H_h, L_l, L_h. But in order to avoid the situation that H_h and L_h are smaller than H_l and L_l, two extra variables R_H and R_L are introduced and defined as follows:

$$\begin{cases} H_l = & H_h * R_H \\ L_l = & L_h * R_L \end{cases} \quad where \quad 0 \leq R_H, R_L \leq 1 \tag{3.12}$$

Table 3.1. The experimental range of control variable

Levels/Factors	H_h	R_H	L_h	R_L
2	4000	0.98	3500	0.98
1	3000	0.74	2750	0.74
0	2000	0.5	2000	0.5
-1	1000	0.26	1250	0.26
-2	0	0.02	50	0.02

Table 3.2. The value of Key parameters

C_h	0.1 $ /unit/period	$r_(\alpha\beta)$	10 periods	λ_h	5
C_h	2 $ /unit/period	$r_(\beta\alpha)$	0.8 period	λ_h	2
C_h	200 $ /batch	p	50 periods	μ	50 units
$u_(max)$	300 unit/period	q	30 periods	σ	5 units

Each variable is divided into 5 levels. According to the theory of CCD, 31 experiments are generated as a result and the value of α is set as 2. Each experiment is replicated for 4 times. So in total, 124 experiments are implemented in this case. The range of each control variables is presented in Table 3.1. For the purpose of our investigation, the key parameters in the simulation stage are defined in Table 3.2: After the simulation, ANOVA is carried out to find the statistical significance of each individual factor and their interactions. As illustrated in Table 3.3, the correlated coefficient is obtained as 95.70% and the adjusted correlated coefficient is around 95% which represents the percent of variance. After taking out the non-significant factors, the corresponding regression function can then be obtained and is provided in Equ.3.13.

Table 3.3. The result of ANOVA

Source	DF	Adj SS	Adj Ms	F-Value	P-Value
H_h	1	32908940	32908940	724.35	0.000
R_H	1	4083193	4083193	89.87	0.000
L_h	1	24964497	24964497	549.48	0.000
R_L	1	39866942	39866942	864.29	0.000
$H_h * H_h$	1	5008950	5008950	110.25	0.000
$R_H * R_H$	1	186280	186280	4.1	0.045
$L_h * L_h$	1	939179	939179	20.67	0.000
$R_L * R_L$	1	6201947	6201947	136.51	0.000
$H_h * R_H$	1	506757	506757	11.15	0.001
$H_h * L_h$	1	5639017	5639017	124.07	0.000
$H_h * R_L$	1	6581056	6581056	144.85	0.000
$L_h * R_H$	1	20027	20027	0.44	0.508
$R_H * R_L$	1	32030	32030	1.37	0.245
$L_h * R_L$	1	11272399	11272399	248.11	0.000
Error	15	6133409	81779	*	*
Total	29	142649653	*	*	*
Summary		R-Sq	R-Sq(adj)	R-sq(pred)	
		95.70%	95.25%	94.52%	

$$
\begin{aligned}
Cost &= 1436 + -.317 * H_h - 798 * R_H - 0.141 * L_h - 4375 * R_L + 0.000196 * H_h^2 \\
&\quad +5447 * R_H^2 - 0.000154 * H_h * L_h - 1.434 * H_h * R_L \\
&\quad +0.502 * L_h * R_L + 854 * R_H^2 + 0.000051 * L_h^2
\end{aligned} \tag{3.13}
$$

The optimal combination of the four control variables can be determined as:

$$(H_h, R_H, L_h, R_L)^* = (1818, 0.464, 1259, 0.583)$$

Hence the values of H_l and L_l can also be obtained as 844 and 734 respectively. The minimum average inventory cost is 174.94. The contour plots between H, R_H and L_h, R_L are provided separately in Figures 3.3 and 3.4.

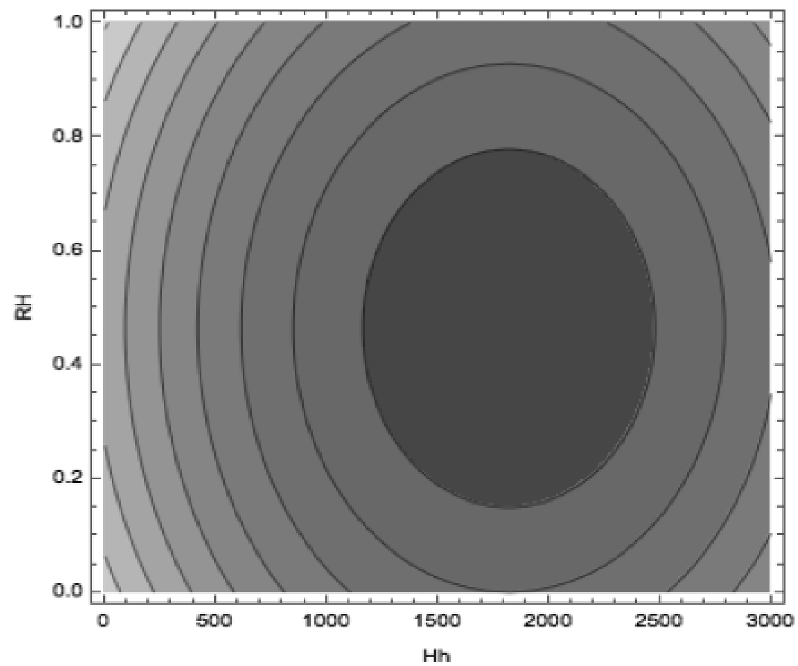

Figure 3.3: Contour plot between H_h and R_H

3.3.2 Case 2: forecast-corrected control policy

Tunc (2011) utilized four categories of demand that can be summarized as stationary, erratic, sinusoidal and life-cycle demand. However, in order to simulate the demand situation and test the corresponding performance of the system, we select two non-stationary demand data sets which represent two types of demand pattern that can often be observed in real cases. The two demand sets are illustrated in Figure 3.5 and Figure 3.6. As can be seen, both demands show non-stationarity, but demand set 2 also has seasonality. So in terms of choosing forecasting methods, apart from Simple Moving Average (SMA), Single Exponential Smoothing (SES) and Autoregressive Integrated Moving Average (ARIMA), the Holt-Winters Seasonal (HWS) method is also chosen for forecasting demand set 2.

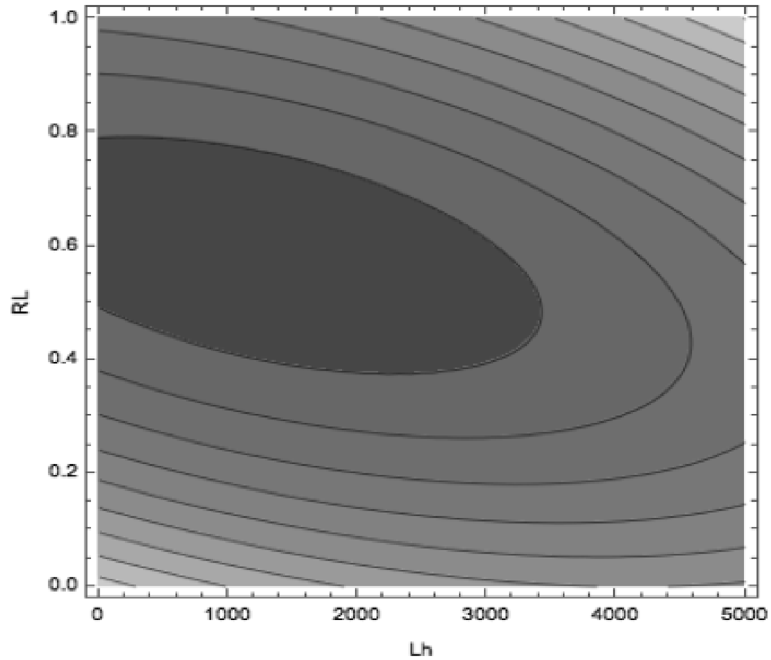

Figure 3.4: Contour plot between L_h and R_L

As mentioned in the previous section, there are also four control variables H,L_h,L_m and L_l which is hedging point, and three lot size values for different demand rates. The integrated simulation, design of experiments and response surface method is again employed to obtain the optimal control policy in both demand sets. The whole numerical experiments consist of 9 different scenarios. For demand set 1, there are four scenarios in total, which consist of scenarios with ARIMA, WMA, SES and one with stationary control policy.While for demand set 2, an extra scenario with HWS is added. The repeated procedure is not shown in detail.

The experimental design and the results of demand set 1 with ARIMA method is presented as an example. Similarly, in order to avoid the situation that $L_h < L_m$ and $L_m < L_l$, R_1 and R_2 are employed as the ratio of $\frac{L_m}{L_h}$ and $\frac{L_l}{L_m}$. The optimal control policy is obtained as $(372, 234, 0.496, 095)$ under the given set of parameters, while the optimal inventory cost is determined as 272.2. It means that the optimal hedging

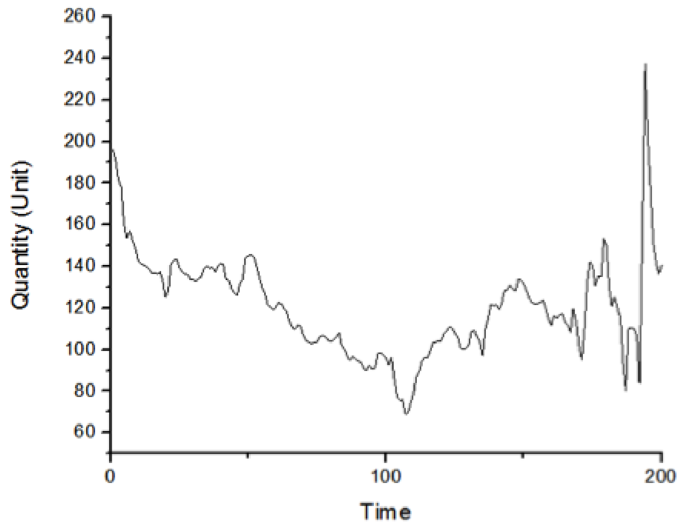

Figure 3.5: Demand Set 1

point is 372 while the optimal lot size for high demand, medium demand and low demand should be set to 234,117 and 111 respectively. The results of other scenarios are also obtained with the same methodology and are used in the following section to analyze the impact of forecasting accuracy on production-inventory system.

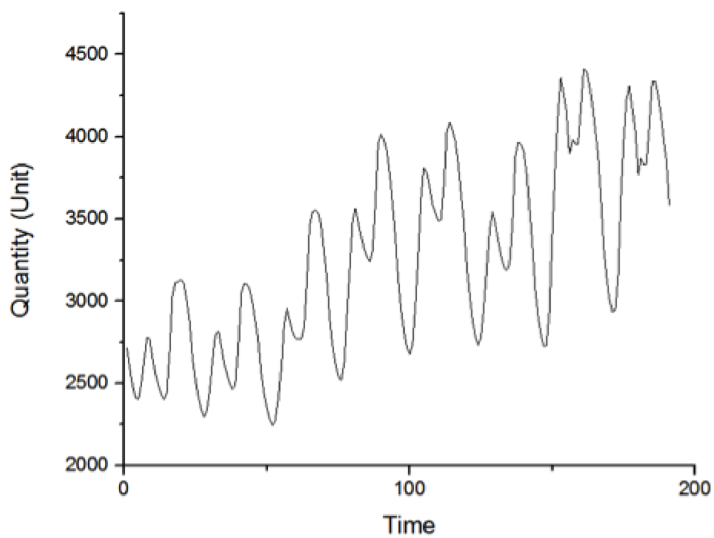

Figure 3.6: Demand Set 2

Table 3.4. The experimental range of control variables in the scenario with ARIMA and demand set 1

Levels/Factors	H	L_h	R_1	R_2
2	800	300	0.98	0.98
1	600	250	0.74	0.74
0	400	200	0.5	0.5
-1	200	150	0.26	0.26
-2	0	100	0.02	0.02

3.4 Impact of Forecasting Accuracy and Proposed Policy on Production-Inventory System

In this section, the impact of using forecasting and forecast-corrected hedging policy control policy is examined first. The relationship between the forecasting errors and cost reduction is then investigated by analyzing the results from all the scenarios in Case 2. A parameter called Percentage Cost Reduction (PCR) is used to measure the cost differences between different scenarios. AC_c and AC_o represent the Average Cost (AC) for forecast-corrected control policy and the original control policy respectively. This parameter helps compare the simulation results generated from forecast-corrected scenarios against the results from the scenarios with original control policy in Equ.(2.1).

$$PCR = \frac{AC_o - AC_c}{AC_c} \tag{3.14}$$

First of all, we start investigation with the accuracy of each forecasting methods. As illustrated in Figure 3.7, the value of MAPE varies and describes the accuracy for each forecasting method. In addition, different demand sets produce various MAPE

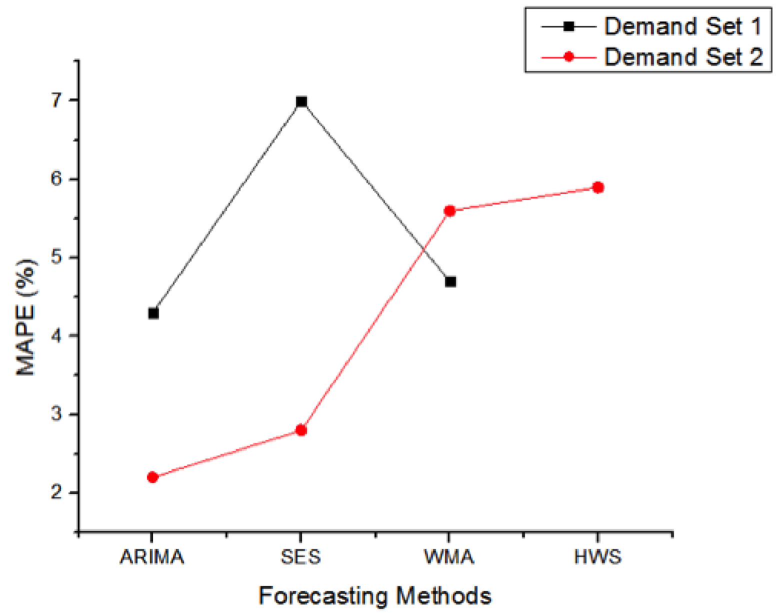

Figure 3.7: The relationship between forecasting methods and MAPE

values when the same forecasting method is applied. For instance, ARIMA yields a value of MAPE around 4 in Demand set 1 and a value about 2 in Demand set 2. A similar situation also occurs in the scenarios with ES and WMA, and shows that the pattern of the demand sets has a significant impact on the forecasting performance.

In terms of a specific demand set, demand set 2 for example, ARIMA can produce the most accurate prediction of the future demand compared with the other two methods. Especially the MAPE value of WMA and Holt-Winter are two times more than the value in ARIMA. In demand set 1, ARIMA is also the best method followed by WMA. Figure 3.8 presents the overall performance of the production and inventory control system which is measured with the average inventory cost. The vertical axis represents the percentage cost reduction when compared with the cost in the stationary scenario. A positive value means the average inventory cost is lower than the value in the stationary scenario. A negative value, on the other hand,

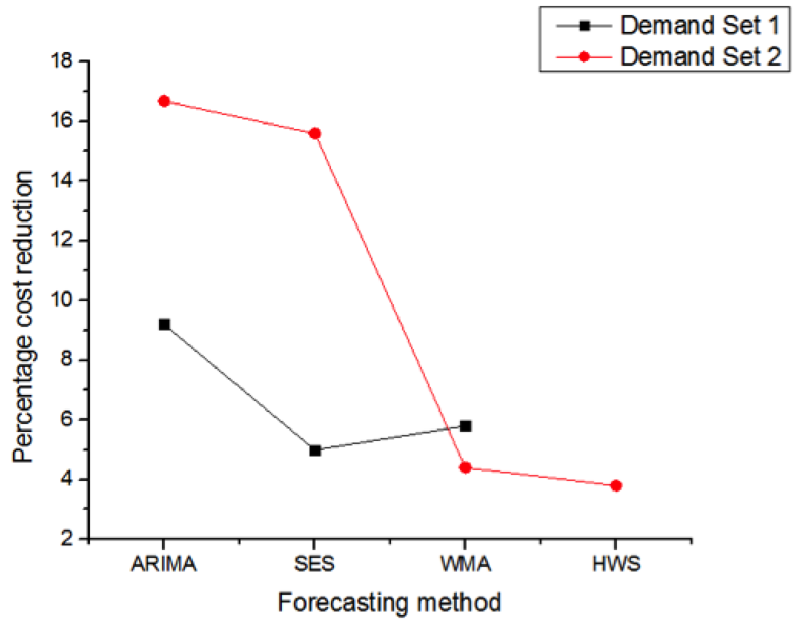

Figure 3.8: The relationship between forecasting methods and PCR

implies that the average inventory cost in this specific scenario is higher than that in the stationary scenario.

The results show that cost has been reduced by using the proposed control policy and forecasting techniques in Demand sets 1 and 2. Take Demand set 2 as an example; ARIMA and SES can help reduce the average inventory cost by around 17 and 16 percent respectively. For WMA and HWS, despite their relatively high forecasting errors, still can reduce the average inventory cost by around 4 percent.

In order to further explain the differences between the demand sets, the relationship between percentage cost reduction and forecasting errors is examined in Figure 3.9. Regardless of the forecasting technique, the relationship between the percentage cost reduction and MAPE is negative. To be more specific, the higher the MAPE is, the lower the percentage cost reduction will be. So if forecasting errors exceeds a certain

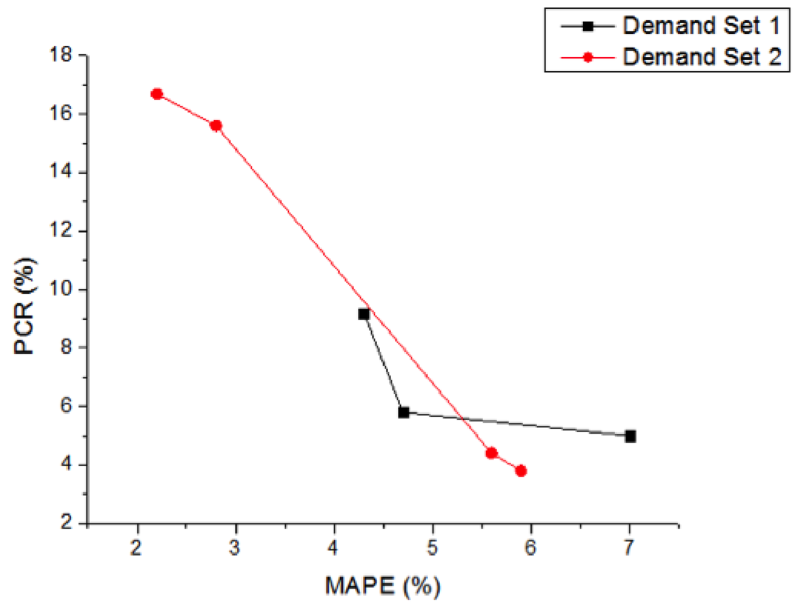

Figure 3.9: The relationship between MAPE and PCR

value, a non-stationary control policy cannot provide better performance compared with original control policy.

3.5 Summary

The traditional hedging-point-based production and inventory control problem normally assumes a stationary demand process. However, stationary demand is not realistic in the real world. Questions such as what is the influence of using stationary control policy when the demand is non-stationary in production and inventory control problem need to be answered. Little work has been reported in the literature in terms of the integration of forecasting and production-inventory control considering a failure-prone manufacturing system and non-stationary demand process and lot sizing.

In our study, we modelled both the quantitative and qualitative forecasting process of non-stationary demand. The Markov-modulated compound Poisson process is utilized first and a corresponding two-level control policy is proposed. Two sets of demand data are employed, together with time series forecasting, to simulate quantitative forecasting process. The forecast-corrected hedging point policy used is modified from the traditional hedging point policy. A large number of simulations and experiments have been conducted, and they prove that the proposed forecast-corrected method can result in a better performance than the traditional stationary control policy under the condition of non-stationary demand.

Chapter 4 An EPQ Model For Deteriorating Production System and Items with Rework

4.1 Notations and Assumptions

In this research, a production system with single-machine and single-product is modelled. In the system, the machine can conduct both production and rework processes. The product is subject to quality deterioration and the machine is assumed to be imperfect and deteriorating in terms of an increasing defective rate. The detailed assumptions made and the notations used in this paper are shown in the following sub-sections:

4.1.1 Notations

Decision Variables

B	Backlog quantity (unit)
Q	Economic production quantity (unit)

Parameters

α_i	The defective rate of time period i*θ
θ	Constant length between each change of defective rate (hour)
δ	Deteriorating ratio

μ	Demand rate (unit/time)
p	Production rate (unit/time)
p_r	Rework rate (unit/time)
M	Total number of invertal θ in normal production period
N	Total number of invertal θ in backlog period
C_{hp}	Holding cost of perfect products (\$/unit/time)
C_{hi}	Holding cost of imperfect products (\$/unit/time)
C_{dc}	Deteriorating cost (\$/unit)
C_p	Production cost (\$/unit)
C_{pr}	Rework cost (\$/unit)
C_b	Penalty cost for backlog (\$/unit/time)
C_s	Fixed setup cost (\$/cycle)
B	Backlog quantity (unit)
Q	Economic production quantity (unit)
I_s	The inventory level at the end of normal production period (unit)
I_m	The inventory level at the end of rework process (unit)
I_{im}	The inventory level of imperfect product (unit)

4.1.2 Assumptions

1. Unsatisfied order will be backlogged and the backlog will be fulfilled at the beginning of the cycle.
2. During the production period, the defective items are produced at a constant rate α_i in the time interval $[(i-1)\theta, i\theta), i \in \mathbb{N}$. We assume that the production system is deteriorating in the sense that the defective rate increases at time $i \times$

$\theta, i \in \mathbb{N}$. Hence we have $\alpha_1 < \alpha_2 < \alpha_3 < \cdots$.Through this paper, we consider a linear relation $\alpha_i = i \times \gamma$, where γ is a constant.

3. To reduce the complexity of the cost function, the normal production run time T_1 and the length of backlog period T_2 are assumed to be the integer multiple of θ since the value of θ is small.
4. Maintenance is carried out after the whole production period, so at the beginning of each cycle, the defective rate is minimized.
5. The imperfect products are reworked after the normal production process with an extra rework cost and the rework is assumed to be perfect.
6. The deterioration only occurs to perfect products with a constant rate δ.
7. The deteriorated products are disposed with cost.
8. Demand rate μ is known and constant.

4.2 Mathematical Modelling

According to the assumptions and description of the production system, a mathematical model for the system has been formulated. The behaviour of the inventory level in one production cycle is shown in Figure 4.1 and Figure 4.2 for perfect and imperfect products respectively.As illustrated in Figure 4.1, the inventory level starts from backlog B and the backlogged orders are satisfied first. During the first time interval θ, the defective rate is maintained as α_1. At the end of the time interval, the defective rate increases to α_2.

After the backlog is made up, the production will be continued until the desired economical production quantity is achieved.All the imperfect products are reworked in T_3 and the whole production process is completed. In T_4 and T_5, the stocks are

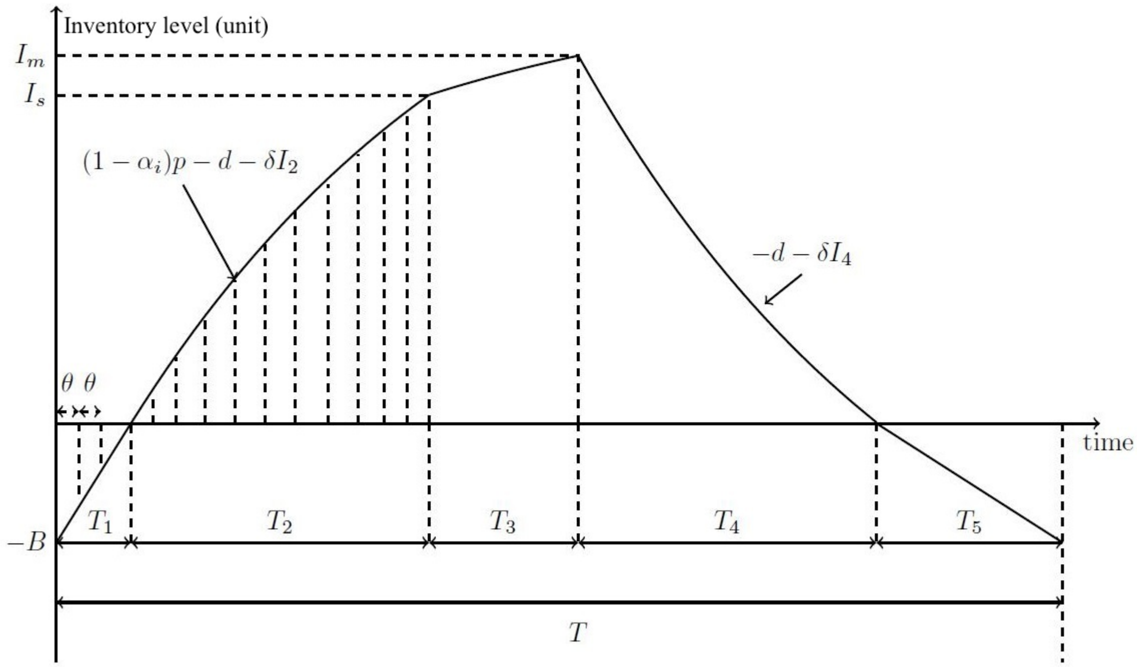

Figure 4.1: Inventory level of perfect items.

consumed by demand and backlog generated during T_5. Similarly in Figure 4.2, the total amount of imperfect products piles up in periods T_1 and T_2 due to the defects produced. The gradient increases along with the rise of the defective rate. So in general, the inventory level can be represented with the following equations. For backlog period $0 \leq t_1 \leq T_1$:

$$I_1'(t_1) = (1-\alpha_i)p - \mu, \quad (i-1)\theta \leq t_1 \leq i\theta \tag{4.1}$$

For i=1,

$$I_1(t_1) = \Big((1-\alpha_1)p - \mu\Big)t_1 - B, \quad 0 \leq t_1 \leq \theta, \tag{4.2}$$

Assume $\lambda_i = (1-\alpha_i)p - \mu$ for simplification and will be used throughout the paper,

$$I_1(\theta) = \lambda_1\theta - B, \tag{4.3}$$

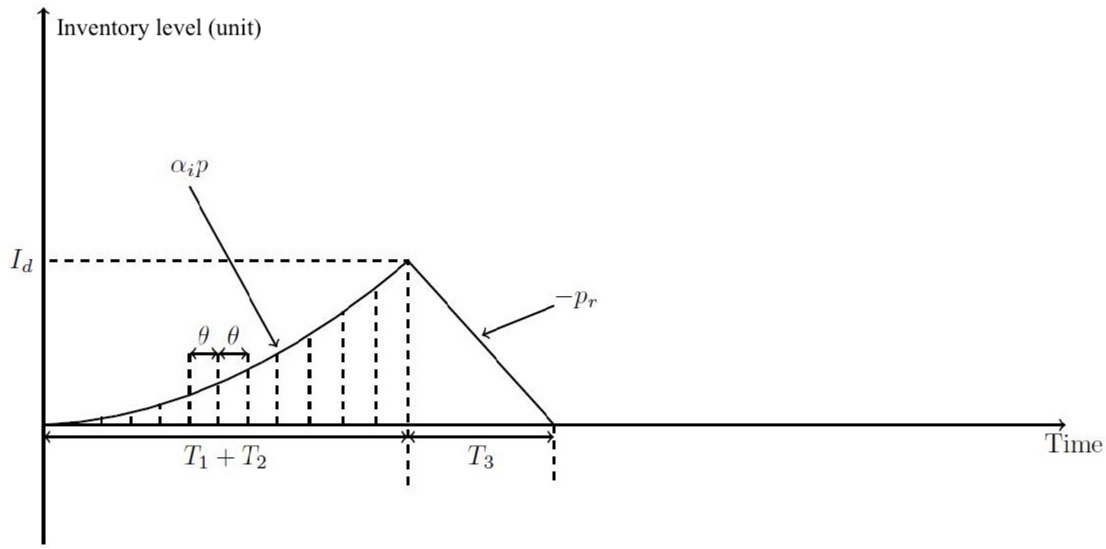

Figure 4.2: Inventory level of imperfect items

For $i = 2, I_2(0) = I_1(\theta) = \lambda_1\theta - B$,

$$I_1(t_1) = \lambda_2 t_1 + \lambda_1\theta - B, \quad \theta \le t_1 \le 2\theta, \tag{4.4}$$

Similarly, the general inventory function in the backlog period can be calculated as follows

$$I_1(t_1) = \begin{cases} \lambda_1 t_1 - B & 0 \le t_1 \le \theta \\ \lambda_2(t_1-\theta) + \lambda_1\theta - B & \theta \le t_1 \le 2\theta \\ \ldots & \\ \lambda_N(t_1-(N-1)\theta) + \sum_{i=1}^{N-1}\lambda_i\theta - B & (N-1)\theta \le t_1 \le T_1 \end{cases} \tag{4.5}$$

For surplus stage, For the first period θ, the inventory function is

$$I_2'(t_2) = \lambda_N - \delta I_2(t_2), \quad 0 \le t_2 \le \theta, \tag{4.6}$$

$$I_2(t_2) = \frac{\lambda_N}{\delta}(1-\exp(-\delta t_2)) \quad 0 \le t_2 \le \theta, \tag{4.7}$$

So for general surplus inventory function can be obtained,

$$I_2(t_2) = \begin{cases} \frac{\lambda_N}{\delta}(1-\exp(-\delta t_2)) & 0 \le t_2 \le \theta \\ \frac{\lambda_N - \lambda_{N+1}}{\delta}\exp(-\delta t_2) - \frac{\lambda_N}{\delta}\exp(-\delta t_2) + \frac{\lambda_N + 1}{\delta} & \theta \le t_2 \le 2\theta \\ \ldots & \\ \sum_{j=1}^{M-N} \frac{\lambda_{N+j-1} - \lambda_{N+j}}{\delta}\exp(-\delta((i-j)\theta) + t_2) & \\ -\frac{\lambda_N}{\delta}\exp(-\delta((M-N-1)\theta + t_2)) + \frac{\lambda_M}{\delta} & (M-1)\theta \le t_2 \le T_2 \end{cases} \tag{4.8}$$

The total production quantity can be calculated as:

$$pM\theta = Q, \tag{4.9}$$

Hence the inventory level at the end of normal production $I_s = I_2(T_2)$ is equal to

$$\begin{aligned} I_s \quad = \quad & \sum_{j=1}^{M-N} \frac{\lambda_{N+j-1} - \lambda_{N+j}}{\delta}\exp(-\delta((M-N-j+1)\theta)) \\ & -\frac{\lambda_N}{\delta}\exp(-\delta((M-N)\theta)) + \frac{\lambda_M}{\delta} \end{aligned} \tag{4.10}$$

After the normal production process,the slope of the inventory level can be represented by:

$$I_3'(t_3) = (p_r - \mu) - \delta I_3(t_3), \quad 0 \le t_3 \le T_3, \tag{4.11}$$

$$I_4'(t_4) = -\mu - \delta I_4(t_4), \quad 0 \le t_4 \le T_4. \tag{4.12}$$

$$I_5'(t_5) = -\mu, \quad 0 \le t_5 \le T_5. \tag{4.13}$$

According to the boundary conditions $I_5(T_5) = -B$, $I_2(T_2) = I_3(0) = I_s$, $I_3(T_3) = I_4(0) = I_m$, $I_4(T_4) = I_5(0) = 0$,the inventory level can be obtained.

$$I_3(t_3) = \Big(I_s - \frac{p_r - \mu}{\delta}\Big)\exp(-\delta t_3) + \frac{p_r - \mu}{\delta}, \quad 0 \leq t_3 \leq T_3, \tag{4.14}$$

$$I_4(t_4) = \Big(I_m + \frac{\mu}{\delta}\Big)\exp(-\delta t_4) - \frac{\mu}{\delta}, \quad 0 \leq t_4 \leq T_4, \tag{4.15}$$

$$I_5(t_5) = -\mu t_5, \quad 0 \leq t_5 \leq T_5. \tag{4.16}$$

The maximum inventory level I_m is equal to

$$I_m = \Big(I_s - \frac{p_r - \mu}{\delta}\Big)\exp(-\delta T_3) + \frac{p_r - \mu}{\delta}, \tag{4.17}$$

and from Eqn.(4.12) and $I_4(T_4) = 0$

$$I_m = \frac{\mu}{\delta}\Big(\exp(\delta T_4) - 1\Big). \tag{4.18}$$

We can also find

$$\sum_{i=1}^{N} \lambda_i \theta = dT_5 = B \tag{4.19}$$

For an imperfect product, the inventory function in normal production time is

$$I_{im}(t_{im}) = \sum_{1}^{i-1} \alpha_i \theta + \alpha_i t, \quad (i-1)\theta \leq t_{im} \leq i\theta. \tag{4.20}$$

$$I_{im}(t_{im}) = \begin{cases} \alpha_1 t_{im} & 0 \leq t_{im} \leq \theta \\ \alpha_1 \theta + \alpha t_{im} & \theta \leq t_{im} \leq 2\theta \\ \dots & \\ \sum_{i=1}^{M-1} \alpha_i \theta + \alpha_M t_{im} & (M-1)\theta \leq t_{im} \leq T_2 + T_1 \end{cases} \tag{4.21}$$

The maximum number of imperfect products is

$$I_d = \sum_{i=1}^{M} \alpha_i p\theta = p_r T_3 \tag{4.22}$$

With respect to the total cost per unit product, it consist of 6 parts: holding cost for both perfect and imperfect product,backlog cost, deterioration cost, cost of production and rework and lastly the fixed setup cost for each cycle run. The aim is to minimize the value of the total cost per unit product.

$$TC = (PHC + IHC + BC + DC + PRC + C_s)/Q \tag{4.23}$$

Whereas the holding cost of a perfect product is

$$PHC = C_{hp}\Big[\int_0^{T_2} I_2(t_2)dt_2 + \int_0^{T_3} I_3(t_3)dt_3 + \int_0^{T_4} I_4(t_4)dt_4\Big] \tag{4.24}$$

The holding cost of imperfect products is:

$$IHC = C_{hi}\Big[\int_0^{T_1+T_2} I_{im}(t_{im})dt + \int_0^{T_3} I_3(t_3)dt_3\Big] \tag{4.25}$$

The backlog cost is:

$$BC = -C_{bc}\Big[\int_0^{T_1} I_1(t_1)dt_1 + \int_0^{T_5} I_5(t_5)dt_5\Big] \tag{4.26}$$

The deteriorating cost is:

$$
\begin{aligned}
DC &= C_{dc}\Big[\frac{1}{2}\lambda_N\theta + \textstyle\sum_{i=N+1}^{M-1}\lambda_i\theta + \frac{1}{2}\lambda_M\theta - I_s\Big] + C_{dc}\Big[(p_r-\mu)T_3 - (I_m - I_s)\Big] \\
&\quad + C_{dc}\Big[I_m - dT_4\Big]
\end{aligned}
\tag{4.27}
$$

The production and rework cost are:

$$
PRC = C_pQ + C_rp_rT_3 \tag{4.28}
$$

By substituting all the inventory functions into the cost equation, the perfect product inventory holding cost *PHC* is

$$
\begin{aligned}
PHC &= C_{hp}\textstyle\sum_{i=1}^{M-N}\int_0^{\theta}\Big[\sum_{j=1}^{i}\frac{\lambda_{N+j-1}-\lambda_{N+j}}{\delta}\exp(-\delta((i-j)\theta)+t_2) \\
&\quad -\frac{\lambda_N}{\delta}\exp(-\delta((i-1)\theta+t_2)) + \frac{\lambda_{N+i}}{\delta}\Big]dt \\
&\quad + C_{hp}\Big[(I_s - \frac{p_r-\mu}{\delta})(\frac{1-\exp(-\delta T_3)}{\delta}) + \frac{(pr-\mu)T_3}{\delta}\Big] \\
&\quad + C_{hp}\Big[(I_m - \frac{\mu}{\delta})(\frac{1-\exp(-\delta T_4)}{\delta}) - \frac{\mu T_4}{\delta}\Big]
\end{aligned}
\tag{4.29}
$$

The holding cost for imperfect product:

$$
IHC = C_{hi}\Big[\textstyle\sum_{i=1}^{M}(\sum_{j=1}^{i}\alpha_j\theta^2 + \frac{1}{2}\alpha_i\theta^2) + \frac{1}{2}p_rT_3^2\Big] \tag{4.30}
$$

The backlog cost is:

$$BC = -C_{bc}\left[\int_0^{\theta}(\lambda_1 t - B)dt\right] - C_{bc}\sum_{i=1}^{N}\left[\int_0^{\theta}\left((\sum_{j=1}^{i-1}\lambda_i\theta - B)\theta + \lambda_i t\right)dt\right]$$
$$-C_{bc}\left[-\frac{1}{2}\mu T_5^2\right] \tag{4.31}$$

The deteriorating cost

$$DC = C_{dc}\left[(\sum_{i=N+1}^{M}\lambda_i\theta) - I_s + (p_r - \mu)T_3 - (I_m - I_s) + I_m - dT_4\right] \tag{4.32}$$

In order to minimize the total cost per product, the optimal combination of M and N are to be determined. So the relationships among M, N and other variables such as time T_i and backlog B are essential for the solution. First of all, according to the assumption, T_1 and T_2 can be represented by

$$T_1 = \theta N \tag{4.33}$$

$$T_2 = \theta(M - N) \tag{4.34}$$

Also since the defective rate is linearly increasing at a constant rate γ, we have

$$\alpha_i = \gamma * i \quad for \quad i \in [0, M] \tag{4.35}$$

Hence

$$\lambda_i = p - \mu - \gamma p i \tag{4.36}$$

and

$$\lambda_i - \lambda_{i+1} = \gamma p \tag{4.37}$$

According to Eq.(4.22) and Eq.(4.35), we have

$$T_3 = \frac{\gamma p \theta (M+1)^2}{2p_r} \tag{4.38}$$

Substitute Eq.(4.35) into the Eq. (4.10), we could obtain the following equation:

$$\begin{aligned} I_s &= \frac{\gamma p}{\delta} \textstyle\sum_{j=1}^{M-N} \exp(-\delta(M-N-j+1)\theta) + \frac{1}{2}\theta \\ &- \frac{(1-\gamma N)p-\mu}{\delta} \exp(-\delta(M-N)\theta) + \frac{(1-\gamma M)p-\mu}{\delta} \end{aligned} \tag{4.39}$$

The above expression can be simplified by using the Taylor series approximation under the assumptions that $\delta(M-N)$, δT_3 and δT_4 are small.This approach can also be found in other research works on deteriorating products such as (Tai, 2013) and (Wee, 1993). So,

$$\exp(-\delta(M-N-j+1)\theta) \approx 1 - \delta(M-N-j+1)\theta + \frac{1}{2}(\delta(M-N-j+1)\theta)^2 \tag{4.40}$$

$$\exp(-\delta T_3) \approx 1 - \delta T_3 + \frac{1}{2}(\delta T_3)^2 \tag{4.41}$$

$$\exp(-\delta T_4) \approx 1 - \delta T_4 + \frac{1}{2}(\delta T_4)^2 \tag{4.42}$$

We simplify the equation of I_s and I_m based on Eq.(4.39) and Eq.(4.17) as

$$I_s = \theta(M-N)(p-\mu-\frac{\gamma p}{2}(M+N+1)) \tag{4.43}$$

$$I_m = \theta(M-N)(p-\mu-\frac{\gamma p}{2}(M+N+1))(1-\frac{\delta\gamma p\theta M^2}{2p_r}) + (p_r-\mu)\frac{\gamma p\theta M^2}{2p_r} \tag{4.44}$$

Since I_m can also be represented with Eq.(4.18), T_4 can be determined as together with Eq.(4.42)

$$T_4 = \frac{1}{\mu}\Big[\theta(M-N)(p-\mu-\frac{\gamma p}{2}(M+N+1))(1-\frac{\delta\gamma p\theta M^2}{2p_r})+(p_r-\mu)\frac{\gamma p\theta M^2}{2p_r}\Big] \tag{4.45}$$

According to Eq.(4.19), B an be expressed as

$$B=(p-d-\frac{1}{2}\gamma pN)\theta N \tag{4.46}$$

and

$$T_5=\frac{(p-d-\frac{1}{2}\gamma pN)\theta N}{\mu} \tag{4.47}$$

Deteriorating cost is reduced to

$$DC = C_{dc}\Big[(p-\mu)(M-N)\theta-\gamma p\theta\tfrac{(M-N)(M-N-1)}{2}+p_rT_3-dT_4\Big] \tag{4.48}$$

Backlog Cost is

$$BC = -C_b\Big[\tfrac{1}{2}(p-\mu)N^2\theta^2-\tfrac{1}{12}\gamma p\theta^2(N-1)(2N+5)N-B\theta N-\tfrac{1}{2}dT5^2\Big] \tag{4.49}$$

Holding cost for perfect products is

$$\begin{aligned} PHC = {} & C_{hp}\Big[\tfrac{1}{2}(p-\mu-\gamma pN)\theta^2((M-N)(M-N+1)-1) \\ & -\tfrac{1}{12}\gamma p\theta^2(2M-2N+1)(M-N)(M-N+1) \\ & +I_s(T_3-\tfrac{1}{2}\delta T_3^2)+\tfrac{1}{2}(p_r-\mu)T_3^2+\tfrac{1}{2}dT_4^2\Big] \end{aligned} \tag{4.50}$$

$$IHC = C_{hi}\Big[\tfrac{1}{12}\gamma p\theta^2 M(M+1)(2M+3)+\tfrac{1}{2}\gamma p\theta M^2 T_3-\tfrac{1}{2}pT_3^2\Big] \tag{4.51}$$

Lastly by substituting T_1,T_2,T_3,T_4,T_5,B and Q with M and N,the cost function can be further reduced. The overall total cost per unit product is obtained. To be notice, the items with second or higher order δ and γ have been removed in order to simplify the equation.

$$\begin{aligned} TC \quad &= A_1+\tfrac{A_2}{M}+A_3M+A_4M^2+A_5M^3+A_6\tfrac{N^3}{M}+A_7N^2+A_8\tfrac{N^2}{M}+A_9N^2M \\ &+A_{10}N+A_{11}\tfrac{N}{M}+A_{12}NM+A_{13}NM^2 \end{aligned} \tag{4.52}$$

where

$$A_1 = C_p+\frac{C_{hp}\theta}{2}(\theta-\frac{\mu}{p}-\frac{\gamma}{6})+\frac{C_{hi}\gamma\theta}{4}$$

$$A_2 = \frac{C_s}{p\theta}-\frac{C_{hp}\theta}{2}(1-\frac{\mu}{p})$$

$$A_3 = \frac{C_r\gamma}{2}+\frac{5C_{hi}\gamma\theta}{12}+\frac{C_{hp}\theta}{2}(\frac{\gamma}{2}-\frac{p\gamma}{\mu}-\frac{1}{2}+\frac{p}{2\mu})$$

$$A_4 = \frac{C_{hi}\gamma\theta}{6}-\frac{C_{hp}\gamma\theta}{6}-\frac{C_{dc}\mu\gamma\delta\theta}{2p_r}(\mu-p)$$

$$A_5 = \frac{C_{hp}\gamma\delta\theta^2}{p_r}(p-\frac{\mu}{2}-\frac{p^2}{2\mu})$$

$$A_6 = \frac{\gamma\theta}{2}(\frac{1}{3}-\frac{p}{\mu})(C_b+C_{hp})$$

$$A_7 = \frac{C_{hp}p\gamma\theta}{2\mu}$$

$$A_8 = C_{dc}\gamma+\frac{C_b\theta}{2}(\frac{\gamma}{2}-1+\frac{p}{\mu})+\frac{C_{hp}\theta}{2}(\frac{p}{\mu}+\frac{3\gamma}{2}-\frac{p\gamma}{\mu}-1)$$

$$A_9 = \frac{C_{hp}\gamma\delta\theta^2}{p_r}(p - \frac{\mu}{2} - \frac{p^2}{\mu})$$

$$A_{10} = C_{dc}\gamma + C_{hp}\theta(\frac{p}{\mu} - 1)(\gamma - 1)$$

$$A_{11} = \frac{C_{hp}\theta}{2}(\frac{\mu}{p} + \frac{7\gamma}{6} - 1) - \frac{5C_b\gamma\theta}{12}$$

$$A_{12} = \frac{C_{dc}\gamma\delta\theta}{2p_r}(\mu - p)$$

$$A_{13} = \frac{C_{hp}\gamma\delta\theta^2}{p_r}(\mu - p + \frac{p^2}{\mu})$$

For optimum values of $TC(N,M)$, we make $\frac{\partial TC(N,M)}{\partial N} = 0$ and $\frac{\partial TC(N,M)}{\partial M} = 0$ which is equivalent to

$$3A_6\frac{N^2}{M} + 2A_7N + 2A_8\frac{N}{M} + 2A_9NM + A_{10} + A_{11}\frac{1}{M} + A_{12}M + A_{13}M^2 = 0 \qquad (4.53)$$

$$A_3 - \frac{A2}{M^2} + 2A_4M + 3A_5M^2 + 2A_12N + A_{11}\frac{N}{M^2} + 2A_{13}MN + A_9N^2 - A_*\frac{N^2}{M^2} - A_6\frac{N^3}{M^2} = 0 \qquad (4.54)$$

The corresponding Hessian Matrix is shown below.

$$H = \begin{pmatrix} H_1 & H_2 \\ H_2 & H_3 \end{pmatrix}$$

Where

$$H_1 = 2A_4 + \frac{2A_2}{M^3} + 6A_5M + 2A_13N + \frac{2A_{11}N}{M^3} + \frac{2A_8N^2}{M^3} + + \frac{2A_6N^3}{M^3}$$

$$H_2 = A_{12} - \frac{A_{11}}{M^2} + 2A_{13}M + 2A_9N - \frac{2A_8N}{M^2} - \frac{3A_6N^2}{M^2}$$

$$H_3 = 2A_7 + \frac{2A_8}{M} + 2A_9M + \frac{6A_6N}{M}$$

According to the optimum condition that if the second order partial derivatives for M and N are positive,then the Hessian matrix is positive definite and the minimum total cost can be found. However, due to the complexity of the cost function, an explicit solution cannot be obtained. Instead, we will use numerical examples to illustrate that the cost function is convex and the optimum result are also determined accordingly.

4.3 Numerical Experiments

Numerical examples and sensitivity are described in the following section. To be more specific, first of all, the plots generated for the cost function are used to demonstrate that the cost function is convex. Then, sensitivity analysis indicates the impact of different parameters on the overall inventory performance.

4.3.1 Numerical examples

In this examples, the values of the parameters are assumed as follows: $\gamma = 0.01, \theta = 0.01, \delta = 0.1, C_{hp} = \$40, C_{hi} = \$30, C_p = \$100; C_r = \$40, C_b = \$60, C_{dc} = \$60, \mu = 100, p = 600, p_r = 300$. The optimum combination of N and M are $(N^*, M^*) = (4, 10)$.

As shown in Figure 4.3 ,the cost function shows its convexity and the optimum pair of (N^*, M^*) is the lowest point. While in Figures 4.4 and 4.5, the convexity is more

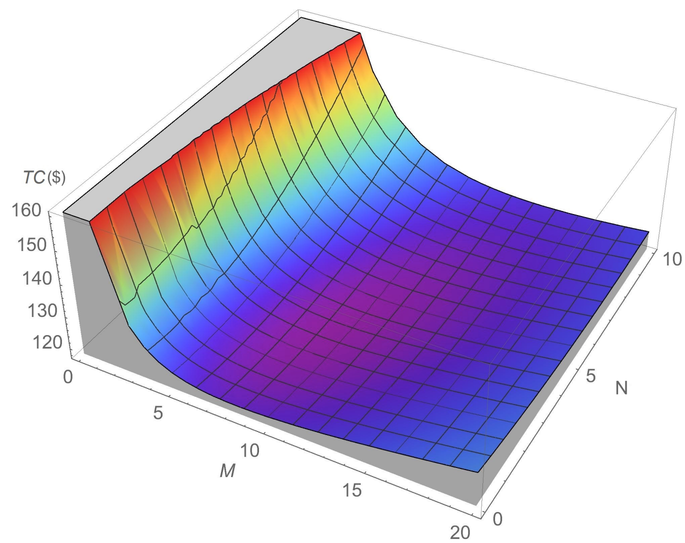

Figure 4.3: The plot of total cost per unit product against M and N

clear when M and N are fixed at its optimum value respectively. Hence the optimal value of T_1^*, T_2^*, T_3^*, T_4^*, T_5^* and T^* are calculated as

$$T_1^* = 0.04, \quad T_2^* = 0.06, \quad T_3^* = 0.01, \quad T_4^* = 0.293, \quad T_5^* = 0.195,$$

The optimal production quantity and backlog quantity are determined as

$$Q^* \approx 60, \quad B^* \approx 20,$$

The optimal total cost per unit product, the optimum production run time and the optimal cycle time are:

$$TC^* = 118.479, \quad T_1^* + T_2^* = 0.1, \quad T^* = 0.598$$

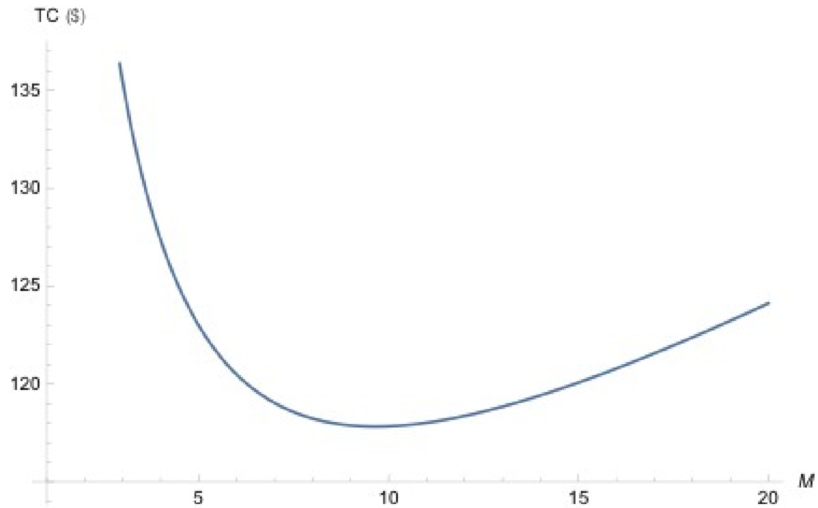

Figure 4.4: Total cost per unit product against *M*

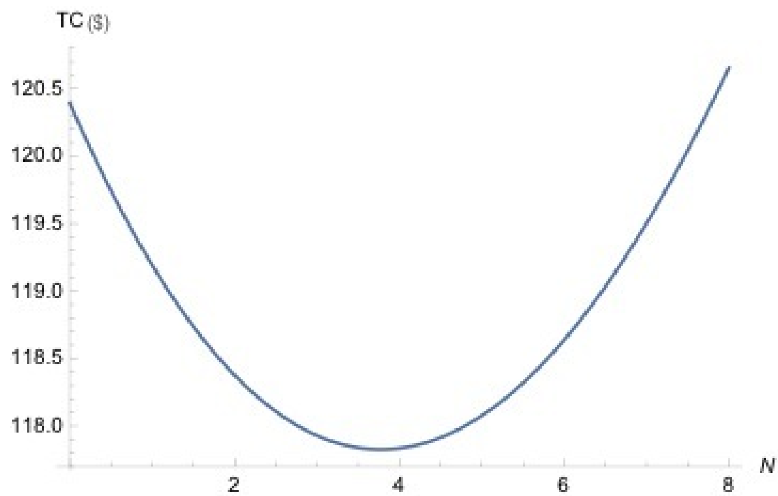

Figure 4.5: Total cost per unit product against *N*

4.3.2 Sensitivity analysis

Sensitivity analysis for the parameters was undertaken and the results and discussion are listed below. As shown in Tables 4.1 and 4.2, for each parameters, four more experiments were carried out with changes of -50%, -25%, 25 % and 50 %. The corresponding optimal values of the total cost per unit product, *M* and *N* are presented

Table 4.1. The sensitivity analysis for different key parameters

Parameter	Optimal values	Changes -50%	-25%	0%	25%	50%
θ	TC^*	120.68	118.824	117.835	117.16	116.765
	M^*	16	12	10	8	7
	N^*	5	4	4	3	3
γ	TC^*	116.116	116.977	117.835	118.569	119.327
	M^*	10	10	10	9	9
	N^*	4	4	4	3	3
δ	TC^*	117.824	117.829	117.835	117.84	117.846
	M^*	10	10	10	10	10
	N^*	4	4	4	4	4
μ	TC^*	124.166	120.033	117.835	116.399	115.43
	M^*	8	9	10	10	10
	N^*	4	3	4	4	3
p	TC^*	119.664	118.479	117.835	117.563	117.193
	M^*	17	12	10	9	6
	N^*	4	4	4	3	2
p_r	TC^*	117.817	117.832	117.835	117.841	117.842
	M^*	10	10	10	10	10
	N^*	4	4	4	4	4

in the tables. It should be noted that, the values of M and N are interpreted as the production run time and backlog quantity respectively in the discussion since they have positive relationships.

For the key parameters:

i. Both the parameters θ and γ,which are related to the imperfect production system, have a significant impact on the total cost per unit product. An increment of θ reduces the cost, while for γ, the lower the value, the lower the cost. It means that slow deterioration of the production quality or a relatively

Table 4.2. The sensitivity analysis for different unit cost

Unit cost	Optimal values	Changes -50%	-25%	0%	25%	50%
C_{hp}	TC^*	115.054	116.687	117.835	118.593	119.234
	M^*	11	10	10	9	9
	N^*	2	3	4	4	5
C_{hi}	TC^*	117.764	117.819	117.835	117.851	117.866
	M^*	10	10	10	10	10
	N^*	4	4	4	4	4
C_{dc}	TC^*	117.07	117.421	117.835	118.035	118.42
	M^*	10	10	10	10	9
	N^*	4	4	4	3	3
C_p	TC^*	67.803	92.8032	117.835	142.898	171.433
	M^*	10	10	10	10	10
	N^*	4	4	4	4	4
C_{pr}	TC^*	116.803	117.303	117.835	118.303	118.753
	M^*	10	10	10	10	9
	N^*	4	4	4	4	4
C_b	TC^*	116.11	117.19	117.835	118.202	118.515
	M^*	10	10	10	9	9
	N^*	7	5	4	3	2
C_s	TC^*	111.898	114.556	117.835	119.908	121.802
	M^*	7	9	10	11	11
	N^*	3	3	4	4	4

steady defective rate helps cut down the total cost per unit product. In addition, high γ shortens production run time which is represented by M.

ii. The product deterioration has positive relationship with the total cost per unit product. However when compared with θ and γ, its influence on the overall system is relatively smaller.

iii. The total cost per unit product is especially sensitive to the changes of the demand rate μ. With a rise of μ, the cost drops rapidly while the total production run time $M\theta$ increases instead. The value of M is heavily influenced by the production rate. During increase of the production rate, M reduces from 17 to as low as 6. On the contrary, the influence brought about by the re-manufacturing rate is not obvious.

For the cost paramters:

i. High holding cost C_{hp} causes an increase in the backlog quantity and a decrease of the holding quantity, meanwhile the value of the backlog quantity drops significantly along with increase of the backlog cost C_b.

ii. The total cost per unit product is affected by the production cost to a large extent, but it does not change the production run time and backlog quantity.

iii. Both the production run time and the backlog quantity increase with higher setup cost C_s. The total cost per unit product is also sensitive to changes in the setup cost.

4.4 Summary

In this chapter, a modified EPQ model, with rework and backlog, has been proposed. Compared with the existing works, the deterioration of the product and production process is taken into account at the same time, which is the main contribution to this research field. To model the deterioration of the production process, we assume that the defective rate increases at constant intervals. Defective products are reworked at the end of normal production process and the rework is viewed as a perfect process. In order to minimize the total cost per unit product, the optimal pair of the total interval number θ in normal period M and in backlog period N are determined. Due to the high complexity of the cost function, we cannot prove the convexity of the function in an analytical way. Instead, numerical experiments are carried out to illustrate the convexity of the cost function and to find the optimal solution. The impact of all different parameters on the system are provided and summarized in the sensitivity analysis.

Chapter 5 A Stochastic Production and Inventory Model in A Two-state Production System with Inventory Deterioration, Rework Process and Backordering

5.1 Notations and Assumptions

The detailed notation and assumptions used in this paper are shown in the following sub-sections:

Control ariables

B Backlog quantity

Q Economic production quantity

Parameters

α The defective rate of production process in in-control state

β The defective rate of production process in out-of-control state

γ The defective rate in rework process

θ Time interval of in-control state

δ Deteriorating rate of inventory

μ Demand rate (unit/time)

p Production rate (unit/time)

p_r Rework rate (unit/time)

C_{hp} Holding cost of perfect products (\$/unit/time)

C_{hi} Holding cost of imperfect products (\$/unit/time)

C_{dc} Deteriorating cost (\$/unit)

C_p Production cost (\$/unit)

C_{dp} Disposal cost of scraped product(\$/unit)

C_{pr} Rework cost of imperfect product(\$/unit)

C_b Penalty cost for backlog (\$/unit/time)

C_s Fixed setup cost (\$/cycle)

I_s The inventory level of perfect product at the end of normal production period

I_m The inventory level of perfect product at the end of rework process

I_r The maximum inventory level of imperfect product

Assumptions

1. Backlog is allowed and at the beginning of each production run, the products are first used to satisfy the backorder.
2. At the end of each production run, maintenance activities are conducted to restore the machine condition. So at the beginning of each production run, the machine is assumed to be in control, and the maintenance cost is included in the setup cost.
3. During the production process, all the manufactured products are inspected instantly and the inspection time is assumed to be negligible. After normal production, imperfect products are reworked together.

4. Only perfect products deteriorate at a constant rate δ, and deteriorated items are screened out and disposed of at the end of a production run.
5. The demand rate μ is a known constant.
6. The process deterioration is assumed to occur in normal production process only, but not in rework processes.

5.2 Mathematical Model

This study models a production system with a single-machine, single-product production system, and both the production and rework processes are operated on the same machine. The products are subject to quality deterioration and the machine is assumed to be imperfect, and deteriorates after a stochastic period of time. The model can be divided into 5 stages, which are represented as T_1, T_2, T_3, T_4 and T_5. During T_1 and T_2, normal production is carried out. In T_1, product is firstly used to satisfy the backlog. And in T_2, products are aggregated and stored in warehouse to meet future demand. Imperfect products generated from T_1 and T_2 are reworked during T_3. Production is stopped in T_4 and demand is satisfied directly from inventory. Finally, during T_5, demand is backlogged and expected to be fulfilled in the next production run.

In addition, according to the time the production system changes from an in-control state to an out-of-control state, three different cases are covered . In Cases 1 and 2, the switch of states occurs in the backordering period and the period with inventory surplus. The defective rate increases from α to β as a result. While in Case 3, the deterioration does not occur during the normal production time, hence, the deterioration does not influence the production at all. The equations for each of the models are presented as follows.

5.2.1 Case 1: the switch occurs within T_1 $\quad 0 \leq \theta \leq T_1$

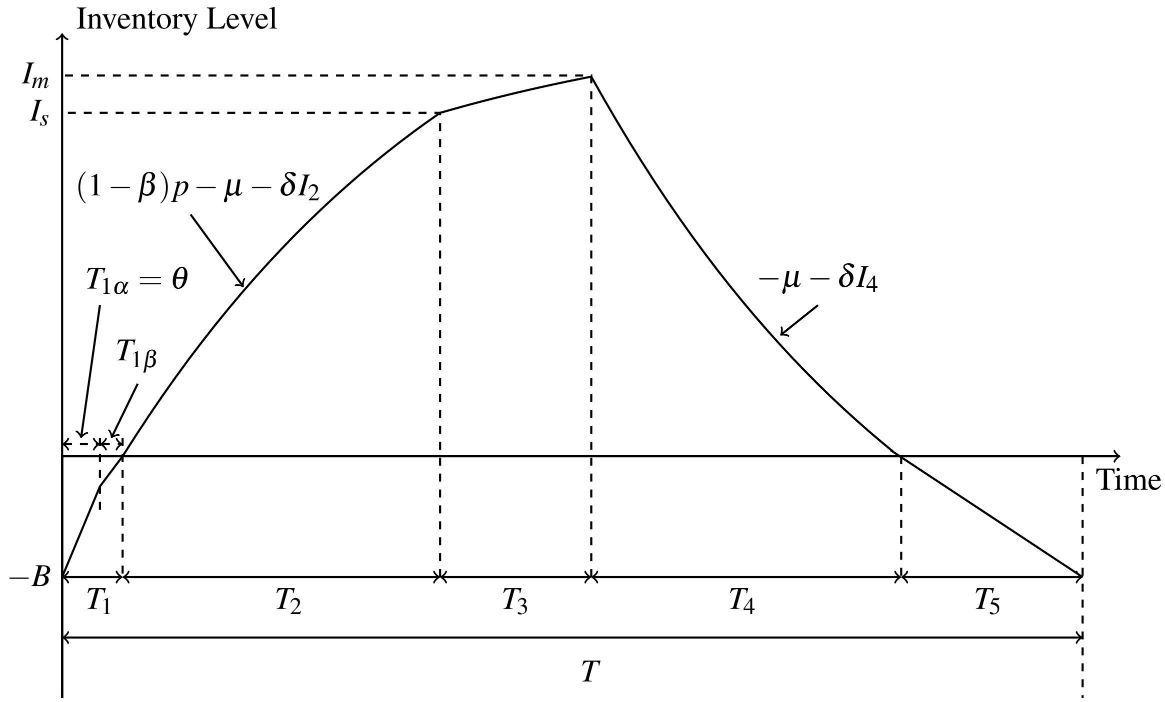

Figure 5.1: Inventory level of perfect product in Case 1.

Figure 5.1 shows the inventory behavior of the perfect products. In this case, T_1 is divided into two separate periods $T_{1\alpha}$ and $T_{1\beta}$. During $T_{1\alpha}$, the defective rate remains as α, while in $T_{1\beta}$, the defective rate changes to β. During T_3, the imperfect products are reworked together, and in T_4 and T_5, the normal production process stops and the current inventory is consumed. The inventory in each time period can be described by the differential equations below:

$$I'_{1\alpha}(t_{1\alpha}) = (1-\alpha)p - \mu, \quad 0 \leq t_{1\alpha} \leq T_{1\alpha}, \tag{5.1}$$

$$I'_{1\beta}(t_{1\beta}) = (1-\beta)p - \mu, \quad 0 \le t_{1\beta} \le T_{1\beta}, \tag{5.2}$$

$$I'_2(t_2) = (1-\beta)p - \mu - \delta I_2(t_2), \quad 0 \le t_2 \le T_2, \tag{5.3}$$

$$I'_3(t_3) = (1-\gamma)p_r - \mu - \delta I_3(t_3), \quad 0 \le t_3 \le T_3, \tag{5.4}$$

$$I'_4(t_4) = -\mu - \delta I_4(t_4), \quad 0 \le t_4 \le T_4, \tag{5.5}$$

$$I'_5(t_5) = -\mu, \quad 0 \le t_5 \le T_5. \tag{5.6}$$

The boundary conditions are $I_{1\alpha}(0) = I_5(T_5) = -B$, $I_{1\alpha}(T_{1\alpha}) = I_{1\beta}(0)$, $I_{1\beta}(T_{1\beta}) = I_2(0) = 0$, $I_2(T_2) = I_3(0) = I_s$, $I_3(T_3) = I_4(0) = I_m$, $I_4(T_4) = I_5(0) = 0$. So the solutions for the above differential equations are:

$$I_{1\alpha}(t_{1\alpha}) = \Big((1-\alpha)p - \mu\Big)t_{1\alpha} - B, \quad 0 \le t_{1\alpha} \le T_{1\alpha}, \tag{5.7}$$

$$\begin{aligned} I_{1\beta}(t_{1\beta}) = \Big((1-\beta)p - \mu\Big)t_{1\beta} + \Big((1-\alpha)p - \mu\Big)T_{1\alpha} - B, \\ 0 \le t_{1\beta} \le T_{1\beta} \end{aligned} \tag{5.8}$$

$$I_2(t_2) = \Big(\frac{(1-\beta)p - \mu}{\delta}\Big)(1 - \exp(-\delta t_2)), \quad 0 \le t_2 \le T_2, \tag{5.9}$$

$$\begin{aligned} I_3(t_3) = \Big(I_s - \frac{(1-\gamma)p - \mu}{\delta}\Big)\exp(-\delta t_3) + \frac{(1-\gamma)p_r - \mu}{\delta}, \\ 0 \le t_3 \le T_3 \end{aligned} \tag{5.10}$$

$$I_4(t_4) = \Big(I_m + \frac{\mu}{\delta}\Big)\exp(-\delta t_4) - \frac{\mu}{\delta}, \quad 0 \le t_4 \le T_4, \tag{5.11}$$

$$I_5(t_5) = -\mu t_5, \quad 0 \le t_5 \le T_5. \tag{5.12}$$

Hence, it can be deduced from Eqn (5.9) when $t_2 = T_2$

$$I_s = \Big(\frac{(1-\beta)p - \mu}{\delta}\Big)(1 - \exp(-\delta T_2)), \tag{5.13}$$

and from Eqn (5.9) and Eqn (5.10), the maximum inventory level for a perfect product can be obtained as

$$I_m = \Big(I_s - \frac{(1-\gamma)p_r - \mu}{\delta}\Big)\exp(-\delta T_3) + \frac{(1-\gamma)p_r - \mu}{\delta}, \tag{5.14}$$

and from Eqn (5.11), I_m can also be represented as

$$I_m = \frac{\mu}{\delta}\Big(\exp(\delta T_4) - 1\Big). \tag{5.15}$$

From Figure 5.1, the backlog quantity B is given by

$$B = \Big((1-\alpha)p - \mu\Big)T_{1\alpha} + \Big((1-\beta)p - \mu\Big)T_{1\beta} \tag{5.16}$$

and the total lot size Q can be obtained as

$$Q = p(T_{1\alpha} + T_{1\beta} + T_2). \tag{5.17}$$

In terms of cost,

$$TC_1 = PHC + IHC + BC + DC + PRC + DPC + C_s, \tag{5.18}$$

where the holding cost of perfect products is

$$\begin{aligned} PHC = \quad & \frac{C_{hp}}{Q}\Big(\int_0^{T_2} I_2(t_2)dt_2 + \int_0^{T_3} I_3(t_3)dt_3 \\ & + \int_0^{T_4} I_4(t_4)dt_4\Big) \end{aligned} \tag{5.19}$$

The holding cost of imperfect products is

$$
\begin{aligned}
IHC = \; & \frac{C_{hi}}{Q}\Big(\int_0^{T_{1\alpha}} (\alpha p t_{1\alpha})\, dt_{1\alpha} + \int_0^{T_{1\beta}} (\beta p t_{1\beta} \\
& + \alpha p T_{1\alpha}) dt_{1\beta} + \int_0^{T_2} (\beta p t_2 + \alpha p T_{1\alpha}) dt_2 \\
& + \int_0^{T_3} (\beta p (T_{1\beta} + T_2) + \alpha p T_{1\alpha} - \gamma p_r t_3) dt_3\Big).
\end{aligned} \tag{5.20}
$$

The backlog cost is

$$
\begin{aligned}
BC = \; & -\frac{C_b}{Q}\Big(\int_0^{T_{1\alpha}} I_{1\alpha}(t_{1\alpha}) dt_{1\alpha} + \int_0^{T_{1\beta}} I_{1\beta}(t_{1\beta}) dt_{1\beta} \\
& + \int_0^{T_5} I_5(t_5) dt_5\Big).
\end{aligned} \tag{5.21}
$$

The deteriorating cost is

$$
\begin{aligned}
DC \; & = \frac{C_{dc}}{Q}\Big((((1-\beta)p - \mu)T_2 + ((1-\gamma)p_r - \mu)T_3 \\
& -I_{max}) + (I_{max} - dT_4)\Big).
\end{aligned} \tag{5.22}
$$

The production and rework costs are

$$
PRC \;=\; C_p + \frac{C_r}{Q}\Big(p_r T_3\Big). \tag{5.23}
$$

The disposal cost is

$$
DPC \;=\; \frac{C_{dp}}{Q}\Big(\gamma p_r T_3\Big). \tag{5.24}
$$

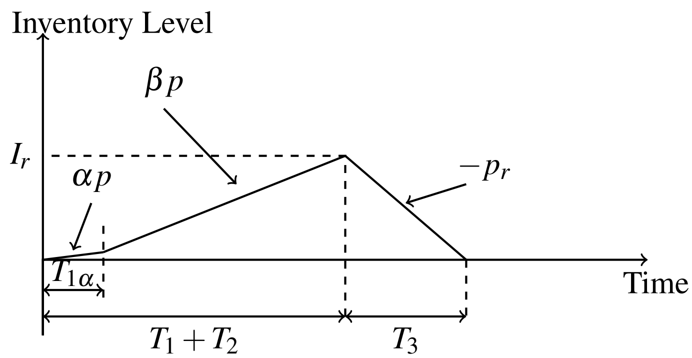

Figure 5.2: Inventory level of imperfect quality items in Case 1.

In order to simplify the equation, we let $a_1 = (1-\alpha)p-\mu$, $a_2 = (1-\beta)p-\mu$, $a_3 = (1-\gamma)p-\mu$. Since the expected in-control time is θ,

$$T_{1\alpha} = \theta. \tag{5.25}$$

According to Eqn (5.16), $T_{1\beta}$ is given by

$$T_{1\beta} = \frac{B - a_1 T_{1\alpha}}{a_2} = \frac{B - a_1\theta}{a_2} \tag{5.26}$$

Hence T_2 can be calculated with Eqn (5.17)

$$T_2 = \frac{Q}{p} - T_{1\alpha} - T_{1\beta} = \frac{Q}{p} - \frac{B}{a_2} - \frac{(a_2-a_1)}{a_2}\theta. \tag{5.27}$$

For the imperfect items, the inventory behavior is presented in Figure 5.2, and the maximum inventory level is given by

$$I_r = \alpha p T_{1\alpha} + \beta p(T_{1\beta} + T_2) = p_r T_3. \tag{5.28}$$

Hence the T_3 can be represented by substituting Eqn (5.25) and Eqn (5.26)

$$T_3 = \frac{1}{p_r}((a_2 - a_1)\theta + Q\beta). \tag{5.29}$$

Next, we express T_4 in terms of T_3 and T_2. From Eqns (5.11), (5.13) and (5.14), we have

$$a_2(1 - exp(-\delta T_2))exp(-\delta T_3) + a_3 = \mu(exp(\delta T_4) - 1). \tag{5.30}$$

The above expression can be simplified by using the Taylor series approximation under the assumption that δT_2, δT_3 and δT_4 are small. After the simplification, as given in Appendix A, T_4 can be represented as

$$T_4 = \frac{a_2}{\mu}T_2 - \frac{a_3}{\mu}(T_3 - \frac{\delta T_3^2}{2}). \tag{5.31}$$

Similarly by using the exponential series approximation, the different cost functions can be simplified as follows:

$$\begin{aligned}
PHC &= \frac{C_{hp}}{Q}(\frac{a_2 T_2^2}{2} + a_2 T_2 T_3 + \frac{a_3 T_3^2}{2} + \frac{\mu T_4^2}{2}) \\
IHC &= \frac{C_{hi}}{Q}(\frac{\alpha p T_{1\alpha}^2}{2} + \frac{1}{2}\beta p(\frac{Q}{p} - T_{1\alpha})^2 + \alpha p\theta(\frac{Q}{p} - T_{1\alpha}) \\
&\quad +(\beta Q - \theta(a_1 - a_2))T_3 + \frac{\gamma p_r T_3^2}{2}) \\
BC &= \frac{C_{bc}}{Q}(\frac{B^2}{2d} + \frac{(B - a_1\theta)^2}{2a_2} + B\theta - \frac{a_1\theta^2}{2}) \\
DC &= \frac{C_{dc}a_3}{Q}(2T_3 - \frac{\delta T_3^2}{2}) \\
PRC &= C_p + \frac{C_r}{Q}\left(p_r T_3\right)
\end{aligned}$$

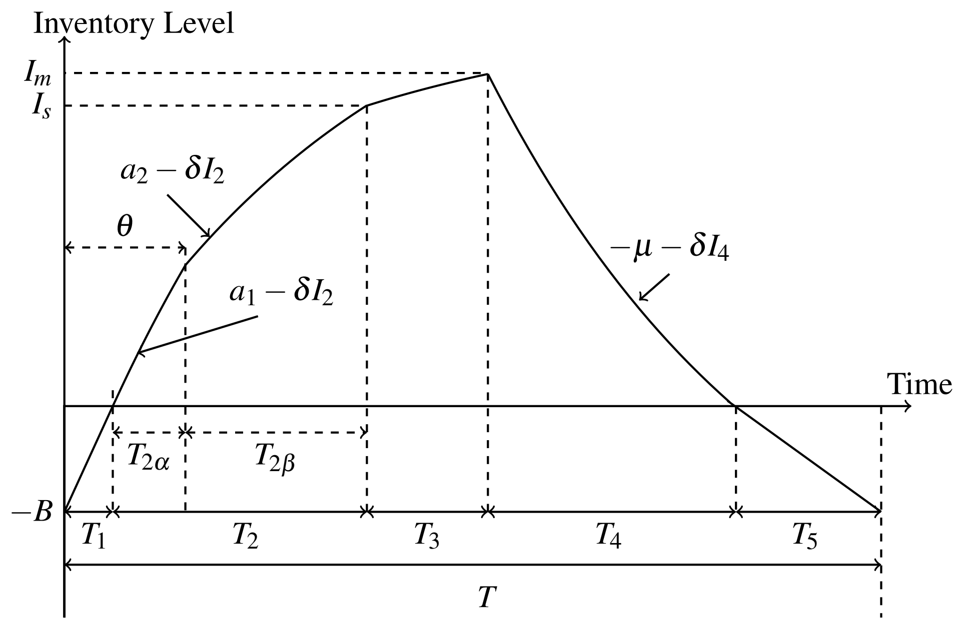

Figure 5.3: Inventory level of perfect items in Case 2

$$DPC = \frac{C_{dp}}{Q}(\gamma p_r T_3)$$

5.2.2 Case 2: the switch occurs within T_1 and T_2 $T_1 \leq \theta \leq T_1 + T_2$

Similar to Case 1, the inventory level in T_2 is represented by two separate equations $I_{2\alpha}$ and $I_{2\beta}$, while I_3, I_4 and I_5 are kept the same. Hence those repeated equations are not listed in the following two cases. Figure 5.3 presents the inventory behavior in Case 2.

$$I_1'(t_1) = (1-\alpha)p - \mu, \quad 0 \leq t_1 \leq T_1, \tag{5.32}$$

$$I_{2\alpha}'(t_{2\alpha}) = (1-\alpha)p - \mu, \quad 0 \leq t_{2\alpha} \leq T_{2\alpha}, \tag{5.33}$$

$$I_{2\beta}'(t_{2\beta}) = (1-\beta)p - \mu, \quad 0 \leq t_{2\beta} \leq T_{2\beta}, \tag{5.34}$$

The boundary conditions are $I_{2\alpha}(T_{1\alpha}) = I_{2\beta}(0)$, $I_{2\beta}(T_{2\beta}) = I_3(0) = I_s$. The solutions for the above differential equations are

$$I_1(t_1) = ((1-\alpha)p - \mu)t_1 - B, \quad 0 \le t_1 \le T_1, \tag{5.35}$$

$$I_{2\alpha}(t_{2\alpha}) = \frac{a_1}{\delta}(1 - \exp(-\delta t_{2\alpha})), \quad 0 \le t_{2\alpha} \le T_{2\alpha}, \tag{5.36}$$

since $I_{2\alpha}(T_{2\alpha}) = I_{2\beta}(0)$. We also have

$$\begin{aligned} I_{2\beta}(t_{2\beta}) = \Big(\frac{a_1 - a_2}{\delta} - \frac{a_1}{\delta}\exp(-\delta T_{2\alpha})\Big)\exp(-\delta t_{2\beta}) + \frac{a_2}{\delta}, \\ 0 \le t_{2\beta} \le T_{2\beta}. \end{aligned} \tag{5.37}$$

Hence, it can be deduced from I_3 that at $t_2 = T_2$

$$I_s = \Big(\frac{a_1 - a_2}{\delta} - \frac{a_1}{\delta}\exp(-\delta T_{2\alpha})\Big)\exp(-\delta T_{2\beta}) + \frac{a_2}{\delta} \tag{5.38}$$

and from Eqn (5.37)

$$I_m = \Big(I_s - \frac{a_3}{\delta}\Big)\exp(-\delta T_3) + \Big(\frac{a_3}{\delta}\Big). \tag{5.39}$$

The backlog quantity B is given by

$$B = a_1 T_1 = d T_5. \tag{5.40}$$

The total lot size Q is given by

$$Q = p(T_1 + T_{2\alpha} + T_{2\beta}). \tag{5.41}$$

Since the expected in-control time is θ and $\theta = T_1 + T_{2\alpha}$, hence

$$T_{2\alpha} = \theta - T_1 = \theta - \frac{B}{a_1}. \tag{5.42}$$

Based on Eqn (5.41),

$$T_{2\beta} = \frac{Q}{p} - \theta. \tag{5.43}$$

According to the imperfect product inventory equation,

$$\alpha p\theta + \beta p T_{2\beta} = p_r T_3. \tag{5.44}$$

So T_3 can be obtained as

$$T_3 = \frac{p}{p_r}\Big(\alpha\theta + \beta T_{2\beta}\Big) = \frac{p}{p_r}\Big(\alpha\theta + \beta\frac{Q}{p} - \theta\Big). \tag{5.45}$$

From Eqns (5.38) and (5.39), we also have

$$\Big(I_s - \frac{a_3}{\delta}\Big)\exp(-\delta T_3) + \frac{a_3}{\delta} = \frac{\mu}{\delta}\Big(\exp(\delta T_4) - 1\Big),$$

$$\Big((\frac{a_1 - a_2}{\delta} - \frac{a_1}{\delta}\exp(-\delta T_{2\alpha}))\exp(-\delta T_{2\beta}) + \frac{a_2}{\delta}$$
$$-\frac{a_3}{\delta}\Big)\exp(-\delta T_3) + \frac{a_3}{\delta} = \frac{\mu}{\delta}(\exp(\delta T_4) - 1).$$

Following the same procedure to simplify the cost function in Case 1, assume $1 - \exp(-\delta T_2) \approx \delta T_2 - \frac{(\delta T_2)^2}{2}$, and the same with T_3 and T_4. The equation for T_4 can be simplified as

$$\begin{aligned} T_4 = \; & \frac{1}{\mu}\Big(\delta(a_1 T_{2\alpha} + a_2 T_{2\beta} + a_3 T_3) - \\ & \delta^2(a_1 T_{2\alpha} T_{2\beta} + a_1 T_{2\alpha} T_3 + a_2 T_{2\beta} T_3)\Big). \end{aligned} \tag{5.46}$$

The cost function can also be reduced accordingly. First of all the holding cost of the perfect product is

$$
\begin{aligned}
PHC = \quad & \frac{C_{hp}}{Q}[\frac{a_1}{\delta}T_{2\alpha} - \frac{a_1}{\delta^2}(\exp(-\delta T_{2\alpha}) - 1) + \frac{a_2}{\delta}T_{2\beta} \\
& +(\frac{a_1 - a_2}{\delta^2} - \frac{a_1}{\delta^2}\exp(-\delta T_{2\alpha}))(1 - \exp(-\delta T_{2\beta})) \\
& +((\frac{a_2 - a_1}{\delta^2} - \frac{a_1}{\delta^2}\exp(-\delta T_{2\alpha}))(\exp(-\delta T_{2\beta})) \\
& +\frac{a_2 - a_3}{\delta})\frac{1 - \exp(-\delta T_3)}{\delta} + \frac{a_3}{\delta}T_3 + \mu \exp(\delta T_4) \\
& -\frac{\mu}{\delta}T_4].
\end{aligned}
$$

By using the Taylor expansion, the equation can be further simplified as

$$
\begin{aligned}
PHC = \quad & \frac{C_{hp}}{Q}\Big(\frac{a_1}{2}T_{2\alpha}^2 + a_1 T_{2\alpha}T_{2\beta} - \frac{a_1 T_{2\beta}^2}{2} + \frac{a_2}{\delta}T_3 \\
& -\frac{a_2 - a_3}{2}T_3^2\Big).
\end{aligned}
$$

The imperfect production holding cost is

$$
IHC \quad = \quad \frac{C_{hi}}{Q}\Big(\frac{1}{2}\alpha\beta\theta^2 + \frac{1}{2}\beta p T_{2\beta}^2 + \alpha p\theta + \frac{1}{2}\gamma p_r T_3^2\Big).
$$

For other cost functions, they are the same as in Case 1, so they are not provided.

5.2.3 Case 3: the switch occurs after T_2, $\quad \theta > T_1 + T_2$

Since the switch occurs outside the normal production run time, the inventory is represented by only one equation in each period, see Figure 5.4. Especially when

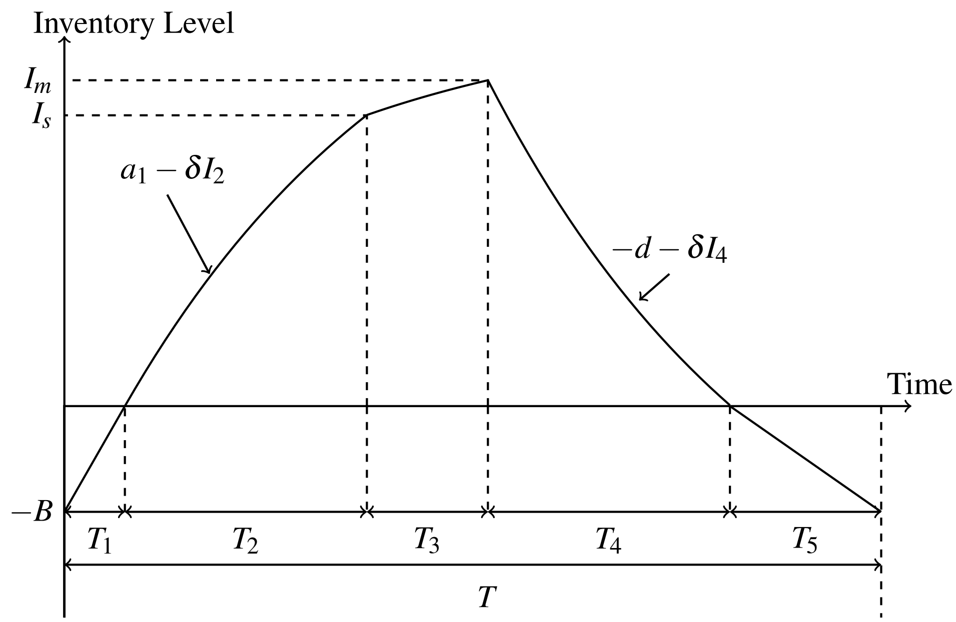

Figure 5.4: Inventory level of perfect items in Case 3.

compared with Case 2, the equation for I_2 is changed to

$$I_2(t_2) = \frac{a_1}{\delta}\Big(1 - \exp(-\delta t_2)\Big), \quad 0 \leq t_2 \leq T_2, \tag{5.47}$$

where the defective rate still remains as α during this period. I_s changes to the following equation according to Eqn (5.47) at $t_2 = T_2$

$$I_s = \frac{a_1}{\delta} - \frac{a_1}{\delta}\exp(-\delta T_2). \tag{5.48}$$

By using the Taylor expansion, the equation of the holding cost for perfect and imperfect products can be simplified as:

$$\begin{aligned} PHC = \;& \frac{C_{hp}}{Q}\Big(\frac{a_1}{2}T_{2\alpha}^2 + a_1 T_{2\alpha}T_{2\beta} - \frac{a_1 T_{2\beta}^2}{2} + \frac{a_2}{\delta}T_3 \\ & - \frac{a_2 - a_3}{2}T_3^2\Big) \end{aligned}$$

$$IHC = \frac{C_{hi}}{Q}\Big(\frac{p_r(p_r+\alpha p)T_3^2}{2\alpha p}\Big)$$

5.2.4 Integrated model

The integrated model takes all the three different cases into consideration. Here, we assume that with mean $1/\lambda$, the transition time of a machine failure θ is stochastic and follows an exponential distribution. An exponential distribution is often used in reliability engineering to model machine failures (Dallery, 1994). While in this study, we assume that the range of the distribution is long enough to cover all the three cases, so all the three cases can occur with certain probabilities.

$$f(\theta) = \quad \lambda e^{-\lambda\theta}, \qquad 0 \le \theta \le \infty. \tag{5.49}$$

For Case 1, θ needs to satisfy

$$-B + ((1-\alpha)p - \mu)\theta \le 0, \tag{5.50}$$

so the boundary condition for θ is

$$\theta \le \frac{B}{a_1}. \tag{5.51}$$

For Case 2, the boundary condition is calculated as

$$\frac{B}{a_1} \le \theta \le \frac{Q}{p}. \tag{5.52}$$

And lastly in Case 3, the boundary condition is

$$\frac{Q}{p} \leq \theta \leq \infty. \tag{5.53}$$

Hence the expected total cost can be expressed as

$$ETC = \lim_{y\to\infty} E\widetilde{T}C \tag{5.54}$$

where

$$\begin{aligned} E\widetilde{T}C = & \int_0^{B/a_1} TC_1(\theta)f(\theta)d\theta + \int_{B/a_1}^{Q/p} TC_2(\theta)f(\theta)d\theta \\ & + \int_{Q/p}^{y} TC_3(\theta)f(\theta)d\theta. \end{aligned} \tag{5.55}$$

The expected total cost is approximated by Eqn (5.55) with a large y. Parameter y is defined as the upper boundary for the integration in order to reduce the integration complexity in the equations and numerical experiments. The value of y has been set to a large number compared with the mean value of the exponential distribution, so it will not affect the overall result too much. This will be proved in the sensitivity analysis on y.

By substituting all the cost functions and T_1, T_2, T_3, T_4 and T_5 with B and Q, the overall total cost per unit product is obtained. Note that the items with second or higher order α, β, δ and γ have been removed in order to simplify the equation.

$$\begin{aligned} E\widetilde{T}C = & \frac{1}{Q}(C_1 + C_2 e^{-\frac{2\lambda}{p}Q} + C_3 e^{-\frac{2\lambda}{a_1}B} + C_4 B e^{-\frac{2\lambda}{a_1}B}) \\ & + Q(C_5 e^{-\frac{\lambda}{a_1}B} + C_6 e^{-\frac{\lambda}{p}Q} B) + C_7 e^{-\frac{\lambda}{a_1}B} \\ & + BC_8 e^{-\frac{2\lambda}{p}Q} + C_9 B^2 e^{-\frac{\lambda}{a_1}B} \end{aligned} \tag{5.56}$$

where C_1 to C_9 are the coefficients without decision variables, and the simplified results are listed in Appendix B. For optimum values of $ETC(Q,B)$, we make $\frac{\partial TC(Q,B)}{\partial B} = 0$ and $\frac{\partial TC(Q,B)}{\partial B} = 0$ which is equivalent to

$$\begin{aligned} &C_5 e^{-\frac{B\lambda}{a_1}} - \frac{C_1}{Q^2} + BC_6 e^{-\frac{Q\lambda}{p}}(1 - \frac{Q\lambda}{p}) \\ &- \frac{2(C_2 + C_8 B)\lambda e^{-\frac{2Q\lambda}{p}}}{p} = 0 \end{aligned} \tag{5.57}$$

and

$$\begin{aligned} &Q(C_6 e^{-\frac{Q\lambda}{p}} - \frac{C_5 \lambda e^{-\frac{B\lambda}{a_1}}}{a_1}) + (C_4 - \frac{2(C_3 + BC_4)\lambda}{a_1})e^{-\frac{2B\lambda}{a_1}} \\ &+(2C_9 B - \frac{(C_7 + B^2 C_9)\lambda}{a_1})e^{-\frac{B\lambda}{a_1}} + C_8 e^{-\frac{2Q\lambda}{p}} = 0 \end{aligned} \tag{5.58}$$

respectively. The corresponding Hessian Matrix is

$$H = \begin{pmatrix} H_1 & H_2 \\ H_2 & H_3 \end{pmatrix}$$

where

$$\begin{aligned} H_1 &= \frac{2C_1}{Q^3} + \frac{4(C_2 + C_8 B)\lambda^2}{p^2} e^{-\frac{2Q\lambda}{p}} + \frac{C_1 1 B\lambda(Q\lambda - 2p)}{p^2} e^{-\frac{Q\lambda}{p}}, \\ H_2 &= (1 - \frac{Q\lambda}{p})C_6 e^{-\frac{Q\lambda}{p}} - \frac{C_5\lambda}{a_1} e^{-\frac{B\lambda}{a_1}} - \frac{2C_8\lambda}{p} e^{-\frac{2Q\lambda}{p}}, \\ H_3 &= \frac{e^{-\frac{2B\lambda}{a_1}}}{a_1^2}((2C_1 8a_1^2 + (C_7 + B^2 C_9 + C_5 Q)\lambda^2 \\ &\quad -4BC_9 a_1 \lambda)e^{-\frac{B\lambda}{a_1}} + 4(C_3 + BC_4)\lambda^2 - 4C_4 a_1 \lambda). \end{aligned}$$

In this model, the production quantity Q and the backlog quantity B are the two decision variables. If the second order Hessian Matrix of this model is positive definite, then the total cost function is convex and the minimum expected total cost per product can be determined. However, an explicit solution for the Hessian matrix cannot be obtained because of the complexity of the cost function. As an alternative approach, numerical examples are applied to show the convexness of the cost function.

5.3 Numerical Experiments

In this section, numerical examples are first provided to prove the convexness of the cost function and to determine the optimal solutions for the proposed model. Sensitivity analysis is then carried out to measure the impact of different parameters on the system.

5.3.1 Numerical examples

In this example, the values of the parameters are taken as: $p = 6000, p_r = 5000, \alpha = 0.15, \beta = 0.3, \gamma = 0.2, \lambda = 0.05, \delta = 0.1, \mu = 3000, y = 800, C_{hp} = \$4, C_{hi} = \$3, C_p = \$25; C_r = \$10, C_b = \$20, C_{dc} = \$40, C_{dp} = \$30, C_s = \$4000$.

According to Figure 5.5, the 3 dimensional plot of the cost function shows convexness and the global optimum solution can be found. In addition, the graphs of the expected total cost per product against both Q and B are shown in Figures 5.6 and 5.7. The two graphs also help prove the convexness of the proposed cost function.

Hence the optimum combination of Q and B are determined as $(Q^*, B^*) = (4223, 456)$, and the optimal total cost per unit product is:

$$ETC^* = 29.3$$

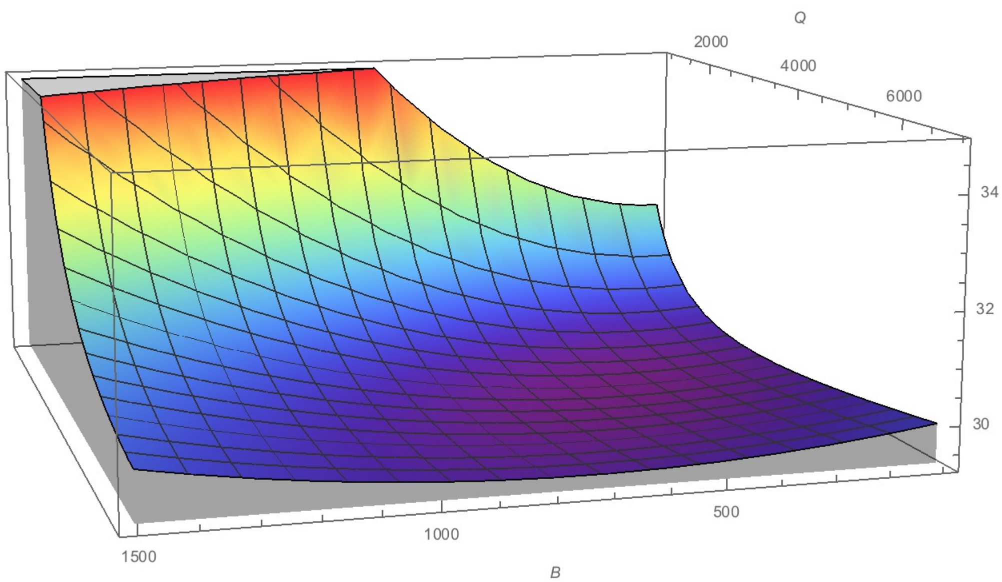

Figure 5.5: The plot of expected total cost per unit product against Q and B

5.3.2 Sensitivity analysis

The sensitivity analysis for the parameters is studied as follows, and mainly consists of three parts. First of all, the inventory performance against system parameters is analyzed, as shown in Table 5.1, then followed by the impact of the cost coefficients on the system performance as in Table 5.2. Lastly, the ratios between Q and B are calculated in each different scenario to examine the relationship between the backlog and total production quantity, as in Table 5.3. The results and sub-conclusions are provided below. In this sensitivity analysis, each parameter listed is varied in four different levels: 50%, -25%, 25% and 50%.

The new optimum solutions (Q^*, B^*) and their corresponding expected total costs per product are calculated and compared with the results from the original optimal solution. In Tables 1 and 2, instead of showing the optimal solutions, the differences

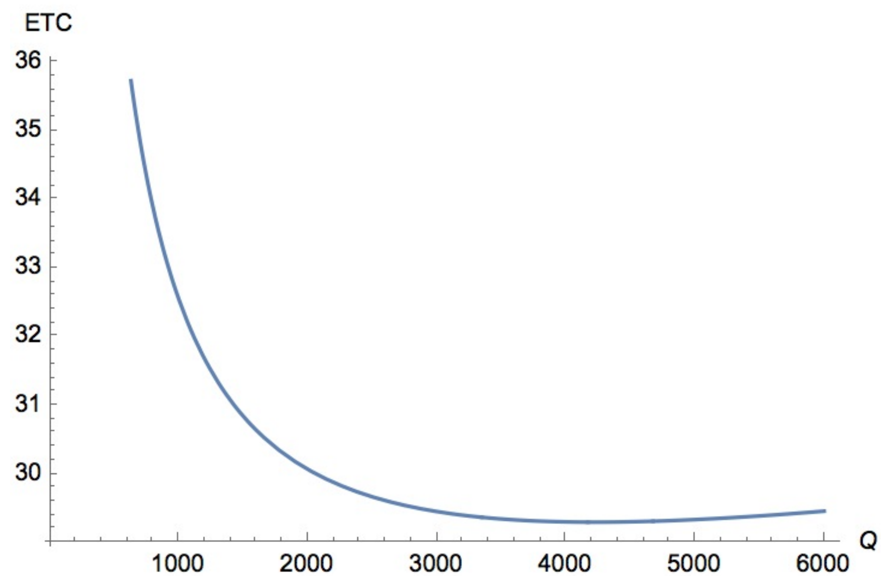

Figure 5.6: ETC against Q

between the original optimal solution and the new solutions are provided for better illustration. In Table 5.3, the values of Q/B are directly provided. Note that the experiments on $0.5 \times p$, $0.5 \times p_r$ and $1.5 \times \mu$ cannot be implemented, otherwise the production speed is lower than the demand rate which makes the model invalid.

Starting with the system parameters, the expected total cost per product is significantly affected by the values of α, λ and μ. α and λ have a positive relationship with the total cost per unit product. λ, as the parameter for exponential distribution, influences the mean time of production with a low defective rate. Hence high λ, which represents low mean time, causes the total cost to rise, but when the mean time can be extended, the total cost can be reduced to a large extent. A high value of α leads to an increasing number of defective products which will result in a higher reproduction cost and backlog cost.

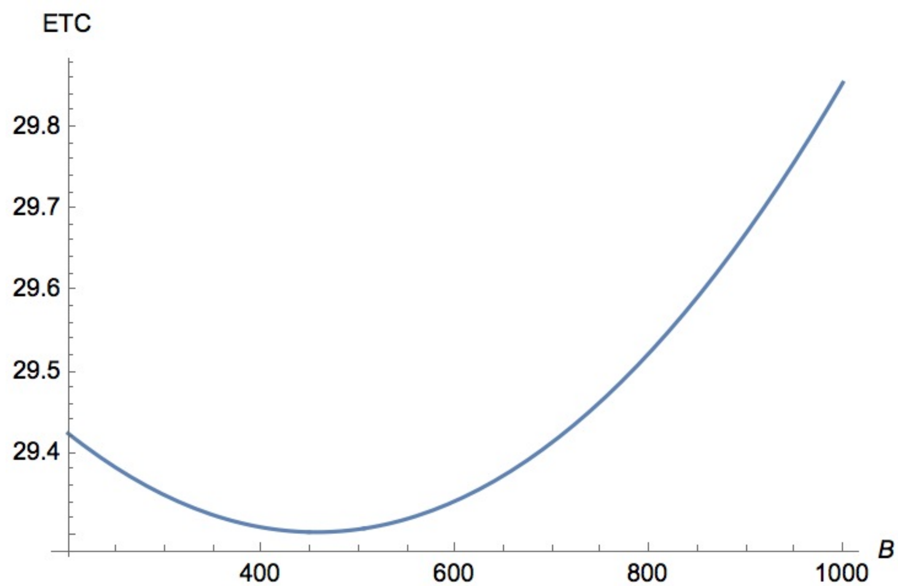

Figure 5.7: ETC against B

The total production quantity is sensitive to the changes in p and μ. The value of Q drops rapidly when the production rate changes from -50% to 50% of the original value. On the contrary, Q rises along with the demand rate because more products need to be produced in order to satisfy the customer demand. The impact of the deterioration rate on system performance is limited, but the increase in the deterioration rate makes the total production quantity smaller and the total cost per unit product higher. In terms of cost parameters, C_{hp}, C_{hi},C_b and C_{dc} show negative relationships with the total production quantity, but both the production quantity and backlog grow with the defect disposal cost and reproduction cost. The impact of the production cost on the total production quantity and backlog can be neglected. A high setup cost will increase both the production quantity and the backlog cost, which can be interpreted as the need to produce more products to share the setup cost.

In Table 5.3, the ratio of Q to B can be interpreted as the relationship between the backlog and inventory surplus which can provide help with the decision making process regarding the backlogging quantity. On the one hand, a high inventory surplus has an excessive inventory holding cost and leads to extra product deterioration. On the other hand, a large backlog leads to a high penalty cost. How to balance the inventory surplus and backlog in different situations is one of the key problems in inventory management. The lower the ratio is, the more the products are used to satisfy the backlog at the beginning of each production run.

According to the results in Table 5.3, the production rate p, C_{hp} and C_{dc} can influence the $\frac{Q}{B}$ ratio to a large extent, especially the production rate. When the value of these three parameters increase, the ratio drops which means that a larger portion of items produced is sent to meet the backlogged demand. On the contrary, the ratio rises significantly with increase of the production defective rate α and β, demand μ and C_b. The changes brought about by the other parameters are relatively smaller. For example, an increase of δ leads to more backlog in the total production quantity since deterioration only occurs in the periods with inventory surplus. In the end, it can be concluded that the value of y does not have a significant influence on the result of the model.

5.4 Summary

On the topic of EPQ with an imperfect production system, a large amount of research has been done. Particularly for deteriorating production systems, the extant literature has provided comprehensive analysis. However, as discussed earlier, in some industries the phenomenon of product deterioration cannot be neglected. What is the inventory behavior under the condition of both product and processes deterioration needs to be examined and analyzed. This study has modeled such a problem, considering rework and backordering at the same time. Both the production and rework process are assumed to be imperfect, but only the normal production process is subject to deterioration. At the end of each production run, maintenance activities are applied to enable the production system to recover to a good condition. The deterioration of production process is categorized into three different cases depending on the occurrence of deterioration in different production stage: Backlog stage, surplus stage and post-production stage. The optimal expected total cost per unit product is obtained by simultaneously determining the total production quantity and backlog quantity in one production run. Numerical experiments are carried out and used to illustrate the performance of the proposed model. Sensitivity analysis shows that the model is sensitive to the changes in different parameters, and their corresponding impact on the inventory performance are discussed and summarized.

Table 5.1. Sensitivity analysis for different parameters

Parameters		Optimal solutions				
		Original setting	Differences with original setting			
		0	−50%	−25%	25%	50%
p	ETC^*	29.3	NA	−0.49	0.16	0.21
	Q^*	4223	NA	1503	−327	−432
	B^*	456	NA	−264	193	359
p_r	ETC^*	29.3	NA	−0.03	0.02	0.04
	Q^*	4223	NA	77	−41	−67
	B^*	456	NA	31	−17	−29
α	ETC^*	29.3	−1.18	−0.59	0.6	1.19
	Q^*	4223	−35	−18	18	34
	B^*	456	87	44	−44	−89
β	ETC^*	29.3	−0.14	−0.07	0.05	0.09
	Q^*	4223	356	151	−107	−184
	B^*	456	73	36	−36	−77
γ	ETC^*	29.3	−0.41	−0.2	0.21	0.42
	Q^*	4223	−75	−39	42	88
	B^*	456	−19	−9	10	20
δ	ETC^*	29.3	−0.11	−0.05	0.06	0.11
	Q^*	4223	166	58	−39	−69
	B^*	456	15	4	−2	−3
λ	ETC^*	29.3	0.17	0.09	−0.07	−0.34
	Q^*	4223	−411	−217	241	850
	B^*	456	−74	−38	42	187
μ	ETC^*	29.3	1.22	0.54	−0.5	NA
	Q^*	4223	−1661	−945	1535	NA
	B^*	456	277	133	−141	NA
y	ETC^*	29.3	0	0	0	0
	Q^*	4223	0	0	0	0
	B^*	456	0	0	0	0

Table 5.2. Sensitivity analysis for different cost parameters

Parameters		Optimal solutions				
		Original setting	Differences with original setting			
		0	−50%	−25%	25%	50%
C_{hp}	ETC^*	29.3	−0.3	−0.13	0.12	0.21
	Q^*	4223	827	335	−243	−429
	B^*	456	−83	−36	30	56
C_{hi}	ETC^*	29.3	−0.1	−0.04	0.05	0.1
	Q^*	4223	239	115	−105	−203
	B^*	456	26	13	−11	−21
C_{dp}	ETC^*	29.3	−0.42	−0.21	0.22	0.43
	Q^*	4223	−53	−27	28	56
	B^*	456	−1	0	1	2
C_p	ETC^*	29.3	−12.5	−6.25	−3.75	12.5
	Q^*	4223	0	0	0	0
	B^*	456	0	0	0	0
C_r	ETC^*	29.3	−0.7	−0.35	0.36	0.71
	Q^*	4223	−88	−44	47	95
	B^*	456	−2	−1	2	3
C_b	ETC^*	29.3	−0.18	−0.07	0.06	0.09
	Q^*	4223	424	159	−105	−181
	B^*	456	270	104	−70	−122
C_s	ETC^*	29.3	−0.55	−0.25	0.23	0.43
	Q^*	4223	−1245	−571	505	963
	B^*	456	−131	−60	53	100
C_{dc}	ETC^*	29.3	−0.08	−0.04	0.04	0.08
	Q^*	4223	194	93	−85	−164
	B^*	456	−10	−4	5	9

Table 5.3. Sensitivity analysis for ratio of Q and B

Parameters	Optimal value	-50%	-25%	0	25%	50%
p	$(Q/B)^*$	NA	29.82	9.26	6	4.65
p_r		NA	8.83		9.53	9.73
α		7.71	8.41		10.29	11.6
β		8.66	8.89		9.8	10.66
γ		9.49	9.36		9.15	9.06
δ		9.32	9.31		9.22	9.17
λ		9.98	9.59		8.97	7.89
μ		3.5	5.57		18.28	NA
y		9.26	9.26		9.26	9.26
C_{hp}		13.54	10.85		8.19	7.41
C_{hi}		9.26	9.25		9.25	9.24
C_{dp}		9.16	9.2		9.3	9.34
C_p		9.26	9.26		9.26	9.26
C_r		9.11	9.18		9.32	9.41
C_b		6.4	7.83		10.67	12.1
C_s		9.16	9.22		9.29	9.33
C_{dc}		9.9	9.55		8.98	8.73

Chapter 6 Discussion

In this chapter, we discuss the proposed models and methodologies used in this research. The results are then analyzed and interpreted in terms of their implications and corresponding managerial insights. In the end, the limitations are highlighted for each proposed model and methodology.

6.1 Integrated Forecasting and Hedging Point Based Control Problem

Chapter 3 presents a study on the integrated forecasting and hedging point based control problem. The demand forecasting process is simulated and categorized into two different cases. First of all, a two-level control policy is proposed to solve the problem with the Markov modulated Poisson demand process, which is often used in qualitative forecasting. Then, the forecasting process, using time series methods, is modeled and a forecast-corrected control policy is proposed accordingly. A simulation-based experiment design and response surface methodology is applied to solve the proposed problem.

In Case 1, a two-level hedging point-lot size control policy is proposed to handle the quantitative demand forecasting. Especially when forecast demand process can be described with two-state Markov process, the two-level hedging point-lot size control policy is suitable to solve this type of problem. The proposed simulation-based experiment design and response surface methodology is proven to be efficient and effective in the study. Compared with analytical and heuristic methods used in previous literature, this method is more practical to solve extremely complex problems. The question of what is the best combination of lot size and hedging point for various levels of low demand period and high demand period can be answered.

In Case 2, a forecast-corrected hedging point control policy is proposed and the impact of different forecasting methods on the performance of control policy is investigated. Results show that proposed forecast-corrected control policy can provide a better performance than the traditional policy. However when the forecasting errors are large enough due to the pattern of demand and forecasting methods, stationary control policy is preferred instead of non-stationary policy. Stationary control policy can then be used to provide a good approximation to the non-stationary control. This study helps us gain some significant insights about which policy should be chosen when the forecasting inaccuracy is known to a company. Managers are able to choose the right control policy and forecasting methods based on the information from demand forecasting. With an accurate forecasting method, non-stationary control policy is recommended to be used in production and inventory control.

Although this study has achieved the objectives stated at the beginning of the study, there are still some limitations and shortcomings in this study. First of all, a single product and single machine is modeled in the system. For multi-product or multi-machine problem, our study is not suitable for them. However the analysis process will become extremely complicated if multi product or multi machine problem are considered. And also a Single product, single machine model can provide a solid theoretical foundation for multi product multi machine one.So this study adopted single part single machine problem as a result.

6.2 EPQ Model with Deterministic and Stochastic Deteriorating Production Process

Chapter 4 and Chapter 5 investigated the economic production quantity model jointly considering inventory, imperfect production process and rework. In the imperfect production system, not only does the machine produce defective product but also the machine subjects to deterioration. There are two main differences between the study in Chapter 4 and 5. First of all, the production process deterioration is modeled with a deterministic piece-wise function in Chapter 4. The defective rate of production process increases after every certain period of time θ by the rate of γ. Hence the longer the production time, the higher the defective rate.

The work in Chapter 5 assumes a stochastic deterioration process which is modeled with an exponential distribution. At the beginning of each production run, the defective rate is low but after a random period of time which follows exponential distribution, the defective rate increases to higher value. And due to introduction of stochastic switch time, the problem should be subdivided into three cases. Expected total cost per product is determined by considering all the three cases at the same time. Hence the complexity of problem in Chapter 5 is higher than that in Chapter 4. Secondly, the rework process is viewed as imperfect in the second model. Scraped products are disposed at the end of the production run with extra dispose cost. However in the first model, the rework process is assumed to be perfect and all the imperfect products are reworked together and become perfect products afterwards.

Results show the proposed models and methodologies have satisfactory performance. Sensitivity analysis illustrated the impact of different parameters on the system performance especially the effect of inventory and production process deterioration. The integration of inventory deterioration with EPQ makes the study applicable to certain specific industries where products have a limited life-time, the food industry, for example. In addition, sensitivity analysis can help decision makers in terms of the right production/backlog quantity and provide insights on how to reduce the total cost in their manufacturing systems through the control of system parameters. Finally, although single product single machine problem are modeled, the two studies are especially suitable for cellular manufacturing systems in which similar products or product families are normally manufactured with fixed or identical routes.

It should be noticed that numerical experiments are utilized in both of these two studies to show the convexness of the two cost functions because it cannot be proved directly through analytical approach, which normally makes use of Hessian matrix to prove the convexness, due to the complexity of the function. Hence the exact solutions cannot be obtained, instead the near-optimal results are determined by using numerical experiments. This method has also been accepted and employed in other studies especially when the problem itself is complicated.

Limitations also exist in these two studies.The deterioration of inventory was assumed to occurs only in normal production period rather than in the rework process. It is because of the relatively short rework time, in which the deterioration during rework is neglected. The demand in these two models are assumed to be constant. The case with stochastic demand could be examined as an extension of this study.

6.3 Future Work

As discussed earlier in the previous chapter, there are some some assumptions and limitations made in this research.Further improvement can be implemented with the following suggestions:

For forecast-corrected hedging point based control policy:

- A single product, single machine problem is modeled in this research. It is especially suitable for cellular manufacturing system. However for other types of manufacturing system such as job-shop manufacturing system, the multi-product or multi-machine model can be a better choice. The same problem can be extended to multi-product and multi-machine version. Lot scheduling problem should be integrated together with forecast corrected hedging point control policy.

- In this research, non-stationary demand and batch production are introduced into the hedging point based control problem. It would be interesting to investigate the impacts of other uncertainties on the system as well as integrating other supply chain activities, such as transportation and replenishment into the model.

For EPQ model with inventory and production process deterioration:

- For EPQ model, we have examined two types of deterioration process for production which are linear piece-wise and stochastic deterioration. However there are other deterioration types that worth looking into. A more general exploration of other types of deterioration process can be done.

- We have only considered production process deterioration in our research. However machine breakdown is also common in reality. Machine breakdown refers to the situation in which machines subject to failures and production process is stopped once the machine is broken. Hence an integrated EPQ problem considering machine breakdown and deterioration is of great interests to be investigated.

- Similar to the hedging point based control problem, the EPQ models proposed in this research can be extended to the case with multi-product and multi-machine. Lot scheduling methods should also be utilized to solve the extended model.

- In the current research, we assumed that the all the unsatisfied demand is backlogged and at the beginning of each production run the backlogged orders are satisfied first. Models with partial backlog or lost sales should also be studied.

- The current models use deterministic demand and defective rate. While in industry, random and price-dependent demand are commonly observed. Especially for price-dependent demands, a pricing strategy can be introduced and can be further integrated with game theory for the problem with multiple suppliers.

Chapter 7 Conclusions and Future Work

This chapter first reviews the background of the this research and then the findings and contribution of our study are summarized. Lastly, the possible future work as a result of this study is discussed.

7.1 Conclusions

Production management has always been a important research topic in both the academic and industrial world and has been divided and extended into many different subtopics due to the everlasting pursuit of improving the time and cost efficiency in production, for instance, production and inventory control, capacity planning, master production schedule, shop floor control, logistics planning and even process design. Among the all the subtopics, production and inventory control is the one of the most popular subtopics. Models such as EPQ, EOQ, ELSP and hedging point based control have been intensively studied and more and more practical factors are considered in extensions of the traditional models. However, despite the advances in the research of production and inventory control, research gaps still exist in the literature.

In the hedging point based control problem, a joint model considering hedging point policy and lot sizing with non-stationary demand is missing. The solutions from previous research are not applicable to the batch production processes. In addition, the impact of forecasting accuracy on hedging point based control policy has not been examined. In most of the existing EPQ models, inventory deterioration and production process deterioration are not considered at the same time even though both types of deterioration are common in real life.

In order to bridge the gaps in literatures and between research and practice, this research first modeled the forecasting-hedging-point-based control problem and provided the mathematical formulation for this problem. An integrated simulation, design of experiment and response surface method is employed to solve problem. The method can provide near-optimal solutions while the problem is extremely hard and time consuming to solve using analytical approaches. Two modified control policies are proposed to handle the forecast demand and the influence of forecasting methods are measured and compared with the traditional control policies.

Later, two modified EPQ models are proposed which considered both inventory and production process deterioration. What's more, two types of process deterioration patterns are modeled to reflect the real machine deterioration process. Results showed that all the three models proposed have meet the objectives of the research mentioned and can generate satisfactory performance. The methods proposed to solve the problems are capable to provide good solutions. This research can contribute to both academia and industry.

The main contribution of this research can be concluded from two different perspectives. First of all, this research fulfills research gaps in the area of production/inventory control as indicated previously. Secondly, managerial insights can be obtained from the research to aid decision making processes in manufacturing companies. For a manager in a company with imperfect production system, factors such as safety stock level, production quantity and backorder quantity are the most important to consider. But at the same time, managers should pay attention to the other factors such as demand uncertainty and product deterioration which can lead to profit losses. How should managers decide what type of control policy to use to tackle non-stationary demand? How should production plan be adjusted to take deteriorating product into consideration? Our research can provide guidance for them to find their own optimal production and inventory control plan. The overall cost in production and inventory can therefore be reduced by using the control policies proposed.

In addition, sensitivity analysis conducted in Study 2 and 3 can provide managers insights about the influence of each parameter on the total cost. It generates decision support in terms of which cost contribute the most to overall cost and which cost should be controlled in order to further improve the performance of system.The contribution of each study can be further summarized as follows:

For hedging point based production and inventory control policy:

- A modified forecast-corrected two-level hedging point control policy is proposed. By using the proposed methods, an optimal solution for determining the optimum control policy under the condition of non-stationary demand and batch production can be obtained. The impact of forecasting on the performance of a manufacturing system subject to failure and non-stationary demand is investigated.

- This study can help decision makers with manufacturing system management, and provide insights on how to reduce the total cost in their systems.

Managers in manufacturing companies are also suggested to utilize non-stationary production-inventory control policy because it leads to a better performance in terms of cost. However, managers should understand under what kind of conditions they can use non-stationary control policy. The impact of forecasting errors on the systems should be given attention by managers. In addition, they should be cautious about the choice of forecasting methods according to the type of demand process, since different forecasting methods generate different accuracies.

For EPQ models:

- From an academic perspective, the study extends the previous literature by taking two types of deterioration process into consideration. Compared with the traditional modeling method, linear piece-wise and stochastic deterioration are more suitable and practical to model the production system. Despite the increasing complexity of the model itself, we have proposed an efficient method to determine the optimal solutions for the control problem.

- From the application perspective, this study looks at the inventory deterioration problem in production and inventory control. Hence, it makes the study applicable to certain specific industries where products have a limited life-time and the storage condition/time has a significant impact on the quality, like food, electronic devices and metals industries. In addition, as a single product problem is modeled, the model in this thesis is especially suitable for cellular manufacturing systems in which similar products or product families are manufactured.

Appendix A: Equations and Coefficients of Cost Function in Chapter 5

According to Eqns (5.14) and (5.15)

$$\Big(I_s - \frac{a_3}{\delta}\Big)\exp(-\delta T_3) + (\frac{a_3}{\delta}) \quad = \quad \frac{\mu}{\delta}\Big(\exp(\delta T_4) - 1\Big) \tag{A.1}$$

Substitute Eqn (5.13) into this equation

$$\begin{aligned} &\Big((\frac{a_2}{\delta})(1-\exp(-\delta T_2)) - \frac{a_3}{\delta}\Big)\exp(-\delta T_3) + \frac{a_3}{\delta} \\ &= (\frac{a_2}{\delta})(1-\exp(-\delta T_2))\exp(-\delta T_3) - \frac{a_3}{\delta}(1-\exp(-\delta T_3)) \\ &= \frac{\mu}{\delta}\Big(\exp(\delta T_4) - 1\Big). \end{aligned} \tag{A.2}$$

Assume δT_2 and δT_3 is small, using Taylor expansion theory

$$\begin{aligned} \exp(-\delta T_2) &\approx 1 - \delta T_2 + \frac{(\delta T_2)^2}{2}, \\ \exp(-\delta T_3) &\approx 1 - \delta T_3 + \frac{(\delta T_3)^2}{2}. \end{aligned}$$

Eqn (A.1) can be simplified to

$$\begin{aligned} &\Big((\frac{a_2}{\delta})(1-\exp(-\delta T_2)) - \frac{a_3}{\delta}\Big)\exp(-\delta T_3) + (\frac{a_3}{\delta}) \\ &= \frac{\mu}{\delta}\Big(\exp(\delta T_4) - 1\Big). \end{aligned} \tag{A.3}$$

To further reduce the complexity, $1-\frac{\delta T_2}{2}\approx 1, 1-\frac{\delta T_3}{2}\approx 1, 1-\frac{\delta T_4}{2}\approx 1$. T_4 is simplified as

$$T_4 = \frac{a_2}{\mu}T_2 - \frac{a_3}{\mu}(T_3 - \frac{\delta T_3^2}{2}). \tag{A.4}$$

In what follows, the coefficients used in Eqn (5.56) are.

$$\begin{aligned}
C_1 = \frac{1}{(a_2\lambda^3\mu p_r^3)}\Big[&3a_1^3a_2a_3C_{hp}\delta(a_3+p_r)+a_1a_2(9a_2^2a_3C_{hp}\delta \\
&(a_3+p_r)-p_r^2\lambda\mu(C_{bc}p_r+2C_{hp}p_r+(2a_3C_{dc}+C_rp_r \\
&+C_{dp}p_r\gamma)\lambda)-2a_2p_r\lambda(C_{hp}a_3^2+C_{hp}p_r(p_r-2\mu) \\
&+(C_{hi}p_r(2+\gamma)-C_{dc}\delta)\mu+a_3C_{hp}(2p_r+\mu))) \\
&+a_1^2(-9a_2^2a_3\delta C_{hp}(a_3+p_r)+(C_{bc}+C_{hp})p_r^3\lambda\mu \\
&+a_2p_r\lambda(a_3^2C_{bc}+C_{bc}p_r(p_r-2\mu)+(C_{hi}p_r(2+\gamma) \\
&-C_{dc}\delta)\mu+a_3C_{hp}(2p_r+\mu)))+a_2(-3a_2^3a_3C_{hp}(a_3+p_r)\delta \\
&+C_{hi}pp_r^3(\beta-\alpha)\lambda\mu)+a_2p_r^2\lambda(C_{hp}p_r+(2a_3C_{dc}+p_r(C_r \\
&+C_{dp}\gamma))\lambda)\mu+a_2^2p_r\lambda(a_3^2C_{hp}+C_{hp}p_r(p_r-2\mu)+(C_{hi}p_r \\
&(2+\gamma)-C_{dc}\delta)\mu+a_3C_{hp}(2p_r+\mu))\Big]+(1-e^{-y\lambda})C_s
\end{aligned}$$

$$\begin{aligned}
C_2 = \frac{1}{(p_r^2\delta\lambda^2)}\Big[&a_2\Big(-C_{dc}p_r^2\delta^2\lambda+C_{hp}p(p(\delta-2\alpha\delta) \\
&-2p_r(-1+\alpha)\lambda)\Big)-\delta\Big(a_3p(C_{hp}(p-2p\alpha)
\end{aligned}$$

$$+C_{dc}p_r(-1+\alpha+\delta)\lambda)+p_r(pp_r(C_r(-1+\alpha)-C_{dp}\gamma)\lambda$$

$$-a_1p_r(2C_{hp}+C_{dc}\delta\lambda)$$

$$+C_{hi}(p_r\alpha\beta+p^2(\gamma-2\alpha\gamma)+pp_r(\beta+\alpha\lambda)))\Big)\Big]$$

$$C_3=\frac{1}{(a_2\lambda^3\mu p_r^3)}\Big[-3a_1^3a_2a_3C_{hp}\delta^2(a_3+p_r)+a_1^2(9a_2^2a_3C_{hp}\delta$$

$$(a_3+p_r)-p_r^3\lambda\mu(C_{bc}+C_{hp})-a_2p_r\lambda(C_{hp}a_3^2+C_{hp}p_r$$

$$(p_r-2\mu)+(C_{hi}p_r(2+\gamma)-C_{dc}\delta)\mu+a_3C_{hp}(2p_r+\mu)))$$

$$+a_1a_2\delta(-9a_2^2a_3C_{hp}(a_3+p_r)+(C_{bc}p_r+(2a_3Cdc$$

$$+p_r(Cr+Cdp\gamma-Cdc\delta))\delta)p_r^2\lambda\mu+2a_2p_r\lambda(a_3^2C_{hp}$$

$$+C_{hp}p_r(p_r-2\mu)+(C_{hi}p_r(2+\gamma)-C_{dc}\delta)\mu$$

$$+a_3C_{hp}(2p_r+\mu)))+a_2(3a_2^3a_3C_{hp}(a_3+p_r)\delta^2$$

$$+C_{hi}pp_r^3(\beta-\alpha)\lambda\mu)+p_r\delta\lambda(a_3p(C_{hp}(p-2p\alpha)$$

$$+C_{dc}p_r(-1+\alpha+\delta)+p_r(pp_r(Cr(-1+\alpha)$$

$$-Cdp\gamma)\lambda+Chi(p_r\alpha\beta+p^2(\gamma-2\alpha\gamma)$$

$$+pp_r\alpha(1+\lambda))))\mu-a_2^2p_r\delta\lambda(a_3^2C_{hp}+C_{hp}p_r$$

$$(p_r-2\mu)+(C_{hi}p_r(2+\gamma)-C_{dc}\delta)\mu+a_3C_{hp}(2p_r+\mu)))\Big]$$

$$C_4=C_r+C_{dp}\gamma+\frac{1}{a_1\lambda\mu}(C_{hp}a_2^2+a_2(C_{hp}+((1-\lambda)C_{dc}$$

$$+C_r+C_{dp}\gamma)\mu+p((C_r(1-\alpha)-C_{dp}\gamma)\lambda$$

$$-C_{hi}\alpha(1+\lambda))\mu)+\frac{1}{a_1p_r\delta\lambda\mu}(a_2(C_{hp}p(\alpha-1)$$

$$-2a_3C_{dc}\delta)\lambda\mu+p\delta(C_{hi}p\gamma+a_3C_{dc}(-1+\alpha+\delta)\lambda)\mu)$$

$$-\frac{(a_2^2+a_1^2)\delta(2a_3C_{hp}+(-2C_{hp}+C_{hi}(2+\gamma))\mu)}{a_1p_r\delta\lambda\mu}$$

$$-\frac{3(a_1-a_2)^3a_3^2C_{hp}\delta}{a_1p_r^3\lambda^2\mu}+\frac{C_{hp}(a_2+\mu)}{\lambda\mu}+\frac{1}{a_1p_r\delta\lambda\mu}(a_1\delta$$

$$(2a_3C_{dc}\lambda\mu+a_2(3a_3C_{hp}+(-3a_3C_{hp}+2C_{hi}(2+\gamma))\mu)))$$

$$-\frac{1}{a_1p_r^2\lambda^2\mu}(C_{hp}(2a_1a_2a_3(4a_2\delta-\lambda(a_3+\mu))+a_1^2a_3$$

$$(-7a_2\delta+\lambda(a_3+\mu))))-\frac{1}{a_1p_r^2\lambda^2\mu}(C_{hp}((2a_1^3-3a_2^3)a_3\delta$$

$$+(a_2-a_3)p^2(1-2\alpha)\lambda\mu+a_2^2a_3\lambda(a_3+\mu)))$$

$$C_5=C_p(1-e^{-y\lambda})+C_r(\beta-\alpha e^{-y\lambda})+\frac{1}{p_r\lambda\mu}(\beta(2a_3C_{dc}\lambda\mu$$

$$-(a_1-a_2)(a_3C_{hp}+(C_{hi}(2+\gamma)-C_{hp})\mu))+\frac{C_{hi}(\alpha-\beta)}{\lambda}$$

$$+\frac{C_{hp}(a_1-a_2)(a_2+\mu)}{p\lambda\mu}+\frac{1}{pp_r^2\lambda^2\mu}(C_{hp}(a_1-a_2)a_3$$

$$(a_1a_2\delta-a_2^2\delta-p\beta\lambda(a_3+\mu)))+\frac{C_{hp}(a_1-a_2)a_2(a_3-\mu)}{pp_r\lambda\mu}$$

$$C_6=\frac{1}{pp_r^2\delta\lambda}((1-\beta)C_r-C_{hi}\alpha+C_{dp}\gamma-\frac{(a_2-a_1\delta)C_{dc}}{p}$$

$$+\frac{((1-\alpha-\beta-\delta)a_3)C_{dc}}{p_r}+\frac{((1-\alpha-\beta)a_2)C_{hp}}{p_r\delta})$$

$$+\frac{p\gamma C_{hi}}{p_r\lambda}+\frac{(1-2\alpha-\beta)(a_2-a_3)pC_{hp}}{p_r^2\lambda}$$

$$C_7 = -\frac{C_{hi}\alpha}{\lambda} + \frac{C_{dc}a_2(1-\delta)}{p} - \frac{C_{hp}(a_1-a_2)a_2}{p\lambda\mu}$$

$$+ \frac{C_{hp}(a_1+a_2)}{p\lambda} + \frac{C_{hp}(a_1a_3 - a_3p + a_2(-a_3+p))\beta}{p_r^2\lambda}$$

$$- \frac{\beta(2C_{hi}a_2 - C_{hp}a_2 + a_1(-2C_{hi} + Chp) + a_3C_{dc}\lambda)}{p_r\lambda}$$

$$+ \frac{C_{hp}(\mu - a_3)(a_1-a_2)a_2}{pp_r\lambda\mu} - \frac{C_{hp}((a_1-a_2)^2a_2a_3)\delta}{pp_r^2\lambda^2\mu}$$

$$+ \frac{C_{hp}(a_1-a_2)a_3\beta}{p_r\lambda\mu} + \frac{C_{hp}(a_1-a_2)a_3^2\beta}{p_r^2\lambda\mu} + \frac{C_{hp}a_2\beta}{p_r\delta}$$

$$C_8 = C_{dc}(\frac{a_2}{a_1} - \delta)$$

$$C_9 = \frac{a_2-a_1}{a_2p_r^2\lambda^2\mu}(a_1a_2a_3C_{hp}\delta - a_2^2a_3C_{hp}\delta$$

$$+ a_2C_{hp}p_r\lambda(a_3+p_r-\mu) + (C_{bc}+C_{hp})pr^2\lambda\mu)$$

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