SELECTED TOPICS IN INTERDISCIPLINARY OPERATIONS MANAGEMENT: BEHAVIORAL PRICING, OMNICHANNEL AND HORIZONTAL MERGER

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Selected Topics in Interdisciplinary Operations Management: Behavioral Pricing, Omnichannel and Horizontal Merger

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CERTIFICATE OF ORIGINALITY

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Abstract

This thesis selects three topics from the interface of operations and marketing. It focuses on the interplay between a firm and its strategic consumers as well as its competitors. It consists of three studies.

In the first study, we investigate the effects of valuation uncertainty and consumers' anticipated regrets on a retailer's pricing decision and strategy. We consider a firm selling two substitutable products in a selling season with two periods: A single product is sold in each period. The firm either commits both prices at the beginning of the selling season (price commitment), or dynamically sets the price at the beginning of each period (dynamic pricing). Besides, a consumer may experience purchase or wait regret. Regret is defined as the disutility of not having chosen the ex post best-forgone alternative. We focus on examining whether and how the retailer can use intertemporal prices and pricing strategies to mitigate consumers' strategic behavior. We find that a firm may need to set a lower price in the first period in order to mitigate strategic waiting. This is true even when the product sold in the first period is more attractive. Besides, the effects of purchase regret and wait regret on the optimal prices may be non-monotone under price commitment. In addition, price commitment always dominates dynamic pricing and the value of commitment depends on the valuation uncertainty and consumers' anticipated regrets.

In the second study, we turn our attention from pricing management to channel management and study how far a retailer should go with omnichannel selling strategy. Specifically, we analyze whether the retailer should keep the traditional selling, step out to use the research online and purchase offline (ROPO) strategy, or go further to adopt the buy online and pick up in store (BOPS) strategy. We find that both the ROPO and BOPS strategies may or may not be optimal for the retailer. We derive the conditions under which the retailer should implement the ROPO or BOPS strategy. For example, we show that if the profit of online retailing is small, then the retailer should adopt the ROPO strategy. By contrast, if the hassle cost of using the BOPS function is low, and the profit of online retailing is big or each consumer brings a great cross-selling benefit, then the BOPS strategy is optimal for the retailer.

Furthermore, since many firms adopt mergers and acquisitions (M&As) as an important strategy in competitive business environment, we look at a big picture and study competing firms' merging decision and strategy in a competitive market. We develop a game-theoretical model, in which multiple firms compete on price and quality and two of them decide whether and how to merge. The post-merger firm achieves cost saving and needs to further decide the degree of post-merger integration, i.e., centralized merger or decentralized merger. We focus on examining whether two competing firms should merge and which merging strategy (i.e., centralized or decentralized merger) is optimal for the post-merger firm when facing competition from the nonparticipant firm in the market. We find that the post-merger firm prefers decentralized merger when market competition is fierce enough; otherwise, it should choose centralized merger. Besides, if both centralized and decentralized mergers are possible, then the competing firms should always choose to merge. However, if centralized merger is not possible, then the participant firms may be worse off after merger when horizontal differentiation level is low.

Publications Arising from the Thesis

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Chapter 1

Introduction

Problems in the operations area do not stand alone and interesting results and insights may be found if we conduct interdisciplinary research. Therefore, this thesis selects three topics from the interface of operations and marketing. It develops three game-theoretical models to study (1) the effects of valuation uncertainty and consumers' anticipated regrets on a seller of substitutable products, (2) the differences between the research online and purchase offline (ROPO) and buy online and pick up in store (BOPS) strategies, and (3) competing firms' merging decision and strategy in a competitive market. Brief introductions of these studies are as follows.

Study 1. Many firms provide consumers with substitutable products which may be launched sequentially over several periods. For example, Bloomsbury Publishing PLC publishs the book Harry Potter and the Philosopher's Stone in different Editions every several years (Bloomsbury 2018). If strategic consumers know that there will be a substitutable product in the near future, they need to make purchase-or-wait decisions at present. However, they may have uncertain valuation of the future product when such decisions are made. Besides, decisions made under uncertainty may lead to regrets when the uncertainty is realized ex post.

Inspired by these observations, in Chapter 2, we investigate the effects of valuation uncertainty of the future product and consumers' anticipated regrets on consumers' behavior and firm's pricing decision and strategy. We develop a game-theoretical model, in which a firm selling two substitutable products in a selling season of two successive periods. A single product is sold in each period. The firm chooses pricing strategy and sets prices to maximize its profit. It either commits both prices at the beginning of the selling season (price commitment) or dynamically sets the price at the beginning of each period (dynamic pricing). Given the firm's pricing strategy and prices, consumers make decisions to maximize their own net surplus with the consideration of anticipated regrets. Each consumer may experience purchase regret if she chooses to purchase in the first period and wait regret if she decides to wait in the first period.

We find that in order to mitigate strategic waiting, a firm may need to set a lower price in the first period. This is true even when the product in this period is more attractive (i.e., the average valuation in this period is higher). Besides, under price commitment, the effect of regret on the optimal price can be non-monotone when the uncertainty level is high. In addition, we show that dynamic pricing is always dominated by price commitment and the value of price commitment depends on the valuation uncertainty and anticipated regrets.

Study 2. Nowadays, many retailers have started to integrate the online and offline channels with the help of ongoing digitalization in marketing and retailing (ForresterResearch 2014, Gao and Su 2016a). There are two typical cross channel strategies, i.e., research online and purchase offline (ROPO) and buy online and pick up in store (BOPS). Different strategies have different features. For instance, the BOPS strategy may cause return in physical store while the ROPO strategy does not. We notice that different retailers adopt different strategies regarding the ROPO and BOPS. For example, GU adopts the ROPO strategy in Hong Kong while Uniqlo adopts the BOPS strategy in both Hong Kong and Mainland China (GUHK 2019, UniqloHK 2019, UniqloChina 2018).

Motivated by the above observations, in Chapter 3, we turn our attention from pricing management to channel management and study how far a retailer should go with omnichannel selling strategy. To be specific, we analyze whether the retailer should keep the traditional selling, step out to use the ROPO strategy, or go further to adopt the BOPS strategy.

We find that both the ROPO and BOPS strategies may or may not be optimal for the retailer. We derive the conditions under which the retailer should implement the ROPO or BOPS strategy. For example, we show that if the profit of online retailing is small, then the retailer should adopt the ROPO strategy. By contrast, if the hassle cost of using the BOPS function is low, and the profit of online retailing is big or each consumer brings a great cross-selling benefit, then the BOPS strategy is optimal for the retailer. Moreover, by analyzing the retailer's optimal inventory level for each selling strategy, we find that, compared to the tradition selling strategy, the retailer holds more inventory in the store under the ROPO strategy, while he may not hold more inventory in the store under the BOPS strategy.

Study 3. Horizontal mergers can be easily found in practice. For example, Hewlett-Packard and Compaq merged in 2001 (Cho 2013); L'ORÉAL merged The Body Shop in 2006 (Galpin 2014), and Coach acquired Kate Spate in 2017 (BusinessWire 2017). Besides, different post-merger firms choose different degrees of post-merger integration: Some post-merger firms choose simple integration that their participants make decentralized price and quality decisions after merger, while others fully integrate their participants and make centralized price and quality decisions. In addition, many mergers happened in competitive markets. For example, although the firms aforementioned have merged with their competitors, they still face competition from nonparticipant firms such as Dell, Estée Lauder, and Michael Kors.

Motivated by the above observations, in Chapter 4, we study competing firms' merging decision and strategy in a competitive market. We develop a game-theoretical model where three firms compete on price and quality. Two of the competing firms decide whether to merge. If they choose to merge, both of them achieve cost synergy and the post-merger firm needs to decide the level of post-merger integration, i.e., decentralized or centralized merger. We analyze and compare different merging decisions and strategies, and discuss the effects on retail price, product design, profit, total consumer utility and social welfare.

We highlight some of our main findings as follows. First, we find that stronger cost synergy may or may not hurt the participant firms in the decentralized merger case, while it always benefits the post-merger firm in the centralized merger case. Second, we find that the post-merger firm prefers decentralized merger when horizontal differentiation level is low and centralized merger otherwise. Third, if both centralized merger and decentralized merger are feasible, then a merger is always beneficial. However, if centralized merger is not possible, then a merger may backfire and hurt the participants. Last, we find that a horizontal merger may not necessarily reduce market competition and result in higher price as well as lower quality. Indeed, under some conditions, a merger may improve the total consumer utility and social welfare.

Chapter 2

Pricing Substitutable Products Under Consumer Regrets

2.1 Introduction

Modest product refinement is one of the basic growth strategies in practice. Many firms launch products which are updated based on the former versions and sell them in different periods. For example, Bloomsbury Publishing PLC published the Harry Potter and the Philosopher's Stone Illustrated Edition in October 2015 (Bloomsbury 2018). And in February 2016 it announced that it would publish the 20th Anniversary Editions (the Hogwarts House Edition) in June 2017 to celebrate the 20th anniversary of publication of this book (mugglenet.com). If a fan of Harry Potter wants to buy a Harry Potter and the Philosopher's Stone after the announcement, what should she do, buy the Illustrated Edition or wait for the 20th Anniversary Edition? Note that her valuation for the 20th Anniversary Edition may be uncertain as there were limited information about the product unavailable in the market.

The aforementioned example leads to several observations: First, time plays a role as the products are launched and sold in different periods. Second, these products are substitutable. Third, a consumer faces the second period valuation uncertainty when she makes the purchase-or-wait decision. The uncertainty may come from personal factors, such as healthy problem, mood, and sudden changes in work schedule. (Shugan and Xie 2005); and product factors, such as size, color, and quality. The uncertainty may lead to uncertain valuations for targets. Moreover, when the uncertainty is realized ex post, decision made under uncertainty may lead to negative emotion, such as regret, which stems from realizing that one's situation is worse than that of the rejected options (Zeelenberg et al. 2000).

Anticipated regret exists in our daily life. For example, a decision-maker who gambles may simultaneously purchase different kinds of insurance (Bell 1982). Literature also shows that anticipated regret affects personal behavior. Indeed, when thinking back, one always regrets and hopes that she has chosen differently (Simonson 1992). Moreover, consumers are able to anticipate negative emotions caused by disappointing results (Baron 1991). And, in anticipation of regret, one tends to spend more time on decision making (Zeelenberg 1999). In other words, anticipation of negative emotion affects one's behavior ex ante (Zeelenberg et al. 2000) and one's purchasing behavior (Simonson 1992). Therefore, it is meaningful to study the effect of anticipated regrets.

Given the fact that consumers face second period valuation uncertainty at the beginning of the selling season and that anticipated regrets may have an impact on consumers purchasing behavior, this chapter focuses on the following research questions: First, what are the effects of valuation uncertainty on the firm's operation? Second, what are the effects of anticipated regret on consumers behavior and firm's prices? Third, should the firm choose price commitment or dynamic pricing?

This chapter develops a game-theoretical model to answer the above questions. We consider a firm selling two substitutable products in a selling season with two periods. A single product is sold in each period. Consumers know that there are two products and they know their valuations of the first product (i.e., the product sold in the first period), but they do not know their valuations of the second product. The game between the seller and the consumers begins with the firm choosing one of the pricing strategies: Price commitment, meaning that both prices are announced at the beginning of the selling season; or dynamic pricing, meaning that the price is determinate by the firm at the beginning of each period. Given the firm's pricing strategy and prices, consumers need to make their purchase-or-wait decisions at the beginning of the selling season in anticipation of either purchase regret or wait regret.

We highlight some of our main results as follows. First, a firm may set a lower price for the first product even when the first product is more attractive (i.e., the average valuation of the first product is higher). Strategic waiting helps consumers to share the uncertainty to the firm.Therefore, the firm has incentive to lower its price in the first period in order to mitigate consumers' strategic waiting.

Second, we find that the effects of anticipated regret on the optimal second period price may be non-monotone and depends on the value of regret and uncertainty parameters. Intuitively, purchase regret increases the second period price and wait regret reduces price. However, when the uncertainty is high and consumers anticipate more purchase regret than wait regret, the second period price can be decreasing in the purchase regret and increasing in the wait regret. The reason is that if uncertainty level is high and consumers experience more purchase regret, most consumers to wait. In this case, the firm should lower its price in the second period in order to capture as many demand as possible.

Third, aversion to purchase regret always harms the firm's profit as it leads consumers to wait and thus share the risks caused by uncertainty to the firm. On the contrary, aversion to wait regret makes the firm better off, which means that the firm should launch more marketing campaigns to evoke potential wait regret.

Finally, price commitment is always better than dynamic pricing as it mitigates strategic waiting. The value of price commitment is determined by the valuation uncertainty and consumers' anticipated regrets.

2.2 Literature Review

This chapter studies the effects of valuation uncertainty and anticipated regrets on consumer behavior and firm's operation. It is closely related to the area regarding the effects of emotions on decision-making. These emotions include regret (e.g., Braun and Muermann 2004, Nasiry and Popescu 2012) and disappointment (e.g., Liu and Shum 2013). Zeelenberg et al. (2000) summarizes the differences between regret and disappointment. It shows that regret results from bad decisions (what one gets is worse than the rejected options), while disappointment is caused by disconfirmed expectancies (what one gets is worse than the expectation). Moreover, because of self-blame, regret may be more intense than disappointment.

Bell (1982) and Loomes and Sugden (1982) develop the Regret Theory, which explains some violations of conventional Expected Utility Theory. Quiggin (1994) extends Regret Theory to general choice sets by defining regret as the disutility of not having chosen the ex post best forgone alternative. Besides, Braun and Muermann (2004) incorporates regret into insurance decision-making, Muermann et al. (2006) and Michenaud and Solnik (2008) consider regret in financial decisions, Filiz-Ozbay and Ozbay (2007) and Engelbrecht-Wiggans and Katok (2008) link regret with auction issues, and Perakis and Roels (2008) extends regret to newsvendor problem.

There are a few papers that study the effect of consumers' anticipated regrets in the operations management area. Nasiry and Popescu (2012) studies the effects of anticipated regret in advance selling context. They model action regret (regret caused by buying in advance) and inaction regret (regret caused by waiting for spot period) into advance selling and find that action regret diminishes the benefits of advance selling while inaction regret makes the firm better off. Diecidue et al. (2012) models buyer's regret and hesitater's regret in a two period advance selling context. This research carefully analyzes the sources of regret. Özer and Zheng (2015) studies the sellers pricing and inventory policies with the consideration of anticipated regrets and consumers' availability mis-perception. It models high-price regret and stockout regret. This research finds that, with anticipated regrets and consumers' availability mis-perception, markdown may be more profitable than everyday low price. Different from the aforementioned works who focus on a monopoly firm, Jiang et al. (2016) studies the effects of consumers' anticipated regrets on competitive firms' profit and product innovation in a competitive context. It models switching regret and repeat-purchase regret and finds that the effects of anticipated regrets on the profit and product innovation are non-monotonic. Besides, Chao et al. (2016) studies the effects of the consumers' ability to anticipate the regret (no, full or partial) on the profit of competitive probabilistic selling. Our study is closely related to Jiang et al. (2016) as both of us consider two products. However, Jiang et al. (2016) considers two firms while we focus on a single firm's optimization problem.

There is extensive literature studying the interaction between strategic consumers and firms' pricing and operation decisions (e.g., Besanko and Winston 1990, Su 2007, Tilson and Zheng 2014, Du et al. 2015, Shum et al. 2016, Dong and Wu 2017). Besanko and Winston (1990) show that prices considering strategic consumers are always lower than that considering myopic consumers. Shum et al. (2016) study the effects of cost reduction on a firm's pricing strategy. They take price commitment, price matching, and dynamic pricing into account, and find that the source of cost reduction affects the firm's pricing strategy. Su and Zhang (2008) study the interaction between commitment and strategic behavior and show that commitment can benefit the firm. Aviv and Pazgal (2008a) and Dasu and Tong (2010) show that price commitment outperforms dynamic pricing in reducing strategic waiting, while Cachon and Swinney (2009b) and Aflaki et al. (2016) find the opposite results that, under certain conditions, dynamic pricing may be better than price commitment. Our study differs from the aforementioned studies as we take anticipated regret into consideration. We contribute to this stream of literature by showing that the value of price commitment is determined by both uncertainty and regret parameters.

2.3 Modeling

We consider a monopolistic firm who sells two substitutable products, denoted by product 1 and product 2, to a mass of infinitesimal consumers over a selling season with two periods. A single product is sold in each period. Without loss of generality, we assume that products 1 and 2 are sold in the first and second periods, respectively.

The firm first decides which pricing strategy to implement: price commitment (superscript c) or dynamic pricing (superscript d). With price commitment, the firm announces both prices at the beginning of the selling season; while with dynamic pricing, the firm dynamically sets the price at the beginning of each period. Dynamic pricing prevails in practice because of its flexibility.

Given the firm's pricing strategy, consumers make their purchase-or-wait decisions at the beginning of the selling season. That is consumers decide whether to buy product 1 in the first period or wait for product 2 in the second period at the beginning of the first period. Each consumer buys at most one product. Each consumer has a heterogeneous and independent valuation for each product and faces a second period valuation uncertainty at the beginning of the first period when the purchase-or-wait decision is made. Valuation uncertainty exists because there is limited information about product 2 before it is sold in the market. Second period valuation uncertainty will be revealed at the beginning of the second period when product 2 is available in the market. Then those who choose to wait in the first period make their decisions of purchase-or-leave and buy product 2 as long as their valuations are higher than the price.

We assume that consumers' valuation of the first and second products are $v_o + v_1$ and $v_o + v_2$, respectively. We use v_o to model the same baseline function of the substitutable products. Besides, we use v_1 and v_2 to model the additional functions of product 1 and 2, respectively. v_1 and v_2 are independent.[2.1] At the beginning of the first period, v_1 is known while v_2 is uncertain. We normalize v_o to zero to simplify the analysis.

Consistent with the behavior literature, we assume v_i $(i = 1, 2)$ follows a uniform distribution on $[0, \overline{v}_i]$ with cumulative distribution function $F_i(\cdot)$ and probability density function $f_i(\cdot)$. Both the firm and consumers know the distribution. Let $\overline{v}_2 = \xi \overline{v}_1$, $\xi \in (\frac{1}{2}, 2)$. From the perspective of consumers, ξ captures the value of the second period valuation uncertainty.

[2.1] Similar assumption can be found in Jiang and Tian (2016), which assumes that consumers can independently obtain different usage valuations in different usage periods.

Following the Regret Theory (Bell 1982, Loomes and Sugden 1982), we assume that consumers are strategic and emotionally rational. Given the firms' prices, they make decisions to maximize their expected surplus, which is the sum of economic surplus and regrets. Following Quiggin (1994) and Braun and Muermann (2004), we define regret as the disutility of not having chosen the ex post best forgone alternative. That is regret is proportional to the difference between one's actual economic surplus and the ex post best economic surplus one could have got in the same state. This study considers two types of regret: Purchase regret is triggered when a consumer purchases the first product and finds out that she could have gained more surplus if she has chosen to wait; while wait regret is triggered when a consumer chooses to wait in the first period but finds out that she could have been better if she has purchased product 1 in the first period. We use α and β to denote the coefficients of purchase regret and wait regret, respectively. We assume $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 1$ for the reason that actual surplus seems to be more valuable than counter-factual surplus (Özer and Zheng 2015).

We assume that the total market size for the two products is deterministic and fixed at 1. Let D_1 and D_2 denote the demands of the first and second periods and let $D_T = D_1 + D_2$ denote the total demand. In addition, this study assumes the availability of each product in its selling period is guaranteedand the marginal costs of both products are normalized to zero.

We first discuss the price commitment strategy in Section 2.4 and then analyze the dynamic pricing strategy in Section 2.5.

2.4 Price Commitment

If the seller implements the price commitment strategy, then the first period valuation v_1, the seller's first period price p_1, and the second period price p_2 are known to each consumer at the beginning of the first period, while the second period valuation v_2 is uncertain at that time and will be realized at the beginning of the second period. The sequence of events are as follows:

i. At the beginning of the first period, the firm announces p_1 and p_2;

ii. All consumers arrive, observe v_1 (consumer dependent), p_1, and p_2, and make their decisions of purchase-or-wait;

iii. At the beginning of the second period, v_2 (consumer dependent) is realized. All waiting consumers make their decisions of purchase-or-leave.

For a particular consumer in the first period, the expected net surplus of purchasing is denoted by S_1, and the expected net surplus of waiting is S_2. S_1 is characterized as follows:

$$S_1 = (v_1 - p_1) - \alpha \int_0^{\overline{v}_2} \big[(v_2 - p_2) - (v_1 - p_1)\big]^+ f_2(v_2)\,\mathrm{d}v_2, \tag{2.1}$$

where $x^+ = \max\{x, 0\}$. The first term in Equation (2.1) is the economic surplus and the second term is the emotional surplus, i.e., negation of the regret triggered by choosing to purchase in the first period. α captures the strength of purchase regret. A consumer experiences purchase regret under the following conditions: (i) $v_1 \geq p_1$; otherwise, the consumer prefers waiting; (ii) $v_2 - p_2 > v_1 - p_1$, which ensures that product 2 is more attractive; and (iii) product 2 is available, which is assured in this study.

Similarly, S_2 is given as follows:

$$\begin{aligned} S_2 = \int_0^{\overline{v}_2} \Big\{ & \Big[(v_2 - p_2) - \beta\big((v_1 - p_1) - (v_2 - p_2)\big)^+\Big]\phi(v_2) \\ & + \Big[-\beta(v_1 - p_1)^+\Big](1 - \phi(v_2))\Big\} f_2(v_2)\,\mathrm{d}v_2, \end{aligned} \tag{2.2}$$

where $\phi(v_2)$ takes the value 1 if $v_2 \geq p_2$, and takes the value 0 if $v_2 < p_2$. S_2 depends on whether the consumer's second period valuation is higher than the price. Note that if product 2 is affordable, the consumer gets positive economic surplus and the corresponding wait regret if she could be better by choosing the other option. By contrast, if product 2 is not affordable, the consumer may also experience wait regret if the product 1 is affordable.

Let $\Delta S = S_1 - S_2$ be the consumer's differential surplus, ΔS is given by:

$$\Delta S = \frac{1}{2\overline{v}_2}\Big\{-(\alpha-\beta)(v_1-p_1)^2 + 2\big[(1+\alpha)\overline{v}_2 - (\alpha-\beta)p_2\big](v_1-p_1) - (1+\alpha)(\overline{v}_2-p_2)^2\Big\}. \tag{2.3}$$

Let v_1^o denotes the cutoff value, which is the value of v_1 when $\Delta S = 0$. v_1^o plays as a threshold rule, i.e., for any given prices and regret parameters, consumers purchase product 1 if and only if $v_1 \geq v_1^o$. v_1^o is shown in the following lemma:

Lemma 2.1. *(i) If $\alpha = \beta$, then $v_1^o = \min\{\frac{(\overline{v}_2 - p_2)^2}{2\overline{v}_2} + p_1, \overline{v}_1\}$. That is, if $\overline{v}_2 \leq \overline{v}_1 - p_1 + p_2 + \sqrt{(\overline{v}_1 - p_1)(\overline{v}_1 - p_1 + 2p_2)}$, then $v_1^o = \frac{(\overline{v}_2 - p_2)^2}{2\overline{v}_2} + p_1$; otherwise, $v_1^o = \overline{v}_1$.*

(ii) If $\alpha \neq \beta$, then $v_1^o = \min\{\frac{(1+\alpha)\overline{v}_2 - (\alpha-\beta)p_2 - \sqrt{(1+\beta)\left[(1+\alpha)\overline{v}_2^2 - (\alpha-\beta){p_2}^2\right]}}{\alpha-\beta} + p_1, \overline{v}_1\}$. That is, if $\overline{v}_2 \leq \overline{v}_1 - p_1 + p_2 + \sqrt{\frac{1+\beta}{1+\alpha}(\overline{v}_1 - p_1)(\overline{v}_1 - p_1 + 2p_2)}$, then $v_1^o = \frac{(1+\alpha)\overline{v}_2 - (\alpha-\beta)p_2 - \sqrt{(1+\beta)\left[(1+\alpha)\overline{v}_2^2 - (\alpha-\beta){p_2}^2\right]}}{\alpha-\beta} + p_1$; otherwise, $v_1^o = \overline{v}_1$.

All proofs can be found in the Appendix. According to Lemma 2.1, if $\alpha \neq \beta$, anticipated regrets make sense. Besides, if the upper bound of the second period valuation is large enough, all consumers prefer waiting and set the maximum of the first period valuation as their cutoff value. The following proposition shows the effects of prices, uncertainty, and anticipated regrets on the cutoff value.

Proposition 2.1. *If $v_1^o < \overline{v}_1$, then*

(i) $\frac{\partial v_1^o}{\partial p_1} > 0$, $\frac{\partial v_1^o}{\partial p_2} < 0$, and $\frac{\partial v_1^o}{\partial \xi} > 0$;

(ii) if $\alpha \neq \beta$, then $\frac{\partial v_1^o}{\partial \alpha} > 0$ and $\frac{\partial v_1^o}{\partial \beta} < 0$.

Proposition 2.1 describes the effects of prices (p_1 and p_2), uncertainty (ξ), and behavioral parameters (α and β) on the cutoff value v_1^o. It shows that v_1^o is increasing in the uncertainty parameter ξ. Under second period valuation uncertainty, purchase behavior in the first period is associated with the risk of being worse than waiting. Given prices, this risk increases in ξ. Therefore, v_1^o increases in ξ. Besides, Proposition 2.1 (ii) shows that, if $\alpha \neq \beta$, aversion to purchase regret drives consumers to wait and thus v_1^o increases while aversion to wait regret lures more shoppers to purchase in the first period. Thus, v_1^o increases in α decreases in β.

v_1^o determines the firm's demands. Specifically, $D_1 = \frac{\overline{v}_1 - v_1^o}{\overline{v}_1}$, $D_2 = \frac{v_1^o}{\overline{v}_1}\frac{\overline{v}_2 - p_2}{\overline{v}_2}$, and $D_T = D_1 + D_2 = \frac{1}{\overline{v}_1}(\overline{v}_1 - v_1^o\frac{p_2}{\overline{v}_2})$. D_T consists of three parts: "Pure first period" consumers (whose valuation of product 1 belongs to $(v_1^o, \overline{v}_1)$), "first-second period" consumes (whose valuations v_1 and v_2 belong to (p_1, v_1^o) and $(p_2, \overline{v}_2)$, respectively), and "pure second period" consumers (whose valuations v_1 and v_2 belong to $(0, p_1)$ and $(p_2, \overline{v}_2)$, respectively). Note that "Pure first period" consumers form D_1, and the last two parts form D_2. We define the demand of "first-second period" consumers as "switching demand" of product 2, which is denoted by $D_{2s} = \frac{v_1^o - p_1}{\overline{v}_1}\frac{\overline{v}_2 - p_2}{\overline{v}_2}$; whereas the demand of "pure second period" consumers as "original demand" of product 2, which is denoted by $D_{2o} = \frac{p_1}{\overline{v}_1}\frac{\overline{v}_2 - p_2}{\overline{v}_2}$. D_{2s} captures consumers' switching behavior while D_{2o} is independent of strategic waiting. The following proposition summarizes the properties of demand.

Proposition 2.2. *If $v_1^o < \overline{v}_1$, then,*

Effect of Prices: (i) D_T is decreasing in p_1; (ii) There exists a threshold $\dot{p}_2$ such that if $p_2 > \dot{p}_2 > p_1$, then D_T is increasing in p_2; otherwise, D_T is decreasing in p_2.

Effect of Uncertainty: (iii) There exists a threshold $\dot{\xi}$ such that if $p_2 > p_1$ and $\xi > \dot{\xi}$, then D_T is decreasing in ξ; otherwise, D_T is increasing in ξ.

Effect of Anticipated Regrets: (iv) If $\alpha \neq \beta$, then D_T is decreasing in α and increasing in β.

Proposition 2.2 shows that the effects of prices, uncertainty, and emotional parameters on D_T are complex. According to the first part of Proposition 2.2, D_T is decreasing in p_1. A higher first period price reduces the attractiveness of product 1 and gives consumers more incentives to wait. However, purchasing behavior in the second period depends on the condition that valuation is larger than price, thus the lost sales in the first period may not be captured by product 2. That is the increase in D_{2o} is less than the decrease in D_1. Besides, D_{2s} is independent of p_1. Consequently, the total demand decreases.

Generally, an increase in p_2 makes product 2 more expensive and thus leads the retailer to lose more demands than the lost sales in the first period. Therefore, the total demand decreases. However, the total demand may increase in p_2 if p_2 is large enough. If p_2 is large enough, the demand of product 2 is already small. In this case, if p_2 increases, consumers have less incentives to wait and more consumers will buy product 1, thus D_1 increases. At the same time, the decrease in D_2 is small. As a result, D_T increases.

According to the third part of Proposition 2.2, D_T could be decreasing in ξ when $p_2 > p_1$ and $\xi > \dot{\xi}$. This is because if $p_2 > p_1$ and $\xi > \dot{\xi}$, most consumers choose to wait in the first period. In this case, a decrease in D_1 outweighs an increase in D_2 and the total demand decreases in ξ.

The last part in Proposition 2.2 justifies the significance of taking consumers' anticipated regret into consideration. The total demand decreases in purchase regret and increases in wait regret, which makes our research different from Özer and Zheng (2015) whose model shows that the total demand is independent of the regret parameters. Purchase regret leads to inertia and thus hurts the total demand as the waiting consumers may leave the market without buying any products if they realize low valuation of product 2. By contrast, wait regret raises the total demand since it drives consumers to buy, rather than to wait, in the first period.

Our further analyses assume that $\xi \in (\frac{1}{2}, 2)$ to ensure that $v_1^o < \overline{v}_1$. The firm's profit maximization problem is given by:

$$\max_{p_1,p_2} \Pi(p_1, p_2) = p_1 D_1 + p_2 D_2. \tag{2.4}$$

2.4.1 Case of $\alpha = \beta$

According to Lemma 2.1, if $\alpha = \beta$, the purchase regret offsets the wait regret and anticipated regrets have no effect on consumer behavior. That is, consumers make their purchase-or-wait decisions as if there is no anticipated regret. The case of $\alpha = \beta$ plays as a benchmark. It captures the effect of valuation uncertainty. Let p_1^{c*} and p_2^{c*} be the optimal prices for product 1 and product 2, respectively. p_1^{c*}

and p_2^{c*} are shown in the following lemma.

Lemma 2.2. *If $\alpha = \beta$, then $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2 - \frac{1}{2}\overline{v}_2^2 + 2\overline{v}_2 p_2^{c*} - \frac{3}{2}(p_2^{c*})^2}{2\overline{v}_2}$ and*
$p_2^{c*} = 2\sqrt{\frac{6+\xi}{3\xi}}\cos(\frac{1}{3}\operatorname{arccos}(-\frac{6}{6+\xi}\sqrt{\frac{3\xi}{6+\xi}}) - \frac{2\pi}{3})\overline{v}_2$.

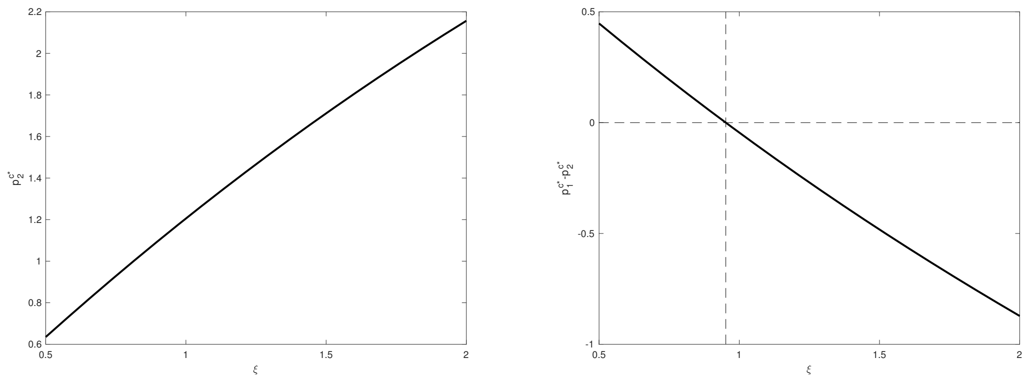
Some properties of the optimal prices are summarized in Proposition 2.3.

Proposition 2.3. *(i) p_1^{c*} is increasing in p_2^{c*};*

(ii) p_2^{c*} is increasing in ξ;

(iii) $p_1^{c*} < p_2^{c*}$ if and only if $\xi > -2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2 \approx 0.9519$;

(iv) With optimal prices, the total demand increases in ξ.



(a) (b)

Figure 2.1: Effect of ξ on Price p_2^{c*} and Price Difference $p_1^{c*} - p_2^{c*}$ ($\alpha = \beta$)

Note: in Figure 2.1, $\overline{v}_1 = 2$.

The first part of Proposition 2.3 shows that the optimal prices have a positive correlation. This is intuitive since these two products are substitutable. Besides, Proposition 2.3 (ii) is because ξ increases both D_2 and the upper bound value $\overline{v}_2$.

If $-2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2 < \xi < 1$, then one may expect a higher price of product 1 since the average valuation of product 1 is larger than that of product 2. Interestingly, the third part of Proposition 2.3 shows that $p_1^{c*} < p_2^{c*}$ when $\xi > -2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2$. Note that ξ denotes the uncertainty in the first period. If ξ is large enough, consumers prefer waiting. However, as mentioned above, waiting consumers may leave the market without any products if they realize low valuations for product 2. In this case, the firm has incentive to lower p_1 in order to lure consumers to purchase in the first period, and thus increases D_1. Since the marginal revenue of purchase in the first period (p_1^{c*}) is larger than the effective marginal revenue of wait ($p_2^{c*}\frac{\overline{v}_2-p_2^{c*}}{\overline{v}_2}$), the increase in D_1 will more than offset the loss in both p_1 and D_2 and therefore make the firm more profitable. In other words, strategic waiting helps consumers to share the risk caused by the uncertainty to the firm. By lowering its price of product 1, the firm moderates the uncertainty faced by the consumers and mitigates strategic waiting, and thus becomes better off. Therefore, $p_1^{c*} < p_2^{c*}$ when $-2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2 < \xi < 1$.

Notice that ξ affects D_2 in two ways. On the one hand, if ξ increases, switching demand increases. However, as mentioned above, these strategic waiting may not be captured by product 2. That is the increase in D_{2s} is less than the decrease in D_1 and the total demand decreases. On the other hand, if ξ increases, the original demand D_{2o} increases. To sum up, the effect of ξ on the total demand depends on the loss in strategic waiting consumers and the increase in the original demand. Proposition 2.3 (iv) shows that if the firm prices its products according to the optimal rule, the total demand increases in ξ.

2.4.2 Case of $\alpha \neq \beta$

If $\alpha \neq \beta$, anticipated regrets play a role in consumers' decisions. Therefore, the case of $\alpha \neq \beta$ captures both the effects of valuation uncertainty and consumers' anticipated regrets. Similar to the analysis of the case of $\alpha = \beta$, we first show the following lemma, which describes the uniqueness of the optimal prices.

Lemma 2.3. *If $\alpha \neq \beta$, then* $p_1^{c*} = \frac{1}{2}\Big\{\overline{v}_1 + p_2^{c*} - \frac{(p_2^{c*})^2}{\overline{v}_2} - \frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2^{c*}-\sqrt{(1+\beta)\left[(1+\alpha)\overline{v}_2^2-(\alpha-\beta)(p_2^{c*})^2\right]}}{\alpha-\beta}\Big\}$, *and there exists a unique optimal solution p_2^{c*} that maximizes $\Pi(p_1, p_2)$ and solves* $2\overline{v}_1\overline{v}_2 - 2\overline{v}_1 p_2 - 2{p_2}^2 + 2\frac{{p_2}^3}{\overline{v}_2} - 2p_2 n + (\overline{v}_2 n - \overline{v}_1\overline{v}_2 + \overline{v}_2 p_2 - {p_2}^2)p_2\sqrt{\frac{1+\beta}{t}} = 0$.

Here, $t = (1+\alpha)\overline{v}_2^2 - (\alpha-\beta){p_2}^2$, and $n = \frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2-\sqrt{(1+\beta)t}}{\alpha-\beta}$. Similar to the case of $\alpha = \beta$, some properties of the optimal prices are summarized as follows.

Proposition 2.4. *(i)* p_1^{c*} *is increasing in* p_2^{c*}*;*

(ii) Generally, p_2^{c*} *is increasing in the purchase regret coefficient* α *and decreasing in the wait regret coefficient* β*. However, if* ξ *is large enough, then given* β*, there exists a threshold* $\hat{\alpha}$ *such that if* $\alpha > \hat{\alpha}$*, then* p_2^{c*} *is decreasing in* α*; while given* α*, there exists a threshold* $\hat{\beta}$ *such that if* $\beta < \hat{\beta}$*,* p_2^{c*} *is increasing in* β*.*

(iii) With optimal prices, the total demand increases in ξ*.*

The first and the last part of Proposition 2.4 are the same as those in the case of $\alpha = \beta$, and their explanations can be found accordingly. Here, we focus on the effects of α and β on the optimal second period price p_2^{c*}, which are shown in the second part of Proposition 2.3. It shows that, depending on the value of ξ, each regret parameter has non-monotone effect on the optimal second period price.

Above all, the uncertainty ξ and the regret parameters α and β affect consumers' decisions. Uncertainty ξ and aversion to purchase regret lead consumers to wait whereas aversion to wait regret results in immediate purchase. Besides, strategic waiting helps consumers to share the risk caused by valuation uncertainty to the firm.

If the risk of uncertainty is moderate, switching demand D_{2s} and original demand D_{2o} is small. Most consumers prefer purchasing in the first period. In this case, the optimal second period price is increasing in α and decreasing in β. Given ξ, any increase in α or decrease in β lures consumers to move from the first period to the second one. Thus, D_1 decreases and D_{2s} increases. However, as mentioned above, product 2 may not capture the lost sales in the first period, and the effective marginal revenue of wait is less than that of purchase. Then, as a result, the firm's profit suffers from any increase in α or decrease in β. Therefore, the seller has incentive to mitigate the strategic waiting behavior. He manages to achieve such objective by increasing p_2 since a higher price in the second period

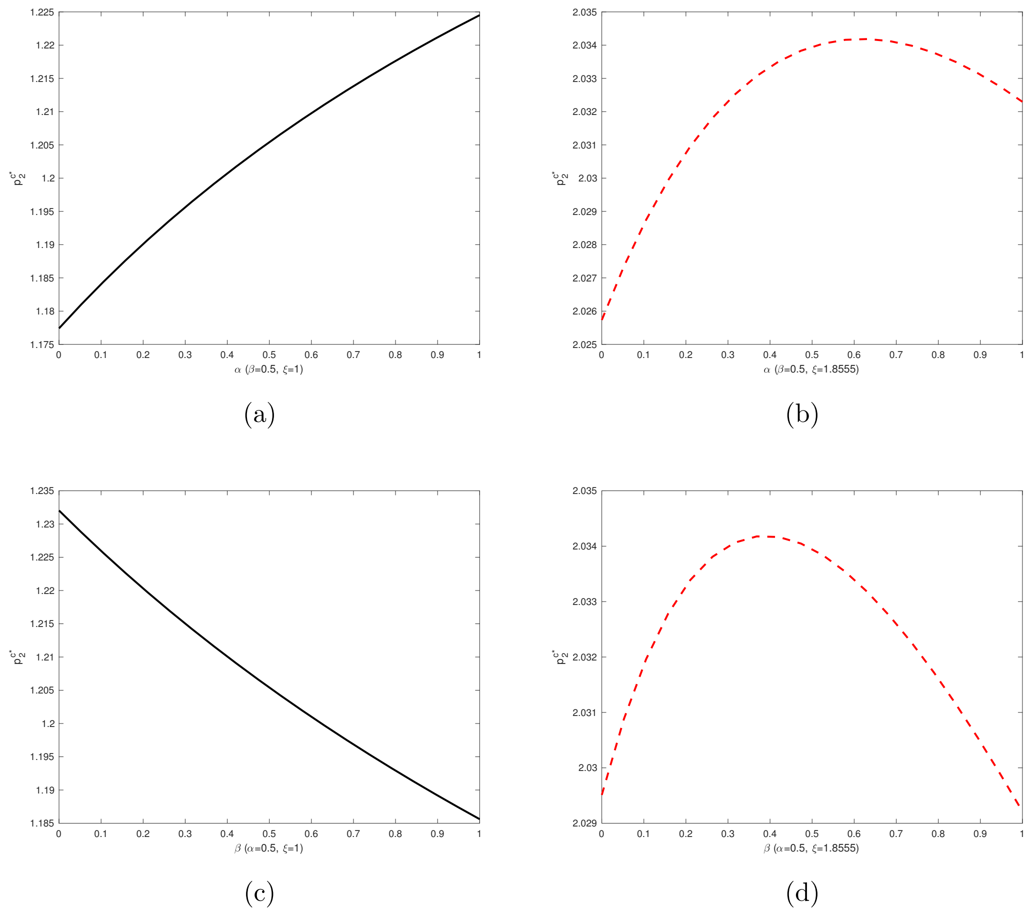


Figure 2.2: Effects of α and β on Price p_2^{c*} $(\alpha \neq \beta)$

Note: in Figure 2.2, $\overline{v}_1 = 2$; in Figure 2.2 (a), $\beta = 0.5$ and $\xi = 1$; in Figure 2.2 (b), $\beta = 0.5$ and $\xi = 1.8555$; in Figure 2.2 (c), $\alpha = 0.5$ and $\xi = 1$; and in Figure 2.2 (b), $\alpha = 0.5$ and $\xi = 1.8555$.

lowers the opportunity cost of purchasing product 1. In this way, the benefit of increased prices and D_1 is more than the decreases in D_2 and the firm becomes better off.

By contrast, if ξ is large enough and $\alpha >> \beta$ such that consumers prefer waiting at the beginning of the selling season, p_2 can be decreasing in α and increasing in β. Note that ξ captures the valuation uncertainty. If ξ is large enough, then both switching demand D_{2s} and original demand D_{2o} are large while D_1 is small. That is most consumers prefer waiting. If there is any increase in α or decrease in β. D_1 decreases and D_{2s} increases. With the same logic as above, the firm's profit decreases. But in this case, lower p_2 is required in order to attract more consumers to buy product 2 since D_2 is large enough that, although the effective marginal of wait is still less than that of purchasing product 1, the increase in D_{2o} offsets such drawbacks. In other words, if there is any increase in α or decrease in β, the firm needs to lower its price in the second period in order to attract more demands.

Proposition 2.4 is depicted graphically in Figure 2.2.

2.5 Dynamic Pricing

Dynamic pricing prevails in both practice and research. With this pricing strategy, only v_1 and p_1 are known at the beginning of the first period, while both v_2 and p_2 are unknown at that time and will be realized at the beginning of the second period. The sequence of events is described as follows:

i. At the beginning of the first period, the firm declares p_1;

ii. All consumers arrive, observe v_1 (consumer dependent) and p_1, and make their decisions of purchase-or-wait;

iii. At the beginning of the second period, the firm chooses p_2 to maximize its profit-to-go;

iv. v_2 (consumer dependent) is revealed. Based on v_2 and p_2, all waiting consumers make their decisions of purchase-or-leave.

The firm's optimization problem is formulated as follows:

$$\max_{p_1,p_2} \Pi(p_1,p_2) = p_1 D_1 + p_2 D_2, \text{ s.t.} p_2 = arg \max_x D_2' x, \tag{2.5}$$

where $D_2' = \frac{1}{\overline{v}_1 \overline{v}_2} v_1^o (\overline{v}_2 - x)$. The difference between price commitment and dynamic pricing is that the former chooses both its first and second period prices at the beginning of the first period; whereas the latter announces its second period price at the beginning of the second period, which means that the firm chooses its second period price in order to maximize its expected profit in the second period. This explains why there is a condition that $p_2 = arg \max_x D_2' x$ in the above Equation. Let p_1^{d*} and p_2^{d*} denote the optimal prices of product 1 and product 2 in the dynamic pricing model. By backward induction, the analysis begins with the firm's decision of p_2. It is straightforward to find that $p_2^{d*} = \frac{1}{2}\overline{v}_2$. This result is interesting as it is independent of the cutoff value v_1^o. Actually, from the perspective of strategic consumers, the price in the second period can be found out ex ante. Thus, the dynamic pricing can be seen as a special case of the price commitment and, in this special case, $p_2^{c*} = \frac{1}{2}\overline{v}_2$. The comparison of the price commitment and dynamic pricing is summarized in Proposition 2.5.

Proposition 2.5. $p_1^{c*} > p_1^{d*}$, $p_2^{c*} > p_2^{d*}$ *and* $\Pi^c > \Pi^d$.

According to Aflaki et al. (2016), from the perspective of mathematics, the profit maximization problem with price commitment is a relaxation of that with dynamic pricing. Therefore, price commitment dominates dynamic pricing. In Section 2.6, we will further show that, although price commitment dominates dynamic pricing, the relative value of price commitment is affected by the uncertainty and behavioral parameters.

2.6 Numerical Study

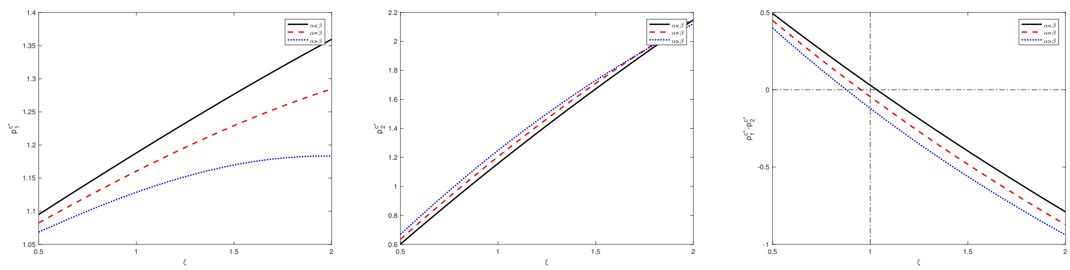
By numerical study, this section furthers our analysis to the effects of uncertainty and anticipated regrets on the firm's prices and profit and on the relative value of price commitment. In this section, we assume $\overline{v}_1 = 2$. Besides, $\alpha > \beta$ means

$\alpha = 0.99$ and $\beta = 0.01$, $\alpha = \beta$ means $\alpha = \beta = 0.5$, and $\alpha < \beta$ means $\alpha = 0.01$ and $\beta = 0.99$.

Figure 2.3 summarizes the effects of the uncertainty and behavioral parameters on the firms prices. According to this figure, we have three observations. First, prices are increasing in the uncertainty parameter ξ. The logic is that, although increasing ξ results in more strategic waiting and may cause lost sales in the first period that may not be captured by product 2, it also increases the upper-bound of the second period valuation and increases the original demand of product 2. Thus the firm can gain more profit by rising its prices.

Second, although we mentioned in Proposition 2.4 that each type of the anticipated regrets has opposite effect on the optimal second period price, here we find that the effects of anticipated regrets on the optimal first period price are monotonic. In other words, the optimal first period price is increasing in the wait regret and decreasing in the purchase regret.

Third, the difference between the two prices depends on the value of both the uncertainty parameter ξ and the emotional parameters α and β. If $\alpha > \beta$, the firm is more likely to set a higher price in the first period, since aversion to wait regret lures consumers to buy in the first period and increases D_1.



(a) (b) (c)

Figure 2.3: Effects of ξ on p_1^{c*}, p_2^{c*}, and $p_1^{c*} - p_2^{c*}$

Note: in Figure 2.3, $\overline{v}_1 = 2$, the solid line refers to $\alpha = 0.01$ and $\beta = 0.99$, the dashed line refers to $\alpha = \beta = 0.5$, and the dotted line refers to $\alpha = 0.99$ and $\beta = 0.01$.

The following figure (Figure 2.4) depicts the impacts of ξ, α and β on the expected maximum profit. First, the profit is increasing in ξ. This is intuitive since the optimal prices are increasing in ξ, and, as shown in Proposition 2.3 (iv) and 4 (iv), if the firm prices according to the optimal rule, the total demand increases in ξ. Second, compared with the case where there is no behavioral effects, purchase regret hurts while wait regret benefits the firm. The reason is that purchase regret causes strategic waiting and helps consumers to share some of the risks caused by uncertainty to the firm. On the contrary, wait regret lures consumers to purchase in the first period. Third, the effects of anticipated regrets on the profit are monotony.

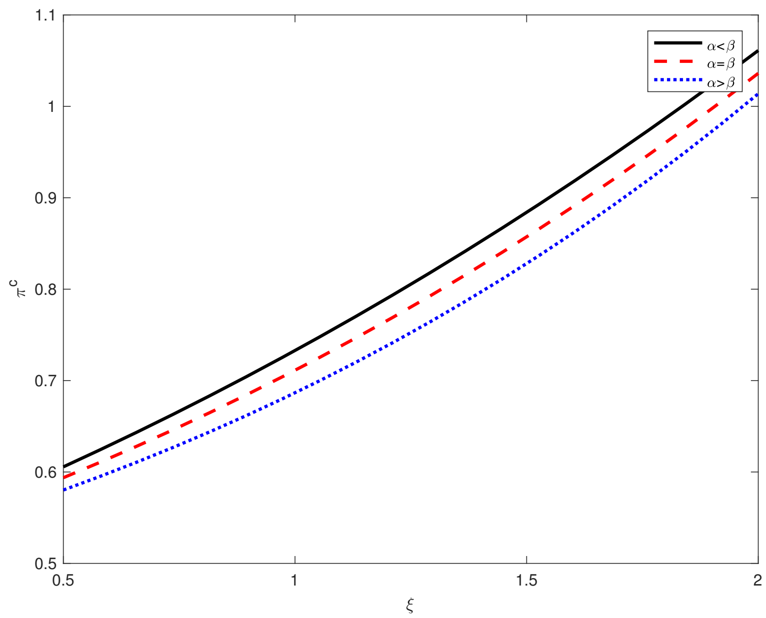


Figure 2.4: Effects of ξ, α, and β on Profit (Π^c)

Note: in Figure 2.4, $\overline{v}_1 = 2$, the solid line refers to $\alpha = 0.01$ and $\beta = 0.99$, the dashed line refers to $\alpha = \beta = 0.5$, and the dotted line refers to $\alpha = 0.99$ and $\beta = 0.01$.

Although we have showed in Section 2.5 that price commitment dominates dynamic pricing, we are interested in the impacts of the behavioral parameters on the relative value of price commitment. Therefore, we compare the profits under these two pricing strategies. Notice that the biggest difference between these two pricing strategies is how they choose the optimal second period price. Therefore, the fact we find that the value of price commitment depends on the value of the uncertainty parameter ξ and the regret parameters α and β can be explained through the parameters' impact on the optimal second period price. For example, when the ξ is close to 1, the value of price commitment hits its maximum since the optimal second period price under commitment is far away from $\frac{1}{2}\overline{v}_2$, which is the optimal second period price under dynamic pricing. Besides, the effects of the emotional parameters follow the same pattern as their effects on the optimal second period price, and the explanation follows the same logic as that behinds Proposition 2.4.

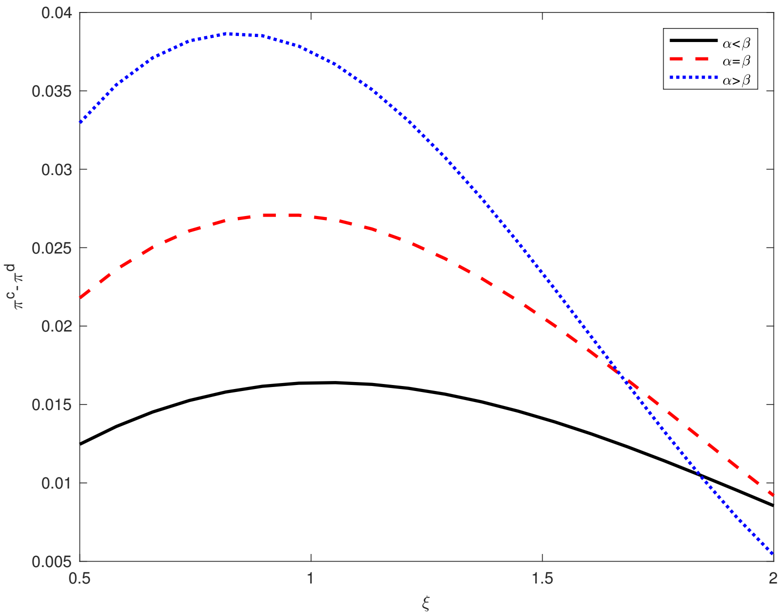


Figure 2.5: Effects of ξ, α, and β on the Value of Commitment ($\Pi^c - \Pi^d$)

Note: in Figure 2.5, $\overline{v}_1 = 2$, the solid line refers to $\alpha = 0.01$ and $\beta = 0.99$, the dashed line refers to $\alpha = \beta = 0.5$, and the dotted line refers to $\alpha = 0.99$ and $\beta = 0.01$.

2.7 Conclusion

This chapter studies the effects of the second period valuation uncertainty and consumers anticipated regret on consumers behavior and firm's prices and pricing strategy. We consider a firm selling two substitutable products to rationally emotional consumers over two periods. A single product is sold in each period. Consumers face the second period valuation uncertainty, and make their purchase-or-wait decisions at the beginning of the first period. The uncertainty may come from personal factors or the product factors, and each action may trigger corresponding regrets. Consumers can anticipate the regrets and also take them into consideration.

We find that the prices of the two substitutable products are positively correlated, which seems like a good news to the seller. However, it is worth to notice that the price of product 1 can be lower than that of product 2 even when the average valuation of product 1 is larger than that of product 2. The reason is that the firm needs to lower its first period price to moderate the uncertainty faced by the consumers and thus mitigates strategic waiting since the effective marginal revenue of strategic waiting is less than that of immediate purchase.

Our research also shows that the effects of anticipated regret on the firm's prices can be non-monotone and depends on the value of the uncertainty. The optimal second period price increases in purchase regret and decreases in wait regret when the uncertainty is moderate. The effects can turn to the opposite if the uncertainty is large enough and consumers anticipate much more aversion to purchase regret. Besides, attention should be paid to the fact that although purchase regret increases the second period price in most cases, it harms the firm's total demand and profit. The reason is that purchase regret results in strategic waiting and thus leads consumers to share some of the risks caused by uncertainty to the firm. This result justifies the existence of return policy in practice as it mitigates the negative impact of purchase regret. On the contrary, wait regret benefits the firm. Therefore, the firm should carry out some marketing campaigns to evoke consumers' aversion to wait regret.

Finally, dynamic pricing is a special case of price commitment and is dominated by the optimal price commitment. And the value of price commitment is affected by the uncertainty and anticipated regrets.

Chapter 3

How Far should a Retailer Go with Omnichannel Selling Strategy?

3.1 Introduction

In the age of the digitalization and networking, retail business models and consumers' shopping behavior have changed dramatically. Traditionally, online channel is added alongside the offline channel as an independent selling channel. Nowadays, many retailers have started to integrate the online and offline channels due to ongoing digitalization in marketing and retailing (ForresterResearch 2014, Gao and Su 2016a). Meanwhile, consumers' shopping journey may cross different channels. Given the integration of online and offline channels, consumers are possible to achieve the seemless shopping experience through two channels. In other words, many retailers are now moving from the multi-channel paradigm to the omnichannel paradigm (Verhoef et al. 2015).

In an omnichannel environment, consumers are sophisticated enough such that they may extensively research the product online and then purchase it in an offline store. Accordingly, many retailers have assessed a cross channel capability that provides store-level inventory availability information and other information (e.g., store location and product price) to drive traffic to the offline store (EbeltoftGroup 2014b). Such strategy is called as research online purchase offline (ROPO), which is one of the major strategies provided by retailers in the omnichannel environment. By auditing 146 retailers in 16 countries, the 2014 Ebeltoft Group Global Cross Channel Report shows that 44% of the retailers have implemented the ROPO strategy (EbeltoftGroup 2014a). Meanwhile, there is another important strategy used by many retailers in the omnichannel environment. That is, many retailers allow consumers to purchase online and pick up in store (BOPS). According to a report by Retail Systems Research, 64% of retailers have implemented the BOPS strategy in 2013 (Rosenblum and Kilcourse 2013).

The ROPO and BOPS strategies can bring benefits to consumers and retailers. Using the ROPO or BOPS strategies, consumers can easily review the product information (e.g., price and material) before visiting the store, by taking the advantage of ubiquitous Internet. Besides, these strategies can generate store traffic and thus increase sales (Bell et al. 2014). Many sales in store are generated by such cross-selling. It is reported that consumers visiting the store will spend $20\% - 25\%$ more than what they intend to pay (Washingtonpost 2015). By contrast, retailers may lose some potential benefits from the cross-selling given the ROPO and BOPS strategies, if the products are out of stock. This is because using the ROPO and BOPS functions, consumers can access the availability information of the products and will not visit the store if the products are out of stocks. Therefore, retailers face a benefit-and-loss trade-off regarding the cross-selling in the omnichannel environment.

In addition, there are some different features between the ROPO and BOPS strategies. Given the ROPO strategy, online channel is only used for displaying the information and consumers cannot purchase the product online. Given the BOPS, consumers can not only access the product information but also purchase the product in the online channel. Besides, given the BOPS function, it is convenient for some consumers to pick up the purchased product without waiting for delivery, and consumers can also avoid the shipping fee for the product purchased online. However, under the BOPS strategy, consumers may return the purchased products to the retailer when they pick them up in store, due to the valuation uncertainty. Such behavior will incur some loss to the retailer, while this will not happen if only the ROPO function is eligible. Although the capability of the ROPO is included in the BOPS (because given the BOPS function, consumers can also research the product online and purchase it in the offline store), different retailers adopt different strategies regarding the ROPO and BOPS. Even for one retailer, different strategies may be adopted in different stores. For example, GU adopts the ROPO strategy in Hong Kong while Uniqlo adopts the BOPS strategy in Hong Kong and Mainland China (GUHK 2019, UniqloHK 2019, UniqloChina 2018). Thus, a key challenge for the retailer is to decide how far to go in an omnichannel environment. That is, if the retailer should just allow the consumers to research online and purchase offline, or should go further to allow the consumers to buy online and pick up in the store?

Motivated by the above observation, in this chapter we study the optimal operations strategies for a retailer selling a single type of products in an omnichannel environment. The retailer has an option to implement the ROPO or BOPS strategy. Under the ROPO strategy, consumers can access the inventory availability information before making the decision to visit the store. Under the BOPS strategy, consumers can not only learn the inventory availability information from the online channel, but also purchase the product online and pick up in the store. One important feature we consider is that consumers' valuation for the product is uncertain. Consumers know their valuations when they visit the store if the product is in-stock. Consequently, under the BOPS strategy, consumers who purchase online may realize a low valuation when picking up the product in the store, and finally return it to the retailer. We consider that the demand is stochastic. Consumers strategically choose the way to buy the product from which channel to maximize her utility. The retailer decides the optimal ordering quantity of the product to maximize his profit. We first consider a base model in which a retailer does not have an online channel, and analyze whether the retailer should adopt the ROPO or BOPS strategy. And then we extend our analysis to start from a setting where a retailer selling the product through both the online and offline channels.

We analyze the consumer equilibrium behavior for each selling strategy. Whether the consumers would visit the store, buy online, or buy online and pick up in store depends on the hassle costs, stock-out probability, consumer valuation parameter etc. A consumer who planned not to visit the store may change to visit it if the omnichannel selling strategies are provided. This is because under the omnichannel selling strategies, consumers make the visiting decision after observing the inventory availability information, and do not need to worry about the stock-out risk when making the visiting decision.

We also derive the retailer's optimal inventory level for each selling strategy. We find that the retailer holds more inventory in the store under the ROPO strategy than that without the omnichannel selling strategy. This is because the retailer may lose some potential cross-selling profit if the product is out of stock, as consumers can access the inventory information under the ROPO strategy. The retailer holds fewer inventory under the BOPS strategy than that under the ROPO strategy, when their unit cross-selling profits are the same. This is because under the BOPS strategy, the retailer can avoid the return loss incurred by consumers who purchase online and realize a low valuation, if the product is out of stock. Interestingly, we find that the retailer may not hold more inventory in the store under the BOPS strategy than that without omnichannel selling, because of the joint effect of the return loss and cross-selling profit.

It is interesting to show that both ROPO and BOPS strategies may not be optimal for the retailer. We derive the conditions under which the retailer should implement the omnichannel selling strategies. We show that if the hassle cost of using the BOPS function is low, and the profits of online retailing are high or each consumer brings a high cross-selling benefit, then adopting the BOPS strategy will be optimal for the retailer. On the other hand, it is also interesting to show that offering consumers more options regarding the omnichannel selling may not benefit to the retailer. In other words, the retailer is able to obtain more profit under the ROPO strategy than that under the BOPS strategy. For example, we show that if the profits of online retailing are low enough, the benefit of the BOPS strategy may be outweighed by its drawback. As a result, adopting the ROPO strategy is more beneficial for the retailer.

We organize the rest of the chapter as follows: In Section 3.2, we review the related literature. In Section 3.3, we introduce a base model, and analyze whether the retailer should adopt the ROPO or BOPS strategy based on this base model. In Section 3.4, we extend our analysis to consider a traditional retailer with multi-channel selling. In Section 3.5, we discuss several extensions. In Section 3.6, we conclude the chapter and provide some future research directions. All proofs are relegated to the Appendix.

3.2 Literature Review

Our research contributes to the literature on the channel management. The traditional literature on the channel management considers that the manufacturer or supplier adds an online direct selling channel alongside the traditional offline channel. Considering that two channels are operated by different parties, most of the literature focuses on the channel conflict of online and offline channels, e.g., Chiang et al. (2003), Tsay and Agrawal (2004), Cattani et al. (2006), Balakrishnan et al. (2014), Ding et al. (2016) and Niu et al. (Forthcoming). In contrast to this research stream, our work considers the channel management in an omnichannel environment where the online and offline channels are operated by a single retailer, and we focus on the integration of the two channels.

Omnichannel retailing has been adopted by many retailers and received much attention in the industry (Brynjolfsson et al. 2013, Bell et al. 2014). There are several papers empirically study the operations of the omnichannel retailing. For example, Gallino and Moreno (2014) study the impact of the BOPS on the online and offline channels. They show that the BOPS strategy will bring lower online sales, higher store sales and higher store traffic, due to the cross-selling effect and channel-shift effect. Bell et al. (2017) investigate the impact of the showroom on the channel management of the retailer. They find that the showroom benefits the retailer from increase of the demand and improvement of the operational efficiency. Gallino et al. (2016) examine the effects of the ship-to-store function and find that the introduction of this function can increase the retailer's sales dispersion. Meanwhile, a few papers theoretically study the channel management in the omnichannel environment. Aflaki and Swinney (2017) study the channel and inventory integration for a retailer selling a seasonal product over two periods. They consider that the strategic consumers who decide whether and when to visit the retailer. They show that the value of channel and inventory integration is significantly affected by consumer behavior. Gao and Su (2016b) explore how information influences consumer behavior and retailer performance with omnichannel retailing. Considering that the information can resolve product value uncertainty and availability uncertainty, they examine the effects of three information mechanisms, i.e., physical showrooms, virtual showrooms and availability information. Differing from the above papers, in this research, we build a stylized model to theoretically study the impacts of the ROPO and BOPS strategies. A closely related paper is Gao and Su (2016a), which studies the implementation of the BOPS in the omnichannel environment. They show that the BOPS can help the retailer expands their market coverage, but may not be suitable for all products. Our work differs from their paper in the following two ways: First, they do not consider the ROPO strategy. Second, they do not consider consumer valuation uncertainty, under which consumers may return the product when they pick it up in the store, resulting in the return loss to the retailer. We show that this is one of the key elements affecting the implementation of the BOPS strategy.

Our research considers that facing the omnichannel retailing, consumers strategically choose the sale channels. So our work compliments the existing literature on the retail management with strategic consumers. There is a large literature on this stream of research with the consideration of various aspects, e.g., advance selling (Zhao and Stecke 2010, Prasad et al. 2011, Yu et al. 2015, Zhao et al. 2016), capacity rationing (Liu and Ryzin 2008), supply chain performance (Su and Zhang 2008), reservation option (Elmaghraby et al. 2009, Osadchiy and

Vulcano 2010, Surasvadi et al. 2017), quick response (Cachon and Swinney 2011, Swinney 2011), pricing strategy (Su 2007, Aviv and Pazgal 2008b, Levin et al. 2010, Kremer et al. 2017), and product rollover (Liang et al. 2014). Majority of these papers consider that consumers choose when to buy the products. However, we consider that facing the omnichannel strategies, consumers strategically choose which channel to buy the product. Besides, they do not consider the operations strategy in the omnichannel environment, which is a salient feature of our study.

If the consumers choose to visit the store directly, they may encounter stockouts. Some literature studies how to reduce the effects of the stockout, e.g., Anderson et al. (2006) and Musalem et al. (2010). By contrast, we consider new ways to share the availability information, that is, the consumers can access the inventory availability information in the omnichannel environment. There are some papers studying the effects of the availability information. For example, DanaJr and Petruzzi (2001) extend the classic newsvendor model by considering that the demand depends on both the price and inventory level. Yin et al. (2009) study the effect of the inventory display format with the consideration of markdown pricing strategy. They consider two inventory display formats, i.e., display all available units and display only one unit at a time. Su and Zhang (2009) examine the effects of product availability on demand by assuming that stockouts are costly to consumers. Huang and Mieghem (2013) investigate the impacts of product availability on customers' incentive to click the website, when the customers know their clicks may be tracked as advance demand information by firms. Cachon and Feldman (2015) study the price commitment strategic, and show that signaling availability and the likelihood of discounts to consumers are poor strategies. Consistent with the above researches, our research considers the role of inventory availability information in the operations decisions. However, we focus on the omnichannel environment and consider that the real-time inventory information can be accessed by consumers when the ROPO and BOPS strategies are implemented.

3.3 Base Model

We start our analysis with a base model, where a traditional retailer does not have an online channel. In other words, the products are only sold at store. Based on this base model, we analyze whether the retailer should provide the ROPO or BOPS strategy. Then we extend our analysis to consider the case that the traditional retailer has both the online and store channels in Section 3.4.

A retailer sells a single type of products in a store at retail price p. The retailer's operation is modeled as a newsvendor problem. The market demand D is a random variable with cumulative distribution function $F(\cdot)$ and probability density function $f(\cdot)$. Let $\mu = E(D)$ denote the mean value of the market demand. The retailer's decision variable is the order quantity q in store. The unit cost of the inventory is $c \in (0, p)$. Without loss of generality, both the shortage cost of unmet demand and the salvage value of leftovers are normalized to zero.

Consumers face uncertain valuations of the products. The uncertainty comes from non-digital attributes. We consider that the consumers are *ex ante* homogeneous. They do not know their valuation beforehand, and will know their valuations only when they examine the products in store. We assume that the valuation follows a Bernoulli distribution, i.e., a fraction of consumers $\theta \in (0, 1)$ has positive value v for the products, and the remaining fraction of consumers $1 - \theta$ has zero value for the products. The former is referred as the high type consumers and the latter is referred as the low type consumers.

The benchmark. We first consider a benchmark situation that the retailer implements neither the ROPO nor BOPS strategy. In such setting, the consumers face both valuation uncertainty and availability uncertainty when making their decisions of visiting the store. If a consumer chooses to visit the store, she first incurs a hassle cost t_s (e.g., cost for traveling to the store). If the product is in stock, then the consumer can realize her valuation for the product. A high type consumer will choose to purchase the product, and a low type consumer will leave the market. If the product is out of stock, the consumer leaves the market without knowing her valuation. According to Deneckere and Peck (1995), DanaJr (2001) and Gao and Su (2016b), we can derive the in-stock probability as $A_o(q) = \frac{E\min\{\phi_o D, \frac{q}{\theta}\}}{E\{\phi_o D\}}$, where $\phi_o > 0$ is the true proportion of consumers visiting the store. If $\phi_o = 0$, then $A_o(q) = 1$. Let $\tilde{\xi}_o$ denote the consumers' beliefs about this probability. In this chapter, we use $\tilde{\cdot}$ to denote the belief. Note that because of the ex ante homogeneity, all consumers form the same common belief of the in-stock probability $\tilde{\xi}_o$. Given belief $\tilde{\xi}_o$, expected utility of the consumer visiting the store can be expressed as $u_s = -t_s + \tilde{\xi}_o\theta(v - p)$. If $u_s > 0$, all consumers visit the store; otherwise, they leave the market without purchasing the product. Besides, we assume that store visiting consumer brings cross-selling, and the net profit of each cross-selling is denoted by k_s.

The retailer anticipates that $\tilde{\phi}_o \in [0, 1]$ of consumers will visit the store. Then his profit can be expressed as follows:

$$\Pi_o(q) = p\theta E\min\{\tilde{\phi}_o D, \frac{q}{\theta}\} - cq + k_s E(\tilde{\phi}_o D). \tag{3.1}$$

The first two terms in Equation (3.1) formulate the classic newsvendor profit. The third term is the expected profit from the cross-selling. The retailer chooses q to maximize his expected profit.

The ROPO strategy. We then turn to the case where the retailer implements the ROPO strategy. In such case, consumers can access the inventory availability information from the retailer's website. It is reported that consumers' shopping trips to Macy's, a retail giant in the U.S., will mostly do some online research first (DHL-Trend-Research 2015). Therefore, we assume that consumers will first check the availability information online and then make their decisions of visiting-or-not based on this information. Note that under the ROPO strategy, online retailing is unavailable. This assumption is consistent with the operation of GU Hong Kong (GUHK 2019). Actually, implementing the ROPO strategy is much easier than adding an online retailing channel alongside a traditional retailer, since the latter faces challenges such as online payment and product delivery.

Similar to the availability information model in Gao and Su (2016b), we assume that the units shown on the website do not matter but the sequence does. It means that once the retailer's website shows that the product is in stock, all consumers with positive expected utility will visit the store no matter how many products are in stock. However, those who arrive late may find that the product is out of stock and then leave the market. The expected utility of the consumer visiting the store can be expressed as $u_s = -t_s + \theta(v - p)$. In other words, if a consumer finds the product available and $u_s > 0$, she will visit the store; otherwise, she will leave the market. The retailer anticipates that $\tilde{\phi}_r \in [0, 1]$ of consumers will visit the store. His profit can be expressed as follows:

$$\Pi_r(q) = p\theta E \min\{\tilde{\phi}_r D, \frac{q}{\theta}\} - cq + k_s E \min\{\tilde{\phi}_r D, \frac{q}{\theta}\}. \tag{3.2}$$

The structure of Equation (3.2) is similar to that of Equation (3.1) except for the third term which shows that the ROPO strategy results in less profit from the cross-selling, since consumers won't visit the store if the product is out of stock.

The BOPS strategy. BOPS provides customers with one more channel choice in addition to the availability information, which makes the BOPS strategy different from the ROPO strategy. Besides, the experience of using the BOPS is different from that of visiting the store directly for both the retailer and consumers. For example, the ordered products are usually prepared well at store for the consumers using the BOPS function before their arrival. If the retailer implements the BOPS strategy, each consumer has three options: visiting the store directly, buying the product online and then picking it up at store (using the BOPS function), or leaving the market. These settings are consistent with the operation of Uniqlo in Mainland China (UniqloChina 2018).

We assume that the consumer incurs a hassle cost t_b by using the BOPS function. In the presence of the BOPS function, a consumer can return the product immediately at the store if she realizes low valuation. There is no extra cost for the low type consumers to return the disliked product. However, since the retailer needs extra efforts to restock the returned products, we consider that the retailer incurs a net loss r_s for each returned product. The retailer anticipates that $\tilde{\phi}_{b1} \in [0, 1]$ of consumers will visit the store directly, and $\tilde{\phi}_{b2} \in [0, 1]$ of consumers will use the BOPS function. Consistent with the assumption under ROPO strategy, we assume that given the BOPS function, the consumers will first check the availability information before visiting the store. Then the expected utility of the consumers visiting the store is $u_s = -t_s + \theta(v - p)$. The expected utility of using the BOPS function is $u_b = -t_b + \theta(v - p)$. A consumer's choice depends on the hassle costs t_s and t_b. If $t_s < t_b$, visiting the store directly is more attractive than using the BOPS. In such case, the BOPS strategy functions as the ROPO strategy, and the retailer's expected profit $\Pi_{b1}(q) = \Pi_r(q)$. If $t_s \geq t_b$, using the BOPS is better for the consumers, that is, the consumer prefers buying the product online and picking up in store to going to the store directly. Besides the above two features of the BOPS strategy, i.e., returns in the store and the hassle cost t_b, we capture two more features of the BOPS strategy as follows: First, as mentioned above, the retailer prepares the ordered products well at store for the consumers using the BOPS function and allows they to pick up within several days. For example, Uniqlo in Mainland China allows its consumers to pick the ordered products within eight days (UniqloChina 2018). The retailer pays for such convenience he provides for consumers by incurring the extra inventory cost $c_e \geq 0$ for each ordered product as he needs separate area in his warehouse to store these products. Second, in order to make sure the profit of the BOPS function is positive, we assume that the net benefit of each cross-selling from consumers using the BOPS function is denoted by $k_b \in (r_s(1-\theta) + c_e, k_s]$. We assume $k_b \leq k_s$ based on the fact that using BOPS function makes visiting more purposeful. Nevertheless, we will extend our results by releasing this assumption in Section 3.5. The retailer's expected profit can be expressed as follows:

$$\begin{aligned}\Pi_{b2}(q) =& p\theta E\min\{\tilde{\phi}_{b2}D, \frac{q}{\theta}\} - cq - r_s(1-\theta)E\min\{\tilde{\phi}_{b2}D, \frac{q}{\theta}\}\\ &+ k_bE\min\{\tilde{\phi}_{b2}D, \frac{q}{\theta}\} - c_eE\min\{\tilde{\phi}_{b2}D, \frac{q}{\theta}\}.\end{aligned} \tag{3.3}$$

In Equation (3.3), the first two terms are the newsvendor expected profit from selling the product in the store. The third term represents the loss caused by the returned products. The last two terms are the expected profits from the cross-selling and the extra inventory costs for the ordered products, respectively.

To study the interaction between the retailer and strategic consumers, we consider the participatory rational expectation (RE) equilibrium, which has been wildly used in the literature (see, e.g., Su and Zhang 2008, Cachon and Swinney 2009a, Gao and Su 2016b,a). We have the following definition for a participatory RE equilibrium:

Definition 3.1. *A participatory RE equilibrium satisfies the following conditions:*

(i) $\phi_i \geq 0$ *and* $q \geq 0$ *(*$i = o$*,* r*,* $b1$ *or* $b2$*).*

(ii) Given $\tilde{\xi}_o$*, (a) for* $i = o$*,* r *and* b_1*, if* $u_s > 0$*, then* $\phi_i = 1$*; otherwise,* $\phi_i = 0$*; (b) for* $i = b_2$*, if* $u_b > 0$*, then* $\phi_i = 1$*; otherwise,* $\phi_i = 0$*.*

(iii) Given $\tilde{\phi}_i$*,* $q_i = \arg\max_q \Pi_i(q)$*.*

(iv) $\tilde{\xi}_o = A_o(q)$*.*

(v) $\tilde{\phi}_i = \phi_i$*.*

The first condition is the non-negative condition. The second and third conditions correspond to the optimal principles, which indicate that under the beliefs $\tilde{\xi}_o$ and $\tilde{\phi}_i$, both the consumers and retailer chooses the optimal decisions. The last two conditions refer to the consistency requirements.

By analyzing the participatory RE equilibria for the above strategies, i.e., the benchmark, the ROPO strategy, and the BOPS strategy, we obtain the following proposition.

Proposition 3.1. *Given* θ*, the participatory RE equilibria in the base model are as follows:*

(i) In the benchmark, if $t_s < A_o(q)\theta(v-p)$*, then* $q_o = \theta \bar{F}^{-1}(\frac{c}{p}) > 0$*; otherwise,* $q_o = 0$*. Here,* $A_o(q) = E\min\{D, \bar{F}^{-1}(\frac{c}{p})\}/\mu$*.*

(ii) Under the ROPO strategy, if $t_s < \theta(v-p)$*, then* $q_r = \theta \bar{F}^{-1}(\frac{c}{p+\frac{k_s}{\theta}}) > 0$*; otherwise,* $q_r = 0$*.*

(iii) Under the BOPS strategy, if $t_s < \min\{\theta(v-p), t_b\}$*, then* $q_b = q_{b1} = q_r$*; if* $t_b < \min\{\theta(v-p), t_s\}$*, then* $q_b = q_{b2} = \theta \bar{F}^{-1}(\frac{c}{p+\frac{k_b - r_s(1-\theta)-c_e}{\theta}}) > 0$*; otherwise,* $q_b = 0$*.*

Proposition 3.1 (i) shows that without implementing the ROPO and BOPS strategies, consumers choose to visit the store only if the hassle cost of visiting the store is low, i.e., $t_s < A_o(q)\theta(v-p)$, under which the retailer will prepare a positive amount of inventory in the store for the consumers.

Similarly, Proposition 3.1 (ii) shows that under the ROPO strategy, consumers choose to visit the store only if the hassle cost is not very high, i.e., $t_s < \theta(v-p)$. However, part (ii) differs from part (i) in two ways. First, the threshold of visiting the store under the ROPO strategy is larger than that in the benchmark, i.e., $\theta(v-p) > A_o(q)\theta(v-p)$. It implies that a consumer who chooses to leave the market in the benchmark may change to visit the store if the retailer provides the ROPO function. This is because under the ROPO strategy, the consumers make the visiting decision after observing the inventory availability information. So they do not worry about the stock-out risk when they make the visiting decision. Another difference between parts (i) and (ii) is the inventory level in the store. This is because the consumers can access the inventory information under the ROPO strategy; and if the product is out of stock, then they will not visit the store and the retailer will lose the cross-selling profit $\frac{k_s}{\theta}$. Consequently, the underage cost increases from $p-c$ to $p-c+\frac{k_s}{\theta}$. As a result, the retailer holds more inventory in the store under the ROPO strategy, i.e., $q_r > q_o$.

Proposition 3.1 (iii) shows that under the BOPS strategy, the consumers will visit the store eventually either if the hassle cost of visiting the store directly or if the hassle cost of using the BOPS is low. Specifically, if the hassle cost of visiting the store directly is low, i.e., $t_s < \min\{\theta(v-p), t_b\}$, the consumers will check the inventory information online first and then visit the store, and the BOPS functions as the ROPO; if the hassle cost of using the BOPS is low, i.e., $t_b < \min\{\theta(v-p), t_s\}$, then the consumers will use the BOPS function; otherwise, no consumer will visit the store and retailer holds no inventory in the store. Part (iii) differs from part (ii) in three ways. First, the consumers have one more choice under the BOPS strategy than the ROPO strategy. That is, besides leaving the market and using the ROPO function, the consumers may use the BOPS function according to their hassle costs. Second, a consumer who chooses to leave the market under the ROPO strategy may still be in the market under the BOPS strategy. Because in this case, if the hassle cost of using the BOPS function is low, i.e., $t_b < \theta(v-p)$, then the consumers will buy the product online and pick it up in the store. Third, the inventory levels in the store with the ROPO and BOPS functions are different. This is because with the BOPS function, if the product is out of stock, then the retailer can avoid the return loss $\frac{r_s(1-\theta)}{\theta}$ caused by the consumer who has bought the product and realized the low valuation and the extra inventory cost $\frac{c_e}{\theta}$. Consequently, the underage cost decreases from $p-c+\frac{k_s}{\theta}$ to $p-c+\frac{k_b}{\theta}-\frac{r_s(1-\theta)}{\theta}-\frac{c_e}{\theta}$. As a result, when the unit cross-selling profits are equal (i.e., $k_s = k_b$), then the retailer holds less inventory in the store if the consumers use the BOPS function than that if they use the ROPO function, i.e., $q_{b2} < q_{b1}$. Figure 3.1 depicts the consumer equilibrium behavior under the three strategies in the base model.

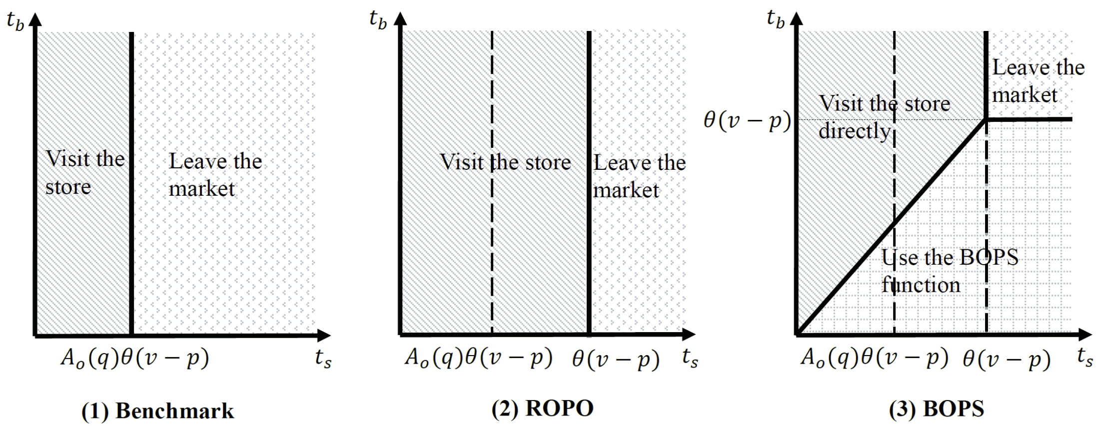


Figure 3.1: Consumer Equilibrium Behavior in the Base Model

Proposition 3.2. *Comparing the three strategies in the base model, we have*

(i) If $t_b > \min\{\theta(v-p), t_s\}$*, then* $\Pi_b = \Pi_{b1} = \Pi_r$.

- *If* $t_s < A_o(q)\theta(v-p)$, $\Pi_o > \Pi_r$;

- *If* $A_o(q)\theta(v-p) < t_s < \theta(v-p)$*,* $\Pi_r > \Pi_o = 0$*;*
- *If* $t_s > \theta(v-p)$*,* $\Pi_r = \Pi_o = 0$*.*

(ii) If $t_b < \min\{\theta(v-p), t_s\}$*, then* $\Pi_b = \Pi_{b2}$*.*

- *If* $t_s < A_o(q)\theta(v-p)$*,* $\Pi_o > \Pi_r > \Pi_b$*;*
- *If* $A_o(q)\theta(v-p) < t_s < \theta(v-p)$*,* $\Pi_r > \Pi_b > \Pi_o = 0$*;*
- *If* $t_s > \theta(v-p)$*,* $\Pi_b > \Pi_r = \Pi_o = 0$*.*

Proposition 3.2 shows the conditions for the strategies being optimal in the base model. According to Proposition 3.2 (i), if the hassle cost of using the BOPS function is large, i.e., $t_b > \min\{\theta(v-p), t_s\}$, and the hassle cost of visiting the store is small, i.e., $t_s < A_o(q)\theta(v-p)$, then the benchmark is optimal for the retailer. This is because under these conditions, the retailer will lose some cross-selling profits if the ROPO strategy or BOPS strategy is adopted by the retailer. If the hassle cost of visiting the store is moderate, i.e., $A_o(q)\theta(v-p) < t_s < \theta(v-p)$, then offering the ROPO function is profitable to the retailer. As shown in Proposition 3.1, under these conditions, the consumers will leave the market in the benchmark but will visit the store if the ROPO function is offered. Meanwhile, the BOPS will function as the ROPO under these conditions. Thus, the ROPO strategy is optimal for the retailer in this case. According to part (ii), the conditions of the BOPS strategy being favorable to the retailer are that it serves consumers with low cost of using the BOPS function, i.e., $t_b < \min\{\theta(v-p), t_s\}$, and at the same time each consumer faces high cost of visiting the store directly, i.e., $t_s > \theta(v-p)$. The BOPS strategy is unfavorable to the retailer when the hassle cost of visiting the store is low as shown in the first two cases in part (ii). This is because in these cases, the benefit of the BOPS function (e.g., lower hassle cost) is outweighed by its drawbacks (e.g., the lower net benefit of cross-selling k_b, the return loss $r_s(1-\theta)$, and the extra inventory cost c_e). Figure 3.2 pictorially shows the conditions for the strategies being optimal, where the profit is labeled in the area if the corresponding strategy is optimal.

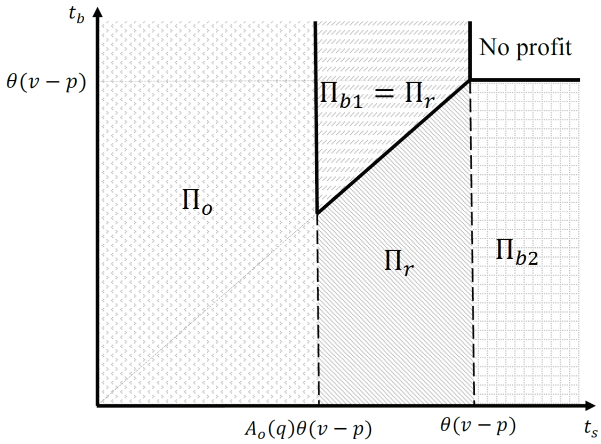


Figure 3.2: Optimal Profits in the Base Model

3.4 Analysis with Online and Offline Sales

In this section, we extend our analysis to consider a traditional retailer selling the product through both online and offline channels. This model is referred as the multi-channel model. We investigate that whether the retailer with the multi-channel selling should implement the ROPO and BOPS strategies. Following Gao and Su (2016a), the online channel is modeled exogenously. That is, we will not determine the ordering decision for the online channel, and the retailer's decision variable is still the order quantity q in the store.

The benchmark. Similar to the base model in Section 3.3, we first consider a benchmark where the retailer implements neither the ROPO nor BOPS strategy. However, the difference is that offered the online channel, the consumer will turn to the online channel if the product is out of stock in the store. If the consumer chooses to buy the product through the online channel, she incurs a hassle cost t_o (e.g., cost of paying shipping fees), and the valuation is realized only if the product is received. The consumer can return the product to the retailer with additional hassle cost t_r, if she realizes the low valuation for it. We assume that it is always better for a low type consumer to return the product rather than keeping it, i.e., $t_r < p$. For each unit sold through the online channel, the retailer obtains a marginal revenue w if it is not returned; otherwise, it incurs a net loss r_o to the retailer. Let $R_o = w\theta - r_o(1-\theta)$ be the net profit of the online channel.

In order to guarantee that the online channel is profitable, we assume $R_o > 0$. However, our analysis can be easily extended to the case where $R_o \leq 0$, which actually is the base model in Section 3.3. The expected utility of the consumer purchasing online can be expressed as $u_o = -t_o + \theta(v-p) + (1-\theta)(-t_r)$. To make sure that the online channel will be considered by the consumers, we assume that $u_o > 0$, i.e., $\theta(v-p) > t_o + (1-\theta)t_r$. Otherwise, this model is reduced to the base model.

Then a consumer has two options in this benchmark: buying online with the expected utility u_o and visiting the store directly with the expected utility $u_s = -t_s + \tilde{\xi}_o\theta(v-p) + (1-\tilde{\xi}_o)u_o$. The retailer forms a belief $\tilde{\phi}_o$ as the show-up rate of consumers in the store. His profit can be expressed as follows:

$$\Pi_o(q) = p\theta E\min\{\tilde{\phi}_o D, \frac{q}{\theta}\} - cq + k_s E(\tilde{\phi}_o D) + R_o E(\tilde{\phi}_o D - \frac{q}{\theta})^+ + R_o E((1-\tilde{\phi}_o)D). \tag{3.4}$$

In Equation (3.4), the first two terms are the newsvendor expected profit from selling the product in the store. The third term is the expected profit from the cross-selling. The fourth term is the expected profit from the consumer who encounters stock-out in the store and buy the product online. The last term is the expected profit from the consumers who buy online directly.

The ROPO strategy. Analysis of the ROPO strategy is the same as that in the base model. This is because, according to the real practice, such as the operations of the ROPO strategy in GU Hong Kong, the online retailing is unavailable if the ROPO strategy is adopted. Thus, consumer's behavior is not affected by the online retailing under the ROPO strategy. The retailer's profit is shown in Equation (3.2).

The BOPS strategy. If the retailer implements the BOPS strategy, each consumer has three options: visiting the store directly, buying the product online and then picking it up at store (using the BOPS function), or purchasing online and waiting for the retailer's delivery. The expected utility of the consumers under these three options are $u_s = -t_s + \theta(v-p)$, $u_b = -t_b + \theta(v-p)$ and u_o, respectively. We can see that if $t_s < t_b$, then visiting store directly is a better option than using the BOPS. In this case, the retailer's profit can be expressed as follows:

$$\begin{aligned}\Pi_{b1}(q) =& p\theta E\min\{\tilde{\phi}_{b1}D, \frac{q}{\theta}\} - cq \\ &+ k_s E\min\{\tilde{\phi}_{b1}D, \frac{q}{\theta}\} \\ &+ R_o E(\tilde{\phi}_{b1}D - \frac{q}{\theta})^+ + R_o E((1-\tilde{\phi}_{b1})D).\end{aligned} \tag{3.5}$$

A remarkable difference between Equations (3.5) and (3.4) is that the accessible availability information leads to less profit for the cross-selling. Besides, Equation (3.5) is different from Equation (3.2), because now the retailer can get profits from online retailing. Thus, if both online and store channels are available, the BOPS strategy never functions as the ROPO strategy.

On the contrary, if $t_b < t_s$, then consumers prefer using the BOPS to visiting the store directly. In this case, the retailer's profit can be expressed as follows:

$$\begin{aligned}\Pi_{b2}(q) =& p\theta E\min\{\tilde{\phi}_{b2}D, \frac{q}{\theta}\} - cq \\ &+ k_b E\min\{\tilde{\phi}_{b2}D, \frac{q}{\theta}\} - r_s(1-\theta)E\min\{\tilde{\phi}_{b2}D, \frac{q}{\theta}\} - c_e E\min\{\tilde{\phi}_{b2}D, \frac{q}{\theta}\} \\ &+ R_o E(\tilde{\phi}_{b2}D - \frac{q}{\theta})^+ + R_o E((1-\tilde{\phi}_{b2})D).\end{aligned} \tag{3.6}$$

The major differences between Equations (3.6) and (3.5) are as follows: First, it will incur return loss for the retailer if the consumers use the BOPS function and realize low valuations for the products at the store. Here, the return loss is presented by the fourth term in Equation (3.6). Second, it will incur the extra inventory cost, as shown by the fifth term in Equation (3.6). Note that, in order to make sure the profit of the BOPS function is positive, we assume that $k_b \in (r_s(1-\theta) + c_e + R_o, k_s]$ in the multi-channel model. Nevertheless, we will extend our results by releasing this assumption in Section 3.5.

Similar to the base model, we consider the participatory RE equilibrium for this multi-channel model. The definition for a participatory RE is the same as Definition 3.1, except for the second condition. Given the online retailing, the second condition is changed to: Given $\tilde{\xi}_o$, (a) for $i = o$, r and b_1, if $u_s > u_o$, then $\phi_i = 1$; otherwise, $\phi_i = 0$; (b) for $i = b_2$, if $u_b > u_o$, then $\phi_i = 1$; otherwise, $\phi_i = 0$.

Proposition 3.3. *Given θ, the participatory RE equilibria in the multi-channel model are as follows:*

(i) In the benchmark, if $t_s < A_o'(q)(t_o + (1-\theta)t_r)$, then $q_o = \theta \bar{F}^{-1}(\frac{c}{p - \frac{R_o}{\theta}}) > 0$; otherwise, $q_o = 0$. Here, $A_o'(q) = E \min\{D, \bar{F}^{-1}(\frac{c}{p-\frac{R_o}{\theta}})\}/\mu$.

(ii) Under the ROPO strategy, if $t_s < \theta(v - p)$, then $q_r = \theta \bar{F}^{-1}(\frac{c}{p+\frac{k_s}{\theta}}) > 0$; otherwise, $q_r = 0$.

(iii) Under the BOPS strategy, if $t_s < \min\{t_b, t_o + (1-\theta)t_r\}$, then $q_b = q_{b1} = \theta \bar{F}^{-1}(\frac{c}{p+\frac{k_s}{\theta}-\frac{R_o}{\theta}}) > 0$; if $t_b < \min\{t_s, t_o + (1-\theta)t_r\}$, then $q_b = q_{b2} = \theta \bar{F}^{-1}(\frac{c}{p+\frac{k_b - r_s(1-\theta)-c_e}{\theta}-\frac{R_o}{\theta}}) > 0$; otherwise, $q_b = 0$.

The structure of the results in Proposition 3.3 is the same as that in Proposition 3.1. Similarly, there exist conditions associated with the hassle costs, under which the consumers choose to visit the store, use the BOPS function, buy online or leave the market. Specifically, Proposition 3.3 (i) shows that in the benchmark, the consumers will visit the store only if the hassle cost of visiting the store is small, i.e., $t_s < A_o'(q)(t_o + (1-\theta)t_r)$. Here, $A_o'(q)$ is the in-stock probability and $t_o + (1-\theta)t_r$ is the expected cost (excluded the retail price) undertaken by consumers when purchasing online. Since the online retailing is unavailable in the ROPO strategy, Proposition 3.3 (ii) is the same as Proposition 3.1 (ii). Part (iii) shows that if the hassle cost of visiting the store directly is low, i.e., $t_s < \min\{t_b, t_o + (1-\theta)t_r\}$, then the consumers will visit the store directly; if the hassle cost of using the BOPS is low, i.e., $t_b < \min\{t_s, t_o + (1-\theta)t_r\}$, then the consumers will use the BOPS function; otherwise, no consumer will visit the store and the retailer will not hold inventory in the store.

Comparing the benchmark and the ROPO strategy, we observe that the threshold of visiting the store under the ROPO strategy is larger than that in the benchmark, i.e., $\theta(v - p) > A_o'(q)(t_o + (1-\theta)t_r)$. This can be explained from two aspects. First, under the ROPO strategy with the availability information, consumers do not worry about the stock-out risk when they make the visiting decision. Second, when making the visiting decision, the consumers will consider the trade-off between the costs of visiting the store and buying online directly in the benchmark; while they will consider the trade-off between the hassle cost of visiting the store and the expected marginal valuation of the product (i.e., $\theta(v-p)$), which is larger than the cost of buying online directly (i.e., $t_o+(1-\theta)t_r$), under the ROPO strategy. The threshold of visiting the store under the BOPS strategy is also different from those under the other two strategies, because of the joint effects of the availability information, cost of buying online directly, and the hassle cost of using the BOPS function.

Besides, the inventory levels in the store under the three strategies are different. We first compare the ROPO strategy with the benchmark. In the benchmark, if the product is out of stock, then the consumers will buy the product online, and the retailer will obtain a profit margin $\frac{R_o}{\theta}$ from the online channel. So the shortage cost is $p-c-\frac{R_o}{\theta}$. On the other hand, under the ROPO strategy, given the availability information, the consumers will leave the market if the product is out of stock. Then the retailer will lose the potential cross-selling profit $\frac{k_s}{\theta}$. So the shortage cost is $p-c+\frac{k_s}{\theta}$. Thus, the shortage cost under the ROPO strategy is larger than that in the benchmark. As a result, the retailer holds more inventory in the store under the ROPO strategy, i.e., $q_r > q_o$. Comparing the BOPS strategy with the benchmark, we observe that $q_{b1} > q_o$. This is also due to the effect of the availability information. With the availability information, the retailer will hold more inventory in the store to avoid the potential loss associated with the cross-selling. It is interesting to show that if the consumers choose to use the BOPS function, with the availability information, the retailer may not hold more inventory in the store than that without the availability information, i.e., q_{b2} may not be larger than q_o. This is because, with the BOPS function, the retailer will incur the return loss from the consumer who has bought the product and realized the low valuation and incur the extra inventory cost for each ordered product. Then, if the product is out of stock, although the retailer will lose the cross-selling profit $\frac{k_b}{\theta}$, he can avoid the return loss $\frac{r_s(1-\theta)}{\theta}$ and the extra inventory cost $\frac{c_e}{\theta}$. Thus, whether the retailer should hold more inventory depends on the value of $\frac{k_b}{\theta} - \frac{r_s(1-\theta)}{\theta} - \frac{c_e}{\theta}$. Comparing the BOPS strategy with the ROPO strategy, we observe that the retailer holds less inventory under the BOPS strategy than that under the ROPO strategy. This is because under the BOPS strategy, the retailer can obtain a profit margin $\frac{R_o}{\theta}$ from the online channel and/or avoid the return loss $\frac{r_s(1-\theta)}{\theta}$ and the extra inventory cost $\frac{c_e}{\theta}$, if the product is out of stock.

Moreover, comparing Propositions 3.3 and 3.1, we find that the inventory levels in the store in the multi-channel model are not larger than those in the base model. In other words, the retailer will hold fewer inventory in the store when the online retailing is offered to consumers. This is because, with the online retailing, the retailer has lower shortage costs, as he can obtain a profit margin from the online channel when encountering the stock-out. In addition, note that we have $A_o'(q) < A_o(q)$ and $t_o + (1-\theta)t_r < \theta(v-p)$. So the thresholds of visiting the store in the multi-channel are not larger than those in the base model. It implies that given the online retailing, the retailer serves fewer consumers in the store. Figure 3.3 depicts the consumer equilibrium behavior under the three strategies in the multi-channel model.

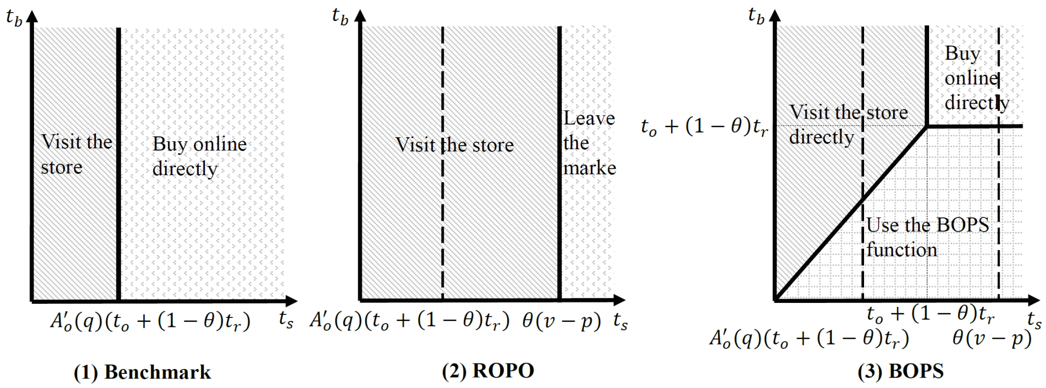


Figure 3.3: Consumer Equilibrium Behavior in the Multi-channel Model

Proposition 3.4. *Comparing the three strategies in the multi-channel model, we have*

(i) If $t_b > \min\{t_s, t_o + (1-\theta)t_r\}$*, then* $\Pi_b = \Pi_{b1}$.

- *If* $t_s < A_o'(q)(t_o + (1-\theta)t_r)$, $\Pi_o > \Pi_b > \Pi_r$*;*

- *If* $A_o'(q)\theta(t_o + (1-\theta)t_r) < t_s < t_o + (1-\theta)t_r$, $\Pi_b > \max\{\Pi_o, \Pi_r\}$*;*
- *If* $t_o + (1-\theta)t_r < t_s < \theta(v-p)$*: then there exists a threshold* $\hat{R}_o$ *such that if* $R_o < \hat{R}_o$*, then* $\Pi_r > \Pi_o = \Pi_b$*; otherwise,* $\Pi_o = \Pi_b \geq \Pi_r$*;*
- *If* $t_s > \theta(v-p)$*,* $\Pi_o = \Pi_b > \Pi_r = 0$*.*

(ii) If $t_b < \min\{t_s, t_o + (1-\theta)t_r\}$*, then* $\Pi_b = \Pi_{b2}$*.*

- *If* $t_s < A_o'(q)(t_o + (1-\theta)t_r)$*, then* $\Pi_o > \max\{\Pi_b, \Pi_r\}$*;*
- *If* $A_o'(q)(t_o + (1-\theta)t_r) < t_s < \theta(v-p)$*: then there exits a threshold* $\tilde{R}_o$ *such that if* $R_o < \tilde{R}_o$*, then* $\Pi_r > \Pi_b > \Pi_o$*; if* $\tilde{R}_o \leq R_o < \hat{R}_o$*, then* $\Pi_b \geq \Pi_r > \Pi_o$*; if* $R_o \geq \hat{R}_o$*, then* $\Pi_b > \Pi_o \geq \Pi_r$*.*
- *If* $t_s > \theta(v-p)$*,* $\Pi_b > \Pi_o > \Pi_r = 0$*.*

Proposition 3.4 shows the conditions for the strategies being optimal in the multi-channel model. Similar to Proposition 3.2 (i), Proposition 3.4 (i) indicates that if the hassle cost of visiting the store is small, i.e., $t_s < A_o'(q)(t_o + (1-\theta)t_r)$, then the benchmark is optimal for the retailer. This is because under these conditions, the retailer will lose some cross-selling profits if the ROPO or BOPS is used by the retailer. Besides, it shows that if the hassle cost of visiting the store is very large, i.e., $t_s > \theta(v-p)$, then $\Pi_o = \Pi_b > \Pi_r = 0$, implying that the benchmark can also be optimal for the retailer. This is because under this condition, all consumers leave the market under the ROPO strategy and some consumers will still purchase online in the benchmark or under the BOPS strategy. We can see similar results when the hassle cost is not very large, i.e., $t_o + (1-\theta)t_r < t_s < \theta(v-p)$, and the profit of online retailing is large enough, i.e., $R_o > \hat{R}_o$. If the hassle cost of visiting the store is moderate, i.e., $A_o'(q)\theta(t_o + (1-\theta)t_r) < t_s < t_o + (1-\theta)t_r$, then the BOPS strategy is optimal for the retailer, while the consumers will choose to buy the product in the store and never use the BOPS function, due to the small hassle cost of visiting the store and the high cost of using the BOPS function. Similar to Proposition 3.2 (ii), Proposition 3.4 (ii) indicates that the conditions of the BOPS strategy being profitable are mainly that it serves consumers with low cost of using the BOPS function, i.e., $t_b < \min\{\theta(v-p), t_s\}$, and at the same time the profit of online retailing is large enough (i.e., $R_o > \tilde{R}_o$) when the hassle cost of visiting the store is not very large (i.e., $A'_o(q)(t_o + (1-\theta)t_r) < t_s < \theta(v-p)$), or the hassle cost of visiting the store is large enough, i.e., $t_s > \theta(v-p)$.

Recalling that if the retailer implements the ROPO strategy, then he will lose the benefits from consumers who encounter stockout in the store, because there is no online retailing providing the purchasing option for these consumers. Meanwhile, the retailer will also lose some potential profit of cross-selling since consumers won't come to the store if they find out stockout from the retailer's website. However, it is interesting to show in Proposition 3.4 that, the ROPO strategy is optimal for the retailer if the offline channel brings more profit than the online channel, i.e., $R_o < \hat{R}_o$ when $t_b > \min\{t_s, t_o + (1-\theta t_r)\}$ and $t_o + (1-\theta)t_r < t_s < \theta(v-p)$, or $R_o < \tilde{R}_o$ when $t_b < \min\{t_s, t_o + (1-\theta t_r)\}$ and $A'_o(q)t_o + (1-\theta)t_r < t_s < \theta(v-p)$. The latter condition is interesting as it shows that, if $R_o < \tilde{R}_o$, the ROPO strategy can be more profitable than the BOPS strategy even when the hassle cost of using the BOPS t_b is small. The reason is that, even the BOPS function can coexist with online retailing and thus gain profit form those consumers who encounter stockout in the store, it may also incur some losses due to joint effect of the lower net profit of each cross-selling k_b, the store return loss $r_s(1-\theta)$ and the extra inventory cost c_e for each ordered product. In other words, when the profit of online retailing is not large enough, i.e., $R_o < \tilde{R}_o$, the benefit of the BOPS function is outweighed by its drawbacks. Figure 3.4 pictorially shows the conditions for the strategies being optimal, where the profit is labeled in the area if the corresponding strategy is optimal.

3.5 Extensions

3.5.1 General k_b

Since consumers who using the BOPS function are more purposeful than those who visit the store directly, we assumed $k_b \leq k_s$ in Sections 3.3 and 3.4. Here we

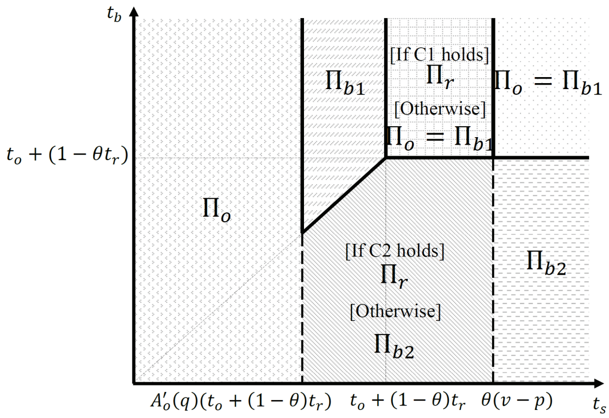


Figure 3.4: Optimal Profits in the Multi-channel Model

Note: in Figure 3.4, conditions C1 and C2 mean $R_o < \hat{R}_o$ and $R_o < \tilde{R}_o$, respectively.

relax this assumption. Then Proposition 3.2 is changed to following proposition:

Proposition 3.5. *For any $k_b > r_s(1-\theta) + c_e$, comparing the three strategies in Proposition 3.1, we have*

(i) If $t_b > \min\{\theta(v-p), t_s\}$, then $\Pi_b = \Pi_{b1} = \Pi_r$.

- *If $t_s < A_o(q)\theta(v-p)$, $\Pi_o > \Pi_r$;*
- *If $A_o(q)\theta(v-p) < t_s < \theta(v-p)$, $\Pi_r > \Pi_o = 0$;*
- *If $t_s > \theta(v-p)$, $\Pi_r = \Pi_o = 0$.*

(ii) If $t_b < \min\{\theta(v-p), t_s\}$, then $\Pi_b = \Pi_{b2}$.

- *If $t_s < A_o(q)\theta(v-p)$: then if $k_b - r_s(1-\theta) - c_e > \frac{k_s}{A_o(q)}$, $\Pi_b > \Pi_o > \Pi_r$; if $k_s < k_b - r_s(1-\theta) - c_e \leq \frac{k_s}{A_o(q)}$, then $\Pi_o \geq \max\{\Pi_b, \Pi_r\}$;*
- *If $A_o(q)\theta(v-p) < t_s < \theta(v-p)$: then if $k_b - r_s(1-\theta) - c_e > k_s$, $\Pi_b > \Pi_r > \Pi_o = 0$; otherwise $\Pi_r \geq \Pi_b > \Pi_o = 0$;*
- *If $t_s > \theta(v-p)$, $\Pi_b > \Pi_r = \Pi_o = 0$.*

Proposition 3.5 further shows the conditions that the BOPS strategy are more profitable than other strategies: Each consumer needs to bring much higher net benefit of each cross-selling, i.e., $k_b > r_s(1-\theta) + c_e + \frac{k_s}{A_o(q)}$ for $t_s < A_o(q)\theta(v-p)$

and $k_b > r_s(1-\theta)+c_e+k_s$ for $A_o(q)\theta(v-p) < t_s < \theta(v-p)$. This result provides an explanation for Walmart's decision to provide more convenience options in addition to the previous kiosks at the existing fuel station and convenience store (CSNews 2017). For example, Walmart started to test a newest fuel station and convenience store combo with $2,500$ square feet, which replaces the previous kiosk at the existing fuel station and provides expanded selection of food and drinks. Besides, Walmart also began testing the Walmart Pickup with Fuel store with $4,000$ square feet, which offers gas pumps and grocery items, with the addition service for pickup grocery orders placed online (CSNews 2017). By offering more options, it is expected to bring more cross-selling benefits from consumers visiting the store.

Similarly, Proposition 3.4 is changed to following proposition:

Proposition 3.6. *For any $k_b > r_s(1-\theta)+c_e+R_o$, comparing the three strategies in the Proposition 3.3, we have*

(i) If $t_b > \min\{t_s, t_o+(1-\theta)t_r\}$, then $\Pi_b = \Pi_{b1}$.

- *If $t_s < A_o'(q)(t_o+(1-\theta)t_r)$, $\Pi_o > \Pi_b > \Pi_r$;*
- *If $A_o'(q)\theta(t_o+(1-\theta)t_r) < t_s < t_o+(1-\theta)t_r$, $\Pi_b > \max\{\Pi_o, \Pi_r\}$;*
- *If $t_o+(1-\theta)t_r < t_s < \theta(v-p)$: then there exists a threshold $\hat{R}_o$ such that if $R_o < \hat{R}_o$, then $\Pi_r > \Pi_o = \Pi_b$; otherwise, $\Pi_o = \Pi_b \geq \Pi_r$;*
- *If $t_s > \theta(v-p)$, $\Pi_o = \Pi_b > \Pi_r = 0$.*

(ii) If $t_b < \min\{t_s, t_o+(1-\theta)t_r\}$, then $\Pi_b = \Pi_{b2}$.

- *If $t_s < A_o'(q)(t_o+(1-\theta)t_r)$: then if $k_b - r_s(1-\theta) - c_e > \frac{k_s}{A_o'(q)}$, $\Pi_b > \Pi_o > \Pi_r$; otherwise $\Pi_o \geq \max\{\Pi_b, \Pi_r\}$;*
- *If $A_o'(q)(t_o+(1-\theta)t_r) < t_s < \theta(v-p)$: then there exits a threshold $\tilde{R}_o$ such that if $R_o < \tilde{R}_o$, then if $k_b < k_b^{(1)}$, $\Pi_r > \Pi_b > \Pi_o$, otherwise, $\Pi_b \geq \Pi_r > \Pi_o$; if $\tilde{R}_o \leq R_o < \hat{R}_o$, then $\Pi_b \geq \Pi_r > \Pi_o$; if $R_o \geq \hat{R}_o$, then $\Pi_b > \Pi_o \geq \Pi_r$. Here, $k_b^{(1)}$ is a threshold value associated with k_b.*

- *If* $t_s > \theta(v-p)$, $\Pi_b > \Pi_o > \Pi_r = 0$.

As mentioned in Section 3.4, when the profit of online retailing is not large enough, i.e., $R_o < \tilde{R}_o$, the benefit of the BOPS function is outweighed by its drawbacks (e.g., the lower cross-selling profit, the return loss and the extra inventory cost). Proposition 3.6 shows that the retailer can change such situation by increasing the net benefit of each cross-selling k_b.

3.5.2 Imperfect Availability Information

Recalling that in the base model and multi-channel model, we consider that given the ROPO function, each consumer can access the availability information from retailer's website. And we assume that she can certainly get the product when visiting the store, if she saw that the product was in stock from the website. However, in practice, the product may be out of stock even when a consumer sees that it is in stock beforehand, since others may buy the product in the store before she gets to the store. Thus, in this extension, we assume that ROPO function provides consumers with imperfect availability information. Each consumer updates her belief of the availability information after checking the inventory level online. Note that without the ROPO and BOPS strategies, the expected number of store visiting consumers who find that the product is in-stock and encounter stockout are $E\min\{\tilde{\phi}_r D, \frac{q}{\theta}\}$ and $E(\tilde{\phi}_{b1} D - \frac{q}{\theta})^+$, respectively. Then, with imperfect availability information, the expected number of consumers visiting the store can be defined as $E\min\{\tilde{\phi}_i D, \frac{q}{\theta}\} + \alpha E(\tilde{\phi}_i D - \frac{q}{\theta})^+$ $(i = r, b1)$, where α, $\alpha \in [0, 1]$, is common knowledge. If $\alpha = 0$, then it means that consumers will visit the store only if they find from the online that the product is in-stock, which is the same as that in the base and multi-channel models; if $\alpha = 1$, then it means that consumers will visit the store even if they find from the online that the product is out of stock. Now, we can derive the in-stock probability $A_i(q) = E\min\{\tilde{\phi}_i D, \frac{q_i}{\theta}\}/(E\min\{\tilde{\phi}_i D, \frac{q_i}{\theta}\} + \alpha E(\tilde{\phi}_i D - \frac{q_i}{\theta})^+)$ $(i = r, b1)$. Differing from the previous models where consumers's beliefs about the in-stock probability is equal to one under the ROPO strategy, here, consumers form their beliefs of the in-stock probability $\tilde{\eta}_r$ after obtaining the imperfect availability information. In other words, if the retailer implements the ROPO strategy, the expected utility of visiting the store when a consumer sees that the store is in stock is: $u_s = -t_s + \tilde{\eta}_r\theta(v - p)$. The retailer's profit function is formulated as follows:

$$\Pi_r(q) = p\theta E\min\{\tilde{\phi}_r D, \frac{q}{\theta}\} - cq + k_s(E\min\{\tilde{\phi}_r D, \frac{q}{\theta}\} + \alpha E(\tilde{\phi}_r D - \frac{q}{\theta})^+).$$

If the retailer implements the BOPS strategy, but consumers prefer visiting the store directly to using the BOPS function, they form their common beliefs in the same way as the ROPO strategy. We highlight the booking function of the BOPS option. That is, consumers can certainly obtain the product if the product is in stock and they choose to buy through BOPS. For this extension, we show that our main results from the comparison of aforementioned three strategies still holds. Detailed discussions of this extension can be found in the Appendix B.

3.5.3 Consumers' Anticipation of Cross-Selling

In the base model and multi-channel model, we assume that consumers make decisions based on the expected utility of the product. This subsection discusses what will happen if the consumers also anticipate their potential expenses of cross-selling. Anticipation of cross-selling may make visiting the store or using the BOPS function more attractive to consumers, if the utility of the cross-selling product is positive. We assume the valuation and price of each cross-selling brought by store visiting consumers to be v_{ks} and p_{ks}, respectively. Similarly, we assume the valuation and price of each cross-selling brought by consumers using the BOPS function to be v_{kb} and p_{kb}, respectively. The cost of each cross-selling is c_k. Then we have $k_s = p_{ks} - c_k$ and $k_b = p_{kb} - c_k$. By defining $t'_s = t_s - (v_{ks} - p_{ks})$ and $t'_b = t_b - (v_{kb} - p_{kb})$, our analyses and results in both the base model and multi-channel model keep unchanged.

3.6 Conclusion

With the help of ubiquitous Internet, consumers' shopping experience crosses different channels, and they strategically choose the channels to purchase the product. Accordingly, many retailers have provided some cross channel capabilities to consumers, and thus transition to the ominichannel operations. In this chapter, we study two typical cross channel strategies, i.e., the ROPO and BOPS, which are implemented by many retailers. We consider a retailer selling a single type of products to consumers in the omnichannel environment. Consumers' valuation for the product is uncertain and demand is stochastic. We first consider a base model in which a traditional retailer does not have an online channel. We analyze whether the retailer should stay in the region with traditional selling, or step out to use the ROPO strategy, or go further to adopt the BOPS strategy. Then we extend our study to consider a traditional retailer with multi-channel selling.

We derive the consumers' equilibrium behavior and retailer's optimal inventory level for each selling strategy. We show that both the ROPO and BOPS strategies may not be optimal for the retailer. Different omnichannel selling strategies should be tailored to different retailers. If the profits of online retailing are low, it could be optimal for the retailer to adopt the ROPO strategy. We also find that in order to successfully adopt the BOPS strategy, the retailer should increase the profits of online retailing or the cross-selling benefit for each store visiting consumers. To achieve this, the retailer may provide expanded selection of the products in the offline store. In addition, we further extend our study to consider imperfect availability information and consumers' anticipation of cross-selling. We show that our key results still hold in these extensions.

Chapter 4

Decentralized or Centralized Merger with Price and Quality Competition

4.1 Introduction

Many firms adopt mergers and acquisitions (M&As) as an important strategy in competitive business environment. According to WilmerHale (2018), there were more than 53 thousand reported M&A transactions in 2017 across the globe, which were valued about 3.26 trillion US dollars. Among all these transactions, some are horizontal mergers which refer to the mergers between competitors in the same business sector, some are vertical mergers happening between supply chain members, and others are conglomerate mergers which are the mergers between irrelevant companies (Amihud and Lev 1981). This study focuses on horizontal mergers. Henceforth, a merger refers to a horizontal merger and a participant refers to a participant of a horizontal merger. Horizontal mergers can be easily found in practice. For example, Hewlett-Packard and Compaq merged in 2001 (Cho 2013), Coach acquired Kate Spate in 2017 (BusinessWire 2017), Unilever acquired Ben & Jerry's in 2002 and L'ORÉAL merged The Body Shop in 2006 (Galpin 2014).

A merger may be motivated by cost synergy (Mazzeo et al. 2014, Pinto and Sibley 2016, Atallah 2015). Indeed, cost synergy is one of the main motivations behind most mergers (Bascle et al. 2008) and is one of the most important factors behind successful mergers (Houston et al. 2001, Delong 2003). Cost synergy can come from multiple sources. After two firms merge, they are able to benefit from economies of scale and produce with lower cost, obtain lower procurement cost through joint purchasing, and reduce R&D cost by sharing R&D personnel and complementary technologies and talents (Cho 2013, Cho and Wang 2016). Coach's acquisition of Kate Spade is a notable example. As said by Coach's Chief Financial Officer, "Due to the complementary nature of our respective businesses, we believe that we can realize a run rate of approximately 50 million US dollars in synergies within three years of the deal closing. These cost synergies will be realized through operational efficiencies, improved scale and inventory management, and the optimization of Kate Spades supply chain network" (BusinessWire 2017).

After merger, the post-merger firm needs to decide the desired degree of post-merger integration, which is an important aspect of M&As (Galpin 2014). In practice, different post-merger firms tend to choose different types of integration. Some choose simple integration and their participants stand alone after merger. We refer to this case as "decentralized merger". In the decentralized merger case, participants make decentralized decisions of their own competitive strategies to maximize their own profits. For example, Ben & Jerry's has been operated as an independent company even though it was acquired by Unilever in 2002 (Galpin 2014). Another example is The Body Shop which was acquired by L'ORÉAL in 2006 and had been remained autonomous about its own decision-making during the time when it was owned by L'ORÉAL (Galpin 2014). In fact, without deep search, it is hard for consumers to find out the relationship between Ben & Jerry's and Unilever and between The Body Shop and L'ORÉAL. By contrast, some post-merger firms fully integrate their participants and seek overall collaboration. We denote this case as "centralized merger". In the centralized merger case, the post-merger firm makes centralized decisions for all the participant firms to maximize the total profit. Norfolk & Southern is an example of centralized mergers (Galpin 2014). Besides, in the case of centralized merger, the post-merger firms are able to change their product lines. Some post-merger firms cut their product lines and offer a single product to the consumers. For example, after DiDi merged with Uber China, Uber China is disappered in the Mainland China. By contrast, some post-merger firms keep all the participant firms' products. For example, Lenovo keeps both brands Think and Lenovo after it acquired IBM's PC Division in 2004 (Spooner 2004). It had tried to make centralized decisions for these two brands before they were split into two business groups in 2013 (Lai 2013).

M&As often hit the headline news due to their significant impacts. Previous literature regarding mergers mainly focuses on their effects on retail price. Conventional wisdom believes that a merger will reduce market competition and raise price. Some researches (e.g., Farrell and Shapiro (1990), Cho (2013), Cho and Wang (2016)) show that, if there is cost synergy, price may fall after a merger. Besides, a merger may raise or reduce the participants' quality or service level. For example, one year after DiDi merged Uber China, an observation was that the company reduced its service level as it had become more difficult for consumers to get a ride and the response rate for rides hailed at busy pickup locations reduced remarkably. This phenomenon was thought, not surprisingly, to be partly caused by the merger (Horwitz 2017).

Besides, many mergers happened in competitive markets. For example, although the firms we mentioned at the beginning of this section merged with their competitors, they still face competition from outside firms such as Dell, Michael Kors, and Shiseido.

Motivated by the above observations, this study focuses on competing firms' merging decision and strategy in a competitive market. We aim to answer the following questions. First, whether and when should two competing firms merge in a competitive market? Second, whether and when should the post-merger firm make centralized or decentralized decision about the participant firms' prices and quality? Third, what are the effects of merger on market competition, price, and quality? Last, how total consumer utility and social welfare are affected by the merger?

In order to answer these research questions, we develop a game-theoretical model where three firms compete in two dimensions: both price and quality. Two of the competing firms decide whether to merge or not. If they choose not to merge, then all firms first choose their quality simultaneously and then, given the quality decisions of all firms, set their prices simultaneously. If these two competing firms decide to merge, both of them achieve cost synergy and the post-merger firm needs to decide the level of post-merger integration, i.e., decentralized or centralized merger. In the case of decentralized merger, each participant operates independently as in the pre-merger case, i.e., it chooses price and quality to maximize its own profit. By contrast, in the centralized merger case, the post-merger firm make centralized decisions of both participants' prices and quality to maximize the total profit. Besides, in the centralized merger case, the post-merger firm may offer two products as in the pre-merger market or offer a single product. We analyze and compare different merging decisions and strategies and discuss the effects of them on prices, product design, firms' profits, total consumer utility, and social welfare.

We highlight some of our main findings as follows. First, we find that stronger fixed cost synergy may or may not hurt the participant firms in the the case of decentralized merger, while it always benefits the participant(s) in the case of centralized merger. This is because stronger cost synergy in fixed cost intensifies the quality competition between the two participants in the case of decentralized merger.

Second, we find that the post-merger firm prefers decentralized merger when horizontal differentiation level is low and centralized merger otherwise. When horizontal differentiation level is low, market competition is fierce. In this case, a centralized post-merger firm with cost benefits may threaten the nonparticipant firm and make the nonparticipant firm to be aggressive by setting the price and quality to take more market share, which may backfire and hurt the participants. By contrast, the market is more balanced in the decentralized merger case. Therefore, the post-merger firm should choose decentralized merger when horizontal differentiation level is low.

Third, if both centralized merger and decentralized merger are possible, then the competing firms should always choose to merge. However, if the post-merger firm wants to maintain the independence of brands or if the centralized merger incurs huge post-merger integration cost, and only decentralized merger is possible, then a merger between some of the competing firms may backfire and hurt the participants as cost synergy in fixed cost may intensify the competition between the two participants in the decentralized merger case.

Last, contrary to the conventional wisdom, a horizontal merger may not necessarily reduce market competition and result in higher price as well as lower quality. As motioned above, market competition may be fastened in the decentralized merger case. Besides, we find that, if there is enough cost synergy, a merger will reduce price and raise quality. In addition, under some conditions, a merger may contribute to higher total consumer utility and higher social welfare.

The remainder of this study is organized as follows. Section 4.2 reviews relevant literature, and Section 4.3 describes the model used in this study. We analyze the subgame equilibrium outcomes in Sections 4.4 and 4.5 and the final equilibrium outcome in Section 4.6. Section 4.7 examines total consumer utility and social welfare. Finally, we conclude in Section 4.8.

4.2 Literature Review

This study is related to the literature on horizontal mergers (see Whinston (2007), Bloch (2005), and Cho and Wang (2016) for a comprehensive review). Some papers in this stream focus on the incentive of mergers. For example, Stigler (1950) and Salant et al. (1983) find that, due to the existence of merger paradox, mergers may not be profitable for the participants when firms in the market engage in quantity competition and there exist some outside firms. By contrast, Deneckere and Davidson (1985) and Levy and Reitzes (1992) show that the participant firms always benefit from mergers when firms in the market engage in Bertrand competition. Some papers check the effects of merger on price. For example, Williamson (1968) articulates the trade-off between improved productivity and reduced market competition. Farrell and Shapiro (1990) characterize the conditions under which price may fall after merger with the consideration of cost synergy. In operations area, Cho (2013) and Cho and Wang (2016) study the effects of merger on price in the context of multi-tier decentralized supply chains and demand uncertainty, respectively. There also exists some literature studying the effects of merger on both price and non-price parameters, such as quality, product variety (e.g., Mazzeo et al. 2014, Milliou and Sandonis 2018, Gabszewicz et al. 2016), location (e.g., Gandhi et al. 2008, Sweeting 2010), and innovation (e.g., Davidson and Ferrett 2007). This study is closely related to the literature studying the effects of merger on both price and quality. The literature in this area are usually simulation study (e.g., Pinto and Sibley 2016) or empirical researches (e.g., Fan 2013, Qiu 2015). Differs from the above literature, this study develops a theoretical model to examine the effects of merger on both price and quality.

This study is also related to the literature using cooperative game theory to study the formation of coalitions or cartels among firms (see Nagarajan and Sošić (2008) for a comprehensive review). Firms behave as entities but stay independent in a coalition. The key issue in this literature stream is that a coalition maximizes the total profit of all firms by reducing competition, but a firm can be better off if it alone deviates from the coalition. Therefore, this literature stream focuses on the stability of coalitions (Nagarajan and Sošić 2007, 2008, 2009, Granot and Yin 2008, Yin 2010, Sošić 2011, Kemahlıoğlu-Ziyaa and Bartholdi 2011, Fang and Cho 2014, Huang et al. 2015). By contrast, we focus on whether two competing firms should merge and which merging strategy is optimal in a competitive market.

There are many studies in operations area focusing on the vertical integration in a supply chain and comparing decentralized system with centralized system (e.g., McGuire and Staelin (1983), Moorthy (1988b), Boyaci and Gallego (2004), Liu and Tyagi (2011), Su and Zhang (2008). For example, McGuire and Staelin

(1983) study the effects of product substitutability on supply chain structure and find that two decentralized supply chains may be better than two integrated ones if product substitutability is high. Su and Zhang (2008) show that, with the existence of strategic consumer behavior, a decentralized supply chain may be more efficient than a centralized one. Different from this literature stream, we compare centralization with decentralization in the context of horizontal merger.

Finally, our research is related to literature concerning price or/and quality competition among differentiated products. For example, Mussa and Rosen (1978) and Moorthy (1984) study a firm's price-quality strategy when consumers are heterogeneous in their preference for quality. By contrast, our research is more related to the literature studying differentiated products in competitive context, such as Shaked and Sutton (1982), Moorthy (1988a) and Ronnen (1991). Gabszewicz et al. (2019) studies the endogenous mergers between vertically differentiated products, while our study focuses on the horizontal differentiated products.

4.3 Model Setup

We consider three firms, firms A, B and C, locating equidistantly in a Salop circular city. Each firm i sells a product i at price p_i, where $i \in \{A, B, C\}$. Each product i consists of two parts: a baseline attribute and some quality-differentiated services.[4.1] Consumers' valuation of the baseline attribute is given by v, which is same for each firm and exogenously given. The service quality offered by firm i is denoted by q_i. The marginal cost of offering the baseline attribute is c. Besides, to provide quality q_i, each firm i needs to incur a fixed cost $\frac{1}{2}kq_i^2$, where $i \in \{A, B, C\}$ and k is the fixed cost factor of service quality for each firm.

Without loss of generality, we assume that firms A and B decide whether to merge or not.[4.2] If they choose not to merge, all firms operate independently, i.e.,

[4.1] The quality-differentiated services can also be interpreted as a new attribute bundled with the product.

[4.2] We disregard the case where all firms merge for two reasons: First, a full merger degenerates to a monopolistic firm and tends to be stopped by the antitrust authorities in practice; second, such case is uninteresting for research since it neglects the outside firm's strategic response.

each firm chooses price and quality to maximize its own profit. We refer to this case as *Pre-merger*, notated by "PM".

If firms A and B decide to merge, we use "AB" to denote the new post-merger firm and we refer to firms A and B as the participant firms. Besides, we refer to firm C as the outside or nonparticipant firm. The post-merger firm AB benefits from the merger by obtaining cost synergies in both marginal and fixed costs. Hence, after merger, the participant firms' marginal and fixed costs are reduced from c to c_m and k to k_m. $0 < c_m \leq c$ and $0 < k_m \leq k$.[4.3]

Depending on the level of post-merger integration, the post-merger firm AB can make decentralized or centralized decisions on the participants' prices and quality.[4.4] If the post-merger integration is simple, the participants operate separately as they do in the pre-merger case. Specifically, participant firms A and B choose their own price and quality independently to maximize each participant's own profit. We refer to this case as *Decentralized Merger*, notated by "DM".

By contrast, if the post-merger integration is deep, the post-merger firm AB makes centralized decisions about participant A's and B's prices and quality to maximize the total profit of these two firms. We refer to this case as *Centralized Merger*, notated by "CM". In this case, the post-merger firm AB is able to change its product line. It may offer a single product or keep both products. We use "CM1" to denote the scenario that the post-merger firm AB offers a single product, and use "CM2" to denote the scenario that the post-merger firm AB provides two products.

Consumers uniformly locate along the circular city. Without loss of generality, we normalize the market size to 1. Each consumer buys at most one unit of product. The utility of a consumer j buying product i is modeled as follows:

$$u_{ji} = v + q_i - p_i - td_{ji}, \tag{4.1}$$

[4.3]For expositional convenience, we do not consider the fixed cost of merging in our model. Note that our main results still hold qualitatively even we include the merging cost and given that it is not too large.

[4.4]Note that in our model, we assume that the cost structures are the same for the decentralized merger and centralized merger. However, our main results hold qualitatively even we relax this assumption by considering that the participants gain different levels of marginal and fixed cost synergy in the decentralized merger and centralized merger.

where v is consumers' baseline valuation of the baseline attribute, t is the horizontal differentiation level and d_{ji} is the shortest arc distance between consumer j and firm i. We normalize each consumer's outside utility to zero. Consumers will purchase the product which brings the greatest nonnegative utility.

The sequence of events is as follows: In stage 0, firms A and B decide whether to merge. If firms A and B choose not to merge (pre-merger case), then they play a two-stage game with firm C. To be specific, all the firms in the market, i.e., firms A, B and C, choose their own quality simultaneously in stage 1, and then, given the quality decisions of all firms, set their own prices simultaneously in stage 2. In stage 3, given all firms' quality and prices, consumers make their purchasing decisions to maximize their utility defined in Equation (4.1). By contrast, in stage 0, if firms A and B choose to merge (post-merger case), then the new firm AB needs to further choose between decentralized merger and centralized merger. If it chooses decentralized merger, then firms A and B play a two-stage game with the outside competitor firm C as they do in the pre-merger case except that firms A and B gain cost synergy from the merger. If the post-merger firm AB chooses centralized merger, then it needs to decide the product line. Given the product line decision, the post-merger firm AB plays a two-stage game with the outside firm C. Figure 4.1 illustrates the sequence of events.

Before analyzing the game, we make three additional assumptions as follows. First, we assume that $t > \underline{t} = \max\{\frac{12+30(c-c_m)k_m}{25k_m}, \frac{5k-k_m+9(c-c_m)kk_m}{9kk_m}\}$ in order to ensure that all firms survive in the equilibrium; second, we assume that $v > \underline{v}$ in order to ensure that the market is fully covered;[4.5] third, we adopt the following tie-breaking rule: in the centralized merger case, when the post-merger firm AB is indifferent between offering a single product and two products, it will offer a single product.[4.6]

For convenience, we summarize the notations used in this study in Table 4.1.

We solve the game by backward induction and discuss the subgame equilib-

[4.5] The lower bound of consumers' baseline valuation of the baseline attribute (i.e., $\underline{v}$) is presented in the Appendix.

[4.6] One can interpret this rule as there is a positive but small enough cost of offering a product.

Table 4.1: Summary of Notations

Notation	Description
p_i	The price of firm $i \in \{A, B, C\}$
v	Consumers' valuation of the common baseline attribute
q_i	The service quality offered by firm $i \in \{A, B, C\}$
c	The marginal cost
k	The fixed cost factor of service quality
c_m	Each participant's post-merger marginal cost, $0 < c_m \leq c$
k_m	Each participant's post-merger fixed cost, $0 < k_m \leq k$
u_{ji}	Consumer j's utility of buying from firm $i \in \{A, B, C\}$
t	The horizontal differentiation level
d_{ji}	The shortest arc distance between consumer j and firm $i \in \{A, B, C\}$
D_i	The demand of firm $i \in \{A, B, C\}$
D'_i	The demand of firm $i \in \{A, C\}$ in the case of centralized merger with a single product
π_i	The profit of firm $i \in \{A, B, C, AB\}$
π'_i	The profit of firm $i \in \{AB, C\}$ in the case of centralized merger with a single product
U	Total consumer utility
SW	Social welfare

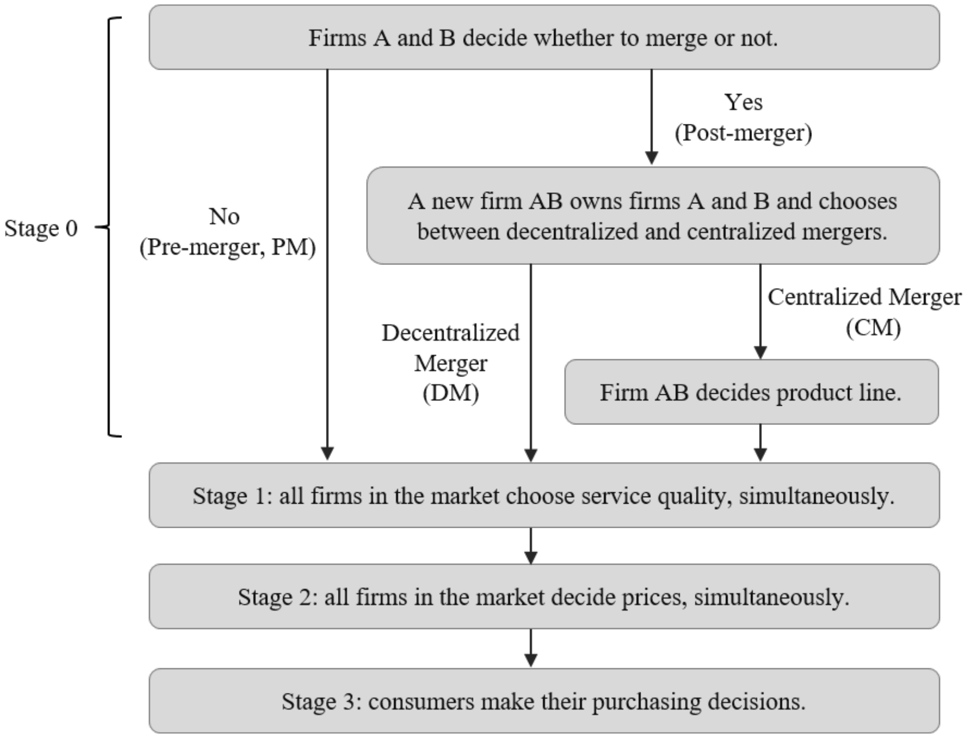


Figure 4.1: Sequence of Events

rium outcome for each case one by one: in Section 4.4, we study the case that firms A and B decide not to merge. We discuss the case that firms A and B choose to merge in Section 4.5. Then Section 4.6 presents the final equilibrium outcome.

4.4 Pre-merger (PM)

In this section, we analyze the case that firms A and B decide not to merge (pre-merger case). In the case of pre-merger, as mentioned in Section 4.3, firms A, B and C play a two-stage game. Each firm chooses first quality and then price independently to maximize its own profit. Given all firms' prices and quality decisions, consumers make their purchasing decisions to maximize their utility defined in Equation (4.1). The demand of product i is shown as follows:

$$D_i = \min\{1, \max\{0, \frac{(q_i - p_i) - (q_{i'} - p_{i'})}{2t} + \frac{1}{6}\} + \max\{0, \frac{(q_i - p_i) - (q_{i''} - p_{i''})}{2t} + \frac{1}{6}\}\}, \tag{4.2}$$

where $i, i', i'' \in \{A, B, C\}$, $i \neq i'$, $i \neq i''$, and $i' \neq i''$. Each firm i's profit is formulated as follows:

$$\pi_i = (p_i - c)D_i - \frac{1}{2}kq_i^2, \text{ for } i \in \{A, B, C\}. \tag{4.3}$$

The first term in Equation (4.3) captures the revenue of selling product i at price p_i and the second term is the fixed cost of providing product i with quality q_i. Let p_i^{PM}, q_i^{PM}, D_i^{PM}, and π_i^{PM} denote the equilibrium price, quality, demand, and profit of firm $i \in \{A, B, C\}$ in the case of pre-merger. We summarize the subgame equilibrium for the case of pre-merger in Lemma 4.1 and Table 4.2.

Lemma 4.1. *Given that firms A and B decide not to merge, then the subgame equilibrium is given as follows:* $p_i^{PM} = c + \frac{t}{3}$, $q_i^{PM} = \frac{4}{15k}$, $D_i^{PM} = \frac{1}{3}$, *and* $\pi_i^{PM} = \frac{1}{225}(25t - \frac{8}{k})$, *where* $i \in \{A, B, C\}$.

All proofs can be found in the Appendix. The equilibrium outcome in Lemma 4.1 is a standard result. In this case, all firms in the market choose the same strategies and gain the same profit since the game in this case is symmetric. Next, we study the effects of some key parameters on the subgame equilibrium for the case of pre-merger and summarize the effect of the horizontal differentiation level t on each firm's profit as follows:[4.7]

Corollary 4.1. *Given that firms A and B decide not to merge, each firm's profit* π_i^{PM} *increases in the horizontal differentiation level (i.e.,* t*), where* $i \in \{A, B, C\}$.

Corollary 4.1 shows that, in the case of pre-merger, each firm's profit π_i^{PM} increases in the the horizontal differentiation level t. This is because, as t increases, market competition among the firms is reduced.

4.5 Post-merger

In this section, we analyze the case that firms A and B decide to merge. Suppose firms A and B have merged and a new firm AB owns them. The post-merger

[4.7] The comprehensive results regarding the effects of key parameters on the subgame equilibrium for the pre-merger case and other cases are presented in the Appendix.

firm AB needs to choose between decentralized merger where firms A and B should make decentralized decisions on their own quality and price to maximize each participant's own profit, and centralized merger where firm AB should make centralized decisions about both firms' quality and prices to maximize the total profit. We first analyze the case of decentralized merger in Section 4.5.1 and the case of centralized merger in Section 4.5.2. Then Section 4.5.3 compares the decentralized merger case with the centralized merger case to identify the post-merger firm's optimal merging strategy.

4.5.1 Decentralized Merger (DM)

In the decentralized merger case, the post-merger firm AB chooses simple integration and firms A and B are autonomous enough to make decentralized decisions to maximize their own profits. In this case, firms A and B play a two-stage game with the outside firm C as they do in the case of pre-merger except that the participant firms A and B gain cost synergy in both marginal and fixed costs. Hench, firm C's problem and profit are the same as the case of pre-merger, while the profit of each participant firm $i \in \{A, B\}$ is given as follows:

$$\pi_i = (p_i - c_m)D_i - \frac{1}{2}k_m q_i^2, \text{ for } i \in \{A, B\}, \tag{4.4}$$

where demand D_i is defined in Equation (4.2), c_m is the post-merger marginal cost, and k_m is the post-merger fixed cost factor of quality. Let p_i^{DM}, q_i^{DM}, D_i^{DM}, and π_i^{DM} denote the equilibrium price, quality, demand, and profit of firm $i \in \{A, B, C\}$ in the case of decentralized merger. We summarize the subgame equilibrium for the case of decentralized merger in Lemma 4.2 and Table 4.2. Note that the expressions of X_{AB}^{DM} and X_C^{DM} are given in Table 4.2.

Lemma 4.2. *Given that firms A and B adopt decentralized merger, the subgame equilibrium is given as follows:*

(i) For the participant firms A and B, $p_A^{DM} = p_B^{DM} = c_m + \frac{t}{3}X_{AB}^{DM}$, $q_A^{DM} = q_B^{DM} = \frac{4}{15k_m}X_{AB}^{DM}$, $D_A^{DM} = D_B^{DM} = \frac{1}{3}X_{AB}^{DM}$, $\pi_A^{DM} = \pi_B^{DM} = \frac{1}{225}(25t - \frac{8}{k_m})X_{AB}^{DM^2}$, *and* $\pi_{AB}^{DM} = \pi_A^{DM} + \pi_B^{DM}$*;*

(ii) *For the nonparticipant firm C,* $p_C^{DM} = c + \frac{t}{3} X_C^{DM}$*,* $q_C^{DM} = \frac{4}{15k} X_C^{DM}$*,* $D_C^{DM} = \frac{1}{3} X_C^{DM}$*, and* $\pi_C^{DM} = \frac{1}{225}(25t - \frac{8}{k}) {X_C^{DM}}^2$*.*

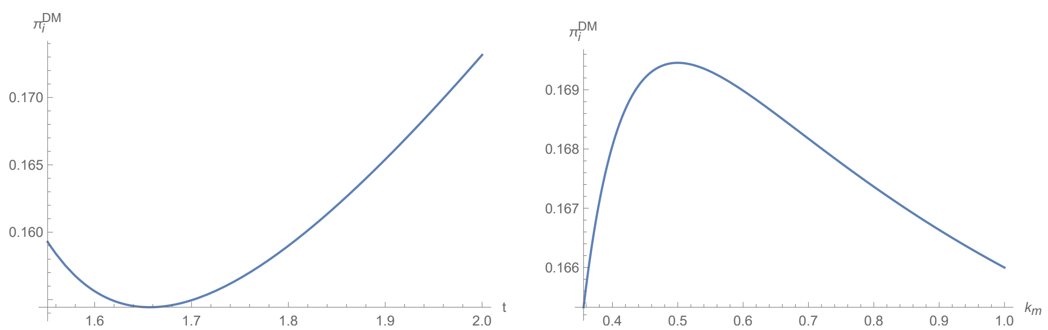
In Lemma 4.2, one can easily see that firms A and B play symmetric strategies and earn the same profit. We further compare the strategies and profits between a participant firm (A or B) and the outside firm C. We find that a participant firm $i \in \{A, B\}$ will provide product with higher quality and earn more profit, i.e., $q_i^{DM} > q_C^{DM}$ and $\pi_i^{DM} > \pi_C^{DM}$. This is because the participants gain cost synergy from the merger and have lower costs. Next, we study the effects of some key parameters on the subgame equilibrium for the case of decentralized merger and show the interesting results in Proposition 4.1 and Figure 4.2.

Proposition 4.1. *Given that firms A and B adopt decentralized merger:*

(i) *Each participant firm's profit (i.e.,* π_A^{DM} *or* π_B^{DM}*) is non-monotone in the horizontal differentiation level (i.e.,* t*);*

(ii) *Each participant firm's profit (i.e.,* π_A^{DM} *or* π_B^{DM}*) decreases in the post-merger marginal cost (i.e.,* c_m*);*

(iii) *Each participant firm's profit (i.e.,* π_A^{DM} *or* π_B^{DM}*) is non-monotone in the post-merger fixed cost (i.e.,* k_m*).*



(a) Effect of the Horizontal Differentiation Level t

(b) Effect of the Post-merger Fixed Cost k_m

Figure 4.2: Effects of t and k_m on Each Participant's Profit in the DM Case

Note: in figure 4.2 (a), $i \in \{A, B\}$, $c = 0.4$, $c_m = 0.25$, $k = 0.4$, and $k_m = 0.35$; and in figure 4.2 (b), $i \in \{A, B\}$, $t = 1.6$, $c = 0.4$, $c_m = 0.25$, and $k = 1$.

Intuitively, we may think that firms' profits should increase in the horizontal differentiation level t since competition is reduced as t increases. This is true for the case of pre-merger. However, when firms A and B adopt decentralized merger, Proposition 4.1 (i) shows that each participant's profit (π_A^{DM} or π_B^{DM}) may increase or decrease in the horizontal differentiation level t. To be more specific, there exits a threshold $\check{t}$ such that, if $t < \check{t}$, then a participant's profit (π_A^{DM} or π_B^{DM}) decreases in t; otherwise, it increases in t. Moreover, we also find that the nonparticipant firm C's profit (π_C^{DM}) increases in t. This is because, in the case of decentrlalized merger, firms A and B have lower marginal and fixed costs compared to firm C due to the cost synergy gained from the merger. Thus, firms A and B are more competitive than firm C in the market. As t increases, there exist a positive effect and a negative effect on each firm's profit. The positive effect is that competition is reduced as products become more differentiated. The negative effect is that consumers' utilities become lower as t increases. In a symmetric market (i.e., in the case of pre-merger), the advantage of reduced competition outweighs the disadvantage of reduced consumers' utilities. However, in an asymmetric market (i.e., in the case of decentralized merger), the advantage of reduced competition for more competitive firms A and B may not be that significant compared to the less competitive firm C, and it could be dominated by the disadvantage of reduced consumers' utilities when t is small. Therefore, firms A and B could be hurt by the increasing differentiation level t in the case of decentralized merger.

One may intuitively think that the participants always benefit from cost synergy since it reduces their costs. This only holds for the cost synergy in marginal cost but not for the cost synergy in fixed cost in the case of decentralized merger. Proposition 4.1 (iii) shows that, if firms A and B adopt decentralized merger, then each participant's profit (π_A^{DM} or π_B^{DM})is non-monotone in the post-merger fixed cost k_m. To be more specific, we find that there exists a threshold $\check{k_m}$ such that, if $k_m < \check{k_m}$, then a participant's profit (π_A^{DM} or π_B^{DM}) increases in k_m; otherwise, it decreases in k_m. The underlying reason is that, in the case of decentralized merger, a participant firm competes with not only the outside firm C but also the other participant firm. As the fixed cost synergy becomes stronger (i.e., k_m decreases), both participants have lower fixed costs and will increase their quality (i..e, q_i^{DM}, $i \in \{A, B\}$, decreases in k_m), which indicates that stronger fixed cost synergy will intensify the quality competition between the two participants. Indeed, when k_m decrease, the quality competition can be so fierce that both the participants may be worse than the case without fixed cost synergy. Therefore, although stronger fixed cost synergy helps reduce cost, it may still make both participants worse off in the case of decentralized merger when k_m is small.

4.5.2 Centralized Merger (CM)

In this section we discuss the case that the post-merger firm AB adopts deep integration and makes centralized decisions about both participant firms' competitive strategies to maximize the total profit. In this case, the post-merger firm AB first needs to decide whether or not to cut its product line. It may offer both products A and B, or a single product. Given firm AB's product line decision, it plays a two-stage game with firm C: both firms first choose their quality simultaneously and then set their prices simultaneously. In the case of centralized merger, we first study the scenario where firm AB offers a single product in Section 4.5.2, and study the scenario where firm AB provides both products A and B in Section 4.5.2. Finally, Section 4.5.2 compares these two scenarios to identify the post-merger firm AB's optimal product line in the case of centralized merger.

4.5.2.1 Centralized Merger with a Single Product (CM1)

In this part, we study the scenario that the post-merger firm AB offers a single product. Without loss of generality, we assume it offers product A and cuts product B. We use superscript "CM1" to denote this scenario. Given the fact that only products A and C are available in the market and the market is fully covered, the demands of products A and C are given as follows:

$$D'_i = \min\{1, \max\{0, \frac{(q_i - p_i) - (q_{i'} - p_{i'})}{2t} + \frac{1}{6}\} + \max\{0, \frac{(q_i - p_i) - (q_{i'} - p_{i'})}{2t} + \frac{1}{3}\}\}, \tag{4.5}$$

where $i, i' \in \{A, C\}$ and $i \neq i'$. Each firm's profit is given in following equations:

$$\pi'_{AB} = (p_A - c_m)D'_A - \frac{1}{2}k_m q_A^2, \text{ and } \pi'_C = (p_C - c)D'_C - \frac{1}{2}kq_C^2. \tag{4.6}$$

Let p_i^{CM1}, q_i^{CM1}, and D_i^{CM1} denote firm i's equilibrium price, quality, and demand, $i \in \{A, C\}$, and let π_{AB}^{CM1} and π_C^{CM1} represent the equilibrium profits of firms AB and C in the case of centralized merger with a single product. We summarize the subgame equilibrium for the CM1 case in Lemma 4.3 and Table 4.2. Note that the expressions for X_{AB}^{CM1} and X_C^{CM1} can be found in Table 4.2.

Lemma 4.3. *Given that firms A and B adopt centralized merger and the post-merger firm AB offers a single product (i.e., product A), the subgame equilibrium is given as follows:*

(i) For the participant firm A, $p_A^{CM1} = c_m + \frac{t}{3}X_{AB}^{CM1}$, $q_A^{CM1} = \frac{2}{9k_m}X_{AB}^{CM1}$, $D_A^{CM1} = \frac{1}{3}X_{AB}^{CM1}$, *and* $\pi_{AB}^{CM1} = \frac{1}{225}(25t - \frac{50}{9k_m}){X_{AB}^{CM1}}^2$*;*

(ii) For the nonparticipant firm C, $p_C^{CM1} = c + \frac{t}{3}X_C^{CM1}$, $q_C^{CM1} = \frac{2}{9k}X_C^{CM1}$, $D_C^{CM1} = \frac{1}{3}X_C^{CM1}$, *and* $\pi_C^{CM1} = \frac{1}{225}(25t - \frac{50}{9k}){X_C^{CM1}}^2$.

In the CM1 case, we find that the participant firm A always provides higher quality and earn more profit than the nonparticipant firm C. This is because the participant firm has cost benefits which reduce its costs. Next, we study the effects of some key parameters on the subgame equilibrium for the CM1 case. We present some of the interesting results in Proposition 4.2 and Figure 4.3.

Proposition 4.2. *Given that firms A and B adopt centralized merger and the post-merger firm AB offers a single product (i.e., product A):*

(i) The post-merger firm's profit (i.e., π_{AB}^{CM1}*) is non-monotone in the horizontal differentiation level (i.e.,* t*);*

(ii) *The post-merger firm's profit (i.e.,* π_{AB}^{CM1}*) decreases in the post-merger marginal cost (i.e.,* c_m*);*

(iii) *The post-merger firm's profit (i.e.,* π_{AB}^{CM1}*) decreases in the post-merger fixed cost (i.e.,* k_m*).*

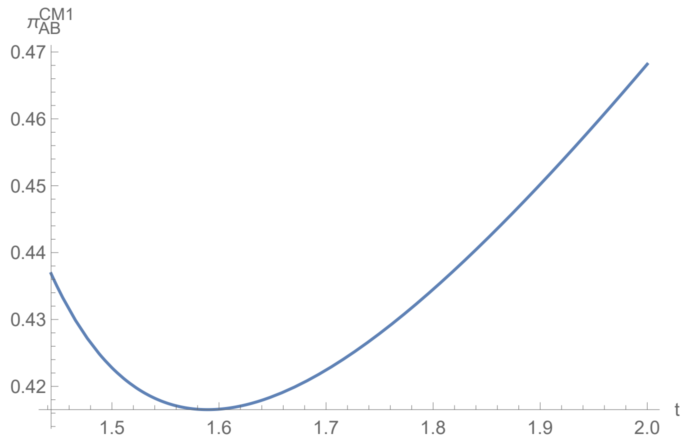


Figure 4.3: Effect of t on the Post-merge Firm's Profit in the CM1 Case

Note: in Figure 4.3, $c = 0.4$, $c_m = 0.25$, $k = 0.4$, and $k_m = 0.38$.

Consistent with the case of decentralized merger, Proposition 4.2 (i) shows that, in the case of centralized merger with a single product, the post-merger firm AB's profit may be increase or decrease in the horizontal differentiation level t. Specifically, there exists a threshold $\hat{t}$ such that, if $t < \hat{t}$, then the post-merger firm AB's profit π_{AB}^{CM1} decreases in t; otherwise, π_{AB}^{CM1} increases in t. The underlying reason follows the same logic as the case of decentralized merger. In the CM1 case, the remaining participant firm A has lower costs than the nonparticipant firm C due to the cost synergy gained from the merger. Therefore, firm A is more competitive than firm C in this case. However, as mentioned in the discussion of Proposition 4.1, in an asymmetric market, the positive effect of increased horizontal differentiation for the more competitive firm A may be less significant and could be outweighed by the negative effect of reduced consumers' utilities when t is small. Thus, in the CM1 case, the post-merger firm AB may be worse off when the horizontal differentiation level t increases.

Different from the case of decentralized merger, Proposition 4.2 (ii) and (iii) show that, if firms A and B adopt centralized merger and the post-merger firm AB decides to offer a single product, then the remaining participant A always benefits from cost synergy. This is because the CM1 case eliminates the quality competition between the participants by cutting one of the products. Therefore, in this case, the post-merger firm's profit decreases in both the post-merger marginal and fixed costs.

4.5.2.2 Centralized Merger with Two Products (CM2)

In this part, we study the scenario that the post-merger firm AB decides to provide both products A and B. We use "CM2" to denote this scenario. In this scenario, firm AB aims to maximize the total profit of these two products by making centralized decisions on their competitive strategies. Thus, firm C's problem is the same as the case of pre-merger, whereas the profit function of firm AB is expressed as follows:

$$\pi_{AB} = (p_A - c_m)D_A - \frac{1}{2}k_m q_A^2 + (p_B - c_m)D_B - \frac{1}{2}k_m q_B^2, \tag{4.7}$$

where D_A and D_B are defined in Equation (4.2). As shown in Equation (4.7), the first two terms capture the profit of the participant firm A while the last two terms is the profit of the other participant firm B.

Let p_i^{CM2}, q_i^{CM2}, and D_i^{CM2} denote the equilibrium price, quality, and demand, $i \in \{A, B, C\}$, and let π_{AB}^{CM2} and π_C^{CM2} denote the equilibrium profits of firms AB and C in the case of centralized merger with two products. We summarizes the subgame equilibrium for the CM2 case in Lemma 4.4 and Table 4.2. Note that the expressions of X_{AB}^{CM2} and X_C^{CM2} are given in Table 4.2.

Lemma 4.4. *Given firms A and B adopt centralized merger and the post-merger firm AB offers two products, the subgame equilibrium is given as follows: if* $t \geq \frac{3}{4k_m}$, *then*

(i) For the post-merger firm AB, $p_A^{CM2} = p_B^{CM2} = c_m + \frac{t}{3}X_{AB}^{CM2}$, $q_A^{CM2} = q_B^{CM2} = \frac{1}{9k_m}X_{AB}^{CM2}$, $D_A^{CM2} = D_B^{CM2} = \frac{1}{6}X_{AB}^{CM2}$, *and* $\pi_{AB}^{CM2} = \frac{1}{225}(25t - \frac{25}{9k_m}){X_{AB}^{CM2}}^2$;

(ii) For the nonparticipant firm C, $p_C^{CM2} = c + \frac{t}{3}X_C^{CM2}$, $q_C^{CM2} = \frac{2}{9k}X_C^{CM2}$, $D_C^{CM2} = \frac{1}{3}X_C^{CM2}$, *and* $\pi_C^{CM2} = \frac{1}{225}(25t - \frac{50}{9k}){X_C^{CM2}}^2$.

In Lemma 4.4, condition $t \geq \frac{3}{4k_m}$ is required to ensure the existence of the pure-strategy equilibrium for the case of centralized merger with two products. Note that, in the CM2 case, the pure-strategy equilibrium does not exist when $t < \frac{3}{4k_m}$. Thus, we assume that when $t < \frac{3}{4k_m}$, given that firms A and B adopt centralized merger, the post-merger firm AB will always offer a single product in the equilibrium. In the following discussion of Section 4.5.2, we only focus on the case that $t \geq \frac{3}{4k_m}$.

Next, we study the effects of some key parameters on the subgame equilibrium for the CM2 case and summarize some interesting results in Proposition 4.3.

Proposition 4.3. *Given that firms A and B adopt centralized merger and the post-merger firm AB offers two products:*

(i) The post-merger firm's profit (i.e., π_{AB}^{CM2}*) increases in the horizontal differentiation level (i.e.,* t*);*

(ii) The post-merger firm's profit (i.e., π_{AB}^{CM2}*) decreases in the post-merger marginal cost (i.e.,* c_m*);*

(iii) The post-merger firm's profit (i.e., π_{AB}^{CM2}*) decreases in the post-merger fixed cost (i.e.,* k_m*).*

Following the same logic as the cases of DM and CM1, we may intuitively think that the post-merger firm's profit should be non-monotone in the horizontal differentiation level t in the case of centralized merger with two products since the market is also asymmetric in this case. However, Proposition 4.3 (i) shows that, if firms A and B adopt centralized merger and the post-merger firm AB decides to offer two products, the post-merger firm's profit always increases in t. The reason underlying this result goes as follows. When t increases, there is a trade-off between reduced competition and reduced consumers' utilities. In the CM2 case, the positive effect of reduced competition can be amplified by the collusion effect, which helps reduce the demand cannibalization between the participants. As a result, the advantage of reduced competition can be so significant that it dominates the disadvantage of reduced consumers' utilities. Therefore, the participants benefit from higher horizontal differentiation level in the CM2 case.

Intuitively, we may think that a post-merger firm with more than one products may be hurt by cost synergy as there exists market cannibalization between them. This is true for the case of decentralized merger. However, Proposition 4.3 (ii) and (iii) show that, in the case of centralized merger with two products, the post-merger firm always benefits from cost synergy as its profit decreases in both the post-merger marginal cost c_m and the post-merger fixed cost k_m. This is because the case of CM2 is accompanied by the collusion effect, which helps soften the competition between the participants. Therefore, the post-merger firm's profit always increases in the cost synergy in this case.

However, although the participants have cost benefits, we find that, in the case of centralized merger with two products, a participant (A or B) could be less profitable than the outside firm C, as shown in the following proposition:

Proposition 4.4. *Given that firms A and B adopt centralized merger and the post-merger firm AB offers two products, merger paradox may happen: there exists a threshold $\tilde{t}$ such that, if $t < \tilde{t}$, then $\pi_A^{CM2} = \pi_B^{CM2} = \frac{1}{2}\pi_{AB}^{CM2} > \pi_C^{CM2}$; otherwise, $\pi_A^{CM2} = \pi_B^{CM2} = \frac{1}{2}\pi_{AB}^{CM2} \leq \pi_C^{CM2}$.*

Intuitively, one may think that each participant should be more profitable than the nonparticipant since it has lower costs. This intuition is true for the cases of decentralized merger and centralized merger with a single product. However, Proposition 4.4 shows that, in the case of centralized merger with two products, a participant firm may earn less profit than the nonparticipant firm when t is large. The underlying reason for this phenomenon is as follows. As t increases, it reduces consumers' utilities. When t is large, as a centralized firm AB, it prefers to soften the demand cannibalization between product A and product B by offering less competitive products rather than provide more attractive products to extend its market share. The outside firm C takes advantage of firm AB's non-aggressive strategy by offering more competitive product, i.e., $q_C - p_C > q_i - p_i$ ($i \in \{A, B\}$), and taking more market share. Thus we find that the nonparticipant firm C may be more profitable than a participant firm (A or B) in the case of centralized merger with two products.

4.5.2.3 One or Two products

In this part, we examine the post-merger firm's optimal product line decision in the centralized merger case by comparing its profits in the cases of a single product and two products. We summarize the result in Proposition 4.5 and Figure 4.4.

Proposition 4.5. *Given that firms A and B adopt centralized merger, there exists a threshold* $\dot{t}$ *such that the post-merger firm offers a single product if* $t \leq \dot{t}$*; otherwise, it provides two products.*

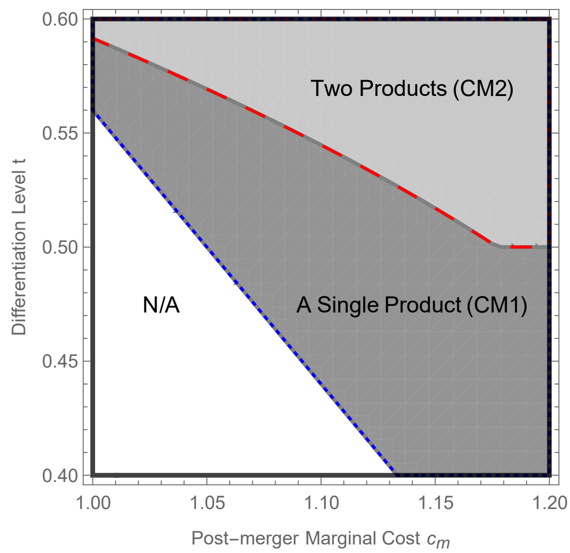


Figure 4.4: Optimal Different Product Line in the CM Case

Note: in Figure 4.4, $c = 1.2$, $k = 2.25$, and $k_m = 1.5$.

Proposition 4.5 shows that, if $t < \dot{t}$, then the post-merger firm AB should cut its product line and offer a single product, rather than keep both products. This is because, when t is small, the horizontal differentiation level is low. Thus offering two products leads to strong market cannibalization and intensifies the competition with firm C, and is less profitable than offering a single product.

Table 4.2: Summary of the Subgame Equilibria under Different Strategies

s	PM	DM	$CM1$	$CM2$
p_A^s or p_B^s	$c + \frac{t}{3}$	$c_m + \frac{t}{3} X_{AB}^{DM}$	$c_m + \frac{t}{3} X_{AB}^{CM1}$	$c_m + \frac{t}{3} X_{AB}^{CM2}$
q_A^s or q_B^s	$\frac{4}{15k}$	$\frac{4}{15k_m} X_{AB}^{DM}$	$\frac{2}{9k_m} X_{AB}^{CM1}$	$\frac{1}{9k_m} X_{AB}^{CM2}$
D_A^s or D_B^s	$\frac{1}{3}$	$\frac{1}{3} X_{AB}^{DM}$	$\frac{1}{3} X_{AB}^{CM1}$	$\frac{1}{6} X_{AB}^{CM2}$
$\pi_A^s + \pi_B^s$ or π_{AB}^s	$\frac{2}{225}(25t - \frac{8}{k})$	$\frac{2}{225}(25t - \frac{8}{k_m}) X_{AB}^{DM^2}$	$\frac{1}{225}(25t - \frac{50}{9k_m}) X_{AB}^{CM1^2}$	$\frac{1}{225}(25t - \frac{25}{9k_m}) X_{AB}^{CM2^2}$
p_C^s	$c + \frac{t}{3}$	$c + \frac{t}{3} X_C^{DM}$	$c + \frac{t}{3} X_C^{CM1}$	$c + \frac{t}{3} X_C^{CM2}$
q_C^s	$\frac{4}{15k}$	$\frac{4}{15k} X_C^{DM}$	$\frac{2}{9k} X_C^{CM1}$	$\frac{2}{9k} X_C^{CM2}$
D_C^s	$\frac{1}{3}$	$\frac{1}{3} X_C^{DM}$	$\frac{1}{3} X_C^{CM1}$	$\frac{1}{3} X_C^{CM2}$
π_C^s	$\frac{1}{225}(25t - \frac{8}{k})$	$\frac{1}{225}(25t - \frac{8}{k}) X_C^{DM^2}$	$\frac{1}{225}(25t - \frac{50}{9k}) X_C^{CM1^2}$	$\frac{1}{225}(25t - \frac{50}{9k}) X_C^{CM2^2}$

Note: (1) $X_{AB}^{DM} = \frac{k_m(-12+15ck-15c_mk+25kt)}{-8k_m+k(-4+25k_mt)}$, $X_C^{DM} = \frac{k(-12-30ck_m+30c_mk_m+25k_mt)}{-8k_m+k(-4+25k_mt)}$, $X_{AB}^{CM1} = \frac{3k_m(-4+6ck-6c_mk+9kt)}{-4k_m+2k(-2+9k_mt)}$, $X_C^{CM1} = \frac{3k(4+6ck_m-6c_mk_m-9k_mt)}{4k_m+k(4-18k_mt)}$, $X_{AB}^{CM2} = \frac{3k_m(-2+3ck-3c_mk+5kt)}{-2k_m+k(-1+9k_mt)}$, and $X_C^{CM2} = \frac{3k(1+3ck_m-3c_mk_m-4k_mt)}{k+2k_m-9kk_mt}$;

(2) $1 < X_{AB}^{DM} < 2$, $0 < X_C^{DM} < 1$, $X_{AB}^{CM1} > 1$, $X_C^{CM1} > 0$, $X_{AB}^{CM2} > 1$, and $X_C^{CM2} > 0$;

(3) Firm B does not exist in the case of centralized merger with a single product (CM1).

Henceforth, we use π_{AB}^{CM} to denote the post-merger firm AB's profit in the case of centralized merger.

4.5.3 Decentralized Merger or Centralized Merger

In this section, we compare the centralized merger case with the decentralized merger case to find out the firm's optimal merging strategy. We present the result in Proposition 4.6 and Figure 4.5.

Proposition 4.6. *Given that firms A and B decide to merge, there exists a threshold t' such that the post-merger firm AB prefers decentralized merger if $t < t'$ and prefers centralized merger otherwise.*

Intuitively, one may think centralized merger to be always better than decentralized merger since it is accompanied by collusion effect which softens the competition between the participants. This will hold if there is no other competing firms in the market (i.e., firm C in our case) or when the firms' price or quality decisions are exogenously given. However, we find that when there exist other competing firms and all firms compete in both price and quality, decentralized merger could be a better strategy than centralized merger. The underlying reason goes as follows. When t is small, the competition level is high. In this case, if the post-merger firm AB is a centralized decision maker and has cost synergy, it will make the non-participant firm C to be aggressive by setting the price and quality in order to take more market share. While in the decentralized merger case, the market is more balanced since firms A and B operate independently. Thus, firm AB could be even better under the case of decentralized merger.

Henceforth, let π_{AB}^{M} to be the post merger firm AB's profit in the case that firms A and B decide to merge.

4.6 Final Equilibrium: Merge or Not Merge

In this section, we compare the pre-merger case with the post-merger case to determine whether the firms should merge or not. We summarize the result in Proposition 4.7.

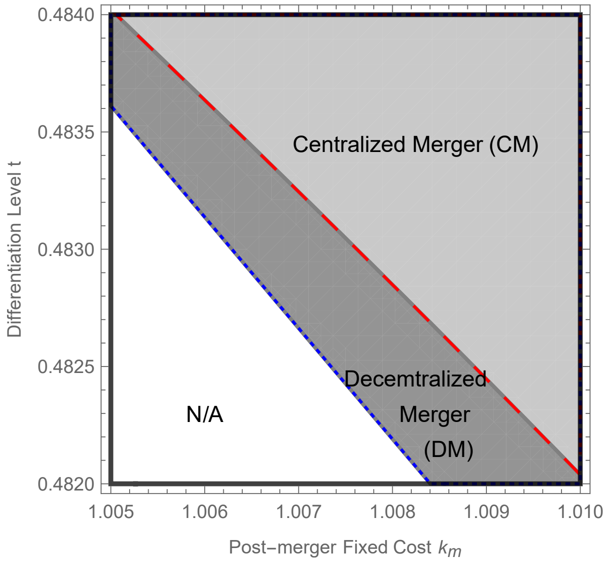


Figure 4.5: Comparison of the DM Case and CM Case

Note: in Figure 4.5, $c = 3.29$, $c_m = 3.285$, and $k = 1.01$.

Proposition 4.7. *If both decentralized merger and centralized merger are feasible, then the final equilibrium is that firms A and B should choose to merge, i.e.,* $\pi^M_{AB} > \pi^{PM}_A + \pi^{PM}_B$.

Proposition 4.7 shows that firms A and B should always choose to merge since they can be more profitable through either decentralized merger or centralized merger. However, if centralized merger is not feasible because the participant firms want to maintain the independence of brands or because centralized merger may incur huge post-merger integration cost, then the participants (i.e., firms A and B) could be worse off after merger, as shown in Proposition 4.8 and Figure 4.6.

Proposition 4.8. *If centralized merger is not possible, then there exists a threshold $\vec{t}$ such that, if $t < \vec{t}$, then $\pi^{DM}_{AB} < \pi^{PM}_A + \pi^{PM}_B$; otherwise, $\pi^{DM}_{AB} \geq \pi^{PM}_A + \pi^{PM}_B$.*

Intuitively, we think that a merger should benefit the participants as it brings cost synergy which lowers the participants' costs and thus makes them more competitive. This is true when centralized merger is possible. However, if centralized merger is not feasible because of the huge post-merger integration cost, Proposition 4.8 shows that the participants may be worse off after merger when the

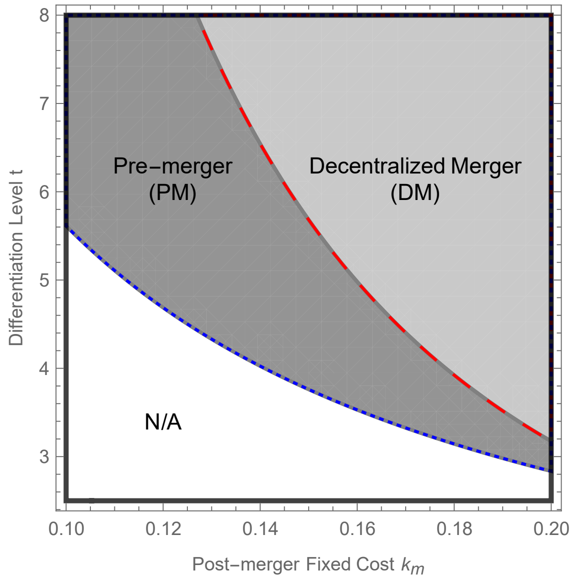


Figure 4.6: Comparison of the PM Case and DM Case

Note: in Figure 4.6, $c = 0.5$, $c_m = 0.35$, and $k = 1.2$.

horizontal differentiation level is low, i.e., $t < \vec{t}$. Decentralized merger may backfire because, as mentioned in the discussion of Proposition 4.1, the participant firms A and B could be hurt by stronger fixed cost synergy because it intensifies the quality competition between the two participants when both of them have lower fixed costs. In addition, when t is small, competition level is high, which may sharpen the already intense quality competition and hurt the participants. Therefore, if the participants can choose only decentralized merger, they may be worse off after merger when t is small.

Next, we study the effects of merger on participant firms' prices and quality. We summarize the results in Proposition 4.9.

Proposition 4.9. *(i) In the case of decentralized merger, the participants' price may be higher or lower while their quality is always higher:*

- *There exists thresholds $\acute{t}$ and $\acute{c_m}$ such that, if $t > \acute{t}$ and $c_m < \acute{c_m}$, then $p_i^{DM} < p_i^{PM}$; otherwise, $p_i^{DM} \geq p_i^{PM}$, $i \in \{A, B\}$;*
- $q_i^{DM} > q_i^{PM}$, $i \in \{A, B\}$.

(ii) In the case of centralized merger with a single product, the post-merger firm's

price may be higher or lower while its quality is always higher:

- *There exists thresholds $\grave{t}$ and $\grave{c_m}$ such that, if $t > \grave{t}$ and $c_m < \grave{c_m}$, then $p_A^{CM1} < p_i^{PM}$; otherwise, $p_A^{CM1} \geq p_i^{PM}$, $i \in \{A, B\}$;*
- $q_A^{CM1} > q_i^{PM}$, $i \in \{A, B\}$.

(iii) In the case of centralized merger with two products, the post-merger firm's price and quality may be higher or lower:

- *There exists a threshold $\hat{c_m}$ such that, if $c_m < \hat{c_m}$, then $p_i^{CM2} < p_i^{PM}$; otherwise, $p_i^{CM2} \geq p_i^{PM}$, $i \in \{A, B\}$;*
- *There exists a threshold $\hat{k_m}$ such that, if $k_m < \hat{k_m}$, then $q_i^{CM2} > p_i^{PM}$; otherwise, $q_i^{CM2} \leq q_i^{PM}$, $i \in \{A, B\}$.*

Proposition 4.9 shows that a horizontal merger does not necessary result in higher price and lower quality. To be specific, if the differentiation level t is large and there is enough marginal cost synergy, then prices fall after merger. Moreover, we find that, the participant firms or the post-merger firm will always increases its quality in the case of decentralized merger and in the case of centralized merger with a single product, while in the case of centralized merger with two products them or it will also provide higher quality if there is enough fixed cost synergy (i.e., $k_m < \hat{k_m}$).

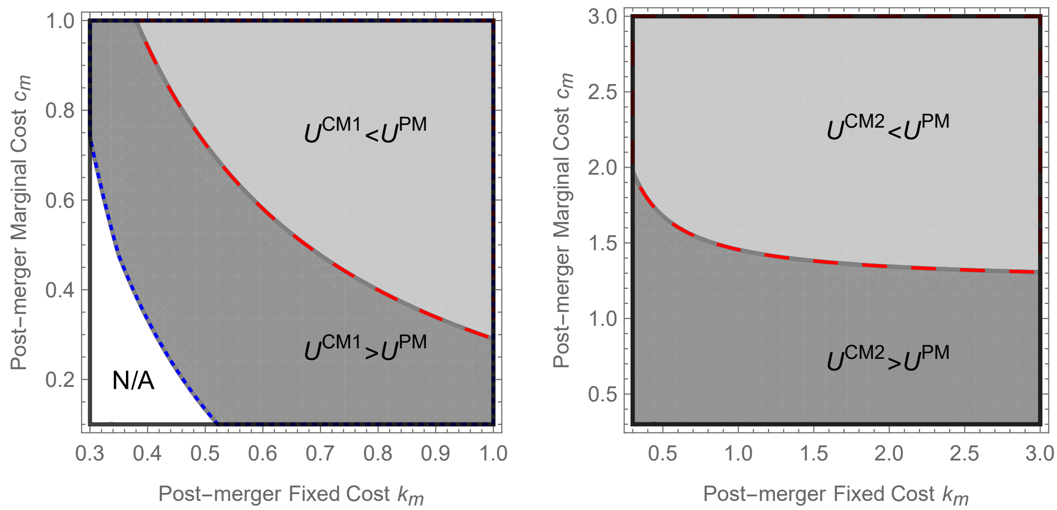
4.7 Total Consumer Utility and Social Welfare

In this section, we study the effects of merger between competing firms on total consumer utility and social welfare. Let U^s denote the total consumer utility in the case of $s \in \{PM, DM, CM1, CM2\}$. The expressions of total consumer utility can be found in the Appendix. Let $SW^{PM} = U^{PM} + \pi_A^{PM} + \pi_B^{PM} + \pi_C^{PM}$ and $SW^s = U^s + \pi_{AB}^s + \pi_C^s$ denote the social welfare in the cases of PM and $s \in \{DM, CM1, CM2\}$, respectively. We first summarize the effect of merger on total consumer utility in Proposition 4.10 and Figure 4.7.

Proposition 4.10. *(i)* $U^{DM} > U^{PM}$;

(ii) There exists a threshold $\check{c_m}$ *(*$\tilde{c_m}$*) such that, if* $c_m < \check{c_m}$ *(*$c_m < \tilde{c_m}$*), then* $U^{CM1} > U^{PM}$ *(*$U^{CM2} > U^{PM}$*); otherwise,* $U^{CM1} \leq U^{PM}$ *(*$U^{CM2} \leq U^{PM}$*);*

(iii) $U^{DM} > U^{CM1}$*; and if* $t > \frac{3}{4k_m}$*,* $U^{DM} > U^{CM2}$.



(a) U^{PM} and U^{CM1} (b) U^{PM} and U^{CM2}

Figure 4.7: Effects of Merger on Total Consumer Utility

Note: in Figure 4.7 (a), $t = 2$, $c = 1$, and $k = 1$; and in Figure 4.7 (b), $t = 5$, $c = 3$, and $k = 3$.

Proposition 4.10 (i) shows that consumers are always better off after decentralized merger. As shown in Proposition 4.9 (i), the participant firms always provide higher quality without raising too much price, which benefits the consumers purchasing from the participants. By contrast, we find that the outside firm C will provide lower quality with lower price, which may benefit or hurt the consumers buying from firm C. Since the participant firms serve more consumers than the outsider firm, the total consumer utility becomes higher in the case of decentralized merger. Proposition 4.10 (ii) shows that, if marginal cost synergy is strong (i.e., c_m is small), then consumers are better off after centralized merger and worse off otherwise. That is, if there are strong marginal cost synergy, the post-merger firm AB will lower its price, which force the outside firm C lower its price and benefit all consumers. Besides, we find that, consumers are better off in the case of decentralized merger compared to the case of centralized merger.

In other words, the intensified competition between participant firms in the case of DM benefits all consumers in the market.

Next, we analyze the effects of the merger between competing firms A and B on social welfare. We summarize the results in Proposition 4.11 and Figure 4.8.

Proposition 4.11. *(i)* $SW^{DM} > SW^{PM}$;

(ii) There exists a threshold $\dot{c_m}$ $(\ddot{c_m})$ *such that, if* $c_m < \dot{c_m}$ $(c_m < \ddot{c_m})$, *then* $SW^{CM1} > SW^{PM}$ $(SW^{CM2} > SW^{PM})$; *otherwise,* $SW^{CM1} \leq SW^{PM}$ $(SW^{CM2} \leq SW^{PM})$;

(iii) There exists thresholds $\bar{\hat{k}}_m$, $\dddot{c_m}$ *and* $\vec{c_m}$ *such that: if* $k_m < \bar{\hat{k}}_m$, $\dddot{c_m} \neq \vec{c_m}$, *and* $c_m < \dddot{c_m}$, *or if* $k_m > \bar{\hat{k}}_m$ *and* $c_m < \dddot{c_m}$, *then,* $SW^{DM} > SW^{CM1}$; *otherwise,* $SW^{DM} \leq SW^{CM1}$;

(iv) if $t > \frac{3}{4k_m}$, $SW^{DM} > SW^{CM2}$.

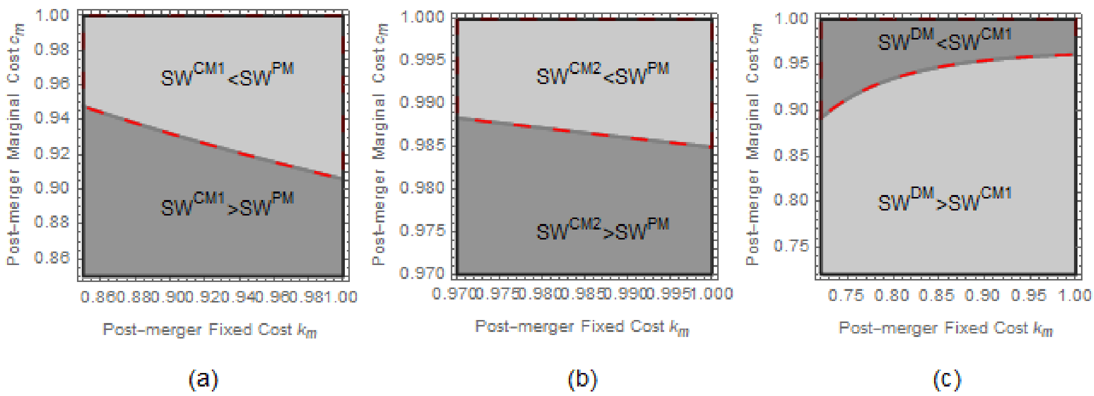


Figure 4.8: Effects of Merger on Social Welfare

Note: in Figure 4.8 (a), $t = 2$, $c = 1$, and $k = 1$; and in Figure 4.8 (b) and (c), $t = 1$, $c = 1$, and $k = 1$.

Proposition 4.11 (i) shows that the merger between firms A and B leads to higher social welfare in the case of decentralized merger compared to the pre-merger case. Therefore, from both consumers' and social viewpoints, decentralized merger is also a desirable merging strategy. Besides, we find that, if marginal cost synergy is strong enough (i.e., c_m is small enough), centralized merger will also result in higher social welfare compared to the pre-merger case, as shown in Proposition 4.11 (ii). In addition, Proposition 4.11 (iii) shows that a centralized merger with a single product may contributes to higher social welfare than a decentralized merger. This is because both the post-merger firm AB and the nonparticipant firm C are significantly better off in the case of centralized merger with a single product.

4.8 Conclusion

In this chapter we study competing firms' merging decision in a competitive market. We develop a game-theoretical model where three firms compete in two dimensions: price and quality. Two of the firms decide whether to merge and how to merge. If they choose to merge, the post-merger firm obtains cost synergy and needs to further decide the level of post-merger integration, i.e., centralized merger or decentralized merger. In the centralized merger case, the post-merger firm make centralized decisions on both participant firms' prices and quality to maximize the total profit. The post-merger firm may offer two products as in the pre-merger market or offer a single product if necessary in the centralized merger case. By contrast, in the decentralized merger case, each participant operates independently and makes its own price and quality decisions to maximize its own profit. This research aims to better understand horizontal mergers in competitive environment and provide insights for firms and antitrust authorities.

We highlight some of our main results. First, stronger cost synergy may or may not hurt the participant firms in the decentralized merger case, while it always benefits the post-merger firm in the centralized merger case. This is because stronger fixed cost synergy may intensify the quality competition between the participants in the decentralized merger case. Second, the post-merger firm prefers decentralized merger when market competition is fierce and centralized merger otherwise. Because a centralized post-merger firm may threat the outside firm and force it to be aggressive when competition level is high, which may backfire and hurt the participant firms. In addition, the participant firms should always choose to merge if both centralized and decentralized mergers are possible.

However, if centralized merger is not possible because of huge post-merger integration cost, then the participants may be worse off after decentralized merger when horizontal differentiation level is low.

This study also provides some insights for the antitrust authorities. To begin with, a horizontal merger does not necessarily reduce market competition and result in both higher price and lower quality. If the post-merger firm adopts decentralized merger, market competition will be fastened. In addition, if cost synergy is strong, the post-merger firm may provide higher quality with lower price. Besides, this study also shows that both the total consumer utility and social welfare may be higher after merger.

Chapter 5

Summary and Future Research

This thesis selects three topics from the interface of operations and marketing. It sets up three game-theoretical models to study: The effects of valuation uncertainty and consumers' anticipated regrets on a seller of substitutable products, the differences between the research online and purchase offline (ROPO) strategy and the buy online and pick up in store (BOPS) strategy, and competiing firms' merging decision and strategy when facing competition from the nonparticipant firm in the market.

The first study explores the effects of second period valuation uncertainty and consumers' anticipated regrets. With our work, there are some research directions valuable for future studies. First, our work focuses on intertemporal pricing decisions and assumes that the availability of each product in its selling period is guaranteed. More results may be found if one consider endogenous price and inventory decisions. Second, in our model, the second period valuation uncertainty exists because the consumers have only limited information for the product unavailable at present. Therefore, it may be interesting to consider the information sharing between the firm and consumers and study the interaction between information disclosure and consumers' anticipated regrets.

One extension of the second study is to consider competing retailers' omnichannel strategies. Initiative of the omnichannel selling may soften retail competition. Besides, in our model, we assume that the retail price is pre-determined. So another future research direction would be to study the pricing strategy with omnichannel selling. Moreover, in this research we focus on operations of the omnichannel selling strategy at the retailer side. It is interesting to extend our work to a supply chain setting, and consider the impacts of the omnichannel selling strategy on both suppliers and retailers.

Furthermore, the third study assumes that firms are symmetric in the pre-merger market. One extension of this study is to investigate the merging decision between asymmetric firms. For example, the firms in the pre-merger market may have different cost efficiency or may be not equidistantly located. Second, our model focuses on price and quality competition and assumes that the firms' locations are fixed. It may be interesting to extend the current model to a more general setting with endogenous location decisions and study the effects of a merger on firms' location choices.

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Appendix A

Proofs for Chapter 2

Proof of Lemma 2.1. According to the definition, v_1^o is determined by $\Delta S = 0$ (i.e., $(\alpha-\beta)(v_1^o - p_1)^2 - 2\big[(\alpha-\beta)(\overline{v}_2 - p_2) + (1+\beta)\overline{v}_2\big](v_1^o - p_1) + (1+\alpha)(\overline{v}_2 - p_2)^2 = 0$). Thus, one can show that $v_1^o = \frac{(\overline{v}_2-p_2)^2}{2\overline{v}_2} + p_1$ if $\alpha = \beta$ and $v_1^o = \frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2 \pm \sqrt{(1+\beta)\left[(1+\alpha)\overline{v}_2^2-(\alpha-\beta){p_2}^2\right]}}{\alpha-\beta} + p_1$ if $\alpha \neq \beta$.

Given p_2, there are three conditions that v_1^o needs to satisfy: $v_1^o \geq p_1$ and the equality holds when $\overline{v}_2 = p_2$, $v_1^o - p_1 < \overline{v}_2 - p_2$, and $v_1^o \leq \overline{v}_1$. Then we get the results shown in Lemma 2.1. $\square$

Proof of Proposition 2.1. If $\alpha = \beta$, by taking derivative with respect to ξ, p_1, and p_2, one can verify that $\frac{\partial v_1^o}{\partial \xi} = \frac{\overline{v}_2^2 - p_2^2}{2\overline{v}_2^2}\overline{v}_1 > 0$, $\frac{\partial v_1^o}{\partial p_1} = 1 > 0$, and $\frac{\partial v_1^o}{\partial p_2} = -\frac{\overline{v}_2 - p_2}{\overline{v}_2} < 0$.

If $\alpha \neq \beta$, let $t = (1+\alpha)\overline{v}_2^2 - (\alpha-\beta){p_2}^2$. Taking derivative with respect to ξ, p_1, p_2, α, and β, one can easily show that $\frac{\partial v_1^o}{\partial \xi} = -\frac{(1+\alpha)(\sqrt{1+\beta}\,\overline{v}_2 - \sqrt{t})}{(\alpha-\beta)\sqrt{t}}\overline{v}_1 > 0$, $\frac{\partial v_1^o}{\partial p_1} = 1 > 0$, $\frac{\partial v_1^o}{\partial p_2} = \frac{\sqrt{1+\beta}\,p_2 - \sqrt{t}}{\sqrt{t}} < 0$, $\frac{\partial v_1^o}{\partial \alpha} = \sqrt{\frac{1+\beta}{t}}\frac{(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)\overline{v}_2^2 - \frac{1}{2}(\alpha-\beta)p_2^2 - \overline{v}_2\sqrt{(1+\beta)t}}{(\alpha-\beta)^2} > 0$, and $\frac{\partial v_1^o}{\partial \beta} = \frac{(1+\alpha)\overline{v}_2\sqrt{(1+\beta)t} - (1+\alpha)(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)\overline{v}_2^2 + \frac{1}{2}(1+\alpha)(\alpha-\beta)p_2^2}{\sqrt{(1+\beta)t}(\alpha-\beta)^2} < 0$. $\square$

Proof of Proposition 2.2. (i) Taking derivative with respect to p_1, α, and β, through straightforward algebraic analysis, one can show that $\frac{\partial D_T}{\partial p_1} < 0$, $\frac{\partial D_T}{\partial \alpha} < 0$, and $\frac{\partial D_T}{\partial \beta} > 0$.

(ii) If $\alpha = \beta$, one can show that

$$\frac{\partial D_T}{\partial \xi} = -\frac{(\overline{v}_1\xi(p_2 - p_1) - p_2^2)p_2}{\overline{v}_1^3\xi^3}. \tag{A.1}$$

If $\alpha \neq \beta$, then we have

$$\frac{\partial D_T}{\partial \xi} = -\frac{p_2}{\overline{v}_1^2 \xi^2}((p_2 - p_1) - \sqrt{1+\beta}\frac{p_2^2}{\sqrt{t}}). \tag{A.2}$$

Define $\dot{\xi} = \frac{p_2^2}{\overline{v}_1(p_2-p_1)}$ if $\alpha = \beta$ and $\dot{\xi} = \sqrt{\frac{\frac{(1+\beta)p_2^4}{(p_2-p_1)^2}+(\alpha-\beta)p_2^2}{(1+\alpha)\overline{v}_1^2}}$ if $\alpha \neq \beta$. Then we have the results shown in Proposition 2.2 (iii).

(iii) Similarly, one can verify that $\frac{\partial D_T}{\partial p_2} = -\frac{1}{\overline{v}_1\overline{v}_2}\frac{(\overline{v}_2-p_2)(\overline{v}_2-3p_2)+2\overline{v}_2p_1}{2\overline{v}_2}$ if $\alpha = \beta$ and $\frac{\partial D_T}{\partial p_2} = -\frac{1}{\overline{v}_1\overline{v}_2}(\frac{\sqrt{1+\beta}p_2^2}{\sqrt{t}} + \frac{(1+\alpha)\overline{v}_2-(\alpha-\beta)p_2-\sqrt{(1+\beta)t}}{\alpha-\beta} + (p_1 - p_2))$ if $\alpha \neq \beta$. In both cases, one can show that there exists a threshold $\dot{p}_2$ such that $\frac{\partial D_T}{\partial p_2} > 0$ if $p_2 > \dot{p}_2$ and $\frac{\partial D_T}{\partial p_2} \leq 0$ otherwise. □

Proof of Lemma 2.2. Given p_2, one can show that $\frac{\partial^2 \Pi(p_1,p_2)}{\partial {p_1}^2} = -\frac{2}{\overline{v}_1} < 0$, that is $\Pi(p_1, p_2)$ is concave in p_1. By solving the first-order condition, we have $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2-\frac{1}{2}\overline{v}_2^2+2\overline{v}_2p_2-\frac{3}{2}{p_2}^2}{2\overline{v}_2}$ and $v_1^o = \frac{(\overline{v}_2-p_2)^2}{2\overline{v}_2} + p_1 < \overline{v}_1$.

Define $H = \overline{v}_2 p_1(\overline{v}_1 - v_1^o) + p_2 v_1^o(\overline{v}_2 - p_2)$. H is proportional to the total profit. By substituting $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2-\frac{1}{2}\overline{v}_2^2+2\overline{v}_2p_2-\frac{3}{2}{p_2}^2}{2\overline{v}_2}$ into H and taking derivative with respect to p_2, we have

$$\frac{\mathrm{d}H}{\mathrm{d}p_2} = \frac{1}{4\overline{v}_2}({p_2}^3 - \overline{v}_2^2 p_2 - 6\overline{v}_1\overline{v}_2 p_2 + 4\overline{v}_1\overline{v}_2^2), \tag{A.3}$$

$$\frac{\mathrm{d}H}{\mathrm{d}p_2}|_{p_2=0} = \frac{1}{4\overline{v}_2}(4\overline{v}_1\overline{v}_2^2) > 0, \text{ and}$$

$$\frac{\mathrm{d}H}{\mathrm{d}p_2}|_{p_2=\overline{v}_2} = \frac{1}{4\overline{v}_2}(-2\overline{v}_1\overline{v}_2^2) < 0.$$

It is easy to verify that $\frac{\mathrm{d}^2H}{\mathrm{d}{p_2}^2} < 0$. That is, the profit is concave in p_2 and there exists an unique solution $p_2^{c*} \in (0, \overline{v}_2)$ such that $\frac{\mathrm{d}H}{\mathrm{d}p_2} = 0$. $\frac{\mathrm{d}H}{\mathrm{d}p_2} = 0$ is a depressed monic cubic equation. Its solutions are given by $t_k = 2\sqrt{\frac{(\overline{v}_2+6\overline{v}_1)\overline{v}_2}{3}}\cos(\frac{1}{3}\arccos(-\frac{6\overline{v}_1\overline{v}_2}{\overline{v}_2+6\overline{v}_1}\sqrt{\frac{3}{(\overline{v}_2+6\overline{v}_1)\overline{v}_2}}) - k\frac{2\pi}{3})$, for $k = 0, 1, 2$. Since $0 < p_2^{c*} < \overline{v}_2$, we know that the optimal solutions are $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2-\frac{1}{2}\overline{v}_2^2+2\overline{v}_2p_2^{c*}-\frac{3}{2}p_2^{c*2}}{2\overline{v}_2}$ and $p_2^{c*} = 2\sqrt{\frac{6+\xi}{3\xi}}\cos(\frac{1}{3}\arccos(-\frac{6}{6+\xi}\sqrt{\frac{3\xi}{6+\xi}}) - \frac{2\pi}{3})\overline{v}_2$. □

Proof of Proposition 2.3. (i) It is easy to show that $\frac{\partial^2\Pi(p_1,p_2)}{\partial p_1 \partial p_2} = \frac{1}{\overline{v}_1\overline{v}_2}(2\overline{v}_2 - 3p_2)$ and $\frac{\mathrm{d}H}{\mathrm{d}p_2}|_{p_2=\frac{2}{3}\overline{v}_2} < 0$, which means that $p_2^{c*} < \frac{2}{3}\overline{v}_2$. Hence, $\frac{\partial^2\Pi(p_1,p_2)}{\partial p_1 \partial p_2} > 0$ and p_1^{c*} is increasing in p_2^{c*}.

(ii) According to the Implicit Function Theorem, $\frac{\partial p_2^{c*}}{\partial \xi} = -\frac{\partial(\frac{\mathrm{d}H}{\mathrm{d}p_2})}{\partial \xi} / \frac{\mathrm{d}^2 H}{\mathrm{d}p_2^2}$, then one can verify that $\frac{\partial p_2^{c*}}{\partial \xi} > 0$.

(iii) $p_1 - p_2 = \frac{1}{4\overline{v}_2}(2\overline{v}_1\overline{v}_2 - \overline{v}_2^2 - 3{p_2}^2)$. One can verify that if ${p_2}^2 < \frac{2\overline{v}_1\overline{v}_2 - \overline{v}_2^2}{3}$, $p_1^{c*} > p_2^{c*}$; otherwise $p_1^{c*} \leq p_2^{c*}$. Replacing p_2 in Equation (A.3) with $\sqrt{\frac{2\overline{v}_1\overline{v}_2 - \overline{v}_2^2}{3}}$, we know that if $27{\overline{v}_1}^2\overline{v}_2 - 32{\overline{v}_1}^3 + \overline{v}_2^3 + 6\overline{v}_1\overline{v}_2^2 \geq 0$, $p_1^{c*} \leq p_2^{c*}$; otherwise, $p_1^{c*} > p_2^{c*}$. Notice that $27{\overline{v}_1}^2\overline{v}_2 - 32{\overline{v}_1}^3 + \overline{v}_2^3 + 6\overline{v}_1\overline{v}_2^2$ is increasing in ξ. Solving $27{\overline{v}_1}^2\overline{v}_2 - 32{\overline{v}_1}^3 + \overline{v}_2^3 + 6\overline{v}_1\overline{v}_2^2 = 0$, we have $\xi = -2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2 \approx 0.9519$. Therefore, we can conclude that $p_1^{c*} < p_2^{c*}$ if and only if $\xi > -2\sqrt{5}\sinh(\frac{1}{3}\operatorname{arsinh}(-\frac{7}{\sqrt{5}})) - 2$.

(iv) Given Equations (A.1) and (A.2), it can be shown that $\frac{\partial D_T}{\partial \xi} > 0$ when $p_1^{c*} = \frac{\overline{v}_1\overline{v}_2 - \frac{1}{2}\overline{v}_2^2 + 2\overline{v}_2 p_2^{c*} - \frac{3}{2}p_2^{c*2}}{2\overline{v}_2}$ and $p_2^{c*} = 2\sqrt{\frac{6+\xi}{3\xi}}\cos(\frac{1}{3}\arccos(-\frac{6}{6+\xi}\sqrt{\frac{3\xi}{6+\xi}}) - \frac{2\pi}{3})\overline{v}_2$. □

Proof of Lemma 2.3. One can verify that, given p_2, $\Pi(p_1, p_2)$ is concave in p_1 and

$$p_1 = \frac{1}{2}\Big\{\overline{v}_1 + p_2 - \frac{p_2^2}{\overline{v}_2} - \frac{(1+\alpha)\overline{v}_2 - (\alpha-\beta)p_2 - \sqrt{(1+\beta)\big[(1+\alpha)\overline{v}_2^2 - (\alpha-\beta)p_2^2\big]}}{\alpha - \beta}\Big\}. \tag{A.4}$$

Let $n = \frac{(1+\alpha)\overline{v}_2 - (\alpha-\beta)p_2 - \sqrt{(1+\beta)t}}{\alpha-\beta}$. $p_1 = \frac{1}{2}(\overline{v}_1 - n + p_2 - \frac{{p_2}^2}{\overline{v}_2})$ and $v_1^o = \frac{1}{2}(\overline{v}_1 + n + p_2 - \frac{{p_2}^2}{\overline{v}_2})$. Next we show that $v_1^o < \overline{v}_1$. $v_1^o - \overline{v}_1 = \frac{1}{2}(-\overline{v}_1 + n + p_2 - \frac{{p_2}^2}{\overline{v}_2}) \propto (\overline{v}_2^2 - p_2^2) - \overline{v}_1\overline{v}_2\frac{\sqrt{t} + \overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$. It can be shown that, if $\alpha < \beta$, $\frac{\sqrt{t} + \overline{v}_2\sqrt{1+\beta}}{\sqrt{t}} \geq 2$, that is $\overline{v}_1\frac{\sqrt{t} + \overline{v}_2\sqrt{1+\beta}}{\sqrt{t}} > \overline{v}_2^2$ and $v_1^o < \overline{v}_1$ if $\alpha < \beta$. If $\alpha > \beta$, we can prove that $(\overline{v}_2^2 - p_2^2) < \overline{v}_1\overline{v}_2\frac{\sqrt{t} + \overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$ by contradiction. We assume that $(\overline{v}_2^2 - p_2^2) > \overline{v}_1\overline{v}_2\frac{\sqrt{t} + \overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$, which indicates that $v_1^o = \overline{v}_1$, $D_1 = 0$, and $D_2 = \frac{\overline{v}_2 - p_2}{\overline{v}_2}$. By solving the firm's maximization problem, we get $p_2^* = \frac{1}{2}\overline{v}_2$, which indicates that $(\overline{v}_2^2 - p_2^2) < \overline{v}_1\overline{v}_2\frac{\sqrt{t} + \overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$. Therefore, $(\overline{v}_2^2 - p_2^2) < \overline{v}_1\overline{v}_2\frac{\sqrt{t} + \overline{v}_2\sqrt{1+\beta}}{\sqrt{t}}$ and $v_1^o < \overline{v}_1$ if $\alpha > \beta$. By combining these two cases, we have $v_1^o < \overline{v}_1$.

Next we show that $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$. The formulation of H is given by:

$$H = \frac{1}{2}\Big[\frac{1}{2}{\overline{v}_1}^2\overline{v}_2 - \overline{v}_1\overline{v}_2 n + \overline{v}_1\overline{v}_2 p_2 - \overline{v}_1{p_2}^2 + \frac{1}{2}\overline{v}_2(n + p_2 - \frac{{p_2}^2}{\overline{v}_2})^2\Big].$$

Taking derivative with respect to p_2, we have $\frac{\mathrm{d}n}{\mathrm{d}p_2} = -1 + p_2\sqrt{\frac{1+\beta}{t}}$, and

$$\frac{\mathrm{d}(2H)}{\mathrm{d}p_2} = 2\overline{v}_1\overline{v}_2 - 2\overline{v}_1 p_2 - 2{p_2}^2 + 2\frac{{p_2}^3}{\overline{v}_2} - 2p_2 n + (\overline{v}_2 n - \overline{v}_1\overline{v}_2 + \overline{v}_2 p_2 - {p_2}^2)p_2\sqrt{\frac{1+\beta}{t}}. \tag{A.5}$$

It can be shown that when $\alpha \to \beta$, Equation (A.5) reduces to that in the case of $\alpha = \beta$. Besides, one can verify that $\frac{\mathrm{d}(2H)}{\mathrm{d}p_2}|_{p_2=0} = 2\overline{v}_1\overline{v}_2 > 0$ and $\frac{\mathrm{d}(2H)}{\mathrm{d}p_2}|_{p_2=\overline{v}_2} = -\overline{v}_1\overline{v}_2 < 0$. Given the assumption that $\xi \in (\frac{1}{2}, 2)$, the following proof will show that $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$.

$$\begin{aligned}
\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} &= -2\overline{v}_1 - 4p_2 + 6\frac{{p_2}^2}{\overline{v}_2} - 2n - 2p_2\frac{\mathrm{d}n}{\mathrm{d}p_2} \\
&\quad + (\overline{v}_2\frac{\mathrm{d}n}{\mathrm{d}p_2} + \overline{v}_2 - 2p_2)p_2\sqrt{\frac{1+\beta}{t}} \\
&\quad + (\overline{v}_2 n - \overline{v}_1\overline{v}_2 + \overline{v}_2 p_2 - {p_2}^2)\sqrt{\frac{1+\beta}{t}} \\
&\quad + (\overline{v}_2 n - \overline{v}_1\overline{v}_2 + \overline{v}_2 p_2 - {p_2}^2)p_2\frac{(\alpha-\beta)p_2}{t}\sqrt{\frac{1+\beta}{t}} \\
&= -2\overline{v}_1 + 6\frac{{p_2}^2}{\overline{v}_2} - \frac{(3+2\alpha+\beta)\overline{v}_2}{\alpha-\beta} \\
&\quad + \Big[\frac{2t}{\alpha-\beta} - 4{p_2}^2 + \frac{1+\alpha}{\alpha-\beta}\overline{v}_2^2 - \frac{1+\alpha}{t}\overline{v}_1\overline{v}_2^3\Big]\sqrt{\frac{1+\beta}{t}}.
\end{aligned}$$

One can easily verify that $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}|_{p_2=0} < 0$ and $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}|_{p_2=\overline{v}_2} < 0$.

Define $u = \sqrt{t} = \sqrt{(1+\alpha)\overline{v}_2^2 - (\alpha-\beta){p_2}^2}$, which lies between $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$. Then we have

$$\begin{aligned}
u^3\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} &= -\frac{6}{\overline{v}_2(\alpha-\beta)}u^5 + \frac{6\sqrt{1+\beta}}{\alpha-\beta}u^4 + \big[-2\overline{v}_1 + \frac{(3+4\alpha-\beta)\overline{v}_2}{\alpha-\beta}\big]u^3 \\
&\quad - \frac{3(1+\alpha)\sqrt{1+\beta}\overline{v}_2^2}{\alpha-\beta}u^2 - (1+\alpha)\sqrt{1+\beta}\overline{v}_1\overline{v}_2^3.
\end{aligned}$$

Let $I = u^3\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}$, then we have

$$\begin{aligned}
\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} &= -\frac{30}{\overline{v}_2}u^3 + 24\sqrt{1+\beta}u^2 + 3\big[-2(\alpha-\beta)\overline{v}_1 + (3+4\alpha-\beta)\overline{v}_2\big]u \\
&\quad - 6(1+\alpha)\sqrt{1+\beta}\overline{v}_2^2.
\end{aligned}$$

If $u = \sqrt{1+\alpha}\overline{v}_2$, then we have

$$\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} = 3\big[6(1+\alpha)^{\frac{1}{2}}(1+\beta)^{\frac{1}{2}}\overline{v}_2 - 2(\alpha-\beta)\overline{v}_1 + (-7-6\alpha-\beta)\overline{v}_2\big](1+\alpha)^{\frac{1}{2}}\overline{v}_2.$$

Let $a = (1+\alpha)^{\frac{1}{2}}$, $b = (1+\beta)^{\frac{1}{2}}$, and $\gamma = \frac{1}{\xi}$, then

$$\begin{aligned}
\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} &= 3\big[6ab - 2(a^2-b^2)\gamma + (-6a^2-b^2)\big](1+\alpha)^{\frac{1}{2}}\overline{v}_2^2 \\
&= -6(\gamma+3)(a - \frac{6-\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b) \\
&\quad * (a - \frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b)(1+\alpha)^{\frac{1}{2}}\overline{v}_2^2.
\end{aligned}$$

This implies that

$$\frac{\mathrm{d}I}{\mathrm{d}u} = -6u(\gamma+3)(a - \frac{6-\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b) * (a - \frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b)\frac{a}{(a^2-b^2)}\overline{v}_2^2.$$

Note that $(a - \frac{6-\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b) > 0$ and $0 < (a - \frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}b) < 1$. Therefore, we have $\frac{\mathrm{d}I}{\mathrm{d}u} < 0$ if $\alpha < \big[\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}\big]^2(1+\beta) - 1$ or $\alpha > \beta$ and $\frac{\mathrm{d}I}{\mathrm{d}u} > 0$ if $\big[\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}\big]^2(1+\beta) - 1 < \alpha < \beta$.

By contrast, if $u = \sqrt{1+\beta}\overline{v}_2$, then we have

$$\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} = 3\big[-2(\alpha-\beta)\overline{v}_1 + (-1+2\alpha-3\beta)\overline{v}_2\big]\sqrt{1+\beta}\overline{v}_2.$$

If $\alpha > \beta$, then one can show that $\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} < 3\big[-(\alpha-\beta)\overline{v}_2 + (-1+2\alpha-3\beta)\overline{v}_2\big]\sqrt{1+\beta}\overline{v}_2 = 3(-1+\alpha-2\beta)\sqrt{1+\beta}\overline{v}_2^2 < 0$. Therefore, $\frac{\mathrm{d}I}{\mathrm{d}u} < 0$ if $\alpha > \beta$. By contrast, if $\alpha < \beta$, then $\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} < 3\big[-4(\alpha-\beta)\overline{v}_2 + (-1+2\alpha-3\beta)\overline{v}_2\big]\sqrt{1+\beta}\overline{v}_2 = 3(-1-2\alpha+\beta)\sqrt{1+\beta}\overline{v}_2^2 < 0$. Therefore, $\frac{\mathrm{d}I}{\mathrm{d}u} > 0$ if $\alpha < \beta$.

Taking derivative with respect to u, we have

$$\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big) = -\frac{90}{\overline{v}_2}u^2 + 48(1+\beta)^{\frac{1}{2}}u + 3\big[-2(\alpha-\beta)\overline{v}_1 + (3+4\alpha-\beta)\overline{v}_2\big].$$

Define $\Delta_2 = 2304(1+\beta) + 12\frac{90}{\overline{v}_2}\big[-2(\alpha-\beta)\overline{v}_1 + (3+4\alpha-\beta)\overline{v}_2\big]$. Note that $-2(\alpha-\beta)\overline{v}_1 + (3+4\alpha-\beta)\overline{v}_2 > -2(\alpha-\beta)\overline{v}_1 + (1.5+2\alpha-0.5\beta)\overline{v}_1 = (1.5+1.5\beta)\overline{v}_1 > 0$. Thus, $\Delta_2 > 0$. Hence, there are two roots θ_1 and θ_2, where $\theta_1 = \frac{48\sqrt{1+\beta}-\sqrt{\Delta_2}}{180}\overline{v}_2 < 0$ and $\theta_2 = \frac{48\sqrt{1+\beta}+\sqrt{\Delta_2}}{180}\overline{v}_2 > 0$.

$$\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\alpha}\overline{v}_2} = -90(1+\alpha)\overline{v}_2 + 48\sqrt{1+\beta}\sqrt{1+\alpha}\overline{v}_2 + 3\big[-2(\alpha-\beta)\overline{v}_1 + (3+4\alpha-\beta)\overline{v}_2\big].$$

If $\alpha > \beta$, then

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\alpha}\overline{v}_2} &< -90(1+\alpha)\overline{v}_2 + 48\sqrt{1+\alpha}\sqrt{1+\alpha}\overline{v}_2 \\ &+ 3(3+4\alpha-\beta)\overline{v}_2 \\ &= 3(-11-10\alpha-\beta)\overline{v}_2 \\ &< 0.\end{aligned}$$

If $\alpha < \beta$, then

$$\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\alpha}\overline{v}_2} < (-90\sqrt{1+\alpha}+48\sqrt{1+\beta})\sqrt{1+\alpha}\overline{v}_2 + 3\big[-4(\alpha-\beta)\overline{v}_2+(3+4\alpha-\beta)\overline{v}_2\big].$$

Note that $-90\sqrt{1+\alpha}+48\sqrt{1+\beta} \leq -90+48\sqrt{2} < 0$. Thus,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\alpha}\overline{v}_2} &< (-90\sqrt{1+\alpha}+48\sqrt{1+\beta})\overline{v}_2+9(1+\beta)\overline{v}_2\\ &\leq (-72+48\sqrt{2})\overline{v}_2\\ &< 0.\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big)\Big|_{u=\sqrt{1+\beta}\overline{v}_2} &= -90(1+\beta)\overline{v}_2+48(1+\beta)\overline{v}_2\\ &\quad+3\big[-2(\alpha-\beta)\overline{v}_1+(3+4\alpha-\beta)\overline{v}_2\big]\\ &= 3\big[-2(\alpha-\beta)\overline{v}_1+(-11+4\alpha-15\beta)\overline{v}_2\big]\\ &\leq 3\big[2\overline{v}_1+(-11+4\alpha-15\beta)\overline{v}_2\big]\\ &< 3\big[4\overline{v}_2+(-11+4\alpha-15\beta)\overline{v}_2\big]\\ &= 3(-7+4\alpha-15\beta)\overline{v}_2\\ &< 0,\end{aligned}$$

which implies that both $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$ are greater than θ_2. Therefore, $\frac{\mathrm{d}}{\mathrm{d}u}\Big(\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u}\Big) < 0$ for all u between $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$. When $\alpha > \big[\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}\big]^2(1+\beta)-1$, it is easy to find that $\frac{\alpha-\beta}{u}\frac{\mathrm{d}I}{\mathrm{d}u} < 0$ for all u between $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$. Hence, if $\alpha < \beta$, then $\frac{\mathrm{d}I}{\mathrm{d}u} > 0$ for all u and if $\alpha > \beta$, then $\frac{\mathrm{d}I}{\mathrm{d}u} < 0$ for all u. Therefore, I is monotonic and $I < 0$. Since $I = u^3\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}$, thus $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}$ is monotonic and $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$. When $\alpha < \big[\frac{6+\sqrt{36+8(\gamma+3)(2\gamma-1)}}{4(\gamma+3)}\big]^2(1+\beta)-1$, one can also verify that $I < 0$ although it is not monotonic. Therefore, $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$ for all u. This completes the proof. □

Proof of Proposition 2.4. (i) If $\alpha \neq \beta$, replacing p_2 in Equation (A.5) with $\frac{1}{2}\overline{v}_2$, then we have,

$$\frac{\mathrm{d}2H}{\mathrm{d}p_2}\Big|_{p_2=\frac{1}{2}\overline{v}_2} = (1-\frac{1}{2}\sqrt{\frac{1+\beta}{1+\frac{3}{4}\alpha+\frac{1}{4}\beta}}) * \Big[\overline{v}_1\overline{v}_2-\frac{1}{4}\overline{v}_2^2-\frac{(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)-\sqrt{(1+\beta)(1+\frac{3}{4}\alpha+\frac{1}{4}\beta)}}{\alpha-\beta}\overline{v}_2^2\Big].$$

One can verify that $1-\frac{1}{2}\sqrt{\frac{1+\beta}{1+\frac{3}{4}\alpha+\frac{1}{4}\beta}} = 1-\sqrt{\frac{1+\beta}{4+3\alpha+\beta}} > 0$ and $1-\Big[\frac{1}{4}+\frac{(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)-\sqrt{(1+\beta)(1+\frac{3}{4}\alpha+\frac{1}{4}\beta)}}{\alpha-\beta}\Big]\xi > 1-2\Big[\frac{1}{4}+\frac{(1+\frac{1}{2}\alpha+\frac{1}{2}\beta)-\sqrt{(1+\beta)(1+\frac{3}{4}\alpha+\frac{1}{4}\beta)}}{\alpha-\beta}\Big] = \frac{1}{2}-\frac{1+\alpha}{2+\alpha+\beta+\sqrt{(1+\beta)(4+3\alpha+\beta)}} > 0$. Therefore, $\frac{\mathrm{d}2H}{\mathrm{d}p_2}\Big|_{p_2=\frac{1}{2}\overline{v}_2} > 0$ and $p_2^{c*} > \frac{1}{2}\overline{v}_2$.

Based on Equation (A.4), taking derivative with respect to p_2, we get $\frac{\mathrm{d}(2p_1^{c*})}{\mathrm{d}p_2^{c*}} = 2-\frac{2}{\overline{v}_2}p_2 - p_2\sqrt{\frac{1+\beta}{t}}$. Next we show that $\frac{\mathrm{d}(2p_1^{c*})}{\mathrm{d}p_2^{c*}} > 0$. Let $Z = \overline{v}_2\frac{\mathrm{d}2p_1^{c*}}{\mathrm{d}p_2^{c*}} = 2(\overline{v}_2 - p_2) - \overline{v}_2 p_2\sqrt{\frac{1+\beta}{t}}$. Oen can show that $\frac{\mathrm{d}Z}{\mathrm{d}p_2} = -2-\overline{v}_2\sqrt{\frac{1+\beta}{t}}\frac{(1+\alpha)\overline{v}_2^2}{t} < 0$, that is Z is decreasing in p_2. Besides, $Z|_{p_2=0} = 2\overline{v}_2 > 0$ and $Z|_{p_2=\overline{v}_2} = -\overline{v}_2 < 0$. Therefore, there exists a ${p_2}^Z \in (0,\overline{v}_2)$ such that $Z|_{p_2={p_2}^Z} = 0$. In addition, $Z|_{p_2=\frac{1}{2}\overline{v}_2} = \overline{v}_2 - \frac{1}{2}\overline{v}_2\sqrt{\frac{1+\beta}{1+\frac{3}{4}\alpha+\frac{1}{4}\beta}} > 0$ and $Z|_{p_2=\frac{2}{3}\overline{v}_2} = \frac{2}{3}\Big(1-\sqrt{\frac{1+\beta}{1+\frac{5}{9}\alpha+\frac{4}{9}\beta}}\Big)\overline{v}_2$. One can show that $Z|_{p_2=\frac{2}{3}\overline{v}_2} > 0$ if $\alpha > \beta$ and $Z|_{p_2=\frac{2}{3}\overline{v}_2} < 0$ if $\alpha < \beta$.

Putting ${p_2}^Z$ into Equation (A.5), we have

$$
\begin{aligned}
\frac{\mathrm{d}2H}{\mathrm{d}p_2}|_{p_2={p_2}^Z} &= -6{p_2}^{Z2} + \frac{4{p_2}^{Z3}}{\overline{v}_2} \\
&\quad + 2(\overline{v}_2 - 2{p_2}^Z)\Big[\frac{(1+\alpha)\overline{v}_2}{\alpha-\beta} - {p_2}^Z - \frac{(1+\beta)\overline{v}_2{p_2}^Z}{2(\alpha-\beta)(\overline{v}_2-{p_2}^Z)}\Big] + 2\overline{v}_2{p_2}^Z \\
&= \frac{(1+\beta)\overline{v}_2{p_2}^Z(\overline{v}_2-2{p_2}^Z)(3{p_2}^Z-2\overline{v}_2)}{2(\alpha-\beta)(\overline{v}_2-{p_2}^Z)^2} \\
&< 0.
\end{aligned}
$$

Therefore, $\frac{\mathrm{d}2H}{\mathrm{d}p_2}|_{Z=0} < 0$. Since both $\frac{\mathrm{d}2H}{\mathrm{d}p_2}$ and Z are decreasing in p_2, $Z|_{p_2^{c*}} > 0$ when $p_2 \in (0,\overline{v}_2)$. Notice that $Z = \overline{v}_2\frac{\mathrm{d}2p_1^{c*}}{\mathrm{d}p_2^{c*}}$, thus we have $\frac{\mathrm{d}2p_1^{c*}}{\mathrm{d}p_2^{c*}} > 0$.

(ii) According to Equation (A.5), $\frac{\partial p_2^{c*}}{\partial\alpha} = -\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha}\Big/\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2}$. Since we have shown that $\frac{\mathrm{d}^2(2H)}{\mathrm{d}{p_2}^2} < 0$ and $\frac{\partial p_2}{\partial\alpha}\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha} > 0$, we then focus on $\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha}$. Following the same logic of Lemma 2.3, one can show that

$$
\begin{aligned}
\frac{2(\alpha-\beta)^2u^3}{\sqrt{1+\beta}p_2}\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha} = &-3u^4 + 6\sqrt{1+\beta}\overline{v}_2u^3 + \big[(\alpha-\beta)\overline{v}_1 - 3(1+\beta)\overline{v}_2\big]\overline{v}_2u^2 \\
&-(\alpha-\beta)(1+\beta)\overline{v}_1\overline{v}_2^3.
\end{aligned}
$$

Notice that u lies between $\sqrt{1+\alpha}\overline{v}_2$ and $\sqrt{1+\beta}\overline{v}_2$. Let $J = \frac{2(\alpha-\beta)^2u^3}{\sqrt{1+\beta}p_2}\frac{\partial(\frac{\mathrm{d}(2H)}{\mathrm{d}p_2})}{\partial\alpha}$. $J > 0$ means that the best second period price p_2^{c*} is increasing in α and vice versa.

$$
\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u} = -12u^2 + 18\sqrt{1+\beta}\overline{v}_2u + 2\big[(\alpha-\beta)\overline{v}_1 - 3(1+\beta)\overline{v}_2\big]\overline{v}_2.
$$

If $16\alpha < 13\beta - 3$ and $\frac{1}{2} < \xi < \frac{8(\beta-\alpha)}{3(1+\beta)}$, then $\Delta_{\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}} < 0$ and $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u} < 0$; otherwise $\Delta_{\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}} \geq 0$. Notice that if $\alpha > \beta$, then $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}|_{u=0} < 0$ and $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}|_{u=\sqrt{1+\beta}\overline{v}_2} > 0$; and if $\alpha < \beta$, then $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}|_{u=\sqrt{1+\beta}\overline{v}_2} < 0$. Let u_1 and u_2 denote the roots of $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u} = 0$ when $\Delta_{\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}} \geq 0$. $u_1 < u_2$. $u_1 = \frac{9\sqrt{1+\beta}\overline{v}_2 - \sqrt{3\left[3(1+\beta)\overline{v}_2 + 8(\alpha-\beta)\overline{v}_1\right]\overline{v}_2}}{12}$ and $u_2 = \frac{9\sqrt{1+\beta}\overline{v}_2 + \sqrt{3\left[3(1+\beta)\overline{v}_2 + 8(\alpha-\beta)\overline{v}_1\right]\overline{v}_2}}{12}$. Next we compare u_1, u_2, $\sqrt{1+\alpha}\overline{v}_2$, and $\sqrt{1+\beta}\overline{v}_2$, and summarize the results in the following table:

No.	$\sqrt{1+\alpha}$	ξ	$\Delta_{\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}}$	$u_1 - \sqrt{1+\alpha}\overline{v}_2$	$u_2 - \sqrt{1+\alpha}\overline{v}_2$
1		$(\frac{1}{2}, \delta_1)$	$-$	N/A	N/A
2		δ_1	0	$+$	$+$
3	$[1, \frac{3}{4}\sqrt{1+\beta})$	(δ_1, δ_2)	$+$	$+$	$+$
4		δ_2	$+$	0	$+$
5		$(\delta_2, 2)$	$+$	$-$	$+$
6		$(\frac{1}{2}, \delta_1)$	$-$	N/A	N/A
7	$\frac{3}{4}\sqrt{1+\beta}$	δ_1	0	0	0
8		$(\delta_1, 2)$	$+$	$-$	$+$
9		$(\frac{1}{2}, \delta_1)$	$-$	N/A	N/A
10		δ_1	0	$-$	$-$
11	$(\frac{3}{4}\sqrt{1+\beta}, \frac{\sqrt{13}}{4}\sqrt{1+\beta})$	(δ_1, δ_2)	$+$	$-$	$-$
12		δ_2	$+$	$-$	0
13		$(\delta_2, 2)$	$+$	$-$	$+$
14		$(\frac{1}{2}, \delta_2)$	$+$	$-$	$-$
15	$[\frac{\sqrt{13}}{4}\sqrt{1+\beta}, \sqrt{1+\beta})$	δ_2	$+$	$-$	0
16		$(\delta_2, 2)$	$+$	$-$	$+$
17		$(\frac{1}{2}, \delta_2)$	$+$	$-$	$+$
18	$(\sqrt{1+\beta}, \frac{5}{4}\sqrt{1+\beta})$	δ_2	$+$	$-$	0
19		$(\delta_2, 2)$	$+$	$-$	$-$
20	$[\frac{5}{4}\sqrt{1+\beta}, \sqrt{2}]$	$(\frac{1}{2}, 2)$	$+$	$-$	$-$

Here, $\delta_1 = \frac{8(\beta-\alpha)}{3(1+\beta)}$ and $\delta_2 = \frac{\sqrt{1+\alpha}+\sqrt{1+\beta}}{3(2\sqrt{1+\alpha}-\sqrt{1+\beta})}$. $\delta_1 < \delta_2$. The table helps to define the sign of $\frac{1}{u}\frac{\mathrm{d}J}{\mathrm{d}u}$ and thus defines the shape of J. Next we complete the proof by analyzing $J|_{u=u_1}$, $J|_{u=\sqrt{1+\alpha}\overline{v}_2}$, and $J|_{u=\sqrt{1+\beta}\overline{v}_2}$. According to the definition of J, $J|_{u=\sqrt{1+\beta}\overline{v}_2} = 0$ and $J|_{u=\sqrt{1+\alpha}\overline{v}_2} = (\sqrt{1+\alpha} - \sqrt{1+\beta})^2\left[(\sqrt{1+\alpha}+\sqrt{1+\beta})^2\overline{v}_1 - 3(1+\alpha)\overline{v}_2\right]\overline{v}_2^3$. Let $\delta_3 = \frac{(\sqrt{1+\alpha}+\sqrt{1+\beta})^2}{3(1+\alpha)}$, if $\xi < \delta_3$, $J|_{u=\sqrt{1+\alpha}\overline{v}_2} > 0$; otherwise, $J|_{u=\sqrt{1+\alpha}\overline{v}_2} \leq 0$. Note that $\delta_2 < \delta_3 < 2$. One can show that $J|_{u=\sqrt{1+\alpha}\overline{v}_2} < 0$ happens only in cases 5, 8, 13, 16, 19 and 20. Specifically, for cases 5, 8, 13 and 16, there is a $u' \in (\sqrt{1+\alpha}\overline{v}_2, u_2)$ such that if $u \in (\sqrt{1+\alpha}\overline{v}_2, u')$, $J < 0$, otherwise $J \geq 0$. Similarly, for cases 19 and 20, there is a $u'' \in (u_2, \sqrt{1+\alpha}\overline{v}_2)$ such that if $u \in (u'', \sqrt{1+\alpha}\overline{v}_2)$, $J < 0$; otherwise, $J \geq 0$. Let $\hat{\alpha}$ be the solution of $J = 0$ in cases 5, 8, 13, 16, 19 and 20. Then, we can conclude that, if ξ is large enough, then there exits a $\hat{\alpha}$ such that the optimal second price p_2^{c*} is decreasing in α if $\alpha > \hat{\alpha}$ and vice versa.

Similarly, one can verify the effects of β on p_2^{c*}. Here we complete the proof of Proposition 2.4. □

Proof of Proposition 2.5. Given that $p_2^{c*} > \frac{1}{2}\overline{v}_2$ and that p_1^{c*} increases in p_2^{c*}, Proposition 2.5 is obvious. □

Appendix B

Proofs and Supplement for Chapter 3

B.1 Proofs

Proof of Proposition 3.1. We start with the benchmark. The participatory RE equilibrium is derived by checking the five conditions in Definition 3.1. First, in the benchmark, a consumer makes decision of visiting-or-not. Her optimal principle requires that, given $\tilde{\xi}_o$, $u_s > 0$, which means that we need $t_s < \tilde{\xi}_o\theta(v-p)$. The retailer's optimal principle asks for that, given $\tilde{\phi}_o$, the optimal inventory decision is $\bar{F}(\frac{q_o}{\tilde{\phi}_o\theta}) = \frac{c}{p}$. Then we check the consistency conditions. If $u_s > 0$, then $\tilde{\phi}_o = \phi_o = 1$, $q_o = \theta\bar{F}^{-1}(\frac{c}{p})$, and $\tilde{\xi}_o = A_o(q) = \frac{E\min\{D,\bar{F}^{-1}(\frac{c}{p})\}}{E\{D\}}$. Therefore, $u_s > 0$ requires $t_s < A_o(q)\theta(v-p)$. To sum up, if $t_s < A_o(q)\theta(v-p)$, all consumers visit the store and $q_o = \theta\bar{F}^{-1}(\frac{c}{p}) > 0$.

The proofs of the participatory RE equilibria of the ROPO and BOPS strategies follow the same logic as above. Under the ROPO strategy, each consumer makes decision of visiting-or-not based on the availability information. Her optimal condition requires $t_s < \theta(v-p)$. The retailer's optimal condition results in $\bar{F}(\frac{q_r}{\tilde{\phi}_r\theta}) = \frac{c}{p+\frac{k_s}{\theta}}$. With the consistency conditions, we know that, if $t_s < \theta(v-p)$, $q_r = \theta\bar{F}^{-1}(\frac{c}{p+\frac{k_s}{\theta}}) > 0$. Under the BOPS strategy, a consumer decides to visit the store, use the BOPS function, or leave the market. If $t_s < \min\{t_b, \theta(v-p)\}$, visiting the store is the optimal choice; and if $t_b < \min\{t_s, \theta(v-p)\}$, using the BOPS function is the best option. The retailer's optimal principles regarding visiting the store and using the BOPS function result in $\bar{F}(\frac{q_{b1}}{\tilde{\phi}_{b1}\theta}) = \frac{c}{p+\frac{k_s}{\theta}}$ and $\bar{F}(\frac{q_{b2}}{\tilde{\phi}_{b2}\theta}) = \frac{c}{p+\frac{k_b - r_s(1-\theta)-c_e}{\theta}}$, respectively. Then we have the results summarized in Proposition 3.1. □

Proof of Proposition 3.2. We prove Proposition 3.2 one by one.

(i) Based on Proposition 3.1, we know that if $t_b > \min\{t_s, \theta(v-p)\}$, then the BOPS strategy functions as the ROPO strategy. In this case, $\Pi_b = \Pi_{b1} = \Pi_r$. If $t_s < A_o(q)\theta(v-p)$, then both Π_o and Π_r are nonnegative. In this case,

$$\begin{aligned}
\Pi_o &= p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + k_s\mu \\
&= p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + k_s(E\min\{D, \frac{q_o}{\theta}\} + E(D - \frac{q_o}{\theta})^+) \\
&\geq p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s(E\min\{D, \frac{q_r}{\theta}\} + E(D - \frac{q_r}{\theta})^+) \\
&> p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_sE\min\{D, \frac{q_r}{\theta}\} \\
&= \Pi_r,
\end{aligned}$$

where the first inequality comes from the fact that q_o is the optimal solution of the newsvendor problem in the benchmark and the second inequality results from the fact that $E(D - \frac{q_r}{\theta})^+ > 0$.

If $A_o(q)\theta(v-p) < t_s < \theta(v-p)$, then $\Pi_o = 0$ and $\Pi_r > 0$. And, if $t_s > \theta(v-p)$, then $\Pi_o = \Pi_r = 0$. Then we have Proposition 3.2 (i).

(ii) If $t_b < \min\{t_s, \theta(v-p)\}$, then, from the perspective of consumers, using BOPS is better than visiting the store or leaving the market. In this case, $\Pi_b = \Pi_{b2}$. If $t_s < A_o(q)\theta(v-p)$, both Π_o, Π_r and Π_b are nonnegative. Based on the first part of this proof, we know that, $\Pi_o > \Pi_r$. Besides,

$$\begin{aligned}
\Pi_r &= p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_sE\min\{D, \frac{q_r}{\theta}\} \\
&\geq p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + k_sE\min\{D, \frac{q_{b2}}{\theta}\} \\
&> p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\} \\
&= \Pi_{b2},
\end{aligned}$$

where the first inequality follows the logic that q_r is the optimal order quantity in this newsvendor problem which aims to maximize Π_r, and the second inequality results from the fact that $k_s > k_b - r_s(1-\theta) - c_e$. Therefore we know that $\Pi_o > \Pi_r > \Pi_{b2}$ if $t_s < A_o(q)\theta(v-p)$.

By contrast, if $A_o(q)\theta(v-p) < t_s < \theta(v-p)$, then $\Pi_o = 0$ and $\Pi_r > 0$. Based on the analyses above, it is obvious to find that $\Pi_r > \Pi_{b2} > \Pi_o = 0$. Lastly, if $t_s > \theta(v-p)$, then $\Pi_{b2} > \Pi_o = \Pi_r = 0$. Then Proposition 3.2 (ii) is proved. □

Proof of Proposition 3.3. The proof of Proposition 3.3 follows the same logic as that of Proposition 3.1. In the benchmark, according to the optimal condition, a consumer visits the store if $u_s > \max\{u_o, 0\}$. Then we need $t_s < \tilde{\xi}_o(t_o+(1-\theta)t_r)$. The retailer's optimal condition results in $\bar{F}(\frac{q_o}{\tilde{\phi}_o\theta}) = \frac{c}{p-\frac{R_o}{\theta}}$. With the consistency conditions, we get Proposition 3.3 (i).

Under the BOPS strategy, $t_s < \min\{t_b, t_o+(1-\theta)t_r\}$ ensures that visiting the store is the optimal choice, whereas $t_b < \min\{t_s, t_o+(1-\theta)t_r\}$ ensures that using the BOPS function is the best option. The retailer's optimal principles regarding visiting the store and using the BOPS function result in $\bar{F}(\frac{q_{b1}}{\tilde{\phi}_{b1}\theta}) = \frac{c}{p+\frac{k_s}{\theta}-\frac{R_o}{\theta}}$ and $\bar{F}(\frac{q_{b2}}{\tilde{\phi}_{b2}\theta}) = \frac{c}{p+\frac{k_b-r_s(1-\theta)-c_e}{\theta}-\frac{R_o}{\theta}}$, respectively. Then Proposition 3.3 is obtained. □

Before providing the Proof of Proposition 3.4, we first show Lemmas B.1 and B.2, which will be useful in the following proofs. Define $L(x) = E(D-x)^+$, $E\min\{D,x\} = \mu - L(x)$, and $G(x) = x\mu - c\bar{F}^{-1}(\frac{c}{x}) - xL(\bar{F}^{-1}(\frac{c}{x}))$. Then we have the following results:

Lemma B.1. *(a) $G(x)$ is increasing in x; (b) $G(c) = 0$, and $G(x)$ will tend to $+\infty$ when x tends to $+\infty$.*

Proof of Lemma B.1. Part (a) holds because $G'(x) = \mu - L(\bar{F}^{-1}(\frac{c}{x})) \geq 0$ and (b) is obvious when we substitute x with the unit cost of the inventory c. □

Lemma B.2. *Given q_r and q_{b2} defined in Proposition 3.3, if $q_r > 0$ and $q_{b2} > 0$,*

(a) there exists a threshold $\hat{R}_o$ such that if $R_o < \hat{R}_o$, then $p\theta E\min\{D,\frac{q_r}{\theta}\} - cq_r + k_sE\min\{D,\frac{q_r}{\theta}\} > R_o\mu$; otherwise, $p\theta E\min\{D,\frac{q_r}{\theta}\} - cq_r + k_sE\min\{D,\frac{q_r}{\theta}\} \leq R_o\mu$;

(b) there exists a threshold $\tilde{R}_o$ such that if $R_o < \tilde{R}_o$, then $p\theta E\min\{D,\frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D,\frac{q_{b2}}{\theta}\} + R_oE(D-\frac{q_{b2}}{\theta})^+ < p\theta E\min\{D,\frac{q_r}{\theta}\} -$

$cq_r + k_s E\min\{D, \frac{q_r}{\theta}\}$; *otherwise*, $p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\} + R_o E(D - \frac{q_{b2}}{\theta})^+ \geq p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\}$.

Proof of Lemma B.2. Let $X = p + \frac{k_s}{\theta}$ and $Y = p + \frac{k_b-(1-\theta)r_s-c_e-R_o}{\theta}$, then $p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\} = \theta G(X)$ and $p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\} + R_o E(D - \frac{q_{b2}}{\theta})^+ = \theta G(Y) + R_o\mu$. Therefore, $p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\} - R_o\mu = \theta G(X) - R_o\mu$ and $p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\} + R_o E(D - \frac{q_{b2}}{\theta})^+ - (p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\}) = \theta G(Y) + R_o\mu - \theta G(X)$. Since $X > Y > c$, we have $G(X) > G(Y) > 0$. If $R_o = 0$, then $\theta G(X) > \theta G(Y) > R_o\mu = 0$. Let $\hat{R}_o = \theta G(X)/\mu$ and $\tilde{R}_o = \theta(G(X) - G(Y))/\mu$. Then the proof is completed. □

Proof of Proposition 3.4. Following the same logic in the Proof of Proposition 3.2, we prove Proposition 3.4 step by step.

(i) Based on the Proposition 3.3, we know that if $t_b > \min\{t_s, t_o + (1-\theta)t_r\}$, then visiting the store directly or purchasing online under the BOPS strategy is a better choice than using the BOPS function. In this case, $\Pi_b = \Pi_{b1}$. If $t_s < A_o'(q)(t_o + (1-\theta)t_r)$, then $\Pi_o = p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + k_s\mu + R_o E(D - \frac{q_o}{\theta})^+$, $\Pi_r = p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\}$, and $\Pi_b = \Pi_{b1} = p\theta E\min\{D, \frac{q_{b1}}{\theta}\} - cq_{b1} + k_s E\min\{D, \frac{q_{b1}}{\theta}\} + R_o E(D - \frac{q_{b1}}{\theta})^+$. Then we have

$$\begin{aligned}
\Pi_o &= p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + k_s\mu + R_o E(D - \frac{q_o}{\theta})^+ \\
&\geq p\theta E\min\{D, \frac{q_{b1}}{\theta}\} - cq_{b1} + k_s\mu + R_o E(D - \frac{q_{b1}}{\theta})^+ \\
&> p\theta E\min\{D, \frac{q_{b1}}{\theta}\} - cq_{b1} + k_s E\min\{D, \frac{q_{b1}}{\theta}\} + R_o E(D - \frac{q_{b1}}{\theta})^+ \\
&= \Pi_{b1} \\
&\geq p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\} + R_o E(D - \frac{q_r}{\theta})^+ \\
&> p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\} \\
&= \Pi_r,
\end{aligned}$$

where the first inequality results from the fact that q_o is the optimal solution in this newsvendor problem which aims to maximize Π_o, the second inequality results from the fact that $\mu > E\min\{D, \frac{q_{b1}}{\theta}\}$, the third one follows the same logic as the first one, and the last one comes from the fact that $R_o E(D - \frac{q_r}{\theta})^+ > 0$.

If $A_o'(q)(t_o+(1-\theta)t_r) < t_s < t_o+(1-\theta)t_r$, then $\Pi_o = R_o\mu$, $\Pi_r = p\theta E\min\{D,\frac{q_r}{\theta}\} - cq_r + k_sE\min\{D,\frac{q_r}{\theta}\}$, and $\Pi_b = \Pi_{b1} = p\theta E\min\{D,\frac{q_{b1}}{\theta}\} - cq_{b1} + k_sE\min\{D,\frac{q_{b1}}{\theta}\} + R_oE(D-\frac{q_{b1}}{\theta})^+$. Then we have

$$\begin{aligned}\Pi_{b1} &= p\theta E\min\{D,\frac{q_{b1}}{\theta}\} - cq_{b1} + k_sE\min\{D,\frac{q_{b1}}{\theta}\} + R_oE(D-\frac{q_{b1}}{\theta})^+\\ &= (p\theta + k_s - w\theta + r_o(1-\theta))E\min\{D,\frac{q_{b1}}{\theta}\} - cq_{b1} + R_o\mu\\ &> R_o\mu,\end{aligned}$$

where the inequality results from the fact that $(p\theta + k_s - w\theta + r_o(1-\theta))E\min\{D,\frac{q_{b1}}{\theta}\} - cq_{b1} > 0$. Based on the analysis in the case that $t_s < A_o'(q)(t_o+(1-\theta)t_r)$, it is obvious that $\Pi_b > \max\{\Pi_o,\Pi_r\}$.

If $t_o+(1-\theta)t_r < t_s < \theta(v-p)$, then $\Pi_o = \Pi_b = R_o\mu$, $\Pi_r = p\theta E\min\{D,\frac{q_r}{\theta}\} - cq_r + k_sE\min\{D,\frac{q_r}{\theta}\}$. Based on Lemma B.2, we have the results shown in the third case of Proposition 3.4 (i).

Lastly, if $t_s > \theta(v-p)$, then $\Pi_r = 0 < \Pi_o = \Pi_b = R_o\mu$.

(ii) If $t_b < \min\{t_s, t_o+(1-\theta)t_r\}$, then visiting store directly or purchasing online under the BOPS strategy is worse than using the BOPS function. In this case $\Pi_b = \Pi_{b2}$. If $t_s < A_o'(q)(t_o+(1-\theta)t_r)$, then $\Pi_b = \Pi_{b2} = p\theta E\min\{D,\frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D,\frac{q_{b2}}{\theta}\} + R_oE(D-\frac{q_{b2}}{\theta})^+$. Besides,

$$\begin{aligned}\Pi_o &= p\theta E\min\{D,\frac{q_o}{\theta}\} - cq_o + k_s\mu + R_oE(D-\frac{q_o}{\theta})^+\\ &\geq p\theta E\min\{D,\frac{q_{b2}}{\theta}\} - cq_{b2} + k_s\mu + R_oE(D-\frac{q_{b2}}{\theta})^+\\ &> p\theta E\min\{D,\frac{q_{b2}}{\theta}\} - cq_{b2} + k_sE\min\{D,\frac{q_{b2}}{\theta}\} + R_oE(D-\frac{q_{b2}}{\theta})^+\\ &> \Pi_{b2},\end{aligned}$$

where the last inequality results from the fact that $k_s \geq k_b$. Then we have the first case in Proposition 3.4 (ii).

If $A_o'(q)(t_o+(1-\theta)t_r) < t_s < \theta(v-p)$, then based on Lemma B.2, we have the results shown in the second case of Proposition 3.4 (ii).

The last case in Proposition 3.4 (ii) is obvious. Then we complete the proof. □

B.2 Detailed Analyses and Proofs for the Extensions

B.2.1 General k_b

Proof of Proposition 3.5. We prove the differences between Propositions 3.5 and 3.2. First, if $t_b < \min\{\theta(v-p), t_s\}$ and $t_s < A_o(q)\theta(v-p)$, then

$$\Pi_o = p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + k_s\mu,$$

$$\Pi_r = p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\}, \text{ and}$$

$$\Pi_{b2} = p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\}.$$

If $k_b - r_s(1-\theta) - c_e > \frac{k_s}{A_o(q)}$, then $(k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_o}{\theta}\} > \frac{k_s}{A_o(q)}E\min\{D, \frac{q_o}{\theta}\} = k_s\mu$. Therefore,

$$\begin{aligned}\Pi_{b2} &> p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_o}{\theta}\}\\ &> p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + \frac{k_s}{A_o(q)}E\min\{D, \frac{q_o}{\theta}\}\\ &= p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + k_s\mu\\ &= \Pi_o.\end{aligned}$$

Similarly, it can be shown that if $k_b - r_s(1-\theta) - c_e \le \frac{k_s}{A_o(q)}$, then $\Pi_{b2} \le \Pi_o$. However, if $k_b - r_s(1-\theta) - c_e \le \frac{k_s}{A_o(q)}$, the relationship between Π_{b2} and Π_r is unclear. Following the same logic as above, it can be shown that, if $k_s < k_b - r_s(1-\theta) - c_e \le \frac{k_s}{A_o(q)}$, then $\Pi_{b2} > \Pi_r$; otherwise, $\Pi_{b2} \le \Pi_r$.

Second, the result in the case where $t_b < \min\{\theta(v-p), t_s\}$ and $A_o(q)\theta(v-p), t_s < \theta(v-p)$ is obvious based on the above analyses. □

Proof of Proposition 3.6. We prove the differences between Propositions 3.6 and 3.4. If $t_b < \min\{t_s, t_o + (1-\theta)t_r\}$ and $A'_o(q)(t_o + (1-\theta)t_r) < t_s < \theta(v-p)$, then

$$\Pi_o = p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + k_s\mu + R_oE(D - \frac{q_o}{\theta})^+,$$

$$\Pi_r = p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s E\min\{D, \frac{q_r}{\theta}\}, \text{ and}$$

$$\Pi_b = p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\} + R_oE(D - \frac{q_{b2}}{\theta})^+.$$

If $k_b - r_s(1-\theta) - c_e > \frac{k_s}{A_o'(q)}$, then $(k_b - r_s(1-\theta) - c_e) E\min\{D, \frac{q_o}{\theta}\} > \frac{k_s}{A_o'(q)} E\min\{D, \frac{q_o}{\theta}\} = k_s\mu$, which means,

$$\begin{aligned}
\Pi_{b2} &\geq p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_o}{\theta}\} + R_oE(D - \frac{q_o}{\theta})^+ \\
&> p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + \frac{k_s}{A_o'(q)}E\min\{D, \frac{q_o}{\theta}\} + R_oE(D - \frac{q_o}{\theta})^+ \\
&= p\theta E\min\{D, \frac{q_o}{\theta}\} - cq_o + k_s\mu + R_oE(D - \frac{q_o}{\theta})^+ \\
&= \Pi_o.
\end{aligned}$$

By contrast, if $k_b - r_s(1-\theta) - c_e \leq \frac{k_s}{A_o'(q)}$, then $\Pi_{b2} \leq \Pi_o$, which means $\Pi_o \geq \max\{\Pi_{b2}, \Pi_r\}$. Here, we have proved the first case in Proposition 3.6 (ii).

Next we prove the second case in part (ii). Recalling that, in the Lemma B.2 and Proof of Proposition 3.4, if $R_o < \tilde{R}_o$, then $\theta G(Y) + R_o\mu < \theta G(X)$, i.e., $\Pi_{b2} < \Pi_r$, for given $k_b \leq k_s$. Since $G(Y)$ is increasing in k_b, $\theta G(Y) + R_o\mu$ could be larger than $\theta G(X)$ if the assumption $k_b \leq k_s$ is released as shown in Proposition 3.6. Let $k_b^{(1)}$ be the solution of $\theta G(Y) + R_o\mu = \theta G(X)$ when $R_o < \tilde{R}_o$, then we complete this proof. □

B.2.2 Imperfect Availability Information

No online retailing. If there is no online retailing, the game between the retailer and the customers in the benchmark keeps unchanged since inventory availability information is unaccessible in the benchmark. Under the ROPO strategy, as mentioned above, consumers make their visit-or-not decisions based on $u_s = -t_s + \tilde{\eta}_r\theta(v-p)$. The firm's profit function is:

$$\Pi_r(q) = p\theta E\min\{\tilde{\phi}_r D, \frac{q}{\theta}\} - cq + k_s(E\min\{\tilde{\phi}_r D, \frac{q}{\theta}\} + \alpha E(\tilde{\phi}_r D - \frac{q}{\theta})^+).$$

By contrast, if the firm implements the BOPS strategy, the expected utility of visiting the store directly is $u_s = -t_s + \tilde{\eta}_{b1}\theta(v-p)$, while that of using BOPS is $u_b = -t_b + \theta(v-p)$. If $u_s > u_b$, BOPS strategy functions as ROPO strategy, and $\Pi_{b1}(q) = \Pi_r(q)$. If $u_b > u_s$, the retailer's profit is the same as Equation (3.3) since customers can successfully book the product by using the BOPS function.

Then we can derive the participatory RE equilibria of the above three strategies, which are shown in the following proposition:

Proposition B.1. *Given θ, the participatory RE equilibria in the base model with imperfect availability information are as follows:*

(i) In the benchmark, if $t_s < A_o(q)\theta(v-p)$, then $q_o = \theta\bar{F}^{-1}(\frac{c}{p}) > 0$; otherwise, $q_o = 0$. Here, $A_o(q) = E\min\{D, \bar{F}^{-1}(\frac{c}{p})\}/\mu$.

(ii) Under the ROPO strategy, if $t_s < A_r(q)\theta(v-p)$, then $q_r = \theta\bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}}) > 0$; otherwise, $q_r = 0$.

(iii) Under the BOPS strategy, if $t_s < \min\{A_r(q)\theta(v-p), t_b-(1-A_r(q))\theta(v-p)\}$, then $q_b = q_{b1} = q_r$; if $t_b < \min\{\theta(v-p), t_s + (1-A_r(q))\theta(v-p)\}$, then $q_b = q_{b2} = \theta\bar{F}^{-1}(\frac{c}{p+\frac{k_b-r_s(1-\theta)-c_e}{\theta}}) > 0$; otherwise, $q_b = 0$.

where $A_r(q) = \frac{E\min\{D,\bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}})\}}{(E\min\{D,\bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}})\}+\alpha E(D-\bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}}))^+)}$.

Necessary proofs are presented in the second part of this appendix. Figure B.1 depicts the consumer equilibrium behavior under the three strategies when there is no online retailing.

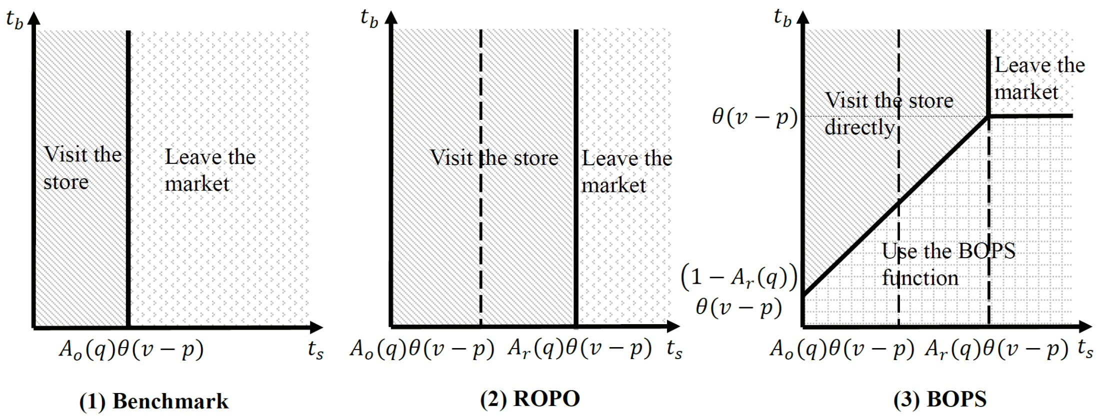

Figure B.1: Consumer Equilibrium Behavior in the Base Model with Imperfect Availability Information

Next, we summarize the comparison of above three strategies as follows.

Proposition B.2. *Comparing the three strategies in the base model with imperfect availability information, we have*

(i) If $t_b > \min\{t_s + (1-A_r(q))\theta(v-p), \theta(v-p)\}$, then $\Pi_b = \Pi_{b1} = \Pi_r$.

- *If* $t_s < A_o(q)\theta(v-p)$, $\Pi_o > \Pi_r$;
- *If* $A_o(q)\theta(v-p) < t_s < A_r(q)\theta(v-p)$, $\Pi_r > \Pi_o = 0$;
- *If* $t_s > A_r(q)\theta(v-p)$, $\Pi_r = \Pi_o = 0$.

(ii) If $t_b < \min\{t_s + (1-A_r(q))\theta(v-p), \theta(v-p)\}$, *then* $\Pi_b = \Pi_{b2}$.

- *If* $t_s < A_o(q)\theta(v-p)$, *then* $\Pi_o > \Pi_r > \Pi_{b2}$;
- *If* $A_o(q)\theta(v-p) < t_s < A_r(q)\theta(v-p)$, $\Pi_r > \Pi_b > \Pi_o = 0$;
- *If* $t_s > A_r(q)\theta(v-p)$, $\Pi_b > \Pi_r = \Pi_o = 0$.

Figure B.2 pictorially shows the conditions for the strategies being optimal, where the profit is labeled in the area if the corresponding strategy is optimal. Therefore, our main results keep unchanged when the availability information is imperfect.

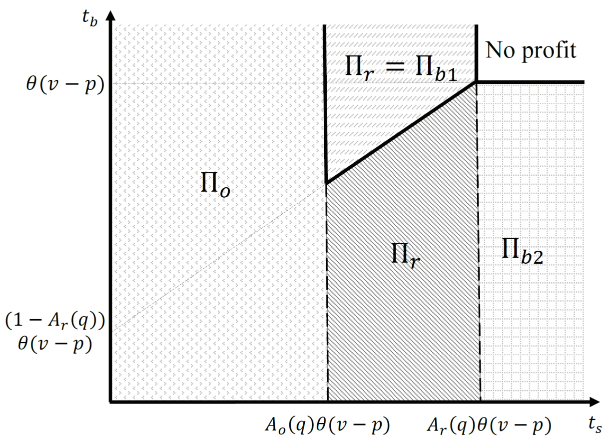

Figure B.2: Optimal Profits in the Base Model with Imperfect Availability Information

Analysis with online and offline sales. Now we extend our analysis to the multi-channel model. The expected utility of consumers purchasing online keeps unchanged and is still $u_o = -t_o + \theta(v-p) + (1-\theta)(-t_r)$. Since imperfect availability information has no effects on the benchmark and the online retailing is unavailable under the ROPO strategy, the analyses of these two strategies can be found in Section 3.4 and Section B.1.1, respectively.

Next we consider the BOPS strategy. Each consumer has three choices under this strategy: purchasing online (u_o), visiting the store directly ($u_s = -t_s + \tilde{\eta}_{b1}\theta(v-p) + (1-\tilde{\eta}_{b1})u_o$), or using the BOPS function ($u_b = -t_b + \theta(v-p)$). Consumer's choice depends on the expected utility of each option. If $u_s > \max\{u_b, u_o\}$, then visiting store directly is a better option than using the BOPS strategy. In such case, the retailer's profit is shown as follows:

$$\begin{aligned}\Pi_{b1}(q) =& p\theta E\min\{\tilde{\phi}_{b1}D, \frac{q}{\theta}\} - cq + k_s(E\min\{\tilde{\phi}_{b1}D, \frac{q}{\theta}\} + \alpha E(\tilde{\phi}_{b1}D - \frac{q}{\theta})^+) \\ &+ R_oE(\tilde{\phi}_{b1}D - \frac{q}{\theta})^+ + R_oE(1-\tilde{\phi}_{b1})D.\end{aligned}$$

On the contrary, if $u_b > \max\{u_s, u_o\}$, consumers prefer to use the BOPS function rather than visit the store directly. In this case, the retailer's profit is the same as that in Equation (3.6). Then, we can derive the participatory RE equilibria, which are summarized as follows.

Proposition B.3. *Given θ, the participatory RE equilibria in the multichannel model with imperfect availability information are as follows:*

(i) In the benchmark, if $t_s < A'_o(q)(t_o + (1-\theta)t_r)$, then $q_o = \theta\bar{F}^{-1}(\frac{c}{p-\frac{R_o}{\theta}}) > 0$; otherwise, $q_o = 0$. Here, $A'_o(q) = E\min\{D, \bar{F}^{-1}(\frac{c}{p-\frac{R_o}{\theta}})\}/\mu$.

(ii) Under the ROPO strategy, if $t_s < A_r(q)\theta(v-p)$, then $q_r = \theta\bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}}) > 0$; otherwise, $q_r = 0$.

(ii) Under the BOPS strategy, if $t_s < \min\{A'_{b1}(q)(t_o + (1-\theta)t_r), t_b - (1 - A'_{b1}(q))(t_o + (1-\theta)t_r)\}$, then $q_b = q_{b1} = \theta\bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}-\frac{R_o}{\theta}}) > 0$; if $t_b < \min\{t_o + (1-\theta)t_r, t_s + (1-A'_{b1}(q))(t_o + (1-\theta)t_r)\}$, then $q_b = q_{b2} = \theta\bar{F}^{-1}(\frac{c}{p+\frac{k_b - r_s(1-\theta) - c_e}{\theta} - \frac{R_o}{\theta}}) > 0$; otherwise, $q_b = 0$. Here, $A'_{b1}(q) = E\min\{D, \frac{q_{b1}}{\theta}\}/(E\min\{D, \frac{q_{b1}}{\theta}\} + \alpha E(D - \frac{q_{b1}}{\theta})^+)$.

where $A_r(q) = \frac{E\min\{D, \bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}})\}}{(E\min\{D, \bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}})\} + \alpha E(D - \bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}}))^+)}$.

Figure B.3 depicts the consumer equilibrium behavior under the three strategies in the multi-channel model with imperfect availability information.

The following proposition shows the comparison of the above three strategies.

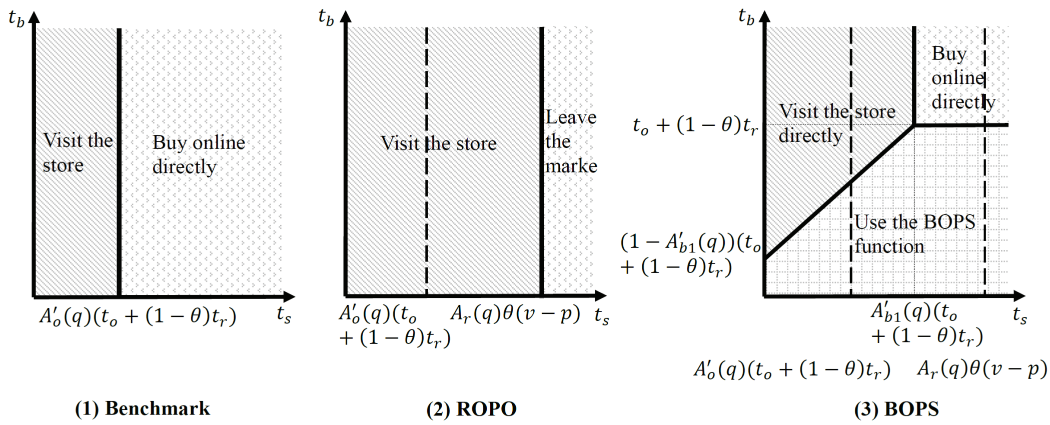

Figure B.3: Consumer Equilibrium Behavior in the Multi-channel Model with Imperfect Availability Information

Proposition B.4. *Comparing the three strategies in the multi-channel model with imperfect availability information, we have*

(i) If $t_b > \min\{t_o + (1-\theta)t_r, t_s + (1 - A'_{b1}(q))(t_o + (1-\theta)t_r)\}$, *then* $\Pi_b = \Pi_{b1}$.

- *If* $t_s < A'_o(q)(t_o + (1-\theta)t_r)$, $\Pi_o > \Pi_b > \Pi_r$.
- *If* $A'_o(q)\theta(t_o+(1-\theta)t_r) < t_s < A'_{b1}(q)(t_o+(1-\theta)t_r)$, $\Pi_b > \max\{\Pi_o, \Pi_r\}$.
- *If* $A'_{b1}(q)(t_o + (1-\theta)t_r) < t_s < A_r(q)\theta(v-p)$*: then there exists a threshold* $\hat{R_o}'$ *such that if* $R_o < \hat{R_o}'$, *then* $\Pi_r > \Pi_o = \Pi_b$*; otherwise,* $\Pi_o = \Pi_b \geq \Pi_r$.
- *If* $t_s > A_r(q)\theta(v-p)$, $\Pi_o = \Pi_b > \Pi_r = 0$.

(ii) If $t_b < \min\{t_o + (1-\theta)t_r, t_s + (1 - A'_{b1}(q))(t_o + (1-\theta)t_r)\}$ *then,* $\Pi_b = \Pi_{b2}$.

- *If* $t_s < A'_o(q)(t_o + (1-\theta)t_r)$, *then* $\Pi_o > \max\{\Pi_b, \Pi_r\}$.
- *If* $A'_o(q)(t_o+(1-\theta)t_r) < t_s < A_r(q)\theta(v-p)$*: then there exits a threshold* $\tilde{R_o}'$ *such that if* $R_o < \tilde{R_o}'$, *then* $\Pi_r > \Pi_b > \Pi_o$*; if* $\tilde{R_o}' \geq R_o < \hat{R_o}'$, *then* $\Pi_b \geq \Pi_r > \Pi_o$, *if* $R_o \geq \hat{R_o}'$, *then* $\Pi_b > \Pi_o \geq \Pi_r$.
- *If* $t_s > A_r(q)\theta(v-p)$, $\Pi_b > \Pi_o > \Pi_r = 0$.

Figure B.4 pictorially shows the conditions for the strategies being optimal, where the profit is labeled in the area if the corresponding strategy is optimal.

Therefore, our main results keep unchanged when the availability information is imperfect.

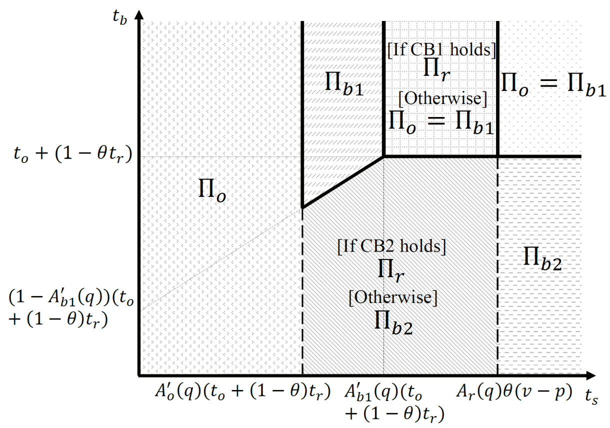

Figure B.4: Optimal Profits in the Multi-channel Model with Imperfect Availability Information

Note: in Figure B.4, conditions CB1 and CB2 mean $R_o < \hat{R_o}'$ and $R_o < \tilde{R_o}'$, respectively.

B.2.3 Proofs of Extensions

Proof of Proposition B.2. It is easy to show that $A_o(q) < A_r(q)$ since $\bar{F}^{-1}(\frac{c}{p}) < \bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}})$ and $\mu > E\min\{D, \bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}})\} + \alpha E(D - \bar{F}^{-1}(\frac{c}{p+(1-\alpha)\frac{k_s}{\theta}}))^+$.

Then we prove the first two cases of Proposition B.2 (ii). Note that the proofs of other cases follow the same logic as that in the Proof of Proposition 3.2).

$$\begin{aligned}
\Pi_r &= p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s(E\min\{D, \frac{q_r}{\theta}\} + \alpha E(D - \frac{q_r}{\theta})^+) \\
&\geq p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + k_s(E\min\{D, \frac{q_{b2}}{\theta}\} + \alpha E(D - \frac{q_{b2}}{\theta})^+) \\
&\geq p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + k_s E\min\{D, \frac{q_{b2}}{\theta}\} \\
&> E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\} \\
&= \Pi_{b2},
\end{aligned}$$

where the first two inequalities result from the fact that q_r is the optimal order quantity in the newsvendor which aims to maximize Π_r and $\alpha \geq 0$, and the third inequality results from the fact that $k_s > k_b - r_s(1-\theta) - c_e$. □

Similar to the Proof of Proposition 3.4, before providing the Proof of Proposition B.4, we first show the following lemma:

Lemma B.3. *Given q_r and q_{b2} defined in Proposition B.3, if $q_r > 0$ and $q_{b2} > 0$, then:*

(a) there exists a threshold $\hat{R}_o'$ such that if $R_o < \hat{R}_o'$, then $p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s(E\min\{D, \frac{q_r}{\theta}\} + \alpha E(D - \frac{q_r}{\theta})^+) > R_o\mu$; otherwise, $p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s(E\min\{D, \frac{q_r}{\theta}\} + \alpha E(D - \frac{q_r}{\theta})^+) \leq R_o\mu$.

(b) there exists a threshold $\tilde{R}_o'$ such that if $R_o < \tilde{R}_o'$, then $p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\} + R_oE(D - \frac{q_{b2}}{\theta})^+ < p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s(E\min\{D, \frac{q_r}{\theta}\} + \alpha E(D - \frac{q_r}{\theta})^+)$; otherwise, $p\theta E\min\{D, \frac{q_{b2}}{\theta}\} - cq_{b2} + (k_b - r_s(1-\theta) - c_e)E\min\{D, \frac{q_{b2}}{\theta}\} + R_oE(D - \frac{q_{b2}}{\theta})^+ \geq p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s(E\min\{D, \frac{q_r}{\theta}\} + \alpha E(D - \frac{q_r}{\theta})^+)$. Here q_r and q_{b2} are given in Proposition B.3.

Proof of Lemma B.3. Let $Z = p + \frac{(1-\alpha)k_s}{\theta}$. Note that, based on Proposition B.3, $p\theta E\min\{D, \frac{q_r}{\theta}\} - cq_r + k_s(E\min\{D, \frac{q_r}{\theta}\} + \alpha E(D - \frac{q_r}{\theta})^+) = \theta G(Z) + \alpha k_s\mu$. (a) If $R_o = 0$, $\theta G(Z) + \alpha k_s\mu > R_o\mu$. Thus, there exists a threshold $\hat{R}_o' = \frac{\theta G(Z)+\alpha k_s\mu}{\mu}$ such that if $R_o < \hat{R}_o'$, then $\theta G(Z) + \alpha k_s\mu > R_o\mu$; (b) If $R_o = 0$ and $\alpha = 0$, then $Z > Y$ and $\theta G(Z) + \alpha k_s\mu > \theta G(Y) + R_o\mu$; by comparison, if $R_o = k_s$ and $\alpha = 1$, then $Z < Y$ and $\theta G(Z) + \alpha k_s\mu < \theta G(Y) + R_o\mu$. Therefore, there exists a threshold $\tilde{R}_o' = \frac{\theta(G(Z)-G(Y))+\alpha k_s\mu}{\mu}$ such that if $R_o < \tilde{R}_o'$, then $\theta G(Z) + \alpha k_s\mu > \theta G(Y) + R_o\mu$. Then we complete the proof of Lemma B.3. □

Proof of Proposition B.4. Here we only show that $A_o'(q)\theta(t_o + (1-\theta)t_r) < A_{b1}'(q)(t_o + (1-\theta)t_r) < A_r(q)\theta(v - p)$. The first inequality is obvious since $q_o < q_{b1}$ and $\mu > E\min\{D, \frac{q_{b1}}{\theta}\} + \alpha E(D - \frac{q_{b1}}{\theta})^+$. Besides, $E\min\{D, \frac{q_{b1}}{\theta}\} + \alpha E(D - \frac{q_{b1}}{\theta})^+ = \alpha\mu + (1-\alpha)E\min\{D, \frac{q_{b1}}{\theta}\}$. Since $q_{b1} < q_r$ and $(t_o + (1-\theta)t_r) < \theta(v - p)$ (i.e., $u_o > 0$), then we have the second inequality. □

Appendix C

Proofs for Chapter 4

In Chapter 4 and Proofs for Chapter 4, we assume that $0 < c_m \leq c$, $0 < k_m \leq k$, $t > \underline{t}$, and $v > \underline{v}$, where $\underline{t}$ and $\underline{v}$ are defined as follows: $\underline{t} = \max\{\frac{12+30(c-c_m)k_m}{25k_m}, \frac{5k-k_m+9(c-c_m)kk_m}{9kk_m}\}$ and

$$\underline{v} = \max\{\frac{-8+30ck+15kt}{30k},$$
$$\frac{4-6ck-12c_mk_m-9kt-16k_mt+27ckk_mt+27c_mkk_mt+36kk_mt^2}{-6k-12k_m+54kk_mt},$$
$$\frac{4-6ck-12c_mk_m-11kt-14k_mt+18ckk_mt+36c_mkk_mt+39kk_mt^2}{-6k-12k_m+54kk_mt},$$
$$\frac{8-12ck-12c_mk_m-19kt-19k_mt+27ckk_mt+27c_mkk_mt+45kk_mt^2}{-12k-12k_m+54kk_mt},$$
$$\frac{96-120ck-240c_mk_m-180kt-200k_mt+300ckk_mt+450c_mkk_mt+375kk_mt^2}{-120k-240k_m+750kk_mt}\}.$$

Proof of Lemma 4.1. Given that firms A and B decide not to merge, all firms in the market make quality and price decisions to maximize their own profits. We solve the subgame equilibrium for the pre-merger case by backward induction. In stage 2, given the quality decisions of all products q_A, q_B, and q_C, the profit of firm i is shown in Equation (4.3), i.e., $\pi_i = (p_i - c)D_i - \frac{1}{2}kq_i^2$, where D_i is given by Equation (4.2), i.e., $D_i = \min\{1, \max\{0, \frac{(q_i-p_i)-(q_{i'}-p_{i'})}{2t} + \frac{1}{6}\} + \max\{0, \frac{(q_i-p_i)-(q_{i''}-p_{i''})}{2t} + \frac{1}{6}\}\}$ for $i, i', i'' \in \{A, B, C\}$, $i \neq i'$, $i \neq i''$, and $i' \neq i''$. Given the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$, that is π_i is concave in p_i. By solving the first-order conditions (i.e., $\frac{\partial \pi_A}{\partial p_A} = 0$, $\frac{\partial \pi_B}{\partial p_B} = 0$, and $\frac{\partial \pi_C}{\partial p_C} = 0$) simultaneously, we can get the equilibrium prices in stage 2 as follows: $p_A^*(q_A, q_B, q_C) = \frac{15c+6q_A-3q_B-3q_C+5t}{15}$,

$p_B^*(q_A, q_B, q_C) = \frac{15c-3q_A+6q_B-3q_C+5t}{15}$, and $p_C^*(q_A, q_B, q_C) = \frac{15c-3q_A-3q_B+6q_C+5t}{15}$.

Let $\pi_i^* = \pi_i|_{p_A=p_A^*(q_A,q_B,q_C),p_B=p_B^*(q_A,q_B,q_C),p_C=p_C^*(q_A,q_B,q_C)}$ denote firm i's profit in stage 2 for given quality levels q_A, q_B, and q_C, where $i \in \{A, B, C\}$. In stage 1, anticipating the price decisions in stage 2, each firm i chooses quality level q_i to maximize its own profit π_i^*. Given the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 \pi_i^*}{\partial q_i^2} < 0$, that is π_i^* is concave in q_i. By solving the first-order conditions (i.e., $\frac{\partial \pi_A^*}{\partial q_A} = 0$, $\frac{\partial \pi_B^*}{\partial q_B} = 0$, and $\frac{\partial \pi_C^*}{\partial q_C} = 0$) simultaneously, we get the equilibrium quality in stage 1 as follows: $q_A^{PM} = q_B^{PM} = q_C^{PM} = \frac{4}{15k}$.

Given the equilibrium quality, we can show the equilibrium price, demand and profit as follows: $p_A^{PM} = p_B^{PM} = p_C^{PM} = c + \frac{t}{3}$, $D_A^{PM} = D_B^{PM} = D_C^{PM} = \frac{1}{3}$, and $\pi_A^{PM} = \pi_B^{PM} = \pi_C^{PM} = \frac{t}{9} - \frac{8}{225k}$. This is the subgame equilibrium for the pre-merger case shown in Lemma 4.1 and Table 4.2. □

Comprehensive Version of Corollary 4.1. Given that firms A and B decide not to merge, each firm's price p_i^{PM} increases in t and c; each firm's quality q_i^{PM} decreases in k; and each firm's profit π_i^{PM} increases in t and k, but is independent of c, where $i \in \{A, B, C\}$. □

Proof of Corollary 4.1. Taking derivative with respect to c, k, and t, one can easily verify that $\frac{\partial p_i^{PM}}{\partial t} = \frac{1}{3}$, $\frac{\partial p_i^{PM}}{\partial c} = 1$, $\frac{\partial q_i^{PM}}{\partial k} = -\frac{4}{15k^2}$, $\frac{\partial \pi_i^{PM}}{\partial t} = \frac{1}{9}$, $\frac{\partial \pi_i^{PM}}{\partial c} = 0$, and $\frac{\partial \pi_i^{PM}}{\partial k} = \frac{8}{225k^2}$, where $i \in \{A, B, C\}$. □

Proof of Lemma 4.2. Given that firms A and B adopt decentralized merger, all firms in the market make quality and price decisions to maximize their own profits as they do in the pre-merger case. We solve the subgame equilibrium for the DM case by backward induction.

In stage 2, given the quality levels of all firms q_A, q_B, and q_C, each firm i chooses price p_i to maximize its own profit π_i, where $i \in \{A, B, C\}$. π_A and π_B are given by Equation (4.4) and π_C is given by Equation (4.3), i.e., $\pi_A = (p_A - c_m)D_A - \frac{1}{2}k_m q_A^2$, $\pi_B = (p_B - c_m)D_B - \frac{1}{2}k_m q_B^2$, and $\pi_C = (p_C - c)D_C - \frac{1}{2}kq_C^2$, where D_i is given by Equation (4.2), i.e., $D_i = \min\{1, \max\{0, \frac{(q_i-p_i)-(q_{i'}-p_{i'})}{2t} + \frac{1}{6}\} + \max\{0, \frac{(q_i-p_i)-(q_{i''}-p_{i''})}{2t} + \frac{1}{6}\}\}$ for $i, i', i'' \in \{A, B, C\}$, $i \neq i'$, $i \neq i''$, and $i' \neq i''$. Given the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$, that is π_i is concave in p_i. By solving the first-order conditions (i.e., $\frac{\partial \pi_A}{\partial p_A} = 0$, $\frac{\partial \pi_B}{\partial p_B} = 0$, and $\frac{\partial \pi_C}{\partial p_C} = 0$) simultaneously, we have the subgame equilibrium prices in stage 2: $p_A^*(q_A, q_B, q_C) = \frac{3c+12c_m+6q_A-3q_B-3q_C+5t}{15}$, $p_B^*(q_A, q_B, q_C) = \frac{3c+12c_m-3q_A+6q_B-3q_C+5t}{15}$, and $p_C^*(q_A, q_B, q_C) = \frac{9c+6c_m-3q_A-3q_B+6q_C+5t}{15}$.

Let $\pi_i^* = \pi_i|_{p_A=p_A^*(q_A,q_B,q_C), p_B=p_B^*(q_A,q_B,q_C), p_C=p_C^*(q_A,q_B,q_C)}$ denote firm i's profit in stage 2 for given quality levels q_A, q_B, and q_C, where $i \in \{A, B, C\}$. In stage 1, in anticipation of the price decisions in stage 2, each firm i chooses quality level q_i to maximize its own profit π_i^*. Given the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 \pi_i^*}{\partial q_i^2} < 0$, that is π_i^* is concave in q_i. By solving the first-order conditions (i.e., $\frac{\partial \pi_A^*}{\partial q_A} = 0$, $\frac{\partial \pi_B^*}{\partial q_B} = 0$, and $\frac{\partial \pi_C^*}{\partial q_C} = 0$) simultaneously, we obtain the equilibrium quality in stage 1 as follows: $q_A^{DM} = q_B^{DM} = \frac{48-60ck+60c_m k-100kt}{60k+120k_m-375kk_m t}$, and $q_C^{DM} = -\frac{4(12+30ck_m-30c_m k_m-25k_m t)}{15(-8k_m+k(-4+25k_m t))}$.

Given the equilibrium quality, we can show the equilibrium prices, demands, and profits as follows:

$$
\begin{aligned}
p_A^{DM} = p_B^{DM} &= \frac{k_m t(-12+15ck+25kt)+12c_m(-2k_m+k(-1+5k_m t))}{3(-8k_m+k(-4+25k_m t))}, \\
p_C^{DM} &= \frac{kt(-12+30c_m k_m+25k_m t)+3c(-8k_m+k(-4+15k_m t))}{3(-8k_m+k(-4+25k_m t))}, \\
D_A^{DM} = D_B^{DM} &= \frac{k_m(-12+15ck-15c_m k+25kt)}{3(-8k_m+k(-4+25k_m t))}, \\
D_C^{DM} &= \frac{k(-12-30ck_m+30c_m k_m+25k_m t)}{3(-8k_m+k(-4+25k_m t))}, \\
\pi_A^{DM} = \pi_B^{DM} &= \frac{k_m(12-15ck+15c_m k-25kt)^2(-8+25k_m t)}{225(-8k_m+k(-4+25k_m t))^2}, \text{ and} \\
\pi_C^{DM} &= \frac{k(-8+25kt)(12+30ck_m-30c_m k_m-25k_m t)^2}{225(-8k_m+k(-4+25k_m t))^2}.
\end{aligned}
$$

Let $X_{AB}^{DM} = \frac{k_m(-12+15ck-15c_m k+25kt)}{-8k_m+k(-4+25k_m t)}$ and $X_C^{DM} = \frac{k(-12-30ck_m+30c_m k_m+25k_m t)}{-8k_m+k(-4+25k_m t)}$. We get the subgame equilibrium shown in Lemma 4.2 and Table 4.2. □

Comprehensive Version of Proposition 4.1. Given that firms A and B adopt decentralized merger, for each participant firm $i \in \{A, B\}$,

(i) p_i^{DM} increases in c and k and decreases in k_m, but may be non-monotone in t, and may increase or decrease in c_m:

- There exists a threshold $\ddot{t}$ such that, if $t < \ddot{t}$, then p_i^{DM} decreases in t; otherwise, p_i^{DM} increases in t;
- There exists a threshold $\acute{t}$ such that, if $t < \acute{t}$, then p_i^{DM} decreases in c_m; otherwise, p_i^{DM} increases in c_m;

(ii) q_i^{DM} increases in c and k, but decreases in t, c_m, and k_m.

(iii) π_i^{DM} increases in c and k and decreases in c_m, but may be non-monotone in t and k_m:

- There exists a threshold $\check{t}$ such that, if $t < \check{t}$, then π_i^{DM} decreases in t; otherwise, π_i^{DM} increases in t;
- There exists a threshold $\check{k}_m$ such that, if $k_m < \check{k}_m$, then π_i^{DM} increases in k_m; otherwise, π_i^{DM} decreases in k_m.

For the outside C,

(iv) p_C^{DM} increases in t, c_m, and k_m, and decreases in k, but may increase or decrease in c:

- There exists a threshold $\dddot{t}$ such that, if $t < \dddot{t}$, then p_C^{DM} decreases in c and vice versa;

(v) q_C^{DM} increases in t, c_m, and k_m, but decreases in c and k;

(vi) π_C^{DM} increases in t, c_m, and k_m, but decreases in c and k.

$\ddot{t}$, $\acute{t}$, $\check{t}$, $\check{k}_m$, and $\dddot{t}$ are defined in the Proof of Proposition 4.1. □

Proof of Comprehensive Version of Proposition 4.1. Taking derivative with respect to t, c, c_m, k, and k_m, through straightforward yet cumbersome algebraic calculation, one can verify that $\frac{\partial \pi_i^{DM}}{\partial c} > 0$, $\frac{\partial \pi_i^{DM}}{\partial k} > 0$, $\frac{\partial \pi_i^{DM}}{\partial c_m} < 0$, $\frac{\partial p_i^{DM}}{\partial c} > 0$, $\frac{\partial p_i^{DM}}{\partial k} > 0$, $\frac{\partial p_i^{DM}}{\partial k_m} < 0$, $\frac{\partial q_i^{DM}}{\partial c} > 0$, $\frac{\partial q_i^{DM}}{\partial k} > 0$, $\frac{\partial q_i^{DM}}{\partial t} < 0$, $\frac{\partial q_i^{DM}}{\partial c_m} < 0$, and

$\frac{\partial q_i^{DM}}{\partial k_m} < 0$, where $i \in \{A, B\}$. Besides, $\frac{\partial \pi_C^{DM}}{\partial t} > 0$, $\frac{\partial \pi_C^{DM}}{\partial c_m} > 0$, $\frac{\partial \pi_C^{DM}}{\partial k_m} > 0$, $\frac{\partial \pi_C^{DM}}{\partial c} < 0$, $\frac{\partial \pi_C^{DM}}{\partial k} < 0$, $\frac{\partial p_C^{DM}}{\partial t} > 0$, $\frac{\partial p_C^{DM}}{\partial c_m} > 0$, $\frac{\partial p_C^{DM}}{\partial k_m} > 0$, $\frac{\partial p_C^{DM}}{\partial k} < 0$, $\frac{\partial q_C^{DM}}{\partial t} > 0$, $\frac{\partial q_C^{DM}}{\partial c_m} > 0$, $\frac{\partial q_C^{DM}}{\partial k_m} > 0$, $\frac{\partial q_C^{DM}}{\partial c} < 0$, and $\frac{\partial q_C^{DM}}{\partial k} < 0$.

Next, we prove the remaining parts in the Comprehensive Version of Proposition 4.1.

(1) We show the effects of t on π_i^{DM}, where $i \in \{A, B\}$. Taking derivative with respect to t, we have $\frac{\partial \pi_i^{DM}}{\partial t} = -\frac{k_m(-12+15ck-15c_mk+25kt)W_1}{9(-8k_m+k(-4+25k_mt))^3}$, where $W_1 = -96k_m^2 + 4kk_m(4+30ck_m-30c_mk_m+75k_mt)+k^2(-64+300k_mt-625k_m^2t^2+15c_mk_m(12-25k_mt)+15ck_m(-12+25k_mt))$. It can be shown that $-\frac{k_m(-12+15ck-15c_mk+25kt)}{9(-8k_m+k(-4+25k_mt))^3} < 0$ under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$. Next we show that W_1 is either negative or changing from positive to negative as t increases from $\underline{t}$ to $+\infty$. Under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 W_1}{\partial t^2} < 0$ and there are following two possibilities:

a) When $W_2 < 0$, $W_1 < 0$ for any $t > \underline{t}$, where $W_2 = k^2k_m^2(-240k_m^2 + 8kk_m(44+105ck_m-105c_mk_m)+k^2(-112+360c_mk_m+225c^2k_m^2+225c_m^2k_m^2-90ck_m(4+5c_mk_m)))$. Define $\check{t} = \underline{t}$ when $W_2 < 0$.

b) When $W_2 \geq 0$, solving $W_1 = 0$ leads to two roots: $t = \frac{12kk_m^2+3k^2k_m(4+5ck_m-5c_mk_m)-\sqrt{W_2}}{50k^2k_m^2}$ and $t = \frac{12kk_m^2+3k^2k_m(4+5ck_m-5c_mk_m)+\sqrt{W_2}}{50k^2k_m^2}$. One can verify that the smaller root is always smaller than $\underline{t}$. Define $\check{t} = \max\{\underline{t}, \frac{12kk_m^2+3k^2k_m(4+5ck_m-5c_mk_m)+\sqrt{W_2}}{50k^2k_m^2}\}$ when $W_2 \geq 0$.

Thus, we have $W_1 > 0$ if $\underline{t} < t < \check{t}$ and $W_1 < 0$ if $t > \check{t}$. Therefore, $\frac{\partial \pi_i^{DM}}{\partial t} < 0$ if $\underline{t} < t < \check{t}$ and $\frac{\partial \pi_i^{DM}}{\partial t} > 0$ if $t > \check{t}$, that is each participant firm's profit (i.e., π_i^{DM}) decreases in the horizontal differentiation level (i.e., t) if $t < \check{t}$ and increases in t otherwise.

(2) We show the effects of k_m on π_i^{DM}, where $i \in \{A, B\}$. Taking derivative with respect to k_m, we have $\frac{\partial \pi_i^{DM}}{\partial k_m} = \frac{32(k-2k_m)(12-15ck+15c_mk-25kt)^2}{225(-8k_m+k(-4+25k_mt))^3}$. Under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can verify that $\frac{32(12-15ck+15c_mk-25kt)^2}{225(-8k_m+k(-4+25k_mt))^3} > 0$. Define $\check{k_m} = \frac{k}{2}$. Thus, $\frac{\partial \pi_i^{DM}}{\partial k_m} > 0$ if $k_m < \check{k_m}$ and $\frac{\partial \pi_i^{DM}}{\partial k_m} < 0$ if $k_m > \check{k_m}$, that is each participant firm's profit (i.e., π_i^{DM}) increases in the post-merger fixed cost (i.e., k_m) if $k_m < \check{k_m}$ and decreases in k_m otherwise.

(3) We show the effects of t on p_i^{DM}, where $i \in \{A, B\}$. Taking derivative with respect to t, we have $\frac{\partial p_i^{DM}}{\partial t} = \frac{k_m(96k_m - 8k(-6+15ck_m - 15c_m k_m + 50k_m t) + k^2(-60c+60c_m+25t(-8+25k_m t)))}{3(-8k_m + k(-4+25k_m t))^2}$. Similar to Proof of Comprehensive Version of Proposition 4.1 (1), under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can verify that $\frac{\partial p_i^{DM}}{\partial t}$ is either positive or changing from negative to positive as t increases from $\underline{t}$ to $+\infty$. Define $\ddot{t} = \max\{\underline{t}, \frac{4k^2+8kk_m+2\sqrt{W3}}{25k^2 k_m}\}$, where $W_3 = k^2(k+2k_m)(-4k_m + k(4+15ck_m - 15c_m k_m)) > 0$. One can further verify that $\frac{\partial p_i^{DM}}{\partial t} < 0$ if $\underline{t} < t < \ddot{t}$ and $\frac{\partial p_i^{DM}}{\partial t} > 0$ if $t > \ddot{t}$, that is, each participant's price (i.e., p_i^{DM}) decreases in t if $t < \ddot{t}$ and increases in t otherwise.

(4) We show the effects of c_m on p_i^{DM}, where $i \in \{A, B\}$. Taking derivative with respect to c_m, we have $\frac{\partial p_i^{DM}}{\partial c_m} = \frac{4(k+2k_m-5kk_m t)}{4k+8k_m-25kk_m t}$. One can show that $4k+8k_m - 25kk_m t < 0$ under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$. Then we focus on $k + 2k_m - 5kk_m t$. Define $\acute{t} = \max\{\underline{t}, \frac{k+2k_m}{5kk_m}\}$. Thus, we have $k + 2k_m - 5kk_m t > 0$ if $\underline{t} < t < \acute{t}$ and $k + 2k_m - 5kk_m t < 0$ if $t > \acute{t}$. Therefore, $\frac{\partial p_i^{DM}}{\partial c_m} < 0$ if $\underline{t} < t < \acute{t}$ and $\frac{\partial p_i^{DM}}{\partial c_m} > 0$ if $t > \acute{t}$, that is each participant's price (i.e., p_i^{DM}) decreases in the post-merger marginal cost (i.e., c_m) if $t < \acute{t}$ and increases in c_m otherwise.

(5) We show the effects of c on p_C^{DM}. Taking derivative with respect to c, we have $\frac{\partial p_C^{DM}}{\partial c} = \frac{4k+8k_m-15kk_m t}{4k+8k_m-25kk_m t}$. Similar to Proof of Comprehensive Version of Proposition 4.1 (4), one can easily verify that $\frac{\partial p_C^{DM}}{\partial c} < 0$ if $\underline{t} < t < \dddot{t}$ and $\frac{\partial p_C^{DM}}{\partial c} > 0$ if $t > \dddot{t}$, where $\dddot{t} = \max\{\underline{t}, \frac{4(k+2k_m)}{15kk_m}\}$. That is the nonparticipant firm C's price (i.e., p_C^{DM}) decreases in c if $t < \dddot{t}$ and increases in c otherwise. $\square$

Proof of Lemma 4.3. Given that firms A and B adopt centralized merger and the post-merger firm AB offers a single product (i.e., product A), firm AB makes quality and price decisions for product A to maximize its profit, while firm C makes quality and price decisions for product C to maximize its profit. We solve the subgame equilibrium for this case by backward induction.

In stage 2, given the quality levels q_A and q_C, firm AB chooses price p_A to maximize profit π'_{AB}, while firm C chooses price p_C to maximize profit π'_C.

π'_{AB} and π'_C are given by Equation (4.6), i.e., $\pi'_{AB} = (p_A - c_m)D'_A - \frac{1}{2}k_m q_A^2$ and $\pi'_C = (p_C - c)D'_C - \frac{1}{2}kq_C^2$, where D'_i is given by Equation (4.5), i.e., $D'_i = \min\{1, \max\{0, \frac{(q_i-p_i)-(q_{i'}-p_{i'})}{2t} + \frac{1}{6}\} + \max\{0, \frac{(q_i-p_i)-(q_{i'}-p_{i'})}{2t} + \frac{1}{3}\}\}$ for $i, i' \in \{A, C\}$ and $i \neq i'$. Given the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 \pi'_{AB}}{\partial p_A^2} < 0$ ($\frac{\partial^2 \pi'_C}{\partial p_C^2} < 0$), that is π'_{AB} (π'_C) is concave in p_A (p_C). By solving the first-order conditions (i.e., $\frac{\partial \pi'_{AB}}{\partial p_A} = 0$ and $\frac{\partial \pi'_C}{\partial p_C} = 0$) simultaneously, we have the equilibrium prices: $p_A^*(q_A, q_C) = \frac{1}{6}(2c + 4c_m + 2q_A - 2q_C + 3t)$ and $p_C^*(q_A, q_C) = \frac{1}{6}(4c + 2c_m - 2q_A + 2q_C + 3t)$.

Let $\pi_i'^* = \pi'_i|_{p_A = p_A^*(q_A,q_C), p_C = p_C^*(q_A,q_C)}$ denote firm i's profit in stage 2 for given quality levels q_A and q_C, where $i \in \{AB, C\}$. In stage 1, anticipating the price decisions in stage 2, firm AB chooses q_A to maximize profit $\pi_{AB}'^*$, while firm C chooses q_C to maximize profit $\pi_C'^*$. Given the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 \pi_{AB}'^*}{\partial q_A^2} < 0$ ($\frac{\partial^2 \pi_C'^*}{\partial q_C^2} < 0$), that is $\pi_{AB}'^*$ ($\pi_C'^*$) is concave in q_A (q_C). By solving the first-order conditions (i.e., $\frac{\partial \pi_{AB}'^*}{\partial q_A} = 0$ and $\frac{\partial \pi_C'^*}{\partial q_C} = 0$) simultaneously, we can get the equilibrium quality in stage 1 as follows: $q_A^{CM1} = \frac{4-6ck+6c_mk-9kt}{6k+6k_m-27kk_mt}$ and $q_C^{CM1} = \frac{4+6ck_m-6c_mk_m-9k_mt}{6k+6k_m-27kk_mt}$.

Given the equilibrium quality, we can show the equilibrium prices, demands and profits as follows:

$$p_A^{CM1} = \frac{k_m t(-4 + 6ck + 9kt) + 4c_m(-k_m + k(-1 + 3k_m t))}{-4k_m + 2k(-2 + 9k_m t)},$$
$$p_C^{CM1} = \frac{k_m t(-4 + 6ck + 9kt) + 4c_m(-k_m + k(-1 + 3k_m t))}{-4k_m + 2k(-2 + 9k_m t)},$$
$$D_A^{CM1} = \frac{k_m(-4 + 6ck - 6c_m k + 9kt)}{-4k_m + 2k(-2 + 9k_m t)},$$
$$D_C^{CM1} = \frac{k(-4 - 6ck_m + 6c_m k_m + 9k_m t)}{-4k_m + 2k(-2 + 9k_m t)},$$
$$\pi_{AB}^{CM1} = \frac{k_m(4 - 6ck + 6c_m k - 9kt)^2(-2 + 9k_m t)}{36(-2k_m + k(-2 + 9k_m t))^2}, \text{ and}$$
$$\pi_C^{CM2} = \frac{k(-2 + 9kt)(4 + 6ck_m - 6c_m k_m - 9k_m t)^2}{36(-2k_m + k(-2 + 9k_m t))^2}.$$

Let $X_{AB}^{CM1} = \frac{3k_m(-4+6ck-6c_mk+9kt)}{-4k_m+2k(-2+9k_mt)}$ and $X_C^{CM1} = \frac{3k(4+6ck_m-6c_mk_m-9k_mt)}{4k_m+k(4-18k_mt)}$. We get the subgame equilibrium shown in Lemma 4.3 and Table 4.2. □

Comprehensive Version of Proposition 4.2. Given that firms A and B adopt centralized merger and the post-merger firm AB offers a single product (i.e., product A), for the post-merger firm AB,

(i) p_A^{CM1} increases in c and k, and decreases in k_m, but may be non-monotone in t, and may increase or decrease in c_m:

- There exists a threshold $\bar{\bar{t}}$ such that, if $t < \bar{\bar{t}}$, then p_A^{CM1} decreases in t; otherwise, p_A^{CM1} increases in t;
- There exists a threshold $\grave{t}$ such that, if $t < \grave{t}$, then p_A^{CM1} decreases in c_m; otherwise, p_A^{CM1} increases in c_m;

(ii) q_A^{CM1} increases in c and k, and decreases in t, c_m, and k_m;

(iii) π_{AB}^{CM1} increases in c and k, and decreases in c_m and k_m, but may be non-monotone in t:

- There exists a threshold $\hat{t}$ such that, if $t < \hat{t}$, then π_{AB}^{CM1} decreases in t and vice versa;

For the outside firm C,

(iv) p_C^{CM1} increases in t, c_m, and k_m, and decreases in k, but may increase or decrease in c:

- If $t < \grave{t}$, then p_C^{CM1} decreases in c and vice versa;

(v) q_C^{CM1} increases in t, c_m, and k_m, and decreases in c and k;

(vi) π_C^{CM1} increases in t, c_m, and k_m, and decreases in c and k.

$\bar{\bar{t}}$, $\grave{t}$, and $\hat{t}$ are defined in the Proof of Proposition 4.2. □

Proof of Comprehensive Version of Proposition 4.2. Taking derivative with respect to t, c, c_m, k, and k_m, through straightforward yet cumbersome algebraic calculation, one can verify that $\frac{\partial \pi_{AB}^{CM1}}{\partial c} > 0$, $\frac{\partial \pi_{AB}^{CM1}}{\partial k} > 0$, $\frac{\partial \pi_{AB}^{CM1}}{\partial c_m} < 0$, $\frac{\partial \pi_{AB}^{CM1}}{\partial K_m} < 0$, $\frac{\partial p_A^{CM1}}{\partial c} > 0$, $\frac{\partial p_A^{CM1}}{\partial k} > 0$, $\frac{\partial p_A^{CM1}}{\partial k_m} < 0$, $\frac{\partial q_A^{CM1}}{\partial c} > 0$, $\frac{\partial q_A^{CM1}}{\partial k} > 0$, $\frac{\partial q_A^{CM1}}{\partial t} < 0$, $\frac{\partial q_A^{CM1}}{\partial c_m} < 0$, and $\frac{\partial q_A^{CM1}}{\partial k_m} < 0$. Besides, $\frac{\partial \pi_C^{CM1}}{\partial t} > 0$, $\frac{\partial \pi_C^{CM1}}{\partial c_m} > 0$, $\frac{\partial \pi_C^{CM1}}{\partial k_m} > 0$, $\frac{\partial \pi_C^{CM1}}{\partial c} < 0$,

$\frac{\partial \pi_C^{CM1}}{\partial k} < 0$, $\frac{\partial p_C^{CM1}}{\partial t} > 0$, $\frac{\partial p_C^{CM1}}{\partial c_m} > 0$, $\frac{\partial p_C^{CM1}}{\partial k_m} > 0$, $\frac{\partial p_C^{CM1}}{\partial k} < 0$, $\frac{\partial q_C^{CM1}}{\partial t} > 0$, $\frac{\partial q_C^{CM1}}{\partial c_m} > 0$, $\frac{\partial q_C^{CM1}}{\partial k_m} > 0$, $\frac{\partial q_C^{CM1}}{\partial c} < 0$, and $\frac{\partial q_C^{CM1}}{\partial k} < 0$.

Next, we prove the remaining parts in the Comprehensive Version of Proposition 4.2.

(1) We show the effects of t on π_{AB}^{CM1}. Taking derivative with respect to t, we have $\frac{\partial \pi_{AB}^{CM1}}{\partial t} = -\frac{k_m(-4+6ck-6c_mk+9kt)E_1}{4(-2k_m+k(-2+9k_mt))^3}$, where $E_1 = -8k_m^2 + 6kk_m^2(2c - 2c_m + 3t) + k^2(4 + 6ck_m - 6c_mk_m - 9k_mt)(-2 + 9k_mt)$. It can be shown that $-\frac{k_m(-4+6ck-6c_mk+9kt)}{4(-2k_m+k(-2+9k_mt))^3} < 0$ under the assumptions that $0 < c_m \le c$, $0 < k_m \le k$, and $t > \underline{t}$. Similar to Proof of Comprehensive Version of Proposition 4.1 (1), one can verify that E_1 is either negative or changing from positive to negative as t increases from $\underline{t}$ to $+\infty$. Define $\hat{t} = \max\{\underline{t}, \frac{kk_m^2+3k^2k_m(1+ck_m-c_mk_m)+\sqrt{W_4}}{9k^2k_m^2}\}$, where $W_4 = k^2k_m^2(-7k_m^2 + 6kk_m(1 + 3ck_m - 3c_mk_m) + k^2(1 + 3ck_m - 3c_mk_m)^2) > 0$. One can further verify that $E_1 > 0$ if $\underline{t} < t < \hat{t}$ and $E_1 < 0$ if $t > \hat{t}$. Therefore, $\frac{\partial \pi_A^{CM1}}{\partial t} < 0$ if $\underline{t} < t < \hat{t}$ and $\frac{\partial \pi_A^{CM1}}{\partial t} > 0$ if $t > \hat{t}$, that is the post-merger firm AB's profit (i.e., π_{AB}^{CM1}) decreases in t if $t < \hat{t}$ and increases in t otherwise.

(2) We show the effects of t on p_A^{CM1}. Taking derivative with respect to t, we have $\frac{\partial p_A^{CM1}}{\partial t} = \frac{k_m(8k_m-4k(-2+3ck_m-3c_mk_m+9k_mt)-3k^2(4c-4c_m+3t(4-9k_mt)))}{2(-2k_m+k(-2+9k_mt))^2}$. Similar to Proof of Comprehensive Version of Proposition 4.1 (1), under the assumptions that $0 < c_m \le c$, $0 < k_m \le k$, and $t > \underline{t}$, one can verify that $\frac{\partial p_A^{CM1}}{\partial t}$ is either positive or changing from negative to positive as t increases from $\underline{t}$ to $+\infty$. Define $\bar{\bar{t}} = \max\{\underline{t}, \frac{2(k^2+kk_m+\sqrt{E_2})}{9k^2k_m}\}$, where $E_2 = k^2(k+k_m)(k-k_m+3ckk_m-3c_mkk_m) > 0$. One can show that $\frac{\partial p_A^{CM1}}{\partial t} < 0$ if $\underline{t} < t < \bar{\bar{t}}$ and $\frac{\partial p_A^{CM1}}{\partial t} > 0$ if $t > \bar{\bar{t}}$, that is the post-merger firm AB's price (i.e., p_A^{CM1}) decreases in t if $t < \bar{\bar{t}}$ and increases in t otherwise.

(3) We show the effects of c_m on p_A^{CM1}. Taking derivative with respect to c_m, we have $\frac{\partial p_A^{CM1}}{\partial c_m} = \frac{2(k+k_m-3kk_mt)}{2k_m+k(2-9k_mt)}$. Under the assumptions that $0 < c_m \le c$, $0 < k_m \le k$, and $t > \underline{t}$, one can verify that $2k_m + k(2 - 9k_mt) < 0$. Next we focus on $k + k_m - 3kk_mt$. Define $\grave{t} = \max\{\underline{t}, \frac{k+k_m}{3kk_m}\}$. Thus, we have $k + k_m - 3kk_mt > 0$ if $\underline{t} < t < \grave{t}$ and $k + k_m - 3kk_mt < 0$ if $t > \grave{t}$. Therefore, $\frac{\partial p_A^{CM1}}{\partial c_m} < 0$ if $\underline{t} < t < \grave{t}$ and $\frac{\partial p_A^{CM1}}{\partial c_m} > 0$ if $t > \grave{t}$, that is the post-merger firm AB's price (i.e., p_A^{CM1}) decreases

in c_m if $t < \grave{t}$ and increases in c_m otherwise.

(4) We show the effects of c on p_C^{CM1}. Taking derivative with respect to c, we have $\frac{\partial p_C^{CM1}}{\partial c} = \frac{\partial p_A^{CM1}}{\partial c_m}$. Therefore, $\frac{\partial p_C^{CM1}}{\partial c} < 0$ if $\underline{t} < t < \grave{t}$ and $\frac{\partial p_C^{CM1}}{\partial c} > 0$ if $t > \grave{t}$. That is the nonparticipant firm C's price (i.e., p_C^{CM1}) decreases in c if $t < \grave{t}$ and increases in c otherwise. □

Proof of Lemma 4.4. Given that firms A and B adopt centralized merger and the post-merger firm AB offers two products (i.e., products A and B), firm AB makes centralized quality and price decisions for products A and B to maximize the total profit, while firm C makes quality and price decisions for product C to maximize its profit. We solve the subgame equilibrium for this case by backward induction.

In stage 2, given the quality levels q_A, q_B, and q_C, firm AB chooses prices p_A and p_B to maximize the total profit π_{AB}, while firm C chooses price p_C to maximize its own profit π_C. π_{AB} and π_C are given by Equations (4.7) and (4.3), respectively, i.e., $\pi_{AB} = (p_A - c_m)D_A - \frac{1}{2}k_m q_A^2 + (p_B - c_m)D_B - \frac{1}{2}k_m q_B^2$ and $\pi_C = (p_C - c)D_C - \frac{1}{2}kq_C^2$, where D_i is given by Equation (4.2), i.e., $D_i = \min\{1, \max\{0, \frac{(q_i-p_i)-(q_{i'}-p_{i'})}{2t} + \frac{1}{6}\} + \max\{0, \frac{(q_i-p_i)-(q_{i''}-p_{i''})}{2t} + \frac{1}{6}\}\}$ for $i, i', i'' \in \{A, B, C\}$, $i \neq i'$, $i \neq i''$, and $i' \neq i''$. Given the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 \pi_{AB}}{\partial p_i^2} < 0$ for $i \in \{A, B\}$, $\frac{\partial^2 \pi_{AB}}{\partial p_A^2}\frac{\partial^2 \pi_{AB}}{\partial p_B^2} - (\frac{\partial^2 \pi_{AB}}{\partial p_A \partial p_B})^2 > 0$, and $\frac{\partial^2 \pi_C}{\partial p_C^2} < 0$, that is π_{AB} is jointly concave in (p_A, p_B) and π_C is concave in p_C. By solving the first-order conditions (i.e., $\frac{\partial \pi_{AB}}{\partial p_A} = 0$, $\frac{\partial \pi_{AB}}{\partial p_B} = 0$, and $\frac{\partial \pi_C}{\partial p_C} = 0$) simultaneously, we obtain the equilibrium prices in stage 2 as follows: $p_A^*(q_A, q_B, q_C) = \frac{12c+24c_m+15qA-3qB-12qC+20t}{36}$, $p_B^*(q_A, q_B, q_C) = \frac{12c+24c_m-3qA+15qB-12qC+20t}{36}$, and $p_C^*(q_A, q_B, q_C) = \frac{12c+6c_m-3qA-3qB+6qC+8t}{18}$.

Let $\pi_i^* = \pi_i|_{p_A=p_A^*(q_A,q_B,q_C), p_B=p_B^*(q_A,q_B,q_C), p_C=p_C^*(q_A,q_B,q_C)}$ denote firm i's profit in stage 2 for given quality levels q_A, q_B, and q_C, where $i \in \{AB, C\}$. In stage 1, in anticipation of the price decisions in stage 2, firm AB chooses quality levels q_A and q_B to maximize profit π_{AB}^*, while firm C chooses quality level q_C to maximize profit π_C^*. Given the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 \pi_{AB}^*}{\partial q_i^2} < 0$ for $i \in \{A, B\}$ and $\frac{\partial^2 \pi_C^*}{\partial q_C^2} < 0$. Besides,

$\frac{\partial^2\pi_{AB}^*}{\partial q_A^2}\frac{\partial^2\pi_{AB}^*}{\partial q_B^2} - (\frac{\partial^2\pi_{AB}^*}{\partial q_A \partial q_B})^2 \geq 0$ if and only if $t \geq \frac{3}{4k_m}$. That is π_C^* is concave in q_C, while π_{AB}^* is jointly concave in (q_A, q_B) if and only if $t \geq \frac{3}{4k_m}$.

Given $t \geq \frac{3}{4k_m}$, by solving the first-order conditions (i.e., $\frac{\partial \pi_{AB}^*}{\partial q_A} = 0$, $\frac{\partial \pi_{AB}^*}{\partial q_B} = 0$, and $\frac{\partial \pi_C^*}{\partial q_C} = 0$) simultaneously, we get the equilibrium quality levels as follows: $q_A^{CM2} = q_B^{CM2} = \frac{2-3ck+3c_mk-5kt}{3k+6k_m-27kk_mt}$, and $q_C^{CM2} = \frac{2+6ck_m-6c_mk_m-8k_mt}{3k+6k_m-27kk_mt}$.

Given the equilibrium quality, we can show the equilibrium prices, demands, and profits as follows:

$$\begin{aligned}
p_A^{CM2} = p_B^{CM2} &= \frac{k_mt(2-3ck-5kt)+c_m(k+2k_m-6kk_mt)}{k+2k_m-9kk_mt}, \\
p_C^{CM2} &= \frac{kt(1-3c_mk_m-4k_mt)+c(k+2k_m-6kk_mt)}{k+2k_m-9kk_mt}, \\
D_A^{CM2} = D_B^{CM2} &= \frac{k_m(-2+3ck-3c_mk+5kt)}{-4k_m+2k(-1+9k_mt)}, \\
D_C^{CM2} &= \frac{k(1+3ck_m-3c_mk_m-4k_mt)}{k+2k_m-9kk_mt}, \\
\pi_{AB}^{CM2} &= \frac{k_m(2-3ck+3c_mk-5kt)^2(-1+9k_mt)}{9(-2k_m+k(-1+9k_mt))^2}, \text{ and} \\
\pi_C^{CM2} &= \frac{k(-2+9kt)(1+3ck_m-3c_mk_m-4k_mt)^2}{9(-2k_m+k(-1+9k_mt))^2}.
\end{aligned}$$

Let $X_{AB}^{CM2} = \frac{3k_m(-2+3ck-3c_mk+5kt)}{-2k_m+k(-1+9k_mt)}$ and $X_C^{CM2} = \frac{3k(1+3ck_m-3c_mk_m-4k_mt)}{k+2k_m-9kk_mt}$. We get the equilibrium shown in Lemma 4.4 and Table 4.2. Note that we can further show that the pure-strategy equilibrium does not exist when $t < \frac{3}{4k_m}$ in the CM2 case and the proof is available from authors upon request. □

Comprehensive Version of Proposition 4.3. Given that firms A and B adopt centralized merger and the post-merger firm AB offers two products, for the post-merger firm AB,

(i) p_i^{CM2} increases in t, c, c_m, and k, and decreases in k_m;

(ii) q_i^{CM2} increases in c and k, and decreases in c_m and k_m, but may increase or decrease in t;

- There exists a threshold $\breve{c_m}$ such that, if $c_m < \breve{c_m}$, then q_i^{CM2} decreases in t and vice versa;

(iii) The post-merger firm AB's profit π_{AB}^{CM2} increases in t, c, and k, but decreases in c_m and k_m.

For the nonparticipant firm C,

(iv) p_C^{CM2} increases in t, c, c_m, and k_m, but decreases in k;

(v) q_C^{CM2} increases in c_m and k_m, and decreases in c and k, but may increase or decrease in t;

- If $c_m < \breve{c_m}$, then q_C^{CM2} increases in t and vice versa;

(vi) π_C^{CM2} increases in t, c_m, and k_m, but decreases in c and k.

Here, $i \in \{A, B\}$, and $\breve{c_m}$ is defined in the Proof of Proposition 4.3. □

Proof of Comprehensive Version of Proposition 4.3. Taking derivative with respect to t, c, c_m, k, and k_m, through straightforward yet cumbersome algebraic calculation, one can verify that $\frac{\partial \pi_i^{CM2}}{\partial t} > 0$, $\frac{\partial \pi_i^{CM2}}{\partial c} > 0$, $\frac{\partial \pi_i^{CM2}}{\partial k} > 0$, $\frac{\partial \pi_i^{CM2}}{\partial c_m} < 0$, $\frac{\partial \pi_i^{CM2}}{\partial k_m} < 0$, $\frac{\partial p_i^{CM2}}{\partial t} > 0$, $\frac{\partial p_i^{CM2}}{\partial c} > 0$, $\frac{\partial p_i^{CM2}}{\partial c_m} > 0$, $\frac{\partial p_i^{CM2}}{\partial k} > 0$, $\frac{\partial p_i^{CM2}}{\partial k_m} < 0$, $\frac{\partial q_i^{CM2}}{\partial c} > 0$, $\frac{\partial q_i^{CM2}}{\partial k} > 0$, $\frac{\partial q_i^{CM2}}{\partial c_m} < 0$, and $\frac{\partial q_i^{CM2}}{\partial k_m} < 0$, where $i \in \{A, B\}$. Besides, $\frac{\partial \pi_C^{CM2}}{\partial t} > 0$, $\frac{\partial \pi_C^{CM2}}{\partial c_m} > 0$, $\frac{\partial \pi_C^{CM2}}{\partial k_m} > 0$, $\frac{\partial \pi_C^{CM2}}{\partial c} < 0$, $\frac{\partial \pi_C^{CM2}}{\partial k} < 0$, $\frac{\partial p_C^{CM2}}{\partial t} > 0$, $\frac{\partial p_C^{CM2}}{\partial c} > 0$, $\frac{\partial p_C^{CM2}}{\partial c_m} > 0$, $\frac{\partial p_C^{CM2}}{\partial k_m} > 0$, $\frac{\partial p_C^{CM2}}{\partial k} < 0$, $\frac{\partial q_C^{CM2}}{\partial c_m} > 0$, $\frac{\partial q_C^{CM2}}{\partial k_m} > 0$, $\frac{\partial q_C^{CM2}}{\partial c} < 0$, and $\frac{\partial q_C^{CM2}}{\partial k} < 0$.

Next, we prove the remaining parts in the Comprehensive Version of Proposition 4.3.

(1) We show the effects of t on q_i^{CM2}, where $i \in \{A, B\}$. Taking derivative with respect to t, we have $\frac{\partial q_i^{CM2}}{\partial t} = -\frac{k(-8k_m+k(5+27ck_m-27c_mk_m))}{3(-2k_m+k(-1+9k_mt))^2}$. Define $\breve{c_m} = \frac{5k-8k_m+27ckk_m}{27kk_m}$. Under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \max\{\underline{t}, \frac{3}{4k_m}\}$, one can verify that $-k(-8k_m + k(5 + 27ck_m - 27c_mk_m)) < 0$ if $c_m < \breve{c_m}$ and $-k(-8k_m + k(5 + 27ck_m - 27c_mk_m)) > 0$ if $c_m > \breve{c_m}$. Therefore, $\frac{\partial q_i^{CM2}}{\partial t} < 0$ if $c_m < \breve{c_m}$ and $\frac{\partial q_i^{CM2}}{\partial t} > 0$ if $c_m > \breve{c_m}$, that is the post-merger firm AB's service quality (i.e., q_i^{CM2}) decreases in t if $c_m < \breve{c_m}$ and increases in t otherwise.

(2) We show the effect of t on q_C^{CM2}. Taking derivative with respect to t, we have $\frac{\partial q_C^{CM2}}{\partial t} = -\frac{2k_m}{k}\frac{\partial q_i^{CM2}}{\partial t}$. Therefore, $\frac{\partial q_C^{CM2}}{\partial t} > 0$ if $c_m < \breve{c_m}$ and $\frac{\partial q_C^{CM2}}{\partial t} < 0$ if

$c_m > \breve{c_m}$, that is the outside firm C's service quality (i.e., q_C^{CM2}) increases in t if $c_m < \breve{c_m}$ and decreases in t otherwise. $\square$

Proof of Proposition 4.4. To show the existence of merger paradox, we compare each participant firm's profit $\pi_A^{CM2} = \pi_B^{CM2} = \frac{1}{2}\pi_{AB}^{CM2}$ with the outsider's profit π_C^{CM2}.

$\frac{1}{2}\pi_{AB}^{CM2} - \pi_C^{CM2} = \frac{W_5}{18(-2k_m+k(-1+9k_m t))^2}$, where $W_5 = 4k_m(-1+9k_m t) + 4k(1 + 9c^2k_m^2 + 9c_m^2k_m^2 - 3k_m t - 29k_m^2t^2 + 3c_m k_m(-3+17k_m t) - 3ck_m(-3+6c_m k_m + 17k_m t)) - k^2(9c^2k_m(1+9k_m t) + 9c_m^2 k_m(1+9k_m t) + 6c_m k_m t(-23+117k_m t) + t(18 - 119k_m t + 63k_m^2 t^2) - 6ck_m(3c_m(1+9k_m t) + t(-23+117k_m t)))$.

Note that W_5 is a polynomial of t with degree 3. Under the conditions that $0 < c_m \le c$, $0 < k_m \le k$, and $t > \max\{\underline{t}, \frac{3}{4k_m}\}$, one can verify that $\frac{\partial^3 W_5}{t^3} < 0$. Solving $W_5 = 0$ yields three roots, one can show that the smallest root and the second root always violate and the largest root may or may not satisfy the assumptions. Let t^{CM2} be this root and let $\tilde{t} = \max\{\underline{t}, \frac{3}{4k_m}, t^{CM2}\}$. Then one can show that $\frac{1}{2}\pi_{AB}^{CM2} > \pi_C^{CM2}$ if $t < \tilde{t}$ and $\frac{1}{2}\pi_{AB}^{CM2} < \pi_C^{CM2}$ if $t > \tilde{t}$. $\square$

Proof of Proposition 4.5. Here, we compare the equilibrium profits π_{AB}^{CM1} with π_{AB}^{CM2} to determine the optimal product line decision in the case of centralized merger under the conditions that $0 < c_m \le c$, $0 < k_m \le k$, and $t > \max\{\underline{t}, \frac{3}{4k_m}\}$.

By straightforward algebra, we have $\pi_{AB}^{CM1} - \pi_{AB}^{CM2} = \frac{k_m H_1}{36(-2k_m+k(-2+9k_m t))^2(-2k_m+k(-1+9k_m t))^2}$, where $H_1 = -64k_m^2 + 32kk_m^2(6c - 6c_m + t(8+9k_m t)) + 4k^3(2 - 27k_m t + 81k_m^2 t^2)(12c(-1+k_m t) - 12c_m(-1+k_m t) + t(-22+37k_m t)) + k^4(2 - 27k_m t + 81k_m^2t^2)(36c^2 + 36c_m^2 + t^2(119 - 171k_m t) + 12c_m t(-11+9k_m t) - 12c(6c_m + t(-11+9k_m t))) - 4k^2(-8 + 36c^2k_m^2 + 36c_m^2k_m^2 + 124k_m t - 478k_m^2t^2 + 819k_m^3t^3 - 12c_m k_m^2 t(8+9k_m t) + 12ck_m^2(-6c_m + t(8+9k_m t)))$. Next we show that H_1 is decreasing in t under the conditions that $0 < c_m \le c$, $0 < k_m \le k$, and $t > \max\{\underline{t}, \frac{3}{4k_m}\}$.

Note that H_1 is a polynomial of t with degree 3. Through straightforward but cumbersome algebraic calculation, we can show that $\frac{\partial^3 H_1}{\partial t^3} < 0$, that is $\frac{\partial^2 H_1}{\partial t^2}$ decreases in t. In addition, we can show that $\frac{\partial^2 H_1}{\partial t^2}|_{t=\max\{\underline{t}, \frac{3}{4k_m}\}} < 0$. Therefore, $\frac{\partial^2 H_1}{\partial t^2} < 0$ for any $t > \max\{\underline{t}, \frac{3}{4k_m}\}$, that is $\frac{\partial H_1}{\partial t}$ decreases in t for $t > \max\{\underline{t}, \frac{3}{4k_m}\}$.

Similarly, we can show that $\frac{\partial H_1}{\partial t}|_{t=\max\{\underline{t},\frac{3}{4k_m}\}} < 0$. Therefore, $\frac{\partial H_1}{\partial t} < 0$ for any $t > \max\{\underline{t}, \frac{3}{4k_m}\}$, that is H_1 decreases in t for $t > \max\{\underline{t}, \frac{3}{4k_m}\}$.

One can verify that H_1 is either negative or changing from positive to negative as t increases from $\max\{\underline{t}, \frac{3}{4k_m}\}$ to $+\infty$. If $H_1 < 0$ for any $t > \max\{\underline{t}, \frac{3}{4k_m}\}$, then define $\dot{t} = \max\{\underline{t}, \frac{3}{4k_m}\}$. Otherwise, there exists a unique $t^{CM1,CM2} > \max\{\underline{t}, \frac{3}{4k_m}\}$ such that $H_1 > 0$ if $t < t^{CM1,CM2}$ and $H_1 < 0$ if $t > t^{CM1,CM2}$. In this case, define $\dot{t} = t^{CM1,CM2}$. Thus, we have $H_1 > 0$ if $\max\{\underline{t}, \frac{3}{4k_m}\} < t < \dot{t}$ and $H_1 < 0$ if $t > \dot{t}$. Therefore, we have $\pi_{AB}^{CM1} > \pi_{AB}^{CM2}$ if $\max\{\underline{t}, \frac{3}{4k_m}\} < t < \dot{t}$ and $\pi_{AB}^{CM1} < \pi_{AB}^{CM2}$ if $t > \dot{t}$. □

Proof of Proposition 4.6. Proposition 4.6 is proved by comparing the post-merger firm AB's profit in the case of decentralized merger (i.e., π_{AB}^{DM}) with the profit of the post-merger firm AB in the case of centralized merger (i.e., π_{AB}^{CM}). By straightforward algebra, we have $\pi_{AB}^{DM} < \pi_{AB}^{CM1}$ and $\pi_{AB}^{DM} < \pi_{AB}^{CM2}$ if $t \geq \frac{3}{4k_m}$. Then we need to compare π_{AB}^{DM} and π_{AB}^{CM1} under the conditions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $\underline{t} < t < \frac{3}{4k_m}$.

$\pi_{AB}^{DM} - \pi_{AB}^{CM1} = \frac{k_m H_2}{900(-2k_m+k(-2+9k_m t))^2(-8k_m+k(-4+25k_m t))^2}$, where $H_2 = -(512k_m^2(-28+225k_m t)+256kk_m(88+254k_m t-3750k_m^2t^2+15ck_m(16-105k_m t)+15c_m k_m(-16+105k_m t))-40k^3(-2+9k_m t)(4500c^2k_m^2t+4500c_m^2k_m^2t+6c(-112+544k_m t-25k_m^2(60c_m-97t)t)+25t(-48+256k_m t+355k_m^2t^2)-6c_m(-112+544k_m t+2425k_m^2t^2))+16k^2(1504-19136k_m t+20606k_m^2t^2+171475k_m^3t^3+450c^2k_m^2(-8+47k_m t)+450c_m^2k_m^2(-8+47k_m t)-60c_m k_m(-32-416k_m t+2795k_m^2t^2)-60ck_m(32+416k_m t-2795k_m^2t^2+15c_m k_m(-8+47k_m t)))+25k^4(-2+9k_m t)(36c^2(-16+44k_m t+175k_m^2t^2)+36c_m^2(-16+44k_m t+175k_m^2t^2)-12c_m t(-176+640k_m t+1125k_m^2t^2)+t^2(-1904+8200k_m t+5625k_m^2t^2)-12c(t(176-640k_m t-1125k_m^2t^2)+6c_m(-16+44k_m t+175k_m^2t^2))))$. Next we show that H_2 decreases in t under the conditions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $\underline{t} < t < \frac{3}{4k_m}$.

Note that H_2 is a polynomial of t with degree 3. Through straightforward but cumbersome algebraic calculation, we can show that $\frac{\partial^3 H_2}{\partial t^3} < 0$, that is $\frac{\partial^2 H_2}{\partial t^2}$ decreases in t. In addition, we can show that $\frac{\partial^2 H_2}{\partial t^2}|_{t=\underline{t}} < 0$. Therefore, $\frac{\partial^2 H_2}{\partial t^2} < 0$ for any $\underline{t} < t < \frac{3}{4k_m}$, that is $\frac{\partial H_2}{\partial t}$ decreases in t for $\underline{t} < t < \frac{3}{4k_m}$. Similarly, we can show that $\frac{\partial H_2}{\partial t}|_{t=\underline{t}} < 0$. Therefore, $\frac{\partial H_2}{\partial t} < 0$ for any $\underline{t} < t < \frac{3}{4k_m}$, that is H_2 decreases in t for $\underline{t} < t < \frac{3}{4k_m}$. In addition, one can easily verify that $H_2|_{t=\frac{3}{4k_m}} < 0$, that is H_2 is either negative or changing from positive to negative as t increases from $\underline{t}$ to $\frac{3}{4k_m}$. We can show that there exists a threshold $t' \in [\underline{t}, \frac{3}{4k_m})$ such that $H_2 > 0$ if $\underline{t} < t < t'$ and $H_2 < 0$ if $t > t'$. Thus, we have shown that $\pi_{AB}^{DM} > \pi_{AB}^{CM}$ if $\underline{t} < t < t'$; otherwise $\pi_{AB}^{DM} \leq \pi_{AB}^{CM}$. □

Proof of Proposition 4.7. Proposition 4.7 is proved by comparing the total profit of firms A and B in the case of pre-merger (i.e., $\pi_A^{PM} + \pi_B^{PM}$) with the profit of the post-merger firm AB in the case of post-merger (i.e., π_{AB}^{M}). By straightforward algebra, one can show that $\pi_A^{PM} + \pi_B^{PM} < \pi_{AB}^{CM1}$ and, if $t \geq \frac{3}{4k_m}$, $\pi_A^{PM} + \pi_B^{PM} < \pi_{AB}^{CM2}$. Therefore, $\pi_A^{PM} + \pi_B^{PM} < \pi_{AB}^{CM}$. Further, $\pi_A^{PM} + \pi_B^{PM} < \max\{\pi_{AB}^{DM}, \pi_{AB}^{CM}\} = \pi_{AB}^{M}$. □

Proof of Proposition 4.8. We derive Proposition 4.8 by comparing the total profit of firms A and B in the case of pre-merger (i.e., $\pi_A^{PM} + \pi_B^{PM}$) with the profit of the post-merger firm AB in the case of decentralized merger (i.e., π_{AB}^{DM}) under the conditions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$.

$\pi_A^{PM} + \pi_B^{PM} - \pi_{AB}^{DM} = \frac{2W_6}{225k(-8k_m + k(-4+25k_m t))^2}$, where $W_6 = -512k_m^2 + 80kk_m(8 + 15k_m t) - 25k^3(-16t + 9c^2k_m(-8+25k_m t) + 9c_m^2 k_m(-8+25k_m t) - 6ck_m(3c_m - 5t)(-8+25k_m t) - 30c_m k_m t(-8+25k_m t)) + 8k^2(45ck_m(-8+25k_m t) - 45c_m k_m(-8+25k_m t) - 8(2+25k_m t))$.

We next show that W_6 is either negative or changing from positive to negative as t increases from $\underline{t}$ to $+\infty$. Under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can verify that $\frac{\partial^2 W_6}{\partial t^2} < 0$. Solving $W_6 = 0$ yields two roots: $t = \frac{I_1 - \sqrt{W_7}}{1500(c-c_m)k^3k_m^2}$ and $t = \frac{I_1 + \sqrt{W_7}}{1500(c-c_m)k^3k_m^2}$, where $I_1 = 48kk_m^2 + 8k^2k_m(-8+45ck_m - 45c_m k_m) + k^3(16 - 240c_m k_m - 225c^2k_m^2 - 225c_m^2k_m^2 + 30ck_m(8+15c_m k_m))$ and $W_7 = k^2(-4k_m + k(4+15ck_m - 15c_m k_m))^2(144k_m^2 - 24kk_m(4+25ck_m - 25c_m k_m) + k^2(16 - 360c_m k_m + 225c^2k_m^2 + 225c_m^2k_m^2 - 90ck_m(-4+5c_m k_m))) > 0$. One can verify that the smaller root is always smaller than $\underline{t}$. Define $\vec{t} = \max\{\underline{t}, \frac{I_1+\sqrt{W_7}}{1500(c-c_m)k^3k_m^2}\}$. Thus, we have $W_6 > 0$ if $\underline{t} < t < \vec{t}$ and $W_6 < 0$ if $t > \vec{t}$. Therefore, we have that $\pi_A^{PM} + \pi_B^{PM} > \pi_{AB}^{DM}$ if $t < \vec{t}$ and $\pi_A^{PM} + \pi_B^{PM} < \pi_{AB}^{DM}$ if $t > \vec{t}$. □

Proof of Proposition 4.9. Proposition 4.9 is proved by comparing the subgame equilibrium prices and quality in the case of pre-merger with those in the cases of decentralized merger, centralized merger with a single product, and centralized merger with two products, under the conditions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$.

(1) By straightforward algebraic calculation, we have $p_i^{PM} - p_i^{DM} = \frac{4((-k+k_m)t+3c_m(k+2k_m-5kk_mt)+3c(-2k_m+k(-1+5k_mt)))}{3(-8k_m+k(-4+25k_mt))}$, $i \in \{A, B\}$. Let $\acute{c_m}$ be the solution of $p_i^{PM} - p_i^{DM} = 0$. $\acute{c_m} = \frac{-3ck-6ck_m-kt+k_mt+15ckk_mt}{3(-k-2k_m+5kk_mt)}$. Since $-8k_m + k(-4 + 25k_mt) > 0$, it can be shown that, if $k + 2k_m - 5kk_mt > 0$, then $\frac{\partial(p_i^{PM}-p_i^{DM})}{\partial c_m} > 0$; and that, if $k+2k_m-5kk_mt < 0$, then $\frac{\partial(p_i^{PM}-p_i^{DM})}{\partial c_m} < 0$. Then we have $p_i^{PM} > p_i^{DM}$ if $t > \acute{t}$ and $c_m < \acute{c_m}$, and $p_i^{PM} < p_i^{DM}$ if $t < \acute{t}$ or if $t > \acute{t}$ and $c_m > \acute{c_m}$. $t > \acute{t}$ is defined in the Proof of Proposition 4.1.

(2) By straightforward algebra, we have $q_i^{PM} - q_i^{DM} = \frac{4}{15}(\frac{1}{k} + \frac{12-15ck+15c_mk-25kt}{-8k_m+k(-4+25k_mt)}) < 0$, $i \in \{A, B\}$.

(3) By straightforward algebraic calculation, we have $p_i^{PM} - p_A^{CM1} = \frac{t(-4k+8k_m-9kk_mt)+12c_m(k+k_m-3kk_mt)+12c(-k_m+k(-1+3k_mt))}{6(-2k_m+k(-2+9k_mt))}$, $i \in \{A, B\}$. Let $\grave{c_m}$ be the solution of $p_i^{PM} - p_A^{CM1} = 0$. $\grave{c_m} = \frac{-12ck-12ck_m-4kt+8k_mt+36ckk_mt-9kk_mt^2}{-12k-12k_m+36kk_mt}$. Since $-2k_m + k(-2 + 9k_mt) > 0$, one can show that, if $k + k_m - 3kk_mt > 0$, then $\frac{\partial(p_i^{PM}-p_A^{CM1})}{\partial c_m} > 0$; and that, if $k + k_m - 3kk_mt < 0$, then $\frac{\partial(p_i^{PM}-p_A^{CM1})}{\partial c_m} < 0$. Then we have $p_i^{PM} > p_A^{CM1}$ if $t > \grave{t}$ and $c_m < \grave{c_m}$ and $p_i^{PM} \leq p_A^{CM1}$ otherwise. $\grave{t}$ is defined in the Proof of Proposition 4.2.

(4) By straightforward algebraic calculation, we have $q_i^{PM} - q_A^{CM1} = \frac{4}{15k} - \frac{4-6ck+6c_mk-9kt}{6k+6k_m-27kk_mt} < 0$, $i \in \{A, B\}$.

(5) By straightforward algebraic calculation, we have $p_i^{PM} - p_i^{CM2} = \frac{c_m(k+2k_m-6kk_mt)-t(k-4k_m+6kk_mt)+3c(-2k_m+k(-1+6k_mt))}{-6k_m+3k(-1+9k_mt)}$, $i \in \{A, B\}$. Let $\hat{c_m}$ be the solution of $p_i^{PM} - p_i^{CM2} = 0$. $\hat{c_m} = \frac{-3ck-6ck_m-kt+4k_mt+18ckk_mt-6kk_mt^2}{-3k-6k_m+18kk_mt} < c$. Since $\frac{\partial p_i^{PM}-p_i^{CM2}}{\partial c_m} < 0$, we have that $p_i^{PM} > p_i^{CM2}$ if $c_m < \hat{c_m}$ and $p_i^{PM} < p_i^{CM2}$ if $c_m > \hat{c_m}$.

(6) By straightforward algebraic calculation, we have $q_i^{PM} - q_i^{CM2} = \frac{4}{15k} + \frac{2-3ck+3c_mk-5kt}{-6k_m+3k(-1+9k_mt)}$, $i \in \{A, B\}$. Let $\hat{k_m}$ be the solution of $q_i^{PM} - q_i^{CM2} = 0$. $\hat{k_m} =$

$\frac{-6k+15ck^2-15c_mk^2+25k^2t}{-8+36kt}$. Since $\frac{\partial(q_i^{PM}-q_i^{CM2})}{\partial k_m} > 0$, it can be shown that $q_i^{PM} < q_i^{CM2}$ if $k_m < \hat{k_m}$ and $q_i^{PM} > q_i^{CM2}$ if $k_m > \hat{k_m}$. □

Proof of Proposition 4.10. We define the total consumer utility U^s in the case of $s \in \{PM, DM, CM2\}$ as follows:

$$\begin{aligned} U^s = &\int_0^a (v + q_A^s - p_A^s - tx)dx + \int_0^b (v + q_A^s - p_A^s - tx)dx \\ &+ \int_0^{\frac{1}{3}-a} (v + q_B^s - p_B^s - tx)dx + \int_0^c (v + q_B^s - p_B^s - tx)dx \\ &+ \int_0^{\frac{1}{3}-b} (v + q_C^s - p_C^s - tx)dx + \int_0^{\frac{1}{3}-c} (v + q_C^s - p_C^s - tx)dx \end{aligned},$$

where $a = \frac{-3p_A^s+3p_B^s+3q_A^s-3q_B^s+t}{6t}$, $b = \frac{-3p_A^s+3p_C^s+3q_A^s-3q_C^s+t}{6t}$, and $c = \frac{-3p_B^s+3p_C^s+3q_B^s-3q_C^s+t}{6t}$.

Besides, the total consumer utility in the CM1 case (i.e., U^{CM1}) is defined as follows:

$$\begin{aligned} U^{CM1} = &\int_0^{\alpha} (v + q_A^{CM1} - p_A^{CM1} - tx)dx + \int_0^{\beta} (v + q_A^{CM1} - p_A^{CM1} - tx)dx \\ &+ \int_0^{\frac{1}{3}-\alpha} (v + q_C^{CM1} - p_C^{CM1} - tx)dx + \int_0^{\frac{2}{3}-\beta} (v + q_C^{CM1} - p_C^{CM1} - tx)dx \end{aligned},$$

where $\alpha = \frac{-3p_A^{CM1}+3p_C^{CM1}+3q_A^{CM1}-3q_C^{CM1}+t}{6t}$ and $\beta = \frac{-3p_A^{CM1}+3p_C^{CM1}+3q_A^{CM1}-3q_C^{CM1}+2t}{6t}$.

We prove Proposition 4.10 by comparing the total consumer utility in the case of pre-merger (i.e., U^{PM}) with those in the cases of decentralized merger (i.e., U^{DM}), centralized merger with a single product (i.e., U^{CM1}), and centralized merger with two products (i.e., U^{CM2}), respectively. Detailed proofs are shown as follows:

(1) We first compare U^{PM} with U^{DM}. By straightforward algebraic calculation, one can verify that $U^{PM} < U^{DM}$ under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$. That is the total consumer utility in the case of DM (i.e., U^{DM}) is always higher than that in the case of PM (i.e., U^{PM}).

(2) We then compare U^{PM} with U^{CM1}. By straightforward algebraic calculation, we have $U^{PM} - U^{CM1} = \frac{H_3}{90k(-2k_m+k(-2+9k_mt))^2}$, where $H_3 = 96k_m^2 - kk_m(48+360ck_m-360c_mk_m+559k_mt)+k^2(-144+1006k_mt+9k_m^2t^2+180ck_m(-2+$

$15k_m t) - 180c_m k_m(-2 + 15k_m t)) + 5k^3 t(61 - 81c^2k_m^2 - 81c_m^2k_m^2 - 387k_m t + 324k_m^2t^2 + 27ck_m(4 + 6c_m k_m - 27k_m t) + 27c_m k_m(-4 + 27k_m t))$. Next we show that H_3 is negative, positive or changing from negative to positive as c_m increases from 0 to c. Under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, one can easily verify that $\frac{\partial^2 H_3}{\partial c_m^2} < 0$. Solving $H_3 = 0$ yields two roots: $c_m = \frac{40kk_m^2+20k^2k_m(2-15k_m t)+15k^3k_m t(-4+6ck_m+27k_m t)\pm\sqrt{W_8}}{90k^3k_m^2 t}$, where $W_8 = 5(k^2k_m^2(-2k-2k_m+9kk_m t)^2(80-384kt+485k^2t^2)) > 0$. One can verify that the larger root is always larger than c. Define $\check{c_m} = \frac{40kk_m^2+20k^2k_m(2-15k_m t)+15k^3k_m t(-4+6ck_m+27k_m t)-\sqrt{W_8}}{90k^3k_m^2 t}$. Thus we have $H_3 < 0$ if $c_m < \check{c_m}$ and $H_3 > 0$ if $c_m > \check{c_m}$. Therefore, $U^{PM} < U^{CM1}$ if $c_m < \check{c_m}$ and $U^{PM} > U^{CM1}$ if $c_m > \check{c_m}$.

(3) Next we compare U^{PM} with U^{CM2}. Given $t \geq \frac{3}{4k_m}$, by straightforward algebraic calculation, we have $U^{PM} - U^{CM2} = \frac{H_4}{90k(-2k_m+k(-1+9k_m t))^2}$, where $H_4 = 96k_m^2 - 4kk_m(6+90ck_m-90c_m k_m+151k_m t)+5k^3t(16-81c^2k_m^2-81c_m^2k_m^2-198k_m t+261k_m^2t^2+54ck_m(1+3c_m k_m-14k_m t)+54c_m k_m(-1+14k_m t))+4k^2(-9+122k_m t+81k_m^2t^2 + 45c_m k_m(1 - 15k_m t) + 45ck_m(-1 + 15k_m t))$. One can verify that H_4 is either positive or changing from negative to positive as c_m increases from 0 to c under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \max\{\underline{t}, \frac{3}{4k_m}\}$. Define $\tilde{c_m} = \frac{I_6-\sqrt{W_9}}{45k^3k_m^2 t}$, where $I_6 = 20kk_m^2+10k^2k_m(1-15k_m t)+15k^3k_m t(-1+3ck_m+14k_m t)$ and $W_9 = 5(k^2k_m^2(-k-2k_m+9kk_m t)^2(20-96kt+125k^2t^2)) > 0$. Thus, we have $H_4 < 0$ if $c_m < \tilde{c_m}$ and $H_4 > 0$ if $c_m > \tilde{c_m}$. Therefore, $U^{PM} < U^{CM2}$ if $c_m < \tilde{c_m}$ and $U^{PM} > U^{CM2}$ if $c_m > \tilde{c_m}$.

(4) We further compare U^{DM} with U^{CM1} and U^{CM2}, respectively. By straightforward algebraic calculation, one can verify that $U^{DM} > U^{CM1}$ under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$, and that $U^{DM} > U^{CM2}$ under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \max\{\underline{t}, \frac{3}{4k_m}\}$. That is the total consumer utility in the case of DM (i.e., U^{DM}) is always higher than that in the cases of CM1 (i.e., U^{CM1}) and CM2 (i.e., U^{CM2}). □

Proof of Proposition 4.11. We define social welfare SW^s in case $s \in \{PM, DM, CM1, CM2\}$ as the total profit of all firms in the market plus the total consumer utility U^s. That is $SW^{PM} = U^{PM} + \pi_A^{PM} + \pi_B^{PM} + \pi_C^{PM}$ and

$SW^s = U^s + \pi^s_{AB} + \pi^s_C$, for $s \in \{DM, CM1, CM2\}$.

Proposition 4.11 is proved by comparing the social welfare in the case of pre-merger (i.e., SW^{PM}) with those in the cases of decentralized merger (i.e., SW^{DM}), centralized merger with a single product (i.e., SW^{CM1}), and centralized merger with two products (i.e., SW^{CM2}), respectively. Detailed proofs are shown as follows:

(1) We first compare SW^{PM} with SW^{DM}. By straightforward algebraic calculation, one can verify that $SW^{PM} < SW^{DM}$ under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \underline{t}$. That is the social welfare in the case of DM (i.e., SW^{DM}) is always higher than that in the case of PM (i.e., SW^{PM}).

(2) We then compare SW^{PM} with SW^{CM1}. By straightforward algebraic calculation, we have $SW^{PM} - SW^{CM1} = \frac{H_5}{450k(-2k_m+k(-2+9k_m t))^2}$, where $H_5 = 288k_m^2 - kk_m(224 + 1800ck_m - 1800c_m k_m + 2267k_m t) + 2k^2(-256 + 450c^2k_m^2 + 450c_m^2k_m^2 + 2179k_m t + 441k_m^2t^2 - 900ck_m(1 + c_m k_m - 9k_m t) - 900c_m k_m(-1 + 9k_m t)) - 25k^3(-729c_m k_m^2 t^2 + 9c^2k_m(-4 + 45k_m t) + 9c_m^2 k_m(-4 + 45k_m t) + t(-13 + 198k_m t - 81k_m^2 t^2) + 9ck_m(81k_m t^2 + c_m(8 - 90k_m t)))$. One can show that $\frac{\partial H_5}{\partial c_m} > 0$ and $H_5 = 0$ has at most one solution under the conditions described in Section 4.3. Let $\dot{c_m}$ be this solution, and $\dot{c_m} = -\frac{-120kk_m^2+120k^2k_m(-1+ck_m+9k_m t)-15k^3k_m(81k_m t^2+c(-8+90k_m t))+\sqrt{W_{10}}}{30k^2k_m(-4k_m+k(-4+45k_m t))}$, where $W_{10} = k^2k_m(-2k_m + k(-2 + 9k_m t))^2(2448k_m + 25k^2t(-52 + 909k_m t) - 4k(-512 + 5185k_m t))$. Then we have $H_5 < 0$ if $c_m < \dot{c_m}$ and $H_5 > 0$ if $c_m > \dot{c_m}$. That is $SW^{PM} < SW^{CM1}$ if $c_m < \dot{c_m}$ and $SW^{PM} > SW^{CM1}$ if $c_m > \dot{c_m}$.

(3) Next we compare SW^{PM} with SW^{CM2}. Given $t \geq \frac{3}{4k_m}$, by straightforward algebraic calculation, we have $SW^{PM} - SW^{CM2} = \frac{H_6}{450k(-2k_m+k(-1+9k_m t))^2}$, where $H_6 = 288k_m^2 - 4kk_m(28 + 450ck_m - 450c_m k_m + 623k_m t) + 4k^2(-32 + 225c^2k_m^2 + 225c_m^2k_m^2 + 526k_m t + 733k_m^2t^2 - 75ck_m(3 + 6c_m k_m - 55k_m t) - 75c_m k_m(-3 + 55k_m t)) - 25k^3(6c_m k_m t(1 - 144k_m t) + 9c^2k_m(-2 + 45k_m t) + 9c_m^2 k_m(-2 + 45k_m t) + t(-4 + 112k_m t - 9k_m^2 t^2) - 6ck_m(t - 144k_m t^2 + 3c_m(-2 + 45k_m t)))$. One can show that $\frac{\partial H_6}{\partial c_m} > 0$ and $H_6 = 0$ has at most one root under the conditions that $0 < c_m \leq c$, $0 < k_m \leq k$, $t > \underline{t}$, and $t \geq \frac{3}{4k_m}$. Let $\ddot{c_m}$ be this solu-

tion. $\ddot{c_m} = -\frac{-60kk_m^2+10k^2k_m(-3+6ck_m+55k_mt)+5k^3k_m(t-144k_mt^2+c(6-135k_mt))+\sqrt{W_{11}}}{45k^3k_m^2t}$, where $W_{11} = k^2k_m(-2k_m + k(-1 + 9k_mt))^2(612k_m + k(256 - 5260k_mt) + 25k^2t(-8 + 261k_mt))$. Then we have $H_6 < 0$ if $c_m < \ddot{c_m}$ and $H_6 > 0$ if $c_m > \ddot{c_m}$. That is $SW^{PM} < SW^{CM2}$ if $c_m < \ddot{c_m}$ and $SW^{PM} > SW^{CM2}$ if $c_m > \ddot{c_m}$.

(4) Next we compare SW^{DM} with SW^{CM1}. By straightforward algebraic calculation, we have $SW^{DM} - SW^{CM1} = \frac{H_7}{450(-2k_m+k(-2+9k_mt))^2(-8k_m+k(-4+25k_mt))^2}$, here $H_7 = 64k_m^3(-368 + 225k_mt) + 16kk_m^2(-2080 + 22748k_mt - 20575k_m^2t^2 + 300ck_m(12 + 5k_mt) - 300c_mk_m(12 + 5k_mt)) - k^2k_m(8704 - 244752k_mt + 1681128k_m^2t^2 - 1643125k_m^3t^3 + 14400c^2k_m^2(2 + 9k_mt) + 14400c_m^2k_m^2(2 + 9k_mt) + 600c_mk_m(144-1424k_mt+959k_m^2t^2)-600ck_m(144-1424k_mt+959k_m^2t^2+48c_mk_m(2+9k_mt))) + 2k^3(512 - 2608k_mt - 169232k_m^2t^2 + 1291675k_m^3t^3 - 1344375k_m^4t^4 + 450c^2k_m^2(-48 + 344k_mt + 229k_m^2t^2) + 450c_m^2k_m^2(-48 + 344k_mt + 229k_m^2t^2) + 300c_mk_m(-48 + 1072k_mt - 6025k_m^2t^2 + 5715k_m^3t^3) - 300ck_m(-48 + 1072k_mt - 6025k_m^2t^2 + 5715k_m^3t^3 + 3c_mk_m(-48 + 344k_mt + 229k_m^2t^2))) + 25k^4(9c^2k_m(-64 + 1232k_mt-5492k_m^2t^2+4275k_m^3t^3)+9c_m^2k_m(-64+1232k_mt-5492k_m^2t^2+4275k_m^3t^3)- 3c_mk_mt(-320+8992k_mt-47880k_m^2t^2+50625k_m^3t^3)+t(-48+1336k_mt+637k_m^2t^2- 42750k_m^3t^3 + 50625k_m^4t^4) - 3ck_m(t(320 - 8992k_mt + 47880k_m^2t^2 - 50625k_m^3t^3) + 6c_m(-64 + 1232k_mt - 5492k_m^2t^2 + 4275k_m^3t^3)))$.

Solving $H_7 = 0$ yields two solutions. Let $\dddot{c_m}$ and $\vec{c_m}$ be these two solutions. Here $\dddot{c_m} = \frac{W_{13}-\sqrt{W_{14}}}{W_{15}}$ and $\vec{c_m} = \frac{W_{13}+\sqrt{W_{14}}}{W_{15}}$, where $W_{13} = 320kk_m^3(12 + 5k_mt) - 40k^2k_m^2(-144 + 1424k_mt - 959k_m^2t^2 + 48ck_m(2 + 9k_mt)) + 40k^3k_m(48 - 1072k_mt + 6025k_m^2t^2-5715k_m^3t^3+3ck_m(-48+344k_mt+229k_m^2t^2))+5k^4k_m(6c(-64+1232k_mt- 5492k_m^2t^2 + 4275k_m^3t^3) + t(-320 + 8992k_mt - 47880k_m^2t^2 + 50625k_m^3t^3))$, $W_{14} = k^2k_m(16k_m^2+2kk_m(12-61k_mt)+k^2(8-86k_mt+225k_m^2t^2))^2(16k_m(656-8448k_mt+ 8725k_m^2t^2) + k(4096 - 93248k_mt + 603488k_m^2t^2 - 560500k_m^3t^3) + 25k^2t(-192 + 6512k_mt - 33724k_m^2t^2 + 33525k_m^3t^3))$ and $W_{15} = 30k^2k_m(-64k_m^2(2 + 9k_mt) + 4kk_m(-48 + 344k_mt + 229k_m^2t^2) + k^2(-64 + 1232k_mt - 5492k_m^2t^2 + 4275k_m^3t^3))$.

Let $\bar{\hat{k}}_m$ be the solution of $\frac{\partial^2 H_7}{\partial c_m^2} = 0$.

It can be shown that, if $k_m < \bar{\hat{k}}_m$, then $\frac{\partial^2 H_7}{\partial c_m^2} < 0$ and $\vec{c_m} \leq \dddot{c_m}$. In this case, if $\vec{c_m} = \dddot{c_m}$, then $H_7 \leq 0$; and if $\vec{c_m} \neq \dddot{c_m}$, then $H_7 > 0$ if $c_m < \dddot{c_m}$ and $H_7 < 0$ if

$c_m > \dddot{c}_m$. Besides, if $k_m > \bar{\hat{k}}_m$, then $\frac{\partial^2 H_7}{\partial c_m^2} > 0$ and $\dddot{c}_m \leq \vec{c}_m$. In this case, $H_7 > 0$ if $c_m < \dddot{c}_m$ and $H_7 < 0$ if $c_m > \dddot{c}_m$. We disregard the case where $k_m = \bar{\hat{k}}_m$. This is because, when $k_m = \bar{\hat{k}}_m$, $E_7 = 0$, which is meaningless.

To sum up, if $k_m < \bar{\hat{k}}_m$, $\dddot{c}_m \neq \vec{c}_m$, and $c_m < \dddot{c}_m$, or if $k_m > \bar{\hat{k}}_m$ and $c_m < \dddot{c}_m$, then the social welfare is higher in the case of decentralized merger than that in the case of centralized merger with a single product, i.e., $SW^{DM} > SW^{CM1}$; otherwise, the social welfare is higher in the case of centralized merger with a single product dominates that in the case of decentralized merger, i.e., $SW^{DM} \leq SW^{CM1}$.

(5) We further compare SW^{DM} with SW^{CM2}. Given $t \geq \frac{3}{4k_m}$, by straightforward algebraic calculation, one can verify that $SW^{DM} > SW^{CM2}$ under the assumptions that $0 < c_m \leq c$, $0 < k_m \leq k$, and $t > \max\{\underline{t}, \frac{3}{4k_m}\}$. That is the social welfare in the case of DM (i.e., SW^{DM}) is always higher than that in the case of CM2 (i.e., SW^{CM2}). $\square$