OPTIMIZATION OF AIRCRAFT HANGAR MAINTENANCE PLANNING UNDER MRO OUTSOURCING MODE

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CERTIFICATE OF ORGINALITY

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Abstract

Aircraft Maintenance, Repair and Overhaul (MRO) activities are critical for the aviation industry. The maintenance checks for aircraft must be conducted periodically to ensure its operational safety and reliability. From the perspective of airlines, the cost of MRO activities, especially hangar maintenance checks, takes a large proportion of their annual operating costs, as carrying out hangar maintenances for aircraft is demanding due to the increasing technical sophistication of the maintenances and the inventory cost on the relevant equipment. In this regard, a trend of outsourcing hangar maintenance activities to an aircraft maintenance service provider has emerged and becomes an attractive operation strategy for airlines, which ensures that the MRO operations continue to meet the safety requirements while reducing the maintenance costs. With the rapid adoption of MRO outsourcing mode among airlines, maintenance service providers receive increasing maintenance demands from multiple clients, and fulfilling those outsourcing demands with the resource constraints becomes challenging. In this connection, enhancing the utilization of maintenance resources and developing an effective maintenance planning optimization approach under the MRO outsourcing mode are significant for the maintenance service provider to handle the fast-growing maintenance demands.

The objective of this study is to analyse the maintenance planning problem arising from the aircraft hangar maintenance service providers under outsourcing mode then develop efficient optimization approach to tackle the proposed problem. Though aircraft maintenance scheduling and planning problems have received much attentions in the literature, the geometric factor is overlooked while such aspect creates a bottleneck in carrying out the hangar maintenance plan under the outsourcing mode. The

maintenance planning problem under such context has to incorporate the variation of the hangar capacity along the planning horizon as well as the impacts of hangar parking layout planning. Specifically, though the dimensions of maintenance hangar are constant, the capacity of the hangar (the number of aircraft that hangar can accommodate) varies according to the physical configurations of respective aircraft along the planning period, while such factor receives little attention in conventional aircraft maintenance scheduling, e.g. aircraft hangar maintenance operated out by airline itself. Therefore, it motivates to analyse the hangar capacity measurement from a spatial aspect under the MRO outsourcing mode. Besides, the additional geometric and manpower resource constraints, including the aircraft's path blocking during the movement operations in the hangar and the multi-skill maintenacne technicain, shall be concurrently incorporated under the outsourcing mode. The outcomes of the integrated solution shall attain trade-offs among aircraft's maintenance scheduling, hangar parking layout planning/movement planning and multi-skill manpower assignment. The interdependent relationships among scheduling, geometric planning and manpower resources shall be carefully analysed and modelled from the MRO outsourcing aspect, which is a pioneer in modeling and optimizing aircraft maintenance scheduling problem.

This research mainly focuses on developing a systematic optimization methodology for the aircraft hangar maintenance planning problem, which fills the research gaps mentioned above. The structure of the methodology is mainly divided as four parts as follows: (i) The first part analyses the impact of aircraft parking stand allocation from a two-dimensional space, which is a critical research element in carrying out aircraft hangar maintenance planning. A mathematical formulation characterizing the variation of hangar capacity in two-dimensional space is developed to cater the aircraft in different sizes to be serviced. A proposed MILP-based heuristic algorithm in carrying high quality layout within a reasonable computation time for practical usage is developed for the static parking stand allocation model; (ii) The second part further analyse the hangar space utilization problem. A heuristic algorithm is developed to provide practical solutions, and the intermediate infeasible solutions identified during searching are utilized to develop valid and approximate inequalities, tightening the optimality gap; (iii) With the foundation of geometric analysis on hangar space utilization in the first and second parts, the third parts incorporate the aircraft movement operations and its blocking impact in aircraft maintenance scheduling, which extends the static aircraft parking stand allocation model in the first part. The original model is enhanced by developing an event-based discrete time model to narrow down the domain of the time-related decision variables. In addition, a rolling horizon approach incorporating the enhanced mathematical model is presented to obtain good quality feasible solutions for large scale instances; (iv) The last part incorporates the multi-skill manpower planning, which is identified as another critical resource in aircraft hangar maintenance, to fulfil the integrated maintenance planning model. A two-stage optimization approach is developed to enhance the computational efficiency by decomposing the integrated model, and the two-stage approach is coordinated by the linkage constrains between geometric and numeric decision-making scattering in the decomposed subproblems. Moreover, systematic computational studies are deployed in respective parts of methodology, using on the problem instances generated based on the data collected from an aircraft hangar maintenance company. The managerial insights on carrying out hangar maintenance planning as well as the analysis on computational efficiency are given for both practical and theoretical uses.

Publications Arising from the Thesis

Journal articles (in reverse chronological order)

Qin, Y., Zhang, J.H., Chan, F.T.S., Chung, S.H., Niu, B., Qu, T. (2019). A two-stage optimization approach for aircraft hangar maintenance planning and staffing assignment problems under MRO outsourcing mode. *Computers & Industrial Engineering*. (Submitted)

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Qin, Y., Chan, F.T.S., Chung, S.H., Qu, T., & Niu, B. (2018). Aircraft parking stand allocation problem with safety consideration for independent hangar maintenance service providers. *Computers & Operations Research*, 91, 225-236. doi:https://doi.org/10.1016/j.cor.2017.10.001

Qin, Y., Wang, Z.X., Chan, F.T.S., Chung, S.H., Qu, T. (2018). A family of heuristic-based inequalities for maximizing overall safety margins in aircraft parking stands arrangement problems. *Mathematical Problems in Engineering*, 2018, Article ID 3525384. https://doi.org/10.1155/2018/3525384

Conference papers (in reverse chronological order)

Qin, Y., Chan, F.T.S., Chung, S.H., Qu, T. (2019). A branch and cut algorithm framework for the integrated aircraft hangar maintenance scheduling and staffing problem. In the 2nd International Conference on Information Science and System (ICISS 2019), 16-19 March 2019, Tokyo, Japan.

Qin, Y., Chan, F.T.S., Chung, S.H., Niu, B. & Qu, T. (2018). Enhanced event-based discrete time model for the integrated aircraft hangar maintenance scheduling and parking layout planning problems. In the 8th International Congress on Engineering and Information (2018 ICEAI), 1-4 May 2018, Sapporo, Japan.

Qin, Y., Chan, F.T.S., Chung, S.H., & Qu, T. (2018). Integrated maintenance scheduling and staff scheduling for an aircraft hangar maintenance problem. In the 4th International Conference on Science, Engineering and Environment, 12-14 November 2018, Nagoya, Japan.

Qin, Y., Chan, F.T.S., Chung, S.H., & Qu, T. (2018). Multi-skill manpower planning in aircraft hangar maintenance scheduling problem. In the 48th International Conference on Computers & Industrial Engineering, 2-5 December 2018, Auckland, New Zealand.

Qin, Y., Chan, F.T.S., Chung, S.H., & Qu, T. (2017). Development of MILP model for integrated aircraft maintenance scheduling and multi-period parking layout planning problems. In 2017 the 4th International Conference on Industrial Engineering and Applications. Nagoya, Institute of Technology, Nagoya, Japan, 21-23 April 2017.

Qin, Y., Chan, F.T.S., Chung, S.H., Qu, T., Wang, X.P., & Ruan, J.H. (2017). MIP models for the hangar space utilization problem with safety consideration. In The 6th International Conference on Mechanics and Industrial Engineering (ICMIE'17), 9-10 June 2017, Roma, Italy.

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List of Abbreviations

2L-SPD	Two-dimensional loading constraints
B&C	Branch-and-Cut
BD	Benders' Decomposition
BDT	Basic Discrete-time formulation
DDT	Disaggregated Discrete-time formulation
ETD	Event-based Discrete-time formulation
ETA	Estimated Time of Arrival
ETD	Estimated Time of Departure
FLP	Facility Layout Problem
GA	Genetic Algorithm
CG	Column Generation
DP	Dynamic Programming
GRASP	Greedy Randomized Adaptive Search Procedure
LIFO	Last-In-First-Out
LS	Local Search
MRO	Maintenance, Repair, and Overhaul
MILP	Mixed-Integer Linear Programming
MIP	Mixed-Integer Programming
MT	Maintenance Time
NFP	No-Fit Polygon
PSAP	Aircraft Parking Stands Arrangement Problem
SA	Simulated Annealing
TSN	Time-space Network
VRP	Vehicle Routing Problem

Chapter 1. Introduction

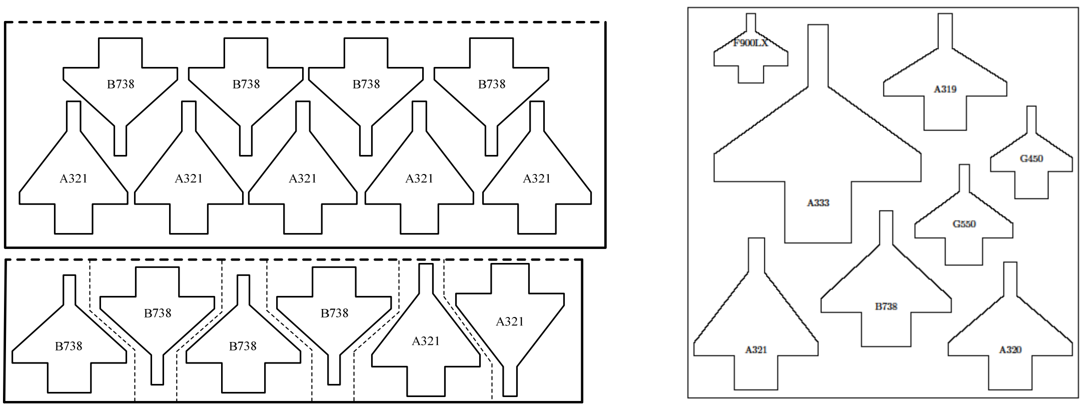
In this chapter, the background of the research is provided. The challenges in handling aircraft hangar maintenance operations under MRO outsourcing mode are illustrated. The difficulties of conducting hangar maintenance operations result from the increasing maintenance demands due to the transformation of aircraft heaving maintenance practice, as well as the conventional planning methodology. The maintenance capacity deficiencies stem from the incapable maintenance scheduling and planning methodology, which can cause delay propagation in meeting the increasing outsourcing maintenance demands. In order to enhance the level of practical usage and the productivity of aircraft hangar maintenance operations, a systematic optimization methodology is indispensable to enhance the maintenance management quality. The hangar maintenance schedule shall fully utilize the limited maintenance resources in face of challenges derived from outsourcing transformation. With respect to the trends of the industrial needs, the capacity of maintenance hangar is identified as a critical maintenance resource and bottleneck in fulfilling maintenance tasks. However, maintenance planning approaches studied from the other perspective, such as airline-operated maintenance planning optimization, are not practical in the outsourcing mode. Moreover, the respective optimization algorithms are inapplicable to be directly applied in the outsourcing context. Therefore, the development of modelling method that accurately measure the maintenance capacity for high-quality decision making can cope with challenges in outsourcing-oriented maintenance mode. This research contributes to the theoretical development of aircraft maintenance planning optimization, scheduling problem with geometric consideration and heuristic algorithm developments to the above-mentioned optimization problems.

1.1. Research background

Aircraft safety and airworthiness are of great importance in the aviation industry and have been gaining greater traction. Strict regulations and requirements of maintenance operations carried out by the aviation authorities form the guideline of safety management in airline companies, aircraft manufacturer, airport operators as well as the maintenance service providers. The commitment to aviation safety from different stakeholders in the aviation industry is the foundation of an efficient and effective aviation operations.

Ensuring the quality of aircraft maintenance performed by maintenance operators is a guarantee of safety and airworthiness of aircraft. It is pointed out that carrying out an appropriate maintenance plans is correlated to ensuring aircraft safety (Yang et al., 2003). Planning a maintenance schedule for a fleet of aircraft is challenging for each airline to achieve their multiple and conflicting goals that relate to the cost of operation and service, profitability, customer satisfaction as well as safety requirements. The cost of aircraft maintenance, repair, and overhaul (MRO) activities represents around 9 percent of the total annual operating cost for airlines (ITAT, 2015), which is the third highest cost center behind the cost of various fuel operations and labor. Operating costs with the fluctuation of passenger demands as well as the competitions among airlines nowadays have forced many airline companies to reconsider their business operation to ensure that their MRO operations continue to meet the safety requirements while maintaining minimum costs, maximum quality and the best lead-time (Eriksson & Steenhuis, 2014; Knotts, 1999). A transformation of aircraft hangar maintenance practice has emerged, which is to outsource the MRO operations to an independent aircraft maintenance service company. By transforming the MRO activities to outsourcing mode, airline companies can reduce the great inventory cost on the sophisticated technical asset for aircraft maintenance and focus more on their high-value commercial flying, customer relationship management as well as marketing. Marcontell (2013) estimates that the percentage of MRO outsourcing has risen 45 percent from the mid-1990s to 2012 and the trend of outsourcing maintenance activities continues as more aircraft are going into service in order to meet the significant growth of air traffic demands.

From the perspective of aircraft hangar maintenance service providers, fulfilling the increasing maintenance requests from clients, e.g. multiple airlines, within a reasonable time becomes a challenging task due to limited resources availability (e.g. limited hangar space, staff, and equipment). As multiple airlines conduct their own fleet's maintenance schedule while carrying out the flight plans, it is possible that the maintenance demands from multiple airlines may congest to a period of time, which creates peaks hours for maintenance service company. To fulfill aircraft maintenance orders from clients efficiently, aircraft maintenance activities for multiple aircraft have to be conducted concurrently in the hangar. The Operation Manager has to carry out a maintenance plan, including the service time of each aircraft, a series of aircraft parking stand allocation plans, and manpower allocation roster align with the service period over the planning horizon. The aircraft parking positions of the maintenance hangar operated by airline (in-house hangar maintenance) are usually predefined into fixed parking stands at the design stage due to the limited types of aircraft models own by single airline company (Figure 1-1 (a)). Differing from the conventional hangar maintenance operated by airlines in which the capacity of hangar can be quantified as constant number, hangar maintenance companies receive maintenance orders from different clients (e.g. full-cost airlines, low-cost airlines, cargo airlines, government flying service as well as private aircraft owners) with their respective desirable maintenance service time windows, which means that the maintenance company needs to flexibly arrange those aircraft in different sizes in the hangar (Figure 1-1 (b)). Therefore, defining and fixing the parking stands at the hangar design stage are not appropriate for independent MRO providers since the parking plan differs from time to time based on the incoming maintenance orders. Moreover, safety considerations, movement blocking of aircraft should be taken into consideration carefully in optimizing the hangar maintenance plan to reduce the risk of collision between aircraft, blocking of movement as well as misalignment with manpower arrangement.



(a) (b)

Figure 1-1 Airline-owned hangar and independent MRO service provider's hangar

1.2. Research motivation

Aircraft maintenance planning problem emerging from the MRO outsourcing mode have been identified as a significant problem in the research field of aircraft maintenance scheduling and planning, with the popularity of MRO outsourcing practice and increasing maintenance demands from multiple airlines and the other clients. Given the different maintenance requests, estimated arrival time of aircraft to the hangar, expected delivery time proposed by each client, aircraft types and the heterogeneous maintneance tasks requiring respective maintenance manpower, the maintenance service provider has to carefully review its maintenance capacity and determine the acceptance of maintenance demands, and the respective maintenance planning for multiple maintenance activities within the hangar, so as to eliminate th epossibility of tardiness and ensure to maintian a certain level of service quality.

Hangar space has been recognized as the one of bottleneck of maintenance capacity. Expanding the hangar space is unlikely given the limitation prescribed by each airport authority and limited space in the airport area. Given this point, accurate measurement of hangar capacity is significant to determine the maintenance service provider’ s maintenance ability. Conventionally, the capacity of the hangar is measured by the type and configuration of aircraft that are going to be maintained, which is applicable in the airline-owned hangar. Such conventional approach adopted by airline-owned hangar is not applicable for maintenance service providers and may lead to low utilization of hangar space that lower the efficiency in fulfilling maintenance requests, or overestimate of hangar capacity that induces tardiness since the some of the aircraft have to wait outside. In this regard, having a proper and accurate hangar capacity measurement approach is significant in carrying out maintenance schedule to make the most of the maintenance capacity. Existing optimization approaches proposed by researchers solve maintenance schedule problems from different approach while the variation of maintenance capacity influenced by the incoming maintenance requests at different times is not incorporated in the approaches given the context of the problem. Therefore, adapting the existing approach to solve the maintenance scheduling optimization problem in the context of maintenance service provider’ s hangar may lead to an unsatisfactory solution or even infeasible solution. In addition, considering the complexity of the maintenance activities associated with the incoming aircraft, arranging capable and suitable group size of maintenance technicians is another critical factor in carrying out the aircraft maintenance plan, as the misalignment of multi-skill maintenance manpower rostering also result in significant tardiness in completing the maintenance tasks. Moreover, the practical constraints of assigning manpower shall be taken into considerations in the integrated model.

Therefore, the main objective of this research is to fill the gaps mentioned above by developing an accurate and efficient optimization approach that tackles the maintenance planning problem in the context of maintenance service providers under the emerging MRO outsourcing mode.

1.3. Research scope and objectives

This research carries out an in-depth analysis of the maintenance planning optimization problem studied from the MRO outsourcing mode. The accurate measurement of hangar capacity acts as a foundation of the subsequent maintenance planning optimization process. The variation of incoming maintenance demands along planning period has impact on the capacity of hangar. Therefore, the hangar capacity measurement approach needs to be flexible and adjustable so as to coordinate with the incoming maintenance requests and scheduling decision. With the foundation of the hangar capacity measurement approach, the subsequent maintenance scheduling methodology incorporating the aircraft movement operations as well as the multi-skill manpower planning is developed.

In order to successfully develop an efficient algorithm for the maintenance planning problem, the objectives of this study can be described as follows:

The first objective is to investigate and analyse the current research status concerning aircraft maintenance scheduling optimization problem, and further investigate the classification scheme of the maintenance scheduling optimization problem in the aviation industry for a better understanding of the studied contexts. Furthermore, the major concerns in formulating the proposed research problem can be identified.

The second objective is to investigate and analyse the application of optimization algorithms, including exact algorithms and approximation algorithms, for solving the maintenance scheduling problem in variant context. Designing appropriate algorithm is significant for the specific problem regarding the features and problem natures.

The third objective is to analyse and model the hangar maintenance planning optimization problem in the context of maintenance service provider under the MRO outsourcing context. This objective can be further divided into the following sub-objective:

- to develop a flexible and dynamic hangar capacity measurement approach that provide accurate measurement of hangar capacity given the different incoming maintenance requests from its clients from time to time.
- to incorporate the hangar capacity measurement approach into maintenance scheduling and planning problem.
- to develop mathematical models with variation of hangar capacity, geometric considerations and the practical constraints of manpower planning.
- to design proper algorithms to solve the proposed problem.
- to conduct computational experiment to examine and validate the effectiveness and efficiency of new models and proposed algorithms

1.4. Thesis structure

After a brief introduction of the research background, motivation, scope, and objective in Chapter 1, the rest of this report is organized as follow:

Chapter 2 provides a literature review on the related works in the field, including the classification of aircraft maintenance checks in industry, the maintenance optimization problem in different contexts, pointing out the research gap in tackling aircraft hangar maintenance planning problem in research field. Moreover, the methodology of modeling and preventing overlapping between aircraft in optimization problem context, and the corresponding proposed optimization algorithms are proposed.

Chapter 3 presents an aircraft parking stand allocation optimization model, which models the hangar parking capacity from a two-dimensional space in an accurate form. A MIP-based heuristic algorithm is developed to provide good quality solutions for large-scale instances. The static aircraft parking stand allocation model presented in this chapter is the foundation of aircraft hangar maintenance planning over multi-period.

Chapter 4 further studies the static aircraft parking stand allocation model developed in Chapter 3. Considering the non-convex irregular shape of aircraft and large number of binary variables associated with the revised NFP, A heuristic algorithm is developed to provide practical solutions, and the intermediate infeasible solutions identified during searching are utilized to develop valid and approximate inequalities, tightening the optimality gap.

Chapter 5 deals with the maintenance scheduling problem and develop a mathematical model to integrate the aircraft allocation approach in Chapter 3, which incorporates the variation of parking capacity. To prepare a series of layouts together aligning with maintenance schedule, the maintenance scheduling and multi-period hangar parking layout problem model the geometric relations between a pair of aircraft for movement process. An efficient rolling horizon approach reducing the complexity of single mathematical model in tackling problems converging long planning period is developed.

Chapter 6 further extends the aircraft maintenance scheduling model with the manpower planning to integrate three core elements in decision makings, i.e. maintenance scheduling, layout & movement path planning and manpower assignment. The maintenance manpower multiple types of maintenance skills, aligning the practice of sophisticated hangar maintenance, is considered. Given the problem complexity, a two-stage optimization approach is developed by composing the original model, which is correlated by the linkage between geometric-related and numeric-related elements in optimization model.

Chapter 7 draws the conclusion, contribution of the study, limitations and the future direction in maintenance planning optimization in the emerging MRO outsourcing mode.

Chapter 2. Literature Review

2.1. Introduction

The number of publications related to aircraft maintenance scheduling optimization has been increasing and gaining popularity as the significant development of aviation industry in the recent year which imposes a great challenge to the operators from different aspects in the industry. This chapter aims to present an overview of the aircraft maintenance scheduling problem. The scope of research on aircraft maintenance scheduling problem arises from different aspects, such as airlines, line maintenance service providers and hangar maintenance service providers, and the maintenance scheduling problem can be incorporated with the other problem for the overall objective within the company. This review aims to classify the research problems on this field systematically first. Then the discussion on hangar maintenance problem is provided later on. Thirdly, a review of the methodology of preventing the irregular polygons from overlapping between each other is presented, which shares the similarities with the aircraft parking stand allocation problem used to measure the capacity of maintenance hangar. A review of the optimization methodology applied in maintenance optimization problems, including approximation algorithms and exact algorithms, is provided later. The research gap is identified at last.

2.2. Classification of aircraft maintenance scheduling problems

Different types of maintenance for aircraft can be roughly classified as: schedule and unscheduled maintenances, line and hangar maintenance, routine and non-routine maintenance, according to the form or the place where the checks are conducted (Van den Bergh et al., 2013). Different terms mentioned above can be used to refer to the same maintenance check. Technically, the maintenance checks can be classified into four types, i.e. A, B, C and D checks, and each type of maintenance check need to be conducted after the aircraft has been flying a certain amount of hour, and the hours are prescribed by the respective aviation authorities around the countries. The four technical maintenance checks can be pre-scheduled or un-scheduled maintenance, according to the actual situation running in business. Therefore, the maintenance checks are not considered as separated ones, but some terms may refer to the same technical checks.

With regard to the four types of technical maintenance checks mentioned above. Each of them is conducted in different frequency, duration, and scope (Gopalan & Talluri, 1998). Different checks are conducted after a certain amount of time or usage, e.g. flight hours or cycles (take-off and landing of an aircraft are regarded as a cycle). To be specific, A check is performed every 400-600 flight hours or 200-300 cycles, which is takes around 50-70-man hours. B check is conducted every 6-8 months and takes several days to complete, and C check is more extensive than B check, which takes one to two weeks to finish. D check is the most comprehensive check that usually takes the whole aircraft apart for thoroughly inspection and overhaul, which can be completed in 2 months. Moreover, the exact maintenance hours required for an aircraft is subject to the aircraft type, flight hours as well as its cycle count. In literature, the term routine and non-routine maintenance are also used. Routine maintenance refers to the schedule maintenance, and the non-routine maintenance checks refer to unscheduled maintenance. Moreover, the other terms, such as layover maintenance referring to the line maintenance conducted at the gate or the apron at the connecting aircraft, predictive maintenance, corrective maintenance and so on, are used according to the authors in their papers. It is the fact that a great variety of terms that are used to refer the different type of technical maintenance checks in the literature related to Operations Research field. Therefore, a general terminology would be convenient for the reader to get the information of the paper. The terms, scheduled, offline, preventive and line maintenance, are generally used ones to explicitly describe the optimization problem that the authors work on. The extensive usage of the term scheduled, preventive and offline indicates the importance of appropriate planning before conducting the maintenance checks. The term corrective scheduling refers to the maintenance scheduling problem conducted after the occurrence of disruption during the process of maintenance checks.

From the perspective of airline companies, minimizing the operational cost can achieve the profit maximization so as to survive in the competitive environment. The operational cost arising from airline company is composed of four problems, i.e. flight scheduling, fleet assignment, maintenance routing as well as crew pairing. Conventionally, the four problems arising in airlines are solved in a sequential manner, which reduces the complexity of the integrated problem substantially while sacrifices the best possible solution (Cohn & Barnhart, 2003; Daskin & Panayotopoulos, 1989; Liang et al., 2011; Mercier et al., 2005). Recently, more attention is devoted to the integrated problem that incorporates two or three problems together (Papadakos, 2009). For example, integrating the maintenance scheduling problem into the flight scheduling problem in order to let the airlines satisfy the mandatory maintenance requirement prescribed by the aviation authorities while carrying out the flight schedule. The integration of the flight scheduling and the maintenance routing has been arising in recent research studies (Gavranis & Kozanidis, 2015; Kozanidis et al., 2012; Kozanidis et al., 2014; Lan et al., 2006), as these two problems significantly influence the profitability of an airline company, the level of customer satisfaction as well their capacity to compete in the competitive market. Moreover, the integration of fleet assignment problem with the maintenance routing is emerging, which determines the route for each aircraft in the fleet that minimizes the cost assigning the aircraft to every flight determined by the flight scheduling problem as well as satisfies the maintenance constraints. Besides, Chen et al. (2017) considered a technicians assignment problem in the context of the maintenance base of the airline, rather than incorporating into flight scheduling problem.

2.3. Hangar maintenance operation

With regard to the classification scheme proposed by Van den Bergh et al. (2013), in which maintenance is categorized into line maintenance and hangar maintenance according to the place where the maintenance check conducted. Line maintenance refers to “on line” maintenance that can be conducted when the aircraft parks at the gate or the apron, and all the other maintenance is categorized as “hangar” maintenance, which needs to be conducted in a maintenance hangar. It is reported that technical delays are the cause of over 20 per cent of disruptions to flight schedules, which is comparable to operational factors (Eriksson & Steenhuis, 2014; Van den Bergh et al., 2013). To minimize the disruption caused by technical problems as well as the cost of maintenance, many airlines have begun to review their MRO operations policy to ensure the airworthiness of each aircraft. Outsourcing of MRO activities is now emerging as a credible solution for airlines to allow them to focus on their high-value added commercial flying activities. It is estimated that the global commercial aviation MRO market will be worth up to USD 60 billion in 2016, and the 10- year demand (from 2016 to 2026) for MRO in the business market is estimated to be USD 121.8 billion (Penton's Aviation Week Network, 2015).

Given the significant development of independent MRO service companies, relatively fewer research studies have considered maintenance problems from aircraft maintenance company's perspective. Chung et al. (2015) provided an extensive review of proactive maintenance planning issues in the aviation industry. Up to date, some studies covered workforce scheduling problems from the aircraft maintenance company's perspective (Belien et al., 2012; Belien et al., 2013). De Bruecker et al. (2015) considered an aircraft maintenance personnel rosters problem from an independent aircraft line maintenance company's aspect, which assumed that the maintenance routing problem was solved and the route was given to several airline companies. Liang et al. (2015) considered an aircraft maintenance routing problem incorporating propagated delay in optimization, and Gavranis and Kozanidis (2015) proposed an exact algorithm to solve a maintenance scheduling problem that maximizes the fleet availability of a unit of military aircraft. The problem we consider here is different from the aircraft stand allocation problem in the airport context (Guepet et al., 2015). In the context of airport operations, the aircraft stand allocation problem mainly focuses on allocating aircraft to the parking gate of a terminal or apron near the terminal for passenger boarding or disembarkation. In such contexts, the accurate physical shapes of aircraft are less considered in scheduling and planning problems as such factors have been incorporated in designing an airport to meet the safety requirements. However, the physical shape of the aircraft becomes the major factor influencing the parking plan in the context of maintenance hangar of MRO service providers, since the parking plan changes flexibly from time to time.

Traditionally, the staffing problem in aircraft maintenance are frequently considered together with the aircraft maintenance routing problem (Belien et al., 2013), as aircraft maintenance activities are conducted by airlines. For example, Chen et al. (2017) considered a technicians assignment optimization problem in the context of an aircraft maintenance hangar operated within single airline company, assuming constant hangar capacity. With the development of MRO outsourcing, some studies covered workforce scheduling problems from the aircraft maintenance company's perspective (Belien et al., 2012; Belien et al., 2013). De Bruecker et al. (2015) considered an aircraft maintenance personnel rosters problem from an independent aircraft line maintenance company serving several airline companies. Liang et al. (2015) considered an aircraft maintenance routing problem incorporating propagated delays in optimization, and Gavranis and Kozanidis (2015) proposed an exact algorithm to solve a maintenance scheduling problem that maximized the fleet availability of a military aircraft unit. From the perspective of maintenance service provider in MRO outsourcing mode, the intensity of workload cannot be changed by revising the maintenance routing decision of fleet in each airline, so as to alleviate or balance the workload during a period of time (Belien et al., 2013). As the maintenance outsourcing decisions are pre-determined by multiple airlines, the maintenance service provider aims to fulfill the maintenance demands within their permissible time windows by utilizing the available maintenance resource. Given the complexity of aircraft maintenance tasks, consideration of multiple skill type and skill levels are commonly adopted and indispensable in the aircraft maintenance staffing optimization. Yan et al. (2004) considered a technician assignment problem in short-term airline maintenance manpower planning, which incorporates multiple types of maintenance skill licenses with flexible management strategies in the mathematical model. Chen et al. (2017) considered a multiple-skill technicians' assignment and problem in an aircraft hangar maintenance operated by a single airline company, with a bi-objective optimization approach to minimize the total labor cost and achieve workload allocation fairness. In literature, most of the staff assignment and rostering problem in aircraft maintenance are correlated to the line maintenance of maintenance company, or hangar maintenance operated by single airline company. The research proposed in this study aims to bridge the gaps between the multi-skill technician assignment problem in MRO industry and hangar maintenance operation under the MRO outsourcing mode.

2.4. Optimization algorithms

As discussed in Section 2.2, integration of problems arising from aviation industry is a possible way to consider the problem in different aspect systematically so as to save the overall operating cost and obtain better global efficiency. However, it is the fact that the integrated scheduling problem are a large-size problem, making the default branch-and-bound algorithm incapable to solve. A review of the solution methods solving the maintenance scheduling problem from different aspects is presented in this subsection.

The classification of solution methods for optimization problem can be categorized into two major groups, namely exact approach, and approximate approach. The approximate approach can be further divided into heuristic and meta-heuristic, and the development of meta-heuristic mainly aims to prevent trapping into local optimum during the searching process. Although the approximate approaches have the advantage of searching for a good solution within a short time, the solution quality obtained by approximate approach is not a guarantee, and the algorithm may be trapped into local optima. Moreover, the approximate approach seldom provides the reference information, i.e. the optimality gap of the obtained solution from the upper bound or the lower bound of the objective function. On the other hand, the exact algorithms refer to the algorithm that always solves an optimization problem to optimality. Given the extensive NP-hard problems in the optimization problems modeled by from the real-life problem. The conventional exact algorithm, such as default branch-and-bound algorithm, becomes incapable to solve the complex optimization problem. In this connection, many problem-specific advanced exact algorithms have been developed to increase the efficiency in solving NP-hard or NP-complete problem.

An efficient maintenance approach is significant for the different operators, such as airline and maintenance service provider, to meet its business objective. Minimizing cost is the most widely used objective while the other terms, such as minimal disruption, tardiness, maximal availability of the fleet and efficient utilization of the resource, are also used to characterize the objective of the optimization problem.

2.4.1. Exact algorithms

A proper mathematical programming formulation, i.e. problem representation, that characterize the optimization problem is a prerequisite for developing an efficient exact algorithm. For example, the flight scheduling problem aims to determine when and where the flights depart and arrives. A typical structure behind the flight scheduling model is a time-space network as the aircraft may depart from home station, visits several out-port stations before finishing its daily operation. Therefore, the foremost framework to formulate this flight scheduling problem is the time-space network (TSN), where nodes in the network represent the possible departure and arrival station, and the arcs in the network represent the possible flights (Clarke et al., 1996; Haouari et al., 2011; Sherali et al., 2010). However, some disadvantages also exist in TSN model as representing the aircraft on the ground is inapplicable for such model. Another widely-used model is mixed-integer multi-commodity flow problem, which overcomes the disadvantages of the TSN model. In mixed-integer multi-commodity flow problem (Clarke et al., 1996; Gabteni & Gronkvist, 2009; Haouari et al., 2009; Rushmeier &

Kontogiorgis, 1997), the commodities represent the fleets, and the constraints in the model make sure the solutions are feasible. Moreover, the set-partitioning based formulation (Cohn & Barnhart, 2003; Gao et al., 2009), job shop problem (Ahire et al., 2000), time-line graph network (Clarke et al., 1996) and connection network (Sarac et al., 2006) are also adopted to formulate the routing problem.

It is clear that the integrated scheduling problems are very larger problem, even the problem instance is in moderate level. Therefore, the decomposition methods from exact algorithms are widely adopted to reduce the complexity of solving the whole problem. Some row-generation-based decomposition approaches, such as Benders' Decomposition (BD) (Rahmaniani et al., 2017), dynamic programming, Dantzig-Wolfe decomposition and branch-and-cut algorithm (Ceria et al., 1998; Marchand et al., 2002), are adopted by researchers. Moreover, the column generation or branch-and-price algorithm are also adopted while the original problem involves a large number of decision variables. Benders' Decomposition algorithm is proposed to solve a class of MILP problem, in which the integer variables are fixed, and the resulting problem comes to the continuous problem solved by dual theory to improve the computational efficiency. Nowadays, BD algorithm has a broad range of extension to solve various of optimization problem, and it also provides a theoretical framework for those intractable problem by conventional approaches (Rahmaniani et al., 2017).

Sherali et al. (2010) studied the integrated flight scheduling and the fleet assignment problem, which consider choosing the optimal flight legs and the assignment of aircraft type. In their model, the optional legs and multi-fare classes are taken into the objective function. After constructing the mixed-integer programming (MIP) model, they conduct a polyhedral analysis to deduce several class of valid inequalities to tighten the formulation, which is solved by Benders' decomposition approach afterward. Haouari et al. (2009) and Haouari et al. (2011) adopted both exact and heuristic approach to tackle the integrated fleet assignment and maintenance routing problem. Haouari et al. (2011) developed an assignment-based as a set partitioning formation solved by Benders' decomposition and a branch-and-price method. Gao et al. (2009) proposed a method to construct an effective approach that provides fleet assignment solution and crew plan with robustness. The solution ensures the number of fleet types that are going to serve an aircraft is within the limit of the specified range. The model is solved by a branch-and-bound technique.

2.4.2. Approximation algorithms

The approximation algorithms can be regarded as a model-free approach, compared with the exact algorithm. One of the advantage of the approximation algorithms is solving the multi-objective optimization problem and generate Pareto-optimal or so-called Pareto-frontier for the operator to make the decision, while the exact algorithm relies on the construction of objective function, where the weight of each term that attributes to the objective value has to be assigned before the implementation of optimization. Therefore, the outcome of the exact algorithm is a single global optimal solution based on the pre-assigned weights on each term, and the outcome of approximation algorithm solving the multi-objective problem is a set of Pareto-optimal solutions, where each solution in Pareto-frontier is equivalent to each other. Quan et al. (2007) developed an aircraft preventive maintenance scheduling problem with a preference-based EA, which obtains Pareto-optimal solution that balances the minimal number of workers, makespan and the preference of airline's preference.

The methodology of obtaining a near optimal solution with avoiding trapping into local optimum heavily relies on the stochastic mechanism for enhancing the searching performance of the meta-heuristic and searching for a better solution based on the knowledge or so-called information derived earlier from previous search (Boussaid et al., 2013). The searching approach utilizes the current solution to perform a trajectory search to obtain a better solution. If a better solution is found, then the currently best-known solution is replaced by the better solution. It is possible that the searching process obtains a better solution at each iteration while approaching to the local optima, which is a common problem of the meta-heuristic approach. To escape from local optimum, the diversification search mechanisms are designed and incorporated into the improved algorithms (Vidal et al., 2013). The stochastic meta-heuristic algorithm can be classified as: neighborhood structure and memory structure. Greedy Randomized Adaptive Search Procedure (GRASP) is a typical stochastic algorithm proposed by (Feo & Resende, 1995), which overcomes the limitation of greedy algorithms from being trapped into a local optimum. The search process of GRASP starts from two phases, i.e. balancing exploitation and exploration, until the looping reaches the limits of iteration. The hill climbing approach is another advanced constructive algorithm that searches with randomized neighborhood structure.

Furthermore, biological evolution is another group of meta-heuristics that utilizes the performance of population solution. The main idea behind the biological evolution is the hereditary through the genetic information or ancestral memories from a group of candidate solutions (Holland, 1992). The mechanism in biological evolution relies on natural selection and genetic variation. The process of natural selection allows evolutionary changes of maintaining certain merits or advantages in a population to adapt to the environment, while genetic variation is the process that an individual is stronger and therefore suitable to live in a situation than the others in the population.

Genetic Algorithm (GA) is a typical biological evolution algorithm that searches based on the process of natural gene selection in heritage (Deb et al., 2002). The solution quality obtained by GA is evaluated by the related objective function, then generate a more favorable solution according to the biological process. Cheung et al. (2005) developed a genetic algorithm to solve the aircraft maintenance scheduling problem with minimal flow time.

The physics-based algorithm is another type of approximation algorithms that represent the natural practice of physical or chemical discipline, such as the reaction in chemistry, the process of heating the iron and so on. One of the most well-known physical-based heuristic algorithms is Simulated Annealing (SA) algorithm that simulates the heat treatment in material science aspect. SA attempts to improve the Local Search (LS) by occasionally allowing the non-improving neighbors to enter the next iteration with some probability, and the probabilities decrease as the number of iteration increase. X. Q. Xu et al. (2013) developed a robust makespan minimization problem under the context of parallel machine scheduling problem, using SA to tackle the large-scale problem. The theory behind the SA is to accept the non-improving solution to escape from the local optimum.

2.5. Research gap

After reviewing the literature related to aircraft maintenance schedule problems from different aspects, the hangar maintenance operations from maintenance service providers' aspect, the methodology of preventing a pair of aircraft from overlapping in two-dimensional space and the optimization algorithms, the following research gaps are identified:

1) Investigation on aircraft maintenance scheduling problem from maintenance service providers' aspect

Given the extensive research studies on maintenance scheduling optimization problem from different aspects, the optimization model from maintenance service provider's aspect is scarce. However, the existing approaches successfully solve the scheduling problem from the other aspects cannot be directly adopted to solve the problem rising from hangar maintenance service provider's perspective since it entails the resource constraint that is not incorporated from the other perspective due to the practice of real operations. Therefore, the maintenance scheduling problem shall be further investigated.

2) Development of hangar capacity measurement approach in the context of maintenance service providers

In literature, the assumption in hangar maintenance scheduling problem is that the capacity of hangar remains constant through the planning horizon. Such assumption is practical and applicable for those hangars owned by airlines as the type of aircraft is known in advanced, and therefore the capacity of hangar does not change during the whole planning period. However, the maintenance hangar operated by maintenance company receives aircraft in a different size. Therefore the capacity of hangar varies according to the incoming demands. As a result, the conventional assumption is impractical in such context, and development of a proper hangar capacity measurement approach is necessary.

3) Development of integrated optimization model for maintenance service provider

The maintenance scheduling optimization model in literature assumes the hangar capacity is unchanged given the predetermined parking stand in the hangar. However, such practice is changed in the context of maintenance service providers. Therefore, a model that integrated the maintenance scheduling problem, the multi-period parking planning and manpower rostering problem that allows the optimization algorithm simultaneously to determine the schedule, layouts and manpwoer roster is significant, as the interdependent relations among these three core elements of decision making shall not be neglected. The qualtify of the solution influeneces the applicability of the obtained solution in real-world operations.

4) Development of algorithm for integrated maintenance planning problem under the MRO outsourcing mode

It is concluded that maintenance planning problem under the MRO outsourcing mode encompasses three core elements in decision-making (service time determination, hangar parking stand allocation & aircraft movement operations and manpower rostering), which have strong interdependent relations among each other. It is justified afterwards that the integrated model is complicated and intractable by the default algorithm provided by the optimizer. Therefore, development of algorithms with efficient searching mechanism is crucial for practical uses and adaptation in real world operations.

2.6. Summary

Aircraft maintenance scheduling problems have received much attention in existing research studies along decades. This chapter presents a review of the literature related to maintenance scheduling problem from different aspects in aircraft maintenance industries and provides fundamental supports for justify the rationale of studying and developing novel maintenance planning optimization approach for the aircraft hangar maintenance under MRO outsourcing mode.

From the literature review, it is noted that the variance of hangar capacity receives little attention in previous studies in aircraft maintenance aspect. Such factor becomes crucial in carrying out the hangar maintenance schedule under the MRO outsourcing mode. Faced with increasing outsourcing demands from multiple airlines and other clients, the maintenance service company has to carefully arrange their limited maintenance resources and construct an efficient maintenance plan. The improper maintenance schedule may result in a series of infeasible parking layout, and manpower assignment. Inappropriate decision on arranging the parking layout and manpower can also induce tardiness in fulfilling the clients' maintenance request in return. Therefore, it is indispensable to integrate the three core elements in decision-making under outsourcing mode to produce an efficient and practical solution for the maintenance service provider, enabling them to remain competitiveness in the market. Furthermore, the outcome of the integrated model is a complex mathematical, and the problem instance from the real-world is expected to be challenging. In this regard, development of an efficient algorithm to tackle the optimization problem is a significant step in the research study.

Chapter 3. Aircraft Parking Stand Allocation Problems

3.1. Introduction of aircraft parking stand allocation problem

An aircraft parking stand allocation problem for aircraft hangar maintenance under MRO outsourcing mode is studied in this chapter. Given a set of maintenance requests on a peak day that exceed the capacity of the maintenance hangar, the service provider has to select and first serve the particular subset of aircraft that maximizes their overall profits and then rearrange the remaining requests later. The objective of the proposed problem is to determine a subset of maintenance orders with maximal overall profits and a feasible parking plan on a peak day. In particular, there is to be no overlap between aircraft, and the risk of collision measured by the shortest distance between each pair of aircraft is to be minimized. To model and tackle the proposed optimization problem, No-Fit Polygon (NFP) construction is adopted to prevent overlap between pairs of aircraft. After modelling the problem, a two-stage MIP approach and heuristic algorithm are introduced in order to improve the efficiency of the branch-and-bound algorithm in the second stage problem. Testing instances in computational results are generated based on the real situation in an aircraft maintenance company, and the effectiveness of the proposed approaches are evaluated through computational experiments afterwards.

From the perspective of aircraft hangar maintenance companies, efficiently fulfilling the increasing maintenance requests from clients becomes a challenging task due to limited resource availability. The limited parking space for accommodating multiple incoming aircraft become challenging. The Operations Manager has to arrange aircraft parking stand allocation plans with a maintenance schedule over a given period of time.

In practice, a maintenance hangar fully owned by an airline is predefined into several fixed parking stands at the design stage due to the limited types of aircraft models they own (Figure 1-1 (a)), and aircraft are then parked at predefined stands during hangar maintenance. Differing from the maintenance hangar owned by airlines in which the capacity of hangar can be easily quantified, independent hangar maintenance companies receive maintenance orders from different clients (e.g. full-cost airlines, low-cost airlines, cargo airlines, government flying services as well as private aircraft owners), which means that the maintenance company needs to flexibly arrange those aircraft in different sizes in the hangar (Figure 1-1 (b)). Moreover, safety considerations should be taken into account when carrying out an aircraft parking plan.

Each client served by an MRO service provider establishes its own maintenance plan, and then each client proposes its desired maintenance period to the MRO service provider. In this chapter, it is regarded that after receiving the maintenance requests, the maintenance company gathers the subset of aircraft with similar proposed arrival times, service requirements and estimated departure times, and then reviews its maintenance capability and notices their availability to the client. If the maintenance company has the capacity to serve such a set of aircraft, these aircraft are grouped into a "batch" then rolled into the hangar together for maintenance (batching mode). The maintenance tasks are finished around the same time, and the aircraft depart from the hangar within a similar timeframe. Although maintenance scheduling problems have been extensively studied in the literature, there is no available approach to quantify the capacity of a maintenance hangar, given the variance of the maintenance requests from time to time in the maintenance company, and manual planning is adopted in those maintenance companies that serve aircraft of difference size. Nowadays, the maintenance requests from different customers are likely to cause crowding on some days, and a situation when the hangar cannot accommodate all the incoming aircraft at the same time over the whole planning period is common, given the increasing outsourcing maintenance requests. Under such circumstances, the maintenance company has to negotiate with some clients to adjust their respective maintenance periods. Given that a set of arriving aircraft may exceed the capacity of the hangar space on peak days, the maintenance company aims to come up with a parking plan that makes the most of their hangar space. The attentions are paid on providing an approach that assists independent hangar maintenance companies in maximizing their overall profits while minimizing the risk of collision in the hangar, particularly in situations where the hangar is not capable of accommodating all the incoming maintenance requests from clients during their peak days. The solution provides reference information for the company management in the process of planning the maintenance schedule and in replying to clients' maintenance requests. According to the best knowledge of the authors, no research to date has addressed the hangar space utilization problem in the context of hangar maintenance for independent MRO service providers.

The remainder of this chapter is divided into the following sections. In Section 3.2, the literature related to the methodology of preventing aircraft from overlapping is described. The concept of the No-Fit Polygon (NFP) that prevents overlap between aircraft is discussed in Section 3.3. In Section 3.4, the proposed two-stage approach is described in detail. In Section 3.5, a heuristic algorithm is proposed as a warm start for the exact algorithm, which is followed by discussion on branching strategies. Section 3.6 presents the results of the numerical experiments and we then analyze the effectiveness of the proposed algorithm. Section 3.7 presents the layouts for selected challenging instances. Finally, concluding summary is presented in Section 3.8.

3.2. Review on non-overlapping constraints development

Some similarities exist between the aircraft parking stand allocation problem and the cutting and packing problem in a two-dimensional fixed dimension container. The two-dimensional packing problem arises whenever one needs to place irregular items inside a container without overlap. According to Dyckhoff (1990), such irregular shape packing problems are referred to as nesting problems and are mainly characterized by the number of relevant dimensions. A survey conducted by Wäscher et al. (2007) provided an improved topology for cutting and packing problems, and the aircraft parking stand allocation problem can be classified as an output maximization problem in fixed dimensions.

Several approaches have been proposed in the literature to cope with the problems of detecting and preventing overlap between two irregular polygons. The most widely used tool for checking whether two polygons overlap is the No-Fit Polygon (NFP) while other approaches, such as the finite-circle method (W. Zhang & Zhang, 2009), are also available. Approaches to generate NFP include the orbiting algorithm proposed by Mahadevan (1984) and improved by Burke et al. (2007); the Minkowski sums approach by Milenkovic et al. (1991), J. A. Bennell et al. (2001), which was updated by J. A. Bennell and Song (2008); and decomposing non-convex polygons into several star-shaped polygons (Daniels et al., 1994) or convex polygons (Agarwal et al., 2002). Julia A. Bennell and Oliveira (2008) and J. A. Bennell and Oliveira (2009) provided a detailed tutorial on how to generate NFP between two non-convex irregular polygons. Moreover, several integer programming formulations that solve nesting problems have been proposed. Gomes and Oliveira (2006) proposed a Simulated Annealing (SA) algorithm and applied a mixed-integer linear formulation in their items compaction phase. Fischetti and Luzzi (2009) introduced the concept of slice that partitions the region outside the NFP. Alvarez-Valdes et al. (2013) introduced a horizontal slice formulation to enhance the formulation of Fischetti and Luzzi (2009). Antonio Martinez-Sykora, Alvarez-Valdes, Bennell, and Tamarit (2015) adopted horizontal slices in their MIP formulation to solve the irregular pieces packing problem with guillotine cuts. Furthermore, A. Martinez-Sykora et al. (2017) applied horizontal slices in the formulation of the bin packing problem with free rotations. Recently, Cherri et al. (2016) proposed two robust mixed-integer formulations for the irregular polygon packing problem that decompose the non-convex polygons into several convex pieces in order to generate NFP.

3.3. No-fit Polygon (NFP) construction

3.3.1. Geometric representation

The geometric shape of the aircraft is characterized as a non-convex polygon (Figure 3-1). We denote the reference point of each aircraft to be the middle point at the bottom of the aircraft, and the coordinates of the reference point of aircraft p_i in two-dimensional space are denoted as (x_i, y_i) .

3.3.2. Generation of NFPs

For a pair of polygons p_i and p_j , the No-fit Polygon NFP_{ij} is the region in which the reference point of polygon p_j cannot be placed if polygon p_i remains relative stationary. The feasible zone for placing polygon p_j without overlap with p_i is the region outside NFP_{ij}. Given these two polygons, the NFP_{ij} is generated by tracing the path of the reference point on p_j as p_j slides around the boundary of p_i such that two polygons always touch but never overlap (Figure 3-2). The Minkowski sums approach (J. A. Bennell et al., 2001; J. A. Bennell & Song, 2008) is a widely used approach generated NFP between two irregular polygons, and is adopted in this chapter to generated NFP between aircraft.

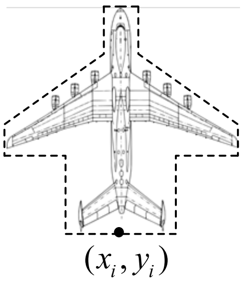


Figure 3-1 Geometric representation and reference point of aircraft

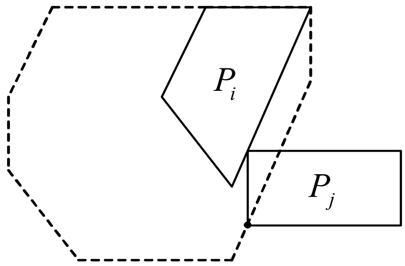
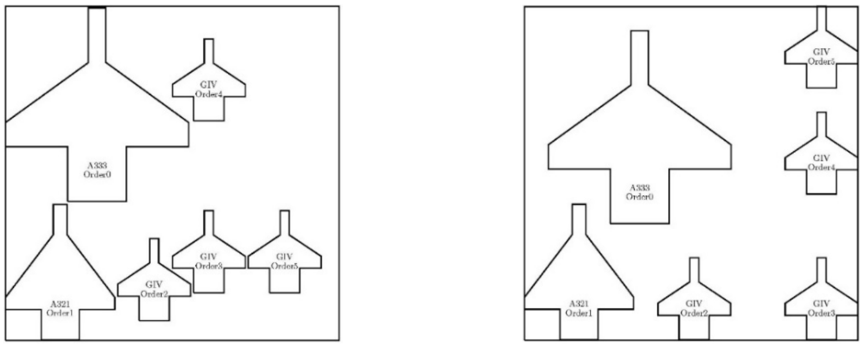


Figure 3-2 No Fit Polygon of P_i and P_j

3.4. Mixed integer formulations

The models developed in this section aim to solve the aircraft parking layout allocation problem for independent hangar maintenance service providers. In particular, the objective is to determine a subset of aircraft maintenance requests with maximal overall profits, as well as to finalize the aircraft parking layout plan with maximal overall safety margins. According to the practice adopted in maintenance service providers, as described in Section 1, the aircraft selected to be served during a short period are batched and rolled into the hangar at the same time, with a predetermined sequence before the maintenance tasks begin, then the aircraft are rolled out after all maintenance tasks are finished in the hangar. In this regard, consideration of rolling in and out of a particular aircraft while conducting the maintenance tasks for the other aircraft is not taken into account under such practice, and the assumptions of the problem are as follows: 1) the position of the aircraft in the hangar remains unchanged once rolled into the hangar; 2) for the sequences of rolling in and out, the aircraft in the inner part of the hangar are rolled in first, followed by the subsequent aircraft arranged near the hangar entrance before maintenance operations; and the roll out operations are conducted in a reverse manner; 3) the maintenance tasks begin after all aircraft are parked in their assigned position, and the aircraft can be rolled out operations until finishing all aircraft maintenance tasks in the hangar. Therefore, the sequence and routing of rolling in and out can be determined according to the outcome of the static parking layout without further stipulation for each case, and the movement operations are not incorporated as a part of the mathematical model for decision making.

We first introduce the concept of Horizontal Slices in Section 4.1, as proposed by Alvarez-Valdes et al. (2013), to prevent overlaps between the polygons in the MIP model. In Phase I, we aim to find the subset of the maintenance order with maximal overall profits. In Phase II, the safety margin of the derived parking plan is considered. It is worth to point out the difference between the problems in these two phases in order to justify the proposed two-stage approach. The objective of the Phase I problem is to find out the subset of aircraft with maximal profits while satisfying the minimal safety margin requirements, therefore, the revised NFPs associated with minimal safety margin are imposed in the model to ensure that aircraft are separated from each other by, at least, the minimal safety distance. The initial parking layout derived from Phase I may be not applicable for practical use. It could be the case in the Phase I solution that aircraft might be placed close to each other or in a concentrated manner in the hangar, even if a large space is left unoccupied and the minimal safety margin is met. In practice, maintenance companies would try to enlarge the distance between pairs of aircraft to minimize the risk of collision during the batch movements, which is different from the cutting and packing problems in the literature that place the items as compactly as possible. The Phase II problem is therefore proposed to provide an exact measurement of the safety margin for aircraft, and finalizes the parking layout. Two figures (a & b) of Figure 3-3 shows an example of the difference between parking layouts derived from the two phases.



(a)
Phase I layout

(b)
Phase II layout

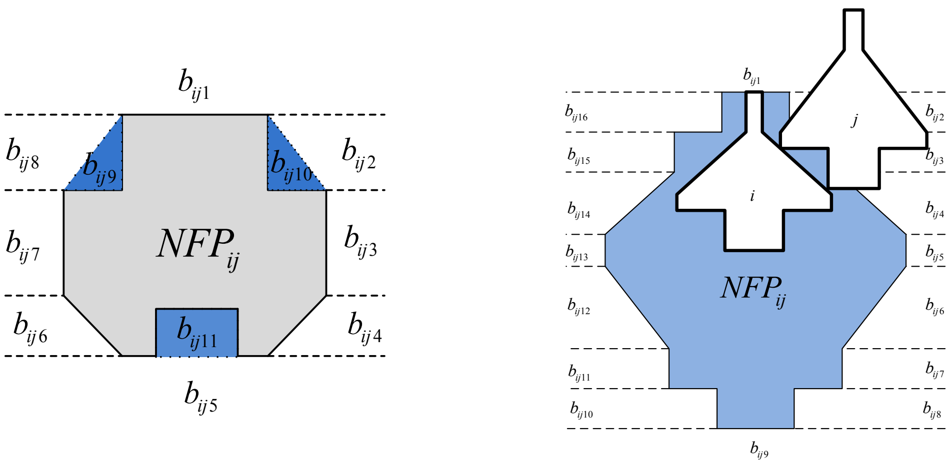
Figure 3-3 Comparison between parking layouts derived from two phases

3.4.1. Concept of horizontal slices

Alvarez-Valdes et al. (2013) improved the approaches of Gomes and Oliveira (2006) and Fischetti and Luzzi (2009) by partitioning the region outside the NFP horizontally (Figure 3-4 (a)). According to Alvarez-Valdes et al. (2013), each horizontal slice is defined by drawing one or two horizontal line(s) outward from each vertex of the NFP, and they are then characterized by one or two horizontal edge(s) as well as the part of the boundary of the NFP. A set of variables b_{ijk} is associated with each horizontal slice and the reference point of p_j is placed in the slice *k* if $b_{ijk}=1$. Therefore, a general form of the constraint preventing overlap is

$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) \le q_{ijk} + M \cdot (1 - b_{ijk}),\ \forall i, j \in P, i \ne j, k = 1,2,\dots,m_{ij}$$

where $\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) = q_{ijk}$ is the equation of the line of the *f*th edge of the *k*th slice in NFP_{ij} and m_{ij} is the number of slices outside the NFP_{ij}. To deal with the concavities in NFPs, Alvarez-Valdes et al. (2013) closed them (the dark shade regions associated with variables b_{ij9}, b_{ij10} and b_{ij11} in Figure 3-4 (a)) until the resulting NFP polygon is convex, then used binary variables to represent each closed concavity. Such treatment is devoted to dealing with the "cave" in the NFP from the vertical direction that is similar to the concavity associated with b_{ij11} in Figure 3-4 (a), which cannot be represented by the horizontal slices. It is found that such a vertical "cave" does not exist in the NFP between two aircraft and all concavities can be represented by the horizontal slicing method. Therefore, the concavities of the NFPs remain unchanged in our problem and the NFP of two aircraft can be found, as in Figure 3-4 (b). Generally, the number of binary variables used to label horizontal slices for an NFP between two aircraft is more than 14.



(a) (b)

Figure 3-4 Horizontal slices outside NFP

3.4.2. Phase I

The objective in this phase is to find the subset of maintenance orders with maximal overall profits given a set of maintenance orders that may exceed the capacity of the hangar in fulfilling all of them, while meeting the minimal safety margin between a pair of aircraft. The methodology to enforce the safety margin between aircraft is described in detailed in Section 3.4.3.1. In general, the profit value of an aircraft is associated with its size, and we prescribe that the profit value for each aircraft is determined by its respective area, since larger aircraft always need more effort in serving and receive more profit, compared with small-sized aircraft. Therefore, the problem is equivalent to minimizing the unused space in the parking plan under such assumption.

3.4.2.1. MIP formulation for Phase I

We first list the parameters, sets and decision variables, along with their associated indices in the Phase I formulation:

Notations

W	width of hangar
H	length of hangar
i	maintenance request associated with aircraft i
P	set of maintenance orders. $1,2,...,i,j \in P$
A_u	a subset of aircraft from P that belongs to the same aircraft type u
v_i	area of aircraft i
w_i	width of aircraft i
h_i	length of aircraft i
NFP_{ij}	NFP of aircraft i and j
s_{ij}^k	kth slice of the region outside the NFP_{ij}
$\alpha_{ij}^{kf}, \beta_{ij}^{kf}, q_{ij}^{kf}$	parameters used to define the fth linear equation of the slice s_{ij}^k outside the NFP_{ij}
m_{ij}	number of slices outside NFP_{ij}
t_{ij}^k	number of linear equations used to define the slice s_{ij}^k
M	large number

Decision Variables

z_i	binary decision variable that takes the value 1 if aircraft i is placed in hangar, and 0 otherwise
x_i	position of reference point of aircraft i on x-axis in two-dimensional space
y_i	position of reference point of aircraft i on y-axis in two-dimensional space
b_{ijk}	binary decision variable that takes the value 1 if the reference point of aircraft j is placed into the slice s_{ij}^{k} of the region outside NFP_{ij} to separate aircraft i and j, and 0 otherwise

$$Objective: \text{Maximize Hangar Utilization} \quad (F1)$$

$$Min\ W \cdot H - \sum_{i \in P} v_i z_i \tag{3-1}$$

$s.t.$

$$x_i + w_i / 2 \le W,\ \forall i \in P \tag{3-2}$$

$$x_i \ge w_i / 2 - M \cdot (1 - z_i),\ \forall i \in P \tag{3-3}$$

$$y_i + h_i \le H,\ \forall i \in P \tag{3-4}$$

$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) \le q_{ij}^{kf} + M \cdot (1 - b_{ijk}),\ \forall i, j \in P, i \ne j,\ \forall k = 1,2,...,m_{ij},\ \forall f = 1,2,...,t_{ij}^{k} \tag{3-5}$$

$$\sum_{k=1}^{m_{ij}} b_{ijk} \le z_i,\ \forall i, j \in P, i \ne j \tag{3-6}$$

$$\sum_{k=1}^{m_{ij}} b_{ijk} \le z_j,\ \forall i, j \in P, i \ne j \tag{3-7}$$

$$\sum_{k=1}^{m_{ij}} b_{ijk} \ge z_i + z_j - 1,\ \forall i, j \in P, i \ne j \tag{3-8}$$

$$z_i \ge z_j,\ \forall i, j \in A_u; i < j \tag{3-9}$$

$$b_{ijk} \in \{0,1\}\ \forall i, j \in P, i \ne j, k = 1,2,...,m_{ij} \tag{3-10}$$

$$z_i \in \{0,1\} \quad \forall i \in P \tag{3-11}$$

$$x_i, y_i \ge 0 \quad \forall i \in P \tag{3-12}$$

The objective function (3-1) minimizes the unused space in the maintenance hangar, which is equivalent to maximizing the overall profits of the parking plan. Constraints

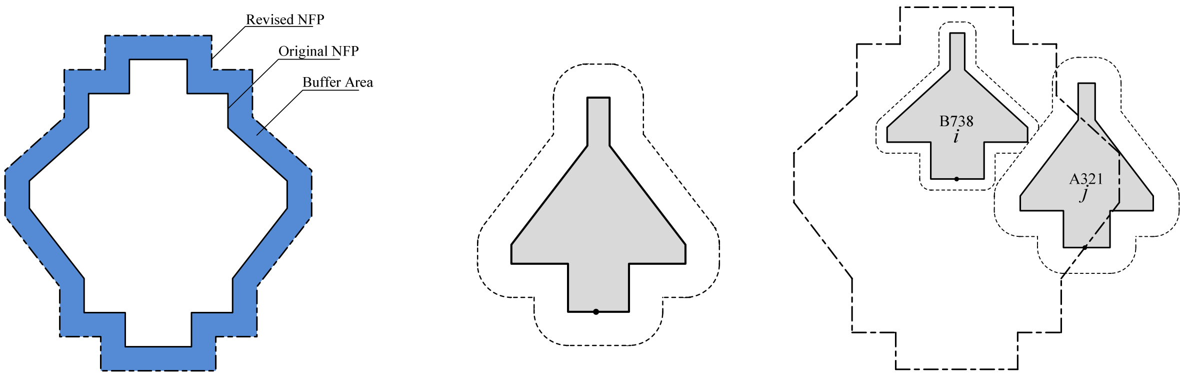
(3-2) - (3-4) are bound constraints that prevent aircraft exceeding the hangar boundary. As the reference point of each aircraft is on the middle point of the bottom, x_i of aircraft i should be equal or larger than $w_i / 2$ to keep the aircraft within the boundary of the hangar (Constraint (3-3)). Constraint (3-5) prevents overlap between each pair of aircraft in the hangar. b_{ijk} are sets of binary variables and one of them must take the value 1 if aircraft i and j are placed in the hangar. Constraints (3-6) - (3-8) indicate that the non-overlapping constraints associated with the minimal safety margin requirement between aircraft i and j are activated only if two aircraft are placed in the hangar. For those aircraft to be placed in the hangar, $z_i = 1$ activates constraints (3-3) and (3-5) – (3-8) to impose boundary and non-overlapping constraints for each aircraft in the hangar, otherwise $z_i = 0$ deactivates non-overlapping constraints for those aircraft that are not going to be placed in the hangar. Constraints (3-10) – (3-11) indicate that b_{ijk} and z_i are binary variables.

It is common that several incoming maintenance requests have the same type of aircraft and we use A_u to denote those sets of aircraft that belong to aircraft type u. If these aircraft belong to the same aircraft type from different clients are considered as different aircraft in the problem, the algorithm has to fathom partial and complete solutions, which are identical to the other solutions already studied in other branch trees. Therefore, Constraint (3-9) sets $z_i \geq z_j (i, j \in A_u; i < j)$ for the aircraft that belong to the same aircraft type in order to prevents duplicate solutions and saves unnecessary computational effort.

3.4.3. Phase II

After the subset of maintenance orders with maximal overall profits is found, the overall safety margins in the hangar are maximized in the Phase II problem. We define the safety margin as the shortest distance between two aircraft in two-dimensional space (Figure 3-5 (c)) and the overall safety margins are calculated based on the weighted sum of the individual safety margin of each aircraft placed in the hangar. As described at the beginning of Section 3.4, the proposed models assume that rolling in and out operations are conducted in a batch before conducting maintenance tasks and after finishing all maintenance tasks in the hangar respectively. With this regard, the manoeuvrability and risk of collision is considered to facilitate the batch movements operations, and the rolling in and out sequences are not in incorporated in the decision-making, as mentioned in Section 3.4. Large-sized aircraft bear more risk of collision than medium-sized and small sized aircraft since larger aircraft are not as manoeuvrable as small aircraft during the batch movement operations. Therefore, the large-sized aircraft are given higher priorities so as to reserve larger safety margin in practice, which enhances the smoothness of the batch movement operations. To meet the practical requirement in maintenance operations, we prescribe that the weight of each aircraft in the objective function in the Phase II problem is determined by the respective aircraft area.

3.4.3.1. Revised NFPs



(a) (b) (c)

Figure 3-5 Revised NFP and safety margin of aircraft

For a pair of aircraft, the distance used to separate the two aircraft is determined by the aircraft with larger safety margin. In Figure 3-5 (c), assume that the safety margin of aircraft i (aircraft type: B738) and j (aircraft type: A321) are 3 meters and 5 meters, respectively. Since aircraft j has a larger safety margin than aircraft i, the shortest distance between the two aircraft is determined by aircraft j. Imposing a safety margin for an aircraft is equivalent to adding a buffer area outside each aircraft in order to prevent other aircraft from entering the buffer area from any direction, and therefore the buffer area outside an aircraft is formed by edges and arcs (Figure 3-5 (b)). According to the definition of NFP, aircraft j touches aircraft i if the reference point of aircraft j is placed on any edge of NFP between aircraft i and j. Moving the edges of NFP for a pair of aircraft outward is equivalent to enlarging the boundary of non-allowable area for the reference point of the relative movable aircraft in that pair, i.e. aircraft j in Figure 3-5 (c). Therefore, to impose a safety margin n between two aircraft, we revise the original NFP to separate the two aircraft by moving each edge of the original NFPs outward by distance n (Figure 3-5 (a) and (c)).

We discretize the safety margin of the aircraft and prescribe the lower bound *lb* and upper bound *ub* of the safety margin *n* ($lb \le n \le ub$) in consideration of practical applications and the tradeoff between the accuracy of the parking plan and the computational efficiency. In actual operation, aircraft are given the coordinates of the parking stands then towed or pushed to the assigned position by a tow truck. Pushing or towing aircraft to the exact position assigned by the planner is rather difficult in reality, and therefore a deviation tolerance between the actual and assigned positions is usually given. Therefore, setting a safety margin as a continuous variable is intended to increase the accuracy of the parking layout, however such an assumption is not that practical in the actual situations. In addition, setting safety margins as continuous also increases the computational burdens on the Phase II problem: we have to construct a significant number of revised NFPs with a set of binary variables. Similarly, the upper bound of the safety margin provides an interpretation in the context of practical operation: separating two aircraft with a safety margin larger than the upper bound of the safety margin does not contribute to the overall safety margin and therefore is not considered in decision-making. Such a limit also controls the scale of the problem in Phase II. As we revise the original NFP, i.e. enlarge the original NFP by removing each edge of the original NFP outwards so as to construct a set of revised NFPs to separate aircraft by a given safety margin *n* in Phase II, activating the revised NFP with safety margin *n* indicates that the algorithm tries to impose the shortest distance between this pair, at least equal to *n*, or larger than *n*.

3.4.3.2. MIP model for Phase II problem

The formulation of the Phase II problem (F2) is similar to that of the Phase I problem (F1) with the modification on activation/deactivation of non-overlapping constraints. The primal decision variables of the Phase II problem are the individual safety margins

z_i^n of the subset of aircraft $i \in P'$ that are derived from the Phase I problem with maximal overall profits. As seen in Figure 3-3, the aircraft parking layout is finalized by enlarging the safety margin of each aircraft in the Phase II problem, with an objective function $Max \sum_{i}^{P'} \cdot \sum_{n=lb}^{ub} z_i^n \cdot n \cdot Area_i$ where z_i^n denotes the binary decision variable associated with admissible safety margin n ($lb \le n \le ub$) of aircraft $i \in P'$ and $Area_i$ refers to the area of the aircraft. NFP_{ij}^n denotes the revised NFPs for a pair of aircraft i and j with safety margin n. Similar to the notation characterizing the non-overlapping constraints in Phase I model, s_{ijn}^k refers to the kth slice outside the region of the revised NFP of aircraft i and j with safety margin n; $\alpha_{ijn}^{kf}, \beta_{ijn}^{kf}, q_{ijn}^{kf}$ are the parameters used to define the fth linear inequality of the slice s_{ijn}^k outside NFP_{ij}^n; m_{ij}^n is the number of horizonal slices outside NFP_{ij}^n and t_{ijn}^k records the number of linear inequalities of the kth slice outside NFP_{ij}^n. b_{ijk}^n is binary decision variable that takes the value 1 if the reference point of aircraft j is placed into slice s_{ijn}^k, and g_{ij}^n is introduced as auxiliary binary decision variable that takes the value 1 if the shortest distance between aircraft i and j is n and NFP_{ij}^n is activated. The coordinates of the aircraft i, i.e. x_i and y_i, are confined by the decision variables mentioned above to finalize their parking positions. Moreover, A_u records a group of aircraft that belong to the same aircraft type u, which is used to created constraints preventing duplicate solutions. The constraints involved in the Phase II model consist of: 1) boundary constraints preventing aircraft from exceeding the boundary of hangar; 2) non-overlapping constraints activated/deactivated by b_{ijk}^n, g_{ij}^n and z_i^n; 3) logical constraints prescribing that each aircraft $\forall i \in P'$ must select a safety margin within the admissible range $\sum_{n=lb}^{ub} z_i^n = 1$, and each pair of aircraft is separated by one revised NFP within the admissible range as well , i.e. $\sum_{n=lb}^{ub} g_{ij}^n = 1, \forall i, j \in P', i \neq j$; and 4) the constraint preventing duplicate solutions for the set of aircraft A_u belong to same type *u*, which prescribes that safety margins for the aircraft belong to the same aircraft type are in a decreasing order, i.e. $\text{margin}_{i1} \geq \text{margin}_{i2} \geq \cdots \geq \text{margin}_{in}$. It may happen that the hangar cannot accommodate the whole subset of aircraft derived from Phase I after imposing safety margins in (F2), if we use the original NFP to solve (F1). To solve this problem, the original NFPs are revised by the lower bound of the safety margins while solving the Phase I problem to impose the minimal safety margin requirement.

3.5. Branch and Bound algorithm

The formulations described in Sections 3.4.2 and 3.4.3 can be solved by a branch-and-bound algorithm. In this section, we focus on improving the efficiency of the branch-and-bound algorithm for the Phase II problem by providing an initial solution from the heuristic as well as introducing branching strategies.

3.5.1. Heuristic algorithm in Phase II problem

Determining the feasibility of a solution in Phase II can be difficult. Specifically, the branch-and-bound has to fathom every possible node of b_{ijk}^n in a given combination of safety margins (set of z_i^n) to prove infeasibility. A heuristic that tightens the upper bound in the Phase II problem (F2) as well as providing a moderate upper bound of safety margin is proposed. Each iteration of the heuristic examines the feasibility of a given combination of safety margins by checking whether the algorithm can feasibly park all aircraft using a MIP model: We construct a MIP model *feasibility check* by revising the formulation of the Phase I problem, i.e. (3-1) – (3-12). In detail, the original NFPs are replaced with the revised NFPs determined by the set of safety margins, and the objective function of the *feasibility check* is $\max \sum_{i \in P'} z_i$, which indicates the number of aircraft placed in hangar. If the optimal objective value of the *feasibility check* model equals the number of aircraft in set P', then the hangar is able to accommodate all the aircraft and such a solution is "feasible" in (F2). Otherwise the optimal value is less than the number of aircraft in set P', which implies that at least one aircraft cannot be placed in the hangar with the given safety margin, and such solution is "infeasible" in (F2). The heuristic algorithm can be described in three steps:

Step 1: We try to place the subset of aircraft derived from Phase I in the hangar with some combinations of safety margins (set of $z_i^n, i \in P'$) by calling the *feasibility check*. The initial individual safety margin for each aircraft equals to the lower bound of the safety margin in this step. If all of those aircraft can be feasibly placed in the hangar with the given safety margins, the respective objective value $Max \sum_{i}^{P'} \cdot \sum_{n=lb}^{ub} z_i^n \cdot n \cdot Area_i$ determined by the heuristic solution can be updated as the lower bound in (F2). After that, the safety margins of all those aircraft are augmented and the *feasibility check* is called again until the infeasible solution returns with a value of safety margin, and this value of safety margin is recorded as the threshold value of the safety margin.

Step 2: When a feasible parking plan cannot be found with the given combination of safety margins, the safety margins of some aircraft must be reduced in order to obtain a feasible parking plan. Two strategies of adjusting the safety margin are introduced during the individual safety margin decrementation and augmentation stage, respectively. In particular, we first assign priorities to each maintenance order according to the area of each aircraft and put all aircraft into a set named *Prioirty_List*, and the priorities are assigned in a decreasing order according the area of each aircraft, i.e. assigning the highest priority to the largest aircraft. The first adjusting strategy is that the safety margin for the aircraft with the lowest priority the *Priority_List* is decreased at first until the *feasibility check* can produce a feasible parking solution, or the aircraft is moved to the waiting list set, i.e. *Aug_List*, for further consideration of the safety margin augmentation later if it reaches the lower bound of the safety margin, and the algorithm selects another aircraft with the lowest priority in *Priority_List* for safety margin decrementation. After finding a feasible solution by decrementing the safety margin of the aircraft, the algorithm selects the aircraft with the highest priority for safety margin augmentation from *Priority_List* again and checks the feasibility. If the infeasible solution returns, then such aircraft is removed from further augmentation consideration. The algorithm continues to select aircraft with highest priority for augmentation until the *Priority_List* is empty.

Step 3: After the iterations in **Step 2**, we again obtain a set of aircraft pending for safety margin augmentation in the waiting list *Aug_List*, and the second adjusting strategy is proposed. In detail, the safety margin for aircraft with higher priorities in the waiting list *Aug_List* are augmented first, then aircraft with lower priorities are augmented later until the *feasibility check* returns infeasible solutions and such aircraft causing infeasibility in this iteration is removed from *Aug_List* for further consideration of safety margin augmentation.

3.5.2. Branching strategies

It is noted that the binary decision variables in both problems have hierarchical structures, and the branching strategies can be adjusted to adopt this feature. Fischetti and Luzzi (2009) proposed a set of branching strategies in open dimension nesting problems and Alvarez-Valdes et al. (2013) conducted an extensive study to compare the efficiency among various branching strategies. The strategy proposed by Fischetti and Luzzi (2009) is first to determine the relative positions of 2 pieces, then those of 3 pieces, then 4 pieces, and so on, by assigning priorities to the variables b_{ijk} in a decreasing order. This strategy avoids visiting the subtrees that produce infeasible solutions before the search process. Similarly, in the Phase I problem, a set of binary variables z_i is used to first decide which aircraft is going to be placed in the hangar, and then set b_{ijk} for each pair of aircraft that is branched to find the relative position for those aircraft with $z_i = 1$. According to Alvarez-Valdes et al. (2013), assigning a higher priority to a larger piece first yields satisfactory results. Similar to their approach, we first assign priority to the variable z_i using Fischetti and Luzzi (2009)'s priorities by ordering pieces by the non-increasing area in (F1), and then we assign priority to variables b_{ijk} that are used to separate each pair of aircraft. Similarly, in the Phase II problem, the decision variables that determine the safety margin for each aircraft z_i^n are branched first, and auxiliary variables g_{ij}^n are determined afterwards. Thirdly, binary variables b_{ijk}^n are branched under given sets of z_i^n and g_{ij}^n.

3.6. Numerical study

This section presents the results of computational experiments that were carried out on instances based on the real-life data provided by an aircraft maintenance company in Hong Kong. All the procedures described in the previous sections are coded in C# in Visual Studio 2010 and run on a computer with an Intel Core i7 processor, at 3.6 GHz with 32 Gb of RAM. The Mixed-Integer Linear Programming is solved by the CPLEX 12.5 serial model.

3.6.1. Description of instances

An aircraft hangar maintenance service provider in Hong Kong, which has over 50 clients, including airlines, business jet companies and utility aircraft companies, is employed as a case study. The company owns a maintenance hangar in the aircraft maintenance area of Hong Kong International Airport, with dimensions are at 110 meters long and 110 meters wide. In the current maintenance scheduling process of the company, the parking stand plans are constructed manually at regular periods. We collected the information of the estimated arrival time (ETA), departure time (ETD), aircraft type and check type of each maintenance order from clients over 157 days from January to May in 2015. We calculated the number of aircraft to be arranged each day by the ETA and ETD of each maintenance order, and the frequencies of the days and number of aircraft to be arranged are presented in Table 3-1. Typically, the maintenance company has to arrange a mix of large-, middle-, and small-sized aircraft in the hangar each day, and the company management stated that planning for 7 aircraft simultaneously by the manual method is challenging. The data in Table 3-1 suggests that over one fifth of the days in these five months are peak days for the maintenance planning; it is estimated that the proportion of peak days is going to increase in the future as outsourcing MRO activities increases. 40 testing instances are generated based on the observed peak-day scenarios and the number of aircraft maintenance orders in those instances ranges from 6 to 12. In detail, the observed peak-day scenarios are incorporated as initial instances in the set at first, then those instances are altered by adjusting the proportion of small-, medium- and large-sized aircraft as well as adding new aircraft in order to create challenging problem instances.

Table 3-1 Frequency of daily handling maintenance requests

Number of aircraft to be arranged in one day	Frequencies	Percentage (%)
0	1	0.6
1	11	7
2	21	13.4
3	23	14.6
4	24	15.3
5	23	14.6
6	19	12.1
7	18	11.5
8	10	6.4
9	7	4.5

The instance set consists of 10 small-sized, 11 medium-sized and 2 large-sized aircraft types, including large-sized civil aircraft, medium-size civil aircraft as well as business jets and the classification of aircraft models is based on their area, with Table 3-2 showing details of the classification. 40 instances are classified into four groups by the proportion of aircraft from the three categories, as follows: 1) the majority of aircraft in the instance are small-sized, 2) the majority of aircraft in the instance are medium-sized, 3) the majority of aircraft in the instance are large-sized and 4) the number of aircraft from different categories in the instance are equal. With regard to the safety margin range, we referred to the practice in the company and prescribed the minimal distance between two aircraft as one meter and the maximal safety margin as eight meters. To avoid the situation in which the hangar cannot accommodate the whole subset of aircraft derived from Phase I after imposing the safety margin in the Phase II problem, all the original NFPs in the Phase I problem are revised by the one-meter safety margin, which accords with the minimum safety requirement between pairs of aircraft adopted in practice.

Table 3-2 Classification of aircraft models

Category	Aircraft Models
Small-sized aircraft (aircraft area < 300 m^2)	G200 CL600 CL605 F900LX F2000EX F2000LX ERJ135 F7X G450 GIV
Medium-sized aircraft (1500 m^2 >aircraft area >= 300 m^2)	GL5T G550 G5000 G6000 G650 A318 ERJ190 A319 A320 B738 A321
Large-sized aircraft (aircraft area>=1500 m^2)	A332 A333

3.6.2. Evaluating the models' performance

3.6.2.1. Computational results of the Phase I problem

Table 3-3 shows the computational results for 40 instances in the Phase I problem. The first column stands for the instance name and the "total_S/M/L" in the second column stands for the total number of aircraft to be arranged in such instance and number of aircraft from the three categories, respectively. The number of binary variables involved in each instance is indicated in the third column. The best objective incumbent values in the Phase I problem is indicated in the fourth column. The column ('CPU) and ('Gap') in the fifth and sixth column report the CPU time elapsed when the termination criterion was met as well as the relative gap, respectively. The seventh column represents the number of aircraft placed in the hangar after the stopping criterion was met and the proportion of used hangar space is presented in the last column. The time limit for each instance was 3,600 seconds.

Table 3-3 Computational results of the Phase I problem

Instance	Aircraft (total_S/M/L)	Binary variables	Objective Function	CPU	Gap	Number of aircraft placed in hangar	Proportion of used hangar space
1	6_4/1/1	400	8430.26	0.46	0	6	0.29
2	6_3/2/1	396	8497.08	0.19	0	6	0.30
3	7_4/1/2	543	7711.13	0.12	0	7	0.35
4	7_3/2/2	583	7261.4	0.19	0	7	0.39
5	8_5/2/1	688	6931.33	0.10	0	8	0.32
6	8_4/2/2	738	6770.9	1.02	0	8	0.44
7	9_7/1/1	993	7884.61	0.45	0	9	0.34
8	10_8/1/1	1236	7461.56	0.54	0	10	0.38
9	11_9/1/1	1469	7409.36	0.93	0	11	0.33
10	12_8/3/1	1644	5333.27	128.06	0	12	0.46
11	6_1/3/2	384	6665.39	0.78	0	6	0.45
12	6_0/4/2	426	7456.76	0.47	0	6	0.38
13	7_0/5/2	567	6251.38	0.98	0	7	0.47
14	8_1/5/2	698	5441.4	904.19	0	7	0.51
15	8_2/4/2	704	6929.64	82.56	0	8	0.45
16	8_1/5/2	734	5965.05	3.55	0	8	0.49
17	9_2/5/2	969	6941.82	1.17	0	9	0.44
18	10_3/7/0	1192	8717.2	0.53	0	10	0.29
19	11_1/9/1	1435	7253.9	115.84	0	10	0.45
20	12_2/9/1	1620	6545.54	3600	13.45	10	0.48
21	4_1/1/2	162	7846.49	0.16	0	4	0.33
22	6_1/2/3	388	6279.82	17.93	0	5	0.44
23	6_2/1/3	392	6315.32	0.28	0	6	0.43
24	7_2/1/4	587	6253.1	17.76	0	6	0.44
25	7_2/2/3	575	6232.82	0.47	0	6	0.44
26	7_1/2/4	384	6697	50.06	0	5	0.44

(Table 3-3 Cont'd)

Instance	Aircraft (total_S/M/L)	Binary variables	Objective Function	CPU	Gap	Number of aircraft placed in hangar	Proportion of used hangar space
27	8_2/2/4	712	6356.49	112.86	0	6	0.46
28	10_3/3/4	1220	5646.73	469.26	0	8	0.50
29	10_3/2/5	1234	5698.1	710.76	0	8	0.48
30	12_2/5/5	1784	4972.67	3600	48.14	9	0.54
31	6_2/2/2	386	8031.06	0.38	0	6	0.36
32	6_2/2/2	414	7473.4	0.39	0	6	0.34
33	6_2/2/2	392	6969.43	0.38	0	6	0.41
34	9_3/3/3	945	5926.93	554.89	0	8	0.50
35	9_3/3/3	971	5565.00	19.61	0	9	0.51
36	9_3/3/3	939	5893.23	2.61	0	8	0.49
37	12_4/4/4	1776	4669.42	3600	36.49	9	0.50
38	12_4/4/4	1742	5641.33	3600	35.30	8	0.45
39	12_4/4/4	1692	5727.48	3600	39.09	8	0.39
40	12_4/4/4	1764	5196.22	3600	33.65	10	0.53

The branch-and-bound algorithm with the revised branching strategy was able to optimally solve 34 instances out of 40. It was found that the highest utilization is 0.54 in the 40 instances the generated layouts demonstrate packed hangars for the instance with high utilization. The irregular shape of aircraft renders a relatively low space utilization compared with the nesting problem containing regular polygons. After examining the solution outcomes, e.g. parking layout of the hangar, it is found that the unoccupied area in the maintenance hangar is not likely to be further utilized and accommodate additional aircraft, and therefore the space utilization is deemed proper under the context. There were 6 instances, i.e. 20, 30, 37, 38, 39 and 40, that were left unsolved to optimal. The generated layouts of these 6 instances were further examined and it was found that the hangar space was almost fully utilized with a compact layout. We further identified the aircraft left unarranged in these 6 instances and found that the majority of unarranged aircraft, i.e. pending aircraft, were large-sized and middle-sized aircraft, and it would be unlikely to park such unarranged aircraft, given the existing layouts. Therefore, the interpretation of the optimality gap in these 6 instances can be provided: the difference between the best-known integer solution and the lower bound corresponds to the area of the subset of the pending aircraft and the gap is updated whenever a pending aircraft is successfully placed in the existing layout or it is proven that one of the pending aircraft cannot be placed in the existing layout. Due to the non-convex shape of aircraft, the number of binary variables involved in each NFP between two aircraft is large compared with the convex shape polygons in the problem instances of the other nesting problems in the literature(Alvarez-Valdes et al., 2013; Cherri et al., 2016; Toledo et al., 2013). Visiting all the possible relative positions for each pair of aircraft before updating the bound is difficult, even the generated layout already demonstrates a satisfactory result for hangar space utilization.

3.6.2.2. Computational results of the Phase II problem

The number of binary variables used to determine the relative positions between each pair of aircraft in the Phase II problem is determined by two factors: 1) the number of aircraft derived from the Phase I problem, and 2) the range of the safety margin in the Phase II problem. It is reported that the branch-and-bound algorithm with the *horizontal slicing* MIP model can optimally solve the open-dimensioned nesting problem with at most 14 pieces, with convex- and non-convex shaped pieces (Alvarez-Valdes et al., 2013). To control the problem size to a moderate level, the upper bound of safety margin is determined by the threshold value derived from the heuristic in this section, and the results are shown in the last column in Table 3-4. Considering that the Phase II problem is similar to the nesting problem with $piece \cdot (ub - lb + 1)$ pieces and such a problem size is challenging, we referred to the stopping criterion adopted in the literature. Alvarez-Valdes et al. (2013) set several time milestones (1h, 2h, 5h and 10h) in solving difficult instances that involved more than 12 complex irregular items. We therefore selected and prescribed the time limit for each instance in Phase II as 18,000 seconds (5h) with the upper bound of the safety margin determined by the threshold value derived from the heuristic. Table 3-4 shows the computational results in the Phase II problem. We found that in many instances, the initial solution provided by the heuristic was proved optimal by the exact algorithm after an exhaustive search in some large instances. Although, in some cases, the objective function value increased in the branch-and-bound algorithm, the searching processes take a long time. In the Phase II problem, the difference between the best integer and upper bound corresponds to the pending safety margin of each aircraft. To update the upper bound of branch-and-bound, one has to prove the infeasibility of the pending safety margins, which corresponds to a single *feasibility check*. Though the final layout demonstrates a satisfactory result from the practice perspective , large gaps have been recorded in instances with packed layouts since updating the bounds in the Phase II problem is far more difficult than that of the Phase I problem, as there is more than one NPF used to separate the aircraft.

Moreover, the performance of the model and the computation time differs, even when comparing two instances from the same instance group with the same number of aircraft to be arranged in the Phase II problem. Taking Instances 23 and 24 as examples, both have 6 aircraft to be arranged in these two instances in the Phase II problem (as determined by the result of the Phase I problem in Section 3.6.2.1) and they belong to the same instance group. Moreover, the threshold values derived from the heuristic are the same, and the number of binary variables involved in the two instances are similar with similar instance settings. The exact algorithm takes much more time to solve instance 23 to optimal than for instance 24. The best-known parking layouts for the 20 instances tested in this section are shown in Section 3.7.

Table 3-4 Computational results of the Phase II problem

Instance	Aircraft (total_S/M/L)	Binary variables	CPLEX + **Threshold**				Heuristic		
			LB	UB	CPU	Gap	CPU	Value	Threshold
1	6_4/1/1	2792	29357.92	29357.92	0.39	0	1.67	29357.92	8
2	6_3/2/1	2732	28823.36	28823.36	0.30	0	1.37	28823.36	8
3	7_4/1/2	3822	35110.96	35110.96	0.53	0	3.45	35110.96	8
4	7_3/2/2	4038	38709.12	38709.12	8.94	0	2.41	38709.10	8
5	8_5/2/1	4832	41349.36	41349.36	0.78	0	4.32	41349.36	8
6	8_4/2/2	4498	34188.06	37303.56	18000	9.11	439.05	32945.88	7
7	9_7/1/1	6888	33723.12	33723.12	99.20	0	5.05	33723.12	8
8	10_8/1/1	8508	31560.05	37107.52	18000	17.58	2314.18	36065.52	8
9	11_9/1/1	10224	30259.88	37525.12	18000	24.01	1252.40	35674.76	8
10	12_8/3/1	7172	N/A	54133.84	18000	N/A	5398.84	27066.92	5
11	6_1/3/2	1356	17756.84	17888.13	1689.97	0.74	85.17	17756.84	4
12	6_0/4/2	2944	35671.30	35671.3	2374.99	0	43.08	34961.92	8
13	7_0/5/2	1984	22436.38	22436.38	11012.39	0	181.61	22366.38	4
14	8_1/5/2	920	N/A	10995.12	18000	N/A	208.63	7643.21	2
15	8_2/4/2	3688	25741.26	31022.16	18000	20.52	401.31	27928.82	6
16	8_1/5/2	1942	17378.80	18404.85	18000	5.90	1548.54	15513.60	3
17	9_2/5/2	5100	27463.44	30949.08	18000	12.69	2466.59	28225.34	6
18	10_3/7/0	8286	27062.56	27062.56	546.80	0	20.80	27062.56	8
19	11_1/9/1	4120	N/A	20296.40	18000	N/A	697.94	19725.02	4
20	12_2/9/1	1966	N/A	11108.92	18000	N/A	371.91	10634.96	2

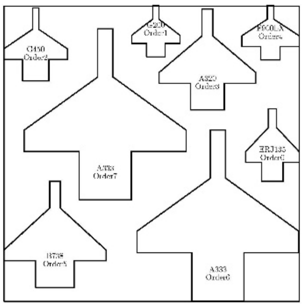
(Table 3-4 Cont'd)

Instance	Aircraft (total_S/M/L)	Binary variables	CPLEX + **Threshold**				Heuristic		
			LB	UB	CPU	Gap	CPU	Value	Threshold
21	4_1/1/2	1136	34028.08	34028.08	0.14	0	0.80	34028.08	8
22	6_1/2/3	741	12882.54	12882.54	35.71	0	15.35	11640.36	3
23	6_2/1/3	1041	12950.94	12950.94	2672.73	0	80.37	11716.44	3
24	7_2/1/4	1107	12962.70	12962.70	1500.32	0	927	11693.80	3
25	7_2/2/3	1101	13023.54	13023.54	2616.64	0	49.60	11734.36	3
26	7_1/2/4	486	5564.0	5564.0	95.91	0	3.08	5564.40	2
27	8_2/2/4	1065	12652.53	12652.53	471.12	0	55.01	11487.02	3
28	10_3/3/4	2058	12783.41	19359.81	18000	51.44	1195.67	12906.54	3
29	10_3/2/5	1998	14263.70	19205.70	18000	34.65	353.64	12803.80	3
30	12_2/5/5	1730	N/A	14254.66	18000	N/A	2563.09	8669.56	2
31	6_2/2/2	2700	34444.88	34444.88	0.95	0	1.64	32551.52	8
32	6_2/2/2	2876	37012.80	37012.80	0.33	0	1.67	37012.80	8
33	6_2/2/2	2736	39290.44	39290.44	739.98	0	168.87	39290.44	8
34	9_3/3/3	1922	16592.67	18519.21	18000	11.61	1888.20	16592.67	3
35	9_3/3/3	1348	10025.78	13392.80	18000	33.58	1271.06	10025.78	2
36	9_3/3/3	1264	N/A	12858.98	18000	N/A	662.18	9080.56	2
37	12_4/4/4	2114	N/A	14861.16	18000	N/A	2249.97	11141.74	2
38	12_4/4/4	1670	N/A	51016.56	18000	N/A	1102.94	12143.07	2
39	12_4/4/4	1672	N/A	12745.04	18000	N/A	1509.96	9553.34	2
40	12_4/4/4	2114	N/A	13897.12	18000	N/A	2008.41	10449.86	2

3.6.2.3. Discussion

The hierarchical structures of binary variables in the two formulations are analysed, and a branching priority to each binary variable is assigned, which is based on the strategies proposed by Fischetti and Luzzi (2009). Moreover, the range of the admissible safety margin is another factor that influences the computational performance of the Phase II problem. For the tradeoff between the solution quality and the computational efficiency, the solution obtained by the heuristic can be considered for practical use, as the accuracy of the safety margins has been significantly improved compared with the manual layout planning method. From a theoretical point of view, some alternate formulations, such as the *semi-continuous* model and the *discrete* (*dotted-board formulation*) model (Leao et al., 2016; Toledo et al., 2013) can be considered to in future studies simplify the geometric complexity of aircraft.

3.7. Supplementary materials: selected best-known parking layouts



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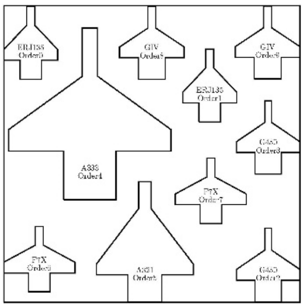

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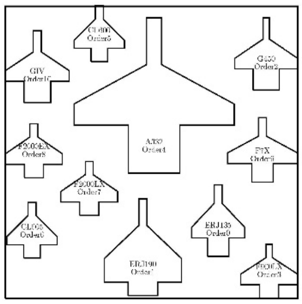

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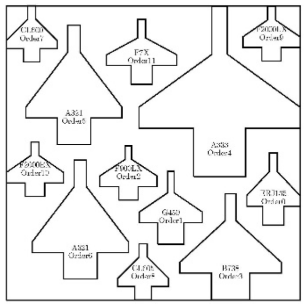

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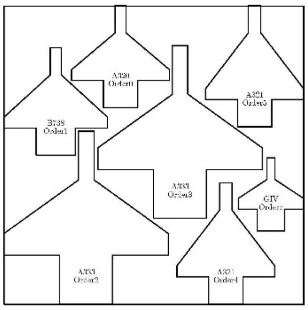

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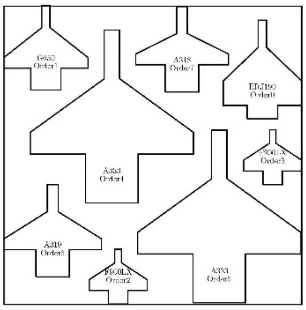

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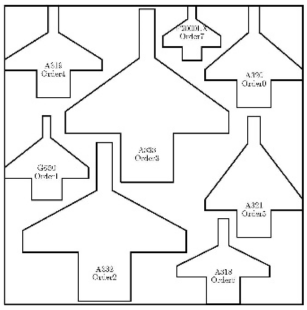

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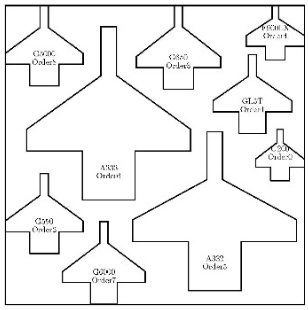

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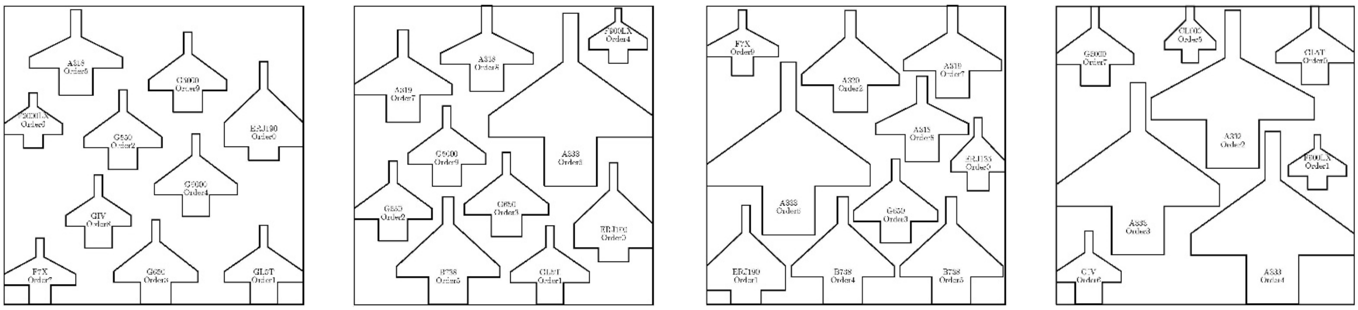

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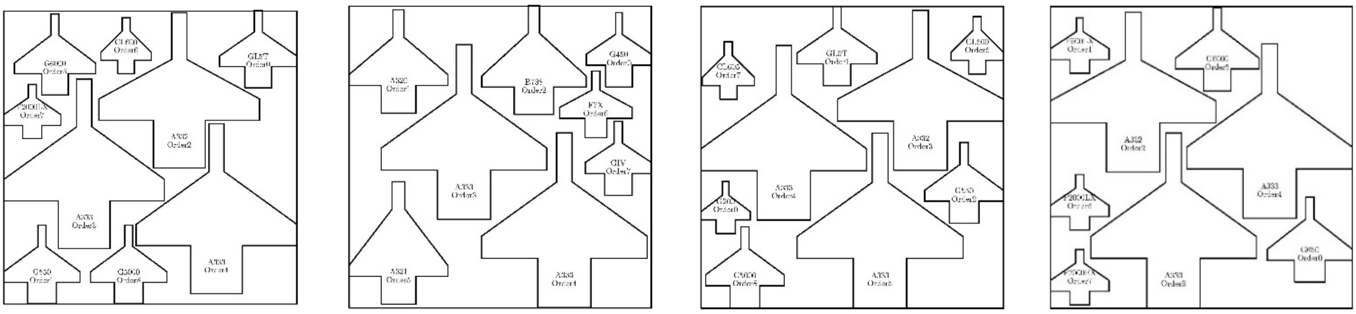

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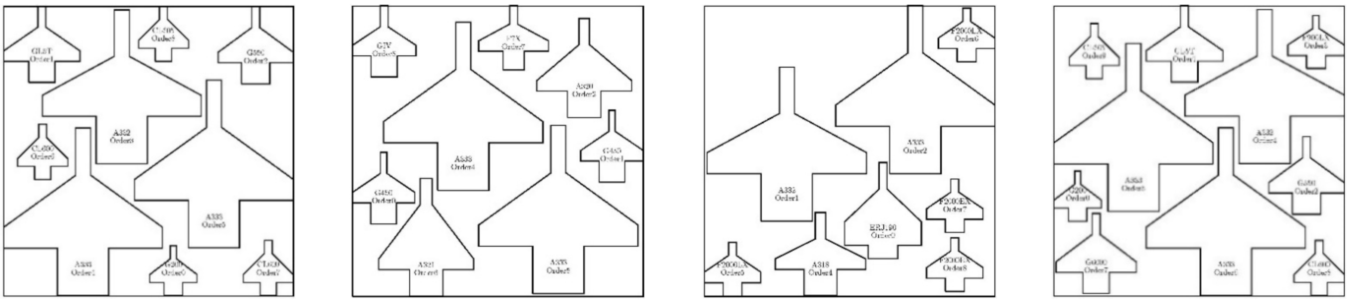


37 38 39 40

3.8. Summary

To assist independent MRO service providers in planning the parking stand allocation plan and in reviewing the hangar capacity, especially on peak days, we propose a two-stage MIP-based approach to solve the problem. The outlined aircraft parking stand allocation problem offers several areas for further work. Concerning hangar space utilization, arranging the aircraft parking stand in a three-dimension space can be considered. For example, small aircraft can be placed under the wings of large aircraft in the real situation. In addition, the analysis of performance has shown that there is still room for improvement in the problem-solving approaches. Given the NP-hardness of this problem, further avenues for future work are the inclusion of additional cut and branching strategies and pruning techniques in the branch-and-bound, as well as the development of efficient metaheuristics. The models we addressed in this work solve a single peak day parking stand allocation problem. Incorporating the proposed problem into the maintenance scheduling problem in the context of aircraft maintenance company under MRO outsourcing mode is presented in Chapter 4.

Chapter 4. A Family of Heuristic-based Inequalities for Maximizing Overall Safety Margins in Aircraft Parking Stands Allocation Problems

4.1. Background introduction

The problem of arranging a set of aircraft in a maintenance hangar at single time point under MRO outsourcing mode is further studied in this chapter, with the foundation in Chapter 3. The overall safety margins of the parking layout need to be maximized within the limited available space, measured by the weighted sum of the individual discrete safety margins of each aircraft. Due to the non-convex irregular shape of aircraft, the model involves a great number of binary variables associated with the revised NFP. The default branch-and-bound algorithm is inefficient in solving such a model as the infeasibility information of the precedent visited solution cannot be directly utilized by the default method to update the bounds. A heuristic algorithm is developed to provide practical solutions, and the intermediate infeasible solutions identified during searching are utilized to develop valid and approximate inequalities, tightening the optimality gap. The computational results demonstrate that the addition of inequalities improves the computational efficiency in solving a wide range of instances and in tightening the optimality gap while the stopping criterion is met.

From the perspective of the aircraft service company under the batching hangar maintenance mode mentioned in Chapter 3, after receiving aircraft maintenance requests from a number of clients, one needs to consider which subset of maintenance requests to serve during a planning period to attain maximal profit, as well as to come up with an aircraft parking layout to minimize the risk of collision during the movement operations of the aircraft. In actual operations, the practitioner typically gathers a subset of aircraft with similar maintenance times in a batch, then arranges the batch in the maintenance hangar to facilitate the maintenance procedures. The aircraft parking stands arrangement problem (PSAP) presented in Chapter 3 aims to maximize the hangar utilization with maximal overall safety margins for the aircraft placed in the hangar. After determining the aircraft to be parked in the hangar, we aim at finalizing the aircraft parking layout by arranging aircraft parking with maximal safety margins (Qin et al., 2018). Given the complexity of the problem, this chapter aim to enhance the computational efficiency of heuristic algorithm used in Chapter 3 by developing a family of inequalities based on the intermediate infeasible solutions during the process of heuristic searching for determining the optimal parking layout with maximal overall safety margins or tightening the optimality gap for the challenging problems. Given a set of aircraft $\dot{P}$ to be served, the objective of the problem is to maximize the overall safety margins measured by the weighted sum of the individual safety margins of each aircraft, preventing the overall risk of collision during the batch movement operation prior to and after the maintenance operations in the hangar.

The PSAP has not been extensively studied in the literature and the most closely related reference we are aware of refers to the irregular item packing problems (Qin et al., 2018), according to the classification of by Wäscher et al. (2007). The problem studied here can be regarded as an extended version of the two-dimensional irregular item packing problem in a fixed dimension container (Martins & Tsuzuki, 2010; Wäscher et al., 2007), since aircraft need to be modelled as irregular polygons to accurately measure the capacity of the hangar accommodating the aircraft. PSAP is different from the general packing problem which aims to arrange the items to be as compact as possible. Instead, PSAP aims to optimally arrange the parking position of the aircraft while reserving a moderate distance between each pair of aircraft. The item cutting &

packing problem in two-dimensional space has been widely studied in the literature because of its practical use in various industries, and such packing problems can be classified as either regular items packing problems and irregular items packing problems(Alves et al., 2012; Amaro et al., 2017; Amaro et al., 2014; Bansal et al., 2006; Caprara et al., 2005; Dyckhoff, 1990; Kenyon & Remila, 2000; Wäscher et al., 2007; Y. X. Xu, 2016). Many mathematical formulations solving irregular items packing problems have been proposed in the literature(Alvarez-Valdes et al., 2013; Cherri et al., 2016; Gomes & Oliveira, 2006; Leao et al., 2016; Antonio Martinez-Sykora, Alvarez-Valdes, Bennell, & Manuel Tamarit, 2015; Taccari, 2016). The No-Fit Polygon (NFP) has been widely used in detecting if two irregular items overlap with each other (J. A. Bennell & Oliveira, 2009; Burke et al., 2007), while the other approaches also exist(W. Zhang & Zhang, 2009). The main difficulty in tackling irregular shape item problems is the great number of binary variables determining the relative position between each pair of aircraft in the container. Generally, solving an instance including more than 10 irregular items becomes challenging and intractable with the current approaches (Cherri et al., 2016). In our problem, we assume that each aircraft has to select an individual safety margin within the admissible discrete range lb and ub. In this regard, there are a number of $\left|P'\right| \cdot (ub - lb + 1)$ NFPs involved in each problem instance for the proposed problem, which are extremely challenging using the existing approach.

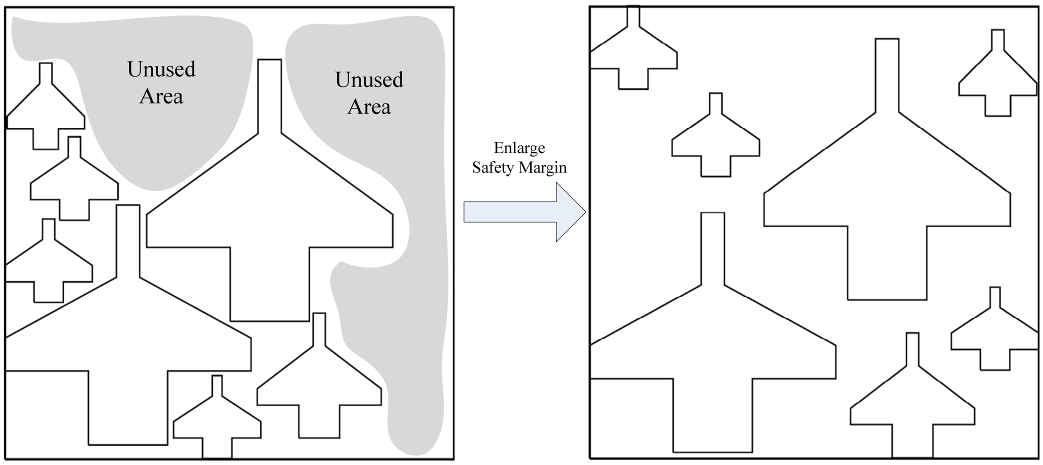
Many heuristic algorithms have been developed to utilize infeasible solutions that are identified during the searching process. For example, some researchers focusing on the Vehicle Routing Problem (VRP) with loading and unloading constraints in a two-dimensional space adopted infeasible solutions to develop heuristics, and there are some similarities with the problem studied in this chapter. Specifically, both these two problems have large numbers of candidate solutions to explore and the feasibility of the candidate solution can be examined only after fixing all decisions. To compare these two problems, the VRP has to determine the routing of each vehicle first, and the parking stand arrangement problem has to determine the individual safety margins for each aircraft first. Afterwards, there is a set of decision variables determining the position of the items to be placed in the container in both two problems afterwards. As the items are arranged in a two-dimensional space in both problems, the feasibility of the solution cannot be directly checked by a single resource constraint as in classic assignment problem, but needs to be examined with a set of geometrical constraints, including non-overlapping constraints and boundary constraints. To utilize these infeasible solutions identified during the searching process, one can eliminate a set of unvisited unpromising candidate solutions that has the same pattern of that identified infeasible solutions. In particular, the identified infeasible solutions are recorded in a list, then the respective inequalities are generated and inputted into the mathematical model according to the information in the list, so as to eliminate unvisited unpromising solutions with the same patterns. For example, Felipe et al. (2011) used the intermediate infeasible solution to diversify the search process in a heuristic algorithm, tackling the vehicle routing problem with precedence and loading constraints. Iori et al. (2007) proposed an exact approach based on a branch-and-cut algorithm to solve the vehicle routing problem with two-dimensional constraints. The infeasible route identified during the feasibility checking process is recorded and converted to a cut, which is added to the original problem. Hokama et al. (2016) developed a branch-and-cut algorithm to deal with the unpackable path generated from the master problem. They used a hash table to record the feasibility information for the route or sub-route visited earlier. The feasibility of the tentative route can be examined by comparing the elements in the route. If the tentative route has the same elements as the infeasible sub-route recorded in the hash table, the tentative route can be inferred to as an infeasible route without further examination by the branch-and-bound algorithm. Some heuristic searching strategies exist to detect the feasibility of a given solution in the packing problem, such as sequence based heuristics (Alvelos et al., 2009), the robust hyper-heuristic algorithm (Beyaz et al., 2015) for the rectangular packing problem and the recursive algorithm for the rectangular guillotine strip packing problem(Yaodong Cui, 2013; Y Cui et al., 2008). However, using these methods may lead to being trapped in local optima as the complexity of the polygons increases. The technical roadmap is organized as follows: a mathematical model is firstly presented to formulate the aircraft parking stands arrangement problem in the context of aircraft maintenance service providers. Due to the large number of binary variables involved, the mathematical model can only solve the small problem instance to optimal within a reasonable time, and the medium- to large-sized instances are intractable, solely by the mathematical model. To provide a warm start and tighten the bounds of the mathematical model, a heuristic algorithm aiming at tackling large-scale instances is developed, which uses the mathematical model as the foundation. In detail, the heuristic algorithm determines a candidate solution first, then a branch-and-bound based feasibility checking approach is incorporated in the heuristic algorithm to examine the feasibility of a given combination of individual safety margins for the set of aircraft to be maintained in the hangar. To examine the feasibility of the tentative solution, the feasibility check approach inputs the tentative solution into the mathematical model with all safety-margin-related decision variables fixed in the model, then implements the model to check whether infeasibility returns. The heuristic adjusts the candidate solution if the previous candidate solutions are infeasible, and the warm start solution for the mathematical model would be determined by the end of the heuristic. During the feasibility checking process, the identified infeasible solutions are recorded to further prune down the set of infeasible solutions by adding several forms of inequalities to exclude infeasible combinations, before initiating running the original model so as to enhance the efficiency of mathematical model. Specifically, a set of inequalities is developed to convert the recorded infeasible solutions that were identified during the heuristic searching process as constraints in the model. The developed approach has been extensively tested on problem instances derived from the actual situation in an aircraft maintenance service company.

The remainder of this chapter is organized as follows. Section 4.2 describes the problem and the notation and formulation of the mathematical model, fundamental to the heuristic and inequalities. Moreover, the complexity of the problem is further analysed in this section. Section 4.3 presents the heuristic approach and the inequalities derived from the infeasible solutions during the heuristic search process. Section 4.4 examines the computational results. Section 4.5 presents the parking layouts for the selected problem instances, and the concluding summary are drawn in Section 4.6.

4.2. Aircraft parking stands allocation problem

4.2.1. Problem description

The problem studied in this chapter can be defined as follows: we are given a subset of aircraft of different shapes to be serviced during a short planning period, and these aircraft can be feasibly arranged in the given maintenance hangar satisfying the minimal safety margin requirements, i.e. the shortest distance between each pair of aircraft is at least equal to or larger than the minimal safety margin. Figure 4-1 presents a typical maintenance hangar operated by an independent aircraft base maintenance company serving different clients and accommodating aircraft of different shapes and sizes. In the problem, each aircraft has to select an individual safety margin to utilize the unused space in the hangar so as to minimize the risk of collision during the movement and maintenance operations. Two hangar layouts both can accommodate the same set of aircraft while the assigned position for each aircraft is different. In Figure 4-1(a), the aircraft can be feasiblely arranged in the maintenance hangar while they are placed in a concentrated manner even there is a lot of unused space (the shadow region in Figure 4-1(a)). In this regard, the problem studied in this chapter aims to enlarge the safety margin of each aircraft so as to make the most of the empty space (Figure 4-1(b)). The safety margin is defined as the shortest distance between two aircraft in the hangar, and the problem aims to have a maximal overall safety margin measured by the weighted sum of the individual safety margins of each aircraft placed in the hangar, as described in Chapter 3. In an actual situation, large-sized aircraft bear more risk of collision, compared with the small- and medium-sized aircraft, as larger aircraft are less manoeuverable than smaller ones. In this regard, the larger aircraft are given higher priority in reserving larger safety margins in practice, and the weight of the individual safety margin in the objective function is associated with the area of each aircraft. In the developed mathematical model, the aircraft safety margin is discretized for the trade-off between accuracy and computational efficiency, and we also prescribe the individual lower bound (lb_i) and the upper bound (lb_i) of the safety margin to represent the minimal safety requirement and the largest safety margin that contribute the overall safety margins respectively, as reserving too high a safety margin does not necessarily contribute to overall safety, but only increases the number of binary variables in solving the problem.



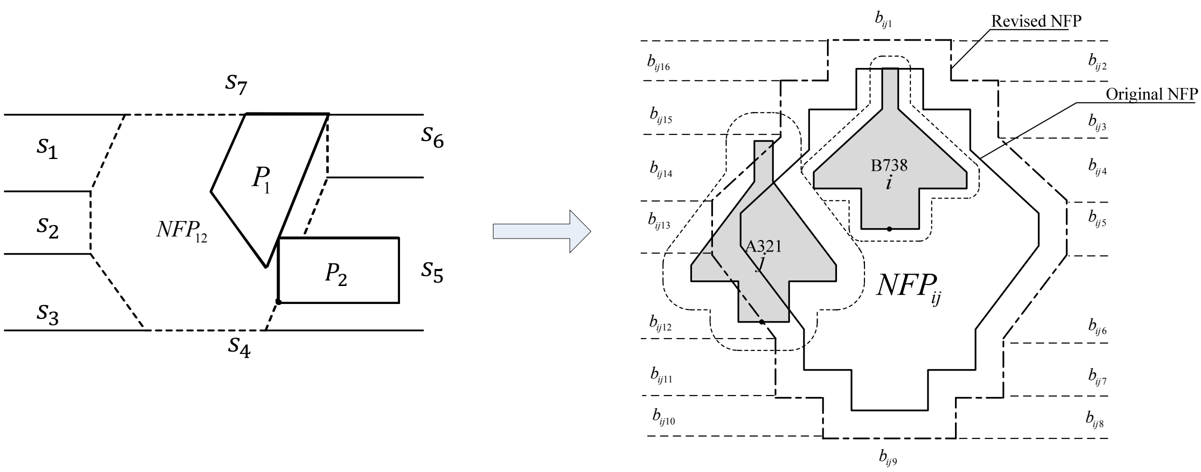
(a) Phase I layout

(b) Phase II layout

Figure 4-1 Comparison between parking layouts derived from two phases

The methodology of preventing irregular items from overlapping refers to the mechanism of No-Fit Polygons(J. A. Bennell et al., 2001; Julia A. Bennell & Oliveira, 2008; J. A. Bennell & Oliveira, 2009; J. A. Bennell & Song, 2008; Burke et al., 2007), as shown in Figure 4-2 (a): P1 and P2 are two simple polygons, and we denote the bottom left corner of polygon P2 as its reference point, for illustration purposes. To generate NFP between P1 and P2, polygon P2 slides along the boundary of polygon P1 while keeping in touch with P1, and the trajectory of the reference point of polygon P2 is recorded as the No-Fit Polygon between these two polygons. To prevent overlap, the reference point of P2 must be placed outside or on the boundary of the No-Fit Polygon. To characterize the NFP in the linear programming model, the area outside the NFP is partitioned into several horizontal slices and each horizonal slice is formed by several lines which can be denoted by a linear equation. Accordingly, the equation $\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) = q_{ij}^{kf}$ is used to denote the *fth* line forming the *kth* slice outside the NFP (Alvarez-Valdes et al., 2013; Gomes & Oliveira, 2006). If the reference point of polygon *j* is placed on slice *k*, then there are several constraints in the form of

$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) \le q_{ij}^{kf}$, denoting that the slice k is activated to impose the relative position requirement. For the No-Fit Polygons between each pair of aircraft, we denote the reference point of the aircraft be the middle of the bottom edge of the aircraft because of its symmetrical shape. To enforce the prescribed safety margin between each pair of aircraft, we move the edges of the original NFP outward, with a distance equal to the prescribed safety margin (Figure 4-2 (b)), and the reference point of aircraft j must be placed outside or on the boundary of the revised NFP. For a pair of aircraft, the safety margin used to separate them is determined by the aircraft with the larger safety margin in that pair, as shown in Figure 4-2(b). Aircraft j has larger individual safety margin than aircraft i and therefore the revised NFP associated with the safety margin of the aircraft is activated to separate the pair.



(a) (b)

Figure 4-2 NFP between two simple polygons and revised NFP for a pair of aircraft

4.2.2. MILP formulation for the problem

We list the notations and decision variables for the formulation of the problem:

Notations

W	width of hangar
H	length of hangar
$Area_i$	area of aircraft i
w_i	width of aircraft i
h_i	length of aircraft i
i	maintenance order associated with aircraft i
P'	subset of aircraft with maximal overall profit and satisfying minimal safety requirement
A_u	a group of aircraft that belong to the same aircraft type u
NFP_{ij}^n	the revised NFP of aircraft i and j with safety margin n
s_{ijn}^k	kth slice outside the region of the revised NFP of aircraft i and j with safety margin n
$\alpha_{ijn}^{kf}, \beta_{ijn}^{kf}, q_{ijn}^{kf}$	parameters used to define the fth linear inequality of the slice s_{ijn}^k outside NFP_{ij}^n
m_{ij}^n	number of slices outside NFP_{ij}^n
t_{ijn}^k	number of linear inequalities of the kth slice outside NFP_{ij}^n
ub_i	upper bound of safety margin of aircraft i
lb_i	lower bound of safety margin of aircraft i
n	admissible safety margin, $\max\{lb_i, lb_j\} \le n \le \max\{ub_i, ub_j\}, \forall i, j \in P'$

Decision Variables

z_i^n	binary decision variable that takes the value 1 if aircraft i is placed in hangar with safety margin n, and 0 otherwise
x_i	position of reference point of aircraft i on x-axis in two-dimensional space
y_i	position of reference point of aircraft i on y-axis in two-dimensional space
b_{ijk}^n	binary decision variable that takes the value 1 if the reference point of aircraft j is placed into slice s_{ijn}^k, and 0 otherwise
g_{ij}^n	auxiliary decision variable that takes value 1 if the shortest distance between aircraft i and j is n and NFP_{ij}^n is activated, and 0 otherwise

Objective : Maximize Overall Safety Margin

$$Max \sum_{i \in P'} \sum_{n=lb}^{ub} z_i^n \cdot n \cdot Area_i \quad (4\text{-}1)$$

s.t.

$$x_i + w_i / 2 \le W,\ \forall i \in P' \quad (4\text{-}2)$$

$$x_i \ge w_i / 2,\ \forall i \in P' \quad (4\text{-}3)$$

$$y_i + h_i \le H,\ \forall i \in P' \quad (4\text{-}4)$$

$$\alpha_{ijn}^{kf}(x_j - x_i) + \beta_{ijn}^{kf}(x_j - x_i) \le q_{ijn}^{kf} + M \cdot (1 - b_{ijk}^n),\ \forall i, j \in P', i \ne j,\ \forall k = 1,2,...,m_{ij}^n,\ \forall f = 1,2,...,t_{ijn}^k, \\ n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}5)$$

$$\sum_{k=1}^{m_{ijn}} b_{ijk}^n = g_{ij}^n,\ \forall i, j \in P', i \ne j, n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}6)$$

$$g_{ij}^n \ge z_i^n - \sum_{n=n+1}^{ub} z_j^n, \forall i, j \in P', i \ne j \quad , n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}7)$$

$$g_{ij}^n \ge z_j^n - \sum_{n=n+1}^{ub} z_i^n, \forall i, j \in P', i \ne j \ , n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}8)$$

$$g_{ij}^n \le 1 - \sum_{n=n+1}^{ub} z_i^n, \forall i, j \in P', i \ne j \ , n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}9)$$

$$g_{ij}^n \le 1 - \sum_{n=n+1}^{ub} z_j^n, \forall i, j \in P', i \ne j \ , n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}10)$$

$$\sum_{n=lb}^{ub} g_{ij}^n = 1, \forall i, j \in P', i \ne j \ , n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}11)$$

$$\sum_{n=lb}^{ub} z_i^n = 1, \forall i \in P' \quad (4\text{-}12)$$

$$1 - z_i^n \ge \sum_{k=n+1}^{ub} z_j^k,\ i, j \in A_u, i < j \ , n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}13)$$

$$b_{ijk}^n \in \{0,1\}\ \forall i, j \in P', i \ne j, k = 1,2,..., m_{ijn}, \quad (4\text{-}14)$$

$$g_{ij}^n \in \{0,1\}\quad \forall i, j \in P', i \ne j, n = \max\{lb_i, lb_j\}, \cdots, \max\{ub_i, ub_j\} \quad (4\text{-}15)$$

$$x_i, y_i \ge 0\ \ \forall i \in P' \quad (4\text{-}16)$$

In the problem formulation, the objective function (4-1) maximizes the overall safety margins. Constraint sets (4-2)-(4-4) are bound constraints. Constraint (4-5) is a non-overlapping constraint that separates each pair of aircraft by the safety margin *n*. Variables b_{ijk}^{n} are binary and one of them must take value 1 if NFP_{ij}^{n} is activated in separating aircraft *i* and *j*. Constraint (4-12) prescribes that each aircraft must take a safety margin value within the prescribed bounds of each pair of aircraft. g_{ij}^{n} is the auxiliary variable used to activate/deactivate the revised NFP with safety margin *n*. Constraint (4-11) imposes the condition that there should be only one revised NFP activated to separate each pair of aircraft. According to the description of the problem, the safety margin used to separate aircraft *i* and *j* is determined by the larger individual safety margin in this pair, and Constraints (4-7) – (4-10) imply that the g_{ij}^{n} associated with the revised NFP with safety margin *n* is activated if and only if one aircraft has a safety margin *n* and the other has a safety margin smaller or equal to *n*, then constraint (4-6) activates/deactivates the respective revised NFP with binary variable g_{ij}^{n} respectively. Constraints (4-14)-(4-15) indicate that b_{ijk}^{n} and g_{ij}^{n} are binary variables. Constraint (4-13) is imposed to avoid duplicate solutions, and prescribes that the safety margins for the aircraft belonging to the same aircraft type are in a decreasing order: $\text{margin}_{i1} \geq \text{margin}_{i2} \geq \cdots \geq \text{margin}_{i3}$.

4.3. Solution methodology

4.3.1. Heuristic algorithm

The feasibility of a tentative solution (a given combination of safety margins z_i^n)) is determined by fixing a set of position-controlling binary variables b_{ijk}^{n} for every pair of aircraft placed in the hangar. When the problem scale becomes larger, the default branch-and-bound provided by a solver such as CPLEX becomes inefficient, as the optimizer tries to find the set of safety margins z_i^n that achieves maximum overall safety margins as the promising tentative solution at first, while such a tentative solution reserves too large a distance between each pair of aircraft that exceeds the capacity of the hangar. However, the infeasibility of the tentative solution can only be confirmed by the branch-and-bound when all b_{ijk}^{n} values for every pair of aircraft have been fathomed, however such search progress is time-consuming for the large-scale instance mentioned above. In this regard, a heuristic algorithm is firstly proposed to provide a practical solution for the problem within a reasonable time in the actual situation, then the mathematical model is tightened by recording the intermediate solutions for later use in the development of inequalities and in providing a moderate upper bound of safety margin, known as threshold of the safety margin, as well. Therefore, the heuristic provides fundamental information for the development of the inequalities discussed later. The notations and the flowchart of the heuristic algorithm are presented in Table 4-1 Notation in the heuristic algorithm and Figure 4-3, respectively.

Table 4-1 Notation in the heuristic algorithm

Heuristic algorithm for problem	
Notations	Meanings
P'	set of aircraft with maximal overall profit and satisfying minimal safety requirement
totalorder	number of aircraft in set *P'*
s(i)	the safety margin of aircraft *i*, $i \in P'$
Prioirty_List	set of aircraft
Aug_List	set of aircraft
j,k	notation of aircraft, *j,k* ∈ *Prioirty_List*
l	notation of aircraft, *l* ;∈ *Aug_List*
best-known fitness	the best-known fitness value
current fitness	the fitness value of the decision under current iteration
ub_i,lb_i	the lower bound and the upper bound of safety margin associated with aircraft *i*, respectively
th	the threshold value of safety margin
u_i^m	safety margin of aircraft *i* under infeasible solution *m*
R	set of infeasible solutions at overall augmentation stage and individual decrementation stage
r	infeasible solution *r*, $r \in R$
Q	set of infeasible solutions at individual augmentation stage
q	infeasible solution *q*, $q \in Q$

Input: Hangar Length *L*, Hangar Width *W*, Geometry Information of Aircraft (length and width of aircraft), Safety Margin Upper Bound ub_i, Safety Margin Lower Bound lb_i, Set Original *NPF*, Set *P'*

Output: Safety Margin *s(i)* for aircraft $i \in P'$, Set of infeasible solution *Q and R,* threshold value *th*

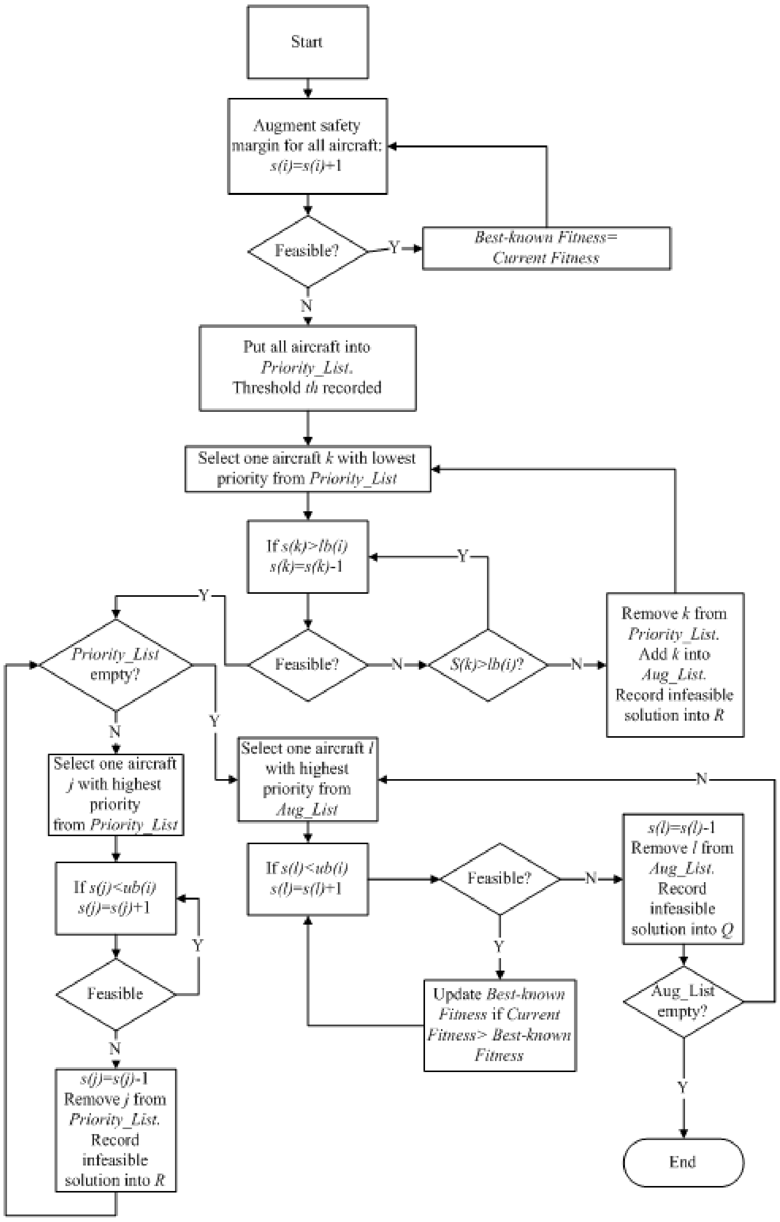


Figure 4-3 Flowchart of heuristic algorithm

The procedures shown in Figure 4-3 can be generalized in three steps. The feasibility of a given tentative solution is examined by creating an MIP model *feasibility check*, which tries to place all aircraft with the revised NFPs determined by a set of safety margins. If the *feasibility check* model is able to accommodate all aircraft in set P', then the hangar is feasible for the problem. Otherwise, such a tentative solution is infeasible because there is at least one aircraft cannot be placed in the hangar with the given safety margin in the tentative solution. The heuristic algorithm consists of three main steps.

Step 1: The algorithm first calls for the *feasibility check* and places the set of aircraft in $P^{'}$ with a combination of safety margins z_i^n for $i \in P^{'}$. The initial individual safety margins for each aircraft is initially set to be the lower bound of the safety margin of individual aircraft *i* (lb_i). If a feasible solution returns, the respective objective value (4-1) determined by the tentative solution examined in this iteration can be updated as the lower bound of the model in Section 4.2.2. After that, the safety margins of all those aircraft are augmented until returning an infeasible solution, with the highest individual safety margin in this infeasible solution defined as the threshold of the safety margin of the current problem.

Step 2: The safety margins of some aircraft need to be reduced in order to obtain a feasible parking plan again. Step 2 is called the individual safety margin decrementation and augmentation stage. Firstly, the priority of each aircraft is assigned according to the physical size of the aircraft (larger aircraft are assigned with higher priorities), and the priority value of each aircraft is kept constant during the heuristic search process. For easy understanding, we can simply regard the value of the priority of each aircraft equals to its size $Area_i$. Afterwards, all aircraft in $P^{'}$ are put into the set $Priority\_List$, and the $Priority\_List$ acts as a sequence of adjustment of individual safety margins. According to the expression of the objective function, the weightiness of the individual aircraft safety margin that each aircraft contributes depends on the physical size ($Area_i$) of each aircraft. Therefore, larger aircraft have higher priority in getting larger safety margin to contribute more to the objective function, and the largest size aircrafts' safety margin should be considered decremented at lastly after decrementing the safety margins of aircraft with lower priorities and no feasible solution returns. In this connection, the first adjusting strategy is that the individual safety margin for the aircraft with the lowest priority in the *Priority_List* is decremented first until a feasible solution returns, or the aircraft is moved to the waiting list set, i.e. *Aug_List*. If it reaches the lower bound of safety margin associated with that aircraft then the algorithm selects another aircraft with lowest priority in the *Prioirty_List* instead. After obtaining a feasible solution after decrementation, the algorithm selects the aircraft with highest priority for safety margin augmentation from the *Priority_List*, and checks the feasibility of the tentative solution. If the infeasible solution returns, such an aircraft is removed from further augmentation consideration. The algorithm in this step continues until the *Priority_List* is empty.

Step 3: After Step 2, the algorithm again obtains a set of aircraft pending for safety margins augmentation again in the waiting list *Aug_List*. The adjustment strategy in this step is that the safety margin for aircraft with the highest priority in *Aug_List* is augmented first, until an infeasible solution returns, and such an aircraft is removed from the *Aug_List* for further consideration of safety margin augmentation.

4.3.2. Inequalities for the problem

4.3.2.1. Valid inequalities

We utilize the infeasible solutions derived from the heuristic search and propose four inequalities to tighten the upper bound of the problem. After presenting the inequalities, we provide examples to illustrate the idea of the respective inequalities accordingly. The sets R and Q in the parameter list of the heuristic denote the infeasible solutions under Step 2 (individual safety margin decrementation/augmentation in *Priority_List* set) and Step 3(individual safety margin augmentation in *Aug_List* set) of heuristic algorithm, respectively. As the inequalities derived from the infeasible solutions of

Step 2 and Step 3 in the heuristic algorithm are presented with different expressions, we use set *R* and *Q* to differentiate the infeasible solutions to be inputted in inequalities 2 and 3, respectively. We find that when the safety margins reach a relatively large value, i.e. a threshold value, during the overall augmentation stage, not all the aircraft in the subset can be feasibly placed in the hangar. Therefore, a safety margin that is larger than or equal to the threshold value cannot obtain a feasible parking plan. Inequality 1 expressed in (4-17), i.e. threshold inequality, is proposed to remove infeasible solutions that exceed the threshold value. Example 1 shows how inequality 1 eliminates the set of infeasible solutions that exceeds the threshold value.

$$\sum_{i \in P'} \sum_{n=th}^{ub_i} z_i^n \leq (totalorder - 1) \tag{4-17}$$

Example 1. Assume that an instance with three aircraft has a feasible parking plan when the safety margin for all aircraft is assigned a value *n*, but the *feasibility check* cannot find a feasible solution when all the safety margins are augmented to a value *n+1*. In this case, the value *n+1* is the threshold for augmentation. Therefore, a bunch of combinations can be excluded from the solution space. Specifically, $\{n+1, n+1, n+1\}$, $\{n+2, n+3, n+1\}$, $\{n+3, n+4, n+2\}$ (the numbers in brackets denote the respective safety margin for three aircraft) can be excluded.

However, there are still large numbers of infeasible solutions left in the solution space since the threshold inequality 1 only eliminates the infeasible solutions such that each safety margin exceeds the threshold value. Therefore, we further propose the two inequalities 2 and 3 expressed in (18-19) to remove infeasible solutions during the individual adjustment stage when an infeasible solution is found.

$$\sum_{i \in P'} \sum_{n=u_i^r}^{ub_i} z_i^n \leq (totalorder - 1), \forall r \in R \tag{4-18}$$

$$\sum_{n=u_l^q}^{ub_l} z_l^n \leq (totalorder - 1) - \sum_{i \in P' \setminus \{q\}} z_i^{u_i^q}, \forall q \in Q \tag{4-19}$$

Inequality 2 is derived during the process of the safety margin decrementation/augmentation stage at **Step 2** of the heuristic. We denote the set of the infeasible solution identified during the individual safety margin decrementaiton/augmentation at **Step 2** as *R*. When the safety margin of each aircraft reaches the threshold value and an infeasible solution returns from the *feasibility check*, the decrementation of the safety margin begins and aircraft *i* with the lowest priority in the *Priority_List* is decremented at first. Therefore, we can infer that before obtaining a feasible solution, any augmentation of the safety margin of any aircraft is infeasible, and we denote *S* as the set of safety margins in that iteration. Similarly, selecting the highest priority aircraft in the *Priority_List* for safety margin augmentation with the infeasible solution in **Step 2** also implies that any augmentation of the safety margin of any aircraft is infeasible. Moreover, inequality 3 is identified at the individual safety margin augmentation in **Step 3**: the safety margin of the aircraft with the highest priority in the *Aug_List* is augmented first (denoted as *l* in (4-29)). If the safety margin of aircraft *i* with the highest priority is augmented to s and the *feasibility check* finds it infeasible, then any safety margin value larger than s for aircraft *i* cannot produce a feasible solution, and we denote *Q* as the set infeasible solutions identified during the individual safety margin augmentation stage. Example 2 shows how inequalities 2 and 3 eliminate the unvisited infeasible solution by utilizing the intermediate visited infeasible solution during the individual safety margin decrementation and augmentation stage in the heuristic algorithm.

Example 2. Assume an instance with three aircraft has a threshold value *n+1*. Taking the safety margin decrementation in **Step 2** as an example, a feasible solution can be found after several individual safety margin decrementations, and the safety margins are assumed to be $\{1,3,n+1\}$. If the priorities of these three aircraft are in increasing order, we can infer that any combination when the safety margin of the third aircraft is $n+1$, and the safety margins of the first and second aircraft are larger than 1 and 3, respectively, are infeasible, and therefore inequality 2 is proposed to remove these combinations as well as the infeasible solutions identified during the safety margin augmentation in **Step 2**. Similarly, inequality 3 is proposed to remove an infeasible solution whenever it is found during the individual augmentation stage in **Step 3**. We assume that the *feasibility check* finds a feasible solution for the decision $\{k-1,2,3\}$ and the priority of the three aircraft are in decreasing order. During the individual augmentation stage in **Step 3**, the safety margin of the first aircraft is augmented to k at first. If the *feasibility check* finds the solution is infeasible after augmentation, i.e. $\{k,2,3\}$, then any other value for the safety margin of the first aircraft larger or equal to k is infeasible, with the safety margins for the other two aircraft remaining unchanged.

4.3.2.2. Approximate inequality

Intuitively, the problem can be viewed as a nesting problem that places “enlarged” aircraft in the hangar, and the enlarged area of the aircraft is determined by the safety margin of each aircraft. Inequality 4 expressed in (4-20) is derived from the implication mentioned above: the sum of the “enlarged” area of aircraft determined by its safety margin cannot exceed the capacity of the hangar. Due to the irregular shape of the aircraft, it is found that the highest utilization of hangar space obtained in the numerical examples is around 50% (i.e. used area of the hangar/ total area of the hangar) as shown in computational results (Table 4-2) in Section 4.4.2. Therefore, restricting the sum of the enlarged area by the hangar area cannot effectively restrict the safety margin of the aircraft. An alternate method to measure the capacity of the hangar is to adopt the threshold value derived from the heuristic algorithm: the capacity of the hangar is approximately equal to the sum of the "enlarged" aircraft area determined by the threshold value, as the *feasibility check* cannot find a feasible solution when all the aircraft safety margins are larger than the threshold value. We would like to point out that the latent assumption in the problem is that overlaps of the "buffer area" of aircraft are allowed since the larger safety margin in a pair activates the respective revised NFP to separate the two aircraft (Figure 4-2(c)). However, inequality 4 regards the buffer area of an aircraft as a part of the aircraft since the inequality calculates the sum of the area of the enlarged aircraft and limits the sum to a defined value (the sum of area determined by the threshold value). Therefore, the interpretation of inequality 4 is as follows: the sum of the area of the enlarged aircraft area cannot exceed its counterpart determined by the threshold value derived from the heuristic in Section 4.4.2. In inequality 4, $aircraft\_revisedarea_i^n$ refers to the area of enlarged aircraft i associated with safety margin n.

$$\sum_{i \in P'} z_i^n \cdot aircraft\_revisedarea_i^n \leq \sum_{i \in P'} aircraft\_revisedarea_i^{threshold} \tag{4-20}$$

4.4. Computational results

This section presents the results of computational experiments that were carried out on instances based on real-life data provided by an aircraft maintenance company in Hong Kong. All the procedures described in the previous sections are coded in C# in Visual Studio 2010 and run on a computer with an Intel Core i7 processor, at 3.6 GHz with 32 Gb of RAM. The Mixed-Integer Linear Programming is solved by the CPLEX 12.7 serial model.

4.4.1. Description of instances

Similar to the computational experiment in Section 3.6 of Chapter 3, we collected data from an aircraft base maintenance service provider in Hong Kong and generated problem instances based on their actual data. The maintenance company we studied has over 50 clients, including airlines, business jet companies and utility aircraft companies. In particular, the maintenance hangar in the aircraft maintenance area of Hong Kong International Airport is operated by the company. The information related to the estimated arrival time (ETA), departure time (ETD), aircraft type and maintenance request of each maintenance order from clients was collected. We further figured out the number of aircraft needed to be arranged each day, the frequencies of the days and number of aircraft to be arranged, as presented in Figure 4-4. A mix of large-, middle-, and small-sized aircraft to be arranged in the hangar each day is typical, and it is reported that planning for 7 aircraft simultaneously by the manual method is challenging. In this regard, we adopted 40 testing instances based on the observed peak-day scenarios and the number of aircraft maintenance orders in those instances ranged from 6 to 12, which was also used in Chapter 3. We refer to Chapter 3 for the complete instance dataset used in the computational experiments. The peak-day scenarios observed in the actual situation were used as initial instances, then the proportion of small-, medium- and large-sized aircraft were adjusted, together with the adding of new maintenance orders in order to create challenging instances in the experiment.

Figure 4-4 Frequency of daily handing maintenance requests

In the instances set, we have 10 small-sized (e.g. G200, CL600, CL605, F900LX, F2000EX, F2000LX, ERJ135, F7X, G450 and GIV), 11 medium-sized (e.g. GL5T, G550, G5000, G6000, G650, A318, ERJ190, A319, A320, B738 and A321) and 2 large-sized (e.g. A332 and A333) aircraft types, which includes large-sized civil aircraft, medium-size civil aircraft as well as business jets. The classification of aircraft models is based on their area. 40 instances are divided into four groups according to the majority of the aircraft type for better presentation of the results, as follows: 1) the majority of aircraft in the instance are small-sized, 2) the majority of aircraft in the instance are medium-sized, 3) the majority of aircraft in the instance are large-sized and 4) the number of aircraft from different categories in the instance are equal. With regard to the safety margin range, we prescribed the minimal individual safety margins lb_i of all small-sized aircraft as 1 meter, medium-sized aircraft as 2 meters and large-sized aircraft as three meters in our computational experiment, which aligns with the idea that larger size aircraft should have higher safety distance. Referring to the practice adopted in the company, the maximal individual safety margin of all types of aircraft is prescribed as eight meters.

4.4.2. Computational results of the problem

The number of binary variables used to determine the relative positions between each pair of aircraft in the problem is determined by two factors: 1) the number of aircraft to be accommodated in the hangar, and 2) the range of the safety margins in the problem. It is reported that the branch-and-bound algorithm with the *horizontal slicing* MIP model can optimally solve the open-dimensioned nesting problem with, at most, 14 pieces, with convex- and non-convex shaped pieces (Alvarez-Valdes et al., 2013). To control the problem size to a moderate level, the upper bound of the safety margin is determined by the threshold value derived from the heuristic in this section, and the results are shown in the fifth column in Table 4-2. Considering that the problem is similar to the nesting problem with $piece \cdot (ub - lb + 1)$ pieces (*ub* and *lb* stands for the upper bound and lower bound of safety margin), such a problem size is challenging, so we referred to the stopping criterion adopted in literature. Alvarez-Valdes et al. (2013) set several time milestones (1h, 2h, 5h and 10h) in solving difficult instances that involve more than 12 complex irregular items. We therefore selected and prescribed the time limit for each instance as 18,000 seconds (5h), with the upper bound of the safety margin determined by the threshold value derived from the heuristic.

Table 4-2 shows the computational results for the problem. The number of aircraft to be placed in the hangar with maximal overall profit and satisfying minimal safety margins is indicated in the third column. As inequalities 2&3 thoroughly utilize the information of the intermediate solutions during the heuristic search, these two are regarded as the most comprehensive and powerful ones to enhance the computational efficiency. Therefore, we analyse the effectiveness of the proposed heuristic with inequalities 2 and 3 used to provide the initial solution and to remove infeasible solutions before the branch-and-bound algorithm. The computational results of CPLEX solving these instances are derived from the previous work in (Qin et al., 2018) for comparison. We are able to optimally solve 21 instances, with the upper bound of the safety margin determined by the threshold from the heuristic. We found that in many instances, the initial solution provided by the heuristic proved optimal by the exact algorithm, after an exhaustive search in some instances. In addition, in 12 instances (those heuristic values in <u>**bold & underlined**</u>) out of 21 that are solved to optimal, the solution provided by the heuristic algorithm is found to be optimal, demonstrating the applicability of the developed heuristic approach. Although in some cases the objective function value increased in the branch-and-bound algorithm, the searching processes took a long time. In the problem, the difference between the best integer and upper bound corresponds to the pending safety margin of each aircraft. To update the upper bound of the branch-and-bound, one has to prove the infeasibility of the pending safety margins, which corresponds to a single *feasibility check*. Though the final layout demonstrates a satisfactory result in regard to operation in practice, large gaps have been recorded in instances with packed layouts, since updating the bounds in the problem we studied is far more difficult than the model involving only one NPFs to separate each pair of aircraft.

Moreover, the performance of the model and the computation time differs a lot, even comparing two instances from same instance group with the same number of aircraft to be arranged in the problem. Taking Instances 23 and 24 as examples, the number of aircraft placed in the hangar satisfying the minimal safety margin requirement and the threshold value derived from the heuristic are the same, and the number of binary variables involved in the two instances are similar, with similar problem settings (three large-size aircraft and 3 small-sized/ medium-sized aircraft), while the exact algorithm takes much more time to solve instance 23 to optimal. It is found out that adding inequalities into the original model can tighten the upper bound but does not necessarily accelerate the searching process of the branch-and-bound algorithm. To ensure the efficiency of the branch-and-cut algorithm, a balance between the generation of the cutting plans and branching must be considered. The searching process might be hindered by adding too many inequalities in the original LPs, although better bounds result in fewer explored nodes (Taccari, 2016), which can also be observed in the computational results in Section 4.4.3.

The computational results in Section 4.4.2 demonstrate that inserting inequalities 2&3 with the warm start provided by the heuristic algorithm is able to shorten the computational time for obtaining an optimal solution, or tighten the optimality gap if the optimal solution cannot be obtained within the time limit, for many instances, compared with the original MIP model, without adding inequalities and providing a warm start by the heuristic algorithm. For the large instances (36-40), the inequalities 2&3 with the heuristic are able to achieve better solution, and instance 38 can be solved to optimal, compared with the original model. Moreover, improvement of the lower bound is recorded after inserting the inequalities, given the lower bound value provided by the heuristic.

Table 4-2 Computational results of the PSAP problem

Instance	Aircrafts in Instance (total_S/M/L)	Number of aircraft placed in the hangar[1]	Proportion of used hangar space	Binary variables	Safety Margin upper bound by heuristic	CPLEX + **Threshold Safety Margin upper bound**				Heuristic Warm Start + Inequalities 2&3 + **Threshold Safety Margin upper bound**				
						LB	UB	CPU	Gap	CPLEX with Heuristic warm start and inequalities 2&3			Heuristic	CPU **CPLEX + Heuristic**
										LB	UB	Gap	Value	
1	6_4/1/1	6	0.29	2792	8	29357.92	29357.92	0.36	0	29357.92	29357.92	0	**29357.92**	1.91
2	6_3/2/1	6	0.30	2732	8	28823.36	28823.36	0.30	0	28823.36	28823.36	0	**28823.36**	2.11
3	7_4/1/2	7	0.35	3822	8	35110.96	35110.96	0.89	0	35110.96	35110.96	0	**35110.96**	3.00
4	7_3/2/2	7	0.39	4038	8	38709.12	38709.12	4.57	0	38709.12	38709.12	0	**38709.10**	4.55
5	8_5/2/1	8	0.32	4832	8	41349.36	41349.36	0.69	0	41349.36	41349.36	0	**41349.36**	4.39
6	8_4/2/2	8	0.44	4498	7	34591.16	37303.56	18000	7.84	34188.06	36522.06	6.83	31711.08	18175.03
7	9_7/1/1	9	0.34	6888	8	33723.12	33723.12	444.99	0	33723.12	33712.12	0	**33723.12**	7.67
8	10_8/1/1	10	0.38	8508	8	33926.74	37107.52	18000	9.38	35037.98	36680.60	4.69	34516.98	19781.78
9	11_9/1/1	11	0.33	10224	8	37334.12	37334.12	717.54	0	37334.12	37334.12	0	35674.76	1484.81
10	12_8/3/1	12	0.46	7172	5	N/A	33833.65	18000	N/A	28269.22	33260.65	17.66	25176.41	18647.12
11	6_1/3/2	6	0.45	1356	4	16836.14	16967.43	4423.36	0.78	16836.14	16836.14	0	**16836.14**	1868.15
12	6_0/4/2	6	0.38	2944	8	35671.28	35671.28	2159.54	0	35671.30	35671.30	0	34943.28	1517.73
13	7_0/5/2	7	0.47	2464	5	22436.38	29243.1	18000	30.34	23591.4	27658.8	17.24	23591.4	18624.09
14	8_1/5/2	7	0.51	920	2	N/A	11069.32	18000	N/A	7643.21	12735.22	66.62	7643.21	19324.23

(Table 4-2 Cont'd)

Instance	Aircrafts in Instance (total_S/M/L)	Number of aircraft placed in the hangar[1]	Proportion of used hangar space	Binary variables	Safety Margin upper bound by heuristic	CPLEX + **Threshold Safety Margin upper bound**				Heuristic Warm Start + Inequalities 2&3 + **Threshold Safety Margin upper bound**				
						LB	UB	CPU	Gap	CPLEX with Heuristic warm start and inequalities 2&3			Heuristic	CPU **CPLEX + Heuristic**
										LB	UB	Gap	Value	
15	8_2/4/2	8	0.45	3688	6	27928.82	31022.16	18000	11.08	27928.82	30109.36	7.81	27928.82	20538.05
16	8_1/5/2	8	0.49	2580	3	N/A	24539.8	18000	N/A	16477.6	22955.5	39.31	17068.8	20110.42
17	9_2/5/2	9	0.44	5100	6	27226.98	30949.08	18000	13.67	27936.34	30360.08	8.68	26945.48	19849.69
18	10_3/7/0	10	0.29	8286	8	27062.56	27062.56	1117.42	0	27062.56	27062.56	0	**27062.56**	41.83
19	11_1/9/1	10	0.45	2534	3	N/A	16240.14	18000	N/A	10628.76	10628.76	0	9491.5	2834.62
20	12_2/9/1	10	0.48	2945	2	N/A	16663.38	18000	N/A	15941.92	16155.38	1.34	13049.46	20368.88
21	4_1/1/2	4	0.33	1136	8	34028.08	34028.08	0.41	0	34028.08	34028.08	0	**34028.08**	1.38
22	6_1/2/3	5	0.44	741	3	12882.54	12882.54	31.57	0	12882.54	12882.54	0	11640.36	330.94
23	6_2/1/3	6	0.43	1041	3	12950.94	12950.94	2042.04	0	12950.94	12950.94	0	11716.44	5521.64
24	7_2/1/4	6	0.44	1107	3	12962.70	12962.70	847.48	0	12962.70	12962.70	0	11693.80	1411.98
25	7_2/2/3	6	0.44	1518	3	13543.04	18485.04	18000	36.49	13023.54	13023.54	0	12323.36	1563.5
26	7_1/2/4	5	0.44	486	2	5564.0	5564.0	29.38	0	5564.40	5564.40	0	**5564.40**	41.92
27	8_2/2/4	6	0.46	1065	3	12652.53	12652.53	5458.65	0	12652.53	12652.53	0	11487.02	4847.19
28	10_3/3/4	8	0.50	2058	3	14417.81	19359.81	18000	34.28	14417.81	18631.81	29.23	12906.54	20630.42

(Table 4-2 Cont'd)

Instance	Aircrafts in Instance (total_S/M/L)	Number of aircraft placed in the hangar[1]	Proportion of used hangar space	Binary variables	Safety Margin upper bound by heuristic	CPLEX + **Threshold Safety Margin upper bound**				Heuristic Warm Start + Inequalities 2&3 + **Threshold Safety Margin upper bound**				
						LB	UB	CPU	Gap	CPLEX with Heuristic warm start and inequalities 2&3			Heuristic	CPU **CPLEX + Heuristic**
										LB	UB	Gap	Value	
29	10_3/2/5	8	0.48	1998	3	14263.70	19205.70	18000	34.65	14263.70	18882.90	32.38	12803.80	19505.05
30	12_2/5/5	9	0.54	1730	2	N/A	14254.66	18000	N/A	9018.16	13890.66	54.03	8574.11	19939.41
31	6_2/2/2	6	0.36	2700	8	34444.88	34444.88	0.28	0	34444.88	34444.88	0	**34444.88**	2.53
32	6_2/2/2	6	0.34	2876	8	37012.80	37012.80	0.33	0	37012.80	37012.8	0	**37012.80**	1.75
33	6_2/2/2	6	0.41	2736	8	39290.44	39290.44	73.59	0	39290.44	39290.44	0	38305.84	169.19
34	9_3/3/3	8	0.50	1922	3	16096.74	18519.21	18000	15.05	16307.07	17948.01	10.06	16307.07	19728.98
35	9_3/3/3	9	0.51	1714	2	N/A	13392.80	18000	33.58	10104.6	13231.40	30.94	10035.1	19351.76
36	9_3/3/3	8	0.49	1956	2	13752.47	18694.47	18000	35.94	13752.47	17966.47	30.64	12462.98	18468.44
37	12_4/4/4	9	0.50	2114	2	N/A	21408.24	18000	N/A	16466.24	21085.44	28.05	14692.84	19059.45
38	12_4/4/4	8	0.45	1368	2	N/A	12895.14	18000	N/A	8592.68	12323.94	43.42	8592.68	18812.17
39	12_4/4/4	8	0.39	1710	2	N/A	12745.04	18000	N/A	7802.04	11168.64	43.15	6931.07	20034.97
40	12_4/4/4	10	0.53	2100	2	N/A	13897.12	18000	N/A	10166.10	13735.72	35.11	10166.1	20101.02

[1]The number of aircraft placed in the hangar is predetermined by maximizing the utilization of hangar space in problem.

4.4.3. Comparing inequalities for the problem

To compare the effectiveness of the proposed inequalities, we selected those instances that were solved to optimality within 18,000 seconds in the problem in Section 4.4.2, with the safety margin upper bound less than 8 meters. In this section, we relaxed the safety margin upper bound to the origin upper bound limit (8 meters) in both the MIP formulation and heuristic algorithm in order to find the improvement of the objective value, as well as the effectiveness of the proposed inequalities in tightening the upper bound. Table 4-3 shows the results of proposed heuristic and a comparison between the four inequalities described in Section 4.3.2. The first column indicates the testing instances that are optimally solved in Section 4.4.2. The performance of each strategy is indicated by four values: the lower bound (best known solution), upper bound, optimality gap and computational time. The time limit for each testing instance is 18,000 seconds, and the layout of the best-known solutions in this section can be found in the Supplementary materials 4.5.

Table 4-3 Comparison among the inequalities in solving the PSAP problem

Instance	Binary Variables	Safety Margin upper bound	Heuristic		CPLEX				CPLEX + Inequality 1			
			CPU	Value	LB	UB	Gap	CPU	LB	UB	Gap	CPU
11	2680		101.14	17661.77	19364.73	27335.08	41.16	18000	19364.73	31771.56	64.07	18000
19	9954		3036.09	10925.18	11644.28	43307.04	271.92	18000	13080.78	42119.04	221.99	18000
22	1976		35.63	14636.04	16553.44	25605.54	54.68	18000	16553.44	16553.44	0	4512.38
23	2776		71.28	16798.74	18033.24	46277.44	156.62	18000	18033.24	40728.34	125.85	18000
24	2930	8	69.21	16777.84	18560.56	46775.20	152.01	18000	18560.56	39957.28	115.28	18000
25	3994		796.99	15516.18	16689.54	49293.44	195.36	18000	16689.54	48147.44	188.49	18000
26	1944		19.21	6532.80	6532.80	25426.2	289.21	18000	6532.80	6532.80	0	9196.88
27	2840		562.34	14022.68	15940.08	45948.08	188.26	18000	15940.08	40990.72	157.16	18000
38	5458		1854.96	8451.79	8451.79	50411.76	496.46	18000	8451.79	48917.54	478.78	18000

N/A: no feasible solution found

(Table 4-3 Cont'd)

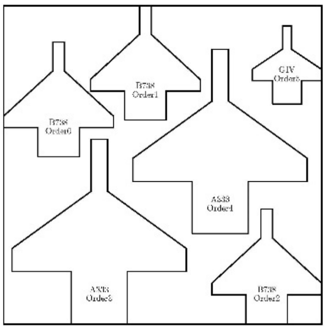
Instance	Binary Variables	Safety Margin Upper bound	CPLEX + Inequalities 2 & 3				CPLEX + Inequality 4			
			LB	UB	Gap	CPU	LB	UB	Gap	CPU
11	2680		18857.68	31059.58	64.71	18000	18857.68	22680.29	20.27	18000
19	9954		11662.50	40568.64	247.86	18000	11349.78	21210.48	86.88	18000
22	1976		16553.44	16553.44	0	10149.44	16553.44	16553.44	0	393.62
23	2776		18033.24	40909.24	126.85	18000	17078.24	19012.05	11.32	18000
24	2930	8	18560.56	45645.40	145.93	18000	17430.76	18907.66	8.47	18000
25	3994		16514.64	46745.44	183.05	18000	16689.54	21132.16	26.62	18000
26	1944		6532.80	11710.20	79.25	18000	6532.80	6532.80	0	3510.80
27	2840		15940.08	36442.28	128.62	18000	15299.70	18456.47	20.63	18000
38	5458		12652.93	48343.69	282.08	18000	9968.73	14404.17	44.49	18000

N/A: no feasible solution found

The results in Table 4-3 show that there is no inequality that is always dominant across all the instances. We find that in some cases the performance of inequality 1 suppresses that of inequalities 2 and 3. We notice that the proposed approximate inequality 4 based on the idea of placing the "enlarged" aircraft into the hangar is able to eliminate a number of combinations of safety margins that significantly exceed the capacity of the hangar and tighten the upper bound in some cases. Although the idea of placing the "enlarged" aircraft in the problem is an approach for controlling the combinations of safety margins and tightening the upper bound, accurate calculation of the sum of the "enlarged" area is required so as to avoid removing a feasible solution, since the overlaps of the "enlarged" part between each pair of aircraft is allowed according to the definition of the safety margin and the methodology applied to separate aircraft. The computational experiments conducted in Section 4.4.3 focus on the challenging instances created by relaxing the upper bound of safety margin for the instances solved to optimal in Section 4.4.2. Therefore, we mainly focus on the final optimality gap recorded after inserting different inequalities. The results have demonstrated that inequality 4 is able to tighten the upper bound of the problem but is not necessarily valid for all cases, and therefore we would only consider inserting inequality 4 to tackle large instances and in providing an approximate estimation of the limit of the maintenance hangar.

4.5. Supplementary materials: layout of the best-known solutions

Best Known Parking Layout for instances tested in Section 4.4.3



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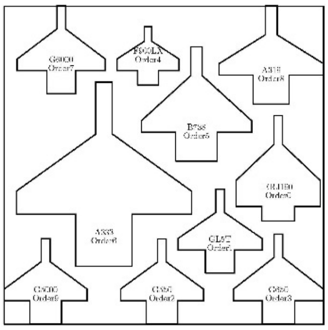

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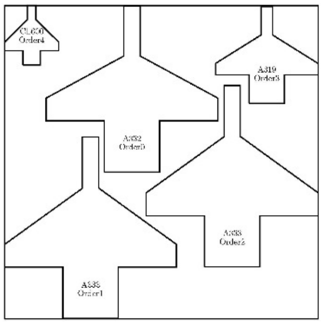

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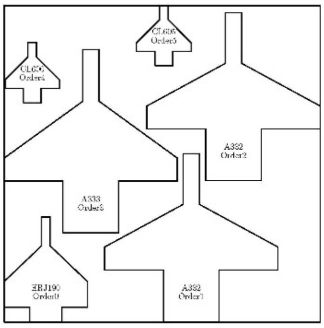

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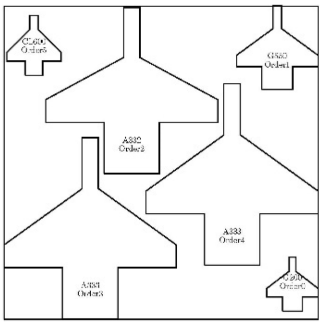

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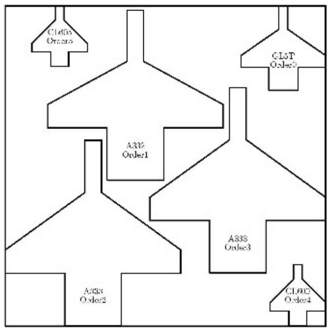

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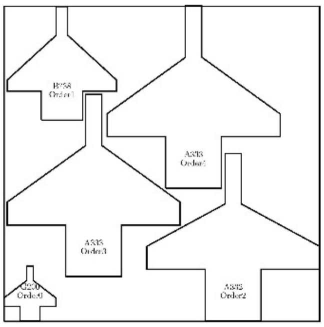

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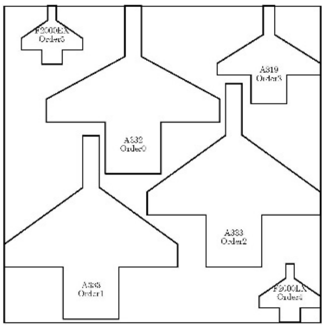

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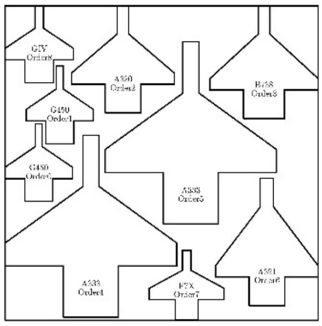

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4.6. Summary

In this chapter, we examined the problem of determining the optimal individual safety margins within the admissible range of safety margins for aircraft to be serviced in a maintenance hangar by a maintenance service company during a planning period in order to minimize the risk of collision during aircraft movement operations as well as during the maintenance processes. This problem arises with the increasing number of outsourced maintenance requests from clients studied in Chapter 3, and the maintenance company has to efficiently utilize their limited maintenance base space to meet the requirements of various clients. We first present a complete mixed integer linear programming model for the aircraft parking stand arrangement problem, considering the accurate shape of aircraft, then incorporate a set of revised NPFs to enforce the discrete safety margin between each pair of aircraft. To tackle the large number of binary variables involved in the model, we developed a heuristic approach to provide a practical solution within a reasonable time. Moreover, a set of inequalities is proposed to convert the recorded infeasible solutions during the heuristic search as cuts to be added in the mathematical model before implementing the branch-and-bound algorithm provided by CPLEX. Problem instances in computational experiments are derived from an aircraft maintenance company in Hong Kong, and the computational results show that the proposed approaches are applicable and beneficial to the problem in practice. The parking stand allocation model is extended to incorporate other realistic considerations, including the aircraft movement operations and the availability of technical staff in the subsequent chapters.

Chapter 5. Rolling Horizon Approach for the Aircraft Hangar Maintenance Scheduling Problem

5.1. Introduction

A multi-period aircraft hangar maintenance scheduling problem is studied in MRO outsourcing context in this chapter. The hangar maintenance scheduling problem consists of determining a maintenance schedule with minimum penalty costs in fulfilling maintenance requests, and a series of hangar parking plans aligned with the maintenance schedule through the planning period. In the model, the variation of parking capacity of the maintenance hangar and the blocking of the aircraft rolling in and out path are considered in the MILP model. Afterwards, the original model is enhanced by narrowing down the domain of the time-related decision variables to the possible rolling in and out operations time of each maintenance request. A rolling horizon approach incorporating the enhanced mathematical model is presented to obtain good quality feasible solutions for large scale instances. The results of computational experiments are reported, showing: (i) the effectiveness of the event-based discrete time MILP model and (ii) the scalability of the rolling horizon approach that is able to provide good feasible solutions for large size instances covering a long planning period.

The maintenance scheduling and parking layout planning problem studied in this chapter considers the daily aircraft hangar maintenance operations under the MRO outsourcing mode, which extends the static aircraft parking stand allocation model proposed in Chapter 3 to multi-period planning to fit the practical operations. Periodic maintenance checks need to be carried out on each aircraft upon meeting operating for a specified number of flying hours. According to different airlines' flight plans, multiple maintenance requests are initiated by their internal maintenance plans. From the perspective of the maintenance service provider, efficiently fulfilling the overwhelming maintenance requests with limited resource availability becomes challenging. One of the major tasks within the maintenance company is to develop a maintenance schedule which involves substantial operational decision making. Such a maintenance plan includes the maintenance schedule for each aircraft (roll in and roll out time) and the parking position of each aircraft in the hangar. Over the planning horizon, the roll in and roll out times of all aircraft should align with the parking plans. The development of such a plan is challenging due to the following considerations: (i) the hangar capacity that accommodates the aircraft varies according to the incoming maintenance requests at different times; (ii) blocking between aircraft occurs whenever there are many incoming maintenance requests arriving at similar times, or the planner makes improper roll in and roll out arrangements. To address these issues and provide a systematic approach to solve the problem, we propose an optimization methodology to develop maintenance plans from the perspective of the independent aircraft maintenance service company. We consider the blocking of aircraft movement operations due to improper hangar planning and overwhelming maintenance requests as a significant bottleneck in fulfilling the maintenance requests, while a such factor has not been incorporated in the other multi-period layout planning problems in the literature. In this regard, the major focus of this chapter falls into the coordination between maintenance scheduling and hangar layout planning. Other practical factors, such as arranging the aircraft's position according to its maintenance type and distance to specific tooling, and manpower limitations, are not incorporated in the scope of the model. The extension of the model to incorporate manpower planning is discussed in subsequent chapter.

The remainder of this chapter is organized as follows. The literature review in Section

5.2 analyses the problem nature and the correlation with layout planning problem. Afterwards, we present problem description in 5.3. Mathematical model and the solution procedures for the proposed problem are discussed in Sections 5.4 and 5.5. The results of computational experiment are reported in Section 5.6. Finally, the concluding summary and future work are discussed in Section 5.7.

5.2. Review on layout planning problem

The problem studied in this chapter involves a dynamic layout planning problem. In the literature, some optimization problems share some similarities in the problem nature and assumptions. The extension of the traditional Vehicle Routing Problem (VRP) incorporating simultaneous picks-up and deliveries, and two-dimensional loading constraints (2L-SPD) belongs to the class of the composite routing-packing optimization problem (Zachariadis et al., 2016). In Vehicle Routing Problem with Two-dimensional Loading and picks-up/deliveries constraints, one has to determine the route of a vehicle that satisfies customers at different demand and delivery points and consider a two-dimensional packing problem for the placing the goods in the vehicle for different customers (Wei et al., 2015), requiring that the routing of the vehicle must satisfy the Last-In-First-Out (LIFO) loading and unloading constraint. In addition, in the literature, the items to be arranged in the vehicle are all rectangle (Cheang et al., 2012; Cherkesly et al., 2015; Cote et al., 2014; Wei et al., 2015; Zachariadis et al., 2016). Moreover, the Facility Layout Problem (FLP) is another classic layout planning problem, which aims to determine the locations of rectangular facilities at different sites, minimizing the material handling costs between the facilities (Anjos & Vieira, 2017; Paes et al., 2017; Solimanpur & Jafari, 2008; Xie & Sahinidis, 2008). Dynamic Facility Layout Problems consider arranging the facilities over a planning period instead of one-time planning (Dunker et al., 2005; J. P. Xu & Song, 2015). Though layout planning problems have been extensively studied in the literature from various perspectives, such as the manufacturing industry (Ahmadi & Jokar, 2016; Bagheri & Bashiri, 2014; Bozer & Rim, 1996; Mohammadi & Forghani, 2014; Tyagi et al., 2016), the relevant approaches cannot be directly applied in our problem due to the following considerations: (i) the shape of an aircraft is irregular. (ii) The Last-In-First-Out constraint can be relaxed as a soft constraint in the maintenance scheduling problem. (iii) Blocking during the aircraft roll in/out operations significantly affects the efficiency and needs to be characterized.

5.3. Problem statement and mathematical formulation

5.3.1. Problem statement

Aircraft heavy maintenance has to be conducted in the aircraft hangar after meeting the flying hours prescribed by the aviation authorities(Van den Bergh et al., 2013). The aircraft is taken out of service and sent to a maintenance service company for heavy maintenance. The maintenance service company receives the maintenance requests initiated by the respective airlines according to their internal flying plans. To fulfill these maintenance requests from clients, the maintenance service company has to determine 1) a maintenance schedule serving the requests, consisting of the timing of movement operations for each aircraft and 2) hangar parking layouts at different times whenever there are any movement of aircraft inducing the changes of the hangar layout along the planning period, subject to the capacity of hangar space. The main goal is to minimize the penalty cost induced in fulfilling the maintenance requests. Figure 5-1 demonstrates a solution of hangar maintenance problem. In particular, Figure 5-1 demonstrates the transitions of the hangar layout plan from time 1 to 10, and it specifies the position assigned for each aircraft and the respective roll in and roll out timings along the planning period. It is note that the hangar layout at Time 2 is omitted as there is not any movement operation conducted at that time, and therefore the layout at Time 2 is kept unchanged and it is the same as Time 1. Specifically, the downward arrow represents that the respective aircraft is rolled into the hangar at current time, and the upward arrow means that the aircraft has finished the maintenance task and is rolled out from the hangar at respective time. The hangar layouts of all times are coherent with respective preceding and subsequent layout plans to ensure the continuity. Moreover, if there are both rolling in and rolling out operation taken place at same point of time, the roll in operation commences after all rolling out operations finish. Take the hangar layout at Time 3 as an example, there is one large aircraft rolling out from the hangar, then two newly arrival small aircraft rolling into the hangar to take up the space vacant from the large departing aircraft.

The contributions of the studied problem can be summarized as twofold: 1) from the perspective of the MRO industry, many researches focused on airline-operated MRO activities' optimizations, which makes the existing approaches inapplicable for the maintenance service company in actual situations. Given the situation that the service company carries out the hangar maintenance schedule manually, in current practice, the developed mathematical model is tailored for the hangar maintenance service company, which significantly increases their planning efficiency and accuracy. 2) from the perspective of academia, it fills the gaps in the literature regarding the aircraft maintenance problem in the context of the MRO service company that have not been addressed yet. Moreover, the problem studied in this chapter extends the multi-period layout planning problem, as the blocking during the facility movements during planning was not regarded as a main bottleneck in the other studies.

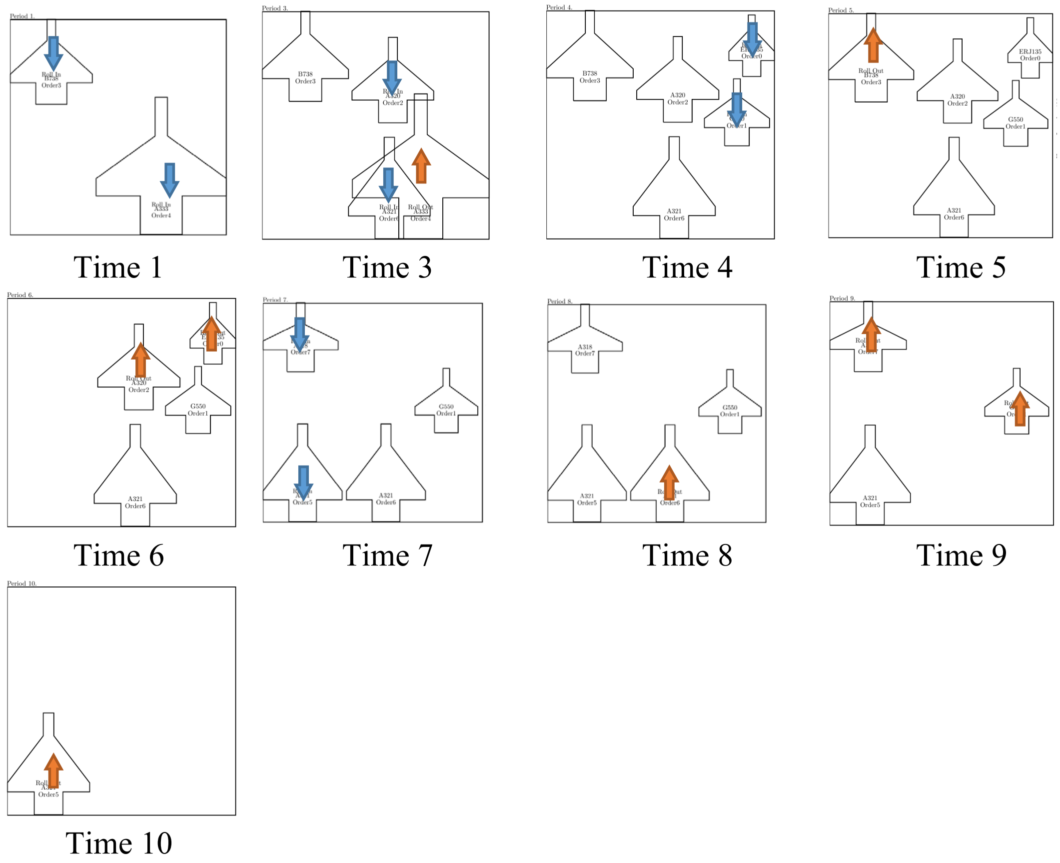


Figure 5-1 Hangar maintenance problem

5.3.2. Aircraft non-overlapping approach and three-dimensional parking

As we consider the physical shape of an aircraft in undertaking the parking planning, appropriate modelling of aircraft is fundamental to fully utilize the hangar space. The non-overlapping approach discussed in this section is incorporated in the mathematical model. Given the geometric shape of an aircraft, it can be characterized as a non-convex polygon (Figure 5-2). We denote the reference point of each aircraft to be the middle point at the bottom of the aircraft, and the coordinates of the reference point of aircraft p_i in two-dimensional space are denoted as (x_i, y_i). For a pair of aircraft p_i and p_j, the No-fit polygon NFP_{ij} is the region in which the reference point of aircraft p_j cannot be placed if aircraft p_i remains stationary since it would overlap aircraft p_i.

A feasible zone for placing aircraft p_j without overlap with p_i is the region outside NFP_{ij}. Given these two polygons, the NFP_{ij} is generated by tracing the path of the reference point on p_j as p_j slides around the boundary of p_i, such that two polygons always touch but never overlap (Figure 5-3). Therefore, if the reference point of *j* moves into the NFP_{ij} then the two polygons overlap, and the interior of the NFP_{ij} represents all overlapping positions. According to Alvarez-Valdes et al. (2013), each horizontal slice is defined by drawing one or two horizontal line(s) outwards from each vertex of the NFP, and they are then characterized by one or two horizontal edge(s) as well as the part of boundary of the NFP (Figure 5-4 (a)).

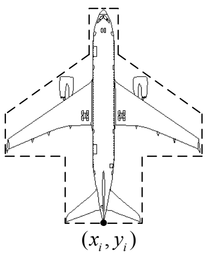


Figure 5-2 Coordinate of aircraft in model

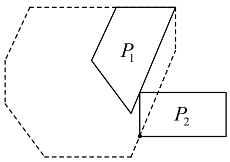
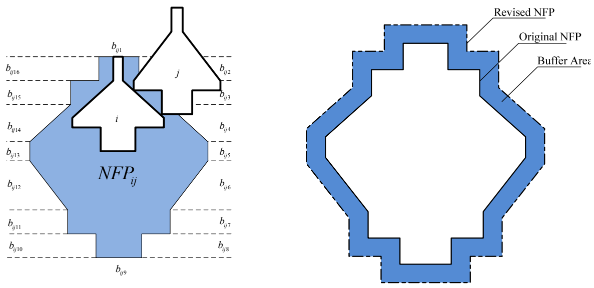


Figure 5-3 NPF between two polygons



(a) (b)

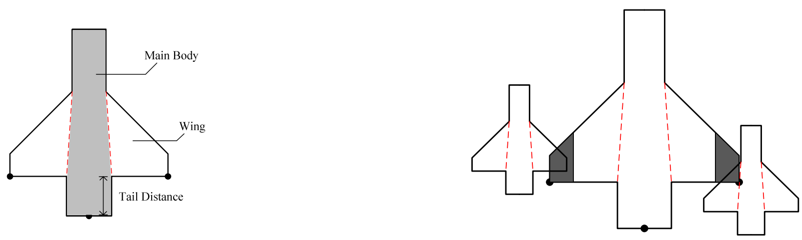
Figure 5-4 Horizontal slicing in mathematical formulation

A set of variables b_{ijk} is associated with each horizontal slice and the reference point of p_j is placed in the slice *k* if $b_{ijk} = 1$. Therefore, a general form of the constraint preventing overlap is

$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) \le q_{ijk} + M \cdot (1 - b_{ijk}),\ \forall i, j \in P, i \ne j, k = 1, 2, ..., m_{ij}$$

where $\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(y_j - y_i) = q_{ijk}$ is the equation of the line of the *f*th edge of the *k*th slice in NFP_{ij} and m_{ij} is the number of slices outside the NFP_{ij}. In a real situation, we cannot allow two aircraft to touch each other during the movement operation. Therefore, a safety margin between aircraft needs to be imposed in NFPs. Imposing a safety margin for an aircraft is equivalent to adding a buffer area outside each aircraft. Moving the edges of NFP for a pair of aircraft outward is equivalent to enlarging the boundary of the non-allowable area for the reference point of the relative movable aircraft in that pair. Each edge of the original NFPs is moved outwards by distance *n* (Figure 5-4 (b)), and the minimum safety distance between two aircraft is prescribed as one meter.

To make the most of the hangar space, the wing of a smaller aircraft can be placed under the wing of a larger aircraft. Such “overlap” of aircraft wings between two aircraft is permissible as the two aircraft’s wings are of different heights, within a safety distance (Figure 5-5), while keeping main bodies of the aircraft separate.



Parts of aircraft

“Three-dimensional” parking with wings “overlap” in two-dimensional view

Figure 5-5 Three-dimensional parking arrangement

After decomposing the aircraft components into the main body and the aircraft, another set of non-overlapping constraints can be derived by using two sets of *NFPs* that separate each pair of aircraft from three-dimensional space, as shown in Figure 5-5: The *Main Body NFP* is used to separate the main body of the two aircraft, and the *Revised Wing NFP* is used to separate the wings of the aircraft with an allowance for “overlap” within a safety margin. For those pairs of aircraft of different wing heights with safety margins between the wings, the non-overlapping constraints (5-4) derived from original *NFP* are replaced by that derived from the *Main Body NFP* and the *Revised Wing NFP*. In this connection, two sets of binary variables that act as a similar function to b_{ijkt} are introduced and are used to separate the main body and the wings of aircraft, respectively.

5.4. Mathematical formulation

5.4.1. Assumptions

The basic assumptions that describe the proposed problem are as follow:

- the estimated time of arrival, estimated time of departure, and required maintenance time are assumed to be deterministic, and the time spent on movement is incorporated in the required maintenance time;
- once the aircraft is rolled into the hangar, its parking position cannot be adjusted until the maintenance task is finished and the aircraft leaves the hangar;
- once the aircraft is rolled into the hangar, the maintenance task must be finished before leaving the hangar. If the planning period ends before finishing the maintenance task (due to the delays of rolling in), such maintenance request is deemed as failed to deliver
- if the arriving aircraft (or the departing aircraft) is blocked by any parked aircraft in the hangar, its movement operations cannot be conducted until its pathway is cleared;
- the moving path of an aircraft is a straight line and turning is not allowed due to safety consideration.
- the aircraft cannot revisit the maintenance hangar after leaving, i.e. the rolling in and rolling out operations can be conducted only once.

- the time spent on roll in and roll out operations are incorporated in the required maintenance time.
- the model applies to planning for regular maintenance. Unexpected events or demands are not considered
- the manpower is assumed to be sufficient to complete the maintenance tasks.

5.4.2. Parameters and decision variables

The given information (parameters) of the problem consists of:

- The specification of each maintenance request, including the aircraft type, the required maintenance services (maintenance check), estimated time of arrival (ETA) to the hangar, and estimated time of departure (ETD), also known as the expected delivery time. The weightiness of each maintenance request.
- The geometric information of the different aircraft types, including the size of the aircraft type and the No-Fit Polygons for each pair of aircraft.
- Different penalty costs induced while fulfilling the maintenance requests.
- The dimensions of the maintenance hangar.

The list of notations for parameters mentioned above are as follows:

Notations

a_t	Set of scheduled arrival maintenance request at time *t*
d_t	Set of schedule departure aircraft in hangar at time *t*
A_t	Set of cumulative scheduled arrival aircraft in hangar from beginning to time *t*. $A_t \in \bigcup_{i=0}^{t} a_t$
D_t	Set of cumulative scheduled departure aircraft in hangar from beginning to time *t*. $D_t \in \bigcup_{i=0}^{t} d_t$

A_T	Set of maintenance requests received during planning horizon
t	Index of time, where T is the length of planning horizon
ETA_i	Estimated time of arrival of maintenance request associated with aircraft i
ETD_i	Estimated time of departure of maintenance request associated with aircraft i
$MTime_i$	Required maintenance time of maintenance request associated with aircraft i
w'_{ij}	Adjusted aircraft width i when aircraft j placed next to it
TD_i	Tail distance of aircraft i
$penalty1$	Penalty of not serving aircraft i during planning period (per request)
$penalty2$	penalty of late delivery of aircraft i during planning period (per minute)
	Penalty of failure to deliver aircraft i during planning period (per request)
$Weightness_i$	Weightiness of maintenance request i
W	width of hangar
H	length of hangar
w_i	width of aircraft i
h_i	length of aircraft i
NFP_{ij}	NFP of aircraft i and j with minimal safety distance
s_{ij}^k	kth slice of the region outside the NFP_{ij}
$\alpha_{ij}^{kf}, \beta_{ij}^{kf}, q_{ij}^{kf}$	parameters used to define the fth linear equation of the slice s_{ij}^k outside the NFP_{ij}
m_{ij}	number of slices outside NFP_{ij}
t_{ij}^k	number of linear equations used to define the slice s_{ij}^k
M	a large number

To determine a maintenance schedule to fulfill the maintenance requests as well as hangar layouts at different times, the following decision variables are introduced, and the uses of auxiliary decision variables in developing specific constraints are discussed in Section 3.3.3 in Chapter 3.

Decision Variables

(x_i, y_i)	position of reference point of aircraft i in the hangar
out_{it}	binary decision variable that takes the value 1 if aircraft i is rolled out at time t, and 0 otherwise

in_{it}	binary decision variable that takes the value 1 if aircraft i is rolled in at time t, and 0 otherwise
out_{iT^*}	binary decision variable that takes the value 1 if fail to deliver aircraft i at the end of planning horizon, and 0 otherwise
p_{it}	binary decision variable that takes the value 1 if aircraft i is parked in hangar at time t, and 0 otherwise
h_{ijt}	binary decision variable that takes the value 1 if aircraft j blocks aircraft i from rolling in or out at time t, and 0 otherwise
L_{ij}	binary decision variable that takes the value 1 if aircraft i is on the left side of aircraft j without overlap, and 0 otherwise
R_{ij}	binary decision variable that takes the value 1 if aircraft i is on the right side of aircraft j without overlap, and 0 otherwise
U_{ij}	binary decision variable that takes the value 1 if aircraft i is above aircraft j without overlap, and 0 otherwise
b_{ijkt}	binary decision variable that takes the value 1 if the reference point of aircraft j is placed into the slice s_{ij}^k of the region outside NFP_{ij} at time t, and 0 otherwise

5.4.3. Objective and constraints

$$Minimize \sum_{\forall i \in A_T} Weightiness_i \cdot \begin{bmatrix} (1-\sum_{t \geq ETA_i} in_{it}) \cdot penalty1_i + \sum_{t \geq ETD_i} out_{it}(t-ETD_i) \cdot penalty2_i \\ +out_{iT^*} \cdot penalty3_i \end{bmatrix}$$

The objective function minimizes the overall penalty costs while fulfilling the maintenance request. It includes the penalty costs of 1) lateness in fulfilling the maintenance requests along the planning horizon; 2) failure to complete the maintenance requests by the end of the planning period and 3) the lost cost in rejecting the maintenance request.

As mentioned earlier, the maintenance hangar operates in a multiperiod context, and the total planning horizon is represented by discrete times along the entire period (Figure 5-6). Each point on the timeline is used to represent the decision and status of the maintenance hangar at time t. The integrated decision at time t involves 1) determining if there are any movement operations conducted at time t (out_{it} and in_{it})

and 2) assigning the position of aircraft (x_i, y_i) parking in the hangar. As the position of an aircraft cannot be changed once it is moved into the hangar, the coordinates of the aircraft are not indexed with time *t*. The other auxiliary decision variables, i.e. out_{iT^*}, p_{it}, h_{ijt}, L_{ij}, R_{ij}, U_{ij} and b_{ijkt}, ensure the outcome of a solution is a logical and rational one. In this regard, it is possible that there may not have any movement operations for a consecutive period, as all maintenance requests are being processed or there are no newly arrival maintenance requests at that time. By combining the integrated decision for each discrete time along the planning horizon, the mathematical model allows us to determine the maintenance schedule and respective aircraft parking arrangement.

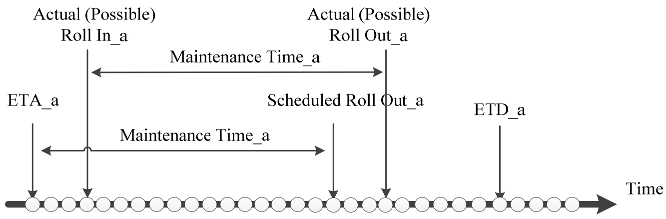


Figure 5-6 Basic discrete-time model

The constraints in the mathematical model can be divided into several functions:

1) Non-overlapping constraint

The aircraft received by the maintenance service company should be served within the boundary of hangar, and the aircraft should be separated with the minimum safety margin while parked in the hangar, using the No-Fit Polygons given in Section 3.2 in Chapter 3.

$$x_i + w_i / 2 \le W, \ \forall i \in A_t \tag{5-1}$$

$$x_i \ge w_i / 2, \ \forall i \in A_t \tag{5-2}$$

$$y_i + h_i \le H,\ \forall i \in A_t, \forall t \ge 0 \tag{5-3}$$

$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(x_j - x_i) \le q_{ij}^{kf} + M \cdot (1 - b_{ijkt}),\ \forall i, j \in A_t,\ \forall k = 1,2,...,m_{ij},\ \forall f = 1,2,...,t_{ij}^{k},, \forall t \ge 0$$

(5-4)

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le p_{it},\ \forall i, j \in A_t, \forall t \ge 0 \tag{5-5}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le p_{jt},\ \forall i, j \in A_t, \forall t \ge 0 \tag{5-6}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le 1 - out_{it},\ \forall i \in D_t, \forall t \ge 0 \tag{5-7}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le 1 - out_{jt},\ \forall j \in D_t, \forall t \ge 0 \tag{5-8}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - 1,\ \forall i, j \in A_t \backslash D_t, \forall t \ge 0 \tag{5-9}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - (out_{it} + out_{jt}) - 1,\ \forall i, j \in D_t, \forall t \ge 0 \tag{5-10}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - out_{it} - 1,\ \forall i \in D_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \tag{5-11}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \ge p_{it} + p_{jt} - out_{jt} - 1,\ \forall i \in A_t \setminus D_t, \forall j \in D_t, \forall t \ge 0 \tag{5-12}$$

Constraints (5-1) – (5-3) ensure that the aircraft are placed within the boundary of the maintenance hangar. No-Fit Polygons between two aircraft are expressed in Constraint (5-4). Constraints (5-4) – (5-12) are entire non-overlapping constraints set for a pair of aircraft parking at time *t*. In particular, the non-overlapping constraint is activated when two aircraft are parked in the hangar simultaneously at time *t* (constraints (5-9) – (5-12)), and the non-overlapping is deactivated if any one of them is not arranged to be parked at time *t* or one of them is rolled out altogether at that time (constraints (5-5) – (5-8)). The auxiliary decision variable p_{it} indicates if aircraft *i* is placed in the hangar at time *t*, activating the non-overlapping constraints. The set of binary variables b_{ijkt} associated with the horizontal slice *k* outside the NFP between aircraft *i* and *j* in constraint (5-4).

2) Movement blocking constraints

During the movement operations of aircraft, there shall not have any obstacles blocking its path of movement. If an aircraft is about to leave or enter the hangar, the other aircraft parking in the hangar should not become the obstacle, blocking the moving aircraft. In this regard, the position between two aircraft need to be determined by the auxiliary decision variables h_{ijt}, L_{ij}, R_{ij}, U_{ij}. If the aircraft about to move at time t is blocked by any other aircraft, its movement operation has to be cancelled at this time.

$$(x_i + w_{ij}^{'} / 2) - (x_j - w_{ji}^{'} / 2) \le M \cdot (1 - L_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{5-13}$$

$$(x_i - w_{ij}^{'} / 2) - (x_j + w_{ji}^{'} / 2) \ge -M \cdot (1 - R_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{5-14}$$

$$(y_i + TD_i) - (y_j + TD_j) \ge -M \cdot (1 - U_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{5-15}$$

$$(1 - h_{ijt}) \ge \frac{1}{6} \cdot \left[L_{ij} + R_{ij} + U_{ij} + in_{jt} + out_{jt} + (1 - p_{jt}) \right] \quad \forall i \in A_t, \forall j \in D_t, \forall t \ge 0 \tag{5-16}$$

$$(1 - h_{ijt}) \le L_{ij} + R_{ij} + U_{ij} + in_{jt} + out_{jt} + (1 - p_{jt}) \quad \forall i \in A_t, \forall j \in D_t, \forall t \ge 0 \tag{5-17}$$

$$(1 - h_{ijt}) \ge \frac{1}{5} \cdot \left[L_{ij} + R_{ij} + U_{ij} + in_{jt} + (1 - p_{jt}) \right] \quad \forall i \in A_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \tag{5-18}$$

$$(1 - h_{ijt}) \le L_{ij} + R_{ij} + U_{ij} + in_{jt} + (1 - p_{jt}) \quad \forall i \in A_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \tag{5-19}$$

Constraints (5-13) – (5-19) indicate and prescribe the correlation between the parking position of the aircraft and the blocking in aircraft movement operations. In particular, binary variables L_{ij}, R_{ij} and U_{ij} prescribe that if they take value 1, then aircraft i is placed on the left-hand side, right-hand side and upper position of aircraft j, respectively, so that aircraft j does not block the movement operations of aircraft i.

The binary variable h_{ijt} reflecting whether aircraft *i* is blocked by aircraft *j* is controlled by constraints (5-16) – (5-19). Specifically, aircraft *j* does not block the movement of aircraft *i* under the following conditions: 1) aircraft *j* undertakes the movement operations at the same time as aircraft *i*; 2) aircraft *j* is not placed in the hangar at time *t*; 3) aircraft *i* is on the the left-hand side, right-hand side or the upper position of aircraft *j*, as indicated by binary variables L_{ij}, R_{ij} and U_{ij}, respectively.

3) Movement Operations and aircraft blocking:

The constraints in this section prescribe that if the movement path of the aircraft rolling in and rolling out is blocked by other aircraft parked in the hangar, the movement actions cannot be conducted. In particular, for an aircraft pending leaving the hangar, the rolling out operation has to wait until the aircraft blocking the path leaves first (or concurrently). For the arrival aircraft, its parking position can be adjusted so that the aircraft can be timely moved in, or the movement operation has to be postponed until the aircraft blocking the pathway leaves the hangar.

$$out_{it} \le 1 - \frac{1}{|A_t \setminus i|} \cdot \sum_{\forall j \in A_t \setminus i} h_{ijt}, \forall i \in D_t, \forall t \ge 0 \tag{5-20}$$

$$in_{it} \le 1 - \frac{1}{|A_t \setminus i|} \cdot \sum_{\forall j \in A_t \setminus i} h_{ijt}, \forall i \in A_t, \forall t \ge 0 \tag{5-21}$$

Constraints (5-20) and (5-21) state that the rolling out and rolling in operations of aircraft i cannot be conducted if it is blocked by any parked aircraft in the hangar at time t. The auxiliary decision variable h_{ijt} indicates the relations between each pair of aircraft at time *t* acting as the mediator between the movement operations decision variable (out_{it}, in_{it}) and the movement blocking constraints (Constraints (5-13)-(5-19)).

4) Staying time requirements:

The duration that each aircraft stays in the hangar should sufficient for conducting the maintenance task. The constrains set in this section ensure the staying time of an aircraft served by the company equals or is longer than its required maintenance. Moreover, the rolling in and rolling out operations for each aircraft can be conducted only once, as the aircraft cannot revisit the hangar during the planning period. The auxiliary decision variable p_{it} acts as a mediator, establishing the relation between the non-overlapping constraint in constraint set 1) and the staying time requirement in this section.

$$(\sum_{t \geq ETD_i} out_{it} \cdot t - \sum_{t \geq ETA_i} in_{it} \cdot t) + M \cdot (1 - \sum_{t \geq ETA_i} in_{it}) + M \cdot (1 - \sum_{t \geq ETD_i} out_{it}) \geq MTime_i, \forall i \in A_T \tag{5-22}$$

$$p_{it} = \sum_{ETA_i \leq m \leq t} in_{im}, \forall i \in A_T, \forall ETA_i \leq t \leq ETD_i \tag{5-23}$$

$$p_{it} = \sum_{ETA_i \leq m \leq t} in_{im} - \sum_{ETD_i \leq m \leq t-1} out_{im}, \forall i \in A_T, \forall t \geq ETD_i + 1 \tag{5-24}$$

$$\sum_{t \geq ETA_i} in_{it} \leq 1, \forall i \in A_T \tag{5-25}$$

$$\sum_{t \geq ETD_i} out_{it} \leq 1, \forall i \in A_T \tag{5-26}$$

$$out_{it} \leq \sum_{ETA_i \leq m < t} in_{im}, \forall i \in A_T, \forall t \geq ETD_i \tag{5-27}$$

$$(1 - out_{iT*}) \leq \sum_{t \geq ETD_i} out_{it} + M \cdot (1 - \sum_{t \geq ETA_i} in_{it}), \forall i \in A_T \tag{5-28}$$

$$(1 - out_{iT*}) \leq \sum_{t \geq ETD_i} out_{it} + M \cdot (1 - \sum_{t \geq ETA_i} in_{it}), \forall i \in D_T \tag{5-29}$$

Constraint (5-22) determines the duration of stay for each aircraft, prescribing that if such aircraft is accepted by the service company then its parking time must equal or be longer than its required maintenance time.

Constraints (5-23) and (5-24) prescribe that p_{it} indicates whether the aircraft is parked in the hangar takes value 1 by the time it rolls into hangar until it rolls out. If the value of p_{it} equals to one, the respective non-overlapping constraints are activated accordingly.

Constraints (5-25) – (5-27) ensure that the rolling in operations happens after the arrival time of the maintenance request (ETA), and rolling out operations are conducted only after the aircraft has been rolled in. Constraints (5-28) – (5-29) imposes that out_{iT^*} equals to one if the aircraft is still parked in the hangar at the end of the planning horizon.

5) Variable domination constraints

$$x_i, y_i \geq 0 \quad \forall i \in A_T \tag{5-30}$$

$$b_{ijkt} \in \{0,1\} \ \forall i, j \in A_t, k = 1,2,...,m_{ij}, \forall t \geq 0 \tag{5-31}$$

$$p_{it} \in \{0,1\} \quad \forall i \in A_t, \forall t \geq 0 \tag{5-32}$$

$$in_{it} \in \{0,1\}, \forall i \in A_t, \forall t \geq 0 \tag{5-33}$$

$$out_{it} \in \{0,1\}, \forall i \in D_t, \forall t \geq 0 \tag{5-34}$$

$$h_{ijt}, L_{ij}, R_{ij}, U_{ij} \in \{0,1\} \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \geq 0 \tag{5-35}$$

Constraint (5-30) ensures that the coordinates of the aircraft are positive, and constraints (5-31) – (5-35) indicate the binary variables in the mathematical model.

6) Tightening the model

To further tighten the mathematical model, we propose the following constraints:

$$L_{ij} + L_{ji} \leq 1, \forall i, j \in A_T, j \neq i \tag{5-36}$$

$$R_{ij} + R_{ji} \leq 1, \forall i, j \in A_T, j \neq i \tag{5-37}$$

$$L_{ij} \leq R_{ji}, \forall i, j \in A_T, j \neq i \tag{5-38}$$

$$R_{ij} \leq L_{ji}, \forall i, j \in A_T, j \neq i \tag{5-39}$$

The feasibility of the tentative solution is examined by firstly determining a feasible maintenance schedule, then fixing the position-related binary variables. After branching on all the position-related variables, the geometry constraints are imposed to examine if such a parking plan is feasible. In this regard, the LP relaxation of the model is not tight, and the updates of the lower bound do not progress well to tighten the optimality gap. Constraints (5-36)-(5-39) impose a side-by-side relation between a pair of aircraft

5.5. Solution approaches

The original model presented in Section 5.4 is inefficient as it relies on a basic discrete time model. In this section, two newly developed solution approaches are discussed to enhance the efficiency in solving the problem.

5.5.1. Event-based discrete time formulation for the problem

Generally, the decision variables in the MILP formulation are indexed by the discrete time to cover the planning horizon (Figure 5-6), such as the basic discrete-time formulation (BDT) (Pritsker et al., 1969) and the disaggregated discrete-time formulation (DDT) (Christofides et al., 1987) in the resource-constrained project scheduling problem, while the number of variables indexed by time increase proportionally with the length of the scheduling horizon T (Koné et al., 2011). Moreover, the setting of the time interval of two consecutive time points along the horizon, e.g. one-minute, 5-minute, 10-minute based, also has an impact on the scale and the accuracy of the problem. The Basic Discrete Time (BDT) model is inefficient and may visit lots of unpromising time points along the horizon. Inspired by the work related to the project scheduling problem(Koné et al., 2011), an event-based discrete time model is developed to identify the possible time point that may trigger roll in and roll out operations along the planning period.

The main idea of reducing the domain of the time-related decision variables along the planning period is to exclude all the points in the timeline that cannot trigger any movement operations so as to leave only the promising time point along the planning period, as many time points in Figure 5-6 cannot trigger movement operations while involving a great number of decision variables in those time points.

Ideally, a maintenance check should commence upon arrival of a new aircraft and the expected roll out time for each aircraft equals $ETA_i + MTime_i$. When blocking occurs due to the insufficient hangar space or improper parking planning, some events (roll in and roll out operations) cannot be triggered at their ideal time, e.g. ETA_i or $ETA_i + MTime_i$ respectively. Under such circumstances, the roll in (roll out) operation can be performed once the blocking is cleared or the hangar has enough space to accommodate the aircraft. The possible roll in / roll out event time can be determined by recursively calculating the possible roll in / roll out time. If the aircraft arrives during the middle of the planning period, it is possible that the hangar capacity has been used up by some earlier arrival aircraft, and the only possible way to accommodate the later arrival aircraft is to wait until the earlier arrival aircraft complete their maintenance tasks and leave the hangar. As a result, the possible roll in time for the arrival aircraft includes its own ETA, or the actual maintenance completion time of the other aircraft parked in the hangar. Similarly, an aircraft arriving during the middle of a planning period can also have a blocking effect on the consequent arrival aircraft. In addition, a set of aircraft with the same *ETA* can also have roll in blocking effects on each other. With regard to the possible roll out time, the later arrival aircraft have effect on the earlier arrival aircraft. It is possible that the earlier arrival aircraft have finished the maintenance, but the later arrival aircraft blocks the roll out path due to the limited hangar space or an improper pakring arrangement. Therefore, the only possible way to move out the aircraft after finishing the maintenance task is to wait until the later arrival aircraft finish their maintenance task. As a result, the possible roll out time for an aircraft includes its own *ETA+MTime*, its actual roll in time plus its *MTime*, or the actual roll out time of the aircraft blocking the movement path.

The detailed procedures of calculating the possible roll in and roll out time for the development of the Event-based Discrete Time model are shown in Algorithm 5-1.

Algorithm 5-1 Calculation of promising event times

Notations	Meanings
M	Set of maintenance requests
$Possible\_RollIn_i$	Set of Possible Roll In time of maintenance request i
$Possible\_RollOut_i$	Set of Possible Roll Out time of maintenance request i
$Rank_i$	The rank of maintenance request i in sorting list
Set_Rank_n	Set of maintenance request at position n. $H = \{1,2,3,...,n_H\}$ be the index set of maintenance request in respective

1: Sort all maintenance requests from M in increasing order according to ETA_i, then derive the position of maintenance request $Rank_i$ according to the result of sorting list.

2: Input the maintenance requests into respective Set_Rank_k according to the $Rank_i$

(Computation of possible roll in time)

3: for $n = 1,2,3,...,n_H$ do

4: For k in Set_Rank_n

5: Include ETA_k into set $Possible\_RollIn_k$

(same position blocking)

6: Calculate the combination d of maintenance request in $Set\_Rank_n \setminus k$ to determine the possible blocking in same position

for the combination of maintenance requests may block k
Possible roll in time of k = possible roll in time of the request in combination for k + respective required maintenance time
If $n \neq 0$
for $n^{'} = 0, 1, 2, ..., n-1$
for m in $Set\_Rank_{n^{'}}$
If the possible roll in time of the previous request + required maintenance time >= ETA_k
Include possible roll in time of the previous request + required maintenance time into $Possible\_RollIn_k$
(Computation of possible roll out time)
for $n = 1, 2, 3, ..., n_H$ do
For k in Set_Rank_n
Include $ETA_k + MTime_k$ into $Possible\_RollOut_k$
Include all entries in $Possible\_RollIn_k$ plus $MTime_k$ into $Possible\_RollOut_k$
while $n^{'} = n+1, n+2, ..., n_H$
For m in $Set\_Rank_{n^{'}}$
For all entries in $Possible\_RollIn_m$
If $Possible\_RollIn_m + MTime_m \geq ETA_k + MTime_k$
Include $Possible\_RollIn_m + MTime_m$ into $Possible\_RollOut_k$

5.5.2. Rolling horizon approach

Though the Event-based Discrete Time (EDT) model presented in Section 5.5.1 significantly reduces the model size and the solution time, it sometimes still requires a long solution time or may be incapable for solving some instances that contain a large number of maintenance requests. One can use the rolling horizon approach to speed up the overall solution process. Inspired by the idea of the rolling horizon approach from (Saddoune et al., 2013), we develop a tailored rolling horizon approach suitable for this problem. The horizon is divided into several sub-problems with *n* maintenance requests in each sub-problem (except the last sub-problem that may include fewer requests), and sub-problem k overlaps with the next one $(k+1)$ if the operation time for some maintenance requests determined in the current sub-problem *k* exceed the planning start time of the next sub-problem $(k+1)$, i.e. the earliest event time of the next subproblem. Let $K=\{1,2,...,n_k\}$ be the index set of those subproblems and $W_k=[B_k,E_k]$ ($B_k=\min_{i\in W_k} ETA_i$, $E_k=\min_{i\in W_{k+1}} ETA_i$, $i\in A_T$). The rolling horizon sequentially solves the subproblems. The pseudo-code can be found in Algorithm 5-2. At each iteration k, a subproblem restricted to the current time domain W_k is solved using the MILP model presented in Section 5.5.1. To ensure continuity in the overall solution, the subsequent subproblem $(k+1)$ includes the initial condition derived from the last sub-problem k, i.e. for the maintenance requests planned to finish after E_k. The initial condition for $(k+1)$ stipulates that the position of the aircraft and the determined roll out time remain unchanged so as to ensure the connectivity of the solution between the previous and current subproblems.

Two strategies for dividing maintenance requests into subproblems are considered:

- First Come First Served (FCFS): Sorting the maintenance requests in increasing order according to ETA, then dividing the maintenance requests into respective subproblems according to the predetermined maximum number of requests to be included in one sub-problem (n).
- Mixed mode: Sorting the maintenance requests in increasing order according to *ETA*, then selecting a set of maintenance requests according to limit r, and determining $W_k=[B_k,E_k]$. Examine the rest of the maintenance requests not included in the subproblem k. If there is any maintenance job with the ETD_i within $W_k=[B_k,E_k]$, include such maintenance job into the present subproblem k.

Optimizing more maintenance requests within one sub-problem usually leads to an overall solution of better quality, as the maintenance scheduling is optimized with a wider local view of the problem (Saddoune et al., 2013)Following a series of preliminary tests to achieve a tradeoff between efficiency and quality, the values of r were set from 5 to 7 in performing the experiments described in Section 5.6.2.

Algorithm 5-2 Rolling horizon approach

Notations	Meanings
M	Set of maintenance requests
r	Number of maintenance requests to be included in one sub-problem
n_k	Number of subproblems
K	Set of subproblem. $K = \{1, 2, ..., n_k\}$
B_k	Beginning time of subproblem k
E_k	Ending time of subproblem k
$earliest(k)$	Earliest ETA_i in subproblem k .
ATD_i	Actual departure time of maintenance request i
$Coordinate_i$	Determined position of maintenance request i

1: Set the number of subproblem as $n_k = \left\lceil \frac{M}{r} \right\rceil$, and divide the request in M into n_k subset according to respective subproblem dividing strategies (FCFS and Mixed)

2: for $k = 1, 2, ..., n_k$ do

3: Take kth subset of maintenance requests containing r (if $k < n_k$) or $M - r*(n_k - 1)$ (if $k = n_k$) maintenance request to establish subproblem k .

$B_k = \min_{i \in W_k} ETA_i$, $E_k = \min_{i \in W_{k+1}} ETA_i$, $i \in A_T$

4: for $k = 1, 2, ..., n_k$ do

5: Solve the subproblem k by MILP model in Section 5.4 or Section 5.5.1

6: For the maintenance request i with $ATD_i \geq E_k$

7: Pass the aircraft i with ATD_i and $Coordinate_i$ to subproblem $k+1$ as initial constraints

For the maintenance request i with $ATA_i \geq E_k$

Pass the aircraft i with ATA_i, ATD_i and $Coordinate_i$ to subproblem $k+1$ as initial constraints

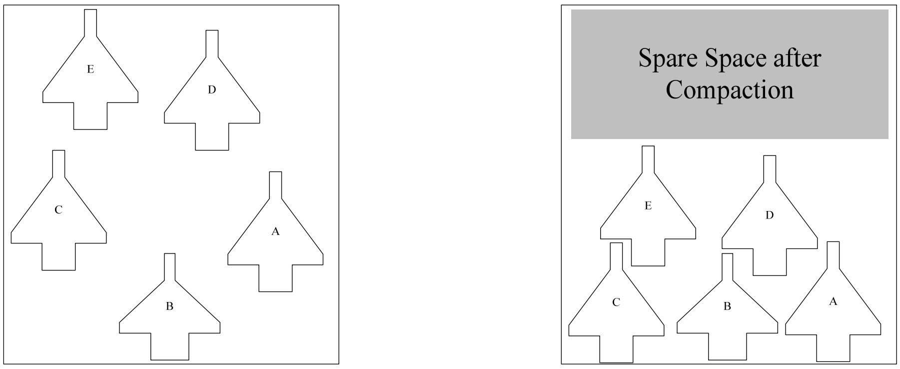
For the maintenance request i has not scheduled to rolled into in subproblem k

Pass the aircraft i with ETA_i, ETD_i to subproblem $k+1$ as ordinary maintenance request

8:	Integrate the solution from $k=1,2,...,n_k$ to produce complete solution along planning horizon

5.5.2.1. Enhancement of rolling horizon approach

While solving the subproblems by the conventional rolling horizon approach mentioned above, the impact of the current parking layout on the subsequent subproblems is not considered. In particular, the subproblem solely focuses on obtaining a feasible aircraft parking layout with an optimal local objective value. It is possible that the earlier arrival aircraft might park at the anterior space of the hangar near the entrance (Figure 5-7 (a)), even if there is a lot of available empty space in the inner area, making the later arrival aircraft in a subsequent subproblem unable to find the parking place due to the blocking at the anterior area. In this regard, we propose an approach for this problem so as to further enhance the algorithm stability. After finding the optimal solution of each subproblem, we supplement a *layout compaction* stage to further tighten the parking layout to spare more space for the subsequent subproblems, without violating the position relation between aircraft. In particular, we compact the aircraft's positions, with the position relations unchanged, by constructing a MILP model with predetermined variable values, i.e. the coordinates in the x-axis x_i and the binary variables, L_{ij}, R_{ij} and U_{ij}, then minimize the coordinates y_i in the *layout compaction* stage (Figure 5-7 (b)) to spare space in the y-axis.



(a) (b)

Figure 5-7 Before and after layout compaction in solving the subproblem

5.6. Computational experiments

This section presents the results of different computational experiments. All the procedures described in the previous sections are coded in C# in Visual Studio 2010 and run on a computer with an Intel Core i7 processor, at 3.6 GHz with 32 Gb of RAM. The Mixed-Integer Linear Programming is solved by the CPLEX 12.7 serial model.

5.6.1. Description of test instances

For our tests, we considered maintenance request data derived from an aircraft hangar maintenance service provider in Hong Kong, serving over 50 clients, including airlines, business jet companies and utility aircraft companies, as a case study. We obtained the information of the estimated arrival time (*ETA*), departure time (*ETD*), aircraft type and check type of each maintenance requests from clients over 157 days from January to May in 2015 to create instances, which were used in our previous chapters. The characteristics of the historical data is presented in Table 5-1, which briefly classify the number of aircraft in three categories, small-, medium- and large-sized, as shown in Table 3-2 in Chapter 3. The shortest distance between two aircraft is prescribed as one meter in conducting the computational experiments given in Section 5.6.2.

Table 5-1 Characteristics of receiving maintenance requests

Month	Number of maintenance requests	Small-sized Aircraft	Medium-size aircraft	Large-size aircraft
January 2015	24	11	12	1
February 2015	19	8	11	0
March 2015	33	12	18	3
April 2015	39	17	27	0
May 2015	33	9	22	2

5.6.2. Computational experiment

We performed three series of computational experiments, with the design of the numerical experiments as follows: the first series in Section 5.6.2.1 compares the computational efficiency of the Basic Discrete Time (BDT) model with different time intervals, the Event-based Discrete Time (EDT) model by solving small- and medium-size instances. After illustrating the superiority of the EDT model, the large instances were solved by the rolling horizon approach incorporating the EDT model, and its performance is reported in Section 5.6.2.2. The weightiness of each maintenance request was regarded as equal, and the unit penalty cost was set as (80, 1, 30) for penalties 1-3 respectively.

5.6.2.1. Model's evaluation

In this section, we compared the effectiveness of the proposed model formulations by presenting the results of solving small- and medium-size aircraft. Table 5-2 Comparisons among basic discrete time and event-based discrete time models shows the results for 8 instances solved by the two models. Since our preliminary experiment in using the BDT model suggested that setting the time interval of the model as less than 20 minutes involves large numbers of binary variables, making the instances intractable. In this regard, we prescribe the time interval in the BDT model as 30 minutes, 45 minutes and 60 minutes. The first column of Table 5-2 Comparisons among basic discrete time and event-based discrete time models stands for the instance name. Data of maintenance requests collected from the maintenance company are organized on a monthly basis, and the monthly maintenance requests are further divided into several sub-sections to create small- and medium size instances. The name of the instance is presented in "month_division_number of requests (number of planning days)" form, e.g. 1_1_3 (7) stands for the instance covering the first subsection of January with 3 maintenance requests covering 7 days. Each instance was solved by the BDT model with three different time interval settings as well as the EDT model. The third column denotes the preprocessing time before CPLEX solves the instance. The preprocessing time includes the initialization of the mathematical model, i.e. defining the decision variables and initializing the constraints, as well as the time spent on calculating the event time, as discussed in Section 5.5.1 for the EDT model. The number of binary variables involved in each model in solving the instance is reported in the fourth column. The best-known solution, lower bound, optimality gap and the CPU time elapsed when the termination criterion was met are recorded from the fifth to eighth columns, respectively. The time limit for each instance was 3600 seconds.

The overall results in Table 5-2 demonstrate the superiority of the Event-based Discrete Time (EDT) model formulation in terms of the number of instances optimally solved, optimality gap and CPU running time. It is noted that the Basic Discrete Time (BDT) model formulation with different time intervals involves a significant number of binary variables in each instance, which grows significantly as the planning period increases. Moreover, setting a large time interval (minute) in the BDT model may eliminate the true optimal solution at the model initialization stage, e.g. for instance 1_1_3 (7), the

BDT (45minutes) and the BDT (60 minutes) models found that the optimal objective value was 90. In the BDT model with a large time interval, the discrete event time in the model may be later than the ETA and the ETD of the maintenance requests, which caused lateness in rolling in and rolling out operations. We further analyzed the performance of the EDT model by solving the medium-size instances and the results are presented in Table 5-3, where it is noted that the number of maintenance requests in each instance is one of determinants of the model scale. However, it is worth to point out that the complexity of the instances does not solely depend on the number of maintenance requests, but also the distribution of arrival time along the planning period and also correlates with the aircraft type in each maintenance request and its arrival time. For example, while solving the medium-size instance set, it is recorded in Table 5-3 that the Instance 3_2_9(9) cannot be solved to optimality within the time limit with optimality gap 100%, while the other instances were solved optimally within an hour. The optimality gap of Instance 3_2_9(9) implies that the event-based discrete time model can only found a feasible solution for this instance without updating the bounds. After further investigating on Instance 3_2_9(9), it is found that the maintenance requests' arrival time concentrate on a short period of time though the length of planning period is moderate, which induces great number of possible rolling in and rolling out time compared with other instances along the planning period. As a result, this instance involves the greatest number of binary variables (406756) among all instances, making it challenging for the event-based discrete time model to tackle within the time limit. When maintenance requests are mainly for medium-size and large-size aircraft, the hangar space becomes limited in accommodating all aircraft, and may induce lateness. In addition, when the number of maintenance requests within an instance approaches 9, the number of possible event times involved in the EDT model grows significantly.

Table 5-2 Comparisons among basic discrete time and event-based discrete time models

Instance	Models	Prepossessing Time (seconds)	Binary Variables	Best-known solution	Lower bound	Gap	CPU (seconds)
1_1_3 (7)	BDT (30)	1.07	26940	0	0	0	9.66
	BDT (45)	0.67	17891	90	90	0	68.55
	BDT (60)	0.51	13513	90	90	0	34.23
	EDT	0.09	431	0	0	0	**0.06**
1_1_4 (8)	BDT (30)	3.18	79265	60	60	0	428.38
	BDT (45)	1.88	52877	120	120	0	964.74
	BDT (60)	1.54	39685	220	120	45.45	3600
	EDT	0.12	1528	0	0	0	**0.09**
1_1_5 (13)	BDT (30)	17.51	350766	160	30	81.25	3600
	BDT (45)	10.40	233927	170	120	29.41	3600
	BDT (60)	6.83	175448	150	150	0	3600
	EDT	0.15	2864	0	0	0	**0.28**
1_2_7 (10)	BDT (30)	39.81	860161	N/A	0	N/A	3600
	BDT (45)	24.36	573353	1485	82.0256	94.48	3600
	BDT (60)	16.23	430221	N/A	230	N/A	3600
	EDT	1.31	31896	0	0	0	**5.40**
2_1_7 (13)	BDT (30)	15.14	323657	160	160	0	1273.03
	BDT (45)	8.68	215615	295	295	0	291.10
	BDT (60)	6.16	161772	310	310	0	209.99
	EDT	0.51	12001	0	0	0	**1.28**
1_1_8 (15)	BDT (30)	68.21	1357537	N/A	0	N/A	3600
	BDT (45)	42.81	904561	N/A	0	N/A	3600
	BDT (60)	28.62	678545	N/A	240	N/A	3600
	EDT	1.43	37689	0	0	0	**5.13**

Table 5-3 Experiments on event-based discrete time model

Instance	Prepossessing Time	Binary Variables	Best-known solution (Upper bound)	Lower Bound	Gap	CPU
1_1_9 (15)	3.63	90688	0	0	0	16.57
1_2_9 (12)	6.73	190168	0	0	0	31.29
1_2_10 (13)	16.77	454004	0	0	0	95.05
2_1_8 (13)	1.51	29358	0	0	0	2.84
2_1_9 (14)	2.34	66384	0	0	0	9.63
2_1_10 (15)	4.64	129259	0	0	0	23.99
2_2_8 (13)	3.51	99135	0	0	0	14.29
3_1_8 (12)	1.82	33438	0	0	0	2.67
3_2_9 (9)	15.06	406756	6010	0	100	3600
3_3_9 (9)	6.96	174871	0	0	0	30.82
4_1_8 (10)	0.65	18245	0	0	0	2.00
4_2_8 (19)	0.55	14799	0	0	0	1.51
4_3_9 (18)	5.06	110869	0	0	0	457.86
4_4_9 (8)	12.42	287555	0	0	0	45.12
5_1_7 (9)	1.42	40192	0	0	0	5.12
5_2_8 (14)	5.66	119019	0	0	0	23.14
5_3_9 (15)	9.08	238335	0	0	0	30.28
5_4_9 (28)	0.28	6271	0	0	0	0.41

5.6.2.2. Rolling horizon approach

Though the EDT model has illustrated its superiority in solving small- and medium-size instances as reported in Section 5.2.1, the monthly maintenance requests received by the service provider ranges from 19 to 39 at present, which are intractable solely using the EDT model. Specifically, the preprocessing time is quite long in solving instances with more than 15 maintenance requests. In this section, we did not have the comparison with the solutions obtained from CPLEX as it was unable to initiate instances with more than 20 maintenance requests. In this regard, CPLEX itself cannot solve the monthly instances tested this section, as the minimum number of maintenance requests is 19 in the problem set. Referring to the computational comparison approach adopted in (Sriram & Haghani, 2003) while tackling large-scale instance, we compared the performance among different rolling horizon strategies in tackling large-scale instances, and demonstrate the advantages of enhanced rolling horizon strategy discussed in Section 5.5.2.1. We examine the performance of the rolling horizon approaches with different strategies in solving large-scale instances with three job dividings. The job limit in each subproblem is described as 5,6 and 7 to examine the differences in computational efficiency and solution quality. The EDT model is embedded in solving each subproblem and the time limit for solving each subproblem is 3600 seconds.

Table 5-4 Comparison among rolling horizon approaches in Table 5-4 reports the computational results of adopting different subproblem dividing strategies in the rolling horizon approach, and we compare the performance among the different strategies and the job limits in the subproblem. The computational time and objective function values are two indicators in comparing the performance of different strategies in this section (Sriram & Haghani, 2003) in solving large-scale problem, so as to demonstrate the advantages of heuristic in tackling challenging problems. A total of ten replications for each instance were conducted to evaluate the average performance of the rolling horizon approach with different strategies and job limits. The rolling horizon approaches with different strategies are able to obtain feasible solutions while prescribing the job limits in the subproblem as 5 and 6. It is noted that all rolling horizon approaches with job limits as 7 cannot tackle Instance 4, as many maintenance requests in the preceding subproblems were passed to the subsequent subproblems that exceed the EDT model, and feasible solution cannot be obtained within the time limits. The advantages of the rolling horizon approach embedding the Mixed strategy with layout compaction method was manifested while solving the instances with high maintenance demands, i.e. Instances 3, 4 & 5. It is recorded that the Mixed strategy with layout compaction method outperforms the FCFS and Mixed strategies while examining the computational efficiency and solution quality in solving complex instances. In particular, the Mixed strategy with compaction method was able to reach the same or better solution measured by objective value (as highlighted in **<u>bold underline</u>**) while require less computational time, which means that it was able to identify a solution with less tardiness in fulfilling maintenance demands. Moreover, the average, Max. and Min. CPU time of Mixed strategy with compaction also showed that the stability of solving time was better than the other two strategies. Therefore, the importance of compacting hangar layout before passing the partial preceding solution to the next subproblem has emerged, as the hangar layout can be tightened to spare more space for the subsequent subproblem's planning so as to improve the solution quality. Through conducting ten replications for each instance, it is observed that the average computational performance of the Mixed strategy with the layout compaction approach is more stable when in investigating the Average, Maximum and Minimum CPU times of ten replications. In particular, the Maximum and Minimum CPUs of the FCFS and Mixed strategies differ quite a lot when solving Instance 5 with the job limit set as 6, and the layout compaction approach spent significantly less time to obtain the same objective value as the FCFS and Mixed strategies. The unstable computational performances of the conventional rolling horizon approaches are caused by the lack of consideration of geometrical factors in the transitions between horizons and the parking position in the beginning planning horizon can be loose. Therefore, the solution outcome, i.e. maintenance schedule and parking layouts, may differ among the ten repetitions, which renders variations of solving times.

Table 5-4 Comparison among rolling horizon approaches

Instance	No. of Maintenance Request	Job Limit	FCFS				Mixed				Mixed w/ Compaction			
			Avg. Obj.	Avg. CPU	Max. CPU	Min. CPU	Avg. Obj.	Avg. CPU	Max. CPU	Min. CPU	Avg. Obj.	Avg. CPU	Max. CPU	Min. CPU
1	24	5	240	18.99	21.77	17.65	240	17.90	20.99	16.75	240	22.84	23.68	22.12
		6	160	39.22	43.69	38.16	160	36.55	37.57	35.94	160	45.93	47.85	45.21
		7	320	373.37	376.64	371.95	320	381.84	399.73	371.43	320	444.82	468.08	440.01
2	19	5	160	8.63	9.28	8.21	160	7.88	8.17	7.61	160	10.89	11.38	10.72
		6	80	20.35	22.26	19.48	80	18.85	19.79	18.53	80	26.42	27.58	25.74
		7	80	127.51	129.15	126.39	80	127.53	128.87	126.37	80	130.93	132.48	129.59
3	33	5	400	280.56	287.07	277.99	400	265.91	270.09	260.46	**320**	**192.49**	196.07	190.48
		6	510	2012.76	2052.96	1990.72	510	2044.74	2101.88	1979.64	**430**	**1893.54**	1909.34	1879.91
		7	320	127.51	129.15	126.39	320	1330.69	1353.38	1301.08	320	1550.78	1525.29	1473.31
4	39	5	640	1264.04	1411.91	1166.15	640	1141.92	1181.14	1117.15	640	**1010.75**	1026.52	985.68
		6	560	1269.84	1288.42	1248.98	560	1358.85	1401.86	1323.09	**480**	**709.49**	718.07	701.37
		7	-	-	-	-	-	-	-	-	-	-	-	-
5	33	5	400	719.72	750.22	687.11	400	685.82	711.85	675.17	**240**	**68.56**	74.48	66.62
		6	220	623.39	4561.03	138.97	220	804.03	3540.54	141.22	220	**108.39**	112.61	107.15
		7	240	203.41	218.54	195.93	240	197.38	204.77	188.70	240	244.29	264.11	233.55

- the subproblem cannot obtain feasible solution within 3600s.

5.6.3. Sensitivity analysis

In this section, a sensitivity analysis is performed to display the impact of the changes on the weightiness value to the objective function using Instance 1 as a case study. The computational experiment in Section 5.6.2 prescribes that the weightiness of all maintenance requests is the same (value 1). The weightiness of each maintenance request was reviewed and evaluated by the maintenance company, with values that fluctuate under different situations. Moreover, the weightiness of each maintenance request given by the maintenance company is kept confidential in actual operations. Nevertheless, we sought the maintenance company's suggestion in assigning the weightiness to design the sensitivity analysis. As suggested by the practitioners, the weightiness of each maintenance request is proportional to the size of the aircraft type in general cases, as larger aircraft usually requires more inputs in maintenance work. We examined the impact of the variations in weightiness by using 6 different settings in the sensitivity analysis, and the difference of the weightiness among small-, medium- and large-size aircraft were adjusted accordingly so as to reflect the preferences and priorities of the different requests. The unit penalty cost was set as (4000, 1, 1000) for penalties 1-3 respectively in this section, so as to amplify the effect of variation of the weightiness for each maintenance request.

We deployed Instances 3 & 5, both involving many maintenance requests, to conduct the sensitivity analysis. Table 5-5 and Table 5-6 reflect the effect of the weightiness changes on the objective value and the optimal solutions. It is found that: 1) the penalty costs have an increasing trend when the weightiness increases; 2) widening the difference of weightiness among three groups of requests or increasing the weightiness do not necessarily increase cumulative delays, and the impact of such changes is reflected on the optimal solutions. 3) it is noted that the optimal solutions under different settings may have the same schedule, and the optimal decisions under different settings prone to reject some same requests, which implies that the demands of maintenance (aircraft type, arrival time, deadline and maintenance) may have larger impact on decision-making compared with the weightiness settings.

Table 5-5 Sensitivity Analysis on Instance 3

Settings	Weightiness			Objective Value	Cumulative Delays (Minutes)	Rejected Requests (Aircraft Type)
	Small-sized	Medium-sized	Large-sized			
1	1	2	4	36540	7050	2 (GL5T, A333)
2	2	4	5	61080	7050	2 (GL5T, A333)
3	3	4	8	50380	4565	2 (GL5T, G550)
4	3	6	8	71550	4565	2 (GL5T, G550)
5	4	7	9	84115	4565	2 (GL5T, G550)
6	4	8	10	122160	7050	2 (GL5T, A333)

Table 5-6 Sensitivity Analysis on Instance 5

Settings	Weightiness			Objective Value	Cumulative Delay (Minutes)	Rejected Requests (Aircraft Type)
	Small-sized	Medium-sized	Large-sized			
1	1	2	4	28740	3000	2 (A332, G5000)
2	2	4	5	45480	3000	2 (A332, G5000)
3	3	4	8	67780	1260	3 (A332, G5000, GL5T)
4	3	6	8	70220	3000	2 (A332, G5000)
5	4	7	9	50260	3180	1 (G5000)
6	4	8	10	90960	3000	2 (A332, G5000)

5.7. Summary

For the maintenance company under outsourcing mode, hangar space is a bottleneck in planning the maintenance schedule, as the movement of aircraft causing blocking and geometric factor for aircraft parking are unique features in hangar maintenance scheduling A hangar maintenance problem is described in this chapter, integrating the scheduling and parking layout planning problems. We develop a mathematical model to accommodate the scheduling practice for hangar maintenance activities. To enhance the efficiency, an event-based discrete time model for reducing the solution space is introduced, which outperforms the basic discrete time model in solving small- and medium-size instances. Moreover, the rolling horizon approach for this problem is developed to provide good quality feasible solutions for large-scale instances. All the developed approaches are tested on a large set of instances, based on real data collected from an aircraft maintenance service company. We assessed the effectiveness of the proposed approaches, then conducted a sensitivity analysis to study the impacts made by the variations of the weightiness of the maintenance requests. Given the difficulties in evaluating the hangar capacity due to unique geometric features involved in the problem, the parking and service capacity varies according to the incoming maintenance requests' demand and the specification of aircraft from time to time. Under different variation of weightiness settings for maintenance requests, the computational results on solving challenging instances have revealed that the congestion of arrival maintenance requests create peak periods requiring much hangar space demand for aircraft parking, but results in rejection of some maintenance requests or lateness in fulfillment since the blocking of movement and insufficient space occurs. In this regard, the negative effects of lateness and rejecting maintenance requests should not be underestimated with the rising maintenance demands, which induces clients' dissatisfaction and adverse profit lost in the company. To enhance the service level of maintenance service provider in serving the increasing demands and fulfill the maintenance requirement of different airlines' fleets, it is recommended that the independent service company and airlines work jointly ahead of time to arrange the maintenance plan, which avoids overwhelming maintenance requests arrive at similar time in a proactive manner. Such tactics enables the service company to have enough time to review their service capacity and carry out the maintenance service plan. When congestions of maintenance request occur, moderate time buffer still allow maintenance company and airlines to negotiate and adjust the maintenance plan in a flexible manner. Further avenues of this research topic include: (1) consideration of additional practical constraints for practitioners in industry, such as the inclusion of repositioning decisions while undergoing the maintenance task; assigning the position according to the maintenance type and distance to the tooling/material stores. (2) the development of exact algorithms, heuristic algorithms and improvement of the existing rolling horizon approaches in solving the more challenging large-scale instances so as to be able to make a theoretical impact on the problem. (3) stochastic modelling incorporating the uncertainties due to unscheduled maintenance requests and material as well as manpower shortage constraints. To incorporate uncertainties, the impact of uncertain arrival time of the maintenance demands on planning multi-period parking layouts becomes one of the focuses, as the movement sequences and position of aircraft may fluctuates significantly given the uncertain range of arrival time is wide. Moreover, the parking position for the aircraft with wide range of uncertain times can be assigned with a specific area in the hangar that minimizes the negative effects on the other aircraft's movements and schedules.

Chapter 6. Two-stage Optimization Approach for the Integrated Hangar Maintenance Planning Problem with Multi-skill Manpower Assignment Consideration

6.1. Introduction

The integrated hangar maintenance planning problem under MRO outsourcing mode is addressed in this chapter. The integrated hangar maintenance scheduling problem fulfil the maintenance scheduling problem studied in Chapter 5, with the multi-skill maintenance technician's assignment problem. The manpower supply is another significant resource factor besides the hangar space. The consideration of maintenance staff with multiple types of maintenance skills aligns with the practice of sophisticated hangar maintenance tasks. Given the complexity of the integrated problem, a two-stage optimization approach is developed by decomposing the original model, which is coordinated by the linkage constrains between geometric and numeric decision-making scattering in the decomposed subproblems. The results and analysis of computational experiments are reported, which shows: (i) the adaptability and effectiveness of two-stage optimization approach and (ii) the scalability of the two-stage optimization approach that is able to provide good feasible solutions for medium- to large- size instances covering various planning period. The impact of manpower supply variation is analysed afterward to provide some managerial insights.

The integrated maintenance plan includes determining the service time of each incoming aircraft, the parking position of each aircraft in the hangar as well as proper maintenance technician assignment to maintenance tasks. Specifically, the service time, rolling operations of aircraft should align with the parking plans over the planning horizon. In addition, the assignment of maintenance staff shall be based on the licenses (also known as skill) of each technician (Chen et al., 2017), as each technician can only perform the particular qualified maintenance task. The licences that the technician holds also relate to the maintenance manpower cost as the senior technician holding advanced license usually involve higher wages. Moreover, other consideration, such as team size and rest time, shall be included while assigning proper technicians to respective maintenance tasks. The development of such a plan is challenging as there exist interdependent relations among the aforementioned three core elements. The number of aircraft that maintenance hangar can accommodate changes along the planning period as the maintenance company receives different size of aircraft from different airlines, and the parking stand is not predetermined as in the conventional maintenance hangar operated by single airlines. In addition, due to the different arrival time, departure time and service time of incoming aircraft, the roll in and out time of each aircraft differ, then the blocking may occur when there are many incoming maintenance requests arriving at similar times, or the improper parking stand allocation is made. Moreover, the assignment of technicians may also influence the service time of maintenance task, which results in the changes of service time windows and fulfilling time of maintenance demands.

To address these issues and provide a systematic approach to solve the problem, we propose an optimization methodology to develop maintenance plans from the perspective of the independent aircraft maintenance service company. The work described in this chapter is developed based on the earlier chapters. Additions of technician assignment problem render a challenging optimization model to tackle than the previous work. The hangar parking capacity, flexible parking assignment, and multi-skill technician assignment are three core difficulties in solving the problem. We focus on the modelling the correlations among maintenance service time scheduling, hangar layout planning and staffing. A Mixed Integer Linear Programming (MILP) model is firstly developed to take in the aforementioned practical factors in hangar maintenance operations under MRO outsourcing mode. Afterwards, a two-stage optimization approach is proposed to provider good quality solution for large-scale instances. The contributions of the studied problem can be summarized as follows: 1) an integrated planning model incorporating the aircraft maintenance scheduling, hangar layout planning and multi-skill technician assignment problem is developed, which is tailored for the hangar maintenance service company under the MRO outsourcing mode. 2) The proposed problem bridges the research gaps in literature regarding the aircraft maintenance problem and multi-skill technician assignment problem with the consideration of MRO outsourcing, which involves geometric factors and practical consideration in staffing. The problem studied in this chapter is an extension of hangar planning model in literature, which fulfilling the lack of understanding in the overall maintenance operations planning problem.

The remainder of this chapter is organized as follows. The problem description, objective and a set of constraints constituting the optimization problem is presented in Section 6.2. Section 6.3 introduce the two-stage optimization approach after analysing the problem structure. The results of computational experiment are reported in Section

0. Finally, the concluding summary and future work of this chapter are discussed in Section 6.5.

6.2. Problem statement and mathematical formulation

6.2.1. Problem statement

Considering the aircraft is temporary taken out of service upon meeting the prescribed flying hours and number of take-off/landing cycles (Van den Bergh et al., 2013), the aircraft has to undergone hangar maintenance, and airline companies have adopted the MRO outsourcing practice. Airline companies send the aircraft in their fleet to an aircraft hangar maintenance company for relevant maintenance service according to each airline's internal maintenance routing plan for fleet. After receiving the multiple hangar maintenance requests from airline companies, an integrated aircraft hangar maintenance plan has to determine the following decisions to fulfil the incoming service requests:

- a service time schedule specifying the maintenance service period of each aircraft, including the rolling in/out timing of each incoming aircraft;
- multi-period hangar layouts arrangement through the entire planning period, which aligns with the maintenance schedule. The hangar parking layouts specify the movement operations of all aircraft that induce the changes of the hangar layouts;
- Staff assignment to each maintenance tasks associated with the incoming aircraft for maintenance service.

The objective of the optimization problem is to minimize the sum of 1) the penalty of tardiness in completing the maintenance service; 2) the penalty of rejecting maintenance request; 3) the penalty cost of failure to complete the maintenance; and 4) the manpower costs associated with the maintenance activities.

6.2.2. MILP model

6.2.2.1. Assumptions in service time scheduling and layout planning

The assumptions in modelling service time scheduling and layout planning are presented as follows:

- The parameters related to the timing of maintenance request are deterministic, including the estimated time of arrival (ETA), estimated time of departure (i.e. desirable delivery/finish time), and maintenance time for each maintenance tasks associated with the aircraft. The duration of aircraft's movement operations (roll in & out) are negligible. The unexpected or unscheduled maintenance requests are not considered;
- Aircraft's parking stand position cannot be changed through its service period. The movement path is a straight line, and turning the direction of movement path is not allowed;
- Aircraft is not allowed to leave the hangar without finishing all maintenance task. The penalty cost of fail to deliver is imposed if the maintenance tasks cannot be finished in the planning horizon. After leaving the hangar, revisit is not allowed for all aircraft;
- If the movement operations, i.e. rolling in & out, are scheduled to conduct at time t, then its movement path cannot have any obstacles created by the other aircraft in the hangar. Otherwise, the schedule, or the parking stand position for the arrival aircraft, have to be revised due to the blockage;

6.2.2.2. Assumptions in manpower planning

- For each incoming aircraft, the complete maintenance request is broken-down into a series of maintenance tasks with precedence relations.
- Each maintenance task requires one or more types of maintenance skills, which correlates to the licenses held by technicians, as shown in **Error! Reference source not found.**.
- At each shift, the exact number of required technicians are assigned if the maintenance task is scheduled to be conducted in the shift.
- Senior maintenance technicians are allowed to conduct maintenance tasks requiring junior-level skills.
- If the precedence relations between two maintenance tasks on one aircraft is imposed, then the later task cannot be conducted before finishing the previous one.
- The assignments of maintenance technician conform with the resting time requirement, i.e. no two consecutives shifts are allowed.

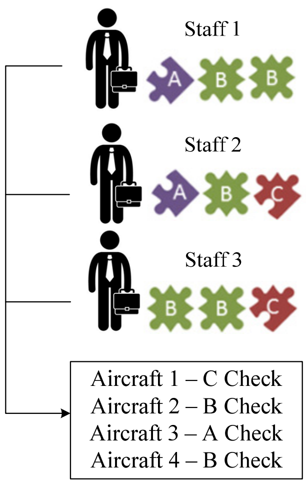


Figure 6-1 Multi-skill maintenance technician assignment

6.2.2.3. Notations, parameters and decision variables

The parameters of hangar maintenance planning include:

- **Maintenance demand's information:** The information of incoming aircraft for hangar maintenance, including aircraft type, the breakdown of maintenance checks with the specifications of maintenance skills and the size of maintenance team. Each maintenance request has its own weightiness (importance level) contributing to the objective function, desired service window, including the estimated time of arrival (ETA), and the desired estimated time of departure (ETD). Delivery after ETA induces a tardiness cost.
- **Non-overlapping constraint's parameters:** The necessary geometric information related to aircraft's dimension, and the No-Fit Polygons for generating non-overlapping constraints, and the dimensions of the maintenance hangar.
- **Manpower information:** Multi-skill maintenance technicians, with licenses held by respective persons, and the available working time a particular subsection and the manpower cost.

The list of notations for parameters mentioned above are as follows:

Notations

a_t	Set of maintenance requests arriving at shift *t*
d_t	Set of scheduled departing aircraft at shift *t*
A_t	Set of cumulative maintenance requests from beginning to shift *t*. $A_t \in \bigcup_{i=0}^{t} a_t$
D_t	Set of aircraft scheduled for departure from beginning of the planning period to shift *t*. $D_t \in \bigcup_{i=0}^{t} d_t$

I	Set of incoming aircraft for maintenance services, $i \in I$
T	Length of planning period
t	Index of shift, $t \in T$
ETA_i	Estimated time of arrival of aircraft i
ETD_i	Estimated time of departure (also desired time of delivery) of aircraft i
$MTime_i$	Aircraft i's minimum staying time in maintenance hangar
$w_{ij}^{'}$	Aircraft i's adjusted width on the occasion that aircraft j is adjacent to i
TD_i	Aircraft i's tail distance
$penalty1$	Penalty of rejecting aircraft i for maintenance service
$penalty2$	Penalty of tardiness in delivering the maintenance service for aircraft i after its ETD (per minute)
$penalty3$	Penalty of failing to finish the maintenance tasks for aircraft i by the end of planning period
$Weightness_i$	Weightiness of aircraft i
W	Hangar Width
H	Hangar Length
w_i	Aircraft i's width
h_i	Aircraft i's length
NFP_{ij}	Aircraft i and j's NFP, which conforms with a minimum safety margin
s_{ij}^k	kth horizontal slice outside the NFP_{ij}
$\alpha_{ij}^{kf}, \beta_{ij}^{kf}, q_{ij}^{kf}$	Parameters related to the fth edge of the horizontal slice s_{ij}^k outside the NFP_{ij}
m_{ij}	Total number of horizonal slices outside NFP_{ij}
t_{ij}^k	Number of edges used to define the horizontal slice s_{ij}^k
MPW	Set of technicians. $m \in MPW$
MPW^{is}	Set of technicians compatible for the maintenance task s associated with aircraft i
θ_m	Manpower cost of maintenance technician m undertaking compatible tasks for one shift
div	Manpower planning division
DIV	Set of manpower planning division, $div \in DIV$
r_{is}	Required working hours by skill s to finish aircraft task s (on aircraft i)
R_{mt}	Availability (hours) of technician m during planning division div, $div \in DIV$
a_{mt}	1, if worker m is available at shift t
S_i	Set of maintenance tasks for aircraft i
S_m	Set of maintenance tasks compatible with technician m
S_m^t	Set of maintenance task compatible with technician m at shift t

h_t	The duration of shift t
τ_{is}	Required number of qualified technicians to perform maintenance task s for aircraft i
PD_{is}	Set of predecessors before conducting task s associated with aircraft i
M	A sufficient large number

To determine a maintenance schedule to fulfill the maintenance requests as well as hangar layouts at different times, the following decision variables are introduced, and the uses of auxiliary decision variables in developing specific constraints are discussed in Section 0.

Continuous Decision Variables

(x_i, y_i)	Coordinates of reference point of aircraft i

Binary Decision Variables

out_{it}	1, if aircraft i is rolled out from the hangar at the beginning of shift t, and 0 otherwise
in_{it}	1, if aircraft i is rolled into the hangar at the beginning of shift t, and 0 otherwise
out_{iT^*}	1, if fail to finish the maintenance tasks for aircraft i by the end of planning horizon, and 0 otherwise
p_{it}	1, if aircraft i is in hangar at shift t, and 0 otherwise
h_{ijt}	1, if aircraft j blocks aircraft i's pending movements at shift t, and 0 otherwise
L_{ij}	1, if aircraft i is on the left side of aircraft j such that aircraft i is not blocked by j, and 0 otherwise
R_{ij}	1, if aircraft i is on the right side of aircraft j such that aircraft i is not blocked by j, and 0 otherwise
U_{ij}	1, if aircraft i is above aircraft j such that aircraft i is not blocked by j, and 0 otherwise
b_{ijkt}	1, if the coordinates of aircraft j are imposed into the region of horizontal slice s_{ij}^{k} at shift t, and 0 otherwise
$z_{mt,is}$	1, if technician m is assigned to task s (belonging to aircraft i) at the beginning of shift t
y_{ist}	1, maintenance task s is conducted at the beginning of shift t (if minimum number of worker is met to conducted maintenance task s (on aircraft i) on shift t & precedence requirement is met)

f_{ist}	1, if working hours of maintenance task s (on aircraft i) is completed by the end of shift t
zl_{mD}	1, if technician m's working hours in division div has met the limit

6.2.2.4. Objective and constraints

$$\min \sum_{\forall i \in A_T} Weightiness_i \cdot \left[(1 - \sum_{t \geq ETA_i} in_{it}) \cdot penalty_1 + \sum_{t \geq ETD_i} out_{it} (t - ETD_i) \cdot penalty2_i + out_{iT^*} \cdot penalty3_i \right]$$
$$+ \sum_{m \in M} \sum_{t \in T} \sum_{s \in S_m} z_{ms,it} \cdot \theta_m$$

The sum of the three penalty costs as well as the manpower cost in servicing the incoming aircraft for hangar maintenance from multiple airlines is minimized in the objective function, which can be broken down into: 1) the penalty costs of tardiness while delivering the maintenance requests; 2) penalty costs of failing to finish and deliver the maintenance request in the planning period; 3) penalty costs of the profit lost cost in rejecting maintenance request; and 4) the manpower cost in conducting maintenance tasks for aircraft.

Maintenance planning's timeline is indexed by the shift through the planning period (Figure 5-6), which is different from our previous work in (Qin et al., 2019) where the timeline is indexed by the possible movements timing of aircraft. Each time point on the timeline represents each shift t in this model. The decisions to be made at shift t involves the movement operations, the parking position, manpower assignments as well as the maintenance tasks' status (whether the task is conducted or finished). The position decision variables are not indexed by shift as the position remain unchanged once rolls in. The other auxiliary decision variables determine the position relation and movement operations, i.e. out_{iT^*}, p_{it}, h_{ijt}, L_{ij}, R_{ij}, U_{ij} and b_{ijkt}, are indexed by shift t to establish the continuity through multiple shifts.

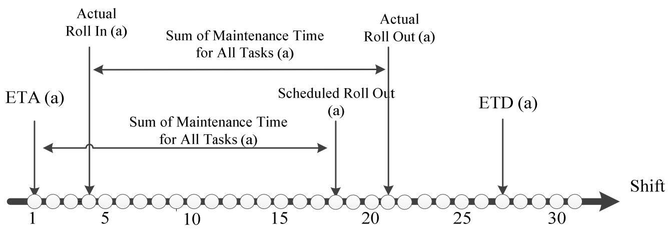


Figure 6-2 Planning Horizon indexed by shifts

For better presentation of the mathematical formulation, the constraints are divided into several subsections according to its functions.

1) Geometric constrains to prevent overlapping

The aircraft have to be parked in maintenance hangar's boundary, and each pair of aircraft to be parked should not overlap with each other and separated by the minimum safety margin, using the No-Fit Polygons given in Chapter 5.

$$x_i + w_i / 2 \le W,\ \forall i \in I \tag{6-1}$$

$$x_i \ge w_i / 2,\ \forall i \in I \tag{6-2}$$

$$y_i + h_i \le H,\ \forall i \in I \tag{6-3}$$

$$\alpha_{ij}^{kf}(x_j - x_i) + \beta_{ij}^{kf}(x_j - x_i) \le q_{ij}^{kf} + M \cdot (1 - b_{ijkt}),\ \forall i, j \in A_t,\ \forall k = 1,2,...,m_{ij},\ \forall f = 1,2,...,t_{ij}^{k},, \forall t \ge 0 \tag{6-4}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le p_{it},\ \forall i, j \in A_t, \forall t \ge 0 \tag{6-5}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \le p_{jt},\ \forall i, j \in A_t, \forall t \ge 0 \tag{6-6}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \leq 1 - out_{it}, \ \forall i \in D_t, \forall t \geq 0 \tag{6-7}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \leq 1 - out_{jt}, \ \forall j \in D_t, \forall t \geq 0 \tag{6-8}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \geq p_{it} + p_{jt} - 1, \ \forall i, j \in A_t \backslash D_t, \forall t \geq 0 \tag{6-9}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \geq p_{it} + p_{jt} - (out_{it} + out_{jt}) - 1, \ \forall i, j \in D_t, \forall t \geq 0 \tag{6-10}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \geq p_{it} + p_{jt} - out_{it} - 1, \ \forall i \in D_t, \forall j \in A_t \setminus D_t, \forall t \geq 0 \tag{6-11}$$

$$\sum_{k=1}^{m_{ij}} b_{ijkt} \geq p_{it} + p_{jt} - out_{jt} - 1, \ \forall i \in A_t \setminus D_t, \forall j \in D_t, \forall t \geq 0 \tag{6-12}$$

The constraint set (6-1) – (6-3) imposes that the entire aircraft should be parked within the hangar boundary. The No-Fit Polygon (NFP) geometric information are converted to the horizonal slicing formulation form expressed in Constraint (6-4), as discussed in Section 3.4.1. The whole set of constraint preventing aircraft parking in the hangar from overlapping is presented in (6-4) – (6-12). To activate the non-overlapping constraint between two aircraft, two aircraft must be parking in the hangar at the same shift t (Constraint (6-9) – (6-12)), otherwise the relevant constraints are deactivated in other scenarios, e.g. any one of the aircraft in that pair is not scheduled to park at the shift, or the aircraft is leaving the hangar at the same shift (Constraints (6-5) – (6-8)). The binary variables b_{ijkt} set is used to denote multiple horizonal slices around the region of NFP_{ij}. If the non-overlapping constraint (Constraint (6-4)) is activated, one of b_{ijkt} in that set must to take value one.

2) Blockings during aircraft's movement operations

Aircraft's movement operations can be conducted under the condition that the other aircraft in the hangar do not block the movement path of the aircraft pending for movement. To determine whether the aircraft pending for movement operations is blocked by the other aircraft, four auxiliary binary decision variables h_{ijt}, L_{ij}, R_{ij}, U_{ij} assist to determine the position relations between the aircraft pending for movement and aircraft in the hangar. If any blocking of aircraft's movement path exists, the movement operations have to be postponed or revised.

$$(x_i + w'_{ij}/2) - (x_j - w'_{ji}/2) \le M \cdot (1 - L_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{6-13}$$

$$(x_i - w'_{ij}/2) - (x_j + w'_{ji}/2) \ge -M \cdot (1 - R_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{6-14}$$

$$(y_i + TD_i) - (y_j + TD_j) \ge -M \cdot (1 - U_{ij}) \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \ge 0 \tag{6-15}$$

$$(1 - h_{ijt}) \ge \frac{1}{6} \cdot \left[L_{ij} + R_{ij} + U_{ij} + in_{jt} + out_{jt} + (1 - p_{jt}) \right] \quad \forall i \in A_t, \forall j \in D_t, \forall t \ge 0 \tag{6-16}$$

$$(1 - h_{ijt}) \le L_{ij} + R_{ij} + U_{ij} + in_{jt} + out_{jt} + (1 - p_{jt}) \quad \forall i \in A_t, \forall j \in D_t, \forall t \ge 0 \tag{6-17}$$

$$(1 - h_{ijt}) \ge \frac{1}{5} \cdot \left[L_{ij} + R_{ij} + U_{ij} + in_{jt} + (1 - p_{jt}) \right] \quad \forall i \in A_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \tag{6-18}$$

$$(1 - h_{ijt}) \le L_{ij} + R_{ij} + U_{ij} + in_{jt} + (1 - p_{jt}) \quad \forall i \in A_t, \forall j \in A_t \setminus D_t, \forall t \ge 0 \tag{6-19}$$

The relations between the aircraft pending for movement and the other aircraft parking in the hangar are characterized in constraint set (6-13) – (6-19). Three auxiliary binary variables (L_{ij}, R_{ij} and U_{ij}) indicate the position relation between the aircraft *i* pending for movement and the other aircraft *j*. If the auxiliary binary variables take value 1, it means that the aircraft *i* pending for movement is on the left side, right side or upper position of the aircraft *j* parking in the hangar, respectively, and its movement path is not blocked by aircraft *j*.

To indicate if the aircraft pending for movement is blocked by the other aircraft parking in the hangar at shift *t*, the movement path status is reflected in the value of the binary variable h_{ijt} and its value is coordinated by constraint set (6-16) – (6-19). If none of the value among L_{ij}, R_{ij} and U_{ij} equals to value one, it means that the movement path of aircraft *i* is blocked by aircraft *j*, and under such situation the h_{ijt} takes value one. Several exceptions exist such that the movement path is clear even the value of h_{ijt} equals to one: 1) aircraft *j* is also pending for movement operations at the same shift *t*, or 2) aircraft *j* is not in the hangar at shift *t*

If the aircraft *i* pending for movement operations is blocked by any one of the other aircraft *j* parking in the hangar, the movement operations for aircraft *i* is detained (Constraints (6-20) – (6-21)). The movement operations have to be suspended until the movement path is cleared afterwards for the departing aircraft. The parking position for the arriving aircraft can be adjusted, or the rolling in time is suspended until the movement path is clear.

$$out_{it} \leq 1 - \frac{1}{|A_t \setminus i|} \cdot \sum_{\forall j \in A_t \setminus i} h_{ijt}, \forall i \in D_t, \forall t \geq 0 \tag{6-20}$$

$$in_{it} \leq 1 - \frac{1}{|A_t \setminus i|} \cdot \sum_{\forall j \in A_t \setminus i} h_{ijt}, \forall i \in A_t, \forall t \geq 0 \tag{6-21}$$

3) Duration of staying in the hangar

The duration of each aircraft staying in the hangar must be long enough to complete its maintenance tasks, and the minimum staying time duration of the aircraft should be the sum of the lead times of all maintenance tasks associated the aircraft, which is denoted as $MTime_i$.

$$(\sum_{t \geq ETD_i} out_{it} \cdot t - \sum_{t \geq ETA_i} in_{it} \cdot t) + M \cdot (1 - \sum_{t \geq ETA_i} in_{it}) + M \cdot (1 - \sum_{t \geq ETD_i} out_{it}) \geq MTime_i, \forall i \in I \tag{6-22}$$

$$p_{it} = \sum_{ETA_i \leq m \leq t} in_{im}, \forall i \in A_T, \forall ETA_i \leq t \leq ETD_i \tag{6-23}$$

$$p_{it} = \sum_{ETA_i \le m \le t} in_{im} - \sum_{ETD_i \le m \le t-1} out_{im}, \forall i \in A_T, \forall t \ge ETD_i + 1 \quad (6\text{-}24)$$

$$\sum_{t \ge ETA_i} in_{it} \le 1, \forall i \in I \quad (6\text{-}25)$$

$$\sum_{t \ge ETD_i} out_{it} \le 1, \forall i \in I \quad (6\text{-}26)$$

$$out_{it} \le \sum_{ETA_i \le m < t} in_{im}, \forall i \in I, \forall t \ge ETD_i \quad (6\text{-}27)$$

$$(1 - out_{iT*}) \le \sum_{t \ge ETD_i} out_{it} + M \cdot (1 - \sum_{t \ge ETA_i} in_{it}), \forall i \in I \quad (6\text{-}28)$$

$$(1 - out_{iT*}) \le \sum_{t \ge ETD_i} out_{it} + M \cdot (1 - \sum_{t \ge ETA_i} in_{it}), \forall i \in A_T \quad (6\text{-}29)$$

It is imposed in Constraint (6-22) that the duration of each aircraft staying in the maintenance hangar need be longer than the minimum maintenance time, which is equal to the sum of lead times of all maintenance tasks associated with the aircraft. Constraint (6-22) is relaxed given the condition that the aircraft is not accepted for maintenance service, or the maintenance tasks cannot be finished within planning horizon.

Binary variable p_{it} indicates whether aircraft i is parking in the hangar or not, and Constraints (6-23) – (6-24) ensure the binary variable takes appropriate value. p_{it} takes value one along the shifts from the time that aircraft i moves into the hangar, until the time it leaves the hangar. Constraint (6-25) prescribes that the aircraft's rolling in operations can be conducted on or after its estimated time of arrival (ETA), and the same logic applies to Constraint (6-26) prescribing the roll out operations' decision-making. Constraint (6-27) makes sure the roll out operations occurs after the aircraft has rolled into the hangar. If the maintenance tasks of aircraft i cannot be finished with planning horizon, the aircraft maintenance company fails to deliver such maintenance request and binary variable out_{iT^*} takes value one by Constraints (6-28) – (6-29).

4) Range of decision variables in maintenance scheduling and layout planning

$$x_i, y_i \geq 0 \quad \forall i \in I \tag{6-30}$$

$$b_{ijkt} \in \{0,1\} \ \forall i, j \in A_t, k = 1,2,...,m_{ij}, \forall t \geq 0 \tag{6-31}$$

$$p_{it} \in \{0,1\} \quad \forall i \in A_t, \forall t \geq 0 \tag{6-32}$$

$$in_{it} \in \{0,1\}, \forall i \in A_t, \forall t \geq 0 \tag{6-33}$$

$$out_{it} \in \{0,1\}, \forall i \in D_t, \forall t \geq 0 \tag{6-34}$$

$$h_{ijt}, L_{ij}, R_{ij}, U_{ij} \in \{0,1\} \quad \forall i \in A_t, \forall j \in A_t \setminus i, \forall t \geq 0 \tag{6-35}$$

The coordinates of aircraft parking in the hangar take positive value (Constraint (6-30)). The binary variables related to the maintenance scheduling and layout planning optimization section are denoted in constraint set (6-31) – (6-35).

$$L_{ij} + L_{ji} \leq 1, \forall i, j \in I, j \neq i \tag{6-36}$$

$$R_{ij} + R_{ji} \leq 1, \forall i, j \in I, j \neq i \tag{6-37}$$

$$L_{ij} \leq R_{ji}, \forall i, j \in I, j \neq i \tag{6-38}$$

$$R_{ij} \leq L_{ji}, \forall i, j \in I, j \neq i \tag{6-39}$$

Constraint set (6-36) – (6-39) tightens the geometric relations between each pair of aircraft while deciding its relative position. For example, only one binary variable in set { L_{ij} L_{ji} }, and set { R_{ij}, R_{ji} } can take value one, which is imposed by Constrains (6-36) – (6-37). Moreover, similar logics apply to Constraints (6-38) – (6-39). For example, if the value of R_{ji} equals to zero, the value of L_{ij} cannot take value 1 for consistency.

5) Staff Assignment Components:

$$y_{ist} \leq p_{it}, \forall i \in I_t, \forall s \in S_i, t \geq ETA_i \tag{6-40}$$

$$y_{ist} \leq \frac{1}{|PD_s|} \cdot \sum_{s' \in PD_{is}} f_{s'd}, \forall s \in S, t \geq ETA_i \tag{6-41}$$

$$y_{ist} \leq \frac{1}{\tau_{is}} \cdot \sum_{m \in MPW^{is}} z_{mt,is}, \forall i \in I, s \in S_i, t \geq ETA_i \tag{6-42}$$

$$y_{ist} \geq \frac{1}{\tau_{is}} \cdot \sum_{m \in MPW^{is}} z_{mt,is}, \forall i \in I, s \in S_i, t \geq ETA_i \tag{6-43}$$

$$\sum_{m \in MPW^{is}} z_{mt,is} \leq \tau_{is}, \forall i \in I, s \in S_i, t \geq ETA_i \tag{6-44}$$

$$\sum_{m \in MPW^{is}} z_{mt,is} \leq \tau_{is} \cdot (1 - f_{ist}), \forall i \in I, s \in S_i, t \geq ETA_i \tag{6-45}$$

$$z_{mt,is} \leq p_{it}, \forall m \in MPW^{is}, \forall i \in I_t, \forall s \in S_i, t \geq ETA_i \tag{6-46}$$

$$M \cdot f_{ist} \geq \sum_{t > t' \geq ETA_i} y_{ist'} \cdot h_{t'} - r_{is}, \forall s \in S_i, \forall i \in I, t \geq ETA_i \tag{6-47}$$

$$-M \cdot (1 - f_{ist}) \leq \sum_{t > t' \geq ETA_i} y_{ist'} \cdot h_{t'} - r_{is}, \forall s \in S_i, \forall i \in I, t \geq ETA_i \tag{6-48}$$

$$\sum_{t \geq ETA_i} y_{ist} \cdot h_t \geq r_{is} - M \cdot (1 - \sum_{t \geq ETA_i} in_{it}), i \in I, s \in S_i \tag{6-49}$$

$$\sum_{s \in S_m^t, i \in I} z_{mt,is} \leq a_{mt}, \forall m \in MPW, \forall t \in T \tag{6-50}$$

$$\sum_{t' \in Div_d, t > t'} \sum_{i \in I} \sum_{s \in S_m^t} z_{mt',is} \cdot h_{t'} \leq R_{md}, \forall m \in MPW, \forall div \in DIV \tag{6-51}$$

$$\sum_{s \in S_t^m} z_{mt,is} + \sum_{s \in S_{t+1}^m} z_{m(t+1),is} \leq 1, \forall m \in MPW, \forall t \in T \tag{6-52}$$

$$out_{it} \leq \frac{1}{|S_i|} \cdot \sum_{s \in S_i} f_{ist}, \forall i \in I, t \geq ETD_i \tag{6-53}$$

$$z_{mt,is} \in \{0,1\}, \forall m \in MPW, \forall t \in T, \forall i \in I, \forall s \in S_i \tag{6-54}$$

The constraints (6-40) – (6-55) are relevant to staff assignment's decision-making, which characterize the assumptions and requirements while forming the maintenance team, assigning maintenance technicians and arranging each individual's maintenance roster. Constraint (6-40) ensures that any maintenance task associated with aircraft *i* can be conducted as long as the aircraft is parking in the hangar. The situation that the aircraft is leaving but the maintenance tasks of aircraft is scheduled to be conducted at the beginning of shift *t*, i.e. $y_{isd} = 1, p_{it} = 1, out_{it} = 1$, is not a possible scenario, since

Constraint (6-54) prescribes that the roll out operation cannot be triggered before completing all maintenance tasks for the aircraft, and therefore Constraint (6-40) is not involved in the aircraft rolling out decision variable. Constraint (6-41) imposes the precedence relations between the maintenance tasks associated with aircraft i, implying that the subsequent maintenance tasks cannot be conducted before the preceding tasks have been finished. Constraints (6-42) – (6-44) ensure that each maintenance task has enough qualified maintenance technicians, and the number of capable technicians does not exceed the required number for a particular task to avoid wastage on the manpower input. Constraint (6-45) prescribes that no more maintenance technicians are assigned to the finished maintenance tasks. Constraint (6-46) ensures that the maintenance technicians cannot be allocated to aircraft not parked in the hangar. Constraints (6-47) – (6-48) determine if the maintenance task s associated with aircraft i have finished by the shift t. Constraint (6-49) prescribes that the maintenance time of each maintenance tasks is equal or larger than the required maintenance time. Constraints (6-50) – (6-54) are the regulations in assigning multi-skill maintenance technicians. Constraints (6-50) and (6-51) impose that a technician can be assigned at shift t if the particular individual is available, and the working time of the individual cannot exceed the prescribed working time limit of that division div. Constraint (6-52) prescribes that an individual maintenance technician cannot undertake maintenance tasks in two consecutive shifts to ensure the technician has sufficient rest. Constraint (6-53) regulates that the aircraft cannot leave the hangar before completing all maintenance tasks associated with that aircraft. Constraint (6-54) prescribes the manpower assignment decision variables as binary.

6.2.3. Branching strategy

Considering the large number of binary variables involved in the mathematical model, difficulties in updating the incumbent solutions and lower bounds are expected during the optimization process as eliminating unpromising solutions and duplicate/symmetric solutions is time-consuming. For example, an improper branching strategy may create a branching tree prescribing the manpower allocation variables on the top of the branching trees, which renders in determining the manpower allocation before fixing the service period of aircraft. Alternatively, an improper branching sequence may indicate the position-related variables at the top of the branching trees, which determines the position of aircraft without confirming their respective service period. Such unwise or default branching strategy may result in the adjustment of relevant decision variables or pruning unpromising subtrees from the top of the branching tree in an inefficient way. Therefore, given the hierarchal structure of the binary decision variables, a branching strategy that caters to the features of mathematical model can be developed to avoid an inefficient default branching strategy.

The hierarchal structure of the binary variables can be listed in a descending order as follows:

1) determine the service period (parking period) of each aircraft with the associated in_{it}, out_{it}, p_{it} and out_{iT^*}. Normally the value of out_{iT^*} for all incoming maintenance demands equals zero, implying that the maintenance demands can be delivered by the end of the planning period;

2) After determining the service period of each incoming aircraft, the non-overlapping constraints and movement path blocking constraints are imposed to

validate the tentative service period. Therefore, the value of binary variables h_{ijt}, L_{ij}, R_{ij}, U_{ij} and b_{ijkt} are branched to determine the coordinates (x_i, y_i) of each aircraft, and examines the hangar capacity and clearance in the movement path in the tentative parking positions;

3) Upon determining the service period and geometric positions of each aircraft, the allocation of multi-skill maintenance technicians to each maintenance tasks associated with aircraft is conducted. In the problem instances with overwhelming maintenance demands or in peak maintenance periods, the negative impact of incoordination among the service time decisions, parking positions and manpower allocations is amplified, which shows in the interdependent relationships among these three core decision issues in the hangar maintenance planning problem.

For these instances with high demand, branching on the binary variables and updating bounds can be trapped for a long time, and therefore a branching strategy tailored for this problem is proposed to assist the branch-and-bound algorithm in searching for the incumbent solutions. The branching priorities assigned to the binary variables follow the hierarchal structure of the mathematical model, i.e. the priorities are assigned to service period-related, position-related and manpower-related binary variables in decreasing order. In this section, we conduct computational experiments to find the performance of the MILP model and the proposed branching strategy in solving small-sized instances.

6.3. A two-stage optimization approach

The MILP model presented in Section 6.2 involves great numbers of geometric constraints, resource constraints and respective decision variables, which makes medium- and large-scale instance intractable by the default solver since a large number of unnecessary branching processes are involved in the branch-and-bound algorithm. In particular, updating bounds for large scale problems can be rather difficult and the default branch-and-bound algorithm do not identify the hierarchal structure of the decision variables. For example, the parking positions of the aircraft in the hangar do not have direct relations with the manpower planning. To strength the interdependent relations among the scheduling decisions, geometrical-related decisions and manpower planning decisions, a two-stage optimization based on model decomposition is presented in this section to provide a good quality solution for tackling the planning problem.

6.3.1. Decomposition of original model

In the original MILP model, the timing constraint, geometric constraints and resources constraints are integrated to reflect the interdependent relations among the three-core decision-making elements, namely the parking period of each aircraft in the hangar, the parking stand position and the assignment of technicians. As the mathematical model involves a larger number of binary decision variables, the branching progress takes up a large amount of time. Updating bounds or finding new incumbent solutions become difficult for the medium- to large-size instances, since the default branching strategies provided by the solver CPLEX are incapable of analyzing the complex relations and practical meaning behind the set of binary variables. Specifically, a hierarchal structure exists in the MILP model, and branching progress may be trapped into investigating an unpromising pending solution for a relatively long time. The hierarchal structure of the mathematical model can be presented as follows: the service period (parking period) of each aircraft is determined according to their ETA and ETD first. Afterwards, the non-overlapping constraints and movement path blocking constraints are imposed to validate the tentative service period, so as to determine if the hangar has enough capacity for aircraft parking and clear movement paths to their dedicated positions. If the infeasible solution returns with the tentative determined service period, then one or more aircraft's service period or parking positions need to be adjusted in order to align with the geometric constraints. Our previous research and computational analysis have revealed that finding feasible parking plans for a single time or multi-periods is challenging while dealing with large numbers of incoming aircraft (Qin et al., 2018; Qin et al., 2019). In this extended model, we incorporate the multi-skill technician assignment problem, which is another resource bottleneck in fulfilling hangar maintenance demand under the MRO mode. The hierarchal structure incorporating multi-skill technician assignment in the extended model makes the branching strategy incapable of tackling the medium-size integrated instance, especially when dealing with an instance with overwhelming maintenance demands and limited maintenance resources (hangar space and manpower). In this regard, we propose a two-stage optimization approach inspired by the decomposition method to reduce the complexity while optimizing the original model.

The original model is decomposed into two subproblems. The first-stage problem consists of all the geometric constraints to determine the parking stands, movement path blocking and service time of each aircraft in the hangar. In the second-stage problem, the resource constraints related to multi-skill technician assignment problem is included.

To ensure the connectivity between two subproblems, an iterative process with linkage constraints is proposed to develop an integrated solution between geometric- and manpower-related decision making. The detailed description of the two-stage optimization approach is discussed in Sections 4.2. The characteristics of the proposed approach is that the service time solution given from the first stage is flexible and can be adjusted during the optimization process of second stage problem. The overview of optimization procedures is presented in Algorithm 6-1.

4.2 First stage problem

The decision variables and constraints related to decision-making in the service period and geometric aspects are included, i.e. to determine the parking period of an aircraft, parking position and movement path clearance. Instead of determining the working schedule of each maintenance task associated with the aircraft, the first stage problem only determines the time period of parking for each aircraft. The first stage problem's optimization process consists of two scenarios:

1) the first scenario is the problem initialization. It is assumed that the manpower supply is sufficient to meet all maintenance tasks at any time, which means that the maintenance tasks for each aircraft can be conducted consecutively during the whole planning period. After determinizing the parking periods, the parking stands and the movement paths for all aircraft, these decisions are passed to the second stage problem for the assignment of maintenance technicians, as well as feasibility checking;

2) the second scenario is the iteration process after the problem initialization. After inputting the initial solution (or the solution in the previous iterations), mismatch of

decisions between the two problems might occur. In the problem initialization stage, it is assumed that the manpower supply is sufficient to meet all maintenance tasks at any time along the planning period. However, the initial solution given by the first stage problem usually does not comply with the staff assignment problem in the second stage at the problem initialization step, which means that the manpower supply cannot meet all maintenance tasks with the predetermined desired time windows given by the first stage problem. Given that the available manpower is not able to meet the desired service period, the service time decision has to be adjusted to align with the manpower supply. A possible way is to extend the parking time of aircraft so as to allow sufficient time to finish the maintenance tasks related to the aircraft. The alignment between the first stage and second stage problem can be found after the adjustment of the parking time of aircraft.

4.3 Second stage problem

The decision variables and constraints related to service time scheduling and multi-skill technician assignment form the second stage problem, which determines the technicians that serve the maintenance tasks within the tentative service period determined in the first stage problem. In the second stage problem, the fulfilment of maintenance tasks is specified, based on the tentative service period determined by the first stage problem. The optimization process of the second stage problem consists of two scenarios:

1) the second stage problem is able to identify a feasible solution with the service period determined in the first stage problem, which means that the manpower available is able

to fulfil all maintenance tasks within the desired service period given by the first stage problem. Service time adjustment is not required, and a feasible solution can be obtained.

2) infeasibility returns after solving the second stage problem with the service period determined in the first stage problem. Under such circumstances, the service periods have to be adjusted in order to impose a longer staying time for the aircraft in the hangar to fulfil its maintenance task.

The auxiliary binary decision variables are introduced in the second stage problem.

out_break_i	1, if the roll out time of aircraft *i* is amended from the outcome of the second stage problem, and 0 otherwise
$out^*\_break_i$	1, if fail to finish the maintenance tasks for aircraft *i* from the outcome of the second stage problem, and 0 otherwise

The service time decisions for all incoming aircraft made in the first stage problem serve as the initial solution for the second stage problem. The initial solution consists of $\{\overline{in_{it}}, \overline{out_{it}}, \overline{out_{iT^*}}\}$ for all aircraft, imposed as "soft constraints" in the second stage problem, which means that the tentative service time decision determined in the first stage problem can be adjusted deemed necessary when the manpower is insufficient to serve all the maintenance tasks within the tentative service period determined in the first stage problem. The relevant constraints are presented as follows:

$$in\_sp_{it} \geq \overline{in_{it}}, \forall i \in A_T, \forall t \geq 0 \tag{6-55}$$

$$out\_sp_{it} \geq \overline{out_{it}} - out\_break_i, \forall i \in D_t, \forall t \geq 0 \tag{6-56}$$

$$out_{T^*}\_sp_{it} \geq \overline{out_{iT^*}}, \forall i \in D_t, \forall t \geq 0 \tag{6-57}$$

Constraints (6-55)-(6-57) prescribe the tentative service time determined by the first stage solution, which are changeable deemed necessary as mentioned above. It is noted that the tentative roll in time from the first stage is not permitted to be revised. The ETA plus MTime for all tasks of incoming aircraft are quite close to the ETD, and we do not allow postponement of the roll in time in the second stage problem. In this regard, the changeable service time decision is the roll out time of the aircraft, which allows extensions of the staying time of aircraft in the hangar to have more time to finish its maintenance tasks.

6.3.2. Linkage constraints for two-stage problem

When misalignment between two problems occurs, the service time adjustment is imposed to extend the staying time of aircraft in the hangar. Intuitively, the extension of aircraft's service time can be expressed as the revised version of the maintenance staying time constraint, as shown in (6-58):

$$(\sum_{t \geq ETD_i} out_{it} \cdot t - \sum_{t \geq ETA_i} in_{it} \cdot t) + M \cdot (1 - \sum_{t \geq ETA_i} in_{it}) + M \cdot (1 - \sum_{t \geq ETD_i} out_{it}) \geq MTime_i^{REVISED}, \forall i \in A_T \tag{6-58}$$

The linkage constraint (6-58) is added into the first stage problem and resolved again in the next iteration. The revised maintenance time $MTime_i^{REVISED}$ can be derived by the revised roll out time of aircraft *i* out_sp_{it} from the second stage problem and the original roll in time $\overline{in_{it}}$ from the first stage problem. $MTime_i^{REVISED} = out\_sp_{it} - \overline{in_{it}}$ for the aircraft needs extension of the staying time in the hangar. After adding the revised maintenance time constraint back to the first stage problem, two possible outcomes of service time decisions are expected: 1) a revised service time period have been found (a revised roll out time is determined), and the aircraft can be serviced during the planning period; 2) after imposing the revised staying time constraint, the aircraft is rejected for maintenance service (the aircraft is not rolled in into the hangar), due to the increase of delay cost induced by the extended staying time or blocking with other aircraft.

Algorithm 6-1 Two-stage optimization approach for the hangar maintenance planning

Notations	Meanings
in_{it}^1, out_{it}^1	Roll in and roll out time decision for aircraft in first stage problem
in_{it}^2, out_{it}^2	Roll in and roll out time decision for aircraft in second stage problem
A_T	All maintenance aircraft during the planning period
out_break_i	Indicator of adjusting roll out time of aircraft *i* in the second stage problem
$RollOutT*_i^2$	Indicator of failure to deliver the aircraft *i* at the end of planning period in the second stage problem
$MTime_i^{REVISED}$	The revised staying time requirement of aircraft *i* after solving the second stage problem

1: Solve the first stage problem and derive the first stage service time decision, including the service time and movement operations decisions.

2: Input the first stage decision into the second stage problem. Solve the second stage problem to determine the staff rosters along the planning period.

3: If the second stage solution is infeasible. The misalignment between service time and manpower supply exists.

4:     For *i* in A_T

5:         If $out\_break_i = 1$

        (The roll out time of aircraft *i* is adjusted)

6:           Calculate the revised staying time required for the aircraft *i*.

$$MTime_i^{REVISED} = \sum_{t \geq ETD_i} out\_sp_{it} \cdot t - \sum_{t \geq ETA_i} \overline{in_{it}} \cdot t)$$

7:           If $MTime_i^{REVISED} > \sum_{t \geq ETD_i} \overline{out_{it}} \cdot t - \sum_{t \geq ETA_i} \overline{in_{it}} \cdot t$

8:             Generate the revised staying time constraint for the aircraft *i*. Add the constraint to the first stage problem

9:         If $RollOutT*_i^2 = 1$

10:	Generate the $RollOut_i^* \geq 1 - M \cdot (1 - \sum_{t \geq ETA_i} in_{it})$ constraint. Add the constraint to the first stage problem
11:	Go to Step 1.
12:	Else if the second stage solution is feasible
13:	The current service time solution can find feasible technician assignment plan. Go to Step 14.
14:	End

6.4. Computational experiments

In this section, we describe the ways of generating problem instances based on real data collected from an aircraft maintenance company, and analysis of the numerical experiment results. The approaches presented in the methodology section were programmed in C# and implemented in Visual Studio 2010, which a computer with an Intel Core i7 processor, at 3.6 GHz with 32 Gb of RAM. The MILP model is solved by the optimizer CPLEX 12.7 serial model.

6.4.1. Description of test instances

The problem instances are generated from the data of maintenance demands derived from an aircraft hangar maintenance service provider based on the problem instances utilized in Chapter 3 and Chapter 5. The necessary information of maintenance demands includes the estimated arrival time (*ETA*), estimated time of departure also known as desirable service completion time (*ETD*), aircraft type and maintenance checks type of each maintenance demands in this chapter. We have utilized these set of data to generate problem instances in our previous study (Qin et al., 2017; Qin et al., 2019). The aircraft to be maintained are classified into three categories according to their physical size as presented in Table 3-2 in Chapter 3.

We refer to Ertogral and Öztürk (2019)'s principle while determining the cost of maintenance technicians, which prescribes the cost of worker on hourly rate basis. As we consider the multi-skill maintenance technician's setup in this problem, it is reasonable to prescribe that the individual technician equipped with more maintenance skill is associated with a higher wage rate, regardless of the maintenance tasks assigned to the technician in actual implementation. Such cost setting encourages assigning senior maintenance technicians, i.e. the individuals equipped with more maintenance skills and senior maintenance licenses, to the maintenance tasks requiring senior technicians to avoid improper utilization of manpower and wastage of manpower. The number of maintenance technicians required for the different tasks for aircraft under different categories are listed in Table 6-1. To examine the performance of proposed two-stage optimization approach, the following parameters are adjusted across the problem instances: 1) the skill levels of maintenance technicians; 2) number of available staff; 3) required maintenance service and associated maintenance tasks and 4) shift settings.

Table 6-1 Generic number of require maintenance technicians

	Small Aircraft	Medium Aircraft	Large Aircraft
Minor Maintenance Task	2	3	5
Medium Maintenance Task	3	4	6
Major Maintenance Task	4	6	9

For the setting of three penalty costs, including chance lose cost (Penalty Cost 1:

Penalty for not serving aircraft i during planning period (per request)), tardiness cost (Penalty Cost 2), as well as the failure to deliver cost upon the end of planning period (Penalty Cost 3), each maintenance demand has its own set of costs according to the maintenance type, aircraft type and the required maintenance skill level. In detail, the chance lose cost (Penalty Cost 1) refers to the situation in which the maintenance company does not have enough maintenance capacity to serve the maintenance demand and reject the demand. Originally, the profit in completing the maintenance service for an aircraft is prescribed as two times of the manpower cost on conducting the task, and the chance lose cost of rejecting the service request is prescribed as the profit of completing maintenance service for the aircraft. The tardiness cost is calculated on a minute basis, which is induced whenever maintenance service for the aircraft is finished after the desirable delivery time (ETD). The cost of failure to deliver the aircraft by the end of planning period is prescribed as a portion of chance lose cost (Penalty 1) in the computational experiments.

6.4.2. Computational results

Two sets of computational experiments are conducted in this section. Section 6.4.2.1 reports the computational results of basic MILP model in small- size instances. Afterwards, the medium- and large-size instances were solved by the two-stage optimization approach, whose performances are reported in Section 6.4.2.2.

6.4.2.1. Model and branching strategy's evaluation

In this section, the performance of the proposed MILP formulation and branching strategy presented in Section 6.2.2 and Section 6.2.3 are examined. The performance comparison between the original model and the model with the branching strategy is shown in Table 6-2. The number of shifts for a single day is prescribed as three, and the length of each shift is 8 hours, which aligns with the normal staff rostering setting. The first column in the table denotes the name of the instance. The maintenance demand data collected from the maintenance company are organized to create different groups of problem instances, which are divided into subsections to create problem instances of different sizes or with different parameters. The maintenance demand data are sorted on a monthly basis, and further divided into multiple sets of problem instances within the month. Instance's name is presented in the form of "number of requests_maintenance tasks's demand level (number of planning days)", e.g. 5_1(14) indicates that such instance includes 5 incoming aircraft to be serviced with the standard manpower demand, which covers 14 days. To investigate the performance of the solution approach, instances covering half a month to two months are created to examine the impact of the planning period length and number of maintenance demands. The original model and the model incorporating branching strategy mentioned in the beginning of this section solve the problem instances. The second column shows the number of binary variables involved in the mathematical model, and the third column denotes the preprocessing time before implementing the branch-and-bound algorithm embedded in CPLEX. To examine the impact of increasing manpower requirement associated with maintenance task (the number of qualified technicians required for maintenance tasks) on the objective value and the computational performance, an adjustment on number of maintenance technicians required for conducting maintenance task is implemented to create a variation of manpower demanding. Specifically, Instance x_1(planning days) denotes the original problem instance with the maintenance team size align with Table 6-1, and Instance x_2(planning days) and Instance x_3(planning days) refers to the problem instances with same maintenance tasks setting on each aircraft as in Instance x_1(planning days), but each maintenance task in Instances x_2(planning days) and x_3(planning days) requires more manpower inputs in a progression manner, i.e. Instance x_3(planning days) prescribes that more maintenance technicians are required to conduct the identical tasks than in Instance x_2(planning days). Therefore, the same group of instances with manpower requirement variation have the same number of binary variables. The preprocessing time refer to time spent on initializing the model formulation in the optimizer, and the stopping criteria for solving each problem instance in both models are prescribed as a time limit 3,600 seconds.

Table 6-2 shows that both models cannot solve the problem instances optimally within one hour. Nevertheless, a minor advantage of the branching strategy is shown, compared with the original model in most instances, in terms of the best-known solution and the optimality gap for some instances. In particular, the branching strategy finds better incumbent solutions or tightens the optimality gap over the original model in the problem instance with a moderate number of maintenance demands or higher manpower requirement, e.g. 8_3(22) and 10_2(32), while such advantages do not appear in solving the challenging instances with large number of maintenance demands & high manpower requirements, e.g. 15_3(63) . It is noted grate number of binary decision variables are created in each instance, and the number of binary variables grows significantly along extending the planning period as well as increase in the number of maintenance tasks. Besides the number of maintenance requests in the instances, the difficulties of tackling instances also lie on the distribution of arrivals, maintenance tasks and precedent relations associated with the aircraft, as well as the manpower inputs requirement of the maintenance tasks. The analysis on the same group of problem instances with a variation of manpower requirements on the maintenance task reveals an increasing trend in the objective value, which is associated with the increase of the manpower cost and rejections of the maintenance requests due to the insufficient manpower supply. The rejection of maintenance requests also reflects in the shrinkages of the optimality gap, as the branching efforts are saved for examining the service period, parking stand allocation, geometric relations with other aircraft and the manpower assignment on the rejected maintenance requests. The minor advantages of the branching strategy demonstrate the hierarchical structure of the mathematical model imposes computational difficulties in tackling the problem instances. However, the results in solving instances also reflect the inefficiency of tackling the problem solely with the branching strategy. It can be inferred that extending the length of the planning period also influences the complexity of solving a single instance as it determines the scale of the time-related decision variables.

Table 6-2 Comparison between original model and model with branching strategy

Instance	Binary Variables	Preprocessing Time (s)	MILP Model without Branching Strategy				MILP Model with Branching Strategy			
			Upper Bound	Lower Bound	Gap (%)	CPU (s)	Upper Bound	Lower Bound	Gap (%)	CPU (s)
5_1(14)	11734	0.48	43719.00	38198.50	12.63	3600	41583.00	35235.32	15.22	3600
5_2(14)		0.40	58266.00	45532.41	21.85	3600	57930.00	43953.50	24.13	3600
5_3(14)		0.42	54234.00	53982.92	0.46	3600	54234.00	50490.13	6.90	3600
8_1(22)	67927	2.52	83595.00	12357.97	85.21	3600	83595.00	12343.76	85.23	3600
8_2(22)		2.76	95115.00	40431.77	57.49	3600	95091.00	40421.21	57.49	3600
8_3(22)		2.69	113091.00	71447.20	36.82	3600	112827.00	111968.31	0.76	3600
10_1(32)	116022	4.44	129351.75	11640.78	91	3600	82311.75	11631.62	85.87	3600
10_2(32)		4.41	176055.75	35495.72	79.84	3600	38577.75	35503.16	7.97	3600
10_3(32)		4.60	194877.75	57955.29	70.26	3600	166002.75	57979.18	65.07	3600
15_1(63)	162657	8.29	195156.00	59608.39	69.46	3600	216510.00	59600.51	72.47	3600
15_2(63)		8.61	413802.00	87353.60	78.89	3600	226206.00	87679.71	61.24	3600
15_3(63)		7.96	490815.00	132279.25	73.05	3600	490815.00	132279.25	73.05	3600

6.4.2.2. Two-stage optimization approach evaluation

The computational results in Section 6.4.2.1 have demonstrated minor advantages of the branching strategy in tackling the instances over the original model. However, it is intractable to deal with the instances solely by the MILP model with the branching strategy as the instances cannot be solved optimally after meeting the stopping criterion. In this section, we further implement the computational experiment to examine the performance of the two-stage optimization approach presented in Section 6.3. In the report of the computational results, the performance of the MILP model incorporating the branching strategy is compared with the two-stage optimization approach. We examine the performance of the two-stage optimization approach with a difference in the setting of shift in one day, i.e. setting 3 shifts in one day and 4 shifts in one day. The MILP model tailored for the two-stage optimization approach is embedded in solving the first stage problem and the second stage problem with branch-and-bound algorithm. The time limit for solving each first and second stage problem is 1,800 seconds.

Table 6-3 shows the computational results of the two-stage optimization approach. The seventh to eighth columns of Table 6-3 show the binary variables involved in the problems in two stages, and the number of iterations, respectively. The CPU and objective values are used to compare the performances between the model with branching strategy and the proposed two-stage approach. The two-stage optimization approach manages to obtain solutions with good quality within around an hour, compared with the MILP model with the branching strategy. The advantages of two-stage approach are reflected in the view of objective value in tackling problem instances with large number of maintenance requests, i.e. problem instances with more than 5 aircraft maintenance requests. It is found that setting 4 shifts per day do not reduce the manpower cost. The required computational time and model scale for solving the same instances increases in the 4-shift per day setting. The strengths of the two-stage optimization approach over the MILP model for large-scale or high demand instances implies the incapability of the MILP model in connecting the three independent core elements of decision-making, as the branch-and-bound algorithm is likely to probe the subtree associated with infeasible solution repeatedly before updating the bounds, and the default branch-and-bound algorithm cannot infer the pattern of unpromising solution from the previous pruned subtrees.

The effectiveness of the two-stage optimization approach reflects that the staying time constraint, i.e. the revised MTime requirement imposed on each incoming aircraft, successfully acts as an efficient connecting bridge between the two stage problem, as the main effect of insufficient manpower supply directly results in the tardiness in fulfilling the maintenance tasks and extension of the service time window. For the maintenance service company, facing hard instances for decision-making within a short period of time is common for real-world operations. To provide solutions for practical use, the two-stage optimization approach can be considered as a reliable heuristic when the service providers are in need of better-quality solutions in less time than the exact method provided by the commercial solver when the allowable computational time is limited.

Table 6-3 Computational results in solving instances by two-stage optimization approach

Instance	Shift Setting	MILP Model with Branching Strategy				Two-stage Optimization Approach					
		Upper Bound	Lower Bound	Gap	CPU	Binary Variables in Stage 1 Problem	Binary Variables in Stage 2 Problem	Iteration	Generated Constraints	Objective Value	CPU
5_1(14)	3	41583.00	35235.32	15.22	3600	5487	6721	1	4	44223.00	1348.66
5_2(14)		57930.00	43953.50	24.13	3600			2	10	59055.00	1884.95
5_3(14)		54234.00	50490.13	6.90	3600			1	7	56442.00	185.29
8_1(22)		83595.00	12343.76	85.23	3600	34100	35347	1	1	21429.00	2268.05
8_2(22)		95091.00	40421.21	57.49	3600			1	4	50733.00	3631.54
8_3(22)		112827.00	111968.31	0.76	3600			2	9	117483.00	2100.73
10_1(32)		82311.75	11631.62	85.87	3600	68320	49953	1	1	29025.75	192.59
10_2(32)		38577.75	35503.16	7.97	3600			1	1	53313.75	2175.83
10_3(32)		166002.75	57979.18	65.07	3600			2	11	127695.75	2090.25
15_1(63)		216510.00	59600.51	72.47	3600	112153	55069	1	13	114303.00	4140.59
15_2(63)		226206.00	87679.71	61.24	3600			2	36	225720.00	2367.86
15_3(63)		490815.00	132279.25	73.05	3600			4	48	260271.00	3896.76

(Table 6-3 Cont'd)

Instance	Shift Setting	MILP Model with Branching Strategy				Two-stage Optimization Approach					
		Upper Bound	Lower Bound	Gap	CPU	Binary Variables in Stage 1 Problem	Binary Variables in Stage 2 Problem	Iteration	Generated Constraints	Objective Value	CPU
5_1(14)	4	51669.00	39097.39	24.33	3600	7372	9045	1	6	49275.00	1894.04
5_2(14)		69900.00	47597.70	31.91	3600			2	12	73626.00	1824.27
5_3(14)		58671.00	51300.52	12.56	3600			1	7	59625.00	847.89
8_1(22)		121539.00	38119.44	68.64	3600	45505	35310	2	6	87267.00	4408.53
8_2(22)		184776.00	57023.04	69.14	3600			3	8	134208.00	1901.62
8_3(22)		207024.00	62777.81	69.68	3600			1	4	146700.00	1850.29
10_1(32)		118938.75	34805.94	70.74	3600	91700	43629	2	18	94057.50	7567.64
10_2(32)		204966.75	55514.02	72.92	3600			1	16	142711.50	1904.96
10_3(32)		335211.00	65667.83	80.41	3600			1	14	166316.25	1880.97
15_1(63)		341013.00	59779.58	82.47	3600	149634	73565	3	31	156357.00	3840.18
15_2(63)		448527.00	85264.98	80.99	3600			3	38	227919.00	3840.54
15_3(63)		510036.00	130365.20	74.44	3600			1	26	318888.00	2295.45

6.4.2.3. Enhancement of maintenance technician's skill

The computational experiment in Sections 6.4.2.1 and 6.4.2.2 focus on the computational efficiency in dealing with the problem instance without considering the variation of manpower supply. We conduct an analysis to discover manpower supply variation's impact on the solution in this section, which provides better understanding for the service provide to better exploit the maintenance capability and realize the profitable portfolio.

In this section, two settings of manpower supply enhancement are prescribed for each. In Table 6-4, the column "Manpower supply enhancement 1" refers to the setting of "minor" manpower supply enhancement, and the "Manpower supply enhancement 2" refers to the "major" manpower supply enhancement, respectively. The specification of maintenance demands, including the estimated arrival time, desirable deliver time, requirement of maintenance tasks (required skill and team size), remains unchanged through the manpower supply enhancement. Table 6-4 reports the computational results of enhancing the manpower supply for the instances. After enhancing the manpower supply progressively, it is found that the objective values of all instances have a decreasing trend while increasing the manpower supply across all skill levels. The iterations and generated additional constraints during the optimization progress also maintain a similar trend, which reflects that the difficulties in finding solution have been reduced. The computational times for the larger size instances are reduced significantly compared with the original manpower supply. After further investigation on the solution outcome, it is found that the major elements contributing to the objective value switch from the tardiness in fulfilling maintenance demands/chance lose cost of rejecting maintenance demands (Penalty 2& Penalty 1, respectively) to the manpower cost. The above findings along the variation of manpower supply reveal that the manpower supply becomes a significant resource bottleneck over the maintenance hangar space in fulfilling the maintenance demands in peak hours, as the penalty costs result from the insufficient manpower supply and such factor contributes significantly to the overall penalty cost. However, these findings of manpower supply do not necessarily imply that having more manpower available at all times benefit the most to the maintenance service provider. The difference of objective values between manpower supply enhancement 1 & 2 has narrowed down, compared with the setting of the original instance, In the mathematical model, the manpower cost is calculated based on an hourly rate as adopted in the relevant literature, while the hiring cost is not incorporated as a part of manpower cost in our analysis. In this regard, strategic planning of manpower determining the number of technicians to be hired can be another important issue besides assigning the technicians to particular maintenance tasks.

Table 6-4 Analysis on the enhancement of maintenance manpower supply

Instance	Before Manpower Supply Enhancements				Manpower Supply Enhancement 1				Manpower Supply Enhancement 2			
	Iteration	Generated Constraints	Objective Value	CPU	Iteration	Generated Constraints	Objective Value	CPU	Iteration	Generated Constraints	Objective Value	CPU
5_1(14)	1	4	44223.00	1348.66	1	1	17520.00	58.12	1	1	15240.00	46.04
5_2(14)	1	10	59055.00	1884.95	1	4	37716.00	919.30	1	2	25476.00	96.38
5_3(14)	1	7	56442.00	185.29	1	7	53010.00	1842.99	1	2	35340.00	142.22
8_1(22)	1	1	21429.00	2268.05	0	0	13821.00	94.69	1	1	12285.00	53.31
8_2(22)	1	4	50733.00	3631.54	0	0	35901.00	540.01	0	0	34605.00	29.28
8_3(22)	2	9	117483.00	2100.73	2	3	53877.00	5519.37	0	0	52509.00	1832.27
10_1(32)	1	1	29025.75	192.59	1	1	13209.75	104.25	0	0	13137.75	38.24
10_2(32)	1	1	53313.75	2175.83	0	0	38193.75	302.58	0	0	36465.75	297.68
10_3(32)	2	11	127695.75	2090.25	1	1	59145.75	3684.18	0	0	58045.75	1843.52
15_1(63)	1	13	114303.00	4140.59	3	9	64605.00	9012.99	2	3	63717.00	5900.67
15_2(63)	2	36	225720.00	2367.86	1	6	12145.00	4183.36	2	7	100246.00	6855.17
15_3(63)	4	48	260271.00	3896.76	5	39	221961.00	4582.51	4	12	167325.00	9722.56

6.5. Summary

Efficient service implementation in fulfilling increasing aircraft hangar maintenance demands has emerged as a crucial factor in the aircraft hangar maintenance service company. An aircraft hangar maintenance planning problem is studied under MRO outsourcing mode as a new problem of aircraft maintenance planning. To fulfill the incoming maintenance demands from multiple airlines and other clients, maintenance planner needs to optimize the service period, multi-period hangar parking plan and the manpower assignment roster altogether. The research work presented in this chapter makes a novel contribution in closing the research gap in the maintenance optimization problem in the aviation industry. A mixed integer linear programming (MILP) model formulating the geometric constraints, manpower assignment constraints is proposed, so as to integrate and characterize the interdependent relations of decision-making. For the maintenance company, developing the hangar parking plan and multi-skill maintenance technicians' roster are bottleneck in fulfilling maintenance demands and implementing maintenance tasks. The limited hangar capacity, flexible parking arrangement, movement blocking together with the multi-skill maintenance technician rostering make the problem intractable by the MILP model with the default branching and bound method, as the scale of model grows significantly. To tackle medium- to large-size problems, a two-stage optimization approach is developed to decompose the original model into two subproblem linked by constraints, and its effectiveness is examined afterwards. The developed two-stage optimization approach is examined by is tested by problem instances generated based on the maintenance demands data collected from industry. It is demonstrated that the proposed approach is able to provide good quality solution within time limits in tackling medium- to large-size instances. Afterwards, an analysis to studying the affect and impact of the variation of the parameters in maintenance demand and manpower supply is carried out. Given the interrelations among three core elements of decision-making, the service capacity fluctuates with the incoming maintenance demands. The challenges in coordinating the maintenance resources is shown in numerical results. Tackling the challenging also reveal that the arrival of maintenance demands congesting within short period results in a high requirement of maintenance resources within short period of time. As a result, tardiness as well as high chance lose cost in rejecting aircraft maintenance requests may occur during the peak hours. From the perspective of management, the negative impact of the lack of coordination of maintenance resources need to be carefully handled, as the incoordination and low utilization of maintenance resource lead to reduce of client's satisfaction rate as well as company profit. Moreover, the arrival pattern of the demands should be carefully analyzed well in advance to prepare the arrangement of maintenance resources supply in real-world practice.

For the future research directions on this topic, it is possible to (1) consider the advanced joint decision making among independent service companies and multiple airlines and understand the arrival pattern proactively, which aims to avoid the congestion of maintenance requests arriving within short period. Joint maintenance planning is able to improve the utilization of maintenance resource and service quality within the maintenance service provider, allowing the MRO service provider to have sufficient time to arrange and adjust their service capacity, especially manpower supply. When congesting arrivals of maintenance demands occurs, the joint decision-making allows the service provider and airline companies to adjust the respective maintenance plans among multiple parties in a flexible manner; (2) incorporate uncertainties in maintenance planning, including unscheduled maintenances and limited information regarding the arrival pattern of aircraft due to uncertain situations (Kenan et al., 2018; Leal de Matos & Powell, 2003; X. Zhang & Mahadevan, 2017), so as to makes the optimization approach close to real operations; (3) developing exact algorithm and improving heuristic algorithms based on the decomposition approaches proposed in this chapter, given the effectiveness of the linkage constraints proposed in the two-stage optimization approach in this chapter.

Chapter 7. Conclusion of the Thesis

7.1. Summary of the thesis

The unifying theme of the thesis is that the tailored optimization approaches enhances the productivity and service capacity level of hangar maintenance operation under MRO outsourcing mode. The increasing hangar maintenance demands from multiple airlines and maintenance capacity deficiencies results in series delays of maintenances service provided by aircraft service providers. The negative effects of improper maintenance planning can lead to a low utilization of hangar maintenance resources, which should not be underestimated given the increasing maintenance demands expected in the future. Considering the conventional hangar maintenance optimization approach is incapable in tackling the overwhelming outsourcing maintenance demands, the optimization approach developed in this thesis systematically characterized the hangar maintenance planning problem, then offered respective algorithms to develop integrated hangar maintenance plans. In order to fully utilize the limited hangar maintenance resources to enhance the service level of maintenance service provider, the geometric factors during the maintenance period are identified as one of critical bottleneck in carrying out the maintenance planning, in addition to manpower resources. To be more specific, this research aims to bridging the research gap between the conventional hangar maintenance scheduling and the layout planning problem under the MRO outsourcing mode, which comprises of (1) utilization of hangar parking space, (2) aircraft movement planning with consideration of possible blocking, (3) the impact of multi-skill manpower supply on conducting various maintenance demands. The expected outcome of this thesis contributes the theory of aircraft maintenance management and the application of hangar maintenance planning for the emerging

MRO outsourcing mode, and the developed models and optimization algorithms are generic ones, which are capable to tackle the real-world hangar maintenance operations under the outsourcing mode.

The results of numerical experiment contribute to the research field. This research project contributes significantly in terms of theory and practical application. The connections among aircraft hangar maintenance scheduling, layout planning, manpower planning is presented in literature, which points out the research gap of maintenance planning in aviation industry. The formulation of hangar maintenance scheduling is inefficient without the consideration of geometric factors, during the course of carrying out the maintenance plan under outsourcing mode.

(a) A literature review on aircraft maintenance scheduling and the relevant optimization problem involved in hangar planning are presented. The aircraft maintenance scheduling studied from various perspectives is discussed, which differentiates the hangar maintenance planning problem under MRO outsourcing mode from the other perspective. The peculiarities of maintenance outsourcing mode are manifested, pointing out the inadequacies of existing modelling approaches in characterizing the hangar planning problem. More realistic constraints and loosening assumptions are essential. The respective optimization algorithms are significant in provided high-quality solution for maintenance planning operations. The summary of literature review demonstrates the rationale and potentials of the proposed research. The complexities in problem modelling and integrations of optimization problems, the use of conventional exact algorithm may not satisfy the computational needs of the geometric-involved and resource-constrained problems. Development of tailored exact methods to solve real-world-size instance is significant. A conclusion can be drawn that the research on hangar maintenance planning with heuristic algorithm can bridge the gap of aircraft maintenance optimization in research field.

(b) Mathematical modelling in aircraft hangar maintenance remains characterizing the maintenance resource in a numerical form. Current publications on the research of hangar maintenance are not applicable from the actual practice under outsourcing mode; a systematic planning approach in outsourcing mode is the a newly emerging research direction, incorporating the geometric factors in problem modelling. To enhance the utilization of resources, an aircraft hangar maintenance planning approach was developed, which is tailored for the MRO outsourcing mode. The research elements in the project are accommodated by the following specific R&D activities from this thesis.

   i. The impact of allocation of aircraft parking stand is studied, as a critical research element in carrying out aircraft hangar maintenance planning. It is manifested that modelling the hangar parking capacity as a constant number during whole planning period is inappropriate under the MRO outsourcing mode, as the sizes of incoming aircraft varies. A mathematical formulation handling the hangar capacity in two-dimensional space is developed to cater the aircraft in different sizes to be serviced. Regarding the solution procedure of parking stand allocation, the proposed MILP-based heuristic algorithm can benefit hangar planner in carrying high quality layout within a reasonable computation time for practical usage. The computational results demonstrated the effectiveness of the proposed algorithm in numerical analysis. Afterwards, a family of inequalities is proposed to enhance the computational performance in solving the large-scale challenging instances to analyse the tractability of the model scale and effectiveness of inequalities in tightening the optimality gaps.

ii. Based on the aircraft parking stand allocation problem studied from a single time point, the aircraft movement operations and its blocking effect are studied. The increasing maintenance demands can result in the propagation of maintenance service delay in the maintenance hangar. In consideration of the difference of the service periods for the demands, the interrelation of aircraft movements timing, service period and parking position are modelled in the multi-period aircraft maintenance scheduling formulation. With the introduction of decision variables for geometric position relations, the hangar planning quality and maintenance capacity can be further improved. After tacking the blocking impact in the maintenance scheduling, the unnecessary movement blocking, and aggregate delays can be alleviated. In addition, a rolling horizon approach to resolve the problem for a long planning period is proposed. The computational analysis illustrates that the performance of proposed heuristic approach provides good quality solution for the practical operation environments.

iii. The hangar maintenance planning approach for outsourcing mode is further extended and fulfilled with incorporation of multi-skill manpower planning and assignment, which is another bottleneck and difficulties. The model incorporates practical consideration in carrying out the maintenance technicians' rostering. The variance of the maintenance technician's skill level and skill type is regarded as a significant factor under the context of aircraft heavy maintenance. Considering the complexity of the integrated mathematical formulation and the intractability of the large-scale instances, a decomposition method is developed, which results in a first stage problem determining the geometric-related decision makings and a second stage problem arranging the manpower supply. The constraint reflecting the relations between the manpower supply and required length of staying time of aircraft to be serviced is introduced to connect the problem from two stages. The computational results suggest that the proposed algorithm outperforms the conventional branch and bound algorithm in terms of solution quality for the medium- to large-size instances within reasonable time.

In conclusion, it is believed that the geometric factors in maintenance hangar and the manpower supply are significant in carrying out the maintenance planning solution under the MRO outsourcing mode. Given the complexity of the mathematical model and large number of binary variables characterizing the geometric relations among aircraft and the manpower assignments, the conventional exact algorithm cannot meet the practical demands and therefore a set of heuristic algorithms is developed to provide good quality solution. It is identified that the negative impact of improper layout planning leads to a propagation of maintenance service delay during the peak operation period. The proposed mathematical model and tailored optimization algorithm are promising in eliminating the impact of aircraft blocking and achieve higher service level under the increasing outsourcing maintenance demands.

7.2. Limitations of the works

Despite the achievements made in the study of aircraft hangar maintenance planning under the MRO outsourcing mode, some limitations exist in the completed works and the followings summarize the limitations for future avenues of the research:

(a) Consideration of uncertainties in maintenance planning is omitted in the course of the development of optimization model. The integrated hangar maintenance plan is optimized under an offline environment with the input deterministic problem data, which can be vulnerable in the real-world implementation while there are any moderate deviations of data. In particular, the disruptions of maintenance plan are possible whenever aircraft's arrival/departure timing are changed, which renders the offline hangar parking layouts inapplicable to align with the real situations.

(b) Joint decision-making among maintenance provider and its clients is not taken into the scope of the research. It is considered that the hangar maintenance service provider carries out the integrated maintenance plan to serve multiple airlines/clients in a passive manner. The maintenance planning is optimized with all the maintenance demands specification (estimated arrival time, desired deliver time and allowable time window) fixed, and the flexibility of the optimized solution is lacking.

(c) The optimization algorithm developments in this research are limited to heuristics based on the respective developed MILP models. The optimality gaps of the incumbent solution are not able to be utilized during the searching process, and therefore the mechanism of searching better incumbent solutions in heuristic cannot benefit from the optimality gaps, which provides accurate information of solution status along the optimization.

7.3. Areas of future research

Several research aspects in aircraft hangar maintenance planning problem and optimisation methods can be considered for future work. These include:

(a) The deterministic aircraft hangar planning decision-making can be extended to incorporate the uncertainties of maintenance demands' arrival and departure time. In real-world practice, airline carries out its own maintenance routing plan for the operating fleet to determine the maintenance time. It is the fact that the actual arrival and departure time can alleviate from the scheduled timing, and the maintenance plan predetermined by the maintenance service company can be interrupted so that the original maintenance plan has to be revised according to the interruption. To enhance the robustness of the maintenance plan, the arrival timing of maintenance demands can be regarded as fall into an interval, which is determined by the uncertain level of the information then enhance the flexibility in carrying out maintenance plan.

(b) The consideration of advanced joint decision making between independent service company and airlines ahead of time to arrange the maintenance plan is recommended, which aims to avoid overwhelming maintenance requests arrive at similar time in a proactive manner. The joint maintenance planning is expected to enhance the service level of maintenance service provider in serving the increasing demands and fulfill the maintenance requirement of different airlines' fleets, allowing MRO service provider to have enough time to review their service capacity and flexibly adjust the maintenance service plan. When congestions of maintenance demands, the joint decision making allow maintenance company and multiple airlines to negotiate and adjust their own maintenance plan in a flexible manner.

(c) The current heuristic approaches for the large-scale problems can be further enhanced to develop the exact algorithms. Given the present decomposition approach utilizes the tailored constraints to connect the problem from two stage, which resembles the structure of iterative approach such as branch-and-cut as well as the benders decomposition algorithm. Therefore, the proposed two stage optimization algorithm can be used as the foundations in developed the decomposition-based approach. The geometric constraints involved optimization problems are commonly found in many research aspects, and the exact algorithm for solving the generic optimization problem is lacking. The exact algorithm is able to provide precise information regarding the optimality status of the optimization process, which provide directions in solving geometric constraints involved optimization problem and make respective theoretical impacts.

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