ISSUES IN ONLINE PLATFORM OPERATIONS: GENDER-BASED SAFETY CONCERNS AND SUPPLIER ENCROACHMENT

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Issues in Online Platform Operations: Gender-based Safety Concerns and Supplier Encroachment

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CERTIFICATE OF ORIGINALITY

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Abstract

The development of technology and e-commerce generates many online platforms that reform the daily life of people. For example, the ride-hailing platforms such as DiDi and Uber allow the drivers to take a better use of their spare time and the riders to call a car whenever needed. The online platforms such as JD.com and Amazon provide more opportunities to sellers and allow them to open a store on the platforms so that the sellers can directly access to the end customers. In this thesis, we focus on the operations of online platforms and study how gender-based safety concerns and supplier encroachment channel should be effectively managed.

In the first topic, we focus on the gender-based safety issue and the operations of ride-hailing platforms. We investigate the performance of two operational systems: a pure pooling system (in which users are matched without considering gender types) and a hybrid system (which contains a pooling subsystem and a female-only subsystem). For each system, we analyze a two-stage queueing game by first determining the respective equilibrium joining behaviors of riders and drivers, and then deriving the platform's optimal pricing and wage decisions. We obtain the following main results. First, we show that in a pooling system, the marginal improvement in the platform's profit increases with the safety confidence on the rider demand side but diminishes with the safety confidence on the driver supply side. Therefore, platforms should improve female riders' safety confidence as much as possible while ensuring that female drivers' safety confidence is sufficiently high. Interestingly, we demonstrate that increasing driver safety confidence may not lead to more female riders joining the pooling system. We find that in a hybrid system, granting flexibility to female drivers may hurt

the platform. By comparing the two system configurations, we show that win-win outcome can be attained, but not everyone in the system is happy with the migrating from a pooling to a hybrid system.

In the second topic, we begin with a supplier (she) who wholesales to a retailer (he), and is considering to encroach into the retail market by opening an independent online/offline store to sell directly to consumers (a direct channel encroachment) or by selling directly to consumers through the online platform of her retailer on commission (a commission channel encroachment). Under the latter encroachment, the retailer may choose to share his private demand information with the supplier, but the supplier must pay the retailer commission fees proportional to her direct sales revenue. In contrast, under the direct channel encroachment, the supplier collects the entire sales revenue but incurs a channel operating cost. We investigate how does a party's role as a Stackelberg quantity leader affect the retailer's information sharing incentive and the supplier's encroachment channel selection. We show that under the commission channel, a quantity leader retailer always shares his demand information with the supplier; however, if the retailer is the quantity follower, he may have no incentive to share his demand information. As to the supplier's encroachment channel selection, we show that for any given commission rate, there exists an upper (lower, resp.) threshold direct channel operating cost, above (below, resp.) which the supplier encroaches via the commission (direct, resp.) channel regardless of who is the quantity leader. When the direct channel operating cost falls between these two thresholds, we have the following: if the commission rate is low, the supplier adopts the commission (direct, resp.) channel encroachment when the retailer is the quantity leader (follower, resp.), while when the commission rate is high, the exact opposite holds. Interestingly, we show that a more accurate demand signal does not necessarily improve the supplier's preference over the commission channel encroachment.

Publications Arising from the Thesis

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Chapter 1

Introduction

The online platforms bring many conveniences to our daily life and reform the operational way of a service facility and the supply chain. On-demand ride-hailing platforms such as Didi, Uber, Lyft, etc. offer convenience for riders and work flexibility for independent drivers. The online platforms (also called e-retailer) such as JD.com and Amazon combine the reselling business and the direct sale business, which provide more opportunities for the suppliers. Because the suppliers not only can wholesale the products to the e-retailers but also can directly sell the products to the end markets. However, the development and operations of those innovative platforms is not so smooth. Uber, Lyft and Didi received complaints from female riders (drivers, respectively) for being harassed sexually by male drivers (riders, respectively) (Feeney 2015). The female users usually have safety concerns about the operational environment of the ride-hailing platforms. Such gender-based safety issue induces a big challenge to the ride-hailing platforms. For the online platforms, allowing the suppliers to directly access to the end market induces a competition between the platforms themselves and the suppliers. Hence, the platforms need to carefully make their operational decisions (such as quantity decision and system configurations).

In Chapter 2, we consider the impact of gender-based safety issue in the ride-hailing platforms. We note that in practice female users usually have safety concerns when they are matched with a male user when they use the ride-hailing platforms and such gender-based safety concern may affect the joining behaviors of the female riders and female drivers, which further affects the profit of the ride-hailing platforms. Currently, many ride-hailing platforms adopt the pooling system, in which all the drivers and riders are matched without considering the gender types. We provide a way to alleviate this problem: providing a choice for the female riders and allow them to select between a pooling subsystem and a female-only system. We call this a hybrid system. We investigate three research questions: (1) Will switching to a hybrid system result in a win-win outcome for all parties (riders, drivers and the platform)? (2) If the current pooling system is kept, how shall platforms work on the user safety to improve their performance? (3) With a hybrid system, female riders would have the flexibility to choose between the pooling and female-only option. Taking into consideration the limited supply of female drivers, should such flexibility also be granted to female drivers?

We obtain the following key insights. In the current pooling system, the platform should put more efforts on enhancing on the rider side safety confidence than that of the driver side. Also, we prove that the number of joined female riders is reduced if the platform takes measure to improve the driver side safety confidence. In a hybrid system, surprisingly, we find that flexibility should not be fully granted to female drivers because it can jeopardize the efficiency of the system. A comparison of the equilibrium outcomes associated with pooling and hybrid systems reveals that when safety concerned female users' safety confidence falls to certain levels, switching from a pooling system to a hybrid system can result in a win-win outcome on the two most important goals, increasing the accessibility for safety-concerned female users and improving the platform's profitability, although male and safety-unconcerned female users might be worse off. Our results shed light on platforms' operational system design, that is, on which side the platform should put more effort into enhancing safety confidence in a pooling system, when to switch to a hybrid system and to what extent the platform should grant female drivers flexibility to choose in a hybrid system. Our analysis also provides a plausible explanation for the adoption of different systems in countries with differing levels of female safety.

In Chapter 3, we explore how a supplier selects the encroachment channel when she has two choices: opening an online/offline channel (which we call direct channel encroachment) and using e-retailer's online platform by paying the platforms commission fees (which we call commission channel encroachment). We consider a supply chain that contains a supplier and a retailer. They compete in the downstream market by making quantity decisions. Also, in the quantity competition, we study two scenarios which differ from each other in the quantity leadership (one is supplier is the quantity leader and the other is retailer plays as a quantity leader). We show that the encroachment channel selection of the supplier depends on the relative relationship between the direct channel encroachment cost and the commission rate in the commission channel. More importantly, the quantity leadership plays an important role in the encroachment channel selection. When the direct channel encroachment cost either high or low, the supplier selects either commission channel or direct channel in both quantity leadership scenarios. However, when the direct channel encroachment cost is medium, the supplier has opposite encroachment channel selection in the two quantity scenarios. In particular, the commission rate further impacts supplier's channel selection when aforementioned cost is medium: The supplier selects the commission channel when she is a quantity follower but the direct channel when she is a leader for a small commission rate; while, she selects the commission channel when she is a quantity leader but the direct channel when she is a follower for a large commission rate.

Chapter 2

Gender-Based Operational Issues Arising from On-Demand Ride-Hailing Platforms

2.1 Introduction

On-demand ride-hailing platforms provide great convenience for riders who need transportation services and work flexibility for self-regulated drivers who work independently. A notable issue facing platforms such as Uber, Lyft and DiDi is that some female users (i.e., riders and drivers) have complained about being sexually harassed by matched male counterparts (Feeney 2015). (Hereafter, "users" refers to riders and drivers on the platform.) Over the past 3 years, there have been a series of reports about female users being sexually assaulted, raped or murdered by male users. For example, Uber drivers were accused of 32 rapes and sex attacks on London passengers in a 12-month period (Samuels 2016). Over 50 DiDi female passengers have been assaulted since 2015 (Zhang 2018). In 2018, a 21-year-old female flight attendant was murdered by a male DiDi driver during her DiDi ride (Grothaus 2018), and another 20-year-old woman was raped and killed by a male DiDi driver (Zhang and Munroe 2018). On the driver side, a 20-year-old female DiDi driver was murdered by a male passenger in 2016 (ChinaDaily 2016). These instances exacerbate the safety concerns of female riders and drivers (Fong 2019). A six-country survey revealed that 64% of surveyed women drivers identify security as a reason that they do not sign up to become Uber drivers (IFC 2018).

These incidents have triggered debates on how to address the safety concerns of female users on ride-hailing platforms Shepherd (2018). Didi now regards safety rather than profitability as the most important issue to consider (Liao 2019). For example, DiDi temporarily suspended its carpooling service in order to develop methods to improve safety (BBC 2018).

To resolve or at least alleviate the safety issue, one approach is to change from the current gender-neutral "pooling system" (that matches riders and drivers without considering gender) to a gender-dedicated system (that only matches riders and drivers of the same gender). Didi announced in 2018 that its carpooling drivers can only pick up riders of the same gender in the early morning and late evening hours (Al-Heeti 2018). Also, to capitalize on female safety concerns, new startups such as SheTaxis, Safr and Chariot for Women (United States), She Cabs (India) and She'Kab (Pakistan) offer female-only ride-hailing services.

However, moving to a gender-dedicated system involves another significant problem: *imbalance between female-rider demand and female-driver supply*. For instance, females account for only 2% of drivers but 60% of customers in the taxi and delivery industry (SheRides 2016). In China, only 10% of registered DiDi drivers are female (about 2.3 million), while half of its riders are female (more than 200 million) (Borak 2018, ChinaNews 2018, AsiaSociety 2017). According to DiDi, if only same-gender users are allowed to be matched, just over 1 in 20 female riders could be successfully served (DidiPublic 2019). In the United States, 48% of Uber's 41.8 million riders are female, but it is estimated that just 14% of its 1 million drivers are female (Iqbal 2019, Muchneeded 2019). Thus, while females' safety concerns are lessened (or absent) in a gender-dedicated system, the adoption of such systems may lead to a significant loss of female riders due to the limited supply of female drivers and the absence of pooling. Essentially, matching users purely based on gender could cause significant profit loss for ride-hailing platforms (one can roughly estimate that such a loss might reach 50%).

Instead of separating users by gender, some suggest that in addition to the current pooling system, DiDi or Uber could provide an option for safety-concerned female users: a female-only subsystem (Buxton 2018). A MoveOn petition calling for this type of operation has collected more than 14,000 signatures (Green and Zimmer 2019). We call such a system a "hybrid system", as it consists of a gender-neutral pooling subsystem and a female-only subsystem. Such a hybrid system could mitigate the loss of the pooling effect associated with a gender-dedicated system. A hybrid system could also exploit the heterogeneity of user safety concerns: not all female riders prefer female drivers. According to a survey, 47% of women have no preference between male and female drivers (IFC 2018). With the flexibility to choose between the subsystems, safety-unconcerned female riders can pick the pooling subsystem, which allows them to enjoy a shorter waiting time due to a larger driver pool, while safety-concerned female riders can choose the female-only subsystem.

The ride-hailing ecosystem consists of multiple parties: riders, drivers and the platform. Intuitively, it is plausible for a hybrid system to result in a win-win outcome for all parties. When female riders are offered more choices, the demand base can be expanded. The female-only subsystem could attract more women to sign on as drivers, helping solve the shortage of women drivers. These changes could help increase a platform's profit. However, the hybrid system cannot fully avoid the loss of the pooling effect. A lengthened waiting time could drive away some riders, and female drivers may not be willing to wait for female riders because it may reduce their income. To provide a holistic evaluation of the two systems, we present a mathematical model to capture different user groups' joining behavior and the platform's pricing and wage decisions. We are particularly interested in answering the following three questions related to ride-hailing service system design:

1. For the current pooling system, how can safety confidence levels on the demand and supply sides be improved?

2. In a hybrid system, female riders are granted flexibility to choose between the pooling subsystem and the female-only subsystem. Should female drivers be granted such flexibility as well?

3. Can moving from a pooling system to a hybrid system result in a win-win outcome for all involved parties?

As an initial attempt to explore the above research questions, we consider a situation in which riders are price- and waiting-time-sensitive and drivers are wage-sensitive. Male users' safety concerns are normalized to zero. Female users are heterogeneous with regard to safety concerns: a fraction of female users exhibit safety concerns when matched with male counterparts and the rest have no such concern. We then construct a two-stage queueing game model (Hassin and Haviv 2003, Hassin 2016) to analyze the performance of the following two systems: 1) a pooling system that matches riders and drivers without considering gender, where the platform adopts a *gender-neutral policy* for its pricing and wage decisions; and 2) a hybrid system consisting of a pooling subsystem and a female-only subsystem, where female riders can freely choose between the two subsystems, female drivers' such flexibility in choosing is controlled by the platform, and the platform adopts a *subsystem-based pricing and wage policy* for its pricing and wage decisions. For each system, we derive the equilibrium joining behaviors of riders and drivers and the platform's optimal pricing and wage decisions.

Regarding the first research question, we show that directly enhancing rider safety confidence has an *increasing* marginal effect on the platform's profit, while doing so on the driver side has a *diminishing* marginal effect. Consequently, with a limited budget, a platform should enhance rider safety confidence as much as possible while keeping driver safety confidence at a certain threshold. This finding is consistent with the current practice implemented by DiDi. Recently, DiDi put great efforts into improving rider safety, installing a one-button emergency call feature in their app, introducing in-trip audio recording and educating drivers (EJinsight 2018, Dai 2018); Uber has also installed an in-app emergency button in its safety toolkit (Uber 2019). Both DiDi and Uber also conduct background checks and screen their drivers to improve rider safety (Shen 2018, Bell 2018). Interestingly, we show that enhancing the safety confidence level of female drivers does not necessarily lead to more female riders joining the pooling system because of the pricing behavior of the platform. With a higher safety confidence level, more female drivers would participate in the ride-hailing service, which would entice more female riders to join the platform due to the positive cross-side externality. Hence, the platform is less worried about maintaining the size of its female driver workforce by attracting enough female riders to join the system and thus has an incentive to increase its price. The negative effect of a higher price can surpass the positive effect of an enlarged female driver pool, resulting in the loss of some female riders.

Regarding the second question, we find that female drivers should not be fully granted the flexibility to choose between the pooling subsystem and the female-only subsystem because of the significant imbalance between female riders and drivers. Offering female riders the flexibility to choose between the subsystems not only helps relieve the demand burden for the female-only subsystem but also eases the dilution of demand for male drivers. However, granting female drivers the flexibility to choose can work in the reverse way: some female drivers may be attracted away by the pooling subsystem, which would worsen the shortage of female drivers and jeopardize the operations of the female-only subsystem.

To address the last question, we investigate the effects of switching from a pooling system to a hybrid system from three aspects: accessibility of safety-concerned female users, other users' utility and platform profit. We find that in general, safety-unconcerned (male and some female) users are unhappy with a hybrid system. First, with the loss of the pooling effect, drivers may face diluted demand and riders may face longer waiting times. Second, in the pooling system, safety-unconcerned female riders gain "privilege" in the queue over safety-concerned female riders, but in the female-only subsystem, they are equal. However, the hybrid system could achieve a win-win result on the other two aspects, namely, increasing the accessibility for safety-concerned female riders and drivers and improving platform's profitability. The condition for such a win-win result is characterized by the safety confidence levels of female drivers and riders. When safety confidence levels are low, the hybrid system is preferred. Noting that females exhibit different safety confidence levels in different countries, our analytical results provide a plausible explanation for why different ride-hailing systems are adopted in different countries. For example, in countries where personal safety is a serious issue for females (such as India, Pakistan and Saudi Arabia (Narayan 2018)), the hybrid system with a female-only subsystem or a gender-dedicated system (such as She Cab) should be adopted.

The remainder of this chapter is organized as follows. We review the related literature in Section 2.2. The model formulation is presented in Section 2.3. We conduct the game-theoretical analysis of the pooling and hybrid systems, respectively, in Sections 2.4 and 2.5. In particular, we derive the associated equilibrium outcomes. We then compare the two systems and conduct the related discussions in Section 2.6. Section 2.7 concludes this chapter. All of the proofs are relegated to Appendix A.1. We discuss the detailed equilibrium derivation and analysis associated with the pooling system and the hybrid system in online Appendices A.2 and A.3, respectively.

2.2 Literature Review

Our work belongs to the emerging research stream that studies on-demand ride-hailing platforms in a two-sided market. For research on two-sided markets, see Armstrong (2006), Rochet and Tirole (2006), Weyl (2010), Hagiu (2014), Hagiu and Wright (2015), Eisenmann et al. (2006) and the references therein. The literature on ride-hailing platforms has investigated issues such as surge pricing (e.g., Banerjee et al. 2015 and Cachon et al. 2017), optimal commission contracts (e.g., Hu and Zhou 2017 and Bai et al. 2019), pricing with cost-sharing consideration (e.g., Jacob and Roet-Green 2017), driver and rider role exchanges (i.e., the roles of riders and drivers are interchangeable, e.g., Gao et al. (2018)), competition between platforms (e.g., Cohen and Zhang 2019) and matching between different types of users (see e.g., Caldentey et al. 2009, Baccara et al. 2018 and Hu and Zhou 2018). For other work, we refer interested readers to the review work of Benjaafar and Hu (2019) and Hu (2019) and the references therein. In this stream of work, ours is closely related to Taylor (2018) and Benjaafar et al. (2019). Taylor (2018) investigates how rider delay sensitivity and driver self-regulation affect a platform's optimal pricing and wage decisions. Benjaafar et al. (2019) investigate the effect of the labor pool size on labor welfare. In Our work, we also consider these price and wage decision issues. To the best of our knowledge, ours is the first research to investigate the gender-based safety concerns associated with the operation of ride-hailing platforms. We examine how to configure a ride-hailing operational system to mitigate such issues. Our work contributes to the literature on ride-hailing platforms by investigating their operations from the angle of user safety instead of pure profit maximization.

Kostami et al. (2017) consider users' gender preferences in a club setting and study the club's profit-maximizing price and capacity allocation decision problem. Our work studies the users' gender preference in a ride-hailing platform setting and investigates the platform's optimal price and wage decisions and the joining and participating behaviors of the riders and drivers. Both Kostami et al. (2017) and Our work consider the following two issues: the externality (excluding congestion effect) brought by one gender type users on the other and using dedicated capacities to separate these two gender types of users to mitigate such externality. However, Our work differs from Kostami et al. (2017) in the following two aspects. One, such externality only occurs within the demand-side users in Kostami et al. (2017) while it occurs between the cross-side users in our context. Specifically, in our work, service quality received by the demand-side rider is affected by the composition of the supply-side drivers and vice versa. Two, the capacity in Kostami et al. (2017) is exogenously given and can be arbitrarily allocated between the two gender types of users. However, in Our work, the capacity is endogenously determined by the drivers' participating behaviors. Moreover, the supply of the two gender types of drivers is capacitated. In particular, the female-driver pool size is very limited, a key issue faced by on-demand ride-hailing platforms. Therefore, in contrast to the two dedicated systems proposed in Kostami et al. (2017), we instead consider a hybrid system that grants safety-unconcerned female riders flexibility to mitigate the demand-supply imbalance issue. Also, note that for the emerging ride-hailing service systems, managers care about not only the platform's profitability but also the growth of its market share. Hence, we investigate both the platform's profit and the access by female users while Kostami et al. (2017) only consider the profit maximization.

Our work is also related to the literature studying the product line design issue when customers are heterogeneous. Chen (2001) considers a manufacturer's product line design when the market contains both green and ordinary customers. The manufacturer needs to determine the optimal product types and qualities for each type. In a recent work, Bellos et al. (2017) study how providing car sharing affects a car manufacturer's driving performance design under a setting that customers have different valuations of driving performance. The most closely related work is Netessine and Taylor (2007) who examine how a manufacturer's product line design is affected by the observed customer type information and production technology. In Netessine and Taylor (2007), the manufacturer faces a key trade-off between exploiting the economics of scale (by providing a composite product for all types of customers) and extracting the higher profit margin (by providing different quality products to customers who have different valuations). In our study, the pooling system can be regarded as the one providing a composite product while the hybrid system with the female-only option as the one providing two different quality products. The pooling effect of our service capacity in spirit is similar to the scale economies of the production cost. However, our service system design problem exhibits several unique key features in comparison to those studies on product line design. First, in our service system, system capacity is endogenously determined by the self-regulated drivers' participating decision. Thus, service capacity is controlled indirectly by the platform through wages in a decentralized way. Nevertheless, the capacity in the aforementioned studies as well as the other classical product line design literature is determined directly by the firm in a centralized way. Second, in the product line design problem, the firm faces no constraint in its capacity decision whereas in our setting, the capacity (i.e., the number of registered drivers) is limited and one key issue is the shortage of female drivers. Third, the capacity cost in the product line design is a function of production volume but here it is also affected by the demand composition, that is, the composition of riders. More female riders joining the ride-hailing platform makes it easier to attract the participation of female drivers and vice versa. Fourth, although the key tradeoff in the product line design problem, namely exploiting the economics of scale by offering a composite product versus extracting the higher profit margin by offering customized products, still works in our service setting, it is mingled with other driving forces. For example, in the hybrid system, the female-only subsystem can be seen as the one offering a "high quality" product. However, in sharp contrast to the product line design problem where high-quality products can be sold in higher prices, the price in this high-quality female-only subsystem could be lower than the one for the pooling subsystem, due to a potential longer waiting time (congestion) in this female-only system.

In our work, we consider the subsystem-based pricing and wage strategy. We note that there exist some studies investigating the price discrimination issue; see, e.g., Choudhary et al. (2015), Ferrell et al. (2018), Trégouët (2015), Horstmann and Krämer (2013), Jayaswal et al. (2011). We refer interested readers to the review work of Mitra and Capella (1997) and Chen (2009) for other related studies in this stream of research.

2.3 Model Setup

Consider an on-demand ride-hailing platform that sets a price rate p (measured in terms of price per service) and a wage rate w (measured in terms of wage per service) to coordinate price- and waiting-time-sensitive riders (i.e., demand) and earning-sensitive independent drivers (i.e., supply) of both genders, female (labeled f) and male (labeled m).

Platform's System Configuration. There are two potential operational systems that a platform can adopt: a pooling system and a hybrid system (consisting of a pooling subsystem and a female-only subsystem); see Figure 2.1 for an illustration. In a pooling system, riders and drivers are matched without considering gender. Hence, the safety concerns of female riders and drivers are present when they are matched with male counterparts. As the pooling system is operated as a single legal entity, gender-based pricing and wages are normally deemed discriminatory and may be illegal. Thus, it suffices to consider a *gender-neutral pricing and wage policy.* In a hybrid system, female users have an option to join a female-only subsystem while male users can only join the pooling subsystem. In the female-only subsystem, female users' safety concerns are absent. In a hybrid system, the platform could operate the two subsystems as separate entities. That is, the platform could set *subsystem-based prices and wages.* (We note that due to legal issues, prices and wages in the pooling subsystem and the female-only subsystem may need to be the same in some areas/countries. Our model can easily be extended to examine such a case with an additional constraint requiring that wages and prices in the subsystems are equal.)

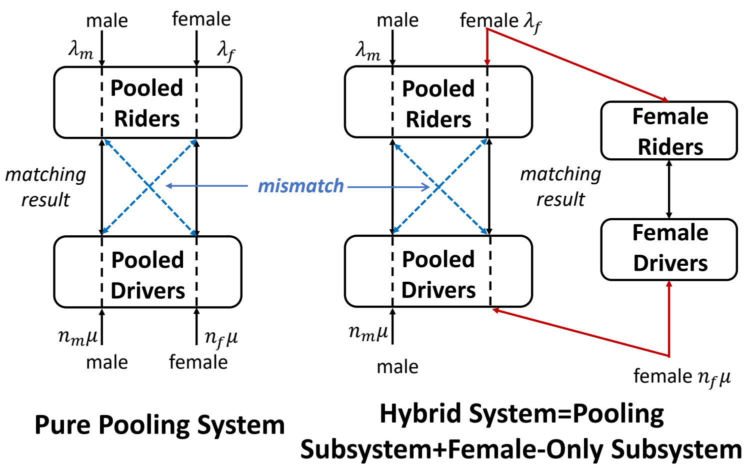


Figure 2.1: Two Operational Systems for the Platform

Rider Characteristics. Potential female and male riders may request on-demand ride-hailing service according to independent Poisson processes with rates Λ_f and Λ_m, respectively. The total potential arrival rate $\Lambda = \Lambda_m + \Lambda_f$. Male riders are homogeneous and have less safety concerns about driver gender than female riders. Without loss of generality, we scale male safety concerns to zero (i.e., no safety concern). Female riders are heterogeneous regarding safety concerns about driver gender. Specifically, δ_R proportion of them have no safety concern regarding driver gender (IFC 2018); that is, they are *safety-unconcerned female* (labelled f_ϕ) riders with a Poisson arrival rate $\Lambda_{f_\phi} = \delta_R \Lambda_f$. The remaining $(1 - \delta_R)$ proportion is concerned about safety and feels uncomfortable when matched with a male driver; that is, they are *safety-concerned female* (labelled f_c) riders with arrival rate $\Lambda_{f_c} = (1 - \delta_R)\Lambda_f$. The total potential arrival rate of *safety-unconcerned riders* consists of male and safety-unconcerned female riders, and we denote it as Λ_ϕ $(= \Lambda_m + \Lambda_{f_\phi})$.

Note that given price p and anticipating waiting cost $c \cdot W$, in which c is unit-time waiting cost and W is expected waiting time, some riders may choose not to request the service. We denote the effective joining rates of female and male riders as λ_f and λ_m, respectively. Then, $\lambda_i \leq \Lambda_i$, $i = f, m$. Furthermore, let λ_{f_c} and λ_{f_ϕ} denote the effective joining rates of safety-concerned and safety-unconcerned females, respectively. Thus, the effective total joining rate of female riders is $\lambda_f = \lambda_{f_c} + \lambda_{f_\phi}$.

To simplify our exposition, we assume that both male and female riders receive the same base reward R from the ride-hailing service. However, when matched with male drivers, safety-concerned female riders receive a lower reward αR, in which $\alpha \in (0, 1)$. The parameter α represents the *safety-concerned female rider's safety confidence level* regarding ride-hailing service offered by male drivers. A larger α indicates a higher degree of safety confidence. This reward discount associated with a gender mismatch may result from anxiety and worries during the trip.

Driver Characteristics. There are N_f female and N_m male registered drivers, each of whom can serve a rider according to an exponential distribution with service rate μ (e.g., the number of riders served per unit of time). Male drivers outnumber female drivers, i.e., $N_f < N_m$, which is consistent with actuality

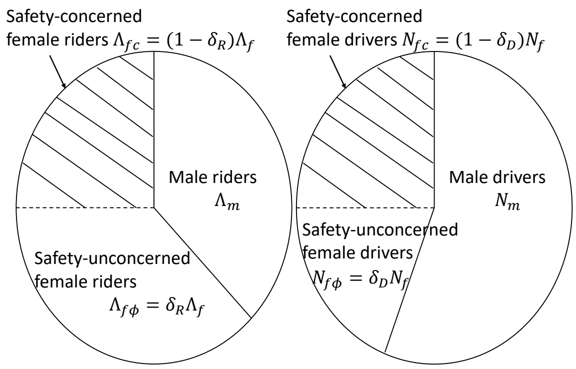


Figure 2.2: The Composition of Riders and Drivers

(SheRides 2016, Borak 2018, ChinaNews 2018, AsiaSociety 2017). Male drivers are homogeneous and less safety-concerned about rider gender than female drivers. Again, we scale males' safety concerns to zero. Female drivers are heterogeneous regarding safety concerns about rider gender. Among the N_f female drivers, δ_D proportion of them do not have safety concerns about rider gender while the remaining $(1-\delta_D)$ proportion have such safety concerns. That is, the number of safety-concerned and unconcerned female drivers are $N_{f_c} = (1-\delta_D)N_f$ and $N_{f_\phi} = \delta_D N_f$, respectively. Hence, the total number of *safety-unconcerned drivers*, including male and safety-unconcerned female drivers, denoted by N_ϕ can be expressed as $N_\phi = N_m + N_{f_\phi}$. For ease of reference, we summarize the composition of rider and driver types in Figure 2.2.

All registered drivers, regardless of gender, have the same reservation price (or opportunity cost) r. They participate and serve if the earning rate is no less than r.[2.1] Denote n_{f_c}, n_{f_ϕ} and n_m as the effective participating number of safety-concerned female drivers, safety-unconcerned female drivers and male drivers, respectively. Then, the effective total participating number of female drivers is $n_f = n_{f_c} + n_{f_\phi}$. Like riders, safety-concerned female drivers discount the earning rate by a factor $\beta \in (0,1)$ when they are matched with male riders. Parameter β represents the *safety-concerned female driver's safety confidence level*, and a

[2.1] Our model can be easily extended to a case in which drivers have heterogeneous reservation prices. Unfortunately, the analysis becomes intractable under such a setting.

larger β indicates a higher degree of safety confidence.

Waiting Time. For tractability, we model the ride-hailing service operation as an M/M/1 queueing system. A similar assumption has been adopted in the literature; see, e.g., Benjaafar et al. (2019). Given an effective rider joining rate, λ, and an effective driver service rate, $n\mu$ (that is, n effective drivers), the expected waiting time in the system $W(\lambda, n)$ is

$$W(\lambda, n) = \begin{cases} \frac{1}{n\mu - \lambda}, & \text{if } \lambda < n\mu \\ +\infty, & \text{otherwise.} \end{cases} \tag{2.1}$$

Sequence of Events. For both the pooling and hybrid systems, the sequence of events is as follows. First, the platform decides the price(s) p and wage(s) w (recall that the platform can set subsystem-based prices and wages in a hybrid system) to maximize its profit

$$\Pi = \lambda(p - w), \tag{2.2}$$

where λ is the effective joining rate of riders. Upon observing the price(s) and wage(s), riders and drivers of both genders respectively decide whether to participate based on their own utility functions. Note that the effective joining rates of different types of riders and the effective participating number of different drivers must be jointly solved through an equilibrium analysis of each player's behavior because their payoffs are determined by their joint behavior.

As the ride-hailing system is often *supply-constrained* (Banerjee et al. 2015, Taylor 2018), $\frac{\Lambda}{N\mu} > 1$ is required, where $N := N_f + N_m$. Also, to reflect the reality that on ride-hailing platforms, female riders account for a large proportion of demand but female drivers account for a only small proportion of supply (SheRides 2016, Borak 2018, ChinaNews 2018), we assume that $\frac{\Lambda_f}{N_f\mu} > 1$. This assumption assures that even when all female drivers participate, they cannot serve all female riders in a steady state. Throughout this chapter, we restrict our attention to the parameter range within which a platform's expected profit under optimal pricing and wage decisions is strictly positive (Taylor 2018). Table 2.1 summarizes the key notation used.

Table 2.1: A List of Key Notation

f, m, f_c, f_ϕ	Female: f; male: m; safety-concerned/-unconcerned female: f_c / f_ϕ
ϕ	Safety-unconcerned users, consisting of males and safety-unconcerned females
$\Lambda_i, i \in \{f_c, f_\phi, f, m\}$	Potential arrival rate of type-i riders with $\Lambda_f = \Lambda_{f_c} + \Lambda_{f_\phi}, \Lambda = \Lambda_m + \Lambda_f$
Λ_ϕ	Potential arrival rate of safety-unconcerned riders with $\Lambda_\phi = \Lambda_m + \Lambda_{f_\phi}$
δ_R	Fraction of safety-unconcerned female riders, $0 < \delta_R = \frac{\Lambda_{f_\phi}}{\Lambda_f} < 1$
$\lambda_i, i \in \{f_c, f_\phi, f, m\}$	Effective joining rate of type-i riders
$N_i, i \in \{f_c, f_\phi, f, m\}$	Number of registered type-i drivers with $N_f = N_{f_c} + N_{f_\phi}, N = N_m + N_f$
N_ϕ	Number of registered safety-unconcerned drivers with $N_\phi = N_m + N_{f_\phi}$
δ_D	Fraction of safety-unconcerned female drivers, $0 < \delta_D = \frac{N_{f_\phi}}{N_f} < 1$
$n_i, i \in \{f_c, f_\phi, f, m, \phi\}$	Effective participating number of type-i drivers
μ	Service rate
r	Reservation price
R	Base service reward per ride
c	Unit-time waiting cost
α	Female rider's safety confidence level, $0 < \alpha < 1$
β	Female driver's safety confidence level, $0 < \beta < 1$
p	Price per service
w	Wage per service

2.4 Analysis of the Pooling System

In this section, we analyze the system performance associated with a gender-neutral pooling system via backward induction. Below, we first characterize the utilities of riders and drivers, and then we derive their equilibrium joining/participating behavior. Based on that, we derive the platform's optimal price and wage decisions.

2.4.1 Users' Utility Functions

In a pooling system, safety-unconcerned female riders behave the same as male riders. Given riders' effective joining rate $\vec{\boldsymbol{\lambda}} = (\lambda_{f_c} + \lambda_{f_\phi} + \lambda_m)$ and drivers' effective participating number $\vec{\boldsymbol{n}} = (n_{f_c} + n_{f_\phi} + n_m)$, the utility of a male rider or a safety-unconcerned female rider joining the ride-hailing service can be written as

$$U_m(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}) = U_{f_\phi}(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}) = R - p - cW(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}), \tag{2.3}$$

where R is service reward, p is price and $cW(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}})$ is the encountered total waiting cost.

A safety-concerned female rider's reward is dependent on being paired with a male or female driver. Given the number of female and male drivers, n_f and n_m, the probability of a safety-concerned female rider being matched with a male driver is $n_m/(n_f + n_m)$, in which situation her reward is discounted by α. Thus, the utility of a safety-concerned female rider can be derived as

$$\begin{aligned} U_{f_c}(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}) &= \frac{n_m}{n_m + n_{f_c} + n_{f_\phi}}\alpha R + \frac{n_{f_c} + n_{f_\phi}}{n_m + n_{f_c} + n_{f_\phi}} R - p - cW(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}) \\ &= \frac{\alpha n_m + n_f}{n_m + n_f} R - p - cW(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}). \end{aligned} \tag{2.4}$$

Similarly, we can derive the net utilities of male drivers, safety-unconcerned female drivers and safety-concerned female drivers participating in the ride-hailing service as follows:

$$S_m(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}) = S_{f_\phi}(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}) = \frac{\lambda_m + \lambda_{f_c} + \lambda_{f_\phi}}{n_{f_c} + n_{f_\phi} + n_m} w - r = \frac{\lambda_m + \lambda_f}{n_f + n_m} w - r, \tag{2.5}$$

$$\begin{aligned} S_{f_c}(\vec{\boldsymbol{\lambda}}, \vec{\boldsymbol{n}}) &= \frac{\lambda_m}{n_{f_c} + n_{f_\phi} + n_m}\beta w + \frac{\lambda_{f_c} + \lambda_{f_\phi}}{n_{f_c} + n_{f_\phi} + n_m} w - r \\ &= \frac{\beta\lambda_m + \lambda_f}{n_f + n_m} w - r. \end{aligned} \tag{2.6}$$

For notational convenience, we define

$$\begin{aligned} d_i(n_{f_c}, n_{f_\phi}, n_m) &:= \frac{\lambda_m + \lambda_f}{n_{f_c} + n_{f_\phi} + n_m} (i = m, f_\phi), \\ d_{f_c}(n_{f_c}, n_{f_\phi}, n_m) &:= \frac{\beta\lambda_m + \lambda_f}{n_{f_c} + n_{f_\phi} + n_m}. \end{aligned} \tag{2.7}$$

Then, $d_i(n_{f_c}, n_{f_\phi}, n_m)$, $i = m, f_\phi$ can be regarded as the *demand rate* of a safety-unconcerned driver, including male drivers and safety-unconcerned female drivers, while $d_{f_c}(n_{f_c}, n_{f_\phi}, n_m)$ is the *safety-concern-adjusted demand rate* of a safety-concerned female driver.

2.4.2 Equilibrium Analysis and Optimal Price and Wage Decisions

After obtaining their utility functions, we can examine the equilibrium joining/participating behaviors of riders/drivers of both genders for the given price and wage. This step is very tedious because on both the demand and supply sides, we have safety-concerned and safety-unconcerned users, and their payoffs are jointly determined by other same- and cross-side players' decisions. Here, we use a fact to facilitate our equilibrium analysis: if some safety-concerned riders join the system, then safety-unconcerned riders must "all join". The same rationale holds for drivers. In other words, safety-unconcerned users gain some "privilege" over same-type safety-concerned users.[2.2]

After obtaining the equilibrium joining behavior of users, we can derive the platform's optimal pricing and wage decisions p^* and w^* by maximizing the platform's profit stated in (2.2). For the sake of brevity and space, we refer interested readers to Appendix A.2 for the detailed equilibrium analysis and Table A.2.1 for the optimal price p^*, wage w^* and platform profit Π^* and the corresponding equilibrium joining/participating behaviors of riders/drivers. (Note that throughout our analyses, we only consider the equilibrium outcomes in which safety-concerned female riders join the system at a non-zero rate. While deriving the equilibrium joining/participating behaviors of riders/drivers, one can easily find some equilibria in which all safety-concerned female riders balk in a pooling/hybrid system. As such equilibrium outcomes deviate from our research motivation, they are not our focus, and we omit such trivial cases.) We present users' equilibrium joining and participating behaviors associated with the platform's optimal (profit-maximizing) price and wage in the following proposition.

Proposition 2.1. *In a pooling system, under the optimal (profit-maximizing) price and wage, the equilibrium joining rates of riders and the number of participating drivers are as follows.*

[2.2] Changing to a hybrid system could cause safety-unconcerned users to lose such privilege and thus make them unhappy with the hybrid system.

1. ***(Demand).*** *All safety-unconcerned riders, i.e., male and safety-unconcerned female riders, join the system. However, only a fraction of safety-concerned female riders join the system. That is,* $\lambda_i^* = \Lambda_i$, $i = m, f_\phi$, *and* $\lambda_{f_c}^* < \Lambda_{f_c}$.

2. ***(Supply).*** *All registered safety-unconcerned drivers, i.e., male and safety-unconcerned female drivers, participate in the system. That is,* $n_i^* = N_i$, $i = m, f_\phi$. *Regarding safety-concerned female drivers, we have:*

   (a) if the number of registered safety-unconcerned drivers N_ϕ *is sufficiently large such that* $\mu N_\phi > \Lambda_\phi$ *and safety-concerned female drivers' safety confidence level* β *is low (i.e.,* $\beta < \widehat{\beta}(\alpha)$*, where* $\widehat{\beta}(\alpha)$ *is characterized by* (A.24) *stated in Appendix A.2), then all safety-concerned female drivers balk from the system, i.e.,* $n_{f_c}^* = 0$*;*

   (b) otherwise, all safety-concerned female drivers participate in the service, i.e., $n_{f_c}^* = N_{f_c}$.

Proposition 2.1 indicates that all safety-unconcerned users, including male and safety-unconcerned female riders and drivers, always join the system as they have no safety concerns. Also, safety-concerned female riders join the system with a certain probability; in contrast, safety-concerned female drivers may "all join" or "never join" the system, a result hinging upon their safety confidence level β and the labor pool size of safety-unconcerned drivers N_ϕ. Such "all join" or "never join" behavior is due to the *positive participating driver externality*, that is, "the equilibrium demand allocated to a driver strictly increases with the number of participating drivers" (Taylor 2018). Thus, under the optimal price and wage, either all registered drivers work or only safety-unconcerned drivers work. Therefore, when making staffing decisions, the platform has to choose between two options: setting a relatively high wage to attract all drivers or setting a relatively low wage to attract only safety-unconcerned drivers. Proposition 2.1 implies that when the labor size of safety-unconcerned drivers is large enough (so that $N_\phi \mu > \Lambda_\phi$ holds), the latter dominates the former as the profit gained from serving more safety-concerned riders cannot surpass the loss encountered due to higher payments to drivers. Proposition 2.1 also implies that the platform may use wages as a tool to screen drivers who are concerned about safety.

2.4.3 Sensitivity Analysis

Here, we first conduct some numerical studies to examine the effect of unit-time waiting cost c on system performance measures. One may expect that when riders are more delay-sensitive (with a higher unit waiting cost), the platform would have more incentive to set a high wage to attract all of the female drivers to participate. Interestingly, Figure 2.3 indicates the opposite: the larger the unit waiting cost, c, the larger the threshold, $\widehat{\beta}(\alpha)$, which indicates that the platform has less incentive to attract safety-concerned female drivers to work. As c increases, less safety-concerned female riders would join the system, increasing the likelihood of a mismatch for safety-concerned female drivers. The platform thus has to increase its wage to maintain the same size pool of safety-concerned female drivers.

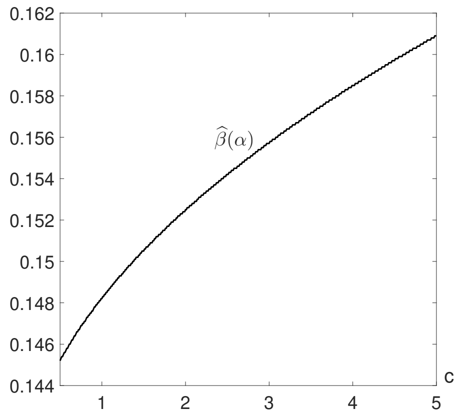


Figure 2.3: The Effect of Unit-time Waiting Cost c on the Threshold $\widehat{\beta}(\alpha)$: $N_m = 1100$, $\Lambda_m = 1000$, $N_f = 300$, $\Lambda_f = 1500$, $\mu = 1.5$, $r = 2$, $R = 10$, $\delta_R = 40\%$, $\delta_D = 50\%$ and $\alpha = 0.9$

We next investigate the effect of α and β, the respective safety confidence levels of female riders and drivers, on system performance. We obtain the following analytical results.

Proposition 2.2. *In a pooling system, when in equilibrium both safety-concerned* *female riders and drivers join at a non-zero rate, which requires that either the number of safety-unconcerned drivers is sufficiently small ($\mu N_\phi \leq \Lambda_\phi$) or that safety-concerned female drivers' safety confidence level is sufficiently high ($\beta > \widehat{\beta}(\alpha)$),*

1. *the optimal price p^* increases while the optimal wage w^* decreases in both α and β. Moreover, p^* is decreasing while w^* is increasing in the driver's reservation price r.*

2. *the effective joining rate of safety-concerned female riders $\lambda^*_{f_c}$ is increasing in α and r but decreasing in β.*

3. *the participating number of safety-concerned female drivers is always N_{f_c}, regardless of the magnitude of α and β.*

4. *the platform's profit Π^* is increasing and convex in α and increasing and concave in β.*

Proposition 2.2 shows that the platform is able to charge a higher price and offer a lower wage when safety-concerned female users' safety confidence levels become higher. This is because a higher safety confidence level makes the safety-concerned female user more likely to join the system. Proposition 2.2 also shows that when the reservation price (or opportunity cost) of a driver r becomes larger, the platform needs to increase its wage and lower its price. It is intuitive that wage increases with a driver's reservation price r but it is counter-intuitive that the price charged to riders decreases with a higher r. One may believe that to compensate for the higher wage, the platform should charge a higher price. However, note that the platform's profit is comprised of two parts: profit margin per ride and number of riders served. Although a lower price reduces the platform's profit margin, it effectively entices many more riders, especially safety-concerned female riders, to join the system, making the platform better off. Therefore, when a government helps provide more job opportunities to drivers (which makes the reservation price r larger), it benefits not only drivers but also riders.

Proposition 2.2 indicates that working on the existing pooling system by improving female users' safety confidence can result in a win-win outcome in terms of increasing accessibility for safety-concerned female users and platform profitability. Indeed, after the tragic incidents, ride-hailing platforms have put much effort into boosting the safety confidence of riders and drivers. For example, both Uber and DiDi now provide users with a one-button emergency call feature in their apps (Uber 2019, EJinsight 2018). Didi also requires in-trip audio recording (Dai 2018). Such actions can enhance the safety confidence levels of both drivers and riders. In addition, DiDi took other actions to enhance female riders' safety confidence, such as educating drivers and conducting driver background checks (DidiPublic 2019, Shen 2018). It is reported that over 300,000 drivers were removed by DiDi due to failing the basic background check (Liao 2019). A close look at Proposition 2.2 further reveals the following implications regarding improving safety in a pooling system.

One, the platform should enhance safety-concerned female riders' safety confidence level α as much as possible. Proposition 2.2 implies that enhancing the safety confidence level α improves safety-concerned female riders' accessibility and the platform's profitability. Proposition 2.2 also shows that there is an increasing marginal improvement as α increases (i.e., convexly increasing). That is, the platform's profit becomes more elastic as α increases. This implies that the platform should put as much effort as possible into improving female riders' safety confidence level α as its improvement benefits both safety-concerned female users and the platform.

Two, the platform should ensure that safety-concerned female drivers' safety confidence level β is sufficiently high. Proposition 2.2 indicates that to participate in the pooling system's ride-hailing service, safety-concerned female drivers need to have a high enough safety confidence level β, in particular, $\beta > \widehat{\beta}(\alpha)$. This requires that the platform take measures to ensure that driver-side safety confidence β reaches at least a minimum threshold $\widehat{\beta}(\alpha)$. In addition, Proposition 2.2 shows that although enhancing the safety confidence level β improves platform profitability, there is diminishing marginal improvement as β increases (i.e., concavely increasing). This implies that if the cost function for safety improvement efforts is linear, there must exist an optimal safety confidence level β that yields the largest profit. Recall that there exists a positive externality among safety-concerned female drivers: the more drivers that participate, the higher payoff each receives because a larger service capacity attracts more demand. Such positive externality greatly relieves the need to improve driver-side safety confidence to attract female drivers. In contrast, no such positive externality exists among female riders due to the imbalance between supply and demand: when drivers all participate to work, a marginal increase in demand only brings negative externality to riders due to longer waiting times. Therefore, improving rider-side safety confidence is more important to attract safety-concerned female riders to join the system.

Three, increasing female drivers' safety confidence could result in fewer joining female riders. Interestingly, Proposition 2.2 shows that enhancing safety-concerned female drivers' safety confidence β could result in fewer safety-concerned female riders joining the system. This is counter-intuitive as one may believe that with greater driver-side safety, more female drivers will join, which would attract more female riders to join. This intuition, however, is distorted by the platform's pricing behavior. When β is low, the platform has to set a sufficiently low price to entice enough safety-concerned female riders to join because keeping sufficient female riders helps ensure the participation of safety-concerned female drivers. Once safety-concerned female drivers' safety confidence level β is high enough ($\beta > \widehat{\beta}(\alpha)$), female drivers always participate in the service. Without worrying about losing female drivers, the platform has an incentive to increase its price, which can result in the loss of some safety-concerned female riders.

These sensitivity analysis results imply that in the face of a limited budget for safety improvement, the efforts the platform puts into improving the two safety confidence levels α and β should be unevenly distributed. The platform should first take actions to improve both α and β. Once the driver-side safety confidence level β achieves a certain threshold, the platform should put effort into further improving the rider-side safety confidence level α. This is consistent with DiDi's practice. According to our contact at the DiDi Safety Department, DiDi indeed puts much more effort into making riders gain confidence in its system relative to the safety measures they have implemented on the driver side.

2.5 Analysis of the Hybrid System

In this section, we analyze the hybrid system. As previously mentioned, a driving force for this system is the flexibility granted to female riders to choose between a pooling subsystem and a female-only subsystem. This flexibility helps mitigate the loss of the pooling effect. A natural question then arises: should female drivers also be granted such flexibility? To examine this system design problem, we consider a control policy Q under which the platform sets an upper limit (i.e., a quota) Q on the number of female drivers who can join the pooling subsystem. For any given control policy Q, we derive the equilibrium joining and participating behaviors of riders and drivers, respectively. Based on that, we then derive the platform's optimal pricing and wage decisions. By checking the optimal control policy Q, we can answer whether flexibility should be granted to female drivers and to what extent.

Recall from Figure 2.1 that in a hybrid system, a female user may be matched with a male user in the pooling subsystem (labelled "M", indicating that the pooling subsystem is *mixed* with male and female users). In contrast, only females users can join the female-only subsystem (labelled "F" to indicate *female only*). Hereafter, for notational convenience, we use subscripts M and F to denote the performance measures associated with the pooling subsystem and female-only subsystem, respectively. Thus, female users' safety concerns are absent in the female-only subsystem but may be present in the pooling subsystem. As a result, in a female-only subsystem with an effective rider joining rate $\vec{\boldsymbol{\lambda}}_F = (\lambda_{f_c,F} + \lambda_{f_\phi,F})$ (from both safety-concerned and -unconcerned female riders) and an effective driver participating number $\vec{\boldsymbol{n}}_F = (n_{f_c,F} + n_{f_\phi,F})$ (from both safety-concerned and -unconcerned female drivers), the joining utilities of the safety-concerned and -unconcerned female riders are the same and can be written as

$$U_{f_c,F}(\vec{\boldsymbol{\lambda}}_F, \vec{\boldsymbol{n}}_F) = U_{f_\phi,F}(\vec{\boldsymbol{\lambda}}_F, \vec{\boldsymbol{n}}_F) = R - p_F - cW(\vec{\boldsymbol{\lambda}}_F, \vec{\boldsymbol{n}}_F), \tag{2.8}$$

where p_F is the price charged in the female-only subsystem. Similarly, the net utilities of safety-concerned and -unconcerned female drivers participating in the service are also the same and can be written as

$$S_{f_c,F}(\vec{\boldsymbol{\lambda}}_F, \vec{\boldsymbol{n}}_F) = S_{f_\phi,F}(\vec{\boldsymbol{\lambda}}_F, \vec{\boldsymbol{n}}_F) = \frac{\lambda_{f_c,F} + \lambda_{f_\phi,F}}{n_{f_c,F} + n_{f_\phi,F}} \cdot w_F - r, \tag{2.9}$$

where w_F is the wage per service and $\frac{\lambda_{f_c,F}+\lambda_{f_\phi,F}}{n_{f_c,F}+n_{f_\phi,F}}$ is the demand rate of a female driver in the female-only subsystem.

In a pooling subsystem with an effective rider joining rate $\vec{\boldsymbol{\lambda}}_M = (\lambda_m + \lambda_{f_\phi,M} + \lambda_{f_c,M})$ and an effective driver participating number $\vec{\boldsymbol{n}}_M = (n_m + n_{f_\phi,M} + n_{f_c,M})$ (including male users and safety-unconcerned and -concerned female users), each safety-unconcerned female rider receives the same utility from joining the system as that of a male rider, which can be written as

$$U_{f_\phi,M}(\vec{\boldsymbol{\lambda}}_M, \vec{\boldsymbol{n}}_M) = U_{m,M}(\vec{\boldsymbol{\lambda}}_M, \vec{\boldsymbol{n}}_M) = R - p_M - cW(\vec{\boldsymbol{\lambda}}_M, \vec{\boldsymbol{n}}_M), \tag{2.10}$$

where p_M is the price charged in the pooling subsystem. As to the safety-concerned female rider, her utility from joining the system can be derived as

$$U_{f_c,M}(\vec{\boldsymbol{\lambda}}_M, \vec{\boldsymbol{n}}_M) = \frac{n_m\alpha + n_{f_c,M} + n_{f_\phi,M}}{n_m + n_{f_c,M} + n_{f_\phi,M}} R - p_M - cW(\vec{\boldsymbol{\lambda}}_M, \vec{\boldsymbol{n}}_M), \tag{2.11}$$

and note that $n_m/(n_m + n_{f_c,M} + n_{f_\phi,M})$ is the probability that she is matched with a male driver. Similarly, we can derive the utilities of drivers. Again, the safety-unconcerned female driver receives the same net utility from participating in the pooling subsystem as that of a male driver, that is,

$$S_{f_\phi,M}(\vec{\boldsymbol{\lambda}}_M, \vec{\boldsymbol{n}}_M) = S_m(\vec{\boldsymbol{\lambda}}_M, \vec{\boldsymbol{n}}_M) = \frac{\lambda_m + \lambda_{f_c,M} + \lambda_{f_\phi,M}}{n_{f_c,M} + n_{f_\phi,M} + n_m} w_M - r, \tag{2.12}$$

with a demand rate of $\frac{\lambda_{f_c,M}+\lambda_{f_\phi,M}+\lambda_m}{n_{f_\phi,M}+n_{f_c,M}+n_m}$. As to the safety-concerned female driver, her net utility from participating in the pooling subsystem can be derived as

$$S_{f_c,M}(\vec{\boldsymbol{\lambda}}_M, \vec{\boldsymbol{n}}_M) = \frac{\lambda_m\beta + \lambda_{f_c,M} + \lambda_{f_\phi,M}}{n_{f_c,M} + n_{f_\phi,M} + n_m} w_M - r, \tag{2.13}$$

with demands rates of $\frac{\lambda_m}{n_{f_c,M}+n_{f_\phi,M}+n_m}$ (from male riders) and $\frac{\lambda_{f_c,M}+\lambda_{f_\phi,M}}{n_{f_c,M}+n_{f_\phi,M}+n_m}$ (from female riders).

Platform's Control Policy Q. Recall that the motivation of moving away from a pooling system to a hybrid system is to eliminate/lessen female users' safety concerns when matched with male counterparts. Also recall that the number of female riders is far more than that of female drivers (SheRides 2016) and not all female users are safety-concerned (IFC 2018). Safety-unconcerned female drivers may have no incentives to join the female-only subsystem. Hence, the flexibility granted to female riders to choose between two subsystems helps alleviate the supply-demand imbalance in the female-only subsystem. However, offering such flexibility to female drivers may act in the reverse. Should female drivers be granted such flexibility? To examine this question, we consider the following control policy Q, under which the platform sets a upper limit (i.e., a quota) on the number of female drivers who can join the pooling subsystem, where $Q \in [0, N_{f_\phi}]$. The maximum upper limit $N_{f_\phi} = \delta_D N_f$ corresponds to the number of registered safety-unconcerned female drivers. Even though the platform cannot identify exactly whether a female driver is safety-concerned or unconcerned, it can easily estimate the fraction of each type, δ_D, via data mining technology or by conducting a survey. Under such a control policy Q, safety-concerned female drivers "all join" the female-only subsystem (which we show later in Proposition 2.3).

Under control policy Q, we first analyze the equilibrium joining/participating behaviors of riders/drivers for the given prices and wages of the two subsystems. Next, we investigate the platform's optimal pricing and wage decisions by maximizing the platform's profit stated in (2.2). We then examine how the control policy Q affects system performance. Again, for the sake of brevity and space, we refer interested readers to Appendix A.3 for the detailed derivation and equilibrium analysis, and Table A.3.1 for the optimal prices $\widetilde{p}^*s$, wages $\widetilde{w}^*s$ and platform profit $\widetilde{\Pi}^*$ and the corresponding equilibrium joining/participating behavior of riders/drivers. For clarity, we use $\widetilde{\cdot}$ to indicate the equilibrium outcomes associated with the hybrid system. Then, we have the following results.

Proposition 2.3. *In a hybrid system, under the optimal prices and wages, in equilibrium,*

1. ***(Demand).*** *depending on control policy Q and the composition of user types, we have that*

   (a) *if the quota allocated to the pooling subsystem Q and the number of registered safety-unconcerned drivers N_ϕ are sufficiently large (i.e., $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$ and $\mu N_\phi > \Lambda_\phi$) and safety-concerned female riders' safety confidence level α is sufficiently high (i.e., $\alpha \geq \widehat{\alpha}$, where $\widehat{\alpha}$ is characterized by (A.29) stated in Appendix A.3), all safety-unconcerned riders, that is, all male and safety-unconcerned female riders, join the pooling subsystem. Regarding safety-concerned female riders, some join the pooling subsystem, some join the female-only subsystem and the rest balk. That is, $\widetilde{\lambda}^*_m = \Lambda_m$, $\widetilde{\lambda}^*_{f_\phi,M} = \Lambda_{f_\phi}$, $\widetilde{\lambda}^*_{f_\phi,F} = 0$, $\widetilde{\lambda}^*_{f_c,M} > 0$, $\widetilde{\lambda}^*_{f_c,F} > 0$, and $\widetilde{\lambda}^*_{f_c,M} + \widetilde{\lambda}^*_{f_c,F} < \Lambda_{f_c}$.*

   (b) *otherwise, some male riders balk. As to safety-unconcerned female riders, some join the pooling subsystem, some join the female-only subsystem and the rest balk. In contrast, safety-concerned female riders either join the female-only subsystem or balk. That is, $0 < \widetilde{\lambda}^*_m + \widetilde{\lambda}^*_{f_\phi,M} < \Lambda_m + \Lambda_{f_\phi}$, $0 < \widetilde{\lambda}^*_{f_\phi,F} + \widetilde{\lambda}^*_{f_\phi,M} \leq \Lambda_{f_\phi}$, and $\widetilde{\lambda}^*_{f_c,M} = 0$ and $0 < \widetilde{\lambda}^*_{f_c,F} < \Lambda_{f_c}$.*

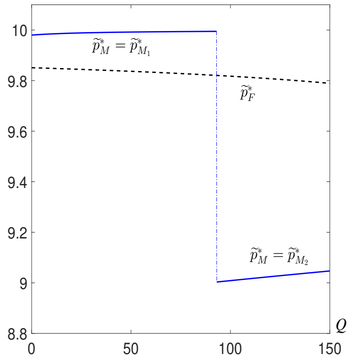
2. ***(Supply).*** *All registered male drivers participate in the pooling subsystem and all registered safety-concerned female drivers participate in the female-only subsystem. Q safety-unconcerned female drivers participate in the pooling subsystem and the others $N_{f_\phi} - Q$ participate in the female-only subsystem. That is, $\widetilde{n}^*_m = N_m$, $\widetilde{n}^*_{f_c,F} = N_{f_c}$, $\widetilde{n}^*_{f_\phi,M} = Q$ and $\widetilde{n}^*_{f_\phi,F} = N_{f_\phi} - Q$.*

Proposition 2.3 reveals that all registered drivers participate in a hybrid system. This is different from a pooling system in which safety-concerned drivers may not participate (see Proposition 2.1). This difference is caused by the existence of the female-only subsystem in the hybrid system. The driver-side safety confidence level β now plays no role as safety-concerned female drivers all join the female-only subsystem. As to the rider side, Proposition 2.3 shows that safety-concerned female riders may also join the pooling subsystem when their safety confidence level α is high ($\alpha \geq \widehat{\alpha}$), the capacity of the pooling subsystem is large (i.e., $\mu N_\phi > \Lambda_\phi$) and enough female drivers are allowed to join the pooling subsystem ($Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$). Otherwise, safety-concerned female riders only join the female-only subsystem.[2.3]

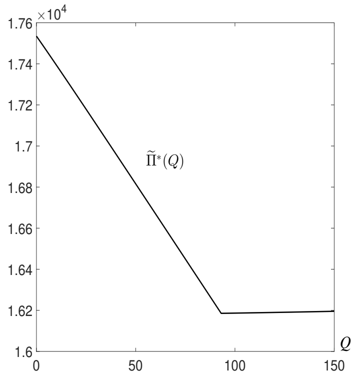
Recall that in the hybrid system, the platform sets subsystem-based prices and wages. One may expect female riders who choose the "women-to-women" service must pay a higher price than those who join the pooling subsystem. We show that this is not always true. Consider a situation in which the number of safety-unconcerned drivers is sufficiently large such that $N_\phi \mu > \Lambda_\phi$. Let $\left(\widetilde{p}^*_{M_1}, \widetilde{w}^*_{M_1}; \widetilde{p}^*_{F_1}, \widetilde{w}^*_{F_1}\right)$ be the optimal prices and wages associated with the pooling and hybrid subsystems when safety-concerned female riders only join the female-only subsystem and $\left(\widetilde{p}^*_{M_2}, \widetilde{w}^*_{M_2}; \widetilde{p}^*_{F_2}, \widetilde{w}^*_{F_2}\right)$ be those when safety-concerned female riders also join the pooling subsystem. We can show that $\widetilde{p}^*_{M_1} > \widetilde{p}^*_{F_1}$ and $\widetilde{p}^*_{M_2} < \widetilde{p}^*_{F_2}$ holds when $\sqrt{R\mu/c}$ is greater than threshold value $F(\alpha, Q)$; see Proposition A.3.4 of Appendix A.3.2. That is, when safety-concerned female riders only join the female-only subsystem, the female-only subsystem charges less than the pooling subsystem ($\widetilde{p}^*_{M_1} > \widetilde{p}^*_{F_1}$); however, if safety-concerned female riders also join the pooling subsystem at a non-zero rate, the female-only subsystem charges more than the pooling subsystem when $\sqrt{R\mu/c}$ is large enough (i.e., $\sqrt{R\mu/c} > F(\alpha, Q)$); see Figure 2.4(a) for an illustration. In other words, female riders who join the female-only subsystem can pay less. The underlying driving force is the limited female driver pool size. Recall that riders are delay-sensitive. When riders join a subsystem with low capacity (e.g., the female-only subsystem), their expected waiting time is long, and the platform must set a lower price

[2.3]We note that multiple equilibria exist; however, they do not affect the platform's optimal pricing and wage decisions. Interested readers can refer to Appendix A.3 for the details.

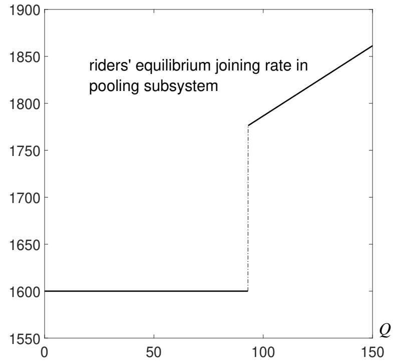
to compensate.



(a) Subsystem-based Prices



(b) Platform's Profit



(c) Demand in the Pooling Subsystem

Figure 2.4: The Impact of Control Policy Q on the Hybrid System: $N_m = 1100$, $\Lambda_m = 1000$, $N_f = 300$, $\Lambda_f = 1500$, $\mu = 1.5$, $r = 2$, $c = 1$, $R = 10$, $\delta_R = 40\%$, $\delta_D = 50\%$, and $\alpha = 0.9$ $(\mu N_\phi > \Lambda_\phi)$

We now investigate how the control policy Q affects platform profitability. Note that a larger Q gives female drivers more *flexibility* in selecting between the pooling and female-only subsystems. When $Q = 0$, the hybrid system degenerates to a dedicated system in which female drivers can only offer female-only service. The sensitivity analysis with respect to Q can provide insight into how much flexibility should be granted to female drivers.

Proposition 2.4. *If the number of registered safety-unconcerned drivers is sufficiently small such that $\mu N_\phi \leq \Lambda_\phi$, the platform's profit $\widetilde{\Pi}^*(Q)$ is increasing in Q. Otherwise, this monotonicity result may not hold.*

Proposition 2.4 provides a sufficient condition under which flexibility granted to female drivers benefits the platform. Note that moving some capacity from the female-only subsystem to the pooling subsystem generates two effects: enhancing pooling in the pooling subsystem but reducing it in the female-only subsystem. Proposition 2.4 shows that when the supply of safety-unconcerned drivers cannot meet the demand from safety-unconcerned riders ($\mu N_\phi \leq \Lambda_\phi$), the pooling system is supply-constrained and hence the potential gain induced by the pooling effect of allowing more female drivers to participate in the pooling subsystem surpasses the potential loss to the female-only subsystem. In this situation, granting female drivers the flexibility of choice benefits the platform. When demand from safety-unconcerned riders is not large enough such that $\Lambda_\phi < \mu N_\phi$, switching capacity to the pooling subsystem does not bring much benefit (as shown in Figure 2.4(c)) but seriously harms the female-only subsystem. As depicted in Figure 2.4(b), the platform obtains the most profit by allocating no female drivers to the pooling subsystem (i.e., setting $Q = 0$) in such a situation. Consequently, the platform should be cautious about the degree of flexibility given to female riders, which hinges upon the magnitude of the labor pool size of safety-unconcerned drivers.

2.6 System Comparison and Discussion

So far, we have derived the equilibrium system performance associated with the pooling and hybrid systems. In this section, we compare the performance of these two systems to investigate whether switching from a pooling system to a hybrid system can lead to a win-win outcome for the platform. We then discuss how our results can provide a plausible explanation for the operation of ride-hailing platforms in different countries/regions.

2.6.1 Comparison between a Pooling System and a Hybrid System

Recall that the key motivation for switching from a gender-neutral pooling system to a hybrid system with a female-only option is to resolve or mitigate female users' safety concerns. However, as the safety confidence level improves, more safety-concerned female riders will join, making the system more congested. In equilibrium, the safety-concerned female rider's utility remains unchanged. Hence, safety-concerned female riders' utility cannot correctly reflect the benefit of adopting a hybrid system. We believe that a more accurate performance indicator is safety-concerned female riders' access of the ride-hailing service, which is measured by the effective joining rate of such riders. We obtain the following results.

Proposition 2.5 (**Pooling vs. Hybrid: Safety-concerned Female Riders' Accessibility**)**.** *Given a control policy Q, $Q \in [0, N_{f_\phi}]$, if the number of safety-unconcerned drivers is sufficiently large such that $N_\phi \mu > \Lambda_\phi$ and the safety confidence levels of female users $(\alpha, \beta) \in \Theta_1$, the effective joining rate of safety-concerned female riders in a hybrid system is (weakly) larger than that in a pooling system. The set Θ_1 is defined as follows:*

1. *if $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$, then $\Theta_1 \equiv \{(\alpha, \beta) : (\bar{\alpha} \leq \alpha \text{ or } \alpha \leq \underline{\alpha}_0) \text{ and } \beta < \widehat{\beta}(\alpha)\}$;*

2. *if $Q \leq \frac{\Lambda_\phi}{\mu} - N_m$, then $\Theta_1 \equiv \{(\alpha, \beta) : \alpha \leq \bar{\alpha}_0 \text{ and } \beta < \widehat{\beta}(\alpha)\}$,*

where the expressions of $\bar{\alpha}_0, \underline{\alpha}_0$ and $\bar{\alpha}$ are provided in (A.12) *and* (A.13) *of Appendix A.1.*

Proposition 2.5 provides a region of the safety confidence level parameters (α, β) so that switching to a hybrid system increases the accessibility for safety-concerned female riders. It is intuitive that this region shall request the safety-concerned female users' safety confidence levels to be low. The interesting part is that in this region, the rider-side safety confidence level can be very high (i.e., $\bar{\alpha} \leq \alpha$). This demonstrates that even when female riders are not so concerned about the gender-based safety issue, it is still sometimes beneficial to change to a hybrid system as it can enlarge female drivers' participation incentives.

As to the labor provision of safety-concerned female drivers, we also adopt the access concept and have the following conclusion.

Proposition 2.6 (**Pooling vs. Hybrid: Safety-concerned Female Drivers' Accessibility**). *The equilibrium participating number of safety-concerned female drivers that provide ride-hailing service is always (weakly) larger in a hybrid system than in a pooling system.*

Proposition 2.6 implies that a hybrid system can always improve the accessibility for safety-concerned female drivers in comparison to a pooling system because in a hybrid system, the existence of a female-only subsystem eliminates the safety concerns of those female drivers.

In a ride-hailing system, there are other parties: male and safety-unconcerned female riders and drivers. Next, we investigate how these users' utilities are affected. Specifically, we compare the individual utilities of all three types of users (male, safety-concerned female and safety-unconcerned female) in the hybrid and pooling systems. In the pooling system, under the optimal price and wage, the individual utility of type-i rider U_i, $i = m, f_c, f_\phi$, can be easily obtained from the foregoing analysis. As to participating drivers, the individual utility of each participating type-i driver providing ride-hailing services can be expressed as

$$\mathbb{U}_i := \frac{\lambda_m^* + \lambda_{f_c}^* + \lambda_{f_\phi}^*}{n_m^* + n_{f_c}^* + n_{f_\phi}^*} \cdot w^*, \quad i = f_\phi, m; \text{ and } \mathbb{U}_{f_c} := \frac{\beta\lambda_m^* + \lambda_{f_c}^* + \lambda_{f_\phi}^*}{n_m^* + n_{f_c}^* + n_{f_\phi}^*} \cdot w^*,$$

in which $\mathbb{U}_{f_c}$ is the safety-concerned female driver's *safety-adjusted utility*, which is less than her monetary income due to the gender-based safety concerns. Similarly, in the hybrid system, we can calculate the individual utility of each type-i rider $\widetilde{U}_i$ and that of each type-i driver $\widetilde{\mathbb{U}}_i$, $i = m, f_c, f_\phi$. We then have the following comparison results.

Proposition 2.7 (**Pooling vs. Hybrid: User Utility**). *Under optimal pricing and wage decisions,*

1. (a) *Safety-concerned female riders obtain the same individual utility in the hybrid system as in the pooling system, i.e.,* $U_{f_c} = \widetilde{U}_{f_c}$*;*

   (b) *Both male and safety-unconcerned female riders obtain a higher individual utility in the hybrid system than in the pooling system, i.e.,*

$\widetilde{U}_i \geq U_i$, $i = f_\phi, m$, when the number of registered safety-unconcerned drivers is sufficiently large such that $\mu N_\phi > \Lambda_\phi$ and safety-concerned female riders' safety confidence level is high ($\alpha \geq \widehat{\alpha}$); otherwise, they obtain a lower individual utility in the hybrid system than in the pooling system.

2. (a) *Participating safety-concerned female drivers obtain the same individual utility in the hybrid system as in the pooling system, i.e., $\mathbb{U}_{f_c} = \widetilde{\mathbb{U}}_{f_c}$;*

   (b) *Participating male and safety-unconcerned female drivers obtain a weakly lower individual utility in the hybrid system than in the pooling system, i.e., $\mathbb{U}_i \geq \widetilde{\mathbb{U}}_i, i = f_\phi, m$.*

Proposition 2.7 shows that not every party in the ride-hailing system is happy about the switching to the hybrid system. Specifically, compared to the pooling system, safety-unconcerned drivers are always unhappy with the hybrid system and safety-unconcerned riders may also be unhappy with the hybrid system. This unhappiness might be caused by two factors. The first is the weakened pooling effect due to splitting riders and drivers between the subsystems. The other is the loss of "privilege": in the pooling system, the pricing and wage decision is made to anchor safety-concerned users, which allows safety-unconcerned users to enjoy a higher utility than safety-concerned users; however, in the female-only subsystem, all female users become equal, allowing the platform to extract more surplus from safety-unconcerned users. In particular, when $\beta < \widehat{\beta}(\alpha)$, the extent of unhappiness for riders is measured by $U_i - \widetilde{U}_i, i = f_\phi, m$ is $\frac{(1-\alpha)N_m}{N_\phi}R$, which is roughly $(1-\alpha)R$. Therefore, safety-unconcerned riders could be rather unhappy with a hybrid system.

Next, we compare platform profit in the two systems and get the following result.

Proposition 2.8 (**Pooling vs. Hybrid: Platform Profitability**). *Given any control policy Q, $Q \in [0, N_{f_\phi}]$, when the safety confidence levels of female users $(\alpha, \beta) \in \Theta_2 \equiv \{(\alpha, \beta) : \alpha \leq \underline{\alpha}(\beta)\}$[2.4], the platform's profit is higher in the hybrid*

[2.4]While Proposition 2.8 focuses on the effect of female riders' safety confidence level α, we *system than in the pooling system, i.e.,* $\widetilde{\Pi}^* \geq \Pi^*$*; otherwise, the platform's profit is higher in the pooling system, i.e.,* $\widetilde{\Pi}^* < \Pi^*$*.*

Proposition 2.8 reveals that the hybrid system yields more profit for the platform when safety-concerned riders' safety confidence level is not high ($\alpha \leq \underline{\alpha}(\beta)$). Although the pooling effect is weakened, the hybrid system resolves or at least mitigates safety concerns for some users, which enables it to expand the supply and demand pools. Recall that platform pricing and wage decisions must be anchored to safety-concerned users. With a hybrid system, the platform can customize its prices and wages for the two subsystems, which allows it to gather more from safety-unconcerned users.

A close look at Propositions 2.5, 2.6 and 2.8 helps us identify conditions under which switching from a pooling system to a hybrid system can induce a win-win outcome in terms of improving the accessibility for safety-concerned female users and increasing platform profit.

Corollary 2.1. *When safety-concerned female users' safety confidence levels* $(\alpha, \beta) \in \Theta_1 \cap \Theta_2$*, the hybrid system reaches a win-win outcome: more safety-concerned female riders and drivers join the system and the platform obtains more profit.*

For illustrative purposes, we conduct some numerical studies, and Figure 2.5 depicts the win-win regions characterized by Propositions 2.5, 2.8 and Corollary 2.1, respectively. In particular, we consider two special cases regarding the platform's control policy Q: granting female drivers no flexibility and maximum flexibility of choosing between the two subsystems, i.e., $Q = 0$, and $Q = N_{f_\phi}$, respectively. Figure 2.5 shows that in both cases, the hybrid system yields a higher profit for the platform than the pooling system when safety confidence levels α and β are low. Figure 2.5 also shows that from the perspective of increasing the accessibility for safety-concerned female riders, the hybrid system is preferred when the driver-side safety confidence level β is low.

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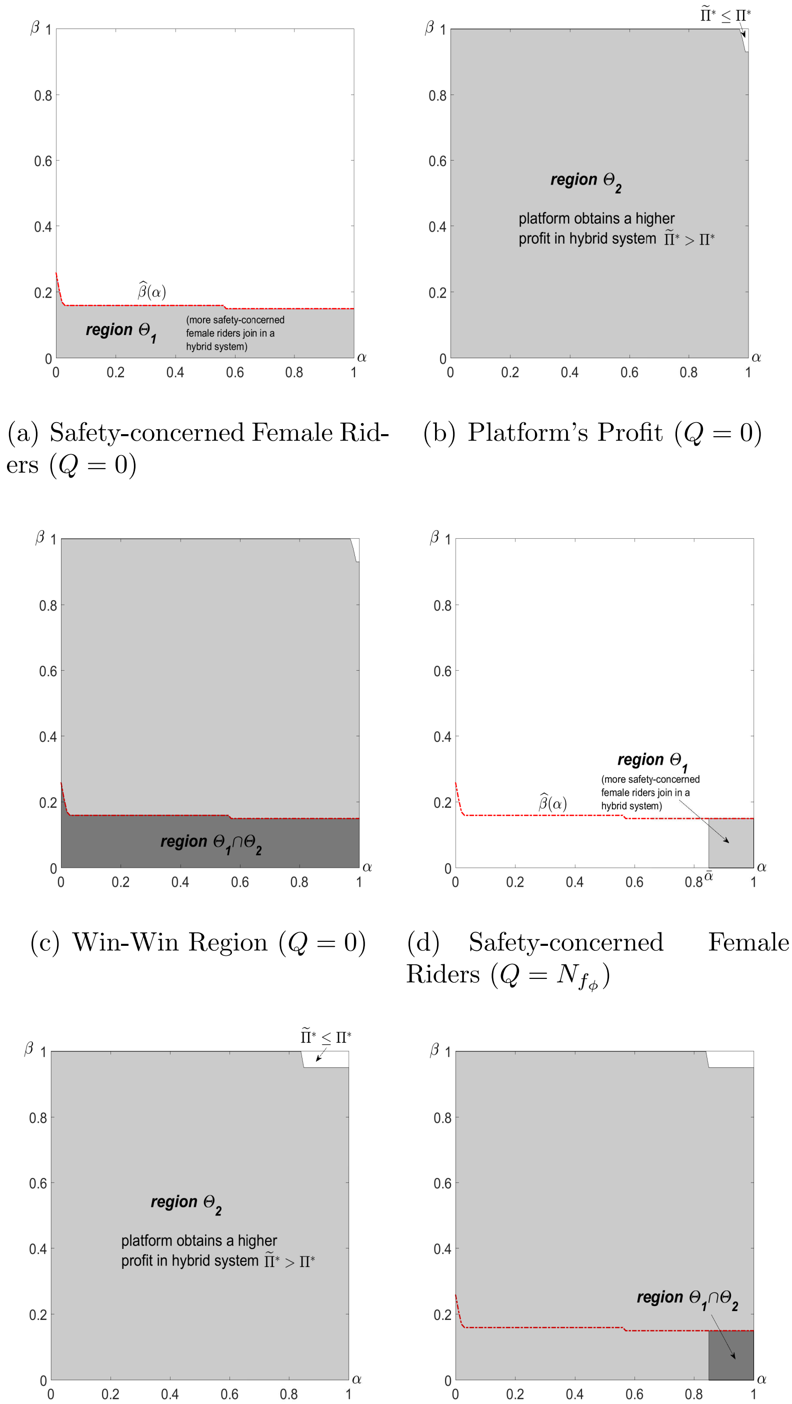
can draw a similar conclusion if we vary female drivers' safety confidence level β. To avoid repetition, we omit details here.



(a) Safety-concerned Female Riders ($Q = 0$)

(b) Platform's Profit ($Q = 0$)

(c) Win-Win Region ($Q = 0$)

(d) Safety-concerned Female Riders ($Q = N_{f_\phi}$)

(e) Platform's Profit ($Q = N_{f_\phi}$)

(f) Win-Win Region ($Q = N_{f_\phi}$)

Figure 2.5: When a Hybrid System Can Achieve Win-Win Compared to a Pooling System: $N_m = 1100$, $\Lambda_m = 1000$, $N_f = 300$, $\Lambda_f = 1500$, $\mu = 1.5$, $r = 2$, $c = 1$, $R = 10$, $\delta_R = 40\%$ and $\delta_D = 50\%$ ($\mu N_\phi > \Lambda_\phi$)

2.6.2 Discussion: Countries and System Adoption

The comparison between hybrid and pooling systems (as shown in Figures 2.5(b) and 2.5(e)) reveals that when female users' safety confidence levels α and β are low, the hybrid system is preferred; otherwise, the pooling system is preferred. This analytic result may help us explain why various ride-hailing systems are observed in different countries.

Some countries have severe female safety problems. According to a Thomson Reuters Foundation survey, the top 10 most dangerous countries for women include India (1st), Saudi Arabia (5th), Pakistan (6th) and the United States (10th) (Narayan 2018). In India, females even sometimes face violence (which may contain sex attacks) from their male family members (Rao et al. 2015). Due to such severe female safety concerns, gender-dedicated ride-hailing services are now provided in certain countries, such as Chariot for Women (United States), She Cabs (India) and She'Kab (Pakistan). In Saudi Arabia, a hybrid system was adopted by Uber which allows its female drivers to serve only female passengers (Kumar 2019).

Some countries are regarded as generally safe for women. For example, according to the 2019 Global Wealth Migration Review conducted by New World Wealth (a global market research group), the five safest countries for women are Australia, Malta, Iceland, New Zealand and Canada (Perper 2019). A pooling system is often adopted by ride-hailing platforms in those countries as females usually have high safety confidence, such as Uber in Australia and Canada and Ola in New Zealand (Barratt et al. 2018, Brail and Donald 2018, Kashyap 2018).

In other countries, such as China, females have a moderate level of safety confidence. Different systems are used by ride-hailing platforms in different time slots. For example, DiDi provides the gender-dedicated service in the early morning and late at night when female users' safety confidence level is relatively low and operates as a pooling system the rest of the time (Al-Heeti 2018). Table 2.2 summarizes the operation of ride-hailing platforms in the aforementioned countries.

Table 2.2: Current System Adoption across Countries

Category	Country	Examples
Pooling System	China	DiDi, DidaChuxing
	Australia	Uber, DiDi
	Canada	Uber, DiDi, Lyft, Grab, Yandex
	New Zealand	Uber, Ola
Hybrid System	Saudi Arabia	Uber Arabia
Dedicated System	India/Pakistan/ United States/China	She Cabs/She'Kab/ DiDi (early morning/late night)

We also note that recently, many ride-hailing platforms have begun to provide different services, which may exhibit different degrees of safety. For example, DiDi runs three business services: DiDi Premier, DiDi Express and DiDi Carpool (Hitch). Platforms can adopt the pooling system for services that are regarded as safer, such as DiDi premier, and consider a hybrid system for services that are regarded as less safe, such as DiDi carpool.

2.7 Conclusion

Some female riders/drivers are safety-concerned when they are matched with male drivers/riders. In this chapter, we consider such gender-based safety concerns and investigate two operational systems for ride-hailing platforms: a pooling system in which riders and drivers are matched without considering gender and a hybrid system consisting of a subsystem in which females can select female-only service. We then derive the equilibrium outcomes for both systems, including the platform's optimal pricing and wage decisions and the respective equilibrium joining and participating behaviors of riders and drivers.

We show that a pooling system is preferred when safety-concerned female users have a high safety confidence level. Our sensitivity analysis result shows that there is a marginally increasing effect to improving the rider-side safety level but a marginally diminishing effect to improving the driver-side safety level. Therefore, the platform should first improve the safety confidence of both female drivers and riders to a certain threshold level; after that, the platform should further improve female riders' safety confidence level as much as possible.

For the hybrid system, we find that male and safety-unconcerned female users can be unhappy with such a system. Their unhappiness comes from two effects: weakening of the pooling effect and the loss of "privilege" over safety-concerned users. In the pooling system, the price and wage must anchor safety-concerned users, and therefore, safety-unconcerned users enjoy a higher utility. Such privilege is lost in the hybrid system as in the female-only subsystem, all females become equal and the platform is entitled to tailor their prices and wages for the different subsystems. Despite these points, a hybrid system can achieve a win-win outcome on two other important measures, increasing the access of both safety-concerned female riders and drivers and improving the platform's profitability, when safety-concerned female users' safety confidence levels fall into certain ranges. The win-win regions in general require safety levels that are not very high.

We note that females' safety confidence level varies across countries. Our analytical results can provide a plausible explanation for the adoption of different operational systems in different countries. For example, in countries where females' safety concerns are severe (such as India and Saudi Arabia), a hybrid system or a gender-dedicated system is observed.

Chapter 3

Encroachment Channel Selection When Retailer Has Private Information: Role of Quantity Leadership

3.1 Introduction

Supplier encroachment is a common phenomenon observed in a wide range of industries (Arya et al. 2007, Li et al. 2014, Huang et al. 2018). It refers to a supplier (she), who already wholesales her product to a retailer, expanding her market demand by direct selling the product in the end market. She may engage in a *direct channel encroachment* via establishing a sales channel such as an online/offline store (e.g., Apple Store, Vmall.com for Huawei Phone). As an example, OnePlus, a Chinese smartphone manufacturer, wholesales its mobile phones to JD.com for reselling, and also sells directly to the end consumers in its own online OnePlus store (https://www.oneplus.com/cn) (JD.com-Corporate-Blog 2018). As another example, Lee Kum Kee, a Hong Kong-based food company, wholesales its products to HKSuning.com. In addition, it also operates its own online store (https://shop.lkk.com/) to sell products directly to consumers. The supplier can also encroach by selling directly through a commission channel provided by an e-commerce giant, such as JD.com, Amazon and HKSuning.com, which charges transaction-based commission fees for using its online platform (namely, the *commission channel encroachment*). Take as an example, Zi Hai

Guo, a Chinese company that produces the convenient self-cooking hotpot, both wholesales the products to JD.com as well as direct sells them via the online platform of JD.com. As both types of encroachment have been observed in practice, it motivates us to examine the following research question: *Through which channel shall the supplier encroach?* While there are studies devoted to supplier encroachment (see, e.g., Arya et al. 2007, Xu et al. 2010, Huang et al. 2018), there are none to our knowledge that consider *encroachment channel selection*.

Information technology can help companies to effectively share data and empirical studies have shown that information sharing can increase supply chain agility, under which the supply chain can improve responsiveness to changing market needs (Swafford et al. 2008). Internet retailers such as JD.com collect big data and are therefore able to tease out demand information (JD.COM. 2019). Moreover, a retailer can dangle its private demand information to motivate a supplier to encroach through the commission channel by using its platform and thus earn commission fees. Alternatively, the supplier can open a direct channel and thus avoid commission fees, but then incur the costs of operating its own offline/online store. Clearly, the retailer's information sharing can improve the attractiveness of the commission channel encroachment, but it may also lead to a fiercer head-to-head competition between the retailer and the supplier in the end market. This makes us wonder *"Will the retailer share the demand information?"*.

When the supplier and the retailer engage in the downstream market competition, they may exhibit different leadership in making decisions (Kadiyali et al. 2000 and Wang et al. 2013). For example, when they are engaged in the Cournot (quantity) competition, one may take the role of the Stackelberg (quantity) leader while the other is the Stackelberg (quantity) follower. The quantity leadership[3.1] between the two parties then affects their respective performance. This motivates us to ask the question: *How does a particular party's leadership role affect the retailer's information sharing incentive and the supplier's encroachment channel*

[3.1] Hereafter, for brevity, we may refer to the quantity leadership as leadership.

selection?

To answer the above three research questions, we consider a supply chain with a retailer (he) and a supplier (she). The retailer is an e-commerce giant (e.g., JD.com) and has private demand information. He procures the product from the supplier and resells it in the market. The supplier has decided to encroach into the retail market by either opening an online/office store (i.e., a direct channel) by herself and incurring the related operating cost or selling directly via the retailer's online platform (i.e, the commission channel), which charges her transaction-based commission fees. If the commission channel is adopted, the retailer may share his demand information with the supplier. The encroaching supplier and the retailer are then engaged in one of the following two Cournot competition games in the end market: a *supplier-as-quantity-leader* game and a *retailer-as-quantity-leader* game.

We show that in the commission channel, when the retailer is the quantity leader, he always shares his demand information with the supplier. This is because such information sharing can, on one hand, eliminate his downward quantity distortion induced by the supplier's rational inference if demand information is asymmetric, and on the other, benefit himself as well as the supplier due to their strengthened cooperative relationship arising from the transfer of revenue-sharing-type commission fees. However, when the supplier is the leader, the retailer shares his demand information only when the competition intensity is low and the commission rate is moderate. This is because the retailer's demand information sharing would further strengthen the supplier's first-mover advantage and may in turn hurt himself, especially when their head-to-head competition is intense. Moreover, a high commission rate stimulates the supplier to focus more on her own direct selling while a low commission rate induces the retailer to focus more on his reselling business, both of which dilute the retailer's sharing incentives.

For any given commission rate, there exists two threshold direct channel operating costs, an upper one and a lower one. The supplier encroaches through the commission channel when the direct channel operating cost is above the upper threshold, but through the direct channel when it is below the lower threshold, regardless of her leadership role. The supplier's encroachment channel selection largely hinges on the trade-off between two driving forces: her revenue loss incurred in the commission channel due to the charged commission fee and the operating cost borne by her for establishing her own direct channel. When the former surpasses the latter, she prefers direct channel encroachment. However, when the direct channel operating cost lies between these two thresholds, then for the low commission rates, the supplier prefers the commission (direct, resp.) channel encroachment if the retailer behaves as the quantity leader (follower, resp.), while for the high commission rates, the exact opposite holds. Under such a circumstance, the quantity leadership, or "who enjoys the first-mover advantage", significantly impacts the supplier's encroachment channel selection. Note that the retailer's revenue comes solely from reselling the supplier's product under the direct channel encroachment, but it comes from both reselling and charging commission fees under the commission channel encroachment. The cooperative relationship between the retailer and the supplier are thus much strengthened in the latter encroachment, leading to a weakened downstream competition. When the commission rate is low, the supplier would encroach through the direct channel only if she can enjoy the first-mover advantage by deciding the wholesale price and direct-selling quantity first, which helps her to compete with the retailer. If she is a quantity follower instead, she has more incentive to encroach through the commission channel so as to weaken the downstream competition. When the commission rate is high, the retailer has less incentive to compete fiercely with the supplier because he can enjoy more revenue from the supplier's direct sales. But in view of high commission rats, the supplier is only willing to select the commission channel when she can freely adjust the wholesale price and the direct-selling quantity in anticipation of the retailer's best response decision, that is, when she is a quantity leader. Interestingly, we show that a more accurate demand signal does not necessarily improve the supplier's preference over the commission channel encroachment, especially when the commission rate is high.

3.2 Literature Review

Our work is closely related to the studies on supplier encroachment and dual channel distribution. Arya et al. (2007) show that a retailer can benefit from its wholesale supplier's encroachment when he has a selling advantage. Xu et al. (2010) study a proprietary component supplier's optimal distribution strategy among three options, only wholesaling the component to an original equipment manufacturer, developing the end product and direct selling exclusively under her own brand name, and both wholesaling the component and direct selling the end product. Khouja et al. (2010) consider a manufacturer who can sell through a direct channel, a manufacturer-owned retail channel, an independent retail channel or any combination thereof. They then identify the manufacturer' optimal distribution channel selection. Ryan et al. (2012) study a supply chain consisting a marketplace firm and a retailer, where the retailer is currently selling the products through its own website.They investigate whether or not the retailer should also sell through the platform provided by the marketplace and if yes, under which prices. Ha et al. (2017) consider quality endogeneity when the supplier encroaches. Guan et al. (2019) further investigate the interaction between the supplier encroachment and the buyer's strategic inventory withholding decision. Yang et al. (2018) study how the capacity limitation affects the supplier's optimal distribution strategy when the supplier may encroach into the market to compete with the buyer. Here, we also investigate the supplier's distribution strategies. However, we focus on examining the supplier's *how to encroach* issue.

The stream of research on vertical information sharing in the presence of competition is related; see, e.g., Chen (2003) for a comprehensive review. Li and Zhang (2008) examine the impact of information confidentiality on the supply chain members' information sharing incentives in a setting with one manufacturer and multiple retailers. Gal-Or et al. (2008) investigate how information sharing affects a manufacturer's wholesale pricing decisions by considering a supply network that contains one manufacturer and two retailers, where all of the supply chain members have private demand information. Ha and Tong (2008) investigate how supply chain contracts affect information sharing. Shang et al. (2016) consider a supply chain consisting of two competing manufacturers and a common retailer. They examine the impact of nonlinear production cost on the retailer's information sharing incentive. Yoon et al. (2020) study a multi-tier supply chain consisting of a manufacturer, a first-tier supplier and a second-tier supplier. The first-tier supplier has access to the second-tier supplier's disruption information and may share such information with the manufacturer. They examine different information sharing contracts under which the manufacturer can obtain the shared information and analyze the impacts on the profits of the manufacturer and the first-tier supplier. Our work complements the above studies by identifying a new driver of information sharing, that is, Stackelberg (quantity) leadership.

Some studies examine both information asymmetry and supplier encroachment. Li et al. (2014) consider a reseller with private information about market size and a supplier encroaching through a direct sales channel. They show that information asymmetry may amplify the double marginalization due to the reseller's downward order distortion behavior and supplier encroachment may hurt both the reseller and the supplier. Li et al. (2015) further extend Li et al. (2014) by considering non-linear pricing. Our work differs from these two studies by endogenizing the retailer's information sharing decision, which may reduce information asymmetry. We also consider different Stackelberg (quantity) leadership scenarios while Li et al. (2014, 2015) assume that the retailer is the Stackelberg leader. Huang et al. (2018) study the interaction between the retailer's information sharing and supplier encroachment.

Our work is also related to works on quantity/pricing leadership. Gal-Or (1987) show that under the market uncertainty, the Stackelberg followership rather than the Stackelberg leadership can be preferred over a wide range of parameter values. Wang et al. (2013) investigate how the quantity timing preferences of an original equipment manufacturer (OEM) and its competitive contract manufacturer are affected by the OEM's outsourcing decision. Fang et al. (2018) examine the effect of price leadership on the profitability of the manufacturer and the retailer. Dagli et al. (2019) study the impact of increasing channel differentiation on the performances of supply chain members under three scenarios: manufacturer is the price leader, retailer is the price leader and they set price simultaneously. They show that increasing channel differentiation always benefit a price leader but may hurt a price follower. Here, we focus on the effect of quantity leadership on the supplier's encroachment channel selection.

3.3 Model

Consider a supply chain with one supplier (she, labeled S) and one retailer (he, labeled R). The retailer buys the product from the supplier at a unit wholesale price w and then resells it in the end market. The supplier has decided to encroach into the retail market by direct selling. One way is to direct sell the product through the retailer's online platform by paying the retailer a transaction-based commission fee, which we name as the "commission channel encroachment" (denoted by $\mathcal{C}$). (For example, Amazon and JD.com are such retailers, who not only operate as a traditional retailer but also allows sellers to sell directly through their platforms.) Denote the unit commission rate charged by the retailer as $\alpha \in (0, 1)$. Another way is to establish her own online/offline store, which we call the "direct channel encroachment" (denoted by $\mathcal{D}$). The supplier incurs an encroachment operating cost c_1 (c_2, resp.) under the commission (direct, resp.) channel encroachment. Generally, encroaching through the commission channel is more cost-effective than encroaching through the direct channel as the former is operated by the retailer instead. This indicates that $c_2 = c_1 + c$, where $c \geq 0$. Without loss of generality, we normalize c_1 at zero and let $c_2 = c$ throughout the this chapter.

The encroaching supplier and the retailer are engaged in one of the following two Cournot competition games in the end market: a supplier-as-quantity-leader game and a retailer-as-quantity-leader game. Both games are possible in reality: in certain industries, production process takes a long lead time and it is hard to adjust the production capacity (Rasmussen 2018); thus, the supplier often announces its production decision first and commits to it, leading the supplier to behave as the quantity leader (Wang et al. 2013). In other industries, production capacity can be more easily adjusted and the supplier cannot commit to her own production decision after receiving the retailer's order, resulting in the retailer to be the quantity leader (Li et al. 2014, 2015). The respective inverse demand functions of the retailer and the supplier are

$$p_i = a + X - q_i - \gamma q_j, i = S, R, \tag{3.1}$$

where a is the base market size, p_i and q_i are the respective retail price and selling quantity of player i, and $\gamma \in (0, 1)$ is a measure of the competition intensity. X is a random variable with mean zero (i.e., $E[X] = 0$) and variance σ^2 (i.e., $Var[X] = \sigma^2$), which represents the demand uncertainty. Such a linear inverse demand function has been widely adopted in the operations management literature; see, e.g., Li and Zhang (2008), Wang et al. (2013), Huang et al. (2018) and references therein.

The retailer holds private information about the market demand. Specifically, he has access to a demand signal x, which is an unbiased estimator of X; that is, $E[x \mid X] = X$. Following Li and Zhang (2008) and Wu and Zhang (2014), we assume that the expectation of X conditional on the signal x is a linear function of the signal.

$$E[X \mid x] = \frac{x}{1+s}, \tag{3.2}$$

based on which we have $E[x^2] = (1+s)\sigma^2$ and $E[x] = 0$. Then, by Ericson (1969), we have $E[Var[x \mid X]] = s\sigma^2$. That is, the reciprocal $1/(s\sigma^2)$ is a measure of signal accuracy. When $1/s \to 0$ or equivalently, $s \to \infty$, it indicates that the retailer has no private information advantage. Table 3.1 summarizes the key notation used.

First we consider the retailer-as-quantity-leader game, referred to as the *retailer-leadership* scenario, and denoted by $\mathcal{RL}$. The sequence of events under this sce-

Table 3.1: A List of Key Notation

$\mathcal{C}$	commission channel;
$\mathcal{D}$	direct channel;
c	operating cost of direct channel,
α	commission rate when using retailer's commission platform service
q_R	retailer's reselling quantity
q_S	supplier's direct selling quantity
γ	competition intensity
a	base market size
X	uncertain part of the market size, $E[X] = 0$, $Var[X] = \sigma^2$
x	private information signal observed by retailer, $E[x\|X] = X$
$1/(s\sigma^2)$	measure of information signal accuracy

nario is sketched in Figure 3.1 and defined as follows: First, the supplier decides whether to encroach through the commission channel or the direct channel. Next, if the commission channel is chosen, the retailer decides whether or not to share his demand information with the supplier and then commits to it. We consider that the supplier's encroachment channel selection is made before the retailer's information sharing decision. Such an assumption is reasonable because encroachment decision is a strategic level decision and may take a long time for the supplier to implement. Similar sequence has been adopted by Li et al. (2014) and Huang et al. (2018), and we refer the interested readers to them for the related discussion. Here, the retailer makes his information sharing decision before he observes the demand signal, which has been commonly assumed in the information sharing literature; see, e.g., Gal-Or et al. (2008), Huang et al. (2018), Guan et al. (2019). However, if the direct channel is chosen, then the retailer has no incentive to share his information. This is consistent with the business practice. For instance, JD.com shares its data analytic tools only with the sellers who have decided to direct sell through its platform. After that, the demand signal x is observed. If the commission channel encroachment is chosen, this signal is revealed to the supplier if the retailer pre-commits to information sharing. Then the supplier decides her unit wholesale price w. Observing the wholesale price w, the retailer then determines his reselling quantity, followed by the supplier deciding her direct-selling quantity. Finally, demands are realized and revenues are collected.

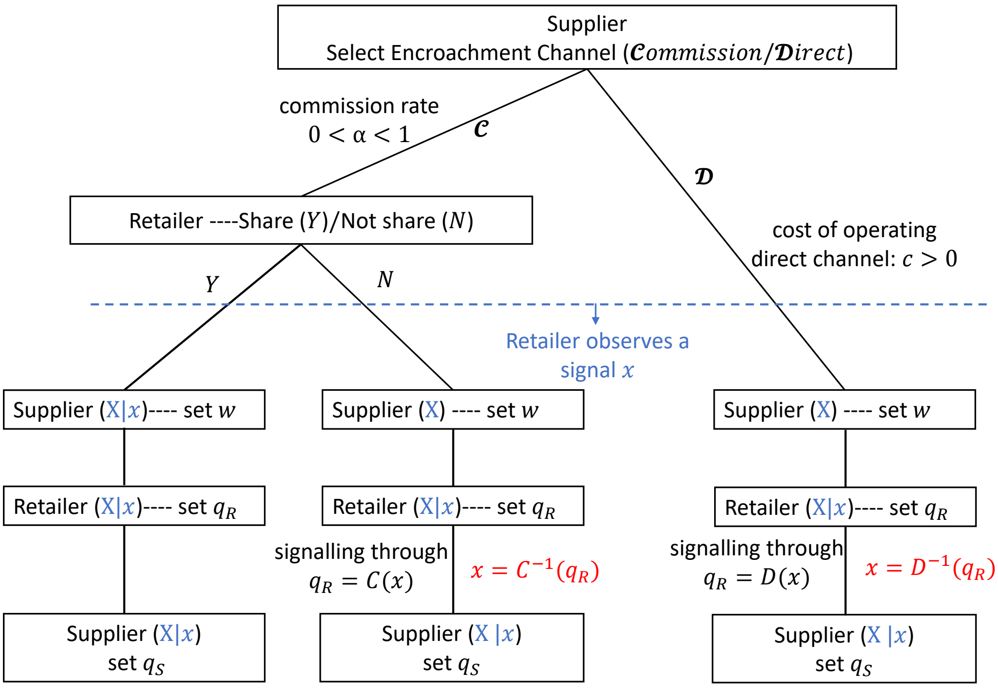


Figure 3.1: Sequences of Events When Retailer Acts as Quantity Leader

Next we consider the supplier-as-quantity-leader game, referred to as the *supplier-leadership* scenario and denoted by $\mathcal{SL}$. The sequence of events remains the same as that under the retailer-leadership scenario except that in the last two stages, the supplier sets her direct-selling quantity before the retailer decides his reselling quantity; see Figure 3.2 .

Both parties are risk neutral and aim to maximize their respective profits. Since the game contains multiple rounds of strategic interactions, backward induction is applied to ensure subgame perfection. Below, we will first derive the supplier's encroachment channel selection under each leadership scenario. We then compare the equilibrium outcomes associated with the two leadership scenarios to investigate the impact of quantity leadership.

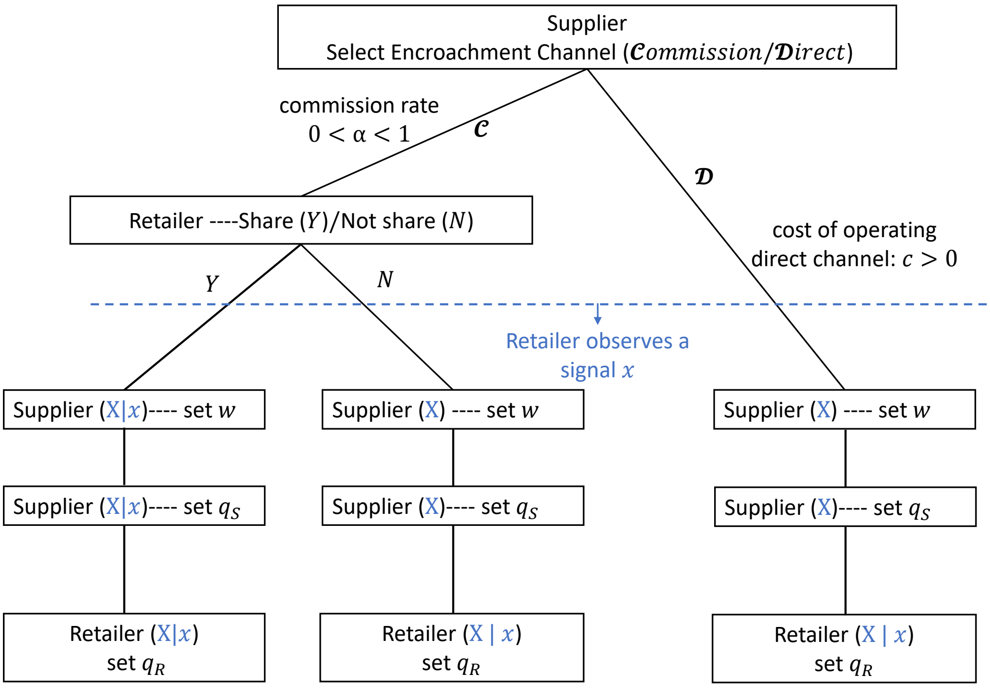


Figure 3.2: Sequences of Events When Supplier Acts as Quantity Leader

3.4 Retailer Acts as Quantity Leader ($\mathcal{RL}$ Scenario)

In this section, we consider the $\mathcal{RL}$ scenario in which the retailer acts as the Stackelberg leader in the quantity decision stage. We first derive the equilibrium outcomes associated with the commission channel encroachment and the direct channel encroachment, respectively. We then compare these equilibrium outcomes to derive the supplier's optimal encroachment channel selection when the retailer is the leader.

3.4.1 Commission Channel Encroachment

Here, we consider that the supplier encroaches via the commission channel, under which the retailer needs to decide whether or not to share his private demand information with the supplier. When the retailer commits to share his demand information, the demand signal x, once observed by the retailer, is revealed to the supplier. Then, given the demand signal x and her own wholesale price w, and after the retailer decides his order quantity q_R, the supplier determines her direct-selling quantity q_S to maximize her expected profit

$$E\left[\Pi_S \mid x\right] = (1-\alpha)\left(a + E[X \mid x] - q_S - \gamma q_R\right) q_S + w q_R, \tag{3.3}$$

which is concave in q_S. Based on the first-order condition, it can be easily shown that

$$q_S(w, q_R) = \frac{1}{2}\left(a + E[X \mid x] - \gamma q_R\right). \tag{3.4}$$

Anticipating the supplier's optimal quantity decision $q_S(w, q_R)$, the retailer then decides his reselling/order quantity q_R by maximizing

$$\begin{aligned} E[\Pi_R \mid x] = &(a + E[X \mid x] - q_R - \gamma q_S(w, q_R) - w) q_R \\ &+ \alpha(a + E[X \mid x] - q_S(w, q_R) - \gamma q_R) q_S(w, q_R), \end{aligned} \tag{3.5}$$

from which we can derive

$$q_R^{\mathcal{C}Y}(w) = \frac{a(2-\alpha\gamma-\gamma) - 2w}{4-(\alpha+2)\gamma^2} + \frac{2-\alpha\gamma-\gamma}{4-(\alpha+2)\gamma^2} E[X \mid x],$$

where the superscript $\mathcal{C}Y$ stands for the scenario *commission channel encroachment with information sharing*. Substituting $q_R^{\mathcal{C}Y}(w)$ into $q_S(w, q_R)$ yields

$$q_S^{\mathcal{C}Y}(w) = \frac{a\left(4-\gamma^2-2\gamma\right) + 2\gamma w}{8-2(\alpha+2)\gamma^2} + \frac{4-\gamma^2-2\gamma}{8-2(\alpha+2)\gamma^2} E[X \mid x].$$

Plugging the order/selling quantities $q_i^{\mathcal{C}Y}(w)$, $i = S, R$, into the supplier's expected profit function $E[\Pi_S \mid x]$, we can show that $E[\Pi_S \mid x]$ is concave in w and the optimal wholesale price

$$w^{\mathcal{C}Y} = \frac{\left(\alpha^2\gamma^3 + 4\alpha\left(\gamma^2-2\right)\gamma + \gamma^3 - 6\gamma^2 + 8\right)\left(a + E[X \mid x]\right)}{2\left(8-(\alpha+5)\gamma^2\right)}.$$

Then, substituting $w^{\mathcal{C}Y}$ and $q_i^{\mathcal{C}Y}(w^{\mathcal{C}Y})$, $i = S, R$, into (3.3) and (3.5), we can get the optimal expected profits $E[\Pi_R^{\mathcal{C}Y} \mid x]$ and $E[\Pi_S^{\mathcal{C}Y} \mid x]$ of the retailer and the supplier conditional on the given signal x, respectively. Taking the expectation with respect to signal x yields the *ex ante* expected profits of the retailer and the supplier, $\Pi_i^{\mathcal{C}Y}$, $i = R, S$. By (3.4), one can show that

$$\frac{dq_S^{\mathcal{C}Y}}{dq_R^{\mathcal{C}Y}} = -\frac{1}{2}\gamma < 0, \tag{3.6}$$

which implies that the supplier's direct-selling quantity q_S^{CY} responds *negatively* to the retailer's order quantity q_R^{CY} when the retailer commits to information sharing.

We now consider the retailer to not commit to voluntarily share his demand information. When the retailer acts as the quantity leader, the supplier can still infer the demand signal x through his order quantity q_R. How she infers the demand information from q_R depends on her belief about the retailer's quantity ordering decision $q_R(x)$ as a function of the demand signal x. Similar to Li and Zhang (2008) and Gal-Or et al. (2008), we assume that the supplier holds the belief or conjectures that the retailer, upon observing a signal x, makes his order decision by adopting the following monotone decision rule:

$$q_R = C(x), \text{ that is, } x = C^{-1}(q_R),$$

where $C(.)$ is a strictly increasing and differentiable function. Given the retailer's order quantity $q_R(x)$ and the supplier's belief $q_R = C(x)$, the supplier decides her direct-selling quantity q_S to maximize her expected profit

$$E[\Pi_S \mid q_R(x)] = (1-\alpha)\left(a + E[X \mid q_R(x)] - q_S - \gamma q_R(x)\right) q_S + w q_R(x),$$

from which we can derive

$$q_S(w, q_R(x)) = \frac{1}{2}\left(a + \frac{C^{-1}(q_R)}{1+s} - \gamma q_R\right) \text{ and } \frac{dq_S}{dq_R} = \frac{1}{2}\left(\frac{1}{1+s} \cdot \frac{1}{C'(x)} - \gamma\right). \tag{3.7}$$

Under this situation, the expression of the retailer's expected profit conditional on x, $E[\Pi_R \mid x]$, shall be the same as the one stated in (3.5) when the retailer shares his information. After substituting $q_S(w, q_R(x))$ into (3.5), we take the first-order derivative of $E[\Pi_R \mid x]$ with respect to q_R. Note that the decision rule $q_R = C(x)$ is an equilibrium if and only if the supplier's belief about $q_R = C(x)$ is fulfilled in equilibrium. That is,

$$C(x) = \arg\max_{q_R} E[\Pi_R \mid x].$$

This requires that the first-order condition of $E[\Pi_R \mid x]$ with respect to q_R must hold if we replace q_R by $C(x)$, that is,

$$\frac{2-\alpha-\alpha\gamma}{2}a - w + \frac{2-(\alpha+1)\gamma}{2(1+s)}x + \frac{(\alpha+2)\gamma^2-4}{2}C(x) - \frac{\gamma}{2(1+s)} \cdot \frac{C(x)}{C'(x)} = 0. \tag{3.8}$$

We then obtain the following result.

Lemma 3.1. *When the retailer is the quantity leader, then under the commission channel encroachment with no information sharing (denoted by $\mathcal{C}N$) and for any given commission rate $\alpha < \frac{2}{\gamma}-2$,*[3.2] *among all the general monotonic relationships between the retailer's order quantity and demand signal x, the linear rule is the unique equilibrium, i. e.,*

$$q_R^{\mathcal{C}N}(w) = C(x) = \frac{((\alpha+2)\gamma-2)(a(2-\alpha\gamma-\gamma)-2w)}{(\alpha\gamma+\gamma-2)\left(4-(\alpha+2)\gamma^2\right)} + \frac{\alpha\gamma+2\gamma-2}{(s+1)\left(\alpha\gamma^2+2\gamma^2-4\right)}x.$$

Lemma 3.1 provides the unique equilibrium one-to-one mapping between the retailer's order quantity and demand signal as long as the charged commission rate is not too high, i.e., $\alpha < \frac{2}{\gamma}-2$. Such a positive linear relationship between the potential demand and order quantity has also been demonstrated in the literature under other settings; see, e.g., Li and Zhang (2008). The requirement in Lemma 3.1 that the commission rate $\alpha < \frac{2}{\gamma}-2$ is actually quite realistic because in practice, the commission rate charged by platforms such as JD.com and Tmall is usually below 10%. For instance, Tmall sets the commission rate at 5% for pet products and home appliances products, and JD.com sets the commission rate for sea food at 3%. Hereafter in §3.4, we restrict our attention to the case $\alpha < \frac{2}{\gamma}-2$. By (3.7) and Lemma 3.1, we can show that

$$\frac{dq_S^{\mathcal{C}N}}{dq_R^{\mathcal{C}N}} = \frac{1}{2}\left(\frac{1}{1+s} \cdot \frac{1}{C'(x)} - \gamma\right) = \frac{2-\gamma}{2-\alpha\gamma-2\gamma} > 0.$$

That is, when the retailer does not share information, the supplier's direct-selling quantity $q_S^{\mathcal{C}N}$ responds *positively* to the retailer's order quantity $q_R^{\mathcal{C}N}$, a result in

[3.2] We can prove that if $\alpha = \frac{2}{\gamma}-2$, the retailer gives up the reselling business and thus the supplier becomes a monopoly. We can further prove that in this situation, the retailer still always shares the information with the supplier; that is, Proposition 3.2, stated later in this section, still holds. If $\alpha > \frac{2}{\gamma}-2$, the retailer would set a negative reselling quantity, which is unrealistic. Please refer to the proof of Lemma 3.1 for a detailed discussion of this assumption.

sharp contrast to that when the retailer commits to information sharing as shown in (3.6). As the retailer's order quantity now conveys the demand information, a larger order quantity from the retailer indicates a higher potential demand, which incentivizes the supplier to direct sell more. A further comparison of the retailer's optimal order quantities under the above two information sharing scenarios indicates that the retailer's order quantity is more *responsive* to the wholesale price changes when he shares information than that when he does not; that is, $\left|\frac{dq_R^{\mathcal{C}Y}(w)}{dw}\right| / \left|\frac{dq_R^{\mathcal{C}N}(w)}{dw}\right| = \frac{2-\gamma-\alpha\gamma}{2-2\gamma-\alpha\gamma} > 1$.

By adopting the linear decision rule stated in Lemma 3.1, we can derive the equilibrium wholesale price $w^{\mathcal{C}N}$ as the supplier's expected profit function is concave in w, and thus the corresponding quantity decisions $q_i^{\mathcal{C}N}$, $i = S, R$. Substituting the equilibrium wholesale price and quantity decisions into the profit functions of the supplier and retailer and then taking the expectation with respect to the demand signal, we can obtain the ex ante expected profits for the retailer and the supplier. Table 3.2 summarizes the equilibrium outcomes associated with "information sharing" and "no information sharing" scenarios.

Next we compare the equilibrium outcomes in the two information scenarios. Note that a supplier's *equilibrium wholesale price* is a function of the demand signal x, a random variable with $E[x] = 0$. We can consider the equilibrium wholesale price from two aspects. One, we can compare the equilibrium wholesale prices conditional on any given demand signal x (i.e., the ex post value). Two, we can check how information sharing affects the equilibrium decisions *on average* by taking expectation with respect to the demand signal x. Similarly, we can compare the equilibrium quantity decisions of the supplier and the retailer from the above two aspects as well. We then obtain the following results.

Proposition 3.1. *When the retailer is the quantity leader, then under the commission channel encroachment, compared with that when the retailer does not share information, the equilibrium outcomes when the retailer commits to information sharing exhibit the following properties:*

1. *the supplier sets a higher wholesale price (i.e., $w^{\mathcal{C}Y}(x) > w^{\mathcal{C}N}(x)$) if and*

Table 3.2: Subgame Equilibria: Commission Channel Encroachment in $\mathcal{RL}$

Information Scenario	Equilibrium Decisions	Ex-ante Profit
Sharing	$w^{\mathcal{C}Y} = w_0^{\mathcal{C}Y}(a + \frac{x}{1+s})$ $q_S^{\mathcal{C}Y} = \frac{\left(8-(\alpha+3)\gamma^2-2\gamma\right)(a+\frac{x}{1+s})}{2(8-(\alpha+5)\gamma^2)}$ $q_R^{\mathcal{C}Y} = \frac{2(1-\gamma)(a+\frac{x}{1+s})}{8-(\alpha+5)\gamma^2}$	$\Pi_S^{\mathcal{C}Y} = \Pi_{S_0}^{\mathcal{C}Y} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right)$ $\Pi_R^{\mathcal{C}Y} = \Pi_{R_0}^{\mathcal{C}Y} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right)$
No Sharing	$w^{\mathcal{C}N} = w_0^{\mathcal{C}N} a$ $q_S^{\mathcal{C}N} = q_{S_0}^{\mathcal{C}N} a + \frac{(2-\gamma)x}{(4-(\alpha+2)\gamma^2)(s+1)}$ $q_R^{\mathcal{C}N} = q_{R_0}^{\mathcal{C}N} a + \frac{(2-(\alpha+2)\gamma)x}{(4-(\alpha+2)\gamma^2)(s+1)}$	$\Pi_S^{\mathcal{C}N} = \Pi_{S_0}^{\mathcal{C}N} a^2 + \frac{(1-\alpha)(2-\gamma)^2\sigma^2}{(4-\alpha\gamma^2-2\gamma^2)^2(s+1)}$ $\Pi_R^{\mathcal{C}N} = \Pi_{R_0}^{\mathcal{C}N} a^2 + \frac{(\alpha+1)(1-\gamma)\sigma^2}{(4-(\alpha+2)\gamma^2)(s+1)}$

Notes:

$w_0^{\mathcal{C}Y} = \frac{\alpha^2\gamma^3+4\alpha\left(\gamma^2-2\right)\gamma+\gamma^3-6\gamma^2+8}{2(8-(\alpha+5)\gamma^2)}$, $\Pi_{S_0}^{\mathcal{C}Y} = \frac{\alpha^2\gamma^2+4\alpha\left(\gamma^2-2\right)-\gamma^2-8\gamma+12}{4(8-(\alpha+5)\gamma^2)}$,

$\Pi_{R_0}^{\mathcal{C}Y} = \frac{\alpha^3\gamma^4+2\alpha^2\left(5\gamma^2-8\right)\gamma^2+\alpha\left(21\gamma^4+8\gamma^3-84\gamma^2+64\right)-8(\gamma-1)^2\left(\gamma^2-2\right)}{4(8-(\alpha+5)\gamma^2)^2}$;

$w_0^{\mathcal{C}N} = \frac{(2-\alpha\gamma-\gamma)\left((\alpha+1)(\alpha+2)\gamma^3-8\alpha\gamma-6\gamma^2+8\right)}{2((\alpha^2+5\alpha+6)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16)}$,

$q_{S_0}^{\mathcal{C}N} = \frac{\left(\alpha^2+3\alpha+2\right)\gamma^3-4(2\alpha+3)\gamma-2\gamma^2+16}{2((\alpha^2+5\alpha+6)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16)}$,

$q_{R_0}^{\mathcal{C}N} = \frac{2(1-\gamma)(2-(\alpha+2)\gamma)}{(\alpha^2+5\alpha+6)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16}$, $\Pi_{S_0}^{\mathcal{C}N} = \frac{(2-\alpha\gamma-\gamma)\left((\alpha+1)(\alpha+2)\gamma^2-8\alpha-10\gamma+12\right)}{4((\alpha^2+5\alpha+6)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16)}$,

$\Pi_{R_0}^{\mathcal{C}N} = \frac{k_0}{4((\alpha^2+5\alpha+6)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16)^2}$.

$k_0 = \alpha^5\gamma^6 + 2\alpha^4\left(5\gamma^2-2\gamma-8\right)\gamma^4 + \alpha^3\left(33\gamma^4-32\gamma^3-96\gamma^2+64\gamma+64\right)\gamma^2+$
$4\alpha^2\left(11\gamma^5-19\gamma^4-42\gamma^3+84\gamma^2+20\gamma-64\right)\gamma+$
$4\alpha\left(5\gamma^6-10\gamma^5-23\gamma^4+76\gamma^3-28\gamma^2-80\gamma+64\right)+32(\gamma-1)^3\left(\gamma^2-2\right)$.

only if the demand signal $x > x_1$; otherwise, $w^{\mathcal{C}Y}(x) \leq w^{\mathcal{C}N}(x)$;

2. *the retailer orders less (i.e., $q_R^{\mathcal{C}Y}(x) < q_R^{\mathcal{C}N}(x)$) if and only if both the demand signal $x > x_2$ and the commission rate $\alpha < \widetilde{\alpha}$; otherwise, $q_R^{\mathcal{C}Y}(x) \geq q_R^{\mathcal{C}N}(x)$;*

3. *the supplier sets a lower direct-selling quantity (i.e., $q_S^{\mathcal{C}Y}(x) < q_S^{\mathcal{C}N}(x)$) if and only if both the demand signal $x > x_3$ and the commission rate $\alpha > \widetilde{\alpha}$; otherwise, $q_S^{\mathcal{C}Y}(x) \geq q_S^{\mathcal{C}N}(x)$.*

The thresholds x_1, x_2, x_3 and $\widetilde{\alpha}$ are all positive, and their detailed expressions are respectively stated in (B.2), (B.4), (B.5) *and* (B.3) *in the Appendix B.1.*

Proposition 3.1 shows that information sharing enables the supplier to increase the wholesale price, thereby improving her wholesaling profit margin when the demand turns out to be large. Proposition 3.1 also shows that the relationship between the optimal quantity decisions under the two information scenarios are jointly affected by the magnitudes of realized demand signal x and the commission rate α. Specifically, the retailer orders less when he does not share his information signal than that when he does, except that when the demand turns out to be extremely high and the commission rate is low. As to the supplier, she also direct sells less without information sharing than that with information sharing except that when both demand signal and the commission rate are very high.

A close look at Table 3.2 reveals that the optimal direct-selling and reselling quantities of the supplier and the retailer are respectively composed of two parts, one being independent of demand signal and the other being linear in the demand signal. Consequently, their respective quantity difference between the 'information sharing' and 'no information sharing' scenarios is also composed of two parts, one containing no information signal and the other containing information signal. Take the retailer's quantity difference under the two information scenarios, $q_R^{\mathcal{C}Y}(x) - q_R^{\mathcal{C}N}(x)$ as an illustration. As shown in the proof of Proposition 3.1, the part containing no demand signal is strictly positive. This positive sign implies that the action "information sharing" itself makes the retailer lose its information advantage, and thus in general he has to order more than that without information sharing by taking advantage of being the first mover. This effect strictly dominates when the demand signal is not so high, resulting in the retailer ordering less without information sharing. However, when the market demand turns out to be very large, the market situation reflected by the demand signal together with the retailer's information sharing choice determine his ordering behavior. Specially, the sign of the part containing demand signal changes from negative to positive as the commission rate increases; see the proof of Proposition 3.1. That is, the effect of the demand signal itself on the retailer's ordering incentives under the two information scenario is reversed as the commission rate becomes high enough. Consequently, compared to that without information sharing, the retailer still orders more when he shares his demand information if the commission rate is high, but would order less if the rate is low. Note that with a low commission rate, the supplier shares a little portion of her direct sales revenue with the retailer and thus can earn more by direct selling. Consequently, when the demand is high, this positive information, if shared, stimulates the supplier to set a much higher wholesale price to dampen the retailer's order incentives so that she can direct sell more. As a result, the retailer orders less if he indeed shares this positive information. In contrast, when the commission rate is very high, the supplier has less incentive to direct sell and would focus more on wholesaling. If the demand signal is shared, knowing for sure that the demand is large, the supplier would on the one hand, further increase her wholesale price to improve her wholesaling profit margin while on the other hand, further reduce her direct-selling quantity so as to motivate the retailer to order more, thereby increasing her revenue. In a similar vein, we can explain the supplier's direst-selling behavior.

Next, we compare the equilibrium outcomes by taking expectation with respect to the demand signal x.

Corollary 3.1. *When the retailer behaves as the quantity leader, then under the commission channel encroachment, compared to that without information sharing, when the retailer commits to information sharing, in equilibrium, the supplier lowers her wholesale price on average; that is, $E[w^{\mathcal{C}Y}(x)] < E[w^{\mathcal{C}N}(x)]$. The supplier/retailer respectively sets a higher direct-selling/reselling quantity on average, that is, $E[q_i^{\mathcal{C}Y}(x)] > E[q_i^{\mathcal{C}N}(x)]$, $i = S, R$.*

Corollary 3.1 implies that the retailer's commitment to information sharing incentivizes the supplier to lower her wholesale price on average (that is, taking the expectation with respect to the demand signal x). The underlying reason is as follows. When the demand signal is not shared, the supplier needs to infer it from the retailer's quantity decision. Anticipating the supplier's information inference, the retailer downward distorts his order quantity in comparison to that when he does share the information ($E[q_R^{\mathcal{C}N}(x)] < E[q_R^{\mathcal{C}Y}(x)]$). Consequently, the market demand inferred by the supplier is smaller than the actually observed signal. This dampens the supplier's direct selling incentive ($E[q_S^{\mathcal{C}N}(x)] < E[q_S^{\mathcal{C}Y}(x)]$). She then increases her wholesale price ($E[w^{\mathcal{C}N}(x)] > E[w^{\mathcal{C}Y}(x)]$) so as to earn more in the wholesaling business. This indicates that information sharing from the retailer can help mitigate the double marginalization on average. Similar results are observed in Li et al. (2014), in which the presence of information asymmetry can reduce the benefit of supplier encroachment as it can exacerbate double marginalization and lead to a less efficient supply chain.

We now derive the retailer's equilibrium information sharing decision by comparing his *ex ante* expected profit under the two information scenarios and obtain the following:

Proposition 3.2. *When the retailer acts as a quantity leader, then under the commission channel encroachment, the retailer always commits to share his information voluntarily, which also benefits the supplier. That is,* $\Pi_i^{\mathcal{C}Y} > \Pi_i^{\mathcal{C}N}$, $i = S, R$*. In addition, the supplier's ex-ante expected profit* $\Pi_S^{\mathcal{C}Y}$ *decreases in the commission rate* α*, while that of the retailer,* $\Pi_R^{\mathcal{C}Y}$*, as well as the total profit of the retailer and the supplier increase in* α*.*

Proposition 3.2 shows that when the retailer acts as the quantity leader, under the commission channel encroachment, voluntary information sharing from the retailer makes both himself and the supplier better off, leading to a *win-win* outcome. Proposition 3.2 also indicates that the increase of the commission rate hurts the supplier's profitability but makes the retailer better off. Here, the retailer's revenue comes from both reselling the supplier's product and sharing her direct selling revenue via charging the transaction-based commission fees, while the supplier's revenue comes from both wholesaling and direct selling the product. Note that a higher commission rate α indicates a larger *indirect* encroachment cost the supplier has to bear as it reduces her direct-selling profit margin, and thus hurting her. In contrast, a higher commission rate increases the retailer's commission revenue and thus makes him better off. It also strengthens the cooperative relationship between the two parties and results in a higher total profit for each.

3.4.2 Direct Channel Encroachment

We now consider the case when the supplier encroaches via the direct channel. Then the retailer does not share his demand information with the supplier. However, the supplier can still infer the information about the demand signal x based on the retailer's order quantity. The analyses are similar to those under the commission channel encroachment. Specifically, the supplier conjectures that the retailer adopts the following general monotonic decision rule when making his order q_R:

$$q_R = D(x), \text{ that is, } D^{-1}(q_R) = x, \tag{3.9}$$

where $D(x)$ is a strictly increasing function of the signal x. Given the retailer's order quantity $q_R(x)$ and the supplier's belief $q_R = D(x)$, the supplier decides the direct-selling quantity q_S to maximize her expected profit

$$E[\Pi_S \mid q_R(x)] = (a + E[X \mid q_R(x)] - q_S + \gamma q_R(x))q_S + wq_R(x) - c,$$

where c is the the *direct channel operating cost* that she has to bear for setting up the direct channel. We can show that

$$q_S(w, q_R(x)) = \frac{1}{2}\left(a + \frac{D^{-1}(q_R)}{1+s} - \gamma q_R\right) \text{ and } \frac{dq_S}{dq_R} = \frac{1}{2}\left(\frac{1}{1+s} \cdot \frac{1}{D'(x)} - \gamma\right). \tag{3.10}$$

As to the retailer, he makes the order decision to maximize his expected profit

$$E[\Pi_R \mid x] = (a + E[X \mid x] - q_R - \gamma q_S(w, q_R(x)) - w)q_R.$$

Upon substituting $q_S(w, q_R(x))$ into the above function, we can derive the first-order derivative of $E[\Pi_R \mid x]$ with respect to q_R. Note that in equilibrium, the supplier's belief $q_R = D(x)$ must be fulfilled. That is, $D(x) = \arg\max_{q_R} E[\Pi_R \mid x]$. This requires that $D(x)$ satisfies the following differential equation in equilibrium:

$$(1+s)\left((2-\gamma)a - 2w\right) + (2-\gamma)x + 2(1+s)(\gamma^2 - 2)D(x) - \frac{\gamma D(x)}{D'(x)} = 0. \tag{3.11}$$

Lemma 3.2. *When the retailer is the quantity leader, then under the direct channel encroachment and among all the general monotonic relationships between* *the retailer's order quantity and demand signal x, the linear rule is the unique equilibrium, i. e.,*

$$q_R^{\mathcal{D}}(w) = D(x) = \frac{(1-\gamma)((2-\gamma)a - 2w)}{\gamma^3 - 2\gamma^2 - 2\gamma + 4} + \frac{1-\gamma}{(1+s)(2-\gamma^2)}x.$$

Based on lemma 3.2, we can derive the optimal direct-selling quantity $q_S^{\mathcal{D}}(w)$ by substituting $q_R^{\mathcal{D}}(w)$ into (3.10). Then, plugging $q_S^{\mathcal{D}}(w)$ and $q_R^{\mathcal{D}}(w)$ into the supplier's expected profit function, we can derive the optimal wholesale price $w^{\mathcal{D}}$. Table 3.3 summarizes the equilibrium outcomes.

Table 3.3: Subgame Equilibria: Direct Channel Encroachment in $\mathcal{RL}$

Equilibrium Decisions	Ex-ante Profit
$w^{\mathcal{D}} = \frac{a(2-\gamma)^3(\gamma+1)}{2(3\gamma^3-5\gamma^2-4\gamma+8)}$	$\Pi_S^{\mathcal{D}} = \frac{a^2(3-\gamma)(\gamma-2)^2}{4(3\gamma^3-5\gamma^2-4\gamma+8)} + \frac{(\gamma-2)^2\sigma^2}{4(\gamma^2-2)^2(s+1)} - c$
$q_S^{\mathcal{D}} = \frac{a(4-\gamma^2-\gamma)(2-\gamma)}{2(3\gamma^3-5\gamma^2-4\gamma+8)} + \frac{(2-\gamma)x}{2(2-\gamma^2)(s+1)}$	$\Pi_R^{\mathcal{D}} = \frac{2a^2(2-\gamma^2)(1-\gamma)^3}{(3\gamma^3-5\gamma^2-4\gamma+8)^2} + \frac{(1-\gamma)\sigma^2}{2(2-\gamma^2)(s+1)}$
$q_R^{\mathcal{D}} = \frac{2a(1-\gamma)^2}{3\gamma^3-5\gamma^2-4\gamma+8} + \frac{(1-\gamma)x}{(2-\gamma^2)(s+1)}$	

3.4.3 Encroachment Channel Selection

Now we are ready to investigate the supplier's optimal encroachment channel s-election when the retailer is the quantity leader. Recall that the retailer always shares his demand information under the commission channel encroachment. The supplier's optimal ex-ante expected profit $\Pi_S^{\mathcal{C}}$ when encroaching via the commission channel then equals $\Pi_S^{\mathcal{C}Y}$. By comparing the supplier's equilibrium ex-ante expected profits under the two encroachment approaches, $\Pi_S^{\mathcal{D}}$ and $\Pi_S^{\mathcal{C}}$, we get the following result:

Proposition 3.3. *When the retailer is the quantity leader, there exists a threshold $\underline{\alpha}(\gamma)$ such that*

1. *when the commission rate $\alpha \le \underline{\alpha}(\gamma)$, the supplier always encroaches via the commission channel;*

2. *otherwise, the supplier encroaches via the commission channel only if the direct channel operating cost c is greater than a threshold $\widehat{c}_1(s)$.*

Furthermore, $\widehat{c}_1(s)$ is decreasing in $1/(s\sigma^2)$, the signal accuracy measure, when $\alpha \in (\underline{\alpha}(\gamma), \bar{\alpha}(\gamma))$, and increasing in $1/(s\sigma^2)$ when $\alpha \geq \bar{\alpha}(\gamma)$, where $\bar{\alpha}(\gamma) < \frac{1}{2}$. The expression of $\widehat{c}_1(s)$ is presented in (B.7), and $\underline{\alpha}(\gamma)$ and $\bar{\alpha}(\gamma)$ are determined by (B.6), as stated in the Appendix B.1.

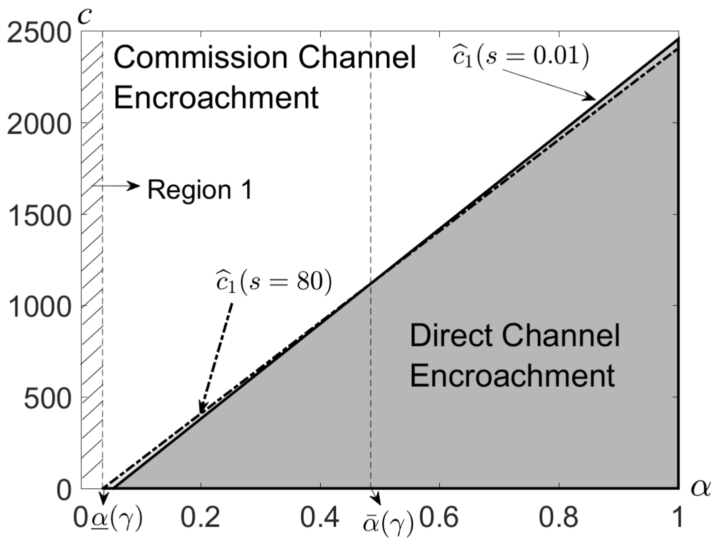


Figure 3.3: Supplier's Encroachment Channel Selection When the Retailer is the Leader: $a = 100, \gamma = 0.2, \sigma = 20$

Proposition 3.3 shows that the supplier's optimal encroachment channel selection highly hinges on the magnitude of the commission rate α and the direct channel operating cost c; see Figure 3.3. Recall that the supplier bears the encroachment cost c when establishing her own direct channel, but not when encroaching via the commission channel, in which case she must share her sales revenue with the retailer in the form of commission fees. One good side of the commission channel encroachment, when compared to the direct channel encroachment, is the cooperative relationship between the parties, and that benefits the retailer further from their revenue-sharing-type commission contract. This weakens the downstream competition between them. A higher α indicates a higher commission channel encroaching cost for the supplier. Which channel the supplier selects to encroach by is determined by the tradeoff between the above-mentioned driving forces. Proposition 3.3 indicates that when the commission rate α is sufficiently low ($\alpha \leq \underline{\alpha}(\gamma)$), the supplier always encroaches via the commission channel, even if it were costless to establish her own direct channel. This is because under this situation, the supplier encounters a small commission channel encroaching cost while the gain from their cooperative relationship is so pronounced that the supplier always prefers the commission channel encroachment. In contrast, when the commission rate α is large, the supplier encroaches via the commission channel only if the encroachment cost difference between the two channels are not so large.

Proposition 3.3 further implies that the demand information accuracy does affect the attractiveness of the commission channel encroachment. In particular, when the commission rate α is moderate (i.e., $\alpha \in (\underline{\alpha}(\gamma), \bar{\alpha}(\gamma))$), a more accurate signal (i.e., a smaller s) makes the supplier more likely to adopt the commission channel encroachment. That is, the commission channel encroachment region depicted in Figure 2.2 will become larger as s decreases. Surprisingly, when α is large ($\alpha \geq \bar{\alpha}(\gamma)$), a more accurate signal actually makes the supplier more likely adopt the direct channel encroachment. The underlying reasons are as follows. Recall that in the commission channel, the retailer shares his demand information freely with the supplier, while in the direct channel, she has to infer the demand signal from the retailer's ordering behavior. A close look at the equilibrium outcomes listed in Tables 3.2 and 3.3 reveal that indeed a more accurate information signal benefits the supplier no matter which channel she encroaches through. However, the supplier's profit under the direct channel is independent of the commission rate α, while in the commission channel, her profit gain brought about by information sharing is decreasing in it [3.3]. That is, the benefit from knowing the information signal is lessened as the commission rate increases. Consequently, the effect of information signal accuracy on the supplier's encroachment channel preference is found to be reversed as the commission rate becomes large enough.

[3.3] That is, $\partial(\Pi_{S_0}^{CY} \cdot \frac{\sigma^2}{1+s})/(\partial\alpha) = -\frac{(\alpha\gamma^2+3\gamma^2+2\gamma-8)(\alpha\gamma^2+7\gamma^2-2\gamma-8)}{4(8-(\alpha+5)\gamma^2-8)^2} \cdot \frac{\sigma^2}{1+s} < 0$.

3.5 Supplier Acts as Quantity Leader ($\mathcal{SL}$ Scenario)

We now consider the $\mathcal{SL}$ scenario in which the supplier acts as the leader in the quantity decision stage. Similar to that in §3.4, we first derive the equilibrium outcomes under the two encroachment approaches. We then conduct the comparison of these equilibrium outcomes to investigate the supplier's endogenous encroachment channel selection decision. For ease of reference, we use the superscript $\tilde{}$ to denote the equilibrium outcomes under the $\mathcal{SL}$ scenario.

3.5.1 Commission Channel Encroachment

Here we analyze the equilibrium outcome under the commission channel encroachment when the supplier is the quantity leader. The derivation and analysis are quite similar to those stated in §3.4.1 except that in the quantity competition stage, the supplier moves first as the Stackelberg leader, followed by the retailer. In this setting, the supplier cannot infer the demand signal through the retailer's order decision. That is, there is no signaling game between them. Given the demand signal x, the wholesale price w and the supplier's direct-selling quantity q_S, the retailer maximizes his expected profit

$$E[\Pi_R \mid x] = (a+E[X \mid x]-q_R-\gamma q_S-w)q_R+\alpha(a+E[X \mid x]-q_S-\gamma q_R)q_S. \quad (3.12)$$

It can be easily shown that his optimal order quantity shall be

$$q_R(q_S, w) = \frac{1}{2}\left(a + E[X \mid x] - w - (\alpha+1)\gamma q_S\right).$$

Anticipating the retailer's order quantity $q_R(q_S, w)$, the supplier maximizes her expected profit conditional on her own information set I_Θ, where $I_\Theta = x$ if the retailer shares his demand information with the supplier and $I_\Theta = \phi$, the empty set, if the retailer does not share. Given the wholesale price w, her expected profit then can be written as

$$\begin{aligned} E[\Pi_S \mid w, I_\Theta] = {} & (1-\alpha)\left(a + E[X \mid I_\Theta] - q_S - \gamma E[q_R(q_S, w) \mid I_\Theta]\right) q_S \\ & + wE[q_R(q_S, w) \mid I_\Theta], \end{aligned} \quad (3.13)$$

where $E[q_R(q_S, w) \mid I_\Theta] = \frac{1}{2}(a + E[X \mid x] - w - (\alpha+1)\gamma q_S)$ and $E[X \mid I_\Theta] = E[X \mid x] = \frac{x}{1+s}$, if the retailer shares the demand information; otherwise, $E[X \mid I_\Theta] = 0$ and $E[q_R(q_S, w) \mid I_\Theta] = \frac{1}{2}(a - w - (\alpha+1)\gamma q_S)$. We then can derive the supplier's optimal quantity

$$q_S(w \mid I_\Theta) = \frac{a(1-\alpha)(2-\gamma) - 2\alpha\gamma w}{2(1-\alpha)\left(2-(\alpha+1)\gamma^2\right)} + \frac{2-\gamma}{2(2-(1+\alpha)\gamma^2)} E[X \mid I_\Theta].$$

Substituting $q_S(w \mid I_\Theta)$ into $q_R(q_S, w)$ yields the retailer's optimal quantity $q_R(w)$. Further substituting $q_S(w \mid I_\Theta)$ and $q_R(w)$ into (3.13) yields the supplier's expected profit function $\Pi_S(w \mid I_\Theta)$.

It can be shown that only when $2\alpha + \gamma < 2$ will the supplier set a strictly *positive* wholesale price that leads to a strictly *positive* direct-selling quantity q_S.[3.4] Hence, hereafter we focus on the case when $2\alpha + \gamma < 2$. Recall that the competition intensity $\gamma < 1$. The condition $2\alpha + \gamma < 2$ generally holds in reality as the commission rate α is often small and less than 10% (see JD.com-Website 2020). Under such a situation, $\Pi_S(w \mid I_\Theta)$ is concave in w, and the optimal equilibrium wholesale price can be derived as follows by solving the first-order condition:

$$\widetilde{w}^{\mathcal{C}i} = \frac{(1-\alpha)\left(2-2\alpha\gamma-\gamma^2\right)(a + E[X \mid I_\Theta])}{2\left(2-2\alpha-\gamma^2\right)}, i = Y, N,$$

where $i = Y$ stands for the 'information sharing' scenario and $i = N$ 'no information sharing'. Substituting $\widetilde{w}^{\mathcal{C}i}$, $i = Y, N$ into $q_S(w \mid I_\Theta)$ and $q_R(w)$ yields the equilibrium quantity decisions of the supplier and the retailer $\widetilde{q}_j^{\mathcal{C}i}$, $j = S, R$ under the two information scenarios. Then, plugging them into (3.12) and (3.13), we can obtain the corresponding ex ante profits of the supplier and the retailer, $\widetilde{\Pi}_j^{\mathcal{C}i}$, $i = Y, N; j = S, R$ under each information scenario by taking the expectation with respect to the demand signal x. Table 3.4 summarizes the equilibrium outcomes under both information scenarios.

A close look at the the equilibrium outcomes listed in Table 3.4 reveals that under the commission channel, when the supplier is the quantity leader, the equilibrium wholesale price and quantity decisions remain the same *on average*

[3.4]The detailed discussion about this requirement can be found in the Appendix B.2.

Table 3.4: Subgame Equilibria: Commission Channel Encroachment in $\mathcal{SL}$

Information Scenario	Equilibrium Decisions	Ex-ante Profit
Share	$\widetilde{w}^{\mathcal{C}Y} = \widetilde{w}_0\left(a + \frac{x}{1+s}\right)$ $\widetilde{q}_S^{\mathcal{C}Y} = \widetilde{q}_{S_0}\left(a + \frac{x}{1+s}\right)$ $\widetilde{q}_R^{\mathcal{C}Y} = \widetilde{q}_{R_0}\left(a + \frac{x}{1+s}\right)$	$\widetilde{\Pi}_S^{\mathcal{C}Y} = \widetilde{\Pi}_{S_0} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right)$ $\widetilde{\Pi}_R^{\mathcal{C}Y} = \widetilde{\Pi}_{R_0} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right)$
Not Share	$\widetilde{w}^{\mathcal{C}N} = \widetilde{w}_0 \cdot a$ $\widetilde{q}_S^{\mathcal{C}N} = \widetilde{q}_{S_0} \cdot a$ $\widetilde{q}_R^{\mathcal{C}N} = \widetilde{q}_{R_0} \cdot a + \frac{x}{2(1+s)}$	$\widetilde{\Pi}_S^{\mathcal{C}N} = \widetilde{\Pi}_{S_0} \cdot a^2$ $\widetilde{\Pi}_R^{\mathcal{C}N} = \widetilde{\Pi}_{R_0} \cdot a^2 + \frac{\sigma^2}{4(1+s)}$

Notes:
$\widetilde{w}_0 = \frac{(1-\alpha)\left(2-2\alpha\gamma-\gamma^2\right)}{2(2-2\alpha-\gamma^2)}$, $\widetilde{q}_{S_0} = \frac{2-2\alpha-\gamma}{2(2-2\alpha-\gamma^2)}$, $\widetilde{q}_{R_0} = \frac{(1-\alpha)(1-\gamma)}{2(2-2\alpha-\gamma^2)}$, $\widetilde{\Pi}_{S_0} = \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4(2-2\alpha-\gamma^2)}$,
$\widetilde{\Pi}_{R_0} = \frac{4\alpha^3+\alpha^2\left(5\gamma^2-2\gamma-7\right)+\alpha\left(2\gamma^3-7\gamma^2+4\gamma+2\right)+(\gamma-1)^2}{4(2-2\alpha-\gamma^2)^2}$.

regardless of whether the retailer shares his demand information or not; that is, $E[\widetilde{w}^{\mathcal{C}Y}(x)] = E[\widetilde{w}^{\mathcal{C}N}(x)]$ and $E[\widetilde{q}_i^{\mathcal{C}Y}(x)] = E[\widetilde{q}_i^{\mathcal{C}N}(x)], i = S, R$. This is in sharp contrast to that stated in Corollary 3.1 when the retailer is the quantity leader instead. This is because now the supplier moves first by setting her direct-selling quantity and thus cannot infer the demand signal from the retailer's order quantity if the retailer does not share his demand information. Thus, the retailer has no incentive to distort the order quantity.

We now derive the retailer's equilibrium information sharing decision and how it impacts the supplier's profitability.

Proposition 3.4. *When the supplier is the quantity leader, then under the commission channel encroachment and for any commission rate $\alpha < 1-\frac{\gamma}{2}$, the retailer shares his demand information with the supplier if, and only if, the competition intensity $\gamma \in (0, \gamma_0)$ and the commission rate $\alpha \in (\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_2(\gamma))$, where*

(i) $\gamma_0 \in (\frac{9}{100}, \frac{1}{10})$ *satisfies* $600\gamma^6 - 1428\gamma^5 + 793\gamma^4 + 298\gamma^3 - 251\gamma^2 - 24\gamma + 4 = 0$;

(ii) $\widetilde{\alpha}_1(\gamma)$ *and* $\widetilde{\alpha}_2(\gamma)$ *are the two feasible roots, i.e., within* $[0, 1]$*, of the equation*

$$\Gamma(\alpha,\gamma) = 4\alpha^3 + \alpha^2\left(5\gamma^2 - 2\gamma - 11\right) + \alpha\left(2\gamma^3 - 11\gamma^2 + 4\gamma + 10\right) - \gamma^4 + 5\gamma^2 - 2\gamma - 3 = 0,$$

and both are greater than $\frac{1}{2}$.

Moreover, this information sharing also benefits the supplier.

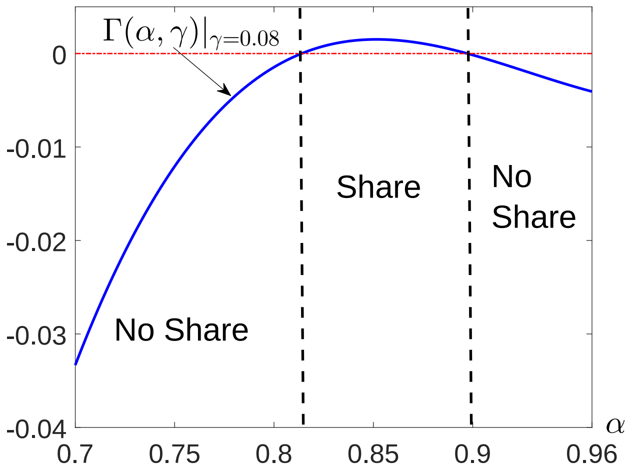


Figure 3.4: The Function $\Gamma(\alpha, \gamma)$

Proposition 3.4 shows that when the supplier is the quantity leader, the retailer's information sharing incentive depends highly on the magnitudes of the commission rate α and the competition intensity γ. This is in a sharp contrast to the case when the retailer is the quantity leader and always shares his demand information; see Proposition 3.2. Specifically, when the competition is intense, the retailer will never share his information with the supplier. This is because the supplier's wholesale price and direct-selling quantity both respond positively to the demand signal. Information sharing will further intensify the downstream competition between the two parties. Only when the competition intensity is sufficiently low ($\gamma < \gamma_0$) will the retailer share his information if the commission rate falls into an intermediate range $(\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_2(\gamma))$. The underlying reason is that under this circumstance, his reselling profit loss from information sharing is not so large as there is less intense competition, while his profit gain from sharing the supplier's direct-selling revenue is much inreased as the supplier now has more accurate demand information. Consequently, information sharing makes the retailer better off.

3.5.2 Direct Channel Encroachment

We now examine the direct channel encroachment when the supplier acts as the quantity leader. Again, we solve the game via backward induction. Given the supplier's direct-selling quantity q_S, the retailer maximizes his expected profit conditional on the demand signal x as given below:

$$E[\Pi_R \mid x] = (a + E[X \mid x] - q_R - \gamma q_S - w)q_R.$$

It can be easily shown that his optimal order quantity

$$q_R(q_S) = \frac{1}{2}\left(a + E[X \mid x] - w - \gamma q_S\right).$$

Anticipating the retailer's order decision $q_R(q_S)$, the supplier then maximizes her expected profit

$$E[\Pi_S] = (a + E[X] - q_S - \gamma E[q_R(q_S)])q_S - c + wE[q_R(q_S)]. \tag{3.14}$$

It can be also easily shown that

$$\widetilde{q}_S^{\mathcal{D}} = \frac{(2-\gamma)a}{2(2-\gamma^2)}.$$

Substituting $\widetilde{q}_S^{\mathcal{D}}$ into (3.14), we can show that the supplier's expected profit is concave in w, from which we can derive the optimal wholesale price

$$\widetilde{w}^{\mathcal{D}} = \frac{a}{2}.$$

Similar to the analysis in the previous subsection, we can obtain the ex ante profits of the retailer and the supplier as given below:

$$\widetilde{\Pi}_S^{\mathcal{D}} = \frac{a^2(3-2\gamma)}{4\left(2-\gamma^2\right)} - c \text{ and } \widetilde{\Pi}_R^{\mathcal{D}} = \frac{a^2(\gamma-1)^2}{4\left(\gamma^2-2\right)^2} + \frac{\sigma^2}{4(s+1)}.$$

3.5.3 Supplier's Encroachment Channel Selection

In this section, we examine the supplier's optimal encroachment channel selection when she is the quantity leader. Specifically, she will compare her equilibrium ex ante profit $\widetilde{\Pi}_S^{\mathcal{D}}$ under the direct channel encroachment with that under the commission channel encroachment, which is $\widetilde{\Pi}_S^{\mathcal{C}Y}$ if the retailer shares his information in equilibrium, and is $\widetilde{\Pi}_S^{\mathcal{C}N}$, otherwise. We then obtain the following result.

Proposition 3.5. *When the supplier acts as a quantity leader, we have two thresholds* $\widehat{c}_2(s) := \left[\frac{a^2(3-2\gamma)}{4(2-\gamma^2)} - \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4(2-2\alpha-\gamma^2)} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right)\right]^+$ *and*
$\widehat{c}_3 := \left[\frac{a^2(3-2\gamma)}{4(2-\gamma^2)} - \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4(2-2\alpha-\gamma^2)} a^2\right]^+$ *such that*

i. when the competition intensity $\gamma \in (0, \gamma_0)$ *and the commission rate* $\alpha \in (\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_2(\gamma))$*, the supplier encroaches through the commission channel only if the direct channel operating cost* $c \geq \widehat{c}_2(s)$*;*

ii. otherwise, the supplier encroaches through the commission channel only if $c \geq \widehat{c}_3$*.*

Moreover, $\widehat{c}_2(s) \leq \widehat{c}_3$*,* $\widehat{c}_2(s)$ *is decreasing in the signal accuracy measure* $1/(s\sigma^2)$*, and* $\widehat{c}_3$ *is independent of* $1/(s\sigma^2)$*.*

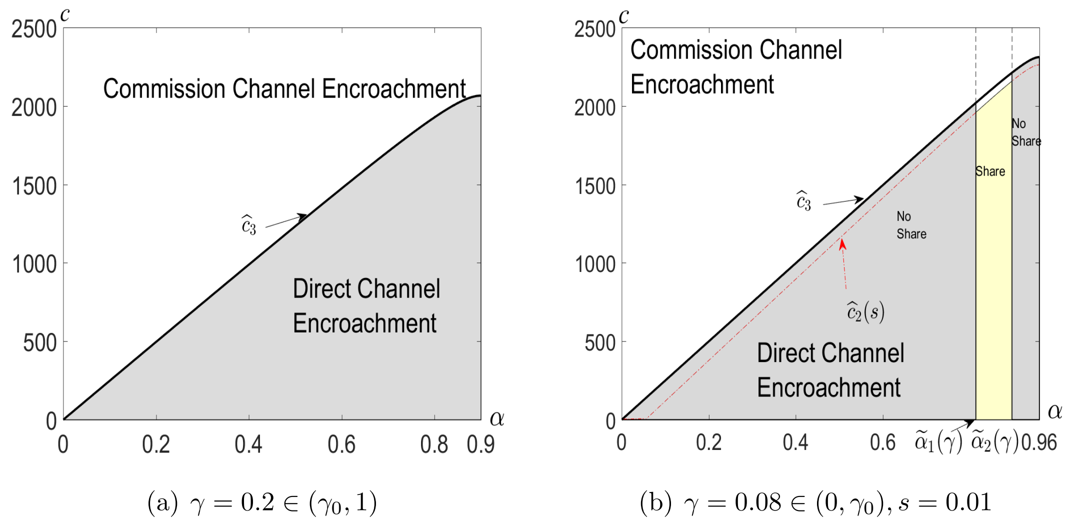


Figure 3.5: Supplier's Encroachment Channel Selection When Supplier is Leader: $a = 100$, $\sigma = 20$

Proposition 3.5 shows that when the supplier is the quantity leader, competition intensity γ together with the commission rate α and the direct channel operating cost c determine the supplier's encroachment channel preference; see Figure 3.5. Again, the high direct channel operating cost hinders the supplier from establishing her own direct channel and thus the supplier is more likely to encroach via the commission channel. Proposition 3.5 implies that when the commission rate falls into a moderate range $(\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_2(\gamma))$, a high competition intensity ($\gamma \geq \gamma_0$) makes the supplier more likely to adopt the direct channel encroachment. This is because under this circumstance, the retailer would share his demand information with the supplier only if the competition intensity is low, i.e., $\gamma < \gamma_0$ as stated in Proposition 3.4. Moreover, in this situation, a higher information signal accuracy enhances the attractiveness of the commission channel encroachment when the competition intensity is low. The reason is that the supplier now can make better quantity decision after receiving the demand information, which makes her better off.

3.6 Impact of Quantity Leadership

So far, we have derived the retailer's information sharing incentive and the supplier's encroachment channel preference under the two leadership scenarios, $\mathcal{RL}$ and $\mathcal{SL}$. The analyses in §3.4 and §3.5 demonstrate that quantity leadership significantly affects the retailer's incentive to share his demand information in the commission channel, which in turn, impacts the supplier's encroachment channel choice. In this section, we are going to conduct a comprehensive comparison of the equilibrium outcomes stated in §3.4 and §3.5 to examine the impact of quantity leadership.

First, a close look at the equilibrium outcomes stated in Propositions 3.2 and 3.4 reveals that the retailer acting as the quantity leader is always willing to share his demand information with the supplier. Instead, when he acts as a quantity follower, he voluntarily shares his demand information if and only if the commission rate is moderate and the competition is less intense. In sum, the first-mover advantage coming from quantity leadership strongly enhances his information sharing incentive.

Next, we compare the equilibrium quantity decisions of the retailer and the supplier in the two leadership scenarios, and obtain the following result:

Proposition 3.6. *Under both the commission and direst channel encroachments, the supplier sets a higher direct-selling quantity in $\mathcal{RL}$ than that in $\mathcal{SL}$, while the retailer sets a higher reselling quantity in $\mathcal{SL}$ than that in $\mathcal{RL}$. That is,*

$q_S^k(x) > \widetilde{q}_S^k(x)$ and $q_R^k(x) < \widetilde{q}_R^k(x)$, $k = \mathcal{C}, \mathcal{D}$.

Interestingly, Proposition 3.6 indicates that the supplier direct sells more aggressively when she is a quantity follower than when she is a leader under both encroachment scenarios. This is somewhat surprising because the common wisdom would suggest that a supplier exploits the first-mover advantage by ordering a large direct-selling quantity. The underlying reasons are as follows. Recall that under the direct channel encroachment, the supplier cannot access the demand information. Thus, she would like to take the advantage of the retailer's information to improve her profit from wholesaling by increasing the wholesale price. This, however, can be achieved only if she can motivate the retailer to order more by committing to lower her direct-selling quantity. Such a direct-selling quantity reduction is credible only when the supplier behaves as the quantity leader and moves first. Indeed, we can verify that the wholesale price under $\mathcal{SL}$ is larger than that under $\mathcal{RL}$, i.e., $\widetilde{w}^{\mathcal{D}} > w^{\mathcal{D}}$. A similar rationale applies to the commission channel encroachment as well.

We now characterize how quantity leadership, commission rate and direct channel operating cost intervene and affect the supplier's encroachment channel selection.

Proposition 3.7. *There exist an upper threshold $\bar{c}$ and a lower threshold $\underline{c}$ such that when the direct channel operating cost $c \leq \underline{c}$, the supplier encroaches through the direct channel, while when $c \geq \bar{c}$, the supplier encroaches through the commission channel. When $\underline{c} < c < \bar{c}$,*

1. *the supplier encroaches through the commission channel when she is the quantity follower but through the direct channel when she is the leader if the commission rate α is less than the threshold $\widehat{\alpha}$;*

2. *otherwise, the supplier encroaches through the commission channel when she is the leader but through the direct channel when she is the follower.*

The detailed expressions of $\widehat{\alpha}$, $\underline{c}$ and $\bar{c}$ are summarized in (B.18) stated in the Appendix B.1.

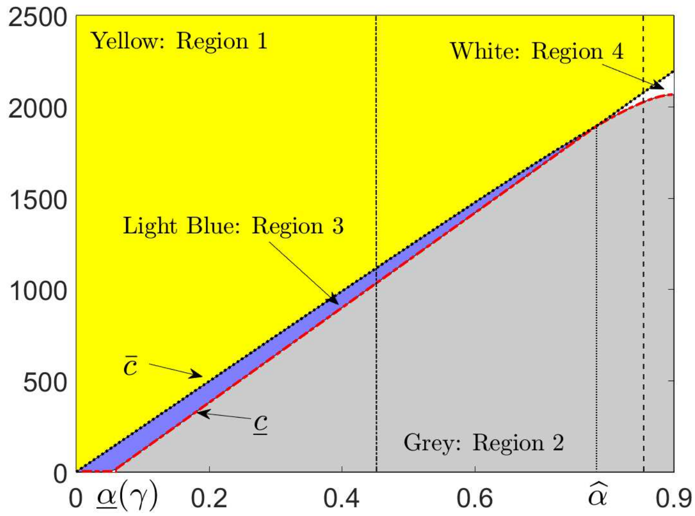


Figure 3.6: Supplier's Equilibrium Encroachment Decision: $a = 100, \sigma = 20, s = 0.01, \gamma = 0.2$, where Regions 1– Always Commission Channel Encroachment; Regions 2– Always Direct Channel Encroachment; Region 3– Commission Channel Encroachment under $\mathcal{RL}$ but Direct Channel Encroachment under $\mathcal{SL}$; and Region 4– Commission Channel Encroachment under $\mathcal{SL}$ but Direct Channel Encroachment under $\mathcal{RL}$.

Proposition 3.7 shows that the supplier always encroaches via the commission (direct, resp.) channel when it is too costly (inexpensive, resp.) to set up her own direct channel, regardless of whether she is the quantity leader or not; see Figure 3.6. There, the black dot line and red dash-dot line represent the upper and lower operating costs $\bar{c}$ and $\underline{c}$, respectively. Note that the lower threshold $\underline{c}$ can be set at zero when the commission rate α is negligibly small. However, when the direct channel operating cost c falls into an intermediate range $(\underline{c}, \bar{c})$, quantity leadership significantly affects the supplier's encroachment channel selection. Specifically, in this situation, the supplier encroaches through the direct channel when she is the quantity leader, but through the commission channel when she is the follower if the commission rate is low ($\alpha \leq \widehat{\alpha}$). When the commission rate is high, the exact opposite holds.

A larger commission rate α implies a higher encroachment cost the supplier needs to bear when encroaching through the commission channel, as she needs to share a higher proportion of her direct sales revenue with the retailer. Nonetheless, the commission channel encroachment also implies a strengthened cooperative relationship between the two parties as the retailer now can obtain revenues from two resources, providing the commission platform service and conducting the reselling business, and thus may have incentive to reduce his reselling quantity. On the contrary, when the supplier establishes her own direct channel, the retailer only has one revenue source, that is, reselling the supplier's product. Thus, the competition between the retailer and the supplier shall be much fiercer under the direct channel encroachment compared to that under the commission channel encroachment. When the commission rate is low, the encroachment cost that the supplier bears if encroaching via the commission channel is not large. Thus, the supplier would encroach through the direct channel only if she is the quantity leader. Under this situation, she can exploit her first-mover advantage to pre-commit the direct-selling quantity to weaken the downstream competition. Instead, if she is the follower, she cannot pre-commit to the selling quantity and thus would encroach through the commission channel to reduce the competition with the retailer. When the commission rate is high, on one hand, the commission channel becomes less attractive to the supplier due to the high commission fees, while on the other, the retailer has less incentive to compete with the supplier fiercely in the downstream market due to their strengthened cooperative relationship. Which channel to encroach through is determined by the tradeoff between the supplier's two revenue sources: the wholesaling revenue and the direct sales revenue. Only when the supplier is the quantity leader can she freely adjust both her wholesale price and direct-selling quantity by anticipating the retailer's order decision to maximize her two revenue sources to mitigate the revenue loss from paying commission fees. Then the commission channel encroachment with a reduced downstream competition makes her better off. Proposition 3.7 indicates that when encroaching into the retail market, the supply chain parties need to take the role of quantity leadership into serious consideration.

3.7 Conclusion

Encroachment channel selection is a strategic decision faced by the supplier when she intends to extend her business by downstream encroachment. Sometimes, the supplier may prefer to independently open a store to access customers directly, while in other situations, the supplier may prefer to open a store on a platform operated by her retailer, especially when the retailer has private information about the market demand. The channel that the supplier will use to encroach hugely affects her own profitability. Whether or not the retailer has the incentive to share his demand information also makes the issue more complicated.

In this chapter, we set up a stylized game-theoretic model to characterize the retailer's information sharing incentive and the supplier's encroachment channel selection under two quantity leadership scenarios: the supplier acts as the Stackelberg quantity leader versus the retailer acts as the Stackelberg quantity leader. We show that quantity leadership greatly affects the retailer's information sharing incentive. Specifically, in the commission channel, the retailer always shares his demand information when acting as a quantity leader; however, he may not do so when acting as a follower. We also show that the supplier's encroachment channel selection highly hinges on whether she is the quantity leader or not, the magnitude of the direct channel operating cost, and the commission rate. For any given commission rate, there exists an upper (lower, resp.) threshold direct channel operating cost, above (below, resp.) which the supplier encroaches via the commission (direct, resp.) channel regardless of her quantity leadership. When the direct channel operating cost falls between these two thresholds, the supplier adopts the commission (direct, resp.) channel encroachment when the retailer is the quantity leader (follower, resp.) if the commission rate is low; otherwise, the exact opposite holds. We also show that increasing the information signal accuracy does not necessarily improve the attractiveness of the commission channel encroachment, especially when the commission rate is high.

Chapter 4

Discussions and Future Work

We focus on two types of online platforms in the thesis. In the first part, we research into a very practical issue that the female users may have safety concern and investigate how such concerns affect a ride-hailing platform's system configurations and pricing and wage decisions. We show that compared with the current pooling system configuration, the hybrid system can simultaneously increase platform's profit and safety-concerned female users' joining probability when the female users have a low level confidence toward the ride-hailing system's safety environment. That is, a win-win outcome can be attained by changing the system configuration. Our results also show that in the current pooling system, when the platform takes measures to improve the safety confidence level of the female users, it should put more efforts on the rider side than that on the driver side. Besides, in a hybrid system, giving the female drivers flexibility to let them select between the pooling subsystem and the female-only subsystem may hurt the profit of the platform.

Can the hybrid system be further extended with more options embedded to cater to demands from other groups? For example, there are also safety concerns for the elderly and children. The demand from these groups is usually low: in the United States, only about 6% of Uber riders are over 55 years old (Iqbal 2019); in China, more than 80% of DiDi riders are between 24-29 years old (CIW 2016). How to provide a safe service for such minority groups remains another important and challenging question, and we leave it for future research. Moreover, we consider subsystem-based prices and wages in a hybrid system. We admit that in practice, due to legal issues, platforms may need to offer the same price and pay the same wage in both the pooling and female-only subsystems. The equilibrium performance of the platform under such a situation would be no better than that when subsystem-based prices and wages are allowed, as the platform loses pricing flexibility. In our study, the number of registered male and female drivers is exogenous. When a platform switches from pooling to a hybrid system, the labor pool of female drivers may become larger, as safety-concerned female drivers can now join the female-only subsystem to serve female riders only. In such a circumstance, the benefit of adopting the hybrid system should be higher than what we have demonstrated. Through our analysis, we find that the shortage of female drivers is a key reason for the difficulty of solving gender-based safety issues in ride-hailing platforms. Note that social norms also affect females' work incentives. Driving is often considered a more appropriate job for males than for females, especially in places such as Egypt and Indonesia (IFC 2018). For instance, 57% of men surveyed say they would be unhappy if a female family member signed up as an Uber driver (IFC 2018). Encouraging more female drivers to participate in ride-hailing services thus requires not only efforts from ride-hailing platforms but also efforts from governments and NGOs to change attitudes toward female drivers.

We then focus on other type of online platform who provides reselling business and allows suppliers to directly sell the products to the end markets. We explore how the quantity leadership in the competition affects the retailer's information sharing and the supplier's encroachment channel selection. Admittedly, our model has some limitations. First, in the current setting, we do not consider the possibility that the supplier can access the demand information if she encroaches through the direct channel due to her small scale and limited data collection ability. Second, we do not make the leadership an endogenized decision of the supplier and the retailer. Instead, we focus on exploring the impact of leadership on the supplier's encroachment channel selection. If the supplier or the retailer has a dominant channel power and can determine which leadership to take, the encroachment channel selection problem becomes much more complicated. For example, we can show that when the supplier encroaches through the direct channel, the retailer always prefers to act as the quantity follower (i.e., $\Pi_R^{\mathcal{D}} < \widetilde{\Pi}_R^{\mathcal{D}}$). That is, if the retailer can unilaterally determine his leadership role, he always prefers the supplier to be the quantity leader under the direct channel encroachment. Anticipating this, the supplier's strategic encroachment channel selection will be undoubtedly affected. We leave these issues as future research topics.

Appendix A

Proofs and Derivations for Chapter 2

A.1 Proofs of Propositions

A.1.1 Proof of Proposition 2.1

This proposition can be easily obtained based on Propositions A.2.1, A.2.2, A.2.3 and A.2.4 stated in Appendix A.2.

A.1.2 Proof of Proposition 2.2

Based on Propositions A.2.1, A.2.2, A.2.3 and A.2.4, when either $N_\phi \mu \leq \Lambda_\phi$ or $\beta > \widehat{\beta}(\alpha)$, in a pooling system, based on Propositions A.2.2 and A.2.4, we know that the interior optimal price p^* satisfies the following first-order condition:

$$\frac{d\Pi(p)}{dp}\Big|_{p=p^*} = \mu N - \frac{c\nu_\alpha R}{(\nu_\alpha R - p^*)^2} + \frac{crN(\beta-1)\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)(\nu_\alpha R - p^*) - c)^2} = 0. \tag{A.1}$$

Then, according to the implicit function theorem, we have

$$\begin{aligned}
\frac{\partial p^*(\alpha,\beta)}{\partial \alpha} = -\frac{\frac{\partial^2 \Pi(p)}{\partial p \partial \alpha}}{\frac{\partial^2 \Pi(p)}{\partial p^2}}\Big|_{p=p^*} &= -\frac{cR\nu_\alpha' \left(\frac{p+\nu_\alpha R}{(\nu_\alpha R - p)^3} + \frac{2rN(1-\beta)\Lambda_m(\mu N + (\beta-1)\Lambda_m)}{((\mu N+(\beta-1)\Lambda_m)(\nu_\alpha R - p) - c)^3}\right)}{\frac{d^2\Pi(p)}{dp^2}}\Bigg|_{p=p^*} \\
&> 0; \\
\frac{\partial p^*(\alpha,\beta)}{\partial \beta} = -\frac{\frac{\partial^2 \Pi(p)}{\partial p \partial \beta}}{\frac{\partial^2 \Pi(p)}{\partial p^2}}\Big|_{p=p^*} &= -\frac{crN\Lambda_m \frac{(\mu N + (1-\beta)\Lambda_m)(\nu_\alpha R - p) - c}{((\mu N+(\beta-1)\Lambda_m)(\nu_\alpha R-p)-c)^3}}{\frac{d^2\Pi(p)}{dp^2}}\Bigg|_{p=p^*} > 0,
\end{aligned}$$

because $(\mu N + (\beta-1)\Lambda_m)(\nu_\alpha R - p) > c$ (see (A.21) of the Appendix A.2) and due to the concavity of the profit function, based on (A.20) of the Appendix A.2

we have

$$\frac{d^2\Pi(p)}{dp^2}\Big|_{p=p^*} = -\left(\frac{2c\nu_\alpha R}{(\nu_\alpha R-p^*)^3}+\frac{2crN(1-\beta)\Lambda_m(\mu N+(\beta-1)\Lambda_m)}{((\mu N+(\beta-1)\Lambda_m)(\nu_\alpha R-p^*)-c)^3}\right)<0.$$

Thus, $p^*(\alpha,\beta)$ increases in both α and β.

Since $p^*<\bar{p}_1<\nu_\alpha R$, we can show that

$$\begin{aligned}-\frac{\partial^2\Pi(p)}{\partial p\partial\alpha}\Big|_{p=p^*} &= -R\nu_\alpha^{'}\left(\frac{c(p^*+\nu_\alpha R)}{(\nu_\alpha R-p^*)^3}+\frac{2crN(1-\beta)\Lambda_m(\mu N+(\beta-1)\Lambda_m)}{((\mu N+(\beta-1)\Lambda_m)(\nu_\alpha R-p^*)-c)^3}\right)\\ &> R\nu_\alpha^{'}\cdot\frac{d^2\Pi(p)}{dp^2}\Big|_{p=p^*}.\end{aligned}$$

Consequently,

$$\frac{\partial p^*(\alpha,\beta)}{\partial\alpha}=-\frac{\frac{\partial^2\Pi(p)}{\partial p\partial\alpha}}{\frac{\partial^2\Pi(p)}{\partial p^2}}\Bigg|_{p=p^*}<\nu_\alpha^{'}R.$$

Then, we can easily obtain that

$$\frac{d\lambda_{f_c}^*}{d\alpha}=\frac{d\left(\mu N-\Lambda_\phi-\frac{c}{\nu_\alpha R-p^*}\right)}{d\alpha}=\frac{c(\nu_\alpha^{'}R-\frac{\partial p^*}{\partial\alpha})}{(\nu_\alpha R-p^*)^2}>0$$

$$\text{and}\quad\frac{d\lambda_{f_c}^*}{d\beta}=\frac{-c\frac{\partial p^*}{\partial\beta}}{(\nu_\alpha R-p^*)^2}<0.$$

That is, $\lambda_{f_c}^*$ increases in α but decreases in β.

Recall that $w^*=\frac{rN}{(\beta-1)\Lambda_m+\mu N-c/(\nu_\alpha R-p^*)}$, we obtain

$$\frac{dw^*}{d\alpha}=-\frac{rNc\frac{(\nu_\alpha^{'}R-\frac{\partial p^*}{d\alpha})}{(\nu_\alpha R-p^*)^2}}{((\beta-1)\Lambda_m+\mu N-c/(\nu_\alpha R-p^*))^2}<0.$$

Thus, w^* is decreasing in α. Note that there exists a one-to-one mapping between the price p and the joining rate $\lambda_{f_c}^e(p)$: $\lambda_{f_c}^e(p)=\mu N-\Lambda_\phi-\frac{c}{\nu_\alpha R-p}$. Thus, at the optimality, we shall also have $\frac{d\Pi(\lambda_{f_c})}{d\lambda_{f_c}}\big|_{\lambda_{f_c}=\lambda_{f_c}^*}=0$. Hereafter, we suppress the superscript "e" for brevity. Plugging $p(\lambda_{f_c})=\nu_\alpha R-\frac{c}{\mu N-\Lambda_\phi-\lambda_{f_c}}$ into $\underline{w}_1(p)$ of (A.17), then the platform's profit function (A.18) can be rewritten as

$$\begin{aligned}\Pi(\lambda_{f_c}) &= (p-\underline{w}_1(p))(\Lambda_m+\Lambda_{f_\phi}+\lambda_{f_c})\\ &= \left(\nu_\alpha R-\frac{c}{\mu N-\Lambda_\phi-\lambda_{f_c}}-\frac{rN}{\beta\Lambda_m+\Lambda_{f_\phi}+\lambda_{f_c}}\right)(\Lambda_m+\Lambda_{f_\phi}+\lambda_{f_c}).\end{aligned}\tag{A.2}$$

Then, we can derive that

$$
\begin{aligned}
\left.\frac{d\Pi(\lambda_{f_c})}{d\lambda_{f_c}}\right|_{\lambda_{f_c}=\lambda^*_{f_c}} &= (\Lambda_\phi+\lambda^*_{f_c})\left(\frac{rN}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^2}-\frac{c}{(\mu N-\Lambda_\phi-\lambda^*_{f_c})^2}\right) \\
&\quad +\nu_\alpha R-\frac{c}{\mu N-\Lambda_\phi-\lambda^*_{f_c}}-\frac{rN}{\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c}} \qquad \text{(A.3)} \\
&= \nu_\alpha R-\frac{c\mu N}{(\mu N-\Lambda_\phi-\lambda^*_{f_c})^2}+\frac{rN(1-\beta)\Lambda_m}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^2} \qquad \text{(A.4)} \\
&= 0.
\end{aligned}
$$

Note that $p^*-w^*=\nu_\alpha R-\frac{c}{\mu N-\Lambda_\phi-\lambda^*_{f_c}}-\frac{rN}{\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c}}>0$, then from (A.3), we know that

$$
\frac{rN}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^2}<\frac{c}{(\mu N-\Lambda_\phi-\lambda^*_{f_c})^2}<\frac{2c\mu N}{(\mu N-\Lambda_\phi-\lambda^*_{f_c})^3}. \qquad \text{(A.5)}
$$

Taking the first derivative of $\frac{d\Pi(\lambda_{f_c})}{d\lambda_{f_c}}$ with respect to β at the point $\lambda_{f_c}=\lambda^*_{f_c}$, we have

$$
\begin{aligned}
\frac{\partial \left.\frac{d\Pi(\lambda_{f_c})}{d\lambda_{f_c}}\right|_{\lambda_{f_c}=\lambda^*_{f_c}}}{\partial\beta} &= -\frac{2c\mu N\cdot\frac{d\lambda^*_{f_c}}{d\beta}}{(\mu N-\Lambda_\phi-\lambda^*_{f_c})^3}-\frac{rN\Lambda_m}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^2} \\
&\quad -\frac{2rN\Lambda_m(1-\beta)(\Lambda_m+\frac{d\lambda^*_{f_c}}{d\beta})}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^3}=0. \qquad \text{(A.6)}
\end{aligned}
$$

Then, based on (A.5) and (A.6), we get

$$
-\frac{d\lambda^*_{f_c}}{d\beta}=\frac{\frac{rN}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^2}+\frac{2rN(1-\beta)\Lambda_m}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^3}}{\frac{2c\mu N}{(\mu N-\Lambda_\phi-\lambda^*_{f_c})^3}+\frac{2rN(1-\beta)\Lambda_m}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^3}}\Lambda_m<\Lambda_m. \qquad \text{(A.7)}
$$

Thus, we can show that

$$
\frac{dw^*}{d\beta}=\frac{d(\frac{rN}{\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c}})}{d\beta}=\frac{-rN}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^2}\left(\Lambda_m+\frac{d\lambda^*_{f_c}}{d\beta}\right)<0.
$$

That is, w^* is decreasing in β as well.

As to the sensitivity analysis with respect to r, first, based on (A.4), we can obtain that

$$\frac{\partial \left.\frac{d\Pi(\lambda_{fc})}{d\lambda_{fc}}\right|_{\lambda_{fc}=\lambda^*_{fc}}}{\partial r} = -\frac{2c\mu N \frac{d\lambda^*_{fc}}{dr}}{\left(N\mu - \Lambda_\phi - \lambda^*_{fc}\right)^3} + N(1-\beta)\Lambda_m \cdot$$

$$\left(\frac{1}{\left(\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}\right)^2} - \frac{2r\frac{d\lambda^*_{fc}}{dr}}{\left(\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}\right)^3}\right) \tag{A.8}$$

$$= \frac{(1-\beta)\Lambda_m N}{\left(\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}\right)^2} - \left(\frac{2c\mu N}{\left(N\mu - \Lambda_\phi - \lambda^*_{fc}\right)^3} + \frac{2r(1-\beta)N\Lambda_m}{\left(\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}\right)^3}\right)\frac{d\lambda^*_{fc}}{dr} \tag{A.9}$$

$$= 0.$$

Based on (A.9), we can easily get that $\frac{d\lambda^*_{fc}}{dr} > 0$, i.e., λ^*_{fc} is increasing in r. Then, the first term in equation (A.8) shall be negative. This implied that the second term of equation (A.8) is positive. Thus,

$$\frac{1}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}} > \frac{2r\frac{d\lambda^*_{fc}}{dr}}{\left(\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}\right)^2}. \tag{A.10}$$

From (A.10), we then have

$$\frac{dw^*}{dr} = \frac{d\left(\frac{rN}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}}\right)}{dr} = N\left(\frac{1}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}} - \frac{r\frac{d\lambda^*_{fc}}{dr}}{\left(\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc}\right)^2}\right) > 0.$$

Regarding the optimal price p^*, as $\frac{d\lambda^*_{fc}}{dr} > 0$, we have

$$\frac{dp^*}{dr} = \frac{d\left(\nu_\alpha R - \frac{c}{\mu N - \Lambda_\phi - \lambda^*_{fc}}\right)}{dr} = -\frac{c}{\left(N\mu - \Lambda_\phi - \lambda^*_{fc}\right)^2} \cdot \frac{d\lambda^*_{fc}}{dr} < 0.$$

By the envelope theorem, from (A.2) and $\nu_\alpha = \frac{\alpha N_m + N_f}{N_m + N_f}$, we can have

$$\begin{aligned} \frac{d\Pi^*}{d\alpha} &= \left.\frac{\partial \Pi(\lambda_{fc}, \alpha)}{\partial \alpha}\right|_{\lambda_{fc}=\lambda^*_{fc}} = \frac{(\Lambda_\phi + \lambda^*_{fc})N_m R}{N_m + N_f} > 0 \text{ and} \\ \frac{d\Pi^*}{d\beta} &= \left.\frac{\partial \Pi(\lambda_{fc}, \alpha)}{\partial \beta}\right|_{\lambda_{fc}=\lambda^*_{fc}} = \frac{rN\Lambda_m(\Lambda_\phi + \lambda^*_{fc})}{(\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{fc})^2} > 0. \end{aligned} \tag{A.11}$$

Furthermore, since $\frac{d\lambda^*_{fc}}{d\alpha} > 0$, $\frac{d\lambda^*_{fc}}{d\beta} < 0$ and $\frac{d\lambda^*_{fc}}{d\beta} + \Lambda_m > 0$ as shwon in (A.7), we have

$$\frac{d^2\Pi^*}{d\alpha^2} = \frac{N_m R}{N_m + N_f}\frac{d\lambda^*_{fc}}{d\alpha} > 0 \text{ and}$$

$$\frac{d^2\Pi^*}{d\beta^2}=\frac{rN\Lambda_m\left((\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})\frac{d\lambda^*_{f_c}}{d\beta}-2(\Lambda_\phi+\lambda^*_{f_c})(\Lambda_m+\frac{d\lambda^*_{f_c}}{d\beta})\right)}{(\beta\Lambda_m+\Lambda_{f_\phi}+\lambda^*_{f_c})^3}<0.$$

Thus, Π^* is increasing and concave in β and increasing and convex in α.

A.1.3 Proof of Proposition 2.3

This proposition can be directly obtained based on Propositions A.3.1, A.3.3 and A.3.4 stated in the Appendix A.3.

A.1.4 Proof of Proposition 2.4

Based on Proposition A.3.2, one can easily derive that

$$\widetilde{\Pi}^*(Q)=\widetilde{\Pi}^*_0(Q)=RN\mu+2c-rN-2\sqrt{cR\mu}\left(\sqrt{N_m+Q}+\sqrt{N_f-Q}\right),$$

from which we can show that $\frac{d\widetilde{\Pi}^*(Q)}{dQ}=\sqrt{cR\mu}\frac{\sqrt{N_m+Q}-\sqrt{N_f-Q}}{\sqrt{(N_m+Q)(N_f-Q)}}>0$ due to $N_f<N_m$. That is, the platform's profit is increasing in Q.

A.1.5 Proof of Proposition 2.5

Consider that $N_\phi\mu>\Lambda_\phi$. Then, according to Proposition 2.1 and Table A.2.1, when $\beta<\widehat{\beta}(\alpha)$, in the pooling system, the equilibrium effective joining rate of safety-concerned female riders under the optimal price and wage is

$$\lambda^*_{f_c}=N_\phi\mu-\Lambda_\phi-\sqrt{\frac{c\mu N_\phi}{\widehat{\nu}_\alpha R}},\ \text{where }\widehat{\nu}_\alpha=\frac{\alpha N_m+N_{f_\phi}}{N_m+N_{f_\phi}}.$$

In the hybrid system, regarding the equilibrium effective joining rate of safety-concerned female riders, we have the following three cases.

Case (1): When $Q\in\left[0,\frac{\Lambda_\phi}{\mu}-N_m\right]$, according to Proposition A.3.3 and (A.27), in the hybrid system, multiple equilibria exist and safety-concerned female riders only join the female-only subsystem. Below, we consider the equilibrium outcome that induces the highest joining rate of safety-concerned female riders; that is,

$$\widetilde{\lambda}^*_{f_c}=\widetilde{\lambda}^*_{f_c,F_1}=(N_f-Q)\mu-\frac{c}{R-\widetilde{p}^*_{F_1}}=(N_f-Q)\mu-\sqrt{\frac{c\mu}{R}(N_f-Q)}.$$

Let $F_{d_2}(\alpha)=\widetilde{\lambda}^*_{f_c}-\lambda^*_{f_c}=(N_f-Q-N_m-N_{f_\phi})\mu-\sqrt{\frac{c\mu}{R}(N_f-Q)}+\Lambda_\phi+\sqrt{\frac{c\mu N_\phi}{\widehat{\nu}_\alpha R}}$. It can be easily shown that $F_{d_2}(\alpha)$ is decreasing in α. Let $\bar{\alpha}_0\in(0,1)$ be the solution of $F_{d_2}(\alpha) = 0$ if it exists, that is,

$$F_{d_2}(\alpha)\Big|_{\alpha=\bar{\alpha}_0} = (N_f - Q - N_m - N_{f_\phi})\mu - \sqrt{\frac{c\mu}{R}(N_f - Q)} + \Lambda_\phi + \sqrt{\frac{c\mu N_\phi}{\widehat{\nu}_\alpha R}} = 0 \quad (A.12)$$

If $F_{d_2}(\alpha)\big|_{\alpha\to 0} < 0$, we let $\bar{\alpha}_0 = 0$, and if $F_{d_2}(\alpha)\big|_{\alpha\to 1} > 0$, we let $\bar{\alpha}_0 = 1$. Then $F_{d_2}(\alpha) > 0$ if and only if $\alpha < \bar{\alpha}_0$. Thus, $\widetilde{\lambda}^*_{f_c} > \lambda^*_{f_c}$, that is, the hybrid system induces a higher effective joining rate for safety-concerned female riders than the pooling system, when $\alpha \le \bar{\alpha}_0$, $\beta < \widehat{\beta}(\alpha)$, $Q \le \frac{\Lambda_\phi}{\mu} - N_m$ and $N_\phi \mu > \Lambda_\phi$.

Case (2): When $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$ and $\alpha \ge \widehat{\alpha}$, according to Propositions A.3.3 and A.3.4 and (A.27), in the hybrid system, the total effective joining rate of safety-concerned female riders is

$$\begin{aligned}\widetilde{\lambda}^*_{f_c} = \widetilde{\lambda}^*_{f_c,F_2} + \widetilde{\lambda}^*_{f_c,M_2} &= N\mu - \Lambda_\phi - \frac{c}{R - \widetilde{p}^*_{F_2}} - \frac{c}{\theta_\alpha R - \widetilde{p}^*_{M_2}} \\ &= N\mu - \Lambda_\phi - \sqrt{\frac{c\mu}{R}(N_f - Q)} - \sqrt{\frac{c\mu}{\theta_\alpha R}(N_m + Q)},\end{aligned}$$

when $\sqrt{\frac{c\theta_\alpha R}{(N_m+Q)\mu}} \ge \frac{c}{(N_m+Q)\mu - \Lambda_\phi}$, where $\theta_\alpha = \frac{\alpha N_m + Q}{N_m + Q}$, or equivalently,

$$\alpha \ge \bar{\alpha}_2 := \left(\frac{c\mu(N_m+Q)^2}{((N_m+Q)\mu - \Lambda_\phi)^2 N_m R} - \frac{Q}{N_m}\right).$$

Otherwise (i.e., when $\alpha < \bar{\alpha}_2$), we have

$$\widetilde{\lambda}^*_{f_c} = (N_f - Q)\mu - \frac{c}{R - \widetilde{p}^*_{F_1}} = (N_f - Q)\mu - \sqrt{\frac{c\mu}{R}(N_f - Q)},$$

under which safety-concerned female riders only join female-only subsystem and the users' behaviors are exactly the same as those when $Q \in \left[0, \frac{\Lambda_\phi}{\mu} - N_m\right]$. Thus, the analysis is the same as that of the foregoing case (1). We then have $\widetilde{\lambda}^*_{f_c} > \lambda^*_{f_c}$ when $\widehat{\alpha} \le \alpha < \min\{\bar{\alpha}_0, \bar{\alpha}_2\}$, $\beta < \widehat{\beta}(\alpha)$, $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$ and $N_\phi \mu > \Lambda_\phi$.

When $\alpha > \bar{\alpha}_2$, let $F_{d_1}(\alpha) = \widetilde{\lambda}^*_{f_c} - \lambda^*_{f_c} = N_{f_c}\mu + \sqrt{\frac{c\mu}{R}}\left(\sqrt{\frac{N_\phi}{\widehat{\nu}_\alpha}} - \sqrt{N_f - Q} - \sqrt{\frac{N_m+Q}{\theta_\alpha}}\right)$. We can derive that

$$\frac{dF_{d_1}}{d\alpha} = \frac{1}{2}N_m\sqrt{\frac{c\mu}{R}}\left(\frac{-(N_m + N_{f_\phi})}{(\alpha N_m + N_{f_\phi})\sqrt{\alpha N_m + N_{f_\phi}}} + \frac{N_m + Q}{(\alpha N_m + Q)\sqrt{\alpha N_m + Q}}\right) \ge 0,$$

where the inequality holds due to the fact that $Q \le N_{f_\phi}$ and the function $f(x) = \frac{N_m + x}{(\alpha N_m + x)^{\frac{3}{2}}}$ is decreasing in x because $f'(x) = \frac{(-3+2\alpha)N_m - x}{2(\alpha N_m + x)^{\frac{5}{2}}} < 0$. That is, $F_{d_1}(\alpha)$

increases in α. Let $\bar{\alpha}_1 \in (0,1)$ be the solution of $F_{d_1}(\alpha) = 0$ if it exists, that is,

$$F_{d_1}(\alpha)\Big|_{\alpha=\bar{\alpha}_1} = N_{f_c}\mu + \sqrt{\frac{c\mu}{R}}\left(\sqrt{\frac{N_\phi}{\widehat{\nu}_\alpha}} - \sqrt{N_f - Q} - \sqrt{\frac{N_m + Q}{\theta_\alpha}}\right) = 0.$$

If $F_{d_1}(\alpha)\big|_{\alpha\to 0} > 0$, we let $\bar{\alpha}_1 = 0$, and if $F_{d_1}(\alpha)\big|_{\alpha\to 1} \leq 0$, we let $\bar{\alpha}_1 = 1$. Then $F_{d_1}(\alpha) \geq 0$ if and only if $\alpha \geq \bar{\alpha}_1$. Based on the above discussion, we can get that $\widetilde{\lambda}^*_{f_c} > \lambda^*_{f_c}$ when $\alpha \geq \max\{\bar{\alpha}_1, \bar{\alpha}_2, \widehat{\alpha}\}$, $\beta < \widehat{\beta}(\alpha)$, $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$ and $N_\phi\mu > \Lambda_\phi$.

Case (3): When $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$ and $\alpha < \widehat{\alpha}$, according to Propositions A.3.3 and A.3.4, the users' behaviors are exactly the same as those when $Q \in \left[0, \frac{\Lambda_\phi}{\mu} - N_m\right]$. Thus, the above analysis in case (1) holds here. We then have $\widetilde{\lambda}^*_{f_c} > \lambda^*_{f_c}$ when $\alpha < \min\{\bar{\alpha}_0, \widehat{\alpha}\}$, $\beta < \widehat{\beta}(\alpha)$, $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$ and $N_\phi\mu > \Lambda_\phi$.

In summary, when $N_\phi\mu > \Lambda_\phi$ and $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$, $\widetilde{\lambda}^*_{f_c} > \lambda^*_{f_c}$ if $\beta < \widehat{\beta}(\alpha)$ and ($\alpha \geq \max\{\bar{\alpha}_1, \bar{\alpha}_2, \widehat{\alpha}\}$ or $\alpha < \min\{\bar{\alpha}_0, \widehat{\alpha}\}$ or $\widehat{\alpha} \leq \alpha < \min\{\bar{\alpha}_0, \bar{\alpha}_2\}$). Note that the condition ($\alpha < \min\{\bar{\alpha}_0, \widehat{\alpha}\}$ or $\widehat{\alpha} \leq \alpha < \min\{\bar{\alpha}_0, \bar{\alpha}_2\}$) is equivalent to $\alpha < \min\{\max\{\widehat{\alpha}, \bar{\alpha}_2\}, \bar{\alpha}_0\}$. Define

$$\bar{\alpha} := \max\{\bar{\alpha}_1, \bar{\alpha}_2, \widehat{\alpha}\}, \underline{\alpha}_0 := \min\{\max\{\widehat{\alpha}, \bar{\alpha}_2\}, \bar{\alpha}_0\}. \tag{A.13}$$

Moreover, note that when $\bar{\alpha}_0 \geq \widehat{\alpha} \geq \max\{\bar{\alpha}_1, \bar{\alpha}_2\}$ or $\bar{\alpha}_0 \geq \bar{\alpha}_2 \geq \max\{\bar{\alpha}_1, \widehat{\alpha}\}$, we have $\bar{\alpha} = \underline{\alpha}_0$; otherwise, $\bar{\alpha} = \bar{\alpha}_1 > \max\{\bar{\alpha}_2, \widehat{\alpha}\} > \bar{\alpha}_0 = \underline{\alpha}_0$.

A.1.6 Proof of Proposition 2.6

The result can be easily obtained by directly comparing the female drivers' equilibrium participating rates in the pooling and hybrid systems as summarized in Tables A.2.1 and A.3.1 of the online Appendices A.2 and A.3.

A.1.7 Proof of Proposition 2.7

First, consider the case that $N_\phi\mu \leq \Lambda_\phi$, i.e, the number of safety-unconcerned drivers is low. In the pooling system, based on Propositions A.2.1 and A.2.2, we know that the safety-concerned female riders' joining utility is zero in equilibrium, that is,

$$U^*_{f_c} = \nu_\alpha R - p^* - cW(\lambda^e_{f_c}(p^*), \Lambda_{f_\phi}, \Lambda_m; N_{f_c}, N_{f_\phi}, N_m) = 0,$$

where $\nu_\alpha = \frac{\alpha N_m + N_f}{N_m + N_f}$. As to male riders and safety-unconcerned female riders, they all join the system and each obtains the following utility:

$$\begin{aligned} U^*_{f_\phi} = U^*_m &= R - p^* - cW(\lambda^e_{f_c}(p^*), \Lambda_{f_\phi}, \Lambda_m; N_{f_c}, N_{f_\phi}, N_m) = (1-\nu_\alpha)R + U^*_{f_c} \\ &= \frac{(1-\alpha)N_m}{N}R. \end{aligned}$$

Regarding the hybrid system, based on Proposition A.3.1 and its proof, we can easily know that all the joining riders obtain a zero utility, that is, $\widetilde{U}^*_{f_c} = \widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = 0$. Based on the above analysis, we can easily obtain that when $N_\phi \mu \leq \Lambda_\phi$, $U^*_{f_c} = \widetilde{U}^*_{f_c} = 0$, and $U^*_{f_\phi} = U^*_m > \widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = 0$.

We now analyze the utility of each participating driver. Based on Propositions A.2.1 and A.3.1, (A.17) and (A.26), we can know that each safety-concerned female driver obtains the reservation price r in both the pooling and the hybrid system. That is, $\mathbb{U}^*_{f_c} = \widetilde{\mathbb{U}}^*_{f_c} = r$. Regarding male drivers and safety-unconcerned female drivers, in the pooling system, each obtains the following utility:

$$\mathbb{U}^*_{f_\phi} = \mathbb{U}^*_m = \frac{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}{N} \cdot w^* = \frac{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}{N} \cdot \frac{rN}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}} > r.$$

While in the hybrid system, each obtains $\widetilde{\mathbb{U}}^*_{f_\phi} = \widetilde{\mathbb{U}}^*_m = r$. Therefore, $\mathbb{U}^*_i > \widetilde{\mathbb{U}}^*_i$ for $i = f_\phi, m$. Moreover, taking the first-order derivation of equation (A.4) stated in the proof of Proposition 2.2 with respect to the unit waiting cost c, we can then easily show that

$$-\frac{\mu N}{\left(\mu N - \Lambda_\phi - \lambda^*_{f_c}\right)^2} = 2\left(\frac{c\mu N}{\left(\mu N - \Lambda_\phi - \lambda^*_{f_c}\right)^3} + \frac{rN(1-\beta)\Lambda_m}{\left(\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}\right)^3}\right) \cdot \frac{\partial \lambda^*_{f_c}}{\partial c} < 0.$$

As $\mu N > \Lambda_\phi + \lambda^*_{f_c}$ (the queuing steady state condition), we have $\frac{\partial \lambda^*_{f_c}}{\partial c} < 0$. Consequently, we have

$$\mathbb{U}^*_i - \widetilde{\mathbb{U}}^*_i = \frac{r(1-\beta)\Lambda_m}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}, i = f_\phi, m$$

is strictly increasing in c.

Next, we consider the case $N_\phi \mu > \Lambda_\phi$, i.e., i.e, the number of safety-unconcerned drivers is low. In the pooling system, according to Propositions A.2.3 and A.2.4 stated in Appendix A.2, we then have that:

(a). When $\beta > \widehat{\beta}(\alpha)$, under the platform's optimal wage and price decision, the system behaves the same as those when $N_\phi \mu \leq \Lambda_\phi$, and the users' joining and participating behaviors are the same as those shown above as well. Thus, we have

$$U^*_{f_c} = 0, U^*_{f_\phi} = U^*_m = \frac{(1-\alpha)N_m}{N} R, \mathbb{U}^*_{f_c} = r, \quad \text{and}$$

$$\mathbb{U}^*_{f_\phi} = \mathbb{U}_m = \frac{\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}}{N} \cdot \frac{rN}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^*_{f_c}} > r.$$

(b). When $\beta \leq \widehat{\beta}(\alpha)$, no safety-concerned female drivers participate to work and in equilibrium, safety-concerned female riders' utility is zero; that is,

$$U^*_{f_c} = \widehat{\nu}_\alpha R - p^* - cW(\lambda^e_{f_c}(p^*), \Lambda_{f_\phi}, \Lambda_m; 0, N_{f_\phi}, N_m) = 0,$$

where $\widehat{\nu}_\alpha = \frac{\alpha N_m + N_{f_\phi}}{N_m + N_{f_\phi}}$. Safety-concerned female drivers get the reservation price $\mathbb{U}^*_{f_c} = r$. Each safety-unconcerned female rider and each male rider obtain the following utility:

$$\begin{aligned} U^*_{f_\phi} = U^*_m &= R - p^* - cW(\lambda^e_{f_c}(p^*), \Lambda_{f_\phi}, \Lambda_m; 0, N_{f_\phi}, N_m) \\ &= (1-\widehat{\nu}_\alpha)R + U^*_{f_c} = \frac{(1-\alpha)N_m}{N_\phi} R. \end{aligned}$$

As only safety-unconcerned drivers participate to work, we can easily get that $\mathbb{U}^*_{f_\phi} = \mathbb{U}^*_m = r$.

In the hybrid system, according to Propositions A.3.3 and A.3.4 of the Appendix A.3.2, when $\alpha < \widehat{\alpha}$, under the platform's optimal wage and price decision, the system behaves the same as those when $N_\phi \mu \leq \Lambda_\phi$. Thus, all the joining riders obtain a zero utility, that is, $\widetilde{U}^*_{f_c} = \widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = 0$. When $\alpha \geq \widehat{\alpha}$, safety-concerned female riders join both the pooling and female-only subsystems and all safety-unconcerned riders join the pooling subsystem. Similarly, we can show that $\widetilde{U}^*_{f_c} = 0$ and $\widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = (1-\theta_\alpha)R = \frac{(1-\alpha)N_m}{N_m+Q} R$. As to the drivers, they always behave the same as that when $N_\phi \mu \leq \Lambda_\phi$. Thus, we have $\widetilde{\mathbb{U}}^*_{f_c} = \widetilde{\mathbb{U}}^*_{f_\phi} = \widetilde{\mathbb{U}}^*_m = r$.

Based on the above discussion, we can easily obtain that when $N_\phi \mu > \Lambda_\phi$, at the driver side, $\mathbb{U}^*_{f_c} = \widetilde{\mathbb{U}}^*_{f_c} = r$ and $\mathbb{U}^*_i \geq \widetilde{\mathbb{U}}^*_i$, $i = f_\phi, m$. At the rider side, $U^*_{f_c} = \widetilde{U}^*_{f_c} = 0$. As both $\frac{(1-\alpha)N_m}{N_\phi} R \leq \frac{(1-\alpha)N_m}{N_m+Q} R$ and $\frac{(1-\alpha)N_m}{N} R < \frac{(1-\alpha)N_m}{N_m+Q} R$, when $\alpha \geq \widehat{\alpha}$, $\widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m \geq U^*_{f_\phi} = U^*_m$; otherwise, $U^*_{f_\phi} = U^*_m > \widetilde{U}^*_{f_\phi} = \widetilde{U}^*_m = 0$.

A.1.8 Proof of Proposition 2.8

When $\alpha, \beta \in (0,1)$, we consider the following two cases.

One, $\mu N_\phi \leq \Lambda_\phi$. According to Tables A.2.1 and A.3.1 and Proposition 2.2, the profit in the pooling system Π^* is increasing in both α and β while the profit in the hybrid system $\widetilde{\Pi}^*$ is independent of both α and β. Thus, the profit difference between the pooling system and the hybrid system, $\Pi^* - \widetilde{\Pi}^*$, is increasing in α.

Two, $\mu N_\phi > \Lambda_\phi$. According to Table A.3.1 and Proposition A.3.4, the platform's profit in the hybrid system $\widetilde{\Pi}^*$ is independent of β. $\widetilde{\Pi}^*$ is dependent of α only when safety-concerned female riders also join the pooling subsystem, that is when $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$, $(\widetilde{p}^*_M, \widetilde{w}^*_M; \widetilde{p}^*_F, \widetilde{w}^*_F) = (\widetilde{p}^*_{M_2}, \widetilde{w}^*_{M_2}; \widetilde{p}^*_{F_2}, \widetilde{w}^*_{F_2})$ and $\widetilde{p}^*_{M_2} = \theta_\alpha R - \sqrt{\frac{c\theta_\alpha R}{(N_m+Q)\mu}}$. Thus, when $\mu N_\phi > \Lambda_\phi$ and $Q \in \left[0, \frac{\Lambda_\phi}{\mu} - N_m\right]$, the platform's profit in the hybrid system $\widetilde{\Pi}^*$ is independent of α while the platform's profit in the pooling system Π^* is increasing in α (see Proposition 2.2 and Table A.2.1). Then, in this situation, the profit difference between the pooling system and the hybrid system, $\Pi^* - \widetilde{\Pi}^*$, is increasing in α.

Below, we show that when $\mu N_\phi > \Lambda_\phi$ and $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$, the profit difference between the pooling system and the hybrid system, $\Pi^* - \widetilde{\Pi}^*$, is increasing in α as well.

Case (1): When $\beta > \widehat{\beta}(\alpha)$, Tables A.2.1, $\Pi^* = \Pi^*_1$. Then, based on (A.11) and (A.28) (of Appendix A.3), we have

$$\frac{d(\Pi^* - \widetilde{\Pi}^*)}{d\alpha} = \frac{d\Pi^*}{d\alpha} - \frac{d\widetilde{\Pi}^*}{d\alpha} = \frac{(\Lambda_\phi + \lambda^*_{f_c})N_m R}{N_m + N_f} - N_m \mu \sqrt{R}\left(\sqrt{R} - \sqrt{\frac{c}{(\alpha N_m + Q)\mu}}\right).$$

Plugging $\Lambda_\phi + \lambda^*_{f_c} = \mu N - \frac{c}{\nu_\alpha R - p^*_1}$ into the above equation, we get

$$\frac{d(\Pi^* - \widetilde{\Pi}^*)}{d\alpha} = N_m \left(\sqrt{\frac{cR\mu}{\alpha N_m + Q}} - \frac{Rc}{N(\nu_\alpha R - p^*_1)}\right).$$

From (A.19) of Appendix A.2, we have

$$\frac{d\Pi(p)}{dp}\Big|_{p=\nu_\alpha R - \frac{\sqrt{cR(\alpha N_m+Q)}}{N\sqrt{\mu}}} = \mu N - \frac{\alpha N_m + N_f}{\alpha N_m + Q}\mu N + \frac{crN(\beta-1)\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)(\nu_\alpha R - p) - c)^2} < 0,$$

where the inequality holds due to $Q < N_f$ and $\beta < 1$. Recall that $p_1^* < \bar{p}_1$, $\frac{d\Pi(p)}{dp}\Big|_{p=p_1^*} = 0$ and the profit in the pooling system $\Pi(p)$ is concave in p over $(0, \bar{p}_1)$. Thus, $p_1^* < \nu_\alpha R - \frac{\sqrt{cR(\alpha N_m+Q)}}{N\sqrt{\mu}}$. Then, we have

$$\frac{d(\Pi^* - \widetilde{\Pi}^*)}{d\alpha} = N_m\left(\sqrt{\frac{cR\mu}{\alpha N_m + Q}} - \frac{Rc}{N(\nu_\alpha R - p_1^*)}\right) > 0.$$

Case (2): When $\beta \leq \widehat{\beta}(\alpha)$, according to Table A.2.1, one can show that

$$\begin{aligned}
\Pi^* &= \Pi_2^* = \left(\widehat{\nu}_\alpha R - \sqrt{\frac{c\widehat{\nu}_\alpha R}{N_\phi \mu}} - \frac{r}{\mu - \sqrt{c\mu/(N_\phi \widehat{\nu}_\alpha R)}}\right)\left(N_\phi \mu - \sqrt{\frac{c\mu N_\phi}{\widehat{\nu}_\alpha R}}\right) \\
&= \widehat{\nu}_\alpha R N_\phi \mu + c - 2\sqrt{c\widehat{\nu}_\alpha R \mu N_\phi} - rN_\phi,
\end{aligned}$$

where $\widehat{\nu}_\alpha = \frac{\alpha N_m + N_{f_\phi}}{N_m + N_{f_\phi}}$ and $N_\phi = N_m + N_{f_\phi}$. We then obtain that

$$\frac{d\Pi^*}{d\alpha} = R\mu N_m - 2\sqrt{cR\mu}\cdot\frac{N_m}{2\sqrt{\alpha N_m + N_{f_\phi}}} = R\mu N_m - N_m\sqrt{\frac{cR\mu}{\alpha N_m + N_{f_\phi}}}.$$

Combining the above equation, (A.28) and $Q \leq N_{f_\phi}$, we obtain that

$$\begin{aligned}
\frac{d(\Pi^* - \widetilde{\Pi}^*)}{d\alpha} &= R\mu N_m - N_m\sqrt{\frac{cR\mu}{\alpha N_m + N_{f_\phi}}} - N_m\mu\sqrt{R}\left(\sqrt{R} - \sqrt{\frac{c}{(\alpha N_m + Q)\mu}}\right), \\
&= N_m\left(\sqrt{\frac{cR\mu}{\alpha N_m + Q}} - \sqrt{\frac{cR\mu}{\alpha N_m + N_{f_\phi}}}\right) \geq 0.
\end{aligned}$$

Based on the above discussion, we can conclude that the profit difference between the two systems $\Pi^* - \widetilde{\Pi}^*$ is always increasing in α. Let $\underline{\alpha}(\beta)$ be the solution of

$$\left(\Pi^* - \widetilde{\Pi}^*\right)\Big|_{\alpha=\underline{\alpha}(\beta)} = 0 \tag{A.14}$$

if it exists. If $(\Pi^* - \widetilde{\Pi}^*)\mid_{\alpha\to 0} > 0$ for a given β, we let $\underline{\alpha}(\beta) = 0$. If $(\Pi^* - \widetilde{\Pi}^*)\mid_{\alpha\to 1} < 0$ for a given β, we let $\underline{\alpha}(\beta) = 1$. Then, $\Pi^* > \widetilde{\Pi}^*$ only when $\alpha > \underline{\alpha}(\beta)$.

Last, we consider a special case when both drivers and riders have full safety confidence, that is, $\alpha = \beta \to 1$. Under such a situation, all the drivers shall join the pooling system as well as the hybrid system due to the abundant demand. Thus, the pooling system is an $M/M/1$ queue with capacity $(N_f + N_m)\mu$ while the hybrid system is a system consisting of two $M/M/1$ queues with capacity

$(N_m + Q)\mu$ and $(N_f - Q)\mu$, respectively. For the pooling system, we can derive that

$$\lambda^e(p) = N\mu - \frac{c}{R-p}, \text{ and } w(p) = \frac{rN}{\lambda^e(p)}.$$

The platform maximizes

$$\Pi(p) = (p - w(p))\lambda^e(p),$$

which is can be easily shown concave in p and the first order condition is

$$\frac{d\Pi(p)}{dp} = \mu N - \frac{cR}{(R-p)^2} = 0.$$

It can be easily shown that the optimal price $p^* = R - \sqrt{\frac{cR}{\mu N}}$. Then we can get the total effective joining rate under optimal price is $\lambda_f^* + \lambda_m^* = \mu N - \frac{c}{R-p^*} = \mu N - \sqrt{\frac{c\mu N}{R}}$.

As for the hybrid system, we can show that the two subsystem adopt the optimal prices

$$\widetilde{p}_M^* = R - \sqrt{\frac{cR}{(N_m + Q)\mu}} \text{ and } \widetilde{p}_F^* = R - \sqrt{\frac{cR}{(N_f - Q)\mu}}.$$

The corresponding equilibrium joining rates in the two subsystems are respectively

$$\lambda_M^e(\widetilde{p}_M^*) = (N_m+Q)\mu - \sqrt{\frac{c(N_m + Q)\mu}{R}}, \text{ and } \lambda_F^e(\widetilde{p}_F^*) = (N_f - Q)\mu - \sqrt{\frac{c(N_f - Q)\mu}{R}}.$$

Since $\sqrt{N} = \sqrt{N_m + Q + N_f - Q} < \sqrt{N_m + Q} + \sqrt{N_f - Q}$ and the riders' effective joining rate in the hybrid system $\widetilde{\lambda}_f^* + \widetilde{\lambda}_m^* = \lambda_M^e(\widetilde{p}_M^*) + \lambda_F^e(\widetilde{p}_F^*)$, we have

$$\lambda_f^* + \lambda_m^* - \left(\widetilde{\lambda}_f^* + \widetilde{\lambda}_m^*\right) = \sqrt{\frac{c\mu}{R}}(\sqrt{N_m + Q} + \sqrt{N_f - Q} - \sqrt{N}) > 0.$$

We then can show that

$$\begin{aligned}
\Pi^* - \widetilde{\Pi}_M^* - \widetilde{\Pi}_F^* &= p^*(\lambda_f^* + \lambda_m^*) - rN - \\
&\quad (\widetilde{p}_M^* \lambda_M^e(\widetilde{p}_M^*) - r(N_m + Q) + \widetilde{p}_F^* \lambda_F^e(\widetilde{p}_F^*) - r(N_f - Q)) \\
&> p^*\left(\lambda_M^e(\widetilde{p}_M^*) + \lambda_F^e(\widetilde{p}_F^*)\right) - \widetilde{p}_M^* \lambda_M^e(\widetilde{p}_M^*) - \widetilde{p}_F^* \lambda_F^e(\widetilde{p}_F^*) > 0,
\end{aligned}$$

because $p^* > \widetilde{p}_j^*, j = F, M$. Thus, $\Pi^* > \widetilde{\Pi}_M^* + \widetilde{\Pi}_F^* = \widetilde{\Pi}^*$.

A.2 The Pooling System: Detailed Analyses

We first present an implication that is useful to understand the equilibrium behaviors of riders and drivers in a pooling system (the logic of this implication can be applied to the pooling subsystem in a hybrid system).

In the pooling system, safety-unconcerned female and male riders and safety-concerned female riders continue to join the system until their utility U_i given in (2.3) and (2.4) hits zero, where $i = f_c, f_\phi, m$. A close look at (2.3) and (2.4) implies that $U_m = U_{f_\phi} \geq U_{f_c}$. Similarly, because safety-unconcerned female and male drivers have no safety concerns, a close look at (2.5) and (2.6) implies that $S_m = S_{f_\phi} \geq S_{f_c}$. These observations yield the following implications.

Implication A.2.1. *In a pooling system, if some safety-concerned female riders/drivers join the system, then all safety-unconcerned riders/drivers will join the system.*

Note that safety-unconcerned riders and drivers, which contains all males and a fraction of safety-unconcerned females, are more eager to join the system than their safety-concerned female counterparts. It is likely to have all $\Lambda_\phi (= \Lambda_m + \Lambda_{f_\phi})$ safety-unconcerned riders and all $N_\phi (= N_m + N_{f_\phi})$ safety-unconcerned drivers joining the system before their safety-concerned female counterparts. Also note that throughout our analyses, we only consider the equilibrium outcomes in which the safety-concerned female riders join the system at a non-zero rate. While deriving the equilibrium joining/participating behaviors of riders/drivers, one can easily find some equilibria in which all the safety-concerned female riders balk in a pooling/hybrid system. As such equilibrium outcomes deviate from our research motivation, they are not our focus and thus we omit such trivial cases. When some safety-concerned female riders join the system, then all Λ_ϕ safety-unconcerned male and female riders join the system (due to Implication A.2.1). Then, some safety-concerned female drivers must participate in the service to ensure the stability of the queuing system when the number of safety-unconcerned drivers are not sufficiently high to serve even just the safety-unconcerned riders, that is, when $\mu N_\phi \leq \Lambda_\phi$. Then, we have the following implication.

Implication A.2.2. *When $\mu N_\phi \leq \Lambda_\phi$, if some safety-concerned female riders join the system, then all safety-unconcerned drivers must participate in the system.*

Below, we derive the equilibrium joining (and participating) behaviors of riders (and drivers) under the two exhaustive and exclusive cases.

A.2.1 When the number of safety-unconcerned drivers is low: $N_\phi \mu \leq \Lambda_\phi$

Based on Implication A.2.2, we can conclude that when $N_\phi \mu \leq \Lambda_\phi$, in order for the platform to retain safety-concerned female riders in the pooling system, it must be the case that all safety-unconcerned riders (and drivers) and some safety-concerned female riders (and drivers) join (and participate in) the system at their potential arrival rates, respectively. This allows us to focus on deriving the joining and participating behaviors of only the female safety-concerned riders and drivers.

Let $\nu_\alpha = \frac{\alpha N_m + N_f}{N}$. Then, $\nu_\alpha \in (0, 1)$ represents the *safety-concern-adjusted reward weight* for a safety-concerned female rider when all the registered drivers participate, and $\nu_\alpha R$ is the *safety-concern-adjusted reward*. Denote λ_i^e as the effective joining rate of type-i riders and n_i^e as the number of participating type-i drivers in equilibrium, $i = f_c, f_\phi, m$. Then, $\lambda_f^e = \lambda_{f_c}^e + \lambda_{f_\phi}^e$. By focusing on the equilibrium outcome that some safety-concerned female riders join the system, we now develop the conditions under which this equilibrium will exist in the following proposition.

Proposition A.2.1. *In a pooling system, if $N_\phi \mu \leq \Lambda_\phi$, the platform sets the price $p \leq \bar{p}_1 := \nu_\alpha R - \frac{c}{N\mu - \Lambda_\phi}$ and the wage $w \geq \underline{w}_1(p) := \frac{rN}{(\beta-1)\Lambda_m + N\mu - c/(\nu_\alpha R - p)}$ to ensure the joining of the safety-concerned female riders. Then, in equilibrium, all registered drivers participate (i.e, $n_m^e = N_m$, $n_{f_\phi}^e = N_{f_\phi}$ and $n_{f_c}^e = N_{f_c}$) and all safety-unconcerned riders (that is, male riders and safety-unconcerned female riders) join the system so that $\lambda_m^e = \Lambda_m$ and $\lambda_{f_\phi}^e = \Lambda_{f_\phi}$. Some safety-concerned* *female riders join the system and the others balk with an effective joining rate* $\lambda^e_{f_c}(p, N_{f_c}) = N\mu - \Lambda_\phi - \frac{c}{\nu_\alpha R - p}$.

Proof of Proposition A.2.1. Based on Implications A.2.1 and A.2.2, we know that when $N_\phi \mu \leq \Lambda_\phi$, if some safety-concerned female riders join the system, then in equilibrium, safety-unconcerned drivers "all participate" and safety-unconcerned riders "all join"; that is, $\lambda^e_m = \Lambda_m$, $\lambda^e_{f_\phi} = \Lambda_{f_\phi}$, $n^e_m = N_m$ and $n^e_{f_\phi} = N_{f_\phi}$. We now analyze the joining and participating behavior of safety-concerned female riders and drivers.

Given n_{f_c}, the number of participating safety-concerned female drivers, to ensure that safety-concerned female riders are willing to join, we should have

$$\begin{aligned} U_{f_c}(0, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = & \frac{\alpha N_m + N_{f_\phi} + n_{f_c}}{N_m + N_{f_\phi} + n_{f_c}} R - p \\ & - cW(0, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) \geq 0, \qquad \text{(A.15)} \end{aligned}$$

where $W(0, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = \frac{1}{(N_m + N_{f_\phi} + n_{f_c})\mu - \Lambda_\phi}$, where $\Lambda_\phi = \Lambda_m + \Lambda_{f_\phi}$. Recall that $\mu N < \Lambda$. Thus, in equilibrium, some safety-concerned female riders must balk. The equilibrium effective joining rate of safety-concerned female riders can be obtained by solving

$$\begin{aligned} U_{f_c}(\lambda_{f_c}, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = & \frac{\alpha N_m + N_{f_\phi} + n_{f_c}}{N_m + N_{f_\phi} + n_{f_c}} R - p \\ & - cW(\lambda_{f_c}, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = 0, \end{aligned}$$

where $W(\lambda_{f_c}, \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = \frac{1}{(N_m + N_{f_\phi} + n_{f_c})\mu - \Lambda_\phi - \lambda_{f_c}}$. It can be shown that safety-concerned female riders' equilibrium joining rate

$$\lambda^e_{f_c}(p, n_{f_c}) = (N_m + N_{f_\phi} + n_{f_c})\mu - \Lambda_\phi - \frac{c}{(\alpha N_m + N_{f_\phi} + n_{f_c})R/(N_m + N_{f_\phi} + n_{f_c}) - p}.$$

Then, we have

$$\begin{aligned} d_{f_c}(n_{f_c}, N_{f_\phi}, N_m) &= \frac{\beta \Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, n_{f_c})}{n_{f_c} + N_{f_\phi} + N_m} \\ &= \frac{(\beta - 1)\Lambda_m + \mu(N_m + N_{f_\phi} + n_{f_c})}{n_{f_c} + N_{f_\phi} + N_m} \\ & \quad - \frac{\frac{c}{(\alpha N_m + N_{f_\phi} + n_{f_c})R/(N_m + N_{f_\phi} + n_{f_c}) - p}}{n_{f_c} + N_{f_\phi} + N_m}. \qquad \text{(A.16)} \end{aligned}$$

Taking the first order derivative with respect to n_{f_c}, we get

$$\frac{\partial d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)}{\partial n_{f_c}} = \frac{(1-\beta)\Lambda_m}{(N_m + N_{f_\phi} + n_{f_c})^2} + \frac{c(R-p)}{\big((\alpha N_m + N_{f_\phi} + n_{f_c})R - p(N_m + N_{f_\phi} + n_{f_c})\big)^2} > 0$$

due to $p < R$ (otherwise, no rider is willing to join). That is, the safety-concern-adjusted demand rate $d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)$ is increasing in the number of participating safety-concerned female drivers n_{f_c}.

Recall that a safety-concerned female driver is willing to participate if and only if her net utility given in (2.6),

$$S_{f_c}(\lambda^e_{f_c}(p, n_{f_c}), \Lambda_{f_\phi}, \Lambda_m; n_{f_c}, N_{f_\phi}, N_m) = \frac{\beta\Lambda_m + \lambda^e_{f_c}(p, n_{f_c}) + \Lambda_{f_\phi}}{n_{f_c} + N_{f_\phi} + N_m} w - r \geq 0.$$

And we just show that $d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)$ increases in n_{f_c}. Following the same logic stated in the Lemma 1 of Taylor (2018), we can get the following result:

$$n^e_{f_c} = \begin{cases} N_{f_c}, & \text{if and only if} \quad w \geq \frac{r(N_m + N_{f_\phi} + N_{f_c})}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})}, \\ 0, & \text{if and only if} \quad w < \frac{r(N_m + N_{f_\phi} + 1)}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, 1)}. \end{cases}$$

Note that when $n^e_{f_c} = 0$, no safety-concerned female drivers participate in the system. Implication A.2.2 then implies that under this situation, no safety-concerned female riders join the system. Thus, to ensure the joining of safety-concerned female riders, the platform shall set the wage $w \geq \frac{r(N_m + N_{f_\phi} + N_{f_c})}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})}$, under which $n^e_{f_c} = N_{f_c}$. Plugging $n^e_{f_c} = N_{f_c}$ into inequality (A.15), we then have that $p \leq \nu_\alpha R - \frac{c}{\mu N - \Lambda_\phi}$ is required. Under such a situation, all drivers participate in the service, i.e., $n^e_i = N_i$, $i \in \{f_c, f_\phi, m\}$. The corresponding equilibrium effective joining rate of safety-concerned female riders for any given price p is $\lambda^e_{f_c}(p, N_{f_c}) = \mu N - \Lambda_\phi - \frac{c}{\nu_\alpha R - p}$. Thus, $w \geq \frac{r(N_m + N_{f_\phi} + N_{f_c})}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})} = \frac{rN}{(\beta-1)\Lambda_m + \mu N - c/(\nu_\alpha R - p)}$ is required. Moreover, it can be further shown that

$$\frac{\partial\left(\frac{\partial d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)}{\partial n_{f_c}}\right)}{\partial \alpha} = \frac{-2c(R-p)RN_m}{\big((\alpha N_m + N_{f_\phi} + n_{f_c})R - p(N_m + N_{f_\phi} + n_{f_c})\big)^3} < 0,$$

$$\frac{\partial\left(\frac{\partial d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)}{\partial n_{f_c}}\right)}{\partial \beta} = \frac{-\Lambda_m}{(N_m + N_{f_\phi} + n_{f_c})^2} < 0.$$

That is, $\frac{\partial d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)}{\partial n_{f_c}}$ decreases in both α and β. From (A.16), we then get

$$\begin{aligned}\frac{\partial d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)}{\partial N_m} &= \frac{(1-\beta)\Lambda_m}{(N_m + N_{f_\phi} + n_{f_c})^2} \\ &\quad + \frac{c(\alpha R - p)}{\left((\alpha N_m + N_{f_\phi} + n_{f_c})R - p(N_m + N_{f_\phi} + n_{f_c})\right)^2}.\end{aligned}$$

Note that when $\beta \to 1$ and $\alpha < p/R$, $\frac{\partial d_{f_c}(n_{f_c}, N_{f_\phi}, N_m)}{\partial N_m} < 0$. □

We now examine the platform's optimal pricing and wage decisions with an aim to maximize its profitability, subject to the constraints $p \leq \bar{p}_1 := \nu_\alpha R - \frac{c}{N\mu - \Lambda_\phi}$ and the wage $w \geq \underline{w}_1(p) := \frac{rN}{(\beta-1)\Lambda_m + N\mu - c/(\nu_\alpha R - p)}$ (which ensures the joining of safety-concerned female riders in the system). Note that $\underline{w}_1(p)$ is the required minimum wage for any given price $p \in (0, \bar{p}_1)$. Clearly, there is no incentive for the platform to offer a wage that is above $\underline{w}_1(p)$. Hence, for a given price p, a rational platform shall set

$$w(p) = \underline{w}_1(p) = \frac{rN}{(\beta - 1)\Lambda_m + N\mu - \frac{c}{\nu_\alpha R - p}}. \tag{A.17}$$

Combining this along with the result stated in Proposition A.2.1, we can formulate the platform's optimization problem as follows:

$$\Pi^* = \max_{\underline{w}_1(p) < p < \bar{p}_1} \Pi(p) = \max_{\underline{w}_1(p) < p < \bar{p}_1} (p - \underline{w}_1(p))(\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})),$$

where $\lambda^e_{f_c}(p, N_{f_c}) = N\mu - \Lambda_\phi - \frac{c}{\nu_\alpha R - p}$.

Proposition A.2.2. *In a pooling system, the platform's profit function $\Pi(p)$ is concave in price p over the range $(0, \bar{p}_1)$. Let p^* be the solution of the first-order condition*

$$\frac{d\Pi(p)}{dp} = N\mu - \frac{c\nu_\alpha R}{(\nu_\alpha R - p)^2} + \frac{crN(\beta - 1)\Lambda_m}{((N\mu + (\beta - 1)\Lambda_m)(\nu_\alpha R - p) - c)^2} = 0.$$

Then, p^* is an interior optimal solution if and only if

$$\frac{d\Pi(p)}{dp}\Big|_{p\to 0} = \eta_1(\alpha, \beta) > 0 \text{ and } \frac{d\Pi(p)}{dp}\Big|_{p\to\bar{p}_1} = \eta_2(\alpha, \beta) < 0,$$

where the detailed expressions of $\eta_1(\alpha, \beta)$ and $\eta_2(\alpha, \beta)$ are provided in (A.22) and (A.23) in the following proof.

Proof of Proposition A.2.2. Recalling that $\Lambda_\phi = \Lambda_m + \Lambda_{f_\phi}$ and plugging $\lambda^e_{f_c}(p, N_{f_c}) = N\mu - \Lambda_\phi - \frac{c}{\nu_\alpha R - p}$ and equation (A.17) into $\Pi(p)$, we get

$$\Pi(p) = (p - \underline{w}_1(p))(\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p)) = p\left(\mu N - \frac{c}{\nu_\alpha R - p}\right) - \frac{rN}{1 + \frac{(\beta-1)\Lambda_m}{\mu N - \frac{c}{\nu_\alpha R - p}}}. \tag{A.18}$$

Then we can derive that

$$\frac{d\Pi(p)}{dp} = \mu N - \frac{c\nu_\alpha R}{(\nu_\alpha R - p)^2} + \frac{crN(\beta-1)\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)(\nu_\alpha R - p) - c)^2}, \tag{A.19}$$

and

$$\frac{d^2\Pi(p)}{dp^2} = -\frac{2c\nu_\alpha R}{(\nu_\alpha R - p)^3} + \frac{2crN(\beta-1)\Lambda_m(\mu N + (\beta-1)\Lambda_m)}{((\mu N + (\beta-1)\Lambda_m)(\nu_\alpha R - p) - c)^3}. \tag{A.20}$$

As $p < \bar{p}_1 = \nu_\alpha R - \frac{c}{\mu N - \Lambda_\phi} = \nu_\alpha R - \frac{c}{\mu N - \Lambda_{f_\phi} - \Lambda_m}$ is required, we have $p < \nu_\alpha R - \frac{c}{\mu N - \Lambda_m} < \nu_\alpha R - \frac{c}{\mu N - \Lambda_m + \beta\Lambda_m}$, which is equivalent to

$$(\mu N + (\beta-1)\Lambda_m)(\nu_\alpha R - p) > c. \tag{A.21}$$

Besides, $\beta < 1$ and $\mu N > \Lambda_m$ (the requirement to ensure that female riders do join the system). Hence, $\frac{d^2\Pi(p)}{dp^2} < 0$, which implies that $\Pi(p)$ is concave in p. Therefore, there must have an interior optimal solution in the range $(0, \bar{p})$ if and only if

$$\frac{d\Pi(p)}{dp}\,|_{p\to 0} > 0 \quad \text{and} \quad \frac{d\Pi(p)}{dp}\,|_{p\to \bar{p}_1} < 0.$$

For ease of notation, let

$$\eta_1(\alpha,\beta) = \frac{d\Pi(p)}{dp}\,|_{p\to 0} = \mu N - \frac{c}{\nu_\alpha R} + \frac{crN(\beta-1)\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)\nu_\alpha R - c)^2}, \tag{A.22}$$

$$\eta_2(\alpha,\beta) = \frac{d\Pi(p)}{dp}\,|_{p\to \bar{p}_1} = \mu N - \frac{\nu_\alpha R(\mu N - \Lambda_\phi)^2}{c} + \frac{Nr\Lambda_m(\beta-1)(N\mu - \Lambda_\phi)^2}{c(\beta\Lambda_m + \Lambda_{f_\phi})^2}. \tag{A.23}$$

Based on those two equations, for any $\alpha < 1$ and $\beta < 1$, we have

$$\frac{\partial \eta_1(\alpha,\beta)}{\partial \alpha} = \frac{cN_m}{NR\nu_\alpha^2} + \frac{-2crN_m(\beta-1)\Lambda_m(\mu N + (\beta-1)\Lambda_m)R}{((\mu N + (\beta-1)\Lambda_m)\nu_\alpha R - c)^3} > 0,$$

$$\frac{\partial \eta_1(\alpha,\beta)}{\partial \beta} = \frac{crN\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)\nu_\alpha R - c)^2} + \frac{2crN\Lambda_m(1-\beta)\nu_\alpha R\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)\nu_\alpha R - c)^3} > 0,$$

$$\frac{\partial \eta_2(\alpha,\beta)}{\partial \alpha} = -\frac{R(\mu N-\Lambda_\phi)^2 N_m}{Nc} < 0 \text{ and}$$
$$\frac{\partial \eta_2(\alpha,\beta)}{\partial \beta} = \frac{Nr\Lambda_m(N\mu-\Lambda_\phi)^2}{c}\cdot\frac{(2-\beta)\Lambda_m+\Lambda_{f_\phi}}{(\beta\Lambda_m+\Lambda_{f_\phi})^3} > 0.$$

That is, $\eta_1(\alpha,\beta)$ increases in both α and β, and $\eta_2(\alpha,\beta)$ decreases in α but increases in β. When $\alpha=\beta\to 1$, we have $\nu_\alpha=\frac{\alpha N_m+N_f}{N}\to 1$. Moreover, to ensure that there exists riders joining an empty system, we must have $R>\frac{c}{N\mu}$, under which $\eta_1(1,1)=\mu N-\frac{c}{R}>0$. Besides, we can show that $\eta_2(1,1)=\mu N-\frac{R(N\mu-\Lambda_\phi)^2}{c}<0$ when $R>\frac{c\mu N}{(\mu N-\Lambda_\phi)^2}$.

The above analysis implies that when $R>\frac{c\mu N}{(\mu N-\Lambda_\phi)^2}$ and $\alpha=\beta\to 1$ (under which $\eta_1(1,1)>0$ and $\eta_2(1,1)<0$), there must exist an interior optimal solution for the optimal price. Similarly, we can construct other ranges of α and β under which the interior optimal solution exists by applying the properties that $\eta_1(\alpha,\beta)$ increases in both α and β and $\eta_2(\alpha,\beta)$ decreases in α but increases in β. □

We can then derive the effective joining rate of safety-concerned female riders $\lambda^*_{f_c}=\lambda^e_{f_c}(p^*,N_{f_c})$ from Proposition A.2.1, the optimal wage w^* from (A.17), and the corresponding optimal profit Π^* by plugging into the optimal price p^* as stated in Proposition A.2.2.

A.2.2 When the number of safety-unconcerned drivers is large: $N_\phi\mu>\Lambda_\phi$

We now analyze the case when $N_\phi\mu>\Lambda_\phi$. Similar to the previous subsection, we first characterize the joining behaviors of drivers and riders for the given price and wage. We then analyze the platforms's optimal price and wage decisions. Again, we shall focus on the equilibrium outcome in which some safety-concerned female riders join the system. We now develop the conditions under which this equilibrium exists in the following proposition.

Proposition A.2.3. *When $N_\phi\mu>\Lambda_\phi$, the safety-concerned female riders join the system at a non-zero rate in equilibrium under the following two cases:*

1. ***(Case $\mathcal{P}1$):*** *the platform sets the price $p\le\bar{p}_1:=\nu_\alpha R-\frac{c}{N\mu-\Lambda_\phi}$ and the wage*

$w \geq \underline{w}_1(p) := \frac{rN}{(\beta-1)\Lambda_m + N\mu - c/(\nu_\alpha R - p)}$, under which the equilibrium outcome stated in Proposition A.2.1 is the equilibrium outcome here.

2. ***(Case $\mathcal{P}2$):*** *the platform sets the price $p \leq \bar{p}_2 := \widehat{\nu}_\alpha R - \frac{c}{N_\phi\mu - \Lambda_\phi}$ and the wage $w \geq \underline{w}_2(p) := \frac{rN_\phi}{N_\phi\mu - c/(\widehat{\nu}_\alpha R - p)}$, where $\widehat{\nu}_\alpha = \frac{\alpha N_m + N_{f_\phi}}{N_m + N_{f_\phi}}$. Then, in equilibrium, all the safety-unconcerned drivers participate in the system but all the safety-concerned female drivers balk, i.e., $n^e_m = N_m$, $n^e_{f_\phi} = N_{f_\phi}$ and $n^e_{f_c} = 0$. All the safety-unconcerned riders join the system so that $\lambda^e_m = \Lambda_m$ and $\lambda^e_{f_\phi} = \Lambda_{f_\phi}$. Some safety-concerned female riders join the system and the others balk with an effective joining rate $\lambda^e_{f_c}(p, 0) = N_\phi\mu - \Lambda_\phi - \frac{c}{\widehat{\nu}_\alpha R - p}$.*

Proof of Proposition A.2.3. Here, we adopt the same logic of proof as that for the proof of Proposition A.2.1. Again, we only focus on the cases where safety-concerned female riders join at a non-zero rate. Then, it must be the case that all the safety-unconcerned riders have joined, that is, $\lambda^e_m = \Lambda_m$ and $\lambda^e_{f\phi} = \Lambda_{f_\phi}$. Since $N_\phi\mu > \Lambda_\phi$, it is possible that no safety-concerned female drivers participate to work in equilibrium. According to the proof of Proposition A.2.1, we know that if the platform sets a wage

$$w \geq \underline{w}_1(p) = \frac{r(N_m + N_{f_\phi} + N_{f_c})}{\beta\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, N_{f_c})},$$

then all the registered drivers would participate to work. Similarly, following the proof of Proposition A.2.1, we can show that if the platform sets a wage

$$w \geq \underline{w}_2(p) := \frac{r(N_m + N_{f_\phi})}{\Lambda_m + \Lambda_{f_\phi} + \lambda^e_{f_c}(p, 0)},$$

where $\lambda^e_{f_c}(p, 0) = N_\phi\mu - \Lambda_\phi - \frac{c}{(\alpha N_m + N_{f_\phi})/(N_m + N_{f_\phi})R - p}$, then all the safety-unconcerned drivers would participate to work.

If $\underline{w}_2(p) \geq \underline{w}_1(p)$, the platform has no incentives to set $w = \underline{w}_2(p)$ because setting this higher wage can only attract a fraction of drivers to participate in the system. If $\underline{w}_2(p) < \underline{w}_1(p)$, then we get the following result, which has the similar structure with that shown in the proof of Proposition A.2.1:

$$(n^e_m, n^e_{f_\phi}, n^e_{f_c}) = \begin{cases} (N_m, N_{f_\phi}, N_{f_c}), & \text{if and only if} \quad w \geq \underline{w}_1(p); \\ (N_m, N_{f_\phi}, 0), & \text{if and only if} \quad \underline{w}_1(p) > w \geq \underline{w}_2(p); \\ (0, 0, 0), & \text{if and only if} \quad w < \frac{r}{\lambda^e(p,1)}, \end{cases}$$

where $\lambda^e(p,1) = \mu - \frac{c}{R-p}$ represents the effective equilibrium joining rate when there is only one driver participating in the system. When $w \geq \underline{w}_1(p)$, the equilibrium outcome of case $\mathcal{P}1$ can be shown to be exactly the same as that stated in Proposition A.2.1. When $\underline{w}_1(p) > w \geq \underline{w}_2(p)$, by adopting the same logic of proof for case $\mathcal{P}1$ stated in Proposition A.2.1, we can easily obtain the equilibrium outcome for case $\mathcal{P}2$. $\square$

Proposition A.2.3 indicates that there may exist two equilibrium outcomes which differ from each other regarding the participating behaviors of safety-concerned female drivers when the given price and wage satisfy both conditions stated in cases $\mathcal{P}1$ and $\mathcal{P}2$. Under such a situation, we follow Taylor (2018) and assume that all the parties (riders, drivers and the platform) work together to coordinate on the equilibrium that has most drivers participating in the system.

We now proceed to analyze the platform's pricing and wage decision. Note that the platform's optimization problem under case $\mathcal{P}1$ is exactly the same as that presented in §A.2.1. Thus, all the analysis and results stated in Proposition A.2.2 hold. Let Π_1^* denote the optimal profit under case $\mathcal{P}1$ and (p_1^*, w_1^*) the associated optimal price and wage.

As to case $\mathcal{P}2$, when the price and wage satisfy its conditions, for a given price p, a rational platform shall set

$$w(p) = \underline{w}_2(p) = \frac{rN_\phi}{N_\phi\mu - c/(\widehat{\nu}_\alpha R - p)},$$

as increasing the wage above $\underline{w}_2(p)$ has no impact on the joining behaviors of drivers. Then, the platform's optimization problem under case $\mathcal{P}2$ can be formulated as

$$\Pi_2^* = \max_{\underline{w}_2(p)<p<\bar{p}_2} \Pi(p) = \max_{\underline{w}_2(p)<p<\bar{p}_2} (p - \underline{w}_2(p))(\Lambda_\phi + \lambda_{f_c}^e(p,0)),$$

where $\lambda_{f_c}^e(p,0) = N_\phi\mu - \Lambda_\phi - \frac{c}{\widehat{\nu}_\alpha R-p}$. Substituting $\underline{w}_2(p)$ and $\lambda_{f_c}^e(p,0)$ into $\Pi(p)$, we can derive that

$$\Pi(p) = \left(p - \frac{rN_\phi}{N_\phi\mu - c/(\widehat{\nu}_\alpha R - p)}\right)(N_\phi\mu - c/(\widehat{\nu}_\alpha R - p)).$$

We can show that

$$\frac{d\Pi(p)}{dp} = N_\phi \mu - \frac{c\widehat{\nu}_\alpha R}{(\widehat{\nu}_\alpha R - p)^2}, \text{ and } \frac{d^2\Pi(p)}{dp^2} = -\frac{2c\widehat{\nu}_\alpha R}{(\widehat{\nu}_\alpha R - p)^3} < 0.$$

Thus, $\Pi(p)$ is concave in p. Denote $(p_2^*, w_2^* = \underline{w}_2(p_2^*))$ as the corresponding optimal price and wage under this case. Then, the optimal p_2^* shall be the solution of $\frac{d\Pi(p)}{dp} = 0$. It can be shown that $p_2^* = \widehat{\nu}_\alpha R - \sqrt{\frac{c\widehat{\nu}_\alpha R}{N_\phi \mu}}$. Correspondingly, $w_2^* = \frac{r}{\mu - \sqrt{c\mu/(N_\phi \widehat{\nu}_\alpha R)}}$ and $\lambda_{fc}^e(p_2^*, 0) = N_\phi \mu - \Lambda_\phi - \sqrt{\frac{c\mu N_\phi}{\widehat{\nu}_\alpha R}}$.

The platform then compares its profits under the two cases, case $\mathcal{P}1$ and case $\mathcal{P}2$ and chooses the one that has a higher profit. That is, the optimal profit of the platform is $\Pi^* = \max\{\Pi_1^*, \Pi_2^*\}$. Π_1^* is increasing in β (stated in Proposition 2.2) and one can easily check that Π_2^* is independent of β. Let $\widehat{\beta}(\alpha)$ is the unique solution of

$$(\Pi_1^* - \Pi_2^*) \,\big|_{\beta = \widehat{\beta}(\alpha)} = 0, \tag{A.24}$$

if it exists. If $(\Pi_1^* - \Pi_2^*) \,|_{\beta \to 0} > 0$, we let $\widehat{\beta}(\alpha) = 0$; while if $(\Pi_1^* - \Pi_2^*) \,|_{\beta \to 1} < 0$, we let $\widehat{\beta}(\alpha) = 1$. We then have the following result.

Proposition A.2.4. *In a pooling system, when $N_\phi \mu > \Lambda_\phi$, there exists a threshold value $\widehat{\beta}(\alpha)$ such that if the safety-concerned female drivers' safety confidence level $\beta > \widehat{\beta}(\alpha)$, the platform sets the optimal price and wage $(p^*, w^*) = (p_1^*, w_1^*)$, the one characterized by Proposition A.2.2 and equation (A.17). Otherwise, the platform sets $(p^*, w^*) = (p_2^*, w_2^*)$, where $p_2^* = \widehat{\nu}_\alpha R - \sqrt{\frac{c\widehat{\nu}_\alpha R}{N_\phi \mu}}$ and $w_2^* = \frac{r}{\mu - \sqrt{c\mu/(N_\phi \widehat{\nu}_\alpha R)}}$.*

For ease of reference, we summarize the platform's optimal price and wage decisions and the corresponding equilibrium user joining behaviors in a pooling system in Table A.2.1.

For ease of reference, we summarize the platform's optimal price and wage decisions and the corresponding equilibrium user joining behaviors in a pooling system in Table A.2.1.

Table A.2.1: Equilibrium Outcomes in a Pooling System

Conditions		$N_\phi\mu < \Lambda_\phi$	$N_\phi\mu > \Lambda_\phi$ $\beta > \widehat{\beta}(\alpha)$	$N_\phi\mu > \Lambda_\phi$ $\beta \le \widehat{\beta}(\alpha)$
Platform	price	p_1^*	p_1^*	p_2^*
	wage	w_1^*	w_1^*	w_2^*
Riders	male	$\lambda_m^* = \Lambda_m$	$\lambda_m^* = \Lambda_m$	$\lambda_m^* = \Lambda_m$
	$f\phi$	$\lambda_{f_\phi}^* = \Lambda_{f_\phi}$	$\lambda_{f_\phi}^* = \Lambda_{f_\phi}$	$\lambda_{f_\phi}^* = \Lambda_{f_\phi}$
	fc	$\lambda_{f_c}^* = N\mu - \Lambda_\phi - \frac{c}{\nu_\alpha R - p_1^*}$		$\lambda_{f_c}^* = N_\phi\mu - \Lambda_\phi - \sqrt{\frac{c\mu N_\phi}{\widehat{\nu}_\alpha R}}$
Drivers	male	$n_m^* = N_m$	$n_m^* = N_m$	$n_m^* = N_m$
	$f\phi$	$n_{f_\phi}^* = N_{f_\phi}$	$n_{f_\phi}^* = N_{f_\phi}$	$n_{f_\phi}^* = N_{f_\phi}$
	fc	$n_{f_c}^* = N_{f_c}$	$n_{f_c}^* = N_{f_c}$	$n_{f_c}^* = 0$

Notes: (p_1^*, w_1^*) is the price and wage that characterized by Proposition A.2.2 and equation (A.17), respectively.
$p_2^* = \widehat{\nu}_\alpha R - \sqrt{\frac{c\widehat{\nu}_\alpha R}{N_\phi\mu}}$ and $w_2^* = \frac{r}{\mu - \sqrt{c\mu/\left(N_\phi\widehat{\nu}_\alpha R\right)}}$.
$\nu_\alpha = \frac{\alpha N_m + N_f}{N_m + N_f}$ and $\widehat{\nu}_\alpha = \frac{\alpha N_m + N_{f_\phi}}{N_m + N_{f_\phi}}$.

A.3 The Hybrid System: Detailed Analyses

In a hybrid system, the platform adopts a control policy Q under which the number of female drivers joining the pooling subsystem is capped at Q, where $Q \in [0, N_{f_\phi}]$. Similar to that in a pooling system, here we also conduct the analysis over the hybrid system by considering two exhaustive and exclusive scenarios, $N_\phi\mu \le \Lambda_\phi$ and $N_\phi\mu > \Lambda_\phi$.

A.3.1 When the number of safety-unconcerned drivers is low: $N_\phi\mu \le \Lambda_\phi$

In a hybrid system, we consider that the platform adopts the subsystem-based pricing and wage policy. That is, the price and wage in the pooling subsystem can be different from that in the female-only subsystem. First, given these two price and wage pairs in the two subsystems, we analyze the equilibrium joining and participating behaviors of riders and drivers. Again, we focus on the equilibrium outcome in which riders join the two subsystems at non-zero rates. We now develop the conditions under which such equilibrium will exist in the following proposition.

Proposition A.3.1. *In a hybrid system, when $N_\phi \mu \le \Lambda_\phi$, given the control policy $Q \in [0, N_{f_\phi}]$, if the platform sets prices and wages satisfying $p_M < \bar{p}_M := R - \frac{c}{\mu(N_m+Q)}$, $w_M \ge \underline{w}_M := \frac{r(N_m+Q)}{(N_m+Q)\mu - c/(R-p_M)}$, $p_F < \bar{p}_F := R - \frac{c}{\mu(N_f-Q)}$ and $w_F \ge \underline{w}_F := \frac{r(N_f-Q)}{(N_f-Q)\mu - c/(R-p_F)}$, then in equilibrium,*

(a). all male drivers and Q safety-unconcerned female drivers join the pooling subsystem, i.e., $n_m^e = N_m$ and $n_{f_\phi,M}^e = Q$. All safety-concerned female drivers and $N_{f_\phi} - Q$ safety-unconcerned female drivers join the female-only subsystem, i.e., $n_{f_c}^e = N_{f_c}$ and $n_{f_\phi,F}^e = N_{f_\phi} - Q$.

(b). Male riders join the pooling subsystem with rate $\lambda_m^e \in (0, \Lambda_m)$. Safety-concerned female riders join the female-only subsystem with rate $\lambda_{f_c,F}^e \in (0, \Lambda_{f_c})$. As to safety-unconcerned female riders, they join both subsystems with rates $\lambda_{f_\phi,j}^e \in (0, \Lambda_{f_\phi}), j = F, M$, respectively. Moreover, those equilibrium effective joining rates satisfy

$$\begin{cases} \lambda_m^e + \lambda_{f_\phi,M}^e = (N_m + Q)\mu - \frac{c}{R-p_M}, \\ \lambda_{f_c,F}^e + \lambda_{f_\phi,F}^e = (N_f - Q)\mu - \frac{c}{R-p_F}. \end{cases} \tag{A.25}$$

Proof of Proposition A.3.1. We first prove that in equilibrium, no safety-concerned female riders join the pooling subsystem, that is, $\lambda_{f_c,M}^e = 0$. We show this by contradiction. Assume that $\lambda_{f_c,M}^e > 0$. In the pooling subsystem, by comparing the safety-unconcerned riders' joining utility stated in (2.10) with that of safety-concerned female riders stated in (2.11), we obtain that $U_{m,M} = U_{f_\phi,M} \ge U_{f_c,M}$. That is, once the safety-concerned female riders join the pooling subsystem at a non-zero rate, it must be the case that all the safety-unconcerned riders have joined the pooling subsystem so that $\lambda_m^e = \Lambda_m$ and $\lambda_{f_\phi,M}^e = \Lambda_{f_\phi}$. In this situation, even though all the N_m male drives and Q female drivers participate to work in the pooling subsystem, we have

$$(N_m + Q)\mu \le (N_m + N_{f_\phi})\mu < (\Lambda_m + \Lambda_{f_\phi} + \lambda_{f_c,M}^e),$$

where the second inequality results from the assumption that $(N_m + N_{f_\phi})\mu = N_\phi \mu < \Lambda_\phi = \Lambda_m + \Lambda_{f_\phi}$. That is, the pooling subsystem is not steady when $\lambda_{f_c,M}^e > 0$. Thus, it must be that $\lambda_{f_c,M}^e = 0$. Moreover, under the control policy Q, safety-concerned female drivers will not join the pooling subsystem.

Thus, $n^e_{f_c,M} = 0$. This implies that when $N_\phi \mu \le \Lambda_\phi$, the pooling subsystem only contains safety-unconcerned users while safety-concerned female users join the female-only subsystem. Besides, both subsystems are supply-constrained as $(N_m + Q)\mu < \Lambda_\phi$ and $(N_f - Q)\mu < \Lambda_f$. Next, we analyze the joining and participating behaviors of riders and drivers in the two subsystems.

We begin with the pooling subsystem. To ensure that at least one safety-unconcerned rider is willing to join the pooling subsystem, we should have

$$U_{i,M}(0,0,0;0,Q,N_m) = R - p_M - cW(0,0,0;0,Q,N_m) = R - p_M - \frac{c}{(Q+N_m)\mu} \ge 0,$$

where $i = m, f_\phi$. That is, $p_M < \bar{p}_M = R - \frac{c}{(N_m+Q)\mu}$ is required. Recall that $(N_m + Q)\mu < \Lambda_\phi$, in equilibrium, some safety-unconcerned riders must balk the system. Given that there are n_m male drivers and q female drivers participating in the pooling subsystem, where $n_m \le N_m$ and $q \le Q$, the effective joining rates of male riders and safety-unconcerned female riders, λ^e_m and $\lambda^e_{f_\phi,M}$ can be obtained by solving

$$U_{i,M}(0,\lambda_{f_\phi,M},\lambda_m;0,q,n_m) = R - p_M - \frac{c}{(n_m+q)\mu - \lambda_{f_\phi,M} - \lambda_m} = 0, i = m, f_\phi.$$

It can be shown that there exist multiple solutions as long as in the pooling subsystem,

$$\lambda^e_M(p_M,n_m,q) := \lambda^e_m(p_M,n_m,q) + \lambda^e_{f_\phi,M}(p_M,n_m,q) = (n_m+q)\mu - \frac{c}{R-p_M}.$$

Applying the same logic used in the proof of Proposition A.2.1, we can show that the average demand allocated to a single driver in this subsystem is

$$\frac{\lambda^e_M(p_M,n_m,q)}{n_m+q} = \mu - \frac{c}{(R-p_M)(n_m+q)},$$

which is obviously increasing in (n_m+q). Recall that a safety-unconcerned driver in the pooling subsystem is willing to participate if and only if her/his net utility given in (2.12),

$$S_{i,M}(0,\lambda^e_{f_\phi,M},\lambda^e_m;0,q,n_m) = \frac{\lambda^e_{f_\phi,M} + \lambda^e_m}{n_m+q} w_M - r = \frac{\lambda^e_M(p_M,n_m,q)}{n_m+q} w_M - r \ge 0,$$

where $i = m, f_\phi$. As $\frac{\lambda_M^e(p_M, n_m, q)}{n_m + q}$ is increasing in $(n_m + q)$, we get the following result:

$$n_m^e + q^e = \begin{cases} N_m + Q, & \text{if and only if} \quad w_M \geq \frac{r(N_m+Q)}{\lambda_M^e(p_M, N_m, Q)}, \\ 0, & \text{if and only if} \quad w_M < \frac{r}{\lambda_M^e(p_M, 1, 0)}. \end{cases}$$

When $n_m^e + q^e = 0$, no drivers participate in the pooling subsystem, and the hybrid system degenerates to a female-only subsystem. This equilibrium outcome is trivial and uninteresting. Thus, below, we restrict our attention to the equilibrium outcome where $n_m^e + q^e = N_m + Q$, under which all the safety-unconcerned riders participate to work in the pooling subsystem. The corresponding total effective joining rate of safety-unconcerned riders is $\lambda_M^e(p_M, N_m, Q) = (N_m + Q)\mu - \frac{c}{R - p_M}$, and the wage needs to satisfy $w_M \geq \underline{w}_M := \frac{r(N_m+Q)}{\lambda_M^e(p_M, N_m, Q)} = \frac{r(N_m+Q)}{(N_m+Q)\mu - c/(R-p_M)}$.

As for the female-only subsystem, similarly, we can show that in equilibrium, all the remaining $N_f - Q$ registered female drives participate in the system, including $N_{f_\phi} - Q$ safety-unconcerned female drivers and all the N_{f_c} safety-concerned female drivers. The price p_F should satisfy $p_F < \bar{p}_F := R - \frac{c}{(N_f - Q)\mu}$ to ensure that there is at least one female rider joining the female-only subsystem. Correspondingly, the effective joining rate in the female-only subsystem is $\lambda_F^e(p_F, N_{f_\phi} - Q, N_{f_c}) := \lambda_{f_\phi, F}^e(p_F, N_{f_\phi} - Q, N_{f_c}) + \lambda_{f_c, F}^e(p_F, N_{f_\phi} - Q, N_{f_c}) = (N_f - Q)\mu - \frac{c}{R - p_F}$, and the wage is required to be $w_F \geq \underline{w}_M := \frac{r(N_f - Q)}{\lambda_F^e(p_F, N_{f_\phi} - Q, N_{f_c})}$. $\square$

Next, we consider the platform's pricing and wage decisions. For the sake of notation simplicity, hereafter we suppress $\lambda_M^e(p_M, N_m, Q)$ and $\lambda_F^e(p_F, N_{f_\phi} - Q, N_{f_c})$ as $\lambda_j^e(p_j)$, $j = M, F$. Note that in each subsystem j, $j = F, M$, for any given p_j, the platform has no incentive to offer a wage above $\underline{w}_j$. Hence, it is optimal for the platform to set

$$w_j(p_j) = \underline{w}_j(p_j), j = F, M. \tag{A.26}$$

Thus, in a hybrid system, the platform's optimization problem becomes

$$\begin{aligned} \widetilde{\Pi}_0^*(Q) &= \max_{\underline{w}_j(p_j) < p_j < \bar{p}_j, j \in \{F,M\}} \sum_{j \in \{F,M\}} (p_j - \underline{w}_j(p_j))\lambda_j^e(p_j) \\ &= (p_M - \underline{w}_M(p)) \left((N_m + Q)\mu - \frac{c}{R - p_M} \right) \end{aligned}$$

$$
\begin{aligned}
&+(p_F - \underline{w}_F(p))\left((N_f - Q)\mu - \frac{c}{R - p_F}\right) \\
= &\max_{\underline{w}_M(p_M) < p_M < \bar{p}_M} \Pi_M(p_M|Q) + \max_{\underline{w}_F(p_F) < p_F < \bar{p}_F} \Pi_F(p_F|Q),
\end{aligned}
$$

where $\Pi_j(p_j|Q)$ is the subsystem j's profit function, $j = M, F$. This indicates that optimizing the total system profit can be derived by optimizing each subsystem's profit individually. Let $\widetilde{p}^*_{j0}$, $j = F, M$, be the optimal price in the subsystem j for the given control policy Q. Then, in the hybrid system the platform's optimal profit is

$$
\widetilde{\Pi}^*_0(Q) = \Pi_M(\widetilde{p}^*_{M_0} \mid Q) + \Pi_F(\widetilde{p}^*_{F_0} \mid Q).
$$

Proposition A.3.2. *In a hybrid system, when $N_\phi \mu \leq \Lambda_\phi$, given the control policy $Q \in [0, N_{f_\phi}]$, the platform sets the optimal prices and wages as follows:*

$$
\begin{cases}
(\widetilde{p}^*_{M_0}, \widetilde{w}^*_{M_0}) = \left(R - \sqrt{\frac{cR}{(N_m+Q)\mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_m+Q)R}}}\right), \\
(\widetilde{p}^*_{F_0}, \widetilde{w}^*_{F_0}) = \left(R - \sqrt{\frac{cR}{(N_f-Q)\mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_f-Q)R}}}\right).
\end{cases}
$$

Moreover, $\widetilde{p}^*_{F_0} < \widetilde{p}^*_{M_0}$.

Proof of Proposition A.3.2. Recall that we can derive the optimal price for each subsystem individually. First, in the pooling subsystem, the platform sets the price p_M to maximize its profit as follows:

$$
\begin{aligned}
\max_{\underline{w}_M(p_M) < p_M < R - \frac{c}{\mu(N_m+Q)}} \Pi_M(p_M) &= (p_M - \underline{w}_M(p_M))\left((N_m + Q)\mu - \frac{c}{R - p_M}\right) \\
&= p_M\left((N_m + Q)\mu - \frac{c}{R - p_M}\right) - r(N_m + Q).
\end{aligned}
$$

It can be easily shown that $\Pi_M(p_M)$ is concave in p_M as $\frac{d^2\Pi_M(p_M)}{dp_M^2} = \frac{-2cR}{(R-p_M)^3} < 0$. Then, based on the first-order condition

$$
\frac{d\Pi_M(p_M)}{dp_M} = (N_m + Q)\mu - \frac{cR}{(R - p_M)^2} = 0,
$$

we obtain the optimal price $\widetilde{p}^*_{M_0} = R - \sqrt{\frac{cR}{(N_m+Q)\mu}}$, which is smaller than $\bar{p}_M$ because $R > \frac{c}{(N_m+Q)\mu}$. Correspondingly, the optimal wage $\widetilde{w}^*_{M_0} = \frac{r(N_m+Q)}{\lambda^e_M(\widetilde{p}^*_M)} = \frac{r}{\mu - \frac{c}{(N_m+Q)(R-\widetilde{p}^*_M)}} = \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_m+Q)R}}}$. The optimal profit of the pooling subsystem is thus

$$
\widetilde{\Pi}^*_{M_0} = \Pi_M(\widetilde{p}^*_{M_0}) = R\mu(N_m + Q) + c - 2\sqrt{R\mu c(N_m + Q)} - r(N_m + Q).
$$

Next, in the female-only subsystem, the platform sets the price p_F to maximize its profit as follows:

$$\max_{\underline{w}_F(p_F) < p_F < R - \frac{c}{\mu(N_f - Q)}} \Pi_F(p_F) = (p_F - \underline{w}_F(p_F))\lambda_F^e(p_F).$$

Similarly, substituting $\underline{w}_F(p_F) = \frac{r(N_f - Q)}{\lambda_F^e(p_F)}$ and $\lambda_F^e(p_F) = (N_f - Q)\mu - \frac{c}{R - p_F}$ into $\Pi_F(p_F)$, one can show that $\Pi_F(p_F)$ is concave, and there exists an interior optimal solution $\left(\widetilde{p}_{F_0}^* = R - \sqrt{\frac{cR}{(N_f - Q)\mu}},\ \widetilde{w}_{F_0}^* = \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_f - Q)R}}}\right)$ because $R > \frac{c}{(N_f - Q)\mu}$. The corresponding profit in the female-only subsystem is

$$\widetilde{\Pi}_{F_0}^* = \Pi_F(\widetilde{p}_{F_0}^*) = R\mu(N_f - Q) + c - 2\sqrt{R\mu c(N_f - Q)} - r(N_f - Q).$$

Furthermore, due to $N_f - Q < N_m + Q$, one can easily show that $\widetilde{p}_{F_0}^* < \widetilde{p}_{M_0}^*$. □

A.3.2 When the number of safety-unconcerned drivers is large: $N_\phi \mu > \Lambda_\phi$

When the number of safety-unconcerned drivers is sufficiently large ($N_\phi \mu > \Lambda_\phi$), the joining and participating behaviors of riders and drivers are much more complicated. To facilitate our analysis, define $\theta_\alpha := \frac{\alpha N_m + Q}{N_m + Q}$. We still focus on the equilibrium outcome in which riders join the two subsystems at non-zero rates. We now develop the conditions under which such equilibrium will exist in the following proposition.

Proposition A.3.3. *In a hybrid system, when $N_\phi \mu \leq \Lambda_\phi$, given the control policy $Q \in [0, N_{f_\phi}]$, we have the following:*

1. *if $Q \in \left[0, \frac{\Lambda_\phi}{\mu} - N_m\right]$, all the results stated in Proposition A.3.1 are applied here.*

2. *if $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$, depending on the magnitude of prices and wages, we further have that*

   *(a) **(Case $\mathcal{H}1$)** when $p_M \in \Omega_1 := \left[R - \frac{c}{(N_m + Q)\mu - \Lambda_\phi}, R - \frac{c}{(N_m + Q)\mu}\right)$, $w_M \geq \underline{w}_M(p)$, $p_F < \bar{p}_F$ and $w_F \geq \underline{w}_F(p)$, in equilibrium, the joining and participating behaviors of riders and drivers are exactly the same as those stated in Proposition A.3.1.*

(b) ***(Case $\mathcal{H}2$)*** *when* $p_M \in \Omega_2 := \left(0, \theta_\alpha R - \frac{c}{(N_m+Q)\mu - \Lambda_\phi}\right]$, $w_M \geq \frac{r(N_m+Q)}{(N_m+Q)\mu - \frac{c}{\theta_\alpha R - p_M}}$, $p_F < \bar{p}_F$ *and* $w_F \geq \underline{w}_F(p)$,

i. *the drivers' equilibrium participating behaviors are exactly the same as those stated in Proposition A.3.1, that is,* $n^e_m = N_m$, $n^e_{f_\phi,M} = Q$, $n^e_{f_c} = N_{f_c}$ *and* $n^e_{f_\phi,F} = N_{f_\phi} - Q$.

ii. Λ_m *male drivers and* Λ_{f_ϕ} *safety-unconcerned female riders all join the pooling subsystem, i.e.,* $\lambda^e_m = \Lambda_m$ *and* $\lambda^e_{f_\phi,M} = \Lambda_{f_\phi}$. *Safety-concerned female riders join the pooling subsystem with rate* $\lambda^e_{f_c,M}(p_M) = (N_m + Q)\mu - \Lambda_\phi - \frac{c}{\theta_\alpha R - p_M}$ *and join the female-only subsystem with rate* $\lambda^e_{f_c,F}(p_F) = (N_f - Q)\mu - \frac{c}{R - p_F}$.

(c) ***(Case $\mathcal{H}3$)*** *for any given* $p_j \geq w_j$, $j = F, M$, *the participating numbers and joining rates* $(\lambda^e_m = \Lambda_m, \lambda^e_{f_\phi,M} = \Lambda_{f_\phi}, \lambda^e_{f_c,M} = 0, \lambda^e_{f_c,F}; n^e_m, n^e_{f_\phi,M}, n^e_{f_\phi,F}, n^e_{f_c,F} = N_{f_c})$ *are an equilibrium outcome if they satisfy the following set of conditions:*

$$\mathcal{H}3 \text{ Conditions}: \begin{cases} S_m = S_{f_\phi,M} = \frac{\Lambda_\phi}{n^e_m + n^e_{f_\phi,M}} w_M - r \geq 0, \\ S_{f_\phi,F} = S_{f_c,F} = \frac{\lambda^e_{f_c,F}}{N_f - n^e_{f_\phi,M}} w_F - r \geq 0, \\ U_m = U_{f_\phi,M} = R - p_M - \frac{c}{\mu(n^e_m + n^e_{f_\phi,M}) - \Lambda_\phi} > 0, \\ U_{f_c,M} = \frac{\alpha n^e_m + n^e_{f_\phi,M}}{n^e_m + n^e_{f_\phi,M}} R - p_M - \frac{c}{\mu(n^e_m + n^e_{f_\phi,M}) - \Lambda_\phi} \leq 0, \\ U_{f_c,F} = R - p_F - \frac{c}{\mu(N_f - n^e_{f_\phi,M}) - \lambda^e_{f_c,F}} = 0, \\ n^e_{f_\phi,M} \leq Q, n^e_{f_\phi,M} + n^e_{f_\phi,F} = N_{f_\phi}. \end{cases}$$

Proof of Proposition A.3.3. If $Q \in \left[0, \frac{\Lambda_\phi}{\mu} - N_m\right]$, or equivalently, $\mu(N_m + Q) < \Lambda_\phi$, we can easily show that even when all the (N_m+Q) drivers participate in the pooling subsystem, no safety-concerned female riders would join the pooling subsystem as the supply of the pooling subsystem cannot even meet the demand of those safety-unconcerned riders. (We prove this by contradiction, similar to that shown in the proof of Proposition A.3.1). Thus, $\lambda^e_{f_c,M} = 0$. Then, following the proof of Proposition A.3.1, we can show that all the results in Proposition A.3.1 are applied here.

Now, we consider the situation $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$. As we focus on the cases where riders join the two subsystems at nonzero rates, we can further classify those cases according to the joining behaviors of female riders. Then, we have the following three cases.

One: safety-concerned female riders join both subsystems at non-zero rates.

When safety-concerned female riders join the pooling subsystem at a non-zero rate, by the same logic used in the proof of Proposition A.2.1, we know that it must be the case that all the safety-unconcerned riders have joined the pooling subsystem. That is, $\lambda_m^e = \Lambda_m$ and $\lambda_{f_\phi,M}^e = \Lambda_{f_\phi}$. To ensure that at least one safety-concerned female rider is willing to join the pooling subsystem, we need to require that her utility of joining is non-negative when all the $N_m + Q$ possible drivers have participated in the service, i.e.,

$$U_{f_c,M}(0, \Lambda_{f_\phi}, \Lambda_m; 0, Q, N_m) = \frac{\alpha N_m + Q}{N_m + Q} R - p_M - cW(0, \Lambda_{f_\phi}, \Lambda_m; 0, Q, N_m) \geq 0.$$

This requires that $p_M \leq \frac{\alpha N_m + Q}{N_m + Q} R - \frac{c}{(N_m+Q)\mu - \Lambda_\phi}$.

Due to the limited supply of drives in the pooling subsystem ($\mu(N_m + Q) < \Lambda_\phi < \Lambda_\phi + \Lambda_{f_c}$), it is impossible that all the safety-concerned female riders join the pooling subsystem in the steady state. Given the number of participating drivers $(n_m + q)$ in the pooling subsystem, where $n_m \leq N_m$ and $q \leq Q$, the equilibrium effective joining rate of safety-concerned female riders can be obtained by solving

$$U_{f_c,M}(\lambda_{f_c,M}^e, \Lambda_{f_\phi}, \Lambda_m; 0, q, n_m) = \frac{\alpha n_m + q}{n_m + q} R - p_M - c \frac{1}{(n_m + q)\mu - \Lambda_\phi - \lambda_{f_c,M}^e} = 0.$$

It can be shown that in the pooling subsystem, the equilibrium total effective joining rate from all riders is

$$\lambda_M^e(p_M, n_m, q) = \Lambda_\phi + \lambda_{f_c,M}^e(p_M, n_m, q) = (n_m + q)\mu - \frac{c}{\frac{\alpha n_m + q}{n_m + q} R - p_M}.$$

Then, we can show that the average demand allocated to a single driver in the pooling subsystem,

$$\frac{\lambda_M^e(p_M, n_m, q)}{n_m + q} = \mu - \frac{c}{(\alpha R - p_M)n_m + (R - p_M)q}$$

is increasing in both q and n_m. Similar to the proof used in Proposition A.3.1, we can conclude that when $w_M \geq \frac{r(N_m+Q)}{\lambda_M^e(p_M, N_m, Q)} = \frac{r(N_m+Q)}{(N_m+Q)\mu - \frac{c}{\theta_\alpha R - p_M}}$, $N_m + Q$ drivers participate to work in the pooling subsystem. Correspondingly, $\lambda_{f_c,M}^e(p_M) =$

$(N_m+Q)\mu - \Lambda_\phi - \frac{c}{\theta_\alpha R - p_M}$. Due to the constrained supply and overwhelming demand, the remaining $N_f - Q$ female drivers will all join the female-only subsystem. Consequently, all the related analyses in the proof of Proposition A.3.1 can be applied here and it can be shown that $\lambda^e_{f_c,F}(p_F) = (N_f - Q)\mu - \frac{c}{R-p_F}$. This leads to the result stated in case $\mathcal{H}2$.

Two: safety-concerned female riders only join the female-only subsystem, and safety-unconcerned female riders join both subsystems.

To ensure that safety-unconcerned female riders join both subsystems, we should have

$$U_{f_\phi,M}(0,\Lambda_{f_\phi},\Lambda_m;0,Q,N_m) = R - p_M - cW(0,\Lambda_{f_\phi},\Lambda_m;0,Q,N_m) \leq 0,$$

implying that if all the safety-unconcerned riders join the pooling subsystem only, they receive a non-positive utility. Under this situation, $p_M \geq R - \frac{c}{(N_m+Q)\mu - \Lambda_\phi}$ is required. As female riders' joining behaviors are exactly the same as those stated in Proposition A.3.1, the analyses in the proof of Proposition A.3.1 all hold here. This leads to the result summarized in case $\mathcal{H}1$.

Three: safety-concerned female riders only join the female-only subsystem, and safety-unconcerned female riders only join the pooling subsystem.

Note that when safety-unconcerned female riders only join the pooling subsystem, they shall receive a positive joining utility. The reason is that if in equilibrium, they receive a utility of zero, then some safety-unconcerned female riders is indifferent between joining the pooling subsystem and balking. Also, note that due to the limited supply of female drivers, the female riders' joining utility in the female-only subsystem is also zero. Under such a case, the safety-unconcerned female riders shall be also indifferent between joining the pooling subsystem and joining the female-only subsystem. This indicates that if safety-unconcerned female riders only join the pooling subsystem, their joining utility must be positive. Thus, safety-unconcerned female riders all join the pooling subsystem, i.e., $\lambda^e_{f_\phi,M} = \Lambda_\phi$. Then, male riders shall all join the pooling system as well as they behave the same as the safety-unconcerned female riders, i.e., $\lambda^e_m = \Lambda_m$. Recall that under this case, $\lambda^e_{f_c,M} = 0$.

Next, we derive the conditions under which such an equilibrium exists. First, it requires the equilibrium participating number of drivers in the pooling subsystem $(n_m^e + n_{f_\phi,M}^e)$ satisfy

$$U_m = U_{f_\phi,M}(0, \Lambda_{f_\phi}, \Lambda_m; 0, n_{f_\phi,M}^e, n_m^e) = R - p_M - \frac{c}{\mu(n_m^e + n_{f_\phi,M}^e) - \Lambda_\phi} > 0;$$

$$U_{f_c,M}(0, \Lambda_{f_\phi}, \Lambda_m; 0, n_{f_\phi,M}^e, n_m^e) = \frac{\alpha n_m^e + n_{f_\phi,M}^e}{n_m^e + n_{f_\phi,M}^e} R - p_M - \frac{c}{\mu(n_m^e + n_{f_\phi,M}^e) - \Lambda_\phi} \leq 0,$$

where the first inequality ensures the joining utility of safety-unconcerned riders is positive and the second utility guarantees that no safety-concerned female rider has incentive to join the pooling subsystem. As to the driver side, it is required that

$$S_m = S_{f_\phi,M} = \frac{\Lambda_\phi}{n_m^e + n_{f_\phi,M}^e} w_M - r \geq 0.$$

Note that the demand rate per each driver, $\frac{\Lambda_\phi}{n_m^e + n_{f_\phi,M}^e}$, now decreases as the participating number of drivers increases. Put differently, the participation of an additional driver hurts all the existing drivers in the system. In this situation, there is a one-to-one mapping between w_M and $n_m^e + n_{f_\phi,M}^e$. The higher the wage, the larger the participating number of drivers in the pooling subsystem. Note that under the platform's control policy Q, $n_{f_\phi,M}^e \leq Q$ is required.

Regarding the female-only subsystem, at most $N_{f_\phi} - n_{f_\phi,M}^e$ safety-unconcerned female drivers join the female-only subsystem, given that $n_{f_\phi,M}^e \leq Q$ of them join the pooling subsystem, Thus, there are at most $(N_{f_\phi} - n_{f_\phi,M}^e + N_{f_c})$ female drivers participating in the female-only subsystem. Due to the limited supply of female drivers and abundant female riders, in equilibrium, all the $N_{f_\phi} - n_{f_\phi,M}^e + N_{f_c} = N_f - n_{f_\phi,M}^e$ drivers shall participate in the service; that is, $n_{f_c,F}^e = N_{f_c}$ and $n_{f_\phi,M}^e = N_{f_\phi} - n_{f_\phi,M}^e$. As to the safety-concerned female riders, below we prove that not all of them join the female-only subsystem. Suppose that all safety-concerned female riders join the female-only subsystem, that is, $\lambda_{f_c,F}^e = \Lambda_{f_c}$. In the steady state, to ensure the stability of the queueing system, we must have $\mu(N_f - n_{f_\phi,M}^e) > \Lambda_{f_c}$ (the female-only subsystem) and that $\mu(n_m^e + n_{f_\phi,M}^e) > \Lambda_m + \Lambda_{f_\phi}$ (the pooling subsystem). This implies that $\mu(N_f + n_m^e) = \mu(n_m^e + n_{f_\phi,M}^e) + \mu(N_f - n_{f_\phi,M}^e) >$

$\Lambda_m + \Lambda_{f_\phi} + \Lambda_{f_c} = \Lambda$, which contradicts our assumption that $\mu N < \Lambda$. Hence, it is impossible that all the safety-concerned female riders join the female-only subsystem. Furthermore, the equilibrium effective joining rate of safety-concerned female riders $\lambda^e_{f_c,F}$ shall satisfy

$$U_{f_c,F} = R - p_F - \frac{c}{\mu(N_f - n^e_{f_\phi,M}) - \lambda^e_{f_c,F}} = 0.$$

In summary, for any given $p_j \geq w_j$, $j = F, M$, the following participating numbers and joining rates of users, $(\lambda^e_m = \Lambda_m, \lambda^e_{f_\phi,M} = \Lambda_{f_\phi}, \lambda^e_{f_c,M} = 0, \lambda^e_{f_c,F}; n^e_m, n^e_{f_\phi,M}, n^e_{f_\phi,F}, n^e_{f_c,F} = N_{f_c})$, are an equilibrium outcome if they satisfy the following set of conditions:

$$\begin{cases} S_m = S_{f_\phi,M} = \frac{\Lambda_\phi}{n^e_m + n^e_{f_\phi,M}} w_M - r \geq 0, \\ S_{f_\phi,F} = S_{f_c,F} = \frac{\lambda^e_{f_c,F}}{N_f - n^e_{f_\phi,M}} w_F - r \geq 0, \\ U_m = U_{f_\phi,M} = R - p_M - \frac{c}{\mu(n^e_m + n^e_{f_\phi,M}) - \Lambda_\phi} > 0, \\ U_{f_c,M} = \frac{\alpha n^e_m + n^e_{f_\phi,M}}{n^e_m + n^e_{f_\phi,M}} R - p_M - \frac{c}{\mu(n^e_m + n^e_{f_\phi,M}) - \Lambda_\phi} \leq 0, \\ U_{f_c,F} = R - p_F - \frac{c}{\mu(N_f - n^e_{f_\phi,M}) - \lambda^e_{f_c,F}} = 0, \\ n^e_{f_\phi,M} \leq Q, n^e_{f_\phi,M} + n^e_{f_\phi,F} = N_{f_\phi}. \end{cases}$$

□

We now consider the platform's pricing and wage decisions. When $Q \leq \frac{\Lambda_\phi}{\mu} - N_m$, as all the results stated in Proposition A.3.1 hold, the optimal prices and wages shall be the same as those stated in Proposition A.3.2. When $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$, we first show that when the platform maximizes its profit, case $\mathcal{H}3$ will be dominated by case $\mathcal{H}1$ under optimization. Note that under case $\mathcal{H}3$, the rider' joining utility in the pooling subsystem shall be positive. However, when maximizing its profit, the platform can always increase its price p_M to reduce the riders' joining utility to zero, under which case $\mathcal{H}3$ degenerates to case $\mathcal{H}1$. In this way, we can focus on the platform's optimal price and wage decisions under cases $\mathcal{H}1$ and $\mathcal{H}2$.

Under case $\mathcal{H}1$, as the joining and participating behaviors of riders and drivers are exactly the same as those stated in Proposition A.3.1, the platform's pricing and wage optimization problem is also similar to that presented in the Appendix

§A.3.1, which can be written as follows:

$$
\begin{aligned}
\widetilde{\Pi}_1^* &= \max \widetilde{\Pi}_1 = \max_{p_M \in \Omega_1} \Pi_M(p_M|Q) + \max_{\underline{w}_F(p_F)<p_F<\bar{p}_F} \Pi_F(p_F|Q), \\
&= \max_{p_M \in \Omega_1} (p_M - \underline{w}_M(p)) \left((N_m + Q)\mu - \frac{c}{R - p_M} \right) \\
&\quad + \max_{\underline{w}_F(p_F)<p_F<\bar{p}_F} (p_F - \underline{w}_F(p)) \left((N_f - Q)\mu - \frac{c}{R - p_F} \right).
\end{aligned}
$$

Let $(\widetilde{p}_{j_1}^*, \widetilde{w}_{j_1}^*)$ be the optimal price and wage of the subsystem j, $j = M, F$ under case $\mathcal{H}1$. Similarly, under case $\mathcal{H}2$, the platform's pricing and wage optimization problem can be derived as

$$
\begin{aligned}
\widetilde{\Pi}_2^* &= \max \widetilde{\Pi}_2 = \max_{p_M \in \Omega_2} \Pi_M(p_M|Q) + \max_{\underline{w}_F(p_F)<p_F<\bar{p}_F} \Pi_F(p_F|Q), \\
&= \max_{p_M \in \Omega_2} \left(p_M - \frac{r(N_m + Q)}{(N_m + Q)\mu - \frac{c}{\theta_\alpha R - p_M}} \right) \left((N_m + Q)\mu - \frac{c}{\theta_\alpha R - p_M} \right) \\
&\quad + \max_{\underline{w}_F(p_F)<p_F<\bar{p}_F} (p_F - \underline{w}_F(p)) \left((N_f - Q)\mu - \frac{c}{R - p_F} \right).
\end{aligned}
$$

Similarly, denote $(\widetilde{p}_{j_2}^*, \widetilde{w}_{j_2}^*)$ as the optimal price and wage of the subsystem j, $j = M, F$ under case $\mathcal{H}2$. The platform compared the optimal profits under the two cases and choose the one that leads to a higher profit. Thus, the platform's profit $\widetilde{\Pi}^* = \max\{\widetilde{\Pi}_1^*, \widetilde{\Pi}_2^*\}$. Analogous to the derivation process shown in the proof of Proposition A.3.2, we can show that the optimal prices and wages are

$$
\begin{cases}
(\widetilde{p}_{F_k}^*, \widetilde{w}_{F_k}^*) = \left(R - \sqrt{\frac{cR}{(N_f - Q)\mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_f - Q)R}}} \right), k = 1, 2. \\
(\widetilde{p}_{M_1}^*, \widetilde{w}_{M_1}^*) = \left(\max\left\{ R - \sqrt{\frac{cR}{(N_m+Q)\mu}}, R - \frac{c}{(N_m+Q)\mu - \Lambda_\phi} \right\}, \frac{r(N_m+Q)}{(N_m+Q)\mu - \frac{c}{R - \widetilde{p}_{M_1}^*}} \right), \\
(\widetilde{p}_{M_2}^*, \widetilde{w}_{M_2}^*) = \left(\min\left\{ \theta_\alpha R - \sqrt{\frac{c\theta_\alpha R}{(N_m+Q)\mu}}, \theta_\alpha R - \frac{c}{(N_m+Q)\mu - \Lambda_\phi} \right\}, \frac{r(N_m+Q)}{(N_m+Q)\mu - \frac{c}{\theta_\alpha R - \widetilde{p}_{M_2}^*}} \right).
\end{cases}
\tag{A.27}
$$

Based on (A.27) and Proposition A.3.3, we can obtain the equilibrium joining rates of all types of users and the corresponding optimal profits under cases $\mathcal{H}1$ and $\mathcal{H}2$. Then, we have the following result.

Proposition A.3.4. *In a hybrid system, when $N_\phi \mu > \Lambda_\phi$ and $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi} \right]$, there exists a threshold $\widehat{\alpha}$ such that if the safety-concerned female riders' safety confidence level $\alpha < \widehat{\alpha}$, the platform sets the optimal price and wage*

$(\widetilde{p}_M^*, \widetilde{w}_M^*; \widetilde{p}_F^*, \widetilde{w}_F^*) = \left(\widetilde{p}_{M_1}^*, \widetilde{w}_{M_1}^*; \widetilde{p}_{F_1}^*, \widetilde{w}_{F_1}^*\right)$. *Otherwise, the platform sets* $(\widetilde{p}_M^*, \widetilde{w}_M^*; \widetilde{p}_F^*, \widetilde{w}_F^*) = \left(\widetilde{p}_{M_2}^*, \widetilde{w}_{M_2}^*; \widetilde{p}_{F_2}^*, \widetilde{w}_{F_2}^*\right)$. *Moreover,* $\widetilde{p}_{M_1} > \widetilde{p}_{F_1}$.
When $\widetilde{p}_{M_2}^* = \theta_\alpha R - \sqrt{\frac{c\theta_\alpha R}{(N_m+Q)\mu}}$ *and* $\sqrt{R\mu/c} > F(\alpha, Q)$, $\widetilde{p}_{M_2}^* < \widetilde{p}_{F_2}^*$.

Proof of Proposition A.3.4. It is easy to check that $\widetilde{\Pi}_1^*$ is independent of α. We next prove that $\widetilde{\Pi}_2^*$ increases in α. It can be shown that

$$\begin{aligned}
\widetilde{\Pi}_2^* &= (\widetilde{p}_{F_2}^* - \widetilde{w}_{F_2}^*) \cdot \lambda_{F_2}^e(\widetilde{p}_{F_2}^*) + (\widetilde{p}_{M_2}^* - \widetilde{w}_{M_2}^*) \cdot \lambda_{M_2}^e(\widetilde{p}_{M_2}^*), \\
&= \widetilde{\Pi}_{F_2}^* + \begin{cases} \left(\theta_\alpha R - \frac{c}{(N_m+Q)\mu - \Lambda_\phi}\right)\Lambda_\phi - r(N_m+Q) \\ \qquad\qquad\qquad \text{if } \widetilde{p}_{M_2}^* = \theta_\alpha R - \frac{c}{(N_m+Q)\mu-\Lambda_\phi}, \\ \theta_\alpha R\mu(N_m+Q) - 2\sqrt{c\theta_\alpha R\mu(N_m+Q)} + c - r(N_m+Q) \\ \qquad\qquad\qquad \text{if } \widetilde{p}_{M_2}^* = \theta_\alpha R - \sqrt{\frac{c\theta_\alpha R}{(N_m+Q)\mu}}. \end{cases}
\end{aligned}$$

where $\widetilde{\Pi}_{F_2}^*$ is independent of α. We then can obtain that

$$\frac{d\widetilde{\Pi}_2^*}{d\alpha} = \begin{cases} \frac{N_m\Lambda_\phi R}{N_m+Q} > 0, & \text{if } \widetilde{p}_{M_2}^* = \theta_\alpha R - \frac{c}{(N_m+Q)\mu-\Lambda_\phi}. \\ \Upsilon := N_m\mu\sqrt{R}\left(\sqrt{R} - \sqrt{\frac{c}{(\alpha N_m+Q)\mu}}\right), & \text{if } \widetilde{p}_{M_2}^* = \theta_\alpha R - \sqrt{\frac{c\theta_\alpha R}{(N_m+Q)\mu}}. \end{cases} \tag{A.28}$$

Not that for any positive optimal price $\widetilde{p}_{M_2}^* = \theta_\alpha R - \sqrt{\frac{c\theta_\alpha R}{(N_m+Q)\mu}} > 0$, it must have that $R > \frac{c}{(\alpha N_m+Q)\mu}$. Thus, $\Upsilon > 0$. Hence, $\frac{d\widetilde{\Pi}_2^*}{d\alpha} > 0$ always holds. Therefore, $\widetilde{\Pi}_2^*$ is increasing in α. Let $\widehat{\alpha}$ is the unique solution of

$$\left(\widetilde{\Pi}_2^* - \widetilde{\Pi}_1^*\right)\Big|_{\alpha=\widehat{\alpha}} = 0, \tag{A.29}$$

if it exists. If $(\widetilde{\Pi}_2^* - \widetilde{\Pi}_1^*)\,|_{\alpha\to 0} > 0$, we let $\widehat{\alpha} = 0$; while if $(\widetilde{\Pi}_2^* - \widetilde{\Pi}_1^*)\,|_{\alpha\to 1} < 0$, we let $\widehat{\alpha} = 1$. Then, if $\alpha \geq \widehat{\alpha}$, $\widetilde{\Pi}_2^* \geq \widetilde{\Pi}_1^*$; otherwise, $\widetilde{\Pi}_2^* < \widetilde{\Pi}_1^*$.

We now compare the above optimal prices of the two subsystems in different cases. Recall from Proposition A.3.2 that $\widetilde{p}_{F_0}^* = R - \sqrt{\frac{cR}{(N_f-Q)\mu}}$, $\widetilde{p}_{M_0}^* = R - \sqrt{\frac{cR}{(N_m+Q)\mu}}$ and $\widetilde{p}_{F_0}^* < \widetilde{p}_{M_0}^*$. Thus, $\widetilde{p}_{F_1}^* = \widetilde{p}_{F_2}^* = \widetilde{p}_{F_0}^*$. As $\widetilde{p}_{M_1}^* \geq R - \sqrt{\frac{cR}{(N_m+Q)\mu}}$, $\widetilde{p}_{M_1}^* \geq \widetilde{p}_{M_0}^* > \widetilde{p}_{F_0}^* = \widetilde{p}_{F_1}^*$.

When $\widetilde{p}_{M_2}^* = \theta_\alpha R - \sqrt{\frac{c\theta_\alpha R}{(N_m+Q)\mu}}$, we have

$$\widetilde{p}_{F_2}^* - \widetilde{p}_{M_2}^* = (1-\theta_\alpha)R - \sqrt{\frac{cR}{\mu}}\left(\frac{1}{\sqrt{N_f - Q}} - \sqrt{\frac{\theta_\alpha}{N_m+Q}}\right).$$

Then, $\widetilde{p}^*_{F_2} - \widetilde{p}^*_{M_2} > 0$ require that

$$\sqrt{R\mu/c} > F(\alpha, Q) := \frac{1}{1-\theta_\alpha}\left(\frac{1}{\sqrt{N_f - Q}} - \sqrt{\frac{\theta_\alpha}{N_m + Q}}\right)$$
$$= \frac{1}{(1-\alpha)N_m}\left(\frac{N_m + Q}{\sqrt{N_f - Q}} - \sqrt{\alpha N_m + Q}\right),$$

where $F(\alpha, Q) > 0$ as

$$\frac{1}{(1-\alpha)N_m}\left(\frac{N_m + Q}{\sqrt{N_f - Q}} - \sqrt{\alpha N_m + Q}\right) > \frac{\sqrt{N_m + Q} - \sqrt{\alpha N_m + Q}}{(1-\alpha)N_m} > 0.$$

□

For ease of reference, we now summarize the equilibrium outcome under the hybrid system in Table A.3.1 based on the above discussions. Note that in Table A.3.1, $\widetilde{\lambda}^*_{m_0} + \widetilde{\lambda}^*_{f_\phi, M_0} = (N_m + Q)\mu - \frac{c}{R - \widetilde{p}^*_{M_0}}$, $\widetilde{\lambda}^*_{f_c, F_0} + \widetilde{\lambda}^*_{f_\phi, F_0} = (N_f - Q)\mu - \frac{c}{R - \widetilde{p}^*_{F_0}}$, $\widetilde{\lambda}^*_{f_\phi, F_0} + \widetilde{\lambda}^*_{f_\phi, M_0} \leq \Lambda_{f_\phi}$; $\widetilde{\lambda}^*_{m_1} + \widetilde{\lambda}^*_{f_\phi, M_1} = (N_m + Q)\mu - \frac{c}{R - \widetilde{p}^*_{M_1}}$, $\widetilde{\lambda}^*_{f_c, F_1} + \widetilde{\lambda}^*_{f_\phi, F_1} = (N_f - Q)\mu - \frac{c}{R - \widetilde{p}^*_{F_1}}$, $\widetilde{\lambda}^*_{f_\phi, F_1} + \widetilde{\lambda}^*_{f_\phi, M_1} \leq \Lambda_{f_\phi}$; and $\widetilde{\lambda}^*_{f_c, M_2} = (N_m + Q)\mu - \Lambda_\phi - \frac{c}{\theta_\alpha R - \widetilde{p}^*_{M_2}}$, $\widetilde{\lambda}^*_{f_c, F_2} = (N_f - Q)\mu - \frac{c}{R - \widetilde{p}^*_{F_2}}$.

Table A.3.1: Equilibrium Outcomes in a Hybrid System

Player	If $N_\phi\mu \leq \Lambda_\phi$ or ($N_\phi\mu > \Lambda_\phi$ and $Q < \frac{\Lambda_\phi}{\mu} - N_m$)
Platform	$(\widetilde{p}^*_{F_0}, \widetilde{w}^*_{F_0}) = \left(R - \sqrt{\frac{cR}{(N_f - Q)\mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_f - Q)R}}}\right)$ $(\widetilde{p}^*_{M_0}, \widetilde{w}^*_{M_0}) = \left(R - \sqrt{\frac{cR}{(N_m + Q)\mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_m + Q)R}}}\right)$ $\widetilde{\Pi}^*_{F_0} = R\mu(N_f - Q) + c - 2\sqrt{R\mu c(N_f - Q)} - r(N_f - Q)$ $\widetilde{\Pi}^*_{M_0} = R\mu(N_m + Q) + c - 2\sqrt{R\mu c(N_m + Q)} - r(N_m + Q)$ $\widetilde{\Pi}^*_0 = RN\mu + 2c - rN - 2\sqrt{cR\mu}\left(\sqrt{N_m + Q} + \sqrt{N_f - Q}\right)$
Riders	Male:join pooling subsystem at rate $\widetilde{\lambda}^*_{m_0}$ Type-f_ϕ female: join female-only subsystem at rate $\widetilde{\lambda}^*_{f_\phi, F_0}$ Type-f_ϕ female: join pooling subsystem at rate $\widetilde{\lambda}^*_{f_\phi, M_0}$ Type-f_c female: join female-only subsystem at rate $\widetilde{\lambda}^*_{f_c, F_0}$
Drivers	All N_m male drivers join pooling subsystem Q safety-unconcerned female drivers join pooling subsystem $N_{f_\phi} - Q$ type-f_ϕ female drivers join female-only subsystem All N_{f_c} safety-concerned female drivers join female-only subsystem

Table A.3.1: Equilibrium Outcomes in a Hybrid System (Continued)

Player	If $N_\phi \mu > \Lambda_\phi$ and $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$: case $\mathcal{H}1$
Platform	$(\widetilde{p}^*_{F_1}, \widetilde{w}^*_{F_1}) = \left(R - \sqrt{\frac{cR}{(N_f-Q)\mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_f-Q)R}}}\right)$ $(\widetilde{p}^*_{M_1}, \widetilde{w}^*_{M_1})$: equation (A.27) $\widetilde{\Pi}^*_{F_1} = R\mu(N_f - Q) + c - 2\sqrt{R\mu c(N_f - Q)} - r(N_f - Q)$ $\widetilde{\Pi}^*_{M_1} = (\widetilde{p}^*_{M_1} - \widetilde{w}^*_{M_1})\left((N_m + Q)\mu - \frac{c}{R - \widetilde{p}^*_{M_1}}\right)$ $\widetilde{\Pi}^*_1 = \widetilde{\Pi}^*_{F_1} + \widetilde{\Pi}^*_{M_1}$
Riders	Male: join pooling subsystem at rate $\widetilde{\lambda}^*_{m_1}$ Type-f_ϕ female: join female-only subsystem at rate $\widetilde{\lambda}^*_{f_\phi, F_1}$ Type-f_ϕ female:join pooling subsystem at rate $\widetilde{\lambda}^*_{f_\phi, M_1}$ Type-f_c female: join female-only subsystem at rate $\widetilde{\lambda}^*_{f_c, F_1}$
Drivers	All three types: same as that in the case $N_\phi \mu \leq \Lambda_\phi$
Player	If $N_\phi \mu > \Lambda_\phi$ and $Q \in \left(\frac{\Lambda_\phi}{\mu} - N_m, N_{f_\phi}\right]$: case $\mathcal{H}2$
Platform	$(\widetilde{p}^*_{F_2}, \widetilde{w}^*_{F_2}) = \left(R - \sqrt{\frac{cR}{(N_f-Q)\mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{(N_f-Q)R}}}\right)$ $(\widetilde{p}^*_{M_2}, \widetilde{w}^*_{M_2})$: equation (A.27) $\widetilde{\Pi}^*_{F_2} = R\mu(N_f - Q) + c - 2\sqrt{R\mu c(N_f - Q)} - r(N_f - Q)$ $\widetilde{\Pi}^*_{M_2} = (\widetilde{p}^*_{M_2} - \widetilde{w}^*_{M_2})\left((N_m + Q)\mu - \frac{c}{\theta_\alpha R - \widetilde{p}^*_{M_2}}\right)$ $\widetilde{\Pi}^*_2 = \widetilde{\Pi}^*_{F_2} + \widetilde{\Pi}^*_{M_2}$
Riders	Male: all join pooling subsystem (i.e., at rate Λ_m) Type-f_ϕ female: all join pooling subsystem(i.e., at rate Λ_{f_ϕ}) Type-f_c female: join female-only subsystem at rate $\widetilde{\lambda}^*_{f_c, F_2}$ Type-f_c female: join pooling subsystem at rate $\widetilde{\lambda}^*_{f_c, M_2}$
Drivers	All three types: same as that in the case $N_\phi \mu \leq \Lambda_\phi$

Appendix B

Proofs and Supplement for Chapter 3

B.1 Proofs of Propositions and Lemmas

B.1.1 Proof of Lemma 3.1

We follow Gal-Or et al. (2008) (appendix TA.1, on pages 2-5) to prove that the linear decision rule is the unique equilibrium among the general monotonic decision rules. We have shown that the decision rule $q_R = C(x)$ must satisfy equation (3.8), which is

$$\frac{2-\alpha-\alpha\gamma}{2}a - w + \frac{2-(\alpha+1)\gamma}{2(1+s)}x + \frac{(\alpha+2)\gamma^2-4}{2}C(x) - \frac{\gamma}{2(1+s)} \cdot \frac{C(x)}{C'(x)} = 0.$$

Denote $T_0 = \frac{2-\alpha-\alpha\gamma}{2}a - w$, $T_1 = \frac{2-(\alpha+1)\gamma}{2(1+s)}$, $T_2 = \frac{4-(\alpha+2)\gamma^2}{2}$, $T_3 = \frac{\gamma}{2(1+s)}$, $y = C(x)$ and $y' = C'(x)$. Then we can rewrite the above differential equation in a simple way, which is

$$y'\left(T_0 + T_1 x - T_2 y\right) - T_3 y = 0. \tag{B.1}$$

Note that this differential equation has the same form with that on page 4 in appendix TA.1 of Gal-Or et al. (2008). Therefore, all their subsequent analysis can be applied. Specifically, by multiplying both sides of above equation with integrating factor $h(y) = ky^{-\frac{T_1+T_3}{T_3}}$ (k is a constant of integration), we obtain an exact differential equation (see pages 95-100 of Boyce and DiPrima (2012) for details). Then we obtain that solution of (B.1) is the following implicit function of y and x.

$$y\left(ky^{T_1/T_3}-\left(\frac{T_3T_0}{T_1}+T_3x+\frac{T_2T_3}{T_3-T_1}y\right)\right)=0.$$

For $y\neq 0$ (i.e., $q_R\neq 0$), the above equation implies that the solution must satisfy

$$m(x,y)=ky^{T_1/T_3}-\left(\frac{T_3T_0}{T_1}+T_3x+\frac{T_2T_3}{T_3-T_1}y\right)=0.$$

Hence, $y'=-\frac{m_x}{m_y}=\frac{T_3}{\frac{T_1}{T_3}ky^{T_1/T_3-1}-\frac{T_2T_3}{T_3-T_1}}$. Note that if $k\neq 0$, when $y^{T_1/T_3-1}>\frac{T_2T_3^2}{k(T_3-T_1)T_1}$ we have $y'>0$ and when $y^{T_1/T_3-1}<\frac{T_2T_3^2}{k(T_3-T_1)T_1}$ we have $y'<0$. Hence, the only way to make sure that y is strictly monotone is $k=0$, under which situation we have $y'=\frac{T_3}{-\frac{T_2T_3}{T_3-T_1}}$. This implies that the decision rule follows the following form

$$y=q_0+\phi x,$$

where $q_0=\frac{((\alpha+2)\gamma-2)(a(\alpha\gamma+\gamma-2)+2w)}{(\alpha\gamma+\gamma-2)((\alpha+2)\gamma^2-4)}$ and $\phi=\frac{\alpha\gamma+2\gamma-2}{(s+1)(\alpha\gamma^2+2\gamma^2-4)}$. Next, we discuss the sign of ϕ.

(1) $\phi=\frac{\alpha\gamma+2\gamma-2}{(s+1)(\alpha\gamma^2+2\gamma^2-4)}>0$, or equivalently, $\alpha<\frac{2}{\gamma}-2$. In this situation, the retailer sets a higher reselling quantity for a more favorite market (because of $y'=\phi>0$), which is intuitive when a retailer is rational and follows the same logic of assumptions that used in the existing literature such as Li et al. (2014) (which writes "It is intuitive that the reseller will order more when the true market size is larger than when the market size is small" in Section 4.2) and Li and Zhang (2008) (they write "we restrict the search for equilibria to the subspace where $P(Y_K)$ is a strictly increasing function function of $E[\theta|Y_K]$" in Section 4.2).

(2) $\phi=0$, or equivalently, $\alpha=\frac{2}{\gamma}-2$. In this situation, we obtain $y=q_0$, which is independent of the information signal x. That is, the retailer sets the same quantity for all the information signal, under which situation the supplier can not obtain any signal from the retailer's ordering quantity (i.e., a pooling equilibrium). We then can derive the equilibrium reselling quantity of the retailer is

$$q_R^{\mathcal{C}N}=y=q_0=\frac{((\alpha+2)\gamma-2)(a(\alpha\gamma+\gamma-2)+2w)}{(\alpha\gamma+\gamma-2)\left((\alpha+2)\gamma^2-4\right)}=0.$$

and the supplier's direct selling quantity is

$$q_S^{\mathcal{C}N}=\frac{1}{2}\left(a-\gamma q_R\right)=\frac{1}{2}a.$$

In other words, for any given wholesale price w, the retailer gives up all the reselling business and focuses on being a pure agency platform and his revenue is only from the commission fees paid by the supplier. The supplier becomes a monopoly in the market, and the profits of the supplier and the retailer are

$$\Pi_S^{\mathcal{C}N} = \frac{1}{4}(1-\alpha)a^2 \text{ and } \Pi_R^{\mathcal{C}N} = \frac{1}{4}\alpha a^2.$$

We can show that

$$\begin{aligned}
\Pi_R^{\mathcal{C}Y} - \Pi_R^{\mathcal{C}N} &= (\frac{\alpha^3\gamma^4 + 2\alpha^2\left(5\gamma^2-8\right)\gamma^2 + \alpha\left(21\gamma^4+8\gamma^3-84\gamma^2+64\right)}{4\left((\alpha+5)\gamma^2-8\right)^2} \\
&\quad - \frac{8(\gamma-1)^2\left(\gamma^2-2\right)}{4\left((\alpha+5)\gamma^2-8\right)^2}) \cdot (a^2 + \frac{\sigma^2}{s+1}) - \frac{1}{4}\alpha a^2 \\
&= \frac{\alpha\left(4\alpha^4+29\alpha^3+64\alpha^2+44\alpha+9\right)\left(a^2(s+1)+\sigma^2\right)}{4\left(2\alpha^2+7\alpha+3\right)^2(s+1)} - \frac{1}{4}\alpha a^2 \\
&= \frac{a^2\left(\alpha^2+3\alpha+2\right)\alpha^2}{4\left(2\alpha^2+7\alpha+3\right)^2} + \frac{\left(4\alpha^4+29\alpha^3+64\alpha^2+44\alpha+9\right)\alpha\sigma^2}{4\left(2\alpha^2+7\alpha+3\right)^2(s+1)} > 0.
\end{aligned}$$

That is, when $\phi = 0$, the retailer is still always share information with the supplier. All the subsequent analyses after Proposition 3.2 in the main text can be applied.

(3) $\phi < 0$, or equivalently, $\alpha > \frac{2}{\gamma} - 2$. Then it is easy to show that $q_0 < 0$. Consequently, for any positive signal x, we have $y = q_0 + \phi x < 0$, which is not possible in practice. Thus, we ignore this case in our main text.

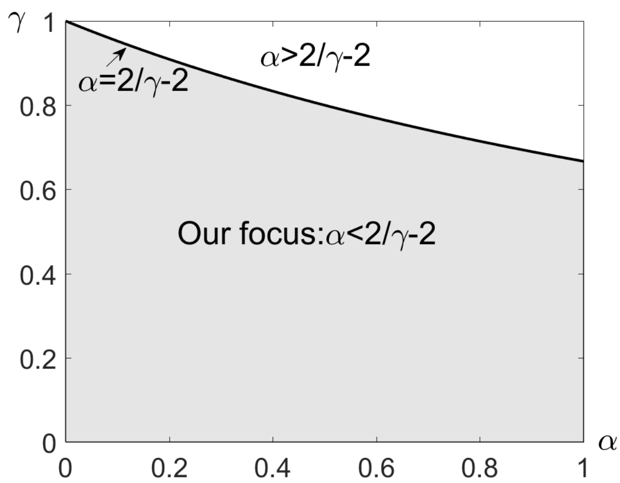


Figure B.1: The Focused Parameter Region in Section 3.4

Based on above discussions, we can conclude that restricting our attention on $\alpha < \frac{2}{\gamma} - 2$ is without loss generality. Also, recall that both γ and α are less than 1 and in practice $\alpha < 0.1$ usually holds, the focused parameter region in our work actually covers most of the practical cases; see the following figure.

B.1.2 Proof of Proposition 3.1 and Corollary 3.1

The proof is based on the results listed in Table 3.2.

(1) Comparison of $E[w^{\mathcal{C}Y}(x)]$ and $E[w^{\mathcal{C}N}(x)]$, and $w^{\mathcal{C}Y}(x)$ and $w^{\mathcal{C}N}(x)$.

First, it can be derived $\frac{E[w^{\mathcal{C}Y}(x)]}{E[w^{\mathcal{C}N}(x)]} = \frac{\left((\alpha^2+5\alpha+6)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16\right)\left(\alpha^2\gamma^3+4\alpha\left(\gamma^2-2\right)\gamma+\gamma^3-6\gamma^2+8\right)}{((\alpha+5)\gamma^2-8)(-2(\alpha^2+6\alpha+5)\gamma^3-4(2\alpha^2+2\alpha-3)\gamma^2+(\alpha+1)^2(\alpha+2)\gamma^4+8(3\alpha+1)\gamma-16)}$, where the numerator minus the denominator equals $2(\alpha-1)(\gamma-1)\gamma^3\left((\alpha+2)\gamma^2-4\right) < 0$. Therefore, $\frac{E[w^{\mathcal{C}Y}(x)]}{E[w^{\mathcal{C}N}(x)]} < 1$. That is, $E[w^{\mathcal{C}Y}(x)] < E[w^{\mathcal{C}N}(x)]$. Furthermore, we have

$$w^{\mathcal{C}Y}(x) - w^{\mathcal{C}N}(x) = E[w^{\mathcal{C}Y}(x)] - E[w^{\mathcal{C}N}(x)] + w_0^{\mathcal{C}Y} \cdot \frac{x}{1+s},$$

where $w_0^{\mathcal{C}Y} > 0$. Thus, there is a unique

$$x = x_1 := \frac{E[w^{\mathcal{C}N}(x)] - E[w^{\mathcal{C}Y}(x)]}{w_0^{\mathcal{C}Y}}(1+s) > 0 \tag{B.2}$$

such that if and only if $x > x_1$, $w^{\mathcal{C}Y}(x) - w^{\mathcal{C}N}(x) > 0$ and otherwise $w^{\mathcal{C}Y}(x) \leq w^{\mathcal{C}N}(x)$.

(2) Comparison of $E[q_R^{\mathcal{C}Y}(x)]$ and $E[q_R^{\mathcal{C}N}(x)]$, and $q_R^{\mathcal{C}Y}(x)$ and $q_R^{\mathcal{C}N}(x)$.

Recall that $E[x] = 0$, then it can be shown that

$$E[q_R^{\mathcal{C}Y}(x)]/E[q_R^{\mathcal{C}N}(x)] = \frac{\left(\alpha^2+5\alpha+6\right)\gamma^3 - 2(\alpha+5)\gamma^2 - 8(\alpha+1)\gamma + 16}{\left((\alpha+2)\gamma-2\right)\left((\alpha+5)\gamma^2-8\right)} > 1,$$

where the inequality results from

$$\begin{aligned}&\left(\alpha^2+5\alpha+6\right)\gamma^3 - 2(\alpha+5)\gamma^2 - \left((\alpha+2)\gamma-2\right)\left((\alpha+5)\gamma^2-8\right) - 8(\alpha+1)\gamma + 16\\&= 2\gamma\left(4-(\alpha+2)\gamma^2\right) > 0.\end{aligned}$$

That is, $E[q_R^{\mathcal{C}Y}(x)] > E[q_R^{\mathcal{C}N}(x)]$. Furthermore, we can show that

$$\frac{d\left(E[q_R^{\mathcal{C}Y}(x)]/E[q_R^{\mathcal{C}N}(x)]\right)}{d\alpha} = \frac{2\gamma^2\left((\alpha+2)^2\gamma^4 - 4(2\alpha+7)\gamma^2 + 6\gamma^3 - 8\gamma + 32\right)}{\left((\alpha+2)\gamma-2\right)^2\left((\alpha+5)\gamma^2-8\right)^2}.$$

Let $f(\alpha) = (\alpha+2)^2\gamma^4 - 4(2\alpha+7)\gamma^2 + 6\gamma^3 - 8\gamma + 32$. It is easy to show that $f'(\alpha) = 2\gamma^2(-4+2\gamma^2+\alpha\gamma^2) < 0$, $f(\alpha)\big|_{\alpha\to 1} = (2+\gamma)(4-3\gamma)(4-3\gamma^2) > 0$ and $f(\alpha)\big|_{\alpha\to\frac{2}{\gamma}-2} = 2(2+\gamma)(2-\gamma)(4-3\gamma) > 0$. Therefore, it must have $f(\alpha) > 0$ for any $\alpha \in \min\{\frac{2}{\gamma}-2, 1\}$. Hence, $\frac{d\left(E[q_R^{\mathcal{C}Y}(x)]\big/E[q_R^{\mathcal{C}N}(x)]\right)}{d\alpha} > 0$. Besides, we have

$$\begin{aligned}
q_R^{\mathcal{C}Y}(x) - q_R^{\mathcal{C}N}(x) &= E[q_R^{\mathcal{C}Y}(x)] - E[q_R^{\mathcal{C}N}(x)] + \frac{2(1-\gamma)\frac{x}{1+s}}{8-(\alpha+5)\gamma^2} - \frac{(2-(\alpha+2)\gamma)\frac{x}{1+s}}{4-(\alpha+2)\gamma^2} \\
&= E[q_R^{\mathcal{C}Y}(x)] - E[q_R^{\mathcal{C}N}(x)] + \frac{g(\alpha)\frac{x}{1+s}}{(4-(2+\alpha)\gamma^2)(8-(5+\alpha)\gamma^2)},
\end{aligned}$$

where $g(\alpha) = 8(1+\alpha)\gamma - 8 + 6\gamma^2 - (6+5\alpha+\alpha^2)\gamma^3$. Obviously, $g(\alpha)$ is concave in α. Combining this with $g'(\alpha)|_{\alpha\to\frac{2}{\gamma}-2} = 8-4\gamma-\gamma^2 > 0$, we know that $g'(\alpha) > 0$, implying that $g(\alpha)$ is increasing in α. Also, $g(\alpha)_{\alpha\to 0} = -2(1-\gamma)(4-3\gamma^2) < 0$ and $g(\alpha)_{\alpha\to\frac{2}{\gamma}-2} = 4(2-\gamma)(1-\gamma) > 0$. Therefore, there is a unique $\alpha = \alpha_3$ solving

$$g(\alpha) = 8(1+\alpha)\gamma - 8 + 6\gamma^2 - (6+5\alpha+\alpha^2)\gamma^3 = 0 \tag{B.3}$$

such that if and only if $\alpha > \alpha_3$, $g(\alpha) > 0$ and otherwise $g(\alpha) < 0$. When $\alpha < \alpha_3$, there is a unique x_2 that equals

$$x_2 = \frac{(4-(2+\alpha)\gamma^2)(8-(5+\alpha)\gamma^2)(1+s)\left(E[q_R^{\mathcal{C}Y}(x)] - E[q_R^{\mathcal{C}N}(x)]\right)}{-g(\alpha)} > 0 \tag{B.4}$$

such that if and only if $x > x_2$, $q_R^{\mathcal{C}Y}(x) < q_R^{\mathcal{C}N}(x)$ and otherwise $q_R^{\mathcal{C}Y}(x) \geq q_R^{\mathcal{C}N}(x)$.

(3) Comparison of $E[q_S^{\mathcal{C}Y}(x)]$ and $E[q_S^{\mathcal{C}N}(x)]$, and $q_S^{\mathcal{C}Y}(x)$ and $q_S^{\mathcal{C}N}(x)$.

We can show that

$$\frac{E[q_S^{\mathcal{C}Y}(x)]}{E[q_S^{\mathcal{C}N}(x)]} = \frac{((\alpha+3)\gamma^2+2\gamma-8)\left((\alpha^2+5\alpha+6)\gamma^3 - 2(\alpha+5)\gamma^2 - 8(\alpha+1)\gamma + 16\right)}{((\alpha+5)\gamma^2-8)\left((\alpha^2+3\alpha+2)\gamma^3 - 4(2\alpha+3)\gamma - 2\gamma^2 + 16\right)},$$

where the numerator minus the denominator equals $4(\gamma-1)\gamma^2\left((\alpha+2)\gamma^2-4\right) > 0$. Thus, we have $E[q_S^{\mathcal{C}Y}(x)]/E[q_S^{\mathcal{C}N}(x)] > 1$. Besides, we have

$$\begin{aligned}
q_S^{\mathcal{C}Y}(x) - q_S^{\mathcal{C}N}(x) &= E[q_S^{\mathcal{C}Y}(x)] - E[q_S^{\mathcal{C}N}(x)] \\
&\quad + \left(\frac{8-(\alpha+3)\gamma^2-2\gamma}{2(8-(\alpha+5)\gamma^2)} - \frac{2-\gamma}{4-(\alpha+2)\gamma^2}\right)\frac{x}{1+s} \\
&= E[q_S^{\mathcal{C}Y}(x)] - E[q_S^{\mathcal{C}N}(x)] \\
&\quad + \frac{-g(\alpha)}{(4-(2+\alpha)\gamma^2)(8-(5+\alpha)\gamma^2)} \cdot \frac{\gamma}{2} \cdot \frac{x}{1+s}.
\end{aligned}$$

The sign of $g(\alpha)$ has been discussed above. $g(\alpha) \leq 0$ if and only if $\alpha \leq \widetilde{\alpha}$. Consequently, $q_S^{\mathcal{C}Y}(x) \geq q_S^{\mathcal{C}N}(x)$ when $\alpha \leq \widetilde{\alpha}$. If $\alpha > \widetilde{\alpha}$, $g(\alpha) > 0$, and there exists a unique x_3 that equals

$$x_3 = \frac{2(4-(2+\alpha)\gamma^2)(8-(5+\alpha)\gamma^2)(1+s)\left(E[q_S^{\mathcal{C}Y}(x)] - E[q_S^{\mathcal{C}N}(x)]\right)}{\gamma g(\alpha)} > 0 \tag{B.5}$$

such that if $x > x_3$, $q_S^{\mathcal{C}Y}(x) < q_S^{\mathcal{C}N}(x)$; otherwise, $q_S^{\mathcal{C}Y}(x) \geq q_S^{\mathcal{C}N}(x)$.

B.1.3 Proof of Proposition 3.2

We first prove that the retailer must share information voluntarily with the supplier. By using the results listed in Table 3.2, we can show that

$$\begin{aligned}\Pi_R^{\mathcal{C}Y} - \Pi_R^{\mathcal{C}N} = & \frac{2(\gamma-1)^2\gamma^2\left(\alpha\gamma^2+2\gamma^2-4\right)k_1 a^2}{\left((\alpha+5)\gamma^2-8\right)^2\left(\left(\alpha^2+5\alpha+6\right)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16\right)^2} \\ & + \frac{k_2\sigma^2}{4\left((\alpha+2)\gamma^2-4\right)\left((\alpha+5)\gamma^2-8\right)^2(s+1)},\end{aligned}$$

where $k_1 = \alpha^3\gamma^4+4\alpha^2\gamma^4-2\alpha^2\gamma^3-8\alpha^2\gamma^2-5\alpha\gamma^4-8\alpha\gamma^3+8\alpha\gamma^2+16\alpha\gamma-18\gamma^4+10\gamma^3+48\gamma^2-16\gamma-32$, and $k_2 = \alpha^4\gamma^6+4\alpha^3\left(3\gamma^2+\gamma-6\right)\gamma^4+\alpha^2(41\gamma^4+52\gamma^3-200\gamma^2-64\gamma+192)\gamma^2+2\alpha\left(17\gamma^6+86\gamma^5-192\gamma^4-224\gamma^3+432\gamma^2+128\gamma-256\right) -4\left(4\gamma^6-33\gamma^5+13\gamma^4+112\gamma^3-80\gamma^2-96\gamma+80\right)$. Recall from Lemma 3.1 that $\alpha < \frac{2}{\gamma}-2$ is required. When $\gamma \in (\frac{2}{3},1)$, $\alpha \in (0,\frac{2}{\gamma}-2)$; and when $\gamma \in (0,\frac{2}{3})$, $\alpha \in (0,1)$. We next consider these two subcases respectively.

Subcase (1): $\gamma \in (0,\frac{2}{3})$. Since

$$\frac{d^2k_1}{d\alpha^2} = 2\gamma^2\left((3\alpha+4)\gamma^2-2\gamma-8\right) < 0 \text{ and } \left.\frac{dk_1}{d\alpha}\right|_{\alpha\to1} = (8-6\gamma^2)(2-\gamma)\gamma > 0,$$

k_1 is increasing in α. Furthermore, $k_1\big|_{\alpha\to1} = -2\left(4-3\gamma^2\right)^2 < 0$. Hence, we have $k_1 < 0$ for any $\alpha \in (0,1)$. Next, we check the sign of k_2. It can be shown that

$$\frac{d^3k_2}{d\alpha^3} = 24\gamma^4\left((\alpha+3)\gamma^2+\gamma-6\right) < 0 \text{ and}$$

$$\frac{d^2k_2}{d\alpha^2} = 2\gamma^2\left(\left(6\alpha^2+36\alpha+41\right)\gamma^4+4(3\alpha+13)\gamma^3-8(9\alpha+25)\gamma^2-64\gamma+192\right).$$

That is, $\frac{d^2k_2}{d\alpha^2}$ is decreasing in α. Besides, we have

$$\left.\frac{d^2k_2}{d\alpha^2}\right|_{\alpha\to1} = 2\gamma^2\left(83\gamma^4+64\gamma^3-272\gamma^2-64\gamma+192\right).$$

Let $k_2^1 = 83\gamma^4 + 64\gamma^3 - 272\gamma^2 - 64\gamma + 192$, it is easily shown that $\frac{dk_2^1}{d\gamma} = 4\left(83\gamma^3 + 48\gamma^2 - 136\gamma - 16\right) < 0$. Therefore, k_2^1 obtains its minimum value when $\gamma \to 1$. We can show that $k_2^1\big|_{\gamma\to 1} = 3 > 0$, implying that $k_2^1 > 0$. It can be concluded that $\frac{d^2k_2}{d\alpha^2}\big|_{\alpha\to 1} > 0$, from which we obtain that $\frac{dk_2}{d\alpha}$ is increasing in α. We have

$$\frac{dk_2}{d\alpha}\big|_{\alpha\to 1} = 4\left(3\gamma^2 - 4\right)\left(13\gamma^4 + 24\gamma^3 - 54\gamma^2 - 16\gamma + 32\right).$$

We can prove that the first order derivative of $13\gamma^4+24\gamma^3-54\gamma^2-16\gamma+32$ equals $4(\gamma-1)(4+31\gamma+13\gamma^2) < 0$ and that $13\gamma^4 + 24\gamma^3 - 54\gamma^2 - 16\gamma + 32 \to \frac{568}{81} > 0$ when $\gamma \to \frac{2}{3}$. Hence, we have $\frac{dk_2}{d\alpha}\big|_{\alpha\to 1} < 0$ for any $\gamma < \frac{2}{2+\alpha}$, which further implies that $\frac{dk_2}{d\alpha} < 0$ for any given $\alpha, \gamma \in (0,1)$ and $\gamma < \frac{2}{2+\alpha}$. We obtain that k_2 is decreasing in α. It can be shown that

$$k_2\big|_{\alpha\to 0} = 4(1-\gamma)(4\gamma^5 - 29\gamma^4 - 16\gamma^3 + 96\gamma^2 + 16\gamma - 80)$$

Denote that $k_2^2 = 4\gamma^5 - 29\gamma^4 - 16\gamma^3 + 96\gamma^2 + 16\gamma - 80$. One can check that the first order derivative of k_2^2 equals $4\left(5\gamma^4 - 29\gamma^3 - 12\gamma^2 + 48\gamma + 4\right) > 0$ and that $k_2^2\big|_{\gamma\to\frac{2}{3}} < k_2^2\big|_{\gamma\to 1} = -9 < 0$. Therefore, $k_2 < 0$. Combining this with $k_1 < 0$, $\alpha\gamma^2 + 2\gamma^2 - 4 < 0$ and $(\alpha+2)\gamma^2 - 4 < 0$, we can conclude that $\Pi_R^{CY} > \Pi_R^{CN}$.

Subcase (2): $\gamma \in (\frac{2}{3}, 1)$. Same to above *subcase (1)*, we can prove that $\frac{d^2k_1}{d\alpha^2} = 2\gamma^2\left((3\alpha+4)\gamma^2 - 2\gamma - 8\right) < 0$, implying that $\frac{dk_1}{d\alpha}$ is decreasing in α. It can be derived that

$$\frac{dk_1}{d\alpha}\Big|_{\alpha\to\frac{2}{\gamma}-2} = -\gamma\left(9\gamma^3 + 8\gamma^2 - 44\gamma + 16\right).$$

Since the first order condition of $9\gamma^3+8\gamma^2-44\gamma+16$ equals $-44+16\gamma+27\gamma^2 < 0$, its maximum is obtained when $\gamma \to \frac{2}{3}$, which equals $-64/9 < 0$. Therefore, $\frac{dk_1}{d\alpha} > \frac{dk_1}{d\alpha}\Big|_{\alpha\to\frac{2}{\gamma}-2} > 0$, implying that k_1 is increasing in α. Due to that $k_1\big|_{\alpha\to\frac{2}{\gamma}-2} = -8(\gamma-2)^2 < 0$, we conclude that $k_1 < 0$. Similar to above subcase, $\frac{d^2k_2}{d\alpha^2}$ can be shown decrease in α. Besides, $\frac{d^2k_2}{d\alpha^2}\big|_{\alpha\to\frac{2}{\gamma}-2} = -2\gamma^2\left(7\gamma^4 - 52\gamma^3 + 8\gamma^2 + 208\gamma - 192\right)$. Let $k_{21} = 7\gamma^4 - 52\gamma^3 + 8\gamma^2 + 208\gamma - 192$. It is easy to show that $k_{21}' = 4\left(7\gamma^3 - 39\gamma^2 + 4\gamma + 52\right) > 0$, implying that k_{21} increases in γ. Since $k_{21}\big|_{\gamma\to 1} = -21 < 0$, $k_{21} < 0$. Therefore, $\frac{d^2k_2}{d\alpha^2} > \frac{d^2k_2}{d\alpha^2}\big|_{\alpha\to\frac{2}{\gamma}-2} > 0$. Consequently, $\frac{dk_2}{d\alpha}$ is increasing in α. It can be derived that

$$\frac{dk_2}{d\alpha}\Big|_{\alpha\to\frac{2}{\gamma}-2} = 2(2-\gamma)\left(9\gamma^5+26\gamma^4-92\gamma^3-16\gamma^2+192\gamma-128\right).$$

Let $k_{22} = 9\gamma^5+26\gamma^4-92\gamma^3-16\gamma^2+192\gamma-128$. We have

$$k_{22}'' = 4\left(45\gamma^3+78\gamma^2-138\gamma-8\right) < 0, k_{22}'\big|_{\gamma\to 1} = 33 > 0 \text{ and } k_{22}\big|_{\gamma\to 1} = -9 < 0.$$

Therefore, $k_{22} < 0$, implying that $\frac{dk_2}{d\alpha} < \frac{dk_2}{d\alpha}\big|_{\alpha\to\frac{2}{\gamma}-2} < 0$. That is, k_2 is decreasing in α. The remaining of the proof in this subcase is exactly the same with the subcase listed above and is omitted. We also can obtain that $\Pi_R^{\mathcal{C}Y} > \Pi_R^{\mathcal{C}N}$.

We next prove the free information sharing always benefit the supplier. Again, based on Table 3.2, we can derive that

$$\Pi_S^{\mathcal{C}Y} - \Pi_S^{\mathcal{C}N} = \frac{8(\gamma-1)^2\gamma\left(4-(\alpha+2)\gamma^2\right)a^2}{4\left(8-(\alpha+5)\gamma^2\right)\left(\left(\alpha^2+5\alpha+6\right)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16\right)} + \frac{\sigma^2\left(\frac{-\alpha^2\gamma^2-4\alpha\left(\gamma^2-2\right)+\gamma^2+8\gamma-12}{(\alpha+5)\gamma^2-8}+\frac{4(\alpha-1)(\gamma-2)^2}{\left((\alpha+2)\gamma^2-4\right)^2}\right)}{4(s+1)}.$$

Let $k_w = \left(\alpha^2+5\alpha+6\right)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16$, it is easy to show that

$$\frac{dk_w}{d\alpha} = \gamma\left((2\alpha+5)\gamma^2-2\gamma-8\right), \frac{d^2k_w}{d\alpha^2} = 2\gamma^3 > 0.$$

That is, the first order derivative of α , $\frac{dk_w}{d\alpha}$, is increasing in α. Same to the analysis of retailer's information sharing decision, we consider two subcases.

<u>Subcase (1): $\gamma \in (0, \frac{2}{3})$.</u> When $\alpha \to 1$, $\frac{dk_w}{d\alpha}\Big|_{\alpha\to 1} = \gamma\left(7\gamma^2-2\gamma-8\right) < 0$. Hence, $\frac{dk_w}{d\alpha} < 0$ for any $\alpha \in (0,1)$, which implies that k_w is decreasing in α. We have that

$$k_w\Big|_{\alpha\to 1} = 4(1-\gamma)(4-3\gamma^2) > 0.$$

We then can conclude that $k_w > 0$.

<u>Subcase (2): $\gamma \in [\frac{2}{3}, 1)$.</u> Since $\frac{dk_w}{d\alpha}\Big|_{\alpha\to\frac{2}{\gamma}-2} = \gamma\left(\gamma^2+2\gamma-8\right) < 0$, it has that $\frac{dk_w}{d\alpha} < 0$ for any $\alpha \in (0, \frac{2}{\gamma}-2)$, which implies that k_w is decreasing in α. Since $k_w\Big|_{\alpha\to\frac{2}{\gamma}-2} = 4(2-\gamma)\gamma > 0$, $k_w > 0$. Consequently, we have

$$\frac{8(\gamma-1)^2\gamma\left(4-(\alpha+2)\gamma^2\right)a^2}{4\left(8-(\alpha+5)\gamma^2\right)\left(\left(\alpha^2+5\alpha+6\right)\gamma^3-2(\alpha+5)\gamma^2-8(\alpha+1)\gamma+16\right)} > 0.$$

It can be shown that

$$\frac{4(\gamma-2)^2}{\left((\alpha+2)\gamma^2-4\right)^2}<1 \text{ and}$$

$$\frac{-\alpha^2\gamma^2-4\alpha\left(\gamma^2-2\right)+\gamma^2+8\gamma-12}{(1-\alpha)\left((\alpha+5)\gamma^2-8\right)}-1=\frac{4(\gamma-1)^2}{(\alpha-1)\left((\alpha+5)\gamma^2-8\right)}>0,$$

we then obtain that

$$\frac{-\alpha^2\gamma^2-4\alpha\left(\gamma^2-2\right)+\gamma^2+8\gamma-12}{(\alpha+5)\gamma^2-8}+\frac{4(\alpha-1)(\gamma-2)^2}{\left((\alpha+2)\gamma^2-4\right)^2}$$

$$=\frac{1}{1-\alpha}\left(\frac{-\alpha^2\gamma^2-4\alpha\left(\gamma^2-2\right)+\gamma^2+8\gamma-12}{(1-\alpha)\left((\alpha+5)\gamma^2-8\right)}-\frac{4(\gamma-2)^2}{\left((\alpha+2)\gamma^2-4\right)^2}\right)>0.$$

Therefore, we have $\Pi_S^{\mathcal{C}Y}>\Pi_S^{\mathcal{C}N}$, implying that the supplier benefits from the retailer's free information sharing.

Based on the results listed in Table 3.2, it is easy to show that

$$\frac{dq_R^{\mathcal{C}Y}}{d\alpha}=\frac{2(1-\gamma)\gamma^2(a+\frac{x}{1+s})}{\left((\alpha+5)\gamma^2-8\right)^2}>0,\ \frac{dq_S^{\mathcal{C}Y}}{d\alpha}=\frac{(\gamma-1)\gamma^3(a+\frac{x}{1+s})}{\left((\alpha+5)\gamma^2-8\right)^2}<0 \text{ and}$$

$$\frac{dw^{\mathcal{C}Y}}{d\alpha}=-\frac{\gamma\left(\left(\alpha^2+10\alpha+19\right)\gamma^4-8(2\alpha+9)\gamma^2+6\gamma^3-8\gamma+64\right)}{2\left((\alpha+5)\gamma^2-8\right)^2}(a+\frac{x}{1+s}).$$

Let $f(\alpha)=\left(\alpha^2+10\alpha+19\right)\gamma^4-8(2\alpha+9)\gamma^2+6\gamma^3-8\gamma+64$. It can be easily shown that $f'(\alpha)=2\gamma^2((5+\alpha)\gamma^2-8)<0$, implying that $f(\alpha)$ decreases in α. Besides, $f(\alpha)\big|_{\alpha\to1}=2(4-3\gamma^2)(8-\gamma-5\gamma^2)>0$ and one can easily check that $f(\alpha)\big|_{\alpha\to\frac{2}{\gamma}-2}=64-40\gamma-36\gamma^2+18\gamma^3+3\gamma^4>0$. That is, $f(\alpha)>0$ for any $\alpha\in(0,\min\{1,\frac{2}{\gamma}-2\})$. Therefore, $\frac{dw^{\mathcal{C}Y}}{d\alpha}<0$. We also can prove that $\frac{d\Pi_S^{\mathcal{C}Y}}{d\alpha}=-\frac{\left(\alpha\gamma^2+3\gamma^2+2\gamma-8\right)\left(\alpha\gamma^2+7\gamma^2-2\gamma-8\right)}{4\left((\alpha+5)\gamma^2-8\right)^2}(a^2+\frac{\sigma^2}{1+s})<0$, and $\frac{d\Pi_R^{\mathcal{C}Y}}{d\alpha}=\frac{\alpha^3\gamma^6+\alpha^2\left(15\gamma^6-24\gamma^4\right)+\alpha\left(79\gamma^6-8\gamma^5-236\gamma^4+192\gamma^2\right)+121\gamma^6+8\gamma^5-604\gamma^4+960\gamma^2-512}{4\left((\alpha+5)\gamma^2-8\right)^3}(a^2+\frac{\sigma^2}{1+s})$. Letting $f_1(\alpha)=\alpha^3\gamma^6+\alpha^2\left(15\gamma^6-24\gamma^4\right)+\alpha\left(79\gamma^6-8\gamma^5-236\gamma^4+192\gamma^2\right)+121\gamma^6+8\gamma^5-604\gamma^4+960\gamma^2-512$. It can be shown that $f_1''(\alpha)=6\gamma^4\left((\alpha+5)\gamma^2-8\right)<0$. Hence, we obtain that $f_1'(\alpha)=\gamma^2\left(\left(3\alpha^2+30\alpha+79\right)\gamma^4-4(12\alpha+59)\gamma^2-8\gamma^3+192\right)$ decreases in α. Combining this with

$$f_1'(\alpha)=\begin{cases}4\gamma^2\left(28\gamma^4-2\gamma^3-71\gamma^2+48\right)>0, & \text{if } \alpha\to1;\\ \gamma^2\left(31\gamma^4+28\gamma^3-128\gamma^2-96\gamma+192\right)>0, & \text{if } \alpha\to\frac{2}{\gamma}-2,\end{cases}$$

we conclude that $f_1'(\alpha)>0$, implying that $f_1(\alpha)$ increases in α. Also, we can show that $f_1(\alpha)\big|_{\alpha\to1}=8\left(3\gamma^2-4\right)^3<0$ and $f_1(\alpha)\big|_{\alpha\to\frac{2}{\gamma}-2}=15\gamma^6+86\gamma^5-208\gamma^4-$

$272\gamma^3+480\gamma^2+384\gamma-512$. Note that the largest value α can attain is $\frac{2}{\gamma}-2$ when $\gamma \in (\frac{2}{3},1)$. One can further check that $f_1(\alpha)\big|_{\alpha\to\frac{2}{\gamma}-2}$ is increasing in $\gamma \in (\frac{2}{3},1)$ and obtains the highest value -27 when $\gamma \to 1$. Therefore, $f_1(\alpha)\big|_{\alpha\to\frac{2}{\gamma}-2} < 0$. Hence, we conclude that $f_1(\alpha) < 0$ for any $\alpha \in (0, \min\{\frac{2}{\gamma}-2, 1\})$. Consequently, $\frac{d\Pi_R^{\mathcal{C}Y}}{d\alpha} > 0$.

Lastly, we have

$$\frac{d\left(\Pi_S^{\mathcal{C}Y}+\Pi_R^{\mathcal{C}Y}\right)}{d\alpha} = \frac{2(\gamma-1)^2\gamma^2\left((\alpha+2)\gamma^2-4\right)}{\left((\alpha+5)\gamma^2-8\right)^3}\left(a^2+\frac{\sigma^2}{1+s}\right) > 0.$$

This completes the proof.

B.1.4 Proof of Lemma 3.2

The proof of this result is very similar to Lemma 3.1 and thus is omitted.

B.1.5 Proof of Proposition 3.3

Note that the supplier's profits under the two channels are respectively:

$$\Pi_S^{\mathcal{C}} = \frac{\left(\alpha^2\gamma^2+4\alpha\left(\gamma^2-2\right)-\gamma^2-8\gamma+12\right)\left(a^2+\frac{\sigma^2}{s+1}\right)}{4\left(8-(\alpha+5)\gamma^2\right)},$$

$$\Pi_S^{\mathcal{D}} = \frac{a^2(3-\gamma)(\gamma-2)^2}{4\left(3\gamma^3-5\gamma^2-4\gamma+8\right)} + \frac{(\gamma-2)^2\sigma^2}{4\left(\gamma^2-2\right)^2(s+1)} - c.$$

We can derive that

$$\begin{aligned}\Pi_S^{\mathcal{D}}-\Pi_S^{\mathcal{C}} =& \frac{1}{4}a^2\left(\frac{(3-\gamma)(\gamma-2)^2}{3\gamma^3-5\gamma^2-4\gamma+8} - \frac{\alpha^2\gamma^2+4\alpha\left(\gamma^2-2\right)-\gamma^2-8\gamma+12}{8-(\alpha+5)\gamma^2}\right)\\ &+\frac{\sigma^2\left(\frac{(\gamma-2)^2}{\left(\gamma^2-2\right)^2}-\frac{\alpha^2\gamma^2+4\alpha\left(\gamma^2-2\right)-\gamma^2-8\gamma+12}{8-(\alpha+5)\gamma^2}\right)}{4(s+1)} - c.\end{aligned}$$

Denote

$$k_4 = \frac{(3-\gamma)(\gamma-2)^2}{3\gamma^3-5\gamma^2-4\gamma+8},\quad k_5 = \frac{(\gamma-2)^2}{\left(\gamma^2-2\right)^2},\ \text{and}$$
$$F(\alpha) = \frac{\alpha^2\gamma^2+4\alpha\left(\gamma^2-2\right)-\gamma^2-8\gamma+12}{8-(\alpha+5)\gamma^2}.$$

Then

$$\Pi_S^{\mathcal{D}}-\Pi_S^{\mathcal{C}} = \frac{1}{4}a^2\left(k_4-F(\alpha)\right)+\frac{\sigma^2}{4(s+1)}\left(k_5-F(\alpha)\right)-c,$$

where k_4 and k_5 both are independent of α. One can derive that $k_4 - k_5 = \frac{(\gamma-2)^4(1-\gamma)(\gamma+1)^2}{(\gamma^2-2)^2(3\gamma^3-5\gamma^2-4\gamma+8)} > 0$, where the inequality holds since $8+3\gamma^3-5\gamma^2-4\gamma > 0$ as $\frac{d(8+3\gamma^3-5\gamma^2-4\gamma)}{d\gamma} = 9\gamma^2-10\gamma-4 < 0$ and its minimum is obtained at $\gamma \to 1$, which is positive. It can be shown that $\frac{dF(\alpha)}{d\alpha} = -\frac{(\alpha^2+10\alpha+21)\gamma^4-4(4\alpha+21)\gamma^2+8\gamma^3+64}{((\alpha+5)\gamma^2-8)^2}$. Let $k_3 = (\alpha^2+10\alpha+21)\gamma^4 - 4(4\alpha+21)\gamma^2+8\gamma^3+64$. Since $\frac{dk_3}{d\alpha} = 2\gamma^2(-8+(5+\alpha)\gamma^2) < 0$, k_3 is decreasing in α. Besides, $k_3\big|_{\alpha\to 1} = 4(-4+\gamma+2\gamma^2)(-4-\gamma+4\gamma^2) > 0$ and $k_3\big|_{\alpha\to\frac{2}{\gamma}-2} = (\gamma^2+4\gamma-8)(5\gamma^2-8) > 0$. Hence, $k_3 > 0$ for any given $\alpha \in (0,1)$, implying that $\frac{dF(\alpha)}{d\alpha} < 0$. We then conclude that $F(\alpha)$ is decreasing in α.

When $\gamma \in (0, \frac{2}{3})$, we have

$$F(0) = \frac{12-\gamma^2-8\gamma}{8-5\gamma^2};\ F(1) = \frac{2(\gamma-1)^2}{4-3\gamma^2} > 0.$$

Furthermore, we have that $F(0) - k_4 = \frac{8(\gamma-1)^2\gamma(\gamma^2-2)}{(5\gamma^2-8)(3\gamma^3-5\gamma^2-4\gamma+8)} > 0$ and $F(1) - k_5 = \frac{-2\gamma^6+4\gamma^5+3\gamma^4-4\gamma^3-8\gamma^2+8}{(\gamma^2-2)^2(3\gamma^2-4)}$. Let $k_6 = -2\gamma^6+4\gamma^5+3\gamma^4-4\gamma^3-8\gamma^2+8$, from which we obtain that

$$\frac{dk_6}{d\alpha} = -4\gamma\left(3\gamma^4-5\gamma^3-3\gamma^2+3\gamma+4\right) = -4\left(3\gamma(1-\gamma)+4-5\gamma^3+3\gamma^4\right) < 0.$$

Thus, $k_6 > k_6\big|_{\gamma\to 1} = 1 > 0$. Hence, we have $F(1) - k_5 < 0$ given that $\gamma \in (0, \frac{2}{3})$.

When $\gamma \in [\frac{2}{3}, 1)$. We have

$$F(0) = \frac{12-\gamma^2-8\gamma}{8-5\gamma^2} \text{ and } F(\frac{2}{\gamma}-2) = \frac{5\gamma^3+8\gamma^2-32\gamma+16}{\gamma(\gamma+2)(3\gamma-4)} > 0.$$

It can be derived that $F(\frac{2}{\gamma}-2) - k_5 = \frac{(\gamma-1)^2(5\gamma^5+18\gamma^4-24\gamma^3-72\gamma^2+32\gamma+64)}{\gamma(\gamma+2)(3\gamma-4)(\gamma^2-2)^2}$. Let $k_{61} = 5\gamma^5+18\gamma^4-24\gamma^3-72\gamma^2+32\gamma+64$. We have $k_{61}''' = 12(25\gamma^2+36\gamma-12) > 0$, $k_{61}''\big|_{\gamma\to 1} = 28 > 0$ and $k_{61}''\big|_{\gamma\to\frac{2}{3}} = -\frac{3088}{27} < 0$ for any given $\gamma \in [\frac{2}{3}, 1)$. Thus, $k_{61}' = 25\gamma^4+72\gamma^3-72\gamma^2-144\gamma+32$ is first decreasing and then increasing in γ. It is easy to check that $k_{61}'\big|_{\gamma\to\frac{2}{3}} = -\frac{5648}{81} < 0$ and $k_{61}'\big|_{\gamma\to 1} = -87 < 0$. Therefore, $k_{61}' < 0$, implying that k_{61} is decreasing in γ. Since $k_{61}\big|_{\gamma\to 1} = 23 > 0$, $k_{61} > 0$. Consequently, $F(\frac{2}{\gamma}-2) - k_5 < 0$.

In summary, we have

$$F(\alpha_m) < k_5 < k_4 < F(0),$$

where $\alpha_m = 1$ for $\gamma \in (0, \frac{2}{3})$ and $\alpha_m = \frac{2}{\gamma} - 2$ otherwise. Therefore, there exist two thresholds $\underline{\alpha}(\gamma) < \bar{\alpha}(\gamma)$, which satisfy

$$F(\underline{\alpha}(\gamma)) = k_4 \text{ and } F(\bar{\alpha}(\gamma)) = k_5. \tag{B.6}$$

Besides, we can show that $F(\frac{1}{2}) = \frac{5\gamma^2-32\gamma+32}{32-22\gamma^2} - k_5 = -\frac{\gamma^2(5\gamma^4-32\gamma^3+34\gamma^2+40\gamma-52)}{2(\gamma^2-2)^2(11\gamma^2-16)} < 0$ and $F(\frac{1}{5}) - k_4 = \frac{71\gamma^5-165\gamma^4+42\gamma^3+186\gamma^2-280\gamma+160}{5(13\gamma^2-20)(3\gamma^3-5\gamma^2-4\gamma+8)} < 0$. Hence, $\bar{\alpha}(\gamma) < \frac{1}{2}$ and $\underline{\alpha}(\gamma) < \frac{1}{5}$. When $\alpha \le \underline{\alpha}(\gamma)$, $F(\alpha) \ge F(\underline{\alpha}(\gamma)) = k_4$, resulting in $\Pi_S^{\mathcal{D}} - \Pi_S^{\mathcal{C}} < 0$ for any given $c \ge 0$. When $\underline{\alpha}(\gamma) < \alpha < \bar{\alpha}(\gamma)$, $k_5 = F(\bar{\alpha}(\gamma)) < F(\alpha) < F(\underline{\alpha}(\gamma)) = k_4$. Define

$$K(s) := \frac{1}{4}a^2\left(k_4 - F(\alpha)\right) + \frac{\sigma^2}{4(s+1)}\left(k_5 - F(\alpha)\right) \text{ and } \widehat{c}_1(s) := \max\{K(s), 0\}. \tag{B.7}$$

Then, if $c \ge \widehat{c}_1(s)$, we have $\Pi_S^{\mathcal{D}} - \Pi_S^{\mathcal{C}} \le 0$; otherwise, $\Pi_S^{\mathcal{D}} - \Pi_S^{\mathcal{C}} > 0$. It is easy to check that $k_5 - F(\alpha) < 0$, which reveals that $K(s)$ is decreasing in $1/s$. So is $\widehat{c}_1(s)$. When $\alpha \ge \bar{\alpha}(\gamma)$, $F(\alpha) < F(\bar{\alpha}(\gamma)) = k_5$. Then, it can be easily verified that if $c \ge \widehat{c}_1(s) = \frac{1}{4}a^2\left(k_4 - F(\alpha)\right) + \frac{\sigma^2}{4(s+1)}\left(k_5 - F(\alpha)\right)$, we have $\Pi_S^{\mathcal{D}} - \Pi_S^{\mathcal{C}} < 0$; otherwise, we have $\Pi_S^{\mathcal{D}} - \Pi_S^{\mathcal{C}} > 0$. In this situation (i.e., $\alpha \ge \bar{\alpha}(\gamma)$), one can check that $k_5 - F(\alpha) > 0$. This means that $K(s)$ is increasing in $1/s$. So is $\widehat{c}_1(s)$. Note that $1/(s\sigma^2)$ has the same monotone property with $1/s$, so the above monotonic property analysis still hold if we conduct them on the $1/(s\sigma^2)$, the signal accuracy measure. The proof is thus completed.

B.1.6 Proof of Proposition 3.4

It is straightforward to show that the supplier is better off when the retailer shares his information as $\widetilde{\Pi}_S^{\mathcal{C}Y} - \widetilde{\Pi}_S^{\mathcal{C}N} = \widetilde{\Pi}_{S_0} \cdot \frac{\sigma^2}{1+s} > 0$. Next, we analyze the retailer's information sharing decision. By using the results listed in Table 3.4, we can obtain that

$$\widetilde{\Pi}_R^{\mathcal{C}Y} - \widetilde{\Pi}_R^{\mathcal{C}N} = \left(\widetilde{\Pi}_{R_0} - \frac{1}{4}\right)\frac{\sigma^2}{1+s} = \frac{\sigma^2\Gamma(\alpha,\gamma)}{4(s+1)\left(2\alpha+\gamma^2-2\right)^2},$$

where

$$\Gamma(\alpha,\gamma) = 4\alpha^3 + \alpha^2\left(5\gamma^2 - 2\gamma - 11\right) + \alpha\left(2\gamma^3 - 11\gamma^2 + 4\gamma + 10\right) - \gamma^4 + 5\gamma^2 - 2\gamma - 3.$$

Obviously, $\widetilde{\Pi}_R^{\mathcal{C}Y} > \widetilde{\Pi}_R^{\mathcal{C}N}$ if and only if $\Gamma(\alpha,\gamma) > 0$; otherwise, $\widetilde{\Pi}_R^{\mathcal{C}Y} \leq \widetilde{\Pi}_R^{\mathcal{C}N}$. We have that $\Gamma(0,0) = -3 < 0$ and $\Gamma(\frac{5}{6},0) = \frac{1}{108} > 0$. We can further show that

$$\begin{aligned}\frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha} =& 12\alpha^2 + 2\alpha\left(5\gamma^2 - 2\gamma - 11\right) + 2\gamma^3 - 11\gamma^2 + 4\gamma + 10,\\ \frac{\partial^2\Gamma(\alpha,\gamma)}{\partial\alpha^2} =& 24\alpha + 2\left(5\gamma^2 - 2\gamma - 11\right) \text{ and } \frac{\partial^3\Gamma(\alpha,\gamma)}{\partial\alpha^3} = 24 > 0.\end{aligned}$$

That is, $\frac{\partial^2\Gamma(\alpha,\gamma)}{\partial\alpha^2}$ is increasing in α. Recall that $\alpha \in (0, 1-\frac{\gamma}{2})$. We can show that

$$\frac{\partial^2\Gamma(\alpha,\gamma)}{\partial\alpha^2}\Big|_{\alpha\to 0} = 2(5\gamma^2 - 2\gamma - 11) < 0 \text{ and } \frac{\partial^2\Gamma(\alpha,\gamma)}{\partial\alpha^2}\Big|_{\alpha\to 1-\frac{\gamma}{2}} = 2\gamma(5\gamma - 8) + 2.$$

It is easy to show that $2\gamma(5\gamma-8)+2 > 0$ when $\gamma \in (0, \frac{4-\sqrt{11}}{5})$ and $2\gamma(5\gamma-8)+2 \leq 0$ when $\gamma \in [\frac{4-\sqrt{11}}{5}, 1)$. Below, we consider these two cases separately.

Case (a): When $\gamma \in (0, \frac{4-\sqrt{11}}{5})$, $\frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha}$ is first decreasing and then increasing in $\alpha \in (0, 1-\frac{\gamma}{2})$. We can show that

$$\frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha}\Big|_{\alpha\to 0} = 2\gamma^3 - 11\gamma^2 + 4\gamma + 10 \text{ and } \frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha}\Big|_{\alpha\to 1-\frac{\gamma}{2}} = (1-\gamma)\gamma(3\gamma-1) < 0.$$

It can be shown that $2\gamma^3 - 11\gamma^2 + 4\gamma + 10$ is increasing in γ for $\gamma \in (0, \frac{4-\sqrt{11}}{5})$ and has a positive value when $\gamma \to 0$. Hence, we have $2\gamma^3 - 11\gamma^2 + 4\gamma + 10 > 0$. Thus, we can conclude that $\Gamma(\alpha,\gamma)$ is unimodal in α, first increasing and then decreasing for any given $\gamma \in (0, \frac{4-\sqrt{11}}{5})$. The unique solution of $\frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha} = 0$ is $\alpha_0 = \frac{1}{12}\left(2\gamma + 11 - 5\gamma^2 - \sqrt{25\gamma^4 - 44\gamma^3 + 26\gamma^2 - 4\gamma + 1}\right)$. Furthermore, we have

$$F(\alpha,\gamma)\big|_{\alpha\to 0} = -\gamma^4 + 4\gamma^2 - 2\gamma - 3 < 0 \text{ and } F(\alpha,\gamma)\big|_{\alpha\to 1-\frac{\gamma}{2}} = -\frac{3}{4}(\gamma-1)^2\gamma^2 < 0.$$

Let $t_0 = \sqrt{25\gamma^4 - 44\gamma^3 + 26\gamma^2 - 4\gamma + 1}$, then

$$\begin{aligned}\Gamma(\alpha_0,\gamma) =& \frac{1}{432}(2\left(125\gamma^6 - 330\gamma^5 + 81\gamma^4 + 292\gamma^3 - 171\gamma^2 - 6\gamma + 1\right) - t_0^3\\ &+ 3\left(25\gamma^4 - 44\gamma^3 + 26\gamma^2 - 4\gamma + 1\right)t_0)\\ =& \frac{1}{432}\left(2\left(125\gamma^6 - 330\gamma^5 + 81\gamma^4 + 292\gamma^3 - 171\gamma^2 - 6\gamma + 1\right) + 2t_0^3\right).\end{aligned}$$

Let $t_1 = t_0^6 - (125\gamma^6 - 330\gamma^5 + 81\gamma^4 + 292\gamma^3 - 171\gamma^2 - 6\gamma + 1)^2$. Then,

$$t_1 = 108(\gamma-1)^2\gamma^2\left(600\gamma^6 - 1428\gamma^5 + 793\gamma^4 + 298\gamma^3 - 251\gamma^2 - 24\gamma + 4\right).$$

The sign of t_1 is determined by

$$t_2 = 600\gamma^6 - 1428\gamma^5 + 793\gamma^4 + 298\gamma^3 - 251\gamma^2 - 24\gamma + 4.$$

It can be shown that $t_2' = 2(\gamma-1)(1800\gamma^4 - 1770\gamma^3 - 184\gamma^2 + 263\gamma + 12)$. We further denote $t_{21} = 1800\gamma^4 - 1770\gamma^3 - 184\gamma^2 + 263\gamma + 12$, which is positive for any given $\gamma \in (0, \frac{4-\sqrt{11}}{5})$. Consequently, we have $t_2' < 0$, which implies that t_2 is decreasing in $\gamma \in (0, \frac{4-\sqrt{11}}{5})$. One can check that $t_2\big|_{\gamma\to 0} = 4 > 0, t_2\big|_{\gamma\to\frac{9}{100}} = \frac{340286987}{5000000000} > 0, t_2\big|_{\gamma\to\frac{1}{10}} = -\frac{27319}{50000} < 0$ and $t_2\big|_{\gamma\to\frac{4-\sqrt{11}}{5}} = -2.995 < 0$. Therefore, there exists a $\gamma_0 \in (\frac{9}{100}, \frac{1}{10})$ that solves

$$t_2\big|_{\gamma=\gamma_0} = 600\gamma^6 - 1428\gamma^5 + 793\gamma^4 + 298\gamma^3 - 251\gamma^2 - 24\gamma + 4 = 0 \qquad \text{(B.8)}$$

such that when $\gamma \in (0, \gamma_0)$ we have $t_2 > 0$ and otherwise, $t_2 < 0$. We can derive that $\gamma_0 \approx 0.0911$. Combining the above discussions, we know that if $\gamma \in (\gamma_0, \frac{4-\sqrt{11}}{5})$, $\Gamma(\alpha_0, \gamma) < 0$. However, if $\gamma \in (0, \gamma_0)$, there exist two thresholds $\widetilde{\alpha}_1(\gamma)$ and $\widetilde{\alpha}_2(\gamma)$ that solve

$$F(\alpha,\gamma) = 4\alpha^3 + \alpha^2\left(5\gamma^2 - 2\gamma - 11\right) + \alpha\left(2\gamma^3 - 11\gamma^2 + 4\gamma + 10\right) - \gamma^4 + 5\gamma^2 - 2\gamma - 3 = 0. \qquad \text{(B.9)}$$

When $\alpha \in (\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_2(\gamma))$, $\Gamma(\alpha,\gamma) > 0$; otherwise, $\Gamma(\alpha,\gamma) \le 0$. In addition, we can show that

$$\Gamma(\alpha,\gamma)\big|_{\alpha\to\frac{1}{2}} = -\frac{1}{4}\left(-2\gamma^2 + \gamma + 1\right)^2 < 0, \text{ and}$$
$$\frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha}\big|_{\alpha\to\frac{1}{2}} = 2\left(\gamma^3 - 3\gamma^2 + \gamma + 1\right) > 0$$

for any $\gamma \in (0, \gamma_0)$. This implies that $\widetilde{\alpha}_1(\gamma) > \frac{1}{2}$.

Case (b): When $\gamma \in (\frac{4-\sqrt{11}}{5}, 1)$, $\frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha}$ is decreasing in $\alpha \in (0, 1-\frac{\gamma}{2})$. Note that

$$\frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha}\big|_{\alpha\to 1-\frac{\gamma}{2}} = (1-\gamma)\gamma(3\gamma-1)$$

is positive when $\gamma \in (\frac{1}{3}, 1)$ but is negative when $\gamma \in (\frac{4-\sqrt{11}}{5}, \frac{1}{3})$. We then further consider the following two cases.

Subcase (b1): When $\gamma \in (\frac{1}{3}, 1)$, we have that $\frac{\partial\Gamma(\alpha,\gamma)}{\partial\alpha} > 0$ for any $\alpha \in (0, 1-\frac{\gamma}{2})$. That is, $\Gamma(\alpha,\gamma)$ is increasing in α. Since $F(\alpha,\gamma)\big|_{\alpha\to 1-\frac{\gamma}{2}} = -\frac{3}{4}(\gamma-1)^2\gamma^2 < 0$, $\Gamma(\alpha,\gamma) < 0$ in this case.

Subcase (b2): When $\gamma \in (\frac{4-\sqrt{11}}{5}, \frac{1}{3})$, we have that $\frac{\partial \Gamma(\alpha,\gamma)}{\partial \alpha}\big|_{\alpha \to 1-\frac{\gamma}{2}} = (1-\gamma)\gamma(3\gamma-1) < 0$. Combining this with $\frac{\partial \Gamma(\alpha,\gamma)}{\partial \alpha}\big|_{\alpha \to 0} = 2\gamma^3 - 11\gamma^2 + 4\gamma + 10 > 0$, we can conclude that $\Gamma(\alpha, \gamma)$ is first increasing and then decreasing in α. By the same logic as that used in Case (a), the sign of $\Gamma(\alpha_0, \gamma)$ is determine by t_2. It can be shown that $t_2 < 0$ for any given $\gamma \in (\frac{4-\sqrt{11}}{5}, \frac{1}{3})$. Therefore, $\Gamma(\alpha_0, \gamma) < 0$, implying that $\Gamma(\alpha, \gamma) < 0$.

Combining the above discussions, we then complete the proof.

B.1.7 Proof of Proposition 3.5

Here , we compare the supplier's ex ante profits under the two encroachment approaches to derive her optimal encroachment channel selection decision. Specifically, under the direct channel encroachment, her ex ante profit is $\widetilde{\Pi}_S^{\mathcal{D}} = \frac{a^2(3-2\gamma)}{4(2-\gamma^2)} - c$; under the commission channel encroachment, her ex ante profit is $\widetilde{\Pi}_S^{\mathcal{C}Y} = \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4(2-2\alpha-\gamma^2)} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right)$ when the retailer shares information and when the retailer does not share information, we have $\widetilde{\Pi}_S^{\mathcal{C}N} = \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4(2-2\alpha-\gamma^2)} a^2$

Case (1): If $\gamma \in (0, \gamma_0)$ and $\alpha \in (\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_2(\gamma))$, then according to Proposition 3.4, the retailer shares information with the supplier. We then have

$$\widetilde{\Pi}_S^{\mathcal{C}} - \widetilde{\Pi}_S^{\mathcal{D}} = \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4\,(2-2\alpha-\gamma^2)} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right) - \frac{a^2(3-2\gamma)}{4\,(2-\gamma^2)} + c.$$

It is straightforward to show that the supplier selects the commission channel encroachment if and only if $c \geq \max\{\widetilde{K}_2(s), 0\}$, where

$$\widetilde{K}_2(s) := \frac{a^2(3-2\gamma)}{4\,(2-\gamma^2)} - \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4\,(2-2\alpha-\gamma^2)} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right).$$

Also, $\widetilde{K}_2(s)$ is decreasing in $1/(s\sigma^2)$. For ease of exposition, we define

$$\widehat{c}_2(s) := \max\{\widetilde{K}_2(s), 0\}. \tag{B.10}$$

Case (2): Otherwise, the retailer does not share information with the supplier. We then have

$$\widetilde{\Pi}_S^{\mathcal{C}} - \widetilde{\Pi}_S^{\mathcal{D}} = \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4\,(2-2\alpha-\gamma^2)} a^2 - \frac{a^2(3-2\gamma)}{4\,(2-\gamma^2)} + c,$$

from which we find that the supplier selects the commission channel encroachment if and only if $c \geq \max\{\widetilde{K}_1, 0\}$, where

$$\widetilde{K}_1 := \frac{a^2(3-2\gamma)}{4\,(2-\gamma^2)} - \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4\,(2-2\alpha-\gamma^2)}a^2.$$

For ease of exposition, we define

$$\widehat{c}_3 := \max\{\widetilde{K}_1, 0\}. \tag{B.11}$$

B.1.8 Proof of Proposition 3.6

Based on the results in Table 3.2 and §3.4.1, we obtain that

$$q_S^{\mathcal{C}} - \widetilde{q}_S^{\mathcal{C}Y} = \frac{(\gamma-1)\gamma\left(\alpha\left(\gamma^2-4\right)+3\gamma^2-4\right)\left(a+\frac{x}{1+s}\right)}{2\left(2\alpha+\gamma^2-2\right)\left((\alpha+5)\gamma^2-8\right)} > 0,$$

$$q_R^{\mathcal{C}} - \widetilde{q}_R^{\mathcal{C}Y} = \frac{\left(\alpha^2+4\alpha-1\right)(\gamma-1)\gamma^2\left(a+\frac{x}{1+s}\right)}{2\left(2\alpha+\gamma^2-2\right)\left((\alpha+5)\gamma^2-8\right)} < 0.$$

Since $\widetilde{q}_S^{\mathcal{C}Y} > \widetilde{q}_S^{\mathcal{C}N}$ and $\widetilde{q}_R^{\mathcal{C}Y} < \widetilde{q}_R^{\mathcal{C}N}$, we obtain that $q_S^{\mathcal{C}} > \widetilde{q}_S^{\mathcal{C}Y} > \widetilde{q}_S^{\mathcal{C}N}$ and $q_R^{\mathcal{C}} < \widetilde{q}_R^{\mathcal{C}Y} < \widetilde{q}_R^{\mathcal{C}N}$. That is, $q_S^{\mathcal{C}} > \widetilde{q}_S^{\mathcal{C}}$ and $q_R^{\mathcal{C}} < \widetilde{q}_R^{\mathcal{C}}$. Furthermore, when $\gamma \in (0, \gamma_0)$ and $\alpha \in (\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_2(\gamma))$, it has that

$$w^{\mathcal{C}} - \widetilde{w}^{\mathcal{C}} = w^{\mathcal{C}} - \widetilde{w}^{\mathcal{C}Y} = \frac{(1-\gamma)\gamma^2\left(\alpha^2\left(\gamma^2-2\right)+4\alpha\left(\gamma^2-1\right)+\gamma^2-2\right)\left(a+\frac{x}{1+s}\right)}{2\left(2\alpha+\gamma^2-2\right)\left((\alpha+5)\gamma^2-8\right)} < 0.$$

Based on the results in Table 3.3 and §3.5.2, it is straightforward to show that

$$w^{\mathcal{D}} - \widetilde{w}^{\mathcal{D}} = -\frac{a(\gamma-1)^2\gamma^2}{2\left(3\gamma^3-5\gamma^2-4\gamma+8\right)} < 0 \text{ and}$$

$$q_S^{\mathcal{D}} - \widetilde{q}_S^{\mathcal{D}} = \frac{(2-\gamma)\left(\frac{a(\gamma-2)(\gamma-1)\gamma(\gamma+1)}{3\gamma^3-5\gamma^2-4\gamma+8}+\frac{x}{s+1}\right)}{2\left(2-\gamma^2\right)} > 0,$$

$$\text{and } q_R^{\mathcal{D}} - \widetilde{q}_R^{\mathcal{D}} = \frac{a(1-\gamma)\gamma\left(\gamma^2+\gamma-4\right)}{2\left(2-\gamma^2\right)\left(3\gamma^3-5\gamma^2-4\gamma+8\right)} - \frac{x\left((2-\gamma)\gamma+\left(2-\gamma^2\right)s\right)}{2\left(2-\gamma^2\right)(s+1)} < 0.$$

B.1.9 Proof of Proposition 3.7

We prove this result by considering two cases: $\alpha \leq \underline{\alpha}(\gamma)$ and $\alpha > \underline{\alpha}(\gamma)$.

Case (1): $\alpha \leq \underline{\alpha}(\gamma)$. According to Propositions 3.3 and 3.4, we have $\alpha \leq \underline{\alpha}(\gamma) < \frac{1}{2} < \widetilde{\alpha}_1(\gamma)$. Then, by Propositions 3.3 and 3.5, we have that the supplier selects commission channel encroachment in the $\mathcal{RL}$ scenario; and she selects the commission channel encroachment if $c \geq \widehat{c}_3$ and otherwise selects the direct channel encroachment in the $\mathcal{SL}$ scenario. In this case, define

$$\underline{c} := 0, \bar{c} := \widehat{c}_3. \tag{B.12}$$

Case (2): $\alpha > \underline{\alpha}(\gamma)$. First, define

$$F_1(\alpha) := \frac{(1-\alpha)(3-2\alpha-2\gamma)}{2-2\alpha-\gamma^2} - F(\alpha) = \frac{(\alpha^2+4\alpha-1)(\gamma-1)^2\gamma^2}{(2-2\alpha-\gamma^2)(8-(\alpha+5)\gamma^2)}.$$

It can be easily shown that for any $\alpha \in (0,1)$,

$$\frac{dF_1(\alpha)}{d\alpha} = \frac{(\gamma-1)^2\gamma^2\left(\alpha^2(\gamma^4-16)+2\alpha(5\gamma^4-16\gamma^2+16)+21\gamma^4-64\gamma^2+48\right)}{(2\alpha+\gamma^2-2)^2(8-(\alpha+5)\gamma^2)^2} > 0,$$

because the numerator is concave in α and equals $48-64\gamma^2+21\gamma^4 > 0$ when $\alpha \to 0$ and $32(\gamma-1)(\gamma+1)(\gamma^2-2) > 0$ when $\alpha \to 1$. This implies that $F_1(\alpha)$ increases in α. Furthermore, when $\alpha < \sqrt{5}-2$, $F_1(\alpha) < 0$; otherwise, $F_1(\alpha) \geq 0$. Besides, when $\alpha \to 1$, $k_4 - \frac{3-2\gamma}{2-\gamma^2} + \frac{(1-\alpha)(3-2\alpha-2\gamma)}{2-2\alpha-\gamma^2} - F(\alpha) = -\frac{(\gamma-1)^2\gamma\left(\gamma^2+\gamma-4\right)}{(\gamma^2-2)(3\gamma^3-5\gamma^2-4\gamma+8)} - F(\alpha)|_{\alpha\to 1} < 0$. Hence, for any $\alpha < 1$,

$$k_4 - \frac{3-2\gamma}{2-\gamma^2} + F_1(\alpha) < 0. \tag{B.13}$$

After knowing the properties of $F_1(\alpha)$ and based on Proposition 3.5, we now further consider the following two subcases.

Case (2a): $\gamma \in (0,\gamma_0)$ and $\alpha \in (\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_2(\gamma))$. First, according to the proof of Proposition 3.5, in the $\mathcal{SL}$ scenario, if $c < \widehat{c}_2(s) = \max\{\widetilde{K}_2(s), 0\}$, the supplier selects the direct channel encroachment; otherwise, she selects the commission channel encroachment. Then, according to Proposition 3.3, in the $\mathcal{RL}$ scenario, if $c < \widehat{c}_1(s) = \max\{K(s), 0\}$, the supplier selects the direct channel encroachment;otherwise, she selects the commission channel encroachment. We now compare these two thresholds. It can be shown that

$$\begin{aligned}
K(s) - \widetilde{K}_2(s) =& \frac{1}{4}a^2\left(k_4 - F(\alpha)\right) + \frac{\sigma^2}{4(s+1)}\left(k_5 - F(\alpha)\right) \\
&- \frac{a^2(3-2\gamma)}{4(2-\gamma^2)} + \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4(2-2\alpha-\gamma^2)} \cdot \left(a^2 + \frac{\sigma^2}{1+s}\right) \\
=& \frac{1}{4}\left(k_4 - \frac{3-2\gamma}{2-\gamma^2} + F_1(\alpha)\right)a^2 + \frac{\sigma^2}{4(1+s)}\left(k_5 + F_1(\alpha)\right).
\end{aligned}$$

Since $\widetilde{\alpha}_1(\gamma) > \frac{1}{2}$ (by Proposition 3.4), we have $\alpha > \widetilde{\alpha}_1(\gamma) > \frac{1}{2} > \sqrt{5}-2$. Hence, $F_1(\alpha) > 0$. Consequently, $k_5 + F_1(\alpha) > 0$. Recall that $F_1(\alpha)$ is increasing in α and $k_4 - \frac{3-2\gamma}{2-\gamma^2} + F_1(\alpha) < 0$, which is shown in (B.13). We can conclude that there exists at most a $\widehat{\alpha}_0$ that solves

$$\left(k_4 - \frac{3-2\gamma}{2-\gamma^2} + F_1(\alpha)\right) a^2 + \frac{\sigma^2}{1+s}\left(k_5 + F_1(\alpha)\right) = 0 \tag{B.14}$$

if it does exist. Note that if $\left(K(s) - \widetilde{K}_2(s)\right)|_{\alpha\to\widetilde{\alpha}_2(\gamma)} < 0$, $\widehat{\alpha}_0 = \widetilde{\alpha}_2(\gamma)$ and if $\left(K(s) - \widetilde{K}_2(s)\right)|_{\alpha\to\widetilde{\alpha}_1(\gamma)} > 0$, $\widehat{\alpha}_0 = \widetilde{\alpha}_1(\gamma)$. Then, $K(s) > \widetilde{K}_2(s)$ if $\alpha > \widehat{\alpha}_0$ and $K(s) \le \widetilde{K}_2(s)$ otherwise. As a result, $\widehat{c}_1(s) > \widehat{c}_2(s)$ only if $\alpha > \widehat{\alpha}_0$.

In this case, define

$$(\underline{c}, \bar{c}) = \begin{cases} \left(\widehat{c}_1(s), \widehat{c}_2(s)\right), & \text{if } \alpha < \widehat{\alpha} := \widehat{\alpha}_0 \\ \left(\widehat{c}_2(s), \widehat{c}_1(s)\right), & \text{otherwise.} \end{cases} \tag{B.15}$$

Then, we can conclude that the supplier selects the commission channel encroachment if $c \ge \bar{c}$ but selects the direct channel encroachment if $c \le \underline{c}$. If $\underline{c} < c < \bar{c}$, when $\alpha < \widehat{\alpha}$ ($\alpha \ge \widehat{\alpha}$, resp.), the supplier selects the commission channel encroachment in the $\mathcal{RL}$ ($\mathcal{SL}$, resp.) scenario but the direct channel encroachment in the $\mathcal{SL}$ ($\mathcal{RL}$, resp.) scenario.

Case (2b):): $\gamma \in (\gamma_0, 1)$ or ($\alpha \le \widetilde{\alpha}_1(\gamma)$ or $\alpha \ge \widetilde{\alpha}_2(\gamma)$). First, according to the proof of Proposition 3.5, in the $\mathcal{SL}$ scenario, if $c < \widehat{c}_3 = \max\{\widetilde{K}_1, 0\}$, the supplier selects the direct channel encroachment; otherwise, she selects the commission channel encroachment. Then, according to Proposition 3.3, in the $\mathcal{RL}$ scenario, if $c < \widehat{c}_1(s) = \max\{K(s), 0\}$, the supplier selects the direct channel encroachment; otherwise, she selects the commission channel encroachment. We now compare the two thresholds. It can be shown that

$$\begin{aligned} K(s) - \widetilde{K}_1 &= \frac{1}{4}a^2\left(k_4 - F(\alpha)\right) + \frac{\sigma^2}{4(s+1)}\left(k_5 - F(\alpha)\right) - \frac{a^2(3-2\gamma)}{4\left(2-\gamma^2\right)} \\ &\quad + \frac{(1-\alpha)(3-2\alpha-2\gamma)}{4\left(2-2\alpha-\gamma^2\right)}a^2 \\ &= \frac{1}{4}\left(k_4 - \frac{3-2\gamma}{2-\gamma^2} + F_1(\alpha)\right)a^2 + \frac{\sigma^2}{4(1+s)}\left(k_5 - F(\alpha)\right). \end{aligned}$$

Recall that $F_1(\alpha)$ is increasing α but $F(\alpha)$ is decreasing in α (by Proposition 3.3). Therefore, $K(s) - \widetilde{K}_1$ is increasing in α. Based on Proposition 3.3, we further have $F(\alpha) > k_5$ if $\alpha < \bar{\alpha}(\gamma)$ and $F(\alpha) \le k_5$ otherwise.

First, consider that $\alpha < \bar{\alpha}(\gamma)$. Then, we can show that $K(s) < \widetilde{K}_1$ because $k_4 - \frac{3-2\gamma}{2-\gamma^2} + F_1(\alpha) < 0$ and $k_5 - F(\alpha) < 0$. Next, consider that $\alpha \geq \bar{\alpha}(\gamma)$. Then, there exists a unique $\widehat{\alpha}_1$ that solves

$$\left(k_4 - \frac{3-2\gamma}{2-\gamma^2} + F_1(\alpha)\right) a^2 + \frac{\sigma^2}{1+s}\left(k_5 - F(\alpha)\right) = 0 \tag{B.16}$$

if it does exist. Note that if $\left(K(s) - \widetilde{K}_1\right)|_{\alpha \to \min\{1, \frac{2}{\gamma}-2\}} \leq 0$, $\widehat{\alpha}_1 = \min\{1, \frac{2}{\gamma} - 2\}$. Then, when $\alpha < \widehat{\alpha}_1$, we have $K(s) < \widetilde{K}_1$; otherwise, we have $K(s) \geq \widetilde{K}_1$. Consequently, if $\alpha < \widehat{\alpha}_1$, we have $\widehat{c}_1(s) \leq \widehat{c}_3$; otherwise, we have $\widehat{c}_1(s) \geq \widehat{c}_3$.

Define

$$(\underline{c}, \bar{c}) = \begin{cases} \left(\widehat{c}_1(s), \widehat{c}_3\right), \text{ if } \alpha < \widehat{\alpha} := \widehat{\alpha}_1 \\ \left(\widehat{c}_3, \widehat{c}_1(s)\right), \text{otherwise.} \end{cases} \tag{B.17}$$

Then, we can conclude that the supplier selects the commission channel encroachment if $c \geq \bar{c}$ but selects the direct channel encroachment if $c \leq \underline{c}$. If $\underline{c} < c < \bar{c}$, when $\alpha < \widehat{\alpha}$ ($\alpha \geq \widehat{\alpha}$, resp.), the supplier selects the commission channel encroachment in the $\mathcal{RL}$ ($\mathcal{SL}$, resp.) scenario but the direct channel encroachment in the $\mathcal{SL}$ ($\mathcal{RL}$, resp.) scenario.

Based on the above discussions, one can find that as α and γ change their parameter values, $\underline{c}$, $\bar{c}$ and $\widehat{\alpha}$ take different expressions, which are defined in equations (B.12),(B.15) and (B.17). Below, we summarize them as follows:

$$(\underline{c}, \bar{c}) = \begin{cases} \left(0, \widehat{c}_3\right); & \alpha \leq \underline{\alpha}(\gamma), \\ \begin{cases} \left(\widehat{c}_1(s), \widehat{c}_2(s)\right), \text{ if } \alpha < \widehat{\alpha} := \widehat{\alpha}_0 \\ \left(\widehat{c}_2(s), \widehat{c}_1(s)\right), \text{if } \alpha \geq \widehat{\alpha} \end{cases} & ; \gamma \in (0, \gamma_0) \text{ and } \alpha \in (\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_1(\gamma)) \\ \begin{cases} \left(\widehat{c}_1(s), \widehat{c}_3\right), \text{ if } \alpha < \widehat{\alpha} := \widehat{\alpha}_1 \\ \left(\widehat{c}_3, \widehat{c}_1(s)\right), \text{if } \alpha \geq \widehat{\alpha} \end{cases} & ; \alpha > \underline{\alpha}(\gamma), \gamma \notin (0, \gamma_0) \cup \alpha \notin (\widetilde{\alpha}_1(\gamma), \widetilde{\alpha}_1(\gamma)) \end{cases} \tag{B.18}$$

B.2 Optimal Wholesale Price under $\mathcal{SL}$ Scenario with Commission Channel Encroachment

According to the process described in §3.5.1, we can derive that

$$\Pi_S(w) = \frac{a^2(\alpha-1)^2(\gamma-2)^2 + 2a(\alpha-1)\left(2w\left(2\alpha\gamma+\gamma^2-2\right)+(\alpha-1)(\gamma-2)^2E[X|\Theta]\right)}{8(1-\alpha)(2-(\alpha+1)\gamma^2)} + \frac{4w^2\left(2\alpha+\gamma^2-2\right)+4wE[X|\Theta]\left(2\alpha^2\gamma+\alpha\left(\gamma^2-2\gamma-2\right)-\gamma^2+2\right)+(\alpha-1)^2(\gamma-2)^2E[X|\Theta]^2}{8(1-\alpha)(2-(\alpha+1)\gamma^2)}.$$

Then it can be shown that

$$\frac{d^2\Pi_S(w)}{dw^2} = \frac{2\alpha + \gamma^2 - 2}{(1-\alpha)\left(2 - (\alpha+1)\gamma^2\right)},$$

and the solution of first-order condition is

$$w_0 = \frac{(1-\alpha)\left(2 - 2\alpha\gamma - \gamma^2\right)\left(a + E[X \mid \Theta]\right)}{2\left(2 - 2\alpha - \gamma^2\right)}.$$

We next focus on the case where retailer does not share information, that is, $E[X \mid \Theta] = 0$. The case that retailer shares information can be derived by a similar logic. Note that when the supplier determines the optimal wholesale price, the quantity should be satisfy $q_S \geq 0$, or equivalently, $w \leq \frac{(1-\alpha)(2-\gamma)}{2\alpha\gamma}a$. Besides, we have

$$2 - (\alpha+1)\gamma^2 > 0$$

must hold since both $\alpha, \gamma \in (0,1)$. The property of $\Pi_S(w)$ determines the optimal wholesale price of the supplier, we next consider three subcases.

1. If $2 - \gamma^2 - 2\alpha\gamma < 0$, or equivalently, $\gamma^2 + 2\alpha\gamma > 2$. Then, it must have that

   $$2 - \gamma^2 - 2\alpha < 2 - \gamma^2 - 2\alpha\gamma < 0.$$

   That is, $w_0 > 0$ and $\frac{d^2\Pi_S(w)}{dw^2} > 0$, which implies that $\Pi_S(w)$ is convex in w. Furthermore, it can be shown that

   $$w_0 - \frac{(1-\alpha)(2-\gamma)}{2\alpha\gamma}a = \frac{a(1-\alpha)(2\alpha+\gamma-2)\left((\alpha+1)\gamma^2 - 2\right)}{2\alpha\gamma\left(2\alpha + \gamma^2 - 2\right)} < 0$$

   due to $2\alpha+\gamma-2 > 2\alpha+\gamma^2-2 > 0, (\alpha+1)\gamma^2-2 < 0$. That is, $\Pi_S(w)$ obtains minimum value when $w = w_0$ and obtains highest profit when $w = 0$ or $w = \frac{(1-\alpha)(2-\gamma)}{2\alpha\gamma}a$. Note that when $w = \frac{(1-\alpha)(2-\gamma)}{2\alpha\gamma}a$, we have $q_S = 0$, which means that the supplier does not encroach. Also, when $w = 0$, it means that supplier sets a wholesale price that equals her production cost and makes no money in the wholesaling business, which is not realistic.

2. If $2 - \gamma^2 - 2\alpha\gamma > 0 > 2 - \gamma^2 - 2\alpha$. That is, $w_0 < 0$ and $\frac{d^2\Pi_S(w)}{dw^2} > 0$. In this case, $\Pi_S(w)$ obtains its maximum value when $w = \frac{(1-\alpha)(1-\gamma)}{2\alpha\gamma}a$, that is, $q_S = 0$, implying that supplier does not encroach.

3. If $2-\gamma^2-2\alpha\gamma > 2-\gamma^2-2\alpha > 0$, then $w_0 > 0$ and $\frac{d^2\Pi_S(w)}{dw^2} > 0$. $\Pi_S(w)$ is concave in w. Note that

$$w_0 - \frac{(1-\alpha)(2-\gamma)}{2\alpha\gamma}a = \frac{a(1-\alpha)(2\alpha+\gamma-2)\left((\alpha+1)\gamma^2-2\right)}{2\alpha\gamma\left(2\alpha+\gamma^2-2\right)} < 0$$

if it further has that $2\alpha+\gamma-2<0$; otherwise, $w_0 - \frac{(1-\alpha)(2-\gamma)}{2\alpha\gamma}a \geq 0$. In the former case, the supplier sets $w = w_0$, which induces a corresponding $q_S > 0$, while in the latter case, the supplier sets $w = \frac{(1-\alpha)(2-\gamma)}{2\alpha\gamma}a$, which implies that $q_S = 0$ and supplier does not encroach.

Based on the above discussion, we conclude that if and only if $2\alpha+\gamma-2<0$ holds, the supplier sets a positive optimal wholesale price that induces a positive quantity q_S. In other cases, the supplier either does not encroach (which is not our focus) or sets a zero wholesale price (which is not realistic). Therefore, in our main text, we assume that $2\alpha+\gamma<2$ holds.

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