COMPETITION IN THE UPSTREAM AND DOWNSTREAM MARKET WITH LEXICOGRAPHIC ALLOCATION POLICY

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Competition in the Upstream and Downstream Market with Lexicographic Allocation Policy

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CERTIFICATE OF ORIGINALITY

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Abstract

In this thesis, we consider a supply chain in which two buyers share the same upstream source of supply that may be insufficient and then engage in quantity competition in the downstream market. When the total order quantity of the buyers exceeds the supplier's total supply, the limited inventory is allocated based on the lexicographic allocation policy, and the priority is given to the buyer who is willing to pay more. In such a setting, a three-stage game-theoretical model is established and solved backward to study the strategic behavior of every supply chain member and the effect of the demand risk level and competition intensity on their optimal strategy. Given the revealed demand, we analyze the pattern for the buyers' order quantity equilibrium as a function of the supplier's inventory level and the wholesale prices. Before the demand is realized, we derive the wholesale price equilibrium and the optimal inventory strategy of the supplier and study the interaction between them. We find that when the wholesale prices are not high enough, the two buyers' total order quantity experiences a drop when the supplier's inventory level crosses a critical value and keeps constant afterward. Therefore, the supplier has the incentive to limit his inventory level and lexicographic policy may now become a factor that causes insufficient supply. Moreover, the wholesale price equilibria are asymmetric with one buyer obtaining the control power while the other one grabbing the benefit of low purchasing cost, even though the two buyers are symmetric.

Keywords: limited inventory allocation, lexicographic allocation policy, upstream and downstream competition, wholesale pricing, inventory strategy

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Chapter 1
Introduction

In the real world, there exist many industries in which the firms not only engage in price or quantity competition in the downstream market, but they also compete for the resource of the same input market when the supply is scarce for multiple reasons. For example, jewelry makers who serve the same retail market also compete for the limited resource of precious stones in the same upstream market. In the input market, the resource is allocated first to the buyer who is willing to pay more, so, firms can bid up and buy all uncut diamonds to gain control of the resource of the upstream market. This is exactly the strategy that has been followed by one company, De Beers, to dominate the diamond market ever since he has been around. And such a phenomenon commonly exists in the input market of the gas (or petrol) retailers as well (Eső, Nocke and White (2010)).

The same strategy of monopolizing the downstream market is also used in the snacks industry. For example, in 1996, Frito-Lay Inc. signed a supply agreement with Procter & Gamble Company's to obtain his new fat-substitute olestra, which is named as Olean. The agreement would enable Frito-Lay to buy far larger quantities of Olean than his competitors could get for a long-term period. So, this agreement effectively excluded the firm's competitor during the period of the contract; and, other snacks companies would be able to get access to large quantities of Olean after the contract expired. Olean was quite controversial and had high-level demand risk, Frito-Lay did this for a purpose that being the monopoly and making a great profit once products made with Olean become popular in the market. And this agreement was signed even before the new manufacturing plant for Olean was build. It is reported that Frito-Lay paid a significant cost for this agreement and he was the only company willing to take the risk (Frank 1996).

In addition to gaining control power, raising products' prices voluntarily can also help buyers to get access to the supplier's inventory in all market conditions and acquire enough quantities of the products when the supply is insufficient due to demand risk.

An industry that matches well with this statement is the semiconductor industry. In the semiconductor industry where OEMs usually procure from the same semiconductor supplier, the mismatch between the supply and the demand often occurs due to the long lead time for both capacity building and product producing, as well as the highly risky market demand. As Karabuk and Wu (2005) illustrated, building the wafer fabs (used for wafer fabrication) normally requires 12 to 18 months and the total manufacturing time for semiconductors is at least 6 to 12 weeks. Such a long lead time makes it hard for the high-tech industry to precisely forecast the volatile demand, and thus, the total order quantity of buyers may exceed the total supply of the semiconductors manufacturer from time to time. To deal with the risk that the supply may be insufficient, it is reported that in 2018, ZTE company raised the purchase price of MOSFET for 20 percent to acquire the supplier's inventories as much as possible. And some of its competitors did the same thing. As a result, the price of this semiconductor component rose at least 20 percent that year.

Motivated by these cases in the real world, in this thesis, we consider that buyers have price-setting power in the input market, and the buyer who proposes a higher wholesale price is prioritized and filled first if the inventory is insufficient. In the scenario of this thesis, the demand risk exists and the supplier prepares the inventory before the demand is realized.

Although raising the proposed price voluntarily is a common practice, does such strategy benefit the buyers in the case that the supply of the upstream market is limited? Actually, on the one hand, raising the wholesale price and obtaining the priority benefit the buyer since the priority can assure the buyer of the access to the supplier's inventory in all market conditions and even enable him to be the monopoly by ordering all the resource in the input market. While on the other hand, from the perspective of the buyer, raising the wholesale price may lead to some harmful outcomes. If the wholesale price is high, the supplier would have the incentive to increase the inventory level and the demand risk is relieved. Besides, the cost of acquiring the supplier's all inventory would be too high to be beneficial, and thereby the buyer who is prioritized would not monopolize the market even though he has gained market power over his competitors.

This indicates that raising the wholesale price becomes meaningless if the proposed wholesale price is too high for the buyer. Therefore, the answer to the question that whether raising the purchase price to be prioritized is beneficial and under what conditions the buyer would like to take this strategy remains unclear and deserves further study. To this end, a three-stage game-theoretic model is established and solved backward.

Specifically, we consider a supply chain where a single supplier sells products to two buyers who engage in quantity competition in the downstream market. The buyers have *price-setting power* in both the upstream and the downstream market, and the buyer who proposes higher wholesale price is prioritized. In such a setting, this thesis aims to study the strategic behavior of each supply chain member. Here are four primary research questions to be addressed in this work:

(i) Given the supplier's inventory level, what are the buyers' optimal order quantities and the pattern of the two buyers' order quantity equilibrium after the market condition is realized? Under what conditions will the prioritized buyer have the incentive to monopolize the market and keep his competitor out of the end market? What is the influence of the wholesale prices?

(ii) With the lexicographic allocation policy, if the cost of producing the products for the supplier is very low, would he prepare as many inventories as possible so that the supply is sufficient in any realized market condition? What is the effect of the wholesale prices proposed by the buyers on the supplier's optimal inventory level?

(iii) Under what condition does the buyer would like to propose a higher wholesale price than his competitor? What are the two buyers' wholesale prices equilibria, are they symmetric or asymmetric?

(iv) How will demand risk level and competition intensity influence the strategic behavior of each supply chain member?

Based on the above questions, we give some of the key conclusions drawn from our thesis as follows.

Given the supplier's inventory level and the revealed demand, the two buyers' order quantity equilibrium is related to the inventory level and the wholesale price proposed by the buyer who has priority. The buyer who is willing to pay more has strong a strong incentive to order all the supplier's inventory to monopolize the end market when the inventory level is low. And his incentive increases with the decrease of the wholesale price he proposed, and therefore, if the wholesale price is low, the buyer would buy up all the inventory even if the total supply exceeds the sum of the two buyers' demand and some of the products cannot be sold. However, if the wholesale price is high, once the inventory level exceeds the total demand that the buyer can acquire as the monopoly, he would quit monopolizing the market, but keeps inflating the order to compete for more demand if the supply is not too large. And, the buyer becomes more aggressive in competing for the limited supply with a larger competition intensity.

Intuitively, the lexicographic allocation mechanism is used by the supplier to allocate the scarce supply. However, we find that using this policy, the supplier would have the incentive to limit his inventory level when the wholesale prices are not high enough, and thus, the lexicographic policy may become a factor that causes insufficient supply. This suggests that there may be an inverse relationship between the use of lexicographic allocation mechanism and the scarce supply. At the same time, as α decreases and δ increases, the supplier would have the incentive to prepare more inventory.

If the cost of obtaining the priority is low, that is: when the buyer's competitor proposes a low wholesale price, he would propose a higher one to gain the priority, otherwise, he would choose to grab the benefit of low purchasing cost and quit the market in some revealed situations. Once the buyer is prioritized, given the wholesale prices, he would prefer a lower inventory level, and sometimes, he has to propose a higher wholesale price to prevent the supplier from preparing too much inventory. However, the buyer would prefer a higher inventory level if he is not given the priority because the probability that he can get access to the supply would be greater with a higher inventory level.

The influence of the demand risk level on the buyer's best-response wholesale price is different depending on whether he is prioritized or not. If the buyer has the priority, as the demand risk level increases, he would be more likely to choose a low wholesale price, otherwise, he would propose a high wholesale price to incentivize the supplier to prepare more inventory.

As for the two buyers' wholesale price equilibrium, we conclude that though the buyers are symmetric, their Nash equilibria are asymmetric. At equilibrium, one buyer obtains control power and grabs the benefit of getting access to the supplier's inventory in all market conditions and being the monopoly sometimes, while the other buyer grabs the benefit of low wholesale price but may not be able to obtain the supplier's products sometimes. This indicates that the discriminative wholesale prices emerge.

Chapter 2

Literature Review

2.1 Capacity allocation policy in the decentralized setting

Considerable works in the OM field have studied *the capacity allocation policies in the decentralized supply chain* in which one single supplier sells products to several buyers. In this chapter, we review studies that are mostly related to our work and explain how our work is different from these papers.

Cachon and Lariviere (1999a, b, c) are among the first few papers analyzing the properties of various limited capacity allocation mechanisms that are commonly used in the practice when the retailers' total order quantity exceeds the supplier's capacity. Specifically, Cachon and Lariviere (1999a) considers the turn-and-earn allocation, in which the supplier's capacity is allocated to the retailers based on their past sales, and concludes that it benefits the supplier at the expense of the retailers' profit and probably the entire supply chain (when the capacity is extremely tight). This work is extended by Lu and Lariviere (2012), which assumes the retailers possess private demand information and obtain some equilibrium behaviors that do not exist in Cachon and Lariviere (1999a). Cachon and Lariviere (1999b) derives the conditions for a limited capacity allocation mechanism to be manipulable or truth-inducing in the case that retailers have private information about their optimal stocking levels. This thesis also studies how an allocation mechanism influences the supplier's capacity choice, the retailers' profits, the supplier's profit, and the entire supply chain's profit; they draw an interesting conclusion that the supplier may prepare a larger capacity under a manipulable mechanism and it can thus benefit all players when the capacity is expensive. Cachon and Lariviere (1999c) extends Cachon and Lariviere (1999b) by deriving Nash equilibria of players' order quantities in the capacity allocation game among retailers with three allocation mechanisms: proportional allocation (manipulable), linear allocation (manipulable) and uniform allocation (truth-inducing). And then the paper compares the retailers' and supply chain's profit across three mechanisms.

Different from the fixed-price mechanisms studied in the above literature, Deshpande and Schwarz (2002) considers a *different-price policy* in which the supplier charges different prices from different buyers. They use a mechanism-design approach to derive the supplier's profit-maximizing allocation policy which ***links both the per-unit purchase price and the quantity allocated to each retailer with retailers' private information of market demand*** and induce retailers to reveal themselves. Furthermore, they design an auction mechanism to implement the optimal policy. Similar to this work, Karabatı and Yalçın (2014) also designs an auction mechanism that can be used to allocate the manufacturer's capacity and induce buyers to disclose their private information on their preferred delivery times truthfully.

A common assumption in the work above is that the buyers only compete for the supplier's scarce capacity but not against each other in the downstream market. However, if the buyers also engage in quantity or price competition in the downstream market, the conclusions may change. For example, Liu (2012) is the very first literature that compares uniform allocation policy and a broad class of individually responsive (IR) allocation rules, such as proportional and linear allocation policies, supposing the competition in the downstream market exists. Compared to Cachon and Lariviere (1999a, b, c) in which uniform allocation policy is classified into truth-inducing mechanisms, Liu (2012) concludes that uniform allocation policy is not necessarily truth-inducing with demand competition. Cho and Tang (2014) extends the study of Liu (2012) by identifying the conditions under which the uniform mechanism cannot erase the gaming effect (the buyers inflate their order quantities); and based on these conditions, deriving an allocation scheme (competitive allocation) that can eliminate the gaming effect. Moreover, an interesting insight from Cho and Tang (2014) is that under competitive allocation (truth-inducing), both the retailers' total profit and the profit of the entire supply chain are certainly higher (this is also different from the conclusions in Cachon and Lariviere (1999a, b, c)).

Among all the works that consider the competition in the downstream market, Chen, Li and Zhang (2013) is the most relevant one with us, and they study a setting where the retailers engage in Cournot competition and analyze the effect of the proportional and lexicographic mechanisms on the supplier's performance. They conclude that the supplier's preference over the two policies is changed by Cournot competition among the retailers. That is to say, the supplier can obtain higher profit under the lexicographic mechanism, which is truth-inducing and leads to less profit for the supplier if the retailers are local monopolistic. The main reasons are i) the lexicographic mechanism can dampen the competition intensity of the downstream market, and thereby encourages the supplier to propose a higher wholesale price; ii) under the lexicographic mechanism, the total order quantity of the retailers is higher given any wholesale price because the prioritized retailer has the chance to monopolize the entire market, and this increases his incentive to order more.

The above literature thoroughly studies the lexicographic allocation policy under the setting that buyers who compete for the limited resource in the upstream market also face each other in the downstream market. This thesis contributes to the existing literature by establishing a novel model in which the buyers have price-setting power in both the upstream and downstream markets and the buyer who proposes the higher wholesale price is prioritized. Based on the model, we study each supply chain member's strategic behavior. Furthermore, our study differentiates with other literature by integrating the wholesale price strategy and the inventory strategy, and we analyze the interaction between the two factors.

Li et al. (2017) and Jain, Hazra and Swaminathan (2019) extend this stream of literature by considering *asymmetric retailers*. Li et al. (2017) studies a setting in which the retailers have asymmetric market bases and the retailer with a greater market base (called high-type retailer) can sell at a higher retail price. They conclude that the performance of the lexicographic mechanism depends on whether the priority is given to the high-type retailer or the low-type one. Furthermore, Jain, Hazra and Swaminathan (2019) considers that the retailers have asymmetric bargaining power and analyzes an allocation policy similar to the lexicographic mechanism in which the priority of obtaining the supplier's capacity is given first to the well-established and more powerful buyer in the case that the unmet demand of one buyer will switch to the other buyer's products if the products of this buyer are out of stock, that is to say, quantity competition between two buyers exists.

In *the multi-channel distribution system* where the supplier sells the products to the market via both the direct channel and the retail channel with the buyer being the intermediary, it is also necessary to analyze the supplier's optimal capacity allocation strategy and the buyer's optimal order quantity when the supplier is capacity-constrained, and this is what Geng and Mallik (2007), Qing, Deng and Wang (2017) and Yang et al. (2018) have studied. The similarities between this body of papers and our work are: i) the supplier, who is also a seller in the end market, and the buyer compete in the downstream market; 2) the allocation rule between the two channels is similar to the lexicographic allocation mechanism, this is, the demand in the direct channel is met first and the buyer would not obtain his optimal order quantity if the quantity of the products kept for the direct channel by the supplier is great.

In addition to these works, Dai and Nu (2020) also considers a multichannel-like system, in which the manufacturer who only has limited capacity enters the product-sharing market and corporates with the sharing platform to offer the rental services to customers in addition to selling products directly to consumers who may also provide rental services on the sharing platform. And they analyze the strategic role of capacity constraint on the manufacturer's optimal capacity allocation strategy between two markets. The difference between this stream of works and our model is that: in our model, given the supplier's inventory, the quantities that the two buyers can finally obtain is determined by their order quantities, however, in the above-mentioned literature, the quantities that the two channels can obtain is decided by one central decision maker.

In the *intra-firm resource allocation* context， when the resource of the firm is scarce, multiple product lines or divisions would compete for the limited capacity, and the issue of allocating resources arises. For example, Harris, Kriebel and Raviv (1982) considers a question of allocating the firm's resource among multiple divisions which possess private information on their productivity. Both Mallik and Harker (2004) and Karabuk and Wu (2005) are inspired by the reality of a major US-based semiconductor manufacturer and study the incentive issues (the manufacturing and product managers lie about their private information) arising in the semiconductor capacity planning and allocation. For example, Karabuk and Wu (2005) develops a two-pronged incentive scheme that can induce the product managers to reveal their demands and allocate the capacity in a way that can maximize the firm's expected profits. At the same time, the scheme can be implemented by using the bonus system which is commonly used in semiconductor firms. Different from Karabuk and Wu (2005), Mallik and Harker (2004) considers that both the product demand and the firm's manufacturing capacity are uncertain. And in the circumstance that the central coordinator of the firm should allocate the uncertain capacities (forecasted by the manufacturing managers) to the different product lines whose demands are forecasted by the product managers for the planning year, Mallik and Harker (2004) states that the forecasts of capacities (demands, resp.) would be deflated by the manufacturing managers to deal with production uncertainties (inflated by the product managers to acquire a greater allocation of the resource, resp.). So, they design a mechanism that consists of a bonus scheme to elicit all managers reporting their forecasts truthfully and an allocation rule to allocate proper capacities to the different products. Similarly, assuming that the manufacturing and marketing managers of the firm will act in their self-interest, Porteus and Whang (1991) use the principal-agent (agency) approach to develop a scheme that could coordinate different divisions and enable the firm to attain the residual returns as much as possible. To conclude, in this body of literature, the mechanism-design approach is used by the central decision maker to seek a scheme that could coordinate the multiple parts of the firm so that he can obtain the maximum profit.

In addition to the literature above, various allocation policies that are used to *allocate demand* to the strategic servers in the queueing system are also studied, e.g., Cachon and Zhang (2007).

2.2 Using auctions to allocate the scarce resource

In the Economics literature, there is a large number of works considering auctions as a method of allocating scarce resources. This stream of work is relevant to our topic because, in our model, the prioritization is decided by an auction-like policy. And these studies have provided many interesting insights. In this section, we will introduce some of them and discuss the contribution of our work.

In the real world, there are probably far more products that are supplied by manufacturers than the amount that retailers can carry given their shelf space, thus, it is commonly observed in the practice in recent years that retailers allocate at least some of their scarce shelf space by auctions via slotting allowances. And this setting has been extensively studied by Shaffer (2005), Sullivan (1997), Lariviere and Padmanabhan (1997), Kuksov and Pazgal (2007) and Marx and Shaffer (2010). Marx and Shaffer (2010) proves that with slotting allowances, the retailer may limit its shelf space tentatively and let the manufacturers compete for the scarce resource. In this thesis, we draw a similar conclusion that even though the supplier's production cost is extremely low, he would limit his inventory level and do not prepare enough products if the wholesale prices are not high enough.

Another body of work, which considers that the buyers who bid for the scarce resource also engage in competition in the downstream market, focuses on studying the interaction between upstream and downstream markets, e.g., Stahl (1988), Yanelle (1997) and Eső, Nocke and White (2010). Among these works, Eső, Nocke and White (2010) is related to our model most and studies a setting in which middlemen acquire capacity allocation from upstream input market by efficient auction or efficient Coasian bargaining among the firms first, and then, compete in the downstream market in a Cournot fashion.

In the setting where a monopolistic supplier sells products to several buyers, Harris and Raviv (1981a) compares three monopolistic pricing schemes, that are: the simple single price strategy, auction, and priority pricing. And they identify the conditions under which the schemes are optimal. Similarly, Harris and Raviv (1981b) explains why auctions are used in certain situations and analyzes which auction mechanism is the most efficient one.

Chapter 3
The Model

We consider a setting with one supplier and two symmetric buyers. The two buyers procure key components that are required to assemble the final products from the supplier, and then sell substitutable products to the end market, assuming that assembling one final product requires one component.

In the end market, every consumer chooses a buyer to visit first and buys one product if the buyer has stock in hand. The retail price p is assumed to be exogenous. This process forms the *local demand* of each buyer. Suppose that the local demand of the two buyers is independent and uncertain, and to be specific, let $D_i, i = 1,2$ be buyer *i*'s local demand, then, $D_i, i = 1,2$ are *i.i.d* random variables with a *two-point discrete distribution* function:

$$D_i = \begin{cases} 1-\delta & \text{with probability } \frac{1}{2} \\ 1+\delta & \text{with probability } \frac{1}{2}, \end{cases}$$

where $\delta(\in [0,1])$ is the standard deviation variance of D_i, and it is used for measuring *the demand risk level*. Here, the expected local demand of each buyer is normalized to one, and we use $d_i, i = 1,2$ to denote the local demand of buyer *i* after it is realized.

Before the local demand of the two buyers is revealed, acting as Stackelberg leaders, the buyers have price-setting power in the upstream market and propose the wholesale prices ($w_i, i = 1,2$) that they want to pay for one product, simultaneously *(stage 1)*. And based on the wholesale prices, the supplier determines an inventory level q_s *(stage 2)*. We assume the fixed cost and the marginal cost for preparing inventory are normalized to zero to highlight the influence of the allocation policy on the supplier's optimal strategy. Then, the local demand is revealed.

After observing the local demand, the two buyers simultaneously determine the order quantities s_i, $i = 1,2$ *(stage 3)*. We assume $s_i \leq q_s$. However, they may not be able to obtain the quantities they have ordered since the supplier's inventory could be insufficient. When the total order quantity of the buyers exceeds the supplier's total supply, that is: the sum of s_1 and s_2 is larger than q_s, the limited inventory is allocated based on *the lexicographic allocation mechanism*. With lexicographic allocation policy, the supplier satisfies the order of the buyer having priority as much as possible, and then fills the other buyer's order with the remaining inventory if any.

The wholesale prices proposed by the two buyers provide the criteria for the supplier to decide which buyer will be prioritized and filled first when the allocation of the inventory is necessary. To be specific, if the two buyers propose different wholesale prices, the priority would be given to the one who is willing to pay more. Otherwise, the supplier gives the priority to one buyer randomly, namely, each buyer has $\frac{1}{2}$ probability to be prioritized. Here, we use $\mathcal{s}_i,\ i = 1{,}2$ to denote the quantities of components that buyers can finally obtain.

The process of allocating the supplier's inventory is as illustrated in Figure 1.

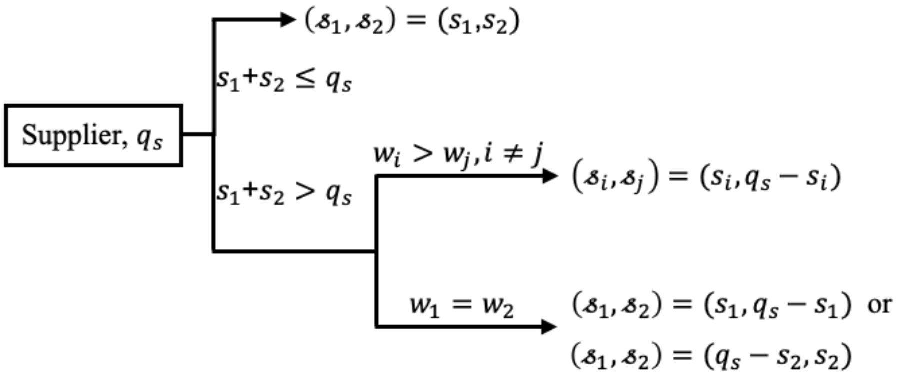


Figure 1 Indication of the allocation process

If $\mathcal{s}_1 < d_1$, some of buyer 1's realized local demand cannot be met by himself and buyer 1 would have *excess demand*; and if $\mathcal{s}_2 \ge d_2$, buyer 2 would have *excess stock* in hand after satisfying his realized local demand. When both the excess demand of buyer 1 and the excess stock of buyer 2 exist, a fraction $\alpha(\in [0,1])$ of the excess demand of buyer 1 would switch to buyer 2 and forms the *switch-over demand* of buyer 2; and then buyer 2 tries his best to satisfy the switch-over demand with the excess stock. By symmetry, we can define buyer 1's switch-over demand. The sum of the local and switch-over demand is referred to as the *effective total demand* of a buyer, and we can express buyer *i*'s effective total demand as:

$$d_i^e = d_i + \alpha(d_j - \mathcal{s}_j)^+,\ i = 1{,}2\ and\ j = 3 - i,$$

where: $(a)^+ = \max\{a, 0\}$.

The local demand and the switch-over demand of the two buyers are as illustrated in Figure 2.

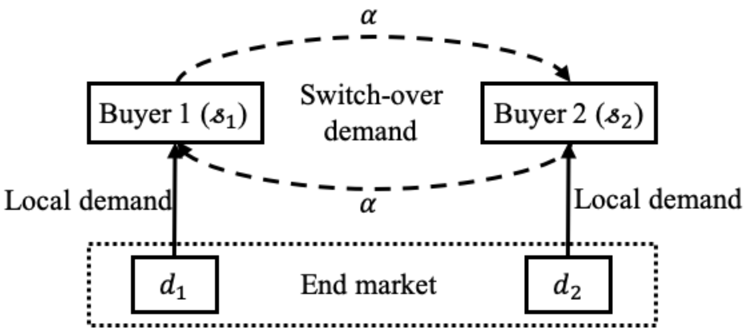


Figure 2 Indication of local and switch-over demand

Here, α is the switch-over intensity, and it can capture the extent that two products can substitute with each other as well as the quantity competition intensity between the two buyers in the following manner:

1) the increase of the buyer *i*'s inventory would result in the decrease of buyer *j*'s switchover demand, and a higher α would lead to a larger extent of such decrease if the buyer *i*'s inventory level is lower than his local demand;

2) with a higher α, the two buyers would have more incentive to propose a higher wholesale price and compete for the priority that enables their order quantities to be satisfied first by the supplier who has only limited inventory;

3) the competition intensity for the limited inventory between the two buyers would increase as α increases because the buyer who has been given the priority would have more incentive to monopolize the market with a higher α.

In this setting, the two buyers compete in two levels:

1) wholesale price competition:

On the one hand, the two buyers compete for the inventory-obtaining priority and the control power, which could enable them to have a chance of monopolizing the market, by proposing a higher wholesale price than the competitor. And the place where our work differs from other reported research, e.g., Chen, Li, and Zhang (2013), is that: the two buyers have to pay for the priority, and this level of competition may lead to asymmetric wholesale prices for symmetric buyers. On the other hand, the two buyers can affect and increase the supplier's inventory level and decrease their competitor's incentive of being the monopoly (According to *Theorem 1* in the following part, the buyer who is given the priority stops ordering the supplier's all inventory if q_s is larger than a threshold; and according to *Theorem 2, 3,* the supplier's optimal inventory level increases with the wholesale price proposed by the buyer who is not prioritized.) by choosing an appropriate price even though they cannot get the priority. To conclude, ***raising the proposed wholesale price can help prevent the buyer's competitor from gaining too much control***.

2) quantity competition:

The two buyers compete for the switch-over demand by ordering more.

To summarize, we illustrate this model in Figure 3.

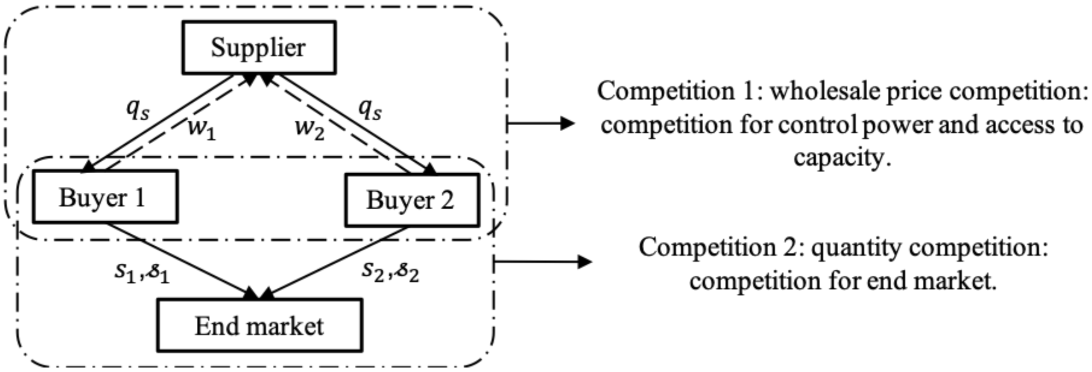


Figure 3 The model

Chapter 4

Model analysis

4.1 Quantity competition game after the demand is realized

Given the wholesale prices (w_1 and w_2) and the supplier's inventory level (q_s), we solve the two buyers' order quantity equilibria (s_1^*, s_2^*) after the demand is realized in this section with the following three steps:

Step 1: analyze the best-response order quantity of the buyer who is given the priority;

Step 2: analyze the best-response order quantity of the buyer who is not given the priority;

Step 3: analyze the two buyers' order quantity equilibria after the demands are realized.

4.1.1 Profit functions

Suppose buyer 1 has the inventory-obtaining priority, this is to say: $w_1 > w_2$ or he is chosen when $w_1 = w_2$. The equilibrium in other cases can be obtained by symmetry.

The buyers' profit functions after the market condition is realized can be written as:

$$\pi_1(s_1|w_1, w_2, q_s, s_2) = \max_{s_1 \le q_s}\{p(d_1^e \wedge s_1) - w_1 s_1\} \tag{1}$$

$$\pi_2(s_2|w_1, w_2, q_s, s_1) = \max_{s_2 \le q_s}\{p(d_2^e \wedge ((q_s - s_1) \wedge s_2)) - w_2((q_s - s_1) \wedge s_2)\} \tag{2}$$

We define: $a \wedge b = \min\{a, b\}$, $a \vee b = \max\{a, b\}$.

Consider buyer *i*'s optimization problem, given s_j, he has two options categorized based on whether or not the capacity constraint is violated or not, that is:

1) if $0 \le s_i \le q_s - s_j$, $\mathscr{s}_i = s_i$ and $\mathscr{s}_j = s_j$;

2) if $q_s - s_j < s_i \le q_s$, $\mathscr{s}_1 = s_1$, $\mathscr{s}_2 = q_s - s_1$ for $i = 1$, and, $\mathscr{s}_2 = q_s - s_1$, $\mathscr{s}_1 = s_1$ for $i = 2$.

And then, buyer *i*'s optimal profit equals to the maximum one between the two greatest profits that can be derived from the above two options. Therefore, we can rewrite equation (1) and (2) as:

$$\pi_1(s_1|w_1, w_2, q_s, s_2) = \max\{\max_{s_1\in[0,q_s-s_2]} G_{11}(s_1, s_2), \max_{s_1\in(q_s-s_2,q_s]} G_{12}(s_1, s_2)\}$$

where:

$G_{11}(s_1, s_2) = p((d_1 + \alpha(d_2 - s_2)^+) \wedge s_1) - w_1 s_1,$

$G_{12}(s_1, s_2) = p((d_1 + \alpha(d_2 - (q_s - s_1))^+) \wedge s_1) - w_1 s_1,$

and,

$$\pi_2(s_2|w_1, w_2, q_s, s_1) = \max\{\max_{s_2\in[0,q_s-s_1]} G_{21}(s_1, s_2), \max_{s_2\in(q_s-s_1,q_s]} G_{22}(s_1, s_2)\}$$

where:

$G_{21}(s_1, s_2) = p((d_2 + \alpha(d_1 - s_1)^+) \wedge s_2) - w_2 s_2,$

$G_{22}(s_1, s_2) = p((d_2 + \alpha(d_1 - s_1)^+) \wedge (q_s - s_1)) - w_2(q_s - s_1).$

4.1.2 Best-response order quantity

We illuminate buyer 1's best response function in *Lemma 1*, and buyer 2's best response function in *Lemma 2*.

As we have mentioned before, there are two subproblems for buyer 1. In subproblem 1, buyer 1 lets buyer 2 obtain s_2 and tries his best to meet the effective total demand $d_1 + \alpha(d_2 - s_2)^+$ with the supplier's left inventory $(q_s - s_2)$, this is to say, buyer 1's order quantity equals to his effective total demand until the demand is larger than $q_s - s_2$, and then he orders $q_s - s_2$, so, there is no wasted inventory for buyer 1.

Different from subproblem 1, in subproblem 2, buyer 1 can compete for more switch-over demand by increasing s_1. If $s_1 \leq q_s - d_2$ $(d_2 \leq q_s - s_1)$, there is no switch over demand for buyer 1, and if $s_1 > q_s - d_2$ $(d_2 > q_s - s_1)$, buyer 1 can obtain α more units switch over demand and earn αp by ordering one more unit with paying w_1. Consequently, ***if $\alpha p \geq w_1$, buyer 1 would like to order more even when the total demand is less than s_1 and has the incentive to monopolize the end market by ordering q_s, otherwise, he would not***. Comparing the optimal profits of two subproblems manifests buyer 1's *tradeoff* between a) ordering less than $q_s - s_2$ but all products ordered can be sold, b) ordering more than $q_s - s_2$ to compete for more effective total demand and even monopolize the end market, but might pay a cost for the products that cannot be sold. We illustrate the two options of the prioritized buyer in Figure 4.

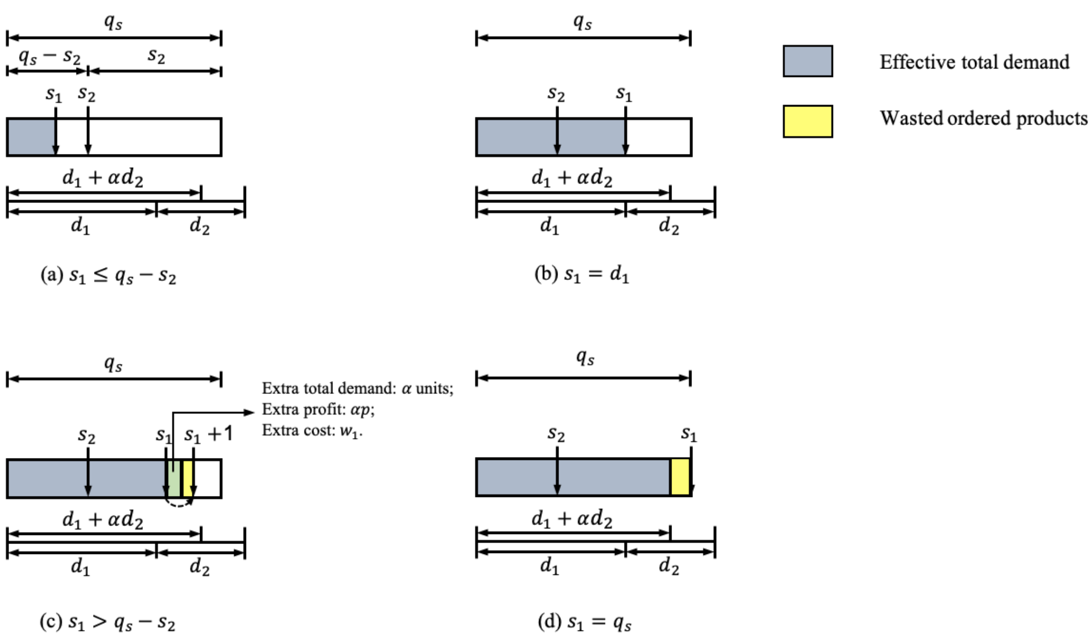


Figure 4 Buyer 1's two options when he has the priority,

$d_1 + \alpha d_2 < q_s \leq d_1 + d_2$ and $q_s - s_2 < d_1$

Lemma 1: Let $s_1(s_2)$ be buyer 1's best response function, if buyer 1 is given inventory-obtaining priority, we can characterize it as follows:

(1) $c \leq w_1 \leq \alpha p$:

if $0 \leq q_s \leq d_1 + \alpha d_2$, $s_1(s_2) = q_s$;

if $d_1 + \alpha d_2 < q_s \leq d_1 + \frac{\alpha p}{w_1} d_2$, then $s_1(s_2) =$

$$\begin{cases} d_1 + \alpha(d_2 - s_2), & 0 \leq s_2 < \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} \\ q_s, & \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} \leq s_2 \leq q_s \end{cases};$$

if $q_s > d_1 + \frac{\alpha p}{w_1} d_2$, then $s_1(s_2) = \begin{cases} d_1 + \alpha(d_2 - s_2), & 0 \leq s_2 < d_2 \\ d_1, & d_2 \leq s_2 \leq q_s \end{cases}$;

(2) $\alpha p < w_1 \leq p$;

if $0 \leq q_s \leq d_1 + \alpha d_2$, $s_1(s_2) = q_s$;

if $d_1 + \alpha d_2 < q_s \leq d_1 + d_2$, then $s_1(s_2) =$

$$\begin{cases} d_1 + \alpha(d_2 - s_2), & 0 \leq s_2 < \frac{q_s-(d_1+\alpha d_2)}{1-\alpha} \\ \frac{d_1+\alpha(d_2-q_s)}{1-\alpha}, & \frac{q_s-(d_1+\alpha d_2)}{1-\alpha} \leq s_2 \leq q_s \end{cases};$$

if $q_s > d_1 + d_2$, then $s_1(s_2) = \begin{cases} d_1 + \alpha(d_2 - s_2), & 0 \leq s_2 < d_2 \\ d_1, & d_2 \leq s_2 \leq q_s \end{cases}$.

It is quite intuitive that: if $q_s \leq d_1 + \alpha d_2$, buyer 1 prefers to monopolize the end market by ordering q_s regardless of the value of w_1 since all the products will be purchased by customers. If $q_s > d_1 + \alpha d_2$, buyer 1 orders more than $q_s - s_2$ to earn strictly more demand than $d_1 + \alpha(d_2 - s_2)^+$ only in the context that q_s is moderate and s_2 is large enough, because:

1) if s_2 is small, on the one hand, buyer 1 already has a high level effective total demand, and competing for more switch over demand becomes unattractive, on the other hand, the left capacity $q_s - s_2$ is already large enough to meet buyer 1's total demand and there is no need for him to order more than $q_s - s_2$;

2) in the case that $w_1 \leq \alpha p$, *buyer 1 has to pay more cost to monopolize the market with a higher* q_s *as there would be more unsold products, however, his effective total demand and the profit he can collect do not change*;

3) in the case that $w_1 > \alpha p$ and q_s is not too large, the inventory left for buyer 2, that is: $q_s - s_1$ becomes less than d_2 and buyer 1's switch over demand alters from zero to be positive when s_1 is too small to meet his local demand, not to mention the effective total demand, therefore, it is profitable for buyer 1 to orders more, and even exceeds $q_s - s_2$, if possible, to meet the demand that cannot be met, and meanwhile, compete for more demand until s_1 equals to buyer 1's effective total demand, nonetheless, if $q_s > d_1 + d_2$, from the perspective of buyer 1, to acquire α more units demand than $d_1 + \alpha(d_2 - s_2)^+$ must be accompanied by ordering at least one more unit inventory and there are at least $1 - \alpha$ units products cannot be sold, which is not profitable in this case.

Lastly, we can conclude that, if q_s is large enough, buyer 1's best response function is just the one in the game without inventory limitation and is independent of the value of w_1.

Lemma 2: Let $s_2(s_1)$ be buyer 2's best response function, if buyer 2 is not given inventory-obtaining priority, we can characterize it as follows:

(1) if $0 \leq q_s \leq d_2 + \alpha d_1$: $s_2(s_1)$ can be any value in the interval $[q_s - s_1, q_s]$;

(2) if $d_2 + \alpha d_1 < q_s \leq d_1 + d_2$: $s_2(s_1) = d_2 + \alpha(d_1 - s_1)$ when $0 \leq s_1 \leq \frac{q_s-(d_2+\alpha d_1)}{1-\alpha}$, and $s_2(s_1)$ can be any value in the interval $[q_s - s_1, q_s]$ when $\frac{q_s-(d_2+\alpha d_1)}{1-\alpha} < s_1 \leq q_s$;

(3) if $q_s > d_1 + d_2$: $s_2(s_1) = d_2 + \alpha(d_1 - s_1)$ when $0 \leq s_1 < d_1$, $s_2(s_1) = d_2$ when $d_1 < s_1 \leq q_s - d_2$, and, $s_2(s_1)$ can be any value in the interval $[q_s - s_1, q_s]$ when $q_s - d_2 < s_1 \leq q_s$.

Buyer 2 is not given inventory-obtaining priority and cannot compete for more switch-over demand by increasing s_2. So, his best-response order quantity equals to his demand $d_2 + \alpha(d_1 - s_1)^+$ until the total demand is larger than $q_s - s_1$, and then he orders $q_s - s_1$.

If $0 \leq q_s \leq d_2 + \alpha d_1$, q_s would be too small so that the supplier's inventory left by buyer 1 is less than buyer 2's total demand no matter how many buyer 1 orders, thereby, he prefers to order at least $q_s - s_1$. However, if $q_s > d_2 + \alpha d_1$, buyer 2's demand $d_2 + \alpha(d_1 - s_1)^+$ is less than the inventory that he can obtain when s_1 is small, therefore, $s_2(s_1)$ equals to $d_2 + \alpha(d_1 - s_1)^+$ if s_1 is small and buyer 2 orders at least $q_s - s_1$ if s_1 is large.

4.1.3 Order quantity equilibrium

Based on the two buyers' best-response order quantities, we can derive the Nash equilibrium of the quantity competition, which is illustrated in *Theorem 1.*

Theorem 1: Let (s_1^*, s_2^*) be the Nash equilibrium of buyers' order quantities and $(\mathcal{s}_1^*, \mathcal{s}_2^*)$ be the quantities that buyers are finally delivered, then we characterize them as follows.

(1) $c \leq w_1 \leq \alpha p$:

if $0 \leq q_s \leq d_1 + \alpha d_2$, then any point in $\{q_s\} \times [0, q_s]$ is a Nash equilibrium with $(\mathcal{s}_1^*, \mathcal{s}_2^*) = (q_s, 0)$;

if $d_1 + \alpha d_2 < q_s \leq d_1 + \frac{\alpha p}{w_1} d_2$, then any point in $\{q_s\} \times [\frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)}, q_s]$ is a Nash equilibrium with $(\mathcal{s}_1^*, \mathcal{s}_2^*) = (q_s, 0)$;

if $q_s > d_1 + \frac{\alpha p}{w_1} d_2$, then $(s_1^*, s_2^*) = (d_1, d_2)$ with $(\mathcal{s}_1^*, \mathcal{s}_2^*) = (d_1, d_2)$ is an unique Nash equilibrium.

(2) $\alpha p < w_1 \le p$:

if $0 \le q_s \le d_1 + \alpha d_2$, then any point in $\{q_s\} \times [0, q_s]$ is a Nash equilibrium with $(\mathcal{s}_1^*, \mathcal{s}_2^*) = (q_s, 0)$;

if $d_1 + \alpha d_2 < q_s \le d_1 + d_2$, then any point in $\left\{\frac{d_1+\alpha(d_2-q_s)}{1-\alpha}\right\} \times [\frac{q_s-(d_1+\alpha d_2)}{1-\alpha}, q_s]$ is a Nash equilibrium with $(\mathcal{s}_1^*, \mathcal{s}_2^*) = (\frac{d_1+\alpha(d_2-q_s)}{1-\alpha}, \frac{q_s-(d_1+\alpha d_2)}{1-\alpha})$;

if $q_s > d_1 + d_2$, then $(s_1^*, s_2^*) = (d_1, d_2)$ with $(\mathcal{s}_1^*, \mathcal{s}_2^*) = (d_1, d_2)$ is an unique Nash equilibrium.

Theorem 1 describes a pattern for the two buyers' total order quantity as a function of the supplier's inventory level and the higher wholesale price proposed by the buyers. If the supplier's inventory is scarce (q_s is less than $d_1 + \frac{\alpha p}{w_1} d_2$ in case (1) and $d_1 + \alpha d_2$ in case (2)), buyer 1, who has been given priority, would order q_s to monopolizes the end market, and this indicates that the buyers' total order quantity increases with q_s in this case. If q_s is large enough (q_s is larger than $d_1 + \frac{\alpha p}{w_1} d_2$ in case (1) and $d_1 + d_2$ in case (2)), the Nash equilibrium is just the one in the game without inventory constraint and both buyers order the amount of their local demand, thus, the buyers' total order quantity does not change with q_s in this case. However, there are also some differences between case (1) and (2): 1) if $w_1 \le \alpha p$, the buyers' total order quantity is not continuous and experiences a ***drop*** from $d_1 + \frac{\alpha p}{w_1} d_2$ to $d_1 + d_2$ as q_s crosses the critical point $d_1 + \frac{\alpha p}{w_1} d_2$; notwithstanding, if $w_1 > \alpha p$, the buyers' total order quantity is continuous and turns into a constant after q_s crosses the critical point $d_1 + d_2$; 2) if $w_1 \le \alpha p$, the Nash equilibrium is either buyer 1 monopolizes the market or buyers' order quantities equals to their local demand, different from this, if $w_1 > \alpha p$, there is a ***transitional state***, in which the total order quantity equals to q_s while $\mathcal{s}_2^*$ is strictly positive, with a moderate inventory level.

In case (1), supposing that w_1 equals to αp, monopolizing is profitable for buyer 1 if and only if q_s is no more than $d_1 + d_2$; with the decrease of w_1, that is: w_1 becomes strictly less than αp, monopolizing becomes less costly, and thus profitable even though q_s is strictly more than $d_1 + d_2$, and then, the drop of the total order quantity emerges. This result is interesting, as intuitively, if $q_s > d_1 + d_2$, there would be no inventory limitation for this quantity game, and buyers would order local demand, however, this only happens when w_1 is larger than αp. Furthermore, buyer 1 becomes more aggressive with a larger α, as on the one hand, the critical point $d_1 + \frac{\alpha p}{w_1} d_2$ in case (1) increases with α and buyer 1 would prefer to monopolize with a higher q_s, on the other hand, the range of interval $(\alpha p, p]$ decreases with α and thus it is more likely for buyer 1 to be in the first case.

Let α be one, then the two buyers are perfectly competitive, our result is consistent with Chen, Li, and Zhang (2013), which studied the capacity allocation problem between two buyers engaged in Cournot competition. Our work extends this study by setting α in the range $[0,1]$ to visualize the Nash equilibria and study the effect of competition intensity in the case that two buyers sell substitutable products and are not perfectly competitive.

The quantity game between the buyers may have multiple equilibria, nonetheless, all equilibria predict that buyer 1 will order q_s and receive the entire inventory from the supplier and it is not important that what buyer 2 orders, and therefore the different equilibria lead to the same profits for the two buyers.

4.2 The optimal inventory strategy of the supplier before demand is realized

Given the wholesale prices proposed by buyers in the first stage, the supplier maximizes her expected profit by determining an inventory level before demand is realized considering the Nash equilibrium of the two buyers' order quantities in the third stage. We analyze this problem in two cases: $w_1 \leq \alpha p$ and $\alpha p < w_1 \leq p$.

Similar to the previous chapters, we hold the assumption here that buyer 1 has been given inventory-obtaining priority and the supplier's decision in the other case (i.e., buyer 2 is given priority) can be obtained by symmetry.

4.2.1 Profit functions

4.2.1.1 The expected profit function when $w_1 \leq \alpha p$

We use $E(\Pi_s(q_s|w_1, w_2))$ to denote the supplier's expected profit, which is given as:

$$E(\Pi_s(q_s|w_1, w_2)) = \max\{\max_{q_s\in[0,q_I^1]} M_1(q_s), \max_{q_s\in(q_I^1,q_I^2]} M_2(q_s), \max_{q_s\in(q_I^2,q_I^3]} M_3(q_s),$$

$$\max_{q_s\in(q_I^3,q_I^4]} M_4(q_s), \max_{q_s\in(q_I^4,\infty)} M_5(q_s)\}.$$

The expected profit function is piecewise with four critical points $q_I^i, i = 1,2,3,4$:

$$q_I^1 = 1 - \delta + \frac{\alpha p}{w_1}(1 - \delta),$$

$$q_I^2 = 1 + \delta + \frac{\alpha p}{w_1}(1 - \delta),$$

$$q_I^3 = 1 - \delta + \frac{\alpha p}{w_1}(1 + \delta),$$

$$q_I^4 = 1 + \delta + \frac{\alpha p}{w_1}(1 + \delta).$$

$M_i(q_s), i = 1, \ldots, 5$ are the profit functions in five ranges of q_s:

$$M_1(q_s) = w_1 q_s;$$

$$M_2(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{3}{4}w_1 q_s;$$

$$M_3(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{1}{2}w_1 q_s;$$

$$M_3(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{1}{4}w_1 q_s;$$

$$M_3(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{w_1(1+\delta)+w_2(1+\delta)}{4}.$$

4.2.1.2 The expected profit function when $\alpha p < w_1 \leq p$

In order to formulate the supplier's expected profit in this case, we first establish some preliminary results by simple algebra in Lemma 3.

Lemma 3:

(1) If $0 \le \delta \le \frac{1-\alpha}{3+\alpha}$, $1-\delta+\alpha(1-\delta) \le 1-\delta+\alpha(1+\delta) \le 1+\delta+\alpha(1-\delta) \le 1+\delta+\alpha(1+\delta) \le 1-\delta+(1-\delta) \le 1-\delta+1+\delta \le 1+\delta+(1+\delta)$;

(2) if $\frac{1-\alpha}{3+\alpha} < \delta \le \frac{1-\alpha}{3-\alpha}$, $1-\delta+\alpha(1-\delta) \le 1-\delta+\alpha(1+\delta) \le 1+\delta+\alpha(1-\delta) \le 1-\delta+(1-\delta) \le 1+\delta+\alpha(1+\delta) \le 1-\delta+1+\delta \le 1+\delta+(1+\delta)$;

(3) if $\frac{1-\alpha}{3-\alpha} < \delta \le \frac{1-\alpha}{1+\alpha}$, $1-\delta+\alpha(1-\delta) \le 1-\delta+\alpha(1+\delta) \le 1-\delta+(1-\delta) \le 1+\delta+\alpha(1-\delta) \le 1+\delta+\alpha(1+\delta) \le 1-\delta+1+\delta \le 1+\delta+(1+\delta)$;

(4) if $\frac{1-\alpha}{1+\alpha} < \delta \le 1$, $1-\delta+\alpha(1-\delta) \le 1-\delta+(1-\delta) \le 1-\delta+\alpha(1+\delta) \le 1+\delta+\alpha(1-\delta) \le 1-\delta+1+\delta \le 1+\delta+\alpha(1+\delta) \le 1+\delta+(1+\delta)$.

We use $q_{II}^i, i = 1, \dots, 7$ to denote the seven critical points $1-\delta+\alpha(1-\delta)$, $1-\delta+\alpha(1+\delta)$, $1+\delta+\alpha(1-\delta)$, $1+\delta+\alpha(1+\delta)$, $1-\delta+(1-\delta)$, $1-\delta+1+\delta$ and $1+\delta+(1+\delta)$, respectively.

(1) If $0 \le \delta \le \frac{1-\alpha}{3+\alpha}$, we have the supplier's piecewise expected profit function as follows:

$$
\begin{aligned}
&E\big(\Pi_s(q_s|w_1,w_2)\big)\\
&= \max\{\max_{q_s\in[0,q_{II}^1]} N_{11}(q_s), \max_{q_s\in(q_{II}^1,q_{II}^2]} N_{12}(q_s), \max_{q_s\in(q_{II}^2,q_{II}^3]} N_{13}(q_s), \max_{q_s\in(q_{II}^3,q_{II}^4]} N_{14}(q_s),\\
&\quad \max_{q_s\in(q_{II}^4,q_{II}^5]} N_{15}(q_s), \max_{q_s\in(q_{II}^5,q_{II}^6]} N_{16}(q_s), \max_{q_s\in(q_{II}^6,q_{II}^7]} N_{17}(q_s), \max_{q_s\in(q_{II}^7,\infty)} N_{18}(q_s)\},
\end{aligned}
$$

where the profit functions in eight ranges of q_s can be described as:

$$N_{11}(q_s) = w_1 q_s;$$

$$N_{12}(q_s) = \frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{3}{4}w_1 q_s;$$

$$
\begin{aligned}
N_{13}(q_s) = {}& \frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} +\\
& \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{1}{2}w_1 q_s;
\end{aligned}
$$

$$
\begin{aligned}
N_{14}(q_s) = {}& \frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} +\\
& \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} +\\
& \frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{1}{4}w_1 q_s;
\end{aligned}
$$

$$N_{15}(q_s)=\frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2(q_s-((1-\delta)+\alpha(1-\delta)))}{4(1-\alpha)}+\frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2(q_s-((1-\delta)+\alpha(1+\delta)))}{4(1-\alpha)}+\frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2(q_s-((1+\delta)+\alpha(1-\delta)))}{4(1-\alpha)}+\frac{w_1\big((1+\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2(q_s-((1+\delta)+\alpha(1+\delta)))}{4(1-\alpha)};$$

$$N_{16}(q_s)=\frac{w_1(1-\delta)+w_2(1-\delta)}{4}+\frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2(q_s-((1-\delta)+\alpha(1+\delta)))}{4(1-\alpha)}+\frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2(q_s-((1+\delta)+\alpha(1-\delta)))}{4(1-\alpha)}+\frac{w_1\big((1+\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2(q_s-((1+\delta)+\alpha(1+\delta)))}{4(1-\alpha)};$$

$$N_{17}(q_s)=\frac{w_1(1-\delta)+w_2(1-\delta)}{4}+\frac{w_1(1-\delta)+w_2(1+\delta)}{4}+\frac{w_1(1+\delta)+w_2(1-\delta)}{4}+\frac{w_1\big((1+\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2(q_s-((1+\delta)+\alpha(1+\delta)))}{4(1-\alpha)};$$

$$N_{18}(q_s)=\frac{w_1(1-\delta)+w_2(1-\delta)}{4}+\frac{w_1(1-\delta)+w_2(1+\delta)}{4}+\frac{w_1(1+\delta)+w_2(1-\delta)}{4}+\frac{w_1(1+\delta)+w_2(1+\delta)}{4};$$

(2) If $\frac{1-\alpha}{3+\alpha}<\delta\le\frac{1-\alpha}{3-\alpha}$, we have:

$$E\big(\Pi_s(q_s|w_1,w_2)\big)$$
$$=\max\{\max_{q_s\in[0,q_{II}^1]}N_{21}(q_s),\max_{q_s\in(q_{II}^1,q_{II}^2]}N_{22}(q_s),\max_{q_s\in(q_{II}^2,q_{II}^3]}N_{23}(q_s),\max_{q_s\in(q_{II}^3,q_{II}^5]}N_{24}(q_s),$$
$$\max_{q_s\in(q_{II}^5,q_{II}^4]}N_{25}(q_s),\max_{q_s\in(q_{II}^4,q_{II}^6]}N_{26}(q_s),\max_{q_s\in(q_{II}^6,q_{II}^7]}N_{27}(q_s),\max_{q_s\in(q_{II}^7,\infty)}N_{28}(q_s)\},$$

where:

$$N_{21}(q_s)=w_1q_s;$$

$$N_{22}(q_s)=\frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2(q_s-((1-\delta)+\alpha(1-\delta)))}{4(1-\alpha)}+\frac{3}{4}w_1q_s;$$

$$N_{23}(q_s)=\frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2(q_s-((1-\delta)+\alpha(1-\delta)))}{4(1-\alpha)}+\frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2(q_s-((1-\delta)+\alpha(1+\delta)))}{4(1-\alpha)}+\frac{1}{2}w_1q_s;$$

$$N_{24}(q_s)=\frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2(q_s-((1-\delta)+\alpha(1-\delta)))}{4(1-\alpha)}+\frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2(q_s-((1-\delta)+\alpha(1+\delta)))}{4(1-\alpha)}+\frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2(q_s-((1+\delta)+\alpha(1-\delta)))}{4(1-\alpha)}+\frac{1}{4}w_1q_s;$$

$$N_{25}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{1}{4}w_1 q_s;$$

$$N_{26}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{w_1\big((1+\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)};$$

$$N_{27}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1+\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)};$$

$$N_{28}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{w_1(1+\delta)+w_2(1+\delta)}{4}.$$

(3) If $\frac{1-\alpha}{3-\alpha} < \delta \le \frac{1-\alpha}{1+\alpha}$, we have:

$$E\big(\Pi_s(q_s|w_1,w_2)\big)$$
$$= \max\{\max_{q_s\in[0,q_{II}^1]} N_{31}(q_s), \max_{q_s\in(q_{II}^1,q_{II}^2]} N_{32}(q_s), \max_{q_s\in(q_{II}^2,q_{II}^5]} N_{33}(q_s), \max_{q_s\in(q_{II}^5,q_{II}^3]} N_{34}(q_s),$$
$$\max_{q_s\in(q_{II}^3,q_{II}^4]} N_{35}(q_s), \max_{q_s\in(q_{II}^4,q_{II}^6]} N_{36}(q_s), \max_{q_s\in(q_{II}^6,q_{II}^7]} N_{37}(q_s), \max_{q_s\in(q_{II}^7,\infty)} N_{38}(q_s)\},$$

where:

$$N_{31}(q_s) = w_1 q_s;$$

$$N_{32}(q_s) = \frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{3}{4}w_1 q_s;$$

$$N_{33}(q_s) = \frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{1}{2}w_1 q_s;$$

$$N_{34}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{1}{2}w_1 q_s;$$

$$N_{35}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{1}{4}w_1 q_s;$$

$$N_{36}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{w_1\big((1+\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)};$$

$$N_{37}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1+\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)};$$

$$N_{38}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{w_1(1+\delta)+w_2(1+\delta)}{4}.$$

(4) If $\frac{1-\alpha}{1+\alpha} < \delta \leq 1$, we have:

$$E\big(\Pi_s(q_s|w_1,w_2)\big)$$
$$= \max\{\max_{q_s\in[0,q_{II}^1]} N_{41}(q_s), \max_{q_s\in(q_{II}^1,q_{II}^5]} N_{42}(q_s), \max_{q_s\in(q_{II}^5,q_{II}^2]} N_{43}(q_s), \max_{q_s\in(q_{II}^2,q_{II}^3]} N_{44}(q_s), \max_{q_s\in(q_{II}^3,q_{II}^6]} N_{45}(q_s), \max_{q_s\in(q_{II}^6,q_{II}^4]} N_{46}(q_s), \max_{q_s\in(q_{II}^4,q_{II}^7]} N_{47}(q_s), \max_{q_s\in(q_{II}^7,\infty)} N_{48}(q_s)\},$$

where:

$$N_{41}(q_s) = w_1 q_s;$$

$$N_{42}(q_s) = \frac{w_1\big((1-\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{3}{4}w_1 q_s;$$

$$N_{43}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{3}{4}w_1 q_s;$$

$$N_{44}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{1}{2}w_1 q_s;$$

$$N_{45}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1-\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1-\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)} + \frac{w_1\big((1+\delta)+\alpha\big((1-\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1-\delta)\big)\big)}{4(1-\alpha)} + \frac{1}{4}w_1 q_s;$$

$$N_{46}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{1}{4}w_1 q_s;$$

$$N_{47}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{w_1\big((1+\delta)+\alpha\big((1+\delta)-q_s\big)\big)+w_2\big(q_s-\big((1+\delta)+\alpha(1+\delta)\big)\big)}{4(1-\alpha)};$$

$$N_{48}(q_s) = \frac{w_1(1-\delta)+w_2(1-\delta)}{4} + \frac{w_1(1-\delta)+w_2(1+\delta)}{4} + \frac{w_1(1+\delta)+w_2(1-\delta)}{4} + \frac{w_1(1+\delta)+w_2(1+\delta)}{4}.$$

4.2.2 The optimal inventory strategy of the supplier when $w_1 \leq \alpha p$

When $w_1 \le \alpha p$, the supplier's expected profit is not continuous and drops at four critical points. Moreover, $M_i(q_s), i = 1, \ldots, 4$ are linearly non-decreasing functions whose slopes decrease successively and $M_5(q_s)$ is constant. The profit function is illustrated in Figure 5.

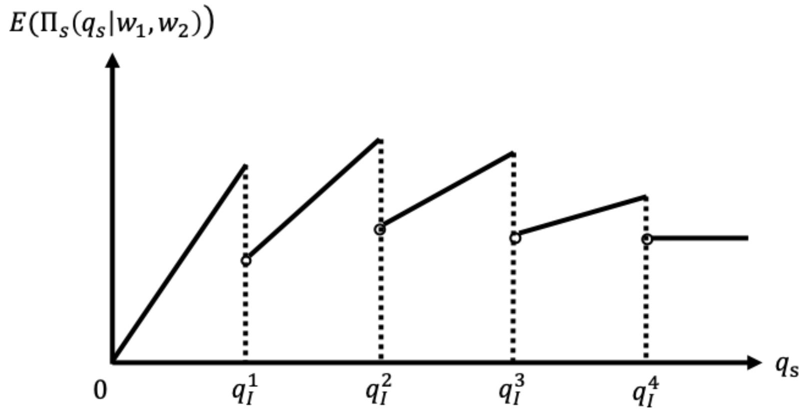


Figure 5 $E\big(\Pi_s(q_s|w_1, w_2)\big)$ if $w_1 \le \alpha p$

The optimal expected profit is the maximum one among $M_1(q_I^1)$, $M_2(q_I^2)$, $M_3(q_I^3)$, and $M_4(q_I^4)$. By comparing the four values, we have:

$M_2(q_I^2) - M_1(q_I^1) = \frac{1}{4}(6\delta w_1 + (1-\delta)w_2 - (1-\delta)\alpha p)$, so, $M_2(q_I^2) - M_1(q_I^1) \ge 0$ iff $w_1 \ge -\frac{1-\delta}{6\delta} w_2 + \frac{1-\delta}{6\delta}\alpha p$;

$M_3(q_I^3) - M_2(q_I^2) = \frac{1}{4}(-4\delta w_1 + (1-\delta)w_2 + (5\delta - 1)\alpha p)$, so, $M_3(q_I^3) - M_2(q_I^2) \ge 0$ iff $w_1 \le \frac{1-\delta}{4\delta} w_2 + \frac{5\delta-1}{4\delta}\alpha p$;

$M_4(q_I^4) - M_3(q_I^3) = \frac{1}{4}(2\delta w_1 + (1+\delta)w_2 - (1+\delta)\alpha p)$, so $M_4(q_I^4) - M_3(q_I^3) \ge 0$ iff $w_1 \ge -\frac{1+\delta}{2\delta} w_2 + \frac{1+\delta}{2\delta}\alpha p$;

$M_4(q_I^4) - M_2(q_I^2) = \frac{1}{2}(-\delta w_1 + w_2 - (1-2\delta)\alpha p)$, so, $M_4(q_I^4) - M_2(q_I^2) \ge 0$ iff $w_1 \le \frac{1}{\delta} w_2 - \frac{1-2\delta}{\delta}\alpha p$;

$M_3(q_I^3) - M_1(q_I^1) = \frac{1}{2}(\delta w_1 + (1-\delta)w_2 - (1-3\delta)\alpha p)$, so, $M_3(q_I^3) - M_1(q_I^1) \ge 0$ iff $w_1 \ge -\frac{1-\delta}{\delta} w_2 + \frac{1-3\delta}{\delta}\alpha p$.

Then, let $Li, i = 1, \ldots, 5$ to be the five linear functions $w_1 = -\frac{1-\delta}{6\delta} w_2 + \frac{1-\delta}{6\delta} \alpha p$, $w_1 = \frac{1-\delta}{4\delta} w_2 + \frac{5\delta-1}{4\delta} \alpha p$, $w_1 = -\frac{1+\delta}{2\delta} w_2 + \frac{1+\delta}{2\delta} \alpha p$, $w_1 = \frac{1}{\delta} w_2 - \frac{1-2\delta}{\delta} \alpha p$, and $w_1 = -\frac{1-\delta}{\delta} w_2 + \frac{1-3\delta}{\delta} \alpha p$, respectively.

Symmetrically, we can analyze the case that buyer 2 is given priority in the same way. We use $Li', i = 1, \ldots, 5$ to denote the inverse functions of $Li, i = 1, \ldots, 5$, respectively, which are: $w_1 = -\frac{6\delta}{1-\delta} w_2 + \alpha$, $w_1 = \frac{4\delta}{1-\delta} w_2 - \frac{5\delta-1}{1-\delta} \alpha p$, $w_1 = -\frac{2\delta}{1+\delta} w_2 + \alpha$, $w_1 = \delta w_2 + (1-2\delta)\alpha p$ and $w_1 = -\frac{\delta}{1-\delta} w_2 + \frac{1-3\delta}{1-\delta} \alpha p$.

We locate $Li, i = 1, \ldots, 5$ together with $Li', i = 1, \ldots, 5$ on the w_2-w_1 plane to divide it into several regions, and then, the optimal inventory strategy is clear in each area as illuminated in *Theorem 2*.

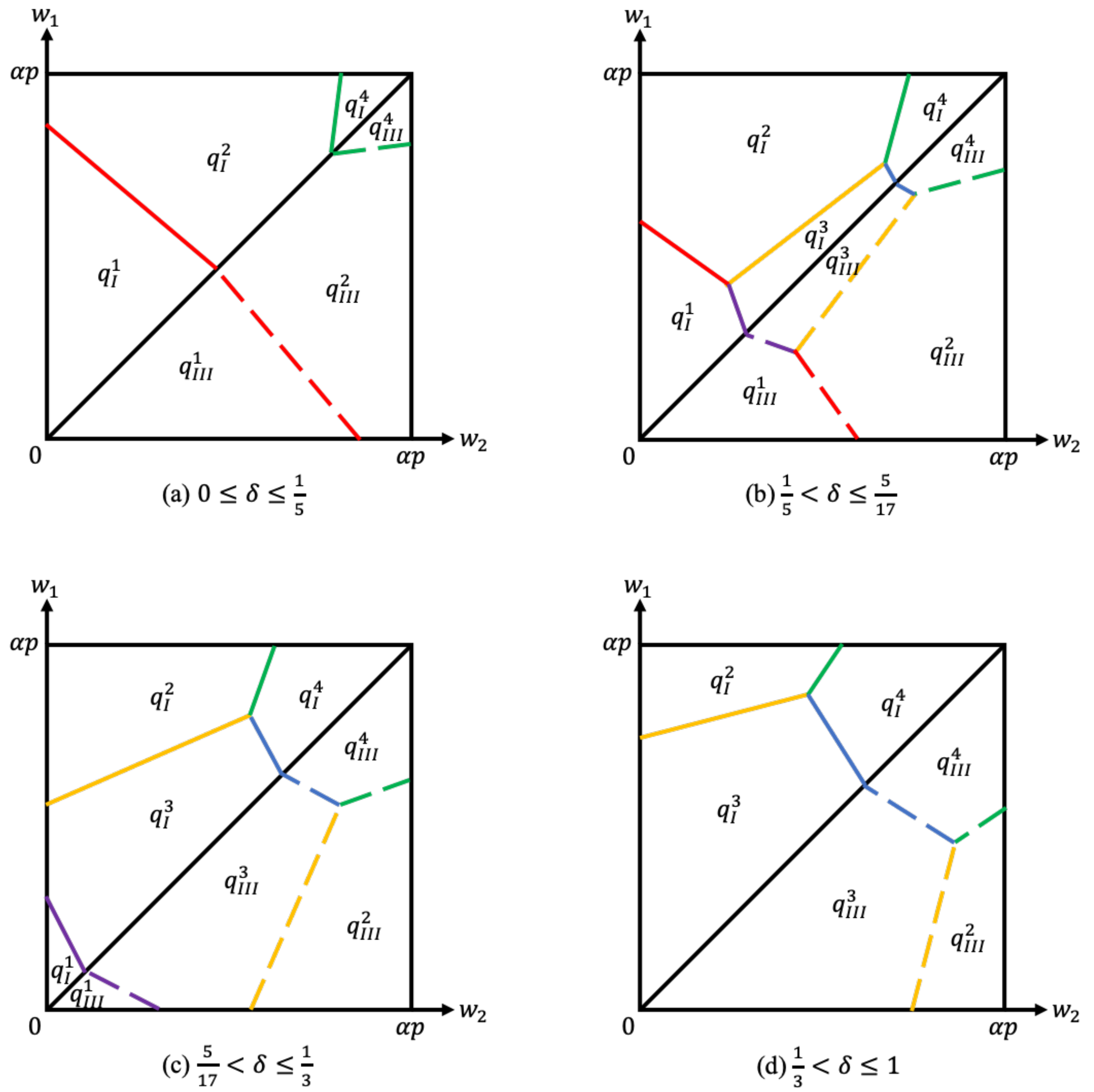
Similar to $q_I^i, i = 1,2,3,4$, we define $q_{III}^i, i = 1,2,3,4$ in Figure 6 as follows:

$q_{III}^1 = 1 - \delta + \frac{\alpha p}{w_2}(1-\delta)$, $q_{III}^2 = 1 + \delta + \frac{\alpha p}{w_2}(1-\delta)$, $q_{III}^3 = 1 - \delta + \frac{\alpha p}{w_2}(1+\delta)$, and, $q_{III}^4 = 1 + \delta + \frac{\alpha p}{w_2}(1+\delta)$.

Theorem 2: If $w_1 \leq \alpha p$, the supplier's optimal inventory strategy is showed in Figure 6.

The supplier faces the tradeoff between ***selling more products to the monopoly*** and ***decreasing the negative influence of adverse equilibrium effect***. For example, compared with $q_s = q_I^1$, even though the supplier can sell more products with $q_s = q_I^2$ when the realized demands $(d_1, d_2) \neq (1-\delta, 1-\delta)$ (i.e., the market is monopolized), she faces an adverse Nash equilibrium of buyers' order quantities, that is: selling less at a lower average wholesale price, when $(d_1, d_2) = (1-\delta, 1-\delta)$.

As δ increases, the supplier would have the incentive to preparing more inventory as observed from Figure 6, where the fraction of region that indicates $q_s^* \geq q_I^i \; for \; i = 1, \ldots, 4$ (q_s^* is the supplier's best strategy) in the plane becomes larger with the increase of δ and $q_s^* \neq q_I^1$ if $\delta > \frac{1}{3}$ regardless of the value of w_1 and w_2. The reason is that the benefit of selling more to the monopoly increases with δ.



Note: 1) Let the solid lines represent $Li, i = 1, \dots, 5$ and the dashed lines represent $Li', i = 1, \dots, 5$;

2) Let red, orange, blue, green, and purple to denote $L1$ ($L1'$),…, $L5$ ($L5'$), respectively.

Figure 6 The supplier's optimal inventory strategy if $w_1 \le \alpha p$

In general, the benefit of selling more products when the market is monopolized increases with w_1 (there is an exception that will be discussed later) and the loss owing to adverse equilibrium effect decreases with w_2, which leads to, as shown in Figure 6, $q_S^* = q_I^1$ and $q_S^* = q_I^4$ locating at the lower left and the upper right corner, respectively, where: w_2 is large (or small) on the top right corner (or the left bottom) and w_1 is large (or small) on the top right corner (or the left bottom), therefore, the benefit of selling more is large enough (or is not large enough) to exceed the limited loss (or the large loss) of the adverse equilibrium.

From Figure 6, we can conclude that: given $w_1 \geq w_2$, the supplier's optimal inventory level increases with w_2. This indicates that buyer 2 can induce the supplier to raise inventory level by proposing a higher w_2 and then decrease the probability of buyer 1 being the monopoly. However, whether increasing w_2 could benefit buyer 2 or not is uncertain so far, and we will discuss this problem in the next section.

What is interesting and unexpected is that: given a moderate w_2, the supplier's optimal strategy is q_I^3 if w_1 is small and q_I^2 if w_1 is large. The behind insight is that: with the increase of w_1, the marginal profit increases while the quantitative superiority of q_I^3 decreases, which leads to a synthetic effect that the benefit of choosing q_I^3 decreases with w_1. Similarly, if w_2 is large, there is a chance that the supplier's optimal strategy would be q_I^2 instead of q_I^4 when w_1 is also large.

Intuitively, as we normalize the supplier's production cost to zero and she does not need to pay for the unsold products, she would like to produce as many products as possible, that is setting q_s^* as q_I^4, to meet the demands in all cases and deal with risks, which is pretty similar to the newsvendor problem. Notwithstanding, the fact is that q_I^4 is optimal if and only if both w_1 and w_2 are large because of the adverse equilibrium effect resulted from the lexicographic mechanism.

4.2.3 The optimal inventory strategy of the supplier when $\alpha p < w_1 \leq p$

If $\alpha p < w_1 \leq p$, the supplier's expected profit function is continuous and piecewise with linear functions $N_{ij}(q_s)$, where $i = 1, \ldots, 4 \; and \; j = 1, \ldots, 8$, at each range of q_s. Here we analyze this optimization problem in four cases.

Case 1: if $0 \leq \delta \leq \frac{1-\alpha}{3+\alpha}$.

We list the slopes of the supplier's expected profit function at each range of q_s as follows: $k_{1j} = \frac{1}{4}\left((j-1)\frac{w_2 - \alpha w_1}{1-\alpha} + (4-j+1)w_1\right)$, $j = 1, \ldots, 5$; $k_{16} = \frac{3(w_2 - \alpha w_1)}{4(1-\alpha)}$; $k_{17} = \frac{w_2 - \alpha w_1}{4(1-\alpha)}$; $k_{18} = 0$.

When $w_2 > \alpha w_1$, the expected profit function is strictly increasing with q_s if $0 \leq q_s \leq q_{II}^7$ and is a constant if $q_s > q_{II}^7$. So, the supplier produces at least q_{II}^7 and we take q_{II}^7 as the optimal point. When $w_2 \leq \alpha w_1$, the profit function is non-increasing with q_s if $q_s > q_{II}^4$ and increasing with q_s if $0 \leq q_s < q_{II}^1$, therefore, the optimal point is less than or equal to q_{II}^4 and is nonzero. Furthermore, the slopes $k_{1j}, j = 1, \dots, 4$ decrease successively, which leads to the optimal point being q_{II}^i, where: $i = \min\{j: k_{1j} \leq 0\} - 1$.

Case 2: if $\frac{1-\alpha}{3+\alpha} < \delta \leq \frac{1-\alpha}{3-\alpha}$.

In this case, the slopes of the supplier's expected profit function are the same with case 1 except for k_{25}, this is to say: $k_{1i} = k_{2i}$ for $i = 1, \dots, 4, 6, \dots, 8$, and $k_{25} = \frac{2w_2 - (3\alpha - 1)w_1}{4(1-\alpha)}$.

Similar to case 1, when $w_2 > \alpha w_1$, the profit function is non-decreasing with q_s and is constant if $q_s > q_{II}^7$, and we thus take q_{II}^7 as the optimal point. When $w_2 \leq \alpha w_1$, we have: $k_{21} \geq k_{22} \geq k_{23} \geq k_{25} \geq k_{24}$ and $k_{2i} \leq 0 \; for \; i = 6,7,8$, thereby, the supplier's optimal strategy can be studied in the following cases:

(1) when $k_{25} \leq 0$, the expected profit function is non-increasing with q_s if q_s is larger than q_{II}^3, and consequently, the optimal q_s is less than or equal to q_{II}^3. Since the slopes $k_{1j}, j = 1, \dots, 4$ decrease successively, we can have the optimal point being q_{II}^i with $i = \min\{j: k_{1j} \leq 0 \; and \; j \leq 4\} - 1$;

(2) when $k_{24} \geq 0$, the optimal point is q_{II}^4;

(3) when $k_{25} \geq 0 \geq k_{24}$, there are two local maximizers, which are q_{II}^3 and q_{II}^4; we have $E\left(\Pi_s(q_{II}^4|w_1, w_2)\right) - E\left(\Pi_s(q_{II}^3|w_1, w_2)\right) = \frac{1-\alpha-3\delta+5\alpha\delta}{4(1-\alpha)}\left(w_2 - w_1 \frac{\alpha(1-\alpha-5\delta+7\alpha\delta)}{1-\alpha-3\delta+5\alpha\delta}\right)$, moreover, $1 - \alpha - 3\delta + 5\alpha\delta \geq 1 - \alpha - \delta(3-\alpha) \geq 0$ in this case, so, $E\left(\Pi_s(q_{II}^4|w_1, w_2)\right) \geq E\left(\Pi_s(q_{II}^3|w_1, w_2)\right)$ iff $w_2 \geq \frac{\alpha(1-\alpha-5\delta+7\alpha\delta)}{1-\alpha-3\delta+5\alpha\delta} w_1$.

Case 3: if $\frac{1-\alpha}{3-\alpha} < \delta \leq \frac{1-\alpha}{1+\alpha}$.

In this case, we have: $k_{2i} = k_{3i}$ for $i = 1, \ldots, 3, 5, \ldots, 8$, and $k_{34} = \frac{w_2 - (3\alpha-2)w_1}{4(1-\alpha)}$. Furthermore, if $w_2 > \alpha w_1$, the optimal point is q_{II}^7, and if $w_2 \leq \alpha w_1$, $k_{31} \geq k_{32} \geq k_{34} \geq k_{33} \geq k_{35}$ and $k_{2i} \leq 0\ for\ i = 6,7,8$. Then, the supplier's optimal strategy can be studied in the following cases:

(1) when $k_{34} \leq 0$, the optimal point is q_{II}^i with $i = \min\{j: k_{1j} \leq 0\ and\ j \leq 3\} - 1$;

(2) when $k_{33} \geq 0$, the optimal point is q_{II}^3 if $k_{35} < 0$ and q_{II}^4 if $k_{35} \geq 0$;

(3) if $k_{34} \geq 0 \geq k_{33}$, there are two local maximizers, which are q_{II}^2 and q_{II}^3; we have: $E\left(\Pi_s(q_{II}^3|w_1,w_2)\right) - E\left(\Pi_s(q_{II}^2|w_1,w_2)\right) = \frac{1-\alpha+\delta-3\alpha\delta}{4(1-\alpha)}(w_2 - w_1\frac{\alpha(1-\alpha)-(4-9\alpha+7\alpha^2)\delta}{1-\alpha+\delta-3\alpha\delta})$, moreover, $1-\alpha+\delta-3\alpha\delta \geq 1-\alpha-\delta(1+\alpha) \geq 0$, and thus, $E\left(\Pi_s(q_{II}^3|w_1,w_2)\right) \geq E\left(\Pi_s(q_{II}^2|w_1,w_2)\right)$ iff $w_2 \geq w_1\frac{\alpha(1-\alpha)-(4-9\alpha+7\alpha^2)\delta}{1-\alpha+\delta-3\alpha\delta}$.

Case 4: if $\frac{1-\alpha}{1+\alpha} < \delta \leq 1$.

In this case, $k_{41} = w_1$, $k_{42} = \frac{w_2-(4\alpha-3)w_1}{4(1-\alpha)}$, $k_{43} = \frac{3}{4}w_1$, $k_{44} = \frac{w_2-(3\alpha-2)w_1}{4(1-\alpha)}$, $k_{45} = \frac{2w_2-(3\alpha-1)w_1}{4(1-\alpha)}$, $k_{46} = \frac{1}{4}w_1$, $k_{47} = \frac{w_2-\alpha w_1}{4(1-\alpha)}$ and $k_{48} = 0$. Similar to the former, if $w_2 > \alpha w_1$, the optimal point is q_{II}^7. If $w_2 \leq \alpha w_1$, there are multiple local maximizers and all $q_{II}^i, i = 1, \ldots, 4$ have chances to be the optimal point, and we can thus derive the supplier's optimal inventory strategy by comparing the four values $E\left(\Pi_s\left(q_{II}^i|w_1,w_2\right)\right), i = 1, \ldots, 4$.

Theorem 3: If $\alpha p < w_1 \leq p$, the supplier's optimal inventory level q_s^* is characterized as follows:

(1) $0 \leq \delta \leq \frac{1-\alpha}{3+\alpha}$:

if $0 \leq w_2 < (4\alpha-3)^+ w_1$: $q_s^* = q_{II}^1$;

if $(4\alpha-3)^+ w_1 \leq w_2 < (2\alpha-1)^+ w_1$: $q_s^* = q_{II}^2$;

if $(2\alpha-1)^+ w_1 \leq w_2 < (\frac{4\alpha-1}{3})^+ w_1$: $q_s^* = q_{II}^3$;

if $(\frac{4\alpha-1}{3})^+ w_1 \leq w_2 \leq \alpha w_1$: $q_s^* = q_{II}^4$;

if $\alpha w_1 < w_2 \le w_1$: $q_s^* = q_{II}^7$.

(2) $\frac{1-\alpha}{3+\alpha} < \delta \le \frac{1-\alpha}{3-\alpha}$:

if $0 \le w_2 < (4\alpha - 3)^+ w_1$: $q_s^* = q_{II}^1$;

if $(4\alpha - 3)^+ w_1 \le w_2 < (2\alpha - 1)^+ w_1$: $q_s^* = q_{II}^2$;

if $(2\alpha - 1)^+ w_1 \le w_2 < \left(\frac{\alpha(1-\alpha-5\delta+7\alpha\delta)}{1-\alpha-3\delta+5\alpha\delta}\right)^+ w_1$: $q_s^* = q_{II}^3$;

if $\left(\frac{\alpha(1-\alpha-5\delta+7\alpha\delta)}{1-\alpha-3\delta+5\alpha\delta}\right)^+ w_1 \le w_2 \le \alpha w_1$: $q_s^* = q_{II}^4$;

if $\alpha w_1 < w_2 \le w_1$: $q_s^* = q_{II}^7$.

(3) $\frac{1-\alpha}{3-\alpha} < \delta \le \frac{1-\alpha}{1+\alpha}$:

if $0 \le w_2 < (4\alpha - 3)^+ w_1$: $q_s^* = q_{II}^1$;

if $(4\alpha - 3)^+ w_1 \le w_2 < (\frac{\alpha(1-\alpha)-(4-9\alpha+7\alpha^2)\delta}{1-\alpha+\delta-3\alpha\delta})^+ w_1$: $q_s^* = q_{II}^2$;

if $(\frac{\alpha(1-\alpha)-(4-9\alpha+7\alpha^2)\delta}{1-\alpha+\delta-3\alpha\delta})^+ w_1 \le w_2 < \left(\frac{3\alpha-1}{2}\right)^+ w_1$: $q_s^* = q_{II}^3$;

if $\left(\frac{3\alpha-1}{2}\right)^+ w_1 \le w_2 \le \alpha w_1$: $q_s^* = q_{II}^4$;

if $\alpha w_1 < w_2 \le w_1$: $q_s^* = q_{II}^7$.

(4) $\frac{1-\alpha}{1+\alpha} < \delta \le \frac{2(1-\alpha)}{2-\alpha}$:

if $0 \le w_2 < \left(\frac{\alpha(1-7\delta)}{1-\delta}\right)^+ w_1$: $q_s^* = q_{II}^1$;

if $\left(\frac{\alpha(1-7\delta)}{1-\delta}\right)^+ w_1 \le w_2 < (3\alpha - 2)^+ w_1$: $q_s^* = q_{II}^2$;

if $(3\alpha - 2)^+ w_1 \le w_2 < \left(\frac{\alpha(1-2\delta)}{1-\delta}\right)^+ w_1$: $q_s^* = q_{II}^3$;

if $\left(\frac{\alpha(1-2\delta)}{1-\delta}\right)^+ w_1 \le w_2 \le \alpha w_1$: $q_s^* = q_{II}^4$;

if $\alpha w_1 < w_2 \le w_1$: $q_s^* = q_{II}^7$.

(5) $\frac{2(1-\alpha)}{2-\alpha} < \delta \le 1$:

if $0 \le w_2 < \left(\frac{\alpha(1-7\delta)}{1-\delta}\right)^+ w_1$: $q_s^* = q_{II}^1$;

if $\left(\frac{\alpha(1-7\delta)}{1-\delta}\right)^+ w_1 \le w_2 < (\alpha - 2\delta + \alpha\delta)^+ w_1$: $q_s^* = q_{II}^2$;

if $(\alpha - 2\delta + \alpha\delta)^+ w_1 \le w_2 < \alpha w_1$: $q_s^* = q_{II}^4$;

if $\alpha w_1 < w_2 \le w_1$: $q_s^* = q_{II}^7$.

As shown in Figure 7, we draw the areas of five cases above on the $\alpha-\delta$ plane and indicate their subcategories which are classified depending on whether each threshold of w_2 in each case being positive or not. Here, the curves that separate area I and II, area II and III, area III and IV, and, area IV and V are $\delta=\frac{1-\alpha}{3+\alpha}$, $\delta=\frac{1-\alpha}{3-\alpha}$, $\delta=\frac{1-\alpha}{1+\alpha}$ and $\delta=\frac{2(1-\alpha)}{2-\alpha}$, respectively. The curves that separate area II-3 and II-4, area III-2 and III-3, and, area V-3 and V-4 are $1-\alpha-5\delta+7\alpha\delta=0$, $(1-\alpha)-(4-9\alpha+7\alpha^2)\delta=0$ and $\delta=\frac{\alpha}{2-\alpha}$, respectively.

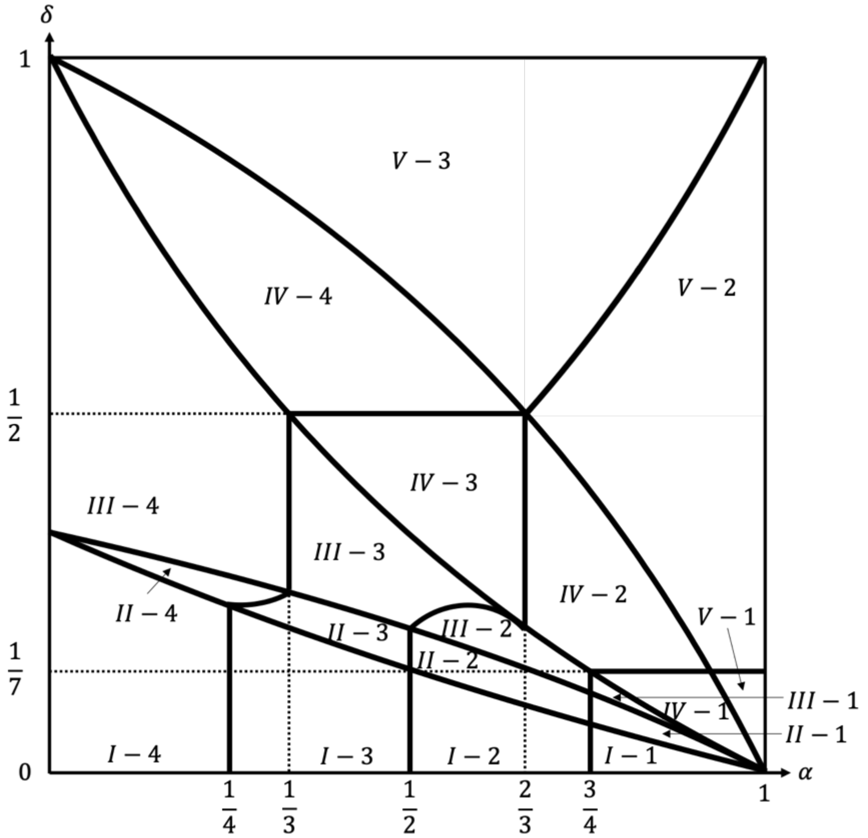


Figure 7 Indication of five cases and their subcategories on the $\alpha-\delta$ plane

Next, for case $K-1, K=I,\ldots,V$, we indicate the supplier's optimal inventory level in different regions of the w_2-w_1 plane as shown in Figure 8. In the region of $w_2\le w_1$, the expressions of solid segments between two areas in each subfigure are the thresholds of its corresponding case in *Theorem 3*, The dashed segments represent the inverse functions of the solid ones.

Given (d_1,d_2), provided $w_2>\alpha w_1$, the supplier's profit is non-decreasing with q_s, and therefore, her optimal strategy is to prepare as many products as possible;

provided $w_2 \le \alpha w_1$, the supplier's profit decreases with q_s and she experiences a loss if q_s is larger than $d_1 + \alpha d_2$, and in this case, the supplier also faces the tradeoff between *selling more products to the monopoly* and *decreasing the negative influence of adverse equilibrium effect* just as we have explained in section 5.2, for example, compared with $q_s = q_{II}^1$, even though the supplier can sell more products with $q_s = q_{II}^2$, she faces a lower average wholesale price in the case where $(d_1, d_2) = (1-\delta, 1-\delta)$.

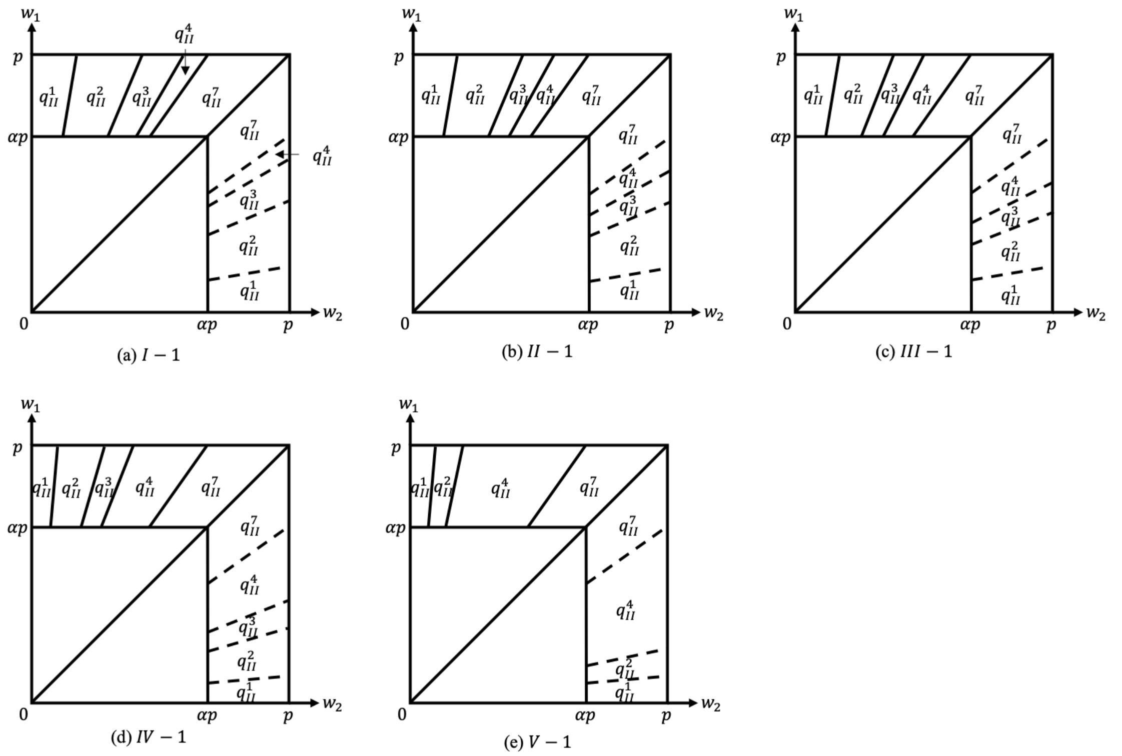


Figure 8 The supplier's optimal inventory strategy if $\alpha p < w_1 \le p$

In Figure 8, the closer from one area to the line $w_1 = w_2$, the higher q_s^* is in this area. This is to say, with the decrease of the absolute value of the difference between w_1 and w_2, the supplier's optimal inventory level increases as the *price superiority* of a lower inventory level decreases, and thus, the loss arising from adverse equilibrium effect decreases.

As α decreases and δ increases, the fraction of region that indicates $q_s^* \ge q_{II}^i\ for\ i = 1, \dots, 4, 7$ is non-decreasing and the supplier has the incentive to prepare more inventory by the following observations: a) in Figure 7, as the region changes from $K-1$ to $K-4$ (or $K-3$ if $K=V$) for $K = I, \dots, V$, the area of $q_s^* = q_{II}^1$, $q_s^* = q_{II}^2$ and $q_s^* = q_{II}^3$ in Figure 8 disappears in sequence, and $q_s^* = q_{II}^4$ also disappear once α

becomes zero; b) in case $K-1$ for $K=I,\dots,V$, as α decreases, the thresholds in *Theorem 3* decreases, so the solid segments (or the dashed segments) are closer to the vertical axis (or horizontal axis), and this fact also happens from (a) to (e) in Figure 5 (from the case I to V, δ increases). It is because 1) as δ increases, the absolute value of the difference between any two possible optimal strategies ($|q_{II}^{i}-q_{II}^{j}|, i\neq j$) is non-decreasing, so, the *quantitative superiority* of a higher inventory level (q_{II}^{i} with higher i) escalates, and thereby the benefit of selling more products by determining a higher inventory level (q_{II}^{i} with higher i) increases; 2) as α decreases, even though the effect on quantitative superiority of a higher inventory level is uncertain, the loss arising from adverse equilibrium effect decreases and dominates, and accordingly, it becomes easier for the benefit of selling more to exceed the loss of lower average wholesale price.

4.3 Wholesale price competition game before demand is realized

In the first stage, buyers engage in wholesale price competition and propose wholesale prices simultaneously to maximize their expected profits before the market condition is realized considering the supplier's optimal inventory level strategy in the second stage and their order quantity equilibrium (after the demand is realized). In this chapter, we assume $\alpha=1$ for analysis simplicity, and the general pattern will not change if we let $\alpha\in(0,1)$.

4.3.1 Profit functions

To formulate buyers' expected profit functions in this section, we first establish a few results by simple algebra in *Lemma 4*.

Lemma 4: In the case that $\alpha=1$, we have:

(1) If $w_1\leq\frac{1-\delta}{1+3\delta}$, $1+\delta+\alpha(1+\delta)\leq 1-\delta+\frac{\alpha p}{w_1}(1-\delta)$;

(2) if $\frac{1-\delta}{1+3\delta}<w_1\leq\frac{1-\delta}{1+\delta}$: $1+\delta+\alpha(1-\delta)\leq 1-\delta+\frac{\alpha p}{w_1}(1-\delta)\leq 1+\delta+\alpha(1+\delta)\leq q_I^2=1+\delta+\frac{\alpha p}{w_1}(1-\delta)$;

(3) if $\frac{1-\delta}{1+\delta} < w_1 \le \frac{1+\delta}{1+3\delta}$: $1-\delta+\alpha(1-\delta) \le 1-\delta+\frac{\alpha p}{w_1}(1-\delta) \le 1-\delta+\alpha(1+\delta) \le 1+\delta+\alpha(1-\delta) \le 1+\delta+\frac{\alpha p}{w_1}(1-\delta) \le 1+\delta+\alpha(1+\delta) \le 1-\delta+\frac{\alpha p}{w_1}(1+\delta)$;

(4) if $\frac{1+\delta}{1+3\delta} < w_1 \le p$: $1-\delta+\alpha(1-\delta) \le 1-\delta+\frac{\alpha p}{w_1}(1-\delta) \le 1-\delta+\alpha(1+\delta) \le 1+\delta+\alpha(1-\delta) \le 1+\delta+\frac{\alpha p}{w_1}(1-\delta) \le 1-\delta+\frac{\alpha p}{w_1}(1+\delta) \le 1+\delta+\alpha(1+\delta) \le 1+\delta+\frac{\alpha p}{w_1}(1+\delta)$.

Let $E\ (\Pi_{\mathrm{b}1}(w_1|w_2))$ be buyer 1's expected profit function given w_2, then we have:

(1) when $w_1 < w_2$:

$E\ (\Pi_{\mathrm{b}1}(w_1|w_2)) = 0$, if the value of w_1 results in $q_s^* = q_{III}^1$; $E\ (\Pi_{\mathrm{b}1}(w_1|w_2)) = \frac{(1-w_1)(1-\delta)}{4}$ if $q_s^* = q_{III}^2$; $E\ (\Pi_{\mathrm{b}1}(w_1|w_2)) = \frac{(1-w_1)(1-\delta)}{2}$ if $q_s^* = q_{III}^3$; and, $E\ (\Pi_{\mathrm{b}1}(w_1|w_2)) = \frac{(1-w_1)(2(1-\delta)+(1+\delta))}{4}$ if $q_s^* = q_{III}^4$;

(2) when $w_1 > w_2$:

if $q_s^* = q_I^1$,

$$E(\Pi_{\mathrm{b}1}(w_1|w_2)) = \begin{cases} \frac{1-\delta}{2}+1+\frac{1+\delta}{2}-w_1(1-\delta+\frac{1}{w_1}(1-\delta)) & w_1 < \frac{1-\delta}{1+3\delta} \\ \frac{1-\delta}{2}+1+(\frac{1}{4}-w_1)(1-\delta+\frac{1}{w_1}(1-\delta)) & \frac{1-\delta}{1+3\delta} \le w_1 < \frac{1-\delta}{1+\delta} \\ \frac{1-\delta}{2}+(\frac{3}{4}-w_1)(1-\delta+\frac{1}{w_1}(1-\delta)) & \frac{1-\delta}{1+\delta} \le w_1 \le 1 \end{cases};$$

if $q_s^* = q_I^2$,

$$E\big(\Pi_{\mathrm{b}1}(w_1|w_2)\big) = \begin{cases} \frac{(1-w_1)(1-\delta)}{4}+1+\frac{1+\delta}{2}-\frac{3}{4}w_1(1+\delta+\frac{1}{w_1}(1-\delta)) & w_1 < \frac{1-\delta}{1+\delta} \\ \frac{(1-w_1)(1-\delta)}{4}+1+(\frac{1}{4}-\frac{3}{4}w_1)(1+\delta+\frac{1}{w_1}(1-\delta)) & \frac{1-\delta}{1+\delta} \le w_1 \le 1 \end{cases};$$

if $q_s^* = q_I^3$,

$$E\big(\Pi_{\mathrm{b}1}(w_1|w_2)\big) = \begin{cases} \frac{1-w_1}{2}+\frac{1}{2}+\frac{1+\delta}{2}-\frac{1}{2}w_1(1-\delta+\frac{1}{w_1}(1+\delta)) & w_1 < \frac{1+\delta}{1+3\delta} \\ \frac{1-w_1}{2}+\frac{1}{2}+(\frac{1}{4}-\frac{1}{2}w_1)(1-\delta+\frac{1}{w_1}(1+\delta)) & \frac{1+\delta}{1+3\delta} \le w_1 \le 1 \end{cases};$$

if $q_s^* = q_I^4$,

$E(\Pi_{b1}(w_1|w_2)) = \frac{(1-w_1)(1-\delta)}{2} + \frac{(1-w_1)(1+\delta)}{4} + \frac{1+\delta}{2} - \frac{1}{4}w_1(1+\delta+\frac{1}{w_1}(1+\delta))$;

(3) when $w_1 = w_2$ and $q_s^* = q_I^i \; for \; i = 1, \dots, 4$ ($q_I^i = q_{III}^i$ in this case):

$E(\Pi_{b1}(w_1|w_2)) = \frac{1}{2}(\overline{E}(\Pi_{b1}(w_1|w_2)) + \underline{E}(\Pi_{b1}(w_1|w_2)))$, where: $\overline{E}(\Pi_{b1}(w_1|w_2))$ is buyer 1's expected profit function provided that he is given priority, and it can be obtained from part (1) (when $w_1 < w_2$), otherwise, his expected profit would be $\underline{E}(\Pi_{b1}(w_1|w_2))$ and can be obtained from part (2) (when $w_1 > w_2$).

Buyer 2's profit functions and best-response wholesale price can be obtained by symmetry.

By analyzing buyer 1's profit functions, we can conclude that: given w_2 and $q_s^* = q_J^i$, $J = I, \text{III}$, and $i = 1, \dots, 4$, buyer 1's expected profit decreases with w_1, except that it is not continuous and drops at $w_1 = w_2$ due to $\overline{E}(\Pi_{b1}(w_1|w_2)) > \underline{E}(\Pi_{b1}(w_1|w_2))$. Therefore, given w_2, buyer 1's best-response wholesale price would be one of the lower bounds of regions in Figure 3, and then the problem of determining buyer 1's optimal wholesale price just becomes choosing a q_s^* that maximizes his expected profit. Furthermore, $w_1 = w_2$ cannot be buyer 1's best-response wholesale price since he can obtain more profit by slightly increasing w_1.

4.3.2 Best-response wholesale price

In this section, for ease of formulating expressions, we first introduce several thresholds values for w_2 and the demand risk level δ.

$w_{2t}^{I} = \frac{8-17\delta+3\delta^2+\delta\sqrt{25-54\delta+33\delta^2}}{4(2-\delta)}$;

$w_{2t}^{II} = \frac{8-9\delta+\delta^2-12\delta^3+\delta\sqrt{25-26\delta+97\delta^2-96\delta^3+144\delta^4}}{4(2+\delta+3\delta^2)}$;

$w_{2t}^{III} = \frac{4-5\delta-2\delta^2+\delta\sqrt{5+2\delta+2\delta^2}}{2(2+\delta)}$;

$w_{2t}^{IV} = \frac{8-17\delta+5\delta^2-4\delta^3+\delta\sqrt{(1-\delta)(25-41\delta+48\delta^2-16\delta^3)}}{4(2-\delta+\delta^2)}$;

$w_{2t}^{V} = \frac{-2+16\delta-30\delta^2+8\delta^3-\delta\sqrt{25-212\delta+550\delta^2-372\delta^3+73\delta^4}}{-2+11\delta-12\delta^2+3\delta^3}$;

$\delta_{t1} \equiv \{\delta \in [0,1]: w_{2t}^{I} = \frac{3+\delta}{4}\}$;

$\delta_{t2} \equiv \left\{\delta \in [0,1]: w_{2t}^{IV} = \frac{1+\delta}{1+3\delta}\right\};$

$\delta_{t3} \equiv \left\{\delta \in [0,1]: w_{2t}^{IV} = w_{2t}^{V} = \frac{3(1-\delta)}{3+\delta}\right\};$

$\delta_{t4} \equiv \left\{\delta \in [0,1]: w_{2t}^{I} = \frac{1+\delta}{1+3\delta}\right\};$

$\delta_{t5} \equiv \left\{\delta \in [0,1]: \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2} = \frac{-2\delta^2+7\delta-1}{\delta(3-\delta)} = \frac{3+\delta}{4}\right\}.$

Buyer 1's best-response wholesale price is illuminated in *Lemma 5*.

Lemma 5: Let $w_1(w_2)$ to denote buyer 1's best-response wholesale price, we can characterize it as follows:

(1) $0 \le \delta \le \frac{1}{5}$:

if $w_2 \in [0, \frac{3(1+\delta)+\sqrt{25+34\delta-23\delta^2}}{8(1+2\delta)} \wedge \frac{1-2\delta}{1-\delta}\Big]$, $w_1(w_2) = w_2 + \varepsilon$, where ε is an infinitely-small positive number which converges to zero;

if $w_2 \in \left(\frac{3(1+\delta)+\sqrt{25+34\delta-23\delta^2}}{8(1+2\delta)} \wedge \frac{1-2\delta}{1-\delta}, \frac{1-2\delta}{1-\delta}\right]$, $w_1(w_2) = -\frac{6\delta}{1-\delta} w_2 + p$;

if $w_2 \in \left(\frac{1-2\delta}{1-\delta}, \left(\frac{1-2\delta}{1-\delta} \vee w_{2t}^{II} \vee w_{2t}^{III}\right) \wedge w_{2t}^{I}\right]$, $w_1(w_2) = \frac{1}{\delta} w_2 - \frac{1-2\delta}{\delta} p$;

if $w_2 \in \left(\left(\frac{1-2\delta}{1-\delta} \vee w_{2t}^{II} \vee w_{2t}^{III}\right) \wedge w_{2t}^{I}, \left(\left(\frac{1-2\delta}{1-\delta} \vee w_{2t}^{II} \vee w_{2t}^{III}\right) \wedge w_{2t}^{I}\right) \vee \frac{3+\delta}{4}\right]$, $w_1(w_2) = w_2 + \varepsilon$;

if $w_2 \in \left(\left(\left(\frac{1-2\delta}{1-\delta} \vee w_{2t}^{II} \vee w_{2t}^{III}\right) \wedge w_{2t}^{I}\right) \vee \frac{3+\delta}{4}, 1\right]$, $w_1(w_2) = \left(-\frac{6\delta}{1-\delta} w_2 + p\right)^+$.

(2) $\frac{1}{5} < \delta \le \frac{9-\sqrt{57}}{6}$:

if $w_2 \in [0, 1-3\delta]$, $w_1(w_2) = w_2 + \varepsilon$;

if $w_2 \in \left(1-3\delta, \frac{3(1-\delta)}{3+\delta} \wedge w_{2t}^{V}\right]$, $w_1(w_2) = \frac{1-\delta}{4\delta} w_2 + \frac{5\delta-1}{4\delta} p$;

if $w_2 \in \left(\frac{3(1-\delta)}{3+\delta} \wedge w_{2t}^{V}, \frac{3(1-\delta)}{3+\delta}\right]$, $w_1(w_2) = w_2 + \varepsilon$;

if $w_2 \in \left(\frac{3(1-\delta)}{3+\delta}, \left(\frac{1+\delta}{1+3\delta} \wedge w_{2t}^{IV}\right) \vee \frac{3(1-\delta)}{3+\delta}\right]$, $w_1(w_2) = \frac{1}{\delta} w_2 - \frac{1-2\delta}{\delta} p$;

if $w_2 \in \left(\left(\frac{1+\delta}{1+3\delta} \wedge w_{2t}^{IV}\right) \vee \frac{3(1-\delta)}{3+\delta}, \frac{1+\delta}{1+3\delta}\right]$, $w_1(w_2) = w_2 + \varepsilon$;

if $w_2 \in \left(\frac{1+\delta}{1+3\delta}, w_{2t}^{I} \vee \frac{1+\delta}{1+3\delta}\right]$, $w_1(w_2) = \frac{1}{\delta} w_2 - \frac{1-2\delta}{\delta} p$;

if $w_2 \in \left(w_{2t}^{I} \vee \frac{1+\delta}{1+3\delta}, \frac{3+\delta}{4} \wedge \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}\right]$, $w_1(w_2) = w_2 + \varepsilon$;

if $w_2 \in \left(\frac{3+\delta}{4} \wedge \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}, \frac{3+\delta}{4} \vee \frac{-2\delta^2+7\delta-1}{\delta(3-\delta)}\right]$, $w_1(w_2) = \delta w_2 + (1-2\delta)p$;

if $w_2 \in \left(\frac{3+\delta}{4} \vee \frac{-2\delta^2+7\delta-1}{\delta(3-\delta)}, 1\right]$, $w_1(w_2) = 0$.

(3) $\frac{9-\sqrt{57}}{6} < \delta \leq 1$:

if $w_2 \in \left(0, \left(\frac{2(1+\delta)}{3+\delta} \wedge \frac{2(1+\delta)}{2+5\delta-\delta^2}\right) \vee \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}\right]$, $w_1(w_2) = w_2 + \varepsilon$;

if $w_2 \in \left(\left(\frac{2(1+\delta)}{3+\delta} \wedge \frac{2(1+\delta)}{2+5\delta-\delta^2}\right) \vee \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}, \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2} \vee \frac{2(1+\delta)}{3+\delta}\right]$, $w_1(w_2) = -\frac{2\delta}{1+\delta} w_2 + p$;

if $w_2 \in \left(\frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2} \vee \frac{2(1+\delta)}{3+\delta}, \frac{-2\delta^2+7\delta-1}{\delta(3-\delta)} \wedge 1\right]$, $w_1(w_2) = \delta w_2 + (1-2\delta)p$;

if $w_2 \in \left(\frac{-2\delta^2+7\delta-1}{\delta(3-\delta)} \wedge 1, 1\right]$, $w_1(w_2) = 0$.

We illustrate the more detailed results in *Table 1*. In the table, the underlined and bold outcomes denote $w_1(w_2)$ less than w_2.

Given w_2, buyer 1 has two selections: 1) obtaining priority to guarantee that he can have access to the supplier's inventory in all market conditions and be the monopoly sometimes, but paying a cost of high wholesale price; 2) proposing a low wholesale price, but may not have access to the supplier's inventory in some market conditions. From Table 1, we can conclude that ***$w_1(w_2)$ is larger than w_2 and buyer 1 prefers to raise the wholesale price to obtain priority until w_2 is larger than a critical point $\widetilde{w_{2t}}$, and then, he would choose to grab the benefit of the low wholesale price and quit the market in some realized situations***, because the benefit of obtaining control power cannot compensate the high wholesale price.

Here, we discuss *the influence of adverse equilibrium effect on buyers*. Assume that buyer 1 is given priority, by comparing q_I^i and q_I^j for $i, j = 1, \dots, 4, and\ i < j$, we conclude that the probability that he can monopolize the end market is less with $q_S^* = q_I^j$ and this result in a loss for buyer 1. However, comparing q_{III}^i and q_{III}^j for $i, j = 1, \dots, 4, and\ i < j$, the probability that he can get access to the supplier's inventory is larger with $q_S^* = q_{III}^j$ if buyer 1 is not given priority and this contributes to buyer 1's expected profit.

Given w_1, $E(\Pi_{\mathrm{b}1}(w_1|w_2))$ with $q_s^* = q_I^i$ is larger than $E(\Pi_{\mathrm{b}1}(w_1|w_2))$ with $q_s^* = q_I^j$ for $i, j = 1, \ldots, 4\ and\ i < j$, since the negative influence of adverse equilibrium effect on buyer 1 dominates and he does not have the incentive to choose a higher q_s^* for making more profit in the situations where he could monopolize. Furthermore, $E(\Pi_{\mathrm{b}1}(w_1|w_2))$ with $q_s^* = q_{III}^i$ is less than $E(\Pi_{\mathrm{b}1}(w_1|w_2))$ with $q_s^* = q_I^j$ for $i, j = 1, \ldots, 4\ and\ i < j$. So, given w_1, buyer 1 prefers to lower inventory level if he is given priority, and sometimes (when w_2 is moderate), he has to raise the wholesale price to prevent the supplier from preparing too much inventory; while buyer 1 would prefer higher inventory level if he is not given priority.

Suppose that $w_2 \leq \widetilde{w_{2t}}$, as δ increases, the buyer who is given priority would be more likely to choose low wholesale cost rather than low inventory level, given the observations that: with the increase of δ, the range of w_2 in which $q_s^* = q_I^3$ and $q_s^* = q_I^4$ are chosen becomes wider, and the range of w_2 in which $q_s^* = q_I^1$ and $q_s^* = q_I^2$ shrinks. This is because: on the one hand, the lower bound of region $q_s^* = q_I^2$ goes up as δ increases, and the wholesale price that motivates the supplier to reduce inventory level is too high, on the other hand, the quantitative advantage of higher inventory level increases with the increase of δ.

Suppose $w_2 > \widetilde{w_{2t}}$, and if δ is small, then $q_s^* = q_{III}^2$ is chosen by buyer 1, and $w_1(w_2)$ is the lower bound of this region, which indicates that buyer 1 determines the lowest price that enables him to have positive expected profit. If δ is moderate, $q_s^* = q_{III}^4$ can also be buyer 1's best choice when w_2 is not too large. If δ is large enough, $q_s^* = q_{III}^4$ is buyer 1's unique best choice and $w_1(w_2)$ is the lower bound of this region. Therefore, the buyer who is not given priority would be more likely to choose higher inventory level rather than low wholesale price with the increase of δ, because the absolute value of the difference between the buyer 1's expected profit if $q_s^* = q_{III}^4$ and the buyer 1's expected profit $q_s^* = q_{III}^2$ (or $q_s^* = q_{III}^3$) is non-decreasing, and at the same time, the lower bound of the region where $q_s^* = q_{III}^4$ descends such that the cost of the high wholesale price is less than the positive influence of adverse equilibrium effect on buyer 1 in region $q_s^* = q_{III}^4$.

4.3.3 Wholesale price equilibrium

Based on buyers' best-response wholesale prices, we can derive the Nash equilibrium of the wholesale price competition, which is illustrated in the Theorem 4.

Theorem 4: Let (w_1^*, w_2^*) be the Nash equilibrium of buyers' wholesale price competition, then we characterize it as follows.

(1) if $\delta \in [0, \delta_{t1}]$, then there exists two Nash equilibria: $w_1^* = \frac{3+3\delta+\sqrt{25+34\delta-23\delta^2}}{8(1+2\delta)} \wedge (w_{2t}^{II} \vee w_{2t}^{III})$, $w_2^* = \left(-\frac{6\delta}{1-\delta} w_1^* + 1\right)^+$; and $w_2^* = \frac{3+3\delta+\sqrt{25+34\delta-23\delta^2}}{8(1+2\delta)} \wedge (w_{2t}^{II} \vee w_{2t}^{III})$, $w_1^* = \left(-\frac{6\delta}{1-\delta} w_2^* + 1\right)^+$;

(2) if $\delta \in (\delta_{t1}, \delta_{t5}]$, then there exists two Nash equilibria: $(w_1^*, w_2^*) = \left(\frac{3+\delta}{4}, 0\right)$; and $\left(0, \frac{3+\delta}{4}\right)$;

(3) if $\delta \in (\delta_{t5}, 2-\sqrt{3}]$, then there exists two Nash equilibria: $(w_1^*, w_2^*) = \left(\frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}, \delta \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2} + (1-2\delta)p\right)$; and $\left(\delta \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2} + (1-2\delta)p, \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}\right)$;

(4) if $\delta \in (2-\sqrt{3}, 1]$, then there exists two Nash equilibria: $(w_1^*, w_2^*) = \left(\frac{2(1+\delta)}{2+5\delta-\delta^2}, -\frac{4\delta(1+\delta)}{(1+\delta)(2+5\delta-\delta^2)} + p\right)$; and $\left(-\frac{4\delta(1+\delta)}{(1+\delta)(2+5\delta-\delta^2)} + p, \frac{2(1+\delta)}{2+5\delta-\delta^2}\right)$.

What is interesting is that even though buyers are symmetric, their Nash equilibria are asymmetric that one buyer obtains control power and grabs the benefit of getting access to the supplier's inventory in all conditions and being the monopoly sometimes, however, the other buyer grabs the benefit of low wholesale price but may not be able to obtain the supplier's products sometimes.

Table 1: Buyer 1's best-response wholesale price

1. $\delta \in [0, \frac{8-\sqrt{37}}{27}]$:

$w_2 \in$	$[0, (3(1+\delta)+\sqrt{25+34\delta-23\delta^2})/8(1+2\delta)]$	$((3(1+\delta)+\sqrt{25+34\delta-23\delta^2})/8(1+2\delta), 1]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\underline{\boldsymbol{-\frac{6\delta}{1-\delta}w_2 + p}}$

2. $\delta \in (\frac{8-\sqrt{37}}{27}, \delta_{t1}]$:

$w_2 \in$	$[0, \frac{1-2\delta}{1-\delta}]$	$\left(\frac{1-2\delta}{1-\delta}, w_{2t}^{II} \vee w_{2t}^{III}\right]$	$(w_{2t}^{II} \vee w_{2t}^{III}, 1]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\frac{1}{\delta}w_2 - \frac{1-2\delta}{\delta}p$	$\underline{\boldsymbol{\left(-\frac{6\delta}{1-\delta}w_2 + p\right)^+}}$

3. $\delta \in (\delta_{t1}, \frac{1}{5}]$:

$w_2 \in$	$[0, \frac{1-2\delta}{1-\delta}]$	$\left(\frac{1-2\delta}{1-\delta}, w_{2t}^{I}\right]$	$(w_{2t}^{I}, \frac{3+\delta}{4}]$	$\left(\frac{3+\delta}{4}, 1\right]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\frac{1}{\delta}w_2 - \frac{1-2\delta}{\delta}p$	$w_2 + \varepsilon$	$\underline{\mathbf{0}}$

4. $\delta \in (\frac{1}{5}, \delta_{t2}]$:

$w_2 \in$	$[0, 1-3\delta]$	$(1-3\delta, \frac{3(1-\delta)}{3+\delta}]$	$\left(\frac{3(1-\delta)}{3+\delta}, w_{2t}^{I}\right]$	$(w_{2t}^{I}, \frac{3+\delta}{4}]$	$\left(\frac{3+\delta}{4}, 1\right]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\frac{1-\delta}{4\delta}w_2 + \frac{5\delta-1}{4\delta}p$	$\frac{1}{\delta}w_2 - \frac{1-2\delta}{\delta}p$	$w_2 + \varepsilon$	$\underline{\mathbf{0}}$

5. $\delta \in (\delta_{t2}, \delta_{t3}]$:

$w_2 \in$	$[0, 1-3\delta]$	$(1-3\delta, \frac{3(1-\delta)}{3+\delta}]$	$\left(\frac{3(1-\delta)}{3+\delta}, w_{2t}^{IV}\right]$	$(w_{2t}^{IV}, \frac{1+\delta}{1+3\delta}]$	$\left(\frac{1+\delta}{1+3\delta}, w_{2t}^{I}\right]$	$(w_{2t}^{I}, \frac{3+\delta}{4}]$	$\left(\frac{3+\delta}{4}, 1\right]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\frac{1-\delta}{4\delta}w_2 + \frac{5\delta-1}{4\delta}p$	$\frac{1}{\delta}w_2 - \frac{1-2\delta}{\delta}p$	$w_2 + \varepsilon$	$\frac{1}{\delta}w_2 - \frac{1-2\delta}{\delta}p$	$w_2 + \varepsilon$	$\underline{\mathbf{0}}$

6. $\delta \in (\delta_{t3}, \delta_{t4}]$:

$w_2 \in$	$[0, 1-3\delta]$	$(1-3\delta, w_{2t}^V]$	$(w_{2t}^V, \frac{1+\delta}{1+3\delta}]$	$(\frac{1+\delta}{1+3\delta}, w_{2t}^I]$	$(w_{2t}^I, \frac{3+\delta}{4}]$	$(\frac{3+\delta}{4}, 1]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\frac{1-\delta}{4\delta} w_2 + \frac{5\delta-1}{4\delta} p$	$w_2 + \varepsilon$	$\frac{1}{\delta} w_2 - \frac{1-2\delta}{\delta} p$	$w_2 + \varepsilon$	$\underline{\mathbf{0}}$

7. $\delta \in (\delta_{t4}, \delta_{t5}]$:

$w_2 \in$	$[0, 1-3\delta]$	$(1-3\delta, w_{2t}^V]$	$(w_{2t}^V, \frac{3+\delta}{4}]$	$(\frac{3+\delta}{4}, 1]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\frac{1-\delta}{4\delta} w_2 + \frac{5\delta-1}{4\delta} p$	$w_2 + \varepsilon$	$\underline{\mathbf{0}}$

8. $\delta \in (\delta_{t5}, \frac{9-\sqrt{57}}{6}]$:

$w_2 \in$	$[0, 1-3\delta]$	$(1-3\delta, w_{2t}^V]$	$(w_{2t}^V, \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}]$	$(\frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}, \frac{-2\delta^2+7\delta-1}{\delta(3-\delta)}]$	$(\frac{-2\delta^2+7\delta-1}{\delta(3-\delta)}, 1]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\frac{1-\delta}{4\delta} w_2 + \frac{5\delta-1}{4\delta} p$	$w_2 + \varepsilon$	$\underline{\boldsymbol{\delta w_2 + (1-2\delta)p}}$	$\underline{\mathbf{0}}$

9. $\delta \in (\frac{9-\sqrt{57}}{6}, 2-\sqrt{3}]$:

$w_2 \in$	$[0, \frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}]$	$(\frac{2(2-\delta)(1-\delta)}{4-3\delta+\delta^2}, \frac{-2\delta^2+7\delta-1}{\delta(3-\delta)}]$	$(\frac{-2\delta^2+7\delta-1}{\delta(3-\delta)}, 1]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\underline{\boldsymbol{\delta w_2 + (1-2\delta)p}}$	$\underline{\mathbf{0}}$

10. $\delta \in (2-\sqrt{3}, 1]$:

$w_2 \in$	$[0, \frac{2(1+\delta)}{2+5\delta-\delta^2}]$	$(\frac{2(1+\delta)}{2+5\delta-\delta^2}, \frac{2(1+\delta)}{3+\delta}]$	$(\frac{2(1+\delta)}{3+\delta}, 1]$
$w_1(w_2) =$	$w_2 + \varepsilon$	$\underline{\boldsymbol{-\frac{2\delta}{1+\delta} w_2 + p}}$	$\underline{\boldsymbol{\delta w_2 + (1-2\delta)p}}$

Chapter 5
Concluding remarks

In this work, we study a setting with one supplier and two symmetric buyers. The two buyers procure key components that are required to assemble the final products from the supplier, and then sell substitutable products to the end market, assuming that assembling one final product requires one component. We consider that the effective total demand of the buyers consists of the local demand and the switch-over demand. The local demand of a buyer represents the number of consumers in the end market who visit him first and this number is random. The switch-over demand is formed by consumers search when one buyer has excess demand and the other buyer has excess inventory.

As the demand is uncertain and the demand risk exists, the supplier's inventory may be insufficient in some market condition and the buyers may not be able to obtain the quantities they have ordered. When the total order quantity of the buyers exceeds the supplier's total supply, the limited inventory is allocated based on the lexicographic allocation mechanism, and the priority under this policy is given to the buyer who is willing to pay more.

Three are three stages in our model: before the local demand of the two buyers is revealed, the supplier lets the buyers propose the wholesale prices ($w_i, i = 1,2$) that they want to pay for one product, simultaneously *(stage 1)*; and based on the wholesale prices, the supplier determines an inventory level *(stage 2)*; then, the local demand is revealed, and after observing the local demand, the two buyers simultaneously determine order quantities *(stage 3)*.

In such a setting, we aim to analyze the strategic behavior of every supply chain member and the effects of the demand risk level and quantity competition intensity on their optimal strategy. The problem is solved backward and we find that: i) the buyer who is willing to pay more has strong incentive to order all the supplier's inventory to monopolize the end market when the inventory level is low, and his incentive increases with the decrease of the wholesale price proposed by himself; ii) with using lexicographic allocation mechanism, the supplier has the incentive to limit his inventory level when the wholesale prices are not high enough, and thus, the lexicographic policy may become a factor that causes insufficient supply; iii) if the cost of gaining the priority is low, the buyers would propose a higher price to obtain the priority, otherwise, they would choose to grab the benefit of low purchasing cost and quit the market in some revealed situations; and, iv) though the buyers are symmetric, their Nash equilibria are asymmetric and the discriminative wholesale prices emerge.

A significant extension of our work is to study some other allocation policies, such as: proportional allocation, linear allocation, uniform allocation, normal lexicographic allocation, and so on. Based on the analysis of the properties, we can compare different allocation mechanisms from multiple aspects and study whether the results will change when the capacity of the supplier is determined by him endogenously. The other extension is from the perspective of pricing schemes. This work chooses a type of discriminating auction to decide wholesale prices, however, there also exist some other pricing schemes, such as: the simple single price strategy and priority pricing strategy. Therefore, a new question arises that whether the auction is the best way to decide the price of scarce supply, and if it is, what is the best auction procedure? We will leave the answer to this question in our future work.

Appendix:

Proof of Lemma 1:

Define sub problem 1 as:

$$\max_{s_1\in[0,q_s-s_2]} G_{11}(s_1,s_2) = \max_{s_1\in[0,q_s-s_2]} p((d_1+\alpha(d_2-s_2)^+)\wedge s_1) - w_1 s_1.$$

If $s_2 < d_2$, $G_{11}(s_1,s_2) = p((d_1+\alpha(d_2-s_2))\wedge s_1) - w_1 s_1$, which is as shown in Figure A1:

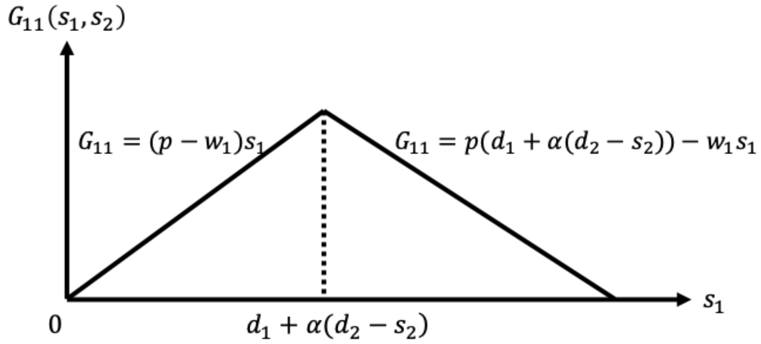


Figure A1: $G_{11}(s_1,s_2)$ if $s_2 < d_2$.

So, $s_{11}^* = q_s - s_2$ when $d_1+\alpha(d_2-s_2) \geq q_s - s_2$ (that is, $s_2 \geq \frac{q_s-(d_1+\alpha d_2)}{1-\alpha}$), and, $s_{11}^* = d_1+\alpha(d_2-s_2)$ when $d_1+\alpha(d_2-s_2) < q_s - s_2$ (that is, $s_2 < \frac{q_s-(d_1+\alpha d_2)}{1-\alpha}$).

Similarly, if $s_2 \geq d_2$, $G_{11}(s_1,s_2) = p(d_1\wedge s_1) - w_1 s_1$, we have: $s_{11}^* = q_s - s_2$ when $d_1 \geq q_s - s_2$ (that is, $s_2 \geq q_s - d_1$), and, $s_{11}^* = d_1$ when $d_1 < q_s - s_2$ (that is, $s_2 < q_s - d_1$).

If $q_s \leq d_1 + d_2$, $\frac{q_s-(d_1+\alpha d_2)}{1-\alpha}$ and $q_s - d_1$ is less than or equal to d_2, and, if $q_s > d_1 + d_2$, $\frac{q_s-(d_1+\alpha d_2)}{1-\alpha}$ and $q_s - d_1$ is larger than d_2. Based on this, sub problem 1 can be solved easily in the following two cases:

(1) if $q_s \leq d_1 + d_2$: we have:

$$s_{11}^* = \arg\max_{0\leq s_1\leq q_s-s_2} G_{11}(s_1,s_2) = \begin{cases} d_1+\alpha(d_2-s_2) & s_2 < (\frac{q_s-(d_1+\alpha d_2)}{1-\alpha})^+ \\ q_s - s_2 & s_2 \geq (\frac{q_s-(d_1+\alpha d_2)}{1-\alpha})^+ \end{cases}, \text{ and,}$$

$$G_{11}^* = \max_{0 \le s_1 \le q_s - s_2} G_{11}(s_1, s_2) = \begin{cases} (p - w_1)(d_1 + \alpha(d_2 - s_2)) & s_2 < (\frac{q_s - (d_1 + \alpha d_2)}{1-\alpha})^+ \\ (p - w_1)(q_s - s_2) & s_2 \ge (\frac{q_s - (d_1 + \alpha d_2)}{1-\alpha})^+ \end{cases};$$

(2) if $q_s > d_1 + d_2$: we have:

$$s_{11}^* = \arg\max_{0 \le s_1 \le q_s - s_2} G_{11}(s_1, s_2) = \begin{cases} d_1 + \alpha(d_2 - s_2) & s_2 < d_2 \\ d_1 & d_2 \le s_2 \le q_s - d_1 \\ q_s - s_2 & s_2 > q_s - d_1 \end{cases}, \text{ and,}$$

$$G_{11}^* = \max_{0 \le s_1 \le q_s - s_2} G_{11}(s_1, s_2) = \begin{cases} (p - w_1)\big(d_1 + \alpha(d_2 - s_2)\big) & s_2 < d_2 \\ (p - w_1)d_1 & d_2 \le s_2 < q_s - d_1 \\ (p - w_1)(q_s - s_2) & s_2 \ge q_s - d_1 \end{cases}.$$

Define sub problem 2 as:

$$\max_{s_1 \in (q_s - s_2, q_s]} G_{12}(s_1, s_2) = p((d_1 + \alpha(d_2 - (q_s - s_1))^+) \wedge s_1) - w_1 s_1.$$

If $d_2 \le q_s - s_1$ (that is: $s_1 \le q_s - d_2$), then: $G_{12}(s_1, s_2) = p(d_1 \wedge s_1) - w_1 s_1$. If $d_2 > q_s - s_1$ (that is: $s_1 > q_s - d_2$), then: $G_{12}(s_1, s_2) = p((d_1 + \alpha(d_2 - (q_s - s_1))) \wedge s_1) - w_1 s_1$, which equals to $(p - w_1)s_1$ if $d_1 + \alpha(d_2 - (q_s - s_1)) \ge s_1$ (that is: $s_1 \le \frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha}$), and equals to $p(d_1 + \alpha d_2 - \alpha q_s) + (\alpha p - w_1)s_1$ if $d_1 + \alpha\big(d_2 - (q_s - s_1)\big) < s_1$ (that is: $s_1 > \frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha}$).

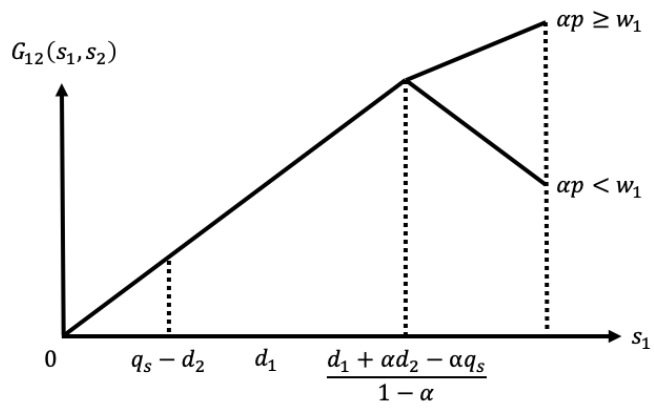
To solve this problem, we can first derive the following results by simple algebra:

(1) if $q_s \le d_1 + d_2$, then: $q_s - d_2 \le d_1 \le \frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha}$;

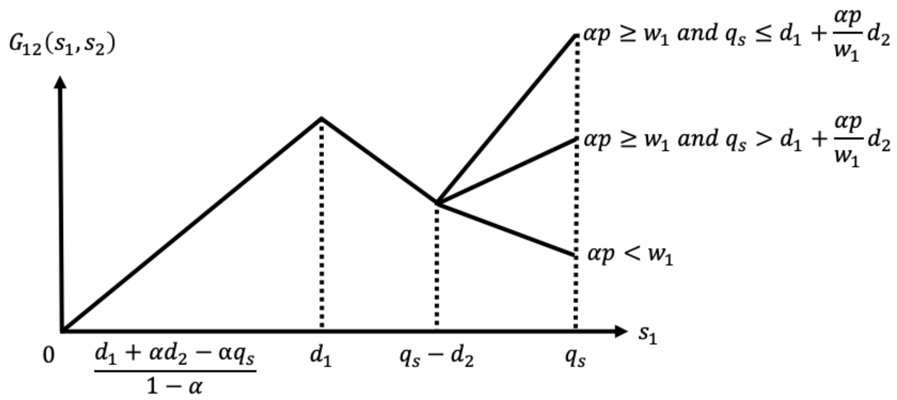
(2) if $q_s > d_1 + d_2$, then: $\frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha} < d_1 < q_s - d_2$;

(3) if $q_s \le d_1 + \alpha d_2$, then: $\frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha} \ge q_s$; and, if $q_s > d_1 + \alpha d_2$, then: $\frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha} < q_s$.

So, the images of $G_{12}(s_1, s_2)$ are as shown in Figure A2 (a) and Figure A2 (b) when $q_s \le d_1 + d_2$ and $q_s > d_1 + d_2$, respectively:



(a) $q_s \leq d_1 + d_2$



(b) $q_s > d_1 + d_2$

Figure A2: $G_{12}(s_1, s_2)$

Then we solve this problem in the following two cases, where, we define $s_{12}^* = \arg\max_{q_s - s_2 < s_1 \leq q_s} G_{12}(s_1, s_2)$, $G_{12}^* = \max_{q_s - s_2 < s_1 \leq q_s} G_{12}(s_1, s_2)$:

(1) $c \leq w_1 \leq \alpha p$:

(1-1) if $0 \leq q_s \leq d_1 + d_2$, we have: $s_{12}^* = q_s$, and, $G_{12}^* = p((d_1 + \alpha d_2) \wedge q_s) - w_1 q_s$;

(1-2) if $d_1 + d_2 < q_s \leq d_1 + \frac{\alpha p}{w_1} d_2$, we have: $s_{12}^* = q_s$, and, $G_{12}^* = p(d_1 + \alpha d_2) - w_1 q_s$;

(1-3) if $q_s > d_1 + \frac{\alpha p}{w_1} d_2$, we have: $s_{12}^* = q_s$ when $\widetilde{s_1} \leq q_s - s_2 \leq q_s$, $s_{12}^* = q_s - s_2$ when $d_1 \leq q_s - s_2 < \widetilde{s_1}$, and $s_{12}^* = d_1$ when $0 \leq q_s - s_2 < d_1$, where: $\widetilde{s_1} = q_s - \frac{\alpha p}{w_1} d_2$ satisfying $p(d_1 + \alpha d_2 - \alpha q_s) + (\alpha p - w_1) q_s = p d_1 - w_1 \widetilde{s_1}$; specifically:

$$s_{12}^* = \begin{cases} q_s & 0 \le s_2 \le \frac{\alpha p}{w_1} d_2 \\ q_s - s_2 & \frac{\alpha p}{w_1} d_2 < s_2 \le q_s - d_1 \\ d_1 & q_s - d_1 < s_2 \le q_s \end{cases}, \text{ and,}$$

$$G_{12}^* = \begin{cases} p(d_1 + \alpha d_2) - w_1 q_s & 0 \le s_2 \le \frac{\alpha p}{w_1} d_2 \\ p d_1 - w_1(q_s - s_2) & \frac{\alpha p}{w_1} d_2 < s_2 \le q_s - d_1 \\ (p - w_1) d_1 & q_s - d_1 < s_2 \le q_s \end{cases};$$

(2) $\alpha p < w_1 \le p$:

(2-1) if $0 \le q_s \le d_1 + \alpha d_2$, we have: $s_{12}^* = q_s$, and, $G_{12}^* = (p - w_1) q_s$;

(2-2) if $d_1 + \alpha d_2 < q_s \le d_1 + d_2$, we have: $s_{12}^* = q_s - s_2$ when $\frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha} < q_s - s_2 \le q_s$, and, $s_{12}^* = \frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha}$ when $0 \le q_s - s_2 \le \frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha}$; specifically:

$$s_{12}^* = \begin{cases} q_s - s_2 & 0 \le s_2 < \frac{q_s - (d_1 + \alpha d_2)}{1-\alpha} \\ \frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha} & \frac{q_s - (d_1 + \alpha d_2)}{1-\alpha} \le s_2 \le q_s \end{cases}, \text{ and,}$$

$$G_{12}^* = \begin{cases} p(d_1 + \alpha(d_2 - s_2)) - w_1(q_s - s_2) & 0 \le s_2 < \frac{q_s - (d_1 + \alpha d_2)}{1-\alpha} \\ (p - w_1) \frac{d_1 + \alpha d_2 - \alpha q_s}{1-\alpha} & \frac{q_s - (d_1 + \alpha d_2)}{1-\alpha} \le s_2 \le q_s \end{cases};$$

(2-3) if $q_s > d_1 + d_2$, we have: $s_{12}^* = q_s - s_2$ when $d_1 < q_s - s_2 \le q_s$, and, $s_{12}^* = d_1$ when $0 \le q_s - s_2 \le d_1$; specifically:

$$s_{12}^* = \begin{cases} q_s - s_2 & 0 \le s_2 < q_s - d_1 \\ d_1 & q_s - d_1 \le s_2 \le q_s \end{cases}, \text{ and,}$$

$$G_{12}^* = \begin{cases} p(d_1 + \alpha(d_2 - s_2)^+) - w_1(q_s - s_2) & 0 \le s_2 < q_s - d_1 \\ (p - w_1) d_1 & q_s - d_1 \le s_2 \le q_s \end{cases}.$$

We first analyze the case where $w_1 \le \alpha p$, if $q_s \le d_1 + d_2$, buyer 1's switch over demand alters from zero to a positive value when s_1 is too small to meet his local demand, not to mention the effective total demand, and therefore, $G_{12}(s_1, s_2)$ increases with s_1; otherwise, s_1 is larger than the effective total demand when buyer 1's switch over demand alters from zero to be positive, (that is, $s_1 = q_s - d_2$), and his total demand increases when $s_1 > q_s - d_2$; so, $G_{12}(s_1, s_2)$'s first order derivative switches from negative to positive when s_1 crosses $q_s - d_2$. Moreover, buyer 1's profit by monopolizing the market is larger than his profit by just ordering d_1 if q_s is not too large (that is, $q_s \le d_1 + \frac{\alpha p}{w_1} d_2$), since buyer 1's effective total demand if he monopolizes the market does not change and he can pay less cost to order all the inventory if q_s is smaller. If $w_1 > \alpha p$, buyer 1 would not allow any wasted inventory, if feasible.

Then we compare G_{11}^* and G_{12}^* to derive buyer 1's best response function in the following cases:

(1) $c \le w_1 \le \alpha p$:

if $0 \le q_s \le d_1 + \alpha d_2$, we have: $G_{11}^* = (p - w_1)(q_s - s_2) \le G_{12}^* = (p - w_1)q_s$, so, $s_1(s_2) = q_s$ and $\pi_1(s_1|w_1, w_2, q_s, s_2) = (p - w_1)q_s$;

if $d_1 + \alpha d_2 < q_s \le d_1 + d_2$, we have: if $s_2 < \frac{q_s-(d_1+\alpha d_2)}{1-\alpha}$, $G_{11}^* - G_{12}^* = -\alpha(p - w_1)s_2 + w_1(q_s - (d_1 + \alpha d_2))$, and, if $s_2 \ge \frac{q_s-(d_1+\alpha d_2)}{1-\alpha}$, $G_{11}^* - G_{12}^* = -(p - w_1)s_2 + p(q_s - (d_1 + \alpha d_2))$; by simple algebra, we can conclude that $\frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} \le \frac{p(q_s-(d_1+\alpha d_2))}{p-w_1} \le \frac{q_s-(d_1+\alpha d_2)}{1-\alpha}$ if $w_1 \le \alpha p$, therefore:

$$s_1(s_2) = \begin{cases} d_1 + \alpha(d_2 - s_2) & 0 \le s_2 \le \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} \\ q_s & \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} < s_2 \le q_s \end{cases}, \text{ and,}$$

$$\pi_1(s_1|w_1, w_2, q_s, s_2) = \begin{cases} (p - w_1)(d_1 + \alpha(d_2 - s_2)) & 0 \le s_2 \le \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} \\ p(d_1 + \alpha d_2) - w_1 q_s & \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} < s_2 \le q_s \end{cases};$$

if $d_1 + d_2 < q_s \le d_1 + \frac{\alpha p}{w_1} d_2$, we have: $\frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} \le d_2$, so if $0 \le s_2 \le \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)}$, $G_{11}^* \ge G_{12}^*$, and if $\frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} < s_2 < d_2$, $G_{11}^* < G_{12}^*$; if $d_2 \le s_2 < q_s - d_1$, $G_{11}^* - G_{12}^* = w_1(q_s - (d_1 + \frac{\alpha p}{w_1} d_2)) \le 0$; if $q_s - d_1 \le s_2 \le q_s$, $\frac{p(q_s-(d_1+\alpha d_2))}{p-w_1} \le q_s - d_1$ in this case, so $G_{11}^* \le G_{12}^*$, and therefore:

$$s_1(s_2) = \begin{cases} d_1 + \alpha(d_2 - s_2) & 0 \le s_2 \le \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} \\ q_s & \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} < s_2 \le q_s \end{cases}, \text{ and,}$$

$$\pi_1(s_1|w_1, w_2, q_s, s_2) = \begin{cases} (p - w_1)(d_1 + \alpha(d_2 - s_2)) & 0 \le s_2 \le \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} \\ p(d_1 + \alpha d_2) - w_1 q_s & \frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} < s_2 \le q_s \end{cases};$$

if $q_s > d_1 + \frac{\alpha p}{w_1} d_2$, if $0 \le s_2 < d_2$, $\frac{w_1(q_s-(d_1+\alpha d_2))}{\alpha(p-w_1)} > d_2$ in this case and $G_{11}^* > G_{12}^*$; if $d_2 \le s_2 < \frac{\alpha p}{w_1} d_2$, $G_{11}^* - G_{12}^* = w_1\left(q_s - \left(d_1 + \frac{\alpha p}{w_1} d_2\right)\right) > 0$; if $\frac{\alpha p}{w_1} d_2 \le s_2 < q_s - d_1$, $G_{11}^* - G_{12}^* = w_1(q_s - s_2 - d_1) > 0$; if $q_s - d_1 \le s_2 \le q_s$, $G_{11}^* - G_{12}^* = (p - w_1)(q_s - s_2 - d_1) \le 0$, therefore:

$$s_1(s_2) = \begin{cases} d_1 + \alpha(d_2 - s_2) & 0 \le s_2 < d_2 \\ d_1 & d_2 \le s_2 \le q_s \end{cases}, \text{ and,}$$

$$\pi_1(s_1|w_1, w_2, q_s, s_2) = \begin{cases} (p - w_1)\big(d_1 + \alpha(d_2 - s_2)\big) & 0 \le s_2 < d_2 \\ (p - w_1)d_1 & d_2 \le s_2 \le q_s \end{cases};$$

(2) $\alpha p < w_1 \le p$:

if $0 \le q_s \le d_1 + \alpha d_2$, $G_{12}^* = (p - w_1)q_s \ge G_{11}^* = (p - w_1)(q_s - s_2)$, so, $s_1(s_2) = q_s$ and $\pi_1(s_1|w_1, w_2, q_s, s_2) = (p - w_1)q_s$;

if $d_1 + \alpha d_2 < q_s \le d_1 + d_2$, if $0 \le s_2 < \frac{q_s-(d_1+\alpha d_2)}{1-\alpha}$, $G_{11}^* - G_{12}^* = w_1(1 - \alpha)\left(\frac{q_s-(d_1+\alpha d_2)}{1-\alpha} - s_2\right) > 0$; if $\frac{q_s-(d_1+\alpha d_2)}{1-\alpha} \le s_2 \le q_s$, $G_{11}^* - G_{12}^* = (p - w_1)(\frac{q_s-(d_1+\alpha d_2)}{1-\alpha} - s_2) \le 0$; so:

$$s_1(s_2) = \begin{cases} d_1 + \alpha(d_2 - s_2) & 0 \le s_2 < \frac{q_s-(d_1+\alpha d_2)}{1-\alpha} \\ \frac{d_1+\alpha d_2-\alpha q_s}{1-\alpha} & \frac{q_s-(d_1+\alpha d_2)}{1-\alpha} \le s_2 \le q_s \end{cases}, \text{ and,}$$

$$\pi_1(s_1|w_1, w_2, q_s, s_2) = \begin{cases} (p - w_1)(d_1 + \alpha(d_2 - s_2)) & 0 \le s_2 < \frac{q_s-(d_1+\alpha d_2)}{1-\alpha} \\ (p - w_1)\frac{d_1+\alpha d_2-\alpha q_s}{1-\alpha} & \frac{q_s-(d_1+\alpha d_2)}{1-\alpha} \le s_2 \le q_s \end{cases};$$

if $q_s > d_1 + d_2$, if $0 \le s_2 < d_2$, $G_{11}^* - G_{12}^* = w_1(1 - \alpha)\left(\frac{q_s-(d_1+\alpha d_2)}{1-\alpha} - s_2\right) > 0$ as $\frac{q_s-(d_1+\alpha d_2)}{1-\alpha} > d_2$ in this case; if $d_2 \le s_2 \le q_s - d_1$, $G_{11}^* - G_{12}^* = w_1(q_s - d_1 - s_2) \ge 0$; and if $q_s - d_1 < s_2 \le q_s$, $G_{11}^* - G_{12}^* = (p - w_1)(q_s - d_1 - s_2) < 0$; so,

$$s_1(s_2) = \begin{cases} d_1 + \alpha(d_2 - s_2) & 0 \le s_2 < d_2 \\ d_1 & d_2 \le s_2 \le q_s \end{cases}, \text{ and,}$$

$$\pi_1(s_1|w_1, w_2, q_s, s_2) = \begin{cases} (p - w_1)\big(d_1 + \alpha(d_2 - s_2)\big) & 0 \le s_2 < d_2 \\ (p - w_1)d_1 & d_2 \le s_2 \le q_s \end{cases}.$$

And the proof is complete.

Proof of Lemma 2:

Define sub problem 1 as:

$$\max_{s_2\in[0,q_s-s_1]} G_{21}(s_1,s_2) = \max_{s_2\in[0,q_s-s_1]} p((d_2+\alpha(d_1-s_1)^+)\wedge s_2) - w_2 s_2,$$

which can be easily solved as:

(1) if $0 \le q_s \le d_1 + d_2$:

$$s_{21}^* = \arg\max_{0\le s_2\le q_s-s_1} G_{21}(s_1,s_2) = \begin{cases} d_2+\alpha(d_1-s_1) & 0\le s_1 < (\frac{q_s-(d_2+\alpha d_1)}{1-\alpha})^+ \\ q_s - s_1 & (\frac{q_s-(d_2+\alpha d_1)}{1-\alpha})^+ \le s_1 \le q_s \end{cases};$$

$$G_{21}^* = \max_{0\le s_2\le q_s-s_1} G_{21}(s_1,s_2) =$$

$$\begin{cases} (p-w_2)(d_2+\alpha(d_1-s_1)) & 0\le s_1 < (\frac{q_s-(d_2+\alpha d_1)}{1-\alpha})^+ \\ (p-w_2)(q_s-s_1) & (\frac{q_s-(d_2+\alpha d_1)}{1-\alpha})^+ \le s_1 \le q_s \end{cases};$$

(2) if $q_s > d_1 + d_2$:

$$s_{21}^* = \arg\max_{0\le s_2\le q_s-s_1} G_{21}(s_1,s_2) = \begin{cases} d_2+\alpha(d_1-s_1) & 0\le s_1 < d_1 \\ d_2 & d_1 < s_1 \le q_s - d_2; \\ q_s - s_1 & q_s - d_2 < s_1 \le q_s \end{cases}$$

$$G_{21}^* = \max_{0\le s_2\le q_s-s_1} G_{21}(s_1,s_2) = \begin{cases} (p-w_2)(d_2+\alpha(d_1-s_1)) & 0\le s_1 < d_1 \\ (p-w_2)d_2 & d_1 < s_1 \le q_s - d_2. \\ (p-w_2)(q_s-s_1) & q_s - d_2 < s_1 \le q_s \end{cases}$$

Define sub problem 2 as:

$$\max_{s_2\in(q_s-s_1,q_s]} G_{22}(s_1,s_2)$$

$$= \max_{s_2\in(q_s-s_1,q_s]} p((d_2+\alpha(d_1-s_1)^+)\wedge(q_s-s_1)) - w_2(q_s-s_1)$$

In this sub problem, $s_{22}^* = \arg\max_{q_s-s_1<s_2\le q_s} G_{22}(s_1,s_2)$ can be any value in the interval $(q_s-s_1, q_s]$ as buyer 2 can only obtain $q_s - s_1$ no matter how much he orders and the objective function is constant given s_1. Buyer 2's profit in this optimization problem is as follows:

(1) if $0 \le q_s \le d_1 + d_2$:

$$G_{22}^* = \max_{q_s-s_1<s_2\le q_s} G_{22}(s_1,s_2) =$$

$$\begin{cases} p(d_2+\alpha(d_1-s_1)) - w_2(q_s-s_1) & 0\le s_1 < (\frac{q_s-(d_2+\alpha d_1)}{1-\alpha})^+ \\ (p-w_2)(q_s-s_1) & (\frac{q_s-(d_2+\alpha d_1)}{1-\alpha})^+ \le s_1 \le q_s \end{cases};$$

(2) if $q_s > d_1 + d_2$:

$$G_{22}^* = \max_{q_s - s_1 < s_2 \leq q_s} G_{22}(s_1, s_2) =$$

$$\begin{cases} p(d_2 + \alpha(d_1 - s_1)) - w_2(q_s - s_1) & 0 \leq s_1 < d_1 \\ pd_2 - w_2(q_s - s_1) & d_1 < s_1 \leq q_s - d_2. \\ (p - w_2)(q_s - s_1) & q_s - d_2 < s_1 \leq q_s \end{cases}$$

Then we compare G_{21}^* and G_{22}^* to derive buyer 2's best response function in the following cases:

(1) if $0 \leq q_s \leq d_2 + \alpha d_1$: $G_{21}^* = G_{22}^* = (p - w_2)(q_s - s_1)$, so, $s_2(s_1)$ can be any value in the interval $[q_s - s_1, q_s]$ with $\pi_2(s_2|w_1, w_2, q_s, s_1) = (p - w_2)(q_s - s_1)$;

(2) if $d_2 + \alpha d_1 < q_s \leq d_1 + d_2$: if $0 \leq s_1 < \frac{q_s - (d_2 + \alpha d_1)}{1 - \alpha}$, $G_{21}^* - G_{22}^* = w_2(1 - \alpha)\left(\frac{q_s - (d_2 + \alpha d_1)}{1 - \alpha} - s_1\right) > 0$, so, $s_2(s_1) = d_2 + \alpha(d_1 - s_1)$ with $\pi_2(s_2|w_1, w_2, q_s, s_1) = (p - w_2)(d_2 + \alpha(d_1 - s_1))$; and if $\frac{q_s - (d_2 + \alpha d_1)}{1 - \alpha} \leq s_1 \leq q_s$, $G_{21}^* = G_{22}^* = (p - w_2)(q_s - s_1)$, so, $s_2(s_1)$ can be any value in the interval $[q_s - s_1, q_s]$ with $\pi_2(s_2|w_1, w_2, q_s, s_1) = (p - w_2)(q_s - s_1)$;

(3) if $q_s > d_1 + d_2$: if $0 \leq s_1 < d_1$, $G_{21}^* - G_{22}^* = w_2(1 - \alpha)\left(\frac{q_s - (d_2 + \alpha d_1)}{1 - \alpha} - s_1\right) > 0$ as $\frac{q_s - (d_2 + \alpha d_1)}{1 - \alpha} > d_1$ in this case, so, $s_2(s_1) = d_2 + \alpha(d_1 - s_1)$ with $\pi_2(s_2|w_1, w_2, q_s, s_1) = (p - w_2)(d_2 + \alpha(d_1 - s_1))$; if $d_1 < s_1 \leq q_s - d_2$, $G_{21}^* - G_{22}^* = w_2(q_s - d_2 - s_1) \geq 0$, so, $s_2(s_1) = d_2$ with $\pi_2(s_2|w_1, w_2, q_s, s_1) = (p - w_2)d_2$; and, if $q_s - d_2 < s_1 \leq q_s$, $G_{21}^* = G_{22}^* = (p - w_2)(q_s - s_1)$, so, $s_2(s_1)$ can be any value in the interval $[q_s - s_1, q_s]$ with $\pi_2(s_2|w_1, w_2, q_s, s_1) = (p - w_2)(q_s - s_1)$.

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