BULK SHIP ROUTING AND SCHEDULING UNDER UNCERTAINTY

LINGXIAO WU

PhD

The Hong Kong Polytechnic University

2020

The Hong Kong Polytechnic University

Department of Logistics and Maritime Studies

Bulk Ship Routing and Scheduling under Uncertainty

LINGXIAO WU

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

March, 2020

CERTIFICATE OF ORIGINALITY

I hereby declare that this thesis is my own work and that, to the best of my knowledge and belief, it reproduces no material previously published or written, nor material that has been accepted for the award of any other degree or diploma, except where due acknowledgement has been made in the text.

_____________________(Signed)

_____LINGXIAO WU_____(Name of student)

Abstract

Bulk shipping contributes to nearly half of the global seaborne transportation volume. In bulk shipping, ships are operated in two different modes: industrial shipping and tramp shipping. In industrial shipping, an industrial corporation owns or controls a fleet of bulk ships and transports cargoes to satisfy its own demand (i.e., the corporation acts as the shipper and the carrier at the same time). In tramp shipping, shipping companies act as carriers that transport cargoes from one port to another by following the orders from the customers (shippers). Seaborne transportation is known for its uncertainties which greatly impact the operations in both industrial and tramp bulk shipping. This thesis focuses on two operations management problems in bulk shipping under uncertainties. Particularly, we consider a bulk ship scheduling problem in industrial shipping in Chapter 2 and a bulk ship routing problem in tramp shipping in Chapter 3.

Chapter 2 explores a ship scheduling problem for an industrial corporation that manages a fleet of bulk ships under stochastic environments. The considered problem is an integration of three interconnected sub-problems from different planning levels: the strategic fleet sizing and mix problem, the tactical voyage planning problem, and the operational stochastic backhaul cargo canvassing problem. To obtain the optimal solution of the problem, this chapter provides a two-step algorithmic scheme. In the first step, the stochastic backhaul cargo canvassing problem is solved by a dynamic programming (DP) algorithm, leading to optimal canvassing strategies for all feasible voyages of all ships. In the second step, a mixed-integer programming (MIP) model that jointly solves the fleet sizing and mix problem and the voyage planning problem is formulated using the results from the first step. To efficiently solve the proposed

MIP model, this chapter develops a tailored Benders decomposition method. Finally, extensive numerical experiments are conducted to demonstrate the applicability and efficiency of the proposed models and solution methods for practical instances.

Chapter 3 presents a robust optimization algorithm to solve a ship routing problem faced by bulk tramp shipping companies. In this problem, the cargo selection behaviors in the settings where a group of cargoes should be treated as a batch are considered. In view of the uncertainties observed in maritime transportation, we formulate the problem in such a way that the solutions are robust against variations in voyage costs. We first provide compact mixed integer linear programming formulations for the problem and then convert them into a strengthened set covering model. A tailored branch-and-price-and-cut algorithm is developed to solve the set covering model. The algorithm is enhanced by a multi-cut generation technique aimed at tightening the lower bounds and a primal heuristic aimed at finding high-quality upper bounds. Extensive computational results show that our algorithm yields optimal or near-optimal solutions to realistic instances within short computing times and that the enhancement techniques significantly improve the efficiency of the algorithm.
Keywords: Bulk shipping operations; Industrial shipping; Tramp shipping; Ship routing and scheduling; Stochastic optimization; Robust optimization

Publications during PhD Study

1. Wu, L., Wang, S., 2018. Joint deployment of quay cranes and yard cranes in container terminals at a tactical level. **Transportation Research Record** 2672, 35-46.
2. Wu, L., Wang, S., 2018. Exact and heuristic methods to solve the parallel machine scheduling problem with multi-processor tasks. **International Journal of Production Economics** 201, 26-40.
3. Wu, L., Pan, K., Wang, S., Yang, D., 2018. Bulk ship scheduling in industrial shipping with stochastic backhaul canvassing demand. **Transportation Research Part B: Methodological** 117, 117-136.
4. Yang, D., Wu, L., Wang, S., Jia, H., Li, K.X., 2019. How big data enriches the shipping research – a critical review of Automatic Identification System (AIS) data applications. **Transport Reviews** 39, 755-773.
5. Wang, S., Yan, R., Wu, L., Yang, D., 2019. Optimal re-allocation of mooring areas for yachts. **Maritime Business Review**, 4, 94-105.
6. Wu, L., Wang, S., 2020. The shore power deployment problem for maritime transportation. **Transportation Research Part E: Logistics and Transportation Review**, 135, 1-12.
7. Wu, L., Yang, D., Wang, S., 2020. Evacuating offshore working barges from a land reclamation site in storm emergencies. **Transportation Research Part E: Logistics and Transportation Review**, 135, 1-29.
8. Jia, S., Wu, L., Meng, Q., 2020. Joint scheduling of vessel traffic and pilots in seaport waters. **Transportation Science**, in press.
9. Wu, L., Jia, S., Wang, S. Pilotage planning in seaports. **European Journal of**

Operational Research, 287, 90-105.

Acknowledgements

There are many people that have earned my gratitude for their contribution to my time at PolyU.

Firstly, I would like to express my sincere gratitude to my supervisor, Dr. Shuaian Wang, for his continuous and invaluable support during the last three years not only in academic research but also in other aspects of life. I have been learning from his broad knowledge, innovative thinking, and the pursuit of excellence in academic research. He is always available and willing to offer help whenever I meet with difficulties in research. This thesis would not have been possible without him. Dr. Wang's dedication to research, passion for work, and great patience in teaching have always been inspiring me. I have been learning from him about how to be a good researcher, a good teacher, and a good person.

Special thanks go to the rest of my thesis examiners: Dr. Miao Song from PolyU, Prof. Dongping Song from the University of Liverpool, and Prof. Xiangtong Qi from HKUST. I am grateful for their precious time and insightful comments on my thesis.

Heartfelt thanks are also due to Dr. Dong Yang and Dr. Kai Pan from PolyU, and Prof. Gilbert Laporte from HEC Montréal. I am grateful for their inspiring discussions, constructive suggestions, and insightful comments on my works. I also would like to thank Prof. Pengfei Guo, Prof. Li Jiang, Dr. Meifeng Luo, Dr. Achim I. Czerny, and many others, for their lectures from which I learned the essential knowledge and skills in conducting research. I am also grateful to the staff in the General Office of LMS and the Research Office for their assistance in my study and life at PolyU.

My friends consist of a great part of my wonderful and exciting life at PolyU.

Special thanks go to Jingxu Chen, Shuai Jia, Hao Lang, Shuli Liu, Wenhao Peng, Kai Wang, Hanxiang Zhang, Wei Zhang, and Dan Zhuge.

Finally, I would like to dedicate this thesis to my parents and my fiancee, Yun Yuan, for their endless love and support.

Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

Ships carry 80% of cargoes by volume around the world, and the dry bulk shipping sector accounts for nearly half of the global seaborne transportation volume, amounting to 5.2 billion tons in 2018 (UNCTAD 2019). There are generally two operational modes in dry bulk shipping: the so-called industrial shipping and tramp shipping. In industrial shipping, an industrial corporation owns or controls a fleet of ships and transport cargoes to satisfy its own demand. In this shipping mode, the shipper is also the carrier. Therefore, the focus in industrial shipping operation is to transport all the required cargoes at the minimum cost. In tramp shipping, shipping companies act as carriers that transport cargoes from one port to another by following the orders from the customers (shippers). A tramp shipping company participates in maritime transportation by owning or controlling a fleet of bulk ships, and profits from the freight gained by transporting cargoes.

In bulk shipping, the operations management problem faced by an industrial shipping or tramp shipping carrier generally consists of three critical decisions, which are fleet mix and sizing, shipment arrangement, and ship routing and scheduling.

Over a given planning horizon, fleet mix and sizing determines which ships to charter in and out. Seaborne bulk transportation is capital intensive, with daily operational costs for a bulk ship amounting to tens of thousands of US dollars (Greiner 2013). Therefore, in order to reduce their costs, bulk shipping carriers generally choose to adjust the composition of their fleet according to the variations in demand.

Shipment arrangement in bulk shipping refers to the decisions of a carrier regarding the selection of cargoes to transport and the weight and timing of each shipment for transporting the cargoes. The cargoes faced by a bulk shipping carrier can generally be classified into two groups: mandatory cargoes and optional cargoes. For the carrier, mandatory cargoes must be transported while he/she has the freedom of choosing optional cargoes. Once the cargoes to be transported are decided, the carriers should arrange the shipments so that the cargoes are transported in a way that meets the requirements from the shippers.

Given the results of the above two decisions, the carrier is then responsible for routing and scheduling the ships so that all the shipments are performed and the objective is to minimize the total transportation cost. This falls into the ship routing and scheduling problem (in its most traditional sense).

It is obvious that these decisions are interconnected and should be tackled in an integrated manner. However, a problem that jointly considers these decisions is very complex and calls for dedicated designs of modeling and solution methodologies. What further complicates the operations management in bulk shipping is the inevitable uncertainties in maritime transportation. For one thing, when making decisions, a carrier may not have the full information of cargoes that will appear from the spot market. For another, although voyage costs account for a large proportion of the total operational cost, they suffer from great randomness in practice.

Operations management problems in bulk shipping have received great attention in the literature. In fact, problems related to fleet mix and sizing, shipment arrangement, and ship routing and scheduling in bulk shipping have solicited great attention from the research community, particularly from the Operations Research (OR) circle. Many OR techniques (including various modeling and solution methods) have been developed to solve these problems. However, most studies consider these decisions in a separate fashion, which leads to sub-optimal solutions from a systematic view. In addition, the inherent uncertainties in maritime transportation are mostly not considered in the literature, making the derived results hard to implement in practice.

To fill the gaps between scientific research and practice, this thesis investigates new modeling and solution methods for solving operations management problems in bulk shipping. In these problems, we jointly consider fleet mix and sizing, shipment arrangement, and ship routing and scheduling faced by a bulk shipping carrier in an uncertain environment. There are many differences between industrial shipping and tramp shipping, particularly when it comes to the arrangements of shipments for transporting cargoes. We, therefore, study the bulk shipping operations management problems under the two different operating modes.

This thesis consists of the following four parts:

(i) In Chapter 1, we introduce the background of the problems that are considered in the thesis.

(ii) In Chapter 2, we address a bulk ship scheduling problem in industrial shipping under stochastic environments. The considered problem is an integration of three interconnected sub-problems from different planning levels: the strategic fleet sizing and mix problem, the tactical voyage planning problem, and the operational stochastic backhaul cargo canvassing problem. We develop a two-stage solution approach for solving the problem. Extensive numerical experiments are performed to test the performance of the solution approach and we also analyze the solution structure through a case study.

(iii) In Chapter 3, we present a branch-and-price-and-cut algorithm to solve a robust bulk ship routing problem in tramp shipping. In this problem, we consider the cargo selection behavior of a tramp shipping company when it is faced with a set of Contracts of Affreightment. We formulate the problem in a robust way such that the shipping company's profitability is protected against the variable voyage costs. Several accelerating techniques are proposed to strengthen the algorithm. We conduct extensive experiments to test the performance of the algorithm and we also analyze the value of robustness in the problem.

(iv) In Chapter 4, we summarize the main findings from the two studies. Some future research directions are discussed.

Chapter 2

Bulk Ship Scheduling in Industrial Shipping with Stochastic Backhaul Canvassing Demand

2.1 Introduction

Industrial shipping is an integral part of the global supply chain for raw materials. According to the estimates by UNCTAD (2017), the trade volumes of the five most common raw materials (i.e., iron ore, grain, coal, bauxite and alumina, and phosphate rock) contributed over 30% of the global seaborne trade. These raw materials are categorized as major bulk cargoes in the shipping market and are generally transported by bulk ships (mostly Capesize or Panamax carriers) in full shiploads from one origin port to one destination port. In 2016, the iron ore trade increased 3.4%, reaching 1.4 billion tons, and more than 70% of iron ores were imported to China from Australia and Brazil using Capesize or Panamax carriers (UNCTAD 2017).

In industrial shipping, the industrial corporation owns or controls a fleet of bulk ships and the focus of the corporation is to minimize the total transportation costs while ensuring that all cargoes are transported to satisfy the demand. In the current shipping market, industrial shipping is widely used in the transportation of these major bulk cargoes. For instance, Baowu Group (formally known as Baosteel), China's largest steel producer, imports iron ore from Brazil by chartering in bulk carriers from a shipping company (Baosteel 2008). Seaborne bulk transportation is capital-intensive, with daily operational costs for a Capesize or Panamax carrier amounting to tens of thousands of US dollars (Greiner 2013). Thus, a proper scheduling of the fleet is of tremendous importance for an industrial shipping operator to reduce costs.

Besides optimally scheduling the fleet to satisfy the demand at the minimum cost, considering the required transportation is one-directional, each ship is able to help further increase the savings by carrying cargoes from the spot market during the return trip from the destination port to the origin one. Take the Brazil-to-China iron ore transportation as an example. The required cargo is transported one-directionally, and after unloading cargoes in China, a ship may return to Brazil in ballast or instead it may carry cargoes during the return trip (our interviews with several managers from different bulk shipping companies reveal that the most common cargoes in the return trip include steam coal from Indonesia to India, and coking coal from Australia to India or from Australia to Europe). In addition, a recent report made by UNECLAC (2018) estimates that with the return cargoes, the Brazil-to-China iron ore transportation cost can be saved up to 20%.

On the one hand, carrying cargoes in the return trip is an appealing opportunity for the industrial corporation to better manage the fleet of ships. On the other, the significant uncertainties in the spot transportation market, the potential delay due to carrying return cargoes, and the restrictive requirements to fulfill the demand on time significantly complicate the whole scheduling job. Therefore, in order to both efficiently satisfy the shipping demand and acquire the benefits through carrying potential return cargoes, advanced scheduling modeling and solution approaches are desired to finally achieve efficient utilization of the transportation capacity of the industrial corporation.

To this end, this chapter considers a stochastic bulk ship scheduling problem in industrial shipping by considering the uncertainties from the spot market. In this problem, we jointly consider three sub-problems from different planning levels. The first sub-problem is the fleet sizing and mix problem which decides the number and size of ships the industrial corporation should charter in to fulfill the transportation demand over the entire planning horizon. The second sub-problem is the voyage planning problem which determines the start and end dates for each voyage completed by each ship. Besides, additional profits can be made if ships are able to carry cargoes from the spot market during the backhaul voyage. Hence, the third sub-problem, i.e., a backhaul cargo canvassing problem, is studied. Given the uncertainties from the spot transportation market, we consider the backhaul cargo canvassing problem under stochastic environments. Since these problems are closely intertwined, to obtain an optimal solution, we propose a two-step solution scheme whose great effectiveness is demonstrated through extensive numerical experiments.

Ship scheduling problems have been well studied in the fields of liner shipping (e.g., Wang and Meng 2012 and Song and Dong 2013), tramp shipping (e.g., Brønmo et al. 2007a and Meng et al. 2015), and industrial shipping (e.g., Ronen 1986 and Tirado et al. 2013). However, in most studies, we noticed that the cargoes (i.e., containers in liner shipping and minor bulk commodities in tramp and industrial shipping) are assumed to be transported among multiple loading and discharging ports in a pickup-and-delivery manner. It follows that the main focus of these studies was to identify optimal sequences for the ships to call at these ports. Besides, since backhaul canvassing faces great uncertainties (including loading and discharging ports, transportation revenues, costs, and detour lengths) in practice and most ship scheduling studies are relied on (at least partially) deterministic demand assumptions, the backhaul canvassing problem has been rarely studied in the literature.

Different from the previous studies, this chapter aims to find an optimal shipping schedule that consists of fleet sizing and mix decision, voyage plan, and backhaul canvassing strategy under a stochastic environment. In particular, our main contributions can be described as follows:

- We consider a stochastic bulk ship scheduling problem in industrial shipping, which has never been well addressed in the literature, through incorporating the consideration of the backhaul cargo canvassing strategy under uncertainty.
- We develop a two-step solution scheme consisting of 1) a dynamic programming model and corresponding polynomial-time algorithm to obtain the op-

timal cargo canvassing strategies, and 2) a tailored Benders decomposition method utilizing the specific problem structure to efficiently solve the mixed-integer programming formulation, leading to optimal integrated fleet sizing and mix decision and voyage plan;

- We conduct extensive numerical experiments to demonstrate that the proposed models and solution methods can well solve the considered problem in various and practical sizes.

The remainder of this chapter is organized as follows. A literature review is given in Section 3.2. Then, Section 3.3 provides a detailed description of the considered problem. In Section 2.4, we propose a DP algorithm to solve the stochastic backhaul cargo canvassing problem. Based on the solution of the stochastic backhaul cargo canvassing problem, an MIP model for the integrated fleet sizing and mix and voyage planning problem is formulated in Section 2.5. In Section 2.6, we propose a tailored Benders decomposition method for the model proposed in Section 2.5. A series of numerical experiments and a case study are conducted in Section 2.7. Finally, we conclude our main findings in Section 2.8. We provide all mathematical proofs in Appendix A.

2.2 Literature Review

Ship scheduling problems have received considerable attention in the literature. Christiansen et al. (2004), Christiansen et al. (2007), and Christiansen et al. (2013) provided an overall review of the problem. As ships are operated in three different modes, i.e., liner, tramp, and industrial shipping, studies on ship scheduling problems can also be generally divided into these three corresponding categories (Christiansen et al. 2013). Since our study focuses on industrial shipping, we concentrate our review on existing research on ship scheduling problems in this regard.

One stream of related studies focuses on the fleet sizing and mix problem, which has been studied in the literature for more than four decades (see the recent reviewing study of Pantuso et al. 2014a). Recent studies include Wang and Meng (2012) and

Ng (2015) for liner shipping, and Fagerholt et al. (2010) and Alvarez et al. (2011a) for tramp and industrial shipping. Research on the fleet sizing and mix problem in industrial shipping starts from the pioneering work conducted by Dantzig and Fulkerson (1954). In this study, the authors addressed a special fleet sizing problem arising in Navy fuel oil transportation where all ships (tankers) were assumed to be identical. The objective was to determine the minimum number of tankers needed to meet the fixed transportation demand. The fleet sizing and mix problem in industrial shipping was also studied by Mehrez et al. (1995). An MIP model was formulated for the considered problem, where the decisions included the number and size of ships chartered in and the voyages made by each chartered ship at each time period in a planning horizon. More recently, Fagerholt et al. (2010) proposed a decision support methodology for strategic planning in industrial and tramp shipping which solves the fleet sizing and mix problem using simulation-based optimization. A robust fleet sizing and deployment problem for industrial or tramp shipping operators was analyzed by Alvarez et al. (2011a). The authors proposed a robust optimization model for the considered problem in which decisions concerning fleet sizing and ship deployment were made in an integrated manner.

The second stream of studies focuses on the ship routing and scheduling problem. Most studies in this stream can be viewed as special applications of the Vehicle Routing Problem, where cargoes are transported among several loading and discharging ports in a pickup-and-delivery manner and are different from the transportation of major bulk cargoes. The objectives of these studies were to identify optimal port calling sequences that satisfy various constraints (e.g., Brønmo et al. 2007b, Song and Dong 2013, Meng et al. 2015, Stålhane et al. 2012a, Tirado et al. 2013). There are also some studies exploring the ship routing and scheduling problem in industrial shipping, where cargoes are transported in full shiploads between a single origin port and a single destination port. For example, Brown et al. (1987) analyzed a tanker scheduling problem for a crude oil company where each tanker traveled between a single loading port and a single discharging port. The study aimed at identifying an optimal schedule of the fleet that minimizes the total cost. The problem was modeled as a set partitioning problem which can be solved efficiently with all feasible schedules generated a prior. A liquefied natural gas (LNG) inventory routing problem was investigated by Stålhane et al. (2012b), where an LNG producer owns a tanker fleet that is heterogeneous and considered as fixed for the planning horizon. In each voyage of a tanker, LNG products are transported in full shiploads between a single loading port and a single discharging port. The corresponding objective was to create an annual delivery program of the fleet that exercises the producer's long-term contracts at minimum cost, while maximizing the revenue from selling LNG to the spot market. Siddiqui and Verma (2015) considered a bi-objective oil-tanker routing and scheduling problem, where both cost and operational risks were considered in the objectives and an MIP model was formulated to solve the problem.

While most studies handle fleet sizing and mix problem and the ship routing and scheduling problem separately, a few of them addressed the two problems in a joint manner. One of them was conducted by Fagerholt and Lindstad (2000). They considered a ship scheduling problem regarding a supply operation in the Norwegian Sea where supplies should be transported using ships from a supply depot to several offshore installations. The objective was to determine the optimal fleet and the corresponding weekly schedules that meet the installations' demands at the minimum cost. A similar problem was considered by Halvorsen-Weare et al. (2012). The study jointly solved the fleet sizing and routing problems of offshore supply ships. Another study from Zeng and Yang (2007) considered a coal shipping problem between a set of supply ports and a set of demand ports.

In practice, besides the required shipments that must be completed in the planning horizon, the optional cargo transportations in backhauls also need to be taken into consideration. The backhaul canvassing problem in industrial shipping has rarely been addressed before. The only literature we found is conducted by Bausch et al. (1998). The author proposed a decision support system for a company to conduct a medium-term (two to three weeks) schedule of coastal tankers and barges that transport liquid bulk products to customers. Note that Bausch et al. (1998) considered the backhaul transportation under deterministic environments.

Our research enriches the existing literature in two aspects. To begin with, we consider the fleet sizing and mix problem and the ship scheduling problem in an integrated way, while most studies analyzed them separately. Besides, to the best of our knowledge, our study is one of the preliminary studies to consider the stochastic backhaul cargo canvassing problem, which makes the study more relevant to real and dynamic situations.

2.3 Problem Description

Suppose an industrial corporation in a country needs to continuously import raw materials from another country within a planning horizon (e.g., a steel plant in China like Baowu Group needs to import iron ore from Brazil). These materials are transported in full shiploads from a single loading port of the exporting country to a single discharging port of the importing country. Meanwhile, the industrial corporation considers utilizing the ships to carry return cargoes during their trips back to the loading port, leading to revenues. To facilitate stable and economic transportation, the corporation chooses to time-charter a fleet of Capesize or Panamax bulk ships from the shipping market. The corporation is responsible for scheduling its controlled fleet to ensure its demand is satisfied and the aim is to minimize the overall net cost (i.e., total shipping cost minus revenues).

In the following part of this section, to describe the whole problem in detail, we will introduce the demand structure in Section 2.3.1, the fleet sizing and mix decision (sub-problem one) in Section 2.3.2, the voyage planning (sub-problem two) in Section 2.3.3, the backhaul cargo canvassing (sub-problem three) in Section 2.3.4, and the assumptions for the whole problem in Section 2.3.5. Meanwhile, a set of decisions made by the corporation are summarized in Section 2.3.6, and a detailed solution procedure is outlined in Section 2.3.7. To facilitate a better understanding of the problem, we will use the example of Baowu Group in the following parts.

2.3.1 Demand Structure

The corporation conducts its production in a continuous process (Wikipedia contributors 2018), which requires stable supplies of raw materials during the entire planning horizon. Accordingly, the demand structure of the corporation can be stated as follows. To begin with, there is a total demand for cargoes that must be satisfied in the entire planning horizon (e.g., Baowu Group needs to import approximately 2.5 million tons of iron ore every year to satisfy its annual production demand). In addition, the planning horizon is further divided into several sub-planning horizons, and to maintain a suitable inventory level (which is decided by the consumption rate and storage capacity of raw materials in the corporation), the corporation also sets lower and upper bounds for the number of cargoes transported to the discharging port at each sub-planning horizon (e.g., Baowu Group requires a stable and even arrival flow of iron ore, and prevents drastic fluctuations in monthly imports). Note that there may be overlaps among different sub-planning horizons. Given that cargoes should be first produced and then transported from inland to the loading port in the exporting country before they can be loaded to ships, and to avoid congestions in port handling, inland transportation, and storage, a minimal time interval between the start times of two consecutive voyages is set (e.g., the current practice in Baowu Group is two weeks).

2.3.2 Fleet Sizing and Mix (Sub-problem One)

Fleet sizing and mix should be decided at the beginning of the planning horizon, as a strategic plan. The corporation time-charters a fleet of ships from a candidate pool composed of ships that have different chartering rates, capacities, operational costs, and speeds. At the beginning of the planning horizon, the corporation should decide the number and types of ships to charter-in. If charted, the ships serve for the entire planning horizon. Besides, in accordance with common practices, we assume that all the chartered ships are ready to load at the loading port and start a voyage at the beginning of the planning horizon. In addition, these ships have to be redelivered at the loading port at the end of the planning horizon. In Baowu Group, 4 Capesize carriers whose chartering contracts are renewed on a yearly basis are rented to fulfill its annual demand.

2.3.3 Voyage Planning (Sub-problem Two)

The cargo transportation in this problem is conducted in a one-origin-one-destination structure. As shown in Figure 3.1, the required cargoes are transported from Port O to Port D (e.g., Brazil's Tubarao Port to China's Shanghai Port). In this chapter, we define that a complete voyage starts when a ship starts loading at the loading port and ends when it returns in empty to the same loading port. In addition, the journey from Port O to Port D is defined as a forward voyage and the journey from Port D back to Port O is defined as a backward voyage or a backhaul. Note that in the following parts of this chapter, unless otherwise specified, a "voyage" is used to refer to a complete journey that includes both the forward voyage and the backhaul.

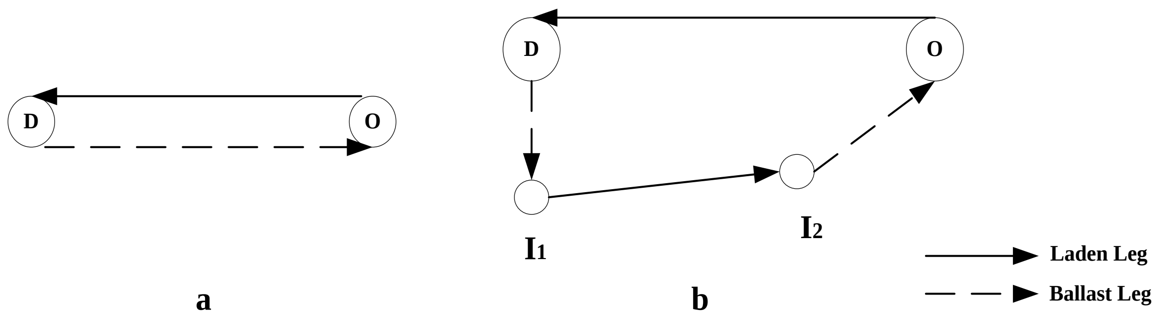

Figure 2.1: Voyage structures in the problem

There are generally two types of voyage structures. While both types have the same forward voyage (i.e., from Port O to Port D), the backward routes can be different. On the one hand, after discharging at Port D, the ship will return to Port O in ballast without carrying any cargoes in the backhaul, leading to the first type of voyage structure (see Figure 3.1a). On the other hand, there may be transportation requests arising in the spot market for carrying cargoes between two ports that locate near the route from Port D to Port O (referring to the iron ore transportation case, the most common backhaul cargoes include steam coal from Indonesia to India and coking coal from Australia to India or from Australia to Europe). A ship can decide to accept a request and carry cargoes between two intermediate ports in the backward voyage (e.g. steam coal from Samarinda Port, Indonesia to Mundra port, India). In such a case, the ship sails in the second type of voyage structure (see Figure 3.1b), e.g., after unloading at Port D, the ship first detours to Port I_1 (e.g. Samarinda), loads cargoes at this port, sails towards Port I_2 (e.g. Mundra), and finally sails to Port O after discharging at Port I_2 (e.g. Mundra).

Note that carrying cargoes in the backhaul may bring additional revenue and meanwhile it also incurs voyage detours and additional time for loading and unloading at intermediate ports. For example, carrying 150 thousand tons steam coal from Samarinda to Mundra using a Capesize carrier generates approximately 900 thousand US dollars' revenue, $12 - 20$ days' detour, and 650 thousand US dollars' additional cost. In this case, compared with sailing to Brazil in ballast, cargo transportation in the backhaul helps save 250 thousand US dollars.

In addition, we define the minimum required and maximum allowed durations for each voyage. The minimum duration is determined by the minimum time a ship needs to complete a voyage and the maximum duration may be set for operational considerations of the corporation. For Baowu Group, a round trip from Brazil to China should take at least 70 days, and normally, no voyages take more than 100 days.

2.3.4 Backhaul Cargo Canvassing (Sub-problem Three)

When sailing on a voyage, a ship may receive transportation requests from the spot market for transporting cargoes in the backhaul. These requests are distinguished by their cargo weights, required detour lengths, and revenues. In particular, they can be divided into different request types based on these features, and more specifically, the transportation requests with the same cargo weights, detour lengths, and revenues belong to the same type. Note that the involved ships are chartered to facilitate the stable transportation of cargoes from Port O to Port D. Therefore, from an economic view, the ships are only possible to accept the types of requests that produce revenues and are of acceptable cargo weights/detour lengths and to accept at most one cargo transportation request from the spot market in a voyage. More specific reasons include (i) ships involved in this problem are Capesize or Panamax sizes ships that have very high operational and port costs and thus not able to carry cargoes multiple times; and (ii) it is not beneficial for these large ships to transport minor bulk cargoes and operate in a pickup-and-delivery manner (even some ports are not large enough to host these ships).

Referring to the Brazil-to-China iron ore transportation case, our interviews with the industrial practitioners indicates that except the commonly recognized backhaul transportation requests (i.e., coal transportation from Indonesia or Australia to India or Europe), it is very rare to see other transportation requests arise in the backhaul that are suitable for Panamax or Capesize ships. This is because requests with too low cargo weights or too long detours are unacceptable due to the relatively low freight rates in the backhaul and the high bunker and port costs. Thus, in practice, a ship can take at most one such transportation request in the backhaul, otherwise, the additional costs outweigh the additional revenues generated in the backhaul.

In addition, during a certain voyage, the ship is open to accept backhaul transportation requests only in a certain period (denoted by the canvassing period thereafter). The canvassing period typically starts when the ship starts leaving Port D and ends when the vessel passes the possible loading areas in the backhaul. The corresponding reasons can be described in two aspects. First, a ship can begin to accept transportation offers from the spot market only if it ensures that it can reach the next loading port on time. Thus, generally, a ship only accepts the backhaul requests after unloading at the current discharging port (i.e., Port D) and becoming immediately ready, as the handling time at Port D can be uncertain. That is also why cargo owners normally do not take a loaded ship as an option for transporting their cargoes. Second, significant detouring cost and time are required for transporting backhaul cargoes if the ship has sailed far away from the region where the main loading ports of backhaul cargoes are located. For example, the typical canvassing period in the backhaul of Brazil-to-China transportation starts from the time when unloading is finished in China and ends at the time when the ship arrives in Singapore for bunkering.

Spot transportation market is quite volatile and it is impossible to estimate the exact types of transportation requests that will arise at a certain time. However, taking advantage of historical data, we can predict the possibility of receiving a particular type of transportation request from the spot market at a particular time. Hence, we handle the canvassing problem in a stochastic environment.

2.3.5 Assumptions

To better analyze the problem, we make the following assumptions:

A1. The planning horizon is divided into a series of discrete unit times and ship scheduling decisions are made at each unit time.

A2. For simplicity, we assume that each voyage should start at the beginning of a unit time and finish at the end of a unit time.

A3. Each ship runs at a constant speed, while the speeds can be different from one ship to another.

A4. Different types of cargo transportation requests in the backhaul arise independently.

A5. The requests for backhaul transportation are presented to a ship at the beginning of a unit time, and the ship should also decide at the beginning of the unit time whether to accept one of them or decline all.

2.3.6 Decisions

This section summarizes the decisions made in the considered problem. As shown in Figure 2.2, there are three types of decisions the corporation should make, with detailed descriptions provided as follows.

- At the beginning of the planning horizon, the strategic decision about which ships should be chartered from the market should be first made.
- Then, for each chartered ship, the corporation should decide, at the tactical level, the number of voyages each ship should complete during the horizon and when each of these voyages should start and end.

- Finally, at the beginning of each unit time of the canvassing period in each voyage, the corporation is able to observe the condition of requests for transporting cargoes in the backhaul from the spot market. Therefore, the operational decision the corporation should make at each unit time of the canvassing period is whether to accept a type of transportation request or not.

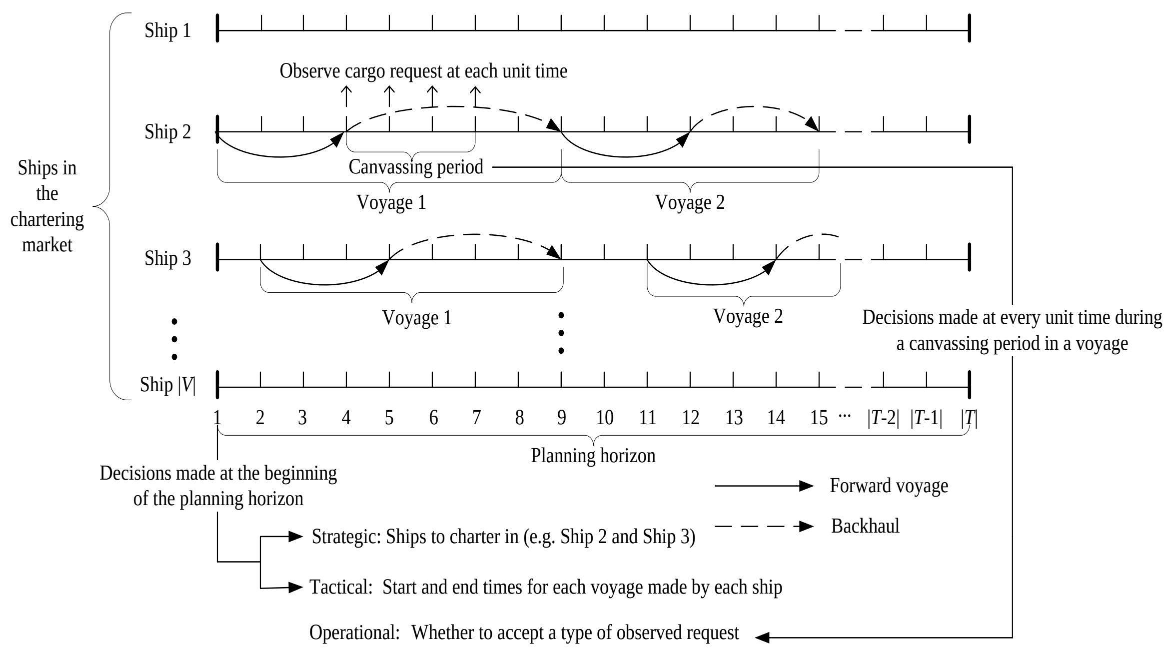

Figure 2.2: Decisions made in the problem

2.3.7 Solution Procedure

In the stochastic bulk ship scheduling problem, we solve the three sub-problems in an integrated manner. The problem is hard to solve. First, all the three sub-problems are difficult combinatorial optimization problems that involve a large number of discrete decision variables. Second, the uncertainties in the backhaul canvassing problem also further complicate the problem. Finally, the three sub-problems from different planning levels are closely intertwined and make the integrated problem even harder to solve.

In order to effectively solve this problem, we develop an exact two-step solution procedure that can generate an optimal solution for the stochastic bulk ship scheduling problem. To begin with, by taking advantage of the predictable probabilities of cargo types in the backhaul, we provide a DP method (see Section 2.4 for details) to handle the stochastic backhaul cargo canvassing problem. The DP method can obtain the optimal backhaul canvassing strategy for a given voyage of a certain ship. Thus, in the first step, we generate the optimal backhaul canvassing strategy for each possible voyage of each ship in the ship pool (i.e., sub-problem three). After this step, the expected revenue generated by a voyage's optimal canvassing strategy can be attached as an attribute for the voyage. In this way, the original problem is reduced from a stochastic system with three sub-problems into a deterministic one with two sub-problems (i.e., the fleet sizing and mix problem and the voyage planning problem), and we then solve them in the second step. The second step tackles these two sub-problems in an integrated manner (see Sections 2.5 and 2.6 for details). An MIP model is first proposed for integrated fleet sizing and mix and the voyage planning problem (see Section 2.5.1 for details). We then further strengthen the model by adding several families of valid inequalities (see Section 2.5.2 for details). Through preliminary experiments, we found that it took a very long time for an off-shelf optimization solver to solve instances with realistic sizes using the MIP model. Therefore, a tailored Benders decomposition algorithm utilizing the problem structure and derived valid inequalities is proposed for solving the MIP model (see Section 2.6 for details).

2.4 DP Model and Algorithm for the Stochastic Backhaul Canvassing Problem

In this section, we focus on the stochastic backhaul canvassing problem for a given voyage of a given ship. The problem aims to determine the optimal canvassing strategy in the canvassing period that satisfies the capacity and detour length constraints to maximize the expected revenue obtained from backhaul cargo transportation. A DP model is first formulated for the problem in Section 2.4.1 and we then propose an algorithm to solve the model in Section 2.4.2.

2.4.1 A DP Model

Dynamic programming has been widely used in optimization problems, including those in the sea transportation area (e.g., Wang et al. 2017 and Zhen et al. 2017). We formulate a DP model to identify the best policy of backhaul cargo canvassing at each unit time under uncertainty. Before presenting the model, we introduce several additional parameters as follows. Suppose a ship starts a voyage at unit time t_s and ends the voyage at t_e and the minimum duration of a voyage conducted by the ship is L (time units). L equals the distance of the complete voyage starting and ending Port O (without carrying cargoes in the backhaul) divided by the ship speed plus the total port times in the origin and destination ports. Note that L is independent of t_s and t_e. Besides, let $\bar{D}$ denote the maximum allowed detour length (time units) the ship is able to make in the current voyage for transporting backhaul cargoes. It is easy to infer that $\bar{D} = t_e - t_s - L + 1$. Obviously, any backhaul transportation request that requires a detour length larger than $\bar{D}$ cannot be accepted. Similarly, requests with cargo weights larger than the capacity of the ship cannot be accepted either. Furthermore, as the information of both the voyage and the ship is known, the corresponding canvassing period can be derived accordingly. That is, we suppose the canvassing period covers N unit times (lasting from time 1 to N).

For the considered voyage, we only need to consider all of the types of transportation requests that enable feasible detour lengths and feasible weights. We let $\mathcal{M}$ denote the set of such types of transportation requests. The revenue for transporting a type-j request ($j \in \mathcal{M}$) is denoted by e_j. We assume that e_j, $j \in \mathcal{M}$ is independent of the time. In addition, we let ρ_{nj} be the probability that type-j transportation requests appear at unit time n, where $0 \leq \rho_{nj} \leq 1$. It is easy to see that at each unit time n, there are $2^{|\mathcal{M}|}$ possible combinations of different requests faced by the ship and one possible combination is called one scenario. We let ξ denote each scenario and the joint probability distribution of all scenarios at unit time n is denoted by P_n. Meanwhile, we let binary parameter $q_j(\xi)$ denote whether type-j requests arise at the beginning of a unit time under scenario ξ ($q_j(\xi)$=1) or not ($q_j(\xi)$= 0). In addition, by letting $p_n(\xi)$ be the probability of the realization of ξ at unit time n, we have

$$p_n(\xi) = \prod_{j \in \mathcal{M}} \Big[q_j(\xi)\rho_{nj} + (1 - q_j(\xi))\,(1 - \rho_{nj}) \Big], \tag{2.1}$$

where the multiplication sign is a result of the independence of the request types, and the term $q_j(\xi)\rho_{nj} + (1 - q_j(\xi))(1 - \rho_{nj})$ is equal to ρ_{nj} if type-j requests arise under scenario ξ (i.e., $q_j(\xi) = 1$) and equal to $1 - \rho_{nj}$ if type j does not arise under scenario ξ (i.e., $q_j(\xi) = 0$).

We are now ready to present the DP model, which is formulated as follows. In particular, we first define the state space and the decision variables of the model, then introduce the state transition equation and the objective function, and finally present the DP formulation.

The model involves N stages, where stage n corresponds to the nth unit time within the canvassing period. For each stage n ($n = 1, 2, \ldots, N$), a corresponding state s_n is defined to reflect whether the ship has made a decision to accept a request before stage n begins ($s_n = 1$) or not ($s_n = 0$). It is obvious that $s_1 = 0$ and if any request is accepted subsequently before some stage n, the corresponding state changes to $s_n = 1$, after which no additional acceptance is allowed.

The decision variable of the model is denoted by $\chi^j_{n,\xi}$, which is binary, to represent whether the ship accepts a type-j request at unit time n under scenario ξ ($\chi^j_{n,\xi} = 1$) or not ($\chi^j_{n,\xi} = 0$). For notation convenience, we define an $|\mathcal{M}|$ − dimensional vector $\boldsymbol{\chi}_{n,\xi} := (\chi^j_{n,\xi},\ j \in \mathcal{M})$ to include the decisions for all types of transportation requests under scenario ξ at the nth unit time. In addition, we use $\mathcal{X}_{n,\xi}(s_n)$ to denote the set of all feasible decisions $\boldsymbol{\chi}_{n,\xi}$ at unit time n under scenario ξ if the state is s_n. It follows that

$$\mathcal{X}_{n,\xi}(s_n) = \left\{ \boldsymbol{\chi}_{n,\xi} \in \{0,1\}^{|\mathcal{M}|} \;\middle|\; \chi^j_{n,\xi} \leq 1 - s_n,\ \chi^j_{n,\xi} \leq q_j(\xi),\ j \in \mathcal{M};\ \sum_{j\in\mathcal{M}} \chi^j_{n,\xi} \leq 1 \right\}, \tag{2.2}$$

which implies that: (i) a type of request can only be accepted if the ship has not accepted any request at previous unit times ($\chi^j_{n,\xi} \leq 1 - s_n$), (ii) a type of request can only be accepted if it arises ($\chi^j_{n,\xi} \leq q_j(\xi)$), and (iii) at most one type of request can be accepted ($\sum_{j\in\mathcal{M}} \chi^j_{n,\xi} \leq 1$).

Besides, corresponding to each decision $\boldsymbol{\chi}_{n,\xi}$ under each scenario ξ at stage n, we let $K_\xi(\boldsymbol{\chi}_{n,\xi})$ denote the immediate revenue (note that we remove s_n in the notation as the immediate revenue is independent of s_n), which can be calculated as follows:

$$K_\xi(\boldsymbol{\chi}_{n,\xi}) = \sum_{j\in\mathcal{M}} \chi^j_{n,\xi} e_j, \quad n = 1, 2, \ldots, N. \tag{2.3}$$

Furthermore, based on the state s_n and the decision $\boldsymbol{\chi}_{n,\xi}$ under scenario ξ at the current stage n, we can derive the state of the next stage (s_{n+1}) by the following state transition equation:

$$s_{n+1}(s_n, \boldsymbol{\chi}_{n,\xi}) = s_n + \sum_{j\in\mathcal{M}} \chi^j_{n,\xi}, \quad n = 1, 2, \ldots, N-1. \tag{2.4}$$

For the objective function, we define $f^\xi_n(s_n, \boldsymbol{\chi}_{n,\xi})$ to represent the total expected revenue from stage n until the end if the system is of state s_n at stage n under scenario ξ, the immediate decision is $\boldsymbol{\chi}_{n,\xi}$ and optimal decisions are made thereafter. In addition, define $\overline{f^\xi_n}(s_n)$ to represent the maximum total expected revenue from stage n until the end if the system is of state s_n at stage n under scenario ξ. That is, we have

$$\overline{f^\xi_n}(s_n) = \max_{\boldsymbol{\chi}_{n,\xi} \in \mathcal{X}_{n,\xi}(s_n)} \left\{ f^\xi_n\left(s_n, \boldsymbol{\chi}_{n,\xi}\right) \right\}. \tag{2.5}$$

Note that $\overline{f^\xi_n}(1) = 0$ since once a request has been accepted in previous stages, no additional revenue can be generated. Our objective is to calculate the maximum expected revenue in the return voyage, i.e., $\mathbb{E}_{P_1}\left[\overline{f_1^{\xi}}(s_1=0)\right]$.

Therefore, a forward DP formulation can be represented as

$$f_n^{\xi}(s_n, \boldsymbol{\chi}_{n,\xi}) = K_{\xi}(\boldsymbol{\chi}_{n,\xi}) + \mathbb{E}_{P_{n+1}}\left[\overline{f_{n+1}^{\xi}}(s_{n+1}(s_n, \boldsymbol{\chi}_{n,\xi}))\right], \quad n = 1, 2, \dots, N-1, \tag{2.6}$$

and the boundary condition is

$$f_N^{\xi}(s_N, \boldsymbol{\chi}_{N,\xi}) = K_{\xi}(\boldsymbol{\chi}_{N,\xi}). \tag{2.7}$$

2.4.2 A Polynomial Time Algorithm

Intuitively, the DP model proposed in Section 2.4.1 can be solved by retrospectively enumerating the value of all $f_n^{\xi}(s_n, \boldsymbol{\chi}_{n,\xi})$'s. In addition, the calculation of each $f_n^{\xi}(s_n, \boldsymbol{\chi}_{n,\xi})$ requires the values of $K_{\xi}(\boldsymbol{\chi}_{n,\xi})$, $\overline{f_n^{\xi}}(s_n)$, and $\mathbb{E}_{P_{n+1}}\left[\overline{f_{n+1}^{\xi}}(s_{n+1})\right]$, which can be obtained in $\mathcal{O}(|\mathcal{M}|)$, $\mathcal{O}(2^{|\mathcal{M}|})$ and $\mathcal{O}(2^{|\mathcal{M}|})$ times, respectively. Therefore, to obtain the optimal solution, we need to enumerate $f_n^{\xi}(s_n, \boldsymbol{\chi}_{n,\xi})$'s under all scenario ξ's, at all stage n's and for all combinations of $\boldsymbol{\chi}_{n,\xi}$ and s_n with regard to $\mathcal{X}_{n,\xi}(s_n)$, leading to $\mathcal{O}(N\,|\mathcal{M}|\,2^{|\mathcal{M}|})$ time in total. Note that $\overline{f_n^{\xi}}(s_n)$ and $\mathbb{E}_{P_{n+1}}\left[\overline{f_{n+1}^{\xi}}(s_{n+1}(s_n, \boldsymbol{\chi}_{n,\xi}))\right]$ do not have to be calculated in each enumeration. Nevertheless, in general, we have $|\mathcal{M}| \geq 100$ and the corresponding computation procedure can be very time-consuming.

To improve the computational efficiency, we propose an algorithm that solves the problem in $\mathcal{O}(N|\mathcal{M}|^2)$ time by utilizing the special structure of the problem. In particular, we have the following property the enables this algorithm.

Proposition 2.1. *Given stage n, $\mathbb{E}_{P_n}\left[\overline{f_n^{\xi}}(s_n)\right]$ can be obtained within $\mathcal{O}(|\mathcal{M}|^2)$ time.*

By taking advantage of Proposition 2.1, to obtain the optimal solution of the DP model, we can retrospectively enumerate $\mathbb{E}_{P_n}\left[\overline{f_n^{\xi}}(s_n)\right]$ for all n's without the tedious calculation of all the corresponding $f_n^{\xi}(s_n, \boldsymbol{\chi}_{n,\xi})$'s. This enables us to solve the problem in $\mathcal{O}(N|\mathcal{M}|^2)$ time. Therefore, we can efficiently generate optimal canvassing strategies and the corresponding optimal expected transportation revenues for all the possible voyages of all ships under consideration. These revenues will be used as a priori knowledge in our development of the solution method for the fleet sizing and mix and voyage planning problems in the following sections.

2.5 An MIP model for the Integrated Fleet Sizing and Mix and Voyage Planning Problem

In this section, we handle the fleet sizing and mix and voyage planning problems in an integrated method. In particular, an integrated MIP model is first formulated in Section 2.5.1 and in Section 2.5.2, the model is further strengthened by adding valid inequalities.

2.5.1 Model Formulation

Given the maximum expected revenue for shipping cargoes in the route from Port D to Port O in each possible voyage, the integrated fleet sizing and mix and voyage planning problem decides which ships should be chartered in and when each voyage should start and end. The problem is formulated as a MIP model. Before presenting the model, we first introduce the notation in Table 3.1.

Table 2.1: Notation

Indices:	
k	Index for ships, arranging in an alphabetical order.
t, h, t_1, t_2	Index for unit times in a planning horizon, arranging in a chronological order.
i	Index for sub-planning horizons.
Sets:	
$\mathcal{V}$	Set of all ships.
$\mathcal{T}$	Set of unit times in a planning horizon, with 1 standing for the first unit time and $\|\mathcal{T}\|$ standing for the last unit time in $\mathcal{T}$ (i.e., $\mathcal{T}$ lasts from the beginning of unit time 1 to the end of unit time $\|\mathcal{T}\|$.)

$\mathcal{T}_i$	Set of unit times in sub-planning horizon i, with $\underline{\mathcal{T}_i}$ and $\overline{\mathcal{T}_i}$ standing for the first and last unit time in $\mathcal{T}_i$, respectively (i.e., $\mathcal{T}_i$ lasts from the beginning of unit time $\underline{\mathcal{T}_i}$ to the end of unit time $\overline{\mathcal{T}_i}$.)
$\mathcal{I}$	Set of sub-planning horizons.
$[a, b]_Z$	Set of integers that are no less than a and no larger than b, where a and b are real numbers.
Parameters:	
$\underline{b_k}$	Minimum duration of ship k to complete a voyage.
$\overline{b_k}$	Maximum duration of ship k to complete a voyage.
$\underline{l}$	Minimum interval between the start times of any two consecutive voyages made by all the chartered ships.
R_k	Chartering rate of ship k in the planning horizon.
v_k	Weight ship k carries from Port O to Port D in each voyage.
c_k	Voyage cost of ship k to complete a voyage.
$\underline{d_i}$	Minimum weight of cargo that must be shipped from Port O in sub-planning horizon i. That is, the summation of the cargo weight shipped from Port O at the beginning of each unit time in sub-planning horizon i should at least reach $\underline{d_i}$.
$\overline{d_i}$	Maximum weight of cargo that can be shipped from Port O in sub-planning horizon i. That is, the summation of the cargo weight shipped from Port O at the beginning of each unit time in sub-planning horizon i should not exceed $\overline{d_i}$.
$\widehat{d}$	Total demand in the planning horizon, which serves as the minimum amount of cargoes that must be shipped from Port O in the whole planning horizon.
$g_k^{t_1,t_2}$	Maximum expected revenue for shipping cargoes in the route from Port D to Port O generated by ship k if it starts the voyage at the beginning of unit time t_1, and ends the voyage at the end of unit time t_2, where $\underline{b_k}-1 \leq t_2-t_1 \leq \overline{b_k}-1$; $g_k^{t_1,t_2}$ is calculated in Section 2.4.
Decision Variables:	
x_k	1 if ship k is chartered and 0, otherwise.
u_k^t	1 if ship k starts a voyage at the beginning of unit time t and 0, otherwise.

$\alpha_k^{t_1,t_2}$	1 if ship k starts a voyage at the beginning of unit time t_1 and ends the voyage at the end of unit time t_2 in a complete voyage, where $\underline{b_k} - 1 \leq t_2 - t_1 \leq \overline{b_k} - 1$ and 0, otherwise.
w_k^t	1 if ship k ends a voyage at the end of unit time t and 0, otherwise.

The mathematical formulation (M1) for the considered problem can be described as follows.

$$\text{(M1)} \quad \min \sum_{k \in \mathcal{V}} R_k x_k + \sum_{k \in \mathcal{V}} \sum_{t=1}^{|\mathcal{T}|-\underline{b_k}+1} c_k u_k^t - \sum_{k \in \mathcal{V}} \sum_{t_1=1}^{|\mathcal{T}|-\underline{b_k}+1} \sum_{t_2=t_1+\underline{b_k}-1}^{\min\{t_1+\overline{b_k}-1, |\mathcal{T}|\}} g_k^{t_1,t_2} \alpha_k^{t_1,t_2} \quad (2.8)$$

$$\text{s.t.} \quad -x_k + u_k^t \leq 0, \ \forall t \in [1, |\mathcal{T}| - \underline{b_k} + 1]_Z, \forall k \in \mathcal{V}, \quad (2.9)$$

$$-\sum_{k \in \mathcal{V}} \sum_{t=\underline{\mathcal{T}_i}}^{\min\{\overline{\mathcal{T}_i}, |\mathcal{T}|-\underline{b_k}+1\}} v_k u_k^t + \underline{d_i} \leq 0, \ \forall i \in \mathcal{I}, \quad (2.10)$$

$$\sum_{k \in \mathcal{V}} \sum_{t=\underline{\mathcal{T}_i}}^{\min\{\overline{\mathcal{T}_i}, |\mathcal{T}|-\underline{b_k}+1\}} v_k u_k^t - \overline{d_i} \leq 0, \ \forall i \in \mathcal{I}, \quad (2.11)$$

$$-\sum_{k \in \mathcal{V}} \sum_{t=1}^{|\mathcal{T}|-\underline{b_k}+1} v_k u_k^t + \widehat{d} \leq 0, \quad (2.12)$$

$$\sum_{t_1=\max\{t-\overline{b_k}+1,1\}}^{t} \sum_{t_2=\max\{t, t_1+\underline{b_k}-1\}}^{\min\{t_1+\overline{b_k}-1, |\mathcal{T}|\}} \alpha_k^{t_1,t_2} \leq 1, \ \forall t \in [1, |\mathcal{T}| - \underline{b_k} + 1]_Z, \forall k \in \mathcal{V}, \quad (2.13)$$

$$\sum_{t_2=t_1+\underline{b_k}-1}^{\min\{t_1+\overline{b_k}-1, |\mathcal{T}|\}} \alpha_k^{t_1,t_2} = u_k^{t_1}, \ \forall t_1 \in [1, |\mathcal{T}| - \underline{b_k} + 1]_Z, \forall k \in \mathcal{V}, \quad (2.14)$$

$$\sum_{t_1=\max\{1, t_2-\overline{b_k}+1\}}^{t_2-\underline{b_k}+1} \alpha_k^{t_1,t_2} = w_k^{t_2}, \ \forall t_2 \in [\underline{b_k}, |\mathcal{T}|]_Z, \forall k \in \mathcal{V}, \quad (2.15)$$

$$\sum_{k\in\mathcal{V}}\sum_{h=t}^{t+\underline{l}-1} u_k^h \le 1,\ \forall t\in[1,|\mathcal{T}|-\underline{l}+1]_Z, \tag{2.16}$$

$$u_k^t = 0,\ \forall t\in[|\mathcal{T}|-\underline{b_k}+2,|\mathcal{T}|]_Z,\ \forall k\in\mathcal{V}, \tag{2.17}$$

$$w_k^t = 0,\ \forall t\in[1,\underline{b_k}-1]_Z, \forall k\in\mathcal{V}, \tag{2.18}$$

$$\alpha_k^{t_1,t_2} = 0,\ \forall t_1\in[|\mathcal{T}|-\underline{b_k}+2,|\mathcal{T}|]_Z, \forall t_2\in\mathcal{T}, \forall k\in\mathcal{V}, \tag{2.19}$$

$$\alpha_k^{t_1,t_2} = 0,\ \forall t_1\in[1,|\mathcal{T}|-\underline{b_k}+1]_Z, \forall t_2\in[1,t_1+\underline{b_k}-2]_Z\bigcup[t_1+\overline{b_k},|\mathcal{T}|]_Z, \forall k\in\mathcal{V}, \tag{2.20}$$

$$\alpha_k^{t_1,t_2} = 0,\ \forall t_2\in[1,\underline{b_k}-1]_Z, \forall t_1\in\mathcal{T}, \forall k\in\mathcal{V}, \tag{2.21}$$

$$u_k^t, w_k^t\in\{0,1\},\ \forall t\in\mathcal{T}, \forall k\in\mathcal{V}, \tag{2.22}$$

$$x_k\in\{0,1\},\ \forall k\in\mathcal{V}, \tag{2.23}$$

$$\alpha_k^{t_1,t_2}\in\{0,1\},\ \forall t_1\in\mathcal{T}, \forall t_2\in\mathcal{T}, \forall k\in\mathcal{V}. \tag{2.24}$$

The objective function (2.8) minimizes the total operational cost, i.e., the total chartering and voyage cost minus the total revenue generated from cargo transportation in the backhaul. Constraint (2.9) ensures that a ship can be used for shipping cargoes only it is chartered. Constraints (2.10) and (2.11) enforce lower and upper bounds for the weight of cargo that can be shipped from Port O in each sub-planning horizon, respectively. Since the durations of all ships traveling from Port O to Port D are similar, these two constraints help maintain the inventory of the corporation at Port D in a suitable level. Constraint (2.12) ensures that the overall demand in the planning horizon can be met. Constraint (2.13) ensures that each ship can sail on at most one voyage at any time t. Constraints (2.14) and (2.15) describe the relationships among α, u and w. In addition, constraints (2.13)-(2.15) also ensure that once ship k starts a voyage at unit time t, to ensure minimum voyage duration, it cannot stop at any time before the end of $t+\underline{b_k}-1$ but must stop before or at the end of $t+\overline{b_k}-1$ or $|\mathcal{T}|$ if $|\mathcal{T}|<t+\overline{b_k}-1$ to meet the requirement of maximum duration limit. Constraint (2.16) ensures the minimum interval between two consecutive voyages. Constraints (2.17)-(2.21) define the values of certain decision variables and constraints (2.22)-(2.24) define binary variables.

2.5.2 Strengthening the Formulation

In this section, to efficiently solve the problem, we will derive several families of valid inequalities to strengthen the proposed MIP formulation (M1). In particular, two families of problem-specific valid inequalities are proposed in Section 2.5.2 and strengthened cover inequalities are derived in Section 2.5.2. In addition, we provide the theoretical validity proofs for the derived valid inequalities and the detailed procedure to generate strengthened cover inequalities, while their significant strength and effectiveness are verified through numerical experiments in Section 2.7.

Problem-specific Valid Inequalities

We first derive two families of valid inequalities as follows by considering the relationships among α, u, and w.

$$\sum_{h=t_1+1}^{t_2} u_k^h \leq (t_2 - t_1)\left(1 - \alpha_k^{t_1 t_2}\right), \quad \forall t_1 \in [1, |\mathcal{T}| - \underline{b_k} + 1]_Z,\ \forall t_2 \in [t_1 + \underline{b_k} - 1, \min\{|\mathcal{T}|, t_1 + \overline{b_k} - 1\}]_Z, \forall k \in \mathcal{V}, \tag{2.25}$$

$$\sum_{h=t_1}^{t_2-1} w_k^h \leq (t_2 - t_1)\left(1 - \alpha_k^{t_1 t_2}\right), \quad \forall t_1 \in [1, |\mathcal{T}| - \underline{b_k} + 1]_Z,\ \forall t_2 \in [t_1 + \underline{b_k} - 1, \min\{|\mathcal{T}|, t_1 + \overline{b_k} - 1\}]_Z,\ \forall k \in \mathcal{V}. \tag{2.26}$$

Proposition 2.2. *Inequality (2.25) is valid for (M1).*

Proposition 2.3. *Constraint (2.26) is valid for (M1).*

Cover Inequalities

We continue to strengthen (M1) by deriving strengthened cover inequalities for the model. In particular, we first introduce the method to theoretically construct the strengthened cover inequalities and then provide the procedure to generate these inequalities selectively for the particular use.

Inequality construction. For (M1), we let polytope P_i be $\{(x_k, u_k^t) \in \mathbb{B}^{|V|} \times \mathbb{B}^{|\mathcal{V}||\mathcal{T}_i|}$: (2.9), (2.11) in (M1)$\}$ for each $i \in \mathcal{I}$ and define conv(P_i) to be the convex hull of P_i. Note that $\mathbb{B}^n$ indicates n-dimensional space consisting of binary vectors and (2.11) is a knapsack constraint that complicates the model significantly. We aim to derive valid inequalities for conv(P_i) to improve the computational efficiency by analyzing the polyhedral structure of P_i. It is worth noting that any valid inequalities for conv(P_i) is also valid for the original formulation (M1).

Let set N include all the well-defined 2-tuple (k, t) for u_k^t in P_i. First, we consider a minimal cover of N, denoted by C (see Wolsey 1998 for the definition of minimal cover), such that $\sum_{(k_j,t_s)\in C} v_{k_j} u_{k_j}^{t_s} > \overline{d_i}$, the cover inequality gives us

$$\sum\nolimits_{(k_j,t_s)\in C} u_{k_j}^{t_s} \leq |C| - 1. \tag{2.27}$$

Next, we strengthen inequality (2.27) in two ways, i.e., (1) considering the problem structure and (2) lifting, eventually leading to a family of strengthened cover inequalities (2.31).

Suppose we have n ships considered in C. For each ship j $(j = 1, 2, \ldots, n)$, we have S_j time period considered to construct the 2-tuple (k_j, t_s) in C, i.e., $s = 1, 2, \ldots, S_j$. For instance, when $j = 1$, we have the 2-tuples $(k_1, t_1), (k_2, t_2), \cdots, (k_1, k_{S_1})$. Thus, we have an equivalent format of (2.27):

$$\sum_{j=1}^{n} \sum_{s=1}^{S_j} u_{k_j}^{t_s} \leq |C| - 1. \tag{2.28}$$

By considering the effects from discrete decisions on chartering a ship or not, i.e., x, we can strengthen the above cover inequality to be

$$\sum_{j=1}^{n} \sum_{s=1}^{S_j} u_{k_j}^{t_s} \leq |C| - 1 + \sum_{j=1}^{n} (S_j - 1) \left(x_{k_j} - 1\right), \tag{2.29}$$

which is stronger than (2.28) since $x_{k_j} - 1 \leq 0$ for all j's. Meanwhile, it is still valid for P_i.

In addition, for cover inequality (2.28) defined on the minimal cover C, it can also be strengthened by lifting to be

$$\sum_{j=1}^{n} \sum_{s=1}^{S_j} u_{k_j}^{t_s} + u_{\overline{k}}^{t} \leq |C| - 1 \tag{2.30}$$

for some $\overline{k}$ such that $v_{\overline{k}} \geq v_{k_j}$ for all $j = 1, 2, \ldots, n$ and some $t \in \mathcal{T}_i$. It follows that we can further strengthen (2.29) to be

$$\sum_{j=1}^{n} \sum_{s=1}^{S_j} u_{k_j}^{t_s} + u_{\overline{k}}^{t} \leq |C| - 1 + \sum_{j \in N_0} (S_j - 1) \left(x_{k_j} - 1 \right), \tag{2.31}$$

where $N_0 \subseteq \{1, 2, \ldots, n\}$ and $v_{k_j} < v_{\overline{k}}$ for all $j \in N_0$. It is easy to check that (2.31) is valid for (M1).

Inequality generation. To enable an efficient generation of the derived strengthened cover inequality (2.31) for particular use like numerical experiments in Section 2.7, here we provide the detailed procedure in two steps: (1) for each sub-planning horizon, a feasible minimal cover is identified; and (2) strengthened cover inequalities are generated based on the minimal cover by using (2.31).

In particular, for each sub-planning horizon i, a feasible minimal cover C is obtained by using the u_k^t's such that $(k, t) \in N$, where N is defined in Section 2.5.2. In addition, to further strengthen these cover inequalities, constraints (2.13) and (2.14) in (M1) are also considered in the procedure to construct the minimal cover C. Furthermore, when constructing C, priorities are given to (k, t)'s corresponding to smaller v_k's in order to generate more effective inequalities. Thus, inequality (2.31) is generated selectively and further improved. The detailed procedure is shown as follows:

The Inequality Generation Procedure.

Initiation: Let the number of the current iteration $\varsigma = 1$; construct a sequence $\Delta =$

$\{K_1, K_2, \cdots, K_i, \cdots, K_{|\mathcal{V}|}\}$ to denote a sequence of all k's, where $v_{K_i} \geq v_{K_{i+1}}$ for all $i \in [1, |\mathcal{V}|-1]_Z$;

Step 1: Initiate the ready time ϕ_k (the time when a ship is ready to start a voyage) for each ship k as $\phi_k = \underline{\mathcal{T}_i}$, define $\underline{v} = v_{K_\varsigma}$ to represent the selection lower bound (i.e., only (k,t)'s with the corresponding $v_k \geq \underline{v}$ can be selected to construct C), let the weight of cargo shipped out in the sub-planning horizon $\sigma = 0$, and let $t = \underline{\mathcal{T}_i}$ and $C = \varnothing$;

Step 2: Identify a set $\mathcal{V}'$ to include all k's such that $t \geq \phi_k$ and $v_k \geq \underline{v}$. If no such k exists, go to Step 4; otherwise, go to Step 3;

Step 3: Let $\underline{k}$ be the index such that $v_{\underline{k}} = \min_{k \in \mathcal{V}'} v_k$, add the corresponding $(\underline{k}, t)$ into C, and update $\sigma = \sigma + v_{\underline{k}}$; update $\phi_{\underline{k}} = t + b_{\underline{k}}$ and $\phi_k = t + \underline{l}$ for each $k \neq \underline{k}$. If $\sigma > \overline{d_i}$ go to Step 5; otherwise, go to Step 4;

Step 4: Update $t = t + 1$;

Step 5: Among all the (k,t)'s in C, identify the one corresponding to the smallest v_k and mark it as $v_{\underline{k}}$. If $\sigma - v_{\underline{k}} < \overline{d_i}$ go to Step 7; otherwise, go to Step 6;

Step 6: Update $\varsigma = \varsigma + 1$. If $\varsigma > |\mathcal{V}|$, output $C = \varnothing$; otherwise, go to Step 1;

Step 7: Output the minimal cover C.

If a non-empty minimal cover is identified by the above procedure for a sub-planning horizon i, we add the corresponding strengthened cover inequality (2.31) into the model.

2.6 A Benders Decomposition Algorithm for the Integrated Fleet Sizing and Mix and Voyage Planning Problem

In this section, to further enable the applicability and effectiveness of our proposed formulation (M1) and valid inequalities at the scale required in the industry, we develop a decomposition algorithm to solve the problem. In particular, a tailored Benders decomposition algorithm will be proposed with our derived valid inequalities embedded. Note that Benders decomposition algorithm (Benders 1962) has been successfully applied to a wide range of difficult optimization problems (e.g., Shen and Chen 2013, An et al. 2014, and Arslan and Karaşan 2016).

Observe that once the values of the $\alpha_k^{t_1,t_2}$ variables are fixed, the values of the u_k^t and w_k^t variables are also fixed. In this case, the u_k^t and w_k^t variables can be relaxed to be continuous variables in the problem, which can be efficiently solved. Based on this observation we develop a Benders decomposition algorithm for solving M1. In the proposed algorithm, the model is divided into a master problem and a sub-problem, both of which are solved iteratively and updated after each iteration. After solving the sub-problem in each iteration with a given solution from the master problem, new constraints (i.e., feasibility and optimality cuts) are added into the master problem, which will be solved again towards the optimal solution. Using the solutions of the master problem and the sub-problem, the lower and upper bounds of the objective value of the original problem are updated, respectively. The algorithm stops when the optimal solution is found or when the gap between the lower and upper bounds of the problem reaches a preset threshold ε. Section 2.6.1 reformulates (M1) into a master problem and a sub-problem. The cutting-plane method used in the algorithm is introduced in Section 2.6.2. In Section 2.6.3, we outline the procedure of the algorithm.

2.6.1 Model Reformulation

In the proposed solution method, model (M1) is divided into a master problem (MP) and a sub-problem (SP). The master problem (MP) is formulated as follows:

$$\text{(MP)} \;\; \min f = \sum_{k\in\mathcal{V}} R_k x_k - \sum_{k\in\mathcal{V}} \sum_{t_1=1}^{|\mathcal{T}|-\underline{b_k}+1} \sum_{t_2=t_1+\underline{b_k}-1}^{\min\{t_1+\overline{b_k}-1,|\mathcal{T}|\}} g_k^{t_1,t_2} \alpha_k^{t_1,t_2} + \eta \tag{2.32}$$

$$\text{s.t.} \quad \sum_{t\in[1,|\mathcal{T}|-\underline{b_k}+1]} u_k^t \geq x_k, \tag{2.33}$$

$$\eta \geq \underline{\eta}, \tag{2.34}$$

$$(2.9), (2.11), (2.13) - (2.16), (2.19) - (2.24), (2.25), (2.26), (2.31),$$
feasibility cuts,
optimality cuts,

where feasibility and optimality cuts are added after solving the sub-problem at each iteration and inequality (2.31) is generated through the inequality generation procedure described in Section 2.5.2. Constraint (2.33) ensures that a vessel chartered must start at least one voyage. In constraint (2.34), $\underline{\eta}$ is a lower bound for η and is calculated by the following equation:

$$\underline{\eta} = \left\lceil \frac{\widehat{d}}{\max_k\{v_k\}} \right\rceil \min_k \{c_k\}. \tag{2.35}$$

In the equation above, $\left\lceil \frac{\widehat{d}}{\max_k\{v_k\}} \right\rceil$ gives the minimum number of voyages needed to fulfill the total demand and $\min_k\{c_k\}$ represents the minimum cost for each of the voyages.

We then describe the sub-problem (SP). For model (M1), we can observe that as long as the variable α is given, we have variables u and w fixed. Therefore, in the following model of the sub-problem, we relax u and w to be continuous, leading to a linear program. Note that when the sub-problem is a linear program, Benders decomposition algorithm theoretically guarantees convergence to optimality after a certain number of iterations (Laporte and Louveaux 1993).

$$\text{(SP)} \ \min \sum_{k \in \mathcal{V}} \sum_{t=1}^{|\mathcal{T}|-\underline{b_k}+1} c_k u_k^t \tag{2.36}$$

$$\text{s.t. } u_k^{t_1} = \sum_{t_2=t_1+\underline{b_k}-1}^{\min\{t_1+\overline{b_k}-1, |\mathcal{T}|\}} \overline{\alpha_k^{t_1,t_2}}, \ \forall t_1 \in [1, |\mathcal{T}| - \underline{b_k} + 1]_Z, \forall k \in \mathcal{V}, \tag{2.37}$$

$$w_k^{t_2} = \sum_{t_1=\max\{1, t_2-\overline{b_k}+1\}}^{t_2-\underline{b_k}+1} \overline{\alpha_k^{t_1,t_2}}, \ \forall t_2 \in [\underline{b_k}, |\mathcal{T}|]_Z, \forall k \in \mathcal{V}, \tag{2.38}$$

$$u_k^t \geq 0, \ \forall t \in \mathcal{T}, \forall k \in \mathcal{V}, \tag{2.39}$$

$$u_k^t \leq 1, \ \forall t \in \mathcal{T}, \forall k \in \mathcal{V}, \tag{2.40}$$

$$w_k^t \geq 0, \ \forall t \in \mathcal{T}, \forall k \in \mathcal{V}, \tag{2.41}$$

$$w_k^t \leq 1, \ \forall t \in \mathcal{T}, \forall k \in \mathcal{V}, \tag{2.42}$$

$$(2.10), (2.12), (2.17), (2.18),$$

where $\overline{\alpha_k^{t_1,t_2}}$ is the optimal solution for $\alpha_k^{t_1,t_2}$ obtained by solving (MP).

The (SP) is then dualized, with dual variables μ_i, σ, λ_{kt}^u, λ_{kt}^w, θ_{kt}^u, θ_{kt}^w, π_{kt}^{u+}, π_{kt}^{u-}, π_{kt}^{w+} and π_{kt}^{w-} corresponding to constraints (2.10), (2.12), (2.37), (2.38), (2.17), (2.18), (2.39), (2.40), (2.41) and (2.42), respectively. Furthermore, let $\mathcal{I}_t^T$ denote the set of sub-planning horizons that contain unit time t. The dual of the sub-problem, i.e., (SPD), is formulated as follows:

$$\text{(SPD)} \ \max \ g = \sum_{i \in \mathcal{I}} \underline{d_i} \mu_i + \widehat{d} \sigma + \sum_{k \in \mathcal{V}} \sum_{t_1=1}^{|\mathcal{T}|-\underline{b_k}+1} \left(\sum_{t_2=t_1+\underline{b_k}-1}^{\min\{t_1+\overline{b_k}-1, |\mathcal{T}|\}} \overline{\alpha_k^{t_1,t_2}} \right) \lambda_{kt_1}^u$$
$$+ \sum_{k \in \mathcal{V}} \sum_{t_2=\underline{b_k}}^{|\mathcal{T}|} \left(\sum_{t_1=\max\{1, t_2-\overline{b_k}+1\}}^{t_2-\underline{b_k}+1} \overline{\alpha_k^{t_1,t_2}} \right) \lambda_{kt_1}^w - \sum_{k \in \mathcal{V}} \sum_{t \in \mathcal{T}} \pi_{kt}^{u-} - \sum_{k \in \mathcal{V}} \sum_{t \in \mathcal{T}} \pi_{kt}^{w-} \tag{2.43}$$

$$\text{s.t.} \ v_k \sum_{i \in \mathcal{I}_t^T} \mu_i + v_k \sigma + \lambda_{kt}^u + \pi_{kt}^{u+} - \pi_{kt}^{u-} \leq c_k,$$
$$\forall t \in [1, \min\{\overline{\mathcal{T}_i}, |\mathcal{T}| - \underline{b_k} + 1\}]_Z, \forall i \in \mathcal{I}, \forall k \in \mathcal{V}, \tag{2.44}$$

$$v_k \sigma + \lambda_{kt}^u + \pi_{kt}^{u+} - \pi_{kt}^{u-} \leq c_k, \ \forall t \in \mathcal{C}_{[1, |\mathcal{T}|-\underline{b_k}+1]_Z} \bigcup_{i \in \mathcal{I}} \mathcal{T}_i, \forall k \in \mathcal{V}, \tag{2.45}$$

$$\lambda_{kt}^w + \pi_{kt}^{w+} - \pi_{kt}^{w-} \leq 0, \ \forall t \in [\underline{b_k}, |\mathcal{T}|]_Z, \forall k \in \mathcal{V}, \tag{2.46}$$

$$\theta_{kt}^u - \pi_{kt}^{u+} - \pi_{kt}^{u-} \leq 0, \ \forall t \in [|\mathcal{T}| - \underline{b_k} + 2, |\mathcal{T}|]_Z, \forall k \in \mathcal{V}, \tag{2.47}$$

$$\theta_{kt}^w - \pi_{kt}^{w+} - \pi_{kt}^{w-} \leq 0, \ \forall t \in [1, \underline{b_k} - 1]_Z, \forall k \in \mathcal{V}, \tag{2.48}$$

$$\mu_i \geq 0, \ \forall i \in \mathcal{I}, \tag{2.49}$$

$$\sigma \geq 0, \tag{2.50}$$

$$\pi_{kt}^{u+} \geq 0,\ \forall t \in \mathcal{T}, \forall k \in \mathcal{V}, \tag{2.51}$$

$$\pi_{kt}^{u-} \geq 0,\ \forall t \in \mathcal{T}, \forall k \in \mathcal{V}, \tag{2.52}$$

$$\pi_{kt}^{w+} \geq 0,\ \forall t \in \mathcal{T}, \forall k \in \mathcal{V}, \tag{2.53}$$

$$\pi_{kt}^{w-} \geq 0,\ \forall t \in \mathcal{T}, \forall k \in \mathcal{V}, \tag{2.54}$$

where the feasible set of constraint (2.45) is defined as the set of t's that belong to the set $[1, |\mathcal{T}| - \underline{b_k} + 1]_Z$ and do not belong to the set $\mathcal{T}_i$ for any $i \in \mathcal{I}$.

2.6.2 Feasibility and Optimality Cuts

In each iteration, if (SPD) is found to be unbounded, a feasibility cut (2.55) is added into (MP):

$$\sum_{i \in \mathcal{I}} \underline{d_i}\overline{\mu_i^+} + \widehat{d}\overline{\sigma} + \sum_{k \in \mathcal{V}} \sum_{t_1=1}^{|\mathcal{T}|-\underline{b_k}+1} \left(\sum_{t_2=t_1+\underline{b_k}-1}^{\min\{t_1+\overline{b_k}-1, |\mathcal{T}|\}} \alpha_k^{t_1,t_2} \right) \overline{\lambda_{kt_1}^u}$$
$$+ \sum_{k \in \mathcal{V}} \sum_{t_2=\underline{b_k}}^{|\mathcal{T}|} \left(\sum_{t_1=\max\{1, t_2-\overline{b_k}+1\}}^{t_2-\underline{b_k}+1} \alpha_k^{t_1,t_2} \right) \overline{\lambda_{kt_1}^w} - \sum_{k \in \mathcal{V}} \sum_{t \in \mathcal{T}} \overline{\pi_{kt}^{u-}} - \sum_{k \in \mathcal{V}} \sum_{t \in \mathcal{T}} \overline{\pi_{kt}^{w-}} \leq 0, \tag{2.55}$$

where $\overline{\mu_i^+}$, $\overline{\sigma}$, $\overline{\lambda_{kt_1}^u}$, $\overline{\lambda_{kt_1}^w}$, $\overline{\pi_{kt}^{u-}}$, and $\overline{\pi_{kt}^{w-}}$ correspond to the extreme ray of (SPD). Otherwise, (SPD) is solved to the optimum and the following optimality cut is generated for (MP):

$$\sum_{i \in \mathcal{I}} \underline{d_i}\overline{\mu_i^+} + \widehat{d}\overline{\sigma} + \sum_{k \in \mathcal{V}} \sum_{t_1=1}^{|\mathcal{T}|-\underline{b_k}+1} \left(\sum_{t_2=t_1+\underline{b_k}-1}^{\min\{t_1+\overline{b_k}-1, |\mathcal{T}|\}} \alpha_k^{t_1,t_2} \right) \overline{\lambda_{kt_1}^u}$$
$$+ \sum_{k \in \mathcal{V}} \sum_{t_2=\underline{b_k}}^{|\mathcal{T}|} \left(\sum_{t_1=\max\{1, t_2-\overline{b_k}+1\}}^{t_2-\underline{b_k}+1} \alpha_k^{t_1,t_2} \right) \overline{\lambda_{kt_1}^w} - \sum_{k \in \mathcal{V}} \sum_{t \in \mathcal{T}} \overline{\pi_{kt}^{u-}} - \sum_{k \in \mathcal{V}} \sum_{t \in \mathcal{T}} \overline{\pi_{kt}^{w-}} \leq \eta, \tag{2.56}$$

where $\overline{\mu_i^+}$, $\overline{\sigma}$, $\overline{\lambda_{kt_1}^u}$, $\overline{\lambda_{kt_1}^w}$, $\overline{\pi_{kt}^{u-}}$, and $\overline{\pi_{kt}^{w-}}$ are optimal values delivered by solving (SPD).

2.6.3 Algorithm Procedure

The detailed procedure for the algorithm can be summarized as follows:

Initialization: Let the lower bound (LB) and the upper bound (UB) of the problem to be $-\infty$ and $+\infty$, respectively; calculate the lower bound $\underline{\eta}$ for variable η using the input problem data;

Step 1: Solve (MP) to optimum and update the optimal values $\overline{a_k^{t_1,t_2}}$ and $\overline{\eta}$ of variables $a_k^{t_1,t_2}$ and η. Update $LB = f$ if $LB < f$;

Step 2: Solve (SPD) using $\overline{a_k^{t_1,t_2}}$ delivered by Step 1 if (SPD) is bounded, go to Step 3; otherwise, go to Step 4.

Step 3: Calculate the optimal solution $\overline{u_k^t}$ for (SPD) and update $UB = f - \overline{\eta} + g$ if $UB > f - \overline{\eta} + g$. In addition, add optimality cut (2.56) into (MP);

Step 4: Add feasibility cut (2.55) into (MP);

Step 5: If $UB - LB < \varepsilon$, the algorithm stops and outputs the best-found solution; otherwise, go to Step 1.

2.7 Numerical Experiments

In this section, we perform extensive computational experiments to verify the applicability and effectiveness of our proposed models and solution methods. In addition, we provide a case study to further investigate the solution structure of the problem. We first generate a set of instances in terms of different input parameters and solve all the instances by the two-step solution method proposed in previous sections. While in the first step, we solve the stochastic backhaul canvassing problem using the DP algorithm proposed in Section 2.4, in the second step, we solve the integrated fleet sizing and mix and voyage planning problem by four different methods: (i) CPLEX using model (M1), (ii) CPLEX using model (M1) with inequalities (2.25) and (2.26) (denoted by M2), (iii) CPLEX using model (M1) with inequalities (2.25) and (2.26) and strengthened cover inequalities generated in Section 2.5.2 (denoted by M3), and

(iv) the Benders decomposition algorithm (denoted as BD). All the experiments are coded in C++ calling CPLEX 12.6 and are conducted on an Intel Core i7 2.50 GHz PC with 8 GB RAM.

2.7.1 Instance Generation

In order to test the performances of the proposed algorithms, we generated 20 instances based on real-world cases. These cases have different settings of the length of the planning horizon ($|\mathcal{T}|$) and the number of ships in the candidate pool ($|\mathcal{V}|$). In particular, $|\mathcal{T}|$ is set to be 60, 90, 120, 150 and 180 unit times and $|\mathcal{V}|$ is set to be 6, 9, 12 and 15. Other input data involving the demand structure, ships in the candidate pool, and backhaul cargoes for these instances are generated as follows.

We first look at the demand structure of these instances. To begin with, for an instance with a $|\mathcal{T}|$-unit-time planning horizon, the total demand ($\widehat{d}$) is set as $\omega\,|\mathcal{T}|$ thousand tons, where ω is randomly generated within the range $[20, 30]$. In addition, a total number of $|\mathcal{T}|/15 - 1$ sub-planning horizons are generated, where the ith ($i = 1, 2, 3, \ldots, |\mathcal{T}|/15 - 1$) sub-planning horizon contains 30 unit times lasting from the $(15i-14)$th to the $15(i+1)$th unit time. Then, for each sub-planning horizon, we set the lower and upper bounds for the amount of cargoes that can be shipped out of Port O to ensure that the cargoes are transported in a stable and balanced manner. Finally, the minimal time interval between the start times of two consecutive voyages are randomly generated within the range $[2, 5]$ (unit times).

As for the data of ships, first, the capacities of the ships are generated within the range $[160, 200]$ (thousand tons). Then, the chartering rates and voyage costs for each ship are generated randomly within reasonable ranges, considering that ships with larger capacities have higher chartering rates and voyage costs. In addition, the minimum durations of voyages of these ships are also generated randomly, within $[17, 20]$ (unit times) (we set differences of minimum durations of voyages of ships in one instance to be less than 2 unit times). Finally, the maximum duration of voyages for each ship is set to be ϖ unit times longer than their minimum durations, where ϖ is randomly generated within the range $[3, 7]$ and we have the same ϖ for all ships in one instance.

In the backhaul cargo part, for each testing instance, we generated 100 types of cargo transportation requests. First, the weights and required detour lengths of the cargoes in these requests are generated randomly within ranges $[120, 180]$ (thousand tons) and $[1, 10]$ (unit times), respectively. In addition, the canvassing period for each voyage is set to be as long as $1/4$ of the minimum voyage duration, starting from the unit time which is $1/2$ of the minimum voyage duration after the start time of the voyage and ending at the unit time which is $3/4$ of the minimum voyage duration after the start time of the voyage. Finally, the freight rates of these backhaul cargoes are generated randomly, considering that voyages with longer durations have higher backhaul transportation freight rates.

2.7.2 Results of Numerical Experiments

We solve these instances by the proposed two-step solution method. Since in the first step, the proposed DP algorithm can solve the stochastic backhaul canvassing problem very efficiently for all instances, we only present the computational results obtained by different methods in the second step. For each algorithm in the second step, we set the optimality gap to be 0.5% and the time limit to be 7200 seconds. The computational results are reported in Table 3.2.

Table 2.2: Computational results

Instance No.	$\|T\|$	$\|\mathcal{V}\|$	M1	Time(s)	M2	Time(s)	M3	Time(s)	BD	Time(s)
1	60	6	14915.36	19.52	14915.36	18.38	14915.36	18.56	14915.36	6.73*
2	60	9	14660.20	209.48	14660.20	200.13	14660.20	45.48	14660.20	13.92*
3	60	12	13924.44	345.44	13924.44	322.39	13924.44	76.13	13924.44	7.22*
4	60	15	16463.36	299.89	16463.36	307.80	16463.36	719.06	16463.36	52.47*
5	90	6	19604.92	52.66	19604.92	51.45	19604.92	51.41	19604.92	12.53*
6	90	9	25976.20	257.77	25976.20	237.05	25976.20	250.94	25976.20	123.41*
7	90	12	18358.68	432.27	18358.68	436.14	18358.68	386.08	18358.68	119.92*
8	90	15	24784.28	3590.73	24784.28	3605.39	24784.28	2382.05	24784.28	209.02*
9	120	6	33055.84	54.61	33055.84	56.94	33055.84	55.09	33055.84	5.48*
10	120	9	27826.76	2994.75	27826.76	3173.50	27826.76	7200.00	27834.60	456.94*
11	120	12	28314.68	5362.13	28314.68	5085.00	28314.68	7200.00	28314.68	626.49*
12	120	15	27879.80	3734.67	27879.80	4114.38	27879.80	2478.61	27879.80	1172.78*
13	150	6	36509.20	2114.08	36509.20	2205.86	36509.20	2108.89	36509.20	715.55*
14	150	9	37187.88	2303.64	37187.88	2303.42	37187.88	2347.14	37187.88	948.83*
15	150	12	31698.92	4196.06	31698.92	4072.67	31698.92	7200.00	31698.92	1873.63*
16	150	15	34242.92	7200.00	34242.92	7200.00	34242.92	7200.00	34266.72	1830.41*
17	180	6	39387.56	7200.00	39387.56	7200.00	39387.56	7200.00	39387.56	457.36*
18	180	9	40674.00	2531.27	40674.00	2486.31	40674.00	5554.31	40674.00	736.92*
19	180	12	39121.80	6486.19	39121.80	6471.00	39121.80	7200.00	39125.80	1694.94*
20	180	15	45043.20	7200.00	45043.20	7200.00	44458.00	7200.00	44285.20	7200.00*
Average			28481.50	2829.26	28481.50	2837.39	28452.24	3343.70	28445.38	913.23

Notes: The units of the solutions are thousand dollars;
The superscript "*" stands for the minimum solution time for an instance.

As shown in Table 3.2, the proposed Benders decomposition algorithm manages to obtain optimal solutions for 19 out of the 20 instances, and the optimality gap for the instance for which the algorithm failed to find an optimal solution is merely 1.05%. In comparison, M1, M2 and M3 can only optimally solve 17, 17, and 13 instances, respectively. As for the solution speed, the proposed Benders decomposition algorithm outperforms all the other methods for solving all instances.

Therefore, the experimental results attest that the proposed Benders decomposition algorithm and our derived valid inequalities can solve the considered problem efficiently.

2.7.3 Case Study

To further analyze the solution structure of the problem and to investigate the impacts of various input parameters, this section presents a case study based on the practices in Baowu Group (see Section 3.3 for details). We show the obtained optimal solution of the instance and further examine the impacts of three key parameters (i.e., the total demand, the maximal detour length, and the backhaul cargo condition) upon the optimal solution.

Instance Data and Solution

Here we let one unit time indicate four days and thus the planning horizon contains 60 unit times (240 days) and total demand is 1500 thousand tons. In addition, the planning horizon is divided into three sub-planning horizons whose covering ranges are $[1, 30]$, $[16, 45]$, and $[30, 59]$ (unit times), respectively. The upper bounds for the weight of cargoes that can be transported in these sub-planning horizons are all 950 thousand tons and the lower bounds are set as 450 thousand tons for the first 2 sub-planning horizons and 0 thousand ton for the last sub-planning horizon. In addition, the minimal time interval between two consecutive voyages is 3 unit times. Moreover, the condition of backhaul cargo transportation requests is generated according to the description in the previous section. In particular, assuming a voyage with the minimum duration d starts at the t_sth unit time, the canvassing period of each voyage lasts from the beginning of the $\lceil 0.5d + t_s \rceil$th unit time to the beginning of the $\lceil 0.75d + t_s \rceil$th unit time. Finally, the input data of the ships in the instance are given in Table 2.3.

Table 2.3: Ship data of the case

Ship	Capacity ($\times 10^3$ tons)	Chartering rate ($\times 10^3$ dollars)	Voyage cost ($\times 10^3$ dollars)	Voyage duration range (unit times)
1	190	3114.72	1127.20	[18, 24]
2	170	2903.04	1050.40	[17, 23]
3	190	3134.88	1134.40	[18, 24]
4	200	3257.52	1178.80	[19, 25]
5	180	3010.56	1089.20	[18, 24]
6	180	3057.60	1106.40	[18, 24]

We solve the instance by the Benders decomposition algorithm, and the obtained optimal total cost is 14.92 million dollars. In the optimal solution, three ships (Ships 1, 3, and 5) are chartered with the total chartering cost equal to 9.26 million dollars. Ships 1 and 3 complete three voyages and Ship 5 completes two voyages during the planning horizon and the detailed voyage planning results for these ships are shown in Table 2.4. Take Ship 1 for example: the ship starts its first voyage at the beginning of the first unit time at Port O and returns to the same port at the end of the 19th unit time. In this voyage, the weight of cargoes transported from Port O to Port D is 190 thousand tons and the voyage cost is 1127.20 thousand dollars. In the backhaul of this voyage, Ship 1 carries cargoes from the spot market and the expected revenue is 263.74 thousand dollars.

Impact of the Total Demand

We first study the impact of the total demand upon the optimal solution. To do this, we gradually change the demand from 1000 to 2000 (thousand tons) and solve the instance with new demand requirements. Note that the lower and upper bounds for the weight of cargoes to be transported in these sub-planning horizons are also changed correspondingly. Table 2.5 shows the optimal solutions for the instance with

Table 2.4: Voyage planning result of the case

Ship	Voyage	Start (unit time)	End (unit time)	Cargo Weight ($\times 10^3$ tons)	Cost ($\times 10^3$ dollars)	Revenue ($\times 10^3$ dollars)[1]
1	v1	1	19	190	1127.20	263.74
	v2	20	38	190	1127.20	364.08
	v3	39	60	190	1127.20	538.38
3	v1	4	22	190	1134.40	322.58
	v2	23	41	190	1134.40	312.20
	v3	42	60	190	1134.40	367.54
5	v1	7	30	180	1089.20	575.12
	v2	31	54	180	1089.20	564.36
		Total		1500	8963.20	3308.00

Notes: [1.] Expected revenue for transporting cargoes in backhauls.

various demands.

Table 2.5: Optimal solutions for the case with various demands

Total demand ($\times 10^3$ tons)	Chartered ships (no. of voyages)	Chartering cost ($\times 10^3$ dollars)	Voyage cost ($\times 10^3$ dollars)	Revenue ($\times 10^3$ dollars)[1]	Total cost ($\times 10^3$ dollars)
1000	2(2),5(2),6(2)	8971.20	6492.00	3400.86	12062.34
1100	1(2),2(2),3(2)	9152.64	6624.00	3400.86	12375.78
1200	2(3),5(2),6(2)	8971.20	7542.40	3886.26	12627.34
1300	1(2),3(2),5(3)	9260.16	7790.80	3674.52	13376.44
1400	2(3),5(3),6(2)	8971.20	8631.60	3795.34	13807.46
1500	1(3),3(3),5(2)	9260.16	8963.20	3308.00	14915.36
1600	1(2),2(3),5(2),6(2)	12085.92	9796.80	5008.80	16873.92
1700	1(3),3(2),4(2),5(2)	12517.68	10186.40	4800.68	17903.40
1800	1(3),2(3),5(2),6(2)	12085.92	10924.00	4926.70	18083.22
1900	1(3),3(3),4(2),5(2)	12517.68	11320.80	4439.36	19399.12
2000	1(2),2(3),4(2),5(2),6(2)	15343.44	12154.40	5928.66	21569.20

Notes: [1]. Expected revenue for transporting cargoes in backhauls.

As can be seen in Table 2.5, the optimal total cost shows a generally increasing trend with the growth of the total demand. To be more specific, the optimal cost slightly increases when the total demand grows from 1000 to 1500 thousand tons. Then, it undergoes a sharp increase when the total demand increases from 1500 thousand tons to 1600 thousand tons. The reason for the sharp increase is that the number of chartered ships increases from three to four and the total number of voyage increases from eight to nine in order to satisfy the strengthened demand requirement. Afterward, the optimal cost gradually increases when the total demand changes from 1600 to 2000 thousand tons. It is worth noting that when the total demand reaches 2000 thousand tons, the corporation needs to charter in one more ship to fulfill the increased demand. However, thanks to the increased revenue obtained in backhaul cargo transportation, the total cost does not increase much.

Impact of the Maximal Detour Length

We then study the impact of the maximal detour length. The maximal detour length for a voyage equals the difference between the maximal duration and the minimal duration of the voyage, and we set congruent maximal detour lengths for all voyages in an instance. We investigate its impact by varying the maximal detour length from 0 to 8 unit times and comparing the corresponding optimal costs obtained by solving the instance. Figure 2.3 shows the relationship between the optimal cost and the maximal detour length. The figure shows that the optimal cost decreases with the increase of the maximal detour length. However, the decreasing speed gradually slows down.

Impact of the Backhaul Cargo Condition

In the last part, we analyze the impact of the backhaul cargo condition upon the optimal solution. In the current setting, it is assumed that the freight rate of transporting cargoes from the same type of request in the backhaul keeps unchanged during the entire planning horizon. However, it is possible that within certain periods of the horizon, the freight rates may fluctuate due to uncertainties in the shipping market.

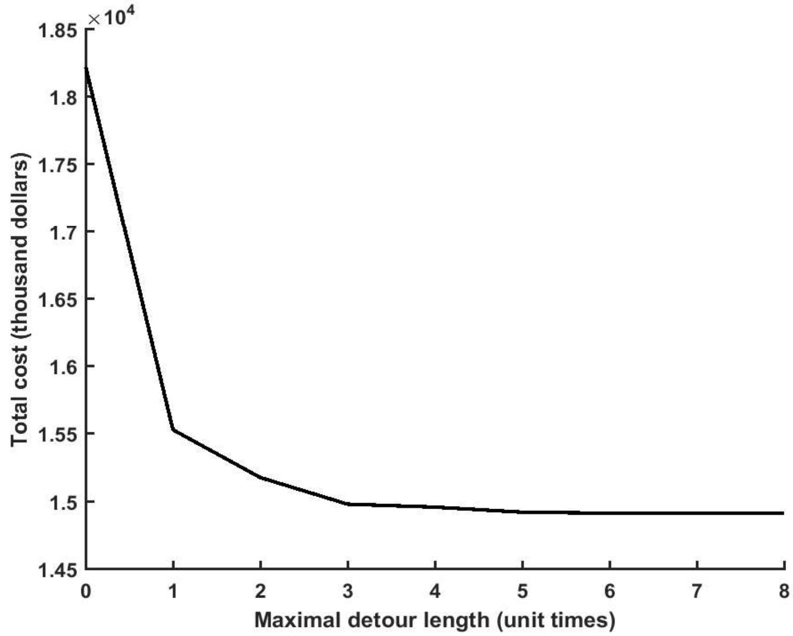

Figure 2.3: Optimal costs with changing maximal detour length

We inspect how such fluctuations would affect the ship scheduling result. To do this, we suppose that the freight rates of transporting all types of cargoes first increase by 30% during the period from the 20th to the 25th unit time and then decrease by 30% from the 40th to the 45th unit time. The instance is then solved under the new settings. The obtained new optimal cost is 14.86 million dollars which is lower than the previous 14.92 million dollars and the new voyage planning result is shown in Table 2.6.

When comparing with Table 2.4, we can find that although the chartered ships remain unchanged, the schedules of them have been adjusted. In particular, the start time of the first voyage of ship 5 is rescheduled from the 7th unit time to the 11th unit time, in order to embrace the increase in freight rates of transportation requests that starts from the 20th unit time. Moreover, the start time of the second voyage of the ship is deferred by 5 unit times to avoid canvassing in periods when freight rates are lower than normal as much as possible.

Table 2.6: Voyage planning result of the case with varied backhaul cargo condition

Ship	Voyage	Start (unit time)	End (unit time)	Cargo Weight ($\times 10^3$ tons)	Cost ($\times 10^3$ dollars)	Revenue ($\times 10^3$ dollars)[1]
1	v1	1	19	190	1127.20	263.74
	v2	20	38	190	1127.20	364.08
	v3	39	60	190	1127.20	538.38
3	v1	4	22	190	1134.40	322.58
	v2	23	41	190	1134.40	312.20
	v3	42	60	190	1134.40	367.54
5	v1	11	34	180	1089.20	655.56
	v2	36	59	180	1089.20	545.86
		Total		1500	8963.20	3369.94

Notes: [1.] Expected revenue for transporting cargoes in backhauls.

2.8 Conclusion

Maritime transportation forms the backbone of the world economy and industrial bulk shipping is an important part of maritime transportation. This chapter studied the stochastic bulk ship scheduling problem in industrial shipping. The problem can be divided into three inter-connected sub-problems from different planning levels, which are the strategic fleet sizing and mix problem, the tactical voyage planning problem, and the operational stochastic backhaul cargo canvassing problem. We proposed a two-step solution method to solve the three sub-problems in an integrated manner. In the first step, a dynamic programming algorithm was proposed to solve the stochastic backhaul cargo canvassing problem and the obtained maximum expected revenues for backhaul cargo transportation of all possible voyages were generated a priori for the second step. In the second step, we formulated a mixed-integer programming model for the integrated fleet sizing and mix and voyage planning problem. To efficiently solve the model, we first strengthened it by adding several families of valid inequalities and then proposed a tailored Benders decomposition method. Extensive numerical experiments were conducted to test the performance of the proposed models and the algorithms and the results demonstrated that our proposed approach can efficiently solve the considered problem. Finally, a case study with sensitivity analysis was conducted and the results were discussed in detail.

For future studies, we find two promising directions. First, in view of the complexity of the problem, it would be interesting to develop efficient heuristic algorithms for solving the considered problem on larger scales. Second, industrial corporations may choose to charter in and redeliver ships at different times within the planning horizons and the consideration of the flexible chartering strategy is a natural extension of the current study.

Chapter 3

The Robust Bulk Ship Routing Problem with Batched Cargo Selection

3.1 Introduction

Ships carry 80% of cargoes by volume around the world, and the dry bulk shipping sector accounts for nearly half of the global seaborne transportation volume, amounting to 5.2 billion tons in 2018 (UNCTAD 2019). Tramp shipping, which operates on customers' callings, is by far the most commonly used transportation mode in dry bulk shipping. In bulk tramp shipping, shipping companies act as carriers that transport cargoes from one port to another by following the orders from the customers (shippers). A tramp shipping company participates in maritime transportation by owning or controlling a fleet of bulk ships, and profits from the freight gained by transporting cargoes.

For a tramp shipping company, fleet adjustment, cargo selection, and ship routing which directly impact its revenue, are the three most important decisions. Over a given planning horizon (e.g., three months), fleet adjustment determines which ships to charter in and out. Seaborne bulk transportation is capital intensive, with daily operational costs for a bulk ship amounting to tens of thousands of US dollars (Greiner 2013). Therefore, in order to reduce their costs, tramp shipping companies generally choose to adjust the composition of their fleet according to the variations in demand. For instance, Pacific Basin (PB), one of the largest bulk tramp shipping companies (Pacific Basin 2018), dynamically adjusts its fleet to meet changes in demand. According to our interviews with managers from PB, the company controls a fleet of 251 ships, among which nearly half (109) are short-term chartered (the chartering periods can be several months).

Cargo selection arises when a set of cargo transportation offers from the shippers are provided, and the company must decide which offers to accept and which ones to reject. It is worth mentioning that in the current bulk shipping, most of the offers come in the form of Contract of Affreightment (COA). Each COA may contain multiple transportation requests covering a period ranging from a few months to several years. The cargoes contained in one COA should be accepted or rejected as a whole. COA has by far become the most commonly used framework for long-term transportation contracts in tramp shipping and generates a large proportion of revenues for most tramp shipping companies, and in some cases, the proportion can be 100% (Fagerholt et al. 2010). Besides cargoes in COAs, cargoes from the spot market serve as another source that brings additional revenues to tramp shipping companies. Unlike those in COAs, cargoes from the spot market can be selected on a one-by-one basis. For a tramp shipping company, selecting the most favorable combination to transport from a set of offers of COAs and spot market cargoes is vital for its profitability. The same problem is also faced by Pacific Basin. For example, in its trans-Pacific business sector alone (i.e., cargo transportation between the west coast of North America and the Far East), PB receives around 20 COA offers every year and currently transports cargoes from 10 COAs between the west coast of North America and the Far East.

Finally, based on the results of the two strategic decisions discussed above, the company should decide how to arrange the ships in its fleet to transport the accepted cargoes. This falls into the well-known ship routing problem (in its most traditional sense), and the objective here is to complete the transportation tasks at the minimum cost.

Maritime transportation suffers from great uncertainties (see the discussion of uncertainties in maritime transportation by Pantuso et al. 2014a). In particular, the cost of a voyage between two ports frequently fluctuates due to uncertain sea conditions (e.g., unpredictable weather and current conditions in the sea) and uncertain ship status (e.g., trim and mechanical conditions of a ship). Nowadays, the dry bulk shipping market is faced with overcapacity and declining demands. The profit margins of bulk shipping companies are thin (typically within 5%) and voyage cost accounts for a large proportion of the total operational cost. For example, according to PB's 2018 Annual Report (Pacific Basin 2018), the profit margin in that year was 5% (the number in 2017 was merely 1%) and the voyage cost accounts for nearly 50% of the total cost. To survive in the harsh business environment, it is therefore preferable if the operations of a tramp shipping company are robust against variations in voyage costs.

Ship routing problems have been considered in numerous papers (see the two recent surveys of Christiansen et al. 2004 and Christiansen et al. 2013). Regarding the cargoes in COAs, they are taken as given parameters in almost all studies in this area (i.e., by assuming a set of COAs has been accepted and cargoes in them must be transported in a planning horizon). In addition, although seaborne transportation is faced with great uncertainties, the majority of studies addressed ship routing problems in a deterministic environment. Interesting exceptions arise in ship scheduling and routing for offshore oil and gas platforms (Kisialiou et al. 2018, 2019), in the context of stochastic demands and weather conditions.

To fill these gaps, we consider the robust bulk ship routing problem with batched cargo selection (RSRPB). In what follows, we first summarize our main contributions and then introduce the structure of the remaining part of the chapter.

3.1.1 Summary of Our Scientific Contributions

Our main scientific contributions can be summarized as follows:

1. We consider a problem that jointly solves three interconnected subproblems in

tramp shipping: the fleet adjustment problem, the batched cargo selection problem, and the ship routing problem. For the batched cargo selection problem, we consider the selection behavior in COA settings. In view of the uncertainties in voyage costs, we solve the problem while ensuring the robustness of the solutions against random voyage costs.

2. We formulate a compact model that is solvable for small-scale instances by using standard MIP solvers and a strengthened set covering model. To solve the problem, we develop a tailored branch-and-price-and-cut (BPC) algorithm. Several enhancement strategies are proposed to improve the efficiency of the algorithm.

3. Results from extensive numerical experiments demonstrate that the algorithm can solve problems with practical sizes to optimality or near-optimality. The efficacy of the enhancement techniques is also attested by the experimental results.

3.1.2 Outline

The remainder of this chapter is structured as follows. We review the relevant studies in Section 3.2, and formally describe the problem in Section 3.3. Compact models for the problem are formulated in Section 3.4. In Section 3.5 we reformulate the problem as a set covering model. The BPC algorithm is described in Section 3.6. Computational results are reported in Section 3.7, followed by conclusions in Section 3.8. We provide all mathematical proofs in Appendix B.

3.2 Literature Review

Ship routing problems have received considerable attention in the scientific literature. Christiansen et al. (2004) and Christiansen et al. (2013) provided an overall review of the field. As ships are operated in three different modes, i.e., liner, tramp, and industrial shipping, studies on ship routing problems can also be generally divided into these three categories (Christiansen et al. 2013). Since our study focuses on tramp shipping, we concentrate our review on the relevant research on tramp shipping operations management problems.

3.2.1 Review of Studies on the Fleet Adjustment Problem

One stream of related research focuses on the fleet adjustment problem. This problem, in its broad definition, falls into two categories: the maritime fleet sizing and mix problem (MFSMP), and the maritime fleet renewal problem (MFRP) (Pantuso et al. 2014a). The MFSMP focuses on the design of a fleet for transporting cargoes in a single period, whereas the MFRP is about the dynamic adjustment of a fleet in multiple periods. The MFSMP in liner shipping has been well studied (e.g., Song and Dong 2013, Ng 2015). Research on the MFSMP in tramp shipping is rooted in the pioneering work of Schwartz (1968) which addresses the fleet sizing and mix problem for a barge service company. The objective is to determine the optimal fleet composition and service schedules for the barges to complete transportation tasks over a planning horizon. Pesenti (1995) considered a MFSMP for a fleet of container ships. They proposed a hierarchical decision model and developed a heuristic algorithm to solve the problem. Zeng and Yang (2007) studied a MFSMP in coal shipping, and solved the problem by tabu search. More recently, Fagerholt et al. (2010) proposed a decision support methodology for strategic planning in industrial and tramp shipping which solves the fleet sizing and mix problem using simulation-based optimization. Wang et al. (2018) considered a stochastic maritime fleet composition and deployment problem where the decisions include the type and number of ships to charter and the length of the chartering periods. They developed a two-stage stochastic programming model to solve the problem.

The MFRP has gained increased attention in recent years (e.g., Bakkehaug et al. 2014, Arslan and Papageorgiou 2017, and Zheng and Chen 2018). As the planning horizons for MFRPs usually cover several years and the seaborne transportation market is known for its volatility, most studies on the MFRP consider uncertainty as an integral part of the problem. One such study was provided by Alvarez et al. (2011b) who investigated the MFRP for industrial or tramp shipping companies. The fleet adjustment decisions considered in the problem include purchase, sale, lay-up, assignment to a market, charter-out, charter-in, and demolition. Uncertainties in the purchase and chartering market were considered. Following Bertsimas and

Sim (2003), the authors formulated the problem as a robust MILP model. Pantuso et al. (2014b) considered a multi-period MFRP for a liner shipping company. In the problem, the fleet of the company can be modified at each period in an uncertain shipping market and the objective is to identify a fleet renewal and deployment plan such that the expected total cost for controlling and operating ships in the planning horizon is minimized. A multi-stage stochastic model was formulated and solved by a decomposition algorithm.

3.2.2 Review of Studies on the Ship Routing Problem

The second stream of research focuses on the ship routing problem. Most studies in this stream can be viewed as special applications of the vehicle routing problem, where cargoes are transported between several loading and discharging ports. In most studies on the ship routing problem, the focus is the routing and scheduling of a fixed fleet of ships to transport a set of cargoes. Decisions are made in a deterministic environment. Both mandatory (from long-term contracts) and optional (from the spot market) cargoes can be considered. The objective can be to maximize the revenue from transporting cargoes from the spot market or to minimize the total transportation cost. Many heuristics have been developed for these problems (e.g., Brønmo et al. 2007a, Korsvik and Fagerholt 2010, and Kosmas and Vlachos 2012).

Among the exact algorithms, a column generation algorithm was proposed by Brønmo et al. (2010) to solve a ship routing problem with flexible cargo sizes. This algorithm was shown to be better than a method that generates all columns a priori. Stålhane et al. (2012a) developed a branch-and-price-and-cut algorithm for a ship routing problem in which cargoes are transported in a pickup and delivery manner. Meng et al. (2015) considered a joint ship routing and bunkering problem and solved it by branch-and-price. In view of the volatile maritime market, Hwang et al. (2008) considered a ship routing problem that limits the variance in profit. The problem was solved by branch-and-price-and-cut.

While most studies in this area assume deterministic parameter settings, a few of them considered uncertain environments. One such study was provided by Hwang et al. (2008) who considered uncertainties in revenues from chartering out ships and from transporting spot market cargoes, as well as costs for transporting cargoes using voyage charters. The uncertainties of these parameters were measured by a mean-variance form. The authors considered a ship routing problem aiming at maximizing the expected revenue of a shipping company under a constraint on the variance in profit. Tirado et al. (2013) addressed a dynamic ship routing problem in which new cargoes arrive stochastically. They proposed three heuristics to solve the problem. Halvorsen-Weare et al. (2012) investigated a ship routing and scheduling problem arising from the liquefied nature gas (LNG) business. In their problem, LNG is transported from a single producer to a set of customers over a planning horizon. All shipments are performed with full shiploads. The authors considered uncertainties in the sailing times and daily production rate of LNG. They proposed several strategies to enhance the robustness of solutions to the problem and they developed a simulation-optimization framework for the problem. Uncertainties of travel times are also considered by Agra et al. (2013) who developed a robust optimization algorithm based on row and column generation.

Our study is also related to the robust vehicle routing problem (RVRP). However, due to their land transportation backgrounds, most RVRPs focus on uncertainties in traveling times or customer demands, and the objectives are to construct robust routes for vehicles in terms of travel time or customer demand satisfaction. Hence, the realizations of the uncertain travel times and demands in these problems are independent of the routes. Examples of studies on such RVRPs include Lee et al. (2012), Gounaris et al. (2013), Agra et al. (2013), and Munari et al. (2019). In our problem, the realization of uncertain voyage costs cannot be considered separately among different shipping routes. Therefore, the considered problem is essentially different from most RVRPs considered in the literature. In particular, if such RVRPs are solved by branch-and-price, the uncertainties in the parameters can be handled independently in the pricing problems for constructing feasible routes (e.g., Lee et al. 2012, Munari et al. 2019). This is, however, not applicable to our problem.

3.2.3 Review of Studies on the Joint Fleet Adjustment and Ship Routing Problem

The interconnection between the fleet adjustment problem and the ship routing problem has been the object of several studies. Some have considered the composition of a fleet to complete transportation tasks within a (short) period; once decided, the fleet remains fixed for the entire planning horizon. Fagerholt and Lindstad (2000) considered a ship scheduling problem regarding a supply operation in the Norwegian Sea where supplies are transported using ships from a supply depot to several offshore installations. The objective is to determine an optimal fleet and the corresponding weekly schedules that meet the installations' demands at minimum cost. Similar studies include those of Zeng and Yang (2007), Fagerholt et al. (2010), and Halvorsen-Weare et al. (2012).

3.2.4 Review of Studies on the Treatment of COAs

Almost all of the papers on the ship routing problem disregard decisions regarding accepting or rejecting COAs. Two exceptions include the studies of Fagerholt et al. (2010) and Laake and Zhang (2016). The former solves a problem that includes fleet mix and sizing and COA cargo selection as strategic decisions, and ship routing as a tactical decision. The authors use a simulation method to solve the problem. The latter considers an integrated fleet renewal and COA cargo selection problem, where the planning horizon can be several years and decisions are made at the beginning of each year.

3.2.5 Contribution of Our Study to the Literature

Our research falls into the group that focuses on the joint fleet adjustment and ship routing problem. We enrich the existing literature in multiple ways. First, to the best of our knowledge, we are among the first to consider cargo selection under COA settings in a ship routing problem. Second, while most relevant studies are based on deterministic parameter assumptions, we consider uncertainties in the problem and solve it in a robust way. Third, to efficiently solve the problem, we develop a novel exact algorithm that can be used to solve other families of routing and scheduling problems with uncertain parameters.

3.3 Problem Description

We consider the case of a bulk tramp shipping company that has to make a series of decisions regarding its operations in a planning horizon that typically covers several months. At the beginning of the planning horizon, the company owns or controls a fleet of heterogeneous ships and has a set of mandatory cargoes that must be transported (i.e., cargoes accepted in the previous planning horizon but not transported yet) and a set of optional cargo transportation offers (i.e., COAs and cargoes in the spot market). The decisions include (i) how to adjust its fleet over the planning horizon, (ii) whether to accept or reject each transportation offer, and (iii) how to route the ships in its fleet to complete the transportation of the mandatory cargoes and of those from the accepted transportation offers. The objective is to maximize the total profit, equal to the revenue from (i) chartering-out ships from the fleet of ships controlled by the company at the beginning of the planning horizon and (ii) freights from optional cargoes, minus the costs of (a) chartering-in ships from the chartering market and (b) transporting cargoes.

In the remainder of this section, we will describe the problem in detail. We will introduce the fleet adjustment in Section 3.3.1, the batched cargo selection in Section 3.3.2, and the routing of ships in Section 3.3.3. The uncertainties in the parameters of the problem are discussed in Section 3.3.4. Our assumptions for the whole problem are presented in Section 3.3.5.

3.3.1 Fleet Adjustment

The fleet is composed of a set of ships denoted by V which belong to one of two subsets V_1 and V_2. The set V_1 consists of ships that are controlled by the shipping company at the beginning of the planning horizon (these ships can either be owned or time-chartered by the company). The set V_2 consists of ships that are in the chartering market and can be chartered in by the company. The ships $v \in V$ are heterogeneous and can have different available times, initial positions, daily chartering rates, capacities, operational costs, and speeds.

For the ships in V_1, the company can choose to charter them out in the planning horizon, which generates revenues. The company may also choose to charter in some ships from V_2 in the planning horizon, and in this case, it has to pay chartering costs.

3.3.2 Batched Cargo Selection

We consider two types of cargoes in the problem. The first type consists of mandatory cargoes coming from COAs that were accepted in previous planning horizons and must be transported in the current planning horizon. We denote the set of cargoes of this type by N^{m}. At the beginning of the planning horizon, the shipping company also has a set of optional cargoes from optional COAs and the spot market which constitutes the second type of cargoes. We denote the set of these cargoes by N^{c}, and let $N := N^{\mathrm{m}} \cup N^{\mathrm{c}}$. Each cargo in N is specified by its weight, loading port, unloading port, loading period (normally known as *laycan*) and the times of cargo handling at the loading and unloading ports. Note that we assume the universal adoption of the "reasonable dispatch" which requires the shipping company to dispatch the cargoes onboard within reasonable times. Therefore, we do not set deadlines for unloading the cargoes.

A COA is a long-term transportation contract between a shipowner (shipping company) and a charterer (shipper). In a COA, the shipping company is required to transport cargoes for a charterer in a specified period. The typical contract period ranges from several months to one or two years. The charterer should specify the weight (range) of cargoes to be transported in the period. Cargoes are transported in different shipments. The loading weight in each shipment is usually at the option of the shipping company but should be within the limits specified by the charterer. A COA also specifies the freight rate for transporting the cargoes, and the actual revenue gained by the shipping company is decided by the actual weight transported. The loading periods of the shipments are generally arranged in two fashions. In the first fashion, the contract provides an agreed upon shipment programme, which specifies the loading periods of these shipments. In the second fashion, the shipping company is only required to make the shipments fairly evenly spread over the period of the contract. For details, the reader may refer to the template contracts for bulk cargo COAs (i.e., VOLCOA and GENCOA) provided by BIMCO (2019), which is the largest international shipping association representing shipowners. It is worth mentioning that charterers who choose to use COAs to import or export their cargoes periodically (e.g., monthly or bimonthly) usually have steady consumption or production rates for these cargoes in the contract period. Besides, cargoes in a COA usually have a fixed origin port and destination port.

Based on these features, we consider the following settings for COAs in the problem. First, for each COA $k \in K$, we first split the cargo to be transported into a set of cargoes denoted by N_k^{c}, each representing a shipment. The number of cargoes in N_k^{c} is either directly specified in a given shipment programme or decided by the shipping company considering (i) the total weight (range) of cargoes to be transported in the contract period, (ii) the loading weight limits for each shipment, and (iii) the requirement of regularity in transportation. For a tramp shipping company, the cargoes in COAs are optional, but can only be accepted or rejected as a batch (i.e., cargoes in one COA should be chosen as a whole). In this sense, a spot market cargo can be treated as a COA k with $|N_k^{\mathrm{c}}| = 1$ in the problem. Second, we set the weight of each cargo in N_k^{c} to be equal to the maximum loading weight specified by the charterer or the maximum total cargo weight divided by the number of cargoes (whichever is smaller). We will discuss in Section 3.4.1 how to handle the cases in which the actual loading weight is less than the maximum weight. Third, COA k is also associated with a revenue parameter p_k which represents the freight revenue for the company by accepting it and transporting the maximum loading weight in each shipment. Fourth, the loading periods for cargoes from a COA are set as follows. If the shipment programme of the COA is given, then the shipping company should follow the loading periods specified in the shipment programme. Otherwise, more freedom is given to the shipping company, and we set relatively longer loading periods for these cargoes. For concrete examples, refer to our problem instances for numerical experiments in Section 3.7. Finally, for a COA k whose contract period exceeds the current planning horizon, we split the cargoes in it into two parts in such a way that the first part contains cargoes to be transported in the current planning horizon, and the second part contains the cargoes to be transported beyond the current planning horizon. Using the cargoes in the first part, a new (shrunk) COA k' is generated and its revenue equals $\frac{p_k|N^{\mathrm{c}}_{k'}|}{|N^{\mathrm{c}}_k|}$. The COA k' will then replace k in the problem, and once k' is accepted, the cargoes contained in the second part of k become mandatory cargoes for the following planning horizons.

3.3.3 Ship Routing

Given the results from the above two decisions, the company is then responsible for arranging the route of each ship in the fleet to transport cargoes in N^{m}, accepted spot market cargoes, and those from the accepted COAs. The routes should be feasible so that each cargo is loaded within the specified loading period and carried by a cargoworthy and seaworthy ship. Being cargoworthy for a cargo means that the capacity of the ship is larger than the (minimum) loading weight of the cargo and the ship has a suitable draft to enter and leave the loading and unloading ports of the cargo. In addition, seaworthiness requires the ship to be able to travel in the voyage from the loading port to the unloading port of the cargo. The costs for the routes of the ships are composed of the port charges and voyage costs associated with the route, and these costs are ship-specific.

In our problem, following the practice in PB, we assume that cargoes can only be transported in full shiploads. The same settings were used in Hwang et al. (2008), Meng et al. (2015) and many other studies. Besides, it is also possible to transport cargoes by using voyage charters from the spot market, which incurs additional costs. For the ease of presentation, we introduce the following definitions.

Definition 3.1. ***Voyage***. *A (ship-specific) voyage refers to a certain ship sailing in the sea from one port to another, either for transporting cargoes or for repositioning an empty ship. A voyage is a* ***laden*** *voyage if the ship sailing in it is loaded, and is a*

ballast *voyage if the ship is empty. Voyages that transport cargoes from their loading ports to their unloading ports are laden voyages, whereas voyages that reposition a ship from its initial location to a loading port of a cargo or from an unloading port of a cargo to the loading port of another cargo are ballast voyages.*

Definition 3.2. ***Trip***. *We define a trip* $(v, i, j), v \in V, i, j \in N$ *as a sequence of cargo handling operations and voyages of ship* v *that starts from the loading port of cargo* i *and ends at the loading port of cargo* j*. A trip includes, in a chronological sequence, loading of cargo* i *at its loading port, sailing in a laden voyage to transport cargo* i *from its loading port to its unloading port, unloading of cargo* i*, and finally sailing in the ballast voyage from the unloading port of cargo* i *to the loading port of cargo* j*.*

In particular, we denote by $(v, 0, j)$ *the trip of ship* v *from its initial position to the loading port of cargo* j*, and by* (v, i, T) *the trip of ship* v *for carrying cargo* i *as its last transported cargo in the planning horizon. Note that trip* (v, i, T) *includes loading of cargo* i *at its loading port, transporting cargo* i *from its loading port to its unloading port, and unloading cargo* i *at the unloading port.*

3.3.4 Uncertainties

The parameters in the RSRPB can be classified into two categories, which are (1) the parameters that are known at the beginning of the planning horizon, and (2) the parameters that can only be estimated at the beginning of the planning horizon. In particular, the parameters involved in the decisions in fleet adjustment and cargo selection fall into the first category. This is because, in the fleet adjustment, the company changes the composition of its fleet by time charters. Information regarding chartering in or out a ship (e.g., the daily chartering rate, details of the ship, and the chartering period) should be specified in a chartering contract that is signed at the beginning of the planning horizon. Similarly, requirements and revenues for transporting cargoes in each COA or a spot market cargo are stipulated in the contracts. For a similar reason, the cost of outsourcing a cargo transportation task to a voyage charter is also deterministic at the beginning of the planning horizon.

There are also some parameters whose exact values are not known at the beginning of the planning horizon. In particular, the time and the cost of sailing between two ports can only be estimated. Further, in bulk shipping, since the time needed by a ship to complete a voyage depends on the sailing speed which is largely controllable by the shipping company, reliable estimates can be made for voyage times. This observation also applies to PB's practices, which takes reliability and punctuality as the most important factors in its services, and delayed loading and unloading are very rare (Pacific Basin 2019). In practice, by dynamically adjusting ships' speed, PB strives to deliver reliable on-time performance even when voyages encounter unforeseen scheduling delays.

In comparison, greater uncertainties lie in the estimation of voyage costs. In our interview with managers from PB, we were told that the cost of a voyage is significantly affected by weather and currents in the sea and hull and engine conditions of the ship, and these conditions are largely unpredictable. In addition, as we discussed, when a voyage encounters unforeseen scheduling delays the ship may choose to speed up in order to reach the load or unloading port by the agreed upon time period. In this case, the cost of a voyage also increases. Therefore, to deal with the uncertain voyage costs in the RSRPB, we seek robust solutions by adopting the concept of the *budget of uncertainty* which was proposed by Bertsimas and Sim (2003). In particular, let $\widetilde{c}_r$ denote the random cost of a ship-specific voyage r. Following the approach proposed by Bertsimas and Sim (2003), we assume that $\widetilde{c}_r$ takes a value in $[\bar{c}_r, \bar{c}_r + d_r]$, where $\bar{c}_r$ is the nominal cost of the voyage (which corresponds to the expected cost of the voyage), and $d_r \geq 0$ is the largest deviation of $\widetilde{c}_r$ from $\bar{c}_r$.

3.3.5 Assumptions

To introduce the assumptions, we make use of the following notations:

R Set of voyages.

$R_{i,j}^v$ Set of voyages contained in trip (v, i, j), $v \in V$, $i \in N \cup \{0\}$, $j \in N \cup \{T\} \setminus \{i\}$. In particular, for $i, j \in N$, if cargo i's unloading port is different from cargo j's unloading port, trip (v, i, j) contains two voyages which are a laden voyage for transporting cargo i from its loading port to its unloading port, and a ballast voyage for repositioning the ship from the unloading port of i to the loading port of j. Meanwhile, if ship v's initial position is different from cargo i's loading port, trip $(v, 0, j)$ contains only one ballast voyage for repositioning ship v from its initial position to the loading port of i. In addition, trip (v, i, T) contains only one laden voyage for transporting cargo i from the loading port to the unloading port.

$t_{i,j}^v$ Time of trip (v, i, j), $v \in V$, $i \in N \cup \{0\}$, $j \in N \cup \{T\} \setminus \{i\}$.

$\hat{c}_i$ Cost of transporting cargo $i \in N$ using a voyage charter.

$C_{i,j}^v$ Nominal cost for trip (v, i, j), $v \in V$, $i \in N \cup \{0\}$, $j \in N \cup \{T\} \setminus \{i\}$. $C_{i,j}^v = g_i^v + \sum_{r \in R_{i,j}^v} \bar{c}_r$, where g_i^v, $v \in V$, $i \in N \cup \{0\}$ is the total port charges for ship v to load and unload cargo i at the loading and unloading ports. Specifically, $g_0^v = 0, v \in V$.

To better analyze the problem, we make the following assumptions:

A1. The triangle inequality holds for the trip times so that $t_{i,j}^v \leq t_{i,k}^v + t_{k,j}^v, i \in N \cup \{0\}, k \in N, j \in N \cup \{T\} \setminus \{i\}, v \in V$.

A2. For each $i, j \in N, i \neq j$ and for each $v \in V$, let r' be the ballast voyage in which ship v sails from the unloading port of i to the loading port of j, we assume $C_{i,j}^v + d_{r'} \leq C_{i,k}^v + C_{k,j}^v, k \in N, k \neq i, k \neq j$.

A3. For each k, we assume that $\sum_{i \in N_k^c} \hat{c}_i > p_k$.

The second assumption ensures that regardless of the realizations of voyage costs, the cost of any ship v to complete trip (v, i, j) never exceeds the cost of completing two consecutive trips (v, i, k) and (v, k, j) (note that $R^v_{i,j} \setminus (R^v_{i,k} \cup R^v_{k,j}) = \{r'\}$). To understand the third assumption, suppose a shipper provides a COA offer k to the shipping company. Then, **A3** states that the cost of the shipper for transporting all cargoes in the COA using voyage charters (which equals $\sum_{i \in N^c_k} \hat{c}_i$) is larger than the cost of transporting them using a COA (which equals the freight the shipper pays to the shipping company, p_k). This is reasonable because otherwise, instead of using a COA, it will be more favorable for the shipper to transport all the cargoes in the COA using voyage charters (i.e., the COA will not exist at all). This assumption also prevents a shipping company from profiting by accepting a spot market cargo and then using a voyage charter to transport it.

3.4 Compact Models

In this section, several compact formulations are developed for the RSRPB. We start by providing a mixed integer linear programming (MILP) model for the deterministic counterpart of the problem in Section 3.4.1. The model will then be converted into a series of robust MIP models in Section 3.4.2. Finally, we discuss the complexity of the problem in Section 3.4.3. Before presenting the models, we summarize the notations in Table 3.1.

Table 3.1: Notation.

Indices:	
v	Index for ships.
k	Index for cargo transportation contracts (offers, COAs).
i, j	Indices for cargoes. We use 0 and T to denote the dummy beginning and ending cargoes for each ship, respectively.
r	Index for voyages. Note that voyages are ship-specific.
Sets:	
V	Set of all ships, $V = V_1 \cup V_2$.

V_1	Set of ships controlled by the tramp shipping company at the beginning of the planning horizon.
V_2	Set of ships available from the chartering market.
K	Set of cargo transportation contracts (offers, COAs).
N	Set of cargoes, $N = N^{\text{m}} \cup N^{\text{c}}$. N does not include the dummy cargoes 0 and T.
N^{m}	Set of mandatory cargoes that must be transported.
N^{c}	Set of cargoes in the contracts in K.
N^{c}_k	Set of cargoes in contract k. $N^{\text{c}} = \bigcup_{k \in K} N^{\text{c}}_k$.
R	Set of voyages.
$R^v_{i,j}$	Set of voyages contained in trip (v, i, j), where $v \in V, i \in N \cup \{0\}$, and $j \in N \cup \{T\} \setminus \{i\}$.
Parameters:	
o_v	Available time of ship $v \in V$.
f_v	Chartering revenue (cost) of ship $v \in V_1$ ($v \in V_2$) in the planning horizon.
p_k	Revenue made for accepting the offer for transporting cargoes in contract $k \in K$.
e_i	Earliest loading start time of cargo $i \in N$.
l_i	Latest loading start time of cargo $i \in N$.
g^v_i	Port charges for ship v to load and unload cargo i at the loading and unloading ports.
$\bar{c}_r$	Nominal cost of voyage r.
$I_{v,i}$	Ship-cargo compatibility index which is obtained by considering the cargoworthness and seaworthness of each ship for transporting each cargo. $I_{v,i}$ equals 1 if cargo $i \in N$ can be carried by ship $v \in V$, and 0, otherwise. Specifically, we let $I_{v,0} = 1$ and $I_{v,T} = 1$, $v \in V$.
$t^v_{i,j}$	Time of trip (v, i, j), where $v \in V$, $i \in N \cup \{0\}$, and $j \in N \cup \{T\} \setminus \{i\}$.
$C^v_{i,j}$	Nominal cost of trip (v, i, j), where $v \in V$, $i \in N \cup \{0\}$, and $j \in N \cup \{T\} \setminus \{i\}$. $C^v_{i,j} = g^v_i + \sum_{r \in R^v_{i,j}} \bar{c}_r$.
$\hat{c}_i$	Cost of transporting cargo $i \in N$ by a voyage charter from the spot market.
M	A large constant.
Decision Variables:	
w_k	1, if the offer of contract k is accepted and 0, otherwise.

x_v	1, if ship $v \in V$ is included in the fleet in the planning horizon and 0, otherwise.
$y^v_{i,j}$	1, if trip (v, i, j) is included in the shipping routes, where $v \in V$, $i \in N \cup \{0\}$, and $j \in N \cup \{T\} \setminus \{i\}$ and 0, otherwise.
z_i,	1, if cargo i is transported using a voyage charter from the spot market and 0, otherwise.
b_i	Time to start loading cargo $i \in N$.

3.4.1 The Deterministic Model

The deterministic model (M1) is as follows:

$$\text{(M1)} \ \underset{b,w,x,y,z}{\text{Maximize}} \ Z_1 = \sum_{k \in K} p_k w_k - \sum_{v \in V} f_v x_v - \sum_{v \in V} \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{T\} \setminus \{i\}} C^v_{i,j} y^v_{i,j} - \sum_{i \in N} \hat{c}_i z_i \tag{3.1}$$

subject to

$$\sum_{v \in V} \sum_{i \in N \cup \{0\}} y^v_{i,j} + z_j = 1, \quad j \in N^{\mathrm{m}} \tag{3.2}$$

$$\sum_{v \in V} \sum_{i \in N \cup \{0\}} y^v_{i,j} + z_j = w_k, \quad j \in N^{\mathrm{c}}_k, k \in K \tag{3.3}$$

$$\sum_{j \in N} y^v_{0,j} = x_v, \quad v \in V \tag{3.4}$$

$$\sum_{j \in N \cup \{0\} \setminus \{i\}} y^v_{j,i} = \sum_{j \in N \cup \{T\} \setminus \{i\}} y^v_{i,j}, \quad i \in N, v \in V \tag{3.5}$$

$$\sum_{i \in N} y^v_{i,T} = x_v, \quad v \in V \tag{3.6}$$

$$b_j \geq (o_v + t^v_{0,j}) y^v_{0,j}, \quad j \in N, v \in V \tag{3.7}$$

$$b_j \geq b_i + t^v_{i,j} + M(y^v_{i,j} - 1), \quad i \in N, j \in N \setminus \{i\}, v \in V \tag{3.8}$$

$$b_i \geq e_i, \quad i \in N \tag{3.9}$$

$$b_i \leq l_i, \quad i \in N \tag{3.10}$$

$$w_k \in \{0,1\}, \quad k \in K \tag{3.11}$$

$$x_v \in \{0,1\}, \quad v \in V \tag{3.12}$$

$$y_{i,j}^v \in \{0,1\}, \quad I_{v,i}=1, I_{v,j}=1, i \in N \cup \{0\}, j \in N \cup \{T\} \setminus \{i\}, v \in V \tag{3.13}$$

$$z_i \in \{0,1\}, \quad i \in N. \tag{3.14}$$

The objective function (3.1) maximizes the profit of the shipping company over the planning horizon (Z_1), which equals the freight revenue ($\sum_{k \in K} p_k w_k$) minus (i) the opportunity cost for not chartering-out ships in the current fleet and the cost for chartering-in ships from the market ($\sum_{v \in V} f_v x_v$), (ii) the cost [$\sum_{v \in V} \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{T\} \setminus \{i\}} C_{i,j}^v y_{i,j}^v$] of transporting cargoes, and (iii) the cost ($\sum_{i \in N} \hat{c}_i z_i$) of using voyage charters. Constraints (3.2) mean that mandatory cargoes must be transported, either by ships in the fleet or by voyage charters. Constraints (3.3) ensure that once a COA is accepted, the cargoes in it must be transported. Constraints (3.4) assign an initial task to each ship included in the fleet. Constraints (3.5) ensure flow balance on the route of each ship. Constraints (3.6) assign a final task to each ship included in the fleet. Constraints (3.7) and (3.8) state that the loading of cargo j can only start after the assigned ship has arrived at the loading port from its initial position or from the port where it finished unloading the previous cargo, respectively. Constraints (3.9) and (3.10) mean that the loading of cargoes should start within the given time windows (*laycans*). Constraints (3.11)–(3.14) define the domains of the variables. Note that Constraints (3.14) also enforce the ship-cargo compatibility.

It is worth commenting on several important aspects of this formulation. First, Constraints (3.8) can be strengthened by replacing M with $M'_{v,i,j} = \max\{0, l_i + t_{i,j}^v - e_j\}$. Second, the model is also capable of handling the cases in which the weights of cargoes come in ranges, e.g., the shipper of a cargo may stipulate the loading weight to be "30,000 tons, 10% more or less", which indicates that the carrier can choose any weight between $[27000, 33000]$ tons to be loaded onboard. Note that in such cases, the actual freight revenue earned by the shipping company is calculated by the actual transported weight times the freight per unit weight. To handle such cases, we calculate p_k based on the maximum loading weight of each cargo $i \in N_i^{\mathrm{c}}$.

Then, if ship v or a ship from a voyage charter cannot load the maximum weight, we add the reduced revenue caused by not carrying the maximum weight of cargo i into $C_{i,j}^v$ or $\hat{c}_i$.

3.4.2 Robust Models

In this section, we convert the deterministic model M1 into robust models. As discussed in Section 3.3.4, the voyage costs are uncertain at the beginning of the planning horizon. Let $\widetilde{c}_r$ denote the random voyage cost of r. Following the approach proposed by Bertsimas and Sim (2003), we assume that $\widetilde{c}_r$ takes a value in $[\bar{c}_r, \bar{c}_r + d_r]$, where $d_r \geq 0$ is the largest deviation of $\widetilde{c}_r$ from $\bar{c}_r$. To avoid over-conservative solutions, Bertsimas and Sim (2003) use a *budget of uncertainty* which allows at most Γ of the $\widetilde{c}_r$'s to deviate from $\bar{c}_r$. The RSRPB is to find a solution whose worst-case profit is maximized. Let H be the decision variable that represents the robust cost (i.e., the largest deviation from the nominal total cost). The robust problem can be formulated in the following model (denoted by M2), where the robustness of the solutions is ensured by Constraints (3.16):

$$\begin{aligned}\text{(M2)} \underset{a,b,w,y,z,H}{\text{Maximize}}\ Z_2 = & \sum_{k \in K} p_k w_k - \sum_{v \in V} f_v x_v - \sum_{v \in V} \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{T\} \setminus \{i\}} C_{i,j}^v y_{i,j}^v \\ & - \sum_{i \in N} \hat{c}_i z_i - H \end{aligned} \tag{3.15}$$

subject to

$$\text{(3.2)–(3.14)}$$

$$H - \sum_{v \in V} \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{T\} \setminus \{i\}} \sum_{r \in G \cap R_{i,j}^v} d_r y_{i,j}^v \geq 0, \quad G \in \Theta, \tag{3.16}$$

where $\Theta := \{G | G \subseteq R, |G| \leq \Gamma\}$.

M2 is equivalent to the following model (M3). Note that in the model, we introduce a set of new decision variables u_r, which decide the proportions of the deviations d_r that should be included in the robust cost:

$$
\begin{aligned}
\text{(M3)} \underset{a,b,w,y,z,u}{\text{Maximize}}\ Z_3 = & \sum_{k\in K} p_k w_k - \sum_{v\in V} f_v x_v - \sum_{v\in V} \sum_{i\in N\cup\{0\}} \sum_{j\in N\cup\{T\}\setminus\{i\}} C^v_{i,j} y^v_{i,j} \\
& - \sum_{i\in N} \hat{c}_i z_i - \max_u \sum_{v\in V} \sum_{i\in N\cup\{0\}} \sum_{j\in N\cup\{T\}\setminus\{i\}} \sum_{r\in R^v_{i,j}} d_r u_r y^v_{i,j}
\end{aligned}
\tag{3.17}
$$

subject to

$$(3.2)\text{–}(3.14)$$

$$\sum_{r\in R} u_r \le \Gamma \tag{3.18}$$

$$0 \le u_r \le 1, \quad r \in R. \tag{3.19}$$

Let θ be the dual variable of Constraint (3.18) and let g_r be the dual variables of Constraints (3.19). By applying strong duality, M3 can be reformulated into the following MILP model (M4). Note that M4 can be solved by an off-the-shelf optimization software (e.g., CPLEX).

$$
\begin{aligned}
\text{(M4)} \underset{a,b,w,y,z,g,\theta}{\text{Maximize}}\ Z_4 = & \sum_{k\in K} p_k w_k - \sum_{v\in V} f_v x_v - \sum_{v\in V} \sum_{i\in N\cup\{0\}} \sum_{j\in N\cup\{T\}\setminus\{i\}} C^v_{i,j} y^v_{i,j} \\
& - \sum_{i\in N} \hat{c}_i z_i - \Gamma\theta - \sum_{r\in R} g_r
\end{aligned}
\tag{3.20}
$$

subject to

$$(3.2)\text{–}(3.14)$$

$$g_r + \theta \ge d_r y^v_{i,j}, \quad r \in R^v_{i,j}, v \in V, i \in N \cup \{0\}, j \in N \cup \{T\} \setminus \{i\} \tag{3.21}$$

$$\theta \ge 0 \tag{3.22}$$

$$g_r \ge 0, \quad r \in R. \tag{3.23}$$

3.4.3 Complexity of the Problem

In this section, we demonstrate the RSRPB is NP-hard in the strong sense. To do so, we show that the decision version of the problem is strongly NP-hard. That is, given settings of ships, contracts, and cargoes, it cannot be determined in polynomial time or even in pseudo-polynomial time whether there exists a solution to the problem whose objective value Z_4 is no smaller than a given constant E unless P=NP.

Proposition 3.1. *The RSRPB is NP-hard in the strong sense.*

3.5 A Strengthened Set Covering Formulation

Because the RSRPB is NP-hard, we propose to solve the problem by branch-and-price-and-cut (BPC). To this end, we reformulate the problem as a set covering model. We first introduce the columns that enable the formulation in Section 3.5.1 and we then present the model in Section 3.5.2. The model is then strengthened in Section 3.5.3.

3.5.1 Columns

In the model, we define a column q as a route of a ship for transporting a set of cargoes. There are in total a set Q of columns for modeling the problem and each column q has a set of attributes which are shown in Table 3.2.

The set Q is partitioned into Q_1, Q_2, and Q_3. The set Q_1 contains a set of columns that are for the purpose of "contract selection". It has $|K|$ elements, each for one contract. For each contract k, the following column q is added into Q_1 where (i) $V_q = \varnothing$, (ii) $c_q = p_k$, (iii) $\Phi_q = N_k^{\mathrm{c}}$, and (iv) $\Xi_q = \varnothing$. Denote by q_k^1 the column corresponding to contract k in Q_1. In the set covering formulation, a contract k will be accepted if and only if $\chi_{q_k^1} = 0$ in the solution. By contrast, a contract k will be rejected when $\chi_{q_k^1} = 1$ in the solution, and the rejection reduces the revenue by p_k.

The set Q_2 contains the set of columns representing the "voyage chartering" choices for all cargoes. In particular, each cargo $i \in N$ corresponds to a column q_i^2

Table 3.2: Parameters of columns.

Sets:	
V_q	Set of ships $v \in V$ used in column q, where $\|V_q\| \leq 1$.
Φ_q	Set of cargoes $i \in N$ covered in column q.
Ξ_q	Set of voyages $r \in R$ covered in column q.
Parameters:	
c_q	Cost of selecting column q.
$\alpha_{q,v}$	1, if $v \in V_q$; 0, otherwise.
$\beta_{q,i}$	1, if $i \in \Phi_q$; 0, otherwise.
$\gamma_{q,r}$	1, if $r \in \Xi_q$; 0, otherwise.
Decision Variables:	
χ_q	1, if column $q \in Q$ is selected and 0, otherwise.
H	Robust cost.

in Q_2 such that (i) $V_{q_i^2} = \varnothing$, (ii) $c_{q_i^2} = \hat{c}_i$, (iii) $\Phi_{q_i^2} = \{i\}$ and (iv) $\Xi_{q_i^2} = \varnothing$. In the set covering formulation, cargo i will be transported by a voyage charter if and only if $\chi_{q_i^2} = 1$ in the solution.

The set Q_3 contains all feasible cargo transportation routes for all ships. Denote a route by h, let v_h be the ship that travels on the route, and let $N_h \neq \varnothing$ be the set of cargoes transported on h. Further, let i_n, $n = 1, 2, ..., |N_h|$ denote the n^{th} transported cargo on the route. Route h is said to be feasible if and only if the following Constraints (3.24)–(3.28) hold:

$$x_{v_h} = 1, \tag{3.24}$$

$$I_{v_h,i} = 1, \quad i \in N_h, \tag{3.25}$$

$$e_i \leq b_i \leq l_i, \quad i \in N_h, \tag{3.26}$$

$$b_{i_1} \geq o_{v_h} + t^{v_h}_{0,i_1}, \tag{3.27}$$

$$b_{i_{n+1}} \geq b_{i_n} + t^{v_h}_{i_n,i_{n+1}}, \quad n \in \{1, 2, ..., |N_h| - 1\}. \tag{3.28}$$

Constraint (3.24) ensures that the ship is included in the fleet in the planning horizon. Constraints (3.25) ensure the ship-cargo compatibility. Constraints (3.26) ensure that each cargo should start loading within a feasible time window. Finally, Constraints (3.27) and (3.28) state that loading a cargo can only start after the assigned ship arrives at the loading port from the initial position of the ship or from the port where the ship finishes unloading the previous cargo.

If a feasible route h has been found, a column q is added into Q_3, such that (i) $V_q = \{v_h\}$, (ii) $c_q = f_{v_h} + C^{v_h}_{0,i_1} + \sum_{n=1}^{|N_h|-1} C^{v_h}_{i_n,i_n+1} + C^{v_h}_{i_{|N_h|},T}$, (iii) $\Phi_q = \{i_1, ..., i_{|N_h|}\}$, and (iv) $\Xi_q = \{r | r \in \bigcup_{n=1}^{|N_h|-1} R^{v_h}_{i_n,i_n+1} \cup R^{v_h}_{0,i_1} \cup R^{v_h}_{i_{|N_h|},T}\}$. Finally, denote by q^3_h the column corresponding to route h in Q_3. In the set covering formulation, a route h will be utilized if and only if $\chi_{q^3_h} = 1$ in the solution.

3.5.2 The Basic Model

We are now ready to formulate the set covering model (M5). Let $\bar{Z}$ denote the total revenue from COAs, which is calculated by $\bar{Z} = \sum_{k \in K} p_k$, and recall that $\Theta := \{G | G \subseteq R, |G| \leq \Gamma\}$. M5 is formulated as follows:

$$\text{(M5) } \underset{\chi,H}{\text{Maximize}}\ Z_5 = \bar{Z} - (\sum_{q \in Q} c_q \chi_q + H) \tag{3.29}$$

subject to

$$\sum_{q \in Q} \alpha_{q,v} \chi_q \leq 1, \quad v \in V \tag{3.30}$$

$$\sum_{q \in Q} \beta_{q,i} \chi_q \geq 1, \quad i \in N \tag{3.31}$$

$$H - \sum_{q \in Q} \sum_{r \in G} d_r \gamma_{q,r} \chi_q \geq 0, \quad G \in \Theta \tag{3.32}$$

$$\chi_q \in \{0, 1\}, \quad q \in Q. \tag{3.33}$$

In the above model, the objective function (3.29) is to maximize the profit of the company, which is calculated by deducting the cost of selected columns and the robust cost (H) from $\bar{Z}$. Constraints (3.30) ensure that each ship travels in at most one route. Constraints (3.31) mean that every cargo should be covered by columns in the solution. We use Constraints (3.32) to ensure the robustness of the solution. The last constraints define the χ_q as binary variables.

Proposition 3.2. *The set covering model M5 is equivalent to the compact models M2, M3, and M4.*

3.5.3 Strengthening the Model

In this section, we strengthen M5 in two ways. First, we derive a family of valid inequalities for the model. Then, we propose a method to reduce the redundancy of the constraints.

Valid Inequalities

For any accepted contract k, cargoes $i \in N_k^c$ can either be transported by a ship $v \in V$ or by using voyage charters. Recall that cargoes in N_k^c have similar weights and share the same loading and unloading ports (while the loading time windows are different). Let L_k denote the lower bound of the number of cargoes in N_k^c that are transported by ships $v \in V$ in an *optimal* solution to the RSRPB. From assumption **A3**, it is clear that $L_k = 1$ is valid for any accepted contract k. We use the procedure given in Algorithm 1 to obtain a tighter L_k.

Algorithm 1 The L_k Calculation Procedure.

Input: p_k, N_k^{c}, $\hat{c}_i, i \in N_k^{\mathrm{c}}$ and $C_{i,T}^v, v \in V, i \in N_k^{\mathrm{c}}$;
Output: L_k;

for $i \in N_k^{\mathrm{c}}$ **do**
    $\bar{V} = \{v | I_{v,i} = 1, o_v + t_{0,i}^v \le l_i, v \in V\}$;
    $MC_i = \min_{v \in \bar{V}} C_{i,T}^v$;
    $DF_i = \hat{c}_i - MC_i$;
end for
$MR = p_k - \sum_{i \in N_k^{\mathrm{c}}} MC_i$;
$N' = N_k^{\mathrm{c}}$;
$SN = 0$;
while $MR > 0 \& N' \neq \varnothing$ **do**
    $i' = \arg\min_{i \in N'} DF_i$;
    $MR = MR - DF_{i'}$;
    $N' = N' \setminus \{i'\}$;
    if $MR > 0$ **then**
        $SN = SN + 1$;
    end if
end while
$L_k = |N_k^{\mathrm{c}}| - SN$.

Lemma 3.1. *Algorithm 1 calculates a lower bound of the number of cargoes from N_k^c that should be transported by ships from V for any accepted contract $k \in K$ in an optimal solution to M5.*

Following Lemma 3.1, we derive the following inequalities:

$$\sum_{q \in Q} n_q^k \chi_q \geq L_k, \quad k \in K, \tag{3.34}$$

where n_q^k is set as follows:

$$n_q^k = \begin{cases} L_k, \text{ if } q = q_k^1, \\ 0, \text{ if } q \neq q_k^1, \end{cases} \quad k \in K, q \in Q_1, \tag{3.35}$$

$$n_q^k = 0, \quad k \in K, q \in Q_2, \tag{3.36}$$

$$n_q^k = |\Phi_q \cap N_k^c|, \quad k \in K, q \in Q_3. \tag{3.37}$$

Proposition 3.3. *Inequalities (3.34) are valid for M5.*

Reducing the Redundancy of Constraints

The number of Constraints (3.32) is extremely large in practice. In principle, to enumerate all constraints, we need to add $A = \sum_{n=1}^{\Gamma} \mathcal{C}_{|R|}^n$ into M5 ($\Gamma \leq |R|$). The following proposition shows that it is safe to reduce the number of Constraints (3.32) from A to $A' = \mathcal{C}_{|R|}^{\Gamma}$.

Proposition 3.4. *Constraints (3.32) can be equivalently replaced by at most A' constraints.*

Considering $\bar{Z}$ is a constant, M5 can be solved by solving the following model (M6), and ${Z_5}^* = \bar{Z} - {Z_6}^*$, where ${Z_6}^*$ is the optimal solution of M6.

$$\text{(M6) } \underset{\chi, H}{\text{Minimize}} \ Z_6 = \sum_{q \in Q} c_q \chi_q + H, \tag{3.38}$$

$$\text{subject to } \ (3.30)\text{–}(3.34).$$

Solving M6 can be very difficult for two reasons. First, the binary variables χ_q require the model to be solved within a branch-and-bound (BB) framework. Second, even solving the linear relaxation of M6 (denoted by LM6) is not easy. The difficulty of solving LM6 stems from its huge number of constraints (rows) and variables (columns). Even with Proposition 3.4, the number of constraints generated by (3.32) is still exponential in $|V|$, $|N|$ and Γ. In addition, the number of columns in Q (Q_3) is also exponential in $|N|$.

Therefore, to deal with these difficulties, we propose a branch-and-price-and-cut (BPC) algorithm to solve M6. In the algorithm, M6 is solved in a branch-and-bound framework, where the rows (for Constraints (3.32)) and columns (from Q_3) are generated dynamically for solving the LM6 at each node $\mathfrak{N}$ in the BB tree (denoted by LM6($\mathfrak{N}$)).

3.6 Branch-and-price-and-cut Algorithm

This section describes the branch-and-price-and-cut algorithm we have developed to solve M6. Before presenting the algorithm, we introduce the following definition.

Definition 3.3. ***Integer Solutions and Fractional Solutions.*** *Let* $(\mathbf{X}, H)$ *be a solution obtained by solving an LM6($\mathfrak{N}$), where* $\mathbf{X}$ *is a* $|Q|$*-dimensional vector of* χ_q*'s. Regardless of the value of* H*, the solution is defined to be integral if and only if all* χ_q*'s in* $\mathbf{X}$ *are integral and is defined to be fractional, otherwise.*

The BPC solves the LM6($\mathfrak{N}$) through column-and-row generation at each node $\mathfrak{N}$ of a BB tree (refer to Sections 3.6.2 and 3.6.3), where columns are generated by solving a pricing problem (denoted by PP) and rows are generated by solving a separation problem (denoted by SP). If the solution to an LM6($\mathfrak{N}$) is integral, a valid upper bound is obtained for M6, otherwise, branching is used to eliminate the current fractional solution (refer to Section 3.6.1). To improve the efficiency of the algorithm, we also develop a primal heuristic that generates upper bounds based on fractional solutions (refer to Section 3.6.4).

The algorithm used to solve LM6($\mathfrak{N}$) at each node $\mathfrak{N}$ is given in Algorithm 2, which solves the problem in a column-and-row generation fashion. At each iteration, the algorithm solves a reduced version of the LM6($\mathfrak{N}$) [denoted by RLM6($\mathfrak{N}$)]. Let $\tilde{\Theta}$ and $\tilde{Q}$ denote the current set of rows for Constraints (3.32) and the current set of columns in the RLM6($\mathfrak{N}$), respectively. It is easy to see that $\tilde{\Theta} \subseteq \Theta$, and $\tilde{Q} \subseteq Q$. After the RLM6($\mathfrak{N}$) has been solved, we check whether there are any columns with negative reduced cost by solving a pricing problem. Let Q' denote the set of columns with negative reduced cost. If $Q' \neq \varnothing$, set $\tilde{Q} = \tilde{Q} \cup Q'$, and solve RLM6($\mathfrak{N}$) again. The procedure will be repeated until $Q' = \varnothing$. Afterwards, a separation problem is solved to check the feasibility of the current solution with regard to (3.32). If new rows (constraints) are separated, we solve a new RLM6($\mathfrak{N}$) with the updated $\tilde{\Theta}$ by a new round of column generation.

When no new rows and columns can be found, the LM6($\mathfrak{N}$) has been solved to optimality. If the solution to the LM6($\mathfrak{N}$) is fractional, then depending on whether its objective value is smaller than the upper bound, the corresponding node is fathomed (if no) or branched to generate new nodes (if yes). If the solution is integral, we then update the upper bound if its objective value is smaller than the current one.

Algorithm 2 The Column-and-Row Generation Procedure.

Input: LM6($\mathfrak{N}$);

Output: The optimal objective value $Z(\mathfrak{N})^*$ of the LM6($\mathfrak{N}$) and an optimal solution (X^*, H^*) to the LM6($\mathfrak{N}$);

```
Row Initiation: Let Θ̃ = Θ_0, where Θ_0 corresponds to the incumbent set of rows;   ▷ For the root node, Θ_0 = ∅, and for a non-root node, Θ_0 is the set of all G's that have been generated in the algorithm.
while True do
    Column Initiation: Let Q̃ = Q_0, where Q_0 is the initial set of columns for the current RLM6(𝔑) (Section 3.6.2);   ▷ Columns are initiated when the algorithm (i) solves the initial RLM6(𝔑) (i.e., Θ̃ = Θ_0) of node 𝔑 and (ii) solves a new RLM6(𝔑) with updated Θ̃ (i.e., new constraints are separated).
    while True do
        Solve the RLM6(𝔑) with the current set of columns (Q̃) and rows (Θ̃);
        Solve the PP of RLM6(𝔑), and obtain Q′ (set of columns with negative reduced cost) (Section 3.6.2);
        if Q′ = ∅ then
            Break the inner while loop;
        end if
        Q̃ = Q̃ ∪ Q′;
    end while
    if Z(𝔑)* < UB then   ▷ UB is the incumbent upper bound for M6.
        Solve the SP, and obtain Θ′ (set of violated rows) (Section 3.6.3);
    end if
    if Θ′ = ∅ then
        Break the outer while loop;
    end if
    Θ̃ = Θ̃ ∪ Θ′;
end while
```

3.6.1 Branching Strategy

This section introduces how the BB tree is explored and extended. It is explored using the best-bound rule, i.e., we select the node with the overall minimum lower bound to branch. When generating new nodes, we adopt a three-level branching strategy which combines, in descending order of their priorities, contract-branch, ship-cargo-branch, and cargo-link-branch. Let $\tilde{Q}$ denote set of columns generated for solving the LM6($\mathfrak{N}$) at node $\mathfrak{N}$. Note that $\tilde{Q} = Q_1 \cup Q_2 \cup \tilde{Q}_3$ where $\tilde{Q}_3 \subseteq Q_3$ is generated dynamically during the solution procedure (refer to Section 3.6.2). Let $(\mathbf{X}^*, H^*)$ denote the optimal solution to the LM6($\mathfrak{N}$). If all ${\chi_q}^*$ in $\mathbf{X}^*$ take binary values, then there is no need to branch. Otherwise, branch decisions are made based on the results of the following calculations.

First, for each trip (v, i, j), we calculate

$$\varphi_{i,j}^{v} = \sum_{q \in \tilde{Q}} \Upsilon_{i,j}^{q,v} {\chi_q}^*,$$

where $\Upsilon_{i,j}^{q,v} = 1$, if (v, i, j) is included in q and 0, otherwise. Then, for each ship-cargo combination (v, i), we calculate

$$\varrho_{v,i} = \sum_{j \in N \cup \{T\}} \varphi_{i,j}^{v}.$$

Finally, we define a couple (i, j) to be a cargo-link on the routes of ships, representing (i) cargo j is the first loaded cargo ($i = 0$, $j \in N$), (ii) cargo j is loaded immediately after transporting cargo i ($i, j \in N$, $i \neq j$), and (iii) cargo i is the last transported cargo ($i \in N$, $j = T$) on the routes. Then for each cargo-link (i, j), we calculate

$$\sigma_{ij} = \sum_{v \in V} \varphi_{i,j}^{v}.$$

We distinguish the following three conditions according to the values of ${\chi_q}^*$'s, $\varrho_{v,i}$'s, and σ_{ij}'s:

Condition (I) Regardless of the ${\chi_q}^*$ for $q \in Q_2 \cup \tilde{Q}_3$, at least one ${\chi_q}^*$ is fractional for $q \in Q_1$;

Condition (II) All $\chi_q{}^*$ are integral for $q \in Q_1$, at least one $\chi_q{}^*$ for $q \in Q_2 \cup \tilde{Q}_3$ is fractional, and at least one $\varrho_{v,i}$ is fractional;

Condition (III) All $\chi_q{}^*$ are integral for $q \in Q_1$, at least one $\chi_q{}^*$ for $q \in Q_2 \cup \tilde{Q}_3$ is fractional, all $\varrho_{v,i}$'s are integral, and at least one σ_{ij} is fractional.

It is easy to see that any fractional solution to LM6($\mathfrak{N}$) satisfies one of these three conditions. To impose the branching results at the nodes, let $B^v_{i,j}$ ($v \in V$, $i \in N \cup \{0\}$, $j \in N \cup \{T\} \setminus \{i\}$) be an indicator which equals 1 (resp., 0) to denote whether ship v can (resp., cannot) travel in trip (v, i, j) in the nodes generated by branching, and let M be a sufficiently large constant. For Condition (I), we identify the $q \in Q_1$ with $\chi_q{}^*$ closest to 0.5 (denoted by q^*), and branch on k^*, which is the contract covered by q^*. For nodes in the left branch (i.e., contract k^* should be accepted), we set $c_{q^1_{k^*}} = M$ ($q^1_{k^*} \in Q_1$); for nodes in the right branch (i.e., contract k^* should be rejected), we set (i) $c_{q^2_i} = M$, $i \in N^c_{k^*}$ ($q^2_i \in Q_2$), and (ii) $B^v_{i,j} = 0$, $i \in N^c_{k^*}, j \in N \cup \{T\} \setminus \{i\}$. For Condition (II), we branch on the (v, i) with $\varrho_{v,i}$ closest to 0.5 (denoted by (v^*, i^*)). For the nodes in the left branch (i.e., ϱ_{v^*,i^*} is fixed at zero, or ship v^* should not be used to transport cargo i^*), we set $B^{v^*}_{i^*,j} = 0, j \in N \cup \{T\} \setminus \{i^*\}$; for the nodes in the right branch (i.e., ϱ_{v^*,i^*} is fixed at one, or cargo i^* must be transported by v^*), we set (i) $c_{q^1_k} = M$ ($q^1_k \in Q_1$), if $i^* \in N^c_k$, (ii) $c_{q^2_{i^*}} = M$ ($q^2_{i^*} \in Q_2$), and (iii) $B^v_{i^*,j} = 0, j \in N \cup \{T\} \setminus \{i^*\}, v \in V \setminus \{v^*\}$. For Condition (III), the node is branched on cargo-link (i, j) with σ_{ij} closest to 0.5 (denoted by (i^*, j^*)). For the nodes in the left branch, we set $B^v_{i^*,j^*} = 0, v \in V$. For those in the right branch, we set (i) $c_{q^1_k} = M$ ($q^1_k \in Q_1$), if $i^* \in N^c_k$ or $j^* \in N^c_k$, (ii) $c_{q^2_{i^*}} = M$, $c_{q^2_{j^*}} = M$ ($q^2_{i^*}, q^2_{j^*} \in Q_2$), and (iii) $B^v_{i,j^*} = 0, i \in N \cup \{0\} \setminus \{i^*, j^*\}, v \in V$, and $B^v_{i^*,j} = 0, j \in N \cup \{T\} \setminus \{i^*, j^*\}, v \in V$.

3.6.2 Column Generation

In this section, we first introduce the method we have developed to generate an initial set of columns for solving an RLM6($\mathfrak{N}$) and then propose the pricing problem (PP) for identifying columns with negative reduced cost. Finally, we propose the labeling algorithm to solve the PP.

Column Initiation

To save the time for generating columns, we propose the following methods for the generation of the initial set of columns (denoted by Q_0) when solving an RLM6($\mathfrak{N}$). At the root node, we set $Q_0 = Q_1 \cup Q_2$. For a non-root node $\mathfrak{N}$, we generate Q_0 as $Q_0 = Q_1 \cup Q_2 \cup \hat{Q}_3$. $\hat{Q}_3 \subseteq Q_3$ comprises two subsets denoted by $\hat{Q}_3^1$ and $\hat{Q}_3^2$. In particular, $\hat{Q}_3^1 := \{q | q \in \tilde{Q}_3', \tilde{\chi}_q' > 0, B_{i,j}^v = 1, (v,i,j) \in \Lambda_q\}$. Here $\tilde{Q}_3'$ and $\tilde{\chi}_q'$'s are the set of generated columns and their solutions for solving the LM6 at the parent node of the $\mathfrak{N}$. In addition, Λ_q is the set of (v,i,j)'s covered in q. Besides, $\hat{Q}_3^2$ is non-empty only when the row generation identifies new constraints for the previous RLM6($\mathfrak{N}$) (i.e., the RLM6($\mathfrak{N}$) without the new constraints found by the row generation this time) and a new RLM6($\mathfrak{N}$) with an updated $\tilde{\Theta}$ is solved by column generation. In this case, $\hat{Q}_3^2 := \{q | q \in \tilde{Q}_3'', \tilde{\chi}_q'' > 0\}$, where $\tilde{Q}_3''$ and $\tilde{\chi}_q''$'s are the set of generated columns and their solutions for solving the previous RLM6($\mathfrak{N}$).

The Pricing Problem

The pricing problem (PP) for solving an RLM6($\mathfrak{N}$) is composed of $|V|$ subproblems, each corresponding to one ship. We denote by PSP$_v$ the subproblem for ship v. To define PSP$_v$, let $\pi_v, v \in V$, $\varpi_i, i \in N$, $\eta_G, G \in \tilde{\Theta}$ ($\tilde{\Theta} \subseteq \Theta$ is the current set of G's separated by row generation) and $\vartheta_k, k \in K$ be the dual values for constraints (3.30), (3.31), (3.32), and (3.34), respectively, after solving the RLM6($\mathfrak{N}$).

The pricing problem for ship $v \in V$ (PSP$_v$) is formulated as follows. The decision variables in this model are (i) $\lambda_{i,j}$ which is set to be 1, if trip (v,i,j) is traveled by the ship and 0, otherwise, and (ii) the loading start time of cargo i (b_i). The model is as follows:

$$
\begin{aligned}
(\mathrm{PSP}_v)\ \underset{b,\lambda}{\text{Minimize}}\ Z_7 = f_v + \pi_v + &\textstyle\sum_{i\in N\cup\{0\}} \sum_{j\in N\cup\{T\}\setminus\{i\}} C^v_{i,j}\lambda_{i,j} \\
&-\textstyle\sum_{i\in N}\varpi_i \sum_{j\in N\cup\{T\}\setminus\{i\}} \lambda_{i,j} + \sum_{G\in\tilde{\Theta}} \eta_G \sum_{r\in G\cap R^v_{i,j}} d_r\lambda_{i,j} \\
&+\textstyle\sum_{k\in K} \vartheta_k \sum_{i\in N^c_k} \sum_{j\in N\cup\{T\}\setminus\{i\}} \lambda_{i,j},
\end{aligned}
\tag{3.39}
$$

subject to

$$\sum_{j\in N\cup\{T\}\setminus\{i\}} \lambda_{i,j} \leq 1, \quad i\in N \tag{3.40}$$

$$\sum_{j\in N} \lambda_{0,j} = 1 \tag{3.41}$$

$$\sum_{j\in N\cup\{0\}\setminus\{i\}} \lambda_{j,i} = \sum_{j\in N\cup\{T\}\setminus\{i\}} \lambda_{i,j}, \quad i\in N \tag{3.42}$$

$$\sum_{i\in N} \lambda_{i,T} = 1 \tag{3.43}$$

$$\lambda_{i,j} \leq B^v_{i,j}, \quad i\in N, j\in N\setminus\{i\} \tag{3.44}$$

$$b_i \geq e_i, \quad i\in N \tag{3.45}$$

$$b_i \leq l_i, \quad i\in N \tag{3.46}$$

$$b_j \geq (o_v + t^v_{0,j})\lambda_{0,j}, \quad j\in N \tag{3.47}$$

$$b_j \geq b_i + t^v_{i,j} + M'_{v,i,j}(\lambda_{i,j}-1), \quad i\in N, j\in N\setminus\{i\} \tag{3.48}$$

$$\lambda_{i,j}\in\{0,1\}, \quad I_{v,i}=1, I_{v,j}=1, i\in N\cup\{0\}, j\in N\cup\{T\}\setminus\{i\}, \tag{3.49}$$

where $M'_{v,i,j}$ are defined in Section 3.4.1.

The objective function (3.39) minimizes the reduced cost of a route for the ship. Constraints (3.40) prevent a cargo from appearing on the route more than once. Equations (3.41)–(3.43) are flow conversation constraints. The branching results at the current node are imposed by Constraints (3.44). Constraints (3.45) and (3.46) ensure the feasibility of the loading time for each cargo. Constraints (3.47) and (3.48) calculate and sequence the loading start times for cargoes on the route. Finally, we define $\lambda_{i,j}$ variables to be binary and guarantee the ship-cargo compatibility in the last set of constraints.

The Labeling Algorithm

The problem formulated in PSP_v for each $v \in V$ can be solved by a labeling algorithm. The algorithm works on a graph $G_v = (\bar{N}_v, \bar{E}_v)$, where $\bar{N}_v$ is a set of nodes, and $\bar{E}_v$ is a set of arcs. $\bar{N}_v$ and $\bar{E}_v$ are defined by

$$\bar{N}_v := \{i \in N, I_{v,i} = 1\} \cup \{0, T\},$$

and

$$\bar{E}_v := \{(i,j) | i,j \in \bar{N}_v, B^v_{i,j} = 1\}.$$

In G_v, travelling on an arc (i,j) is equivalent to ship v travelling in a trip (v,i,j). The time of travelling on an arc (i,j) is defined to be $t^v_{i,j}$. We now derive the cost of travelling on an arc (i,j) in graph G_v (denoted by $\zeta_{i,j}$).

Then, $\zeta_{i,j}$ can be calculated by the following equation:

$$\zeta_{i,j} = \begin{cases} f_v + C^v_{i,j} + \pi_v - \varpi_j + \sum_{G \in \tilde{\Theta}} \eta_G \sum_{r \in G \cap R^v_{i,j}} d_r - \sum_{k \in K} \vartheta_k \mathbf{I}^{\mathrm{K}}(j,k), i = 0, j \in N, \\ C^v_{i,j} - \varpi_j + \sum_{G \in \tilde{\Theta}} \eta_G \sum_{r \in G \cap R^v_{i,j}} d_r - \sum_{k \in K} \vartheta_k \mathbf{I}^{\mathrm{K}}(j,k), i,j \in N, \\ \sum_{G \in \tilde{\Theta}} \eta_G \sum_{r \in G \cap R^v_{i,j}} d_r, i \in N, j = T, \end{cases} \tag{3.50}$$

where $\mathbf{I}^{\mathrm{K}}(i,k)$ is an indicator function which equals 1 if $i \in N^c_k$ and 0, otherwise.

Let P denote a path in G_v. We define a P to be feasible if it satisfies the following three conditions: (1) P starts from node 0 (i.e, the initial position of ship v) at time o_v, (2) P is elementary (i.e., visits each node $i \in \bar{N}_v$ at most once), and (3) each node i in P is visited by the ship within the time window $[e_i, l_i]$. Let $\mathcal{P}_v$ be the set of all feasible paths in G_v. Moreover, the cost of P, denoted by ξ_P, is calculated by adding the $\zeta_{i,j}$ of all arc (i, j) contained in the path.

Therefore, the PSP_v can be defined as finding a path $P^* \in \mathcal{P}_v$ such that

$$P^* \in \arg\min_{P \in \mathcal{P}_v} \xi_P.$$

The PSP_v can be taken as an elementary shortest path problem with time windows (ESPPTW), which can be solved by a labeling algorithm. In the labeling algorithm for the ESPPTW defined on $G_v = (\bar{N}_v, \bar{E}_v)$, labels are used to represent partial paths starting from node 0. Corresponding to each label ς, let n_ς, $\mathbf{s}_\varsigma$, t_ς, and c_ς denote the last visited node $i \in \bar{N}_v$, the set of previously visited nodes in the path, the time when ship v is ready to load cargoes at node n_ς and the cost of the path, respectively.

The algorithm starts by creating an initial label ς at node 0, such that $n_\varsigma = 0$, $\mathbf{s}_\varsigma = \varnothing$, $t_\varsigma = o_v$, and $c_\varsigma = 0$. To find the path with the minimum reduced cost, the labels are extended forwardly. In particular, for a label ς with $n_\varsigma = i$, the algorithm extends the corresponding partial path from i to any $j \in \bar{N}_v$ such that (i) $j \neq i$, (ii) $j \notin \mathbf{s}_\varsigma$, (iii) $(i, j) \in \bar{E}_v$ and (iv) $t_\varsigma + t^v_{i,j} \leq l_j$. For any feasible j, a new label ς' such that $n_{\varsigma'} = j$, $\mathbf{s}_{\varsigma'} = \mathbf{s}_\varsigma \cup \{i\}$, $t_{\varsigma'} = \max\{t_\varsigma + t^v_{i,j}, e_j\}$, and $c_{\varsigma'} = c_\varsigma + \zeta_{i,j}$ is created. The algorithm terminates when no new labels can be created. Besides, a label ς_1 is dominated by another label ς_2 if (i) $n_{\varsigma_1} = n_{\varsigma_2}$, (ii) $\mathbf{s}_{\varsigma_1} \subseteq \mathbf{s}_{\varsigma_2}$, (iii) $t_{\varsigma_1} \geq t_{\varsigma_2}$, and (iv) $c_{\varsigma_1} \geq c_{\varsigma_2}$. Dominated labels are discarded to accelerate the algorithm.

Algorithm Refinements

In the following sections, we improve the efficiency of the labeling algorithm in three ways.

Shrinking the graph. To further narrow down the searching space, we define a set of infeasible arcs for ship v (IA_v), such that

$$\begin{aligned} IA_v := \{(i,j)|i,j \in \bar{N}_v,\\ o_v + t^v_{0,j} > l_j, \text{if } i = 0,\\ e_i + t^v_{i,j} > l_j, \text{if } i \neq 0\}. \end{aligned}$$

Then G_v for PSP_v can be reduced into $G'_v = (\bar{N}_v, \bar{E}'_v)$, where $\bar{E}'_v = \bar{E}_v \setminus IA_v$.

q-route relaxation. To enable stronger dominance relationships among labels, we relax the constraint that the path generated by the algorithm should be elementary. Instead, we allow cycles to appear in the paths. Hence, when extending a label ς to j, the algorithm does not check whether j has been included in the path corresponding to ς. Besides, when checking the dominance between two labels, the algorithm does not compare the sets of nodes visited in the two paths corresponding to the labels.

Enhancement using branching results. We also speed up the labeling algorithm using the branching results. For PSP_v, let $\bar{F}_v$ be the set of nodes that must be visited by v due to the branching decisions at the current path. Further, let F'_ς denote the set of nodes in $\bar{F}_v$ that have not been visited in the corresponding partial path. The algorithm is enhanced as follows. Every time a new label ς is generated, we check whether the label can still be extended to each i in F'_ς. If any $i \in F'_\varsigma$ cannot be visited by extending the current partial path, the label will be discarded. Meanwhile, as for the dominance check, we require $F'_{\varsigma_2} \subseteq F'_{\varsigma_1}$ to be a necessary condition for label ς_1 to be dominated by ς_2. In addition, for any v, we only generate columns q such that $\bar{F}_v \subseteq \Phi_q$, which reduces the likelihood of generating fractional solutions for $q \in Q_3$.

3.6.3 Row Generation

Rows are generated when the RLM6($\mathfrak{N}$) of node $\mathfrak{N}$ has been solved to optimality by column generation (i.e., the inner while loop of Algorithm 2 has been completed). In this section, we first formulate the problem for separating violated constraints and then propose an algorithm that solves the separation problem in polynomial time. By taking advantage of the special structures of the problem we propose a multi-cut generation technique to further strengthen the cuts.

The Separation Problem

Suppose an RLM6($\mathfrak{N}$) has been solved to optimality by column generation. To check the feasibility of the current solution, a separation problem (SP) is solved. If violations are identified, new constraints are generated for the RLM6($\mathfrak{N}$) (i.e., a set of G's is added into $\tilde{\Theta}$). Let ${\chi_q}^*, q \in \tilde{Q}$, H^* be the optimal solution to the current RLM6($\mathfrak{N}$). We separate constraints for Equation (3.32) of the LM6($\mathfrak{N}$) by solving the following separation problem (SP). In SP, the decision variable is ρ_r, which is binary and decides whether d_r should be included as part of the robust cost:

$$\text{(SP) Maximize } Z_8 = \sum_{r \in R} \rho_r d_r \sum_{q \in \tilde{Q}} \gamma_{q,r} {\chi_q}^* - H^* \tag{3.51}$$

subject to

$$\sum_{r \in R} \rho_r = \Gamma \tag{3.52}$$

$$\rho_r \in \{0, 1\}, \quad r \in R, \tag{3.53}$$

where Equation (3.52) is derived from Proposition 3.4.

Let ${Z_8}^*$ and ${\rho_r}^*$ be the optimal objective value and the optimal solution to the SP. If ${Z_8}^* > 0$ in the optimal solution to the SP, then a violation of Equation (3.32) is detected and we add $G := \{r | {\rho_r}^* = 1, r \in R\}$ into $\tilde{\Theta}$. In this case, a new RLM6($\mathfrak{N}$) with the updated $\tilde{\Theta}$ will again be solved by column generation. If ${Z_8}^* \leq 0$, then the LM6($\mathfrak{N}$) has been solved to optimality.

A Polynomial-time Algorithm for Solving the SP

We solve the SP by Algorithm 3 whose correctness in given in Proposition 3.5.

Proposition 3.5. *Algorithm 3 solves the SP in* $O(|R||\tilde{Q}| + |R| \log |R|)$ *time.*

Algorithm 3 The Row Generation Procedure.

Input: ${\chi_q}^*, q \in \tilde{Q}$, and H^*;
Output: ${Z_8}^*$ and G;

Generate $\bar{G} := \{r | d_r \sum_{q \in \tilde{Q}} \gamma_{q,r} {\chi_q}^* > 0, r \in R\}$;
if $|\bar{G}| \leq \Gamma$ **then**
    Generate $G = \bar{G}$;
end if
if $|\bar{G}| > \Gamma$ **then**
    Generate G by selecting from $\bar{G}$ the r's with $|\Gamma|$ largest $d_r \sum_{q \in \tilde{Q}} \gamma_{q,r} {\chi_q}^*$;
end if
${Z_8}^* = \sum_{r \in G} d_r \sum_{q \in \tilde{Q}} \gamma_{q,r} {\chi_q}^* - H^*$.

Multi-cut Generation

In the row generation procedure, supposing a new constraint is separated, then the following constraint is generated for the RLM6($\mathfrak{N}$) and all the LM6($\mathfrak{N}$)'s at nodes generated later in the algorithm:

$$H - \sum_{q \in Q} \sum_{r \in G} d_r \gamma_{q,r} \chi_q \geq 0, \tag{3.54}$$

where G is a set of voyages (r) returned by Algorithm 3. Each voyage r in G corresponds to a ship sailing from an origin to a destination. Let $\bar{R}_r$ denote the set of voyages that share the same origin and the same destination with r (while the ships sailing in them can be different).

We can tighten (3.54) to be

$$H - \sum_{q \in Q} \sum_{r' \in \bar{G}} d_{r'} \gamma_{q,r'} \chi_q \geq 0, \tag{3.55}$$

where $\bar{G} := \bigcup_{r \in G} \bar{R}_r$.

Proposition 3.6. *Constraint (3.55) is valid for M6.*

Remedy of Infeasible Integer Solutions

We consider the situation in which the ${\chi_q}^*$ variables in the solution delivered by the column generation procedure are all integral. Then, if ${Z_8}^* \leq 0$, the solution is feasible for M6. In this case, ${Z_6}^*$ is a feasible upper bound (denoted by UB') for M6 (${Z_6}^*$ is the optimal objective value of the RLM6($\mathfrak{N}$)). Meanwhile, if ${Z_8}^* > 0$, the solution is infeasible for M6, but we can easily construct a feasible upper bound for M6, which is calculated as $UB' = {Z_6}^* + {Z_8}^*$. In either case, if UB' is smaller than the incumbent upper bound (denoted by UB) for M6, we update $UB = UB'$.

3.6.4 A Primal Heuristic

A high-quality upper bound for M6 helps to prune nodes in the B&B tree in early stages. In order to quickly identify high-quality upper bounds, we develop a heuristic to construct a feasible integer solution using a fractional solution delivered by the algorithm. The construction is completed in three phases, these being the route-construction phase, the cargo-check phase, and the robustness-check phase. We run the heuristic if the obtained solution is fractional when an RLM6($\mathfrak{N}$) has been solved to optimum by column generation.

The Route-construction Phase

In the first phase, we construct routes for ships that will be used to construct an integer solution. The routes are constructed based on a set of columns $\tilde{Q}'$ such that $\tilde{Q}' := \{q|\chi_q > 0, q \in \tilde{Q} \cap Q_3\}$, where $\tilde{Q}$ is the set of columns generated for solving the RLM6($\mathfrak{N}$). During the construction, we first prioritize each column, and columns with higher priorities have greater chances of being selected to be included in the integer solution. The priorities for the columns are assigned according to three strategies (ι's), and we will construct an integer solution $\mathcal{S}_\iota$ for each ι.

To describe each strategy, the following additional notations will be used. First, let nr_q^1 denote the "Net revenue I" of column q, which is calculated by $nr_q^1 = \sum_{i \in \Phi_q} \hat{c}_i - c_q$. Second, let nr_q^2 denote the "Net revenue II" of column q, which is calculated by $nr_q^2 = \sum_{i\in\Phi_q\cap N^{\mathrm{m}}} \hat{c}_i + \sum_{k\in K} p_k \frac{|\Phi_q\cap N_k^{\mathrm{c}}|}{|N_k^{\mathrm{c}}|} - c_q$. The following three strategies are adopted to prioritize columns in $\tilde{Q}'$:

Strategy 1 Prioritize the columns in a sequence with non-decreasing nr_q^1's;

Strategy 2 Prioritize the columns in a sequence with non-decreasing nr_q^2's;

Strategy 3 Prioritize the columns in a sequence with non-decreasing χ_q's, and for two columns with the same χ_q, give higher priority to the one with the larger nr_q^1.

The detailed procedure to construct the columns in Q_3 under each strategy ι (denoted by $\hat{Q}_\iota, \iota = 1, 2, 3$) is demonstrated in Algorithm 4 in B.2.1.

The Cargo-check Phase

In the first phase, we have identified three sets of columns each constructed based on one prioritizing strategy. Each column q in these sets $\hat{Q}_\iota$ represents a route traveled by a ship (i.e., $q \in Q_3$). To generate a complete solution, we need to further construct solutions for columns from Q_1 and Q_2. To this end, we first decide whether a contract $k \in K$ should be accepted or not. Then for each $i \in N^{\mathrm{m}}$ and $i \in N_k^{\mathrm{c}}$ such that k is decided to be accepted, we need to decide whether it should be transported by a ship $v \in V$ or by a voyage charter. Further, columns in $\hat{Q}_\iota$ require modification based on these decisions. Algorithm 5 presented in B.2.2 is applied for these purposes.

The Robustness-check Phase

In the last phase, we identify the minimum feasible robust costs H_ι's for the constructed integer solutions. In particular, for each ι, let $Q^\iota = \check{Q}_1^\iota \cup \check{Q}_2^\iota \cup \check{Q}_3^\iota$. To calculate H_ι, we solve the SP (Section 3.6.3) by letting ${\chi_q}^* = 1, \forall q \in Q^\iota$ and $H^* = 0$, and set $H_\iota = {Z_8}^*$. This completes the whole construction procedure.

Finally, the objective value (Z_{6_ι}) for each $\mathcal{S}_\iota$ is calculated by $Z_{6_\iota} = \sum_{q\in Q^\iota} c_q + H_\iota$. The current upper bound for M6 (UB) is updated to be $UB = \min_{\iota=1}^3\{Z_{6_\iota}\}$ if $UB > \min_{\iota=1}^3\{Z_{6_\iota}\}$.

3.7 Numerical Experiments

We now perform extensive computational experiments in order to confirm the applicability and effectiveness of our models and algorithms. The experiments include three parts. In the first part, we examine the performance of a standard MIP solver for model M4 and we compare it with the BPC algorithm. The impact of the multi-cut generation technique is also tested in the first part. In the second part, we analyze the effects of using the strengthened set covering model and the primal heuristic on the performance of the BPC algorithm. In the third part, we evaluate the value of robust optimization in the ship routing problem with batched cargo selection.

3.7.1 Algorithmic Settings and Computational Platform

In the numerical experiments, we will compare the performances of five different algorithms for solving the instances of the RSRPB. These algorithms are (i) the branch-and-cut algorithm in CPLEX for solving model M4 (denoted by CPLEX), (ii) the branch-and-price-and-cut algorithm using the basic set-covering model (M5) without multi-cut generation and the primal heuristic (we denote this algorithm by OBPC), (iii) the branch-and-price-and-cut algorithm using the basic set-covering model (M5) and the multi-cut generation technique without the primal heuristic (we denote this algorithm by OBPC+MC), (iv) the branch-and-price-and-cut algorithm using the strengthened set-covering model (M6) and the multi-cut generation technique without the primal heuristic (we denote this algorithm by OBPC+MC+SM), and (v) the branch-and-price-and-cut algorithm using the strengthened set-covering model (M6), the multi-cut generation technique, and the primal heuristic (we denote this algorithm by OBPC+MC+SM+PH).

In all experiments, unless otherwise specified, we use the following algorithmic settings. First, the time limits for the algorithms to solve the instances are all set to 3,600 seconds. Second, we set the optimality tolerance level to be 0.1%. That is, when solving an instance, we allow the algorithm to stop when the optimality gap between the upper bound and the lower bound is no larger than 0.1%.

All experiments are coded in C++ and performed on an Intel Core i7 2.20 GHz

PC with 32 GB RAM. For ease of comparison, all experiments are conducted in a single thread environment. CPLEX 12.6 is used as the MIP solver for model M4 and also the LP solver in the branch-and-price-and-cut algorithms.

3.7.2 Instance Generation

We generated the instances based on PB's Pacific Ocean business sector (i.e., cargo transportation along or between the west coast and the east coast of the Pacific Ocean). The instances have different settings for the number of ships $|V|$ in the candidate pool and the number of COA offers $|K|$ faced by the company. In particular, $|V|$ is set to 10, 20 and 30, and $|K|$ is set to 10 and 20. We also consider three planning horizon lengths (L): 90, 120, and 150 days. In our tests, we consider 14 different combinations of L, $|V|$ and $|K|$. For each combination we generate five random instances, yielding 70 instances in total. The n^{th} ($n = 1, 2, 3, 4, 5$) instance with L days' planning horizon, $|V|$ ships, and $|K|$ offers is denoted by $(L, |V|, |K|)$-n.

We first generate a set of loading and unloading ports in the instances. As shown in Figure 3.1, cargoes are mainly transported between six areas in PB's Pacific Ocean business sector. To generate the ports, we identify a center point in each area and then generate 20 ports that are randomly located in a disk with 500nm radius centered at the point (nm is the abbreviation for nautical mile). Associated with each port p is a port charge rate p_p^r which is randomly generated in $[0.2, 0.5]$ (dollars per ton per day).

The ship parameters are as follows. Half of the ships in V are controlled by the company at the beginning of the planning horizon, and half come from the chartering market (i.e., $|V_1| = |V_2| = |V|/2$). The capacity (sc_v) and the speed of each ship are randomly generated in $[25, 65]$ (thousand tons) and $[12, 16]$ (knots), respectively. In addition, for the ships in V_1, we set $o_v \in [1, 30]$, and $f_v \in [(L - o_v)(3 + 0.04sc_v), (L - o_v)(3 + 0.07sc_v)]$ (thousand dollars). For those in V_2, we set $o_v \in [1, 60]$, and $f_v \in [(L - o_v)(3.15 + 0.04sc_v), (L - o_v)(3.15 + 0.07sc_v)]$ (thousand dollars). The initial position (port) for ship v (denoted by p_v^{s}) is randomly selected from the 120 ports by considering the historical distribution of cargoes among the

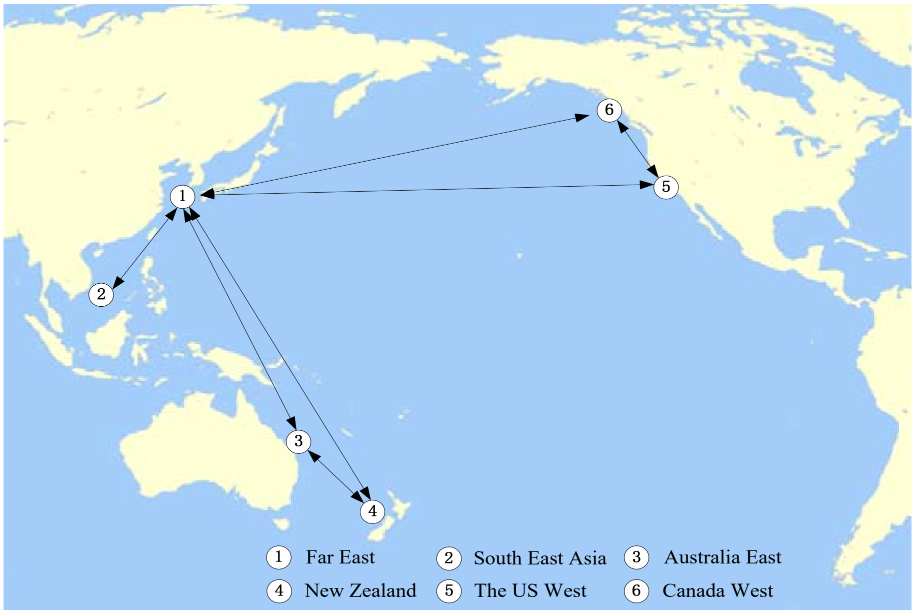

Figure 3.1: Main trading areas around the Pacific Ocean.

trade links.

In the instances, we distinguish between three types of cargoes: mandatory cargoes, spot market cargoes, and COA cargoes. Note that a spot market cargo is treated as a COA with a single cargo in the model and in the algorithm. The cargoes are generated as follows. First, in an instance, the number of mandatory cargoes is set to be $|N^{\mathrm{m}}| = \lceil |V_1|L/30 \rceil$. We set the number of cargoes from the spot market to be $\lceil 0.5|V_1|L/30 \rceil$. The number of cargoes contained in a COA within the planning horizon is $\lceil L/30 \rceil$. Second, the loading and unloading ports for each cargo are generated as follows. We first identify 14 main trade links (each link corresponds to a particular loading and unloading area), as shown in Figure 3.1. Before generating the exact loading and unloading ports for a cargo, we first randomly allocate the cargo to a particular trade link. In all instances, the possibility of a spot market, a mandatory cargo or the cargoes in a COA belonging to each link is set according to the historical distribution of cargoes among these links. After the trade link of a cargo is decided, we then generate its loading (unloading) port by randomly selecting one port from the 20 ports in the corresponding loading (unloading) area. We assume that cargoes in the same COA share the same loading (unloading) port. Third, the weight of a cargo (denoted by cw_i) is randomly generated in $[0.8\underline{sc}, \overline{sc}]$, where $\underline{sc}$ and $\overline{sc}$ represent the minimum and maximum capacities of ships in V, and cargoes in the same COA have the same weight. Fourth, for each mandatory cargo or spot market cargo, the start time of its loading period (e_i) is randomly generated in $[1, L-15]$ (day). Meanwhile, the e_i of cargoes in the same COA are uniformly distributed over the planning horizon. In addition, for each cargo, we let $l_i = e_i + DD_i$, where DD_i is a random number in $[1, 3]$ (days) for spot market cargoes and in $[3, 10]$ (days) for mandatory cargoes and cargoes in COAs. We also let cargoes that belong to the same COA share an identical DD_i. The freight revenue obtained from a COA k is calculated as $p_k = \sum_{i\in N_k^{\mathrm{c}}} cp_i$, and $cp_i = 10,000 + uf_i \cdot cw_i \cdot vd_i$ (dollars), where uf_i is randomly generated in $[35, 50]$ and vd_i is the distance (nm) for transporting the cargo which equals the Euclidean distance between the loading and unloading ports. The freight revenue for a spot market cargo i is set similarly, by letting $N_k^{\mathrm{c}} = \{i\}$. Finally, as for the ship-cargo compatibility ($I_{v,i}$), we set $I_{v,i} = 1$ if $sv_v \geq cw_i$ and 0, otherwise (it is assumed that all ships $v \in V$ can sail between any two ports).

We set the parameters of voyages and trips as follows. First, in all instances, the unit time is set to one day. Second, for cargo i, let p_i^{l} and p_i^{u} denote the loading and unloading ports of it. The handling times at the loading and unloading ports, $pt_{i,p_i^{\mathrm{l}}}$ and $pt_{i,p_i^{\mathrm{u}}}$ are both randomly generated in $[1, 3]$ (days). Then, the port charge of a ship v for loading (unloading) cargo i at port p is calculated as $pc_{i,p}^v = pr_p \cdot sc_v \cdot pt_{i,p}$. Second, we let $t_{i,j}^v = pt_{i,p_i^{\mathrm{l}}} + pt_{i,p_i^{\mathrm{u}}} + st_{i,j}^v$, where $st_{i,j}^v$ is the time for ship v to sail in the laden voyage from port p_i^{l} to port p_i^{u} and the ballast voyage from port p_i^{u} to port p_j^{l}. In addition, $t_{0,j}^v = st_{0,j}^v$ and $t_{i,T}^v = pt_{i,p_i^{\mathrm{l}}} + pt_{i,p_i^{\mathrm{u}}} + st_{i,T}^v$, where $st_{0,j}^v$ is the time for ship v to sail in the ballast voyage from port p_v^{s} (the initial port of ship v) to port p_j^{l} and $st_{i,T}^v$ is the time for the ship to sail in the laden voyage between ports p_i^{l} and p_i^{u}. $st_{i,j}^v$, $st_{0,j}^v$, and $st_{i,T}^v$ are obtained by dividing the Euclidean distances between the ports by the daily sailing distance and making the results integral by applying the ceiling operator. Third, we set $C_{i,j}^v = pc_{i,p_i^{\mathrm{l}}}^v + pc_{i,p_i^{\mathrm{u}}}^v + sc_{i,j}^v$ (in thousand dollars), where $sc_{i,j}^v$ denotes the sailing cost of the laden voyage for transporting cargo i and the ballast voyage to reposition the ship to load cargo j. In addition, we let $C^v_{0,j} = sc^v_{0,j}$ and $C^v_{i,T} = pc_{i,p^{\rm l}_i} + pc_{i,p^{\rm u}_i} + sc^v_{i,T}$, where $sc^v_{0,j}$ is the cost for ship v to sail from port $p^{\rm s}_v$ to port $p^{\rm l}_j$ and $sc^v_{i,T}$ is the cost for the ship to sail in the laden voyage for transporting cargo i. Moreover, $sc^v_{0,j}$, $sc^v_{i,j}$, and $sc^v_{i,T}$ are calculated using the daily bunker costs of ship v multiplied by the corresponding sailing times in laden and ballast voyages. In particular, when a ship is sailing in laden voyages, its daily bunker cost is randomly generated in $[9, 24]$ (thousand dollars) and we let ships with larger capacities have higher daily bunker costs. Meanwhile, the daily bunker cost of the same ship in ballast voyages is set to be 90% of that in laden voyages. The cost for using a voyage charter to transport cargo i is set to be $b_i = \underline{b_i} + vf_i \cdot cw_i \cdot vd_i$ (dollars), where $\underline{b_i}$ and vf_i are randomly generated in $[15,000, 20,000]$ and $[52.5, 100]$, respectively, and vd_i is the distance (nm) for transporting cargo i.

Finally, for the uncertainty budget, we let $\Gamma = |N^{\rm m}|$ and $d_r = rd \cdot \bar{c}_r$, where rd is randomly generated in $[0, 1]$. For each generated instance, we verify that the three assumptions **A1** to **A3** hold and make modifications if necessary.

3.7.3 Comparison of CPLEX, OBPC, and OBPC+MC

We first compare the performances of CPLEX (solving model M4), OBPC (the branch-and-price-and-cut algorithm using the basic set covering model without multi-cut generation and primal heuristic), and OBPC+MC (which is OBPC with multi-cut generation). We tested the three solution methods by solving 20 instances with 90 days' planning horizon, 10 or 20 ships, and 10 or 20 COAs. The results are presented in Table 3.3. Column 2 shows the total number of cargoes ($|N|$) in an instance. Columns 3 and 4 show the optimality gap (in percentage) delivered by CPLEX for solving M4 and the associated computational time. Columns 5 and 6 report the optimality gap and the computational time of OBPC. The optimality gap and the computational time of OBPC+MC are reported in Columns 7 and 8, respectively. Note that we report the optimality gap as "0.10%" if the resulting optimality gap of an algorithm for solving an instance does not exceed a tolerance of 0.10%. Similarly, if the computational time of an algorithm for solving an instance is less than or equal to one second, we report it as "1s" in the table. In "Average" Rows, we report the average optimality gaps and the computational times of the algorithms for solving 10 instances that share the same planning horizon length and the same number of ships.

Table 3.3: Comparison of the three algorithms.

Instance	$\lvert N \rvert$	CPLEX		OBPC		OBPC+MC	
		Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)
(90,10,10)-1	50	0.10	2	0.10	12	0.10	1
(90,10,10)-2	50	0.10	3	0.10	19	0.10	1
(90,10,10)-3	50	0.10	2	0.10	1	0.10	1
(90,10,10)-4	50	0.10	7	0.10	128	0.10	1
(90,10,10)-5	50	0.10	1	0.10	1	0.10	1
(90,10,20)-1	80	0.10	10	0.10	1	0.10	1
(90,10,20)-2	80	0.10	1	0.10	8	0.10	1
(90,10,20)-3	80	0.10	2	0.10	1	0.10	1
(90,10,20)-4	80	0.10	5	0.10	103	0.10	2
(90,10,20)-5	80	0.10	2	0.10	2	0.10	1
Average		0.10	4	0.10	28	0.10	1
(90,20,10)-1	70	11.54	3600	58.96	3600	0.10	6
(90,20,10)-2	70	1.98	3600	31.98	3600	0.10	3
(90,20,10)-3	70	5.25	3600	2.05	3600	0.10	2
(90,20,10)-4	70	5.18	3600	1.73	3600	0.10	2
(90,20,10)-5	70	8.84	3600	40.95	3600	0.10	1
(90,20,20)-1	100	6.82	3600	29.75	3600	0.10	1
(90,20,20)-2	100	11.75	3600	3.13	3600	0.10	4
(90,20,20)-3	100	6.43	3600	27.24	3600	0.10	8
(90,20,20)-4	100	15.61	3600	2.44	3600	0.10	11
(90,20,20)-5	100	6.83	3600	39.59	3600	0.10	2
Average		8.02	3600	23.68	3600	0.10	4

We see from Table 3.3 that all the three algorithms can provide optimal solutions to all instances with 10 ships in short times. CPLEX is able to solve each instance in no more than 10s. Compared with CPLEX, OBPC requires more time to solve five out of the 10 instances and the average computational time (28s) is also longer than that of CPLEX (4s). The OBPC+MC outperforms the other two methods when solving the first 10 instances, which reports the shortest solution time for each instance.

Both CPLEX and OBPC fail to deliver optimal solutions to all instances with 20 ships within 3,600s, and the average optimality gaps delivered by CPLEX and OBPC are 8.02% and 23.68%, respectively. In comparison, OBPC+MC is able to provide optimality certificates for all of the 10 instances, and the average computational time is only 4s.

The results in Table 3.3 demonstrate that the branch-and-cut algorithm used by CPLEX can well solve instances with small scales but cannot provide solutions with proven high qualities to instances with slightly larger scales. In addition, the great superiority of OBPC+MC against OBPC demonstrates the efficacy of the multi-cut generation technique in improving algorithm efficiency.

3.7.4 Comparison of OBPC+MC, OBPC+MC+SM, and OBPC +MC+SM+PH

In order to evaluate the impacts of the strengthened set covering model and the primal heuristic on the performance of the BPC algorithm, we compare the performances of three algorithms, i.e., OBPC+MC, OBPC+MC+SM, and OBPC+MC+SM+PH in this section. We use the three algorithms to solve 50 instances with 90 to 150 days' planning horizon, 20 or 30 ships and 10 or 20 COAs.

We report the computational results in Tables 3.4 and 3.5. Column 2 reports the total number of cargoes in an instance. Columns 3 and 4 present the optimality gaps (in percentage) and the computational times (in seconds) of OBPC+MC for solving the instances. Columns 5 to 8 present the similar results of the algorithm after incrementally including the usage of the strengthened set covering model M6 (which becomes OBPC+MC+SM) and the primal heuristic (which becomes OBPC+MC+SM+PH). Note that we report the optimality gap as "0.10%" if the resulting optimality gap of an algorithm for solving an instance does not exceed a tolerance of 0.10%. Similarly, if the computational time of an algorithm for solving an instance is less than or equal to one second, we report it as "1s" in the table.

In “Average” Rows, we report the average optimality gaps and the computational times of the algorithms for solving 10 instances that share the same planning horizon length and the same number of ships.

We see from Tables 3.4 and 3.5 that both the strengthened set covering model and the primal heuristic significantly improve the performance of the BPC algorithm. In the 50 instances, OBPC+MC+SM and OBPC+MC+SM+PH are able to solve one more instance to optimum (i.e., instance (120,30,20)-3) when compared with OBPC+MC. For instances that can be solved to the optimum by all the three methods, the OBPC+MC+SM+PH generally reports the least computational times followed by OBPC+MC+SM and OBPC+MC. For instances that cannot be solved to the optimum by any of the three methods, OBPC+MC+SM+PH also generally reports the smallest computational gaps followed by OBPC+MC+SM and OBPC+MC.

The importance of primal heuristic for the BPC algorithm is especially obvious for solving instances with large scales. Particularly, the average optimality gap obtained by the algorithms for solving the instances with 150 days’ planning horizon and 30 ships decreases from 26.54% to 1.04% after incorporating the primal heuristic.

Table 3.4: Comparison of the three BPC algorithms (part I).

Instance	$\|N\|$	OBPC+MC		OBPC+MC+SM		OBPC+MC+SM+PH	
		Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)
(90,30,10)-1	90	0.10	6	0.10	6	0.10	6
(90,30,10)-2	90	0.10	1	0.10	1	0.10	1
(90,30,10)-3	90	0.10	332	0.10	54	0.10	39
(90,30,10)-4	90	0.10	393	0.10	398	0.10	100
(90,30,10)-5	90	0.10	378	0.10	97	0.10	29
(90,30,20)-1	120	0.10	20	0.10	8	0.10	8
(90,30,20)-2	120	0.10	63	0.10	30	0.10	32
(90,30,20)-3	120	0.10	1	0.10	1	0.10	1
(90,30,20)-4	120	0.10	2833	0.10	1295	0.10	1208
(90,30,20)-5	120	0.10	1	0.10	1	0.10	1
Average		0.10	403	0.10	189	0.10	143
(120,20,10)-1	100	0.10	1684	0.10	1077	0.10	572
(120,20,10)-2	100	0.67	3600	0.55	3600	0.51	3600
(120,20,10)-3	100	0.10	3	0.10	4	0.10	4
(120,20,10)-4	100	0.10	3	0.10	3	0.10	1
(120,20,10)-5	100	0.10	37	0.10	32	0.10	25
(120,20,20)-1	140	0.10	20	0.10	16	0.10	16
(120,20,20)-2	140	0.10	39	0.10	31	0.10	32
(120,20,20)-3	140	0.24	3600	0.18	3600	0.14	3600
(120,20,20)-4	140	0.10	166	0.10	108	0.10	58
(120,20,20)-5	140	0.10	317	0.10	251	0.10	254
Average		0.17	947	0.15	872	0.15	816
(120,30,10)-1	130	0.47	3600	0.44	3600	0.24	3600
(120,30,10)-2	130	0.32	3600	0.17	3600	0.15	3600
(120,30,10)-3	130	0.10	21	0.10	20	0.10	7
(120,30,10)-4	130	0.10	350	0.10	284	0.10	177
(120,30,10)-5	130	0.10	35	0.10	4	0.10	4
(120,30,20)-1	170	64.12	3600	64.19	3600	1.06	3600
(120,30,20)-2	170	0.10	457	0.10	496	0.10	489
(120,30,20)-3	170	0.25	3600	0.10	1960	0.10	1976
(120,30,20)-4	170	0.28	3600	0.30	3600	0.28	3600
(120,30,20)-5	170	58.92	3600	58.81	3600	0.47	3600
Average		12.48	2246	12.44	2076	0.27	2065

Table 3.5: Comparison of the three BPC algorithms (part II).

Instance	$\|N\|$	OBPC+MC		OBPC+MC+SM		OBPC+MC+SM+PH	
		Gap(%)	Time(s)	Gap(%)	Time(s)	Gap(%)	Time(s)
(150,20,10)-1	120	0.10	10	0.10	10	0.10	10
(150,20,10)-2	120	0.10	158	0.10	64	0.10	56
(150,20,10)-3	120	0.47	3600	0.90	3600	0.46	3600
(150,20,10)-4	120	0.10	122	0.10	116	0.10	57
(150,20,10)-5	120	0.10	1205	0.10	1055	0.10	1005
(150,20,20)-1	170	1.15	3600	1.47	3600	1.24	3600
(150,20,20)-2	170	0.10	1646	0.10	809	0.10	633
(150,20,20)-3	170	0.10	41	0.10	27	0.10	27
(150,20,20)-4	170	0.10	140	0.10	68	0.10	57
(150,20,20)-5	170	0.10	85	0.10	73	0.10	52
Average		0.24	1061	0.32	942	0.25	910
(150,30,10)-1	155	70.08	3600	1.06	3600	0.70	3600
(150,30,10)-2	155	72.16	3600	71.45	3600	1.23	3600
(150,30,10)-3	155	57.78	3600	57.44	3600	1.28	3600
(150,30,10)-4	155	0.10	1173	0.10	675	0.10	590
(150,30,10)-5	155	0.10	459	0.10	438	0.10	201
(150,30,20)-1	205	1.40	3600	1.26	3600	1.05	3600
(150,30,20)-2	205	45.94	3600	1.36	3600	1.35	3600
(150,30,20)-3	205	38.38	3600	38.15	3600	1.21	3600
(150,30,20)-4	205	45.96	3600	45.65	3600	1.62	3600
(150,30,20)-5	205	48.46	3600	48.86	3600	1.75	3600
Average		38.04	3043	26.54	2991	1.04	2959

3.7.5 Value of Robustness

In this section, we evaluate the value of robustness (VOR) in the ship routing problem with batched cargo selection.

We derive VOR of an instance with $\Gamma = n$ as follows. First, let ${Z_6}^*(n)$ be the optimal objective value delivered by the BPC algorithm. Second, based on ${Z_6}^*(n)$, we calculate the "robust optimal operational cost" (in thousand dollars) $rc(n)$ by $rc(n) = {Z_6}^*(n) - \sum_{k \in K} p_k$. Note that $rc(n)$ equals the cost paid by the shipping company for operating the ships in its fleet (we approximate the cost for operating ship v by f_v), transporting cargoes (either using ships in the fleet or using voyage charters) and the robust cost, minus the revenue obtained from freight collected from COAs and spot cargoes. Note that the cost of operating the ships in its fleet includes the cost of chartering-in ships from the market and the cost of managing the ships in the fleet of a shipping company but does not include the operating cost of ships that are chartered-out or the voyage costs of the ships in the fleet. Third, by solving the same instance with $\Gamma = 0$, we obtain the optimal solution to the nominal case of the instance where all voyage costs are at their nominal values. Let ${Z_6}^*(0)$ denote the optimal objective value of the nominal case. Fourth, given an optimal solution to the nominal case, by supposing that the costs of at most n voyages in the solution can deviate from their nominal values, we can calculate a worst-case robust cost which is denoted by $H(n)$. Fifth, the worst-case operational cost (in thousand dollars) derived from the optimal solution to the nominal case (denoted by $dc(n)$) is obtained by $dc(n) = {Z_6}^*(0) - \sum_{k \in K} p_k + H(n)$. Finally, we define the value of robustness (VOR) for an instance with $\Gamma = n$ as the gap (in percentage) between $rc(n)$ and $dc(n)$, which is calculated by $100(dc(n) - rc(n))/dc(n)$.

To evaluate the value of robustness, we solve 20 instances with 90 days' planning horizon and 10 or 20 ships under different values of Γ. For a fair comparison, we set the optimality gap in the BPC algorithm to be 0% when solving the instances in this part and all instances (with different Γ) were solved to their optimum.

As shown by Table 3.6, the VOR increases with the value of Γ. The average VORs for instances with different Γ ranges from 0.25% to 2.10%. In extreme cases,

Table 3.6: The Value of Robustness under Different Values of Γ.

Instance	Γ = 5			Γ = 10			Γ = 15			Γ = 20		
	dc(5)	rc(5)	VOR	dc(10)	rc(10)	VOR	dc(15)	rc(15)	VOR	dc(20)	rc(20)	VOR
(90,10,10)-1	7045	7045	0.00	7552	7531	0.28	7920	7868	0.66	8138	8081	0.70
(90,10,10)-2	9375	9375	0.00	10016	9970	0.46	10327	10112	2.08	10503	10155	3.31
(90,10,10)-3	7671	7657	0.18	8351	8252	1.19	8758	8559	2.27	8922	8644	2.89
(90,10,10)-4	9160	9160	0.00	9836	9751	0.86	10298	10063	2.28	10601	10222	3.58
(90,10,10)-5	8614	8597	0.20	8906	8889	0.19	9126	9109	0.19	9295	9278	0.18
(90,10,10)-1	8450	8239	2.50	9278	8925	3.80	9678	9305	3.85	9872	9467	4.10
(90,10,10)-2	9251	9204	0.51	9585	9480	1.10	9724	9576	1.52	9801	9640	1.64
(90,10,10)-3	9670	9668	0.02	10272	10266	0.06	10684	10544	1.31	10887	10763	1.14
(90,10,10)-4	9375	9375	0.00	9960	9907	0.53	10358	10214	1.39	10607	10459	1.40
(90,10,10)-5	7795	7777	0.23	8354	8315	0.47	8791	8677	1.30	9042	8859	2.02
Average	8641	8610	0.36	9211	9129	0.89	9566	9403	1.69	9767	9559	2.10

Instance	Γ = 10			Γ = 20			Γ = 30			Γ = 40		
	dc(10)	rc(10)	VOR	dc(20)	rc(20)	VOR	dc(30)	rc(30)	VOR	dc(40)	rc(40)	VOR
(90,20,10)-1	11685	11639	0.39	13036	12986	0.38	14126	14058	0.48	14817	14591	1.53
(90,20,10)-2	17431	17422	0.05	18910	18830	0.42	19928	19541	1.94	20463	19973	2.39
(90,20,10)-3	12929	12929	0.00	14115	14076	0.28	14910	14780	0.87	15320	15089	1.51
(90,20,10)-4	12751	12694	0.45	13978	13878	0.72	14649	14544	0.72	14969	14897	0.48
(90,20,10)-5	13868	13820	0.35	15080	14986	0.62	15874	15580	1.85	16307	15823	2.97
(90,20,10)-1	13594	13551	0.32	14640	14551	0.61	15355	15197	1.03	15805	15667	0.87
(90,20,10)-2	13191	13150	0.31	14601	14543	0.40	15704	15553	0.96	16473	16064	2.48
(90,20,10)-3	16419	16372	0.29	17838	17692	0.82	18788	18336	2.41	19309	18697	3.17
(90,20,10)-4	9102	9074	0.31	10475	10407	0.65	11462	11340	1.06	12187	11942	2.01
(90,20,10)-5	10032	10032	0.00	11392	11261	1.15	12378	12101	2.24	12988	12659	2.53
Average	13100	13068	0.25	14407	14321	0.60	15317	15103	1.36	15864	15540	1.99

the VOR of an instance can be over 4%. Considering the thin revenue margins of tramp bulk shipping companies, savings of even small proportions of the operational costs can be critical to the profitability of the companies. Therefore, by using robust optimization, a shipping company is capable of securing its profitability in the face of uncertain voyage costs.

3.8 Conclusion

We have studied a robust bulk ship routing problem with fleet adjustment, batched cargo selection, and uncertain voyage costs. In the problem, we allow the target shipping company to dynamically adjust its fleet composition. We also require that cargoes in one COA should be accepted and rejected as a batch. The uncertainties of voyage costs are also considered and are handled through an uncertainty budget. We have developed MIP models for the problem and showed that it is strongly NP-hard. To solve the problem, we have developed a tailored branch-and-price-and-cut algorithm. We have also proposed several acceleration strategies to improve the performance of the algorithm. Extensive computational results have shown that the proposed BPC algorithm outperforms a state-of-the-art MIP optimization solver and that the acceleration techniques provide significant enhancements for the algorithm.

Chapter 4

Summary and Future Research

4.1 Conclusions

This thesis has investigated the bulk ship routing and scheduling problem under uncertainties. It comprises two main parts. In the first part, we have considered a bulk ship scheduling problem in industrial shipping with stochastic backhaul canvassing demand. In the problem, an industrial corporation is responsible for the transportation of its raw materials or products. We have jointly solved three subproblems from different decision levels: the strategic fleet sizing and mix problem, the tactical voyage planning problem, and the operational stochastic backhaul cargo canvassing problem. In industrial shipping, the required transportation is one-directional and various constraints regarding the arrangement of shipments for transporting the cargoes should be respected. To generate additional revenues, the ships are allowed to canvass from the spot market in the backhaul and we have considered the uncertainties in this spot market. To solve this complicated problem, a tailored two-step solution approach has been developed. The first step solves the stochastic backhaul cargo canvassing problem using a DP algorithm. Based on the results from the first step, we have formulated the remaining two subproblems as an integrated MIP model which has been solved by a Benders decomposition algorithm. Extensive numerical experiments have been performed and the results have demonstrated the great efficiency of our solution approach.

The second part focuses on a practical bulk ship routing problem in tramp shipping. In this problem, we have considered the batched cargo selection behavior of a tramp shipping company when it is faced with COAs. Considering the random voyage costs, we have proposed a robust optimization model for the problem. The problem has been formulated as a compact MILP and we have then reformulated it as a strengthened set covering model. To solve the problem, we have developed a tailored branch-and-price-and-cut algorithm. Several acceleration strategies have been proposed to further improve the performance of the algorithm. We have performed extensive numerical experiments and the results have demonstrated that our algorithm can well solve instances with practical sizes and that the enhancement techniques greatly improve the efficiency of the algorithm. We have also compared the solutions to instances under different robustness settings, and the results have indicated that our solution method can secure the profitability of a shipping company in uncertain environments.

The contributions of this thesis are two-folded. From the industrial perspective, we have proposed two research problems that are of great practical importance but have not been well studied in the literature. Therefore, our research can provide references to industrial and tramp shipping companies and can help improve their current operations. From the academic perspective, our research has developed two novel and effective exact algorithms for solving the considered problems. We believe that the frameworks of the two algorithms and the acceleration techniques used in them can be adopted for solving other routing and scheduling problems under uncertainty.

4.2 Future Research

Several future research directions related to the above studies are introduced as follows.

First, in the two studies, we developed algorithms that can solve the studied problems exactly. It is interesting to identify strategies that can further enhance the performances of these algorithms. For the Benders decomposition algorithm in the first study, future studies can try to design additional cuts that help tighten the lower bounds in the algorithm. For the branch-and-price-and-cut algorithm, it is interesting to look at the impact of stabilization on the column generation procedure. Another promising direction is to identify strategies that can reduce the number of pricing subproblems to be solved in the algorithm. The studies demonstrated that our proposed algorithms are able to solve instances with practical sizes. However, real applications may require solving instances with even larger sizes that cannot be well solved by our algorithm. Therefore, future research can consider how to design efficient heuristic algorithms that can obtain high-quality solutions to the problems in short computational times.

Second, in both studies presented in this thesis, we assume that the speed of each ship is constant. In practice, a carrier may choose to dynamically adjust the speeds of ships to achieve better utilization of these ships. Therefore, incorporating speed optimization in ship routing and scheduling problems should be a very interesting extension of the current studies. Besides, all the cargoes are "exogenous" (i.e., the weights and timings for each shipment are given as inputs) in the first study. Since the shipper is also the carrier in industrial shipping, to further improve the efficiency of an industrial shipping operator, one can solve the inventory management problem and the ship routing and scheduling problem in an integrated manner. This leads to the inventory routing problem, and how to solve this problem in the presence of uncertain backhaul cargo demand would be an interesting extension of the current study.

Finally, uncertainties affect the operations of all sectors in maritime transportation, including dry bulk shipping, liner shipping, liquid bulk shipping, as well as port operations. The recent advances in the global vessel tracking system (i.e., Automatic Identification System, AIS) have brought new opportunities for handling uncertainties in problems in maritime transportation. By leveraging the large-volume vessel traffic data provided by AIS, one can solve these problems in a "Smart Predict, then Optimize" manner. Future studies can consider how to develop data-driven optimization approaches to solve OR problems arising in shipping and port operations.

Appendix A

Mathematical Proofs for Chapter 2

This appendix presents proofs to propositions in Chapter 2.

A.1 Proof of Proposition 2.1

Proof. Proof of Proposition 2.1. We discuss the following two possible cases in terms of the value of s_n:

(i) If $s_n = 1$, this directly follows that $\mathbb{E}_{P_n}\left[\ \overline{f_n^{\xi}}(1)\right] = 0$, and the computation takes no time.

(ii) If $s_n = 0$, we can calculate the value of $\mathbb{E}_{P_n}\left[\ \overline{f_n^{\xi}}(0)\right]$ as follows. Let $\Delta = \{J_1,\ J_2, \cdots, J_i, \cdots, J_{|\mathcal{M}|}\}$ denote a sequence of all j's, where $e_{J_i} \geq e_{J_{i+1}},\ i = 1, \ldots, |\mathcal{M}| - 1$. Note that (1) at most one transportation request can be accepted at each stage and that (2) the rising possibility ρ_{nj}'s are independent from one to another. Therefore, for $1 \leq n \leq N-1$ we have:

$$\begin{aligned}\mathbb{E}_{P_n}\left[\ \overline{f_n^{\xi}}(0)\right] =& p_{nJ_1} \max\left\{e_{J_1}, \mathbb{E}_{P_{n+1}}\left[\ \overline{f_{n+1}^{\xi}}(0)\right]\right\} \\ &+ \sum_{i=2}^{\mathcal{M}} p_{nJ_i} \prod_{k=1}^{i-1}\left(1 - p_{nJ_k}\right) \max\left\{e_{J_i}, \mathbb{E}_{P_{n+1}}\left[\ \overline{f_{n+1}^{\xi}}(0)\right]\right\}, \quad \text{(A.1)}\end{aligned}$$

and for $n = N$, we have:

$$\mathbb{E}_{P_N}\left[\ \overline{f_N^{\xi}}(0)\right] = p_{NJ_1}e_{J_1} + \sum_{i=2}^{\mathcal{M}} p_{NJ_i}\prod_{k=1}^{i-1}\Big(1 - p_{NJ_k}\Big)e_{J_i}. \tag{A.2}$$

In (A.1), p_{nJ_1} and $p_{nJ_i}\prod_{k=1}^{i-1}\Big(1-p_{nJ_k}\Big)$, $i = 2, \ldots, |\mathcal{M}|$ give the possibility that the type of transportation requests with the ith ($i = 1, \ldots, |\mathcal{M}|$) highest revenue for the ship arise and no ones with higher revenues arise at stage n. Since a ship can choose to accept at most one transportation request from the spot market in the backhaul, it can accept at most one request at any stage n which corresponds to the nth unit time in the canvassing period. Further, when type-J_i requests generate the highest revenue for the ship at stage n, the best s-trategy for accepting a request at this stage is to accept one type-J_i request and it thus has no incentive to accept other types of requests at this stage. In addition, $\max\Big\{e_{J_i}, \mathbb{E}_{P_{n+1}}\left[\ \overline{f_{n+1}^{\xi}}(0)\right]\Big\}$, $i = 1, \ldots, |\mathcal{M}|$, gives the revenue (can be expected) corresponding to the ship's optimal decision if type-J_i requests arise as the requests with the highest revenue at stage n. That is, if the revenue for the ship by accepting one of the type-J_i requests, i.e., e_{J_i}, is larger than the expected revenue that the ship can obtain by rejecting these requests and making optimal decisions at subsequent stages, i.e., $\mathbb{E}_{P_{n+1}}\left[\ \overline{f_{n+1}^{\xi}}(0)\right]$, the ship should accept a type-J_i request, and the corresponding revenue is e_{J_i}; otherwise, the ship should reject type-J_i requests and the corresponding maximum expected revenue is $\mathbb{E}_{P_{n+1}}\left[\ \overline{f_{n+1}^{\xi}}(0)\right]$. Furthermore, the calculation of $p_{nJ_i}\prod_{k=1}^{i-1}\Big(1-p_{nJ_k}\Big)$ for each $i = 2, \ldots, |\mathcal{M}|$ takes $\mathcal{O}(|\mathcal{M}|)$ time, and corresponding to $|M|$ types of requests, the total operation in (A.1) should be done $|\mathcal{M}|$ times. Hence, the computation of $\mathbb{E}_{P_n}\left[\ \overline{f_n^{\xi}}(0)\right]$ can be done in $\mathcal{O}(|\mathcal{M}|^2)$ time.

Therefore, the proposition is proved. □

A.2 Proof of Proposition 2.2

Proof. Proof of Proposition 2.2. We discuss the following two possible cases in terms of the value of $\alpha_k^{t_1 t_2}$:

(i) If $\alpha_k^{t_1 t_2} = 0$, then (2.25) converts to $\sum_{h=t_1+1}^{t_2} u_k^h \leq (t_2 - t_1)$, which is valid since $u_k^h \leq 1$ due to constraint (2.22).

(ii) if $\alpha_k^{t_1 t_2} = 1$, then (2.25) converts to $\sum_{h=t_1+1}^{t_2} u_k^h \leq 0$. To show this inequality holds, we will prove $u_k^h = 0$ for any $h \in [t_1 + 1, t_2]_Z$. We do this by contradiction: assume there exists $u_k^{t_3} = 1$ where $t_3 \in [t_1 + 1, t_2]_Z$. Then, due to constraint (2.14), there must exist an $\alpha_k^{t_3,t_4}$ such that

$$\sum_{t_4=t_3+\underline{b_k}-1}^{\min\{t_3+\overline{b_k}-1,|\mathcal{T}|\}} \alpha_k^{t_3,t_4} = 1 \tag{A.3}$$

Considering $t_1 \geq \max\{t_3 - \overline{b_k} + 1, 1\}$, $t_1 \leq t_3$, $t_2 \geq \max\{t_3, t_1 + \underline{b_k} - 1\}$, $t_2 \leq \min\{t_1+\overline{b_k}-1, |\mathcal{T}|\}$, $t_4 \geq \max\{t_3, t_3+\underline{b_k}-1\}$, and $t_4 \leq \min\{t_3+\overline{b_k}-1, |\mathcal{T}|\}$ we can easily infer that the following equation must hold:

$$\sum_{t_5=\max\{t_3-\overline{b_k}+1,1\}}^{t_3} \sum_{t_6=\max\{t_3,t_5+\underline{b_k}-1\}}^{\min\{t_5+\overline{b_k}-1,|\mathcal{T}|\}} \alpha_k^{t_5,t_6} \geq \alpha_k^{t_1,t_2} + \alpha_k^{t_3,t_4} = 2. \tag{A.4}$$

which contradicts constraint (2.13) and thus the proof is complete.

□

A.3 Proof of Proposition 2.3

Proof. Proof of Proposition 2.3. The proof is similar to that in Proposition 2.2 and thus omitted here. □

Appendix B

Proofs and Supplement for Chapter 3

B.1 Mathematical Proofs

This appendix presents proofs to lemmas and propositions in Chapter 3.

B.1.1 Proof of Proposition 3.1

Proof. Proof of Proposition 3.1. We prove the strong NP-hardness of the RSRPB by reducing a well-known strongly NP-hard problem, the Travelling Salesman Problem (TSP), to a decision version of the RSRPB. The decision version of the TSP can be stated as follows. Let $G^{TSP} = (V^{TSP}, A^{TSP})$ be a graph where $V^{TSP} = \{v_1^{TSP}, ..., v_n^{TSP}\}$ is a set of vertices and A^{TSP} is a set of arcs. Let $c_{i,j}^{TSP} > 0$ be the travel cost associated with each arc $(v_i^{TSP}, v_j^{TSP}) \in A^{TSP}$. The TSP asks whether there exists a path $\mathcal{P}$ in G^{TSP} that starts from v_1^{TSP}, visits all vertices (except v_1^{TSP}) exactly once, and returns to v_1^{TSP} such that the total traveling cost of $\mathcal{P}$ is no larger than a given constant C^{TSP}.

Given an arbitrary instance of TSP, we construct a corresponding instance of the RSRPB as follows. There is a set $N^{\mathrm{m}} = \{1, ..., |N^{\mathrm{m}}|\}$ of mandatory cargoes (i's) to be transported by a single ship v (i.e., $|V| = 1$). Specifically, we set other parameters as follows:

$$|N^{\mathrm{m}}| = |V^{TSP}| + 1, \tag{B.1}$$

$$K = N^{\mathrm{c}} = \varnothing, \tag{B.2}$$

$$f_v = 0, \tag{B.3}$$

$$o_v = 0, \tag{B.4}$$

$$e_i = 0, i \in N^{\mathrm{m}} \setminus \{|N^{\mathrm{m}}|\}, \tag{B.5}$$

$$e_{|N^{\mathrm{m}}|} = C^{TSP}, \tag{B.6}$$

$$l_i = C^{TSP}, i \in N^{\mathrm{m}} \setminus \{1\}, \tag{B.7}$$

$$l_1 = 0, \tag{B.8}$$

$$I_{v,i} = 1, i \in N^{\mathrm{m}}, \tag{B.9}$$

$$t^v_{i,j} = c^{TSP}_{i,j}, i \in N^{\mathrm{m}} \setminus \{|N^{\mathrm{m}}|\}, j \in N^{\mathrm{m}} \setminus \{i, |N^{\mathrm{m}}|\}, \tag{B.10}$$

$$t^v_{0,1} = 0, \tag{B.11}$$

$$t^v_{i,|N^{\mathrm{m}}|} = c^{TSP}_{i,1}, i \in N^{\mathrm{m}} \setminus \{1, |N^{\mathrm{m}}|\}, \tag{B.12}$$

$$t^v_{1,|N^{\mathrm{m}}|} = C^{TSP}, \tag{B.13}$$

$$C^v_{i,j} = 0, i \in N^{\mathrm{m}} \cup \{0\}, j \in N^{\mathrm{m}} \cup \{T\} \setminus \{i\}, \tag{B.14}$$

$$d_r = 0, r \in R, \tag{B.15}$$

$$\Gamma = 0, \tag{B.16}$$

$$\hat{c}_i = 1, i \in N^{\mathrm{m}}, \tag{B.17}$$

$$E = 0. \tag{B.18}$$

Clearly, this transformation can be conducted in polynomial time. We will show that there exists a feasible solution to the constructed instance of the RSRPB if and only if the answer to the TSP is “yes”.

First, we prove the "if" part. Suppose the answer to the TSP is "yes". Then consider the following solution ($\mathbb{S}$) to the constructed instance of the RSRPB. First, all cargoes in N^{m} are transported by ship v ($z_i = 0, i \in N^{\mathrm{m}}$). Second, ship v transports cargo 1 first after leaving its initial position, then transports cargo i ($1 < i \leq |V^{TSP}|$) in the same sequence of v_i^{TSP} being visited in $\mathcal{P}$, and finally transports cargo $|N^{\mathrm{m}}|$ after all the other cargoes have been transported. Third, the ship loads cargo 1 at time 0. Afterwards, let $\tilde{C}_i$, $1 < i \leq |V^{TSP}|$ be the total traveling cost when the salesman arrives at v_i^{TSP} in $\mathcal{P}$, then ship v starts loading cargo $i, 1 < i \leq |V^{TSP}|$ at time $b_i = \tilde{C}_i$. Finally, the ship starts loading cargo $|N^{\mathrm{m}}|$ at time $b_{|N^{\mathrm{m}}|} = C^{TSP}$.

The feasibility of $\mathbb{S}$ to the instance of the RSRPB can be verified as follows. To begin with, Equations (B.9) indicate that ship v is cargoworthy and seaworthy for each $i \in N^{\mathrm{m}}$. Then, given Equations (B.10)–(B.12), it is easy to show by induction that the ship v is able to transport all cargoes $i \in N^{\mathrm{m}}$ in the specific sequence and start loading each cargo i at time b_i. Then, given Equations (B.5)–(B.8) and considering that $b_1 = 0$, $b_i = \tilde{C}_i \leq C^{TSP}, 1 < i \leq |V^{TSP}|$, and $b_{|N^{\mathrm{m}}|} = C^{TSP}$, all cargoes are loaded within their loading time windows in the $\mathbb{S}$. Besides, Equations (B.2), (B.3), and (B.14)–(B.17) indicate that the objective value Z_4 of $\mathbb{S}$ which is calculated in Equation (3.20) can be equivalently calculated by $Z_4 = -\sum_{i \in N^{\mathrm{m}}} z_i$. Since $z_i = 0, i \in N^{\mathrm{m}}$, we have $Z_4 = 0 \geq E$. Therefore, $\mathbb{S}$ is feasible to the instance of the RSRPB.

Conversely, for the "only if part", suppose that there exists a feasible solution to the constructed instance of the RSRPB such that $Z_4 \geq E = 0$. Since $Z_4 = -\sum_{i \in N^{\mathrm{m}}} z_i$, it is easy to infer that $z_i = 0, i \in N^{\mathrm{m}}$. Hence, there must exist a feasible path ($\mathcal{P}'$) for ship v such that (i) all cargoes $i \in N^{\mathrm{m}}$ are transported by v exactly once, (ii) cargo 1 starts loading at time 0 and cargo $|N^{\mathrm{m}}|$ starts loading at time C^{TSP}, and (iii) the ship starts loading each cargo $i \in N^{\mathrm{m}} \setminus \{1, |N^{\mathrm{m}}|\}$ within the time window $[0, C^{TSP}]$. Note that (i) is a result of assumptions **A1** and **A2**. Besides, from (ii) and (iii) it follows that $|N^{\mathrm{m}}|$ is the last cargo transported by v and that the total traveling time of ship v to reach the loading port of cargo $|N^{\mathrm{m}}|$ in $\mathcal{P}'$ is no larger than C^{TSP}.

Then consider constructing a path ($\mathcal{P}''$) for the TSP such that (a) the path starts from node v_1^{TSP}, (b) v_i^{TSP} is visited in the same sequence in which cargo $1 < i \leq |V^{TSP}|$ is transported by ship v, and (c) the path returns to v_1^{TSP} after visiting the last node $v_i^{TSP} \in V^{TSP} \setminus \{v_1^{TSP}\}$. The feasibility of $\mathcal{P}''$ can be verified as follows. First, from (i), we have that except v_1^{TSP}, each node in $\mathcal{P}''$ is visited exactly once. Second, considering Equations (B.10)–(B.12) it is easy to infer that $\mathcal{P}''$ is a feasible path for the TSP such that the total traveling cost of $\mathcal{P}''$ is no larger than C^{TSP}. Therefore, $\mathcal{P}''$ is a feasible solution to the TSP. This completes the proof. □

B.1.2 Proof of Proposition 3.2

Proof. Proof of Proposition 3.2. Recall that in Section 3.4.2 we have shown that model M2, M3 and M4 are equivalent. Therefore, it is sufficient to show that M5 is equivalent to M2. Let ${Z_2}^*$ and ${Z_5}^*$ denote the optimal objective values of M2 and M5, respectively. It is sufficient to show that ${Z_2}^* = {Z_5}^*$. Suppose ${x_v}^*$, ${b_i}^*$, ${w_k}^*$, ${y_{i,j}^v}^*$, ${z_i}^*$, and H^* comprise the optimal solution to M2, then ${Z_2}^* = \sum_{k \in K} p_k {w_k}^* - \sum_{v \in V} f_v {x_v}^* - \sum_{v \in V} \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{T\} \setminus \{i\}} C_{i,j}^v {y_{i,j}^v}^* - \sum_{i \in N} \hat{c}_i {z_i}^* - H^*$. Let $V^* \subseteq V$ denote set of ships that are used (i.e., $V^* = \{v | {x_v}^* = 1, v \in V\}$) in the optimal solution. It is easy to infer that ${Z_2}^* = \sum_{k \in K} p_k {w_k}^* - \sum_{v \in V^*} f_v - \sum_{v \in V^*} \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{T\} \setminus \{i\}} C_{i,j}^v {y_{i,j}^v}^* - \sum_{i \in N} \hat{c}_i {z_i}^* - H^*$.

Correspondingly, we can construct a solution [denoted by $(\mathbf{X}, H)$] to M5 as follows. Note that $\mathbf{X}$ is a Q-dimensional vector consisting of χ_q's. First, let $Q_v := \{q | V_q = \{v\}, q \in Q\}$ (note that $\bigcup_{v \in V} Q_v = Q_3$). Then, for each $v \in V^*$, generate the route h of ship v by (i) constructing N_h (i.e., cargoes transported on the route) using cargo i's such that $\sum_{j \in N \cup \{T\} \setminus \{i\}} {y_{i,j}^v}^* = 1$ and transporting the cargoes using the same sequence as specified by ${y_{i,j}^v}^*$'s (i.e., by letting the ship travel in all trips (v, i, j) such that ${y_{i,j}^v}^* = 1$), and (ii) setting $b_i = {b_i}^*, i \in N_h$. Then set $\chi_{q_h^3} = 1$ and $\chi_q = 0$, $q \in Q_v \setminus \{q_h^3\}$. In addition, for each $v \in V \setminus V^*$, set $\chi_q = 0$, $q \in Q_v$. The cost for selecting these columns in Q_3 is $C_3 = \sum_{v \in V^*} \sum_{q \in Q_v} c_q \chi_q = \sum_{v \in V^*} f_v + \sum_{v \in V^*} \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{T\} \setminus \{i\}} C_{i,j}^v {y_{i,j}^v}^*$. Then, for each ${z_i}^*$, set $\chi_{q_i^2} = 1$ if ${z_i}^* = 1$, and $\chi_{q_i^2} = 0$ otherwise. The cost for selecting these columns in Q_2 is

$C_2 = \sum_{i \in N} \hat{c}_i {z_i}^*$. Finally, for each k, set $\chi_{q_k^1} = 1$ if ${w_k}^* = 0$, and $\chi_{q_k^1} = 0$ otherwise. The cost for selecting these columns in Q_1 is $C_1 = \sum_{k \in K} p_k(1 - {w_k}^*)$. Finally, let $H = H^*$.

Next, check the feasibility of the $(\mathbf{X}, H)$ for M5. First, we have $\sum_{j \in N} {y_{0,j}^v}^* \leq 1, v \in V$ in M2, which implies that at most one column q such that $\alpha_{q,v} = 1$ is selected in the solution, hence Constraints (3.30) are satisfied. Besides, as implied by Constraints (3.2) and (3.3), each cargo from N^{m} or $\bigcup_{k \in K: {w_k}^* = 1} N_k^{\mathrm{c}}$ is associated with $\sum_{v \in V} \sum_{j \in N \cup \{T\} \setminus \{i\}} {y_{i,j}^v}^* = 1$ or ${z_i}^* = 1$. This implies that these cargoes are covered in selected columns from Q_2 and Q_3. In addition, each $i \in \bigcup_{k \in K: {w_k}^* = 0} N_k^{\mathrm{c}}$ is covered in selected columns from Q_1. Therefore, each cargo $i \in N$ is covered at least once by the selected columns, which implies that Constraints (3.31) are also satisfied. In addition, considering the two solutions contain the same set of voyages and $H = H^*$, it can be easily verified that Constraints (3.16) and (3.32) are equivalent. This follows that Constraints (3.32) are also satisfied. Therefore $(\mathbf{X}, H)$ is feasible for M5. It follows that ${Z_5}^* \leq Z_5[(\mathbf{X}, H)] = \bar{Z} - C_1 - C_2 - C_3 - H^* = \sum_{k \in K} p_k - \sum_{v \in V^*} f_v - \sum_{v \in V^*} \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{T\} \setminus \{i\}} (C_{i,j}^v {y_{i,j}^v}^*) - \sum_{k \in K} p_k(1 - {w_k}^*) - \sum_{i \in N} \hat{c}_i {z_i}^* - H^* = {Z_2}^*$, that is, ${Z_5}^* \leq {Z_2}^*$.

We can also convert an optimal solution to M5 into a feasible solution to M2, by reversing the above logic. This gives us that ${Z_2}^* \leq {Z_5}^*$. It follows immediately that ${Z_2}^* = {Z_5}^*$, which completes the proof. □

B.1.3 Proof of Lemma 3.1

Proof. Proof of Lemma 3.1. To begin with, it is easy to see that $C_{i,T}^v$ is the lower bound of the cost incurred by transporting cargo i using ship v. It follows that MC_i is the lower bound of the cost incurred by transporting cargo i using ships in V. Therefore, the MR calculated in Line 6 in Algorithm 1 gives the upper bound of the profit brought by accepting contract k and transporting all the cargoes in it using ships in V. Then, from Line 9 to 16, the algorithm identifies the largest number of cargoes (SN) that can be transported using spot charters such that MR is positive. Finally, $L_k = |N_k^{\mathrm{c}}| - SN$. Hence, for any accepted k, transporting any number $N' < L_k$ cargoes from N_k^c using ships from V incurs net cost for the shipping company. Then considering assumptions **A1** and **A2**, it is easy to infer that in any optimal solution to the RSRPB, a contract k is accepted only when it is possible to bring (positive) profit to the shipping company. Therefore, it is readily seen that at least L_k cargoes from N_k^c should be transported by using ships in V if k is accepted in an optimal solution to M5. □

B.1.4 Proof of Proposition 3.3

Proof. Proof of Proposition 3.3. Considering Equations (3.35)–(3.37), we have $\sum_{q\in Q} n_q^k \chi_q = L_k \chi_{q_k^1} + \sum_{q\in Q_3} n_q^k \chi_q, k \in K$. We discuss the following two possible cases in terms of the value of $\chi_{q_k^1}$:

(i) If $\chi_{q_k^1} = 1$, then (3.34) converts to $L_k + \sum_{q\in Q_3} n_q^k \chi_q \geq L_k$, which is valid since $n_q^k \geq 0$.

(ii) If $\chi_{q_k^1} = 0$, then (3.34) converts to $\sum_{q\in Q_3} n_q^k \geq L_k$. In this case, contract k is accepted. Therefore, from Lemma 3.1, at least L_k cargoes from N_k^c should be transported by ships from V. Meanwhile, Equation (3.37) infers that $\sum_{q\in Q_3} n_q^k \chi_q$ calculate the number of cargoes from k that are transported using ships from V. Thus the inequality is valid.

This completes the proof. □

B.1.5 Proof of Proposition 3.4

Proof. Proof of Proposition 3.4. It is sufficient to show that we can reduce $\Theta := \{G | G \subseteq R, |G| \leq \Gamma\}$ to $\Theta' := \{G | G \subseteq R, |G| = \Gamma\}$, without changing the optimal solution to M5. Consider an arbitrary constraint C_1 of Constraints (3.32) with $G = G_1$ such that $|G_1| < \Gamma$. Given C_1, we can construct a valid constraint C_2 for M5 with $G = G_2$ such that $G_1 \subseteq G_2$, and $|G_2| = \Gamma$. The different between the right-hand sides of C_2 and C_1 is calculated by $\sum_{q\in Q} \sum_{r\in G_2} d_r \gamma_{q,r} \chi_q - \sum_{q\in Q} \sum_{r\in G_1} d_r \gamma_{q,r} \chi_q$ $= \sum_{r\in G_2 \setminus G_1} d_r \gamma_{q,r} \chi_q - \sum_{r\in G_1 \setminus G_2} d_r \gamma_{q,r} \chi_q = \sum_{r\in G_2 \setminus G_1} d_r \gamma_{q,r} \chi_q \geq 0$. Therefore C_2 is at least as tight as C_1. Hence, it is safe to replace C_1 with C_2. Since C_1 is arbitrarily chosen, we can replace any constraint with $|G| < \Gamma$ with a constraint with $|G| = \Gamma$. Therefore, it is safe to only use constraints with $|G| = \Gamma$ for Constraints (3.32). Note that all constraints with $|G| = \Gamma$ belong to the set Θ'. □

B.1.6 Proof of Proposition 3.5

Proof. Proof of Proposition 3.5. To begin with, the SP solves a maximization problem. Therefore, it can be solved by only considering r's such that $d_r \sum_{q\in\tilde{Q}} \gamma_{q,r}{\chi_q}^* > 0$. Let $\bar{G} := \{r | d_r \sum_{q\in\tilde{Q}} \gamma_{q,r}{\chi_q}^* > 0\}$. Then if $|\bar{G}| \leq \Gamma$, we have $G = \bar{G}$. Otherwise, we construct G using voyages r that have the $|\Gamma|$ largest $d_r \sum_{q\in\tilde{Q}} \gamma_{q,r}{\chi_q}^*$. Finally, ${Z_8}^* = \sum_{r\in G} d_r \sum_{q\in\tilde{Q}} \gamma_{q,r}{\chi_q}^* - H^*$.

Generating $\bar{G}$ and takes $O(|R||\tilde{Q}|)$ time and given $\bar{G}$, G can be constructed in $O(|R|\log|R|)$ time. Therefore, the algorithm solves the SP in $O(|R||\tilde{Q}| + |R|\log|R|)$ time. □

B.1.7 Proof of Proposition 3.6

Proof. Proof of Proposition 3.6. To show that (3.55) is valid for M6, it is equivalent to show that (3.55) (i) makes the current solution to the RLM6($\mathfrak{N}$) at node $\mathfrak{N}$ infeasible and (ii) does not change the optimal solution to M6.

First, we show that (i) holds. Let ${\chi_q}^*, q \in \tilde{Q}$, and H^* be the current optimal solution to the RLM6($\mathfrak{N}$). For any separated Constraint (3.54), we have $H^* < \sum_{q\in\tilde{Q}} \sum_{r\in G} d_r\gamma_{q,r}{\chi_q}^*$. Considering $G \subseteq \bar{G}$, we have $\sum_{q\in\tilde{Q}} \sum_{r\in G} d_r\gamma_{q,r}{\chi_q}^* \leq \sum_{q\in\tilde{Q}} \sum_{r'\in\bar{G}} d_{r'}\gamma_{q,r'}{\chi_q}^*$, which implies $H^* < \sum_{q\in\tilde{Q}} \sum_{r'\in\bar{G}} d_{r'}\gamma_{q,r'}{\chi_q}^*$. Therefore, the current solution is infeasible.

Second, given assumptions **A1** and **A2**, it is safe to claim that in any optimal solution to M6 (denoted by ${\chi_q}^{**}, q \in Q$ and H^{**}), $\sum_{r'\in\bar{R}_r} \sum_{q\in Q} \gamma_{q,r'}{\chi_q}^{**} \leq 1$ (i.e., at most one ship sail from a specific origin to a specific destination). This implies $\sum_{r'\in\bar{G}} \sum_{q\in Q} \gamma_{q,r'}{\chi_q}^{**} = \sum_{r\in G} \sum_{r'\in\bar{R}_r} \sum_{q\in Q} \gamma_{q,r'}{\chi_q}^{**} \leq |G| \leq \Gamma$. Hence, in Constraint (3.55), the costs of at most Γ voyages can deviate from their nominal values. Therefore, it is readily seen that (ii) holds as well. □

B.2 Pseudo-codes for Algorithms in the Primal Heuristic

This appendix presents the pseudo-codes of the primal heuristic proposed in Section 3.6.4 in Chapter 3.

B.2.1 The Pseudo-code for the Route-construction Procedure

For ease of presentation, we make use of the following notations in the pseudo-code. We start by using $pr_q^\iota, q \in \tilde{Q}', \iota = \{1, 2, 3\}$ to indicate the priority of column q assigned by strategy ι such that a larger pr_q^ι indicates higher priority. Then, $VD_v, v \in V$ and $CD_i, i \in N$ are used to indicate whether ship v has been used in the columns in $\hat{Q}_\iota$ to transport cargoes ($VD_v = 1$) or not ($VD_v = 0$) and whether cargo i has been transported in the solution ($CD_i = 1$) or not ($CD_i = 0$), respectively. Besides, $QD_q, q \in \tilde{Q}'$ denotes whether column q has been checked in the procedure ($QD_q = 1$) or not ($QD_q = 0$).

Algorithm 4 generates five sets of columns based on different prioritizing strategies ($\hat{Q}_\iota$'s). These sets are generated iteratively. In particular, at each iteration, we first update the set of feasible columns ($\tilde{Q}''$). A column q is feasible if (i) it has not been checked ($QD_q = 0$), (ii) the ship used in q has not been used by any column in $\hat{Q}_\iota$ ($V'_q \neq \varnothing$) and (iii) at least one cargo has not been transported by columns in $\hat{Q}_\iota$ ($N'_q \neq \varnothing$) (Lines 9 and 10). Then from $\tilde{Q}''$, we select the column (q^*) that has the minimum number of cargoes that have been transported by columns in $\hat{Q}_\iota$, and if there is more than one such column, select the one with the highest priority (Lines 12–14). After that, a column q is generated by removing cargoes that have been transported by columns in $\hat{Q}_\iota$ and reconstructing a route using the remaining cargoes (Line 16). We will add q into $\hat{Q}_\iota$ if its cost c_q is less than the cost of transporting all cargoes in N_q by using voyage charters. The state parameters (i.e., QD_q, VD_v, and CD_i) are updated in each iteration (Lines 15 and 19). The construction completes when $\tilde{Q}'' = \varnothing$.

Algorithm 4 The Route-construction Procedure.

Input: $\tilde{Q}'$;

Output: Set of selected columns $\hat{Q}_\iota, \iota = 1, 2, 3$;

for $\iota \in \{1, 2, 3\}$ **do**

  Set $\hat{Q}_\iota = \varnothing$ and set $VD_v = 0, \forall v \in V$ and $CD_i = 0, \forall i \in N$, $QD_q = 0, \forall q \in \tilde{Q}'$;

  Initiate the set of feasible columns $\tilde{Q}'' = \tilde{Q}'$;

  while $\tilde{Q}'' \neq \varnothing$ **do**

    $UVD := \{v | VD_v = 0, v \in V\}$; $UCD := \{i | CD_i = 0, i \in N\}$; ▷ UVD and UCD are the set of ships and the set of cargoes that have not been covered in $\tilde{Q}'$, respectively.

    for $q \in \tilde{Q}''$ **do**

      $V'_q := V_q \cap UVD$; $N'_q := N_q \cap UCD$; $\overline{N'_q} := N_q \setminus UCD$;

    end for

    $\tilde{Q}''' = \{q | QD_q = 0, V'_q \neq \varnothing, N'_q \neq \varnothing, q \in \tilde{Q}''\}$;

    $\tilde{Q}'' = \tilde{Q}'''$;

    if $\tilde{Q}'' \neq \varnothing$ **then**

      $NCD = \min_{q \in \tilde{Q}''} \{|\overline{N'_q}|\}$;

      $\check{Q} := \{q || \overline{N'_q}| = NCD, q \in \tilde{Q}''\}$;

      $q^* = \arg\max_{q \in \check{Q}} pr^\iota_q$;

      $QD_{q^*} = 1$;

      Construct a column q based on q^* by (i) letting $V_q = V_{q^*}$ and $N_q = N'_{q^*}$, (ii) generating Ξ_q by transporting cargoes in N_q using ship $v \in V_q$ (V_q is a singleton) according to the same sequence as in N_{q^*} and (iii) calculating $c_q = f_v + C^v_{0,i_1} + \sum_{n=1}^{|N_q|-1} C^v_{i_n,i_n+1} + C^v_{i_{|N_q|},\mathrm{T}}$. Here, i_n is the n-th cargo transported by v in N_q.

      if $c_q < \sum_{i \in N_q} \hat{c}_i$ **then**

        $\hat{Q}_\iota = \hat{Q}_\iota \cup \{q\}$;

        $VD_v = 1$; $CD_i = 1, \forall i \in N_q$;

      end if

    end if

  end while

end for

B.2.2 The Pseudo-code for the Cargo-check Procedure

The notations used in the pseudo-code are as follows. First, for each set $\hat{Q}_\iota$, let $\hat{N}_\iota := \bigcup_{q \in \hat{Q}_\iota} N_q$. Then, given $\hat{N}_\iota$, we identify the following sets of cargoes: (i) $\bar{N}^{\mathrm{m}}_\iota := \{i | i \notin \hat{N}_\iota, i \in N^{\mathrm{m}}\}$ and (ii) $\bar{N}^{\mathrm{c}}_{\iota,k} := \{i | i \notin \hat{N}_\iota, i \in N^{\mathrm{c}}_k\}, k \in K$. Outputs of the algorithm are $\check{Q}^\iota_1 \subseteq Q_1$, $\check{Q}^\iota_2 \subseteq Q_2$, and $\check{Q}^\iota_3 \subseteq Q_3$, and the columns from these three sets will be used to further construct the integer solution under strategy ι.

Algorithm 5 The Cargo-check Procedure.

Input: $\hat{Q}_\iota$, N;
Output: $\check{Q}_1^\iota$, $\check{Q}_2^\iota$, $\check{Q}_3^\iota$;

$\check{Q}_1^\iota = \varnothing$, $\check{Q}_2^\iota = \varnothing$, $\check{Q}_3^\iota = \varnothing$;
Initiate the set of contracts to reject as $\bar{K} = \varnothing$;
Initiate the set of cargoes transported by ships $v \in V$ as $N^{\mathrm{v}} = \varnothing$;
Initiate the set of cargoes that need to be transported as $N^{\mathrm{t}} = N^{\mathrm{m}}$;
for $k \in K$ **do**
  $pc_k = \sum_{i \in \bar{N}_{\iota,k}} \hat{c}_i$;
  if $pc_k \geq p_k$ **then**
    $\bar{K} = \bar{K} \cup \{k\}$;
    $\check{Q}_1^\iota = \check{Q}_1^\iota \cup \{q_k^1\}$; ▷ Recall that q_k^1 is the column corresponding to contract k in Q_1.
  else
    $N^{\mathrm{t}} = N^{\mathrm{t}} \cup N_k^{\mathrm{c}}$;
  end if
end for
$\bar{N}^{\mathrm{c}} := \bigcup_{k \in \bar{K}} N_k^{\mathrm{c}}$;
for $q \in \hat{Q}_\iota$ **do**
  $N_q' := N_q \cap \bar{N}^{\mathrm{c}}$;
  if $N_q' = \varnothing$ **then**
    $\check{Q}_3^\iota = \check{Q}_3^\iota \cup q$;
    $N^{\mathrm{v}} = N^{\mathrm{v}} \cup N_q$;
  else
    Construct a column q' based on q by (i) letting $V_{q'} = V_q$ and $N_{q'} = N_q \setminus N_q'$, (ii) generating $\Xi_{q'}$ by transporting cargoes in $N_{q'}$ using ship $v \in V_{q'}$ ($V_{q'}$ is a singleton) according to the same sequence as in N_q and (iii) calculating $c_q = f_v + C_{0,i_1}^v + \sum_{n=1}^{|N_{q'}|-1} C_{i_n,i_n+1}^v + C_{i_{|N_{q'}|},\mathrm{T}}^v$. Here, i_n is the n-th cargo transported by v in $N_{q'}$.
    if $c_{q'} < \sum_{i \in N_{q'}} \hat{c}_i$ **then**
      $\check{Q}_3^\iota = \check{Q}_3^\iota \cup \{q'\}$;
      $N^{\mathrm{v}} = N^{\mathrm{v}} \cup N_{q'}$;
    end if
  end if
end for
for $i \in N^{\mathrm{t}} \setminus N^{\mathrm{v}}$ **do**
  $\check{Q}_2^\iota = \check{Q}_2^\iota \cup q_i^2$ (q_i^2 is the column corresponding to cargo i in Q_2);
end for

References

Agra, A., Christiansen, M., Figueiredo, R., Hvattum, L. M., Poss, M., Requejo, C., 2013. The robust vehicle routing problem with time windows. Computers & Operations Research 40 (3), 856–866.

Alvarez, J. F., Tsilingiris, P., Engebrethsen, E. S., Kakalis, N. M., 2011a. Robust fleet sizing and deployment for industrial and independent bulk ocean shipping companies. INFOR: Information Systems and Operational Research 49 (2), 93–107.

Alvarez, J. F., Tsilingiris, P., Engebrethsen, E. S., Kakalis, N. M. P., 2011b. Robust fleet sizing and deployment for industrial and independent bulk ocean shipping companies. INFOR: Information Systems and Operational Research 49 (2), 93–107.

An, Y., Zeng, B., Zhang, Y., Zhao, L., 2014. Reliable P-median facility location problem: two-stage robust models and algorithms. Transportation Research Part B: Methodological 64, 54–72.

Arslan, A. N., Papageorgiou, D. J., 2017. Bulk ship fleet renewal and deployment under uncertainty: a multi-stage stochastic programming approach. Transportation Research Part E: Logistics and Transportation Review 97, 69–96.

Arslan, O., Karaşan, O. E., 2016. A Benders decomposition approach for the charging station location problem with plug-in hybrid electric vehicles. Transportation Research Part B: Methodological 93, 670–695.

Bakkehaug, R., Eidem, E. S., Fagerholt, K., Hvattum, L. M., 2014. A stochastic programming formulation for strategic fleet renewal in shipping. Transportation Research Part E: Logistics and Transportation Review 72, 60–76.

Baosteel, 2008. Baosteel sets up Haibao Shipping Company with China Shipping. http://www.baosteel.com/plc_e/02news/ShowArticle.asp?ArticleID=1262, Accessed June 18, 2018.

Bausch, D. O., Brown, G. G., Ronen, D., 1998. Scheduling short-term marine transport of bulk products. Maritime Policy & Management 25 (4), 335–348.

Benders, J. F., 1962. Partitioning procedures for solving mixed-variables programming problems. Numerische Mathematik 4 (1), 238–252.

Bertsimas, D., Sim, M., 2003. Robust discrete optimization and network flows. Mathematical Programming 98 (1-3), 49–71.

BIMCO, 2019. BIMCO Contracts. https://www.bimco.org/contracts-and-clauses/bimco-contracts, Accessed December 8, 2019.

Brønmo, G., Christiansen, M., Fagerholt, K., Nygreen, B., 2007a. A multi-start local search heuristic for ship scheduling–a computational study. Computers & Operations Research 34 (3), 900–917.

Brønmo, G., Christiansen, M., Nygreen, B., 2007b. Ship routing and scheduling with flexible cargo sizes. Journal of the Operational Research Society 58 (9), 1167–1177.

Brønmo, G., Nygreen, B., Lysgaard, J., 2010. Column generation approaches to ship scheduling with flexible cargo sizes. European Journal of Operational Research 200 (1), 139–150.

Brown, G. G., Graves, G. W., Ronen, D., 1987. Scheduling ocean transportation of crude oil. Management Science 33 (3), 335–346.

Christiansen, M., Fagerholt, K., Nygreen, B., Ronen, D., 2007. Maritime transportation. Handbooks in operations research and management science 14, 189–284.

Christiansen, M., Fagerholt, K., Nygreen, B., Ronen, D., 2013. Ship routing and scheduling in the new millennium. European Journal of Operational Research 228 (3), 467–483.

Christiansen, M., Fagerholt, K., Ronen, D., 2004. Ship routing and scheduling: status and perspectives. Transportation Science 38 (1), 1–18.

Dantzig, G., Fulkerson, D., 1954. Minimizing the number of tankers to meet a fixed schedule. Naval Research Logistics 1 (3), 217–222.

Fagerholt, K., Christiansen, M., Hvattum, L. M., Johnsen, T. A., Vabø, T. J., 2010. A decision support methodology for strategic planning in maritime transportation. Omega 38 (6), 465–474.

Fagerholt, K., Lindstad, H., 2000. Optimal policies for maintaining a supply service in the Norwegian Sea. Omega 28 (3), 269–275.

Gounaris, C. E., Wiesemann, W., Floudas, C. A., 2013. The robust capacitated vehicle routing problem under demand uncertainty. Operations Research 61 (3), 677–693.

Greiner, R., 2013. Ship operating costs: current and future trends. Tech. rep., Moore Stephens LLP, Singapore, Accessed December 4, 2019.
URL http://www.propellerclub.gr/files/Richard_Greiner.pdf

Halvorsen-Weare, E. E., Fagerholt, K., Nonås, L. M., Asbjørnslett, B. E., 2012. Optimal fleet composition and periodic routing of offshore supply vessels. European Journal of Operational Research 223 (2), 508–517.

Hwang, H.-S., Visoldilokpun, S., Rosenberger, J. M., 2008. A branch-and-price-and-cut method for ship scheduling with limited risk. Transportation Science 42 (3), 336–351.

Kisialiou, Y., Gribkovskaia, I., Laporte, G., 2018. Robust supply vessel routing and scheduling. Transportation Research Part C: Emerging Technologies 90, 366 – 378.

Kisialiou, Y., Gribkovskaia, I., Laporte, G., 2019. Supply vessel routing and scheduling under uncertain demand. Transportation Research Part C: Emerging Technologies 104, 305–316.

Korsvik, J. E., Fagerholt, K., 2010. A tabu search heuristic for ship routing and scheduling with flexible cargo quantities. Journal of Heuristics 16 (2), 117–137.

Kosmas, O., Vlachos, D., 2012. Simulated annealing for optimal ship routing. Computers & Operations Research 39 (3), 576–581.

Laake, J., Zhang, A., 2016. Joint optimization of strategic fleet planning and contract analysis in tramp shipping. Applied Economics 48 (3), 203–211.

Laporte, G., Louveaux, F. V., 1993. The integer L-shaped method for stochastic integer programs with complete recourse. Operations Research Letters 13 (3), 133–142.

Lee, C., Lee, K., Park, S., 2012. Robust vehicle routing problem with deadlines and travel time/demand uncertainty. Journal of the Operational Research Society 63 (9), 1294–1306.

Mehrez, A., Hung, M. S., Ahn, B. H., 1995. An industrial ocean-cargo shipping problem. Decision Sciences 26 (3), 395–423.

Meng, Q., Wang, S., Lee, C.-Y., 2015. A tailored branch-and-price approach for a joint tramp ship routing and bunkering problem. Transportation Research Part B: Methodological 72, 1–19.

Munari, P., Moreno, A., De La Vega, J., Alem, D., Gondzio, J., Morabito, R., 2019. The robust vehicle routing problem with time windows: compact formulation and branch-price-and-cut method. Transportation Science 53 (4), 1043–1066.

Ng, M.-W., 2015. Container vessel fleet deployment for liner shipping with stochastic dependencies in shipping demand. Transportation Research Part B: Methodological 74, 79–87.

Pacific Basin, 2018. Pacific basin annual report 2018. Tech. rep., Pacific Basin, Hong Kong, China, Accessed January 4, 2020.
URL https://www.pacificbasin.com/upload/en/ir/financial_disclosure/report/2018/e2343_AnnualReport2018.pdf

Pacific Basin, 2019. PB's Customer Focus. https://www.pacificbasin.com/en/service/focus.php, Accessed December 4, 2019.

Pantuso, G., Fagerholt, K., Hvattum, L. M., 2014a. A survey on maritime fleet size and mix problems. European Journal of Operational Research 235 (2), 341–349.

Pantuso, G., Fagerholt, K., Wallace, S. W., 2014b. Solving hierarchical stochastic programs: application to the maritime fleet renewal problem. INFORMS Journal on Computing 27 (1), 89–102.

Pesenti, R., 1995. Hierarchical resource planning for shipping companies. European Journal of Operational Research 86 (1), 91–102.

Ronen, D., 1986. Short-term scheduling of vessels for shipping bulk or semi-bulk commodities originating in a single area. Operations Research 34 (1), 164–173.

Schwartz, N., 1968. Discrete programs for moving known cargos from origins to destinations on time at minimum bargeline fleet cost. Transportation Science 2 (2), 134–145.

Shen, S., Chen, Z., 2013. Optimization models for differentiating quality of service levels in probabilistic network capacity design problems. Transportation Research Part B: Methodological 58, 71–91.

Siddiqui, A. W., Verma, M., 2015. A bi-objective approach to routing and scheduling maritime transportation of crude oil. Transportation Research Part D: Transport and Environment 37, 65–78.

Song, D.-P., Dong, J.-X., 2013. Long-haul liner service route design with ship deployment and empty container repositioning. Transportation Research Part B: Methodological 55, 188–211.

Stålhane, M., Andersson, H., Christiansen, M., Cordeau, J.-F., Desaulniers, G., 2012a. A branch-price-and-cut method for a ship routing and scheduling problem with split loads. Computers & Operations Research 39 (12), 3361–3375.

Stålhane, M., Rakke, J. G., Moe, C. R., Andersson, H., Christiansen, M., Fagerholt, K., 2012b. A construction and improvement heuristic for a liquefied natural gas inventory routing problem. Computers & Industrial Engineering 62 (1), 245–255.

Tirado, G., Hvattum, L. M., Fagerholt, K., Cordeau, J.-F., 2013. Heuristics for dynamic and stochastic routing in industrial shipping. Computers & Operations Research 40 (1), 253–263.

UNCTAD, 2017. Review of maritime transportation 2017. In: Paper presented at the United Nations Conference on Trade and Development. New York and Geneva,

Accessed June 12, 2018.
URL http://unctad.org/en/PublicationsLibrary/rmt2017_en.pdf

UNCTAD, 2019. Review of maritime transportation 2019. In: Paper presented at the United Nations Conference on Trade and Development. New York and Geneva, Accessed December 30, 2019.
URL http://unctad.org/en/PublicationsLibrary/rmt2019_en.pdf

UNECLAC, 2018. The transport of natural resources from latin america and the caribbean to asia-pacific. Tech. rep., Santiago, Accessed June 18, 2018.
URL https://www.cepal.org/sites/default/files/news/files/escap.pdf

Wang, K., Wang, S., Zhen, L., Qu, X., 2017. Cruise service planning considering berth availability and decreasing marginal profit. Transportation Research Part B: Methodological 95, 1–18.

Wang, S., Meng, Q., 2012. Liner ship route schedule design with sea contingency time and port time uncertainty. Transportation Research Part B: Methodological 46 (5), 615–633.

Wang, X., Fagerholt, K., Wallace, S. W., 2018. Planning for charters: a stochastic maritime fleet composition and deployment problem. Omega 79, 54–66.

Wikipedia contributors, 2018. Continuous production — Wikipedia, the free encyclopedia. https://en.wikipedia.org/w/index.php?title=Continuous_production&oldid=838993501, Accessed 18-July-2018.

Wolsey, L. A., 1998. Integer Programming. Wiley, New York.

Zeng, Q., Yang, Z., 2007. Model integrating fleet design and ship routing problems for coal shipping. Computational Science–ICCS 2007, 1000–1003.

Zhen, L., Wang, S., Zhuge, D., 2017. Dynamic programming for optimal ship refueling decision. Transportation Research Part E: Logistics and Transportation Review 100, 63–74.

Zheng, S., Chen, S., 2018. Fleet replacement decisions under demand and fuel price uncertainties. Transportation Research Part D: Transport and Environment 60, 153–173.