COMPETITION AND COOPERATION AMONG TELECOM ENTERPRISES

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Ph.D

The Hong Kong Polytechnic University

This programme is jointly offered by

The Hong Kong Polytechnic University and Zhejiang University

2021

The Hong Kong Polytechnic University

Department of Logistics and Maritime Studies

Zhejiang University

Department of Data Science and Engineering Management

Competition and Cooperation among Telecom Enterprises

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A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

June 2020

CERTIFICATE OF ORIGINALITY

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Abstract

Telecommunications industry is a strategic, precursory and basic public service industry, and it plays an important role in the national economy. The quality of telecommunications networks and the healthy development of the telecommunications market are the basis for ensuring that the telecommunications industry fully exerts the role of "booster" for economic growth. To address the issue of poor interconnection among Internet service providers (ISPs) and the "dumb pipe" dilemma faced by mobile operators, this thesis models and analyzes the horizontal relationship among ISPs and the vertical relationship between ISP and content providers (CPs).

Chapter 2 designs an interconnection settlement that can motivate ISPs to interconnect with each other through Network Access Point (NAP). How to coordinate ISPs with different interests via a reasonable interconnection settlement mechanism thus to achieve effective interconnection among ISPs has always been an important issue of concern to the government and the industry. To cope with this issue, a cooperative game framework is adopted to study the profit allocation among multiple ISPs. We propose a Characterized Profit Allocation (CPA) that meet the principles of fairness and the principle of win-win, and we also design an interconnection settlement mechanism based on CPA to enable the ISPs to act independently but achieve global optimality. Analytical results and numerical experiments show that CPA and its corresponding settlement rule can stimulate ISPs to interconnect with each other through NAP in a variety of realistic situations, effectively improve social welfare, and provide theoretical basis for the design of the interconnection settlement at NAPs.

Chapter 3 investigates ISP's and CP's competition strategies and profits when sponsored data services are offered from a supply chain perspective. Sponsored data is an innovative business model that allows content providers to pay for the data traffic generated by users while visiting the sponsoring CP's content. Sponsored data changes user's traditional payment model. It has a great impact on the benefits of all parties

including CP, users and ISP, and it also brings a lot of controversy. This study applies a game model to study the equilibrium decisions and profits of the ISP and the CP under sponsored data in different situations. We classify CPs as subscription CPs and platform CPs and carry out analysis for these two types of CPs respectively. Our analysis suggests that social welfare is enhanced when a platform CP participates in sponsored data services, and the result is mixed when the sponsoring CP is a subscription CP. For the ISP, offering sponsored data services enhances its profit no matter the sponsoring CP is of which type. For the CP, sponsored data services reduce its profit with a few exceptions. These results provide managerial insights and are useful for ISPs and CPs when making relevant decisions.

Chapter 4 investigates the competition strategies and profits of ISP and CP when sponsored data services are offered in a market where both horizontal competition between ISPs and vertical competition between ISP and CP exist simultaneously. We also apply a game model to study the equilibrium decisions and profits of two ISPs and one CP under different market structures when sponsored data services are offered. Results show that even in a market where two ISPs compete, ISPs can still profit from sponsored data, and unintegrated ISP can improve its weak position by implementing sponsored data in a market where there is vertical integration between ISP and CP. CP’s profit is reduced in most situations, and only CP with strong profitability can improve its profit by offering subsidy to users of both ISPs. ISP and CP should take full consideration of their strategic goals of both market share and profit when choosing cooperation method and integration strategy. From a social welfare perspective, sponsored data has a positive effect in most cases with some exceptions, and the positive effect is more pronounced when the competition between ISPs is intense. These findings provide practical guidance for telecom enterprises’ integration strategies and sponsored data decisions, and provide a theoretical basis for the regulation of the telecommunications market.

Acknowledgements

First and foremost, I would like to express my deepest appreciation to my chief supervisors, Dr. Hengqing Ye and Dr. Weihua Zhou. Their patient guidance and critical way of thinking have inspired me and helped me overcome many difficulties in research. Without their dedicated involvement in every step throughout the process, this thesis would have never been accomplished. Besides research, they also give me invaluable advice and support on my career and support in every aspect, thank you!

I would also like to thank professors in LMS for giving enlightening lectures. I am also grateful to Dr. Qingwei Jin and Dr. Yi Yang at Zhejiang University, for the seminars and discussions that helped me develop research ability.

My special thanks go to my friends, Peng Liao, Yunjuan Kuang, Yanli Tang, Guoxuan Huang, Yuxuan He, Wenxin Zhou, Yefei Wang, Claudio He, Weijiao Wang, Yuxiao Ye and many others. Their companion enriched my life in Hong Kong.

Finally, I would take this opportunity to thank my parents, my husband and my cat, for their endless love and support.

Table of Contents

List of Tables

List of Figures

1 Introduction

As a strategic and basic public service industry which plays an import role in national economy, telecommunications industry demonstrates distinctive features. First, it is an industry strongly driven by technological innovations, such as telegraph, telephone, communications satellites, Internet and several generations of mobile network. The advent of these technologies not only brought us new ways of communication and expanded telecom operators' lines of business, but also have changed the structure of telecom service supply chain and poses new operational problems which inspired this thesis.

The provision of telecommunications services relies heavily on multilateral cooperation. On one hand, telecommunications networks exhibit prominent network externality, and a vast network exemplified by the Internet which connects as more people and devices as possible can bring the most value to the users. Thus, different ISPs (Internet Service Providers) must cooperate and interconnect with each other to form a national or global network. On the other hand, the cooperation among different types of service providers is also needed. Especially with the development of Internet and mobile Internet, users have generated great demand for various kinds of services other than just an access to the network. Therefore, only a combination of network access services and content services, which are offered by ISPs and CPs (Content Providers) respectively, can fully meet users' needs.

There are many other players on the telecom service supply chain: CDN (Content Delivery Network) that efficiently distribute the service to end-users located in different regions, OEM (Original Equipment Manufacturer) that produce terminal devices, etc. Despite that the telecom service supply chain includes so many participants, this thesis focuses only on ISPs and CPs and analyze the interactions among them. By extracting management science problems from our observations of the telecom market and building up mathematical models to solve them, this thesis aims at settling some problems that arise in telecom operations and providing theoretical guidance to practitioners and policymakers in telecommunications industry.

The remainder of this section introduces the background and provides an overview of the three projects in this thesis.

1.1 Interconnection and Settlement Design

Internet users perceive the Internet as seamless and global, while behind the scene there exist many individual networks connected with each other instead of a sole massive network. ISPs connect their networks to each other by interconnection arrangements in order to enable communication among end-users from different networks. There are three interconnection modes: (1) public peering, i.e., two or more ISPs exchange traffic through a public NAP (Network Access Point), also termed as NAP peering; (2) private peering, i.e., two ISPs exchange traffic through a direct physical connection between them; (3) transit, i.e., an ISP (the buyer) purchase Internet connectivity from another ISP (the provider) and visit other networks through the provider. When an ISP decides to connect itself to other networks, it has to consider the impact of the interconnection arrangements on its cost and profit and decide its interconnection strategy.

In China, ISPs, such as China Netcom and China Telecom, interconnect with each other mainly through private peering and NAP peering. There are only three national NAPs located at Beijing, Shanghai and Guangzhou before 2014.Since then, ten more national NAPs have been put to use, and currently there are thirteen national NAPs located at major cities in China, which is still sparse compared to the vast territory of China. The quality of communication between networks and the data transmission efficiency across different regions are relatively low due to the limited number of NAPs. To facilitate data exchange and extenuate traffic congestion at national NAP, regional NAPs have been built successively in several cities since 2004. This kind of local Internet exchange point has been proven beneficial to local Internet ecosystems, especially in developing countries and regions such as Latin America (Weller &

Woodcock, 2013; Galperín, 2015). Similarly, the construction of these regional NAPs in China has greatly reduced the interconnection costs and improved the network response speed, data exchange quality and safety.

However, new problems arose as regional NAPs were put into use. For example, Shanghai Network Access Point SHNAP (SHNAP) initially adopted a non-settlement rule for the data exchange between its member ISPs. Under the non-settlement rule, ISPs do not charge each other for data exchange. Since SHNAP can save great money for ISPs for interconnection, it attracted 16 ISPs to connect to it and the data exchange volume increased very fast in the first few years. The non-settlement rule, however, began to show its inefficiency after several years of implementation, as it can hardly reflect the cost and revenue to peer in SHNAP for each ISP. As a result, it caused an imbalance of profit allocation among member ISPs, and ISPs, especially those large ones, had no incentive to connect to SHNAP and would rather peer with other ISPs privately. Similar problem in Argentina has also been documented by Galperin (2015). Furthermore, ISPs who have already connected to SHNAP were reluctant to invest in capacity to improve the interconnection quality. At this stage, the introduction of a rational settlement rule, which allows fair profit allocation and stimulates ISPs to exchange traffic through regional NAP, becomes the key to promote regional NAP peering and enhance the value of local network.

Chapter 2 attempts to solve this problem by proposing an interconnection settlement rule under which the ISPs will voluntarily participate in peering at the NAP, thereby improving the utilization of regional NAPs and enhancing the interconnection efficiency and overall social welfare. To achieve this goal, we need to guarantee that the ISPs can profit more by peering at the NAP instead of other interconnection modes and get a fair payback when the proposed settlement rule is implemented.

To start with, we model the network externality as a quadratic term of the network size and set up the demand function and profit function of each ISP. Then a cooperative game framework is built to examine three profit allocation rules: non-settlement profit allocation, Shapley-value based profit allocation and characterized profit allocation (CPA) we propose, to see whether they are in the core of the game and are fair in profit distribution. Theoretical analysis and numerical results suggest that non-settlement profit allocation is not in the core, while the other two profit allocation rules are both in the core and CPA can more effectively reflect each ISP's contribution to the grand coalition.

Based on above analysis, we devise a settlement rule under which ISPs' independent pricing decisions can lead to optimal total profit and each ISP will gain a profit exactly the same with its profit allocation under CPA. Moreover, we examine the effectiveness of the proposed settlement rule in three extensions where (1) ISPs make both pricing and interconnection quality decisions; (2) consider the direct competition between ISPs; (3) consider linear network externality, and results show that the settlement rule and its modified version perform well in these scenarios. It can effectively encourage ISPs to peer at NAP with higher link quality and provide important managerial implication for settlement mechanism to NAPs across the world.

1.2 Mobile Operators' "Dump Pipe" Dilemma and Sponsored Data Services

With the proliferation of smart devices and the rapid development of mobile telecommunications technology, mobile Internet has penetrated into many aspects of people's life. Various Internet-based apps provides us with much convenience, even changes our behavior patterns: we do not need to bring cash or credit cards, merchants can just scan QR codes on consumers' smart phones to complete the payments; we do not have to have our own vehicles, taxi-calling apps and bicycle-sharing apps give us access to nearest shared vehicles efficiently. These benefits owe much to mobile network's evolution from 1G to 5G, which greatly improves the speed and reliability of the mobile Internet to support a wide range of Internet-based services. On one hand, the huge data traffic generated by the use of mobile Internet has become mobile operators' (MO) major revenue source, on the other hand, those mobile Internet-based services have largely replaced the traditional voice service and value-added services (VAS), which impairs MO's channel power, increases MO's network infrastructure construction and maintenance cost, and lowers MO's profit margin.

In the traditional mobile telecom supply chain, mobile operator and value-added service provider (VASP) join hands to provide users with various value-added services (VAS), including ring back tones, news subscriptions, mobile mails, location-based services, etc. MO naturally is the leader of the telecom supply chain, as it is the sole link between VASPs and users. It charges the users for using VASs first, and then share a portion of the revenue with the VASPs. The income from distributing VASs accounted for a large proportion of MO's revenue in early years, until the advent of 3G and 4G technology. Nowadays with the prevalence of high-speed mobile Internet enabled by 3G and 4G, more and more Service Providers choose to provide Internet-based services that establish close relationships with users, bypassing the traditional distribution of MOs. This type of services is termed as over-the -top (OTT) services. Users typically turn to third-party application stores like Apple Store and Google Play to purchase OTT applications directly. In-app purchases also are charged by these third-party stores or other third-party payment applications.

As OTT services cover a wide range of services from instant messaging to public-shared bicycles, most traditional telecom services can find their counterparts in OTT. MOs' revenue from VASs is declining year by year, and their role in the mobile telecommunications market has been marginalized to a "dumb pipe". Take China's largest mobile telecom company, China Mobile, as an example, Figure 1.1 shows the its revenue composition from 2009 to 2016. We can see that there is a clear decline in the revenue from voice and SMS and MMS services, while the revenue from wireless data traffic is booming, exceeding the sum of revenue from the other two traditional services for the first time in 2016. In contrast to the rapid growth of data service, China Mobile's profit margin rate is continuously decreasing, reflecting an imbalance between data traffic revenue and the corresponding infrastructure construction and maintenance costs.

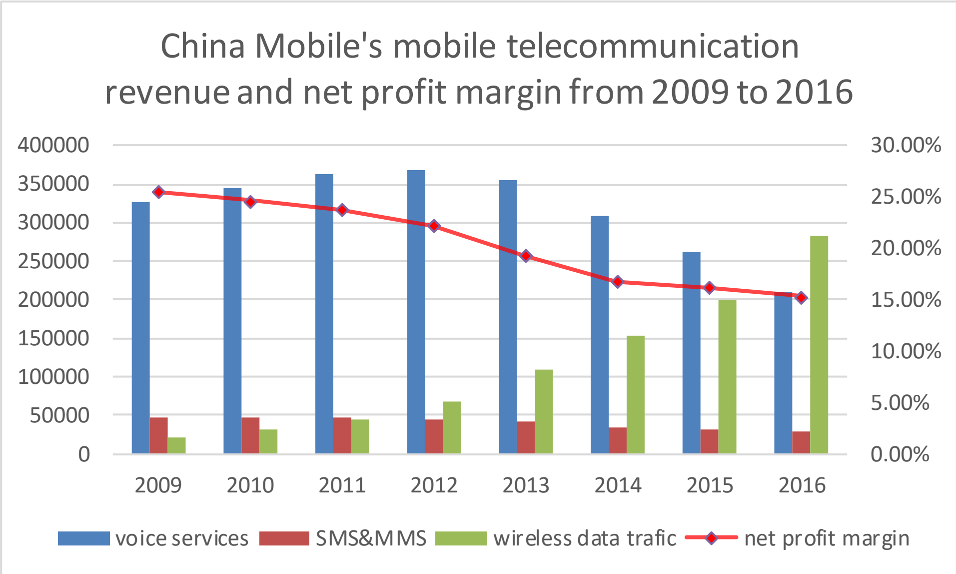


Figure 1.1. China Mobile's mobile telecommunications revenue and net profit margin from 2009 to 2016

Data source: China Mobile's annual report

In such a critical situation, mobile operators need to adjust their revenue model to enhance profitability. People have different views with respect to this issue. Some think that MOs should enter the valuable OTT business and develop their own OTT applications, while some people think that MOs should direct resources to their advantageous business, improving network infrastructure and enhancing service level. Ernst & Young's (2015) survey suggested that major telecom companies' top strategic priorities are not direct competition with OTT services, but rather upgrading their infrastructures, improving cost efficiency and selling the data traffic efficiently.

Among the various practices proposed, sponsored data service is one that attracts much attention. SDP allows OTT providers to subsidize their users by paying for the data traffic generated, so that users can enjoy their services freely without worry about exceeding their data caps. Though seemingly a beneficial practice for each party, how will SDP affect mobile operator, OTT provider and users is still unclear.

Chapter 3 attempts to understand the impact of SDP on each participant of the telecom service supply chain and the overall social welfare by considering a market with monopolist ISP and monopolist CP. CPs are divided into two types, subscription CP and platform CP, according to their revenue model. Subscription CPs, such as Hulu and Netflix, profit from collecting subscription fees from their users, while platform CPs, such as Facebook and Taobao, do not charge their users directly but profit from advertising and other value-added services of the platform.

A Stackelberg game model where the ISP acts as the leader and the CP acts as the follower is built to analyze ISP's pricing decision and CP's subsidization decision. In the basic model, we assume that the CP is a subscription CP and all parameters are common knowledge. The impact of sponsored data is investigated by comparing the equilibrium outcome with and without a sponsored data contract between ISP and CP. Similar analysis is carried out in three extended scenarios:(1) CP's cost is its private information, (2) the quality of service is a decision of CP, and (3) the CP is a platform CP, to enrich our results about the impact of sponsored data practice. Results suggest that the ISP benefits from offering sponsored data service in general, except when it does not possess the subscription CP's cost information and the CP's actual cost is high. A subscription CP's profit decreases when it provides data subsidization for its users, although a subscription CP with high profit margin can turn the tide by hiding its actual cost or indicating possible variations in its content quality. The impact of sponsored data on social welfare is positive when a platform participates in sponsored data and is mixed when the sponsoring CP is a subscription CP, depending on the profitability of the CP.

Chapter 4 adopts a two-stage Stackelberg-Nash game model to model the decision-making behavior of two ISPs and one CP under sponsored data plan. Results show that even in a market where two ISPs compete, ISPs can still profit from sponsored data, and unintegrated ISP can improve its weak position by implementing sponsored data in a market where there is vertical integration between ISP and CP. CP's profit is reduced in most situations, and only CP with strong profitability can improve its profit by offering subsidy to users of both ISPs. ISP and CP should take full consideration of their strategic goals of both market share and profit when choosing cooperation method and integration strategy. From a social welfare perspective, sponsored data has a positive effect in most cases with some exceptions, and the positive effect is more pronounced when the competition between ISPs is intense. These findings provide practical guidance for telecom enterprises' integration strategies and sponsored data decisions, and provide a theoretical basis for the regulation of the telecommunications market.

In summary, this thesis studies ISPs' interconnection settlement mechanism and sponsored data mode by modeling the competition and cooperation among ISPs and CPs, aiming at promoting high-quality interconnection and the healthy development of the telecommunications industry. Table 1.1 summarizes the research subjects and methodologies of each study.

Table 1.1 Summary of the research subjects and methodologies

Research Problem	**Research Subject**	**Methodology**
Cooperative Interconnection Settlement Design at Network Access Point (Chapter 2)	Horizontal competition and cooperation among ISPs	Cooperative game Mechanism design
Impact of Sponsored Data on ISP, CP and Social Welfare (Chapter 3)	Vertical competition between ISP and CP	Stackelberg game
Impact of Sponsored Data in a Competitive Market (Chapter 4)	Horizontal competition between ISPs Vertical competition between ISP and CP	Stackelberg-Nash game

2 Cooperative Interconnection Settlement Design at Network Access Point

2.1 Introduction

The research of this chapter is inspired by the interconnection dilemma faced by Shanghai NAP. SHNAP is one of the first regional NAPs set up in China to facilitate data exchange across local network and extenuate traffic congestion at national NAPs. SHNAP initially adopted a non-settlement rule, which achieved rapid growth of access members and average daily traffic in the first few years. However, as non-settlement rule can hardly reflect the cost and revenue for ISPs to peer at SHNAP, an imbalance of profit allocation among member ISPs occurred, and it demotivated the ISPs to peer at SHNAP.

In response to the problem observed at SHNAP and similar phenomenon documented in Argentina (Galperín, 2016) we propose the main research problem of this chapter. We consider a regional Internet market with n ISPs of different size. Each ISP has its own potential user base, and its realized demand is affected by the pricing and value of its Internet access service. The value of an ISP's service perceived by users depends on the size of the accessible network the ISP can provide, so interconnection with other ISPs must increase the ISP's service value. However, as ISPs involved in an interconnected network have different sizes of potential user base and network, the benefits and costs they get from the interconnection are also different. Therefore, an ISP need to make a set of decisions, including the pricing strategy, whether to interconnect and with whom to interconnect, to maximize its profit. From the perspective of the NAP, we need an interconnection settlement rule that encourages ISPs to spontaneously peer with each other at the regional NAP and make jointly optimal decisions, so as to promote the development of the regional NAPs and to improve the interconnection quality of regional network.

To achieve this goal, the profit allocation induced by this settlement rule must be fair in profit distribution and can bring more profit to each ISP. To be specific, the settlement rule should meet following two principles: (1) win-win principle, i.e., any ISP, no matter large or small in network size, can profit more if it participates in the multilateral peering at the NAP instead of other methods (such as private peering); (2) fair principle, i.e., the profit allocation each ISP gets by peering at the NAP can objectively reflect its contribution to the whole network. Of course, the settlement rule should be easy to interpret and implement. As NAPs are continually being set up all over the world for both regional interconnection and global interconnection (such as London Internet Exchange which has members from 40 countries), this chapter can provide valuable managerial insights on how to improve NAP peering efficiency not only for SHNAP, but also for practitioners worldwide.

The remainder of this chapter is organized as follows. Section 2.2 reviews the related literature studying Internet interconnection, especially those who endeavored to approach a cooperative settlement rule. Section 2.3 justifies and builds the basic model, including demand function and profit function. Section 2.4 builds the cooperative game framework to analyze and examine whether different profit allocation rules (i.e., non-settlement profit allocation, the Shapley-value based profit allocation and the Characterized Profit Allocation (CPA) we devise) meet the win-win principle and fair principle proposed above, and then proposes a settlement rule to implement the CPA. In section 2.5, we extend the basic model in three directions: consider the interconnection quality choice of ISPs, introduce competition in the market, and consider linear network externality, and the effectiveness of the proposed settlement rule and its modified versions is examined in each scenario. Numerical experiments are conducted in section 2.6 to support our theoretical results. Section 2.7 summarizes the findings and discusses future research directions. All proofs are given in section 2.8

2.2 Related Literature

Under non-cooperative game analysis framework, there is a large body of literature studying ISP interconnection strategies, including determining compatibility and access charges. Cremer et al. (2000) develop their research on basis of Katz and Shapiro's model of network externalities (Katz & Shapiro, 1985) and study the strategies of Internet backbone providers. They use a Cournot-cum-installed-bases model and show that compared with small Internet Backbone Provider (IBP), larger IBP prefers a lower interconnection quality. This result is robust even if customer can connect to several IBPs (multi-homing). Foros and Hansen (2001) follow their work by modeling network externalities in a two-stage game where the two ISPs choose compatibility level at stage 1 and compete over market shares a la Hotelling at stage 2. They find that ISPs can reduce competitive pressure in stage 2 by increasing compatibility, so that a higher compatibility in stage 1 will be achieved. While previous studies (Cremer et al., 2000; Foros & Hansen, 2001; Foros et al., 2005; Matsubayashi & Yamada, 2008) all model network externality as a linear function of the number of customers, it seems the utility a user get from the Internet is already beyond a linear term with the evolution from web 2.0 to web 4.0. In this chapter, we model network externality as a quadratic function of network size. Some more recent studies of Jahn and Prüfer (2008) and Badasyan and Chakrabarti (2008) also use simple game-theoretic model to analyze ISPs' interconnection choices. These models vary in complexity, with the former dealing with asymmetric networks and the latter dealing with different cost structures, which is essential to evaluating potential peering arrangements (Motiwalaet al., 2012). In terms of pricing, He and Walrand (2005) show that non-cooperative pricing strategies may result in unfair profit distribution. They propose a fair allocation policy based on the weighted proportional fairness criterion. López (2011) extends Laffont et al.'s (2003) analysis to asymmetric but reciprocal access pricing in the presence of an arbitrary number of network operators. He shows that the configuration of interconnection charges has important implications for the market structure: If the reciprocal access charge of a pair of networks departs away from a given symmetric access charge, then the two networks are driven out of one side of the market (consumers/websites).

Several studies address the profit allocation among interconnected ISPs under non-cooperative game analysis framework. Huston (1998) suggests that ISP interconnection settlement can base on inbound traffic volume, on outbound traffic volume, on a hybrid of inbound and outbound traffic volume, or on the line capacity regardless of volume. Weiss and Shin (2004) believe that the cost of ISP interconnection is a function of traffic, and the traffic volume is a function of a market share. Thus, they address the interconnection settlement problem with knowledge of inbound and outbound traffic flows. Tan et al. (2006) propose a more complex pricing scheme that considers network utilization, link capacity, and the cost structure of the interconnecting ISPs. They show that a usage-based, utilization-adjusted interconnection agreement could align the costs and revenues of the providers while allowing them to achieve higher service levels.

Cooperative game theory is currently a hot topic in operational research (Chen & Chen, 2013; Hu et al., 2013; Lozano et al., 2013; Karsten & Basten, 2014; Borkotokey et al., 2015; Kimms & Kozeletskyi, 2016), and there is also a growing trend of using cooperative game theory to approach a profit allocation that encourages network-wide interconnection. Cheung et al. (2008) find that with information of global topology and traffic information for each ISP tier, there exist prices that can make the revenue division under bilateral settlement equal to that calculated by Shapley value. Shapley value, a classic solution concept in cooperative game theory, indicates the marginal contribution each agent makes to the coalition. Shapley value is known to be in the core of a convex cooperative game (Shapley & Roth, 1988), Ma et al. (2010) also adopt a Shapley-value approach and show that profit model based on Shapley value can make ISPs' selfish behavior result in global optimal routing and interconnecting decisions. Following this study, Ma et al. make several attempts to implement this Shapley-value rule. They first use a content-eyeball model to show that Shapley-value revenue distribution can be implemented by bilateral payment between eyeball and content ISPs (Ma et al, 2008), and then extend their model to include transit ISPs (Ma et al., 2011). In those studies, Shapley value is calculated as the cost of handling traffic, and it is strongly affected by ISP topology structure and routing strategies. Mycek et al. (2009) extend the concept in Ma et al. (2008) with a fair income distribution policy, coupling a routing decomposition optimization framework that deals with multiple connections. They use decomposition result parameters to solve the complex issue of computing Shapley values, while Misra et al. (2010) present another method based on fluid approximation, which can also effectively reduce the complexity of computing Shapley values. Furthermore, Singh et al. (2012) study the similar problem of cooperative profit sharing in wireless network markets. They model such cooperation using the theory of transferable payoff cooperative game and propose a set of payoffs that are commensurate with the resource the network providers invest and the wealth they generate. They also develop an algorithm to obtain the optimal resource allocation and corresponding profit-sharing rule. The authors numerically show that cooperation can tremendously enhance network providers' profits. Indeed, cooperative game theory is a powerful tool in analyzing the behavior and interaction of the individual nodes in various communication networks (Matsubayashi et al., 2005; Saad et al., 2009; Liu et al., 2013). Most of the previous researches pay attention to sharing of the physical resources, such as base stations, and propose the profit allocation based on the Shapley value. In our study, we extract the fact that interconnection can bring positive network externality and focus on profit allocation rules other than the Shapley value.

Our study adds to interconnection literature in two ways. First, previous researches focus on private peering and transit, and little attention is spared to NAP peering. However, as more and more global or local traffic are exchanged at NAPs, the operational model of these NAPs needs more exploration, and our study takes a step forward with a discussion of a rational settlement rule. Second, most of existing literature model network externality as a linear function, which we think is inadequate to appropriately reflect the network value according to our analysis in section 2.5.3. We believe that our analytical approach and the proposed Characterized Profit Allocation can be used in other networks with network externality, such as telecommunication network and logistic network.

2.3 The Model

Consider a set of ISPs $N = \{ISP_i, i = 1, \ldots, n\}, (n \geq 2)$. Each ISP is characterized by two parameters. The first is the intrinsic demand potential D_i, which is related to the coverage area of ISP_i and is considered to be exogenous. The second parameter is the installed network size e_i, which can be composed of the number of installed end-users and the richness of contents (Xu, 2007). The e_i in our model mainly reflects the richness of contents, and decision-makers can use the number of ISP_i's installed websites weighted by popularity factors to measure e_i (Ma et al, 2008). A consumer makes purchase decision based on the price and associated network size of the ISP and can only subscribe to one ISP. Let y_i denote the associated network size of ISP_i, which is the total size of network interconnected with ISP_i. The associated network size of ISP_i is equal to its installed network size e_i if ISP_i does not interconnect with any other ISPs. Otherwise, y_i equals $\sum_{j \in S} e_j$, where S is the set of ISPs interconnected with ISP_i including ISP_i itself. As all ISPs in a set S have same associated network size ($y_i = y_j, i, j \in S$), we write $\sum_{j \in S} e_j$ as E_S in the remainder for simplification.

ISP_i first decides the set S, i.e., which ISPs to interconnect with, and then decide its service price p_i. In the equilibrium analysis where interconnection settlement is absent, we take the interconnection decision S as exogenous and solve ISP_i's optimal pricing decision when it interconnect with other ISPs in S. Given ISP_i participates in the interconnection of set S, its realized demand is denoted by $d_i(S)$. The demand $d_i(S)$ is formulated as a function of intrinsic demand potential D_i, price $p_i(S)$ and associated network size y_i:

$$d_i(S) = D_i - \alpha p_i(S) + \beta y_i^2 \tag{2-1}$$

where α, β are positive constant coefficients denoting the demand variation responsive to the price and network size respectively. The actual demand decreases with price and increases with the associated network size.

Consistent with previous studies (Foros & Hansen, 2001; Foros et al., 2005; Matsubayashi & Yamada, 2008), we assume that the realized demand of ISP_i is linear to its price. Despite that previous studies assume a linear network externality, we assume the network externality to be βy_i^2 ($\beta > 0$), i.e., the realized demand increases quadratically with the associate network size (linear network externality is also discussed in section 2.5.3). This assumption is made based on an extension of Metcalfe's Law (Metcalfe, 1995), which states that the value of a network is proportional to the square of the number of connected users of the system. The implicit assumption of Metcalfe's law is that the larger the network, the more users one can communicate with, so the utility of the network for each user is linear with the number of users in the network. In today's Internet, users can not only communicate peer-to-peer with other users, but also can create content by themselves, interact with multiple users at the same time, and extract positive utility from the content created by other users' interaction process. These new features of Internet compared to the telephone and fax networks have made Internet more valuable than a communication network that only allows peer-to-peer communication. We further assume that there is no direct competition among ISPs in the market for simplification and to focus on our main problem. This assumption makes sense if different ISPs' networks do not overlap with each other or they target at different groups of users. The China Education and Research Network (CERN), for example, provides Internet service only to educational and research institutes. Additionally, regional ISP monopoly is not a rare phenomenon in both China and United States. According to Federal Communications Commission's report (2014), 35% of American households have two or less options of ISP that provides at least 10 Mbps downstream and at least 1.5 Mbps upstream connectivity service in their residential locations. Later, we relax this assumption in an extension where two ISPs interconnect and also compete with each other.

An ISP's revenue comes from the access fee charged to users, and its cost includes fixed costs of network infrastructure construction, variable costs incurred by each user's connection, and interconnection costs incurred by data transmission. As long as the fixed cost is smaller than the ISP's equilibrium revenue minus variable cost, the fixed cost has no effect on the equilibrium. To simplify the exposition, we assume that the fixed cost equals zero. Without loss of generality, we also take the variable cost to be zero. Following previous researches (Weiss & Shin, 2004; Badasyan & Chakrabarti, 2008) that consider interconnection cost as a function of traffic, we model the interconnection cost as the data transmission cost between users and resources in the associated network. For simplification, we assume that each customer has one unit demand for each resource in the associated network. Let c_o and c_t denote the unit cost of data transmission for originating network and terminating network respectively, and $c_o + c_t = c$. Therefore, the profit function of ISP_i is formulated as:

$$\pi_i(S) = p_i(S)d_i(S) - c_o e_i \sum_{j\in S} d_j(S) - c_t d_i(S) E_S \qquad \text{(2-2)}$$

The first term is revenue, the second term is the transmission cost of outbound traffic for all users in the interconnected network accessing ISP_i's resources, and the third term is the transmission cost of inbound traffic for ISP_i's users accessing resources in the whole interconnected network. Typically, the terminating network bears most of the transmission cost, i.e., $c_t > c_o$, due to the 'hot potato' routing strategy (Laffont et al., 2001, 2003). Each ISP makes the pricing decision to maximize its own profit.

We present the results of a non-interconnection system, where ISPs do not interconnect with each other, as a benchmark. It is straightforward to verify that $\pi_i(p_i)$ is concave. Therefore, we have the following theorem.

Theorem 2.1 In a non-interconnection system, for ISP_i, the optimal price p_i^*, the corresponding demand d_i^* and the optimal profit π_i^* are given by following equations:

$$p_i^* = \frac{D_i+\beta e_i^2+\alpha c e_i}{2\alpha},\ d_i^* = \frac{D_i+\beta e_i^2-\alpha c e_i}{2},\ \pi_i^* = \frac{\left(D_i+\beta e_i^2-\alpha c e_i\right)^2}{4\alpha}.$$

Here we assume that $D_i + \beta e_i^2 - \alpha t e_i > 0$ and $2\beta e_i - \alpha t > 0$ hold for any $ISP_i \in N$. The former ineuation makes sure that each ISP has positive demand, and the latter assures that the demand of an ISP is increasing in its network size.

2.4 Interconnection Settlement Design

When an ISP interconnects with other ISPs (see Figure 2.1), its subscribers can visit networks interconnected with it. On one hand, more consumers will be attracted to the ISP since they can visit more resources, and the ISP can generate more revenue by serving more end-users. On the other hand, interconnection will also incur an extra cost, as the data transmission cost increases with the increasing number of end-users and the expanded associated network size.

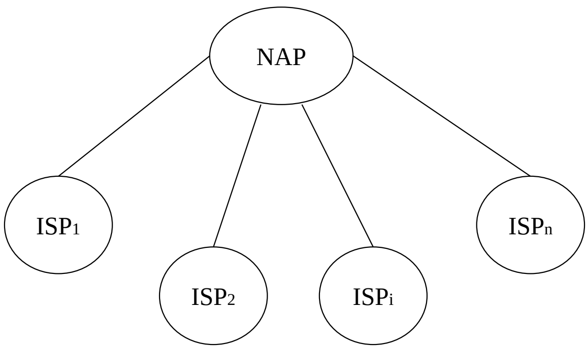


Figure 2.1 Public peering through NAP

With the demand function defined in section 2.3, the total profit of coalition S, denoted as $\Pi(S)$, can be formulated as $\Pi(S) = \sum_{ISP_i \in S}[p_i(S)d_i(S) - td_i(S)E_S]$. It is straightforward to check that $\Pi(S)$ is separable in p_i, and is concave in each p_i. Therefore, we have the following theorem.

Theorem 2.2 In an ISP coalition S, for $ISP_i \in S$, the jointly optimal price $p_i^*(S)$, the corresponding demand $d_i^*(S)$ and the optimal profit $\Pi^*(S)$ are given by following equations:

$$p_i^*(S) = \frac{D_i + \beta E_S^2 + \alpha c E_S}{2\alpha},\ d_i^*(S) = \frac{D_i + \beta E_S^2 - \alpha c E_S}{2},\ \Pi^*(S) = \frac{\sum_{i \in S}\left(D_i + \beta E_S^2 - \alpha c E_S\right)^2}{4\alpha}.$$

Comparing the optimal total profit above with the results in Theorem, it is easy to see that interconnection among ISPs elevates the total profit from $\frac{\sum_{i \in S}\left(D_i + \beta e_i^2 - \alpha c e_i\right)^2}{4\alpha}$ to $\frac{\sum_{i \in S}\left(D_i + \beta E_S^2 - \alpha c E_S\right)^2}{4\alpha}$. This increase in profit due to interconnection is called cooperative surplus, and a positive cooperative surplus affirms the viability of implementing profit allocation rules to encourage interconnection.

2.4.1 Cooperative Game Framework

We then use cooperative game theory to discuss the profit allocation problem among all ISPs. We refer to the subset S $(S \subseteq N)$ of ISPs who interconnected with each other as coalition S and to the set N as the grand coalition. Thus, we formulate our problem as a cooperative game $(N, \Pi^*(S))$, in which $\Pi^*(S)$ is the characteristic function specifying the optimal total profit associated with coalition S. According to Shapley's (1971) definition, it can be easily proved that $(N, \Pi^*(S))$ is a convex cooperative game.

A vector $r = (r_1, \dots, r_n)$ is called an allocation and each element r_i corresponds to the portion of total profit of grand coalition that ISP_i should get. If $\sum_{i=1}^{n} r_i = \Pi^*(N)$, then the allocation is said to be *efficient*. An allocation is said to be *individually rational* if $r_i \geq \pi_i^*$ and to be *stable* for a coalition S if $\sum_{i \in S} r_i \geq \Pi^*(S)$. Altogether, an allocation is said to be in the core if it satisfies the following two conditions:

(i) Efficiency: $\sum_{i \in N} r_i = \Pi^*(N)$;

(ii) Coalitional rationality: $\sum_{i \in S} r_i \geq \Pi^*(S), \forall\, S \subseteq N$.

When an allocation is in the core, no subset of ISPs would secede from the grand coalition to form smaller coalitions, including being on their own. In addition to the requirement of being in the core, it is desirable for an allocation to be perceived as *fair* (for a more elaborate discussion of fairness in cost allocation rules see Moulin, 1995). To be clear, the allocation of the total profit should reflect the value of each ISP's network: ISPs with larger network size should earn a larger proportion of total profit. Thus, the benefit of every ISP can be guaranteed, and ISPs should have incentive to expand their networks to increase profit. As a result, ISP interconnection is encouraged, and the development of Internet market is boosted.

In what follows, we analyze three different profit allocation rules. The first is the **non-settlement profit allocation**, in which each ISP invoices its user for the services, but no financial settlement is made across ISPs. The non-settlement profit allocation was adopted by SHNAP. The second allocation is the widely researched **Shapley-value based profit allocation**. The third is the **Characterized Profit Allocation** we proposed, in which each ISP has to pay the other ISPs for accessing their networks.

2.4.2 Non-settlement Profit Allocation

The non-settlement profit allocation implies that there is no side-payments among ISPs. That is, an ISP does not pay or charge other ISPs for network accessing. Non-settlement allocation can be regarded as a special profit allocation rule, where each ISP simply connects to other networks but no financial settlement is payable.

Substitute the optimal price $p_i^*(N)$ given in Theorem 2.2 into equation (2-2), we can obtain ISP_i's optimal profit $\pi_i^*(N)$ when it joins the grand coalition, which is also the profit allocation it can get under non-settlement profit allocation rule. Let $r_A = (r_{A1}, \dots, r_{An})$ denote the profit allocation vector under non-settlement allocation, we have r_{Ai} as follows:

$$r_{Ai} = \frac{[D_i+\beta E_N^2-\alpha(c_o+c_t)E_N][D_i+\beta E_N^2+\alpha(c_o-c_t)E_N]}{4\alpha} - \frac{c_o e_i}{2}\sum_{j\epsilon N}\left(D_j + \beta E_N^2 - \alpha c E_N\right) \tag{2-3}$$

Proposition 2.3 Non-settlement profit allocation is not in the core of the game $(N, \Pi^*(S))$.

Proposition 2.3 implies that some ISPs may not voluntarily participate in the interconnection at the NAP. In particular, when a set of ISPs possess networks of relatively large size but small intrinsic demand potentials, they may find joining in the NAP less profitable than forming a coalition by themselves through private peering.

Furthermore, non-settlement does not make a fair allocation. According to the Metcalfe's Law, the ISP with larger network size adds more value to the interconnected network than the ISP with smaller network size. But under non-settlement allocation, an ISP with larger network size receives less profit allocation from the grand coalition given that all ISPs have the same intrinsic demand potential. So, r_A cannot truly reflect how much contribution each ISP has made to the grand coalition. If an ISP decides to expand its network, it may benefit other ISPs more than itself. Thus, ISPs would be reluctant to invest in their networks or interconnect with other ISPs if the non-settlement allocation was implemented. Thus, the non-settlement profit allocation may restrain the development of internet interconnection.

2.4.3 Shapley-value Based Profit Allocation

Shapley value is an important concept in cooperative game, and it is well-known for its fairness property. The payoff to a player under Shapley-value based profit allocation is calculated as the marginal contribution of a player averaged over joining orders of the coalition. To be specific, it can be formulated as:

$$\varphi_i(N, \Pi^*) = \frac{1}{|N|!} \sum_{\zeta \in \mathrm{Z}} \Delta_i(\Pi^*, P(\zeta, i)), \forall i \in N \tag{2-4}$$

where $\Delta_i(\Pi^*, S) = \Pi^*(S \cup \{i\}) - \Pi^*(S)$, Z is the set of all $|N|!$ orderings of N, and $P(\zeta, i)$ is the set of ISPs preceding ISP_i in the ordering ζ.

Let $r_B = (r_{B1}, \dots, r_{Bn}) = (\varphi_1, \dots, \varphi_n)$ denote the profit allocation vector under Shapley-value based allocation. As $(N, \Pi^*(S))$ is a convex cooperative game, Shapely value is consequently in the core.

Though in the core and satisfying desirable properties such as symmetry, linearity, additivity, and most importantly, efficiency and fairness, Shapley value is difficult to calculate as the number of players in the coalition becomes large. In addition, the complex structure of Shapley value may thwart ISPs from understanding the rules properly and make this profit allocation difficult to be implemented at NAP.

2.4.4 Characterized Profit Allocation

Network size is a critical factor for an interconnected network to attract more end-users and make greater profit, so it is important for an allocation to reward ISPs who invest more in network construction and make bigger contributions to the grand coalition with larger size of network. Thus, the Characterized Profit Allocation (CPA) is developed to meet this end. Under this proposed allocation, the total profit of all interconnected ISPs is redistributed in a way that reflects their specific characteristics, including the network size and intrinsic demand potential.

Let $r_C = (r_{C1}, \dots, r_{Cn})$ denote the profit allocation vector under the CPA. Based on the structure of the optimal total profit, we design r_{Ci} as:

$$r_{Ci} = \frac{1}{4\alpha}\left[D_i^2 + e_i(\beta E_N - \alpha c)\sum_{j\in N}\left(2D_j + \beta E_N^2 - \alpha c E_N\right)\right] \quad (2\text{-}5)$$

An ISP's profit allocation in the grand coalition under CPA can be divided into two parts: the first part is linear to D_i^2, which reflects the value of its market coverage; the second part is linear to e_i, which reflects the value of ISP's network.

Proposition 2.4 The Characterized Profit Allocation is in the core of the game $(N, \Pi^*(S))$ and is a fair allocation.

As we can see, ISP_i's investment in network expansion will bring extra profit to other ISPs who make no effort as well as to itself, so we are particularly interested in how the total profit is allocated among ISP_i and other ISPs. To check that, the marginal profit of each ISP as e_i increases is calculated as follows:

$$\frac{\partial r_{Ci}}{\partial e_i} = \frac{1}{4\alpha}\Big[(\beta E_N - \alpha c)\sum_{k\in N}(2D_k + \beta E_N^2 - \alpha c E_N) + n e_i(\beta E_N - \alpha c)(2\beta E_N - \alpha c) + \beta e_i \sum_{j\in N}\left(2D_j + \beta E_N^2 - \alpha c E_N\right)\Big] \quad (2\text{-}6)$$

$$\frac{\partial r_{Cj}}{\partial e_i} = \frac{1}{4\alpha}\left[n e_j(\beta E_N - \alpha c)(2\beta E_N - \alpha c) + \beta e_j \sum_{k\in N}(2D_k + \beta E_N^2 - \alpha c E_N)\right] \quad (2\text{-}7)$$

Comparing equation (2-6) and (2-7), we find that, despite that all ISPs benefit from ISP_i's investment in the network size, ISP_i can get an extra part of profit compared to other ISPs. This property ensures that the ISP who invests in its own network benefits more than other ISPs from the expansion of the entire interconnected network, which again demonstrates the fairness of CPA.

2.4.5 CPA-based Settlement Rule

From the analysis above, we conclude that the non-settlement allocation is not in the core of the cooperative game $(N, \Pi^*(S))$, and shows no fairness in profit allocation either. The Shapley-value based profit allocation is a traditional fair allocation rule and is in the core, but its complicated and obscure formulation cripples its practical implementation. The CPA preserves fairness and is also in the core, so it is desirable that ISPs interconnected through NAP can achieve the profit allocation under this allocation rule. Therefore, the primary goal of this section is to develop a settlement rule under which ISPs can make decisions independently and achieve CPA.

Cooperative game assumes a central decision-maker that makes jointly optimal decisions for the grand coalition, and then distributes the optimal total profit among players in a way that makes sure each player is willing to join the grand coalition. In the cooperative game framework, each player only has the choice of whether to join the cooperation, other decisions are all made by the central decision-maker. In practice, ISPs are unlikely to make pricing decisions together to achieve global optimum, and they usually make their pricing decisions independently. Thus, the introduction of a settlement rule under which ISPs will independently make the jointly optimal pricing decisions is very essential. It can effectively motivate ISPs to cooperate, i.e., connecting to the NAP to exchange traffic with each other.

Based on CPA, we propose a settlement rule under which each ISP connecting to NAP receives a subsidy (or pay a side-payment) while it makes its pricing decision independently. For ISP_i, the subsidy $s_i(N)$ is composed as follows:

$$s_i(N) = c_o\left(e_i \sum_{j\epsilon N} d_j - d_i E_N\right) + \frac{1}{4\alpha}\left[e_i(\beta E_N - \alpha c) \sum_{j\epsilon N}\left(2D_j + \beta E_N^2 - \alpha c E_N\right) - (2D_i + \beta E_N^2 - \alpha c E_N)(\beta E_N^2 - \alpha c E_N)\right] \quad (2\text{-}8)$$

The first part of equation (2-8) adjusts the data transmission cost, subsidizing ISP_i for the transmission cost generated by other ISPs' users visiting resources in ISP_i's network and making ISP_i pay extra cost for its own users visiting other ISPs' network. The second part adjusts ISP_i's profit on basis of its intrinsic demand potential and network size, in order to reflect ISP_i's contribution to the whole interconnected network. In the following, we use the subsidiary vector $\boldsymbol{s}$ to denote the settlement rule we proposed. Under settlement rule $\boldsymbol{s}$, ISP_i's profit function π_i^c will be the sum of

$\pi_i(N)$ and $s_i(N)$, written as:

$$\pi_i^{c} = p_i d_i - c d_i E_N + \frac{1}{4\alpha}\left[e_i(\beta E_N - \alpha c)\sum_{j\epsilon N}\left(2D_j + \beta E_N^2 - \alpha c E_N\right) - (2D_i + \beta E_N^2 - \alpha c E_N)(\beta E_N^2 - \alpha c E_N)\right] \quad (2\text{-}9)$$

Proposition 2.5 If settlement rule $\boldsymbol{s}$ is implemented at the NAP, ISP_i 's equilibrium pricing decision coincides with the jointly optimal pricing, and its optimal profit π_i^{c*} is exactly the same as r_{Ci}.

Under settlement rule $\boldsymbol{s}$, each ISP gets more profit in the grand coalition than in any other coalitions, so they will not split from the grand coalition, and the aim of encouraging network interconnection through NAP can be achieved. An ISP can get extra profit than others under CPA when it expands its network, so the proposed settlement rule $\boldsymbol{s}$ can also encourage ISPs to constantly invest in their network construction. Moreover, settlement rule $\boldsymbol{s}$ shows certain robustness as it can efficiently allocate the total profit among ISPs in the coalition even if there are some ISPs deviating from optimal pricing decisions. Thus, implementing settlement rule $\boldsymbol{s}$ at NAP can pull the NAP peering market onto the track of quick development. In section 2.6, we conduct a series of numerical experiments to demonstrate the effectiveness of CPA-based settlement rule.

2.5 Extensions

2.5.1 Incorporating Quality Decision in Interconnection Settlement

In practice, ISPs with asymmetric network sizes usually have asymmetric incentives to provide interconnection quality, which will determine how useful and efficient the interconnection is. To address this issue, we extend our model to consider ISPs' interconnection quality decision along with pricing decision. Some scholars have looked into the similar price and quality-based competition problem in the interconnection market. Matsubayashi and Yamada (2008) study how the asymmetry in consumer loyalty affects firms' price and quality competition. Le Cadre et al. (2011) analyze the price and quality choice of each player in a vertically integrated autonomous system under four types of contract, and their numerical illustration suggests that grand coalition cooperation contract is efficient when consumers' QoS sensitivity is relatively low.

The basic model is extended as follows. When an ISP connects to a NAP to exchange traffic, it has to decide its access bandwidth, and it can take some technical measures such as uplink and downlink bandwidth limits to control the transmission quality for inbound and outbound traffics. We extract two decision variables, γ_i^U and γ_i^D, denoting the uplink quality and downlink quality respectively. Here γ_i^U, $\gamma_i^D \in [\underline{\gamma}, 1]$, $\underline{\gamma} \in (0,1)$ is the lowest interconnection quality provided at the NAP, and 1 represents perfect interconnection quality. An ISP with large network and rich resources may tend to limit the number of routing paths available and decrease the access bandwidth, i.e., choosing a low level of interconnection quality, to avoid superabundant traffic load. As a result of imperfect interconnection, end-users cannot equally access to resources that belong to different ISPs in Internet, so end-users subscribing to different ISPs perceive the size of the interconnected network differently. In accordance with Cremer et al. (2000), the resulting perceived network quality (i.e., QoS level of ISP_i) is constructed as a function of its own network size, the network size of other ISPs and the quality of interconnection decisions of all ISPs connected to the NAP. For example, if there are three ISPs connecting to NAP, as shown in Figure 2.2, then ISP_1's subscribers can visit its own resources with perfect quality and visit the other two ISPs' resources with quality determined by ISP_1's downlink quality and ISP_2 and ISP_3's uplink quality, so the QoS level of ISP_1 is $\gamma_1^D(\gamma_2^U e_2 + \gamma_3^U e_3) + e_1$.

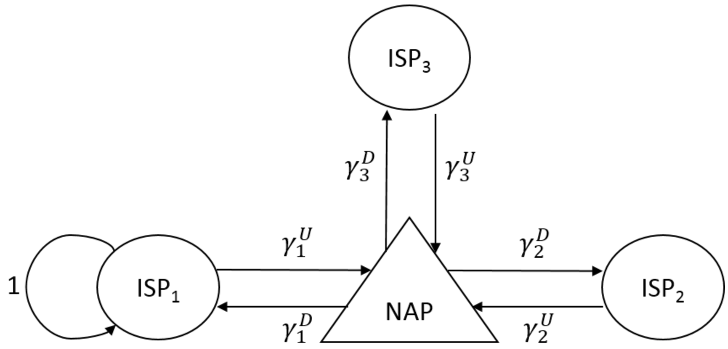


Figure 2.2 Factors determining ISPs' QoS level in a market of three ISPs

In general, after introducing the interconnection quality decisions, the demand function changes to $d_i(S) = D_i - \alpha p_i(S) + \beta \hat{y}_i^2$, where $\hat{y}_i = \gamma_i^D \sum_{j \neq i} \gamma_j^U e_j + e_i$ denotes the QoS level of ISP_i.The profit function remains to be $\pi_i(S) = p_i(S) d_i(S) - c_o e_i \sum_{j \in S} d_j - c_t d_i E_S$. For ISP_i, its uplink quality does not affect its own QoS level, but has a positive effect on other ISPs' QoS level. When ISP_i improves its uplink quality, other ISPs' end-users will have a better experience visiting resources in ISP_i's network, and thus a better perception of the interconnected network, so more users will be attracted to subscribe to these ISPs while the number of ISP_i's subscribers remains the same. The increase in the number of other ISPs' end-users raises the total data transmission cost ISP_i bears, and leads to a lower level of ISP_i's uplink quality choice. Indeed, ISP_i's equilibrium uplink quality decision is $\underline{\gamma}$, suggesting that ISPs will choose the lowest level of uplink quality when making decisions independently in order to optimize their own profit. On the other hand, ISPs will choose perfect downlink quality at equilibrium, and its pricing decision is given in Theorem 2.6.

Theorem 2.6 When ISPs make decisions independently, the equilibrium uplink quality, downlink quality and price are ${\gamma_i^U}^* = \underline{\gamma}$, ${\gamma_i^D}^* = 1$, $p_i^*(S) = \frac{D_i + \beta\left(\sum_{j \neq i} \underline{\gamma} e_j + e_i\right)^2 + \alpha c_o e_i + \alpha c_t E_S}{2\alpha}$ respectively.

Theorem 2.7 The jointly optimal uplink and downlink quality decisions and pricing decision are $\gamma_i^U = \gamma_i^D = 1$ and $p_i^*(S) = \frac{D_i + \beta E_S^2 + \alpha t E_S}{2\alpha}$ respectively.

Theorem 2.7 shows when interconnected ISPs cooperate with each other and make jointly optimized decisions, they will all set their downlink and uplink quality levels simultaneously at 1, to maximize the network externality effect and to attract as more end-users as they can. As the optimal choice of γ_i^D and γ_i^U are both 1, this problem reduces to the basic model we have discussed in section 2.3, so the optimal pricing decision is the same as in Theorem 2.2. The analysis of the three profit distribution rules also holds, and the CPA still works the best among the three allocation rules. We just have to do a slight modification to the settlement rule $\boldsymbol{s}$ to get the new settlement rule $\mathbf{s^Q}$, which can induce ISPs to make jointly optimal decisions.

Under $\mathbf{s^Q}$, the subsidy $s_i^Q(N)$ an ISP receives (pays) is:

$$s_i^Q(N) = c_o\left(e_i \sum_{j\epsilon N} d_j - d_i E_N\right) + \frac{1}{4\alpha}\left[\gamma_i^U e_i(\beta E_N - \alpha c)\sum_{j\epsilon N}\left(2D_j + \beta E_N^2 - \alpha c E_N\right) - (2D_i + \beta E_N^2 - \alpha c E_N)(\beta E_N^2 - \alpha c E_N)\right] \quad (2\text{-}10)$$

Similar with $s_i(N)$, the first part of $s_i^{\mathrm{Q}}(N)$ adjusts the data transmission cost ISP_i bears, and the second part adjusts ISP_i's profit on basis of its uplink quality, intrinsic demand potential and network size, in order to reflect ISP_i's contribution to the whole interconnected network. Under settlement rule $\boldsymbol{s^Q}$, ISP_i's profit function π_i^{Qc} will be the sum of $\pi_i(N)$ and $s_i^Q(N)$, written as:

$$\pi_i^{Qc} = p_i d_i - c d_i E_N + \frac{1}{4\alpha}\left[\gamma_i^U e_i(\beta E_N - \alpha c)\sum_{j\epsilon N}\left(2D_j + \beta E_N^2 - \alpha c E_N\right) - (2D_i + \beta E_N^2 - \alpha c E_N)(\beta E_N^2 - \alpha c E_N)\right] \quad (2\text{-}11)$$

Proposition 2.8 If settlement rule $\boldsymbol{s^Q}$ is implemented at the NAP, ISP_i's equilibrium uplink quality, downlink quality and pricing decisions coincide with the jointly optimal decisions, and its optimal profit $\pi_i^{\mathrm{Qc}*}$ is exactly the same as r_{Ci}.

As ISPs obtain higher profits which fairly reflect their network values and contributions to the network when they exchange traffic with other ISPs through NAP, they have no incentive switching to private peering. In the meantime, the proposed settlement rule $\mathbf{s^Q}$ can incentivize ISPs to choose high interconnection quality, as a high level of uplink quality can bring an ISP more profit. The efficiency of NAP will be improved by the increased number of ISPs connecting to it and the enhanced interconnection quality, and Internet users will have a better experience in terms of the availability of resources and the access speed when browsing the Internet.

However, the efficiency of settlement rule $\mathbf{s}^{\mathbf{Q}}$ is not robust, i.e., when an ISP is not fully rational, or has other considerations and make decisions deviating from optimal, $\mathbf{s}^{\mathbf{Q}}$ cannot efficiently allocate all profit to ISPs. Luckily, $\sum_{i\epsilon N} \pi_i^{\mathrm{Qc}}$ will not exceed $\sum_{i\epsilon N} \pi_i(N)$ causing profit imbalance, and we believe ISPs will converge to optimal decisions after periods of strategy adjustments.

2.5.2 Competition while Cooperation

We consider two ISPs in a specific region and relax the assumption that there is no competition between the two ISP. As in the basic model, each ISP has a certain coverage area and corresponding intrinsic demand potential, but end-users located in ISP_i's coverage area may be attracted to subscribe to ISP_j if ISP_j has a price advantage over ISP_i. To account for the price competition, we extend the demand function to be $d_i = D_i - \alpha_1 p_i + \alpha_2 p_j + \beta y_i^2,\ i,j \in \{1,2\}, i \neq j$. The composition of the demand in competitive market is similar to that of Matsubayashi and Yamada (2008) except that there is no quality competition in our model.

We derive the pricing decision of each ISP in four scenarios respectively:

1) Non-interconnection: ISPs do not interconnect with each other and decide on prices independently;

2) Private peering: ISPs interconnect with each other via private peering and decide on prices independently, and there is no payment between the two ISPs;

3) NAP peering: ISPs interconnect with each other via NAP and make pricing decisions independently according to the settlement rule $\mathbf{s}$ implemented at the NAP.

4) Cooperative peering: ISPs interconnect with each other and decide on prices cooperatively to achieve optimal total profit. This scenario serves as a benchmark.

Theorem 2.9

(a) Unique Nash equilibria exist for first three scenarios respectively. Equilibrium pricing decisions are presented as follows:

1) Non-interconnection

$$p_i^* = \frac{2\alpha_1 D_i + \alpha_2 D_j + 2\alpha_1^2 te_i + \alpha_1\alpha_2 te_j + 2\alpha_1 \beta e_i^2 + \alpha_2 \beta e_j^2}{(2\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)}$$

2) Private peering

$$p_1^* = \frac{2\alpha_1 D_1 + \alpha_2 D_2 + \alpha_2 c_o(\alpha_2-\alpha_1)(e_1-e_2)}{(2\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)} + \frac{\beta(e_1+e_2)^2 + \alpha_1 c_t(e_1+e_2) - (\alpha_2-\alpha_1)c_o e_1}{2\alpha_1-\alpha_2}$$

$$p_2^* = \frac{2\alpha_1 D_2 + \alpha_2 D_1 + 2\alpha_1 c_o(\alpha_2-\alpha_1)(e_1-e_2)}{(2\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)} + \frac{\beta(e_1+e_2)^2 + \alpha_1 c_t(e_1+e_2) - (\alpha_2-\alpha_1)c_o e_1}{2\alpha_1-\alpha_2}$$

3) NAP peering

$$p_i^* = \frac{\alpha_2 D_j + 2\alpha_1 D_i}{(2\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)} + \frac{\beta(e_1+e_2)^2 + \alpha_1 c(e_1+e_2)}{2\alpha_1-\alpha_2}$$

(b) The optimal pricing decision p_i^* under cooperative peering is $\frac{\alpha_j(D_2-D_1)}{2(\alpha_1+\alpha_2)(2\alpha_1-\alpha_2)} + \frac{D_1+\beta(e_1+e_2)^2+c(\alpha_1-\alpha_2)(e_1+e_2)}{2(\alpha_1-\alpha_2)}$.

By comparing the jointly optimal pricing decision and the optimal pricing decision under NAP peering, we find that the proposed settlement rule $\boldsymbol{s}$ fails to induce ISPs to make jointly optimal pricing decisions. However, numerical experiments (see Section 2.6.5) suggest that the total profit of two ISPs under NAP peering is close to the jointly optimal total profit and is fairly apportioned to the two ISPs.

2.5.3 Linear Network Externality

Though one of the theoretical contributions of this chapter is to suggest that the network externality should be modeled as a quadratic function of network size in order to capture the characteristic of the Internet, previous studies in interconnection field modeled the network externality as a linear term. To be consistent with these studies, we also adopt the linear network externality assumption and reformulate the demand function in basic model as $d_i(S) = D_i - \alpha p_i(S) + \beta y_i$. Under this linear network externality assumption, we derive the optimal price $p_i^*(S)$, the corresponding demand $d_i^*(S)$ and the optimal profit $\Pi^*(S)$ of ISPs in coalition S as follows:

$p_i^*(S) = \frac{D_i+\beta E_S+\alpha c E_S}{2\alpha}$, $d_i^*(S) = \frac{D_i+\beta E_S-\alpha c E_S}{2}$, and $\Pi^*(S) = \frac{\sum_{i\in S}(D_i+\beta E_S-\alpha c E_S)^2}{4\alpha}$.

Then the three profit allocation rules are:

1) Non-settlement profit allocation:

$r_{Ai} = \frac{[D_i+\beta E_N-\alpha(c_o+c_t)E_N][D_i+\beta E_N+\alpha(c_o-c_t)E_N]}{4\alpha} - \frac{c_o e_i}{2}\sum_{j\epsilon N}\left(D_j + \beta E_N - \alpha c E_N\right)$;

2) Shapley-value based profit allocation:

$r_{Bi} = \frac{1}{|N|!}\sum_{\zeta\in Z}\Delta_i(\Pi^*, P(\zeta,i))$, $\forall i \in N$, where $\Delta_i(\Pi^*, S) = \Pi^*(S \cup \{ISP_i\}) - \Pi^*(S)$, Z is the set of all $|N|!$ orderings of ISPs, and $P(\zeta, i)$ is the set of ISPs preceding ISP_i in the ordering ζ;

3) Characterized Profit Allocation:

$r_{Ci} = \frac{1}{4\alpha}\left[D_i^2 + e_i(\beta - \alpha t)\sum_{j\in N}\left(2D_j + \beta E_N - \alpha c E_N\right)\right]$.

As for the core analysis, the conclusions are similar to that of quadratic-network-externality model. Non-settlement allocation is not in the core when there exist ISPs with relatively large network size and small intrinsic demand potential, while Shapley-value based profit allocation and CPA are in the core. As for the fairness property analysis, on the other hand, we present a numerical experiment in Section 2.6.3, and results show that linear network externality is inadequate to reflect the contribution of ISPs made to the interconnected network.

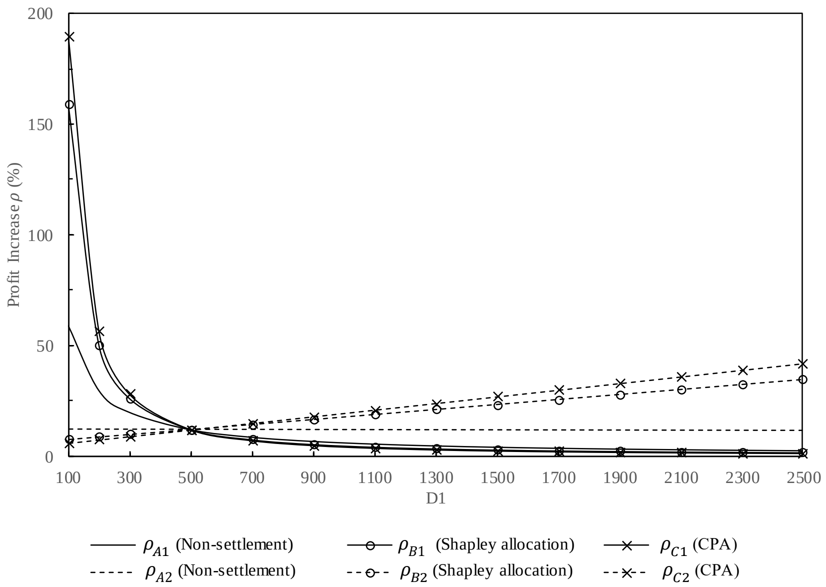
2.6 Numerical Experiments

In section 2.6.1 to section 2.6.4 numerical experiments are conducted to illustrate the effectiveness of CPA in the basic model as well as the model with linear network externality. In Section 2.6.5, we provide numerical evidence of the effectiveness of the proposed settlement rule **s** in a competitive market with two players.

2.6.1 The Benefit of Interconnection

We have learned that interconnecting with other ISPs through NAP can bring more profit. In this section, we explore how the profit increase under three allocations rules (non-settlement allocation, Shapley-value based allocation and characterized allocation) through interconnection are affected by different parameters. We use ρ_{ji} to indicate the percentage increase of profit of ISP_iunder allocation j ($j = A, B, or\ C$) compared with the non-interconnection case. According to the definition we have $\rho_{ji} = \frac{r_{ji}-\pi_i^*}{\pi_i^*} * 100\%$, where π_i^* is the optimal profit of ISP_i in the case of non-interconnection. Numerical experiments are carried out in the case of two ISPs.

First, we check how the benefit of interconnection for ISP_1 and ISP_2 are affected by ISP_1's intrinsic demand potential D_1. As shown in Figure 2.3, it is clear that interconnection can always bring profit improvement for both ISPs, no matter which allocation is implemented.



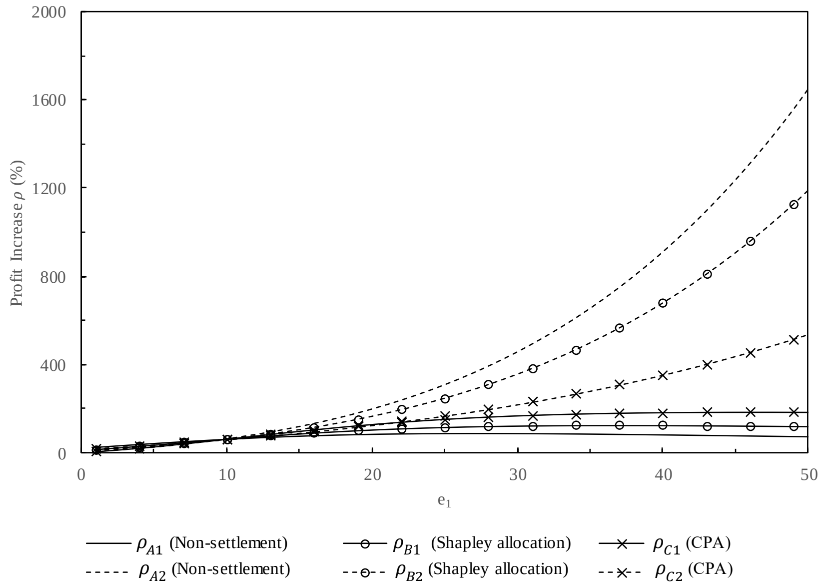
* $N = \{ISP_1, ISP_2\}, e_1 = e_2 = 10, D_2 = 500, \alpha = 0.8, \beta = 0.1, t = 0.1, c_0 = \frac{1}{3}t, c_t = \frac{2}{3}t.$

Figure 2.3. Sensitivity analysis on intrinsic demand potential

It is interesting to notice that when D_1 is small, ISP_1's profit increase is much more significant. The reason is twofold. First, the effect of network externality on profit increase measured in percentage is more evident for ISP with lower independent optimal profit π_i^*, despite that both ISPs enjoy the same amount of network externality. Second, under CPA and Shapley allocation an ISP enjoys part of the other ISP's profit for providing resources to end-users of the other network. Thus, when D_1 is small, ISP_1 can receive more compensation from ISP_2 than it has to pay to ISP_2. This rationale also explains why ρ_{B1} and ρ_{C1} is higher than ρ_{A1} when D_1 is smaller than D_2.

Conclusively, all three allocations rules favor ISP with relatively smaller intrinsic demand potential, and the gap between two ISPs' percentage increase of profit decreases following the sequence of CPA, Shapley allocation, and non-settlement allocation.

Second, we check how benefits of interconnection for ISP_1 and ISP_2 are affected by ISP_1's network size e_1. As shown in Figure 2.4, under all three allocations rules, ISP_2's percentage of profit increase climbs up high as e_1 increases, while ISP_1's percentage of profit increase is concave but not monotone increasing in e_1.



$* \ N = \{ISP_1, ISP_2\}, D_1 = D_2 = 100, \mathrm{e}_2 = 10, \alpha = 0.8, \beta = 0.1, t = 0.1, c_0 = \frac{1}{3}t, c_t = \frac{2}{3}t.$

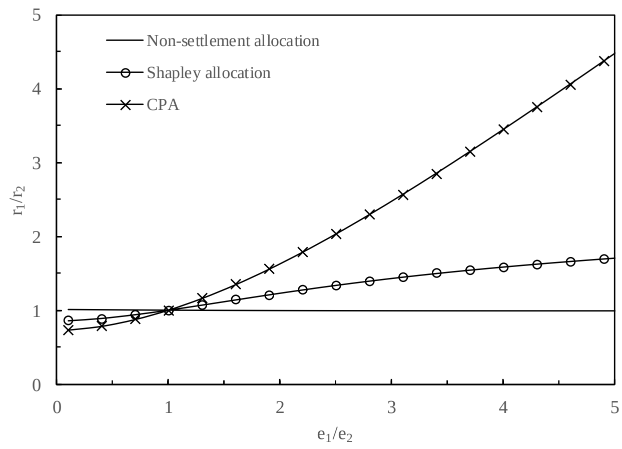
Figure 2.4. Sensitivity analysis on network size

Actually, increase in e_1 can benefit ISPs in two aspects. First, it can enlarge the total network size of the grand coalition, which in turn can generate higher total revenue by attracting more end-users. Second, larger e_1 assures ISP_1 a bigger proportion of the total profit under CPA and Shapley allocation. However, as e_1 increases further to a fairly large extent compared to ISP_2, ISP_1 can make a considerable amount of profit operating by itself without any interconnection. Therefore, when e_1 is fairly large, the profit increase of ISP_1 brought by interconnection starts to decrease in percentage, while ISP_2's percentage of profit increase becomes fairly large because π_2^* stays constant.

Another interesting observation is that, in terms of percentage of profit increase, non-settlement allocation and Shapley allocation always favor the ISP with relatively smaller network size, while CPA favors ISP with relatively larger network size except when there is too large a difference between the sizes of two ISPs' networks. This fact to some extent demonstrates that CPA can effectively compensate for an ISP's network investment.

2.6.2 Fairness of Different Allocation Rules under Quadratic Network Externality

This subsection compares the degree of fairness of these three allocation rules. In the case of two ISPs, we plot the profit ratio $(\frac{r_{j_1}}{r_{j_2}}, j = A, B, or\ C)$ curves against network size ratio $(\frac{e_1}{e_2})$ to see the degree of fairness of different allocations rules. We conduct this numerical experiment for the model with quadratic network externality (the basic model) and the model with linear network externality (described in section 2.5.3) respectively.



$* \ N = \{ISP_1, ISP_2\}, D_1 = D_2 = 100, e_2 = 10, \alpha = 0.8, \beta = 0.1, t = 0.1, c_0 = \frac{1}{3}t, c_t = \frac{2}{3}t.$

Figure 2.5 Fairness of different allocation rules under quadratic network externality

The result for the basic model is presented in Figure 2.5. The profit ratio curve for non-settlement allocation ($\frac{r_{A1}}{rA_2}$) is a near-horizontal line with a downward slope, indicating that non-settlement allocation does not reward ISPs for bringing more resources to the interconnected network. The profit ratio of CPA and Shapley allocation both increase with the network size ratio, indicating that these two allocations rules reward the ISP with larger network size more. It is clear to tell from Figure 2.5 that the slope of the curve corresponding to CPA is the closest to 1, suggesting that it is the fairest allocation among these three.

2.6.3 Fairness of Different Allocation Rules under Linear Network Externality

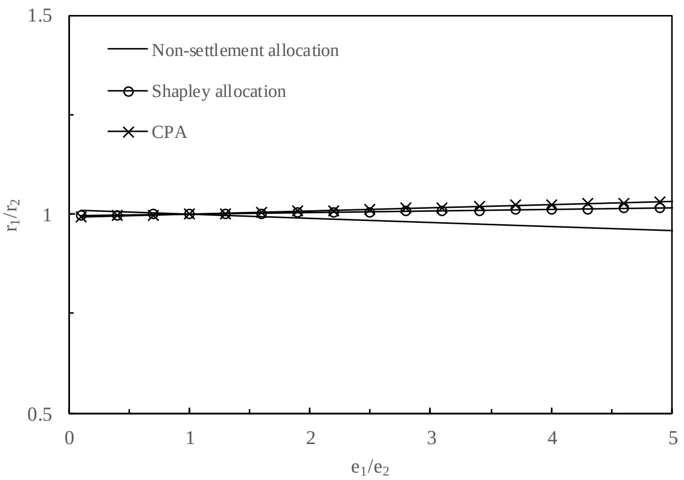


Figure 2.6 Fairness of different allocations rules under linear network externality

The same experiment with section 2.6.2 is conducted to see the performance of different allocations rules under linear network externality. In Figure 2.6, we can see that as the network size ratio of ISP_1 to ISP_2 increases, the profit ratios under all three allocations rules almost remain unchanged, with Shapley allocation and CPA showing an increasing trend and non-settlement allocation showing a decreasing trend. As the change in profit ratio is highly disproportionate to the change in network size ratio, and the difference between the profits of two ISPs is very small, we conclude the linear network externality is inadequate to reflect the network value. If we cannot properly recognize the value of network externality, it will be hard to devise any settlement rule that can appropriately reward the contributions ISPs make to the interconnection coalition, which will result in a situation where large ISPs are reluctant to peer with other ISPs.

2.6.4 Different Internet Network Structures

To check the effectiveness of the profit allocations rules under different network structures, we carry out numerical experiments in the case of three ISPs, i.e., $N = \{ISP_1, ISP_2, ISP_3\}$, where the three ISPs have the same intrinsic demand potential. We are interested in the magnitude of percentage increase in profit for each ISP under three allocations rules respectively. The results of percentage increase in profit for each ISP in various network structures are presented in Table 2.1 Percentage increase in profit under three allocations rules in different network structuresTable 2.1 .

Table 2.1 Percentage increase in profit under three allocations rules in different network structures

Network structure			Non-settlement allocation			CPA			Shapley allocation		
e_1	e_2	e_3	ISP_1	ISP_2	ISP_3	ISP_1	ISP_2	ISP_3	ISP_1	ISP_2	ISP_3
1	2	7	19.61	19.08	9.91	5.73	11.01	28.98	11.85	15.01	19.67
1	4	5	19.61	16.65	14.78	5.73	20.00	23.62	11.40	18.63	19.54
2.5	3.5	4	18.38	17.17	16.39	13.47	17.97	20.00	15.56	17.59	18.32
3.33	3.33	3.33	17.22	17.22	17.22	17.22	17.22	17.22	17.22	17.22	17.22
5	2.5	2.5	14.11	17.95	17.95	23.62	13.47	13.47	19.35	15.68	15.68
7.5	1.5	1	7.45	18.27	18.47	29.92	8.43	5.73	19.38	13.73	12.05

* $N = \{ISP_1, ISP_2, ISP_3\}, D_1 = D_2 = D_3 = 100, \alpha = 0.8, \beta = 0.1, t = 0.1, c_0 = \frac{1}{3}t, c_t = \frac{2}{3}t.$

Table 2.1 reveals that no matter in the market where three ISPs are equal in network size, or in the market where one ISP is particularly large and the others are small, or in any other network structure, the profit for all three ISPs will increase considerably under all three allocations rules. Non-settlement allocation benefits ISPs with smaller network size the most, while the other two allocations rules favor ISPs with larger network size. Furthermore, CPA allocates more profit to ISPs with relatively large network than Shapely allocation.

These results indicate that CPA appreciates the value of network the most, and gives the most reward to ISPs with larger network among three allocations rules analyzed in this chapter. Thus, if the proposed settlement rule $\mathbf{s}$ that leads to CPA instead of non-settlement is implemented at NAP, ISPs will be motivated to expand their network to pursue higher profit.

2.6.5 Competitive Market

In this subsection, we first carry out a numerical test to show that the total profit of two competing and interconnecting ISPs under proposed settlement rule $\mathbf{s}$ is close to the jointly optimal total profit, and then illustrate the fairness performance of the proposed settlement rule $\mathbf{s}$ in competitive market.

200 problem instances are generated randomly. For each problem instance, the price coefficients, network size coefficient, unit data transmission cost, the intrinsic demand potential vector **D** and the network size vector **e** are randomly assigned. For each instance, we have $\alpha_2 \leq \frac{1}{2}\alpha_1$ to limit the degree of competition. Table 2.2 summarizes the rules to generate random parameters in the numerical test.

Table 2.2 Rules used to generate random parameters

Price coefficient: α_1	$\alpha_1 \sim Uni[0.1,1]$
Price coefficient: α_2	$\alpha_2 = \rho\alpha_1, \rho \sim Uni[0.1,0.5]$
Network size coefficient: β	$\beta \sim Uni[0.1,1]$
Unit data transmission cost: t	$t \sim Uni[0, 0.1]$
Intrinsic demand potential: **D**	$D_i \sim Uni[100,2000]$
Network size: **e**	$e_i \sim Uni[5,50]$

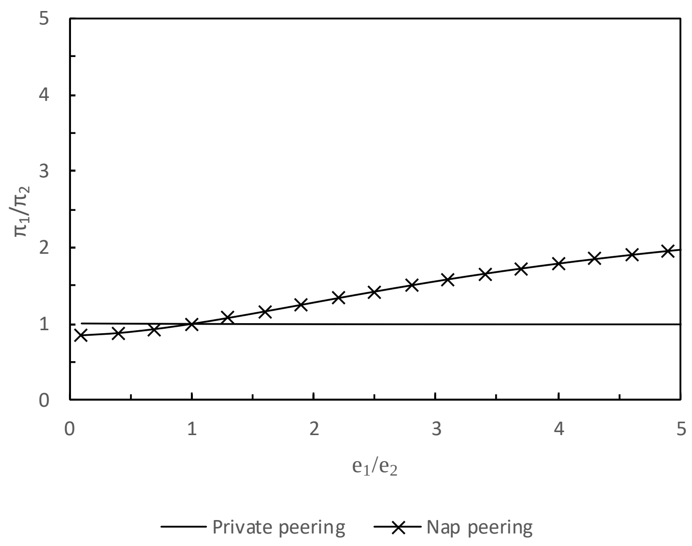
Table 2.3 gives the results of the numerical test. The "Average gap" measures the gap between the profit under settlement rule $\mathbf{s}$ and the optimal total profit on average over the 200 randomly generated instances. The "Median gap" and "80% percentile

gap” list the median and the 80% percentile result among these instances respectively. For example, if the “80% Percentile” is 7%, it indicates that 80% of the instances have a gap no larger than 7%. The “Max gap” measures the maximum gap among these instances. The results in Table 2.3 demonstrate that the proposed settlement rule $\mathbf{s}$ performs well and can lead to a total profit close to optimum.

Table 2.3 The Gap between the profit under proposed settlement rule and the optimal profit

Average gap	Median gap	80% percentile gap	Max gap
2.94%	1.95%	5.56%	10.55%

Then we illustrate the fairness performance of the proposed settlement rule $\mathbf{s}$ in competitive market. Profit ratio curves similar to that in section 2.6.2 are used to reflect the changes in profit distribution among two competing ISPs as network size ratio increases. The results in Figure 2.7 show that as network size ratio e_1/e_2 increases, the profit ratio π_1/π_2 decreases slightly when ISPs interconnect via private peering and no payments is paid between them. On the contrary, the profit ratio increases with network size ratio when ISPs interconnect via NAP and make decisions according to proposed settlement rule $\mathbf{s}$. This suggests that our proposed settlement rule can transfer part of the benefits of interconnection from ISP with smaller network size to ISP with larger network size, so it can serve the competitive market as well by fairly apportioning the total profit to ISPs.



* $N = \{ISP_1, ISP_2\}, D_1 = D_2 = 100, e_2 = 10, \alpha = 0.8, \alpha_1 = 0.6, \alpha_2 = 0.2, \beta = 0.1, c_0 = \frac{1}{30}, c_t = \frac{1}{15}$

Figure 2.7. Percentage increase in profit under private peering and NAP peering

To sum up, the numerical experiments in section 2.6 verify that the proposed CPA can most effectively increase ISPs' profit and fairly distribute the total profit among three allocations rules analyzed in this chapter, and the corresponding settlement rule **s** can encourage ISPs to exchange traffic through NAP in a wide range of settings, including markets with different market structures and competitive market.

2.7 Discussion and Conclusion

ISPs in a specific Internet market exchanging data through NAP can give rise to the welfare of the whole society. For the Internet users, they can visit more resources with shorter delay. For ISPs, as long as there is a rational settlement rule, exchanging data with other ISPs through NAP will also increase their profit. If there is no such settlement rule, ISPs' profit could be hindered, and would rather choose to operate alone or form small coalitions with some of the other ISPs. In our study, we have designed such a settlement rule **s**. For the whole society, if all the ISPs access to the NAP, the size of the total network will increase remarkably, and the value of the Internet will be enhanced.

This chapter analyzes the profit allocation among ISPs in a cooperative game framework, and proposes a Characterized Profit Allocation that has a much more brief and comprehensible formulation than Shapley-value based profit allocation, and can increase all ISPs' profit in all circumstances. Implementing the settlement rule $\mathbf{s}$ that induces ISPs to make jointly optimal pricing decisions and leads to Characterized Profit Allocation, the aim of encouraging ISPs to interconnect with each other through NAP can be achieved, and the network response speed and data exchange quality of the Internet can be greatly improved. We also show that with a little adjustment, the settlement rule $\mathbf{s^Q}$ can lead to jointly optimal pricing and interconnection quality decisions when taking into account the asymmetric incentives of ISPs to provide interconnection quality. Our settlement rule $\mathbf{s}$ works well for a competitive market with two ISPs as well.

The theoretical contribution of this chapter is to introduce quadratic network externality and show that linear network externality is inappropriate in Internet interconnection context, and the CPA and the corresponding settlement rule $\mathbf{s}$ we propose is instructive for NAPs in operation across the world. For future research, as Internet users are becoming important resources themselves as we are entering an era of "We media", we suggest to include installed customer base in the demand function. Second, studies that extend our two-player competition to a general case should be fruitful. Practically, measurement items for network size also need to be specified to facilitate the enforcement of the settlement rule $\mathbf{s}$.

2.8 Proofs in Chapter 2

2.8.1 Proof of Proposition 2.3

As $\sum_{i\in N} r_{Ai} = \sum_{i\in N}\left\{\frac{[D_i+\beta E_N^2-\alpha(c_o+c_t)E_N][D_i+\beta E_N^2+\alpha(c_o-c_t)E_N]}{4\alpha} - \frac{c_o e_i}{2}\sum_{j\in N}\left(D_j + \beta E_N^2 - \alpha c E_N\right)\right\} = \frac{\sum_{i\in N}\left(D_i+\beta E_N^2-\alpha c E_N\right)^2}{4\alpha} = \Pi^*(N)$, r_A satisfies the efficiency principle.

Then we rewrite the total allocation a set of ISPs can get from the grand coalition as $\sum_{i\in S} r_{Ai} = \frac{\sum_{i\in S}\left\{\left(D_i+\beta E_N^2-\alpha c E_N\right)^2+2\alpha c_o\left[E_N\left(D_i+\beta E_N^2-\alpha c E_N\right)-e_i\sum_{j\epsilon N}\left(D_j+\beta E_N^2-\alpha c E_N\right)\right]\right\}}{4\alpha}$. By observing $\Pi^*(S) = \frac{\sum_{i\in S}\left(D_i+\beta E_S^2-\alpha c E_S\right)^2}{4\alpha}$ and $\sum_{i\in S} r_{Ai}$, we can conclude the sign of $\sum_{i\in S} r_{Ai} - \Pi^*(S)$ is not definite, and it depends on the value of e_i and D_i of ISPs in set S. Indeed, when ISPs have large e_i and small D_i, the profit allocation they can get from the grand coalition is less than what they can earn by deviating from the grand coalition and forming associated network by themselves. For example, when there are two ISPs in the market, and the parameters are as follows: $N = \{ISP_1, ISP_2\}, (e_1, e_2) = (10, 1), (D_1, D_2) = (100, 1000), \alpha = 0.8, \beta = 0.1, c = 0.1, c_0 = \frac{1}{3}c, c_t = \frac{2}{3}c$. In this setting, ISP_1 's profit is 3726 when operating independently and 3699 when interconnecting with ISP_2 through NAP, which is 27 less than the independent profit.

2.8.2 Proof of Proposition 2.4

On one hand, characterized profit allocation can be written as $r_{Ci} = \frac{1}{4\alpha}\left[D_i^2 + e_i\sum_{j\in N}(\beta E_N - \alpha c)\left(2D_j + \beta E_N^2 - \alpha c E_N\right)\right]$. On the other hand, grand coalition's optimal total profit is $\Pi^*(N) = \frac{\sum_{i\in N}\left(D_i+\beta E_N^2-\alpha t E_N\right)^2}{4\alpha}$, which can be rewritten as $\frac{1}{4\alpha}\left[\sum_{ISP_i\in N} D_i^2 + \sum_{ISP_i\in N} E_N(\beta E_N - \alpha t)(2D_i + \beta E_N^2 - \alpha t E_N)\right]$. It is easy to see that $\sum_{ISP_i\in N} r_{Ci} = \Pi^*(N)$, so efficiency principle is satisfied.

Denote the smallest network size as e_{min}. According to our assumption that $2\beta e_i - \alpha c > 0$ for any $ISP_i \in N$, as long as $n \geq 2$, we have:

$$\sum_{i\in S} r_{Ci} = \frac{1}{4\alpha}\left[\sum_{i\in S} D_i^2 + E_S\sum_{j\in N}(\beta E_N - \alpha c)\left(2D_j + \beta E_N^2 - \alpha c E_N\right)\right]$$

$$> \frac{1}{4\alpha}\left[\sum_{i\in S} D_i^2 + E_S\sum_{j\in S}(\beta E_S - \alpha c)\left(2D_j + \beta E_S^2 - \alpha c E_S\right)\right] = \Pi^*(S)$$

Thus, coalition rationality principle that $\sum_{i\in S} r_{Ci} \geq \Pi^*(S)$ is satisfied for any coalition $S \subseteq N$. Together with efficiency principle, we can conclude that r_C is in the core of the game.

As r_{Ci} is an increasing function of e_i, the ISP with larger network size can share a larger portion of profit under this allocation rule. Therefore, the CPA also preserves the fairness property.

2.8.3 Proof of Proposition 2.5

As $\frac{\partial^2 \pi_i^c}{\partial p_i^2} = -2a < 0$, π_i^c is concave in p_i. Solving the first order condition $\frac{\partial \pi_i^c}{\partial p_i} = D_i - 2ap_i + \beta E_N^2 + acE_N = 0$ yields ISP_i 's optimal pricing $p_i^* = \frac{D_i + \beta E_N^2 + acE_N}{2a}$, which is equal to the jointly optimal pricing decision in Theorem 2.2. Plugging $p_i^* = \frac{D_i + \beta E_N^2 + acE_N}{2a}$ into π_i^c, we get ISP_i's profit as $\frac{1}{4\alpha}\left[D_i^2 + e_i(\beta E_N - \alpha c)\sum_{j\in N}\left(2D_j + \beta E_N^2 - \alpha c E_N\right)\right]$, which is exactly the same with r_{Ci}, so proposition 2.5 is proved.

2.8.4 Proof of Theorem 2.6

As $\frac{\partial \pi_i(S)}{\partial \gamma_i^{\mathrm{U}}} = -\mathrm{c_o} e_i^2 \sum_{j\epsilon S, j\neq i} 2\beta \hat{y}_j < 0$, ISPs will set the equilibrium uplink quality $\gamma_i^{\mathrm{U}*}$ at lower bound $\underline{\gamma}$.

Assume $p_i(S) \geq c_o e_i + c_t E_S$ for each ISP_i.

As $\frac{\partial \pi_i(S)}{\partial \gamma_i^{\mathrm{D}}} = \left(2\beta \underline{\gamma} \hat{y}_i \sum_{j\epsilon S, j\neq i} e_j\right)[p_i(S) - c_o e_i + c_t E_S] \geq 0$, the equilibrium downlink quality γ_i^{D*} is the upper bound 1.

And we have $\frac{\partial \pi_i(S)}{\partial p_i(S)} = D_i - 2\alpha p_i(S) + \beta \hat{y}_i^2 + \alpha c_o e_i + \alpha c_t E_S$, $\frac{\partial^2 \pi_i(S)}{\partial p_i^2(S)} = -2\alpha$, so we use first order condition to obtain the equilibrium price $p_i^*(S)$. We can calculate that $p_i^*(S)$ is $\frac{D_i + \beta\left(\sum_{j\neq i} \underline{\gamma} e_j + e_i\right)^2 + \alpha c_o e_i + \alpha c_t E_S}{2\alpha}$, and the corresponding equilibrium demand $d_i^*(S)$ is $\frac{D_i + \beta\left(\sum_{j\neq i} \underline{\gamma} e_j + e_i\right)^2 - (\alpha c_o e_i + \alpha c_t E_S)}{2}$. Because demand should be no less than zero, $D_i + \beta\left(\sum_{j\neq i} \underline{\gamma} e_j + e_i\right)^2 - (\alpha c_o e_i + \alpha c_t E_S) > 0$. So $p_i^*(S) = \frac{D_i + \beta\left(\sum_{j\neq i} \underline{\gamma} e_j + e_i\right)^2 + \alpha c_o e_i + \alpha c_t E_S}{2\alpha} > c_o e_i + c_t E_S$, which is in accordance with our assumption $p_i(S) \geq c_o e_i + c_t E_S$.

2.8.5 Proof of Theorem 2.7

Assume $p_i(S) \geq cE_S$ for each ISP_i.

We have $p_i(S) \in \left[cE_S, \frac{D_i+\beta E_S^2}{\alpha}\right]$ and $\gamma_i^{\mathrm{U}}, \gamma_i^{\mathrm{D}} \in \left[\underline{\gamma}, 1\right]$, so the feasible set $(\boldsymbol{\gamma_i^U}, \boldsymbol{\gamma_i^D}, \boldsymbol{p}) \in \mathcal{B} \subset \mathrm{R}^{\mathrm{S}} \times R^S \times R^S$ is compact. Furthermore, as $\Pi(S) = \sum_{i\in S}[p_i(S)d_i(S) - td_i(S)E_S] = \sum_{i\in S}\left\{(p_i(S) - cE_S)\left[D_i - \alpha p_i(S) + \beta\left(\gamma_i^D \sum_{j\neq i}\gamma_j^U e_j + e_i\right)^2\right]\right\}$ is continuous on $\mathcal{B}$, we can conclude that a maximum exists on $\mathcal{B}$ with an appeal to the Weierstrass theorem. As $\frac{\partial\Pi(S)}{\partial\gamma_i^U} = 2\beta e_i \sum_{j\neq i}\gamma_j^D\left(\gamma_j^D \sum_{k\neq j}\gamma_k^U e_k + e_j\right)\left[p_j(S) - cE_S\right] \geq 0$ and $\frac{\partial\Pi(S)}{\partial\gamma_i^U} = 2\beta\left(\gamma_i^D \sum_{j\neq i}\gamma_j^U e_j + e_i\right)\sum_{j\neq i}\gamma_j^U e_j\left[p_j(S) - cE_S\right] \geq 0$ by assumption, $\Pi(S)$ is increasing in γ_i^U and γ_i^D, so the maximum is achieved at the boundary where $\gamma_i^U = \gamma_i^D = 1$ for $i\epsilon S$.

Substituting $\boldsymbol{\gamma^U} = \boldsymbol{\gamma^D} = \mathbf{1}$ to $\Pi(S)$, the maximization problem of $\Pi(S)$ is exactly the same as in section 2.3, so the optimal price is $\frac{D_i+\beta E_S^2+\alpha c E_S}{2\alpha}$. As there are at least two ISPs in a coalition, $p_i^*(S) = \frac{D_i+\beta E_S^2+\alpha c E_S}{2\alpha} \geq \frac{D_i+(2\beta e_{min}+\alpha c)E_S}{2\alpha} > \frac{D_i+2\alpha c E_S}{2\alpha} > cE_S$, which is in accordance with our assumption $p_i(S) \geq cE_S$.

2.8.6 Proof of Proposition 2.8

Under settlement rule $\boldsymbol{s^Q}$, ISP_i's profit function becomes $\pi_i^{Qc} = (p_i - cE_N)\left\{D_i - \alpha p_i + \beta\left[\gamma_i^D \sum_{j\neq i}\gamma_j^U e_j + e_i\right]^2\right\} + \frac{1}{4\alpha}\left[\gamma_i^U e_i(\beta E_N - \alpha c)\sum_{j\epsilon N}\left(2D_j + \beta E_N^2 - \alpha c E_N\right) - (2D_i + \beta E_N^2 - \alpha c E_N)(\beta E_N^2 - \alpha c E_N)\right]$ after plugging in the demand function. It is easy to see that π_i^{Qc} increases with both γ_i^U and γ_i^D, so $\gamma_i^{U*} = \gamma_i^{D*} = 1$.

Substituting $\gamma_i^{U*} = \gamma_i^{D*} = 1$ into π_i^{Qc}, we get $\pi_i^{\mathrm{Qc}} = (p_i - cE_N)(D_i - \alpha p_i + \beta E_N^2)$. Solving the first order condition of π_i^{Qc} gives the optimal pricing decision $p_i^* = \frac{D_i+\beta E_N^2+\alpha t E_N}{2\alpha}$, which coincides with the jointly optimal pricing decision in Theorem 2.7.

2.8.7 Proof of Theorem 2.9

For part (a), by simultaneously solving the first order condition of each ISP's profit function in each scenario, we can derive the optimal pricing decision for each ISP. For part (b), as the Hessian matrix of the total profit function $\Pi = d_1[p_1 - c(e_1 + e_2)] + d_2[p_2 - c(e_1 + e_2)]$ is negative definite, Π reaches a global maximum at the critical point, which is solved as p_i^* in part (b).

3 Impact of Sponsored Data on Internet Service Provider, Content Provider and Social Welfare

3.1 Introduction

In an attempt to explore smart data pricing for the development of new revenue sources, numerous Internet service providers (ISPs) have introduced sponsored data service. Sponsored data services allow content providers (CPs) to subsidize their users' data consumption by paying ISPs such that their content does not count toward their users' data caps. When users do not have to pay for the data usage generated while visiting the content of the sponsoring CP, this practice is also termed as zero-rating. In January 2014, AT&T was the first mobile operator in the United States to launch a sponsored data service, which was named "Sponsored Data." This service enables users to visit any content with a "Sponsored Data" icon without incurring data charges. T-Mobile's "Binge On" program has subsequently emerged as the most popular sponsored data service. This service allows users unlimited video streaming from several CPs, such as Netflix, YouTube, Amazon, and HBO, without affecting their data plan. Although AT&T charges related CPs for the data usage of their users, T-Mobile does not charge CPs and instead requires them to meet certain technical conditions. Facebook, a CP, offers Free Basics, a mobile app that claims to provide affordable Internet access to people in developed markets. With this app, users who otherwise cannot afford Internet access can now browse a range of selected websites and Internet services, including Facebook itself. Free Basics now serves 50 million people in 65 countries. Among these users, more than 19 million accessed the Internet for the first time in February 2016[1]. Sponsored data services are experiencing growth in the global market (Strategy Analytics, 2015). In China, three major mobile operators, China

[1] https://www.cbsnews.com/news/facebook-mark-zuckerberg-presses-on-with-global-Internet-goal-free-basics/

Unicom, China Mobile, and China Telecom, have cooperated with different CPs and offered end users with various wireless cell phone plans that allow the use of specific apps without counting toward the monthly data cap. For example, China Unicom users who subscribe to the "DaWangKa" plan can use apps from the Tencent group without any data charge.

Sponsored data has provoked considerable controversy despite its apparent benefit to each party. Critics assert that sponsored data violates net neutrality rules and has undesirable effects. One concern is that sponsored data services can prevent users from accessing small CPs that cannot afford to subsidize data usage and thus impair competition in the content market. Another concern is that sponsored data services may induce the ISP to set artificially low data caps to increase their desirability as a consumer attraction provided by CPs; this approach may de facto limit consumer choice to only sponsoring CPs (Kimball, 2015). As a result, sponsored data services have been banned in several countries, such as India and Egypt. Meanwhile, under certain circumstances, sponsored data services are compatible with net neutrality in accordance with the guidelines on the implementation of European net neutrality rules issued by Body of European Regulators for Electronic Communications. In the United States, the Federal Communications Commission (FCC) closed its investigation into Verizon, AT&T, and T-Mobile's zero-rating offers in February 2017. FCC's Chairman Ajit Pai concluded, "These free-data plans have proven to be popular among consumers, particularly low-income Americans, and have enhanced competition in the wireless marketplace."[2]

Given that a consensus on the legitimacy of sponsored data services has not been reached and data showing the impact of this innovative business model remain inadequate, policymakers need a comprehensive understanding of sponsored data to issue applicable regulations. Without touching upon the issue of whether sponsored data

[2]Chairman Pai statement on free data program, https://apps.fcc.gov/edocs_public/attachmatch/DOC-343345A1.pdf

violates net neutrality, in this study, we employ an economic framework to investigate the impact of sponsored data on each participant of the telecom service supply chain. Specifically, we focus on the impact of sponsored data on the vertical competition between ISP and CP, consumer surplus, and social welfare. We aim to determine if sponsored data empowers ISP with excessive monopoly power and hurt other parties of the telecom service supply chain. We also identify possible situations wherein sponsored data is beneficial to supply chain participants other than the ISP. Various scenarios are examined to understand these problems thoroughly. First, we analyze a simple telecom service supply chain that is composed of one ISP and one subscription CP that profits from the subscription fees collected from its users. By comparing the cases of sponsored data service and conventional practice, we show that sponsored data services always benefit the ISP but hinder the CP from profiting. Furthermore, the whole supply chain, as well as consumers, can benefit from sponsored data services if the market price for the service is high and the cost of the CP is low, i.e., when the CP has a large profit margin. Thus, in contrast to the claims of the proponents of sponsored data, sponsored data services do not necessarily improve social welfare, which defined as the sum of supply chain profit and consumer surplus. To enhance our understanding of the impact of sponsored data, we investigate three other common scenarios: (1) CP's cost is its private information; (2) The quality of service is a decision of CP; (3) Instead of a subscription CP, the CP is a platform CP, which does not charge its users but generates value by providing various types of platforms. Given that the ISP only elects to provide a sponsored data contract when beneficial, its profit always increases in all three scenarios. In the first two scenarios, a CP with high profit margin can benefit from sponsored data services, and social welfare is improved in most cases. In the third scenario, social welfare benefits from sponsored data services, and the profit of the platform CP increases when its profitability is moderate and decreases otherwise.

Our analysis of the vertical competition between the ISP and the CP shows that sponsored data services indeed enable the ISP to extract revenue from the CP and thus increase its revenue. Although a portion of the revenue from extra consumers who are attracted by data subsidization is transferred to the ISP, a CP with high profitability can still benefit from sponsoring its consumers if its cost information is unknown to the ISP or if it simultaneously makes quality and subsidization decisions. Few studies have investigated the impact of sponsored data on the revenue of CPs. Wong et al. (2015) concluded that CPs always benefit from subsidizing their users, whereas Ma (2014) claimed that sponsored data programs only benefit CPs with high profitability. Our results coincide with those of Ma (2014). Moreover, we provide further insights into circumstances wherein CPs can profit from participating in sponsored data programs. We further demonstrate that in line with the results of Xiong et al. (2018), who showed that network effects enhance the benefits of sponsored data, platform CPs derive more benefit from sponsored data services than subscription CPs. Previous results indicated that sponsored data services positively affect social welfare (Ma, 2014; Wong et al., 2015; Zhang et al., 2015a, Andrews et al., 2013). However, our results suggest that social welfare is enhanced when a platform CP participates in sponsored data services, and the result is mixed when the sponsoring CP is a subscription CP, depending on the profitability of the CP. Similarly, Kies (2017) and Somogyi (2017) revealed that sponsored data services exert mixed effects on social welfare by accounting for the network capacity constraint of the ISP.

The remainder of this chapter is organized as follows. A review of related literature is provided in Section 3.2. The basic model used in this chapter and its analysis are presented in section 3.3. Studies on two extended models are described in section 3.4.1 and 3.4.2, and an investigation on a telecom service supply chain with a platform CP is discussed in section 3.4.3. The main results and implications of this study and future research directions are given in section 3.5.

3.2 Related literature

Sponsored data is an emerging innovative business model in the mobile broadband industry and has attracted considerable attention from the industry and academia. A large body of literature (Frieden, 2016; Layton et al., 2015) has been generated by the heated debate on whether sponsored data services should be regulated in accordance with the net neutrality principle and provides a discussion of the pros and cons of sponsored data services based on observation or logical deduction. Critics (Kimball, 2015; Schewick, 2015) claimed that sponsored data services impede competition and innovation in the CP market by prioritizing the content of participating CPs and hindering startups from accessing Internet users. Kimball (2015) argued that sponsored data services are meaningless without data capping, which by itself is an unfair restriction on Internet usage. Combined data capping and data-cap exemption practices, such as sponsored data, serve as a tool for mobile carriers to extract revenue from data growth while adversely affecting competition among content providers. Howell and Layton (2016) held a tentative attitude toward the issue of sponsored data. By carefully examining the potential trade-offs in the Internet ecosystem, they formulated five questions to guide the decision of regulators to regulate or ban zero-rating practices in certain cases. Eisenach (2015) and Rogerson (2016) explicitly supported the practices of data capping and zero-rating. In contrast to the opponents of sponsored data, Rogerson believed that regulations against zero-rating impede competition and innovation because they deprive firms of the ability to compete with others with innovative practices. For a recent review of the approaches several countries have taken to regulate sponsored data, refer to Yoo (2017).

An emerging stream of literature has employed theoretical models to analyze interactions among ISPs, CPs, and users under sponsored data. Xiong et al. (2020) model the interactions among ISP, CP and users as a hierarchical Stackelberg game, and investigate the equilibrium strategies in three different scenarios, i.e., sequential competition, simultaneous competition and cooperation. Several studies have focused on the effects of sponsored data services on competition among CPs (Garmani et al., 2019, 2020; Vyavahare et al., 2019). Cho et al. (2016) studied the network management strategy of a monopolistic ISP when faced with two CPs that are differentiated by their profit margins. The ISP will allow only the competitive CP to subsidize when competition is fierce and its rival's profitability is relatively low. This strategy may drive the low-profitability CP out of the market and deviate from the social optimum. Joewong et al. (2015) showed that sponsored data, despite its positive effect on the benefits of all parties (ISP, CPs, and consumers), favors CPs with large profit margins. However, the results of their simulations revealed that when faced with heterogeneous consumers, which is often the case in reality, the profits of the CPs can be evenly distributed. Zhang and Wang (2014) examined the subsidization strategy of CPs and the competition between two CPs over the short and long run. They reported that if the difference between the numbers of users of the two CPs is substantial, then the small CP will opt for sponsored data and will benefit from zero-rating. By contrast, the market share of the large CP will increase and enlarge the gap between the two CPs in the long run. Zhao et al. (2020) considers users' demand for content variety and show that CP's benefits of subsidizing its users will be greatly reduced if users prioritize content variety more over price.

Another stream of the related literature has examined the impact of sponsored data on network capacity investment and social welfare. Zhang et al. (2015a) stated that when the ISP has sufficient capacity to deliver all contents at best quality of service (QoS), sponsored data services will prevent the ISP from enlarging data caps and diminish the ISP's incentives to invest in service-level improvement. Subsequently, these effects greatly hurt the CPs' and consumers' interests in the long run and may call for regulatory intervention. When capacity is insufficient, the ISP prefers a small data cap to extract revenue from the CPs that participate in sponsored data programs. Ma's (2014) analysis suggested that data subsidization positively affects the mobile network ecosystem. He concluded that sponsored data programs improve the utilization of network capacity and enhance the revenue of ISPs. These effects, in turn, strengthen the ISP's investment incentives. Certain CPs whose users are price insensitive or have low profitability may suffer from a decrease in throughput. This phenomenon, however, is not mainly caused by data subsidization but by high access price. Thus, access price is the only component that requires regulation if insufficient competition exists in the ISP market. Similarly, Jullien and Zantman (2018) suggested that sponsored data can improve social welfare if subjected to regulations to a certain extent, e.g., a price cap. Banning the innovative practice of charging CPs will yield suboptimal outcomes. Somogyi (2017) characterized conditions for the ISP to render different sponsoring decisions and suggested that sponsored data has mixed effects on social welfare. Jeitschko et al. (2017) examined incentives for ISPs to offer sponsored data regardless of monetary transfer. By considering the differentiation between the services of two CPs, they characterized the equilibrium in each scenario and concluded that sponsored data is always welfare-enhancing when monetary transfer occurs. Hanawal et al. (2018) studied the investment incentive of CPs in a service capacity. The QoS for users can degrade when sponsored data is allowed, and the profit of the CP with low QoS level can increase by sponsoring additional data. Kies's (2017) study is most closely related to the present work. He identified investment and choice distortions as two welfare-distorting effects that prevent sponsored data from achieving social welfare maximization. He concluded that social welfare decreases in most cases if sponsored data is implemented, except when the network costs and the profitability of CP are sufficiently high. Few studies have examined the vertical competition between ISPs and CPs. Zhang et al. (2015a) showed that the ISP's strategy to optimize its profit always works against the CP's benefit. Andrews et al. (2013) studied a specific form of contract to implement sponsored data and showed that the ISP can achieve an optimal total profit and split it with the CP at any ratio by adjusting contract parameters.

This work is loosely related to the stream of literature on the effect of paid prioritization, a practice that allows CPs to pay ISPs to prioritize the transmission of their data packets over the data packets of other CPs. Paid prioritization is similar to sponsored data but clearly violates the net neutrality principle. Many studies have investigated this phenomenon from various perspectives, including its effect on the profit of each party (Guo et al., 2017), the market innovation of CPs (Guo et al., 2012; Guo & Easley, 2016), and the investment incentive of the ISP network (Cheng et al., 2011; Ma et al., 2017; Pil Choi & Kim, 2010). Guo et al. (2010) examined the effect of priority delivery in the presence of vertical integration. Although priority is an extensively investigated issue, the above topics warrant exploration in the setting of sponsored data.

The few existing studies that have explicitly shown the impact of sponsored data on each party have provided mixed results. Previous analyses are largely restricted to their specific model set-ups, and the inconsistent results on the impact of sponsored data from almost every perspective are unsurprising. By considering a simple telecom service supply chain with one ISP and one CP in some specific but common scenarios, we provide a thorough understanding of the impact of sponsored data.

3.3 The Basic Model

We consider a two-tier telecom service supply chain that consists of an ISP and a CP. We denote the potential number of users in the market as a. Users who want to access to certain Internet content must pay the CP for the content and the ISP for the data usage generated while browsing the CP's content. When sponsored data plans are implemented, the CP can choose to subsidize a certain amount of its users' data usage by paying the ISP, such that users only pay part of the data usage cost. When the CP subsidizes all data usage, users only pay the CP for the content. Figure 3.1 illustrates the cash flow in the telecom service supply chain.

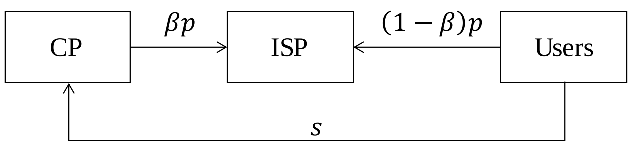


Figure 3.1 Cash flow in the telecom service supply chain

A two-stage Stackelberg game is formulated to model the decision-making process, and the sequence of events is as follows. In stage 1, the ISP decides whether to offer sponsored data plans and then sets the access price. Upon observing the access price, CP decides the proportion of data to sponsor or to do nothing if a sponsored data contract is not offered in stage 2.

Market demand is realized as access price and subsidization plans are revealed to potential users. For simplicity and analytical tractability, users are assumed to consume a certain amount of data packets per month. Thus, each user is charged a flat rate of p, which is the access price decided by the ISP in stage 1. Users pay the CP a subscription fee s to access its content and services. We take s as exogenously given because fierce competition and competitive pricing exist in the content market. For example, the lowest monthly charge of major video-streaming service providers (Netflix, Hulu, and Amazon Video) in the United States is approximately $8/month and that of major video-streaming service providers in China is approximately 15 yuan/month. s is observable by all parties and represents the market price of a type of service. However, several CPs adopt an alternative business model in which they do not directly collect fees from their users. Instead, they benefit from ad revenues that are dependent on the usefulness of their service platform. Given that a different revenue model may lead to the CP's different subsidization decision, we investigate this kind of revenue model and provide the results of our investigation in section 4.3.

The realized demand depends on access price and subscription fee. A linear demand structure $D = a - b[(1-\beta)p + s]$ is adopted, where b is a positive coefficient that denotes the responsiveness of demand to price variation, and $a - bs > 0$ is assumed to guarantee a positive demand. The linear demand assumption is general, and our demand model is equivalent to models in previous studies (Cho et al., 2016; Guo et al., 2012, 2016), wherein consumer heterogeneity was modeled by the distribution of the valuation of Internet services. $\beta \in [0,1]$ is the CP's subsidization decision, with $\beta = 0$ indicating that the CP does not subsidize and $\beta = 1$ indicating that the CP subsidizes all data traffic its users generate. The CP's revenue comes from its collected subscription fees, and costs include unit cost m to serve a user and subsidization cost $\beta \cdot p$. m is assumed as a constant distributed on $(0, s)$ and a small m corresponds to high profit margin and vice versa. To conclude, the CP's profit π_{CP} can be formulated as $(s - m - \beta p)D$. The ISP's profit is simply the revenue from data charges, i.e., $\pi_{ISP} = p \cdot D$, and its marginal cost is normalized to zero. Finally, the optimization problem in each stage can be characterized as follows:

Stage 1: $\max_{p} \pi_{ISP} = p \cdot D, \text{s.t.} p > 0,\ \pi_{CP} \geq 0$ (3-1)

Stage 2: $\max_{\beta} \pi_{CP} = (s - m - \beta p)D, \text{s.t.} 0 \leq \beta \leq 1$ (3-2)

Table 3.1 provides a list of the notations used in this chapter.

Table 3.1 List of notations

Notation	Description
p	flat-rate price of the ISP for Internet access service
s	market price of the CP's service
m	unit cost of service provision by the CP
$f(m)$	probability density function of the CP's cost
a	potential market size
b	responsiveness of demand to price
c	responsiveness of demand to content quality
ϵ	cost parameter of the CP's quality investment
D	realized demand of the CP and ISP
β	subsidization decision of the CP
q	quality decision of the CP
π_{ISP}, π_{CP}	profit of the ISP and profit of the CP, respectively
ϕ	magnitude of the network externality of the platform CP
r	profitability of the platform CP

3.3.1 Impact of sponsored data with complete information

We first examine the equilibrium outcome when the ISP does not offer a sponsored data contract to the CP. We refer to this case as the benchmark case. In the benchmark case, the ISP sets the access price, the CP has no decision to make, and the consumers decide whether to purchase the service. Lemma 3.1 summarizes the equilibrium outcome in the benchmark case.

Lemma 3.1 When a sponsored data contract is not offered, the optimal access price set by the ISP is $p^0 = \frac{a-bs}{2b}$; the corresponding realized demand is $D^0 = \frac{a-bs}{2}$; and the optimal profits of the ISP and CP are $\pi_{ISP}^0 = \frac{(a-bs)^2}{4b}$ and $\pi_{CP}^0 = \frac{(s-m)(a-bs)}{2}$, respectively.

We further examine the case when a sponsored data contract is offered. We solve the Stackelberg game backwards and summarize the results in Lemma 3.2.

Lemma 3.2 The equilibrium subsidization proportion and access price when a sponsored data contract is offered are stated in the following table:

Table 3.2 Equilibrium outcomes under sponsored data in a supply chain with complete information

m	β^*	p^*	D^*
$\frac{a-\sqrt{2}(a-bs)}{3b} \le m < s$	0	$\frac{a-bs}{2b}$	$\frac{a-bs}{2}$
$\frac{4bs-3a}{b} \le m < \frac{a-\sqrt{2}(a-bs)}{3b}$	$\frac{4bs-3bm-a}{2(a-bm)}$	$\frac{a-bm}{2b}$	$\frac{a-bm}{4}$
$0 < m < \frac{4bs-3a}{b}$	1	$\frac{2bs-a-bm}{b}$	$a-bs$

As shown in the first column of Table 3.2, when $s \le \frac{(2-\sqrt{2})a}{2b}$, the thresholds $\frac{a-\sqrt{2}(a-bs)}{3b}$ and $\frac{4bs-3a}{b}$ become nonpositive. This change suggests that for a service with relatively low market price, the CP is incapable of subsidizing users' data cost. When $\frac{(2-\sqrt{2})a}{2b} < s \le \frac{3a}{4b}$, the threshold $\frac{4bs-3a}{b}$ remains nonpositive. Therefore, for a service with moderate market price, the CP is never capable of fully subsidizing its users' data cost. When $s > \frac{3a}{4b}$, all three equilibrium results are possible: When the CP's unit cost of serving a customer is high, it will elect not to sponsor, and the equilibrium in this case is equivalent to that in the benchmark case. When the CP's cost is moderate, it will partially subsidize its users' data traffic. The CP will pay all data costs for its users when its cost is lower than $\frac{4bs-3a}{b}$. In summary, given that the CP's opportunity cost of losing a customer increases with its profit margin, the CP with high profit margin has strong subsidization incentive.

To analyze the impact of sponsored data plans on the ISP, the CP, and users, we calculate the corresponding profits of the ISP and CP, as well as the realized demand for each case. The results for these cases are shown in Table 3.2 and are compared with those for the benchmark case. Consumer surplus can be calculated from the realized demand and the demand curve through simple algebra. Given that high demand is equivalent to high consumer surplus, we use the realized demand as a proxy to describe consumer surplus. Our findings are summarized in Proposition 3.1.

Proposition 3.1 (Complete information) The ISP is always willing to offer a sponsored data contract. When $s < \frac{(2-\sqrt{2})a}{2b}$ or $\frac{a-\sqrt{2}(a-bs)}{b} \leq m < s$, the CP's optimal strategy is not to sponsor ($\beta^* = 0$). Thus, this case is equivalent to the benchmark case wherein a sponsored data contract is not offered. When $m < \frac{a-\sqrt{2}(a-bs)}{b}$, the CP elects to sponsor. Compared with that in the benchmark case, the impact of sponsored data services on members of the telecom service supply chain in this case is as follows:

(a) The profit of the ISP increases.

(b) The profit of the CP decreases.

(c) The total profit of the supply chain increases when $s > \frac{a}{2b}$ and $m < \frac{2bs-a}{b}$ and decreases when $s > \frac{a}{2b}$ and $m > \frac{2bs-a}{b}$ or when $s < \frac{a}{2b}$.

(d) Consumer surplus increases when $s > \frac{a}{2b}$ and $m < \frac{2bs-a}{b}$ and decreases when $s > \frac{a}{2b}$ and $m > \frac{2bs-a}{b}$ or when $s < \frac{a}{2b}$.

When the CP has a high unit cost, i.e., $m \geq \frac{a-\sqrt{2}(a-bs)}{b}$, the ISP will be convinced that the CP is incapable of subsidizing its users and will set an access price that is equivalent to that set in the benchmark case. Then, we consider the scenario when $m < \frac{a-\sqrt{2}(a-bs)}{b}$ and sponsored data is implemented. In this case, the ISP can generate additional profit, whereas the profit of the CP always decreases because the ISP knows that a CP with a high profit margin, i.e., low unit cost m, has increased incentive to subsidize its users to attract additional demand and generate revenue. Hence, the ISP utilizes its monopolist pricing power and first mover advantage to set the access price at a high level. Thus, the CP needs to pay a high price to attract additional end consumers. This approach decreases profit. Proposition 3.1 (c, d) examines the effect of sponsored data services from a systematic perspective. If the market price of the service and CP's profit margin are high, i.e., $\frac{a}{2b} < s < \frac{a}{b}$ and $0 < m < \frac{2bs-a}{b}$, the total profit of the supply chain and the consumer surplus benefit from sponsored data. Otherwise, when the market price of the service is low or when the market price of service is high but the CP's profit margin is small, the total profit of the supply chain and consumer surplus decreases, and thus social welfare decreases. This result reveals that sponsored data, a practice that claims to provide Internet access to more people, actually can do just the opposite. Moreover, the CP's profit drastically drops in this case and could not be offset by the increase of ISP's profit. Thus, the total profit of the supply chain decreases. These detrimental consequences are undesirable to the CP and social planners. Given that sponsored data services are currently examined in a case-by-case manner, regulators should be careful that the market condition does not incur the detrimental effects of sponsored data.

To conclude, the analysis of the basic model shows that sponsored data services always reduce the CP's profit and benefit ISP, and their effects on social welfare depends on the market condition. In the next section, we extend the models to account for practical considerations and explore the effects of sponsored data in these situations.

3.4 Impact of sponsored data in various scenarios

3.4.1 Cost as the private information of CP

As with most existing studies, we assume in the basic model that the CP's cost information and other market parameters are common knowledge. These parameters are crucial for the ISP, as the market leader, to extract the maximum profit from the CP. As a result, the CP is always worse off when sponsored data programs are implemented under complete information. In practice, although the CP's subscription price can be observed by the public, the CP usually keeps its unit cost as private information. For example, we cannot obtain the cost information of CPs that do not reveal financial statements to the public or those of that are subsidiaries of large Internet companies. The revenue and cost information of such subsidiary CPs are mixed with those of the other businesses of the parent company. In these situations, the ISP needs to make pricing decisions with an estimation of the CP's actual cost.

We relax the complete information assumption, and the ISP only knows that the CP's unit cost m is uniformly distributed on $[0, s]$ with a density function $f(m) = \frac{1}{s}$. The ISP's objective now is to maximize its expected profit. The sequence of events in this case remains the same as that in the complete information case, and we solve the Stackelberg game similarly by backward induction.

The CP profit maximization problem is the same as that in the basic model. Thus, the optimal subsidization proportion in stage 2 remains $\beta^* = \frac{-a-bm+bp+2bs}{2bp}$, and β^* equals 1 when $m < 2s - p - \frac{a}{b}$ and equals 0 when $m > 2s + p - \frac{a}{b}$. Given that the ISP's pricing decision in stage 1 affects the CP's subsidization decision β, the ISP's expected profit consists of three parts that each corresponds to the CP's decision in one scenario:

$$
\begin{aligned}
\mathrm{E}(\pi_{ISP}) = & \int_0^{\min\left\{s,\max\left\{0,2s-p-\frac{a}{b}\right\}\right\}} pD(p,\beta=1)f(m)dm + \\
& \int_{\min\left\{s,\max\left\{0,2s-p-\frac{a}{b}\right\}\right\}}^{\min\left\{s,\max\left\{0,2s+p-\frac{a}{b}\right\}\right\}} D\left(p,\beta=\frac{-a-bm+bp+2bs}{2bp}\right)f(m)dm + \\
& \int_{\min\left\{s,\max\left\{0,2s+p-\frac{a}{b}\right\}\right\}}^{s} pD(p,\beta=0)f(m)dm \\
= & \int_0^{\min\left\{s,\max\left\{0,2s-p-\frac{a}{b}\right\}\right\}} \frac{(a-bs)p}{s}dm + \\
& \int_{\min\left\{s,\max\left\{0,2s-p-\frac{a}{b}\right\}\right\}}^{\min\left\{s,\max\left\{0,2s+p-\frac{a}{b}\right\}\right\}} \frac{[a-b(m+p)]p}{2s}dm + \\
& \int_{\min\left\{s,\max\left\{0,2s+p-\frac{a}{b}\right\}\right\}}^{s} \frac{[a-b(p+s)]p}{s}dm \qquad (3\text{-}3)
\end{aligned}
$$

Given that m should be no less than 0 and no larger than s, the integral intervals above are formulated to ensure meaningful integrals, and the value of these intervals depends on the value of market price s and the ISP's pricing decision p. The ISP's expected profit function $\mathrm{E}(\pi_{ISP})$ is a piecewise function of p, and the formulation of this piecewise function depends on the value of s. We summarize $\mathrm{E}(\pi_{ISP})$ for different values of s in section 3.6.4. By solving the ISP profit maximization problem in different cases of s, we can derive the ISP's optimal pricing strategies and the corresponding expected profit.

Comparing the optimal expected profit with the optimal profit in the benchmark case wherein the ISP does not provide a sponsored data contract, we find that the ISP's expected profit under sponsored data weakly increases. Hence, the ISP is always willing to provide a sponsored data program even without the accurate cost information of the CP. Upon observing the ISP's access price, the CP renders the subsidization decision in accordance with the actual cost.

Lemma 3.3 summarizes the ISP's optimal pricing decision and the CP's subsidization decision when the ISP lacks accurate information for the CP's cost.

Lemma 3.3 (Incomplete Information) The ISP's optimal price and its corresponding optimal expected profit and the CP's subsidization decision when a sponsored data contract is offered to the CP are given in Table 3.3.

Table 3.3 Optimal decisions of the ISP and CP under sponsored data with incomplete information

s	p^*	$E^*(\pi_{ISP})$	m	β^*
$0 < s \leq \frac{a}{3b}$	$\frac{a-bs}{2b}$	$\frac{(a-bs)^2}{4b}$	$0 < m < s$	0
$\frac{a}{3b} < s \leq \frac{2a}{3b}$	$\frac{a}{3b}$	$\frac{a^3}{27b^2 s}$	$0 < m \leq \frac{6bs-2a}{3b}$	$\frac{6bs-3bm-2a}{2a}$
			$\frac{6bs-2a}{3b} < m < s$	0
$\frac{2a}{3b} < s < \frac{a}{b}$	$\frac{a}{b} - s$	$\frac{(a-bs)^2(2bs-a)}{b^2 s}$	$0 < m \leq \frac{3bs-2a}{b}$	1
			$\frac{3bs-2a}{b} < m < s$	$\frac{b(s-m)}{2(a-bs)}$

By substituting the equilibrium decisions in each case in Table 3.3 into the profit functions of the ISP and CP, we can derive the equilibrium profit for each case. By comparing the equilibrium results in the incomplete information case with those in the benchmark case, we obtain the following proposition:

Proposition 3.2 (Incomplete Information) The ISP is always willing to offer a sponsored data contract. When $0 < s \leq \frac{a}{3b}$, the CP's optimal strategy is not to sponsor ($\beta^* = 0$). Thus, this case is equivalent to the benchmark case where a sponsored data contact is not offered. When $s > \frac{a}{3b}$, the CP elects to sponsor. Compared with that in the benchmark case, the impact of sponsored data services on members of the telecom service supply chain in this case is as follows:

(a) The profit of the ISP increases when $\frac{a}{3b} < s < \frac{2a}{3b}$ and $m \leq \frac{-5a^2+18abs-9b^2s^2}{6ab}$ or when $\frac{2a}{3b} < s < \frac{a}{b}$ and $m \leq \frac{3bs-a}{2b}$ and decreases otherwise.

(b) The profit of the CP increases when $s > \frac{2a}{3b}$ and $m \leq \frac{3bs-2a}{b}$ and decreases otherwise.

(c) The total profit of the supply chain increases when $\frac{a}{3b} < s < \frac{2a}{3b}$ and $m < \frac{3bs-a}{3b}$ or when $\frac{2a}{3b} < s < \frac{a}{b}$ and $m \leq \frac{2bs-a}{b}$ and decreases otherwise.

(d) Consumer surplus increases when $\frac{a}{3b} < s < \frac{2a}{3b}$ and $m < \frac{3bs-a}{3b}$ or when

$\frac{2a}{3b} < s < \frac{a}{b}$ and $m \leq \frac{2bs-a}{b}$ and decreases otherwise.

Figure 3.2 provides a summary of Proposition 3.2 (a,b) and an illustration of the effects on the profits of ISP and CP when sponsored data services are allowed. We use $ij\ (i,j \in \{W,E,B\},\ W\ for\ worse,\ E\ for\ equal, B\ for\ better)$ to denote the outcome, where i represents the outcome for the ISP, and j represents the outcome for the CP. When information asymmetry is considered, even a monopolistic ISP cannot effectively extract revenue from the CP by implementing a sponsored data program and may be hampered by sponsored data if the CP's profit margin is low. However, if the market price for the service is high, i.e., $s > \frac{2a}{3b}$, a CP with high profit margin can benefit from sponsored data. Particularly, the CP is likely to benefit from sponsored data when the market price of the service increases. Proposition 3.2 (c,d) shows that sponsored data positively affects the supply chain profit and consumer surplus when the CP has a relatively high profit margin.

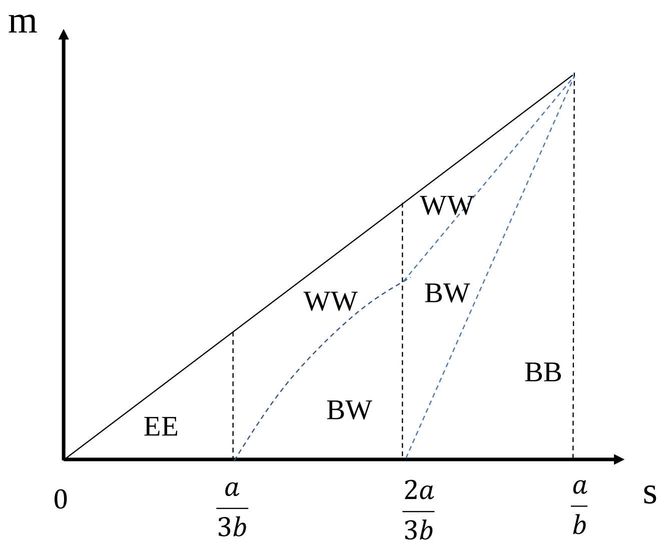


Figure 3.2 Impact of sponsored data on ISP and CP with incomplete information

The equilibrium analysis presented in this section suggests that when the complete information assumption is relaxed, the ISP's profit could be hampered and the CP's profit could increase if a sponsored data contract is implemented. As *s* increases, even a CP with a relatively small profit margin can benefit from sponsored data because of the ISP's conservative pricing strategy when the ISP does not know the CP's actual cost information. Similar to that in the basic model, the impact of sponsored data programs on social welfare, including supply chain profit and consumer surplus, is dependent on consumer valuation and the CP's profit margin, but is now positive over a broadened range. Specifically, $s > \frac{a}{2b}$ and $m < \frac{2bs-a}{b}$ are required to ensure the positive impact of sponsored data plan on social welfare in the complete information case, and the counterpart requirement in the incomplete information case is $\frac{a}{3b} < s < \frac{2b}{3a}$ and $m < \frac{3bs-a}{3b}$ or $s > \frac{2b}{3a}$ and $m < \frac{2bs-a}{b}$.

3.4.2 Quality as the decision of CP

The CP's content quality is its core competence. For example, a video-streaming site that provides popular TV series or access to exclusive films can attract high numbers of subscriptions even if its price is a slightly higher than that of its competitors. In 2017, Netflix spent $6.3 billion on original and acquired content. This strategy has resulted in 20 Emmy awards and 23.78 million new subscriptions. Hulu released *The Handmaid's Tale*, which won the 2017 Emmy for best drama series, in April 2017, and then saw a 98% rise in daily sign-ups from March to September. When the ISP attempts to implement sponsored data programs with a high access price to extract revenue from the CP, the CP can always choose to sponsor less and use their budget on improving content quality as an alternative to attract additional consumers. The ISP's awareness of the fact that the CP will simultaneously make subsidization and quality decisions may prevent it from increasing access price, thus providing the CP with a competitive advantage.

In this section, we explicitly model the CP's quality decision and investigate the impact of sponsored data programs in this scenario. The sequence of the game is the same as that in the basic model. In stage 2, however, the subscription CP simultaneously decides the subsidization proportion and the content quality. Demand is affected by access price and content quality q, and the demand function is formulated as $D = a -$

$b[(1-\beta)p+s]+cq$, where c is a positive coefficient that denotes the users' responsiveness to content quality. In reference to previous works that addressed quality decisions, we use a quadratic term ϵq^2 to model the cost of quality investment and assume that $\epsilon > \frac{c^2}{2b}$. The CP optimization problem in stage 2 can be written as:

$$\max_{0\leq\beta\leq1,q\geq0} \pi_{CP} = (s-m-\beta p)D - \epsilon q^2 \qquad (3\text{-}4)$$

We solve for the CP's decisions in stage 2 by applying Karush–Kuhn–Tucker (KKT) multipliers and summarize the results in Lemma 3.4.

Lemma 3.4 The CP's subsidization decision and quality investment decision in stage 2 are given in Table 3.4 and Table 3.5.

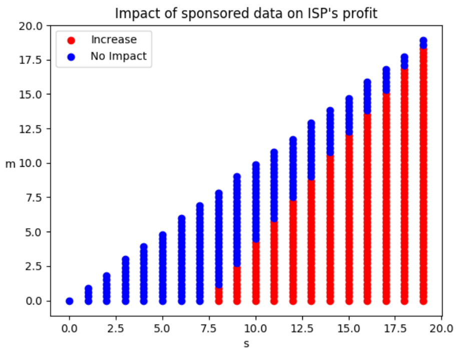
Table 3.4 Optimal decision of the CP when $p \leq \frac{a}{b} - s$

m	β^{S*}	q^*
$0 < m \leq \frac{c^2(p-s)+2\epsilon(2bs-a-bp)}{2b\epsilon-c^2}$	1	$\frac{c(s-m-p)}{2\epsilon}$
$\frac{c^2(p-s)+2\epsilon(2bs-a-bp)}{2b\epsilon-c^2} < m \leq \frac{2b\epsilon(p+2s)-2a\epsilon-c^2 s}{2b\epsilon-c^2}$	$\frac{c^2(m-s)+2\epsilon[b(p+2s-m)-a]}{p^2(4b\epsilon-c^2)}$	$\frac{c(a-bm-bp)}{4b\epsilon-c^2}$
$\frac{2b\epsilon(p+2s)-2a\epsilon-c^2 s}{2b\epsilon-c^2} < m \leq s$	0	$\frac{c(s-m)}{2\epsilon}$

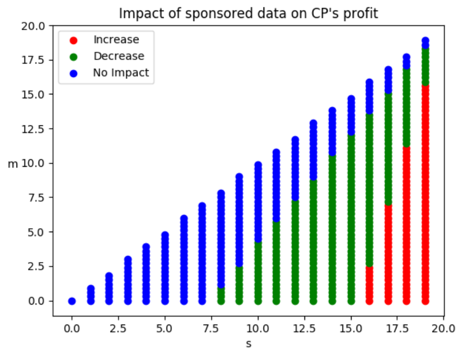
Table 3.5 Optimal decision of the CP when $p > \frac{a}{b} - s$

m	β^{S*}	q^*
$0 < m \leq \frac{c^2(p-s)+2\epsilon(2bs-a-bp)}{2b\epsilon-c^2}$	1	$\frac{c(s-m-p)}{2\epsilon}$
$\frac{c^2(p-s)+2\epsilon(2bs-a-bp)}{2b\epsilon-c^2} < m \leq \frac{a-bp}{b}$	$\frac{c^2(m-s)+2\epsilon[b(p+2s-m)-a]}{p^2(4b\epsilon-c^2)}$	$\frac{c(a-bm-bp)}{4b\epsilon-c^2}$
$\frac{a-bp}{b} < m \leq s$	$\frac{-a+b(2s+p-m)}{2bp}$	0

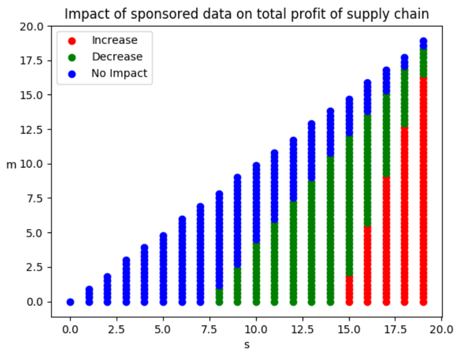
By comparing the results in Table 3.4 and Table 3.5, we find that if the CP has a large profit margin, it will fully subsidize its users. If the CP's profit margin is moderate, it will partially subsidize its users and increase its investment in content quality. If its profit margin is small, it will choose to use its limited budget on data subsidization when the access price is high and on quality investment when the access price is low. Substituting the CP's optimal decisions and the corresponding conditions in stage 2 into the ISP's profit maximization in stage 1 yields a set of constrained maximization problems that each corresponds to one set of the CP's reactions (See Appendix 3). To investigate the ISP's optimal pricing decision, we solve these optimization problems and compare the optimal profit in each case to identify the access price that maximizes the ISP's profit. Comparing these profits and analytically deriving the ISP's optimal pricing decision in all cases are impossible given the high number of parameters. Thus, we employ numerical experiments to calculate the equilibrium profit of the ISP and CP and compare the results with the results for the case wherein sponsored data contracts are not offered. The parameters used in the numerical experiment are $a = 100, b = 5$, $c = 3$, and $\epsilon = 1$. The results are illustrated in Figure 3.3.



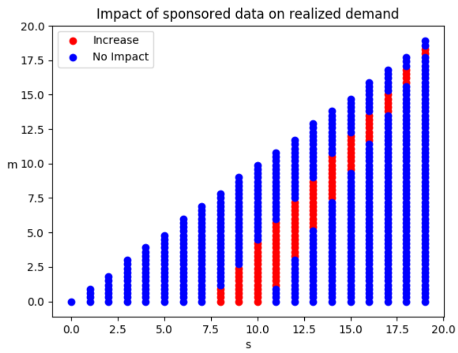
(a) ISP's profit



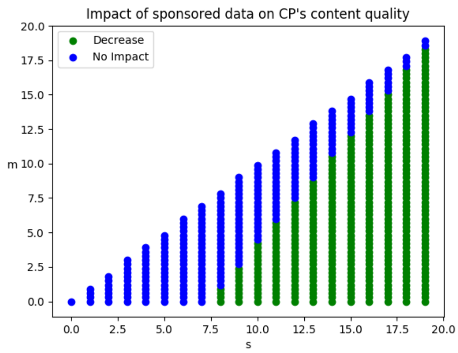
(b) CP's profit



(c) total profit



(d) demand



(e) content quality

Figure 3.3 Impact of sponsored data when CP jointly optimizes content quality and subsidization decisions

Figure 3.3(a) suggests that the ISP's profit weakly increases under sponsored data and that the ISP is always willing to offer sponsored data contracts. The effect on the CP's profit in this case is similar to that in the incomplete information case. That is, when the content price is sufficiently high, a CP with relatively high profit margin benefits from sponsored data, whereas the profit of a CP with lower profit margin is hampered. Figure 3.3(c–e) show that while the impact of sponsored data on the total profit of the supply chain remains dependent on market conditions, consumer surplus weakly increases at the cost of content quality degradation.

3.4.3 Platform CP

To differentiate CPs with different types of revenue models, we refer to the previously investigated CP and the CP in this section as subscription CP and platform CP, respectively. In contrast to subscription CPs, platform CPs do not directly charge their users. Instead, they provide their users with different kinds of platforms that cater to different needs, such as social connections and trading. Clicks and visits to the platform generate great value and bring revenue related to advertising and other value-added services to CPs. Thus, the profitability of a platform CP largely depends on the value of its "platform" or "network." Facebook, YouTube, and Uber are good examples of platform CPs. The service of a platform CP usually demonstrates network externality, i.e., its users derive increased utility from the service as additional users join the platform. To reflect these characteristics, the demand function is characterized as $D = a - b(1 - \beta p) + \phi D$, where $\phi \in (0,1)$ is the magnitude of network externality. Many previous studies, such as those of Foros and Hansen (2001) and Jahn and Prüfer (2008), have modeled network externality with a term linear in the expected consumer base. Under the assumption that users can form a rational expectation of the equilibrium demand, the realized demand can be written as $D = \frac{a-b(1-\beta)p}{1-\phi}$. The revenue of the platform CP is usually derived from advertisements, which in turn depends on the value of its user network. Thus, we invoke Metcalfe's law, which states that a network's value is proportional to the square of the number of its connected users, and formulate the profit function as $\pi_{CP} = rD^2 - \beta pD$, where r denotes the profitability of the platform CP. Given that r cannot be easily improved over the short term, we assume it is a constant and satisfies $r < \frac{1-\phi}{b}$.

To justify the CPs' revenue models, we collect the revenue data of Netflix and

Facebook, the representative companies for subscription CP and platform CP, respectively, for the past few years. As shown in Figure 3.4, Facebook's revenue increases quadratically with its number of users, whereas Netflix's revenue grows linearly with its number of subscribers. Empirical studies have utilized revenue data from companies, such as Facebook and Tencent, to validate that a network's value is best characterized by Metcalfe's law (Metcalfe, 2013; Zhang et al., 2015b).

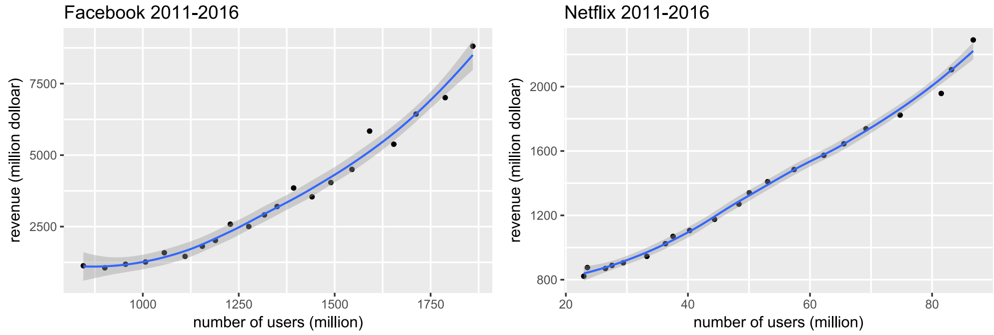


Figure 3.4 Number of users and revenue of Facebook and Netflix for the period of 2011–2016

We first examine the equilibrium outcome when the platform CP does not offer a sponsored data contract and refer to this case as the benchmark case. In this case, the ISP sets the access price p, and the CP has no decision to make. Lemma 3.5 summarizes the equilibrium outcome.

Lemma 3.5 (Platform CP) When a sponsored data contract is not offered, the optimal access price set by ISP is $p^0 = \frac{a}{2b}$; the corresponding realized demand is $D^0 = \frac{a}{2(1-\phi)}$; the optimal profits of the ISP and CP are $\pi_{ISP}^0 = \frac{a^2}{4b(1-\phi)}$ and $\pi_{\mathrm{CP}}^0 = \frac{a^2 r}{4b(1-\phi)^2}$, respectively.

Lemma 3.6 (Platform CP) The equilibrium subsidization proportion and access price when a sponsored data contract is offered are presented in Table 3.6.

Table 3.6. Equilibrium decisions in a telecom supply chain with a platform CP

Conditions	β^*	p^*
$0 < r \le \frac{1-\phi}{2b}$	0	$\frac{a}{2b}$
$\frac{1-\phi}{2b} < r \le \frac{3(1-\phi)}{4b}$	$\frac{2br+\phi-1}{2(1-br-\phi)}$	$\frac{a}{2b}$
$\frac{3(1-\phi)}{4b} < r < \frac{1-\phi}{b}$	1	$\frac{a(2br+\phi-1)}{b(1-\phi)}$

Table 3.6 shows that a platform CP's subsidization incentive increases with its profitability. Specifically, when the CP's profitability is low, the CP will not sponsor its users, and this case is equivalent to the benchmark case. When the CP's profitability is moderately low, it will subsidize a portion of its users' data usage. Interestingly, in this case, the ISP sets an optimal access price that is equal to that set in the benchmark case wherein a sponsored data contract is not offered. Given that the access price is the same and the CP's profitability level is high, the CP can profitably sponsor its users, and its profit increases compared with that in the benchmark case. When the CP's profitability is moderately high or high, it will subsidize all data traffic, whereas the ISP will charge a price that leaves CP zero profit in the latter case.

To analyze the impact of sponsored data on the ISP, CP, and users, we calculate the corresponding profits of the ISP and CP, as well as the realized demand for each case. The results are shown in Table 3.6 and are compared with the results for the benchmark case. Our findings are summarized in Proposition 3.3.

Proposition 3.3 (Platform CP) For a telecom service supply chain with a platform CP, the ISP is always willing to offer a sponsored data contract. When $r \le \frac{1-\phi}{2b}$, the CP's optimal strategy is not to sponsor ($\beta^* = 0$), and offering a sponsored data contract is equivalent to the benchmark case wherein a sponsored data contract is not offered. When $r > \frac{1-\phi}{2b}$, the CP elects to sponsor, and the impact of sponsored data on the members of the telecom supply chain is as follows:

(a) The profit of the ISP increases.

(b) The profit of the CP increases when $r \leq \frac{4(1-\phi)}{5b}$ and decreases otherwise.

(c) The total profit of the supply chain increases.

(d) Consumer surplus increases.

In a telecom supply chain with a platform CP, the ISP will always profit when offering a sponsored data contract. Under the sponsored data contract, the profit of the platform CP weakly increases if its profitability is not too high, i.e., $r \leq \frac{4(1-\phi)}{5b}$. When the profitability of the CP is high, its marginal revenue of attaining an extra user is high such that it must attempt to win every customer, thus giving ISP the power to charge a high price that leaves the CP little to no profit. From a social planner's perspective, sponsored data programs are beneficial because the total profit of supply chain and consumer surplus increase. Xiong et al. (2018) studied sponsored data in a setting wherein the content service shows network effects and revealed that network effects enhance the utility of all three parties. Similarly, our results indicate that network effects play a positive role in the sponsored data scheme.

3.5 Discussion and Conclusion

Table 3.7 summarizes the results of our analysis on the impact of sponsored data. When a sponsored data contract is offered to a subscription CP, the ISP benefits from the sponsored data contract except when it does not have accurate information on the subscription CP's cost and the CP actually has a high unit cost. The profit of subscription CPs usually decreases under a sponsored data contract. However, subscription CPs with large profit margins could take advantage of the subsidization mechanism and generate additional profit by hiding their cost information or indicating possible variations in their content quality. Our analysis indicates that the implementation of sponsored data programs has similar and mixed effects on the total profit of supply chain and consumer surplus. Thus, the impact of sponsored data on social welfare is mixed and is most likely positive when the content price is high and CP cost is low.

Existing works have mainly focused on subscription CPs with linearly related revenue and demand. Thus, we attempted to enrich the results in this field by focusing on platform CPs. When the ISP provides sponsored data to a platform CP, the profit of the ISP increases and that of the CP could increase if its profitability is not excessively high. Social welfare always increases when data subsidization is implemented.

Table 3.7 Impact of sponsored data in different settings

CP type	Setting	ISP Profit	CP Profit	Supply Chain Profit	Consumer Surplus
Subscription CP	complete information	weakly increases	weakly decreases	increases when market price is high and CP cost is low and decreases otherwise	increases when market price is high and CP cost is low and decreases otherwise
	incomplete information	increases when CP cost is low relative to market price and decreases otherwise	increases when market price is high and CP cost is low and decreases otherwise	increases when CP cost is low relative to market price and decreases otherwise	increases when CP cost is low relative to market price and decreases otherwise
	combined quality decision	weakly increases	increases when market price is high and CP cost is low and decreases otherwise	increases when market price is high and CP cost is low and decreases otherwise	weakly increases
Platform CP	complete information	weakly increases	increases when CP profitability is moderate and decreases otherwise	weakly increases	weakly increases

In general, sponsored data is a business model innovation that enhances an ISP's profitability. By charging content providers instead of users for data usage, the ISP could regain control over applications and share a part of its revenue. Although CPs usually can be hurt by sponsored data, they could adopt certain tactics to improve profitability. Our results on the effects of sponsored data on social welfare provide guidance for regulators when they investigate specific sponsored data schemes. Regulators should be more cautious in cases wherein social welfare is likely to be hampered and may apply restrictions on access price to provide benefits to all parties.

Many possible research directions can be pursued to understand the impact of sponsored data. First, previous studies have examined competition among CPs but not among ISPs. When multiple ISPs are in the market, their ability to extract revenue from CPs should be weakened and CPs should benefit. Second, we can investigate the impact of sponsored data in the case of vertical integration between the ISP and CP. Examples can be found in practice: Comcast holds a majority of NBC Universal's stake, and AT&T is in the process of acquiring Time Warner. The impact of sponsored data in this context has additional managerial implications. Third, the emerging trend wherein the ISP and CP establish a close relationship by issuing customized SIM card deserves careful examination. For example, China Unicom has issued a series of SIM cards with different CPs that each offer a specific package and exempt users from data usage charges for certain content.

3.6 Proofs in Chapter 3

3.6.1 Proof of Lemma 3.1

When a sponsored data contract is not offered, the ISP's profit function is reduced to $p[a-b(p+s)]$, which is a concave function in p. The optimal p^0 can be obtained by solving the first-order condition $a-2bp-bs=0$. Substituting $p^0=\frac{a-bs}{2b}$ into the demand function and profit function of each player provides the results presented in Lemma 3.1.

3.6.2 Proof of Lemma 3.2

First, we solve the CP profit maximization problem in stage 2. Given that $\frac{d^2\pi_{CP}}{d\beta^2} = -2bp^2 < 0$, π_{CP} is concave in β, and $\beta^* = \frac{-a-bm+bp+2bs}{2bp} = \frac{1}{2} + \frac{2bs-a-bm}{2bp}$ by solving the first-order condition. Note that β^* should be bounded by 0 and 1. Thus, β^* equals 1 when $0 < p \leq \frac{2bs-a-bm}{b}$ and equals 0 when $0 < p \leq \frac{-2bs+a+bm}{b}$.

(a) When $\mathrm{m} \leq \frac{\mathrm{2bs-a}}{b}$, β^* is always larger than 0. Hence, the ISP pricing problem in stage 1 has two subcases, as follows:

$$\max_{p \geq \frac{2bs-a-bm}{b}} \frac{p[a-b(m+p)]}{2} \text{ and } \max_{0 \leq p \leq \frac{2bs-a-bm}{b}} p(a-bs).$$

(b) When $m > \frac{2bs-a}{b}$, β^* is always less than 1. Hence, the ISP pricing problem in stage 1 has two subcases, as follows:

$$\max_{p \geq \frac{-2bs+a+bm}{b}} \frac{p[a-b(m+p)]}{2} \text{ and } \max_{0 \leq p \leq \frac{-2bs+a+bm}{b}} p[a-b(p+s)].$$

By calculating the optimal p and the corresponding profit of the ISP in each subcase in (a) and (b), we can derive the equilibrium price p^* for different values of m by selecting the price that maximizes the profit of the ISP. The results are given in Table 3.8.

Table 3.8 Optimal price of the ISP in a supply chain with complete information

Conditions	ISP's profit maximization problem	Optimal outcome in each subcase	Optimal ISP price
$m \leq \frac{2bs-a}{b}$	$\max\limits_{p \geq \frac{2bs-a-bm}{b}} \frac{p[a-b(m+p)]}{2}$	When $0 < m \leq \frac{4bs-3a}{b}, p^* = \frac{2bs-bm-a}{b}, \pi_{ISP}^* = \frac{(2bs-a-bm)(a-bs)}{b}$; When $\frac{4bs-3a}{b} < m \leq \frac{2bs-a}{b}, p^* = \frac{a-bm}{2b}, \pi_{ISP}^* = \frac{(a-bm)^2}{8b}$	When $0 < m \leq \frac{4bs-3a}{b}, p^* = \frac{2bs-bm-a}{b}$ When $\frac{4bs-3a}{b} < m \leq \frac{2bs-a}{b}, p^* = \frac{a-bm}{2b}$
	$\max\limits_{0 \leq p \leq \frac{2bs-a-bm}{b}} p(a-bs)$	$p^* = \frac{2bs-bm-a}{b}$ $\pi_{ISP}^* = \frac{(2bs-a-bm)(a-bs)}{b}$	
$m > \frac{2bs-a}{b}$	$\max\limits_{p \geq \frac{-2bs+a+bm}{b}} \frac{p[a-b(m+p)]}{2}$	When $\frac{2bs-a}{b} < m \leq \frac{4bs-a}{3b}, p^* = \frac{a-bm}{2b}, \pi_{ISP}^* = \frac{(a-bm)^2}{8b}$; When $m > \frac{4bs-a}{3b}, p^* = \frac{a+bm-2bs}{b}, \pi_{ISP}^* = (s-m)[a+b(m-2s)]$	When $\frac{2bs-a}{b} < m \leq \frac{a-\sqrt{2}(a-bs)}{b}$, $p^* = \frac{a-bm}{2b}$ When $m > \frac{a-\sqrt{2}(a-bs)}{b}$, $p^* = \frac{a-bs}{2b}$
	$\max\limits_{0 \leq p \leq \frac{-2bs+a+bm}{b}} p[a-b(p+s)]$	When $\frac{2bs-a}{b} < m \leq \frac{3bs-a}{2b}, p^* = \frac{a+bm-2bs}{b}, \pi_{ISP}^* = (s-m)[a+b(m-2s)]$ When $m > \frac{3bs-a}{2b}, p^* = \frac{a-bs}{2b}, \pi_{ISP}^* = \frac{(a-bs)^2}{4b}$	

The equilibrium subsidization proportion β^* can then be obtained by substituting p^* into $\beta^* = \frac{-a-bm+bp+2bs}{2bp}$.

3.6.3 Proof of Proposition 3.1

By substituting the equilibrium decision of the ISP and CP into their profit function, we obtain their equilibrium demand and profits, which are given in Table 3.9. Thus, the results presented in Proposition 3.1 can be obtained by comparing π_{ISP}^*, π_{CP}^*, and D^* with π_{ISP}^0, π_{CP}^0, and D^0.

Table 3.9 Equilibrium ISP's profit and subscription CP's profit under sponsored data in a supply chain with complete information

m	π_{ISP}^*	π_{CP}^*	D^*
$\frac{a-\sqrt{2}(a-bs)}{3b} \le m < s$	$\frac{(a-bs)^2}{4b}$	$\frac{(s-m)(a-bs)}{2}$	$\frac{a-bs}{2}$
$\frac{4bs-3a}{b} \le m < \frac{a-\sqrt{2}(a-bs)}{3b}$	$\frac{(a-bm)^2}{8b}$	$\frac{(a-bm)^2}{16b}$	$\frac{a-bm}{4}$
$0 < m < \frac{4bs-3a}{b}$	$\frac{(2bs-a-bm)(a-bs)}{b}$	$\frac{(a-bs)^2}{b}$	$a-bs$

3.6.4 Proof of Lemma 3.3

The ISP profit maximization problem in stage 1 can be formulated as $\max_{p>0} E(\pi_{ISP}) = \int_0^{2s-p-\frac{a}{b}} \frac{(a-bs)p}{s}\, dm + \int_{2s-p-\frac{a}{b}}^{2s+p-\frac{a}{b}} \frac{[a-b(m+p)]p}{2s}\, dm + \int_{2s+p-\frac{a}{b}}^{s} \frac{[a-b(p+s)]p}{s}\, dm$ when $0 \le 2s-p-\frac{a}{b} \le 2s+p-\frac{a}{b} \le s$. If the two thresholds of m: $2s-p-\frac{a}{b}$ and $2s+p-\frac{a}{b}$ are not in the interval $(0, s)$, then the ISP's expected profit function will take on different forms. Therefore, the ISP's expected profit function is a piecewise function, and the possible boundary points are $0, 2s-\frac{a}{b}$, $\frac{a}{b}-s$, and $\frac{a}{b}-2s$. In addition, the value of s determines the relationship among these boundary points and consequently determines the composition of the piecewise function. For example, when $0 < s < \frac{a}{2b}$, $2s-\frac{a}{b} < 0 < \frac{a}{b}-2s < \frac{a}{b}-s$, $E(\pi_{ISP})$ has three segments:

$$E(\pi_{ISP}) = \int_0^s p \cdot D(\beta = 0) f(m) dm = p(a-b(p+s)), if\ \ 0 < p \le \frac{a}{b} - 2s;$$

$$\mathrm{E}(\pi_{ISP}) = \int_0^{2s+p-\frac{a}{b}} p\cdot D\left(\beta=\frac{-a-bm+bp+2bs}{2bp}\right) f(m)dm + \int_{2s+p-\frac{a}{b}}^{s} p\cdot D(\beta=0)f(m)dm$$

$$= \frac{p(a-bp)^2}{4bs}, if\ \frac{a}{b}-2s < p \le \frac{a}{b}-s;$$

$$\mathrm{E}(\pi_{ISP}) = \int_0^{s} p\cdot D\left(\beta=\frac{-a-bm+bp+2bs}{2bp}\right) f(m)dm$$

$$= \frac{p[2a-b(2p+s)]}{4}, \text{if p} > \frac{a}{b}-s.$$

Similarly, we can derive the ISP's expected profit function in the other two cases, where $\frac{a}{2b} < s \le \frac{2a}{3b}$ (implies $\frac{a}{b}-2s < 0 < 2s-\frac{a}{b} < \frac{a}{b}-s$) and $\frac{2a}{3b} < s < \frac{a}{b}$ (implies $\frac{a}{b}-2s < 0 < \frac{a}{b}-s < 2s-\frac{a}{b}$). We summarize the ISP's expected profit function as follows:

(a). When $0 < s \le \frac{a}{2b}$,

$$\mathrm{E}(\pi_{ISP}) = \begin{cases} p\big(a-b(p+s)\big), & if\ 0 < p \le \frac{a}{b}-2s \\ \frac{p(a-bp)^2}{4bs}, & if\ \frac{a}{b}-2s < p \le \frac{a}{b}-s \\ \frac{p[2a-b(2p+s)]}{4}, & if\ \text{p} > \frac{a}{b}-s \end{cases}$$

(b). When $\frac{a}{2b} < s \le \frac{2a}{3b}$,

$$\mathrm{E}(\pi_{ISP}) = \begin{cases} \frac{p(p-s)(bs-a)}{s}, & if\ 0 < p \le 2s-\frac{a}{b} \\ \frac{p(a-bp)^2}{4bs}, & if\ 2s-\frac{a}{b} < p \le \frac{a}{b}-s \\ \frac{p[2a-b(2p+s)]}{4}, & if\ \text{p} > \frac{a}{b}-s \end{cases}$$

(c). When $\frac{2a}{3b} < s < \frac{a}{b}$,

$$\mathrm{E}(\pi_{ISP}) = \begin{cases} \frac{p(p-s)(bs-a)}{s}, & if\ 0 < p \le \frac{a}{b}-s \\ \frac{-p[a^2+2ab(p-3s)+b^2(p^2-2ps+5s^2)]}{4bs}, & if\ \frac{a}{b}-s < p \le 2s-\frac{a}{b} \\ \frac{p[2a-b(2p+s)]}{4}, & if\ \text{p} > 2s-\frac{a}{b} \end{cases}$$

For each case, we can solve the ISP's expected profit maximization problem by calculating the optimal price and corresponding profit in each segment and selecting the pricing strategy that maximizes the ISP's expected profit among all segments. For example, for case (a), we can calculate that in the first segment, the ISP's optimal price is $\frac{a-bs}{2b}$ and the corresponding $\mathrm{E}(\pi_{ISP})$ is $\frac{(a-bs)^2}{4b}$. In the second segment, the ISP's optimal price is $\frac{a}{b}-2s$ when $0<s\leq\frac{a}{3b}$ and $\frac{\mathrm{a}}{3b}$ when $\frac{a}{3b}<s\leq\frac{a}{2b}$, and the corresponding $\mathrm{E}(\pi_{ISP})$ are $s(a-2bs)$ and $\frac{a^3}{27b^2s}$, respectively. In the third segment, the ISP's optimal price is $\frac{2a-bs}{4b}$ and the corresponding $\mathrm{E}(\pi_{ISP})$ is $\frac{(a-bs)s}{4}$. By comparing the optimal expected profits, we conclude that ISP's optimal pricing strategy is $\frac{a-bs}{2b}$ when $0<s\leq\frac{a}{3b}$ and $\frac{\mathrm{a}}{3b}$ when $\frac{a}{3b}<s\leq\frac{a}{2b}$. The optimal price in other two cases can be derived similarly, and the equilibrium subsidization proportion β^* can then be obtained by substituting p^* into $\beta^*=\frac{-a-bm+bp+2bs}{2bp}$.

3.6.5 Proof of Proposition 3.2

By substituting the equilibrium decision of the ISP and CP into their profit function, we obtain their equilibrium profit as follows:

When $0<s\leq\frac{a}{3b}$, $\beta^*=\frac{-a-2bm+3bs}{2(a-bs)}$, $\pi_{CP}^*=\frac{(s-m)(a-bs)}{2}$, $\pi_{ISP}^*=\frac{(a-bs)^2}{4b}$, $D^*=\frac{a-bs}{2}$;

When $\frac{\mathrm{a}}{3\mathrm{b}}<s\leq\frac{2a}{3b}$, if $0<m\leq\frac{6bs-2a}{3b}$, then $\beta^*=\frac{6bs-3bm-2a}{2a}$, $\pi_{\mathrm{CP}}^*=\frac{(2a-3bm)^2}{36b}$, $\pi_{\mathrm{ISP}}^*=\frac{\mathrm{a}(2a-3bm)}{18b}$, $\mathrm{D}^*=\frac{2a-3bm}{6}$; if $\frac{6bs-2a}{3b}<m<s$, then $\beta^*=0$, $\pi_{\mathrm{CP}}^*=\frac{(s-m)(2a-3bs)}{3}$, $\pi_{\mathrm{ISP}}^*=\frac{a(2a-3bs)}{9b}$, $\mathrm{D}^*=\frac{2a-3bs}{3}$;

When $\frac{2\mathrm{a}}{3\mathrm{b}}<\mathrm{s}<\frac{\mathrm{a}}{\mathrm{b}}$, if $0<m\leq\frac{3bs-2a}{b}$, then $\beta^*=1$, $\pi_{\mathrm{CP}}^*=\frac{[(2s-m)b-a](a-bs)}{b}$, $\pi_{\mathrm{ISP}}^*=\frac{(a-bs)^2}{b}$, $\mathrm{D}=\mathrm{a}-\mathrm{bs}$; if $\frac{3\mathrm{bs}-2\mathrm{a}}{\mathrm{b}}<\mathrm{m}<\mathrm{s}$, then $\beta^*=\frac{b(s-m)}{2(a-bs)}$, $\pi_{\mathrm{CP}}^*=\frac{b(s-m)^2}{4}$, $\pi_{\mathrm{ISP}}^*=\frac{(s-m)(a-bs)}{2}$, $\mathrm{D}^*=\frac{b(s-m)}{2}$.

The results in Proposition 2 can be obtained by comparing π_{ISP}^*, π_{CP}^*, D^* with π_{ISP}^0, π_{CP}^0, D^0.

3.6.6 Proof of Lemma 3.4

The Hessian matrix of the CP's profit function is $\begin{pmatrix} -2bp^2 & -cp \\ -cp & -2\epsilon \end{pmatrix}$, which is negative definite. Thus, π_{CP} is concave. Given that the constraints $0 \le \beta \le 1$ and $q \ge 0$ are continuously differentiable convex functions, KKT conditions are necessary and sufficient for finding the optimal solution.

The Lagrangian of the CP profit maximization problem is as follows:

$L(\pi_{CP}) = (s - m - \beta p)D - \epsilon q^2 + \mu_1 q + \mu_2 \beta + \mu_3(1 - \beta).$

The corresponding KKT conditions are listed below:

$$\frac{\partial L(\pi_{CP})}{\partial \beta} = 0,\ \frac{\partial L(\pi_{CP})}{\partial q} = 0,\ \mu_1 q = 0,\ \mu_2 \beta = 0,\ \mu_3(1 - \beta) = 0.$$

Solving the first two first-order conditions yields $\beta = \frac{c^2 p(m-s) - 2p\epsilon[a + b(m-p-2s)] - \mu_1 cp + 2\mu_2\epsilon - 2\mu_3\epsilon}{p^2(4b\epsilon - c^2)}$ and $q = \frac{cp(a-bm-bp) + 2\mu_1 bp - \mu_2 c + \mu_3 c}{p^2(4b\epsilon - c^2)}$. Given that the KKT multipliers can be either positive or zero, we have following scenarios to examine:

1) When $\mu_1 = \mu_2 = \mu_3 = 0$, we have the optimal decisions $\beta = \frac{c^2 p(m-s) - 2p\epsilon[a + b(m-p-2s)]}{p^2(4b\epsilon - c^2)}$ and $q = \frac{cp(a-bm-bp)}{p^2(4b\epsilon - c^2)}$. β should be between 0 and 1, and q should be nonnegative. These requirements result in conditions $p < \frac{a}{b} - s$ and $\frac{c^2(p-s) + 2\epsilon(2bs-a-bp)}{2b\epsilon - c^2} < m \le \frac{2b\epsilon(p+2s) - 2a\epsilon - c^2 s}{2b\epsilon - c^2}$ or $p > \frac{a}{b} - s$ and $\frac{c^2(p-s) + 2\epsilon(2bs-a-bp)}{2b\epsilon - c^2} < m \le \frac{a-bp}{b}$.
2) When $\mu_1 = \mu_2 = 0$ and $\mu_3 > 0$, we have $\beta = 1$, $q = \frac{c(s-m-p)}{2\epsilon}$, and $\mu_3 = \frac{p\{c^2(m+p-s) - 2\epsilon[a + b(m+p-2s)]\}}{2\epsilon}$. q should be nonnegative, and μ_3 should be positive. These requirements result in the condition $0 < m \le \frac{c^2(p-s) + 2\epsilon(2bs-a-bp)}{2b\epsilon - c^2}$.

3) When $\mu_1 = \mu_3 = 0$ and $\mu_2 > 0$, we have $\beta = 0$, $q = \frac{c(s-m)}{2\epsilon}$, and $\mu_2 = \frac{p\{c^2(s-m)-2\epsilon[a+b(m-p-2s)]\}}{2\epsilon}$. q should be nonnegative, and μ_2 should be positive. These requirements result in conditions $p < \frac{a}{b} - s$ and $\frac{2b\epsilon(p+2s)-2a\epsilon-c^2 s}{2b\epsilon-c^2} < m \leq s$.

4) When $\mu_2 = \mu_3 = 0$ and $\mu_1 > 0$, we have $\beta = \frac{-a+b(2s+p-m)}{2bp}$, $q = 0$, and $\mu_1 = \frac{c(-a+bm+bp)}{2b}$. β should be between 0 and 1, and μ_1 should be positive. These requirements result in conditions $p > \frac{a}{b} - s$ and $\frac{a-bp}{b} < m \leq s$.

5) When $\mu_2 = 0$ and $\mu_1, \mu_3 > 0$, we have $\beta = 1$, $q = 0$, $\mu_1 = c(m+p-s)$, and $\mu_3 = p(-a-bm-bp+2bs)$. μ_1 and μ_3 should be positive. These requirements do not result in any possible condition.

6) When $\mu_3 = 0$ and $\mu_1, \mu_2 > 0$, we have $\beta = 0$, $q = 0$, $\mu_1 = c(m-s)$, and $\mu_2 = p(a+bm-bp-2bs)$. μ_1 and μ_2 should be positive. These requirements do not result in any possible condition.

Summarizing the above conditions in each case yields the results shown in Table 3.4 and Table 3.5.

3.6.7 Summarization of ISP's profit maximization problems

Case 1: $\beta = 0, q = \frac{c(s-m)}{2\epsilon}$.

To induce this set of CP reactions, the ISP's price should ensure that the following conditions hold:

1) $m \leq \frac{c^2(p-s)+2\epsilon(2bs-a-bp)}{2b\epsilon-c^2}$;

2) $p \leq \frac{a}{b} - s$;

3) $\pi_{CP} \geq 0$ (Substituting CP's decisions into its profit function yields $\pi_{CP} = c^2(s-m) + 4\epsilon[a-b(p-s)]$.

We can calculate that the above three conditions hold when $0 < p <$

$\frac{c^2(s-m)+2\epsilon(a+bm-2bs)}{2b\epsilon}$. Hence, the ISP profit maximization problem in this case can be written as:

$$\max_p \; p \cdot D\left(\beta = 0, q = \frac{c(s-m)}{2\epsilon}\right)$$
$$\text{s.t.}\, 0 < \text{p} < \frac{\text{c}^2(s-m) + 2\epsilon(a+bm-2bs)}{2b\epsilon}$$

Similarly, we can derive the ISP profit maximization problems in the other three cases as follows:

Case 2: $\max_p \; p \cdot D\left(\beta = \frac{-\text{a}+\text{b}(2s+p-m)}{2bp}, q = 0\right)$

$\text{s.t. p} > \frac{a-bm}{b}$

Case 3:

a) If $m \leq \frac{4b\epsilon s-c^2 s-2a\epsilon}{2b\epsilon-c^2}$:

$$\max_p \; p \cdot D\left(\beta = \frac{c^2(m-s)+2\epsilon[b(p+2s-m)-a]}{\text{p}^2(4b\epsilon-c^2)}, q = \frac{\text{c}(a-bm-bp)}{4b\epsilon-c^2}\right)$$

$$\text{s.t.}\, \frac{\text{c}^2(m-s)+2\epsilon(2bs-a-bm)}{2b\epsilon-c^2} < \text{p} < \frac{\text{a}-\text{bm}}{b}$$

b) If $m > \frac{4\text{b}\epsilon\text{s}-\text{c}^2 s-2a\epsilon}{2b\epsilon-c^2}$

$$\max_p \; p \cdot D\left(\beta = \frac{c^2(m-s) + 2\epsilon[b(p+2s-m)-a]}{\text{p}^2(4b\epsilon-c^2)}, q = \frac{\text{c}(a-bm-bp)}{4b\epsilon-c^2}\right)$$

$$\text{s.t.}\, \frac{\text{c}^2(s-m)+2\epsilon(a+bm-2bs)}{2b\epsilon} < \text{p} < \frac{\text{a}-\text{bm}}{b}$$

Case 4:

a) If $m \leq s - \frac{(a-bs)\epsilon}{c^2}$:

$$\max_p \; p \cdot D\left(\beta = 1, q = \frac{\text{c}(s-m-p)}{2\epsilon}\right)$$

$$\text{s.t.}\, 0 < \text{p} < s-m$$

b) If $m > s - \frac{(a-bs)\epsilon}{c^2}$:

$$\max_p \; p \cdot D\left(\beta = 1, q = \frac{\text{c}(s-m-p)}{2\epsilon}\right)$$

$$\text{s.t.}\, 0 < p < \frac{\text{c}^2(m-s)+2\epsilon(2bs-a-bm)}{2b\epsilon-c^2}$$

Applying KKT conditions to each above maximization problem yields the optimal price of the ISP in each case:

Case 1: When $m \leq \frac{(6b\epsilon-c^2)s-2a\epsilon}{4b\epsilon-c^2}$, the optimal price of the ISP is $p^* = \frac{c^2(s-m)+2\epsilon(a+bm-2bs)}{2b\epsilon}$; and when $m > \frac{(6b\epsilon-c^2)s-2a\epsilon}{4b\epsilon-c^2}$, the optimal price of the ISP $p^* = \frac{c^2(s-m)+2\epsilon(a-bs)}{2b\epsilon}$.

Case 2: For any m, the optimal price of the ISP is $p^* = \frac{a-bm}{b}$.

Case 3: When $m \leq \frac{ac^2-2bc^2s-6ab\epsilon+8b^2\epsilon s}{b(2b\epsilon-c^2)}$, the optimal price of the ISP is $p^* = \frac{c^2(m-s)+2\epsilon(2bs-a-bm)}{2b\epsilon-c^2}$; when $\frac{ac^2-2bc^2s-6ab\epsilon+8b^2\epsilon s}{b(2b\epsilon-c^2)} < m \leq \frac{4b\epsilon s-a\epsilon-c^2s}{3b\epsilon-c^2}$, the optimal price of the ISP is $p^* = \frac{2b\epsilon(a-bm)}{4b^2\epsilon}$; and when $m > \frac{4b\epsilon s-a\epsilon-c^2s}{3b\epsilon-c^2}$, the optimal price of the ISP is $p^* = \frac{c^2(s-m)+2\epsilon(a+bm-2bs)}{2b\epsilon}$.

Case 4: When $m \leq s - \frac{2\epsilon(a-bs)}{c^2}$, the optimal price of the ISP is $p^* = \frac{c^2(s-m)+2\epsilon(a-bs)}{2c^2}$; when $s - \frac{2\epsilon(a-bs)}{c^2} < m \leq s - \frac{\epsilon(a-bs)}{c^2}$, the optimal price of the ISP is $p^* = s - m$; and when $m > s - \frac{\epsilon(a-bs)}{c^2}$, the optimal price of the ISP is $p^* = \frac{c^2(m-s)+2\epsilon(2bs-a-bm)}{2b\epsilon-c^2}$.

Given a value of m, we can solve the optimal price in each of the four cases and select the price that maximizes the ISP's profit as the ISP's equilibrium price in stage 1. After obtaining the equilibrium price, we can then calculate the CP's subsidization and quality decision, as well as the ISP's and CP's profits.

3.6.8 Proof of Lemma 3.5

In the absence of a sponsored data contract, the ISP's profit function is reduced to $\frac{p(a-bp)}{1-\phi}$, which is a concave function in p, and the optimal $\mathbf{p^0}$ can be obtained by solving the first-order condition $a - 2bp = 0$. Substituting $\mathbf{p^0}$ into the demand function and profit functions yields the results presented in Lemma 3.5.

3.6.9 Proof of Lemma 3.6

First, we solve the CP profit maximization problem in stage 2. Given that $\frac{d^2\pi_{CP}}{d\beta^2} = \frac{2bp^2(br+\phi-1)}{(1-\phi^2)} < 0$, π_{CP} is concave in β, and $\beta^* = \frac{(a-bp)(2br+\phi-1)}{2bp(1-br-\phi)}$ by solving the first-order condition. Note that β^* should be bounded by 0 and 1. Thus, we substitute the optimal β^* for different values of p into the ISP pricing problem in stage 1 and obtain the following subcases:

a) When $0 < r \le \frac{1-\phi}{2b}$, the ISP's profit maximization problem is

$$\max_{p\ge\frac{a}{b}} \frac{p(a-bp)}{2(1-br-\phi)} \text{ or } \max_{\frac{a(2br+\phi-1)}{b(1-\phi)}\le p<\frac{a}{b}} \frac{p(a-bp)}{1-\phi}.$$

b) When $\frac{1-\phi}{2b} < r < \frac{1-\phi}{b}$, the ISP's profit maximization problem is

$$\max_{\frac{a(2br+\phi-1)}{b(1-\phi)}\le p<\frac{a}{b}} \frac{p(a-bp)}{2(1-br-\phi)} \text{ or } \max_{p<\frac{a(2br+\phi-1)}{b(1-\phi)}} \frac{ap}{1-\phi} \text{ or } \max_{p\ge\frac{a}{b}} \frac{p(a-bp)}{1-\phi}.$$

Calculating the optimal p and the corresponding ISP's profit in each subcase in a) and b), we can derive the equilibrium price p^* for different values of *r* by selecting the price that maximize the ISP's profit. Then equilibrium subsidization proportion β^* can then be obtained by substituting p^* into $\beta^* = \frac{(a-bp)(2br+\phi-1)}{2bp(1-br-\phi)}$.

3.6.10 Proof of Proposition 3.3

By substituting the equilibrium decision of the ISP and the CP into their profit function, we obtain the equilibrium profit given in Table 3.10. The results in Proposition 3.3 can be obtained by comparing π_{ISP}^*, π_{CP}^*, and D^* with π_{ISP}^0, π_{CP}^0, and D^0.

Table 3.10 Equilibrium ISP's and platform CP's profits under sponsored data in a supply chain with complete information

r	D^*	π_{ISP}^*	π_{CP}^*
$0 < r \le \frac{1-\phi}{2b}$	$\frac{a}{2(1-\phi)}$	$\frac{a^2}{4b(1-\phi)}$	$\frac{a^2 r}{4(1-\phi)^2}$
$\frac{1-\phi}{2b} < r \le \frac{3(1-\phi)}{4b}$	$\frac{a}{4(1-br-\phi)}$	$\frac{2br+\phi-1}{2(1-br-\phi)}$	$\frac{a}{2b}$
$\frac{3(1-\phi)}{4b} < r < \frac{1-\phi}{b}$	$\frac{a}{1-\phi}$	1	$\frac{a(2br+\phi-1)}{b(1-\phi)}$

4 Impact of Sponsored Data in a Competitive Market

4.1 Introduction

In recent years, China's mobile operators have begun to provide various sponsored data services on a large scale. Guangzhou Unicom first initiated sponsored data mode in 2013 by promoting WeChat SIM card, and a surge of sponsored data services came with “DaWangKa” cooperatively launched by China Unicom and Tencent company in October 2016. A year later, China’s three major operators launched over 30 this kind of cooperation plans. China Unicom and China Telecom are of strong momentum launching many types of cooperation plans. Meanwhile, China Mobile, the leader of the 4G market, is slow in this competition and is currently losing. The number of 4G users of China Mobile maintains a rapid growth since 2014, but the business report released by China Mobile in April 2018 shows that the number of 4G users decreased for the first time with a monthly loss of 2.427 million users.

Intuitively, in addition to enabling the ISP to take advantage of the close relationship between content providers and users in the OTT era for their own benefits, it can also increase the attractiveness of ISPs in the network access market to gain a certain competitive advantage over other ISPs. However, in a market with multiple ISPs, how should ISPs and CPs choose partners for sponsored data services and make appropriate pricing and subsidy decisions and how the implementation of the sponsored data modes affects various stakeholders in the telecommunications service supply chain lack a clear theoretical analysis. Therefore, this chapter aims to study the following in a market with two competing ISPs. First, what ISP’s and CP’s competition strategies are under sponsored data mode. Second, for ISPs, whether the introduction of sponsored data service can enable them to gain advantages in the competition with other ISPs. Third, for CP, how data subsidies provided for users will affect their own revenue. Fourth, what impact sponsored data has on total market demand and total profit of telecommunications service supply chain. In this chapter, we analyze the ISP's pricing decision and CP's subsidy decision in three scenarios, namely, no sponsored data, sponsored data service provided by one of the ISPs, and sponsored data service provided by both two ISPs. Moreover, this chapter summarizes the influencing mechanism of sponsored data services on the telecommunication service supply chain by comparing the equilibrium results in the three scenarios. Finally, to take advantage of CP's relationship with users thoroughly, some ISPs directly acquire a CP as their subsidiary company. This scenario is also studied in Section 4.4, where the effect of sponsored data implementation is explored when there is vertical integration between ISP and CP.

4.2 Related Literature

Although there have been many studies focusing on competition between CPs in sponsored data context, few literatures examine the impact of sponsored data in the presence of ISP competition. Kamiyama (2014) used a three-stage Stackelberg game model to analyze how ISP's share of CP's revenue would affect all parties in a market with two ISPs and a CP. In his model, there is no competition between the two ISPs in the end-user market, and only one ISP charges the CP according to the amount of content distributed, while the other ISP only charges the CP a traditional transfer fee. Kamiyama's numerical experiment results show that the when the ISP charges CP on content distribution, both the ISP and the CP may benefit from this practice, depending on the interconnection quality of the two ISPs and the market share gap between them. Maillé and Tuffin (2018) investigates how a CP decides the proportion to subsidize for each ISP in a market with ISP competition, and found that the introduction of sponsored data service, especially when CP's subsidization proportion for each ISP must be the same, has a positive effect on ISPs' profit and consumers' utility. The limitation of their study is that the pricing decision of ISP is not considered, and the main conclusions are all obtained through numerical experiments without theoretical results. There are also literatures that consider ISP competition in the area of paid priority. For example, Bourreau et al. (2015) shows that the implementation of paid priority can always improve the overall social welfare, CP's content innovation and ISP's network investment. But when the competition for end-users between ISPs is fierce, ISPs' profit may be damaged.

4.3 The model

We consider a two-tier telecom service supply chain that consists of two competing ISPs and one CP. If a user wants to access the CP's contents and services, she has to purchase Internet access service first from either of the two ISPs. It is assumed that the CP does not charge users for browsing its content but profits from other sources such as advertising, so users only have to pay the ISP for data traffic generated while visiting the CP. For simplicity and tractability, all users are assumed to consume a constant amount of data packets per month, so that they are charged a constant monthly access price set by their ISPs. If sponsored data services are implemented and CP subsidizes a certain portion of users' data usage, then users only have to pay for part of the data usage cost, and the other part is paid by the sponsoring CP. It can be seen that the introduction of sponsored data services can reduce the actual data usage cost of the users to a certain extent, so that the ISP who provides this kind of service gains a price advantage in the market competition.

To investigate the impact of sponsored data services on each participant of the telecom service supply chain, we first establish a two-stage Stackelberg-Nash game framework to analyze the two ISPs' pricing decision and the CP's subsidization decision. In the first stage, ISP_1 and ISP_2 simultaneously set the access price p_1 and p_2. In the second stage, if ISP_1 and ISP_2 provide sponsored data contracts to the CP, then the CP decides the amount of subsidization s_1 and s_2 it is willing to offer to users of the ISP_1 and ISP_2 respectively. If ISP_i does not provide sponsored data contract, then s_i is automatically set to 0. After the access price of the two ISPs and CP's subsidization plan are revealed to the potential users, market demand is realized.

Since the two ISPs are competing with each other, the actual demand faced by an ISP is determined by the access price subtracting subsidization $p_i - s_i$, which we refer to as ISP_i's effective price of both ISPs. We use a linear function to characterize ISP_i's demand:

$$D_i = a_i - b(p_i - s_i) + \theta\left[\left(p_j - s_j\right) - (p_i - s_i)\right], j \neq i \qquad (4\text{-}1)$$

where $a_i > 0, b > \theta \geq 0$. a_i is ISP_i's potential market size, b is a positive coefficient denoting the responsiveness of an ISP's demand to its own price variation, and θ is a non-negative coefficient implying the intensity of competition. The formulation of demand function (4-1) guarantees that the total demand remains unchanged when the competition intensity θ changes (Choi, 1991; Tsay & Agrawal, 2000). ISP_i's revenue comes from the Internet access fee charged to users and the CP, and its marginal cost is normalized to zero. Thus, ISP_i's profit function can be written as:

$$\pi_{ISP_i} = p_i D_i \qquad (4\text{-}2)$$

The CP does not charge users directly, but instead it monetizes the traffic via advertising and other value-added services. Assume that each user brings the CP a profit margin r. The CP's cost is composed of user-related cost, such as customer acquisition cost and customer retention cost, and subsidization cost. Since user-related cost can be marginalized to zero without loss of generality, CP's profit function can be formulated as:

$$\pi_{CP} = r \sum_{i=1,2} D_i - \sum_{i=1,2} s_i D_i \qquad (4\text{-}3)$$

To simplify the analysis, we assume that the two ISPs have the same potential market size, i.e., $a_1 = a_2 = a$, and that all parameters are common knowledge to all players. In the following sections, we analyze the equilibrium results in three scenarios respectively: (1) neither ISP provides sponsored data contract; (2) only ISP_1 provides sponsored data contract; (3) both ISPs provide sponsored data contracts.

4.3.1 No sponsored data

When neither ISP provides sponsored data services, the CP has no decision to make, and ISP's profit function reduces to $\pi_{ISP_i} = p_i[a - bp_i + \theta(p_j - p_i)]$. We refer to this case as the benchmark case. As $\frac{\partial^2 \pi_{ISP_i}}{\partial p_i^2} = -2(b+\theta) < 0$, π_{ISP_i} is concave in p_i, so ISP_i's equilibrium pricing decision can be obtained by solving following first order conditions:

$$\frac{\partial}{\partial p_i}\pi_{ISP_i} = a - 2bp_i + (-2p_i + p_j)\theta = 0 \ (i, j = 1,2, i \neq j) \quad (4\text{-}4)$$

Solving (4-4) yields ISPs' equilibrium access price $p_1^0 = p_2^0 = \frac{a}{2b+\theta}$, where the superscript "0" denotes the no sponsored data scenario. The equilibrium market demand and each party's profit can be calculated subsequently by substituting above equilibrium prices into the demand functions and profit functions.

Lemma 4.1 When neither ISP offers a sponsored data contract, the equilibrium access prices and other equilibrium results are summarized in Table 4.1.

Table 4.1 Equilibrium access prices and other equilibrium results when no sponsored data contract is offered

p_i^0	D_i^0	$\pi_{ISP_i}^0$	π_{CP}^0
$\frac{a}{2b+\theta}$	$\frac{a(b+\theta)}{2b+\theta}$	$\frac{a^2(b+\theta)}{(2b+\theta)^2}$	$\frac{2ar(b+\theta)}{2b+\theta}$

4.3.2 Only ISP_1 provides sponsored data

In current telecom market practices, it is not rare to see cases where a CP only provides data subsidization for users of a certain ISP. For example, China Unicom and Tencent jointly launched a data plan called " DaWangKa ", allowing China Unicom users who subscribe to this data plan to use applications developed by Tencent, including WeChat, QQ music and etc., without any data charge. China Telecom also collaborates with Alibaba and launched a similar data plan called "Ali Yu Ka". In this section, we conduct equilibrium analysis in the scenario where only one of the ISP provides sponsored data contract to the CP and then examine the impact of sponsored data services on the telecom service supply chain.

As ISP_1 and ISP_2 both has a potential market size a and are symmetric, we can without loss of generality assume that only ISP_1 offers sponsored data option to the CP. As ISP_2 does not offer sponsored data services, $s_2 = 0$. Substituting $s_2 = 0$ into function (4-1) and (4-3), ISP_1's and ISP_2's demand functions become$D_1 = a - b(p_1 - s_1) + \theta[p_2 - (p_1 - s_1)]$ and $D_2 = a - bp_2 + \theta[(p_1 - s_1) - p_2]$ respectively, and the profit function of the CP reduces to $\pi_{CP} = (r - s_1)D_1 + rD_2$.

The subgame perfect equilibrium (SPE) of this game is solved backwards. First, given the ISPs access price p_1 and p_2, we solve for the CP's optimal subsidization decision s_1. Since $\frac{\partial^2 \pi_{CP}}{\partial s_1^2} = -2(b + \theta) < 0, \pi_{CP}$ is concave in s_1. Solving the first order condition $\frac{\partial}{\partial s_1}\pi_{CP} = -a + b(p_1 + r - 2s) + (p_1 - p_2 - 2s)\theta = 0$, we get that the CP's optimal unconstrained subsidization decision is $s_1 = \frac{-a+b(p_1+r)+(p_1-p_2)\theta}{2(b+\theta)}$. Plugging in this result into the ISPs' demand functions, the first-order derivative of the ISP's demand versus its own price can be calculated as $\frac{dD_1}{dp_1} = -\frac{(b+\theta)}{2}$ and $\frac{dD_2}{dp_2} = -\left(b + \theta + \frac{\theta}{2(b+\theta)}\right)$. Compared to the first-order derivative of demand versus price $\frac{dD_1}{dp_1} = \frac{dD_2}{dp_2} = -(b + \theta)$ when no sponsored data services is offered, we can see that the implementation of sponsored data at ISP_1 reduce the decrease of demand caused by price increase for both ISP_1 and ISP_2, implying that the introduction of sponsored data services enhances the pricing power of both ISPs.

As CP's subsidization decision s_1 is bounded on the interval $[0, p_1]$, CP's optimal response to ISPs' pricing decisions can be formulated as the following piecewise function:

$$s_1^*(p_1, p_2) = \begin{cases} 0, & if\ \frac{-a+b(p_1+r)+(p_1-p_2)\theta}{2(b+\theta)} \le 0 \\ \frac{-a+b(p_1+r)+(p_1-p_2)\theta}{2(b+\theta)}, & if\ 0 < \frac{-a+b(p_1+r)+(p_1-p_2)\theta}{2(b+\theta)} \le p_1 \\ p_1, & if\ \frac{-a+b(p_1+r)+(p_1-p_2)\theta}{2(b+\theta)} > p_1 \end{cases} \quad (4\text{-}5)$$

Then we analyze ISPs' equilibrium pricing decisions in the first stage. Each ISP takes the CP's best response to ISPs' pricing decisions in the first stage as well as the rival ISP's pricing strategy into account while setting the access price. Since the CP's optimal response s_1^* is a piecewise function, ISP's profit function should be a piecewise function of the access price, of which the composition depends on the access price of the rival ISP. Thus, we are not able to prove that π_{ISP_i} is a concave function, nor can we prove that ISP_i's best response function is a contraction, so the existence of Nash equilibrium can not be established. Moreover, it is very difficult, if not impossible, to derive the two ISPs' best response function to each other due to the interdependency of their profit functions.

To obtain the Nash equilibrium in the first stage without deriving ISPs' best response function, we adopt a procedure of calculating the potential equilibria and then validating their stability. The logic of our method is as follows. If there exists a Nash equilibrium, then the two ISPs must hold the same expectation about the CP's subsidization decision in the second stage in equilibrium. As the CP has three possible types of decision (not subsidize, partially subsidize, and fully subsidize), correspondingly there are three possible types of equilibrium. We derive and examine these three types of equilibrium one by one to obtain the final equilibrium outcome. The algorithm of finding Nash equilibrium in the first stage can be described as follows:

1) Substitute the CP's three possible types of subsidization decision ($s_1 = 0,\ s_1 = \frac{-a+b(p_1+r)+(p_1-p_2)\theta}{2(b+\theta)},\ s_1 = p_1$) into $\pi_{ISP_i}\ (i = 1,2)$ respectively, and solve the two ISPs' profit maximization problem to obtain their best response functions in each case.

2) In each case, solve the two ISPs' best response functions simultaneously to obtain the equilibrium decisions and regard them as a potential equilibrium. Specifically, we use E^{1N}, E^{1P}, and E^{1F} to denote the equilibrium of no subsidization, partial subsidization and full subsidization when only ISP_1 offers sponsored data contract respectively. The superscript "1" denotes the scenario where only ISP_1 provides sponsored data.

3) In each case, substitute the potential equilibrium into the CP's best response

function $s_1^*(p_1, p_2)$ to obtain CP's subsidization decision, see if it is consistent with the presumption when deriving this equilibrium, and calculate the market condition on which this potential equilibrium may exist. The results are showed in Table 4.2.

4) For each potential equilibrium, check whether ISP_i's access price p_i^1 is an optimal decision in response to the ISP_j's access price p_j^1. If yes, then it is indeed an equilibrium. Substituting the equilibrium prices into $s_1^*(p_1, p_2)$ yields CP's equilibrium subsidization decision, and other equilibrium results can be obtained easily. All proof can be found in section 4.6.

Table 4.2 Potential equilibrium outcome in stage 1 when only ISP_1 offers sponsored data contract

Type of potential equilibrium E^{1j}	E^{1N} (no subsidization)	E^{1P} (Partial subsidization)	E^{1F} (Full subsidization)
p_1^{1j}	$\frac{a}{2b+\theta}$	$\frac{br(4b^2+8b\theta+\theta^2)+a(4b^2+10b\theta+5\theta^2)}{(b+\theta)(8b^2+16b\theta+3\theta^2)}$	$\frac{2br(b+\theta)-a(2a+3\theta)}{2(b+\theta)^2}$
p_2^{1j}	$\frac{a}{2b+\theta}$	$\frac{4ba+7\theta a-b\theta r}{8b^2+16b\theta+3\theta^2}$	$\frac{a}{2(b+\theta)}$
Market condition r	$\left(0, \frac{a(b+\theta)}{b(2b+\theta)}\right]$	$\left(\frac{a(4b^2+10b\theta+5\theta^2)}{b(12b^2+24b\theta+5\theta^2)}, \frac{3a(4b^2+10b\theta+5\theta^2)}{b(4b^2+8b\theta+3\theta^2)}\right)$	$\left[\frac{a(2b+3\theta)}{2b(b+\theta)}, \infty\right)$

Lemma 4.2 When only ISP_1 offers sponsored data contract, the two ISPs' equilibrium pricing decisions, the CP's subsidization decision and other equilibrium results under different market conditions are summarized in Table 4.3.

Table 4.3 Equilibrium outcome when only ISP_1 offers sponsored data contract

Type of equilibrium E^{1j}	E^{1N} (no subsidization)	E^{1P} (Partial subsidization)
p_1^1	$\frac{a}{2b+\theta}$	$\frac{br(4b^2+8b\theta+\theta^2)+a(4b^2+10b\theta+5\theta^2)}{(b+\theta)(8b^2+16b\theta+3\theta^2)}$
p_2^1	$\frac{a}{2b+\theta}$	$\frac{4ba+7\theta a-b\theta r}{8b^2+16b\theta+3\theta^2}$
s_1	0	$\frac{br(12b^2+24b\theta+5\theta^2)-a(4b^2+10b\theta+5\theta^2)}{2(b+\theta)(8b^2+16b\theta+3\theta^2)}$
D_1^1	$\frac{a(b+\theta)}{2b+\theta}$	$\frac{br(4b^2+8b\theta+\theta^2)+a(4b^2+10b\theta+5\theta^2)}{16b^2+32b\theta+6\theta^2}$
D_2^1	$\frac{a(b+\theta)}{2b+\theta}$	$\frac{(4ba+7\theta a-b\theta r)(2b^2+4b\theta+\theta^2)}{2(b+\theta)(8b^2+16b\theta+3\theta^2)}$
$\pi_{ISP_1}^1$	$\frac{a^2(b+\theta)}{2(b+\theta)^2}$	$\frac{\left[br(4b^2+8b\theta+\theta^2)+a(4b^2+10b\theta+5\theta^2)\right]^2}{2(b+\theta)(8b^2+16b\theta+3\theta^2)^2}$
$\pi_{ISP_2}^1$	$\frac{a^2(b+\theta)}{2(b+\theta)^2}$	$\frac{(4ba+7\theta a-b\theta r)^2(2b^2+4b\theta+\theta^2)}{2(b+\theta)(8b^2+16b\theta+3\theta^2)^2}$
π_{CP}^1	$\frac{2ar(b+\theta)}{2b+\theta}$	π_{CP}^{1P}
Market condition r	$r \le r_{1L}$	$r_{1M} \le r \le r_{1H}$

* $r_{1L} < r_{1M} < r_{1H}$, where $r_{1L} = \frac{2(\sqrt{2}-1)a(b+\theta)}{b(2b+\theta)}$, $r_{1M} = \frac{2a(4b^2+10b\theta+5\theta^2)}{b\left[8(1+\sqrt{2})b^2+16(1+\sqrt{2})b\theta+(4+3\sqrt{2})\theta^2\right]}$,

$$r_{1H} = \frac{a\left[16b^3+60b^2\theta+64b\theta^2+14\theta^3-\sqrt{2(2b^2+4b\theta+\theta^2)}(8b^2+16b\theta+3\theta^2)\right]}{2b\theta(2b^2+4b\theta+\theta^2)},$$

$$\pi_{CP}^{1P} = \frac{a^2(4b^2+10b\theta+5\theta^2)^2+b^2r^2(16b^4+96b^3\theta+200b^2\theta^2+156b\theta^3+25\theta^4)+2ar(80b^5+472b^4\theta+1008b^3\theta^2+920b^2\theta^3+323b\theta^4+36\theta^5)}{4(b+\theta)(8b^2+16b\theta+3\theta^2)^2}$$

Lemma 4.2 shows that, when the CP's profit margin is low, i.e., $r \le r_{1L}$, the two ISPs' pricing decisions will reach equilibrium E^{1N}, under which the CP will not choose to subsidize, and this situation is equivalent to that in the benchmark case. When the CP's profit margin is moderate, i.e., $r_{1M} \le r \le r_{1H}$, the two ISPs pricing decisions will reach equilibrium E^{1P}, where CP provide partial subsidization for the users of ISP_1. Comparing the two ISPs' profit, the CP's profit, total demand and the total profit of the supply chain in Table 4.3 to their counterparts in the benchmark case, we conclude the impact of ISP_1's adoption of sponsored data program and summarize it in Proposition 4.1.

Proposition 4.1 In the scenario where only ISP_1 offers sponsored data contract to the CP, if CP's profit margin is low ($r \leq r_{1L}$), the telecom market will reach an equilibrium where the CP does not provide subsidization, and this case is equivalent to the benchmark case where no sponsored data contract is offered. If CP's profit margin is moderate ($r_{1M} \leq r \leq r_{1H}$), the telecom market will reach an equilibrium where the CP provides partial subsidization. When the CP indeed provides subsidization to ISP_1's users, the impact of sponsored data services on members of the telecom service supply chain is as follows:

(1) ISP_1's profit increases.

(2) ISP_2's profit increases when $r_{1M} \leq r < \hat{r}^1$ and decreases when $\hat{r}^1 < \mathrm{r} \leq \mathrm{r}_{1\mathrm{H}}$.

(3) CP's profit decreases.

(4) The total demand decreases when $r_{1M} \leq r < \bar{r}^1$ and increases when $\bar{r}^1 < r \leq \mathrm{r}_{1\mathrm{H}}$.

(5) The total profit of the supply chain decreases when $r_{1M} \leq r < \tilde{r}^1$ and increases when $\tilde{r}^1 < r \leq \mathrm{r}_{1\mathrm{H}}$.

where $\hat{r}^1 = a\big[16b^4 + 68b^3\theta + 94b^2\theta^2 + 46b\theta^3 + 7\theta^4 - \sqrt{2b^2 + 4b\theta + \theta^2}\big(8\sqrt{2}b^3 + 24\sqrt{2}b^2\theta + 19\sqrt{2}b\theta^2 + 3\sqrt{2}\theta^3\big)\big]/[b\theta(2b+\theta)(2b^2 + 4b\theta + \theta^2)]$, $\bar{r}^1 = \frac{a(8b^3+28b^2\theta+34b\theta^2+17\theta^3)}{b(8b^3+24b^2\theta+20b\theta^2+5\theta^3)}$, $\tilde{r}^1 = a\big[(l) + 2(m)\sqrt{(n)}\big]/[b(2b+\theta)^2(o)]$

$(l) = 64b^6 + 480b^5\theta + 1440b^4\theta^2 + 2152b^3\theta^3 + 1624b^2\theta^4 + 556b\theta^5 + 65\theta^6$

$(m) = 16b^3 + 40b^2\theta + 22b\theta^2 + 3\theta^3$

$(n) = 16b^6 + 96b^5\theta + 232b^4\theta^2 + 304b^3\theta^3 + 251b^2\theta^4 + 134b\theta^5 + 36\theta^6$

$(o) = 48b^4 + 224b^3\theta + 348b^2\theta^2 + 196b\theta^3 + 29\theta^4$

As illustrated by Proposition 4.1, the introduction of sponsored data services can always improve ISP_1's profit, and ISP_2's profit can benefit when the sponsoring CP's profit margin is low due to the enhanced pricing power. As the CP's profit margin further increases, it can profit more from a larger user base, so it is prone to subsidize more of the users' access price. This largely reduces ISP_1's effective price and put ISP_2 at a disadvantage in the market competition, resulting in a lower market share and a decrease in profit. Proposition 4.1 (3,4) summarize the impact of sponsored data services on consumer welfare and supply chain profit. When the CP's profit margin is relatively small, the number of users that are willing to purchase Internet access decreases as well as the supply chain profit decrease when sponsored data services are offered. When the sponsoring CP's profit margin is high, consumer welfare and supply chain profit will both increase. Comparing the results in Proposition 4.1 to those in Proposition 3.1 reveals that the impact of sponsored data services on the telecom service supply chain and each participant of it when only ISP_1 offers sponsored data is similar to that in a market with a monopolistic ISP.

Proposition 4.2 In the scenario where only ISP_1 offers sponsored data contract to the CP, as the competition intensity θ between the two ISPs increases,

(1) ISP_1 gets greater profit increase from the implementation of sponsored data;

(2) the CP's profit loss due to the participation in sponsored data decreases

(3) the increase/loss in the total demand due to the implementation of sponsored data increases/decreases.

Proposition 4.2 shows that, the more intense the ISP competition is, the greater the benefits are for the participants of sponsored data practices. In a market where ISPs compete fiercely in price, offering sponsored data services can more effectively improve an ISP's profit, and the CP who provides subsidization for users' data consumption can also do it at a lower profit loss. Moreover, a CP with a relatively high profit margin ($r > \bar{r}^1$) can obtain more users by providing subsidization. These results imply that introducing sponsored data practice in a competitive market is generally beneficial to the healthy development of the telecom market.

Another point worth noting is that no equilibrium exists when $r_{1L} < r < r_{1M}$ or $r > r_{1L}$. To examine the possible outcome under these conditions, we run a Python program to calculate the optimal response of ISP_1 (ISP_2) to $ISP_2's$ ($ISP_1's$) various possible pricing decisions and then delineate ISP_1's and ISP_2's best response function.

Figure 4.1 and Figure 4.2 respectively illustrate ISP_1's and ISP_2's best response to each other under two conditions: $r = 10.3$ ($r_{1L} < 10.3 < r_{1M}$) and $r = 75$ ($75 > r_{1H}$). Parameters are set at $a = 100$, $b = 5, \theta = 3$. We also run this numerical experiment under other sets of parameters, and the results are similar to the pattern illustrated in Figure 4.1 and Figure 4.2, which suggest that these patterns are representative.

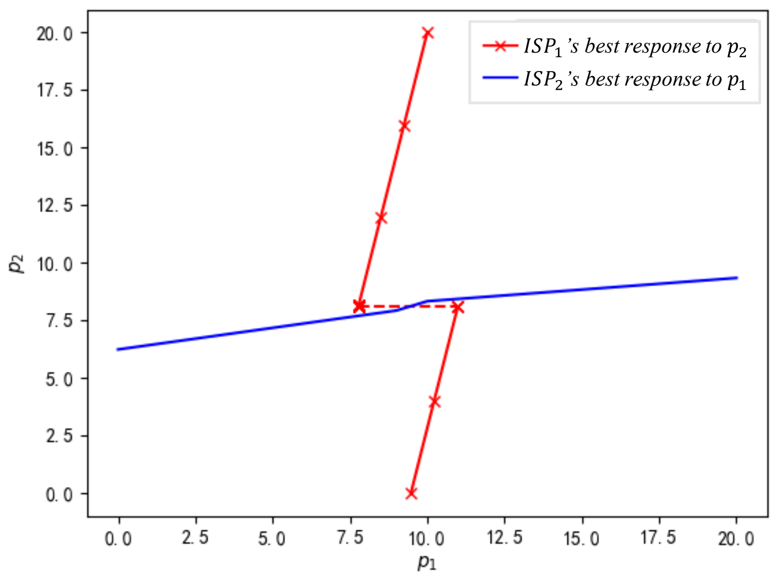


Figure 4.1 Best response function of ISP_1 and ISP_2 when $r = 10.3$

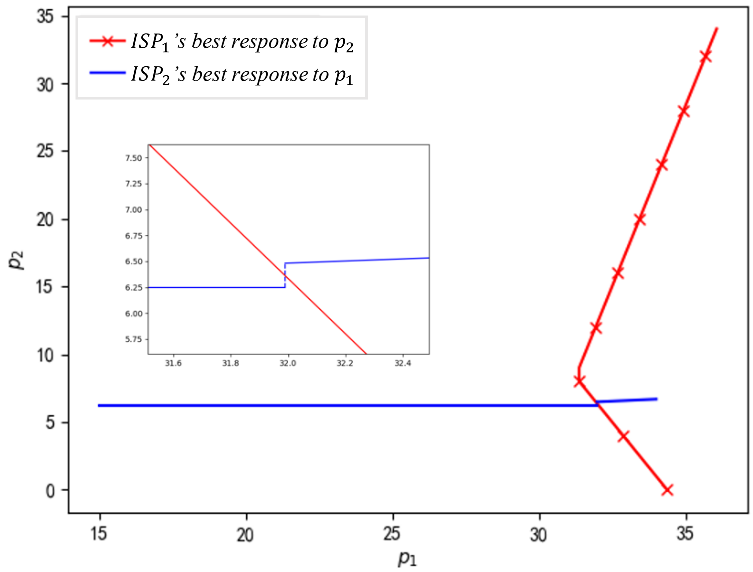


Figure 4.2 Best response function of ISP_1 and ISP_2 when $r = 75$

From Figure 4.1, we can see that ISP_1's best response function is discontinuous around $p_2 = 8.1$, resulting in no intersection of the best response function of ISP_1 and ISP_2. Actually, our numerical experiments under other sets of parameters show that there is always a jump in ISP_1's best response function when the CP's profit margin falls in interval (r_{1L}, r_{1M}), so no pure strategy Nash equilibrium exists. This is primarily because that the CP's profit margin does not allow it to subsidize much of the users' access cost, so that ISP_1 cannot raise the access price without losing too much users if it offers sponsored data services. In this situation, whether sponsored data services can enhance ISP_1's profit depends on ISP_2's pricing strategy: if ISP_2 sets a relatively high access price, then ISP_1 can raise its access price to improve its profit at a cost of losing only a few of subscribers due to the CP's subsidization; upon observing its rival's high price strategy, ISP_2 will lower the access price to gain more competitive edges, which will in turn cause ISP_1 to follow ISP_2 and reduce its access price to avoid more customer churn. However, numerical results show that the interval (r_{1L}, r_{1M}) covers only a rather narrow range, so we do not have to worry too much about the difficulty to predict ISPs' pricing strategy due to the nonexistence of equilibrium in this interval.

Similarly, Figure 4.2 shows that there is a jump in ISP_2's best response function when the CP's profit margin is high, i.e., $r > r_{1H}$, so no pure strategy Nash equilibrium exists in this interval neither. On the other hand, the magnitude of the jump in ISP_2's best response function is small, so that we can roughly predict the pricing strategy of ISP_1 and ISP_2 despite that their prices cannot converge to an equilibrium in theory. When the CP's profit margin is high, ISP_1 will take full advantage of CP's ability to subsidize and raise the access price to a level at which the CP will cover almost all the access cost for its users. To compete with ISP_1's low effective price after subsidization, ISP_2 will set a low access price.

4.3.3 Two ISPs provide sponsored data

Despite those sponsored data programs (such as Tencent Wang Ka) exclusive for users of a certain ISP, there are also sponsored data programs in which the CP offers subsidization for multiple ISPs' subscribers. For example, NetEase Cloud Music, a music streaming service provider in China, offers "data free" service to subscribers of all three major mobile operators at a price around 9 yuan/month. By purchasing this service, users can stream music freely without worrying about additional data usage charge from the ISP at a price much lower than the normal data price. Many other content providers, such as iQiyi and Youku, also launched similar programs to encourage users to spend more time on their apps.

To examine the impact of sponsored data services when both ISPs provide sponsored data contract to the CP, we first analyze the equilibrium decisions of both ISPs and the CP. For simplicity, we assume that the CP provides equal subsidization for users of ISP_1 and ISP_2, i.e., $s_1 = s_2 = s$, and $0 \leq s \leq min\{p_1, p_2\}$. The demand function is now $D_i = a - b(p_i - s) + \theta[p_j - p_i]$. The CP's profit function is $\pi_{CP} = (r - s)(D_1 + D_2)$, and ISP_i's profit function remains to be $\pi_{ISP_i} = p_i D_i$. Similar to the analysis in section 3.1.1, we first solve for the CP's optimal subsidization decision in the second stage. Since $\frac{\partial^2 \pi_{CP}}{\partial s_1^2} = -4b < 0$, π_{CP} is concave in s. Solving the first order condition $\frac{\partial}{\partial s_1}\pi_{CP} = -2a + b(p_1 + p_2 + 2r - 4s) = 0$, we obtain that the CP's optimal unconstrained subsidization is $s = \frac{-2a+b(p_1+p_2+2r)}{4b}$. As s is bounded on the interval $[0, min\{p_1, p_2\}]$, CP's optimal response function can be characterized as follows:

(1) when $p_1 \leq p_2$:

$$s^* = \begin{cases} 0, & if \ \frac{-2a+b(p_1+p_2+2r)}{4b} \leq 0 \\ \frac{-2a+b(p_1+p_2+2r)}{4b}, & if \ 0 < \frac{-2a+b(p_1+p_2+2r)}{4b} \leq p_1 \\ p_1, & if \ \frac{-2a+b(p_1+p_2+2r)}{4b} > p_1 \end{cases} \quad (4\text{-}6)$$

(2) when $p_1 > p_2$:

$$s^* = \begin{cases} 0, & if \ \frac{-2a+b(p_1+p_2+2r)}{4b} \leq 0 \\ \frac{-2a+b(p_1+p_2+2r)}{4b}, & if \ 0 < \frac{-2a+b(p_1+p_2+2r)}{4b} \leq p_2 \\ p_2, & if \ \frac{-2a+bp(p_1+p_2+2r)}{4b} > p_2 \end{cases} \quad (4\text{-}7)$$

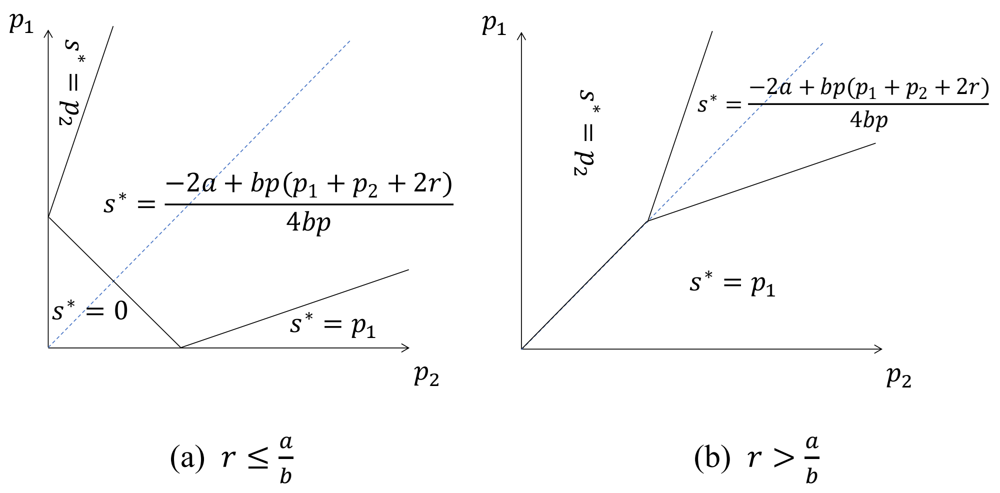


Figure 4.3 CP's optimal subsidization decision in stage 2 when both ISPs offer sponsored data contract

Figure 4.3 visually demonstrates the CP's optimal subsidization decision after observing p_1 and p_2. As Figure 4.3(a) shows, a CP with a low profit margin ($r \leq \frac{a}{b}$)

will not subsidize when the access prices of the two ISPs are similar and low, because the total market demand is already high in this scenario. If the access prices of the ISPs are similar and high, then the CP will provide partial subsidization for users of both ISPs. If the prices of the two ISPs differ greatly, the CP will fully subsidize the users of the cheaper ISP and partially subsidize the other one. Figure 4.3(b) illustrates the results for CP with a high profit margin ($r > \frac{a}{b}$). If the access prices of the two ISPs are comparable and high, then the CP will subsidize part of the users' data cost. Otherwise the CP will fully subsidize the users of the cheaper ISP and partially subsidize the other one.

Then we solve for the Nash equilibrium of ISP_1 and ISP_2 in stage 1. As the two ISPs have the same potential market size, and the CP's subsidization for the two ISPs are equal, the Nash game in stage 1 is a symmetric game. In the following, we focus only on the symmetric equilibrium where ISP_1 and ISP_2 set the access price at same level. The algorithm described in 3.1.1 is adopted to find the potential equilibria and final equilibrium outcome, which are summarized in Table 4.4 and Lemma 4.3 respectively. The superscript "2" denotes scenario where both ISPs provide sponsored data.

Table 4.4 Potential equilibrium outcome in stage 1 when both ISPs offer sponsored data contract

Type of potential equilibrium E^{2j}	E^{2N} (no subsidization)	E^{2P} (Partial subsidization)	E^{2F} (Full subsidization)
p_i^{2j}	$\frac{a}{2b+\theta}$	$\frac{2(a+br)}{5b+4\theta}$	$\left[0, r-\frac{a}{b}\right]$
Market condition r	$\left(0, \frac{a(b+\theta)}{b(2b+\theta)}\right]$	$\left(\frac{a(3b+4\theta)}{b(7b+4\theta)}, \frac{a(7b+4\theta)}{b(3b+4\theta)}\right)$	$\left[\frac{a}{b}, \infty\right)$

Lemma 4.3 When both ISPs offer sponsored data contract, their equilibrium

pricing decisions are summarized in Table 4.5.

Table 4.5 Equilibrium access prices in stage 1 when both ISPs offer sponsored data contract

Type of equilibrium E^{2j}	E^{2N} (no subsidization)	E^{2P} (Partial subsidization)	E^{2F} (Full subsidization)
p_i^2	$\frac{a}{2b+\theta}$	$\frac{2(a+br)}{5b+4\theta}$	$\left[\frac{a}{b+\theta}, min\left\{r-\frac{a}{b}, \frac{a}{\theta}\right\}\right]$
Market condition r	$r \leq \bar{r}_{2L}$	$\underline{r}_{2M} \leq r \leq \bar{r}_{2M}$	$r \geq \underline{r}_{2H}$

* $\underline{r}_{2M} < \bar{r}_{2L} < \underline{r}_{2H} < \bar{r}_{2M}$, where $\bar{r}_{2L} = \frac{a(-5b-6\theta+4\sqrt{3b^2+7b\theta+4\theta^2})}{2b(2b+\theta)}$, $\underline{r}_{2M} = \frac{a[-6b^2-9b\theta-2\theta^2+(5b+4\theta)\sqrt{3b^2+7b\theta+4\theta^2}]}{2b(3b^2+7b\theta+3\theta^2)}$, $\bar{r}_{2M} = \frac{a(7b+4\theta)}{b(3b+4\theta)}$, $\underline{r}_{2H} = \frac{a(2b+\theta)}{b(b+\theta)}$.

Depending on the CP's profit margin, the Nash equilibrium could be of three type, denoting by E^{2N}, E^{2P} and E^{2F} respectively. When CP's profit margin r is lower than r_{2L}, the two ISP's pricing strategy will converge to E^{2N}, and the CP will not subsidize. When CP's profit margin is moderate and falls in $[\underline{r}_{2M}, \bar{r}_{2M})$, ISPs will reach equilibrium E^{2P} where the CP will subsidize part of the users' data cost. Full subsidization equilibrium E^{2F} is only possible when $r \geq \underline{r}_{2H}$. It is worth noting that any price in the interval $\left[\frac{a}{b+\theta}, min\left\{r-\frac{a}{b}, \frac{a}{\theta}\right\}\right]$ can form equilibrium, so there are infinite sets of equilibrium in this scenario. Moreover, since the market conditions of the three types of equilibrium overlap, there are multiple equilibria in the overlapping interval as well. To see to which equilibrium the game will converge when there are multiple equilibria, we use different initial prices and iteratively calculate two ISPs' best response until it converges. Results show that ISPs' pricing strategy always converge to the price nearest to the initial price, implying that in the short run, the equilibrium access price depends on current market price. In the long run, two ISPs may reach an equilibrium that maximizes their profit. In order to calculate each party's profit and concisely analyze the impact of sponsored data services in this scenario, we make the assumption that ISPs' access price will eventually converge to the profit maximizing equilibrium. The equilibrium outcome of the Stackelberg game is characterized in Table 4.6.

Table 4.6 Equilibrium outcome when both ISPs offer sponsored data contract

Type of equilibrium E^{2j}	E^{2N} (no subsidization)	E^{2P} (Partial subsidization)	E^{2F} (Full subsidization)	E^{2F} (Full subsidization)
p_i^2	$\frac{a}{2b+\theta}$	$\frac{2(a+br)}{5b+4\theta}$	$r-\frac{a}{b}$	$\frac{a}{\theta}$
s	0	$\frac{br(7b+4\theta)-a(3b+4}{2b(5b+4\theta)}$	$r-\frac{a}{b}$	$\frac{a}{\theta}$
D_i^2	$\frac{a(b+\theta)}{2b+\theta}$	$\frac{(a+br)(3b+4\theta)}{10b+8\theta}$	a	a
$\pi_{ISP_i}^2$	$\frac{a^2(b+\theta)}{2(2b+\theta)^2}$	$\frac{(a+br)^2(3b+4\theta)}{(5b+4\theta)^2}$	$\frac{a(br-a)}{b}$	$\frac{a^2}{\theta}$
π_{CP}^2	$\frac{2ar(b+\theta)}{2b+\theta}$	$\frac{(a+br)^2(3b+4\theta)^2}{2b(5b+4\theta)^2}$	$\frac{2a^2}{b}$	$2a\left(r-\frac{a}{\theta}\right)$
Market condition r	$r \le \underline{r}_{2M}$	$\underline{r}_{2M} \le r \le \bar{r}_{2M}$	$\bar{r}_{2M} \le r \le \bar{r}_{2H}$	$r > \bar{r}_{2H}$

* $\bar{r}_{2H} = \frac{a(b+\theta)}{b\theta}$

Comparing the equilibrium results in Table 4.6 to those in Table 4.1, one can obtain the impact of sponsored data services when two ISPs in the market both participate in this kind of practice as summarized in Proposition 4.3.

Proposition 4.3 In the scenario where both ISPs offer sponsored data contract to the CP, if CP's profit margin is low ($r \le \bar{r}_{2L}$), the telecom market will reach an equilibrium where the CP does not provide subsidization, and this case is equivalent to the benchmark case where no sponsored data contract is offered. If CP's profit margin is moderate ($\underline{r}_{2M} \leq r \leq \bar{r}_{2M}$), the telecom market will reach an equilibrium where the CP provides partial subsidization. If the CP's profit margin is high ($r \geq \underline{r}_{2H}$), the telecom market will reach an equilibrium where CP provides full subsidization. When the CP indeed provides subsidization to both ISPs' users, the impact of sponsored data services on members of the telecom service supply chain is as follows:

(1) ISP_1's and ISP_2's profits increase.

(2) CP's profit decreases when $r < \frac{a(2b+\theta)}{b\theta}$ and increases when $r \geq \frac{a(2b+\theta)}{b\theta}$.

(3) The total demand decreases when $r < \frac{a(4b^2+7b\theta+4\theta^2)}{b(6b^2+11b\theta+4\theta^2)}$ and increases when $r > \frac{a(4b^2+7b\theta+4\theta^2)}{b(6b^2+11b\theta+4\theta^2)}$.

(4) The total profit of the supply chain decreases when $r < \frac{a(4b^2+7b\theta+4\theta^2)}{b(6b^2+11b\theta+4\theta^2)}$ and increases when $r > \frac{a(4b^2+7b\theta+4\theta^2)}{b(6b^2+11b\theta+4\theta^2)}$.

Proposition 4.3 summarizes the changes in the profits of each party on the telecom service supply chain when both ISPs offer sponsored data services. It can be seen that ISPs can effectively extract part of the CP's profit through sponsored data even under competition, so as to improve their profits. For the CP, if both ISPs in the market offer sponsored data contract, then whether it can profitably subsidize the users depends on if its profit margin is high enough. The reason lies in the fact that the CP's subsidization cannot exceed the access price of the cheaper ISP, which virtually enhances the mutual restriction between the two ISPs' access prices. The price restriction effect is most prominent when the CP's profit margin is high: when $r > \bar{r}_{2H}$, ISPs' equilibrium access price is $\frac{a}{\theta}$, indicating that an ISP cannot improve its access price with the CP's profit margin in order to extract the most profit from the CP as it does in monopolistic scenario or in the scenario where it is the only ISP that provides sponsored data services. In fact, if an ISP observes that the CP has a high margin and attemps to set an access price higher than $\frac{a}{\theta}$, then its rival ISP will set a price lower than $\frac{a}{\theta}$ to compete for more users, resulting in a profit decrease for the former ISP. As the portion of CP's profit margin above $\frac{a}{\theta}$ can not be extracted by the ISPs through sponsored data contract, it is possible for a CP with large profit margin to benefit from subsidizing the users' data cost. When $r < \bar{r}_{2H}$, the price restriction effect is relatively weak compared to ISPs' pricing power enhanced by the sponsored data services, so their equilibrium access prices increase with r, and the CP suffers a profit loss by participating in sponsored data. At last, how total demand and the total supply chain profit are affected when both ISPs provide sponsored data services depends on how large the CP's profit margin is.

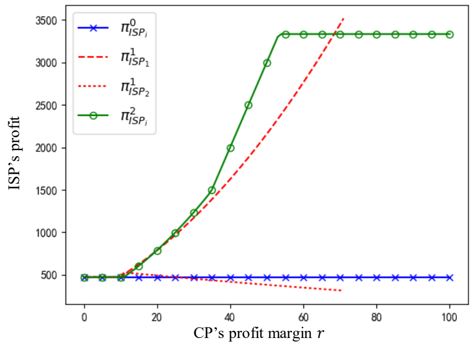
4.3.4 Numerical experiments and discussion

Section 4.3.2 and 4.3.3 theoretically analyze the decisions and profits of each member on the telecom service supply chain in scenario where only ISP_1 provides sponsored data and both ISPs provide sponsored data respectively, and this section employs numerical experiments to graphically illustrate the theoretical results in previous sections. Figure 4.4 depicts how equilibrium results change with the CP's profit margin in scenarios where neither ISP provides sponsored data, only ISP_1 provides sponsored data and both ISPs provide sponsored data (denoted by superscripts "0", "1" and "2" respectively), and the model parameters are set at $a = 100, b = 5, \theta = 3, r \in (0,70]$.

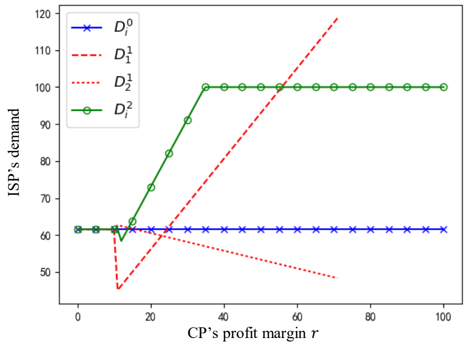
Figure 4.4(a,b) shows that, ISP_1 can gain substantially more profit from offering sponsored data when it is the only ISP in the market that offers this kind of service. However, ISP_1's market share would decline if the CP's profit margin is relatively low, so ISP_1 should carefully balance the tradeoff between profitability and market share when considering whether to introduce sponsored data services especially when the CP's profitability is low. Comparing $\pi^1_{ISP_1}$ and $\pi^2_{ISP_1}$ in Figure 4.4(a) and D^1_1 and D^2_1 in Figure 4.4(b) gives us another important insight, that an ISP should reach exclusive sponsored data agreement with an CP in order to achieve higher profit and market share if the CP's profitability is high, otherwise it is more beneficial to offer sponsored data services together with its rival ISP.

Figure 4.4(c) shows that, a less profitable CP suffers smaller profit loss if it subsidizes users of both ISPs, and a CP with moderate profit margin is better off subsidizing users of only one of the ISPs in the market. As CP's profit margin further increases, the loss of sponsoring both ISPs shrinks, and a CP with very high profit margin can even earn more profit by offering subsidization to all users in the market compared to the benchmark case where there is no sponsored data practice. From the perspective of expanding user base, offering subsidization to users of both ISP can always attract more users than subsidizing users of only one ISP.

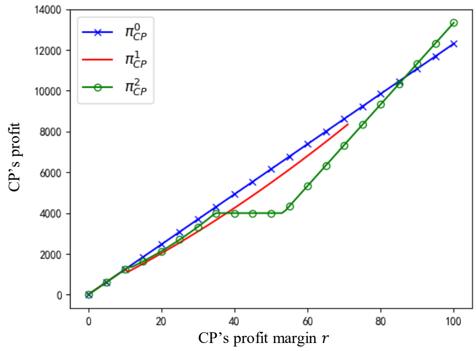
From Figure 4.4(d,e), it can be concluded that two ISPs simultaneously offering sponsored data services is beneficial for the users and the whole telecom service supply chain in most situations. Policymakers should pay more attention to cases where only one ISP provides sponsored data services when they investigate a specific zero-rating or sponsored data program, as this may hurt social welfare.



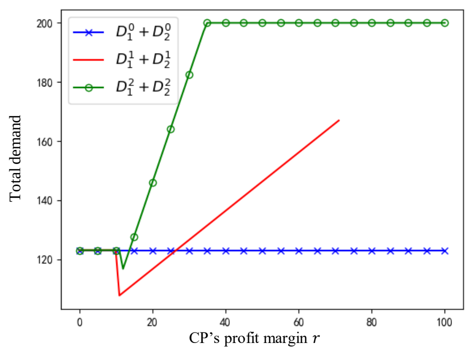
(a) ISP's profit



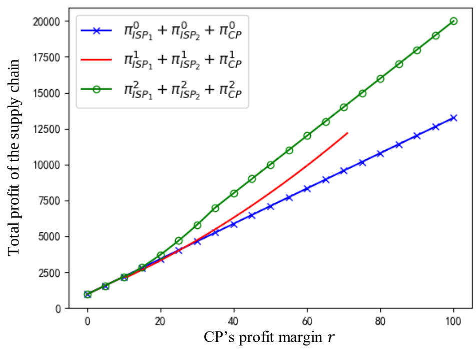
(b) ISP's demand



(c) CP's profit



(d) Total demand



(e) Total profit of the supply chain

Figure 4.4 Impact of sponsored data when there is no vertical integration

4.4 Vertical integration

As the explosion of OTT services has posed great threat to ISPs' traditional primary business, ISPs are actively seeking ways to adapt to this trend, an some of them choose to acquire popular CP to achieve vertical integration. Merging with CPs can bring an ISP extra sources of revenue, and more importantly, it also can increase the ISP's attractiveness as an mobile operator and enhance the ISP's market competitiveness. In recent years, there have been more and more cases of mobile operators acquiring content providers or developing autonomous content services: U.S. operator Verizon acquired AOL and Yahoo! successively in 2015 and 2017; AT&T completed the acquisition of Time Warner in 2018 to bring in its premium content; China Mobile established a subsidiary Migu which specialized in digital content field. These vertical integrated firms may subsidize their network users of the traffic charges generated while browsing the content of the subsidiary CP to attract more network users and to improve the overall profit. For example, AT&T announced that their network users can freely stream DirecTV Now without impacting their data plans after acquiring DirecTV. The subsidiary CP of the integrated firm may also subsidize other ISPs' customers. For example, DirecTV Now is also on the list of the BingeON program, a zero-rating program provided by T-mobile , one of AT&T's major competitor in the U.S.

This section mainly discusses the interaction between vertical integration and sponsored data services and provides decision support for ISPs' acquisition strategy and sponsored data plans. Specifically, we analyze how the competition between the two ISPs is affected when one of the ISP integrates with the CP and possibly provides data subsidization for its rival ISP. We also examine the effects of vertical integration of one ISP and the CP when sponsored data services are already offered by comparing the equilibrium results in this section and those in section 4.3. As the two ISPs are symmetric, we assume that ISP_1 and the CP integrate without loss of generality, and refer to the integrated firm as IF.

Since the CP and ISP_1 belong to a same firm, there is no payment issue between them, and there is no need for the CP to explicitly subsidize ISP_1's users as this practice only transfers profit from the CP to the ISP. Instead, the CP's subsidization for ISP_1's users is directly reflected in the access price p_1. The sequence of the game between IF and ISP_2 is as follows: in stage 1, IF and ISP_2 simultaneously decide the access price p_1 and p_2; in stage 2, if ISP_2 provides sponsored data services, then IF decides the amount of subsidization s_2 ($s_2 \in [0, p_2]$), otherwise no decisions have to be made in this stage. Market demand realizes after the access price and the subsidization plan are revealed to the public. IF and ISP_2 make decisions to maximize their own profits.

IF has two group of users: the network access users (D_1) of its subsidiary ISP_1 and the content users $(D_1 + D_2)$ of its subsidiary CP, and its revenue also consists of two parts: revenue from network access services and content services. IF's cost comes from its subsidization for ISP_2's users, so its profit function can be formulated as

$$\pi_{IF} = p_1 D_1 + r(D_1 + D_2) - s_2 D_2 \tag{4-10}$$

where the demand function D_1 and D_2 can be obtained by substituting $s_1 = 0$ into equation (4-1). ISP_2's profit function remains as $\pi_{ISP_2} = p_2 D_2$.

4.4.1 ISP_2 does not offer sponsored data

When ISP_2 does not offer sponsored data, i.e., $s_2 = 0$, IF's and ISP_2's profit function can be written as $\pi_{IF} = a(p_1 + 2r) - b(p_1^2 + p_1 r + p_2 r) + \theta p_1(p_2 - p_1)$ and $\pi_{ISP_2} = p_2[a - bp_2 + \theta(p_1 - p_2)]$ respectively. This case is referred to as the benchmark case in vertical integration scenario. As $\frac{\partial^2 \pi_{IF}}{\partial p_1^2} = \frac{\partial^2 \pi_{ISP_2}}{\partial p_2^2} = -2(b + \theta) < 0$, π_{IF} and π_{ISP_2} are both concave, so solving their first order conditions $\begin{cases} a - b(2p_1 + r) + \theta(p_2 - 2p_1) = 0 \\ a - 2bp_2 + \theta(p_1 - 2p_2) = 0 \end{cases}$ together yields the unconstrained optimal pricing decisions $p_1 = \frac{a(2b+3\theta) - 2br(b+\theta)}{4b^2 + 8b\theta + 3\theta^2}$ and $p_2 = \frac{a(2b+3\theta) - b\theta r}{4b^2 + 8b\theta + 3\theta^2}$. Accounting for the nonnegativity of the access price, the equilibrium access price of IF and ISP_2 can be written as:

$$p_1^{V0}=\begin{cases}\frac{a(2b+3\theta)-2br(b+\theta)}{4b^2+8b\theta+3\theta^2}, \text{if} < \frac{a(2b+3\theta)}{2b(b+\theta)}\\ 0, \quad \text{if } r \geq \frac{a(2b+3\theta)}{2b(b+\theta)}\end{cases} \quad (4\text{-}8)$$

$$p_2^{V0}=\begin{cases}\frac{a(2b+3\theta)-b\theta r}{4b^2+8b\theta+3\theta^2}, \quad \text{if } r < \frac{a(2b+3\theta)}{2b(b+\theta)}\\ \frac{a}{2(b+\theta)}, \quad \text{if } r \geq \frac{a(2b+3\theta)}{2b(b+\theta)}\end{cases} \quad (4\text{-}9)$$

The superscripts "V" and "0" represents the vertical integration scenario and the no sponsored data scenario respectively.

Table 4.7 summarizes the equilibrium access prices and other equilibrium results in different market conditions. Comparing the results in Table 4.7 to those in Lemma 4.1 gives us following conclusions.

Proposition 4.4 Comparing with the no vertical integration scenario, the impact of the integration of ISP_1 and CP on members of the telecom service supply chain is as follows:

(1) The total profit of ISP_1 and CP decreases after integration when $r < \frac{2ab\theta^2+3a\theta^3}{b(4b^3+16b^2\theta+20b\theta^2+7\theta^3)}$ and increases otherwise.

(2) ISP_2's profit decreases.

(3) The access prices of ISP_1 and ISP_2 both decrease, and the total market demand increases.

Proposition 4.4 shows that, ISP_1 will set a more competitive access price if it vertically integrate with the CP, so as to achieve higher overall profit at the expense of part of the network access service revenue. Under the pressure of competition, ISP_2 can only reduce the access price, resulting in lower profits. In general, as long as the CP's profitability is not too low, the vertical integration of ISP_1 and CP can always enhance their total profit as well as elevate consumer welfare.

Table 4.7 (vertical integration)Equilibrium outcome when ISP_2 does not offer sponsored data contract

Market condition	$r < \frac{a(2b+3\theta)}{2b(b+\theta)}$	$r \geq \frac{a(2b+3\theta)}{2b(b+\theta)}$
p_2^{V0}	$\frac{a(2b+3\theta)-b\theta r}{4b^2+8b\theta+3\theta^2}$	$\frac{a}{2(b+\theta)}$
D_1^{V0}	$\frac{br(2b^2+4b\theta+\theta^2)+a(2b^2+5b\theta+3\theta^2)}{(2b+\theta)(2b+3\theta)}$	$\frac{a(2b+3\theta)}{2(b+\theta)}$
D_2^{V0}	$\frac{(b+\theta)(2ab+3a\theta-br\theta)}{(2b+\theta)(2b+3\theta)}$	$\frac{a}{2}$
π_{IF}^{V0}	$\frac{a^2(b+\theta)(2b+3\theta)^2+b^2r^2(4b^3+16b^2\theta+20b\theta^2+7\theta^3)+ar(16b^4+72b^3\theta+114b^2\theta^2+75b\theta^3+18\theta^4)}{(2b+\theta)^2(2b+3\theta)^2}$	$\frac{ar(3b+4\theta)}{2(b+\theta)}$
$\pi_{ISP_2}^{V0}$	$\frac{(b+\theta)(2ab+3a\theta-br\theta)^2}{(2b+\theta)^2(2b+3\theta)^2}$	$\frac{a^2}{4(b+\theta)}$

Similar to the case in no vertical integration scenario, ISP_2 can gain the ability to capture part of the CP's revenue by employing sponsored data services. In the following, section 4.4.2 investigates whether sponsored data services can help ISP_2 deal with the competitive pressure brought by the integration of its rival ISP and the CP, and analyzes the impact of sponsored data services in the vertical integration scenario.

4.4.2 ISP_2 offers sponsored data

When ISP_2 offers sponsored data services, the profit function of IF and ISP_2 can be formulated as $\pi_{IF} = (p_1 + r)\big[a - bp_1 + \theta[p_2 - s_2 - p_1]\big] + (r - s_2)[a - b(p_2 - s_2) + \theta(p_1 - p_2 + s_2)]$ and $\pi_{ISP_2} = [a - b(p_2 - s_2) + \theta(p_1 - p_2 + s_2)]$ respectively.

We solve the game backwards and start from the IF's subsidization decision in stage 2. Since $\frac{\partial^2 \pi_{IF}}{\partial s_2^2} = -2(b+\theta) < 0$, π_{IF} is concave in s_2, so solving the first order condition $\frac{\partial \pi_{IF}}{\partial s_2} = 0$ yields IF's unconstrained optimal subsidization $s_2 = \frac{-a+(b+\theta)p_2-2\theta p_1+br}{2(b+\theta)}$. IF's optimal subsidization decision is $s_2^*(p_1, p_2) =$

$min\left\{\left(\frac{-a+(b+\theta)p_2-2\theta p_1+br}{2(b+\theta)}\right)^+, p_2\right\}$ as $s_2 \in [0, p_2]$.

To solve the equilibrium prices of IF and ISP_2 in stage 1, we first derive their best response functions to each other's pricing strategy.

Lemma 4.4 (Vertical Integration) When ISP_2 offers sponsored data contract, IF's and ISP_2's best response functions in stage 1 are summarized in Table 4.8 and Table 4.9 respectively.

Table 4.8 IF's best response function to ISP_2's access price

Market condition	IF's best response function $p_1^*(p_2)$
$0 < r < \frac{a}{b}$	$\begin{cases} \frac{a-br+\theta p_2}{2(b+\theta)}, if\ 0 < p_2 \le \frac{a-br}{b} \\ \frac{a-br}{2b}, \quad if\ p_2 > \frac{a-br}{b} \end{cases}$
$r \ge \frac{a}{b}$	0

Table 4.9 ISP_2's best response function to IF's access price

Market condition	ISP_2's best response function $p_2^*(p_1)$
$0 < r < \frac{a}{4b}$	$\frac{a+\theta p_1}{2(b+\theta)}$
$\frac{a}{4b} \le r < \frac{a}{3b}$	$\begin{cases} \frac{a-br+2\theta p_1}{b+\theta}, if\ 0 < p_1 \le \frac{4br-a}{5\theta} \\ \frac{a+\theta p_1}{2(b+\theta)}, \quad if\ p_1 > \frac{4br-a}{5\theta} \end{cases}$
$\frac{a}{3b} \le r < \frac{3a}{b}$	$\begin{cases} \frac{a+br}{2(b+\theta)}, \quad if\ 0 < p_1 \le \frac{3br-a}{4\theta} \\ \frac{a-br+2\theta p_1}{b+\theta}, if\ \frac{3br-a}{4\theta} < p_1 \le \frac{4br-a}{5\theta} \\ \frac{a+\theta p_1}{2(b+\theta)}, \quad if\ p_1 > \frac{4br-a}{5\theta} \end{cases}$
$r \ge \frac{3a}{b}$	$\begin{cases} \frac{br-a-2\theta p_1}{b+\theta}, \quad if\ 0 < p_1 \le \frac{br-3a}{4\theta} \\ \frac{a+br}{2(b+\theta)}, if\ \frac{br-3a}{4\theta} < p_1 \le \frac{3br-a}{4\theta} \\ \frac{a-br+2\theta p_1}{b+\theta}, if\ \frac{3br-a}{4\theta} < p_1 \le \frac{4br-a}{5\theta} \\ \frac{a+\theta p_1}{2(b+\theta)}, \quad if\ p_1 > \frac{4br-a}{5\theta} \end{cases}$

Under different market conditions, we solve IF's and ISP_2's best response functions simultaneously to obtain their respective access price p_1 and p_2, and then verify whether $p_1(p_2)$ is indeed the best response to $p_2(p_1)$, i.e., whether the set of price is stable. If the set of price is stable, then it is an equilibrium, otherwise equilibrium does not exist. By solving each possible pair of best response functions and verifying the resulting pair of price one by one, we can obtain the equilibrium access prices under various market conditions in stage 1, and IF's equilibrium subsidization decision can be obtained by substituting the equilibrium access prices into $s_2^*(p_1, p_2)$. Table 4.10 summarizes the equilibrium decisions, and other pertinent equilibrium results listed in Table 4.11 can be calculated easily.

Table 4.10 Equilibrium access prices and subsidization when ISP_2 offers sponsored data contract

Market condition	p_1^V	p_2^V	s_2
$0 < r \le r_1$	$\frac{a(2b+3\theta)-2br(b+\theta)}{4b^2+8b\theta+3\theta^2}$	$\frac{a(2b+3\theta)-b\theta r}{4b^2+8b\theta+3\theta^2}$	0
$r_1 < r < r_2$	/	/	/
$r_2 \le r \le r_3$	$\frac{a-br}{2b}$	$\frac{a-br}{b}$	0
$r_3 < r \le r_4$	$\frac{a-br}{2b}$	$\frac{a+br}{2(b+\theta)}$	$\frac{br(3b+2\theta)-a(b+2\theta)}{4b(b+\theta)}$
$r_4 < r \le r_5$	0	$\frac{a+br}{2(b+\theta)}$	$\frac{3br-a}{4(b+\theta)}$
$r > r_5$	0	$\frac{br-a}{b+\theta}$	$\frac{br-a}{b+\theta}$

* $r_1 = \frac{a(2b^2+9b\theta+9\theta^2)}{b(8b^2+21b\theta+11\theta^2)}$, $r_2 = \frac{a(2b+5\theta)}{b(8b+5\theta)}$, $r_3 = \frac{a(b+2\theta)}{b(3b+2\theta)}$, $r_4 = \frac{a}{b}$, $r_5 = \frac{3a}{b}$

Table 4.11 Equilibrium demands and profits when ISP_2 offers sponsored data contract

Market condition	D_1^V	D_2^V	π_{IF}^V	$\pi_{ISP_2}^V$
$0 < r \le r_1$	$\frac{br(2b+4b\theta+\theta^2)\ a(2b^2+5b\theta+3\theta^2}{(2b+\theta)(2b+3\theta)}$	$\frac{(b+\theta)(2ab+3a\theta-}{(2b+\theta)(2b+3\theta}$	π_{IF}^{V1}	$\frac{(b+\theta)(2ab+3a\theta-b\theta r}{(2b+\theta)^2(2b+3\theta)^2}$
$r_1 < r < r_2$	/	/	/	/
$r_2 \le r \le r_3$	$\frac{br(b-\theta)+a(b+\theta}{2b}$	$\frac{br(2b+\theta)-a\theta}{2b}$	$\frac{2abr(b-\theta)+a^2(b+\theta)+b^2r^2(5b+\theta)}{4b^2}$	$\frac{(a-br)(2b^2r-a\theta+b\theta}{2b^2}$
$r_3 < r \le r_4$	$\frac{(a+br)(2b+3\theta)}{4(b+\theta)}$	$\frac{(a+br)}{4}$	$\frac{(a+br)^2(5b+8\theta)}{16b(b+\theta)}$	$\frac{(a+br)^2}{8(b+\theta)}$
$r_4 < r \le r_5$	$\frac{4ab+7a\theta-b\theta r}{4(b+\theta)}$	$\frac{(a+br)}{4}$	$\frac{a^2+b^2r^2+2ar(9b+16\theta)}{16b(b+\theta)}$	$\frac{(a+br)^2}{8(b+\theta)}$
$r > r_5$	a	a	$\frac{a[a+r(b+2\theta)]}{b+\theta}$	$\frac{a(br-a)}{b+\theta}$

$$* \ \pi_{IF}^{V1} = \frac{[a^2(b+\theta)(2b+3\theta)^2+b^2r^2(4b^3+16b^2\theta+20b\theta^2+7\theta^3)+ar(16b^4+72b^3\theta+114b^2\theta^2+75b\theta^3+18\theta^4)]}{(2b+\theta)^2(2b+3\theta)^2}$$

Proposition 4.5 (Vertical Integration) In the scenario where ISP_2 offers sponsored data contract to IF's subsidiary CP, if the CP's profit margin is very low ($r \le r_1$), the telecom market will reach an equilibrium where the IF does not provide subsidization, and this case is equivalent to the benchmark case where ISP_2 does not offer sponsored data. If $r_2 \le r \le r_3$, ISP_2 will raise its access price while IF still will not subsidize ISP_2's users. As CP's profit margin increases, ISP_2 will raise its access price to induce the IF to subsidize. If $r_3 < r \le r_5$, the telecom market will reach an equilibrium where the IF provides partial subsidization. If $r > r_5$, the telecom market will reach an equilibrium where the IF provides full subsidization. When the IF indeed provides subsidization to ISP_2's users, the impact of sponsored data services on members of the telecom service supply chain is as follows:

(1) IF's profit decreases.

(2) ISP_2's profit decreases when $r < \hat{r}^V$ and increases when $r > \hat{r}^V$.

(3) The total demand decreases when $r < \bar{r}^V$ and increases when $r > \bar{r}^V$.

(4) The total profit of the supply chain decreases when $r < \tilde{r}^V$ and increases

when $r > \tilde{r}^V$.

$$* \ \hat{r}^V = \frac{a(2b+3\theta)[8(-1+\sqrt{2})b^3+28(-1+\sqrt{2})b^2\theta+2(-15+14\sqrt{2})b\theta^2+(-11+8\sqrt{2})\theta^3]}{b(16b^4+64b^3\theta+80b^2\theta^2+32b\theta^3+\theta^4)},$$

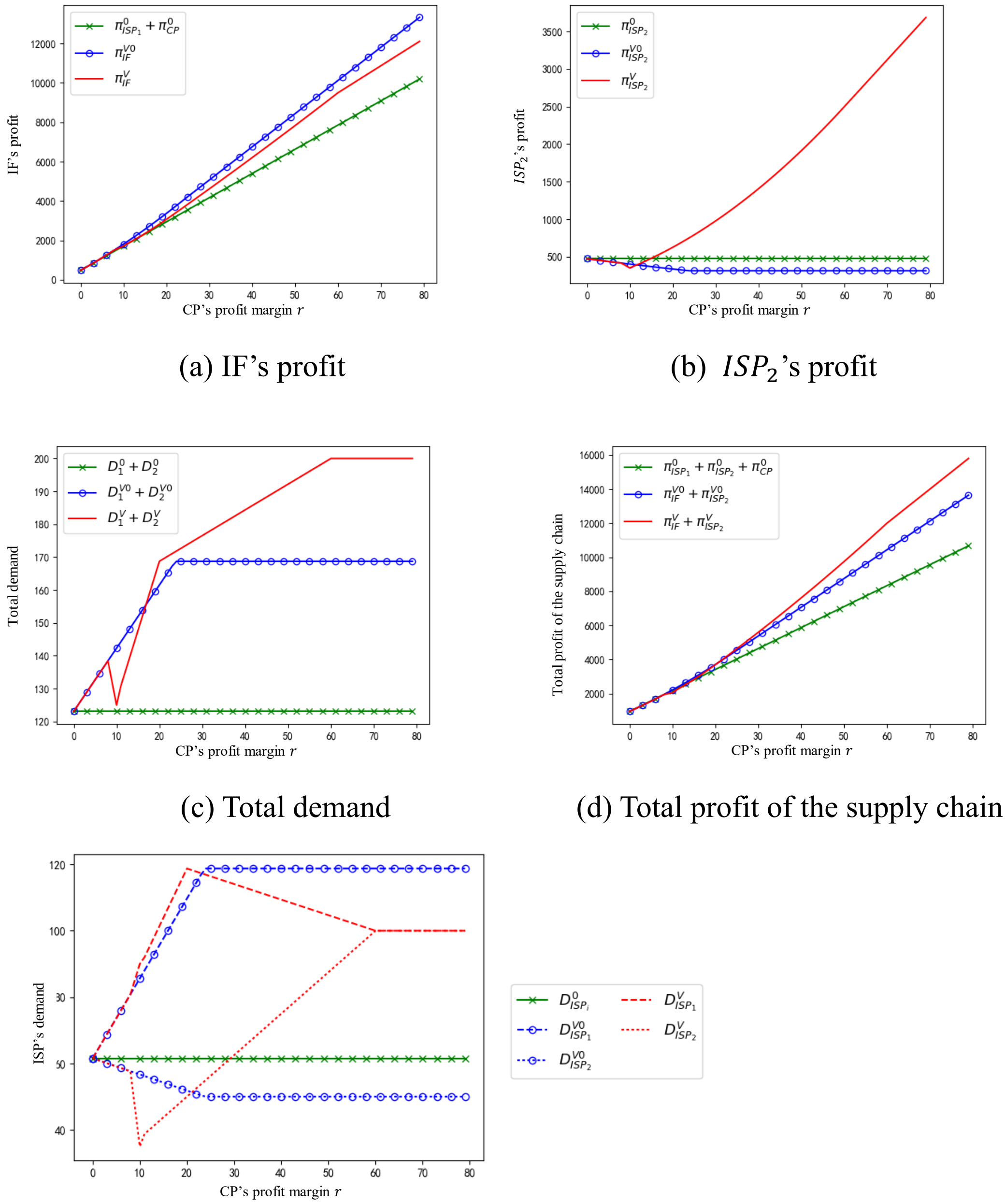
$$\bar{r}^V = \frac{a(2b^2+5b\theta+4\theta^2)}{b(2b^2+7b\theta+4\theta^2)},$$

$$\tilde{r}^V = \frac{a[48b^4+288b^3\theta+616b^2\theta^2+552b\theta^3+171\theta^4-4\sqrt{4b^2+8b\theta+5\theta^2}(4b^3+16b^2\theta+19b\theta^2+6\theta^3)]}{b(16b^4+128b^3\theta+328b^2\theta^2+320b\theta^3+101\theta^4)},$$

and $\hat{r}^V < \bar{r}^V < \tilde{r}^V$.

Conclusions in Proposition 4.5 can be obtained by comparing the equilibrium results in Table 4.11 and Table 4.7. As the formulation of the thresholds in Proposition 4.5 are quite complicated, we use numerical experiments to illustrate how equilibrium demands and profits change with the subsidiary CP's profitability in three scenarios (no vertical integration and no sponsored data, vertical integration but no sponsored data, vertical integration and ISP_2 provides sponsored data, denoted by superscripts "0", "V0" and "V" respectively), in order to show the impact of sponsored data in vertical integration scenario. Other model parameters are set at $a = 100, b = 5, \theta = 3$, and Figure 4.5 demonstrates the results.

Figure 4.5(a) shows that, providing data subsidization for its rival ISP will reduce the IF's profit, despite that IF's overall profit is still higher than the total profit of ISP_1 and the CP when they operate independently. From Figure 4.5(b), it could be observed that ISP_2 can increase its profit by providing sponsored data services if $r > \hat{r}^V$. In particular, if $r > \frac{a[2(\sqrt{2}-1)b+(2\sqrt{2}-1)\theta]}{bp(2b+\theta)}$, ISP_2's profit is even higher than its counterpart when ISP_1 and CP does not integrate. As the two thresholds above are relatively low, ISP_2 can benefit from offering sponsored data services in most situations. To conclude, offering sponsored data contract to IF's subsidiary CP is an effective means for other unintegrated ISPs to win back revenue.



(a) IF's profit

(b) ISP_2's profit

(c) Total demand

(d) Total profit of the supply chain

(e) ISP's demand

Figure 4.5 Impact of ISP_2 offering sponsored data when ISP_1 and the CP integrate

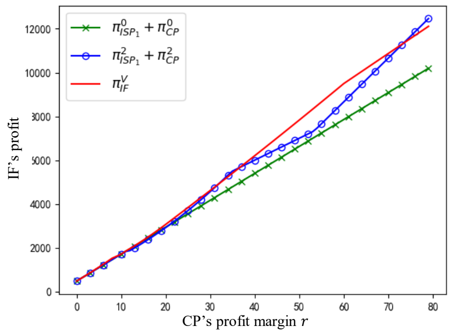
Figure 4.5(c) and Figure 4.5(d) show that the implementation of sponsored data can further raise supply chain efficiency, in terms of consumer welfare and total profit of the supply chain, except when the subsidiary CP's profit margin is low.

Figure 4.5(e) reveals that vertical integration can effectively increase the market demand of the IF's network access service, and that providing data subsidization for ISP_2's users can even further enhance IF's market share in network access market if its subsidiary CP's profitability is relatively low. However, with the increase of the CP's profitability, the importance of revenue from content service increases, and the IF will be more motivated to subsidize the users of its rival ISP so as to attract more content users, resulting in a smaller market share in the network access market. Therefore, the IF should take both the subsidiary CP's profit margin and its primary business goal into consideration when making subsidization decision: if its goal is the optimize the overall profit, do not subsidize; if its goal is to increase the number network access users, subsidize when the CP's profit margin is relatively low and do not subsidize otherwise; if its goal is to increase the number of content users, subsidize when the CP's profit margin is relatively high and do not subsidize otherwise.

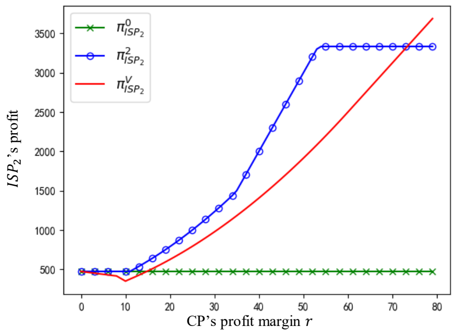
In the following, we conduct similar numerical experiments under the same parameter conditions to show the impact of vertical integration on a telecom market where sponsored data services are already prevalent. Figure 4.6 shows the results and depicts how equilibrium demands and profits change with the subsidiary CP's profitability in three scenarios (no vertical integration and no sponsored data, no vertical integration but both ISPs provide sponsored data, vertical integration and ISP_2 provides sponsored data, denoted by superscripts "0", "2" and "V" respectively).

Figure 4.6(e) indicates that ISP_1 can improve its market share in the network access market by integrating with the CP. In terms of the impact on profit, we combine Figure 4.6(a) and Figure 4.6(b) and find that ISP_1's vertical integration with the CP can only improve the overall profit when CP's profitability is relatively high while substantially reduce the profit of ISP_2, resulting in an unchanged or lower total profit of the supply chain as shown in Figure 4.6(d).

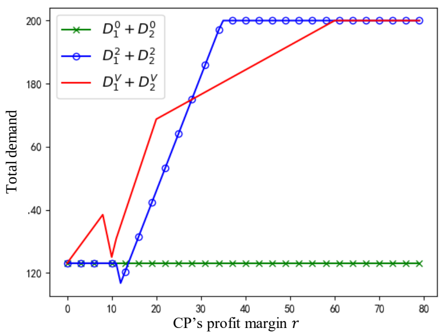
Figure 4.6(c) implies that the impact of vertical integration on the total demand depends on the CP's profitability: total demand increases when CP's profitability is low and decreases when CP's profitability is moderate. When CP's profitability is high, total demand will always reach maximum due to the abundant subsidization offered to the users regardless of whether ISP_1 integrates with the CP or not. Therefore, in a market where sponsored data services are prevalent, regulators should pay special attention to cases which involves ISP integrating with CP with relatively high profit margin, as this could possibly reduce social welfare. Conversely, ISP's integration with CP of relatively low profit margin can enhance social welfare.



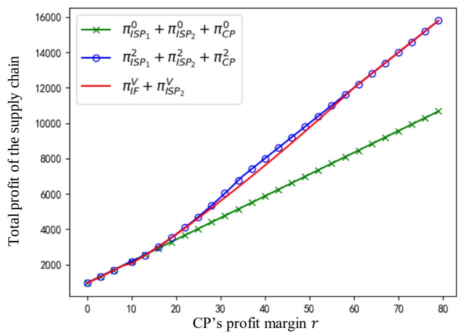
(a) IF's profit



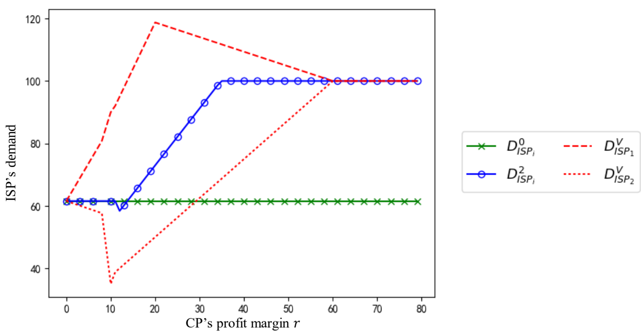
(b) ISP_2's profit



(c) Total demand



(d) Total profit of the supply chain



(e) ISP's demand

Figure 4.6 Impact of ISP_1's vertical integration with the CP when both ISPs offer sponsored data

4.5 Discussion and conclusion

This chapter considers a telecom market with two ISPs and one CP and analyzes their equilibrium decisions under sponsored data contract. By comparing each player's demand and profit in equilibrium across three different scenarios (no sponsored data, only ISP_1 offers sponsored data, both ISPs offer sponsored data), we conclude what impact sponsored data services have on the ISPs, the CP and the users. Since most extant literature on sponsored data model competition among CPs but lack the consideration of competition on the ISP side, this study fills this research gap. This chapter also studies sponsored data in a market where one of the ISP vertically integrates with the CP and analyzes the equilibrium results when the unintegrated ISP do/do not provides sponsored data services. These analysis help us understand how sponsored data services and vertical integration, two practices of great concern, interact with each other, and provides practical guidance for regulation of the telecom market. Remainder of this section summarizes the main findings of this chapter.

In a market where there is no vertical integration and ISPs and the CP operate independently, following insights can be generated from our analysis.

(1) ISPs can use sponsored data services as an effective means to enhance profit, especially when the competition in the network access market is intense. If an ISPs primary business goal is to improve market share, then offering sponsored data services may not serve its goal: when the CP is not profitable enough, allowing it to subsidize the network users' data usage will instead decrease the demand for network access service. In addition, an ISP should fully consider the profitability of the CP when deciding how to cooperate with the CP: if the CP's profit margin is high, then it is better to sign an exclusive sponsored data contract with the CP so as to achieve higher profit as well as larger market share; if the CP's profit margin is moderate or low, then providing sponsored data contract to the CP simultaneously with rival ISPs turns out to be a better strategy.

(2) The CP's profit is usually hurt by sponsored data services, but there are exceptions. An extremely profitable CP can improve its profit by providing subsidization to users of both ISPs. CPs with different profit margin have different

preference when deciding whether to cooperate with one ISP exclusively to offer sponsored data services. The least profitable and the most profitable CPs are more willing to sponsor users of both ISPs, while CPs with moderate profit margin are prone to subsidizing only one of the ISP. Combining ISPs' and CPs' strategies when choosing sponsored data service partner, it can be observed that their preferences are consistent to a certain extent. This theoretical result can partly explain the phenomenon that there are cases where a CP sponsors multiple ISPs and also cases where a CP sponsors one ISP in current telecom market. For example, video streaming service providers Youku and iQiyi, both offer "data free" service to subscribers of all three major mobile operators, while Tencent and Alibaba, two of the largest integrated content providers in China, cooperate with China Unicom and China Telecom respectively and launched their own customized data plans that subsidize the data usage generated by using their services and content.

(3) In regards to social welfare, sponsored data services have positive effects on the overall profit of the whole telecom supply chain and the consumers in most cases, except when the CP's profit margin is very low. One situation that requires special attention from regulators in telecom market is when a less profitable CP offers subsidization to users of only one ISP in the market, which will result in a decrease in number of network access users and harm consumer welfare.

In a market where one of the ISP vertically integrates with the CP, analysis in this chapter offers following implications.

(1) Vertical integration itself can increase the total market demand, enabling more users to access the mobile Internet. When the CP's profit margin is not too low, total profit of the supply chain is enhanced as well, resulting in a higher social welfare. On the other hand, in a telecom market where all ISPs provide sponsored data services already, an ISP's integration with a less profitable CP could raise social welfare, while its integration with a more profitable CP may reduce social welfare.

(2) In a market with vertical integration of ISP and CP, how the implementation of

sponsored data services affects each participant on the telecom service supply chain is similar to that in the no vertical integration scenario: the unintegrated ISP could raise its profit by offering sponsored data services, which helps reverse its disadvantageous position in the ISP market to some extent. From a systematic perspective, the introduction of sponsored data services can increase the unintegrated ISP's profit while lower the profit of the integrated firm, adding up to an enhanced total profit of the supply chain.

(3) When the integrated firm's subsidiary CP's profitability is weak, providing subsidization for the users of the rival ISP will lower the total demand for content, but its market share in the network access market will expand. When the integrated firm's subsidiary CP's profitability is strong, providing subsidization for the users of the rival ISP will do just the opposite: improve total demand for content and reduce market share in the network access market. Therefore, the integrated firm need to carefully evaluate the CP's profitability and its own strategic goal when determining whether to offer subsidization for its rival ISP's users' data usage. This finding may explain why DirecTV NOW, a content provider purchased by AT&T, is also on the list of T-mobile's zero-rating program.

In summary, in a market where there is competition among ISPs, sponsored data services still can effectively improve ISPs' profit. CP's profit is hindered in most cases, and only CP with a high profit margin can benefit from participating in sponsored data services when it provides data subsidization for users of both ISPs in the market. In addition, the positive impact of sponsored data services is more prominent in market where competition among ISPs is more intense. When choosing with whom to cooperate to offer sponsored data services, ISPs and CPs should thoroughly consider their strategic goals in both market share and profit. In terms of social welfare, sponsored data services could improve social welfare in most cases, except when a CP with relatively low profit margin subsidize only one ISP. Another circumstance under which the social welfare will be reduced is that an ISP integrates with a highly profitable CP in a market where sponsored data services are common practice. Therefore, regulators in the telecom market should pay special attention to above two situations when investigating particular sponsored data programs and mergers and acquisitions.

4.6 Proofs in Chapter 4

4.6.1 Proof of Lemma 4.2

We first prove the potential equilibria of the first stage ISP pricing game and the corresponding market conditions given in Table 4.2 when only ISP_1 provides sponsored data services, and then verify whether these potential equilibria are indeed in equilibrium.

(a) Potential equilibria and their corresponding market conditions in Table 4.2

(1) Potential equilibrium E^{1N}: $s_1 = 0$

Substitute $s_1 = 0$ into the demand function of Formula (4-1) and the ISP profit function of Formula (4-2), and obtain $\pi_{ISP_i} = p_i\left[a - bp_i + \theta\left(p_j - p_i\right)\right]$, which is equivalent to the circumstance of no sponsored data. Therefore, the equilibrium pricing of ISP_1 and ISP_2 are $\frac{a}{2\mathrm{b}+\theta}$.

Substitute $p_1^{1N} = p_2^{1N} = \frac{a}{2b+\theta}$ into Formula (4-5) and enable not to be greater than zero. Obtain the market condition where this potential equilibrium E^{1N} may exist is $r \le \frac{a(b+\theta)}{b(2b+\theta)}$.

(2) Potential equilibrium E^{1P}: $s_1 = \frac{-a+b(p_1+r)+(p_1-p_2)\theta}{2(b+\theta)}$.

Substitute $s_1 = \frac{-a+b(p_1+r)+(p_1-p_2)\theta}{2(b+\theta)}$ into the demand function of Formula (4-1) and the ISP profit function of Formula (4-2), and obtain $\pi_{ISP_1} = \frac{p_1}{2}[a - b(p_1 - r) - \theta(p_1 - p_2)]$, $\pi_{ISP_2} = p_2\left(a - bp_2 + \frac{\theta[a+b(p_1-2p_2-r)+\theta(p_1-p_2)]}{2(b+\theta)}\right)$. The equilibrium pricing of ISP_1 and ISP_2 can be obtained from the simultaneous first-order conditions $\frac{\partial}{\partial p_1}\pi_{ISP_1} = \frac{1}{2}[a + b(r - 2p_1) + \theta(p_2 - 2p_1)] = 0$ and $\frac{\partial}{\partial p_2}\pi_{ISP_2} = \frac{-4b^2p_2 + b\theta(p_1 - 8p_2 - r) + \theta^2(p_1 - 2p_2) + a(2b+3\theta)}{2(b+\theta)} = 0$. Substitute the solved equilibrium pricing $p_1^{1P} = \frac{br(4b^2+8b\theta+\theta^2) + a(4b^2+10b\theta+5\theta^2)}{(b+\theta)(8b^2+16b\theta+3\theta^2)}$ and $p_2^{1P} = \frac{4ba+7\theta a - b\theta r}{8b^2+16b\theta+3\theta^2}$ into Equation (4-5) to obtain $s_1 = \frac{-a(4b^2+10b\theta+5\theta^2)+br(12b^2+24b\theta+5\theta^2)}{2(b+\theta)(8b^2+16b\theta+3\theta^2)}$. Set s_1 to be greater than 0 and less than 1. The market conditions that may exist for this potential equilibrium E^{1P} are $\frac{a(4b^2+10b\theta+5\theta^2)}{b(12b^2+24b\theta+5\theta^2)} < r < \frac{3a(4b^2+10b\theta+5\theta^2)}{b(4b^2+8b\theta+3\theta^2)}$.

（3）Potential equilibrium E^{1F}: $s_1 = p_1$

Substitute $s_1 = p_1$ into the demand function of Formula (4-1) and the ISP profit function of Formula (4-2), and obtain $\pi_{ISP_1} = p_1(a + \theta p_2)$, $\pi_{ISP_2} = p_2[a - p_2(b + \theta)]$. Due to $\frac{\partial}{\partial p_1}\pi_{ISP_1} = (a + \theta p_2) > 0$, ISP_1's profit is monotonically increasing with p_1. As $\frac{\partial^2 \pi_{ISP_2}}{\partial p_2^2} = -2(b+\theta) < 0$, the optimal pricing of ISP_2 can be obtained from solving the first-order condition $\frac{\partial}{\partial p_2}\pi_{ISP_2} = a - 2p_2(b+\theta) = 0$, which yields $p_2^{1F} = \frac{a}{2(b+\theta)}$.

Substitute $p_2^{1F} = \frac{a}{2(b+\theta)}$ into Equation (4-5) to obtain $s_1 = \frac{-a(2b+3\theta)+2(b+\theta)[b(p_1+r)+p_1\theta]}{4(b+\theta)^2}$. Therefore, ISP_1 will set the price to the appropriate level of $s = p_1$, $p_1^{1F} = \frac{2br(b+\theta) - a(2a+3\theta)}{2(b+\theta)^2}$. Set $p_1^{1F} \geq 0$ and obtain the market conditions that may exist for this potential equilibrium E^{1F} are $r \geq \frac{a(2b+3\theta)}{2b(b+\theta)}$.

(b) Table 4.3. The first-stage equilibrium pricing when sponsored data services are provided by $\mathbf{ISP_1}$

(1) Verify that when $r \leq \frac{a(b+\theta)}{b(2b+\theta)}$, the potential equilibrium E^{1N} is the subgame perfect equilibrium:

(1.a) First verify that given $p_2 = p_2^{1N}$, the optimal pricing for $\mathrm{ISP_1}$ is p_1^{1N}:

Substitute $p_2 = p_2^{1N} = \frac{a}{2b+\theta}$ into Formula (4-6) to obtain $s_1 = \frac{-2a(b+\theta)+(2b+\theta)[b(p_1+r)+p_1\theta]}{2(b+\theta)(2b+\theta)}$. As of $0 \le s_1 \le p_1$, the optimal response function of CP can be obtained as:

$$s_1^* = \begin{cases} 0,\ if\ 0 \le p_1 < \frac{2a(b+\theta)-br(2b+\theta)}{(b+\theta)(2b+\theta)} \\ \frac{-2a(b+\theta)+(2b+\theta)[b(p_1+r)+p_1\theta]}{2(b+\theta)(2b+\theta)},\ if\ p_1 \ge \frac{2a(b+\theta)-br(2b+\theta)}{(b+\theta)(2b+\theta)} \end{cases}.$$

Correspondingly, $\mathrm{ISP_1}$'s profit function is:

$$\pi_{ISP_1} = \begin{cases} p_1\left[a-(b+\theta)p_1+\frac{\theta a}{2b+\theta}\right], \text{If } 0 \le p_1 < \frac{2a(b+\theta)-br(2b+\theta)}{(b+\theta)(2b+\theta)} \\ \frac{p_1\{2a(b+\theta)-(2b+\theta)[b(p_1-r)+\theta p_1]\}}{2(2b+\theta)}, \text{If } p_1 \ge \frac{2a(b+\theta)-br(2b+\theta)}{(b+\theta)(2b+\theta)} \end{cases}.$$

To obtain the optimal pricing of ISP_1, the optimal pricing and the corresponding profit for each segment of the segmented function π_{ISP_1} are needed to be solved at first, and then the pricing decision that makes the highest profit on all segments is selected.

In the first segment of π_{ISP_1}, ISP_1's optimal pricing and corresponding profit are $\frac{a}{2b+\theta}$ and $\frac{a^2(b+\theta)}{(2b+\theta)^2}$, respectively. In the second segment, when $0 < r \le \frac{2a(b+\theta)}{3b(2b+\theta)}$, ISP_1's the optimal pricing and corresponding profit are $\frac{2ab-2b^2r+2a\theta-br\theta}{(b+\theta)(2b+\theta)}$ and $\frac{br[2a(b+\theta)-br(2b+\theta)]}{(b+\theta)(2b+\theta)}$, respectively; when $r > \frac{2a(b+\theta)}{3b(2b+\theta)}$, ISP_1's optimal pricing and corresponding profit are $\frac{2a(b+\theta)+br(2b+\theta)}{2(b+\theta)(2b+\theta)}$ and $\frac{[2a(b+\theta)+br(2b+\theta)]^2}{8(b+\theta)(2b+\theta)^2}$, respectively. Comparison of the profits in two segments can obtain that when $0 < r \le \frac{2a(b+\theta)}{3b(2b+\theta)}$, $\frac{a^2(b+\theta)}{(2b+\theta)^2} > \frac{br[2a(b+\theta)-br(2b+\theta)]}{(b+\theta)(2b+\theta)}$; when $\frac{2a(b+\theta)}{3b(2b+\theta)} < r \le \frac{2(\sqrt{2}-1)a(b+\theta)}{b(2b+\theta)}$, $\frac{a^2(b+\theta)}{(2b+\theta)^2} > \frac{[2a(b+\theta)+br(2b+\theta)]^2}{8(b+\theta)(2b+\theta)^2}$; when $r > \frac{2(\sqrt{2}-1)a(b+\theta)}{b(2b+\theta)}$, $\frac{a^2(b+\theta)}{(2b+\theta)^2} < \frac{[2a(b+\theta)+br(2b+\theta)]^2}{8(b+\theta)(2b+\theta)^2}$. Therefore, the optimal pricing of ISP_1 is concluded as:

$$p_1^* = \begin{cases} \frac{a}{2b+\theta}, \quad \text{if } 0 < r \le \frac{2(\sqrt{2}-1)a(b+\theta)}{b(2b+\theta)} \\ \frac{2a(b+\theta)+br(2b+\theta)}{2(b+\theta)(2b+\theta)},\ \text{if } \frac{2(\sqrt{2}-1)a(b+\theta)}{b(2b+\theta)} < r \le \frac{2a(b+\theta)}{3b(2b+\theta)} \end{cases}.$$

Therefore, $ISP_1's$ optimal pricing is $p_1^{1N}=\frac{a}{2b+\theta}$ when $0<r\le\frac{2(\sqrt{2}-1)a(b+\theta)}{b(2b+\theta)}$ is verified.

(1.b) Then verify the given $p_1=p_1^{1N}$, ISP_2's optimal pricing is $p_2^{1N}=\frac{a}{2b+\theta}$:

Substitute $p_1=p_1^{1N}=\frac{a}{2b+\theta}$ into Formula (4-5) to obtain $s_1=\frac{-ab+(2b+\theta)(br-\theta p_2)}{2(b+\theta)(2b+\theta)}$. As of $0\le s_1\le p_1$, the optimal response function of CP can be obtained as:

$$s_1^*=\begin{cases}\frac{-ab+(2b+\theta)(br-\theta p_2)}{2(b+\theta)(2b+\theta)},\ \text{if}\ 0\le p_2<\frac{b(-a+2br+\theta r)}{\theta(2b+\theta)}\\ 0,\ \text{if}\ p_2\ge\frac{b(-a+2br+\theta r)}{\theta(2b+\theta)}\end{cases}.$$

Correspondingly, ISP_2's profit function is:

$$\pi_{ISP_2}=\begin{cases}p_2\left\{a-bp_2+\frac{\theta[a+b(p_1-2p_2-r)+\theta(p_1-p_2)]}{2(b+\theta)}\right\},\text{if}\ 0\le p_2\le\frac{b(-a+2br+\theta r)}{\theta(2b+\theta)}\\ p_2[a-bp_2+\theta(p_1-p_2)],\ \text{if}\ p_2>\frac{b(-a+2br+\theta r)}{\theta(2b+\theta)}\end{cases}.$$

To obtain the optimal pricing of ISP_2, the optimal pricing and the corresponding profit for each segment of the segmented function π_{ISP_2} are needed to be solved at first, and then the pricing decision that makes the highest profit on all segments is selected.

In the first segment of π_{ISP_2}, ISP_2's optimal pricing and corresponding profit are $\frac{b(-a+2br+\theta r)}{\theta(2b+\theta)}$ and $\frac{b(b+\theta)[a-r(2b+\theta)][br(2b+\theta)-a(b+2\theta)]}{\theta^2(2b+\theta)^2}$, respectively; In the second segment, ISP_2's optimal pricing and corresponding profit are $\frac{a}{2b+\theta}$ and $\frac{a^2(b+\theta)}{(2b+\theta)^2}$, respectively. The comparison of the optimal profits of the function in two segments shows that $\frac{b(b+\theta)[a-r(2b+\theta)][br(2b+\theta)-a(b+2\theta)]}{\theta^2(2b+\theta)^2}<\frac{a^2(b+\theta)}{(2b+\theta)^2}$. This proves that when $0<r\le\frac{a(b+\theta)}{b(2b+\theta)}$, $ISP_2's$ optimal pricing is $p_2^{1N}=\frac{a}{2b+\theta}$.

Finally, the results in (1.a) and (1.b) reveal that when $0<r\le\frac{2(\sqrt{2}-1)a(b+\theta)}{b(2b+\theta)}$, the subgame perfect equilibrium E^{1N} exists.

(2) Verify that when $\frac{a(4b^2+10b\theta+5\theta^2)}{b(12b^2+24b\theta+5\theta^2)} < r \leq \frac{3a(4b^2+10b\theta+5\theta^2)}{b(4b^2+8b\theta+3\theta^2)}$, the potential equilibrium E^{1P} is the subgame refined equilibrium:

Similar to (1), the following results can be obtained by calculation:

(2.a) Given $p_2 = p_2^{1P}$, ISP_1's optimal pricing is :

$$p_1^* = \begin{cases} \frac{-b\theta^2 r+2a(4b^2+10b\theta+5\theta^2)}{2(b+\theta)(8b^2+16b\theta+3\theta^2)}, if \ \frac{a(4b^2+10b\theta+5\theta^2)}{b(12b^2+24b\theta+5\theta^2)} < r < r_{1M} \\ \frac{br(4b^2+8b\theta+\theta^2)+a(4b^2+10b\theta+5\theta^2)}{(b+\theta)(8b^2+16b\theta+3\theta^2)}, \text{if } r_{1M} \leq r \leq \frac{3a(4b^2+10b\theta+5\theta^2)}{b(4b^2+8b\theta+3\theta^2)}, \end{cases}$$

where $r_{1M} = \frac{2a(4b^2+10b\theta+5\theta^2)}{b[8(1+\sqrt{2})b^2+16(1+\sqrt{2})b\theta+(4+3\sqrt{2})\theta^2]}$. Therefore, when $r_{1M} \leq r \leq \frac{3a(4b^2+10b\theta+5\theta^2)}{b(4b^2+8b\theta+3\theta^2)}$, ISP_1's optimal pricing is verified as $p_1^{1P} = \frac{br(4b^2+8b\theta+\theta^2)+a(4b^2+10b\theta+5\theta^2)}{(b+\theta)(8b^2+16b\theta+3\theta^2)}$,

(2.b) Given $p_1^{1P} = \frac{br(4b^2+8b\theta+\theta^2)+a(4b^2+10b\theta+5\theta^2)}{(b+\theta)(8b^2+16b\theta+3\theta^2)}$, $ISP_2$the optimal pricing is :

$$p_2^* = \begin{cases} \frac{4ba+7\theta a-b\theta r}{8b^2+16b\theta+3\theta^2}, \text{if } \frac{a(4b^2+10b\theta+5\theta^2)}{b(12b^2+24b\theta+5\theta^2)} < r \leq r_{1H} \\ \frac{a}{2(b+\theta)}, \text{if } r_{1H} < r \leq \frac{3a(4b^2+10b\theta+5\theta^2)}{b(4b^2+8b\theta+3\theta^2)} \end{cases},$$

where $r_{1H} = \frac{a\left[16b^3+60b^2\theta+64b\theta^2+14\theta^3-\sqrt{2(2b^2+4b\theta+\theta^2)}(8b^2+16b\theta+3\theta^2)\right]}{2b\theta(2b^2+4b\theta+\theta^2)}$. Therefore, when $\frac{a(4b^2+10b\theta+5\theta^2)}{b(12b^2+24b\theta+5\theta^2)} < r \leq r_{1H}$, ISP_2's optimal pricing is verified as $p_2^{1P} = \frac{4ba+7\theta a-b\theta r}{8b^2+16b\theta+3\theta^2}$,

Finally, the results in (2.a) and (2.b) uncover that when $r_{1M} \leq r \leq r_{1H}$, the subgame perfect equilibrium E^{1P} exists.

(3) Verify that when $r \geq \frac{a(2b+3\theta)}{2b(b+\theta)}$, the potential equilibriumE^{1F} is the subgame perfect equilibrium:

(3.a) Given $p_2^{1F} = \frac{a}{2(b+\theta)}$, ISP_1's optimal pricing is :

$$p_1^* = \begin{cases} \frac{-b\theta^2 r+2a(4b^2+10b\theta+5\theta^2)}{2(b+\theta)(8b^2+16b\theta+3\theta^2)}, \text{if } \frac{a(2b+3\theta)}{2b(b+\theta)} \leq r < \frac{3a(2b+3\theta)}{2b(b+\theta)} \\ \frac{2br(b+\theta)-a(2a+3\theta)}{2(b+\theta)^2}, \text{if } r \geq \frac{3a(2b+3\theta)}{2b(b+\theta)} \end{cases}.$$

Therefore, when $r \geq \frac{3a(2b+3\theta)}{2b(b+\theta)}$, ISP_1's optimal pricing is verified as $p_1^{1\,F} = \frac{2br(b+\theta)-a(2a+3\theta)}{2(b+\theta)^2}$.

(3.b) Given $p_1^{1\,F} = \frac{2br(b+\theta)-a(2a+3\theta)}{2(b+\theta)^2}$,$ISP_2$'s optimal pricing is :

$$p_2^* = \begin{cases} \frac{-4ba+4b^2r-5\theta a+4b\theta r}{2\theta(b+\theta)}, \text{ if } \frac{a(2b+3\theta)}{2b(b+\theta)} \leq r < \frac{a(16b^3+56b^2\theta+56b\theta^2+13\theta^3)}{8b(2b^3+6b^2\theta+5b\theta^2+\theta^3)} \\ \frac{a(4b^2+8b\theta+3\theta^2)}{4(b+\theta)(2b^2+4b\theta+\theta^2)}, \text{ if } r \geq \frac{a(16b^3+56b^2\theta+56b\theta^2+13\theta^3)}{8b(2b^3+6b^2\theta+5b\theta^2+\theta^3)} \end{cases}.$$

Therefore, ISP_2's optimal response to $p_1^{1\,F}$ is not $p_2^{1\,F}$

Finally, the results in (3.a) and (3.b) unveil that $E^{1\,F}$ is not a subgame perfect equilibrium.

4.6.2 Proof of Proposition 4.2

(1) $\frac{\partial}{\partial\theta}\left(\pi_{ISP_1}^1 - \pi_{ISP_1}^0\right) = [-b^2r^2(2b+\theta)^3(128b^6+832b^5\theta+1904b^4\theta^2+1808b^3\theta^3+640b^2\theta^4+80b\theta^5+3\theta^6) - 2abr(2b+\theta)^3(64b^6+448b^5\theta+1160b^4\theta^2+1392b^3\theta^3+806b^2\theta^4+220b\theta^5+15\theta^6) + a^2\theta(1024b^8+8192b^7\theta+26432b^6\theta^2+44064b^5\theta^3+40320b^4\theta^4+19744b^3\theta^5+4562b^2\theta^6+322b\theta^7-21\theta^8)]/[2(b+\theta)^2(2b+\theta)^3(8b^2+16b\theta+3\theta^2)^3]$, it can be verified that it is a positive number on $[r_M, r_L]$.

(2) $\frac{\partial}{\partial\theta}(\pi_{CP}^1 - \pi_{CP}^0) = [-5a^2\theta^3(2b+\theta)^2(8b^3+32b^2\theta+40b\theta^2+15\theta^3) + b^2r^2(2b+\theta)^2(128b^6+704b^5\theta+1168b^4\theta^2+240b^3\theta^3-868b^2\theta^4-536b\theta^5-75\theta^6) + 2abr(256b^8+1024b^7\theta+544b^6\theta^2-1760b^5\theta^3-40b^4\theta^4+5344b^3\theta^5+5328b^2\theta^6+1748b\theta^7+183\theta^8)]/[4(b+\theta)^2(2b+\theta)^2(8b^2+16b\theta+3\theta^2)^3]$, it can be verified that it is a positive number on $[r_M, r_L]$.

(3) $\frac{\partial}{\partial\theta}(D_1^1+D_2^1-D_1^0-D_2^0)=-\mathrm{b}[64b^2r+352b^6\theta r-51a\theta^6+3b\theta^5(-68a+5\theta r)+24b^2\theta^4(-2a+5\theta r)+32b^4\theta^2(12a+23\theta r)+8b^5\theta(12a+91\theta r)+2b^3\theta^3(204a+203\theta r)]/[2(b+\theta)^2(2b+\theta)^2(8b^2+16b\theta+3\theta^2)^2]$, it can be verified that it is positive for $r>0$.

4.6.3 Proof of Lemma 4.3

We first prove the potential equilibria of the first stage ISP pricing game and the corresponding market conditions given in Table 4.4 when both ISP_1 and ISP_2 provide sponsored data services, and then verify whether these potential equilibria are indeed in equilibrium.

(a) Potential equilibria and their corresponding market conditions in Table 4.4.

(1) Potential equilibrium E^{2N}: $s=0$

Substitute $s=0$ into the demand function of Formula (4-1) and the ISP profit function of Formula (4-2), and obtain $\pi_{ISP_i}=p_i\left[a-bp_i+\theta\left(p_j-p_i\right)\right]$, which is equivalent to the circumstance of no sponsored data. Therefore, the equilibrium pricing of ISP_1 and ISP_2 are $\frac{a}{2b+\theta}$.

Substitute $p_1^{2N}=p_2^{2N}=\frac{a}{2b+\theta}$ into Formula (4-6) and enable not to be greater than zero. Obtain the market condition where potential equilibrium E^{2N} may exist is $r\leq\frac{a(b+\theta)}{b(2b+\theta)}$.

(2) Potential equilibrium E^{2P}: $s=\frac{-2a+bp(p_1+p_2+2r)}{4bp}$

Substitute $s=\frac{-2a+\mathrm{bp}(\mathrm{p}_1+\mathrm{p}_2+2r)}{4\mathrm{bp}}$ into the demand function of Formula (4-1) and the ISP profit function of Formula (4-2), and obtain $\pi_{ISP_i}=\frac{p_i}{4}\left[2a-b\left(3p_i-p_j-2r\right)-4\theta\left(p_i-p_j\right)\right], i,j\in\{1,2\}$ and $i\neq j$.Equilibrium pricing strategies of ISP_1 and ISP_2 can be obtained by simultaneously solving first-order conditions $\frac{\partial}{\partial p_i}\pi_{ISP_i}=$

$\frac{1}{4}\left[2a+b\left(2r-6p_i+p_j\right)+4\theta\left(p_j-2p_i\right)\right]=0$. Substitute the solved equilibrium pricing $p_i^{2P}=\frac{2(a+br)}{5b+4\theta}$ into Equation (4-6) to obtain $s=\frac{-3ab+7b^2r-4a\theta+4b\theta r}{2b(5b+4\theta)}$. Set s to be greater than 0 and less than 1, the market conditions that may exist for potential equilibrium E^{2P} are $\frac{a(3b+4\theta)}{b(7b+4\theta)}<r<\frac{a(7b+4\theta)}{b(3b+4\theta)}$.

（3）Potential equilibrium E^{2F}： $s=min\{p_1,p_2\}$

ISPs' pricing decision in E^{2F} can be solved in three cases as follows:

(3.1) If $p_1<p_2$, then $s=p_1$. Substitute $s=p_1$ into the profit function of two ISPs and solve the first-order conditions in parallel to obtain $p_1=\frac{a(2b+3\theta)}{3\theta(b+\theta)}$ and $p_2=\frac{a(b+3\theta)}{3\theta(b+\theta)}$. As $p_1>p_2$ is contradictory to our premises, equilibrium with $s=p_1<p_2$ does not exist.

(3.2) If $p_1>p_2$, then $s=p_2$. Similar to (3.1), equilibrium with $s=p_2<p_1$ does not exist.

(3.3) In summary, the potential equilibrium E^{2F} wherein CP fully subsidies may only exist when $p_1=p_2$. Substitute $s=p_1=p_2$ into the profit function of two ISPs and obtain $\pi_{ISP_i}=p_i a$, and π_{ISP_i} is monotonically increasing with p_i. Combined with the CP's optimal response s^*, we know that any pricing combination that satisfies $p_1=p_2\le r-\frac{a}{b}$ is a potential equilibrium. As ISP pricing cannot be below zero, set $\frac{br-a}{b}\ge 0$, obtain the market condition that may exist for potential equilibrium E^{2F} is $r\ge\frac{a}{b}$.

(b) Table 4.5. The first-stage equilibrium pricing when sponsored data services are provided by both ISP_1 and ISP_2

We adopt the same verification procedure as in the scenario where sponsored data is provided by one ISP to verify the three potential equilibriums.

(1) Verify that when $r\le\frac{a(b+\theta)}{b(2b+\theta)}$, the potential equilibrium E^{2N} is indeed in equilibrium:

First verify that given $p_2^{2N}=\frac{a}{2b+\theta}$, the optimal pricing for ISP_1 is $p_1^{1N}=\frac{a}{2b+\theta}$: Substitute $p_2^{2N}=\frac{a}{2b+\theta}$ into Formula (4-6) to obtain $s=\frac{b(p_1+2r)(2b+\theta)-a(3b+2\theta)}{4b(2b+\theta)}$. As of $0\le s\le min\{p_1,p_2\}$, the optimal response function of CP can be obtained as:

$$s^*=\begin{cases}0,\ \text{if } 0\le p_1<\frac{a(3b+2\theta)-2br(2b+\theta)}{b(2b+\theta)}\\ \frac{b(p_1+2r)(2b+\theta)-a(3b+2\theta)}{4b(2b+\theta)},\text{if } \frac{a(3b+2\theta)-2br(2b+\theta)}{b(2b+\theta)}\le p_1\le\frac{a(7b+2\theta)-2br(2b+\theta)}{b(2b+\theta)}\\ p_2^{2N},\ \text{if } p_1>\frac{a(7b+2\theta)-2br(2b+\theta)}{b(2b+\theta)}\end{cases}.$$

Correspondingly, ISP_1's profit function is:

$$\pi_{\mathrm{ISP}_1}=$$

$$\begin{cases}p_1\left[a-(b+\theta)p_1+\frac{\theta a}{2b+\theta}\right],\ \text{if } 0\le \mathrm{p}_1<\frac{a(3b+2\theta)-2br(2b+\theta)}{b(2\mathrm{b}+\theta)}\\ \frac{p_1\{a(5b+6\theta)-(2b+\theta)[b(3p_1-2r)+4\theta p_1]\}}{4(2b+\theta)},\text{if } \frac{a(3b+2\theta)-2br(2b+\theta)}{b(2\mathrm{b}+\theta)}\le \mathrm{p}_1\le\frac{a(7b+2\theta)-2br(2b+\theta)}{b(2\mathrm{b}+\theta)}\\ \frac{p_1\left[a(3b+2\theta)-p_1(2b^2+3b\theta+\theta^2)\right]}{2b+\theta},\text{if } \mathrm{p}_1>\frac{a(7b+2\theta)-2br(2b+\theta)}{b(2\mathrm{b}+\theta)}\end{cases}$$

To obtain the optimal pricing of ISP_1, the optimal pricing and the corresponding profit for each segment of the segmented function π_{ISP_1} are needed to be solved at first, and then the pricing decision that makes the highest profit on all segments is selected. As the calculation process is similar to the proof process in Table 4.3, the intermediate steps are omitted and the optimal pricing of ISP_1 is directly given as:

$$p_1^*=\begin{cases}\frac{a}{2b+\theta},\text{if } 0<r\le\frac{-a\left(5b+6\theta-4\sqrt{3b^2+7b\theta+4\theta^2}\right)}{2b(2b+\theta)}\\ \frac{2br(2b+\theta)+a(5b+6\theta)}{12b^2+22b\theta+8\theta^2},\ \text{if } \frac{-a\left(5b+6\theta-4\sqrt{3b^2+7b\theta+4\theta^2}\right)}{2b(2b+\theta)}<r\le\frac{a(b+\theta)}{b(2b+\theta)}\end{cases},$$

where $\bar{r}_{2L}=\frac{-a\left(5b+6\theta-4\sqrt{3b^2+7b\theta+4\theta^2}\right)}{2b(2b+\theta)}$. Therefore, ISP_1's optimal pricing is $p_1^{2N}=\frac{a}{2b+\theta}$ when $0<r\le\bar{r}_{2L}$. The symmetry reveals that ISP_2's optimal response function p_2^* to p_1^{2N} is equivalent to p_1^* in the above formula. Therefore, there exists a subgame perfect equilibrium E^{2N} when $0<r\le\bar{r}_{2L}$,

where $\bar{r}_{2L} = \frac{-a\left(5b+6\theta-4\sqrt{3b^2+7b\theta+4\theta^2}\right)}{2b(2b+\theta)}$.

(2) Verify that when $\frac{a(3b+4\theta)}{b(7b+4\theta)} < r < \frac{a(7b+4\theta)}{b(3b+4\theta)}$, the potential equilibrium E^{2P} is indeed in equilibrium:

Similar to (1), we calculate that given $p_2 = p_2^{2P}$, ISP_1's optimal pricing is:

$$p_1^* = \begin{cases} \frac{5ab+6a\theta+2b\theta r}{2(b+\theta)(5b+4\theta)}, \text{if } \frac{a(3b+4\theta)}{b(7b+4\theta)} < r < \frac{a\left[-6b^2-9b\theta-2\theta^2+(5b+4\theta)\sqrt{3b^2+7b\theta+4\theta^2}\right]}{2b(3b^2+7b\theta+3\theta^2)} \\ \frac{2(a+br)}{5b+4\theta}, \text{if } \frac{a\left[-6b^2-9b\theta-2\theta^2+(5b+4\theta)\sqrt{3b^2+7b\theta+4\theta^2}\right]}{2b(3b^2+7b\theta+3\theta^2)} \le r \le \frac{a(7b+4\theta)}{b(3b+4\theta)} \end{cases}.$$

Therefore, ISP_1's optimal pricing is $p_1^{2P} = \frac{2(a+br)}{5b+4\theta}$ when $\frac{a\left[-6b^2-9b\theta-2\theta^2+(5b+4\theta)\sqrt{3b^2+7b\theta+4\theta^2}\right]}{2b(3b^2+7b\theta+3\theta^2)} \le r \le \frac{a(7b+4\theta)}{b(3b+4\theta)}$. The symmetry shows that ISP_2's optimal response function p_2^* to p_1^{2P} is equivalent to p_1^* in the above formula. Therefore, there exists a subgame perfect equilibrium E^{2P} when $\underline{r}_{2M} \le r \le \bar{r}_{2M}$, and $\bar{r}_{2M} = \frac{a(7b+4\theta)}{b(3b+4\theta)}$.

(3) Verify that when $r \ge \frac{a}{b}$, the potential equilibrium E^{2F} is the subgame perfect equilibrium:

Potential equilibrium E^{2F} contains a set of infinite equilibrium prices, that is, $p_1 = p_2, p_1, p_2 \in \left[0, r - \frac{a}{b}\right]$ are all potential equilibrium. To verify whether these potential equilibria really constitute an equilibrium, ISP_1's optimal pricing given $p_2 = r - \frac{a}{b}$ is first calculated as follows:

$$p_1^* = \begin{cases} \frac{br(b+\theta)-a\theta}{2b(b+\theta)}, \text{ if } \frac{a}{b} \le r < \frac{a(2b+\theta)}{b(b+\theta)} \\ \frac{br-a}{b}, \text{ if } \frac{a(2b+\theta)}{b(b+\theta)} \le r \le \frac{a(b+\theta)}{b\theta} \\ \frac{a(b-\theta)+b\theta r}{2b\theta}, \text{if } r > \frac{a(b+\theta)}{b\theta} \end{cases}$$

It can be obtained that when $\frac{a(2b+\theta)}{b(b+\theta)} \le r \le \frac{a(b+\theta)}{b\theta}$, ISP_1's optimal pricing is $p_1 = \frac{br-a}{b}$. When $r < \frac{a(2b+\theta)}{b(b+\theta)}$, ISP_1's optimal pricing not equal to $p_2 = \frac{br-a}{b}$. Therefore, the lower bound of the equilibrium price is $\frac{a}{b+\theta}$. When $r > \frac{a(b+\theta)}{b\theta}$, ISP_1's optimal pricing is not equal to $p_2 = \frac{br-a}{b}$. Therefore, the upper bound of the equilibrium price is $\frac{a}{\theta}$. Combining the upper and lower bounds of the above equilibrium price and the interval $\left[0, r-\frac{a}{b}\right]$, it is proven that when $r \geq \underline{r}_{2H}$ there exists a subgame perfect equilibrium E^{2P} wherein CP fully subsidies. $\underline{r}_{2H} = \frac{a(2b+\theta)}{b(b+\theta)}$, and the values within the range $\left[\frac{a}{b+\theta}, min\left\{r-\frac{a}{b}, \frac{a}{\theta}\right\}\right]$ all constitute the equilibrium price.

4.6.4 Proof of Lemma 4.4

(a) Table 4.8. IF's optimal response function to ISP_2's pricing

Write IF's optimal subsidy decision $s_2^* = min\left\{\left(\frac{-a+(b+\theta)p_2-2\theta p_1+br}{2(b+\theta)}\right)^+, p_2\right\}$ of the second stage as the following piecewise function of p_1:

$$s_2^* = \begin{cases} p_2, & \text{if } 0 < p_1 \leq \frac{br-a-(b+\theta)p_2}{2\theta} \\ \frac{-a+(b+\theta)p_2-2\theta p_1+br}{2(b+\theta)}, & \text{if } \frac{br-a-(b+\theta)p_2}{2\theta} < p_1 \leq \frac{br-a+(b+\theta)p_2}{2\theta} \\ 0, & \text{if } p_1 > \frac{br-a+(b+\theta)p_2}{2\theta} \end{cases}$$

The sign of the dividing point of the above piecewise function depends on the values of *r* and p_2. Therefore, the corresponding profit function of IF for *r* and p_2 in different value ranges will be discussed.

(1) When $r \leq \frac{a}{b}$:

If $0 \leq p_2 < \frac{a-br}{b+\theta}$, $\pi_{IF} = a(p_1+2r) - b(p_1^2+rp_1+rp_2) + \theta p_1(p_2-p_1)$. It can be verified that π_{IF} is a concave function. The optimal pricing of IF is obtained as $p_1^* = \frac{a-br+\theta p_2}{2(b+\theta)}$ from the first-order conditions;

If $p_2 \geq \frac{a-br}{b+\theta}$,

$$\pi_{IF} = \begin{cases} \frac{a^2+b^2[(p_2-r)^2-4p_1(p_1+r)]-2b\theta[4p_1(p_1+r)+\theta p_2(r-p_2)]+\theta^2 p_2^2+a[2b(2p_1-p_2+3r)+2\theta(4p_1-p_2+4r)]}{4(b+\theta)}, & if\ 0 \leq p_1 \leq \frac{br-a+(b+\theta)p_2}{2\theta} \\ a(p_1+2r) - b(p_1^2+rp_1+rp_2) + \theta p_1(p_2-p_1), & if\ p_1 > \frac{br-a+(b+\theta)p_2}{2\theta} \end{cases}$$

IF's optimal pricing decision on the two-stage function is gained and the optimal pricing is obtained through comparison as:

$$p_1^* = \begin{cases} \frac{a-br+\theta p_2}{2(b+\theta)}, \text{if } \frac{a-br}{b+\theta} \le p_2 \le \frac{a-br}{b} \\ \frac{a-br}{2b}, \quad \text{if } p_2 > \frac{a-br}{b} \end{cases}$$

Ultimately, when $r \le \frac{a}{b}$, the IF's optimal response function for ISP_2 pricing is:

$$p_1^*(p_2) = \begin{cases} \frac{a-br+\theta p_2}{2(b+\theta)}, \text{if } 0 \le p_2 \le \frac{a-br}{b} \\ \frac{a-br}{2b}, \quad \text{if } p_2 > \frac{a-br}{b} \end{cases}.$$

(2) When $r > \frac{a}{b}$:

If $0 \le p_2 < \frac{br-a}{b+\theta}$,

$$\pi_{IF} = \begin{cases} (r-p_2)(a+\theta p_1) + (p_1+r)[a-p_1(b+\theta)], \quad \text{if } 0 \le p_1 \le \frac{br-a-(b+\theta)p_2}{2\theta} \\ \frac{a^2+b^2[(p_2-r)^2-4p_1(p_1+r)]-2b\theta[4p_1(p_1+r)+\theta p_2(r-p_2)]+\theta^2 p_2^2+ a[2b(2p_1-p_2+3r)+2\theta(4p_1-p_2+4r)]}{4(b+\theta)}, \text{ if } \frac{br-a-(b+\theta)p_2}{2\theta} \le p_1 \le \frac{br-a+(b+\theta)p_2}{2\theta}. \\ a(p_1+2r) - b(p_1^2+rp_1+rp_2) + \theta p_1(p_2-p_1), \quad \text{if } p_1 > \frac{br-a+(b+\theta)p_2}{2\theta} \end{cases}$$

IF's optimal pricing decision on the three-stage function is gained and the optimal pricing is obtained through comparison as $p_1^* = 0$.

If $p_2 \ge \frac{br-a}{b+\theta}$,

$$\pi_{IF} = \begin{cases} \frac{a^2+b^2[(p_2-r)^2-4p_1(p_1+r)]-2b\theta[4p_1(p_1+r)+\theta p_2(r-p_2)]+\theta^2 p_2^2+ a[2b(2p_1-p_2+3r)+2\theta(4p_1-p_2+4r)]}{4(b+\theta)}, \text{ if } 0 \le p_1 \le \frac{br-a+(b+\theta)p_2}{2\theta} \\ a(p_1+2r) - b(p_1^2+rp_1+rp_2) + \theta p_1(p_2-p_1), \text{if } p_1 > \frac{br-a+(b+\theta)p_2}{2\theta}. \end{cases}$$

IF's optimal pricing decision on the two-stage function is gained and the optimal pricing is obtained through comparison as $p_1^* = 0$

In summary, when $r > \frac{a}{b}$, IF's optimal response function to ISP_2's pricing is $p_1^*(p_2) = 0$.

(b) Table 4.9. IF's optimal response function to ISP_2's pricing

Write IF's optimal subsidy decision $s_2^* = min\left\{\left(\frac{-a+(b+\theta)p_2-2\theta p_1+br}{2(b+\theta)}\right)^+, p_2\right\}$ in the second stage as the following piecewise function of p_2:

$$s_2^* = \begin{cases} p_2, \quad \text{if } 0 \le p_2 < \left(\frac{br-a-2\theta p_1}{b+\theta}\right)^+ \\ 0, \ \text{if } 0 \le p_2 < \left(-\frac{br-a-2\theta p_1}{b+\theta}\right)^+ \\ \frac{-a+(b+\theta)p_2-2\theta p_1+br}{2(b+\theta)}, \quad \text{if } p_2 \ge max\left\{\frac{br-a-2\theta p_1}{b+\theta}, -\frac{br-a-2\theta p_1}{b+\theta}\right\} \end{cases}.$$

Similar to the proof for Table 4.7, the sign of the cutoff point of the piecewise function in the above formula depends on the values of *r* and p_1. Therefore, the solution process is similar, and is omitted here.

5 Conclusions and Future Studies

5.1 Conclusions

This thesis focuses on operational problems in telecommunications industry, analytically models the competition and cooperation among ISPs and CPs from a supply chain perspective, and provides managerial insights and guidance for practitioners and policymakers.

To deal with the interconnection settlement problem at NAP, Chapter 2 employs a cooperative game framework to analyze the profit allocation among multiple ISPs. A quadratic term of the network size is included in the demand model to fully reflect ISPs' contribution to the interconnected network, and a Characterized Profit Allocation rule that allocate the total profit to each ISP according to their contribution to the interconnection is proposed. An interconnection settlement mechanism is designed based on the Characterized Profit Allocation to enable the ISPs to act independently but achieve global optimality. Analytical results and numerical experiments show that the proposed settlement mechanism can stimulate ISPs to interconnect with each other through NAP in a variety of realistic situations, effectively improve social welfare, and provide theoretical basis for the design of the interconnection settlement at NAPs. We believe this settlement mechanism can also be applied in other network structures, such as logistic network, that show network externality.

Chapter 3 develops a two-stage Stackelberg game to investigate the impact of sponsored data service on participants of the telecom service supply chain. Results show that sponsored data service is indeed an innovative business model that can improve ISP's profitability. By shifting from charging users to charging CPs for the data traffic, ISPs can essentially share part of CP's profit and re-participate in the mobile service value chain while acting as the "data pipe". Subscription CPs usually suffers a decrease in revenue if they provide data subsidization for their users, but subscription CPs with large profit margins could take advantage of the subsidization mechanism and improve profit by taking hiding their cost information or indicating possible variations in their content quality. Platform CPs can benefit from sponsored data services as long as they have moderate profitability. Finally, our analysis suggests that social welfare is enhanced when a platform CP participates in sponsored data services, and the result is mixed when the sponsoring CP is a subscription CP. Policymakers should carefully evaluate market conditions to determine the impact of certain sponsored data service on social welfare. When social welfare may be damaged, measures such as data price control should be taken to achieve results that are beneficial to society.

Chapter 4 studies the impact of sponsored data service in the presence of ISP competition. A two-stage Stackelberg-Nash game is developed to model the decision-making behavior of two competing ISPs and one CP in the telecom service supply chain. In a market where ISPs compete with each other, sponsored data service can still effectively improve ISPs' profits, and the benefits increase with the intensity of competition. CP's profits are impaired in most cases, but a CP with strong profitability can increase its profit by providing subsidization to both ISPs' users. In regards of choosing with whom to cooperate to offer sponsored data services, ISPs and CPs have the same preference in some cases: when CP's profit margin is relatively low, it will tend to subsidize both ISPs, and ISPs have no incentive to reach an exclusive agreement; when CP's profit margin is relatively high, it will be more willing to reach an exclusive contract with one of the ISP, and so is the ISP. These results can help ISPs and CPs to make rational sponsored data decisions. In terms of social welfare, sponsored data services could improve social welfare in most cases, except when a CP with relatively low profit margin subsidize only one ISP. Another circumstance under which the social welfare will be reduced is that an ISP integrates with a highly profitable CP in a market where sponsored data services are common practice. Therefore, regulators in the telecom market should pay special attention to above two situations when investigating particular sponsored data programs and or merger and acquisition cases.

In summary, the interconnection settlement mechanism designed in Chapter 2 can effectively encourage ISPs to peer with each other through NAP, thus improve the operating efficiency of NAPs, alleviate network congestion and enhance network quality. Chapters 4 and 5 study the impact of sponsored data service to provide managerial insights and guidance for ISPs and CPs and the policymakers in the telecom market.

5.2 Future Studies

First, this study did not consider the competition between ISPs when designing the interconnection settlement mechanism, and only conducted a preliminary exploration with a model of two competing ISPs in the extension. Extending the model of competition between two ISPs to the model of competition among multiple ISPs for analysis should yield richer results. Moreover, we also need to clarify the measurements for the network size to facilitate the implementation of the settlement mechanism.

Second, in the research of sponsored data service, we assume that ISP's pricing for CP is the same as that for end users for the convenience of analysis. In practice, ISP’s pricing for users is relatively difficult to change (especially raising prices), and ISP can adopt different pricing strategies for CP and users in order to maximize revenue. Future research can consider the model where ISP simultaneously makes two pricing decisions, for the CP and the users respectively, and analyze the impact of sponsored data in this scenario.

Furthermore, we did not consider ISP's network capacity limitation in our model, assuming that the network capacity is unlimited and can fully meet user's needs. In most cases, the introduction of the sponsored data service will bring about an increase in market demand, and whether the ISP’s network can carry the traffic surge may become the bottleneck of ISP’s service quality. Therefore, taking ISP’s and CP’s network investment decisions as endogenous decision variables can provide more comprehensive practical guidance for the implementation of the sponsored data service.

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