# OPTIMAL PRICING STRATEGIES OF MULTINATIONAL SAAS FIRMS UNDER DUAL DISTRIBUTION CHANNELS

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Optimal Pricing Strategies of Multinational SaaS Firms under Dual Distribution Channels

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CERTIFICATE OF ORIGINALITY

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# Abstract

Software as a service (SaaS) is a web-based software delivery model that is licensed on a subscription basis and is centrally managed. SaaS products have been covering every aspect of business and life. Specific examples can be services such as *Apple iCloud, Dropbox*, *Gmail* and *Salesforce*. In light of the recent remote working trend worldwide, SaaS nowadays becomes the most intensively competed place in a B2B sense. In rivalry for market share, SaaS giants adopt price discrimination strategies and deliver different services to every corner of the world. Nonetheless, we notice that there exist few studies regarding pricing strategies of differentiated SaaS products in B2B market. We therefore study the optimal pricing strategies of multinational firms (MNFs) under dual distribution channels. In our model, the MNF provides both standard (low-end) and customized (high-end) SaaS products across borders. The standard products are offered directly by the MNF headquarters while the customized products are offered by MNF's retailing divisions that locate in the foreign countries. When products are sold across borders, they inevitably face different value added tax rates. We incorporate the concern into our model and discuss the scenarios respectively, when MNF is selling from low tax rate region to high tax rate region and vice versa. We find out that for pricing sequence, it is optimal for the MNF to firstly price standard applications and then the retailing divisions to price customized applications as internal pricing sequence, when this MNF sells software from a region with relatively lower tax rate to regions with relatively higher tax rate. On the contrary, when this MNF sells software from a region with relatively higher tax rate to regions with relatively lower tax rate, it is optimal for the MNF to firstly price customized applications and then its retailing division to price standard applications as internal pricing sequence. These findings still hold when market size is expanded by the network effect in the long run. Another key finding is that it is never an optimal strategy for the MNF and retailing division to determine standard applications' and customized applications' prices simultaneously, regardless of the relative tax differentials. This finding is also robust under the impact of network effect.

*Keywords: tax planning, software as a service, self-competition, pricing strategy*

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**Optimal Pricing Strategies of Multinational SaaS Firms under Dual Distribution Channels**

## 1. Introduction

Software as a service (SaaS), one of the most popular forms of cloud computing, is a web-based software delivery model that is licensed on a subscription basis and is centrally managed. SaaS products have been covering every aspect of business and life. They can be classified as any cloud-based software. Examples can be services such as *Apple iCloud*, *Dropbox*, *Gmail* and *Salesforce* (Anselmi et al., 2014). Particularly in a B2B sense, enterprises can now enjoy cloud-based software services on a subscription basis via internet browsers for enterprise resource planning (ERP) or customer relationship management (CRM) purpose (Sun et al, 2008).

Contrary to our common sense that all SaaS products are easy-accessible and can merely be subscribed online, SaaS giants nowadays tend to diversify the products and delivery modes. Many choose to provide differentiated versions of one product through multiple distribution channels. These differentiated versions can be mainly classified into two categories, "standard" and "customized". Industry examples can be seen from *Salesforce* and *Dropbox Business* who provide differentiated editions of cloud services. We illustrate the version differences with the help of package prices of *Salesforce* as shown in Figure 1. In Figure 1, the *Essential*s and *Professional* versions are what we call "standard" SaaS. Standard SaaS applications are designed to serve as many customers as possible. They are one-size-fits-all and have little customization. On the contrary, *Enterprise* and *Unlimited* versions are "customized" SaaS. For customized SaaS applications, software vendors provide customers with tailor-made services by extending and modifying standardized functions, catering for specific needs of certain industries and companies.

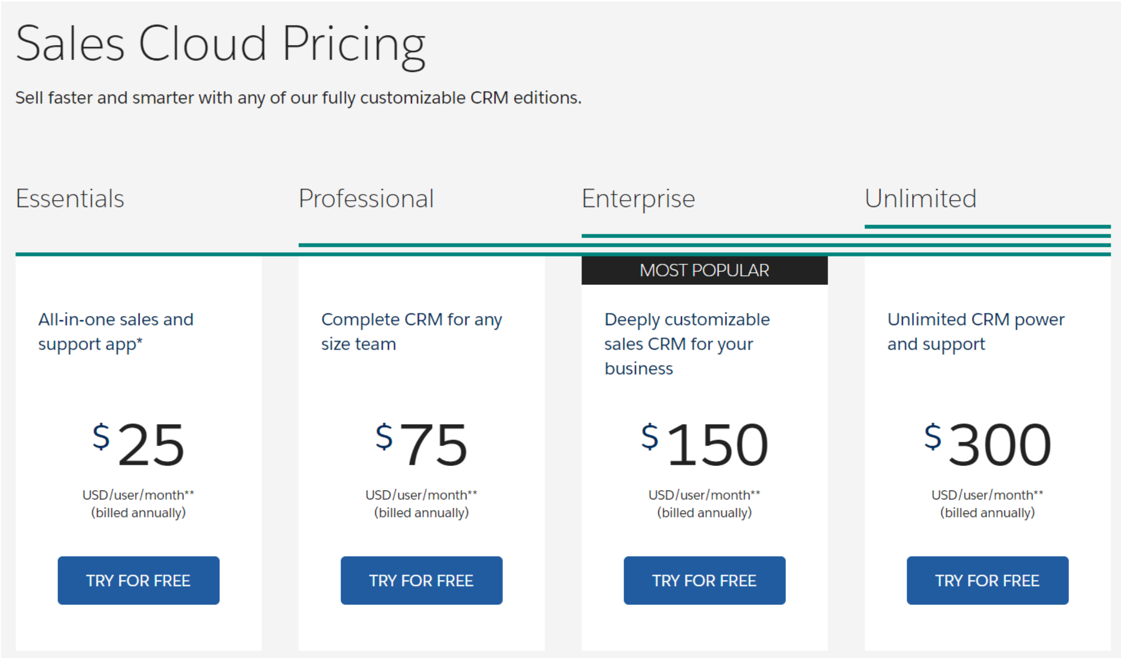



Figure 1. Package Prices of "Sales Cloud"

*(Source: https://www.salesforce.com/products/sales-cloud/pricing/)*

These two versions of products are not delivered through identical distribution channel. Standard versions can be directly ordered online from SaaS vendor' s website. This delivery model is referred to as "direct channel". Conversely, companies that demand customized services need to contact software vendor's retailing division on purchasing, as tailor-made services ask for communication in details and re-programming of products by developers. The corresponding distribution channel is referred to as "retailing channel" or "indirect channel". Through the retailing channel, the retailing division builds, operates, serves, supports and also invoices the customer. It then pays a license fee or transfer price back to MNF for each unit of service it offers to customers as regulated by Arm's Length Principle (Samuelson, 1982).

Particularly, motivated by managerial purpose, the multinational firm (MNF) that offers SaaS services often locate headquarters in its homeland, and locate their retailing divisions at regions worldwide for better customer relationship management. *Salesforce*, for example, is headquartered in San Francisco, U.S and has 67 retail offices across 28 countries. In dealing with the cross-border business, however, the multinational firm might take regional tax rate differential problems into consideration (Shunko et al., 2014), as

SaaS products are now taxable in over 40 countries of the world, including major economies such as EU, U.S and China (Wilkinson, 2019). In EU, an increasing number of digital service tax (DST) plans and similar taxes have been introduced unilaterally and multilaterally to cover more digitalized business (Eggert et al., 2019). In the U.S, SaaS products are taxed differently according to each state's specific regulations. Most commonly they are taxed in the category of "software" or "service" (Dunn, 2020). China now regards SaaS as one kind of software authorization, and correspondingly deduct 6% of value-added tax and 10% of withholding income tax ("Taxation and digital," 2017). For the MNFs, operating in different regions will lead them to different tax rates. While taxation consideration plays an increasingly crucial role in MNFs' transfer pricing and managerial motivations, few studies investigate tax planning strategies regarding information goods.

Moreover, pricing strategies of differentiated goods can differ. In Figure 1, *Salesforce* takes a simultaneous pricing strategy. The standard and customized products are priced simultaneously. In other words, prices of each version are displayed directly and together on the same page. In Figure 2 of *Dropbox*'s example, we find a sequential pricing strategy. While the prices of standard products (*individuals* version and *teams* version) are listed out directly, prices of customized products are labeled as "contact sales staff for pricing". In this scenario, Dropbox company adopts a sequential pricing strategy. Customized products are priced after the standard products, as we can directly observe standard prices and we have to take one further step of contacting the staff members for pricing specifics of customized products.

| | Suitable for individuals | | Suitable for teams | |
|---|---|---|---|---|
| | **Professional**<br>US$16.58 per month<br>Free trial<br>Or buy now | **Professional + eSign**<br>US$24.99 per month<br>Buy now | **Standard**<br>US$12.50 per user per month<br>Free trial<br>Or buy now | **Advanced**<br>US$20 per user per month<br>Free trial<br>Or buy now |
| **Core Dropbox features** | | | | |
| storage | 3TB (3,000GB) | 3TB (3,000GB) | 5TB (5,000GB) | Enough space required |
| First-class synchronization technology | ✔ | ✔ | ✔ | ✔ |
| Access anytime, anywhere | ✔ | ✔ | ✔ | ✔ |
| Easy and secure sharing | ✔ | ✔ | ✔ | ✔ |
| 256-bit AES and SSL/TLS encryption | ✔ | ✔ | ✔ | ✔ |
| Legally binding electronic signatures in Dropbox | Up to 3 per month | Unrestricted | | |
| 5 custom electronic signature templates | | ✔ | | |
| Industry-leading electronic signature security and privacy standards | | ✔ | | |

Dropbox Enterprise

Customized solutions and personalized support services to assist IT professionals in large-scale management

Contact sales staff for pricing

Figure 2. Package Price of "Dropbox"

*(Source: https://www.dropbox.com/business/plans-comparison)*

Observing both tax planning and pricing strategy concerns, we hence come up with our research questions: 1. What is the optimal pricing strategy for a SaaS MNF with two distribution channels under the consideration of tax planning? 2. Will the relative tax differentials affect the optimal pricing strategy?

We develop an analytical model to answer our questions. We derive optimal pricing strategies for a monopoly multinational SaaS firm that offers standard SaaS applications through direct channel and customized applications through indirect channel. We list three types of strategies and compare their profitability under two contrasting taxation scenarios. The three strategies are expressed in pricing sequence, where MNF headquarters and its retailing division take turns to be price leaders. Strategy *lh* is for the MNF to be the pricing leader. In strategy *lh*, MNF sets standard prices (prices for low-end goods) in the first step and retailing division determines retail prices (prices of high-end goods) in the second step. Strategy *ss* is the simultaneous game. Under this strategy, MNF and the retailing division determine standard price and customized prices simultaneously. Strategy *hl* is for the retailing division to be the pricing leader. In this case, the retailing division settle the customized prices before MNF determines standard price. Two tax scenarios include cases when MNF provides services from high tax rate region to low tax rate regions and when MNF provides services from low tax rate region to high tax rate regions.

We find out that when the tax rate of home country is lower than that of customers' country, it maximizes total profit for the MNF to be the pricing leader and to price standard products firstly. Specifically, transfer price is highest in *lh* strategy. Market price for both standard and customized products is highest in *hl* strategy. Demand for products depends on price difference customized and standard products. For standard SaaS, demand is highest in strategy *lh* when standard products have high quality matching degree and price difference is large. Demand for customized SaaS is always highest in *ss* strategy. For the opposite tax rate scenario when the tax rate of customers' country is lower than the tax rate of MNF's country, it maximizes total profit for the retailing division to be the pricing leader and price customized products firstly. Another key finding is that it is never an optimal strategy for the MNF and its retailing division to determine standard applications' and customized applications' prices simultaneously, regardless of the relative tax differentials. These findings still hold when market size is influenced by the network effect in the long run.

The rest of the context is organized as follows. In section 2 we review the past literature concerning tax effective supply chain, pricing strategies and vertical differentiation market. In section 3 we discuss about the optimal pricing strategies of MNF when it distributes its segmented software applications both through direct and indirect channel. In section 4 we re-examine the model of optimal pricing strategies when the market size is influenced by the network effect in the long run. In section 5 we summarize our conclusions.

## 2. Literature Review

We review the past literature mainly from three flows, tax planning in multinational firms, pricing strategies and vertical differentiation market.

## 2.1 Tax Planning in Multinational Firms

Our research firstly relates to a growing amount of literature on tax planning in multinational firms. Extensive research has been performed for MNFs' managerial decisions regarding relative tax differentials, such as Horst (1971), Samuelson (1982), Eden (1983), Baldenius (2004), Choe and Hyde (2007), Hsu and Zhu (2010), Webber (2011), Shunko et al. (2014), Shunko et al. (2017), Wu and Lu (2018), Kim et al. (2018), Hsu et al. (2019), Niu et al. (2019), Niu et al. (2019), Lu and Wu (2020), and Hsu and Hu (2020). Horst (1971) presents optimal transfer pricing strategies according to government's regulations, assuming taxation an exogenously given factor. Samuelson (1982) and Eden (1983) endogenize transfer prices for tax purpose as MNFs' decision variables. Analytical analyses follow over the years, concerning issues such as transfer price, material sourcing, supply chain decentralization and supply chain financing. For example, Baldenius (2004) and Choe and Hyde (2007) analyze multinational firm's decisions on transfer prices restricted by tax and managerial motivations. Hsu and Zhu (2010) particularly study the influence of a series of China's export-oriented tax and tariff regulations. They investigate the resulting optimal supply chain scheme and strategies for firms that sell both to domestic and to foreign markets. Webber (2011) recommends that MNFs should connect income tax with supply chain constructions rather than merely focus on minimize pretax cost. Shunko et al. (2014) point out MNFs' tradeoffs between profiting incentives and taxation concerns when setting transfer prices. Shunko et al. conduct another study in 2017. They find out that, by locating parts of their supply chain at low tax districts, MNFs earn more via transferring profits along the chain and meanwhile bear higher risks of inefficiency. Wu and Lu (2018) provide insight of tax effective supply chain model regarding tax asymmetry and analyze two corresponding transfer pricing strategies, cost-plus strategy and resale-price strategy. Kim et al. (2018) and Hsu et al. (2019) report that two features of tax planning, tax rate differential and transfer price restricted by Arm's Length Principle, exert substantial impact on MNF's cross-border selling strategies. Niu et al. (2019) investigate MNFs' sourcing timing when the retailer and manufacturer locate in areas of different tax rates. Niu et al. (2019) again weigh between MNF's tax planning profits and channel decentralization costs in the setting of a chain-to-chain competition model. Lu and Wu (2020) pinpoint the relationship of tax planning and supply chain financing. Hsu and Hu (2020) investigate tax planning schemes when an MNF has retailers both at high tax rate region and low tax rate region.

Our findings differ from theirs in several aspects. First, we study scenarios when information goods face relative tax differentials across regions, contributing to SaaS taxation literature. Second, we merge the taxation issues with MNFs' pricing strategies and fill in the literature blank by taking both tax planning and pricing strategies into consideration.

**2.2 Pricing Strategies**

Our research also has connection with pricing strategies. There exists literature that considers pricing strategies within a single channel. Mishra and Prasad (2004) examine cases under information asymmetry when the firm either determines prices independently or delegates the responsibility to the salesforce. Xu (2009) evaluates a joint pricing and product quality decision problem in a distribution channel, in which a manufacturer sells a product through a retailer. Karray (2013) casts doubt on the prevailing assumption that manufacturers and retailers resolve pricing and marketing strategies simultaneously. She pinpoints the optimal pricing sequence and marketing efforts determinations for a distribution channel.

Perspectives concerning dual distribution channels are not uncommon, either. For example, Weng (1997) suggests the optimal coordinated pricing and production policies of the distribution channel that consists one manufacturer and one distributor. Yan (2008) develops a game theory model to determine the optimal pricing strategies for the company with online and traditional retail channels. Mantena et al. (2010) focus on competition between platform-based information goods between vendors and of platforms influenced by indirect network effect. Guo et al. (2013) demonstrate the optimal dynamic pricing strategy in a segmented market for service products, using the online distribution channel. Niu et al. (2015) explain the price competition between an OEM and its competitive ODM and perform the endogenous timing game to examine firms' price leadership preferences. Ding et al. (2016) prove a hierarchical pricing decision process and put forward the joint optimal strategies for three types of prices which include the wholesale price, the retail price of traditional channel, and the selling price of direct channel. Chen et al. (2017) explain price and quality decisions in both centralized and decentralized chains, and show that introducing a new channel can enhance product quality. Chen et al. (2020) adopt game theory models in agency selling and reselling model analysis and put forward dynamic pricing strategies for promotional purpose. Wu and Chamnisampan (2021) study the entry strategies of two platforms in two-sided market competition. We add to this literature by studying inter-firm pricing strategies of dual channels and apply them into differentiated SaaS service delivery.

### 2.3 Vertical Differentiation Market

Another related stream of research is vertical differentiation market. These studies can be categorized primarily into two branches, the physical goods market and digital goods market. For the market of physical goods, we observe a trend in studying the mutual conversion phenomenon between high-end and low-end materials. For example, Bansal and Transchel (2014) study high-tech firms' downward and upward substitution strategies for demand diverting. This phenomenon occurs in a customer segmented market when the companies face market stockouts. Lu et al. (2019) discuss a framework where differentiated co-products of several manufacturers are sold to quality-oriented consumers through one independent distributor. Zhou et al. (2020) investigate manufacturers' optimal collective input quantity, downward conversion approach and pricing strategies in a co-production scheme comprising two vertical differentiated products. Bundling is also an issue in differentiated physical goods market. Ma and Mallik (2017) measure the equilibrium outcome of retailer bundling and manufacturer bundling scenarios when one manufacturer produces products of basic quality and premium quality. With respect to digital product market, the issue that firms launch products of divergent quality to obtain network effect has been densely researched through years. Conner (1995) matches software products of differentiated quality as "clone" version and original version. He reports that the firm's profit decreases in the quality reduction of products with lower value. Haruvy and Prasad (1998) analyze the optimal strategies for a software firm that provides two compatible products of differentiated qualities. The market is a segmented and consumers hold either high or low valuation towards the products. The authors thus derive optimal conditions on when to introduce a low-quality version. Shy and Thisse (1999) demonstrate the strategic reasons for firms' not protecting their software against pirate versions in a duopoly market of differentiated goods. Faugere and Tayi (2007) set up a vertically differentiated game-theoretic model that explains the issue of free trial software designs for obtainning succeeding sales. Cheng and Tang (2010) explain the tradeoffs between network effect and encroachment effect regarding afore-mentioned free trial strategies. Nan et al. (2018) conduct a research similar to Cheng and Tang's. They instead study the tradeoffs between consumer uncertainty reduction and demand cannibalization. In this study, the free version of lower quality aims to defend against piracy. Driving forces other than network effect are also considered. Yu et al. (2011) study the effect of digital content of devices on firms' profitability under both horizontal and vertical product differentiation settings. Bhargava and Choudhary (2015) emphasize on the vertical differentiation phenomenon in physical goods market and describe conditions when the differentiation strategy may not be optimal for information goods. Roger (2017) demonstrates competition in differentiated products between two-sided platforms.

Among the vast literature regarding the issue of vertically differentiated market, the study that is most relevant to ours is conducted by Li et al. in 2018, where the authors analyze and compare the optimum profitability of direct and indirect distribution channels of differentiated enterprise software. They specifically investigate SaaS applications distributed through direct channel and customized equipment through indirect channel. The SaaS applications serve as lower quality products and the customized applications function as higher quality products. They find out that when unfit cost is higher than customization cost, the software company will optimize its profit by adopting a dual channel strategy, and the firm will turn to SaaS channel strategy under the converse condition. One of our key findings shares the similar assumption of customization cost and discount factor for the dual delivery channel to coexist. The major driving force that distinguish our results with Li et al.'s study is the relative tax rate differential, which plays an essential role in pricing strategies according to our results.

**3. The Model**

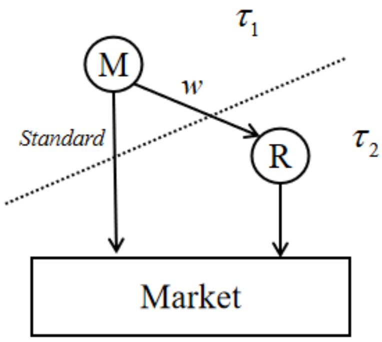



Figure 3. The Global Distribution Channel Structure

MNF's optimal pricing strategies are modeled in this section. Figure 3 illustrates the global distribution channel structure. We consider a MNF that provides differentiated SaaS products through dual distribution channels, the direct and indirect (retailing) channel. Specifically, standard (low-end) products are delivered by MNF through direct channel, while customized (high-end) products are offered by MNF's retailing division through retailing channel.

Even though the products come from the same MNF, the retailing division specifically deals with enterprise users and customizes products accordingly. MNF and its retailing division locate at different countries and the original products are designed by MNF. As Arm's Length Principle (ALP) regulates, the retailing division pays back a transfer price $w$ to MNF for each unit of customized products it provides to customers. Prices deservedly differ for standard and customized products. We denote $p_L$ as the market price of standard version and $p_H$ as the market price of customized version. Here $L$ and $H$ represent "low-end" and "high-end" respectively. It should be specified that, while $w$ and $p_L$ and determined by MNF directly, $p_H$ is independently determined by retailing division according to customization specifics.

Consumers, as mentioned above, have their own valuation $v$, that is uniformly distributed in the range [0,1] towards the commodities. For consumers who purchase standard products through direct channel, they do not enjoy the tailor-made benefit, and in turn suffer from an unfit cost of valuation, which is denoted by the discount factor $\theta$ that ranges between 0 to 1. Conversely, for the retailing channel, through which made-to-measure services are provided, customers thus bear a customization cost $c$ in exchange for better fit of use.

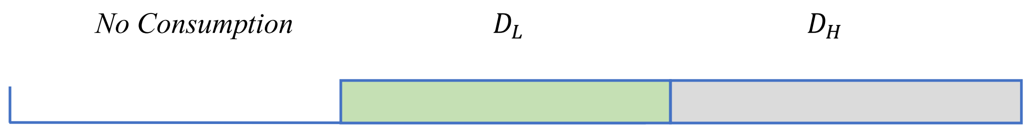
We hence model customers' utility under retailing channel as:

$$U_H = v - p_H - c \tag{1}$$

And we model the utility of customers under standard SaaS channel as:

$$U_L = \theta v - p_L \tag{2}$$

The margin that customers are indifferent of between buying nothing and consuming through standard SaaS channel is $v = \frac{p_L}{\theta}$ by letting equation (1) = 0. The margin that consumers are indifferent between standard SaaS and customized SaaS is thus $v_H = \frac{p_H + c - p_L}{1-\theta}$ by letting equation (1) = (2). Demand of the two channels is expressed as $D_L = \frac{\theta(p_H + c) - p_L}{\theta(1-\theta)}$ and $D_H = 1 - \frac{p_H + c - p_L}{1-\theta}$. In Figure 4 we illustrate the market segmentation under mixed channel situation.



0 $\frac{p_L}{\theta}$ $\frac{p_H+c-p_L}{1-\theta}$ 1

Figure 4. Market Segmentation under Mixed Distribution Channels

We also consider the firm to face different tax rate levels when trading cross borders. The tax rate in home country where the MNF head quarter locates is denoted as $\tau_1$ while in foreign country where the retailing division locates as $\tau_2$. $\Delta = \frac{1-\tau_1}{1-\tau_2}$ refers to the relative tax rate differential of home country to foreign country. To simplify expression, here we assume MNF to locate at home country and its retailing divisions locate in foreign markets. The MNF has two profit channels. One is the direct channel run by the headquarters. The other is the retailing channel managed by the retailing division. It should be noted that the profit from the retailing channel is included in MNF's total profit. We thus derive two objective functions. For the MNF, the objective is to maximize the total profit from both channels. In other words, the MNF makes decisions on $p_L$ and $w$ out of a more comprehensive concern. Instead of intensifying price competition and grabbing market share, the MNF aims to allocate the market share reasonably to maximize total revenue. The objective profit function of MNF is thus described in equation (3):

$$\pi_M = p_L D_L(1-\tau_1) + wD_H(1-\tau_1) + (p_H - w)D_H \Delta(1-\tau_1) \quad (3)$$

As we mentioned above, $\pi_M$ comprises profit from both channels. $p_L D_L(1-\tau_1) + wD_H(1-\tau_1)$ consist of MNF headquarters' income and $(p_H - w)D_H \Delta(1-\tau_1)$ refers to the income of retailing division. Specifically, for the retailing division that only provides customized products, the profit function is defined as equation (4):

$$\pi_r = (p_H - w)D_H \Delta(1-\tau_1) \quad (4)$$

As there are more than one decision maker that sets the price, pricing strategies can differ when pricing sequences change. In the analysis part, we introduce three pricing strategies. Strategy *"lh"* is for the MNF to determine price of standard SaaS products $p_L$ at the first step and the retailing division to determine price of customized SaaS products $p_H$ at the second step. Strategy *"ss"* is for the MNF and its retailing division to determine $p_L$ and $p_H$ simultaneously. Strategy *"hl"* is for the retailing division to determine $p_H$ at the first step and MNF to decide on $p_L$ at the second step. It should be noticed that $w$ is always set ahead of $p_H$, based on supply sequence. The retailing division, as regulated by "ALP", cannot adopt original products from MNF for free. Instead, it purchases the original products from MNF at the market price $w$. After purchase, the retailing division can embark on the customization process and sell the customized products at the price $p_H$. The order of $w$ and $p_H$ is therefore not influenced by strategy differences. The sequence of $p_H$ and $p_L$ inevitably vary in three strategies. We plot the event sequence of the afore mentioned strategies in Figure 5 with the help of timeline.

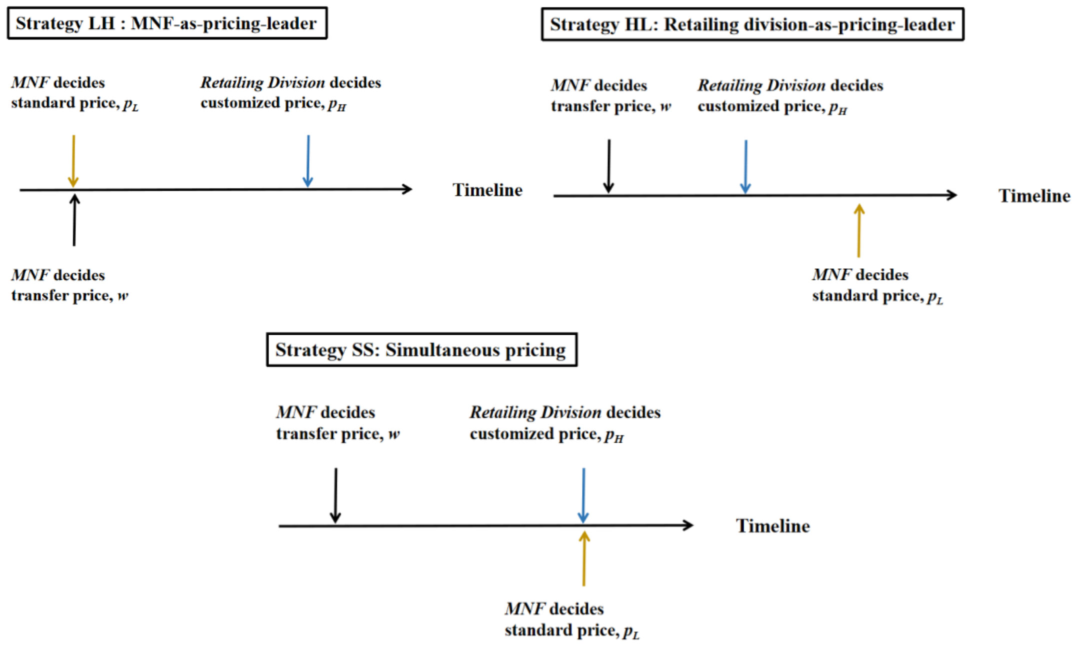



Figure 5. Event Sequence of Three Pricing Strategies

In the short term, we discuss and compare these three cases under two scenarios, when cross-border tax ratio $\Delta$ is smaller than one and when $\Delta$ is greater than one. In the long term, with the influence of network effect $\gamma$, we re-examine the impact of relative tax rate differential on these three cases. In Table 1 we present a summary of notations.

Table 1. Notation Summary

| Notation | Meaning |
|---|---|
| M | Multinational SaaS Firm |
| R | Retailing Division |
| $\tau_1$ | Tax rate in home country |
| $\tau_2$ | Tax rate in foreign country |
| $\Delta$ | Relative tax rate differential of home country to foreign country |
| $p_H$ | Price under retail channel |
| $p_L$ | Price under SaaS channel |
| $w$ | Transfer price under retail channel |
| $\theta$ | Valuation discount factor under SaaS channel |
| $c$ | Customization cost under retail channel |
| $v$ | Customers' valuation towards services |
| $\gamma$ | Intensity of network effect, marginal perceived valuation |

## 4. Outcomes and Analysis

### 4.1 Comparison: From Low to High

We discuss cases when $\Delta < 1$ and $\Delta > 1$ respectively. In the following section, we focus on scenarios when $\Delta < 1$ first.

Under this condition, the MNF locates at a region of relatively lower tax rate and sells to markets of relatively higher tax rate. In light of the relative tax rate differentials worldwide, we restrict $\frac{1}{2} < \Delta < 1$. To insure the non-negativity of prices and demand, we restrict $0 < \theta < 1 - c$.

#### 4.1.1 Optimal Outcomes

We reach the outcomes through backward induction. Below we summarize the optimal outcomes under three pricing strategies respectively in Table 2 - Table 4.

**Table 2: Outcomes for Strategy *lh***

$$w^* = \frac{2(-1+c)(-1+\Delta)+\Delta\theta}{4-2\Delta}$$

$$p_L^* = \frac{\theta}{2}$$

$$p_H^* = \frac{3-3c-2\Delta+2c\Delta-\theta+\Delta\theta}{4-2\Delta}$$

$$\pi_r^* = \frac{\Delta(-1+c+\theta)^2(-1+\tau_1)}{4(-2+\Delta)^2(-1+\theta)}$$

$$\pi_M^* = \frac{(c^2+2c(-1+\theta)+(-1+\theta)(-1+(-1+\Delta)\theta))(-1+\tau_1)}{4(2-\Delta)(-1+\theta)}$$

**Table 3: Outcomes for Strategy *ss***

$$w^* = \frac{8(-1+c)(-1+\Delta)+2\Delta(1+c+\Delta-c\Delta)\theta-(-1+\Delta^2)\theta^2}{-8(-2+\Delta)+2(-1+\Delta)^2\theta}$$

$$p_L^* = \frac{\theta(10+2c(-1+\Delta)-\theta+\Delta(-6+\Delta\theta))}{-8(-2+\Delta)+2(-1+\Delta)^2\theta}$$

$$p_H^* = \frac{12-8\Delta+2(-1+\Delta^2)\theta-(-1+\Delta)^2\theta^2-2c(6+\theta+\Delta(-4+(-2+\Delta)\theta))}{-8(-2+\Delta)+2(-1+\Delta)^2\theta}$$

$$\pi_r^* = \frac{\Delta(-1+c+\theta)^2(-2+(-1+\Delta)\theta)^2(-1+\tau_1)}{(-1+\theta)(8-4\Delta+(-1+\Delta)^2\theta)^2}$$

$$\pi_M^* = \frac{(4c^2+8c(-1+\theta)-(-1+\theta)(2+\theta-\Delta\theta)^2)(-1+\tau_1)}{4(-1+\theta)(8-4\Delta+(-1+\Delta)^2\theta)}$$

**Table 4: Outcomes for Strategy *hl***

$$w^* = \frac{-2(-1+\Delta)(4+(1+\Delta)(-3+\theta)\theta)+c(-8+8\Delta+6\theta+2(2-3\Delta)\Delta\theta-(1+\Delta)^2\theta^2)}{2(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)}$$

$$p_L^* = \frac{\theta(10-6\Delta-8\theta-c(-1+\Delta)(-2+\theta+\Delta\theta)+2\theta(\theta+\Delta(-2+2\Delta+\theta)))}{2(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)}$$

$$p_H^* = \frac{4(-1+c)(-3+2\Delta)+2((1+\Delta)(-5+3\Delta)+c(3+(4-3\Delta)\Delta))\theta-(-2+2(-2+\Delta)\Delta+c(1+\Delta)^2)\theta^2}{2(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)}$$

$$\pi_r^* = -\frac{2\Delta(-1+c+\theta)^2(-1+\Delta\theta)^2(-2+\theta+\Delta\theta)(-1+\tau_1)}{(-1+\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)^2}$$

$$\pi_M^* = \frac{(-4(-1+\theta)(-1+\Delta\theta)^2+c^2(-2+\theta+\Delta\theta)^2+2c(-1+\theta)(-2+\theta+\Delta\theta)^2)(-1+\tau_1)}{4(-1+\theta)(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)}$$

### 4.1.2 Analysis of prices

To analyze the intuition behind the optimal strategies, we compare prices in terms of $w$, $p_L$ and $p_H$. The footnotes, $lh$, $ss$ and $hl$, correspond to strategies $lh$, $ss$ and $hl$. Intuitively, each decision maker benefits from second-mover advantage in pricing competition, because the second mover can directly observe competitors' price and adjust its own price accordingly. In strategy $lh$, MNF moves before its retailing division. Retailing division has the second mover advantage. In strategy $hl$, retailing division moves ahead of MNF. MNF in turn has the second mover advantage. In strategy $ss$, the two parties set prices simultaneously and no one enjoys second mover advantage. We define the second-mover advantage as channel power. In strategy $lh$, retailing division has stronger channel power while in strategy $hl$ MNF does. In simultaneous game, the two players have identical channel power. We summarize channel power comparison for two decision markers in Table 5. Detailed explanation is as follows.

Table 5. Channel Power Comparison for MNF and Retailing Division

| ***Strategy*** / ***Decision Maker*** | ***lh*** | ***ss*** | ***hl*** |
|---|---|---|---|
| ***Second Mover/ More Powerful Party*** | ***Retailing Division*** | ***Same*** | ***MNF*** |

**Proposition 1.** *(from Low to High, Transfer Price Comparison)* When the MNF provides services from low tax rate region to high tax rate region, the transfer price under strategy $lh$ is the highest, followed by transfer prices under strategy $ss$ and $hl$ in descending order. To sum up, $w^{lh} > w^{ss} > w^{hl}$.

In most cases, when price competition takes place between two independent companies, each decision maker spares no effort to lower the price. Nonetheless, in our setting, competition takes place between MNF and its retailing division. The retailing division's profit is included in MNF's total profit. Contrary to competition between independent companies, the MNF headquarters may not wish to always defeat the retailing division, because fierce competition may reduce total profit if retailing division earns too little. We verify this self-competition situation by comparing the three strategies.

First, in $lh$ strategy, retailing division is the second mover and is more powerful in supply chain. It intuitively grabs the second-mover advantage by charging a low $p_H$. The MNF, as a rational player, predicts that fierce price competition will hurt total profit if both sides minimize prices. It should be noted that MNF is less powerful in $lh$ strategy. At an inferior position, MNF adopts both $w$ and $p_L$ weapon to resist competition from retailing division and to balance between profits from low-end goods and total profits. Transfer price weapon is performed to ease competition. A high $w^{lh}$ effectively increases retailing division's cost. Facing higher transfer price cost, the retailing division cannot set too low a $p_H$. At the same time, with a higher $w^{lh}$, MNF retains a larger share of profit in a region of lower tax rate.

Second, in strategy *ss*, the retailing division and MNF have identical channel power. In a simultaneous game, no one can observe the competitor's decision and make effective adjustments. For the MNF, a high $w^{ss}$ cannot help to ease the competition. A high transfer price will only boost retailing division's $p_H$ and reduce total demand. The optimal $w^{ss}$ is in turn lower than $w^{lh}$.

Lastly, in strategy $hl$, MNF has the second-mover advantage and the strongest channel power. Unlike the retailing division who merely concerns about self-interest, MNF cares about total profit and does not wish to intensify competition. In other words, MNF will not choose to profit by hurting its retailing division's interest. Since a relatively low $w$ reduces retailing division's cost, MNF sets $w^{hl}$ the lowest among the three strategies. The counterintuitive results are justified.

We may conclude that, as headquarters, MNF pays attention to the interests of the whole. Instead of merely competing for more market share, the MNF concerns about market segmentation and total profit. The retailing division, as an overseas department, make decisions according to its own lights and in its own interests. It behaves more aggressively in competition.

**Proposition 2.** *(from Low to High, Product Price Comparison)* When the MNF provides services from low tax rate region to high tax rate region, prices of SaaS products are the lowest under strategy $lh$, followed by prices under strategy *ss* and *hl* in ascending order. To sum up, $p_N^{lh} < p_N^{ss} < p_N^{hl},\ N \in \{L, H\}$.

In strategy $lh$, the retailing division is more powerful. It exhibits aggressiveness and sets a low $p_H^{lh}$. To compete with retailing division, the MNF in turn sets a low $p_L^{lh}$.

In strategy *ss* when two parties have similar channel power, severe price competition will only hurt total profit margin. Foreseeing this situation, both players reach higher $p_L^{ss}$ and $p_H^{ss}$.

In strategy *hl*, the MNF is more powerful in channel and it makes use of the two pricing weapons to the extreme. As a non-aggressive second mover, MNF maximizes total profits with moderate competition. It sets a high $p_L^{hl}$ along with a low $w^{hl}$. A high $p_L^{hl}$ increases its own profit margin. A low $w^{hl}$ decreases cost of its retailing division. Foreseeing MNF's high market price and low transfer price, the retailing division seizes the opportunity to maximize self-profit by setting the highest $p_H^{hl}$.

**4.1.3 Analysis of channel demand**

We define $\theta_1 = \frac{5+2\Delta-\Delta^2}{2(1+\Delta)^2} - \frac{1}{2}\sqrt{\frac{9-4\Delta-6\Delta^2+4\Delta^3+\Delta^4}{(1+\Delta)^4}}$. We summarize the comparison results in Proposition 3a and Proposition 3b.

**Proposition 3a.** *(From Low to High, Customized SaaS Product Demand Comparison)* When the MNF provides services from low tax rate region to high tax rate region, demand of customized SaaS products is the highest under strategy *ss*, followed by demand of strategy *lh* and *hl* in descending order. In equality form, $D_H^{ss} > D_H^{lh} > D_H^{hl}$.

**Proposition 3b.** *(From Low to High, Standard SaaS Product Demand Comparison)* When the MNF provides services from low tax rate region to high tax rate region, demand of standard SaaS products is the lowest under strategy *ss*. Specifically, demand for standard SaaS is highest in strategy $hl$ when $0 < \theta < \theta_1$. Demand for standard SaaS is highest in strategy $lh$ when $\theta_1 < \theta < 1$. In equality form, $D_L^{hl} > D_L^{lh} > D_L^{ss}$ when $\theta_1 <$

$\theta < 1$. $D_L^{lh} > D_L^{hl} > D_L^{ss}$ when $0 < \theta < \theta_1$.

We start the analysis with Proposition 3a. It can be recalled that, in the model setting, $D_H = 1 - \frac{p_H + c - p_L}{1-\theta}$. $D_H$ is negatively influenced by the difference between $p_H$ and $p_L$, i. e. $p_H - p_L$. When prices of customized products exceed the standard products to a large extent, the demand for customized products shrinks. As indicated in Proposition 3a, $D_H^{ss} > D_H^{lh} > D_H^{hl}$. $D_H^{hl}$ is the lowest, resulting from the largest difference between $p_H^{hl}$ and $p_L^{hl}$. It should be recalled that, the second mover has the incentives to set a relatively low price for more market share. In strategy $hl$, MNF is the second mover and sets a relatively low $p_L^{hl}$. A relatively low $p_L^{hl}$ increases the price difference. Conversely, in strategy $lh$, retailing division is the second mover and is inclined to set a relatively low $p_H^{lh}$. A relatively low $p_H^{lh}$ decreases the price difference. In simultaneous game, two players move simultaneously and have identical channel power, the price difference is the smallest. The demand for high-end goods, $D_H^{ss}$, is the largest.

We then turn to Proposition 3b. Similarly, $D_L = \frac{\theta(p_H + c) - p_L}{\theta(1-\theta)}$. Contrary to $D_H$, $D_L$ is positively influenced by the price difference between $p_H$ and $p_L$. When price difference is large, demand for standard products will increase. This relationship strengthens our assumption in Proposition 3a. When low-end products become cheaper on a relative basis, customers' tolerance on mismatching degrees increases and consumption of low-end goods increases accordingly.

We firstly note that $D_L^{ss}$ is always the lowest among the three strategies, due to the smallest price difference in simultaneous game. The result is in accordance with what we reach for $D_H^{ss}$.

In sequential game, $D_L$ depends on $\theta$, the value discount factor of standard products. When $\theta$ is relatively large, the quality of standard products is more in line with customers' needs. Customers become less sensitive to mismatch degrees and more sensitive to price differences. Price difference plays a dominant role in package selecting. Therefore, when $\theta$ is relatively large, we conclude that $D_L^{hl} > D_L^{lh}$ as a result of smaller price difference in strategy *hl*. Nonetheless, the influence of price difference is offset by larger mismatching degrees when $\theta$ is relatively small. In this scenario, $D_L^{lh} > D_L^{hl}$ because customers are willing to pay more for customized services.

The price difference law applies in managerial decisions. When price difference is small, low-end products have the largest competitiveness because they are more cost-effective. Customers pay a small amount more and enjoy the customized products. When price difference is large, in other words, the high-end products are much more expensive than the low-end products, customers care less about mismatching degrees and prefer low-end goods.

**4.2.3 Analysis of firms' profits**

Combing the decision variables together, we investigate the profitability of each channel. It should be recalled that the MNF has two profiting channels, the direct channel and indirect channel. The MNF sells standard products through direct channel. We refer standard SaaS products as "low-end goods" in the model and denote them with footnote "$L$". The retailing division provides customized services from indirect channel. We refer customized SaaS products as "high-end goods" in the model and denote them with footnote "*H*". Below we compare the profitability of direct and indirect channel respectively.

We define $\theta_2$ to be the minimum solution of the equation (5):

$$24 - 24c - 20\Delta + 20c\Delta + 4\Delta^2 - 4c\Delta^2 + (-48 + 32c - 16\Delta + 8c\Delta + 28\Delta^2 - 20c\Delta^2 - 4\Delta^3 + 4c\Delta^3)\theta + (33 - 14c + 48\Delta - 19c\Delta + 10\Delta^2 + 3c\Delta^2 - 20\Delta^3 + 7c\Delta^3 + \Delta^4 - c\Delta^4)\theta^2 + (-10 + 2c - 24\Delta + 5c\Delta - 24\Delta^2 + 3c\Delta^2 - 4\Delta^3 - c\Delta^3 + 6\Delta^4 - c\Delta^4)\theta^3 + (1 + 4\Delta + 6\Delta^2 + 4\Delta^3 + \Delta^4)\theta^4 = 0$$

(5)

We present the findings in Proposition 4a and Proposition 4b.

**Proposition 4a.** *(From Low to High, Comparison of Customized Channel's Profitability)* When the MNF provides services from low tax rate region to high tax rate region, profitability of customized channel is largest under strategy *ss*, followed by that under strategy *lh* and three in descending order. To sum up, $\pi_H^{ss} > \pi_H^{lh} > \pi_H^{hl}$.

**Proposition 4b.** *(From Low to High, Comparison of Standard SaaS Channel's Profitability)* When the MNF provides services from low tax rate region to high tax rate region, profitability of standard SaaS channel is the lowest under strategy *ss*. Channel profitability of strategy *lh* is larger than that of strategy *hl* when $0 < \theta < \theta_2$. Channel profitability of strategy *lh* is smaller than that of strategy *hl* when $\theta_2 < \theta < 1 - c$. To sum up, $\pi_L^{lh} > \pi_L^{ss}$ and $\pi_L^{hl} > \pi_L^{ss}$. $\pi_L^{lh} > \pi_L^{hl}$ when $0 < \theta < \theta_2$. $\pi_L^{lh} < \pi_L^{hl}$ when $\theta_2 < \theta < 1 - c$.

We start the analysis with $\pi_H$, i. e. Proposition 4a, the profitability of retailing or the direct channel. From Proposition 2b and Proposition 3b, we note that even though $p_H^{ss}$ is not the largest (i.e. $p_H^{lh} < p_H^{ss} < p_H^{hl}$), the benefit from the largest demand $D_H^{ss}$ (i.e. $D_H^{ss} > D_H^{lh} > D_H^{hl}$) offset the loss from the relatively lower retailing price. $\pi_H^{ss}$ thus becomes the largest among the three retailing channels. Conversely, even though for strategy *hl*, the profit margin $p_H^{hl}$ is the largest, the effect of smaller demand $D_H^{hl}$ counterbalances the revenue from higher retailing price. $\pi_H^{hl}$ therefore brings the smallest retailing profit.

We next analyze the profitability of direct channel $\pi_L$, i. e. Proposition 4b. The profit margin for low-end products is more sensitive to price effect. Firstly we discuss about $\pi_L^{lh} > \pi_L^{ss}$. Similar to the situation in retailing channel, even though profit margin $p_L^{lh}$ is smaller than $p_L^{ss}$, the effect of larger market share $D_L^{lh}$ makes up for the loss from prices. As for $\pi_L^{hl} > \pi_L^{ss}$, because both price $p_L^{ss}$ and demand $D_L^{ss}$ are smaller than those under strategy *hl*, profit from strategy *ss* is obviously smaller. When comparing $\pi_L^{lh}$ and $\pi_L^{hl}$, we take $\theta$ into consideration, as demand depends on the factor. Specifically, when $\theta$ is in a smaller range ($0 < \theta < \theta_2$), the larger demand $D_L^{lh}$ has stronger impact regardless of smaller profit margin $p_L^{lh}$. In this case $\pi_L^{lh}$ is greater than $\pi_L^{hl}$. When $\theta$ is in a larger range ($\theta_2 < \theta < 1 - c$), demand $D_L^{lh}$ is smaller than $D_L^{hl}$. The loss of lower price $p_L^{lh}$ cannot be offset by a small $D_L^{lh}$. Therefore $\pi_L^{lh}$ is smaller than $\pi_L^{hl}$.

As we have discussed, when two channels have similar power and moderate price competition, customers prefer the high-end products out of cost efficiency concerns. The demand effect proves to be strong, because larger demand of high-end products offsets the loss from lower prices. The profit for high-end goods is highest under strategy $ss$. On the contrary, profit for low-end goods is lowest under strategy $ss$.

We finally compare the total profits of the MNF under the three strategies. Total profit is the sum of profits from both direct and indirect channel, i. e. $\pi = \pi_L + \pi_H$. Summarizing the results after comparison, we reach Proposition 5：

***Proposition 5*** *(Sell from Low to High, Total Profit Comparison).* When MNF sells SaaS products from low tax rate regions to high tax rate regions, it maximizes MNF's profit for it to determine $p_L$ for standard SaaS products at the first move and its retailing division to decide on $p_H$ for customized SaaS products at the second move. In inequality equation form, $\pi^{lh*} > \pi^{hl*} > \pi^{ss*}$.

Total profit is the summation of profits generated from both low-end product channel and high-end product channel. Because neither channel can dominate total profit, the total profit is the jointly determined by two channels. Overall, the MNF can derive the highest total profit under strategy $lh$. On the contrary, simultaneous strategy $ss$, brings the lowest total profit for MNF. We may conclude that, when an MNF sells products from low tax rate region to high tax rate region under dual distribution channels, it maximizes total profit to first price low-end products and next to price high-end products. Simultaneous pricing strategy on the contrary provides the least benefit for total profit.

**4.2 Comparison: From High to Low**

**4.2.1 Analysis of firms' profits**

In this section, we focus on scenarios when $\Delta > 1$. Now the MNF locates at a high tax rate region and sells to low tax rate regions. The retailing division stands at low tax region accordingly. Define $\Delta_1 = \frac{2+3\theta-\theta^2-2\sqrt{1-3\theta+3\theta^2-\theta^3}}{3\theta+\theta^2}$, where $\Delta_1 > 1$. The comparison of profitability is presented in Proposition 6.

**Proposition 6.** *(Sell from High to Low, Total Profit Comparison)* When $1 < \Delta < \Delta_1$, the MNF sells SaaS products from high tax rate regions to low tax rate regions. It maximizes MNF's profit to adopt strategy three. Strategy one brings the MNF second most profit, and strategy two is the least profitable. In inequality equation form, $\pi^{hl*} > \pi^{lh*} > \pi^{ss*}$.

We restrict the delta to be within $1 < \Delta < \Delta_1$ because the profit level will turn infinite in math and thus have no practical meaning once $\Delta$ exceeds $\Delta_1$. We illustrate this statement with the help of numerical analysis. We tried all the possible combinations and selected the most representative example. We assign $c = 0.05$, $\theta = 0.8$ and $\tau_1 = 0.2$. The profit levels are listed in Lemma 1. Specifically, we plot the impact of tax rate differentials on MNF's profitability of three strategies in Figure 6.

**Lemma 1.** MNF's Total Profit of Three Strategies (Numerical Analysis).

$$\pi^{lh*} = \frac{4}{25} - \frac{9}{400(-2+\Delta)}$$

$$\pi^{ss*} = \frac{4}{25} + \frac{9}{80(11+(-7+\Delta)\Delta)}$$

$$\pi^{hl*} = \frac{1937+4\Delta(-779+313\Delta)}{400(29+\Delta(-47+19\Delta))}$$

We restrict $\Delta$ to be within the first discontinuity point ($\Delta = 1.178$), as shown in Figure 6. This tax rate differential value falls in the common range worldwide. More importantly, when $\Delta$ exceeds the first discontinuity point, the profit goes infinite, contradicting to common sense. With the restriction, we can conclude strategy *hl* to be dominant. Strategy *lh* still brings the MNF second most profit, while strategy two ranks last in profitability. This order is in accordance with the constrained analytical results.

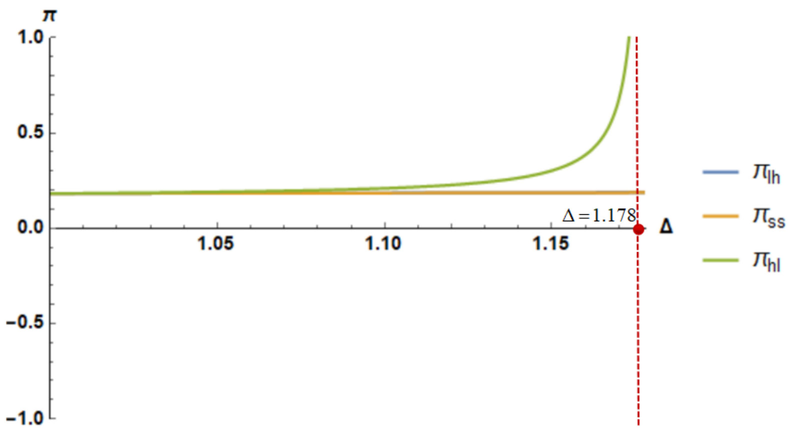



Figure 6. MNF's Profit Comparison (from High to Low)

## 5. Extension: The impact of network effect

We consider the impact of network effect in the long run. Because consumers can benefit from the increasing trading transparency, data efficiency and market comprehensiveness brought by network effect as the number of consumers of one particular application inflates, they will in turn increase their valuation over this application. An example may be *Salesforce's* shared database, where the enterprises can anonymize and share their data with all other *Salesforce's* customers and depict a whole picture of the economy together. The network effect intensity $\gamma$, which represents how much each addition to the number of buyers boosts the software's perceived value, will thus expand the total market size by $\gamma(D_H + D_L)$.

The customers' utility under customized channel is still $U_H = v - p_H - c$ and under direct channel is $U_L = \theta v - p_L$ similar to previous case. The indifferent margins are still $v_H = \frac{p_H + c - p_L}{1-\theta}$ and $v_L = \frac{p_L}{\theta}$ respectively. Due to the network effect, the market size and segmentation is now depicted in Figure 7.

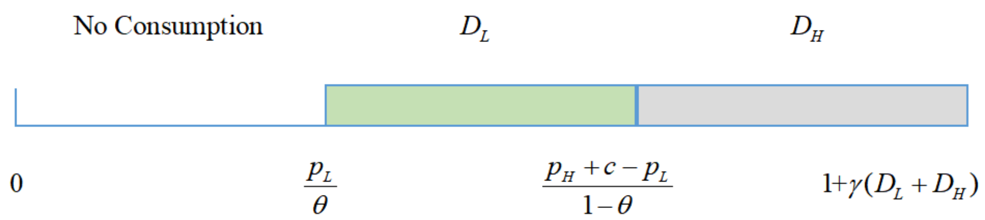



Figure 7. Market Segmentation under Mixed Distribution Channels with Network Effect

The demand for each channel with network effect now becomes $D_L = \frac{p_H+c-p_L}{1-\theta} - \frac{p_L}{\theta} = \frac{-\theta(c+p_H)+p_L}{(-1+\theta)\theta}$ and $D_H = \frac{1+\gamma D_L - \frac{p_H+c-p_L}{1-\theta}}{1-\gamma} = \frac{(1+c(-1+\gamma)-\theta)\theta+(-1+\gamma)\theta p_H+(-\gamma+\theta)p_L}{(-1+\gamma)(-1+\theta)\theta}$. The profit functions of MNF and retailing division are still $\pi_M = p_L D_L(1-\tau_1) + w D_H(1-\tau_1) + (p_H - w)D_H \Delta(1-\tau_1)$ and $\pi_r = (p_H - w)D_H \Delta(1-\tau_1)$. Because there are no analytical results for optimum with network effect, we perform numerical analysis as well. We assign $c = 0.05$, $\theta = 0.8, \tau_1 = 0.2$ and $\gamma = 0.2$. By backward induction, we reach the optimal outcomes under three strategies and summarize the total profit of MNF in Lemma 2.

**Lemma 2.**

$$\pi^{lh*} = \frac{516-240\Delta}{3195-1600\Delta}$$

$$\pi^{ss*} = \frac{3(-13+5\Delta)(-229+45\Delta)}{20(-27+5\Delta)(-103+45\Delta)}$$

$$\pi^{hl*} = \frac{12(-32+25\Delta)(-512+351\Delta)}{5(-240+163\Delta)(-1040+837\Delta)}$$

### 5.1 Total Profit Comparison: From Low to High

We start the analysis for cases when MNF provides services from low tax rate region to high tax rate region. By comparing total profits, we present the findings in Proposition 7.

**Proposition 7.** *(MNF's Optimal Pricing Strategy, from Low to High, with Network Effect)* With the influence of network effect, when $\frac{1}{2} < \Delta < 1$, it maximizes MNF's profit to adopt strategy *lh*, followed by strategy *hl* and *ss* in descending order. In inequality equation form, $\pi^{lh*} > \pi^{hl*} > \pi^{ss*}$.

By comparing the optimal profits when the tax differential ranges from $\frac{1}{2}$ to 1, we find that $\pi^{lh*} > \pi^{hl*} > \pi^{ss*}$, and thus the results from previous analysis still hold. The optimal total profit of strategy *lh* is still greater than those of other two strategies. In other words, it benefits the MNF most to price standard SaaS application first with the influence of network effect. Another finding is that simultaneous game produces least profit among the three strategies. In Figure 8(a) we plot the strategy comparison results under the influence of network effect, when the MNF sells from a low tax rate region to high tax rate regions. From the figure, we can observe the dominant position of strategy *lh*.

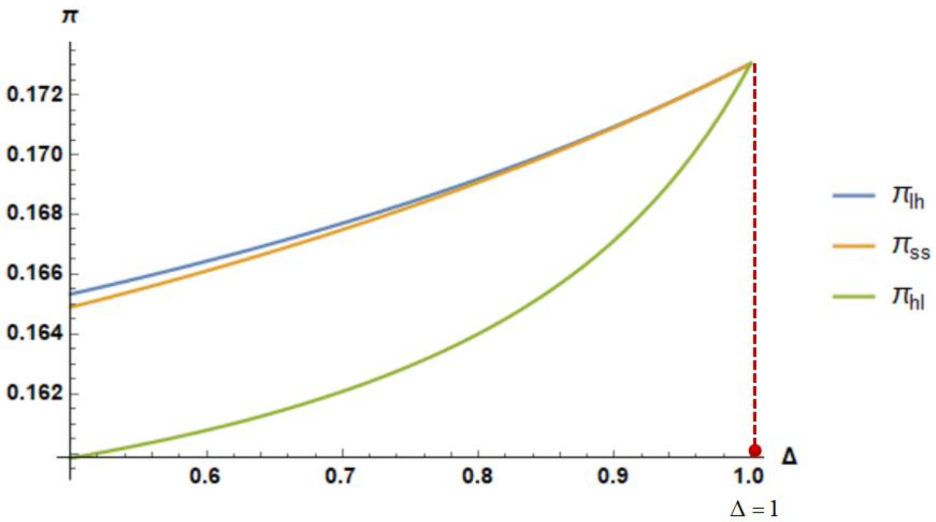



Figure 8(a). MNF's Profit Comparison with Network Effect (from Low to High)

### 5.2 Total Profit Comparison: From High to Low

Symmetrically, when we investigate cases when relative tax differential ranges between 1 and 1.24254, the pattern follows the previous results that are without network effect. We present the findings in Proposition 8.

**Proposition 8** *(MNF's Optimal Pricing Strategy, from High to Low, with Network Effect).*

When $1 < \Delta < 1.24253$, it maximizes MNF's total profit to adopt strategy *hl*, while strategy *lh* and *ss* produce MNF less profit in descending. In inequality equation form, $\pi^{hl*} > \pi^{lh*} > \pi^{ss*}$ when $1 < \Delta < 1.24253$. We plot the profit comparison in Figure 8(b), when firms provide services from high tax rate regions to low tax rate regions. This figure illustrates visually the dominant position for strategy *hl* when $\Delta > 1$.

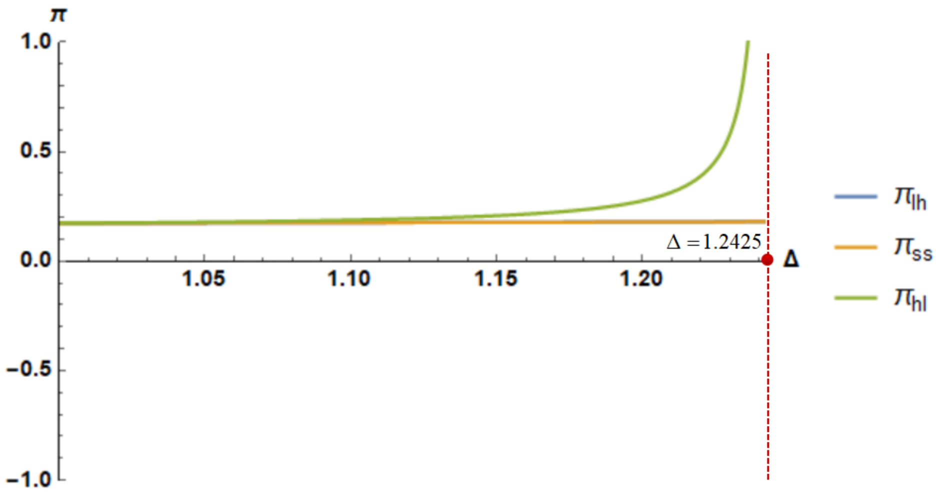



Figure 8(b). MNF's Profit Comparison with Network Effect (from High to Low)

To sum up, with the impact of network effect in the long run, the results we reach for the short run still hold. When an MNF sells products from a low tax rate region to a high tax rate region, it is optimal for the MNF to price low-end products and then to price high-end products. Conversely, when an MNF sells products from high tax rate region to low tax rate region, it is optimal for the MNF to price high-end products and then to price low-end products.

## 6. Conclusions

Our results point out that, for a multinational SaaS firm that provides standard SaaS applications through direct channel and customized applications through indirect channel, it is optimal for the MNF to firstly price standard applications and then the retailing divisions to price customized applications as internal pricing sequence, when this MNF sells software from a region with relatively lower tax rate to regions with relatively higher tax rate. For price and demand specifics, transfer price is highest for a retailing division when then firm chooses to price standard products first. On the contrary, market prices for both standard and customized products are highest when the MNF chooses to price customized products first. As for demand, we notice that it depends on price difference customized and standard products. For standard SaaS products, demand is highest in when standard products are priced first. The conditions for the conclusion are that standard products have high quality matching degree and price difference is large. For customized SaaS products, demand is always highest in simultaneous pricing condition. In the opposite tax rate differential scenario, when this MNF sells software from a region with relatively higher tax rate to regions with relatively lower tax rate, it is optimal for the MNF to firstly price customized applications and then its retailing division to price standard applications as internal pricing sequence. These findings still hold when market size is expanded by the network effect in the long run.

Another key finding is that it is never an optimal strategy for the MNF and retailing division to determine standard applications' and customized applications' prices simultaneously, regardless of the relative tax differentials. This finding is also robust under the impact of network effect.

There are several perspectives of the extending this research. First, we consider the market to be in a monopoly situation. When competition exists, nonetheless, the pricing strategies may alter accordingly. Second, multi-period charging can be an attractive issue. While SaaS products are charged on a subscription basis, subscription fees can vary between periods. The fees for standard products may slightly vary across periods. For customized products, whilst the customization costs can be large initially, the following subscription fees can decrease relatively (Li et al., 2018).

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## Appendix

**Proof of the outcomes in Table 2.**

The proof is based on the demand functions and the objective functions (3) and (4).

We adopt backward induction to solve the problem.

In step one, the retailing division solves the following problem to maximize its profit:

$$\max_{p_H} \pi_r = (p_H - w) D_H \Delta (1 - \tau_1)$$

We yield $p_H = \frac{1}{2}(1 - c + w - \theta + p_L)$.

In step two, the MNF maximizes its total profit by solving:

$$\max_{p_L} \pi_M = p_L D_L (1 - \tau_1) + w D_H (1 - \tau_1) + (p_H - w) D_H \Delta (1 - \tau_1)$$

Transfer price is calculated by solving:

$$\max_{w} \pi_M = p_L D_L (1 - \tau_1) + w D_H (1 - \tau_1) + (p_H - w) D_H \Delta (1 - \tau_1)$$

By bringing $p_H = \frac{1}{2}(1 - c + w - \theta + p_L)$ into $\pi_M$, we yield $w^* = \frac{2(-1+c)(-1+\Delta)+\Delta\theta}{4-2\Delta}$ and $p_L^* = \frac{\theta}{2}$.

We substitute $w^*$ and $p_L^*$ into the equation of $p_H$. We derive $p_H^* = \frac{3-3c-2\Delta+2c\Delta-\theta+\Delta\theta}{4-2\Delta}$.

By bringing the price results back to the equations of quantities and profits, we derive the optima:

$D_L^* = \frac{c+(-1+\Delta)(-1+\theta)}{2(-2+\Delta)(-1+\theta)}$ , $D_H^* = -\frac{-1+c+\theta}{2(-2+\Delta)(-1+\theta)}$ , $\pi_r^* = \frac{\Delta(-1+c+\theta)^2(-1+\tau_1)}{4(-2+\Delta)^2(-1+\theta)}$ and $\pi_M^* = \frac{(c^2+2c(-1+\theta)+(-1+\theta)(-1+(-1+\Delta)\theta))(-1+\tau_1)}{4(2-\Delta)(-1+\theta)}$.

**Proof of the outcomes in Table 3.**

In step one, the retailing division and MNF simultaneously solve the following problem to maximize their profits:

$$\max_{p_H} \pi_r = (p_H - w) D_H \Delta (1 - \tau_1)$$

$$\max_{p_L} \pi_M = p_L D_L (1 - \tau_1) + w D_H (1 - \tau_1) + (p_H - w) D_H \Delta (1 - \tau_1)$$

We yield $p_H = \frac{-2(1+w)-c(-2+\theta)+(2+w(-1+\Delta))\theta}{-4+\theta+\Delta\theta}$ and $p_L = \frac{(w(-3+\Delta)+c(-1+\Delta)+(1+\Delta)(-1+\theta))\theta}{-4+\theta+\Delta\theta}$.

In step two, the MNF sets transfer price by solving:

$$\max_{p_H} \pi_r = (p_H - w)D_H\Delta(1-\tau_1)$$

By bringing $p_H = \frac{-2(1+w)-c(-2+\theta)+(2+w(-1+\Delta))\theta}{-4+\theta+\Delta\theta}$ and $p_L = \frac{(w(-3+\Delta)+c(-1+\Delta)+(1+\Delta)(-1+\theta))\theta}{-4+\theta+\Delta\theta}$ into $\pi_M$, we yield $w^* = \frac{8(-1+c)(-1+\Delta)+2\Delta(1+c+\Delta-c\Delta)\theta-(-1+\Delta^2)\theta^2}{-8(-2+\Delta)+2(-1+\Delta)^2\theta}$.

We substitute $w^*$ into the equation of $p_H$ and $p_L$. We derive $p_L^* = \frac{\theta(10+2c(-1+\Delta)-\theta+\Delta(-6+\Delta\theta))}{-8(-2+\Delta)+2(-1+\Delta)^2\theta}$ and $p_H^* = \frac{12-8\Delta+2(-1+\Delta^2)\theta-(-1+\Delta)^2\theta^2-2c(6+\theta+\Delta(-4+(-2+\Delta)\theta))}{-8(-2+\Delta)+2(-1+\Delta)^2\theta}$.

By bringing the price results back to the equations of quantities and profits, we derive the optimums:

$D_L^* = \frac{2c(-3+\Delta)+(-1+\Delta)(-1+\theta)(-2+(-1+\Delta)\theta)}{2(-1+\theta)(8-4\Delta+(-1+\Delta)^2\theta)}$, $D_H^* = \frac{(-1+c+\theta)(-2+(-1+\Delta)\theta)}{(1-\theta)(8-4\Delta+(-1+\Delta)^2\theta)}$,

$\pi_r^* = \frac{\Delta(-1+c+\theta)^2(-2+(-1+\Delta)\theta)^2(-1+\tau_1)}{(-1+\theta)(8-4\Delta+(-1+\Delta)^2\theta)^2}$ and $\pi_M^* = \frac{(4c^2+8c(-1+\theta)-(-1+\theta)(2+\theta-\Delta\theta)^2)(-1+\tau_1)}{4(-1+\theta)(8-4\Delta+(-1+\Delta)^2\theta)}$.

**Proof of the outcomes in Table 4.**

In step one, the MNF solves the following problem to maximize its profit:

$$\max_{p_L} \pi_M = p_L D_L(1-\tau_1) + wD_H(1-\tau_1) + (p_H - w)D_H\Delta(1-\tau_1)$$

We yield $p_L = \frac{(w(-3+\Delta)+c(-1+\Delta)+(1+\Delta)(-1+\theta))\theta}{-4+\theta+\Delta\theta}$.

In step two, the retailing division sets retailing price by solving:

$$\max_{p_H} \pi_r = (p_H - w)D_H\Delta(1-\tau_1)$$

In step three, MNF sets transfer price by solving:

$$\max_{w} \pi_M = p_L D_L(1-\tau_1) + wD_H(1-\tau_1) + (p_H - w)D_H\Delta(1-\tau_1)$$

By bringing $p_L = \frac{(w(-3+\Delta)+c(-1+\Delta)+(1+\Delta)(-1+\theta))\theta}{-4+\theta+\Delta\theta}$ into $\pi_r$ and $\pi_M$, we yield

$w^* = \frac{-2(-1+\Delta)(4+(1+\Delta)(-3+\theta)\theta)+c(-8+8\Delta+6\theta+2(2-3\Delta)\Delta\theta-(1+\Delta)^2\theta^2)}{2(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)}$ and

$p_H^* = \frac{4(-1+c)(-3+2\Delta)+2((1+\Delta)(-5+3\Delta)+c(3+(4-3\Delta)\Delta))\theta-(-2+2(-2+\Delta)\Delta+c(1+\Delta)^2)\theta^2}{2(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)}$.

We substitute $w^*$ and $p_H^*$ into the equation of $p_L$.

We derive $p_L^* = \frac{\theta(10-6\Delta-8\theta-c(-1+\Delta)(-2+\theta+\Delta\theta)+2\theta(\theta+\Delta(-2+2\Delta+\theta)))}{2(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)}$.

By bringing the price results back to the equations of quantities and profits, we derive the optimums:

$D_H^* = \frac{(-1+c+\theta)(-1+\Delta\theta)(-2+\theta+\Delta\theta)}{(-1+\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}$, $D_L^* = \frac{2(-1+\Delta)(-1+\theta)(-1+\Delta\theta)-c(-2+\theta+\Delta\theta)(-3+\Delta+\theta+\Delta\theta)}{2(-1+\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}$,

$\pi_r^* = -\frac{2\Delta(-1+c+\theta)^2(-1+\Delta\theta)^2(-2+\theta+\Delta\theta)(-1+\tau_1)}{(-1+\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)^2}$

$\pi_M^* = \frac{(-4(-1+\theta)(-1+\Delta\theta)^2+c^2(-2+\theta+\Delta\theta)^2+2c(-1+\theta)(-2+\theta+\Delta\theta)^2)(-1+\tau_1)}{4(-1+\theta)(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)}$.

**Proof of Proposition 1.**

The proof is based on the outcomes in Table 2-4. According to the assumptions in Section 4.1, we have $0 < c < 1$, $\frac{1}{2} < \Delta < 1$ and $0 < \theta < 1 - c$.

When we calculate $w^{lh} - w^{ss}$, we can easily have $-\frac{(-1+\Delta)\theta(-1+c+\theta)}{(-2+\Delta)(8-4\Delta+(-1+\Delta)^2\theta)} > 0$. Similarly, based on restrictions, we calculate $w^{ss} - w^{hl} = -\frac{(1+\Delta)\theta(-1+c+\theta)(8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2)}{2(8-4\Delta+(-1+\Delta)^2\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}$. It is easily observed that $(1+\Delta)\theta > 0$, $-1+c+\theta < 0$ and $8-4\Delta+(-1+\Delta)^2\theta > 0$.

For $8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2$, we regard it as a quadratic function of $\theta$. We apply quadratic function root formula and calculate the negativity of $(-5+3(-2+\Delta)\Delta)^2-4(1+\Delta)^2(8-4\Delta)$. If the function is negative, there is no root for $8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2=0$.

We assume that $((-5+3(-2+\Delta)\Delta))^2-4(1+\Delta)^2(8-4\Delta) < 0$. If so, $(-5+3(-2+\Delta)\Delta)^2 < 4(1+\Delta)^2(8-4\Delta)$. Expanding both sides, we get $25+60\Delta+6\Delta^2-36\Delta^3+9\Delta^4 < 32+48\Delta-16\Delta^3$. The function can be reduced to $(-1+\Delta)^3(7+9\Delta) < 0$. Because $\frac{1}{2} < \Delta < 1$, the inequality holds on the domain. We prove that $((-5+3(-2+\Delta)\Delta))^2-4(1+\Delta)^2(8-4\Delta) < 0$.

We therefore conclude that there is no root for quadratic function $8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2$. Because quadratic coefficient $(1+\Delta)^2$ is positive, the function does not intersect with X axis and its value is always larger than zero.

Similarly, we regard $8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2$ as a quadratic function with negative coefficient of quadratic term $\theta$.

We apply quadratic function root formula and calculate the negativity of $[6-2\Delta(4+3\Delta)]^2-4(-1+\Delta)(1+\Delta)^2(8\Delta)$. If it is negative, we conclude that there is no solution for function $8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2=0$.

We assume that $[6-2\Delta(4+3\Delta)]^2-4(-1+\Delta)(1+\Delta)^2(8\Delta)<0$, $[6-2\Delta(4+3\Delta)]^2<4(-1+\Delta)(1+\Delta)^2(8\Delta)$. However, $[6-2\Delta(4+3\Delta)]^2>0$ while $4(-1+\Delta)(1+\Delta)^2(8\Delta)<0$. The inequality does not hold. There are roots for $8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2$ with respect to $\theta$.

We calculate the ranges of $8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2$.When $\theta=0$, $8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2=8\Delta>0$. When $\theta=1$, $8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2=\Delta^3-5\Delta^2\Delta+5$. We take the second order derivatives of $\Delta^3-5\Delta^2-\Delta+5$ with respect to $\Delta$. We get $6\Delta-10<0$. $\Delta^3-5\Delta^2-\Delta+5$ is monotonically decreasing on the domain of definition. $\Delta^3-5\Delta^2-\Delta+5\in(0,\frac{27}{8})$ when $\Delta\in(\frac{1}{2},1)$.

We prove that $8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2>\mathbf{0}$.

Taking the formulas together, we get $w^{ss}-w^{hl}=-\frac{(1+\Delta)\theta(-1+c+\theta)(8\Delta+6\theta-2\Delta(4+3\Delta)\theta+(-1+\Delta)(1+\Delta)^2\theta^2)}{2(8-4\Delta+(-1+\Delta)^2\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}>0$. $w^{ss}>w^{hl}$.

We thus have $w^{lh}>w^{ss}>w^{hl}$.

**Proof of Proposition 2 ($p_L$)**

The proof is based on the outcomes in Table 2-4. According to the assumptions in Section 4.1, we have $0<c<1$, $\frac{1}{2}<\Delta<1$ and $0<\theta<1-c$.

When we calculate $p_L^{lh}-p_L^{ss}$, we have $-\frac{(-1+\Delta)\theta(-1+c+\theta)}{8-4\Delta+(-1+\Delta)^2\theta}<0$.

Similarly, based on restrictions, we calculate $p_L^{ss}-p_L^{hl}=\frac{(-1+\Delta^2)\theta^2(-1+c+\theta)(-4+(3+\Delta^2)\theta)}{2(8-4\Delta+(-1+\Delta)^2\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}$. In the equation, $-1+\Delta^2<0$, $-1+c+\theta<0$, $-4+$

$(3+\varDelta^2)\theta < 0$ and $8-4\varDelta+(-1+\varDelta)^2\theta < 0$. $8-4\varDelta+(-5+3(-2+\varDelta)\varDelta)\theta+(1+\varDelta)^2\theta^2 > 0$ is proved before. We have $p_L^{ss}-p_L^{hl} < 0$.

We thus have $p_L^{lh} < p_L^{ss} < p_L^{hl}$.

**Proof of Proposition 2 ($p_H$)**

The proof is based on the outcomes in Table 2-4. According to the assumptions in Section 4.1, we have $0 < c < 1$, $\frac{1}{2} < \varDelta < 1$ and $0 < \theta < 1-c$.

When we calculate $p_H^{lh}-p_H^{ss}$, we have $-\frac{(-1+\varDelta)^2\theta(-1+c+\theta)}{2(-2+\varDelta)(8-4\varDelta+(-1+\varDelta)^2\theta)} < 0$. Similarly, based on restrictions, we calculate $p_H^{ss} < p_H^{hl} = -\frac{(-1+\varDelta^2)\theta(-1+c+\theta)(8-8\varDelta\theta+(-1+\varDelta^2)\theta^2)}{2(8-4\varDelta+(-1+\varDelta)^2\theta)(8-4\varDelta+(-5+3(-2+\varDelta)\varDelta)\theta+(1+\varDelta)^2\theta^2)} < 0$. We thus have $p_H^{lh} < p_H^{ss} < p_H^{hl}$.

**Proof of Proposition 3a.**

The proof is based on the outcomes in Table 2-4. According to the assumptions in Section 4.1, we have $0 < c < 1$, $\frac{1}{2} < \varDelta < 1$ and $0 < \theta < 1-c$.

When we calculate $D_H^{lh}-D_H^{ss}$, we have $\frac{(-3+\varDelta)(-1+\varDelta)\theta(-1+c+\theta)}{2(-2+\varDelta)(-1+\theta)(8-4\varDelta+(-1+\varDelta)^2\theta)} < 0$. $D_H^{lh} < D_H^{ss}$. Similarly, based on restrictions, we calculate $D_H^{lh}-D_H^{hl} = -\frac{(-1+\varDelta^2)\theta(-1+c+\theta)(4+\theta(-2+\theta+\varDelta(-4+\varDelta\theta)))}{(-1+\theta)(8-4\varDelta+(-1+\varDelta)^2\theta)(8-4\varDelta+(-5+3(-2+\varDelta)\varDelta)\theta+(1+\varDelta)^2\theta^2)} > 0$. We thus have $D_H^{ss} > D_H^{lh} > D_H^{hl}$.

**Proof of Proposition 3b.**

The proof is based on the outcomes in Table 2-4. According to the assumptions in Section 4.1, we have $0 < c < 1$, $\frac{1}{2} < \varDelta < 1$ and $0 < \theta < 1-c$.

Comparing $D_L$ under different strategies, we calculate $D_L^{lh}-D_L^{ss}$ first. $D_L^{lh}-D_L^{ss} = \frac{(-1+\varDelta)(4+\varDelta(-2+\theta)-\theta)(-1+c+\theta)}{2(-2+\varDelta)(-1+\theta)(8-4\varDelta+(-1+\varDelta)^2\theta)}$. According to restrictions, we can easily obtain $D_L^{lh}-D_L^{ss} > 0$. In other words, $D_L^{lh} > D_L^{ss}$.

Next we compare $D_L^{ss}$ and $D_L^{hl}$. $D_L^{ss} - D_L^{hl} = \frac{(-1+\Delta^2)\theta(-1+c+\theta)(4+\theta(3-\theta+\Delta(-8+\Delta+\Delta\theta)))}{2(-1+\theta)(8-4\Delta+(-1+\Delta)^2\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)} < 0$. $D_L^{ss} < D_L^{hl}$.

Finally we compare $D_L^{lh}$ and $D_L^{hl}$. $D_L^{lh} - D_L^{hl} = \frac{(-1+\Delta)(-1+c+\theta)(4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}{2(-2+\Delta)(-1+\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}$. Solving $D_L^{lh} - D_L^{hl} > 0$, we have conditions for the inequality to hold. We can prove that $-1+\Delta < 0$, $-1+c+\theta < 0$, $-2+\Delta < 0$, $-1+\theta < 0$ and $8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2 > 0$.

To calculate the value $4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2$, we regard it as a quadratic function of $\theta$. We apply quadratic function root formula and calculate the negativity of $(-5+(-2+\Delta)\Delta)^2-4(1+\Delta)^2(4-2\Delta)$. If the function is negative, there are no roots for $4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2 = 0$.

We assume that $(-5+(-2+\Delta)\Delta)^2-4(1+\Delta)^2(4-2\Delta) < 0$. Expanding the left-hand side, we have $9-4\Delta-6\Delta^2+4\Delta^3+\Delta^4 < 0$. We group the polynomials as $4-4\Delta+5-6\Delta^2+4\Delta^3+\Delta^4 < 0$. $4-4\Delta > 0$. $\Delta^4 > 0$. We let $y = 5-6\Delta^2+4\Delta^3$. By taking the second order derivatives of function y with respect to $\Delta$, we get $24\ \Delta - 12 > 0$ on the domain of $\Delta$. Function y monotonically increasing on the definition range of $\Delta$. The minimum value of function y is 4 when $\Delta = \frac{1}{2}$. We thus prove $5-6\Delta^2+4\Delta^3 > 0$. Taking three groups together, we reject the assumption and conclude that $(-5+(-2+\Delta)\Delta)^2 - 4(1+\Delta)^2(4-2\Delta) < 0$. In other words, there are roots for $4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2 = 0$.

Solving $4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2 = 0$, we get $\theta^a = \frac{5+2\Delta-\Delta^2}{2(1+\Delta)^2} - \frac{1}{2}\sqrt{\frac{9-4\Delta-6\Delta^2+4\Delta^3+\Delta^4}{(1+\Delta)^4}}$ and $\theta^b = \frac{5+2\Delta-\Delta^2}{2(1+\Delta)^2} + \frac{1}{2}\sqrt{\frac{9-4\Delta-6\Delta^2+4\Delta^3+\Delta^4}{(1+\Delta)^4}}$.

We assume that $\theta^b > 1$. By simplification, we get $3\Delta^4-7\Delta^3+4\Delta^2-\Delta-1 < 0$. We take the second order derivative of $y = 3\Delta^4-7\Delta^3+4\Delta^2-\Delta-1$ to get the maximum value of $y$. $y$ decreases at $\Delta \in$

$(\frac{1}{2}, \frac{1}{12}(7+\sqrt{17})$ and increases at $\Delta \in (\frac{1}{12}(7+\sqrt{17}, 1)$.The maximum value of $y$ is $-1.1875$ when $\Delta < \frac{1}{2}$. We thus prove that $\theta^b > 1$ and we neglect $\theta^b$.

We assume that $\theta^a < 1$. Similarly, by simplification, we have $8(1+\Delta)^2(-1+\Delta+3\Delta^2) > 0$. It holds in the domain of $\Delta$. We thus take $\theta^a$ as an effective root of $4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2$. Specifically, $4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2 > 0$ when $\theta \in (0, \theta^a)$ and $4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2 < 0$ when $\theta \in (\theta^a, 1)$.

We assign $\theta_1 = \theta^a = \frac{5+2\Delta-\Delta^2}{2(1+\Delta)^2} - \frac{1}{2}\sqrt{\frac{9-4\Delta-6\Delta^2+4\Delta^3+\Delta^4}{(1+\Delta)^4}}$. Taking the value of other items together, we thus prove that $D_L^{lh} - D_L^{hl} = \frac{(-1+\Delta)(-1+c+\theta)(4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}{2(-2+\Delta)(-1+\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)} > 0$ when $\theta \in (0, \theta_1)$ and $D_L^{lh} - D_L^{hl} = \frac{(-1+\Delta)(-1+c+\theta)(4-2\Delta+(-5+(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)}{2(-2+\Delta)(-1+\theta)(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)} < 0$ when $\theta \in (\theta_1, 1)$.

**Proof of Proposition 4a.**

The proof is based on the outcomes in Table 2-4. According to model setting in Section 3, we have $0 < \tau_1 < 1$. According to the assumptions in Section 4.1, we have $0 < c < 1$, $\frac{1}{2} < \Delta < 1$ and $0 < \theta < 1-c$.

we compare $\pi_L^{lh}$ and $\pi_L^{ss}$. $\pi_L^{lh} - \pi_L^{ss} = \frac{(-3+\Delta)(-1+\Delta)\theta(-1+c+\theta)(4(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2)(-1+\tau_1)}{4(-2+\Delta)(-1+\theta)(8-4\Delta+(-1+\Delta)^2\theta)^2}$.

It can be easily observed that $-3+\Delta < 0$, $-1+\Delta < 0$, $-1+c+\theta < 0$, $-2+\Delta < 0$, $-1+\tau_1 < 0$, $-1+\theta < 0$ and $(8-4\Delta+(-1+\Delta)^2\theta)^2 > 0$. For $4(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2$, because $(-1+c)(-2+\Delta) > 0$, we have $4(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2 > 0$. Taking together, we prove that $\pi_L^{lh} - \pi_L^{ss} > 0$. $\pi_L^{lh} > \pi_L^{ss}$.

In the second step we compare $\pi_L^{ss}$ and $\pi_L^{hl}$. $\pi_L^{ss} - \pi_L^{hl} = \frac{\theta(-1+\tau_1)}{4(-1+\theta)} \times$ $(\frac{(-2(-1+\Delta)(-1+\theta)(-1+\Delta\theta)+c(-2+\theta+\Delta\theta)(-3+\Delta+\theta+\Delta\theta))(-10+6\Delta+8\theta+c(-1+\Delta)(-2+\theta+\Delta\theta)-2\theta(\theta+\Delta(-2+2\Delta+\theta)))}{(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)^2}$ $-\frac{(2c(-3+\Delta)+(-1+\Delta)(-1+\theta)(-2+(-1+\Delta)\theta))(10+2c(-1+\Delta)-\theta+\Delta(-6+\Delta\theta))}{(8-4\Delta+(-1+\Delta)^2\theta)^2})$. Powered by Mathematica, we have $\pi_L^{ss} - \pi_L^{hl} < 0$. $\pi_L^{ss} < \pi_L^{hl}$.

In the third step we compare $\pi_L^{lh}$ and $\pi_L^{hl}$. $\pi_L^{lh} - \pi_L^{hl} =$

$$\frac{\theta\left(-\frac{c+(-1+\Delta)(-1+\theta)}{-2+\Delta}+\frac{(-2(-1+\Delta)(-1+\theta)(-1+\Delta\theta)+c(-2+\theta+\Delta\theta)(-3+\Delta+\theta+\Delta\theta))(-10+6\Delta+8\theta+c(-1+\Delta)(-2+\theta+\Delta\theta)-2\theta(\theta+\Delta(-2+2\Delta+\theta)))}{(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)^2}\right)(-1+\tau_1)}{4(-1+\theta)}.$$

We know $\frac{\theta(-1+\tau_1)}{4(-1+\theta)} > 0$. Solving $-\frac{c+(-1+\Delta)(-1+\theta)}{-2+\Delta} + \frac{(-2(-1+\Delta)(-1+\theta)(-1+\Delta\theta)+c(-2+\theta+\Delta\theta)(-3+\Delta+\theta+\Delta\theta))(-10+6\Delta+8\theta+c(-1+\Delta)(-2+\theta+\Delta\theta)-2\theta(\theta+\Delta(-2+2\Delta+\theta)))}{(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)^2} = 0$, we get the following condition. We define $\theta_2$ to be the minimum solution of the equation.

$$24-24c-20\Delta+20c\Delta+4\Delta^2-4c\Delta^2+(-48+32c-16\Delta+8c\Delta+28\Delta^2-20c\Delta^2-4\Delta^3+4c\Delta^3)\theta$$
$$+\quad(33-14c+48\Delta-19c\Delta+10\Delta^2+3c\Delta^2-20\Delta^3+7c\Delta^3+\Delta^4-c\Delta^4)\theta^2+(-10+2c-24\Delta+5$$
$$-24\Delta^2+3c\Delta^2-4\Delta^3-c\Delta^3+6\Delta^4-c\Delta^4)\theta^3+(1+4\Delta+6\Delta^2+4\Delta^3+\Delta^4)\theta^4=0$$

As we have the restriction that $0 < \theta < 1-c$, $\theta_2$ is the unique solution on the value range. $\pi_L^{lh} > \pi_L^{hl}$ when $0 < \theta < \theta_2$. $\pi_L^{lh} < \pi_L^{hl}$ when $\theta_2 < \theta < 1-c$.

**Proof of Proposition 4b.**

The proof is based on the outcomes in Table 2-4. According to model setting in Section 3, we have $0 < \tau_1 < 1$. According to the assumptions in Section 4.1, we have $0 < c < 1$, $\frac{1}{2} < \Delta < 1$ and $0 < \theta < 1-c$.

We compare $\pi_H^{lh}$ and $\pi_H^{ss}$. $\pi_H^{lh} - \pi_H^{ss} = \frac{(-1+c+\theta)\left(\frac{1-c}{-2+\Delta}-\frac{2(-2+(-1+\Delta)\theta)(4(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2)}{(8-4\Delta+(-1+\Delta)^2\theta)^2}\right)(-1+\tau_1)}{4(-1+\theta)}$.

Because $-1+c+\theta > 0$, $-1+\tau_1 < 0$, $4(-1+\theta) < 0$ and $\frac{1-c}{-2+\Delta} > 0$. For $\frac{2(-2+(-1+\Delta)\theta)(4(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2)}{(8-4\Delta+(-1+\Delta)^2\theta)^2}$, we can observe the numerator is positive. $-2+(-1+\Delta)\theta < 0$ because $\Delta < 1$. $(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2 > 0$ because $(-1+c)(-2+\Delta) > 0$. Taking the items together, we have $-\frac{2(-2+(-1+\Delta)\theta)(4(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2)}{(8-4\Delta+(-1+\Delta)^2\theta)^2} > 0$. We prove that $\pi_H^{lh} - \pi_H^{ss} < 0$, and thus $\pi_H^{ss} > \pi_H^{lh}$.

We then compare $\pi_H^{lh}$ and $\pi_H^{hl}$. Powered by Mathematica, we have:

$$\pi_H^{lh} - \pi_H^{hl} = \frac{(-1+c+\theta)\left(\frac{1-c}{-2+\Delta}-\frac{2(-1+\Delta\theta)(-2+\theta+\Delta\theta)(-8+4\Delta+6\theta+c(-2+\theta+\Delta\theta)(-4+\theta+\Delta(2+\theta))-2\theta(\theta+\Delta(-2+\Delta+\Delta\theta)))}{(8-4\Delta+(-5+3(-2+\Delta)\Delta)\theta+(1+\Delta)^2\theta^2)^2}\right)(-1+\tau_1)}{4(-1+\theta)} > 0.$$

We thus have $\pi_H^{lh} > \pi_H^{hl}$.

**Proof of Proposition 5.**

The proof is based on the outcomes in Table 2-4. According to model setting in Section 3, we have $0 < \tau_1 < 1$. According to the assumptions in Section 4.1, we have $0 < c < 1$, $\frac{1}{2} < \Delta < 1$ and $0 < \theta < 1 - c$.

We compare $\pi^{lh*}$and $\pi^{hl*}$. Powered by Mathematica, we have:

$$\pi^{lh*} - \pi^{hl*} = \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{(1-c)(-1+c+\theta)}{-2+\Delta} - \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{(4(-1+\Delta)(-1+\theta)(-1+\Delta\theta)^2(8+(1+\Delta)(-5+\theta)\theta)}{(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2} - \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{2c(-1+\theta)(-2+\theta+\Delta\theta)(16(-1+\Delta)+2(9+(8-13\Delta)\Delta)\theta}{(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2} - \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{(-7+\Delta(-15+\Delta(-5+11\Delta)))\theta^2+(1+\Delta)^3\theta^3)}{(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2} - \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{c^2(32-32\Delta-52\theta+4\Delta(-8+17\Delta)\theta+16(2+4\Delta-3\Delta^3)\theta^2}{(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2} - \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{(1+\Delta)(-9+\Delta(-19+\Delta(-7+11\Delta)))\theta^3+(1+\Delta)^4\theta^4)}{(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2} > 0.$$

We have $\pi^{lh*} > \pi^{hl*}$. Similarly, we compare $\pi^{ss*}$ and $\pi^{hl*}$ by $\pi^{ss*} - \pi^{hl*}$. Powered by Mathematica, we have:

$$\pi^{ss*} - \pi^{hl*} = \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{2(-1+c+\theta)(-2+(-1+\Delta)\theta)\left(4(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2\right)}{(8-4\Delta+(-1+\Delta)^2\theta)^2}$$
$$- \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{2(-1+c+\theta)(-2+(-1+\Delta)\theta)(4(-1+c)(-2+\Delta)+(-1+\Delta)^2\theta^2)}{(4(-1+\Delta)(-1+\theta)(-1+\Delta\theta)^2(8+(1+\Delta)(-5+\theta)\theta)(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2}$$
$$- \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{2c(-1+\theta)(-2+\theta+\Delta\theta)(16(-1+\Delta)+2(9+(8-13\Delta)\Delta)\theta}{(4(-1+\Delta)(-1+\theta)(-1+\Delta\theta)^2(8+(1+\Delta)(-5+\theta)\theta)(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2}$$
$$- \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{(-7+\Delta(-15+\Delta(-5+11\Delta)))\theta^2+(1+\Delta)^3\theta^3)}{(4(-1+\Delta)(-1+\theta)(-1+\Delta\theta)^2(8+(1+\Delta)(-5+\theta)\theta)(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2}$$
$$- \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{c^2(32-32\Delta-52\theta+4\Delta(-8+17\Delta)\theta+16(2+4\Delta-3\Delta^3)\theta^2}{(4(-1+\Delta)(-1+\theta)(-1+\Delta\theta)^2(8+(1+\Delta)(-5+\theta)\theta)(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2}$$
$$- \frac{(-1+\tau_1)}{4(-1+\theta)} \times \frac{(1+\Delta)(-9+\Delta(-19+\Delta(-7+11\Delta)))\theta^3+(1+\Delta)^4\theta^4)}{(4(-1+\Delta)(-1+\theta)(-1+\Delta\theta)^2(8+(1+\Delta)(-5+\theta)\theta)(8-4\Delta-5\theta+3(-2+\Delta)\Delta\theta+(1+\Delta)^2\theta^2)^2} < 0.$$

$\pi^{ss*} < \pi^{hl*}$. We thus have $\pi^{lh*} > \pi^{hl*} > \pi^{ss*}$.