CONSUMERS' INFORMATION SEARCHING BEHAVIOR OF CHANNEL INTEGRATION FOR NEW RETAILING ENTERPRISES

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Consumers' Information Searching Behavior of Channel Integration for New Retailing Enterprises

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CERTIFICATE OF ORIGINALITY

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Abstract

In both product and service markets, quality or other characteristics regarding the offerings are the sellers' private information. Consumers are able to learn these inherent attributes via either omni-channels, sellers' price signaling or online review forums. All the accesses help consumers better understand the product and service industries which in turn determine sellers' selling strategies (i.e., pricing strategy, product assortment strategy, effort level strategy) to a certain extent. In this thesis, we mainly focus on consumers information searching behavior in both product and service markets to study its impact on sellers' marketing policies.

In Chapter 2, we examine a monopolist selling two products with both horizontal and vertical heterogeneity through dual channels and our results show that the optimal pricing strategy reflects a complement relationship between online and offline product. Meanwhile the consumers demand is highly related to the online return cost which finally influence the seller's optimal profit. Moreover, the quality performances of both offline product and online product will have great influence on the seller's decision regarding the product design feature, then induce different demands between two products. In Chapter 3, we further investigate an oligopoly market with two sellers competing to sell four products through dual channels. Based on the model, we find that prices of online products and offline products are both influenced by return cost although with different monotonicity. The sellers choose optimal product placement strategies by considering the unit misfit cost of products' horizontal feature and the return cost of online products simultaneously. In Chapter 4, we turn to examine the service market by considering a two-period model with service provider's price signaling strategy to convey her cost efficiency or quality certification. And our results show that the existence of separating and pooling equilibrium are highly related to the composition ratio of high-type consumers and the prior probability of cost-efficient service provider. In Chapter 5, we develop an intertemporal decision model under the collaborative service market with review forums which can help consumers acquire service quality information ex ante. Results show that when the reviewing effect is high and the collaborating effect is approaching two end values, the review process can help improve service provider's profit. Meanwhile, when the tendency for consumers to

make review become lower, namely, consumers are more prudent to post reviews, the review system performs more helpfully, which is also embodied in the refinement of rating scales.

All our works in this thesis contribute to the literatures on the interface between operations management and marketing with a full profile to depict consumers' information searching and sharing behavior in both product and service markets via channel integration. Our implications help sellers make their marketing tactics more sensibly. As no matter the consumers need to pay for products or services, the primary task for them is to acquire relevant information, after which they make the decision of whether to purchase and which one to purchase. Costs (including products return cost, service cost efficiency, etc) play an important role in consumers' purchasing process. When we explore the influence of costs on the sellers marketing strategy, we further discover the significance of the foundation in respect of information dissemination platform, which makes a difference to sellers' improvement of their profitability.

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Chapter 1

Introduction

Consumers in their retail practice are attaching great importance to the information searching behavior through all available channels. They take use of both online platform and offline retail channels readily, which help them search for and transmit information in their purchasing experience. In order to be better adapted to this new environment, retailers of all industries are reexamining their strategies for delivering both information and products (or services) to their target consumers. A 2018 annual report on retailers' channel integration and development in China shows that what consumers purchasing via online platform value most is the genuine guarantee of products, while price is put to the second place. As to service, consumers are more dependent on the online platform to gather information before their purchasing decision. Information streams via all channels cover several consumption scenarios and realize the mesh surrounding of consumers. Our work makes the sellers not only the decision maker of the pricing strategy, but also the core competence to optimize their product placement strategy or effort level strategy. All of these marketing tactics can help the sellers gain advantage in the wave of market competition. Studies on the consumers' information dissemination behavior in the new era of retail are mainly focusing on the empirical analyses, while theoretical models are limited. This kind of consumers' seamless information searching behaviors are embedded in the new retailing enterprises via channel integration. They appeal us to explore the sellers' operation mechanism behind the information asymmetry. Namely, consumers' information searching and transmission behaviors will disclose partial information and alleviate the information asymmetry problem to a certain extent. However, the information contents regarding both products and services are diverse and disperse. As for the product market, information contents regarding products are mainly divided into two dimensions, i.e., vertical differentiation for product quality performance and horizontal differentiation for product feature (Lacourbe, Loch, & Kavadias, 2009). As for the service market, the effort level strategies are not known to outliers ex ante, and meanwhile the inherent competence of service providers to offer the service is also unknown (Sun & Xu, 2018). The information asymmetry between product or service producers and consumers makes distinguishing the superior sellers an important problem worth exploring. Taking into consideration both the information categories and model structures, we can summarize the characteristics of each chapter in Table 1. This table clearly describes the main contents of information asymmetry in products and services between firms and consumers.

	Information content		Model structure	
	Ex-ante unobservable	Ex-ante observable	Single period	Two period
Chapter 2	quality	fitness	√	
Chapter 3	partial fitness	partial fitness	√	
Chapter 4	effort level; cost efficiency	cost efficiency; quality		√
Chapter 5	quality	effort level		√

Table 1. Brief Classification of the Chapters

We differentiate each chapter mainly by the concrete type of asymmetric information. However, they are connected by the common theme in the background of channel integration for new retailing enterprises. To be more specific, amid fast advance in technology and information, firms have initiated various means to reach out to consumers. Omni-channel is just one of the many new market makers, which tend to reshape the business arena. The latest initiatives give rise to new issues, including asymmetric information, integration of multiple channels, and system coordination. All these cause consumers to alter their purchase behavior and drive managers to rethink their selling strategies. We thus tackle a series of issues, through modeling and analysis.

We investigated the behaviors of both consumers and firms, and generated insights on how firms should design products and services and distribute in various supply chain structures. Our focus is on how the firms should leverage the information uncertainty to maximize profit performances, attending to the behaviors that consumers search for unknown information in different means and across various channels. Concretely speaking, in the product market, we investigate the selling strategy of firms when facing consumers' cross-channel information searching, after which we further derive conditions when the traditional retailers should consider this clicks and mortar by earning more from channel integration. Moreover, in the parallel scenario, we consider the service market by exploring the collaborative service providers' optimal pricing or effort level strategy. Under the background where they are confronted with consumers' information dissemination and transmission behaviors intertemporally, we also probe into conditions when the traditional seller can benefit herself from participating in the information revelation through online review platform, given the availability/positivity of customer reviews and the level of participation required from consumers. Our results shed light on the strategic firms' selling policies when the new era of retailing has begun. We focus our research attention on addressing the following problems: what extent of the impacts that consumers' cross-channel searching behaviors will have on firms' information uncertainty; when the product/service producers should embrace the promotion of the Internet and big data; how enterprises can actively reformed and innovated through the upgrading of business models.

Based on this theme, we next make further analyses of each project in the following sections with detailed introduction.

1.1. Consumers' Information Searching Behavior in Product Market

1.1.1. Monopolistic Market

A business report points out that among the consumers conducting omnichannel behavior, 53% of them start researching digitally, while 47% start gathering information in-store (Oracle Bronto. 2018). These kinds of consumers' searching behavior are actually of great help for consumers to gather product information from multidimensions. Meanwhile, this phenomenon gives a challenge to retailers, as they need to design their selling strategy under various circumstances via this kind of cross-channel shopping platform, which is so-called omnichannel selling strategies. We are motivated by the complexity of consumers information searching behaviors in the wave of omnichannel, and we would like to explore the following problems regarding retailers' omnichannel selling strategies: firstly, what fraction of information will be disclosed by consumers' omnichannel information searching behavior; secondly, how the retailers should adjust their product placement and pricing strategy when facing consumers' cross channel shopping behavior; finally, what the influence the omnichannel strategies will have on both consumers' purchasing decision and retailers' revenue.

We study these issues by developing a theoretical model with a monopoly seller selling two products differentiated in both their quality performances and horizontal locations. The quality performance demonstrates the post-purchase experienced attribute in customers' valuations for the product, which is only realized after the consumer receives the product upon product experience. While the horizontal features depict the product's ex-ante observable characteristics, which can be explained as product features that are resolved prior purchase. What we can obtain from our model is in four aspects. Firstly, the two products placed by the seller via both online channel and offline channel are competitive in their market share, however, both products' attributes are more transparent in the omnichannel market, which will finally result in the same direction movement of their prices. They are more likely to be complement goods rather than substitutes. Secondly, the "partial keep" scenario exists when the return cost is low. While the "all keep" scenario exits when the return cost coefficient is not too low. Thirdly, the horizonal dominance only exists when the offline product is better in its quality compared with the online product. Meanwhile, the misfit cost is relatively high compared to the return cost. Finally, when the offline product quality is better than the expected quality of online product, consumers have incentive to purchase online even if the product may be defective in its quality. However, when the offline product quality is worse than the online expected quality, consumers only purchase offline as the online purchasing is faced with more uncertainty.

1.1.2. Oligopolistic Market

After our exploration of consumer' information searching behavior in a monopolistic product market, what also attract our attentions are the competitive pricing and product placement strategies among retailers. They sell similar kinds of products, with the only difference in their assortment methods via both online and offline channels. We assume consumers are heterogenous in their horizonal fitness with respect to different products, while they are common in the return probability facing deceptive product due to quality dissatisfaction. We consider a competitive omnichannel selling market with four horizontally differentiated products sold by two competing retailers. Consumers are heterogeneous in their taste of the product with an intrinsic preference parameter, which is comprised of two main parts: an observable component and an unobservable component prior purchase. Consumers' purchase decision of whether to purchase through physical store or online store and making purchase from which retailer depend on not only the product assortment strategies across competitive retailers, but also the return cost that the consumers are faced with if product return happens.

From our analyses of the model analyses, we can derive the following three insights. Firstly, no matter what the placement strategy both sellers choose, the optimal prices of products sold through online channel are first increasing in the return cost of online product and then decreasing in the online product return cost; while the optimal prices of products sold via offline are always increasing in the online product return cost. Secondly, no matter what the placement strategy sellers will choose, the optimal profits of both sellers is first decreasing in the return cost of online product and then increasing in it. Thirdly, if we consider the three placement strategies given the optimal equilibrium results, the sellers choose the three cases by considering the unit misfit cost of products' horizontal feature and the return cost of online product simultaneously. We further make comparisons between the one-seller market and the two-seller market, our results show that the in the market without competition, the optimal pricing strategies of online product and offline product are in difference with a constant and they change in the same direction.

1.2. Consumers' Information Searching Behavior in Service Market

1.2.1. Signaling Framework

Among all types of service provision, a fraction of them that specifically attract our research interest are the knowledge payment service and online career or interest training service. For instance, teaching a certain instrument, helping students pass language tests or other vocational qualification examinations, coaching body builders to keep fit and so on. As the rapid growth of service industry, it is estimated that the global knowledge payment service market will reach nearly 68 billion RMB by 2021(data. iimedia.cn). Given the complexity of the online training service and its enduring influence, it is of great importance for us to figure out the effort contribution mechanism between the service provider and consumers, meanwhile it is intriguing to study the strategic interaction of both parties under information intertemporal transmission. There are some properties of this kind of so-called “collaborative services” (Roles 2014). Firstly, the outcome of the service depends on both the service provider’s effort contribution and the consumers’ effort input. Furthermore, the service provider makes her own effort level strategy and pricing strategy, while the consumer can decide his effort contribution to the whole service provision process. Both effort level strategies are not prominent to outliers, thus moral hazard might exist in the service process. Both the service provider and the consumer strategically choose their effort contribution accordingly. Besides the unobservable effort levels, the inherent competence of service providers’ cost efficiency is also unobservable, making differentiating the efficient service provider another significant problem worth studying. Meanwhile, we incorporate another asymmetric information regarding the service provider’s quality certification, which makes the problem a two-dimension informational structure.

We consider a monopolistic service provider offer a kind of collaborative service by means of both online and offline channels. Consumers in the market can collect information regarding the service seamlessly across channels. The early consumers who have determined to pay for the service will learn the true effort level of the service provider and disclose it to followers via customer reviews through either online platform or offline word of mouth. At the beginning of the second period, the reviews are delivered to the public. Therefore, the follower consumers will make purchase decisions by considering the service provider’s true effort level revealed through early consumers’ reviews and her second period pricing strategy. Based on our analyses of the signaling model regarding the collaborative service, we can obtain several conclusions as follows. Firstly, the optimal effort level of the service provider is decreasing in the service provider’s cost coefficient parameter, namely, the optimal effort level of the cost-efficient service provider is greater than that of the cost-inefficient service provider. Meanwhile, the optimal profit of the cost-efficient service provider is greater than that of the cost-inefficient one. Secondly, in either case of the separating equilibrium, the optimal effort level strategy of the cost-efficient service provider is greater than that of cost-inefficient service provider. While the optimal effort level strategy of the early consumers when facing the cost-efficient service provider is greater than that facing the cost-inefficient service provider. Thirdly, as the prior probability of cost-efficient service provider increases, the expected revenue of the service provider at pooling equilibrium also increases. Finally, when both the cost efficiency and quality certification information are unobservable to consumers before purchase, the cost-efficient service provider prefers the uniform pricing strategy in respect of the quality dimension to any differential pricing strategies. While the cost-inefficient service provider will adopt the differential pricing strategy in respect of the quality dimension.

1.2.2. Review Platform

Consumers review websites such as Yelp in overseas market and Dazhongdianping in China have become increasingly popular over the past decades, and now exist in nearly every service type. The functions of them in service industries are just like the Alibaba in product industries, for example, Yelp contains more than 70 million reviews in respect of restaurants, education institutions, fitness clubs and other services. Moreover, there is increasingly strong evidence indicating that these reviews posting by consumers via online platforms directly influence the service providers' sales volume (Chevalier & Mayzlin, 2006). Motivated by this phenomenon, we consider a model structure depicting a service market where each transaction party simultaneously owns some private information that influences both parties' payoffs. For example, in our model, consumers possess qualitative information observing the online reviews in respect of service providers' service outcome, while service providers possess private information regarding their quality type. Thus, consumers can only infer the service provider's quality type via the information embedded in the online reviews. By referring to the online reviews posting by early consumers truthfully, followers finally choose whether to pay for the service. Different consumers will conceive different views of the service outcome when facing the same review, as each of them possesses some idiosyncratic elements that might probably influence outcomes of the service.

Based on our model, we consider an intertemporal model where there is a monopolistic service provider providing a kind of collaborative service to consumers that are review dependent via online platform over two consecutive periods. Our conclusions are in four aspects. Firstly, as for the optimal effort levels, in the presence of review process, the optimal effort level of the service provider is always greater than the optimal effort level in the absence of review process. The joint influence of both the *collaborating effect* and the *reviewing effect* on the optimal consumers' effort level is that the follower consumer's optimal effort level is lower than that of the early consumer only if the reviewing effect is weak and the collaborating effect is in a middle range. Secondly, the service provider adopts the decreasing pricing plan when the collaborating effect is in the middle range and the reviewing effect is in a low range. Thirdly, we further explore that as the review informational influence parameter increases from zero, the optimal profit is changing more rapidly in the work allocation parameter, which reflects a "mutual promotion" mechanism between the reviewing effect and collaborating effect. Finally, as the refinement of rating scales, the influence of review process is intensified. The reviews are more helpful for the service provider to make her optimal pricing strategies and optimal effort level strategy. This finally results in the improvement of the overall optimal profit.

Chapter 2

The Omnichannel Selling Strategy in a Monopolistic Market

2.1. Introduction

Consumers in their retail practice are now attaching great importance to the omnichannel behavior in their perspective. They are ready to take use of both online and offline retail channels in their searching behavior of product information. In order to be better adapted to this new environment, retailers of all industries are reexamining their strategies for delivering both information and products to their target consumers through channels. Under the omnichannel environment, a portion of consumers start their inspection of product information via the physical store before their purchase decisions, which is demonstrated as showrooming strategy (Emma et al. 2017). In the meanwhile, the other portion of consumers begin their product information gathering via the online store, which is called webrooming strategy. Webrooming is the opposite behavior of showrooming strategy. That is to say, showrooming strategy means consumers start to gather information in respect of their target products through the physical store, and then they can reach the online store for a reference of final purchase decisions. However, under the webrooming strategy, the retailer is confronted with the challenge that consumers only browse through the online website for product information gathering, after which they can reach the brick-and-mortar store to try the products in person for a finalized evaluation (Emma et al. 2017, Gary 2018). A business report points out that among the consumers conducting omnichannel behavior, 53% of them start researching digitally, while 47% start gathering information in-store, and the two proportions are almost the same (Oracle Bronto. 2018). These kinds of consumers' searching behavior are of great help for consumers to gather product information from multidimensions in practice. Meanwhile, this phenomenon gives a challenge to retailers, as they need to design their selling strategies including both pricing strategy and product assortment strategy via this kind of channel integration shopping platform, which is so-called omnichannel selling strategies.

A 2018 annual report on retailers' omnichannel integration and development in China shows that what consumers purchasing via online platform value most is the genuine guarantee of products, while price is put to the second place. Omnichannel covers several consumption scenarios and realizes the mesh surrounding of consumers. Under this background, what sellers care about most is which product will satisfy consumers' needs. Therefore, our work makes the retailers not only the decision maker of the pricing strategy, but also the core competence to optimize their production and product placement strategy. All of these strategies can help retailers gain competitive advantage in the wave of omnichannel.

We study this issue by focusing on the consumers' information searching behavior in the omnichannel environment under both the showrooming strategy and webrooming strategy. The consumers will gather information of the product items that are placed in the physical store with information fully revealed, which can help them update the information of the online products' post-purchase attributes. In the case of omnichannel strategy, the consumers will search information through all available channels. We mainly focus on the extent of information revelation through different channels. What we can obtain from our model is in four aspects. Firstly, although the two products placed by the seller via both online channel and offline channel are competitive in their market share, however, under the omnichannel searching environment, their vertical information regarding the quality performance can be updated through consumers' offline inspection. Thus, both products' attributes are more transparent in the omnichannel market, which will finally result in the same direction movement of optimal online product price and offline product price, as they are more likely to be complement goods rather than substitutes. Secondly, the "partial keep" scenario exists when the return cost is low. The relatively low return cost will make consumers consider returning the online product if it is defective in quality as the small resistance it puts on return behavior. While the "all keep" scenario exists when the return cost coefficient is not too low, this high return cost will influence the consumers' online post purchase return behavior as if it becomes a resistance when the product is not good enough in its quality. The consumers will keep the defective product without undertaking the relatively high return cost. Thirdly, the horizonal dominance only exists when the offline product is better in its quality performance compared with the online product. Meanwhile, the misfit cost is relatively high compared to the return cost. The latter condition guarantees the importance of the differentiation regarding the seller's horizontal feature decision. The former condition makes sure the seller's optimal product placement strategy is that the online products are mainstream, while the offline products are niche. Finally, when the offline product quality is better than that of the expected online product, consumers have incentive to purchase online even if the product may be defective in its quality. However, when the offline product quality is worse than the online expected quality, consumers only purchase offline as the online purchasing is faced with more uncertainty. Therefore, the seller should balance or develop proper product assortment methods via both online and offline stores. As the consumers' offline inspection is essential in consumers omnichannel searching behavior, the products placed in the physical store play an important role for consumers to judge the product performance before purchase. It also has great relations with consumers' confidence in online purchase, as consumers are faced with more uncertainty of product performance and the challenge of undertaking return cost when shopping online.

2.2. Literature Review

Previous studies have focused on the competitive sellers in the multi-channel market from various perspectives, such as supply chain management, marketing and information system. However, they have not incorporated or systematically studied consumers' behaviors under the omnichannel environment. Researchers (Gupta, Koulamas, & Kyparisis, 2009; Tsay & Agrawal, 2004) discuss whether the manufacturer with independent retailers should open its own online channel. Another stream of studies is based on price setting models. For example, Chiang, Chhajed, and Hess (2003) build a pricing model structure between a manufacturer and its independent retailer. Their studies show that introducing a direct channel will be in favor of the manufacturer by cutting down the double marginalization. This conclusion is further explained by Arya, Mittendorf, and Sappington (2007). Similarly, Cattani, Gilland, Heese, and Swaminathan (2006) set up a model where a manufacturer opens a direct channel in competition with the traditional one, and results show that this will benefit the manufacturer by segmenting the market. Besides, Bernstein, Song, and Zheng (2008) consider an oligopoly setting whose results make a difference to the monopoly setting. They show that clicks-and-mortar only performs as a strategic necessity in the equilibrium channel structure. Moreover, studies (Bernstein, Song, and Zheng, 2009; K.-Y. Chen, Kaya, and Ozer, 2008; Dumrongsiri, Fan, Jain, and Moinzadeh, 2008) also analyze what factors influence the manufacturer's foundation of an online channel. For example, Bernstein, Song, and Zheng (2009) show that relative channel costs can determine whether the manufacturer should set online channel; K.-Y. Chen, Kaya, and Ozer (2008) identify the optimal dual channel strategy is dependent on several factors, including the cost of managing a direct channel, retailer inconvenience and other product characteristics; Dumrongsiri, Fan, Jain, and Moinzadeh (2008) show that the manufacturer's motivation for opening a direct channel is affected by both the difference in marginal costs of channels and the demand variability.

Furthermore, researchers in the field of marketing address and resolve problems regarding the competitive independent retailers. In their model structures, the competitive retailers sell products via alternate channels. For example, Balasubramanian (1998) analyzes how the direct retailer's participance influences the selling activity, and how the market coverage choice affects the competition among traditional retailers; Viswanathan (2005) makes extension to this model by considering the stylized spatial differentiation, and examines how the difference in channel flexibility, network externalities and switching costs influence the competing pricing strategy in a dual channel setting. However, none of these studies incorporate consumers' valuation uncertainty about products caused by their searching behaviors in the channel integration model structure, which is central of our research.

Some of the other papers take into consideration the consumers' behaviors to visit multiple sellers before their purchase decisions. They mainly focus on how the strategy to open clicks and mortar store influences sellers' pricing strategy, as well as their decisions of whether to offer information services via the online channel. Lal and Sarvary (1999) set up a competitive model structure between two sellers selling horizontally differentiated products. They find that the access to online channel augments the browsing cost and thus promotes sellers to raise their prices. Wu, Ray, Geng, and Whinston (2004) build the model with competitive sellers offering horizontally differentiated products as well. However, in their model structure, the sellers can choose to provide information service. After consumers receive the information service, they can free ride by purchasing with lower price from other sellers without any information service. Their results show that sellers can make positive profit by providing information service. Shin (2007) further examines the competition between two sellers with one of them providing information service, and this can resolve consumers' valuation uncertainty in respect of product features. Nevertheless, the aforementioned papers are different from our study in both the research targets and model assumptions regarding consumers information searching costs. They mainly focus on channel competitions rather than channel integration. Besides, the most important difference is that they never consider consumers' product return behaviors during the shopping process. Therefore, we build up a richer model setting to fill this research gap by taking consumers information searching behaviors between channels into account. Meanwhile, we allow consumers' return behaviors via online store to further conduct investigation of how the seller can benefit from this omnichannel shopping environment.

2.3. Model Setting

The consumers' information searching behavior can be divided into showrooming strategy and webrooming strategy in an omnichannel environment. Researchers (Nageswaran, Cho, & Scheller-Wolf, 2020) point out that the consumers' information searching behavior including either showrooming (i.e., purchase online after visiting the physical store) or webrooming (i.e., visit the brick-and-mortar store after checking online). They are both accommodated in the consumers' omnichannel searching behavior with the inspection of items before making a purchase through different channels. In an omnichannel shopping environment, consumers can search the product information via various channels, thus, the information of either product is based on the information provision of all channels. For example, consumers stand in the physical store can further search through online channel to get more information about either the quality or design feather attribute. And the fraction of consumers who only search through online channel are the ones that are not making full use of the information provision channels in an omnichannel environment, as they still stick to the shopping behavior in the background of sellers' web-only strategy (Nageswaran et al., 2020). This kind of behavior is not an omnichannel behavior and will not be considered in our main model. Thus, under the assumption of the omnichannel information searching behavior, the webrooming searching behavior will degenerate to the showrooming strategy because the online information signal is based on the offline fully revealed information signal.

2.3.1. Assumptions and Notations

We model a monopoly seller selling two products differentiated in both their quality performances for vertical attributes (i.e., m_i or M_i) and design features for horizontal locations (i.e., f_i) through both online and offline channel, respectively. For instance, in a smartphone industry, design features represent the color or screen size of the mobile phone, which only vary in consumers' idiosyncratic preferences and do not influence the overall product to be "better" or "worse". We assume they can be discerned by consumers prior their purchase in the omnichannel environment. However, the quality performances represent the battery capacity or operating speed of the mobile phone, which can only be recognized after the consumer receives the product upon product experience. Thus, we assume the quality performance demonstrates the post-purchase experienced attribute in customers' valuations for the product. Consumers may attain either a high or low value of the product post-purchase experienced attribute when shopping through both online and offline channels. We assume the online product quality performance is in the set $\odot_1 = \{m_1, M_1\}$ and the offline product quality performance is in the set $\odot_2 = \{m_2, M_2\}$. To be specific, m_i is the bad post-purchase perception of product quality performance after product experience, while M_i is its good counterpart. According to several works in marketing and supply chain management, the online purchase and offline purchase both have distinct characteristics in essence. For example, consumers' online purchase should require waiting for delivery, whereas the offline purchase provides consumers with instant satisfactions of product ownership (Chen et al. 2008, Gupta et al. 2004). However, online purchase is more convenient as it saves time for transportation. It is also more flexible as online orders can be placed at any time of the day without any restriction on business hours. But retailers conduct the offline selling should take into consideration the costs for shopping assistance from sale clerks (Ofek et al. 2011). These strengths and weaknesses of either channel in the consumers' purchasing decision are also captured in the value of product post-purchase attribute. Generally speaking, the main disadvantage of online purchase is that consumers cannot "touch and feel" the product prior purchase. Thus, consumers' information searching behavior can help alleviate the information uncertainty regarding the post-purchase attribute by either showrooming or webrooming strategy. Once the online product is received, the consumer will inspect the product and decide whether to keep or return it after the post-purchase attribute is resolved.

We consider the consumers' information searching behavior in the omnichannel environment via both the showrooming strategy and webrooming strategy. It means the consumers will gather information through the product items that are placed in the physical store with information fully revealed. This can help them update the information of online products' post-purchase experienced attributes. In the case of omnichannel strategy, the consumers will search information through all available channels. What we mainly focus on is the extent of information revelation through different channels, thus we assume that all consumers undertake zero cost to reach the seller's bricks-and-mortar store or to purchase from the online store. This can abstract away the influence of channel accessibility on the seller's product assortment strategies in the omnichannel environment (Z. Gu & Tayi, 2017).

Before we further consider the state-dependent expected utility of the online product, we first figure out the market space of consumers' profiles (b, α) in detail. The market space is comprised of two dimensions. They represent the consumers' valuations on both horizontal features and vertical quality performances of the product. Consumers are heterogeneous in both information dimensions in respect of the horizontal tastes and the vertical quality valuations. The horizontal dimension depicts the prior-purchase feature expected by the consumer in respect of the product. Each consumer possesses his ideal feature location as $b \in [0,1]$. He will undertake a disutility in quadratic form when the product is in deviation from his horizontal ideal feature (Lacourbe, Loch, & Kavadias, 2009). To be specific, the horizontal feature ranges from zero to one, just as the color of the smartphone ranges from white to black. Namely, this market dimension does not indicate the rank order valuations of consumers' product experiences, but it is only an approach to represent consumers' various options on the horizontal features. For instance, a smartphone with its color valuation of 0.2 does not mean it has less design features or worse performance than a smartphone with color valuation of 0.6, but it only owns a color closer to white.

The vertical dimension (i.e., $\alpha\epsilon[0,1]$) depicts the consumers' sensitivity to the quality performance of the product. All consumers prefer better quality performance, meaning this market dimension indicates the rank order valuations of the product. For instance, a smartphone with its quality valuation of 0.6 indeed owns better quality performance than a smartphone with quality valuation of only 0.2. Meanwhile, consumers are heterogeneous in their sensitivity coefficient α to a unit change of product quality performance.

In general, we demonstrate the market space of consumers' profiles with the set $Y \in \{(b,\alpha): b\epsilon[0,1], \alpha\epsilon[0,1]\}$. We further depict the consumers' various valuations for the product horizontal features and vertical quality performances as an ex-ante observable joint probability density distribution. To be specific, the probability density function of consumers' market profile over the market space Y is uniformly distributed as $y(b,\alpha) = 1$, $\forall b \in [0,1], \forall \alpha \in [0,1]$. Note that each individual consumer is endowed with a unique sensitivity coefficient α to the quality performance of the product, which can only be experienced after purchase, and an idiosyncratic preference b over the product's ex-ante observable characteristics, which has been explained as product features that are resolved prior purchase.

We assume the consumer with preference b to the horizontal features and sensitivity coefficient α to the vertical quality performance obtains utility $U_2 = \alpha M_2 V - p_2 - t(b - f_2)^2$ from purchasing offline product, where the quality performance is high in this case. The item $\alpha M_2 V$ demonstrates the rank order valuations of consumers regarding the product vertical quality performance, where V denotes the reservation value of consumers' utility gain. The item $t(b - f_2)^2$ depicts the disutility if the product deviates from the consumer's ideal horizontal feature. The unit deviation cost t measures the marginal disutility of a unit misfit in the horizontal features and we assume it to be common over products. Finally, the item p_2 is the price of the product charged by the seller.

We then take into consideration consumers' information searching behavior and the corresponding information updating process to further explain their utility function of online product. The consumer will factor his uncertainty about the online product quality performance in his purchasing decision after observing or trying the offline product. Specifically, the consumer anticipates that with probability τ_1, the online product will perform well after purchase, which will result in a consumer utility as $\alpha M_1 V - p_1 - t(b - f_1)^2$. Whereas with probability $1 - \tau_1$, the online product will perform bad after purchase, which will result in a consumer utility as $\alpha m_1 V - p_1 - t(b - f_1)^2$. Namely, the information updating process of the consumer's information searching behavior after his offline inspection can be demonstrated as the two probabilities: $p(\odot_1 = m_1 | \odot_2 = M_2) = 1 - \tau_1, p(\odot_1 = M_1 | \odot_2 = M_2) = \tau_1$. After the purchase of online product and upon product experience, the consumer makes a second choice, to either keep or return the purchased online product. In case of return, they will have to undertake a return cost rV $(0 \leq r \leq 1)$, which contains not only financial expenditures caused by returns but also psychological burden and time cost undertaken by consumers. When the offline product utility is $U_2 = \alpha m_2 V - p_2 - t(b - f_2)^2$, which means the quality performance is low, the only change it will bring about to our model analysis is the difference in probability τ_i, which measures the online product information updating probabilities. To be specific, consumers in this case will form the online product post-purchase perception of quality as $p(\odot_1 = m_1 | \odot_2 = m_2) = 1 - \tau_2$, $p(\odot_1 = M_1 | \odot_2 = m_2) = \tau_2$.

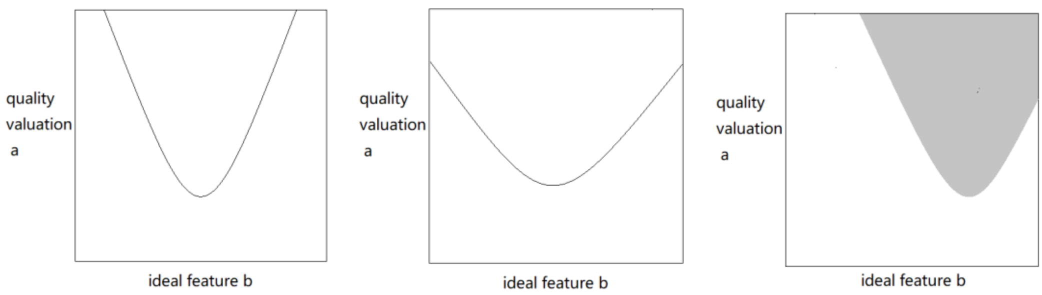
The consumer will then take shape of his prior purchase expected utility of the online product by anticipating his behaviors after purchase rationally (Z. Gu & Tayi, 2017). Particularly, if the online product performs well after experience, the consumer will consider whether to keep it with a utility gain of $\alpha M_1 V - p_1 - t(b - f_1)^2$ or to return it by undertaking a disutility $-rV$. The consumer will choose to keep the online product as long as the utility gain is higher than the disutility: $\alpha M_1 V - p_1 - t(b - f_1)^2 > -rV$, and return it without any purchase otherwise. Specifically, we define the curve $L = \{(b, \alpha) \epsilon Y | \alpha M_1 V - p_1 - t(b - f_1)^2 = -rV\}$ as the indifference line in the consumer market space of keeping and returning process when the post-purchase experienced attribute is good, i.e., all consumers located along the line are indifferent between keeping and returning the online product when the quality performance is high. By solving this indifference curve, we can derive the expression of the indifferent sensitivity coefficient as $\alpha = \frac{-rV+p_1+t(b-f_1)^2}{VM_1}$. Therefore, the consumers with market profiles (b,α) below the indifference curve L, i.e., $\alpha < \frac{-rV+p_1+t(b-f_1)^2}{VM_1}$, will return the online product even if a disutility is generated. Otherwise, the consumers above the indifference curve L, i.e., $\alpha > \frac{-rV+p_1+t(b-f_1)^2}{VM_1}$, will keep the online product. As their sensitivity coefficient to the quality performance of the product is quite high, they have higher level utility gain of unit improvement regarding quality performance. Similarly, if the online product doesn't perform well after experience, consumers will keep it as long as $\alpha m_1 V - p_1 - t(b-f_1)^2 > -rV$, and return it otherwise. In the same way, we can define the curve $L' = \{(b,\alpha)\epsilon Y | \alpha m_1 V - p_1 - t(b-f_1)^2 = -rV\}$ as the indifference line in the consumer market space of keeping and returning process when the post-purchase experienced attribute is not good. The indifference curve is $\alpha = \frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$ under this circumstance, thus the consumers with market profiles (b,α) below the indifference curve L', i.e., $\alpha < \frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$, will return the online product without any purchase even if a disutility is generated. While the consumers above the indifference curve L', i.e., $\alpha > \frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$, will still keep the online product even if it doesn't perform well after experience. As this fraction of consumers are quite high in their sensitivity coefficient regarding quality performance, that is to say, they have sufficiently high level of unit utility gain in respect of quality performance.

Considering consumers' post purchase actual utility in each case as shown above, we can derive the online product prior purchase expected utility as

$$EU_1 = \begin{cases} -rV, & \alpha \leq \frac{-rV+p_1+t(b-f_1)^2}{VM_1} \\ \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(-rV), & \frac{-rV+p_1+t(b-f_1)^2}{Vm_1} > \alpha \geq \frac{-rV+p_1+t(b-f_1)^2}{VM_1} \\ \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(\alpha m_1 V - p_1 - t(b-f_1)^2), & \alpha \geq \frac{-rV+p_1+t(b-f_1)^2}{Vm_1} \end{cases}, \tag{1}$$

where $\alpha \leq \frac{-rV+p_1+t(b-f_1)^2}{VM_1}$ represents the fraction of consumers with market profiles (b,α) below the curve $\frac{-rV+p_1+t(b-f_1)^2}{VM_1}$. They will return the product even if the post-purchase experienced attribute is good, as they have quite low level of sensitivity coefficient α to the quality performance. And consumers with profiles between the curve $\frac{-rV+p_1+t(b-f_1)^2}{VM_1}$ and $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$ will keep the online product if its post-purchase experienced attribute performs well with probability τ_1 and will still return the product if its post-purchase experienced attribute is not good with probability $1-\tau_1$. Finally, when the consumers' quality performance sensitivity α is above the curve $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$, the consumer will always choose to keep the product regardless of the potential possibility of product return, as they have quite high level of unit utility gain regarding quality performance.

Notice that consumers will purchase the online or offline product if and only if they obtain nonnegative expected utilities in each case. Before proceeding our analyses, we clarify several potential cases in which the demand area formed by the nonnegative expected utility of online product should fit into $Y \in \{(b,\alpha): b\epsilon[0,1], \alpha\epsilon[0,1]\}$. Namely, there exist three possible ways of how the demand area intersects with side boundaries of the market space. We define two side boundaries as $S_1 = \{(b,\alpha)\epsilon Y: b = 0\}$ and $S_2 = \{(b,\alpha)\epsilon Y: b = 1\}$. Then, the three cases can be described as: (i) the demand area does not intersect with either side boundaries (i.e., $L \cap S_1 = \emptyset$ and $L \cap S_2 = \emptyset$); (ii) the demand area intersects with both side boundaries (i.e., $L \cap S_1 \neq \emptyset$ and $L \cap S_2 \neq \emptyset$); (iii) the demand area intersects with one of the side boundaries (i.e., $L \cap S_1 \neq \emptyset$ and $L \cap S_2 = \emptyset$).



Case (i) Case (ii) Case (iii)

Figure 1. Three Cases of the Demand Area

However, Case (iii) cannot be an optimal product location in the two-dimensional market space. To be specific, if we move the shaded area along the horizontal feature line, meanwhile we keep the product price and quality performance fixed, the shaded area inside the market space, which can also be explained as consumers' demand, will amplify without further expenses until we obtain the demand in Case (i) or Case (ii). Because the only difference between Case (iii) and Case (i) or Case (ii) is the horizonal dimension's location regarding the product design feature, which is not related to the product cost amplification. This kind of design feature's effect on the product cost is negligible, as it demonstrates the horizontal feature adjustment that reflects consumers' idiosyncratic preference over the product's ex-ante observable characteristics and does not influence the product's quality performance (Lacourbe, Loch, & Kavadias, 2009). We can further derive the condition under which Case (i) and Case (ii) hold respectively, i.e., Case (i) holds when $2\sqrt{\frac{VM_1-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}} \leq 1$ and $2\sqrt{\frac{V(\tau_1 M_1+(1-\tau_1)m_1)-p_1}{t}} \leq 1$; Case (ii) holds when $2\sqrt{\frac{VM_1-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}} > 1$ and $2\sqrt{\frac{V(\tau_1 M_1+(1-\tau_1)m_1)-p_1}{t}} > 1$. To be concrete, the former condition holds in each case when the online product prior purchase expected utility is $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(-rV)$, while the latter condition holds when $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(\alpha m_1 V - p_1 - t(b-f_1)^2)$.

2.3.2. Demand Generation Process for Both Channels

Following the above definitions of our main model, we first consider the demand of online product, which can be derived via the ex-ante online product expected utility. It is generated by consumers' information searching behavior in the omnichannel environment as the figure shows below.

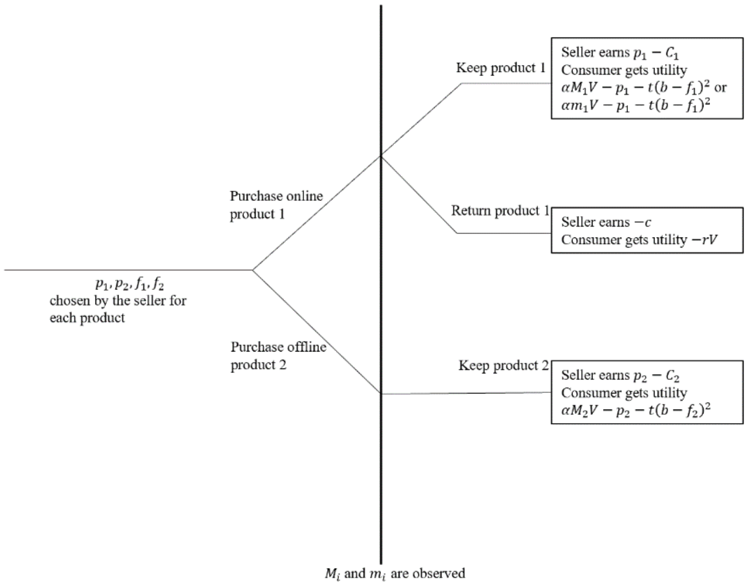


Figure 2. Sequence of Events and Payoffs for Each Party

We can derive the online product demand by considering the actual utility of consumers after receiving the product as figure 2 shows. Consumers will generally consider the probability of returning and keeping the online product before any purchase. Then they set their original purchase strategy based on the expected prior-purchase utility by considering each probable post-purchase behavior. In particular, the online product demand and return quantities are

$$D_1 = \begin{cases} \displaystyle\int_{f_1-\sqrt{\frac{VM_1-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}}}^{f_1+\sqrt{\frac{VM_1-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}}} \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V}\,db & if\ \ p_1 < rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1} \\ \displaystyle\int_{f_1-\sqrt{\frac{V(\tau_1 M_1+(1-\tau_1)m_1)-p_1}{t}}}^{f_1+\sqrt{\frac{V(\tau_1 M_1+(1-\tau_1)m_1)-p_1}{t}}} \frac{p_1+t(b-f_1)^2}{\tau_1 M_1 V+(1-\tau_1)m_1 V}\,\mathrm{d}b & if\ \ p_1 \geq rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1} \end{cases};$$

$$R = \begin{cases} (1-\tau_1)\left(\displaystyle\int_{b_1}^{b_2} \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V}\,db - \int_{b_1}^{b_2} \frac{-rV+p_1+t(b-f_1)^2}{Vm_1}\,db\right) & if p_1 < rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1} \\ 0 & if\ \ p_1 \geq rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1} \end{cases}. \tag{2}$$

The above the demand function marks the scenario when the demand area of online product doesn't intersect with the market side boundaries as Case (i) shows. However, if the online product demand area intersects with the market side boundaries as Case (ii) shows, then the following demand function and return quantities should be satisfied:

$$D_1 = \begin{cases} \int_0^1 \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V} db & if\ p_1 < rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1} \\ \int_0^1 \frac{p_1+t(b-f_1)^2}{\tau_1 M_1 V+(1-\tau_1)m_1 V} \mathrm{d}b & if\ p_1 \geq rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1} \end{cases};$$

$$R = \begin{cases} (1-\tau_1)\left(\int_0^1 \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V} db - \int_0^1 \frac{-rV+p_1+t(b-f_1)^2}{Vm_1} db\right) if p_1 < rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1} \\ 0 \quad if\ p_1 \geq rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1} \end{cases}. \tag{3}$$

Concretely speaking, the nonnegative expected utility of online product $EU_1 > 0$ should be satisfied before purchase decision. Therefore, the consumers with market profiles below the curve $\frac{-rV+p_1+t(b-f_1)^2}{VM_1}$ will exit the market, as their expected utility of online purchase is negative $-rV$. This fraction of consumers has sufficient low level of sensitivity coefficient to unit quality improvement, thus their expected utility is always negative even if the online product is expected to perform well under this circumstance. However, when the consumers market profile is between the curve $\frac{-rV+p_1+t(b-f_1)^2}{VM_1}$ and $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$, the consumers take shape of the prior purchase expected utility as $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(-rV)$. The nonnegative expected utility condition $EU_1 > 0$ should be satisfied when the consumers market profile is above the curve $\alpha = \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V}$ in the two-dimensional market structure. Moreover, consumers in this market profile will return the online product with probability $1-\tau_1$, when the post-purchase experienced attribute of online product is not good. And the return quantity only exists if the online product price is not too high: $p_1 < rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1}$. However, if the online product price is higher than the threshold: $p_1 \geq rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1}$, the return fraction of consumers will not exist. The disutility of online product return $-rV$ is relatively lower than the utility for consumers to keep the online product even if it doesn't perform well after purchase, then all consumers will keep the product with the prior purchase expected utility $EU_1 = \tau_1(\alpha M_1 V - p_1 -$

$t(b-f_1)^2) + (1-\tau_1)(\alpha m_1 V - p_1 - t(b-f_1)^2)$. The nonnegative expected utility condition $EU_1 > 0$ should be satisfied when the consumers are located in the market profile above the curve $\alpha = \frac{p_1 + t(b-f_1)^2}{\tau_1 M_1 V + (1-\tau_1) m_1 V}$.

After our clarifications of online product demands based on the prior purchase expected utility, we next take offline product utility into consideration in the omnichannel environment. The consumer will only purchase the offline product when it renders greater actual utility than the online counterpart, i.e., $U_2 > EU_1$. To be specific, in the case $U_2 = \alpha M_2 V - p_2 - t(b-f_2)^2$, the quality performance of the offline product is high. The consumer will factor his uncertainty about the online product quality performance as the two aforementioned probabilities: $p(\odot_1 = m_1 | \odot_2 = M_2) = 1 - \tau_1$, $p(\odot_1 = M_1 | \odot_2 = M_2) = \tau_1$. When the consumers market profile is between the curve $\frac{-rV + p_1 + t(b-f_1)^2}{VM_1}$ and $\frac{-rV + p_1 + t(b-f_1)^2}{Vm_1}$, the consumers take shape of the prior purchase expected utility as $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(-rV)$. Therefore, $U_2 > EU_1$ should be satisfied when the consumers market profile is above the curve $\alpha = \frac{p_2 + t(b-f_2)^2 - \tau_1 p_1 - \tau_1 t(b-f_1)^2 - (1-\tau_1) rV}{M_2 V - M_1 \tau_1 V}$ in the two dimensional market structure. However, when the consumers' quality sensitivity is above the curve $\frac{-rV + p_1 + t(b-f_1)^2}{Vm_1}$, the consumers will always choose to keep the online product regardless of the potential possibility of product return, thus their ex ante expected utility of online product is $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(\alpha m_1 V - p_1 - t(b-f_1)^2)$. Therefore, $U_2 > EU_1$ should be satisfied when the consumers market profile is above the line $\frac{-p_1 + p_2 + tf_2{}^2 - tf_1{}^2 - 2(f_2 - f_1)tb}{M_2 V - (\tau_1 M_1 V + (1-\tau_1) m_1 V)}$. As the expressive complexity of the specific forms, the demands and return quantities in respect of both offline product and online product in each scenario are elaborated in Appendix B of this chapter.

2.3.3. Important Concepts in Demand Generation Process

Based on the above demand generation process of both products, in this section, we further clarify some details in our model analyses. For expression simplicity, we use μ_i to demonstrate the expected value of online product's post-purchase attribute. For example, when the offline product utility is $U_2 = \alpha M_2 V - p_2 - t(b-f_2)^2$, the consumers form the expected probabilities of online product post-purchase attribute as $p(\odot_1 = m_1 | \odot_2 = M_2) = 1 - \tau_1$, $p(\odot_1 = M_1 | \odot_2 = M_2) = \tau_1$, thus we define $\mu_1 = \tau_1 M_1 + (1 - \tau_1) m_1$ under this circumstance. Similarly, we use μ_2 to demonstrate the online product's expected post-purchase attribute when the offline product utility is $U_2 = \alpha m_2 V - p_2 - t(b - f_2)^2$. As consumers form the post-purchase perception of online product quality as $p(\odot_1 = m_1 | \odot_2 = m_2) = 1 - \tau_2$, $p(\odot_1 = M_1 | \odot_2 = m_2) = \tau_2$, we define $\mu_2 = \tau_2 M_1 + (1 - \tau_2) m_1$ under this circumstance. Nevertheless, if we consider the webrooming strategy separately, the consumer can only form his belief of online product expected utility through the online distorted information. It will render him a probability which we assume as τ_3 to expect the online product is of high-quality performance, meaning it performs well after purchase. Under the assumption that $\tau_3 \geq \tau_1$, it marks the online information will raise consumers' expectation of online product's post-purchase attribute in several cases. For example, the online information is in a reasonable level (H. Sun & Xu, 2018); the seller can manipulate the online signal (i.e., online reviews) by posting fake anonymous ratings that praise her product (Dellarocas, 2006); most of the consumers' reviews posted in the early periods are systematically positively biased (Li & Hitt, 2008). However, when the online distorted information is less sufficient or the online information is too rich which makes it deviate from the reasonable level (H. Sun & Xu, 2018); or when the consumers prefer to rely on the opportunity to "touch and feel" the product through the physical store (Letizia, 2012), the consumers will lower their expectation of online product's performance, which means $\tau_3 < \tau_1$ in these cases. This kind of uncertainty regarding online distorted information will add complexity to our model analyses. Before the further simplicity of our model setting in order to get analytical solutions, we define the information sets via both online and offline channels. We assume under the webrooming strategy, the online product's expected post-purchase attribute is $\mu_3 = \tau_3 M_1 + (1 - \tau_3) m_1$ and the offline product's expected post-purchase attribute is $\mu_4 = \tau_3 M_2 + (1 - \tau_3) m_2$. Therefore, we denote the offline product post-purchase attribute signal set as $\Delta_2 = \{M_2, m_2, \mu_4\}$, while the online product post-purchase attribute signal set as $\Delta_1 = \{\mu_1, \mu_2, \mu_3\}$.

If we follow the demand generation process to combine the webrooming consumers with the showrooming consumers, the only difference between consumers' different information searching behavior is their expected probability of online product performance, which will finally influence their expectations of online product post-purchase attribute. However, under the assumption of the omnichannel information searching behavior, the webrooming searching behavior will degenerate to the showrooming strategy because the online information is based on the offline fully revealed information. If we follow the above demand generation process to combine the webrooming consumers with the showrooming consumers, the only difference is the change of the elements in the signal set. However, this combination will introduce more parameters in the profit function and will not help us to derive any analytical results.

To be more specific, in an omnichannel shopping environment, consumers can search the information through different channels, thus, the information of either product is based on the information provision of all channels. For example, consumers stand in the physical store can further search through online channel to get more information about either the quality or design feather attribute. And the fraction of consumers who only search through online channel are the ones that are not making full use of the information provision channels in an omnichannel environment. Namely, they still stick to the shopping behavior in the background of the seller's web-only strategy (Nageswaran et al., 2020).

In following parts, we mainly consider the case when post-purchase attribute signal set satisfy $\Delta_2 = M_2$ and $\Delta_1 = \mu_1$. Our results show that the optimal selling strategy is highly related to the relations between the products' signal set Δ_1 and Δ_2. Also, we demonstrate the case when the return quantity exists as "partial keep" scenario, while the case when the return quantity doesn't exist as "all keep" scenario.

2.4. Model Analyses

2.4.1. The Seller's Profit Maximization Problem

In our monopolistic model setting, the seller first chooses the placement strategy of the product location f_1 and f_2 through online and offline channels simultaneously. Next, the monopolistic seller makes her pricing strategy p_1 and p_2 of the products, respectively. We assume that the seller makes the product design feature strategy before the pricing strategy because it is generally supposed that the pricing strategy is more flexible and easier to change than the product design feature strategy. Therefore, the pricing strategy possesses a shorter time horizon than the product design feature strategy. Namely, the seller sets her selling strategy of both online product and offline product to maximize her profit

$$\max_{p_1,p_2,f_1,f_2} \pi = (p_1 - C_1) * D_1 + (p_2 - C_2) * D_2 - c * R, \tag{4}$$

where C_1 and C_2 demonstrate the production cost of each product; D_1 and D_2 are the demand of online product and offline product respectively, and R is the return quantity of online product with c the unit return cost undertaken by the seller.

2.4.2. Properties of the Equilibrium Results with Omnichannel Consumers

The equilibrium results under each scenario can be obtained following the derivation process in Appendix B of this chapter and can be illustrated as follows. We demonstrate the scenario when return fraction exists as "partial keep" scenario and the scenario when return fraction doesn't exist as "all keep" scenario. We first conclude from the overall equilibrium results with the quantitative relationships between the online product optimal pricing strategy p_1^* and offline product optimal pricing strategy p_2^* as the following proposition shows.

Proposition 2.1. *In "all keep" scenario, the optimal pricing strategy of the seller satisfies the quantitative relationship under all circumstances:* $p_2^* = \frac{-VM_2+M_2p_1^*+V\mu_1}{\mu_1}$*; while in "partial keep" scenario, the optimal pricing strategy of the seller satisfies another quantitative relationship under all circumstances:* $p_2^* = \frac{-VM_2-rVM_2+M_2p_1^*+m_1p_1^*+V\mu_1+rV\mu_1-p_1^*\mu_1}{m_1}$*, where* p_1^* *is the optimal price of online product and* p_2^* *is that of offline product.*

This conclusion can be derived from both scenarios when $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$ and $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$. It means the quantitative relationships between online product optimal price p_1^* and offline product optimal price p_2^* are always hold, with the only difference whether there exists return behavior in the market. When there exists no return behavior in the market, $p_2^* = \frac{-VM_2+M_2p_1^*+V\mu_1}{\mu_1}$ holds. We can furthermore analyze the elasticity between the optimal pricing strategy. It reflects how much a unit change in the offline product price will make the online product price change correspondingly. Namely, the elasticity can be expressed as $\frac{\partial p_2^*}{\partial p_1^*}=\frac{M_2}{\mu_1}$, which is always positive. When there exists return behavior in the market, the optimal pricing strategy of the seller satisfies $p_2^*=\frac{-VM_2-rVM_2+M_2p_1^*+m_1p_1^*+V\mu_1+rV\mu_1-p_1^*\mu_1}{m_1}$. The elasticity coefficient between prices is demonstrated as $\frac{\partial p_2^*}{\partial p_1^*}=\frac{M_2+m_1-\mu_1}{m_1}$, which is also always positive as $\mu_1-M_2<m_1$ always holds. This optimal pricing elasticity coefficient in our model measures the interactive mechanism between online product price and its offline counterpart in the omnichannel environment. That is to say, although the two products placed by the seller via both online channel and offline channel are competitive in their market share, however, under the omnichannel searching environment, their vertical information regarding the quality performance can be updated through consumers' offline inspection. Thus, both products' attributes are more transparent in the omnichannel market. The market fairness makes the seller have no incentive to execute channel competition strategy within her own products. The intention of channel integration is more preferred, which finally results in the same direction of variation between p_1^* and p_2^*. Namely, the products sold by the seller in the omnichannel environment are more likely to be complement goods rather than substitutes. This is intuitive since the seller with dual channels should guarantee her products in each channel to remain consistency, so as to avoid internal competition, which will not benefit the seller from expanding her market share.

After consumers' offline inspection of products in the omnichannel environment, the online product expected quality performance μ_1 should be greater or smaller, depending on the expected probabilities of online product post-purchase attribute $p(\odot_1=m_1|\odot_2=M_2)=1-\tau_1, p(\odot_1=M_1|\odot_2=M_2)=\tau_1$. If the online product expected quality performance is greater, both $\frac{\partial p_2^*}{\partial p_1^*}=\frac{M_2}{\mu_1}$ and $\frac{\partial p_2^*}{\partial p_1^*}=\frac{M_2+m_1-\mu_1}{m_1}$ are smaller. This indicates that when the online product is better in its expected post-purchase attribute after offline inspection, the rate of change between online product price and offline product price is decreasing. To be specific, when the seller raises her price of offline product p_2^* as before, the increase of online product price p_1^* is cut down although its value is still higher than before, which is due to the complemental effect of omnichannel environment. However, it is counterintuitive that the amplitude of variation regarding prices is smaller. As the online product is expected to be better in its quality before purchase, it will to some extent raise the seller's confidence to raise price.

After explaining the optimal pricing strategy of the seller, we then consider the following conclusion with all situations taking into consideration.

Proposition 2.2. *In case of* $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$, *there is no return in the market when the online product return cost coefficient* $r > \frac{3(V-C_2)(m_1-\mu_1)}{5VM_2}$ *or* $r < \frac{t(m_1-\mu_1)}{V\mu_1}$; *there is consumers' return behavior in the market when* $\frac{t(m_1-\mu_1)}{V\mu_1} \le r \le \frac{3(V-C_2)(m_1-\mu_1)}{5VM_2}$. *Meanwhile, in case of* $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$, *there is no return when* $r > \frac{(V-C_2)(m_1-\mu_1)}{VM_2}$; *otherwise consumers' return behavior exist in the market when* $r \le \frac{(V-C_2)(m_1-\mu_1)}{VM_2}$.

Before we explain the above proposition, we first illustrate the equilibrium results under all circumstances as the tables and figures shown below.

When $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$ which means the offline products post-purchase attribute is better than the expected online products post-purchase attribute, the equilibrium results can be demonstrated in the following table.

	Price	Fit	Profit
Region 1	$p_1 = \frac{1}{24}(-t + 24V + \frac{12(-V+C_2)\mu_1}{m_2})$ $p_2 = \frac{1}{2}(V + C_2) - \frac{tm_2}{24\mu_1}$	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2}$	$\pi = \frac{(tm_2 + 12(-V+C_2)\mu_1)^2}{576 m_2 \mu_1^2}$
Region 2	$p_1 = \frac{5Vm_2 - 3V\mu_1 + 3C_2\mu_1}{5m_2}$ $p_2 = \frac{1}{5}(2V + 3C_2)$	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2}$	$\pi = \frac{8\sqrt{\frac{3}{5}}t^2 m_2 (\frac{(V-C_2)\mu_1}{tm_2})^{5/2}}{25\mu_1^2}$
Region 3	$p_2 = \frac{-Vm_2 - rVm_2 + m_2p_1 + M_1p_1 + V\mu_1 + rV\mu_1 - p_1\mu_1}{M_1}$ p_1^*	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2}$	π^*
Region 4	$p_2 = \frac{-Vm_2 - rVm_2 + m_2p_1 + M_1p_1 + V\mu_1 + rV\mu_1 - p_1\mu_1}{M_1}$	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2}$	complex
Region 5	$p_1 = V - rV\frac{\mu_1}{M_1 - \mu_1}$ $p_2 = V(1 + r + \frac{rM_1}{-M_1 + \mu_1})$	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2} \pm \sqrt{\frac{2(tV + \sqrt{t^2(V-p_1)(V-p_2)}) - t(p_1+p_2)}{t^2}}$	complex

Region 6	p_1 $=\frac{-(((-2t+24V+24C_1)m_2+\mu_1(12C_1-12C_2+(t-12(1+r)V)(-1+\tau_1))(}{12(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1)}$ p_2 $=\frac{-((m_2(-3t+36V+24rV+24C_2+12C_1(-1+\tau_1)+(t-12(1+2r)V)\tau_1)+}{}$	$f_1=\frac{1}{2}$ $f_2=\frac{1}{2}$	π $=\frac{-(t-12V+12C_1)^2m_2+(t-12V+12C_1)^2m_1\tau_1-12\mu_1(-C_1+C_2+rV(-1+\tau_1))(t-12(1+r)V+}{144\mu_1(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1)}$
Region 7	p_1 $=\frac{m_1\tau_1(12c-t-12(-1+r)V+12C_2(-1+\tau_1)-(12c+t-12(1+r)V)\tau_1^2+}{12\tau_1(-4m_2+}$ p_2 $=\frac{m_1\tau_1(1+\tau_1)(-t+12V+12C_2\tau_1)+m_2(12rV+3(4c+t-4(3+2r)V+4C_1}{12\tau_1(-4m_2+m_1(1+\tau_1)}$	$f_1=\frac{1}{2}$ $f_2=\frac{1}{2}$	π $=\frac{(t-12V+12C_2)m_1\tau_1(12rV+12(c+C_1-C_2)\tau_1+(t-12(1+r)V+12C_1)\tau_1^2-12c\tau_1^3)-m_2(12rV}{144m_1\tau_1^2(-4m_2+m_1(1+\tau_1)^2)}$

Table 2. Equilibrium Prices and Fits when $\Delta_2=\{m_2,M_2\}>\Delta_1=\{\mu_1,\mu_2\}$

	Demand of offline product	Demand of online product	Return of online product
Region 1	$\frac{V-C_2}{2m_2}-\frac{t}{24\mu_1}$	0	0
Region 2	$\frac{4\sqrt{\frac{3}{5}}t(\frac{(V-C_2)\mu_1}{tm_2})^{3/2}}{5\mu_1}$	0	0
Region 3	$\frac{tm_1(-m_2+\mu_1)\tau_1+12m_1M_1\tau_1(c+C_1-C_2-c\tau_1)+M_1^2(12rV-(12c+t}{24M_1(M_1^2+m_1(m_2-\mu_1))\tau_1}$	$\frac{-12rVM_1(M_1^2+m_1(2m_2+M_1-2\mu_1))+(m_1-M_1)((12c+t-12(1+r)V+12C_1)M_1^2}{24m_1M_1(M_1^2+m_1(m_2-\mu}$	$\frac{(-1+\tau_1)(12rVM_1(M_1^2+m_1(2m_2+M_1-2\mu_1))+(m_1-M_1)(-(12c+t-12(1}{24m_1M_1(M_1^2}$
Region 4	complex	complex	complex
Region 5	$(2rV(m_2^3(\mu_1\sqrt{-\frac{rtV(m_2+\mu_1)^2}{\mu_1(-m_2+\mu_1)^2(-M_1+\mu_1)}}-t\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}})+\mu_1^3(-\mu_1\sqrt{-}$	$-((2rV(m_2+\mu_1)^2(\mu_1(-\mu_1\sqrt{-\frac{rtV(m_2+\mu_1)^2}{\mu_1(-m_2+\mu_1)^2(-M_1+\mu_1)}}$	0
Region 6	$-\frac{-24C_2+(t-12(1+2r)V)(-1+\tau_1)+12C_1(1+\tau_1)}{12(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1)}$	$\frac{2(t-12V+12C_1)m_2-2(t-12V+12C_1)m_1\tau_1+\mu_1(-t+12(1+r)V-12C_2(1+\tau_1)}{12\mu_1(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1}$	0
Region 7	$\frac{-12rV+\tau_1(-12c+t-12V+24C_2-12C_1(1+\tau_1)+\tau_1(-t+12(1+r)}{12\tau_1(-4m_2+m_1(1+\tau_1)^2)}$	$\frac{-(t-12V+12C_2)m_1\tau_1(1+\tau_1)+2m_2(12rV+\tau_1(12c+t-12(1+r)V+12C_1-12c}{12m_1\tau_1(-4m_2+m_1(1+\tau_1)^2)}$	$\frac{(-1+\tau_1)((t-12V+12C_2)m_1\tau_1(1+\tau_1)+2m_2(-12rV-(12c+t-12(1+r)}{12m_1\tau_1(-4m_2+m_1(1+\tau_1)^2)}$

Table 3. Equilibrium Demands and Return Quantities when $\Delta_2=\{m_2,M_2\}>\Delta_1=\{\mu_1,\mu_2\}$

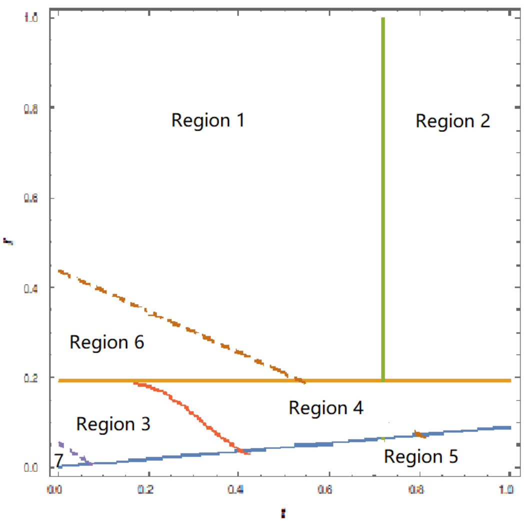


Figure 3. Equilibrium Regions in $r-t$ Plane when $\Delta_2=\{m_2,M_2\}>\Delta_1=\{\mu_1,\mu_2\}$

Notes: The scenarios in each region are summarized as below:

Region 1: All keep & vertical dominance & demand intersects with market boundary

Region 2: All keep & vertical dominance & demand doesn't intersect with market boundary

Region 3: Partial keep & vertical dominance & demand intersects with market boundary

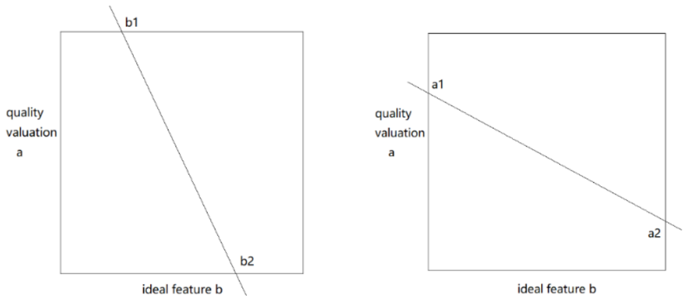
Region 4: Partial keep & vertical dominance & demand doesn't intersect with market boundary

Region 5: All keep & horizontal dominance & demand doesn't intersect with market boundary

Region 6: All keep when $\Delta_2 = \{m_2, M_2, \mu_2\} < m_1\tau_1$

Region 7: Partial keep when $\Delta_2 = \{m_2, M_2, \mu_2\} < m_1\tau_1$

We next elaborate the two new concepts presented in the equilibrium results: vertical dominance and horizontal dominance. When the offline product demand is the common market space above the line $\frac{-p_1+p_2+tf_2{}^2-tf_1{}^2-2(f_2-f_1)tb}{M_2V-\mu_1V}$ and the curve $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$, there are two ways how the line $\alpha = \frac{-p_1+p_2+tf_2{}^2-tf_1{}^2-2(f_2-f_1)tb}{M_2V-\mu_1V}$ intersects the side boundaries of market space $Y \in \{(b,\alpha): b\epsilon[0,1], \alpha\epsilon[0,1]\}$.



Horizontal dominance Vertical dominance

Figure 4. Horizontal Dominance and Vertical Dominance

In this two-dimensional market space, consumers have preferences for the product's horizonal design feature as well as for its vertical quality performance. We introduce a term "vertical-horizontal ratio" to denote which dimension of the product is preferred more by the consumer in the two-dimensional market space. For example, a low vertical-horizontal ratio means that the consumer has a greater preference for the product's horizontal design feature. Namely, his disutility from purchasing a product that does not perfectly match his ideal taste is high. On the other hand, a high vertical-horizontal ratio means that the vertical quality performance is more significant to the consumer compared with the horizontal design feature, or the disutility from the mismatch between product design feature and his ideal taste is low. We can express the vertical-horizontal ratio in our main model as $\gamma = \frac{\Delta_2 - \Delta_1}{t}$, where Δ_1 and Δ_2 are the signal sets of the quality performance in respect of both products and t is the unit misfit cost measuring a unit deviation of the product design feature from the consumer's ideal taste. A higher value of t indicates higher disutility from product's horizontal attribute and therefore there is a lower vertical-horizontal ratio. The slope of the indifference line $\alpha = \frac{-p_1+p_2+tf_2{}^2-tf_1{}^2-2(f_2-f_1)tb}{M_2V-\mu_1V}$ in respect of the offline product demand exactly depends on the vertical-horizontal ratio. On account of how the indifference line intersects with the market space Y, we can denote the demands for both offline product and online product in two ways. To be concrete, if the indifference line intersects both $\alpha = 0$ and $\alpha = 1$ boundaries on the market space, then each product captures consumers for all values of α. Under this circumstance, the consumer's preference for product horizontal design feature dominates his preference for vertical quality performance, which can be perceived as "horizonal dominance". Horizonal dominance depicts a type of the two-dimensional market structure where products are differentiated in their horizontal design features and consumers should undertake high utility loss due to mismatch between the product feature and their ideal tastes (i.e., the value of t is high). However, if the indifference line intersects both $b = 0$ and $b = 1$ boundaries on the market space, the product above the line captures all consumers who have a higher preference for vertical quality performance and the product below the line captures all consumers who have a lower preference for quality performance. Under this circumstance, the consumer's preference for product vertical quality performance dominates his preference for horizontal design feature, which is perceived as "vertical dominance". Vertical dominance depicts a type of the two-dimensional market structure where products are relatively homogeneous in their horizontal design features (i.e., the value of t is low) and products are mainly distinct in their vertical quality performance (Kwark, Chen, & Raghunathan, 2014). These two kinds of consumers' preference regarding the product attributes are reflected in our equilibrium solutions.

What we have demonstrated in the above proposition is that under the condition $\Delta_2 =$

$\{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$, which means the offline products post-purchase attribute is better than the expected online products post-purchase attribute, there is no return in the market when the online product return cost coefficient $r > \frac{3(V-C_2)(m_1-\mu_1)}{5VM_2}$ or $r < \frac{t(m_1-\mu_1)}{V\mu_1}$; there is consumers' return behavior in the market when $\frac{t(m_1-\mu_1)}{V\mu_1} \leq r \leq \frac{3(V-C_2)(m_1-\mu_1)}{5VM_2}$. In "all keep" scenarios, when $r > \frac{3(V-C_2)(m_1-\mu_1)}{5VM_2}$, the cases are all vertical dominance which describes a characteristic of markets where products are relatively homogeneous in their horizontal feature and products primarily differentiate on quality. The product horizontal feature decisions are the same for both online and offline products under the vertical dominance scenario. As the relatively low value of the unit misfit cost compared with the high return cost results in that the products mainly differentiate in their quality performance, the horizontal feature decisions seem less important. This will make all product homogeneous in their horizontal locations. Also note that when the "all keep" scenario exists if the return cost coefficient is not too low, this high return cost will more or less have influence on the consumers' online post purchase return behavior. The reason is that the return cost becomes a resistance when the product is not good enough in its quality, the consumers will still keep the defective product rather than undertaking the relatively high return cost. However, when $r < \frac{t(m_1-\mu_1)}{V\mu_1}$, there also exists a region of "all keep" scenario with the difference as the above region, that is, it is horizontal dominance. It describes a characteristic of markets where products are differentiated and consumers experience a huge disutility due to misfit, due to the relatively high value of the unit misfit cost compared to the low return cost. Thus, the offline product and online product are distinct in their horizontal feature strategies. To be more specific, the online product's optimal horizontal location is in the middle of the unit line, while the offline product's optimal horizontal location deviates from the middle. The relatively high value of the unit misfit cost results in the horizontal location decisions becoming prominent and significant, thus the seller will consider assorting the product with some special features via brick-and-mortar store. It will satisfy the consumer's personalized and customized demand, and horizontal dominance only exists in this region 5. While the online product is still the mainstream product with the general feature, this will reduce the possibility of product return to some extent. Therefore, the "horizontal dominance" strategy is implemented, and this strategy helps the seller earns more revenue from the product differentiated assortment policy. The practical examples include the household appliance industry, etc, we will give a detailed clarification afterwards.

We then consider the "partial keep" scenario when the return cost is low ($r \leq \frac{3(V-C_2)(m_1-\mu_1)}{5VM_2}$) and the unit misfit cost is not too high ($\frac{t(m_1-\mu_1)}{V\mu_1} \leq r$) at the same time. The relatively low return cost will make consumer consider returning the online product if it is defective in quality, as the small resistance it puts on return behavior. Thus, the return behavior resulting from the poor quality will occur under this circumstance. Meanwhile, the unit misfit cost is not too high, which makes the vertical dominance exist in the market. Because the post purchase quality performance is more important than the prior purchase horizontal feature when the unit misfit cost is relatively low compared with return cost, the seller will not obtain more revenue from the horizontally differentiated product assortment strategy. Thus, the product horizontal feature decisions are the same for both online and offline products.

We then consider the opposite case. When $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$, it means the offline products post-purchase attribute is worse than the expected online products post-purchase attribute. The equilibrium results can be demonstrated in the following table.

	Price	Fit	Profit
Region 8	$p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ $p_2 = p_1 - \frac{(m_2-\mu_1)(V+rV-p_1)}{M_1}$	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2}$	$\pi = -\frac{(rVm_2-(V-C_2)(M_1-\mu_1))(-tM_1+(t+12rV)\mu_1)}{12(M_1-\mu_1)^2\mu_1}$
Region 9	$p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ $p_2 = p_1 - \frac{(m_2-\mu_1)(V+rV-p_1)}{M_1}$	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2}$	$\pi = -\frac{4rV(rVm_2-(V-C_2)(M_1-\mu_1))\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}}}{3(M_1-\mu_1)^2}$
Region 10	$p_1 = V - \frac{t}{4} - \frac{1-\tau_1}{\tau_1}rV$ $p_2 = p_1 - \frac{(m_2-\mu_1)(V+rV-p_1)}{M_1}$	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2}$	$\pi = \frac{1}{24m_1M_1^2\tau_1^2}(-tM_1^2\tau_1(4rV+\tau_1(4c+t-4(1+r)V+4C_1-4c\tau_1))+m_1(6rV+t\tau_1)(4M_1\tau_1(c+C_1-C_2$ $-c\tau_1)-m_2(4rV+t\tau_1)+\mu_1(4rV+t\tau_1)))$
Region 11	$p_1 = V - \frac{t}{4} - \frac{1-\tau_1}{\tau_1}rV$ $p_2 = p_1 - \frac{(m_2-\mu_1)(V+rV-p_1)}{M_1}$	$f_1 = \frac{1}{2}$ $f_2 = \frac{1}{2}$	$\pi = \frac{1}{24m_1M_1^2\tau_1^2}(-tM_1^2\tau_1(4rV+\tau_1(4c+t-4(1+r)V+4C_1-4c\tau_1))+m_1(6rV+t\tau_1)(4M_1\tau_1(c+C_1-C_2$ $-c\tau_1)-m_2(4rV+t\tau_1)+\mu_1(4rV+t\tau_1)))$

Table 4. Equilibrium Prices and Fits when $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$

	Demand of offline product	Demand of online product	Return of online product
Region 8	$\frac{rV}{M_1-\mu_1} - \frac{t}{12\mu_1}$	0	0

Region 9	$\frac{4rV\sqrt{\frac{rV\mu_1}{tM_1 - t\mu_1}}}{3(M_1-\mu_1)}$	0	0
Region 10	$\frac{6rV + t\tau_1}{6M_1\tau_1}$	$\frac{1}{6}(\frac{t}{m_1} - \frac{6rV + t\tau_1}{M_1\tau_1})$	$\frac{(-1+\tau_1)(6rVm_1 + t(m_1 - M_1)\tau_1)}{6m_1M_1\tau_1}$
Region 11	$\frac{6rV + t\tau_1}{6M_1\tau_1}$	$\frac{1}{6}(\frac{t}{m_1} - \frac{6rV + t\tau_1}{M_1\tau_1})$	$\frac{(1-\tau_1)(4rVm_1 + t(m_1 - M_1)\tau_1)^2}{6m_1^2M_1^2\tau_1^2\sqrt{\frac{t(m_1-M_1)(4rVm_1 + t(m_1-M_1)\tau_1)}{m_1^2M_1^2\tau_1}}}$

Table 5. Equilibrium Demands and Return Quantities when $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$

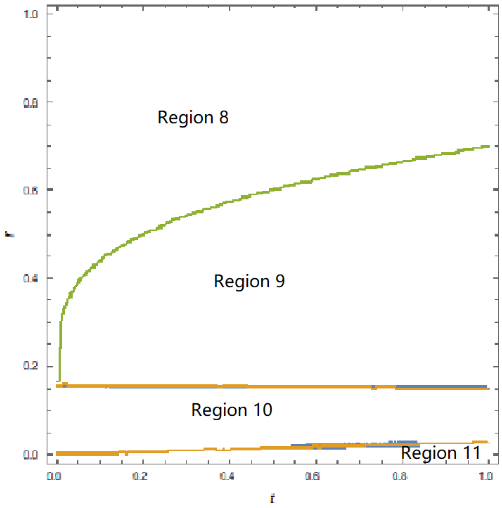


Figure 5. Equilibrium Regions in $r - t$ Plane when $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$

Notes: The scenarios in each region are

Region 8: All keep & vertical dominance & demand intersects with market boundary

Region 9: All keep & vertical dominance & demand doesn't intersect with market boundary

Region 10: Partial keep & vertical dominance & demand intersects with market boundary

Region 11: Partial keep & vertical dominance & demand doesn't intersect with market boundary

What we have demonstrated in the above proposition is that under the condition $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$, there is no return when $r > \frac{(V-C_2)(m_1-\mu_1)}{VM_2}$; otherwise consumers' return behavior exist in the market when $r \leq \frac{(V-C_2)(m_1-\mu_1)}{VM_2}$. The same reason is behind this phenomenon. The high return cost acts as a resistance for the consumers to make return decisions when they are faced with the product defective in its quality.

Almost the same conclusion regarding the existence of all keep and partial keep scenario follows as the case when $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$, with the only difference that there is no existence of horizontal dominance when the offline products post-purchase attribute is worse than the expected online products post-purchase attribute ($\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$). As the offline products have been low in its quality performance, there is no incentive for the seller to consider placing the product with special features in the brick-and-mortar store, which will only intensify the disutility of consumers when facing the niche product. It will ultimately weaken the offline demand of the seller and harm the channel integration policy. Thus, the products are assorted in similar horizontal features without any deviation. Following the above equilibrium results, the corollaries are summarized as below.

Corollary 2.1. *Online product horizonal fitness strategy is always* $f_1 = \frac{1}{2}$, *while the offline product fitness strategy only differs from it when* $r < \frac{t(m_1 - \mu_1)}{V\mu_1}$ *(Region 5) and* $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$.

We have derived from the equilibrium results that the horizonal dominance only exists when the offline product is better in its quality compared with the online product. Meanwhile, the misfit cost is relatively high compared to the return cost. The latter condition guarantees the importance of the differentiation regarding the seller's horizontal feature decision. The former condition makes sure the seller's optimal product placement strategy is that the online products are mainstream, while the offline products are niche. As the offline product has good performance in its quality, the seller has incentive to consider assorting products with special features in the physical store, which will not worsen the utility of consumers too much. It guarantees the offline demand and the seller's overall revenue. At the same time, it will satisfy the consumer's personalized product requirements. While the online product is still the mainstream product with the general feature, this will balance the online product utility and reduce the possibility of product return to some extent, as it is low in its expected quality performance prior purchase.

In practice, the household appliance industry is a good example corresponding to this differential product assortment phenomenon. That is to say, the seller will choose to place the appliances with hot style via online channel, while put the niche style in the physical store. A specific example goes for the security door industry. That is, security door retailers will choose to assort the products with mainstream features (i.e., type and specification) via online stores, while the customized products with niche features are assorted via brick-and-mortar store. Back to the household appliance industry, many consumers have noticed that the online hot-selling products are in mid-low end. The seller can take use of the online products to attract more consumers in order to enhance her sales volume. Nowadays, many sellers start to adopt the differential product assortment strategy. Their products can be divided into mall version and e-commerce version. The e-commerce version is in low quality performance compared with the mall version. Because of the large demand of online consumers, the seller needs to make some adjustments to the existing models, by which way she can decrease the product costs to adapt to the online sales volume. As the online products' quality can only be resolved after purchase, this selling strategy will to some extent impair consumers' interests. The other example in clothing industry such as Li Ning can also verify the differential product assortment strategy. There are many shoes or clothes of Li Ning on sale, while only 50% types of them are similar in their design features via online stores and physical stores. The others are differentiated in their horizontal design features.

Corollary 2.2. *Whether the demand area intersects with the market side boundaries doesn't influence the scenario to be "all keep" or "partial keep" when either* $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$ *or* $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$.

As we have taken into consideration both case (i) $L \cap S_1 = \emptyset$ and $L \cap S_2 = \emptyset$, and case (ii) $L \cap S_1 \neq \emptyset$ and $L \cap S_2 \neq \emptyset$, they depict whether the demand area intersects with the market side boundaries or not. The equilibrium results in the table show that this market structure doesn't have any influence on the consumers' return behavior regarding the online product. This is because that the customer returns are mainly influenced by the value of product return cost. The relative cost in case (i) and case (ii) remains unchanged thus will not affect the consumers' second choice of whether to keep or return the product. However, the specific value of both products' overall demand and the return quantities are changed, as the demand area in each case requires different range of value regarding online product price. To be specific, case (i) holds when $2\sqrt{\frac{VM_1 - p_1 - \frac{1-\tau_1}{\tau_1} rV}{t}} \leq 1$ and $2\sqrt{\frac{V(\tau_1 M_1 + (1-\tau_1)m_1) - p_1}{t}} \leq 1$; case (ii) holds when $2\sqrt{\frac{VM_1 - p_1 - \frac{1-\tau_1}{\tau_1} rV}{t}} > 1$ and $2\sqrt{\frac{V(\tau_1 M_1 + (1-\tau_1)m_1) - p_1}{t}} > 1$. Thus, when the demand area does not intersect with the market side boundaries as case (i) shows, the optimal online product price is in high range of value. This will induce lower expected utility of online product prior consumers' purchase, which will finally result in lower demands in this case. While the demand area intersects with the market side boundaries as case (ii) shows, the optimal online product price is in low range of value. This results in greater prior purchase expected utility of online product and the demands of both online product and offline product are higher in this case.

After explaining the common points under the two parallel conditions between the signal sets Δ_2 and Δ_1, we then explore the difference between them as the below proposition shows.

Proposition 2.3. *In case of* $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$, *there exists only offline demand in the market without any online purchase when the online product return cost coefficient* $r > \frac{3(V - C_2)(m_1 - \mu_1)}{5VM_2}$ *and* $\Delta_2 = \{m_2, M_2\} > m_1\tau_1$*; otherwise, online demand and offline demand coexist in the market. In case of* $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$, *there exists only offline demand in the market when* $r > \frac{(V - C_2)(m_1 - \mu_1)}{VM_2}$ *("all keep" scenarios); otherwise, online demand and offline demand coexist in the market when* $r \leq \frac{(V - C_2)(m_1 - \mu_1)}{VM_2}$ *("partial keep" scenarios).*

After our observations of the demands regarding online and offline products in each region, the above conclusion can be obtained. The "all keep" scenario includes the situations where all demands are generated from the offline product and the demands are generated from both products without return. In the case when the offline product's post-purchase attribute is better than the expected online products post-purchase attribute, the "all keep" scenario belongs to the former case (i.e., all demands are generated from the offline product) when the return cost is relatively high and the offline product's post-purchase attribute is much better than its online counterpart. As the condition $\Delta_2 = \{m_2, M_2\} > m_1\tau_1 > \Delta_1 = \{\mu_1, \mu_2\}$ is stricter than before, this guarantees the offline product's performance to be good enough so as all consumers choose offline product after inspection. However, when $\Delta_1 = \{\mu_1, \mu_2\} < \Delta_2 = \{m_2, M_2\} < m_1\tau_1$, region 6 depicts the "all keep" scenario in the latter case (i.e., the demands are generated from both products without return). The offline product's performance is better than the expected online product's performance, however, the difference between the two products' quality is not too distinct. Thus, consumers still make comparison between offline and online product, which finally results in the demands generating from both products. Meanwhile, the return cost ($r > \frac{3(V-C_2)(m_1-\mu_1)}{5VM_2}$) becomes a resistance, as the consumers will still keep the defective online product rather than undertaking the relatively high return cost.

In the case when the offline product's post-purchase attribute is worse than the expected online products post-purchase attribute, the "all keep" scenario belongs to the former case (i.e., all demands are generated from the offline product) when the return cost is relatively high ($r > \frac{(V-C_2)(m_1-\mu_1)}{VM_2}$). This high return cost just acts as a threat for consumers to purchase online. Even if the offline product performs no better than its online counterpart, consumers still only purchase via offline channel to eliminate the uncertainty of return. However, when the return cost is low ($r \leq \frac{(V-C_2)(m_1-\mu_1)}{VM_2}$), return behavior always exists in the "partial keep" scenario. The "all keep" scenario in the latter case (i.e., the demands are generated from both products without return) does not exist when $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$.

This shows us the difference of consumers behavior under the two conditions of the signal sets. When the offline product quality is better than that of the expected online product, consumers have incentive to purchase online even if the product may be defective in its quality. However, when the offline product quality is worse than the online expected quality, consumers only purchase offline as the online purchasing is faced with more uncertainty. Thus, the seller should consider to balance her product assortment strategy via both online and offline stores. For example, if the seller wants the market shares of both online and offline channels to be more even, she should consider assort the product with better performance in the physical store. However, if she does not care about the sales via online channel, she can assort the products that are not good in their performance in the brick-and-mortar store. As the consumers' offline inspection is essential in consumers omnichannel searching behavior, the products placed in the physical store play an important role for consumers to judge the product performance before purchase. It also has great relations with consumers' confidence in online purchase, as consumers are faced with more uncertainty of product quality performance and the challenge of undertaking return cost when shopping online.

Proposition 2.4. *When* $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$, *the seller's optimal profit under each circumstance is increasing in the online product return cost coefficient* r.

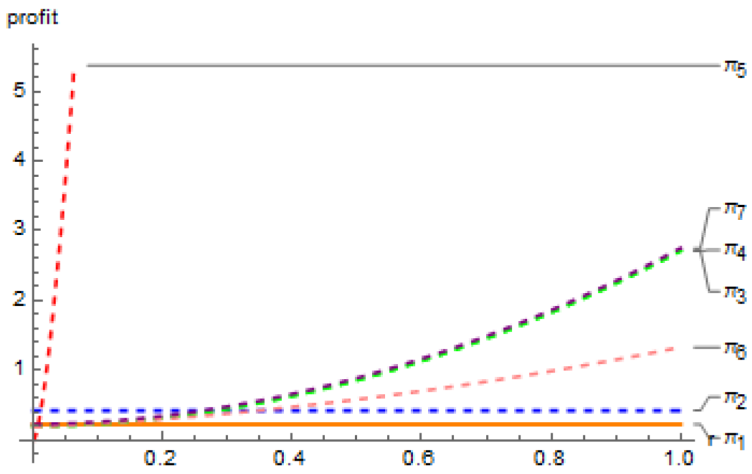


Figure 6. The Impact of r on Optimal Profits when $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$

We further consider the scenarios when the optimal internal equilibrium results exist, i.e., when $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$, which means the offline products post-purchase attribute is better than the expected online products post-purchase attribute. The seller's optimal profit is increasing in the online product return cost coefficient no matter whether the return behavior exists in the market. As the expressions of a portion profits are complex in forms, we thus resort to numerical analyses. To be specific, in the case when return behavior exists, the corresponding profits are π_3, π_4 and π_7, while all the others are cases without any return. We can learn from our numerical study that in the cases with the existence of return behavior, the profits change more rapidly in r than that in the cases without any return. It is intuitive since the return cost coefficient r only plays a role of boundary condition in the case without return, while r enters into the expected utility function and has influence on the demands of both products when the return behavior indeed exists. What make us surprised is that the value of π_5 increases rapidly in r which corresponds to the case when horizonal dominance exists. That is to say, under this circumstance, products are differentiated in their horizontal features and consumers experience a huge disutility due to misfit (high value of t). The reason behind this phenomenon is that the boundary condition when "all keep" exists (i.e., $\alpha \geq \frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$) is related to both the return cost coefficient r and the unit misfit cost t. In the case when the horizonal dominance exists in the market with high value of t, the amplification function of t on the boundary condition is quite great and cannot be ignored. Likewise, a unit variation of return cost coefficient r will make the relative magnitude in the growth of the boundary condition even higher than before. Thus, the demands of both products are growing rapidly as the increase of the return cost coefficient r. This results in the phenomenon that under this "all keep" scenario, the profit increases rapidly in r.

2.4.3. Properties of the Equilibrium Results with Online-only Consumers

What we have mentioned in our model assumption is that in the omnichannel environment, consumers will make use of all available channels to search for information. If we further take into consideration the fraction of consumers that are not making full use of the omnichannel strategy, namely, they only search online for product information, we need to add some new assumptions to our main model as follows.

Considering the consumers that only search online without any in-store inspection before making purchase, they are uncertain about the idiosyncratic preference over the product's ex-ante observable characteristics b. It is explained as the product feature that is resolved prior purchase for consumers who are making full use of information searching channels. However, under this circumstance of online-only consumers, we assume they are heterogeneous in their taste β. It is comprised of two main parts: an observable component and an unobservable component prior purchase. To be more specific, $\beta = b + \varepsilon$, where $b \sim U[0,1]$ is perceived by consumers prior to their original purchase decision and ε is uniformly distributed over $[-\delta, \delta]$, which is a common knowledge prior purchase.

Then the online product expected utility for the online-only consumers can be demonstrated as below following the same procedures, as we have explained in our original model:

$$EU_1 =$$

$$\begin{cases} -rV, & \alpha \leq \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{VM_1} \\ \tau_2(\alpha M_1 V - p_1 - t(E[\beta]-f_1)^2) + (1-\tau_2)(-rV), & \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{Vm_1} > \alpha \geq \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{VM_1}. \\ \tau_2(\alpha M_1 V - p_1 - t(E[\beta]-f_1)^2) + (1-\tau_2)(\alpha m_1 V - p_1 - t(E[\beta]-f_1)^2), & \alpha \geq \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{Vm_1} \end{cases}$$

(5)

The specific form of this prior purchase expected utility of online product is determined on the range of δ. It represents the extent of uncertainty brought about by the online-only inspection behavior. When $-\delta > f_1 - \frac{\sqrt{t(rV+\alpha m_1-p_1)}}{t}$ and $-\delta > f_1 - \frac{\sqrt{t(rV+\alpha M_1-p_1)}}{t}$, the online product prior purchase expected utility when we take into consideration the online-only inspection can be simplified to

$$EU_1 = \begin{cases} -rV, & \alpha \leq \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{VM_1} \\ \tau_2\left(\alpha M_1 V - p_1 - t\left(b+\frac{-\delta+\gamma-b}{2}-f_1\right)^2\right)+(1-\tau_2)(-rV), & \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{Vm_1} > \alpha \geq \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{VM_1} \\ \tau_2\left(\alpha M_1 V - p_1 - t\left(b+\frac{-\delta+\gamma'-b}{2}-f_1\right)^2\right)+(1-\tau_2)\left(\alpha m_1 V - p_1 - t\left(b+\frac{-\delta+\gamma'-b}{2}-f_1\right)^2\right), & \alpha \geq \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{Vm_1} \end{cases},$$

(6)

where $\gamma = min\, f_1 + \frac{\sqrt{t(rV+\alpha m_1-p_1)}}{t}$ and $\gamma' = min f_1 + \frac{\sqrt{t(rV+\alpha M_1-p_1)}}{t}$.

Then the demand and return quantity under both “all keep” scenario and “partial keep” scenario can be derived in Appendix A of this chapter.

However, when $-\delta < f_1 - \frac{\sqrt{t(rV+\alpha m_1-p_1)}}{t}$ and $-\delta < f_1 - \frac{\sqrt{t(rV+\alpha M_1-p_1)}}{t}$, the online product prior purchase expected utility when we take into consideration the online-only inspection can be simplified to

$$EU_1 = \begin{cases} -rV, & \alpha \leq \frac{-rV+p_1}{VM_1} \\ \tau_2(\alpha M_1 V - p_1)+(1-\tau_2)(-rV), & \frac{-rV+p_1}{Vm_1} > \alpha \geq \frac{-rV+p_1}{VM_1} \\ \tau_2(\alpha M_1 V - p_1)+(1-\tau_2)(\alpha m_1 V - p_1), & \alpha \geq \frac{-rV+p_1}{Vm_1} \end{cases}. \tag{7}$$

It means the consumers are identical in their attribute of horizontal dimension when browse through online channel only. Then the demand and return quantity under both “all keep” scenario and “partial keep” scenario can also be derived in Appendix A of this chapter.

We assume the ratio between online-only consumers and omnichannel consumers is N. Without loss of generality, we normalize the population of omnichannel consumers to be one and that of online-only consumers to be N. Following the similar procedures of the derivation process, we can obtain the equilibrium results, which are tedious in forms. However, we can summarize from these results to get the following conclusion.

Proposition 2.5. *When we take both online-only consumers and omnichannel consumers into consideration, what makes changes to our original results is the value of* p_1 *and* p_2*, the relationship between* p_1 *and* p_2 *does not make change:*

in "all keep" scenario, the optimal pricing strategy of the seller satisfies the quantitative relationship under all circumstances: $p_2^* = \frac{-VM_2+M_2p_1^*+V\mu_1}{\mu_1}$*; while in "partial keep" scenario, the optimal pricing strategy of the seller satisfies another quantitative relationship under all circumstances:* $p_2^* = \frac{-VM_2-rVM_2+M_2p_1+m_1p_1^*+V\mu_1+rV\mu_1-p_1\mu_1}{m_1}$*, where* p_1^* *is the optimal price of online product while* p_2^* *is that of offline product.*

This conclusion is similar to proposition 2.1. It means the quantitative relationships between online product optimal price p_1^* and offline product optimal price p_2^* are always hold, no matter whether there are online-only consumers in the market or not. The online-only consumers represent the fraction of consumers that are not making full use of the omnichannel strategy. They only stick to purchase online without any in-store inspection. Although their behaviors will influence the overall demand of online product, the behaviors of omnichannel consumers are not affected. That is, the relatively transparent market information under the omnichannel environment still makes the seller have no incentive to execute channel competition strategy within her own products. The intention of channel integration is more preferred, which finally results in the same direction of variation between p_1^* and p_2^*. As both products' attributes are more overt in the omnichannel market, the online and offline products are more likely to be complement goods rather than substitutes. This is intuitive since the seller with dual channels should guarantee her products' demand in each channel to remain selling consistency, so as to avoid internal competition, which will not benefit the seller from expanding her market share.

2.4.4. One-Dimensional Model Structure

In this section, we mainly focus on the one-dimensional model by considering products' vertical quality separately. The objective is to explore whether there are any differences or similarities in the conclusions between the one-dimensional model structure and our two-dimensional one. All assumptions remain the same as our main model. However, we consider the market space with consumers differentiated in their vertical quality performance. Each individual consumer is endowed with a unique sensitivity coefficient α to the quality performance of the product, where α is uniformly distributed over $[0,1]$. We assume there is no product's ex-ante observable characteristics, then the consumers' prior purchase expected utility of online product can be expressed as

$$EU_1 = \begin{cases} -rV, & \alpha \le \frac{-rV+p_1}{VM_1} \\ \tau_1(\alpha M_1 V - p_1) + (1-\tau_1)(-rV), & \frac{-rV+p_1}{Vm_1} > \alpha \ge \frac{-rV+p_1}{VM_1} \\ \tau_1(\alpha M_1 V - p_1) + (1-\tau_1)(\alpha m_1 V - p_1), & \alpha \ge \frac{-rV+p_1}{Vm_1} \end{cases}. \tag{8}$$

The offline product utility is $U_2 = \alpha M_2 V - p_2 - t(b-f_2)^2$. The consumers form the expected probabilities of online product post-purchase attribute as $p(\odot_1 = m_1 | \odot_2 = M_2) = 1-\tau_1$, $p(\odot_1 = M_1 | \odot_2 = M_2) = \tau_1$, thus we still define $\mu_1 = \tau_1 M_1 + (1-\tau_1)m_1$ under this circumstance. Then, we follow the same demand generation process as our main model, the demand and return in both "partial keep" scenario and "all keep" scenario can be derived as follows.

Demands and return in "partial keep" scenario of offline product and online product:

$$D_2 = \frac{p_2 + \tau_1 V - \tau_1 p_1 - (1-\tau_1)rV}{M_2 V - M_1 \tau_1 V},$$
$$D_1 = \frac{\tau_1 p_1 + (1-\tau_1)rV}{\tau_1 M_1 V} - \frac{p_2 - \tau_1 p_1 - (1-\tau_1)rV}{M_2 V - M_1 \tau_1 V},$$
$$R = (1-\tau_2)\left(\frac{p_1 + \frac{1-\tau_1}{\tau_1} rV}{M_1 V} - min\left\{\frac{-rV+p_1}{m_1 V}, 0\right\}\right). \tag{9}$$

Demands in "all keep" scenario of offline product and online product:

$$D_2 = \frac{p_1 - p_2}{-M_2 V + \mu_1 V},$$
$$D_1 = \frac{p_1}{\mu_1 V} - \frac{p_1 - p_2}{-M_2 V + \mu_1 V}. \tag{10}$$

Then the seller sets her selling strategy of both online product and offline product to maximize her profit

$$\max_{p_1, p_2} \pi = (p_1 - C_1) * D_1 + (p_2 - C_2) * D_2 - c * R, \tag{11}$$

where C_1 and C_2 demonstrate the production cost of each product; D_1 and D_2 are the demand of online product and offline product respectively, and R is the return quantity of online product with c the unit return cost undertaken by the seller.

The equilibrium results under both "partial keep" scenario and "all keep" scenario can be obtained. We can then summarize the similar conclusions from the results in this one-dimensional model structure.

Proposition 2.6. *In "all keep" scenario, the optimal pricing strategy of the seller satisfies:* $p_2^* = \frac{1}{2}(C_2 - \frac{M_2(V+C_1-2p_1^*)}{\mu_1})$*; while in "partial keep" scenario, the optimal pricing strategy of the seller satisfies:* $p_2^* = \frac{1}{2}(-C_1 + C_2 + p_1^*(1+\tau_1))$*, where* p_1^* *is the optimal price of online product while* p_2^* *is that of offline product.*

When there exists no return behavior in the market, $p_2^* = \frac{1}{2}(C_2 - \frac{M_2(V+C_1-2p_1^*)}{\mu_1})$ holds. We can furthermore analyze the elasticity between the optimal pricing strategy. The elasticity can be expressed as $\frac{\partial p_2^*}{\partial p_1^*} = \frac{M_2}{\mu_1}$, which is always positive. When there exists return behavior in the market, the optimal pricing strategy of the seller satisfies $p_2^* = \frac{1}{2}(-C_1 + C_2 + p_1^*(1+\tau_1))$. The elasticity coefficient between prices is demonstrated as $\frac{\partial p_2^*}{\partial p_1^*} = \frac{1}{2}(1+\tau_1)$, which is also always positive. This optimal pricing elasticity coefficient in our model measures the interactive mechanism between online product price and its offline counterpart in the omnichannel environment. That is to say, although the two products placed by the seller via both online channel and offline channel are competitive in their market share, however, under the omnichannel searching environment, their vertical information regarding the quality performance can be updated through consumers' offline inspection. Thus, both products' attributes are more transparent in the omnichannel market. This finally results in the same direction of variation between p_1^* and p_2^*. Namely, they are more likely to be complement goods rather than substitutes.

Proposition 2.7. *The seller's optimal profit is increasing in the online product return cost coefficient* r.

This is intuitive since the increase of return cost r will guarantee the online product sales from avoiding consumers' arbitrary return behavior. Thus, the profit increases in the return cost. Besides, we don't consider the exchange behavior of consumers. When $r = 0$, there is no restriction to return behavior of consumers. It will reduce the online product demand, but it will not increase offline product demand. Therefore, the optimal profit is approaching zero when the return cost is zero.

2.5. When the Traditional Retailer Should Consider Clicks and Mortar

In this section, we first consider a traditional retailer's optimal selling strategy, which means the retailer only opens the brick-and-mortar store to sell her products. There are also two scenarios of the two-products market structure, i.e., vertical dominance and horizontal dominance. The demand generation process follows that the consumers make comparison between the two offline products' actual utility.

Under the circumstance of vertical dominance, the objective function of the traditional retailer should be

$$\pi(p_1, p_2, f_1, f_2) = (p_2 - C_2)\left(\int_0^1 \frac{p_1 - p_2 - t(b - f_2)^2 + t(b - f_1)^2}{M_2V - M_1V} db\right)$$
$$+(p_1 - C_1)\left(1 - \int_0^1 \frac{p_1 - p_2 - t(b - f_2)^2 + t(b - f_1)^2}{M_2V - M_1V} db\right). \quad (12)$$

We follow the envelope theorem as our main model's solving procedure, then the equilibrium results are

$$f_1^* = \frac{1}{2}; f_2^* = \frac{1}{2}; p_1^* = \frac{1}{2}(C_1 - C_2 + M_1V); p_2^* = \frac{M_2V}{2}. \quad (13)$$

The optimal profit is

$$\pi^* = \frac{1}{4}\left(-2C_1 - 2C_2 + M_1V + M_2V + \frac{(C_1 - C_2)^2}{M_1V - M_2V}\right). \quad (14)$$

However, under the circumstance of horizontal dominance, the objective function of the traditional retailer should be

$$\pi(p_1, p_2, f_1, f_2) = (p_2 - C_2)\left(\int_0^1 \frac{tf_1^2 - tf_2^2 - \alpha M_1V + \alpha M_2V + p_1 - p_2}{2tf_1 - 2tf_2} d\alpha\right)$$
$$+(p_1 - C_1)\left(1 - \int_0^1 \frac{tf_1^2 - tf_2^2 - \alpha M_1V + \alpha M_2V + p_1 - p_2}{2tf_1 - 2tf_2} d\alpha\right). \quad (15)$$

We follow the envelope theorem as our main model's solving procedure, then the equilibrium results are

$$f_1^* = \frac{1}{2}; f_2^* = \frac{1}{2} + \frac{\sqrt{t(M_1V - M_2V - 2p_1 + 2p_2)}}{\sqrt{2}t};$$

$$p_1^* = \frac{1}{18}\left(-t + \sqrt{t(t - 12C_1 + 12C_2 + 6M_1V - 6M_2V)}\right.$$

$$\left. + 6\left(C_1 - C_2 + M_1V - M_2V + 3\left(V - \frac{m_2}{2}\right)\right)\right);$$

$$p_2^* = \frac{M_2V}{2}. \tag{16}$$

The optimal profit is

$$\pi^* = \frac{A - B}{2\sqrt{2}\sqrt{t(t - 6C_1 + 6C_2 - 3m_1 + 3m_2 - \sqrt{t(t - 12C_1 + 12C_2 - 6m_1 + 6m_2)})}}.$$

where

$A = 3(\frac{1}{9}(-V + C_2 + \frac{m_2}{2})(2t - 12C_1 + 12C_2 - 6m_1 + 6m_2 -$

$2\sqrt{t(t - 12C_1 + 12C_2 - 6m_1 + 6m_2)} +$

$3\sqrt{2}\sqrt{t(t - 6C_1 + 6C_2 - 3m_1 + 3m_2 - \sqrt{t(t - 12C_1 + 12C_2 - 6m_1 + 6m_2)})})$ and

$B = \frac{1}{162}(-t + 18V - 12C_1 - 6C_2 - 6m_1 - 3m_2 +$

$\sqrt{t(t - 12C_1 + 12C_2 - 6m_1 + 6m_2)})(-2t + 12C_1 - 12C_2 + 6m_1 - 6m_2 +$

$2\sqrt{t(t - 12C_1 + 12C_2 - 6m_1 + 6m_2)} +$

$3\sqrt{2}\sqrt{t\left(t - 6C_1 + 6C_2 - 3m_1 + 3m_2 - \sqrt{t(t - 12C_1 + 12C_2 - 6m_1 + 6m_2)}\right)}\Big).$ (17)

We next analyze when the traditional retailer should consider opening an online store to implement the clicks and mortar selling strategy or still sticking to her single channel selling strategy. Our conclusions can be depicted in the following propositions.

Proposition 2.8. *When* $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$, *the traditional retailer sticks to the single channel strategy if* $\frac{t(M_1-\mu_1)}{V\mu_1} < r < \bar{r}$ *and* $t > \frac{3072(V-C_2)^5(M_1-M_2)^2\mu_1}{3125M_2^3(C_1^2+(C_2+M_1)^2-2C_2M_2-M_2^2+4V(-M_1+M_2)-2C_1(C_2-M_1+M_2))^2}$; *otherwise, the retailer prefers the clicks and mortar selling strategy.*

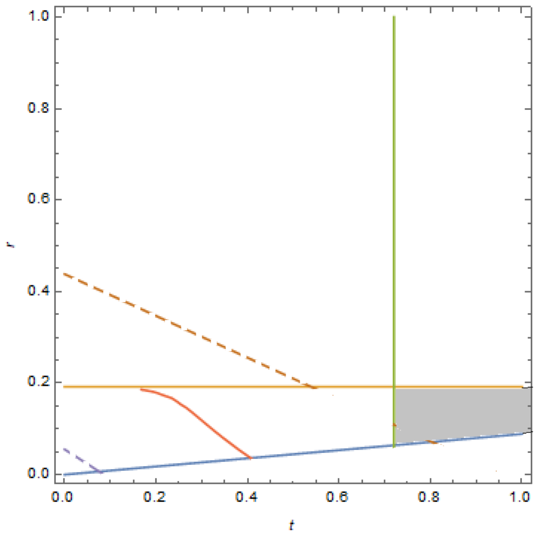


Figure 7. Optimal Selling Channel Strategy when $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$

The traditional retailer will still choose her original single channel selling strategy only if the unit misfit cost of horizontal feature is high, and the return cost of online product is in an intermediate range. Otherwise, the clicks and mortar selling strategy is more attractive to the retailer. The reason is that when the misfit cost of product fitness is high, the relative disutility of the misfit in horizontal feature is large. The consumers' prior purchasing utility has been greatly cut down in respect of the horizontal dimension. As consumers' online purchase is also faced with uncertainty regarding vertical quality, the traditional retailer is difficult to benefit from the clicks and mortar selling strategy especially when the online product return cost is not too high. The online product return cost is relatively low compared with the horizontal misfit cost, thus, the returns of online product can not be avoided with the return cost restriction (i.e., "partial keep" scenario exists). When both conditions are satisfied, namely, online selling is not beneficial for the retailer to expand her market share (i.e., the return cost is not too high) and consumers experience a huge disutility due to misfit (i.e., the unit misfit cost is high), the traditional retailer still stick to her original selling strategy without opening online stores.

However, in all other cases, the clicks and mortar selling strategy dominates the single channel selling strategy. The disutility due to misfit is low in other parameter regions, thus, the consumers can undertake the cost of uncertainty regarding online purchase. Besides, the online product return cost is relatively high compared with the horizontal misfit cost, which helps the retailer expand her market share by avoiding arbitrary online returns. Also note that the conclusion only holds when the offline product's post-purchase attribute is better than the expected online product's post-purchase attribute. This condition guarantees the market share of the traditional retailer's physical store not to be affected too seriously by the opening of online store. That is, the offline product quality is better than online product expected quality, and meanwhile the online purchase is faced with more uncertainty regarding the quality performance. The strength and weaknesses render consumers be more prudent to balance the two products utility gains in the omnichannel environment. This will in turn make the overall marketplace run and operate smoothly. We next consider the opposite case as below.

Proposition 2.9. *When* $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$, *the traditional retailer prefers clicks and mortar selling strategy if* $t < \bar{t}$; *otherwise, the retailer sticks to single channel selling strategy.*

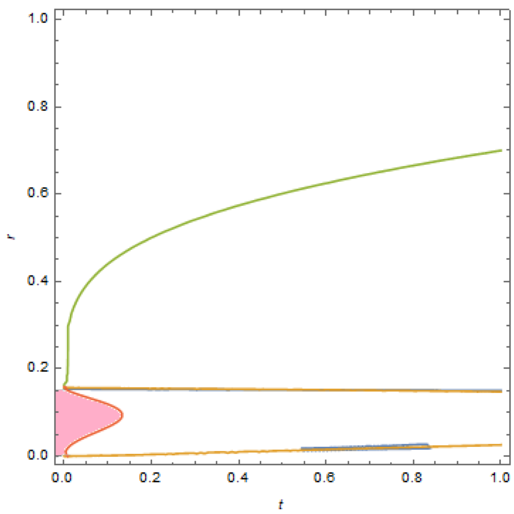


Figure 8. Optimal Selling Channel Strategy when $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$

When the offline product's post-purchase attribute is worse than the expected online product's post-purchase attribute, the traditional retailer will choose to open online store only if the misfit cost of horizontal feature is quite low. Otherwise, she always sticks to the original single channel selling strategy. In this case, the offline product is poor in its quality performance in the omnichannel selling strategy. If the horizontal misfit cost is not low enough, the offline channel will not attract any sales from consumers due to utility loss from both horizontal feature deviation and defective vertical quality. This will make the traditional retailer be prudent to open an online store. However, if the misfit cost is low, the offline product will still own its market share even if its quality is relatively poor compared with online counterpart, as the disutility from horizontal feature is negligible. The quality performances of both products are the primary concern when the consumers set their purchase strategies. The seller prefers clicks and mortar to adapt to consumers' information updating regarding quality performances via channel integration. Also note that the omnichannel selling strategies in all regions belong to the "partial keep" scenario as the return cost is not high enough in this range, thus, online product returns can not be avoided due to dissatisfactory post-purchase quality performance.

From the two parallel propositions above, we can also conclude that when the offline product's post-purchase attribute is better than the expected online product's post-purchase attribute, the traditional retailer has more incentive to open online store to conduct the clicks and mortar selling strategy. However, when the offline product is poor in its quality performance, the traditional retailer prefers to open online store only if the horizontal misfit cost is low enough.

In practice, Belle of shoe industry sets up as a good example in channel integration. This seller pays high attention to her construction of retailing channel network. In the beginning, Belle takes advantage of her multi brands to open physical shops in department stores. The strengths of traditional offline selling reflect in that consumers can try the products before any purchase. That is to say, the brick-and-mortar store selling gives expression to consumers' tiny and emotional information, thus consumers have high unit misfit cost under this circumstance. After the seller does some research on the products that are not sold well after consumers' trying or consulting, she can try her best to make small changes to her products according to consumers' suggestions. Then these relaunching products with tiny adjustments can be placed through both online and offline channels, and they become hot styles soon. This kind of selling strategy adjustment is corresponding to the aforementioned conclusion. When the unit misfit cost of consumers is high, the traditional seller chooses the single offline channel strategy to gather consumers' information. While when the unit misfit cost of consumers is low, the seller prefers the clicks and mortar selling strategy.

2.6. Concluding Remarks

We consider the consumers' information searching behavior in the omnichannel strategy through both the showrooming strategy and webrooming strategy. It means the consumers will gather information through the product items that are placed in the physical store. This fully revealed information will help them update the information of the online products' post-purchase experienced attributes. In the case of omnichannel strategy, the consumers will search information through all available channels where we mainly focus on the extent of information revelation through different channels.

What we can obtain from our model is in four aspects. Firstly, the optimal pricing elasticity coefficient in our model measures the interactive mechanism between online product price and its offline counterpart in the omnichannel environment. Although the two products placed by the seller via both online channel and offline channel are competitive in their market share, under the omnichannel searching environment, their vertical information regarding the quality performance can be updated through consumers' offline inspection. Thus, both products' attributes are more transparent in the omnichannel market, which will finally result in the same direction movement of optimal online product price and offline product price, as they are more likely to be complement goods rather than substitutes.

Secondly, the "partial keep" scenario exists when the return cost is low. The relatively low return cost will make consumer consider returning the online product if it is defective in quality as the small resistance it puts on return behavior. While the "all keep" scenario exists when the return cost coefficient is not too low. This high return cost will influence the consumers' online post purchase return behavior as if it becomes a resistance when the product is not good enough in its quality. The consumers will keep the defective product without undertaking the relatively high return cost.

Thirdly, the horizonal dominance only exists when the offline product is better in its quality compared with the online product. Meanwhile, the misfit cost is relatively high compared to the return cost. The latter condition guarantees the importance of the differentiation regarding the seller's horizontal feature decision. The former condition makes sure the seller's optimal product placement strategy is that the online products are mainstream, while the offline products are niche. As the offline product has good performance in its quality, the seller has incentive to consider assorting products with special features in the physical store, which will not worsen the utility of consumers too much. At the same time, it will satisfy the consumer's personalized demand. While the online product is still the mainstream product with the general feature, this will reduce the possibility of product return to some extent, as it is low in its expected quality performance prior purchase.

Finally, when the offline product quality is better than that of the expected online product, consumers have incentive to purchase online even if the product may be defective in its quality. However, when the offline product quality is worse than the online expected quality, consumers only purchase offline as the online purchasing is faced with more uncertainty. The seller should pay attention to her product assortment strategy for better market division. For example, if the seller wants the market shares of both online and offline channels to be more even, she should consider assort the product with better performance in the physical store.

Chapter 3

The Omnichannel Selling Strategy in an Oligopolistic Market

3.1. Introduction

Consumers in their retail practice are now attaching importance to omnichannel behavior in their perspective. They are ready to take use of both online and offline retail channels in their searching behavior of product information. In order to be better adapted to this new environment, retailers of all industries are reexamining their strategies for delivering both information and products to their target consumers through channels. A business report points out that among the consumers conducting omnichannel behavior, 53% of them start researching digitally while 47% start gathering information in-store, and the two proportions are almost the same (Oracle Bronto. 2018). These kinds of consumers' searching behavior are of great help for consumers to gather product information from multidimensions in practice. Meanwhile, they give challenges to retailers to design their selling strategies including the pricing strategy and product placement strategy via the channel integration shopping platform, in order to devise their omnichannel selling strategies under the cross-channel shopping environment.

What attract our attentions are the competitive pricing and product placement strategies among retailers selling similar kinds of products, with the only difference in their assortment methods via both online and offline channels. We assume consumers are heterogenous in their horizonal fitness with regard to different products. While they are common in the return probability facing deceptive product due to quality dissatisfaction. To be more specific, we study the issue by focusing on the difference remaining in consumers' online and offline shopping behavior, namely, the expected cost occurring during selling process. Consumers' purchase decision of whether to purchase through physical store or online store and making purchase from which retailer depend on not only the product assortment strategies across competitive retailers, but also the return cost that the consumers are faced with if product return happens. We assume the return costs include not only financial expenditures, but psychological burdens and time cost undertaken by consumers. Results show that the online product return cost plays an important role in the retailers' optimal pricing strategy design and product assortment strategy design. Our analyses depict a two-dimensional market structure by considering sellers' return cost and consumers' misfit cost to investigate the optimal selling strategies under the cross-channel shopping platform.

From our analyses of the model depicting a competing market with two sellers selling products through both online and offline channels, we can derive the following three insights. Firstly, no matter what the placement strategy both sellers choose, the optimal prices of products that sold through online channel are first increasing in the return cost of online product and then decreasing in the online product return cost; while the optimal prices of products sold via offline are always increasing in the online product return cost. The online product return cost acts as a resistance for the consumers to make their final purchase decisions between the four products via both channels. When the return cost is quite small, the seller can raise her online product prices. Since the return cost indeed exists and cannot be avoided, it can be small enough for the consumers to ignore the disadvantages of purchasing online (i.e., uncertain about the preference parameter which will cause product exchanges). However, when the online product return cost is relatively great, the consumers will be more prudent to realize their purchases via web stores. Thus, the seller has to cut down her online product prices in order to retain consumers to accomplish their purchases via web stores. Secondly, no matter what the placement strategy sellers will choose, the optimal profits of both sellers is first decreasing in the return cost of online product and then increasing in it. When the online product return cost is quite small, the profits of both sellers decrease rapidly in value as this return cost can not hinder the consumers' intention of returning products. As the increase of return cost, the online product sales are guaranteed from avoiding consumers' arbitrary returning or exchanging behavior. Thus, the profits are increasing in the return cost then. Thirdly, if we consider the three placement strategies given the optimal equilibrium results, the sellers choose the three cases by considering the unit misfit cost of products horizontal feature and the return cost of online product simultaneously. The placement strategy that both online products or both offline products are differentiated to the maximum extent, while for a certain seller, the online product and offline product she sells are adjacent in their horizontal locations will dominate others when the return cost of online product is quite low or quite high. Then, no matter what the value of unit product misfit cost is, this product placement strategy dominates the other two. While when the return is in an intermediate range, the seller will choose this product placement strategy only if the misfit cost is quite low. The performances of other two placement strategies are also related to both unit misfit cost and the return cost of online product.

3.2. Literature Review

The first stream of literatures highly related to our research in this chapter are those studying sellers' product assortment strategy across channels. Brynjolfsson, Hu, and Smith (2003) empirically study the influence of assortment reduction via traditional seller and assortment expansion via online channel. Their results show that this strategy can increase consumer surplus. Dukes, Geylani, and Srinivasan (2009) argue from the perspective of the competitive incentives regarding assortment decisions. Results show that the strategic assortment reduction of traditional sellers however can cut down consumer surplus. Bhatnagar and Syam (2014) set up a model to study the product allocation for a hybrid retailer with both online and offline store. In their model setting, the products can be withdrawn from the offline stores and placed exclusively at online stores to save inventory costs. Nevertheless, all the preceding studies focus on the supply side factors impelling sellers' cross-channel product assortment strategies and none of them consider the sellers' product design feature strategies. Our research adds another motivation to sellers' product assortment strategies by considering the demand side factors of consumers omnichannel information searching behaviors. Meanwhile, we make the sellers as decision makers of product features, which can dynamically depict the transformation of omnichannel sellers' product placement strategy.

Another stream of studies that are highly related to our research in this chapter are those regarding product returns. Some of them examine this topic from the perspective of supply chain management. For instance, Majumder and Groenevelt (2001) develop a two-period model of competitive market to study the impact of remanufacturing cost on competing returned products; M. E. Ferguson and Toktay (2006) set up models to support a manufacturer's recovery strategy in the competitive remanufactured product market; Savaskan, Bhattacharya, and Van Wassenhove (2004) show that the simple coordination mechanisms can be designed to obtain the same level of retailer effort and supply chain profits as the centrally coordinated system; Savaskan and Van Wassenhove (2006) focus on the interaction between a manufacturer's reverse channel choice to collect postconsumer products and the forward channel strategy to determine prices in a competitive market. Besides, Cachon (2003) reviews the supply chain coordination with contracts in respect of inventory decisions and return contracts. Other researchers mainly focus on investigating the buyback pricing strategy in the durable products' market (Desai, Koenigsberg, & Purohit, 2004; Shulman & Coughlan, 2007; Yin, Ray, Gurnani, & Animesh, 2010). Meanwhile, product returns are also brought about by the lack of information regarding product quality. It mainly results in the warranty returns of damaged or low-quality products (Balachander, 2001; M. Ferguson, Guide, & Souza, 2006; Moorthy & Srinivasan, 1995). However, we focus our study on the product returns brought about by consumers' lack of information regarding both their preferences of product design feature and product quality performance.

In the field of marketing, researchers also examine consumers' return behaviors in multichannel shopping environment. Sarvary, Katona, and Ofek (2011) study a competitive market with dual channels and investigate how the pricing strategies and the assistance levels in physical store can change with the foundation of online channel. Studies (Che, 1996; Davis, Gerstner, & Hagerty, 1995) also examine the influences of money-back guarantees on retailers' profits and social welfare. Other researchers mainly focus on the study in respect of return policies. Davis, Hagerty, and Gerstner (1998) develop a theoretical model to analyze when the retailer should offer low hassle cost return policy compared with no refund and full refund return policies. Others (Chu, Gerstner, & Hess, 1998; Hess, Chu, & Gerstner, 1996) further analyze the monopolist's optimal pricing and restocking fee strategies by taking into consideration the speculators who purchase and return products for free renting. Shulman, Coughlan, and Savaskan (2010) employ an analytical model with a bilateral monopoly to investigate the influence of reverse channel structure on equilibrium return policies. However, we mainly focus our research on the competing sellers' pricing strategy and equilibrium profits with the interactive relations between the product demands and consumers' return behaviors.

Besides, we assume consumers are heterogenous in their preferences regarding product design features. This assumption leads to different implications compared with previous studies in respect of return policies. For example, Xie and Gerstner (2007) study the benefits of consumers' escape from pre-purchased service contracts. Their results show that the refund policy for cancellations can reduce the demand and improve the capacity utilization. Guo (2009) develops a model based on the preceding work to investigate how the competition influences the equilibrium profits and refund policies through advance and spot selling. Result shows that competing sellers only adopt the partial refund policy for advanced selling if there is sufficiently constrained capacity. However, our model makes the return cost an exogeneous given value, and we mainly focus on the competition between retailers under the assumption of consumers' heterogeneity in their initial valuations. Meanwhile we take into consideration the product assortment strategy via its influence on competing sellers' pricing strategies.

3.3. Model Setting

We consider a competitive omnichannel selling market with four horizontally differentiated products sold by two competing retailers. We refer to the locations of product j as x_j which is assumed evenly spaced out along a unit circle (Salop 1979). This assumption helps us get analytical results in our main model setting. However, in more general cases where the product spaces nonuniformly along the circle, the consumers with nonuniform preferences are introduced, which makes the model intractable. In order to eliminate the technical problems, we first consider the model by focusing on the interactions between sellers in the market. Each of the two competitive sellers owns two out of the four products and she can choose to place one product online and the other offline in the omnichannel environment. By taking into account overall four products in the competitive market structure, we introduce consumers' returning behavior of each good and exchanging behavior between goods. That is to say, consumers in the competitive market can not only return their unsatisfied original goods but also can swap it for a more satisfactory one. Each product has a common marginal cost of production c in the vertically integrated systems. Moreover, we assume the products selling in the market are a kind of experience goods. Experience goods represent those products that consumers can only know whether their preferences match with or not after they purchase the product or have a try in person (Nelson 1970). We assume this kind of consumers who observe the product through online website without any personal inspection of product fitness before purchase as online consumers, which account for ω of the overall consumers. Namely, this fraction of consumers is not sure if the product is a good fit with their preferences before purchase. We assume consumers are heterogeneous in their taste of the product with an intrinsic preference parameter ϑ_i, which is comprised of two main parts: an observable component and an unobservable component prior purchase. To be more specific, $\vartheta_i = \theta_i + \varepsilon_i$, where $\theta_i \sim U[0,1]$ is perceived by consumers prior to their original purchase decision and ε_i is uniformly distributed over $[-\delta, \delta]$, which is a common knowledge prior purchase. But the specific value of the unobservable component ε_i is resolved only after the consumers obtain the experience products. Note that the value of δ is assumed to be less than $\frac{1}{8}$ in the market with four products in order to ensure that the uncertain component of consumers' preference will not affect the final judgment of consumers' purchase decision. For instance, when the consumer is located at the exact position of a certain product with no misfit value in respect of θ_i, then no matter what the value of ε_i is, he will always unambiguously choose this product rather than its adjacent counterparts. As for those consumers with store inspection before purchase that account for $1-\omega$ of overall consumers, we call them offline consumers. This kind of consumers understand the value of ϑ_i without any uncertainty after they try the experience products or observe the features of the goods such as colors or sizes. This finally resolves the uncertain component in consumers' taste before they make purchase decision.

In addition to the uncertainty about products' horizontal design feature to consumers' ideal preferences, there also exists an uncertain factor in terms of consumers' reservation value v_i, which is the utility gain a consumer obtains after consumption of a product. We can also demonstrate this factor as the vertical quality performance of the product that can augment consumers' utility gain with a rank-ordered preference. Both consumers with and without store inspection are uncertain about this reservation value because it will only be resolved after consumers' consumption of the product. This is the reason why many retailers set up the return policy for free return after seven days of usage or other return warrants like quality guarantee for one year of usage. Concretely, the consumer obtains zero utility gain from possessing any one of the products offered by the retailers through either channel with probability α, i.e., $v_i = 0$. It corresponds to the scenario where the product is defective in quality after proper usage. However, with probability $1-\alpha$, the consumer obtains positive utility gain with $v_i = v$ and the consumer's consumption value equals to $v - t|x_j - \vartheta_i|$ when the product is located at x_j. The parameter t measures the unit misfit cost regarding the difference between the product design feature and consumers' ideal preferences. Both v and t are common knowledge to the consumers. Nevertheless, online purchase and offline purchase are differentiated in the cost if the behaviors of returning the defective goods or exchanging the misfit products occur. Specifically, the return cost of online products includes the return freight insurance, the waiting cost of time or the transportation cost, and we assume it to be r. While as for the offline products, this cost is the hassle cost involved with arguing with the salesclerks or shoe-leather cost which is assumed to be h.

3.3.1. Sequence of the Game

In the competitive setting, each seller first chooses the placement strategy of the product location x_j through online and offline channels simultaneously. Note that product locations are assumed evenly spaced out along a unit circle. We assume there are two placement strategies for two competing sellers: with one seller's products on adjacent locations along the circle or one seller's products on the opposite locations along the circle. Specifically, we suppose one firm sells $x_1 = \frac{1}{4}$ (online) and $x_0 = 0$ (offline), the other firm sells $x_2 = \frac{2}{4}$ (online) and $x_3 = \frac{3}{4}$ (offline) which can be noted as Case (i) or one firm sells $x_1 = \frac{1}{4}$ (online) and $x_0 = \frac{3}{4}$ (offline), the other firm sells $x_2 = \frac{2}{4}$ (online) and $x_3 = 0$ (offline) which is noted as Case (ii). However, if we further consider the online and offline product assortment strategy as well, there is one more product placement strategy: one firm sells $x_1 = \frac{1}{4}$ (online) and $x_0 = 0$ (offline), the other firm sells $x_2 = \frac{3}{4}$ (online) and $x_3 = \frac{2}{4}$ (offline) which is noted as Case (iii). Next, each firm sets her pricing strategy p_j of the products she possesses in each case. We assume that the firm makes the product assortment strategy before the pricing strategy because it is generally supposed that the pricing strategy is more flexible and easier to change than the product assortment strategy. Therefore, the pricing strategy possesses a shorter time horizon than the product assortment strategy.

3.3.2. Demand Generation Process for Online Consumers

Each consumer makes his original purchase decision that maximizes his expected utility on account of his observation of the known part regarding preference parameter: θ_i. We focus on studying the cases in which the two sellers are direct competitors in the market. Thus, all consumers make their original purchase decisions and can possess at most one good out of the four choices. This assumption will hold naturally when the value of v is high enough. However, a consumer will obtain zero utility gain from possessing any one of the products with probability α. He can return this product with deceptive quality performance. On the other hand, consumers will obtain the utility gain of value v with probability $1-\alpha$ from the quality dimension. They will choose to keep their original purchased product or exchange it for a more preferred one after the purchase has been made and they have observed the value of ε_i (Shulman, Coughlan, & Savaskan, 2011).

We first examine the demand and return behavior of online consumers when the sellers' pricing and product placement strategy are given. We assume consumers are forward looking. They will take into consideration the chance of returning their original purchase or exchanging it for another at the beginning of their purchase decision. That is, consumers set their original purchase strategies on the strength of the expected utility by taking each probable post purchase behavior into account. We can make use of the backward induction method to figure out which product will optimize each consumer's expected utility gain. After the consumer determines which product to buy initially, he obtains the product and have a try on it afterwards. The consumer then makes the post-purchase return or exchange strategy based on the actual utility gain he obtains from consuming the product.

We consider the demand generation process of case (i) as an example, and that of case (ii) and case (iii) can be derived in the similar method. In the scenario of case (i), we assume seller one sells product 1 and product 0, with product 1 through online channel and product 0 through offline channel. Meanwhile, seller two sells product 2 and product 3, with product 2 through online store and product 3 through offline store simultaneously. The consumers' information searching behavior can be demonstrated in the figure below.

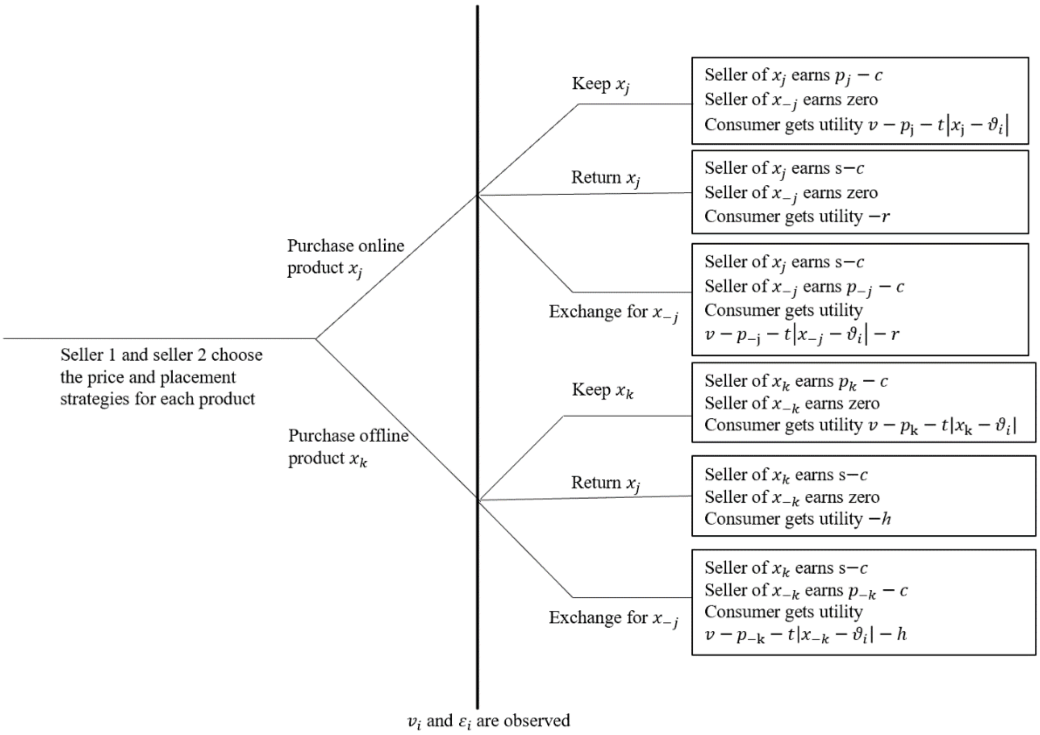


Figure 9. The Sequence of Events and Payoffs for Each Party

Following the demand generation process with the detailed analyses in Appendix A of this chapter, we can obtain the online consumers' initial demands, return quantities and exchange quantities for each product from case (i) to case (iii). We only list those of case (i) as follows, and other cases can be found in appendix.

The sellers' product placement strategies in case (i) are: one firm sells $x_1 = \frac{1}{4}$ (online) and $x_0 = 0$ (offline), the other firm sells $x_2 = \frac{2}{4}$ (online) and $x_3 = \frac{3}{4}$ (offline).

The initial demand of each product can be derived as below, with the subscript denoting the product number:

$$D_1 = \frac{2p_0 + \frac{(h-r)(-(h+r)(-1+\alpha) - 4t(1+\alpha)\delta) + 2(h+r)(-1+\alpha)(-2p_1 + p_2 - tx_0 + tx_2)}{(h+r)(-1+\alpha)}}{4t},$$

$$D_0 = \frac{-4p_0 + \frac{(h-r)((h+r)(-1+\alpha) + 4t(1+\alpha)\delta) + 2(h+r)(-1+\alpha)(p_1 + p_3 + tx_1 - tx_3)}{(h+r)(-1+\alpha)}}{4t},$$

$$D_2 = \frac{2p_1 + \frac{(h-r)(-(h+r)(-1+\alpha) - 4t(1+\alpha)\delta) + 2(h+r)(-1+\alpha)(-2p_2 + p_3 - tx_1 + tx_3)}{(h+r)(-1+\alpha)}}{4t},$$

$$D_3 = \frac{2p_0 + \frac{(h-r)((h+r)(-1+\alpha) + 4t(1+\alpha)\delta) + 2(h+r)(-1+\alpha)(p_2 - 2p_3 + tx_0 - tx_2)}{(h+r)(-1+\alpha)}}{4t}. \tag{18}$$

The exchange quantities of each product are derived as below, with the subscript denoting the exchange behavior happening between the two products number. To be more specific, the exchanges are from the left product number to the right one. Besides, the return quantities are derived as follows. Detailed analyses of consumers' post-purchase behaviors can be found in the appendix of this chapter.

$$e_{01} = \frac{(h+r)(-1+\alpha)\delta}{2t} - \frac{2(h-r)(1+\alpha)\delta^2}{h+r};$$
$$e_{10} = 0;$$
$$e_{32} = \frac{(h+r)(-1+\alpha)\delta}{2t} - \frac{2(h-r)(1+\alpha)\delta^2}{h+r};$$
$$e_{23} = 0;$$
$$R_j = \alpha D_j. \tag{19}$$

3.3.3. Demand Generation Process for Offline Consumers

We next examine the demand and return behavior of offline consumers when the sellers' pricing and product placement strategy are given. The consumers who take in-store inspection will have no uncertainty regarding the preference parameter ϑ_i as they can try the product prior purchase, while all other behaviors are not affected compared with online consumers. We can also make use of the backward induction method to examine which product will optimize the consumers' expected utility gain. That is, consumers set their original purchase strategies on the strength of the expected utility by taking each probable post purchase behavior into account. After their original purchase decision, the consumers obtain the product and have a try on it afterwards. They then determine whether to return it on account of the actual utility they obtain from consuming the product. The derivation process of Case (i) can be seen as follows.

The consumers who take in-store inspection are uncertain only about v_i. The consumer will purchase the online product 1 if he can obtain his optimal utility from it under rational expectation: $(1-\alpha)(v-p_1-t|x_1-\vartheta_i|)-\alpha\, r$. It means the expected utility of purchasing product 1 should be greater than the expected utility of product 2 and product 0 simultaneously: both $(1-\alpha)(v-p_1-t|x_1-\vartheta_i|)-\alpha\, r>(1-\alpha)(v-p_2-t|x_2-\vartheta_i|)-\alpha\, r$ and $(1-\alpha)(v-p_1-t|x_1-\vartheta_i|)-\alpha\, r>(1-\alpha)(v-p_0-t|x_0-\vartheta_i|)-\alpha\, h$ should be satisfied. Therefore, the total demand of product 1 for offline consumers is $D_1'=\frac{-p_1+p_2+t(x_1+x_2)}{2t}-\frac{(h-r)\alpha-(-1+\alpha)p_0+(-1+\alpha)p_1+t(-1+\alpha)(x_0+x_1)}{2t(-1+\alpha)}$. And the corresponding returns of product 1 is $\alpha D_1'$.

With the same method, we can derive the total demand of product 0 as $D_0'=\frac{(h-r)\alpha-(-1+\alpha)p_0+(-1+\alpha)p_1+t(-1+\alpha)(x_0+x_1)}{2t(-1+\alpha)}-\frac{-p_3+p_0+t(x_0+x_3)}{2t}$. And the corresponding returns of product 0 is $\alpha D_0'$. Also note that the offline consumer has no uncertainty about his preference, thus no exchange behavior will happen when he takes in-store inspection before purchase.

The offline consumers' initial demands and return quantities for each product of case (i) can be obtained as below. Demand generation process of Case (ii) and Case (iii) can also be derived in the similar method, and results of them can be found in Appendix A of this chapter.

The sellers' product placement strategies in case (i) are: one firm sells $x_1=\frac{1}{4}$ (online) and $x_0=0$ (offline), the other firm sells $x_2=\frac{2}{4}$ (online) and $x_3=\frac{3}{4}$ (offline).

The demand and return quantity of each product can be derived as below, with the subscript denoting the product number:

$$D_1'=\frac{-h\alpha+r\alpha+(-1+\alpha)p_0-2(-1+\alpha)p_1-p_2+\alpha p_2+tx_0-t\alpha x_0+t(-1+\alpha)x_2}{2t(-1+\alpha)},$$

$$D_0'=-\frac{-h\alpha+r\alpha+2(-1+\alpha)p_0+p_1+p_3+tx_1-\alpha(p_1+p_3+tx_1)+t(-1+\alpha)x_3}{2t(-1+\alpha)},$$

$$D_2' = \frac{-h\alpha + r\alpha + (-1+\alpha)p_1 - 2(-1+\alpha)p_2 - p_3 + \alpha p_3 + tx_1 - t\alpha x_1 + t(-1+\alpha)x_3}{2t(-1+\alpha)},$$

$$D_3' = \frac{h\alpha - r\alpha + (-1+\alpha)p_0 + (-1+\alpha)p_2 + 2p_3 - 2\alpha p_3 - tx_0 + t\alpha x_0 - t(-1+\alpha)x_2}{2t(-1+\alpha)},$$

$$R_j' = \alpha D_j'. \quad (20)$$

After we have obtained the demand of both online consumers and offline consumers in all three cases of the product placement strategy, we then further analyze the equilibrium results of each case. That is, we calculate the optimal pricing strategy in the condition that we are first given all possible placement strategies.

3.4. Model Analyses

We examine a market where there are two competing sellers each selling two products that are horizontally differentiated from each other through either online or offline channel. The objective function of each seller can be shown as below where the product placement strategy has been divided into the three cases we have demonstrated. Meanwhile, we have listed the demand quantities, exchange quantities and return quantities of each case in the aforementioned section. Thus, we can derive the equilibrium results in each case from the following profit maximization problem, which is exactly the objective profit function of Case (i). The objective profit functions of Case (ii) and (iii) are similar to Case (i) with the only change of the sources in respect of product exchange quantities.

$$\max_{p_1,p_0,x_1,x_0} \omega[(p_1-c)(D_1+e_{01}+e_{21})-(c-s)(e_{10}+e_{12}+R_1) + (p_0-c)(D_0+e_{10}+e_{30})-(c-s)(e_{01}+e_{03}+R_0)] + (1-\omega)[(p_1-c)D_1'-(c-s)R_1'+(p_0-c)D_0'-(c-s)R_0'];$$

$$\max_{p_2,p_3,x_2,x_3} \omega[(p_2-c)(D_2+e_{12}+e_{32})-(c-s)(e_{21}+e_{23}+R_2)+(p_3-c)(D_3+e_{03}+e_{23})-(c-s)(e_{30}+e_{32}+R_3)]+(1-\omega)[(p_2-c)D_2'-(c-s)R_2'+(p_3-c)D_3'-(c-s)R_3']. \quad (21)$$

3.4.1. Equilibrium Pricing Strategy for Both Sellers

We obtain the analytical results from the profit maximization problem. The equilibrium results are tedious in their expressions thus we only put the results of Case (i) here, and others can be found in Appendix B of this chapter for reference. The equilibrium results in Case (i) are:

Firm 1: $x_1 = \frac{1}{4}$ (online) and $x_0 = 0$ (offline)

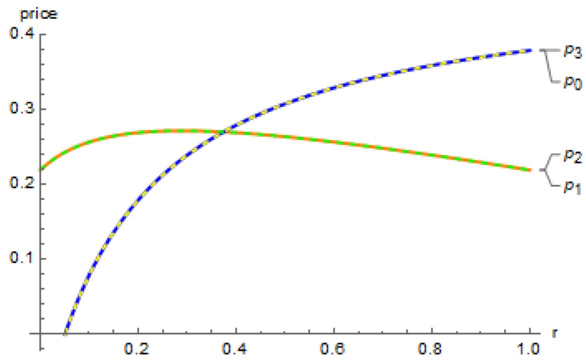
Firm 2: $x_2 = \frac{2}{4}$ (online) and $x_3 = \frac{3}{4}$ (offline)

$$
\begin{aligned}
p_1 = \frac{1}{10(h+r)(-1+\alpha)}\Big(&(h+r)\big(t(-1+\alpha)+2\alpha(-h+r+5s-5s\alpha)\\
&+10c(-1+\alpha^2)\big)\\
&+\Big(h^2(1+\alpha+6(-1+\alpha)^2\delta)\\
&-4h\delta\big(-3r(-1+\alpha)^2+t(1+\alpha+6(-1+\alpha^2)\delta)\big)\\
&+r\big(-r(1+\alpha)+6r(-1+\alpha)^2\delta+4t\delta(1+\alpha+6(-1+\alpha^2)\delta)\big)\Big)\omega\Big);\\
p_0 = \frac{1}{10(h+r)(-1+\alpha)}\Big(&(h+r)\big(t-t\alpha+2\alpha(h-r+5s-5s\alpha)+10c(-1+\alpha^2)\big)\\
&+\Big(h^2(-1-\alpha+4(-1+\alpha)^2\delta)\\
&+r\big(4t(1+\alpha)\delta(-1+4(-1+\alpha)\delta)+r(1+\alpha+4(-1+\alpha)^2\delta)\big)\\
&+4h\delta\big(2r(-1+\alpha)^2+t(1+\alpha+4\delta-4\alpha^2\delta)\big)\Big)\omega\Big);\\
p_2 = \frac{1}{10(h+r)(-1+\alpha)}\Big(&(h+r)\big(t(-1+\alpha)+2\alpha(-h+r+5s-5s\alpha)\\
&+10c(-1+\alpha^2)\big)\\
&+\Big(h^2(1+\alpha+6(-1+\alpha)^2\delta)\\
&-4h\delta\big(-3r(-1+\alpha)^2+t(1+\alpha+6(-1+\alpha^2)\delta)\big)\\
&+r\big(-r(1+\alpha)+6r(-1+\alpha)^2\delta+4t\delta(1+\alpha+6(-1+\alpha^2)\delta)\big)\Big)\omega\Big);\\
p_3 = \frac{1}{10(h+r)(-1+\alpha)}\Big(&(h+r)\big(t(1-\alpha)+2\alpha(h-r+5s-5s\alpha)+10c(-1+\alpha^2)\big)\\
&+\Big(h^2(-1-\alpha+4(-1+\alpha)^2\delta)\\
&+4h\delta\big(2r(-1+\alpha)^2+t(1+\alpha+4(1-\alpha^2)\delta)\big)\\
&+r\big(4t(1+\alpha)\delta(-1+4(-1+\alpha)\delta)+r(1+\alpha+4(-1+\alpha)^2\delta)\big)\Big)\omega\Big).
\end{aligned} \tag{22}
$$

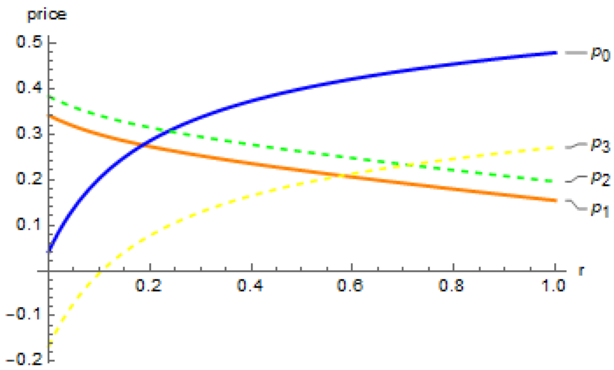
3.4.2. Properties of the Equilibrium Prices and Optimal Profits

With the results we have obtained from all three cases, we can further derive the following several propositions. Our conclusions are mainly focusing on the optimal pricing strategies and optimal profit values. Besides, we also take use of a two-dimensional figure to illustrate the optimal product assortment strategy. We next clarify each result in detailed analyses.

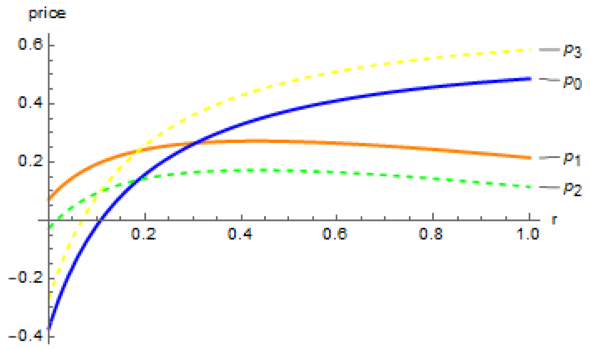
Proposition 3.1. *No matter what the placement strategy sellers will choose, the prices of products that sold through online channel (Product 1 and Product 2) are first increasing in the return cost of online product (i.e.,* r*) and then decreasing in the return cost of online product* r*; while the prices of products that sold via offline channel (Product 0 and Product 3) are always increasing in the return cost of online product* r.



Optimal pricing strategy of Case (i)



Optimal pricing strategy of Case (ii)



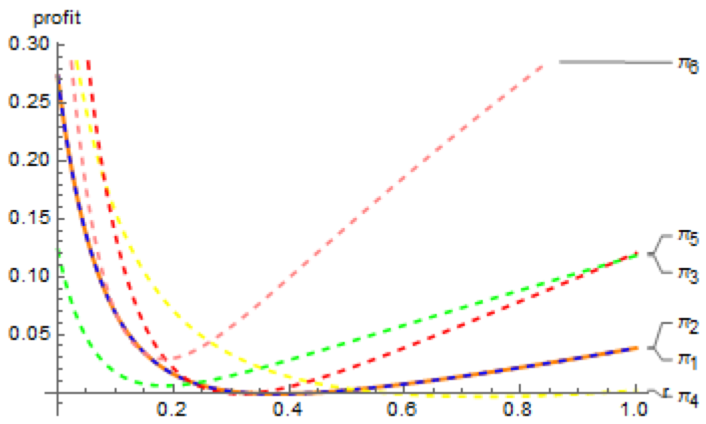
Optimal pricing strategy of Case (iii)

Figure 10. Equilibrium Prices from Case (i) to Case (iii)

This proposition demonstrates that in all three placement cases, the optimal pricing strategy of online product and that of offline product are changing in the same tendency with respect to the online product return cost, respectively. No matter which seller sells the specific product, as long as it is sold through a certain channel, the optimal price of this product is in the trajectory of change as depicted in the above figures. We can see clearly that the prices of online products are decreasing in the online product return cost, although there is a little interval where r is low, the prices are increasing in it. However, it will not change the overall tendency of online products' prices being decreasing in r. On the contrary, the prices of offline products are increasing in the online product return cost no matter which seller sells them. This is quite intuitive since the online product return cost acts as a resistance for the consumers to make their purchase decisions of online product when considering which product to buy. Thus, when the return cost r is quite small, the seller can raise her online product prices. Since the return cost indeed exists and cannot be avoided, but it can be small enough for the consumers to ignore the disadvantages of purchasing online (i.e., uncertain about the preference parameter which will cause product exchanges). However, when the online product return cost r is relatively great, the consumers will be more prudent to realize their purchases via web stores. Thus, the seller should strive to cut down her online product prices in order to appeal consumers to accomplish their purchases via web stores. Otherwise, the operation cost of online channel can not be covered by its revenue when there are no consumers purchasing online, and it will result in a waste of vacancy channel.

Meanwhile, as the augmentation of online product return cost, the prices of their offline counterparts are increasing too. The offline inspection helps consumers eliminate concerns of product exchanges due to the uncertainty on their preference when making online purchase. The seller that makes their products sold in the brick-and-mortar store will always have incentive to raise their offline product prices, as consumers will accept the high price in order to avoid the possible exchange or return behavior that may happen through online purchase. We next analyze the optimal profits of each case.

Proposition 3.2. *No matter what placement strategy the sellers will choose, the optimal profits of both sellers are first decreasing in the return cost of online product (i.e., r) and then increasing in r.*



Optimal profit value of each cases (Profit 1-2 corresponds to Case (i); Profit 3-4 corresponds to Case (ii); Profit 5-6 corresponds to Case (iii))

Figure 11. Optimal Profits from Case (i) to Case (iii)

This proposition shows us the property of the optimal profits regarding both sellers in all three placement strategies. They are first decreasing in the online product return cost r over a small interval and then increasing in it afterwards. As the above figure depicts, when the online product return cost is quite small, the profits of both sellers decrease rapidly in value as this return cost can not hinder the consumers' intention of returning products. It will impair the seller's profit as the possible negative effects brought about by the occurance of exchanging and returning product. Thus, the return cost should not be too low for the seller to choose the online product selling strategy as long as the online return cost is not approaching zero. What is also intriguing is that when the return cost is approaching zero, which means the return cost is almost a nonexistence, the profits of both sellers are approaching positive infinity. As under this circumstance, the online selling goes smoothly like offline selling without any cost of product exchange, thus the market is degenerated to a transparent market with seamless product transaction. That is to say, any product without maximized fitness or good quality will be eliminated in the market which will result in a market with no deceptive products. However, this is not true in practice. We focus our attention on the reality by considering that the optimal profits then increase in the online product return cost after the rapid decrease when r is low. This is intuitive since the increase of return cost r will guarantee the online product sales from avoiding consumers' arbitrary returning or exchanging behavior. Namely, consumers that make online purchase will have to balance their expected utility from exchanging a misfit product or returning a deceptive product with the utility of keeping the original product, which probably does not match well with their preference or even is a product with poor quality. This gives us the reason why in practice, the online product's return or exchange should satisfy several conditions. These conditions will be demonstrated by sellers before consumers' online purchase. The restrictions of return or exchange make consumers consider their purchase more seriously, which avoids vicious or intentional online product return or exchange behavior. Meanwhile, they guarantee the sellers' profit to some extent.

3.4.3. Optimal Product Placement Strategy

We then consider the product placement strategies by further analyzing the sellers' optimal profit in each case (Case (i) to Case (iii)), and the optimal placement strategy can be derived in the following proposition.

Proposition 3.3. *If we consider the three placement strategies given the optimal equilibrium results, the sellers choose the three cases by considering the unit misfit cost of products horizontal feature (i.e., t) and the return cost of online product (i.e., r) simultaneously.*

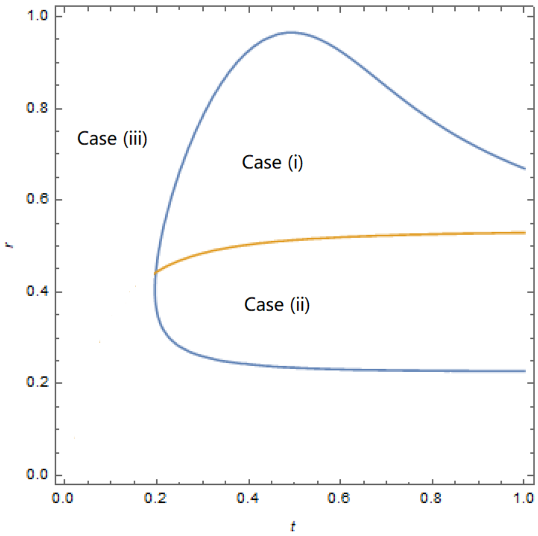


Figure 12. Optimal Product Placement Strategies in $r - t$ Plane

This proposition illustrates the relationship between the product placement strategy with the two costs containing in our model, i.e., the unit product misfit cost and the online product return cost. To be more specific, Case (i) depicts the scenario when both online products (equivalent to both offline products) are adjacent in their horizontal locations and meanwhile as for a certain seller, the online product and offline product she sells are also adjacent in their horizontal feature. In respect to Case (ii), it describes a scenario when both online products (equivalent to both offline products) are adjacent in their horizontal locations while a seller sells the online product and offline product that are differentiated to the maximum extent, i.e., the two goods are placed on the opposite locations along the unit circle. As for Case (iii), both online products or both offline products are differentiated to the maximum extent, while for a certain seller, the online product and offline product she sells are adjacent in their horizontal locations. We first explain the Case (iii), as in the above figure, it shows that when the return cost of online product is quite low or quite high, no matter what the value of unit product misfit cost is, this product placement strategy dominates the other two. While when r is in an intermediate range, the seller will choose this product placement strategy only if t is quite low. The reason behind this phenomenon is that when r is high or low, what we have derived from Proposition 2 has shown that the optimal profits for both sellers are higher than that when r is in an intermediate range. Meanwhile, the optimal profit of both sellers in Case (iii) dominates that in Case (i) and Case (ii). As in this case, the seller sells similar products in their horizontal feature, while the online products of both sellers (equivalently the offline products of both sellers) are differentiated in their horizontal feature. This will make each seller focus on selling the products with similar feature, thus resulting in more exchanges between the seller's products through online and offline channel. However, the exchanges between different sellers' products placed online or offline will not happen then. Thus, the *promotion effect* of a certain seller's optimal profit brought about by the change of online product return cost is amplified in this scenario. Since the return cost highly affects the consumers' product exchange behavior within a certain seller rather than between the two sellers' products.

As for the other cases (Case (i) and Case (ii)), the main difference between them is that the two products sold by one seller is similar in Case (i), while they are differentiated a lot in Case (ii). Note that in both cases, the online products or offline products sold by both sellers are similar in their horizontal feature. Thus, the change of return cost will affect the exchange quantities between both online products and offline products. It also affects the consumer's exchange behavior within a certain seller in Case (i). Nevertheless, its influence on the exchanges within a specific seller in Case (ii) is tiny as both sellers sell differentiated products in Case (ii) and exchanges will not happen between two differentiated products in our setting. Thus, when the unit misfit cost t is not too small, which means the mismatch between product horizontal feature and the consumers' preference is influential, Case (i) dominates the others when the return cost of online product r is exerting positive effect on seller's optimal profit (i.e., r is greater than the threshold when the optimal profit is lowest in the change of r). Otherwise, Case (ii) dominates the other two cases as r has little influence on the optimal profits of both sellers in this range.

3.5. The Effect of Competing

In this section, we mainly focus on the influence of competition in our main model. As we have modeled an oligopolistic setting with two sellers, we then take the benchmark setting of one seller into consideration. All assumptions remain the same as the main model. However, we analyze a monopolistic scenario with one seller managing two products, with one of them via online store and the other via brick-and-mortar store. Both products are evenly spaced out along a unit circle. Therefore, we can follow the same demand derivation process as our main model.

The equilibrium results of the monopolist's profit maximization problem should satisfy the following equations:

$$p_1^* = \frac{C - D}{4(1-\alpha)\delta((h+r)^2(-1+\alpha) - 4(h-r)t(1+\alpha)\delta)} + p_0^*,$$

where $C = (h-r)(h+r)t(1+\alpha) + 2(t((h+r)^2 + 2(-h+r)t)$ and $D = ((h-r)(h+r)^2 + 2(h+r)^2 t + 2(h-r)t^2)\alpha + (h+r)^2(h-r+t)\alpha^2)\delta + 8(h-r)t(1+\alpha)(t-(h-r+t)\alpha)\delta^2$. (23)

Meanwhile, the optimal profit should be:

$$\pi_m^* = \frac{E + F}{16t(1-\alpha)\delta^2((h+r)^2(-1+\alpha) - 4(h-r)t(1+\alpha)\delta)(-(h+r)^2(-1+\alpha)^2 + 4(h-r)t(-1+\alpha^2)\delta)},$$

where $E = 16(c-s)t(1-\alpha)^3\delta^2((h+r)^2(-1+\alpha) - 4(h-r)t(1+\alpha)\delta)^2$ and $F = ((h-r)(h+r)t(1+\alpha) + 2(t((h+r)^2 + 2(-h+r)t) - ((h-r)(h+r)^2 + 2(h+r)^2 t + 2(h-r)t^2)\alpha + (h+r)^2(h-r+t)\alpha^2)\delta + 8(h-r)t(1+\alpha)(t-(h-r+t)\alpha)\delta^2)^2$. (24)

The monopolist's product placement strategy is: $x_0 = 0$ (offline) and $x_1 = \frac{1}{2}$ (online).

After our comparisons between the equilibrium results of the two sellers' structure and the one seller structure, we can obtain several conclusions. These differences and similarities reflect the influence of competition.

Proposition 3.4. *In the market without competition, the optimal pricing strategies of online product and offline product are in difference with a constant and change in the same direction.*

This conclusion regarding optimal prices is different from the pricing strategy in the market with competition. However, it is similar to the conclusion in the monopolistic setting in Chapter 2. Namely, the online product and offline product are more likely to be complements in their prices than substitutes.

That is to say, although the two products placed by the seller via both online channel and offline channel are competitive in their market share, however, we allow the exchange and return behavior after online purchase or in-store inspection. Thus, both products' characteristics are more transparent in the omnichannel selling market. This finally results in the same changing direction between p_1^* and p_0^*. Namely, they are more likely to be complement goods rather than substitutes. This is intuitive since the seller with dual channels should guarantee her products in each channel to remain consistency, so as to avoid internal competition, which will not benefit the seller from expanding her market share.

While, in the competitive setting, the two online products and two offline products are changing in the opposite directions in respect of return cost. The reason behind this phenomenon is that we allow exchange behavior in our oligopolistic setting, thus, the sellers will balance the demands in each channel in case of the existence of vacancy channels. The influence of return cost on online and offline products' prices reflects the sellers' objective to attract consumers' demand in each channel. Otherwise, the operation cost of online store or physical store can not be covered when there is no consumer's purchase in that channel. However, the two sellers' products in the same channel are changing in the same direction in respect of return cost. This reflects the products selling through a certain channel have synergistic effect. They are not in malicious differentiated price competition, which will not benefit both sellers in long run.

Although the pricing strategy in the market without competition is different from that with competition, the optimal profits are not affected by the inducement of competition.

Proposition 3.5. *In the market without competition, the optimal profit of the seller is increasing in r when r is higher than a certain threshold (i.e., $r > \bar{r}$).*

This result in respect of the optimal profit is the same as that with competition. That is to say, the optimal profits are always increasing in the online product return cost, as long as the return cost is higher than a threshold.

This is intuitive since the increase of return cost r will guarantee the online product sales from avoiding consumers' arbitrary returning or exchanging behavior. Namely, consumers that make online purchase will have to balance their expected utility from exchanging a misfit product or returning a deceptive product with the utility of keeping the original product. This gives us the reason why in practice, the online product's return or exchange should satisfy several conditions. These conditions will be demonstrated by sellers before consumers' online purchase. The restrictions of return or exchange behavior make consumers' considering their purchase more seriously, which avoids vicious or intentional online product return or exchange behavior. Meanwhile, they guarantee the sellers' profit to some extent.

3.5.1. Comparisons between Chapter 2 and Chapter 3

If we consider the model setting and corresponding conclusions in chapter 2 and chapter 3 simultaneously, we can summarize the differences in their modeling assumptions and results in this section.

There are several significant differences in the monopolistic setting and oligopolistic setting. Firstly, in the monopolistic setting, we consider a product market with two-dimensional information structure, i.e., the product horizontal features and quality performances. The horizontal features are assumed to be known prior purchase for omnichannel consumers. The quality performances can only be perceived after purchase, which leads to product returns. However, in the oligopolistic setting, in order to get analytical results, we consider a product market with uncertainty only in product's horizontal features, while the defective probability of products in their quality remains the same. That is to say, there is no "all keep" scenarios in this setting, and product returns always exist with a fixed proportion for online product. Secondly, in the monopolistic setting, there is only one product placed in each channel. Thus, we assume the consumers make purchase between online and offline product with one-time decision. Also, we make the retailer as the decision maker of product features in both channels. The online product and offline product can be the same in their horizontal features. That is to say, there is no exchange happening between the two products due to fitness discrepancy. However, in the oligopolistic setting, there are two online products assorted via online channel. For consumers purchasing online without any in-store inspection before purchase, they are uncertain about the fitness of each product prior purchase. Thus, the exchange between online products and offline products always exists for online consumers.

In general, in the monopolistic setting, we mainly focus on the consumers' return behavior brought about by the uncertainty regarding products' post-purchase attribute (i.e., quality performance). While, in the oligopolistic setting, we make the return quantities as a fixed ratio of demand. And we mainly focus on the exchange behavior brought about by the uncertainty in products' horizontal features. The exchanges of products always exist as we assume each product is different in its horizontal fitness. These assumptions help us get analytical results in the four products market. After the clarifications of the differences in the model settings, we then can summarize the following differences in pricing strategies.

In terms of the monopolistic setting, under "all keep" scenarios, the online product price and offline product price have no relations with return cost; under "partial keep" scenarios, when $\Delta_2 = \{m_2, M_2\} > \Delta_1 = \{\mu_1, \mu_2\}$, the online product price and offline product price increase in return cost, while when $\Delta_2 = \{m_2, M_2\} < \Delta_1 = \{\mu_1, \mu_2\}$, the online product price and offline product price decrease in return cost. In terms of the oligopolistic setting, there is no "all keep" scenario; in "partial keep" scenarios, the online product prices decrease in return cost, while the offline product prices increase in return cost.

This conclusion shows that in our monopolistic setting, the pricing strategy is related to the post-purchase attribute of both products. The influence of online product return cost on the pricing strategy depends on whether the offline product is better in its quality or not. However, the changing directions of both products' prices are the same in the monopolistic setting. As we have clarified in Chapter 2 that the monopolist regards the two products as complements but not substitutes. We here elaborate this complemental pricing strategy of the seller with an example such as the high-end luxury market (i.e., SKP in Beijing). This kind of product market always stick to the operation principle with the same direction of pricing adjustment to similar products through channels. This pricing strategy will not induce the vicious price competition between the similar products of the same brand via online and offline stores. On the contrary, the same direction of pricing variation strategy will maintain consumers' brand loyalty and improve the seller's selling performance.

While, in the oligopolistic setting, the two online products and two offline products are changing in the opposite directions in respect of return cost. The reason behind this phenomenon is that we allow exchange behavior in our oligopolistic setting, thus, the sellers will balance the demands in each channel in case of the existence of vacancy channels. The influence of return cost on online and offline products' prices reflects the sellers' objective to attract consumers' demand in each channel. Otherwise, the operation cost of online store or physical store can not be covered when there is no consumer's purchase in that channel. However, the two sellers' products in the same channel are changing in the same direction in respect of return cost. This reflects that the products sold through a certain channel have synergistic effect in the seller's selling performance. They are not in malicious differentiated price competition, which will not benefit both sellers in long run. In practice, the online product selling is always focusing on the pursuit of high-performance-cost ratio. With the augmentation of online product's return cost, the online product prices are decreasing significantly. Online selling should depend on providing products with good performance of low return risk, meanwhile, the reduced prices compared with offline products also attract consumers to purchase through online stores. On the contrary, offline selling should focus on improving the products' tastes and performance rather than competing with online products' prices. For many consumers, they stick to purchasing through physical store by taking advantage of offline inspection and fitting experience with high-end products such as clothes. What they mainly focus on is whether the clothes fit them or not, while price is put to the second place. Thus, these kinds of experience products with high value are more suitable for consumers to purchase through physical store, and the products' prices can be set to increase with the return difficulty of their online counterparts. Besides the high-end clothes market, many luxuries such as LV/Channel also stick to their offline selling by raising prices of their bags or watches, while they still have great demands in the physical store. That is to say, the offline selling should not compete with online selling in the pricing strategy, but it should consider providing consumers with better purchasing experience and introduce high-end products with high prices to attract more consumers especially when the products are hard to return via online channel.

As for the optimal profits in both settings, the overall changing tendency of the sellers' optimal profits are increasing in online product return cost. However, there is a little difference when the return cost is approaching zero. We summarize the difference as follows. In terms of the monopolistic setting, the seller's optimal profits increase in the online product return cost; in terms of the oligopolistic setting, the optimal profits of both sellers first decrease in online product return cost over small intervals and then increase in it.

As in the monopolistic setting, we don't consider the exchange behavior of consumers. When $r = 0$, there is no restriction to return behavior of consumers. It will reduce the online product demand, but it will not increase offline product demand. Therefore, the optimal profit is approaching zero when the return cost is zero. As for the oligopolistic setting, we allow for the exchange behavior. When the return cost is approach zero, the profits of both sellers are approaching positive infinity, which is not true in practice. We have clarified it in the aforementioned section in this chapter.

In practice, we try to understand the variation of sellers' optimal profits in the online product return cost with some actions taken by sellers. By the year 2022, the sales via online e-commerce are expected to account for nearly 35% of fashion retails (Forrester, 2021), and clothing is the most popular type among them. As for the sellers in this industry, it is significant for them to seek an efficient return process and avoid the resource consumption brought about by product returns. In the past five years, the return rate in e-commerce is increasing rapidly by 95% (Payments journal), many retailers are reexamining their strategies to reduce return quantities. The key point for retailers to reduce returns is to learn about the reasons of return behaviors and to satisfy clients' real demand. According to a survey made by WBE, 59% consumers return products as for the damaged goods, while 42% of them return for the reason of regret and 29% of them for the reason of information misleading. Some retailers consider increasing the barrier in the return process, such as charging for the returns or shortening the return periods. These strategies will reduce the arbitrary returns of many illogical consumer behaviors. Other retailers such as Zara, H&M and Aday resort to provide consumers with better service and detailed description of their products in order to help consumers make sensible purchasing decisions (Vogue Business). All these actions can reduce the arbitrary product return behaviors and improve retailers' revenue performance in the long run.

3.6. Single Channel vs Dual Channel Selling Strategy

In this section, we analyze when the single channel retailer should stick to her original selling strategy, and when she should consider the dual channel selling strategy as our main model depicts. We separate the single channel retailer from the web-only retailer and the store-only retailer. Nevertheless, we still take into account the competitive market structure with four goods sold by two firms respectively. Therefore, the product placement strategy for the single channel retailer can be divided into two cases. Case (1), the two products sold by one retailer are adjacent in their horizontal locations; case (2), the two products sold by one retailer are differentiated in their horizontal locations.

We first take into account the web-only sellers with all their products selling via online shops, and the objective functions of each web-only seller can be obtained as follows:

$$\max_{p_1,p_0,x_1,x_0} (p_1 - c)(D_1 + e_{01} + e_{21}) - (c - s)(e_{10} + e_{12} + \alpha D_1)$$
$$+ (p_0 - c)(D_0 + e_{10} + e_{30}) - (c - s)(e_{01} + e_{03} + \alpha D_0);$$
$$\max_{p_2,p_3,x_2,x_3} (p_2 - c)(D_2 + e_{12} + e_{32}) - (c - s)(e_{21} + e_{23} + \alpha D_2) + (p_3 - c)(D_3 +$$
$$e_{03} + e_{23}) - (c - s)(e_{30} + e_{32} + \alpha D_3). \quad (25)$$

We next analyze the store-only sellers' optimal selling strategy with all their products selling via brick-and-mortar shop, and the objective functions of each store-only seller are as follows:

$$\max_{p_1,p_0,x_1,x_0} (p_1 - c)D_1' - (c - s)\alpha D_1' + (p_0 - c)D_0' - (c - s)\alpha D_0';$$
$$\max_{p_2,p_3,x_2,x_3} (p_2 - c)D_2' - (c - s)\alpha D_2' + (p_3 - c)D_3' - (c - s)\alpha D_3'. \quad (26)$$

After the demand generation process for both types of retailers, we can derive the equilibrium results for web-only retailers in case (1) as:

Firm 1: $x_1 = \frac{1}{4}$ (online) and $x_0 = 0$ (online); $p_1^* = c + \frac{t}{10} + c\alpha - s\alpha$ and $p_0^* = c - \frac{t}{10} + c\alpha - s\alpha$; $\pi^* = \frac{3t}{100}$.

Firm 2: $x_2 = \frac{2}{4}$ (online) and $x_3 = \frac{3}{4}$ (online); $p_2^* = c + \frac{t}{10} + c\alpha - s\alpha$ and $p_3^* = c - \frac{t}{10} + c\alpha - s\alpha$; $\pi^* = \frac{3t}{100}$. (27)

The equilibrium results for web-only retailers in case (2) are as follows:

Firm 1: $x_1 = \frac{1}{4}$ (online) and $x_0 = \frac{3}{4}$(online); $p_1^* = c + \frac{t}{12} + c\alpha - s\alpha$ and $p_0^* = c + \frac{t}{12} + c\alpha - s\alpha$; $\pi^* = \frac{t}{72}$.

Firm 2: $x_2 = \frac{2}{4}$ (online) and $x_3 = 0$ (online); $p_2^* = c + \frac{t}{6} + c\alpha - s\alpha$ and $p_3^* = c - \frac{t}{3} +$

$c\alpha - s\alpha;\ \pi^* = \frac{5t}{36}$. (28)

The equilibrium results for store-only retailers are similar to that of the web-only retailers, thus we omit them here and only put them in the Appendix B of this chapter.

We next make comparisons between the optimal equilibrium results when the retailers choose single channel and dual channel selling strategy. We can obtain the conclusion by clarifying the condition when the retailers prefer clicks and mortar and when they stick to the single channel selling strategy.

Proposition 3.8. *The sellers stick to the single channel selling strategy if and only if* $\underline{r} \le r \le \overline{r}$ *and* $t \ge \overline{t}$*; otherwise, they prefer the dual channel selling strategy.*

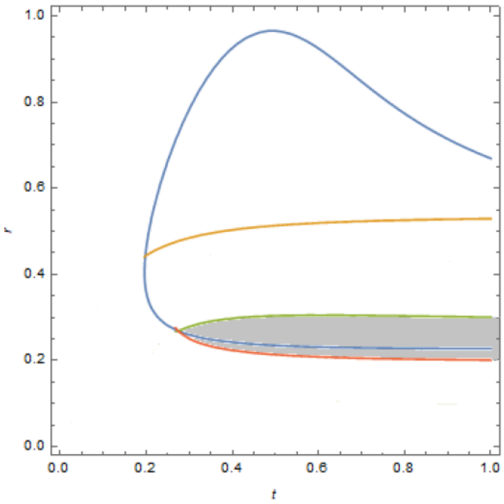


Figure 13. Optimal Selling Channel Strategy in $r - t$ Plane

This conclusion is similar to the one we have analyzed in chapter 2. The web-only or store-only retailer will still choose her original single channel selling strategy only if the unit misfit cost of horizontal feature is high, and the return cost of online product is in an intermediate range. Otherwise, the clicks and mortar selling strategy is more attractive to the retailer. The reason is that when the misfit cost of product fitness is high, the relative disutility of the misfit in horizontal feature is large. The consumers' prior purchasing utility has been greatly cut down in respect of the horizontal dimension. As consumers' online purchase is also faced with uncertainty regarding fitness prior purchase, which will result in the exchanges between offline products and online products, the single channel retailer is difficult to benefit from the clicks and mortar selling strategy especially when the online product return cost is not too high. The online product return cost is relatively low compared with the horizontal misfit cost, thus, the returns or exchanges of online product can not be avoided with the limited return cost restriction. When both conditions are satisfied, namely, dual channel selling strategy is not beneficial for the retailer to expand her market share (i.e., the return cost is not too high) and consumers experience a huge disutility due to misfit (i.e., the unit misfit cost is high), the single channel retailer still stick to her original selling strategy without taking use of clicks and mortar strategy.

However, in all other cases, the clicks and mortar selling strategy dominates the single channel selling strategy. The disutility due to misfit is low in other parameter regions, thus, the consumers can undertake the cost of uncertainty regarding the exchanges that may occur between online purchase and offline purchase. Besides, the online product return cost is relatively high compared with the horizontal misfit cost, which helps the retailer expand her market share by avoiding arbitrary exchanges between offline and online purchase occurring in the clicks and mortar selling strategy.

3.7. Concluding Remarks and Discussions

From our analyses of the model depicting a competing market with two sellers selling products through both online and offline channels, we can derive the following three main conclusions as shown in our model analyses.

Firstly, no matter what the placement strategy both sellers choose, the optimal prices of products that sold through online channel are first increasing in the return cost of online product and then decreasing in the online product return cost; while the optimal prices of products sold via offline are always increasing in the online product return cost. The online product return cost acts as a resistance for the consumers to make their final purchase decisions between the four products via both channels. When the return cost is quite small, the seller can raise her online product prices. Since the return cost indeed exists and cannot be avoided, it can be small enough for the consumers to ignore the disadvantages of purchasing online (i.e., uncertain about the preference parameter which will cause product exchanges). However, when the online product return cost is in great level, the consumers will be more prudent to realize their purchases via web stores. Thus, the seller should take into account the method of cutting down her online product prices in order to retain consumers via web stores.

Secondly, no matter what the placement strategy sellers will choose, the optimal profits of both sellers is first decreasing in the return cost of online product and then increasing in it. When the online product return cost is quite small, the profits of both sellers decrease rapidly in value as this return cost can not hinder the consumers' intention of returning products. As the increase of return cost, the online product sales are guaranteed from avoiding consumers' arbitrary returning or exchanging behavior. Thus, the profits are increasing in the return cost.

Thirdly, if we consider the three placement strategies given the optimal equilibrium results, the sellers choose the three cases by considering the unit misfit cost of products horizontal feature and the return cost of online product simultaneously. The placement strategy that both online products or both offline products are differentiated to the maximum extent, while for a certain seller, the online product and offline product she sells are adjacent in their horizontal locations will dominate others when the return cost of online product is quite low or quite high. Then, no matter what the value of unit product misfit cost is, this product placement strategy dominates the other two. While when the return cost is in an intermediate range, the seller will choose this product placement strategy only if the misfit cost is quite low. The performances of other two placement strategies are also related to both unit misfit cost and the return cost of online product.

There are several significant differences in the monopolistic setting and oligopolistic setting. Firstly, in the monopolistic setting, we consider a product market with two-dimensional information structure. However, in the oligopolistic setting, in order to get analytical results, we consider a product market with uncertainty only in product's horizontal features, while the defective probability of products in their quality remains the same. Secondly, in the monopolistic setting, there is only one product placed in each channel. Thus, we assume the consumers make purchase between online and offline product with one-time decision. However, in the oligopolistic setting, there are two online products assorted via online channel. For consumers purchasing online without any in-store inspection before purchase, they are uncertain about the fitness of each product prior purchase. These differences in model assumptions will make the conclusions in chapter 2 and 3 distinct.

Moreover, we would like to make some discussions on our model setting and conclusions with other studies in this section. According to recent research on seller's omnichannel selling strategies (Z. Gu & Tayi, 2017), they mainly focus on the optimal product assortment strategy by multichannel consideration. In their setting, the probabilty of product fitness is exogenously given. This induces the resulting conclusion that the optimal pricing strategy and product assortment stategy depend on two elements: the online product return cost (which is the same as our conclusion) and the fittness probability (which is distinct from ours). However, their assumptions are quite restricted as they failed to demonstrate the product attributes in a two-dimensional market structure, and the information revelation behaviors of consumers are over-simplified in their model setting. We thus generalize a more abundant model structure by depicting the consumer market profile with both vertical and horiziontal feature locations. Our conclusions in both monopolistic and duopolistic model settings are consistent in respect of the optimal product assortment strategies. Besides, according to the research on how competitive sellers should manage consumer returns (Shulman, Coughlan, & Savaskan, 2011), their research emphesis is on the optimal pricing and restocking fee strategies of competitive sellers. They mainly consider the horizontally differentiated goods with the exogenously given probability of product return, which is distinct from our monopolistic setting but is similar to our duopolistic setting. What we mainly care about is the product assortment strategy, while what they focus on is the equilibirum product prices and resocking fees, among which the latter one we do not take into consideration in our model structure. We plan to include the restocking fee strategies with the full image of return policy in our future study.

Chapter 4

Collaborative Service Provision under Signaling Framework

4.1. Introduction

Among all types of service provision, a fraction of them that specifically attract our research interest are the knowledge payment service and online career or interest training service. For instance, teaching a certain instrument, helping students pass language tests or other vocational qualification examinations, coaching body builders to keep fit and so on. As the rapid growth of service industry, it is estimated that the global knowledge payment service market will reach nearly 68 billion RMB by 2021(data. iimedia.cn). The knowledge payment service is in prosperity and development around the world, and numerous knowledge suppliers are taking part in the market of knowledge payment services. For instance, Zhihu in China and Skillshare in America. They are gradually altering the life style of the public for knowledge sharing. The market of knowledge payment services in China is rapidly growing. The year of 2016 is honored as “year one of knowledge payment service”. According to a business study (iresearch, 2018), the market size of the knowledge payment service in China has reached nearly 5 billion RMB in the year of 2017. The business volume in respect of the knowledge payment service is at a growth rate of more than two hundred percent in the year of 2017. This phenomenon suggests that the knowledge payment service market in China has huge and strong growth potential in the long run.

Given the complexity of the knowledge payment service and its enduring influence, it is of great importance for us to figure out the effort contribution mechanism between the service provider and consumers, meanwhile it is intriguing to study the strategic interaction of both parties under information intertemporal transmission. There are some properties of this kind of so-called “collaborative services” (Roles 2014). Firstly, the outcome of the service depends on both the service provider’s effort contribution and the consumers’ effort input. For instance, the online fitness training sessions offer the consumers with courses explaining and showing the body building movements, however, the success of the service provision that helps consumers keep fit cannot be separated from the consumers' effort in trying to do exercises themselves. Furthermore, the service provider makes her own effort level strategy and pricing strategy, while the consumer can decide his effort contribution to the whole service provision process. Both effort contributions are not directly observable to consumers ex ante, thus moral hazard may exist in the service process. Both the service provider and the consumer strategically choose their effort contributions accordingly. Besides the unobservable strategies, the inherent competence of a service provider's cost efficiency is also unobservable, making differentiating the efficient service provider another significant problem worth studying. Meanwhile, we incorporate another asymmetric information regarding the service provider's quality certification, which makes the problem a two-dimension information structure.

Although motivated from the background of knowledge payment service, our model and conclusions have pervasive inspirations for other kinds of services with similar properties, only if the service is collaborative in essence. Namely, the service provider and consumers both have the right to make the effort level decisions on their own respectively. Similar service provisions include advisory services (e.g., legal consulting service requiring cooperation of each party, i.e., lawyers and consultants), medical services (e.g., physical rehabilitation and therapy) and professional services (e.g., investment and finance) (H. Sun & Xu, 2018). We can call them as knowledge-intensive business service which are typically high-end services in a complex B2B context.

Back to the knowledge payment services, we have studied a literature review about this field systematically (Qi, T., Wang et al. 2019). In this review article, it points out that most previous research is limited to the traditional online service market with digital content, while the research on the knowledge payment services has just begun. According to the introduction on this knowledge payment field, it discovers that the study conclusions of the existing research are always in conflict with each other. Moreover, researchers often study this topic independently. Therefore, a cohesive theoretical framework should be founded to integrate elements including knowledge supplier (i.e., service provider), knowledge demander (i.e., consumers) and knowledge payment platform (i.e., online review platform). We thus resolve to the signaling model framework to study this issue with a comprehensive understanding about this kind of collaborative service provision.

Based on our analyses of the signaling model regarding the collaborative service, we can obtain several conclusions as follows. Firstly, the optimal effort level of the service provider is decreasing in the service provider's cost coefficient parameter. Namely, the optimal effort level of the cost-efficient service provider is greater than that of the cost-inefficient service provider. Meanwhile, the optimal profit of the cost-efficient service provider is greater than that of the cost-inefficient one. Secondly, in either case of the separating equilibrium, the optimal effort level strategy of the early consumers when facing the cost-efficient service provider is greater than that facing the cost-inefficient service provider. The optimal effort level strategies of the late consumers are equal when facing both cost-efficient and cost-inefficient service provider. Thirdly, as the prior probability of cost-efficient service provider increases, the expected revenue of the service provider at pooling equilibrium also increases. This indirectly causes the expected pooling prices to increase at the same time. The growing prices directly result in the additional profits in each case under the pooling equilibrium, without any additional effort level contributions of both parties to the service. Finally, when both the cost efficiency and quality certification information are unobservable to consumers before purchase, our analyses show that the cost-efficient service provider prefers the uniform pricing strategy in respect of the quality dimension to any differential pricing strategies. While the cost-inefficient service provider will adopt the differential pricing strategy in respect of the quality dimension.

4.2. Literature Review

There is a stream of literatures regarding social learning that is highly related to our study in this chapter. Banerjee (1992) first explores a sequential strategic model where each party of decision refers to the foregoing strategies to manage his individual strategy. Based on this study, Bikhchandani, Hirshleifer, and Welch (1992) further analyze the function of informational cascades. Bose, Orosel, Ottaviani, and Vesterlund (2008) investigate how a monopolist with dynamic pricing strategy can control the information amount that can be deduced by future consumers. These theoretical models are common in the assumption that the social learning is based on reviews which can reveal consumers' ex-post experiences, rather than their ex-ante private information. For example, Bergemann and Välimäki (1997) analyze the demand of a new product with valuation uncertainty in a duopolistic market, where each party learns the true value of the new product from the experiences of early consumers; Ifrach, Maglaras, Scarsini, and Zseleva (2019) study the monopoly pricing with the binary reviews of consumers and they use Bayesian updating to infer quality performance of the product; He, Chen, and Righter (2020) study consumers' learning behaviors in service operations systems and they mainly focus on the social learning on quantity during a new product's launch period. The above studies do not take into consideration the consumers' strategic behavior to decide whether to purchase the product in the early period or future period. However, a more relaxed assumption is that consumers can choose to wait to purchase in the future period. They can gain information from product reviews by revisiting the market in a later period. A recent study by Yu, Debo, and Kapuscinski (2016) analyze such dynamic pricing strategy in the presence of strategic consumers. Papanastasiou and Savva (2017) compare the results between preannounced and responsive pricing strategies. However, our model takes the consumers' type into consideration, which can determine their purchasing behavior in each time period. We disregard the strategic consumers' behaviors in order to gain analytical results and focus our research on the interactions between the service provider's pricing and effort level strategies over periods.

Besides, there are many studies related to the quality-signaling model which is another way to depict the social learning process. Jin and Kato (2006) show that a seller with private information of her product quality can signal it by pricing strategies. While others examine the advertising-signaling of product quality (Feng & Xie, 2012) and a combination of price and advertising signaling of product quality (Milgrom & Roberts, 1986). Moorthy and Srinivasan (1995) study how money-back guarantees can signal product quality in the direct selling market. Moorthy (2012) demonstrates that the separating equilibrium relies on off-equilibrium beliefs that are poorly motivated while the pooling equilibrium is more preferred. Moreover, B. Jiang, Ni, and Srinivasan (2014) investigate a signaling game regarding credence goods, where consumers are uncertain about their own treatment costs as well as the service provider's quality even after the service. Y.-H. Chen and Jiang (2021) develop a dynamic model to study seller's dynamic spot-pricing and price commitment with new experience products. The previous studies in respect of signaling model focus on exploring the seller's means of signaling her exogenously endowed quality performance. While B. Jiang and Yang (2019) models the seller's endogenous quality and pricing decisions for her new experience products with the setting of two-dimensional asymmetric information, where both her quality decision and cost-efficiency are the seller's private information. We also make the effort level strategy of the service provider as an endogenous decision and consider the signaling game when neither the cost-efficiency nor quality certification is observable as our benchmark model.

Furthermore, research on collaborative service also shed light on our study in this chapter. In the view of contracting research, researcher studies the moral hazard in multiagent setting resulting from collaboration (Holmstrom, 1982). Others prove that the optimal effort level contracts are linear when both agents are risk neutral (Bhattacharyya & Lafontaine, 1995). Double moral hazard framework is extended to analyze a broader class of effort cost functions (Corbett, DeCroix, & Ha, 2005), however, the nonlinear contract is best when the entities are risk-averse in this framework (S. K. Kim & Wang, 1998). In the field of supply chain management, many researchers study the contracting process where the total expected cost is dependent on both the consumer's and provider's internal resources (Iyer, Schwarz, & Zenios, 2005). Others investigate the optimal relational contract in a dynamic system with double moral hazard (Plambeck & Taylor, 2006). Xue and Field (2008) study the pricing schedule and effort allocation in collaborative services by taking information stickiness into consideration. Roels, Karmarkar, and Carr (2010) analyze the optimal contracts in the collaborative service environment and identify that the service process design can improve contract efficiency. White and Badinelli (2012) employ a model in respect of resource-integration decision for a service process. Building on their work, Roels (2014) further examines a CES production function by designing the optimal joint production service model and also discusses the implications for service process reengineering. Our study refers to these studies on collaborative service by attributing the service outcome to both the effort level decision of the service provider and the consumers, and we make further analyze regarding the influence of price signals on the optimal pricing and effort level strategies.

4.3. Model Setup

4.3.1. Model Description

We consider a monopolistic service provider offering a kind of collaborative service by means of both online and offline channels. Consumers in the market can collect information regarding the service seamlessly across channels. Without loss of generality, we normalize the total number of consumers making purchase decisions in the first period to 1. The relative total number of consumers in the second period is normalized to m. Each consumer can purchase at most one unit of the collaborative service offered by the service provider. In each time period, we take use of a representative consumer to denote the mass of consumers. To be specific, an early consumer is in the first period and a late consumer is in the second period. Namely, the information transmission between periods in this setting represents an interpersonal interaction. The service provider and consumers in the market are taking part in a kind of collaborative service. That is, the service outcome relies on the effort level strategies of both the service provider and the consumer. We express the effort level decision of the service provider as x and that of the consumer in each period as y_t, where $t = 1,2$ is denoted as the effort level decision in each representative time period. For analytical simplicity, we normalize the boundary of each effort level decision over the range between zero and one, i.e., $x, y_t \in [0,1]$.

At the beginning of the first period, early consumers observe the service provider's first-period pricing strategy. They make their purchase decisions based on their prior beliefs regarding the effort level decision of the service provider. The early consumers who have determined to pay for the service will observe the true effort level of the service provider, and they then disclose it to followers via customer review platforms including either online forums or offline word of mouth. At the beginning of the second period, the reviews are delivered to both parties. Therefore, the follower consumers will make their purchase decisions based on both the service provider's true effort level revealed through early consumers' reviews and the service provider's second period pricing strategy. We use "she" to denote the service provider and "he" to denote the representative consumer for our interpretation convenience. Note that at the beginning of each period $t = 1,2$, the consumer has the right to make his own effort level decision y_t to maximize his expected prepurchase utility in the corresponding period. Moreover, we assume the service provider is differentiated in her marginal effort level cost $c_i x^2$, where x is the true effort level of the service provider and c_i is a constant that we use to denote the service provider's cost efficiency. That is to say, the service provider's heterogeneity in respect of her cost efficiency can be divided into two types, i.e., $c_i = c_e$ with probability $\gamma > 0$, and $c_i = c_{in}$ with probability $1 - \gamma$. Without loss of generality, we assume $c_e < c_{in}$ where the service provider with marginal effort level cost $c_e x^2$ is more cost efficient than that with marginal effort level cost $c_{in} x^2$. Namely, the former type of service provider owns lower marginal cost given the same effort level compared with the latter one. Another dimension of the service provider's heterogeneity can be expressed as her quality or certification. We denote the utility gain of the service with different quality as the reservation value v_j in the consumer's expected utility function. We explain it as the augmented value that the consumer can earn from the service brought about by the service provider. We assume the two types in respect of the service provider's quality also follows the two-point distribution, i.e., $v_j = v_h$ with probability β, and $v_j = v_l$ with probability $1 - \beta$, where $v_h > v_l$ without loss of generality.

The consumer's net utility in period $t = 1,2$ after observing the pricing strategy and effort level strategy of the service provider can then be derived as $U_t = v_j + \mu_\theta (a x^r + (1 - a) y_t^r)^{\frac{1}{r}} - w y_t^2 - p_t$, where μ_θ stands for the consumer's willingness to pay for the service which is highly related to the consumer's own type and p_t is the price charged by the service provider in period t. The term $(a x^r + (1 - a) y_t^r)^{\frac{1}{r}}$ demonstrated the combination contribution of the service provider and the consumer's effort level on the whole service provision (Roles 2014). It is a function of the weighted arithmetic expectation of effort contributions from both parties, and the weight is proportional to the contribution value of each party respectively (Hardy et al. 1952). In the aforementioned generalized model expression, the parameter a, $0 < a < 1$, is used to denote the corresponding weight of the service provider's effort level in the combination value brought about from the collaborative service. Especially when a is approaching 0, the service combination value is a function of the consumer's effort level. While when a is approaching 1, it is a function of only the service provider's effort contribution. Namely, the allocation of effort contribution is depicted as the relative value of a and we thus call this parameter as "work allocation". Meanwhile, there is a parameter r in the combination value of service which we can refer to as the substitution parameter of both efforts. We further illustrate several scenarios of the term $(ax^r + (1-a)y_t^r)^{\frac{1}{r}}$ under different value of r. Firstly, when r is approaching negative infinity, the combination effort contribution is $\min\{x, y\}$ which refer to both effort levels as perfect complements; secondly, when r is approaching zero, it is simplified as a Cobb-Douglas function $x^a y^{1-a}$; thirdly, when r equals to one, then the term has its form in $ax + (1-a)y$ which marks the effort levels as perfect substitutes; finally, when r is approaching positive infinity, the combination effort contribution is $\max\{x, y\}$ which makes both effort levels redundant. The distinct forms of combination effort contribution show that r is an increasing function of the substitution elasticity between effort levels of the service provider and the consumers. To be more specific, effort levels are complements when r is relatively low and they are substitutes when r is relatively high. Besides the aforementioned combination value of service which can benefit the consumers' utility, there is also utility cost of consumers from exerting effort contribution which is referred to as wy_t^2 in period $t = 1,2$, where y_t is the effort level determined by consumers in each period and w is the consumer's effort cost coefficient. We take use of this convex cost function to capture the increasing marginal characteristic of consumers' effort level cost as the increase of their effort contribution. A representative consumer will choose to make purchase of the service only if his expected utility is nonnegative. We can also explain it as the value of utility function should be larger than or equal to zero, i.e., his outside option of no purchase decision. We assume that the consumer only purchases in the corresponding period and will exit the market if no purchase occurs. This is reasonable in practice for many service marketplaces. For instance, preschool curriculum is adapted to children's learning in some specific ages or online vocational education is for adults to manage their career, which will lose value if they are not taken in the particular time period. Furthermore, consumers are heterogenous in μ_θ, which represents the consumer's willingness to pay for the service and we use this parameter to differentiate the consumers' type in each period. Concretely, we assume a portion $\rho\epsilon(0,1)$ of consumers are high-type and the corresponding type-parameter satisfies $\mu_\theta = 1$. The remaining portion of consumers (i.e., $1-\rho$) are low-type and the corresponding type-parameter satisfies $\mu_\theta = \mu \in [0,1]$. That is to say, the high-type consumers are more likely to pay for the service than the low-type ones given the same service provider's effort level and pricing strategies. Moreover, we assume the consumers' type is in discrete form in order to derive the analytical results for our model setting.

4.3.2. Sequence of the Game

In this section, we emphasize the sequence of the events in our model setting. There exist two dimensions of asymmetric information regarding the service provider's own type. Firstly, the service provider owns asymmetric information in respect of her cost efficiency. In our benchmark setting, we assume consumers can directly observe this information before purchase decision. We further make extensions to the benchmark setting by considering the scenario where this information is unobservable. And the prior probability of this information regarding cost efficiency is common knowledge, i.e., it follows a two-point distribution as $\Pr\{c_i = c_e\} = \gamma$ and $\Pr\{c_i = c_{in}\} = 1-\gamma$. Secondly, the service provider also influences the consumer's reservation value in his utility gain by the asymmetric information in respect of her quality or certification. We also assume this information can be observed by consumers in our benchmark setting and the prior probability follows two-point distribution as $\Pr\{v_j = v_h\} = \beta$ and $\Pr\{v_j = v_l\} = 1-\beta$.

In order to depict the decision sequence of the game more clearly, we list them in the following several stages:

Stage 1. Nature determines the service provider's cost type i and quality type j, then the service provider obtains the private information regarding her own type.

Stage 2. In the beginning of first period, the service provider makes her effort level decision x through periods. Then the service provider makes her first period pricing strategy p_1, where the subscript indicates the time period of the corresponding strategic span.

Stage 3. The first period consumers determine his own first period effort level y_1, and will make purchase decisions based on his expected utility after observing the first period pricing strategy. After making purchase decision of the service, the first period consumers obtain the information regarding the effort level of the service provider and disclose it via online review forums or offline word-of-mouth.

Stage 4. In the beginning of second period, the service provider sets her second period pricing strategy p_2.

Stage 5. The second period consumers determine his own second period effort level y_2, and make purchase decisions based on his expected utility after observing the second period pricing strategy.

We assume that the service provider makes the effort level decision before the pricing decision because it is generally supposed that the pricing decision is more flexible and easier to change than the effort level decision. Therefore, the pricing strategy possesses a shorter time horizon than the effort level strategy.

4.4. Model Analysis

Before we begin our model analyses in scenarios with different informational structure of the service provider, we first clarify the function of first period consumers' review in our model formation. If there is no review disclosed, the information in respect of service provider's effort level will not be transformed across periods to follower consumers, which finally results in the market for the service provision breaking down. That is to say, the service provider will have the intention to provide minimal effort level, which is quite intuitive. With no consumer reviews regarding the effort level revealed in the first period, consumers then will have the same information set intertemporally and always hold the same belief regarding the service provider's effort level. This substantially results in both periods being independent and identical to all parties.

Provided that consumers in the market have the belief that the service provider's effort level is positive after observing the service price in the first period, in the subsequent time period, the service provider can always cut down her devotion of effort level to promote her revenue, but still set the same pricing strategy. As a consequence, under any sensible beliefs of consumers, no positive effort level strategy can be sustained. Thus, in the equilibrium setting, any types of service providers will always set the zero-effort level strategy, which is quite a fraud market that we will not further study on. We mainly focus our research on the market where the effort level of service provider can be perfectly revealed through early consumers' reviews and meanwhile the first period price acts as a signal to inform the service provider's own type to follower consumers. Namely, in the market with review disclosing the service provider's effort level information intertemporally, the service provider has the incentive to offer positive effort level to the whole service provision. To be more specific, the service provider will trade off between the profit of offering service with higher effort contribution to the follower consumers and the benefit derived from offering service with lower effort contribution to cheat the early consumers. We assume early consumers can make rational inference of the service provider's effort level from her first period pricing plan (B. Jiang & Yang, 2019).

As the service provider are differentiated in two dimensional intrinsic properties, i.e., cost and quality, we assume the information structure of the market can be divided into several circumstances: both private information are observed by the market, one of them is observed by consumers and neither of them is observed by consumers.

4.4.1. Benchmark: Service Market with Known Cost Efficiency & Quality Certification

In this subsection, we assume that the service provider's cost efficiency c_i and her quality certification v_j are common knowledge and both can be observed by consumers prior purchase. In equilibrium, the service provider will make the pricing strategy and effort level strategy of herself, while the consumer has the right to decide how much the effort contribution of himself to devote in the beginning of each period.

The early consumers' purchase decisions are formally dependent on their rational inferences of the service provider's effort level decision from her first period pricing strategy. Besides, if the consumer's demand for the service which is a function of the service provider's pricing strategy is given, the service provider will not have an incentive to deviate from her effort level strategy and pricing strategy in equilibrium. Nevertheless, there may exist several perfect Bayesian equilibria, and we only demonstrate the most profitable equilibrium result for the service provider. It is equivalent to discovering the most profitable effort level and pricing strategy profile that is reliable to both parties in the game.

The service provider's objective function can be demonstrated as:

$$\pi_{ij}(p_{1,ij}, p_{2,ij}, x_i) = D_1(p_{1,ij} - c_i x_i^2) + D_2(p_{2,ij} - c_i x_i^2). \tag{29}$$

The service provider first determines her optimal effort level strategy over two periods, then she determines the demand of each period (i.e., D_1 and D_2) by setting her optimal pricing strategy. Meanwhile, the consumer can determine his own effort level strategy in each period by optimizing his expected utility. We can derive the equilibrium results in each case as below.

Lemma 4.1. *When both the service provider's cost efficiency and quality certification are common knowledge, her optimal effort level strategy are as follows:*

Range of ρ	$D_1 = \rho$					$D_1 = 1$				
	D_2	π_{ij}^*	x_i^*	$y_{1,i}^*$	$y_{2,i}^*$	D_2	π_{ij}^*	x_i^*	$y_{1,i}^*$	$y_{2,i}^*$
$(0, \rho_1)$	m	Case (2)	$\frac{am\mu}{2(m+\rho)c_i}$	$\frac{1-a}{2w}$	$\frac{(1-a)\mu}{2w}$	m	Case (4)	$\frac{am\mu}{2(1+m)c_i}$	$\frac{(1-a)\mu}{2w}$	$\frac{(1-a)\mu}{2w}$
(ρ_1, ρ_2)	ρm	Case (1)	$\frac{am}{2(1+m)c_i}$	$\frac{1-a}{2w}$	$\frac{1-a}{2w}$	m	Case (4)	$\frac{am\mu}{2(1+m)c_i}$	$\frac{(1-a)\mu}{2w}$	$\frac{(1-a)\mu}{2w}$
$(\rho_2, 1)$	ρm	Case (1)	$\frac{am}{2(1+m)c_i}$	$\frac{1-a}{2w}$	$\frac{1-a}{2w}$	ρm	Case (3)	$\frac{am\rho}{2c_i + 2m\rho c_i}$	$\frac{(1-a)\mu}{2w}$	$\frac{1-a}{2w}$

Table 6. Equilibrium Effort Levels and Profits when Cost and Quality are Known

The optimal profits in Case (1) to Case (4) are listed as bellow where the superscript of the profit function represents the number of the case, details can be found in Appendix A:

$$\pi_{ij}^{(1)} = \frac{\rho(a^2m(2+m)w+(1+m)^2c_i((-1+a)^2+4wv_j))}{4(1+m)wc_i};$$

$$\pi_{ij}^{(2)} = \frac{a^2mw\mu(m\mu+2\rho)+(m+\rho)c_i((-1+a)^2(m\mu^2+\rho)+4w(m+\rho)v_j)}{4w(m+\rho)c_i};$$

$$\pi_{ij}^{(3)} = \frac{a^2mw\rho(2\mu+m\rho)+(1+m\rho)c_i((-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_j)}{4w(1+m\rho)c_i};$$

$$\pi_{ij}^{(4)} = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}. \tag{30}$$

Based on the equilibrium results we derive under different cases, we discover several universalities regarding the optimal strategies of the service provider no matter what the demand in each period should be. Conclusions are summarized in the following propositions.

Proposition 4.1. *The optimal effort level of the service provider (x_i^*) is increasing in the work allocation parameter a; while the optimal effort level of the consumers in both periods ($y_{1,i}^*$*

and $y_{2,i}^{*}$*) is decreasing in* a*.*

This conclusion shows that no matter what the pricing strategy the service provider chooses to set and thereby results in any kind of demands in both periods, the optimal effort level of the service provider is always increasing as the relative weight of her effort level to the whole service provision is increasing. While that of the consumers is decreasing in a, but also increasing in $1-a$ which is also the relative weight of the consumers' effort level to the whole service provision. Namely, both parties that participate in the service provision process will raise his or her effort level when the entity's relative weight of effort level to the whole service provision is augmenting. It is intuitive since the relative weight represents the participant's contribution to the service which will finally result in the change of consumers' utility and affect the demand in both periods. The corresponding party that accounts more for the service provision will raise his or her effort level to guarantee the demand of the service market. While when the work allocation parameter is more approaching the middle value, both parties' effort levels are not reaching their maximum value but contribute to the whole service with reservation. We can refer to this phenomenon as the "free riding" behavior of both parties in the service provision process.

Proposition 4.2. *The optimal effort level of the service provider (*x_i^**) is decreasing in the service provider's cost coefficient parameter* c_i*, namely, the optimal effort level of the cost-efficient service provider (*x_e^**) is greater than that of the cost-inefficient service provider (*x_{in}^**). Meanwhile, the optimal profit of the cost-efficient service provider* π_{ej}^* *is greater than that of the cost-inefficient one* π_{inj}^**.*

The results always hold in the case when there is no private information regarding the service provider in the market. Thus, the cost-efficient service provider has incentive to provide higher effort level than the cost-inefficient one mainly because the incremental cost per unit in the effort level provision is lower in the cost-efficient case. This result gives us inspirations to explain the phenomenon in practice why many service providers strive to be cost efficient such as opening online courses with the existence of offline stores or opening the take-out service for some restaurants. All these actions help to reduce the unit cost of effort level thus will in the end promote the service providers to raise their effort level, which finally results in the overall improvement of profits.

Proposition 4.3. *The optimal pricing strategies in both periods under the four cases are all first decreasing in the work allocation parameter* α *and then increasing in it.*

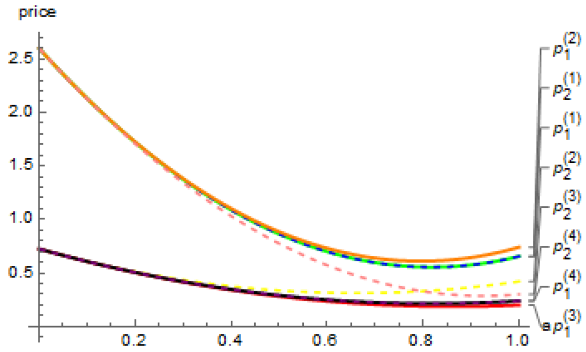


Figure 14. Then Impact of α on Equilibrium Prices when Cost and Quality are Known

The figure above depicts this conclusion where the superscript of the prices represents the number of cases (1-4) while the subscript of the prices represents the time period (1-2). This result is mainly due to the combination effect of the work allocation parameter on the effort level of both the service provider and consumers. To be specific, when either party's effort level is increasing to its maximum value (i.e., when α is approaching 0 or 1), the price will rise simultaneously. However, we can see that the minimum values of prices are obtained when α deviates from the middle and approaches 1. This is because the pricing strategy, which is obtained from the concept of market clear, containing the utility gain of both party's effort level and the disutility of the consumers' effort level cost. This will to some extent amplify the influence of the consumers' effort level strategy on the pricing strategy. Therefore, the optimal pricing strategy is more likely to follow the path of the optimal consumer's effort level with a relatively large interval of reduction section as the increase of α. While the increasing trend is positioned in a relatively small interval, only when the work allocation parameter approaches 1 (i.e., the influence of service provider's effort contribution is magnified). Thus, in a market with work allocation parameter α approaching zero, which means the service provider's effort level is zero while the consumers' effort level is maximized, the service provider on the contrary will raise her prices in both periods. Namely, the consumers' participation is magnified under this circumstance. After the consumer makes the high-level effort contribution to the service provision, he then has less incentive to exit the service market. This will finally result in the service provider's bold action to raise price and meanwhile with no loss of consumers.

4.4.2. Service Market with Unknown Cost Efficiency & Known Quality Certification

We next consider the case when the cost efficiency is the private information of the service provider while the quality is still common knowledge and is observed by consumers prior purchase. The service provider will choose either the separating equilibrium to signal her type via first period prices or the pooling equilibrium without any type information in her pricing strategy. And our results show that whether the service provider separates or pools herself depends on both the fraction of high type consumers in the market and the prior probability of cost-efficient service provider. Our results are demonstrated in the following propositions.

Proposition 4.4. *The separating equilibrium in respect of the cost efficiency dimension exists in two parallel scenarios:*

(A) When the fraction of high type consumers $\rho < min\{\rho_1, \rho_3\}$ *and the prior probability of cost- efficient service provider* $\gamma < \frac{mc_e(-c_i+c_{in})}{2c_i(c_e-c_{in})}$*, the separating equilibrium exists;*

(B) When the fraction of high type consumers $\rho_2 < \rho < \rho_4$ *and the prior probability of cost-efficient service provider* $\gamma < \frac{m\rho c_e(-c_i+c_{in})}{2\mu c_i(c_e-c_{in})}$*, the separating equilibrium exists.*

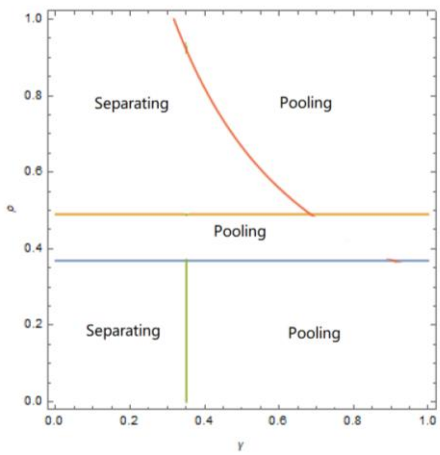


Figure 15. Separating and Pooling Equilibrium in $\rho - r$ Plane

We can conclude from our detailed analyses under the circumstance where the service provider holds her private information regarding her cost efficiency in Appendix A of this chapter. Our results show that only in a certain fraction of the high type consumers (ρ) in the market, the service provider would aim at both types of consumers in the first period. Then she is able to signal her true type through aiming at lower fraction of consumers by raising her early-period price.

There are two intervals that satisfy the above condition. When ρ is in a low interval ($\rho < min\ \{\rho_1, \rho_3\}$), the service provider aims at both high and low types of consumers in both periods, while when ρ is in a high interval ($\rho_2 < \rho < \rho_4$), the service provider aims at both types of consumers in the first period but only aims at high types of consumers in the second period. No matter what the demand of second period consumers is, as long as the first period demand is the whole market share, the cost-efficient service provider will have incentive and ability to raise her first period price by aiming at fewer consumers in the first period. However, there also exists the pooling equilibrium in both aforementioned intervals of ρ. And results show in the same interval of ρ, the separating equilibrium only exists when the prior probability of cost-efficient service provider is in a low range. That is to say, when the market has relatively low value in its expected prior probability of cost efficiency ($\gamma c_e + (1-\gamma)c_{in}$), the cost-efficient service provider is more likely to signal her type by separating herself from the inefficient one. On the contrary, when the market has high value in the expected cost efficiency, the cost-efficient service provider will pool herself with the inefficient one. This phenomenon is mainly due to the reason that when the prior probability of cost-efficient service provider is low, which means there are small number of cost-efficient ones in the market as this type of service provider is in an inferior position. They are more eager and sincere in their selling periods to prove their uniqueness by signal their superiority in cost. Thus, they can set a higher price by aiming at only high-end consumers to signal their uniqueness. However, once there are more cost-efficient service providers in the market, the original ones' cost efficiency is not competitive in the market anymore, which will prompt cost-efficient service providers to lower their price until the same as the cost-inefficient ones as a result of pooling with each other. We then list the corresponding results of the separating equilibrium in the following proposition.

Proposition 4.5. *We first assume the range of the cost efficiency difference as* $c_e = \xi c_{in}$, *when* *case (A) of separating equilibrium exists, if* $\frac{2a^2m(1+m)w\mu^2\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi < \frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$ *, the optimal effort level and corresponding optimal profit for cost efficient service provider are* $x_e^* = \frac{am\mu}{2(m+\rho)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{(1-a)\mu}{2w}$, $\pi_e^{(sep2)} = \frac{a^2m^2w\mu^2+\frac{c_e(a^2mw\mu^2(m+(2+m)\rho)+(1+m)^2(m+\rho)c_{in}((-1+a)^2\mu^2+4wv_j))}{(1+m)c_{in}}}{4w(m+\rho)c_e}$; *those for cost inefficient service provider are* $x_{in}^* = \frac{am\mu}{2(1+m)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{(1-a)\mu}{2w}$, $\pi_{in}^{(4)} = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$;

if $\frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)} < \xi < -\frac{a^2mw\mu(-m\mu-2\rho+m(-2+\mu)\rho)}{(m+\rho)(2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$ *, the optimal effort level and corresponding optimal profit for cost efficient service provider are* $x_e^* = \frac{am\mu}{2(m+\rho)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{(1-a)\mu}{2w}$, $\pi_e^{(2)} = \frac{a^2mw\mu(m\mu+2\rho)+(m+\rho)c_e((-1+a)^2(m\mu^2+\rho)+4w(m+\rho)v_j)}{4w(m+\rho)c_e}$*; those for cost inefficient service provider remain unchanged* $x_{in}^* = \frac{am\mu}{2(1+m)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{(1-a)\mu}{2w}$, $\pi_{in}^{(4)} = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$*. Similar results can be obtained in case (B) and we put them in the Appendix A of this chapter.*

As this proposition shows, in both case (A) and case (B) of the separating equilibrium which we have defined in proposition 4.4, there are two kinds of equilibrium results mainly due to the range of the cost efficiency difference. It influences the concrete value of the service provider's first period prices under each circumstance. As when ξ is in a lower range which means the extent of cost efficiency is more distinct, the optimal price of the service provider can not be obtained in its maximized value by the cost-efficient service provider, but only a corner solution of the early price can be derived. This is mainly because when ξ is low, the valuation difference between c_e and c_{in} is large. It makes the efficient service provider more easily to separate herself from others, thus her first period pricing strategy may not be optimal under this circumstance. However, when the valuation difference between c_e and c_{in} is small, only if the efficient service provider optimally sets her first period price will she be able to separate herself from others. This finally results in when ξ is high, the optimal pricing strategy of both periods can be obtained. We then discuss the optimal effort level strategies in the separating equilibrium by making comparison between the effort levels of both types service providers.

Corollary 4.1. *In either case of the separating equilibrium, the optimal effort level strategy of the cost-efficient service provider is greater than that of cost-inefficient service provider(i.e., $x_e^* > x_{in}^*$), while the optimal effort level strategy of the early consumers faced with the cost-efficient service provider is greater than that confronted with the cost-inefficient service provider (i.e., $y_{1,e}^* > y_{1,in}^*$) and the optimal effort level strategy of the late consumers are equal when facing both cost-efficient and cost-inefficient service provider(i.e., $y_{2,e}^* = y_{2,in}^*$).*

The above corollary is drawn by taking the separating equilibrium in both case (A) and case (B) into consideration simultaneously. The equilibrium results show that the optimal effort level strategies of either the service provider or the consumers in cost-efficient case is no less than that in the cost-inefficient case. It means that in the first period of the game, the cost-efficient service provider will enlarge her effort level to reach a certain objective targeting consumers and meanwhile the early consumers should also raise his effort level in order to coordinate with the service provider's separating action in the first period. This finally results in the phenomenon that both service provider and the early consumers are providing higher effort level in the cost-efficient case, while that of late consumers remain unchanged in the separating equilibrium.

In practice, we consider the example in cloud service. IBM is awarded as the most cost-efficient cloud service provider in 2021, as it strives to improve its digital transformation with the optimal cost performance. This is exactly an example of the above separating equilibrium in practice. According to the statistics announced by Flexera, in the recent price evaluation in respect of 67 cloud calculation scenarios, IBM defeated the others including Microsoft, Google and AWS to win the most cost-efficient cloud service provider. To be specific, IBM separates itself from the others by its pricing strategy and effort level strategy. According to IBM's operation principle, enterprises should manage their costs to achieve higher revenues. Therefore, IBM Cloud devotes great effort contribution to its service provision process. For example, it is committed to help clients optimize their cloud computing in order to make itself be cost-efficient, easy-to-use and innovative.

Corollary 4.2. *The optimal profits in both Case (A) and Case (B) are first decreasing in the work allocation parameter* a *and then increasing in it.*

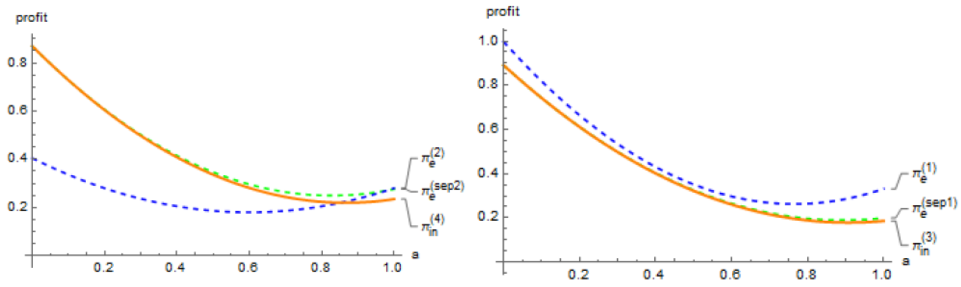


Figure 16. The Impact of a on Optimal Profits in Separating Equilibrium

The above two figures illustrate the variation tendency of the profits in both separating cases with the work allocation parameter, where $\pi_e^{(sep2)}$, $\pi_e^{(2)}$ and $\pi_{in}^{(4)}$ are the optimal profits in Case (A) while $\pi_e^{(sep1)}$, $\pi_e^{(1)}$ and $\pi_{in}^{(3)}$ are the optimal profits in Case (B).

We can still obtain the similar conclusions as that of the first-rank pricing strategy in Proposition 4.3. As in the expressions of the optimal profits, not only the optimal prices but also the disutility of service provider's cost will influence the value of profits. Also note that the minimum values of profits are obtained when a deviates from the middle and approaches 1. It is similar to the optimal pricing strategy path. Therefore, the optimal profits are more likely to follow the path of the optimal consumer's effort level with relatively large interval of reduction section as the increase of a. While the increasing trend is positioned in a relatively small interval only when the work allocation parameter approaches 1 (i.e., the influence of service provider's optimal effort level strategy is revealed or disclosed). Thus, in a market with work allocation parameter a approaching zero, which means the service provider's effort level is zero while the consumers' effort level is maximized, the service provider on the contrary will get her maximized profit values. This is mainly due to that the service provider's maximized pricing strategy is obtained when a approaching zero and meanwhile her effort level disutility vanishes as her effort level is zero. This finally results in this profit changing pattern. We next make further analyses of the consumer surplus under each circumstance.

Lemma 4.2. *We first assume the range of the cost efficiency difference as* $c_e = \xi c_{in}$, *when both the quality certification and cost efficiency are observable in the service market, the consumer surplus is* $\rho(\frac{1}{2}(-1+\mu)\mu(-\frac{(-1+a)^2}{w}+\frac{a^2m(\gamma(-1+\xi)-\xi)}{(1+m)\xi c_{in}})+v_j)$ *if* $\rho < min\{\rho_1,\rho_3\}$*;*

the consumer surplus is $\frac{1}{2w\xi(1+m\rho)c_{in}}\rho(a^2m(1+m)w(-1+\mu)(\gamma(-1+\xi)-\xi)\rho+\xi(1+m\rho)c_{in}(-(-1+a)^2(-1+\mu)(m+\mu)+2(1+m)wv_j))$ *if* $\rho_2 < \rho < \rho_4$.

We first consider the consumer surplus when the service market has the characteristic of known quality certification and cost efficiency. The consumer surplus in each case can be derived by calculating the difference between the price that consumers are willing to pay and the price they actually pay for the service. What we are mainly interested in is the difference of consumer surplus when the separating equilibrium exists and that when the information is observable to consumers. Results of the consumer surplus in each corresponding case can be seen in Lemma 4.2 and Lemma 4.3.

Lemma 4.3. *When case (A) of separating equilibrium exists (i.e.,* $\rho < min\{\rho_1,\rho_3\}$*), if* $\frac{2a^2m(1+m)w\mu^2\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi < \frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$*, the consumer surplus is* $\frac{a^2mw\mu(G+H)}{4(1+m)w\xi(m+\rho)c_{in}}$ *where* $G = -m\gamma\mu\xi+m(2\gamma+(2+\gamma(-2+\mu)-2\mu)\xi)\rho+2\rho(\gamma-\gamma\mu\xi+(-1+\gamma)(-1+\mu)\xi\rho))$ *and* $H = (1+m)\xi(m+\rho)c_{in}((-1+a)^2(-2(-1+\mu)\mu\rho+\gamma(\rho+\mu(-\mu+2(-1+\mu)\rho)))+4w(-\gamma+\rho)v_j$*;*

if $\frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)} < \xi < -\frac{a^2mw\mu(-m\mu-2\rho+m(-2+\mu)\rho)}{(m+\rho)(2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$*, the consumer surplus is* $(1-\gamma)\rho(-\frac{(-1+a)^2(-1+\mu)\mu}{2w}-\frac{a^2m(-1+\mu)\mu}{2(1+m)c_{in}}+v_j)$.

When case (B) of separating equilibrium exists (i.e., $\rho_2 < \rho < \rho_4$*), if* $\frac{2a^2mw\mu\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi < \frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$*, the consumer surplus is* $\frac{I+J}{4(1+m)\xi(1+m\rho)c_{in}}$ *where* $I = \rho(a^2m\gamma(2-2\mu\xi+m(2\rho-\xi(-1+2\mu+\rho)))+$

$\frac{1}{w\rho}(2a^2mw(-1+\mu)\rho(-m\gamma+(-m^2\gamma+(1+m)^2(-1+\gamma)\xi)\rho)$ *and* $J=(1+m)\xi(1+m\rho)c_{in}((-1+a)^2(-2(-1+\mu)(m+\mu)\rho+\gamma(\rho+\mu(-\mu+2(-1+\mu)\rho)))+4w(-\gamma+\rho+m\rho)v_j)))$ *;*

if $\frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}<\xi<\frac{a^2mw\rho(2+m+m\rho)}{(1+m)(2a^2mw\mu\rho+(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$, *the consumer surplus is* $\rho(\frac{1}{2}(-1+\mu)(-\frac{(-1+a)^2(m+\mu-\gamma\mu)}{w}+\frac{a^2m(-m\gamma+(-m^2\gamma+(1+m)^2(-1+\gamma)\xi)\rho)}{(1+m)\xi(1+m\rho)c_{in}})+(1+m-\gamma)v_j)$.

The aforementioned two lemmas list the consumer surplus under each circumstance. If we make comparisons between them in each case, we can obtain the conclusion that the consumer surplus is high in the case when cost information is unobservable to consumers if and only if the cost-efficient service provider and the cost-inefficient service provider are distinct in their cost efficiency. Otherwise, the consumer surplus is high when all information regarding quality and cost is observable to consumers if and only if the cost efficiency is non-significant.

Proposition 4.6. *When* $\rho<min\{\rho_1,\rho_3\}$, *the consumer surplus is higher when the cost efficiency is unobservable to consumers than when it is observable if and only if*

$$\xi<min\{\frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)},\frac{2a^2mw\mu\rho(1+m\mu+(-1+\mu)\rho)}{a^2mw\mu^2(m+(2+m)\rho)-(1+m)(m+\rho)c_{in}((-1+a)^2(\rho+\mu(-\mu+2(-1+\mu)\rho))-4wv_j)}\};$$

When $\rho_2<\rho<\rho_4$, *the consumer surplus is higher when the cost efficiency is unobservable to consumer than when it is observable if and only if*

$$\xi<min\{\frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)},\frac{2a^2mw\rho(1+m-m\mu+(-1+\mu+m(-1+2\mu))\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))-(1+m)(1+m\rho)c_{in}((-1+a)^2(\rho+\mu(-\mu+2(-1+\mu)\rho))-4wv_j}\}.$$

The proposition shows us that under each case when the separating equilibrium exists, the consumer surplus is higher when the cost information is unobservable to consumers only if the cost efficiency of the service provider is quite significant. This conclusion characterizes the condition when the consumers can obtain higher consumer surplus. When the cost-efficient service provider has great advantage over her cost efficiency, she can make the consumers earn quite high surplus over the service consumption process. Otherwise, when the cost efficiency of the two types service provider is quite similar, the consumers are better under the case when all information is observable. The resulting phenomenon is that the cost-efficient service provider can enlarge the social welfare to separate herself from the cost-inefficient one only if the cost of her service provision is quite low. As she dominates the cost-inefficient service provider with quite low unit cost of effort level contribution, she has the incentive to separate herself from others by targeting at fewer consumers with higher prices.

After our clarifications of the separating equilibrium, we next consider the most-efficient pooling equilibrium. Results can be seen in the following proposition.

Proposition 4.7. *The pooling equilibrium in respect of the cost efficiency dimension exists in tree scenarios:*

(C) When the fraction of high type consumers $\rho < min\{\rho_1, \rho_3\}$ *and the prior probability of cost- efficient service provider* $\gamma > \frac{mc_e(-c_i+c_{in})}{2c_i(c_e-c_{in})}$*, the pooling equilibrium exists. The optimal pooling profits are* $\pi_p^{(2)} = \frac{a^2m^2w\mu^2+c_i((-1+a)^2(m+\rho)(m\mu^2+\rho)+2w(\frac{a^2m\gamma\mu\rho}{c_e}-\frac{a^2m(-1+\gamma)\mu\rho}{c_{in}}+2(m+\rho)^2v_j))}{4w(m+\rho)c_i}$ *and* $\pi_p^{(4)} = \frac{a^2m^2w\mu^2+c_i((-1+a)^2(1+m)^2\mu^2+2w(\frac{a^2m\gamma\mu^2}{c_e}-\frac{a^2m(-1+\gamma)\mu^2}{c_{in}}+2(1+m)^2v_j))}{4(1+m)wc_i}$.

(D) When the fraction of high type consumers $\rho_2 < \rho < \rho_4$ *and the prior probability of cost-efficient service provider* $\gamma > \frac{m\rho c_e(-c_i+c_{in})}{2\mu c_i(c_e-c_{in})}$*, the pooling equilibrium exists. The optimal pooling profits are* $\pi_p^{(1)} = \frac{\rho(a^2m^2w+\frac{c_i(2a^2mw\gamma c_{in}+c_e(-2a^2mw(-1+\gamma)+(1+m)^2c_{in}((-1+a)^2+4wv_j)))}{c_ec_{in}})}{4(1+m)wc_i}$

and $\pi_p^{(3)} = \frac{a^2m^2w\rho^2+\frac{c_i(2a^2mw\gamma\mu\rho c_{in}+c_e(-2a^2mw(-1+\gamma)\mu\rho+(1+m\rho)c_{in}((-1+a)^2(\mu^2+m\rho)+4w(1+m\rho)v_j)))}{c_ec_{in}}}{4w(1+m\rho)c_i}$.

(E) When the fraction of high type consumers $\rho_1 < \rho < \rho_2$ *and the prior probability of cost-efficient service provider* $\gamma \in [0,1]$*, the pooling equilibrium exists. The optimal pooling profits are* $\pi_p^{(1)} = \frac{\rho(a^2m^2w+\frac{c_i(2a^2mw\gamma c_{in}+c_e(-2a^2mw(-1+\gamma)+(1+m)^2c_{in}((-1+a)^2+4wv_j)))}{c_ec_{in}})}{4(1+m)wc_i}$ *and*

$\pi_p^{(4)} = \frac{a^2m^2w\mu^2+c_i((-1+a)^2(1+m)^2\mu^2+2w(\frac{a^2m\gamma\mu^2}{c_e}-\frac{a^2m(-1+\gamma)\mu^2}{c_{in}}+2(1+m)^2v_j))}{4(1+m)wc_i}$.

All pooling equilibrium profits are increasing in γ*, to be more specific,* $\frac{\partial\pi_p^{(2)}}{\partial\gamma} > \frac{\partial\pi_p^{(4)}}{\partial\gamma} > 0$ *in case (C);* $\frac{\partial\pi_p^{(1)}}{\partial\gamma} > \frac{\partial\pi_p^{(3)}}{\partial\gamma} > 0$ *in case (D).*

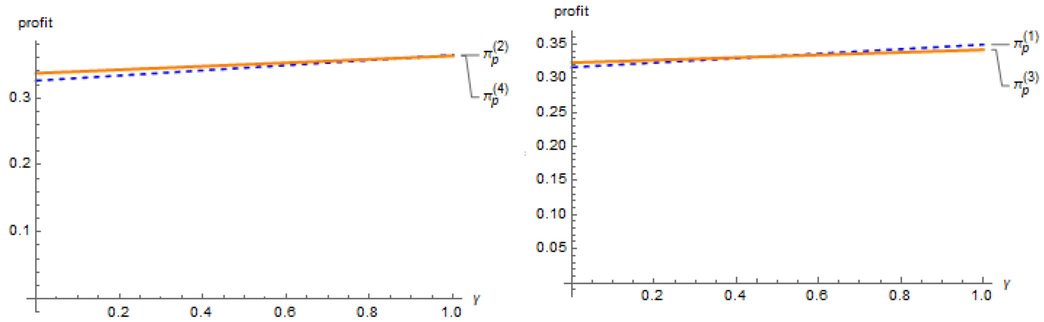


Figure 17. The Impact of γ on Optimal Profits in Pooling Equilibrium

This proposition characterizes the remaining sections depicted in Figure 10 where the pooling equilibrium exists. The results show that in the interval of ρ where the separating equilibrium exists too (Case (C) and Case (D)), the pooling equilibrium only exists when the prior probability of cost-efficient service provider γ is in a high range. That is to say, when the market has relatively high value in its expected prior probability of cost efficiency ($\gamma c_e + (1 - \gamma)c_{in}$), the cost-efficient service provider is more likely to pool herself with the inefficient one. This phenomenon is mainly due to the reason that if there are more cost-efficient service providers in the market, the original ones' cost efficiency is not competitive in the market anymore, thus she has no incentive to separate herself from others by improving effort level and raising price in first period. This will prompt cost-efficient service providers to lower their price until the same as the cost-inefficient ones as a result of pooling with each other.

What we can further derive from the pooling equilibrium is that all the resulting pooling profits are increasing in the prior probability of cost-efficient service provider γ but with the only difference in their rate of change. That is to say, when the pooling equilibrium exists, the values of profits are greater if the prior probability of cost-efficient service provider γ is larger. This is intuitive since as the increase of γ, the expected prior probability of cost efficiency ($\gamma c_e + (1 - \gamma)c_{in}$) is also increasing, which indirectly causes the expected pooling prices to increase at the same time. The growing prices directly result in the additional profits in each case under the pooling equilibrium, without any additional effort level contributions of both parties to the service. Although all pooling profits are increasing in γ, the changing rates are different. To be more specific, the profit in pooling equilibrium is changing more rapidly in γ when $D_1 = \rho$ (i.e., $\pi_p^{(2)}$and $\pi_p^{(1)}$), however when $D_1 = 1$, the pooling profit will change in a slower pace (i.e., $\pi_p^{(4)}$ and $\pi_p^{(3)}$) which we can find from Figure 12. The reason behind this phenomenon is that when the first period demand only generates from high type consumers, the service provider's first period prices is higher than that when the first period demand is the whole market share, with the only difference in the multiplier changing from 1 to $\mu < 1$(i.e., the low type consumer's willingness to pay for the service). This multiplier will be taken into the profit function and finally be expressed in the first order conditions of profits with respect to γ. In the case when $D_1 = \rho$, the first order conditions own higher value than that when $D_1 = 1$. Thus, the pooling equilibrium profits in the former case are changing more rapidly in γ than that in the latter case.

After clarifying the relative changing rate of the pooling equilibrium profits, we can make a comparison between them in magnitude given the profits are monotonically increasing in γ. The corresponding corollary follows easily.

Corollary 4.3. *In the pooling equilibrium of case (C),*

when $\frac{mc_e(-c_i+c_{in})}{2c_i(c_e-c_{in})} < \gamma < \frac{(1+m)(m+\rho)c_e c_{in}\left(\frac{(-1+a)^2(\mu^2-\rho)}{w}+\frac{a^2m^2\mu^2(-1+\rho)}{(1+m)(m+\rho)c_i}+\frac{2a^2m\mu\left(\frac{\mu}{1+m}-\frac{\rho}{m+\rho}\right)}{c_{in}}-4(-1+\rho)v_j\right)}{2a^2m\mu(m\mu+(-1-m+\mu)\rho)(c_e-c_{in})}$, $\pi_p^{(4)} = \frac{a^2m^2w\mu^2+c_i\left((-1+a)^2(1+m)^2\mu^2+2w\left(\frac{a^2m\gamma\mu^2}{c_e}-\frac{a^2m(-1+\gamma)\mu^2}{c_{in}}+2(1+m)^2v_j\right)\right)}{4(1+m)wc_i} > \pi_p^{(2)} = \frac{a^2m^2w\mu^2+c_i\left((-1+a)^2(m+\rho)(m\mu^2+\rho)+2w\left(\frac{a^2m\gamma\mu\rho}{c_e}-\frac{a^2m(-1+\gamma)\mu\rho}{c_{in}}+2(m+\rho)^2v_j\right)\right)}{4w(m+\rho)c_i}$*,thus,* $D_1 = 1$*; otherwise,*

when $1 > \gamma > \frac{(1+m)(m+\rho)c_e c_{in}\left(\frac{(-1+a)^2(\mu^2-\rho)}{w}+\frac{a^2m^2\mu^2(-1+\rho)}{(1+m)(m+\rho)c_i}+\frac{2a^2m\mu\left(\frac{\mu}{1+m}-\frac{\rho}{m+\rho}\right)}{c_{in}}-4(-1+\rho)v_j\right)}{2a^2m\mu(m\mu+(-1-m+\mu)\rho)(c_e-c_{in})}$*,* $D_1 = \rho$.

In the pooling equilibrium of case (D),

when $\frac{m\rho c_e(-c_i+c_{in})}{2\mu c_i(c_e-c_{in})} < \gamma < \frac{c_e c_{in}\left(\frac{a^2m^2(-1+\rho)\rho}{c_i}+\frac{2a^2mw\rho(-1+\mu+m\mu-m\rho)+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}{wc_{in}}\right)}{2a^2m\rho(-1+\mu+m\mu-m\rho)(c_e-c_{in})}$, $\pi_p^{(3)} = \frac{a^2m^2w\rho^2+\frac{c_i(2a^2mw\gamma\mu\rho c_{in}+c_e(-2a^2mw(-1+\gamma)\mu\rho+(1+m\rho)c_{in}((-1+a)^2(\mu^2+m\rho)+4w(1+m\rho)v_j)))}{c_e c_{in}}}{4w(1+m\rho)c_i} > \pi_p^{(1)} = \frac{\rho\left(a^2m^2w+\frac{c_i(2a^2mw\gamma c_{in}+c_e(-2a^2mw(-1+\gamma)+(1+m)^2c_{in}((-1+a)^2+4wv_j)))}{c_e c_{in}}\right)}{4(1+m)wc_i}$*,thus,* $D_1 = 1$*; otherwise, when*

$$1 > \gamma > \frac{c_e c_{in}\left(\frac{a^2 m^2(-1+\rho)\rho}{c_i} + \frac{2a^2 m w\rho(-1+\mu+m\mu-m\rho)+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}{w c_{in}}\right)}{2a^2 m\rho(-1+\mu+m\mu-m\rho)(c_e-c_{in})}, \ D_1 = \rho.$$

After we demonstrate the separating and pooling equilibrium in the scenario when the service provider has private information on her cost efficiency, we finally consider the scenario when both the cost efficiency and quality certification are unknown to the consumers.

4.4.3. Unknown Cost Efficiency & Unknown Quality Certification

We finally consider the circumstance when both the cost efficiency and quality certification are the service provider's private information and can not be observed by consumers before their purchase decisions. We begin by investigating the case when the quality certification is private information while the cost efficiency is common knowledge whose analyses generally lays the foundation of our future analyses under the case when both dimensions of information can not be observed before purchase.

Since the service provider's cost efficiency is common knowledge, we first need to discuss the pricing strategy of the cost efficient and cost inefficient service provider separately. In order to facilitate the analyses and obtain useful information, we mainly focus on the case when the extent of service providers' cost efficiency is sufficiently distinct. For expressional convenience, we use p^* to demonstrate the price charging by the service provider with the expected reservation value as $E[v_j] = \beta v_h + (1-\beta)v_l$ when taking the unobservable quality certification into consideration. We further use p^l to demonstrate the price with low reservation value $v_j = v_l$, while p^h demonstrates the price with the high reservation value $v_j = v_h$.

Firstly, let's consider the case of cost-efficient service provider whose marginal profit is $p - c_e x^2$, which is always positive with sufficiently low value of c_e when the service provider charges a positive price. Therefore, the cost-efficient service provider's incentives to serve consumers at price p^* with both ex-ante and ex-post incentives are coordinated. We then show the service provider's maximum profit can be obtained by setting her pricing strategy at p^*, which results in consumers' purchasing behavior as section 4.4.1 when both cost efficiency and quality certification are common knowledge. The result can be proved by contradiction. Let's consider the case where the service provider can obtain higher profit with a differentiated pricing strategy regarding her quality certification, that is to say, the service provider can obtain higher expected profit than the case when she sets price p^*. It means that at least one member of the consumer groups is willing to pay more than the price p^* for the service in equilibrium. Therefore, a separating equilibrium with respect to the quality certification dimension should be generated as a new equilibrium, as in a pooling equilibrium all consumers agree on the unified price setting p^* without any deviation. However, in any separating equilibrium, the maximized expected profit cannot be larger than that she obtains with price p^* no matter whether the information of quality certification is common knowledge. In conclusion, the cost-efficient service provider's maximized profit can be obtained with the uniform pricing strategy p^* regardless of whether her quality certification is high or low.

Now we turn to the case of cost-inefficient service provider. Let's first assume that the cost-inefficient service provider set a single pricing strategy no matter whether her quality is high or low. Then if this single price is higher than p^*, the expected utility of all consumers is negative thus no one will choose to purchase; while if this single price is higher than p^l but lower than p^*, only a fraction of consumers will choose to purchase when their expected utility is positive; if the single price is lower than p^l, the consumers will always choose to purchase the service. Therefore, the cost-inefficient service provider's optimal uniform pricing strategy is p^l. We next consider the probable differentiated pricing strategy $\{p_l, p_h\}$ set by the cost-inefficient service provider where p_l is lower than p_h. Besides, p_l is lower than or equal to p^l, otherwise no consumer will choose to purchase and p_h is lower than or equal to p^h for the same reason. Meanwhile, p_l and p_h should be both larger than or equal to $c_{in}x^2$ for the reason that the cost-inefficient service provider's optimal pricing strategy should be at least as large as her effort level cost. The cost-inefficient service provider will always set the pricing strategy p_h when she is high in her quality certification as she will bring about a revenue loss when setting p_l with high quality certification. However, when she is low in her quality certification, the cost-inefficient service provider always set the pricing strategy p_l.

Our analyses show the results that the cost-efficient service provider prefers the uniform pricing strategy regardless of her quality certification to any differential pricing strategies.

Therefore, the cost-efficient service provider will choose some uniform price $p_e^* \geq p^l$ and consumers will make purchase decisions as long as $p_e^* \leq p^*$. The cost-inefficient service provider will adopt the differential pricing strategy $\{p_l^* = p^l, p_h^* = p^h\}$.

We then consider the case when both the cost efficiency and quality certification information are unobservable to consumers before purchase. The above investigations are mainly based on the qualitative deduction of the equilibrium results. In order to obtain some analytical solutions, we next consider the cases when the cost-efficient service provider is high in her quality certification, while the cost-inefficient service provider is low in her quality certification. In practice, we often regard the cost-efficient service provider as the one with high quality certification, for example, many user-generated-content platforms such as Bilibili and Youtube are more efficient in their cost if the network spreads to more users, meanwhile, the overall quality of the videos they provide to consumers are higher than cost-inefficient ones. Other examples like the taxi platform (i.e., Didi Chuxing) or takeaway platform (i.e., Meituan) also conform to the same logic in the relations between the cost efficiency and quality certification. This assumption restricts the type of service providers from four types to two types, which is beneficial for us to make analyses of the separating equilibrium results. We can derive the analytical equilibrium solutions under this assumption as below.

Proposition 4.8. *We first assume the range of the cost efficiency difference as* $c_e = \xi c_{in}$ *and the range of the quality certification difference as* $v_h = \tau v_l (\tau > 1)$, *when case (A) of separating equilibrium exists (i.e.,* $\rho < min\{\rho_1, \rho_3\}$ *), if* $\frac{2a^2 m(1+m) w\mu^2\rho}{a^2 mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi < \frac{2a^2 m(1+m) w\mu\rho}{a^2 mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$ *and* $\frac{a^2 mw\mu(2(-1+\mu)\rho+m(\mu+(-2+\mu)\rho))+(1+m)(m+\rho)c_{\mathrm{in}}((-1+a)^2(\mu^2-\rho)+4wv_l)}{4(1+m)w\rho(m+\rho)c_{\mathrm{in}}v_l} < \tau < \frac{a^2 mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{\mathrm{in}}((-1+a)^2(\mu^2+\rho)+4wv_l)}{4(1+m)w\rho(m+\rho)c_{\mathrm{in}}v_l}$, *the optimal effort level and corresponding optimal profit for cost efficient service provider are* $x_e^* = \frac{am\mu}{2(m+\rho)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{(1-a)\mu}{2w}$, $\pi_e^{(sep2)} = \frac{a^2m^2w\mu^2+(m+\rho)c_e((-1+a)^2(1+m)\mu^2+\frac{a^2mw\mu^2(m+(2+m)\rho)}{(1+m)(m+\rho)c_{\mathrm{in}}}+4w(mv_h+v_l))}{4w(m+\rho)c_e}$; *those for cost inefficient* *service provider are* $x_{in}^* = \frac{am\mu}{2(1+m)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{(1-a)\mu}{2w}$, $\pi_{in}^{(4)} = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_{\text{in}}((-1+a)^2\mu^2+4wv_l)}{4(1+m)wc_{\text{in}}}$;

if $\frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)} < \xi < -\frac{a^2mw\mu(-m\mu-2\rho+m(-2+\mu)\rho)}{(m+\rho)(2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$ *and* $1 < \tau < \frac{a^2mw\mu(2(-1+\mu)\rho+m(\mu+(-2+\mu)\rho))+(1+m)(m+\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)+4wv_l)}{4(1+m)w\rho(m+\rho)c_{\text{in}}v_l}$, *the optimal effort level and corresponding optimal profit for cost efficient service provider are* $x_e^* = \frac{am\mu}{2(m+\rho)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{(1-a)\mu}{2w}$, $\pi_e^{(2)} = \frac{a^2mw\mu(m\mu+2\rho)+(m+\rho)c_e((-1+a)^2(m\mu^2+\rho)+4w(m+\rho)v_h)}{4w(m+\rho)c_e}$*; those for cost inefficient service provider remain unchanged* $x_{in}^* = \frac{am\mu}{2(1+m)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{(1-a)\mu}{2w}$, $\pi_{in}^{(4)} = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_{\text{in}}((-1+a)^2\mu^2+4wv_l)}{4(1+m)wc_{\text{in}}}$.

When case (B) of separating equilibrium exists (i.e., $\rho_2 < \rho < \rho_4$ *),*

if $\frac{2a^2mw\mu\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi < \frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$ *and* $\frac{-a^2mw\rho(2-2\mu+m(1-2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)+4wv_l)}{4(1+m)w\rho(1+m\rho)c_{\text{in}}v_l} < \tau < \frac{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2+\rho)+4wv_l)}{4(1+m)w\rho(1+m\rho)c_{\text{in}}v_l}$ *, the optimal effort level and corresponding optimal profit for cost efficient service provider are* $x_e^* = \frac{am}{2(1+m)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{1-a}{2w}$, $\pi_e^{(sep1)} = \frac{\rho(a^2m^2w+(1+m)c_e(\frac{a^2mw(2\mu+m(-1+2\mu+\rho))}{(1+m)(1+m\rho)c_{\text{in}}}+\frac{(-1+a)^2(\mu^2+m\rho)+4w(m\rho v_h+v_l)}{\rho}))}{4(1+m)wc_e}$;

those for cost inefficient service provider are $x_{in}^* = \frac{am\rho}{2(1+m\rho)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{1-a}{2w}$, $\pi_{in}^{(3)} = \frac{a^2mw\rho(2\mu+m\rho)+(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_l)}{4w(1+m\rho)c_{\text{in}}}$;

if $\frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)} < \xi < \frac{a^2mw\rho(2+m+m\rho)}{(1+m)(2a^2mw\mu\rho+(1+m\rho)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$ *and* $1 < \tau < \frac{-a^2mw\rho(2-2\mu+m(1-2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)+4wv_l)}{4(1+m)w\rho(1+m\rho)c_{\text{in}}v_l}$ *, the optimal effort level and corresponding optimal profit for cost efficient service provider are* $x_e^* = \frac{am}{2(1+m)c_e}$, $y_{1,e}^* =$

$\frac{1-a}{2w}$, $y^*_{2,e} = \frac{1-a}{2w}$, $\pi^{(1)}_e = \frac{\rho(a^2m(2+m)w+(1+m)^2c_e((-1+a)^2+4wv_h))}{4(1+m)wc_e}$*; those for cost inefficient service provider remain unchanged* $x^*_{in} = \frac{am\rho}{2(1+m\rho)c_{in}}$, $y^*_{1,in} = \frac{(1-a)\mu}{2w}$, $y^*_{2,in} = \frac{1-a}{2w}$, $\pi^{(3)}_{in} = \frac{a^2mw\rho(2\mu+m\rho)+(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_l)}{4w(1+m\rho)c_{\text{in}}}$.

This proposition points out the difference between the separating equilibrium under the assumption that both dimensions of information are unobservable and what we have analyzed in section 4.4.2 when only cost efficiency information is unobservable. The results show that the range of cost efficiency difference and quality certification difference both determine the separating equilibrium. When the two types of service provider are quite similar in their types, the results of the separating equilibrium are similar to those when both dimensions of information are observable. However, when the two types of service provider are distinct in their types, the results of the separating equilibrium are different from those when both dimensions of information are observable. This is mainly because when ξ is low and τ is high, the valuation difference between both cost efficiency and quality certification are large. It makes the efficient and high-quality service provider more easily to separate herself from others, thus her first period pricing strategy may not be optimal under this case. However, when the valuation difference between both cost efficiency and quality certification are small, only if the efficient and high-quality service provider optimally sets her first period price will she be able to separate herself from others. Nevertheless, the optimal effort level strategies are not affected by the range of information differences.

4.5. Concluding Remarks

Based on our analyses of the signaling model regarding the collaborative service of the private information owner, we can obtain several conclusions as follows.

Firstly, the optimal effort level of the service provider is increasing in the work allocation parameter; while the optimal effort levels of the consumers in both periods are decreasing in the work allocation parameter. The optimal effort level of the service provider is decreasing in the service provider's cost coefficient parameter, namely, the optimal effort level of the cost-efficient service provider is greater than that of the cost-inefficient service provider. Meanwhile, the optimal profit of the cost-efficient service provider is greater than that of the cost-inefficient one.

Secondly, in either case of the separating equilibrium, the optimal effort level strategy of the cost-efficient service provider is greater than that of cost-inefficient service provider. While the optimal effort level strategy of the early consumers when facing the cost-efficient service provider is greater than that facing the cost-inefficient service provider. The optimal effort level strategies of the late consumers are equal when facing both cost-efficient and cost-inefficient service provider.

Thirdly, as the prior probability of cost-efficient service provider increases, the expected revenue of the service provider at pooling equilibrium also increases for the reason that the expected prior probability of cost efficiency is enlarged. This indirectly causes the expected pooling prices to increase at the same time. The growing prices directly result in the additional profits in each case under the pooling equilibrium without any additional effort level contributions of both parties to the service.

Finally, when both the cost efficiency and quality certification information are unobservable to consumers before purchase. Our analyses show that the cost-efficient service provider prefers the uniform pricing strategy in respect of the quality dimension to any differential pricing strategies. While the cost-inefficient service provider will adopt the differential pricing strategy in respect of the quality dimension.

Chapter 5

Collaborative Service Provision under Online Review Platform: Implications for the Service Provider and Consumers

5.1. Introduction

In this chapter, we consider a model structure depicting a service market where each transaction party simultaneously owns some private information that influences both parties' payoffs. For example, in our model, consumers possess qualitative information observing the online reviews in respect of service providers' service outcome, while service providers possess private information regarding their quality type. A high-quality service provider tends to deliver the service more efficiently than a low-quality service provider preset the same effort levels. Thus, consumers can only infer the service provider's quality type via the information embedded in the online reviews.

Consumers review websites such as Yelp (https://www.yelp.com/) in overseas market and Dazhongdianping (https://www.dianping.com/) in China have become increasingly popular over the past decades, and now exist in nearly every type of service industry. The functions of them in service industries are just like the Alibaba in product industries, for example, Yelp contains more than 70 million reviews in respect of restaurants, education institutions, beauty salons and other services. Moreover, there is increasingly strong evidence indicating that these reviews posting by consumers via online platforms directly influence the service providers' sales volume (Chevalier & Mayzlin, 2006). As the popularity of these online review platforms has grown, the credibility of reviews is also confronted with challenges as it may be undermined by sellers' manipulating reviews behavior. This will mislead consumers and their competitors to a wrong belief regarding the seller's characteristics (Luca & Zervas, 2015). In our model, we disregard this possibility by focusing on the fact that the consumer will have a formation in his mind of the service provider's service ability and service outcome. By referring to the online reviews posting by early consumers truthfully, the consumer finally chooses whether to pay for the service. Different consumers will conceive different views of the service outcome when facing the same review, as each of them possesses some idiosyncratic elements that might probably influence the service outcome.

What we mainly focus on is the type of collaborative service, which means the outcome of the service depends on both the effort contribution of the service provider and the consumer. Moreover, both parties have the right to make his (i.e., the consumer) or her (i.e., the service provider) own effort level decision respectively, for example, in the knowledge payment service industry or fitness industry, the service providers produce the online education courses while the consumers decide the time spent on following these courses. Although motivated from the background of knowledge payment service, our model and conclusions have pervasive inspirations for other kinds of services with similar properties, only if the service is collaborative in essence. Namely, the service provider and consumers both have the right to make the effort level decisions on their own respectively. For example, the knowledge-intensive business services which are high-end services in a complex B2B context (i.e., lawyers, consultants, etc).

Based on our model, we consider an intertemporal model where there is a monopolistic service provider providing a kind of collaborative service to consumers that are review dependent via online platform over two consecutive periods. The problem we try to resolve is that what strategic actions (e.g., in terms of pricing or effort level) the service provider and consumers should undertake given the level of participation required from consumers and given the availability/positivity of online reviews in order to maximize profits. Our conclusions are in four aspects. Firstly, in the presence of review process, there exists a degree of the positivity of online consumer reviews such that the follower consumers' optimal effort level is lower than the early consumers' optimal effort level, if and only if this availability/positivity of online consumer reviews is lower than this threshold. The joint influence of both the level of participation required from consumers and the availability of online consumer reviews on the optimal consumers' effort level is that the follower consumer's optimal effort level is lower than that of the early consumer, only if the availability of reviews is weak and the level of consumers' participation is in middle range. Secondly, the service provider adopts the pricing plan with lower prices in the second period compared with the first period when the level of consumers' participation is in the middle range and the availability of reviews is in a low range. That is to say, in a fairer public opinion environment (the availability of reviews is in a high range), the service provider can raise her second period price higher than the first period price, if follower consumers learn more about her service quality level. Thirdly, we further explore that as the availability of reviews increases from the case in the absence of review process, the optimal profit is changing more rapidly in the level of participation required from consumers, which reflects a "mutual promotion" mechanism between the level of consumers' participation and the availability of reviews. Finally, when the threshold for a certain early consumer to post his review is rising, which results in the overall quality of the reviews posted online is improving, the reviews are more helpful for the service provider to make her optimal pricing strategies and consumers effort level strategy. This finally results in the promotion of optimal profit. Moreover, as the number of reviews level increases with rank refinement, the influence of review process is intensified, which finally results in the improvement of the overall optimal profit.

5.2. Literature Review

We first summarize the literatures on collaborative services or joint production, then the analytical framework for online reviews, which are two main topics that are highly related to our study in this chapter.

In the field of collaborative service, researchers start investigating this topic from early on. Fuchs (1968) firstly puts forward this concept. Then Chase (1981) further studies the service system design with this concept in the background of operations management. Maglio, Vargo, Caswell, and Spohrer (2009) then extend the research on service-system abstraction to study the co-created value in the integrated science of service. Their results indicate that the joint production in service has been widespread and nearly every kind of service contains co-created value to some extent. Furthermore, globally distributed organization design of work teams draws researchers' attention (Kumar, van Fenema, & Von Glinow, 2005). Meanwhile, a systematic framework for collaboration with interorganizational systems technologies is put forward (Chi & Holsapple, 2005). Nevertheless, the conventional top-down method to manage design system becomes useless as the popularity of collaborative problems in complex environment. Thus, new methods should be introduced to cope with this problem. To be specific, Kogan and Muller (2006) describe how knowledge workers in their areas of expertise can develop their own strategies for getting their work done in collaborative environment; Hill, Yates, Jones, and Kogan (2006) point out that users have relied on ad hoc collaboration tools to coordinate their work in business processes. They all show that a majority of collaborative works often depend on individual information administration instruments. Thus, conventional methods are not enough to analyze the collaboration system, while system dynamics with a group modelling approach should be used to promote the collaborative design process (Elf, Putilova, Von Koch, & Öhrn, 2007).

In the context of operations management, the collaborative service appears as the form of value cocreation, and it arises in many research fields. For example, Buzacott (2004) develops models to study the teamwork of manufacturing techniques; Gurvich and Van Mieghem (2015) further study networks in workflows where collaboration imposes constraints on the process capacity. Bhaskaran and Krishnan (2009) formulate the joint development of products involving two firms with different capabilities; Iyer et al. (2005) further analyze a principal-agent model for product specification and joint product development; while Baiman, Fischer, and Rajan (2000) analyze the relation between product quality, quality cost and contracting information. Our work takes into consideration the collaborative service under the background of the interface between operations management and marketing, and we mainly analyze the collaborative nature in respect of the service provider's effort level strategy and pricing strategy.

There is another stream of literatures regarding online reviews that is also highly related to our study in this chapter. Unlike our focus on the rationale of service reviews system, the existing literature mainly aims at online reviews of products that are in different walks of life. For example, Chevalier and Mayzlin (2006) empirically examine the effect of consumer reviews on sales of books and Liu (2006) uses real word-of-mouth information to study its impact on movie industry. Dellarocas, Zhang, and Awad (2007) reconcile some inconsistencies among previous studies regarding the influence of online reviews on entertainment good sales. Zhu and Zhang (2010) further examine the impacts of online consumer reviews on the sales regarding computer games. Chintagunta, Gopinath, and Venkataraman (2010) empirically analyze the influences of online consumer reviews on box office of films. And others (B. Gu, Park, & Konana, 2012) analyze the impact of word-of-mouth on sales for high involvement products such as digital cameras.

All of the aforementioned studies are using empirical approach to analyze the influence of online product reviews on products sales amount in various industries. Meanwhile, many researchers also investigate theoretical models to study the impacts of consumers' reviews. In the theoretical research on online reviews, there are mainly two methods taken into use to depict how the reviews can transmit product information. One type of method (Y. Chen & Xie, 2005, 2008; Dellarocas, 2006; Kwark, Jianqing, et al., 2014) models product reviews as an exogenous source of information transmission. For example, Li and Hitt (2010) build up a model to study the impact of price-influenced reviews on product sales; Dellarocas (2006) models the review generation process as an exogneeous source to transfer product quality infomation; Y. Chen and Xie (2008) develop a normative model to study how online consumer reviews influence sellers' marketing strategies; Kwark, Jianqing, and Raghunathan (2014) consider a manufacturer competition with the effect of online product reviews. The other type of method (Y. Jiang & Guo, 2015; M. Sun, 2012) considers review generation process as an endogenous source of information transmission by taking use of a deterministic consumer's utility function. For example, M. Sun (2012) builds a theoretical model to examine the informational role of product ratings; Y. Jiang and Guo (2015) further develop the sellers' review system to study the product pricing strategies and endogenously take into consideration the product features such as product valuation, product mainstream level. Consequently, consumers with rational expectation can induce the product fit or quality information from the observable review information such as the mean or variance of reviews. Moreover, other researchers (Kuksov & Xie, 2010; H. Sun & Xu, 2018) not only take use of the information updating method to depict the endogenous review generation process, but also explain the influences of unobservable strategic actions and noises involved in the reviews. To be specific, Kuksov and Xie (2010) develop a two-period model to portray the review process by transferring the hidden product quality information that all parties are unobservable of; H. Sun and Xu (2018) study the service reviews model by taking into consideration the influence of both review and service outcome on the optimal effort level decisions. What we mainly focus on is that we not only endogenously model the review generation process, but also investigate the service provider and consumers' effort level decisions in the collaborative service provision model structure. Moreover, we combine the two effects regarding both reviews and collaborative services together to investigate the interactive relationships between the two processes.

5.3. Model Description

We consider an intertemporal model where there is a monopolistic service provider providing a kind of collaborative service to consumers that are review dependent via online platform over two consecutive periods. Without loss of generality, we normalize the total number of consumers making purchase decisions in the first period to 1. The relative total number of consumers in the second period is normalized to m. Each consumer can purchase at most one unit of the collaborative service offered by the service provider. In each time period, we take use of a representative consumer to denote the mass of consumers. To be specific, an early consumer (i.e., review writer) is in the first period and a late consumer (i.e., review reader) is in the second period. Thus, the information transmission between periods in this setting represents an interpersonal interaction. We assume that the consumer only purchases in the corresponding period and will exit the market if no purchase occurs. For instance, preschool curriculum is adapted to children's learning in some specific ages or online vocational education is for adults to manage their career, which will lose value if they are not taken in the specific time period. The service provider and consumers in the market are taking part in a kind of collaborative service. That is, the service outcome relies on the effort level strategies of both the service provider and the consumer. We express the effort level of the service provider as x and that of the consumer in each period as y_t, where $t = 1,2$ is denoted as the effort level decision in each representative time period. For analytical simplicity, we normalize the boundary of each effort level decision over the range between zero and one, i.e., $x, y_t \in [0,1]$. Besides, the outcome of the service is dependent on the inherent capacity of the service provider to offer the service as well. We refer to this kind of capacity as the quality or certification of the service provider. We assume the service provider owns different quality levels in the range over a continuous distribution. A service provider with higher quality level tends to offer the service more efficiently than a service provider with lower one, provided that the same effort level strategy of the service provider and consumers are prescribed. As a consequence, we assume the service outcome O to be a binary random variable (i.e., 0 and 1). To be specific, the value 1 corresponds to a success of the service (e.g., achieving the prior target, completing the qualification exam) and the value 0 corresponds to a failure of the service. Moreover, we assume the specific value of the service outcome is dependent on a hidden variable which is denoted as $u_t, t = 1,2$. Its value reflects the combination of the overall factors that may influence the outcome of the service in each period t. Concretely, in case of $u_t > 0$, we obtain the service outcome as $O = 1$; otherwise, in case of $u_t \leq 0$, we obtain this value as $O = 0$. In particular, the form of this variable u_t can be demonstrated as below.

$$u_t = \mu_\theta(ax + (1-a)y_t) + \gamma, t = 1,2. \tag{31}$$

The term $ax + (1-a)y_t$ denotes the combination of effort contributions from both the service provider and consumers on the overall service provision, which has been widely used in the collaborative service literatures (Roels, 2014; Roels et al., 2010). The parameter a here denotes the portion of the service provider's effort level contribution to the whole service outcome, while $1-a$ is that of the consumers' own effort level contribution. Especially when a is approaching 0, the service combination value is a function of the consumer's effort level. While when a is approaching 1, it is a function of only the service provider's effort contribution. Namely, the allocation of effort contribution is depicted as the relative value of a and we thus call this parameter as "work allocation". In our setting, μ_θ stands for the service provider's quality or certification which is known to the service provider but not to consumers prior their purchase decisions. Consumers only share a common prior belief over μ_θ which follows a uniform distribution between zero (the lowest quality level) and one (the highest quality level), i.e., $\mu_\theta \sim U[0,1]$. Some other idiosyncratic factors of the consumers are reflected in the last term γ of this hidden variable. These factors include all probable elements that could influence the outcome of the service. We assume the term γ as a random variable and refer to its cumulative distribution function (i.e., CDF) as $G(\gamma)$. In order to obtain clear analytical solutions, we assume γ obeys a uniform distribution over the fixed support $[-1,0]$. Therefore, the expression of its cumulative distribution function is $G(\gamma) = \gamma + 1$. We set the support of this distribution based on both the possible range of u_t and the implications of γ. To be more specific, when the deterministic term of u_t (i.e., $\mu_\theta(ax + (1-a)y_t)$) is minimized in its value as zero, we obtain $u_t(\gamma) \le 0$ for all probable valuations of γ; otherwise, when $\mu_\theta(ax + (1-a)y_t)$ is maximized in its value as one, we obtain $u_t(\gamma) \ge 0$ for any valuation of γ. Consequently, we can derive the probability of the service outcome to be a success conditional on the true quality level of the service provider and the effort level strategies in each period as below

$$\Pr\{O = 1|\mu_\theta, x, y_t\} = \Pr\{u_t(\gamma) > 0|\mu_\theta, x, y_t\} = 1 - G\big(-\mu_\theta(ax + (1-a)y_t)\big)$$
$$= \mu_\theta(ax + (1-a)y_t), \mu_\theta \sim U[0,1], t = 1,2. \tag{32}$$

To eliminate the possible effects of the assumption regarding the distribution of γ, we further assume it follows the uniform distribution over arbitrary supports in Appendix A of this chapter, and results show that the main conclusions in our base model always hold.

Following the aforementioned assumptions, we can further derive the net utility of consumers as $U_t = r1_{O=1} - wy_t{}^2 - p_t$ in each time period $t = 1,2$. r here denotes the consumers' utility benefit brought about by the success of the service outcome and $1_{O=1}$ is an indicator function whose value depends on the service outcome. That is, $1_{O=1}$ takes its value as one when the service outcome satisfies $O = 1$, and the consumers will earn positive utility benefit r; otherwise, $1_{O=1}$ takes its value as zero when the service outcome satisfies $O = 0$, then the consumers only obtain zero utility benefit. Besides, the consumers in the corresponding period make their effort level decision y_t and will incur utility cost denoted as $wy_t{}^2$. Meanwhile, the utility costs also include the price that consumers need to pay for the service after their purchase decision in that period as p_t. After our clarifications of the consumers utility function, we next interpret the formation of the review process and how consumers perceive information from the posted reviews.

5.3.1. First Period Consumers' Review Process

At the beginning of the first period, the service provider has possessed the information in respect of her true quality or certification (i.e., μ_θ) and determines her own effort level x over the consecutive time periods. Meanwhile, the customer sets his own effort level strategy in the first period y_1. We use "she" to denote the service provider and "he" to denote the representative consumer for our interpretation convenience. Based on our assumption regarding the distribution of μ_θ, the expected quality perceived by the early consumer can be derived as $E[\mu_\theta|0 < \mu_\theta < 1] = \frac{1}{2}$. This expectation regarding the service provider's quality will subsequently lay foundation to the early consumer's purchasing decision.

When the service provider makes use of the online forum or platform to collect consumers' reviews, consumers in the first period can choose to post their reviews regarding the service based on their service outcomes. That is, consumers tend to post positive (or equivalently negative) reviews regarding the service if and only if their service outcomes are valued as one (or equivalently zero). In order to further depict the review process, we assume the review levels regarding the service can be normalized to ratings in a unit line segment between zero (the lowest review level) and one (the highest review level). Meanwhile, the intermediate review levels are spaced at even intervals along the unit line segment. Therefore, under an online review platform based on review levels numbered as s, we can map the review levels available for consumers' choices and needs to the corresponding points (i.e., 0, $\frac{1}{s-1}$, …, $\frac{s-2}{s-1}$, 1) along this unit line segment. Then we take advantage of the logistic function which is commonly used in consumer choices literatures (Franses & Paap, 2001; Malhotra, 1984) to associate consumers' post service outcome in the first period (i.e., the hidden variable $u_1(\gamma) \in \mathbb{R}$) with the outcome rating score (i.e., $\tau_1(\gamma) \in [0,1]$). That is to say, with the tranformation $\tau_1(\gamma) = \frac{e^{u_1(\gamma)}}{e^{u_1(\gamma)}+1} = \frac{1}{1+e^{-u_1(\gamma)}} = \frac{1}{1+e^{-\mu_\theta(ax+(1-a)y_1)-\gamma}}$, the consumers' service outcome will become the new variable marked as the outcome rating score, and first period consumers will post their review levels in respect of the service outcome based on this rating score directly.

We next consider the condition under which the first period consumers will choose to post their rating score via the online review platform. As for a representative consumer, only when he makes his purchase decision in the first period will he have a chance to consider whether to post the review regarding the service. We then define the review decision process made by first period consumers as a reviewing utility function: $V(\gamma) = \alpha 1_{P=1} - \beta \left|\frac{k}{s-1} - \tau_1(\gamma)\right| - \varphi$, where $k = arg \min_{i=0,\dots,s-1}\{|\frac{i}{s-1} - \tau_1(\gamma)|\}$ which is dependent on the specific realization of the random factor μ_θ. We next explain each component respectively under the main rationale in respect of the reviewing decision process: consumers should make a trade-off between the benefit and cost of doing so. To be specific, α in the reviewing utility function represents the benefit or utility gain of consumer's decision to review the service. Consumers may obtain the review benefit originating from several aspects. Many researchers have studied this topic from two main fields: the traditional offline word-of-mouth (Sundaram, Mitra, and Webster, 1998) and online word-of-mouth communication (Hennig-Thurau, Gwinner, Walsh, and Gremler, 2004). And their results show that consumers are mainly motivated to express their opinions regarding the service by the social communication needs, economic rewards provided by the review platform. Campbell, Mayzlin, and Shin (2017) further developed a theoretical model to study the incentives of consumers' willingness to engage in review process due to reputational concerns and self-enhancement probabilities.

$1_{P=1}$ is an indicator function such that $1_{P=1} = 1$ if early consumers make purchase decision in the first period, and $1_{P=0} = 0$ if early consumers exit the market without any purchase. It means a representative consumer derives positive utility gain from reviewing the service only if he makes purchase in the first period, otherwise he has no chance to consider whether to post a review thus zero utility gain is obtained.

In addition to the aforementioned reviewing benefit, there also exist costs in terms of the reviewing process. As for a representative consumer in the first period, he will make a choice among all review levels to determine one level $\frac{k}{s-1}$ which is the closest to his outcome rating score $\tau_1(\gamma)$. In this decision process, a comparison cost denoted as φ is generated by consumers' comprehensive evaluation regarding the service outcome. Researchers in the field of consumer behavior (Bettman, Luce, & Payne, 1998; Shugan, 1980) make specialized study on consumers' choice modelling. Their results show that the number of options highly influence the choice difficulty and the comparision cost should be nondecreasing in the number of review levels (i.e., s). Nevertheless, the most fitted review level $\frac{k}{s-1}$ might not perfectly match the consumers' outcome rating score $\tau_1(\gamma)$ without any deviation. Thus, a misfit cost is caused due to this utility loss which we demonstrate as $\beta\left|\frac{k}{s-1}-\tau_1(\gamma)\right|$, where β is a unit misfit cost parameter. We make this assumption by referring to studies on horizontal feature distinction (Anderson, 1992). This research gives us inspiration on depicting consumer's behavior when he incurs a misfit cost between the closest available review level and his outcome rating score. Moreover, the utility loss brought about by this misfit increases in the distance between them.

With all of the above assumptions, the consumer in the first period makes his review decision based on the expected reviewing utility function $EV(\gamma) = E\left[\alpha 1_{P=1} - \beta\left|\frac{k}{s-1} - \tau_1(\gamma)\right| - \varphi\right] \geq 0$. We introduce the parameter $\Delta = \frac{E[\alpha 1_{P=1}]-\varphi}{\beta} > 0$ to denote consumers' tendency to post their reviews. That is, when the deviation between the service outcome rating score and the closest review level satisfies $\left|\frac{k}{s-1} - \tau_1(\gamma)\right| \leq \Delta$, the consumer earns nonnegative reviewing utility and will post $\frac{k}{s-1}$ as his service review level via online platforms. As a result, we demonstrate the review level function by depicting reviewing behaviors of first period consumers as:

$$R(\gamma) = \begin{cases} \frac{k}{s-1}, & when \left|\frac{k}{s-1} - \tau_1(\gamma)\right| \leq \Delta \\ does\ not\ review, & otherwise \end{cases}. \tag{33}$$

The review level function maps the correspondence between each consumer's idiosyncratic factors that could influence the outcome of the service and the review level he posts in the first period. A consumer will choose to review the service if and only if his expected utility of posting reviews is nonnegative, that is $EV(\gamma) \geq 0$. Consequently, not all consumers review the service. As if the utility gain of consumer's reviewing the service is lower than its costs for some consumers, this will result in a kind of consumer behavior which we can name as "purchase but not review".

The comprehensive consequences brought about by the review process can be demonstrated as the mean of reviews $\bar{R}(x, y_1)$ and the number of reviews $N(x, y_1)$. What we have observed in practice is that the online reviews mainly appear as the average of personal reviews or the percentage of positive reviews. For instance, Dazhongdianping.com lets consumers to choose from one star to five stars in its review platform and displays the average stars as the mean of reviews. Whereas Meituan.com or Missfresh.com ask the consumers a question “whether you are satisfied or unsatisfied with the service”, which is exactly a form of binary review levels. We take use of the binary review levels (i.e., $s = 2$) to conduct our analyses mostly because of the model tractability, and we will make extensions to this basic assumption after our clarifications of the basic setting.

5.3.2. Second Period Consumers’ Review Interpretation Process

At the beginning of the second period, the early consumers’ reviews are observable to both the service provider and the late consumers. Consumers in the second period will update their beliefs regarding the service provider’s quality level μ_θ based on the information containing in the reviews. More precisely, their updated belief on the service provider’s quality level is on the strength of both their prior belief without reviews $\mu_{\theta NR}$ and their posterior belief with reviews $\mu_{\theta R}$. Studies on information system (Y. Jiang & Guo, 2015) for which they built consumers’ review systems of product industries give us theoretical basis for the review interpretation process. The late consumers’ belief on the service provider’s quality level remains unchanged in the absence of reviews, therefore, the prior belief without reviews always holds as $\mu_{\theta NR} = E[\mu_\theta | 0 < \mu_\theta < 1] = \frac{1}{2}$. However, in terms of the posterior belief with reviews, it is dependent on the information containing in the first period consumers’ reviews. We can denote this belief in the form $\mu_{\theta R} = \mu_{\theta min} + \bar{R}(x, y_1)(\mu_{\theta max} - \mu_{\theta min})$, where $\mu_{\theta min} = 0$ and $\mu_{\theta max} = 1$ as $\mu_\theta \sim U[0,1]$ in our earlier assumption. Early consumers base their behaviors of posting reviews regarding the service on their service outcomes after purchase. Meanwhile, late consumers are aware of the fact that $\bar{R}(x, y_1)$ (i.e., the mean of reviews) should reflect the expected service outcome of early consumers. If we keep all other factors fixed, we can derive the correspondence between the mean of reviews and the service outcome.

That is, a higher mean of reviews marks a higher expected service outcome, meanwhile, it signals a higher service provider's quality or certification.

According to the research on social learning (Papanastasiou & Savvab, 2017), we can denote the posterior expectation of the service provider's quality updated via late consumers' review interpretation process by a weighted average between the prior belief without reviews and the posterior belief with reviews. It can be demonstrated as follows:

$$E\left[\mu_\theta \middle| \bar{R}(x,y_1), N(x,y_1)\right] = \frac{\rho N(x,y_1)}{\rho N(x,y_1)+1}\mu_{\theta R} + \frac{1}{\rho N(x,y_1)+1}\mu_{\theta NR},$$

$$\text{where } \mu_{\theta R} = \bar{R}(x,y_1) \text{ and } \mu_{\theta NR} = \tfrac{1}{2}. \quad (34)$$

The weight set by second-period consumers on the posterior belief $\mu_{\theta R}$ grows with both the number of reviews $N(x,y_1)$ and ρ which we refer to as the review reliance parameter. The value of ρ can also reflect the informational influence of reviews.

To be specific, for a given ρ, more weight will be placed to the posterior belief with reviews if there are more reviews posted via online platforms. That is, greater number of consumer reviews make the posterior belief with reviews seem more reliable. Researchers in the field of information system have identified several justifications for this kind of consumers' rational expectations, such as preciseness and consciousness (Duan, Gu, & Whinston, 2008b; Liu, 2006) as well as credibleness and trustfulness (Clemen, 1989; Wong, 2000). As for the informational influence parameter ρ, it acts as a measurement of overall effects brought about by the review process. That is, larger value of ρ indicates that the review process is more influential in the formation of late consumers' posterior expectation regarding the quality. We generate the above belief interpretation process from research on combination forecasting models (Clemen, 1989; Wong, 2000) and similar methods in theoretical modeling have been commonly used in product reviews research (Kwark, Chen, et al., 2014).

5.3.3. Sequence of the Game

The service provider makes her effort level decision x at the beginning of the first period, which will remain unchanged intertemporally. She then makes her first period pricing decision p_1. We assume that the service provider makes the effort level decision before the pricing decision because it is generally supposed that the pricing decision is more flexible and easier to change than the effort level decision. Therefore, the pricing strategy possesses a shorter time horizon than the effort level strategy. The consumer makes his effort level decision y_t at the beginning of each period $t = 1,2$ to maximize his expected prepurchase utility in that corresponding period. Then early consumer arrives in the market and makes purchase decision after which he chooses whether to post reviews about his experience of the service via online platform. At the beginning of the second period, the reviews are released to both the service provider and consumers, meanwhile, the service provider offers her second period price p_2. The follower consumer arrives in the market and makes purchase decision based on his expected prepurchase utility. Note that in our basic model, we ignore the consumers' strategic behavior for their intertemporal purchase decisions mainly because of analytical tractability. We will alter this assumption and make further analyses in the model extension.

5.4. Model Analysis

In this section, we first analyze the equilibrium pricing strategy and effort level strategy under the circumstance in the absence of review process. Then we further take into consideration the review process by analyzing the equilibrium results of the service provider's pricing strategy and effort level strategy. Both scenarios have the same sequence of events, with the only difference in the second period consumers' belief over the quality level of the service provider.

5.4.1. Benchmark: Equilibria without Review Process

Under the benchmark case with no review process, consumers in each period only make their purchase decisions. That is, the first period consumers will not make review decisions anymore, as a result, the second period consumers can maintain the original belief over the quality level of the service provider, which is exactly captured by the limit case $\rho \to 0$ compared with the case with review process in our general setting.

In the first period, the service provider sets her effort level strategy x and the first period price p_1, and the early consumer sets his effort level in the first period y_1 simultaneously. Consumers form their expected quality level of the service provider which remain unchanged in the absence of review process, that is, $E[\mu_\theta | 0 < \mu_\theta < 1] = \frac{1}{2}$. The early consumer will make a purchase if and only if $EU_1 = E[r1_{O=1} - wy_1{}^2 - p_1] \geq 0$. In the second period, the service provider offers her second period price p_2, and the follower consumer makes the effort level decision in the second period y_2 at the same time, after which the follower consumer makes purchase decision and will purchase the service if and only if $EU_2 = E[r1_{O=1} - wy_2{}^2 - p_2] \geq 0$. The service provider's ex post payoff is then given by the following function:

$$\pi(p_1, p_2, x) = \mathbf{1}(EU_1 \geq 0)(p_1 - cx^2) + \mathbf{1}(EU_2 \geq 0)m(p_2 - cx^2), \tag{35}$$

while the consumers' expected utilities in each period are maximized by setting their own effort level as $EU_1(y_1) = E[r1_{O=1} - wy_1{}^2 - p_1] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - wy_1{}^2 - p_1$ for the first period consumer and $EU_2(y_2) = E[r1_{O=1} - wy_2{}^2 - p_2] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_2)r - wy_2{}^2 - p_2$ for the second period consumer. Thus, the optimal pricing strategy and effort level strategy are as follows.

Proposition 5.1. *In the absence of review process, there exists a unique equilibrium in pure strategies. The unique optimal pricing strategy and effort level strategy are*

$$\hat{p}_1^* = \hat{p}_2^* = \frac{r^2((-1+a)^2c + 2a^2w)}{16cw},$$

$$\hat{x}^* = \frac{ar}{4c},$$

$$\textit{and } \hat{y}_1^* = \hat{y}_2^* = \frac{(1-a)r}{4w}.$$

Furthermore, $\hat{p}_1^*(\hat{p}_2^*)$ *is decreasing in* a *when* $a < \frac{c}{c+2w}$, *and increasing in* a *otherwise;* $\hat{x}^*$ *is increasing in* a *while* $\hat{y}_1^*(\hat{y}_2^*)$ *is decreasing in* a*; and the service provider's profit is*

$$\hat{\pi}^* = \frac{(1+m)r^2((-1+a)^2c + a^2w)}{16cw},$$

which is decreasing in a *when* $a < \frac{c}{c+w}$, *and increasing in* a *otherwise.*

The above equilibrium results show that in the absence of review process the service provider maintains the same pricing strategy over periods, meanwhile, the early consumer and the late consumer make the same effort level strategy spanning two periods. Although all equilibrium results are independent of the review informational influence parameter ρ, they have closely connection with the work allocation parameter a, which we refer to as a measurement of the effort level contributions that both the service provider and the consumers make to the whole collaborative service. According to the aforementioned proposition, effort levels of the service provider and the consumers change in the opposite direction as the increase of work allocation a. As to achieve the objective of maximizing the expected profit, the service provider will enlarge her optimal effort level strategy when the specific weight of the service provider's effort level accounting for the outcome of collaborative service increases. The same rationale goes for the consumer's optimal effort level strategy to maximize his expected prepurchase utility throughout periods. However, prices $\hat{p}_1^*$ and $\hat{p}_2^*$ first decrease as work allocation a approaches $\frac{c}{c+2w}$ and increase afterwards. The same tendency holds for the optimal profit $\hat{\pi}^*$ with the only difference in interval boundary $\frac{c}{c+w}$. The reason behind this phenomenon is that when the work allocation a approaches the extreme boundary value 0 or 1, at least one of the parties (the service provider or the consumer) will adopt her or his maximized effort level strategy throughout the entire range of work allocation a, which will augment the resulting optimal value of prices and profit. Nevertheless, when the work allocation a is in the middle range, both parties cut down their effort level strategy from the maximized values, and the resulting overall effort level contribution are reduced by two parties input reduction. We can call the actions taken by both parties as "free riding" on each other when the work participations required from them are more even. As the so-called collaborative service require the effort contribution from both entities, either one of them has the incentive to contribute more only if the level of participation required from him is significant. Otherwise, both parties will deviate from his maximized effort contribution as the overall service outcome is dependent on the average participation level.

5.4.2. Service Provider's Strategy with Review Process

Let's now return to the general setting, where the first period consumer makes the review decision in addition to the purchase decision. In the first period, the service provider sets her effort level strategy and the first period price, then the early consumer expects the service quality to be $E[\mu_\theta | 0 < \mu_\theta < 1] = \frac{1}{2}$, which determines his expected service outcome and finally influence his expected net utility. That is, he will purchase the service if and only if $EU_1 = E[r1_{O=1} - wy_1{}^2 - p_1] \geq 0$, where the expected service outcome is the expectation of a two-point distribution. If the early consumer decides to purchase the service, after consumption, he forms his outcome score and chooses to report it truthfully through online review platform if and only if his expected utility from reviewing the service is nonnegative, that is $EV(\gamma) = E\left[\alpha 1_{t=1} - \beta \left|\frac{k}{s-1} - \tau_1(\gamma)\right| - \varphi\right] \geq 0$. At the beginning of second period, the follower observes the reviews posted by the early consumer and makes reference to online reviews to update his quality expectation, which is exactly $E[\mu_\theta | \bar{R}(x, y_1), N(x, y_1)]$. The late consumer then sets his purchase strategy in respect of the service if and only if $EU_2 = E[r1_{O=1} - w{y_2}^2 - p_2] \geq 0$. Anticipating the consumer's purchasing behavior over two periods, the service provider then determines the prices in each period to maximize her ex-post payoff:

$$\pi = \mathbf{1}(EU_1 \geq 0)(p_1 - cx^2) + \mathbf{1}(EU_2 \geq 0)m(p_2 - cx^2), \tag{36}$$

while the consumers' expected utilities in each period are maximized by setting their own effort level as $EU_1(y_1) = E[r1_{O=1} - w{y_1}^2 - p_1] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - w{y_1}^2 - p_1$ for the first period consumer and $EU_2(y_2) = E[r1_{O=1} - w{y_2}^2 - p_2] = E\left[\mu_\theta \middle| \bar{R}(x, y_1), N(x, y_1)\right](ax + (1-a)y_2)r - w{y_2}^2 - p_2$ for the second period consumer. It can be shown that the optimal prices over two periods are market clearing prices and the equilibrium results can be summarized as following propositions.

Proposition 5.2. *In the presence of review process, there exists a unique equilibrium in pure strategies when the informational influence parameter* $\rho \in \left[0, \min\left\{\frac{c(1+m)}{c\left(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)+a^2mr\mu_\theta}, \frac{2c(1+m)w}{2c\left(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)w+a^2mrw\mu_\theta+\sqrt{a^2mr^2w((-1+a)^2c(1+m)+a^2mw)\mu_\theta^2}}\right\}\right]$.

The unique optimal effort level strategy is

$x^* = \frac{ar(K+L)}{2(4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2-a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho+4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))}$ *where* $K = \frac{(-1+a)^4mr^2\mu_\theta^2\rho^2}{2w} + (-1+a)^2mr\mu_\theta\rho(2+\rho-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)$ *and* $L = 2w(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(-1-m+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))\rho)$, *and* $y_1^* = \frac{(1-a)r}{4w}$, $y_2^* = \frac{(-1+a)r(2c(1+m)(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(T-W)}{4w(-4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2+a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho+4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))}$ *where* $T = -(-1+a)^2r\mu_\theta\rho + w(-2+4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho))$ *and* $W = a^2r\mu_\theta\rho((-1+a)^2mr\mu_\theta\rho -$

$2w(1 - m + \rho - 2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho + 2\,ln\left(\frac{1-\Delta}{\Delta}\right)m\rho))$.

x^* is increasing in both ρ and a; y_1^* and y_2^* are both decreasing in a while y_2^* is decreasing in ρ when $\rho < \rho_1$, and increasing in ρ otherwise.

In order to clarify our equilibrium analyses and distinguish the influences brought about by distinct sources, we define two effects in our model framework. Firstly, *the reviewing effect* is mainly measured by the review informational influence parameter ρ, which we use to denote the availability/positivity of online consumer reviews. In our model setting, ρ is an exogeneous factor that evaluates the relative importance of the review process, the degree of which is greater indicating the reviewing effect is more influential and the review process accounting more in shaping the follower consumers' expected quality level of the service provider. Secondly, *the collaborating effect* is depicted by the work allocation parameter a and mainly helps to figure out the contribution of each party makes to the whole service provision. We use it to denote the level of participation required from consumers. When the extent of a approaches boundary values (1 or 0), the collaborative service is more likely to be a single principal service provision; however, as a falls into the middle range of its value, the collaborative attribute of the service is expanded, which somehow will not always be a profitable event.

The above proposition characterizes the equilibrium effort level strategies setting by both the service provider and the consumers in the presence of review process. When the review informational influence parameter ρ is within a certain range, the service provider and the consumers are both able to set their optimal effort level strategies thus the maximized profit and utilities are achieved. The reason lies in that if ρ is much greater than the above threshold, the reviewing effect dominates the second period consumers' posterior belief. Thus, no matter what the number of first period consumers making review decision is, that is, $N(x, y_1)$ doesn't count anymore, the follower consumers will always abandon their prior belief, which is not reasonable in practice. The optimal effort level strategies however vary in a quite opposite direction in respect of distinct entities. The service provider tends to raise her effort level provision as the enhancement of reviewing effect, because the review process helps follower consumers learn better of the service provider's quality level rather than always hold on to their prior belief. The service provider has more incentive to improve her own effort level in a more transparent public opinion environment. As for the collaborating effect regarding the service provider, she will raise her effort level when the success of service provision is attributed to her own in a greater extent. Thus, as the increase of work allocation parameter from 0 to 1, the importance of service provider's effort contribution is enhanced, which finally results in the increase of her optimal effort level provision. The collaborating effect with regard to the consumers is the same as that of the service provider. That is, as the increase of a from 0 to 1, the relative weight of consumers' effort contribution to the whole service outcome is impaired, the consumer has more incentive to have a free ride on the service provider's contribution but to lower his own effort level provision in each period. What intriguing us most is that the reviewing effect does not influence the follower consumer's effort provision monotonously. Specifically speaking, the reviewing effect first cuts down the second period consumer's optimal effort level in a primitive low range of ρ, while after which raises his effort level as the increase of ρ. This is because the reviewing effect to some extent helps second period consumers have a better understanding of the service provider's true quality level, which avoids the moral hazard brought about by the information asymmetry. It hence enhances the service provider's effort provision from the beginning of reviewing effect. However, as the second period consumer's misunderstanding of the reason why the service provider raises her effort level, he will first continue free riding on the service provider's effort contribution to lower his own effort level, for the reason he attributes this increase to collaborating effect. However, as the strengthening of reviewing effect, the public opinion environment becomes fairer for the consumer, which finally reminds the follower consumer to improve his effort contribution to the whole service provision. We further examine the influence of the existence of the review process by first exploring the optimal effort level strategies with and without review. The following proposition demonstrates the optimal effort level strategy in service provider's respect with and without review process.

Proposition 5.3. *In the presence of review process, the optimal effort level of the service provider* x^* *is always greater than the optimal effort level* $\hat{x}^*$ *in the absence of review process if and only if the work allocation parameter* $a > 0$ *and the review informational influence* *parameter* $\rho > 0$.

As our aforementioned propositions, the optimal effort level of the service provider with review process increases monotonously in both collaborating effect and reviewing effect. While without review process, only collaborating effect will strengthen her optimal effort level. However, no matter how much the increase contribution the service provider makes in her effort provision in the absence of review process, the optimal effort level strategy she sets in the presence of review process is always greater than that without review process. This is because in the presence of review process, second period consumers know better about the service provider's quality level owing to the early consumers' reviews regarding their service outcomes. Meanwhile, the reviewing effect dominates the collaborating effect, which mainly leads the service provider contributes more to the success of service provision, as less benefit the service provider can derive from the moral hazard due to information asymmetry. We further explore the optimal effort level strategy of consumers in both period with and without review process.

Proposition 5.4. *In the presence of review process, there exists a degree of review informational influence parameter* $\rho_2 = \frac{(1+m)(2cw-(-1+a)^2cr\mu_\theta-a^2rw\mu_\theta)}{a^2(1+2m-2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))rw\mu_\theta+c(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right))(1+m)(2w-(-1+a)^2r\mu_\theta)}$, *such that the follower consumers' optimal effort level is lower than or equal to the early consumers' optimal effort level (i.e.,* $y_2^* \le y_1^*$*) if and only if* $\rho \in [0, \rho_2]$, *otherwise,* $y_2^* > y_1^*$. *Also, it is easily verified that the early consumers' optimal effort level* y_1^* *in the presence of review process is the same as the that in the absence of review process* $\hat{y}_1^*(\hat{y}_2^*)$.

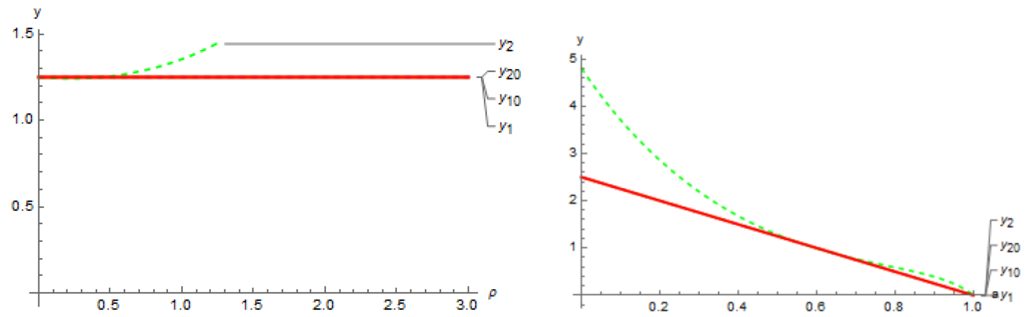


Figure 18. The Impact of ρ and a on Consumers' Optimal Effort Levels

Firstly, note that in any time period with no review taken into consideration of the consumer's expected purchase utility, the consumer will always choose his optimal effort level strategy as that in the absence of review process (i.e., $\frac{(1-a)r}{4w}$). From the expression we can conclude that it is independent of review informational influence parameter ρ. However, as for the second period consumer's optimal effort level in the presence of review process, we have demonstrated that this value first decreases with ρ when there is less extensive reviewing effect in shaping consumers' expectation of the service provider's quality level, then increases with ρ as the reviewing effect intensifying. There exists a certain review informational degree ρ_2 such that the follower consumer's optimal effort level in the presence of review process is thus greater than that of the early consumer without review reference. The reviewing effect helps amplify the consumer's effort provision only if its influence is no less than the opposite force exerted by the collaborating effect. That is to say, the review process not only makes the service provider enhance her effort level in a more transparent public opinion environment, but also promotes the consumer himself to contribute more effort to the whole service provision. The increased effort level of both parties will finally result in an overall improvement of the service provider's expected profit, which we will make detailed analyses subsequently.

Corollary 5.1. *In the presence of review process, there exist the degree of work allocation parameter* $a_1 = 1$, $a_2 = -\frac{(c(1+m)(2w-r\mu_\theta)(-1+(-1+2\,ln(\frac{1-\Delta}{\Delta}))\rho))}{(c(1+m)r\mu_\theta\left(-1+\left(-1+2\,ln(\frac{1-\Delta}{\Delta})\right)\rho\right)-A+B)}$ *(where* $A = \sqrt{}(c(1+m)rw\mu_\theta(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(2c(1+m)(-1-\rho+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)$ *and* $B = (2w-r\mu_\theta)(-1-m+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)+2(-1+ln\left(\frac{1-\Delta}{\Delta}\right))m)\rho))$ *) and* $a_3 = -\frac{\left(c(1+m)(2w-r\mu_\theta)\left(-1+\left(-1+2\,ln(\frac{1-\Delta}{\Delta})\right)\rho\right)\right)}{(c(1+m)r\mu_\theta\left(-1+\left(-1+2\,ln(\frac{1-\Delta}{\Delta})\right)\rho\right)+C+D)}$ *(where* $C = \sqrt{}(c(1+m)rw\mu_\theta(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(2c(1+m)(-1-\rho+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)$ *and* $D = (2w-r\mu_\theta)(-1-m+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)+2(-1+ln\left(\frac{1-\Delta}{\Delta}\right))m)\rho))$ *), such that the optimal effort level of the follower consumers* y_2^* *is greater than the early consumers' optimal effort level* y_1^* *if and only if* $a \in [0, a_2] \cup [a_3, a_1]$.

If we further consider the collaborating effect separately to analyze the influence of the variation of the work allocation parameter brought about to the consumer's optimal effort level, we find that when the reviewing effect is not too much higher, there exist some intervals of a

such that the follower consumer's optimal effort level is even lower than that in the absence of review process (i.e., $[a_2, a_3]$). More specifically, when the work allocation parameter is in a middle range, it makes both parties' contribution to the whole service provision more equal, the follower consumer has less incentive to improve his effort level; however, if the work allocation parameter is approaching both ends of the value, the follower consumer is more likely to enhance his effort level to an extent greater than that of the early consumer. Note that we have analyzed in the previous proposition the optimal effort level strategies of both early consumers and followers are decreasing in a no matter what the relative values of them are. Namely, the reduction rate of the early consumers' optimal effort level in a is a constant, while that of the followers varies in a. Concretely, when a is quite low over 0 to a_2, the contribution of consumers make to the whole service provision is relatively high. It to some extent amplifies the reduction rate of the followers' optimal effort level in a, resulting in the relative value of $y_2^* > y_1^*$ in $[0, a_2]$ changing to $y_2^* < y_1^*$ in $[a_2, a_3]$. However, in the middle range of a where both parties are more equally weighted in the service outcome combination, the reduction rate of followers' optimal effort level in a becomes flat thus finally resulting in $y_2^* > y_1^*$ in $[a_3, 1]$.

The following figure 19 in a two-dimensional plane depicts more clearly the joint influence of both the collaborating effect (a) and the reviewing effect (ρ). We can see the region where the follower consumer's optimal effort level is lower than that of the early consumer only if the reviewing effect is weak and the work allocation parameter is in a middle range; otherwise, the follower consumer contributes more to the service provision than the early consumer, where the shaded area in the figure marks the increasing consumer effort level plan in two consecutive periods.

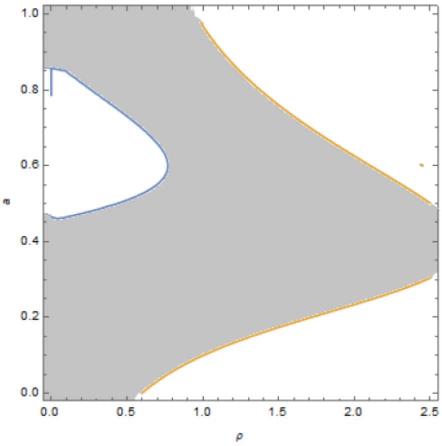


Figure 19. Follower Consumers' Optimal Effort Level in $a-\rho$ Plane

We next consider the optimal pricing strategy of the service provider with review process in each period. We can draw the conclusion as follows that holds in the presence of review process.

Proposition 5.5. *In the presence of review process, there exists a unique equilibrium in pure strategies as long as the informational influence parameter* ρ *is lower than* $\rho_2 = \frac{(1+m)(2cw-(-1+a)^2 cr\mu_\theta - a^2 rw\mu_\theta)}{a^2\left(1+2m-2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m)\right)rw\mu_\theta + c\left(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)(2w-(-1+a)^2 r\mu_\theta)}$. *The unique optimal pricing strategy is* p_1^* *and* p_2^*. *Note that* p_1^* *and* p_2^* *are both increasing in* ρ. *Furthermore, there exist thresholds* $\tilde{a}_1(\rho) \in [0,1]$ *and* $\tilde{a}_2(\rho) \in [0,1]$ *such that* p_1^* *is decreasing in* a *when* $a < \tilde{a}_1(\rho)$, *and increasing in* a *otherwise;* p_2^* *is decreasing in* a *when* $a < \tilde{a}_2(\rho)$, *and increasing in* a *otherwise*

The above proposition clarifies the impacts of both collaborating effect and reviewing effect on the equilibrium pricing strategy. Results show that both optimal first period price and second period price are increasing as the intensifying of reviewing effect. While the influence of collaborating effect on prices is non-monotonic. This is because the pricing strategy is a combination of effort levels from two parties. The variation tendency of prices due to collaborating effect and reviewing effect is also resulted from the efficacy that both effects exerting on optimal effort levels. Also note that the optimal pricing strategy takes the effort level cost from consumers into consideration. It will weaken the influence of collaborating effect and reviewing effect on the optimal consumers' effort level to a certain extent. Then the price strategies should reflect the impacts of both effects on the optimal service provider's effort level more obviously. Therefore, the optimal pricing decisions set by the service provider in both periods are growing with the strengthening of the reviewing effect, which is just like the input effort level of the service provider improving in the reviewing effect. As for the collaborating effect, although the optimal prices in both periods are first decreasing in a, this decreasing interval is quite small compared with the increasing interval in a. This is mainly due to that the collaborating effect on the optimal service provider's effort level is up-trend as a increases from 0 to 1, while that on the consumer's effort level is down-trend. The effect of a on consumer's effort level is also cut down in the overall consideration of the optimal pricing strategy. Thus, the increasing interval of collaborating effect (like the service provider) on optimal prices is larger than the decreasing interval (like the consumers). We then make a comparison between the optimal prices in the two consecutive periods and can draw the following conclusion.

Corollary 5.2. *There exist thresholds* $\tilde{a}_3(\rho) \in [0,1]$ *and* $\tilde{a}_4(\rho) \in [0,1]$ *such that if* $\tilde{a}_3(\rho) \leq a \leq \tilde{a}_4(\rho)$*, the service provider's optimal first period price* p_1^* *is greater than her optimal second period price* p_2^**, that is, the service provider adopts the decreasing pricing plan; otherwise, the service provider adopts the increasing pricing plan.*

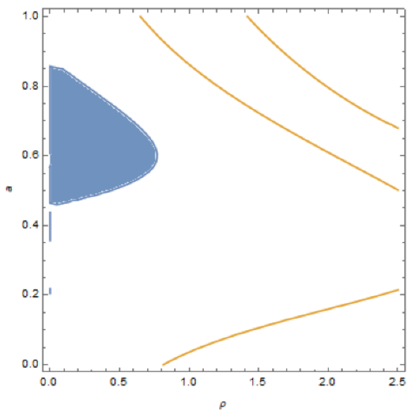


Figure 20. Service Provider's Optimal Pricing Strategy in $a-\rho$ Plane

We illustrate the optimal pricing strategy in a two-dimensional market structure to clarify the joint influence of both the collaborating effect (a) and the reviewing effect (ρ) on the optimal pricing plan.

The decreasing pricing plan marked in the shade area (i.e., the pricing strategy set by the seller is decreasing as time goes by) is similar to the decreasing optimal effort level of consumers over two periods which we have mentioned before. This mainly shows us that the decreasing pricing strategy over periods is only adopted by the service provider when the reviewing effect is mild (i.e., ρ is low) and the collaborating effect to a certain extent makes both parties cut down their effort levels from two ends value (i.e., a is in a middle range). Or else, the service provider will always take the increasing pricing strategy especially when the reviewing effect is quite intensified. That is to say, in a fairer public opinion environment, the service provider can raise her second period price higher than the first period price if follower consumers learn more about her service quality level. It will benefit both consumers and the service provider, thus in the end make the service market just and sound. The following proposition characterizes the variation review process brought about to the optimal pricing strategy compared with that in the absence of review process.

Proposition 5.6. *In the presence of review process, the optimal first period price (second period price) of the service provider* $p_1^*(p_2^*)$ *is always greater than the optimal first period price (second period price) in the absence of review process* $\hat{p}_1^*(\hat{p}_2^*)$ *if and only if the work allocation parameter* $a > 0$ *and the review informational influence parameter* $\rho > 0$.

This conclusion is almost the same as that comparison between the optimal service provider's effort level with and without review process. Although both effort levels of the service provider and the consumers contribute to the formation of the optimal pricing strategy, the contribution of the consumer's effort level is cut down due to his effort level cost being considered into the prices. It finally results in that the optimal pricing strategy just follows the same relative magnitude with and without review process as the optimal service provider's effort level. It is also intuitive that in the presence of review process, the transparent public opinion environment benefits the service provider to raise her prices in both periods compared with that in the absence of review process. However, this pricing enhancing strategy doesn't affect the market demand because consumers will still accept the service as before. Nevertheless, the review information regarding the service quality transforming over periods could make consumers discern the true quality level of the service provider better than that without any review, it induces the service provider adopting both higher effort level strategy and setting higher pricing strategy, which in turn makes the service market operate more efficiently. After our clarifications of optimal strategies taken by both parties, we finally arrive at the corresponding properties in respect of the optimal profit.

Proposition 5.7. *In the presence of review process, the service provider's optimal profit is* $\pi^* = \frac{r^2(a^2w((-1+a)^4m(1+m)r^2\mu_\theta^2\rho^2-4(-1+a)^2m(1+m)rw\mu_\theta\rho(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)+E))}{16w^2(4c(1+m)w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2-a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))}$ where $E = 4w^2(1+m+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)m\rho)^2)+(-1+a)^2c(1+m)((-1+a)^4mr^2\mu_\theta^2\rho^2-4(-1+a)^2mrw\mu_\theta\rho(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)+4w^2(m\left(1-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho\right)^2+\left(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho\right)^2$*, which is decreasing in the informational influence parameter* ρ *when* $\rho < \frac{(1+m)(2cw-(-1+a)^2cr\mu_\theta-a^2rw\mu_\theta)}{a^2(1+2m-2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))rw\mu_\theta+c(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right))(1+m)(2w-(-1+a)^2r\mu_\theta)} = \rho_2$ *, and increasing in* ρ *otherwise; furthermore, there exists a threshold of the work allocation parameter* $\tilde{a}_5(\rho) \in [0,1]$*, such that* π^* *is decreasing in* a *when* $a < \tilde{a}_5(\rho)$*, and increasing in* a *otherwise.*

Firstly, note that the threshold of the review informational influence parameter ρ_2 when the service provider's profit with review process varies from a declining range to an increasing range is the same as the threshold that the follower consumers' optimal effort level varies from lower than the early consumers' optimal effort level (i.e., $y_2^* \le y_1^*$) to greater than that. It is quite intuitive since the optimal profit depends on effort contributions from both the service provider and the consumers. As the optimal effort level of the service provider is monotonously growing as the reviewing effect strengthening, what affects the monotonicity of the profit in the reviewing effect is mainly the variation tendency of the optimal effort level of the consumers in ρ. That is to say, when the follower consumers' optimal effort level is lower than that of the early consumers (i.e., $\rho \in [0, \rho_2]$), the optimal profit is decreasing in ρ as well, which is principally due to the wrecking policy of follower consumers to the whole second period service provision. As the reviewing effect is relatively weaker than the opposite force exerted by the collaborating effect. When the follower consumer raises his effort level greater than the early consumer, the overall profit is increasing in ρ, as both effort levels of the follower consumer and the service provider are increasing in ρ.

As for the work allocation parameter a, the optimal profit is declining when the collaborating effect makes both parties in seemingly equal contribution to the whole service provision, while the profit is extensively increasing as one of the parties dominates the other in work allocation. This is mainly on account of both parties' free riding behavior due to collaborating effect. When both of them share more equal weight in the service outcome, collaborating effect makes both sides diminish their optimal effort levels from the corresponding maximum values, which results in the overall reduction of optimal profit. Otherwise, the profit is growing if either one of the parties dominates the other. The entity has more initiative to contribute as much as possible to the success of service outcome and meanwhile benefits the service provider in the form of the profit improvement. What follows is the comparison of optimal profits in the presence and in the absence of review process where we can further sketch the region that review process helps improve the optimal profit.

Proposition 5.8. *In the presence of review process, there exists a degree of review informational influence parameter* ρ_3*, such that the service provider achieves greater expected profit than she achieves in the absence of review process (i.e.,* $\pi^* \geq \hat{\pi}^*$ *) if and only if* $\rho \in [\rho_3, min\{\frac{c(1+m)}{c\left(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)+a^2mr\mu_\theta}, \frac{2c(1+m)w}{2c\left(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)w+a^2mrw\mu_\theta+\sqrt{a^2mr^2w((-1+a)^2c(1+m)+a^2mw)\mu_\theta^2}}$ *, otherwise,* $\pi^* < \hat{\pi}^*$*. Furthermore, there exist thresholds* $\tilde{a}_6(\rho) \in [0,1]$ *and* $\tilde{a}_7(\rho) \in [0,1]$ *such that if* $a \leq \tilde{a}_6(\rho)$ *or* $a \geq \tilde{a}_7(\rho)$*, the service provider achieves greater expected profit in the presence of review process than she achieves in the absence of review process.*

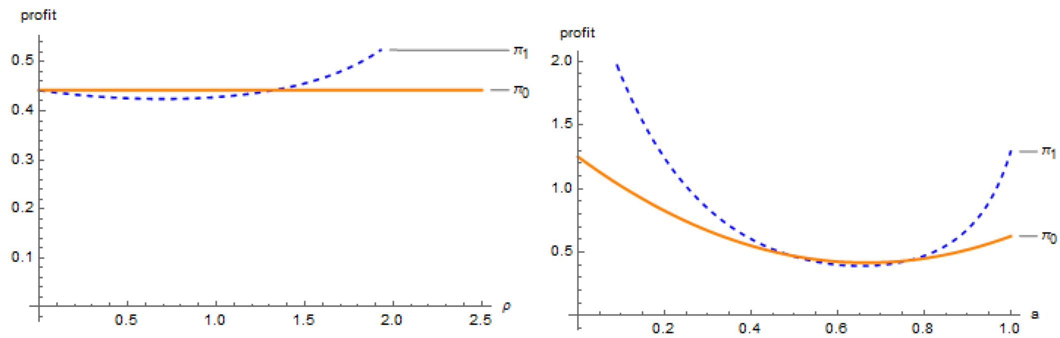


Figure 21. The Impact of ρ and a on Optimal Profits

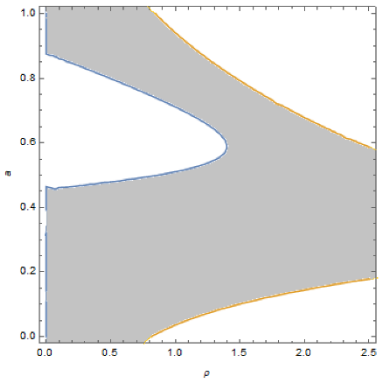


Figure 22. Service Provider's Optimal Profits in $a - \rho$ Plane

This conclusion elaborates the condition when the existence of review process can make the service provider earn more profit. Figure 22 shows the region when the presence of review process improves the profit. Under the prerequisite that makes the equilibrium results hold, the reviewing effect generally helps the service provider earn more as long as review informational influence parameter is not too low, and meanwhile the collaborating effect tends to allocate the service to either one of both parties more than the other. That is, the work allocation parameter a is approaching the two ends value (i.e., $[0, \tilde{a}_6(\rho)]$ and $[\tilde{a}_7(\rho), 1]$). This result is a comprehensive consideration of optimal effort level strategy and optimal pricing strategy in respect of both reviewing effect and collaborating effect. Namely, the profit improvement region is quite similar to the follower consumers' effort level enhancement region and the intertemporal pricing strategy increasing region. The importance of the online reviews is measured by the review informational influence parameter ρ. An investigation by BrightLocal shows that the significance of reviews is increasing in nearly every field of service. For example, the method to enhance the review importance can be the algorithms developed by Google, which can increase the visibility of consumers' reviews in searching process. Besides, the new platforms for review censorship and new media forms such as videos are springing up. All these methods in practice reflect the increase of the informational influence parameter ρ with the innovation of technology.

As for the example that can verify our conclusion, we refer to the knowledge payment service such as Zhihu Live Platform. In this kind of collaborative service, the keynote speaker acts as the service provider to offer their lectures to consumers. Some empirical studies (Shun

Cai et al. 2019) have shown that the consumers' reviews in the knowledge payment service have great influence on the sales volume of the service and the profit of the service provider. Their result points out that as the importance of reviews is increasing, the service provider can earn more from the service process. To achieve this goal, the keynote speaker should interact and communicate with the audience during the Live, for example, she can pay close attention to audience's questions and try her best to provide answers. Moreover, she can encourage audience to give feedbacks and reviews on her performance. All these actions can increase the significance of review process and finally benefit the service provider from obtaining more profits. Nevertheless, the scenario before the Live is similar to the case where the work allocation parameter is approaching to 1. As under this circumstance, only the service provider can make preparations for the service. The keynote speaker can take active part in the community activities and provide high-quality contents to enhance her reputations. For example, she can answer more questions in respect of the Live or publish more relevant influential articles. All these actions make the service provider devote more effort contributions and can finally benefit herself from earning more revenue.

Furthermore, what is also intriguing is the interactive influence of reviewing effect and collaborating effect on the optimal profit, which impels us to derive the following corollary.

Corollary 5.3. *We further explore that as the review informational influence parameter* ρ *increases from 0 (which is exactly the case in the absence of review process),* π^* *is changing more rapidly in the work allocation parameter* α *as the increase of* ρ*.*

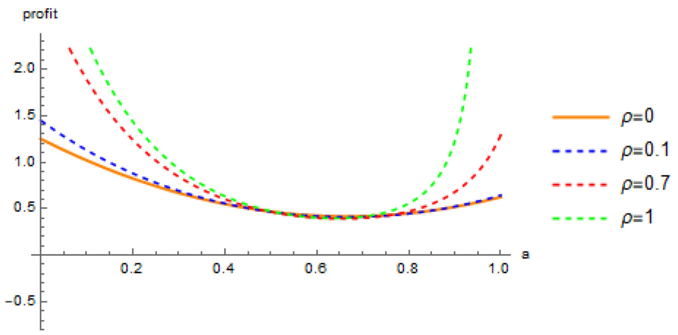


Figure 23. The Mutual Promotion Mechanism between α and ρ

Figure 23 graphically shows us the profit is changing with steeper slope owing to the collaborating effect when the reviewing effect is synchronously strengthening. This result can be considered as the description of the interaction relationship between the above two effects and we can conclude this interaction as “mutual promotion” mechanism. This so called “mutual promotion” mechanism helps us explain the phenomenon in online education or online consulting service. If the information of previous clients’ service outcome is more adequate (for example, the grade changes of students’ academic achievements after taking the online courses), namely, the public opinion environment is more transparent, the teachers (or consultants) have more incentive to devote more effort than before especially when they account more for the whole service outcome. This finally results in a positive feedback cycle of the service provision process. Meanwhile, when the service outcome is highly related to the clients (for example, keeping fit via online courses or consulting difficult legal cases), consumers’ own effort contribution to the service succuss is remarkable. Under this circumstance, more information provision regarding the service outcome, especially that of the similar cases, helps clients know their situation very well in their heart. Thus they may choose to devote more effort and strive to achieve an optimal solution regarding the service, which can also be considered as a positive feedback cycle of the service provision process.

5.5. Model Extensions

Our systematic analyses under the basic model setting, where the consumers’ rating scale is two, is generally manifested on the review platform by asking the consumers a question “whether you are satisfied or unsatisfied with the service”. It is a binary problem by answering yes or no. We thus have adopted this binary rating example (i.e., $s = 2$) as our basic model, mostly because it is analytically tractable. We further consider the model robustness. As in practice, there are also many online review platforms asking consumers to express their feelings after taking the service by the options like “good, general and bad”, which is a ternary question. This can be depicted as a ternary rating problem in our main model by setting the rating scale $s = 3$, which we next consider as our model extension.

5.5.1. Rating Level $s = 3$

We consider the case of ternary rating when $s = 3$. Following the same analytical procedure as the original model, we can obtain the similar conclusions in the main model, which prove the robustness of our basic model.

The benchmark case without review process is the same as that in our original model, the only difference lies in the case with review process. The second-period consumers' expected quality level of the service provider is changing to $E\left[\mu_\theta \middle| \bar{R}(x,y_1), N(x,y_1)\right] = \frac{-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2(\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)+ax\mu_\theta)\rho+2(-1+a)\mu_\theta\rho y_1}{-2+(-2+4\ln\left(\frac{1-\Delta}{\Delta}\right)-4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right))\rho}$ when $s=3$. All other analytical methods follow as before, we thus can derive the following equilibrium results in the presence of review process:

$$x^* = \frac{F+G}{2(-4c(1+m)w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)^2+a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)))}$$

where $F = am(r-ar)^2\mu\rho(1-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho+\frac{(-1+a)^2r\mu\rho}{2w})$ *and* $G = arw(2+(2-4\ln\left(\frac{1-\Delta}{\Delta}\right)+4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right))\rho)(1+m-(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m)-2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)(1+m))\rho+\frac{(-1+a)^2mr\mu\rho}{2w})$,

$$y_1^* = \frac{(1-a)r}{4w},$$

$$y_2^* = \frac{(-1+a)r(\mathrm{H}+\mathrm{I})}{4w(-4c(1+m)wJ+a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)))}$$

where $H = -a^2r\mu\rho\left(2(-1+m)w+\left(\left(-2-4\ln\left(\frac{1-\Delta}{\Delta}\right)(-1+m)+4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)(-1+m)\right)w+(-1+a)^2mr\mu\right)\rho\right)$; $I = 2c(1+m)(-1+(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)-2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right))\rho)(-(-1+a)^2r\mu\rho+w\left(-2+4\ln\left(\frac{1-\Delta}{\Delta}\right)\rho-4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho\right)$; $\mathrm{J} = (1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)^2$.

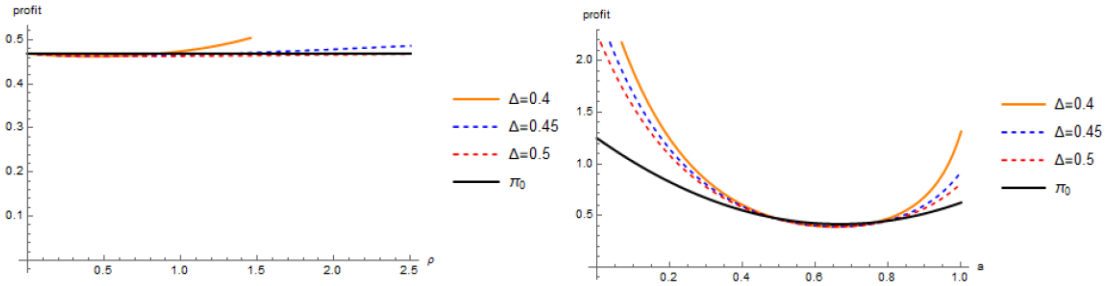
(37)

The optimal prices in each period $p_1^*(p_2^*)$ and the optimal expected profit π^* are all analytically solvable but are tedious in expressions so we don't intend to present them here. We thus resolve to numerical analysis to help us obtain some conclusions, and results show that the same inferences can be derived as the analytical results in our original model. Details can be seen in Appendix B of this chapter.

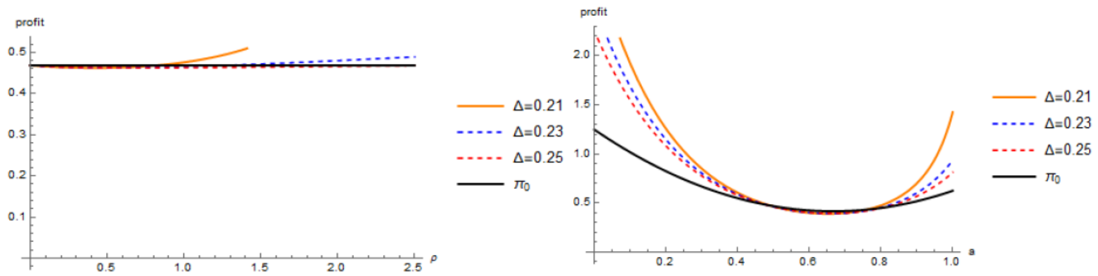
5.5.2. The Function of Review System

Apart from the results we have mentioned above, we are also interested in the factors affecting the function of review process, which can be demonstrated in the following conclusions.

Corollary 5.4. *The influence of review process is intensified as the decrease of* $\Delta = \frac{E[\alpha 1_{P=1}] - \varphi_1}{\beta}$, *whose value represents the consumers tendency to review the service:*



when $s = 2$*;*



when $s = 3$.

Figure 24. The Impact of Δ on the Review Process

Our results show that as the decrease of the value Δ, the consumers tendency to review the service become more prudent because the gap between the utility score and the rating level should be small enough for the consumer to choose his proper rating level. We can further discover that the decrease of Δ results in the shrinking of review volume $N(x, y_1)$, as fewer early consumers choose to post their reviews online unless their true service outcome score $\tau_1(\gamma)$ is more approximate to the closest rating level $\frac{k}{s-1}$. Furthermore, the decrease of value Δ is brought about by the reduction of expected reviewing benefit $E[\alpha 1_{P=1}]$ and the amplification of reviewing cost φ and β. That is to say, no matter the benefit of reviewing is decreasing or the cost of reviewing is increasing, the threshold for a certain early consumer to post his review is improving, which results in the overall quality of the reviews posted online is improving. This is intuitive since the noise or error brought about by the reviewing system where the rating scale can not perfectly match consumers' utility score should be reduced as the decrease of Δ. This finally promotes the overall function of review process and makes the reviews quite helpful for the service provider to make her optimal pricing strategy and effort level strategy (see Appendix B of this chapter), and then results in the promotion of optimal profit.

We next take into account the influence of reviews by the refinement of rating scales when s increases from 2 or 3 to more rating levels, and the analytical solutions under each case of rating level can be obtained as below.

When $s=4$, the second-period consumers' expected quality level of the service provider is changing to $E\left[\mu_\theta \middle| \bar{R}(x,y_1), N(x,y_1)\right] = \frac{-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2(\ln\left(\frac{1+3\Delta}{1-3\Delta}\right)+\ln\left(\frac{2+3\Delta}{2-3\Delta}\right)+ax\mu_\theta)\rho+2(-1+a)\mu_\theta\rho y_1}{-2+(-2+4\,ln\left(\frac{1-\Delta}{\Delta}\right)-4(\ln\left(\frac{1+3\Delta}{1-3\Delta}\right)+\ln\left(\frac{2+3\Delta}{2-3\Delta}\right)))\rho}$. All other analytical methods follow as before, we thus can derive the equilibrium results in the presence of review process in Appendix B of this chapter.

When $s=5$, the second-period consumers' expected quality level of the service provider is changing to $E\left[\mu_\theta \middle| \bar{R}(x,y_1), N(x,y_1)\right] = \frac{-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2(\ln\left(\frac{1+4\Delta}{1-4\Delta}\right)+\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)+\ln\left(\frac{3+4\Delta}{3-4\Delta}\right)+ax\mu_\theta)\rho+2(-1+a)\mu_\theta\rho y_1}{-2+(-2+4\,ln\left(\frac{1-\Delta}{\Delta}\right)-4(\ln\left(\frac{1+4\Delta}{1-4\Delta}\right)+\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)+\ln\left(\frac{3+4\Delta}{3-4\Delta}\right)))\rho}$. All other analytical methods follow as before. The analytical equilibrium results in the presence of review process are quite tedious, thus we omit to present them here.

When $s=6$, the second-period consumers' expected quality level of the service provider is changing to $E\left[\mu_\theta \middle| \bar{R}(x,y_1), N(x,y_1)\right] = \frac{-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2(\ln\left(\frac{1+5\Delta}{1-5\Delta}\right)+\ln\left(\frac{2+5\Delta}{2-5\Delta}\right)+\ln\left(\frac{3+5\Delta}{3-5\Delta}\right)+\ln\left(\frac{4+5\Delta}{4-5\Delta}\right)+ax\mu_\theta)\rho+2(-1+a)\mu_\theta\rho y_1}{-2+(-2+4\,ln\left(\frac{1-\Delta}{\Delta}\right)-4(\ln\left(\frac{1+5\Delta}{1-5\Delta}\right)+\ln\left(\frac{2+5\Delta}{2-5\Delta}\right)+\ln\left(\frac{3+5\Delta}{3-5\Delta}\right)+\ln\left(\frac{4+5\Delta}{4-5\Delta}\right)))\rho}$. All other analytical methods follow as before. The analytical equilibrium results in the presence of review process are quite tedious, thus we omit to present them here.

Based on the equilibrium results in each case of rating scale, we can further explore the influence of the rating refinement on the optimal service provider's profit as the proposition follows.

Proposition 5.9. *As the rating scale* s *is more refined, the influence of review process is intensified:*

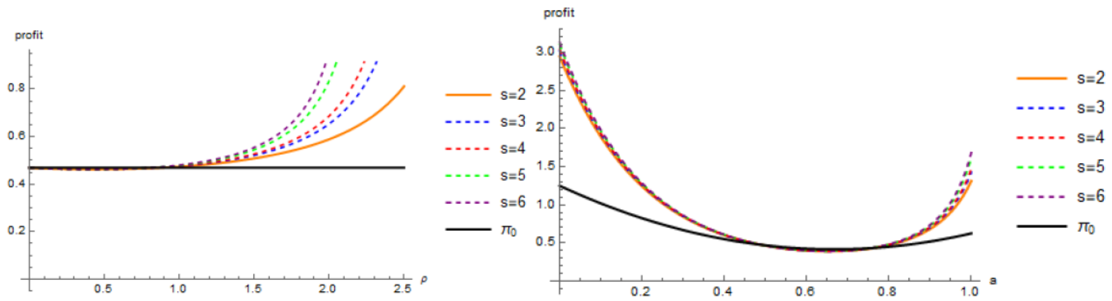


Figure 25. The Impact of s on the Review Function

This result is corresponding to the aforementioned **Corollary 5.4**. As the rating scale is more refined, the review volume $N(x, y_1)$ is more complicated in its expression. Meanwhile the value of Δ should be lower for the review volume to fall into the range $[0,1]$, only in which way can make the review system function well, i.e., the review volume should be nonnegative. The decrease of value Δ is brought about by the reduction of expected reviewing benefit $E[\alpha 1_{P=1}]$ and the amplification of reviewing cost φ and β. We have assumed the comparison cost φ is nondecreasing in rating scale s. Thus, as the rating scale is refined in its number, the comparison cost induces the value of Δ to reduce just as the above corollary shows. Therefore, the resulting effect is demonstrated as when the rating level increases in its number, the influence of review process is intensified. It will have the reviews quite helpful for the service provider to make her optimal pricing strategy and effort level strategy, which finally results in the promotion of the overall optimal profit.

Based on this conclusion, one may argue that as the rating level is more refined, the reviews are more significant in its function to promote service providers' revenue, why not we consider dividing the rating scale to infinite segments? In practice, however, what we confront with most is the number limited in its rating scale, such as 2, 3, 5, 10 etc. The reason behind this phenomenon is that when the consumers are making their reviewing decision, the comparison cost φ becomes higher as the refinement of rating scale s. Let's consider an extreme case, when the rating scale is larger than a certain threshold (i.e., $s > \bar{s}$), the comparison cost increases larger than the expected benefit of consumers' reviewing the service (i.e., $\varphi > E[\alpha 1_{P=1}]$). This will make it detrimental for consumers to post reviews as $EV(\gamma) < 0$ for all consumers. Thus, no consumers in the first period will post reviews anymore, which will result in the same situation as the benchmark, i.e., the service market without review process. This outcome will make the review system perform practically no function. Therefore, the service provider should consider the trade-off between the refinement of rating scale and the control of the upper limit regarding it, which to some extent ensures the comparison cost to increase no more than the expected utility gain. We will consider in our future study about the endogeny of the rating scale choice to help the service provider make scientific decisions.

5.5.3. Strategic Consumers' Behavior

We next consider the case when the consumers are differentiated in their valuation difference regarding the success of service. To be more specific, r here denotes the consumers' utility gain brought about by the service where we assume as a constant in our basic model. The consumers are heterogenous in this utility gain under the circumstance of strategic consumer assumption, namely, we assume consumers are uniformly distributed in r with the support $[0, \bar{r}]$. Following our derivation of the consumers demand by considering their waiting behavior, consumers' expected purchase utility function is endogenous. Meanwhile, consumers' expected review utility where the two-point distribution regarding consumers purchasing probability in the first period also become endogenous, which is also the coefficient of α that we use to represent the benefit or utility gain of consumer's reviewing the service. To be more specific, the review decision process made by first period consumers as a reviewing utility function is: $V(\gamma) = \alpha 1_{t=1} - \beta \left|\frac{k}{s-1} - \tau_1(\gamma)\right| - \varphi$, where $1_{t=1}$ is an indicator function such that $1_{t=1} = 1$ if consumers choose to purchase the service in the first period, and $1_{t=2} = 0$ if consumers postpone the purchase to the second period. It means a representative consumer derives positive utility gain from reviewing the service only if he makes purchase in the first period, that is, he is an early consumer, otherwise he has no chance to consider whether to post a review thus zero utility gain is obtained. This finally results in $\Delta = \frac{E[\alpha 1_{t=1}] - \varphi}{\beta} = \frac{[\Pr(t=1)*1 + \Pr(t=2)*0]\alpha - \varphi}{\beta} = \frac{[\Pr(EU_1 \geq EU_2 \geq 0)*1 + \Pr(EU_1 < EU_2)*0]\alpha - \varphi}{\beta}$ (which we use to denote consumers' tendency to review the service) becomes endogenous too. The analytical results are hard to obtain, and we thus will resolve to numerical simulation to get the conclusions in our future study.

5.6. Model Robustness Test

In this section, we consider the robustness of our model assumption in respect of both the distribution assumption and utility form assumption. The core principle lies in that the review interpretation process should maintain the same distribution regarding the mean rating $\bar{R}(x, y_1)$ and μ_θ, which denotes the service provider's quality and is known to the service provider but not to consumers. Namely, the existence of review process should guarantee the distribution of expected quality to be the same as the original assumption. We first test the robustness of the distribution assumption by following the original setting with the only change of the distribution. Then, we test the robustness of utility form by changing the utility function as well.

5.6.1. The Robustness of the General Setting when γ Follows a Normal Distribution

If we still follow the original assumption of u_t, that is

$$u_t = \mu_\theta(ax + (1-a)y_t) + \gamma, t = 1,2,$$

with the only difference in the assumption regarding the distribution of μ_θ and γ by changing from uniform distribution to normal distribution, i.e., $\mu_\theta \sim N[\hat{\mu}, \sigma_\mu^2]$ and $\gamma \sim N[\hat{\gamma}, \sigma_\gamma^2]$.

Then, we follow the same derivation process of second period expected quality (see Appendix A of this chapter), we can derive that when $s = 2$, which means the binary rating is taken in the service provider's review platform, the number of reviews can be simplified as $N(x, y_1) = 1 + Erf\left[\frac{\hat{\gamma} - \ln\left[\frac{1-\Delta}{\Delta}\right] + \mu_\theta(ax + (1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right] + Erf\left[\frac{\hat{\gamma} + \ln\left[\frac{1-\Delta}{\Delta}\right] + \mu_\theta(ax + (1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right]$, the mean of reviews can be simplified as $\bar{R}(x, y_1) = \frac{1 + Erf\left[\frac{\hat{\gamma} - \ln\left[\frac{1-\Delta}{\Delta}\right] + \mu_\theta(ax + (1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right]}{1 + Erf\left[\frac{\hat{\gamma} - \ln\left[\frac{1-\Delta}{\Delta}\right] + \mu_\theta(ax + (1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right] + Erf\left[\frac{\hat{\gamma} + \ln\left[\frac{1-\Delta}{\Delta}\right] + \mu_\theta(ax + (1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right]}$. The mean of reviews does not follow a normal distribution anymore, thus we can not derive any closed form results from the new assumption regarding the distribution of μ_θ and γ.

5.6.2. The Robustness of the Utility Function when γ Follows a Normal Distribution

We assume u_t is modeled as follows

$$u_t = (\mu_\theta + \gamma)(ax + (1-a)y_t), t = 1,2,$$

where $\mu_\theta \sim N[\hat{\mu}, \sigma_\mu^2]$ and $\gamma \sim N[\hat{\gamma}, \sigma_\gamma^2]$.

As a result, the probability of the service outcome being a success can be derived as

$$\Pr\{O = 1|\mu_\theta, x, y_t\} = \Pr\{u_t(\gamma) > 0|\mu_\theta, x, y_t\} = 1 - F(-\mu_\theta)$$

$$= 1 - \frac{1}{\sigma_\gamma\sqrt{2\pi}}\int_{-\infty}^{-\mu_\theta} e^{-\frac{(t-\hat{\gamma})^2}{2\sigma_\gamma^2}} dt, \mu_\theta \sim N[\hat{\mu}, \sigma_\mu^2], t = 1,2.$$

Then the derivation of the second period consumers' expected service quality under normal distribution assumption can be seen in Appendix A of this chapter. We can derive that when $s = 2$, which means the binary rating is taken in the service provider's review platform, the number of reviews can be simplified as $N(x, y_1) = 1 + Erf\left[\frac{\hat{\gamma} - \frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1} + \mu_\theta}{\sqrt{2}\sigma_\gamma}\right] + Erf\left[\frac{\hat{\gamma} + \frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1} + \mu_\theta}{\sqrt{2}\sigma_\gamma}\right]$, the mean of reviews can be simplified as $\bar{R}(x, y_1) = \frac{1 + Erf\left[\frac{\hat{\gamma} - \frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1} + \mu_\theta}{\sqrt{2}\sigma_\gamma}\right]}{1 + Erf\left[\frac{\hat{\gamma} - \frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1} + \mu_\theta}{\sqrt{2}\sigma_\gamma}\right] + Erf\left[\frac{\hat{\gamma} + \frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1} + \mu_\theta}{\sqrt{2}\sigma_\gamma}\right]}$. The mean of reviews does not follow a normal distribution anymore, thus we can not derive any closed form results from the new assumption regarding the utility function and the distribution of μ_θ and γ.

The above robustness test regarding our model assumption further verifies the robustness of our model setting. That is to say, our model setting can guarantee the expected quality that second period consumers infer from the review platform to hold its original distribution's property, and meanwhile we can obtain closed form equilibria from our model setup.

5.7. Concluding Remarks and Discussions

Based on our model, we consider an intertemporal model where there is a monopolistic service provider providing a kind of collaborative service to consumers that are review dependent via online platform over two consecutive periods. Our conclusions are in four aspects.

Firstly, as for the optimal effort levels, in the presence of review process, the optimal effort level of the service provider is always greater than the optimal effort level in the absence of review process. In the presence of review process, there exists a degree of review informational influence parameter such that the follower consumers' optimal effort level is lower than or equal to the early consumers' optimal effort level if and only if review informational influence parameter is lower than this threshold. Also, the early consumers' optimal effort level in the presence of review process is the same as the that in the absence of review process. The joint influence of both the collaborating effect and the reviewing effect on the optimal consumers' effort level is that the follower consumer's optimal effort level is lower than that of the early consumer only if the reviewing effect is weak and the collaborating effect is in middle range.

Secondly, the service provider adopts the decreasing pricing plan when the collaborating effect is in the middle range and the reviewing effect is in a low range. Also, in the presence of review process, the optimal first period price (second period price) of the service provider is always greater than the optimal first period price (second period price) in the absence of review process. It is also intuitive that in the presence of review process, the transparent public opinion environment benefits the service provider to raise her prices in both periods compared with that in the absence of review process. However, this pricing enhancing strategy doesn't affect the market demand. As the review information regarding the service quality transforming over periods could make consumers discern the true quality level of the service provider better than that without any review. It induces the service provider adopting both higher effort level strategy and higher pricing strategy.

Thirdly, in the presence of review process, there exists a degree of review informational influence parameter, such that the service provider achieves greater expected profit than she achieves in the absence of review process if and only if the review informational parameter is high than this threshold. Moreover, when the work allocation parameter is approaching two end values, the service provider achieves greater expected profit in the presence of review process than she achieves in the absence of review process. To achieve this goal, the service provider should interact and communicate with the consumers during the service provision process, for example, she can pay close attention to consumers' questions and try her best to provide answers. Moreover, she can encourage consumers to give feedbacks and reviews on her performance. All these actions can increase the significance of review process and finally benefit the service provider from obtaining more profits. Nevertheless, the scenario before the service provision is similar to the case where the work allocation parameter is approaching to 1. As under this circumstance, only the service provider can make preparations for the service. The service provider can take active part in the community activities and provide high-quality contents to enhance her reputations. For example, she can answer more questions in respect of the service or publish more relevant influential articles. All these actions make the service provider devote more effort contributions and can finally benefit herself from earning more revenue.

Finally, when the threshold for a certain early consumer to post his review is rising, which results in the overall quality of the reviews posted online is improving, the reviews are more helpful for the service provider to make her optimal pricing strategies and effort level strategy. This finally results in the promotion of optimal profit. Moreover, as the increase of rating scale, the influence of review process is intensified. It finally results in the improvement of the overall optimal profit. Thus, the service provider should consider making a tradeoff between the refinement of rating scales and the increase of consumers' comparison cost when posing reviews.

Besides, we would like to make some discussions about our results against prior research in this part. According to the study on social learning (Papanastasiou & Savvab, 2017), they established model with strategic consumers' information searching behaviors. What they mainly focus on is the strategic interaction between a monopolistic seller and consumers when a preannounced pricing or responsive pricing strategy is executed by the seller. They found that in the absence of social learing process, the seller would like to choose the decreasing price plans, while in the presence of this consumers' information updating, the price path may either rise or decline intertemporally depending on the specific pricing strategy. As for our model structure, we depict the consumers' information searching and dissemination behaviors with a detailed review process via analyses of followers' interpresation from earliers' messages. This give us implications when the seller should choose to participate in the review platform for information revelation. Our results shows that whether the seller choose the decreasing or increaing price plans depend on two factors: the availability of online reviews and the level of participation required from consumers. These two elements also determine the seller's profit gain through this review information implementation, which in turn influence the seller's motivation to use online forum by earing more revenue. Moreover, what we mainly study is the review process in a specific industry called "collaborative service". Many researchers have studied the collaborative service with detailed theoretical modeling description of the effort level contribution from both parties (Roels, 2014). We demonstrate a more concrete scenario by taking into consideration customer reviews. The same idea is also adopted by H. Sun and Xu (2018) in their study. While they establish a signal jamming model structure by introducing the influence of both review and service outcome on the optimal effort level decisions. They take use of the review and service outcome as two signals separately, which is distinct from ours. What we are mainly concerned is that we not only endogenously model the review generation process to analyze its influence on optimal effort levels, but also investigate service provider's pricing and profit gain in the collaborative service provision model structure with review revelations. In our future study, we will further add the strategic consumers' behaviors with the possible purchasing delays into our model analyses, thus the preannounced or responsive pricing may be taken by the seller and we will make comparisons between them.

Chapter 6

Appendix

6.1. Appendix for Chapter 2

6.1.1. Appendix A: Online-only Consumers' Demand Generation Process

The online product expected utility can be demonstrated as:

$$EU_1=\begin{cases} -rV, & \alpha\le\frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{VM_1} \\ \tau_2(\alpha M_1V-p_1-t(E[\beta]-f_1)^2)+(1-\tau_2)(-rV), & \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{Vm_1}>\alpha\ge\frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{VM_1} \\ \tau_2(\alpha M_1V-p_1-t(E[\beta]-f_1)^2)+(1-\tau_2)(\alpha m_1V-p_1-t(E[\beta]-f_1)^2), & \alpha\ge\frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{Vm_1} \end{cases}.$$

(1) When $-\delta>f_1-\frac{\sqrt{t\left(rV+\alpha m_1-p_1\right)}}{t}$ and $-\delta>f_1-\frac{\sqrt{t\left(rV+\alpha M_1-p_1\right)}}{t}$, the utility function can be simplified to

$$EU_1=\begin{cases} -rV, & \alpha\le\frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{VM_1} \\ \tau_2\left(\alpha M_1V-p_1-t\left(b+\frac{-\delta+\gamma-b}{2}-f_1\right)^2\right)+(1-\tau_2)(-rV), & \frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{Vm_1}>\alpha\ge\frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{VM_1} \\ \tau_2\left(\alpha M_1V-p_1-t\left(b+\frac{-\delta+\gamma'-b}{2}-f_1\right)^2\right)+(1-\tau_2)\left(\alpha m_1V-p_1-t\left(b+\frac{-\delta+\gamma'-b}{2}-f_1\right)^2\right), & \alpha\ge\frac{-rV+p_1+t(b+\varepsilon-f_1)^2}{Vm_1} \end{cases},$$

where $\gamma=min\ f_1+\frac{\sqrt{t(rV+\alpha m_1-p_1)}}{t}$ and $\gamma'=minf_1+\frac{\sqrt{t(rV+\alpha M_1-p_1)}}{t}$.

Partial keep demand and return for online only consumers are

$$\int_{\frac{(t(\delta+f_1)-\sqrt{t(V+rV-m_1-p_1)})\tau_2-2\sqrt{t\tau_2(-rV+(V+rV-p_1)\tau_2)}}{t\tau_2}}^{\frac{(t(\delta+f_1)-\sqrt{t(V+rV-m_1-p_1)})\tau_2+2\sqrt{t\tau_2(-rV+(V+rV-p_1)\tau_2)}}{t\tau_2}}\left(-\frac{-rV(-1+\tau_2)-V\tau_2+\frac{\left(t(b-\delta-f_1)+\sqrt{t(V+rV-m_1-p_1)}\right)^2\tau_2}{4t}+p_1\tau_2}{m_1\tau_2}\right)db$$

$$=\frac{8(t\tau_2(-rV+(V+rV-p_1)\tau_2))^{3/2}}{3t^2m_1\tau_2^3}$$

$$(1-\tau_2)*\int_{f_1+\frac{t\delta-2\sqrt{t(V+rV-p_1)}-\sqrt{t(V+rV-m_1-p_1)}}{t}}^{f_1+\frac{t\delta+2\sqrt{t(V+rV-p_1)}-\sqrt{t(V+rV-m_1-p_1)}}{t}}\left(\frac{V+rV-\frac{\left(t(b-\delta-f_1)+\sqrt{t(V+rV-m_1-p_1)}\right)^2}{4t}-p_1}{M_1}\right)db=\frac{8(t(V+rV-p_1))^{3/2}(1-\tau_2)}{3t^2M_1}$$

or

$$\int_0^1\left(-\frac{-rV(-1+\tau_2)-V\tau_2+\frac{\left(t(b-\delta-f_1)+\sqrt{t(V+rV-m_1-p_1)}\right)^2\tau_2}{4t}+p_1\tau_2}{m_1\tau_2}\right)db$$

$$=-\frac{rV-V\tau_2-rV\tau_2+\frac{(-(-t(\delta+f_1)+\sqrt{t(V+rV-m_1-p_1)})^3+(t-t(\delta+f_1)+\sqrt{t(V+rV-m_1-p_1)})^3)\tau_2}{12t^2}+p_1\tau_2}{m_1\tau_2}$$

$$(1-\tau_2)*\int_0^1\left(\frac{V+rV-\frac{\left(t(a-\delta-f_1)+\sqrt{t(V+rV-m_1-p_1)}\right)^2}{4t}-p_1}{M_1}\right)da=\frac{(V+rV-\frac{-(-t(\delta+f_1)+\sqrt{t(V+rV-m_1-p_1)})^3+(t-t(\delta+f_1)+\sqrt{t(V+rV-m_1-p_1)})^3}{12t^2}-p_1)(1-\tau_2)}{M_1}.$$

All keep demand for online only consumers are

$$\int_{f_1+\frac{t\delta-2\sqrt{t(V-p_1)}-\sqrt{t(V+rV-M_1-p_1)}}{t}}^{f_1+\frac{t\delta+2\sqrt{t(V-p_1)}-\sqrt{t(V+rV-M_1-p_1)}}{t}}\left(-\frac{-V+\frac{\left(t(b-\delta-f_1)+\sqrt{t(V+rV-M_1-p_1)}\right)^2}{4t}+p_1}{\mu_3}\right)db=\frac{8(t(V-p_1))^{3/2}}{3t^2\mu_3}$$

or

$$\int_0^1\left(-\frac{-V+\frac{\left(t(b-\delta-f_1)+\sqrt{t(V+rV-M_1-p_1)}\right)^2}{4t}+p_1}{\mu_3}\right)db=-\frac{-V+\frac{-(-t(\delta+f_1)+\sqrt{t(V+rV-M_1-p_1)})^3+(t-t(\delta+f_1)+\sqrt{t(V+rV-M_1-p_1)})^3}{12t^2}+p_1}{\mu_3},$$

where $\mu_3=\tau_2M_1+(1-\tau_2)m_1$.

(2) When $-\delta<f_1-\frac{\sqrt{t(rV+\alpha m_1-p_1)}}{t}$ and $-\delta<f_1-\frac{\sqrt{t(rV+\alpha M_1-p_1)}}{t}$, the utility function is

$$EU_1=\begin{cases}-rV, & \alpha\le\frac{-rV+p_1}{VM_1}\\ \tau_2(\alpha M_1V-p_1)+(1-\tau_2)(-rV), & \frac{-rV+p_1}{Vm_1}>\alpha\ge\frac{-rV+p_1}{VM_1}\\ \tau_2(\alpha M_1V-p_1)+(1-\tau_2)(\alpha m_1V-p_1), & \alpha\ge\frac{-rV+p_1}{Vm_1}\end{cases}.$$

.

It means the consumers are identical in their attribute of horizontal dimension when browse through online channel only.

Partial keep demand and return for online only consumers are

$K=\frac{p_1+\frac{1-\tau_2}{\tau_2}rV}{M_1V}$ and $R=(1-\tau_2)(\frac{p_1+\frac{1-\tau_2}{\tau_2}rV}{M_1V}-min\{\frac{rV+p_1}{m_1V},0\})$.

All keep demand for online only consumers is $K=\frac{p_1}{\tau_2M_1V+(1-\tau_2)m_1V}$

We assume the ratio between online-only consumers and omnichannel consumers is N, which without loss of generality, we normalize the population of omnichannel consumers to be one and that of online-only consumers to be N. When we take both online only consumers and omnichannel consumers into consideration, we follow the derivation process stated in Appendix B to obtain equilibrium results, what makes changes to our original results is the value of p_1 and p_2, but the relationship between p_1 and p_2 does not make change:

$p_2=\frac{-VM_2+M_2p_1+V\mu_1}{\mu_1}$ where $\mu_1=\tau_1M_1+(1-\tau_1)m_1$.

6.1.2. Appendix B: Derivation of Equilibrium Results

We first elaborate the demand generation process of our main model at the beginning.

The nonnegative expected utility of online product $EU_1>0$ should be satisfied before purchase decision. Therefore, the consumers with market profiles below the curve $\frac{-rV+p_1+t(b-f_1)^2}{VM_1}$ will exit the market, as their expected utility of online purchase is negative $-rV$. This fraction of consumers has sufficient low level of sensitivity coefficient to unit quality improvement. However, when the consumers market profile is between the curve

$\frac{-rV+p_1+t(b-f_1)^2}{VM_1}$ and $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$, they take shape of the prior purchase utility as $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(-rV)$. The nonnegative expected utility condition $EU_1 > 0$ should be satisfied when the consumers market profile is above the curve $\alpha = \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V}$ in the two-dimensional market structure. Namely, the demand of online product is $D_1 = \int_0^1 \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V} \mathrm{d}b$ when the product demand area intersects with the market side boundaries as Case (ii) shows. While the demand of online product is $D_1 = \int_{f_1-\sqrt{\frac{VM_1-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}}}^{f_1+\sqrt{\frac{VM_1-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}}} \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V} \mathrm{d}b$ when the demand area doesn't intersect with the market side boundaries as Case (i) shows. Moreover, consumers in this market profile will return the online product with probability $1-\tau_1$, when the post-purchase experienced attribute of online product is not good. That is to say, the return quantity of online product is $R = (1-\tau_1)(\int_0^1 \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V} \mathrm{d}b - \int_0^1 \frac{-rV+p_1+t(b-f_1)^2}{Vm_1} \mathrm{d}b)$, which marks the ideal case when the consumers with market profile above the curve $\alpha = \frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$ is included in the market profile above the curve $\alpha = \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V}$. Also, the product return quantity area under this circumstance intersects with the market side boundaries as Case (ii) depicts. Otherwise, in more general cases, the return quantity of online product is $R = (1-\tau_1)(\int_{b_1}^{b_2} \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V} \mathrm{d}b - \int_{b_1}^{b_2} \frac{-rV+p_1+t(b-f_1)^2}{Vm_1} \mathrm{d}b)$, where b_1 and b_2 are the intersection points of the curve $\alpha = \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V}$ and $\alpha = \frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$. We further derive the condition for the existence of consumers' return faction. It can be demonstrated as the case when the online product price is not too high: $p_1 < rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1}$, only when there exists consumers' market profile between the curve $\alpha = \frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$ and $\alpha = \frac{\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{\tau_1 M_1 V}$.

However, when the online product price is higher than the threshold: $p_1 \geq rV\frac{\tau_1 M_1+(1-\tau_1)m_1}{(\tau_1 M_1+(1-\tau_1)m_1)-m_1}$, the return fraction of consumers will not exist. The disutility of online product return $-rV$ is relatively lower than the utility for consumers to keep the online product that doesn't perform well after purchase, then all consumers will keep the product with the prior purchase expected utility $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(\alpha m_1 V - p_1 - t(b-f_1)^2)$. The nonnegative expected utility condition $EU_1 > 0$ should be satisfied when the consumers are located in the market profile above the curve $\alpha = \frac{p_1+t(b-f_1)^2}{\tau_1 M_1 V+(1-\tau_1)m_1 V}$.

Namely, the demand of online product is $D_1 = \int_0^1 \frac{p_1+t(b-f_1)^2}{\tau_1 M_1 V+(1-\tau_1)m_1 V} \mathrm{d}b$ when the product demand area intersects with the market side boundaries as Case (ii) depicts. While the demand of online product is $D_1 = \int_{f_1-\sqrt{\frac{V(\tau_1 M_1+(1-\tau_1)m_1)-p_1}{t}}}^{f_1+\sqrt{\frac{V(\tau_1 M_1+(1-\tau_1)m_1)-p_1}{t}}} \frac{p_1+t(b-f_1)^2}{\tau_1 M_1 V+(1-\tau_1)m_1 V} \mathrm{d}b$ when the demand area doesn't intersect with the market side boundaries as Case (i) depicts.

After our clarifications of online product demands based on the prior purchase expected utility, we next take offline product utility into consideration in the omnichannel environment. The consumer will only purchase the offline product when it renders greater actual utility than its online counterpart, i.e., $U_2 > EU_1$. To be specific, in the case $U_2 = \alpha M_2 V - p_2 - t(b-f_2)^2$, the quality performance of the offline product is high. The consumer will factor his uncertainty about the online product quality performance as the two aforementioned probabilities: $p(\odot_1 = m_1 | \odot_2 = M_2) = 1-\tau_1$, $p(\odot_1 = M_1 | \odot_2 = M_2) = \tau_1$. When the consumers market profile is between the curve $\frac{-rV+p_1+t(b-f_1)^2}{VM_1}$ and $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$, the consumers take shape of the prior purchase utility in respect of the online product as $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(-rV)$. Therefore, $U_2 > EU_1$ should be satisfied when the consumers market profile is above the curve $\alpha = \frac{p_2+t(b-f_2)^2-\tau_1 p_1-\tau_1 t(b-f_1)^2-(1-\tau_1)rV}{M_2 V-M_1\tau_1 V}$ in the two dimensional market structure.

However, when the consumers' quality sensitivity is above the curve $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$, the consumers will always choose to keep the online product regardless of the potential possibility of product return, thus their ex ante expected utility of online product is $EU_1 = \tau_1(\alpha M_1 V - p_1 - t(b-f_1)^2) + (1-\tau_1)(\alpha m_1 V - p_1 - t(b-f_1)^2)$ .Therefore, $U_2 > EU_1$ should be satisfied when the consumers market profile is above the line $\frac{-p_1+p_2+tf_2^{\,2}-tf_1^{\,2}-2(f_2-f_1)tb}{M_2 V-(\tau_1 M_1 V+(1-\tau_1)m_1 V)}$. There are also two parallel cases regarding the offline product demand when consumers are located in this market profile with sufficiently high quality sensitivity. To be specific, if the online product price and offline product price satisfy the condition

$$min\left\{\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}\right\} < max\left\{\frac{-p_1+p_2+tf_2^{\,2}-tf_1^{\,2}-2(f_2-f_1)tb}{M_2 V-(\tau_1 M_1 V+(1-\tau_1)m_1 V)}\right\},$$

the offline product demand area is the common market space above the line $\frac{-p_1+p_2+tf_2^{\,2}-tf_1^{\,2}-2(f_2-f_1)tb}{M_2 V-(\tau_1 M_1 V+(1-\tau_1)m_1 V)}$ and the curve $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$.

However, when the online product price and offline product price satisfy the opposite condition:

$$min\left\{\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}\right\} < max\left\{\frac{-p_1+p_2+tf_2^{\,2}-tf_1^{\,2}-2(f_2-f_1)tb}{M_2 V-(\tau_1 M_1 V+(1-\tau_1)m_1 V)}\right\},$$

we can combine the scenario when the consumers' quality sensitivity above the curve $\frac{-rV+p_1+t(b-f_1)^2}{Vm_1}$ with that below the curve, thus the offline product demand area is the market space below the curve $\alpha = \frac{p_2+t(b-f_2)^2-\tau_1 p_1-\tau_1 t(b-f_1)^2-(1-\tau_1)rV}{M_2 V-M_1\tau_1 V}$ in the two dimensional market structure. For expressive complexity, the specific forms of demands regarding both offline product and online product in each scenario are elaborated in the below cases.

Following the demand generation process demonstrated in our main model, we can derive the equilibrium results in each case by considering "all keep scenario" and "partial keep scenario" respectively.

We demonstrate the case when return quantity exists as "partial keep" scenario while the case when return quantity doesn't exist as "all keep" scenario.

That is, when $p_1 < V - rV\frac{\mu_1}{M_1-\mu_1}$, $max\left\{\frac{V(1+r)-p_1-t(b-f_1)^2}{M_1}\right\} < max\left\{\frac{V-p_1-t(b-f_1)^2}{\mu_1}\right\} < max\left\{\frac{\tau_1 V-\tau_1 p_1-\tau_1 t(b-f_1)^2-(1-\tau_1)rV}{\tau_1 m_1}\right\}$, return faction of online product exists and we refer to it as "partial keep" scenario.

When $p_1 > V - rV\frac{\mu_1}{M_1-\mu_1}$, return faction of online product doesn't exist, and we refer to it as "all keep".

(a) Partial keep ($p_1 < V - rV\frac{\mu_1}{M_1-\mu_1}$)

(1) Vertical dominance

Case (i)

When $2\sqrt{\frac{V-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}} > 1$ holds, the online product demand area intersects with the market boundary as Case (i) shows.

The profit function is $\pi = (p_1 - C_1) * \left(\int_0^1 \frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}db - \left(\int_0^{h1}\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db + \int_{h1}^{h2}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db + \int_{h2}^{1}\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db\right)\right) + (p_2 - C_2) * \left(\int_0^{h1}\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db + \int_{h1}^{h2}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db + \int_{h2}^{1}\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db\right) - c * (1-\tau_1) * \left(\int_0^1 \frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}db - \int_0^1\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db\right)$ where $h_1 = f_1 - \frac{M_1\sqrt{t\left(\frac{V+rV-p_1+\frac{M_1(t(f_1-f_2)^2-p_1+p_2)}{m_2-\mu_1}}{M_1^2}+\frac{t(f_1-f_2)^2}{(m_2-\mu_1)^2}\right)}}{t} + \frac{(f_1-f_2)M_1}{m_2-\mu_1}$ and $h_2 = f_1 + \frac{M_1\sqrt{t\left(\frac{V+rV-p_1+\frac{M_1(t(f_1-f_2)^2-p_1+p_2)}{m_2-\mu_1}}{M_1^2}+\frac{t(f_1-f_2)^2}{(m_2-\mu_1)^2}\right)}}{t} + \frac{(f_1-f_2)M_1}{m_2-\mu_1}$ are the intersection points of $\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ and $\frac{V+rV-p_1-t(b-f_1)^2}{M_1}$. It is difficult for us to derive the optimal solutions directly, thus we resort to the envelope theorem to first figure out the relationships between f_1 and f_2 under optimal condition. That is, if we take FOC of the profit with respect to f_i,

$$\frac{\partial \pi}{\partial f} = \left[(p_2 - C_2)\frac{\partial D_2}{\partial f} + (p_1 - C_1)\frac{\partial D_1}{\partial f} - c\frac{\partial R}{\partial f}\right] + \frac{\partial \pi}{\partial p}\frac{\partial p}{\partial f},$$

Where $\frac{\partial \pi}{\partial p} = 0$ under the optimal condition. Also $\frac{\partial \pi}{\partial f}$ should also equal 0 under optimal condition, thus we can derive the results of design feature f_1 and f_2 as $f_1 = \frac{1}{2}; f_2 = \frac{1}{2}$.

Next, we further derive the seller's pricing strategy under the FOC $\frac{\partial \pi}{\partial p} = 0$, where $p_2 = \frac{-Vm_2 - rVm_2 + m_2p_1 + M_1p_1 + V\mu_1 + rV\mu_1 - p_1\mu_1}{M_1}$ and $p_1 = \frac{12rVM_1^2 + tm_1m_2\tau_1 - 24Vm_1m_2\tau_1 - 24rVm_1m_2\tau_1 + 12cm_1M_1\tau_1 + 12C_1m_1M_1\tau_1 - 12C_2m_1M_1\tau_1 - 12cM_1^2\tau_1 + tM_1^2\tau_1 - 12VM_1^2\tau_1 - 12rVM_1^2\tau_1 - 12C_1M_1^2\tau_1 - tm_1\mu_1\tau_1 + 24Vm_1\mu_1\tau_1 + 24rVm_1\mu_1\tau_1 - 12cm_1M_1\tau_1^2 + 12cM_1^2\tau_1^2}{24(-m_1m_2 - M_1^2 + m_1\mu_1)\tau_1}$.

This inner solution only exists when the Hessian matrix is negative definitive, that is, $\frac{\partial^2 \pi}{\partial {p_1}^2} \leq 0$, $\frac{\partial^2 \pi}{\partial {p_2}^2} \leq 0$ and $\frac{\partial^2 \pi}{\partial {p_1}^2}\frac{\partial^2 \pi}{\partial {p_2}^2} - (\frac{\partial^2 \pi}{\partial p_1 \partial p_2})^2 \geq 0$ which only set up when $m_2 < \mu_1$, then the profit is $\pi = \frac{t^2m_1^2(m_2-\mu_1)^2\tau_1^2 + 24tm_1^2M_1(m_2-\mu_1)(-C_1+C_2+c(-1+\tau_1))\tau_1^2 - 24m_1M_1^3(-C_1+C_2+c(-1+\tau_1))\tau_1(12rV-(12c+t-12(1+r)V+12C_1)\tau_1+12c\tau_1^2) + 2m_1M_1^2\tau_1(12rV(24c+t-24(1+r)V+24C_1)(m_2-\mu_1)+(72(c+C_1-C_2)^2m_1+(t(12c+t)-12(24cr+t+rt)V+12tC_1)(m_2-\mu_1))\tau_1-12c(12(c+C_1-C_2)m_1+t(m_2-\mu_1))\tau_1^2+72c^2m_1\tau_1^3)+M_1^4(12rV+\tau_1(12c+t-12(1+r)V+12C_1-12c\tau_1))^2}{576m_1M_1^2(M_1^2+m_1(m_2-\mu_1))\tau_1^2}$

Also, we can derive the demand of both products as the offline product demand is $\frac{tm_1(-m_2+\mu_1)\tau_1 + 12m_1M_1\tau_1(c+C_1-C_2-c\tau_1) + M_1^2(12rV-(12c+t-12(1+r)V+12C_1)\tau_1+12c\tau_1^2)}{24M_1(M_1^2+m_1(m_2-\mu_1))\tau_1}$ while the online product demand is $\frac{-12rVM_1(M_1^2+m_1(2m_2+M_1-2\mu_1))+(m_1-M_1)((12c+t-12(1+r)V+12C_1)M_1^2+m_1(tm_2-12(c+C_1-C_2)M_1-t\mu_1))\tau_1+12c(m_1-M_1)^2M_1\tau_1^2}{24m_1M_1(M_1^2+m_1(m_2-\mu_1))\tau_1}$ and the return fraction of online product is $\frac{(-1+\tau_1)(12rVM_1(M_1^2+m_1(2m_2+M_1-2\mu_1))+(m_1-M_1)(-(12c+t-12(1+r)V+12C_1)M_1^2+m_1(-tm_2+12(c+C_1-C_2)M_1+t\mu_1))\tau_1-12c(m_1-M_1)^2M_1\tau_1^2)}{24m_1M_1(M_1^2+m_1(m_2-\mu_1))\tau_1}$.

However, when $m_2 > \mu_1$, only the corner solution exists for both p_1 and p_2, that is $p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ or $p_1 = V - \frac{t}{4} - \frac{1-\tau_1}{\tau_1}rV$ and meanwhile $p_2 = p_1 - \frac{(m_2-\mu_1)(V+rV-p_1)}{M_1}$ under which condition $\frac{V+rV-p_1}{M_1} = \frac{p_1-p_2}{m_2-\mu_1}$.

The offline product demand is $-\frac{t}{12M_1} + \frac{rV}{M_1-\mu_1}$ when $p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ while the online product demand is $\frac{\frac{(m_1-M_1)((t-12rV)M_1-t\mu_1)}{M_1(M_1-\mu_1)} - \frac{12rV}{\tau_1}}{12m_1}$ and the return fraction of online product is $\frac{(-1+\tau_1)(12rVM_1(M_1-\mu_1)-(-m_1+M_1)(-(t-12rV)M_1+t\mu_1)\tau_1)}{12m_1M_1(M_1-\mu_1)\tau_1}$ which finally results in the profit $\pi = \frac{1}{12m_1M_1(M_1-\mu_1)^2\tau_1}(M_1^3(-c + V - C_1 + c\tau_1)(-12rV - (t - 12rV)\tau_1) + tm_1\mu_1\tau_1(-rVm_2 + \mu_1(-c + rV - C_1 + C_2 + c\tau_1)) + M_1^2((t - 12rV)m_1(-C_1 + C_2 + c(-1 + \tau_1))\tau_1 + \mu_1(12rV(-2c + (2 + r)V - 2C_1) + (-2ct + (36cr + (2 + r)t)V - 12r(1 + r)V^2 - 2(t - 6rV)C_1)\tau_1 + 2c(t - 6rV)\tau_1^2)) - M_1(rV(-t + 12rV)m_1m_2\tau_1 + \mu_1^2(-c + V + rV - C_1 + c\tau_1)(12rV + t\tau_1) + m_1\mu_1\tau_1(rV(t - 12rV) - 2c(t - 6rV) + 2(t - 6rV)(-C_1 + C_2 + c\tau_1))))$.

The offline product demand is $\frac{6rV+t\tau_1}{6M_1\tau_1}$ when $p_1 = V - \frac{t}{4} - \frac{1-\tau_1}{\tau_1}rV$ and the online product demand is $\frac{1}{6}(\frac{t}{m_1} - \frac{6rV+t\tau_1}{M_1\tau_1})$ and the return fraction of online product is

$\frac{(-1+\tau_1)(6rVm_1+t(m_1-M_1)\tau_1)}{6m_1M_1\tau_1}$ which results in the profit $\pi = \frac{1}{24m_1M_1^2\tau_1^2}(-tM_1^2\tau_1(4rV + \tau_1(4c + t - 4(1+r)V + 4C_1 - 4c\tau_1)) + m_1(6rV + t\tau_1)(4M_1\tau_1(c + C_1 - C_2 - c\tau_1) - m_2(4rV + t\tau_1) + \mu_1(4rV + t\tau_1)))$.

Case (ii)

When $2\sqrt{\frac{V-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}} < 1$ holds, the online product demand area doesn't intersect with the market boundary as Case (ii) shows.

The profit function is $\pi = (p_1 - C_1) * \left(\int_{f_1-\frac{\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}}^{\frac{tf_1\tau_1+\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}} \frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}db - \left(\int_{f_1-\frac{\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}}^{h1} \frac{V+rV-p_1-t(b-f_1)^2}{M_1}db + \int_{h1}^{h2} \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db + \int_{h2}^{\frac{tf_1\tau_1+\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}} \frac{V+rV-p_1-t(b-f_1)^2}{M_1}db\right)\right) + (p_2 - C_2) * \left(\int_{f_1-\frac{\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}}^{h1} \frac{V+rV-p_1-t(b-f_1)^2}{M_1}db + \int_{h1}^{h2} \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db + \int_{h2}^{\frac{tf_1\tau_1+\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}} \frac{V+rV-p_1-t(b-f_1)^2}{M_1}db\right) - c * (1-\tau_1) * \left(\int_{k_1}^{k_2} \frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}db - \int_{k_1}^{k_2} \frac{V+rV-p_1-t(b-f_1)^2}{M_1}db\right)$, where $h_1 = f_1 - \frac{M_1\sqrt{t(\frac{V+rV-p_1+\frac{M_1(t(f_1-f_2)^2-p_1+p_2)}{m_2-\mu_1}}{M_1^2}+\frac{t(f_1-f_2)^2}{(m_2-\mu_1)^2})}}{t} + \frac{(f_1-f_2)M_1}{m_2-\mu_1}$ and $h_2 = f_1 + \frac{M_1\sqrt{t(\frac{V+rV-p_1+\frac{M_1(t(f_1-f_2)^2-p_1+p_2)}{m_2-\mu_1}}{M_1^2}+\frac{t(f_1-f_2)^2}{(m_2-\mu_1)^2})}}{t} + \frac{(f_1-f_2)M_1}{m_2-\mu_1}$ are the intersection points of $\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ and $\frac{V+rV-p_1-t(b-f_1)^2}{M_1}$; $k_1 = f_1 + \frac{m_1M_1\sqrt{\frac{t(m_1-M_1)(rVM_1+(m_1-M_1)(V+rV-p_1)\tau_1)}{m_1^2M_1^2\tau_1}}}{t(m_1-M_1)}$ and $k_2 = f_1 - \frac{m_1M_1\sqrt{\frac{t(m_1-M_1)(rVM_1+(m_1-M_1)(V+rV-p_1)\tau_1)}{m_1^2M_1^2\tau_1}}}{t(m_1-M_1)}$ are the intersection points of $\frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}$ and $\frac{V+rV-p_1-t(b-f_1)^2}{M_1}$. Under the envelope theorem, the optimal design feature policy can be derived as $f_1 = \frac{1}{2}; f_2 = \frac{1}{2}$.

Next, we further derive the seller's pricing strategy under the FOC $\frac{\partial\pi}{\partial p} = 0$, where $p_2 =$

$\frac{-Vm_2-rVm_2+m_2p_1+M_1p_1+V\mu_1+rV\mu_1-p_1\mu_1}{M_1}$ and $p_1 = p_1^*$. This inner solution only exists when the Hessian matrix is negative definitive, that is, $\frac{\partial^2\pi}{\partial {p_1}^2} \leq 0$, $\frac{\partial^2\pi}{\partial {p_2}^2} \leq 0$ and $\frac{\partial^2\pi}{\partial {p_1}^2}\frac{\partial^2\pi}{\partial {p_2}^2} - (\frac{\partial^2\pi}{\partial p_1 \partial p_2})^2 \geq 0$ which only set up when $m_2 < \mu_1$. Given the complexity of the profit function, the demand of both products and the return fraction are also quite complex.

However, when $m_2 > \mu_1$, only the corner solution exists for both p_1 and p_2, that is $p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ or $p_1 = V - \frac{t}{4} - \frac{1-\tau_1}{\tau_1}rV$ and meanwhile $p_2 = p_1 - \frac{(m_2-\mu_1)(V+rV-p_1)}{M_1}$ under which condition $\frac{V+rV-p_1}{M_1} = \frac{p_1-p_2}{m_2-\mu_1}$.

The offline product demand is $\frac{2rV\sqrt{\frac{rtV(\mu_1+M_1(-1+\tau_1))\tau_1}{M_1-\mu_1}}(-\mu_1+M_1(1+2\tau_1))}{3tM_1(M_1-\mu_1)\tau_1^2}$ when $p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ while the online product demand is $\frac{2rV\sqrt{\frac{rtV(\mu_1+M_1(-1+\tau_1))\tau_1}{M_1-\mu_1}}(2M_1(\mu_1+M_1(-1+\tau_1))+m_1(\mu_1-M_1(1+2\tau_1)))}{3tm_1M_1(M_1-\mu_1)\tau_1^2}$ and the return fraction of online product is $\frac{4rVM_1(-1+\tau_1)(\mu_1+M_1(-1+\tau_1)-m_1\tau_1)\sqrt{\frac{rtV(m_1-M_1)(-\mu_1-M_1(-1+\tau_1)+m_1\tau_1)}{m_1^2M_1(M_1-\mu_1)\tau_1}}}{3t(m_1-M_1)(M_1-\mu_1)\tau_1}$ which finally results in the profit complex in its form.

The offline product demand is $\frac{6rV+t\tau_1}{6M_1\tau_1}$ when $p_1 = V - \frac{t}{4} - \frac{1-\tau_1}{\tau_1}rV$ and the online product demand is $\frac{1}{6}(\frac{t}{m_1} - \frac{6rV+t\tau_1}{M_1\tau_1})$ and the return fraction of online product is $-\frac{(-1+\tau_1)(4rVm_1+t(m_1-M_1)\tau_1)^2}{6m_1^2M_1^2\tau_1^2\sqrt{\frac{t(m_1-M_1)(4rVm_1+t(m_1-M_1)\tau_1)}{m_1^2M_1^2\tau_1}}}$ which results in the profit $\pi = \frac{1}{24m_1M_1^2\tau_1^2}(-tM_1^2\tau_1(4rV + \tau_1(4c + t - 4(1+r)V + 4C_1 - 4c\tau_1)) + m_1(6rV + t\tau_1)(4M_1\tau_1(c + C_1 - C_2 - c\tau_1) - m_2(4rV + t\tau_1) + \mu_1(4rV + t\tau_1)))$.

(2) Horizontal dominance

Case (i)

When $2\sqrt{\frac{V-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}} \leq 1$ holds, the online product demand area doesn't intersect with the market boundary as Case (i) shows.

The profit function is $\pi = (p_1 - C_1) * \left(\int_{f_1-\frac{\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}}^{\frac{tf_1\tau_1+\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}} \frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1} db - \left(\int_{f_1-\frac{\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}}^{\mathrm{h1}} \frac{V+rV-p_1-t(b-f_1)^2}{M_1} db + \int_{\mathrm{h1}}^{\mathrm{b2}} \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1} db\right)\right) + (p_2 - C_2) *$

$\left(\int_{f_1-\frac{\sqrt{t\tau_1(-rV+(V+rV-p_1)\tau_1)}}{t\tau_1}}^{\mathrm{h1}}\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db+\int_{\mathrm{h1}}^{\mathrm{b2}}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db\right)-c*(1-\tau_1)*$

$\left(\int_{k_1}^{k_2}\frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}db-\int_{k_1}^{k_2}\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db\right)$, where $h_1=f_1-\frac{M_1\sqrt{t\left(\frac{V+rV-p_1+\frac{M_1(t(f_1-f_2)^2-p_1+p_2)}{m_2-\mu_1}}{M_1^2}+\frac{t(f_1-f_2)^2}{(m_2-\mu_1)^2}\right)}}{t}+\frac{(f_1-f_2)M_1}{m_2-\mu_1}$ is the intersection point of $\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ and $\frac{V+rV-p_1-t(b-f_1)^2}{M_1}$; $k_1=f_1+\frac{m_1M_1\sqrt{\frac{t(m_1-M_1)(rVM_1+(m_1-M_1)(V+rV-p_1)\tau_1)}{m_1^2M_1^2\tau_1}}}{t(m_1-M_1)}$ and $k_2=f_1-\frac{m_1M_1\sqrt{\frac{t(m_1-M_1)(rVM_1+(m_1-M_1)(V+rV-p_1)\tau_1)}{m_1^2M_1^2\tau_1}}}{t(m_1-M_1)}$ are the intersection points of $\frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}$ and $\frac{V+rV-p_1-t(b-f_1)^2}{M_1}$. $b_2=\frac{tf_1^2-tf_2^2+p_1-p_2}{2tf_1-2tf_2}$ is the intersection point of the line $\alpha=\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ when $\alpha=0$. It is difficult for us to derive the optimal solutions directly, thus we resort to the envelope theorem to first figure out the relationships between f_1 and f_2 under optimal condition. That is, if we take FOC of the profit with respect to f_i,

$$\frac{\partial\pi}{\partial f}=\left[(p_2-C_2)\frac{\partial D_2}{\partial f}+(p_1-C_1)\frac{\partial D_1}{\partial f}-c\frac{\partial R}{\partial f}\right]+\frac{\partial\pi}{\partial p}\frac{\partial p}{\partial f},$$

where $\frac{\partial\pi}{\partial p}=0$ under the optimal condition. Also $\frac{\partial\pi}{\partial f}$ should also equal 0 under optimal condition, thus we can derive the results of design feature f_1 and f_2 as $f_1=\frac{1}{2}$; $f_2=\frac{1}{2}\pm\sqrt{\frac{2((1+r)tV+\sqrt{t^2(V+rV-p_1)(V+rV-p_2)})-t(p_1+p_2)}{t^2}}$.

Under the optimal condition $\frac{\partial\pi}{\partial p}=0$, while no inner solution can be found, thus we resolve to the corner solution under the condition $b_2=\frac{tf_1^2-tf_2^2+p_1-p_2}{2tf_1-2tf_2}\in[0,1]$ with the corner solution $p_1=\frac{1}{4}(-t+4V+4rV)$ and $p_2=\frac{1}{4}(-t+4V+4rV)$ which doesn't satisfy the condition $f_2=\frac{1}{2}\pm\sqrt{\frac{2((1+r)tV+\sqrt{t^2(V+rV-p_1)(V+rV-p_2)})-t(p_1+p_2)}{t^2}}\in[0,1]$ and should be deleted. The other corner solution $p_1=V-rV\frac{\mu_1}{M_1-\mu_1}, p_2=V(1+r+\frac{rM_1}{-M_1+\mu_1})$ should also guarantee the condition $b_2=\frac{tf_1^2-tf_2^2+p_1-p_2}{2tf_1-2tf_2}\in[0,1]$ and $f_2=\frac{1}{2}\pm\sqrt{\frac{2((1+r)tV+\sqrt{t^2(V+rV-p_1)(V+rV-p_2)})-t(p_1+p_2)}{t^2}}\in[0,1]$ which only holds when $r<\frac{t(M_1-\mu_1)}{16VM_1}$.

The offline product demand, the online product demand and the profit are complex in form. However, under the condition that $p_1=V-rV\frac{\mu_1}{M_1-\mu_1}$, the return fraction will not exist as this case will converge to the "All keep" scenario.

Case (ii)

When $2\sqrt{\frac{V-p_1-\frac{1-\tau_1}{\tau_1}rV}{t}}>1$ holds, the online product demand area intersects with the market boundary as Case (ii) shows.

The profit function is $\pi=(p_1-C_1)*\left(\left(\int_0^1\frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}db-\left(\int_0^{\mathrm{h1}}\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db+\int_{\mathrm{h1}}^{\mathrm{b2}}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db\right)\right)\right)+(p_2-C_2)*\left(\int_0^{\mathrm{h1}}\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db+\int_{\mathrm{h1}}^{\mathrm{b2}}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db\right)-c*(1-\tau_1)*\left(\left(\int_0^1\frac{V-p_1-t(b-f_1)^2-\frac{1-\tau_1}{\tau_1}rV}{m_1}db-\int_0^1\frac{V+rV-p_1-t(b-f_1)^2}{M_1}db\right)\right)$, where $h_1=f_1-\frac{M_1\sqrt{t\left(\frac{V+rV-p_1+\frac{M_1(t(f_1-f_2)^2-p_1+p_2)}{m_2-\mu_1}}{M_1^2}+\frac{t(f_1-f_2)^2}{(m_2-\mu_1)^2}\right)}}{t}+\frac{(f_1-f_2)M_1}{m_2-\mu_1}$ is the intersection point of $\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ and $\frac{V+rV-p_1-t(b-f_1)^2}{M_1}$, $b_2=\frac{tf_1^2-tf_2^2+p_1-p_2}{2tf_1-2tf_2}$ is the intersection point of the line $\alpha=\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ when $\alpha=0$. However, there is no real solution to f_i under the optimal condition $\frac{\partial\pi}{\partial f_i}=0$.

(b) All keep ($p_1>V-rV\frac{\mu_1}{M_1-\mu_1}$)

(1) Vertical dominance

Case (i)

When $2\sqrt{\frac{V-p_1}{t}}\le 1$ holds, the online product demand area doesn't intersect with the market boundary as Case (i) shows.

The profit function is $\pi=(p_2-C_2)*\left(\int_{f_1-\sqrt{\frac{V-p_1}{t}}}^{h1}\frac{V-p_1-t(b-f_1)^2}{\mu_1}db+\int_{h1}^{h2}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db+\int_{h2}^{f_1+\sqrt{\frac{V-p_1}{t}}}\frac{V-p_1-t(b-f_1)^2}{\mu_1}db\right)+(p_1-C_1)*\left(\int_{f_1-\sqrt{\frac{V-p_1}{t}}}^{f_1+\sqrt{\frac{V-p_1}{t}}}\frac{V-p_1-t(b-f_1)^2}{\mu_1}db-\left(\int_{f_1-\sqrt{\frac{V-p_1}{t}}}^{h1}\frac{V-p_1-t(b-f_1)^2}{\mu_1}db+\int_{h1}^{h2}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db+\int_{h2}^{f_1+\sqrt{\frac{V-p_1}{t}}}\frac{V-p_1-t(b-f_1)^2}{\mu_1}db\right)\right)$ where $h_1=\frac{f_1m_2-f_2\mu_1}{m_2-\mu_1}-\frac{\mu_1\sqrt{\frac{t(m_2^2(V-p_1)+m_2(-2V+t(f_1-f_2)^2+p_1+p_2)\mu_1+(V-p_2)\mu_1^2)}{(m_2-\mu_1)^2\mu_1^2}}}{t}$ and $h_2=\frac{f_1m_2-f_2\mu_1}{m_2-\mu_1}+\frac{\mu_1\sqrt{\frac{t(m_2^2(V-p_1)+m_2(-2V+t(f_1-f_2)^2+p_1+p_2)\mu_1+(V-p_2)\mu_1^2)}{(m_2-\mu_1)^2\mu_1^2}}}{t}$ are the intersection points of $\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ and $\frac{V-p_1-t(b-f_1)^2}{\mu_1}$. It is difficult for us to derive the optimal solutions directly, thus we resort to the envelope theorem to first figure out the relationships between f_1 and f_2 under optimal condition. That is, if we take FOC of the profit with respect to f_i,

$$\frac{\partial\pi}{\partial f} = \left[(p_2 - C_2)\frac{\partial D_2}{\partial f} + (p_1 - C_1)\frac{\partial D_1}{\partial f} - c\frac{\partial R}{\partial f}\right] + \frac{\partial\pi}{\partial p}\frac{\partial p}{\partial f},$$

where $\frac{\partial\pi}{\partial p} = 0$ under the optimal condition. Also $\frac{\partial\pi}{\partial f}$ should also equal 0 under optimal condition, thus we can derive the results of design feature f_1 and f_2 as $f_1 = \frac{1}{2}; f_2 = \frac{1}{2}$.

Under the optimal condition $\frac{\partial\pi}{\partial p} = 0$, where $p_2 = \frac{m_2 p_1}{\mu_1} - \frac{V(m_2-\mu_1)}{\mu_1} = \frac{1}{5}(2V + 3C_2)$ and $p_1 = \frac{5Vm_2-3V\mu_1+3C_2\mu_1}{5m_2}$. This inner solution only exists when the Hessian matrix is negative definitive, that is, $\frac{\partial^2\pi}{\partial {p_1}^2} \le 0$, $\frac{\partial^2\pi}{\partial {p_2}^2} \le 0$ and $\frac{\partial^2\pi}{\partial {p_1}^2}\frac{\partial^2\pi}{\partial {p_2}^2} - (\frac{\partial^2\pi}{\partial p_1 \partial p_2})^2 \ge 0$ which only set up when $m_2 < \mu_1$, then the profit is $\pi = \frac{8\sqrt{\frac{3}{5}}t^2 m_2(\frac{(V-C_2)\mu_1}{tm_2})^{5/2}}{25\mu_1^2}$. Also, we can derive the demand of both products as the offline product demand is $\frac{4\sqrt{\frac{3}{5}}t(\frac{(V-C_2)\mu_1}{tm_2})^{3/2}}{5\mu_1}$ while the online product demand is 0.

However, when $m_2 > \mu_1$, only the corner solution exists for both p_1 and p_2, that is $p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ or $p_1 = V - \frac{t}{4}$ and meanwhile $p_2 = p_1 - \frac{(m_2-\mu_1)(V+rV-p_1)}{M_1}$ under which condition $\frac{V+rV-p_1}{M_1} = \frac{p_1-p_2}{m_2-\mu_1}$.

The offline product demand is $\frac{4rV\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}}}{3(M_1-\mu_1)}$ when $p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ while the online product demand is 0 which results in the profit $\pi = -\frac{4rV(rVm_2-(V-C_2)(M_1-\mu_1))\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}}}{3(M_1-\mu_1)^2}$.

The offline product demand is $\frac{(t+4rV)\mu_1^2\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}}+tM_1(t-\mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}})}{6tM_1\mu_1}$ when $p_1 = V - \frac{t}{4}$ and the online product demand is $\frac{\mu_1^2(\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2})^{3/2}}{6t^2}$ which results in the profit $\pi = \frac{1}{24tM_1^2\mu_1}((t+4rV)^2\mu_1^3\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}} + (t+4rV)M_1\mu_1(t^2 - (t - 4C_1 + 4C_2)\mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}}) - tM_1^2(t^2 - 4tV + 4C_1\mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}} + 4C_2(t - \mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}})) - (t+4rV)m_2((t + 4rV)\mu_1^2\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}} + tM_1(t - \mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}})))$.

Case (ii)

When $2\sqrt{\frac{V-p_1}{t}} > 1$ holds, the online product demand area intersects with the market boundary as

Case (ii) shows. The profit function is $\pi = (p_2 - C_2) * (\int_0^{h1} \frac{V-p_1-t(b-f_1)^2}{\mu_1} db + \int_{h1}^{h2} \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1} db + \int_{h2}^{1} \frac{V-p_1-t(b-f_1)^2}{\mu_1} db) + (p_1 - C_1) * (\int_0^{1} \frac{V-p_1-t(b-f_1)^2}{\mu_1} db - (\int_0^{h1} \frac{V-p_1-t(b-f_1)^2}{\mu_1} db + \int_{h1}^{h2} \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1} db + \int_{h2}^{1} \frac{V-p_1-t(b-f_1)^2}{\mu_1} db))$ where $h_1 = \frac{f_1 m_2 - f_2 \mu_1}{m_2-\mu_1} - \frac{\mu_1 \sqrt{\frac{t(m_2^2(V-p_1)+m_2(-2V+t(f_1-f_2)^2+p_1+p_2)\mu_1+(V-p_2)\mu_1^2)}{(m_2-\mu_1)^2\mu_1^2}}}{t}$ and $h_2 = \frac{f_1 m_2 - f_2 \mu_1}{m_2-\mu_1} + \frac{\mu_1 \sqrt{\frac{t(m_2^2(V-p_1)+m_2(-2V+t(f_1-f_2)^2+p_1+p_2)\mu_1+(V-p_2)\mu_1^2)}{(m_2-\mu_1)^2\mu_1^2}}}{t}$ are the intersection points of $\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ and $\frac{V-p_1-t(b-f_1)^2}{\mu_1}$. Under the envelope theorem, the optimal design feature policy can be derived as $f_1 = \frac{1}{2}; f_2 = \frac{1}{2}$.

Under the optimal condition $\frac{\partial \pi}{\partial p} = 0$, where $p_2 = \frac{m_2 p_1}{\mu_1} - \frac{V(m_2-\mu_1)}{\mu_1} = \frac{1}{2}(V + C_2) - \frac{t m_2}{24\mu_1}$ and $p_1 = \frac{1}{24}(-t + 24V + \frac{12(-V+C_2)\mu_1}{m_2})$. This inner solution only exists when the Hessian matrix is negative definitive, that is, $\frac{\partial^2 \pi}{\partial {p_1}^2} \leq 0$, $\frac{\partial^2 \pi}{\partial {p_2}^2} \leq 0$ and $\frac{\partial^2 \pi}{\partial {p_1}^2}\frac{\partial^2 \pi}{\partial {p_2}^2} - (\frac{\partial^2 \pi}{\partial p_1 \partial p_2})^2 \geq 0$ which only set up when $m_2 < \mu_1$, then the profit is $\pi = \frac{(t m_2 + 12(-V+C_2)\mu_1)^2}{576 m_2 \mu_1^2}$. Also, we can derive the demand of both products as the offline product demand is $\frac{V-C_2}{2m_2} - \frac{t}{24\mu_1}$ while the online product demand is 0.

However, when $m_2 > \mu_1$, only the corner solution exists for both p_1 and p_2, that is $p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ or $p_1 = V - \frac{t}{4}$ and meanwhile $p_2 = p_1 - \frac{(m_2-\mu_1)(V+rV-p_1)}{M_1}$ under which condition $\frac{V+rV-p_1}{M_1} = \frac{p_1-p_2}{m_2-\mu_1}$.

The offline product demand is $\frac{rV}{M_1-\mu_1} - \frac{t}{12\mu_1}$ when $p_1 = V - rV\frac{\mu_1}{M_1-\mu_1}$ while the online product demand is 0 which results in the profit $\pi = -\frac{(rVm_2-(V-C_2)(M_1-\mu_1))(-tM_1+(t+12rV)\mu_1)}{12(M_1-\mu_1)^2\mu_1}$.

The offline product demand is $\frac{(t+4rV)\mu_1^2\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}}+tM_1(t-\mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}})}{6tM_1\mu_1}$ when $p_1 = V - \frac{t}{4}$ and the online product demand is $\frac{\mu_1^2(\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2})^{3/2}}{6t^2}$ which results in the profit

$\pi = \frac{1}{24tM_1^2\mu_1}((t + 4rV)^2\mu_1^3\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}} + (t + 4rV)M_1\mu_1(t^2 - (t - 4C_1 + 4C_2)\mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}}) - tM_1^2(t^2 - 4tV + 4C_1\mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}} + 4C_2(t - \mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}})) - (t + 4rV)m_2((t +$

$4rV)\mu_1^2\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}}+tM_1(t-\mu_1\sqrt{\frac{t(tM_1-(t+4rV)\mu_1)}{M_1\mu_1^2}}))).$

(2) Horizontal dominance

Case (i)

When $2\sqrt{\frac{V-p_1}{t}}\leq 1$ holds, the online product demand area doesn't intersect with the market boundary as Case (i) shows.

The profit function is $\pi=(p_2-C_2)*(\int_{f_1-\sqrt{\frac{V-p_1}{t}}}^{h1}\frac{V-p_1-t(b-f_1)^2}{\mu_1}db+\int_{h1}^{b2}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db)+(p_1-C_1)*(\int_{f_1-\sqrt{\frac{V-p_1}{t}}}^{f_1+\sqrt{\frac{V-p_1}{t}}}\frac{V-p_1-t(b-f_1)^2}{\mu_1}db-(\int_{f_1-\sqrt{\frac{V-p_1}{t}}}^{h1}\frac{V-p_1-t(b-f_1)^2}{\mu_1}db+\int_{h1}^{b2}\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}db))$ where $h_1=f_1-\frac{M_1\sqrt{t(\frac{V+rV-p_1+\frac{M_1(t(f_1-f_2)^2-p_1+p_2)}{m_2-\mu_1}}{M_1^2}+\frac{t(f_1-f_2)^2}{(m_2-\mu_1)^2})}}{t}+\frac{(f_1-f_2)M_1}{m_2-\mu_1}$ is the intersection point of $\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ and $\frac{V-p_1-t(b-f_1)^2}{\mu_1}$, $b_2=\frac{tf_1^2-tf_2^2+p_1-p_2}{2tf_1-2tf_2}$ is the intersection point of the line $\alpha=\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ when $\alpha=0$. It is difficult for us to derive the optimal solutions directly, thus we resort to the envelope theorem to first figure out the relationships between f_1 and f_2 under optimal condition. That is, if we take FOC of the profit with respect to f_i,

$$\frac{\partial\pi}{\partial f}=\left[(p_2-C_2)\frac{\partial D_2}{\partial f}+(p_1-C_1)\frac{\partial D_1}{\partial f}-c\frac{\partial R}{\partial f}\right]+\frac{\partial\pi}{\partial p}\frac{\partial p}{\partial f},$$

where $\frac{\partial\pi}{\partial p}=0$ under the optimal condition. Also $\frac{\partial\pi}{\partial f}$ should also equal 0 under optimal condition, thus we can derive the results of design feature f_1 and f_2 as $f_1=\frac{1}{2}$; $f_2=\frac{1}{2}\pm\sqrt{\frac{2(tV+\sqrt{t^2(V-p_1)(V-p_2)})-t(p_1+p_2)}{t^2}}$.

Under the optimal condition $\frac{\partial\pi}{\partial p}=0$, while no inner solution can be found, thus we resolve to the corner solution under the condition $b_2=\frac{tf_1^2-tf_2^2+p_1-p_2}{2tf_1-2tf_2}\in[0,1]$ with the corner solution $p_1=\frac{1}{4}(-t+4V)$ and $p_2=\frac{1}{4}(-t+4V)$ which doesn't satisfy the condition $f_2=\frac{1}{2}\pm\sqrt{\frac{2(tV+\sqrt{t^2(V-p_1)(V-p_2)})-t(p_1+p_2)}{t^2}}\in[0,1]$ and should be deleted. The other corner solution $p_1=V-rV\frac{\mu_1}{M_1-\mu_1}, p_2=V(1+r+\frac{rM_1}{-M_1+\mu_1})$ should also guarantee the condition $b_2=\frac{tf_1^2-tf_2^2+p_1-p_2}{2tf_1-2tf_2}\in[0,1]$ and $f_2=\frac{1}{2}\pm\sqrt{\frac{2(tV+\sqrt{t^2(V-p_1)(V-p_2)})-t(p_1+p_2)}{t^2}}\in[0,1]$ which only holds when $r<\frac{t(M_1-\mu_1)}{16V\mu_1}$.

The offline product demand is

$(2rV(m_2^3(\mu_1\sqrt{-\frac{rtV(m_2+\mu_1)^2}{\mu_1(-m_2+\mu_1)^2(-M_1+\mu_1)}} - t\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}}) + \mu_1^3(-\mu_1\sqrt{-\frac{rtV(m_2+\mu_1)^2}{\mu_1(-m_2+\mu_1)^2(-M_1+\mu_1)}} + 3t\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}}) - m_2\mu_1^2(\mu_1\sqrt{-\frac{rtV(m_2+\mu_1)^2}{\mu_1(-m_2+\mu_1)^2(-M_1+\mu_1)}} + 3t\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}}) + m_2^2\mu_1(\mu_1\sqrt{-\frac{rtV(m_2+\mu_1)^2}{\mu_1(-m_2+\mu_1)^2(-M_1+\mu_1)}} + 9t\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}})))/(3t(m_2-\mu_1)^3(-M_1+\mu_1))$ the online product demand is $-((2rV(m_2+\mu_1)^2(\mu_1(-\mu_1\sqrt{-\frac{rtV(m_2+\mu_1)^2}{\mu_1(-m_2+\mu_1)^2(-M_1+\mu_1)}} + t\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}}) + m_2(\mu_1\sqrt{-\frac{rtV(m_2+\mu_1)^2}{\mu_1(-m_2+\mu_1)^2(-M_1+\mu_1)}} + t\sqrt{\frac{rV\mu_1}{tM_1-t\mu_1}})))/(3t(m_2-\mu_1)^3(-M_1+\mu_1)))$ and the profit is complex in form.

Case (ii)

When $2\sqrt{\frac{V-p_1}{t}} > 1$ holds, the online product demand area intersects with the market boundary as Case (ii) shows.

The profit function is $\pi = (p_2 - C_2) * (\int_0^{h1} \frac{V-p_1-t(b-f_1)^2}{\mu_1} db + \int_{h1}^{b2} \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1} db) + (p_1 - C_1) * (\int_0^1 \frac{V-p_1-t(b-f_1)^2}{\mu_1} db - (\int_0^{h1} \frac{V-p_1-t(b-f_1)^2}{\mu_1} db + \int_{h1}^{b2} \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1} db))$ where $h_1 = f_1 - \frac{M_1\sqrt{t(\frac{V+rV-p_1+\frac{M_1(t(f_1-f_2)^2-p_1+p_2)}{m_2-\mu_1}}{M_1^2} + \frac{t(f_1-f_2)^2}{(m_2-\mu_1)^2})}}{t} + \frac{(f_1-f_2)M_1}{m_2-\mu_1}$ is the intersection point of $\frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ and $\frac{V-p_1-t(b-f_1)^2}{\mu_1}$, $b_2 = \frac{tf_1^2-tf_2^2+p_1-p_2}{2tf_1-2tf_2}$ is the intersection point of the line $\alpha = \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-\mu_1}$ when $\alpha = 0$. However, there is no real solution to f_i under the optimal condition $\frac{\partial\pi}{\partial f_i} = 0$.

Further, we consider the case when the straight line $\alpha = \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-(\tau_1 m_1+(1-\tau_1)M_1)}$ has all its value larger than the points on the curve $\alpha = \frac{V+rV-p_1-t(b-f_1)^2}{M_1}$, we can combine the scenario when $\alpha < \frac{V+rV-p_1-t(b-f_1)^2}{M_1}$ with $\alpha \geq \frac{V+rV-p_1-t(b-f_1)^2}{M_1}$ thus the offline product demand is the region below the curve $\alpha = \frac{V-p_2-t(b-f_2)^2-\tau_1 V+\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{m_2-m_1\tau_1}$. This case only exists when the vertical dominance holds for the straight line $\alpha = \frac{p_1-p_2-t(b-f_2)^2+t(b-f_1)^2}{m_2-(\tau_1 m_1+(1-\tau_1)M_1)}$ which means $f_1 = \frac{1}{2}; f_2 = \frac{1}{2}$ should be satisfied, that is, $\frac{V+rV-p_1}{M_1} < \frac{p_1-p_2}{m_2-\mu_1}$.

(a) Partial keep ($p_1 < V - rV\frac{\mu_1}{M_1-\mu_1}$)

The profit function is $\pi = (p_2 - C_2) * \int_0^1 \frac{V-p_2-t(b-f_2)^2-\tau_1 V+\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{m_2-\tau_1 m_1} db + (p_1 - C_1) * (\int_0^1 \frac{V-p_1-t(b-f_1)^2-\frac{1-\tau}{\tau}rV}{m_1} db - \int_0^1 \frac{V-p_2-t(b-f_2)^2-\tau_1 V+\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{m_2-\tau_1 m_1} db) - c * (1 - \tau_1) * (\int_0^1 \frac{V-p_1-t(b-f_1)^2-\frac{1-\tau}{\tau}rV}{m_1} db - \int_0^1 \frac{V+rV-p_1-t(b-f_1)^2}{M_1} db - (\int_0^1 \frac{V-p_2-t(b-f_2)^2-\tau_1 V+\tau_1 p_1+\tau_1 t(b-f_1)^2+(1-\tau_1)rV}{m_2-\tau_1 m_1} db - \int_0^1 \frac{V+rV-p_1-t(b-f_1)^2}{M_1} db))$ with $f_1 = \frac{1}{2}; f_2 = \frac{1}{2}$.

Under the optimal FOC $\frac{\partial \pi}{\partial p} = 0$, where $p_1 = \frac{m_1\tau_1(12c-t-12(-1+r)V+12C_2(-1+\tau_1)-(12c+t-12(1+r)V)\tau_1^2+12C_1(1+\tau_1))+2m_2(12rV+\tau_1(-12c+t-12(1+r)V-12C_1+12c\tau_1))}{12\tau_1(-4m_2+m_1(1+\tau_1)^2)}$ and $p_2 = \frac{m_1\tau_1(1+\tau_1)(-t+12V+12C_2\tau_1)+m_2(12rV+3(4c+t-4(3+2r)V+4C_1-8C_2)\tau_1-(24c+t-12(1+r)V+12C_1)\tau_1^2+12c\tau_1^3)}{12\tau_1(-4m_2+m_1(1+\tau_1)^2)}$.

This inner solution only exists when the Hessian matrix is negative definitive, that is, $\frac{\partial^2 \pi}{\partial {p_1}^2} \le 0$, $\frac{\partial^2 \pi}{\partial {p_2}^2} \le 0$ and $\frac{\partial^2 \pi}{\partial {p_1}^2}\frac{\partial^2 \pi}{\partial {p_2}^2} - (\frac{\partial^2 \pi}{\partial p_1 \partial p_2})^2 \ge 0$ which only set up when $m_2 > m_1\tau_1$, then the profit is $\pi = \frac{(t-12V+12C_2)m_1\tau_1(12rV+12(c+C_1-C_2)\tau_1+(t-12(1+r)V+12C_1)\tau_1^2-12c\tau_1^3)-m_2(12rV+\tau_1(12c+t-12(1+r)V+12C_1-12c\tau_1))^2}{144m_1\tau_1^2(-4m_2+m_1(1+\tau_1)^2)}$. Also, we can derive the demand of both products as the offline product demand is $\frac{-12rV+\tau_1(-12c+t-12V+24C_2-12C_1(1+\tau_1)+\tau_1(-t+12(1+r)V+12c\tau_1))}{12\tau_1(-4m_2+m_1(1+\tau_1)^2)}$ while the online product demand is $\frac{-(t-12V+12C_2)m_1\tau_1(1+\tau_1)+2m_2(12rV+\tau_1(12c+t-12(1+r)V+12C_1-12c\tau_1))}{12m_1\tau_1(-4m_2+m_1(1+\tau_1)^2)}$ and the return fraction of online product is $\frac{(-1+\tau_1)((t-12V+12C_2)m_1\tau_1(1+\tau_1)+2m_2(-12rV-(12c+t-12(1+r)V+12C_1)\tau_1+12c\tau_1^2))}{12m_1\tau_1(-4m_2+m_1(1+\tau_1)^2)}$.

(b) All keep ($p_1 > V - rV\frac{\mu_1}{M_1-\mu_1}$)

The profit function is $\pi = (p_2 - C_2) * \int_0^1 \frac{V-p_2-t(b-f_2)^2-\tau_1V+\tau_1p_1+\tau_1t(b-f_1)^2+(1-\tau_1)rV}{m_2-\tau_1m_1} db + (p_1 - C_1) * (\int_0^1 \frac{V-p_1-t(b-f_1)^2}{\mu_1} db - \int_0^1 \frac{V-p_2-t(b-f_2)^2-\tau_1V+\tau_1p_1+\tau_1t(b-f_1)^2+(1-\tau_1)rV}{m_2-\tau_1m_1} db)$ with $f_1 = \frac{1}{2}; f_2 = \frac{1}{2}$.

Next, under the optimal first order condition of the seller $\frac{\partial \pi}{\partial p} = 0$, where $p_1 = \frac{-(((-2t+24V+24C_1)m_2+\mu_1(12C_1-12C_2+(t-12(1+r)V)(-1+\tau_1))(-1+\tau_1)+2(t-12V-12C_1)m_1\tau_1)}{12(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1)}$ and $p_2 = \frac{-((m_2(-3t+36V+24rV+24C_2+12C_1(-1+\tau_1)+(t-12(1+2r)V)\tau_1)+m_1\tau_1(3(t-4(3+2r)V+4C_1-8C_2)-(t-12(1+2r)V+12C_1)\tau_1)+\mu_1(-1+\tau_1)(-t+12(1+r)V+(t-12(1+r)V+12C_1-12C_2)\tau_1))}{12(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1)}$.

This inner solution only exists when the Hessian matrix is negative definitive, that is, $\frac{\partial^2 \pi}{\partial {p_1}^2} \le 0$, $\frac{\partial^2 \pi}{\partial {p_2}^2} \le 0$ and $\frac{\partial^2 \pi}{\partial {p_1}^2}\frac{\partial^2 \pi}{\partial {p_2}^2} - (\frac{\partial^2 \pi}{\partial p_1 \partial p_2})^2 \ge 0$ which only set up when $m_2 > m_1\tau_1$, then the profit is $\pi = \frac{-(t-12V+12C_1)^2m_2+(t-12V+12C_1)^2m_1\tau_1-12\mu_1(-C_1+C_2+rV(-1+\tau_1))(t-12(1+r)V+12C_2-(t-12(1+r)V+12C_1)\tau_1)}{144\mu_1(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1)}$. Also, we can derive the demand of both products as the offline product demand is $-\frac{-24C_2+(t-12(1+2r)V)(-1+\tau_1)+12C_1(1+\tau_1)}{12(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1)}$ while the online product demand is $\frac{2(t-12V+12C_1)m_2-2(t-12V+12C_1)m_1\tau_1+\mu_1(-t+12(1+r)V-12C_2(1+\tau_1)+\tau_1(t-12V+24C_1-12rV\tau_1))}{12\mu_1(-4m_2+\mu_1(-1+\tau_1)^2+4m_1\tau_1}$.

6.2. Appendix for Chapter 3

6.2.1. Appendix A: Demand derivation process for both Online and Offline Consumers

Case (i): We first consider the demand of online consumers.

We first take into account a consumer i whose prior purchase preference is located as $\theta_i \epsilon [x_1, x_2]$. It means he needs to decide between the two online products. We have assumed in our main model setting that $\delta < \frac{1}{2*4} = \frac{1}{8}$. Thus, this representative consumer located at $\theta_i \epsilon [x_1, x_2]$ will make his initial purchase decision only between product 1 and product 2. Otherwise, if this consumer with prior purchase taste satisfying $\theta_i \epsilon [x_1, x_2]$ prefers product 0 to product 1 (or equivalently prefers product 3 to product 2), no consumers will ever buy product 1 (or equivalently product 2) as a consequence.

In the beginning, we first consider the ex-post purchase strategy if a consumer's original purchase decision is online product 1, which is sold by seller one through online channel. Then we consider the ex-post purchase strategy if the consumer's original purchase is the adjacent product 0 or product 2. With probability $1 - \alpha$ of good quality product, the representative consumer i, whose initial purchase is product 1, will choose to keep it rather than exchange it for an adjacent product 2, if and only if the utility satisfies $v - p_1 - t|x_1 - (\theta_i + \varepsilon_i)| > v - p_2 - r - t|x_2 - (\theta_i + \varepsilon_i)|$. Namely, his preference should satisfy $\theta_i + \varepsilon_i < \beta \equiv \frac{p_2 - p_1 + r}{2t} + \frac{x_1 + x_2}{2}$.

We next take into account the prior purchase expected utility of the consumer located at θ_i, whose original decision is to buy the online product one. That is, the overall prior purchase expected utility of this consumer is $E_{1i} = (1-\alpha)\sum_{k=1,2} P_{1ki} E_{1ki} - \alpha r$. In this function, P_{1ki} is the probability that product k is finally kept by consumer i given product 1 is the original purchase for him; E_{1ki} is the expected utility that product k is finally kept by consumer i given online product 1 is the original purchase. The consumer with the observation of his location θ_i makes his initial purchase of product 1 will optimize his utility via keeping online product 1, only when the uncertain component of his preference ε_i belongs to the interval $[-\delta, \beta - \theta_i]$. Otherwise, he will optimize his utility by exchanging online product 1 for online product 2, only when this uncertainty component of his preference ε_i belongs to $[\beta - \theta_i, \delta]$. We have assumed that ε_i is uniformly distributed over $[-\delta, \delta]$, therefore, the probabilities that online product 1 and online product 2 are ultimately kept are $P_{11i} = max\,\{min\left\{\frac{1}{2\delta}(\beta - \theta_i + \delta), 1\right\}, 0\}$ and $P_{12i} = max\,\{min\left\{\frac{1}{2\delta}(\delta - \beta + \theta_i), 1\right\}, 0\}$, respectively. Meanwhile, the actual utility derived from keeping online product 1 is $v - p_1 - t|x_1 - (\theta_i + \varepsilon_i)|$, where θ_i is observable prior purchase and ε_i belongs to the interval $[-\delta, \beta - \theta_i]$. The actual utility the consumers will obtain from exchanging online product 1 for online product 2 is $v - p_2 - r - t|x_2 - (\theta_i + \varepsilon_i)|$, where θ_i is observable prior purchase and ε_i belongs to $[\beta - \theta_i, \delta]$. Thus, the expected utilities that online product 1 and online product 2 are ultimately kept can then be demonstrated as $E_{11i} = v - p_1 - t(\theta_i + \frac{-\delta+\beta-\theta_i}{2} - x_1)$ and $E_{12i} = v - p_2 - r - t(x_2 - \theta_i - \frac{\delta+\beta-\theta_i}{2})$. As a result, consumer i's prior purchase expected utility when his initial purchase is online product 1, can be derived as $E_{1i} = (1-\alpha)\left[\frac{1}{2\delta}(\beta - \theta_i + \delta)\left(v - p_1 - t\left(\theta_i + \frac{-\delta+\beta-\theta_i}{2} - x_1\right)\right) + \frac{1}{2\delta}(\delta - \beta + \theta_i)\left(v - p_2 - r - t\left(x_2 - \theta_i - \frac{\delta+\beta-\theta_i}{2}\right)\right)\right] - \alpha r$ if $-\delta < \beta - \theta_i < \delta$.

Consumer i's prior purchase expected utility when his initial purchase is online product 2, where his prior location satisfies $\theta_i \epsilon [x_1, x_2]$, can be derived in similar method. We use $\gamma \equiv \frac{p_2-p_1-r}{2t} + \frac{x_1+x_2}{2}$ to demonstrate the post purchase preference (i.e., ϑ_i) of the consumer who is indifferent between keeping his initial purchase online product 2 and exchanging it for the adjacent online product 1. Then, the consumer's prior purchase expected utility when his original decision is to buy online product 2, can be further derived as $E_{2i} = (1-\alpha)\left[\frac{1}{2\delta}(-\gamma + \theta_i + \delta)\left(v - p_2 - t\left(x_2 - \theta_i - \frac{\delta+\gamma-\theta_i}{2}\right)\right) + \frac{1}{2\delta}(\delta + \gamma - \theta_i)\left(v - p_1 - r - t\left(\theta_i + \frac{-\delta+\gamma-\theta_i}{2} - x_1\right)\right)\right] - \alpha r$ if $-\delta < \gamma - \theta_i < \delta$. Therefore, the consumer who is indifferent between purchasing online product 1 and purchasing online product 2 satisfies $\theta^1 = \frac{p_2-p_1}{2t} + \frac{x_1+x_2}{2}$. That is to say, consumers with the prior purchase preference satisfying $\theta_i \epsilon [x_1, \theta^1]$ will purchase online product 1 in the beginning. Consumers with the prior preference satisfying $\theta_i \epsilon [\theta^1, x_2]$ will buy online product 2 otherwise.

With the similar method of utility formation process, a consumer i whose prior location θ_i belongs to $[x_0, x_1]$, which means he needs to decide between seller one's online product and offline product, will take shape of his expected utility when his initial purchase is product 1 as $E_{1i} = (1-\alpha)\left[\frac{1}{2\delta}(-\gamma + \theta_i + \delta)\left(v - p_1 - t\left(x_1 - \theta_i - \frac{\delta+\gamma-\theta_i}{2}\right)\right) + \frac{1}{2\delta}(\delta + \gamma - \theta_i)\left(v - p_0 - r - t\left(\theta_i + \frac{-\delta+\gamma-\theta_i}{2} - x_0\right)\right)\right] - \alpha r$. In this utility function, $\gamma \equiv \frac{p_1-p_0-r}{2t} + \frac{x_1+x_0}{2}$ denote the ex-post preference of the consumers who is indifferent between keeping online product 1 and exchanging online product 1 for offline product 0 with a return cost r. Consumer i's ex-ante expected utility when his original purchase decision is offline product 0 can also be derived as $E_{0i} = (1-\alpha)\left[\frac{1}{2\delta}(\beta - \theta_i + \delta)\left(v - p_0 - t\left(\theta_i + \frac{-\delta+\beta-\theta_i}{2} - x_0\right)\right) + \frac{1}{2\delta}(\delta - \beta + \theta_i)\left(v - p_1 - h - t\left(x_1 - \theta_i - \frac{\delta+\beta-\theta_i}{2}\right)\right)\right] - \alpha r$, where $\beta \equiv \frac{p_1-p_0+h}{2t} + \frac{x_1+x_0}{2}$ denote the ex-post location of the consumers who is indifferent between keeping offline product 0 and exchanging offline product 0 for online product 1 with a hassle cost h. The consumer with horizontal preference indifferent between physical store product 0 and online store product 1 (i.e., θ_i s.t. $E_{1i} = E_{0i}$) should satisfy $\theta^0 =$

$\frac{(h-r)((h+r)(-1+\alpha)+4t(1+\alpha)\delta)+2(h+r)(-1+\alpha)(-p_0+p_1+t(x_0+x_1))}{4(h+r)t(-1+\alpha)}$. Generally speaking, consumers with prior purchase preference satisfying $\theta_i \epsilon [x_0, \theta^0]$ will purchase offline product 0 in the beginning. Consumers with prior preference satisfying $\theta_i \epsilon [\theta^0, x_1]$ will buy online product 1 in the beginning. Thus, online product 1 has its initial demand generate from two consumer preference intervals, that is, $\theta_i \epsilon [x_1, \theta^1]$ and $\theta_i \epsilon [\theta^0, x_1]$. Namely, the overall initial demand of online product 1 is exactly $\theta^1 - \theta^0$. It can be simplified as the demand function:

$$D_1 = \frac{2p_0 + \frac{(h-r)(-(h+r)(-1+\alpha)-4t(1+\alpha)\delta)+2(h+r)(-1+\alpha)(-2p_1+p_2-tx_0+tx_2)}{(h+r)(-1+\alpha)}}{4t}.$$

We can take the similar method to derive that the consumer indifferent between purchasing offline product 3 and offline product 0 is located at $\theta^3 = \frac{-p_3+p_0+t(x_0+x_3)}{2t}$. Consumers with prior preference satisfying $\theta_i \epsilon [\theta^3, x_0]$ will buy offline product 0 at the beginning. Consumers with prior preference satisfying $\theta_i \epsilon [x_3, \theta^3]$ will buy offline product 3 at the beginning. Thus, offline product 0 has its initial demand generate from two consumer preference intervals, $\theta_i \epsilon [x_0, \theta^0]$ and $\theta_i \epsilon [\theta^3, x_0]$. Namely, the overall initial demand of offline product 0 is exactly $\theta^0 - \theta^3$. It can be simplified as the demand function $D_0 = \frac{-4p_0 + \frac{(h-r)((h+r)(-1+\alpha)+4t(1+\alpha)\delta)+2(h+r)(-1+\alpha)(p_1+p_3+tx_1-tx_3)}{(h+r)(-1+\alpha)}}{4t}$.

A consumer will return a certain product without any following purchase if he realizes it is defective in quality, i.e., $v_i = 0$. This happens with probability α when his original purchase is product j which we assume without loss of generality. Therefore, the return quantity of this product should be αD_j.

We next consider the possible exchange behaviors that will happen among consumers' purchase decision. A representative consumer i whose prior preference θ_i belongs to $[x_1, x_2]$ will exchange online product 1 for online product 2 if and only if the following three conditions are met simultaneously: firstly, online product 1 brings higher expected utility gain prior purchase; secondly, online product 2 brings higher utility gain post purchase, namely, his post purchase preference should satisfy $\theta_i + \varepsilon_i > \beta$; thirdly, either product generates nonnegative utility gain with the fixed probability $1-\alpha$. We thus examine the consumers' interval with prior preference satisfying $\beta - \varepsilon_i < \theta_i < \theta^1$, which is exactly $[\theta^1 - (\beta - \varepsilon_i)]^+$ for a given arbitrary ε_i. We use e_{12} to denote the quantity of consumers with prior preference $\theta_i \epsilon [x_1, x_2]$ who purchase online product 1 at the beginning and exchange it for online product 2 afterwards. We can derive the exchange quantity as $e_{12} = (1-\alpha)\int_{-\delta}^{\delta} [\theta^1 - (\beta - \varepsilon_i)]^+ d\varepsilon_i$. Similarly, the exchange quantity of consumers with prior preference $\theta_i \epsilon [x_1, x_2]$ who will exchange online product 2 for online product 1 can be demonstrated as $e_{21} = (1-\alpha)\int_{-\delta}^{\delta} [(\gamma - \varepsilon_i) - \theta^1]^+ d\varepsilon_i$.

Following the same rationale, we can derive the quantity of consumers with prior preference $\theta_i \epsilon [x_0, x_1]$ who will exchange offline product 0 for online product 1 as $e_{01} = (1-\alpha)\int_{-\delta}^{\delta} [\theta^0 - (\beta - \varepsilon_i)]^+ d\varepsilon_i$, and those who will exchange online product 1 for offline product 0 can be demonstrated as $e_{10} = (1-\alpha)\int_{-\delta}^{\delta} [(\gamma - \varepsilon_i) - \theta^0]^+ d\varepsilon_i$.

If we follow the same method of demand generation as the aforementioned case (i), we can derive the online consumers' initial demands, return quantities and exchange quantities for each product of case (ii) and case (iii).

Case (ii): one firm sells $x_1 = \frac{1}{4}$ (online) and $x_0 = \frac{3}{4}$(offline), the other firm sells $x_2 = \frac{2}{4}$ (online) and $x_3 = 0$ (offline);

$$D_1 = \frac{-4p_1 + \frac{(h-r)(-(h+r)(-1+\alpha) - 4t(1+\alpha)\delta) + 2(h+r)(-1+\alpha)(p_2 + p_3 + tx_2 - tx_3)}{(h+r)(-1+\alpha)}}{4t};$$

$$D_0 = \frac{-4p_0 + \frac{(h-r)((h+r)(-1+\alpha) + 4t(1+\alpha)\delta) + 2(h+r)(-1+\alpha)(p_2 + p_3 - tx_2 + tx_{33})}{(h+r)(-1+\alpha)}}{4t};$$

$$e_{31} = e_{02} = \frac{(h+r)(-1+\alpha)\delta}{2t} - \frac{2(h-r)(1+\alpha)\delta^2}{h+r};$$

$$e_{13} = e_{20} = 0;$$

$$D_2 = \frac{2p_0 + \frac{(h-r)(-(h+r)(-1+\alpha) - 4t(1+\alpha)\delta) + 2(h+r)(-1+\alpha)(p_1 - 2p_2 + tx_0 - tx_1)}{(h+r)(-1+\alpha)}}{4t};$$

$$D_3 = \frac{2p_0 + \frac{(h-r)((h+r)(-1+\alpha) + 4t(1+\alpha)\delta) + 2(h+r)(-1+\alpha)(p_1 - 2p_3 + t(-x_0 + x_1 + x_3 - x_{33}))}{(h+r)(-1+\alpha)}}{4t};$$

$$e_{31} = e_{02} = \frac{(h+r)(-1+\alpha)\delta}{2t} - \frac{2(h-r)(1+\alpha)\delta^2}{h+r};$$

$$e_{13} = e_{20} = 0;$$

$$R_j = \alpha D_j.$$

Case (iii): one firm sells $x_1 = \frac{1}{4}$ (online) and $x_0 = 0$ (offline), the other firm sells $x_2 = \frac{3}{4}$ (online) and $x_3 = \frac{2}{4}$ (offline);

$$D_1 = \frac{(h-r)(-(h+r)(-1+\alpha) - 4t(1+\alpha)\delta) + (h+r)(-1+\alpha)(p_0 - 2p_1 + p_3 - tx_0 + tx_3)}{2(h+r)t(-1+\alpha)};$$

$$D_0 = \frac{1}{2(h+r)t(-1+\alpha)}\Big((h-r)\big((h+r)(-1+\alpha) + 4t(1+\alpha)\delta\big) - 2(h+r)(-1+\alpha)p_0 + (h+r)(-1+\alpha)(p_1 + p_2 + tx_1 - tx_2)\Big);$$

$$e_{01} = e_{31} = e_{02} = \frac{(h+r)(-1+\alpha)\delta}{2t} - \frac{2(h-r)(1+\alpha)\delta^2}{h+r};$$

$$e_{10} = e_{13} = e_{20} = 0;$$

$$D_2 = \frac{(h-r)(-(h+r)(-1+\alpha) - 4t(1+\alpha)\delta) + (h+r)(-1+\alpha)(p_0 - 2p_2 + p_3 + tx_0 - tx_3)}{2(h+r)t(-1+\alpha)};$$

$$D_3 = \frac{(h-r)((h+r)(-1+\alpha) + 4t(1+\alpha)\delta) + (h+r)(-1+\alpha)(p_1 + p_2 - 2p_3 - tx_1 + tx_2)}{2(h+r)t(-1+\alpha)};$$

$$e_{32} = e_{31} = e_{02} = \frac{(h+r)(-1+\alpha)\delta}{2t} - \frac{2(h-r)(1+\alpha)\delta^2}{h+r};$$

$$e_{23} = e_{13} = e_{20} = 0;$$

$$R_j = \alpha D_j.$$

We then consider the demand of offline consumers.

The consumers who take in-store inspection are uncertain only about v_i. It means that the consumer will make purchase of online product indexed as one, if he can obtain his first-rank expected utility from purchasing it: $(1-\alpha)(v - p_1 - t|x_1 - \vartheta_i|) - \alpha\, r$ .Which means

$(1-\alpha)(v-p_1-t|x_1-\vartheta_i|)-\alpha\, r>(1-\alpha)(v-p_2-t|x_2-\vartheta_i|)-\alpha\, r$ and meanwhile $(1-\alpha)(v-p_1-t|x_1-\vartheta_i|)-\alpha\, r>(1-\alpha)(v-p_0-t|x_0-\vartheta_i|)-\alpha\, h$. The total demand of product 1 for offline consumers is $D_1'=\frac{-p_1+p_2+t(x_1+x_2)}{2t}-\frac{(h-r)\alpha-(-1+\alpha)p_0+(-1+\alpha)p_1+t(-1+\alpha)(x_0+x_1)}{2t(-1+\alpha)}$, which can be simplified as $\frac{-h\alpha+r\alpha+(-1+\alpha)p_0-2(-1+\alpha)p_1-p_2+\alpha p_2+tx_0-t\alpha x_0+t(-1+\alpha)x_2}{2t(-1+\alpha)}$.And the corresponding returns of product 1 is $\alpha D_1'$.

With the same method, we can derive the total demand of product 0 as $D_0'=\frac{(h-r)\alpha-(-1+\alpha)p_0+(-1+\alpha)p_1+t(-1+\alpha)(x_0+x_1)}{2t(-1+\alpha)}-\frac{-p_3+p_0+t(x_0+x_3)}{2t}$, which can be simplified as $-\frac{-h\alpha+r\alpha+2(-1+\alpha)p_0+p_1+p_3+tx_1-\alpha(p_1+p_3+tx_1)+t(-1+\alpha)x_3}{2t(-1+\alpha)}$.And the corresponding returns of product 0 is $\alpha D_0'$. Also note that the offline consumer has no uncertainty regarding his preference parameter, thus no exchange behavior will happen when he takes in-store inspection before purchase.

Demand derivation process of Case (ii) and Case (iii) can also be derived in the similar method as the above demand generation process by taking the demand of online consumers and offline consumers into consideration. We list the other two cases as follows.

Case (ii): one firm sells $x_1=\frac{1}{4}$ (online) and $x_0=\frac{3}{4}$(offline), the other firm sells $x_2=\frac{2}{4}$ (online) and $x_3=0$ (offline);

$$D_1'=-\frac{h\alpha-r\alpha+2(-1+\alpha)p_1+p_2+p_3+tx_2-\alpha(p_2+p_3+tx_2)+t(-1+\alpha)x_3}{2t(-1+\alpha)};$$

$$D_0'=\frac{h\alpha-r\alpha-2(-1+\alpha)p_0+(-1+\alpha)p_2-p_3+\alpha p_3+tx_2-t\alpha x_2+t(-1+\alpha)x_{33}}{2t(-1+\alpha)};$$

$$D_2'=\frac{-h\alpha+r\alpha+(-1+\alpha)p_0+(-1+\alpha)p_1+2p_2-2\alpha p_2-tx_0+t\alpha x_0-t(-1+\alpha)x_1}{2t(-1+\alpha)};$$

$$D_3'=\frac{p_0+\frac{(h-r)\alpha+(-1+\alpha)p_1-2(-1+\alpha)p_3+t(-1+\alpha)(-x_0+x_1+x_3-x_{33})}{-1+\alpha}}{2t};$$

$$R_j'=\alpha D_j'.$$

Case (iii): one firm sells $x_1=\frac{1}{4}$ (online) and $x_0=0$ (offline), the other firm sells $x_2=\frac{3}{4}$ (online) and $x_3=\frac{2}{4}$ (offline);

$$D_1'=\frac{-2h\alpha+2r\alpha+(-1+\alpha)p_0-2(-1+\alpha)p_1-p_3+\alpha p_3+tx_0-t\alpha x_0+t(-1+\alpha)x_3}{2t(-1+\alpha)};$$

$$D_0'=-\frac{-2h\alpha+2r\alpha+2(-1+\alpha)p_0+p_1+p_2+tx_1-\alpha(p_1+p_2+tx_1)+t(-1+\alpha)x_2}{2t(-1+\alpha)};$$

$$D_2'=-\frac{2h\alpha-2r\alpha-(-1+\alpha)p_0+2(-1+\alpha)p_2+p_3+tx_0-\alpha(p_3+tx_0)+t(-1+\alpha)x_3}{2t(-1+\alpha)};$$

$$D_3' = \frac{2h\alpha - 2r\alpha + (-1+\alpha)p_1 + (-1+\alpha)p_2 + 2p_3 - 2\alpha p_3 + tx_1 - t\alpha x_1 + t(-1+\alpha)x_2}{2t(-1+\alpha)};$$

$$R_j' = \alpha D_j'.$$

6.2.2. Appendix B: Derivation of Equilibrium Results

We examine a market where there are two competing sellers each selling two products that are horizontally differentiated from each other through either online or offline channel. The objective function of each seller can be shown as below where the product placement strategy has been divided into the three cases we have demonstrated. Meanwhile, the demand quantities, exchange quantities and return quantities in each case have been analyzed in Appendix A, thus, the equilibrium results in each case can be derived by examining the FOCs and SOCs of the profit function with respect to prices of each product.

$$\max_{p_1,p_0,x_1,x_0} \omega[(p_1-c)(D_1+e_{01}+e_{21}) - (c-s)(e_{10}+e_{12}+\alpha D_1) + (p_0-c)(D_0+e_{10}+e_{30}) - (c-s)(e_{01} + e_{03}+\alpha D_0)] + (1-\omega)[(p_1-c)D_1' - (c-s)\alpha D_1' + (p_0-c)D_0' - (c-s)\alpha D_0'];$$

$$\max_{p_2,p_3,x_2,x_3} \omega[(p_2-c)(D_2+e_{12}+e_{32}) - (c-s)(e_{21}+e_{23}+\alpha D_2) + (p_3-c)(D_3+e_{03}+e_{23}) - (c-s)(e_{30} + e_{32}+\alpha D_3)] + (1-\omega)\left[\left(p_2-c\right)D_2' - (c-s)\alpha D_2' + \left(p_3-c\right)D_3' - (c-s)\alpha D_3'\right].$$

Case (i)

Before setting the FOCs of the profit function with respect to prices to zero, we analyze that the SOCs of each price are all $-\frac{2}{t}$ which is negative thus the optimal profits can be achieved by setting the FOCs to zero. The equilibrium results are:

Firm 1: $x_1 = \frac{1}{4}$ (online) and $x_0 = 0$(offline)

Firm 2: $x_2 = \frac{2}{4}$ (online) and $x_3 = \frac{3}{4}$ (offline)

$$p_1 = \frac{1}{10(h+r)(-1+\alpha)}\Big((h+r)\big(t(-1+\alpha) + 2\alpha(-h+r+5s-5s\alpha) + 10c(-1+\alpha^2)\big) + \Big(h^2(1+\alpha+6(-1+\alpha)^2\delta) - 4h\delta\big(-3r(-1+\alpha)^2 + t(1+\alpha+6(-1+\alpha^2)\delta)\big) + r\big(-r(1+\alpha) + 6r(-1+\alpha)^2\delta + 4t\delta(1+\alpha+6(-1+\alpha^2)\delta)\big)\Big)\omega\Big);$$

$$p_0 = \frac{1}{10(h+r)(-1+\alpha)}\Big((h+r)\big(t - t\alpha + 2\alpha(h-r+5s-5s\alpha) + 10c(-1+\alpha^2)\big) + \Big(h^2(-1-\alpha+4(-1+\alpha)^2\delta) + r\big(4t(1+\alpha)\delta(-1+4(-1+\alpha)\delta) + r(1+\alpha+4(-1+\alpha)^2\delta)\big) + 4h\delta\big(2r(-1+\alpha)^2 + t(1+\alpha+4\delta-4\alpha^2\delta)\big)\Big)\omega\Big);$$

$$p_2 = \frac{1}{10(h+r)(-1+\alpha)}\Big((h+r)\big(t(-1+\alpha)+2\alpha(-h+r+5s-5s\alpha)+10c(-1+\alpha^2)\big)$$

$$+\Big(h^2(1+\alpha+6(-1+\alpha)^2\delta)-4h\delta\big(-3r(-1+\alpha)^2+t(1+\alpha+6(-1+\alpha^2)\delta)\big)$$

$$+r\big(-r(1+\alpha)+6r(-1+\alpha)^2\delta+4t\delta(1+\alpha+6(-1+\alpha^2)\delta)\big)\Big)\omega\Big);$$

$$p_3 = \frac{1}{10(h+r)(-1+\alpha)}\Big((h+r)\big(t-t\alpha+2\alpha(h-r+5s-5s\alpha)+10c(-1+\alpha^2)\big)$$

$$+\Big(h^2(-1-\alpha+4(-1+\alpha)^2\delta)$$

$$+r\big(4t(1+\alpha)\delta(-1+4(-1+\alpha)\delta)+r(1+\alpha+4(-1+\alpha)^2\delta)\big)$$

$$+4h\delta\big(2r(-1+\alpha)^2+t(1+\alpha+4\delta-4\alpha^2\delta)\big)\Big)\omega\Big).$$

The expressions of each seller's optimal profit are quite complex in forms and we just omit to paste them here.

Case (ii)

The SOCs of each price are all $-\frac{2}{t}$ which is negative thus the optimal profits can be achieved by setting the FOCs to zero. The equilibrium results are:

Firm 1: $x_1 = \frac{1}{4}$ (online) and $x_0 = \frac{3}{4}$(offline)

Firm 2: $x_2 = \frac{2}{4}$ (online) and $x_3 = 0$ (offline)

$$p_1 = \frac{1}{24(h+r)(-1+\alpha)}\Big(-2(h+r)\big(t-t\alpha+3\big(h-r+4s(-1+\alpha)\big)\alpha-12c(-1+\alpha^2)\big)$$

$$+3\Big(h^2(1+\alpha+3(-1+\alpha)^2\delta)-2h\delta\big(-3r(-1+\alpha)^2+2t(1+\alpha+3(-1+\alpha^2)\delta)\big)$$

$$+r\big(-r(1+\alpha)+3r(-1+\alpha)^2\delta+4t\delta(1+\alpha+3(-1+\alpha^2)\delta)\big)\Big)\omega\Big);$$

$$p_0 = \frac{1}{24(h+r)(-1+\alpha)}\Big(2(h+r)(t(-1+\alpha)+3(h-r)\alpha)+24(h+r)(-1+\alpha)(c+c\alpha-s\alpha)$$

$$+3\Big(h^2(-1-\alpha+(-1+\alpha)^2\delta)$$

$$+r\big(4t(1+\alpha)\delta(-1+(-1+\alpha)\delta)+r(1+\alpha+(-1+\alpha)^2\delta)\big)$$

$$+2h\delta\big(r(-1+\alpha)^2+2t(1+\alpha+\delta-\alpha^2\delta)\big)\Big)\omega\Big);$$

$$p_2 = \frac{1}{24(h+r)(-1+\alpha)}\Big(2(h+r)\big(2t(-1+\alpha)+3\alpha(-h+r+4s-4s\alpha)+12c(-1+\alpha^2)\big)$$

$$+3\Big(h^2(1+\alpha+3(-1+\alpha)^2\delta)-2h\delta\big(-3r(-1+\alpha)^2+2t(1+\alpha+3(-1+\alpha^2)\delta)\big)$$

$$+r\big(-r(1+\alpha)+3r(-1+\alpha)^2\delta+4t\delta(1+\alpha+3(-1+\alpha^2)\delta)\big)\Big)\omega\Big);$$

$$p_3 = \frac{1}{24(h+r)(-1+\alpha)}\Big(2(h+r)(-4t(-1+\alpha)+3(h-r)\alpha) + 24(h+r)(-1+\alpha)(c+c\alpha-s\alpha)$$

$$+ 3\left(h^2(-1-\alpha+(-1+\alpha)^2\delta)\right.$$

$$+ r\big(4t(1+\alpha)\delta(-1+(-1+\alpha)\delta) + r(1+\alpha+(-1+\alpha)^2\delta)\big)$$

$$\left.+ 2h\delta\big(r(-1+\alpha)^2 + 2t(1+\alpha+\delta-\alpha^2\delta)\big)\right)\omega\Big).$$

The expressions of each seller's optimal profit are quite complex in forms and we just omit to paste them here.

Case (iii)

The SOCs of each price are all $-\frac{2}{t}$ which is negative thus the optimal profits can be achieved by setting the FOCs to zero. The equilibrium results are:

Firm one: $x_1 = \frac{1}{4}$ online and $x_0 = 0$ offline

Firm two: $x_2 = \frac{3}{4}$ online and $x_3 = \frac{2}{4}$ offline

$$p_1 = \frac{1}{210(h+r)(-1+\alpha)}\Big(-3(h+r)(-7t(-1+\alpha)+20(h-r)\alpha) + 210(h+r)(-1+\alpha)(c+c\alpha-s\alpha)$$

$$+ 30\left(h^2(1+\alpha+8(-1+\alpha)^2\delta) - 4h\delta\big(-4r(-1+\alpha)^2 + t(1+\alpha+8(-1+\alpha^2)\delta)\big)\right.$$

$$\left.+ r\big(-r(1+\alpha) + 8r(-1+\alpha)^2\delta + 4t\delta(1+\alpha+8(-1+\alpha^2)\delta)\big)\right)\omega\Big);$$

$$p_0 = \frac{1}{210(h+r)(-1+\alpha)}\Big(3(h+r)(-7t(-1+\alpha)+20(h-r)\alpha) + 210(h+r)(-1+\alpha)(c+c\alpha-s\alpha)$$

$$+ 30\left(h^2(-1-\alpha+6(-1+\alpha)^2\delta)\right.$$

$$+ r\big(4t(1+\alpha)\delta(-1+6(-1+\alpha)\delta) + r(1+\alpha+6(-1+\alpha)^2\delta)\big)$$

$$\left.+ 4h\delta\big(3r(-1+\alpha)^2 + t(1+\alpha+6\delta-6\alpha^2\delta)\big)\right)\omega\Big);$$

$$p_2 = \frac{1}{210(h+r)(-1+\alpha)}\Big(-3(h+r)(7t(-1+\alpha)+20(h-r)\alpha) + 210(h+r)(-1+\alpha)(c+c\alpha-s\alpha)$$

$$+ 30\left(h^2(1+\alpha+8(-1+\alpha)^2\delta) - 4h\delta\big(-4r(-1+\alpha)^2 + t(1+\alpha+8(-1+\alpha^2)\delta)\big)\right.$$

$$\left.+ r\big(-r(1+\alpha) + 8r(-1+\alpha)^2\delta + 4t\delta(1+\alpha+8(-1+\alpha^2)\delta)\big)\right)\omega\Big);$$

$$p_3 = \frac{1}{210(h+r)(-1+\alpha)}\Big(3(h+r)(7t(-1+\alpha)+20(h-r)\alpha) + 210(h+r)(-1+\alpha)(c+c\alpha-s\alpha)$$

$$+ 30\left(h^2(-1-\alpha+6(-1+\alpha)^2\delta)\right.$$

$$+ r\big(4t(1+\alpha)\delta(-1+6(-1+\alpha)\delta) + r(1+\alpha+6(-1+\alpha)^2\delta)\big)$$

$$\left.+ 4h\delta\big(3r(-1+\alpha)^2 + t(1+\alpha+6\delta-6\alpha^2\delta)\big)\right)\lambda\Big).$$

The expressions of each seller's optimal profit are quite complex in forms and we just omit to paste them here.

We next consider the single channel selling strategy and the corresponding optimal pricing strategy and profits.

We first consider the decision process of **web-only retailers**. When case (1) follows, which means one retailer assorts her products with adjacent horizontal location, the objective functions of each web-only retailer are as follows:

$$\max_{p_1,p_0,x_1,x_0} (p_1-c)(D_1+e_{01}+e_{21})-(c-s)(e_{10}+e_{12}+\alpha D_1)+(p_0-c)(D_0+e_{10}+e_{30})-(c-s)(e_{01}+e_{03}+\alpha D_0);$$

the product placement strategy is $x_1=\frac{1}{4}$ (online) and $x_0=0$(online);

$$\max_{p_2,p_3,x_2,x_3} (p_2-c)(D_2+e_{12}+e_{32})-(c-s)(e_{21}+e_{23}+\alpha D_2)+(p_3-c)(D_3+e_{03}+e_{23})-(c-s)(e_{30}+e_{32}+\alpha D_3);$$

the product placement strategy is $x_2=\frac{2}{4}$ (online) and $x_3=\frac{3}{4}$ (online).

Before setting the FOCs of the profit function with respect to prices to zero, we analyze that the SOCs of each price are all $-\frac{2}{t}$ which is negative, thus the optimal profits can be achieved by setting the FOCs to zero. The equilibrium results in case (1) are:

Firm 1: $p_1^*=c+\frac{t}{10}+c\alpha-s\alpha$ and $p_0^*=c-\frac{t}{10}+c\alpha-s\alpha$; $\pi^*=\frac{3t}{100}$.

Firm 2: $p_2^*=c+\frac{t}{10}+c\alpha-s\alpha$ and $p_3^*=c-\frac{t}{10}+c\alpha-s\alpha$; $\pi^*=\frac{3t}{100}$.

We then analyze the equilibrium results for web-only retailers in case (2), which means one retailer assorts her products with differentiated horizontal location. The objective functions of each web-only retailer are as follows:

$$\max_{p_1,p_0,x_1,x_0} (p_1-c)(D_1+e_{01}+e_{21})-(c-s)(e_{10}+e_{12}+\alpha D_1)+(p_0-c)(D_0+e_{10}+e_{30})-(c-s)(e_{01}+e_{03}+\alpha D_0);$$

the product placement strategy is $x_1=\frac{1}{4}$ (online) and $x_0=\frac{3}{4}$(online);

$$\max_{p_2,p_3,x_2,x_3} (p_2-c)(D_2+e_{12}+e_{32})-(c-s)(e_{21}+e_{23}+\alpha D_2)+(p_3-c)(D_3+e_{03}+e_{23})-(c-s)(e_{30}+e_{32}+\alpha D_3);$$

the product placement strategy is $x_2=\frac{2}{4}$ (online) and $x_3=0$ (online).

We analyze that the SOCs of each price are all $-\frac{2}{t}$ which is negative, thus the optimal profits can be achieved by setting the FOCs to zero. The equilibrium results in case (2) are:

Firm 1: $p_1^*=c+\frac{t}{12}+c\alpha-s\alpha$ and $p_0^*=c+\frac{t}{12}+c\alpha-s\alpha$; $\pi^*=\frac{t}{72}$.

Firm 2: $p_2^*=c+\frac{t}{6}+c\alpha-s\alpha$ and $p_3^*=c-\frac{t}{3}+c\alpha-s\alpha$; $\pi^*=\frac{5t}{36}$.

We next take into consideration the optimal pricing strategy taken by the **store-only retailers** with all their products selling via brick-and-mortar stores. When case (1) follows, which means one retailer assorts her products with adjacent horizontal location, the objective functions of each store-only retailer are as follows:

$$\max_{p_1,p_0,x_1,x_0}(p_1-c)D_1'-(c-s)\alpha D_1'+(p_0-c)D_0'-(c-s)\alpha D_0';$$

the product placement strategy is $x_1=\frac{1}{4}$ (offline) and $x_0=0$(offline);

$$\max_{p_2,p_3,x_2,x_3}(p_2-c)D_2'-(c-s)\alpha D_2'+(p_3-c)D_3'-(c-s)\alpha D_3';$$

the product placement strategy is $x_2=\frac{2}{4}$ (offline) and $x_3=\frac{3}{4}$ (offline).

Before setting the FOCs of the profit function with respect to prices to zero, we analyze that the SOCs of each price are all $-\frac{2}{t}$ which is negative, thus the optimal profits can be achieved by setting the FOCs to zero. The equilibrium results in case (1) are:

Firm 1: $p_1^*=c+\frac{t}{10}+c\alpha-s\alpha$ and $p_0^*=c-\frac{t}{10}+c\alpha-s\alpha$; $\pi^*=\frac{3t}{100}$.

Firm 2: $p_2^*=c+\frac{t}{10}+c\alpha-s\alpha$ and $p_3^*=c-\frac{t}{10}+c\alpha-s\alpha$; $\pi^*=\frac{3t}{100}$.

We then analyze the equilibrium results for store-only retailers in case (2), which means one retailer assorts her products with differentiated horizontal location. The objective functions of each store-only retailer are as follows:

$$\max_{p_1,p_0,x_1,x_0}(p_1-c)D_1'-(c-s)\alpha D_1'+(p_0-c)D_0'-(c-s)\alpha D_0';$$

the product placement strategy is $x_1=\frac{1}{4}$ (offline) and $x_0=\frac{3}{4}$(offline);

$$\max_{p_2,p_3,x_2,x_3}(p_2-c)D_2'-(c-s)\alpha D_2'+(p_3-c)D_3'-(c-s)\alpha D_3';$$

the product placement strategy is $x_2=\frac{2}{4}$ (offline) and $x_3=0$ (offline).

We analyze that the SOCs of each price are all $-\frac{2}{t}$ which is negative, thus the optimal profits can be achieved by setting the FOCs to zero. The equilibrium results in case (2) are:

Firm 1: $p_1^*=c+\frac{t}{12}+c\alpha-s\alpha$ and $p_0^*=c+\frac{t}{12}+c\alpha-s\alpha$; $\pi^*=\frac{t}{72}$.

Firm 2: $p_2^*=c+\frac{t}{6}+c\alpha-s\alpha$ and $p_3^*=c-\frac{t}{3}+c\alpha-s\alpha$; $\pi^*=\frac{5t}{36}$.

6.3. Appendix for Chapter 4

6.3.1. Appendix A: Derivation Process of the Optimal Equilibrium Solutions

6.3.1.1 The equilibrium results when cost efficiency and quality certification are known

We first clarify the equilibrium results derivation process when the cost efficiency and quality certification are both observable to consumers prior purchase.

Firstly, we determine the specific form of the combination effort level strategy regarding both parties $(ax^r+(1-a)y_t^r)^{\frac{1}{r}}$. We have demonstrated in our model description that when r is approaching negative infinity, the combination effort contribution is $\min\{x,y\}$ which refer to both effort levels as perfect complements; secondly, when r is approaching zero, it is simplified as a Cobb-Douglas function $x^a y^{1-a}$; thirdly, when r equals to one, then the term has its form in $ax+(1-a)y$ which marks the effort levels as perfect substitutes; finally, when r is approaching positive infinity, the combination effort contribution is $\max\{x,y\}$ which makes both efforts redundant.

Considering the feasibility of assumption in practice, the possibility that the substitution parameter of both efforts r is approaching negative infinity or positive infinity is quite unrealistic with the too extreme hypothesis, thus we mainly focus our attention on the assumption when r is approaching zero or when r equals to one.

Nevertheless, the assumption when $r \to 0$, $(ax^r+(1-a)y^r)^{\frac{1}{r}} \to x^a y^{1-a}$, which is a Cobb-Douglas function, makes most our equilibrium results as nonreal numbers thus cannot sustain our further analyses, we finally decide the form of the combination effort level $(ax^r+(1-a)y_t^r)^{\frac{1}{r}}$ under the assumption when $r=1$, $(ax^r+(1-a)y^r)^{\frac{1}{r}} = ax+(1-a)y$, i.e., efforts are perfect substitutes.

The **equilibrium derivation process** can be listed below with four main scenarios taken into consideration different consumer's demand for the service across the two time periods, which is a function of the service provider's pricing strategy. The service provider first makes her optimal effort level strategy though period, then she sets her price to determine the demand in each period. However, the consumer in the first period can't observe the effort level of the service provider, and can only make purchase decision based on his expected utility on the observation of the first period price. We use backward induction to derive the equilibrium results in each case.

Case (1)

The service provider aims at the fraction $\rho\epsilon(0,1)$ of high type consumers in the first period and aims at the fraction $\rho\epsilon(0,1)$ of high type consumers in the second period as well. The profit function given the consumers' demand over two periods is:

$$\pi_{ij} = \rho\left(p_H - c_i x_i^2\right) + m\rho\left(v_j + ax_i + (1-a)y_{2,i} - w{y_{2,i}}^2 - c_i x_i^2\right),$$

where subscript j represents the service provider's quality or certification; subscript i represents the service provider's cost efficiency; $t=1,2$ represents the time period of consumers' effort level strategy; p_H represents the price charged by the service provider to facilitate the first period demand when only high type consumers are willing to pay for the service.

The optimal effort level strategy of the service provider can be derived by taking FOC of the profit function with respect to her effort level where $\frac{d\pi_{ij}}{dx_i} = am\rho - 2(1+m)x_i\rho c_i$; while the optimal effort level strategies of the consumers in each period are derived by taking FOC of the expected utility of consumers with respect to his effort level, where $Eu_2 = v_j + ax_i + (1-a)y_{2,i} - wy_{2,i}{}^2 - p_{2,i}$ and $Eu_1 = v_j + ax_i + (1-a)y_{1,i} - wy_{1,i}{}^2 - p_{1,i}$ and FOCs are $\frac{dEu_2}{dy_{2,i}} = 1 - a - 2wy_{2,i}$ and $\frac{dEu_1}{dy_{1,i}} = 1 - a - 2wy_{1,i}$. We next check the second-order derivations of the results respectively, where $\frac{d^2\pi_{ij}}{dx_i^2} = -2(1+m)\rho c_i < 0$, $\frac{d^2Eu_2}{dy_{2,i}{}^2} = -2w < 0$ and $\frac{d^2Eu_1}{dy_{1,i}{}^2} = -2w < 0$, that is, all the equilibrium results satisfy the Hessian being negative definitive thus the unique solution of the FOCs are global optimum.

The optimal effort levels are $x_i^* = \frac{am}{2(1+m)c_i}$, $y_{1,i}^* = \frac{1-a}{2w}$, $y_{2,i}^* = \frac{1-a}{2w}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{\rho(a^2m^2w + (1+m)c_i((-1+a)^2m + 4w(p_H + mv_j)))}{4(1+m)wc_i}$.

Case (2)

The service provider aims at the fraction $\rho\epsilon(0,1)$ of high type consumers in the first period and aims at both types of consumers in the second period. The profit function given the consumers' demand over two periods is:

$$\pi_{ij} = \rho\left(p_H - c_i x_i^2\right) + m\left(v_j + \mu\left(ax_i + (1-a)y_{2,i}\right) - wy_{2,i}{}^2 - c_i x_i^2\right).$$

The optimal effort level strategy of the service provider can be derived by taking FOC of the profit function with respect to her effort level where $\frac{d\pi_{ij}}{dx_i} = am\mu - 2x_i(m+\rho)c_i$; while the optimal effort level strategies of the consumers in each period are derived by taking FOC of the expected utility of consumers with respect to his effort level, where $Eu_2 = v_j + \mu(ax_i + (1-a)y_{2,i}) - wy_{2,i}{}^2 - p_{2,i}$ and $Eu_1 = v_j + ax_i + (1-a)y_{1,i} - wy_{1,i}{}^2 - p_{1,i}$ and FOCs are $\frac{dEu_2}{dy_{2,i}} = \mu - a\mu - 2wy_{2,i}$ and $\frac{dEu_1}{dy_{1,i}} = 1 - a - 2wy_{1,i}$. We next check the second-order derivations of the results respectively, where $\frac{d^2\pi_{ij}}{dx_i^2} = -2(m+\rho)c_i < 0$, $\frac{d^2Eu_2}{dy_{2,i}{}^2} = -2w < 0$ and $\frac{d^2Eu_1}{dy_{1,i}{}^2} = -2w < 0$, that is, all the equilibrium results satisfy the Hessian being negative definitive thus the unique solution of the FOCs are global optimum.

The optimal effort levels are $x_i^* = \frac{am\mu}{2(m+\rho)c_i}$, $y_{1,i}^* = \frac{1-a}{2w}$, $y_{2,i}^* = \frac{(1-a)\mu}{2w}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2m^2w\mu^2 + (m+\rho)c_i((-1+a)^2m\mu^2 + 4w\rho p_H + 4mwv_j)}{4w(m+\rho)c_i}$.

Case (3)

The service provider aims at both types of consumers in the first period and aims at the fraction $\rho\epsilon(0,1)$ of high type consumers in the second period. The profit function given the consumers' demand over two periods is:

$$\pi_{ij} = \left(p_L - c_i x_i^2\right) + m\rho\left(v_j + a x_i + (1-a) y_{2,i} - w {y_{2,i}}^2 - c_i x_i^2\right),$$

where p_L represents the price charged by the service provider to facilitate the first period demand when both types of consumers are willing to pay for the service.

The optimal effort level strategy of the service provider can be derived by taking FOC of the profit function with respect to her effort level where $\frac{d\pi_{ij}}{dx_i} = am\rho - 2(x_i + m x_i \rho)c_i$; while the optimal effort level strategies of the consumers in each period are derived by taking FOC of the expected utility of consumers with respect to his effort level, where $Eu_2 = v_j + a x_i + (1-a) y_{2,i} - w {y_{2,i}}^2 - p_{2,i}$ and $Eu_1 = v_j + \mu(a x_i + (1-a) y_{1,i}) - w {y_{1,i}}^2 - p_{1,i}$ and FOCs are $\frac{dEu_2}{dy_{2,i}} = 1 - a - 2w y_{2,i}$ and $\frac{dEu_1}{dy_{1,i}} = \mu - a\mu - 2w y_{1,i}$. We next check the second-order derivations of the results respectively, where $\frac{d^2\pi_{ij}}{dx_i^2} = -2(1+m\rho)c_i < 0$, $\frac{d^2 Eu_2}{d{y_{2,i}}^2} = -2w < 0$ and $\frac{d^2 Eu_1}{d{y_{1,i}}^2} = -2w < 0$, that is, all the equilibrium results satisfy the Hessian being negative definitive thus the unique solution of the FOCs are global optimum.

The optimal effort levels are $x_i^* = \frac{am\rho}{2c_i + 2m\rho c_i}$, $y_{1,i}^* = \frac{(1-a)\mu}{2w}$, $y_{2,i}^* = \frac{1-a}{2w}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2 m^2 w \rho^2 + (1+m\rho)c_i(4w p_L + m\rho((-1+a)^2 + 4w v_j))}{4w(1+m\rho)c_i}$.

Case (4)

The service provider aims at both types of consumers in the first period and aims at both types of consumers in the second period as well. The profit function given the consumers' demand over two periods is:

$$\pi_{ij} = \left(p_L - c_i x_i^2\right) + m\left(v_j + \mu\left(a x_i + (1-a) y_{2,i}\right) - w {y_{2,i}}^2 - c_i x_i^2\right).$$

The optimal effort level strategy of the service provider can be derived by taking FOC of the profit function with respect to her effort level where $\frac{d\pi_{ij}}{dx_i} = am\mu - 2(1+m)x_i c_i$; while the optimal effort level strategies of the consumers in each period are derived by taking FOC of the expected utility of consumers with respect to his effort level, where $Eu_2 = v_j + \mu(a x_i + (1-a) y_{2,i}) - w {y_{2,i}}^2 - p_{2,i}$ and $Eu_1 = v_j + \mu(a x_i + (1-a) y_{1,i}) - w {y_{1,i}}^2 - p_{1,i}$ and FOCs are $\frac{dEu_2}{dy_{2,i}} = \mu - a\mu - 2w y_{2,i}$ and $\frac{dEu_1}{dy_{1,i}} = \mu - a\mu - 2w y_{1,i}$. We next check the second-order derivations of the results respectively, where $\frac{d^2\pi_{ij}}{dx_i^2} = -2(1+m)c_i < 0$, $\frac{d^2 Eu_2}{d{y_{2,i}}^2} = -2w < 0$ and $\frac{d^2 Eu_1}{d{y_{1,i}}^2} = -2w < 0$, that is, all the equilibrium results satisfy the Hessian being negative definitive thus the unique solution of the FOCs are global optimum.

The optimal effort levels are $x_i^* = \frac{am\mu}{2(1+m)c_i}$, $y_{1,i}^* = \frac{(1-a)\mu}{2w}$, $y_{2,i}^* = \frac{(1-a)\mu}{2w}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2 m^2 w \mu^2 + (1+m)c_i((-1+a)^2 m\mu^2 + 4w(p_L + m v_j))}{4(1+m)w c_i}$.

Moreover, we make a summary of the above subgame equilibrium results by making comparison of the optimal profit, given the first period consumers' demand and the highest price that the consumers are willing to pay in each period charged by the service provider.

Specifically, when the service provider charges p_H to aim at high type consumers in the first period, her final decision of whether to still aim at high type consumers in the second period or transform to aim at both consumers in the second period depends on the fraction of high type consumers $\rho \epsilon (0,1)$.

(1) The service provider will aim at both types of consumers in the second period when the high type consumers' fraction is lower than a certain threshold:

$$\rho < \frac{-a^2m^2w + (1+m)c_i(-(-1+a)^2(m-\mu^2) - 4(-1+m)wv_j) + \sqrt{(a^4m^2w^2(m^2+4(1+m)\mu^2) + (1+m)c_i((1+m)c_i((-1+a)^2(m+\mu^2)+4(1+m)wv_j)^2 + 2a^2mw((-1+a)^2(m^2+(2+3m)\mu^2)+4(1+m)w(m+2\mu^2)v_j)))}}{2(a^2mw + (1+m)c_i((-1+a)^2+4wv_j))},$$

the optimal effort levels are $x_i^* = \frac{am\mu}{2(m+\rho)c_i}$, $y_{1,i}^* = \frac{1-a}{2w}$, $y_{2,i}^* = \frac{(1-a)\mu}{2w}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2m^2w\mu^2+(m+\rho)c_i((-1+a)^2m\mu^2+4w\rho p_H+4mwv_j)}{4w(m+\rho)c_i}$, which is exactly Case (2) in our former study.

(2) The service provider will aim at high type consumers in the second period when the high type consumers' fraction is greater than a certain threshold:

$$\rho > \frac{-a^2m^2w + (1+m)c_i(-(-1+a)^2(m-\mu^2) - 4(-1+m)wv_j) + \sqrt{(a^4m^2w^2(m^2+4(1+m)\mu^2) + (1+m)c_i((1+m)c_i((-1+a)^2(m+\mu^2)+4(1+m)wv_j)^2 + 2a^2mw((-1+a)^2(m^2+(2+3m)\mu^2)+4(1+m)w(m+2\mu^2)v_j)))}}{2(a^2mw + (1+m)c_i((-1+a)^2+4wv_j))},$$

The optimal effort levels are $x_i^* = \frac{am}{2(1+m)c_i}$, $y_{1,i}^* = \frac{1-a}{2w}$, $y_{2,i}^* = \frac{1-a}{2w}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{\rho(a^2m^2w+(1+m)c_i((-1+a)^2m+4w(p_H+mv_j)))}{4(1+m)wc_i}$, which is exactly Case (1) in our former study.

When the service provider charges p_L to aim at both types of consumers in the first period, her final decision of second period pricing strategy still depends on the fraction of high type consumers.

(3) The service provider will aim at both types of consumers in the second period when the high type consumers' fraction is lower than a certain threshold:

$$\rho < \frac{a^2m^2w\mu^2 + (1+m)c_i((-1+a)^2(-1+m\mu^2) + 4(-1+m)wv_j) + \sqrt{(4m(1+m)(a^2w + c_i((-1+a)^2+4wv_j))(a^2mw\mu^2 + (1+m)c_i((-1+a)^2\mu^2+4wv_j)) + (a^2m^2w\mu^2 + (1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j))^2)}}{2m(1+m)(a^2w + c_i((-1+a)^2+4wv_j))},$$

The optimal effort levels are $x_i^* = \frac{am\mu}{2(1+m)c_i}$, $y_{1,i}^* = \frac{(1-a)\mu}{2w}$, $y_{2,i}^* = \frac{(1-a)\mu}{2w}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2m^2w\mu^2+(1+m)c_i((-1+a)^2m\mu^2+4w(p_L+mv_j))}{4(1+m)wc_i}$, which is exactly Case (4) in our former analyses.

(4) The service provider will aim at high type consumers in the second period when the high type consumers' fraction is greater than a certain threshold:

$$\rho > \frac{a^2m^2w\mu^2 + (1+m)c_i((-1+a)^2(-1+m\mu^2) + 4(-1+m)wv_j) + \sqrt{(4m(1+m)(a^2w + c_i((-1+a)^2+4wv_j))(a^2mw\mu^2 + (1+m)c_i((-1+a)^2\mu^2+4wv_j)) + (a^2m^2w\mu^2 + (1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j))^2)}}{2m(1+m)(a^2w + c_i((-1+a)^2+4wv_j))},$$

The optimal effort levels are $x_i^* = \frac{am\rho}{2c_i+2m\rho c_i}$, $y_{1,i}^* = \frac{(1-a)\mu}{2w}$, $y_{2,i}^* = \frac{1-a}{2w}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2m^2w\rho^2+(1+m\rho)c_i(4wp_L+m\rho((-1+a)^2+4wv_j))}{4w(1+m\rho)c_i}$, which is exactly Case (3) in our former analyses.

We can further summarize the above equilibrium results by classifying the different scenarios with demand diversity in each period where D_1 represents the demand in the first period while D_2 represents the demand in the second period in the following table; besides, the threshold of high type consumers' fraction can be denoted as $\rho_1 = \frac{a^2m^2w\mu^2+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j)+\sqrt{4m(1+m)(a^2w+c_i((-1+a)^2+4wv_j))(a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j))+(a^2m^2w\mu^2+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j))^2}}{2m(1+m)(a^2w+c_i((-1+a)^2+4wv_j))}$

and $\rho_2 = \frac{a^2m^2w\mu^2+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j)+\sqrt{4m(1+m)(a^2w+c_i((-1+a)^2+4wv_j))(a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j))+(a^2m^2w\mu^2+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j))^2}}{2m(1+m)(a^2w+c_i((-1+a)^2+4wv_j))}$.

Range of ρ	$D_1 = \rho$					$D_1 = 1$				
	D_2	π_{ij}^*	x_i^*	$y_{1,i}^*$	$y_{2,i}^*$	D_2	π_{ij}^*	x_i^*	$y_{1,i}^*$	$y_{2,i}^*$
$(0,\rho_1)$	m	Case (2)	$\frac{am\mu}{2(m+\rho)c_i}$	$\frac{1-a}{2w}$	$\frac{(1-a)\mu}{2w}$	m	Case (4)	$\frac{am\mu}{2(1+m)c_i}$	$\frac{(1-a)\mu}{2w}$	$\frac{(1-a)\mu}{2w}$
(ρ_1,ρ_2)	ρm	Case (1)	$\frac{am}{2(1+m)c_i}$	$\frac{1-a}{2w}$	$\frac{1-a}{2w}$	m	Case (4)	$\frac{am\mu}{2(1+m)c_i}$	$\frac{(1-a)\mu}{2w}$	$\frac{(1-a)\mu}{2w}$
$(\rho_2,1)$	ρm	Case (1)	$\frac{am}{2(1+m)c_i}$	$\frac{1-a}{2w}$	$\frac{1-a}{2w}$	ρm	Case (3)	$\frac{am\rho}{2c_i+2m\rho c_i}$	$\frac{(1-a)\mu}{2w}$	$\frac{1-a}{2w}$

We can further discover in detail between the two parallel cases as the above table shown in the same range of ρ by considering the specific value of p_H and p_L in each case. The results can be seen as follows:

(a) When $\rho\epsilon(0,\rho_1)$, the service provider aims at both types of consumers in the second period. When $\rho > \rho_3 = \frac{1}{2(1+m)c_i((-1+a)^2+4wv_j)}(a^2mw\mu(-2(1+m)+(2+m)\mu)+(1+m)c_i(-(-1+a)^2(m-\mu^2)-4(-1+m)wv_j)+\sqrt{(a^4m^2w^2\mu^2(-2(1+m)+(2+m)\mu)^2+(1+m)^2c_i^2((-1+a)^2(m+\mu^2)+4(1+m)wv_j)^2-2a^2m(1+m)w\mu c_i((-1+a)^2(-2m(1+m)+m^2\mu+2(1+m)\mu^2-(2+m)\mu^3)+4(1+m)w(2+m(-2+\mu)-2\mu)v_j)})$, the service provider will choose the Case (2) strategy. She will aim at high type consumers in the first period with first period pricing strategy set as $p_H = \frac{2a^2mw\mu+(m+\rho)c_i((-1+a)^2+4wv_j)}{4w(m+\rho)c_i}$.

However, when $\rho < \rho_3$, the service provide will choose the Case (4) strategy where she will aim at both types of consumers in the first period with first period pricing strategy as $p_L = \frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$.

(b) When $\rho\epsilon(\rho_2,1)$, the service provider aims at high type of consumers in the second period, When $\rho > \rho_4 = \frac{a^2mw(-2-m+2(1+m)\mu)+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j)+\sqrt{4m(1+m)c_i((-1+a)^2\mu^2+4wv_j)(a^2mw+(1+m)c_i((-1+a)^2+4wv_j))+(a^2mw(2+m-2(1+m)\mu)-(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j))^2}}{2m(a^2mw+(1+m)c_i((-1+a)^2+4wv_j))}$

, the service provider will choose the Case (1) strategy where she will aim at high type consumers in the first period with first period pricing strategy set as $p_H = \frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$.

However, when $\rho < \rho_4$, the service provide will choose the Case (3) strategy where she will aim at both types of consumers in the first period with first period pricing strategy as $p_L = \frac{2a^2mw\mu\rho+(1+m\rho)c_i((-1+a)^2\mu^2+4wv_j)}{4w(1+m\rho)c_i}$.

(c) When $\rho\epsilon(\rho_1,\rho_2)$, the service provider aims at the same types of consumers in two periods, specifically, when $\rho > \rho_5 = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_i((-1+a)^2\mu^2+4wv_j)}{a^2m(2+m)w+(1+m)^2c_i((-1+a)^2+4wv_j)}$, the service provider will choose the Case (1) strategy where she will aim at high type consumers in both periods. However, when $\rho < \rho_5 = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_i((-1+a)^2\mu^2+4wv_j)}{a^2m(2+m)w+(1+m)^2c_i((-1+a)^2+4wv_j)}$, the service provider will choose the Case (4) strategy where she will aim at both types of consumers in both periods.

We divide the above equilibrium results under different value intervals of the high type consumers' fraction parameter (i.e., ρ) as the summary below:

(A) When $\rho < \min\{\rho_1, \rho_3\}$, Case (4) follows. The optimal effort levels are $x_i^* = \frac{am\mu}{2(1+m)c_i}$, $y_{1,i}^* = \frac{(1-a)\mu}{2w}$, $y_{2,i}^* = \frac{(1-a)\mu}{2w}$; the optimal prices are $p_{1,i}^* = \frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$, $p_{2,i}^* = \frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$; consumer demands then follow as the service provider aims at both types of consumers in both periods.

When $\rho_3 < \rho < \rho_1$, Case (2) follows. The optimal effort levels are $x_i^* = \frac{am\mu}{2(m+\rho)c_i}$, $y_{1,i}^* = \frac{1-a}{2w}$, $y_{2,i}^* = \frac{(1-a)\mu}{2w}$; the optimal prices are $p_{1,i}^* = \frac{2a^2mw\mu+(m+\rho)c_i((-1+a)^2+4wv_j)}{4w(m+\rho)c_i}$, $p_{2,i}^* = \frac{2a^2mw\mu^2+(m+\rho)c_i((-1+a)^2\mu^2+4wv_j)}{4w(m+\rho)c_i}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2mw\mu(m\mu+2\rho)+(m+\rho)c_i((-1+a)^2(m\mu^2+\rho)+4w(m+\rho)v_j)}{4w(m+\rho)c_i}$; consumer demands then is generated as the service provider aims at high consumers in the first period and aims at both consumers in the second period.

(B) When $\rho > \max\{\rho_2, \rho_4\}$, Case (1) follows where the optimal effort levels are $x_i^* = \frac{am}{2(1+m)c_i}$, $y_{1,i}^* = \frac{1-a}{2w}$, $y_{2,i}^* = \frac{1-a}{2w}$; the optimal prices are $p_{1,i}^* = \frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$, $p_{2,i}^* = \frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{\rho(a^2m(2+m)w+(1+m)^2c_i((-1+a)^2+4wv_j))}{4(1+m)wc_i}$; consumers demands then follow as the service provider aims at high type consumers in both periods.

When $\rho_2 < \rho < \rho_4$, Case (3) follows where the optimal effort levels are $x_i^* = \frac{am\rho}{2c_i+2m\rho c_i}$, $y_{1,i}^* = \frac{(1-a)\mu}{2w}$, $y_{2,i}^* = \frac{1-a}{2w}$; the optimal prices are $p_{1,i}^* = \frac{2a^2mw\mu\rho+(1+m\rho)c_i((-1+a)^2\mu^2+4wv_j)}{4w(1+m\rho)c_i}$, $p_{2,i}^* = \frac{2a^2mw\rho+(1+m\rho)c_i((-1+a)^2+4wv_j)}{4w(1+m\rho)c_i}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2mw\rho(2\mu+m\rho)+(1+m\rho)c_i((-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_j)}{4w(1+m\rho)c_i}$; consumer demands then is generated as the service provider aims at both types of consumers in the first period and aims at high type consumers in the second period.

(C) When $\rho_1 < \rho < \min\{\rho_2, \rho_5\}$, Case (4) follows where the optimal effort levels are

$x_i^* = \frac{am\mu}{2(1+m)c_i}$, $y_{1,i}^* = \frac{(1-a)\mu}{2w}$, $y_{2,i}^* = \frac{(1-a)\mu}{2w}$; the optimal prices are $p_{1,i}^* = \frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$, $p_{2,i}^* = \frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$; consumer demands then follow as the service provider aims at both types of consumers in both periods.

When $\max\{\rho_1, \rho_5\} < \rho < \rho_2$, Case (1) follows where the optimal effort levels are $x_i^* = \frac{am}{2(1+m)c_i}$, $y_{1,i}^* = \frac{1-a}{2w}$, $y_{2,i}^* = \frac{1-a}{2w}$; the optimal prices are $p_{1,i}^* = \frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$, $p_{2,i}^* = \frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$; meanwhile the optimal profit $\pi_{ij}^* = \frac{\rho(a^2m(2+m)w+(1+m)^2c_i((-1+a)^2+4wv_j))}{4(1+m)wc_i}$; consumers demands then follow as the service provider aims at high type consumers in both periods.

6.3.1.2 The equilibrium results when cost efficiency is unknown and quality certification is known

We next consider the case when the cost efficiency is private information of the service provider, how will the service provider of different types choose their pricing strategy and effort level strategy?

Section 1. Pure-strategy separating equilibrium

We focus our attention on the pure-strategy separating equilibrium and pooling equilibrium by taking the no-deviation conditions of the separating equilibrium into consideration at first. As the service provider with the efficient cost ($c_e < c_{in}$) as private information will set different prices to the early consumers, in order to signal her cost information and separate herself from the service provider with inefficient cost. Thus, the different first period prices correspond to different types and finally result different early consumers' demands. We can assume there are two circumstances of early consumers' demand in respect of different service provider's type: $D_{1,e} = \rho, D_{1,in} = 1$ or $D_{1,e} = 1,\ D_{1,in} = \rho$.

We suppose the demands of efficient service provider and inefficient service provider are $D_{1,e} = 1,\ D_{1,in} = \rho$ respectively, that is to say, the early consumer's demand of efficient service provider is greater than that of inefficient service provider, therefore, the first period prices should satisfy $p_{1,e} < p_{1,in}$. We then consider the no-deviation conditions under different ranges of ρ as follows.

(1) When $\rho\epsilon(\rho_2, 1)$, if the prices follow $p_{1,e} < p_{1,in}$, then for the cost-efficient service provider, her no-deviation condition should satisfy $\pi_{e\to in} = \frac{\rho(a^2m^2w+(1+m)c_e((-1+a)^2m+4w(p_{1,in}^*+mv_j)))}{4(1+m)wc_e} < \pi_e = \frac{a^2m^2w\rho^2+(1+m\rho)c_e(4wp_{1,e}^*+m\rho((-1+a)^2+4wv_j))}{4w(1+m\rho)c_e}$ and the results show that $\rho p_{1,in}^* - p_{1,e}^* < \frac{a^2m^2(-1+\rho)\rho}{4(1+m)(1+m\rho)c_e}$;

for the cost-inefficient service provider, her no-deviation condition should satisfy $\pi_{in\to e} =$

$\frac{a^2m^2w\rho^2+(1+m\rho)c_{in}(4wp_{1,e}^*+m\rho((-1+a)^2+4wv_j))}{4w(1+m\rho)c_{in}} < \pi_{in} =$

$\frac{\rho(a^2m^2w+(1+m)c_{in}((-1+a)^2m+4w(p_{1,in}^*+mv_j)))}{4(1+m)wc_{in}}$ and thus the prices should satisfy $\rho p_{1,in}^* - p_{1,e}^* > \frac{a^2m^2(-1+\rho)\rho}{4(1+m)(1+m\rho)c_{in}}$;

however, the two conditions for prices cannot sustain simultaneously, thus the assumption $D_{1,e} = 1,\ D_{1,in} = \rho$ will not hold.

(2) When $\rho\epsilon(\rho_1,\rho_2)$, if the prices follow $p_{1,e} < p_{1,in}$, then for the cost-efficient service provider, her no-deviation condition should satisfy $\pi_{e\to in} = \frac{\rho(a^2m^2w+(1+m)c_e((-1+a)^2m+4w(p_{1,in}^*+mv_j)))}{4(1+m)wc_e} < \pi_e =$

$\frac{a^2m^2w\mu^2+(1+m)c_e((-1+a)^2m\mu^2+4w(p_{1,e}^*+mv_j))}{4(1+m)wc_e}$ and results show the prices should satisfy

$\rho p_{1,in}^* - p_{1,e}^* < \frac{m(a^2mw(\mu^2-\rho)+(1+m)c_e((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}{4(1+m)wc_e}$;

for the cost-inefficient service provider, her no-deviation condition should satisfy $\pi_{in\to e} = \frac{a^2m^2w\mu^2+(1+m)c_{in}((-1+a)^2m\mu^2+4w(p_{1,e}^*+mv_j))}{4(1+m)wc_{in}} < \pi_{in} =$

$\frac{\rho(a^2m^2w+(1+m)c_{in}((-1+a)^2m+4w(p_{1,in}^*+mv_j)))}{4(1+m)wc_{in}}$, thus the prices satisfy $\rho p_{1,in}^* - p_{1,e}^* >$

$\frac{m(a^2mw(\mu^2-\rho)+(1+m)c_{in}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}{4(1+m)wc_{in}}$; however, the two conditions for prices cannot sustain simultaneously, thus the assumption $D_{1,e} = 1,\ D_{1,in} = \rho$ will not hold.

(3) When $\rho\epsilon(0,\rho_1)$, if the prices follow $p_{1,e} < p_{1,in}$, then for the cost-efficient service provider, her no-deviation condition should satisfy $\pi_{e\to in} = \frac{a^2m^2w\mu^2+(m+\rho)c_e((-1+a)^2m\mu^2+4w\rho p_{1,in}^*+4mwv_j)}{4w(m+\rho)c_e} < \pi_e =$

$\frac{a^2m^2w\mu^2+(1+m)c_e((-1+a)^2m\mu^2+4w(p_{1,e}^*+mv_j))}{4(1+m)wc_e}$, thus the prices satisfy $\rho p_{1,in}^* - p_{1,e}^* <$

$\frac{a^2m^2\mu^2(-1+\rho)}{4(1+m)(m+\rho)c_e}$;

for the cost-inefficient service provider, her no-deviation condition should satisfy $\pi_{in\to e} = \frac{a^2m^2w\mu^2+(1+m)c_{in}((-1+a)^2m\mu^2+4w(p_{1,e}^*+mv_j))}{4(1+m)wc_{in}} < \pi_{in} =$

$\frac{a^2m^2w\mu^2+(m+\rho)c_{in}((-1+a)^2m\mu^2+4w\rho p_{1,in}^*+4mwv_j)}{4w(m+\rho)c_{in}}$, thus the prices satisfy $\rho p_{1,in}^* - p_{1,e}^* >$

$\frac{a^2m^2\mu^2(-1+\rho)}{4(1+m)(m+\rho)c_{in}}$;

however, the tow conditions for prices cannot sustain simultaneously, thus the assumption $D_{1,e} = 1,\ D_{1,in} = \rho$ will not hold.

The above proof by contradiction shows that our former assumption $D_{1,e} = 1,\ D_{1,in} = \rho$

is not true, therefore the demand should follow $D_{1,e} = \rho$, $D_{1,in} = 1$, that is to say, the early consumer's demand of the efficient service provider is lower than that of the inefficient service provider, therefore, the first period prices satisfy $p_{1,e}^{*} > p_{1,in}^{*}$.

We next give the detailed explanation of the specific separating equilibrium results.

Given $D_{1,e} = \rho$, $D_{1,in} = 1$, the separating equilibrium only exists in a certain range of ρ when the service provider aims at both types of consumers in the first period when her type is known, i.e., $\rho < \min\{\rho_1, \rho_3\}$ or $\rho_2 < \rho < \rho_4$. That is to say, when the service provider aims at both types of consumers in the first period, the cost efficient service provider has the incentive to separate herself from the cost inefficient one by charging the first period consumers a higher price and aims at only high type consumers in that period. We thus can derive the profitable separating equilibrium following the step 1 and 2 in each probable range of ρ.

When $\rho < \min\{\rho_1, \rho_3\}$, both types of service provider aim at both types of consumers in the second period, but the first period demand is distinct by the incentive of separating equilibrium. In step 1 of the derivation process, we analyze the no-deviation condition of both types of service provider to derive the range of optimal pricing strategy. In step 2, we obtain the optimal value of prices by considering the specific range of the cost efficiency difference.

Step 1:

(1) The cost efficient service provider's no-deviation condition:

$\boldsymbol{\pi_e} = \frac{a^2m^2w\mu^2+(m+\rho)c_e((-1+a)^2m\mu^2+4w\rho p_{1,e}+4mwv_j)}{4w(m+\rho)c_e} > \boldsymbol{\pi_{e\to in}} = \frac{a^2m^2w\mu^2+(1+m)c_e\left((-1+a)^2m\mu^2+4w\left(p_{1,in}+mv_j\right)\right)}{4(1+m)wc_e}$ where the prices should satisfy $p_{1,in} < \rho p_{1,e} + \frac{a^2m^2\mu^2(1-\rho)}{4(1+m)(m+\rho)c_e}$.

(2) The cost inefficient service provider's no-deviation condition:

$\boldsymbol{\pi_{in}} = \frac{a^2m^2w\mu^2+(1+m)c_{in}\left((-1+a)^2m\mu^2+4w\left(p_{1,in}+mv_j\right)\right)}{4(1+m)wc_{in}} > \boldsymbol{\pi_{in\to e}} = \frac{a^2m^2w\mu^2+(m+\rho)c_{in}\left((-1+a)^2m\mu^2+4w\rho p_{1,e}+4mwv_j\right)}{4w(m+\rho)c_{in}}$ where the prices should satisfy $p_{1,in} > \rho p_{1,e} + \frac{a^2m^2\mu^2(1-\rho)}{4(1+m)(m+\rho)c_{in}}$.

(3) $\frac{(-1+a)^2(-1+2\mu)}{4w} + \frac{a^2m\mu^2}{2(m+\rho)c_e} + v_j = v_j + \mu(ax + (1-a)y_1) - wy_1{}^2 \le \boldsymbol{p_{1,e}} \le v_j + ax + (1-a)y_1 - wy_1{}^2 = \frac{2a^2mw\mu+(m+\rho)c_e((-1+a)^2+4wv_j)}{4w(m+\rho)c_e}$.

(4) $\boldsymbol{p_{1,in}} = v_j + \mu(ax + (1-a)y_1) - wy_1{}^2 = \frac{2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$.

Step 2:

Given the concrete value of $p_{1,in} = \frac{2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$ and the constraint condition between $p_{1,in}$ and $p_{1,e}$: $p_{1,in} < \rho p_{1,e} + \frac{a^2m^2\mu^2(1-\rho)}{4(1+m)(m+\rho)c_e}$ and $p_{1,in} >$

$\rho p_{1,e}+\frac{a^2m^2\mu^2(1-\rho)}{4(1+m)(m+\rho)c_{in}}$, we can first derive the range of the cost efficiency difference (where we assume $c_e=\xi c_{in}$ with $\xi\in(0,1)$) should satisfy: $\frac{2a^2m(1+m)w\mu^2\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{\mathrm{in}}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)}<\xi<$ $-\frac{a^2mw\mu(-m\mu-2\rho+m(-2+\mu)\rho)}{(m+\rho)(2a^2mw\mu^2+(1+m)c_{\mathrm{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$.

We then derive the range value of the cost efficiency difference when the maximized value of $p_{1,e}$ can not be obtained at $v_j+ax+(1-a)y_1-wy_1{}^2=\frac{2a^2mw\mu+(m+\rho)c_e((-1+a)^2+4wv_j)}{4w(m+\rho)c_e}$, but can only value along the line $p_{1,in}=\rho p_{1,e}+\frac{a^2m^2\mu^2(1-\rho)}{4(1+m)(m+\rho)c_{in}}$ when $p_{1,in}=\frac{2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$. That is to say, the maximized value is $p_{1,e}=\frac{\frac{a^2m\mu^2(m+(2+m)\rho)}{m+\rho}+\frac{(1+m)c_{\mathrm{in}}((-1+a)^2\mu^2+4wv_j)}{w}}{4(1+m)\rho c_{\mathrm{in}}}$ when the constraint condition between $p_{1,in}$ and $p_{1,e}$ satisfy: $\rho p_{1,e}+\frac{a^2m^2\mu^2(1-\rho)}{4(1+m)(m+\rho)c_{in}}<p_{1,in}<\rho p_{1,e}+\frac{a^2m^2\mu^2(1-\rho)}{4(1+m)(m+\rho)c_{in}}$. We can then derive the range of the cost efficiency difference as: $\frac{2a^2m(1+m)w\mu^2\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{\mathrm{in}}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)}<\xi<$ $\frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{\mathrm{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$.

However, the maximized value of $p_{1,e}$ can be obtained at $\frac{2a^2mw\mu+(m+\rho)c_e((-1+a)^2+4wv_j)}{4w(m+\rho)c_e}$ when $\frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{\mathrm{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}<\xi<$ $-\frac{a^2mw\mu(-m\mu-2\rho+m(-2+\mu)\rho)}{(m+\rho)(2a^2mw\mu^2+(1+m)c_{\mathrm{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$.

When $\rho_2<\rho<\rho_4$, both types of service provider aim at high type consumers in the second period, but the first period demand is distinct by the incentive of separating equilibrium. Following the same steps as the above derivation process, we can obtain the optimal value of prices by considering the specific range of the cost efficiency difference.

Step 1:

(1) The cost efficient service provider's no-deviation condition:

$\boldsymbol{\pi_e}=\frac{\rho(a^2m^2w+(1+m)c_e((-1+a)^2m+4w(p_{1,e}+mv_j)))}{4(1+m)wc_e}>\boldsymbol{\pi_{e\to in}}=$ $\frac{a^2m^2w\rho^2+(1+m\rho)c_e(4wp_{1,in}+m\rho((-1+a)^2+4wv_j))}{4w(1+m\rho)c_e}$ where the prices should satisfy $p_{1,in}<\rho p_{1,e}+\frac{a^2m^2(1-\rho)\rho}{4(1+m)(1+m\rho)c_e}$.

(2) The cost inefficient service provider's no-deviation condition:

$\boldsymbol{\pi_{in}} = \frac{a^2m^2w\rho^2+(1+m\rho)c_{in}(4wp_{1,in}+m\rho((-1+a)^2+4wv_j))}{4w(1+m\rho)c_{in}} > \boldsymbol{\pi_{in\to e}} = \frac{\rho(a^2m^2w+(1+m)c_{in}((-1+a)^2m+4w(p_{1,e}+mv_j)))}{4(1+m)wc_{in}}$ where the prices should satisfy $p_{1,in} > \rho p_{1,e} + \frac{a^2m^2(1-\rho)\rho}{4(1+m)(1+m\rho)c_{in}}$.

(3) $\frac{(-1+a)^2(-1+2\mu)}{4w} + \frac{a^2m\mu}{2(1+m)c_e} + v_j = v_j + \mu(ax+(1-a)y_1) - wy_1{}^2 \leq \boldsymbol{p_{1,e}} \leq v_j + ax + (1-a)y_1 - wy_1{}^2 = \frac{2a^2mw+(1+m)c_e((-1+a)^2+4wv_j)}{4(1+m)wc_e}$.

(4) $\boldsymbol{p_{1,in}} = v_j + \mu(ax+(1-a)y_1) - wy_1{}^2 = \frac{2a^2mw\mu\rho+(1+m\rho)c_{in}((-1+a)^2\mu^2+4wv_j)}{4w(1+m\rho)c_{in}}$.

Step 2:

Given the concrete value of $p_{1,in} = \frac{2a^2mw\mu\rho+(1+m\rho)c_{in}((-1+a)^2\mu^2+4wv_j)}{4w(1+m\rho)c_{in}}$ and the constraint condition between $p_{1,in}$ and $p_{1,e}$: $p_{1,in} < \rho p_{1,e} + \frac{a^2m^2(1-\rho)\rho}{4(1+m)(1+m\rho)c_e}$ and $p_{1,in} > \rho p_{1,e} + \frac{a^2m^2(1-\rho)\rho}{4(1+m)(1+m\rho)c_{in}}$, we can first derive the range of the cost efficiency difference (where we assume $c_e = \xi c_{in}$ with $\xi \in (0,1)$) should satisfy: $\frac{2a^2mw\mu\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi < \frac{a^2mw\rho(2+m+m\rho)}{(1+m)(2a^2mw\mu\rho+(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$.

We then derive the range value of the cost efficiency difference when the maximized value of $p_{1,e}$ can not be obtained at $v_j + ax + (1-a)y_1 - wy_1{}^2 = \frac{2a^2mw+(1+m)c_e((-1+a)^2+4wv_j)}{4(1+m)wc_e}$, but can only value along the line $p_{1,in} = \rho p_{1,e} + \frac{a^2m^2(1-\rho)\rho}{4(1+m)(1+m\rho)c_{in}}$ when $p_{1,in} = \frac{2a^2mw\mu\rho+(1+m\rho)c_{in}((-1+a)^2\mu^2+4wv_j)}{4w(1+m\rho)c_{in}}$. That is to say, the maximized value is $p_{1,e} = \frac{\frac{a^2m\rho(2\mu+m(-1+2\mu+\rho))}{1+m}+\frac{(1+m\rho)c_{\text{in}}((-1+a)^2\mu^2+4wv_j)}{w}}{4\rho(1+m\rho)c_{\text{in}}}$ when the constraint condition between $p_{1,in}$ and $p_{1,e}$ satisfy: $\rho p_{1,e} + \frac{a^2m^2(1-\rho)\rho}{4(1+m)(1+m\rho)c_{in}} < p_{1,in} < \rho p_{1,e} + \frac{a^2m^2(1-\rho)\rho}{4(1+m)(1+m\rho)c_{in}}$. We can then derive the range of the cost efficiency difference as: $\frac{2a^2mw\mu\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi < \frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$.

However, the maximized value of $p_{1,e}$ can be obtained at $\frac{2a^2mw+(1+m)c_e((-1+a)^2+4wv_j)}{4(1+m)wc_e}$ when $\frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)} < \xi <$

$\frac{a^2mw\rho(2+m+m\rho)}{(1+m)(2a^2mw\mu\rho+(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$.

We can next conclude from the above derivation process by listing the separating equilibrium results as shown below:

When $\rho < \min\{\rho_1, \rho_3\}$, the separating equilibrium outcome can be classified with the range of ξ, where $\frac{2a^2m(1+m)w\mu^2\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{\text{in}}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi < \frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$, the equilibrium results for cost efficient service provider are $x_e^* = \frac{am\mu}{2(m+\rho)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{(1-a)\mu}{2w}$, $p_{1,e}^* = \frac{\frac{a^2m\mu^2(m+(2+m)\rho)}{m+\rho}+\frac{(1+m)c_{\text{in}}((-1+a)^2\mu^2+4wv_j)}{w}}{4(1+m)\rho c_{\text{in}}}$, $p_{2,e}^* = \frac{2a^2mw\mu^2+(m+\rho)c_e((-1+a)^2\mu^2+4wv_j)}{4w(m+\rho)c_e}$, $\pi_e^* = \frac{a^2m^2w\mu^2+\frac{c_e(a^2mw\mu^2(m+(2+m)\rho)+(1+m)^2(m+\rho)c_{\text{in}}((-1+a)^2\mu^2+4wv_j))}{(1+m)c_{\text{in}}}}{4w(m+\rho)c_e}$; the equilibrium results for cost inefficient service provider are $x_{in}^* = \frac{am\mu}{2(1+m)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{(1-a)\mu}{2w}$, $p_{1,in}^* = \frac{2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$, $p_{2,in}^* = \frac{2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$, $\pi_{in}^* = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$.

When $\frac{2a^2m(1+m)w\mu\rho}{a^2mw\mu^2(m+(2+m)\rho)+(1+m)(m+\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)} < \xi < -\frac{a^2mw\mu(-m\mu-2\rho+m(-2+\mu)\rho)}{(m+\rho)(2a^2mw\mu^2+(1+m)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$, the equilibrium results for cost efficient service provider are $x_e^* = \frac{am\mu}{2(m+\rho)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{(1-a)\mu}{2w}$, $p_{1,e}^* = \frac{2a^2mw\mu+(m+\rho)c_e((-1+a)^2+4wv_j)}{4w(m+\rho)c_e}$, $p_{2,e}^* = \frac{2a^2mw\mu^2+(m+\rho)c_e((-1+a)^2\mu^2+4wv_j)}{4w(m+\rho)c_e}$, $\pi_e^* = \frac{a^2mw\mu(m\mu+2\rho)+(m+\rho)c_e((-1+a)^2(m\mu^2+\rho)+4w(m+\rho)v_j)}{4w(m+\rho)c_e}$; the equilibrium results for cost inefficient service provider remains unchanged $x_{in}^* = \frac{am\mu}{2(1+m)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{(1-a)\mu}{2w}$, $p_{1,in}^* = \frac{2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$, $p_{2,in}^* = \frac{2a^2mw\mu^2+(1+m)c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$, $\pi_{in}^* = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$.

Similarly, when $\rho_2 < \rho < \rho_4$, the separating equilibrium outcome can be classified with the range of ξ, where $\frac{2a^2mw\mu\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2+\rho-2\mu\rho)-4w(-1+\rho)v_j)} < \xi <$

$\frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}$, the equilibrium results for cost efficient service provider are $x_e^* = \frac{am}{2(1+m)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{1-a}{2w}$, $p_{1,e}^* = \frac{\frac{a^2m\rho(2\mu+m(-1+2\mu+\rho))}{1+m}+\frac{(1+m\rho)c_{\text{in}}((-1+a)^2\mu^2+4wv_j)}{w}}{4\rho(1+m\rho)c_{\text{in}}}$, $p_{2,e}^* = \frac{2a^2mw+(1+m)c_e((-1+a)^2+4wv_j)}{4(1+m)wc_e}$, $\pi_e^* = \frac{\rho(a^2m^2w+(1+m)c_e(\frac{a^2mw(2\mu+m(-1+2\mu+\rho))}{(1+m)(1+m\rho)c_{\text{in}}}+\frac{(-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_j}{\rho}))}{4(1+m)wc_e}$; the equilibrium results for cost inefficient service provider are $x_{in}^* = \frac{am\rho}{2(1+m\rho)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{1-a}{2w}$, $p_{1,in}^* = \frac{2a^2mw\mu\rho+(1+m\rho)c_{in}((-1+a)^2\mu^2+4wv_j)}{4w(1+m\rho)c_{in}}$, $p_{2,in}^* = \frac{2a^2mw\rho+(1+m\rho)c_{in}((-1+a)^2+4wv_j)}{4w(1+m\rho)c_{in}}$, $\pi_{in}^* = \frac{a^2mw\rho(2\mu+m\rho)+(1+m\rho)c_{in}((-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_j)}{4w(1+m\rho)c_{in}}$.

When $\frac{2a^2mw\rho(1+m\rho)}{a^2mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)} < \xi < \frac{a^2mw\rho(2+m+m\rho)}{(1+m)(2a^2mw\mu\rho+(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j))}$, the equilibrium results for cost efficient service provider are $x_e^* = \frac{am}{2(1+m)c_e}$, $y_{1,e}^* = \frac{1-a}{2w}$, $y_{2,e}^* = \frac{1-a}{2w}$, $p_{1,e}^* = \frac{2a^2mw+(1+m)c_e((-1+a)^2+4wv_j)}{4(1+m)wc_e}$, $p_{2,e}^* = \frac{2a^2mw+(1+m)c_e((-1+a)^2+4wv_j)}{4(1+m)wc_e}$, $\pi_e^* = \frac{\rho(a^2m(2+m)w+(1+m)^2c_e((-1+a)^2+4wv_j))}{4(1+m)wc_e}$; the equilibrium results for cost inefficient service provider remains unchanged $x_{in}^* = \frac{am\rho}{2(1+m\rho)c_{in}}$, $y_{1,in}^* = \frac{(1-a)\mu}{2w}$, $y_{2,in}^* = \frac{1-a}{2w}$, $p_{1,in}^* = \frac{2a^2mw\mu\rho+(1+m\rho)c_{in}((-1+a)^2\mu^2+4wv_j)}{4w(1+m\rho)c_{in}}$, $p_{2,in}^* = \frac{2a^2mw\rho+(1+m\rho)c_{in}((-1+a)^2+4wv_j)}{4w(1+m\rho)c_{in}}$, $\pi_{in}^* = \frac{a^2mw\rho(2\mu+m\rho)+(1+m\rho)c_{in}((-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_j)}{4w(1+m\rho)c_{in}}$.

Section 2. Pooling equilibrium analyses

After deriving the profitable separating equilibrium results, we can then consider the existence of pooling equilibrium under different value intervals of the high type consumers' fraction parameter ρ.

(1) When $\rho\epsilon(0,\rho_1)$, the service provider aims at both types of consumers in the second period.

When $D_1 = \rho$, the pooling equilibrium results are $x_{pool}^* = \frac{am\mu}{2(m+\rho)c_i}$, $y_{1,pool}^* = \frac{1-a}{2w}$, $y_{2,pool}^* = \frac{(1-a)\mu}{2w}$; however the pricing strategy is $p_{1,pool}^* = v_j + ax + (1-a)y_1 - wy_1{}^2$ where $x = \frac{am\mu}{2(m+\rho)}(\frac{\gamma}{c_e}+\frac{1-\gamma}{c_{\text{in}}})$ and $y_1 = \frac{1-a}{2w}$, $p_{2,pool}^* = \frac{2a^2mw\mu^2+(m+\rho)c_i((-1+a)^2\mu^2+4wv_j)}{4w(m+\rho)c_i}$; meanwhile the optimal profit $\pi_{poolj}^* =$

$\frac{a^2m^2w\mu^2+c_i((-1+a)^2(m+\rho)(m\mu^2+\rho)+2w(\frac{a^2m\gamma\mu\rho}{c_e}-\frac{a^2m(-1+\gamma)\mu\rho}{c_{\text{in}}}+2(m+\rho)^2v_j))}{4w(m+\rho)c_i}$;

When $D_1=1$, the pooling equilibrium results are $x^*_{pool}=\frac{am\mu}{2(1+m)c_i}$, $y^*_{1,pool}=\frac{(1-a)\mu}{2w}$, $y^*_{2,pool}=\frac{(1-a)\mu}{2w}$; however the pricing strategy is $p^*_{1,pool}=v_j+\mu(ax+(1-a)y_1)-wy_1{}^2$ where $x=\frac{am\mu}{2(1+m)}(\frac{\gamma}{c_e}+\frac{1-\gamma}{c_{\text{in}}})$ and $y_1=\frac{(1-a)\mu}{2w}$, $p^*_{2,pool}=\frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$; meanwhile the optimal profit $\pi^*_{poolj}=\frac{a^2m^2w\mu^2+c_i((-1+a)^2(1+m)^2\mu^2+2w(\frac{a^2m\gamma\mu^2}{c_e}-\frac{a^2m(-1+\gamma)\mu^2}{c_{\text{in}}}+2(1+m)^2v_j))}{4(1+m)wc_i}$.

We further consider when will the two pooling equilibria exist under the value interval of the probability when the service provider is cost efficient $\gamma\in[0,1]$. That is to say, when $\gamma<\frac{(1+m)(m+\rho)c_ec_{\text{in}}(\frac{(-1+a)^2(\mu^2-\rho)}{w}+\frac{a^2m^2\mu^2(-1+\rho)}{(1+m)(m+\rho)c_i}+\frac{2a^2m\mu(\frac{\mu}{1+m}-\frac{\rho}{m+\rho})}{c_{\text{in}}}-4(-1+\rho)v_j)}{2a^2m\mu(m\mu+(-1-m+\mu)\rho)(c_e-c_{\text{in}})}$, $\pi^*_{poolj}=\frac{a^2m^2w\mu^2+c_i((-1+a)^2(1+m)^2\mu^2+2w(\frac{a^2m\gamma\mu^2}{c_e}-\frac{a^2m(-1+\gamma)\mu^2}{c_{\text{in}}}+2(1+m)^2v_j))}{4(1+m)wc_i}>\pi^*_{poolj}=\frac{a^2m^2w\mu^2+c_i((-1+a)^2(m+\rho)(m\mu^2+\rho)+2w(\frac{a^2m\gamma\mu\rho}{c_e}-\frac{a^2m(-1+\gamma)\mu\rho}{c_{\text{in}}}+2(m+\rho)^2v_j))}{4w(m+\rho)c_i}$,thus, $D_1=1$; otherwise, when $\gamma>\frac{(1+m)(m+\rho)c_ec_{\text{in}}(\frac{(-1+a)^2(\mu^2-\rho)}{w}+\frac{a^2m^2\mu^2(-1+\rho)}{(1+m)(m+\rho)c_i}+\frac{2a^2m\mu(\frac{\mu}{1+m}-\frac{\rho}{m+\rho})}{c_{\text{in}}}-4(-1+\rho)v_j)}{2a^2m\mu(m\mu+(-1-m+\mu)\rho)(c_e-c_{\text{in}})}$, $D_1=\rho$.

Furthermore, we take the existence of separating equilibrium into consideration when $\rho\epsilon(0,\rho_1)$. We explore that when $\rho<\min\{\rho_1,\rho_3\}$, the separating equilibrium only exists if $\gamma<\frac{mc_e(-c_i+c_{\text{in}})}{2c_i(c_e-c_{\text{in}})}$; however, when $\rho_3<\rho<\rho_1$, only the pooling equilibrium exists.

(2) When $\rho\epsilon(\rho_2,1)$, the service provider aims at high type consumers in the second period.

When $D_1=\rho$, the pooling equilibrium results are $x^*_{pool}=\frac{am}{2(1+m)c_i}$, $y^*_{1,pool}=\frac{1-a}{2w}$, $y^*_{2,pool}=\frac{1-a}{2w}$; however the pricing strategy is $p^*_{1,pool}=v_j+ax+(1-a)y_1-wy_1{}^2$ where $x=\frac{am}{2(m+1)}(\frac{\gamma}{c_e}+\frac{1-\gamma}{c_{\text{in}}})$ and $y_1=\frac{1-a}{2w}$, $p^*_{2,pool}=\frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$; meanwhile the optimal profit $\pi^*_{poolj}=\frac{\rho(a^2m^2w+\frac{c_i(2a^2mw\gamma c_{\text{in}}+c_e(-2a^2mw(-1+\gamma)+(1+m)^2c_{\text{in}}((-1+a)^2+4wv_j)))}{c_ec_{\text{in}}})}{4(1+m)wc_i}$.

When $D_1=1$, the pooling equilibrium results are $x^*_{pool}=\frac{am\rho}{2c_i+2m\rho c_i}$, $y^*_{1,pool}=\frac{(1-a)\mu}{2w}$, $y^*_{2,pool}=\frac{1-a}{2w}$; however the pricing strategy is $p^*_{1,pool}=v_j+\mu(ax+(1-a)y_1)-$

$wy_1{}^2$ where $x = \frac{am\rho}{2+2m\rho}(\frac{\gamma}{c_e} + \frac{1-\gamma}{c_{\text{in}}})$ and $y_1 = \frac{(1-a)\mu}{2w}$, $p^*_{2,pool} = \frac{2a^2mw\rho+(1+m\rho)c_i((-1+a)^2+4wv_j)}{4w(1+m\rho)c_i}$; meanwhile the optimal profit $\pi^*_{poolj} = \frac{a^2m^2w\rho^2+\frac{c_i(2a^2mw\gamma\mu\rho c_{\text{in}}+c_e(-2a^2mw(-1+\gamma)\mu\rho+(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2+m\rho)+4w(1+m\rho)v_j)))}{c_ec_{\text{in}}}}{4w(1+m\rho)c_i}$.

We further consider when will the two pooling equilibria exist under the value interval of the probability when the service provider is cost efficient ($\gamma \in [0,1]$). That is to say, when $\gamma < \frac{c_ec_{\text{in}}(\frac{a^2m^2(-1+\rho)\rho}{c_i}+\frac{2a^2mw\rho(-1+\mu+m\mu-m\rho)+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}{wc_{\text{in}}})}{2a^2m\rho(-1+\mu+m\mu-m\rho)(c_e-c_{\text{in}})}$, $\pi^*_{poolj} = \frac{a^2m^2w\rho^2+\frac{c_i(2a^2mw\gamma\mu\rho c_{\text{in}}+c_e(-2a^2mw(-1+\gamma)\mu\rho+(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2+m\rho)+4w(1+m\rho)v_j)))}{c_ec_{\text{in}}}}{4w(1+m\rho)c_i} > \pi^*_{poolj} = \frac{\rho(a^2m^2w+\frac{c_i(2a^2mw\gamma c_{\text{in}}+c_e(-2a^2mw(-1+\gamma)+(1+m)^2c_{\text{in}}((-1+a)^2+4wv_j)))}{c_ec_{\text{in}}})}{4(1+m)wc_i}$,thus, $D_1 = 1$; otherwise, when $\gamma > \frac{c_ec_{\text{in}}(\frac{a^2m^2(-1+\rho)\rho}{c_i}+\frac{2a^2mw\rho(-1+\mu+m\mu-m\rho)+(1+m)(1+m\rho)c_{\text{in}}((-1+a)^2(\mu^2-\rho)-4w(-1+\rho)v_j)}{wc_{\text{in}}})}{2a^2m\rho(-1+\mu+m\mu-m\rho)(c_e-c_{\text{in}})}$, $D_1 = \rho$.

Furthermore, we take the existence of separating equilibrium into consideration when $\rho\epsilon(\rho_2, 1)$. We discover that when $\rho_2 < \rho < \rho_4$, the separating equilibrium only exists if $\gamma < \frac{m\rho c_e(-c_i+c_{\text{in}})}{2\mu c_i(c_e-c_{\text{in}})}$; however, when $\rho > \max\{\rho_2, \rho_4\}$, only the pooling equilibrium exists.

(3) When $\rho\epsilon(\rho_1, \rho_2)$, the service provider aims at the same type of consumers in both periods, and no separating equilibrium exist in this range of ρ.

Thus, we can derive the pooling equilibria respectively as when $\rho_1 < \rho < \min\{\rho_2, \rho_5\}$, the equilibrium pooling effort level strategy are $x^*_{pool} = \frac{am\mu}{2(1+m)c_i}$, $y^*_{1,pool} = \frac{(1-a)\mu}{2w}$, $y^*_{2,pool} = \frac{(1-a)\mu}{2w}$; however the pricing strategy is $p^*_{1,pool} = v_j + \mu(ax + (1-a)y_1) - wy_1{}^2$ where $x = \frac{am\mu}{2(1+m)}(\frac{\gamma}{c_e} + \frac{1-\gamma}{c_{\text{in}}})$ and $y_1 = \frac{(1-a)\mu}{2w}$, $p^*_{2,pool} = \frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$; meanwhile the optimal profit $\pi^*_{poolj} = \frac{a^2m^2w\mu^2+c_i((-1+a)^2(1+m)^2\mu^2+2w(\frac{a^2m\gamma\mu^2}{c_e}-\frac{a^2m(-1+\gamma)\mu^2}{c_{\text{in}}}+2(1+m)^2v_j))}{4(1+m)wc_i}$; and the consumer demand follows as $D_1 = 1$ and $D_2 = 1$.

When $\max\{\rho_1, \rho_5\} < \rho < \rho_2$, the equilibrium pooling effort level strategy are $x^*_{pool} = \frac{am}{2(1+m)c_i}$, $y^*_{1,pool} = \frac{1-a}{2w}$, $y^*_{2,pool} = \frac{1-a}{2w}$; however the pricing strategy is $p^*_{1,pool} = v_j + ax + (1-a)y_1 - wy_1{}^2$ where $x = \frac{am}{2(m+1)}(\frac{\gamma}{c_e} + \frac{1-\gamma}{c_{\text{in}}})$ and $y_1 = \frac{1-a}{2w}$, $p^*_{2,pool} =$

$\frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$; meanwhile the optimal profit $\pi^*_{poolj} = \frac{\rho(a^2m^2w+\frac{c_i(2a^2mw\gamma c_{\text{in}}+c_e(-2a^2mw(-1+\gamma)+(1+m)^2c_{\text{in}}((-1+a)^2+4wv_j)))}{c_ec_{\text{in}}})}{4(1+m)wc_i}$; and the consumer demand follows as $D_1 = \rho$ and $D_2 = \rho$.

6.3.2. Appendix B: Proofs of Structure Properties

Proof of Proposition 4.1:

This conclusion can be easily derived from the table of equilibrium results as all the FOCs of the optimal service provider's effort level with respect to the work allocation parameter are positive: $\frac{\partial x_i^*}{\partial a} > 0$;while all the FOCs of the optimal consumers' first period effort level and second period effort level with respect to the work allocation parameter are negative: $\frac{\partial y_{1,i}^*}{\partial a} < 0$ and $\frac{\partial y_{2,i}^*}{\partial a} < 0$. Thus, in all cases, the optimal effort level of the service provider (x_i^*) is increasing in the work allocation parameter a; while the optimal effort level of the consumers in both periods ($y_{1,i}^*$ and $y_{2,i}^*$) is decreasing in a.

Proof of Proposition 4.2:

In case (1), the FOC of the optimal service provider's effort level with respect to the cost coefficient parameter is $\frac{\partial x_i^*}{\partial c_i} = -\frac{am}{2(1+m)c_i^2} < 0$; in case (2), $\frac{\partial x_i^*}{\partial c_i} = -\frac{am\mu}{2(m+\rho)c_i^2} < 0$; in case (3), $\frac{\partial x_i^*}{\partial c_i} = -\frac{am\rho}{2(1+m\rho)c_i^2} < 0$; in case (4), $\frac{\partial x_i^*}{\partial c_i} = -\frac{am\mu}{2(1+m)c_i^2} < 0$; thus in all cases, the optimal effort level of the service provider (x_i^*) is decreasing in the service provider's cost coefficient parameter c_i. And as $c_e < c_{in}$, $x_e^* > x_{in}^*$ always holds.

As for the optimal profit under the four cases:

$\pi_{ij}^{(1)} = \frac{\rho(a^2m(2+m)w+(1+m)^2c_i((-1+a)^2+4wv_j))}{4(1+m)wc_i}$, $\frac{\partial \pi_{ij}^{(1)}}{\partial c_i} = -\frac{a^2m(2+m)\rho}{4(1+m)c_i^2} < 0$;

$\pi_{ij}^{(2)} = \frac{a^2mw\mu(m\mu+2\rho)+(m+\rho)c_i((-1+a)^2(m\mu^2+\rho)+4w(m+\rho)v_j)}{4w(m+\rho)c_i}$, $\frac{\partial \pi_{ij}^{(2)}}{\partial c_i} = -\frac{a^2m\mu(m\mu+2\rho)}{4(m+\rho)c_i^2} < 0$;

$\pi_{ij}^{(3)} = \frac{a^2mw\rho(2\mu+m\rho)+(1+m\rho)c_i((-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_j)}{4w(1+m\rho)c_i}$, $\frac{\partial \pi_{ij}^{(3)}}{\partial c_i} = -\frac{a^2m\rho(2\mu+m\rho)}{4(1+m\rho)c_i^2} < 0$;

$\pi_{ij}^{(4)} = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$, $\frac{\partial \pi_{ij}^{(4)}}{\partial c_i} = -\frac{a^2m(2+m)\mu^2}{4(1+m)c_i^2} < 0$; thus in all cases, the optimal profits of the service provider (π_{ij}^*) is decreasing in the service provider's cost coefficient parameter c_i. And as $c_e < c_{in}$, $\pi_{ej}^* > \pi_{inj}^*$ always holds.

Proof of Proposition 4.3:

In case (1), the optimal first period price and optimal second period price are $p_1^{(1)} = \frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$ and $p_2^{(1)} = \frac{2a^2mw+(1+m)c_i((-1+a)^2+4wv_j)}{4(1+m)wc_i}$, and the FOCs of prices with respect to the work allocation parameter a are $\frac{\partial p_1^{(1)}}{\partial a} = \frac{-1+a}{2w} + \frac{am}{(1+m)c_i}$ which is negative when $a < \frac{(1+m)c_i}{2mw+(1+m)c_i}$ but positive when $a > \frac{(1+m)c_i}{2mw+(1+m)c_i}$ and $\frac{\partial p_2^{(1)}}{\partial a} = \frac{-1+a}{2w} + \frac{am}{(1+m)c_i}$ which is negative when $a < \frac{(1+m)c_i}{2mw+(1+m)c_i}$ but positive when $a > \frac{(1+m)c_i}{2mw+(1+m)c_i}$.

In case (2), $p_1^{(2)} = \frac{2a^2mw\mu+(m+\rho)c_i((-1+a)^2+4wv_j)}{4w(m+\rho)c_i}$ and $p_2^{(2)} = \frac{2a^2mw\mu^2+(m+\rho)c_i((-1+a)^2\mu^2+4wv_j)}{4w(m+\rho)c_i}$, and FOCs are $\frac{\partial p_1^{(2)}}{\partial a} = \frac{-1+a}{2w} + \frac{am\mu}{(m+\rho)c_i}$ which is negative when $a < \frac{(m+\rho)c_i}{2mw\mu+(m+\rho)c_i}$ but positive when $a > \frac{(m+\rho)c_i}{2mw\mu+(m+\rho)c_i}$; $\frac{\partial p_2^{(2)}}{\partial a} = \frac{1}{2}\mu^2(\frac{-1+a}{w} + \frac{2am}{(m+\rho)c_i})$ which is negative when $a < \frac{(m+\rho)c_i}{2mw+(m+\rho)c_i}$ but positive when $a > \frac{(m+\rho)c_i}{2mw+(m+\rho)c_i}$.

In case (3), $p_1^{(3)} = \frac{2a^2mw\mu\rho+(1+m\rho)c_i((-1+a)^2\mu^2+4wv_j)}{4w(1+m\rho)c_i}$ and $p_2^{(3)} = \frac{2a^2mw\rho+(1+m\rho)c_i((-1+a)^2+4wv_j)}{4w(1+m\rho)c_i}$, and FOCs are $\frac{\partial p_1^{(3)}}{\partial a} = \frac{(-1+a)\mu^2}{2w} + \frac{am\mu\rho}{c_i+m\rho c_i}$ which is negative when $a < \frac{\mu(1+m\rho)c_i}{2mw\rho+(\mu+m\mu\rho)c_i}$ but positive when $a > \frac{\mu(1+m\rho)c_i}{2mw\rho+(\mu+m\mu\rho)c_i}$; $\frac{\partial p_2^{(3)}}{\partial a} = \frac{-1+a}{2w} + \frac{am\rho}{c_i+m\rho c_i}$ which is negative when $a < 1 - \frac{2mw\rho}{2mw\rho+c_i+m\rho c_i}$ but positive when $a > 1 - \frac{2mw\rho}{2mw\rho+c_i+m\rho c_i}$.

In case (4), $p_1^{(4)} = \frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$ and $p_2^{(4)} = \frac{2a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_i}$, and FOCs are $\frac{\partial p_1^{(4)}}{\partial a} = \frac{1}{2}\mu^2(\frac{-1+a}{w} + \frac{2am}{(1+m)c_i})$ which is negative when $a < \frac{(1+m)c_i}{2mw+(1+m)c_i}$ but positive when $a > \frac{(1+m)c_i}{2mw+(1+m)c_i}$; $\frac{\partial p_2^{(4)}}{\partial a} = \frac{1}{2}\mu^2(\frac{-1+a}{w} + \frac{2am}{(1+m)c_i})$ which is negative when $a < \frac{(1+m)c_i}{2mw+(1+m)c_i}$ but positive when $a > \frac{(1+m)c_i}{2mw+(1+m)c_i}$.

Proof of Proposition 4.4:

The proof of separating equilibrium can be found in **Appendix A** and thus is omitted here. Note that $\rho_1 =$

$$\frac{a^2m^2w\mu^2+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j)+\sqrt{(4m(1+m)(a^2w+c_i((-1+a)^2+4wv_j))(a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j))+(a^2m^2w\mu^2+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j))^2)}}{2m(1+m)(a^2w+c_i((-1+a)^2+4wv_j))},$$

$\rho_2 =$

$$\frac{a^2m^2w\mu^2+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j)+\sqrt{(4m(1+m)(a^2w+c_i((-1+a)^2+4wv_j))(a^2mw\mu^2+(1+m)c_i((-1+a)^2\mu^2+4wv_j))+(a^2m^2w\mu^2+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j))^2)}}{2m(1+m)(a^2w+c_i((-1+a)^2+4wv_j))},$$

$$\rho_3 = \frac{1}{2(1+m)c_i((-1+a)^2+4wv_j)}(a^2mw\mu(-2(1+m)+(2+m)\mu)+(1+m)c_i(-(-1+a)^2(m-\mu^2)-4(-1+m)wv_j)+\sqrt{(a^4m^2w^2\mu^2(-2(1+m)+(2+m)\mu)^2+(1+m)^2c_i^2((-1+a)^2(m+\mu^2)+4(1+m)wv_j)^2-2a^2m(1+m)w\mu c_i((-1+a)^2(-2m(1+m)+m^2\mu+2(1+m)\mu^2-(2+m)\mu^3)+4(1+m)w(2+m(-2+\mu)-2\mu)v_j))},$$

$\rho_4 =$

$$\frac{a^2mw(-2-m+2(1+m)\mu)+(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j)+\sqrt{(4m(1+m)c_i((-1+a)^2\mu^2+4wv_j)(a^2mw+(1+m)c_i((-1+a)^2+4wv_j))+(a^2mw(2+m-2(1+m)\mu)-(1+m)c_i((-1+a)^2(-1+m\mu^2)+4(-1+m)wv_j))^2)}}{2m(a^2mw+(1+m)c_i((-1+a)^2+4wv_j))}$$

Proof of Proposition 4.5:

The proof of separating equilibrium can be found in **Appendix A** and thus is omitted here.

Proof of Corollary 4.1:

In case (A), $x_e^* = \frac{am\mu}{2(m+\rho)c_e} > x_{in}^* = \frac{am\mu}{2(1+m)c_{in}}$; $y_{1,e}^* = \frac{1-a}{2w} > y_{1,in}^* = \frac{(1-a)\mu}{2w}$; $y_{2,e}^* = y_{2,in}^* = \frac{(1-a)\mu}{2w}$; while in case (B), $x_e^* = \frac{am}{2(1+m)c_e} > x_{in}^* = \frac{am\rho}{2(1+m\rho)c_{in}}$; $y_{1,e}^* = \frac{1-a}{2w} > y_{1,in}^* = \frac{(1-a)\mu}{2w}$; $y_{2,e}^* = y_{2,in}^* = \frac{(1-a)}{2w}$.

Proof of Corollary 4.2:

In the separating equilibrium of case (A), $\pi_e^{(sep2)} = \frac{a^2m^2w\mu^2+\frac{c_e(a^2mw\mu^2(m+(2+m)\rho)+(1+m)^2(m+\rho)c_{in}((-1+a)^2\mu^2+4wv_j))}{(1+m)c_{in}}}{4w(m+\rho)c_e}$, $\pi_e^{(2)} = \frac{a^2mw\mu(m\mu+2\rho)+(m+\rho)c_e((-1+a)^2(m\mu^2+\rho)+4w(m+\rho)v_j)}{4w(m+\rho)c_e}$, $\pi_{in}^{(4)} = \frac{a^2m(2+m)w\mu^2+(1+m)^2c_{in}((-1+a)^2\mu^2+4wv_j)}{4(1+m)wc_{in}}$. The FOCs of these profit functions with respect of the work allocation parameter are $\frac{\partial\pi_e^{(sep2)}}{\partial a} = \frac{\mu^2(am^2w+(-1+a)(1+m)(m+\rho)c_e+\frac{amw(m+(2+m)\rho)c_e}{(1+m)c_{in}})}{2w(m+\rho)c_e}$ which is negative when $a < \frac{(1+m)^2(m+\rho)c_ec_{in}}{m^2(1+m)wc_{in}+c_e(mw(m+(2+m)\rho)+(1+m)^2(m+\rho)c_{in})}$ but positive when $a > \frac{(1+m)^2(m+\rho)c_ec_{in}}{m^2(1+m)wc_{in}+c_e(mw(m+(2+m)\rho)+(1+m)^2(m+\rho)c_{in})}$; $\frac{\partial\pi_e^{(2)}}{\partial a} = \frac{1}{2}(\frac{(-1+a)(m\mu^2+\rho)}{w} + \frac{am\mu(m\mu+2\rho)}{(m+\rho)c_e})$ which is negative when $a < \frac{(m+\rho)(m\mu^2+\rho)c_e}{mw\mu(m\mu+2\rho)+(m+\rho)(m\mu^2+\rho)c_e}$ but positive when $a > \frac{(m+\rho)(m\mu^2+\rho)c_e}{mw\mu(m\mu+2\rho)+(m+\rho)(m\mu^2+\rho)c_e}$; $\frac{\partial\pi_{in}^{(4)}}{\partial a} = \frac{\mu^2(\frac{(-1+a)(1+m)^2}{w}+\frac{am(2+m)}{c_{in}})}{2(1+m)}$ which is negative when $a < \frac{(1+m)^2c_{in}}{m(2+m)w+(1+m)^2c_{in}}$ but positive when $a > \frac{(1+m)^2c_{in}}{m(2+m)w+(1+m)^2c_{in}}$.

In the separating equilibrium of case (B), $\pi_e^{(sep1)} = \frac{\rho(a^2m^2w+(1+m)c_e(\frac{a^2mw(2\mu+m(-1+2\mu+\rho))}{(1+m)(1+m\rho)c_{in}}+\frac{(-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_j}{\rho}))}{4(1+m)wc_e}$, $\pi_e^{(1)} = \frac{\rho(a^2m(2+m)w+(1+m)^2c_e((-1+a)^2+4wv_j))}{4(1+m)wc_e}$, $\pi_{in}^{(3)} = \frac{a^2mw\rho(2\mu+m\rho)+(1+m\rho)c_{in}((-1+a)^2(\mu^2+m\rho)+4(w+mw\rho)v_j)}{4w(1+m\rho)c_{in}}$. The FOCs of these profit functions with respect of the work allocation parameter are $\frac{\partial\pi_e^{(sep1)}}{\partial a} = \frac{\rho(2am^2w+2c_e(\frac{(-1+a)(1+m)(\mu^2+m\rho)}{\rho}+\frac{amw(2\mu+m(-1+2\mu+\rho))}{(1+m\rho)c_{in}}))}{4(1+m)wc_e}$ which is negative when $a <$

$\frac{(1+m)(1+m\rho)(\mu^2+m\rho)c_e c_{\text{in}}}{m^2 w\rho(1+m\rho)c_{\text{in}}+c_e(mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)(\mu^2+m\rho)c_{\text{in}})}$ but positive when $a > \frac{(1+m)(1+m\rho)(\mu^2+m\rho)c_e c_{\text{in}}}{m^2 w\rho(1+m\rho)c_{\text{in}}+c_e(mw\rho(2\mu+m(-1+2\mu+\rho))+(1+m)(1+m\rho)(\mu^2+m\rho)c_{\text{in}})}$; $\frac{\partial\pi_e^{(1)}}{\partial a} = \frac{\rho(\frac{(-1+a)(1+m)^2}{w}+\frac{am(2+m)}{c_e})}{2(1+m)}$ which is negative when $a < \frac{(1+m)^2 c_e}{m(2+m)w+(1+m)^2 c_e}$ but positive when $a > \frac{(1+m)^2 c_e}{m(2+m)w+(1+m)^2 c_e}$; $\frac{\partial\pi_{in}^{(3)}}{\partial a} = \frac{1}{2}(\frac{(-1+a)(\mu^2+m\rho)}{w} + \frac{am\rho(2\mu+m\rho)}{(1+m\rho)c_{\text{in}}})$ which is negative when $a < \frac{(1+m\rho)(\mu^2+m\rho)c_{\text{in}}}{mw\rho(2\mu+m\rho)+(1+m\rho)(\mu^2+m\rho)c_{\text{in}}}$ but positive when $a > \frac{(1+m\rho)(\mu^2+m\rho)c_{\text{in}}}{mw\rho(2\mu+m\rho)+(1+m\rho)(\mu^2+m\rho)c_{\text{in}}}$.

Proof of Proposition 4.7:

The proof of pooling equilibrium can be found in **Appendix A** and thus is omitted here. While considering the FOCs of these equilibrium profits with respect to the prior probability of cost-efficient service provider γ, we can derive $\frac{\partial\pi_p^{(2)}}{\partial\gamma} = \frac{a^2 m\mu\rho(-c_e+c_{in})}{2(m+\rho)c_e c_{in}} > \frac{\partial\pi_p^{(4)}}{\partial\gamma} = \frac{a^2 m\mu^2(-c_e+c_{in})}{2(1+m)c_e c_{in}} > 0$ in case (C); $\frac{\partial\pi_p^{(1)}}{\partial\gamma} = \frac{a^2 m\rho(-c_e+c_{\text{in}})}{2(1+m)c_e c_{\text{in}}} > \frac{\partial\pi_p^{(3)}}{\partial\gamma} = \frac{a^2 m\mu\rho(-c_e+c_{\text{in}})}{2(1+m\rho)c_e c_{\text{in}}} > 0$ in case (D).

6.4. Appendix for Chapter 5

6.4.1. Appendix A: Derivation of the Expected Quality and Model Robustness Test

Given an service outcome score $\tau_1(\gamma)$ generated by the first period consumer's posting service outcome, the inverse function can be denoted as $\tau_1^{-1}(\tau) = -\mu_\theta(ax + (1-a)y_1) - \ln\left(\frac{1-\tau_1}{\tau_1}\right)$, which depicts the corresponding consumer's idiosyncratic factors that affect the service outcome (γ). We mainly focus on the scenario where the consumer review levels are evenly distributed between the range of zero to one $\{0, \frac{1}{s-1}, \dots, \frac{s-2}{s-1}, 1\}$. Therefore, the mapping relation from the service outcome score $\tau_1(\gamma)$ to the consumer's idiosyncratic factors affecting the service outcome γ is as follows: consumers posting the highest rating score (1) own an idiosyncratic outcome influencing factor in $[\tau_1^{-1}(1-\Delta), 0]$, where $\tau_1^{-1}(1-\Delta) = -\mu_\theta(ax + (1-a)y_1) + \ln\left(\frac{1-\Delta}{\Delta}\right)$; consumers posting the lowest rating score (0) own an idiosyncratic outcome influencing factor in $[-1, \tau_1^{-1}(\Delta)]$, where $\tau_1^{-1}(\Delta) = -\mu_\theta(ax + (1-a)y_1) - \ln\left(\frac{1-\Delta}{\Delta}\right)$; consumers posting $\frac{i}{s-1}$ have an idiosyncratic outcome influencing factor in $[\tau_1^{-1}\left(\frac{i}{s-1} + \Delta\right), \tau_1^{-1}\left(\frac{i}{s-1} - \Delta\right)]$, where $\tau_1^{-1}\left(\frac{i}{s-1} + \Delta\right) = -\mu_\theta(ax + (1-a)y_1) - \ln\left[\frac{(1-\Delta)(s-1)-i}{i+\Delta(s-1)}\right]$ and $\tau_1^{-1}\left(\frac{i}{s-1} - \Delta\right) = -\mu_\theta(ax + (1-a)y_1) - \ln\left[\frac{(1+\Delta)(s-1)-i}{i-\Delta(s-1)}\right]$. We consider the scenario where all available ratings should be significant and taken into account, therefore, the two boundary conditions should be satisfied simultaneously: $\tau_1^{-1}(1-\Delta) \leq 0$ and $\tau_1^{-1}(\Delta) \geq -1$, i.e., $-\mu_\theta(ax + (1-a)y_1) + \ln\left(\frac{1-\Delta}{\Delta}\right) \leq 0$ and $-\mu_\theta(ax + (1-a)y_1) - \ln\left(\frac{1-\Delta}{\Delta}\right) + 1 \geq 0$.

When all available ratings are significant, we can derive the number of reviews posting by first period consumers as $N(x, y_1) = \int_{-1}^{\tau_1^{-1}(\Delta)} g(\gamma)d\gamma + \sum_{i=1}^{s-2}\left[\int_{\tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right)}^{\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right)} g(\gamma)d\gamma\right] + \int_{\tau_1^{-1}(1-\Delta)}^{0} g(\gamma)d\gamma$, where $\tau_1^{-1}(\Delta) = -\mu_\theta(ax + (1-a)y_1) - \ln\left(\frac{1-\Delta}{\Delta}\right)$, $\tau_1^{-1}\left(\frac{i}{s-1} + \Delta\right) = -\mu_\theta(ax + (1-a)y_1) - \ln\left[\frac{(1-\Delta)(s-1)-i}{i+\Delta(s-1)}\right]$, $\tau_1^{-1}\left(\frac{i}{s-1} - \Delta\right) = -\mu_\theta(ax + (1-a)y_1) - \ln\left[\frac{(1+\Delta)(s-1)-i}{i-\Delta(s-1)}\right]$, $\tau_1^{-1}(1-\Delta) = -\mu_\theta(ax + (1-a)y_1) + \ln\left(\frac{1-\Delta}{\Delta}\right)$ and we have assumed that γ follows a uniform distribution over the support $[-1,0]$, so $g(\gamma) = 1$. Therefore, we further substitute the corresponding items into the aforementioned expression regarding the number of reviews, we can obtain the results as $N(x, y_1) = 1 - 2\left(\ln\left(\frac{1-\Delta}{\Delta}\right) + \sum_{i=1}^{s-2}\ln\left[\frac{i-\Delta(s-1)}{i+\Delta(s-1)}\right]\right)$ for $s \geq 3$. When $s = 2$, which means the binary rating is taken in the service provider's review platform, the number of reviews can be simplified as $N(x, y_1) = 1 - 2\ln(\frac{1-\Delta}{\Delta})$. In our model extension to further consider the case of ternary rating when $s = 3$, the number of reviews can be simplified as $N(x, y_1) = 1 - 2\ln\left(\frac{1-\Delta}{\Delta}\right) + 2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)$ which is a fixed number. We can make further analysis to derive the interval range of $\Delta = \frac{\alpha-\varphi_1}{\beta}$ in our base model setting. That is to say, the number of reviews should be nonnegative and meanwhile should not be greater than the sum of consumers in the first period which is normalized to 1 in our assumption. Therefore, from the observation $0 \le N(x, y_1) \le 1$ we can derive the scope of Δ as $\frac{1}{\sqrt{e}+1} \le \Delta \le \frac{1}{2}$.

When all available rating levels are significant, we can derive the mean of reviews posting by first period consumers as $\bar{R}(x, y_1) = \frac{1}{N(x,y_1)}[\sum_{i=1}^{s-2}(\frac{i}{s-1})\int_{\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right)}^{\tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right)} g(\gamma)d\gamma + \int_{\tau_1^{-1}(1-\Delta)}^{0}(1)g(\gamma)d\gamma]$ which is quite complicated in form. When we consider the binary rating ($s = 2$), the mean of reviews can be simplified as $\bar{R}(x, y_1) = \frac{\ln\left(\frac{1-\Delta}{\Delta}\right) - (a(x-y_1)+y_1)\mu_\theta}{-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)}$. When we further consider the case of ternary rating ($s = 3$), the mean of reviews can be simplified as $\bar{R}(x, y_1) = \frac{-\ln\left(\frac{1-\Delta}{\Delta}\right)+\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)+(a(x-y_1)+y_1)\mu_\theta}{1-2\ln\left(\frac{1-\Delta}{\Delta}\right)+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)}$ which follows uniform distribution.

The second-period consumers' expected quality level of the service provider is given by $E[\mu_\theta|\bar{R}(x, y_1), N(x, y_1)] = \frac{\rho N(x,y_1)}{\rho N(x,y_1)+1}\mu_{\theta R} + \frac{1}{\rho N(x,y_1)+1}\mu_{\theta NR}$ which also follows uniform distribution where $\mu_{\theta R} = \mu_{\theta min} + \bar{R}(x, y_1)(\mu_{\theta max} - \mu_{\theta min})$ and $\mu_{\theta NR} = E[\mu_\theta|0 < \mu_\theta < 1] = \frac{1}{2}$. Taking into consideration the case of binary rating and ternary rating respectively by substituting $N(x, y_1)$ and $\bar{R}(x, y_1)$ back to $E[\mu_\theta|\bar{R}(x, y_1), N(x, y_1)]$ under each circumstance, the resulting second-period expectations of the service provider's quality level are yields as $E[\mu_\theta|\bar{R}(x, y_1), N(x, y_1)] = \frac{-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho - 2ax\mu_\theta\rho + 2(-1+a)\mu_\theta\rho y_1}{-2+\left(-2+4\ln\left(\frac{1-\Delta}{\Delta}\right)\right)\rho}$ when $s = 2$ and $E[\mu_\theta|\bar{R}(x, y_1), N(x, y_1)] = \frac{-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho - 2\left(\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)+ax\mu_\theta\right)\rho + 2(-1+a)\mu_\theta\rho y_1}{-2+\left(-2+4\ln\left(\frac{1-\Delta}{\Delta}\right)-4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\right)\rho}$ when $s = 3$.

Proof of the robustness of the general setting: γ follows a uniform distribution over an arbitrary support:

We let γ follow a uniform distribution over an arbitrary support $[-b_1, b_2]$,where in original model is the exceptional case by setting $b_1 = 1$ and $b_2 = 0$. The PDF and CDF of γ are $g(\gamma) = \frac{1}{b_2+b_1}$ and $G(\gamma) = \frac{\gamma+b_1}{b_2+b_1}$. We can then derive the probability of the service outcome to be a success conditional on the true quality level of the service provider and the effort level strategies in respect of both the service provider and consumers in each time period as

$$\Pr\{O=1|\mu_\theta,x,y_t\}=\Pr\{u_t(\gamma)>0|\mu_\theta,x,y_t\}=1-G\big(-\mu_\theta(ax+(1-a)y_t)\big)$$
$$=\frac{\mu_\theta(ax+(1-a)y_t)+b_2}{b_2+b_1},\mu_\theta\sim U[0,1],t=1,2.$$

Furthermore, when all available rating levels are significant with review process, we can derive the number of reviews posting by first period consumers as $N(x,y_1)=\int_{-b_1}^{\tau_1^{-1}(\Delta)}g(\gamma)d\gamma+\sum_{i=1}^{s-2}\left[\int_{\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right)}^{\tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right)}g(\gamma)d\gamma\right]+\int_{\tau_1^{-1}(1-\Delta)}^{b_2}g(\gamma)d\gamma$ where $g(\gamma)=\frac{1}{b_2+b_1}$, and the mean of reviews posting by first period consumers as $\bar{R}(x,y_1)=\frac{1}{N(x,y_1)}[\sum_{i=1}^{s-2}(\frac{i}{s-1})\int_{\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right)}^{\tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right)}g(\gamma)d\gamma+\int_{\tau_1^{-1}(1-\Delta)}^{0}(1)g(\gamma)d\gamma]$ where $g(\gamma)=\frac{1}{b_2+b_1}$. When we consider the binary rating ($s=2$), the number of reviews can be simplified as $N(x,y_1)=1-\frac{2\ln\left(\frac{1-\Delta}{\Delta}\right)}{b_1+b_2}$ and the mean of reviews can be simplified as $\bar{R}(x,y_1)=\frac{-\ln\left(\frac{1-\Delta}{\Delta}\right)+ax\mu_\theta+b_2+(\mu_\theta-a\mu_\theta)y_1}{-2\ln\left(\frac{1-\Delta}{\Delta}\right)+b_1+b_2}$, which will result in the second-period consumers' expected quality level of the service provider as $E\left[\mu_\theta\middle|\bar{R}(x,y_1),N(x,y_1)\right]=-\frac{b_1+(1+2\rho)b_2-2\rho(\ln\left(\frac{1-\Delta}{\Delta}\right)-ax\mu_\theta+(-1+a)\mu_\theta y_1)}{4\ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2(1+\rho)b_1-2(1+\rho)b_2}$.

The expected profit function of the service provider can be demonstrated as

$$\pi(p_1,p_2,x)=\mathbf{1}(EU_1\geq 0)(p_1-cx^2)+\mathbf{1}(EU_2\geq 0)m(p_2-cx^2).$$

In the case without review process just as the benchmark case in our original model, the optimal prices over two periods are market clearing prices and can be derived as $p_1=\frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_1)+b_2}{b_2+b_1}r-wy_1{}^2$ and $p_2=\frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_2)+b_2}{b_2+b_1}r-wy_2{}^2$. The service provider's ex post payoff function can be transforms into the following function and the service provider's objective is to maximize her payoff by setting her optimal effort level strategy $\hat{x}^*$:

$$\pi(x)=\left(\frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_1)+b_2}{b_2+b_1}r-wy_1{}^2-cx^2\right)$$
$$+m\left(\frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_2)+b_2}{b_2+b_1}r-wy_2{}^2-cx^2\right).$$

At the same time, the early consumer's (the follower consumer's) objective is to maximize his expected utility by setting his optimal effort level strategy $\hat{y}_1^*(\hat{y}_2^*)$:

$EU_1(y_1)=E[r1_{O=1}-wy_1{}^2-p_1]=\frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_1)+b_2}{b_2+b_1}r-wy_1{}^2-p_1$ and

$EU_2(y_2)=E[r1_{O=1}-wy_2{}^2-p_2]=\frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_2)+b_2}{b_2+b_1}r-wy_2{}^2-p_2$.

We can get clear analytical results

$$\hat{x}^*=\frac{ar}{4cb_1+4cb_2},$$
$$\hat{y}_1^*=\hat{y}_2^*=\frac{r-ar}{4wb_1+4wb_2},$$

$$\hat{p}_1^* = \hat{p}_2^* = \frac{r((-1+a)^2cr + 2a^2rw + 16cwb_2(b_1+b_2))}{16cw(b_1+b_2)^2},$$
$$\hat{\pi}^* = \frac{(1+m)r((-1+a)^2cr + a^2rw + 16cwb_2(b_1+b_2))}{16cw(b_1+b_2)^2}.$$

Following the same methods taking in original model, we can get the monotonicity of the above equilibrium results with regards to the work allocation parameter a with the similar properties.

As for the case with review process just as the main model in our original setting, the optimal prices over two periods are market clearing prices and can be derived as $p_1 = \frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_1)+b_2}{b_2+b_1}r - w{y_1}^2$ and $p_2 = \frac{E[\mu_\theta|\bar{R}(x,y_1),N(x,y_1)](ax+(1-a)y_1)+b_2}{b_2+b_1}r - w{y_2}^2$. The service provider's ex post payoff function can be transforms into the following function and the service provider's objective is to maximize her payoff by setting her optimal effort level strategy x^*:

$$\pi(x) = \left(\frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_1)+b_2}{b_2+b_1}r - w{y_1}^2 - cx^2\right) + m\left(\frac{E[\mu_\theta|\bar{R}(x,y_1),N(x,y_1)](ax+(1-a)y_2)+b_2}{b_2+b_1}r - w{y_2}^2 - cx^2\right).$$

At the same time, the early consumer's (the follower consumer's) objective is to maximize his expected utility by setting his optimal effort level strategy $y_1^*(y_2^*)$:

$$EU_1(y_1) = E[r1_{O=1} - w{y_1}^2 - p_1] = \frac{E[\mu_\theta|0<\mu_\theta<1](ax+(1-a)y_1)+b_2}{b_2+b_1}r - w{y_1}^2 - p_1$$

and $EU_2(y_2) = E[r1_{O=1} - w{y_2}^2 - p_2] = \frac{E[\mu_\theta|\bar{R}(x,y_1),N(,y_1)](ax+(1-a)y_2)+b_2}{b_2+b_1}r - w{y_2}^2 - p_2$. Note that $E[\mu_\theta|\bar{R}(x,y_1),N(x,y_1)] = -\frac{b_1+(1+2\rho)b_2-2\rho(\ln(\frac{1-\Delta}{\Delta})-ax\mu_\theta+(-1+a)\mu_\theta y_1)}{4\ln(\frac{1-\Delta}{\Delta})\rho-2(1+\rho)b_1-2(1+\rho)b_2}$ when $s=2$, following the above derivation of the second period consumers' expected service quality. Thus, following the same method in our main model, we can derive the optimal effort level strategy that can maximize each party's objective function. The equilibrium results are

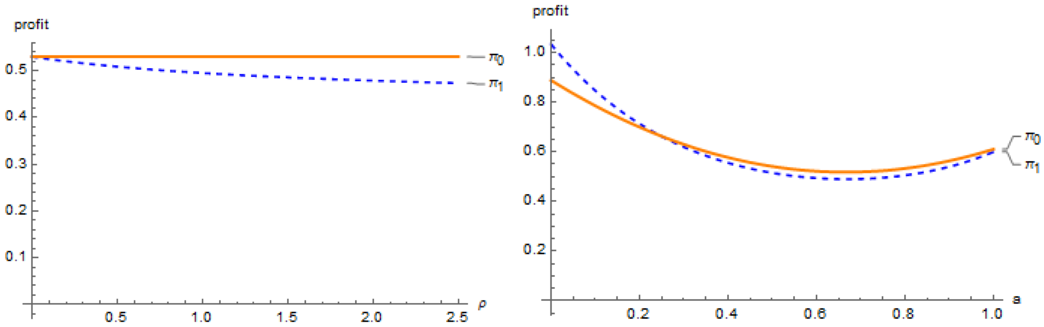
$$x^* = \frac{am(r-ar)^2\mu\rho(2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho - b_1 - (1+2\rho)b_2 - \frac{(-1+a)^2r\mu\rho}{2w(b_1+b_2)}) - 2arw(b_1+b_2)(2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho - (1+\rho)b_1 - (1+\rho)b_2)(2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m)\rho - (1+m+\rho)b_1 - (1+m+\rho+2m\rho)b_2 - \frac{(-1+a)^2mr\mu\rho}{2w(b_1+b_2)})}{2(b_1+b_2)^2(-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho + (1+\rho)b_1 + (1+\rho)b_2)^2(4c(1+m)w - \frac{a^2mr\mu\rho((-1+a)^2r\mu\rho + 4w(b_1+b_2)(-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho + (1+\rho)b_1 + (1+\rho)b_2))}{(b_1+b_2)^2(-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho + (1+\rho)b_1 + (1+\rho)b_2)^2})}$$

$$y_1^* = \frac{r-ar}{4wb_1+4wb_2},$$

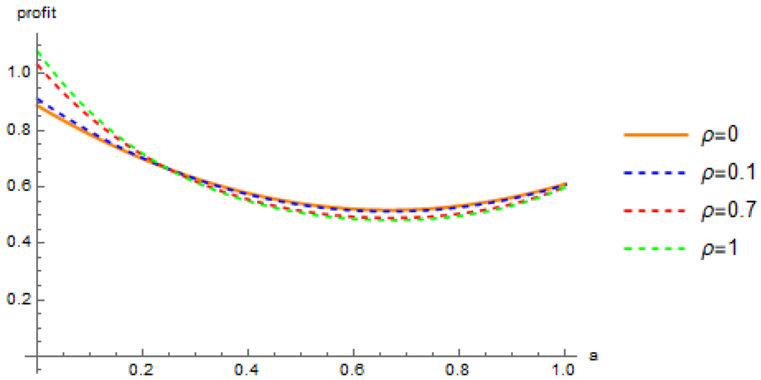
$$y_2^* = \frac{(-1+a)r(-\frac{a^2r\mu\rho^2((-1+a)^2mr\mu - 4\ln\left(\frac{1-\Delta}{\Delta}\right)(-1+m)w(b_1+b_2))}{b_1+b_2} + 2(-2(-1+a)^2c\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m)r\mu\rho^2 + 2c(1+m)w(1+\rho)b_1^3 - 2c(1+m)wb_1^2(2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho(2+\rho) - (1+\rho)(3+2\rho)b_2) + b_2(\rho(a^2rw\mu(1-m+\rho-2m\rho) + c(1+m)(8\ln\left(\frac{1-\Delta}{\Delta}\right)^2w\rho + (-1+a)^2r\mu(1+\rho))) + 2c(1+m)wb_2(-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho(2+3\rho) + (1+\rho)(1+2\rho)b_2)) + b_1(\rho(a^2rw\mu(1-m+\rho) + c(1+m)(8\ln\left(\frac{1-\Delta}{\Delta}\right)^2w\rho + (-1+a)^2r\mu(1+\rho))) + 2c(1+m)w(1+\rho)b_2(-8\ln\left(\frac{1-\Delta}{\Delta}\right)\rho + (3+4\rho)b_2))))}{4w((-1+a)^2a^2mr^2\mu^2\rho^2 - 4w(b_1+b_2)(-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho + (1+\rho)b_1 + (1+\rho)b_2)(-a^2mr\mu\rho + c(1+m)(b_1+b_2)(-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho + (1+\rho)b_1 + (1+\rho)b_2)))}$$

They are quite complicated in forms, even though we can substitute the above effort level strategy into the price function to get the optimal pricing strategy and finally derive the optimal expected profit, all the equilibrium results are analytically solvable except for the tedious expressions. We thus resolve to numerical analysis to obtain the same analytical results as the original model.

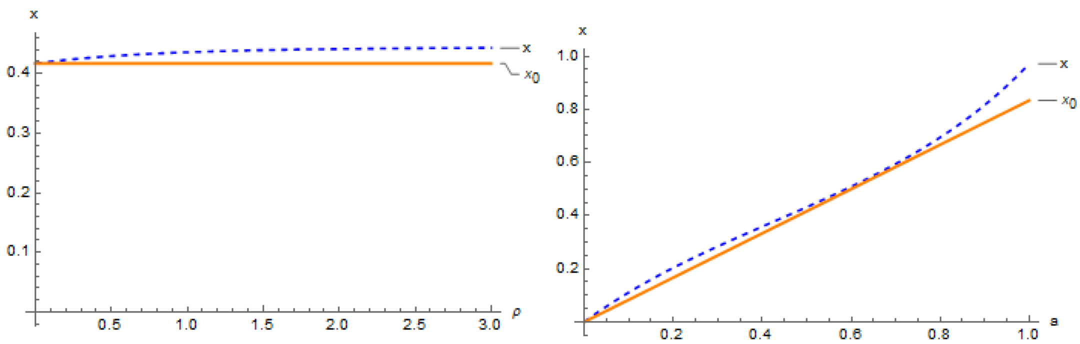
The following several figures demonstrate the main conclusions that still hold with the change of the support regarding γ.



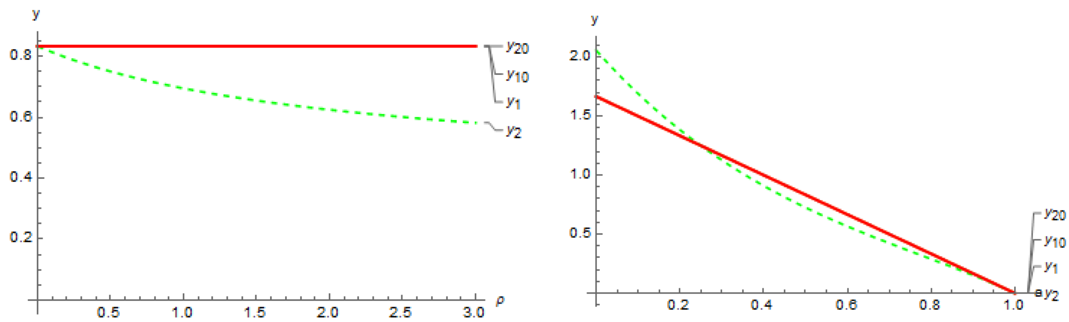
(a) The Impact of ρ and a on Optimal Profits



(b) The Mutual Promotion Mechanism between a and ρ



(c) The Impact of ρ and a on Optimal Service Provider's Effort Level



(d) The Impact of ρ and a on Consumers' Optimal Effort Levels

Figure 26. Equilibrium Comparisons when $\gamma \sim U[-b_1, b_2]$

Proof of the robustness of the utility function when γ follows a normal distribution:

We assume u_t is modeled as follows

$$u_t = (\mu_\theta + \gamma)(ax + (1-a)y_t), t = 1,2,$$

Where $\mu_\theta \sim N[\hat{\mu}, \sigma_\mu^2]$ and $\gamma \sim N[\hat{\gamma}, \sigma_\gamma^2]$.

We can then derive the probability of the service outcome to be a success as

$$\Pr\{O = 1|\mu_\theta, x, y_t\} = \Pr\{u_t(\gamma) > 0|\mu_\theta, x, y_t\} = 1 - F(-\mu_\theta)$$

$$= 1 - \frac{1}{\sigma_\gamma\sqrt{2\pi}}\int_{-\infty}^{-\mu_\theta} e^{-\frac{(t-\hat{\gamma})^2}{2\sigma_\gamma^2}} dt, \mu_\theta \sim N[\hat{\mu}, \sigma_\mu^2], t = 1,2.$$

Then the derivation of the second period consumers' expected service quality under normal distribution assumption can be derived as follows:

We first transform consumers' posting service outcome in the first period $u_1(\gamma) \in \mathbb{R}$ to the outcome rating score $\tau_1(\gamma) \in (0,1)$: $\tau_1(\gamma) = \frac{e^{u_1(\gamma)}}{e^{u_1(\gamma)}+1} = \frac{1}{1+e^{-u_1(\gamma)}} = \frac{1}{1+e^{-(\mu_\theta+\gamma)(ax+(1-a)y_1)}}$, which facilitates that consumers' outcome rating scores have the same scale as the service reviews from first period consumers.

Given an service outcome score $\tau_1(\gamma)$ generated by the first period consumer's posting service outcome, the inverse function can be denoted as $\tau_1^{-1}(\tau) = -\frac{\ln\left(\frac{1-\tau_1}{\tau_1}\right)}{ax+(1-a)y_1} - \mu_\theta$, which depicts the corresponding consumer's idiosyncratic factors that affect the service outcome (γ). We mainly focus on the scenario where the consumer ratings are evenly distributed between the range of zero to one $\{0, \frac{1}{s-1}, \dots, \frac{s-2}{s-1}, 1\}$. Therefore, the mapping relation from the service outcome score $\tau_1(\gamma)$ to the consumer's idiosyncratic factors affecting the service outcome γ is as follows: consumers posting the highest rating score (1) own an idiosyncratic outcome influencing factor in $[\tau_1^{-1}(1-\Delta), +\infty]$, where $\tau_1^{-1}(1-\Delta) = \frac{\ln\left(\frac{1-\Delta}{\Delta}\right)}{ax+(1-a)y_1} - \mu_\theta$; consumers posting the lowest rating score (0) own an idiosyncratic outcome influencing factor in $[-\infty, \tau_1^{-1}(\Delta)]$, where $\tau_1^{-1}(\Delta) = -\frac{\ln\left(\frac{1-\Delta}{\Delta}\right)}{ax+(1-a)y_1} - \mu_\theta$; consumers posting $\frac{i}{s-1}$ have an idiosyncratic outcome influencing factor in $[\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right), \tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right)]$, where $\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right) = -\frac{\ln\left[\frac{(1-\Delta)(s-1)-i}{i+\Delta(s-1)}\right]}{ax+(1-a)y_1} - \mu_\theta$ and $\tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right) = -\frac{\ln\left[\frac{(1+\Delta)(s-1)-i}{i-\Delta(s-1)}\right]}{ax+(1-a)y_1} - \mu_\theta$. We consider the scenario where all available ratings should be significant and taken into account, therefore, the two boundary conditions should be satisfied simultaneously: $\tau_1^{-1}(1-\Delta) \leq +\infty$ and $\tau_1^{-1}(\Delta) \geq -\infty$.

When all available ratings are significant, we can derive the number of reviews posting by first period consumers as $N(x, y_1) = \int_{-\infty}^{\tau_1^{-1}(\Delta)} f(\gamma)d\gamma + \sum_{i=1}^{s-2}\left[\int_{\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right)}^{\tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right)} f(\gamma)d\gamma\right] + \int_{\tau_1^{-1}(1-\Delta)}^{+\infty} f(\gamma)d\gamma$, where $\tau_1^{-1}(\Delta) = -\frac{\ln\left(\frac{1-\Delta}{\Delta}\right)}{ax+(1-a)y_1} - \mu_\theta$, $\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right) = -\frac{\ln\left[\frac{(1-\Delta)(s-1)-i}{i+\Delta(s-1)}\right]}{ax+(1-a)y_1} -$

μ_θ, $\tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right)=-\frac{\ln\left[\frac{(1+\Delta)(s-1)-i}{i-\Delta(s-1)}\right]}{ax+(1-a)y_1}-\mu_\theta$, $\tau_1^{-1}(1-\Delta)=\frac{\ln\left(\frac{1-\Delta}{\Delta}\right)}{ax+(1-a)y_1}-\mu_\theta$ and we have assumed that γ follows a normal distribution where $f(\gamma)=\frac{1}{\sigma_\gamma\sqrt{2\pi}}e^{-\frac{(\gamma-\hat{\gamma})^2}{2\sigma_\gamma^2}}$. When $s=2$, which means the binary rating is taken in the service provider's review platform, the number of reviews can be simplified as $N(x,y_1)=1+Erf\left[\frac{\hat{\gamma}-\frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1}+\mu_\theta}{\sqrt{2}\sigma_\gamma}\right]+Erf\left[\frac{\hat{\gamma}+\frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1}+\mu_\theta}{\sqrt{2}\sigma_\gamma}\right]$.

When all available ratings are significant, we can derive the mean of reviews posting by first period consumers as $\bar{R}(x,y_1)=\frac{1}{N(x,y_1)}\left[\sum_{i=1}^{s-2}\left(\frac{i}{s-1}\right)\int_{\tau_1^{-1}\left(\frac{i}{s-1}+\Delta\right)}^{\tau_1^{-1}\left(\frac{i}{s-1}-\Delta\right)}f(\gamma)d\gamma+\int_{\tau_1^{-1}(1-\Delta)}^{+\infty}(1)f(\gamma)d\gamma\right]$. When we consider the binary rating ($s=2$), the mean of reviews can be simplified as $\bar{R}(x,y_1)=\frac{1+Erf\left[\frac{\hat{\gamma}-\frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1}+\mu_\theta}{\sqrt{2}\sigma_\gamma}\right]}{1+Erf\left[\frac{\hat{\gamma}-\frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1}+\mu_\theta}{\sqrt{2}\sigma_\gamma}\right]+Erf\left[\frac{\hat{\gamma}+\frac{\ln\left[\frac{1-\Delta}{\Delta}\right]}{ax+(1-a)y_1}+\mu_\theta}{\sqrt{2}\sigma_\gamma}\right]}$ which does not follow a normal distribution anymore, thus we can not derive any closed form results from the new assumption regarding the utility function u_t and the distribution of μ_θ and γ.

Proof of the robustness of the general setting when γ follows a normal distribution:

If we still follow the original assumption of u_t, that is

$$u_t=\mu_\theta(ax+(1-a)y_t)+\gamma, t=1,2,$$

with the only difference in the assumption regarding the distribution of μ_θ and γ by changing from uniform distribution to normal distribution, i.e., $\mu_\theta \sim N[\hat{\mu},\sigma_\mu^2]$ and $\gamma \sim N[\hat{\gamma},\sigma_\gamma^2]$. If we follow the same derivation process of second period expected quality, we can derive that when $s=2$, which means the binary rating is taken in the service provider's review platform, the number of reviews can be simplified as $N(x,y_1)=1+Erf\left[\frac{\hat{\gamma}-\ln\left[\frac{1-\Delta}{\Delta}\right]+\mu_\theta(ax+(1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right]+Erf\left[\frac{\hat{\gamma}+\ln\left[\frac{1-\Delta}{\Delta}\right]+\mu_\theta(ax+(1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right]$, the mean of reviews can be simplified as $\bar{R}(x,y_1)=\frac{1+Erf\left[\frac{\hat{\gamma}-\ln\left[\frac{1-\Delta}{\Delta}\right]+\mu_\theta(ax+(1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right]}{1+Erf\left[\frac{\hat{\gamma}-\ln\left[\frac{1-\Delta}{\Delta}\right]+\mu_\theta(ax+(1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right]+Erf\left[\frac{\hat{\gamma}+\ln\left[\frac{1-\Delta}{\Delta}\right]+\mu_\theta(ax+(1-a)y_1)}{\sqrt{2}\sigma_\gamma}\right]}$ which does not follow a normal distribution anymore, thus we can not derive any closed form results from the new assumption regarding the distribution of μ_θ and γ.

6.4.2. Appendix B: Proofs of Structural Properties

Proof of Proposition 5.1:

The expected profit function of the service provider can be demonstrated as

$$\pi(p_1, p_2, x) = \mathbf{1}(EU_1 \geq 0)(p_1 - cx^2) + \mathbf{1}(EU_2 \geq 0)m(p_2 - cx^2),$$

where $EU_1 = E[r1_{O=1} - wy_1{}^2 - p_1] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - wy_1{}^2 - p_1 \geq 0$ and $EU_2 = E[r1_{O=1} - wy_2{}^2 - p_2] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_2)r - wy_2{}^2 - p_2 \geq 0$ are the purchase conditions for the early consumers and follower consumers, respectively. Thus, the optimal prices over two periods are market clearing prices and can be derived as $p_1 = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - wy_1{}^2$ and $p_2 = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_2)r - wy_2{}^2$. The service provider's ex post payoff function can be transforms into the following function and the service provider's objective is to maximize her payoff by setting her optimal effort level strategy $\hat{x}^*$:

$$\pi(x) = (E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - wy_1{}^2 - cx^2) + m(E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_2)r - wy_2{}^2 - cx^2).$$

At the same time, the early consumer's (the follower consumer's) objective is to maximize his expected utility by setting his optimal effort level strategy $\hat{y}_1^*(\hat{y}_2^*)$:

$EU_1(y_1) = E[r1_{O=1} - wy_1{}^2 - p_1] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - wy_1{}^2 - p_1$ and $EU_2(y_2) = E[r1_{O=1} - wy_2{}^2 - p_2] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_2)r - wy_2{}^2 - p_2$.

The first order conditions of aforementioned functions are

$$\frac{\partial \pi(x)}{\partial x} = \frac{1}{2}(1+m)(ar - 4cx),$$
$$\frac{\partial EU_1(y_1)}{\partial y_1} = \frac{1}{2}(r - ar - 4wy_1),$$
$$\frac{\partial EU_2(y_2)}{\partial y_2} = \frac{1}{2}(r - ar - 4wy_2).$$

We then check the SOCs of each variable as follows

$$\frac{\partial^2 \pi(x)}{\partial x^2} = -2c(1+m) < 0,$$
$$\frac{\partial^2 EU_1(y_1)}{\partial y_1^2} = \frac{\partial^2 EU_2(y_2)}{\partial y_2^2} = -2w < 0.$$

Thus, be setting each of the above first order condition to zero, we can derive the optimal effort level strategy that can maximize each party's objective function. The equilibrium results are

$$\hat{x}^* = \frac{ar}{4c},$$
$$\hat{y}_1^* = \hat{y}_2^* = \frac{(1-a)r}{4w}.$$

The equilibrium pricing strategy can be derived by taking the above optimal effort level into $p_1 = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - wy_1{}^2$ and $p_2 = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_2)r - wy_2{}^2$, therefore, the optimal prices of each period is then

$$\hat{p}_1^* = \hat{p}_2^* = \frac{r^2((-1+a)^2 c + 2a^2 w)}{16cw},$$

which will finally result in the service provider's expected profit to be

$$\hat{\pi}^* = \frac{(1+m)r^2((-1+a)^2 c + a^2 w)}{16cw}.$$

As for the monotonicity of the above equilibrium results with regards to the work allocation parameter a, we can further take derivations of each equilibrium result as shown below $\frac{\partial \hat{p}_1^*}{\partial a} = \frac{\partial \hat{p}_2^*}{\partial a} = \frac{r^2((-1+a)c+2aw)}{8cw}$, which is negative when $a < \frac{c}{c+2w}$, and positive when $a > \frac{c}{c+2w}$; $\frac{\partial \hat{x}^*}{\partial a} = \frac{r}{4c}$, which is always positive; $\frac{\partial \hat{y}_1^*}{\partial a} = \frac{\partial \hat{y}_2^*}{\partial a} = -\frac{r}{4w}$, which is always negative; $\frac{\partial \hat{\pi}^*}{\partial a} = \frac{(1+m)r^2((-1+a)c+aw)}{8cw}$, which is negative when $a < \frac{c}{c+w}$, and positive when $a > \frac{c}{c+w}$.

Proof of Proposition 5.2:

In the presence of review process, the expected profit function of the service provider can be demonstrated as

$$\pi(p_1, p_2, x) = \mathbf{1}(EU_1 \geq 0)(p_1 - cx^2) + \mathbf{1}(EU_2 \geq 0)m(p_2 - cx^2),$$

where $EU_1 = E[r1_{O=1} - w{y_1}^2 - p_1] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - w{y_1}^2 - p_1 \geq 0$ and $EU_2 = E[r1_{O=1} - w{y_2}^2 - p_2] = E\left[\mu_\theta \middle| \bar{R}(x, y_1), N(x, y_1)\right](ax + (1-a)y_2)r - w{y_2}^2 - p_2 \geq 0$ are the purchase conditions for the early consumers and follower consumers, respectively. Thus, the optimal prices over two periods are market clearing prices and can be derived as $p_1 = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - w{y_1}^2$ and $p_2 = E\left[\mu_\theta \middle| \bar{R}(x, y_1), N(x, y_1)\right](ax + (1-a)y_2)r - w{y_2}^2$. The service provider's ex post payoff function can be transforms into the following function and the service provider's objective is to maximize her payoff by setting her optimal effort level strategy x^*:

$$\begin{aligned}\pi(x) = (&E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - w{y_1}^2 - cx^2) \\ &+ m\left(E\left[\mu_\theta \middle| \bar{R}(x, y_1), N(x, y_1)\right](ax + (1-a)y_2)r - w{y_2}^2 - cx^2\right).\end{aligned}$$

At the same time, the early consumer's (the follower consumer's) objective is to maximize his expected utility by setting his optimal effort level strategy $y_1^*(y_2^*)$:

$EU_1(y_1) = E[r1_{O=1} - w{y_1}^2 - p_1] = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - w{y_1}^2 - p_1$ and $EU_2(y_2) = E[r1_{O=1} - w{y_2}^2 - p_2] = E\left[\mu_\theta \middle| \bar{R}(x, y_1), N(x, y_1)\right](ax + (1-a)y_2)r - w{y_2}^2 - p_2$. Note that $E\left[\mu_\theta \middle| \bar{R}(x, y_1), N(x, y_1)\right] = \frac{-1 + 2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho - 2ax\mu_\theta\rho + 2(-1+a)\mu_\theta\rho y_1}{-2 + (-2 + 4\ln\left(\frac{1-\Delta}{\Delta}\right))\rho}$ when $s = 2$ following the above derivation of the second period consumers' expected service quality.

The FOCs with respect to effort levels are

$$\frac{\partial \pi(x)}{\partial x} = \frac{-4a^2 mrx\mu_\theta\rho - 4c(1+m)x(-1 + (-1 + 2\ln\left(\frac{1-\Delta}{\Delta}\right))\rho) + ar(-1 - m + (-1 + 2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))\rho) + 2(-1+a)amr\mu_\theta\rho(y_1 + y_2)}{-2 + (-2 + 4\ln\left(\frac{1-\Delta}{\Delta}\right))\rho},$$

$$\frac{\partial EU_1(y_1)}{\partial y_1} = \frac{1}{2}(r - ar - 4wy_1),$$

$$\frac{\partial EU_2(y_2)}{\partial y_2} = \frac{(-1+a)r(1-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho + 2ax\mu_\theta\rho - 2(-1+a)\mu_\theta\rho y_1)}{-2+(-2+4\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho} - 2wy_2.$$

We then check the SOCs with respect to each variable as follows

$$\frac{\partial^2\pi(x)}{\partial x^2} = \frac{-4a^2mr\mu_\theta\rho - 4c(1+m)(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)}{-2+(-2+4\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho},$$

$$\frac{\partial^2 EU_1(y_1)}{\partial y_1^2} = \frac{\partial^2 EU_2(y_2)}{\partial y_2^2} = -2w < 0,$$

$$\frac{\partial^2\pi(x)}{\partial x\partial y_2} = \frac{(-1+a)amr\mu_\theta\rho}{-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho},$$

$$\frac{\partial^2 EU_2(y_2)}{\partial y_2\partial x} = \frac{(-1+a)ar\mu_\theta\rho}{-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho},$$

$$\frac{\partial^2\pi(x)}{\partial x^2}\frac{\partial^2 EU_2(y_2)}{\partial y_2^2} - \frac{\partial^2\pi(x)}{\partial x\partial y_2}\frac{\partial^2 EU_2(y_2)}{\partial y_2\partial x}$$

$$= \frac{4c(1+m)w\left(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho\right)^2 - a^2mr\mu_\theta\rho\left((-1+a)^2r\mu_\theta\rho + 4w\left(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho\right)\right)}{\left(-1+\left(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\right)\rho\right)^2}.$$

In order to make sure that the Hessian is negative definitive which is the necessary and sufficient condition to derive the unique solutions of the FOCs as a global optimum, the informational influence parameter ρ should not be greater than a certain threshold:

$$\rho \in \left[0, \min\left\{\frac{c(1+m)}{c\left(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)+a^2mr\mu_\theta}, \frac{2c(1+m)w}{2c\left(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)w+a^2mrw\mu_\theta+\sqrt{a^2mr^2w((-1+a)^2c(1+m)+a^2mw)\mu_\theta^2}}\right\}\right].$$

Thus, be setting each of the above first order condition to zero, we can derive the optimal effort level strategy that can maximize each party's objective function. The equilibrium results are

$$x^* = \frac{ar(\frac{(-1+a)^4mr^2\mu_\theta^2\rho^2}{2w} + (-1+a)^2mr\mu_\theta\rho(2+\rho-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho) + 2w(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(-1-m+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))\rho))}{2(4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2 - a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho + 4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))},$$

$$y_1^* = \frac{(1-a)r}{4w},$$

$$y_2^* = \frac{(-1+a)r(2c(1+m)(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(-(-1+a)^2r\mu_\theta\rho + w(-2+4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)) - a^2r\mu_\theta\rho((-1+a)^2mr\mu_\theta\rho - 2w(1-m+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\,ln\left(\frac{1-\Delta}{\Delta}\right)m\rho)))}{4w(-4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2 + a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho + 4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))}$$

As for the monotonicity of the above equilibrium results with respect to both the informational influence parameter ρ and the work allocation parameter a, we can further take derivations of each equilibrium result as shown below

$$\frac{\partial x^*}{\partial\rho} = \frac{ar(-4cm(1+m)(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)))\rho)((-1+a)^4r^2\mu^2\rho-2(-1+a)^2rw\mu(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)+w^2(-2-2\rho+4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho))+2a^2mrw\mu(4(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2+(-1+a)^2r\mu\rho(2+2\rho-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho+m(2+3\rho-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho))))}{2(-4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2+a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))^2},$$

$$\frac{\partial x^*}{\partial a} =$$

$$\frac{r(\frac{1}{2w}a^2mr\mu\rho(-(-1+a)^5(1+a)mr^3\mu^3\rho^3+2(-1+a)^3mr^2w\mu^2\rho^2(-4(1+a)+(-3-5a+8(1+a)\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)-4(-1+a)rw^2\mu\rho(-1-\rho+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)(-1+3a-5m-am+(-1+3a+2\,ln\left(\frac{1-\Delta}{\Delta}\right)-6a\,ln\left(\frac{1-\Delta}{\Delta}\right)+2(-1-a+(5+a)\,ln\left(\frac{1-\Delta}{\Delta}\right))m)\rho)-16w^3(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2(-1-m+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))(1+m))\rho))+2c(1+m)(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2((-1+a)^3(-1+5a)mr^2\mu^2\rho^2-2(-1+a)(-1+3a)mrw\mu\rho(-2-\rho+4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)+4w^2(-1-\rho+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)(-1-m+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))(1+m))\rho)))}{2(-4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2+a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)))^2},$$

Results show that both $\frac{\partial x^*}{\partial \rho}$ and $\frac{\partial x^*}{\partial a}$ are positive, which means $x^*$is increasing in both ρ and a. Furthermore, $\frac{\partial y_1^*}{\partial a} = -\frac{r}{4w}$;

$$\frac{\partial y_2^*}{\partial a} = \frac{r(8c^2(1+m)^2w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)^3(-3(-1+a)^2r\mu\rho+w(-2+4\ln(\frac{1-\Delta}{\Delta})\rho))-a^4mr^2\mu^2\rho^2((-1+a)^4mr^2\mu^2\rho^2+8w^2(-1-\rho+2\ln(\frac{1-\Delta}{\Delta})\rho)(1-m+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho+2\ln(\frac{1-\Delta}{\Delta})m\rho)-2(-1+a)^2rw\mu\rho(-1-5m+(-1+2\ln(\frac{1-\Delta}{\Delta})-6m+10\ln(\frac{1-\Delta}{\Delta})m)\rho))-2ac(1+m)r\mu\rho(-1+(-1+2\ln(\frac{1-\Delta}{\Delta}))\rho)(-(-2+a)(-1+a)^4mr^2\mu^2\rho^2-2(-1+a)^2mrw\mu\rho(-4(1+a)+(-2-7a+8(1+a)\ln(\frac{1-\Delta}{\Delta}))\rho)+4w^2(-1+(-1+2\ln(\frac{1-\Delta}{\Delta}))\rho)(4\ln(\frac{1-\Delta}{\Delta})(1+m)\rho-2(1+m+\rho)+a(3-m+(3+2\ln(\frac{1-\Delta}{\Delta})(-3+m))\rho)))))}{4w(-4c(1+m)w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)^2+a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)))^2}$$

Results show that both $\frac{\partial y_1^*}{\partial a}$ and $\frac{\partial y_2^*}{\partial a}$ are negative, which means y_1^* and y_2^* are both decreasing in a. Meanwhile $\frac{\partial y_1^*}{\partial \rho} = 0$, while $\frac{\partial y_2^*}{\partial \rho} =$

$$\frac{(-1+a)r(-2a^4mr^2w\mu^2(-4mw+(-1+a)^2(1+m)r\mu)\rho^2+8c^2(1+m)^2w(2w-(-1+a)^2r\mu)(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)^2-2a^2c(1+m)r\mu(-2(-1+a)^2mrw\mu\rho^2+(-1+a)^4mr^2\mu^2\rho^2+4w^2(-1-\rho+2\ln(\frac{1-\Delta}{\Delta})\rho)(-1-m+(-1-3m+2\ln(\frac{1-\Delta}{\Delta})(1+m))\rho)))}{4w(-4c(1+m)w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)^2+a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)))^2}$$

which is negative when $\rho < \rho_1$, and positive when $\rho > \rho_1$, where $\rho_1 = \frac{2(\sqrt{(-(-1+a)^2a^2cm(1+m)^2r^2w\mu^2(4cw^2((-1+a)^2c(1+m)+a^2mw)-4rw((-1+a)^2c+a^2w)((-1+a)^2c(1+m)+a^2mw)\mu+(-1+a)^2(1+m)r^2((-1+a)^2c+a^2w)^2\mu^2))}-2(-1+a)c(1+m)w(a^2(-1-2m+2C1(1+m))rw\mu+c(-1+2\ln(\frac{1-\Delta}{\Delta}))(1+m)(-2w+(-1+a)^2r\mu)))}{(1-a)(-4c^2(1-2\ln(\frac{1-\Delta}{\Delta}))^2(1+m)^2w(2w-(-1+a)^2r\mu)+a^4mr^2w\mu^2(-4mw+(-1+a)^2(1+m)r\mu)+a^2c(1+m)r\mu(4(-1+2\ln(\frac{1-\Delta}{\Delta}))(-1-3m+2\ln(\frac{1-\Delta}{\Delta})(1+m))w^2-2(-1+a)^2mrw\mu+(-1+a)^4mr^2\mu^2))}$.

Proof of Proposition 5.3:

In order to make a comparison between the optimal effort level of the service provider the presence of review process x^* with the optimal effort level in the absence of review process $\hat{x}^*$, we try to calculate the intersection point of

$$x^* = \frac{ar(\frac{(-1+a)^4mr^2\mu_\theta^2\rho^2}{2w}+(-1+a)^2mr\mu_\theta\rho(2+\rho-4\ln(\frac{1-\Delta}{\Delta})\rho)+2w(-1+(-1+2\ln(\frac{1-\Delta}{\Delta}))\rho)(-1-m+(-1+2\ln(\frac{1-\Delta}{\Delta})(1+m))\rho))}{2(4c(1+m)w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)^2-a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho+4w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)))}$$

and $\hat{x}^* = \frac{ar}{4c}$, results shows that only if $\rho = 0$ or $a = 0$ which is the **nonnegative real solution** in the situation when $x^* = \hat{x}^*$, otherwise, because both $\frac{\partial x^*}{\partial \rho}$ and $\frac{\partial x^*}{\partial a}$ are positive, which means $x^*$is increasing in both ρ and a, x^* is always greater than $\hat{x}^*$ when the work allocation parameter $a > 0$ and the review informational influence parameter $\rho > 0$.

Proof of Proposition 5.4:

In order to make a comparison between the optimal effort level of early consumers (follower consumers) in the presence of review process $y_2^*(y_1^*)$ with the optimal effort level of early consumers (follower consumers) in the absence of review process $\hat{y}_1^*(\hat{y}_2^*)$, we try to calculate the intersection point of

$$y_2^* = \frac{(-1+a)r(2c(1+m)(-1+(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(-(-1+a)^2r\mu_\theta\rho+w(-2+4\ln\left(\frac{1-\Delta}{\Delta}\right)\rho))-a^2r\mu_\theta\rho((-1+a)^2mr\mu_\theta\rho-2w(1-m+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1-\Delta}{\Delta}\right)m\rho)))}{4w(-4c(1+m)w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2+a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))}$$

and $y_1^* = \hat{y}_1^* = \hat{y}_2^* = \frac{(1-a)r}{4w}$, results shows that only if $\rho = 0$ and $\rho_2 = \frac{(1+m)(2cw-(-1+a)^2cr\mu_\theta-a^2rw\mu_\theta)}{a^2(1+2m-2\ln(\frac{1-\Delta}{\Delta})(1+m))rw\mu_\theta+c(-1+2\ln(\frac{1-\Delta}{\Delta}))(1+m)(2w-(-1+a)^2r\mu_\theta)}$ will $y_2^* = y_1^*$; at the same time, $\frac{\partial y_2^*}{\partial \rho}$ is negative when $\rho < \rho_1$, and positive when $\rho > \rho_1$, where $\rho_1 < \rho_2$, we can see that when $\rho \in [0, \rho_2]$, y_2^* is lower than y_1^*.

Proof of Corollary 5.1:

Furthermore, we examine the optimal effort level of early consumers in the presence of review process y_1^* and that of follower consumers y_2^* from the perspective of work allocation parameter a, such that the intersection points of y_1^* and y_2^* are $a_1 = 1$,

$$a_2 = -\frac{(c(1+m)(2w - r\mu_\theta)(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho))}{(c(1+m)r\mu_\theta\left(-1+\left(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\right)\rho\right) - \sqrt{(c(1+m)rw\mu_\theta(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(2c(1+m)(-1-\rho+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)+(2w-r\mu_\theta)(-1-m+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)+2(-1+ln\left(\frac{1-\Delta}{\Delta}\right))m)\rho)))}}$$

and

$$a_3 = -\frac{\left(c(1+m)(2w - r\mu_\theta)\left(-1+\left(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\right)\rho\right)\right)}{(c(1+m)r\mu_\theta\left(-1+\left(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\right)\rho\right) + \sqrt{c(1+m)rw\mu_\theta\left(-1+\left(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\right)\rho\right)\left(2c(1+m)\left(-1-\rho+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho\right)+(2w-r\mu_\theta)\left(-1-m+\left(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)+2\left(-1+ln\left(\frac{1-\Delta}{\Delta}\right)\right)m\right)\rho\right)\right)})}.$$

As $\frac{\partial y_1^*}{\partial a}$ and $\frac{\partial y_2^*}{\partial a}$ are negative, which means y_1^* and y_2^* are both decreasing in a, when $a \in [0, a_2] \cup [a_3, a_1]$, y_2^* is greater than y_1^*; otherwise, when $a_2 < a < a_3$, y_2^* is lower than y_1^*.

Proof of Proposition 5.5:

Following the results derived in Proposition 2, the equilibrium pricing strategy can be derived by taking the above optimal effort levels into $p_1 = E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - w{y_1}^2$ and $p_2 = E[\mu_\theta | \bar{R}(x,y), N(x,y)](ax + (1-a)y_2)r - w{y_2}^2$, therefore, the optimal prices of each period is then

$$p_1^* = \frac{1}{16}r(-\frac{(-1+a)^2 r}{w} + 2r(\frac{(-1+a)^2}{w} + \frac{2a^2(\frac{(-1+a)^4 mr^2\mu_\theta^2\rho^2}{2w} + (-1+a)^2 mr\mu_\theta\rho(2+\rho-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho) + 2w(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(-1-m+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))\rho))}{4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2 - a^2 mr\mu_\theta\rho((-1+a)^2 r\mu_\theta\rho + 4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho))}$$

and

$$p_2^* = \frac{r^2(4(-1+a)^2c^2(1+m)^2(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2((-1+a)^2r\mu_\theta\rho+w(2-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho))^2 + 8a^2c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2((-1+a)^2r\mu_\theta\rho+w(2-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho))((-1+a)^2(1+m)r\mu_\theta\rho+2w(1+m+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)m\rho)) - a^4r\mu_\theta\rho(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)(-2(1+m)w+((-2+4\,ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))w-(-1+a)^2mr\mu_\theta)\rho)(-(-1+a)^2r\mu_\theta\rho+w(-4-4\rho+8\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho))(-(-1+a)^2mr\mu_\theta\rho+2w(1-m+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\,ln\left(\frac{1-\Delta}{\Delta}\right)m\rho)))}{16w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)(-4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2+a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho+4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))^2}$$

As for the monotonicity of the above equilibrium prices with respect to both the informational influence parameter ρ and the work allocation parameter a, we can further take derivations of each equilibrium result as shown below

$$\frac{\partial p_1^*}{\partial \rho} = \frac{a^2r^2(-4cm(1+m)(-1+(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right))\rho)((-1+a)^4r^2\mu^2\rho - 2(-1+a)^2rw\mu(-1+2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho) + w^2(-2-2\rho+4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)) + 2a^2mrw\mu(4(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2 + (-1+a)^2r\mu\rho(2+2\rho-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho + m(2+3\rho-4\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho))))}{4(-4c(1+m)w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2 + a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\,ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))^2}$$

$$\frac{\partial p_2^*}{\partial \rho} = \text{[illegible]}$$

Results show that both $\frac{\partial p_1^*}{\partial \rho}$ and $\frac{\partial p_2^*}{\partial \rho}$ are positive, which means both p_1^* and p_2^* are increasing in ρ. Furthermore, we assume $\frac{\partial p_1^*}{\partial a} = 0$ when $a = \tilde{a}_1(\rho)$, thus we can derive $\frac{\partial p_1^*}{\partial a} < 0$ when $a < \tilde{a}_1(\rho)$ and $\frac{\partial p_1^*}{\partial a} > 0$ when $a > \tilde{a}_1(\rho)$, which means p_1^* is decreasing in a when $a < \tilde{a}_1(\rho)$, and increasing in a otherwise; meanwhile, we assume $\frac{\partial p_2^*}{\partial a} = 0$ when $a = \tilde{a}_2(\rho)$, thus we can derive $\frac{\partial p_2^*}{\partial a} < 0$ when $a < \tilde{a}_2(\rho)$ and $\frac{\partial p_2^*}{\partial a} > 0$ when $a > \tilde{a}_2(\rho)$, which means p_2^* is decreasing in a when $a < \tilde{a}_2(\rho)$, and increasing in a otherwise.

Proof of Corollary 5.2:

In order to make a comparison between the service provider's optimal first period price and her optimal second period price, we try to calculate the intersection point of p_1^* and p_2^* which we assume as $\tilde{a}_3(\rho)$ and $\tilde{a}_4(\rho)$. As $\tilde{a}_3(\rho) < \tilde{a}_1(\rho)(\tilde{a}_2(\rho))$, p_1^* and p_2^* are both decreasing when they intersect each other at $\tilde{a}_3(\rho)$, meanwhile $\left|\frac{\partial p_2^*}{\partial a}\right| > \left|\frac{\partial p_1^*}{\partial a}\right|$ which means p_2^* decreases faster than p_1^*, thus before p_1^* and p_2^* intersect at $\tilde{a}_4(\rho) > \tilde{a}_1(\rho)(\tilde{a}_2(\rho))$, the value of p_2^* is lower than p_1^*, which means if $\tilde{a}_3(\rho) \le a \le \tilde{a}_4(\rho)$, then the service provider's optimal first period price p_1^* is greater than her optimal second period price p_2^*, that is, the service provider adopts the decreasing pricing plan.

Proof of Proposition 5.6:

We further make comparison between the service provider's optimal prices in the presence of review process with that in the absence of review process in the first period and second period respectively, results shows that only if $\rho = 0$ or $a = 0$ which is the **nonnegative real solution** in the situation when $p_1^* = \hat{p}_1^*$ and $p_2^* = \hat{p}_2^*$, otherwise, because both $\frac{\partial p_1^*}{\partial \rho}$ and $\frac{\partial p_2^*}{\partial \rho}$ are positive, which means both p_1^* and p_2^* are increasing in ρ, and $\frac{\partial \hat{p}_1^*}{\partial \rho} = \frac{\partial \hat{p}_2^*}{\partial \rho} = 0$, the optimal first period price (second period price) of the service provider $p_1^*(p_2^*)$ is always greater than the optimal first period price (second period price) in the absence of review process $\hat{p}_1^*(\hat{p}_2^*)$ when the review informational influence parameter $\rho > 0$.

Proof of Proposition 5.7:

We substitute the above equilibrium results back into the profit function in the presence of review process

$$\pi(x) = (E[\mu_\theta | 0 < \mu_\theta < 1](ax + (1-a)y_1)r - w{y_1}^2 - cx^2) + m(E[\mu_\theta | \bar{R}(x,y), N(x,y)](ax + (1-a)y_2)r - w{y_2}^2 - cx^2),$$

which results in the optimal profit

$$\pi^* = \frac{r^2(a^2w((-1+a)^4m(1+m)r^2\mu_\theta^2\rho^2 - 4(-1+a)^2m(1+m)rw\mu_\theta\rho(-1+2\ln(\frac{1-\Delta}{\Delta})\rho) + 4w^2(1+m+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho - 2\ln(\frac{1-\Delta}{\Delta})m\rho)^2) + (-1+a)^2c(1+m)((-1+a)^4mr^2\mu_\theta^2\rho^2 - 4(-1+a)^2mrw\mu_\theta\rho(-1+2\ln(\frac{1-\Delta}{\Delta})\rho) + 4w^2(m(1-2\ln(\frac{1-\Delta}{\Delta})\rho)^2 + (1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)^2)))}{16w^2(4c(1+m)w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)^2 - a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho + 4w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)))}$$

.We then take derivation with respect to both the informational influence parameter ρ and the work allocation parameter a to analyze the monotonicity of the profit

$$\frac{\partial \pi^*}{\partial \rho} = \frac{mr^2(-(-1+a)^2c^2(1+m)^2(-2w+(-1+a)^2r\mu)(-1+(-1+2\ln(\frac{1-\Delta}{\Delta}))\rho)((-1+a)^2r\mu\rho + w(2-4\ln(\frac{1-\Delta}{\Delta})\rho)) - a^4rw^2\mu(-1-m+(-1-2m+2\ln(\frac{1-\Delta}{\Delta})(1+m))\rho)((-1+a)^2(1+m)r\mu\rho + 2w(1+m+\rho-2C1\rho-2\ln(\frac{1-\Delta}{\Delta})m\rho)) - a^2c(1+m)w(4w^2(-1+(-1+2\ln(\frac{1-\Delta}{\Delta}))\rho)(-1-m+(-1+2\ln(\frac{1-\Delta}{\Delta})(1+m))\rho) + (-1+a)^4r^2\mu^2\rho(-2(1+m)+(-2-3m+4\ln(\frac{1-\Delta}{\Delta})(1+m))\rho) - 2(-1+a)^2rw\mu(2(1+m) - 2(-1+4C1)(1+m)\rho + (-m+4\ln(\frac{1-\Delta}{\Delta})(-1+2\ln(\frac{1-\Delta}{\Delta}))(1+m))\rho^2)))}{2w(-4c(1+m)w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)^2 + a^2mr\mu\rho((-1+a)^2r\mu\rho + 4w(1+\rho-2\ln(\frac{1-\Delta}{\Delta})\rho)))^2}$$

which is negative when $\rho < \frac{(1+m)(2cw-(-1+a)^2cr\mu_\theta - a^2rw\mu_\theta)}{a^2(1+2m-2\ln(\frac{1-\Delta}{\Delta})(1+m))rw\mu_\theta + c(-1+2\ln(\frac{1-\Delta}{\Delta}))(1+m)(2w-(-1+a)^2r\mu_\theta)}$ and is positive otherwise. Furthermore, we assume $\frac{\partial \pi^*}{\partial a} = 0$ when $a = \tilde{a}_5(\rho)$, thus we can derive $\frac{\partial \pi^*}{\partial a} < 0$ when $a < \tilde{a}_5(\rho)$ and $\frac{\partial \pi^*}{\partial a} > 0$ when $a > \tilde{a}_5(\rho)$, which means π^* is decreasing in a when $a < \tilde{a}_5(\rho)$, and increasing in a otherwise.

Proof of Proposition 5.8:

In order to make a comparison between the optimal expected profit in the presence of review process π^* with the optimal expected profit in the absence of review process $\hat{\pi}^*$, we try to calculate the intersection point of

$$\pi^* = \frac{r^2(a^2w((-1+a)^4m(1+m)r^2\mu_\theta^2\rho^2-4(-1+a)^2m(1+m)rw\mu_\theta\rho(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)+4w^2(1+m+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)m\rho)^2)+(-1+a)^2c(1+m)((-1+a)^4mr^2\mu_\theta^2\rho^2-4(-1+a)^2mrw\mu_\theta\rho(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)+4w^2(m(1-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2+(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2)))}{16w^2(4c(1+m)w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)^2-a^2mr\mu_\theta\rho((-1+a)^2r\mu_\theta\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho)))}$$

and $\hat{\pi}^* = \frac{(1+m)r^2((-1+a)^2c+a^2w)}{16cw}$, results shows that only if $\rho = 0$ and $\rho_3 = \frac{4(1+m)w((-1+a)^2c+a^2w)(2cw-(-1+a)^2cr\mu_\theta-a^2rw\mu_\theta)}{(-1+a)^2c^2(1+m)(-2w+(-1+a)^2r\mu_\theta)((2-8\ln\left(\frac{1-\Delta}{\Delta}\right))w+(-1+a)^2r\mu_\theta)+a^4(1+m)rw^2\mu_\theta((4-8\ln\left(\frac{1-\Delta}{\Delta}\right))w+(-1+a)^2r\mu_\theta)+2a^2cw((8\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m)-2(2+m))w^2-2(-1+a)^2(-1+4\ln\left(\frac{1-\Delta}{\Delta}\right))(1+m)rw\mu_\theta+(-1+a)^4(1+m)r^2\mu_\theta^2)}$ will $\pi^* = \hat{\pi}^*$; at the same time, $\frac{\partial \pi^*}{\partial \rho}$ is negative when $\rho < \frac{(1+m)(2cw-(-1+a)^2cr\mu_\theta-a^2rw\mu_\theta)}{a^2(1+2m-2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))rw\mu_\theta+c(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right))(1+m)(2w-(-1+a)^2r\mu_\theta)}$, and positive otherwise, where $\frac{(1+m)(2cw-(-1+a)^2cr\mu_\theta-a^2rw\mu_\theta)}{a^2(1+2m-2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m))rw\mu_\theta+c(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right))(1+m)(2w-(-1+a)^2r\mu_\theta)} < \rho_3$, we can see that when $\rho \in [0, \rho_3]$, π^* is lower than $\hat{\pi}^*$; otherwise, when $\rho \in [\rho_3, min\{\frac{c(1+m)}{c\left(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)+a^2mr\mu_\theta}, \frac{2c(1+m)w}{2c\left(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\right)(1+m)w+a^2mrw\mu_\theta+\sqrt{a^2mr^2w((-1+a)^2c(1+m)+a^2mw)\mu_\theta^2}}\}]$, $\pi^* \geq \hat{\pi}^*$.

We then further analyze the region where the presence of review process can improve the profit.

We try to calculate the intersection point of π^* and $\hat{\pi}^*$which we assume as $\tilde{a}_6(\rho)$ and $\tilde{a}_7(\rho)$. As $\tilde{a}_6(\rho) < \tilde{a}_5(\rho)$ and $\frac{c}{c+w}$, π^* and $\hat{\pi}^*$ are both decreasing when they intersect each other at $\tilde{a}_6(\rho)$, meanwhile $\left|\frac{\partial \pi^*}{\partial a}\right| > \left|\frac{\partial \hat{\pi}^*}{\partial a}\right|$ which means π^* decreases faster than $\hat{\pi}^*$ with respect to work allocation parameter, thus before π^* and $\hat{\pi}^*$ intersect at $\tilde{a}_7(\rho) > \tilde{a}_5(\rho)$ and $\frac{c}{c+w}$, the value of $\pi^*$is lower than $\hat{\pi}^*$, which means if $\tilde{a}_6(\rho) \leq a \leq \tilde{a}_7(\rho)$, then the service provider's expected profit in the presence of review process π^* is lower than her expected profit in the absence of review process $\hat{\pi}^*$; otherwise, when $a < \tilde{a}_6(\rho)$ or $a > \tilde{a}_7(\rho)$, the service provider achieves greater expected profit in the presence of review process π^* than she achieves in the absence of review process $\hat{\pi}^*$. There exists a region of the shaded area in the picture below that marks when the presence of review process can improve the profit.

Proof of Corollary 5.3:

The absolute value of this derivation $\left|\frac{\partial \pi^*}{\partial a}\right|$ measures the rate of change of the expected profit with respect to work allocation parameter. We have numerically found that as the review informational influence parameter ρ increases from zero (which is exactly the case in the absence of review process), $\left|\frac{\partial \pi^*}{\partial a}\right|$ is also increases which means π^* is changing more rapidly in the work allocation parameter a. That is to say, $\frac{\partial\left|\frac{\partial \pi^*}{\partial a}\right|}{\partial \rho} > 0$ always holds. Figures can be found below.

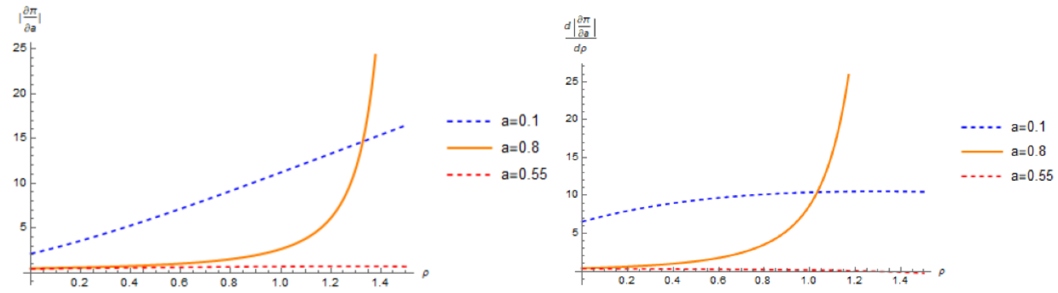


Figure 27. The Impact of ρ on $\left|\frac{\partial \pi^*}{\partial a}\right|$ and $\frac{\partial \left|\frac{\partial \pi^*}{\partial a}\right|}{\partial \rho}$

Proof of Model extension when $s = 3$:

We consider the case of ternary rating when $s = 3$. Following the same analytical procedure as the original model, we can obtain the similar conclusions in the main model, which prove the robustness of our basic model.

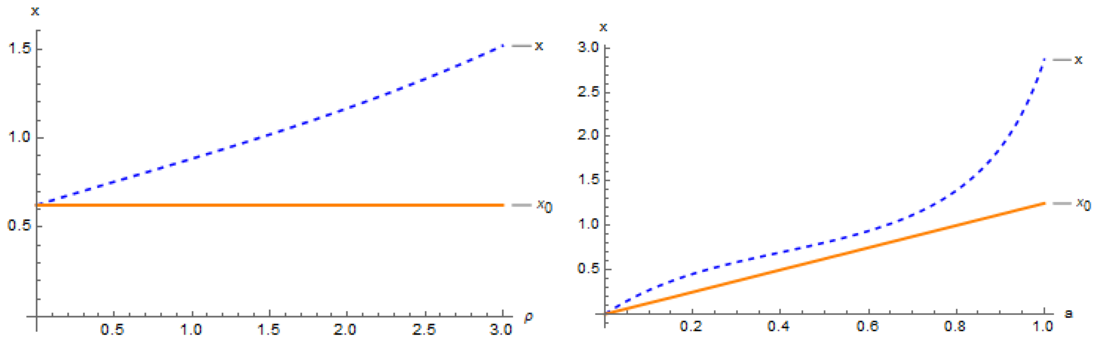
The benchmark case without review process is the same as that in our original model, the only difference lies in the case with review process, the second-period consumers' expected quality level of the service provider is changing to $E\left[\mu_\theta \middle| \bar{R}(x, y_1), N(x, y_1)\right] = \frac{-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho-2\left(\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)+ax\mu_\theta\right)\rho+2(-1+a)\mu_\theta\rho y_1}{-2+\left(-2+4\ln\left(\frac{1-\Delta}{\Delta}\right)-4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\right)\rho}$ when $s = 3$. All other analytical methods follow as before, we thus can derive the following equilibrium results in the presence of review process:

$$x^* = \frac{am(r-ar)^2\mu\rho(1-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho+\frac{(-1+a)^2r\mu\rho}{2w})+arw(2+(2-4\ln\left(\frac{1-\Delta}{\Delta}\right)+4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right))\rho)(1+m-(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m)-2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)(1+m))\rho+\frac{(-1+a)^2mr\mu\rho}{2w})}{2(-4c(1+m)w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)^2+a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)))}$$

$$y_1^* = \frac{(1-a)r}{4w},$$

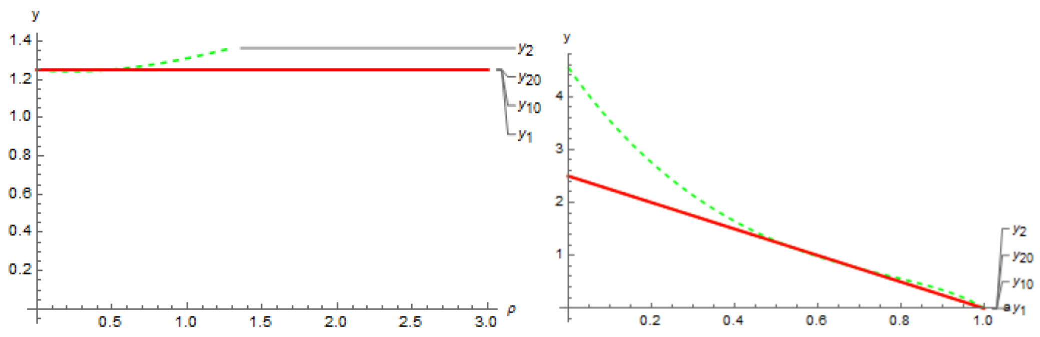
$$y_2^* = \frac{(-1+a)r(-a^2r\mu\rho(2(-1+m)w+((-2-4\ln\left(\frac{1-\Delta}{\Delta}\right)(-1+m)+4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)(-1+m))w+(-1+a)^2mr\mu)\rho)+2c(1+m)(-1+(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)-2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right))\rho)(-(-1+a)^2r\mu\rho+w(-2+4\ln\left(\frac{1-\Delta}{\Delta}\right)\rho-4\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)))}{4w(-4c(1+m)w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)^2+a^2mr\mu\rho((-1+a)^2r\mu\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)\rho)))}$$

The optimal prices in each period $p_1^*(p_2^*)$ and the optimal expected profit π^* are all analytically solvable but are tedious in expressions so we don't intend to present them here. We thus resolve to numerical analysis to help us obtain some conclusions and results show that the same inferences can be derived as the analytical results in our original model.
The following several figures demonstrate that the main conclusions still hold with the change of rating scale $s = 3$.



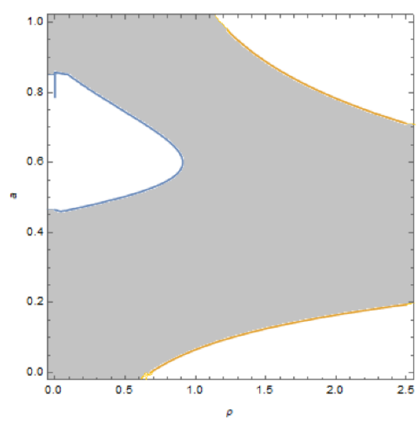
(a) The impact of ρ and a on Optimal Service Provider's Effort Level

Figure above illustrates in the presence of review process, the optimal effort level of the service provider is always greater than the optimal effort level in the absence of review process if and only if the work allocation parameter $a > 0$ and the review informational influence parameter $\rho > 0$.



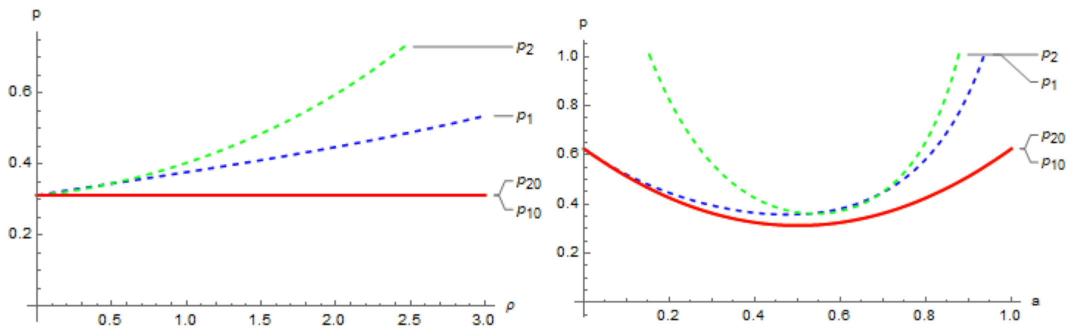
(b) The impact of ρ and a on Consumers' Optimal Effort Levels

Figure above illustrates that in the presence of review process, there exists a degree of review informational influence parameter such that the follower consumers' optimal effort level is lower than or equal to the early consumers' optimal effort level if and only if review informational influence parameter is lower than this threshold. Also, it is easily verified that the early consumers' optimal effort level in the presence of review process is the same as the that in the absence of review process. As for the work allocation parameter, there exist the degree of work allocation parameter such that the optimal effort level of the follower consumers is greater than the early consumers' optimal effort level if and only if the work allocation parameter is approaching two end values.



(c) Follower Consumers' Optimal Effort Level in $a-\rho$ Plane

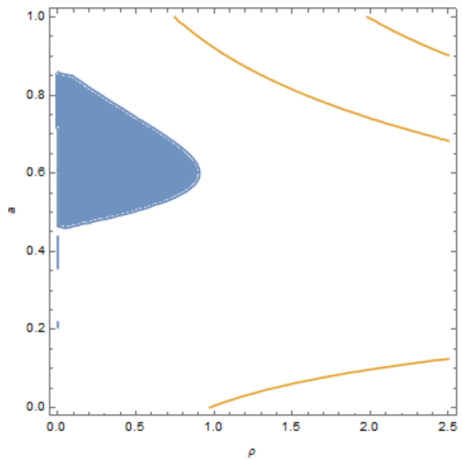
The above figure in a two-dimensional plane depicts more clearly the joint influence of both the collaborating effect and the reviewing effect, from which we can see the region where the follower consumer's optimal effort level is lower than that of the early consumer only if the reviewing effect is weak and the work allocation parameter is in a middle range; otherwise, the follower consumer contributes more to the service provision than the early consumer, where the shaded area in the figure marks the increasing consumer effort level plan in two consecutive periods.



(d) The Impact of ρ and a on Service Provider's Optimal Prices

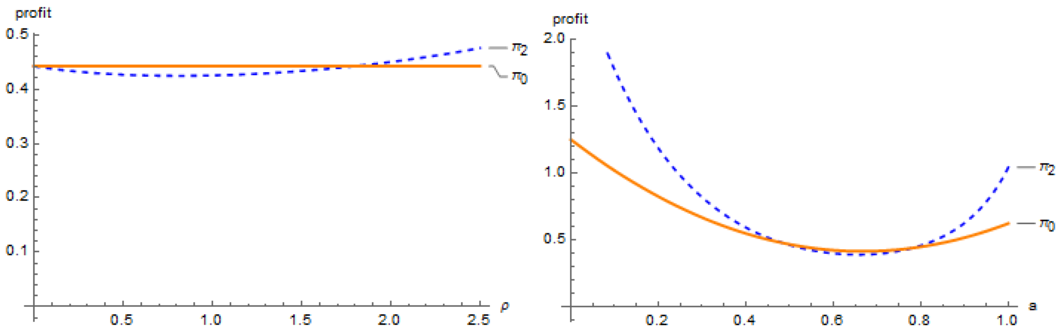
Figures demonstrates that in the presence of review process, the optimal first period price (second period price) of the service provider is always greater than the optimal first period price (second period price) in the absence of review process if and only if the work allocation parameter is nonnegative and the review informational influence parameter is nonnegative. Also note that the optimal prices in the presence of review process are both increasing in the review informational parameter. And the optimal prices with and without review process are all first decreasing in the work allocation parameter and then increasing in it.

The following figure illustrates the area where the service provider's optimal first period price is greater than her optimal second period price, that is, the service provider adopts the decreasing pricing plan; otherwise, the service provider adopts the increasing pricing plan.

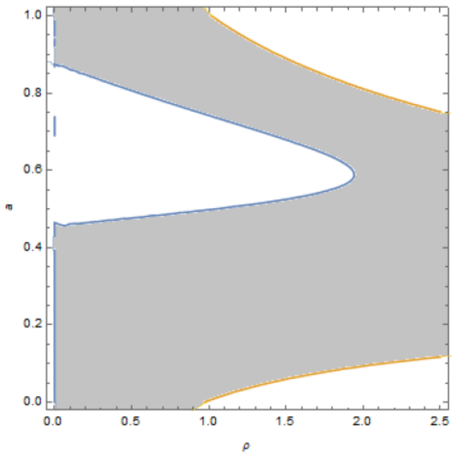


(e) Service Provider's Optimal Pricing Strategy in $a-\rho$ Plane

Finally, the conclusions on the optimal profits also hold. In the presence of review process, the service provider's optimal profit is decreasing in the review informational parameter and then increasing in it then. Furthermore, it first decreases in the work allocation parameter and then increases in it. In the presence of review process, there exists a degree of review informational influence parameter, such that the service provider achieves greater expected profit than she achieves in the absence of review process if and only if the review informational parameter is high than this threshold. Moreover, when the work allocation parameter is approaching two end values, the service provider achieves greater expected profit in the presence of review process than she achieves in the absence of review process. There exists a region of the shaded area that marks when the presence of review process can improve the profit.

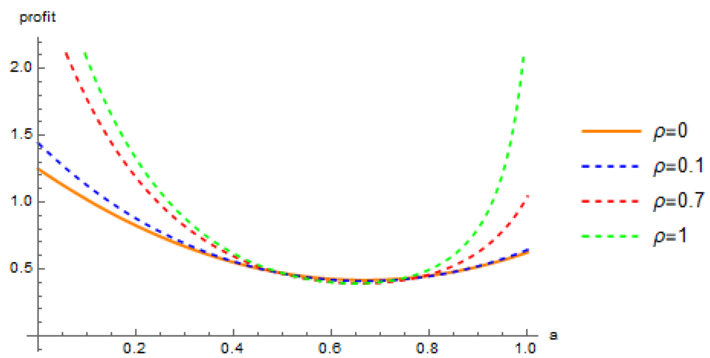


(f) The Impact of ρ and a on Optimal Profits



(g) Service Provider's Optimal Profits in $a-\rho$ Plane

We further explore that as the review informational influence parameter increases from zero (which is exactly the case in the absence of review process), the optimal profit is changing more rapidly in the work allocation parameter as the increase of review informational influence parameter.



(h) The Mutual Promotion Mechanism between α and ρ

Figure 28. Equilibrium Comparisons when $s = 3$

Proof of Corollary 5.4:

We further numerically derived the pricing strategy and effort level strategy with the variation of the value Δ which can further strengthen our conclusion.

When $s = 2$,

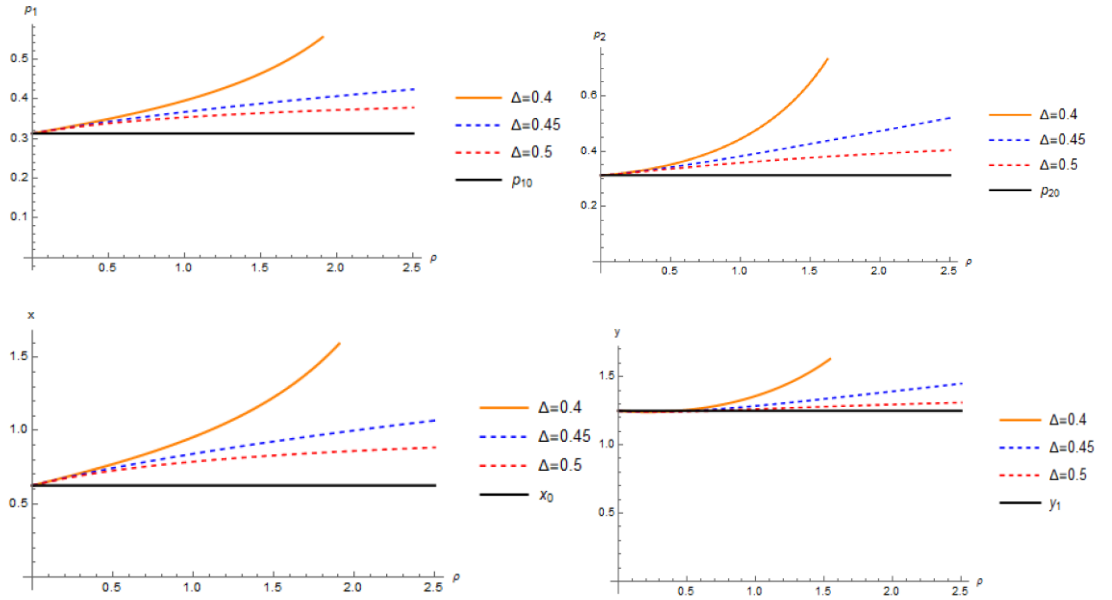


Figure 29. The Impact of Δ on the Prices and Effort Levels when $s = 2$

Where we can see that as the decrease of the value Δ, the review process has more influence on the pricing strategy and the effort level strategy.

When $s = 3$, the same results can be derived:

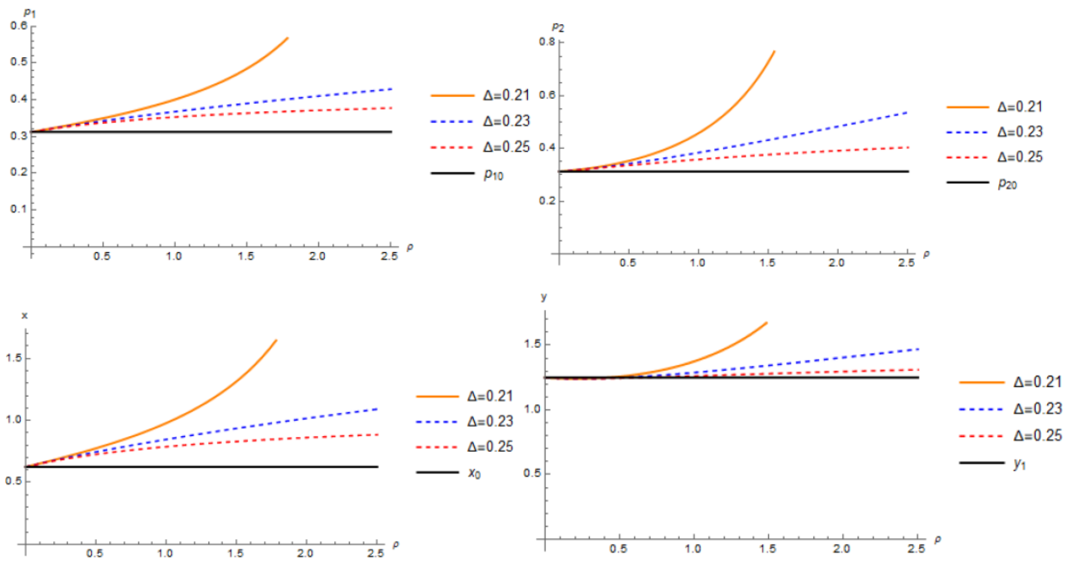


Figure 30. The Impact of Δ on the Prices and Effort Levels when $s = 3$

Proof of Proposition 5.9:

When $s = 4$, the number of reviews can be simplified as $N(x, y_1) = 1 - 2\ln\left(\frac{1-\Delta}{\Delta}\right) + 2\ln\left(\frac{1+3\Delta}{1-3\Delta}\right) + 2\ln\left(\frac{2+3\Delta}{2-3\Delta}\right)$, the mean of reviews can be simplified as $\bar{R}(x, y_1) = \frac{-\ln\left(\frac{1-\Delta}{\Delta}\right)+\ln\left(\frac{1+3\Delta}{1-3\Delta}\right)+\ln\left(\frac{2+3\Delta}{2-3\Delta}\right)+(a(x-y_1)+y_1)\mu_\theta}{1-2\ln\left(\frac{1-\Delta}{\Delta}\right)+2\ln\left(\frac{1+3\Delta}{1-3\Delta}\right)+2\ln\left(\frac{2+3\Delta}{2-3\Delta}\right)}$;

All other analytical methods follow as before, we thus can derive the following equilibrium results in the presence of review process:

$$x^* = \frac{am(r-ar)^2\mu\rho(1-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2(\ln[\frac{1+3\Delta}{1-3\Delta}]+\ln[\frac{2+3\Delta}{2-3\Delta}])\rho+\frac{(-1+a)^2 r\mu\rho}{2w})+arw(2+(2-4\ln\left(\frac{1-\Delta}{\Delta}\right)+4(\ln[\frac{1+3\Delta}{1-3\Delta}]+\ln[\frac{2+3\Delta}{2-3\Delta}]))\rho)(1+m-(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)(1+m)-2(\ln[\frac{1+3\Delta}{1-3\Delta}]+\ln[\frac{2+3\Delta}{2-3\Delta}])(1+m))\rho+\frac{(-1+a)^2 mr\mu\rho}{2w})}{2(-4c(1+m)w\left(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\left(\ln\left[\frac{1+3\Delta}{1-3\Delta}\right]+\ln\left[\frac{2+3\Delta}{2-3\Delta}\right]\right)\rho\right)^2+a^2mr\mu\rho((-1+a)^2 r\mu\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2(\ln[\frac{1+3\Delta}{1-3\Delta}]+\ln[\frac{2+3\Delta}{2-3\Delta}])\rho)))}$$

$$y_1^* = \frac{(1-a)r}{4w},$$

$$y_2^* = \frac{(-1+a)r(-a^2 r\mu\rho(2(-1+m)w+((-2-4\ln\left(\frac{1-\Delta}{\Delta}\right)(-1+m)+4(\ln[\frac{1+3\Delta}{1-3\Delta}]+\ln[\frac{2+3\Delta}{2-3\Delta}])(-1+m))w+(-1+a)^2 mr\mu)\rho)+2c(1+m)(-1+(-1+2\ln\left(\frac{1-\Delta}{\Delta}\right)-2(\ln[\frac{1+3\Delta}{1-3\Delta}]+\ln[\frac{2+3\Delta}{2-3\Delta}]))\rho)(-(-1+a)^2 r\mu\rho+w(-2+4\ln\left(\frac{1-\Delta}{\Delta}\right)\rho-4(\ln[\frac{1+3\Delta}{1-3\Delta}]+\ln[\frac{2+3\Delta}{2-3\Delta}])\rho)))}{4w(-4c(1+m)w\left(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2\left(\ln\left[\frac{1+3\Delta}{1-3\Delta}\right]+\ln\left[\frac{2+3\Delta}{2-3\Delta}\right]\right)\rho\right)^2+a^2mr\mu\rho((-1+a)^2 r\mu\rho+4w(1+\rho-2\ln\left(\frac{1-\Delta}{\Delta}\right)\rho+2(\ln[\frac{1+3\Delta}{1-3\Delta}]+\ln[\frac{2+3\Delta}{2-3\Delta}])\rho)))}$$

When $s = 5$, the number of reviews can be simplified as $N(x, y_1) = 1 - 2\ln\left(\frac{1-\Delta}{\Delta}\right) + 2\ln\left(\frac{1+4\Delta}{1-4\Delta}\right) + 2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right) + 2\ln\left(\frac{3+4\Delta}{3-4\Delta}\right)$, the mean of reviews can be simplified as $\bar{R}(x, y_1) = \frac{-\ln\left(\frac{1-\Delta}{\Delta}\right)+\ln\left(\frac{1+4\Delta}{1-4\Delta}\right)+\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)+\ln\left(\frac{3+4\Delta}{3-4\Delta}\right)+(a(x-y_1)+y_1)\mu_\theta}{1-2\ln\left(\frac{1-\Delta}{\Delta}\right)+2\ln\left(\frac{1+4\Delta}{1-4\Delta}\right)+2\ln\left(\frac{1+2\Delta}{1-2\Delta}\right)+2\ln\left(\frac{3+4\Delta}{3-4\Delta}\right)}$;

When $s = 6$, the number of reviews can be simplified as $N(x, y_1) = 1 - 2\ln\left(\frac{1-\Delta}{\Delta}\right) + 2\ln\left(\frac{1+5\Delta}{1-5\Delta}\right) + 2\ln\left(\frac{2+5\Delta}{2-5\Delta}\right) + 2\ln\left(\frac{3+5\Delta}{3-5\Delta}\right) + 2\ln\left(\frac{4+5\Delta}{4-5\Delta}\right)$, the mean of reviews can be simplified as $\bar{R}(x, y_1) = \frac{-\ln\left(\frac{1-\Delta}{\Delta}\right)+\ln\left(\frac{1+5\Delta}{1-5\Delta}\right)+\ln\left(\frac{2+5\Delta}{2-5\Delta}\right)+\ln\left(\frac{3+5\Delta}{3-5\Delta}\right)+\ln\left(\frac{4+5\Delta}{4-5\Delta}\right)+(a(x-y_1)+y_1)\mu_\theta}{1-2\ln\left(\frac{1-\Delta}{\Delta}\right)+2\ln\left(\frac{1+5\Delta}{1-5\Delta}\right)+2\ln\left(\frac{2+5\Delta}{2-5\Delta}\right)+2\ln\left(\frac{3+5\Delta}{3-5\Delta}\right)+2\ln\left(\frac{4+5\Delta}{4-5\Delta}\right)}$.

Following the same equilibrium derivation process we have taken use of, we can obtain the optimal profit by taking into consideration of the optimal pricing strategy and effort level strategy, which are complicated in forms and we thus resolve to numerical analyze the obtain some useful conclusions as stated in Proposition 5.9.

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