Dual Sourcing in the Presence of Quality Uncertainty When Consumers Are Fairness Concerned

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CERTIFICATE OF ORIGINALITY

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Abstract

Dual sourcing, a strategy frequently employed by original equipment manufacturers (OEMs), may result in ex-post quality heterogeneity among the final products. This heterogeneity is typically a result of the unpredictability of the production processes of various suppliers. When ex-post quality heterogeneity exists, a customer may feel unfairly treated when she pays the same price but gets a lower-quality product than that received by a peer customer. Our paper will investigate how these issues interact and impact the supply chain parties.

In this study, we examine a supply chain in which an original equipment manufacturer sources from two suppliers and sells the final products to customers with peer-induced fairness concerns. The suppliers are ex-ante identical but may be ex-post heterogeneous with respect to the realized quality (either high or low) and quantity levels, with the quantity heterogeneity resulting from yield uncertainty: their production processes are unreliable and modeled by correlated proportional random yields. The suppliers compete on wholesale price for the OEM's order and the OEM decides the order quantities to each supplier. Customers are fairness-concerned and incur psychological disutility if they pay the same price as their peers but receive a product of inferior quality.

We find that, when the degree of fairness concern becomes higher, the OEM is more willing to source from a single supplier. Moreover, suppliers' wholesale prices are non-increasing in fairness concern, and in particular, wholesale prices will be equal to their marginal cost in the end. Interestingly, as the consumer's fairness concern increases, we show that the consumer surplus first decreases and then increases.

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Chapter 1

Introduction

Outsourcing helps an original equipment manufacturer (OEM) to focus on value-added activities such as research and development. When outsourcing their manufacturing operations, OEMs often adopt dual sourcing, as this approach can help them mitigate supply risks (Tomlin 2006 and Wang et al. 2010) and enhance their bargaining power (Maurer et al. 2004 and Li and Wan 2017). However, a notable consequence of dual sourcing is heterogeneity in ex post quality among the final products from different suppliers. In October 2015, Apple encountered a problem known as the "iPhone 6s chipgate" (Tyrone 2015). It was reported that not all iPhone 6s units had the same battery capacity. The underlying reason was that the A9 chip processors installed on the iPhone 6s were sourced from two suppliers, namely Samsung and Taiwan Semiconductor Manufacturing Company (TSMC). Testing results revealed that iPhones with TSMC A9 chips had longer battery life (by about two hours) than those with Samsung chips under certain usage conditions (Dremali 2015). Since battery life is one key measure of the performance of a smartphone, this caused a great deal of complaints and dissatisfaction among customers who bought iPhones with Samsung chips (PatentlyApple 2015).

The essential reason for the "iPhone 6s chipgate" was Apple's dual sourcing strategy from both Samsung and TSMC for its A9 chip processors. Apple's adoption of this dual sourcing strategy is to mitigate the uncertain yield issue through diversification and induce supplier competition (Panzarino 2015, Dillet 2016,Tayal 2017). The semiconductor industry is famous for its random yield issue; i.e., the number of qualified units from a manufacturing process is randomly distributed. By sourcing from two suppliers, Apple can ensure a steady supply of iPhone 6s (Panzarino 2015). Besides, inducing two suppliers to compete with each other can help Apple drive the wholesale price down (Dillet 2016).

The advantages of diversification and supplier competition through dual sourcing have been well documented in the literature; see Tomlin (2006), Babich et al. (2007), Wang et al. (2010) and Li et al. (2017) and the instances described therein. However, the potential drawback of such dual sourcing strategy, that is, inconsistent quality in the delivered components from the two suppliers, have not been considered. One reason is that such quality inconsistency between the suppliers' components is usually not a problem for manufacturers because the differences are normally well within the manufacturing tolerances. In the *chipgate* incident, Samsung and TSMC used different processes with different characteristics because of their independent research and development (Smith 2015). The *realized* qualities of their final products could exhibit certain differences. Nevertheless, Apple claimed that the differences are well below the tolerance level, and Apple argued that the large battery life differences shown by some third parties were tested under unrealistic conditions (Panzarino 2015). Apple further claimed that its own testing and data gathered from customers showed 2-3% difference in the actual battery life. *Techcrunch* noted that such a difference was far too low to be noticeable in real-world usage, and even two iPhones with the same processor can vary more than 3% (Panzarino 2015). Another independent test conducted by *Consumer Reports* revealed only 1% difference in battery life between iPhones with Samsung chips and those with TSMC chips (Gikas 2016).

Despite knowing that the battery life difference in real-world usage was almost unnoticeable, many customers still complained fiercely about getting an iPhone with a Samsung chip, and some even wished to return it (PatentlyApple 2015, Leong 2015). One user on an online forum stated the following (Macrumors 2015): "I paid a pretty big lump sum to have the newest and greatest! Not to have a 2-3% less than someone who paid the same price I did!" This statement explains the logic behind a lot of customers: They are displeased with the fact that their products are inferior to those of others who have paid the same amount, even though the differences are barely noticeable. Such a behavior can be explained as a concern for *peer-induced fairness*; that is, customers dislike being treated unfairly relative to peer customers (Ho and Su 2009). This paper experimentally verify that a player incurs a disutility if the player's payoff is *lower* than the player's peers. Because of this peer-induced fairness concern, although the quality difference induced by dual sourcing does not affect the product performance in a noticeable way, it still causes unhappiness/dissatisfaction among customers if they can and do perceive it. It is worth mentioning that even two units of iPhone 6s with identical components from the same supplier can vary in their battery performance (Panzarino 2015), which, however, is hardly noticeable by users given that individual usage patterns can be quite difficult to compare. In contrast, if such quality difference is due to the components from different suppliers, then customers are able to perceive it through identifying the supplier of the component used in their purchased products. For example, some apps can help identify the supplier of the A9 processor for each iPhone 6s. Such a quality difference and the subsequent consumer fairness concern might potentially affect an OEM's dual sourcing practice. Their impact and the underlying implications, however, are not well understood. Therefore, it is essential to characterize the effect of the ex-post product quality heterogeneity induced by components from different suppliers and the resulting consumer fairness concern on the original equipment manufacturer's (OEM's) sourcing strategy selection, the supplier's wholesale pricing, and the OEM's optimal ordering decision. Specifically, we are going to investigate the following research questions that previous studies have not yet adequately explored:

(1). When the suppliers face random yields and quality uncertainty, how shall the OEM place the orders between the two suppliers?

(2). How do the random yields and quality uncertainty affect the suppliers' pricing?

(3). How does customers' fairness concern (in the sense that they dislike paying the same price but receive products of uneven quality) affect the OEM' sourcing decision? Under what conditions will the OEM single (dual) source?

(4). What are the overall effect of consumer fairness concern arising from the potential ex-post product quality heterogeneity and the supplier's yield uncertainty on the system performance?

To answer these questions, we consider a supply chain in which an OEM sources from two suppliers and sells the end products to customers with peer-induced fairness concerns. The suppliers are ex ante identical whose product quality is uncertain but maybe ex post heterogeneous with respect to their realized qualities. The suppliers face yield uncertainty: their production processes are unreliable and modeled by correlated proportional random yields. The suppliers compete on wholesale price for the OEM's order and the OEM decides the order quantities to each supplier. Note that under dual sourcing, the OEM orders positively from both suppliers. Customers are fairness-concerned and incur psychological disutility if they pay the same price as their peers but receive a product of inferior quality. We adopt the backward induction to derive the equilibrium outcomes including the optimal wholesale price and order quantities. Specifically, we first characterize how fairness concern induced consumer disutility from the ex post quality heterogeneity affects the OEM's profit. We then examine the OEM's sourcing strategy and the supplier's wholesale pricing decisions.

Following is an outline of the remainder of this study. In chapter 2 we review some of the most relevant studies. The model formulation and assumptions are presented in chapter 3. In chapter 4, we derive the equilibrium outcomes for the OEM and suppliers as well as fairness-seeking consumers under both single sourcing and dual sourcing and analyze the OEM's optimal sourcing strategies selection. Conclusions are presented in chapter 5. All proofs are included in chapter A.

Chapter 2

Literature Review

A growing literature has examined the impact of fairness concerns on firms' strategic decisions and this study contributes to that literature. One strand focuses on distributional fairness, which describes a player's behavior of comparing its payoff with that of other players. Fehr and Schmidt (1999) propose a distributional fairness concern model where a player incurs a disutility if its payoff is *different* from other players'. This model has been widely used to study the roles of distributional fairness concern between supply chain members (Haitao Cui et al. 2007, Wu and Niederhoff 2014), and between retailers and consumers (Guo 2015, Guo and Jiang 2016, Yi et al. 2018). We are closely involved with the area of research that focuses on peer-induced fairness, in which an agent's perception of fairness is closely related to their peers' rewards (Campbell 1999, Kukar-Kinney et al. 2007, Ho and Su 2009, Chen and Cui 2013). Through experimental studies, Haws and Bearden (2006) show the possible negative impact of price difference under dynamic pricing methods for price fairness concerned consumers. Ho and Su (2009) experimentally verify that a player incurs a disutility if the player's payoff is *lower* than the player's peers. According to their estimates, peer-induced fairness concerns can be twice as strong as distributional fairness concerns. Chen and Cui (2013) use consumers' peer-induced fairness concern to explain the frequently observed uniform pricing of a firm for its horizontally differentiated products. Li and Jain (2016) demonstrate that price discrimination between consumers based on preferences learned from histories can help a retailer generate a higher profit if consumers exhibit strong peer-induced fairness concerns.

It is closely related to research that investigates the issue of supplier diversification in supply-uncertainty situations; see, e.g., Kazaz (2004), Tomlin and Wang (2005), Tomlin (2009), Li et al. (2013), Hu and Kostamis (2015) and the references therein. Wang et al. (2010) consider that a firm can either use dual sourcing or make efforts to improve supplier reliability to mitigate supply uncertainty. Tang and Kouvelis (2011) investigate the benefit from supplier diversification when a manufacturer faces competition in the end market. They find that dual sourcing still benefits manufacturers by reducing random yield inefficiency, but a manufacturer does not necessarily aim to minimize output variability. Calvo and Martínez-de Albéniz (2016) consider that a buyer sells short-life-cycle products. They find that dual sourcing performs worse than single sourcing in terms of supply chain efficiency and buyer profitability. Different from aforementioned studies, we consider a situation in which dual sourcing may lead to ex post product quality heterogeneity (in the components/products produced by different suppliers) and provoke peer-induced fairness concerns among consumers. We then investigate how this affects an OEM's sourcing decision.

Our work is also related to those studies on supplier competition when the buyer diversifies its supply source. For instance, Babich et al. (2007) consider supplier diversification when a monopolist buyer is concerned about supplier default risk, They show that increased correlation in supplier default risks enhances price competitiveness, resulting in a higher profit for the sourcing firm. Yang et al. (2012) analyze dual sourcing in a procurement contract setting where suppliers hold private information about their probability of disruption. They find that with better information the buyer benefits more from diversification but less from competition. Li and Wan (2017) show there can be both positive and negative effects of competition on suppliers' incentives to improve, depending on the information structure. The aforementioned studies focus on the diversification and cost reduction advantages of a supplier portfolio with competing supplier. They do not consider that dual sourcing may lead to ex post product quality heterogeneity, which, however, is our focus.

The purpose of this study is to investigate the interaction between consumer fairness concerns and the OEM's sourcing strategy. Supplementing the aforementioned studies, we demonstrate that consumers' fairness-seeking behavior can significantly influence firms' sourcing decisions and that sole sourcing can be superior to dual sourcing when there are strong fairness-minded consumers present.

Chapter 3

Model Setup

Consider a supply chain in which an original equipment manufacturer (OEM) purchases a critical component from two suppliers (denoted by i, $i \in \{1, 2\}$) and sells products to consumers with fairness concerns about qualities. We assume that the two suppliers are *ex-ante identical* in the sense that they are symmetric in their cost structure (each supplier has a positive unit production cost c), yield distribution, and quality distribution, but *ex-post heterogeneous* regarding the delivered quantity and the realized quality. Specifically, the supplier's production process is unreliable so that the quantity delivered may not be equal to the ordered size, and the realized quality s_i of each supplier might be different. Following (Tang and Kouvelis 2011), for a given order of size q_i received by supplier i, we assume the actual quantity delivered is $Y_i q_i$. Y_i is a random variable with support on $[0, 1]$, mean μ, and stand deviation σ. In addition, we assume that $g(\cdot)$ and $G(\cdot)$ are the probability density function and cumulative distribution function, respectively. Their yields from the two suppliers are correlated with each other, and the correlation coefficient of Y_1 and Y_2 is denoted by $\rho \in [-1, 1]$. We assume that $\mu \geq \sigma$ to ensure that $\mathbb{E}[Y_1 Y_2] = \mu^2 + \rho\sigma^2 \geq 0$. This means that both delivered order quantities are non-negative. Moreover, due to the compatibility of a supplier's technology and facility settings with the product design, the realized qualities of the suppliers may differ from each other. That is, the quality from supplier i might be *relatively* higher than that from supplier j, where $j \in \{1, 2\}$, $j \neq i$. Clearly, this cannot be determined before the mass production. The random S_i is characterized as follows:

$$S_i = \begin{cases} s_H, & \text{with probability } \beta; \\ s_L, & \text{with probability } 1-\beta. \end{cases}$$

Here, $\beta \in [0,1]$ and $s_H \geq s_L$. Throughout the paper, we assume $s_L \geq c$ to ensure a non-negative profit of a supplier. Define $\bar{s} = \beta s_H + (1-\beta)s_L$, which denotes the average quality level; and $\Delta = s_H - s_L$, which represents the quality difference. Moreover, we assume the expected delivered ration is not too small to ensure that it is profitable for the OEM to source from supplier; specifically, we assume $\mu\bar{s} > c$. Notice that $\bar{s}$ is the expected quality. Since the upper bound of valuation v is one, $\bar{s}$ is also the maximum valuation. Multiplying this term with the average yield μ results in the maximum expected revenue from a consumer. The expression $\mu\bar{s} - c$ is the maximum expected margin from a consumer. Thus, this condition ensures that the maximum margin should be positive. The objective of supplier i is to determine its wholesale price $w_i \geq c$ to maximize its profit.

The OEM decides the order quantity q_i to supplier i to maximize the expected profit. We assume the OEM pays to the suppliers based on an order quantity. That is, it pays $w_i q_i$ to supplier i for the procurement. It is common that suppliers, in certain hi-tech and biotech industries where there is high yield uncertainty, overestimate the amount of product being produced in order to meet all the ordered quantity; buyers bear a portion of this extra cost, with managers in such industries commenting that "almost always, we pay for the (supplier's) startup volume, rather than just the output." In the literature on random yields (Babich et al. 2007, Tang and Kouvelis 2011, etc.), this is not an uncommon assumption. After the qualities are realized and receiving the delivered quantities, the OEM sells the product to the end consumers at the market-clearing price p. This assumption is widely used in supply chain yields (Jung and Kouvelis 2022, Kong et al. 2013, ect.).

Consumers have heterogeneous valuations on the product quality. In particular, a type-v consumer can gain vs_i reward for a received product from supplier i. We assume that v is uniformly distributed on $[0,1]$. All consumers are perfectly rational decision makers, and each consumer purchases at most one unit product.

Without loss of generality, the population of consumers is normalized to 1. When the product is released to the market, a third party tests and reveals the quality realization from different suppliers to consumers. Consumers know the quality distribution among the product before purchase, but they learn the quality of their own units only after the purchase. If qualities of all the unites are ex-post equal ($s_1 = s_2 = s_H$ or $s_1 = s_2 = s_L$), then consumers have no fairness concerns and obtain a utility $U = vs_k - p,\ k \in \{H, L\}$. However, if part of the units are of high quality, and the remaining units are of low quality, say, $s_1 = s_H$ and $s_2 = s_L$, then consumers who receive a product with high quality obtains a utility $vs_H - p$; whereas a consumer who receives a product with low quality obtains $vs_L - p - n\alpha\Delta$, where $\alpha > 0$ measures a consumer's psychological disutility from fairness concerns, and n is the number of consumers who receive a high-quality product (note that n is random and determined by the distributions of Y_i and S_i). Clearly, the larger the quality difference Δ, the higher the disutility induced by fairness concern(i.e., $\alpha\Delta$). A consumer purchases the product if and only if the ex-ante expected utility, which is jointly affected by the quantity and quality distributions, is non-negative (we will describe the ex-ante expected utility in detail in the subsequent analysis of each model). Table 3.1 summarizes the key notation used in our paper.

The sequence of events is illustrated in Figure 3. First, the supplier i announces the wholesale price w_i. Then, the OEM orders quantity q_i to supplier i. Next, the quantity and quality are realized. The OEM releases the product to the market. After that, a third party tests and reveals the product quality to consumers. Finally, consumers purchase the product by paying p, where p is a market-clearing price and endogenously determined by the realized quantities and consumers' types.

Table 3.1: A List of Key Notation

Symbol	Description;
i	Subscript for the two suppliers, $i \in \{1, 2\}$
c	The unit production cost for the symmetric suppliers
Y_i	Random variables with support on $[0, 1]$, the realized proposition of the quantity order of supplier i
$g(\cdot)$	The probability density function of Y_i
$G(\cdot)$	The cumulative distribution of Y_i
μ	The mean of Y_i
σ	The standard deviation of Y_i
$\rho \in [-1, 1]$	The correlation coefficient between Y_1 and Y_2
S_i	Random variable, product's quality of supplier i
β	The probability that the realized quality is high
s_H	High quality level
s_L	Low quality level
Δ	$\Delta = s_H - s_L \geq 0$, the quality difference between high- and low-level of product
$\bar{s}$	The average quality level, $\bar{s} = \beta s_H + (1 - \beta) s_L$
α	Fairness concern parameter
Π_M	OEM's profit
Π_i	Supplier i's profit
w_i	Supplier i's wholesale price

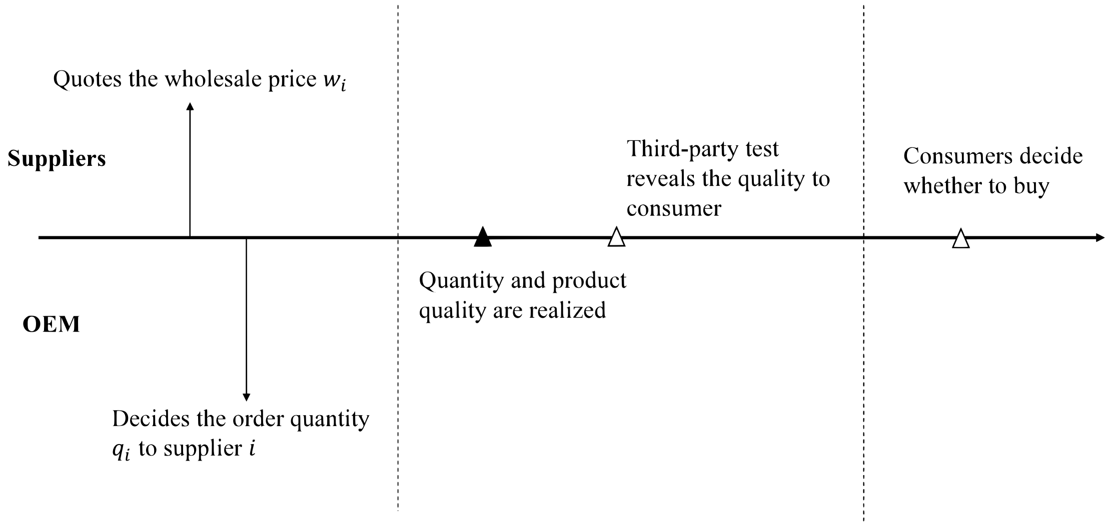

Figure 3.1: Sequence of events

Chapter 4

Results and Analysis

4.1 A Benchmark: Single Sourcing

In this section, we analyze a benchmark where the OEM adopts the single sourcing strategy. The OEM places an order q after the supplier sets w, and the actual delivered quantity is Yq, where Y has the same distribution with Y_i under the dual sourcing model. Clearly, the fairness concerns are absent because there is only one source of the products, which have equal quality level. Denote $k \in \{H, L\}$ by the realization of product's quality state. For any given market-clearing price p_k and quality s_k, a type-v consumer obtains a utility $U = vs_k - p_k$. If and only if $U \geq 0$, the consumer would buy the product. The endogenous market-clearing price p_k is determined by the following equation

$$D_k = 1 - \frac{p_k}{s_k} = Yq,$$

from which we obtain $p_k = (1-Yq)s_k$. The OEM maximizes the ex-ante expected profit function

$$\Pi_M = \beta\mathbb{E}[(1-Yq)s_H \cdot Yq] + (1-\beta)\mathbb{E}[(1-Yq)s_L \cdot Yq] - wq \tag{4.1}$$

by selecting $q^*(w)$ for any given wholesale price w. The supplier sets w to maximize

$$\Pi_S = (w-c)q^*(w). \tag{4.2}$$

We use superscript "b" (for benchmark) to denote the results associated with the single sourcing case. The following proposition summarizes supplier's and OEM's optimal decisions and expected profits.

Proposition 4.1. *Under single sourcing, the supplier's optimal wholesale price and the OEM's optimal order quantity are respectively,*

$$w^{b*} = \frac{\mu\bar{s} + c}{2},\ q^{b*} = \frac{\mu\bar{s} - c}{4(\mu^2 + \sigma^2)\bar{s}},$$

The supplier's optimal profit and the OEM's optimal expected profit are respectively,

$$\Pi_S^{b*} = \frac{(\mu\bar{s} - c)^2}{8\bar{s}(\mu^2 + \sigma^2)},\ \Pi_M^{b*} = \frac{(\mu\bar{s} - c)^2}{16\bar{s}(\mu^2 + \sigma^2)}.$$

Since there is only one supplier and all products on the market are of the same quality, all equilibrium outcomes in the proposition are indifferent in terms of consumers' fairness concerns. We will discuss the model with two suppliers in the next section.

4.2 Dual Sourcing Model

In this section, we analyze the case in which the OEM sources from two suppliers. We use "H" and "L" to denote the cases with different realizations of the products' quality state. For example, "HL" means the case in which supplier 1's realized quality is s_H, and supplier 2's is s_L. In the "HH" case, for a given market-clearing price p_{HH}, the consumer's utility function is similar to the benchmark model: $U_{HH} = vs_H - p_{HH}$. Similarly, in the "LL" case, the utility function is $U_{LL} = vs_L - p_{LL}$. In the "HL" or "LH" case, however, consumers have fairness concerns, which affect the utility function. We define $\theta = \frac{Y_1 q_1}{Y_1 q_1 + Y_2 q_2}$, which is the probability of getting a product with the component from supplier 1.

Before purchasing, a consumer's expected utilities in these cases are

$$U_{HL} = \theta(vs_H - p_{HL}) + (1 - \theta)[vs_L - p_{HL} - Y_1 q_1 \alpha\Delta],$$
$$U_{LH} = (1 - \theta)(vs_H - p_{LH}) + \theta[vs_L - p_{LH} - Y_2 q_2 \alpha\Delta].$$

If and only if $U \geq 0$, the consumer would bue the product. The endogenous market demand D is as follows:

$$D_{HH} = 1 - \frac{p_{HH}}{s_H}, \quad D_{LL} = 1 - \frac{p_{LL}}{s_L},$$
$$D_{HL} = 1 - \frac{p_{HL} + (1-\theta)Y_1 q_1 \alpha\Delta}{\theta s_H + (1-\theta)s_L}, \quad D_{LH} = 1 - \frac{p_{LH} + \theta Y_2 q_2 \alpha\Delta}{\theta s_L + (1-\theta)s_H}, \tag{4.3}$$

The endogenous market-clearing price is determined by the equation $D = Y_1 q_1 + Y_2 q_2$, from which we obtain:

$$p_{HH} = s_H (1 - Y_1 q_1 - Y_2 q_2), \quad p_{LL} = s_L (1 - Y_1 q_1 - Y_2 q_2),$$
$$p_{HL} = [\theta s_H + (1-\theta)s_L] (1 - Y_1 q_1 - Y_2 q_2) - (1-\theta)Y_1 q_1 \alpha\Delta,$$
$$p_{LH} = [\theta s_L + (1-\theta)s_H] (1 - Y_1 q_1 - Y_2 q_2) - \theta Y_2 q_2 \alpha\Delta. \tag{4.4}$$

The OEM maximizes the ex-ante expected profit function

$$\begin{aligned}\Pi_M =& \mathbb{E}[\beta^2 p_{HH} D_{HH} + (1-\beta)^2 p_{LL} D_{LL} + \beta(1-\beta) p_{HL} D_{HL} + (1-\beta)\beta p_{LH} D_{LH}] \\ &- w_1 q_1 - w_2 q_2,\end{aligned} \tag{4.5}$$

by selecting $q_1^*(w_1, w_2)$ and $q_2^*(w_1, w_2)$ from any given wholesale prices w_1, w_2.

Substituting the expressions of market demand in (4.3) and market-clearing price in (4.4) into the expression of the expected profit in (4.5), we can obtain the following problem for the OEM:

$$\begin{aligned}\max_{q_1, q_2} \ \Pi_M =& \mu\bar{s}(q_1 + q_2) - (q_1 w_1 + q_2 w_2) - \bar{s}(q_1^2 + q_2^2)(\mu^2 + \sigma^2) \\ &- 2q_1 q_2 (\mu^2 + \rho\sigma^2)[\bar{s} + \alpha\Delta\beta(1-\beta)] \\ \text{s.t.} \quad & q_i \geq 0, i \in \{1, 2\}.\end{aligned} \tag{4.6}$$

For simplicity of discussion, we define $Z = \alpha\Delta\beta(1-\beta)$, which can be interpreted as a measure of the fairness concerns. We present OEM's equilibrium order quantities for any given wholesale price w_1, w_2 in the following lemma.

Proposition 4.2. *For given wholesale prices* (w_1, w_2) *under dual sourcing, the*

OEM's optimal order quantity $q_i^*(w_1, w_2)$ to supplier i is as follows:

$$q_i^* = \begin{cases} \frac{(\mu\bar{s}-w_i)\bar{s}(\mu^2+\sigma^2)-(\mu\bar{s}-w_j)(Z+\bar{s})(\mu^2+\rho\sigma^2)}{2\{[\bar{s}(\mu^2+\sigma^2)]^2-[(Z+\bar{s})(\mu^2+\rho\sigma^2)]^2\}}, & \text{if } w_i, w_j < \mu\bar{s},\ Z < \bar{Z}(w_1, w_2); \\ \frac{\mu\bar{s}-w_i}{2\bar{s}(\mu^2+\sigma^2)}, & \text{if } w_i < \mu\bar{s} \leq w_j, \\ & \text{or } (w_i < w_j < \mu\bar{s},\ Z \geq \bar{Z}(w_1, w_2)); \\ 0, & \text{if } w_i \geq \mu\bar{s}, \\ & \text{or } (w_j < w_i < \mu\bar{s},\ Z \geq \bar{Z}(w_1, w_2)), \end{cases} \tag{4.7}$$

where $j = 3 - i$ and $\bar{Z}(w_1, w_2) = \frac{(\mu\bar{s}-\max\{w_1,w_2\})\bar{s}(\mu^2+\sigma^2)}{(\mu\bar{s}-\min\{w_1,w_2\})(\mu^2+\rho\sigma^2)} - \bar{s}$. Especially, if $w_1 = w_2 \leq \mu\bar{s}, Z \geq \bar{Z}(w_1, w_2)$, we can show the OEM only orders $\frac{\mu\bar{s}-w_i}{2\bar{s}(\mu^2+\sigma^2)}$ from supplier 1 with probability 0.5, and from supplier 2 with the same probability.

Proposition 4.2 shows that the order quantity is always zero when the corresponding supplier quotes a very high wholesale price, i.e. $w_i \geq \mu\bar{s}$. This result is trivial. Accordingly, we will only discuss the case in which $w_i < \mu\bar{s}$.

Then, we will analyze suppliers' decisions. The supplier i sets w_i to maximizes $\Pi_i = (w_i - c)q_i^*(w_1, w_2)$. We use superscript "$d$" (for dual sourcing) to denote the results associated with the dual sourcing case. Their optimal decisions are summarized in the following proposition.

Proposition 4.3. *Under dual sourcing, the optimal wholesale prices charged by the suppliers are*

$$w^{d*} = w_1^{d*} = w_2^{d*} = \begin{cases} \frac{\bar{s}(\mu^2+\sigma^2)(\mu\bar{s}+c)-(Z+\bar{s})(\mu^2+\rho\sigma^2)\mu\bar{s}}{2\bar{s}(\mu^2+\sigma^2)-(Z+\bar{s})(\mu^2+\rho\sigma^2)}, & \text{if } Z < \hat{Z}, \\ c, & \text{if } Z \geq \hat{Z}, \end{cases} \tag{4.8}$$

where $\hat{Z} = \frac{\bar{s}(\mu^2+\sigma^2)}{(\mu^2+\rho\sigma^2} - \bar{s}$.

Proposition 4.3 shows that suppliers will set wholesale prices as their marginal production costs when consumers' concerns regarding fairness are significant. There is a principal reason for this result because the OEM prefers to purchase from only one supplier, and then two suppliers must lower their wholesale prices to obtain the order. It resembles the well-known model "Bertrand Competition".

Next, we present the OEM's equilibrium order quantities in the following corollary.

Corollary 4.1. *The OEM's equilibrium order quantities are as follows:*

If $Z < \hat{Z}$*, then*

$$q_1^{d*} = q_2^{d*} = \frac{\bar{s}(\mu^2+\sigma^2)(\mu\bar{s}-c)}{2[2\bar{s}(\mu^2+\sigma^2)-(Z+\bar{s})(\mu^2+\rho\sigma^2)][(Z+\bar{s})(\mu^2+\rho\sigma^2)+\bar{s}(\mu^2+\sigma^2)]}; \tag{4.9}$$

otherwise, the OEM only orders $\frac{\mu\bar{s}-c}{2\bar{s}(\mu^2+\sigma^2)}$ *from supplier 1 with probability* 0.5*, and from supplier 2 with the same probability.*

Corollary 4.1 shows that the OEM will purchase components from a single supplier when consumers are strongly fairness-minded. It well explains Apple's actions in adopting a sole sourcing strategy after the chipgate.

Finally, after we obtain the OEM's optimal order quantities and suppliers' optimal wholesale prices, we can derive their profits. The following proposition summarizes the OEM's and suppliers' equilibrium profits.

Proposition 4.4. *The optimal expected profits of the OEM and two suppliers are:*

1. *If* $Z < \hat{Z}$, $\Pi_M^{d*} = \frac{(\bar{s}\mu-c)^2[\bar{s}(\mu^2+\sigma^2)]^2}{2[\bar{s}(\mu^2+\sigma^2)+(Z+\bar{s})(\mu^2+\rho\sigma^2)][2\bar{s}(\mu^2+\sigma^2)-(Z+\bar{s})(\mu^2+\rho\sigma^2)]^2}$*,*
   and $\Pi_1^{d*} = \Pi_2^{d*} = \frac{(\bar{s}\mu-c)^2\bar{s}(\mu^2+\sigma^2)[\bar{s}(\mu^2+\sigma^2)-(Z+\bar{s})(\mu^2+\rho\sigma^2)]}{2[]\bar{s}(\mu^2+\sigma^2)+(Z+\bar{s})(\mu^2+\rho\sigma^2)][2\bar{s}(\mu^2+\sigma^2)-(Z+\bar{s})(\mu^2+\rho\sigma^2)]^2}$*;*

2. *If* $Z \geq \hat{Z}$, $\Pi_M^{d*} = \frac{(\bar{s}\mu-c)^2}{4\bar{s}(\mu^2+\sigma^2)}, \Pi_1^{d*} = \Pi_2^{d*} = 0$*.*

In the next section, we will examine the effect of market factors, such as consumer fairness concerns, on the performance of the OEM and suppliers.

4.3 Sensitivity Analysis

Here, we investigate the effect of α, the consumer fairness concern on system performance. As their performances under benchmark setting are unaffected by α, we focus on these results under the dual sourcing model. First, we rewrite the condition $Z < \hat{Z}$ in Proposition 4.3 and 4.4 as $\alpha < \hat{\alpha}$, where $\hat{\alpha} = \frac{\bar{s}(\mu^2+\sigma^2)}{\Delta\beta(1-\beta)(\mu^2+\rho\sigma^2} - \frac{\bar{s}}{\Delta\beta(1-\beta)}$. Note that the consumer fairness concern does not affect OEM's and suppliers' performance when $\alpha \geq \hat{\alpha}$. Therefore, we focus on the case in which

$\alpha < \hat{\alpha}$ when we discuss the sensitivity. Moreover, for simplicity of discussion, we will analyze the total order quantities $q^{d*}(q^{d*} = q_1^{d*} + q_2^{d*})$ instead of q_1^{d*} and q_2^{d*}.

Because the OEM's order quantity is influenced not only by the fairness issue, but also by wholesale prices charged by suppliers, we should analyze how the fairness concern affects order quantity when wholesale prices are fixed. According to proposition 4.3, the equilibrium wholesale prices of two suppliers are always the same. Hence, the focus of our discussion is $q^{d*}(w_1, w_2)$ when $w_1 = w_2$, where $q^{d*}(w_1, w_2) = q_1^{d*}(w_1, w_2) + q_2^{d*}(w_1, w_2)$. From proposition 4.2, we can show $q^{d*}(w_1, w_2) = \frac{(\mu\bar{s} - w_1)}{\bar{s}(\mu^2+\sigma^2)+(Z+\bar{s})(\mu^2+\rho\sigma^2)}$. The following lemma illustrates how the fairness concern and wholesale price affect OEM's order quantities.

Lemma 4.1. *For given wholesale prices, the order quantities $q^*(w_1, w_2)$ is decreasing in wholesale price and fairness concern.*

We will now discuss the final equilibrium results. The following proposition presents our results.

Proposition 4.5. *1. The OEM's optimal total order quantities q^{d*} decreases in α when $\alpha < \underline{\alpha}$ while increases when $\alpha \geq \underline{\alpha}$, where $\underline{\alpha} = \frac{\bar{s}[2(\mu^2+\rho\sigma^2)-(\mu^2+\sigma^2)]}{2\Delta\beta(1-\beta)(\mu^2+\rho\sigma^2)}$.*

2. The optimal wholesale price w^{d*} is decreasing in α.

3. The OEM's profit Π_M^{d*} increases in α.

4. The two suppliers' profits Π_1^{d*} and Π_2^{d*} both are decreasing in α.

Figure 4.1 summarizes results in Proposition 4.5. According to proposition 4.5, suppliers charge a lower price when consumers' fairness concern levels becomes higher. This is because a higher level of fairness leads the OEM to purchase from only one supplier, and then it strengthens the competition between suppliers. Proposition 4.5 also shows the total order quantities decrease in the consumer fairness concern at first, then increase. This is because there are two forces driving OEM's decision. On the one hand, OEM tends to order fewer quantities when consumers are more sensitive to fairness. On the other hand, OEM tends to order more when the wholesale price becomes lower(see Lemma 4.1).

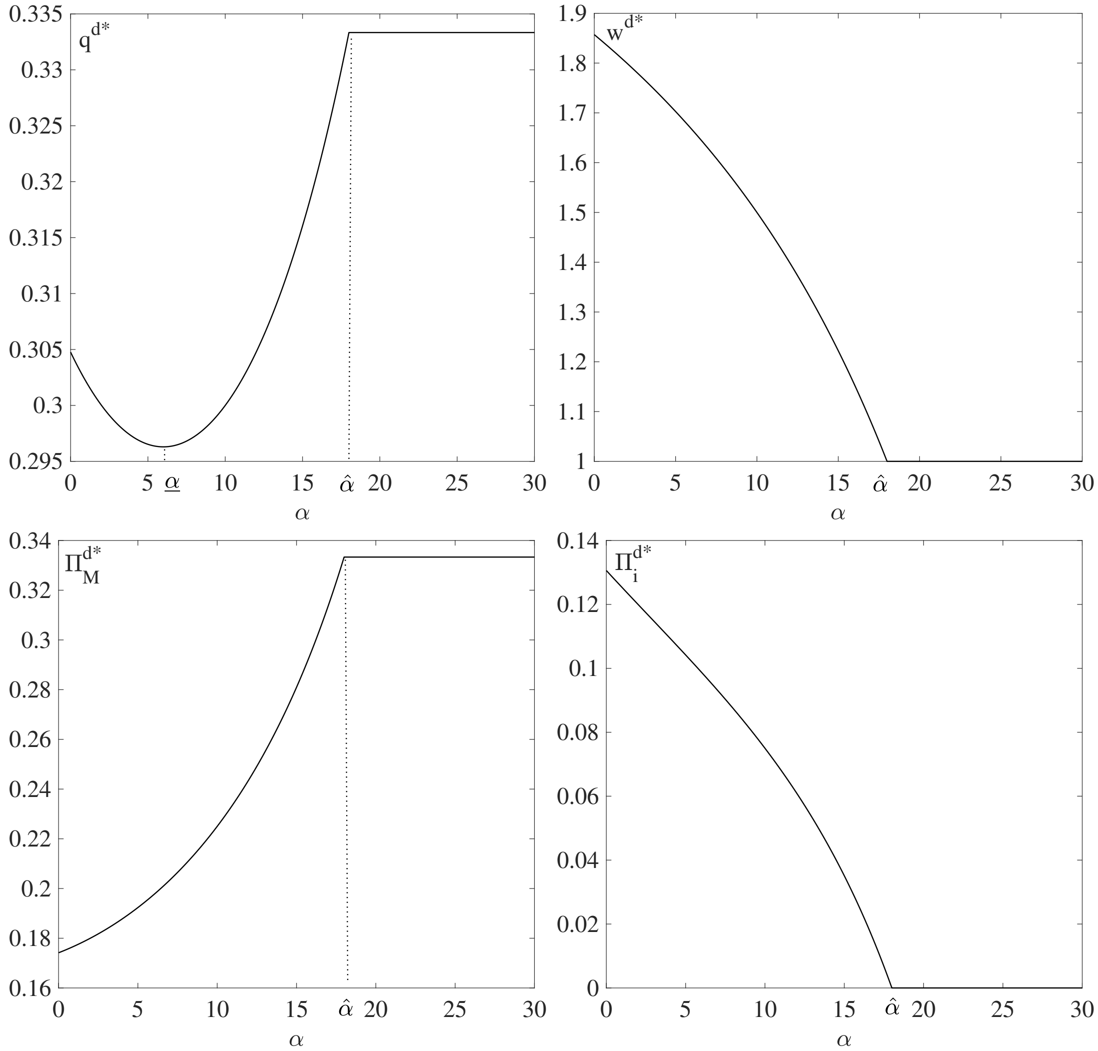

Figure 4.1: The impact of consumer fairness seeking on system performance

When α is small, the second force is dominated by the former, the optimal total order quantities q^{d*} is decreasing in α. When α is large, however, it reverses. Proposition 4.5 implies that by enhancing the fairness concern, OEM is more profitable, but suppliers are less. These sensitivity analysis results imply that the fairness concern actually strengthens supplier competition. In next section, we will compare the benchmark model and the dual sourcing model.

4.4 Comparison between the Benchmark model and the Dual Sourcing Model

Here, we compare the OEM's profit under the benchmark setting and the dual sourcing setting. We want to know which model benefits the OEM more. The following proposition summarizes the results.

Proposition 4.6. *Even if consumers are fairness seeking, the OEM always prefer the dual sourcing model. Moreover, there are* $q^{b*} \leq q^{d*}$*, and* $w^{b*} \geq w^{d*}$*.*

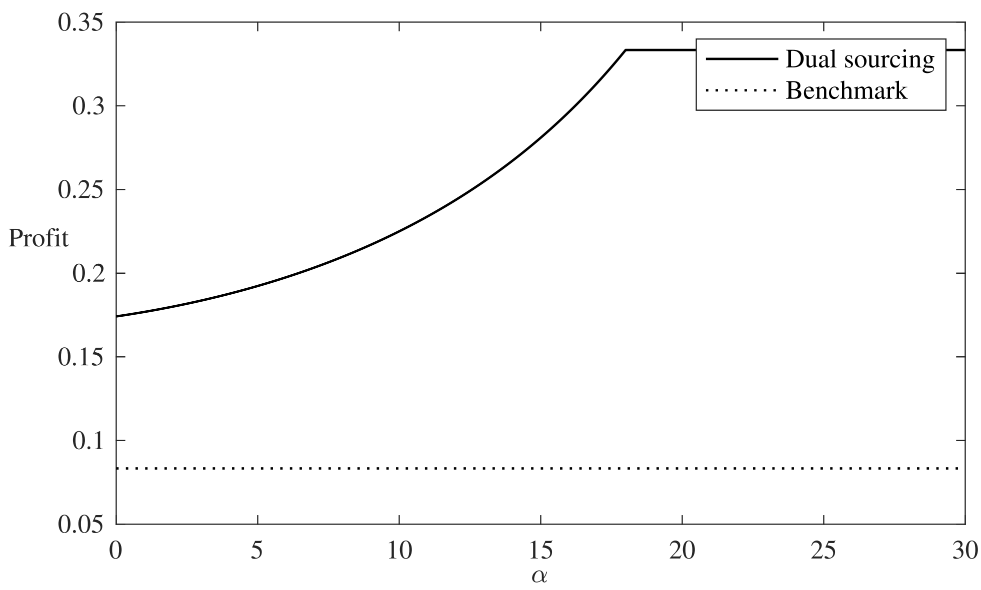

Figure 4.2: Benchmark vs. Dual Sourcing

As figure 4.2 shows, the OEM is more profitable under the dual sourcing model. This is because the supplier competition is intense when the OEM purchases from two suppliers. Although dual sourcing may decrease OEM profits due to consumer disutility, the lower wholesale price will fully compensate for this loss. According to proposition 4.6, the OEM always prefers to bring in another supplier when consumers are fairness-seeking.

4.5 Analysis of Consumer surplus

In this section, we will focus on how the fairness concern affects consumer surplus. As their performances under benchmark setting are unaffected by α, we focus on these results under the dual sourcing model. For simplicity of analysis, we need to make extra assumption on the distribution of supply uncertainty. Following (Babich et al. 2007), we consider a special case in which the delivered quantity $Y_i q_i$ only has two results 0 and q_i. In other words, suppliers provide either complete orders or nothing to the OEM. The joint distribution of Y_1, Y_2 is determined by the probabilities

$$r^{y_1 y_2} = P[Y_1 = y_1, Y_2 = y_2], y_i \in \{0, 1\}, i \in \{1, 2\}.$$

We will indicate marginal probabilities by replacing appropriate indices of $r^{y_1 y_2}$ by $*$. The joint probabilities, $r^{00}, r^{01}, r^{10}, r^{11}$, satisfying the following equations.

$$\begin{aligned} r^{00} + r^{01} + r^{10} + r^{11} &= 1, \\ r^{11} + r^{10} &= r^{1*}, \\ r^{11} + r^{01} &= r^{*1}. \end{aligned} \tag{4.10}$$

For the purpose of characterization of the joint distribution, these three parameters can be used: r^{1*}, r^{*1}, and r^{11}, i.e., the marginal completely delivered probabilities for each supplier as well as the probability of exactly two completely orders from suppliers. The supplier correlation can be modeled in this way in an easy manner. Consequently, if $r^{11} = r^{1*} = r^{*1}$, then $r^{01} = r^{10} = 0$, meaning that the yields are perfectly positive correlated. By contrast, if $r^{11} = 0$ and $r^{1*} + r^{*1} = 1$, then the yields are perfectly negatively correlated, since $r^{00} = 0$, $r^{01} = r^{*1}$, and $r^{10} = r^{1*}$.

Additionally, we can characterize the different correlations by keeping π_1 and π_2 constant and varying r^{11}. For example, as r^{11} increases so does (and thus r^{01} and r^{10} decrease), the correlation increases. Therefore, in order to capture the full range from perfect negative correlation to perfect positive correlation in this way, r^{1*} and r^{1*} should be equal to $\frac{1}{2}$ and to allow r^{11} to vary between 0 and $\frac{1}{2}$. Under this setting, we can show $\mu = \sigma = \frac{1}{2}$ and $\rho = 4p_{11} - 1$.

Since the OEM only purchases from two suppliers when $\alpha < \hat{\alpha}$, we will focus on this case. Next, we will show how to derive the consumer surplus. For example, when the realization of the products' quality state is "HL", the utility function is

$$U_{HL} = \theta(vs_H - p_{HL}) + (1-\theta)[vs_L - p_{HL} - Y_1 q_1 \alpha\Delta],$$

where $\theta = \frac{Y_1 q_1}{Y_1 q_1 + Y_2 q_2}$. According to corollary 4.1, we can show $q_1 = q_2 = q^{d*}$. Moreover, the expected utility is

$$\mathbb{E}[U_{HL}] = \frac{(v-1)(3-\rho)(s_H+s_L) + 4q^{d*}(s_H+s_L)}{8}.$$

Because a consumer purchases the product if and only if the ex-ante expected utility is non-negative, we can show $\mathbb{E}[U_{HL}] \geq 0$ only and if only $v \geq \frac{3-\rho-4q^{d*}}{3-\rho}$. By integral, the consumer surplus in "HL" case is $\frac{(q^{d*})^2(s_H+s_L)}{3-\rho}$. Keeping the same logic, the consumer surplus in other cases can be derived. Finally, we conclude that the ex-ante consumer surplus is

$$CS = (q^{d*})^2\left[\frac{\beta^2 s_H + (1-\beta^2)s_L}{2} + \frac{2\beta(1-\beta)(s_H+s_L)}{3-\rho}\right].$$

The following proposition summarizes the impact of fairness concerns on consumer surplus.

Proposition 4.7. *When the OEM purchase from two suppliers, the consumer surplus decreases in α if $\alpha < \underline{\alpha}$ and then increases in it.*

As a result of this proposition, it is shown that consumers do not always benefit from their concerns about fairness. Consumer surplus decreases in α when fairness concerns are not significant (see figure 4.3). In other words, when consumers become concerned about fairness, they may harm themselves. Despite the fact that this analytical result is derived under a strong assumption, numerical experiments have shown that it also holds for other common distributions. For example, we illustrate in figure 4.4 how the consumer surplus is affected by α when suppliers' yields follow truncated normal distribution.

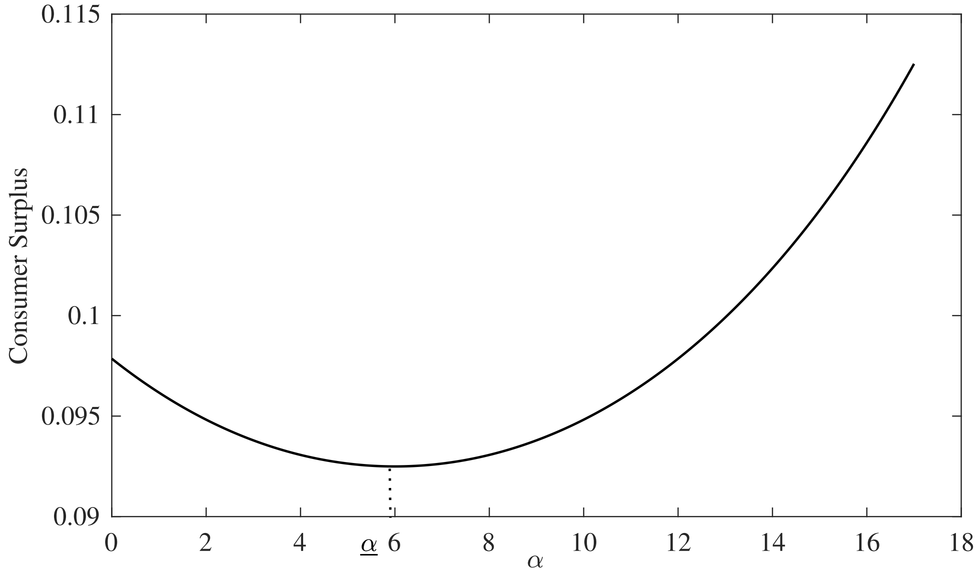

Figure 4.3: The impact of fairness seeking on consumer surplus

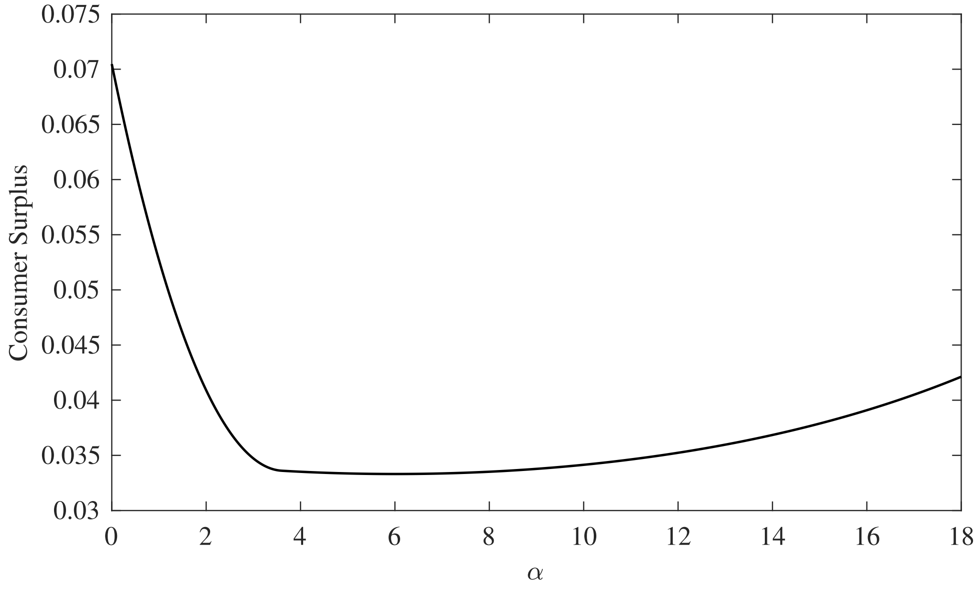

Figure 4.4: The impact of fairness seeking on consumer surplus under truncated normal distribution

Chapter 5

Conclusion and Future Work

Original equipment manufacturers (OEMs) can concentrate on value-added activities such as research and development thanks to outsourcing. Dual sourcing is frequently employed by original equipment manufacturers (OEMs) when outsourcing their manufacturing operations, as it can help them mitigate supply risks and increase their bargaining power. However, there is one notable consequence of dual sourcing — heterogeneity in terms of quality between the final products sourced from different suppliers. While the quality difference caused by dual sourcing does not affect the product performance in a noticeable manner, it still causes consumers unhappiness/dissatisfaction due to peer-induced fairness concerns. It is possible that an OEM's dual sourcing practice could be adversely affected by the quality difference and consumer fairness concerns. However, the extent of their impact and the implications underlying them are not fully understood. It is important to elucidate how ex-post quality heterogeneity due to components sourced from different suppliers, as well as consumer fairness concerns, influence the OEM's sourcing decisions and the supplier's wholesale pricing.

To this end, we set up a model to characterize the OEM's sourcing strategy selection and investigate how the fairness concern affects the supply chain performances. We obtain the following main insights. One, the OEM purchases components from only one supplier when fairness concerns are significant. Two, suppliers set wholesale prices as marginal costs and get zero profit when the OEM only chooses one supplier. We also show that the OEM's optimal total order quantities decrease in fairness concern at first and then increase. Two forces are primarily responsible for OEM's decision-making regarding quantities: disutility due to fairness concerns and competition between two suppliers. Lastly, we investigate the specific effect of the fairness concern on the consumer surplus. Interestingly, when consumers are not very sensitive to fairness concerns, they might be hurt by a larger degree of fairness concern. However, when consumers are significantly fairness-seeking, consumer surplus increases in the degree of fairness concern. As a result of our findings, we believe it is crucial to incorporate consumers' fairness concerns into the selection of sourcing strategies.

However, in practice, there is another reason why OEMs choose to source from two or multiple suppliers. To be more specific, a single supplier may not be able to provide enough components to the OEM. In other words, suppliers' production capacity is limited. The limited capacity of suppliers can soften competition among suppliers when the degree of fairness concern increases. From this aspect, we may explore how such production capacity limitation influences the OEM's sourcing decision when consumers are fairness-seeking. Furthermore, this thesis sets a very strong assumption when discussing the impact of fairness concerns on consumer surplus. In the future, we will examine some more general distributions of yield uncertainty.

Appendix A

Proofs and Derivations for Chapter 4

A.1 Proof of Proposition 4.1

Recall the OEM's ex-ante expected profit function in (4.1). It is concave in q. Hence, given any wholesale price w, the OEM's optimal order quantity is

$$q^{b*}(w) = \frac{\{\mu\bar{s} - w\}}{2\bar{s}(\mu^2 + \sigma^2)}. \tag{A.1}$$

Substituting (A.1) into the supplier's profit function (4.2), we can show the function is also concave, and then we obtain the equilibrium wholesale price is $\frac{\mu\bar{s}+c}{2}$. Substituting the equilibrium wholesale price into (A.1), we obtain the equilibrium order quantity is $q^{b*} = \frac{\mu\bar{s}-c}{4(\mu^2+\sigma^2)\bar{s}}$. According to the equilibrium order quantity and wholesale price, we derive the OEM's and supplier's equilibrium profits easily.

A.2 Proof of Proposition 4.2

Substituting the expressions of market demand in (4.3) and market clearing price in (4.4) into the expression of the expected profit in (4.5), we can obtain the following problem for the OEM:

$$\begin{aligned} \max \ \Pi_M(q_1, q_2) = &\bar{s}\left[(q_1 + q_2)\mu - (q_1^2 + q_2^2)(\mu^2 + \sigma^2) + 2q_1q_2(\mu^2 + \rho\sigma^2)\right] \\ &- 2q_1q_2(\mu^2 + \rho\sigma^2)Z - (q_1w_1 + q_2w_2) \\ \text{s.t.} \qquad &q_i \geq 0, i \in \{1, 2\}. \end{aligned}$$

We can derive the Hessian matrix:

$$H = \begin{bmatrix} 2\bar{s}\left(\mu^2+\sigma^2\right) & 2(Z+\bar{s})\left(\mu^2+\rho\sigma^2\right) \\ -2(Z+\bar{s})\left(\mu^2+\rho\sigma^2\right) & -2\bar{s}\left(\mu^2+\sigma^2\right) \end{bmatrix}.$$

We can show the definiteness of H depends on $\left[\bar{s}^2\left(\mu^2+\sigma^2\right)^2-(Z+\bar{s})^2\left(\mu^2+\rho\sigma^2\right)^2\right]$. Hence, we next analyze the following cases:

(i) $\left[\bar{s}^2\left(\mu^2+\sigma^2\right)^2-(Z+\bar{s})^2\left(\mu^2+\rho\sigma^2\right)^2\right]>0$;

(ii) $\left[\bar{s}^2\left(\mu^2+\sigma^2\right)^2-(Z+\bar{s})^2\left(\mu^2+\rho\sigma^2\right)^2\right]=0$;

(iii) $\left[\bar{s}^2\left(\mu^2+\sigma^2\right)^2-(Z+\bar{s})^2\left(\mu^2+\rho\sigma^2\right)^2\right]<0$.

(Case i)

By changing terms, this condition $\left[\bar{s}^2\left(\mu^2+\sigma^2\right)^2-(Z+\bar{s})^2\left(\mu^2+\rho\sigma^2\right)^2\right]>0$ can be rewritten as $\rho<\hat{\rho}$, where

$$\hat{\rho}=\frac{\bar{s}(\mu^2+\sigma^2)}{(Z+\bar{s})\sigma^2}-\frac{\mu^2}{\sigma^2}. \tag{A.2}$$

Because H is negative definite, the profit function Π_M is concave. We can use K.K.T conditions to solve this problem. The K.K.T. conditions are

$$\begin{pmatrix} \bar{s}[2q_1^*(\mu^2+\sigma^2)+2q_2^*(\mu^2+\rho\sigma^2)]-\mu\bar{s}+2q_2^*(\mu^2+\rho\sigma^2)Z+w_1 \\ \bar{s}[2q_2^*(\mu^2+\sigma^2)+2q_1^*(\mu^2+\rho\sigma^2)]-\mu\bar{s}+2q_1^*(\mu^2+\rho\sigma^2)Z+w_2 \end{pmatrix}$$

$$+\lambda_1\begin{pmatrix}-1\\0\end{pmatrix}+\lambda_2\begin{pmatrix}0\\-1\end{pmatrix}=0, \tag{A.3}$$

$$\lambda_i\geq 0,\ i\in\{1,2\}, \tag{A.4}$$

$$q_i^*\geq 0,\ i\in\{1,2\}, \tag{A.5}$$

$$-\lambda_1 q_1^*=0, \tag{A.6}$$

$$-\lambda_2 q_2^*=0. \tag{A.7}$$

Next, we focus on the case in which $w_1\leq w_2$. Since the two firms are symmetric, the results for the case in which $w_1\geq w_2$ can be obtained just by switching the index. We next analyze the following four cases: (case i-1) $\lambda_1=0$ and $\lambda_2=0$; (case i-2) $\lambda_1>0$ and $\lambda_2=0$; (case -3) $\lambda_1=0$ and $\lambda_2>0$; (case i-4) $\lambda_1>0$ and $\lambda_2>0$.

(Case i-1)

By solving equation (A.3), we have

$$q_1^*(w_1, w_2) = \frac{[(w_1 - \mu\bar{s})\bar{s}(\mu^2 + \sigma^2) + (\mu\bar{s} - w_2)(Z + \bar{s})(\mu^2 + \rho\sigma^2)]}{2\{[(Z + \bar{s})(\mu^2 + \rho\sigma^2)]^2 - \bar{s}^2(\mu^2 + \sigma^2)^2\}},$$
$$q_2^*(w_1, w_2) = \frac{[(w_2 - \mu\bar{s})\bar{s}(\mu^2 + \sigma^2) + (\mu\bar{s} - w_1)(Z + \bar{s})(\mu^2 + \rho\sigma^2)]}{2\{[(Z + \bar{s})(\mu^2 + \rho\sigma^2)]^2 - \bar{s}^2(\mu^2 + \sigma^2)^2\}}. \quad \text{(A.8)}$$

Clearly, conditions (A.4), (A.6), and (A.7) are satisfied. We will verify condition (A.5) to ensure the solution above is optimal. Since the condition (A.5) depends on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \leq w_2$, we discuss the following scenarios: (Scenario i) $w_1 < \mu\bar{s}$ and $w_2 < \mu\bar{s}$, (Scenario ii) $w_1 < \mu\bar{s}$ and $w_2 \geq \mu\bar{s}$, and (Scenario iii) $w_1 \geq \mu\bar{s}$ and $w_2 \geq \mu\bar{s}$.

(Scenario i)

Because the condition (A.2), the denominator of q_i^* is always negative. We can show $q_1^* \geq 0$ if and only if

$$(\mu\bar{s} - w_2)(Z + \bar{s})(\mu^2 + \rho\sigma^2) - (\mu\bar{s} - w_1)\bar{s}(\mu^2 + \sigma^2) \leq 0 \quad \text{(A.9)}$$

In addition, we can show $q_2^* \geq 0$ if and only if

$$(\mu\bar{s} - w_1)(Z + \bar{s})(\mu^2 + \rho\sigma^2) - (\mu\bar{s} - w_2)\bar{s}(\mu^2 + \sigma^2) \leq 0 \quad \text{(A.10)}$$

Because $w_1 < \mu\bar{s}$ and $w_2 < \mu\bar{s}$, we can rewrite conditions (A.9) and (A.10) as $\rho \leq \bar{\rho}_1$ and $\rho \leq \bar{\rho}_2$, respectively, where,

$$\bar{\rho}_1 = \frac{(\mu\bar{s} - w_1)\bar{s}(\mu^2 + \sigma^2)}{(\mu\bar{s} - w_2)(Z + \bar{s})\sigma^2} - \frac{\mu^2}{\sigma^2}, \quad \text{(A.11)}$$
$$\bar{\rho}_2 = \frac{(\mu\bar{s} - w_2)\bar{s}(\mu^2 + \sigma^2)}{(\mu\bar{s} - w_1)(Z + \bar{s})\sigma^2} - \frac{\mu^2}{\sigma^2}. \quad \text{(A.12)}$$

Because $\bar{\rho}_2 < \bar{\rho}_1$, we can show $\min\{q_1^*, q_2^*\} \geq 0$ if $\rho \leq \bar{\rho}_2$. And we can show $\bar{\rho}_2 < \hat{\rho}$.

In (Scenario i), we show that the solution in (A.8) is feasible optimal if $\rho \in [-1, \bar{\rho}_2]$.

(Scenario ii)

In this case, the condition (A.10) can be rewritten as $\rho \leq \bar{\rho}_2$. We can show $\bar{\rho}_2 \leq -1$ because

$$\bar{\rho}_2 \leq 1 - \frac{(\mu^2 + \sigma^2)[(\mu\bar{s} - w_1)\bar{s} + Z(\mu\bar{s} - w_1)]}{(\bar{s} + Z)(\mu\bar{s} - w_1)\sigma^2} = -\frac{\mu^2}{\sigma^2} \leq -1.$$

The first inequality is from $w_1 < \mu\bar{s} \leq w_2$, and the second inequality is from $\sigma \leq \mu$. Hence, q_2^* always negative and the solution in (A.8) is not optimal.

(Scenario iii)

In this case, we first consider the situation when $w_1 = w_2 = \mu\bar{s}$. Obviously, both (A.9) and (A.10) hold and the optimal solution $q_1^* = q_2^* = 0$. Then we consider the situation when $\mu\bar{s} = w_1 < w_2$. The condition (A.10) cannot hold. Finally, we consider it when $\mu\bar{s} < w_1 \leq w_2$. We can rewrite the condition (A.9) as $\rho \geq \bar{\rho}_1$. Because $\rho < \hat{\rho} < \bar{\rho}_1$, this condition cannot hold.

In (Case i-1), we conclude that the solution in (A.8) is optimal when $w_1 < \mu\bar{s}$ and $w_2 < \mu\bar{s}$ or $w_1 = w_2 = \mu\bar{s}$.

(Case i-2)

By solving equation (A.3) and (A.6), we have

$$\begin{aligned} q_1^*(w_1, w_2) &= 0, \\ q_2^*(w_1, w_2) &= \frac{(\mu\bar{s} - w_2)}{2\bar{s}(\mu^2 + \sigma^2)}, \\ \lambda_1 &= \frac{(\mu\bar{s} - w_2)(\mu^2 + \rho\sigma^2)}{\bar{s}(\mu^2 + \sigma^2)}(\bar{s} + Z) + w_1 - \mu\bar{s}. \end{aligned} \tag{A.13}$$

Since signs of $q_2^*(w_1, w_2)$ and λ_1 in (A.13) depend on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \leq w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, and (Scenario iii) $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$.

(Scenario i) With the condition $\mu\bar{s} \geq w_2$, we can show $q_2^*(w_1, w_2) \geq 0$. We also need to verify $\lambda_1 \geq 0$ (the condition in (A.4)), which is equivalent to $\rho \geq \bar{\rho}_1$. In addition, we can show $\bar{\rho}_1 \geq \hat{\rho}$. Because the condition (A.2), λ_1 always negative in this case. This solution is not optimal.

(Scenario ii) With the condition $\mu\bar{s} < w_2$, we can show $q_2^*(w_1, w_2) < 0$. Hence, we prove the solution in (A.13) is not optimal.

(Scenario iii) This case is same as (Scenario ii).

In (Case 2), we conclude that the solution in (A.13) is not optimal.

(Case i-3)

By solving (A.7) and (A.3), we have

$$
\begin{aligned}
q_1^*(w_1, w_2) &= \frac{(\mu\bar{s} - w_1)}{2\bar{s}(\mu^2 + \sigma^2)}, \\
q_2^*(w_1, w_2) &= 0, \\
\lambda_2 &= \frac{(\mu\bar{s} - w_1)(\mu^2 + \rho\sigma^2)}{\bar{s}(\mu^2 + \sigma^2)}(\bar{s} + Z) + w_2 - \mu\bar{s}.
\end{aligned} \tag{A.14}
$$

Since signs of $q_1^*(w_1, w_2)$ and λ_1 in (A.14) depend on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \leq w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, and (Scenario iii) $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$.

(Scenario i) With the condition $w_1 \leq \mu\bar{s}$, we can show $q_1^*(w_1, w_2) \geq 0$. We also need to verify $\lambda_2 \geq 0$ (the condition in (A.4)), which is equivalent to $\rho \geq \bar{\rho}_2$. Hence, we prove that $q_i \geq 0$ and the solution in (A.14) is optimal if $\rho \in [\bar{\rho}_2, \hat{\rho})$.

(Scenario ii) With the condition $w_1 \leq \mu\bar{s}$, we can show $q_1^*(w_1, w_2) \geq 0$. In addition, because $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, we can show $\lambda_2 \geq 0$ in (A.14) always holds. Hence, we prove that $q_i \geq 0$ and the solution in (A.14) is optimal in this case.

(Scenario iii) With the condition $\mu\bar{s} < w_1$, we can show $q_1^*(w_1, w_2) < 0$. Hence, we prove the solution in (A.14) is not optimal.

In (Case 3), we conclude that the solution in (A.14) is optimal if $\rho \in [\bar{\rho}_2, \hat{\rho})$ in (Scenario i) and is optimal for all ρ in (Scenario ii).

(Case i-4)

We get the solution $q_1^* = 0$ and $q_2^* = 0$. By solving (A.3), we have $\lambda_1 = w_1 - \mu\bar{s}$ and $\lambda_2 = w_2 - \mu\bar{s}$. Since signs of λ_1 and λ_2 depend on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \leq w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, and (Scenario iii) $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$.

(Scenario i) Obviously, with conditions $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, this solution is optimal if and only if $w_i = \mu\bar{s}$.

(Scenario ii) Obviously, with the condition $w_1 \leq \mu\bar{s}$, this solution is optimal if and only if $w_1 = \mu\bar{s}$.

(Scenario iii) When $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$, we can show $\lambda_1 > 0, \lambda_2 > 0$ In this case, this solution which $q_1^* = 0$ and $q_2^* = 0$ is optimal.

In (case i), we conclude the OEM's optimal order quantity q_i^* to supplier i is as follows:

$$q_i^* = \begin{cases} \frac{\left[(w_i-\mu\bar{s})\bar{s}(\mu^2+\sigma^2)+(\mu\bar{s}-w_j)(Z+\bar{s})(\mu^2+\rho\sigma^2)\right]}{2\{[(Z+\bar{s})(\mu^2+\rho\sigma^2)]^2-\bar{s}^2(\mu^2+\sigma^2)^2\}}, & \text{if } w_i, w_j < \mu\bar{s},\ \rho \le \bar{\rho}; \\ \frac{(\mu\bar{s}-w_i)}{2\bar{s}(\mu^2+\sigma^2)}, & \text{if } w_i < w_j < \mu\bar{s},\ \rho > \bar{\rho}, \text{ or } w_i < \mu\bar{s} \le w_j; \\ 0, & \text{if } w_j < w_i < \mu\bar{s},\ \rho > \bar{\rho}, \text{ or } w_i \ge \mu\bar{s}, \end{cases} \tag{A.15}$$

where $j = 3 - i$ and $\bar{\rho} = 1 - \frac{(\mu^2+\sigma^2)[|w_2-w_1|\bar{s}+Z(\mu\bar{s}-\min\{w_1,w_2\})]}{(\bar{s}+Z)(\mu\bar{s}-\min\{w_1,w_2\})\sigma^2}$.

(Case ii)

In this case, we have this condition $\left[\bar{s}^2\left(\mu^2+\sigma^2\right)^2 - (Z+\bar{s})^2\left(\mu^2+\rho\sigma^2\right)^2\right] = 0$. And we still focus on the case in which $w_1 \le w_2$. Since the two firms are symmetric, the results for the case in which $w_1 \ge w_2$ can be obtained just by switching the index.

We can rewrite the problem (4.6) by a matrix form as follows:

$$\begin{aligned} \max\ \Pi_M(q_1, q_2) &= \begin{bmatrix} q_1 & q_2 \end{bmatrix} A \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \mathbf{b} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \\ \text{s.t.} \qquad & q_i \ge 0, i \in \{1, 2\}, \end{aligned} \tag{A.16}$$

where

$$A = \begin{bmatrix} -\bar{s}(\mu^2+\sigma^2) & -\bar{s}(\mu^2+\sigma^2) \\ -\bar{s}(\mu^2+\sigma^2) & -\bar{s}(\mu^2+\sigma^2) \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} \mu\bar{s} - w_1 \\ \mu\bar{s} - w_2 \end{bmatrix}$$

According the property of quadratic function, when A is semi-definite, the function has a local maximum value if and only if $\mathbf{b}$ is the image of A. And then the unconstrained optimizer is $-\frac{\mathbf{v}}{2}$ such that $A\mathbf{v} = \mathbf{b}$. We can show $\mathbf{b}$ is the image of A when $w_1 = w_2$. This means that we can find an optimal solution such that $A[-2q_1^*,\ -2q_2^*]^T = \mathbf{b}$ when $w_1 = w_2$. By solving this system, we can show the optimal condition is $q_1^* + q_2^* = \frac{(\mu\bar{s}-w_1)}{2\bar{s}(\mu^2+\sigma^2)}$. Let $q_2^* = 0$, we can show one of optimal solutions.

$$q_1^* = \begin{cases} \frac{(\mu\bar{s}-w_1)}{2\bar{s}(\mu^2+\sigma^2)}, & \text{if } w_1 = w_2 < \mu\bar{s}, \\ 0, & \text{if } w_1 = w_2 \ge \mu\bar{s}, \end{cases}, \text{and } q_2^* = 0. \tag{A.17}$$

When $w_1 < w_2$, $\mathbf{b}$ is not the image of A. There is no local maximum value if the problem is unconstrained. Because the K.K.T conditions are the necessary conditions of the optimal solution, we can find some solutions and then compare them to obtain the optimal one. The K.K.T. conditions are

$$\begin{pmatrix} 2\bar{s}\left(\mu^2+\sigma^2\right)\left(q_1+q_2\right)-\mu q+w_1 \\ 2\bar{s}\left(\mu^2+\sigma^2\right)\left(q_1+q_2\right)-\mu q+w_2 \end{pmatrix}+\lambda_1\begin{pmatrix} -1 \\ 0 \end{pmatrix}+\lambda_2\begin{pmatrix} 0 \\ -1 \end{pmatrix}=0, \tag{A.18}$$

$$\lambda_i \geq 0,\ i \in \{1,2\}, \tag{A.19}$$

$$q_i^* \geq 0,\ i \in \{1,2\}, \tag{A.20}$$

$$-\lambda_1 q_1^*=0, \tag{A.21}$$

$$-\lambda_2 q_2^*=0. \tag{A.22}$$

We next analyze the following four cases: (case ii-1) $\lambda_1=0$ and $\lambda_2=0$; (case ii-2) $\lambda_1 \geq 0$ and $\lambda_2=0$; (case ii-3) $\lambda_1=0$ and $\lambda_2 \geq 0$; (case ii-4) $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$.

(Case ii-1) Because $w_1<w_2$, there do not exist a solution such that

$$\begin{pmatrix} 2\bar{s}\left(\mu^2+\sigma^2\right)\left(q_1+q_2\right)-\mu q+w_1=0 \\ 2\bar{s}\left(\mu^2+\sigma^2\right)\left(q_1+q_2\right)-\mu q+w_2=0 \end{pmatrix}.$$

Hence, we cannot find a solution satisfying K.K.T conditions in this case.

(Case ii-2)

By solving equation (A.3) and (A.6), we have

$$\begin{aligned} q_1^*(w_1,w_2)&=0, \\ q_2^*(w_1,w_2)&=\frac{(\mu\bar{s}-w_2)}{2\bar{s}(\mu^2+\sigma^2)}, \\ \lambda_1&=w_1-w_2. \end{aligned} \tag{A.23}$$

Since signs of $q_2^*(w_1,w_2)$ and λ_1 in (A.23) depend on the sign of $(\mu\bar{s}-w_i)$, with the condition $w_1<w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2>\mu\bar{s}$, and (Scenario iii) $w_1>\mu\bar{s}$ and $w_2>\mu\bar{s}$.

(Scenario i) With the condition $\mu\bar{s} \geq w_2$, we can show $q_2^*(w_1,w_2) \geq 0$. However, because $w_1<w_2$, λ_1 is negative. This means that the solution is not satisfying K.K.T conditions.

(Scenario ii) With the condition $\mu\bar{s}<w_2$, we can show $q_2^*(w_1,w_2)<0$. In other words, the solution is not feasible.

(Scenario iii) This case is same as (Scenario ii).

(Case ii-3)

By solving (A.7) and (A.3), we have

$$\begin{aligned} q_1^*(w_1, w_2) &= \frac{(\mu\bar{s} - w_1)}{2\bar{s}(\mu^2 + \sigma^2)}, \\ q_2^*(w_1, w_2) &= 0, \\ \lambda_2 &= w_2 - w_1. \end{aligned} \tag{A.24}$$

Since signs of $q_1^*(w_1, w_2)$ and λ_1 in (A.24) depend on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 < w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, and (Scenario iii) $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$.

(Scenario i) With the condition $w_1 \leq \mu\bar{s}$, we can show $q_1^*(w_1, w_2) \geq 0$. Because $w_1 < w_2$, we can show $\lambda_1 > 0$ always holds. Hence, the solution is feasible and satisfying K.K.T conditions.

(Scenario ii) This case is same as (Scenario i).

(Scenario iii) With the condition $\mu\bar{s} < w_1$, we can show $q_1^*(w_1, w_2) < 0$. Hence, the solution is not feasible.

(Case ii-4)

We get the solution $q_1^* = 0$ and $q_2^* = 0$. By solving (A.3), we have $\lambda_1 = w_1 - \mu\bar{s}$ and $\lambda_2 = w_2 - \mu\bar{s}$. Since signs of λ_1 and λ_2 depend on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \leq w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, and (Scenario iii) $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$.

(Scenario i) Obviously, with conditions $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, this solution is feasible if and only if $w_i = \mu\bar{s}$.

(Scenario ii) Obviously, with the condition $w_1 \leq \mu\bar{s}$, this solution is feasible if and only if $w_1 = \mu\bar{s}$.

(Scenario iii) When $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$, we can show $\lambda_1 > 0, \lambda_2 > 0$ In this case, this solution which $q_1^* = 0$ and $q_2^* = 0$ is feasible.

By combining all these conditions in (case ii), we can conclude the OEM's optimal order quantity q_i^* to supplier i is as follows:

$$q_i^* = \begin{cases} \frac{(\mu\bar{s}-w_i)}{2\bar{s}(\mu^2+\sigma^2)}, & \text{if } w_i \le w_j \le \mu\bar{s}, \ , \text{ or } w_i \le \mu\bar{s} < w_j; \\ 0, & \text{if } w_j < w_i \le \mu\bar{s}, \ , \text{ or } w_i > \mu\bar{s}, \end{cases} \tag{A.25}$$

where $j = 3 - i$.

(Case iii)

By changing terms, this condition $\left[\bar{s}^2\left(\mu^2+\sigma^2\right)^2 - (Z+\bar{s})^2\left(\mu^2+\rho\sigma^2\right)^2\right] < 0$ can be rewritten as $\rho > \hat{\rho}$. In this case, because the Hessian matrix of profit function is indefinite, there is no local maximum value of the problem is unconstrained. Because the K.K.T conditions are the necessary conditions of the optimal solution, we can find some solutions and then compare them to obtain the optimal one. The K.K.T. conditions are the same as (case i).

Next, we focus on the case in which $w_1 \le w_2$. Since the two firms are symmetric, the results for the case in which $w_1 \ge w_2$ can be obtained just by switching the index. We next analyze the following four cases: (case 1) $\lambda_1 = 0$ and $\lambda_2 = 0$; (case 2) $\lambda_1 > 0$ and $\lambda_2 = 0$; (case 3) $\lambda_1 = 0$ and $\lambda_2 > 0$; (case 4) $\lambda_1 > 0$ and $\lambda_2 > 0$.

(Case iii-1) By solving equation (A.3), we have

$$\begin{aligned} q_1^*(w_1,w_2) &= \frac{[(w_1-\mu\bar{s})\bar{s}(\mu^2+\sigma^2)+(\mu\bar{s}-w_2)(Z+\bar{s})(\mu^2+\rho\sigma^2)]}{2\left\{[(Z+\bar{s})(\mu^2+\rho\sigma^2)]^2-\bar{s}^2(\mu^2+\sigma^2)^2\right\}}, \\ q_2^*(w_1,w_2) &= \frac{[(w_2-\mu\bar{s})\bar{s}(\mu^2+\sigma^2)+(\mu\bar{s}-w_1)(Z+\bar{s})(\mu^2+\rho\sigma^2)]}{2\left\{[(Z+\bar{s})(\mu^2+\rho\sigma^2)]^2-\bar{s}^2(\mu^2+\sigma^2)^2\right\}}. \end{aligned} \tag{A.26}$$

Clearly, conditions (A.4), (A.6), and (A.7) are satisfied. We will verify condition (A.5). Since the condition (A.5) depends on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \le w_2$, we discuss the following scenarios: (Scenario i) $w_1 < \mu\bar{s}$ and $w_2 < \mu\bar{s}$, (Scenario ii) $w_1 < \mu\bar{s}$ and $w_2 \ge \mu\bar{s}$, and (Scenario iii) $w_1 \ge \mu\bar{s}$ and $w_2 \ge \mu\bar{s}$.

(Scenario i)

In (case iii), the denominator of q_i^* is always positive. We can show $q_1^* \ge 0$ if and only if

$$(\mu\bar{s} - w_2)(Z+\bar{s})(\mu^2+\rho\sigma^2) - (\mu\bar{s}-w_1)\bar{s}(\mu^2+\sigma^2) \ge 0 \tag{A.27}$$

In addition, we can show $q_2^* \geq 0$ if and only if

$$(\mu\bar{s} - w_1)(Z + \bar{s})(\mu^2 + \rho\sigma^2) - (\mu\bar{s} - w_2)\bar{s}(\mu^2 + \sigma^2) \geq 0 \tag{A.28}$$

Because $w_1 < \mu\bar{s}$ and $w_2 < \mu\bar{s}$, we can rewrite conditions (A.9) and (A.10) as $\rho \geq \bar{\rho}_1$ and $\rho \geq \bar{\rho}_2$, respectively.

Because $\bar{\rho}_2 < \bar{\rho}_1$, we can show $\min\{q_1^*, q_2^*\} \geq 0$ if $\rho \geq \bar{\rho}_1$. And we can show $\bar{\rho}_1 > \hat{\rho}$.

In (Scenario i), we show that the solution in (A.26) is feasible solution if $\rho \in [\bar{\rho}_1, 1]$.

(Scenario ii)

In this case, the condition (A.27) cannot hold because $(\mu\bar{s} - w_2) \leq 0$, $-(\mu\bar{s} - w_1) < 0$, and other terms in (A.27) all are non-negative. Hence, the solution in (A.26) is not feasible in this scenario.

(Scenario iii)

In this case, we first consider the situation when $w_1 = w_2 = \mu\bar{s}$. Obviously, both (A.27) and (A.28) hold and the optimal solution $q_1^* = q_2^* = 0$. Then we consider the situation when $\mu\bar{s} = w_1 < w_2$. The condition (A.27) cannot hold. Finally, we consider it when $\mu\bar{s} < w_1 \leq w_2$. We can rewrite the condition (A.28) as $\rho \leq \bar{\rho}_2$. Because $\bar{\rho}_2 < \hat{\rho} < \rho$, this condition cannot hold.

In (Case iii-1), we conclude that the solution in (A.8) is a feasible when $w_1 < \mu\bar{s}$ and $w_2 < \mu\bar{s}$ or $w_1 = w_2 = \mu\bar{s}$.

(Case iii-2)

By solving equation (A.3) and (A.6), we have

$$\begin{aligned} q_1^*(w_1, w_2) &= 0, \\ q_2^*(w_1, w_2) &= \frac{(\mu\bar{s} - w_2)}{2\bar{s}(\mu^2 + \sigma^2)}, \\ \lambda_1 &= \frac{(\mu\bar{s} - w_2)(\mu^2 + \rho\sigma^2)}{\bar{s}(\mu^2 + \sigma^2)}(\bar{s} + Z) + w_1 - \mu\bar{s}. \end{aligned} \tag{A.29}$$

Since signs of $q_2^*(w_1, w_2)$ and λ_1 in (A.29) depend on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \leq w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, and (Scenario iii) $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$.

(Scenario i) With the condition $\mu\bar{s} \geq w_2$, we can show $q_2^*(w_1, w_2) \geq 0$. We also need to verify $\lambda_1 \geq 0$ (the condition in (A.4)), which is equivalent to $\rho \geq \bar{\rho}_1$. In addition, we can show $\bar{\rho}_1 \geq \hat{\rho}$. In this scenario, we can show $q_2^* \geq 0$ and $\lambda_1 \geq 0$ when $\rho \in [\bar{\rho}_1, 1]$. This means that the solution is feasible if $\rho \in [\bar{\rho}_1, 1]$.

(Scenario ii) With the condition $\mu\bar{s} < w_2$, we can show $q_2^*(w_1, w_2) < 0$ and the solution is not feasible.

(Scenario iii) This case is same as (Scenario ii).

(Case iii-3)

By solving (A.7) and (A.3), we have

$$\begin{aligned} q_1^*(w_1, w_2) &= \frac{(\mu\bar{s} - w_1)}{2\bar{s}(\mu^2 + \sigma^2)}, \\ q_2^*(w_1, w_2) &= 0, \\ \lambda_2 &= \frac{(\mu\bar{s} - w_1)(\mu^2 + \rho\sigma^2)}{\bar{s}(\mu^2 + \sigma^2)} (\bar{s} + Z) + w_2 - \mu\bar{s}. \end{aligned} \tag{A.30}$$

Since signs of $q_1^*(w_1, w_2)$ and λ_1 in (A.30) depend on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \leq w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, and (Scenario iii) $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$.

(Scenario i) With the condition $w_1 \leq \mu\bar{s}$, we can show $q_1^*(w_1, w_2) \geq 0$. In (case iii), because $\rho > \hat{\rho}$, by changing terms, we can show $\frac{\mu^2+\rho\sigma^2}{\mu^2+\sigma^2} > \frac{\bar{s}}{Z+\bar{s}}$. Hence, we can show

$$\lambda_2 > \frac{(\mu\bar{s} - w_1)}{\bar{s}} \frac{\bar{s}}{Z + \bar{s}} (\bar{s} + Z) + w_2 - \mu\bar{s} = w_2 - w_1.$$

The first inequality from $\frac{\mu^2+\rho\sigma^2}{\mu^2+\sigma^2} > \frac{\bar{s}}{Z+\bar{s}}$. Because $w_1 \leq w_2$, we can show $\lambda_2 \geq 0$ always hold. In other words, the solution is always feasible in this scenario.

(Scenario ii) It is same as (scenario i).

(Scenario iii) With the condition $\mu\bar{s} < w_1$, we can show $q_1^*(w_1, w_2) < 0$. Hence, we prove the solution in (A.14) is not feasible.

(Case iii-4)

We get the solution $q_1^* = 0$ and $q_2^* = 0$. By solving (A.3), we have $\lambda_1 = w_1 - \mu\bar{s}$ and $\lambda_2 = w_2 - \mu\bar{s}$. Since signs of λ_1 and λ_2 depend on the sign of $(\mu\bar{s} - w_i)$, with the condition $w_1 \leq w_2$, we discuss the following scenarios: (Scenario i) $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, (Scenario ii) $w_1 \leq \mu\bar{s}$ and $w_2 > \mu\bar{s}$, and (Scenario iii) $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$.

(Scenario i) Obviously, with conditions $w_1 \leq \mu\bar{s}$ and $w_2 \leq \mu\bar{s}$, this solution is feasible if and only if $w_i = \mu\bar{s}$.

(Scenario ii) Obviously, with the condition $w_1 \leq \mu\bar{s}$, this solution is feasible if and only if $w_1 = \mu\bar{s}$.

(Scenario iii) When $w_1 > \mu\bar{s}$ and $w_2 > \mu\bar{s}$, we can show $\lambda_1 > 0, \lambda_2 > 0$ In this case, this solution which $q_1^* = 0$ and $q_2^* = 0$ is feasible. We see the solution in (case iii-4) is the only feasible solution satisfying K.K.T conditions when $w_1 \geq \mu\bar{s}$ and $w_2 \geq \mu\bar{s}$. Hence, if both $w_i \geq \mu\bar{s}$, then ordering nothing is the optimal for the OEM. In addition, the solution in (case iii-3) is the feasible solution satisfying K.K.T conditions when $w_1 < \mu\bar{s}$ and $w_2 \geq \mu\bar{s}$, and then it is optimal. Especially, when both $w_i < \mu\bar{s}$, these solutions we obtained in (case iii-1), (case iii-2), and (case iii-3) all are feasible, and we refer them as $\mathbf{l}_1$, $\mathbf{l}_2$, and $\mathbf{l}_3$, respectively.

Next, we will compare $\mathbf{l}_1$, $\mathbf{l}_2$, and $\mathbf{l}_3$ and derive the optimal one. We can show $\mathbf{l}_3$ is feasible for all $\rho \in (\hat{\rho}, 1]$ but $\mathbf{l}_2$ and $\mathbf{l}_1$ are feasible only when $\rho \in [\bar{\rho}_1, 1]$. Hence, when $\rho \in (\hat{\rho}, \bar{\rho}_1)$, the $\mathbf{l}_1$ is the only one feasible solution satisfying K.K.T conditions, and then it is the optimal solution. When $\rho \in [\bar{\rho}_1, 1]$, we need to compare $\Pi_M(\mathbf{l}_1)$, $\Pi_M(\mathbf{l}_2)$ and $\Pi_M(\mathbf{l}_3)$. Obviously, because $w_1 \leq w_2$, we can show $\Pi_M(\mathbf{l}_3) \geq \Pi_M(\mathbf{l}_2)$. By substituting $\mathbf{l}_3$ and $\mathbf{l}_1$ into the profit function, we can show $\Pi_M(\mathbf{l}_3) \geq \Pi_M(\mathbf{l}_1)$ if $\rho \geq \bar{\rho}_1$. Hence, when both $w_i < \mu\bar{s}$, we can show $\mathbf{s}_l$ is the optimal solution. By combining these results, in (case iii), we can conclude the OEM's optimal order quantity q_i^* to supplier i is as follows:

$$q_i^* = \begin{cases} \frac{(\mu\bar{s}-w_i)}{2\bar{s}(\mu^2+\sigma^2)}, & \text{if } w_i < w_j < \mu\bar{s}, \text{ , or } w_i < \mu\bar{s} \leq w_j; \\ 0, & \text{if } w_j < w_i < \mu\bar{s}, \text{ , or } w_i \geq \mu\bar{s}, \end{cases} \tag{A.31}$$

where $j = 3 - i$.

Combining (case i), (case ii), and (case iii), we can conclude the OEM's optimal order quantity q_i^* to supplier i is as follows:

$$q_i^* = \begin{cases} \frac{\left[(w_i-\mu\bar{s})\bar{s}(\mu^2+\sigma^2)+(\mu\bar{s}-w_j)(Z+\bar{s})(\mu^2+\rho\sigma^2)\right]}{2\{[(Z+\bar{s})(\mu^2+\rho\sigma^2)]^2-\bar{s}^2(\mu^2+\sigma^2)^2\}}, & \text{if } w_i, w_j < \mu\bar{s},\ \rho \leq \bar{\rho}; \\ \frac{(\mu\bar{s}-w_i)}{2\bar{s}(\mu^2+\sigma^2)}, & \text{if } w_i < w_j < \mu\bar{s},\ \rho > \bar{\rho}, \\ & \text{ or } w_i < \mu\bar{s} \leq w_j; \\ 0, & \text{if } w_j < w_i < \mu\bar{s},\ \rho > \bar{\rho}, \text{ or } w_i \geq \mu\bar{s}, \end{cases} \tag{A.32}$$

where $j = 3 - i$ and $\bar{\rho} = 1 - \frac{(\mu^2+\sigma^2)[|w_2-w_1|\bar{s}+KM(\mu\bar{s}-\min\{w_1,w_2\})]}{(\bar{s}+KM)(\mu\bar{s}-\min\{w_1,w_2\})\sigma^2}$. Especially, if $w_1 = w_2 < \mu\bar{s}, \rho > \bar{\rho}$, we can show the OEM only orders $\frac{M(\mu\bar{s}-w_i)}{2\bar{s}(\mu^2+\sigma^2)}$ from supplier 1 with probability 0.5, and from supplier 2 with the same probability. Moreover, by changing terms, we can rewrite $\rho < \bar{\rho}$ as $Z < \bar{Z}$.

A.3 Proof of Proposition 4.3

In this proof, given the OEM's best response function, we are going to derive the equilibrium wholesale prices of both two suppliers. According to Proposition 4.2, we can show that the profit of suppliers will be zero if they choose a wholesale price w that is greater than $\mu\bar{s}$, because the order quantity is zero. However, if supplier i's wholesale price is between $\mu\bar{s}$ and c, i.e. $c \leq w_i < \mu\bar{s}$, he always obtains a non-negative profit. Hence, we only need to consider the case with $w_i < \mu\bar{s}$.

Since suppliers 1 and 2 are symmetric, we focus on the best response function of supplier 1. The best response function of supplier 2 can be obtained by following the same procedure. To prepare for the proof, we need to change the constraint in proposition 4.2 to be based on w_1 instead of ρ. Notice that the expression of $\bar{\rho}$ will change when the relation between w_1 and w_2 changes. Hence, for better analyzing, we discuss the following two cases: (case i) $w_1 \leq w_2$ and (case ii) $w_1 > w_2$.

(Case i)

According to proposition 4.2, the function of order quantity from supplier 1 can be rewritten as follows:

$$q_1^* = \begin{cases} q_1^{(d)}, & \text{if } w_1 \geq t^{(1)}; \\ q_1^{(s)}, & \text{if } w_1 \leq t^{(1)}, \end{cases} \tag{A.33}$$

where $q_1^{(d)} = \frac{[(w_1-\mu\bar{s})\bar{s}(\mu^2+\sigma^2)+(\mu\bar{s}-w_2)(Z+\bar{s})(\mu^2+\rho\sigma^2)]}{2\{[(Z+\bar{s})(\mu^2+\rho\sigma^2)]^2-\bar{s}^2(\mu^2+\sigma^2)^2\}}$, $q_1^{(s)} = \frac{(\mu\bar{s}-w_1)}{2\bar{s}(\mu^2+\sigma^2)}$, and

$$t^{(1)} = \mu\bar{s} - \frac{(\mu\bar{s}-w_2)\bar{s}(\mu^2+\sigma^2)}{(Z+\bar{s})(\mu^2+\rho\sigma^2)}. \tag{A.34}$$

Based on (A.33), we discuss two sub-cases: (case i-1) $w_1 \geq t^{(1)}$ and (case i-2) $w_1 \leq t^{(1)}$, respectively.

(Case i-1)

We can show the supplier 1's pricing problem is as follows:

$$\max_{w_1} \Pi_1 = (w_1 - c)q_1^{(d)} \tag{A.35}$$
$$\text{s.t. } w_1 \leq w_2,$$
$$w_1 \geq c,$$
$$w_1 \geq t^{(1)}.$$

Because there are three inequalities, we want to find whether the feasible region exists, and if it exists, what it is. Apparently, $w_2 \geq c$. In addition, we can show $t^{(1)} \leq w_2$ only when $\rho \leq \hat{\rho}$. This means that there is no feasible solution when $\rho > \hat{\rho}$. Hence, we conclude that there is no feasible solution when $\rho > \hat{\rho}$ and the feasible region exists only when $\rho \leq \hat{\rho}$.

Now, we focus on the case with existing feasible solutions. Then we need to compare $t^{(1)}$ with c. We can show $t^{(1)} \geq c$ when $w_2 \geq w^{(1)}$, where

$$w^{(1)} = \mu\bar{s} - \frac{(\mu\bar{s}-c)(Z+\bar{s})(\mu^2+\rho\sigma^2)}{\bar{s}(\mu^2+\sigma^2)}. \tag{A.36}$$

Next, we discuss these two scenarios: (scenario i) $t^{(1)} \geq c$ and (scenario ii) $t^{(1)} < c$.

(Scenario i)

In this scenario, we obtain the feasible region is that $t^{(1)} \leq w_1 \leq w_2$. The problem (A.35) can be rewritten as:

$$\max_{w_1} \Pi_1 = (w_1 - c)q_1^{(d)} \tag{A.37}$$
$$\text{s.t. } t^{(1)} \leq w_1 \leq w_2.$$

We can show the profit function is concave. According to FOC, we can derive the unconstrained optimizer $w_1^{(i)}$, where

$$w_1^{(i)} = \frac{\mu\bar{s}+c}{2} - \frac{(\mu\bar{s}-w_2)(Z+\bar{s})(\mu^2+\rho\sigma^2)}{2\bar{s}(\mu^2+\sigma^2)}. \tag{A.38}$$

Next, we need to verify whether $w_1^{(i)}$ is feasible. If it is feasible, then it is the optimal feasible solution. Because of the convexity, if $w_1^{(i)}$ is larger than the upper bound, then the upper bound w_2 is the optimal feasible solution. On the other hand, when $w_1^{(i)}$ is less than the lower bound, then the lower bound $t^{(1)}$ is optimal feasible.

We can show $w_1^{(i)} \leq w_2$ only when $w_2 \geq w^{(2)}$,where

$$w^{(2)} = \frac{\bar{s}(\mu\bar{s} + c)(\mu^2 + \sigma^2) - \mu\bar{s}(Z + \bar{s})(\mu^2 + \rho\sigma^2)}{2\bar{s}(\mu^2 + \sigma^2) - (Z + \bar{s})(\mu^2 + \rho\sigma^2)}. \tag{A.39}$$

In addition, if $w_2 \leq w^{(3)}$, then $w_1^{(i)} \geq t^{(1)}$, where

$$w^{(3)} = \bar{s} - \frac{(\bar{s} - c)\bar{s}(\mu^2 + \sigma^2)(Z + \bar{s})(\mu^2 + \rho\sigma^2)}{2[(Z + \bar{s})(\mu^2 + \rho\sigma^2)]^2 - [\bar{s}(\mu^2 + \sigma^2)]^2}. \tag{A.40}$$

We can show $w^{(2)} \leq w^{(1)} \leq w^{(3)}$(See the Appendix for the proof), where $w^{(1)}$ is from (A.36). Notice that we only consider $w_2 \geq w^{(1)}$ because it is the scenario condition. Hence, if $w_2 \leq w^{(3)}$, the solution to the problem (A.37) is $w_1^{(i)}$; otherwise, the solution is the lower bound of the feasible set, $t^{(1)}$.; i.e. the best response function is

$$w_1^*(w_2) = \begin{cases} w_1^{(i)}, & w^{(1)} \leq w_2 \leq w^{(3)}, \\ t^{(1)}, & w_2 \geq w^{(3)}. \end{cases} \tag{A.41}$$

(Scenario ii)

In this scenario, we obtain the feasible set is that $c \leq w_1 \leq w_2$. The problem (A.35) can be rewritten as follows:

$$\begin{aligned} &\max_{w_1} \Pi_1 = (w_1 - c)q_1^{(d)} \\ &\text{s.t. } c \leq w_1 \leq w_2. \end{aligned} \tag{A.42}$$

In this problem, the unconstrained optimizer is still w_1^i. The procedure is the same as (scenario i). We need to verify whether $w_1^{(i)}$ is feasible. If it is feasible, then it is the optimal feasible solution. Because of the convexity, if $w_1^{(i)}$ is larger than the upper bound, then the upper bound w_2 is the optimal feasible solution. On the other hand, when $w_1^{(i)}$ is less than the lower bound, then the lower bound c is optimal.

We can show $w_1^i \geq c$ when $w_2 \geq w^{(4)}$, where

$$w^{(4)} = \frac{\bar{s}\left[c\left(\mu^2+\sigma^2\right)+Z\mu\left(\mu^2+\rho\sigma^2\right)+\mu\bar{s}(\rho-1)\sigma^2\right]}{\left(Z+\bar{s}\right)\left(\mu^2+\rho\sigma^2\right)} \tag{A.43}$$

In addition, we can show $w_1^{(i)} \leq w_2$ when $w_2 \geq w^{(2)}$. We can show $w^{(4)} \leq c \leq w^{(2)}$(see the Appendix for the proof). Hence, if $w_2 \geq w^{(2)}$, the solution to the problem (A.37) is $w_1^{(i)}$; otherwise, the solution is the upper bound of the feasible set, w_2.; i.e. the best response function is

$$w_1^*(w_2) = \begin{cases} w_2, & c \leq w_2 \leq w^{(2)}, \\ w_1^{(i)}, & w^{(2)} \leq w_2 \leq w^{(1)}. \end{cases} \tag{A.44}$$

Combining all conditions in (case i-1), we can conclude that if $\rho > \hat{\rho}$, there do not exist feasible solution; otherwise, the best response function is

$$w_1^*(w_2) = \begin{cases} w_2, & c \leq w_2 \leq w^{(2)}, \\ w_1^{(i)}, & w^{(2)} \leq w_2 \leq w^{(3)}, \\ t^{(1)}, & w_2 \geq w^{(3)}. \end{cases} \tag{A.45}$$

Case i-2

Recall the case condition $w_1 \leq t^{(1)}$. Because the OEM takes mixed strategy in this case when $w_1 = w_2$, we show that the supplier 1 expected sales is

$$\mathbb{E}[q_1] = \begin{cases} q_1^{(s)}, & w_1 < w_2, \\ \frac{q_1^{(s)}}{2}, & w_1 = w_2. \end{cases}$$

In this case, we can show the supplier 1's pricing problem is as follows:

$$\begin{aligned} \max_{w_1} \ & \mathbb{E}[\Pi_1] = (w_1 - c)\mathbb{E}[q_1] \\ \text{s.t. } & w_1 \leq w_2, \\ & w_1 \geq c, \\ & w_1 \leq t^{(1)}. \end{aligned} \tag{A.46}$$

Keeping the same procedure in (case i-1), we want to find whether the feasible region exists, and if it exists, what it is. Apparently, $w_2 \geq c$. In addition, we can show $t^{(1)} < w_2$ only when $\rho < \hat{\rho}$. Then we need to compare $t^{(1)}$ with c. We can show $t^{(1)} \geq c$ when $w_2 \geq w^{(1)}$. Hence, we discuss these three scenarios: (scenario

i) $c \leq t^{(1)} < w_2$, (scenario ii) $t^{(1)} < c$ and $t^{(1)} < w_2$, (scenario iii) $t^{(1)} \geq w_2$ and $t^{(1)} \geq c$, and (scenario iv) $t^{(1)} \geq w_2$ and $t^{(1)} < c$.

(Scenario i)

Because $\mathbb{E}[q_1]$ is not continuous, we divide the problem into two parts, $w_1 < w_2$ and $w_1 = w_2$. Notice that $w_1 < w_2$ always holds in this scenario. We rewrite the condition $c \leq t^{(1)} < w_2$ as $w_2 \geq w^{(1)}$ and $\rho < \hat{\rho}$. In this scenario, we obtain the feasible set is that $c \leq w_1 \leq t^{(1)}$. The problem (A.46) can be rewritten as follows:

$$\begin{aligned} &\max_{w_1} \Pi_1 = (w_1 - c)q_1^{(s)} \\ &\text{s.t. } c \leq w_1 \leq t^{(1)}. \end{aligned} \tag{A.47}$$

We can show the profit function is concave. According to FOC, we can derive the unconstrained optimizer $w^{(i\prime)}$, where

$$w^{(i\prime)} = \frac{c + \mu\bar{s}}{2}. \tag{A.48}$$

Next, we need to verify whether $w^{(i\prime)}$ is feasible. If it is feasible, then it is the optimal feasible solution. Because of the convexity, if $w^{(i\prime)}$ is larger than the upper bound, then the upper bound $t^{(1)}$ is the optimal feasible solution. On the other hand, when $w^{(i\prime)}$ is less than the lower bound, then the lower bound c is optimal feasible.

From the assumption $\mu\bar{s} > c$, we can show $w^{(i\prime)} \geq c$ always hold. And we can show $w^{(i\prime)} \leq t^{(1)}$ when $w_2 \geq w^{(5)}$, where

$$w^{(5)} = \frac{\mu\bar{s}^2(\mu^2 + \sigma^2) + (c - \mu\bar{s})(Z + \bar{s})(\mu^2 + \rho\sigma^2)}{2\bar{s}\,(\mu^2 + \sigma^2)} \tag{A.49}$$

We can show $w^{(5)} \geq w^{(1)}$ (See Appendix for the proof). Hence, if $w_2 \geq w^{(5)}$, the solution to the problem (A.47) is $w^{(i\prime)}$; otherwise, the solution is the upper bound of the feasible set, $t^{(1)}$.; i.e. the best response function is

$$w_1^*(w_2) = \begin{cases} t^{(1)}, & w^{(1)} \leq w_2 \leq w^{(5)}, \\ w^{(i\prime)}, & w_2 \geq w^{(5)}. \end{cases} \tag{A.50}$$

(Scenario ii)

If $t^{(1)} < c$, then the feasible set of the problem is empty. Hence, in this scenario, there do not exist any solution.

(Scenario iii)

Because $\mathbb{E}[q_1]$ is not continuous, we divide the problem into two parts, $w_1 < w_2$ and $w_1 = w_2$. We can show $w_1 = w_2$ is dominated by $w_1 < w_2$. Hence, we only consider the case in which $w_1 < w_2$. We rewrite the condition $c \leq t^{(1)}$ and $t^{(1)} \geq w_2$ as $w_2 \geq w^{(1)}$ and $\rho \geq \hat{\rho}$, respectively.

In this scenario, we obtain the feasible set is that $c \leq w_1 < w_2$. The problem (A.46) can be rewritten as follows:

$$\max_{w_1} \Pi_1 = (w_1 - c)q_1^{(s)} \tag{A.51}$$
$$\text{s.t. } c \leq w_1 < w_2.$$

We can show the profit function is concave. In this problem, the unconstrained optimizer is still $w_1^{i\prime}$. The procedure is the same as (scenario i). Next, we need to verify whether $w^{(i\prime)}$ is feasible. If it is feasible, then it is the optimal feasible solution. Because of the convexity, if $w^{(i\prime)}$ is larger than the upper bound, then the upper bound w_2 is the optimal feasible solution. On the other hand, when $w^{(i\prime)}$ is less than the lower bound, then the lower bound c is optimal feasible.

From the assumption $\mu\bar{s} > c$, we can show $w^{(i\prime)} \geq c$ always hold. Notice that $w^{(i\prime)}$ is a constant. We can show $w^{(i\prime)} \geq c \geq w^{(1)}$ when $\rho \geq \hat{\rho}$ (See the Appendix for the proof).

Hence, if $w_2 \geq w^{(i\prime)}$, the solution to the problem (A.47) is $w^{(i\prime)}$; otherwise, the solution is the upper bound of the feasible set, w_2.; i.e. the best response function is

$$w_1^*(w_2) = \begin{cases} \max\{w_2 - \varepsilon, c\}, & c \leq w_2 \leq w^{(i\prime)}, \\ w^{(i\prime)}, & w_2 > w^{(i\prime)}, \end{cases} \tag{A.52}$$

where ε is a minor positive real number.

(Scenario iv)

In this scenario, the feasible set of the problem is empty.

Combining all conditions in (case i-1), we can conclude that if $\rho > \hat{\rho}$, the best response function is

$$w_1^*(w_2) = \begin{cases} \max\{w_2 - \varepsilon, c\}, & c \leq w_2 \leq w^{(i\prime)}, \\ w^{(i\prime)}, & w_2 > w^{(i\prime)}; \end{cases} \tag{A.53}$$

otherwise, the best response function is

$$w_1^*(w_2) = \begin{cases} \emptyset, & c \le w_2 < w^{(1)}, \\ t^{(1)}, & w^{(1)} \le w_2 \le w^{(5)}, \\ w^{(i\prime)}, & w_2 \ge w^{(5)}. \end{cases} \tag{A.54}$$

Case ii

According to proposition 4.2, the function of order quality to supplier 1 can be rewritten as follows:

$$q_1^* = \begin{cases} q_1^{(d)}, & \text{if } \ w_1 \le t^{(2)}; \\ 0, & \text{if } \ w_1 \ge t^{(2)}, \end{cases} \tag{A.55}$$

where $t^{(2)} = \mu\bar{s} - \frac{(\mu\bar{s}-w_2)(Z+\bar{s})(\mu^2+\rho\sigma^2)}{\bar{s}(\mu^2+\sigma^2)}$. Based on (A.55), we discuss two sub-cases: (case ii-1) $w_1 \le t^{(2)}$, and (case ii-2) $w_1 \ge t^{(2)}$, respectively.

Case ii-1

We can show the supplier 1's pricing problem is as follows:

$$\begin{aligned} \max_{w_1} \ & \Pi_1 = (w_1 - c)q_1^{(d)} \\ \text{s.t. } & w_1 \ge w_2, \\ & w_1 \le t^{(2)}. \end{aligned} \tag{A.56}$$

Obviously, if $t^{(2)} \ge w_2$, the feasible set is not empty. We can show that if $\rho \le \hat{\rho}$, then $t^{(2)} \ge w_2$ holds; otherwise, the feasible set is empty. Now, we focus on the case with existing feasible solutions.

We can show the profit function is concave. According to FOC, we can still obtain the unconstrained optimizer w_1^i. The procedure is the same as before cases. We need to verify whether $w_1^{(i)}$ is feasible. If it is feasible, then it is the optimal feasible solution. Because of the convexity, if $w_1^{(i)}$ is larger than the upper bound, then the upper bound $t^{(2)}$ is the optimal feasible solution. On the other hand, when $w_1^{(i)}$ is less than the lower bound, then the lower bound w_2 is optimal.

We can show $w_1^i \le t^{(2)}$ if $w_2 \ge w^{(4)}$. In addition, if $w_2 \le w^{(2)}$, then we have $w_1^{(i)} \ge w_2$. In this case, we can show $w^{(4)} \le c \le w^{(2)}$.

Hence, if $w_2 \le w^{(2)}$, the solution to the problem (A.47) is $w_1^{(i}$; otherwise, the solution is the lower bound of the feasible set, w_2.; i.e. the best response function is

$$w_1^*(w_2) = \begin{cases} w_1^{(i)}, & c \le w_2 \le w^{(2)}, \\ w_2, & w_2 > w^{(2)}. \end{cases} \tag{A.57}$$

Case ii-2

We can show the supplier 1's pricing problem is as follows:

$$\begin{aligned} \max_{w_1} \Pi_1 &= 0 \\ \text{s.t. } w_1 &\ge w_2, \\ w_1 &\ge t^{(2)}. \end{aligned} \tag{A.58}$$

We want to find the feasible set. Furthermore, we have known $t^{(2)} \ge w_2$ can be rewritten as $\rho < \hat{\rho}$. Hence, if $\rho < \hat{\rho}$, the feasible set is $w_1 \ge t^{(2)}$; otherwise, it is $w_1 \ge w_2$. Without of loss generality, let the lower bound as the optimal solution. Hence, if $\rho < \hat{\rho}$, the best response function is $w_1 = t^{(2)}$, otherwise, it is $w_1 = w_2$.

According to all results in (case i) and (case ii), we find the best response function depends on ρ. That means that we have one best response function when $\rho \le \hat{\rho}$ and another one when $\rho > \hat{\rho}$. Hence, we will discuss the case when $\rho \le \hat{\rho}$ at first. According to (A.45), (A.54), and (A.57), we can see there are five different intervals for w_2:$[c, w^{(2)}]$, $[w^{(2)}, w^{(1)}]$, $[w^{(1)}, w^{(3)}]$, $[w^{(3)}, w^{(5)}]$, and $[w^{(5)}, \mu\bar{s})$. Notice that we can show $c \le w^{(2)} \le w^{(1)} \le w^{(3)} \le w^{(5)} < \mu\bar{s}$ in this case.(see Appendix for proof). For the 1st interval $w_2 \in [c, w^{(2)}]$, we derive the best response function from (A.45), (A.54), and (A.57):

$$w_1^*(w_2) = \begin{cases} \emptyset, & w_1 \le t^{(1)}, \\ w_2, & t^{(1)} < w_1 < w_2, \\ w_1^{(i)}, & w_2 \le w_1 \le t^{(2)}, \\ t^{(2)}, & w_1 > t^{(2)}. \end{cases} \tag{A.59}$$

When $t^{(1)} < w_1 < w_2$ and $w_1 > t^{(2)}$, we can show corresponding optimal solutions are its upper bound and lower bound, respectively. Hence, we only need to consider the range between $t^{(1)} < w_1 < w_2$ and $w_1 > t^{(2)}$. In the range $w_2 \le w_1 \le t^{(2)}$, the best response is $w_1^{(i)}$. The best response function for $w_2 \in [c, w^{(2)}]$ is as follows:

$$w_1^*(w_2) = w_1^{(i)}. \tag{A.60}$$

For the 2nd interval $w_2 \in [w^{(2)}, w^{(1)}]$, we derive the best response function from (A.45), (A.54), and (A.57):

$$w_1^*(w_2) = \begin{cases} \emptyset, & w_1 \leq t^{(1)}, \\ w_1^{(i)}, & t^{(1)} < w_1 < w_2, \\ w_2, & w_2 \leq w_1 \leq t^{(2)}, \\ t^{(2)}, & w_1 > t^{(2)}. \end{cases} \tag{A.61}$$

When $w_2 < w_1 \leq t^{(2)}$ and $w_1 > t^{(2)}$, we can show corresponding optimal solutions both are lower bound. Hence, we only need to consider the range $t^{(1)} < w_1 < w_2$. In the range, the best response is $w_1^{(i)}$. The best response function for $w_2 \in [w^{(2)}, w^{(1)}]$ is the same as (A.60).

For the 3rd interval $w_2 \in [w^1, w^{(3)}]$, we derive the best response function from (A.45), (A.54), and (A.57):

$$w_1^*(w_2) = \begin{cases} t^{(1)}, & w_1 \leq t^{(1)}, \\ w_1^{(i)}, & t^{(1)} < w_1 < w_2, \\ w_2, & w_2 \leq w_1 \leq t^{(2)}, \\ t^{(2)}, & w_1 > t^{(2)}. \end{cases} \tag{A.62}$$

When $w_1 \leq t^{(1)}$, $w_2 \leq w_1 \leq t^{(2)}$, and $w_1 > t^{(2)}$, we can show corresponding optimal solutions are its upper bound, lower bound, and lower bound, respectively. Hence, we only need to consider the range between them. In the range $t^{(1)} < w_1 < w_2$, the best response is $w_1^{(i)}$. The best response function for $w_2 \in [w^{(1)}, w^{(3)}]$ is the same as (A.60).

For the 4th interval $w_2 \in [w^{(3)}, w^{(5)}]$, we derive the best response function from (A.45), (A.54), and (A.57):

$$w_1^*(w_2) = \begin{cases} t^{(1)}, & w_1 \leq t^{(1)}, \\ t^{(1)}, & t^{(1)} < w_1 < w_2, \\ w_2, & w_2 \leq w_1 \leq t^{(2)}, \\ t^{(2)}, & w_1 > t^{(2)}. \end{cases} \tag{A.63}$$

When $w_2 \leq w_1 \leq t^{(2)}$, and $w_1 > t^{(2)}$, we can show corresponding optimal solutions both are lower bounds. Hence, we only need to consider $c \leq w_1 < w_2$. In the range, the best response is $t^{(1)}$. The best response function for $w_2 \in [w^{(3)}, w^{(5)}]$ is as follows:

$$w_1^*(w_2) = t^{(1)}. \tag{A.64}$$

For the 5th interval $w_2 \in [w^{(5)}, \mu\bar{s})$, we derive the best response function from (A.45), (A.54), and (A.57):

$$w_1^*(w_2) = \begin{cases} w^{(i\prime)}, & w_1 \leq t^{(1)}, \\ t^{(1)}, & t^{(1)} < w_1 < w_2, \\ w_2, & w_2 \leq w_1 \leq t^{(2)}, \\ t^{(2)}, & w_1 > t^{(2)}. \end{cases} \tag{A.65}$$

When $t^{(1)} < w_1 < w_2$, $w_2 \leq w_1 \leq t^{(2)}$, and $w_1 > t^{(2)}$, we can show corresponding optimal solutions both are lower bounds. Hence, we only need to consider $c \leq w_1 \leq t^{(1)}$. In the range, the best response is $w^{(i\prime)}$. The best response function for $w_2 \in [w^{(5)}, \mu\bar{s})$ is as follows:

$$w_1^*(w_2) = w^{(i\prime)}. \tag{A.66}$$

By combining all the scenarios above and the best response functions in (A.60), (A.64), and (A.66), we can obtain the best response function for supplier 1 as follows:

$$w_1^*(w_2) = \begin{cases} w_1^{(i)}, & w_2 \leq w^{(3)}, \\ t^{(1)}, & w^{(3)} < w_2 \leq w^{(5)}, \\ w^{(i\prime)}, & w_2 > w^{(5)}. \end{cases} \tag{A.67}$$

See (A.38), (A.34), and (A.48) for the expressions of $w_1^{(i)}$, $t^{(1)}$, and $w^{(i\prime)}$.

Then we focus on the best response function when $\rho > \hat{\rho}$. By combining (A.53) and (case ii-2), we can see there are two different intervals for w_2: $[c, w^{(i\prime)}]$ and $[w^{(i\prime)}, \mu\bar{s})$. For the 1st interval $w_2 \in [c, w^{(i\prime)}]$, we derive the best response function from (A.53) and results in (case i-1) and (case ii):

$$w_1^*(w_2) = \begin{cases} \max\{w_2 - \varepsilon, c\}, & c \leq w_1 \leq w_2, \\ w_2, & w_1 > w_2. \end{cases} \tag{A.68}$$

From (A.68), we can show the best response function for supplier 1 is as follows:

$$w_1^*(w_2) = \max\{w_2 - \varepsilon, c\}. \tag{A.69}$$

For the 2nd interval $w_2 \in [w^{(i\prime)}, \mu\bar{s})$, we derive the best response function from (A.53) and results in (case i-1) and (case ii):

$$w_1^*(w_2) = \begin{cases} w^{(i\prime)}, & c \leq w_1 \leq w_2, \\ w_2, & w_1 > w_2. \end{cases} \tag{A.70}$$

When $w_1 \geq w_2$, we can show the corresponding optimal solution is the lower bound. Hence, we only need to consider $c \leq w_1 \leq w_2$. From (A.68), we can show the best response function for supplier 1 is as follows:

$$w_1^*(w_2) = w^{(i\prime)}. \tag{A.71}$$

By combining the best response functions in (A.69) and (A.71), we can obtain the final best response function for supplier 1 is as follows:

$$w_1^*(w_2) = \begin{cases} \max\{w_2 - \varepsilon, c\}, & c \leq w_2 < w^{(i\prime)}, \\ w^{(i\prime)}, & w_2 \geq w^{(i\prime)}. \end{cases} \tag{A.72}$$

Finally, by combining (A.67) and (A.72), we conclude the supplier 1's best response function is as follows: if $\rho < \hat{\rho}$, we have

$$w_1^*(w_2) = \begin{cases} w_1^{(i)}, & w_2 \leq w^{(3)}, \\ t^{(1)}, & w^{(3)} < w_2 \leq w^{(5)}, \\ w^{(i\prime)}, & w_2 > w^{(5)}; \end{cases} \tag{A.73}$$

otherwise, we have

$$w_1^*(w_2) = \begin{cases} \max\{w_2 - \varepsilon, c\}, & c \leq w_2 < w^{(i\prime)}, \\ w^{(i\prime)}, & w_2 \geq w^{(i\prime)}. \end{cases} \tag{A.74}$$

The best response function for supplier 2 can be obtained by following the same procedure: if $\rho < \hat{\rho}$, we have

$$w_2^*(w_1) = \begin{cases} \frac{\mu\bar{s}+c}{2} - \frac{(\mu\bar{s}-w_1)(Z+\bar{s})(\mu^2+\rho\sigma^2)}{2\bar{s}(\mu^2+\sigma^2)}, & w_1 \leq w^{(3)}, \\ \mu\bar{s} - \frac{(\mu\bar{s}-w_1)\bar{s}(\mu^2+\sigma^2)}{(Z+\bar{s})(\mu^2+\rho\sigma^2)}., & w^{(3)} < w_1 \leq w^{(5)}, \\ w^{(i\prime)}, & w_1 > w^{(5)}; \end{cases} \tag{A.75}$$

otherwise, we have

$$w_2^*(w_1) = \begin{cases} \max\{w_2 - \varepsilon, c\}, & c \leq w_2 < w^{(i\prime)}, \\ w^{(i\prime)}, & w_2 \geq w^{(i\prime)}. \end{cases} \tag{A.76}$$

By solving (A.73) and (A.75) simultaneously, we can show the equilibrium wholesale price when $\rho < \hat{\rho}$,

$$w_1^* = w_2^* = \frac{\bar{s}[\mu\bar{s}(1-\rho)\sigma^2 + c(\mu^2+\sigma^2) - Z\mu(\mu^2+\sigma^2)]}{\bar{s}[\mu^2 + (2-\rho)\sigma^2 - KM(\mu^2+\rho\sigma^2)]}.$$

In addition, by solving (A.74) and (A.76) simultaneously, we can show the equilibrium wholesale price when $\rho \geq \hat{\rho}$,

$$w_1^* = w_2^* = c.$$

In conclusion, we show the equilibrium wholesale price is as follows:

$$w_1^* = w_2^* = \begin{cases} \frac{\bar{s}[\mu\bar{s}(1-\rho)\sigma^2 + c(\mu^2+\sigma^2) - Z\mu(\mu^2+\sigma^2)]}{\bar{s}[\mu^2+(2-\rho)\sigma^2 - KM(\mu^2+\rho\sigma^2)]}, & \text{if } \rho < \hat{\rho}, \\ c, & \text{if } \rho \geq \hat{\rho} \end{cases}$$

By changing terms, we can rewrite $\rho < \hat{\rho}$ as $Z < \hat{Z}$.

Additional proof of Proposition 4.3

When $\rho \leq \hat{\rho}$

We can show $w^{(4)} \leq c \leq w^{(2)} \leq w^{(1)} \leq w^{(3)} \leq w^{(5)} < \mu\bar{s}$.

We want to show $w^{(4)} \leq c$. Let

$$h_{4c}(c) = w^{(4)} - c. \tag{A.77}$$

We can show $h_{4c}(c) = 0$ if and only if $\mu\bar{s} = c$. And the slope of $\frac{\bar{s}(\mu^2+\sigma^2)}{(Z+\bar{s})(\mu^2+\rho\sigma^2)} - 1$, which is larger than 0. From the assumption $\mu\bar{s} > c$, we find that $h_{4c} < 0$ always holds. Hence, we have $w^{(4)} \leq c$.

We want to show $w^{(2)} \geq c$. Let

$$h_{2c}(c) = w^{(2)} - c. \tag{A.78}$$

We can show $h_{2c}(c) = 0$ if and only if $\mu\bar{s} = c$. And the slope of $h_{2c}(c)$ is $\frac{\bar{s}(\mu^2+\sigma^2)}{2\bar{s}(\mu^2+\sigma^2)-(Z+\bar{s})(\mu^2+\rho\sigma^2)} - 1$, which is less than 0. From the assumption $\mu\bar{s} > c$, we find that $h_{2c} > 0$ always holds. Hence, we have $w^{(2)} \geq c$.

We want to show $w^{(2)} \leq w^{(1)}$. Let

$$h_{21}(c) = w^{(2)} - w^{(1)}. \tag{A.79}$$

We can show $h_{21}(c) = 0$ if and only if $\mu\bar{s} = c$. From the assumption $\mu\bar{s} > c$, we find that $h_{21} < 0$ always holds. Hence, we have $w^{(2)} \leq w^{(1)}$.

We want to show $w^{(1)} \leq w^{(3)}$. Let

$$h_{13}(c) = w^{(1)} - w^{(3)}. \tag{A.80}$$

We can show $h_{13}(c) = 0$ if and only if $\mu\bar{s} = c$. And $\frac{dh_{15}}{dc} = \frac{\left(\mu^2+\rho\sigma^2\right)\left(\frac{\left(\mu^2+\sigma^2\right)^2}{\left(\mu^2+\rho\sigma^2\right)^2-2\left(\mu^2+\sigma^2\right)^2}+1\right)}{\mu^2+\sigma^2} > 0$. From the assumption $\mu\bar{s} > c$, we find that $h_{13} < 0$ always holds. Hence, we have $w^{(1)} \leq w^{(3)}$.

We want to show $w^{(3)} \leq w^{(5)}$ Let

$$h_{53}(c) = w^{(5)} - w^{(3)}. \tag{A.81}$$

We can show $h_{53}(c) = 0$ if and only if $\mu\bar{s} = c$. And $\frac{dh_{53}}{dc}$ is less than 0. From the assumption $\mu\bar{s} > c$, we find that $h_{53} < 0$ always holds. Hence, we have $w^{(3)} \leq w^{(5)}$.

We want to show $w^{(5)} \leq \mu\bar{s}$. Let

$$h_{5\mu\bar{s}}(c) = w^{(5)} - \mu\bar{s}. \tag{A.82}$$

We can show $h_{5\mu\bar{s}}(c) = 0$ if and only if $\mu\bar{s} = c$. And $\frac{dh_{5\mu\bar{s}}}{dc} = \frac{\mu^2+\rho\sigma^2}{2(\mu^2+\sigma^2)}$ is larger than 0. From the assumption $\mu\bar{s} > c$, we find that $h_{3\mu\bar{s}} < 0$ always holds. Hence, we have $w^{(5)} \leq \mu\bar{s}$.

A.4 Proof of Corollary 4.1

Using Proposition 4.2 and 4.3, we can easily obtain this corollary.

A.5 Proof of Proposition 4.4

We can easily obtain this corollary using Proposition 4.3 and Corollary 4.1.

A.6 Proof of Proposition 4.5

For part (1):

We can rewrite $\rho \geq \hat{\rho}$ as $\alpha \geq \overline{\alpha}$, where $\overline{\alpha} = \frac{\bar{s}(1-\rho)\sigma^2}{\Delta\beta(1-\beta)(\mu^2+\rho\sigma^2)}$. If $\alpha \geq \overline{\alpha}$, we can show $q^* = \frac{\mu\bar{s}-c}{2Z}$ that does not depend on α. Then, we consider the case which $\alpha < \overline{\alpha}$. In this case, $q^* = \frac{Z(\mu\bar{s}-c)}{(2Z-A)(A+Z)}$. We can show that:

$$\frac{dq^*}{d\alpha} = -\frac{(\beta-1)\beta\Delta\bar{s}\left(\mu^2+\sigma^2\right)(\mu\bar{s}-c)\left(\mu^2+\rho\sigma^2\right)}{\left(-\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)+2\mu^2\bar{s}+(\rho+1)\bar{s}\sigma^2\right)^2} \cdot \frac{\left(\bar{s}\left(\mu^2+(2\rho-1)\sigma^2\right)-2\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)\right)}{\left(\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)+\bar{s}\left(\mu^2-(\rho-2)\sigma^2\right)\right)^2}.$$

According to the equation, we can show $\frac{dq^*}{d\alpha} \leq 0$ if and only if $\alpha \leq \underline{\alpha}$, where $\underline{\alpha} = \frac{\bar{s}[2(\mu^2+\rho\sigma^2)-(\mu^2+\sigma^2)]}{2\Delta\beta(1-\beta)(\mu^2+\rho\sigma^2)}$.

For part (2):

We can show the optimal wholesale price $w^* = c$ if $\alpha \geq \overline{\alpha}$. When $\alpha < \overline{\alpha}$, we can show

$$\frac{dw^*}{d\alpha} = -\frac{(1-\beta)\beta\Delta\bar{s}\left(\mu^2+\sigma^2\right)\left(\mu\bar{s}-c\right)\left(\mu^2+\rho\sigma^2\right)}{\left(\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)+\bar{s}\left(\mu^2-(\rho-2)\sigma^2\right)\right)^2} \leq 0$$

For part (3): When $\alpha < \overline{\alpha}$, we can show

$$\begin{aligned}\frac{d\Pi_M^d}{d\alpha} &= \frac{3(1-\beta)\beta\Delta\bar{s}^2\left(\mu^2+\sigma^2\right)^2(c-\mu\bar{s})^2\left(\mu^2+\rho\sigma^2\right)^2\left(\bar{s}+\alpha(1-\beta)\beta\Delta\right)}{2\left[-\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)+2\mu^2\bar{s}+(\rho+1)\bar{s}\sigma^2\right]^2} \\ &\cdot \frac{1}{\left[\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)+\bar{s}\left(\mu^2-(\rho-2)\sigma^2\right)\right]^3} > 0.\end{aligned}$$

For part (4): When $\alpha < \hat{\alpha}$, we can show

$$\begin{aligned}\frac{d\Pi_1^d}{d\alpha} &= \frac{-(1-\beta)\beta\Delta s\left(\mu^2+\sigma^2\right)(c-\mu s)^2\left(\mu^2+\rho\sigma^2\right)}{\left(-\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)+2\mu^2 s+(\rho+1)s\sigma^2\right)^2} \\ &\cdot \left\{\frac{\alpha^2(\beta-1)^2\beta^2\Delta^2\left(\mu^2+\rho\sigma^2\right)^2+s^2\left(\mu^2(\rho+1)\sigma^2+\mu^4+\left(\rho^2-\rho+1\right)\sigma^4\right)}{\left(\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)+s\left(\mu^2-(\rho-2)\sigma^2\right)\right)^3}\right. \\ &\left.+\frac{\alpha(1-\beta)\beta\Delta s\left(\mu^2(3\rho-1)\sigma^2+\mu^4+\rho(2\rho-1)\sigma^4\right)}{\left(\alpha(\beta-1)\beta\Delta\left(\mu^2+\rho\sigma^2\right)+s\left(\mu^2-(\rho-2)\sigma^2\right)\right)^3}\right\} < 0.\end{aligned}$$

A.7 Proof of Proposition 4.6

Based on Proposition 4.1 and 4.4, we can obtain this proposition.

A.8 Proof of Proposition 4.7

Recall the ex-ante consumer surplus is

$$(q^{d*})^2\left[\frac{\beta^2 s_H+(1-\beta^2)s_L}{2}+\frac{2\beta(1-\beta)(s_H+s_L)}{3-\rho}\right].$$

Because the consumer surplus is affect by α only from the term q^{d*} and q^{d*} are non-negative, the monotonicity is the same as q^{d*}.

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