FASHION KNOCKOFFS AND COUNTERFEITING PROBLEMS IN THE FASHION INDUSTRY

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CERTIFICATE OF ORIGINALITY

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Abstract

Due to rapid development of efficient global logistics and thriving e-business operations, the phenomena of counterfeiting and copycat are getting more and more popular recently, especially in the fashion industry. Fashion knockoffs, which refer to the copycat behaviors of some brands in fashion apparel, are widely seen. It is commonly believed that the presence of fashion knockoffs harms the original-designer-label (ODL) product seller. However, real world observations show that some ODLs seem to be fine with fashion knockoffs and they co-exist. Moreover, facing the potential threats of fashion knockoffs, ODL product sellers tend to be risk sensitive (i.e., risk averse or risk seeking) rather than risk neutral. Although prior studies have explored the impacts of fashion knockoffs on the ODL product seller, they did not consider the common manufacturer scenario and risk attitude of the ODL product seller. Hence, in this thesis, we aim to fill the research gaps by incorporating these two factors to examine the impacts of fashion knockoffs on the ODL product supply chain and its agents. Fashion knockoffs also play a critical role in the context of consumer-to-consumer product exchange (C2C-PE) and logo design strategy of luxury fashion brands. It is widely considered that the popularity of C2C-PE can reduce customer's demand for new products, which hurts original fashion brand's sales and profit. However, this perspective overlooks the impact of C2C-PE on the knockoff trading, a noticeable challenge the fashion brands is encountering. Taking a fresh perspective, we intend to explore the impacts of C2C-PE on the original product supply chain, its members and the consumers in the presence of fashion knockoffs. Lastly, we observe that different luxury brands use different logo design strategies, e.g., LV usually shows prominent logos in its products, while a blatant logo can hardly be found in a Hermès bag. Real world practices show that these luxury fashion brands (with big logo or no logo) have different vulnerabilities to fashion knockoffs and counterfeits. Hence, we aim to explore the impacts of fashion knockoffs and counterfeits on the logo design strategy of luxury fashion brands.

Motivated by the popularity of fashion knockoffs and its potential impacts on the ODL product supply chain in the context of (i) a common manufacturer; (ii) C2C-PE; and (iii) logo design strategy, we build game theoretical models in this thesis to explore (i) impacts of fashion knockoffs on the ODL product supply chain and its agents considering a common manufacturer and risk attitude of the ODL product seller; (ii) impacts of C2C-PE on the original product supply chain, its members and the consumers in the presence of fashion knockoffs; and (iii) impacts of fashion knockoffs and counterfeits on the logo design strategy of a luxury fashion brand. By addressing the research objectives, we have obtained some important findings:

First, we interestingly find that the presence of fashion knockoffs benefits the ODL product supply chain and its agents when the ODL product seller is risk averse and the ratio of demand uncertainty is relatively small, or the ODL product seller is risk seeking and the ratio of demand uncertainty is sufficiently large. The findings remain robust when we consider a non-standard markup wholesale pricing policy and different manufacturers' scenario. Moreover, we discover that when the ODL product seller is risk sensitive, a common manufacturer scenario is more likely to bring a win-win outcome to the manufacturer and the ODL product seller, which may explain why a common manufacturer producing for both an ODL and a knockoff product seller is widely observed in the fashion industry.

Second, we theoretically find that the presence of C2C-PE benefits the original supply chain, its members and the consumers, while harms the knockoff supply chain, its members and the consumers. The findings are robust when considering: (i) strategic quality decision; (ii) price dependent C2C-PE utility and (iii) consumers' conspicuous behavior. In addition, we discover that in the presence of C2C-PE, members of the original supply chain tend to produce a higher-price and superior-quality product, while members of the knockoff supply chain incline to sell a lower-price knockoff product. The original brand will encroach some of the knockoff brand's demand.

Third, we uncover that when the negative effects differential caused by counterfeits and copycats is relatively small, showing a big logo is more beneficial to the luxury fashion supply chain, its members and the consumers; otherwise, showing no logo is more beneficial. The results remain robust when considering risk attitude of the luxury fashion brand and implementation of the blockchain technology. By comparing the optimal logo design strategies among different cases, we uncover that the big logo strategy is more prone to be optimal when implementing the blockchain technology. Moreover, when the luxury fashion brand is risk sensitive, whether the big logo strategy is more likely to be optimal depends on risk attitude of the luxury fashion brand and consumers' status disparity.

Based on the analytical findings, we provide ODL product sellers with managerial implications regarding (i) strategies towards fashion knockoffs (i.e., whether to allow, ignore or deter the presence of fashion knockoffs); (ii) adoption of the C2C-PE scheme (e.g., how to increase the C2C-PE utility); and (iii) logo design strategy (i.e., whether to use big logo or no logo in luxury fashion products). Finally, future research directions are proposed.

Publications and working papers arising from this thesis

1. **Wang, Y.**, Lin, J., & Choi, T. M. (2020). Gray market and counterfeiting in supply chains: A review of the operations literature and implications to luxury industries. *Transportation Research Part E: Logistics and Transportation Review*, *133*, 101823.
2. **Wang, Y.**, Fan, D., Fung, Y. N., & Luo, S. (2022). Consumer-to-consumer product exchanges for original fashion brands in the sharing economy: Good or bad for fashion knockoffs?. *Transportation Research Part E: Logistics and Transportation Review*, *158*, 102599.
3. **Wang, Y.**, Xu, X., Choi, T. M., & Shen, B. (2022). Will the presence of "fashion knockoffs" benefit the original-designer-label product supply chain? A mean-risk analysis. Under journal review.
4. **Wang, Y.**, Xu, X., Siqin, T., & Choi, T. M. (2022). Show big logos or not in products of luxury fashion supply chains? Impacts of counterfeits and copycats. Under journal review.

Conference presentations

1. **Wang, Y.**, Xu, X., Siqin, T., & Choi, T. M. (2022). Show big logos or not in products of luxury fashion supply chains? Impacts of counterfeits and copycats. *POMS 32nd Annual Conference 2022 in Orlando, FL, USA, April 2022.*
2. **Wang, Y.**, Fan, D., Fung, Y. N., & Luo, S. (2021). Consumer-to-consumer product exchanges for original fashion brands in the sharing economy: Good or bad for fashion knockoffs? *INFORMS Annual Conference 2021 in Anaheim, CA, USA, October 2021.*
3. **Wang, Y.**, Xu, X., Choi, T. M., & Shen, B. (2021). Will the presence of "fashion knockoffs" benefit the original-designer-label product supply chains? A mean-risk analysis. *MSOM Conference 2021 at Indiana University, USA, June 2021.*

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Chapter 1 Introduction

1.1 Background

Due to rapid development of efficient global logistics and thriving e-business operations, the phenomena of counterfeiting and copycat are getting more and more popular recently, especially in the fashion industry. It is reported that the global trade in copied products will reach US$4.2 trillion by 2022[1], with fashion products like clothes and shoes being one of the most replicated items. In 2020 alone, the copied products have caused a loss of over US$50 billion to the fashion industry[1]. Counterfeits are illegal, low-priced and often inferior-quality products that infringe on the intellectual property rights of the authentic brand (Cordell et al. 1996; Wilcox et al. 2009)[2]. A similar concept is copycat, which refers to legal products which imitate the substance, name, shape, form, package design, meaning or intent to a leading brand's product to free ride on the leading brand's positive associations and marketing efforts (Lai and Zaichkowsky 1999; Van Horen and Pieters 2012). The major difference between these two concepts is that counterfeit is illegal while copycat is legal. The copycat business model is so popular in the fashion industry that it even "arguably" has a specific market segment: "fast fashion" (Pun and DeYong 2017). The fast fashion's products which intentionally copy the original designer label (ODL)'s design at a fraction of the cost are called fashion knockoffs (Dahlén 2012; Miller 2017). Most often cited fashion knockoffs include *Zara* and *Forever 21* (Pun and DeYong 2017). Martinez (2015) reports that "*Zara* has been widely criticized for its design model, which seems nothing short of copycat"[3]. Similarly, Frost (2014) describes that "*Forever21* can have a much more wallet-friendly copycat version" of the dresses from *Givenchy*.

From the observed industrial cases, in general, we can see that fashion knockoffs commonly exhibit the following features:

1. *High similarity level.* By definition, fashion knockoffs exhibit a high similarity level to ODL products in terms of "trademark, logo, name, packaging or product design features" (Gao et al. 2017b).

2. *Lower marginal production cost.* Due to the high production standards, better craftmanship and the superior materials used by ODL products, they are usually produced at a higher cost compared with their knockoff counterparts. Taking *Louis Vuitton* (*LV*) as an example, it is reported that each *LV* bag needs to go through more than 100 steps for assembly, and each bag is stitched and edge-glazed

[1] https://www.redpoints.com/blog/fashion-counterfeit-impact/ [Accessed on 16 Feb 2022]
[2] Please refer to Tables 1.1 and 1.2 for the detailed definitions of counterfeit, copycat and fashion knockoff.
[3] https://thecourtroom.org/how-big-fashion-brands-commonly-steal-designs-and-get-away-with-it/ [Accessed on 16 Feb 2022]

by hand in the last stage of production, which adds to the production cost[4]. Moreover, *LV* always selects superior materials (e.g., leather or faux leather) which are expensive[5].

3. *Lower fixed operational cost*. The fixed operational cost for fashion brands usually includes (i) product design and development costs; (ii) advertising and communications costs; (iii) selling costs; and (iv) general and administrative costs. ODL product sellers usually spend substantially in the fixed operational cost. For example, *Prada* spent 409 million euros in "product design and development" and "advertising and communications" in 2021, taking up 12.2% of the net revenues[6]. Similarly, *LVMH* spent 7291 million euros in advertising and promotions in 2021, which accounted 11% of the total revenues[7]. However, most fast fashion brands (which are arguably fashion knockoffs) spend much less in the fixed operational cost, especially for the "product design and development cost" and the "advertisement cost". For instance, *Zara* only spends "0.3%" of sales on advertising, whereas its competitors (e.g., *H&M*) commonly spend "3%-5%" of sales on advertising (Kumar and Steenkamp 2007).

4. Lower selling price. Because of the lower marginal production cost and the fixed operational cost, fashion knockoffs are usually sold at a lower selling price compared with ODL products.

5. Non-deceptive. Fashion knockoffs are non-deceptive counterfeits that consumers can identify before purchasing (Cho et al. 2015).

6. *Rapid product launch.* Owing to the efficient supply chain and quick response, many fast fashion brands can copy the runway design and sell the product in the same season as the ODL product (Gao et al. 2017b). For example, it only takes *Zara* 10 to 15 days to go from design to sale (Ferdows et al. 2004).

Due to the price advantage and efficient supply chains, fashion knockoffs usually apply intense pressure to the ODL product sellers. Hence, it is commonly believed that the presence of fashion knockoffs harms the ODL product seller. However, real world observations show that some ODLs seem to be fine with fashion knockoffs and they co-exist (e.g., the garment manufacturing giant *TAL* produces for both the designer-labels such as *Tommy Hilfiger* and some fast fashion brands which are arguably the "knockoffs"). Moreover, facing the potential threats of fashion knockoffs, ODL product sellers tend to be risk sensitive (i.e., risk averse or risk seeking) rather than risk neutral. Motivated by the industrial observations, we build analytical models to explore whether the presence of fashion

[4] https://www.budgetandthebees.com/why-is-louis-vuitton-so-expensive/ [Accessed on 16 Feb 2022]
[5] https://hotfashionzone.com/how-much-does-it-cost-to-make-a-louis-vuitton-bag/ [Accessed on 16 Feb 2022]
[6]https://www.pradagroup.com/content/dam/pradagroup/documents/Shareholderinformation/2022/inglese/annual-report-2021/e-Annual%20Report%202021.df.pdf [Accessed on 16 Feb 2022]
[7] https://r.lvmh-static.com/uploads/2022/02/lvmh-comptes-consolides-2021-va.pdf [Accessed on 16 Feb 2022]

knockoffs benefits the ODL product supply chain considering a common manufacturer and risk attitude of the ODL product seller.

Fashion knockoffs also play a critical role in the context of consumer-to-consumer product exchange (C2C-PE) and logo design strategy, which are both important problems in the fashion industry. It is noticeable that the fashion industry is particularly affected by the sharing economy (Choi and Shen 2017; Shen et al. 2017; Choi and He 2019; Feng et al. 2020). The number of users of top C2C fashion e-commerce platforms in May 2020 exceeds 60 million[8]. It is widely considered that the popularity of C2C-PE can reduce customer's demand for new products, which hurts original fashion brand's sales and profit. However, will this argument still hold when considering the presence of fashion knockoffs is largely unknown.

With respect to the logo design strategy, it is commonly seen that different luxury brands use different logo design strategies, e.g., *LV* usually shows prominent logos in its products, while a blatant logo can hardly be found in a *Hermès* bag. Real world practices show that these luxury fashion brands (with big logo or no logo) have different vulnerabilities to fashion knockoffs and counterfeits. It is proposed by Han et al. (2010) that the louder a luxury brand, the more likely it is to be copied by counterfeiters. Accordingly, we aim to analytically explore the impacts of fashion knockoffs and counterfeits on the logo design strategy of luxury fashion brands.

Table 1.1. Definitions of counterfeit, copycat and fashion knockoff from prior studies.

	Studies	**Definitions**
Counterfeit	Cordell et al. (1996)	"Any unauthorized manufacturing of goods whose special characteristics are protected as intellectual property rights (trademarks, patents, and copyrights) constitutes produce counterfeiting."
	Lai and Zaichkowsky (1999)	"A counterfeit is a 100% direct copy usually having inferior quality, although not always."
	Bian and Moutinho (2009)	"Counterfeit brands are those bearing a trademark that is identical to, or indistinguishable from, a trademark registered to another party and infringes on the right of the holder of the mark."
	Staake et al. (2009)	"Counterfeit trade is the trade in goods that, be it due to their design, trademark, logo, or company name, bear without authorization a reference to a brand, a manufacturer, or any organization that warrants for the quality or standard conformity of the goods in such a way that the counterfeit merchandise could, potentially, be confused with goods that rightfully use this reference."

[8] https://www.statista.com/statistics/712248/c2c-fashion-unique-visitors/ [Accessed on 13 Oct 2021]

	Wilcox et al. (2009)	"Counterfeit goods are illegal, low-priced and often lower-quality replicas of products that typically possess high brand value."
Copycat	Balabanis and Craven (1997)	"A new generation of own brand products that have similar packaging characteristics to leading brands products."
	Lai and Zaichkowsky (1999)	"Product or service, though not identical, is viewed as similar in substance, name, shape, form, meaning or intent to an acknowledged and widely known product or service currently in the marketplace."
	Chaudhry et al. (2009)	"Copycat products look the same as branded products, but they do not abuse the intellectual property, or patents and trademarks, of any manufacturer."
	Van Horen and Pieters (2012)	"Copycats imitate the name, logo, and/or package design of a leading national brand to take advantage of the latter's positive associations and marketing efforts."
Fashion knockoff	Dahlén (2012)	"Fashion knockoffs are broadly defined as the intentional replication of fashion apparel designs between competing brands--fuel and rob the fashion apparel industry of its creative uniqueness as well as lead to social and environmental issues within society."
	Miller (2017)	"A ready-to-wear garment which offers the consumer the cachet of couture styling at a fraction of the cost."
	Sponsiello (2019)	"Fashion knockoff indicates a fashion product with a design very similar, but not equal, to the one created by another brand's stylist."

Table 1.2. Definitions of counterfeit, copycat and fashion knockoff in this thesis.

Terminologies	Definitions
Counterfeit	Counterfeits are illegal, low-priced and often inferior-quality products that infringe on the intellectual property rights of the authentic brand.
Copycat	Copycats are legal products which imitate the substance, name, shape, form, package design, meaning or intent to a leading brand's product to free ride on the leading brand's positive associations and marketing efforts.
Fashion knockoff	Fashion knockoff is a kind of copycat in the fashion industry. Fashion knockoff products intentionally copy ODL's designs at a fraction of the cost while remain legal.

1.2 Research Objectives

Motivated by the popularity of fashion knockoffs and its potential impacts on the ODL product supply chain in the context of (i) a common manufacturer; (ii) C2C-PE; and (iii) logo design strategy, this thesis aims to solve the following research objectives:

- To study the impacts of fashion knockoffs on the ODL product supply chains and its agents under different supply chain configurations with the considerations of different risk attitude of the ODL product seller. (This research objective is addressed in Chapter 3).
- To explore the impacts of C2C-PE on the original product supply chain, its members and the consumers in the presence of fashion knockoffs. The cases with (i) strategic quality decision; (ii) price dependent C2C-PE utility and (iii) consumers' conspicuous behavior, are examined. (This research objective is achieved in Chapter 4).
- To examine the impacts of fashion knockoffs and counterfeits on the optimal logo design strategy of a luxury fashion brand. The effects of luxury fashion brand's risk attitude and blockchain implementation on the optimal logo design strategy are analytically studied. (This research objective is examined in Chapter 5).

1.3 Contribution Statements

This thesis not only contributes to the existing literature on fashion knockoffs and counterfeits, but also provides ODL product sellers (e.g., luxury fashion brands) with important managerial insights. Explicitly, for academic contribution, this doctoral thesis is the first to analytically explore (i) impacts of fashion knockoffs on the ODL product supply chain considering a common manufacturer and risk attitude of the ODL product seller; (ii) impacts of C2C-PE on the original product supply chain in the presence of fashion knockoffs; and (iii) impacts of fashion knockoffs and counterfeits on the logo design strategy of luxury fashion brands. We have derived some important findings and filled in the research gaps to some extent. For managerial contribution, all the problems explored in this thesis are well motivated by real world observations in the fashion industry. Moreover, we conduct various interviews with the fashion industrialists to acquire first-hand knowledge of the problem and to validate our analytical findings. Hence, we provide ODL product sellers with rigorous and practical guidelines concerning (i) strategies towards fashion knockoffs; (ii) adoption of the C2C-PE scheme; and (iii) logo design strategy.

1.4 Thesis Outline

As shown in Figure 1.1, this thesis is arranged as follows. We first illustrate the introduction in Chapter 1. A detailed literature review on "copycats and counterfeits", "risk sensitive supply chain agents", "sharing economy", "conspicuous consumption" and "brand prominence" is provided in Chapter 2. Three analytical studies are conducted to solve the above-mentioned research objectives in Chapter 3-5 respectively. Explicitly, Chapter 3 explores whether the presence of fashion knockoffs benefits the

ODL product supply chain considering risk attitude of the ODL product seller, Chapter 4 examines whether C2C-PE for original fashion brands in the sharing economy is good or bad for fashion knockoffs, Chapter 5 studies whether to show big logos or not in products of luxury fashion supply chains considering the impacts of fashion knockoffs and counterfeits. Chapter 6 concludes this thesis by providing major findings, managerial implications and future research agenda.

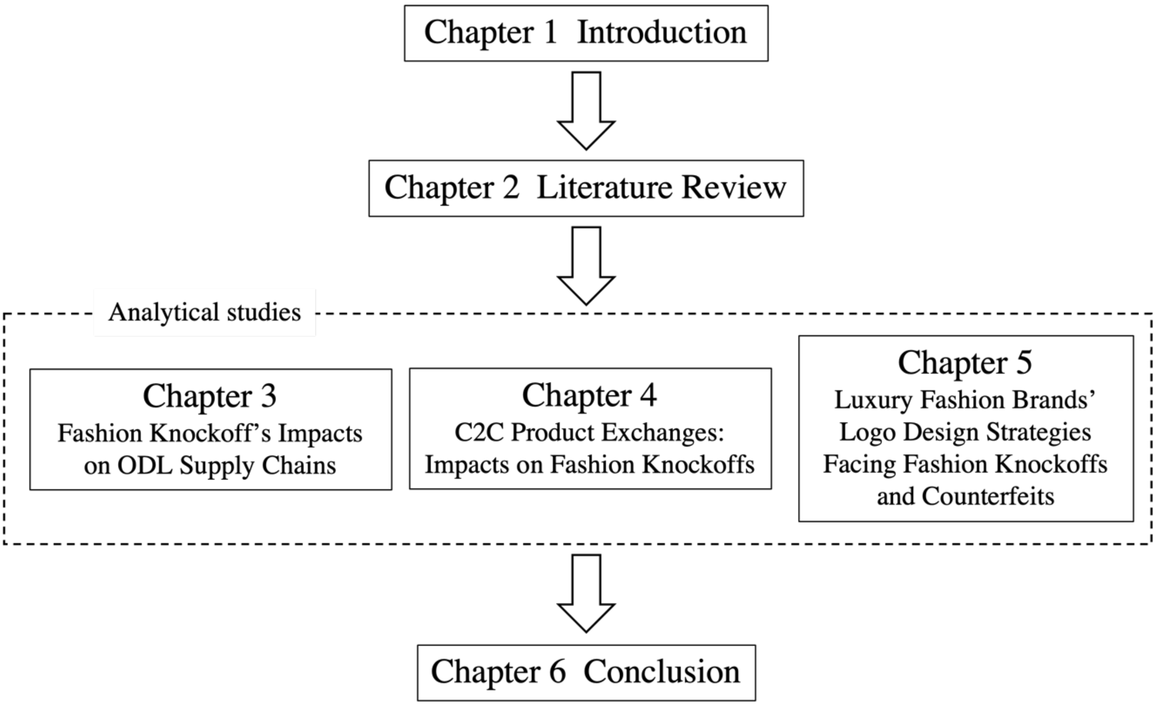

Figure 1.1. The outline of this thesis.

Chapter 2 Literature Review

2.1 Copycats and Counterfeits

There is extensive research which explores the impacts of counterfeits/copycats and the deterrence strategies. For example, Cho et al. (2015) build game theoretical models to explore the impacts of anticounterfeiting strategies on a brand name company, its counterfeiter and the consumers. They discover that the effectiveness of the strategies is dependent on the counterfeiter type (i.e., deceptive or nondeceptive). Zhang and Zhang (2015) construct vertical differentiation models to study the optimal supply chain structure of a brand name company when encountering deceptive counterfeits. They interestingly find that selling through the general channel (popular with counterfeits) can still be beneficial for the brand name company under certain conditions. Gao et al. (2017a) build game theoretical models to examine the implications of potential entry of copycats. They uncover that the entry of copycat can enhance consumer surplus and social welfare under certain circumstances. Gao et al. (2017b) further explore the impacts of copycat entry by incorporating two important features, which are physical resemblance and product quality. They analytically reveal that a copycat with high resemblance and lower quality to the original one is more prone to successfully enter the market.

Pun et al. (2017) analytically study the decision making of an authentic manufacturer and a copycat firm in the presence of strategic consumers. They surprisingly find that lowering quality level of the original product may increase price and profit of the manufacturer. Hou et al. (2020) analytically explore the effectiveness of a fighter brand in deterring copycats. They recommend the manufacturer to launch a fighter brand when the copycat is less resemblant to the authentic one. Ghamat et al. (2021) analytically investigate the effectiveness of intellectual property (IP) agreement in deterring supplier or third-party copycatting. They interestingly find that even though the IP agreement is cost-free, there exist conditions under which signing an IP agreement harms the manufacturer. As revealed by the prior literature, the existence of counterfeits/copycats will inevitably affect the consumers and supply chain's decisions, which will be captured in our model settings. Particularly, we are the first study considering the impacts of counterfeits/copycats on the luxury fashion brand's logo strategy. Yi et al. (2022) build game theoretical models to study the implications of counterfeits on a global supply chain. They uncover that, compared with battling the counterfeits itself, it is wiser for the manufacturer to induce the retailer to battle counterfeits.

Among all the anticounterfeiting strategies, the blockchain technology has gained extra attention from both academics and practitioners. Choi (2019) build utility driven operations models to explore the value of blockchain in diamond authentication and certification. They analytically derive the conditions under which the blockchain supported selling platform outperforms the traditional selling platform. Pun et al. (2021) analytically study the effectiveness of blockchain in combating deceptive counterfeits by considering the role of government and consumers' privacy concern. They suggest that the manufacturer should implement blockchain technology when the counterfeit is in an intermediate quality or the consumers moderately distrust the product in the market. Shen et al. (2021) analytically study the impact of quality inspection and blockchain adoption on deterring counterfeit masks. They reveal that blockchain technology is a preferred strategy for the authentic mask seller when the Covid 19 is mildly spread. Shen et al. (2022) establish game theoretical models to examine the impacts of blockchain technology in combatting copycats. The authors consider two segments of consumers, i.e., novice consumers and expert consumers. They uncover that implementing blockchain technology is beneficial to the brand name company only when the proportion of novice consumers is sufficiently large. Adding to the above studies, we also analytically explore the implications of blockchain technology in combatting deceptive counterfeits.

2.2 Risk Sensitive Supply Chain Agents

There is a stream of research which studies supply chain coordination where supply chain agents are risk sensitive. Gan et al. (2004) explore the coordination of supply chains with risk averse agents. The authors derive the corresponding Pareto-optimal solutions and contracting schemes under each explored case. Choi et al. (2008) study the implementation of returns policy to coordinate a supply chain with a manufacturer and a retailer taking mean-variance (MV) objectives. Both the centralized and decentralized supply chain structures are examined. Huang and Goetschalckx (2014) evaluate the strategic robust supply chain design based on the Pareto-optimal with the consideration of both the efficiency and the risk. In the model settings, the authors depict the risk issue by solving Mean-Standard Deviation Robust Design Problem (MSD-RDP) and extend the model to consider Mean–Variance Robust Design Problem (MV-RDP). Li et al. (2014) explore the use of return policy to coordinate the fast fashion supply chain with the consideration of stock-out. The authors employ the MV approach to study both the single-retailer and multi-retailer settings. They surprisingly find that the return policy can effectively coordinate the fast fashion supply chain.

Vedantam and Iyer (2021) study a revenue sharing contracting problem between a supplier and a third party remanufacturer (3PR) who disposes the used electronic equipment. The authors use the VaR (value at risk) measure to model the 3PR's risk aversion towards the uncertain quality of the used equipment. They derive the conditions under which unique contracts exist. Zhang et al. (2020) analytically study the newsvendor problem by considering the newsvendor's preferences on profit's mean, variance, skewness, and kurtosis (MVSK). They uncover that when considering the MVSK objective, both achievability of supply chain coordination and flexibility of coordinating contracts will be affected.

There is another stream of research which focuses on exploring the impacts of risk attitudes on operational decisions of supply chain agents. For instance, Agrawal and Seshadri (2000) examine the pricing and quantity decisions of a risk averse retailer under the single-period newsvendor setting. The authors consider two scenarios where the price affects the scale of demand distribution and the location of demand distribution respectively. They reveal that, in the first scenario, the retailer sets a higher price and orders less; while in the second scenario, the retailer sets a lower price. Xie et al. (2011) explore the joint decisions of price and quality made by a global supply chain with risk averse agents. They uncover that risk sensitivity coefficient and supply chain strategies are governing factors for quality and pricing decisions.

Li et al. (2020b) study the manufacturer's financing provision strategies with the consideration of risk aversion and market competition. They discover that the magnitude of risk aversion largely affects the equilibrium financing provision mode. Xue et al. (2020) examine the value of inventory pooling for a firm with limited demand information and risk averse attitude. They measure the firm's utility using the CVaR approach and find that inventory pooling is always beneficial to the firm in terms of CVaR. Similar to the above reviewed papers, this study also explores how risk attitudes of an ODL product seller affect his pricing decision. Differently, this study innovatively examines the pricing decision of a risk neutral knockoff product seller and the impacts of risk attitudes on the ODL product supply chain performance. As a remark, there are various approaches to measure the agent's risk attitude. In this thesis, we adopt the MSD approach since it is widely used in the literature and analytically tractable.

2.3 Sharing Economy[9]

With the increasing popularity of the sharing economy, C2C product exchange has been widely explored over the past decade (Plouffe 2008). There is a stream of studies explicitly examine the impacts of C2C-PE. Jiang et al. (2017) develop an analytical two-period model to study the impacts of P2P (peer-to-peer) marketplaces on the supply chain members, consumer surplus and social welfare. The authors theoretically uncover that the product's unit cost is a critical factor governing whether the supply chain members and consumers can be benefited by the P2P marketplaces. Jiang and Tian (2018)

[9] A part of this chapter has been published in Wang, Y., Fan, D., Fung, Y. N., & Luo, S. (2022). Consumer-to-consumer product exchanges for original fashion brands in the sharing economy: Good or bad for fashion knockoffs?. Transportation Research Part E: Logistics and Transportation Review, 158, 102599.

construct a product sharing model in a two-period setting to examine how C2C-PE affects the firm's profit, consumers and social welfare. The authors discover that, if the firm strategically decides on the retail price, product sharing will either yield a "lose-lose" or "win-win" situation to the firm and consumers, depending on the marginal cost. They further uncover that if the firm considers price and quality decisions simultaneously, the firm will benefit while consumers will suffer.

Tian and Jiang (2018) develop an analytical framework to examine the effects of C2C-PE on the distribution channel. The authors discover that the manufacturer's optimal capacity first increases and then decreases with the capacity cost coefficient. Moreover, the C2C-PE is more likely to benefit the downstream retailer than the upstream manufacturer. Benjaafar et al. (2019) construct an equilibrium model to explore the impacts of peer-to-peer collaborative consumption (P2P-CC) on ownership, usage levels, consumer surplus, and social welfare. The authors analytically uncover that P2P-CC can lead to either high or low ownership and usage levels, mediated by the cost of ownership. Moreover, the P2P-CC always benefits the consumers. Feng et al. (2020) build game-theoretical models to explore the impacts of a fashion rental platform on the luxury fashion brand, consumers and the society. They uncover that the presence of a rental platform benefits the luxury fashion brand and the results remain robust when they consider consumers' conspicuous behavior. Bian et al. (2021) analytically explore the decision of a C2C sharing platform to launch its own product or not. They find that when the proportion of product owners is small and consumer acceptance for platform's self-owned product is high, it is beneficial for the platform to launch its own product.

Prior studies that are closest to ours are Choi and He (2019) and Choi et al. (2019). Choi and He (2019) analytically investigate the impacts of P2P-CC on the fashion brand and consumers. The authors find that both the fashion brand and the consumers are benefited by the presence of P2P-CC. Moreover, they uncover that it is more profitable for the platform to employ the revenue sharing scheme rather than the fixed charging scheme. Although we also consider the exchange of fashion products, we did not consider the presence of platforms. Choi et al. (2019) construct stylized models to explore the impacts of C2C-PE on the digital product developers (DPDs) and consumers. They analytically prove that the presence of C2C-PE always benefits the DPDs and consumers regardless of their risk attitudes. They also consider the case where the C2C-PE utility is price dependent. Different from this paper which considers the exchange of digital products, we consider the exchange of fashion products. Following this stream of research, we also build product sharing models to analytically explore the impacts of C2C-PE on the original brand and the consumers. While totally different, we consider the context with fashion knockoffs in the market and how the C2C-PE affects the supply chain members and the consumers.

There is another stream of research which is devoted to exploring the platform operations in the sharing economy. For example, Dhanorkar et al. (2021) conduct research about the product reuse issues from industry level. The authors uncover that the expert service is critical for industrial exchanging platform. They put forward the critical issues including commodity mix and operations feature. They conduct an empirical experiment to demonstrate the importance of the proposed factors with the adoption of a real-world transaction dataset. Their findings shed light on online reuse platform economy. On-demand service platforms have been analytically explored in OM and attract more and more attention, Choi et al. (2020a) present a study about the pricing decision of on-demand platform economy by taking customers' risk preferences into consideration. The authors leverage mean-risk methodology to examine the effect of different risk attitudes. They further highlight the value of related blockchain technology.

In the work of Benjaafar and Hu (2020), the authors propose an OM study about platform economy. They take advantage of three existing hot "canonical" applications to emphasize the differentiating traits of various platform economy mechanisms. They build the relationship between classical theory and innovative platform systems, and put forward promising research avenues. Most recently, Hu (2020) considers both the platform economy concept and the emerging industry. The author focuses on the relationship and gap between traditional systems and innovative solutions. He establishes a new proposal for further studies. Following this stream of studies, our study falls into the platform economy domain. But different from the works mentioned above, we concentrate on investigating the impacts of C2C-PE on knockoffs in the fashion area. Compared to the industrial level, our study mainly concerns the product reuse issues from personal perspective.

2.4 Conspicuous Consumption

There is an extensive literature which incorporates the psychological and sociological realism in formal economic analysis. Amaldoss and Jain (2005a,b) are among the first to analytically study consumers' conspicuous consumption. The authors introduce the framework containing two segments of consumers: snobs, who value exclusivity, and conformists, who value conformity. Amaldoss and Jain (2005a,b) respectively consider a monopoly and a duopoly that sell conspicuous products. They interestingly find that snobs would demand more as price increases only in the presence of conformists. And consumers' higher desire for uniqueness (conformity) would lead to higher (lower) prices and profits. Amaldoss and Jain (2008) build a two-period game theoretical model to examine the impacts of reference groups on a luxury firm's pricing, product design and target consumer strategies. They find that the reference group effects may encourage the firm to add costly features or to adopt a scarcity strategy under certain conditions. Amaldoss and Jain (2010) develop a discrete game of Amaldoss and Jain (2008) to experimentally study how reference groups affect product line decisions. They uncover that a luxury firm can be better off by offering multiple product lines or limited editions. Tereyağoğlu and Veeraraghavan (2012) build newsvendor based analytical models to study a luxury firm's pricing, production and sourcing decisions. The authors consider the presence of conspicuous consumption and consumers' strategic behavior. They derive the conditions under which a scarcity strategy is beneficial to the firm.

Rao and Schaefer (2013) supplement Amaldoss and Jain (2008) by jointly considering the impacts of status utility and intrinsic quality on marketing conspicuous goods. They interestingly find that a higher intrinsic quality can generate a higher exclusivity, hence generating higher social payoffs for the consumers. Amaldoss and Jain (2015) analytically explore how social effects and competitive effects affect the branding of luxury firms. They interestingly find that the umbrella branding strategy outperforms the individual branding strategy when consumers exhibit a higher desire for uniqueness in a duopoly market. Agrawal et al. (2016) analytically study the planned obsolescence strategy for luxury firms. They uncover that when considering consumers' conspicuous consumption and desire for exclusivity, a luxury firm can benefit from offering a product with higher durability, higher price and a lower volume. Gao et al. (2016) develop a two-period game theoretical model to explore the entry of copycats and its implications. They discover that a copycat with high similarity and lower quality can successfully enter the market. Chiu et al. (2018) analytically study the advertising budget allocation strategy for a luxury fashion company serving conspicuous consumers. They interestingly find that a polarized strategy can be optimal for the firm.

Li (2019) builds game theoretical models to explore the vertical line extensions of luxury firms when considering consumers' preferences for status and a potential entry of lower quality product. They uncover that in the presence of status preferences, originally unprofitable extensions can be profitable. Based on the empirical data, Li and Liu (2019) analytically study the aesthetic design strategy of firms providing status goods. They interestingly find that an asymmetric design strategy can benefit the competing firms. Arifoğlu et al. (2020) build game-theoretical models to explore the markdown and rationing strategies of a luxury fashion retailer selling to strategic and exclusivity-seeking consumers. They reveal that the markdown policy is largely dependent on consumers' desire for exclusivity. Shen et al. (2020) analytically study the impacts of preordering information on a luxury fashion supply chain, its members and the conspicuous consumers. They surprisingly find that additional demand information may bring harm to the retailers. Adding to the above stream of research, we also consider consumers' conspicuous consumption while focusing on the luxury fashion brand's logo design strategy.

2.5 Brand Prominence

The field of conspicuous consumption is revolutionized by Han et al. (2010) through introducing the concept of brand prominence. Consumers who prefer prominent luxury product are considered as engaging in a louder form of conspicuous consumption, whereas consumers who prefer discreet luxury product are viewed as engaging in a quieter form of conspicuous consumption. In addition to the variation of brand visibility, Meyer and Manika (2017) reveal another two variations of brand prominence, which are brand frequency and brand distribution. Most of the prior literature on brand prominence has an empirical root which employs the experimental or survey approach.

Among the studies, a large number of them explore the factors determining consumers' brand prominence preferences. For instance, Cheah et al. (2015) conduct a survey to explore the antecedents of brand prominence. They find that luxury brand values, social influence and vanity are three motives of brand prominence. Song et al. (2017) conduct experiments to test the relationship between embarrassment and preferences for brand conspicuousness. They uncover that when consumers feel embarrassed, a high (low) self-esteem consumer prefers a more (less) conspicuous product. Kauppinen-Räisänen et al. (2018) quantitatively study how consumers' personality and social traits affect their preferences for brand prominence. They discover that consumers with a high need for uniqueness prefer a loud luxury brand, while consumers with a high self-monitoring prefer a quiet luxury brand. Greenberg et al. (2019) adopt a mixed methods approach to explore motives of brand prominence. They find that extraversion can lead to a higher preference for brand prominence via the construct of need for status. Siahtiri and Lee (2019) employ the survey approach to test the impacts of materialism on consumers' preferences for brand prominence in developing countries. They reveal that materialism positively affect brand prominence through consumers' fashion consciousness and quality consciousness.

Another stream of research is devoted to examining the outcomes of brand prominence. For example, Butcher et al. (2016) conduct surveys to investigate the impacts of brand prominence on consumers' purchase intention. They discover that brand prominence indirectly affects purchase intention for luxury products via emotional value. Pino et al. (2019) conduct a cross-market study to explore how brand prominence as well as status consumption jointly affect consumers' purchase intension towards luxury accessories. They uncover that consumers in the emerging (mature) markets are more willing to buy prominently (subtly) branded products. Aw et al. (2021) experimentally study the impact of brand prominence on luxury products purchase intention. They uncover that the effect of brand prominence via self-congruence and value-for-money perception is conditional on consumers' power distance belief. Following this stream of research, we consider that brand prominence affects consumers' purchase intention for luxury products. Although brand prominence has been well explored using empirical approach, none of the prior research is analytical based. Hence, in this thesis, we aim to bridge this research gap by analytically exploring the implications of brand prominence on luxury fashion supply chain's profitability and consumer surplus. This is an important theoretical contribution.

Chapter 3 Fashion Knockoff's Impacts on ODL Supply Chains

3.1 Introduction

3.1.1 Background and Motivation

In the presence of fashion knockoffs, the ODL product seller generally faces three levels of risk: (i) The knockoff product seller is capable of launching a knockoff product in the same season as the original one, which weakens the competitiveness of the ODL product seller and triggers the market volatility. (ii) Consumers may unknowingly purchase a knockoff product that is inferior in quality, which harms the ODL product seller's reputation and reduces the potential market demand greatly. In 2017, the fast fashion brand *Forever 21* was sued by the luxury brand *Gucci* for knocking off *Gucci*'s trademark stripe webbing, as the consumers may mistakenly regard these products as a collaboration with *Gucci*[10]. (iii) With the booming of e-commerce platforms, the intellectual property of the ODL brand is more vulnerable to be infringed by the knockoff brand, which raises the potential negative effects like demand reduction and reputation damage for the ODL brand. Hence, managers of the ODL brand tend to be risk sensitive (i.e., risk averse or risk seeking) in the presence of fashion knockoffs.

It is confirmed by the interviewee that some luxury fashion brands like *LV* and *Gucci* even establish a specific team to monitor the copying level of the knockoff brands. For those risk averse ODL product sellers, once the copying level reaches some point, they will sue the knockoff companies to avoid potential risks. Moreover, some ODL product sellers become more cautious about market testing under the potential threat of fashion knockoffs. Rather than putting a product into the regular product line, the ODL product seller may launch the product as a seasonal product and wait for consumers' and competitors' (e.g., knockoff product sellers) reactions first. Industrial data also reveal the risk averse attitude of ODL brand managers. In a study involving 250 British companies, more than half of the brand managers described themselves as having one of the following risk averse characteristics, which are "wary, prudent, or deliberate"[11].

This is also supported by the existing literature. Kros et al. (2019) empirically find that managers of ODL brands are commonly risk averse towards counterfeits. Although risk aversion seems to be a dominant risk attitude taken by ODL product sellers, there are conditions under which the ODL product seller exhibits a risk seeking attitude towards fashion knockoffs. From our interview, the interviewee pointed out that, for some well-established companies like *Supreme* and *RIMOWA*, they are not afraid

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[10] https://www.forbes.com/sites/barbarathau/2017/07/11/gucci-versus-forever-21-legal-fashion-experts-disagree-on-alleged-knockoff-drama/?sh=b7eae8c6e786 [Accessed on 16 Feb 2022]
[11] https://www.forbes.com/sites/theyec/2013/04/26/risk-aversion-is-more-common-in-entrepreneurs-than-you-think/?sh=4a3144a61dc2 [Accessed on 16 Feb 2022]

of being copied, instead, they may like being copied since the existence of fashion knockoffs can help them promote their products and brand name, which may even benefit them. This is evidenced by the literature that risk seeking may do more good than harm for the retailers in some cases (Choi et al. 2018; 2020b). Motivated by the industrial observations and the prior literature, we explore how the ODL product seller's risk attitude affects the impacts of fashion knockoffs on the ODL product supply chain and its agents.

Another common practice observed in the fashion industry is that, in many cases, the manufacturer of the knockoff product seller is the same as the manufacturer of the ODL product seller. For example, the garment manufacturing giant *TAL* produces for both the designer-labels such as *Tommy Hilfiger* and some fast fashion brands which are arguably the "knockoffs". The interviewee supplemented that the common manufacturer scenario is especially usual for manufacturers producing T-shirts. For example, *Esquel* is producing T-shirts for both a luxury brand and a knockoff brand.

Although prior studies have investigated the impacts of fashion knockoffs on the ODL product seller (e.g., Gao et al. 2017a, b; Pun and DeYong 2017), they did not consider (i) the risk associated with fashion knockoffs and different risk attitudes of the ODL product seller, and (ii) the common manufacturer scenario, which are all important. Hence, in this study, we fill the research gaps by incorporating these two important factors to explore the impacts of fashion knockoffs on the ODL product supply chain and its agents.

3.1.2 Research Questions

- RQ1: When the ODL product and the knockoff product are produced by a common manufacturer (i.e., basic model), will the presence of fashion knockoffs benefit the ODL product supply chain and its agents? If yes, under what conditions?
- RQ2: When the wholesale price is set based on the non-standard markup wholesale pricing policy (i.e., different markup rates for the ODL product seller and the knockoff product seller), will the results in the basic model still hold?
- RQ3: When we consider another configuration of the supply chain (i.e., when the ODL product and the knockoff product are produced by different manufacturers), will the results derived under the basic model change?
- RQ4: Will the risk attitude (i.e., risk averse, risk seeking, risk neutral) of the ODL product seller affect the impacts of fashion knockoffs on the ODL product supply chain and its agents?

3.1.3 Research Methodology

This study combines the analytical modelling method with industrial interviews (as shown in Figure 3.1). To be specific, motivated by the industrial observations (e.g., popularity of fashion knockoffs, risk attitude of the ODL product seller, common manufacturer scenario), we build analytical models to solve the research questions. Before model building, we conduct a pre-analysis interview to acquire first-hand knowledge of the problem explored, scenarios, assumptions, etc. After obtaining the analytical results and numerical findings, we conduct a post-analysis interview to further validate our major findings. With the two research methods applied together, the research rigor of this study is enhanced, and we can generate more practical insights for industrialists (Singhal et al. 2014; Tang 2016).

Explicitly, we conduct an open-ended interview with an experienced former manager in *Li and Fung*, who used to cooperate with retailers like *Michael Kors*, *Macy's*, *Walmart*, etc., on November 14, 2021. The results of the pre-analysis interview and the corresponding model setting are summarized in Table 3.1. We mainly pay attention to three aspects including (i) features of fashion knockoffs, (ii) supply chain settings regarding common manufacturer scenario, standard markup wholesale pricing policy, interrelated demands and fixed operational cost and (iii) risk attitude of the ODL product seller.

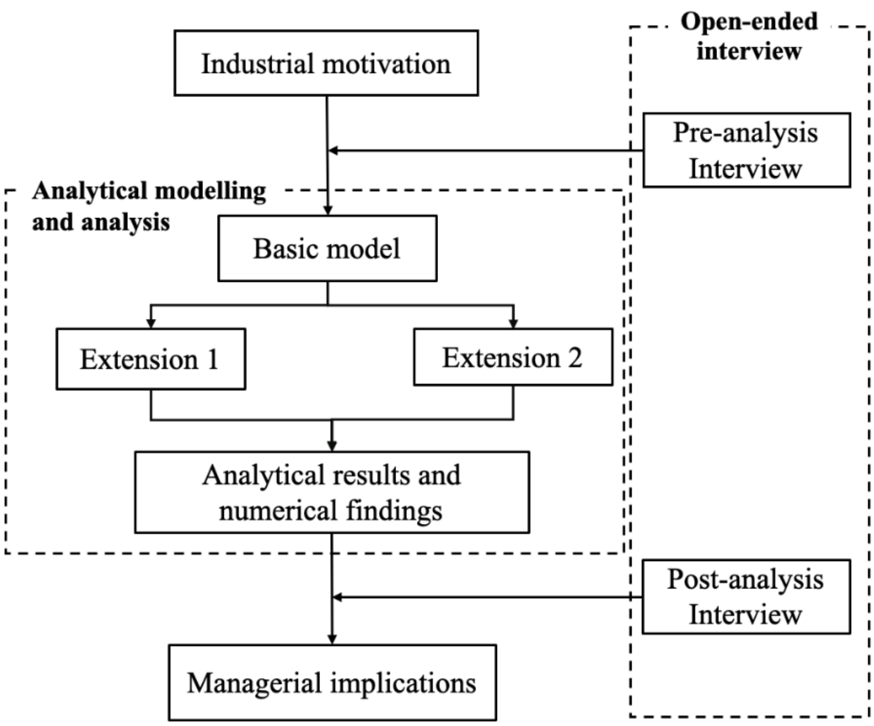


Figure 3.1. The research methodology adopted in the study of this chapter.

Table 3.1. Results of the pre-analysis interview and the corresponding model setting[12].

Aspects	Interview results	Model and major assumptions
Features of fashion knockoffs	"When talking about fashion knockoffs, I think of low quality and cheap products."	The knockoff product is produced at a lower production cost and sold at a lower selling price compared to the ODL product.
	"The ODL product and its knockoff product can coexist in the market. Some knockoff brands, especially fast fashion brands may steal the ideas from the runway and launch the products in the same season as the ODL product."	We consider that the ODL product seller and the knockoff product seller make pricing decisions simultaneously.
Supply chain settings	"It is very common that a manufacturer produces for the ODL product seller and the knockoff product seller at the same time, especially for the manufacturers producing T-shirts."	In our basic model, we consider a common manufacturer producing for both the ODL product seller and the knockoff product seller.
	"The standard markup wholesale pricing policy is commonly observed in the fashion industry."	We consider that the wholesale price is exogenously given in the basic model.
	"Normally speaking, demand for the ODL product will positively affect demand for the knockoff product, while demand for the knockoff product usually has a negative effect on the ODL product."	We capture the inter-related demands of the ODL product and knockoff product using $-\phi$ and β (see Eq.(3.7) and Eq.(3.8)).
	"It is for sure that the ODL product sellers will incur a much larger operational cost, because they have to invest in product research and development (R&D) cost. While for knockoff product seller, the only cost is to purchase an ODL product, copy its design and then produce."	We consider that the fixed operational cost for the ODL is much larger than that of the knockoff product seller. And to better capture the free riding behaviour of the knockoff product seller, we normalize its fixed operational cost to zero.
Risk attitude	"I think the ODL product seller tends to be risk averse/seeking rather than risk neutral with the potential threat of fashion knockoffs."	We adopt the mean-risk theory to capture the supply chain agents' risk sensitive decision making.

3.2 Basic Model

In the basic model, we consider a two-tier supply chain where a common manufacturer (e.g., *TAL*) produces for both an ODL product seller (e.g., *Tommy Hilfiger*) and a knockoff product seller (e.g., a

[12] Please refer to Appendix E for more details.

certain fast fashion brand)[13]. The manufacturer charges the ODL product seller with a unit wholesale price w_O and the knockoff product seller with w_K. There are two independent markets serving consumers buying ODL product (market 1) and knockoff product (market 2) respectively. As we have discussed earlier in Section 1, the knockoff product is produced at a lower production cost, and sold at a lower selling price compared with its ODL counterpart (i.e., $m_K < m_O$ and $p_K < p_O$). Moreover, freeriding on the ODL product seller who incurs a much larger fixed operational cost, the knockoff product seller rarely engages in R&D and marketing campaigns. Hence, we normalize its fixed operational cost to be zero. (i.e., $F_O \gg F_K = 0$). The manufacturer also incurs a fixed operational cost (F_M) in its operations which contains rents, equipment depreciation and staff salaries[14].

We first consider the case where the wholesale price is exogenously given. There are two reasons behind: First, manufacturers in the fashion industry widely use the "standard markup wholesale pricing policy"[15] where the profit margin is relatively stable, ranging from 30% to 50%[16], which results in a relatively stable wholesale price. This is also supported by the interviewee who stated that the markup rate is retailer dependent. For mass market retailers like *Giordano*, the markup rate is usually around 20-30%, while for giant retailers like *Walmart*, the markup rate is around 10%. Second, an exogenously given wholesale price enables us to derive closed-form results and obtain fruitful insights. Note that, when the wholesale price is determined by the non-standard markup wholesale pricing policy, the qualitative results remain unchanged (see Section 3.4.1 for more details). The basic model is illustrated in Figure 3.2. please refer to Table 3.2 for the notation used in this study.

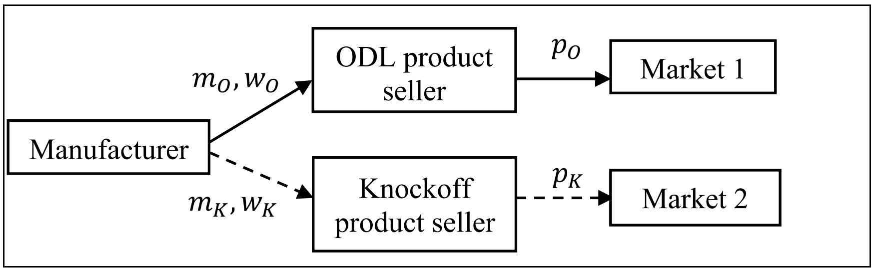


Figure 3.2. The basic model[17].

[13] To avoid confusion, we use "seller" to represent the brand owner and "manufacturer" to represent the factory in this study.
[14] https://smallbusiness.chron.com/costing-fashion-industry-77191.html [Accessed on 16 Feb 2022]
[15] The mechanism is that the manufacturer sets the wholesale price by multiplying a profit margin with the cost of goods manufactured (COGM). COGM usually includes (i) material cost, (ii) labor cost, and (iii) additional costs and overhead, which can be easily figured out by the manufacturer. The importance of standard markup policy is also highlighted in the literature. As pointed out by Noble and Gruca (1999), the standard markup wholesale pricing policy is one of the most widely used pricing strategies in the retailing industry. The intuition behind is that a normal markup over cost provides a simple pricing guideline to profitability (Wang et al. 2013).
[16] https://en.shopify.hk/retail/product-pricing-for-wholesale-and-retail [Accessed on 16 Feb 2022]
[17] The ODL product supply chain consists of the common manufacturer and the ODL product seller.

Table 3.2. List of notation for Chapter 3.

Notation	Meaning
O	ODL product seller
K	Knockoff product seller
M	Manufacturer
SC	Supply chain
p_i	Unit retail price for product $i \in (O, K)$
w_i	Unit wholesale price for product $i \in (O, K)$
m_i	Unit production cost for product $i \in (O, K)$
F_j	Fixed cost for member $j \in (O, K, M)$
a	Market base for the ODL product
α	Market base for the knockoff product
ϕ	The coefficient which reflects the impact of the knockoff product on the ODL product
β	The coefficient which reflects the impact of the ODL product on the knockoff product
b	The price-demand sensitivity coefficient ($b > 0$)
ε_i	Independent random variable with zero mean and standard derivation of σ_i, $i \in (O, K)$
k	Risk sensitivity coefficient
CN	Common manufacturer, risk neutral ODL product seller
DS	Different manufacturers, risk sensitive ODL product seller
DN	Different manufacturers, risk neutral ODL product seller
D_i	The demand function for product $i \in (O, K)$
π_j	The profit function for member $j \in (O, K, M)$
$E[\pi_j]$	The expected profit function for member $j \in (O, K, M)$
$SD[\pi_j]$	The standard derivation of profit function for member $j \in (O, K, M)$
$U[\pi_j]$	The mean-standard deviation (MSD) objective function for member $j \in (O, K, M)$

3.2.1 Without Fashion Knockoffs

We first consider the case where there is no fashion knockoff in the market. Following Luo et al. (2009) and Shi et al. (2013), we adopt the price dependent linear demand function as shown in Eq.(3.1), where a is the market base for the ODL product, and ε_O is a random variable (with zero mean and standard derivation σ_O) which captures the demand uncertainty for the ODL product. Note that we use an overbar to represent the functions and optimal decisions under the case without fashion knockoffs.

$$\bar{D}_O = a - \bar{p}_O + \varepsilon_O. \tag{3.1}$$

We can then derive the profit functions for the ODL product seller and the manufacturer as

$$\bar{\pi}_O = (\bar{p}_O - w_O)(a - \bar{p}_O + \varepsilon_O) - F_O, \tag{3.2}$$

$$\bar{\pi}_M = (w_O - m_O)(a - \bar{p}_O + \varepsilon_O) - F_M. \tag{3.3}$$

To capture the supply chain agents' risk sensitive decision making, we use the classic mean risk theory (Gan et al. 2004; Chen et al. 2014; Chiu et al. 2018; Zhang et al. 2020). Mean risk is a standard and classic approach. It is based on the Nobel prize winning theory by Harry Markowitz and the classic risk hedging approach in practice[18]. The theory is powerful as it can be applied to problems in economics, finance, and business operations. For analytical traceability, we use the Mean-Standard Deviation (MSD) theory as expressed in Eq.(3.4), where $U[\pi]$ is the MSD objective function, $E[\pi]$ is the expected profit, $SD[\pi]$ is the standard derivation of profit, and k is the risk sensitivity coefficient. The agent is risk averse when $k > 0$; risk seeking when $k < 0$; risk neutral when $k = 0$, and its objective is to maximize the expected profit, i.e., $U[\pi] = E[\pi]$.

$$U[\pi] = E[\pi] - kSD[\pi] \tag{3.4}$$

Risk sensitive ODL product seller

We can easily derive the objective functions for the ODL product seller and the manufacturer:

$$U[\bar{\pi}_O] = (\bar{p}_O - w_O)(a - k\sigma_O - \bar{p}_O) - F_O. \tag{3.5}$$

$$E[\bar{\pi}_M] = (w_O - m_O)(a - \bar{p}_O) - F_M. \tag{3.6}$$

Maximizing Eq.(3.5) with respect to $\bar{p}_O$ generates the optimal retail price $\bar{p}_O^*$. Through substitution, we obtain the optimal solutions: $\bar{p}_O^* = \frac{a+w_O-k\sigma_O}{2}$, $\bar{D}_O^{MSD*} = \frac{a-w_O-k\sigma_O}{2}$, $U[\bar{\pi}_O^*] = \left(\frac{a-w_O-k\sigma_O}{2}\right)^2 - F_O$, $E[\bar{\pi}_M^*] = \frac{(w_O-m_O)(a-w_O+k\sigma_O)}{2} - F_M$. To guarantee that demand for the ODL product is always positive, we have $|k| < K_1$, where $K_1 = \frac{a-w_O}{\sigma_O}$.

We further conduct sensitivity analysis on the optimal decisions and objectives, and summarize the findings in Table 3.3. It is intuitive that with a larger market size (a), demand for the ODL product is larger. Meanwhile, the ODL product seller tends to set a higher retail price, so that both the ODL product seller and the manufacturer are better off. When the manufacturer raises the wholesale price (w_o), the ODL product seller raises the retail price accordingly, demand for the ODL product decreases, the ODL product seller is hence worse off. Regarding the manufacturer, when the wholesale price is not too high, increase in the wholesale price can compensate the loss in consumer demand, so the manufacturer is still better off; when the wholesale price is sufficiently large, the increase in wholesale price cannot counteract the decrease in demand, which harms the manufacturer's profit.

When the ODL product seller is more risk averse (i.e., $k \uparrow$), its perceived demand (i.e., demand considering risk) decreases. In this case, the ODL product seller will lower the retail price to stimulate market demand. It is interesting to find that the risk averse attitude of the ODL product seller (k) harms the ODL product seller while benefits the manufacturer. The standard deviation σ_O has an opposite effect on the optimal decisions for different risk attitudes of the ODL product seller. Specifically, if

[18] Note that there are other alternative optimization objectives which can also be employed to model risk related optimization objective. See Shi et al. (2011) for profit satisficing objective and others (Shi et al. 2010). We choose the mean-risk objective for its nice analytical feature as well as being widely applied in practice (e.g., in risk hedging) and used in the literature (Chiu and Choi 2013).

the ODL product seller is risk averse, when demand for the ODL product is more volatile, the ODL product seller tends to set a lower selling price, demand for the ODL product decreases, the ODL product seller is harmed while the manufacturer is benefited. If the ODL product seller is risk seeking, an increase in demand volatility increases the retail price, demand and profit of the ODL product seller while decreases the profit of the manufacturer.

Table 3.3. Sensitivity analysis for the optimal decisions and objectives in the basic model without fashion knockoffs and with risk sensitive ODL product seller.

		$\bar{p}_O^*$	$\bar{D}_O^{MSD*}$	$U[\bar{\pi}_O^*]$	$E[\bar{\pi}_M^*]$
$a\uparrow$		↑	↑	↑	↑
$w_o\uparrow$		↑	↓	↓	↑ if $m_o < w_o < \overline{w_O}$ ↓ if $w_o \geq \overline{w_O}$
$k\uparrow$		↓	↓	↓	↑
$\sigma_O\uparrow$	Risk averse	↓	↓	↓	↑
	Risk seeking	↑	↑	↑	↓
Remark: $\overline{w_O} = \frac{a+k\sigma_O+m_o}{2}$, which is increasing in k.					

Risk neutral ODL product seller

In this case, the optimization objective is solely on expected profit maximization and hence "$k = 0$". Note that we use the superscript CN (common manufacturer, risk neutral ODL product seller) to represent the functions and optimal decisions derived in this subsection. We can easily derive the following results by setting $k = 0$: $\bar{p}_O^{CN*} = \frac{w_o+a}{2}$, $E[\bar{D}_O^{CN*}] = \frac{a-w_o}{2}$, $E[\bar{\pi}_O^{CN*}] = (\frac{a-w_o}{2})^2 - F_O$, $E[\bar{\pi}_M^{CN*}] = \frac{(w_o-m_o)(a-w_o)}{2} - F_M$.

We further conduct sensitivity analysis on the optimal decisions and expected profits, and summarize the results in Table 3.4. Comparing the results with the case with risk sensitive ODL product seller (Table 3.3), we find that the impacts of market base a and wholesale price w_o on the optimal solutions remain unchanged.

Table 3.4. Sensitivity analysis for the optimal decisions and expected profits in the basic model without fashion knockoffs and with risk neutral ODL product seller.

	$\bar{p}_O^{CN*}$	$E[\bar{D}_O^{CN*}]$	$E[\bar{\pi}_O^{CN*}]$	$E[\bar{\pi}_M^{CN*}]$
$a\uparrow$	↑	↑	↑	↑
$w_o\uparrow$	↑	↓	↓	↑ if $m_o < w_o < \frac{a+m_o}{2}$ ↓ if $\frac{a+m_o}{2} < w_o < a$

3.2.2 With Fashion Knockoffs

We now consider the case with fashion knockoffs in the market. In order to capture the feature of quick copying of the knockoff product seller, we consider that the ODL product seller and the knockoff product seller arrive in the market during the same season and make pricing decisions simultaneously. Following Chou et al. (2012) and Chiu et al. (2018), we construct the demand functions for the ODL product and the knockoff product as shown in Eq.(3.7) and Eq.(3.8), where α is the market base for the knockoff product, and b is the price-demand sensitivity coefficient ($b > 0$). A larger b means that demand for the knockoff product is more sensitive with respect to its price. Considering the social influence, demands for the ODL product and the knockoff product are inter-related. This feature is reflected in the coefficients of ϕ and β. A higher demand for the ODL product induces a higher demand for the knockoff product, while a higher demand for the knockoff product reduces demand for the ODL product. ε_O and ε_K are two independent random variables, which capture the demand uncertainties for the ODL product and the knockoff product respectively, with zero mean and standard derivation of σ_O and σ_K.

$$D_O = a - p_O - \phi D_K + \varepsilon_O, \tag{3.7}$$

$$D_K = \alpha - bp_K + \beta D_O + \varepsilon_K. \tag{3.8}$$

We can then derive the profit functions for the ODL product seller and the knockoff product seller as

$$\pi_O = (p_O - w_O)D_O - F_O, \tag{3.9}$$

$$\pi_K = (p_K - w_K)D_K. \tag{3.10}$$

Since the manufacturer produces for both the ODL product seller and the knockoff product seller, its profit comes from two parts as shown in Eq.(3.11).

$$\pi_M^* = (w_O - m_O)D_O^* + (w_K - m_K)D_K^* - F_M. \tag{3.11}$$

Risk Sensitive ODL product seller

We can easily derive the objective functions for the ODL product seller and the knockoff product seller:

$$U[\pi_O] = (p_O - w_O)\frac{a-\phi\alpha-p_O+\phi b p_K-k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{1+\phi\beta} - F_O. \tag{3.12}$$

$$E[\pi_K] = (p_K - w_K)\frac{a\beta+\alpha-\beta p_O-bp_K}{1+\phi\beta}. \tag{3.13}$$

Maximizing Eq.(3.12) and Eq.(3.13) with respect to p_O and p_K respectively yields the optimal prices for the original product p_O^* and the knockoff product p_K^*. Through substitution, we obtain the optimal solutions: $p_O^* = \frac{(2+\phi\beta)a+(bw_K-\alpha)\phi+2w_O-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{4+\phi\beta}$,

$p_K^* = \frac{(2+\phi\beta)\alpha+(a-w_O)\beta+2w_K b+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{b(4+\phi\beta)}$, $D_O^{MSD*} = \frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}$,

$E[D_K^*] = \frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}$,

$$U[\pi_O^*] = \frac{1}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(4+\phi\beta)}\right]^2 - F_O,$$

$$E[\pi_K^*] = \frac{1}{b(1+\phi\beta)}\left[\frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(4+\phi\beta)}\right]^2, \quad E[\pi_M^*] = (w_O - m_O)$$

$$\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)+(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right] + (w_K - m_K)\left[\frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right] -$$

F_M.

To guarantee that demand for the ODL product is always positive, we have $|k| < K_2$, where $K_2 = \frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{2\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}$.

We further conduct sensitivity analysis on the optimal solutions and summarize the results in Table 3.5. When the primary market size for the ODL product a is larger, demand for the ODL product is larger. The ODL product seller can set a higher selling price, hence getting more profits. Since a higher demand for the ODL product will lead to an increased demand for the knockoff product, the knockoff product seller can set a higher selling price, hence also getting better off. Meanwhile, the common manufacturer which produces for the ODL product seller and the knockoff product seller is benefited. Similarly, when the market size for the knockoff product α is larger, the knockoff product seller is benefited by setting a higher selling price and gaining a larger consumer demand. Due to the negative demand inter-relationship ϕ, with the increase in knockoff product's demand, demand for the ODL product decreases. The ODL product seller hence is worse off. When the ratio of manufacturer's profit margin ($\frac{w_O-m_O}{w_K-m_K}$) is relatively small (i.e., manufacturer's profit margin from the ODL product seller is relatively small), the manufacturer can still benefit from producing for the knockoff product seller; while as this ratio keeps increasing, the manufacturer can no longer be benefited due to the demand loss for the ODL product.

When the price-demand sensitivity coefficient b is larger, which means that consumers for the knockoff product are more sensitive to price, demand for the knockoff product decreases, and the knockoff product seller is worse off. With the decrease in knockoff product's demand, demand for the

ODL product increases, so the ODL product seller is benefited. When the ratio of manufacturer's profit margin is sufficiently large, the manufacturer can benefit from producing the ODL product; while when the ratio is getting smaller, manufacturer's profit gain from the ODL product seller can no longer counteract its profit loss from the knockoff product seller. We thus suggest that the manufacturer increase its profit margin from the ODL product seller (or decrease its profit margin from the knockoff product seller) if the price-demand sensitivity coefficient b is higher.

Besides, when the manufacturer raises the wholesale price w_O for the ODL product, the ODL product seller will raise the selling price sequentially. Demands for the ODL and knockoff products decrease, hence both the ODL product seller and knockoff product seller are worse off. However, for the manufacturer, when the ODL product seller is more risk averse and when the wholesale price for the ODL product is relatively small, the manufacturer can still benefit from producing the two products; when the wholesale price for the ODL product is getting higher, the gain from wholesale price cannot compensate the demand loss of the two products, the manufacturer hence is worse off; while interestingly, when the ODL product seller is less risk averse, raising the wholesale price always harms the manufacturer. Similarly, when the manufacturer raises the wholesale price for the knockoff product (w_K), the knockoff product seller raises the selling price. As a result, demand for the knockoff product decreases sequentially, and the knockoff product seller is worse off. With the decrease in knockoff's demand, demand for the ODL product increases, so the ODL product seller is better off.

For the manufacturer, when the wholesale price is relatively small, the profit gain from the ODL product seller surpasses the profit loss from the knockoff product seller, so the manufacturer is benefited; when the wholesale price is sufficiently large, the manufacturer is harmed due to the significant loss from the knockoff product seller. The threshold governing whether the manufacturer is benefited or not is increasing in the risk sensitivity coefficient of the ODL product seller (k), that is, when the ODL product seller is more risk averse, the manufacturer is more prone to benefit from raising the wholesale price for the knockoff product. The findings suggest that the manufacturers who are producing for both the ODL product seller and knockoff product seller should consider the risk attitude of the ODL product seller when making the wholesale price decision. Specifically, it is unwise for the manufacturer to blindly pursue higher wholesale prices, especially when the ODL product seller is less risk averse. Note that the demand inter-relationship coefficients ϕ and β are key to explain the above findings.

For risk and uncertainty related parameters, when the ODL product seller is more risk averse ($k \uparrow$), its perceived demand decreases, it tends to set a lower selling price to stimulate demand, ODL product seller is hence worse off. We interestingly find that, compared with the social influence, risk attitude of the ODL product seller exerts a larger effect on the demand for knockoff product. That is, although a decrease in ODL product's demand decreases the knockoff's demand to some extent, as the ODL product seller gets more risk averse, its perceived demand for the knockoff product still increases. For the common manufacturer, its profit gain from the knockoff product seller exceeds its profit loss from the ODL product seller, thus it is always benefited.

Moreover, the effects of demand uncertainty (σ_O and σ_K) are determined by the risk attitude of the ODL product seller. When the ODL product seller is risk averse, a more volatile market demand benefits the knockoff product seller and the manufacturer while it harms the ODL product seller; when the ODL product seller is risk seeking, a more volatile market demand harms the knockoff product seller and the manufacturer while it benefits the ODL product seller. The findings indicate that different risk attitudes of the ODL product seller can lead to totally opposite results. It is interesting to find that, when the ODL product seller is risk seeking rather than risk averse towards the demand uncertainty, the ODL product seller can be better off.

Table 3.5. Sensitivity analysis for the optimal decisions and objectives in the basic model with fashion knockoffs and with risk sensitive ODL product seller.

		p_O^*	p_K^*	D_O^{MSD*}	$E[D_K^*]$	$U[\pi_O^*]$	$E[\pi_K^*]$	$E[\pi_M^*]$
$a\uparrow$		↑	↑	↑	↑	↑	↑	↑
$\alpha\uparrow$		↓	↑	↓	↑	↓	↑	↑ if $0<\frac{w_O-m_O}{w_K-m_K}<\frac{2+\beta\phi}{\phi}$ ↓ if $\frac{w_O-m_O}{w_K-m_K}>\frac{2+\beta\phi}{\phi}$
$b\uparrow$		↑	↓	↑	↓	↑	↓	↑ if $\frac{w_O-m_O}{w_K-m_K}>\frac{2+\beta\phi}{\phi}$ ↓ if $0<\frac{w_O-m_O}{w_K-m_K}<\frac{2+\beta\phi}{\phi}$
$w_O\uparrow$		↑	↓	↓	↓	↓	↓	$-K_2<k<K_4$: ↓ $K_4<k<K_2$: ↑ if $m_O<w_O<\widetilde{w_O}$; ↓ if $w_O>\widetilde{w_O}$
$w_K\uparrow$		↑	↑	↑	↓	↑	↓	↑ if $m_K<w_K<\widetilde{w_K}$ ↓ if $w_K>\widetilde{w_K}$
$k\uparrow$		↓	↑	↓	↑	↓	↑	↑
$\sigma_O(\sigma_K)\uparrow$	Risk averse	↓	↑	↓	↑	↓	↑	↑
	Risk seeking	↑	↓	↑	↓	↑	↓	↓

Remarks: $\widetilde{w_O}=\frac{1}{2}\left(a+\frac{m_O(2+\phi\beta)+\phi(bw_K-\alpha)-\beta(w_K-m_K)}{2+\phi\beta}+k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}\right)$,

$\widetilde{w_K}=\frac{(2+\phi\beta)(\alpha+bm_K)+\beta(a-w_O)+b\phi(w_O-m_O)+k\beta\sqrt{\phi^2\sigma_K^2+\sigma_O^2}}{2b(2+\beta\phi)}$, which are increasing in k.

$K_4 = \frac{-(a-m_O)(2+\phi\beta)-\phi\,(bw_K-\alpha)+\beta\,(w_K-m_K)}{(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}$.

We summarize the main findings regarding the wholesale price in Proposition 3.1.

Proposition 3.1. *When* $-K_2 < k < K_4$[19], $E[\pi_M^*]$ *is decreasing in* w_O*; When* $K_4 < k < K_2$, $E[\pi_M^*]$ *is concave in* w_O*, where* $K_2 = \frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{2\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}$ *and* $K_4 = \frac{-(a-m_O)(2+\phi\beta)-\phi\,(bw_K-\alpha)+\beta\,(w_K-m_K)}{(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}$.

We interestingly find that, when there is no knockoff product in the market, the expected profit of the manufacturer is strictly concave in the wholesale price of the ODL product w_o, irrespective of the risk attitude of the ODL product seller. However, when there is knockoff product in the market, the risk attitude of the ODL product seller can decide the monotonicity of $E[\pi_M^*]$ (concave or decreasing). To be specific, when the ODL product seller is less risk averse, an increase in the wholesale price for ODL product always leads to a decrease in the expected profit of the manufacturer; when the ODL product seller is more risk averse, an increase in the wholesale price for ODL product first increases and then decreases the expected profit of the manufacturer.

This can be explained as follows: the threshold governing whether the manufacturer is benefited or harmed ($\widetilde{w_O}$) is increasing in the risk sensitivity coefficient of the ODL product seller (k). When the ODL product seller is more risk averse, the manufacturer is more prone to be benefited. The findings have very good implications: in contrast to the conventional wisdom that raising the wholesale price always benefits the manufacturer, we find that the manufacturer should be strategic in deciding the wholesale price when the ODL product seller possesses a risk sensitive attitude. To be specific, when the ODL product seller is more risk averse, there exist cases where the manufacturer benefits from raising the wholesale price; whereas when the ODL product seller is less risk averse, raising the wholesale price always harms the manufacturer.

Risk neutral ODL product seller

We now consider the case where the ODL product seller is risk neutral. We can easily derive the following results from the basic model by setting $k = 0$: $p_O^{CN*} = \frac{(2+\phi\beta)a+(bw_K-\alpha)\phi+2w_O}{4+\phi\beta}$,

$p_K^{CN*} = \frac{(2+\phi\beta)\alpha+(a-w_O)\beta+2w_K b}{b(4+\phi\beta)}$, $E[D_O^{CN*}] = \frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{(1+\phi\beta)(4+\phi\beta)}$,

$E[D_K^{CN*}] = \frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)}{(1+\phi\beta)(4+\phi\beta)}$, $E[\pi_O^{CN*}] = \frac{1}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{(4+\phi\beta)}\right]^2 - F_O$,

[19] We analytically find that K_4 can be positive, negative, or even zero. When $K_4 = 0$, it means that when the ODL product seller is risk seeking (risk averse), the optimal expected profit of the manufacturer is decreasing (concave) in the wholesale price of the ODL product.

$E[\pi_K^{CN*}] = \frac{1}{b(1+\phi\beta)}\left[\frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)}{(4+\phi\beta)}\right]^2, E[\pi_M^{CN*}] = (w_O - m_O)\frac{(a-w_O)(2+\phi\beta)+\phi(bw_K-\alpha)}{(1+\phi\beta)(4+\phi\beta)} + (w_K - m_K)\frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)}{(1+\phi\beta)(4+\phi\beta)} - F_M$.

We further conduct sensitivity analysis on the optimal decisions and expected profits, and summarize the results in Table 3.6. Comparing the results with the case with risk sensitive ODL product seller (Table 3.5), we find that most of the results are consistent. The major difference lies in the impact of w_O on $E[\pi_M^{CN}]^*$. When the production cost for the ODL product is relatively small, and when the wholesale price is comparatively small, demand reduction for the ODL product is not significant, so the manufacturer can still be better off; while when the production cost is sufficiently large, with an increase in the wholesale price, demand for the ODL product decreases remarkably. Consequently, the manufacturer can no longer be benefited.

Table 3.6. Sensitivity analysis for the optimal decisions and objectives in the basic model with fashion knockoffs and with risk neutral ODL product seller.

	p_O^{CN*}	p_K^{CN*}	$E[D_O^{CN*}]$	$E[D_K^{CN*}]$	$E[\pi_O^{CN*}]$	$E[\pi_K^{CN*}]$	$E[\pi_M^{CN*}]$	
$a\uparrow$	↑	↑	↑	↑	↑	↑	↑	
$\alpha\uparrow$	↓	↑	↓	↑	↓	↑	↑ if $0 < \frac{w_O-m_O}{w_K-m_K} < \frac{2+\beta\phi}{\phi}$ ↓ if $\frac{w_O-m_O}{w_K-m_K} > \frac{2+\beta\phi}{\phi}$	
$b\uparrow$	↑	↓	↑	↓	↑	↓	↑ if $\frac{w_O-m_O}{w_K-m_K} > \frac{2+\beta\phi}{\phi}$ ↓ if $0 < \frac{w_O-m_O}{w_K-m_K} < \frac{2+\beta\phi}{\phi}$	
$w_O\uparrow$	↑	↓	↓	↓	↓	↓	if $m_O < \tilde{w}_O^{CN}$	↑ if $m_O < w_O < \tilde{w}_O^{CN}$ ↓ if $w_O > \tilde{w}_O^{CN}$
							if $m_O > \tilde{w}_O^{CN}$	↓
$w_K\uparrow$	↑	↑	↑	↓	↑	↓	↑ if $m_K < w_K < \tilde{w}_K^{CN}$ ↓ if $w_K > \tilde{w}_K^{CN}$	

Remarks: $\tilde{w}_O^{CN} = \frac{1}{2}\left(a + \frac{m_O(2+\phi\beta)+\phi(bw_K-\alpha)-\beta(w_K-m_K)}{2+\phi\beta}\right), \tilde{w}_K^{CN} = \frac{(2+\phi\beta)(\alpha+bm_K)+\beta(a-w_O)+b\phi(w_O-m_O)}{2b(2+\beta\phi)}$.

3.3 Impacts of Fashion Knockoffs

After getting the results in Section 3.2, we now proceed to explore the impacts of fashion knockoffs on the ODL product supply chain and its agents. For a notational purpose, we define $\triangle\,\pi_O = U[\pi_O^*] - U[\bar{\pi}_O^*]$, $\triangle\,\pi_M = E[\pi_M^*] - E[\bar{\pi}_M^*]$.

Risk sensitive ODL product seller

We first consider the case with risk sensitive ODL product seller. By comparing the optimal MSD objectives of the ODL product seller with and without the presence of fashion knockoffs, we have

$$\triangle \pi_O = \frac{1}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(4+\phi\beta)}\right]^2 - \left(\frac{a-w_O-k\sigma_O}{2}\right)^2. \tag{3.14}$$

We then compare the optimal expected profits of the manufacturer with and without the presence of fashion knockoffs and have the following:

$$\triangle \pi_M = (w_O - m_O)\left[\frac{(a-w_O)(2+\phi\beta)+\phi(bw_K-\alpha)}{(1+\phi\beta)(4+\phi\beta)} + \frac{(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right]$$

$$+(w_K - m_K)\left[\frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)}{(1+\phi\beta)(4+\phi\beta)} + \frac{\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right] - \frac{(w_O-m_O)(a-w_O+k\sigma_O)}{2}. \tag{3.15}$$

When $\triangle \pi_O > 0$ (resp. $\triangle \pi_M > 0$), the presence of fashion knockoffs benefits the ODL product seller (resp. the manufacturer); otherwise, the presence of fashion knockoffs harms the ODL product seller (resp. the manufacturer).

To guarantee that demands for the ODL product and the knockoff product are both positive, we have $|k| < \min\{K_1, K_2\}$. It means that the ODL product seller's risk attitude is within a reasonable range, i.e., neither extremely risk seeking nor extremely risk averse. The following analyses are all based on this precondition. We summarize the key results in Proposition 3.2. Please refer to Appendix A Table A1 for the expressions of thresholds shown in propositions and corollaries.

Proposition 3.2. *Under the basic model, we have:*
(a) When $0 < \frac{\sigma_K}{\sigma_O} < S_1$*, the presence of fashion knockoffs benefits the ODL product seller if and only if* $k > K_3 > 0$ *(risk averse); when* $\frac{\sigma_K}{\sigma_O} > S_1$*, the presence of fashion knockoffs benefits the ODL product seller if and only if* $k < K_3 < 0$ *(risk seeking). (b) When* $0 < \frac{\sigma_K}{\sigma_O} < S_2$*, the presence of fashion knockoffs benefits the manufacturer if and only if* $0 < k < G$ *(risk averse) or* $k < \min\{G, 0\}$[20] *(risk seeking); when* $\frac{\sigma_K}{\sigma_O} > S_2$*, the presence of fashion knockoffs benefits the manufacturer if and only if* $k > \max\{G, 0\}$ *(risk averse) or* $G < k < 0$ *(risk seeking).*

We denote $\frac{\sigma_K}{\sigma_O}$ as the ratio of demand uncertainty, which captures the disparity of demand uncertainties between the knockoff product and the ODL product. When the ratio is relatively small ($0 < \frac{\sigma_K}{\sigma_O} < S_1$), the presence of fashion knockoffs benefits the ODL product seller when the ODL product seller is sufficiently risk averse; when the ratio is sufficiently large ($\frac{\sigma_K}{\sigma_O} > S_1$), the presence of fashion knockoffs benefits the ODL product seller when the ODL product seller is sufficiently risk

[20] We analytically find that G can be either positive or negative.

seeking. The findings highlight the important role of risk attitudes when making decisions. In order to benefit from the presence of fashion knockoffs, the ODL product seller is advised to carefully target the market based on its risk attitude. To be specific, for the risk averse ODL product seller, it is wise to target the market with relatively large demand uncertainty (e.g., seasonal or fashionable products); while for the risk seeking ODL product seller, it is more beneficial to target the market with relatively small demand uncertainty (e.g., classic products). The findings also reveal that risk seeking can do more good than harm in some cases.

The impact of fashion knockoffs on the manufacturer is also determined by the ratio of demand uncertainty and the risk attitude of the ODL product seller. Specifically, when the ratio of demand uncertainty is relatively small ($0 < \frac{\sigma_K}{\sigma_O} < S_2$), and when the ODL product seller is less risk averse or more risk seeking, the presence of fashion knockoffs benefits the manufacturer; when the ratio of demand uncertainty is sufficiently large ($\frac{\sigma_K}{\sigma_O} > S_2$), and when the ODL product seller is more risk averse or less risk seeking, the presence of fashion knockoffs benefits the manufacturer. The findings reveal that the impact of fashion knockoffs on the manufacturer is not only decided by the ODL product seller's type of risk attitude, but also its magnitude of risk attitude. Hence, the manufacturer should carefully decide whether to produce for the knockoff product seller based on the above findings; otherwise, it may even be worse off.

Now we examine the conditions under which the presence of fashion knockoffs benefits the ODL product supply chain. It is clear that when the ODL product seller and the manufacturer are benefited at the same time (Proposition 3.2 (a) and (b) are both satisfied), the presence of fashion knockoffs benefits the ODL product supply chain. We summarize the key results in Corollary 3.1.

Corollary 3.1. *Under the basic model, we have:*

(a) When $0 < \frac{\sigma_K}{\sigma_O} < S_2$*, the presence of fashion knockoffs benefits the ODL product supply chain and its agents if and only if* $0 < K_3 < k < G$ *(risk averse).*

(b) When $S_2 < \frac{\sigma_K}{\sigma_O} < S_1$*, the presence of fashion knockoffs benefits the ODL product supply chain and its agents if and only if* $k > \max\{G, K_3\} > 0$ *(risk averse).*

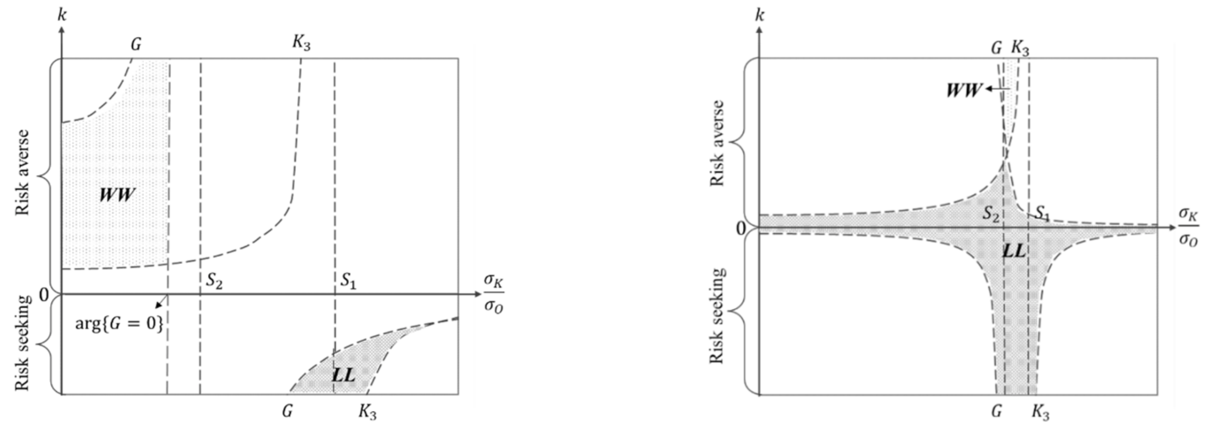
(c) When $\frac{\sigma_K}{\sigma_O} > S_1$*, the presence of fashion knockoffs benefits the ODL product supply chain and its agents if and only if* $G < k < K_3 < 0$ *(risk seeking).*

We interestingly find that the presence of fashion knockoffs can benefit the ODL product supply chain and its agents simultaneously, while it is under very strict conditions (considering ratio of demand uncertainty and type and magnitude of the ODL product seller's risk attitude). To be specific, when the ratio of demand uncertainty is relatively small ($0 < \frac{\sigma_K}{\sigma_O} < S_2$), the presence of fashion knockoffs benefits the ODL product supply chain and its agents when the ODL product seller is moderately risk averse; when the ratio of demand uncertainty is moderate ($S_2 < \frac{\sigma_K}{\sigma_O} < S_1$), the presence of fashion knockoffs benefits the ODL product supply chain and its agents when the ODL product seller is sufficiently risk averse; when the ratio of demand uncertainty is sufficiently large ($\frac{\sigma_K}{\sigma_O} > S_1$), the presence of fashion knockoffs benefits the ODL product supply chain and its agents when the ODL product seller is moderately risk seeking.

The findings suggest that the ODL product seller should carefully target the market based on both its type of risk attitude and magnitude of risk attitude when encountering fashion knockoffs. Specifically, for the risk averse ODL product seller, it is wise to target the market with relatively large demand uncertainty (e.g., seasonal or fashionable products), and its risk averse attitude can benefit the ODL product supply chain and its agents in most cases; while for the risk seeking ODL product seller, it is more beneficial to target the market with relatively small demand uncertainty (e.g., classic products). Notice that, the ODL product seller should be very careful with the level of risk seeking; it is unadvisable to hire a manager with either too mild or too strong risk seeking attitude, as it will harm the ODL product supply chain.

To show a clear picture of the impacts of fashion knockoffs on the ODL product seller and the manufacturer, we depict Figure 3.3. All the parameters we set satisfy the model assumptions and fit the physical meanings. Figure 3.3 reveals two important findings. First, when the ratio of manufacturer's profit margin ($\frac{w_O - m_O}{w_K - m_K}$) is relatively small, it is easier for the ODL product seller and the manufacturer to achieve win-win from the presence of fashion knockoffs; while when the ratio is sufficiently large, the ODL product seller and the manufacturer are more prone to be lose-lose from the presence of fashion knockoffs. The findings provide the manufacturer with feasible pricing strategy when encountering fashion knockoffs. Specifically, the manufacturer is advised to set a relatively low (high) wholesale price for the ODL (knockoff) product when there is knockoff product in the market.

Second, we observe that the outcome of win-win is easier to be achieved when the ODL product seller is risk averse; whereas the outcome of lose-lose is easier to be achieved when the ODL product seller is risk seeking. The findings imply that the risk attitude of the ODL product seller is a critical factor governing whether the presence of fashion knockoffs can benefit the ODL product seller and the manufacturer at the same time. More specifically, when there is knockoff product in the market, a risk averse ODL product seller (rather than a risk seeking one) is more likely to achieve a win-win outcome in the ODL product supply chain.



(a) when $\frac{w_O - m_O}{w_K - m_K}$ is small. (b) when $\frac{w_O - m_O}{w_K - m_K}$ is large.

Figure 3.3. Impacts of fashion knockoffs on the ODL product seller and the manufacturer (P.S.: We let $a = 0.9$, $\alpha = 0.6, \phi = 0.3$, $\beta = 0.5$, $b = 1.5$, $w_O = 0.6$, $m_O = 0.3$, $m_K = 0.1$, $\sigma_O = 2, \sigma_K \in [0, 4.8]$, $\frac{\sigma_K}{\sigma_O} \in [0, 2.4]$, $w_K = 0.2$ in (a) and $w_K = 0.12$ in (b). Region WW represents the win-win outcome where $\triangle\, \pi_O > 0$ and $\triangle\, \pi_M > 0$, and Region LL represents the lose-lose outcome where $\triangle\, \pi_O < 0$ and $\triangle\, \pi_M < 0$).

Risk neutral ODL product seller

We now consider the case where the ODL product seller is risk neutral. Comparing the expected profits of the ODL product seller (or manufacturer) with and without the presence of fashion knockoffs, we have:

$$\triangle\, \pi_O^{CN} = \frac{1}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{(4+\phi\beta)}\right]^2 - \left(\frac{a-w_O}{2}\right)^2. \tag{3.16}$$

$$\triangle\, \pi_M^{CN} = \frac{(w_O-m_O)\big((2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)\big)}{(1+\phi\beta)(4+\phi\beta)} + \frac{(w_K-m_K)\big(\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)\big)}{(1+\phi\beta)(4+\phi\beta)} - \frac{(w_O-m_O)(a-w_O)}{2}. \tag{3.17}$$

When $\triangle\, \pi_O^{CN} > 0$ (resp. $\triangle\, \pi_M^{CN} > 0$), the presence of fashion knockoffs benefits the ODL product seller (resp. the manufacturer); otherwise, the presence of fashion knockoffs harms the ODL product seller (resp. the manufacturer). The findings are summarized in Proposition 3.3. Note that we only show the most general case where $m_O < \min\{\frac{-E-\sqrt{E^2-4DF}}{2D}, \frac{-E'-\sqrt{E'^2-4D'F'}}{2D'}\}$.

Proposition 3.3. *Under Model CN, we have:*

(a) The presence of fashion knockoffs always harms the ODL product seller.

(b) The presence of fashion knockoffs benefits the manufacturer if and only if $m_O < w_O < \frac{-E-\sqrt{E^2-4DF}}{2D}$ *or* $w_O > \frac{-E+\sqrt{E^2-4DF}}{2D}$*; the presence of fashion knockoffs harms the manufacturer if and only if* $\frac{-E-\sqrt{E^2-4DF}}{2D} < w_O < \frac{-E+\sqrt{E^2-4DF}}{2D}$.

(c) The presence of fashion knockoffs benefits the ODL product supply chain if and only if $m_O < w_O < \frac{-E'-\sqrt{E'^2-4D'F'}}{2D'}$ *or* $w_O > \frac{-E'+\sqrt{E'^2-4D'F'}}{2D'}$*; the presence of fashion knockoffs harms the ODL product supply chain if and only if* $\frac{-E'-\sqrt{E'^2-4D'F'}}{2D'} < w_O < \frac{-E'+\sqrt{E'^2-4D'F'}}{2D'}$.

Proposition 3.3 shows that when the ODL product seller is risk neutral towards the demand uncertainty, the presence of fashion knockoffs always harms the ODL product seller, while it can still benefit the common manufacturer and the ODL product supply chain under certain conditions. To be specific, when the wholesale price is relatively low or sufficiently high, the presence of fashion knockoffs benefits the manufacturer and the ODL product supply chain; when the wholesale price is moderate, the presence of fashion knockoffs harms the manufacturer and the ODL product supply chain. The results can be explained as follows: when the manufacturer sets a relatively low wholesale price for the ODL product, demand for the ODL product increases, which indirectly increases demand for the knockoff product. Hence, the manufacturer benefits from producing more products (both the ODL and knockoff products).

When the manufacturer sets a sufficiently high wholesale price for the ODL product, although both demands for the ODL product and knockoff product decrease, the manufacturer can still benefit from gaining a higher profit margin from the ODL product seller. When the profit gain from the manufacturer exceeds the profit loss from the ODL product seller, the ODL product supply chain is benefited. The results imply that in order to benefit from the presence of fashion knockoffs, the manufacturer should set its wholesale price for the ODL product very carefully. Comparing the results with Proposition 3.2 (risk sensitive ODL product seller), we uncover that risk attitude of the ODL product seller is a critical factor governing whether the ODL product seller can benefit from the presence of fashion knockoffs. Specifically, either a risk averse or risk seeking attitude can be beneficial.

3.4 Extended Models

3.4.1 Non-standard Markup Wholesale Pricing Policy

As motivated by the interview results that the manufacturer in fashion industry usually sets different markup rates for the ODL product seller and the knockoff product seller (see Appendix E), we define this markup policy as non-standard markup wholesale pricing policy and explore this policy in this subsection[21]. Following Taylor and Plambeck (2007), we consider that the common manufacturer shares the ex post gain $(\bar{p}_O' - m_O)(a - \bar{p}_O' + \varepsilon_O)$ with the ODL product seller according to the generalized Nash bargaining solution, where λ proportion of the gain is allocated to the ODL product seller, and the remaining $1 - \lambda$ is allocated to the manufacturer. Hence, in equilibrium, the wholesale price for the ODL product is $w_O = \lambda m_O + (1 - \lambda)p_O$. Similarly, wholesale price for the knockoff product is expressed as $w_K = \mu m_K + (1 - \mu)p_K$, where λ and μ are the markup rates for the ODL product seller and the knockoff product seller, respectively.

Risk sensitive ODL product seller

When there is no presence of fashion knockoffs, the objective functions for the ODL product seller and the manufacturer are

$$U[\bar{\pi}_O'] = \lambda(\bar{p}_O' - m_O)(a - \bar{p}_O' - k\sigma_O) - F_O, \tag{3.18}$$

$$E[\bar{\pi}_M'] = (1 - \lambda)(\bar{p}_O' - m_O)(a - \bar{p}_O') - F_M. \tag{3.19}$$

Maximizing $U[\bar{\pi}_O']$ with respect to $\bar{p}_O'$ generates the optimal decision $\bar{p}_O'^* = \frac{a+m_O-k\sigma_O}{2}$. Through substitution, we obtain the optimal solutions: $U[\bar{\pi}_O'^*] = \lambda(\frac{a-m_O-k\sigma_O}{2})^2 - F_O$, $E[\bar{\pi}_M'^*] = \frac{(1-\lambda)((a-m_O)^2-(k\sigma_O)^2)}{4} - F_M$.

Likewise, we derive the objective functions for the ODL product seller, knockoff product seller and the manufacturer as follows:

$$U[\pi_O'] = \lambda(p_O' - m_O)\left(\frac{a-\phi\alpha}{1+\phi\beta} - \frac{p_O'}{1+\phi\beta} + \frac{\phi b}{1+\phi\beta}p_K' - \frac{k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{1+\phi\beta}\right) - F_O, \tag{3.20}$$

$$[\pi_K'] = \mu(p_K' - m_K)\left[\frac{a\beta+\alpha}{1+\phi\beta} - \frac{\beta}{1+\phi\beta}p_O' - \frac{b}{1+\phi\beta}p_K'\right], \tag{3.21}$$

$$E[\pi_M'^*] = (1 - \lambda)(p_O' - m_O)E[D_O'^*] + (1 - \mu)(p_K' - m_K)E[D_K'^*] - F_M. \tag{3.22}$$

Maximizing $U[\pi_O']$ and $E[\pi_K']$ with respect to p_O' and p_K', respectively, we obtain the optimal decisions: $p_O'^* = \frac{(2+\phi\beta)a+(bm_K-\alpha)\phi+2m_O-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{4+\phi\beta}, p_K'^* = \frac{(2+\phi\beta)\alpha+(a-m_O)\beta+2m_K b+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{b(4+\phi\beta)}$.

Through substitution, we derive the optimal solutions as:

$$U[\pi_O'^*] = \frac{\lambda}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-m_O)+\phi(bm_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{4+\phi\beta}\right]^2 - F_O,$$

[21] We also consider the case where the wholesale price is endogenously given in Appendix C. We did not put it in the mainbody since the results are too messy and cannot derive clear insights.

$$E[\pi_M'^*] = (1-\lambda)\left[\frac{(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{4+\phi\beta}\right]\left[\frac{(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)+(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right]$$

$$+(1-\mu)\left[\frac{\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{b(4+\phi\beta)}\right]\left[\frac{\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right]-F_M.$$

Define $\triangle\, \pi_O' = U[\pi_O'^*] - U[\bar{\pi}_O'^*]$, $\triangle\, \pi_M' = E[\pi_M'^*] - E[\bar{\pi}_M'^*]$.

When $\triangle\, \pi_O' > 0$ ($resp.\triangle\, \pi_M' > 0$), the presence of fashion knockoffs benefits the ODL product seller (resp. the manufacturer); otherwise, the presence of fashion knockoffs harms the ODL product seller (resp. the manufacturer). We summarize the main results in Proposition 3.4.

Proposition 3.4. *Under the extended Model GNB, when the ODL product seller is risk sensitive, we have:*

(a) When $0 < \frac{\sigma_K}{\sigma_O} < S_1$, *the presence of fashion knockoffs benefits the ODL product seller if and only if* $k > K_3' > 0$ *(risk averse); when* $\frac{\sigma_K}{\sigma_O} > S_1$, *the presence of fashion knockoffs benefits the ODL product seller if and only if* $k < K_3' < 0$ *(risk seeking).*

(b) When $0 < \frac{\sigma_K}{\sigma_O} < S_2'$ *or* $0 < b < b'$, *the presence of fashion knockoffs benefits the manufacturer if and only if* $\left(0 < k < \frac{-Y-\sqrt{Y^2-4XZ}}{2X} \text{ or } k > \max\left\{\frac{-Y+\sqrt{Y^2-4XZ}}{2X}, 0\right\}\right)$ *(risk averse) or* $\left(k < \min\left\{\frac{-Y-\sqrt{Y^2-4XZ}}{2X}, 0\right\} \text{ or } \frac{-Y+\sqrt{Y^2-4XZ}}{2X} < k < 0\right)$*(risk seeking); when* $\frac{\sigma_K}{\sigma_O} > S_2'$ *and* $b > b'$, *the presence of fashion knockoffs benefits the manufacturer if and only if* $\max\left\{0, \frac{-Y+\sqrt{Y^2-4XZ}}{2X}\right\} < k < \frac{-Y-\sqrt{Y^2-4XZ}}{2X}$ *(risk averse) or* $\frac{-Y+\sqrt{Y^2-4XZ}}{2X} < k < \min\left\{\frac{-Y-\sqrt{Y^2-4XZ}}{2X}, 0\right\}$ *(risk seeking).*

Proposition 3.4 indicates that, under non-standard markup wholesale pricing policy, when the ODL product seller is risk sensitive, the presence of fashion knockoffs can benefit the ODL product seller and the manufacturer under certain conditions. The results are qualitatively consistent with the basic model. To be specific, the beneficial conditions for the ODL product seller remain the same as in the basic model. For the manufacturer, whether it is benefited or not is governed by the ratio of demand uncertainty ($\frac{\sigma_K}{\sigma_O}$) and the price demand sensitivity coefficient for the knockoff product (b). More specifically, when the ratio of demand uncertainty is relatively small ($0 < \frac{\sigma_K}{\sigma_O} < S_2'$) or the price demand sensitivity coefficient is comparatively small ($0 < b < b'$), the presence of fashion knockoffs benefits the manufacturer if the ODL product seller owns a mild or strong risk attitude (either risk averse or risk seeking); when the ratio of demand uncertainty is relatively large ($\frac{\sigma_K}{\sigma_O} > S_2'$) and the price demand sensitivity coefficient is relatively large ($b > b'$), the presence of fashion knockoffs benefits the manufacturer if the ODL product seller owns a moderate risk attitude (either risk averse or risk seeking). Exploring further on the threshold b', we uncover that with an increase in λ or a decrease in μ, b' will be increased, which means that it is easier for the manufacturer to benefit from the presence of fashion knockoffs. Hence, the manufacturer is advised to set a higher markup rate for the ODL product seller (λ) or a lower markup rate for the knockoff product seller (μ) in the presence of fashion knockoffs.

Now we examine the conditions under which the presence of fashion knockoffs benefits the ODL product supply chain. It is clear that when the ODL product seller and the manufacturer are benefited at the same time (Proposition 3.4 (a) and (b) are both satisfied), the presence of fashion knockoffs benefits the ODL product supply chain. The key results are summarized in Corollary 3.2.

Corollary 3.2. *Under the extended Model GNB, when the ODL product seller is risk sensitive, we have:*

(a) If $0 < b < b'$, *the presence of fashion knockoffs benefits the ODL product supply chain and its agents if and only if* $k < \frac{-Y-\sqrt{Y^2-4XZ}}{2X}$ *or* $k > \frac{-Y+\sqrt{Y^2-4XZ}}{2X}$ *(risk averse or risk seeking).*

(b) If $b > b'$, *the presence of fashion knockoffs benefits the ODL product supply chain and its agents if and only if one of the following conditions is satisfied:*

(i) When $0 < \frac{\sigma_K}{\sigma_O} < \min\{S_2', S_1\}$, *the presence of fashion knockoffs benefits the ODL product supply chain and its agents if and only if* $K_3' < k < \frac{-Y-\sqrt{Y^2-4XZ}}{2X}$ *or* $k > \max\left\{\frac{-Y+\sqrt{Y^2-4XZ}}{2X}, K_3'\right\}$ *(risk averse).*

(ii) When $\min\{S_1, S_2'\} < \frac{\sigma_K}{\sigma_O} < \max\{S_1, S_2'\}$, *, the presence of fashion knockoffs benefits the ODL product supply chain and its agents if and only if* $\max\left\{K_3', \frac{-Y+\sqrt{Y^2-4XZ}}{2X}\right\} < k < \frac{-Y-\sqrt{Y^2-4XZ}}{2X}$*(risk averse) or* $\left(k < \min\left\{\frac{-Y-\sqrt{Y^2-4XZ}}{2X}, K_3'\right\}\right.$ *or* $\left.\frac{-Y+\sqrt{Y^2-4XZ}}{2X} < k < K_3'\right)$ *(risk seeking).*

(iii) When $\frac{\sigma_K}{\sigma_O} > \max\{S_2', S_1\}$, *the presence of fashion knockoffs benefits the manufacturer if and only if* $\frac{-Y+\sqrt{Y^2-4XZ}}{2X} < k < \min\left\{\frac{-Y-\sqrt{Y^2-4XZ}}{2X}, K_3'\right\}$ *(risk seeking).*

From Corollary 3.2, we find that the presence of fashion knockoffs is likely to benefit the ODL product supply chain and its agents under the non-standard markup wholesale pricing policy. The results are consistent with the basic model. The beneficial conditions are determined by the price demand sensitivity coefficient for the knockoff product (b) and the ratio of demand uncertainty ($\frac{\sigma_K}{\sigma_O}$).

Specifically, when the price demand sensitivity coefficient for the knockoff product is relatively small ($0 < b < b'$), the presence of fashion knockoffs benefits the manufacturer if the ODL product seller is sufficiently risk seeking or risk averse; when the price demand sensitivity coefficient for the knockoff product is comparatively large ($b > b'$), the beneficial conditions for the manufacturer are much more complex and are governed by the ratio of demand uncertainty.

To be specific, when the ratio of demand uncertainty is relatively small ($0 < \frac{\sigma_K}{\sigma_O} < \min\{S_2', S_1\}$), the presence of fashion knockoffs benefits the manufacturer if the ODL product seller is moderately or sufficiently risk averse; when the ratio of demand uncertainty is moderate ($\min\{S_1, S_2'\} < \frac{\sigma_K}{\sigma_O} < \max\{S_1, S_2'\}$), the presence of fashion knockoffs benefits the manufacturer if the ODL product seller is moderately risk averse or sufficiently (or moderately) risk seeking; when the ratio of demand uncertainty is sufficiently large ($\frac{\sigma_K}{\sigma_O} > \max\{S_2', S_1\}$), the presence of fashion knockoffs benefits the manufacturer if the ODL product seller is moderately risk seeking. The findings reveal that when the ratio of demand uncertainty is relatively small or sufficiently large, only one kind of risk attitude of the ODL product seller (either risk averse or risk seeking) can benefit the ODL product supply chain and its agents; while when the ratio of demand uncertainty is moderate, both kinds of risk attitude of the ODL product seller can benefit the ODL product supply chain and its agents.

Risk neutral ODL product seller

Comparing the profits of the ODL product seller (manufacturer) with and without the presence of fashion knockoffs, we have

$$\Delta \pi_O^{CN'} = \frac{\lambda}{1+\phi\beta}\left[\frac{(2+\phi\beta)(a-m_O)+\phi(bm_K-\alpha)}{4+\phi\beta}\right]^2 - \lambda\left(\frac{a-m_O}{2}\right)^2. \tag{3.23}$$

$$\Delta \pi_M^{CN'} = \frac{(1-\lambda)[(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)]^2}{(1+\phi\beta)(4+\phi\beta)^2}$$
$$+\frac{(1-\mu)[\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)]^2}{b(1+\phi\beta)(4+\phi\beta)^2} - \frac{(1-\lambda)(a-m_O)^2}{4}. \tag{3.24}$$

When $\Delta \pi_O^{CN'} > 0$ (resp. $\Delta \pi_M^{CN'} > 0$), the presence of fashion knockoffs benefits the ODL product seller (resp. the manufacturer); otherwise, the presence of fashion knockoffs harms the ODL product seller (resp. the manufacturer). The findings are summarized in Proposition 3.5.

Proposition 3.5. *Under the extended Model GNB, when the ODL product seller is risk neutral, we have:*

(a) The presence of fashion knockoffs always harms the ODL product seller.

(b) The presence of fashion knockoffs benefits the manufacturer if and only if $0 < \mu < 1 - \frac{(1-\lambda)M}{N}$.

(c) The presence of fashion knockoffs benefits the ODL product supply chain if and only if $0 < \mu < 1 - \frac{M}{N}$.

Proposition 3.5 reveals that under the non-standard markup wholesale pricing policy, when the ODL product seller is risk neutral, the presence of fashion knockoffs always harms the ODL product seller; while it can benefit the manufacturer and the ODL product supply chain under certain conditions. The results are qualitatively consistent with the basic model. To be specific, when the markup rate for the knockoff product (μ) is relatively small, the presence of fashion knockoffs benefits the manufacturer, and the threshold of μ (i.e., $1 - \frac{(1-\lambda)M}{N}$) is positively correlated with λ, which means that with a larger markup rate for the ODL product (λ), it is easier for the manufacturer to benefit from the presence of fashion knockoffs.

Moreover, we discover that the presence of fashion knockoffs can benefit the ODL product supply chain when the markup rate for the knockoff product is relatively small, regardless of the markup rate for the ODL product. Based on these interesting findings, we hence encourage the manufacturer to set a higher markup rate for the ODL product. By doing so, the manufacturer is more likely to be benefited while without hurting the interest of the whole ODL product supply chain. Comparing the results with Proposition 3.4 where the ODL product seller is risk sensitive, we discover that under the non-standard markup wholesale pricing policy, the presence of fashion knockoffs benefits the ODL product seller only when the ODL product seller is risk sensitive, which further highlights the importance of risk attitude of the ODL product seller.

3.4.2 Different Manufacturers

Recall that in the basic model, we consider a common manufacturer producing for both the knockoff product seller and ODL product seller. In this section, we consider another configuration of the supply chain, i.e., there are two manufacturers producing for the ODL product seller and the knockoff product seller respectively (as depicted in Figure 3.4). Explicitly, rather than produce for both the ODL product seller and the knockoff product seller, we consider that manufacturer 1 only produces for the ODL product seller and explore how the satisfying conditions derived under the basic model change.

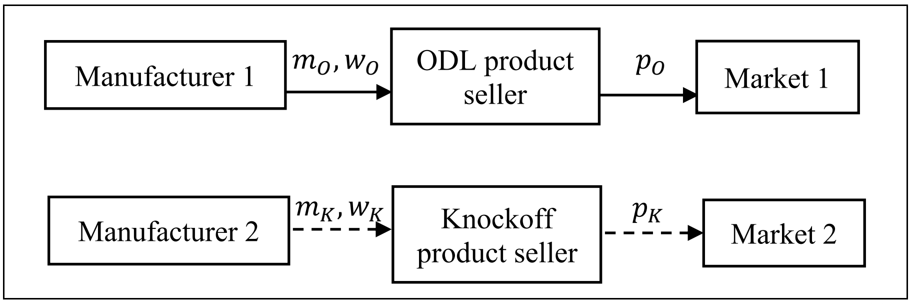


Figure 3.4. The supply chain system with different manufacturers[22].

Risk sensitive ODL product seller

We use the superscript DS (different manufacturers, risk sensitive ODL product seller) to represent the new functions or results derived in this subsection. We can easily extend the findings of the ODL product seller in the basic model into this extended model.

When considering the manufacturer, we have

$$E[\pi_{M1}^{DS*}] = (w_O - m_O)E[D_O^*] - F_M =$$

$$(w_O - m_O)\left[\frac{(a-w_O)(2+\phi\beta)+\phi(bw_K-\alpha)}{(1+\phi\beta)(4+\phi\beta)} + \frac{(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right] - F_M. \tag{3.25}$$

Comparing the optimal expected profits for manufacturer 1 with and without the presence of fashion knockoffs, we have:

$$\Delta\,\pi_{M1}^{DS} = (w_O - m_O)\left[\frac{-\phi\beta(a-w_O)(3+\phi\beta)-2\phi(\alpha-bw_K)}{2(1+\phi\beta)(4+\phi\beta)} + \frac{(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)} - \frac{k\sigma_O}{2}\right]. \tag{3.26}$$

The main findings of the manufacturer are summarized in Proposition 3.6.

Proposition 3.6. *Under the extended Model DS, we have:*

(a) When $0 < \frac{\sigma_K}{\sigma_O} < S_1$*, the presence of fashion knockoffs benefits the ODL product seller if and only if* $k > K_3 > 0$ *(risk averse); when* $\frac{\sigma_K}{\sigma_O} > S_1$*, the presence of fashion knockoffs benefits the ODL product seller if and only if* $k < K_3 < 0$ *(risk seeking).*

(b) When $0 < \frac{\sigma_K}{\sigma_O} < S_2^{DS}$*, the presence of fashion knockoffs benefits the manufacturer if and only if* $k < G^{DS} < 0$ *(risk seeking); when* $\frac{\sigma_K}{\sigma_O} > S_2^{DS}$*, the presence of fashion knockoffs benefits the manufacturer if and only if* $k > G^{DS} > 0$ *(risk averse).*

Proposition 3.6 indicates that when there are different manufacturers producing for the ODL product seller and the knockoff product seller, there exist conditions under which the presence of fashion knockoffs benefits the ODL product seller or the manufacturer. It is intuitive that the beneficial

[22] The ODL product supply chain consists of manufacturer 1 and the ODL product seller.

conditions for the ODL product seller remain the same as in the basic model; while for the manufacturer, we notice some differences. To be specific, in the basic model where there is a common manufacturer producing for both the ODL product seller and the knockoff product seller, for a certain ratio of demand uncertainty (either large or small), both types of risk attitude (either risk averse or risk seeking) of the ODL product seller can benefit the manufacturer; while in this extended model, there is a strict correspondence between the ratio of demand uncertainty and the ODL product seller's type of risk attitude.

More specifically, when the ratio of demand uncertainty is relatively small ($0 < \frac{\sigma_K}{\sigma_O} < S_2^{DS}$), only the risk seeking attitude of the ODL product seller can benefit the manufacturer in the presence of fashion knockoffs; whereas when the ratio of demand uncertainty is sufficiently large ($\frac{\sigma_K}{\sigma_O} > S_2^{DS}$), only the risk averse attitude of the ODL product seller can benefit the manufacturer in the presence of fashion knockoffs. The findings imply that it is critical for the manufacturer to observe the risk attitude of the ODL product seller and demand uncertainty for the ODL product when deciding whether to produce for the knockoff product seller or not. Particularly, in the presence of fashion knockoffs, it is advisable for the manufacturer to produce only for the ODL product seller when the ODL product seller is more aggressive in ordering fashionable products (which implies that the ODL product seller tends to be risk seeking and demand uncertainty for the ODL product is relatively large), or when the ODL product seller is very careful in ordering classic products (which implies that the ODL product seller tends to be risk averse and demand uncertainty for the ODL product is relatively small).

Now we examine the conditions under which the presence of fashion knockoffs benefits the ODL product supply chain. It is clear that when the ODL product seller and the manufacturer are benefited at the same time (Proposition 3.6 (a) and (b) are both satisfied), the presence of fashion knockoffs benefits the ODL product supply chain. We summarize the key results in Corollary 3.3.

Corollary 3.3. *Under the extended Model DS, we have: When* $S_2^{DS} < \frac{\sigma_K}{\sigma_O} < S_1$, *the presence of fashion knockoffs benefits the ODL product supply chain and its agents if and only if* $k > \max\{F^{DS}, K_3\} > 0$ *(risk averse).*

We interestingly find that, when there are different manufacturers producing for the ODL product seller and the knockoff product seller, the presence of fashion knockoffs can benefit the ODL product supply chain and its agents under certain conditions. That is, when the ratio of demand uncertainty is moderate ($S_2^{DS} < \frac{\sigma_K}{\sigma_O} < S_1$) and when the ODL product seller is sufficiently risk averse, the ODL product supply chain and its agents can be benefited. Comparing the results with the basic model (common manufacturer case), we uncover that when the manufacturer is only producing for the ODL product seller, only the risk averse attitude of the ODL product seller can benefit the manufacturer, and there is a strict requirement for the ratio of demand uncertainty. In other words, when the ODL product seller is risk seeking, the ODL product seller and the manufacturer can never achieve a win-win outcome, irrespective of the level of demand uncertainty.

To show a clear picture of the impacts of fashion knockoffs on the ODL product seller and the manufacturer, we depict Figure 3.5. All the parameters are set to be the same as in the basic model (Figure 3.3). Figure 3.5 shows that only when the ODL product seller is risk averse, the ODL product seller and the manufacturer can achieve a win-win outcome in the presence of fashion knockoffs, and the win-win outcome is much more difficult to be achieved compared with the lose-lose outcome. Comparing with the basic model (Figure 3.3), we have two major findings. First, unlike the basic model, the conditions satisfying win-win or lose-lose outcomes are irrelevant to the ratio of manufacturer's profit margin ($\frac{w_O-m_O}{w_K-m_K}$). Second, the conditions satisfying the win-win outcome is much stricter than that in the basic model, which means that, when the manufacturer only produces for the ODL product seller, it is easier to get hurt from the presence of fashion knockoffs. The findings also explain why the phenomenon of a common manufacturer producing for both an ODL product seller and a knockoff product seller is widely observed in the fashion industry.

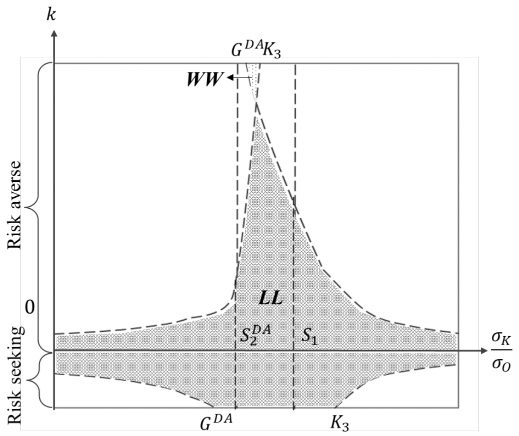


Figure 3.5. Impacts of fashion knockoffs on the ODL product seller and the manufacturer (P.S.: We let $a = 0.9$, $\alpha = 0.6, \phi = 0.3$, $\beta = 0.5$, $b = 1.5$, $w_O = 0.6$, $m_O = 0.3$, $m_K = 0.1$, $\sigma_O = 2, \sigma_K \in [0, 4.8]$, $\frac{\sigma_K}{\sigma_O} \in [0, 2.4]$, $w_K = 0.2$ in (a) and $w_K = 0.12$ in (b). Region WW represents the win-win outcome where $\Delta\,\pi_O^{DS} > 0$ and $\Delta\,\pi_{M1}^{DS} > 0$, and Region LL represents the lose-lose outcome where $\Delta\,\pi_O^{DS} < 0$ and $\Delta\,\pi_{M1}^{DS} < 0$).

Risk neutral ODL product seller

We use the superscript DN (different manufacturers, risk neutral ODL product seller) to represent the new functions or results derived in this subsection. We can easily extend the findings of the ODL product seller in model CN into this extended model.

Then, by exploring the impacts of fashion knockoffs on manufacturer 1, we have:

$$\Delta \pi_M^{DN} = (w_O - m_O)\left[\frac{-\phi\beta(a-w_O)(3+\phi\beta)-2\phi(\alpha-bw_K)}{2(1+\phi\beta)(4+\phi\beta)}\right]. \tag{3.27}$$

We summarize the main findings in Proposition 3.7.

Proposition 3.7. *Under the extended Model DN, the presence of fashion knockoffs always harms the ODL product supply chain and its agents.*

Proposition 3.7 shows that when there are different manufacturers producing for the ODL product seller and the knockoff product seller, the presence of fashion knockoffs always harms the ODL product supply chain and its agents if the ODL product seller is risk neutral. Comparing the results with the one derived under Model CN, we discover that the ODL product seller with risk neutral attitude is always harmed from the presence of fashion knockoffs, irrespective of the supply chain structure. This result also suggests that the manufacturer should produce for both the ODL product seller and the knockoff product seller when the ODL product seller is risk neutral. Compared with the results derived under Model DS, the findings indicate that when the manufacturer only produces for the ODL product seller, it is wise for the ODL product seller to be risk sensitive (risk averse or risk seeking) rather than risk neutral.

3.5 Chapter Conclusion

Motivated by the fact that the ODL product seller exhibits different risk attitudes and the presence of fashion knockoffs normally harms the ODL product seller, we establish game-theoretical models to examine the effects of fashion knockoffs on the ODL product supply chain and its agents with the consideration of demand uncertainty. We first conduct an interview with the industrialist to get industrial inputs which help formulate our model as well as research questions.

In the basic model, we consider a common manufacturer producing for both a knockoff product seller and an ODL product seller taking an MSD objective. We derive the conditions under which the presence of fashion knockoffs benefits the ODL product supply chain and its agents. We interestingly uncover that with different risk attitudes of the ODL product seller, the impacts of fashion knockoffs can be totally different. To enhance robustness, we further extend the basic model to consider (i) the non-standard markup wholesale pricing policy, and (ii) the case with different manufacturers. The major findings are summarized in Table 3.7.

Table 3.7. Major findings.

<table>
<tr><th>Models</th><th>Risk attitude of the ODL product seller</th><th>ODL product seller</th><th>Manufacturer</th><th>ODL product supply chain</th></tr>
<tr><td rowspan="3">Basic model
(common manufacturer)</td><td>Risk averse</td><td colspan="3">✓</td></tr>
<tr><td>Risk seeking</td><td colspan="3">✓</td></tr>
<tr><td>Risk neutral</td><td>×</td><td colspan="2">✓</td></tr>
<tr><td rowspan="3">Non-standard markup wholesale pricing policy</td><td>Risk averse</td><td colspan="3">✓</td></tr>
<tr><td>Risk seeking</td><td colspan="3">✓</td></tr>
<tr><td>Risk neutral</td><td>×</td><td colspan="2">✓</td></tr>
<tr><td rowspan="3">Different manufacturers</td><td>Risk averse</td><td colspan="3">✓</td></tr>
<tr><td>Risk seeking</td><td colspan="2">✓</td><td>×</td></tr>
<tr><td>Risk neutral</td><td colspan="3">×</td></tr>
<tr><td colspan="5">Remarks: ✓means that the presence of fashion knockoffs is beneficial in some cases; ×means that the presence of fashion knockoffs is harmful in all cases.</td></tr>
</table>

Table 3.7 reveals that, unlike the conventional wisdom that the presence of fashion knockoffs always harms the ODL product seller, we interestingly find that the presence of fashion knockoffs can benefit the ODL product supply chain and its agents under certain conditions, and risk attitude of the ODL product seller is a critical governing factor therein. More specifically, when considering a common manufacturer producing for both an ODL product seller and a knockoff product seller (i.e., the basic model), the presence of fashion knockoffs can contribute to an all-win situation, i.e., it benefits both the ODL product supply chain and its agents when the ODL product seller is risk sensitive (either risk averse or risk seeking), while always harms the ODL product seller when the ODL product seller is risk neutral (RQ1 and RQ4). The results remain robust when we consider a non-standard markup wholesale pricing policy (RQ2). However, when we consider another configuration of the supply chain (i.e., different manufacturers), we find that the presence of fashion knockoffs benefits the ODL product supply chain and its agents when the ODL product seller is risk averse; always harms the ODL product supply chain and its agents when the ODL product seller is risk neutral; and benefits the ODL product seller and the manufacturer while harms the ODL product supply chain when the ODL product seller is risk seeking (RQ3).

In order to validate our major findings, we further seek inputs from the interviewee. We find that the interview results are generally consistent with our major findings. The detailed interview results are summarized in Table 3.8.

Table 3.8. Post-analysis interview results with respect to the major findings[23].

Models	**Major findings**	**Interview results**
Risk attitude	(i) We analytically find that a risk sensitive (rather than risk neutral) attitude is more likely to benefit the ODL product seller in the presence of fashion knockoffs. (ii) We analytically find that a risk seeking attitude can do more good than harm in some cases.	(i) This partially explains why "For some big companies like *LV* or *Gucci*, they usually have a specific team which monitors the copying level of the knockoff brands". One reason behind is that they benefit from the presence of fashion knockoffs when they are risk sensitive. (ii) "For some well-established companies like *Supreme* and *RIMOWA*, they are not afraid of being copied, instead, they may like being copied since the existence of fashion knockoffs can help them promote their products and brand name, which may even benefit them."
Basic model	Our results indicate that the presence of fashion knockoffs may benefit the ODL product supply chain and its agents under certain conditions.	"In real world, the ratio of knockoff product sellers being sued by the ODL product seller is very low. One possible explanation is that the ODL product seller actually benefits from the presence of fashion knockoffs."
Non-standard markup wholesale pricing policy	We analytically find that the manufacturer is advised to set a higher markup rate for the ODL product seller or a lower markup rate for the knockoff product seller in the presence of fashion knockoffs.	"It is common that the manufacturer sets a higher markup rate for the ODL product seller than for the knockoff product seller."
Different manufacturers	We analytically find that compared with producing only for the ODL product seller, when producing for both the ODL product seller and the knockoff product seller, the manufacturer is more likely to be benefited.	"This can be true. When the manufacturer produces for both the ODL product seller and the knockoff product seller, it is more likely to be benefited ."

[23] You may refer to Appendix E for more details.

Chapter 4 C2C Product Exchanges: Impacts on Fashion Knockoffs[24]

4.1 Introduction

4.1.1 Background and Motivation

Sharing economy refers to the situation where peers share the access of underutilized products or services, prioritizing accessibility and utilization over ownership (Schor and Fitzmaurice 2015). Sharing in the form of bartering has long been existed, whereas technological advances distinguish the present "sharing economy" from traditional sharing contexts (Hamari et al. 2016). The prevailing use of the online platforms, mobile platforms, digital payments, and mutual rating systems has facilitated the product exchange (PE). Various industries have evidenced the concept and popularity of the sharing economy. For instance, Spinlister for bikes, Getaround for cars, NeighborGoods for household items, Airbnb for accommodations, Landshare for gardens, JustPark for parking, etc. It is noticeable that the fashion industry is particularly affected by the sharing economy (Choi and Shen 2017; Shen et al. 2017; Choi and He 2019; Feng et al. 2020). The number of users of top C2C fashion e-commerce platforms in May 2020 exceeds 60 million[25]. The C2C-trading is further expanded during the Covid-19 crisis. It is reported that fashion product is the most popular category being exchanged during Covid-19[26].

The PEs facilitate products sharing among consumers, thus prolonging the product life and reducing waste. However, fashion brands may concern that the C2C-PE can reduce consumer demand for new products, leading to a reduced sales and profitability (e.g., Thomas 2003; Cooper and Gutowski 2017). We argue that this view overlooks that C2C-PE can also reduce demand for knockoffs who the original brands are competing with. This perspective motivates us to ask a question: could original brand be benefiting from C2C-PE because of striking knockoffs?

As stated by Chevalier and Goolsbee (2009), with the anticipation of product resale in the secondary market, consumers' valuation of the product will be enhanced in the primary market. This is called the value-enhancement effect. Since the product sharing market and the secondary market share some conceptual similarities, the value-enhancement effect is also applicable to the product sharing market (Jiang and Tian 2018). With the anticipation of future product exchange, consumers'

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[24] A part of this chapter has been published in Wang, Y., Fan, D., Fung, Y. N., & Luo, S. (2022). Consumer-to-consumer product exchanges for original fashion brands in the sharing economy: Good or bad for fashion knockoffs?. Transportation Research Part E: Logistics and Transportation Review, 158, 102599.

[25] https://www.statista.com/statistics/712248/c2c-fashion-unique-visitors/ [Accessed on 13 Oct 2021]

[26] https://www.mckinsey.com/industries/technology-media-and-telecommunications/our-insights/c2c-ecommerce-could-a-new-business-model-sell-more-old-goods [Accessed on 13 Oct 2021]

valuation of the product is enhanced when they are making purchase decisions. Thus, the consumers considered knockoffs may change to purchase the original products. In this study, we intend to analytically verify the idea of whether the presence of C2C-PE would change the game between the original brand and the knockoff.

Note that the C2C-PE explored in this study is different from the common secondary market in a number of ways: First, there is no need to pay a fee for platform or middleman. Second, it is convenient, quick and almost always guaranteed to be traded out given the popularity of social media platforms such as Facebook and Instagram. Third, it is easy to be done by every individual. This study does not consider the presence of platforms, since we aim to focus on exploring the impacts of C2C-PE, which may be affected by the presence of platforms. Specially, we consider that a consumer can exchange the product with another consumer via social media platforms like Facebook and Instagram.

Although the area of C2C-PE is well explored by the prior academics, how the C2C-PE performs in the presence of knockoff product is largely unknown. In this study, we build game theoretical models to analytically explore the impacts of C2C-PE on the fashion supply chain members and the consumers.

4.1.2 Research Questions

- RQ1: Will the C2C-PE benefit the original supply chain, its members and consumers? How about the knockoff supply chain, its members and consumers?
- RQ2: When the manufacturer of the original brand decides on both the quality and price of the original product, will the results in the basic model still hold?
- RQ3: When the C2C-PE utility (the utility consumers gained through exchanging the product) is price dependent, how will the results change?
- RQ4: Does it make a difference when considering consumers' conspicuous behavior?

4.2 Basic Model

We consider a two-echelon fashion supply chain where there are different manufacturers producing for the original brand and the knockoff brand (as shown in Figure 4.1). Since the knockoff brand copies the runway offerings of the original brand very quickly, it can offer the product in the same season as the original one. In our model, we consider that the original brand and the knockoff brand coexist in the market and make pricing decisions simultaneously. Manufacturer 1 produces for the original brand (OB) with a unit wholesale price w, and manufacturer 2 produces for the knockoff brand (CC) with a unit wholesale price c. Since the knockoff brand free rides on the original brand's marketing, advertising, research and development, the knockoff products are usually sold at a much lower price ($p_{CC} < p_{OB}$). For mathematical tractability, we normalize all production costs to zero.

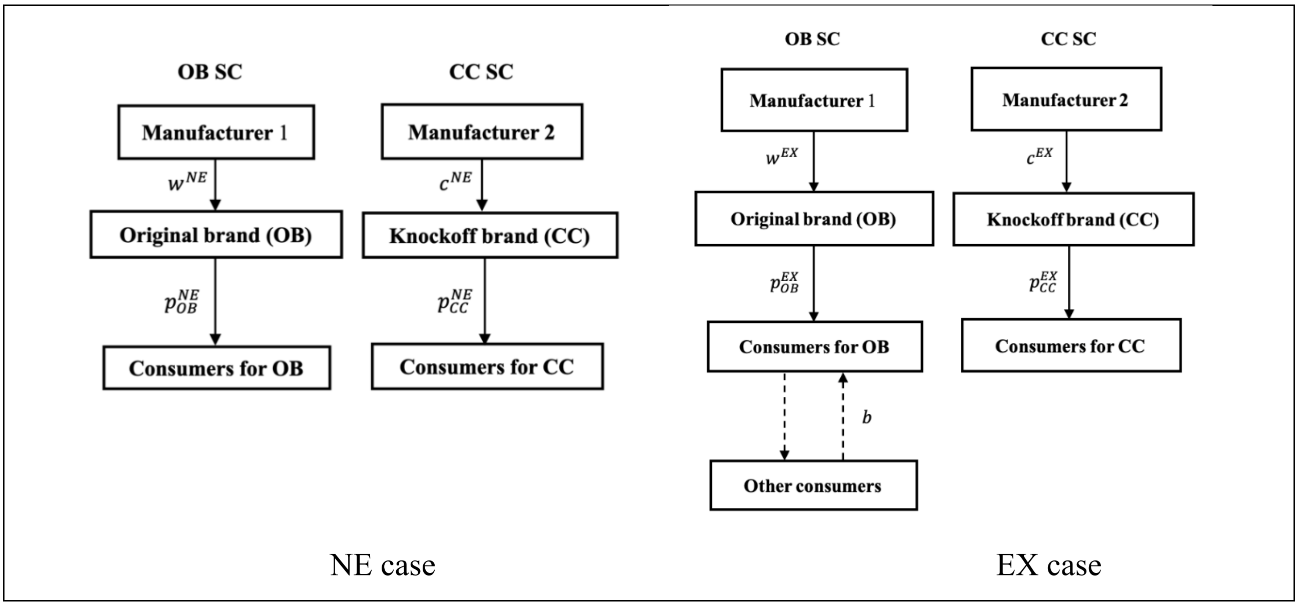

Figure 4.1. The basic model.

The original brand and the knockoff brand serve for the same market. Without loss of generality, we normalize the market size to one. Each consumer in the market has three options: buying the original product, buying the knockoff product or buying nothing, depending on their valuations towards the product. We consider that consumers are heterogonous in their product valuation v, which follows a probability distribution function $f(v)$ and a cumulative distribution function $F(v)$. Here, we follow the mainstream research in this area (e.g., Feng et al. 2020; Wen and Siqin 2020; Bian et al. 2021) and consider $f(v)$ as a uniform distribution with support between 0 and 1. Since knockoffs are usually in an inferior quality compared to their original counterparts, consumers' valuation for knockoff product is discounted, which is denoted as θv, θ is consumer's acceptance for knockoff product. Each consumer will purchase the product (at most one unit) which generates the highest utility. Without C2C-PE (NE case), consumers simply keep the product for private use (i.e., $b = 0$). With C2C-PE (EX case), consumers who bought the original product can exchange it with other consumers after usage, which generates an extra value b (C2C-PE utility) for the consumer (see Figure 4.1).

Similar to Gao et al. (2017b), we consider that consumers will make purchase decisions following the threshold purchasing policy (as shown in Figure 4.2). Considering the general case with product exchange, the threshold purchasing policy is characterized by τ_{OB} and τ_{CC}, where $0 < \tau_{CC} < \tau_{OB} < 1$. Consumers will: (1) buy the original product if $v \in [\tau_{OB}, 1]$; (2) buy the knockoff product if $v \in [\tau_{CC}, \tau_{OB})$; and (3) buy nothing if $v \in [0, \tau_{CC})$.

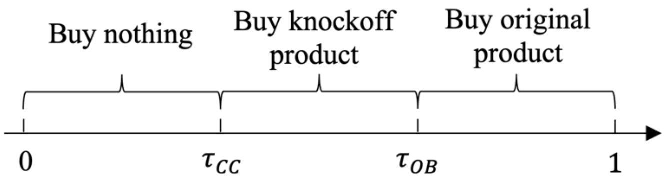

Figure 4.2. The threshold purchasing policy.

We can then derive the respective utility functions for the consumers in the presence of C2C-PE:

$$U_{OB}^{EX} = v - p_{OB}^{EX} + b,$$
$$U_{CC}^{EX} = \theta v - p_{CC}^{EX},$$
$$U_{NO} = 0. \tag{4.1}$$

Following the standard approach to derive the demand functions, we obtain the respective demand functions for the original product and the knockoff product: $D_{OB}^{EX} = 1 - \tau_{OB} = 1 - \frac{p_{OB}^{EX} - b - p_{CC}^{EX}}{1-\theta}$, $D_{CC}^{EX} = \tau_{OB} - \tau_{CC} = \frac{\theta(p_{OB}^{EX} - b) - p_{CC}^{EX}}{\theta(1-\theta)}$, where $\tau_{OB} = \frac{p_{OB}^{EX} - b - p_{CC}^{EX}}{1-\theta}$, $\tau_{CC} = \frac{p_{CC}^{EX}}{\theta}$.

The gaming interaction between the manufacturer and the original (knockoff) brand is captured by the Stackelberg game. The manufacturer decides on the wholesale price as the leader. After observing the decision of the manufacturer, the original (knockoff) brand makes his price decision as a follower. We use backward induction to analyze the sequential game.

We first derive the profit functions for the original brand and the knockoff brand:

$$\pi_{OB}^{EX} = (p_{OB}^{EX} - w^{EX})D_{OB}^{EX}, \tag{4.2}$$
$$\pi_{CC}^{EX} = (p_{CC}^{EX} - c^{EX})D_{CC}^{EX}. \tag{4.3}$$

Maximizing π_{OB}^{EX} (π_{CC}^{EX}) with respect to p_{OB}^{EX} (p_{CC}^{EX}), we obtain the optimal selling prices for the original product (p_{OB}^{EX*}) and the knockoff product (p_{CC}^{EX*}). Substituting p_{OB}^{EX*} and p_{CC}^{EX*} back into the demand functions, we have the optimal demands D_{OB}^{EX*} and D_{CC}^{EX*}.

We now consider the manufacturers. The profit functions for the manufacturers are:

$$\pi_{M1}^{EX} = w^{EX} D_{OB}^{EX*}, \tag{4.4}$$
$$\pi_{M2}^{EX} = c^{EX} D_{CC}^{EX*}. \tag{4.5}$$

Maximizing π_{M1}^{EX} (π_{M2}^{EX}) with respect to w^{EX} (c^{EX}), we obtain the optimal wholesale prices for the original product and the knockoff product w^{EX*} and c^{EX*}. Through substitution, we obtain the optimal prices, demands and profits.

Considering profits of the supply chain, we have

$$\pi_{SC,OB}^{EX*} = \pi_{OB}^{EX*} + \pi_{M1}^{EX*}, \tag{4.6}$$
$$\pi_{SC,CC}^{EX*} = \pi_{CC}^{EX*} + \pi_{M2}^{EX*}. \tag{4.7}$$

The consumer surplus for buying the original product and the knockoff product are as follows:

$$CS_{OB}^{EX} = \int_{\tau_{OB}}^{1} (v - p_{OB}^{EX} + b) f(v)\, dv, \tag{4.8}$$

$$CS_{CC}^{EX} = \int_{\tau_{CC}}^{\tau_{OB}} (\theta v - p_{CC}^{EX}) f(v)\, dv\,. \tag{4.9}$$

The equilibrium results are summarized in Appendix F, Lemma A1. We further conduct sensitivity analysis on the equilibrium results with respect to the C2C-PE utility (b) and consumer's acceptance for knockoff product (θ). The findings are summarized in Propositions 4.1 and 4.2.

Proposition 4.1. *Under the basic model NE case, we have:*

(a) For the original supply chain and the consumers: (i) w^{NE*}, p_{OB}^{NE*} and π_{OB}^{NE*} decrease in θ; (ii) D_{OB}^{NE*} and CS_{OB}^{NE*} increase in θ; (iii) π_{M1}^{NE*} and $\pi_{SC,OB}^{NE*}$ are concave in θ.

(b) For the knockoff supply chain and the consumers: (i) D_{CC}^{NE*} and CS_{CC}^{NE*} increase in θ; (ii) c^{NE*}, p_{CC}^{NE*} and π_{CC}^{NE*} are concave in θ; (iii) π_{M2}^{NE*} and $\pi_{SC,CC}^{NE*}$ first increase and then decrease in θ.

When consumers have a higher acceptance (θ) for knockoff product, their utility gained from buying the knockoff product is higher. More consumers will buy the knockoff product. Observing the increase in demand, the knockoff brand will raise the price accordingly. At the same time, the original brand has to lower its price to compete with the knockoff brand. Demand for the original product increases. While the increase in demand cannot counteract the decrease in price, the original brand is worse off. When the wholesale price is not very low, increase in demand for the original product can benefit manufacturer 1. While with the decrease in wholesale price, the increase in demand can no longer compensate the loss caused by the wholesale price. Hence, the profit for manufacturer 1 first increases and then decreases. With the increase in demand for the original product, the knockoff brand has to lower its selling price. Hence, profit for the knockoff brand and manufacturer 2 first increase and then decrease. We interestingly find that increase in consumer's acceptance for knockoff product not only benefits consumers for knockoff product but also consumers for original product. Demand for both products increase, the aggregate market demand increases, which implies that the parameter θ has a market expansion effect.

Table 4.1. Sensitivity analysis on the equilibrium results under the basic model NE case.

Original supply chain	w^{NE*}	p_{OB}^{NE*}	D_{OB}^{NE*}	π_{OB}^{NE*}	π_{M1}^{NE*}	$\pi_{SC,OB}^{NE*}$	CS_{OB}^{NE*}
$\theta \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	concave	concave	$\uparrow$
Knockoff supply chain	c^{NE*}	p_{CC}^{NE*}	D_{CC}^{NE*}	π_{CC}^{NE*}	π_{M2}^{NE*}	$\pi_{SC,CC}^{NE*}$	CS_{CC}^{NE*}
$\theta \uparrow$	concave	concave	$\uparrow$	concave	first $\uparrow$ then $\downarrow$	first $\uparrow$ then $\downarrow$	$\uparrow$

Proposition 4.2. *Under the basic model EX case, we have:*

(a) For the original supply chain and the consumers: (i) all the equilibrium results increase linearly in b; (ii) $w^{EX*}$and p_{OB}^{EX*}decrease in θ; (iii) D_{OB}^{EX*} and CS_{OB}^{EX*} increase in θ.

(b) For the knockoff supply chain and the consumers: (i) all the equilibrium results decrease linearly in b; (ii) c^{EX*} and p_{CC}^{EX*} are concave in θ.

The effects of θ under the EX case are generally consistent with the NE case. While there are two major differences: (1) With an increase in θ, demand for the knockoff product and the respective consumer surplus strictly increase under the NE case, while they may decrease under the EX Case. (2) An increase in θ definitely harms the original brand under the NE case, while may benefit the original brand under the EX case. The most interesting finding is that an increase in consumer's acceptance for knockoff product actually increases demand for the original product and benefits the respective consumers under both the NE case and the EX case.

With a higher C2C-PE utility (b), consumers obtain a higher utility from buying the original product, some consumers shift from buying the knockoff product to buying the original product (D_{OB}^{EX*} ↑ while D_{CC}^{EX*} ↓). Due to the increased C2C-PE utility, agents of the original supply chain tend to increase their prices, hence improving their profits. The original supply chain is better off. While the increase in b has an opposite effect on the knockoff supply chain and the consumers. Facing the competition of the original brand, agents of the knockoff supply chain have to lower their prices. The knockoff supply chain is worse off. Although the decrease in price increases each consumer's utility for buying the knockoff product, demand for the knockoff product decreases in the meantime, the total consumer surplus for the knockoff product still decreases. The findings indicate that the agents of the original supply chain have the incentive to increase the C2C-PE utility of the original product. In practice, the manufacturer is encouraged to produce more durable products, and the original brand is advised to provide more exclusive products.

Table 4.2. Sensitivity analysis on the equilibrium results under the basic model EX case.

Original supply chain	w^{EX*}	p_{OB}^{EX*}	D_{OB}^{EX*}	π_{OB}^{EX*}	π_{M1}^{EX*}	$\pi_{SC,OB}^{EX*}$	CS_{OB}^{EX*}
θ ↑	↓	↓	↑				↑
b ↑	↑	↑	↑	↑	↑	↑	↑
Knockoff supply chain	c^{EX*}	p_{CC}^{EX*}	D_{CC}^{EX*}	π_{CC}^{EX*}	π_{M2}^{EX*}	$\pi_{SC,CC}^{EX*}$	CS_{CC}^{EX*}
θ ↑	concave	concave					
b ↑	↓	↓	↓	↓	↓	↓	↓

In order to explore the impacts of C2C-PE, we now compare the optimal decisions and demands under the EX case and the NE case and summarize the results in Proposition 4.3.

Proposition 4.3. *Under the basic model, we have:*

(a) $w^{EX*} > w^{NE*}$, $p_{OB}^{EX*} > p_{OB}^{NE*}$, $D_{OB}^{EX*} > D_{OB}^{NE*}$;

(b) $c^{EX*} < c^{NE*}$, $p_{CC}^{EX*} < p_{CC}^{NE*}$, $D_{CC}^{EX*} < D_{CC}^{NE*}$.

From Proposition 4.3, we uncover that, in the presence of C2C-PE, agents of the original supply chain raise their prices while agents of the knockoff supply chain lower their prices. Demand for the original product increases while for the knockoff product decreases, which implies that, with C2C-PE, the original brand encroaches some of the knockoff brand's demand.

We further examine the impacts of C2C-PE on the original (knockoff) supply chain, its members and the respective consumers. The findings are summarized in Proposition 4.4.

Proposition 4.4. *Under the basic model, the presence of C2C-PE (a) always benefits the original supply chain, its members and consumers, while (b) always harms the knockoff supply chain, its members and consumers.*

In the presence of C2C-PE, consumers buying the original product can gain a higher utility. It is intuitive that the respective consumers are benefited, while consumers for the knockoff product are harmed. It is interesting to find that the presence of C2C-PE not only affects the consumers but also the supply chain members. The C2C-PE is a win-win scheme to the original supply chain and the respective consumers.

4.3 Extended Models

4.3.1 Strategic Quality Decision

With the popularity of sharing economy, firms may make more strategic decisions in anticipation of consumers' product sharing (Jiang and Tian 2018). In this section, we consider that manufacturer of the original brand (manufacturer 1) not only decides on the wholesale price, but also quality of the product. Given the same price, consumers have a higher valuation for a higher quality product. Without loss of generality, we normalize the quality of knockoff product as 0, and quality of the original product as q. We use the notation with a ^ to denote the functions and solutions in this section.

Similar to the basic model, we can derive the utility functions for the consumers buying the original product, knockoff product and buying nothing respectively:

$$\hat{U}_{OB}^{EX} = v - \hat{p}_{OB}^{EX} + b + \hat{q}^{EX},$$
$$\hat{U}_{CC}^{EX} = \theta v - \hat{p}_{CC}^{EX},$$
$$\hat{U}_{NO} = 0. \tag{4.10}$$

We can then derive the respective demand functions: $\hat{D}_{OB}^{EX} = 1 - \hat{\tau}_{OB}, \hat{D}_{CC}^{EX} = \hat{\tau}_{OB} - \hat{\tau}_{CC}$, where $\hat{\tau}_{OB} = \frac{b+\hat{q}^{EX}+\hat{p}_{CC}^{EX}-\hat{p}_{OB}^{EX}}{\theta-1}, \hat{\tau}_{CC} = \frac{\hat{p}_{CC}^{EX}}{\theta}$.

The profit functions for the original brand and the knockoff brand are:

$$\hat{\pi}_{OB}^{EX} = (\hat{p}_{OB}^{EX} - \hat{w}^{EX})\hat{D}_{OB}^{EX}, \tag{4.11}$$

$$\hat{\pi}_{CC}^{EX} = (\hat{p}_{CC}^{EX} - \hat{c}^{EX})\hat{D}_{CC}^{EX}. \tag{4.12}$$

For manufacturer 1, producing a product at quality q engenders a fixed cost (e.g., investment in machines/quality control). The fixed cost is expressed using the commonly adopted quadratic cost function $\frac{kq^2}{2}$, where k is the cost coefficient for quality (Taylor 2002).

The profit functions for the manufacturers are:

$$\hat{\pi}_{M1}^{EX} = \hat{w}^{EX}\hat{D}_{OB}^{EX} - \frac{k\hat{q}^{EX^2}}{2}, \tag{4.13}$$

$$\hat{\pi}_{M2}^{EX} = \hat{c}^{EX}\hat{D}_{CC}^{EX}. \tag{4.14}$$

Considering profits of the supply chain, we have

$$\hat{\pi}_{SC,OB}^{EX*} = \hat{\pi}_{OB}^{EX*} + \hat{\pi}_{M1}^{EX*}, \tag{4.15}$$

$$\hat{\pi}_{SC,CC}^{EX*} = \hat{\pi}_{CC}^{EX*} + \hat{\pi}_{M2}^{EX*}. \tag{4.16}$$

The consumer surplus for buying the original product and knockoff product are

$$\widehat{CS}_{OB}^{EX} = \int_{\hat{\tau}_{OB}}^{1} (v - \hat{p}_{OB}^{EX} + b + \hat{q}^{EX}) f(v)\, dv, \tag{4.17}$$

$$\widehat{CS}_{CC}^{EX} = \int_{\hat{\tau}_{CC}}^{\hat{\tau}_{OB}} (\theta v - \hat{p}_{CC}^{EX}) f(v)\, dv\,. \tag{4.18}$$

The equilibrium results are summarized in Appendix F, Lemma A2. The results under the NE case can be simply obtained by setting $b = 0$. We further conduct sensitivity analysis on the equilibrium results with respect to the C2C-PE utility (b) and the cost coefficient for quality (k). Due to mathematical complexity, the sensitivity analysis with respect to θ is not shown here. The findings are summarized in Propositions 4.5 and 4.6.

Proposition 4.5. *With strategic quality decision, under the NE case, we have:*

(a) For the original supply chain and the consumers, all the equilibrium results decrease in k.

(b) For the knockoff supply chain and the consumers, all the equilibrium results increase in k.

When it becomes more expensive for manufacturer 1 to improve quality of the original product, it tends to make less quality improvement effort. Due to the decrease in product quality, consumer's utility for buying the original product is declined. Some consumers who originally would buy the original product shift to buying the knockoff product ($\hat{D}_{OB}^{NE*} \downarrow$ while $\hat{D}_{CC}^{NE*} \uparrow$). In order to compensate for the loss in demand, the original brand lowers its retail price. The original supply chain and its members are worse off. On the contrary, the knockoff supply chain, its members and the respective consumers are all benefited by the increase in original product's quality improvement cost. The findings are consistent with Wen and Siqin (2020), while differently we consider the supply chain setting and how the cost coefficient for quality affects the knockoff supply chain and the consumers. The findings highlight the importance for the manufacturer to reduce the quality improvement cost in order to achieve win-win for the original supply chain and the respective consumers.

Table 4.3. Sensitivity analysis on the equilibrium results with strategic quality decision NE case.

Original supply chain	$\widehat{w}^{NE*}$	$\hat{p}_{OB}^{NE*}$	$\hat{q}^{NE*}$	$\widehat{D}_{OB}^{NE*}$	$\hat{\pi}_{OB}^{NE*}$	$\hat{\pi}_{M1}^{NE*}$	$\hat{\pi}_{SC,OB}^{NE*}$	$\widehat{CS}_{OB}^{NE*}$
$k\uparrow$	↓	↓	↓	↓	↓	↓	↓	↓
Knockoff supply chain	$\hat{c}^{NE*}$	$\hat{p}_{CC}^{NE*}$		$\widehat{D}_{CC}^{NE*}$	$\hat{\pi}_{CC}^{NE*}$	$\hat{\pi}_{M2}^{NE*}$	$\hat{\pi}_{SC,CC}^{NE*}$	$\widehat{CS}_{CC}^{NE*}$
$k\uparrow$	↑	↑		↑	↑	↑	↑	↑

Proposition 4.6. *With strategic quality decision, under the EX case, we have:*

(a) For the original supply chain and the consumers: (i) all the equilibrium results increase linearly in b*; (ii) all the equilibrium results decrease in* k.

(b) For the knockoff supply chain and the consumers: (i) all the equilibrium results decrease linearly in b*; (ii) all the equilibrium results increase in* k.

The effects of k under the EX case remain the same as in the NE case, which indicates that reducing the quality improvement cost is a critical task for the manufacturer irrespective of the presence of C2C-PE.

Comparing with the basic model, we uncover that the impacts of C2C-PE utility (b) remain unchanged. When manufacturer of the original brand (manufacturer 1) decides on both the price and quality of the product, a higher C2C-PE utility benefits the original supply chain, its members and consumers, while harms the knockoff supply chain, its members and consumers. The findings provide more incentives for the members of the original supply chain to improve the C2C-PE utility of the original product.

Table 4.4. Sensitivity analysis on the equilibrium results with strategic quality decision EX case.

Original supply chain	$\widehat{w}^{EX*}$	$\hat{p}_{OB}^{EX*}$	$\hat{q}^{EX*}$	$\widehat{D}_{OB}^{EX*}$	$\hat{\pi}_{OB}^{EX*}$	$\hat{\pi}_{M1}^{EX*}$	$\hat{\pi}_{SC,OB}^{EX*}$	$\widehat{CS}_{OB}^{EX*}$
$b\uparrow$	↑	↑	↑	↑	↑	↑	↑	↑
$k\uparrow$	↓	↓	↓	↓	↓	↓	↓	↓
Knockoff supply chain	$\hat{c}^{EX*}$	$\hat{p}_{CC}^{EX*}$		$\widehat{D}_{CC}^{EX*}$	$\hat{\pi}_{CC}^{EX*}$	$\hat{\pi}_{M2}^{EX*}$	$\hat{\pi}_{SC,CC}^{EX*}$	$\widehat{CS}_{CC}^{EX*}$
$b\uparrow$	↓	↓		↓	↓	↓	↓	↓
$k\uparrow$	↑	↑		↑	↑	↑	↑	↑

Exploring further on the impacts of C2C-PE, we have Propositions 4.7 and 4.8.

Proposition 4.7. *With strategic quality decision, we have:*

(a) $\widehat{w}^{EX*} > \widehat{w}^{NE*}$, $\hat{p}_{OB}^{EX*} > \hat{p}_{OB}^{NE*}$, $\hat{q}^{EX*} > \hat{q}^{NE*}$, $\widehat{D}_{OB}^{EX*} > \widehat{D}_{OB}^{NE*}$;

(b) $\hat{c}^{EX*} < \hat{c}^{NE*}$, $\hat{p}_{CC}^{EX*} < \hat{p}_{CC}^{NE*}$, $\widehat{D}_{CC}^{EX*} < \widehat{D}_{CC}^{NE*}$.

When manufacturer 1 decides on both the wholesale price and quality of the original product, it tends to set a higher wholesale price and create a higher quality product in the presence of C2C-PE. The original brand raises the selling price accordingly. On the contrary, manufacturer 2 and the knockoff brand lower the price of the knockoff product in the presence of C2C-PE. The original brand encroaches some of the knockoff brand's demand. The findings are qualitatively consistent with the basic model.

Proposition 4.8. *With strategic quality decision, the presence of C2C-PE (a) always benefits the original supply chain, its members and consumers, while (b) always harms the knockoff supply chain, its members and consumers.*

Proposition 4.8 reveals that irrespective of the strategic quality decision, the C2C-PE is a win-win scheme to the original supply chain and the respective consumers, while a lose-lose scheme to the knockoff supply chain and the respective consumers. Different from Jiang and Tian (2018) which uncovers that the presence of sharing market increases the firm's profit while lowers consumer surplus, we discover that when considering fashion knockoffs in the market, sharing market can also benefit the consumers for original product.

4.3.2 Price Dependent C2C-PE Utility

In the basic model, we simply consider that the utility gained from exchanging the original product is a constant. While a more realistic case is that the C2C-PE utility is increasing in the selling price of the product. Following Choi et al. (2019), we consider that the C2C-PE utility is denoted as $b = h_1 + h_2 p_{OB}$, where h_1 is the basic C2C-PE utility which lies on the intrinsic features of the product, h_2 is the price sensitivity coefficient which captures how sensitive the C2C-PE utility is to the retail price. For example, a higher-tier LV bag usually has a higher h_1 compared with a lower-tier one. And h_2 is considered larger if a consumer concerns more about price rather than other aspects when exchanging an original product. We use the notation with a $\sim$ to denote the functions and solutions in this section.

The utility functions for the consumers buying the original product, knockoff product and buying nothing respectively:

$$\tilde{U}_{OB}^{EX} = v - \tilde{p}_{OB}^{EX} + (h_1 + h_2 \tilde{p}_{OB}^{EX}),$$
$$\tilde{U}_{CC}^{EX} = \theta v - \tilde{p}_{CC}^{EX},$$
$$\tilde{U}_{NO} = 0. \tag{4.19}$$

We can then derive the respective demand functions: $\tilde{D}_{OB}^{EX} = 1 - \tilde{\tau}_{OB}, \tilde{D}_{CC}^{EX} = \tilde{\tau}_{OB} - \tilde{\tau}_{CC}$, where $\tilde{\tau}_{OB} = \frac{h_1 + \tilde{p}_{CC}^{EX} + (h_2 - 1)\tilde{p}_{OB}^{EX}}{\theta - 1}, \tilde{\tau}_{CC} = \frac{\tilde{p}_{CC}^{EX}}{\theta}$.

The profit functions for the original brand and the knockoff brand are:

$$\tilde{\pi}_{OB}^{EX} = (\tilde{p}_{OB}^{EX} - \tilde{w}^{EX})\tilde{D}_{OB}^{EX}, \tag{4.20}$$

$$\tilde{\pi}_{CC}^{EX} = (\tilde{p}_{CC}^{EX} - \tilde{c}^{EX})\tilde{D}_{CC}^{EX}. \tag{4.21}$$

The profit functions for the manufacturers are:

$$\tilde{\pi}_{M1}^{EX} = \tilde{w}^{EX}\tilde{D}_{OB}^{EX*}, \tag{4.22}$$

$$\tilde{\pi}_{M2}^{EX} = \tilde{c}^{EX}\tilde{D}_{CC}^{EX*}. \tag{4.23}$$

Considering profits of the supply chain, we have

$$\tilde{\pi}_{SC,OB}^{EX*} = \tilde{\pi}_{OB}^{EX*} + \tilde{\pi}_{M1}^{EX*}, \tag{4.24}$$

$$\tilde{\pi}_{SC,CC}^{EX*} = \tilde{\pi}_{CC}^{EX*} + \tilde{\pi}_{M1}^{EX*}. \tag{4.25}$$

The consumer surplus for buying the original product and knockoff product are

$$\widetilde{CS}_{OB}^{EX} = \int_{\tilde{\tau}_{OB}}^{1} (v - \tilde{p}_{OB}^{EX} + h_1 + h_2\tilde{p}_{OB}^{EX}) f(v)\, dv, \tag{4.26}$$

$$\widetilde{CS}_{CC}^{EX} = \int_{\tilde{\tau}_{CC}}^{\tilde{\tau}_{OB}} (\theta v - \tilde{p}_{CC}^{EX}) f(v)\, dv\,. \tag{4.27}$$

The equilibrium results are summarized in Appendix F, Lemma A3. The results under the NE case can be simply obtained by setting $h_1 = h_2 = 0$. Since it is the same as the basic model NE case, we neglect the sensitivity analysis here. We further conduct sensitivity analysis on the equilibrium results under the EX case and summarize the findings in Proposition 4.9.

Proposition 4.9. *With price dependent C2C-PE utility, under the EX case, we have:*

(a) For the original supply chain and the consumers: (i) $\tilde{w}^{EX*}$and $\tilde{p}_{OB}^{EX*}$decrease in θ; (ii) $\tilde{D}_{OB}^{EX*}$ and $\widetilde{CS}_{OB}^{EX*}$ increase in θ; (iii) all the equilibrium results increase in h_1; (iv) all the equilibrium results except $\tilde{D}_{OB}^{EX*}$ and $\widetilde{CS}_{OB}^{EX*}$ increase in h_2.

(b) For the knockoff supply chain and the consumers: (i) $\tilde{c}^{EX*}$ and $\tilde{p}_{CC}^{EX*}$ are concave in θ; (ii) all the equilibrium results decrease in h_1 while are irrelevant to h_2.

Comparing the results with the basic model (Table 4.2), we find that the impacts of θ remain the same, and h_1(the intrinsic C2C-PE utility) and *b* have the same impact on the equilibrium results. When consumers have a stronger belief that a higher price product can generate a higher C2C-PE utility, it is wise for the original brand to raise the price of the product. With the increase in selling price, the original supply chain and its members are all better off. Interestingly, consumers' belief will not affect their purchasing decision on the original product, which implies that consumers' purchasing decision mainly depends on the intrinsic features of the product (h_1). Another interesting finding is that consumers' belief on the original product will not affect their purchasing decision on the knockoff product. The findings have good implications. The original brand is encouraged to find out how consumers perceive the impacts of price on the C2C-PE utility or how many consumers in the market hold this kind of belief. In this way, the original brand can better adjust its pricing strategy.

Table 4.5. Sensitivity analysis on the equilibrium results with price dependent C2C-PE utility EX case.

Original supply chain	$\tilde{w}^{EX*}$	$\tilde{p}_{OB}^{EX*}$	$\tilde{D}_{OB}^{EX*}$	$\tilde{\pi}_{OB}^{EX*}$	$\tilde{\pi}_{M1}^{EX*}$	$\tilde{\pi}_{SC,OB}^{EX*}$	$\widetilde{CS}_{OB}^{EX*}$
$\theta \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$				$\uparrow$
$h_1 \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$h_2 \uparrow$	$\uparrow$	$\uparrow$	NA	$\uparrow$	$\uparrow$	$\uparrow$	NA
Knockoff supply chain	$\tilde{c}^{EX*}$	$\tilde{p}_{CC}^{EX*}$	$\tilde{D}_{CC}^{EX*}$	$\tilde{\pi}_{CC}^{EX*}$	$\tilde{\pi}_{M2}^{EX*}$	$\tilde{\pi}_{SC,CC}^{EX*}$	$\widetilde{CS}_{CC}^{EX*}$
$\theta \uparrow$	concave	concave					
$h_1 \uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$h_2 \uparrow$	NA	NA	NA	NA	NA	NA	NA

Exploring further on the impacts of C2C-PE, we have Propositions 4.10 and 4.11.

Proposition 4.10. *With price dependent C2C-PE utility, we have:*

(a) $\tilde{w}^{EX*} > \tilde{w}^{NE*}$, $\tilde{p}_{OB}^{EX*} > \tilde{p}_{OB}^{NE*}$, $\tilde{D}_{OB}^{EX*} > \tilde{D}_{OB}^{NE*}$*;*

(b) $\tilde{c}^{EX*} < \tilde{c}^{NE*}$, $\tilde{p}_{CC}^{EX*} < \tilde{p}_{CC}^{NE*}$, $\tilde{D}_{CC}^{EX*} < \tilde{D}_{CC}^{NE*}$.

Proposition 4.10 indicates that when the C2C-PE utility gained from exchanging the original product is price dependent (i.e., linearly increasing in price), manufacturer 1 and the original brand tend to increase the price of the original product, while manufacturer 2 and the knockoff brand incline to decrease the price of the knockoff product. The original brand seizes some demand from the knockoff brand. The results are consistent with the basic model.

Proposition 4.11. *With price dependent C2C-PE utility, the presence of C2C-PE (a) always benefits the original supply chain, its members and consumers, while (b) always harms the knockoff supply chain, its members and consumers.*

We discover that no matter the C2C-PE utility is price dependent or not, the scheme of C2C-PE is always beneficial to the original supply chain and the consumers while harmful to the knockoff supply chain and the consumers. The findings derived under the basic model remain robust. Comparing the findings with Choi et al. (2019) which also considers a price dependent C2C-PE utility, we uncover that the C2C-PE benefits the original brand and the respective consumers irrespective of the presence of fashion knockoffs.

4.3.3 Consumers' Conspicuous Behavior

In this section, we consider an important consumer behavior, which is conspicuous consumption. When the original product for exchange is a luxury fashion product (e.g., a *Burberry* trench coat, a *LV* bag), consumers tend to buy these products to display their social status (Amaldoss and Jain 2015). With the number of consumers who bought the product (both original and knockoff) increases, their utility gained from purchasing the original product decreases accordingly. Following Amaldoss and Jain (2005b), we consider that consumers who bought the original product can gain a utility of $v - p_{OB} - \delta(D_{OB} + D_{CC})$, where δ is consumers' need for uniqueness. We use the notation with a - to denote the functions and solutions in this section.

The utility functions for consumers who bought the original product, knockoff product and nothing are:

$$\bar{U}_{OB}^{EX} = v - \bar{p}_{OB}^{EX} + b - \delta(\bar{D}_{OB}^{EX} + \bar{D}_{CC}^{EX}),$$
$$\bar{U}_{CC}^{EX} = \theta v - \bar{p}_{CC}^{EX},$$
$$\bar{U}_{NO} = 0. \tag{4.28}$$

We can then derive the respective demand functions: $\bar{D}_{OB}^{EX} = 1 - \bar{\tau}_{OB}, \bar{D}_{CC}^{EX} = \bar{\tau}_{OB} - \bar{\tau}_{CC}$, where $\bar{\tau}_{OB} = \frac{\bar{p}_{OB}^{EX} - b + \delta(\bar{D}_{OB}^{EX} + \bar{D}_{CC}^{EX}) - \bar{p}_{CC}^{EX}}{1-\theta}, \bar{\tau}_{CC} = \frac{\bar{p}_{CC}^{EX}}{\theta}$.

The profit functions for the original brand and the knockoff brand are:

$$\bar{\pi}_{OB}^{EX} = (\bar{p}_{OB}^{EX} - \bar{w}^{EX})\bar{D}_{OB}^{EX}, \tag{4.29}$$
$$\bar{\pi}_{CC}^{EX} = (\bar{p}_{CC}^{EX} - \bar{c}^{EX})\bar{D}_{CC}^{EX}. \tag{4.30}$$

The profit functions for the manufacturers are:

$$\bar{\pi}_{M1}^{EX} = \bar{w}^{EX}\bar{D}_{OB}^{EX*}, \tag{4.31}$$
$$\bar{\pi}_{M2}^{EX} = \bar{c}^{EX}\bar{D}_{CC}^{EX*}. \tag{4.32}$$

Considering profits of the supply chain, we have

$$\bar{\pi}_{SC,OB}^{EX} = \bar{\pi}_{OB}^{EX*} + \bar{\pi}_{M1}^{EX*}, \tag{4.33}$$
$$\bar{\pi}_{SC,CC}^{EX} = \bar{\pi}_{CC}^{EX*} + \bar{\pi}_{M2}^{EX*}. \tag{4.34}$$

The consumer surplus for buying the original product and knockoff product are

$$\overline{CS}_{OB}^{EX} = \int_{\bar{\tau}_{OB}}^{1} \left(v - \bar{p}_{OB}^{EX} + b - \delta(\bar{D}_{OB}^{EX} + \bar{D}_{CC}^{EX})\right) f(v)\, dv, \tag{4.35}$$

$$\overline{CS}_{CC}^{EX} = \int_{\bar{\tau}_{CC}}^{\bar{\tau}_{OB}} (\theta v - \bar{p}_{CC}^{EX}) f(v)\, dv\,. \tag{4.36}$$

The equilibrium results are summarized in Appendix F, Lemma A4. We further conduct sensitivity analysis on the equilibrium results with respect to the C2C-PE utility (b), consumer's acceptance for knockoff product (θ) and consumers' need for uniqueness (δ). The findings are summarized in Propositions 4.12 and 4.13.

Proposition 4.12. *With consumers' conspicuous behavior, in the NE case, we have:*

(a) For the original supply chain and the consumers: (i) $\bar{w}^{NE*}$ *and* $\bar{p}_{OB}^{NE*}$ *and* $\bar{\pi}_{OB}^{NE*}$ *decrease in* θ*; (ii) all the equilibrium results decrease in* δ.

(b) For the knockoff supply chain and the consumers: (i) $\bar{D}_{CC}^{NE*}$ *and* $\overline{CS}_{CC}^{NE*}$ *increase in* θ*; (ii)* $\bar{c}^{NE*}$ *and* $\bar{p}_{CC}^{NE*}$ *are concave in* θ*; (iii) all the equilibrium results except* $\bar{c}^{NE*}$ *and* $\bar{p}_{CC}^{NE*}$ *increase in* δ*; (iv)* $\bar{c}^{NE*}$ *is concave in* δ.

Comparing the results with the basic model (Table 4.1), we interestingly find that an increase in consumer's acceptance for knockoff product (θ) may not lead to an increase in demand for the original product and a higher consumer surplus for consumers buying the original product. The reasons are as follows: with an increase in consumer's acceptance for knockoff product, consumers buying the knockoff product can gain a higher utility, hence more consumers would buy the knockoff product. With the number of consumers buying the knockoff product increases, consumers buying the original product suffer a larger utility loss due to their need for uniqueness, hence demand for the original product may not increase, consumer surplus may not increase accordingly.

For the impacts of consumer's need for uniqueness (δ), it is intuitive that when consumers have a higher need for uniqueness, their utility gained from buying the original product is lower, fewer consumers will buy the original product. The original brand has to lower its price accordingly. The original supply chain and its members are all worse off. On the contrary, some consumers who originally would buy the original product shift to buying the knockoff product, demand for the knockoff product increases, the knockoff supply chain and its members are all better off.

Table 4.6. Sensitivity analysis on the equilibrium results with consumers' conspicuous behavior NE case.

Original supply chain	$\bar{w}^{NE*}$	$\bar{p}_{OB}^{NE*}$	$\bar{D}_{OB}^{NE*}$	$\bar{\pi}_{OB}^{NE*}$	$\bar{\pi}_{M1}^{NE*}$	$\bar{\pi}_{SC,OB}^{NE*}$	$\overline{CS}_{OB}^{NE*}$
$\theta\uparrow$	$\downarrow$	$\downarrow$		$\downarrow$			
$\delta\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Knockoff supply chain	$\bar{c}^{NE*}$	$\bar{p}_{CC}^{NE*}$	$\bar{D}_{CC}^{NE*}$	$\bar{\pi}_{CC}^{NE*}$	$\bar{\pi}_{M2}^{NE*}$	$\bar{\pi}_{SC,CC}^{NE*}$	$\overline{CS}_{CC}^{NE*}$
$\theta\uparrow$	concave	concave	$\uparrow$				$\uparrow$
$\delta\uparrow$	concave		$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

Proposition 4.13. *With consumers' conspicuous behavior, in the EX case, we have:*

(a) For the original supply chain and the consumers: (i) $\bar{w}^{EX*}$ *and* $\bar{p}_{OB}^{EX*}$ *decrease in* θ*; (ii) all the equilibrium results increase linearly in* b *while decrease in* δ.

(b) For the knockoff supply chain and the consumers: (i) $\bar{c}^{EX*}$ *and* $\bar{p}_{CC}^{EX*}$ *are concave in* θ*; (ii) all the equilibrium results decrease linearly in* b*; (iii) all the equilibrium results except* $\bar{c}^{EX*}$ *and* $\bar{p}_{CC}^{EX*}$ *increase in* δ*; (iv)* $\bar{c}^{EX*}$ *is concave in* δ.

Comparing the results with the NE case (Table 4.6), we find that when there is C2C-PE, an increase in consumer's acceptance for knockoff product (θ) may not lead to a decrease in original brand's profit and an increase in knockoff product's demand. We have the following explanation: when there is C2C-PE, consumers buying the original product can gain a higher utility. Their utility gain in exchanging the original product partially counteracts their utility loss caused by the need for uniqueness. Hence, the original brand is not always worse off, and demand for the knockoff product would not increase continuously.

The findings have good practical implications. Irrespective of the C2C-PE, an increase in consumer's need for uniqueness (δ) always harms the original supply chain, its members and consumers while benefits the knockoff supply chain, its members and consumers. Hence, it is critical for the original brand to learn more about consumers' level of need for uniqueness. The findings also highlight the importance of increasing the C2C-PE utility for the original brand.

Table 4.7. Sensitivity analysis on the equilibrium results with consumers' conspicuous behavior EX case.

Original supply chain	$\bar{w}^{EX*}$	$\bar{p}_{OB}^{EX*}$	$\bar{D}_{OB}^{EX*}$	$\bar{\pi}_{OB}^{EX*}$	$\bar{\pi}_{M1}^{EX*}$	$\bar{\pi}_{SC,OB}^{EX*}$	$\overline{CS}_{OB}^{EX*}$
$\theta\uparrow$	↓	↓					
$b\uparrow$	↑	↑	↑	↑	↑	↑	↑
$\delta\uparrow$	↓	↓	↓	↓	↓	↓	↓
Knockoff supply chain	$\bar{c}^{EX*}$	$\bar{p}_{CC}^{EX*}$	$\bar{D}_{CC}^{EX*}$	$\bar{\pi}_{CC}^{EX*}$	$\bar{\pi}_{M2}^{EX*}$	$\bar{\pi}_{SC,CC}^{EX*}$	$\overline{CS}_{CC}^{EX*}$
$\theta\uparrow$	concave	concave					
$b\uparrow$	↓	↓	↓	↓	↓	↓	↓
$\delta\uparrow$	concave		↑	↑	↑	↑	↑

Proposition 4.14. *With consumers' conspicuous behavior, we have:*

(a) $\bar{w}^{EX*} > \bar{w}^{NE*}$, $\bar{p}_{OB}^{EX*} > \bar{p}_{OB}^{NE*}$, $\bar{D}_{OB}^{EX*} > \bar{D}_{OB}^{NE*}$;

(b) $\bar{c}^{EX*} < \bar{c}^{NE*}$, $\bar{p}_{CC}^{EX*} < \bar{p}_{CC}^{NE*}$, $\bar{D}_{CC}^{EX*} < \bar{D}_{CC}^{NE*}$.

We uncover that, when considering consumers' conspicuous behavior, manufacturer 1 and the original brand would still raise the price of the original product while manufacturer 2 and the knockoff brand would lower the price of the knockoff product in the presence of C2C-PE. Demand for the original product increases whereas demand for the knockoff product decreases. The results are qualitatively consistent with the basic model.

Proposition 4.15. *With consumers' conspicuous behavior, the presence of C2C-PE (a) always benefits the original supply chain, its members and consumers, while (b) always harms the knockoff supply chain, its members and consumers.*

Proposition 4.15 indicates that when considering consumers' conspicuous behavior, the presence of C2C-PE can still benefit the original supply chain and the consumers while harm the knockoff supply chain and the consumers. The findings verify the robustness of the results under the basic model. Comparing the results with Feng et al. (2020), we uncover that C2C-PE benefits the original brand when considering consumers' conspicuous behavior irrespective of the presence of fashion knockoffs.

4.4 Chapter Conclusion

Due to the rapid development of technological advances and consumers' increasing awareness of sustainability, sharing economy has become a prevalent concept in recent years. Fashion products are among the top items being exchanged by consumers. Another noticeable phenomenon in the fashion industry is the knockoff products. How the scheme of C2C-PE would perform in the presence of knockoff products is largely unknown. In this study, we build game theoretical models to explore the impacts of C2C-PE on the fashion supply chain members and their consumers. By comparing between the cases with C2C-PE and without C2C-PE, we theoretically find that the presence of C2C-PE benefits the original supply chain, its members and consumers, while harms the knockoff supply chain, its members and consumers (RQ1). We further verify that the results remain robust under three scenarios: (i) when manufacturer of the original brand makes strategic quality decision (RQ2); (ii) when the C2C-PE utility is price dependent (RQ3) and (iii) when considering consumers' conspicuous behavior (RQ4). We uncover that in the presence of C2C-PE, members of the original supply chain tend to produce a higher-price and superior-quality product, while members of the knockoff supply chain incline to sell a lower-price knockoff product. The original brand will encroach some of the knockoff brand's demand.

Comparing the findings with prior literature, we uncover that: (i) Different from Jiang and Tian (2018) which finds that the presence of sharing market increases the firm's profit while lowers consumer surplus, we discover that when considering fashion knockoffs in the market, sharing market can also benefit the consumers for original product when the manufacturer makes pricing and quality decisions simultaneously; (ii) Consistent with Choi et al. (2019), we uncover that C2C-PE benefits the original brand and the respective consumers. Besides, we interestingly find that the presence of C2C-PE harms the knockoff supply chain and its consumers; (iii) Consistent with Feng et al. (2020), we find that C2C-PE benefits the original brand when considering consumers' conspicuous behavior. We further discover that C2C-PE harms the knockoff brand and the consumers when consumers have conspicuous behavior.

We also obtain the following findings with respective to the three important factors considered in this study: (1) We interestingly find that an increase in consumer's acceptance for knockoff product not only benefits consumers for knockoff product but also consumers for original product irrespective of the presence of C2C-PE; (2) We uncover that an increase in quality improvement cost for the original product harms the original supply chain, its members and consumers while benefits the knockoff supply chain, its members and consumers; (3) We discover that an increase in consumer's need for uniqueness harms the original supply chain, its members and consumers while benefits the knockoff supply chain, its members and consumers.

Chapter 5 Luxury Fashion Brands' Logo Design Strategies Facing Fashion Knockoffs and Counterfeits

5.1 Introduction

5.1.1 Background and motivation

The revenue of global luxury market reaches 310 billion U.S. dollars in 2021, which is expected to hit 387 billion dollars in 2025[27]. Luxury products differ from the ordinary ones that the mere display of the product conveys social status of the product owners (Choi and Shen 2017). Consumers who buy luxury products to signal their wealth and social status are considered to be engaging in conspicuous consumption (Amaldoss and Jain 2005a). Depending on the logo design of the luxury product, luxury brands can be classified into two categories, which are loud luxury and quiet luxury, respectively (Han et al. 2010). Under loud luxury, the products feature prominent markings that allow consumers to readily recognize the brand. Examples are *Louis Vuitton*'s monogram print and *Gucci*'s "double G" logo which can be seen in a selection of their collections. Quiet luxury, on the other hand, sells products which carry subtle logos or even no markings. Examples are *Hermès* and *Bottega Veneta* which intentionally do not show any explicit logo or brand name (see Figure 5.1). Unlike loud luxury, only fashion gurus and elite consumers (or consumers "in the know") can recognize the quiet luxury. These often ultra-wealthy consumers can identify the quiet luxury via House Codes. Well known House Codes include *LV*'s bags with leather reinforced corners or *Chanel*'s bags with interlaced chains[28].

LV Neverfull

Hermès Birkin

Figure 5.1. Loud luxury (LV) and quiet luxury (Hermès) (as defined in this study).

[27] https://www.statista.com/statistics/1063757/global-personal-luxury-goods-market-value-forecast/ [Accessed on 9 Feb 2022]

[28] https://www.thedrum.com/opinion/2020/02/26/shout-or-whisper-dissecting-quiet-and-loud-luxury [Accessed on 9 Feb 2022]

Han et al. (2010) define brand prominence as "the extent to which a product has visible markings that help ensure observers recognize the brand". Consumers have different preferences for brand prominence. Specifically, wealthy consumers who have a high need for status prefer a more prominent product (i.e., big logo product); while those who have a low need for status prefer a less prominent product (i.e., no logo product). The former consumer segment is called "parvenus" and the latter is called "patricians" (Han et al. 2010). These two consumer segments have different association and dissociation preferences. Explicitly, parvenus tend to use big logo product to associate with patricians and to dissociate with less affluent consumers, whereas patricians opt for no logo product to associate with their same group of consumers, and to prevent being considered as part of the parvenus. A comparison between the patricians and parvenus is summarized in Table 5.1. Understanding the nuances between the two consumer segments is critical for luxury brand marketers to tailor their marketing strategies (e.g., logo design strategy).

Although consumers' preferences for loud/quiet luxury are relatively stable, they are interchangeable in some cases. For example, with the recent return of logomania where the product is full of big logos, some patricians also purchase loud luxury simply to make a bold fashion statement[27], which results in the uncertain consumer segment. In this case, the luxury fashion brand naturally faces risk and would be risk sensitive in making the logo design and pricing decisions. Note that different luxury fashion brands may exhibit different risk attitudes (i.e., either risk averse or risk seeking) towards the uncertain consumer segment. For example, *Gucci* follows the trend of logomania by launching the collections with *Balenciaga*, *adidas* and *North Face* respectively, where jackets, t-shirts, and coats are all covered[29], which suggests that *Gucci* is probably risk seeking towards consumer segment uncertainty. On the contrary, some luxury fashion brands which are uncertain about showing big logos on their clothes may launch logomania collections on accessories first[30], which suggests that they are probably risk averse towards consumer segment uncertainty. Note that the risk sensitive decision making is also supported by many prior studies such as Chiu et al. (2018) and Wang et al. (2022). Accordingly, we take the luxury fashion brand's risk attitude into consideration in this study.

Table 5.1. Comparison between patricians and parvenus.

	Patricians	**Parvenus**
Wealthy?	Yes	Yes
Need for status	Low	High
Logo preference	No logo	Big logo

[29] https://www.gucci.com/uk/en_gb/st/capsule/the-north-face-gucci?sitelink=purebrand_en_2&gclid=Cj0KCQjw1N2TBhCOARIsAGVHQc7s_UBFweUkHi6kWYRWtndxgWFJ9vRIhQwnXU2k1zHp7V7CWeGHjcQaAvroEALw_wcB&gclsrc=aw.ds [Accessed on 29 Mar 2022]
[30] https://www.suitelife.blog/the-logo-mania-trend/ [Accessed on 29 Mar 2022]

Association and dissociation preferences	Associate with patricians and dissociate from parvenus	Associate with patricians and dissociate from less affluent consumers

When making the logo design decision, the luxury fashion brands should also consider the impacts of counterfeits and copycats. It is reported that fake luxury products account for 60% to 70% of the total fake trade (around US$4.5 trillion), far beyond pharmaceutical and entertainment industries[31]. One major reason behind is the prominent logos of luxury brands which can be easily copied by counterfeiters. Thus, Fontana et al. (2019) comment that: "Luxury firms will need to emphasize a style and quality that is tough to replicate and is independent of the logo". Similarly, Han et al. (2010) point out that the louder a luxury fashion brand is, the more likely it is to be copied by counterfeiters. Real world observations also support this point. *LV*, which is a well known loud luxury, is regarded as one of the most affected luxury brands by counterfeits[32]. It is reported that fake *LV* bag operations worth US$15.4 million have been forced to shut down in China, where suppliers copy labels, warrant certificates, and dust bags of the authentic *LV* bag[33]. Another case is that retailers like *Target* or *Steve Madden* offer lookalikes of *Hermès*'s Oran sandals at a much lower price to compete with the authentic ones[34]. These cases reveal that loud luxury (e.g., *LV*) and quiet luxury (e.g., *Hermès*) usually suffer different kinds of threats. To be specific, loud luxury is more vulnerable to counterfeits which bear a trademark identical to the authentic one, while quiet luxury is more sensitive to copycats which have similar packaging characteristics to the authentic one (Le Roux et al. 2016). Therefore, the luxury fashion brand adopting big logo strategy should be devoted to defending counterfeits; whereas the one with no logo strategy should focus more on combating copycats.

To deter counterfeits and copycats, luxury fashion brands have implemented a series of advanced technologies. For instance, *LVMH* have collaborated with *Cartier* and *Prada* to form the first global luxury blockchain called "*Aura Blockchain Consortium*". Due to the trust, immutability and transparency features of the blockchain technology, luxury fashion brands in the consortium can use blockchain system to provide consumers with transparent and trustworthy product information. Once consumers purchase the product, a specific digital ID will be sent to their mobile phone automatically. Using the ID code, consumers can have access to product history information and authenticity

[31] https://hbr.org/2019/05/how-luxury-brands-can-beat-counterfeiters [Accessed on 9 Feb 2022]
[32] https://www.thefashionlaw.com/gucci-rolex-louis-vuitton-top-list-of-brands-most-frequently-targeted-by-counterfeiters-on-tiktok/ [Accessed on 9 Feb 2022]
[33] https://www.scmp.com/news/people-culture/article/3125158/fake-louis-vuitton-luxury-bag-operation-china-worth-us154 [Accessed on 9 Feb 2022]
[34] https://www.thefashionlaw.com/copycat-versions-of-herms-oran-sandals-are-everywhere-so-why-isnt-the-brand-suingnbsp/ [Accessed on 9 Feb 2022]

certificate through the entire product value chain, from production to reselling. While using the blockchain technology is not free. Luxury fashion brands participating in the *Aura Blockchain Consortium* need to pay an annual licensing fee and a volume fee for using the blockchain technology[35]. See Figure 5.2 for the detailed transaction processes supported by the blockchain technology. Since blockchain can not only ensure the authenticity of luxury brands but also protect their customers from purchasing counterfeits/copycats, it is considered to be a win-win strategy for both the luxury brand and the consumers[32]. Hence, in this study, we analytically study how the implementation of blockchain technology can combat counterfeits and copycats, and its implications on the luxury fashion supply chain and the consumers.

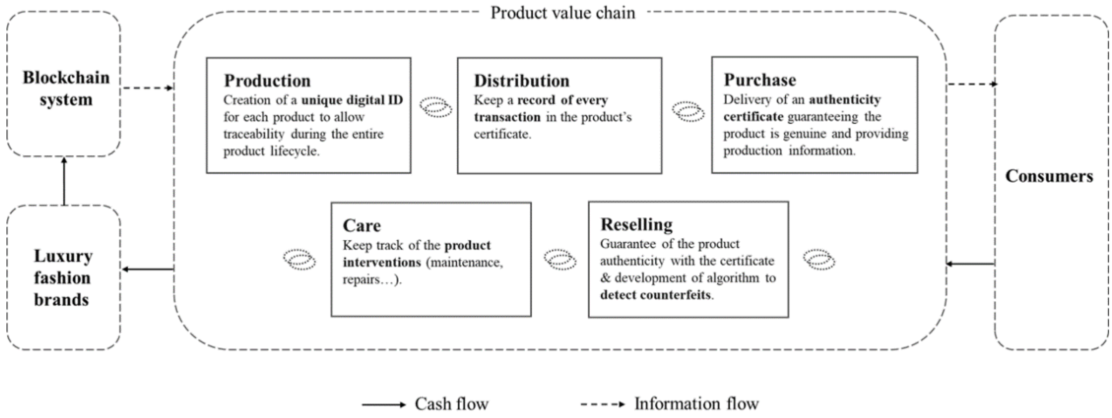

Figure 5.2. Transaction processes supported by the blockchain technology (based on the real practices of Aura Blockchain Consortium)[36].

5.1.2 Research Questions

- RQ1: When should the luxury fashion brand show big logo or no logo? What are the governing factors therein?
- RQ2: When considering risk attitude (i.e., risk averse or risk seeking) of the luxury fashion brand, how would the results derived in the basic model change?
- RQ3: When implementing the blockchain technology to fight against counterfeits and copycats, will the optimal logo design strategy adopted by the luxury fashion brand change?

[35] https://www.forbes.com/sites/pamdanziger/2021/04/22/lvmh-cartier-and-prada-partner-to-fight-counterfeits-and-invite-other-luxury-brands-to-join/?sh=debf2c730723 [Accessed on 29 Mar 2022]
[36] The figure of "product value chain" is sourced from https://auraluxuryblockchain.com/?cli_action=1648522561.312#experience.

5.2 Preliminaries: Model Descriptions

We consider a monopoly market where a luxury fashion brand gets the product from a manufacturer with a wholesale price w and sells them to the consumers with a retail price p. There is one unit of consumers (i.e., we normalize the market population to be 1) in the market, where α ($0 \leq \alpha \leq 1$) proportion are patricians, and the remaining $1 - \alpha$ are parvenus (Tereyağoğlu and Veeraraghavan 2012; Fruchter et al. 2021). Consumers hold heterogeneous valuation v towards the product, which follows a uniform distribution $f(v)$ between 0 and 1 (Cho et al. 2015; Gao et al. 2017b). The luxury fashion brand decides whether to use a big logo or no logo on the product, and the corresponding retail price. Due to the different logo preferences of patricians and parvenus (see Table 5.1), they have different status utilities according to the logo design strategy adopted by the luxury fashion brand. Explicitly, if the luxury fashion brand adopts a big logo, the parvenus who prefer big logo product gain a status utility of θ, while the patricians who prefer no logo product suffer a status disutility of l; if the luxury fashion brand adopts a no logo, the patricians gain a status utility of ϕ, while the parvenus suffer a status disutility of k.

Another critical factor which cannot be ignored when making logo decisions is the presence of counterfeits/copycats. When the luxury fashion brand uses a big logo, he faces the potential threat of counterfeits; while when the luxury fashion brand uses no logo, he faces the threat of copycats (Le Roux et al. 2016). We capture the disutility that a consumer unknowingly purchasing a counterfeit and copycat by λ and τ, respectively. Since counterfeits may pose a health or safety threat to the consumers (Cho et al. 2015), we consider its negative effect to be larger than that of a copycat (i.e., $\lambda > \tau$). Note that the only difference between the big logo product and the no logo product is the logo prominence, hence the production costs for the two products are considered the same. We consider that the luxury fashion brand and the manufacturer play a Stackelberg game, where the manufacturer plays as the leader to decide the wholesale price first, and the luxury fashion brand as a follower decides the logo prominence and the corresponding retail price sequentially. We use backward induction to solve the problem, which is standard in the literature. Please refer to Table 5.2 for the list of notation used in this study.

Table 5.2. List of notation for Chapter 5.

Notation	Meaning
i	Big logo ($= BL$), no logo ($= NL$)
j	Luxury fashion brand (i.e., Retailer) ($= R$), manufacturer ($= M$), supply chain ($= SC$)
v	Consumer's product valuation, $v \sim U[0,1]$
α	The proportion of patricians in the market, $\alpha \in [0,1]$

θ	The status utility gained by parvenus when big logo is adopted
l	The status disutility suffered by patricians when big logo is adopted
λ	The disutility suffered by consumers who unknowingly purchase a counterfeit
ϕ	The status utility gained by patricians when no logo is adopted
k	The status disutility suffered by parvenus when no logo is adopted
τ	The disutility suffered by consumers who unknowingly purchase a copycat
p_i	Unit retailing price for product i
w_i	Unit wholesaling price for product i
m	Unit production cost
$\hat{\alpha}$	Random variable with mean α_0 and standard derivation σ
β	Risk sensitivity coefficient
D_i	The demand function for product i
π_i^j	The profit function for member j when strategy i is adopted
$E[\pi_i^j]$	The expected profit function for member j when strategy i is adopted
$SD[\pi_i^j]$	The standard derivation of profit function for member j when strategy i is adopted
$U[\pi_i^j]$	The mean-standard deviation (MSD) objective function for member j when strategy i is adopted
CS_i	The total consumer surplus when strategy i is adopted
γ	Effectiveness of blockchain
t	Per unit cost when implementing the blockchain
F	Fixed operational cost when implementing the blockchain

5.2.1 Big Logo

We first consider the case where the luxury fashion brand decides to use a big logo. We use the subscript BL to represent the functions and decisions derived in this section. We can easily derive the utility functions for patricians and parvenus as: $U_{BL}^{Patrician} = v - p_{BL} - l - \lambda$, $U_{BL}^{Parvenu} = v - p_{BL} + \theta - \lambda$. Based on the utility functions, we can then derive the total number of consumers who purchase the big logo product:

$$D_{BL} = \alpha \int_{p_{BL}+l+\lambda}^{1} f(v)\, dv + (1-\alpha) \int_{p_{BL}-\theta+\lambda}^{1} f(v)\, dv = 1 - p_{BL} - \lambda - \alpha l + (1-\alpha)\theta. \quad (5.1)$$

We can then derive the profit function for the luxury fashion brand:

$$\pi_{BL}^{R} = (p_{BL} - w_{BL})D_{BL} = (p_{BL} - w_{BL})(1 - p_{BL} - \lambda - \alpha l + (1-\alpha)\theta). \quad (5.2)$$

Maximizing Eq. (5.2) with respect to p_{BL}, we obtain the optimal retail price for the product as $p_{BL}^{*} = \frac{1}{2}(1 - \lambda - \alpha l + (1-\alpha)\theta + w_{BL})$. Substituting it back into Eq.(5.1), we have the optimal demand for the big logo product: $D_{BL}^{*} = \frac{1}{2}(1 - \lambda - \alpha l + (1-\alpha)\theta - w_{BL})$.

Considering profits of the manufacturer and the luxury fashion supply chain, we have:

$$\pi_{BL}^{M} = (w_{BL} - m)D_{BL}^{*} = \frac{1}{2}(w_{BL} - m)(1 - \lambda - \alpha l + (1-\alpha)\theta - w_{BL}). \quad (5.3)$$

$$\pi_{BL}^{SC*} = \pi_{BL}^{R*} + \pi_{BL}^{M*}. \quad (5.4)$$

Maximizing Eq.(5.3) with respect to w_{BL}, we obtain the optimal wholesale price $w_{BL}^* = \frac{1}{2}(1 - \lambda - \alpha l + (1 - \alpha)\theta + m)$.

We also consider the corresponding consumer surplus in Eq.(5.5), where the first term is the consumer surplus for the patricians and the second term is the consumer surplus for the parvenus. Through substitution, we obtain the optimal decisions, profits and consumer surplus as summarized in Lemma 5.1.

$$CS_{BL} = \alpha \int_{p_{BL}+l+\lambda}^{1} (v - p_{BL} - l - \lambda) f(v)\, dv + (1 - \alpha) \int_{p_{BL}-\theta+\lambda}^{1} (v - p_{BL} + \theta - \lambda) f(v)\, dv. \quad (5.5)$$

Lemma 5.1. *Under big logo strategy, the optimal decisions, demand, profits of the luxury fashion supply chain and its members, and the corresponding consumer surplus are given as follows:*
$w_{BL}^* = \frac{1}{2}(A + m),\ p_{BL}^* = \frac{1}{4}(3A + m),\ D_{BL}^* = \frac{1}{4}(A - m),\ \pi_{BL}^{R*} = \frac{1}{16}(A - m)^2,\ \pi_{BL}^{M} = \frac{1}{8}(A - m)^2,$
$\pi_{BL}^{SC} = \frac{3}{16}(A - m)^2, CS_{BL}^* = \frac{1}{32}[\alpha(4 - 3A - m - 4l - 4\lambda)^2 + (1 - \alpha)(4 - 3A - m + 4\theta - 4\lambda)^2],$
$where\ A = 1 - \lambda - \alpha l + (1 - \alpha)\theta.$

Lemma 5.1 presents the neat expressions of the optimal decisions, profits, and the corresponding consumer surplus. We further conduct sensitivity analysis on the optimal decisions and summarize the results in Table 5.3. It is intuitive that with a larger production cost, both the wholesale price and the retail price increase, while the demand decreases. Since the markup cannot compensate the loss in demand, the luxury fashion supply chain and its members are all worse off. The consumer surplus is decreased at the same time. Denote A as the unit net benefit to consumers brought by the big logo product. Exploring further on A, we uncover that when (i) the status disutility suffered by patricians decreases (i.e., $\alpha \downarrow$ or $l \downarrow$); (ii) the status utility gained by parvenus increases (i.e., $1 - \alpha \uparrow$ or $\theta \uparrow$); or (iii) the disutility caused by counterfeits decreases (i.e., $\lambda \downarrow$), the unit net benefit to consumers brought by the big logo product A increases, which results in a higher wholesale price and retail price, and higher profits for the luxury fashion supply chain and its members.

Regarding the consumer surplus, we uncover that when the proportion of patricians is sufficiently large ($\alpha > \bar{\alpha}_{BL}$), an increase in α harms both the luxury fashion supply chain and the consumers; while when the proportion of patricians α is relatively small ($\alpha < \bar{\alpha}_{BL}$), an increase in α harms the luxury fashion supply chain whereas benefits the consumers. Looking closer at the threshold $\bar{\alpha}_{BL}$, we uncover that when the production cost m increases or the disutility caused by counterfeits λ increases, $\bar{\alpha}_{BL}$ increases, which suggests that an increase in the proportion of patricians is more likely to bring a lose-lose outcome to the luxury fashion supply chain and the corresponding consumers. Additionally, we interestingly find that when the status disutility suffered by patricians is sufficiently large (i.e., $l > \bar{l}$), an increase in l increases the total consumer surplus. This is because as l increases, p_{BL}^* decreases as a result. Although patricians suffer a higher disutility from the big logo product, their monetary gain surpasses the status disutility, they can still be benefited.

Table 5.3. Sensitivity analysis for the optimal solutions under big logo strategy.

	w_{BL}^*	p_{BL}^*	D_{BL}^*	$\pi_{BL}^{R*^*}$	π_{BL}^{M}	π_{BL}^{SC}	CS_{BL}^*
$m\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\alpha\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow if\ \alpha < \bar{\alpha}_{BL}$ $\downarrow if\ \alpha > \bar{\alpha}_{BL}$
$l\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow if\ l < \bar{l}$ $\uparrow if\ l > \bar{l}$
$\theta\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\lambda\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

Remarks: $\bar{\alpha}_{BL} = \frac{8l+m+7\theta+\lambda-1}{15(l+\theta)}, \bar{l} = \frac{1-m-15(1-\alpha)\theta-\lambda}{16-15\alpha}$.

5.2.2 No Logo

We now consider the case where the luxury fashion brand decides to use no logo. We use the subscript NL to represent the functions and decisions derived in this section. The utility functions for the patricians and parvenus are $U_{NL}^{Patrician} = v - p_{NL} + \phi - \tau$ and $U_{NL}^{Parvenu} = v - p_{NL} - k - \tau$. Accordingly, we can derive the total number of consumers who purchase the no logo product:

$$D_{NL} = \alpha \int_{p_{NL}-\phi+\tau}^{1} f(v)\, dv + (1-\alpha) \int_{p_{NL}+k+\tau}^{1} f(v)\, dv = 1 - p_{NL} - \tau + \alpha\phi - (1-\alpha)k. \quad (5.6)$$

We can then derive the profit function for the luxury fashion brand:

$$\pi_{NL}^{R} = (p_{NL} - w_{NL})(1 - p_{NL} - \tau + \alpha\phi - (1-\alpha)k). \quad (5.7)$$

Maximizing Eq.(5.7) with respect to p_{NL}, we obtain the optimal retail price for the product as $p_{NL}^* = \frac{1}{2}(1 - \tau + \alpha\phi - (1-\alpha)k + w_{NL})$. Substituting it back into Eq.(5.6), we have the optimal demand for the no logo product: $D_{NL}^* = \frac{1}{2}(1 - \tau + \alpha\phi - (1-\alpha)k - w_{NL})$.

Considering profits of the manufacturer and the luxury fashion supply chain, we have:

$$\pi_{NL}^{M} = (w_{NL} - m)D_{NL}^* = \frac{1}{2}(w_{NL} - m)(1 - \tau + \alpha\phi - (1-\alpha)k - w_{NL}). \quad (5.8)$$

$$\pi_{NL}^{SC*} = \pi_{NL}^{R*} + \pi_{NL}^{M*}. \quad (5.9)$$

Maximizing Eq.(5.8) with respect to w_{NL}, we obtain the optimal wholesale price $w_{NL}^* = \frac{1}{2}(1 - \tau + \alpha\phi - (1-\alpha)k + m)$.

We then derive the total consumer surplus in Eq.(5.10), where the first term is the consumer surplus for the patricians and the second term is the consumer surplus for the parvenus. Through substitution, we obtain the optimal decisions, profits and consumer surplus as summarized in Lemma 5.2.

$$CS_{NL} = \alpha \int_{p_{NL}-\phi+\tau}^{1} (v - p_{NL} + \phi - \tau) f(v)\, dv + (1-\alpha) \int_{p_{NL}+k+\tau}^{1} (v - p_{NL} - k - \tau) f(v)\, dv. \quad (5.10)$$

Lemma 5.2. *Under no logo strategy, the optimal decisions, demand, profits of the luxury fashion supply chain and its members, and the corresponding consumer surplus are given as follows:*

$w_{NL}^{*} = \frac{1}{2}(B+m),\ p_{NL}^{*} = \frac{1}{4}(3B+m),\ D_{NL}^{*} = \frac{1}{4}(B-m),\ \pi_{NL}^{R*} = \frac{1}{16}(B-m)^2,\ \pi_{NL}^{M} = \frac{1}{8}(B-m)^2,$ $\pi_{NL}^{SC} = \frac{3}{16}(B-m)^2,\ CS_{NL}^{*} = \frac{1}{32}[\alpha(4-3B-m+4\phi-4\tau)^2 + (1-\alpha)(4-3B-m-4k-4\tau)^2], where\ B = 1-\tau+\alpha\phi-(1-\alpha)k.$

Lemma 5.2 shows a similar result as in Lemma 5.1. The effects of the production cost m remain the same. Likewise, we denote B as the unit net benefit to consumers brought by the no logo product. Exploring further on B, we uncover that when (i) the status utility gained by patricians increases (i.e., $\alpha \uparrow$ or $\phi \uparrow$); (ii) the status disutility suffered by parvenus decreases (i.e., $1-\alpha \downarrow$ or $k \downarrow$); or (iii) the disutility caused by copycats decreases (i.e., $\tau \downarrow$), the unit net benefit to consumers brought by the no logo product B increases, which results in a higher wholesale price and retail price, and higher profits for the luxury fashion supply chain and its members. Considering the consumer surplus, we discover that an increase in the proportion of patricians (i.e., $\alpha \uparrow$) not always leads to an increase in the consumer surplus.

The rationale is that when the proportion of patricians is relatively large (i.e., $\alpha > \bar{\alpha}_{NL}$), price of the no logo product is significantly large, patricians' status gain can no longer offset the monetary loss, the total consumer surplus decreases as a result. And when the production cost m decreases or the disutility caused by copycat τ decreases (which results in an increase in $\bar{\alpha}_{NL}$), an increase in the proportion of patricians is more prone to bring a win-win outcome to the luxury fashion supply chain and the consumers. We also interestingly find that an increase in parvenus' status disutility (i.e., $k \uparrow$) can bring a higher consumer surplus under certain conditions. Explicitly, when the parvenus' status disutility is sufficiently large (i.e., $k > \bar{k}$), consumers' monetary gain exceeds their status loss, the total consumer surplus is still benefited.

Table 5.4. Sensitivity analysis for the optimal solutions under no logo strategy.

	w_{NL}^{*}	p_{NL}^{*}	D_{NL}^{*}	$\pi_{NL}^{R*\,*}$	π_{NL}^{M}	π_{NL}^{SC}	CS_{NL}^{*}
$m \uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\alpha \uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow if\ \alpha < \bar{\alpha}_{NL}$ $\downarrow if\ \alpha > \bar{\alpha}_{NL}$

$\phi\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$k\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow if\ k<\bar{k}$ $\uparrow if\ k>\bar{k}$
$\tau\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Remarks: $\bar{\alpha}_{NL}=\frac{1+7k-m-\tau+8\phi}{15(k+\phi)}, \bar{k}=\frac{1-m-\tau-15\alpha\phi}{1+15\alpha}$.							

5.2.3 When to Show Big Logos

By comparing the results under big logo and no logo strategies, we derive the conditions under which one strategy outperforms the other and summarize the results in Propositions 5.1 and 5.2.

Proposition 5.1. *(i) $D_{BL}^*>0$ if and only if $m<m_{BL}$*[37]*; $D_{NL}^*>0$ if and only if $m<m_{NL}$, where $m_{BL}=A=1-\lambda-\alpha l+(1-\alpha)\theta$ and $m_{NL}=B=1-\tau+\alpha\phi-(1-\alpha)k$; (ii) $m_{BL}>m_{NL}$ if and only if $\lambda-\tau<\xi$, where $\xi=(1-\alpha)(\theta+k)-\alpha(l+\phi)$.*

We denote $\lambda-\tau$ as the negative effects differential caused by counterfeits and copycats, which captures the comparative negative effects of counterfeits and copycats. Proposition 5.1 reveals two important findings. First, only when the production cost is not significantly large, can demand for the big logo product or the no logo product be positive. Specifically, for the big logo product, with a smaller proportion of patricians α, lower status disutility for patricians θ, higher status utility for parvenus l, or lower disutility of counterfeits λ, demand for the big logo product is more likely to be positive; for the no logo product, on the contrary, with a larger proportion of patricians α, higher status utility for patricians ϕ, lower status disutility for parvenus k, or lower disutility of copycats τ, demand for the no logo product is more inclined to be positive. Another critical finding is that, if and only if the negative effects differential is relatively small (i.e., $\lambda-\tau<\xi$), adopting the big logo strategy is more likely to bring a positive demand compared with the no logo strategy. The finding highlights the impacts of counterfeits and copycats in making the logo design decision.

Proposition 5.2. *(i) Showing big logo is more beneficial to the luxury fashion supply chain and its members if and only if $\lambda-\tau<\xi$, where $\xi=(1-\alpha)(\theta+k)-\alpha(l+\phi)$; otherwise, showing no logo is more beneficial. (ii) ξ increases linearly in θ and k while decreases linearly in l, ϕ and α.*

Proposition 5.2 uncovers an important finding that it is the comparative negative effects caused by counterfeits and copycats (i.e., $\lambda-\tau$) rather than the single impact of counterfeits (λ) or copycats (τ) that decides on the optimal logo design decision. Explicitly, when the negative effects differential is relatively small (i.e., $\lambda-\tau<\xi$), showing a big logo is more beneficial to the luxury fashion supply

[37] Although it is an implied condition that demand should always be positive, we still keep it here to examine the conditions under which demand is more likely to be positive. In addition, by comparing the results with the ones derived under the extended models (i.e., Propositions 4.1 and 4.5), we uncover the cases under which a positive demand is more likely to be achieved.

chain and its members; otherwise, adopting the no logo strategy is more beneficial. The finding suggests that luxury fashion brands should consider the joint impacts of counterfeits and copycats in making the logo design decision. For example, for some luxury fashion brands (e.g., *LV*) which are severely affected by counterfeits compared with copycats (i.e., the negative effects differential is relatively large), they are advised to launch more products with no logo to prevent the encroachment of counterfeits.

Exploring further on the threshold of ξ, we discover that with an increase in parvenus' status utility ($\theta \uparrow$) or a decrease in patricians' status disutility ($l \downarrow$) when adopting the big logo strategy, or an increase in parvenus' status disutility ($k \uparrow$) or a decrease in patricians' status utility ($\phi \downarrow$) when adopting the no logo strategy, or a decrease in the ratio of patricians in the market ($\alpha \downarrow$), ξ increases, which suggests that it is more prone for the big logo strategy to be optimal. The findings provide several crucial insights. First, luxury fashion brands are advised to collect more customer information regarding their preferences for brand prominence (i.e., consumers' status utility or disutility when adopting the big or no logo strategy). Second, we recommend luxury fashion brands make logo design decisions based on their target markets, i.e., providing more products with big logo in emerging markets (where the ratio of patricians is relatively small) while providing more products with no logo in mature markets (where the ratio of patricians is relatively large). This recommendation is supported by many real world practices. For example, *Hermès* adopted this strategy to launch a collection of highly visible luxury accessories in the Indian market[38]. Likewise, *Coach* developed some luxury leather accessories with discreet logo in the U.S. market (Pino et al. 2019).

5.3 Extended Models and Analyses

5.3.1 Risk Analyses

The business risk derived from demand uncertainty is usually a critical factor that will affect the supply chain's decisions and performance (Choi et al. 2020a; Johari et al. 2022). In this subsection, we examine the case where the market segment is uncertain (Li et al. 2020a), and the luxury fashion brand is risk sensitive (i.e., either risk averse or risk seeking) when facing the demand uncertainty (use "^" to denote). To be specific, we suppose that the proportion of consumer segment $\hat{\alpha}$ is a random variable with mean α_0 (which equals α in the basic model) and standard derivation σ; the luxury fashion brand will take its risk attitude into consideration when making logo and pricing decisions. Following mainstream operations management (OM) papers (e.g., Chiu et al. 2018; Zhang et al. 2020; Zhu et al.

[38] https://www.forbes.com/sites/worldviews/2011/10/12/hermes-is-now-selling-saris-in-india-and-its-a-big-deal/?sh=46f937ff5f24 [Accessed on 28 Mar 2022]

2021), we adopt the classic mean risk theory to capture the luxury fashion brand's risk attitude in this study. This theory is well-established and commonly used in conducting risk related analysis. For analytical traceability, we use Mean-Standard Deviation (MSD) theory to depict the luxury fashion brand's objective function which is shown in Eq.(5.11):

$$U[\hat{\pi}_i^R] = E[\hat{\pi}_i^R] - \beta SD[\hat{\pi}_i^R], \tag{5.11}$$

where $i = BL \; or \; NL$, $E[\hat{\pi}_i^R]$ is the luxury fashion brand's expected profit, $SD[\hat{\pi}_i^R]$ is the standard derivation of profit, and β is the risk sensitivity coefficient. Note that, a positive β means that the luxury fashion brand is risk averse (e.g., the brands hesitating to follow the logomania trend); a negative β means that the luxury fashion brand is risk seeking (e.g., *Gucci*, which launches a series of logomania collections); if β equals zero, the luxury fashion retail is risk neutral and his objective is to maximize the expected profit (i.e., basic model). The manufacturer is risk neutral and decides its wholesale price to maximize the expected profit $E[\hat{\pi}_i^M]$.

Big Logo

We first consider the case where the luxury fashion brand adopts the big logo strategy.

Substituting Eq.(5.2) into Eq.(5.11), we can easily derive the MSD objective function for the luxury fashion brand:

$$U[\hat{\pi}_{BL}^R] = (\hat{p}_{BL} - \widehat{w}_{BL})(1 - \hat{p}_{BL} - (l + \theta)(\beta\sigma + \alpha_0) + \theta - \lambda). \tag{5.12}$$

Maximizing Eq(5.12) with respect to $\hat{p}_{BL}$, we obtain the optimal retail price for the big logo product $\hat{p}_{BL}^* = \frac{1}{2}(1 + \theta - \lambda - (l + \theta)(\beta\sigma + \alpha_O) + w_{\mathrm{BL}})$. Substituting $\hat{p}_{BL}^*$ into Eq.(5.1) and take expectation, we obtain the optimal expected demand: $E[\widehat{D}_{BL}^*] = \frac{1}{2}(1 + \theta - \lambda - (l + \theta)(\beta\sigma + \alpha_O) - w_{BL})$.

We can then derive the expected profit function for the manufacturer:

$$E[\hat{\pi}_{BL}^M] = \frac{1}{2}(\widehat{w}_{BL} - m)(1 + \theta - \lambda - (l + \theta)(\beta\sigma + \alpha_O) - w_{BL}). \tag{5.13}$$

Maximizing Eq(5.13) with respect to $\widehat{w}_{BL}$, we obtain the optimal wholesale price for the big logo product $\widehat{w}_{BL}^*$. Through substitution, we summarize the optimal decisions, profits and consumer surplus in Lemma 5.3.

For a notational purpose, we define: $\hat{A} = 1 - \lambda - \alpha_0 l + (1 - \alpha_0)\theta$, and we let $G = \beta(l + \theta)\sigma$.

Lemma 5.3. *When considering the luxury fashion brand's risk attitude, under big logo strategy, the optimal decisions, demand, profits of the luxury fashion supply chain and its members, and the corresponding consumer surplus are given as follows:*

$\hat{w}_{BL}^* = \frac{1}{2}\left(\hat{A} + G + m\right),\ \hat{p}_{BL}^* = \frac{1}{4}\left(3\hat{A} - G + m\right),\ E[\hat{D}_{BL}^*] = \frac{1}{4}\left(\hat{A} + G - m\right),\ U[\hat{\pi}_{BL}^{R*}] = \frac{1}{16}\left(\hat{A} - 3G - m\right)^2, E[\hat{\pi}_{BL}^{M*}] = \frac{1}{8}\left(\hat{A} + G - m\right)^2, E[\hat{\pi}_{BL}^{SC*}] = \frac{1}{16}(3\hat{A} - G - 3m)(\hat{A} + G - m),$
$E[\widehat{CS}_{BL}^*] = \frac{1}{32}\left[\alpha_0\left(4 - 3\hat{A} + G - m - 4l - 4\lambda\right)^2 + (1 - \alpha_0)\left(4 - 3\hat{A} + G - m + 4\theta - 4\lambda\right)^2\right].$

Lemma 5.3 shows the neat expressions of the optimal decisions, profits, and the corresponding consumer surplus in this case. As we can observe, the luxury fashion brand's risk attitude can have a significant impact on the supply chain members' decisions and performance. Denoting $|G|$ as the unit net impact brought by the risk of demand uncertainty for the big logo product, we find that both the patricians' and parvenus' status disutility/utility (i.e., l and θ) will enlarge this kind of impact (i.e., $\frac{d|G|}{dl} > 0, \frac{d|G|}{d\theta} > 0$). Moreover, whether the demand volatility (σ) can positively or negatively affect the supply chain performance depends on the luxury fashion brand's risk attitude. That is, when the luxury fashion brand is risk averse (i.e., $\beta > 0$), the expected demand can be increased by the demand volatility (i.e., $\frac{dE[\hat{D}_{BL}^*]}{d\sigma} > 0$) as the luxury fashion brand tends to lower its retail price (i.e., $\frac{d\hat{p}_{BL}^*}{d\sigma} < 0$) to attract more consumers; hence, the risk of demand uncertainty can be beneficial to the luxury fashion brand as long as the production cost is not too small (i.e., $\frac{dU[\hat{\pi}_{BL}^{R*}]}{d\sigma} > 0$ if an only if $m > \hat{A} - 3G$). Meanwhile, the manufacturer tends to set a higher wholesale price (i.e., $\frac{d\hat{w}_{BL}^*}{d\sigma} > 0$) under the risk, which helps itself to earn more profit from the luxury fashion brand (i.e., $\frac{dE[\hat{\pi}_{BL}^{M*}]}{d\sigma} > 0$).

While when the luxury fashion brand is risk seeking (i.e., $\beta < 0$), the impacts brought by the demand uncertainty is completely opposite. Thus, when facing the risk of demand uncertainty, the supply chain members should carefully make their decisions according to the luxury fashion brand's risk attitude. Besides, in terms of the consumer surplus, we interestingly uncover that it is always convex in the demand uncertainty (i.e., $\frac{d^2E[\hat{\pi}_{BL}^{M*}]}{d\sigma^2} > 0$), irrespective of the luxury fashion brand's risk attitude. This is because the consumers can enjoy an extremely low retail price under the extremely high (resp. low) demand uncertainty from the risk averse (resp. seeking) luxury fashion brand, so that they can be benefited by the demand uncertainty.

No Logo

We now consider the case where the luxury fashion brand adopts the no logo strategy.

Substituting Eq.(5.7) into Eq.(5.11), we can easily derive the MSD objective function for the luxury fashion brand:

$$U[\hat{\pi}_{NL}^R] = (\hat{p}_{NL} - \hat{w}_{NL})\left(1 - \hat{p}_{NL} - k - \tau + (\phi + k)(\alpha_0 - \beta\sigma)\right). \quad (5.14)$$

Maximizing Eq(5.14) with respect to $\hat{p}_{NL}$, we obtain the optimal retail price for the no logo product $\hat{p}_{NL}^* = \frac{1}{2}(1 - k - \tau + (\phi + k)(\alpha_0 - \beta\sigma) + w_{NL})$. Substituting $\hat{p}_{NL}^*$ into Eq.(5.6) and take expectation, we obtain the optimal expected demand: $E[\widehat{D}_{NL}^*] = \frac{1}{2}(1 - k - \tau + (\phi + k)(\alpha_0 + \beta\sigma) - w_{NL})$.

We can then derive the expected profit function for the manufacturer:

$$E[\hat{\pi}_{NL}^M] = \frac{1}{2}(\widehat{w}_{NL} - m)(1 - k - \tau + (\phi + k)(\alpha_0 + \beta\sigma) - w_{NL}). \tag{5.15}$$

Maximizing Eq(5.15) with respect to $\widehat{w}_{NL}$, we obtain the optimal wholesale price for the no logo product $\widehat{w}_{NL}^*$. Through substitution, we summarize the optimal decisions, profits and consumer surplus in Lemma 5.4.

We define $\hat{B} = 1 - \tau + \alpha_0\phi - (1 - \alpha_0)k$, and we let $H = \beta(\phi + k)\sigma$.

Lemma 5.4. *When considering the luxury fashion brand's risk attitude, under no logo strategy, the optimal decisions, demand, profits of the luxury fashion supply chain and its members, and the corresponding consumer surplus are given as follows:*

$\widehat{w}_{NL}^* = \frac{1}{2}(\hat{B} + H + m)$, $\hat{p}_{NL}^* = \frac{1}{4}(3\hat{B} - H + m)$, $E[\widehat{D}_{NL}^*] = \frac{1}{4}(\hat{B} + H - m)$, $U[\hat{\pi}_{NL}^{R*}] = \frac{1}{16}(\hat{B} - 3H - m)^2$, $E[\hat{\pi}_{NL}^{M*}] = \frac{1}{8}(\hat{B} + H - m)^2$, $E[\hat{\pi}_{NL}^{SC*}] = \frac{1}{16}(3\hat{B} - H - 3m)(\hat{B} + H - m)$, $E[\widehat{CS}_{NL}^*] = \frac{1}{32}[\alpha_0(4 - 3\hat{B} + H - m + 4\phi - 4\tau)^2 + (1 - \alpha_0)(4 - 3\hat{B} + H - m - 4k - 4\tau)^2]$.

Lemma 5.4 presents the neat expressions of the optimal decisions, profits, and the corresponding consumer surplus in the no logo case. Similar to the findings derived in the big logo case (i.e., Lemma 5.3), we also suggest the supply chain members to carefully make their decisions based on the luxury fashion brand's risk attitude when facing the risk of demand uncertainty, as different risk attitudes (i.e., risk averse and risk seeking) will have totally different impacts on the results. The corresponding impacts are the same with the big logo case.

When to Show Big Logos

Then, we proceed to explore the value of big logo and no logo strategies in Propositions 5.3 and 5.4.

Proposition 5.3. *(i)* $E[\widehat{D}_{BL}^*] > 0$ *if and only if* $m < \widehat{m}_{BL}$; $E[\widehat{D}_{NL}^*] > 0$ *if and only if* $m < \widehat{m}_{NL}$, *where* $\widehat{m}_{BL} = \hat{A} + G$ *and* $\widehat{m}_{NL} = \hat{B} + H$. *(ii)* $\widehat{m}_{BL} > \widehat{m}_{NL}$ *if and only if* $\lambda - \tau < (1 - \alpha_0)(\theta + k) - \alpha_0(l + \phi) + \beta\sigma(l + \theta - k - \phi)$.

The major findings derived in Proposition 5.3 are qualitatively consistent with the ones shown in the basic model (i.e., Proposition 5.1): (i) Only when the production cost is not significantly large, can demand for the big logo product or the no logo product be positive. (ii) Only when the negative effects differential is relatively small, the big logo strategy is more likely to result in a positive demand than the no logo strategy. Furthermore, we notice that the existence of demand uncertainty can bring advantages to the luxury fashion supply chain. Specifically, the demand for both big logo and the no logo products are more likely to be positive under the existence of demand uncertainty (i.e., $\widehat{m}_{BL} > m$ and $\widehat{m}_{NL} > m$) if the luxury fashion brand is risk averse (i.e., $\beta > 0$). This finding is interesting, as it is generally believed that the risk of demand uncertainty should be unwelcomed by the risk averse luxury fashion brand; while in fact, the existence of demand uncertainty induces the luxury fashion brand to reduce its retail price and consequently attract more consumers.

Proposition 5.4. *When considering the luxury fashion brand's risk attitude, showing big logo is more beneficial to (i) the luxury fashion brand if and only if* $\lambda - \tau < \xi^R$*, where* $\xi^R = (1-\alpha_0)(\theta + k) - \alpha_0(l + \phi) + 3\beta\sigma(k + \phi - l - \theta)$*, and (ii) the manufacturer if and only if* $\lambda - \tau < \xi^M$*, where* $\xi^M = (1-\alpha_0)(\theta + k) - \alpha_0(l + \phi) + \beta\sigma(l + \theta - k - \phi)$*; otherwise, showing no logo is more beneficial.*

Proposition 5.4 also presents a similar insight as the one derived in the basic model, that is, showing a big logo is more recommended if the negative effects differential caused by counterfeits and copycats ($\lambda - \tau$) is relatively small. We hence prove that the major findings shown in the basic model are robust. While differently, when considering the existence of risk and the luxury fashion brand's risk attitude, the luxury fashion brand and manufacturer may have different preferences for the logo strategy ($\xi^R \neq \xi^M$). Under this circumstance, the win-win outcome is less likely to be achieved, which reveals the negative impact brought by the risk. Then, in order to explore how the risk of demand uncertainty will influence the optimal logo strategy, we further analyze the thresholds (ξ^R and ξ^M) in details in Proposition 5.5.

Proposition 5.5. ξ^R *(resp.* ξ^M*) increases (resp. decreases) in* σ *if and only if (*$\beta < 0$ *and* $0 < \frac{\phi - l}{\theta - k} < 1$*) or (*$\beta > 0$ *and* $\frac{\phi - l}{\theta - k} > 1$*); otherwise,* ξ^R *(resp.* ξ^M*) decreases (resp. increases) in* σ*.*

By conducting sensitivity analysis on the thresholds with respect to the demand volatility σ, Proposition 5.5 provides implications to the luxury fashion brand on how to decide its optimal logo decision with respect to the demand uncertainty. We first define $\frac{\phi - l}{\theta - k}$ as the ratio of status disparity, which jointly affects the luxury fashion brand's decision. This ratio can be regarded as the status sensitivity of patricians compared with parvenus. A higher ratio (i.e., $\frac{\phi - l}{\theta - k} > 1$) means that the patricians care more about their status compared with the parvenus, and a lower ratio (i.e., $0 < \frac{\phi - l}{\theta - k} < 1$) means that the parvenus are more sensitive about their status compared with the patricians.

Our analytical results imply that (i) if the luxury fashion brand is risk seeking (i.e., $\beta < 0$), with a higher demand volatility (σ), the luxury fashion brand (manufacturer) is more (less) likely to be benefited from showing big logo when the ratio of status disparity is relatively low, and (ii) if the luxury fashion brand is risk averse (i.e., $\beta > 0$), with a higher demand volatility (σ), the luxury fashion brand (manufacturer) is more (less) likely to be benefited from showing big logo when the ratio of status disparity is relatively high. This finding indicates that the risk of demand uncertainty will cause a reverse effect for the luxury fashion brand and manufacturer, which results in the conflict of interest in the luxury fashion supply chain. We further compare the thresholds of ξ^R and ξ^M with the one derived under the basic model (i.e.,ξ) and summarize the results in proposition 5.6.

Proposition 5.6. *$\xi^R > \xi$ (resp. $\xi^M < \xi$) if and only if ($\beta < 0$ and $0 < \frac{\phi-l}{\theta-k} < 1$) or ($\beta > 0$ and $\frac{\phi-l}{\theta-k} > 1$); otherwise, $\xi^R \leq \xi$ (resp. $\xi^M \geq \xi$).*

By comparing the thresholds with the basic model, we reveal the impact of the luxury fashion brand's risk attitude on the optimal logo strategy. To be specific, (i) when the ratio of status disparity is relatively small (i.e., $0 < \frac{\phi-l}{\theta-k} < 1$), a risk seeking luxury fashion brand (rather than risk neutral) is more likely to be benefited from adopting the big logo strategy; while (ii) when the ratio of status disparity is sufficiently large (i.e., $\frac{\phi-l}{\theta-k} > 1$), a risk averse luxury fashion brand (rather than risk neutral) is more prone to be benefited from adopting the big logo strategy. The findings suggest that the ratio of status disparity is a critical factor governing the impacts of risk attitudes on the optimal logo strategy. Moreover, similar to Proposition 5.5, Proposition 5.6 also reveals the conflict of interest of the luxury fashion brand and the manufacturer. To better illustrate the impacts of risk attitude on the luxury fashion brand's optimal logo strategy as well as providing clear managerial insights for the luxury fashion supply chain, we summarize the above insights in Theorem 5.1.

Theorem 5.1. *When the luxury fashion brand is risk sensitive, the big logo (no logo) strategy is more likely to be optimal for the luxury fashion brand (manufacturer) if one of the following conditions can be satisfied:*

(i) The luxury fashion brand is risk seeking and the ratio of consumers' status disparity is relatively low.

(ii) The luxury fashion brand is risk averse and the ratio of consumers' status disparity is relatively high.

Our findings may partially explain why some brands like *Gucci* actively launch logomania collections while others hesitate to do so. One explanation is that, in terms of *Gucci*'s consumers, the parvenus are less sensitive about their status compared with the patricians. In this case, parvenus tend to buy big-logo *Gucci* products to show their social status whereas patricians who are less concerned about social status simply buy big-logo *Gucci* products to join the logomania trend. As a result, *Gucci* can be successful in the trend of logomania. On the contrary, for the brands hesitating to follow the logomania trend, their consumers are probably not ready for the trend (i.e., the patricians care more about social status rather than joining the logomania trend). To conclude, when a luxury fashion brand decides whether to follow the logomania trend, it should first consider consumers' acceptance for this trend (i.e., the ratio of consumers' status disparity); otherwise, it will fall prey to the logomania trend.

Next, we pay attention to the performance of the entire luxury fashion supply chain and consumer surplus and depict Figures 5.3 and 5.4. As we can observe, the luxury fashion brand's risk attitude will directly influence the results. First, in terms of the entire luxury fashion supply chain's profit (i.e., Figure 5.3), the big logo strategy dominates the no logo strategy when the luxury fashion brand is risk averse; and when the luxury fashion brand is risk seeking, showing big logo is more profitable only if the proportion of patricians who dislike big logos (α_0) is relatively small, which is consistent with the findings in the basic model. Then, as regards to the consumer surplus (Figure 5.4), no matter the luxury fashion brand is risk averse or risk seeking, there exist conditions under which showing big logo is a better choice. To be specific, when the luxury fashion brand is risk averse, the big logo strategy is more welcomed by the consumers if the proportion of patricians who dislike big logos (α_0) is extremely small; while when the luxury fashion brand is risk seeking, the consumers would prefer the big logo strategy if the proportion of patricians who dislike big logos (α_0) is relatively large.

Combining the results in Figures 5.3, 5.4 and Proposition 5.4, we notice that there should exist an all-win outcome for the entire luxury fashion supply chain, its members, and the consumers (i.e., when the luxury fashion brand is risk averse and α_0 is relatively small), which is in line with the findings in the basic model; but the likelihood should be much smaller, as there exists a conflict of interest between the consumers and the luxury fashion supply chain when the luxury fashion brand is risk seeking. To illustrate, we further depict Figure 5.5 which shows the all-win outcomes. Our finding has very good managerial implications: in order to achieve an all-win outcome for the luxury fashion supply chain, its members and the consumers, (i) a risk averse luxury fashion brand is advised to adopt the big logo strategy when the ratio of patricians is relatively small; while (ii) a risk seeking luxury fashion brand is recommended to adopt the no logo strategy when the ratio of patricians is comparatively large.

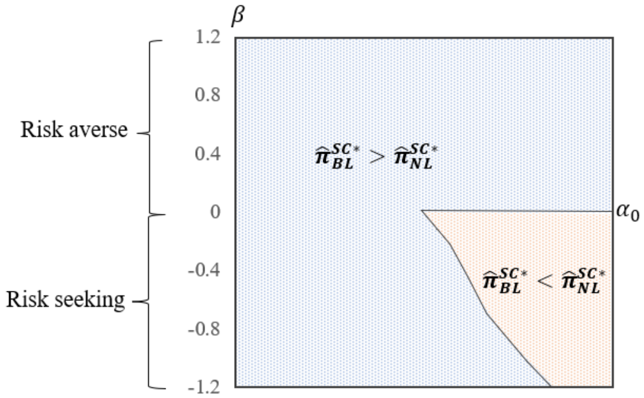

Figure 5.3. Comparison between showing big logo and no logo for the luxury fashion supply chain (We let $l = 0.1$, $\theta = 0.1$, $\phi = 0.2$, $k = 0.4$, $\lambda = 0.2$, $\tau = 0.1$, $m = 0.2$, $\sigma = 1$, $\alpha_0 \in [0,1]$, and $\beta \in R$).

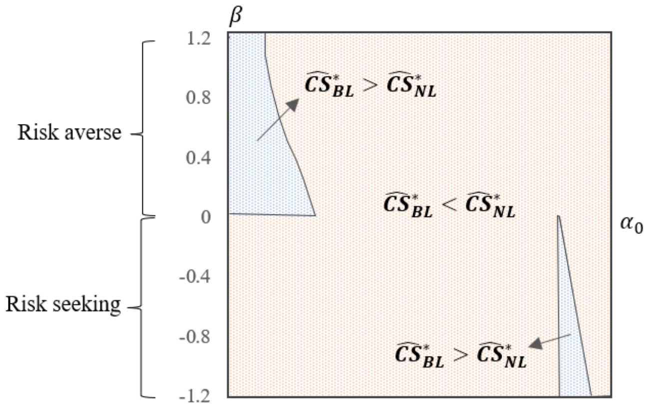

Figure 5.4. Comparison between showing big logo and no logo for consumer surplus (We let $l = 0.1$, $\theta = 0.1$, $\phi = 0.2$, $k = 0.4$, $\lambda = 0.2$, $\tau = 0.1$, $m = 0.2$, $\sigma = 1$, $\alpha_0 \in [0,1]$, and $\beta \in R$).

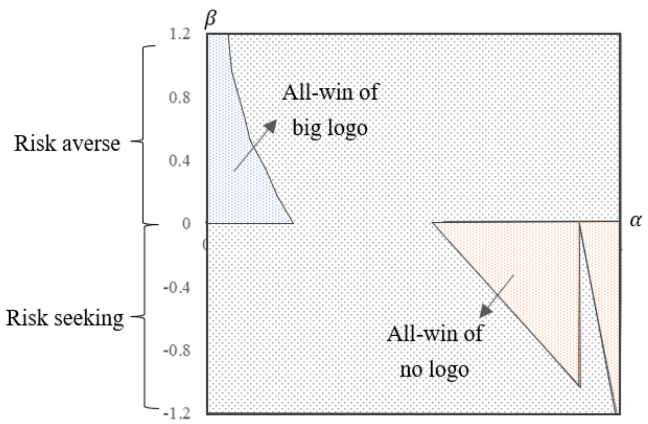

Figure 5.5. All-win outcomes for the luxury fashion supply chain, its members and the consumers. (We let $l = 0.1$, $\theta = 0.1$, $\phi = 0.2$, $k = 0.4$, $\lambda = 0.2$, $\tau = 0.1$, $m = 0.2$, $\sigma = 1$, $\alpha_0 \in [0,1]$, and $\beta \in R$).

5.3.2 The Implementation of Blockchain

Blockchain, as an emerging technology, has been widely adopted to monitor the authenticity and reliability of product information for supply chain management (Choi 2019; Hastig and Sodhi 2019). In the fashion industry, fashion brands usually implement this technology to combat counterfeits and copycats (Pun et al. 2021; Shen et al. 2021). As we have discussed earlier in Section 5.1, with the implementation of blockchain technology, consumers of *LVMH* can have access to the entire product value chain. In this case, their probability of buying counterfeit or copycat luxury products is largely reduced. Hence, we construct the disutilities caused by consumers unknowingly purchasing a counterfeit or copycat as $\frac{\lambda}{\gamma}$ and $\frac{\tau}{\gamma}$, respectively, where $\gamma > 1$ is the effectiveness of blockchain. With the increase of blockchain effectiveness (i.e., increased γ), disutilities brought by counterfeit and copycat will be reduced. Note that, the implementation of blockchain requires a fixed cost $F > 0$ and a per-unit cost $t \in (0,1)$ (Pun et al. 2021). This is in line with the practice of the *Aura Blockchain Consortium*, where participants (e.g., luxury fashion brands) need to pay an annual licensing fee (i.e., the fixed cost) and a volume fee (i.e., the per-unit cost) for using the blockchain technology[39]. In the following, we examine the cases where the luxury fashion brand adopts big logo and no logo strategies, respectively. We use dots marked overhead (e.g., $\ddot{A}$) to denote the functions and decisions derived in this section.

Big Logo

With the implementation of blockchain, consumers' disutility caused by counterfeits can be reduced. Market demand from patricians and parvenus thus changes to be Eq. (5.16).

$$\ddot{D}_{BL} = \alpha \int_{\ddot{p}_{BL}+l+\frac{\lambda}{\gamma}}^{1} f(v)\,dv + (1-\alpha)\int_{\ddot{p}_{BL}-\theta+\frac{\lambda}{\gamma}}^{1} f(v)\,dv = 1 - \ddot{p}_{BL} - \frac{\lambda}{\gamma} - \alpha l + (1-\alpha)\theta. \quad (5.16)$$

We can then derive the profit function for the luxury fashion brand:

$$\ddot{\pi}_{BL}^{R} = (\ddot{p}_{BL} - \ddot{w}_{BL} - t)D_{BL} - F = (\ddot{p}_{BL} - \ddot{w}_{BL} - t)\left(1 - \ddot{p}_{BL} - \frac{\lambda}{\gamma} - \alpha l + (1-\alpha)\theta\right) - F. \quad (5.17)$$

Maximizing Eq. (5.17) with respect to $\ddot{p}_{BL}$, we can obtain the optimal selling price of the big logo product as $\ddot{p}_{BL}^{*} = \frac{(1+t+\ddot{w}_{BL}+\theta-\alpha(l+\theta))}{2} - \frac{\lambda}{2\gamma}$. Substituting the optimal decision into Eq. (5.16), we obtain the optimal demand when the big logo strategy is adopted: $\ddot{D}_{BL}^{*} = \frac{(1-t-\ddot{w}_{BL}-l\alpha+(1-\alpha)\theta)}{2} - \frac{\lambda}{2\gamma}$. Then,

[39] https://www.forbes.com/sites/pamdanziger/2021/04/22/lvmh-cartier-and-prada-partner-to-fight-counterfeits-and-invite-other-luxury-brands-to-join/?sh=debf2c730723 [Accessed on 29 Mar 2022]

solving the manufacturer's problem Eq. (5.3) by substituting $\ddot{D}^*_{BL}$, we obtain the optimal wholesale price $\ddot{w}^*_{BL} = \frac{(1+m-t-\alpha l+(1-\alpha)\theta))}{2} - \frac{\lambda}{2\gamma}$. Similar to basic model, it is straightforward to derive the luxury fashion supply chain's optimal profit and consumer surplus. In Lemma 5.5 we summarize the optimal decisions, profits, and consumer surplus.

Lemma 5.5. *When the blockchain technology is implemented, under big logo, the optimal decisions, demand, profits for the luxury fashion supply chain and its members, and consumer surplus are shown as follows:*

$\dddot{w}^*_{BL} = \frac{\ddot{A}+m-t}{2} - \frac{\lambda}{2\gamma}$, $\dddot{p}^*_{BL} = \frac{3\ddot{A}+m+t}{4} - \frac{3\lambda}{4\gamma}$, $\dddot{D}^*_{BL} = \frac{\ddot{A}-m-t}{4} - \frac{\lambda}{4\gamma}$, $\dddot{\pi}^{R*}_{BL} = \frac{(\gamma(\ddot{A}-m-t)-\lambda)^2}{16\gamma^2} - F$, $\dddot{\pi}^{M*}_{BL} = \frac{(\gamma(\ddot{A}-m-t)-\lambda)^2}{8\gamma^2}$, $\dddot{\pi}^{SC*}_{BL} = \frac{3(\gamma(\ddot{A}-m-t)-\lambda)^2}{16\gamma^2} - F$, $\ddot{CS}^*_{BL} = \frac{1}{32\gamma^2}(\alpha(\gamma\left(3\ddot{A}+m+t-4(1-l)\right)+\lambda)^2 + (1-\alpha)(\gamma\left(3\ddot{A}+m+t-4(1+\theta)\right)+\lambda)^2)$, *where* $\ddot{A} = (1-\alpha l+(1-\alpha)\theta)$.

Lemma 5.5 presents the neat expressions of the optimal results when blockchain technology is implemented. Observing them, we find that the effectiveness of blockchain and the operation cost of the blockchain-based system are crucial for the luxury fashion supply chain and its members. When blockchain becomes more effective to alleviate the disutility of copycat or counterfeit, the luxury fashion brand can be benefited by increasing the retail price due to the increased market demand (i.e., $\frac{\partial \ddot{D}^*_{BL}}{\partial \gamma}$>0). In this case, the manufacturer is encouraged to raise the wholesale price to get a higher profit (i.e., $\frac{\partial \dddot{\pi}^{M*}_{BL}}{\partial \gamma}$>0). It means that an increase in blockchain effectiveness can improve the performance of luxury fashion supply chain in obtaining more profit. With regard to the consumer surplus, it will always be increased when facing higher blockchain effectiveness. This is because consumers can be satisfied by the lower disutility of the counterfeits (i.e., $\frac{\partial \frac{\lambda}{\gamma}}{\partial \gamma} > 0$) owing to the more effective blockchain, which offsets the negative impacts of the price increases.

On the other hand, the operation cost of blockchain has an opposite impact on the optimal results. The luxury fashion supply chain and its members' optimal decisions and performances are irrelevant to the fixed operation cost (i.e., F), while affected by the unit-cost t. The luxury fashion brand will be hurt by the increase of unit-cost for implementing blockchain (i.e., $\frac{\partial \ddot{\pi}^{R*}_{BL}}{\partial t} < 0$). It is because facing the higher unit-cost, the luxury fashion brand is more likely to increase the retail price (i.e., $\frac{\partial \dddot{p}^*_{BL}}{\partial t}$) to alleviate the loss of marginal profit. The higher price results in loss of demand ($\frac{\partial \ddot{D}^*_{BL}}{\partial t}$) and decrease of profit. In this case, the manufacturer tends to stimulate the market by reducing the wholesale price (i.e.,

$\frac{\partial \dddot{w}_{BL}^{*}}{\partial t}$), however, it fails to complement the loss induced by the loss of market demand and leads to a profit decrease (i.e., $\frac{\partial \dddot{\pi}_{BL}^{M*}}{\partial t} < 0$). Considering the consumer surplus, it is decreased in the unit-cost of blockchain because consumers experience a higher retail price.

<u>No Logo</u>

Now, we consider the case where the luxury fashion brand develops the no logo design. With the implementation of blockchain technology, consumer demand from patricians and parvenus can be presented as Eq. (5.18).

$$\dddot{D}_{NL} = \alpha \int_{\dddot{p}_{NL}-\phi+\frac{\tau}{\gamma}}^{1} f(v)\,dv + (1-\alpha)\int_{\dddot{p}_{NL}+k+\frac{\tau}{\gamma}}^{1} f(v)\,dv = 1 - \dddot{p}_{NL} - \frac{\tau}{\gamma} + \alpha\phi - (1-\alpha)k. \quad (5.18)$$

We can then obtain the luxury fashion brand's profit function as Eq. (5.19).

$$\dddot{\pi}_{NL}^{R} = (\dddot{p}_{NL} - \dddot{w}_{NL} - t)D_{NL} - F = (\dddot{p}_{NL} - \dddot{w}_{NL} - t)\left(1 - \dddot{p}_{NL} - \frac{\tau}{\gamma} + \alpha\phi - (1-\alpha)k\right) - F. \quad (5.19)$$

Maximizing Eq. (5.19) with respect to $\dddot{p}_{NL}$, we obtain the luxury fashion brand's optimal decision as $\dddot{p}_{NL}^{*} = \frac{(1+t+\dddot{w}_{NL}-k(1-\alpha)+\alpha\phi)}{2} - \frac{\tau}{2\gamma}$. Substituting it back into Eq. (5.18), we have the optimal demand for the no logo product $\dddot{D}_{NL}^{*} = \frac{(1-t-\dddot{w}_{NL}-k(1-\alpha)+\alpha\phi)}{2} - \frac{\tau}{2\gamma}$. Then, we solve the manufacturer's problem by maximizing Eq. (5.8) with $\dddot{D}_{NL}^{*}$, and the optimal wholesale price $\dddot{w}_{NL}^{*} = \frac{(1+m-t-k(1-\alpha)+\alpha\phi)}{2} - \frac{\tau}{2\gamma}$ is obtained. Similarly, we derive the luxury fashion supply chain's optimal profit and consumer surplus and provide the summary of optimal decisions, profits, and consumer surplus in Lemma 5.6 as follows.

Lemma 5.6. *When the blockchain technology is implemented, under no logo, the optimal decisions, demand, profits for luxury fashion supply chain and its members, and consumer surplus are shown as follows:*

$\dddot{w}_{NL}^{*} = \frac{\ddot{B}+m-t}{2} - \frac{\tau}{2\gamma}$, $\dddot{p}_{NL}^{*} = \frac{3\ddot{B}+m+t}{4} - \frac{3\tau}{4\gamma}$, $\dddot{D}_{NL}^{*} = \frac{\ddot{B}-m-t}{4} - \frac{\tau}{4\gamma}$, $\dddot{\pi}_{NL}^{R*} = \frac{(\gamma(\ddot{B}-m-t)-\tau)^2}{16\gamma^2} - F$, $\dddot{\pi}_{NL}^{M*} = \frac{(\gamma(\ddot{B}-m-t)-\tau)^2}{8\gamma^2}$, $\dddot{\pi}_{NL}^{SC*} = \frac{3(\gamma(\ddot{B}-m-t)-\tau)^2}{16\gamma^2} - F$, $\dddot{CS}_{NL}^{*} = \frac{1}{32\gamma^2}(\alpha(\gamma\left(3\ddot{B}+m+t-4(1+\phi)\right)+\tau)^2 + (1-\alpha)(\gamma\left(3\ddot{B}+m+t-4(1-k)\right)+\tau)^2)$, *where* $\ddot{B} = (1-(1-\alpha)k+\alpha\phi)$.

Lemma 5.6 shows the neat expressions of the optimal results for this extension under no logo. Similar to the findings derived from the case where big log strategy is adopted, it is critical for the luxury fashion brand and manufacturer to pay attention to the effects of blockchain when making optimal decisions. Moreover, the performances of this luxury fashion supply chain and its members are significantly affected by the blockchain effectiveness and operation costs. Their impacts follow the same pattern as the ones under the big logo case.

When to Show Big Logos

Proposition 5.7. *(i)* $\dddot{D}_{BL}^{*} > 0$ *if and only if* $m < \dddot{m}_{BL}$*;* $\dddot{D}_{NL}^{*} > 0$ *if and only if* $m < \dddot{m}_{NL}$*, where* $\dddot{m}_{BL} = \dddot{A} - t - \frac{\lambda}{\gamma}$ *and* $\dddot{m}_{NL} = \dddot{B} - t - \frac{\tau}{\gamma}$*. (ii)* $\dddot{m}_{BL} > \dddot{m}_{NL}$ *if and only if* $\lambda - \tau < \dddot{\xi}$*, where* $\dddot{\xi} = (1-\alpha)(\theta + k)\gamma - \alpha\gamma(l + \phi)$.

In Proposition 5.7, we uncover the demand performances under big logo and no logo strategies when blockchain is implemented. The major findings are consistent with the basic model. Specifically, (i) only when the production cost is relatively small, product demand under big logo or small logo strategy can be positive; (ii) only when the negative effects differential is not too large, adopting big logo is more likely to bring business to the luxury fashion supply chain compared with no logo. In addition, by comparing with the basic model, we find that implementing blockchain is more likely to bring business to the luxury fashion supply chain (i.e., $\dddot{m}_{BL} > m_{BL}$ *or* $\dddot{m}_{NL} > m_{NL}$) when the unit-cost of blockchain is relatively small (i.e., $t < \lambda - \frac{\lambda}{\gamma}$ or $t < \tau - \frac{\tau}{\gamma}$). Such finding reveals that it is important for the luxury fashion brand to consider implementing blockchain technology to make profit.

Proposition 5.8. *When the blockchain technology is implemented, (i) showing big logo is more beneficial to the luxury fashion supply chain and its members if and only if* $\lambda - \tau < \dddot{\xi}$*, where* $\dddot{\xi} = (1-\alpha)(\theta + k)\gamma - \alpha\gamma(l + \phi)$*; otherwise, showing no logo is optimal. (ii)* $\dddot{\xi}$ *increases linearly in* θ, k *and* γ *while decreases linearly in* l*,* ϕ *and* α*. (iii)* $\dddot{\xi} > \xi$ *always holds.*

Proposition 5.8 uncovers the optimal logo strategy with the adoption of blockchain technology. We interestingly find that the results remain robust as in the basic model. That is, only when the negative effects differential caused by counterfeits and copycats (i.e., $\lambda - \tau$) is relatively small, can big logo outperform the no logo strategy. Exploring further on the threshold of $\dddot{\xi}$, we find that the impacts of consumers' status utility/disutility (i.e., θ, k, l, ϕ) and the ratio of patricians (α) follow the same pattern as in the basic model. Moreover, we discover that with an increase in the effectiveness of blockchain (i.e., γ), the threshold increases, which implies that adopting the big logo strategy is more likely to benefit the luxury fashion supply chain and its members. The finding hence encourages the luxury fashion brand to disclose more product information to improve the effectiveness of blockchain.

Lastly, our results reveal that it is more advantageous to adopt the big logo strategy when blockchain technology is implemented. The rationale is that, with blockchain technology, the disutility caused by consumers unknowingly purchasing a counterfeit is reduced largely than a copycat (i.e., $\frac{\lambda}{\gamma} > \frac{\tau}{\gamma}$). Hence, luxury fashion brands (e.g., *LV*, *Prada*) which carry big logo are more advised to implement the blockchain technology. To better present the impacts of blockchain implementation on the optimal logo strategy and luxury fashion supply chain, we refine the qualitative findings of Proposition 4.6 and generate Theorem 5.2.

Theorem 5.2. *When blockchain is implemented, showing big logo is more prone to be optimal for the luxury fashion supply chain and its members.*

Theorem 5.2 provides luxury fashion brands with practical guidelines on whether to adopt the blockchain technology. We suggest that luxury fashion brands which exhibit a big logo (rather than small logo) consider adopting the blockchain technology. This finding also partially explains why *LV*, the well-known loud luxury has joined the "*Aura Blockchain Consortium*" to combat counterfeits.

Next, to investigate the performance of consumer surplus under different logo strategies, we numerically present the findings as depicted in Figure 5.6. We can observe that showing big logo can bring a higher consumer surplus only when the proportion of patricians is relatively small, which is consistent with the finding derived under the basic model. It means that the implementation of blockchain will not affect the pattern of comparison for consumer surplus. Furthermore, we notice that the big logo strategy is more welcomed by the consumers when the effectiveness of blockchain (γ) is higher, which also shows the efficiency of using blockchain to combat counterfeits from the consumer's perspective.

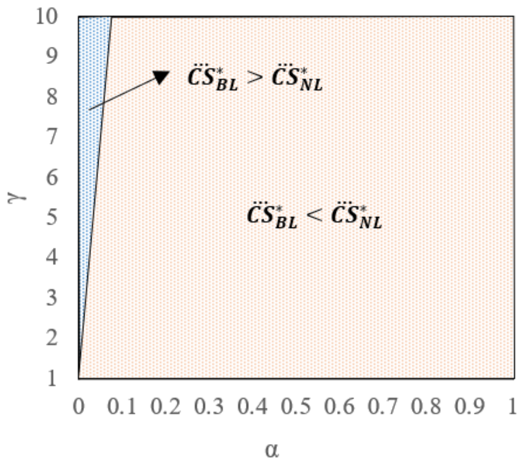

Figure 5.6. Comparison between showing big logo and no logo for consumer surplus. (We let $l = 0.1$, $\theta = 0.1$, $\phi = 0.2$, $k = 0.4$, $\lambda = 0.2$, $\tau = 0.1$, $m = 0.2$, $t = 0.1$).

5.4 Chapter Conclusion

Motivated by the industrial observation that different luxury brands adopt different logo design strategies (either big or no logo) and a lack of analytical studies in this domain, we build consumer utility based game theoretical models in this study to explore the optimal logo design strategy of the luxury fashion brand, and its implications on the entire supply chain and the consumers.

Han et al. (2010) empirically uncover that a louder luxury brand is more likely to be copied by counterfeiters, while they have not explored the impacts of counterfeits on luxury brands. Our work fills in this research gap by further studying the impacts of counterfeits and copycats on the luxury fashion supply chain, its members (i.e., the luxury fashion brand and manufacturer), and the consumers. To be specific, in the basic model, we consider a risk neutral luxury fashion brand who buys products from a manufacturer and serves both segments of consumers (i.e., patricians and parvenus) in the market. We find that when the negative effects differential caused by counterfeits and copycats is relatively small, showing big logo in luxury fashion products will bring an all-win outcome to the luxury fashion supply chain, its members and the consumers; otherwise, it is more beneficial to show no logo (RQ1). We further extend the basic model to the cases with (i) risk sensitive luxury fashion brand and (ii) the implementation of blockchain technology. We uncover that the results remain robust as in the basic model. Additionally, we discover that risk attitude of the luxury fashion brand is a critical factor governing the optimal logo design strategy. More specifically, when the negative effects differential caused by counterfeits and copycats is relatively small (large), adopting the big (no) logo strategy is more likely to result in an all-win outcome by a risk averse (seeking) luxury fashion brand.

We also compare the optimal logo design strategies under the three cases (i.e., basic model, risk sensitive luxury fashion brand, and the implementation of blockchain) and find that, the big logo strategy is more prone to be optimal when implementing the blockchain technology (RQ3). While cases become more complex when considering risk attitude of the luxury fashion brand. We analytically find that when the luxury fashion brand is risk seeking (averse) and the ratio of status disparity is comparatively small (large), adopting the big logo strategy is more inclined to be optimal for the luxury fashion brand. However, there exists a conflict of interest between the luxury fashion brand and the manufacturer, which implies that the all-win outcome is more difficult to be achieved when the luxury fashion brand is risk sensitive (RQ2).

Chapter 6 Conclusions and Suggestions for Future Research

6.1 Concluding Remarks and Major Findings

Motivated by the popularity of fashion knockoffs and its potential impacts on the ODL product supply chain, C2C-PE scheme and logo design strategy of the original fashion brand, we build game theoretical models in this thesis to explore (i) impacts of fashion knockoffs on the ODL product supply chain and its agents; impacts of C2C-PE on the original product supply chain in the presence of fashion knockoffs; and (iii) impacts of fashion knockoffs and counterfeits on the logo design strategy of luxury fashion brand. By addressing the research objectives, we have obtained some interesting findings.

First, in Chapter 3 where we explore the impacts of fashion knockoffs on the ODL product supply chain, we interestingly find that the presence of fashion knockoffs benefits the ODL product supply chain and its agents when the ODL product seller is risk averse and the ratio of demand uncertainty is relatively small, or the ODL product seller is risk seeking and the ratio of demand uncertainty is sufficiently large. The findings remain robust when we consider a non-standard markup wholesale pricing policy and different manufacturers' scenario. Moreover, we discover that when the ODL product seller is risk sensitive, a common manufacturer scenario is more likely to bring a win-win outcome to the manufacturer and the ODL product seller, which may explain why a common manufacturer producing for both an ODL and a knockoff product seller is widely observed in the fashion industry. (Please refer to Section 3.5 for more details).

Second, in Chapter 4 where we examine the impacts of C2C-PE on the original product supply chain, we theoretically find that the presence of C2C-PE benefits the original supply chain, its members and the consumers, while harms the knockoff supply chain, its members and the consumers. The findings are robust when considering: (i) strategic quality decision; (ii) price dependent C2C-PE utility and (iii) consumers' conspicuous behavior. In addition, we discover that in the presence of C2C-PE, members of the original supply chain tend to produce a higher-price and superior-quality product, while members of the knockoff supply chain incline to sell a lower-price knockoff product. The original brand will encroach some of the knockoff brand's demand. (Please refer to Section 4.4 for more details).

Third, in Chapter 5 where we study the impacts of fashion knockoffs and counterfeits on the luxury fashion brand's logo design strategy, we uncover that when the negative effects differential caused by counterfeits and copycats is relatively small, showing a big logo is more beneficial to the luxury fashion supply chain, its members and the consumers; otherwise, showing no logo is more beneficial.

The results remain robust when considering risk attitude of the luxury fashion brand and implementation of the blockchain technology. By comparing the optimal logo design strategies among different cases, we uncover that the big logo strategy is more prone to be optimal when implementing the blockchain technology. Moreover, when the luxury fashion brand is risk sensitive, whether the big logo strategy is more likely to be optimal depends on risk attitude of the luxury fashion brand and consumers' status disparity. (Please refer to Section 5.4 for more details).

6.2 Managerial Implications

Based on the derived analytical findings (i.e., Section 6.1), we provide fashion industrialists with the following managerial insights:

6.2.1 Fashion Knockoff's Impacts on ODL Supply Chains

- **Strategies towards fashion knockoffs:** We interestingly find that there are conditions under which the presence of fashion knockoffs benefits the ODL product supply chain and its agents, which implies that it is unwise for the ODL product seller to blindly deter the knockoff products in all cases. Instead, the ODL product seller should be strategic when encountering knockoff products in the market. In other words, whether to allow, ignore or deter the presence of fashion knockoffs is an important strategic decision, which relies on the ratio of demand uncertainty and the risk attitude of the ODL product seller. The findings are in line with our interview results. As revealed by the interviewee, the ratio of knockoff product sellers being sued by the ODL product seller is very low, which implies that the ODL product sellers just "let the knockoff go" in most cases. One explanation is that these ODL product sellers actually benefit from the presence of fashion knockoffs. Moreover, with the prevalence of fashion knockoffs, fashion trends like seasonal color, textiles etc., are being promoted. In consequence, demand for the ODL product sellers may increase accordingly, and hence benefit the ODL product seller.

  Additionally, based on Proposition 3.4, we have explicit suggestions for manufacturers adopting the non-standard markup wholesale pricing policy: In the presence of fashion knockoffs, the manufacturer is advised to set a higher markup rate for the ODL product seller or a lower markup rate for the knockoff product seller. It is explained by the interviewee that, due to the complex production procedure of the ODL product (i.e., high production cost), the manufacturer has to set a higher markup rate for the ODL product seller; while for the knockoff product seller which has a lower production cost, the manufacture can set a lower markup rate.
- **Risk attitude of the ODL product seller:** As indicated by our findings, the presence of fashion knockoffs always harms the ODL product seller when it holds a risk neutral attitude, while can

benefit the ODL product seller when it is risk sensitive (either risk averse or risk seeking) irrespective of the supply chain structure. One implication based on this finding is: When we find that some ODL product sellers are "happy" with the presence of fashion knockoffs, a bit counterintuitively, they probably exhibit a risk averse/seeking attitude. On the contrary, for the ODL product sellers who are very much concerned about the presence of fashion knockoffs (e.g., those who will launch serious lawsuits to sue the fashion knockoffs), they are probably risk neutral. This is consistent with our interview results. The interviewee stated that: "For some well-established companies like *Supreme* and *RIMOWA*, they are not afraid of being copied (which implies that they are likely to be risk seeking), instead, they may like being copied since the existence of fashion knockoffs can help them promote their products and brand name, which may even benefit them."

Despite the importance of risk attitude, we suggest the ODL product seller jointly consider the factors of (i) demand uncertainty, and (ii) supply chain structure when encountering fashion knockoffs in the market. The reason is that either of these two factors can affect the impacts of risk attitudes on the ODL product supply chain. To be specific, the ODL product seller should carefully target the market (i.e., seasonal or fashionable products with relatively large demand uncertainty, or classic products with relatively small demand uncertainty) based on its risk attitude. When there is a mismatch between the ratio of demand uncertainty and the ODL product seller's type (or magnitude) of risk attitude, the ODL product supply chain and its agents will be hurt in the presence of fashion knockoffs. We hence suggest the managers make strategic decisions based on our analytical findings, i.e., results in propositions. Moreover, we uncover that it is more beneficial for the manufacturer to produce both the ODL product and the knockoff product than producing the ODL product solely regardless of the ODL product seller's risk attitude. This finding partially explains the reason why the phenomenon of a common manufacturer (e.g., *TAL*) producing for both an ODL product seller (e.g., *Tommy Hilfiger*) and a knockoff product seller (e.g., a certain fast fashion brand) is very common in the fashion industry.

6.2.2 C2C Product Exchanges: Impacts on Fashion Knockoffs

- **C2C-PE scheme:** As revealed by the findings, when there is knockoff product in the market, C2C-PE is a win-win scheme for the original supply chain and its consumers while a lose-lose scheme for the knockoff supply chain and its consumers. This finding challenges the conventional wisdom of C2C-PE hurting the profitability of original brands. The results suggest that C2C-PE can be an effective way to reduce consumer demand for knockoffs, which benefits original brands. Therefore, original brand is advised to encourage their consumers to exchange the original products after usage.

- **C2C-PE utility:** According to the findings, increasing the C2C-PE utility always benefits the original supply chain, its members and consumers, while harms the knockoff supply chain, its members and consumers. Hence, both the original brand and the manufacturer have the incentive to enhance the C2C-PE utility. In practice, the original brand is encouraged to provide more exclusive, limited or prestigious products while the manufacturer is encouraged to produce more durable products.
- **Consumer's acceptance for knockoff product:** we interestingly find that an increase in consumer's acceptance for knockoff product usually benefits the consumers for original products while harms the original brand. Hence, it is important for the original brand to balance its own profit and the respective consumer surplus.
- **Quality improvement cost:** As suggested by the findings, when it is expensive to improve the quality of the original product, the original supply chain suffers while the knockoff supply chain benefits. Hence, it is crucial for manufacturer of the original supply chain to reduce the quality improvement cost.
- **Consumer's need for uniqueness:** According to the findings, an increase in consumer's need for uniqueness always harms the original supply chain, its members and consumers while benefits the knockoff supply chain, its member and consumers. Hence, the original brand is advised to conduct survey or interview on their target customers to learn more about their levels of need for uniqueness. Accordingly, they can better design products to meet consumers' preferences.

6.2.3 Luxury Fashion Brands' Logo Design Strategies Facing Fashion Knockoffs and Counterfeits

- **Which logo strategy to adopt?** As revealed by our analytical findings, either a big logo or no logo strategy can be optimal, while it is dependent on the negative effects differential caused by counterfeits and copycats, or the status disutility suffered by patricians, or the ratio of patricians in the market. Correspondingly, we provide luxury fashion brands with the following three suggestions when making the logo design decision. (i) First, luxury fashion brands which are heavily affected by counterfeits compared with copycats (e.g., *LV*) are advised to launch more product collections with no logo to prevent the encroachment of counterfeits. (ii) Second, we encourage luxury fashion brands to collect more customer information regarding their preferences for brand prominence (i.e., big logo or no logo). This accords with the real-world practices that most luxury fashion brands, e.g., *Burberry*, *LV*, *Montblanc*, are keen on collecting and analyzing consumer data in their operations (Silva et al. 2019). (iii) Lastly, we recommend luxury fashion brands make logo design decisions based on their target markets, i.e., providing more products with big logo in emerging markets while providing more products with no logo in mature markets. This is in line with Pino et al. (2017) which also suggests different branding strategies in different markets. While different from their work, we further consider the operations of luxury fashion supply chain and risk attitude of the luxury fashion brand. We discover that when the luxury fashion brand is risk sensitive (either risk averse or risk seeking), his logo preference may differ from that of the manufacturer, which highlights the importance of risk attitude in affecting the optimal logo design decision.

- **How will risk attitude affect the optimal logo strategy?** Our findings suggest that luxury fashion brands make logo design decisions based on their risk attitudes. Explicitly, when a luxury fashion brand is risk averse, it is more advised to adopt a big logo strategy in order to achieve an all-win outcome (i.e., benefit the luxury fashion supply chain, its members and the consumers); while when a luxury fashion brand is risk seeking, the "no logo" strategy is more recommended. Our findings also imply that in the context of luxury logo design, risk seeking can do "more good than harm" in some cases, which is consistent with Choi et al. (2020b)'s proposals (even though their context is in logistics which is very different). In addition, our findings provide luxury fashion brands with recommendations concerning the logomania trend (i.e., whether or not to follow the logomania trend depends on the ratio of consumers' status disparity).
- **When to implement the blockchain technology?** Another important insight is that a luxury fashion brand who uses big logo (compared with no logo) is more recommended to implement the blockchain technology, which can not only benefit the luxury fashion brand but also satisfy the consumers. This partially explains why *LV*, the well-known loud luxury has joined the "*Aura Blockchain Consortium*" to combat counterfeits. Moreover, we encourage loud luxury brands who have already implemented the blockchain technology (e.g., *LV*, *Prada*) to disclose more information of the product (e.g., origin, components, environmental and ethical information). In this way, they are more likely to be benefited from the blockchain technology.

6.3 Future Studies

Based on the studies completed in this thesis, we propose the following paths to continue:

In the study concerning impacts of fashion knockoffs on the ODL product supply chain, we mainly propose three directions. First, this study explores the risk attitude of the ODL product seller, it would be interesting for future studies to explore the risk attitude of the knockoff product seller or the manufacturer. Second, this study examines the supply chain structures of a common manufacturer and different manufacturers, while other configurations of the supply chain can also be explored (Shan et al. 2022). Third, this study considers the decentralized supply chain, future studies can explore the centralized supply chain setting using supply contracts (Leng and Zhu 2009; Feng et al. 2017). Fourth, this study only interviews one fashion industrialist. It would generate more insights if more industrial data or practices can be obtained. Finally, this study mainly follows a standardized process to construct and analyze the models. Some important features like status utility (Gao et al. 2017b) or strategic consumer behavior (Pun and DeYong 2017) can be further considered in modelling the ODL supply chain. In this way, the theoretical contribution of this study can be strengthened.

There are some limitations in the study of C2C-PE. First, we did not consider the presence of platforms, while original brands who have incentive to combat knockoffs may consider running its own platform to encourage the business mode of C2C-PE. Hence, in future studies, we will further consider the role of platforms. Second, we simply normalize all production costs to zero in this study, while production costs for the original product and the knockoff product are normally not the same. Thus, we will explore more on the different production costs to generate more insights in future studies. Third, we only consider the exchange value for the original product in this study, we may further consider the case where exchanging the knockoff product also generates an extra utility and see how the findings change. Fourth, this study only considers consumers' conspicuous behavior, other consumer behaviors like forward-looking can be further explored (Zha et al. 2022). Fifth, this study did not consider the risk attitudes of the supply chain members, it is promising to explore how risk attitudes affect the impacts of C2C-PE on the fashion knockoffs. Lastly, it would enhance the contribution of this study by conducting more numerical analyses.

We propose some research directions for the study of logo design strategy. First, as revealed by our analytical findings that a conflict of interest exists between the manufacturer and the luxury fashion brand, hence it is important to examine whether supply chain coordination can be achieved using supply chain contracts (Xu et al. 2015). Second, this study did not consider the presence of manufacturer encroachment, which is a common phenomenon in the luxury fashion industry and may affect our theoretical findings (Zhang et al. 2021a). Third, this study simply considers a monopoly luxury fashion brand, it is meaningful for future studies to explore the role of competition (e.g., two luxury fashion brands) in affecting the optimal logo design strategy (Feng et al. 2017; Pun et al. 2020). Lastly, our study only focuses on the specific anticounterfeiting strategy of blockchain. Other strategies such as launching a fighter brand, or using RFID technology can be explored in future studies.

In addition to the above future research directions, there are several paths to proceed with this thesis. First, it is promising to explore the collaboration between the original brand and the knockoff brand. For example, Lu et al. (2021) analytically find that sourcing from an overseas supplier (which is potentially a counterfeiter) can bring a win-win outcome to the original brand and the supplier. Gao (2022) interestingly discover that it would benefit the original brand if it open sources its green technology for free to an unsustainable knockoff brand. Following these studies, we intend to further examine the collaboration between the original brand and the knockoff brand. Second, Cho et al. (2015) uncover how the effectiveness of anticounterfeiting strategies is affected by licit or illicit supply chains. Based on this study, we can further explore the legal issues concerning fashion knockoffs and counterfeits. Thirdly, supplier or third-party encroachment is a severe problem existing in different regions in the world, which is worth exploring (Zhang et al. 2021a). Lastly, it would be interesting to study consumers' loss aversion concerning fashion knockoffs and counterfeits (Zhang et al. 2021b).

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Appendix A: Thresholds for Chapter 3

Table A1. Critical thresholds identified in the analyses.

Notation	Expressions
K_1	$\frac{a-w_O}{\sigma_O}$
K_2	$\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{2\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}$
K_3	$\frac{(a-w_O)[2(2+\phi\beta)-\sqrt{1+\phi\beta}(4+\phi\beta)]-2\phi(\alpha-bw_K)}{4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta)}$
K_4	$\frac{-(a-m_O)(2+\phi\beta)-\phi\,(bw_K-\alpha)+\beta\,(w_K-m_K)}{(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}$
G	$\frac{-2(w_O-m_O)\big((a-w_O)(2+\phi\beta)+\phi(bw_K-\alpha)\big)-2(w_K-m_K)\big(\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)\big)+(w_O-m_O)(a-w_O)(1+\phi\beta)(4+\phi\beta)}{2\big((w_O-m_O)(2+\phi\beta)+\beta(w_K-m_K)\big)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-(w_O-m_O)\sigma_O(1+\phi\beta)(4+\phi\beta)}$
S_1	$\sqrt{\frac{\phi^2\beta^3+9\phi\beta^2+24\beta}{16\phi}}$
S_2	$\sqrt{\left(\frac{(w_O-m_O)(1+\phi\beta)(4+\phi\beta)}{2\phi\big((w_O-m_O)(2+\phi\beta)+\beta(w_K-m_K)\big)}\right)^2-\frac{1}{\phi^2}}$ $(S_2 < S_1)$
K_3'	$\frac{(a-m_O)[2(2+\phi\beta)-\sqrt{1+\phi\beta}(4+\phi\beta)]-2\phi(\alpha-bm_K)}{4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta)}$
S_2'	$\sqrt{\frac{\phi\beta(\lambda-1)(16+\beta\phi(9+\beta\phi))b+4\beta^2(\mu-1)}{8\phi^2(\lambda-1)(2+\beta\phi)b-4\phi^2\beta^2(\mu-1)}}$
b'	$\frac{\beta^2(1-\mu)}{2(1-\lambda)(2+\beta\phi)}$
G^{DS}	$\frac{\phi\beta(a-w_O)(3+\phi\beta)+2\phi(\alpha-bw_K)}{2(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O(1+\phi\beta)(4+\phi\beta)}$
S_2^{DS}	$\sqrt{\frac{\beta(3+\beta\phi)(8+\beta\phi(7+\beta\phi))}{4\phi(2+\beta\phi)^2}}$ $(S_2^{DS} < S_1)$
D	$\frac{\phi\beta\,(3+\phi\beta)}{2(1+\phi\beta)(4+\phi\beta)}$ $(D > 0)$
E	$-\frac{\phi\beta(m_O+a)(3+\phi\beta)+2\beta(w_K-m_K)+2\phi(\alpha-bw_K)}{2(1+\phi\beta)(4+\phi\beta)}$ $(E < 0)$
F	$\frac{\phi m_O[a\beta(3+\phi\beta)+2(\alpha-bw_K)]+2(w_K-m_K)[a\beta+(\alpha-bw_K)(2+\phi\beta)]}{2(1+\phi\beta)(4+\phi\beta)}$ $(F > 0)$
D'	$\frac{\beta\phi(16+\beta\phi(9+\beta\phi))}{4(1+\beta\phi)(4+\beta\phi)^2}$ $(D' > 0)$
E'	$\frac{\beta\big(2\phi^2(\alpha-bw_K)-2(w_K-m_K)(4+\beta\phi)-\phi\big(2a(2+\beta\phi)+m_O(3+\beta\phi)(4+\beta\phi)\big)\big)}{2(1+\beta\phi)(4+\beta\phi)^2}$
F'	$-\frac{16(w_K-m_K)(2bw_K-2\alpha-a\beta)+4\big(2a^2\beta+2(bw_K-\alpha)(2m_O+3(w_K-m_K)\beta)+a\big(-4bw_K+4\alpha+\beta(-6m_O-w_K\beta+m_K\beta)\big)\big)\phi-(4(-bw_K+\alpha)^2+4(2a-m_O)(bw_K-\alpha)\beta+(-5a^2+14am_O-4(w_K-m_K)(bw_K-\alpha))\beta^2)\phi^2+a(a-2m_O)\beta^3\phi^3}{4(1+\beta\phi)(4+\beta\phi)^2}$
X	$\frac{4\phi^2\sigma_K^2(2b(\lambda-1)(2+\beta\phi)-\beta^2(\mu-1))+\beta\sigma_O^2(\phi^2\beta(4-4\mu-b(\lambda-1)(9+\beta\phi))-16b(\lambda-1)\phi)}{4(4+\beta\phi)^2(b+b\beta\phi)}$
Y	$\frac{\beta(\alpha(2+\beta\phi)-b^2m_Ok(\lambda-1)\phi^2-2(\mu-1)((a-m_O)\beta)+b(2m_Ok(\mu-1)(2+\beta\phi)+\phi(\lambda-1)((2+\beta\phi)(m_O-a)+\alpha\phi)))\sqrt{\phi^2\sigma_K^2+\sigma_O^2}}{(4+\beta\phi)^2(b+b\beta\phi)}$
Z	$\frac{-4b^3m_Ok^2(\lambda-1)\phi^2-4(\mu-1)\big((a-m_O)\beta+\alpha(2+\beta\phi)\big)^2+b(8m_Ok(\mu-1)(2+\beta\phi)((2+\beta\phi)\alpha+(a-m_O)\beta)+(\lambda-1)\phi(8(a-m_O)(2\alpha+a\beta-m_O\beta)-(2\alpha+a\beta-m_O\beta)(2\alpha-5a\beta+5m_O\beta)\phi+(a-m_O)^2\beta^3\phi^2))+4b^2m_Ok(2(\lambda-1)\phi((2+\beta\phi)(m_O-a)+\alpha\phi)-m_Ok(\mu-1)(2+\beta\phi)^2)}{4(4+\beta\phi)^2(b+b\beta\phi)}$
M	$b(a-m_O)^2(1+\phi\beta)(4+\phi\beta)^2-4b[(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)]^2$ $(M > 0)$
N	$4[\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)]^2$ $(N > 0)$

Appendix B: All Proofs for Chapter 3

Derivation of the optimal solutions without fashion knockoffs and with risk sensitive ODL product seller:

Taking the second order derivative of Eq.(3.5) with respect to $\bar{p}_O$, we have: $\frac{\partial^2 U[\bar{\pi}_O]}{\partial \bar{p}_O{}^2} = -2 < 0$. Hence, $U[\bar{\pi}_O]$ is concave in $\bar{p}_O$. By solving the first order derivative of Eq.(3.5) with respect to $\bar{p}_O$, we have $\bar{p}_O^* = \frac{a+w_O-k\sigma_O}{2}$. Substituting $\bar{p}_O^*$ back into Eq.(3.1), Eq.(3.5) and Eq.(3.6), we have: $\bar{D}_O^{MSD*} = \frac{a-w_O-k\sigma_O}{2}$, $U[\bar{\pi}_O^*] = \left(\frac{a-w_O-k\sigma_O}{2}\right)^2 - F_O$, $E[\bar{\pi}_M^*] = \frac{(w_O-m_O)(a-w_O+k\sigma_O)}{2} - F_M$.

To guarantee that demand for the ODL product is always positive, we have $\bar{D}_O^{MSD*} > 0$ and $E[\bar{D}_O^*] > 0$, solving them, we have $|k| < K_1$, where $K_1 = \frac{a-w_O}{\sigma_O}$. (Q.E.D.)

Derivation of the optimal solutions with fashion knockoffs and risk sensitive ODL product seller:

Taking the second order derivative of Eq.(3.12) with respect to p_O, we have: $\frac{\partial^2 U[\pi_O]}{\partial p_O{}^2} = -\frac{2}{1+\beta\phi} < 0$. Hence, $U[\pi_O]$ is concave in p_O. By solving the first order derivative of Eq.(3.12) with respect to p_O, we have: $p_O^* = \frac{1}{2}(a + w_O - \alpha\phi + b\phi p_K - k\sqrt{\phi^2\sigma_K^2 + \sigma_O^2})$. Similarly, taking the second order derivative of Eq.(3.13) with respect to p_K, we have: $\frac{\partial^2 E[\pi_K]}{\partial p_K{}^2} = -\frac{2b}{1+\beta\phi} < 0$. Hence, $E[\pi_K]$ is concave in p_K. By solving the first order derivative of Eq.(3.13) with respect to p_K, we have: $p_K = \frac{bw_K+\alpha+a\beta-\beta p_O}{2b}$. Solving $p_O^* = \frac{1}{2}(a + w_O - \alpha\phi + b\phi p_K - k\sqrt{\phi^2\sigma_K^2 + \sigma_O^2})$ and $p_K^* = \frac{bw_K+\alpha+a\beta-\beta p_O}{2b}$, we have $p_O^* = \frac{(2+\phi\beta)a+(bw_K-\alpha)\phi+2w_O-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{4+\phi\beta}$ and $p_K^* = \frac{(2+\phi\beta)\alpha+(a-w_O)\beta+2w_K b+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{b(4+\phi\beta)}$.

Substituting them back into Eq.(3.7) and Eq(3.8), we have the optimal demands D_O^{MSD*} and D_K^{MSD*}. Substituting them back into Eq.(3.11) and taking expectation, the optimal expected profit for the manufacturer $E[\pi_M^*]$ is derived. Substituting p_O^* and p_K^* into Eq.(3.12) and Eq.(3.13), we obtain the optimal utility for the ODL product seller $U[\pi_O^*]$ and the optimal expected profit for the knockoff product seller $E[\pi_K^*]$.

To guarantee that demands for the ODL product and the knockoff product are both positive, we have $D_O^{MSD*} > 0, E[D_O^*] > 0$ and $E[D_K^*] > 0$, solving them, we have $|k| < K_2$, where $K_2 = \frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{2\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}$. (Q.E.D.)

Proof of Proposition 3.1:

Taking the second order derivative of $E[\pi_M^*]$ with respect to w_O, we have: $\frac{\partial^2 E[\pi_M^*]}{\partial {w_O}^2} = -\frac{2(2+\beta\phi)}{(1+\beta\phi)(4+\beta\phi)} < 0$. Hence, $E[\pi_M^*]$ is strictly concave in w_O. By solving the first order derivative of $E[\pi_M^*]$ with respect to w_O, we have $w_O^* = \widetilde{w_O} = \frac{1}{2}\left(a + \frac{m_O(2+\phi\beta)+\phi(bw_K-\alpha)-\beta(w_K-m_K)}{2+\phi\beta} + k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}\right)$. When $m_O > \widetilde{w_O}$, we have $w_O > m_O > \widetilde{w_O}$, $E[\pi_M^*]$ *is decreasing in* w_O*;* When $m_O < \widetilde{w_O}$, $E[\pi_M^*]$ *is concave in* w_O. *Solving* $m_O > \widetilde{w_O}$ *, we have* $k < K_4$ *; Solving* $m_O < \widetilde{w_O}$ *, we have* $k > K_4$ *, where* $K_4 = \frac{-(a-m_O)(2+\phi\beta)-\phi\,(bw_K-\alpha)+\beta\,(w_K-m_K)}{(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}$. (Q.E.D.)

Proof of Proposition 3.2(a):

When the ODL product seller is benefited by the presence of fashion knockoffs, we have:

$$\Delta\,\pi_O = \frac{1}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(4+\phi\beta)}\right]^2 - \left(\frac{a-w_O-k\sigma_O}{2}\right)^2 > 0.$$

Define $K_3 = \frac{(a-w_O)[2(2+\phi\beta)-\sqrt{1+\phi\beta}(4+\phi\beta)]-2\phi(\alpha-bw_K)}{4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta)}$, $S_1 = \sqrt{\frac{\phi^2\beta^3+9\phi\beta^2+24\beta}{16\phi}}$.

(1) When $4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta) > 0$, we have $k < K_3 < 0$.

Solving $4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta) > 0$, we have $\frac{\sigma_K}{\sigma_O} > S_1$.

(2) When $4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta) < 0$, we have $k > K_3 > 0$.

Solving $4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta) < 0$, we have $0 < \frac{\sigma_K}{\sigma_O} < S_1$. (Q.E.D.)

Proof of Proposition 3.2(b):

When the manufacturer is benefited by the presence of knockoff, we have:

$$\Delta\,\pi_M = (w_O - m_O)\left[\frac{(a-w_O)(2+\phi\beta)+\phi(bw_K-\alpha)}{(1+\phi\beta)(4+\phi\beta)} + \frac{(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right] + (w_K - m_K)\left[\frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)}{(1+\phi\beta)(4+\phi\beta)} + \frac{\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right] - \frac{(w_O-m_O)(a-w_O+k\sigma_O)}{2} > 0.$$

For a notational purpose, we define the following:

$$G = \frac{-2(w_O-m_O)\big((a-w_O)(2+\phi\beta)+\phi(bw_K-\alpha)\big)-2(w_K-m_K)\big(\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)\big)+(w_O-m_O)(a-w_O)(1+\phi\beta)(4+\phi\beta)}{2\big((w_O-m_O)(2+\phi\beta)+\beta(w_K-m_K)\big)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-(w_O-m_O)\sigma_O(1+\phi\beta)(4+\phi\beta)},$$

$S_2 = \sqrt{\left(\frac{(w_O-m_O)(1+\phi\beta)(4+\phi\beta)}{2\phi\left((w_O-m_O)(2+\phi\beta)+\beta(w_K-m_K)\right)}\right)^2 - \frac{1}{\phi^2}}$.

(1) When $2\left((w_O - m_O)(2+\phi\beta) + \beta(w_K - m_K)\right)\sqrt{\sigma_O^2 + \phi^2\sigma_K^2} - (w_O - m_O)\sigma_O(1+\phi\beta)$ $(4+\phi\beta) > 0$, we have: $k > G$.

Solving $2\left((w_O - m_O)(2+\phi\beta) + \beta(w_K - m_K)\right)\sqrt{\sigma_O^2 + \phi^2\sigma_K^2} - (w_O - m_O)\sigma_O(1+\phi\beta)$ $(4+\phi\beta) > 0$, we have: $\frac{\sigma_K}{\sigma_O} > S_2$.

(2) When $2\left((w_O - m_O)(2+\phi\beta) + \beta(w_K - m_K)\right)\sqrt{\sigma_O^2 + \phi^2\sigma_K^2} - (w_O - m_O)\sigma_O(1+\phi\beta)$ $(4+\phi\beta) < 0$, we have: $k < G$.

Solving$2\left((w_O - m_O)(2+\phi\beta) + \beta(w_K - m_K)\right)\sqrt{\sigma_O^2 + \phi^2\sigma_K^2} - (w_O - m_O)\sigma_O(1+\phi\beta)$ $(4+\phi\beta) < 0$, we have:$0 < \frac{\sigma_K}{\sigma_O} < S_2$. (Q.E.D.)

Proof of Corollary 3.1:

Comparing the thresholds of S_1 and S_2, we uncover that $S_2 < S_1$ always holds.

It is intuitive that when the ODL product seller and the manufacturer are benefited at the same time (Proposition 3.2 (a) and (b) are both satisfied), the presence of fashion knockoffs benefits the ODL product supply chain. (Q.E.D.)

Proof of Proposition 3.3(a):

Under model CN, comparing the profits for the ODL product seller with and without the presence of fashion knockoffs, we have: $\Delta\, \pi_O^{CN} = \frac{1}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(4+\phi\beta)}\right]^2 - \left(\frac{a-w_O-k\sigma_O}{2}\right)^2$.

Rearrange terms, we have:

$\Delta\, \pi_O^{CN} = \frac{(a-w_O)^2\left[4\,(2+\phi\beta)^2-(1+\phi\beta)(4+\phi\beta)^2\right]+4\phi(bw_K-\alpha)[2(2+\phi\beta)(a-w)+\phi(bw_K-\alpha)]}{4(1+\phi\beta)(4+\phi\beta)^2}$.

We find that $\Delta\, \pi_O^{CN} < 0$ always holds. (Q.E.D.)

Proof of Proposition 3.3(b):

Under model CN, comparing the profits for the manufacturer with and without the presence of fashion knockoffs, we have

$\Delta\, \pi_M^{CN} = (w_O - m_O)\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{(1+\phi\beta)(4+\phi\beta)}\right] + (w_K - m_K)\left[\frac{\beta(a-w_O)+(2+\phi\beta)(\alpha-bw_K)}{(1+\phi\beta)(4+\phi\beta)}\right] - \frac{(w_O-m_O)(a-w_O)}{2}$.

For a notational purpose, we define the following:

$D = \frac{\phi\beta\,(3+\phi\beta)}{2(1+\phi\beta)(4+\phi\beta)}$, $(D > 0)$,

$E = -\frac{\phi\beta(m_O+a)(3+\phi\beta)+2\beta(w_K-m_K)+2\phi(\alpha-bw_K)}{2(1+\phi\beta)(4+\phi\beta)}$, $(E < 0)$,

$F = \frac{\phi m_O[a\beta(3+\phi\beta)+2(\alpha-bw_K)]+2(w_K-m_K)[a\beta+(\alpha-bw_K)(2+\phi\beta)]}{2(1+\phi\beta)(4+\phi\beta)}$, $(F > 0)$.

We can rewrite the above equation as $\triangle\,\pi_M^{CN} = Dw^2 + Ew + F$.

Solving the quadratic equation yields two roots. (Q.E.D)

Proof of Proposition 3.3(c):

When the presence of fashion knockoffs benefits the ODL product supply chain, we have: $\triangle\,\pi_{SC}^{CN} = \triangle\,\pi_O^{CN} + \triangle\,\pi_M^{CN}$.

For a notational purpose, we define the following:

$D' = \frac{\beta\phi(16+\beta\phi(9+\beta\phi))}{4(1+\beta\phi)(4+\beta\phi)^2}$, $(D > 0)$,

$E' = \frac{\beta\left(2\phi^2(\alpha-bw_K)-2(w_K-m_K)(4+\beta\phi)-\phi\left(2a(2+\beta\phi)+m_O(3+\beta\phi)(4+\beta\phi)\right)\right)}{2(1+\beta\phi)(4+\beta\phi)^2}$,

$F' =$

$$-\frac{16(w_K-m_K)(2bw_K-2\alpha-a\beta)+4\left(2a^2\beta+2(bw_K-\alpha)(2m_O+3(w_K-m_K)\beta)+a\left(-4bw_K+4\alpha+\beta(-6m_O-w_K\beta+m_K\beta)\right)\right)\phi-(4(-bw_K+\alpha)^2+4(2a-m_O)(bw_K-\alpha)\beta+(-5a^2+14am_O-4(w_K-m_K)(bw_K-\alpha))\beta^2)\phi^2+a(a-2m_O)\beta^3\phi^3}{4(1+\beta\phi)(4+\beta\phi)^2}$$

We can rewrite the above equation as $\triangle\,\pi_{SC}^{CN} = D'w^2 + E'w + F'$.

Solving the quadratic equation yields two roots. (Q.E.D)

Proof of Proposition 3.4(a):

Under the extended Model GNB, when the ODL product seller is risk sensitive, comparing the profits for the manufacturer with and without the presence of fashion knockoffs, we have:

$$\triangle\,\pi_O' = \frac{\lambda}{1+\phi\beta}\left[\frac{(2+\phi\beta)(a-m_O)+\phi(bm_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{4+\phi\beta}\right]^2 - \lambda\left(\frac{a-m_O-k\sigma_O}{2}\right)^2 > 0.$$

Define $K_3' = \frac{(a-m_O)[2(2+\phi\beta)-\sqrt{1+\phi\beta}(4+\phi\beta)]-2\phi(\alpha-bm_K)}{4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta)}$, $S_1 = \sqrt{\frac{\phi^2\beta^3+9\phi\beta^2+24\beta}{16\phi}}$.

(1) When $4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta) > 0$, we have $k < K_3' < 0$.

Solving $4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta) > 0$, we have $\frac{\sigma_K}{\sigma_O} > S_1$.

(2) When $4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta) < 0$, we have $k > K_3' > 0$.

Solving $4\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O\sqrt{1+\phi\beta}(4+\phi\beta)<0$, we have $0<\frac{\sigma_K}{\sigma_O}<S_1$. (Q.E.D.)

Proof of Proposition 3.4(b):

Under the extended Model GNB, when the ODL product seller is risk sensitive, comparing the profits for the manufacturer with and without the presence of fashion knockoffs, we have $\Delta\pi'_M=$

$$(1-\lambda)\left[\frac{(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(4+\phi\beta)}\right]\left[\frac{(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)+(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right]+$$

$$(1-\mu)\left[\frac{\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{b(4+\phi\beta)}\right]\left[\frac{\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)+\beta k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right]-$$

$$\frac{(1-\lambda)[(a-m_O)^2-(k\sigma_O)^2]}{4}.$$

For a notational purpose, we define the following:

$$X=\frac{4\phi^2\sigma_K^2(2b(\lambda-1)(2+\beta\phi)-\beta^2(\mu-1))+\beta\sigma_O^2(\phi^2\beta(4-4\mu-b(\lambda-1)(9+\beta\phi))-16b(\lambda-1)\phi)}{4(4+\beta\phi)^2(b+b\beta\phi)},Y=$$

$$\frac{\beta(\alpha(2+\beta\phi)-b^2m_Ok(\lambda-1)\phi^2-2(\mu-1)((a-m_O)\beta)+b(2m_Ok(\mu-1)(2+\beta\phi)+\phi(\lambda-1)((2+\beta\phi)(m_O-a)+\alpha\phi)))\sqrt{\phi^2\sigma_K^2+\sigma_O^2}}{(4+\beta\phi)^2(b+b\beta\phi)},\ Z=$$

$$\frac{\begin{array}{c}-4b^3m_Ok^2(\lambda-1)\phi^2-4(\mu-1)\big((a-m_O)\beta+\alpha(2+\beta\phi)\big)^2+b(8m_Ok(\mu-1)(2+\beta\phi)((2+\beta\phi)\alpha+(a-m_O)\beta)+(\lambda-1)\phi(8(a-m_O)(2\alpha+a\beta-m_O\beta)-\\(2\alpha+a\beta-m_O\beta)(2\alpha-5a\beta+5m_O\beta)\phi+(a-m_O)^2\beta^3\phi^2))+4b^2m_Ok(2(\lambda-1)\phi((2+\beta\phi)(m_O-a)+\alpha\phi)-m_Ok(\mu-1)(2+\beta\phi)^2)\end{array}}{4(4+\beta\phi)^2(b+b\beta\phi)}.$$

We can rewrite the above equation as $\Delta\pi'_M=Xk^2+Yk+Z$.

When $X>0$, we have $0<b<\frac{\beta^2(1-\mu)}{2(1-\lambda)(2+\beta\phi)}\ or\ 0<\frac{\sigma_K}{\sigma_O}<S'_2$;

When $X<0$, we have $\frac{\sigma_K}{\sigma_O}>S'_2\ and\ b>\frac{\beta^2(1-\mu)}{2(1-\lambda)(2+\beta\phi)}$.

Solving the quadratic equation yields two roots. (Q.E.D.)

Proof of Corollary 3.2:

It is intuitive that when the ODL product seller and the manufacturer are benefited at the same time (Proposition 3.4 (a) and (b) are both satisfied), the presence of fashion knockoffs benefits the ODL product supply chain. (Q.E.D.)

Proof of Proposition 3.5(a):

Under Model GNB and when the ODL product seller is risk neutral, comparing the profits of the ODL product seller with and without the presence of fashion knockoffs, we have:

$\triangle\, \pi_O^{CN\prime} = \frac{\lambda}{1+\phi\beta}\left[\frac{(2+\phi\beta)(a-m_O)+\phi(bm_K-\alpha)}{4+\phi\beta}\right]^2 - \lambda\left(\frac{a-m_O}{2}\right)^2.$

Since $\triangle\, \pi_O^{CN\prime}$ is in a similar form as $\triangle\, \pi_O^{CN}$($\triangle\, \pi_O^{CN} = \frac{1}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{(4+\phi\beta)}\right]^2 - \left(\frac{a-w_O}{2}\right)^2$), and we have already proved that $\triangle\, \pi_O^{CN} < 0$ in Proposition 3.3, we have $\triangle\, \pi_O^{CN'} < 0$. (Q.E.D.)

Proof of Proposition 3.5(b):

Under Model GNB and when the ODL product seller is risk neutral, comparing the profits of the manufacturer with and without the presence of fashion knockoffs, we have:

$\triangle\, \pi_M^{CN\prime} = \frac{(1-\lambda)[(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)]^2}{(1+\phi\beta)(4+\phi\beta)^2} + \frac{(1-\mu)[\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)]^2}{b(1+\phi\beta)(4+\phi\beta)^2} - \frac{(1-\lambda)(a-m_O)^2}{4}.$

Define $M = b(a-m_O)^2(1+\phi\beta)(4+\phi\beta)^2 - 4b[(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)]^2,\ M > 0,$
$N = 4[\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)]^2, N > 0.$

We can rewrite the above equation as: $\triangle\, \pi_M^{CN\prime} = \frac{(1-\mu)N-(1-\lambda)M}{4b(1+\phi\beta)(4+\phi\beta)^2}.$

Solving $\triangle\, \pi_M^{CN'} > 0$, we have: $0 < \mu < 1 - \frac{(1-\lambda)M}{N}.$ (Q.E.D.)

Proof of Proposition 3.5(c):

Under Model GNB and when the ODL product seller is risk neutral, considering the ODL product supply chain, we have: $\triangle\, \pi_{SC}^{CN'} = \triangle\, \pi_O^{CN'} + \triangle\, \pi_M^{CN'} = \frac{[(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)]^2}{(1+\phi\beta)(4+\phi\beta)^2} +$
$\frac{(1-\mu)[\beta(a-m_O)+(2+\phi\beta)(\alpha-bm_K)]^2}{b(1+\phi\beta)(4+\phi\beta)^2} - \frac{(a-m_O)^2}{4}.$

We can rewrite $\triangle\, \pi_{SC}^{CN\prime}$ as: $\triangle\, \pi_{SC}^{CN\prime} = \frac{(1-\mu)N-M}{4b(1+\phi\beta)(4+\phi\beta)^2}.$

Solving $\triangle\, \pi_{SC}^{CN'} > 0$, we have $\mu < 1 - \frac{M}{N}.$ (Q.E.D.)

Proof of Proposition 3.6(a): the same as the proof of Proposition 3.2(a).

Proof of Proposition 3.6(b):

When the manufacturer is benefited by the presence of fashion knockoffs, we have:

$$\triangle\, \pi_{M1}^{DS} = (w_O - m_O)\left[\frac{-\phi\beta(a-w_O)(3+\phi\beta)-2\phi(\alpha-bw_K)}{2(1+\phi\beta)(4+\phi\beta)} + \frac{(2-\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)} - \frac{k\sigma_O}{2}\right] > 0.$$

Define: $G^{DS} = \frac{\phi\beta(a-w_O)(3+\phi\beta)+2\phi(\alpha-bw_K)}{2(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O(1+\phi\beta)(4+\phi\beta)}$, $S_2^{DS} = \sqrt{\frac{\beta(3+\beta\phi)(8+\beta\phi(7+\beta\phi))}{4\phi(2+\beta\phi)^2}}.$

(1) When $2(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2} - \sigma_O(1+\phi\beta)(4+\phi\beta) > 0$, we have: $k > G^{DS} > 0.$

Solving $2(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O(1+\phi\beta)(4+\phi\beta)>0$, we have $\frac{\sigma_K}{\sigma_O}>S_2^{DS}$.

(2) When $2(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O(1+\phi\beta)(4+\phi\beta)<0$, we have: $k<G^{DS}<0$.

Solving $2(2+\phi\beta)\sqrt{\sigma_O^2+\phi^2\sigma_K^2}-\sigma_O(1+\phi\beta)(4+\phi\beta)<0$, we have $0<\frac{\sigma_K}{\sigma_O}<S_2^{DS}$. (Q.E.D.)

Proof of Corollary 3.3:

Comparing the thresholds of S_1 and S_2^{DS}, we uncover that $S_2^{DS}<S_1$ always holds.

It is intuitive that when the ODL product seller and the manufacturer are benefited at the same time (Proposition 3.6 (a) and (b) are both satisfied), the presence of fashion knockoffs benefits the ODL product supply chain. (Q.E.D.)

Proof of Proposition 3.7:

Under model DN, comparing the profits for the manufacturer with and without the presence of fashion knockoffs, we have: $\triangle\,\pi_M^{DN}=(w_O-m_O)\left[\frac{-\phi\beta(a-w_O)(3+\phi\beta)-2\phi(\alpha-bw_K)}{2(1+\phi\beta)(4+\phi\beta)}\right]$.

We can easily find that $\triangle\,\pi_M^{DN}<0$ always holds. (Q.E.D.)

Proof of Proposition A1(a): the same as the proof of Proposition 3.4(a).

Proof of Proposition A1(b):

Under the extended Model GNB, when the ODL product seller is risk sensitive and when there are different manufacturers, comparing the profits for the manufacturer with and without the presence of fashion knockoffs, we have

$$\triangle\,\pi_M^{DS'}=(1-\lambda)\left[\frac{(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(4+\phi\beta)}\right]\left[\frac{(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)+(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right]$$
$$-\frac{(1-\lambda)\left[(a-m_O)^2-(k\sigma_O)^2\right]}{4}.$$

For a notational purpose, we define the following:

$$X'=\frac{\phi\left(-8\phi(2+\beta\phi)\sigma_K^2+\beta\left(16+\beta\phi(9+\beta\phi)\right)\sigma_O^2\right)}{4(4+\beta\phi)^2(1+\beta\phi)},$$

$$Y'=\frac{4\left((a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)\right)\phi\beta\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{4(4+\beta\phi)^2(1+\beta\phi)},\ (Y'>0)$$

$$Z'=\frac{4\left((a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)\right)^2-(a-m)^2(1+\phi\beta)(4+\phi\beta)^2}{4(4+\beta\phi)^2(1+\beta\phi)},\ (Z'<0).$$

We can rewrite the above equation as $\triangle\,\pi_M^{DS'}=X'k^2+Y'k+Z'$.

When $X' > 0$, we have $0 < \frac{\sigma_K}{\sigma_O} < S_2''$;

When $X' < 0$, we have $\frac{\sigma_K}{\sigma_O} > S_2''$, where $S_2'' = \sqrt{\frac{\beta(16+\beta\phi(9+\beta\phi))}{8\phi(2+\beta\phi)}}$.

Solving the quadratic equation yields two roots. (Q.E.D.)

Proof of Corollary A1:

It is intuitive that when the ODL product seller and the manufacturer are benefited at the same time (Proposition A1 (a) and (b) are both satisfied), the presence of fashion knockoffs benefits the ODL product supply chain. (Q.E.D.)

Proof of Proposition A2:

Under the extended Model GNB, when the ODL product seller is risk sensitive and when there are different manufacturers, comparing the profits for the manufacturer with and without the presence of fashion knockoffs, we have: $\triangle\, \pi_M^{DN'} = \frac{(1-\lambda)[(a-m)(2+\phi\beta)+\phi(b\widehat{m}-\alpha)]^2}{(1+\phi\beta)(4+\phi\beta)^2} - \frac{(1-\lambda)(a-m)^2}{4}$.

Since $\triangle\, \pi_M^{DN'}$ is in a similar form as $\triangle\, \pi_O^{CN}$($\triangle\, \pi_O^{CN} = \frac{1}{(1+\phi\beta)}\left[\frac{(2+\phi\beta)(a-w_O)+\phi(bw_K-\alpha)}{(4+\phi\beta)}\right]^2 - \left(\frac{a-w_O}{2}\right)^2$), and we have already proved that $\triangle\, \pi_O^{CN} < 0$ in Proposition 3.3, we have $\triangle\, \pi_M^{DN'} < 0$. (Q.E.D.)

Appendix C: Endogenous Wholesale Price for Chapter 3

In this section, we consider that the wholesale price is endogenously given. That is, the manufacturer and the ODL product seller (or the knockoff product seller) play a Stackelberg game, the manufacturer first decides on the wholesale prices for the two product sellers. After observing the decision of the manufacturer, the sellers decide on the retail prices accordingly. The optimal decisions under the case with fashion knockoffs are shown as follows:

$$w_O^* =$$

$$\frac{8ab+8bm_O-2\alpha\beta-a\beta^2+(-2b\alpha+9b(a+m_O)\beta-\alpha\beta^2)\phi-b(bm_O+\beta(\alpha-2(a+m_O)\beta))\phi^2+b(\beta+b\phi)(2+\beta\phi)m_K+k(-\beta^2+b(8+\beta\phi(9+2\beta\phi)))\sqrt{\phi^2\sigma_K^2+\sigma_O^2}}{-\beta^2+b(16+\phi(-b\phi+2\beta(9+2\beta\phi)))},$$

$$w_K^* = \frac{\alpha(8+\phi(-b\phi+\beta(9+2\beta\phi)))+(-\beta^2+b(8+\beta\phi(9+2\beta\phi)))m_K+(\beta+b\phi)(2+\beta\phi)(a-m_O+k\sqrt{\phi^2\sigma_K^2+\sigma_O^2})}{-\beta^2+b(16+\phi(-b\phi+2\beta(9+2\beta\phi)))},$$

$$p_O^* = \frac{-\alpha\beta+a(-\beta^2+b(12+\beta\phi(15+4\beta\phi)))+b(\beta+3b\phi+2b\beta\phi^2)m_K-b(\alpha\phi(3+2\beta\phi)+m_O(-4-3\beta\phi+b\phi^2)+k(4+3\beta\phi-b\phi^2)\sqrt{\phi^2\sigma_K^2+\sigma_O^2})}{-\beta^2+b(16+\phi(-b\phi+2\beta(9+2\beta\phi)))},$$

$$p_K^* = \frac{\alpha(12+\phi(-b\phi+\beta(15+4\beta\phi)))+(-\beta^2+b(4+3\beta\phi))m_K+(b\phi+\beta(3+2\beta\phi))(a-m_O+k\sqrt{\phi^2\sigma_K^2+\sigma_O^2})}{-\beta^2+b(16+\phi(-b\phi+2\beta(9+2\beta\phi)))}.$$

Since the optimal decisions are too messy, traceable optimal profits cannot be derived.

Appendix D: Non-standard Markup Wholesale Pricing Policy with Different Manufacturers for Chapter 3

For "non-standard markup wholesale pricing policy", we also consider the case where there are different manufacturers producing for the ODL product seller and the knockoff product seller.

Risk sensitive ODL product seller

We first consider the case where the ODL product seller is risk sensitive. The results about the ODL product seller under the common manufacturer scenario can be easily extended into this scenario.

Comparing profits of the manufacturer with and without the presence of fashion knockoffs, we have

$$\triangle \pi_M^{DS'} = (1-\lambda)\left[\frac{(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)-2k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(4+\phi\beta)}\right]$$

$$\left[\frac{(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)+(2+\phi\beta)k\sqrt{\sigma_O^2+\phi^2\sigma_K^2}}{(1+\phi\beta)(4+\phi\beta)}\right]-\frac{(1-\lambda)[(a-m_O)^2-(k\sigma_O)^2]}{4}.$$

When $\triangle \pi_M^{DS'} > 0$, the presence of fashion knockoffs benefits the manufacturer. For a notational purpose, we define $S_2'' = \sqrt{\frac{\beta(16+\beta\phi(9+\beta\phi))}{8\phi(2+\beta\phi)}}$. We summarize the results in Proposition A1.

Proposition A1. *Under the extended Model GNB, when the ODL product seller is risk sensitive and when there are different manufacturers, we have:*

(a) When $0 < \frac{\sigma_K}{\sigma_O} < S_1$*, the presence of fashion knockoffs benefits the ODL product seller if and only if* $k > K_3' > 0$ *(risk averse); when* $\frac{\sigma_K}{\sigma_O} > S_1$*, the presence of fashion knockoffs benefits the ODL product seller if and only if* $k < K_3' < 0$ *(risk seeking).*

(b) When $0 < \frac{\sigma_K}{\sigma_O} < S_2''$*, the presence of fashion knockoffs benefits the manufacturer if and only if* $k > \frac{-Y'+\sqrt{Y'^2-4X'Z'}}{2X'}$ *(risk averse) or* $k < \frac{-Y'-\sqrt{Y'^2-4X'Z'}}{2X'}$ *(risk seeking); when* $\frac{\sigma_K}{\sigma_O} > S_2''$*, the presence of fashion knockoffs benefits the manufacturer if and only if* $\frac{-Y'+\sqrt{Y'^2-4X'Z'}}{2X'} < k < \frac{-Y'-\sqrt{Y'^2-4X'Z'}}{2X'}$ *(risk averse).*

Proposition A1 reveals that under the non-standard markup wholesale pricing policy, with a risk sensitive ODL product seller and different manufacturers, the presence of fashion knockoffs can benefit the ODL product seller and the manufacturer. The beneficial conditions for the ODL product seller remain the same as in model GNB common manufacturer scenario (Proposition 3.4). For the manufacturer, when the ratio of demand uncertainty is relatively small and when the ODL product seller is sufficiently risk averse or risk seeking, the presence of fashion knockoffs benefits the manufacturer; when the ratio of demand uncertainty is relatively large and when the ODL product seller is moderately risk averse, the presence of fashion knockoffs benefits the manufacturer. Comparing the results with Proposition 3.6 where the wholesale price is exogenously given, the major difference is that, when the manufacturer can decide the wholesale price of the product, and when demand uncertainty for the ODL product is sufficiently large, both a risk averse and risk seeking attitude of the ODL product seller can benefit the manufacturer in the presence of fashion knockoffs.

Now we examine the conditions under which the presence of fashion knockoffs benefits the ODL product supply chain. It is clear that when the ODL product seller and the manufacturer are benefited at the same time (Proposition A1(a) and (b) are both satisfied), the presence of fashion knockoffs benefits the ODL product supply chain. We summarize the key results in Corollary A1.

Corollary A1. *Under the extended Model GNB, when the ODL product seller is risk sensitive and when there are different manufacturers, we have:*

(a) When $0 < \frac{\sigma_K}{\sigma_O} < S_2''$*, the presence of fashion knockoffs benefits the ODL product supply chain and its members if and only if* $k > \max\{K_3', \frac{-Y+\sqrt{Y^2-4XZ}}{2X}\}$ *(risk averse).*

(b) When $S_2'' < \frac{\sigma_K}{\sigma_O} < S_1$*, the presence of fashion knockoffs benefits the ODL product supply chain and its members if and only if* $\max\left\{\frac{-Y'+\sqrt{Y'^2-4X'Z'}}{2X'}, K_3'\right\} < k < \frac{-Y'-\sqrt{Y'^2-4X'Z'}}{2X'}$ *(risk averse).*

From Corollary A1, we uncover that under the non-standard markup wholesale pricing policy, with a risk sensitive ODL product seller and different manufacturers, there exist conditions under which the ODL product supply chain and its members are all benefited in the presence of fashion knockoffs. To be specific, when the ratio of demand uncertainty is relatively small, the presence of fashion knockoffs benefits the ODL product supply chain and its members if the ODL product seller is sufficiently risk averse; when the ratio of demand uncertainty is moderate, the presence of fashion knockoffs benefits the ODL product supply chain and its members if the ODL product seller is moderately risk averse. We interestingly find that a risk seeking attitude of the ODL product seller will never benefit the manufacturer. The results are consistent with the case under standard markup wholesale pricing policy (Corollary 3.3).

Risk neutral ODL product seller

We now consider the case where the ODL product seller is risk neutral. The results about the ODL product seller under the common manufacturer scenario can be easily extended into this scenario.

For the manufacturer, comparing its profits with and without the presence of fashion knockoffs, we have:

$$\Delta\, \pi_M^{DN'} = \frac{(1-\lambda)[(a-m_O)(2+\phi\beta)+\phi(bm_K-\alpha)]^2}{(1+\phi\beta)(4+\phi\beta)^2} - \frac{(1-\lambda)(a-m_O)^2}{4}.$$

The main results are summarized in Proposition A2.

Proposition A2. *Under the extended Model GNB, when the ODL product seller is risk neutral and when there are different manufacturers, the presence of fashion knockoffs always harms the ODL product supply chain and its members.*

Proposition A2 shows that, under the non-standard markup wholesale pricing policy, when there are different manufacturers producing for the ODL product seller and the knockoff product seller, the presence of fashion knockoffs always harms the ODL product supply chain and its members if the ODL product seller is risk neutral. The results are consistent with Proposition 3.7 where the wholesale price is exogenously given. Comparing with Proposition A1 where the ODL product seller is risk sensitive, we can conclude that risk sensitive attitude of the ODL product seller always outperforms the risk neutral attitude in terms of the profits of the ODL product supply chain and its members. Hence, the ODL product seller is advised to take risk attitude into consideration when making decisions.

Appendix E: Interview for Chapter 3

Assumptions:

<u>Features of fashion knockoffs</u>

1. Have you heard about fashion knockoffs? Is it popular in the fashion industry? What is your impression on it? What is its difference from ODL product? (in terms of retail price, production cost, quality, etc.)

Yes, of course. It is very popular in the fashion industry. For example, many fast fashion brands are famous for copying luxury fashion brands. When talking about copycat product, I think of low quality and cheap products. Although there do exist high quality copycat products which may be even in a higher quality to the original one, it is not the common case.

2. Have you heard about runway fashion knockoffs? Do you think it is possible that consumers have access to both the ODL product and its knockoff product at the same time?

Yes, it is a very popular phenomenon. Of course, the ODL product and its knockoff product can coexist in the market. There even exists the case where the knockoff product is launched before the original one. Normally speaking, designers will exhibit the products in the fashion shows two seasons before its official launch. For example, it is now spring/summer of 2021, the designers are exhibiting the products which will be launched officially in spring/summer 2022. While some brands, especially fast fashion brands may steal the ideas from the runway and launch the products months before the ODL product seller.

<u>Supply chain settings</u>

1. Have you heard the situation that the ODL product seller and its knockoff product seller have the common manufacturer? Can you name some examples in the fashion industry?

Yes, and actually it is quite common, especially for the manufacturers producing for t-shirts. For example, Esquel is producing T-shirts for both a luxury brand and a knockoff product seller. Sometimes, the manufacturer may only change the button design for the knockoff product seller.

2. Is it common in the fashion industry that the wholesale price is determined by the market, that is, the manufacturer simply offers a standard markup with the markup rate determined by the industrial norm?

Yes, I think it is common, and the markup rate depends on the ordering size of the retailer. For mass market retailers like *Giordano*, the markup rate is usually around 20-30%, while for giant retailers like *Walmart* whose ordering size is much larger, the markup rate is around 10%.

3. From your perspective, do you think demand for the ODL product will affect demand for its knockoff product? And in which form? How about the other way round?

Of course. Normally speaking, demand for the ODL product will positively affect demand for the knockoff product. While demand for the knockoff product usually has a negative effect on the ODL product. The reasons are as follows: (1) Due to the existence of conspicuous consumption, with the number of consumers purchasing the knockoff product increases, consumers who initially would buy the ODL product may decide not to buy for the sake of social status; (2) When the knockoff product is very prevalent in the market, the ODL product seller may switch to produce another product rather than the existing one; (3) A group of price-sensitive consumers may buy a knockoff product which has a similar design as the original one to satisfy their needs for status. In this way, the knockoff product seizes the market share from the ODL product.

4. As you know, both the ODL product seller and its knockoff product seller incur a fixed cost in their operations (e.g., product research and development cost), which one do you think will incur a larger operational cost?

It is for sure that the ODL product sellers will incur a much larger operational cost, because they have to invest in product research and development cost. While for knockoff product seller, the only cost is to purchase an ODL product, copy its design and then produce. For some fast fashion brands (who are fashion knockoffs), they don't even have a marketing department. Their major cost for marketing is on visual merchandising and new product photographing.

Risk attitude

1. Do you think the ODL product seller tend to be risk sensitive or risk neutral in the presence of fashion knockoffs?

I think the ODL product seller tends to be risk sensitive. In the potential threat of fashion knockoffs, the ODL product seller is more cautious about market testing. Rather than putting a product into the regular product line, the ODL product seller may first launch the product as a seasonal product and wait for consumers and competitors (e.g., knockoff product sellers)'reactions. For some big companies like *LV* or *Gucci*, they even have a specific team which monitors the copying level of the knockoff product sellers. When the copying level reaches some point, they will sue the knockoff companies to avoid potential risks.

2. Do you think the ODL product seller can be risk seeking in the presence of fashion knockoffs?

Yes, there do exist the case where the ODL product seller is risk seeking in the presence of fashion knockoffs, but it is very rare. For some well-established companies like *Supreme* and *RIMOWA*, they are not afraid of being copied, instead, they may like being copied since the existence of fashion knockoffs can help them promote their products and brand name, which may even benefit them.

Findings:

<u>Basic model</u>

Our results indicate that the presence of fashion knockoffs may benefit the ODL product supply chain and its members, does it make sense? Do you think in real world cases, the ODL will intentionally allow the existence of fashion knockoffs?

Yes, although some ODL product sellers tend to be risk sensitive, the ratio of knockoff product sellers being sued is very low, which implies that the ODL product sellers just "let the knockoff go" in most cases. One explanation is that these ODL product sellers are actually benefited by the presence of fashion knockoffs. With the prevalence of fashion knockoffs, fashion trends like seasonal color, textiles etc., are being promoted. In consequence, demand for the ODL product sellers may increase accordingly, and hence benefit the ODL product seller.

<u>Non-standard Markup Wholesale Pricing Policy</u>

Will the manufacturer set different markup rates for the ODL product seller and the knockoff product seller? We analytically find that the manufacturer is advised to set a higher markup rate for the ODL product seller or a lower markup rate for the knockoff product seller in the presence of fashion knockoffs. Is it true?

Yes, it is common that the manufacturer sets different markup rates for the ODL product seller and the knockoff product seller, and the markup rate for the ODL product seller is usually higher than that of the knockoff product seller. For ODL product sellers (esp. luxury brands), the production procedure is rather complex and the wasting caused by cutting is substantial, which leads to a higher production cost. Besides, for ODL products which are produced overseas, extra logistics fee is caused. In this case, the manufacturer inclines to set a higher markup rate to cover the substantial cost for the ODL product seller. On the contrary, for knockoff products, the manufacturer may skip some production procedures, and most of the knockoff products are produced domestically. Moreover, knockoff product sellers usually order in large batches, which creates the economy of scale, the manufacturer can hence set a lower markup rate for the knockoff product seller.

Different Manufacturers

We analytically find that compared with producing only for the ODL product seller, when producing for both the ODL product seller and the knockoff product seller, the manufacturer is more likely to be benefited. Is it true?

Yes, it is true. When the manufacturer only produces for the ODL product seller, due to substantial wasting and the lack of economy of scale, the manufacturer is less likely to be benefited. While when the manufacturer produces for both the ODL product seller and the knockoff product seller, the primarily invested research and development cost, facilities, etc., can also be used for producing the knockoff products. Moreover, due to the economy of scale, the manufacturer is more likely to be benefited.

Appendix F: Equilibrium Results for Chapter 4

Lemma A1. The equilibrium results under the basic model are summarized in Tables A2 and A3. The results under the NE case can be simply obtained by setting $b = 0$. All the equilibrium results are positive when demand is positive.

Table A2. The equilibrium results for the original supply chain and the consumers under the basic model.

OB		**EX case**	**NE case**
Optimal decisions	w^*	$\frac{(\theta-1)(3\theta-8)+b(8+\theta(2\theta-9))}{16+\theta(4\theta-17)}$	$\frac{(\theta-1)(3\theta-8)}{16+\theta(4\theta-17)}$
	p_{OB}^*	$\frac{2(\theta-3)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))}{(\theta-4)(16+\theta(4\theta-17))}$	$\frac{2(\theta-3)(\theta-1)(3\theta-8)}{(\theta-4)(16+\theta(4\theta-17))}$
Optimal solutions	D_{OB}^*	$-\frac{(\theta-2)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$-\frac{(\theta-2)(\theta-1)(3\theta-8)}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	CS_{OB}^*	$\frac{(\theta-2)^2((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))(8+\theta+\theta^2(4\theta-13)+b(8+\theta(4\theta-13)))}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$	$\frac{(\theta-2)^2(\theta-1)(3\theta-8)(8+\theta+\theta^2(4\theta-13))}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$
	π_{OB}^*	$-\frac{(\theta-2)^2((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$	$-\frac{(\theta-2)^2((\theta-1)(3\theta-8))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$
	π_{M1}^*	$-\frac{(\theta-2)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))^2}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2}$	$-\frac{(\theta-2)((\theta-1)(3\theta-8))^2}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2}$
	$\pi_{SC,OB}^*$	$-\frac{(2(\theta-3)(\theta-2)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2)}$	$-\frac{(2(\theta-3)(\theta-2)((\theta-1)(3\theta-8))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2)}$

Table A3. The equilibrium results for the knockoff supply chain and the consumers under the basic model.

CC		**EX case**	**NE case**
Optimal decisions	c^*	$\frac{b(\theta-2)\theta+2(\theta-3)(\theta-1)\theta}{16+\theta(4\theta-17)}$	$\frac{2(\theta-3)(\theta-1)\theta}{16+\theta(4\theta-17)}$
	p_{CC}^*	$\frac{2(\theta-3)(b(\theta-2)\theta+2(\theta-3)(\theta-1)\theta)}{(\theta-4)(16+\theta(4\theta-17))}$	$\frac{4(\theta-3)^2(\theta-1)\theta}{(\theta-4)(16+\theta(4\theta-17))}$
Optimal solutions	D_{CC}^*	$-\frac{(\theta-2)(b(\theta-2)+2(\theta-3)(\theta-1))}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$-\frac{2(\theta-2)(\theta-3)(\theta-1)}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	CS_{CC}^*	$\frac{(b(\theta-2)+2(\theta-3)(\theta-1))^2(\theta-2)^2\theta}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$	$\frac{(2(\theta-3)(\theta-1))^2(\theta-2)^2\theta}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$
	π_{CC}^*	$-\frac{(\theta-2)^2(b(\theta-2)+2(\theta-3)(\theta-1))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$	$-\frac{(\theta-2)^2(2(\theta-3)(\theta-1))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$
	π_{M2}^*	$-\frac{(b(\theta-2)+2(\theta-3)(\theta-1))^2(\theta-2)\theta}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2}$	$-\frac{(2(\theta-3)(\theta-1))^2(\theta-2)\theta}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2}$
	$\pi_{SC,CC}^*$	$-\frac{2(b(\theta-2)+2(\theta-3)(\theta-1))^2(\theta-3)(\theta-2)\theta}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$	$-\frac{2(2(\theta-3)(\theta-1))^2(\theta-3)(\theta-2)\theta}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$

Lemma A2. The equilibrium results with strategic quality decision are summarized in Tables A4 and A5. The results under the NE case can be simply obtained by setting $b = 0$. All the equilibrium results are positive when demand is positive.

Table A4. The equilibrium results for the original product supply chain and the consumers with strategic quality decision.

OB		**EX case**	**NE case**
Optimal decisions	$\widehat{w}^*$	$\frac{k(\theta-4)(\theta-1)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$\frac{k(\theta-4)(\theta-1)^2(3\theta-8)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	$\widehat{q}^*$	$-\frac{(\theta-2)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$-\frac{(\theta-2)(\theta-1)(3\theta-8)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	$\widehat{p}_{OB}^*$	$\frac{2k(\theta-3)(\theta-1)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$\frac{2k(\theta-3)(\theta-1)^2(3\theta-8)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
Optimal solutions	$\widehat{D}_{OB}^*$	$-\frac{k(\theta-2)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$-\frac{k(\theta-2)(\theta-1)(3\theta-8)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	$\widehat{CS}_{OB}^*$	$\frac{k(\theta-2)^2((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))(2(\theta-2)\theta+k(8+\theta+\theta^2(4\theta-13)+b(8+\theta(4\theta-13))))}{2((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$	$\frac{k(\theta-2)^2(\theta-1)(3\theta-8)(2(\theta-2)\theta+k(8+\theta+\theta^2(4\theta-13)))}{2((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$
	$\widehat{\pi}_{OB}^*$	$-\frac{k^2(\theta-2)^2(\theta-1)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))^2}{((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$	$-\frac{k^2(\theta-2)^2(\theta-1)^3(3\theta-8)^2}{((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$
	$\widehat{\pi}_{M1}^*$	$-\frac{k(\theta-2)(2k(\theta-4)(\theta-1)+\theta-2)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))^2}{2((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(+\theta-1)(16+\theta(4\theta-17)))^2}$	$-\frac{k(\theta-2)(2k(\theta-4)(\theta-1)+\theta-2)((\theta-1)(3\theta-8))^2}{2((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(+\theta-1)(16+\theta(4\theta-17)))^2}$
	$\widehat{\pi}_{SC,OB}^*$	$-\frac{k(\theta-2)(4k(\theta-3)(\theta-1)+\theta-2)((\theta-1)(3\theta-8)+b(8+\theta(2\theta-9)))^2}{2((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$	$-\frac{k(\theta-2)(4k(\theta-3)(\theta-1)+\theta-2)((\theta-1)(3\theta-8))^2}{2((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$

Table A5. The equilibrium results for the knockoff product supply chain and the consumers with strategic quality decision.

CC		**EX case**	**NE case**
Optimal decisions	$\widehat{c}^*$	$\frac{(\theta-4)(\theta-1)\theta(bk(\theta-2)+2k(\theta-3)(\theta-1)+\theta-2)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$\frac{(\theta-4)(\theta-1)\theta(2k(\theta-3)(\theta-1)+\theta-2)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	$\widehat{p}_{CC}^*$	$\frac{2(\theta-3)(\theta-1)\theta(bk(\theta-2)+2k(\theta-3)(\theta-1)+\theta-2)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$\frac{2(\theta-3)(\theta-1)\theta(2k(\theta-3)(\theta-1)+\theta-2)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
Optimal solutions	$\widehat{D}_{CC}^*$	$-\frac{(\theta-2)(bk(\theta-2)+2k(\theta-3)(\theta-1)+\theta-2)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$-\frac{(\theta-2)(2k(\theta-3)(\theta-1)+\theta-2)}{(\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	$\widehat{CS}_{CC}^*$	$\frac{(\theta-2)^2\theta(bk(\theta-2)+2k(\theta-3)(\theta-1)+\theta-2)^2}{2((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$	$\frac{(\theta-2)^2\theta(2k(\theta-3)(\theta-1)+\theta-2)^2}{2((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$
	$\widehat{\pi}_{CC}^*$	$-\frac{(\theta-2)^2(\theta-1)\theta(bk(\theta-2)+2k(\theta-3)(\theta-1)+\theta-2)^2}{((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$	$-\frac{(\theta-2)^2(\theta-1)\theta(2k(\theta-3)(\theta-1)+\theta-2)^2}{((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$
	$\widehat{\pi}_{M2}^*$	$-\frac{(\theta-4)(\theta-2)(\theta-1)\theta(bk(\theta-2)+2k(\theta-3)(\theta-1)+\theta-2)^2}{((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$	$-\frac{(\theta-4)(\theta-2)(\theta-1)\theta(2k(\theta-3)(\theta-1)+\theta-2)^2}{((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$
	$\widehat{\pi}_{SC,CC}^*$	$\frac{(6-2\theta)(\theta-2)(\theta-1)\theta(bk(\theta-2)+2k(\theta-3)(\theta-1)+\theta-2)^2}{((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$	$\frac{(6-2\theta)(\theta-2)(\theta-1)\theta(2k(\theta-3)(\theta-1)+\theta-2)^2}{((\theta-2)(8+\theta(2\theta-9))+k(\theta-4)(\theta-1)(16+\theta(4\theta-17)))^2}$

Lemma A3. The equilibrium results with price dependent C2C-PE utility are summarized in Tables A6 and A7. The results under the NE case can be simply obtained by setting $h_1 = h_2 = 0$. All the equilibrium results are positive when demand is positive.

Table A6. The equilibrium results for the original product supply chain and the consumers with price dependent C2C-PE utility.

OB		**EX case**	**NE case**
Optimal decisions	$\tilde{w}^*$	$\frac{11\theta-3\theta^2-8+((9-2\theta)\theta-8)h_1}{(16+\theta(4\theta-17))(h_2-1)}$	$-\frac{11\theta-3\theta^2-8}{16+\theta(4\theta-17)}$
	$\tilde{p}_{OB}^*$	$-\frac{2(\theta-3)((\theta-1)(3\theta-8)+(8+\theta(2\theta-9))h_1)}{(\theta-4)(16+\theta(4\theta-17))(h_2-1)}$	$\frac{2(\theta-3)(\theta-1)(3\theta-8)}{(\theta-4)(16+\theta(4\theta-17))}$
Optimal solutions	$\widetilde{D}_{OB}^*$	$-\frac{(\theta-2)((\theta-1)(3\theta-8)+(8+\theta(2\theta-9))h_1)}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$-\frac{(\theta-2)(\theta-1)(3\theta-8)}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	$\widetilde{CS}_{OB}^*$	$\frac{(\theta-2)^2((\theta-1)(3\theta-8)+(8+\theta(2\theta-9))h_1)(8+\theta-13\theta^2+4\theta^3+(8+\theta(4\theta-13))h_1)}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$	$\frac{(\theta-2)^2(\theta-1)(3\theta-8)(8+\theta-13\theta^2+4\theta^3)}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$
	$\tilde{\pi}_{OB}^*$	$\frac{(\theta-2)^2((\theta-1)(3\theta-8)+(8+\theta(2\theta-9))h_1)^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2(h_2-1)}$	$-\frac{(\theta-2)^2((\theta-1)(3\theta-8))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$
	$\tilde{\pi}_{M1}^*$	$\frac{(\theta-2)((\theta-1)(3\theta-8)+(8+\theta(2\theta-9))h_1)^2}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2(h_2-1)}$	$-\frac{(\theta-2)((\theta-1)(3\theta-8))^2}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2}$
	$\tilde{\pi}_{SC,OB}^*$	$\frac{2(\theta-3)(\theta-2)((\theta-1)(3\theta-8)+(8+\theta(2\theta-9))h_1)^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2(h_2-1)}$	$-\frac{2(\theta-3)(\theta-2)((\theta-1)(3\theta-8))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$

Table A7. The equilibrium results for the knockoff product supply chain and the consumers with price dependent C2C-PE utility.

CC		**EX case**	**NE case**
Optimal decisions	$\tilde{c}^*$	$\frac{2(\theta-3)(\theta-1)\theta+(\theta-2)\theta h_1}{16+\theta(4\theta-17)}$	$\frac{2(\theta-3)(\theta-1)\theta}{16+\theta(4\theta-17)}$
	$\tilde{p}_{CC}^*$	$\frac{2(\theta-3)\theta(2(\theta-3)(\theta-1)+(\theta-2)h_1)}{(\theta-4)(16+\theta(4\theta-17))}$	$\frac{4\theta(\theta-3)^2(\theta-1)}{(\theta-4)(16+\theta(4\theta-17))}$
Optimal solutions	$\widetilde{D}_{CC}^*$	$-\frac{(\theta-2)(2(\theta-3)(\theta-1)+(\theta-2)h_1)}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$	$-\frac{2(\theta-3)(\theta-2)(\theta-1)}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))}$
	$\widetilde{CS}_{CC}^*$	$\frac{(\theta-2)^2\theta(2(\theta-3)(\theta-1)+(\theta-2)h_1)^2}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$	$\frac{(\theta-2)^2\theta(2(\theta-3)(\theta-1))^2}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$
	$\tilde{\pi}_{CC}^*$	$-\frac{(\theta-2)^2\theta(2(\theta-3)(\theta-1)+(\theta-2)h_1)^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$	$-\frac{(\theta-2)^2\theta(2(\theta-3)(\theta-1))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$
	$\tilde{\pi}_{M2}^*$	$-\frac{(\theta-2)\theta(2(\theta-3)(\theta-1)+(\theta-2)h_1)^2}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2}$	$-\frac{(\theta-2)\theta(2(\theta-3)(\theta-1))^2}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2}$
	$\tilde{\pi}_{SC,CC}^*$	$-\frac{2(\theta-3)(\theta-2)\theta(2(\theta-3)(\theta-1)+(\theta-2)h_1)^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$	$-\frac{2(\theta-3)(\theta-2)\theta(2(\theta-3)(\theta-1))^2}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$

Lemma A4. The equilibrium results when considering consumers' conspicuous consumption are summarized in Tables A8 and A9. The results under the NE case can be simply obtained by setting $b = 0$. All the equilibrium results are positive when demand is positive.

Table A8. The equilibrium results for the original product supply chain and the consumers with consumers' conspicuous consumption.

OB		**EX case**
Optimal decisions	$\bar{w}^*$	$\frac{8-\delta(-5+\delta(2+\delta))-11\theta-7\delta\theta+(3+\delta)\theta^2+b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))}{16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2}$
	$\bar{p}_{OB}^*$	$\frac{2(3+2\delta-\theta)(8-\delta(-5+\delta(2+\delta))-11\theta-7\delta\theta+(3+\delta)\theta^2+b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta)))}{(4+3\delta-\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)}$
Optimal solutions	$\bar{D}_{OB}^*$	$\frac{1}{9}(\frac{2(3+b+2\delta)}{4+3\delta-\theta}+\frac{-b+\delta}{-1+\theta}+\frac{3(4+2b(2+\delta-\theta)-\theta+\delta(3+\delta+\theta))}{16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2})$
	$\overline{CS}_{OB}^*$	$\frac{((2+\delta-\theta)^2(8-\delta(-5+\delta(2+\delta))+\theta+\delta(17+2\delta(7+\delta))\theta-(13+5\delta(5+2\delta))\theta^2+4(1+\delta)\theta^3+b(8+7\delta+\delta^2-(13+\delta(11+2\delta))\theta+2(2+\delta)\theta^2))(-8+\delta(-5+\delta(2+\delta))+11\theta+7\delta\theta-(3+\delta)\theta^2-b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))))}{-(2(-1+\theta)^2(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}$
	$\bar{\pi}_{OB}^*$	$-\frac{(2+\delta-\theta)^2(8-\delta(-5+\delta(2+\delta))-11\theta-7\delta\theta+(3+\delta)\theta^2+b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta)))^2}{((-1+\theta)(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}$
	$\bar{\pi}_{M1}^*$	$-\frac{(2+\delta-\theta)(8-\delta(-5+\delta(2+\delta))-11\theta-7\delta\theta+(3+\delta)\theta^2+b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta)))^2}{(4+3\delta-\theta)(-1+\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}$
	$\bar{\pi}_{SC,OB}^*$	$-\frac{2(2+\delta-\theta)(3+2\delta-\theta)(8-\delta(-5+\delta(2+\delta))-11\theta-7\delta\theta+(3+\delta)\theta^2+b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta)))^2}{(-1+\theta)(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}$

Table A9. The equilibrium results for the knockoff product supply chain and the consumers with consumers' conspicuous consumption.

CC		**EX case**
Optimal decisions	$\bar{c}^*$	$\frac{\theta(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2+b(-2-\delta+\theta))}{16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2}$
	$\bar{p}_{CC}^*$	$\frac{2(3+2\delta-\theta)\theta(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2+b(-2-\delta+\theta))}{(4+3\delta-\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)}$
Optimal solutions	$\bar{D}_{CC}^*$	$-\frac{(1+\delta)(2+\delta-\theta)(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2+b(-2-\delta+\theta))}{(4+3\delta-\theta)(-1+\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)}$
	$\overline{CS}_{CC}^*$	$\frac{(1+\delta)^2(2+\delta-\theta)^2\theta(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2+b(-2-\delta+\theta))^2}{2(-1+\theta)^2(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}$
	$\bar{\pi}_{CC}^*$	$-\frac{(1+\delta)(2+\delta-\theta)^2\theta(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2+b(-2-\delta+\theta))^2}{(-1+\theta)(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}$
	$\bar{\pi}_{M2}^*$	$-\frac{(1+\delta)(2+\delta-\theta)\theta(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2+b(-2-\delta+\theta))^2}{(4+3\delta-\theta)(-1+\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}$
	$\bar{\pi}_{SC,CC}^*$	$-\frac{2(1+\delta)(2+\delta-\theta)(3+2\delta-\theta)\theta(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2+b(-2-\delta+\theta))^2}{(-1+\theta)(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}$

Appendix G: All Proofs for Chapter 4

Proof of Lemma A1:

The profit functions for the original brand and the knockoff brand are $\pi_{OB}^{EX} = (p_{OB}^{EX} - w^{EX})\left(1 - \frac{p_{OB}^{EX} - b - p_{CC}^{EX}}{1-\theta}\right)$, $\pi_{CC}^{EX} = (p_{CC}^{EX} - c^{EX})\frac{\theta(p_{OB}^{EX} - b) - p_{CC}^{EX}}{\theta(1-\theta)}$. We find that π_{OB}^{EX} (π_{CC}^{EX}) is concave in p_{OB}^{EX} (p_{CC}^{EX}). By solving the first order derivative of π_{OB}^{EX} (π_{CC}^{EX}) with respect to p_{OB}^{EX} (p_{CC}^{EX}), we obtain the optimal selling prices p_{OB}^{EX*} and p_{CC}^{EX*}. Substituting them back into the demand functions, we have D_{OB}^{EX*} and D_{CC}^{EX*}. Substituting D_{OB}^{EX*} and D_{CC}^{EX*} into the profit functions of the manufacturers, we have: $\pi_{M1}^{EX} = w^{EX}\frac{2+2b+c^{EX}-2w^{EX}-2\theta-b\theta+w^{EX}\theta}{4-5\theta+\theta^2}$, $\pi_{M2}^{EX} = c^{EX}\frac{c^{EX}(\theta-2)-\theta(b-w^{EX}-1+\theta)}{(\theta-4)(\theta-1)\theta}$. We find that π_{M1}^{EX} (π_{M2}^{EX}) is concave in w^{EX} (c^{EX}). Solving the first order derivative of π_{M1}^{EX} (π_{M2}^{EX}) with respect to w^{EX} (c^{EX}), we obtain the optimal wholesale prices w^{EX*} and c^{EX*}.

Through substitution, we obtain the optimal decisions and solutions as summarized in Tables A2 and A3. Note that the results under the NE case can be simply obtained by setting $b = 0$. (Q.E.D.)

Proof of Proposition 4.1:

To ensure the coexistence of the original product and knockoff product in the market under both the EX case and the NE case, we have $D_{OB}^{EX*} > 0$, $D_{CC}^{EX*} > 0$, $D_{OB}^{NE*} > 0$ and $D_{CC}^{NE*} > 0$. Solving them, we obtain the following condition: (1) $0 < b < 1$ and $0 < \theta \leq \frac{7-\sqrt{17}}{4}$; or (2) $0 < b < \frac{-2(\theta^2-4\theta+3)}{\theta-2}$ and $\frac{7-\sqrt{17}}{4} < \theta < 1$.

To conduct sensitivity analysis, we take first order derivative and second order derivative on the equilibrium results with respect to θ. For the original supply chain and the consumers, we have:

$\frac{\partial w^{NE*}}{\partial\theta} < 0$, $\frac{\partial p_{OB}^{NE*}}{\partial\theta} < 0$, $\frac{\partial \pi_{OB}^{NE*}}{\partial\theta} < 0$; $\frac{\partial D_{OB}^{NE*}}{\partial\theta} > 0$, $\frac{\partial CS_{OB}^{NE*}}{\partial\theta} > 0$; $\frac{\partial^2 \pi_{M1}^{NE*}}{\partial\theta^2} < 0$, $\frac{\partial^2 \pi_{SC,OB}^{NE*}}{\partial\theta^2} < 0$.

For the knockoff supply chain and the consumers, we have:

$\frac{\partial D_{CC}^{NE*}}{\partial\theta} > 0$, $\frac{\partial CS_{CC}^{NE*}}{\partial\theta} > 0$; $\frac{\partial^2 c^{NE*}}{\partial\theta^2} < 0$, $\frac{\partial^2 p_{CC}^{NE*}}{\partial\theta^2} < 0$, $\frac{\partial^2 \pi_{CC}^{NE*}}{\partial\theta^2} < 0$; first $\frac{\partial \pi_{M2}^{NE*}}{\partial\theta} > 0$ and then $\frac{\partial \pi_{M2}^{NE*}}{\partial\theta} < 0$, first $\frac{\partial \pi_{SC,CC}^{NE*}}{\partial\theta} > 0$ and then $\frac{\partial \pi_{SC,CC}^{NE*}}{\partial\theta} < 0$. (Q.E.D.)

Proof of Proposition 4.2:

Under the EX case, we take first order derivative and second order derivative on the equilibrium results with respect to θ and b respectively. For the original supply chain and the consumers, we have:

$\frac{\partial D_{OB}^{EX*}}{\partial \theta} > 0, \frac{\partial CS_{OB}^{EX*}}{\partial \theta} > 0; \frac{\partial w^{EX*}}{\partial \theta} < 0, \frac{\partial p_{OB}^{EX*}}{\partial \theta} < 0; \frac{\partial w^{EX*}}{\partial b} > 0, \frac{\partial p_{OB}^{EX*}}{\partial b} > 0, \frac{\partial D_{OB}^{EX*}}{\partial b} > 0, \frac{\partial \pi_{OB}^{EX*}}{\partial b} > 0, \frac{\partial \pi_{M1}^{EX*}}{\partial b} > 0,$
$\frac{\partial \pi_{SC,OB}^{EX*}}{\partial b} > 0, \frac{\partial CS_{OB}^{EX*}}{\partial b} > 0.$

For the knockoff supply chain and the consumers, we have:

$\frac{\partial^2 c^{EX*}}{\partial \theta^2} < 0, \frac{\partial^2 p_{CC}^{EX*}}{\partial \theta^2} < 0; \frac{\partial c^{EX*}}{\partial b} > 0, \frac{\partial p_{CC}^{EX*}}{\partial b} > 0, \frac{\partial D_{CC}^{EX*}}{\partial b} > 0, \frac{\partial \pi_{CC}^{EX*}}{\partial b} > 0, \frac{\partial \pi_{M2}^{EX*}}{\partial b} > 0, \frac{\partial \pi_{SC,CC}^{EX*}}{\partial b} > 0, \frac{\partial CS_{CC}^{EX*}}{\partial b} >$
$0.$ (Q.E.D.)

Proof of Proposition 4.3:

Comparing the optimal decisions and demands under the EX case and the NE case, we find:

$$w^{EX*} - w^{NE*} = \frac{b(8+\theta(-9+2\theta))}{16+\theta(-17+4\theta)} > 0,$$

$$p_{OB}^{EX*} - p_{OB}^{NE*} = \frac{2b(-3+\theta)(8+\theta(-9+2\theta))}{(-4+\theta)(16+\theta(-17+4\theta))} > 0,$$

$$D_{OB}^{EX*} - D_{OB}^{NE*} = -\frac{b(-2+\theta)(8+\theta(-9+2\theta))}{(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} > 0,$$

$$c^{EX*} - c^{NE*} = \frac{b(-2+\theta)\theta}{16+\theta(-17+4\theta)} < 0,$$

$$p_{CC}^{EX*} - p_{CC}^{NE*} = \frac{2b(-3+\theta)(-2+\theta)\theta}{(-4+\theta)(16+\theta(-17+4\theta))} < 0,$$

$$D_{CC}^{EX*} - D_{CC}^{NE*} = -\frac{b(-2+\theta)^2}{(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} < 0.$$ (Q.E.D.)

Proof of Proposition 4.4:

Define: $\triangle \pi_i = \pi_i^{EX*} - \pi_i^{NE*}; \triangle \pi_{Mj} = \pi_{Mj}^{EX*} - \pi_{Mj}^{NE*}; \triangle \pi_{SC,i} = \pi_{SC,i}^{EX*} - \pi_{SC,i}^{NE*}; \triangle CS_i = CS_i^{EX*} - CS_i^{NE*},$

$i = OB, CC; \ j = 1,2.$

From Table A2, we can simply have the following:

$$\triangle \pi_{OB} = -\frac{b(\theta-2)^2(8(2+b)-(22+9b)\theta+2(3+b)\theta^2)(8+\theta(2\theta-9))}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2},$$

$$\triangle \pi_{M1} = -\frac{b(\theta-2)(8(2+b)-(22+9b)\theta+2(3+b)\theta^2)(8+\theta(2\theta-9))}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2},$$

$$\triangle \pi_{SC,OB} = -\frac{2b(\theta-3)(\theta-2)(8(2+b)-(22+9b)\theta+2(3+b)\theta^2)(8+\theta(2\theta-9))}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2},$$

$$\triangle CS_{OB} = \frac{b(\theta-2)^2(b(8+\theta(2\theta-9))(8+\theta(4\theta-13))+2(\theta-1)(-64+\theta(64+\theta(13+\theta(4\theta-21)))))}{(2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2)}.$$

We find that $\triangle \pi_{OB}$, $\triangle \pi_{M1}$, $\triangle \pi_{SC,OB}$ and $\triangle CS_{OB}$ are all positive under the condition where both demands are positive.

Similarly, from Table A3, we can simply have the following:

$\triangle\,\pi_{CC} = -\frac{b(b(\theta-2)+4(\theta-3)(\theta-1))(\theta-2)^3\theta}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$,

$\triangle\,\pi_{M2} = -\frac{b(b(\theta-2)+4(\theta-3)(\theta-1))(\theta-2)^2\theta}{(\theta-4)(\theta-1)(16+\theta(4\theta-17))^2}$,

$\triangle\,\pi_{SC,CC} = -\frac{2b(b(\theta-2)+4(\theta-3)(\theta-1))(\theta-3)(\theta-2)^2\theta}{(\theta-4)^2(\theta-1)(16+\theta(4\theta-17))^2}$,

$\triangle\,CS_{CC} = \frac{b(b(\theta-2)+4(\theta-3)(\theta-1))(\theta-2)^3\theta}{2(\theta-4)^2(\theta-1)^2(16+\theta(4\theta-17))^2}$.

We find that $\triangle\,\pi_{CC}$, $\triangle\,\pi_{M2}$, $\triangle\,\pi_{SC,CC}$ and $\triangle\,CS_{CC}$ are all negative under the condition where both demands are positive. Proposition 4.4 is proved. (Q.E.D)

Proof of Lemma A2:

Similar to the basic model, we use backward induction to solve the problem. The profit functions for the original brand and the knockoff brand are $\hat{\pi}_{OB}^{EX} = (\hat{p}_{OB}^{EX} - \hat{w}^{EX})(1 + \frac{b+\hat{q}^{EX}+\hat{p}_{CC}^{EX}-\hat{p}_{OB}^{EX}}{\theta-1})$, $\hat{\pi}_{CC}^{EX} = (\hat{p}_{CC}^{EX} - \hat{c}^{EX})\frac{\hat{p}_{CC}^{EX}+\theta(b+\hat{q}^{EX}-\hat{p}_{OB}^{EX})}{(-1+\theta)\theta}$. We find that $\hat{\pi}_{OB}^{EX}$ ($\hat{\pi}_{CC}^{EX}$) is concave in $\hat{p}_{OB}^{EX}$ ($\hat{p}_{CC}^{EX}$). By solving the first order derivative of $\hat{\pi}_{OB}^{EX}$ ($\hat{\pi}_{CC}^{EX}$) with respect to $\hat{p}_{OB}^{EX}$ ($\hat{p}_{CC}^{EX}$), we obtain the optimal selling prices $\hat{p}_{OB}^{EX*}$ and $\hat{p}_{CC}^{EX*}$. Substituting them back into the demand functions, we have $\widehat{D}_{OB}^{EX*}$ and $\widehat{D}_{CC}^{EX*}$.

Substituting $\widehat{D}_{OB}^{EX*}$ and $\widehat{D}_{CC}^{EX*}$ into the profit functions of the manufacturers, we have: $\hat{\pi}_{M1}^{EX} = \hat{w}^{EX}\frac{2+2b+\hat{c}^{EX}+2\hat{q}^{EX}-2\hat{w}^{EX}-(2+b+\hat{q}^{EX}-\hat{w}^{EX})\theta}{(-4+\theta)(-1+\theta)} - \frac{k\hat{q}^{EX^2}}{2}$, $\hat{\pi}_{M2}^{EX} = \hat{c}^{EX}\frac{\hat{c}^{EX}(-2+\theta)-\theta(-1+b+\hat{q}^{EX}-\hat{w}^{EX}+\theta)}{(-4+\theta)(-1+\theta)\theta}$. By checking the Hessian matrix, we find that $\hat{\pi}_{M1}^{EX}$ is jointly concave in $\hat{w}^{EX}$ and $\hat{q}^{EX}$. Solving the first order derivative of $\hat{\pi}_{M1}^{EX}$ with respect to $\hat{w}^{EX}$ and $\hat{q}^{EX}$ respectively, we obtain $\hat{w}^{EX*}$ and $\hat{q}^{EX*}$. By checking the second order derivative of $\hat{\pi}_{M2}^{EX}$, we find that $\hat{\pi}_{M2}^{EX}$ is concave in $\hat{c}^{EX}$. Solving the first order derivative of $\hat{\pi}_{M2}^{EX}$ with respect to $\hat{c}^{EX}$, we obtain $\hat{c}^{EX*}$. Through substitution of the optimal decisions, we summarize the equilibrium results in Tables A4 and A5. Note that the results under the NE case can be simply obtained by setting $b = 0$. (Q.E.D.)

Proof of Proposition 4.5:

To ensure the coexistence of the original product and knockoff product in the market under both the EX case and the NE case, we have $\widehat{D}_{OB}^{EX*} > 0$, $\widehat{D}_{CC}^{EX*} > 0$, $\widehat{D}_{OB}^{NE*} > 0$ and $\widehat{D}_{CC}^{NE*} > 0$. Solving them, we obtain the following condition: $0 < b < 1, 0 < \theta < \frac{7-b}{4} - \frac{1}{4}\sqrt{17+2b+b^2}$ and $\frac{2-\theta}{6-2b-8\theta+b\theta+2\theta^2} < k < 1$.

To conduct sensitivity analysis, we take first order derivative and second order derivative on the equilibrium results with respect to k. For the original supply chain and the consumers, we have:

$$\frac{\partial \widehat{w}^{NE*}}{\partial k} < 0, \frac{\partial \hat{p}_{OB}^{NE*}}{\partial k} < 0, \frac{\partial \hat{q}^{NE*}}{\partial k} < 0, \frac{\partial \widehat{D}_{OB}^{NE*}}{\partial k} < 0, \frac{\partial \widehat{\pi}_{OB}^{NE*}}{\partial k} < 0, \frac{\partial \widehat{\pi}_{M1}^{NE*}}{\partial k} < 0, \frac{\partial \widehat{\pi}_{SC,OB}^{NE*}}{\partial k} < 0, \frac{\partial \widehat{CS}_{OB}^{NE*}}{\partial k} < 0.$$

For the knockoff supply chain and the consumers, we have:

$$\frac{\partial \hat{c}^{NE*}}{\partial k} > 0, \frac{\partial \hat{p}_{CC}^{NE*}}{\partial k} > 0, \frac{\partial \widehat{D}_{CC}^{NE*}}{\partial k} > 0, \frac{\partial \widehat{\pi}_{CC}^{NE*}}{\partial k} > 0, \frac{\partial \widehat{\pi}_{M2}^{NE*}}{\partial k} > 0, \frac{\partial \widehat{\pi}_{SC,CC}^{NE*}}{\partial k} > 0, \frac{\partial \widehat{CS}_{CC}^{NE*}}{\partial k} > 0. \qquad \text{(Q.E.D.)}$$

Proof of Proposition 4.6:

Under the EX case, we take first order derivative and second order derivative on the equilibrium results with respect to b and k respectively. For the original supply chain and the consumers, we have:

$$\frac{\partial \widehat{w}^{EX*}}{\partial b} > 0, \frac{\partial \hat{p}_{OB}^{EX*}}{\partial b} > 0, \frac{\partial \hat{q}^{EX*}}{\partial b} > 0, \frac{\partial \widehat{D}_{OB}^{EX*}}{\partial b} > 0, \frac{\partial \widehat{\pi}_{OB}^{EX*}}{\partial b} > 0, \frac{\partial \widehat{\pi}_{M1}^{EX*}}{\partial b} > 0, \frac{\partial \widehat{\pi}_{SC,OB}^{EX*}}{\partial b} > 0, \frac{\partial \widehat{CS}_{OB}^{EX*}}{\partial b} > 0;$$

$$\frac{\partial \widehat{w}^{EX*}}{\partial k} < 0, \frac{\partial \hat{p}_{OB}^{EX*}}{\partial k} < 0, \frac{\partial \hat{q}^{EX*}}{\partial k} < 0, \frac{\partial \widehat{D}_{OB}^{EX*}}{\partial k} < 0, \frac{\partial \widehat{\pi}_{OB}^{EX*}}{\partial k} < 0, \frac{\partial \widehat{\pi}_{M1}^{EX*}}{\partial k} < 0, \frac{\partial \widehat{\pi}_{SC,OB}^{EX*}}{\partial k} < 0, \frac{\partial \widehat{CS}_{OB}^{EX*}}{\partial k} < 0.$$

For the knockoff supply chain and the consumers, we have:

$$\frac{\partial \hat{c}^{EX*}}{\partial b} < 0, \frac{\partial \hat{p}_{CC}^{EX*}}{\partial b} < 0, \frac{\partial \widehat{D}_{CC}^{EX*}}{\partial b} < 0, \frac{\partial \widehat{\pi}_{CC}^{EX*}}{\partial b} < 0, \frac{\partial \widehat{\pi}_{M2}^{EX*}}{\partial b} < 0, \frac{\partial \widehat{\pi}_{SC,CC}^{EX*}}{\partial b} < 0, \frac{\partial \widehat{CS}_{CC}^{EX*}}{\partial b} < 0;$$

$$\frac{\partial \hat{c}^{EX*}}{\partial k} > 0, \frac{\partial \hat{p}_{CC}^{EX*}}{\partial k} > 0, \frac{\partial \widehat{D}_{CC}^{EX*}}{\partial k} > 0, \frac{\partial \widehat{\pi}_{CC}^{EX*}}{\partial k} > 0, \frac{\partial \widehat{\pi}_{M2}^{EX*}}{\partial k} > 0, \frac{\partial \widehat{\pi}_{SC,CC}^{EX*}}{\partial k} > 0, \frac{\partial \widehat{CS}_{CC}^{EX*}}{\partial k} > 0. \qquad \text{(Q.E.D.)}$$

Proof of Proposition 4.7:

Comparing the optimal decisions and demands under the EX case and the NE case, we find:

$$\widehat{w}^{EX*} - \widehat{w}^{NE*} = \frac{bk(-4+\theta)(-1+\theta)(8+\theta(-9+2\theta))}{(-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} > 0,$$

$$\hat{p}_{OB}^{EX*} - \hat{p}_{OB}^{NE*} = \frac{2bk(-3+\theta)(-1+\theta)(8+\theta(-9+2\theta))}{(-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} > 0,$$

$$\hat{q}_{OB}^{EX*} - \hat{q}_{OB}^{NE*} = -\frac{b(-2+\theta)(8+\theta(-9+2\theta))}{(-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} > 0,$$

$$\widehat{D}_{OB}^{EX*} - \widehat{D}_{OB}^{NE*} = -\frac{bk(-2+\theta)(8+\theta(-9+2\theta))}{(-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} > 0,$$

$$\hat{c}^{EX*} - \hat{c}^{NE*} = \frac{bk(-4+\theta)(-2+\theta)(-1+\theta)\theta}{(-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} < 0,$$

$$\hat{p}_{CC}^{EX*} - \hat{p}_{CC}^{NE*} = \frac{2bk(-3+\theta)(-2+\theta)(-1+\theta)\theta}{(-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} < 0,$$

$$\widehat{D}_{CC}^{EX*} - \widehat{D}_{CC}^{NE*} = -\frac{bk(-2+\theta)^2}{(-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} < 0. \qquad \text{(Q.E.D.)}$$

Proof of Proposition 4.8:

From Table A4, we can simply have the following:

$$\triangle\, \widehat{\pi}_{OB} = -\frac{bk^2(-2+\theta)^2(-1+\theta)(8(2+b)-(22+9b)\theta+2(3+b)\theta^2)(8+\theta(-9+2\theta))}{((-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta)))^2},$$

$\triangle\,\hat{\pi}_{M1} = -\frac{bk(-2+\theta)(-2+2k(-4+\theta)(-1+\theta)+\theta)(8(2+b)-(22+9b)\theta+2(3+b)\theta^2)(8+\theta(-9+2\theta))}{2((-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta)))^2}$,

$\triangle\,\hat{\pi}_{SC,OB} = -\frac{bk(-2+\theta)(-2+4k(-3+\theta)(-1+\theta)+\theta)(8(2+b)-(22+9b)\theta+2(3+b)\theta^2)(8+\theta(-9+2\theta))}{2((-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta)))^2}$,

$\triangle\,\widehat{CS}_{OB} =$

$\frac{bk(-2+\theta)^2(2(-2+\theta)\theta(8+\theta(-9+2\theta))+k(b(8+\theta(-9+2\theta))(8+\theta(-13+4\theta))+2(-1+\theta)(-64+\theta(64+\theta(13+\theta(-21+4\theta))))))}{2((-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta)))^2}$.

We find that $\triangle\,\hat{\pi}_{OB}$, $\triangle\,\hat{\pi}_{M1}$, $\triangle\,\hat{\pi}_{SC,OB}$ and $\triangle\,\widehat{CS}_{OB}$ are all positive under the condition where both demands are positive.

Similarly, from Table A5 we can simply have the following:

$\triangle\,\hat{\pi}_{CC} = -\frac{bk(2(-2+\theta)+bk(-2+\theta)+4k(-3+\theta)(-1+\theta))(-2+\theta)^3(-1+\theta)\theta}{((-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta)))^2}$,

$\triangle\,\hat{\pi}_{M2} = -\frac{bk(2(-2+\theta)+bk(-2+\theta)+4k(-3+\theta)(-1+\theta))(-4+\theta)(-2+\theta)^2(-1+\theta)\theta}{((-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta)))^2}$,

$\triangle\,\hat{\pi}_{SC,CC} = -\frac{2bk(2(-2+\theta)+bk(-2+\theta)+4k(-3+\theta)(-1+\theta))(-3+\theta)(-2+\theta)^2(-1+\theta)\theta}{((-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta)))^2}$,

$\triangle\,\widehat{CS}_{CC} = \frac{bk(2(-2+\theta)+bk(-2+\theta)+4k(-3+\theta)(-1+\theta))(-2+\theta)^3\theta}{2((-2+\theta)(8+\theta(-9+2\theta))+k(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta)))^2}$.

We find that $\triangle\,\hat{\pi}_{CC}$, $\triangle\,\hat{\pi}_{M2}$, $\triangle\,\hat{\pi}_{SC,CC}$ and $\triangle\,\widehat{CS}_{CC}$ are all negative under the condition where both demands are positive. Proposition 4.8 is proved. (Q.E.D)

Proof of Lemma A3:

The profit functions for the original brand and the knockoff brand are $\tilde{\pi}_{OB}^{EX} = (\tilde{p}_{OB}^{EX} - \widetilde{w}^{EX})(1 - \frac{h_1+\tilde{p}_{CC}^{EX}+(-1+h_2)\tilde{p}_{OB}^{EX}}{-1+\theta})$, $\tilde{\pi}_{CC}^{EX} = (\tilde{p}_{CC}^{EX} - \tilde{c}^{EX})\frac{\theta h_1+\tilde{p}_{CC}^{EX}+\theta(-1+h_2)\tilde{p}_{OB}^{EX}}{(-1+\theta)\theta}$. We find that $\tilde{\pi}_{OB}^{EX}$ ($\tilde{\pi}_{CC}^{EX}$) is concave in $\tilde{p}_{OB}^{EX}$ ($\tilde{p}_{CC}^{EX}$). By solving the first order derivative of $\tilde{\pi}_{OB}^{EX}$ ($\tilde{\pi}_{CC}^{EX}$) with respect to $\tilde{p}_{OB}^{EX}$ ($\tilde{p}_{CC}^{EX}$), we obtain the optimal selling prices $\tilde{p}_{OB}^{EX*}$ and $\tilde{p}_{CC}^{EX*}$. Substituting them back into the demand functions, we have $\widetilde{D}_{OB}^{EX*}$ and $\widetilde{D}_{CC}^{EX*}$.

Substituting $\widetilde{D}_{OB}^{EX*}$ and $\widetilde{D}_{CC}^{EX*}$ into the profit functions of the manufacturers, we have:

$\tilde{\pi}_{M1}^{EX} = \widetilde{w}^{EX}\frac{2+\tilde{c}^{EX}-2\widetilde{w}^{EX}+(-2+\widetilde{w}^{EX})\theta-(-2+\theta)h_1-\widetilde{w}^{EX}(-2+\theta)h_2}{(-4+\theta)(-1+\theta)}$,

$\tilde{\pi}_{M2}^{EX} = \tilde{c}^{EX}\frac{\tilde{c}^{EX}(-2+\theta)+(1+\widetilde{w}^{EX}-\theta)\theta-\theta(h_1+\widetilde{w}^{EX}h_2)}{(-4+\theta)(-1+\theta)\theta}$.

We find that $\tilde{\pi}_{M1}^{EX}$ ($\tilde{\pi}_{M2}^{EX}$) is concave in $\widetilde{w}^{EX}$ ($\tilde{c}^{EX}$). Solving the first order derivative of $\tilde{\pi}_{M1}^{EX}$ ($\tilde{\pi}_{M2}^{EX}$) with respect to $\widetilde{w}^{EX}$ ($\tilde{c}^{EX}$), we obtain the optimal wholesale prices $\widetilde{w}^{EX*}$ and $\tilde{c}^{EX*}$. Through substitution of the optimal decisions, we summarize the equilibrium results in Tables A6 and A7. Note that the results under the NE case can be simply obtained by setting $h_1 = h_2 = 0$. (Q.E.D.)

Proof of Proposition 4.9:

Under the EX case, we take first order derivative and second order derivative on the equilibrium results with respect to θ, h_1 and h_2 respectively. For the original supply chain and the consumers, we have: $\frac{\partial \widetilde{D}_{OB}^{EX*}}{\partial \theta} > 0, \frac{\partial \widetilde{CS}_{OB}^{EX*}}{\partial \theta} > 0, \frac{\partial \widetilde{w}^{EX*}}{\partial \theta} < 0, \frac{\partial \widetilde{p}_{OB}^{EX*}}{\partial \theta} < 0; \frac{\partial \widetilde{w}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{p}_{OB}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{D}_{OB}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{\pi}_{OB}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{\pi}_{M1}^{EX*}}{\partial h_1} > 0,$ $\frac{\partial \widetilde{\pi}_{SC,OB}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{CS}_{OB}^{EX*}}{\partial h_1} > 0; \frac{\partial \widetilde{w}^{EX*}}{\partial h_2} > 0, \frac{\partial \widetilde{p}_{OB}^{EX*}}{\partial h_2} > 0, \frac{\partial \widetilde{\pi}_{OB}^{EX*}}{\partial h_2} > 0, \frac{\partial \widetilde{\pi}_{M1}^{EX*}}{\partial h_2} > 0, \frac{\partial \widetilde{\pi}_{SC,OB}^{EX*}}{\partial h_2} > 0.$

For the knockoff supply chain and the consumers, we have:

$\frac{\partial^2 \tilde{c}^{EX*}}{\partial \theta^2} < 0, \frac{\partial^2 \widetilde{p}_{CC}^{EX*}}{\partial \theta^2} < 0; \frac{\partial \tilde{c}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{p}_{CC}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{D}_{CC}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{\pi}_{CC}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{\pi}_{M2}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{\pi}_{SC,CC}^{EX*}}{\partial h_1} > 0, \frac{\partial \widetilde{CS}_{CC}^{EX*}}{\partial h_1} > 0.$ (Q.E.D.)

Proof of Proposition 4.10:

Comparing the optimal decisions and demands under the EX case and the NE case, we find:

$$\widetilde{w}^{EX*} - \widetilde{w}^{NE*} = \frac{(-8+(9-2\theta)\theta)h_1-(-1+\theta)(-8+3\theta)h_2}{(16+\theta(-17+4\theta))(-1+h_2)} > 0,$$

$$\widetilde{p}_{OB}^{EX*} - \widetilde{p}_{OB}^{NE*} = -\frac{2(-3+\theta)((8+\theta(-9+2\theta))h_1+(-1+\theta)(-8+3\theta)h_2)}{(-4+\theta)(16+\theta(-17+4\theta))(-1+h_2)} > 0,$$

$$\widetilde{D}_{OB}^{EX*} - \widetilde{D}_{OB}^{NE*} = -\frac{(-2+\theta)(8+\theta(-9+2\theta))h_1}{(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} > 0,$$

$$\tilde{c}^{EX*} - \tilde{c}^{NE*} = \frac{(-2+\theta)\theta h_1}{16+\theta(-17+4\theta)} < 0,$$

$$\widetilde{p}_{CC}^{EX*} - \widetilde{p}_{CC}^{NE*} = \frac{2(-3+\theta)(-2+\theta)\theta h_1}{(-4+\theta)(16+\theta(-17+4\theta))} < 0,$$

$$\widetilde{D}_{CC}^{EX*} - \widetilde{D}_{CC}^{NE*} = -\frac{(-2+\theta)^2 h_1}{(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))} < 0. \qquad \text{(Q.E.D.)}$$

Proof of Proposition 4.11:

From Table A6, we can simply have the following:

$$\triangle\, \widetilde{\pi}_{OB} = \frac{(-2+\theta)^2\left((8-11\theta+3\theta^2)^2+\frac{((-1+\theta)(-8+3\theta)+(8+\theta(-9+2\theta))h_1)^2}{-1+h_2}\right)}{(-4+\theta)^2(-1+\theta)(16+\theta(-17+4\theta))^2},$$

$$\triangle\, \widetilde{\pi}_{M1} = \frac{(-2+\theta)\left((8-11\theta+3\theta^2)^2+\frac{((-1+\theta)(-8+3\theta)+(8+\theta(-9+2\theta))h_1)^2}{-1+h_2}\right)}{(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))^2},$$

$$\triangle\, \widetilde{\pi}_{SC,OB} = \frac{2(-3+\theta)(-2+\theta)((8+\theta(-9+2\theta))h_1(2(-1+\theta)(-8+3\theta)+(8+\theta(-9+2\theta))h_1)+(8-3\theta)^2(-1+\theta)^2h_2)}{(-4+\theta)^2(-1+\theta)(16+\theta(-17+4\theta))^2(-1+h_2)},$$

$$\triangle\, \widetilde{CS}_{OB} = \frac{(-2+\theta)^2h_1(2(-1+\theta)(-64+\theta(64+\theta(13+\theta(-21+4\theta))))+(8+\theta(-9+2\theta))(8+\theta(-13+4\theta))h_1)}{2(-4+\theta)^2(-1+\theta)^2(16+\theta(-17+4\theta))^2}.$$

We find that $\triangle\, \widetilde{\pi}_{OB}$, $\triangle\, \widetilde{\pi}_{M1}$, $\triangle\, \widetilde{\pi}_{SC,OB}$ and $\triangle\, \widetilde{CS}_{OB}$ are all positive under the condition where both demands are positive.

Similarly, from Table A7, we can simply have the following:

$\triangle\,\tilde{\pi}_{CC} = -\frac{(-2+\theta)^3\theta h_1(4(-3+\theta)(-1+\theta)+(-2+\theta)h_1)}{(-4+\theta)^2(-1+\theta)(16+\theta(-17+4\theta))^2}$,

$\triangle\,\tilde{\pi}_{M2} = -\frac{(-2+\theta)^2\theta h_1(4(-3+\theta)(-1+\theta)+(-2+\theta)h_1)}{(-4+\theta)(-1+\theta)(16+\theta(-17+4\theta))^2}$,

$\triangle\,\tilde{\pi}_{SC,CC} = -\frac{2(-3+\theta)(-2+\theta)^2\theta h_1(4(-3+\theta)(-1+\theta)+(-2+\theta)h_1)}{(-4+\theta)^2(-1+\theta)(16+\theta(-17+4\theta))^2}$,

$\triangle\,\widetilde{CS}_{CC} = \frac{(-2+\theta)^3\theta h_1(4(-3+\theta)(-1+\theta)+(-2+\theta)h_1)}{2(-4+\theta)^2(-1+\theta)^2(16+\theta(-17+4\theta))^2}$.

We find that $\triangle\,\tilde{\pi}_{CC}$, $\triangle\,\tilde{\pi}_{M2}$, $\triangle\,\tilde{\pi}_{SC,CC}$ and $\triangle\,\widetilde{CS}_{CC}$ are all negative under the condition where both demands are positive. Proposition 4.11 is proved. (Q.E.D)

Proof of Lemma A4:

The profit functions for the original brand and the knockoff brand are $\bar{\pi}_{OB}^{EX} = (\bar{p}_{OB}^{EX} - \bar{w}^{EX})\frac{-((\delta+\theta)\bar{p}_{CC}^{EX})+\theta(-1-b+\delta+\theta+\bar{p}_{OB}^{EX})}{(-1+\theta)\theta}$, $\bar{\pi}_{CC}^{EX} = (\bar{p}_{CC}^{EX} - \bar{c}^{EX})\frac{(1+\delta)\bar{p}_{CC}^{EX}+\theta(b-\delta-\bar{p}_{OB}^{EX})}{(-1+\theta)\theta}$. We find that $\bar{\pi}_{OB}^{EX}$ ($\bar{\pi}_{CC}^{EX}$) is concave in $\bar{p}_{OB}^{EX}$ ($\bar{p}_{CC}^{EX}$). By solving the first order derivative of $\bar{\pi}_{OB}^{EX}$ ($\bar{\pi}_{CC}^{EX}$) with respect to $\bar{p}_{OB}^{EX}$ ($\bar{p}_{CC}^{EX}$), we obtain the optimal selling prices $\bar{p}_{OB}^{EX*}$ and $\bar{p}_{OB}^{EX*}$. Substituting them back into the demand functions, we have $\bar{D}_{OB}^{EX*}$ and $\bar{D}_{CC}^{EX*}$.

Substituting $\bar{D}_{OB}^{EX*}$ and $\bar{D}_{CC}^{EX*}$ into the profit functions of the manufacturers, we have:

$\bar{\pi}_{M1}^{EX} = \bar{w}^{EX}\frac{\bar{c}^{EX}(1+\delta)(\delta+\theta)-\theta\left(-2+\delta^2+\bar{w}^{EX}(2+\delta-\theta)+(2+\delta)\theta+b(\theta-2-\delta)\right)}{(-1+\theta)\theta(-4-3\delta+\theta)}$,

$\bar{\pi}_{M2}^{EX} = \bar{c}^{EX}\frac{(1+\delta)(\bar{c}^{EX}(2+\delta)-(1-b+\bar{c}^{EX}+\bar{w}^{EX}+\delta)\theta+\theta^2)}{(4+3\delta-\theta)(-1+\theta)\theta}$.

We find that $\bar{\pi}_{M1}^{EX}$ ($\bar{\pi}_{M2}^{EX}$) is concave in $\bar{w}^{EX}$ ($\bar{c}^{EX}$). Solving the first order derivative of $\bar{\pi}_{M1}^{EX}$ ($\bar{\pi}_{M2}^{EX}$) with respect to $\bar{w}^{EX}$ ($\bar{c}^{EX}$), we obtain the optimal wholesale prices $\bar{w}^{EX*}$ and $\bar{c}^{EX*}$. Through substitution of the optimal decisions, we summarize the equilibrium results in Tables A8 and A9. Note that the results under the NE case can be simply obtained by setting $b = 0$. (Q.E.D.)

Proof of Proposition 4.12:

To ensure the coexistence of the original product and knockoff product in the market under both the EX case and the NE case, we have $\bar{D}_{OB}^{EX*} > 0$, $\bar{D}_{CC}^{EX*} > 0$, $\bar{D}_{OB}^{NE*} > 0$ and $\bar{D}_{CC}^{NE*} > 0$. Solving them, we obtain the following condition: $Max\left[\frac{-8+\delta(-5+\delta(2+\delta))+11\theta+7\delta\theta-(3+\delta)\theta^2}{8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta)},0\right] < b < Min\left[4+3\delta-\frac{2(1+\delta)^2}{2+\delta-\theta}-2\theta,1\right]$, $0 < \delta < 1$ and $0 < \theta < \frac{11+7\delta-(1+\delta)\sqrt{25+12\delta+4\delta^2}}{2(3+\delta)}$.

To conduct sensitivity analysis, we take first order derivative and second order derivative on the equilibrium results with respect to θ, b and δ. For the original supply chain and the consumers, we have:

$\frac{\partial \bar{w}^{NE*}}{\partial \theta} < 0$, $\frac{\partial \bar{p}_{OB}^{NE*}}{\partial \theta} < 0$, $\frac{\partial \bar{\pi}_{OB}^{NE*}}{\partial \theta} < 0$; $\frac{\partial \bar{w}^{NE*}}{\partial \delta} < 0$, $\frac{\partial \bar{p}_{OB}^{NE*}}{\partial \delta} < 0$, $\frac{\partial \bar{D}_{OB}^{NE*}}{\partial \delta} < 0$, $\frac{\partial \overline{CS}_{OB}^{NE*}}{\partial \delta} < 0$, $\frac{\partial \bar{\pi}_{OB}^{NE*}}{\partial \delta} < 0$, $\frac{\partial \bar{\pi}_{M1}^{NE*}}{\partial \delta} < 0$, $\frac{\partial \bar{\pi}_{SC,OB}^{NE*}}{\partial \delta} < 0$.

For the knockoff supply chain and the consumers, we have:

$\frac{\partial \bar{D}_{CC}^{NE*}}{\partial \theta} > 0$, $\frac{\partial \overline{CS}_{CC}^{NE*}}{\partial \theta} > 0$, $\frac{\partial^2 \bar{c}^{NE*}}{\partial \theta^2} < 0$, $\frac{\partial^2 \bar{p}_{CC}^{NE*}}{\partial \theta^2} < 0$; $\frac{\partial \bar{D}_{CC}^{NE*}}{\partial \delta} > 0$, $\frac{\partial \bar{\pi}_{CC}^{NE*}}{\partial \delta} > 0$, $\frac{\partial \bar{\pi}_{M2}^{NE*}}{\partial \delta} > 0$, $\frac{\partial \bar{\pi}_{SC,CC}^{NE*}}{\partial \delta} > 0$, $\frac{\partial \overline{CS}_{CC}^{NE*}}{\partial \delta} > 0$, $\frac{\partial^2 \bar{c}^{NE*}}{\partial \delta^2} < 0$. (Q.E.D.)

Proof of Proposition 4.13:

Under the EX case, we take first order derivative and second order derivative on the equilibrium results with respect to θ, b and δ. For the original supply chain and the consumers, we have:

$\frac{\partial \bar{w}^{EX*}}{\partial \theta} < 0$, $\frac{\partial \bar{p}_{OB}^{EX*}}{\partial \theta} < 0$; $\frac{\partial \bar{w}^{EX*}}{\partial b} > 0$, $\frac{\partial \bar{p}_{OB}^{EX*}}{\partial b} > 0$, $\frac{\partial \bar{D}_{OB}^{EX*}}{\partial b} > 0$, $\frac{\partial \bar{\pi}_{OB}^{EX*}}{\partial b} > 0$, $\frac{\partial \bar{\pi}_{M1}^{EX*}}{\partial b} > 0$, $\frac{\partial \bar{\pi}_{SC,OB}^{EX*}}{\partial b} > 0$, $\frac{\partial \overline{CS}_{OB}^{EX*}}{\partial b} > 0$; $\frac{\partial \bar{w}^{EX*}}{\partial \delta} < 0$, $\frac{\partial \bar{p}_{OB}^{EX*}}{\partial \delta} < 0$, $\frac{\partial \bar{D}_{OB}^{EX*}}{\partial \delta} < 0$, $\frac{\partial \bar{\pi}_{OB}^{EX*}}{\partial \delta} < 0$, $\frac{\partial \bar{\pi}_{M1}^{EX*}}{\partial \delta} < 0$, $\frac{\partial \bar{\pi}_{SC,OB}^{EX*}}{\partial \delta} < 0$, $\frac{\partial \overline{CS}_{OB}^{EX*}}{\partial \delta} < 0$.

For the knockoff supply chain and the consumers, we have:

$\frac{\partial^2 \bar{c}^{EX*}}{\partial \theta^2} < 0$, $\frac{\partial^2 \bar{p}_{CC}^{EX*}}{\partial \theta^2} < 0$; $\frac{\partial \bar{c}^{EX*}}{\partial b} < 0$, $\frac{\partial \bar{p}_{CC}^{EX*}}{\partial b} < 0$, $\frac{\partial \bar{D}_{CC}^{EX*}}{\partial b} < 0$, $\frac{\partial \bar{\pi}_{CC}^{EX*}}{\partial b} < 0$, $\frac{\partial \bar{\pi}_{M2}^{EX*}}{\partial b} < 0$, $\frac{\partial \bar{\pi}_{SC,CC}^{EX*}}{\partial b} < 0$, $\frac{\partial \overline{CS}_{CC}^{EX*}}{\partial b} < 0$; $\frac{\partial^2 \bar{c}^{EX*}}{\partial \delta^2} < 0$, $\frac{\partial \bar{D}_{CC}^{EX*}}{\partial \delta} > 0$, $\frac{\partial \bar{\pi}_{CC}^{EX*}}{\partial \delta} > 0$, $\frac{\partial \bar{\pi}_{M2}^{EX*}}{\partial \delta} > 0$, $\frac{\partial \bar{\pi}_{SC,CC}^{EX*}}{\partial \delta} > 0$, $\frac{\partial \overline{CS}_{CC}^{EX*}}{\partial \delta} > 0$. (Q.E.D.)

Proof of Proposition 4.14:

Comparing the optimal decisions and demands under the EX case and the NE case, we find:

$\bar{w}^{EX*} - \bar{w}^{NE*} = \frac{b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))}{16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2} > 0$,

$\bar{p}_{OB}^{EX*} - \bar{p}_{OB}^{NE*} = \frac{2b(3+2\delta-\theta)(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))}{(4+3\delta-\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)} > 0$,

$\bar{D}_{OB}^{EX*} - \bar{D}_{OB}^{NE*} = \frac{b(2+\delta-\theta)(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))}{(-1+\theta)(-4-3\delta+\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)} > 0$,

$\bar{c}^{EX*} - \bar{c}^{NE*} = \frac{b\theta(-2-\delta+\theta)}{16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2} < 0$,

$\bar{p}_{CC}^{EX*} - \bar{p}_{CC}^{NE*} = -\frac{2b(2+\delta-\theta)(3+2\delta-\theta)\theta}{(4+3\delta-\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)} < 0$,

$\bar{D}_{CC}^{EX*} - \bar{D}_{CC}^{NE*} = \frac{b(1+\delta)(2+\delta-\theta)^2}{(4+3\delta-\theta)(-1+\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)} < 0$. (Q.E.D)

Proof of Proposition 4.15:

From Table A8, we can simply have the following:

$$\triangle\,\bar{\pi}_{OB}=\frac{b(2+\delta-\theta)^2(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))(2(-8+\delta(-5+\delta(2+\delta))+11\theta+7\delta\theta-(3+\delta)\theta^2)-b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta)))}{(-1+\theta)(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2},$$

$$\triangle\,\bar{\pi}_{M1}=\frac{b(2+\delta-\theta)(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))(2(-8+\delta(-5+\delta(2+\delta))+11\theta+7\delta\theta-(3+\delta)\theta^2)-b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta)))}{(4+3\delta-\theta)(-1+\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2},$$

$$\triangle\,\bar{\pi}_{SC,OB}=$$

$$\frac{2b(2+\delta-\theta)(3+2\delta-\theta)(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))(2(-8+\delta(-5+\delta(2+\delta))+11\theta+7\delta\theta-(3+\delta)\theta^2)-b(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta)))}{(-1+\theta)(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2},$$

$$\begin{aligned}\triangle\,\overline{CS}_{OB}=\ &(b(2+\delta-\theta)^2(-2(8+\delta(7+\delta))(-8+\delta(-5+\delta(2+\delta)))+4(-64+\delta(-57+\delta(30+\delta(42+\delta(12+\delta)))))\\&\theta-2\Big(-51+\delta\Big(81+\delta\big(186+\delta(89+11\delta)\big)\Big)\Big)\theta^2+4\Big(17+\delta\big(72+\delta(61+14\delta)\big)\Big)\theta^3\\&-2(25+\delta(48+19\delta))\theta^4+8(1+\delta)\theta^5-b(-8-\delta(7+\delta)+13\theta+\delta(11+2\delta)\theta\\&-2(2+\delta)\theta^2)(8+\delta^2+\delta(7-5\theta)+\theta(-9+2\theta))))/\\&(2(-1+\theta)^2(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2).\end{aligned}$$

We find that $\triangle\,\bar{\pi}_{OB}$, $\triangle\,\bar{\pi}_{M1}$, $\triangle\,\bar{\pi}_{SC,OB}$ and $\triangle\,\overline{CS}_{OB}$ are all positive when both demands are positive.

Similarly, from Table A9, we can simply have the following:

$$\triangle\,\bar{\pi}_{CC}=-\frac{b(1+\delta)(2+\delta-\theta)^3\theta\big(b(2+\delta-\theta)-2(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2)\big)}{(-1+\theta)(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2},$$

$$\triangle\,\bar{\pi}_{M2}=\frac{b(1+\delta)(2+\delta-\theta)^2\theta(b(-2-\delta+\theta)+2(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2))}{(4+3\delta-\theta)(-1+\theta)(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2},$$

$$\triangle\,\bar{\pi}_{SC,CC}=-\frac{2b(1+\delta)(2+\delta-\theta)^2(3+2\delta-\theta)\theta(b(2+\delta-\theta)-2(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2))}{(-1+\theta)(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2},$$

$$\triangle\,\overline{CS}_{CC}=\frac{b(1+\delta)^2(2+\delta-\theta)^3\theta(b(2+\delta-\theta)-2(6+\delta(6+\delta)-8\theta-5\delta\theta+2\theta^2))}{2(-1+\theta)^2(-4-3\delta+\theta)^2(16+3\delta(5+\delta)-17\theta-9\delta\theta+4\theta^2)^2}.$$

We find that $\triangle\,\bar{\pi}_{CC}$, $\triangle\,\bar{\pi}_{M2}$, $\triangle\,\bar{\pi}_{SC,CC}$ and $\triangle\,\overline{CS}_{CC}$ are all negative when both demands are positive.

Proposition 4.15 is proved. (Q.E.D)

Appendix H: All Proofs for Chapter 5

Proof of Lemma 5.1:

By checking the second order derivative of π_{BL}^{R} with respect to p_{BL}, we find that π_{BL}^{R} is strictly concave in p_{BL}. Solving the first order derivative of π_{BL}^{R} with respect to p_{BL}, we obtain the optimal retail price for the big logo product: $p_{BL}^{*} = \frac{1}{2}(1 - \lambda - \alpha l + (1 - \alpha)\theta + w_{BL})$. Substituting it back into Eq.(5.1), we have the optimal demand for the big logo product: $D_{BL}^{*} = \frac{1}{2}(1 - \lambda - \alpha l + (1 - \alpha)\theta - w_{BL})$. Substituting D_{BL}^{*} into Eq.(5.3) generates the profit function for the manufacturer $\pi_{BL}^{M} = \frac{1}{2}(w_{BL} - m)(1 - \lambda - \alpha l + (1 - \alpha)\theta - w_{BL})$. After checking the concavity of π_{BL}^{M}, solving the first order derivative of π_{BL}^{M} with respect to w_{BL} generates the optimal wholesale price $w_{BL}^{*} = \frac{1}{2}(1 - \lambda - \alpha l + (1 - \alpha)\theta + m)$. Through substitution of the optimal decisions, we summarize the results in Lemma 5.1. (Q.E.D.)

Proof of Lemma 5.2:

By solving the first order derivative of π_{NL}^{R} with respect to p_{NL}, we obtain the optimal retail price for the no logo product: $p_{NL}^{*} = \frac{1}{2}(1 - \tau + \alpha\phi - (1 - \alpha)k + w_{NL})$. Substituting it back into Eq.(5.6), we have the optimal demand for the no logo product: $D_{NL}^{*} = \frac{1}{2}(1 - \tau + \alpha\phi - (1 - \alpha)k - w_{NL})$. Substituting D_{NL}^{*} into Eq.(5.8) generates the profit function for the manufacturer $\pi_{NL}^{M} = \frac{1}{2}(w_{NL} - m)(1 - \tau + \alpha\phi - (1 - \alpha)k - w_{NL})$. By solving the first order derivative of π_{NL}^{M} with respect to w_{NL}, we obtain the optimal wholesale price $w_{NL}^{*} = \frac{1}{2}(1 - \tau + \alpha\phi - (1 - \alpha)k + m)$. Through substitution of the optimal decisions, we summarize the results in Lemma 5.2. (Q.E.D.)

Proof of Proposition 5.1:

From Lemmas 5.1 and 5.2, we have derived the optimal demands for the big logo and no logo product: $D_{BL}^{*} = \frac{1}{4}(1 - \lambda - \alpha l + (1 - \alpha)\theta - m)$ and $D_{NL}^{*} = \frac{1}{4}(1 - \tau + \alpha\phi - (1 - \alpha)k - m)$. By solving $D_{BL}^{*} > 0$ and $D_{NL}^{*} > 0$ respectively, we obtain $m < m_{BL}$ and $m < m_{NL}$ respectively, where $m_{BL} = 1 - \lambda - \alpha l + (1 - \alpha)\theta$ *and* $m_{NL} = 1 - \tau + \alpha\phi - (1 - \alpha)k$. By solving $m_{BL} > m_{NL}$, we obtain $\lambda - \tau < \xi$, *where* $\xi = (1 - \alpha)(\theta + k) - \alpha(l + \phi)$. (Q.E.D.)

Proof of Proposition 5.2:

By comparing the respective optimal solutions under the big logo case (Lemma 5.1) and the no logo case (Lemma 5.2), we can easily find that only when $A > B$, we have $\pi_{BL}^{R*} > \pi_{NL}^{R*}, \pi_{BL}^{M*} > \pi_{NL}^{M*}$ and $\pi_{BL}^{SC*} > \pi_{NL}^{SC*}$, which implies that showing big logo is more beneficial to the luxury fashion supply chain and its members. By solving $A > B$, we have $\lambda - \tau < \xi, where\ \xi = (1-\alpha)(\theta + k) - \alpha(l + \phi)$. Rearranging terms, we can obtain the remaining results in Proposition 5.2. Taking the first order derivative of ξ with respect to θ, k, l, ϕ and α respectively, we find that $\frac{\partial \xi}{\partial \theta} > 0, \frac{\partial \xi}{\partial k} > 0, \frac{\partial \xi}{\partial l} < 0, \frac{\partial \xi}{\partial \phi} < 0, \frac{\partial \xi}{\partial \alpha} < 0$. (Q.E.D.)

Proof of Lemma 5.3:

By solving the first order derivative of $U[\hat{\pi}_{BL}^{R}]$ with respect to $\hat{p}_{BL}$, we obtain the optimal retail price for the big logo product: $\hat{p}_{BL}^{*} = \frac{1}{2}(1 + \theta - \lambda - (l + \theta)(\beta\sigma + \alpha_O) + w_{\mathrm{BL}})$. Substituting $\hat{p}_{BL}^{*}$ into Eq.(5.1) and take expectation, we obtain the optimal expected demand: $E[\widehat{D}_{BL}^{*}] = \frac{1}{2}(1 + \theta - \lambda - (l + \theta)(\beta\sigma + \alpha_O) - w_{BL})$. We can then derive the expected profit function for the manufacturer: $E[\hat{\pi}_{BL}^{M}] = \frac{1}{2}(\widehat{w}_{BL} - m)(1 + \theta - \lambda - (l + \theta)(\beta\sigma + \alpha_O) - w_{BL})$. Solving the first order derivative of $E[\hat{\pi}_{BL}^{M}]$ with respect to $\widehat{w}_{BL}$ generates the optimal wholesale price for the big logo product. Through substitution of the optimal decisions, we summarize the results in Lemma 5.3. (Q.E.D.)

Proof of Lemma 5.4:

By solving the first order derivative of $U[\hat{\pi}_{NL}^{R}]$ with respect to $\hat{p}_{NL}$, we obtain the optimal retail price for the no logo product: $\hat{p}_{NL}^{*} = \frac{1}{2}(1 - k - \tau + (\phi + k)(\alpha_0 - \beta\sigma) + w_{NL})$. Substituting $\hat{p}_{NL}^{*}$ into Eq.(5.6) and take expectation, we obtain the optimal expected demand: $E[\widehat{D}_{NL}^{*}] = \frac{1}{2}(1 - k - \tau + (\phi + k)(\alpha_0 + \beta\sigma) - w_{NL})$. We can then derive the expected profit function for the manufacturer: $E[\hat{\pi}_{NL}^{M}] = \frac{1}{2}(\widehat{w}_{NL} - m)(1 - k - \tau + (\phi + k)(\alpha_0 + \beta\sigma) - w_{NL})$. Solving the first order derivative of $E[\hat{\pi}_{NL}^{M}]$ with respect to $\widehat{w}_{NL}$ generates the optimal wholesale price for the no logo product. Through substitution of the optimal decisions, we summarize the results in Lemma 5.4. (Q.E.D.)

Proof of Proposition 5.3:

From Lemmas 5.3 and 5.4, we have derived the optimal expected demands for the big logo and no logo product with risk sensitive luxury fashion brand: $E[\widehat{D}_{BL}^{*}] = \frac{1}{4}\left(\hat{A} + G - m\right)$ and $E[\widehat{D}_{NL}^{*}] = \frac{1}{4}\left(\hat{B} + H - m\right)$. By solving $E[\widehat{D}_{BL}^{*}] > 0$ and $E[\widehat{D}_{NL}^{*}] > 0$ respectively, we obtain $m < \widehat{m}_{BL}$ and $m <$

$\widehat{m}_{NL}$ respectively, where $\widehat{m}_{BL} = \hat{A} + G$ *and* $\widehat{m}_{NL} = \hat{B} + H$. By solving $\widehat{m}_{BL} > \widehat{m}_{NL}$, we obtain $\lambda - \tau < (1-\alpha_0)(\theta + k) - \alpha_0(l + \phi) + \beta\sigma(l + \theta - k - \phi)$. (Q.E.D.)

Proof of Proposition 5.4:

By comparing the respective optimal solutions under the big logo case (Lemma 5.3) and the no logo case (Lemma 5.4) with risk sensitive luxury fashion brand, we can easily find that only when $\hat{A} - 3G > \hat{B} - 3H$, we have $U[\hat{\pi}_{BL}^{R*}] > U[\hat{\pi}_{NL}^{R*}]$, which implies that showing big logo is more beneficial to the luxury fashion brand. By solving $\hat{A} - 3G > \hat{B} - 3H$, we have $\lambda - \tau < \xi^R$, *where* $\xi^R = (1-\alpha_0)(\theta + k) - \alpha_0(l + \phi) + 3\beta\sigma(k + \phi - l - \theta)$. Similarly, we notice that only when $\hat{A} + G > \hat{B} + H$, we have $U[\hat{\pi}_{BL}^{M*}] > U[\hat{\pi}_{NL}^{M*}]$, which means that showing big logo is more beneficial to the manufacturer. By solving $\hat{A} + G > \hat{B} + H$, we have $\lambda - \tau < \xi^M$, *where* $\xi^M = (1-\alpha_0)(\theta + k) - \alpha_0(l + \phi) + \beta\sigma(l + \theta - k - \phi)$. (Q.E.D.)

Proof of Proposition 5.5:

Taking the first order derivative of ξ^R with respect to σ, we have $\frac{d\xi^R}{d\sigma} = 3\beta(k - l - \theta + \phi)$. Solving $3\beta(k - l - \theta + \phi) > 0$, we have $(\beta < 0 \; and \; 0 < \frac{\phi - l}{\theta - k} < 1)$ *or* $(\beta > 0 \; and \; \frac{\phi - l}{\theta - k} > 1)$. Similarly, taking the first order derivative of ξ^M with respect to σ, we have $\frac{d\,\xi^M}{d\sigma} = \beta(l + \theta - \phi - k)$. Solving $\beta(l + \theta - \phi - k) < 0$, we have $(\beta < 0 \; and \; 0 < \frac{\phi - l}{\theta - k} < 1)$ *or* $(\beta > 0 \; and \; \frac{\phi - l}{\theta - k} > 1)$. (Q.E.D.)

Proof of Proposition 5.6:

Comparing the thresholds of ξ^R and ξ^M with the one derived under the basic model (i.e., ξ) respectively, we have $\xi^R - \xi = 3\beta\sigma(k + \phi - l - \theta)$ and $\xi^M - \xi = \beta\sigma(l + \theta - k - \phi)$, by solving $\xi^R - \xi > 0$ (or $\xi^M - \xi < 0$), we have $(\beta < 0 \; and \; 0 < \frac{\phi - l}{\theta - k} < 1)$ *or* $(\beta > 0 \; and \; \frac{\phi - l}{\theta - k} > 1)$. (Q.E.D.)

Proof of Lemma 5.5:

By solving the first order derivative of $\ddot{\pi}_{BL}^R$ with respect to $\ddot{p}_{BL}$, we obtain the optimal retail price for the big logo product: $\ddot{p}_{BL}^* = \frac{(1 + t + \ddot{w}_{BL} + \theta - \alpha(l + \theta))}{2} - \frac{\lambda}{2\gamma}$. Substituting it back into Eq.(5.16), we have the optimal demand for the big logo product: $\ddot{D}_{BL}^* = \frac{(1 - t - \ddot{w}_{BL} - l\alpha + (1-\alpha)\theta)}{2} - \frac{\lambda}{2\gamma}$. Substituting $\ddot{D}_{BL}^*$ into Eq.(5.3) generates the profit function for the manufacturer $\ddot{\pi}_{BL}^M = (\ddot{w}_{BL} - m)(\frac{(1 - t - \ddot{w}_{BL} - l\alpha + (1-\alpha)\theta)}{2} -$

$\frac{\lambda}{2\gamma}$). By solving the first order derivative of $\dddot{\pi}_{BL}^{M}$ with respect to $\dddot{w}_{BL}$, we obtain the optimal wholesale price $\dddot{w}_{BL}^{*} = \frac{(1+m-t-\alpha l+(1-\alpha)\theta))}{2} - \frac{\lambda}{2\gamma}$. Through substitution of the optimal decisions, we summarize the results in Lemma 5.5. (Q.E.D.)

Proof of Lemma 5.6:

By solving the first order derivative of $\dddot{\pi}_{NL}^{R}$ with respect to $\dddot{p}_{NL}$, we obtain the optimal retail price for the no logo product: $\dddot{p}_{NL}^{*} = \frac{(1+t+\dddot{w}_{NL}-k(1-\alpha)+\alpha\phi)}{2} - \frac{\tau}{2\gamma}$. Substituting it back into Eq.(5.18), we have the optimal demand for the no logo product: $\dddot{D}_{NL}^{*} = \frac{(1-t-\dddot{w}_{NL}-k(1-\alpha)+\alpha\phi)}{2} - \frac{\tau}{2\gamma}$. Substituting $\dddot{D}_{NL}^{*}$ into Eq.(5.8) generates the profit function for the manufacturer $\dddot{\pi}_{NL}^{M} = (\dddot{w}_{NL} - m)(\frac{(1-t-\dddot{w}_{NL}-k(1-\alpha)+\alpha\phi)}{2} - \frac{\tau}{2\gamma})$. By solving the first order derivative of $\dddot{\pi}_{NL}^{M}$ with respect to $\dddot{w}_{NL}$, we obtain the optimal wholesale price $\dddot{w}_{NL}^{*} = \frac{(1+m-t-k(1-\alpha)+\alpha\phi)}{2} - \frac{\tau}{2\gamma}$. Through substitution of the optimal decisions, we summarize the results in Lemma 5.6. (Q.E.D.)

Proof of Proposition 5.7:

From Lemmas 5.5 and 5.6, we have derived the optimal demands for the big logo and no logo product: $\dddot{D}_{BL}^{*} = \frac{\dddot{A}-m-t}{4} - \frac{\lambda}{4\gamma}$ and $\dddot{D}_{NL}^{*} = \frac{\dddot{B}-m-t}{4} - \frac{\tau}{4\gamma}$. By solving $\dddot{D}_{BL}^{*} > 0$ and $\dddot{D}_{NL}^{*} > 0$ respectively, we obtain $m < \dddot{m}_{BL}$ and $m < \dddot{m}_{NL}$ respectively, where $\dddot{m}_{BL} = \dddot{A} - t - \frac{\lambda}{\gamma}$ *and* $\dddot{m}_{NL} = \dddot{B} - t - \frac{\tau}{\gamma}$. By solving $\dddot{m}_{BL} > \dddot{m}_{NL}$, we obtain $\lambda - \tau < \dddot{\xi}$, *where* $\dddot{\xi} = (1-\alpha)(\theta + k)\gamma - \alpha\gamma(l + \phi)$. (Q.E.D.)

Proof of Proposition 5.8:

By comparing the respective optimal solutions under the big logo case (Lemma 5.5) and the no logo case (Lemma 5.6) with the implementation of blockchain technology, we can easily find that only when $\gamma(\dddot{A} - m - t) - \lambda > \gamma(\dddot{B} - m - t) - \tau$, we have $\dddot{\pi}_{BL}^{R*} > \dddot{\pi}_{NL}^{R*}, \dddot{\pi}_{BL}^{M*} > \dddot{\pi}_{NL}^{M*}$ *and* $\dddot{\pi}_{BL}^{SC*} > \dddot{\pi}_{NL}^{SC*}$, which implies that showing big logo is more beneficial to the luxury fashion supply chain and its members. By solving $\gamma(\dddot{A} - m - t) - \lambda > \gamma(\dddot{B} - m - t) - \tau$, we have $\lambda - \tau < \dddot{\xi}$, *where* $\dddot{\xi} = (1-\alpha)(\theta + k)\gamma - \alpha\gamma(l + \phi)$.

Comparing the threshold of $\dddot{\xi}$ with the one derived under the basic model (i.e., ξ), we have $\frac{\dddot{\xi}}{\xi} = \gamma$, since $\gamma > 1$, it is intuitive that $\dddot{\xi} > \xi$ always holds. Similar to the proof of Proposition 5.2, $\dddot{\xi}$ increases linearly in θ and k while decreases linearly in l, ϕ and α. Taking the first order derivative of $\ddot{\xi}$ with respect to γ, we find that $\frac{\partial \ddot{\xi}}{\partial \gamma} > 0$. (Q.E.D.)