SELLING STRATEGIES SELECTION OF SERVICE GOODS IN TWO-SIDED MARKET

QIANQIAN CHEN

PhD

The Hong Kong Polytechnic University

This programme is jointly offered by The Hong Kong Polytechnic University and Zhejiang University

2022

The Hong Kong Polytechnic University
Department of Logistics and Maritime Studies

Zhejiang University
School of Management

Selling Strategies Selection of Service Goods in Two-sided Market

Qianqian CHEN

A thesis submitted in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

June 2022

CERTIFICATE OF ORIGINALITY

I hereby declare that this thesis is my own work and that, to the best of my knowledge and belief, it reproduces no material previously published or written, nor material that has been accepted for award of any other degree or diploma, except where due acknowledgement has been made in the text.

____________________ (Signed)

Qianqian CHEN (Name of Student)

ABSTRACT

Two-sided market is a marketplace where participants on demand- and supply-side transact with each other through a platform to create economic values. How to select selling strategies to improve the matching efficiency of service goods has become the key to service providers. In this paper, we compare post pricing and k-double auction mechanism, and explore the role of upgrading and opaque selling mechanism in disposing of leftovers. How to manage the pricing, selling strategies selection and resource allocation problems of above mechanisms in two-sided market is the focus of this paper.

Firstly, we compare dynamic pricing with k-double auction in the presence of strategic providers. Under dynamic pricing mechanism, the platform sets prices, customers and providers are matched randomly. Under k-double auction mechanism, the platform announces a matching policy, participants on two sides are matched based on a bidding priority. To capture providers' strategic behaviour, we construct a two-period model in which a sub-game Nash equilibrium is characterized. Results indicate that the lower the demand-supply intensity and the higher the bidding power, the more likely that the platform chooses post pricing. Because the transaction price is higher in post pricing than in k-double auction, customers are better off in k-double auction than in dynamic pricing. Both providers and the whole society are always better off under k-double auction except Buyer's Bid Double Auction. By considering static post pricing, we show that the platform with more pricing flexibility prefers post pricing to k-double auction. And by considering one-period model, we find that providers' strategic behaviour motivates the platform to adopt k-double auction.

Secondly, we examine the relationship between two typical probabilistic mechanisms, upgrading and opaque selling, against a backdrop of vertical differentiated markets. In upgrading mechanism, high-quality capacities are offered as upgrades with a much lower price to customers who have purchased low-quality capacity. In opaque selling mechanism, high- and low-quality capacities are offered as an opaque mix with the same selling price. To capture different roles of two probabilistic mechanisms played in the seller's salvage value generation process, we construct a two-stage model, including a regular and salvage stage, to integrate these two mechanisms into a unified framework. Result shows even though the price discrimination and demand segmentation effect make upgrading dominate opaque selling, the two mechanisms are either complementary or substitutable. That is, the two probabilistic mechanisms are substitutes when high-quality capacity is extremely small when no upgrading platform participates, are either complementary or substitutable when high-quality capacity is in the medium level, or are substitutes when upgrading comes first or complements when opaque selling comes first if high-quality capacity is rather large.

This paper provides managerial insights for service providers and platforms regarding operations decisions, and lays a foundation for the follow-up research on mechanism design in two-sided market.

Acknowledgments

Firstly, I would like to express my sincere gratitude to Prof. Li Jiang, my supervisor in The Hong Kong Polytechnic University, and Prof. Yi Yang, my supervisor in Zhejiang University. Their constructive suggestions and profound knowledge enlighten me a lot in my research study, and their incredibly carefulness and rigorous attitude towards research has influenced me a lot and will guide me to become a qualified scholar in the near future.

Secondly, I greatly acknowledge other academic staff in and out of the Department of Logistics and Maritime Studies of PolyU and the School of Management of Zhejiang University. Their wonderful courses deepens my understanding of professional knowledge. I would like to express my thanks to the Professors who provided kind suggesstions to my research work.

Thirdly, I want to thank all my dear friends and roommates. Thank you for bringing so much happiness and laughter. Without your sincere friendship, I would not have a completed and wonderful journey in my PhD study.

Lastly, I would like to express my deepest thanks to my family for their consistent and unconditional support. Words are powerless to express my special thanks to my boyfriend, thank you for everything.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Recognizing that unused service goods at the end of the selling season has no salvage value, we find that many researchers have examined different pricing mechanisms, such as dynamic pricing, probabilistic selling, strategic stockout, and reservations, to extract consumer surplus as much as possible (Georgiadis and Tang 2014). In this paper, we explore the optimal selling strategies of service goods with limited capacity in two-sided market. In what follows, we first introduce the background, recognize the research gap, then identify the main research questions, and finally point out main conclusions and contributions of this work.

Background

The platform acts as an intermediary to effectively match customers on the demand side with providers on the supply side. It involves in daily life, such as transportation, accommodation, online shopping, catering consumption, investment and financing, etc. The platform has developed rapidly in recent years. For example, Amazon generated net sales of 280.5 billion in 2019, exceeding 102.64 billion of 2017. The transaction volume of Taobao reached 268.4 billion yuan during double 11 in 2019, which is more than 5368 times of the one in 2009. Platforms can be divided into different types based on the main businesses it conducts, such as sharing economy platform (i.e., Uber, Lyft, Airbnb), tourism reservation platform (i.e., Hotwire, Priceline), task rabbit platform (i.e., WeWork), retail platform (i.e., Amazon), and communication platform (i.e., WhatsApp). Among which service platforms play a vital role in the platform economy. Service providers can use their own skills, resources, or creativity to serve customers so as to generate economic values. Because service goods cannot be reproduced and utilized and have low residual value, mitigating demand-supply mismatch is vital in operations management. In this paper, we investigate the optimal selling strategies of service providers, such as the third-party platform or the seller in two-sided market.

Pricing and auction mechanisms are widely used in two-sided market to coordinate demand with supply. For example, Didi Chuxing adopted both auction and pricing mechanism before 2017. In pricing mechanism, drivers have no choice but to accept service assignments from the platform. In auction mechanism, drivers choose whether and when to serve customers. In addition, Yahoo! Shopping and Yahoo! Auction are two independent online platforms for customers to choose. List pricing and NYOP (name-your-own-price) auction mechanism coexist on Priceline. On eBay, the second price sealed bid and post pricing are used. Based on the coexistence of two pricing mechanisms and distinct features of two-sided market, we explore the platform's optimal mechanism between post pricing and k-double auction mechanism in the presence of strategic providers.

Selling strategy with a probabilistic nature has become a common way to dispose of leftovers in online travel agencies. Opaque selling and upgrading are two probabilistic selling mechanisms with capacity offering uncertainties. In opaque selling mechanism, capacities with similar attributes or differentiated quality are sold collectively, and the probability of customers obtaining a specific type of capacity depends on the service provider's decision. Amazon's lucky bag, Germanwings's blind booking, and blind box are typical opaque selling examples. In upgrading mechanism, high-quality capacities are offered to fulfill demand from customers who have purchased low-quality ones, and the probability of customers getting upgraded successfully also depends on the service provider's decision. Upgrades include conditional upgrades, such as eStandby, and last-minute upgrades, such as front-desk upgrading. Both upgrading and opaque selling mechanisms act on the salvage value generation process of service goods. We compare upgrading and opaque selling mechanism in a vertical differentiated market to clarify the relationship between two probabilistic mechanisms in disposing of leftovers.

The two-sided market proposed by (Rochet and Tirole 2003) is the basis for the study of platform economy. In this paper, we first compare post pricing and k-double auction mechanism, and discuss the optimal selling strategy of homogeneous service goods. Then, we compare upgrading and opaque selling mechanism, and discuss the optimal selling strategy of heterogeneous service goods.

Brief Research Overview

Platform economy is also known as two-sided market economy. Many researchers have discussed the impact of peer-to-peer and business-to-business selling mechanisms on customers' participation and service provider's fulfillment strategies (Benjaafar et al. 2022), resource allocation within time and space dimensions, and information disclosure (Jin et al. 2018, Ke et al. 2017) or signaling (Allon et al. 2017). Topics regarding pricing strategy in two-sided market include the exploration of the optimal pricing decision of two-sided market, the optimal contract design (Halaburda et al. 2018), the optimal matching mechanism design (Hu and Zhou 2022), and the choice of open or closed selling strategies of the platform (Hagiu and Wright 2015, Johnson 2020). We compare post pricing and two-sided auction, which differ in both the price determination and the matching policy.

Opaque selling is widely explored in marketing and operations management. Research topics include the economic impact of opaque selling mechanism on platform's operation management (Post 2010), and comparative analysis of opaque selling and other selling mechanisms, such as markdown pricing, advance selling and last-minute selling. Upgrading research topic is mainly about pricing and capacity allocation of upgrades (Ceryan et al. 2018). Recognizing that there are few studies studying probabilistic selling mechanism with vertical differentiation, we integrate opaque selling and upgrading into a unified framework so as to clarify the complementary or substitutable role of these two mechanisms.

Research Questions

Based on the popularity of two-sided market economy and the research gaps, we want to explore the optimal pricing decision, the optimal selling strategy selection, and the optimal resource allocation of service goods in two-sided market.

Firstly, based on the characteristics of two-sided market, we investigate the optimal selling strategy between post pricing and k-double auction in the presence of strategic service providers. We aim to highlight the important role of demand-supply intensity, the platform's pricing flexibility, pricing entity and providers' strategic behavior on the platform's selling strategy selection. The specific research questions are as follows: What are the optimal prices under post pricing and k-double auction? What is the optimal participation behavior of customers and providers under two pricing mechanisms? Which pricing mechanism, dynamic pricing or k-double auction, is more beneficial to the sharing platform, customers, and the whole society? Which parameter, such as the pricing flexibility, the bidding power of customers, or the demand-supply intensity, explains the dominance?

Secondly, recognizing the fact that opaque selling and upgrading both act on the seller's salvage value generation process, we integrate these two probabilistic selling mechanisms into a vertical market, and explore the condition, such as the high-quality capacity level, the platform's participation or the adoption sequence, that permits the complementary or substitutable role of opaque selling and upgrading. The specific research questions are as follows: What is the fundamental difference between pure opaque selling and pure upgrading? Which pure probabilistic selling mechanism is more beneficial to the seller? In terms of the mixed use of two probabilistic mechanisms, are opaque selling and upgrading complements or substitutes? Which parameter, such as the high-quality capacity level, the platform's intervention, or the adoption sequence, that explains the complementary, and/or substitutable role of two probabilistic mechanisms?

Brief Introduction of Research Content

First, for the selling strategy selection between post pricing and k-double auction mechanism in two-sided market, we build a two-period model to capture providers' strategic service fulfillment behavior. Customers arrive in each period while the number of providers is fixed over the selling season. The platform determines a transaction price under post pricing mechanism, and announces a matching policy under k-double auction mechanism. We formulate the interaction between the platform and participants on two sides as a sequential game, and derive the sub-game Nash equilibrium and mixed fulfillment equilibrium by solving the nonlinear optimization problem under each mechanism.

Second, for the selling strategy selection of two probabilistic selling strategies with capacity offering uncertainties, we build a two-stage model to capture the role of probabilistic selling mechanism in the seller's salvage value generation process. The seller with limited capacity makes pricing decisions in the regular stage and cooperates with a third-party platform or not to deal with leftovers in the salvage stage. The platform, if any, makes pricing decisions. Customers coming at each stage are myopic. We compare scenarios with pure and mixed use of opaque selling and upgrading. A rational expectation equilibrium is characterized in opaque selling mechanism, and backward induction is employed to solve a sub-game Nash equilibrium under each scenario.

Research Innovation

As for the work on selling strategies of homogeneous service capacity in two-sided market, we consider strategic service providers who have work flexibility. We also consider k-double auction mechanism, which is in line with the characteristics of multi-party participation in two-sided market. By comparing post pricing and k-double auction mechanism, we identify the key factors including pricing entity, pricing flexibility, demand-supply intensity and providers' strategic behavior that determine the optimal selling strategy of the sharing platform. We extend the model to another two-sided auction so called bid-ask mechanism, to enrich the analysis of auction mechanism in two-sided market.

As for the work on the probabilistic selling strategies of heterogeneous service capacity in two-sided market, we are among the first to compare opaque selling and upgrading with vertical differentiation. By comparing scenarios where opaque selling and upgrading are adopted singly or jointly, we identify that the adoption sequence between two probabilistic mechanisms, the platform's participation and the high-quality capacity level are the key elements that determine the complementary or substitutable role of two mechanisms. We also characterize the optimal capacity offering, the optimal pricing and the optimal resource allocation strategies of limited capacity within both time and channel dimensions.

Structure of Thesis

The reminder of this paper proceeds as follows. Chapter 2 details the model settings and model assumptions, summarizes related literature, characterizes the equilibrium prices under post pricing and k-double auction mechanisms, and uncovers the parameters that explain the dominant pricing mechanism. Chapter 3 introduces the model descriptions of opaque selling and upgrading mechanisms, summarizes related literature, compares these two probabilistic selling mechanisms in pure use and mixed use, and reveals the conditions under which two probabilistic mechanisms are complementary, substitutable, or either complementary or substutitable. Chapter 4 summarizes this paper and points out several avenues for future research. All mathematical proofs are referred to the appendix.

CHAPTER 2

POST PRICING VS. DOUBLE AUCTION IN TWO-SIDED MARKET

2.1 Introduction

The intervention of an online platform is a fundamental characteristic of two-sided market. How to match demand with supply efficiently is a key to the sharing platform. One of the main strategies that the platform adopts is post pricing mechanism, such as Uber's surge pricing and static pricing mechanism. Another strategy is auction mechanism, such as the second price sealed auction on eBay, and double auction in call market. Empirical study reveals that auction mechanism yields a higher likelihood of successful sales while pricing mechanism yields higher transaction prices (Einav et al. 2018). Even though auction achieves a revenue dominance (Hammond 2010), the shift from auctions to post pricing is widely observed. For instance, eBay was a pure online auction site till 2002, after that time post pricing prevails. Didi Chuxing previously allowed drivers' cherry-picking behavior, but now drivers have no choice but to fulfill assignments from the online platform. Motivated by the trend from auction to pricing, we aim to investigate the performance of these two pricing mechanisms from the sharing platform's perspective.

In two-sided market, there are several sellers who hold items for sale or rental and several buyers who consider buying or renting these items. Examples are stock exchanges, used-car markets, emission trading markets, and Internet advertisements. A double auction (DA) is a mechanism for organizing a two-sided market-deciding who will buy, who will sell and at what prices (Segal-Halevi et al. 2018). Double auction (also called two-sided auction) which is widely used in financial markets, call markets and automated control (McAfee 1992, Chu and Shen 2006) can incorporate the dynamic interaction between customers and providers, and the intermediary role of the platform as well. For markets with finitely many traders, the best-known mechanism of this kind is the k-DA, such as the uniform k-DA or discriminatory k-DA, which provides an explicit formula for calculating market-clearing prices (Jantschgi et al. 2022). Choosing a mechanism with the most economic efficiency is not the focus of our paper. Instead, we compare post pricing with k-double auction in main analysis. The price setting entity is shifted from the platform in pricing mechanism to participants on two sides in auction mechanism, and the corresponding matching rule is shifted from random rationing to priority bidding.

Recognizing that researchers have separately examined the economic impact of pricing in two-sided market and double auction mechanism design in stock markets, we aim to uncover whether there is a dominant selling strategy between post pricing and k-double auction in the presence of strategic providers. Questions we want to pursue are as follows:

(i) Which pricing mechanism, post pricing or k-double auction, is more beneficial to the sharing platform, customers, and the whole society? Which parameter (i.e., demand-supply intensity, bidding power, pricing flexibility, or providers' strategic behavior) explains the dominance?

(ii) What is the optimal pricing policy for the sharing platform under post pricing and k-double auction?

(iii) Does providers' fulfillment equilibrium strategy differ in two mechanisms? Do the optimal transaction prices and the optimal transaction volumes differ in two mechanisms?

To answer these questions, we build a two-period model in which a pool of providers are available over two periods, while a stream of customers arrive in each period. In post pricing mechanism, the platform sets prices, service providers decide whether and when to work, and customers at the same time choose whether to participate. Customers and providers are randomly rationed. In k-double auction mechanism, the platform announces the matching and price determination rules before the start of the selling season. Providers who plan to fulfill demand at that time period and customers propose bids simultaneously.

The higher the customer's (resp., provider's) bid, the higher (resp., lower) the matching probability. We formulate the interaction between the platform and participants on two sides as a sequential game, and backward induction is employed to seek the subgame perfect Nash equilibria. Moreover, a mixed strategy[1] is employed to characterize service providers' demand fulfillment decision.

We have the following main insights: (a) The platform's pricing flexibility, customers' bidding power, demand-supply intensity and providers' strategic behavior all play vital roles in influencing platform's strategy selection between post pricing and k-double auction. Specifically, if the demand-supply intensity is low, and the bidding power is high, then the likelihood of using post pricing is high. If the platform owns more pricing flexibility, then the platform is more likely to use post pricing. And providers' strategic behavior is a strong incentive for the platform to use k-double auction. (b) Because the transaction price is higher under both dynamic and static pricing than under k-DA, hence, only customers are better off under k-DA than under post pricing. Both providers and the whole society are better off under SODA and GDA. Under BBDA, providers are worse off for the price distortion effect is more evident than the demand expansion effect in BBDA, and the whole society is better off only when there are few last-minute customers and more regular customers. (c) In k-DA, no matter what the transaction price is (i.e., a market-clearing price in uniform k-DA, and prices vary with matching pairs in discriminatory k-DA), providers (resp., customers) bid at their discounted reservation prices (resp., valuations) if customers have (resp., do not have) the bidding power. In addition, the optimal linear bidding policies on demand and supply sides show the same structure in general case in uniform k-DA.

The reminder of this study proceeds as follows. The following part of this section introduces related works. Section 2.2 details the model setting. Section 2.3 and 2.4 are devoted to equilibrium characterization in dynamic pricing and k-double auction, respec-

1. A mixed strategy equilibrium is defined as the probability of joining in period one such that given one provider's optimal participation strategy, the best response of other providers is also to choose the same probability of participation.

tively. Section 2.5 compares two mechanisms and section 2.6 uncovers two extensions. Section 2.7 summarizes this work and points out several avenues for future research. All mathematical proofs are referred to the appendix.

2.1.1 Literature Review

Our work relates to three streams of literature: dynamics of sharing economy, double auction mechanism design, and mechanisms comparison between pricing and auction.

An extensive literature examines the dynamics of sharing economy. A body of this research explores the economic impact of sharing economy on sellers' operation strategies. (Benjaafar et al. 2021) investigate the impact of car owners choosing to share their cars through the platform (i.e., business-to-customer service, under which full-time drivers fulfill demand even though it is not driven by their personal needs) or not (i.e., peer-to-peer service, under which drivers share cars with riders when they fulfill their own transportation needs) on the congestion effect, and they claim that if the ratio of the ownership to usage is low (resp., high), peer-to-peer (resp., business-to-customer) service emerges in equilibrium. (Benjaafar et al. 2019) explore the impact of collaborative consumption on consumers' endogenous choice between renters and owners. They claim that low-use individuals choose to be renters and high-use individuals choose to be owners. (Jiang and Tian 2018) and (Tian and Jiang 2018) study the impacts of consumer-to-consumer product sharing on consumer's owning or renting choice and retailers and manufacturers' profits, respectively. They conclude that product sharing can be a win-win or lose-lose situation depending the transaction cost (Jiang and Tian 2018) or the capacity building cost (Tian and Jiang 2018).

Another body of this research investigates the role of the sharing platform in balancing demand and supply over time and space. (Taylor 2018) studies how delay sensitivity and agent independence impact the platform's per-service price and wage. With uncertainty incorporated neither in the customers' valuation or in agents' opportunity cost, price decreases while wage increases with the delay sensitivity. While incorporated uncertainty reverses the results. (Bai et al. 2019) use a queueing model to figure out the optimal price, wage and payout ratio when considering price and waiting-time sensitive consumers and earnings sensitive providers. They claim that the optimal price increases with demand while is not monotonic with capacity or waiting cost.(Hu and Zhou 2020) study the performance of the fixed commission contract. (Gurvich et al. 2019) examine how a firm balance the tradeoff between maintaining an adequate pool of agents on a long-term basis and attracting enough agents for each time interval over a short time horizon. The staffing problem is modeled as a newsvendor problem, and the optimal decision is a variant of the critical-fractile equation. (Bimpikis et al. 2019) derive the pricing and compensation strategy which facilitates the demand-supply matching in space, and they show that the price and compensation achieve maximum when balanced demand pattern arises over the network's locations. Other topics include regulation control (Benjaafar et al. 2022, Yu et al. 2020), information asymmetry (Jin et al. 2018, Ke et al. 2017, Allon et al. 2017), matching mechanism design (Hu and Zhou 2022) and competition (Cohen and Zhang 2017) are also explored.

All these papers consider a single pricing scenario, while we explore the implications of sharing platform by unifying post pricing (i.e., dynamic and static pricing) with double auction (i.e., uniform and discriminatory k-double auction) in two-sided market.

The following papers relate to our work regarding dynamic pricing in two-sided market. (Cachon et al. 2017) consider a two-period model to investigate the performance of surge pricing mechanism. Self-scheduled providers make joining decision before demand realization or cost revelation in the first period, and make participation decision in the second period. Result implies that both customers and providers are better off under surge pricing. (Guda and Subramanian 2019) investigate the optimal time and space to implement surge pricing. They consider a two-period model in which customers and providers interact with the platform over two market zones. They show that surge pricing employed with excess supply motivates workers to move across zones. No research work except (Chen and Hu 2020) considers participants' forward-looking behavior, they examine whether a fixed or dynamic pricing policy is optimal in two-sided market. They confirm managerial insights of (Cachon et al. 2017). In contrast, we are among the first to study the strategic interactions among customers, providers and the sharing platform in a dynamic environment featured with providers' strategic demand fulfillment.

Our work also connects to the literature on double auction mechanism design. A lot of papers explore the efficient double auction mechanism that satisfies the properties of individual rationality, balanced budget, incentive compatibility and economic efficiency (McAfee 1992, Chu and Shen 2006, Chu and Shen 2007, 2008). Other research works regarding k-double auction focus on the existence and the rate of convergence to efficiency of equilibria. For instance, (Williams 1991) studies buyer's bid double auction with $k = 1$. (Satterthwaite and Williams 1989a) consider k-double auction with a single buyer and a single seller. (Rustichini 1990) generalize the model to k-double auction and extend the analysis to unequal number of customers and sellers. Results show that traders' bids converge to their reservation values when the number of traders is rather large. Mechanism design is not the focus of our paper, we consider k-double auction and bid-ask mechanism, which are feasible in capturing two-sided market features. Our aim is to explore the underlying incentive that explains the dominance of post pricing/k-double auction.

Our paper is built off of the literature on auction and pricing comparison. One part of this stream compares auction and posted price theoretically. (Wang 1993) compares posted pricing mechanism (which involves displaying cost) and second-price sealed bid auction (which involves storing and displaying cost) in an offline market. He proves that auctioning dominates pricing when conducting auction is costless for displaying is usually cheaper than storing. Moreover, if auction cost is considered, auctions are more attractive in occasions of more dispersed valuation distribution. (Ziegler and Lazear 2003) focus on the role of customers' waiting cost and product's discount rate on the seller's revenues. They claim that stores are superior than auctions when products deteriorate quickly. (Etzion et al. 2006) explore the optimal design of dual channel (i.e., posted price, auction duration and quantity) where posted price and seal-bid $q + 1$-price auction are employed online simultaneously. They prove that customers with valuation lower than the posed price have no choice but to bid and those with high valuation can choose between bidding and pricing based on a threshold policy. With this market segmentation, short one-unit auctions and long multi-unit auctions are optimal. They also claim that joint adoption dominates single post pricing mechanism. Different from (Etzion et al. 2006), (Caldentey and Vulcano 2007) consider supply scarcity and a multiplicative utility function instead of an additive one. They examine how customers behave when the seller only controls auction format or manages auction and post price channels. They find that the number of units offered to sale is a nonmonotonic function of the posted price if the seller controls only the auction format, and dual channel reduces to single posted price channel if the seller's capacity is small or the discount factor is rather large.

Another part of this stream conducts numerical analysis. A set of empirical papers show that the dominance may depends on market shareholders and product features, such as research cost in pricing and revenue risk together with monitoring cost in auction (Zeithammer and Liu 2006) and sellers with valuable outside options (Bauner 2015). (Hammond 2013) also shows that seller's choice of mechanisms is based on the opportunity cost of selling, sellers with a valuable alternative use favor the posted-price mechanism. (Einav et al. 2018) prove that customers' inconvenience cost and reservation utility have important effects on the seller's mechanism choice.

Our paper falls into the selling strategies comparison, customers make separate participation decisions in two pricing strategies. To our knowledge, we are the first to prove the importance of the pricing entity, demand-supply intensity, pricing flexibility and providers' strategic behavior in shaping the performance of post pricing and k-double auction mechanism.

2.2 Model Setup

Consider a two-period model in which the sharing platform chooses two selling strategies, post pricing or k-double auction, to maximize its profit. Participants include customers on the demand side and providers on the supply side are payoff maximizers. Each customer (she) submits one fulfillment request, and each provider (he) serves one customer during each time period.

Customers' Behavior

Customers are heterogeneous in valuation v, which is uniformly distributed over interval $[0, 1]$, with cumulative distribution function (cdf) $G(.)$ and corresponding probability density function (pdf) $g(.)$. This heterogeneous valuation can be interpreted as the maximum price the sharing platform can post in post pricing mechanism and the maximum price that customers can bid in double auction mechanism. To capture the feature that multiple transactions take place on the sharing platform during a time period, we consider m_i customers joining in each period, where the subscript $i \in \{1, 2\}$ indicates the time period. Here m_i can be regarded as the number of customers who would like to request the service when it is offered for free. We consider varied demand across periods (i.e., m_1 and m_2 are not always equal), because demand depends on multiple sources of variability, such as weather, holidays, or- morning and evening peak periods (Hu and Zhou 2020). If a customer joins the platform and being matched successfully, she will incur a utility which is defined as the difference between the valuation and the service payment. If she turns to other alternatives, the corresponding utility is normalized to zero. Customers are price-takers in post pricing and bidders in double auction mechanism. Unmatched customers at the end of each period are lost.

Service Providers' Behavior

Providers are heterogeneous in their reservation price c, which is uniformly distributed over interval $[0, 1]$, and the cdf and pdf are denoted by $F(.)$ and $f(.)$, respectively. Here the reservation price represents the minimum price that providers are willing to serve for the platform (i.e., the cost of participation). There are n providers available on the platform over two periods. The supporting evidence is the government's regulatory policy, there always exists a finite pool of independent drivers in ride-hailing industry (J. J. Yu et al. 2020). The relationship between demand across periods is given by $m_2 = \delta m_1$, and the demand-supply relationship is captured by $n = \beta m_1$, where $\beta > \frac{1}{2}$ to ensure equilibrium exists and $\delta > 0$. If a provider participates and fulfills demand successfully, he obtains a utility which is defined as the payment from the platform less than the reservation value. Unmatched providers at the end of period one are left to the second period, while unmatched providers at the end of the selling season are lost. Providers make fulfillment decisions based on the transaction prices and the successful matching probabilities, and a symmetric mixed equilibrium such that all participating providers are indifferent between in serving in period one and period two can be realized.

The Sharing Platform's Role

The sharing platform determines a commission rate γ, where $\gamma \in (0, 1]$, for each successful transaction. For instance, Uber and StubHub charge service providers a percentage fee (usually 15%-25%) for each successful matching.

(a) Post pricing mechanism.

The platform sets price p_i at the beginning of period i in dynamic pricing. Random rationing rule is employed in demand-supply mismatch (Hu and Zhou 2020, Cachon et al. 2017). We assume that customers are aggregated over a certain time period, and transactions take place at the end of that time period.

(b) Double auction mechanism.

The platform determines a matching policy including the matching and the transaction price determination rules, in which customers along with providers' bids are ordered in an ascending order, a market-clearing price equals the convex combination of the $m-th$ and $(m+1)-th$ lowest bidding/asking price. Matchings take place among customers with bidding prices no less than the market-clearing price and providers with asking prices no more than this price. Specifically, the customer with the highest bidding price is matched with the provider with the lowest asking price, the customer with the second highest bidding price is assigned to the provider with the second lowest asking price, so on and so forth. The matching process terminates when there are lack of customers or providers.

To conduct game-theoretic analysis, we approximate that sales can occur in fractions. Moreover, we call customers (resp., providers) planning to join, available to be matched and being matched successfully as potential, effective and successful demand (resp., supply), respectively. The number of successful matches is defined as the minimum number of effective demand and effective supply. We assume that the sharing platform employs either pure dynamic pricing (abbreviated as DP) or pure double auction (abbreviated as DA) scenario. In what follows, we analyze two scenarios subsequently.

2.3 Dynamic Pricing

The sequence of events is summarized as follows, as is also depicted in Figure 2.1: First the sharing platform declares a price at the beginning of period one, upon observing the posted price, customers make participation decisions and providers decide whether to serve or to wait till the next period based on a mixed strategy. Unmatched providers linger in the system while unmatched customers are lost. At the beginning of period two, the platform sets another price, unmatched providers and providers waiting till this time period, and newly come customers make participation decisions simultaneously. The unmatched on both sides leave the market at the end of period two.

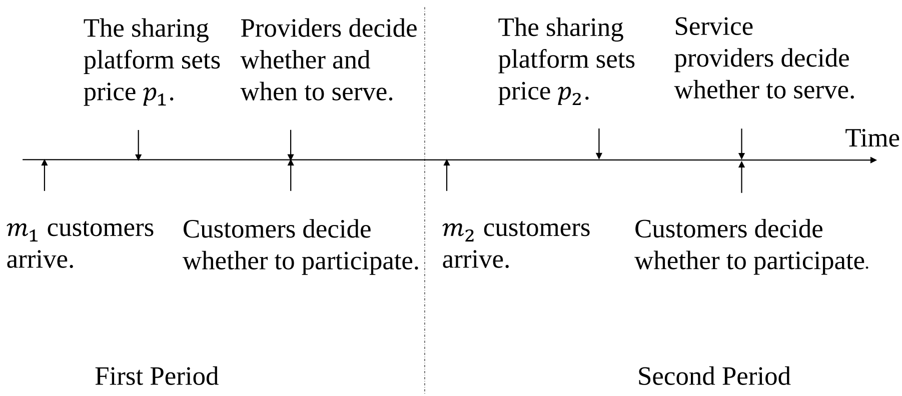

Figure 2.1: Sequence of Events in Dynamic Pricing Mechanism

Customers will seek service upon arrivals if $v - p_i \geq 0$, then the effective demand equals $m_i\bar{G}(p_i)$. Providers will participate in period i if $\gamma p_i - c \geq 0$. The demand fulfilling decision of the supply side is described as: $nF(\gamma p_1)$ providers make decisions about when to serve upon observing price p_1. Suppose that the fraction of providers choosing period one is given by α, then providers including $nF(\gamma p_1)(1-\alpha)$ who choose period two, $nF(\gamma p_1)\alpha - \min\{nF(\gamma p_1)\alpha, m_1\bar{G}(p_1)\}$ unmatched ones and $n\bar{F}(\gamma p_1)$ who have a negative payoff when participating in period one defer their service fulfillments. Denote $nF(\gamma p_1)\alpha$ by s, we have supply-demand state $(s, m_1\bar{G}(p_1))$ in period one and $((n-\min\{s, m_1\bar{G}(p_1)\})F(\gamma p_2), m_2\bar{G}(p_2))$ in period two. Then, the indifference function of providers is described as equation 2.1:

$$\min\{1, \frac{m_1\bar{G}(p_1)}{s}\}\gamma p_1 = \min\{1, \frac{m_2\bar{G}(p_2)}{(n-\min\{s, m_1\bar{G}(p_1)\})F(\gamma p_2)}\}\gamma p_2. \tag{2.1}$$

The probability of being matched successfully is defined as the fraction of effective customers over effective providers, which is no more than one. No heterogeneity is considered in shaping providers' fulfillment decisions. This is in line with the concern that once providers choose to join the sharing platform, they give up outside option, that is, the participation cost is a sunk cost no matter which period they will serve.

2.3.1 Optimization Problem of Period Two

Given providers' participation behavior and price p_1, the profit function for the sharing platform in the second period equals:

$$\pi_2^{DP}(p_2) = (1-\gamma)p_2 \min\{(n-\min\{s, m_1\bar{G}(p_1)\})F(\gamma p_2), m_2\bar{G}(p_2)\}. \tag{2.2}$$

The following lemma characterizes the platform's optimal price and profit of period two. Lemma 2.3.1(Optimal Solutions of Period Two) Given price p_1 and the number of providers s joining in period one, the platform's optimal price of the second period equals

$$p_2^* = \begin{cases} \frac{1}{2} & if (n-\min\{s, m_1\bar{G}(p_1)\})\gamma > m_2, \\ \frac{m_2}{m_2+\gamma(n-\min\{s, m_1\bar{G}(p_1)\})} & otherwise. \end{cases}$$

The platform's optimal profit equals

$$\pi_2^*(p_2^*) = \begin{cases} \frac{(1-\gamma)m_2}{4} & if (n-\min\{s, m_1\bar{G}(p_1)\})\gamma > m_2, \\ \frac{(\gamma-\gamma^2)m_2^2(n-\min\{s, m_1\bar{G}(p_1)\})}{(m_2+\gamma(n-\min\{s, m_1\bar{G}(p_1)\}))^2} & otherwise. \end{cases}$$

Lemma 2.3.1 reveals that when the effective demand is less than the effective supply, then it is optimal for the platform to propose price $\frac{1}{2}$ to extract the maximum surplus from the demand side. Otherwise, the platform increases the posted price to shut a proportion of customers out of the market.

2.3.2 Optimization Problem of Period One

With the optimal solution of period two, the sharing platform announces price p_1 to maximize the total profit π^{DP} over two periods:

$$\pi^{DP}(p_1) = (1-\gamma)p_1 \min\{s, m_1\bar{G}(p_1)\} + \pi_2^*(p_2^*). \tag{2.3}$$

Proposition 2.3.2 summarizes the optimal pricing policy under dynamic pricing.

Proposition 2.3.2 (Optimal Strategy Under Dynamic Pricing Mechanism)

(i) If $m_2 > \gamma n - \frac{\gamma m_1}{2}$, then the optimal strategy is a stable pricing policy $p_1^* = p_2^* = A_1^{DP} > \frac{1}{2}$, the optimal fraction of providers joining in period one equals $\alpha^* = \frac{m_1^2}{m_2 n}(A_1^{DP} + \frac{n}{m_1} - 1)$, and the number of successful transactions of two periods equal $D_1^* = s^* = m_1 A_2^{DP}$ and $D_2^* = m_2 A_2^{DP}$.

(ii) If $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \leq m_2 \leq \gamma n - \frac{\gamma m_1}{2}$, then the optimal strategy is a stable pricing policy $p_1^* = p_2^* = \frac{1}{2}$, the optimal service fulfillment decision equals $\alpha^* = \frac{2m_1 n - m_1^2}{2m_2 n}$, the equilibrium number of providers choosing period one equals $s^* = \frac{2\gamma m_1 n - \gamma m_1^2}{4m_2}$, and the number of successful transactions of period i equals $D_i^* = \frac{m_i}{2}$.

(iii) If $0 < m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$, then the optimal strategy is a markup pricing policy $p_1^* = A_3^{DP} < p_2^* = \frac{1}{2}$, the equilibrium fraction of providers joining in period one equals $\alpha^* = \frac{2m_1^2}{m_2 n}(A_3^{DP} + \frac{n}{m_1} - 1 - \frac{m_2}{2\gamma m_1})$, and the number of successful transactions of two periods equal $D_1^* = s^* = m_1 A_4^{DP}$ and $D_2^* = \frac{m_2}{2}$.

Proposition 2.3.2 shows that if the second-period demand is much lower than the first-period demand ($\delta < \frac{\gamma}{2} - \frac{\gamma}{4\beta}$),then a mark-up pricing policy is employed (Area (III) in Figure 2.2). Otherwise, a stable pricing policy is employed (Areas (I) and (II) in Figure

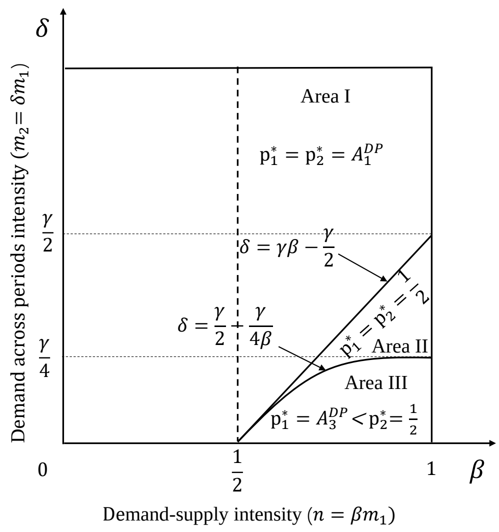

Figure 2.2: The Optimal Pricing Strategy of Dynamic Pricing

2.2). If supply is less than the realized market demand, then the supply is completely cleared, this price price is referred to as the supply clearance price. If supply is more than the realized market demand, then there will be some unmatched providers. In this case, the price will drop to the level such that all customers will be served, and the price is so called as the maximum demand coverage price.

Part (i) of Proposition 2.3.2 reveals that all providers choosing to fulfill demand in period one are matched successfully. The total number of providers leaving the market equals $n - (m_1 + m_2)A_2^{DP}$. The number of customers seeking outside options in period i equals $m_i(1 - A_2^{DP})$. In equilibrium, the transaction volumes over two periods depend on the effective supply. If the number of customers at a certain time period increases, then competition on the demand side is fierce. So, the platform will raise price to retain customers with high valuation so as to match providers with customers who value the service most ($\frac{\partial p_i^*}{\partial m_i} > 0$). High prices also attract more providers to join because of anticipation effect ($\frac{\partial s^*}{\partial m_1} > 0$, $\frac{\partial s^*}{\partial m_2} < 0$). In the other period, in order to retain more providers, the platform increases prices correspondingly. When the total supply increases, the competition effect on the demand side is mitigated because of the enlarged providers pool, so, prices decrease ($\frac{\partial p_i^*}{\partial n} < 0$).

Part (ii) indicates that there are unmatched participants on both demand- and supply side over two periods. The number of unmatched providers equals $\frac{2\gamma m_1 n-\gamma m_1^2-2m_1m_2}{4m_2}$ in period one and equals $n-\frac{m_1+m_2}{2}$ in period two. The number of unmatched customers equals $\frac{m_i}{2}$ in period i. In equilibrium, the transaction volumes over two periods depend on the effective demand. So, the platform charges the price that extracts the maximum surplus from customers.

Part (iii) highlights that all providers choosing to fulfill demand in period one are matched successfully. The total number of unmatched providers equals $n-(m_1A_4^{DP}+\frac{m_2}{2})$. The number of customers seeking outside options in period one (two) equals $m_1(1-A_4^{DP})$ ($\frac{m_2}{2}$). In equilibrium, the transaction volume of period one depends on the effective supply while the one of period two depends on the demand side. So the impact of the market size on the posted price of period one and period two can be explained by the logic described in Part (i) and Part (ii), respectively.

Proposition 2.3.2 also indicates that the larger the market size of period two, the lower the number of providers choosing to fulfill demand in period one: $s^{(i)} < s^{(ii)} < s^{(iii)}$. The higher the posted price in period one, the higher the probability of providers choosing period one: $\alpha^{(i)} > \alpha^{(ii)} > \alpha^{(iii)}$. Moreover, the number of unmatched providers (customers) becomes larger and larger when there are fewer and fewer providers available on the platform, that is, $n-(m_1+m_2)A_2^{DP} > n-\frac{m_1+m_2}{2} > n-(m_1A_4^{DP}+\frac{m_2}{2})$. The total unmatched probability, which is defined as the ratio of all unmatched participants over all participants, decreases as n increases ($\frac{n+(m_1+m_2)(1-2A_2^{DP})}{n+m_1+m_2} > \frac{n}{n+m_1+m_2} > \frac{n+m_1(1-2A_4^{DP})}{n+m_1+m_2}$).

Detailed analysis that explains the impacts of model parameters on equilibrium outcomes (price and demand) is summarized as Corollary 2.3.1.

Corollary 2.3.1

(i) If $m_2 > \gamma n-\frac{\gamma m_1}{2}$, then the impact of m_1, m_2, n and γ on the optimal prices and transaction volumes: $\frac{\partial p_i^*}{\partial m_1} > 0$, $\frac{\partial p_i^*}{\partial m_2} > 0$, $\frac{\partial p_i^*}{\partial n} < 0$, and $\frac{\partial p_i^*}{\partial \gamma} < 0$; $\frac{\partial D_1^*}{\partial m_1} > 0$, $\frac{\partial D_1^*}{\partial m_2} < 0$, $\frac{\partial D_1^*}{\partial n} > 0$, $\frac{\partial D_1^*}{\partial \gamma} > 0$, and $\frac{\partial D_2^*}{\partial m_1} > 0$, $\frac{\partial D_2^*}{\partial m_2} > 0$, $\frac{\partial D_2^*}{\partial n} > 0$ and $\frac{\partial D_2^*}{\partial \gamma} > 0$.

(ii) If $\frac{\gamma m_1}{2}-\frac{\gamma m_1^2}{4n} \leq m_2 \leq \gamma n-\frac{\gamma m_1}{2}$, then the optimal prices are independent of those

platform parameters, and the optimal transaction volume only increases with m_i: $\frac{\partial D_i^*}{\partial m_i} > 0$.

(*iii*) If $m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$, then the impact of m_1, m_2, n and γ on p_1^* and the optimal transaction volumes: $\frac{\partial p_1^*}{\partial m_1} > 0$, $\frac{\partial p_1^*}{\partial m_2} > 0$, $\frac{\partial p_1^*}{\partial n} < 0$ and $\frac{\partial p_1^*}{\partial \gamma} < 0$; $\frac{\partial D_1^*}{\partial m_1} > 0$, $\frac{\partial D_1^*}{\partial m_2} < 0$, $\frac{\partial D_1^*}{\partial n} > 0$, $\frac{\partial D_1^*}{\partial \gamma} > 0$ and $\frac{\partial D_2^*}{\partial m_2} > 0$.

We also summarize how fulfillment equilibrium change with parameters as Corollary 2.3.2.

Corollary 2.3.2

(i) The impact of m_1, m_2, n and γ on the fulfillment equilibrium is described as: $\frac{\partial s^*}{\partial m_1} > 0$, $\frac{\partial s^*}{\partial m_2} < 0$, $\frac{\partial s^*}{\partial n} > 0$, and $\frac{\partial s^*}{\partial \gamma} > 0$.

(ii) The impact of m_1, m_2, n and γ on α^*: $\frac{\partial \alpha^*}{\partial m_2} < 0$; $\frac{\partial \alpha^*}{\partial n} > 0$; $\frac{\partial \alpha^*}{\partial m_1} > 0$ if $m_2 \geq \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$ and $m_1 < n$ or $\gamma n - \frac{\gamma n^2}{2m_1} < m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$; $\frac{\partial \alpha^*}{\partial \gamma} < 0$ if $m_2 > \gamma n - \frac{\gamma m_1}{2}$, $\frac{\partial \alpha^*}{\partial \gamma} = 0$ if $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \leq m_2 \leq \gamma n - \frac{\gamma m_1}{2}$, or $\frac{\partial \alpha^*}{\partial \gamma} > 0$ if $\gamma n - \frac{\gamma n^2}{2m_1} < m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$.

Corollary 2.3.2 indicates that large number of regular (last-minute) customers leads to large (small) number of providers in period one, this is due to the positive network effect. And because of the market expansion effect induced by the labour pool size increment, or by the payment increment from the sharing platform, the number of providers serving in period one also increases.

As for the impact of system parameters on the fulfillment probability, the logic behind $\frac{\partial \alpha^*}{\partial m_2} < 0$ and $\frac{\partial \alpha^*}{\partial n} > 0$ is the same as the one regarding s^*. When there are more and more customers in period one, both the number of providers choosing period one and the optimal posted price increase. When the number of regular customers exceeds the total supply, the effective supply is more than the effective demand, providers will find that the probability of being matched successfully decreases, so, they are more willing to wait for the second period.

2.4 K-Double Auction

In uniform k-double auction, all bidding and asking prices at a specific time period are aggregated and ordered in an ascending order, then the order statistics of m customers along with n providers' bids is described as $T_{(1)} \leq ... \leq T_{(m)} \leq T_{(m+1)} \leq ... \leq T_{(m+n)}$, the transaction price equals $(1-k)T_{(m)}+kT_{(m+1)}$, where $k \in [0, 1]$. Parameter k represents the bidding power of customers, the higher the value, the stronger the bidding power. And k is usually exogenous, each different choice of k represents different mechanisms. Moreover, the available supply equals available demand when $T_{(m)} \neq T_{(m+1)}$, this is because the sum of the number of asking and bidding prices no more than the transaction price equals m, which is the total number of bidding prices. So, the number of asking prices no more than $T_{(m)}$ equals the number of bidding prices no less than $T_{(m+1)}$. Shortages/surpluses in demand (supply) may exist at this clearing price if $T_{(m)} = T_{(m+1)}$. For model tractability, we assume that demand equals supply in equilibrium no matter whether condition $T_{(m)} \neq T_{(m+1)}$ holds. This is consistent with (Rustichini 1990), and consistent with the assumption that large labour pool and demand size is considered in matching game.

For participation behavior, we consider symmetric bidding strategy such that bidders' bidding strategies follow the same structure. Because participants only know the distributions of others' private values, they bid against others' strategies by choosing their bids as functions of their private values. Consistent with the argument that bidding policy must be nondecreasing with valuations (Chatterjee and Samuelson 1983), we assume a linear bidding structure. That is, customers make bidding decisions based on $B(v) = a_c + b_c v$, and providers propose asking prices based on $S(c) = a_p + b_p c$, where $b_c > 0$ and $b_p > 0$ (refer to the appendix for proof).

The order of play in the double auction unfolds as follows, as is also depicted in Figure 2.3: First, the sharing platform declares a matching policy including matching and transaction price determination rule. Then, in period one, customers arrive and each one submits a bidding price based on the aforementioned bidding strategy, at the same time, a subset of providers anticipating a lower price or higher matching probability in period one plan to serve and propose asking prices. Customers and providers are matched based on predetermined matching rule. Unmatched customers turn to other alternatives while unmatched providers will linger in the platform till the next period. In period two, newly come customers propose bidding prices and providers who postpone their service fulfillment along with those unmatched ones from previous period submit asking prices simultaneously. Matches take place among available customers and providers, and unmatched participants on both sides are lost forever at the end of period two.

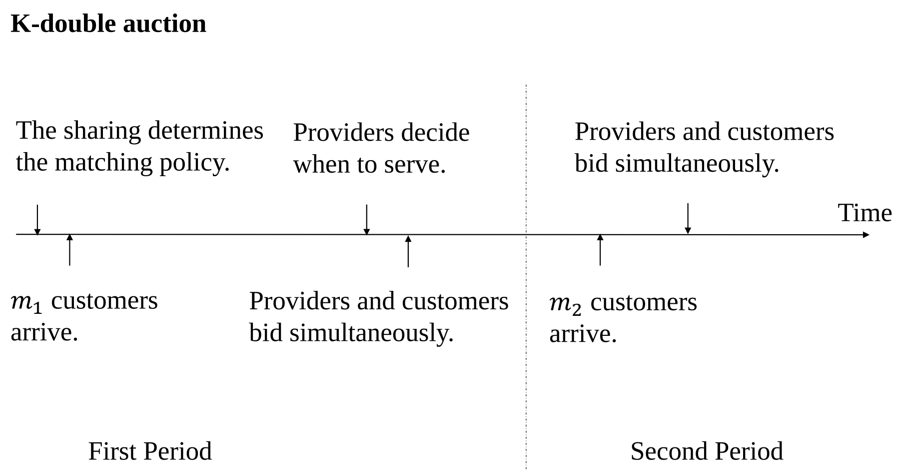

Figure 2.3: Sequence of Events In Double Auction

Given these preliminaries, we solve the Bayesian Nash equilibrium for the optimal bidding decisions of participants on two sides, that is, every player participating in the platform adopts the bidding strategy to maximize their utility given other players propose their own optimal strategies. And we solve the mixed strategy equilibrium for providers' fulfillment decision.

2.4.1 Customers and Providers' Bidding Equilibrium

Because of endogenous transaction price and unknown bids from others, a participant at the time of submitting a bid will weigh the likelihood that his/her bid can affect the transaction price, which in turn will influence the expected gain or loss.

Specifically, a specific customer weighs two factors as she raises her bidding price from b to $b + \Delta b$, the first factor is that if her bidding price determines the transaction price, that is, $(1 - k)T_{(m)} + kb$, then, raising her bid a little bit increases the price she pays (provided that $b+\Delta b$ still determines the transaction price when Δb is arbitrarily small). The second factor is that the customer with bidding price $b + \Delta b$ makes a profitable transaction that she fails to make with bidding price b. In other words, the customer's bidding price b is between $T_{(m-1)}$ and $T_{(m)}$ among total $n + m - 1$ bids and bidding price $b + \Delta b$ must surpass $T_{(m)}$. Bid $T_{(m)}$ specified here can be a bidding or an asking price.

To this end, the probability of a specific customer's bidding price is within interval $(T_{(m)}, T_{(m+1)})$ in the pool of $m - 1$ customers with bidding strategy B and n providers with bidding strategy S (or equivalently, the probability that b is the $(m + 1) - th$ lowest bid in the order statistics of all $m + n$ bids) is denoted by

$$P_1(b) = \sum_{0 \leq i \leq m-1, 0 \leq j \leq n}^{i+j=m} \begin{pmatrix} n \\ j \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} G(v)^i F(c)^j (1 - G(v))^{m-1-i} (1 - F(c))^{n-j},$$

where $v = \frac{b-a_c}{b_c}$ and $c = \frac{b-a_p}{b_p}$. The term $G(v)^i F(c)^j$ implies that the total number of providers together with customers whose bids less than b is exactly m (for $i + j = m$) because b is the $(m + 1) - th$ lowest bid among all $n + m$ bids. On the contrary, the term $(1 - G(v))^{m-1-i}(1 - F(c))^{n-j}$ indicates that there are $n - 1$ participants whose bids are more than b (for $m - 1 - i + n - j = n - 1$). The expected loss of customer increasing her bid equals $kP_1\Delta b$ (i.e., $(v - ((1 - k)T_{(m)} + k(b + \Delta b))) - (v - ((1 - k)T_{(m)} + kb)) = -k\Delta b$).

If a customer's bid is too small to make her get matched successfully, then increasing her bid may lead her to surpass the bid of a customer or a provider to move to the pool of successful matches. That is, if the customer with bid $b+\Delta b$ surpasses a provider, then the probability equals $\frac{f(c)\Delta b}{S'(c)}$, which is derived by $P(S \in (b, b+\Delta b)) = P(S \leq b+\Delta b) - P(S \leq b) = P(c \leq \frac{b+\Delta b - a_p}{b_p}) - P(c \leq \frac{b-a_p}{b_p}) = \frac{\Delta b}{b_p}$. Let P_2 denote the probability that exactly $m-1$ of the remaining $n+m-2$ bids (excluding the specified customer with bidding price $b/b + \Delta b$ and the specified provider whose bid is the $m - th$ lowest among $n + m - 1$ bids) including bidding prices from $m - 1$ customers and asking prices from $n - 1$ providers are no more than b, we have the following:

$$P_2(b) = \sum_{0\le i\le m-1, 0\le j\le n-1}^{i+j=m-1} \begin{pmatrix} n-1 \\ j \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} G(v)^i F(c)^j (1-G(v))^{m-1-i} (1-F(c))^{n-1-j}.$$

The marginal gain of customers increasing her bid from b to $b+\Delta b$ is given by $v-b$. The reason behind this can be explained by the following: suppose that the bid within interval $(b, b+\Delta b)$ is denoted by $b^\bullet$, the selected customer with bid b can not be matched because the transaction price is equal to $(1-k)b+kb^\bullet$ and $(1-k)b+kb^\bullet > b$. If she increases her bid by Δb, then she will be matched with transaction price $(1-k)b^\bullet + k(b+\Delta b)$ for $(1-k)b^\bullet + k(b+\Delta b) < b+\Delta b$. The payoff from such a bidding price is between $v-b$ and $v-(b+\Delta b)$ and is reduced to $v-b$ when Δb is close to zero. Another explanation is that the transaction price is within the interval $(b, b+\Delta b)$ when the second factor accounts. Correspondingly, the expected marginal gain for the customer surpassing a provider is denoted by $nP_2 \frac{f(c)\Delta b}{S'(c)}(v-b)$ because of randomness in choosing a specific provider among n providers.

Analogously, if a bidding price lies in $(b, b+\Delta b)$, then we have $P(B \in (b, b+\Delta b)) = \frac{g(v)\Delta b}{B'(c)} = \frac{\Delta b}{b_c}$. The corresponding gain is denoted by $(m-1)P_3 \frac{g(v)\Delta b}{B'(c)}(v-b-k\Delta b)$, where the probability that exactly $m-1$ of the remaining $n+m-2$ bids less than or equal to b equals

$$P_3(b) = \sum_{0\le i\le m-2, 0\le j\le n}^{i+j=m-1} \begin{pmatrix} n \\ j \end{pmatrix} \begin{pmatrix} m-2 \\ i \end{pmatrix} G(v)^i F(c)^j (1-G(v))^{m-2-i} (1-F(c))^{n-j}.$$

To sum up, the marginal expected gain of a specific customer is given by Equation 2.4:

$$\lim_{\Delta b\to 0} \frac{r_c(b+\Delta b) - r_c(b)}{\Delta b} = (v-b)(nP_2 \frac{f(c)}{S'(c)} + (m-1)P_3 \frac{g(v)}{B'(v)}) - kP_1. \tag{2.4}$$

The marginal payoff function here can be interpreted as the difference between the marginal gains and loss from changing customer's bid b with $b+\Delta b$. Through mathematical transformation, we have the reduced form of Equation 2.4:

$$\lim_{\Delta b\to 0} \frac{r_c(b+\Delta b) - r_c(b)}{\Delta b} =(v-b)(\frac{1}{b_p F(c)} \sum_{i}^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} (m-i) A^i$$

$$+\frac{1}{b_cG(v)}\sum_{i}^{m-1}\left(\begin{array}{c}n\\m-i\end{array}\right)\left(\begin{array}{c}m-1\\i\end{array}\right)iA^i)-k\sum_{i}^{m-1}\left(\begin{array}{c}n\\m-i\end{array}\right)\left(\begin{array}{c}m-1\\i\end{array}\right)A^i,$$

where A is defined as $\frac{G(v)(1-F(c))}{F(c)(1-G(v))}$.

The analysis of deriving the marginal expected payoff of a specific provider is similar to that of a customer (the logic is summarized in Figure 2.4). By increasing the asking

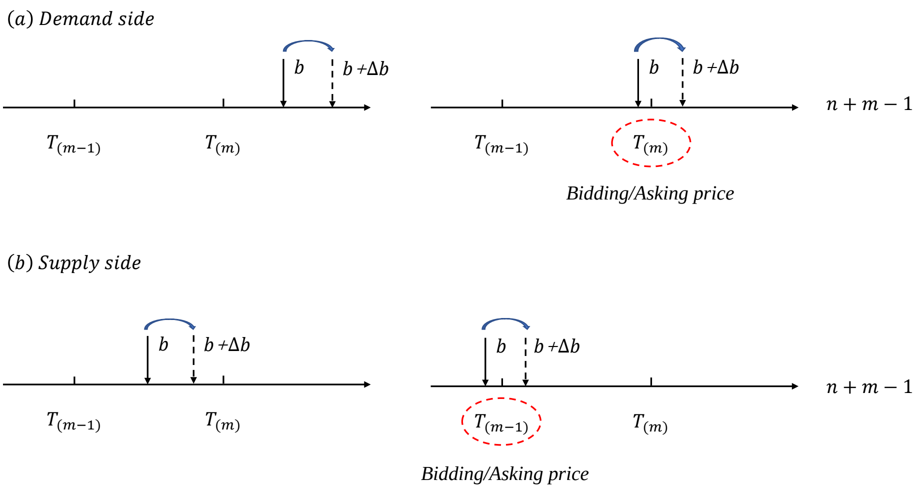

Figure 2.4: Bidding Strategy Characterization

price, the provider will encounter two possibilities: First, the provider will still be matched successfully with a higher transaction price. Second, the provider will surpass a provider's asking price or a customer's bidding price and will not be matched anymore at this increased bid. The gain is given by $\gamma((1-k)(b+\Delta b)+kT_{(m+1)})-c-(\gamma((1-k)b+kT_{(m+1)})-c)=\gamma(1-k)\Delta b$, while for the loss, we know that the provider can be matched when the transaction price is equal to $\gamma((1-k)b+kb^{\diamond})$ (or equivalently, the asking price is between $T_{(m-1)}$ and $T_{(m)}$ among $n+m-1$ bids), where $b^{\diamond}$ denotes the bidding price between b and $b+\Delta b$. And he will not be matched when he increases his bidding price to $b+\Delta b$ for the transaction price becomes $\gamma((1-k)b^{\diamond}+k(b+\Delta b))$ (the bidding price must surpass the bid $T_{(m)}$ specified here). Thus, the marginal net loss equals $\gamma b-c$. Let P_4 be the probability that the bid is between the $(m-1)-th$ and $m-th$ lowest bids within a pool of m customers and $n-1$ sellers, P_5 be the probability that the bid lies between $(m-1)-th$ and $m-th$ lowest bids among total $n+m-1$ bids including bidding prices from m customers and asking prices from $n-1$ sellers. The specific expressions are given in the appendix. The differential equation on the supply side is given by:

$$\lim_{\Delta b \to 0} \frac{r_p(b+\Delta b) - r_p(b)}{\Delta b} = -(\gamma b - c)((n-1)P_4 \frac{f(c)}{S'(c)} + mP_2 \frac{g(v)}{B'(v)}) + \gamma(1-k)P_5. \quad (2.5)$$

Similarly, we have simplified form for equation 2.5:

$$\begin{aligned}\lim_{\Delta b \to 0} \frac{r_p(b+\Delta b) - r_p(b)}{\Delta b} =& -(\gamma b - c)(\frac{1}{b_p(1-F(c))} \sum_i^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} (n-m+i)A^i \\ &+ \frac{1}{b_c(1-G(v))} \sum_i^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} (m-i)A^i) \\ &+ \gamma(1-k) \sum_i^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} A^i.\end{aligned}$$

To solve above best-response functions, we choose bids $B(v) = S(c) = b$ to form linked differential equations. Note that the first-order approach will fail if $B(v) \neq S(c)$ (Yang et al. 2017). The necessary and sufficient conditions of the bidding equilibrium are characterized by Lemma 2.4.1.

Lemma 2.4.1(Necessary and Sufficient Conditions of the Bidding equilibrium)

(a) Necessary condition: if B together with S characterizes an equilibrium, then $B'(v)$ satisfies the equation $\frac{1}{B'(v)} = \frac{kP_1}{(v-b)(m-1)P_3 g(v)} - \frac{nP_2 f(c)}{S'(c)(m-1)P_3 g(v)}$, and $S'(c)$ satisfies the equation $\frac{1}{S'(C)} = \frac{\gamma(1-k)P_5}{(\gamma b - c)(n-1)P_4 f(c)} - \frac{mP_2 g(v)}{B'(v)(n-1)P_4 f(c)}$, where $v = \frac{b-a_c}{b_c}$ and $c = \frac{b-a_p}{b_p}$;

(b) Sufficient condition: if $B'(v)$ and $S'(c)$ respectively satisfy above equations, then B along with C characterizes an equilibrium.

2.4.1.1 Optimal Bidding Strategy

For model tractability, we first introduce the dynamics of k-double auction with two special cases, then turn to the general case. The increase of k makes the fraction of the profit go from the demand side to the supply side. Case $k = 1$ represents the situation where service providers own more fraction of the profit, case $k = 0$ represents the situation where customers own more fraction of the profit, and case $k = \frac{1}{2}$ represents the situation where customers and providers own the same fraction of the profit. Moreover, known as double auction with average mechanism when $k = \frac{1}{2}$, uniform k-DA exhibits individual rationality, budget balance, and economic efficiency properties, and only incentive compatibility is lacking (Lin et al. 2019). We believe that these three cases are representative.

Lemma 2.4.1.1 indicates that customer's bidding strategy converges to their valuations when the number of customers is rather large, and it is optimal for providers to bid $\frac{c}{\gamma}$ under case $k = 1$, which we call buyer's bid double auction. When a specific provider find that his bid determines the transaction price (i.e., his bid is on the RHS of the convex combination), then increasing the bid a little bit does not change the transaction price for his bid does not account for the price anymore when $k = 1$.

Lemma 2.4.1.1 (Optimal Bidding Strategies When $k = 1$) Under buyer's bid double auction, providers bid $\frac{c}{\gamma}$, and customers bid $\frac{m}{m+1}v$, where m denotes the number of customers at that time period.

Similarly, when $k = 0$, which we call seller's offer double auction, then providers have the pricing power, so the optimal strategy for customers is to truthfully report their valuations. The underlying reason is similar to BBDA: customers' bids do not account for the price anymore when $k = 0$. As for providers, their bidding policy follows the structure $a_p + b_p = 1$ and both a_p and b_p are functions of the number of available providers at that time period. In particular, it is optimal for providers to bid their reservation prices when the amount of supply is extremely large and $\gamma = 1$. Details are summarized as Lemma 2.4.1.2.

Lemma 2.4.1.2 (Optimal Bidding Strategies When $k = 0$) Under seller's offer double auction, customers bid v, and providers' optimal bidding strategy follows structure $a_p + b_p = 1$, $a_p = \frac{\gamma(nb+b-1)-nb}{\gamma(nb+b-1)-n}$, $b_p = \frac{nb-n}{\gamma(nb+b-1)-n}$, and $b = a_b + b_p c$. Especially, providers bid $\frac{1}{n+1} + \frac{n}{n+1}c$ when $\gamma = 1$. By Lemmas 2.4.1.1 and 2.4.1.2, the higher the value of k, the lower the bidding prices of customers, this confirms the claim that the higher the value of k, the stronger the customers' bidding power.

Case $k \in (0,1)$ implies that both customers and providers have the bidding power, and the optimal bidding strategy is as Lemma 2.4.1.3 shows.

Lemma 2.4.1.3 (Optimal Bidding Strategies When $k \in (0,1)$) Under general k-double auction, with the number of customers and providers satisfy $m > 1$ and $n > 1$, the optimal bidding strategies satisfy $a_c = a_p$ and $b_c = b_p$, where $a_c = a_p = \frac{(1-k)(k+m)\gamma b_p^2 - (1-k)\gamma m b_p}{b_p((k-1)\gamma m + \gamma n k) + mn - \gamma mn}$, and $b_c = b_p > 0$.

Lemma 2.4.1.3 reveals that in general double auction with $k \in (0,1)$, the optimal bidding policy of customers and providers follows the same structure: equal intercepts and equal slopes. Moreover, the bidding parameters is closely related. Especially, $b_c = b_p = \frac{m}{k+m}$ when $a_c = a_p = 0$ and $a_c = a_p = \frac{(1-k)\gamma k}{(k-1)\gamma m + \gamma n k + mn - \gamma mn}$ when $b_c = b_p = 1$. All these are consistent with the optimal solutions of special cases $k = 1$ and $k = 0$.

To sum up, when the transaction price depends on the demand (supply) side, providers' asking prices (customers' bidding price) do not change the final transaction price, then providers (customers) will bid as low (high) as possible to increase the matching probability so as to maximize their expected utility. This conclusion is still valid when considering discriminatory k-DA (see Appendix A.2 for details). When the transaction price counts on both demand- and supply-side, then customers (providers) want to decrease (increase) the transaction price, and the decrement and increment must be the same.

2.4.2 Provider's Demand Fulfilling Equilibrium

Denote the probability and the number of providers joining in period one by α and s, respectively. Then providers joining in period two include $n - s$ providers planning to serve in period two and unmatched ones from period one. With the optimal bidding strategy of customers (resp., providers) in period one and two denoted by $a_{c_1} + b_{c_1} v$ (resp., $a_{p_1} + b_{p_1} c$) and $a_{c_2} + b_{c_2} v$ (resp., $a_{p_2} + b_{p_2} c$), respectively, equating effective supply with effective demand in two periods yields

$$sF(\frac{p_1 - a_{p_1}}{b_{p_1}}) = m_1\bar{G}(\frac{p_1 - a_{c_1}}{b_{c_1}}), \tag{2.6}$$

$$(n - sF(\frac{p_1 - a_{p_1}}{b_{p_1}}))F(\frac{p_2 - a_{p_2}}{b_{p_2}}) = m_2\bar{G}(\frac{p_2 - a_{c_2}}{b_{c_2}}). \tag{2.7}$$

In equilibrium, providers are indifferent between serving in two periods:

$$\gamma p_1 F(\frac{p_1 - a_{p_1}}{b_{p_1}}) = \gamma p_2 F(\frac{p_2 - a_{p_2}}{b_{p_2}}). \tag{2.8}$$

The corresponding platform's profit equals:

$$\pi^{DA} = (1-\gamma)(p_1 m_1 \bar{G}(\frac{p_1 - a_{c_1}}{b_{c_1}}) + p_2 m_2 \bar{G}(\frac{p_2 - a_{c_2}}{b_{c_2}})). \tag{2.9}$$

2.4.3 The Optimal Strategy Characterization

In what follows, we derive the equilibrium results of BBDA, SODA and GDA one by one.

2.4.3.1 BBDA

Plugging the number of potential customers and providers into their bidding strategies and Equations 2.6, 2.7 and 2.8 simultaneously yields the optimal pricing strategy. The optimal results under BBDA are summarized as Proposition 2.4.3.1.

Proposition 2.4.3.1 In BBDA ($k = 1$), equilibrium exists if $\frac{m_1(1+\gamma n)^2 + m_1^2(1+\gamma n - \gamma^2 n)}{(1+\gamma n + m_1)(1+\gamma n)} \leq m_2 \leq m_1 + \gamma m_1 n$. The optimal strategy is a stable pricing policy $p_1^* = p_2^* = \frac{2m_2}{E}$, the equilibrium fraction of providers joining in period one equals $\alpha^* = \frac{Em_1 - 2m_2(m_1+1)}{2\gamma m_2 n}$, and the equilibrium transaction volumes equal $D_1^* = m_1 - \frac{2m_2(m_1+1)}{E}$ and $D_2^* = m_2 - \frac{2m_2(m_2+1)}{E}$.

Proposition 2.4.3.1 shows that there exists an equilibrium when the demand intensity across periods is not significantly large or small, and the optimal prices over two periods are stable. This is because large demand gap causes providers to join the period with higher demand. To this end, prices across periods are not equal in equilibrium. Neither all available customers or providers at a time period are matched successfully: the number of unmatched customers (providers) equals $\frac{2m_2(m_1+m_2+2)}{E}$ ($\frac{(n-m_1-m_2)E+2m_2(m_1+m_2+2)}{E}$), and the total unmatched proability equals $\frac{n-m_1-m_2}{n+m_1+m_2}$. When compared with Proposition 2.3.2, the total matching probability is larger under BBDA than under DP when the number of last-minute customers is not rather large, and both the optimal prices and transaction volumes are larger under DP than under BBDA otherwise.

Moreover, when the number of regular customers increases, then the order statistics of the transaction price at that time period increases, so, the endogenously determined transaction price also increases ($\frac{\partial p_1^*}{\partial m_1} > 0$). High price attracts more service providers to fulfill demand in the first period. So, the transaction volume increases correspondingly $\frac{\partial D_1^*}{\partial m_1} > 0$. With fewer providers waiting for the second period, the order statistics of the transaction price increases, so the transaction price also increases ($\frac{\partial p_2^*}{\partial m_1} > 0$). Because effective demand equals effective supply in equilibrium, so the transaction volume in the second period decreases when s increases.

When the number of last-minute customers increases, then because of the order statistics changing effect, the transaction price at that time period increases ($\frac{\partial p_2^*}{\partial m_2} > 0$). Similarly, high prices lead to more providers pour into the second period ($\frac{\partial s^*}{\partial m_2} < 0$). The transaction volume increases correspondingly. When the number of providers fulfilling demand in the first period decreases, the order statistics of the transaction price increases, and the successful transaction volume is positively related to the amount of effective supply.

When the number of service providers becomes large, the order statistic of transaction prices over two periods decrease, so as the optimal transaction prices ($\frac{\partial p_i^*}{\partial n} < 0$). Because the positive relationship between the successful demand and successful supply, the increased number of providers available on the platform enhances the transaction volume of each period ($\frac{\partial D_i^*}{\partial n} > 0$).

All these comparative analyses are summarized as Corollary 2.4.3.1.

Corollary 2.4.3.1

(i) The impact of m_1, m_2, and n on the optimal prices and transaction volumes: $\frac{\partial p_i^*}{\partial m_1} >$

0, $\frac{\partial p_i^*}{\partial m_2} > 0$, $\frac{\partial p_i^*}{\partial n} < 0$; $\frac{\partial D_1^*}{\partial m_1} > 0$, $\frac{\partial D_1^*}{\partial n} > 0$, $\frac{\partial D_1^*}{\partial m_2} < 0$, $\frac{\partial D_2^*}{\partial m_1} < 0$, $\frac{\partial D_2^*}{\partial m_2} > 0$, $\frac{\partial D_2^*}{\partial n} > 0$.

(ii) The impact of m_1, m_2, and n on the fulfillment equilibrium: $\frac{\partial s^*}{\partial m_1} > 0$ when m_2 is low, $\frac{\partial s^*}{\partial m_2} < 0$, $\frac{\partial s^*}{\partial n} > 0$.

When compared with Corollaries 2.3.1 and 2.3.2, Corollary 2.4.3.1 shows that the stable pricing strategy in BBDA is attributed to the dynamics of the order statistics changing effect and the anticipation effect instead of the dynamics of the competition effect and anticipation effect in DP.

2.4.3.2 SODA

In seller's offer double auction (with $k = 0$), we focus on case $\gamma = 1$ for mathematical tractability. Given the number of potential players in two periods (i.e., (m_1, s) and $(m_2, n - (s+1)p_1 + 1)$), providers' bidding policy in period one and two are denoted by $S_1(c) = \frac{1}{s+1} + \frac{s}{s+1}c$, and $S_2(c) = \frac{1}{n-(s+1)p_1+2} + \frac{n-(s+1)p_1+1}{n-(s+1)p_1+2}c$, respectively. Proposition 2.4.3.2 summarizes the optimal outcomes.

Proposition 2.4.3.2 In SODA ($k = 0$, $\gamma = 1$), equilibrium exists if

$2\sqrt{\delta(1+m_1)(1+\delta m_1)} \leq F \leq \frac{2\delta(1+m_1+\delta m_1)(1+\delta m_1)}{\sqrt{\delta(1+m_1)(1+\delta m_1)}}$. The optimal transaction prices equal $p_1^* = \frac{F-2(1-m_1+\delta m_1+\beta m_1)}{2m_1}$ and $p_2^* = \frac{2(1+\delta m_1)}{F}$, the optimal fraction of providers joining in period one equals $\alpha^* = \frac{\sqrt{\delta(1+m_1)(1+\delta m_1)}F - 2\delta(1+m_1)(1+\delta m_1)}{2\delta(1+\delta m_1)\delta m_1}$, and the equilibrium transaction volumes equal $D_1^* = \frac{2(\beta m_1+\delta m_1+1)-F}{2}$ and $D_2^* = \frac{m_2 F - 2m_2(\delta m_1+1)}{F}$.

Note that $p_1^* - p_2^* = \frac{2(\sqrt{\delta(1+m_1)(1+\delta m_1)}-\delta m_1-1)}{F}$, numerical analysis shows that the platform's optimal strategy under SODA approximates to a stable pricing policy ($p_1^* \approx p_2^*$).

By comparing Propositions 2.4.3.1 and 2.4.3.2, we find that the impacts of cross-demand intensity and demand-supply intensity on the optimal prices and transaction volumes are consistent under two double auction mechanisms (i.e., $\frac{\partial p_i^*}{\partial \delta} > 0$, $\frac{\partial p_i^*}{\partial \beta} < 0$; and $\frac{\partial D_1^*}{\partial \delta} < 0$, $\frac{\partial D_i^*}{\partial \beta} > 0$), the main difference lies in the impact of the first-period demand on the price and transaction volume (i.e., $\frac{\partial p_i^*}{\partial m_1} < 0$ and $\frac{\partial D_i^*}{\partial m_1} > 0$) and the impact of cross-demand intensity on the second-period demand ($\frac{\partial D_2^*}{\partial \delta} < 0$). This can be attributed to the driving forces of providers' fulfillment equilibrium. That is, providers making demand fulfillment decision have to take the matching probability and the transaction price into account under SODA. While the matching probability does not count in BBDA.

2.4.3.3 GDA

With system state given by (m_1, s) in period one and $(m_2, n - \frac{m_1 s}{m_1+s})$ in period two, we have results summarized in Proposition 2.4.3.3.

Proposition 2.4.3.3 In GDA ($k \in (0,1)$), the optimal transaction prices equal $p_1 = \frac{m_1 s b_{p_1}}{m_1+s} + \frac{(1-k)\gamma((k+m_1)b_{p_1}^2 - m_1 b_{p_1})}{b_{p_1}((k-1)\gamma m_1 + \gamma sk) + m_1 s - \gamma m_1 s}$ and $p_2 = \frac{m_2(m_1+s)b_{p2}}{(n+m_2)(m_1+s)-m_1 s}$ $+\frac{(1-k)\gamma(m_1+s)((k+m_2)b_{p2}^2 - m_2 b_{p2})}{(\gamma k(m_1 n - m_1 s + ns) - \gamma m_2(1-k)(m_1+s))b_{p2} + m_2(1-\gamma)(m_1 n - m_1 s + ns)}$, where $\frac{m_1}{m_1+s}p_1 = \frac{m_2(m_1+s)}{(n+m_2)(m_1+s)-m_1 s}p_2$, the optimal fraction of providers joining in period one equals $\alpha^* = \frac{s}{n}$, and the equilibrium transaction volumes equal $D_1^* = \frac{m_1 s}{m_1+s}$ and $D_2^* = \frac{m_2(n(m_1+s)-m_1 s)}{(n+m_2)(m_1+s)-m_1 s}$.

Proposition 2.4.3.3 introduces the optimal pricing structure for the sharing platform. By deriving the optimal solutions under case that $a_p = a_c = 0$, $b_p = b_c = \frac{m}{k+m}$, and $k = \frac{1}{2}$, we find that the optimal prices over two periods approximate to be stable ($p_1^* \approx p_2^*$) (details are referred to the appendix).

By comparing Propositions 2.4.3.1 and 2.4.3.3, we find that the transaction price is higher under BBDA than under GDA ($p^{BBDA} > p^{GDA}$), so, the transaction volume is higher under GDA than under BBDA ($\sum_{i\in\{1,2\}} m_i(1 - \frac{m_i+k}{m_i}p_i^{GDA}) > \sum_{i\in\{1,2\}} m_i(1 - \frac{m_i+k}{m_i}p_i^{BBDA})$).

2.5 Mechanisms Comparison

In this section, we compare Scenarios DP and DA, with an emphasis on identifying conditions under which the platform, customers and providers can (individually and jointly) enjoy higher benefits.

2.5.1 Profit Comparison

2.5.1.1 BBDA vs. DP

Note that when equilibria of dynamic pricing and BBDA coexist, the optimal pricing strategy in pricing mechanism is a stable pricing strategy. Theorem 2.5.1.1 summarizes the comparison results.

Theorem 2.5.1.1 (BBDA vs. DP Regarding Platform's Profit) Two mechanisms coexist if $\min\{\frac{m_1(1+\gamma n)^2+m_1^2(1+\gamma n-\gamma^2 n)}{(1+\gamma n+m_1)(1+\gamma n)}, \gamma n - \frac{\gamma m_1}{2}\} < m_2 \leq \gamma m_1 n + m_1$, and $\pi^{BBDA} < \pi^{DP}$.

Theorem 2.5.1.1 indicates that the pricing entity (the bidding power) is a key in characterizing the relationship between two selling mechanisms. That is, the platform earns higher revenues when it has price setting power than does not have. In equilibrium, the successful transaction price in BBDA is always smaller than the one in DP ($p_i^{DP} > p_i^{BBDA}$). The optimal transaction volume is larger in BBDA than in DP if $\frac{m_1(1+\gamma n)^2+m_1^2(1+\gamma n-\gamma^2 n)}{(1+\gamma n+m_1)(1+\gamma n)} < m_2 < \gamma n - \frac{\gamma m_1}{2}$. Hence, the dominance of dynamic pricing mechanism is attributed to the evident price increment effect when last-minute customers pool is not large, or is attributed to both the price increment and demand expansion effect otherwise.

2.5.1.2 SODA vs. DP

Note that SODA and DP coexists if $2\sqrt{\delta}\sqrt{m_1+1}\sqrt{\delta m_1+1} \leq F \leq \frac{2\sqrt{\delta}\sqrt{\delta m_1+1}(\delta m_1+m_1+1)}{\sqrt{m_1+1}}$. There is no difference between dynamic pricing and SODA when $\gamma = 1$. With the following parameter setting: $m_1 = 3000$, $\gamma = 0.25$, $\beta = 2$, $\delta \in (0, 0.09375)$, $\delta \in (0.09375, 0.375)$, $\delta \in (0.375, 2)$, where domains of δ represents the dynamic pricing scenario of (3), (2) and (1) in Proposition 2.3.2, respectively. Numerical analysis shows that the platform has higher transaction volumes in SODA than in DP, and higher transtion prices in DP than in SODA (i.e., $\Delta p_i^{SODA-DP} < 0$ in Figure 2.5 and $\sum_{i\in\{1,2\}} \Delta D_i^{SODA-DP} > 0$ in Figure 2.6).

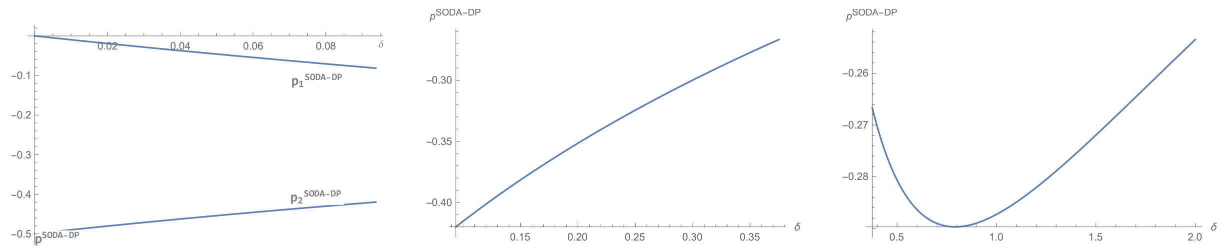

Figure 2.5: Differences in Price between SODA and DP

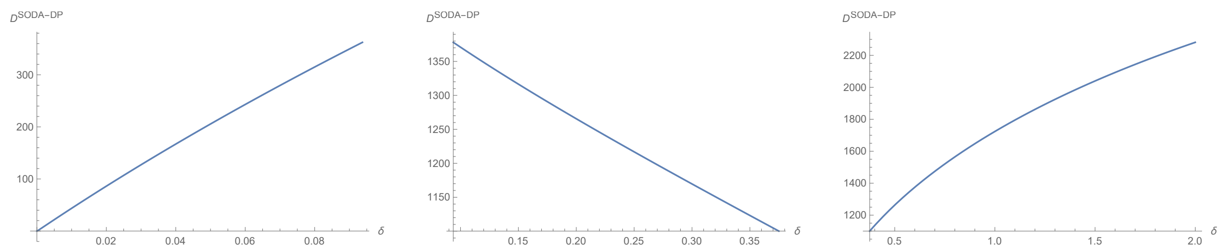

Figure 2.6: Differences in Demand between SODA and DP

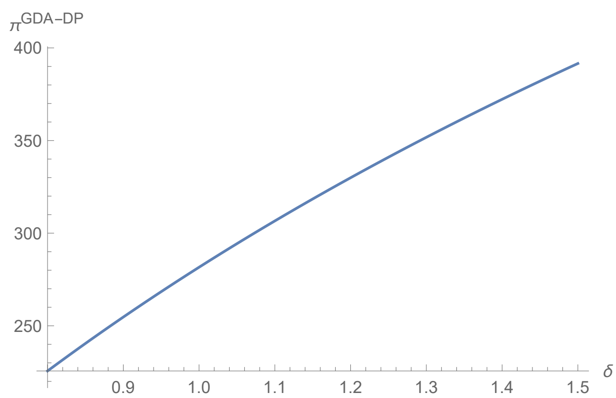

Figure 2.7: Differences in Platform's Profit Between Mechanisms GDA and DP

2.5.1.3 GDA vs. DP

When GDA and DP coexist, the optimal pricing strategy in dynamic pricing is $p_i^* = A_1^{DP}$. With the following parameter setting: $m_1 = 2000$, $\gamma = 0.25$, $\beta = 1$, $\delta \in (0.8, 1.5)$, results are illustrated by Figures 2.7 and 2.8. Numerical analysis shows that the platform earns higher revenue in GDA than in DP, and the optimal prices and transaction volumes satisfy $\Delta p_i^{GDA-DP} < 0$, and $\sum\limits_{i \in \{1,2\}} \Delta D_i^{GDA-DP} > 0$. Hence, the priority of GDA over DP is attributed to the market expansion effect.

Note that the mechanism under which the platform (resp., participants rather than the platform) determines the transaction price is so called as the closed (resp., open)

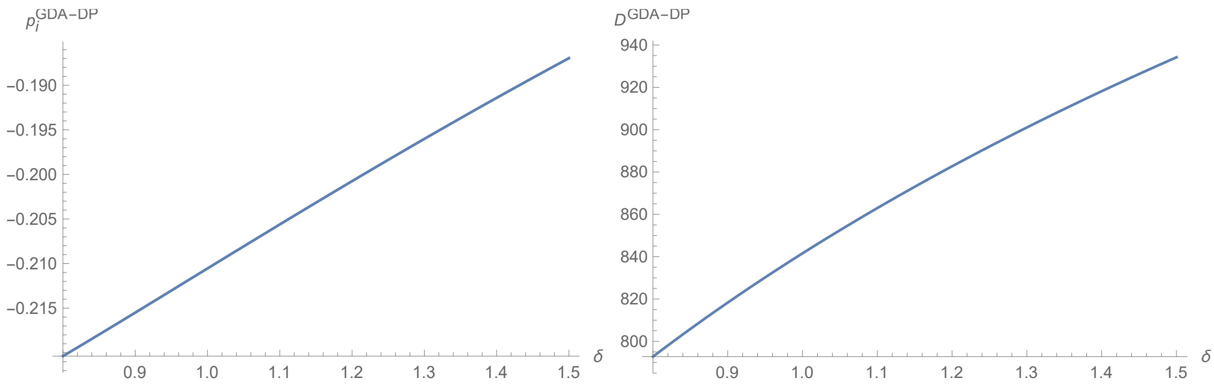

Figure 2.8: The Equilibrium Price and Demand under BBDA and DP

platform strategy (Economides and Katsamakas 2006,Hagiu 2007,Rysman 2009,Ryan et al. 2012,Hagiu and Wright 2015,Johnson 2020). (Hagiu and Wright 2015) shows that the open or closed strategy depends on the platform's control rights over pricing, advertising, and service fulfillment responsibility. For instance, great discounts, such as no delivery fee and large number of coupons, adopted on JD.com in early stages, at which it is a closed B2C platform. While we prove that the transaction price is higher on closed platform (with dynamic pricing mechanism) than on open platform (with k-double auction mechanism). We also claim the importance of pricing entity in influencing the open or closed strategy of the sharing platform (i.e., $\pi^{GDA} > \pi^{DP}$ and $\pi^{BBDA} < \pi^{DP}$).

2.5.2 Surpluses and Social Welfare Comparison

Because of controversial practices, such as racial discrimination, safety concerns, private information and labor law problems in ride-haling industry, platforms have to take into account consumer/provider surplus. In what follows, we compare total consumer surplus (abbreviated as CS), which is defined as the cumulative integration of the net utility over valuation across two periods, provider surplus (abbreviated as PS), which is defined as the cumulative integration of the net reservation value across two periods, and social welfare (abbreviated as SW), which is defined as the sum of consumer surplus, provider surplus and platform's profit. Formulas are as follows:

$$CS = \min\{s, m_1\bar{G}(p_1^*)\} \int_{p_1^*}^{1} (v - p_1^*) g(v) dv$$

$$+\min\{(n-\min\{s,m_1\bar{G}(p_1^*)\})F(\gamma p_2^*),m_2\bar{G}(p_2^*)\}\int_{p_2^*}^{1}(v-p_2^*)g(v)dv.$$

$$PS=\min\{s,m_1\bar{G}(p_1^*)\}\int_{0}^{\gamma p_1^*}(\gamma p_1^*-c)f(c)dc$$

$$+\min\{(n-\min\{s,m_1\bar{G}(p_1^*)\})F(\gamma p_2^*),m_2\bar{G}(p_2^*)\}\int_{0}^{\gamma p_2^*}(\gamma p_2^*-c)f(c)dc.$$

$$SW=\pi^*+\min\{s,m_1\bar{G}(p_1^*)\}(\int_{p_1^*}^{1}(v-p_1^*)g(v)dv+\int_{0}^{\gamma p_1^*}(\gamma p_1^*-c)f(c)dc)$$

$$+\min\{(n-\min\{s,m_1\bar{G}(p_1^*)\})F(\gamma p_2^*),m_2\bar{G}(p_2^*)\}(\int_{p_2^*}^{1}(v-p_2^*)g(v)dv+\int_{0}^{\gamma p_2^*}(\gamma p_2^*-c)f(c)dc)$$

where p_i refers to the optimal posted price of period i in post pricing mechanism and the successful transaction price in k-double auction.

2.5.2.1 BBDA vs. DP

Expressions of CS, PS and SW are complicated in BBDA and DP which makes theoretical analyses difficult. We numerically investigate how the pricing entity changed from the platform to the participants on two sides affects different stakeholders' benefits with varying system parameters. Figures 2.9, 2.10, and 2.11 present the value of $CS^{BBDA-DP}$, $PS^{BBDA-DP}$ and $SW^{BBDA-DP}$, respectively, when a focal parameter δ changes. Other parameters take the following values: $m_1 = 1500$, $\gamma = 0.25$, $\beta = 4.5$, which ensure the conditions in Propositions 2.3.2 and 2.4.3.1 hold and non-trivial equilibrium results exist.

Consumer Surplus.

By Figure 2.9, BBDA always benefits customers, the rationale lies in the formulation of consumer surplus. In both mechanisms, consumer surpluses are composed of two segments: (i) the net utility $v-p_i^*$, and (ii) the total transaction volumes D_i^*. Recall that the optimal price is smaller under BBDA than under DP, and the price effect is more evident when the optimal transaction volume under BBDA is smaller than under DP. Hence, customers are better off when they have the power of determining the transaction price.

Provider Surplus.

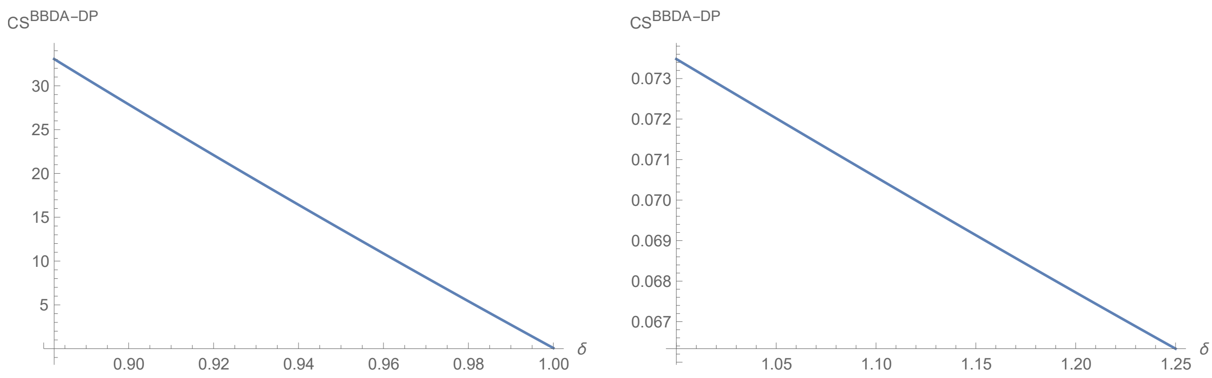

Figure 2.9: Differences in Consumer Surplus Between Mechanisms BBDA and DP

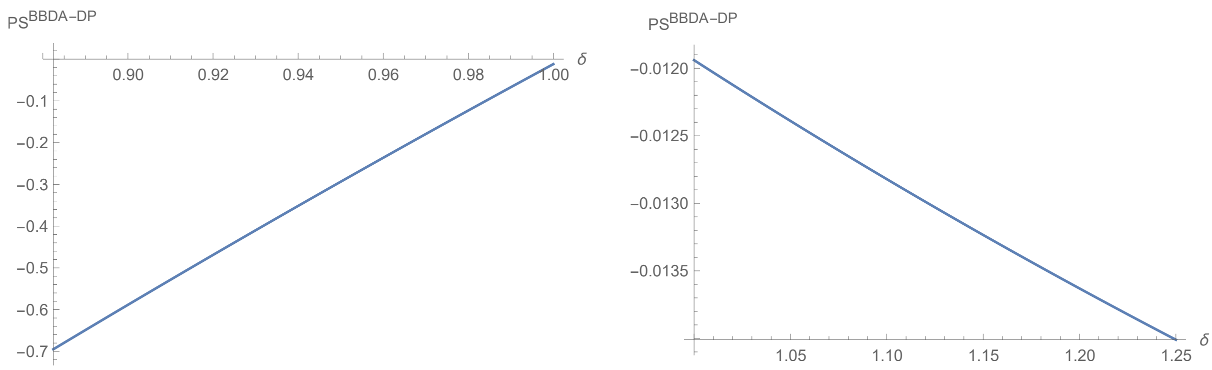

Figure 2.10: Differences in Provider Surplus Between Mechanisms BBDA and DP

By Figure 2.10, DP always benefits providers, the rationale also lies in the formulation of provider surplus. In both mechanisms, provider surpluses are composed of two segments: (i) the net utility $\gamma p_i^* - c$, and (ii) the total transaction volumes D_i^*. Recall that the optimal price under BBDA is smaller than the one under DP, the optimal transaction volume under BBDA is higher than the one under DP, and the price distortion effect is more evident than the demand expansion effect in BBDA. Hence, providers are better off under dynamic pricing mechanism.

Social Welfare.

By Figure 2.11, social welfare gap $SW^{BBDA-DP}$ changes with demand-intensity across periods. When the number of last-minute customers is rather low, social welfare gap is positive. This is because the difference in social welfare $SW^{BBDA-DP}$ is mainly driven by $CS^{BBDA-DP}$. When the number of last-minute customers increases, negative provider surplus and profit gap as well as reduced consumer surplus leads to negative social welfare.

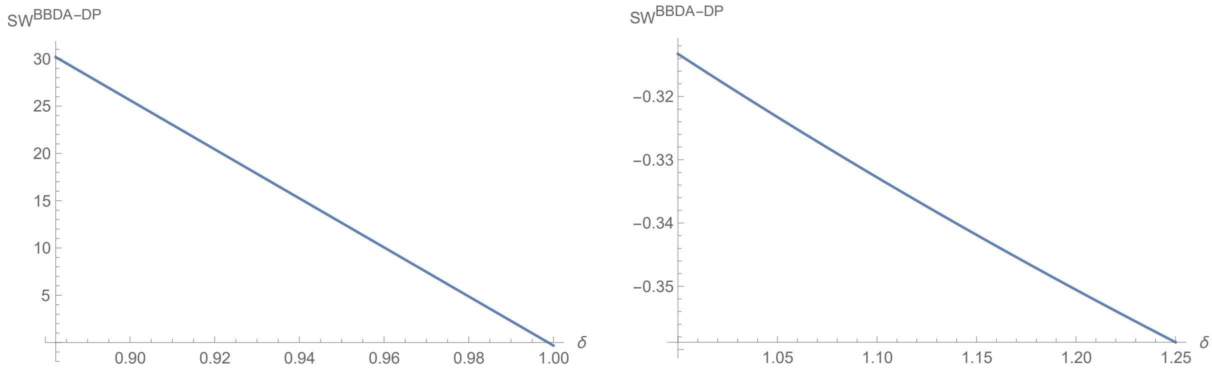

Figure 2.11: Differences in Social Welfare Between Mechanisms BBDA and DP

2.5.2.2 SODA vs. DP

Resemblances are found between SODA and BBDA: consumer surplus is higher under SODA than under DP for $\Delta p^{SODA-DP} < 0$, $\Delta D_i^{SODA-DP} > 0$. As for provider surplus, the dominance depends on the tradeoff between price distortion (enhancement) effect and demand expansion (contraction) effect. By conducting numerical analysis, we have $PS^{SODA-DP} > 0$ (see Figure 2.12). Hence, SODA is more beneficial to the whole society when compared with DP.

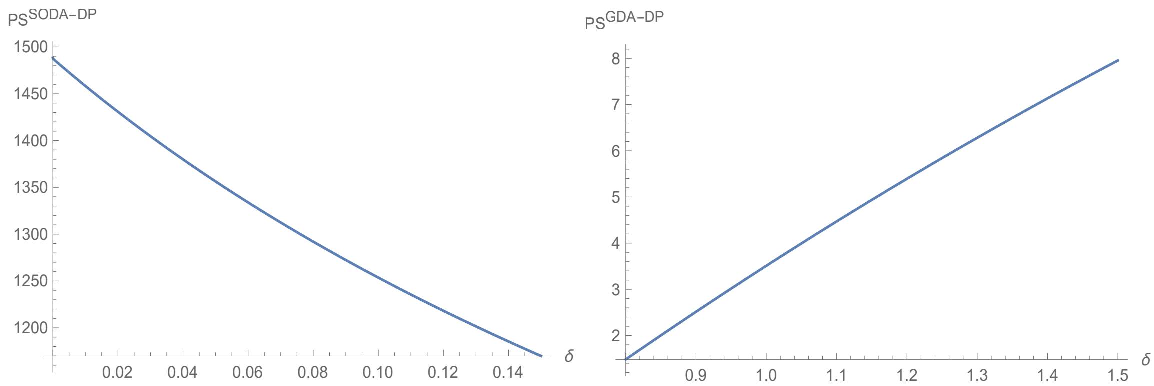

Figure 2.12: Differences in Provider Surplus Between Mechanisms SODA/GDA and DP

2.5.2.3 GDA vs. DP

Similar to SODA, customers and providers are better off under GDA than under DP, and GDA is more beneficial to the whole society when compared with DP.

2.6 Extensions

In this section, we test the robustness of our model and generate new insights by considering other forms of selling strategies, and extending analysis to a one-period model.

2.6.1 Other Forms of Selling Strategies

Unfairness emerges when implementing price discrimination, and transaction prices of each matching pair are unequal in practice. Here we consider static pricing (abbreviated as SP) and discriminatory mechanism (refer to Appendix A.2) to capture more realistic features.

2.6.1.1 Static Pricing Policy

With price p announced at the beginning of period one, the effective demand in period i is $m_i\bar{G}(p)$ and the effective supply over two periods is $nF(\gamma p)$. The number of transactions in period one and two are defined as $\min\{nF(\gamma p)\alpha, m_1\bar{G}(p)\}$ and $\min\{nF(\gamma p)(1-\alpha) + (nF(\gamma p)\alpha - m_1\bar{G}(p))^+, m_2\bar{G}(p)\}$, respectively. Providers are indifferent between serving in period one or waiting for the next period if

$$\min\{1, \frac{m_1\bar{G}(p)}{nF(\gamma p)\alpha}\} = \min\{1, \frac{m_2\bar{G}(p)}{nF(\gamma p)(1-\alpha) + (nF(\gamma p)\alpha - m_1\bar{G}(p))^+}\}.$$

The platform's profit equals $\pi^{SP}(p) = (1-\gamma)p(\min\{nF(\gamma p)\alpha, m_1\bar{G}(p)\} + \min\{nF(\gamma p)(1-\alpha) + (nF(\gamma p)\alpha - m_1\bar{G}(p))^+, m_2\bar{G}(p)\})$. Proposition 2.6.1.1 characterizes the optimal solutions.

Proposition 2.6.1.1 (Optimal Strategy Under Static Pricing Mechanism)

(*i*) If $m_2 > \gamma n - m_1$, then $p^* = \frac{m_2}{\gamma n(1-\alpha^*)+m_2}$, $D_1^* = \frac{\gamma n\alpha m_2}{\gamma n(1-\alpha)+m_2}$, $D_2^* = \frac{\gamma n(1-\alpha)m_2}{\gamma n(1-\alpha)+m_2}$ if $\alpha^* \in [0, \frac{m_1}{m_1+m_2})$; or $p^* = \frac{m_1}{\gamma n\alpha^*+m_1}$, $D_1^* = \frac{\gamma n\alpha m_1}{\gamma n\alpha+m_1}$ and $D_2^* = \frac{\gamma n(1-\alpha)m_1}{\gamma n\alpha+m_1}$ if $\alpha^* \in [\frac{m_1}{m_1+m_2}, 1]$;

(*ii*) If $m_2 \leq \gamma n - m_1$, then $p^* = \frac{1}{2}$, $\alpha^* = \frac{m_1 n\gamma - m_1^2}{m_2 n\gamma}$ and $D_i^* = \frac{m_i}{2}$.

Proposition 2.6.1.1 indicates that when the number of customers is large ($m_1 + m_2 > \gamma n$), then there is no differece in serving two periods for providers. When there are more customers than providers in the first period ($\alpha^* \in [0, \frac{m_1}{m_1+m_2})$), as more providers pour in, the platform will increase the posted price to attract more providers ($\frac{\partial p^*}{\partial \alpha^*} > 0$). When there are more providers than customers in the first period ($\alpha^* \in [\frac{m_1}{m_1+m_2}, 1]$), as more providers pour in, the platform will decrease the posted price to shut providers out. When supply exceeds demand ($m_1 + m_2 \leq \gamma n$), providers are more willing to serve in the period with higher demand, and maximum demand coverage price is proposed.

Dynamic Pricing vs. Static Pricing

Theorem 2.6.1.1 (Dynamic Pricing vs. Static Pricing)

(i) If $m_2 \leq \min\{\gamma n - m_1, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}\}$, then $\pi^{DP} < \pi^{SP}_{max}$.

(ii) If $\min\{\gamma n - m_1, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}\} < m_2 < \max\{\gamma n - m_1, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}\}$, then there is no absolute dominance between two pricing mechanisms.

(iii) If $m_2 \geq \max\{\gamma n - m_1, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}\}$, then $\pi^{DP} > \pi^{SP}$.

Theorem 2.6.1.1 shows that when $m_2 \leq \min\{\gamma n - m_1, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}\}$, the platform's maximum profit in SP is higher than the platform's profit in DP, the corresponding optimal price is higher in SP than in DP, and the optimal transaction volume is lower in SP than in DP ($A_3^{DP} < \frac{1}{2} < A_4^{DP}$). Because the number of customers is rather small, the profit increment of static pricing induced by price enhancement effect exceeds the profit increment of dynamic pricing induced by transaction volume enhancement effect.

When $m_2 \geq \max\{\gamma n - m_1, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}\}$, the platform earns higher profit in DP than in SP, and DP (resp., SP) has an advantage over the transaction volume (resp., transaction price), because the number of customers is rather large, then the profit increment of DP induced by transaction volume enhancement effect exceeds the profit increment of SP induced by price enhancement effect.

When $\min\{\gamma n - m_1, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}\} < m_2 < \max\{\gamma n - m_1, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}\}$, there is no absolute dominance between two mechanisms. Specifically, if $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} < m_2 < \gamma n - m_1$, then the platform earns the same profit under two mechanisms. Otherwise, the platform is better off under SP if and only if the number of customers is rather small.

K-Double Auction vs. Static Pricing

We first compare BBDA with SP, then conduct numerical analysis to compare SODA with SP and GDA with SP.

Theorem 2.6.1.2 (BBDA vs. SP) Two mechanisms coexist if $\min\{\frac{m_1(1+\gamma n)^2+m_1^2(1+\gamma n-\gamma^2 n)}{(1+\gamma n+m_1)(1+\gamma n)}, \gamma n - m_1\} < m_2 \leq \gamma m_1 n + m_1$. Moreover, SP dominates BBDA if $\frac{m_1(1+\gamma)(\gamma n+1)+m_1^2(1+\gamma n-\gamma^2)}{(1+\gamma)m_1+(1+\gamma)^2} < m_2 < \gamma n - m_1 + 2\sqrt{2\gamma n}$. BBDA dominates SP if $\max\{\frac{m_1(1+\gamma n)^2+m_1^2(1+\gamma n-\gamma^2 n)}{(1+\gamma n+m_1)(1+\gamma n)}, \gamma n - m_1 + 2\sqrt{2\gamma n}\} < m_2 \leq \gamma m_1 n + m_1$.

By Theorem 2.6.1.2, there is no absolute dominance between pricing and double auction when the platform does not have enough pricing flexibility. In addition, the posted price is higher in SP than in BBDA, while the transaction volume is lower in SP than in BBDA, this is in consistent with (Hammond 2010). By comparing Theorems 2.5.1.1 and 2.6.1.2, we find that both static and dynamic pricing dominates buyer's bid double auction when the number of customers is small, while when the number of customers is large and demand gap across periods is small, static pricing is inferior to buyer's bid double auction. This is consistent with the practice that license tag auction is prevalent in the market with multiple demand requests, such as in Shanghai.

Numerical analysis shows that comparable results between SODA and SP and GDA and SP are consistent with the ones under dynamic setting. Specifically, with the following parameters setting: $m_1 = 3000$, $\gamma = 0.25$, $\beta = 2$, $\delta \in [0.375, 2]$, the optimal prices and transaction volume in SODA satisfy: $p^{SP} > p_i^{SODA}$ and $D_1^{SP} + D_2^{SP} < D_1^{SODA} + D_2^{SODA}$. With the following parameters setting: $m_1 = 2000$, $\gamma = 0.25$, $\beta = 1$, $\delta \in [0.8, 1.5]$, the optimal prices, the transaction volume and the platform's profit in GDA satisfy: $p^{SP} > p_i^{GDA}$, $D_1^{SP} + D_2^{SP} < D_1^{GDA} + D_2^{GDA}$, and $\pi^{SP} < \pi^{GDA}$ (Figure 2.13).

To sum up, by putting all comparisons together, we claim that the pricing flexibility, pricing entity and demand-supply intensity play a vital role in influencing platform's strategy selection. The platform with more pricing flexibility are more willing to use post pricing. The lower the demand-supply intensity and the higher the bidding power, the higher the likelihood that the platform uses post pricing.

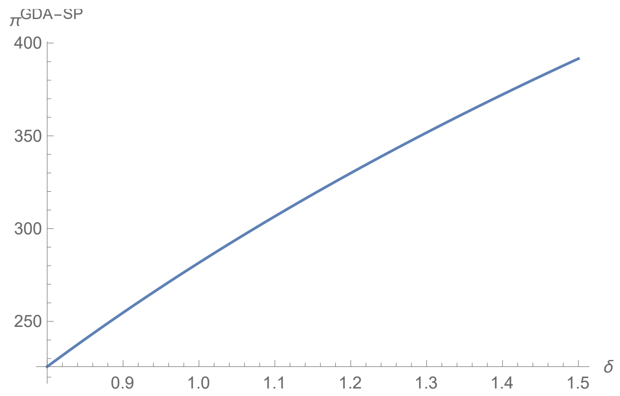

Figure 2.13: Differences in Platform's Profit Between Mechanisms GDA and SP

2.6.2 One-period Model without Strategic Behavior of Providers

We incorporate providers' strategic behavior in the main analysis, we want to explore whether this strategic behavior influences the platform's strategy selection by considering a one-period model.

2.6.2.1 Post Pricing & K-double Auction

In post pricing (abbreviated PP), the effective demand and supply equals $m\bar{G}(p)$ and $nF(\gamma p)$, respectively, and the platform's profit equals $\pi(p) = (1-\gamma)p\min\{m\bar{G}(p), nF(\gamma p)\}$. The optimal solutions are summarized as the following lemma.

Lemma 2.6.2.1 (Optimal Solutions of Post Pricing) The optimal solutions of the platform equal $p^* = \frac{1}{2}$, $D^* = \frac{m}{2}$, and $\pi^*(p^*) = \frac{(1-\gamma)m}{4}$ if $m < \gamma n$, or $p^* = \frac{m}{m+\gamma n}$, $D^* = \frac{\gamma mn}{m+\gamma n}$, $\pi^*(p^*) = \frac{(1-\gamma)\gamma nm^2}{(m+\gamma n)^2}$ otherwise.

In double auction mechanism, the effective demand equals effective supply: $nF(\frac{p-a_p}{b_p}) = m\bar{G}(\frac{p-a_c}{b_c})$. The optimal results are described as the following lemma.

Lemma 2.6.2.2 (Optimal Solutions of k-Double Auction)

(*i*) If $k = 1$, then the optimal solutions equal $p^* = \frac{m}{\gamma n+m+1}$, $D^* = \frac{\gamma mn}{\gamma n+m+1}$, and $\pi^* = \frac{(1-\gamma)\gamma m^2 n}{(m+1+\gamma n)^2}$;

(*ii*) If $k = 0$ and $\gamma = 1$, then the optimal solutions equal $p^* = \frac{m+1}{n+1+m}$, $D^* = \frac{mn}{n+1+m}$, and $\pi^* = 0$;

(*iii*) If $k \in (0,1)$, then the optimal solutions equal $p^* = \frac{m^2}{(m+n)(k+m)}$, $D^* = \frac{mn}{n+m}$, and $\pi^* = \frac{(1-\gamma)m^3 n}{(k+m)(m+n)^2}$ when $a_p = a_c = 0$.

By Lemmas Lemma 2.6.2.1 and Lemma 2.6.2.2, post pricing does not have price advantage in the absence of providers' strategic behavior: except SODA, both BBDA and GDA generate lower prices than PP. And only GDA generates higher transaction volume than PP.

2.6.2.2 Post Pricing vs. K-double Auction

Proposition 2.6.2.2 (Post Pricing vs. K-double Auction) PP dominates BBDA. PP dominates GDA if $n > \min\{\frac{m}{\gamma}, \frac{m(1-\gamma)\sqrt{\gamma m(k+m)}-\gamma mk}{\gamma k+\gamma m-\gamma^2 m}\}$ when $a_c = a_p = 0$. Proposition 2.6.2.2 shows that the platform still earns higher revenue in PP than in BBDA, and PP is superior than GDA if there are enough providers. This is because GDA (PP) has an advantage over the transaction volume (transaction price), and as more providers join in, the demand expansion effect of GDA becomes less evident (i.e., $\frac{\partial(D^{GDA}-D^{PP})}{\partial n} < 0$ if $n > \frac{\sqrt{\gamma}m}{\gamma}$). Proposition 2.6.2.2 confirms that the pricing entity and demand-supply intensity are the keys in explaining performance of post pricing and k-double auction.

By the comparable results of DP vs. DA, SP vs. DA and PP vs. DA, we claim that the presence of providers' strategic behavior is a strong incentive for the platform to use k-double auction rather than post pricing. This is because the platform can mitigate the demand-supply mismatch induced by providers' strategic behavior by giving up its pricing power.

2.7 Concluding Remarks

The sharing platform can use various pricing mechanisms to mitigate demand-supply mismatch in two-sided market. We examine the optimal pricing mechanism, dynamic pricing or k-double auction, for the sharing platform in the presence of strategic providers. We consider a two-period model, in which the platform sets prices in dynamic pricing mechanism, or determines the transaction policy in k-double auction. Results show that there is no absolute dominant strategy for the sharing platform, the pricing entity, the pricing flexibility and the demand-supply intensity are the keys that explain the performance of pricing and k-double auction. Pricing dominates Buyer's Bid Double Auction if the platform has pricing flexibility, or when there are many customers if the platform does not have pricing flexibility. General Double Auction dominates pricing no matter whether the platform has pricing flexibility or not. Moreover, customers are always better off under k-double auction, providers and the whole society are always better off under k-double auction except BBDA.

There are several avenues for our work extension. First, we consider rational and risk-neutral participants, behavior economics, such as risk attitude or overconfidence, is worth attention. Second, asymmetric information is also worth investigation. Third, the two-period matching model is not applicable to some online sharing platforms, and the two-period model we consider is a simplification of a multi-period model setting, we can extend our model to a multi-period one to capture more realistic features. Finally, competition among platforms also needs further discussion, such as the single- and multi-homing behavior of participants on two sides. More complicated but possibly insightful results can be derived and we leave the analysis to our future work.

CHAPTER 3

AN INVESTIGATION INTO PROBABILISTIC SELLING: OPAQUE SELLING OR UPGRADING

3.1 Introduction

The travel industry has been sold excess capacity with a discount over the recent years. Examples include the weekend getaways program on starwoodhotels.com and last-minute travel deals on Hotwire prevail in hotel industry. Selling mechanism with probabilistic nature has become a popular way to handle excess capacity. Opaque selling and upgrading are two typical probabilistic selling mechanisms with capacity offerings uncertainties.

Opaque intermediaries, exemplified by Priceline, Hotwire and Amazon, have emerged as main channels to sell excess capacities. For instance, Express Deals and Pricebreakers on Priceline.com and Hot Rate Hotels on Hotwire.com offer a discount price by hiding the identity of the item, such as the location of a hotel room, or an airline rout of an airline ticket. Customers are not informed of this information until payment is submitted. Capacities within an opaque mix are horizontally or vertically differentiated. For example, a lucky bag consisting of at least two products varied in colors is put on sale. Hotwire and Priceline provide hotel rooms which differ in brands, such as 4-star opaque hotel including Hyatt, Holiday Inn and Sheraton. The final capacity allocated to customers depends on the provider's decisions. Opaque selling may bring about benefits or losses to different stakeholders. For instance, price discrimination facilitation, demand-supply mismatches coordination, consumer segmentation, or inventory utilization effects make opaque selling mechanism beneficial. While cannibalization effect and the presence of opaque selling platforms taking a revenue share make aforementioned beneficial effects less evident.

Upgrading, which refers to the replacement of an old product (i.e., a low-quality product) with a new one (i.e., a high-quality product), is also prevalent in travel industry.

Nor.1.com and Optiontown.com are two typical websites that handle upgrading business. Different from opaque selling, upgrading involves vertical differentiation. For instance, hotels and airlines handle excess business-class fares and luxury rooms with a front-desk upselling mechanism, under which customers who have booked economic-class fares or standard rooms now get upgraded by paying an additional fee. eStandby upgrades are available at the booking time in hotel industry, customers who accept this upgrade will pay this offer once the upgrading fulfillment is successfully completed at the check-in time. The probability of customers getting upgraded successfully in either last-minute and conditional upgrade also depends on provider's decisions. Upgrading offers the seller another avenue in price discrimination, risk pooling and demand uncertainty mitigating. While concerns over unfairness and illegal capacity hoarding, and prohibitive cost induced by additional software and staff can not be ignored.

Both upgrading and opaque selling involve at least two kinds of capacities to create opacity or to provide upgrades, and both mechanisms are targeted for low-valuation customers. That is, customers with low valuation participate in upgrading/opaque selling mechanism anticipate a possibility of obtaining high-quality capacities with a discounted price. While upgrading (resp., opaque selling) is employed mainly for high-quality (resp., low-quality) capacities disposal. Specifically, high-quality capacities used as upgrades are offered to customers having purchased low-quality capacities. High- and low-quality capacities are mixed together and offered as an opaque mix with the same price. Moreover, opaque selling can be managed by a third-party platform in a more organized manner (Fay 2008). While parties that manipulate regular selling without probabilistic nature and the ones operating upgrading can be the same or different. Opaque selling and upgrading differ in pricing, capacity offerings, timing, market manipulation and positioning. Hence, performance of these two mechanisms on the seller deserves in-depth explorations. We aim to explore how a monopolist seller with limited capacity can employ two mechanisms individually or jointly to manage excess capacity offerings in vertical differentiated markets.

Previous studies have examined the economic impacts of opaque selling and upgrading separately. The incorporation of two mechanisms in a unified framework is rarely seen in the extant literature and the reality as well. Recognizing this research gap, we aim to answer the following questions:

(i) Under pure use, which probabilistic selling mechanism is more beneficial to the seller? Do the optimal prices and transaction volumes of probabilistic selling and upgrading differ? What is the main difference between two probabilistic mechanisms?

(ii) Under mixed use, whether opaque selling and upgrading are complementary or substitutable? What is the main driving force, the adoption sequence between two mechanisms, the capacity level, or a third-party platform's intervention?

We construct a two-stage model, in which two types of capacities are sold by a seller in the regular stage and leftovers are sold through or not through probabilistic mechanisms in the salvage stage. Under pure pricing mechanism, the seller makes pricing decisions to sell capacities to customers coming in each stage. Under pure upgrading mechanism, a proportion of unsold high-quality capacities are offered as upgrades to regular customers having purchased low-quality ones. After successful upgrading, all unsold capacities are sold regularly to last-minute customers. Under pure opaque selling mechanism, all unsold capacities are offered by an opaque platform as an opaque mix to target last-minute customers. Under the joint adoption of two probabilistic mechanisms, the seller first determines the number of high-quality capacities used for upgrades, then the opaque platform mixes all remaining ones to sell as opaque capacities when upgrading comes first, or the opaque platform first sells capacities collectively then the seller offers remaining high-quality capacities as upgrades when opaque selling comes first. We use backward induction to seek sub-game Nash equilibrium to solve the sequential game in each scenario.

Our main management insights are summarized as follows: (i) Under pure use, upgrading is superior than pricing, and pricing is superior than opaque selling. This is attributed to the pricing flexibility and demand segmentation optimization of upgrading, and high-quality sales cannibalization effect of opaque selling. (ii) Under mixed use, two probabilistic mechanisms are substitutes when the high-quality capacity level is extreme low without upgrading platform's participation, while the complementary and substitutable relationships are mixed when the high-quality capacity is in the medium level. If the seller owns large amount of high-quality capacities, then the two mechanisms are substitutes when upgrading comes first or complements when opaque selling comes first. (iii) In addition, we analyze several variants of the model to check the robustness of our main results. First, we introduce conditional upgrading, and results show that the optimal solutions under the joint adoption with conditional upgrade mimick the one of pure opaque selling mechanism.

The reminder of this study proceeds as follows. The following part of Section 3.1.1 introduces some related works. Section 3.2 details model setting. Sections 3.3 and 3.4 are devoted to equilibrium characterization of pure use of probabilistic selling mechanisms and mixed use of probabilistic mechanisms, respectively. Section 3.5 unfolds model extension. Section 3.6 summarizes the study and points out several avenues for future research. All mathematical proofs are referred to the appendix.

3.1.1 Literature Review

Our work relates to three streams of literature: opaque selling mechanism, upgrading mechanism, and vertical product line design.

An extensive literature examines the dynamics of opaque selling. One part of this stream studies the impact of opaque selling on a firm's marketing and operations decisions. (Elmachtoub et al. 2015) explore the role of opaque selling in saving ordering and holding costs in supply chains, and they claim that opaque selling improves the retailer's profit because of significant cost reduction. (Fay and Xie 2015) compare PS-Early (where the seller makes assignments before demand realization) and PS-Late (where the seller makes assignments after demand realization), they show that PS-Early is superior than PS-Late, because the seller can charge higher price when consumers value opaque products more in PS-Late with random allocation than in PS-Early with determined allocation.

(Zhang et al. 2015) characterize the market expansion effect of opaque selling in vertical differentiated markets, and they claim that opaque selling is beneficial for the implementation of opaque selling introduces a new type of products with quality in-between high- and low-level.

Above papers assume that customers are rational, while customers' behavior economics play a vital role in capturing the optimality of opaque selling. For example, (Huang and Yu 2014) examine the role of bounded rationality (in the sense of anecdotal reasoning) on the implementation of opaque selling, and they highlight the benefit of opaque selling in softening price competition in the presence of customers' bounded rationality. (Chao et al. 2016) focus on anticipated regret, which implies that consumers will experience post-purchase regret when the opaque product turns out to be a low-type one. They prove that anticipate regret makes probabilistic selling more attractive for the firm, because this regret increases product differentiation, which increases the firm's price. (Zheng et al. 2019) study the impact of context-dependent preference (i.e., salient thinking), which refers to the practice that consumers evaluate each choice taking into account it's absolute and relative position in an option with several choices, on the pricing and product line design problems of probabilistic products. Because probabilistic selling mechanism makes consumers' choice set in a more favorable way, the seller obtains benefit in the presence of salient thinkers.

Another part of this stream is to compare opaque selling with other selling mechanisms. (Fay and Xie 2010) compare opaque selling and advance selling which both involve buyer uncertainty, and they find that the dominance between two mechanisms depends on buyers' heterogeneity. (Jerath et al. 2010) build a two-stage model, where transparent selling is employed in the first stage and last-minute selling or opaque selling is prevalent in the second stage. They compare opaque selling and last-minute selling in the presence of strategic customers in horizontal differentiated market. They show that opaque selling yields higher profits than last-minute selling for the seller, because opaque selling attracts customers who would not have purchased in last-minute selling to purchase in opaque selling channel now. (R. R. Chen et al. 2014) construct a two-period model to compare PP (posted price) and NYOP (name-your-own-price) mechanism in the context of opaque selling. They show that the dominance of PP over NYOP depends on the power of extracting postponers' expected surplus. Under PP, the retailer sets price while under NYOP the retailer has no choice but to accept or reject bids.

(Ren and Huang 2017) is the most relevant work to ours. They investigate how the firm makes pricing and inventory decisions when employing opaque selling in vertical differentiated markets. Unlike (Zhang et al. 2015) who consider simultaneous use of transparent and opaque selling, (Ren and Huang 2017) introduce transparent selling (in the regular-season) and opaque selling (in the sales-season) in a sequential manner, and they clarify that opaque selling increases regular-season revenue because of inter-temporal cannibalization reduction. Similar to (Ren and Huang 2017), probabilistic mechanisms are employed as capacity clearance policy in our model, while inter-temporal cannibalization is not effective in our model with myopic customers. We highlight the price discrimination and demand segmentation of upgrading and demand fulfillment flexibility of opaque selling. Moreover, our paper aims to explore the complementary or substitutable role between two probabilistic mechanisms in vertical differentiated markets. To our knowledge, we are the first to incorporate opaque selling and upgrading in disposing of leftovers in the context of vertical differentiation.

Our paper also connects to the literature on upgrade, which can be classified as firm-driven upgrade and consumer driven upgrade. Firm-driven upgrade is provided at the discretion of the firm to manage stockout demand, and customers do not have to pay (Ceryan et al. 2018; Y. Yu et al. 2015; Çakanyıldırım et al. 2020). As for consumer-driven upgrade, (Yılmaz et al. 2017) prove that conditional upgrading, which can be act as a price discrimination tool, is beneficial to the Hotel in the presence of myopic customers. (Cui et al. 2019) study the optimal pricing problem of conditional upgrade, they conduct both theoretical and empirical analysis to verify the optimality of upgrades for the firm. By introducing upgrades, customers will pour in as the firm will decrease the price of base product (admission effect), and as customers value the purchase more with the upgrading availability (valuation effect). Both effects boost revenues. (Cui et al. 2018) explore the economic impact of condition upgrade in the presence of strategic customers, and they show that the inventory allocation and demand segmentation effects of condition upgrades utilize demand-supply matching, which generates more revenues for the firm. All these papers focus on the pricing and role exploration of upgrades, whereas we integrate opaque selling and upgrading to identify conditions that permit the complementary or substitutable role of two mechanisms.

Moreover, our paper is built off of the literature on vertical product line design. Topics can be separated into two sides. On the supply side, research focuses include the optimal product line design under monopoly and duopoly setting (Desai 2001), simultaneous or sequential product introduction, variety design in co-products (Y.-J. Chen et al. 2013) and resources management with allocation flexibility (Yayla-Küllü et al. 2021). On the demand side, information asymmetry in customers' valuation (Biyalogorsky and Koenigsberg 2014), and behavior economies including uninformed quality perception (Guo and Zhang 2012) or anticipated regret (Zou et al. 2020) are explored. We extend literature on product variety management to consider capacity allocation within time and channel dimensions.

3.2 Model Setup

Consider a monopolist sells vertical differentiated capacities to two streams of customers over a two-stage selling season, which includes stage 1, termed the regular stage, and stage 2, termed the salvage stage. In the regular stage, capacities are sold regularly by the seller through pricing. In the salvage stage, unsold capacities are disposed of, either by the seller through upgrading, pricing or by a third-party platform through opaque selling. Capacities are service goods, which are not consumed till the end of the selling season. Stakeholders including the seller (he), the platform (it), and each customer (she) are payoff maximizers.

Customers' Behavior

D_i customers arrive in stage i, where $i \in \{1, 2\}$. Here assuming two separate streams of demand allows us to capture situations such that not all customers are aware of the salvage selling process. Customers are heterogeneous in their preference (denoted by θ) towards quality, which is uniformly distributed over interval $[0, 1]$, and does not change with customers' types. Customers are myopic, and each of them consumes at most one unit of capacity so as to maximize her individual surplus, we model the surplus of capacity with quality q as directly proportional to her valuation level θ, so that the consumption surplus equals $U = \theta q - p$. The consumption surplus is normalized to zero for customers without purchasing.

The Seller's Behavior

The seller owns K_H high-quality capacity with quality level q_H and K_L low-quality capacity with quality level q_L. We assume that capacities are exogenously given in the main analysis, because building and expanding capacity is time- and money-consuming, such as hotel room amenities construction. We will explore the dynamics under endogenous capacity in the extension part. Quality levels are also exogenous. For notation convenience, we normalize low-quality level to one and high-quality level to δ times of low-quality level. We assume that the relative quality level satisfies $\delta \in (1, 1 + \frac{3D_1D_2+3D_2^2}{D_1^2})$[1].

In the regular stage, the seller sets prices p_{1H} and p_{1L} for high- and low-quality capacities, respectively. In the salvage stage, the seller manages upgrading and pricing by himself, or seeks help from a third-party platform in employing opaque selling mechanism. We assume that deliberate capacity hoarding is absent in different selling formats. Detailed dynamics are as follows.

Upgrading

Under pure upgrading (denoted by U), in the situation where high-quality capacities have leftovers in the salvage stage, the seller offers upgrades to customers having purchased low-quality capacities in the regular stage by charging an additional price p (the actual price satisfies $p_U = p + p_{1L}$) or determining the upgrading quantity S[2]. In the situation where capacities remain unsold after upgrading, then the seller sets prices to sell through

1. We clarify that our main results are robust when we allow $\delta \geq 1 + \frac{3D_1D_2+3D_2^2}{D_1^2}$.

2. Pricing and quantity decision are perfect substitute for the seller, refer to Section 3.3.2 for details.

pricing. The case that the seller does not offer upgrades is equivalent to pure pricing mechanism (denoted by P). And we assume that unsold capacities and abandoned ones from low-valuation customers at the end of the selling season have no value to the seller.

Opaque Selling

Under pure opaque selling (denoted by O), a third-party platform manages the opaque selling mechanism in disposing of high- and low-quality capacities jointly. And a two-part tariff contract (namely, a two-way revenue sharing contract) is used to capture the interaction between the seller and the platform. Specifically, the seller collects γ_O ($\gamma_O \in [0, 1]$) proportion of revenues from each transaction, and pays the rest to the platform. And the seller pays a long-term fee F_O to the platform to encourage it's participation. This revenue sharing contract is widely used in practice. Common examples include cover charges and per-drink prices at bars, registration fees and per-ride fees at amusement parks, and wholesale sourcing memberships. The sequence of events in the salvage stage is described as: Before the start of the salvage stage, the third-party platform determines a demand fulfilling strategy ϕ (i.e., the probability that the platform will assign high-quality capacity to customers when they purchase the opaque mix), where $\phi \in [0, 1]$, together with an opaque price p_O. Last-minute customers decide whether to consume opaque capacities anticipating the expected quality of the opaque mix. Finally, the platform shares revenues with the seller based on the predetermined contract.

The sequence of events in pure adoption of opaque selling/upgrading is depicted in Figure 3.1.

Upgrading & Opaque Selling

Upon joint adoption of upgrading and opaque selling, the sequence between upgrading and opaque selling and a third-party platform's participation give rise to different market structures. For instance, UO stands for the system where the seller manages upgrading and the platform manages opaque selling sequentially to dispose of leftovers, system OU is the opposite to UO. In this paper, we assume exogenous market structures for our core interest is to explore how different system structures affect equilibrium outcomes rather than to investigate system emergence per se.

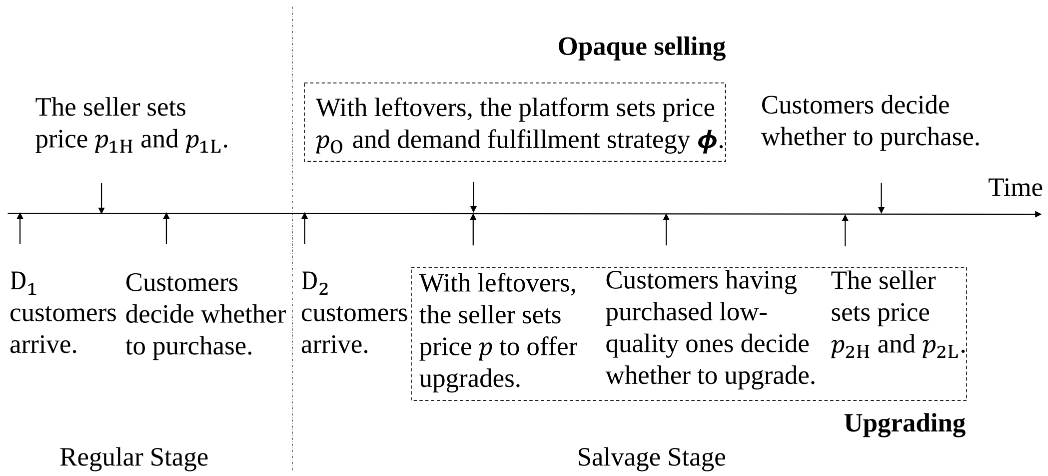

Figure 3.1: Sequence of Events in Pure Adoption of Probabilistic Mechanisms

If upgrading mechanism comes first, remaining low-quality capacities from the regular stage, abandoned low-quality capacities from customers accepting upgrades and high-quality leftovers from upgrading mechanism form the opaque mix. This is consistent with (Cui et al. 2018) and (Çakanyıldırım et al. 2020), in their models, the same amount of regular capacities are released for possible future sales when customers requesting upgrades get upgraded. If opaque selling mechanism comes first, then all unsold high-quality capacities from opaque selling mechanism are sent directly to the upgrading mechanism.

The sequence of events in the joint adoption scenario that upgrading mechanism comes first is depicted in Figure 3.2: The seller in the regular stage makes pricing decisions for both high- and low-quality capacities, regular customers arrive and decide whether and which type of capacities to purchase upon observing the posted prices. At the end of regular stage, in the situation where capacities left unsold, the seller first disposes of high-quality capacities through an upgrading mechanism with an additional price charged for upgrades, then disposes of all remaining capacities if any through an opaque platform. The platform decides the opaque price and the demand fulfillment strategy. Last-minute customers anticipating the fulfillment probability decide whether to consume. The sequence of events in the joint adoption scenario that opaque selling comes first mimicks the one aforementioned, and we omit the details.

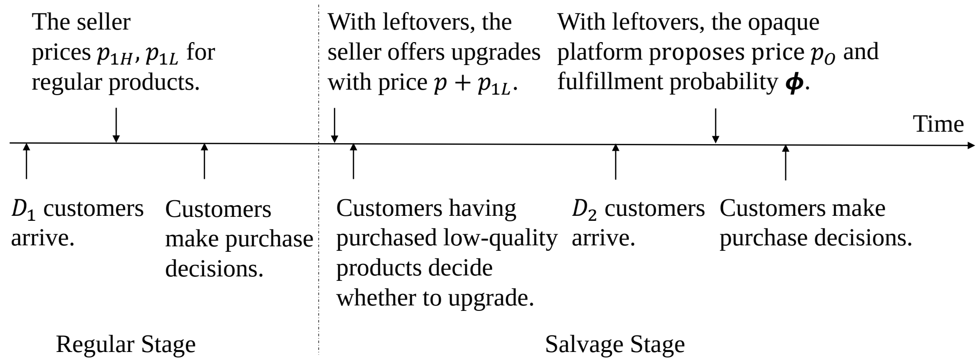

Figure 3.2: Sequence of Events in Joint Adoption

Given these preliminaries, we seek a subgame perfect Nash equilibrium of the sequential game in each scenario, and rational expectation equilibrium of customers' anticipation on the fulfillment strategy.

3.3 Pure Use of Pricing, Upgrading and Opaque Selling

High-quality capacity is a prerequisite for the existence of upgrading mechanism, and opaque selling mechanism is infeasible when either type of capacities are out of stock (Li et al. 2020; Ren and Huang 2017). To study the implications of probabilistic selling mechanisms, we assume that the capacity is sufficient to cover demand in the regular stage[3].

The seller's pricing decisions can be decoupled into (i) choosing the price range to decide the types of capacities to be sold in each stage in a given scenario, and (ii) choosing prices within the range to maximize the total profit under that capacity offering. Hence, our analysis follows two steps: first enumerate all capacity offerings, then compare profit under all capacity offerings.

We define $(H^{\Omega}, L^{\Omega}; H^{\Omega}, L^{\Omega})$ for capacity offerings, where the first (resp., third) term

3. Specific constraints are proposed in each mechanism, details are referred to Section 3.3.1, 3.3.2, 3.3.3, 3.4.1, and 3.4.2.

represents the capacity offering of high-quality capacity in the regular (resp., salvage) stage, and the corresponding Ω includes capacity type $\{P, \varnothing\}$ (resp., $\{P, U, O, U+P, U+O, \varnothing\}$), the second (resp., fourth) term represents the capacity offering of low-quality capacity in the regular (resp., salvage) stage, and the corresponding Ω includes capacity type $\{P, \varnothing\}$ (resp., $\{P, O, \varnothing\}$), note that type $\varnothing$ represents the case that no capacities are sold in equilibrium.

To facilitate scenarios comparison, we assume $K_H \leq K_L$ and $t = \frac{D_2}{D_1} < 1$, where $t > 0$ captures the demand gap across two types. Condition $K_H \leq K_L$ is consistent with the evidence that the seller keeps more low-quality capacities than high-quality ones for procurement or production cost consideration, and $t < 1$ means that last-minute customers are fewer than regular customers. In travel industry, price insensitive customers book in advance while price sensitive customers wait for discounts. The number of the former is more than that of the latter.

3.3.1 Pure Pricing Mechanism

In pure pricing mechanism, tie-breaking rules are as follows: (i) Customers will purchase if they are indifferent between purchasing and no purchasing. (ii) Customers will purchase high-quality capacities if they are indifferent between buying two types of capacities.

With customer's utility in purchasing j-type capacity in stage i denoted by $U_j = \theta q_j - p_{ij}$, where $i \in \{1, 2\}$, $j \in \{H, L\}$, customers in stage i purchase high-quality capacities if $\theta\delta - p_{iH} \geq (\theta - p_{iL})^+$ or choose low-type ones if $\theta - p_{iL} > (\theta\delta - p_{iH})^+$, where $x^+ = \max\{x, 0\}$. Define $\theta_{iH} = \frac{p_{iH} - p_{iL}}{\delta - 1}$ and $\theta_{iL} = p_{iL}$, customers with preference higher than or equal to θ_{iH} choose high-quality capacities, and customers with preference no less than θ_{iL} and less than θ_{iH} choose low-quality capacities. The high-quality effective demand is $D_i(1 - \theta_{iH})$ and low-quality effective demand is $D_i(\theta_{iH} - \theta_{iL})$ in stage i.

Given θ_{1L} and θ_{1H} and the one-to-one correspondence between prices and indifference thresholds, the seller's optimization problem of stage two is written as

$$\max\ \pi_{2P}(\theta_{2H},\theta_{2L}) = (\theta_{2L}+\theta_{2H}(\delta-1))\min\{D_2(1-\theta_{2H}), K_H - D_1(1-\theta_{1H})\}$$
$$+\,\theta_{2L}\min\{D_2(\theta_{2H}-\theta_{2L}), K_L - D_1(\theta_{1H}-\theta_{1L})\}$$
$$s.t.\quad 0\le\theta_{2L}\le\theta_{2H}\le 1,$$

where term $\theta_{2L}+\theta_{2H}(\delta-1)$ (resp., θ_{2L}) represents high (resp., low) posted price, and the transaction volume is defined as the minimum of available demand and capacity. The constraint is imposed to guarantee well-defined consumer segments without loss of generality (Pan and Honhon 2012). That is, the θ line parts must form a mutually exclusive and collectively exhaustive partition of θ line (Kornish 2001). The seller sets prices in the regular stage anticipating the outcomes in the salvage stage to optimize the profit over the selling season

$$\max\ \pi_P(\theta_{1H},\theta_{1L}) = (\theta_{1L}+\theta_{1H}(\delta-1))D_1(1-\theta_{1H}) + \theta_{1L}D_1(\theta_{1H}-\theta_{1L}) + \pi^*_{2P}(\theta^*_{2H},\theta^*_{2L})$$

$$s.t.\quad \begin{cases} K_H - D_1(1-\theta_{1H})\ge 0,\\ K_L - D_1(\theta_{1H}-\theta_{1L})\ge 0,\\ 0\le\theta_{1L}\le\theta_{1H}\le 1.\end{cases}$$

The first and second constraint means that high- and low-quality capacities remain unsold at the beginning of the salvage stage, respectively.

Lemma 3.3.1 is formally restated in the appendix, and we present the equilibrium capacity offerings and corresponding properties under pricing mechanism as follows:

Lemma 3.3.1 Under pure pricing mechanism, the equilibrium strategy can be divided into three areas as shown in Figure 3.3. The seller's optimal capacity offerings over the whole selling season must be $(H^P, L^\varnothing; H^P, L^\varnothing)$ (Area (I)) or $(H^P, L^P; H^P, L^P)$ (Areas (II) and (III)). In equilibrium,

(i) The optimal prices of a certain type of capacity over two stages are equal: $p^*_{1H} = p^*_{2H} > p^*_{1L} = p^*_{2L}$;

(ii) Either maximum demand ($\frac{D_1+D_2}{2}$) is covered if $K_H + K_L > \frac{D_1+D_2}{2}$ or all supply ($K_H + K_L$) is cleared if $K_H + K_L \le \frac{D_1+D_2}{2}$.

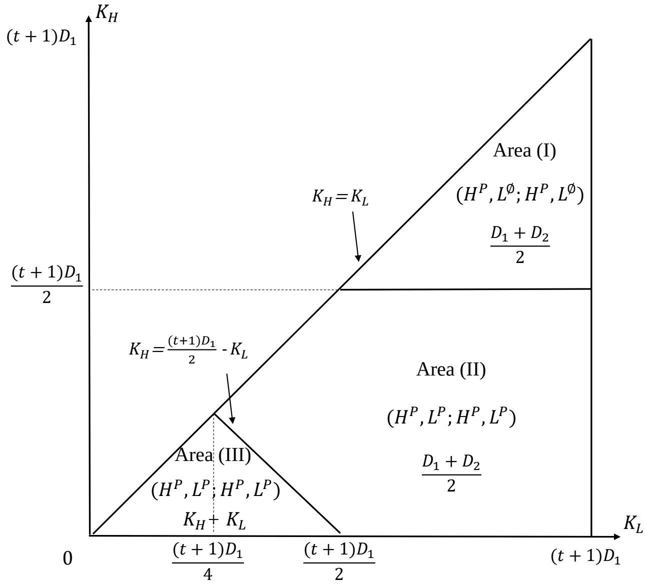

Figure 3.3: State-dependent Generation Process Under Pricing Mechanism

By part (i) of Lemma 3.3.1, when there are both types of capacities at the beginning of each stage, and demand over stages are independent, then the optimal prices of capacities with certain types charged at each stage are stable (i.e., $p^*_{1H} = p^*_{2H}$ and $p^*_{1L} = p^*_{2L}$). And the seller has no incentive to differentiate the capacity offerings in two stages. That is, the seller prices higher to sell only high-quality capacities or prices lower to sell both types of capacities. Moreover, we confirm two standard properties regarding price competition in terms of vertical differentiation as (Shaked and Sutton 1982): The price charged for high-quality capacity is higher than the low posted price, and the seller's total profit increases with the vertical differentiation parameter δ.

By part (ii) of Lemma 3.3.1, when the amount of high-quality capacities is sufficient in Area (I) (i.e., the capacity level is more than the maximum demand coverage $K_H > \frac{D_1+D_2}{2}$), the seller only sells high-quality capacities to customers who value the capacity most, that is, the optimal capacity offering is $(H^P, L^\varnothing; H^P, L^\varnothing)$. The amount of high-quality capacities sold with price $\frac{\delta}{2}$, at which we call the maximum coverage price, in stage i equals $\frac{D_i}{2}$.

When the amount of high-quality capacities is less than the maximum demand coverage, while the amount of total capacity is more than the maximum demand coverage in Area (II) (i.e., $K_H < \frac{D_1+D_2}{2}$ and $K_H + K_L > \frac{D_1+D_2}{2}$), the seller sells both types of capacities in each stage, that is, the optimal capacity offering is $(H^P, L^P; H^P, L^P)$. $\frac{D_i K_H}{D_1+D_2}$ high-quality capacities are sold with price $\delta - \frac{1}{2} - \frac{(\delta-1)K_H}{D_1+D_2}$ and $\frac{D_i}{2} - \frac{D_i K_H}{D_1+D_2}$ low-quality capacities are sold with price $\frac{1}{2}$ in stage i. Note that both revenues from high-quality capacities and revenues over the whole selling season increase with K_H, while revenues from low-quality capacities decrease with K_H. This is because when the seller owns more high-quality capacities, high posted price decreases, then customers who plan to buy low-quality capacities now turn to purchase high-quality ones. To this end, when the high-quality capacity level increases, the seller abandons low-quality sales, and the optimal capacity offering reduces to $(H^P, L^\varnothing; H^P, L^\varnothing)$.

When the total capacity is less than the maximum demand coverage in Area (III) (i.e., $K_H + K_L < \frac{D_1+D_2}{2}$), the seller sells out all capacities $K_H + K_L$, both types of capacities are sold in each stage, that is, the optimal capacity offering is $(H^P, L^P; H^P, L^P)$. $\frac{D_i K_H}{D_1+D_2}$ high-quality capacities are sold with price $\delta - \frac{\delta K_H + K_L}{D_1+D_2}$ and $\frac{D_i K_L}{D_1+D_2}$ low-quality capacities are sold with price $1 - \frac{K_H+K_L}{D_1+D_2}$ in stage i. The price at which all capacities are completely depleted is so called the supply clearance price. Note that revenues from high-quality capacities increase (resp., decrease) with capacity level K_H (resp., K_L) while revenues from low-quality capacities decrease (resp., increase) with K_H (resp., K_L). When both K_H and K_L increase a little bit, the seller's revenues from high-quality capacities remain unchanged. This is because the increment driven by the increase of K_H is mitigated by the decrement induced by the increase of K_L. When total capacity level is above the threshold $\frac{D_1+D_2}{2}$, the seller's profit only increases with K_H while is independent of K_L, and the optimal capacity offering reduces to $(H^P, L^\varnothing; H^P, L^\varnothing)$ when K_H increases.

3.3.2 Pure Upgrading Mechanism

In what follows, we first introduce the dynamics of pure upgrading model, then compare Scenarios P and U with an emphasis on identifying conditions under which the seller can enjoy higher benefits.

3.3.2.1 Pure Upgrading Model

In pure upgrading mechanism, with remaining high-quality capacity $K_H - D_1(1-\theta_{1H})$ and low-quality capacity $K_L - D_1(\theta_{1H} - \theta_{1L})$, the seller offers S high-quality capacities (which are unobservable to customers) as last-minute upgrades and sets price $p_{1L} + p$ to target customers with preferences within interval $[\theta_{1L}, \theta_{1H})$. Since pricing and rationing are strategic substitutes, rationing is less likely to happen (Maglaras and Meissner 2006), there is a one-to-one correspondence between pricing and allocation.

Customers' utility of accepting the upgrade and getting upgraded successfully is defined as $U_U = \theta\delta - (p + p_{1L})$, and we assume that customers are more willing to accept upgrades than to keep with low-quality capacities when the two yields the same utility. Hence, customers having consumed low-quality capacities will accept the upgrade if $Pr(\theta\delta - p_{1L} - p) + (1 - Pr)(\theta - p_{1L}) \geq \theta - p_{1L}$ (or equivalently, $\theta \geq \theta_U = \frac{p}{\delta-1}$), where Pr denotes the probability of getting upgraded successfully.

Note that the amount of high-quality capacities used for upgrades is capped by the number of customers having purchased low-quality capacities and high-quality capacities in the regular stage: $S \in [0, \min\{D_1(\theta_{1H} - \theta_{1L}), K_H - D_1(1 - \theta_{1H})\}]$. Hence, the number of successful transactions equals $\min\{S, D_1(\theta_{1H} - \theta_U)\}$ provided that $\theta_U \in [\max\{\theta_{1L}, 1 - \frac{K_H}{D_1}\}, \theta_{1H}]$.

In the salvage stage, the seller's problem after upgrading is described as:

$$\begin{aligned}
&\max\ \pi_{2U}(\theta_{2H}, \theta_{2L}) = (\theta_{2L} + \theta_{2H}(\delta - 1)) \min\{D_2(1 - \theta_{2H}), K_H - D_1(1 - \theta_U)\} \\
&+ \theta_{2L} \min\{D_2(\theta_{2H} - \theta_{2L}), K_L - D_1(\theta_U - \theta_{1L})\} \\
&s.t. \quad 0 \leq \theta_{2L} \leq \theta_{2H} \leq 1,
\end{aligned}$$

The seller's problem in the salvage stage is described as:

$$\begin{aligned}
&\max\ \pi_2(\theta_U) = D_1(\delta - 1)(\theta_{1H} - \theta_U)\theta_U + \pi^*_{2U}(\theta^*_{2H}, \theta^*_{2L}) \\
&s.t. \quad \max\{\theta_{1L}, 1 - \frac{K_H}{D_1}\} \leq \theta_U \leq \theta_{1H}.
\end{aligned}$$

The constraint means that the price charged for the upgrade can not exceed the high posted price charged in the regular stage. And the number of upgrades is no less than either the amount of customers having purchased low-quality capacities or the amount of remaining high-quality capacities from the regular stage.

In the regular stage, the seller anticipating the optimal solutions of the salvage stage solves the following optimization problem:

$$\max \ \pi_U(\theta_{1H}, \theta_{1L}) = (\theta_{1L} + \theta_{1H}(\delta - 1))D_1(1 - \theta_{1H}) + \theta_{1L}D_1(\theta_{1H} - \theta_{1L}) + \pi_2^*(\theta_U^*)$$

$$s.t. \quad \begin{cases} 0 \leq \theta_{1L} \leq \theta_{1H} \leq 1, \\ K_H \geq D_1(1 - \theta_{1H}), \\ K_L \geq D_1(\theta_{1H} - \theta_{1L}). \end{cases}$$

The equilibrium capacity offerings and properties regarding the transaction prices and transaction volumes under pure upgrading mechanism are as follows:

Lemma 3.3.2.1 Under pure upgrading mechanism, the equilibrium strategy can be divided into four areas as shown in Figure 3.4 (the blank area is not applicable (N_A)). The seller's optimal capacity offerings over the whole selling season must be $(H^P, L^P; H^{U+P}, L^P)$ (Areas (I) and (II)), $(H^P, L^{\varnothing}; H^{U+P}, L^P)$ (Area (III)) or $(H^P, L^{\varnothing}; H^{U+P}, L^{\varnothing})$ (Area (IV)). In equilibrium,

(i) The optimal prices charged for high- and low-quality capacities satisfy: $p_{1H}^* > p_{2H}^* > p_U^*$, $p_{1L}^* \leq p_{2L}^*$, $p_{1H}^* + p_U^* = 2p_{2H}^*$, and $p_{1H}^* + p_{1L}^* < 2p_U^*$.

(ii) The amount of high-quality capacities sold in the regular stage and those offered as upgrades are equal in size: $D_1(1 - \theta_{1H}^*) = D_1(\theta_{1H}^* - \theta_U^*)$.

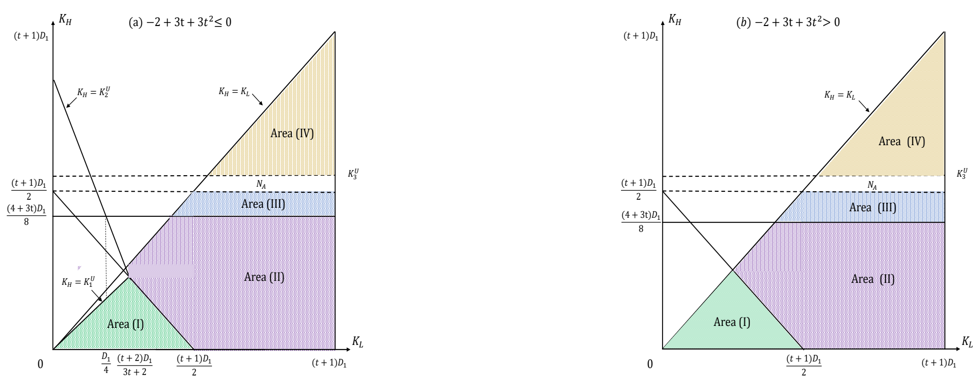

Figure 3.4: State-dependent Generation Process Under Upgrading Mechanism

Lemma 3.3.2.1 reveals that it is neither optimal for the seller to dispose of high-quality leftovers not through pricing in the salvage stage, or only through pricing in the salvage stage. This is because high-quality leftovers sold regularly in the salvage stage will be transacted with a higher price than sold as upgrades ($p^*_{2H} > p^*_U$). In addition, upgrades help the seller separate regular customers with high valuation inter-temporally by selling high-quality capacities sequentially with a premium price in regular stage and a discounted price in salvage stage ($p^*_{1H} > p^*_U$), and upgrades also offer another lever of price discrimination ($p^*_{1H} > p^*_{2H}$ and $p^*_{1L} \leq p^*_{2L}$).

Specifically, when the total capacity level is low as shown in Area (I), the profit from high-quality (resp., low-quality) capacities increases (resp., decreases) with K_H while decreases (resp., increases) with K_L. The total profit increases with both K_H and K_L. The underlying reason is explained as follows: When high-quality capacity level increases, both high and low posted prices decrease, and the downward trend of the high posted price is more evident, then customers who plan to purchase low-quality capacities now choose to purchase high-quality capacities. When low-quality capacity level increases, high and low posted price decreases with the same degree, but customers who can not afford previously now choose to purchase low-quality capacities. When only the high-quality capacity level is low as shown in Areas (II) and (III), the the profit from high-quality (resp., low-quality) capacities increases (resp., decreases) with K_H while is independent of K_L. The seller's total profit increases with K_H while is independent of K_L. With high-quality capacity increases, high posted price increases while low posted price remains unchanged. So, purchasing high-quality capacities becomes more attractive to customers who plan to purchase low-type ones. When the total capacity level is rather high as shown in Area (IV), then the seller's total profit is independent of the capacity level.

When compared with pure pricing mechanism, we also find that the seller's total profit increases with the vertical differentiation parameter ($\frac{\partial \pi^*_U}{\partial \delta} > 0$). And if the seller prices stable prices for two types of capacities over stages ($p^*_{1H} + p^*_U = 2p^*_{2H}$ and $p^*_{1L} = p^*_{2L}$), then the optimal capacity offerings are symmetric (i.e., $(H^P, L^P; H^{U+P}, L^P)$ and $(H^P, L^{\varnothing}; H^{U+P}, L^{\varnothing})$) given that the amount of high-quality capacities sold regularly and the number of upgrades are equal in size. While the emergence of $(H^P, L^{\varnothing}; H^{U+P}, L^P)$ can be attributed to a higher price margin of low-quality capacities sold in the salvage stage than in the regular stage ($p^*_{1L} < p^*_{2L}$). Moreover, if $K_H < \frac{4D_1+3D_2}{8}$, then the seller fulfills only part of the upgrading demand, and the total transaction volumes are the same in Scenarios U and P. Otherwise, the seller fulfills all the upgrading demand, and the total transaction volumes are higher in Scenario U than in Scenario P.

3.3.2.2 Pricing v.s. Pure Upgrading

Theorem 3.3.2.2 (Pricing vs. Pure Upgrading) Pure upgrading dominates pricing mechanism: $\pi^*_U > \pi^*_P$.

Theorem 3.3.2.2 shows that introducing upgrades sequentially helps the seller collects higher revenues ($\Delta\pi^{U-P} > 0$) by managing the capacity through the ex post availability-based substitution tool (i.e., upgrading) in addition to the ex ante price-based substitution tool (i.e., pricing). And the profit gap between upgrading and pricing mechanisms gets larger as the high-quality capacity level increases ($\frac{\partial\Delta\pi^{U-P}}{\partial K_H} \geq 0$). Furthermore, upgrading benefits the seller in both types of capacities ($\Delta\pi^{U-P}_H > 0$, $\Delta\pi^{U-P}_L \geq 0$). Corollary 3.3.2.2 shows how the equilibrium prices and transaction volumes vary under two mechanisms.

Corollary 3.3.2.2 (Transaction Price and Transaction Volume Comparison (U-P))

(i) For high-quality capacities, the revenue generated in Scenario U is higher than in Scenario P, moreover, $\Delta p^{U-P}_{1H} > 0$, $p^U_{1H} + p^*_U - 2p^P_{1H} \geq 0$, $\Delta D^{U-P}_{1H} < 0$, $\Delta p^{U-P}_{2H} \geq 0$, and $\Delta D^{U-P}_{2H} > 0$.

(i) For low-quality capacities, the revenue generated in Scenario U is no less than in Scenario P, moreover, $\Delta p^{U-P}_{1L} \leq 0$, $\Delta D^{U-P}_{1L} \leq 0$, $\Delta p^{U-P}_{2L} = 0$, and $\Delta D^{U-P}_{2L} \geq 0$.

By Corollary 3.3.2.2, upgrading gives the seller a finer segmentation of high-quality demand and helps the seller attract more low-valuation customers by decreasing low posted price. That is, the revenue increment of upgrading can be attributed to the demand segmentation and pricing flexibility of high-quality capacities and price discrimination of low-quality capacities. Details are as follows.

When the high-quality capacity is rather large ($K_H > \frac{D_1+D_2}{2}$), only high-quality capacities are sold. In the regular stage, prices charged for high-quality capacities in two scenarios are the same ($p_{1H}^U + p_U^* = 2p_{1H}^P$), while the transaction volume is higher in Scenario U than in Scenario P. In the salvage stage, both the price and transaction volume are equal in two scenarios.

When the high-quality capacity is neither large or small ($\frac{4D_1+3D_2}{8} < K_H \leq \frac{D_1+D_2}{2}$), all upgrading demand is fulfilled in Scenario U. The total transaction volumes of high-quality capacities and prices charged for low-quality capacity in the salvage stage under two scenarios are the same, while the total transaction volume of low-quality capacities, and prices charged for high-quality capacities in both stages are higher in Scenario U than in Scenario P.

When the high-quality capacity is small ($K_H \leq \frac{4D_1+3D_2}{8}$), the seller offers the same capacity offerings and has the same sales composition. Prices charged for high-quality capacities in both stages are higher Scenario U than in Scenario P, while prices charged for low-quality capacities in two stages are the same.

3.3.3 Pure Opaque Selling Mechanism

In this subsection, we first explore the dynamics of pure opaque selling model followed by the influences of pure opaque selling by comparing the equilibrium results in Scenarios O and P.

3.3.3.1 Pure Opaque Selling Model

In pure opaque selling mechanism, we assume no regular selling after opaque selling process. This is consistent with the claim that opaque selling is mainly used for leftovers disposal (Jerath et al. 2010 and references therein).

In the salvage stage, the platform's fulfillment strategy depends on the remaining capacity. Customers make purchase decisions upon observing the posted price, they do not know the demand fulfilling strategy but develop expectations, under rational expectation equilibrium, rational expectation is fulfilled in equilibrium.[4] Hence, the utility is defined as $U_O = \theta(\phi\delta+1-\phi)-p_O$ provided that their requests will be fulfilled. They will purchase if $\theta \geq \theta_O = \frac{p_O}{\phi\delta+1-\phi}$.

For mathematical tractability, we assume $\gamma_O = 1$, and the fixed cost F_O does not influence the pricing decisions given any capacity configuration. Hence, the seller's optimization problem is given by

$$\pi_{2O} = \theta_O(\phi\delta + 1 - \phi)\min\{D_2(1-\theta_O), K_H + K_L - D_1(1-\theta_{1L})\}$$
$$s.t. \quad 0 \leq \theta_O, \phi \leq 1.$$

For the composition of the total supply, there are $K_H - D_1(1-\theta_{1H}) - \min\{D_1(\theta_{1H}-\theta_U), S\}$ high-quality and $K_L - D_1(\theta_{1H} - \theta_{1L}) + \min\{D_1(\theta_{1H} - \theta_U), S\}$ low-quality capacities.

The following lemma characterizes the unique Nash equilibria of the salvage stage.

Lemma 3.3.3.1 (Optimal Solutions of the Opaque Selling Mechanism) Given θ_{1H} and θ_{1L}, in equilibrium,

(i) The platform's pricing decision

$$p_O^* = \begin{cases} \dfrac{\phi\delta + 1 - \phi}{2} & if K_H - D_1(1-\theta_{1L}) + K_L > \dfrac{D_2}{2}, \\ (1 - \dfrac{K_H - D_1(1-\theta_{1L}) + K_L}{D_2})(\phi\delta + 1 - \phi) & otherwise. \end{cases}$$

(ii) The seller's corresponding profit equals

$$\pi_{2O}^*(p_O^*) = \begin{cases} \dfrac{D_2}{4}(\phi\delta + 1 - \phi) - F_O, \\ (K_H - D_1(1-\theta_{1L}) + K_L - \dfrac{(K_H - D_1(1-\theta_{1L}) + K_L)^2}{D_2})(\phi\delta + 1 - \phi) - F_O. \end{cases}$$

4. Note that without uncertainty or unobservable decision variable (the remaining capacity is a state rather than decision variable), we analyze the rational expectation equilibrium by replacing participants' beliefs with actual outcomes.

(iii) The probability that customers will be assigned by the high-quality capacities is given by

$$\phi^* = \begin{cases} \dfrac{K_H - D_1(1-\theta_{1H})}{K_H + K_L - D_1(1-\theta_{1L})} & if D_2 \geq K_H + K_L - D_1(1-\theta_{1L}), \\ \dfrac{K_H - D_1(1-\theta_{1H})}{D_2} & if D_2 < K_H + K_L - D_1(1-\theta_{1L}), K_H - D_1(1-\theta_{1H}) > \dfrac{D_2}{2}, \\ \dfrac{1}{2} & if D_2 < K_H + K_L - D_1(1-\theta_{1L}), K_H - D_1(1-\theta_{1H}) \leq \dfrac{D_2}{2}. \end{cases}$$

By Lemma 3.3.3.1, when capacity is insufficient to satisfy demand, the platform will increase the posted price to sell all capacities collectively to customers who value the capacity most. The price that clears all capacities is so called as the supply-clearance price, and the probability of obtaining high-quality capacity equals the proportion of high-quality leftovers over the total remaining capacities. Otherwise, the platform prices $\frac{1}{2}$ to extract maximum surplus from the demand side. When the high-quality capacity is ample to fulfill one half demand, the probability of obtaining high-quality capacity is more than $\frac{1}{2}$. Otherwise, the platform will allocate two types of capacities equally.

In the regular stage, the seller's optimization problem is given by

$$\pi_O(\theta_{1H}, \theta_{1L}) = \theta_{1H}(\delta - 1)D_1(1-\theta_{1H}) + \theta_{1L}D_1(1-\theta_{1L}) + \pi^*_{2O}(\theta^*_O)$$

$$s.t. \quad \begin{cases} K_H - D_1(1-\theta_{1H}) \geq 0, \\ K_L - D_1(\theta_{1H} - \theta_{1L}) \geq 0, \\ 0 \leq \theta_{1L} \leq \theta_{1H} \leq 1. \end{cases}$$

The equilibrium results under pure opaque selling mechanism are as follows:

Lemma 3.3.3.2 Under pure opaque mechanism, the equilibrium results can be divided into four areas as shown in Figure 3.5. The seller's optimal capacity offerings over the whole selling season must be $(H^P, L^P; H^O, L^O)$ (Areas (I), (II) and (IV)) or $(H^P, L^{\varnothing}; H^O, L^O)$ (Areas (III)). Note that Areas (III) and (IV) exist if and only if $D_1 < 8D_2$. In equilibrium,

(i) The optimal prices charged for low-quality capacities satisfy: $p^*_{1L} < p^*_O$.

(ii) The effective demand of a specific capacity across stages satisfy: $\sum_{j\in\{H,L\}} D^*_{1j} >$

$\sum_{j\in\{H,L\}} D^*_{2j}$; $\sum_{i\in\{1,2\}} D^*_{iH} = K_H < \sum_{i\in\{1,2\}} D^*_{iL} = K_L$ if $K_H < \frac{D_1}{2}$, or $\sum^{j\in\{H,L\}}_{i\in\{1,2\}} D^*_{ij} = \frac{D_1+D_2}{2}$ and $\sum_{i\in\{1,2\}} D^*_{iH} > \sum_{i\in\{1,2\}} D^*_{iL}$ otherwise.

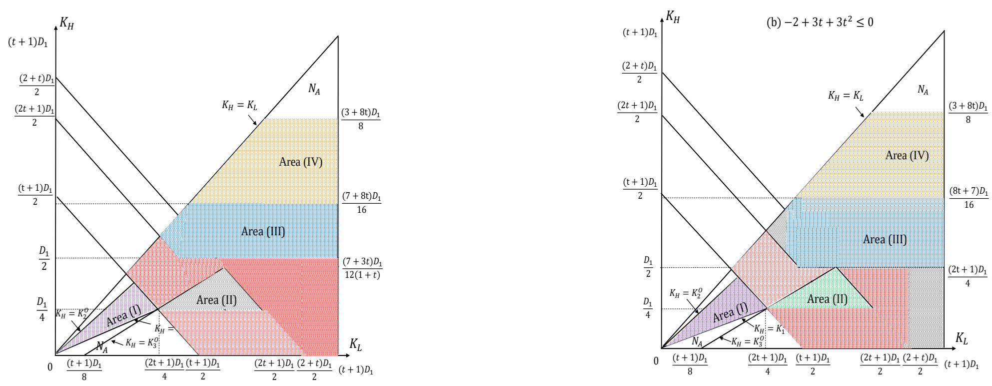

Figure 3.5: State-dependent Generation Process Under Opaque Selling Mechanism

Lemma 3.3.3.2 claims that the regular posted price of low-quality capacity is less than the opaque selling price ($p^*_{1L} < p^*_O$), this is because with the opaque capacity quality no less than the low quality, the seller suffers loss if leftovers are sold through the opaque platform when $p^*_{1L} > p^*_O$. And because of the higher demand in the regular stage than in the salvage stage and the presence of price discrimination effect in selling individually, the seller sells more capacities individually in the regular stage than collectively in the salvage stage.

If high-quality capacity is small ($K_H < \frac{D_1}{2}$), then the probability of purchasing an opaque mix to obtain a high-quality one can be higher or lower than $\frac{1}{2}$ ($\phi^*_I = \frac{-(D_1+2D_2)K_H+D_1K_L}{(\delta-1)D_1K_H-2D_2(K_H+K_L)}$ in Area (I) and $\phi^*_{II} = \frac{4K_H-D_1}{2D_2}$ in Area (II)). The seller's total profit decreases with K_L, while increases with K_H in Area (I) and increases with K_H only when the vertical differentiation is evident ($\delta > \frac{4(K_H+K_L)+D_1-2D_2}{D_1}$) in Area (II). The underlying reasons are as follows.

In Area(I), in the regular stage, when high-quality capacity increases, low-posted price decreases, and the high- and low-posted price gap first decreases then increases ($\frac{\partial(p^*_{1H}-p^*_{1L})}{\partial K_H} < 0$ if $\delta < 1 + \frac{2D_2}{D_1}$). Hence, new customers always pour into the market. And high-quality capacity becomes less attractive when vertical differentiation is not evident ($\delta < 1 + \frac{2D_2}{D_1}$) So, low-quality demand always increases, while high-quality demand first increases then decreases ($\frac{\partial D^*_{1H}}{\partial K_H} > 0$ if $\delta < 1 + \frac{2D_2}{D_1}$). When low-quality capacity increases, low-posted price first decreases then increases ($\frac{\partial p_{1L}^*}{\partial K_L} < 0$ if $\delta < 1 + \frac{4D_2}{D_1}$), and price gap $p_{1H}^* - p_{1L}^*$ always decreases. Hence, more customers will choose to buy high-quality capacities, while low-quality demand first increases then decreases ($\frac{\partial D_{1L}^*}{\partial K_L} > 0$ if $\delta < 1 + \frac{2D_2}{D_1}$). In the salvage stage, when low-quality capacity increases, opaque selling price decreases ($\frac{\partial p_O^*}{\partial K_L} < 0$), more and more last-minute customers will pour in ($\frac{\partial (D_{2H}^* + D_{2L}^*)}{\partial K_L} > 0$). When high-quality capacity increases, opaque selling price decreases if vertical differentiation is not evident ($\frac{\partial p_O^*}{\partial K_H} < 0$ if $\delta < 1 + \frac{2D_2}{D_1}$). Because all capacities are sold out in equilibrium, so, the higher the capacity level of a specific type, the higher (resp., lower) the transaction volume of the same capacity type (resp., the other capacity type): $\frac{\partial D_{2H}^*}{\partial K_H} = \frac{\partial D_{2L}^*}{\partial K_L} > 0$, $\frac{\partial D_{2H}^*}{\partial K_L} < 0$, $\frac{\partial D_{2L}^*}{\partial K_H} < 0$. In Area(II), in the regular stage, high- and low-posted prices decrease with K_H and K_L with the same degree, so, when capacity of a specific type increases, new customers will join to purchase low-quality capacity ($\frac{\partial D_{1L}^*}{\partial K_H} = \frac{\partial D_{1L}^*}{\partial K_L} > 0$). In the salvage stage, opaque selling price only increases with high-quality capacity. With total transaction volume equals $\frac{D_2}{2}$, we have $\frac{\partial D_{2H}^*}{\partial K_H} > 0$ and $\frac{\partial D_{2L}^*}{\partial K_H} < 0$.

If high-quality capacity is high, then the probability of buying an opaque capacity to obtain a high-quality one is no less than $\frac{1}{2}$ ($\phi_{III}^* = \frac{1}{2}$ and $\phi_{IV}^* = \frac{8K_H - 3D_1}{8D_2}$). The seller's total profit is independent of K_L, while is independent of K_H in Area (III) and increases with K_H in Area (IV). This is because all posted prices except opaque selling price in Area (IV) are independent of the capacity level. That is, the opaque selling price increases with high-quality capacity. So, sales from high-quality (low-quality) capacities in the opaque mix increases (decreases) for the total transaction volume in the salvage stage equals $\frac{D_2}{2}$.

When compared with pure pricing mechanism, we find that all capacities are used up when the high-quality capacity is less than $\frac{D_2}{2}$ rather than when the total capacity is less than $\frac{D_1 + D_2}{2}$. When the high-quality capacity is large, the total transaction volume equals $\frac{D_1 + D_2}{2}$. The difference highlights the importance of high-quality capacities to opaque selling mechanism.

3.3.3.2 Pricing v.s. Pure Opaque Selling

Theorem 3.3.3.2 (Pure Pricing vs. Pure Opaque Selling) Pure pricing dominates pure opaque selling mechanism: $\pi_P^* > \pi_O^*$.

Theorem 3.3.3.2 claims that opaque selling with segmentation inflexibility can bring losses to the seller. This complements the literature on contrasting mechanisms of opaque selling in different settings. (Zhang et al. 2015) shows that the seller should always offer opaque capacities (alongside transparent capacities) in vertical differentiated markets, because opaque selling helps to segment the market by introducing a new capacity type with quality in-between high and low level. Moreover, (Jerath et al. 2010) shows that the seller can increase the sales-season revenue in horizontally differentiated markets, because opaque selling helps to salvage leftovers more effectively. If the regular channel is still available in the salvage stage, then opaque selling dominates pure pricing. Opaque selling makes providers avoid head-to-head competition for regular customers. This is analogy to the sequential game between pricing and opaque selling in the presence of strategic customers. Opaque selling reduces the quality of capacities offered to high-valuation customers and thus makes them more willing to buy in the regular stage.

Corollary 3.3.3.2 shows how the equilibrium prices and transaction volumes vary under two mechanisms.

Corollary 3.3.3.2 (Transaction Price and Transaction Volume Comparison)

(i) If $\frac{3D_1+4D_2}{8} < K_H \leq \frac{3D_1+8D_2}{8}$, then $\Delta p_{1H}^{O-P} > 0$, $\Delta D_{1H}^{O-P} < 0$, $\Delta p_{1L}^{O-P} = 0$, $\Delta D_{1L}^{O-P} > 0$, $\Delta p_{2H}^{O-P} < 0$, $\Delta D_{2H}^{O-P} < 0$, $\Delta p_{2L}^{O-P} > 0$, and $\Delta D_{2L}^{O-P} > 0$.

(ii) If $\min\{\frac{D_1+2D_2}{2} - K_L, \frac{D_1}{2}\} < K_H \leq \frac{3D_1+4D_2}{8}$, then $\Delta p_{1H}^{O-P} > 0$, $\Delta D_{1H}^{O-P} < 0$, $\Delta p_{1L}^{O-P} = 0$, $\Delta D_{1L}^{O-P} < 0$, $\Delta p_{2H}^{O-P} < 0$, $\Delta D_{2H}^{O-P} < 0$, $\Delta p_{2L}^{O-P} > 0$, and $\Delta D_{2L}^{O-P} > 0$.

(iii) Otherwise, then Δp_{1H}^{O-P} depends, $\Delta D_{1H}^{O-P} < 0$ if K_H is above a threshold, $\Delta p_{1L}^{O-P} < 0$, $\Delta D_{1L}^{O-P} > 0$ if K_H is above a threshold, $\Delta p_{2H}^{O-P} < 0$, $\Delta D_{2H}^{O-P} < 0$ if K_H is below a threshold, $\Delta p_{2L}^{O-P} > 0$, and $\Delta D_{2L}^{O-P} > 0$ if K_H is below a threshold.

Corollary 3.3.3.2 indicates that because of the fulfillment flexibility of opaque selling, the seller can sell more capacities and reshape the demand composition (capacity utilization). With the absence of price discrimination effect and reduced quality of high-quality capacities sold as opaque mix, opaque selling cannibalizes high-quality sales. Hence, introducing opaque selling makes the seller suffer losses when compared with pricing mechanism. Details are as follows.

When high-quality capacity is large ($K_H > \frac{D_1+D_2}{2}$), then the seller sells the same quantity ($\frac{D_1+D_2}{2}$) in two scenarios, while the seller only sells high-quality capacities in Scenario P, and uses high-quality capacities to dispose of low-quality capacities in Scenario O. Low-quality sales cannibalizes high-quality one ($\Delta\pi_H^{O-P} < 0$ and $\Delta\pi_L^{O-P} > 0$).

When $\frac{7D_1+8D_2}{16} < K_H \leq \frac{D_1+D_2}{2}$, the seller has the same capacity offerings and the same sales quantity in two scenarios. In the regular stage, high posted-price is higher in Scenario O than in Scenario P, while low posted-prices in two scenarios are the same. And the transaction volume of high-quality (resp., low-quality) capacity is lower (resp., higher) in Scenario O than in Scenario P. In the salvage stage, both price and transaction volume of high-quality (resp., low-quality) capacities are lower (resp., higher) in Scenario O than in Scenario P. Low-quality sales cannibalizes high-quality one ($\Delta\pi_H^{O-P} < 0$ and $\Delta\pi_L^{O-P} > 0$).

When $\min\{\frac{D_1+2D_2}{2} - K_L, \frac{D_1}{2}\} < K_H \leq \frac{7D_1+8D_2}{16}$, then the seller sells the same quantity ($\frac{D_1+D_2}{2}$) in two scenarios, while the seller only sells high-quality capacities in the regular stage in Scenario O. In the regular (resp., salvage) stage, the seller obtains gains from high-quality (resp., low-quality) capacity and suffers losses from low-quality (resp., high-quality) capacity in Scenario O, and gains in the regular stage are mitigated by losses in the salvage stage.

When the high-quality capacity is small, the seller has the same capacity offerings in two scenarios, and the sales quantity in Scenario O is no less than in Scenario P. Opaque selling lowers the low-posted price in the regular stage ($p_{1L}^{O-P} < 0$) to attract more low-valuation customers, and the opaque selling price is in-between high- and low-posted prices in the salvage in Scenario P ($p_{2L}^P < p_O^* < p_{2H}^P$).

3.3.3.3 Pure Upgrading v.s. Pure Opaque Selling

By Theorems 3.3.2.2 and 3.3.3.2, we have the following corollary.

Corollary 3.3.3.3 (Pure Upgrading v.s. Pure Opaque Selling) The seller earns higher profit in pure upgrading than in pure opaque selling: $\pi_U^* > \pi_O^*$. Moreover, $p_{2L}^{U-O} < 0$ and $D_{2H}^{U-O} > 0$.

Corollary 3.3.3.3 claims that pure upgrading dominates pure opaque selling. Reports show that the dominance of upgrading can be attributed to the lower cost of maintaining current customers when compared with new customers, increasing customer lifetime value makes the seller benefit under upgrading[5]. And the seller sells more high-quality capacities in the upgrading channel than in the opaque selling channel ($\Delta D_{2H}^{U-O} > 0$), while the price of low-quality capacity sold regularly in the upgrading mechanism is lower than sold collectively in opaque selling mechanism ($\Delta p_{2L}^{U-O} < 0$), this further confirms that the superiority of upgrading can be attributed to the demand segmentation optimization and price discrimination effect.

Reports show that the dominance of upgrading can be attributed to the lower cost of maintaining current customers when compared with new customers, increasing customer lifetime value makes the seller benefit under upgrading[6].

3.4 Mixed Use of Opaque Selling and Upgrading

Note that upgrading targets regular customers, and opaque selling targets last-minute customers, so we use a sequential adoption sequence to integrate upgrading and opaque selling, and focus on the optimal allocation of limited capacity across different stages and different probabilistic mechanisms.

5. https://hoteltechreport.com/news/upselling-tactics.

6. https://hoteltechreport.com/news/upselling-tactics.

In this section, we first introduce the joint adoption model with upgrading comes first, then introduce the joint adoption model with opaque selling comes first. By comparing pure and mixed use, we aim to characterize the conditions that permit the complementary or substitutable role of two probabilistic mechanisms.

3.4.1 Mixed Use with Upgrading Comes First

The mixed use of two probabilistic mechanisms is similar to pure upgrading mechanism except that last-minute customers are offered with opaque mix. In what follows, we conduct analysis by backward induction.

3.4.1.1 Optimization Problem in the Salvage Stage

We start analysis with the optimization problem of opaque selling mechanism, then turn to derive upgrading price, and find the optimal allocation quantity in upgrading mechanism.

Optimal Decision in Opaque Selling Mechanism In the salvage stage, the seller's problem under the opaque selling mechanism is formulated as

$$\max\ \pi_{2O}^{J}(\theta_O) = \theta_O(\phi\delta + 1 - \phi)\min\{K_H + K_L - D_1(1-\theta_{1L}), D_2(1-\theta_O)\} - F_O$$
$$s.t.\quad 0 \leq \theta_O, \phi \leq 1.$$

Following the results aforementioned, we have the optimal solutions in opaque selling mechanism described as Lemma 3.3.3.1, the only difference lies in the composition of opaque mix, and the probability that customers will be assigned by the high-quality capacities equals

$$\phi^* = \begin{cases} \dfrac{K_H - D_1(1-\theta_U)}{K_H + K_L - D_1(1-\theta_{1L})} & if D_2 \geq K_H + K_L - D_1(1-\theta_{1L}), \\ \dfrac{K_H - D_1(1-\theta_U)}{D_2} & if D_2 < K_H + K_L - D_1(1-\theta_{1L}), K_H - D_1(1-\theta_U) > \dfrac{D_2}{2}, \\ \dfrac{1}{2} & if D_2 < K_H + K_L - D_1(1-\theta_{1L}), K_H - D_1(1-\theta_U) \leq \dfrac{D_2}{2}. \end{cases}$$

Optimal Decision in Upgrading Mechanism The seller's problem in the salvage stage is formulated as

$$\max\ \pi_2^J(S,p) = p\min\{D_1(\theta_{1H} - \theta_U), S\} + \pi_{2O}^*(\theta_O^*)$$

$$s.t. \quad \begin{cases} p \leq p_{1H} - p_{1L}, \\ 0 \leq S \leq \min\{D_1(\theta_{1H} - \theta_{1L}), K_H - D_1(1-\theta_{1H})\}. \end{cases}$$

Pure opaque selling mechanism is a special case of the joint adoption of upgrading and opaque selling (i.e., when $\theta_U = \theta_{1H}$), and case $\theta_U = 1 - \frac{K_H}{D_1}$ indicates that opaque selling mechanism is unavailable.

3.4.1.2 Optimization Problem in the Regular Stage

In the regular stage, the optimization problem of the seller is denoted by:

$$\pi_J(\theta_{1H}, \theta_{1L}) = (\theta_{1H}(\delta - 1) + \theta_{1L})D_1(1 - \theta_{1H}) + \theta_{1L}D_1(\theta_{1H} - \theta_{1L}) + \pi_2^*(S^*, p^*),$$

subject to $0 \leq \theta_{1L} \leq \theta_{1H} \leq 1$, $K_H - D_1(1 - \theta_{1H}) \geq 0$, and $K_L - D_1(\theta_{1H} - \theta_{1L}) \geq 0$.

The equilibrium capacity offerings and properties regarding the optimal prices and transaction volumes under joint adoption are described as the following proposition.

Proposition 3.4.1.2 Under joint upgrading and opaque selling mechanism, the seller's optimal capacity offerings over the whole selling season must be $(H^P, L^P; H^O, L^O)$, $(H^P, L^P; H^U, L^{\varnothing})$, $(H^P, L^{\varnothing}; H^{O+U}, L^O)$, or $(H^P, L^P; H^{O+U}, L^O)$. In equilibrium,

(i) The optimal prices charged for high- and low-quality capacities, the upgrading price and the opaque selling price satisfy: $p_{1H}^* \geq p_U^* > p_{1L}^*$, $p_{1H}^* + p_{1L}^* < 2p_U^*$, and $p_{1L}^* < p_O^*$;

(ii) The number of high-quality capacities sold in the regular stage and those offered as upgrades are equal in size: $D_1(1 - \theta_H^*) = D_1(\theta_H^* - \theta_U^*)$.

Part (i) and (ii) of Proposition 3.4.1.2 demonstrates that properties regarding the posted prices and the transaction volumes are consistent in pure and mixed use.

Proposition 3.4.1.2 also reveals that pure upgrading without opaque selling is an equilibrium strategy even though the fixed fee paid to the opaque platform is regarded as a sunk cost. Capacity offering $(H^P, L^P; H^U, L^{\varnothing})$ emerges for both the profit generated from upgrades and the seller's total profit increase with the high-quality capacity level (details are summarized as the following table). This means that the benefit from offering upgrades is more evident when there are more and more high-quality capacities, and there is no need for the seller to seek help from the opaque platform. In terms of other capacity offerings with available opaque selling, conditions $\frac{\partial(p_U D_U)}{\partial K_H} > 0$ and $\frac{\partial \pi^J}{\partial K_H} > 0$ do not hold simultaneously.

Table 3.1: Relationship Characterization of Partial Derivatives of Upgrading and Total Profit

Capacity Offerings	$p_U D_U$	$\frac{\partial(p_U D_U)}{\partial K_H}$	$\frac{\partial \pi^J}{\partial K_H}$
$(H^P, L^P; H^U, L^{\varnothing})$	$\frac{(\delta-1)(D_1 K_H - K_H^2)}{2D_1}$	> 0	> 0
$(H^P, L^P; H^{O+U}, L^O)$	$\frac{4D_1\delta+2D_1+3(D_2-2(K_H+K_L))}{36}$	< 0	> 0 if $K_H + K_L < \frac{D_1+D_2}{2}$, or $\delta > \frac{4(K_H+K_L)-(D_1+2D_2)}{D_1}$ & $\frac{D_1+D_2}{2} < K_H < \frac{2D_1^2+3D_2^2+5D_1D_2}{4D_1}$
$(H^P, L^{\varnothing}; H^{O+U}, L^O)$	$\frac{(\delta^2+\delta)\delta D_1}{(3\delta+1)^2}$	$= 0$	$= 0$
$(H^P, L^{\varnothing}; H^{O+U}, L^O)$	$\frac{\delta D_1}{8}$	$= 0$	> 0

By considering an upgrading platform's participation in the joint adoption with upgrading comes first, we claim that cooperating with the platform is always optimal for the seller, and details are referred to Appendix B.2.1.

3.4.2 Mixed Use with Opaque Selling Comes First

When opaque selling mechanism is employed before upgrading mechanism in the salvage stage, then the seller first seeks help from the opaque platform to dispose of remaining capacities to last-minute customers. In the situation where high-quality capacity remains unsold after opaque selling, the seller uses those high-quality capacities as upgrades.

The demand formulation is the same as the main analysis. In the opaque selling mechanism, all remaining capacities form the opaque mix, and the platform makes pricing decision p_O to maximize $\pi_{2O} = (\phi\delta + 1 - \phi)\theta_O \min\{D_2(1-\theta_O), K_H + K_L - D_1(1-\theta_{1L})\}$. Among total transaction volume $\min\{D_2(1-\theta_O), K_H + K_L - D_1(1-\theta_{1L})\}$, ϕ proportion are high-quality sales, which is denoted by D_{OH}. If $K_H - D_1(1-\theta_{1H}) - D_{OH} > 0$, suppose that the amount of high-quality capacity used as upgrades is given by S, where $S \leq \min\{D_1(\theta_{1H} - \theta_{1L}), K_H - D_1(1-\theta_{1H}) - D_{OH}\}$ if allocation comes first or $S \leq \min\{D_1(\theta_{1H} - \theta_U), K_H - D_1(1-\theta_{1H}) - D_{OH}\}$ if pricing comes first. Hence, the seller's profit generated in the upgrading mechanism equals $\pi_{2U} = (\delta - 1)\theta_U \min\{S, D_1(\theta_{1H} - \theta_U)\}$ or $\pi_{2U} = (\delta - 1)\theta_U S$.

The optimal solutions under joint adoption with opaque selling first and upgrading second are summarized as Proposition 3.4.2.

Proposition 3.4.2 Upon joint adoption with opaque selling comes first, the seller's optimal capacity offerings over the whole selling season must be $(H^P, L^P; H^O, L^O)$, $(H^P, L^P; H^{O+U}, L^O)$, or $(H^P, L^{\varnothing}; H^{O+U}, L^O)$. In equilibrium,

(i) The optimal prices charged for high- and low-quality products, the upgrading price and the opaque selling price satisfy: $p^*_{1H} \geq p^*_U > p^*_{1L}$, $p^*_{1H} + p^*_{1L} < 2p^*_U$, and $p^*_{1L} < p^*_O$;

(ii) The number of high-quality capacities sold in the regular stage and those offered as upgrades are not always equal in size.

By Proposition 3.4.2, we find that the fixed payment for the opaque platform can be regarded as a sunk cost and incentivizes the seller to dispose of leftovers through opaque selling mechanism. By Propositions 3.4.1.2 and 3.4.2, we conclude that when a third-party platform managing a probabilistic selling mechanism moves first in the sequential adoption, then the seller has an incentive to corporate with this platform to dispose of leftovers.

Proposition 3.4.2 also claims that properties regarding posted prices are consistent in sequential adoption of two probabilistic selling mechanisms [7]. While the amount of high-quality capacity sold in the regular stage does not always equal the amount of high-quality capacity sold as upgrades when opaque selling mechanism comes first. This is attributed to the differentiated changing patterns of $\frac{\partial p^*_O}{\partial K_H}$ and $\frac{\partial p^*_U}{\partial K_H}$. Specifically, if opaque selling price is independent of the high-quality capacity level, then the number of high-quality capacity sold regularly in the regular stage and the number of high-quality capacity sold as upgrades in the salvage stage are equal in size ($D^*_{1H} = D^*_U$). If opaque selling price increases with

7. If there are only high-quality leftovers, then properties regarding prices and transaction volumes are no longer consistent. Refer to Appendix B.2.2 for details.

K_H while upgrading price decreases with K_H, then there are fewer customers purchasing opaque capacities and more customers accepting upgrades. As the high-quality capacity level increases, then the amount gap first becomes positive then negative ($D^*_{1H} > D^*_U$ if $K_H < \frac{D_1}{2}$). If only opaque selling price increases with the high-quality capacity level, then more high-quality capacities will be left to the upgrading process ($D^*_{1H} < D^*_U$).

3.4.3 Complementary or Substitutable Role Characterization

Note that two mechanisms are complementary if the joint adoption generates a supper-additive benefit (Cachon and Swinney 2011; Huang et al. 2018): the incremental value of a joint adoption of two mechanisms is higher than the combined incremental values of the pure upgrading and opaque selling mechanism in isolation. That is, $\pi^*_{J/J1} - \pi^*_P \geq \pi^*_U - \pi^*_P + \pi^*_O - \pi^*_P$, or equivalently, $\pi^*_{J/J1} + \pi^*_P \geq \pi^*_U + \pi^*_O$.

3.4.3.1 Relationship Investigation with Upgrading Comes First

Theorem 3.4.3.1 offers insights about whether the complementary/substitutable role of upgrading and opaque selling depends, monotonically, on the high-quality capacity level.

Theorem 3.4.3.1 (The Relationship between Upgrading and Opaque Selling)

(i) When the high-quality capacity is either large or small, then upgrading and opaque selling are substitutable.

(ii) When the high-quality capacity is in the medium level, then upgrading and opaque selling mechanisms are either complements or substitutes.

Producing and maintaining the operation of high-quality capacities is more time- and money-consuming than low-quality ones. So, from the perspective of the seller, the higher amount of high-quality capacities, the seller's brand is famous and the scale is large.

When the high-quality capacity level is high, then the joint adoption with upgrading mechanism comes first is inferior to pure opaque selling. The underlying reason is attributed to the upgrading process, which cannibalizes the sales of high-quality capacities sold to regular customers. In the opaque selling channel, the release of low-quality capacities from successful upgrading increases the ratio of low-quality capacities to the total capacities in the opaque mix which in turn results in low opaque selling price. Pure opaque selling dominates joint adoption and pure upgrading dominates pure pricing. Hence, opaque selling and upgrading are substitutes. Because pure upgrading dominates pure opaque selling, adopting upgrading mechanism makes the seller have more economic advantages than adopting opaque selling mechanism.

When the high-quality capacity is in the medium level, then the demand segmentation optimization and price discrimination effect of upgrading interact with the capacity utilization effect of opaque selling. Hence, the complementary and substitutable relationships between two probabilistic mechanisms are mixed. Moreover, when the high-quality capacity level is high, then two mechanisms are complements. Otherwise, they are substitutes.

This is consistent with the phenomenon that it is costly for midscale and economy hotels to manage upgrading through software or additional staff for the limited service, on-site amenities, and room types [8]. For large brand sellers, their customers are loyal or with high valuation. Therefore, when there are high-quality leftovers, they are more willing to provide upgrades to customers who have purchased in the early stage rather than to sell to customers who are price sensitive and arrive in later stage. For instance, Hyatt reported that it would like to launch loyalty program (World of Hyatt) to drive more direct bookings through its own website rather than through OTAs, such as Expedias channels including Hotel.com, Travelocity, and Hotwire[9].

When the high-quality capacity level is low, then the joint adoption with upgrading

8. Refer to https://hoteltechreport.com/news/oracle-nor1-machine-learning.

9. Refer to https://skift.com/2017/06/15/is-hyatt-playing-hardball-with-expedia-over-contract-negotiations/.

mechanism comes first reduces to pure opaque selling mechanism. So, opaque selling and upgrading are substitutable for the pure upgrading mechanism dominates pure pricing mechanism. The seller does not need to use opaque selling when the amount of high-quality capacities is low.

3.4.3.2 Relationship Investigation with Opaque Selling Comes First

We identify the conditions that permit the complementary or substitutable role of two probabilistic selling mechanisms by comparing the joint adoption scenario with opaque selling comes first, pure upgrading, pure opaque selling and pure pricing scenario, details are articulated in Theorem 3.4.3.2.

Theorem 3.4.3.2 (The Relationship between Upgrading and Opaque Selling With Alternative Sequence)

(i) When the high-quality capacity level is rather high, then upgrading and opaque selling are complements.

(ii) When the high-quality capacity level is in the medium level, then upgrading and opaque selling mechanisms are either complements or substitutes.

(iii) When the high-quality capacity level is rather low, then upgrading and opaque selling are substitutes.

Consistent with Theorem 3.4.3.1, Theorem 3.4.3.2 shows that adopting both opaque selling and upgrading strategies to dispose of leftovers is not beneficial to the seller when high-quality capacity level is low. This is because small amount of high-quality capacity is not sufficient to fulfill both upgrading demand and opaque selling requests. Two probabilistic mechanisms are either complementary or substitutable when high-quality capacity is in the medium level, the underlying reason mimicks the one aforementioned in Theorem 3.4.3.1.

Note that upgrading and opaque selling are complementary rather than substitutable in joint adoption with upgrading comes first when high-quality capacity level is rather high. This is because the seller has an incentive to cooperate with the opaque platform when opaque selling comes first.

(Gao and Su 2017) shows that two mechanisms are complementary if these two mechanisms have overlaps in how they impact customer behavior. Results in Theorems 3.4.3.1 and 3.4.3.2 claim that these two mechanisms have overlaps when the high-quality capacity level is in the medium level. Opaque selling and upgrading have overlaps in customers' purchase behavior and the seller together with the platform's pricing policy. From the perspective of customers, low-valuation customers are the target customers of both mechanisms. They aim to obtain the high-quality capacities with discounts. From the seller and platform's perspective, both mechanisms are adopted to generate sales from the salvage selling stage, while upgrading is disposed of high-quality leftovers and opaque selling is disposed of low-quality capacities.

3.5 Extensions

In this section, we check the robustness of our main results by considering endogenous capacity and introducing conditional upgrades.

3.5.1 Endogenous Capacity

Due to perishable service and unstable market demand, service providers can realize the sale of multi-level fare structure not only through probabilistic selling, but also through capacity expansion, such as seats adjustment within business and economy class in aviation industry. Hence, we consider endogenous capacity. The sequence of events with endogenous capacity is described as follows: first, the seller makes production decision before the start of the selling season. Second, the decision sequence is the same as the one with exogenous capacity.

Theorem 3.5.1 confirms that upgrading dominates opaque selling when the seller can make production decision, and Corollary 3.5.1 gives the comparative analysis on upgrading and pricing, and opaque selling and pricing.

Theorem 3.5.1 (Pure Probabilistic Mechanisms vs. Pricing Under Endogenous Capacity) Pure upgrading always dominates pricing mechanism, pricing always dominates pure opaque selling mechanism.

Corollary 3.5.1 (Transaction Price and Transaction Volume Comparison (U-P))

(i) Upgrading vs. pricing:

(a) For high-quality capacities, revenue generated in Scenario U is higher than in Scenario P, moreover, $\Delta p_{1H}^{U-P} > 0$, $p_{1H}^{U} + p_{U}^{*} - 2p_{1H}^{P} \geq 0$, $\Delta D_{1H}^{U-P} < 0$, $\Delta p_{2H}^{U-P} \geq 0$, and $\Delta D_{2H}^{U-P} > 0$.

(b) For low-quality capacities, revenue generated in Scenario U is no less than in Scenario P, moreover, $\Delta p_{1L}^{U-P} \leq 0$, $\Delta D_{1L}^{U-P} \leq 0$, $\Delta p_{2L}^{U-P} = 0$, and $\Delta D_{2L}^{U-P} \geq 0$.

(ii) Opaque selling vs. pricing:

(a) For high-quality capacities, revenue generated in Scenario P is more than in Scenario O, moreover, $p_{1H}^{O-P} \geq 0$, $\Delta D_{1H}^{O-P} \leq 0$, $p_O - p_{2H}^{P} \leq 0$, and $\Delta D_{2H}^{O-P} \leq 0$.

(b) For low-quality capacities, revenue generated in Scenario P is less than in Scenario O, moreover, $p_{1L}^{O-P} \leq 0$, $\Delta D_{1L}^{O-P} \geq 0$, $p_O - p_{2L}^{P} > 0$, and $\Delta D_{2L}^{O-P} \geq 0$.

Theorem 3.5.2 shows that our results are robust under the joint adoption scenario that upgrading comes first with exogenous capacity. Moreover, two probabilistic mechanisms are substitutes when K_H is small. Under the joint adoption scenario that opaque selling comes first with exogenous capacity, Theorem 3.5.2 highlights that opaque selling and upgrading mechanisms are substitutes when the high-quality capacity level is high. This is because the seller's profits in pure upgrading and pricing mechanisms are independent of K_H or first increase then decrease with K_H and Scenario P's inflection point comes before Scenario U, and the seller's profits in both pure opaque selling and joint adoption mechanisms are increasing with the high-quality capacity level under this circumstance. While the maximum production that the seller can reach in the joint adoption mechanism

($K_H^* = \frac{D_1+8D_2}{8}$) is smaller than in the pure opaque selling mechanism ($K_H^* = \frac{3D_1+8D_2}{8}$), and the increase in the output of high-quality capacity leads to a more obvious increase in revenue under pure opaque selling mechanism (or equivalently, $\Delta\pi^{J1-O}$ decreases from $\frac{(\delta-1)(25\delta+7)D_1}{64(3\delta+1)}$ to $\frac{(\delta-1)(13\delta+3)D_1}{64(3\delta+1)}$).

Theorem 3.5.2 (Relationship Between Opaque Selling and Upgrading Mechanisms Under Endogenous Capacity)

(i) If $K_H > \frac{D_1+D_2}{2}$, or $K_H < \frac{D_1+D_2}{2} - K_L$, then upgrading and opaque selling are substitutes.

(ii) If $\frac{D_1+D_2}{2} - K_L \leq K_H \leq \frac{D_1+D_2}{2}$, then upgrading and opaque selling are either complements or substitutes. Moreover, two probabilistic mechanisms are substitutes when K_H is small.

3.5.2 Conditional Upgrading

In conditional upgrading, customers purchasing regular capacities are offered with an additional price charged for upgrades, and upgrading demand are fulfilled at the end of the selling season if there are some high-quality capacities left, customers who accept upgrades have to bear the risk of not obtaining the high-quality capacities, which is denoted by $1-\xi$, where $\xi \in [0,1]$.

In the regular stage, the seller announces price p_{1H}, p_{1L} and $p + p_{1L}$ for high-quality, low-quality capacities and upgrades, respectively, where $p < p_{1H} - p_{1L}$. Upon observing the posted price, regular customers make purchase decisions among high-quality capacity with utility $\theta\delta - p_{1H}$, low-quality ones with upgrade with utility $\xi(\theta\delta - p - p_{1L}) + (1-\xi)(\theta - p_L)$, low-quality capacity without upgrade with utility $\theta - p_{1L}$ and not purchasing with utility 0.

We denote demand of capacity H, U and L in the regular stage by D_H, D_U and D_L. The seller's total profit equals $\pi^C(p_{1H}, p_{1L}, p) = p_{1H}D_H + p_{1L}(D_L + D_U) + \pi_2^*$, where $\pi_{2U}(S) = pS$ and $\pi_2^* = \pi_{2U}(S) + \pi_{2O}(\theta_O)$ if opaque selling is available, $\pi_2^* =$

$\pi_{2U}(S)+\pi_{2P}(\theta_{2H}, \theta_{2L})$ if pricing is available, or $\pi_2^* = \pi_{2U}(S)$ if neither strategy is available. Details are as follows.

In the salvage stage, the seller determines the amount of high-quality leftovers S to satisfy upgrading demand, where $S \in [0, \min\{D_U, K_H - D_H\}]$. Hence, the probability of receiving upgrades equals $\xi = \frac{S}{D_U}$, and the seller's corresponding profit equals $\pi_{2U}(S) = pS$. The number of high- and low-quality leftovers after upgrading equal $K_H - D_H - S$ and $K_L - D_L - D_U + S$, respectively. If opaque selling mechanism is available, then the seller's corresponding profit equals $\pi_{2O} = p_O \min\{D_2(1-\theta_O), K_H - D_H + K_L - D_L - D_U\} - F_O$ provided that $0 \le \theta_O \le 1$. If regular selling mechanism is available, then the seller's profit equals $\pi_{2P} = (\theta_{2H}(\delta - 1) + \theta_{2L}) \min\{K_H - D_H - S, D_2(1-\theta_{2H})\} + \theta_{2L} \min\{K_L - D_L - D_U + S, D_1(\theta_{2H} - \theta_{2L})\}$ provided that $0 \le \theta_{2L} \le \theta_{2H} \le 1$. If neither opaque or regular selling is available, then all unsold capacities are lost.

Results in conditional upgrading followed by an opaque selling mechanism are summarized as Proposition 3.5.2.

Proposition 3.5.2 Under conditional upgrading mechanism, the seller's optimal capacity offering must be $(H^U, L^P; H^O, L^O)$, $(H^P, L^P; H^\varnothing, L^\varnothing)$, or $(H^U, L^\varnothing; H^O, L^O)$. In equilibrium,

(i) The optimal prices charged for high- and low-quality capacity, the upgrading price and opaque selling price satisfy: $p_{1H}^* \ge p_U^* > p_{1L}^*$, and $p_{1L}^* < p_O^*$;

(ii) The amount of high-quality capacity offered as upgrades is more than the amount offered as opaque mix: $D_1(1-\theta_U^*) > D_2(1-\theta_O^*)\phi$.

Proposition 3.5.2 shows that when introducing conditional upgrade, all high-quality capacities are sold as conditional upgrades if opaque selling is available. This is equivalent to the capacity offering, the total transaction volume clarified in Lemma 3.3.3.1. To this end, conditional upgrading and opaque selling are substitutes rather than complements, and details are articulated in Theorem 3.5.2.

Theorem 3.5.2 (The Relationship between Upgrading and Opaque Selling

With Conditional Upgrading) Upgrading and opaque selling are substitutes under conditional upgrading.

3.6 Concluding Remarks

Perishable leftovers incur revenue losses for service marketers. Opaque selling and upgrading are two probabilistic mechanisms that help the seller manage the salvage value generation process. We consider a two-stage model in which the seller sells capacities regularly in the regular stage, and employs opaque selling and upgrading singly/jointly to dispose of leftovers in the salvage stage. We highlight the vital role of the high-quality capacity level, the adoption sequence, and the platform's participation in characterizing the complementary/substitutable relationship between opaque selling and upgrading.

Results show that when high-quality capacity is rather small, opaque selling and upgrading are substitutes. This is because low high-quality capacity is insufficient to fulfill both upgrading demand and opaque selling demand. When high-quality capacity is in the medium level, two mechanisms are either complementary or substitutable, and the relationship depends on the tradeoff between beneficial effects of upgrading and cannibalization effect of opaque selling. When high-quality capacity is rather large, two mechanisms are complements if opaque selling comes first or substitutes if upgrading comes first. We also show that the seller has a strong incentive to cooperate with a third-party platform who manages the probabilistic selling mechanism and moves first in the salvage value generation process.

There are several possible avenues for our work extension. First, we consider a monopolistic setting, and we can extend our analysis to allow for competition within multiple sellers and platforms. Moreover, since contract design within service providers plays a vital role in shaping pricing decisions, it will be interesting to verify whether our key findings are robust under richer contract mechanisms. Second, we ignore operational costs of probabilistic selling mechanisms. Third, consumer economics, such as anticipated regret and bounded rationality, may yield interesting results. Last but not least, information asymmetry, such as the lack of detailed sales data, may arise within service providers and different selling formats. All these complicate the analysis and we leave the analysis to our future work.

CHAPTER 4

CONCLUDING REMARKS

4.1 Summary of Research Work

First, we explore the optimal pricing decisions under post pricing and k-double auction mechanism and compare the platform's profits. To capture service providers' strategic fulfillment decision, we construct a two-stage model in which the platform plays different roles under two mechanisms. Under post pricing mechanism, the platform sets prices, under k-double auction mechanism, the platform determines the matching policy including the matching and price determination rule. We use a mixed strategy equilibrium to analyze strategic service providers' fulfillment decision, and use backward induction to solve the sequential game between the platform and the participants on demand- and supply-side. Specifically, under post pricing, we construct differential equations to solve the nonlinear optimization problem. Under k-double auction mechanism, we use order statistics to construct differential equations so as to derive customers and service providers' optimal bidding policies.

Results of post pricing vs. k-double auction in two-sided market are summarized as follows: First, we find that the optimal price under post pricing is higher than under k-double auction mechanism. And there is no dominant strategy between post pricing and k-double auction, and the pricing entity, the demand-supply intensity, the pricing flexibility and providers' strategic behavior play a vital role. That is, The lower the demand-supply intensity and the higher the bidding power, the higher the likelihood that the platform uses post pricing; the platform prefers post pricing with more pricing flexibility; and the presence of providers' strategic behavior is a strong incentive for the platform to use k-double auction rather than post pricing. Second, in both k-double auction and bid-ask double auction mechanisms, when customers (resp., service providers) have the pricing power, then service providers (resp., customers) propose bid which equals their discount reservation prices in equilibrium. In addition, in k-double auction, when both customers and providers determine the transaction price, then the optimal bidding strategy of customers and providers shows the same structure, that is, the same slope and same intercept of the linear bidding functions.

We are the first to study the optimal selling strategy of the platform in the presence of strategic service providers in two-sided market, we are also the first to propose that customers' bidding power, the platform's pricing flexibility, the demand-supply intensity, and providers' strategic behavior all play important roles in the selling strategies selection of the sharing platform.

Second, we unify opaque selling and upgrading into a vertical differentiated framework to investigate the role of these two mechanisms in the seller's salvage value generation process. To highlight the role of probabilistic selling mechanisms in disposing of leftovers, we construct a two-stage model. In the regular stage, the seller sells two types of capacities regularly. In the salvage stage, the seller with remaining capacities sells through opaque selling and upgrading individually or jointly. The seller can seek help from a third-party platform who uses high-quality leftovers along with low-quality ones to create an opaque mix in opaque selling mechanism. Or, the seller can use high-quality leftovers to upgrade demand from customers having purchased low-quality capacities in previous stage in upgrading mechanism. We use backward induction to characterize the sub-game Nash equilibrium, and use a rational expectation equilibrium to explore customers' anticipation towards platform's demand fulfillment decision.

Results of opaque selling and upgrading in the seller' salvage value generation process are summarized as follows: pure upgrading is superior to pure dynamic pricing mechanism due to demand segmentation and price discrimination effect, while high-quality sales cannibalization makes pure opaque selling inferior to pure dynamic pricing mechanism. When high-quality capacity is rather small, opaque selling and upgrading are substitutes. This is because low high-quality capacity is insufficient to fulfill both upgrading demand and opaque selling demand. When high-quality capacity is in the medium level, two mechanisms are either complementary or substitutable, and the relationship depends on the trade-off between beneficial effects of upgrading and cannibalization effect of opaque selling. When high-quality capacity is rather large, two mechanisms are complements if opaque selling comes first or substitutes if upgrading comes first. We also show that the seller has a strong incentive to cooperate with a third-party platform who manages the probabilistic selling mechanism and moves first in the salvage value generation process.

We are the first to compare upgrading mechanism and opaque selling mechanism under vertical competition. We highlight the demand segmentation and price discrimination role of upgrading, and high-quality sales cannibalization effect of opaque selling. We are also the first to integrate upgrading and opaque selling mechanism into a sequential model to characterize that the adoption sequence, the high-quality capacity level and the participation of platforms are the key elements in determining the complementary or substitutable role between two mechanisms.

4.2 Future Research Work

There are several avenues for our first work extension. First, we consider rational and risk-neutral participants, customers, providers and the sharing platform are rational and are all payoff maximizers. Behavior economics, such as risk attitude or overconfidence, is worth attention. For instance, service providers may overestimate the transaction price and the matching probability, or they may be risk averse when they make fulfillment decisions. Overconfidence makes service providers more likely to wait for the second period, while risk aversion makes them more likely to fulfill demand without delay. Second, asymmetric information is also worth investigation. For instance, demand over stages in real life are relevant under certain circumstances. In the ride-hailing industry, the number of customers in peak and off peak hours of holidays will be more than that in non holidays. to this end, providers making service fulfilling decisions can infer the second period demand from previous one. Finally, competition among platforms also needs further discussion, such as multi-homing behavior of participants on two sides, and loyalty programs design including reward timing, reward types and discount amount to enlarge loyal customers' lifetime value.

There are several possible avenues for our second work extension. First, we consider a monopolistic setting, while in the hotel industry, there exists almost no monopoly on the market. We can extend our analysis to allow for competition within multiple sellers and platforms. Moreover, contract design within service providers also plays a vital role in shaping pricing decisions, it will be interesting to verify whether our key findings are robust under richer contract mechanisms, such as the bargaining framework. Second, we ignore operational costs of probabilistic selling mechanisms, and our analysis can be classified as the pricing problem in revenue management. The production cost is a major concern when we consider endogenous capacity decision. Third, consumer economics including anticipated regret and bounded rationality are widely considered in customers' anticipation, such as customers' anticipation towards the platform's demand fulfillment decision. Last but not least, information asymmetry, such as the lack of detailed sales data, may arise within the seller and platforms and sellers in different selling formats.

More complicated but possibly insightful results can be derived from aforementioned directions and we leave the analysis to our future work.

CHAPTER 5

REFERENCES

Allon, G., A. Bassamboo, and E. B. Çil. 2017. "Skill Management in Large-Scale Service Marketplaces." *Production and Operations Management* 26 (11): 2050–2070.

Bai, J., K. C. So, C. S. Tang, X. Chen, and H. Wang. 2019. "Coordinating supply and demand on an on-demand service platform with impatient customers." *Manufacturing & Service Operations Management* 21 (3): 556–570.

Bauner, C. 2015. "Mechanism choice and the buy-it-now auction: A structural model of competing buyers and sellers." *International Journal of Industrial Organization* 38:19–31.

Benjaafar, S., H. Bernhard, C. Courcoubetis, M. Kanakakis, and S. Papafragkos. 2021. "Drivers, riders, and service providers: The impact of the sharing economy on mobility." *Management Science.*

Benjaafar, S., J.-Y. Ding, G. Kong, and T. Taylor. 2022. "Labor welfare in on-demand service platforms." *Manufacturing & Service Operations Management* 24 (1): 110–124.

Benjaafar, S., G. Kong, X. Li, and C. Courcoubetis. 2019. "Peer-to-peer product sharing: Implications for ownership, usage, and social welfare in the sharing economy." *Management Science* 65 (2): 477–493.

Bimpikis, K., O. Candogan, and D. Saban. 2019. "Spatial pricing in ride-sharing networks." *Operations Research* 67 (3): 744–769.

Biyalogorsky, E., and O. Koenigsberg. 2014. "The design and introduction of product lines when consumer valuations are uncertain." *Production and Operations Management* 23 (9): 1539–1548.

Cachon, G. P., K. M. Daniels, and R. Lobel. 2017. "The role of surge pricing on a service platform with self-scheduling capacity." *Manufacturing & Service Operations Management* 19 (3): 368–384.

Cachon, G. P., and R. Swinney. 2011. "The value of fast fashion: Quick response, enhanced design, and strategic consumer behavior." *Management science* 57 (4): 778–795.

Çakanyıldırım, M., Ö. Özer, and X. Zhang. 2020. "Dynamic pricing and timing of upgrades." *Available at SSRN 3056060.*

Caldentey, R., and G. Vulcano. 2007. "Online auction and list price revenue management." *Management Science* 53 (5): 795–813.

Casella, G., and R. L. Berger. 2021. *Statistical inference.* Cengage Learning.

Ceryan, O., I. Duenyas, and O. Sahin. 2018. "Dynamic pricing and replenishment with customer upgrades." *Production and Operations Management* 27 (4): 663–679.

Chao, Y., L. Liu, and D. Zhan. 2016. "Vertical probabilistic selling under competition: The role of consumer anticipated regret." *Available at SSRN 2848543.*

Chatterjee, K., and W. Samuelson. 1983. "Bargaining under incomplete information." *Operations research* 31 (5): 835–851.

Chen, R. R., E. Gal-Or, and P. Roma. 2014. "Opaque distribution channels for competing service providers: Posted price vs. name-your-own-price mechanisms." *Operations research* 62 (4): 733–750.

Chen, Y.-J., B. Tomlin, and Y. Wang. 2013. "Coproduct technologies: Product line design and process innovation." *Management Science* 59 (12): 2772–2789.

Chen, Y., and M. Hu. 2020. "Pricing and matching with forward-looking buyers and sellers." *Manufacturing & Service Operations Management* 22 (4): 717–734.

Chu, L. Y., and Z.-J. M. Shen. 2006. "Agent competition double-auction mechanism." *Management Science* 52 (8): 1215–1222.

———. 2007. "Trade reduction vs. multi-stage: A comparison of double auction design approaches." *European journal of operational research* 180 (2): 677–691.

———. 2008. "Truthful double auction mechanisms." *Operations research* 56 (1): 102–120.

Cohen, M., and R. P. Zhang. 2017. "Competition and coopetition for two-sided platforms." *Available at SSRN 3028138.*

Cui, Y., I. Duenyas, and O. Sahin. 2018. "Pricing of conditional upgrades in the presence of strategic consumers." *Management Science* 64 (7): 3208–3226.

Cui, Y., A. Y. Orhun, and I. Duenyas. 2019. "How price dispersion changes when upgrades are introduced: Theory and empirical evidence from the airline industry." *Management Science* 65 (8): 3835–3852.

Desai, P. S. 2001. "Quality segmentation in spatial markets: When does cannibalization affect product line design?" *Marketing Science* 20 (3): 265–283.

Economides, N., and E. Katsamakas. 2006. "Two-sided competition of proprietary vs. open source technology platforms and the implications for the software industry." *Management science* 52 (7): 1057–1071.

Einav, L., C. Farronato, J. Levin, and N. Sundaresan. 2018. "Auctions versus posted prices in online markets." *Journal of Political Economy* 126 (1): 178–215.

Elmachtoub, A. N., Y. Wei, and Y. Zhou. 2015. "Retailing with opaque products." *Available at SSRN 2659211.*

Etzion, H., E. Pinker, and A. Seidmann. 2006. "Analyzing the simultaneous use of auctions and posted prices for online selling." *Manufacturing & Service Operations Management* 8 (1): 68–91.

Fay, S. 2008. "Selling an opaque product through an intermediary: The case of disguising one's product." *Journal of Retailing* 84 (1): 59–75.

Fay, S., and J. Xie. 2010. "The economics of buyer uncertainty: Advance selling vs. probabilistic selling." *Marketing Science* 29 (6): 1040–1057.

———. 2015. "Timing of product allocation: Using probabilistic selling to enhance inventory management." *Management Science* 61 (2): 474–484.

Gao, F., and X. Su. 2017. "Online and offline information for omnichannel retailing." *Manufacturing & Service Operations Management* 19 (1): 84–98.

Georgiadis, G., and C. S. Tang. 2014. "Optimal reservation policies and market segmentation." *International Journal of Production Economics* 154:81–99.

Guda, H., and U. Subramanian. 2019. "Your uber is arriving: Managing on-demand workers through surge pricing, forecast communication, and worker incentives." *Management Science* 65 (5): 1995–2014.

Guo, L., and J. Zhang. 2012. "Consumer deliberation and product line design." *Marketing Science* 31 (6): 995–1007.

Gurvich, I., M. Lariviere, and A. Moreno. 2019. "Operations in the on-demand economy: Staffing services with self-scheduling capacity." In *Sharing economy,* 249–278. Springer.

Hagiu, A. 2007. "Merchant or two-sided platform?" *Review of Network Economics* 6 (2).

Hagiu, A., and J. Wright. 2015. "Marketplace or reseller?" *Management Science* 61 (1): 184–203.

Halaburda, H., M. Jan Piskorski, and P. Yıldırım. 2018. "Competing by restricting choice: The case of matching platforms." *Management Science* 64 (8): 3574–3594.

Hammond, R. G. 2010. "Comparing revenue from auctions and posted prices." *International Journal of Industrial Organization* 28 (1): 1–9.

———. 2013. "A structural model of competing sellers: Auctions and posted prices." *European Economic Review* 60:52–68.

Hu, M., and Y. Zhou. 2020. "Price, wage, and fixed commission in on-demand matching." *Available at SSRN 2949513.*

———. 2022. "Dynamic type matching." *Manufacturing & Service Operations Management* 24 (1): 125–142.

Huang, T., C. Liang, and J. Wang. 2018. "The value of "bespoke": Demand learning, preference learning, and customer behavior." *Management Science* 64 (7): 3129–3145.

Huang, T., and Y. Yu. 2014. "Sell probabilistic goods? A behavioral explanation for opaque selling." *Marketing Science* 33 (5): 743–759.

Jantschgi, S., H. H. Nax, B. Pradelski, and M. Pycia. 2022. "On market prices in double auctions."

Jerath, K., S. Netessine, and S. K. Veeraraghavan. 2010. "Revenue management with strategic customers: Last-minute selling and opaque selling." *Management science* 56 (3): 430–448.

Jiang, B., and L. Tian. 2018. "Collaborative consumption: Strategic and economic implications of product sharing." *Management Science* 64 (3): 1171–1188.

Jin, C., K. Hosanagar, and S. K. Veeraraghavan. 2018. "Do ratings cut both ways? Impact of bilateral ratings on platforms." *Impact of Bilateral Ratings on Platforms (March 14, 2018).*

Johnson, J. P. 2020. "The agency and wholesale models in electronic content markets." *International Journal of Industrial Organization* 69:102581.

Ke, T. T., B. Jiang, M. Sun, et al. 2017. "Peer-to-peer markets with bilateral ratings." In *MIT Sloan Research Paper No. 5236-17; NET Institute Working Paper No,* 17–101.

Kornish, L. J. 2001. "Pricing for a durable-goods monopolist under rapid sequential innovation." *Management Science* 47 (11): 1552–1561.

Li, Q., C. S. Tang, and H. Xu. 2020. "Mitigating the Double-Blind Effect in Opaque Selling: Inventory and Information." *Production and Operations Management* 29 (1): 35–54.

Lin, J., M. Pipattanasomporn, and S. Rahman. 2019. “Comparative analysis of auction mechanisms and bidding strategies for P2P solar transactive energy markets.” *Applied energy* 255:113687.

Maglaras, C., and J. Meissner. 2006. “Dynamic pricing strategies for multiproduct revenue management problems.” *Manufacturing & Service Operations Management* 8 (2): 136–148.

McAfee, R. P. 1992. “A dominant strategy double auction.” *Journal of economic Theory* 56 (2): 434–450.

Pan, X. A., and D. Honhon. 2012. “Assortment planning for vertically differentiated products.” *Production and Operations Management* 21 (2): 253–275.

Post, D. 2010. “Variable opaque products in the airline industry: A tool to fill the gaps and increase revenues.” *Journal of Revenue and Pricing Management* 9 (4): 292–299.

Ren, H., and T. Huang. 2017. “Opaque selling and last-minute selling: Revenue management in vertically differentiated markets.” *Available at SSRN 3060001.*

Rochet, J.-C., and J. Tirole. 2003. “Platform competition in two-sided markets.” *Journal of the european economic association* 1 (4): 990–1029.

Rustichini, A. 1990. *Convergence to price-taking behavior in a simple market.* Technical report. Discussion paper.

Ryan, J. K., D. Sun, and X. Zhao. 2012. “Competition and coordination in online marketplaces.” *Production and Operations Management* 21 (6): 997–1014.

Rysman, M. 2009. “The economics of two-sided markets.” *Journal of economic perspectives* 23 (3): 125–43.

Satterthwaite, M. A., and S. R. Williams. 1989a. “Bilateral trade with the sealed bid k-double auction: Existence and efficiency.” *Journal of Economic Theory* 48 (1): 107–133.

———. 1989b. “The rate of convergence to efficiency in the buyer’s bid double auction as the market becomes large.” *The Review of Economic Studies* 56 (4): 477–498.

Segal-Halevi, E., A. Hassidim, and Y. Aumann. 2018. “Muda: A truthful multi-unit double-auction mechanism.” In *Proceedings of the AAAI Conference on Artificial Intelligence,* vol. 32. 1.

Shaked, A., and J. Sutton. 1982. “Relaxing price competition through product differentiation.” *The review of economic studies,* 3–13.

Silvey, S. D. 2017. *Statistical inference.* Routledge.

Taylor, T. A. 2018. "On-demand service platforms." *Manufacturing & Service Operations Management* 20 (4): 704–720.

Tian, L., and B. Jiang. 2018. "Effects of consumer-to-consumer product sharing on distribution channel." *Production and operations management* 27 (2): 350–367.

Wang, R. 1993. "Auctions versus posted-price selling." *The American Economic Review,* 838–851.

Williams, S. R. 1991. "Existence and convergence of equilibria in the buyer's bid double auction." *The Review of Economic Studies* 58 (2): 351–374.

Yang, L., L. Debo, and V. Gupta. 2017. "Trading time in a congested environment." *Management Science* 63 (7): 2377–2395.

Yayla-Küllü, H. M., J. K. Ryan, and J. M. Swaminathan. 2021. "Product line flexibility for agile and adaptable operations." *Production and Operations Management* 30 (3): 725–737.

Yılmaz, Ö., P. Pekgün, and M. Ferguson. 2017. "Would you like to upgrade to a premium room? Evaluating the benefit of offering standby upgrades." *Manufacturing & Service Operations Management* 19 (1): 1–18.

Yu, J. J., C. S. Tang, Z.-J. Max Shen, and X. M. Chen. 2020. "A balancing act of regulating on-demand ride services." *Management Science* 66 (7): 2975–2992.

Yu, Y., X. Chen, and F. Zhang. 2015. "Dynamic capacity management with general upgrading." *Operations Research* 63 (6): 1372–1389.

Zeithammer, R., and P. Liu. 2006. "When is auctioning preferred to posting a fixed selling price?" *University of Chicago.*

Zhang, Z., K. Joseph, and R. Subramaniam. 2015. "Probabilistic selling in quality-differentiated markets." *Management Science* 61 (8): 1959–1977.

Zheng, Q., X. A. Pan, and J. E. Carrillo. 2019. "Probabilistic selling for vertically differentiated products with salient thinkers." *Marketing Science* 38 (3): 442–460.

Ziegler, A., and E. P. Lazear. 2003. *The dominance of retail stores.*

Zou, T., B. Zhou, and B. Jiang. 2020. "Product-line design in the presence of consumers' anticipated regret." *Management Science* 66 (12): 5665–5682.

APPENDIX A

PROOF OF CHAPTER 2

A.1 Proof of Main Results

Proof 1 *Proof of Lemma Lemma 2.3.1.*

(i) *If* $(n - \min\{s, m_1\bar{G}(p_1)\})F(\gamma p_2) > m_2\bar{G}(p_2)$ *(or equivalently,* $p_2 > \frac{m_2}{m_2+\gamma(n-\min\{s,m_1\bar{G}(p_1)\})}$*), then the optimization problem is to find* $p_2^* = \arg\max\{(1-\gamma)m_2p_2\bar{G}(p_2)\}$*. The profit function is concave w.r.t* p_2 *and reaches maximum at* $p_2^* = \frac{1}{2}$ *if* $\frac{m_2}{m_2+\gamma(n-\min\{s,m_1\bar{G}(p_1)\})} < \frac{1}{2}$*. Correspondingly,* $\pi_2^*(p_2^*) = \frac{(1-\gamma)m_2}{4}$*.*

(ii) *If* $(n - \min\{s, m_1\bar{G}(p_1)\})F(\gamma p_2) \leq m_2\bar{G}(p_2)$ *(or equivalently,* $p_2 \leq \frac{m_2}{m_2+\gamma(n-\min\{s,m_1\bar{G}(p_1)\})}$*), then the optimization problem is to find* $p_2^* = \arg\max\{(1-\gamma)p_2(n-\min\{s, m_1\bar{G}(p_1)\})F(\gamma p_2)\}$*. The profit function increases with* p_2 *and reaches maximum at* $p_2^* = \frac{m_2}{m_2+\gamma(n-\min\{s,m_1\bar{G}(p_1)\})}$*. Correspondingly,* $\pi_2^*(p_2^*) = \frac{(\gamma-\gamma^2)m_2^2(n-\min\{s,m_1\bar{G}(p_1)\})}{(m_2+\gamma(n-\min\{s,m_1\bar{G}(p_1)\}))^2}$*.*

Proof 2 *Proof of Proposition 2.3.2.*

(i) *If* $m_1(1-p_1) \geq s$*, and* $m_2 > (n-s)\gamma$*, then* $p_2^* = \frac{m_2}{m_2+\gamma(n-s)}$*,* $p_1 = p_2^*$*, and* $s = n + \frac{m_2}{\gamma} - \frac{m_2}{\gamma p_1}$*, the platform's problem over two periods is described as*

$$\max\ \pi^{DP}(p_1) = \frac{-(1-\gamma)\gamma m_2 p_1^2 + (1-\gamma)(m_2+\gamma m_2+\gamma n)p_1 - (1-\gamma)m_2}{\gamma}$$

$$s.t. \quad \begin{cases} \frac{1}{2} < p_1 \leq 1, \\ m_1p_1 - \frac{m_2}{\gamma p_1} \leq m_1 - n - \frac{m_2}{\gamma}. \end{cases}$$

The effective domain for price p_1 *is given by* $(\frac{1}{2}, -\frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}]$ *provided that* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} > 0$*.*

Because $\frac{d^2\pi^{DP}(p_1)}{dp_1^2} < 0$, *the profit function* $\pi^{DP}(p_1)$ *increases in* p_1 *if* $p_1 < \frac{n}{2m_2} + \frac{1}{2} + \frac{1}{2\gamma}$ *and decreases otherwise. Because* $\frac{n}{2m_2} + \frac{1}{2} + \frac{1}{2\gamma} > 1$, *so,* $\pi^{DP}(p_1)$ *increases with* p_1 *over interval* $[0,1]$. *Hence,* $p_1^* = -\frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}$, *and* $\pi_{DP}^* = (1-\gamma)((n + \frac{m_2}{\gamma} + \frac{m_2 n}{m_1} + \frac{m_2^2}{\gamma m_1})(-\frac{n}{2m_1} - \frac{m_2}{2\gamma m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}) - \frac{m_2^2}{\gamma m_1} - \frac{m_2}{\gamma})$. *The optimal value of* $s^* = n + \frac{m_2}{\gamma} - \frac{m_2}{\gamma p_2^*}$ *equals* $s^* = \frac{m_2 + \gamma(m_1+n)}{2\gamma} - m_1\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}$, *and* $\alpha^* = \frac{s^*}{\gamma n p_1^*}$ *equals* $\alpha^* = \frac{m_1^2}{m_2 n}(-\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}})$.

To ensure that equilibrium exists, we have $0 \le s^* \le n$. *Note that condition* $0 < s^* = m_1 B < n$ *always holds.*

(ii) If $m_1(1-p_1) < s$, *and* $m_2 > (n - m_1(1-p_1))\gamma$, *then* $p_1 \frac{m_1(1-p_1)}{s} = p_2^*$, $p_2^* = \frac{m_2}{m_2 + \gamma(n - m_1(1-p_1))}$, *and* $s = \frac{m_1 p_1(1-p_1)(m_2 + \gamma n - \gamma m_1(1-p_1))}{m_2}$. *Hence, the platform's problem over two periods is described as*

$$\max\ \pi^{DP}(p_1) = (1-\gamma)(m_1 p_1(1-p_1) + \gamma(n - m_1(1-p_1))(\frac{m_2}{m_2 + \gamma(n - m_1(1-p_1))})^2)$$

$$s.t. \quad \begin{cases} \frac{m_2 p_1 + \gamma p_1(n - m_1(1-p_1))}{m_2} > 1, \\ p_1 < 1 - \frac{n}{m_1} + \frac{m_2}{\gamma m_1}. \end{cases}$$

The effective domain for price p_1 *is given by* $(-\frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}, \min\{1 - \frac{n}{m_1} + \frac{m_2}{\gamma m_1}, 1\})$ *if* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} > 0$.

Because $\frac{d^2\pi^{DP}(p_1)}{dp_1^2} < 0$, *the profit* $\pi^{DP}(p_1)$ *is concave with* p_1, *and we assume that it decreases with* p_1 *over the effective domain for model tractability. Note that* $\frac{d\pi^{DP}(p_1)}{dp_1}\Big|_{p_1 = -\frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}} = (1-\gamma)h_1(\gamma)$, *where*

$$h_1(\gamma) = \frac{-\sqrt{\frac{\gamma^2(n-m_1)^2 + 2\gamma m_2(m_1+n) + m_2^2}{\gamma^2 m_1^2}} + m_2 + \gamma n}{\gamma} - \frac{4\gamma m_1 m_2^2\left(\gamma m_1\left(\sqrt{\frac{\gamma^2(n-m_1)^2 + 2\gamma m_2(m_1+n) + m_2^2}{\gamma^2 m_1^2}} - 1\right) - 3m_2 + \gamma n\right)}{\left(\gamma m_1\left(\sqrt{\frac{\gamma^2(n-m_1)^2 + 2\gamma m_2(m_1+n) + m_2^2}{\gamma^2 m_1^2}} - 1\right) + m_2 + \gamma n\right)^3},$$

$h_1(\gamma)$ *increases with* γ *for* $\frac{\partial p_1}{\partial \gamma} < 0$, $\frac{d^2\pi^{DP}(p_1)}{dp_1^2} < 0$. *Functions* $h_1(\gamma)$ *and* $h_2(\gamma) = \frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1}$ *interact at* $(\frac{2m_2}{2n - m_1})$. *Hence,* $\frac{d\pi^{DP}(p_1)}{dp_1}\Big|_{p_1 = -\frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}} < 0$. *So, there is no optimal solution.*

(iii) *If* $m_1(1-p_1) \geq s$*, and* $m_2 \leq (n-s)\gamma$*, then* $p_2^* = \frac{1}{2}$*,* $p_1 = \frac{m_2}{2\gamma(n-s)}$*, and* $s = n - \frac{m_2}{2\gamma p_1}$*. The platform's problem over two periods is described as*

$$\max \ \pi^{DP}(p_1) = (1-\gamma)(np_1 - \frac{m_2}{2\gamma} + \frac{m_2}{4})$$

$$s.t. \quad \begin{cases} m_1 p_1 - \frac{m_2}{2\gamma p_1} \leq m_1 - n, \\ 0 \leq p_1 \leq \frac{1}{2}. \end{cases}$$

The effective domain for p_1 *is given by* $[0, \min\{-\frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}, \frac{1}{2}\}]$*. Because* $\frac{d\pi_{p_1}^{DP}}{dp_1} > 0$*, hence,* $p_1^* = \min\{-\frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}, \frac{1}{2}\}$*. If* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} \leq 0$*, then* $\pi_{DP}^*(p_1^*) = (1-\gamma)(-\frac{n^2}{2m_1} + \frac{n}{2} + n\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}} - \frac{m_2}{2\gamma} + \frac{m_2}{4})$*,* $s^* = \frac{m_1+n}{2} - m_1\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}$*, and* $\alpha^* = \frac{2m_1}{m_2}(\frac{n}{2m_1} - \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}) - \frac{2m_1^2}{m_2 n}(\frac{n}{2m_1} - \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}})^2$*; otherwise,* $\pi_{DP}^*(p_1^*) = (1-\gamma)(\frac{n}{2} - \frac{m_2}{2\gamma} + \frac{m_2}{4})$*,* $s^* = n - \frac{m_2}{\gamma}$*, and* $\alpha^* = \frac{2}{\gamma} - \frac{2m_2}{n\gamma^2}$*.*

To ensure equilibrium exists, we have $0 \leq s^* \leq n$*. When* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} \leq 0$*, we have* $\frac{m_1+n}{2} - m_1\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}} > 0$ *if* $2\gamma n > m_2$ *and* $s^* < n$ *is always true. When* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} > 0$*, condition* $s^* < n$ *always holds and condition* $s^* > 0$ *holds if* $\gamma n > m_2$*.*

(iv) *If* $m_1(1-p_1) < s$*, and* $m_2 \leq (n - m_1(1-p_1))\gamma$*, then* $\frac{m_1(1-p_1)p_1}{s} = \frac{m_2}{2\gamma(n-m_1(1-p_1))}$*,* $p_2 = \frac{1}{2}$*, and* $s = \frac{2\gamma m_1 p_1(1-p_1)(n-m_1(1-p_1))}{m_2}$*. The platform's problem over two periods is described as*

$$\max \ \pi^{DP}(p_1) = (1-\gamma)(m_1 p_1(1-p_1) + \frac{m_2}{4})$$

$$s.t. \quad \begin{cases} p_1 \geq 1 - \frac{n}{m_1} + \frac{m_2}{\gamma m_1}, \\ 2\gamma m_1 p_1^2 + (2\gamma n - 2\gamma m_1)p_1 - m_2 > 0. \end{cases}$$

The effective domain for p_1 *is given by* $[1 - \frac{n}{m_1} + \frac{m_2}{\gamma m_1}, 1]$ *if* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} > 0$ *and* $\gamma n > m_2$ *or* $(-\frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}, 1]$ *if* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} \leq 0$ *and* $2\gamma n > m_2$*.*

If $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} > 0$*, then* $p_1^* = 1 - \frac{n}{m_1} + \frac{m_2}{\gamma m_1}$*,* $\pi_{DP}^* = (1-\gamma)(\frac{m_2}{4} + n - \frac{n^2}{m_1} - \frac{m_2^2}{\gamma^2 m_1} - \frac{m_2}{\gamma} +$

$\frac{2m_2n}{\gamma m_1})$, $s^* = 2n - \frac{2m_2}{\gamma} - \frac{2n^2}{m_1} + \frac{4m_2n}{\gamma m_1} - \frac{2m_2^2}{\gamma^2 m_1}$ *and* $\alpha^* = \frac{2\gamma n - 2m_2}{\gamma^2 n}$. *If* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} \le 0$, *then* $p_1^* = \frac{1}{2}$, $\pi_{DP}^* = (1-\gamma)\frac{m_1+m_2}{4}$, $s^* = \frac{2\gamma m_1 n - \gamma m_1^2}{4m_2}$, *and* $\alpha^* = \frac{2m_1 n - m_1^2}{2m_2 n}$.

To ensure equilibrium exists, we have $0 \le s^* \le n$. *If* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} > 0$, *then* $s^* = \frac{2(m_2+\gamma m_1-\gamma n)(\gamma n - m_2)}{\gamma^2 m_1} > 0$ *if* $m_2 \in (\gamma n - \frac{\gamma m_1}{2}, \gamma n)$, *and* $s^* < n$ *is always true for* $s^* < n$ *reduces to* $-2m_2^2 - 2\gamma(m_1-2n)m_2 + \gamma^2 n(m_1-2n) < 0$, *which is true if* $m_1 < 2n$. *If* $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} \le 0$, *condition* $s^* > 0$ *holds if* $m_1 < 2n$ *and* $s^* < n$ *if* $m_2 \in (\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}, 2\gamma n]$.

To sum up,

(i) If $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} > 0$, *then*

(a) $p_1^* = p_2^* = -\frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}$, $\pi_{DP}^* = (1-\gamma)((n + \frac{m_2}{\gamma} + \frac{m_2 n}{m_1} + \frac{m_2^2}{\gamma m_1})(-\frac{n}{2m_1} - \frac{m_2}{2\gamma m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}) - \frac{m_2^2}{\gamma m_1} - \frac{m_2}{\gamma})$, *and* $\alpha^* = \frac{m_1^2}{m_2 n}(-\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}})$;

(b) $p_1^* = 1 - \frac{n}{m_1} + \frac{m_2}{\gamma m_1}$, $p_2^* = \frac{1}{2}$, $\pi_{DP}^* = (1-\gamma)(\frac{m_2}{4} + n - \frac{n^2}{m_1} - \frac{m_2^2}{\gamma^2 m_1} - \frac{m_2}{\gamma} + \frac{2m_2 n}{\gamma m_1})$, *and* $\alpha^* = \frac{2\gamma n - 2m_2}{\gamma^2 n}$ *given that* $m_1 < 2n$ *and* $m_2 \in (\gamma n - \frac{\gamma m_1}{2}, \gamma n)$;

(c) $p_1^* = p_2^* = \frac{1}{2}$, $\pi_{DP}^* = (1-\gamma)(\frac{n}{2} - \frac{m_2}{2\gamma} + \frac{m_2}{4})$, *and* $\alpha^* = \frac{2\gamma n - 2m_2}{\gamma^2 n}$ *provided that* $\gamma n > m_2$.

(ii) If $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} \le 0$, *then*

(a) $p_1^* = -\frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}$, $p_2^* = \frac{1}{2}$, $\pi_{DP}^* = (1-\gamma)(-\frac{n^2}{2m_1} + \frac{n}{2} + n\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}} - \frac{m_2}{2\gamma} + \frac{m_2}{4})$, *and* $\alpha^* = \frac{2m_1}{m_2}(\frac{n}{2m_1} - \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}) - \frac{2m_1^2}{m_2 n}(\frac{n}{2m_1} - \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}})^2$ *given that* $m_2 < 2\gamma n$;

(b) $p_1^* = p_2^* = \frac{1}{2}$, $\pi_{DP}^* = (1-\gamma)\frac{m_1+m_2}{4}$ *and* $\alpha^* = \frac{2m_1 n - m_1^2}{2m_2 n}$ *given that* $m_2 \in [\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}, 2\gamma n]$.

Note that (a) dominates (b), and (b) dominates (c) if $\frac{1}{4} - \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1} > 0$; *or (b) dominates*

Table A.1: The Equilibrium Number of Unmatched Providers and Customers

Participants	Providers			Customers	
range of m_2	$s^* - D_1^*$	$n - D_1^* - D_2^*$	α^*	$m_1 - D_1^*$	$m_2 - D_2^*$
$(\gamma n - \frac{\gamma m_1}{2}, \gamma n)$	0	$n - (m_1 + m_2)A_2^P$	$\frac{m_1^2}{m_2 n}(A_1^P + \frac{n}{m_1} - 1)$	$m_1(1 - A_2^P)$	$m_2(1 - A_2^P)$
$[\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}, \gamma n - \frac{\gamma m_1}{2}]$	$\frac{2\gamma m_1 n - \gamma m_1^2 - 2m_1 m_2}{4m_2}$	$n - \frac{m_1+m_2}{2}$	$\frac{2m_1 n - m_1^2}{2m_2 n}$	$\frac{m_1}{2}$	$\frac{m_2}{2}$
$(0, \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n})$	0	$n - (m_1 A_4^P + \frac{m_2}{2})$	$\frac{2m_1^2}{m_2 n}(A_3^P + \frac{n}{m_1} - 1 - \frac{m_2}{2\gamma m_1})$	$m_1(1 - A_4^P)$	$\frac{m_2}{2}$

(a) if $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \le m_2 \le \gamma n - \frac{\gamma m_1}{2}$. *Define*

$$A_1^{DP} = -\frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}},$$

$$A_2^{DP} = \frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} + \frac{1}{2} - \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}},$$

$$A_3^{DP} = -\frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}},\ A_4^{DP} = \frac{n}{2m_1} + \frac{1}{2} - \sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}},$$

then the optimal strategies are given by

(i) *If* $m_2 > \gamma n - \frac{\gamma m_1}{2}$, *then* $p_1^* = p_2^* = A_1^{DP}$, $\pi_{DP}^* = (1-\gamma)(m_1+m_2)A_1^{DP}A_2^{DP}$, $\alpha^* = \frac{m_1^2}{m_2 n}(A_1^{DP} + \frac{n}{m_1} - 1)$, $D_1^* = s^* = m_1 A_2^{DP}$, *and* $D_2^* = m_2 A_2^{DP}$.

(ii) *If* $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \le m_2 \le \gamma n - \frac{\gamma m_1}{2}$, *then* $p_1^* = p_2^* = \frac{1}{2}$, $\pi_{DP}^* = (1-\gamma)\frac{m_1+m_2}{4}$, $\alpha^* = \frac{2m_1 n - m_1^2}{2m_2 n}$, $s^* = \frac{2\gamma m_1 n - \gamma m_1^2}{4m_2}$, *and* $D_i^* = \frac{m_i}{2}$.

(iii) *If* $0 < m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$, *then* $p_1^* = A_3^{DP}$, $p_2^* = \frac{1}{2}$, $\pi_{DP}^* = (1-\gamma)(m_1 A_3^{DP} A_4^{DP} + \frac{m_2}{4})$, $\alpha^* = \frac{2m_1^2}{m_2 n}(A_3^{DP} + \frac{n}{m_1} - 1 - \frac{m_2}{2\gamma m_1})$, $D_1^* = s^* = m_1 A_4^{DP}$, *and* $D_2^* = \frac{m_2}{2}$.

Recall that

$$\begin{aligned} CS =& \min\{s, m_1(1-p_1^*)\}(\frac{1}{2}(p_1^*)^2 - p_1^* + \frac{1}{2}) \\ &+ \min\{(n - \min\{s, m_1(1-p_1^*)\})\gamma p_2^*, m_2(1-p_2^*)\}(\frac{1}{2}(p_2^*)^2 - p_2^* + \frac{1}{2}), \\ PS =& \min\{s, m_1(1-p_1^*)\}\frac{\gamma^2}{2}(p_1^*)^2 + \min\{(n - \min\{s, m_1(1-p_1^*)\})\gamma p_2^*, m_2(1-p_2^*)\}\frac{\gamma^2}{2}(p_2^*)^2, \\ SW =& \pi_{DP}^* + \min\{s, m_1(1-p_1^*)\}\frac{(\gamma^2+1)(p_1^*)^2 - 2p_1^* + 1}{2} \\ &+ \min\{(n - \min\{s, m_1(1-p_1^*)\})\gamma p_2^*, m_2(1-p_2^*)\}\frac{(\gamma^2+1)(p_2^*)^2 - 2p_2^* + 1}{2}. \end{aligned}$$

(i) If $m_2 > \gamma n - \frac{\gamma m_1}{2}$, then $p_1^* = p_2^* = A_1^{DP}$, $D_1^* = \frac{m_2+\gamma(m_1+n)}{2\gamma} - m_1\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}} = m_1 A_2^{DP}$ and $D_2^* = \frac{m_2^2+\gamma m_2(m_1+n)}{2\gamma m_1} - m_2\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}} = m_2 A_2^{DP}$, hence, $CS = \frac{(m_1+m_2)(A_2^{DP})^3}{2}$, $PS = \frac{\gamma^2(m_1+m_2)(A_1^{DP})^2 A_2^{DP}}{2}$, and $SW = CS + PS + \pi_{DP}^* = \frac{(m_1+m_2)A_2^{DP}((A_2^{DP})^2+\gamma^2(A_1^{DP})^2+2(1-\gamma)A_1^{DP})}{2}$, where $\pi_{DP}^* = (1-\gamma)(m_1+m_2)(-\frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} + \frac{1}{2} + \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}})(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} + \frac{1}{2} - \sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}})$.

(ii) If $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \leq m_2 \leq \gamma n - \frac{\gamma m_1}{2}$, then $p_1^* = p_2^* = \frac{1}{2}$ and $D_i^* = \frac{m_i}{2}$, hence, $CS = \frac{m_1+m_2}{16}$, $PS = \frac{\gamma^2(m_1+m_2)}{16}$. and $SW = \frac{(\gamma^2-4\gamma+5)(m_1+m_2)}{16}$.

(iii) If $m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$, then $p_1^* = A_3^{DP}$, $p_2^* = \frac{1}{2}$, $D_1^* = m_1 D$, and $D_2^* = \frac{m_2}{2}$. Hence, $CS = \frac{m_1(A_4^{DP})^3}{2} + \frac{m_2}{16}$, $PS = \frac{\gamma^2 m_1 (A_3^{DP})^2 A_4^{DP}}{2} + \frac{\gamma^2 m_2}{16}$, and $SW = \frac{m_1((A_4^{DP})^3+\gamma^2(A_3^{DP})^2 A_4^{DP}+2(1-\gamma)A_3^{DP}A_4^{DP})}{2} + \frac{(\gamma^2-4\gamma+5)m_2}{16}$.

Proof 3 *Proof of Corollary 2.3.1.*

(i) $m_2 > \gamma n - \frac{\gamma m_1}{2}$.

Deriving the FOCs of the optimal prices w.r.t m_1, m_2, n and γ yields:

$$\frac{\partial p_1^*}{\partial m_1} = \frac{\partial p_2^*}{\partial m_1} = \frac{(m_2+\gamma n)(\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}} + \frac{1}{2} - \frac{n}{2m_1} - \frac{m_2}{2\gamma m_1}) - m_2}{2\gamma m_1^2\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}} > 0,$$

$$\frac{\partial p_1^*}{\partial m_2} = \frac{\partial p_2^*}{\partial m_2} = \frac{-\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}} + \frac{1}{2} + \frac{n}{2m_1} + \frac{m_2}{2\gamma m_1}}{2\gamma m_1\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}} > 0,$$

$$\frac{\partial p_1^*}{\partial n} = \frac{\partial p_2^*}{\partial n} = -\frac{\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}} + \frac{1}{2} - \frac{n}{2m_1} - \frac{m_2}{2\gamma m_1}}{2m_1\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}} < 0,$$

$$\frac{\partial p_1^*}{\partial \gamma} = \frac{\partial p_2^*}{\partial \gamma} = \frac{m_2(\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}} - \frac{1}{2} - \frac{n}{2m_1} - \frac{m_2}{2\gamma m_1})}{2\gamma m_1^2\sqrt{(\frac{m_2}{2\gamma m_1} + \frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{\gamma m_1}}} < 0.$$

Deriving the FOCs of the optimal transaction volume of period one w.r.t m_1, m_2, n, and γ yields:

$$\frac{\partial D_1^*}{\partial m_1} = \frac{\gamma(-\gamma m_1 - m_2 + \gamma n) + \gamma\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}}{2\gamma\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}} > 0,$$

$$\frac{\partial D_1^*}{\partial m_2} = \frac{-\gamma(m_1+n) + \sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2} - m_2}{2\gamma\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}} < 0,$$

$$\frac{\partial D_1^*}{\partial n} = \frac{\gamma\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2} - \gamma(-\gamma m_1 + m_2 + \gamma n)}{2\gamma\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}} > 0,$$

$$\frac{\partial D_1^*}{\partial \gamma} = \frac{m_2\left(\gamma m_1 - \sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2} + m_2 + \gamma n\right)}{2\gamma^2\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}} > 0.$$

Deriving the FOCs of the optimal transaction volume of period two w.r.t m_1, m_2, n, and γ yields:

$$\frac{\partial D_2^*}{\partial m_1} = \frac{m_2\left(\gamma m_1(m_2 - \gamma n) - (m_2 + \gamma n)\left(\sqrt{\gamma^2(n - m_1)^2 + 2\gamma m_2(m_1 + n) + m_2^2} - m_2 + \gamma(-n)\right)\right)}{2\gamma m_1^2\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}} > 0,$$

$$\frac{\partial D_2^*}{\partial m_2} = \frac{\left(\gamma m_1 - \sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2} + m_2 + \gamma n\right)\left(\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2} - m_2\right)}{2\gamma m_1\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}} > 0,$$

$$\frac{\partial D_2^*}{\partial n} = \frac{m_2\left(\gamma\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2} - \gamma(-\gamma m_1 + m_2 + \gamma n)\right)}{2\gamma m_1\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}} > 0,$$

$$\frac{\partial D_2^*}{\partial \gamma} = \frac{m_2^2\left(\gamma m_1 - \sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2} + m_2 + \gamma n\right)}{2\gamma^2 m_1\sqrt{4\gamma m_1 m_2 + (-\gamma m_1 + m_2 + \gamma n)^2}} > 0.$$

(ii) $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \le m_2 \le \gamma n - \frac{\gamma m_1}{2}$. *The optimal prices are independent of system parameters and* $\frac{\partial D_i^*}{\partial m_i} > 0$.

(iii) $m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$.

The FOCs of the optimal price of period one w.r.t m_1, m_2, n, and γ are given as follows:

$$\frac{\partial p_1^*}{\partial m_1} = \frac{\gamma n\left(\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}} + \frac{1}{2} - \frac{n}{2m_1}\right) - \frac{m_2}{2}}{2\gamma m_1^2\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}} > 0,\ \frac{\partial p_1^*}{\partial m_2} = \frac{1}{4\gamma m_1\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}} > 0,$$

$$\frac{\partial p_1^*}{\partial n} = \frac{-\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}} - \frac{1}{2} + \frac{n}{2m_1}}{2m_1\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}} < 0,\ \frac{\partial p_1^*}{\partial \gamma} = \frac{-m_2}{4\gamma^2 m_1\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}}} < 0.$$

Deriving the FOCs of the optimal transaction volume of period one w.r.t m_1, m_2, n, and γ yields:

$$\frac{\partial D_1^*}{\partial m_1} = \frac{\gamma(-\gamma m_1 - m_2 + \gamma n) + \gamma\sqrt{\gamma(\gamma(n - m_1)^2 + 2m_1 m_2)}}{2\gamma\sqrt{\gamma(\gamma(n - m_1)^2 + 2m_1 m_2)}} > 0,\ \frac{\partial D_1^*}{\partial m_2} = -\frac{m_1}{2\sqrt{\gamma(\gamma(n - m_1)^2 + 2m_1 m_2)}} < 0,$$

$$\frac{\partial D_1^*}{\partial n} = \frac{\gamma^2(m_1 - n) + \gamma\sqrt{\gamma(\gamma(n - m_1)^2 + 2m_1 m_2)}}{2\gamma\sqrt{\gamma(\gamma(n - m_1)^2 + 2m_1 m_2)}} > 0,\ \frac{\partial D_1^*}{\partial \gamma} = \frac{m_1 m_2}{2\gamma\sqrt{\gamma(\gamma(n - m_1)^2 + 2m_1 m_2)}} > 0.$$

The optimal price of period two is independent of system parameters and $\frac{\partial D_2^*}{\partial m_2} > 0$.

Proof 4 *Proof of Corollary 2.3.2.*

(i) If $\frac{1}{4}-\frac{n}{2m_1}+\frac{m_2}{2\gamma m_1}>0$*, then deriving the FOCs of the equilibrium number of providers joining in period one w.r.t* m_1*,* m_2*,* n*, and* γ *yields:*

$$\frac{\partial s^*}{\partial m_1}=\frac{\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}-\frac{1}{2}+\frac{n}{2m_1}-\frac{m_2}{2\gamma m_1}}{2\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}}>0,$$

$$\frac{\partial s^*}{\partial m_2}=\frac{\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}-(\frac{1}{2}+\frac{n}{2m_1}+\frac{m_2}{2\gamma m_1})}{2\gamma\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}}<0,$$

$$\frac{\partial s^*}{\partial n}=\frac{\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}+\frac{1}{2}-\frac{n}{2m_1}-\frac{m_2}{2\gamma m_1}}{2\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}}>0,$$

$$\frac{\partial s^*}{\partial \gamma}=\frac{m_2(-\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}+\frac{1}{2}+\frac{n}{2m_1}+\frac{m_2}{2\gamma m_1})}{2\gamma^2\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}}>0.$$

(ii) If $\frac{\gamma m_1}{2}-\frac{\gamma m_1^2}{4n}\leq m_2\leq\gamma n-\frac{\gamma m_1}{2}$*, then* $s^*=\frac{\gamma m_1(2n-m_1)}{4m_2}$*. Hence,* $\frac{\partial s^*}{\partial m_1}>0$*,* $\frac{\partial s^*}{\partial m_2}<0$*,* $\frac{\partial s^*}{\partial n}>0$*, and* $\frac{\partial s^*}{\partial \gamma}>0$*.*

(iii) If $m_2<\frac{\gamma m_1}{2}-\frac{\gamma m_1^2}{4n}$*, then deriving the FOCs of* s^* *w.r.t* m_1*,* m_2*,* n*, and* γ *yields:*

$$\frac{\partial s^*}{\partial m_1}=\frac{\gamma m_1(\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}+\frac{n}{2m_1}-\frac{1}{2})-\frac{m_2}{2}}{2\gamma m_1\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}}>0,\quad \frac{\partial s^*}{\partial m_2}=\frac{-1}{4\gamma\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}}<0,$$

$$\frac{\partial s^*}{\partial n}=\frac{m_2}{4\gamma^2\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}}>0,\quad \frac{\partial s^*}{\partial \gamma}=\frac{m_1(\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}-\frac{n}{2m_1}+\frac{1}{2})}{2m_1\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}}>0.$$

Recall that $\alpha=\frac{s^*}{\gamma np_1^*}$*, we deriving the FOCs of* α^* *w.r.t* m_1*,* m_2*,* n *and* γ *if any yields:*

(i) If $\frac{1}{4}-\frac{n}{2m_1}+\frac{m_2}{2\gamma m_1}>0$*, then*

$$\frac{\partial \alpha^*}{\partial m_1}=\frac{2(\gamma m_1^2-\gamma m_1 n)\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}+(2m_1n-m_1^2-n^2)\gamma-m_1m_2-m_2n}{4\gamma m_2^2 n\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}},$$

$$\frac{\partial \alpha^*}{\partial m_2}=\frac{-2\gamma m_1(m_2+\gamma(2m_1-n))\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}+(2m_1^2-3m_1n+n^2)\gamma^2}{4\gamma^2 m_1m_2n\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}}$$

$$+\frac{(3m_1m_2+2m_2n)\gamma+m_2^2}{4\gamma^2 m_1m_2n\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}}<0,$$

$$\frac{\partial \alpha^*}{\partial n}=\frac{2\gamma m_1(m_2+\gamma m_1)\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}+(m_1n-m_1^2)\gamma^2-(2m_1m_2+m_2n)\gamma-m_2^2}{4\gamma^2 m_2n^2\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}}>0,$$

$$\frac{\partial\alpha^*}{\partial\gamma} = \frac{m_1(\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}-(\frac{1}{2}+\frac{n}{2m_1}+\frac{m_2}{2\gamma m_1}))}{2\gamma^2 n\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}}} < 0.$$

As for $\frac{\partial\alpha^*}{\partial m_1}$*, the simplified form of the numerator* $\sqrt{(\frac{m_2}{2\gamma m_1}+\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}} + \frac{-(m_1-n)^2\gamma-(m_1+n)m_2}{2\gamma m_1(m_1-n)}$ *is positive if* $m_1 < n$*, or is negative otherwise. So,* $\frac{\partial\alpha^*}{\partial m_1} > 0$ *if* $m_1 < n$ *or* $\frac{\partial\alpha^*}{\partial m_1} < 0$ *if* $m_1 > n$*.*

(ii) If $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \le m_2 \le \gamma n - \frac{\gamma m_1}{2}$*, then* $\alpha^* = \frac{m_1(2n-m_1)}{2m_2 n}$*, hence,* $\frac{\partial\alpha^*}{\partial m_1} > 0$ *if* $m_1 < n$*,* $\frac{\partial\alpha^*}{\partial m_2} < 0$*, and* $\frac{\partial\alpha^*}{\partial n} > 0$*.*

(iii) If $m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$*, then*

$$\frac{\partial\alpha^*}{\partial m_1} = \frac{\left(\sqrt{\gamma\left(\gamma\left(n-m_1\right)^2+2m_1m_2\right)}-\gamma m_1\right)\left(-\gamma m_1+\sqrt{\gamma\left(\gamma\left(n-m_1\right)^2+2m_1m_2\right)}-m_2+\gamma n\right)}{\gamma m_2 n\sqrt{\gamma\left(\gamma\left(n-m_1\right)^2+2m_1m_2\right)}},$$

$$\frac{\partial\alpha^*}{\partial m_2} = \frac{(2\gamma m_1^2-2\gamma m_1 n)\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}-(\gamma n^2+\gamma m_1^2-2\gamma m_1 n+m_1m_2)}{2\gamma m_2^2 n\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}} < 0,$$

$$\frac{\partial\alpha^*}{\partial n} = \frac{(m_1^2+\frac{m_1m_2}{\gamma})\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}-(\frac{m_1^2}{2}-\frac{m_1 n}{2}+\frac{m_1m_2}{\gamma})}{m_2 n^2\sqrt{(\frac{n}{2m_1}-\frac{1}{2})^2+\frac{m_2}{2\gamma m_1}}} > 0,$$

$$\frac{\partial\alpha^*}{\partial\gamma} = \frac{m_1\left(\sqrt{\gamma\left(\gamma\left(n-m_1\right)^2+2m_1m_2\right)}-\gamma m_1\right)}{\gamma^2 n\sqrt{\gamma\left(\gamma\left(n-m_1\right)^2+2m_1m_2\right)}}.$$

As for $\frac{\partial\alpha^*}{\partial m_1}$*, the term in the second bracket of the numerator is positive. Hence,* $\frac{\partial\alpha^*}{\partial m_1}$ *and* $\frac{\partial\alpha^*}{\partial\gamma}$ *show the same pattern: the numerators are positive if* $m_2 > \gamma n - \frac{\gamma n^2}{2m_1}$*. Hence,* $\frac{\partial\alpha^*}{\partial m_1} > 0$ *if* $\gamma n - \frac{\gamma n^2}{2m_1} < m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$ *or* $\frac{\partial\alpha^*}{\partial m_1} < 0$ *otherwise.*

Proof 5 *Proof of Increasing Bidding Policy on the Demand Side.*

Consider customers with valuation v' *and* v''*, where* $v'' > v'$*, and let* $B(v')$ *and* $B(v'')$ *be customer's respective bidding prices, then we have*

$$r_c(v', B(v')) - r_c(v', B(v'')) \ge 0,\ r_c(v'', B(v'')) - r_c(v'', B(v')) \ge 0.$$

Adding above two inequalities yields

$$r_c(v'', B(v'')) - r_c(v', B(v'')) + r_c(v', B(v')) - r_c(v'', B(v')) \ge 0,$$

where $r_c(v, B(v)) = (v-p)Pr\{B(v)\}$*. Note that the transaction price is a market-clearing price and probabilities for customers with the same bids being matched are equal, the above inequality reduces to* $(v'' - v')(Pr\{B(v'')\} - Pr\{B(v')\}) \geq 0$*. If* $v'' > v'$*, then* $Pr\{B(v'')\} \geq Pr\{B(v')\}$*. Hence,* $b_c > 0$ *and* $b_p > 0$*.*

Proof 6 *Simplification of Differential Equations.*

(i) Simplification of Equation 2.4*.*

To simplify P_1*,* P_2 *and* P_3*, we rewrite using only the index* i*:*

$$P_1(b) = \sum_i^{m-1} \binom{n}{m-i} \binom{m-1}{i} G(v)^i F(c)^{m-i} (1-G(v))^{m-1-i} (1-F(c))^{n-m+i},$$

$$P_2(b) = \sum_i^{m-1} \binom{n-1}{m-1-i} \binom{m-1}{i} G(v)^i F(c)^{m-1-i} (1-G(v))^{m-1-i} (1-F(c))^{n-m+i},$$

$$P_3(b) = \sum_i^{m-2} \binom{n}{m-1-i} \binom{m-2}{i} G(v)^i F(c)^{m-1-i} (1-G(v))^{m-2-i} (1-F(c))^{n-m+i+1}.$$

Substituting $\binom{n-1}{m-1-i} = \frac{m-i}{n} \binom{n}{m-i}$*, and* $\binom{m-2}{i-1} = \frac{i}{m-1} \binom{m-1}{i}$ *into* P_2 *and* P_3*, respectively, and replacing index* i *with* $i-1$ *in* P_3 *yield:*

$$P_1(b) = \sum_i^{m-1} \binom{n}{m-i} \binom{m-1}{i} (\frac{G(v)(1-F(c))}{F(c)(1-G(v))})^i F(c)^m (1-G(v))^{m-1} (1-F(c))^{n-m},$$

$$P_2(b) = \sum_i^{m-1} \binom{n}{m-i} \binom{m-1}{i} (\frac{G(v)(1-F(c))}{F(c)(1-G(v))})^i \frac{m-i}{n} F(c)^{m-1} (1-G(v))^{m-1} (1-F(c))^{n-m},$$

$$P_3(b) = \sum_i^{m-1} \binom{n}{m-i} \binom{m-1}{i} (\frac{G(v)(1-F(c))}{F(c)(1-G(v))})^i \frac{i}{m-1} G(v)^{-1} F(c)^m (1-G(v))^{m-1} (1-F(c))^{n-m}.$$

Correspondingly, denoted $\frac{G(v)(1-F(c))}{F(c)(1-G(v))}$ *by* A *and equation* 2.4 *reduces to:*

$$\lim_{\Delta b \to 0} \frac{r_c(b+\Delta b) - r_c(b)}{\Delta b} = (v-b)[\frac{1}{b_p F(c)} \sum_i^{m-1} \binom{n}{m-i} \binom{m-1}{i} (m-i) A^i + \frac{1}{b_c G(v)} \sum_i^{m-1} \binom{n}{m-i} \binom{m-1}{i} i A^i] - k \sum_i^{m-1} \binom{n}{m-i} \binom{m-1}{i} A^i.$$

(ii) Simplification of Equation 2.5*.*

The following two equations define the formula of P_4 *and* P_5 *respectively:*

$$P_4(b) = \sum_{0 \leq i \leq m, 0 \leq j \leq n-2}^{i+j=m-1} \binom{n-2}{j} \binom{m}{i} G(v)^i F(c)^j (1-G(v))^{m-i} (1-F(c))^{n-2-j},$$

$$P_5(b) = \sum_{0\le i\le m, 0\le j\le n-1}^{i+j=m-1} \begin{pmatrix} n-1 \\ j \end{pmatrix} \begin{pmatrix} m \\ i \end{pmatrix} G(v)^i F(c)^j (1-G(v))^{m-i}(1-F(c))^{n-1-j}.$$

Similarly, we have the simplified forms of P_4 and P_5:

$$P_4(b) = \sum_i^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} A^i \frac{m(n-m+i)}{n(n-1)} F(c)^{m-1}(1-G(v))^m (1-F(c))^{n-m-1},$$

$$P_5(b) = \sum_i^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} A^i \frac{m}{n} F(c)^{m-1}(1-G(v))^m (1-F(c))^{n-m}.$$

Plugging the simplified forms of P_2, P_4 and P_5 into equation 2.5 *yields:*

$$\begin{aligned}\lim_{\Delta b\to 0}\frac{r_p(b+\Delta b)-r_p(b)}{\Delta b} =& -(\gamma b-c)\Big(\frac{1}{b_p(1-F(c))}\sum_i^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} (n-m+i)A^i \\ &+\frac{1}{b_c(1-G(v))}\sum_i^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} (m-i)A^i\Big) \\ &+\gamma(1-k)\sum_i^{m-1} \begin{pmatrix} n \\ m-i \end{pmatrix} \begin{pmatrix} m-1 \\ i \end{pmatrix} A^i.\end{aligned}$$

Proof 7 *Proof of Lemma 2.4.1. Suppose that the bidding policies B and S characterize an equilibrium, then the necessary condition is obtained by solving two differential equations simultaneously:*

$$\frac{1}{B'(v)} = \frac{kP_1}{(v-b)(m-1)P_3 g(v)} - \frac{nP_2 f(c)}{S'(c)(m-1)P_3 g(v)}, \frac{1}{S'(C)} = \frac{\gamma(1-k)P_5}{(\gamma b-c)(n-1)P_4 f(c)} - \frac{mP_2 g(v)}{B'(v)(n-1)P_4 f(c)}.$$

We prove the necessary and sufficient conditions based on Satterthwaite and Williams 1989b and Figure 2.4.

(a) Necessary Part.

On the demand side, let $y = T_{(m)}$, $z = T_{(m+1)}$ and $f(y,z)$ denote the joint density of y and z. If customers' bid satisfies $b \in (y,z)$, then the utility equals $v-(1-k)y-kb$, if customers' bid satisfies $b > z$, then the utility equals $v-(1-k)y-kz$. Hence, customers' expected utility equals

$$r_c = \int_b^1\int_0^b (v-(1-k)y-kb)f(y,z)dydz + \int_0^b\int_0^z (v-(1-k)y-kz)f(y,z)dydz.$$

By double integral derivation theorem, differentiating the utility function w.r.t b yields

$$\frac{dr_c}{db} = -\int_0^b (v-(1-k)y-kb)f(y,b)dy + \int_b^1 (v-(1-k)b-kb)f(b,z)dz$$

$$-k\int_b^1\int_0^b f(y,z)dydz+\int_0^b (v-(1-k)y-kb)f(y,b)dy.$$

Note that the first and the fourth terms of the above equation are cancelled out, hence, the differential equation reduces to $\frac{dr_c}{db}=\int_b^1 f(b,z)dz(v-b)-k\int_b^1\int_0^b f(y,z)dydz$, *where* $\int_b^1 f(b,z)dz$ *(resp.,* $\int_b^1\int_0^b f(y,z)dydz$*) is the probability that the customer increases her bidding price with gains (resp., losses) and being matched successfully (refer to theorem 5.4.4 in (Casella and Berger 2021), p.229). Specifically,* $\int_b^1 f(b,z)dz$ *is the density of the order statistic* y *judged at* b*, and* $\int_b^1\int_0^b f(y,z)dydz$ *(i.e.,* $Pr\{y<b<z\}$*) is the probability that the customer's bidding price satisfies* $b\in(T_{(m)},T_{(m+1)})$ *in the pool of* $n+m-1$ *bids.*

Similarly, a provider with bid b *can be matched if* $x<b<y$ *or* $b<x<y$*, where* $x=T_{(m-1)}$*. Let* $g(x,y)$ *denote the joint density of* x *and* y*, we have the expected utility of providers given by*

$$r_p=\int_x^1\int_b^1(\gamma((1-k)x+ky)-c)g(x,y)dxdy+\int_b^1\int_0^b(\gamma((1-k)b+ky)-c)g(x,y)dxdy.$$

Differentiating the utility function w.r.t b *yields*

$$\frac{dr_p}{db}=-\int_b^1(\gamma((1-k)b+ky)-c)g(b,y)dy+\int_b^1(\gamma((1-k)b+ky)-c)g(b,y)dy$$
$$-\int_0^b(\gamma((1-k)b+kb)-c)g(x,b)dx+\gamma(1-k)\int_0^b\int_b^1 g(x,y)dxdy.$$

The first and second term of the above equation are cancelled out, hence, the differential equation on the supply side reduces to $\frac{dr_p}{db}=-(\gamma b-c)\int_0^b g(x,b)dx+\gamma(1-k)\int_0^b\int_b^1 g(x,y)dxdy$*, where* $\int_0^b g(x,b)dx$ *and* $\int_0^b\int_b^1 g(x,y)dxdy$ *corresponds to the first and second probabilities defined in equation* 2.5*, respectively.*

(b) *Sufficient Part.*

We need to prove that the payoff function of customers (resp., providers) is maximized at $b=B(v)$ *(resp.,* $b=S(c)$*).*

On the demand side, (a) $\frac{dr_c}{db}=F(v,b,B)(v-b)-kP_1$*, where* $K(v,b,B)=nP_2\frac{f(c)}{S'(c)}+(m-1)P_3\frac{g(v)}{B'(v)}$*. Note that* $\frac{dr_c}{db}=0$ *if* $b=B(v)$*:* $K(v,B(v),B)(v-B(v))-kP_1=0$*.*

(b) Because $K(v,b,B) = nP_2\frac{f(c)}{S'(c)} + (m-1)P_3\frac{g(v)}{B'(v)}$*, then* $\frac{dr_c}{db} = K(B^{-1}(b),b,B)(v - B^{-1}(b)) + K(B^{-1}(b),b,B)(B^{-1}(b)-b) - kM$*. If* $v = B^{-1}(b)$*, then the FOC reduces to* $\frac{dr_c}{db} = K(B^{-1}(b),b,B)(v - B^{-1}(b))$*. Note that the marginal expected payoff of customers is positive, it reaches zero at* b *and changes from positive to negative as* $b = B(v)$ *increases. Thus, customer's payoff function is maximized at* b*. On the supply side, the analysis is similar, we omit the details for simplicity.*

Proof 8 *Simplification of Bidding Equilibrium Derivation.*

From the simplified forms of differential equations, we obtain the vector field:

$$\frac{1}{b_c} = \frac{G(v)k - G(v)\frac{(v-b)(m-i)}{b_pF(c)}}{(v-b)i}, \ \frac{1}{b_p} = \frac{-\frac{(m-i)(\gamma b-c)(1-F(c))}{b_c(1-G(v))} + \gamma(1-k)(1-F(c))}{(n-m+i)(\gamma b - c)},$$

where $c = \frac{b-a_p}{b_p}$*,* $v = \frac{b-b_c}{b_c}$*.*

By rearranging the terms (the LHS are reduced to two terms with and without the index and RHS is reduced to zero, then both terms on LHS are equal to zero), we have equations labeled as equation 1 to 4:

$$\frac{1}{b_c}(\frac{b-a_c}{b_c} - b) = \frac{b-a_c}{b_c}\frac{\frac{b-a_c}{b_c} - b}{b-a_p},$$

$$\frac{b-a_c}{b_c}(k - m\frac{\frac{b-a_c}{b_c} - b}{b-a_p}) = 0,$$

$$(1 - \frac{b-a_p}{b_p})(\gamma(1-k) - m\frac{\gamma b - \frac{b-a_p}{b_p}}{b_c - b + a_c}) = (n-m)\frac{1}{b_p}(\gamma b - \frac{b-a_p}{b_p}),$$

$$(1 - \frac{b-a_p}{b_p})\frac{\gamma b - \frac{b-a_p}{b_p}}{b_c - b + a_c} = \frac{1}{b_p}(\gamma b - \frac{b-a_p}{b_p}).$$

Note that $\frac{b-a_c}{b_c} \neq 0$ *and* $1 - \frac{b-a_p}{b_p} \neq 0$ *for equation* $\frac{b-a_c}{b_c} = 0$ *leads to* $-\frac{b}{b_c} = 0$ *and equation* $1 - \frac{b-a_p}{b_p} = 0$ *results in* $\gamma b - \frac{b-a_p}{b_p} = 0$ *which are not reasonable. Moreover,* $\frac{b-a_c}{b_c} - b = 0$ *and* $\gamma b - \frac{b-a_p}{b_p} = 0$ *can not hold simultaneously for the first (resp., second) equation indicates that* $k = 0$ *(resp.,* $k = 1$*), and two cases violate each other.*

So, condition $\frac{b-a_c}{b_c} - b \neq 0$ *along with* $\gamma b - \frac{b-a_p}{b_p} = 0$*,* $\frac{b-a_c}{b_c} - b = 0$ *together with* $\gamma b - \frac{b-a_p}{b_p} \neq 0$*, and* $\frac{b-a_c}{b_c} - b \neq 0$ *together with* $\gamma b - \frac{b-a_p}{b_p} \neq 0$ *corresponds to case* $k = 1$*,* $k = 0$ *and* $k \in (0,1)$*, respectively.*

Proof 9 *Proof of Lemma 2.4.1.1.*

(a) *Supply Side.*

Equation $\gamma b - \frac{b-a_p}{b_p} = 0$ *implies that* $b = \gamma c$, *so,* $a_p = 0$ *and* $b_p = \frac{1}{\gamma}$. *This can be verified by the following logic: Providers' bidding policy includes* b^0, b *and* b^1, *where*

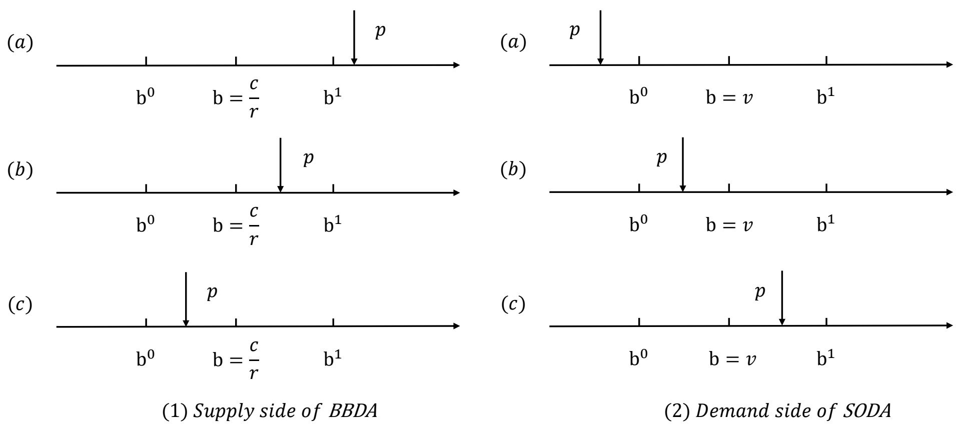

Figure A.1: Possible Consideration of Case (a) in Lemma 4 & 5

$b^0 < b = \frac{c}{\gamma} < b^1$ *when* $k = 1$ *(refer to Figure A.1 of (1)). If the transaction price is larger than* b^1, *then the provider will be successfully matched with* b^1, b *and* b^0, *while the net surplus* $\gamma b - c$ *of proposing bid* b^0 *is negative. If the transaction price is within* (b, b^1), *then providers with bidding price* b *will be matched instead of* b^1. *If the transaction price is within interval* (b^0, b), *providers with bidding price* b^0 *have the chance of being matched while the net surplus is negative. Hence, it is optimal for providers to bid the discounted reservation prices in BBDA.*

(b) *Demand Side.*

Plugging $a_p = 0$ *and* $b_p = \frac{1}{\gamma}$ *into simplified equations of 1 and 2 (i.e.,* $a_c = a_p$ *and* $1 - m\frac{\frac{b-a_c}{b_c} - b}{b} = 0$*) yields* $a_c = 0$ *and* $b_c(\frac{1}{m} + 1) = 1$ *correspondingly. Hence,* $B(v) = \frac{m}{m+1}v$.

Proof 10 *Proof of Lemma 2.4.1.2.*

(a) *Demand Side.*

Equation $\frac{b-a_c}{b_c} - b = 0$ *implies that* $b = v$*, so,* $a_c = 0$ *and* $b_c = 1$*. This can also be verified by the following logic: Customers' bidding policy includes* b^0*,* b *and* b^1*, where* $b^0 < b = v < b^1$ *when* $k = 0$ *(refer to Figure A.1 of (2)). If the transaction price is smaller than* b^0*, then the provider will be successfully matched with* b^0*,* b *and* b^1*, while the net surplus* $v - b$ *of proposing bid* b^1 *is negative. If the transaction price is within* (b^0, b)*, then providers with bid* b *will be matched instead of with bid* b^0*. If the price is within interval* (b, b^1)*, providers with bid* b^1 *have the chance of being matched while the net surplus is negative. Hence, it is optimal for customers to bid their valuation under SODA.*

(b) *Supply Side.*

Replacing v *with* b*, and* k *with* 0 *into the equations 3 and 4 leads to* $(\gamma b - \frac{b-a_p}{b_p})n = \gamma(a_p + b_p - b)$ *and* $a_p + b_p = a_c + b_c$*, respectively, solving these two equations yields* $a_p = \frac{1}{n+1}$ *and* $b_p = \frac{n}{n+1}$ *when* $\gamma = 1$ *or* $a_p + b_p = 1$*,* $a_p = \frac{\gamma(nb+b-1)-nb}{\gamma(nb+b-1)-n}$*, and* $b_p = \frac{nb-n}{\gamma(nb+b-1)-n}$ *when* $\gamma \in (0, 1)$*, where* $b = a_b + b_p c$*,* $c \in [0, 1]$*.*

Proof 11 *Proof of Lemma 4.2.1.3. To solve the aforementioned four equations, we focus on conditions* $\gamma b - \frac{b-a_p}{b_p} \neq 0$ *and* $\frac{b-a_c}{b_c} - b \neq 0$*, for* $\frac{b-a_c}{b_c} - b = 0$ *indicates that* $k = 0$ *and* $\gamma b - \frac{b-a_p}{b_p} = 0$ *indicates that* $k = 1$*, while* $k \in (0, 1)$*.*

If $\gamma b - \frac{b-a_p}{b_p} \neq 0$ *and* $\frac{b-a_c}{b_c} - b \neq 0$*, then equation* 1 *and* 4 *reduce to* $a_c = a_p$ *and* $a_c + b_c = a_p + b_p$*, respectively. Equation* 2 *reduces to* $b = \frac{a_p b_c k - a_c m}{b_c(k+m)-m}$ *and equation 3 reduces to* $b = \frac{b_p^2(\gamma-\gamma k)+a_p b_p(\gamma-\gamma k)-a_p n}{-n+b_p(\gamma-\gamma k+\gamma n)}$*. By replacing* a_c *(resp.,* b_c*) with* a_p *(resp.,* b_p*) and equating* $\frac{a_p b_c k - a_c m}{b_c(k+m)-m}$ *with* $\frac{b_p^2(\gamma-\gamma k)+a_p b_p(\gamma-\gamma k)-a_p n}{-n+b_p(\gamma-\gamma k+\gamma n)}$*, we have* $a_p = \frac{(1-k)\gamma((k+m)b_p^2 - m b_p)}{b_p((k-1)\gamma m+\gamma n k)+mn-\gamma mn}$*.*

Proof 12 *Proof of Proposition 2.4.3.1. Given* (m_1, s) *in period one, and* $(m_2, n - \gamma s p_1)$ *in period two, customers' bidding policies are given by* $B_1(v) = \frac{m_1}{m_1+1}v$ *in period one and* $B_2(v) = \frac{m_2}{m_2+1}v$ *in two periods. Plugging bidding policies of both sides into Equations* 2.6*,* 2.7 *and* 2.8 *leads to:*

$$\gamma s p_1 = m_1(1 - \frac{m_1+1}{m_1}p_1),\ (n - \gamma s p_1)\gamma p_2 = m_2(1 - \frac{m_2+1}{m_2}p_2),\ p_1 = p_2.$$

Table A.2: The Equilibrium Number of Unmatched Providers and Customers

Participants	Equilibrium number	Outcomes
Providers	$s - D_1^*$	$\frac{m_1 E^2 - 2m_2(m_1+1+\gamma m_1)E + 4\gamma m_2^2(m_1+1)}{2\gamma m_2 E}$
	$n - (D_1^* + D_2^*)$	$\frac{(n-m_1-m_2)E - 2m_2(m_1+m_2+2)}{E}$
	α^*	$\frac{m_1 E - 2m_2(m_1+1)}{2\gamma m_2 n}$
Customers	$m_1 - D_1^*$	$\frac{2m_2(m_1+1)}{E}$
	$m_2 - D_2^*$	$\frac{2m_2(m_2+1)}{E}$

Solving three equations leads to $s^* = \frac{m_1 - 2m_2 - m_1 m_2 - \gamma m_1^2 + \gamma m_1 n + m_1\sqrt{m_2^2 + (1+\gamma n - \gamma m_1)^2 + 2m_2(1+\gamma(2+m_1+n))}}{2\gamma m_2}$. *Hence,* $p_1^* = p_2^* = \frac{2m_2}{1+m_2-\gamma m_1+\gamma n+\sqrt{m_2^2+(1+\gamma n-\gamma m_1)^2+2m_2(1+\gamma(2+m_1+n))}}$, $D_1^* = m_1 - \frac{2m_2(m_1+1)}{E}$, $D_2^* = m_2 - \frac{2m_2(m_2+1)}{E}$, *and* $\pi^* = \frac{2m_2(1-\gamma)(E(m_1+m_2)-4m_2-2m_2^2-2m_1 m_2)}{E^2}$, *where* $E = 1 + m_2 - \gamma m_1 + \gamma n + \sqrt{m_2^2 + (1+\gamma n - \gamma m_1)^2 + 2m_2(1+\gamma(2+m_1+n))}$.

Because $s^* \in [0, n]$, *that is,* $0 \leq \frac{Em_1 - 2m_2(m_1+1)}{2\gamma m_2} \leq n$. *Or equivalently,* $\frac{m_1(1+\gamma n)^2 + m_1^2(1+\gamma n - \gamma^2 n)}{(1+\gamma n + m_1)(1+\gamma n)} \leq m_2 \leq m_1 + \gamma m_1 n$.

Social Surpluses and Social Welfare.

$CS = \frac{(E-2m_2)^2(E(m_1+m_2)-2m_2(2+m_1+m_2))}{2E^3}$, $PS = \frac{2\gamma^2 m_2^2(E(m_1+m_2)-2m_2(2+m_1+m_2))}{E^3}$, *and* $SW = \frac{(E^2-4\gamma m_2 E+4(1+\gamma^2)m_2^2)(E(m_1+m_2)-2m_2(2+m_1+m_2))}{2E^3}$.

Proof 13 *Proof of Corollary 2.4.3.1.*

(i) The FOCs of the optimal price w.r.t m_1, m_2, *and* n *are denoted by:*

$$\frac{\partial p_1^*}{\partial m_1} = \frac{2\gamma m_2\left(1 - \frac{m_2-(1+\gamma n-\gamma m_1)}{\sqrt{m_2^2+(1+\gamma n-\gamma m_1)^2+2m_2(1+\gamma(2+m_1+n))}}\right)}{E^2} > 0,$$

$$\frac{\partial p_1^*}{\partial m_2} = \frac{2((1-\gamma m_1+\gamma n)E + 2\gamma m_2(m_1+1))}{\sqrt{m_2^2+(1+\gamma n-\gamma m_1)^2+2m_2(1+\gamma(2+m_1+n))}E^2} > 0,$$

$$\frac{\partial p_1^*}{\partial n} = -\frac{2\gamma m_2}{\sqrt{m_2^2+(1+\gamma n-\gamma m_1)^2+2m_2(1+\gamma(2+m_1+n))}E} < 0.$$

(ii) The FOCs of the optimal transaction volumes w.r.t m_1, m_2, *and* n *are denoted by:*

$$\frac{\partial D_1^*}{\partial m_1} = \frac{-\gamma m_1 + \sqrt{(-\gamma m_1+\gamma n+1)^2 + 2m_2(\gamma(m_1+n+2)+1) + m_2^2} - m_2 + \gamma n + 1}{2\sqrt{(-\gamma m_1+\gamma n+1)^2 + 2m_2(\gamma(m_1+n+2)+1) + m_2^2}} > 0,$$

$$\frac{\partial D_1^*}{\partial m_2} = \frac{-\gamma(m_1+n+2) + \sqrt{(-\gamma m_1+\gamma n+1)^2 + 2m_2(\gamma(m_1+n+2)+1) + m_2^2} - m_2 - 1}{2\gamma\sqrt{(-\gamma m_1+\gamma n+1)^2 + 2m_2(\gamma(m_1+n+2)+1) + m_2^2}} < 0,$$

$$\frac{\partial D_1^*}{\partial n} = \frac{\gamma m_1 + \sqrt{(-\gamma m_1+\gamma n+1)^2 + 2m_2(\gamma(m_1+n+2)+1) + m_2^2} - m_2 + \gamma(-n) - 1}{2\sqrt{(-\gamma m_1+\gamma n+1)^2 + 2m_2(\gamma(m_1+n+2)+1) + m_2^2}} > 0,$$

$$\frac{\partial D_2^*}{\partial m_1} = \frac{2\gamma m_2 (m_2+1)\left(\frac{\gamma m_1+m_2+\gamma(-n)-1}{\sqrt{(-\gamma m_1+\gamma n+1)^2+2m_2(\gamma(m_1+n+2)+1)+m_2^2}}-1\right)}{\left(-\gamma m_1+\sqrt{(-\gamma m_1+\gamma n+1)^2+2m_2(\gamma(m_1+n+2)+1)+m_2^2}+m_2+\gamma n+1\right)^2} < 0,$$

$$\frac{\partial D_2^*}{\partial m_2} = -\frac{2m_2}{-\gamma m_1+\sqrt{(-\gamma m_1+\gamma n+1)^2+2m_2(\gamma(m_1+n+2)+1)+m_2^2}+m_2+\gamma n+1}$$

$$+\frac{2(m_2+1)m_2\left(\frac{\gamma m_1+m_2+\gamma(n+2)+1}{\sqrt{(-\gamma m_1+\gamma n+1)^2+2m_2(\gamma(m_1+n+2)+1)+m_2^2}}+1\right)}{\left(-\gamma m_1+\sqrt{(-\gamma m_1+\gamma n+1)^2+2m_2(\gamma(m_1+n+2)+1)+m_2^2}+m_2+\gamma n+1\right)^2}$$

$$-\frac{2(m_2+1)}{-\gamma m_1+\sqrt{(-\gamma m_1+\gamma n+1)^2+2m_2(\gamma(m_1+n+2)+1)+m_2^2}+m_2+\gamma n+1}+1>0,$$

$$\frac{\partial D_2^*}{\partial n} = \frac{2\gamma m_2(m_2+1)}{F\sqrt{(-\gamma m_1+\gamma n+1)^2+2m_2(\gamma(m_1+n+2)+1)+m_2^2}} > 0.$$

(iii) The FOCs of s^* w.r.t m_1, m_2, and n are denoted by:

$$\frac{\partial s^*}{\partial m_1} = \frac{(E-2m_2)(E-1-m_2-\gamma n)}{2\gamma m_2\sqrt{m_2^2+(1+\gamma n-\gamma m_1)^2+2m_2(1+\gamma(2+m_1+n))}},$$

$$\frac{\partial s^*}{\partial m_2} = \frac{-m_1(2\gamma m_2(1+m_1)+(1-\gamma m_1+\gamma n)E)}{2\gamma m_2^2\sqrt{m_2^2+(1+\gamma n-\gamma m_1)^2+2m_2(1+\gamma(2+m_1+n))}} < 0,$$

$$\frac{\partial s^*}{\partial n} = \frac{m_1 E}{2m_2\sqrt{m_2^2+(1+\gamma n-\gamma m_1)^2+2m_2(1+\gamma(2+m_1+n))}} > 0,$$

$\frac{\partial s^*}{\partial m_1} > 0$ *if* $\sqrt{\gamma^2 m_1^2+4\gamma(m_1+1)(1+\gamma+\gamma n)}-(1+\gamma(2+m_1+n)) < m_2 < \gamma n+1.$

Proof 14 *Proof of Proposition 2.4.3.2. Given (m_1, s) in period one, and $(m_2, n-(s+1)p_1+1)$ in period two, then providers' bidding policies are denoted by $S_1(c) = \frac{1}{s+1}+\frac{s}{s+1}c$, and $S_2(c) = \frac{1}{(n-(s+1)p_1+2)}+\frac{n-(s+1)p_1+1}{(n-(s+1)p_1+2)}c$. Plugging the bidding policies of both sides into Equations 2.6, 2.7 and 2.8 leads to:*

$$(s+1)p_1-1 = m_1(1-p_1),\ (n-(s+1)p_1+2)p_2-1 = m_2(1-p_2),\ p_1\frac{(s+1)p_1-1}{s} = p_2\frac{(n-(s+1)p_1+2)p_2-1}{n-(s+1)p_1+1}.$$

Replacing m_2 with δm_1 and n with βm_1 into the equilibrium conditions leads to:

$$s^* = \frac{-2\delta(1+m_1)(\delta m_1+1)+(1+(-1+\delta+\beta)m_1)\sqrt{\delta(1+m_1)(1+\delta m_1)}}{2\delta(1+\delta m_1)}$$

$$+\frac{\sqrt{(1+m_1)\delta(1+\delta m_1)((1+(-1+\delta+\beta)m_1)^2+4m_1\sqrt{\delta(1+m_1)(1+\delta m_1)})}}{2\delta(1+\delta m_1)},$$

$$p_1^* = \frac{\sqrt{((1+(-1+\delta+\beta)m_1)^2+4m_1\sqrt{\delta(1+m_1)(1+\delta m_1)})}-(1+(-1+\delta+\beta)m_1)}{2m_1},$$

$$p_2^* = \frac{2(\delta m_1 + 1)}{\sqrt{((1 + (-1 + \delta + \beta)m_1)^2 + 4m_1\sqrt{\delta(1 + m_1)(1 + \delta m_1)})} + \beta m_1 + \delta m_1 - m_1 + 1}.$$

Let $F = \sqrt{((1 + (-1 + \delta + \beta)m_1)^2 + 4m_1\sqrt{\delta(1 + m_1)(1 + \delta m_1)})} + \beta m_1 + \delta m_1 - m_1 + 1$*, hence,* $s^* = \frac{\sqrt{\delta(1+m_1)(1+\delta m_1)}F - 2\delta(1+m_1)(1+\delta m_1)}{2\delta(1+\delta m_1)}$, $p_1^* = \frac{F-2(\beta m_1+\delta m_1-m_1+1)}{2m_1}$, $p_2^* = \frac{2(\delta m_1+1)}{F}$, $D_1^* = \frac{2(\beta m_1+\delta m_1+1)-F}{2}$, $D_2^* = \frac{m_2F-2m_2(\delta m_1+1)}{F}$, $\pi^* = 0$, $CS = \frac{m_1(-(p_1^*)^3+3(p_1^*)^2-3p_1^*+1)}{2} + \frac{m_2(-(p_2^*)^3+3(p_2^*)^2-3p_2^*+1)}{2} = \frac{1}{2}\left(\frac{m_2(F-2\delta m_1-2)^3}{F^3} + \frac{(-F+2m_1(\beta+\delta)+2)^3}{8m_1^2}\right)$, $PS = \frac{\gamma^2 m_1((p_1^*)^2-(p_1^*)^3)}{2} + \frac{\gamma^2 m_2((p_2^*)^2-(p_2^*)^3)}{2} = \frac{F^3\left(2\left(\gamma^2+3\right)m_1(F-2m_1(\beta+\delta-1)-2)^2-\left(\gamma^2+1\right)(F-2m_1(\beta+\delta-1)-2)^3-12m_1^2(F-2m_1(\beta+\delta-1)-2)+8m_1^3\right)}{16F^3m_1^2} + \frac{8m_1^2m_2\left(F^3-6F^2(\delta m_1+1)+4\left(\gamma^2+3\right)F(\delta m_1+1)^2-8\left(\gamma^2+1\right)(\delta m_1+1)^3\right)}{16F^3m_1^2}$*, and* $SW = \frac{m_1(-(\gamma^2+1)(p_1^*)^3+(3+\gamma^2)(p_1^*)^2-3p_1^*+1)}{2} + \frac{m_2(-(\gamma^2+1)(p_2^*)^3+(3+\gamma^2)(p_2^*)^2-3p_2^*+1)}{2}$.

Because $0 \le s^* \le n$*, that is,* $\frac{2\delta(1+m_1)(1+\delta m_1)}{\sqrt{\delta(1+m_1)(1+\delta m_1)}} \le F \le \frac{2\delta(1+m_1+\delta m_1)(1+\delta m_1)}{\sqrt{\delta(1+m_1)(1+\delta m_1)}}$

Proof 15 *Proof of Proposition 2.4.3.3. Because* $a_{p1} = a_{c1}$, $b_{p1} = b_{c1}$, $a_{p2} = a_{c2}$ *and* $b_{p2} = b_{c2}$*, equations capturing fulfillment equilibrium are simplified into the following:*

$$s\frac{p_1 - a_{p1}}{b_{p1}} = m_1(1 - \frac{p_1 - a_{p1}}{b_{p1}}),\ (n - s\frac{p_1 - a_{p1}}{b_{p1}})\frac{p_2 - a_{p2}}{b_{p2}} = m_2(1 - \frac{p_2 - a_{p2}}{b_{p2}}),\ p_1\frac{p_1 - a_{p1}}{b_{p1}} = p_2\frac{p_2 - a_{p2}}{b_{p2}}.$$

The first and second equation leads to $\frac{p_1-a_{p1}}{b_{p1}} = \frac{m_1}{m_1+s}$ *and* $\frac{p_2-a_{p2}}{b_{p2}} = \frac{m_2(m_1+s)}{(n+m_2)(m_1+s)-m_1s}$*, respectively. Correspondingly,* $D_1^* = \frac{m_1s}{m_1+s}$ *and* $D_2^* = \frac{m_2(n(m_1+s)-m_1s)}{(n+m_2)(m_1+s)-m_1s}$.

Give system state (m_1, s) *in period one and* $(m_2, n - \frac{m_1s}{m_1+s})$ *in period two, then*

$$p_1 = \frac{m_1sb_{p_1}}{m_1 + s} + \frac{(1-k)\gamma((k + m_1)b_{p_1}^2 - m_1b_{p_1})}{b_{p_1}((k-1)\gamma m_1 + \gamma sk) + m_1s - \gamma m_1s},$$

$$p_2 = \frac{m_2(m_1 + s)b_{p2}}{(n + m_2)(m_1 + s) - m_1s} + \frac{(1-k)\gamma(m_1 + s)((k + m_2)b_{p2}^2 - m_2b_{p2})}{(\gamma k(m_1n - m_1s + ns) - \gamma m_2(1-k)(m_1 + s))b_{p2} + m_2(1-\gamma)(m_1n - m_1s + ns)}.$$

Hence, $\frac{m_1}{m_1+s}p_1 = \frac{m_2(m_1+s)}{(n+m_2)(m_1+s)-m_1s}p_2$*, and* $\pi^{GDA} = (1-\gamma)(\frac{m_1s}{m_1+s}p_1 + \frac{m_2(n(m_1+s)-m_1s)}{(n+m_2)(m_1+s)-m_1s}p_2)$.

To obtain insightful results, we consider one specific case: $a_p = a_c = 0$, $b_p = b_c = \frac{m}{k+m}$*, and* $k = \frac{1}{2}$*. By conducting numerical analysis, we find that parameter k taking value at* $(0, 1)$ *does not influence the optimal transaction prices of both periods. Solving equations* $p_1 = \frac{m_1^2}{(m_1+k)(m_1+s)}$, $p_2 = \frac{(m_1\delta)^2(m_1+s)}{((m_1\delta+\beta m_1)(m_1+s)-m_1s)(m_1\delta+k)}$*, and* $\frac{m_1}{m_1+s}p_1 - \frac{m_2(m_1+s)}{(n+m_2)(m_1+s)-m_1s}p_2 = 0$ *simultaneously yields*

$$s^* = \frac{m_1\left((\beta-1)\sqrt{2\delta m_1 + 1} - 2\delta^{3/2}\sqrt{2m_1 + 1} + \delta\sqrt{2\delta m_1 + 1}\right)}{2\delta^{3/2}\sqrt{2m_1 + 1}}$$

$+\frac{1}{2}\sqrt{\frac{m_1^2\left((\beta+\delta-1)^2+2\delta m_1(\beta+\delta-1)^2+4\delta^{3/2}\sqrt{2m_1+1}\sqrt{2\delta m_1+1}\right)}{\delta^3\left(2m_1+1\right)}},$

$$p_1^*=\frac{4\delta^{3/2}m_1^2}{\sqrt{2m_1+1}\left(\sqrt{m_1^2\left((\beta+\delta-1)^2+2\delta m_1(\beta+\delta-1)^2+4\delta^{3/2}\sqrt{2m_1+1}\sqrt{2\delta m_1+1}\right)}+m_1(\beta+\delta-1)\sqrt{2\delta m_1+1}\right)},$$

$$p_2^*=\frac{4\delta^2m_1^2}{\sqrt{2\delta m_1+1}\sqrt{m_1^2\left((\beta+\delta-1)^2+2\delta m_1(\beta+\delta-1)^2+4\delta^{3/2}\sqrt{2m_1+1}\sqrt{2\delta m_1+1}\right)}+2\delta m_1^2(\beta+\delta-1)+m_1(\beta+\delta-1)},$$

$D_1^*=\frac{1}{2}\left(m_1(\beta+\delta+1)-\frac{\sqrt{m_1^2\left((\beta+\delta-1)^2+2\delta m_1(\beta+\delta-1)^2+4\delta^{3/2}\sqrt{2m_1+1}\sqrt{2\delta m_1+1}\right)}}{\sqrt{2\delta m_1+1}}\right)$, *and* $D_2^*=\frac{\sqrt{\delta}m_1\left((\beta-1)\sqrt{2\delta m_1+1}+2\sqrt{\delta}\sqrt{2m_1+1}+\delta\sqrt{2\delta m_1+1}\right)}{2\sqrt{2m_1+1}}-\frac{\sqrt{m_1^2\left((\beta+\delta-1)^2+2\delta m_1(\beta+\delta-1)^2+4\delta^{3/2}\sqrt{2m_1+1}\sqrt{2\delta m_1+1}\right)}}{2\sqrt{2m_1+1}}$.

Proof 16 *Proof of Theorem 2.5.1.1. Because* $m_1+\gamma m_1 n>\gamma n-\frac{\gamma m_1}{2}$ *and* $\frac{m_1(1+\gamma)(\gamma n+1)+m_1^2(1+\gamma n-\gamma^2)}{(1+\gamma)m_1+(1+\gamma)^2}>\frac{\gamma m_1}{2}-\frac{\gamma_1^2 m_1}{4n}$. *So, the optimal pricing strategy in DP and BBDA are stable prices, and two mechanisms coexist if* $\min\{\frac{m_1(1+\gamma)(\gamma n+1)+m_1^2(1+\gamma n-\gamma^2)}{(1+\gamma)m_1+(1+\gamma)^2},\gamma n-\frac{\gamma m_1}{2}\}\le m_2\le\gamma m_1 n+m_1$.

(i) If $\frac{m_1(1+\gamma)(\gamma n+1)+m_1^2(1+\gamma n-\gamma^2)}{(1+\gamma)m_1+(1+\gamma)^2}<m_2<\gamma n-\frac{\gamma m_1}{2}$, *then*

$$\begin{aligned}\pi^{BBDA}-\pi^{DP}=&\frac{2m_2(1-\gamma)(E(m_1+m_2)-4m_2-2m_2^2-2m_1m_2)}{E^2}-\frac{1}{4}(1-\gamma)\left(m_1+m_2\right)\\=&\frac{-(m_1+m_2)E^2+8m_2(m_1+m_2)E-16m_2(2m_2+m_2^2+m_1m_2)}{4E^2},\end{aligned}$$

The numerator is negative for the discriminant formula of the quadratic function of E *is negative* $\Delta=-128m_2^2\left(m_1+m_2\right)<0$. *So,* $\pi^{BBDA}-\pi^{DP}<0$. *In addition,* $p^{BBDA}-p^{DP}<0$. *By conducting numerical analysis, we have* $D_1^{BBDA}+D_2^{BBDA}>D_1^{DP}+D_2^{DP}$.

(ii) If $\max\{\frac{m_1(1+\gamma)(\gamma n+1)+m_1^2(1+\gamma n-\gamma^2)}{(1+\gamma)m_1+(1+\gamma)^2},\gamma n-\frac{\gamma m_1}{2}\}\le m_2\le\gamma m_1 n+m_1$, *then*

$$\begin{aligned}\pi^{BBDA}-\pi^{DP}=&\frac{2m_2(1-\gamma)(E(m_1+m_2)-4m_2-2m_2^2-2m_1m_2)}{E^2}-(1-\gamma)\left(m_1+m_2\right)A_1^{DP}A_2^{DP}\\=&(1-\gamma)\frac{-(m_1+m_2)A_1^{DP}A_2^{DP}E^2+2m_2(m_1+m_2)E-4m_2(2m_2+m_2^2+m_1m_2)}{E^2}.\end{aligned}$$

The discriminant of the quadratic function in the numerator is always positive, that is, $\Delta=4m_2^2\left(-4A_1^{DP}A_2^{DP}\left(m_1+m_2+2\right)+m_1+m_2\right)=4(m_1+m_2+2)(\frac{n}{2m_1}+\frac{m_2}{2\gamma m_1}-\sqrt{(\frac{n}{2m_1}+\frac{m_2}{2\gamma m_1}-\frac{1}{2})^2+\frac{m_2}{\gamma m_1}})^2-2>0$. *The roots for this function are equal to* $E=\frac{(m_1+m_2)m_2-m_2\sqrt{\left(-4A_1^{DP}A_2^{DP}(m_1+m_2+2)+m_1+m_2\right)}}{(m_1+m_2)A_1^{DP}A_2^{DP}}$ *and* $E=\frac{(m_1+m_2)m_2+m_2\sqrt{\left(-4A_1^{DP}A_2^{DP}(m_1+m_2+2)+m_1+m_2\right)}}{(m_1+m_2)A_1^{DP}A_2^{DP}}$.

Numerical analysis shows that $p^{BBDA} - p^{DP} < 0$, $D_1^{BBDA} + D_2^{BBDA} < D_1^{DP} + D_2^{DP}$, *and* $\pi^{BBDA} < \pi^{DP}$.

Proof 17 *Proof of Proposition 2.6.1.1.*

(i) *If* $nF(\gamma p)\alpha \le m_1\bar{G}(p)$ *and* $nF(\gamma p)(1-\alpha) \le m_2\bar{G}(p)$, *then* $\pi(p) = \gamma(1-\gamma)np^2$. *The profit function increases with* p *over interval* $[0, \min\{\frac{m_1}{\gamma n\alpha + m_1}, \frac{m_2}{\gamma n(1-\alpha)+m_2}\}]$. *Hence,* $p^* = \min\{\frac{m_1}{\gamma n\alpha+m_1}, \frac{m_2}{\gamma n(1-\alpha)+m_2}\}$, *and* $\pi^*(p^*) = \gamma(1-\gamma)n(\min\{\frac{m_1}{\gamma n\alpha+m_1}, \frac{m_2}{\gamma n(1-\alpha)+m_2}\})^2$, *where* $\alpha \in [0,1]$.

The profit function increases with α *if* $\alpha < \frac{m_1}{m_1+m_2}$, *and decreases with* α *otherwise. Hence,* $\pi^{SP}_{max} = \frac{(1-\gamma)\gamma n(m_1+m_2)^2}{(\gamma n+m_1+m_2)^2}$ *if* $p^* = \frac{m_1+m_2}{\gamma n+m_1+m_2}$ *and* $\alpha^* = \frac{m_1}{m_1+m_2}$.

(ii) *If* $nF(\gamma p)\alpha \le m_1\bar{G}(p)$ *and* $nF(\gamma p)(1-\alpha) > m_2\bar{G}(p)$, *then* $\alpha = 1$ *and* $\pi(p) = (1-\gamma)((n\gamma\alpha - m_2)p^2 + m_2 p)$, *which is concave with* p *if* $n\gamma\alpha < m_2$. *Hence,* $p^* = \frac{m_2}{2(m_2 - n\alpha\gamma)}$ *provided that* $p^* \le \frac{m_1}{\gamma n\alpha+m_1}$ *and* $p^* > \frac{m_2}{\gamma n(1-\alpha)+m_2}$. *While these two constraints contradict with each other when* $\alpha = 1$.

(iii) *If* $nF(\gamma p)\alpha > m_1\bar{G}(p)$ *and* $nF(\gamma p) - m_1\bar{G}(p) \le m_2\bar{G}(p)$, *then* $\alpha = 0$ *and* $\pi(p) = (1-\gamma)\gamma np^2$, *which increases with* p, *hence,* $p^* = \frac{m_1+m_2}{\gamma n+m_1+m_2}$ *provided that* $p^* \in (\frac{m_1}{\gamma n\alpha+m_1}, \frac{m_1+m_2}{\gamma n+m_1+m_2}]$. *While the price is more than one when* $\alpha = 0$.

(iv) *If* $nF(\gamma p)\alpha > m_1\bar{G}(p)$ *and* $nF(\gamma p) - m_1\bar{G}(p) > m_2\bar{G}(p)$, *then* $\alpha = \frac{(m_1 n\gamma + m_1^2)p - m_1^2}{m_2 n\gamma p}$ *and* $\pi(p) = (1-\gamma)(m_1+m_2)(p-p^2)$, *which reaches maximum at point* $\frac{1}{2}$ *if* $\max\{\frac{m_1}{\gamma n\alpha+m_1}, \frac{m_1+m_2}{\gamma n+m_1+m_2}\} < \frac{1}{2}$. *Hence,* $\alpha^* = \frac{m_1 n\gamma - m_1^2}{m_2 n\gamma}$ *and* $\pi^* = (1-\gamma)\frac{m_1+m_2}{4}$. *The constraints are simplified into* $\frac{1}{2} > \max\{\frac{m_2}{m_2+n\gamma-m_1}, \frac{m_1+m_2}{\gamma n+m_1+m_2}\}$. *Note that when* $\frac{m_2}{m_2+n\gamma-m_1} > \frac{m_1+m_2}{\gamma n+m_1+m_2}$ *(or equivalently,* $\frac{1}{2\gamma} + \frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} > 0$*),* $\frac{m_2}{m_2+n\gamma-m_1} > \frac{1}{2}$, *thus the optimization problem has no optimal solution. If* $\frac{m_2}{m_2+n\gamma-m_1} \le \frac{m_1+m_2}{\gamma n+m_1+m_2}$ *(or equivalently,* $\frac{1}{2\gamma} + \frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} \le 0$*), then* $\frac{m_1+m_2}{\gamma n+m_1+m_2} < \frac{1}{2}$, *thus the platform's profit reaches maximum at* $p = \frac{1}{2}$.

To sum up,

(i) *If* $\frac{1}{2\gamma} + \frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} \le 0$, *then* $p^* = \frac{1}{2}$, $\alpha^* = \frac{m_1 n\gamma - m_1^2}{m_2 n\gamma}$ *and* $\pi^* = (1-\gamma)\frac{m_1+m_2}{4}$;

(ii) *Otherwise,* $p^* = \frac{m_2}{\gamma n(1-\alpha)+m_2}$ *and* $\pi^* = \gamma(1-\gamma)n(\frac{m_2}{\gamma n(1-\alpha)+m_2})^2$ *if* $0 \leq \alpha^* < \frac{m_1}{m_1+m_2}$ *or* $p^* = \frac{m_1}{\gamma n\alpha+m_1}$ *and* $\pi^* = \gamma(1-\gamma)n(\frac{m_1}{\gamma n\alpha+m_1})^2$ *if* $\frac{m_1}{m_1+m_2} \leq \alpha^* \leq 1$.

Consumer surplus and social welfare

(i) *If* $\frac{1}{2\gamma}+\frac{m_2}{2\gamma m_1}-\frac{n}{2m_1} \leq 0$, *then* $CS = \frac{m_1+m_2}{16}$, $PS = \frac{\gamma^2(m_1+m_2)}{16}$ *and* $SW = \frac{(\gamma^2-4\gamma+5)(m_1+m_2)}{16}$, *respectively.*

(ii) *If* $\frac{1}{2\gamma} + \frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} > 0$, *then* $CS = \frac{(1-\alpha)^2 m_2 n^3\gamma^3}{2(m_2+\gamma n(1-\alpha))^3}$, $PS = \frac{m_2^3 n\gamma^3}{2(m_2+\gamma n(1-\alpha))^3}$ *and* $SW = \frac{m_2 n\gamma(2(1-\alpha)m_2 n(1-\gamma)\gamma+(1-\alpha)^2n^2\gamma^2+m_2^2(\gamma^2-2\gamma+2))}{2(m_2+\gamma n(1-\alpha))^3}$ *when* $0 \leq \alpha^* < \frac{m_1}{m_1+m_2}$, *or* $SW = \frac{(m_1 n\gamma(2\alpha m_1 n(1-\gamma)\gamma+\alpha^2n^2\gamma^2+m_1^2(\gamma^2-2\gamma+2)))}{2(m_1+\gamma n\alpha)^3}$, $CS = \frac{\alpha^2 m_1 n^3\gamma^3}{2(m_1+\gamma n\alpha)^3}$, *and* $PS = \frac{m_1^3 n\gamma^3}{2(m_1+\gamma n\alpha)^3}$ *when* $\frac{m_1}{m_1+m_2} \leq \alpha^* \leq 1$.

Because the surpluses and social welfare depends on α *when* $\frac{1}{2\gamma} + \frac{m_2}{2\gamma m_1} - \frac{n}{2m_1} > 0$, *we explore the impact of* α *on surpluses and social welfare in the following part.*

(i) *Consumer Surplus.*

Consumer surplus increases with α *over interval* $[0, 1-\frac{2m_2}{\gamma n}]$ *and decreases with* α *over interval* $(1-\frac{2m_2}{\gamma n}, \frac{m_1}{m_1+m_2})$ *if* $0 \leq \alpha^* < \frac{m_1}{m_1+m_2}$. *Or consumer surplus increases (resp., decreases) with* α *over interval* $[\frac{m_1}{m_1+m_2}, \frac{m_1}{\gamma n}]$ *(resp.,* $(\frac{m_1}{\gamma n}, 1]$*) otherwise. Hence,* $CS|_{\alpha=\frac{m_1}{m_1+m_2}} = \frac{n^3\gamma^3(m_1+m_2)}{2(m_1+m_2+\gamma n)^3}$, $CS|_{\alpha=0} = \frac{n^3\gamma^3 m_2}{2(m_2+\gamma n)^3}$ *and* $CS|_{\alpha=1} = \frac{n^3\gamma^3 m_1}{2(m_1+\gamma n)^3}$, $CS|_{\alpha=1-\frac{2m_2}{\gamma n}} = \frac{2\gamma n}{27}$, *and* $CS|_{\alpha=\frac{m_1}{\gamma n}} = \frac{\gamma n}{16}$.

(ii) *Provider Surplus.*

Provider surplus increases with α *over interval* $[0, \frac{m_1}{m_1+m_2})$ *while decreases over* $[\frac{m_1}{m_1+m_2}, 1]$. *Hence,* $PS|_{\alpha=\frac{m_1}{m_1+m_2}} = \frac{(m_1+m_2)^3 n\gamma^3}{2(\gamma n+m_1+m_2)^3}$, $PS|_{\alpha=0} = \frac{m_2^3 n\gamma^3}{2(\gamma n+m_2)^3}$ *and* $PS|_{\alpha=1} = \frac{m_1^3 n\gamma^3}{2(\gamma n+m_1)^3}$.

(iii) *Social Welfare.*

Social welfare increases with α *over* $[0, \frac{m_1}{m_1+m_2})$ *while decreases over* $[\frac{m_1}{m_1+m_2}, 1]$. *Hence,* $SW|_{\alpha=\frac{m_1}{m_1+m_2}} = \frac{(m_1+m_2)\gamma n(2(m_1+m_2)n(1-\gamma)\gamma+n^2\gamma^2+(m_1+m_2)^2(\gamma^2-2\gamma+2))}{2(m_1+m_2+\gamma n)^3}$. *Boundary solutions equal* $SW|_{\alpha=0} = \frac{m_2\gamma n(2m_2 n(1-\gamma)+n^2\gamma^2+m_2^2(\gamma^2-2\gamma+2))}{2(m_2+\gamma n)^3}$ *and* $SW|_{\alpha=1} = \frac{m_1\gamma n(2m_1 n(1-\gamma)+n^2\gamma^2+m_1^2(\gamma^2-2\gamma+2))}{2(m_1+\gamma n)^3}$.

Proof 18 *Proof of Theorem 2.6.1.1.*

(i) *If* $\gamma n - m_1 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$, *there are four cases.*

(a) *When* $m_2 \leq \gamma n - m_1$, *then* $\pi^{SP} = (1-\gamma)\frac{m_1+m_2}{4}$, $\pi^{DP} = (1-\gamma)(m_1 A_3^{DP} A_4^{DP} + \frac{m_2}{4})$, *and* $\pi^{DP} - \pi^{SP} = -(1-\gamma)m_1(\sqrt{(\frac{n}{2m_1} - \frac{1}{2})^2 + \frac{m_2}{2\gamma m_1}} - \frac{n}{2m_1})^2 < 0$.

(b) *When* $\gamma n - m_1 < m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$, *then* $\pi^{DP} = (1-\gamma)(m_1 A_3^{DP} A_4^{DP} + \frac{m_2}{4})$ *and* $\pi^{SP} = (1-\gamma)\gamma n(\frac{m_1+m_2}{\gamma n + m_1 + m_2})^2$. *Numerical results show that* $\pi^{DP} - \pi^{SP} < 0$ *if* $m_2 < m_2^{DS}$, *where* m_2^{DS} *solves equation* $\pi^{DP} - \pi^{SP} = 0$.

(c) *When* $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \leq m_2 \leq \gamma n - \frac{\gamma m_1}{2}$, *then* $\pi^{DP} = (1-\gamma)\frac{m_1+m_2}{4}$ *and* $\pi^{SP} = \gamma(1-\gamma)n(\frac{m_2}{\gamma n(1-\alpha)+m_2})^2$ *if* $0 \leq \alpha^* < \frac{m_1}{m_1+m_2}$ *or* $\pi^{SP} = \gamma(1-\gamma)n(\frac{m_1}{\gamma n\alpha + m_1})^2$ *if* $\frac{m_1}{m_1+m_2} \leq \alpha^* \leq 1$. *Proposition* 2.6.1.1 *reveals that the maximum profit in static pricing (i.e.,* $\pi^{SP} = (1-\gamma)\gamma n(\frac{m_1+m_2}{\gamma n+m_1+m_2})^2$*) is no more than the profit under dynamic setting:* $\pi^{DP} > \pi^{SP}_{max} > \pi^{SP}_{min}$.

(d) *When* $m_2 > \gamma n - \frac{\gamma m_1}{2}$, *then* $\pi^{DP} = (1-\gamma)(m_1+m_2)A_1^{DP}A_2^{DP}$, $\pi^{SP} = (1-\gamma)\gamma n(\frac{m_1+m_2}{\gamma n+m_1+m_2})^2$, *and* $\pi^{DP} - \pi^{SP}_{max} = \frac{1}{(m_1+m_2+\gamma n)^2}(2m_1^2(m_1+m_2)\gamma^3 n + (m_1 + m_2 + \gamma n)^2(\frac{m_2}{2\gamma m_1} + \frac{1}{2} + \frac{n}{2m_1} - \sqrt{(\frac{m_2}{2\gamma m_1} - \frac{1}{2} + \frac{n}{2m_1})^2 + \frac{m_2}{\gamma m_1}}))(-\frac{m_2}{2\gamma m_1} + \frac{1}{2} - \frac{n}{2m_1} + \sqrt{(\frac{m_2}{2\gamma m_1} - \frac{1}{2} + \frac{n}{2m_1})^2 + \frac{m_2}{\gamma m_1}}) = \frac{1}{(m_1+m_2+\gamma n)^2}(2m_1^2(m_1+m_2)\gamma^3 n + (m_1+m_2+\gamma n)^2 A_1^{DP})A_2^{DP} > 0$. *For price comparison, suppose that* $A_1^{DP} - p^* < 0$, *then* $-\frac{n((1-\gamma)(m_1+m_2)+\gamma n)}{(m_1+m_2+\gamma n)^2} < 0$. *For transaction volume gap, suppose that* $(m_1 + m_2)A_2^{DP} - \frac{\gamma n(m_1+m_2)}{m_1+m_2+\gamma n} < 0$, *then* $\frac{n((1-\gamma)(m_1+m_2)+\gamma n)}{(m_1+m_2+\gamma n)^2} > 0$.

(ii) *If* $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \leq \gamma n - m_1 < \gamma n - \frac{\gamma m_1}{2}$, *there are also four cases needed further discussion.*

(a) *When* $m_2 < \frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n}$, *then* $\pi^{SP} = (1-\gamma)\frac{m_1+m_2}{4}$, $\pi^{DP} = (1-\gamma)(m_1 A_3^{DP} A_4^{DP} + \frac{m_2}{4})$. *The profit gap is negative, which is the same as condition (a) of case (i).*

(b) *When* $\frac{\gamma m_1}{2} - \frac{\gamma m_1^2}{4n} \leq m_2 \leq \gamma n - m_1$, *then* $\pi^{SP} = \pi^{DP} = (1-\gamma)\frac{m_1+m_2}{4}$.

(c) *When* $\gamma n - m_1 < m_2 \leq \gamma n - \frac{\gamma m_1}{2}$, *then* $\pi^{DP} = (1-\gamma)\frac{m_1+m_2}{4}$, $\pi^{SP} = (1-\gamma)\gamma n(\frac{m_1+m_2}{\gamma n+m_1+m_2})^2$. *The comparison is the same as condition (c) of case (i).*

(d) *When* $m_2 > \gamma n - \frac{\gamma m_1}{2}$*, then* $\pi^{DP} = (1-\gamma)(m_1+m_2)A_1^{DP}A_2^{DP}$*,* $\pi^{SP} = (1-\gamma)\gamma n(\frac{m_1+m_2}{\gamma n+m_1+m_2})^2$*. The comparison is the same as condition (d) of case (i).*

Proof 19 *Proof of Theorem 2.6.1.2. BBDA and SP coexist if* $\min\{\frac{m_1(1+\gamma)(\gamma n+1)+m_1^2(1+\gamma n-\gamma^2)}{(1+\gamma)m_1+(1+\gamma)^2}, \gamma n - m_1\} \leq m_2 \leq \gamma m_1 n + m_2$.

(i) *If* $\frac{m_1(1+\gamma)(\gamma n+1)+m_1^2(1+\gamma n-\gamma^2)}{(1+\gamma)m_1+(1+\gamma)^2} \leq m_2 \leq \gamma n - m_1$*, then*

$$\pi^{BBDA} - \pi^{SP} = \frac{2m_2(1-\gamma)(E(m_1+m_2) - 4m_2 - 2m_2^2 - 2m_1m_2)}{E^2} - \frac{1}{4}(1-\gamma)(m_1+m_2) < 0.$$

(ii) *If* $\max\{\frac{m_1(1+\gamma)(\gamma n+1)+m_1^2(1+\gamma n-\gamma^2)}{(1+\gamma)m_1+(1+\gamma)^2}, \gamma n - m_1\} \leq m_2 \leq \gamma m_1 n + m_1$*, then*

$$\begin{aligned}\pi^{BBDA} - \pi^{SP} =& \frac{2m_2(1-\gamma)(E(m_1+m_2) - 4m_2 - 2m_2^2 - 2m_1m_2)}{E^2} - (1-\gamma)\gamma n(\frac{m_1+m_2}{\gamma n+m_1+m_2})^2\\ =&(1-\gamma)\frac{-\gamma n(\frac{m_1+m_2}{\gamma n+m_1+m_2})^2E^2 + 2m_2(m_1+m_2)E - 4m_2(2m_2+m_2^2+m_1m_2)}{E^2}.\end{aligned}$$

The numerator is negative if the discriminant formula of the quadratic function of E *is negative, that is,* $\Delta = 4m_2^2(m_1+m_2)^2\left(1 - \frac{4\gamma(m_1+m_2+2)n}{(m_1+m_2+\gamma n)^2}\right) < 0$*, or equivalently,* $\gamma n - m_1 - 2\sqrt{2\gamma n} < m_2 < \gamma n - m_1 + 2\sqrt{2\gamma n}$*. So,* $\pi^{BBDA} - \pi^{SP} \geq 0$ *if* $\frac{m_2(\gamma n+m_1+m_2)^2 - m_2(\gamma n+m_1+m_2)\sqrt{(m_1+m_2-\gamma n)^2-8\gamma n}}{\gamma n(m_1+m_2)} \leq E \leq \frac{m_2(\gamma n+m_1+m_2)^2 + m_2(\gamma n+m_1+m_2)\sqrt{(m_1+m_2-\gamma n)^2-8\gamma n}}{\gamma n(m_1+m_2)}$, *which is always true. Moreover,* $p^{SP} > p^{BBDA}$*,* $D_1^{SP} + D_2^{SP} < D_1^{BBDA} + D_2^{BBDA}$*.*

Proof 20 *Proof of Lemma 2.6.2.1.*

(i) *If* $n\gamma p > m(1-p)$ *(i.e.,* $p > \frac{m}{m+\gamma n}$*), then* $\pi = (1-\gamma)pm(1-p)$*. Hence,* $p^* = \frac{1}{2}$ *and* $\pi^* = \frac{(1-\gamma)m}{4}$ *if* $\frac{1}{2} > \frac{m}{m+\gamma n}$*.*

(ii) *If* $n\gamma p \leq m(1-p)$ *(i.e.,* $p \leq \frac{m}{m+\gamma n}$*), then* $\pi(p) = (1-\gamma)\gamma np^2$*. Hence,* $p^* = \frac{m}{m+\gamma n}$ *and* $\pi^* = \frac{(1-\gamma)\gamma nm^2}{(m+\gamma n)^2}$*.*

Proof 21 *Proof of Lemma 2.6.2.2.*

(i) *When* $k = 1$*, we have* $a_p = 0$*,* $b_p = \frac{1}{\gamma}$*,* $a_c = 0$ *and* $b_c = \frac{m}{m+1}$*. Hence,* $p^* = \frac{m}{\gamma n+m+1}$*,* $D^* = \frac{\gamma mn}{\gamma n+m+1}$*, and* $\pi^* = \frac{(1-\gamma)\gamma m^2 n}{(m+1+\gamma n)^2}$*.*

(ii) *When* $k = 0$ *and* $\gamma = 1$, *then* $a_p = \frac{1}{n+1}$, $b_p = \frac{n}{n+1}$, $a_c = 0$ *and* $b_c = 1$. *Hence,* $p^* = \frac{m+1}{n+1+m}$ *and* $\pi^* = 0$.

(iii) *When* $k \in (0,1)$, *if* $a_p = a_c = 0$, *then* $b_c = b_p = \frac{m}{k+m}$. *Hence,* $p^* = \frac{mb_p}{m+n}$ *and* $\pi^* = \frac{(1-\gamma)m^3 n}{(k+m)(m+n)^2}$.

Proof 22 *Proof of Proposition 2.6.2.2.*

(i) *Post pricing vs. Buyer's Bid Double Auction. If* $m < \gamma n$, *then* $\pi^{PP-BBDA} = \frac{(1-\gamma)m}{4} - \frac{(1-\gamma)\gamma m^2 n}{(m+1+\gamma n)^2} = \frac{m(1-\gamma)((m-\gamma n)^2+2(m+\gamma n)+1)}{4(1+m+\gamma n)^2} > 0$; *if* $m \geq \gamma n$, *then* $\pi^{PP-BBDA} = \frac{(1-\gamma)\gamma n m^2}{(m+\gamma n)^2} - \frac{(1-\gamma)\gamma m^2 n}{(m+1+\gamma n)^2} = \frac{(1-\gamma)\gamma m^2 n(2\gamma n+2m+1)}{(m+1+\gamma n)^2(m+\gamma n)^2} > 0$. *So, post pricing dominates BBDA.*

(ii) *Post pricing vs. Seller's Offer Double Auction. The platform's profits both equal zero when* $\gamma = 1$.

(iii) *Post pricing vs. General Double Auction. If* $m < \gamma n$, *then* $\pi^{PP-GDA} = \frac{(1-\gamma)m}{4} - \frac{(1-\gamma)m^3 n}{(k+m)(m+n)^2} = \frac{(1-\gamma)m}{4}\frac{(k+m)n^2+2(k-m)mn+km^2+m^3}{(k+m)(m+n)^2}$; *if* $m \geq \gamma n$, *then* $\pi^{PP-GDA} = \frac{(1-\gamma)\gamma n m^2}{(m+\gamma n)^2} - \frac{(1-\gamma)m^3 n}{(k+m)(m+n)^2} = m^2 n(1-\gamma)\frac{(\gamma k+\gamma m-\gamma^2 m)n^2+2\gamma kmn+m^3(-1+\gamma)+k\gamma m^2}{(m+\gamma n)(k+m)(m+n)^2}$. *The numerator of the first profit gap is always positive for the discriminant of the quadratic function about* n *(i.e.,* $(k+m)n^2+2(k-m)mn+km^2+m^3$*) is negative (i.e.,* $-16km^3 < 0$*). The numerator of the second profit gap is positive if* $\frac{m(1-\gamma)\sqrt{\gamma m(k+m)}-\gamma mk}{\gamma k+\gamma m-\gamma^2 m} < n \leq \frac{m}{\gamma}$ *or is negative if* $\frac{m}{\gamma} \leq \frac{m(1-\gamma)\sqrt{\gamma m(k+m)}-\gamma mk}{\gamma k+\gamma m-\gamma^2 m}$.

A.2 Discriminatory k-DA Mechanism

The difference between two double auction mechanisms lies in ways of ordering and the transaction price formation: We list the order statistics of providers as $S_{(1)} \leq S_{(2)} \leq ... \leq S_{(n)}$ *and of customers as* $B_{(1)} \geq B_{(2)} \geq ... \geq B_{(m)}$ *in a certain period with n providers and m customers. A successful matching takes place if and only if the bidding price is no less than the asking price in a matching pair, which refers to a customer with the same order statistic with the provider. Each matching pair is transacted with a transaction price, which is a convex combination of the asking and bidding prices of that matching pair.*

Note that customers and providers propose symmetric bidding policy (i.e., $S(c) = a_p + b_p c$ *and* $B(v) = a_c + b_c v$*), denote the respective cdf and pdf of customer's bidding strategy by functions* $F_1(B)$ *and* $f_1(B)$*, and the cdf and pdf of provider's bidding strategy by* $F_2(S)$ *and* $f_2(S)$*, then the pdf of* $B_{(i)}$ *and of* $S_{(j)}$ *(Silvey 2017) are given by*

$$f(B_{(i)}) = \frac{m!}{(i-1)!\,(m-i)!} f_1(B)[F_1(B)]^{m-i}[1 - F_1(B)]^{i-1},$$

$$f(S_{(j)}) = \frac{n!}{(j-1)!\,(n-j)!} f_2(S)[F_2(S)]^{j-1}[1 - F_2(S)]^{n-j}.$$

where $B_{(i)}$ *is the* $i-th$ *highest (or the* $(m-i+1)-th$ *lowest) bidding price and* $S_{(j)}$ *is the* $(n-j+1)-th$ *lowest (or the* $j-th$ *highest) asking price.*

The Expected Utility Function.

For a specific customer, we assume that her bidding price $B_{(x)}$ *is the* $x-th$ *highest among all bids, where* $x \in \{1, 2, ..., m\}$*,* m *here denotes* m_1 *in period one or* m_2 *in period two. The probability that there are* $m-1$ *competitors is denoted by* $P_6(m, x) = \begin{pmatrix} m-1 \\ x-1 \end{pmatrix} G(v)^{m-x}(1 - G(v))^{x-1}$*, where* $v = \frac{B_{(x)} - a_c}{b_c}$*. A customer's expected utility is defined as her valuation less the transaction price given that her bid is greater than or equal to the corresponding provider's asking price. Denote the asking price of the specific provider in the matching pair by* $S_{(x)}$*, we have:*

$$r_c(B_{(x)}) = (v - ((1-k)E[S_{(x)}|B_{(x)} \geq S_{(k)}] + kB_{(x)}))P_6(m, x)Pr\{B_{(x)} \geq S_{(x)}\},$$

where

$$Pr\{B_{(x)} \geq S_{(x)}\} = \int_{a_p}^{B_{(x)}} \frac{n!}{(x-1)!\,(n-x)!} f_2(S)[F_2(S)]^{x-1}[1-F_2(S)]^{n-x}dS.$$

The term $v - ((1-k)E[S_{(x)}|B_{(x)} \geq S_{(k)}] + kB_{(x)})$ *is the net surplus of the customer, and the term* $P_6(m,x)Pr\{B_{(x)} \geq S_{(x)}\}$ *captures the probability that the customer will be matched successfully.*

For a specific provider, we assume that the order statistic of his asking price is $S_{(y)}$*, where* $y \in \{1, 2, ..., n\}$*. The probability that there are* $n-1$ *competitors such that* $y-1$ *have lower asking prices than his asking price is denoted by* $P_7(n,y) = \begin{pmatrix} n-1 \\ y-1 \end{pmatrix} F(c)^{y-1}(1-F(c))^{n-y}$*, where* $c = \frac{S_{(y)}-a_p}{b_p}$*. Hence the provider proposes asking price* $S_{(y)}$ *in order to maximize his expected utility:*

$$r_p(S_{(y)}) = (\gamma((1-k)S_{(y)} + kE[B_{(y)}|B_{(y)} \geq S_{(y)}]) - c)P_7(n,y)Pr\{B_{(y)} \geq S_{(y)}\},$$

where

$$Pr\{B_{(y)} \geq S_{(y)}\} = \int_{S_{(y)}}^{a_c+b_c} \frac{m!}{(y-1)!\,(m-y)!} f_1(B)[F_1(B)]^{m-y}[1-F_1(B)]^{y-1}dB,$$

$$E[B_{(y)}|B_{(y)} \geq S_{(y)}] = \frac{\int_{S_{(y)}}^{a_c+b_c} \frac{m!}{(y-1)!(m-y)!} f_1(B)[F_1(B)]^{m-y}[1-F_1(B)]^{y-1}BdB}{Pr\{B_{(y)} \geq S_{(y)}\}}.$$

Note that the order statistics of a specific matching pair can not exceed the minimum number of customers and providers, that is, both x *and* y *take values over* $\{1, 2, ..., \min\{m, n\}\}$*.*

The Optimal Bidding Strategy.

The following lemma characterizes the optimal bidding policy of both customers and providers.

Lemma A.2 (Optimal Bidding Strategy of Both Sides In Discriminatory k-DA Mechanism) *Under bid-ask mechanism,*

(i) Customers' unique optimal bidding strategy $B_{(k)}$ *solves the equation* $(v-B_{(x)})(\frac{B_{(x)}-a_p}{b_p})^{x-1}(1-\frac{B_{(x)}-a_p}{b_p})^{n-x} - k\int_0^{\frac{B_{(x)}-a_p}{b_p}} [F_2(S)]^{x-1}[1-F_2(S)]^{n-x}dF_2(S) = 0$ *provided that* $B_{(x)} \in (\frac{B_1-\sqrt{B_1^2-4(n+k)B_2}}{2(n+k)}, \frac{B_1+\sqrt{B_1^2-4(n+k)B_2}}{2(n+k)})$*, where* $x \in \{1, 2, ..., \min\{m, n\}\}$*.*

(i) *Providers' unique optimal bidding strategy* $S_{(y)}$ *solves the equation* $-(\gamma S_{(y)}-c)(\frac{S_{(y)}-a_c}{b_c})^{m-y}(1-\frac{S_{(y)}-a_c}{b_c})^{y-1}+\gamma(1-k)\int_{\frac{S_{(y)}-a_c}{b_c}}^{1}[F_1(B)]^{m-y}[1-F_1(B)]^{y-1}dF_1(B)=0$ *provided that*

$S_{(x)}\in(\frac{S_1-\sqrt{S_1^2-4\gamma(m+k-1)S_2}}{2\gamma(m+k-1)},\frac{S_1+\sqrt{S_1^2-4\gamma(m+k-1)S_2}}{2\gamma(m+k-1)})$, *where* $y\in\{1,2,...,\min\{m,n\}\}$.

Results in Lemma A.2 show that it is optimal for providers to uniformly bid $\frac{c}{\gamma}$ *when the transaction price of a matching pair equals the bidding price, and it is optimal for customers to uniformly bid* v *when the price is equal to the asking price of that matching pair. While the optimal bidding strategy on the demand-side (resp., supply-side) when* $k=1$ *(resp.,* $k=0$*) depends on the number of participants on the other side, their own order statistics at that time period, and the commission rate as well. Details are as Corollary A.2, in which part (i) and (ii) confirms the optimal bidding policy of provider side in Lemma 2.4.1.1 and of customer side in Lemma 2.4.1.2, respectively.*

Corollary A.2

(i) *If* $k=1$, *providers bid* $\frac{c}{\gamma}$, *and customers' bid* $B_{(k)}$ *satisfies* $(v-B_{(x)})(\gamma B_{(x)})^{x-1}(1-\gamma B_{(x)})^{n-x}-\int_0^{\gamma B_{(x)}}[F_2(S)]^{x-1}[1-F_2(S)]^{n-x}dF_2(S)=0$ *provided that*

$B_{(x)}\in(\frac{1+x-\gamma v+\gamma nv-\sqrt{(-1-x+\gamma v-\gamma nv)^2+4(\gamma+\gamma n)(v-xv)}}{2(\gamma+\gamma n)},\frac{1+x-\gamma v+\gamma nv+\sqrt{(-1-x+\gamma v-\gamma nv)^2+4(\gamma+\gamma n)(v-xv)}}{2(\gamma+\gamma n)})$, *where* $x\in\{1,2,...,\min\{m,n\}\}$.

(ii) *If* $k=0$, *customers bid* v, *and providers' unique optimal bidding strategy* $S_{(y)}$ *satisfies* $-(\gamma S_{(y)}-c)(S_{(y)})^{m-y}(1-S_{(y)})^{y-1}+\gamma\int_{S_{(y)}}^{1}[F_1(B)]^{m-y}[1-F_1(B)]^{y-1}dF_1(B)=0$ *provided that* $S_{(y)}\in(\frac{c}{\gamma},\frac{m-y}{m-1})$ *if* $\frac{c}{\gamma}<\frac{m-y}{m-1}$ *or* $S_{(y)}\in(\frac{m-y}{m-1},\frac{c}{\gamma})$ *otherwise, where* $y\in\{1,2,...,\min\{m,n\}\}$.

Providers' Fulfillment Equilibrium.

Suppose s *providers join in period one, then the number of successful transactions equals* z, *where* $z=\max\{j\in[0,\min\{m_1,s\}]|S_{(j)}\leq B_{(j)}\}$. *Denote the effective number of transactions in period two by* w, *where* $w=\max\{j\in[0,\min\{m_2,n-z\}]|S_{(j)}\leq B_{(j)}\}$. *Hence, the demand-supply state is* (m_1,s) *in period one and* $(m_2,n-z)$ *in period two. Suppose a provider with order statistic* x *in period one and* y *in period two, then there is* *no difference for him to serve in two periods if*

$$P_7(s,x)\gamma((1-k)S_{(x)}+kB_{(x)})Pr\{B_{(x)} \geq S_{(x)}\} = P_7(n-z,y)\gamma((1-k)S_{(y)}+kB_{(y)})Pr\{B_{(y)} \geq S_{(y)}\}.$$

The platform's profit is defined as the multiplier of $1-\gamma$ *and the sum of effective transaction prices over two periods, that is, from* $(1-k)S_{(1)}+kB_{(1)}$ *to* $(1-k)S_{(z)}+kB_{(z)}$ *in period one and from* $(1-k)S_{(1)}+kB_{(1)}$ *to* $(1-k)S_{(w)}+kB_{(w)}$ *in period two.*

Proof 23 *Proof of Lemma A.2.* P_6 *and* P_7 *do not affect participants' bidding equilibrium (bidders proposing bids care about the bid with the same order statistics on the other side instead of bids on their same side). Hence,*

$$r_c(B_{(x)}) = (v - ((1-k)E[S_{(x)}|B_{(x)} \geq S_{(x)}] + kB_{(x)}))Pr\{B_{(x)} \geq S_{(x)}\},$$
$$r_p(S_{(y)}) = (\gamma((1-k)S_{(y)} + kE[B_{(y)}|B_{(y)} \geq S_{(y)}]) - c)Pr\{B_{(y)} \geq S_{(y)}\},$$

where

$$E[S_{(x)}|B_{(x)} \geq S_{(x)}] = \frac{\int_{a_p}^{B_{(x)}} \frac{n!}{(x-1)!(n-x)!} f_2(S)[F_2(S)]^{x-1}[1-F_2(S)]^{n-x}SdS}{Pr\{B_{(x)} \geq S_{(x)}\}},$$
$$E[B_{(y)}|B_{(y)} \geq S_{(y)}] = \frac{\int_{S_{(y)}}^{a_c+b_c} \frac{m!}{(m-y)!(y-1)!} f_1(B)[F_1(B)]^{m-y}[1-F_1(B)]^{y-1}BdB}{Pr\{B_{(y)} \geq S_{(y)}\}}.$$

By applying the the formula of changing element of definite integral, we have the interval of the cdf of $F_1(B)$ *denoted by* $[\frac{S_{(y)}-a_c}{b_c}, 1]$ *when* $B \in [S_{(y)}, a_c+b_c]$*. Similarly, the interval for the cdf of* $F_2(S)$ *is given by* $[0, \frac{B_{(x)}-a_p}{b_p}]$ *when* $S \in [a_p, B_{(x)}]$*. Replacing* B *with* $b_cF_1(B)+a_c$ *for* $F_1(B) = \frac{B-a_c}{b_c}$ *yields*

$$r_c(B_{(x)}) = (v - kB_{(x)} - (1-k)a_p) \int_0^{\frac{B_{(x)}-a_p}{b_p}} \frac{n!}{(x-1)!\,(n-x)!}[F_2(S)]^{x-1}[1-F_2(S)]^{n-x}dF_2(S)$$
$$- (1-k)b_p \int_0^{\frac{B_{(x)}-a_p}{b_p}} \frac{n!}{(x-1)!\,(n-x)!}[F_2(S)]^{x}[1-F_2(S)]^{n-x}dF_2(S),$$
$$r_p(S_{(y)}) = (\gamma(1-k)S_{(y)} - c + \gamma k a_c) \int_{\frac{S_{(y)}-a_c}{b_c}}^1 \frac{m!}{(y-1)!\,(m-y)!}[F_1(B)]^{m-y}[1-F_1(B)]^{y-1}dF_1(B)$$
$$+ \gamma k b_c \int_{\frac{S_{(y)}-a_c}{b_c}}^1 \frac{m!}{(y-1)!\,(m-y)!}[F_1(B)]^{m-y+1}[1-F_1(B)]^{y-1}dF_1(B).$$

By deriving the FOCs and SOCs of customers and providers' payoff functions, we have the effective domain on the demand side given by $(n+k)B_{(x)}^2 - ((n+2k+1)a_p + (k+x)b_p + (n-1)v)B_{(x)} + (k+1)a_p(a_p+b_p) + (n-1)a_pv + (x-1)b_pv < 0$ *(or equivalently,* $B_{(x)} \in (\frac{B_1-\sqrt{B_1^2-4(n+k)B_2}}{2(n+k)}, \frac{B_1+\sqrt{B_1^2-4(n+k)B_2}}{2(n+k)})$*, where* $B_1 = a_p+b_px+a_pn+2ka_p+kb_p-v+nv$ *and* $B_2 = a_p^2 + a_pb_p + ka_p^2 + ka_pb_p - a_pv - b_pv + b_pxv + a_pnv$*), and the effective domain on the supply side given by* $\gamma(m+k-1)S_{(y)}^2 - (\gamma(m+2k-1)a_c + \gamma(m+k-l)b_c + (m-y)c)S_{(y)} + \gamma ka_c(a_c+b_c) + ((m-1)a_c + (m-y)b_c)c < 0$ *(or equivalently,* $S_{(y)} \in (\frac{S_1-\sqrt{(S_1)^2-4\gamma(m+k-1)S_2}}{2\gamma(m+k-1)}, \frac{S_1+\sqrt{S_1^2-4\gamma(m+k-1)S_2}}{2\gamma(m+k-1)})$*, where* $S_1 = -cy+cm-\gamma a_c-\gamma b_cy+\gamma a_cm+\gamma b_cm+2\gamma ka_c+\gamma kb_c$ *and* $S_2 = -a_cc - b_ccy + a_ccm + b_ccm + \gamma ka_c^2 + \gamma ka_cb_c$*).*

Proof 24 *Proof of Corollary A.2.*

(a) *If* $k = 1$*, then* $S_{(y)} = \frac{c}{\gamma}$*, and* $B_{(x)}^*$ *satisfies equation* $(v - B_{(x)})(\gamma B_{(x)})^{x-1}(1-\gamma B_{(x)})^{n-x} - \int_0^{\gamma B_{(x)}}[F_2(S)]^{x-1}[1-F_2(S)]^{n-x}dF_2(S) = 0$ *provided that* $(\gamma n+\gamma)B_{(x)}^2 + ((1-n)\gamma v - x - 1)B_{(x)} + (x-1)v < 0$*. Solving equation* $(\gamma n+\gamma)B_{(x)}^2 + ((1-n)\gamma v - x - 1)B_{(x)} + (x-1)v = 0$ *yields* $B_{(x)} = \frac{1+x-\gamma v+\gamma nv-\sqrt{(-1-x+\gamma v-\gamma nv)^2+4(\gamma+\gamma n)(v-xv)}}{2(\gamma+\gamma n)}$ *or* $B_{(x)} = \frac{1+x-\gamma v+\gamma nv+\sqrt{(-1-x+\gamma v-\gamma nv)^2+4(\gamma+\gamma n)(v-xv)}}{2(\gamma+\gamma n)}$*. Hence,*

$(\frac{1+x-\gamma v+\gamma nv-\sqrt{(-1-x+\gamma v-\gamma nv)^2+4(\gamma+\gamma n)(v-xv)}}{2(\gamma+\gamma n)}, \frac{1+x-\gamma v+\gamma nv+\sqrt{(-1-x+\gamma v-\gamma nv)^2+4(\gamma+\gamma n)(v-xv)}}{2(\gamma+\gamma n)})$.

(b) *If* $k = 0$*, then* $B_{(x)}^* = v$*, and* $S_{(y)}^*$ *satisfies equation* $\gamma \int_{S_{(y)}}^1 [F_1(B)]^{m-y}[1-F_1(B)]^{y-1}dF_1(B) - (\gamma S_{(y)} - c)(S_{(y)})^{m-y}(1-S_{(y)})^{y-1} = 0$ *provided that* $S_{(y)} \in (\frac{c}{\gamma}, \frac{m-y}{m-1})$ *if* $\frac{c}{\gamma} < \frac{m-y}{m-1}$ *or* $S_{(y)} \in (\frac{m-y}{m-1}, \frac{c}{\gamma})$ *otherwise.*

APPENDIX B

PROOF OF CHAPTER 3

B.1 Proof of Main Results

The analysis follows three steps: (i) *First check whether the objective functions are (jointly) concave with the indifference thresholds on a convex set (i.e., the Hessian matrix is negative definite) which permits the FOCs together with feasible conditions to characterize the optimal solutions.* (ii) *Then check feasible conditions labeled as condition* (a) *to* (e)*:* (a) $0 \leq \theta_{iL} \leq \theta_{iH} \leq 1$, (b) $K_H - D_1(1-\theta_{1H}) \geq 0$, (c) $K_L - D_1(\theta_{1H} - \theta_{1L}) \geq 0$, (d) $0 \leq \theta_O, \phi \leq 1$, (e) $\max\{\theta_{1L}, 1 - \frac{K_H}{D_1}\} < \theta_U < \theta_{1H}$. *To verify whether those constraints are satisfied when* $\delta \in (1, 1 + \frac{3D_1D_2 + 3D_2^2}{D_1^2})$. (iii) *Finally figure out equilibrium selling strategies which strictly dominate other selling strategies in the same feasible region.*

Proof 25 *Proof of Lemma* 3.3.1. *The optimal solutions satisfy conditions* (a), (b), *and* (c) *mentioned above.*

(i) If $D_2(1-\theta_{2H}) < K_H - D_1(1-\theta_{1H})$ *and* $D_2(\theta_{2H} - \theta_{2L}) < K_L - D_1(\theta_{1H} - \theta_{1L})$, *then the seller's profit in the salvage stage equals* $\pi_{2P}(\theta_{2H}, \theta_{2L}) = \theta_{2H}(\delta - 1)D_2(1-\theta_{2H}) + \theta_{2L}D_2(1-\theta_{2L})$. *Because it's Hessian matrix is negative definite, equating the FOCs to zero yields* $\theta_{2H}^* = \theta_{2L}^* = \frac{1}{2}$. *The seller's optimal profit in the salvage stage equals* $\pi_{2P}^* = \frac{D_2}{4}\delta$. *The seller's total profit equals*

$$\pi_P(\theta_{1H}, \theta_{1L}) = \theta_{1H}(\delta - 1)D_1(1-\theta_{1H}) + \theta_{1L}D_1(1-\theta_{1L}) + \frac{D_2}{4}\delta.$$

Because the Hessian matrix of π_P *is negative definite, equating the FOCs to zero yields* $\theta_{1H}^* = \theta_{1L}^* = \frac{1}{2}$. *Hence,* $\pi_P^* = \frac{D_1 + D_2}{4}\delta$ *provided that* $D_1 + D_2 < 2K_H$.

(ii) If $D_2(1-\theta_{2H}) \geq K_H - D_1(1-\theta_{1H})$ *and* $D_2(\theta_{2H} - \theta_{2L}) \geq K_L - D_1(\theta_{1H} - \theta_{1L})$, *then the seller's profit in the salvage stage equals* $\pi_{2P}(\theta_{2H}, \theta_{2L}) = \theta_{2H}(\delta - 1)(K_H - D_1(1-\theta_{1H})) + \theta_{2L}(K_H + K_L - D_1(1-\theta_{1L}))$. *By deriving FOCs and SOCs w.r.t*

θ_{2H} and θ_{2L}, we find that π_{2P} increases with both θ_{2H} and θ_{2L}, hence, $\theta^*_{2H} = 1 - \frac{K_H - D_1(1-\theta_{1H})}{D_2}$,$\theta^*_{2L} = 1 - \frac{K_H + K_L - D_1(1-\theta_{1L})}{D_2}$, and the seller's total profit equals

$$\pi_P(\theta_{1H}, \theta_{1L}) = \theta_{1H}(\delta-1)D_1(1-\theta_{1H}) + (1 - \frac{K_H + K_L - D_1(1-\theta_{1L})}{D_2})(K_H + K_L - D_1(1-\theta_{1L}))$$
$$+ \theta_{1L}D_1(1-\theta_{1L}) + (1 - \frac{K_H - D_1(1-\theta_{1H})}{D_2})(\delta-1)(K_H - D_1(1-\theta_{1H})).$$

Because the Hessian matrix of π_P is negative definite, equating the FOCs w.r.t θ_{1H} and θ_{1L} to zero yields $\theta^*_{1H} = 1 - \frac{K_H}{D_1+D_2}$ and $\theta^*_{1L} = 1 - \frac{K_H+K_L}{D_1+D_2}$. Hence, $\theta^*_{2H} = 1 - \frac{K_H}{D_1+D_2}$, $\theta^*_{2L} = 1 - \frac{K_H+K_L}{D_1+D_2}$, and $\pi^*_P = K_H(1-\frac{K_H}{D_1+D_2})(\delta-1) + (K_H+K_L)(1-\frac{K_H+K_L}{D_1+D_2})$ provided that $D_1 + D_2 \geq K_H + K_L$.

(iii) If $D_2(1-\theta_{2H}) \geq K_H - D_1(1-\theta_{1H})$ and $D_2(\theta_{2H} - \theta_{2L}) < K_L - D_1(\theta_{1H} - \theta_{1L})$, then the seller's profit in the salvage stage equals $\pi_{2P}(\theta_{2H}, \theta_{2L}) = \theta_{2H}(\delta-1)(K_H - D_1(1-\theta_{1H})) + \theta_{2L}(K_H - D_1(1-\theta_{1H})) + \theta_{2L}D_2(\theta_{2H} - \theta_{2L})$. By deriving FOCs and SOCs w.r.t θ_{2H} and θ_{2L}, we find that π_{2P} increases with θ_{2H} and is concave with θ_{2L}. Hence, $\theta^*_{2H} = 1 - \frac{K_H - D_1(1-\theta_{1H})}{D_2}$ and $\theta^*_{2L} = \frac{1}{2}$, and the seller's total profit equals

$$\pi_P(\theta_{1H}, \theta_{1L}) = \theta_{1H}(\delta-1)D_1(1-\theta_{1H}) + \theta_{1L}D_1(1-\theta_{1L})$$
$$+ (1 - \frac{K_H - D_1(1-\theta_{1H})}{D_2})(\delta-1)(K_H - D_1(1-\theta_{1H})) + \frac{D_2}{4}.$$

Because the Hessian matrix of π_P is negative definite, hence, $\theta^*_{1H} = \theta^*_{2H} = 1 - \frac{K_H}{D_1+D_2}$, $\theta^*_{1L} = \theta^*_{2L} = \frac{1}{2}$, and $\pi^*_P = K_H(1-\frac{K_H}{D_1+D_2})(\delta-1) + \frac{D_1+D_2}{4}$ provided that $\frac{D_1+D_2}{2} - K_L < K_H \leq \frac{D_1+D_2}{2}$.

(iv) If $D_2(1-\theta_{2H}) < K_H - D_1(1-\theta_{1H})$ and $D_2(\theta_{2H} - \theta_{2L}) \geq K_L - D_1(\theta_{1H} - \theta_{1L})$, then the seller's profit of the salvage stage equals $\pi_{2P}(\theta_{2H}, \theta_{2L}) = \theta_{2H}(\delta-1)D_2(1-\theta_{2H}) + \theta_{2L}(K_L - D_1(\theta_{1H} - \theta_{1L})) + \theta_{2L}D_2(1-\theta_{2H})$. By deriving FOCs and SOCs w.r.t θ_{2H} and θ_{2L}, we find that π_{2P} increases with θ_{2L} and is concave with θ_{2H}. Hence, $\theta^*_{2H} = \frac{D_2(\delta-1) + K_L - D_1(\theta_{1H} - \theta_{1L})}{D_2(2\delta-1)}$, $\theta^*_{2L} = \frac{D_2(\delta-1) - 2(K_L - D_1(\theta_{1H} - \theta_{1L}))(\delta-1)}{D_2(2\delta-1)}$, and the seller's total profit equals

$$\pi_P(\theta_{1H}, \theta_{1L}) = \theta_{1H}(\delta-1)D_1(1-\theta_{1H}) + \theta_{1L}D_1(1-\theta_{1L}) + \frac{(\delta-1)(D_2^2\delta^2 - 4\delta(K_L - D_1(\theta_{1H} - \theta_{1L}))^2)}{D_2(1-2\delta)^2}$$
$$+ \frac{(\delta-1)(K_L - D_1(\theta_{1H} - \theta_{1L}))(-D_2 + 3(K_L - D_1(\theta_{1H} - \theta_{1L})))}{D_2(1-2\delta)^2}.$$

The Hessian matrix of π_P is negative definite. Hence, $\theta^*_{1H} = \frac{4D_1\delta^2 + 4D_2\delta^2 - 3D_1\delta - 4D_2\delta + 8K_L\delta + 2D_2 - 6K_L}{2(4D_1\delta^2 + 4D_2\delta^2 - 3D_1\delta - 4D_2\delta + D_2)}$, $\theta^*_{1L} = \frac{4D_1\delta^2 + 4D_2\delta^2 - 8K_L\delta^2 - 3D_1\delta - 5D_2\delta + 14K_L\delta + 2D_2 - 6K_L}{2(4D_1\delta^2 + 4D_2\delta^2 - 3D_1\delta - 4D_2\delta + D_2)}$, $\theta^*_{2H} = \frac{4D_1\delta^2 + 4D_2\delta^2 - 5D_1\delta - 6D_2\delta + 4K_L\delta + 2D_2 - 2K_L}{2(4D_1\delta^2 + 4D_2\delta^2 - 3D_1\delta - 4D_2\delta + D_2)}$,

$\theta_{2L}^* = \frac{2D_1\delta^2+2D_2\delta^2-4K_L\delta^2-2D_1\delta-3D_2\delta+6K_L\delta+D_2-2K_L}{4D_1\delta^2+4D_2\delta^2-3D_1\delta-4D_2\delta+D_2}$, *and* $\pi_P^* = \frac{-4\delta(D_2-7K_L)K_L+4K_L(D_2-3K_L)}{4(4D_1\delta^2+4D_2\delta^2-3D_1\delta-4D_2\delta+D_2)} + \frac{4\delta^3(D_1+D_2)^2-\delta^2(3D_1^2+7D_1D_2+4(D_2^2+4K_L^2))}{4(4D_1\delta^2+4D_2\delta^2-3D_1\delta-4D_2\delta+D_2)}$.

Because $\theta_{2L}^* \leq \theta_{2H}^*$ *reduces to* $f_1^P(\delta) = 8K_L\delta^2 - (D_1+8K_L)\delta + 2K_L > 0$, *and* $\theta_{1H}^* \leq 1$ *reduces to* $f_2^P(\delta) = 4(D_1+D_2)\delta^2 - (3D_1+4D_2+8K_L)\delta + 6K_L \geq 0$ *when* $\delta \in (1, 1+\frac{3D_1D_2+3D_2^2}{D_1^2})$. *Note that* $f_1^P(\delta) > 0$ *if* $K_L > \frac{D_1}{2}$, *and* $f_2^P(\delta) \geq 0$ *if* $K_L \leq \frac{D_1}{2}$. *The two violates each other. Hence, there are no optimal solutions.*

The profit gap between cases (i) and (ii) equals $\Delta\pi_P^{i-ii} = \frac{\delta(D_1+D_2-2K_H)^2-4K_L(D_1+D_2-2K_H)+4K_L^2}{4(D_1+D_2)} > 0$, *and cases (iii) and (ii) equals* $\Delta\pi_P^{iii-ii} = \frac{(D_1+D_2-2(K_H+K_L))^2}{4(D_1+D_2)} > 0$. *Hence, the optimal prices are denoted by*

$$(p_{iH}^*, p_{iL}^*) = \begin{cases} (\frac{\delta}{2}, \frac{1}{2}) & if K_H > \frac{D_1+D_2}{2}, \\ (\frac{(2\delta-1)(D_1+D_2)-2(\delta-1)K_H}{2(D_1+D_2)}, \frac{1}{2}) & if \frac{D_1+D_2}{2} - K_L < K_H \leq \frac{D_1+D_2}{2}, \\ (\delta - \frac{\delta K_H + K_L}{D_1+D_2}, 1 - \frac{K_H+K_L}{D_1+D_2}) & otherwise. \end{cases}$$

The seller's profit equals

$$\pi_P^* = \begin{cases} \frac{D_1+D_2}{4}\delta & if K_H > \frac{D_1+D_2}{2}, \\ K_H(1-\frac{K_H}{D_1+D_2})(\delta-1) + \frac{D_1+D_2}{4} & if \frac{D_1+D_2}{2} - K_L < K_H \leq \frac{D_1+D_2}{2}, \\ K_H(1-\frac{K_H}{D_1+D_2})(\delta-1) + (K_H+K_L)(1-\frac{K_H+K_L}{D_1+D_2}) & otherwise. \end{cases}$$

Lemma 25 *uncovers the optimal fraction of customers accepting upgrades.*

Lemma (Optimal Solutions in Upgrading Mechanism) *Given* θ_{1H} *and* θ_{1L} *and* $\theta_U^* \in [\max\{\theta_{1L}, 1-\frac{K_H}{D_1}\}, \theta_{1H}]$, *the seller prices* $p^* = \theta_U^*(\delta-1)$, *where the cutoff value such that customers are indifferent between accepting and not accepting upgrades equals*

$$\theta_U^* = \begin{cases} \frac{\theta_{1H}}{2} & if \frac{\theta_{1H}}{2} > 1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1} \& \frac{\theta_{1H}}{2} < \theta_{1L} + \frac{K_L}{D_1}, \\ \frac{D_2(1+(1-2\delta)^2\theta_{1H}) + 2(-3+4\delta)(K_L+D_1\theta_{1L})}{2((-3+4\delta)D_1 + (1-2\delta)^2D_2)} & if \theta_U^* > \frac{(D_1-K_H)(2\delta-1)+\delta D_2 - K_L - D_1\theta_{1L}}{2D_1(\delta-1)}, \\ \frac{2D_1+D_2-2K_H+D_2\theta_{1H}}{2(D_1+D_2)} & otherwise. \end{cases}$$

Proof 26 *Proof of Lemma 25. We focus on case $K_H \geq D_1(1-\theta_{1H})$ and $K_L \geq D_1(\theta_{1H}-\theta_{1L})$, and the optimal solutions satisfy conditions (a), (b), (c), and (e) mentioned above.*

(i) If $D_2(1-\theta_{2H}) < K_H - D_1(1-\theta_U)$ and $D_2(\theta_{2H}-\theta_{2L}) < K_L - D_1(\theta_U - \theta_{1L})$, then the seller's profit selling through pricing equals $\pi_{2U} = D_2(1-\theta_{2H})(\theta_{2L}+\theta_{2H}(\delta-1)) + D_2(\theta_{2H}-\theta_{2L})\theta_{2L}$. The Hessian matrix of π_{2U} is negative definite. Equating the FOCs of π_{2U} w.r.t θ_{2H} and θ_{2L} to zero yields $\theta_{2H}^* = \theta_{2L}^* = \frac{1}{2}$. Hence, $\pi_{2U}^* = \frac{D_2\delta}{4}$, and the seller's profit in the salvage stage equals $\pi_2^U = (\delta-1)\theta_U D_1(\theta_{1H}-\theta_U) + \frac{D_2\delta}{4}$ provided that $\theta_U > 1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1}$ and $\theta_U < \theta_{1L} + \frac{K_L}{D_1}$. Because π_{2U}^* is concave with θ_U, hence, $\theta_U^* = \frac{\theta_{1H}}{2}$ provided that $\frac{\theta_{1H}}{2} \in (\max\{\theta_{1L}, 1-\frac{K_H}{D_1}\}, \theta_{1H})$.

(ii) If $D_2(1-\theta_{2H}) \geq K_H - D_1(1-\theta_U)$ and $D_2(\theta_{2H}-\theta_{2L}) \geq K_L - D_1(\theta_U-\theta_{1L})$, then the seller's profit selling through pricing equals $\pi_{2U} = (K_H - D_1(1-\theta_U))(\theta_{2L}+\theta_{2H}(\delta-1)) + (K_L - D_1(\theta_U-\theta_{1L}))\theta_{2L}$, which increases with θ_{2H} and θ_{2L}. Hence, $\theta_{2H}^* = 1 - \frac{K_H - D_1(1-\theta_U)}{D_2}$, $\theta_{2L}^* = 1 - \frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2}$, and $\pi_2^U = \pi_{2U}^* + (\delta-1)\theta_U D_1(\theta_{1H}-\theta_U)$. Because $\frac{\partial^2 \pi_2^U}{\partial \theta_U^2} < 0$, hence, $\theta_U^* = \frac{2D_1+D_2-2K_H+D_2\theta_{1H}}{2(D_1+D_2)}$.

(iii) If $D_2(1-\theta_{2H}) \geq K_H - D_1(1-\theta_U)$ and $D_2(\theta_{2H}-\theta_{2L}) < K_L - D_1(\theta_U-\theta_{1L})$, then the seller's profit selling through pricing equals $\pi_{2U} = (K_H - D_1(1-\theta_U))(\theta_{2L} + \theta_{2H}(\delta-1)) + D_2(\theta_{2H}-\theta_{2L})\theta_{2L}$, which increases with θ_{2H} while is concave with θ_{2L}. Hence, $\theta_{2H}^* = 1 - \frac{K_H-D_1(1-\theta_U)}{D_2}$, $\theta_{2L}^* = \frac{D_2\theta_{2H}+K_H-D_1(1-\theta_U)}{2D_2} = \frac{1}{2}$ provided that $\theta_{1L} > 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$. Because $\frac{\partial^2\pi_2}{\partial\theta_U^2} < 0$, hence, $\theta_U^* = \frac{2D_1+D_2-2K_H+D_2\theta_{1H}}{2(D_1+D_2)}$ provided that $\theta_U \in (\max\{\theta_{1L}, 1-\frac{K_H}{D_1}\}, \theta_{1H})$.

(iv) If $D_2(1-\theta_{2H}) < K_H - D_1(1-\theta_U)$ and $D_2(\theta_{2H}-\theta_{2L}) \geq K_L - D_1(\theta_U-\theta_{1L})$, then the seller's profit selling through pricing equals $\pi_{2U} = D_2(1-\theta_{2H})(\theta_{2L} + \theta_{2H}(\delta-1)) + (K_L - D_1(\theta_U-\theta_{1L}))\theta_{2L}$, which is concave with θ_{2H} and increases with θ_{2L}. Hence, $\theta_{2H}^* = \frac{-D_2+D_2\delta+K_L+D_1\theta_{1L}-D_1\theta_U}{D_2(2\delta-1)}$ and $\theta_{2L}^* = \frac{(\delta-1)(D_2-2K_L-2D_1\theta_{1L}+2D_1\theta_U)}{D_2(2\delta-1)}$ provided that $\theta_U > \frac{(D_1-K_H)(2\delta-1)+\delta D_2-K_L-D_1\theta_{1L}}{2D_1(\delta-1)}$. Because $\frac{\partial^2\pi_2}{\partial\theta_U^2} < 0$, hence, $\theta_U^* = \frac{D_2(1+(1-2\delta)^2\theta_{1H})+2(-3+4\delta)(K_L+D_1\theta_{1L})}{2((-3+4\delta)D_1+(1-2\delta)^2D_2)}$.

Boundary solutions: $\theta_U^* = \theta_{1L}$ means that upgrades are sent to all customers who have *purchased low-quality capacities in the regular stage, $\theta_U^* = \theta_{1H}$ means that no upgrading is available, and $\theta_U^* = 1 - \frac{K_H}{D_1}$ means that all unsold high-quality capacities are offered as upgrades.*

Proof 27 *Proof of Lemma* 3.3.2.1 *. Following Lemma* 25*, we derive the optimal solutions in the regular stage taking into account the optimal solutions in the salvage stage.*

(i) If $D_2(1-\theta_{2H}) < K_H - D_1(1-\theta_U)$ and $D_2(\theta_{2H}-\theta_{2L}) < K_L - D_1(\theta_U - \theta_{1L})$, then the Hessian matrix of the seller's total profit $\pi_U = D_1(1-\theta_{1H})(\theta_{1L}+\theta_{1H}(\delta-1)) + D_1(\theta_{1H}-\theta_{1L})\theta_{1L} + \pi_2^*$ is negative definite. Equating the FOCs to zero yields $\theta_{1H}^* = \frac{2}{3}$ and $\theta_{1L}^* = \frac{1}{2}$. While $\theta_U^* = \frac{1}{3}$ violates condition $\theta_U^* \geq \theta_{1L}^*$.

***Boundary solution** $\theta_U = \theta_{1L}$.*

If $\theta_U > 1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1}$ and $\frac{\theta_{1H}}{2} < \theta_{1H}$, then $\theta_U = \theta_{1L}$. The Hessian matrix of π_{2U} is negative definite. Hence, $\theta_{2H}^* = \theta_{2L}^* = \frac{1}{2}$ and $\pi_{2U}^* = \frac{D_2\delta}{4}$, and the seller's total profit equals $\pi_U(\theta_{1H},\theta_{1L}) = D_1(\delta-1)(\theta_{1H} - \theta_{1H}^2 + \theta_{1H}\theta_{1L} - \theta_{1L}^2) + D_1(\theta_{1L} - \theta_{1L}^2) + \frac{D_2\delta}{4}$. The Hessian matrix of π_U is negative definite, hence, $\theta_{1H}^* = \frac{2\delta+1}{3\delta+1}$, $\theta_U^* = \theta_{1L}^* = \frac{\delta+1}{3\delta+1}$, and $\pi_U^* = \frac{\delta^2 D_1}{3\delta+1} + \frac{D_2\delta}{4}$ provided that $f_1^U(\delta) = (6K_H - 4D_1 - 3D_2)\delta + 2K_H - D_2 \geq 0$ and $f_2^U(\delta) = (3K_L - D_1)\delta + K_L \geq 0$. Note that $f_1^U(\delta) \geq 0$ if $K_H \geq \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3}{8D_1^2+18D_1D_2+18D_2^2}$, and $f_2^U(\delta) \geq 0$ if $K_L \geq \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$. So, the effective domain is $K_H \geq \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3}{8D_1^2+18D_1D_2+18D_2^2}$ and $K_L \geq \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$.

(ii) If $D_2(1-\theta_{2H}) \geq K_H - D_1(1-\theta_U)$ and $D_2(\theta_{2H}-\theta_{2L}) \geq K_L - D_1(\theta_U - \theta_{1L})$, the Hessian matrix of π_U is negative definite. Equating the FOCs w.r.t θ_{1H} and θ_{1L} to zero yields $\theta_{1H}^* = \frac{4D_1+3D_2-2K_H}{4D_1+3D_2}$, $\theta_{1L}^* = \theta_{2L}^* = \frac{D_1+D_2-(K_H+K_L)}{D_1+D_2}$, $\theta_U^* = \frac{4D_1+3D_2-4K_H}{4D_1+3D_2}$, $\theta_{2H}^* = \frac{4D_1+3D_2-3K_H}{4D_1+3D_2}$, and $\pi_U^* = \frac{\delta(D_1+D_2)(4D_1+3D_2-3K_H)K_H+4D_1^2K_L+3D_2(D_2-2K_H-K_L)K_L}{(D_1+D_2)(4D_1+3D_2)} - \frac{D_1(K_H^2-7D_2K_L+8K_HK_L+4K_L^2)}{(D_1+D_2)(4D_1+3D_2)}$ provided that $K_H < \min\{D_1+D_2-K_L, \frac{4D_1D_2+3D_2^2}{2D_1^2+D_1D_2}K_L, \frac{4D_1+3D_2}{D_2}K_L\}$. If we assume $K_H < K_L$, then the effective domain is given by $K_H < \min\{D_1+D_2-K_L, \frac{4D_1D_2+3D_2^2}{2D_1^2+D_1D_2}K_L\}$.

***Boundary solution** $\theta_U = \theta_{1L}$.*

If $D_2(1-\theta_{2H}) \geq K_H - D_1(1-\theta_{1L})$ and $D_2(\theta_{2H}-\theta_{2L}) \geq K_L$, then $\pi_{2U} = (\theta_{2H}(\delta -$

$1) + \theta_{2L})(K_H - D_1(1-\theta_{1L})) + \theta_{2L}K_L$, *which increases with* θ_{2H} *and* θ_{2L}. *Hence,* $\theta_{2H} = 1 - \frac{K_H - D_1(1-\theta_{1L})}{D_2}$, $\theta_{2L} = \theta_{2H} - \frac{K_L}{D_2}$, *and* $\pi_U = D_1(\delta-1)(\theta_{1H} - \theta_{1H}^2 + \theta_{1H}\theta_{1L} - \theta_{1L}^2) + D_1(\theta_{1L} - \theta_{1L}^2) + \pi_{2U}^*$. *The Hessian matrix of* π_U *is negative definite. Equating the FOCs of* π_U *w.r.t* θ_{1H} *and* θ_{1L} *to zero yields* $\theta_{1H}^* = \frac{(4D_1+3D_2-2K_H)\delta+D_2-2K_L}{4\delta D_1+D_2+3\delta D_2}$, $\theta_{1L}^* = \theta_U^* = \frac{(4D_1+3D_2-4K_H)\delta+D_2-4K_L}{4\delta D_1+D_2+3\delta D_2}$, $\theta_{2H}^* = \frac{(4D_1+3D_2-3K_H)D_2\delta+D_2^2-D_2K_H+4D_1K_L}{D_2(4\delta D_1+D_2+3\delta D_2)}$, $\theta_{2L}^* = \frac{(4D_1D_2-4D_1K_L+3D_2^2-3D_2K_H-3D_2K_L)\delta+D_2^2+4D_1K_L-D_2(K_H+K_L)}{D_2(4\delta D_1+D_2+3\delta D_2)}$, *and* $\pi_U^* = \frac{D_2(4D_1+3D_2-3K_H)K_H\delta^2}{D_2(4D_1\delta+D_2+3D_2\delta)} + \frac{K_L(D_2^2+4D_1K_L-D_2(2K_H+K_L))+(-4D_1K_L^2+D_2^2(K_H+3K_L)-D_2(K_H^2-4D_1K_L+6K_HK_L+3K_L^2))\delta}{D_2(4D_1\delta+D_2+3D_2\delta)}$ *provided that* $K_H > \frac{D_1}{D_2}K_L$, $K_H > \frac{D_1}{D_1+2D_2}K_L$, $K_H > \frac{4D_1+3D_2}{D_2}K_L$, $f_3^U(\delta) = (4D_1 + 3D_2 - 4K_H)\delta + D_2 - 4K_L \geq 0$, $f_4^U(\delta) = (4D_1(D_2-K_L) + 3D_2(D_2 - K_H - K_L))\delta + D_2^2 + 4D_1K_L - D_2(K_H+K_L) \geq 0$, *and* $f_5^U(\delta) = (-2D_1K_H + 4D_1K_L + 3D_2K_L)\delta + (-2D_1 + D_2)K_L \geq 0$. *Note that* $f_3^U(\delta) \geq 0$ *if* $K_H \leq \min\{D_1+D_2-K_L, \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3-4D_1^2K_L}{4D_1^2+12D_1D_2+12D_2^2}\}$, $f_4^U(\delta) \geq 0$ *if* $K_H \leq \min\{D_1+D_2-K_L, \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3-(16D_1^2+21D_1D_2+9D_2^2)K_L}{4D_1^2+9D_1D_2+9D_2^2}\}$, *and* $f_5^U(\delta) \geq 0$ *if* $K_H \leq \min\{\frac{2D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3}{2D_1^3+6D_1^2D_2+6D_1D_2^2}K_L, \frac{D_1+2D_2}{D_1}K_L\}$. *Note that* $\frac{4D_1+3D_2}{D_2}K_L < K_H$ *violates assumption* $K_H < K_L$. *So, there are no optimal solutions.*

Boundary solution $\theta_U = \theta_{1H}$.

If $\frac{2D_1+D_2-2K_H+D_2\theta_{1H}}{2(D_1+D_2)} > \theta_{1H}$, *or equivalently,* $\theta_{1H} < 1 - \frac{2K_H}{2D_1+D_2}$, *then* $\theta_U = \theta_{1H}$. *The Hessian matrix of* π_U *is negative definite. Hence,* $\theta_{1H}^* = 1 - \frac{K_H}{D_1+D_2}$, *which is larger than* $1 - \frac{2K_H}{2D_1+D_2}$. *So,* $\theta_U^* \neq \theta_{1H}$.

Boundary solution $\theta_U = 1 - \frac{K_H}{D_1}$.

If $\frac{2D_1+D_2-2K_H+D_2\theta_{1H}}{2(D_1+D_2)} < 1 - \frac{K_H}{D_1}$ *and* $1 - \frac{K_H}{D_1} > \theta_{1L}$, *then* $\theta_U = 1 - \frac{K_H}{D_1}$, *the Hessian matrix of* π_U *is negative definite. Hence,* $\theta_{1H}^* = 1 - \frac{K_H}{2D_1}$, $\theta_{1L}^* = \theta_{2L}^* = 1 - \frac{K_H+K_L}{D_1+D_2}$ *and* $\theta_{2H}^* = 1$. *Note that* $\frac{2D_1+D_2-2K_H+D_2\theta_{1H}^*}{2(D_1+D_2)} < 1 - \frac{K_H}{D_1}$ *does not hold. So,* $\theta_U^* \neq 1 - \frac{K_H}{D_1}$.

(iii) If $D_2(1-\theta_{2H}) \geq K_H - D_1(1-\theta_U)$ *and* $D_2(\theta_{2H} - \theta_{2L}) < K_L - D_1(\theta_U - \theta_{1L})$, *then the Hessian matrix of* π_U *is negative definite, equating the FOCs of* π_U *w.r.t* θ_{1H} *and* θ_{1L} *to zero yields* $\theta_{1H}^* = \frac{4D_1+3D_2-2K_H}{4D_1+3D_2}$, *and* $\theta_{1L}^* = \theta_{2L}^* = \frac{1}{2}$. *Hence,* $\theta_U^* = \frac{4D_1+3D_2-4K_H}{4D_1+3D_2}$, $\theta_{2H}^* = \frac{4D_1+3D_2-3K_H}{4D_1+3D_2}$, *and* $\pi_U^* = \frac{4D_1^2+D_1(7D_2+16(\delta-1)K_H)+3(D_2^2+4(\delta-1)D_2K_H-4(\delta-1)K_H^2)}{4(4D_1+3D_2)}$ *provided that* $K_H > \frac{D_1+D_2}{2} - K_L$, $K_H < \frac{4D_1+3D_2}{8}$ *and* $K_H \geq \frac{4D_1^2+3D_1D_2-(8D_1+6D_2)K_L}{4D_1}$.

Boundary solution $\theta_U = \theta_{1L}$.

Note that $\pi_{2U} = ((\delta-1)\theta_{2H} + \theta_{2L})(K_H - D_1(1-\theta_{1L})) + \theta_{2L}D_2(\theta_{2H} - \theta_{2L})$ *increases with* θ_{2H} *and is concave with* θ_{2L}. *So,* $\theta_{2H} = 1 - \frac{K_H - D_1(1-\theta_{1L})}{D_2}$ *and* $\theta_{2L} = \frac{1}{2}$. *The Hessian matrix of* π_U *is negative definite, equating the FOCs of* π_U *w.r.t* θ_{1H} *and* θ_{1L} *to zero yields* $\theta^*_{1H} = \frac{4D_1(\delta-1)+3\delta D_2+2K_H-2\delta K_H}{4\delta D_1+D_2+3\delta D_2-4D_1}$, $\theta^*_U = \theta^*_{1L} = \frac{4D_1(\delta-1)+3\delta D_2+4K_H-4\delta K_H-D_2}{4\delta D_1+D_2+3\delta D_2-4D_1}$, $\theta^*_{2H} = \frac{D_1(4\delta-2)+(3\delta+1)(D_2-K_H)}{4\delta D_1+D_2+3\delta D_2-4D_1}$, $\theta^*_{2L} = \frac{1}{2}$, *and* $\pi^*_U = \frac{(D_2-2K_H)^2+4\delta^2(4D_1+3D_2-3K_H)K_H}{4(4\delta D_1+D_2+3\delta D_2-4D_1)} + \frac{\delta(3D_2^2+4D_1(D_2-4K_H)-8D_2K_H+8K_H^2)}{4(4\delta D_1+D_2+3\delta D_2-4D_1)}$ *provided that* $K_H > \frac{D_1}{2}$, $K_H > \frac{D_1}{4}$, $K_H > \frac{4D_1+3D_2}{8}$, $f_6^U(\delta) = (4D_1+3D_2-4K_H)\delta - 4D_1 - D_2 + 4K_H \geq 0$, $f_7^U(\delta) = (4D_1+3D_2-6K_H)\delta + D_2 - 2K_H \geq 0$, $f_8^U(\delta) = (-2D_1K_H+4D_1K_L+3D_2K_L)\delta - D_1D_2 + 2D_1K_H - 4D_1K_L + D_2K_L \geq 0$, *and* $f_9^U(\delta) = (-4D_1D_2 + 8D_1K_L - 3D_2^2 + 6D_2K_H + 6D_2K_L)\delta - D_2^2 - 8D_1K_L + 2D_2(K_H + K_L) > 0$. *Note that* $f_6^U(\delta) \geq 0$ *if* $K_H \leq \frac{14D_1^2+21D_1D_2+9D_2^2}{12D_1+12D_2}$, $f_7^U(\delta) \geq 0$ *if* $K_H \leq \frac{D_1+D_2}{2}$, $f_8^U(\delta) \geq 0$ *if* $K_H \leq \frac{-D_1^3+(16D_1^2+21D_1D_2+9D_2^2)K_L}{6D_1^2+6D_1D_2}$ *and* $K_L \geq \frac{D_1}{4}$, *and* $f_9^U(\delta) > 0$ *if* $K_H > \max\{\frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3-(32D_1^2+42D_1D_2+18D_2^2)K_L}{8D_1^2+18D_1D_2+18D_2^2}, \frac{D_1+D_2}{2} - K_L\}$. *So, the effective domain is* $\frac{4D_1+3D_2}{8} < K_H < \min\{\frac{D_1+D_2}{2}, \frac{-D_1^3+(16D_1^2+21D_1D_2+9D_2^2)K_L}{6D_1^2+6D_1D_2}\}$.

Boundary solution $\theta_U = \theta_{1H}$.

By solving the FOCs of π_U *w.r.t* θ_{1H} *and* θ_{1L}, *we have* $\theta^*_{1H} = 1 - \frac{K_H}{D_1+D_2}$, *where* $1 - \frac{K_H}{D_1+D_2} > 1 - \frac{2K_H}{2D_1+D_2}$. *So,* $\theta^*_U \neq \theta_{1H}$.

Boundary solution $\theta_U = 1 - \frac{K_H}{D_1}$.

If $D_2(\theta_{2H} - \theta_{2L}) < K_H + K_L - D_1(1-\theta_{1L})$, *then* $\pi_{2U} = \theta_{2L}D_2(\theta_{2H} - \theta_{2L})$ *increases with* θ_{2H} *and is concave with* θ_{2L}, *so,* $\theta^*_{2H} = 1$, $\theta^*_{2L} = \frac{1}{2}$. *The Hessian matrix of* $\pi_U = D_1(\delta-1)(\theta_{1H} - \theta_{1H}^2) + D_1(\theta_{1L} - \theta_{1L}^2) + \frac{D_2}{4} + D_1(\delta-1)(1-\frac{K_H}{D_1})(\theta_{1H} - 1 + \frac{K_H}{D_1})$ *is negative definite, by deriving the FOCs of* π_U *w.r.t* θ_{1H} *and* θ_{1L}, *we have* $\theta^*_{1H} = 1 - \frac{K_H}{2D_1}$, $\theta^*_{1L} = \frac{1}{2}$, *and* $\pi^*_U = \frac{D_1^2-3(\delta-1)K_H^2+D_1(D_2+4(\delta-1)K_H)}{4D_1}$ *provided that* $K_H < \frac{D_1}{2}$, $K_H > \frac{D_1+D_2}{2} - K_L$, *and* $K_H > D_1 - 2K_L$. *While condition* $\frac{2D_1+D_2-2K_H+D_2\theta_{1H}}{2(D_1+D_2)} < 1 - \frac{K_H}{D_1}$ *does not hold. So,* $\theta^*_U \neq 1 - \frac{K_H}{D_1}$.

(iv) If $D_2(1-\theta_{2H}) < K_H - D_1(1-\theta_U)$ *and* $D_2(\theta_{2H} - \theta_{2L}) \geq K_L - D_1(\theta_U - \theta_{1L})$, *then the Hessian matrix of* π_U *is negative definite. By equating the FOCs of* π_U *w.r.t* θ_{1H} *and* θ_{1L} *to zero, we have* $\theta^*_{1H} = \frac{(-3-2\delta+8\delta^2)D_1+(3-8\delta+8\delta^2)D_2+2(4\delta-3)K_L}{(-3-5\delta+12\delta^2)D_1+3(1-2\delta)^2D_2}$, $\theta^*_{1L} =$

$\frac{2(-3+\delta+4\delta^2)D_1+3((2-5\delta+4\delta^2)D_2-2(3-7\delta+4\delta^2)K_L)}{2((-3-5\delta+12\delta^2)D_1+3(1-2\delta)^2D_2)}$, $\theta^*_U = \frac{(-3+\delta+4\delta^2)D_1+(3-4\delta+4\delta^2)D_2+4(-3+4\delta)K_L}{(-3-5\delta+12\delta^2)D_1+3(1-2\delta)^2D_2}$, $\theta^*_{2H} = \frac{3((-2-3\delta+4\delta^2)D_1+2(-1+2\delta)((-1+\delta)D_2+K_L))}{2((-3-5\delta+12\delta^2)D_1+3(1-2\delta)^2D_2)}$, $\theta^*_{2L} = \frac{(\delta-1)((3+4\delta)D_1+3(-1+2\delta)(D_2-2K_L))}{(-3-5\delta+12\delta^2)D_1+3(1-2\delta)^2D_2}$, *and* $\pi^*_U = \frac{4\delta^3(4D_1^2+7D_1D_2+3D_2^2)-12K_L(D_1-D_2+3K_L)+4\delta K_L(7D_1-3D_2+21K_L)}{4((-3-5\delta+12\delta^2)D_1+3(1-2\delta)^2D_2)}$ $+ \frac{\delta^2(12D_1^2+25D_1D_2+12D_2^2+16D_1K_L+48K_L^2)}{4((-3-5\delta+12\delta^2)D_1+3(1-2\delta)^2D_2)}$.

Because $\theta^*_{2H} \geq \theta^*_{2L}$, *and* $\theta^*_U < \theta^*_{1H}$ *reduce to* $f^U_{10}(\delta) = 4(D_1 + 6K_L)\delta^2 - (7D_1 + 24K_L)\delta + 6K_L \geq 0$ *and* $f^U_{11}(\delta) = 4(D_1 + D_2)\delta^2 - (3D_1 + 4D_2 + 8K_L)\delta + 6K_L > 0$, *respectively. Note that* $f^U_{10}(\delta) \geq 0$ *if* $K_L \geq \frac{D_1}{2}$, *and* $f^U_{11}(\delta) > 0$ *if* $K_L < \frac{D_1}{2}$. *So, there are no optimal solutions.*

Boundary solution $\theta_U = \theta_{1L}$.

Note that $\pi_{2U} = ((\delta-1)\theta_{2H} + \theta_{2L})D_2(1-\theta_{2H}) + \theta_{2L}K_L$ *is concave with* θ_{2H} *and increases with* θ_{2L}, *hence,* $\theta_{2H} = \frac{D_2(\delta-1)+K_L}{(2\delta-1)D_2}$ *and* $\theta_{2L} = \frac{D_2(\delta-1)-2K_L(\delta-1)}{(2\delta-1)D_2}$. *The Hessian matrix of* π_U *is negative definite, by deriving the FOCs of* π_U *w.r.t* θ_{1H} *and* θ_{1L}, *we have* $\theta^*_{1H} = \frac{2\delta+1}{3\delta+1}$, $\theta^*_{1L} = \frac{\delta+1}{3\delta+1}$, $\theta^*_{2H} = \frac{D_2(1-\delta)-K_L}{D_2(1-2\delta)}$, $\theta^*_{2L} = \frac{(\delta-1)(D_2-2K_L)}{D_2(2\delta-1)}$, *and* $\pi^*_U = \frac{\delta^4D_2(4D_1+3D_2)+(D_2-3K_L)K_L+2\delta(D_2-K_L)K_L}{(1-2\delta)^2(D_2+3\delta D_2)} + \frac{-2\delta^3(2D_1D_2+D_2^2+6K_L^2)+\delta^2(D_1D_2-D_2^2-3D_2K_L+17K_L^2)}{(1-2\delta)^2(D_2+3\delta D_2)}$.

Because $\theta^*_{1L} > \frac{D_2(1+(1-2\delta)^2\theta^*_{1H})+2(-3+4\delta)(K_L+D_1\theta^*_{1L})}{2((-3+4\delta)D_1+(1-2\delta)^2D_2)}$ *reduces to* $f^U_{12}(\delta) = 4(D_2-6K_L)\delta^2 + (-7D_2+10K_L)\delta+6K_L > 0$. *Because* $f^U_{12}(\delta)\big|_{\delta=1} = -3D_2-8K_L < 0$, *which indicates that* $f^U_{12}(\delta) > 0$ *does not always hold when* $\delta \in [1, 1 + \frac{3D_1D_2+3D_2^2}{D_1^2}]$. *So, there are no optimal solutions.*

Boundary solution $\theta_U = \theta_{1H}$.

The analysis is the same as case (iv) in Lemma 3.3.1, and there are no optimal solutions.

Boundary solution $\theta_U = 1 - \frac{K_H}{D_1}$.

If $D_2(\theta_{2H} - \theta_{2L}) \geq K_H + K_L - D_1(1-\theta_{1L})$, *then* $\pi_{2U} = \theta_{2L}(K_H + K_L - D_1(1-\theta_{1L}))$ *increases with* θ_{2L}, *so,* $\theta_{2H} = 1$ *and* $\theta_{2L} = 1 - \frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2}$. *The Hessian matrix of* π_U *is negative definite. By deriving the FOCs of* π_U *w.r.t* θ_{1H} *and* θ_{1L}, *we have* $\theta^*_{1H} = 1 - \frac{K_H}{2D_1}$, $\theta^*_{1L} = 1 - \frac{K_H+K_L}{D_1+D_2}$, $\theta^*_{2H} = 1$, $\theta^*_{2L} = 1 - \frac{K_H+K_L}{D_1+D_2}$, *and* $\pi^*_U = \frac{\delta(D_1+D_2)(4D_1-3K_H)K_H+3D_2K_H^2+4D_1^2K_L-D_1(K_H^2+8K_HK_L+4K_L(-D_2+K_L))}{4D_1(D_1+D_2)}$.

Because $1 - \frac{K_H}{D_1} > \frac{D_2(1+(1-2\delta)^2\theta^*_{1H})+2(-3+4\delta)(K_L+D_1\theta^*_{1L})}{2((-3+4\delta)D_1+(1-2\delta)^2D_2)}$ *reduces to* $f^U_{13}(\delta) = 4(D_1 +$

$D_2)(2D_1-3K_H)\delta^2-4(2D_1^2-3D_2K_H+D_1(2D_2+K_H+4K_L))\delta+9D_1K_H-3D_2K_H+12D_1K_L>0$. *Note that* $f_{13}^U(\delta)\big|_{\delta=1}=-7D_1K_H-3D_2K_H-4D_1K_L<0$, *which means that* $f_{13}^U(\delta)>0$ *does not always hold when* $\delta\in[1,1+\frac{3D_1D_2+3D_2^2}{D_1^2}]$. *So, there are no optimal solutions.*

If $D_1>D_2$ *and* $K_H<K_L$, *then we have the optimal solutions summarized as:*

(i) *If* $K_H\geq\frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3}{8D_1^2+18D_1D_2+18D_2^2}$ *and* $K_L\geq\frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$, *then* $\theta_{1H}^*=\frac{2\delta+1}{3\delta+1}$, $\theta_U^*=\theta_{1L}^*=\frac{\delta+1}{3\delta+1}$, $\theta_{2H}^*=\theta_{2L}^*=\frac{1}{2}$, *and* $\pi_U^*=\frac{\delta^2D_1}{3\delta+1}+\frac{D_2\delta}{4}$.

(ii) *If* $K_H<\min\{D_1+D_2-K_L,\frac{4D_1D_2+3D_2^2}{2D_1^2+D_1D_2}K_L\}$, *then* $\theta_{1H}=\frac{4D_1+3D_2-2K_H}{4D_1+3D_2}$, $\theta_{1L}^*=\theta_{2L}^*=\frac{D_1+D_2-(K_H+K_L)}{D_1+D_2}$, $\theta_U^*=\frac{4D_1+3D_2-4K_H}{4D_1+3D_2}$, $\theta_{2H}^*=\frac{4D_1+3D_2-3K_H}{4D_1+3D_2}$, *and* $\pi_U^*=\frac{\delta(D_1+D_2)(4D_1+3D_2-3K_H)K_H}{(D_1+D_2)(4D_1+3D_2)}+\frac{4D_1^2K_L+3D_2(D_2-2K_H-K_L)K_L-D_1(K_H^2-7D_2K_L+8K_HK_L+4K_L^2)}{(D_1+D_2)(4D_1+3D_2)}$.

(iii) *If* $K_H>\max\{\frac{4D_1^2+3D_1D_2-(8D_1+6D_2)K_L}{4D_1},\frac{D_1+D_2}{2}-K_L\}$ *and* $K_H<\frac{4D_1+3D_2}{8}$, *then* $\theta_{1H}^*=\frac{4D_1+3D_2-2K_H}{4D_1+3D_2}$, $\theta_{1L}^*=\theta_{2L}^*=\frac{1}{2}$, $\theta_U^*=\frac{4D_1+3D_2-4K_H}{4D_1+3D_2}$, $\theta_{2H}^*=\frac{4D_1+3D_2-3K_H}{4D_1+3D_2}$, *and* $\pi_U^*=\frac{4D_1^2+D_1(7D_2+16(\delta-1)K_H)+3(D_2^2+4(\delta-1)D_2K_H-4(\delta-1)K_H^2)}{4(4D_1+3D_2)}$.

(iv) *If* $\frac{4D_1+3D_2}{8}<K_H<\min\{\frac{D_1+D_2}{2},\frac{-D_1^3+(16D_1^2+21D_1D_2+9D_2^2)K_L}{6D_1^2+6D_1D_2}\}$, *then* $\theta_{1H}^*=\frac{4D_1(\delta-1)+3\delta D_2+2K_H-2\delta K_H}{4\delta D_1+D_2+3\delta D_2-4D_1}$, $\theta_U^*=\theta_{1L}^*=\frac{4D_1(\delta-1)+3\delta D_2+4K_H-4\delta K_H-D_2}{4\delta D_1+D_2+3\delta D_2-4D_1}$, $\theta_{2H}^*=\frac{D_1(4\delta-2)+(3\delta+1)(D_2-K_H)}{4\delta D_1+D_2+3\delta D_2-4D_1}$, $\theta_{2L}^*=\frac{1}{2}$, *and* $\pi_U^*=\frac{(D_2-2K_H)^2+4\delta^2(4D_1+3D_2-3K_H)K_H+\delta(3D_2^2+4D_1(D_2-4K_H)-8D_2K_H+8K_H^2)}{4(4\delta D_1+D_2+3\delta D_2-4D_1)}$.

Overlap Characterization

Cases (ii) and (iii): $\Delta\pi^{ii-iii}<0$, *so, case (iii) dominates case (ii).*

Cases (ii) and (iv): The coefficient of $\delta-1$ *of the numerator of* $\Delta\pi^{ii-iv}$ *is a quadratic function of* K_H, *its discriminator and roots are* $\Delta^U=16D_1\left(D_1+D_2\right)\left(4D_1+3D_2\right)^2\left(D_2-8K_L\right)^2>0$, $K_H^{1U}=\frac{(4D_1+3D_2)(3D_2(D_1+D_2)-2(4D_1+3D_2)K_L)-\sqrt{D_1(D_1+D_2)(4D_1+3D_2)^2(D_2-8K_L)^2}}{2D_2(8D_1+9D_2)}<0$, *and* $K_H^{2U}=\frac{(4D_1+3D_2)(3D_2(D_1+D_2)-2(4D_1+3D_2)K_L)+\sqrt{D_1(D_1+D_2)(4D_1+3D_2)^2(D_2-8K_L)^2}}{2D_2(8D_1+9D_2)}<\frac{4D_1+3D_2}{8}$, *respectively. Recall that the effective domian of the overlap is given by* $\frac{4D_1+3D_2}{8}<K_H<\min\{\frac{D_1+D_2}{2},D_1+D_2-K_L\}$. *So, the numerator decreases with* δ *and the maximum value obtained at* $\delta=1$ *is negative. So, case (iv) dominates case (ii).*

Define $K_1^U = \frac{4D_1D_2+3D_2^2}{2D_1^2+D_1D_2}K_L$, $K_2^U = \frac{4D_1^2+3D_1D_2-(8D_1+6D_2)K_L}{4D_1}$, *and* $K_3^U = \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3}{8D_1^2+18D_1D_2+18D_2^2}$, *then the region* $\frac{D_1+D_2}{2} < K_H < \min\{K_3^U, K_L\}$ *in Figure 3.4 is not applicable.*

Proof 28 *Proof of Theorem* 3.3.2.2 . *By comparing Lemmas 3.3.1 and 3.3.2.1, we have*

$$\Delta\pi^{U-P} = \begin{cases} \frac{(\delta^2-\delta)D_1}{4(3\delta+1)} > 0 & if \frac{(4+16t+21t^2+9t^3)D_1}{8+18t+18t^2} < K_H < K_L, \\ \frac{(\delta-1)D_1(4(-5+\delta)K_H^2+16(D_1+D_2)K_H-(4D_1^2+7D_1D_2+3D_2^2))}{4(D_1+D_2)(4(\delta-1)D_1+(3\delta+1)D_2)} > 0 & if \frac{4D_1+3D_2}{8} < K_H < \min\{\frac{D_1+D_2}{2}, K_L\}, \\ \frac{(\delta-1)D_1K_H^2}{(D_1+D_2)(4D_1+3D_2)} > 0 & if \frac{D_1+D_2}{2} - K_L < K_H < \min\{K_L, \frac{4D_1+3D_2}{8}\}, \\ \frac{(\delta-1)D_1K_H^2}{(D_1+D_2)(4D_1+3D_2)} > 0 & if K_H < \min\{K_L, \frac{D_1+D_2}{2} - K_L\}. \end{cases}$$

Proof 29 *Proof of Corollary* 3.3.2.2.

Table B.1: Comparisons Between Scenarios U and P

Range of K_H	Volume gap	Capacity offering	H in 1	L in 1	H in 2	L in 2
$[0, \frac{4D_1+3D_2}{8}]$	$\sum_{j\in\{H,L\}}^{i\in\{1,2\}} D_{ij}^{U-P} = 0$	$(H^P, L^P; H^{U+P}, L^P)$ $(H^P, L^P; H^P, L^P)$	$\Delta p_{1H}^{U-P} > 0$ $\Delta D_{1H}^{U-P} < 0$	$\Delta p_{1L}^{U-P} = 0$ $\Delta D_{1L}^{U-P} < 0$	$\Delta p_{2H}^{U-P} > 0$ $\Delta D_{2H}^{U-P} > 0$	$\Delta p_{2L}^{U-P} = 0$ $\Delta D_{2L}^{U-P} > 0$
$[\frac{4D_1+3D_2}{8}, \frac{D_1+D_2}{2}]$	$\sum_{j\in\{H,L\}}^{i\in\{1,2\}} D_{ij}^{U-P} > 0$	$(H^P, L^\varnothing; H^{U+P}, L^P)$ $(H^P, L^P; H^P, L^P)$	$\Delta p_{1H}^{U-P} > 0$ $\Delta D_{1H}^{U-P} < 0$	$\Delta p_{1L}^{U-P} < 0$ $\Delta D_{1L}^{U-P} < 0$	$\Delta p_{2H}^{U-P} > 0$ $\Delta D_{2H}^{U-P} > 0$	$\Delta p_{2L}^{U-P} = 0$ $\Delta D_{2L}^{U-P} > 0$
$[\frac{D_1+D_2}{2}, K_L]$	$\sum_{j\in\{H,L\}}^{i\in\{1,2\}} D_{ij}^{U-P} > 0$	$(H^P, L^\varnothing; H^{U+P}, L^\varnothing)$ $(H^P, L^\varnothing; H^P, L^\varnothing)$	$\Delta p_{1H}^{U-P} > 0$ $\Delta D_{1H}^{U-P} < 0$	$\Delta p_{1L}^{U-P} < 0$ $\Delta D_{1L}^{U-P} = 0$	$\Delta p_{2H}^{U-P} = 0$ $\Delta D_{2H}^{U-P} > 0$	$\Delta p_{2L}^{U-P} = 0$ $\Delta D_{2L}^{U-P} = 0$

If $K_H > \frac{(4+16t+21t^2+9t^3)D_1}{8+18t+18t^2}$ *&* $K_H \leq K_L$, *then* $\Delta\pi_H^{U-P} = \frac{(\delta-1)\delta D_1}{12\delta+4} > 0$, $\Delta\pi_L^{U-P} = 0$.

If $\frac{4D_1+3D_2}{8} < K_H < \min\{\frac{D_1+D_2}{2}, K_L\}$, *then* $\Delta\pi_H^{U-P} = \frac{(\delta-1)D_1K_H(2(D_1+D_2)+(\delta-5)K_H)}{(D_1+D_2)(4(\delta-1)D_1+(3\delta+1)D_2)} > 0$, $\Delta\pi_L^{U-P} = -\frac{(\delta-1)D_1(4D_1+3D_2-8K_H)}{4(4(\delta-1)D_1+(3\delta+1)D_2)} > 0$.

If $\frac{D_1+D_2}{2} - K_L < K_H < \min\{K_L, \frac{4D_1+3D_2}{8}\}$, *then* $\Delta\pi_H^{U-P} = \frac{(\delta-1)D_1K_H^2}{(D_1+D_2)(4D_1+3D_2)} > 0$, $\Delta\pi_L^{U-P} = 0$.

If $K_H < \min\{K_L, \frac{D_1+D_2}{2} - K_L\}$, *then* $\Delta\pi_H^{U-P} = \frac{(\delta-1)D_1K_H^2}{(D_1+D_2)(4D_1+3D_2)} > 0$, $\Delta\pi_L^{U-P} = 0$.

Proof 30 *Proof of Lemma* 3.3.3.1. *The optimal solutions satisfy conditions* (*a*), (*b*), (*c*), *and* (*d*) *mentioned above.*

(*a*) *Pricing decision.*

If $K_H - D_1(1-\theta_{1L}) + K_L \leq D_2(1 - \frac{p_O}{\phi\delta+1-\phi})$, then π^*_{2O} increases with p_O, thus, the maximum π^*_{2O} equals $\pi^*_{2O}|_{p^*_O=(1-\frac{K_H-D_1(1-\theta_{1L})+K_L}{D_2})(\phi\delta+1-\phi)} = (K_H - D_1(1-\theta_{1L}) + K_L - \frac{(K_H-D_1(1-\theta_{1L})+K_L)^2}{D_2})(\phi\delta+1-\phi) - F_O$. If $K_H - D_1(1-\theta_{1L}) + K_L > D_2(1-\frac{p_O}{\phi\delta+1-\phi})$, then π^*_{2O} is concave with p_O and the maximum π^*_{2O} equals $\pi^*_{2O}|_{p^*_O=\frac{1}{2}} = \frac{D_2}{4}(\phi\delta+1-\phi) - F_O$.

(b) *Demand fulfillment decision.*

The seller's optimal profit increases with ϕ.

If $D_2 \geq K_H + K_L - D_1(1-\theta_{1L})$, then all capacity are sold out, and $\phi = \frac{K_H-D_1(1-\theta_U)}{K_H+K_L-D_1(1-\theta_{1L})}$ (resp., $\phi = \frac{K_H-D_1(1-\theta_{1H})-S}{K_H+K_L-D_1(1-\theta_{1L})}$).

If $D_2 < K_H + K_L - D_1(1-\theta_{1L})$, then for random allocation policy, $\phi = \frac{1}{2}$. For policy of high-quality capacity with priority (i.e., fulfill demand with high-quality capacity first), $\phi = \min\{\frac{K_H-D_1(1-\theta_U)}{D_2}, 1\}$ (resp., $\phi = \min\{\frac{K_H-D_1(1-\theta_{1H})-S}{D_2}, 1\}$). For policy of low-quality capacity with priority, $1-\phi = \min\{\frac{K_L-D_1(\theta_U-\theta_{1L})}{D_2}, 1\}$ (resp., $1-\phi = \min\{\frac{K_L-D_1(\theta_{1H}-\theta_{1L})+S}{D_2}, 1\}$). Because $\frac{K_H-D_1(1-\theta_U)}{D_2} > 1 - \frac{K_L-D_1(\theta_U-\theta_{1L})}{D_2}$ (resp., $\frac{K_H-D_1(1-\theta_{1H})-S}{D_2} > 1 - \frac{K_L-D_1(\theta_{1H}-\theta_{1L})+S}{D_2}$), hence, $\phi = \frac{K_H-D_1(1-\theta_U)}{D_2}$ (resp., $\phi = \frac{K_H-D_1(1-\theta_{1H})-S}{D_2}$) if $K_H - D_1(1-\theta_U) > \frac{D_2}{2}$ (resp., $K_H - D_1(1-\theta_{1H}) - S > \frac{D_2}{2}$) or $\phi = \frac{1}{2}$ otherwise.

Proof 31 *Proof of Lemma 3.3.3.2.*

(i) *If $\theta_{1L} \leq 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, then $\pi_O = D_1(\delta-1)(\theta_{1H}-\theta^2_{1H}) + D_1(\theta_{1L}-\theta^2_{1L}) + (K_H\delta + K_L - D_1(1-\theta_{1L}) - D_1(\delta-1)(1-\theta_{1H}))(1-\frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2}) - F_O$. The Hessian matrix of π_O is negative definite. By deriving the FOCs of π_O w.r.t θ_{1H} and θ_{1L}, we have $\theta^*_{1H} = \frac{-D_1^2\delta+D_1K_H\delta+D_1^2+4D_1D_2+4D_2^2-D_1K_H-2D_2K_H-2D_2K_L}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$ and $\theta^*_{1L} = \frac{-D_1^2\delta+D_1K_H\delta+D_1^2+4D_1D_2+4D_2^2-D_1K_H-2D_2K_H-4D_2K_L-2D_2K_H\delta+D_1K_L\delta-D_1K_L}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$. Consequently, $\theta^*_O = 2\theta^*_{1H} - 1 = \frac{(2D_1K_H-D_1^2)(\delta-1)+4D_2(D_1+D_2-(K_H+K_L))}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$, $\phi^* = \frac{(-D_1-2D_2)K_H+D_1K_L}{(\delta-1)D_1K_H-2D_2(K_H+K_L)}$, where $0 \leq \phi^* = \frac{K_H-D_1(1-\theta^*_{1H})}{K_H+K_L-D_1(1-\theta^*_{1L})} \leq 1$, and $\pi^*_O = \frac{\delta^2D_1K_H(D_1-K_H)}{-4(D_1+D_2)D_2+D_1^2(\delta-1)}$*

$+ \frac{\delta(D_1^2(-K_H+K_L)+4D_2K_H(-D_1-D_2+K_H+K_L)-D_1(-K_H^2+2K_HK_L+K_L^2))}{-4(D_1+D_2)D_2+D_1^2(\delta-1)}$

$+ \frac{K_L(-D_1^2+4D_2(-D_1-D_2+K_H+K_L)+D_1(2K_H+K_L))}{-4(D_1+D_2)D_2+D_1^2(\delta-1)} - F_O$ *provided that* $f_1^O(\delta) = (4D_1K_H -$

$D_1^2)(\delta-1)+4D_2(D_1+D_2-2(K_H+K_L))\geq 0$, $f_2^O(\delta)=(-D_1K_L+2D_2K_H)\delta+D_1K_L+2D_2K_L\geq 0$, $f_3^O(\delta)=-D_1K_H(\delta-1)+2D_2(K_H+K_L)\geq 0$, $f_4^O(\delta)=(-D_1^2+D_1K_H-2D_2K_H+D_1K_L)(\delta-1)+4D_2(D_1+D_2-(K_H+K_L))\geq 0$, $f_5^O(\delta)=2D_2K_H+D_1(K_H-K_L)\geq 0$, $f_6^O(\delta)=-D_1K_H\delta+D_1K_L+2D_2K_L\geq 0$, *and* $f_7^O(\delta)=(2D_1K_H-D_1^2)(\delta-1)+4D_2(D_1+D_2-(K_H+K_L))\geq 0$. *Note that* $f_1^O(\delta)\geq 0$ *if* $\frac{2D_1}{D_1+3D_2}K_L-\frac{D_1(D_1+D_2)}{4D_1+12D_2}\leq K_H\leq\frac{D_1+D_2}{2}-K_L$ *provided that* $D_1+2D_2-4K_L\geq 0$, $f_2^O(\delta)\geq 0$ *if* $K_H\geq\frac{D_1^2+3D_1D_2}{2D_1^2+6D_1D_2+6D_2^2}K_L$, $f_3^O(\delta)\geq 0$ *if* $K_H\leq\frac{2D_1}{D_1+3D_2}K_L$, $f_4^O(\delta)\geq 0$ *if* $K_H\leq\min\{\frac{(3D_1D_2-D_1^2)K_L+D_1^3+D_1^2D_2}{D_1^2+3D_1D_2+6D_2^2}, D_1+D_2-K_L\}$, $f_5^O(\delta)\geq 0$ *if* $K_H\geq\frac{D_1}{D_1+2D_2}K_L$, $f_6^O(\delta)\geq 0$ *if* $K_H\leq\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L$, *and* $f_7^O(\delta)\geq 0$ *if* $\frac{2D_1}{D_1+3D_2}K_L-\frac{D_1^2+D_1D_2}{2D_1+6D_2}\leq K_H\leq D_1+D_2-K_L$ *provided that* $D_1+2D_2-2K_L\geq 0$. *So, the effective domain is given by* $\frac{D_1}{2D_2+D_1}K_L<K_H\leq\min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L,\frac{D_1+D_2}{2}-K_L\}$.

Boundary solution.

If $\theta_{1L}^*>1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}$, *or equivalently,* $(4D_1K_H-D_1^2)(\delta-1)+4D_2(D_1+D_2-2(K_H+K_L))<0$, *then* $\theta_{1H}^*=\frac{3}{4}$, $\theta_{1L}^*=1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}$, $\theta_O^*=\frac{1}{2}$, $\phi^*=\frac{4K_H-D_1}{2D_2}$, *and* $\pi_O^*=\frac{(D_1^2+8D_1K_H)\delta-D_1^2-4D_1D_2+8D_1K_H+16D_1K_L-4(D_2-2(K_H+K_L))^2}{16D_1}-F_O$ *provided that* $\frac{D_1}{4}<K_H<\min\{\frac{8D_1K_L-D_1(D_1+D_2)}{4D_1+12D_2},\frac{2D_1+D_2}{2}-K_L,\frac{D_1+2D_2}{4}\}$.

(ii) *If* $1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}<\theta_{1L}\leq 1+\frac{D_2}{D_1}-\frac{K_H+K_L}{D_1}$, *then* $\pi_O=D_1(\theta_{1L}-\theta_{1L}^2)+D_1(\delta-1)(\theta_{1H}-\theta_{1H}^2)+\frac{D_2}{4}\frac{K_H-D_1(1-\theta_{1H})}{K_H-D_1(1-\theta_{1L})+K_L}(\delta-1)+\frac{D_2}{4}$. *Because* $\frac{\partial^3\pi_O}{\partial\theta_{1L}^3}<0$, *and* $\left.\frac{\partial^2\pi_O}{\partial\theta_{1L}^2}\right|_{\theta_{1L}=1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1},\theta_{1H}=\frac{3}{4}}=\frac{D_1\left(-(\delta-1)D_1(D_1-4K_H)-2D_2^2\right)}{D_2^2}<0$ *if* $K_H<\frac{3D_1^2+5D_1D_2}{12(D_1+D_2)}$. *So, the optimal solutions are obtained by solving the FOCs w.r.t* θ_{1H} *and* θ_{1L} *simultaneously:*

$$4D_2(K_H+K_L)(2\theta_{1H}^*-1)-2D_1D_2(2\theta_{1H}^*-1)-D_2^2-8D_1(\delta-1)(K_H-D_1(1-\theta_{1H}^*))(2\theta_{1H}^*-1)^3=0,$$

$$\theta_{1H}^*=\frac{1}{2}+\frac{D_2}{8(K_H+K_L-D_1(1-\theta_{1L}^*))}.$$

Note that $\frac{\partial\pi_O}{\partial\theta_{1L}}<0$ *if* $\theta_{1L}\geq\frac{1}{2}$. *So, if* $\theta_{1L}=1+\frac{D_2}{2D_1}-\frac{K_H}{D_1}\leq\frac{1}{2}$ *(or equivalently,* $D_1+D_2\geq 2(K_H+K_L)$*), there are no optimal solutions. Hence, the effective domain is given by* $\frac{D_1+D_2}{2}-K_L<K_H<\frac{3D_1^2+5D_1D_2}{12(D_1+D_2)}$. *Because* $\theta_O^*=\frac{1}{2}$, $\theta_{1H}^*\neq\theta_{1L}^*$, *and* $\theta_{1H}^*\neq 1$, *so the optimal capacity offering is given by* $(H^P,L^P;H^O,L^O)$.

Boundary solution.

The optimal interior solution is a convex combination of the boundary solutions.

(a) $\theta_{1L}^* = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = \frac{3}{4}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{4K_H-D_1}{2D_2}$, *and* $\pi_O^* = \frac{(\delta-1)D_1^2+D_1(-4D_2+8(\delta+1)K_H+16K_L)-4(D_2-2(K_H+K_L))^2}{16D_1} - F_O$. *The effective domain is given by* $\max\{\frac{D_1}{4}, \frac{D_1+2D_2}{4} - K_L\} \leq K_H \leq \min\{\frac{D_1+2D_2}{4}, \frac{2D_1+D_2}{2} - K_L\}$.

(b) $\theta_{1L}^* = 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = \frac{5}{8}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{8K_H-3D_1}{8D_2}$, *and* $\pi_O^* = \frac{9(\delta-1)D_1^2+16D_1(-3D_2+(\delta+3)K_H+4K_L)-64(-D_2+K_H+K_L)^2}{64D_1} - F_O$. *The effective domain is given by* $\max\{\frac{3D_1}{8}, \frac{3D_1+8D_2}{8} - K_L\} \leq K_H \leq \min\{\frac{3D_1+8D_2}{8}, D_1 + D_2 - K_L\}$.

(iii) *If* $D_2 < K_H - D_1(1-\theta_{1L}) + K_L, K_H - D_1(1-\theta_{1H}) > \frac{D_2}{2}$, *then the Hessian matrix of* π_O *is negative definite, hence,* $\theta_{1H}^* = \frac{5}{8}$, $\theta_{1L}^* = \frac{1}{2}$, $\phi^* = \frac{8K_H-3D_1}{8D_2}$, *and* $\pi_O^* = \frac{(9D_1+16K_H)\delta-16K_H+7D_1+16D_2}{64} - F_O$ *provided that* $2(K_H+K_L) > D_1 + 2D_2$, $8K_L \geq D_1$ *and* $3D_1 + 4D_2 \leq 8K_H \leq 3D_1 + 8D_2$.

(iv) *Otherwise,* $\theta_{1H}^* = \theta_{1L}^* = \theta_O^* = \frac{1}{2}$, $\phi^* = \frac{1}{2}$, *and* $\pi_O^* = \frac{2D_1\delta+D_2(\delta+1)}{8} - F_O$ *provided that* $D_1 \leq 2K_H \leq D_1 + D_2$ *and* $2(K_H+K_L) > D_1 + 2D_2$.

To summarize,

(i) *If* $\frac{D_1}{2D_2+D_1}K_L < K_H \leq \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2} - K_L\}$, *then*

$\theta_O^* = \frac{(2D_1K_H-D_1^2)(\delta-1)+4D_2(D_1+D_2-(K_H+K_L))}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$, $\theta_{1H}^* = \frac{-D_1^2\delta+D_1K_H\delta+D_1^2+4D_1D_2+4D_2^2-D_1K_H-2D_2K_H-2D_2K_L}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$,

$\theta_{1L}^* = \frac{-D_1^2\delta+D_1K_H\delta+D_1^2+4D_1D_2+4D_2^2-D_1K_H-2D_2K_H-4D_2K_L-2D_2K_H\delta+D_1K_L\delta-D_1K_L}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$,

$\pi_O^* = \frac{\delta^2D_1K_H(D_1-K_H)+\delta(D_1^2(-K_H+K_L)+4D_2K_H(-D_2+K_H+K_L)-D_1(4D_2K_H-K_H^2+2K_HK_L+K_L^2))}{-4(D_1+D_2)D_2+D_1^2(\delta-1)} + \frac{K_L(-D_1^2+4D_2(-D_2+K_H+K_L)+D_1(-4D_2+2K_H+K_L))}{-4(D_1+D_2)D_2+D_1^2(\delta-1)} - F_O$, *and* $\phi^* = \frac{(-D_1-2D_2)K_H+D_1K_L}{(\delta-1)D_1K_H-2D_2(K_H+K_L)}$.

(ii) *If* $\frac{D_1}{4} < K_H < \min\{\frac{8D_1K_L-D_1(D_1+D_2)}{4D_1+12D_2}, \frac{2D_1+D_2}{2} - K_L, \frac{D_1+2D_2}{4}\}$, *then* $\theta_{1H}^* = \frac{3}{4}$, $\theta_{1L}^* = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta_O^* = \frac{1}{2}$, $\pi_O^* = \frac{(D_1^2+8D_1K_H)\delta-D_1^2-4D_1D_2+8D_1K_H+16D_1K_L-4(D_2-2(K_H+K_L))^2}{16D_1} - F_O$, *and* $\phi^* = \frac{4K_H-D_1}{2D_2}$.

(iii) *If* $\max\{\frac{D_1+2D_2}{2} - K_L, \frac{3D_1+4D_2}{8}\} < K_H \leq \frac{3D_1+8D_2}{8}$ *and* $K_L \geq \frac{D_1}{8}$, *then* $\theta_{1H}^* = \frac{5}{8}$, $\theta_{1L}^* = \frac{1}{2}$, $\theta_O^* = \frac{1}{2}$, $\pi_O^* = \frac{(9D_1+16K_H)\delta-16K_H+7D_1+16D_2}{64} - F_O$, *and* $\phi^* = \frac{8K_H-3D_1}{8D_2}$.

(iv) If $\max\{\frac{D_1+2D_2}{2} - K_L, \frac{D_1}{2}\} < K_H \le \frac{D_1+D_2}{2}$*, then* $\theta^*_{1H} = \theta^*_{1L} = \theta^*_O = \frac{1}{2}$*,* $\pi^*_O = \frac{2D_1\delta + D_2(\delta+1)}{8} - F_O$*, and* $\phi^* = \frac{1}{2}$*.*

Overlap Characterization

Cases (iii) and (iv): case (iii) dominates case (iv) if $K_H > \frac{7D_1+8D_2}{16}$*.*

To simplify notation, we have $K^O_1 = \frac{D_1}{2D_2+D_1}K_L$*,* $K^O_2 = \frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L$*, and* $K^O_3 = \frac{8D_1K_L - D_1(D_1+D_2)}{4D_1+12D_2}$*. Hence, the region* $K_H + K_L \le \frac{D_1+D_2}{2}$ *except Area (I) and region* $K_H > \frac{3D_1+8D_2}{8}$ *in Figure 3.5 are not applicable.*

Proof 32 *Proof of Theorem* 3.3.3.2*. We compare the seller's profit based on Figure 3.3 and 3.5.*

(i) $K_H \le \min\{K_L, \frac{D_1(1+t)}{2} - K_L\}$.

If $\frac{D_1}{D_1+2D_2}K_L < K_H < \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2} - K_L\}$*, then*

$$\Delta\pi^{O-P} = \pi^*_O - \pi^*_P = \frac{D_2(\delta-1)(D_1\delta K_H^2 - 2(D_1+2D_2)K_H + D_1K_L^2)}{(D_1+D_2)(4D_1D_2 + 4D_2^2 - (\delta-1)D_1^2)} - F_O < 0,$$

(ii) $\frac{D_1(1+t)}{2} - K_L < K_H \le \min\{K_L, \frac{D_1(1+t)}{2}\}$

(a) If $\frac{D_1}{4} < K_H < \min\{\frac{D_1+2D_2}{4}, K_L\}$*, then*

$$\Delta\pi^{O-P} = \frac{(\delta-5)D_1^3 + ((\delta-13)D_2 - 8(\delta-3)K_H + 16K_L)D_1^2 - 4D_2(D_2 - 2(K_H+K_L))^2}{16D_1(D_1+D_2)}$$
$$-\frac{4(3D_2^2 + 2((\delta-5)K_H - 4K_L)D_2 - 4(\delta-2)K_H^2 + 8K_HK_L + 4K_L^2)D_1}{16D_1(D_1+D_2)} - F_O < 0.$$

(b) If $\frac{D_1}{2} < K_H < \min\{\frac{7D_1+8D_2}{16}, K_L\}$*, then*

$$\Delta\pi^{O-P} = \frac{(\delta-1)(2D_1^2 + 3D_1D_2 + D_2^2 - 8D_1K_H - 8D_2K_H + 8K_H^2)}{8(D_1+D_2)} - F_O < 0.$$

(c) If $\frac{7D_1+8D_2}{16} < K_H < \min\{\frac{D_1+D_2}{2}, K_L\}$*, then*

$$\Delta\pi^{O-P} = \frac{(\delta-1)(9D_1^2 + 9D_1D_2 - 48D_1K_H - 48D_2K_H + 64K_H^2)}{64(D_1+D_2)} - F_O < 0.$$

(iii) $\frac{D_1(1+t)}{2} < K_H \le K_L$.

$$\Delta\pi^{O-P} = \frac{(\delta-1)(16K_H - (7D_1+16D_2))}{64} - F_O < 0.$$

Proof 33 *Proof of Corollary 3.3.3.2. The comparison between Scenarios P and O are as follows.*

(i) If $K_H > \frac{D_1+D_2}{2}$ *&* $K_H \leq K_L$,

$$\Delta\pi_{1H}^{O-P} = -\frac{1}{64}(\delta+3)D_1 < 0, \quad \Delta\pi_{1L}^{O-P} = \frac{D_1}{16} > 0.$$

$$\Delta\pi_{2H}^{O-P} = \frac{(3D_1+8D_2-8K_H)\left(3(\delta-1)D_1-8\delta\left(D_2+K_H\right)+8K_H\right)}{256D_2} > 0,$$

$$\Delta\pi_{2L}^{O-P} = \frac{(3D_1+8D_2-8K_H)\left((\delta-1)\left(8K_H-3D_1\right)+8D_2\right)}{256D_2} > 0,$$

(ii) If $\frac{7D_1+8D_2}{16} < K_H < \min\{\frac{D_1+D_2}{2}, K_L\}$,

$$\Delta\pi_{1H}^{O-P} = \frac{D_1\left(3D_1+3D_2-8K_H\right)\left((5\delta-1)D_1+(5\delta-1)D_2-8(\delta-1)K_H\right)}{64\left(D_1+D_2\right)^2} < 0,$$

$$\Delta\pi_{1L}^{O-P} = \frac{1}{16}D_1\left(\frac{8K_H}{D_1+D_2}-3\right) > 0,$$

$$\Delta\pi_{2H}^{O-P} = \frac{D_2K_H\left((1-2\delta)D_1+(1-2\delta)D_2+2(\delta-1)K_H\right)}{2\left(D_1+D_2\right)^2} + \frac{(8K_H-3D_1)\left((\delta-1)\left(8K_H-3D_1\right)+8D_2\right)}{256D_2} < 0,$$

$$\Delta\pi_{2L}^{O-P} = \frac{(3D_1+8D_2-8K_H)\left((\delta-1)\left(8K_H-3D_1\right)+8D_2\right)}{256D_2} + \frac{1}{4}D_2\left(\frac{2K_H}{D_1+D_2}-1\right) > 0 if D_1 < 4D_2.$$

(iii) If $\min\{\frac{D_1+2D_2}{2}-K_L, \frac{D_1}{2}\} < K_H < \min\{K_L, \frac{7D_1+8D_2}{16}\}$,

$$\Delta\pi_{1H}^{O-P} = \frac{D_1\left(D_1+D_2-2K_H\right)\left(\delta\left(D_1+D_2\right)-2(\delta-1)K_H\right)}{4\left(D_1+D_2\right)^2} > 0, \quad \Delta\pi_{1L}^{O-P} = \frac{1}{4}D_1\left(\frac{2K_H}{D_1+D_2}-1\right) < 0.$$

$$\Delta\pi_{2H}^{O-P} = \frac{1}{16}D_2\left(\delta+\frac{8K_H\left((1-2\delta)\left(D_1+D_2\right)+2(\delta-1)K_H\right)}{\left(D_1+D_2\right)^2}+1\right) < 0,$$

$$\Delta\pi_{2L}^{O-P} = \frac{1}{16}D_2\left(\delta+\frac{8K_H}{D_1+D_2}-3\right) > 0.$$

(iv) If $\min\{\frac{D_1+D_2}{2}-K_L, \frac{D_1}{4}\} < K_H < \min\{\frac{8D_1K_L-D_1(D_1+D_2)}{4D_1+12D_2}, \frac{2D_1+D_2}{2}-K_L, \frac{D_1+2D_2}{4}\}$,

$$\Delta\pi_{1H}^{O-P} = \frac{1}{16}\left(D_1\left(3\delta+\frac{8K_H\left((1-2\delta)\left(D_1+D_2\right)+2(\delta-1)K_H\right)}{\left(D_1+D_2\right)^2}+1\right)+2\left(D_2-2\left(K_H+K_L\right)\right)\right) < 0,$$

$$\Delta\pi_{1L}^{O-P} = \frac{1}{8}\left(-\frac{2\left(D_2-2\left(K_H+K_L\right)\right)^2}{D_1}-5\left(D_2-2\left(K_H+K_L\right)\right)+D_1\left(\frac{4K_H}{D_1+D_2}-4\right)\right),$$

$$\Delta\pi_{2H}^{O-P} = \frac{(D_1-4K_H)\left((\delta-1)D_1-2D_2-4(\delta-1)K_H\right)}{16D_2} - \frac{D_2K_H\left((2\delta-1)D_1+(2\delta-1)D_2-2(\delta-1)K_H\right)}{2\left(D_1+D_2\right)^2},$$

$$\Delta\pi_{2L}^{O-P} = \frac{1}{16}\left(\frac{(D_1+2D_2-4K_H)\left((1-\delta)D_1+2D_2+4(\delta-1)K_H\right)}{D_2}+4D_2\left(\frac{2K_H}{D_1+D_2}-1\right)\right).$$

$\Delta\pi_{1L}^{O-P} < 0$ *if* $K_H < \frac{7(D_1+D_2)}{32}$. *Otherwise,* $\Delta\pi_{1L}^{O-P} > 0$ *if*

Table B.2: The optimal price and transaction volume comparison under O and P

p and D	Case	Gap (O-P)	Relationship
Δp^*_{1H}	1	$\frac{\delta-1}{8}$	>0
	2	$\frac{\delta-1}{8}(\frac{8K_H}{D_1+D_2}-3)$	>0
	3	$-\frac{\delta}{2}+\frac{(\delta-1)K_H}{D_1+D_2}+\frac{1}{2}$	>0
	4	$\frac{1}{4}\left(-\delta+\frac{4(\delta-1)K_H}{D_1+D_2}+\frac{2(D_2-2(K_H+K_L))}{D_1}+3\right)$	>0 if $K_L<-\frac{1}{4}(\delta-3)D_1+\frac{((\delta-2)D_1-D_2)K_H}{D_1+D_2}+\frac{D_2}{2}$
	5	$-\frac{(\delta-1)D_2(\delta D_1K_H-(D_1+2D_2)K_L)}{(D_1+D_2)\left((\delta-1)D_1^2-4D_2D_1-4D_2^2\right)}$	<0
ΔD^*_{1H}	1	$-\frac{D_1}{8}$	<0
	2	$\frac{1}{8}D_1\left(3-\frac{8K_H}{D_1+D_2}\right)$	<0
	3	$\frac{D_1}{2}(1-\frac{2K_H}{D_1+D_2})$	<0
	4	$\frac{1}{4}D_1\left(1-\frac{4K_H}{D_1+D_2}\right)$	<0 if $K_H>\frac{D_1+D_2}{4}$
	5	$\frac{D_1D_2(D_1((\delta+1)K_H-2K_L)+2D_2(K_H-K_L))}{(D_1+D_2)\left((\delta-1)D_1^2-4D_2D_1-4D_2^2\right)}$	<0 if $\delta>\frac{(-D_1-2D_2)K_H+2(D_1+D_2)K_L}{D_1K_H}$
Δp^*_{1L}	1	0	
	2	0	
	3	0	
	4	$\frac{D_1+D_2-2(K_H+K_L)}{2D_1}$	<0
	5	$\frac{(\delta-1)D_2(D_1(K_H-K_L)+2D_2K_H)}{(D_1+D_2)\left((\delta-1)D_1^2-4D_2D_1-4D_2^2\right)}$	<0
ΔD^*_{1L}	1	$\frac{D_1}{8}$	>0
	2	$\frac{D_1}{8}(\frac{8K_H}{D_1+D_2}-3)$	>0
	3	$\frac{D_1}{2}(\frac{2K_H}{D_1+D_2}-1)$	<0
	4	$D_1\left(\frac{K_H}{D_1+D_2}-\frac{3}{4}\right)-\frac{D_2}{2}+K_H+K_L$	>0 if $K_H>\frac{-4D_1K_L-4D_2K_L+3D_1^2+5D_2D_1+2D_2^2}{4(2D_1+D_2)}$
	5	$\frac{D_1D_2(((\delta+1)D_1+2D_2)K_L-2\delta(D_1+D_2)K_H)}{(D_1+D_2)\left((\delta-1)D_1^2-4D_2D_1-4D_2^2\right)}$	>0 if $\delta>\frac{(D_1+2D_2)K_L}{2(D_1+D_2)K_H-D_1K_L}$
$p^*_O-p^*_{2H}$	1	$-\frac{(\delta-1)(3D_1+8D_2-8K_H)}{16D_2}$	<0
	2	$-\frac{(\delta-1)\left(D_1(19D_2-8K_H)+8D_2(2D_2-3K_H)+3D_1^2\right)}{16D_2(D_1+D_2)}$	<0
	3	$\frac{1}{4}(\delta-1)\left(\frac{4K_H}{D_1+D_2}-3\right)$	<0
	4	$-\frac{(\delta-1)\left(D_1(5D_2-4K_H)+4D_2(D_2-2K_H)+D_1^2\right)}{4D_2(D_1+D_2)}$	<0
	5	$-\delta+\frac{\delta K_H+K_L}{D_1+D_2}+p^*_O$	<0
ΔD^*_{2H}	1	$\frac{1}{16}(-3D_1-8D_2+8K_H)$	<0
	2	$\frac{1}{2}\left(1-\frac{2D_2}{D_1+D_2}\right)K_H-\frac{3D_1}{16}$	<0
	3	$\frac{1}{4}D_2\left(1-\frac{4K_H}{D_1+D_2}\right)$	<0
	4	$\frac{1}{4}D_1\left(\frac{4K_H}{D_1+D_2}-1\right)$	>0 if $K_H>\frac{D_1+D_2}{4}$
	5	$D_2\left(\frac{2(D_1(K_H-K_L)+2D_2K_H)}{(1-\delta)D_1^2+4D_2(D_1+D_2)}-\frac{K_H}{D_1+D_2}\right)$	<0 if $\delta<\frac{(-D_1-2D_2)K_H+2(D_1+D_2)K_L}{D_1K_H}$
$p^*_O-p^*_{2L}$	1	$-\frac{(\delta-1)(3D_1-8K_H)}{16D_2}$	>0
	2	$-\frac{(\delta-1)(3D_1-8K_H)}{16D_2}$	>0
	3	$\frac{\delta-1}{4}$	>0
	4	$-\frac{(\delta-1)(D_1-4K_H)}{4D_2}$	>0
	5	$p^*_O+\frac{K_H+K_L}{D_1+D_2}-1$	>0
ΔD^*_{2L}	1	$\frac{1}{16}(3D_1+8D_2-8K_H)$	>0
	2	$\frac{(D_2-D_1)K_H}{2(D_1+D_2)}+\frac{3D_1}{16}$	>0
	3	$\frac{1}{4}D_2\left(\frac{4K_H}{D_1+D_2}-1\right)$	>0
	4	$\frac{1}{4}D_1\left(1-\frac{4K_H}{D_1+D_2}\right)$	>0 if $K_H<\frac{D_1+D_2}{4}$
	5	$D_2\left(\frac{2(D_1(K_L-\delta K_H)+2D_2K_L)}{(1-\delta)D_1^2+4D_2(D_1+D_2)}-\frac{K_L}{D_1+D_2}\right)$	>0 if $\delta>\frac{(D_1+2D_2)K_L}{2(D_1+D_2)K_H-D_1K_L}$

Notes. case 1: $K_H>\frac{7D_1+8D_2}{16}$ & $K_H\leq K_L$, case 2: $\frac{3D_1+4D_2}{8}<K_H<\min\{\frac{7D_1+8D_2}{16},K_L\}$, case 3: $\min\{\frac{D_1+2D_2}{2}-K_L,\frac{D_1}{2}\}<K_H<\min\{K_L,\frac{3D_1+4D_2}{8}\}$, case 4: $\min\{\frac{D_1+D_2}{2}-K_L,\frac{D_1}{4}\}<K_H<\min\{\frac{8D_1K_L-D_1(D_1+D_2)}{4D_1+12D_2},\frac{2D_1+D_2}{2}-K_L,\frac{D_1+2D_2}{4}\}$, and case 5: $\frac{D_1}{D_1+2D_2}K_L<K_H<\min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L,\frac{D_1+D_2}{2}-K_L\}$.

$p^*_O=\frac{((1-\delta)D_1(D_1-2K_H)+4D_2(D_1+D_2-K_H-K_L))(2D_2(\delta K_H+K_L)+(1-\delta)D_1K_L)}{\left((1-\delta)D_1^2+4D_2(D_1+D_2)\right)((1-\delta)D_1K_H+2D_2(K_H+K_L))}$

$\frac{1}{8}\left(4\left(D_2-2K_H\right)+D_1\left(5-\frac{\sqrt{32K_H-7(D_1+D_2)}}{\sqrt{D_1+D_2}}\right)\right) < K_L <$
$\frac{1}{8}\left(4\left(D_2-2K_H\right)+D_1\left(\frac{\sqrt{32K_H-7(D_1+D_2)}}{\sqrt{D_1+D_2}}+5\right)\right)$. $\Delta\pi_{2H}^{O-P} < 0$ *if* $K_H > \frac{D_1+D_2}{4}$.

$\Delta\pi_{2L}^{O-P} > 0$ *if* $K_H < \frac{D_1+D_2}{4}$. *Otherwise,* $\Delta\pi_{2L}^{O-P} > 0$ *if* $\delta > 1+\frac{2D_1D_2(D_1+D_2-4K_H)}{(D_1+D_2)(D_1-4K_H)(D_1+2D_2-4K_H)}$.

(v) *If* $\frac{D_1}{2D_1+D_2}K_L < K_H < \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2}-K_L\}$, *then expressions of* $\Delta\pi_{ij}^{O-P}$ *are more complicated than their counterparts in other cases. Note that if* $\delta > \frac{(-D_1-2D_2)K_H+2(D_1+D_2)K_L}{D_1K_H}$, *then* $\pi_{1H}^{O-P} < 0$; *if* $\delta < \frac{(-D_1-2D_2)K_H+2(D_1+D_2)K_L}{D_1K_H}$, *then* $\pi_{2H}^{O-P} < 0$. *If* $\delta < \frac{(D_1+2D_2)K_L}{2(D_1+D_2)K_H-D_1K_L}$, *then* $\pi_{1L}^{O-P} < 0$; *if* $\delta > \frac{(D_1+2D_2)K_L}{2(D_1+D_2)K_H-D_1K_L}$, *then* $\pi_{2L}^{O-P} > 0$.

Proof 34 *Proof of Corollary* 3.3.3.3. *By Theorems 3.3.2.2 and 3.3.3.2, we have* $\pi_O^* < \pi_U^*$, $\Delta p_{2L}^{U-O} < 0$, *and* $\Delta D_{2H}^{U-O} > 0$.

Before analyzing the seller's optimal decision over the whole selling period, we propose a lemma characterizing the optimal decisions of the upgrading mechanism.

Lemma 34 (Optimal Solutions of the Upgrading Mechanism) *Given* θ_{1H} *and* θ_{1L}, *and* $\theta_U^* \in [\max\{\theta_{1L}, 1-\frac{K_H}{D_1}\}, \theta_{1H}]$, *the optimal additional fee charged for the upgrade equals* $p^* = (\delta-1)\theta_U^*$, *where*

$$\theta_U^* = \begin{cases} \frac{\theta_{1H}}{2}+\frac{1}{2}-\frac{K_H-D_1(1-\theta_{1L})+K_L}{2D_2} & \text{if} D_2 \geq 2(K_H-D_1(1-\theta_{1L})+K_L), \\ \frac{\theta_{1H}}{2}+\frac{D_2}{8(K_H-D_1(1-\theta_{1L})+K_L)} & \text{if} K_H-D_1(1-\theta_{1L})+K_L \leq D_2 < 2(K_H-D_1(1-\theta_{1L})+K_L), \\ \frac{\theta_{1H}}{2}+\frac{1}{8} & \text{if} D_2 < K_H-D_1(1-\theta_{1L})+K_L, \frac{D_2}{2} < K_H-D_1(1-\theta_U) \leq D_2 \\ \frac{\theta_{1H}}{2} & \text{otherwise.} \end{cases}$$

Proof 35 *Proof of Lemma* 34.

(i) $D_1(\theta_{1H}-\theta_U) \leq S$. *Case* $D_1(\theta_{1H}-\theta_U) \leq S$ *means that the seller makes pricing decision.*

(a) $D_2 \geq 2(K_H-D_1(1-\theta_{1L})+K_L)$.

Equating the FOC of π_2 *to zero yields* $\theta_U^* = \frac{\theta_{1H}}{2} + \frac{1}{2} - \frac{K_H - D_1(1-\theta_{1L}) + K_L}{2D_2}$. *The seller's optimal profit in the salvage stage equals* $\pi_2^* = \frac{4K_H - 3D_1 + 2D_1\theta_{1H} + D_1\theta_{1H}^2}{4}(\delta - 1) + \frac{D_1(\delta-1) - 4D_2}{4D_2^2}(K_H - D_1(1-\theta_{1L}) + K_L)^2 + \frac{2D_2 - (2K_H - D_1 + D_1\theta_{1H})(\delta-1)}{2D_2}(K_H - D_1(1-\theta_{1L}) + K_L) - F_O$.

(b) $K_H - D_1(1-\theta_{1L}) + K_L \leq D_2 < 2(K_H - D_1(1-\theta_{1L}) + K_L)$.

Equating the FOC of π_2 *to zero yields* $\theta_U^* = \frac{\theta_{1H}}{2} + \frac{D_2}{8(K_H - D_1(1-\theta_{1L}) + K_L)}$, *which is within the interval* $[\max\{\theta_{1L}, 1 - \frac{K_H}{D_1}\}, \theta_{1H}]$. *The seller's optimal profit in the salvage stage equals* $\pi_2^* = \frac{D_2(2K_H - 2D_1 + D_1\theta_{1H})(\delta-1)}{8(K_H - D_1(1-\theta_{1L}) + K_L)} + \frac{D_1 D_2^2(\delta-1)}{64(K_H - D_1(1-\theta_{1L}) + K_L)^2} + \frac{D_2 q_L + D_1(\delta-1)\theta_{1H}^2}{4} - F_O$.

(c) $D_2 < K_H + K_L - D_1(1-\theta_{1L})$ *&* $K_H - D_1(1-\theta_U) > \frac{D_2}{2}$.

Equating the FOC of π_2 *to zero yields* $\theta_U^* = \frac{\theta_{1H}}{2} + \frac{1}{8}$, *provided that* $\theta_U^* \in [\max\{\theta_{1L}, 1 - \frac{K_H}{D_1}\}, \theta_{1H}]$ *and* $\phi = \frac{K_H - D_1(1-\theta_U)}{D_2} \leq 1$. *The seller's optimal profit in the salvage stage equals* $\pi_2^* = \frac{(\delta-1)(16D_1\theta_{1H}^2 + 8D_1\theta_{1H} - 15D_1 + 16K_H)}{64} + \frac{D_2}{4} - F_O$.

(d) $D_2 < K_H + K_L - D_1(1-\theta_{1L})$ *&* $K_H - D_1(1-\theta_U) \leq \frac{D_2}{2}$.

Equating the FOC of π_2 *to zero yields* $\theta_U^* = \frac{\theta_H}{2}$. *The seller's optimal profit in the salvage stage equals* $\pi_2^* = \frac{(\delta-1)D_1\theta_{1H}^2}{4} + \frac{D_2(\delta+1)}{8} - F_O$.

Boundary solutions: $\theta_U^* = \max\{\theta_{1L}, 1 - \frac{K_H}{D_1}\}$ *or* $\theta_U^* = \theta_{1H}$.

(ii) $D_1(\theta_{1H} - \theta_U) \geq S$.

Case $D_1(\theta_H - \theta_U) \geq S$ *means that the seller first makes allocation then the upgrade pricing decision.*

(a) $D_2 \geq 2(K_H - D_1(1-\theta_{1L}) + K_L)$. *The seller's profit function of the salvage stage* $\pi_2(S) = (1 - \frac{K_H - D_1(1-\theta_{1L}) + K_L}{D_2})(\delta - 1)(K_H - D_1(1-\theta_{1H}) - S) + (\delta-1)(\theta_{1H} - \frac{S}{D_1})S - F_O$ *is concave w.r.t.* S, *hence,* $S^* = \frac{D_1\theta_{1H}}{2} - \frac{D_1}{2} + \frac{D_1(K_H - D_1(1-\theta_{1L}) + K_L)}{2D_2}$, *and* $\theta_U^* = \frac{\theta_{1H}}{2} + \frac{1}{2} - \frac{K_H - D_1(1-\theta_{1L}) + K_L}{2D_2}$.

(b) $K_H - D_1(1-\theta_{1L}) + K_L \leq D_2 < 2(K_H - D_1(1-\theta_{1L}) + K_L)$. *The seller's profit function of the salvage stage* $\pi_2(S) = \frac{D_2(\delta-1)}{4}\frac{K_H - D_1(1-\theta_{1H}) - S}{K_H - D_1(1-\theta_{1L}) + K_L} + (\delta - 1)(\theta_{1H} -$

$\frac{S}{D_1})S - F_O$ *is concave w.r.t.* S, *hence,* $S^* = \frac{D_1\theta_{1H}}{2} - \frac{D_1 D_2}{8(K_H - D_1(1-\theta_{1L}) + K_L)}$, *and* $\theta_U^* = \frac{\theta_{1H}}{2} + \frac{D_2}{8(K_H - D_1(1-\theta_{1L}) + K_L)}$.

(c) $D_2 < K_H + K_L - D_1(1-\theta_{1L})$ & $K_H - D_1(1-\theta_U) > \frac{D_2}{2}$. *The seller's profit function of the salvage stage* $\pi_2(S) = \frac{D_2(\delta-1)}{4}\frac{K_H - D_1(1-\theta_{1H}) - S}{D_2} + (\delta-1)(\theta_{1H} - \frac{S}{D_1})S - F_O$ *is concave w.r.t.* S, *hence,* $S^* = \frac{D_1\theta_H}{2} - \frac{D_1}{8}$ *and* $\theta_U^* = \frac{\theta_{1H}}{2} + \frac{1}{8}$.

(d) $D_2 < K_H + K_L - D_1(1-\theta_{1L})$ & $K_H - D_1(1-\theta_U) \le \frac{D_2}{2}$. $S^* = \frac{D_1\theta_{1H}}{2}$ *and* $\theta_U^* = \frac{\theta_{1H}}{2}$.

To sum up, case $D_1(\theta_{1H} - \theta_U) \le S$ *and case* $D_1(\theta_{1H} - \theta_U) > S$ *are equivalent.*

Proof 36 *Proof of Proposition* 3.4.1.2. *There are at most* 4×4 *cases (four possible cases in each mechanism) in the joint adoption, and the optimal solutions satisfy conditions* (a) *to* (e) *mentioned above.*

Special case. $\theta_U = \theta_{1H}$

If $\theta_U = \theta_{1H}$, *then the optimal outcomes are equivalent to Lemma* 3.3.3.1 *in pure opaque selling with condition* $\theta_U = \theta_{1H}$ *verified. Specifically, in case (i) of Lemma* 3.3.3.1, $\theta_U = \frac{\theta_{1H}}{2} + \frac{1}{2} - \frac{K_H + K_L - D_1(1-\theta_{1L})}{2D_2}$ *reduces to* $\theta_U = \theta_{1H}$ *for* $1 - \theta_{1H} = \frac{K_H + K_L - D_1(1-\theta_{1L})}{2D_2}$. *Similarly,* θ_U *reduces to* θ_{1H} *for* $\frac{1}{2} - \theta_{1H} + \frac{D_2}{8(K_H + K_L - D_1(1-\theta_{1L}))} = 0$ *in case (ii) of Lemma* 3.3.3.1. *While* $\theta_U \neq \theta_{1H}$ *in cases (iii) and (iv) of Lemma* 3.3.3.1.

Special case. $\theta_U = 1 - \frac{K_H}{D_1}$

If $\theta_U = 1 - \frac{K_H}{D_1}$, *then* $\pi_J(\theta_{1H}, \theta_{1L}) = D_1(\delta-1)(\theta_{1H} - \theta_{1H}^2) + D_1(\theta_{1L} - \theta_{1L}^2) + (\delta-1)(1 - \frac{K_H}{D_1})(K_H - D_1(1-\theta_{1H}))$. *By solving the FOCs, we have* $\theta_{1H}^* = 1 - \frac{K_H}{2D_1}$, $\theta_{1L}^* = \frac{1}{2}$, $S^* = \frac{K_H}{2}$, $\theta_U^* = 1 - \frac{K_H}{D_1}$ *and* $\pi_J^* = \frac{D_1}{4} + K_H(\delta-1)(1 - \frac{3K_H}{4D_1})$ *provided that* $K_H \le \frac{D_1}{2}$ *and* $\frac{K_H}{2} + K_L \ge \frac{D_1}{2}$. *Moreover, conditions that permit the existence of this boundary solution are given by* $1 - \frac{K_H}{D_1} > \frac{\theta_{1H}}{2} + \frac{1}{2} - \frac{K_H + K_L - D_1(1-\theta_{1L})}{2D_2}$, $1 - \frac{K_H}{D_1} > \frac{\theta_{1H}}{2} + \frac{D_2}{8(K_H + K_L - D_1(1-\theta_{1L}))}$, $1 - \frac{K_H}{D_1} > \frac{\theta_{1H}}{2} + \frac{1}{8}$, *or* $1 - \frac{K_H}{D_1} > \frac{\theta_{1H}}{2}$, *and the reduced forms are* $K_L > \frac{(3D_2 - 2D_1)K_H}{2D_1} + \frac{D_1}{2}$, $K_L > \frac{2D_1^2 + D_1D_2 - 7D_1K_H + 6K_H^2}{4D_1 - 6K_H}$, $K_H < \frac{D_1}{2}$, *or* $K_H < \frac{2D_1}{3}$. *So, the effective domain is given by* $K_H \le \frac{D_1}{2}$ *together with* $K_L \ge \max\{\frac{D_1 - K_H}{2}, \frac{2D_1^2 + D_1D_2 - 7D_1K_H + 6K_H^2}{4D_1 - 6K_H}\}$.

General case.

(i) $D_2 \geq 2(K_H - D_1(1-\theta_{1L}) + K_L)$.

Condition 1. $\theta_U = \frac{\theta_{1H}}{2} + \frac{1}{2} - \frac{K_H+K_L-D_1(1-\theta_{1L})}{2D_2}$

If $\theta_{1L} \leq 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, *the Hessian matrix of* π_J *is negative definite. Hence, by solving the FOCs, we have* $\theta^*_{1H} = \frac{(2D_1^2-D_1K_H)(\delta-1)-6D_1D_2-6D_2^2+2D_2K_H+2D_2K_L}{2(D_1^2\delta-D_1^2-3D_1D_2-3D_2^2)}$, *and* $\theta^*_{1L} = \frac{(2D_1^2-2D_1K_H+3D_2K_H-2D_1K_L)(\delta-1)-6D_1D_2-6D_2^2+6D_2K_H+6D_2K_L}{2(D_1^2\delta-D_1^2-3D_1D_2-3D_2^2)}$. *Correspondingly,* $\theta^*_U = 2\theta^*_{1H} - 1 = \frac{(D_1^2-D_1K_H)(\delta-1)-3D_1D_2-3D_2^2+2D_2K_H+2D_2K_L}{D_1^2\delta-D_1^2-3D_1D_2-3D_2^2}$, $\phi^* = \frac{-2D_1K_H-6D_2K_H+4D_1K_L}{3(\delta-1)D_1K_H-6D_2(K_H+K_L)}$, $\theta^*_O = \frac{(2D_1^2-3D_1K_H)(\delta-1)-6D_1D_2-6D_2^2+6D_2K_H+6D_2K_L}{2(D_1^2\delta-D_1^2-3D_1D_2-3D_2^2)}$, *where* $\phi^* = \frac{K_H-D_1(1-\theta^*_U)}{K_H+K_L-D_1(1-\theta^*_{1L})}$ *always satisfies* $0 \leq \phi^* \leq 1$, *and* $\pi^*_J = \frac{-4D_1^2K_L+12D_2K_L(K_H+K_L-D_2)+D_1(K_H^2+8K_HK_L+4K_L(-3D_2+K_L))}{4(D_1^2\delta-D_1^2-3D_1D_2-3D_2^2)} + \frac{D_1K_H(4D_1-3K_H)\delta^2-2(6D_2K_H(D_2-K_H-K_L)+2D_1^2(K_H-K_L)+D_1(6D_2K_H-K_H^2+4K_HK_L+2K_L^2))\delta}{4(D_1^2\delta-D_1^2-3D_1D_2-3D_2^2)} - F_O$.

Because $\theta^*_{1H} \leq 1$ *and* $K_H - D_1(1-\theta_{1H}) \geq 0$ *reduce to* $D_1K_H(\delta-1) - 2D_2(K_H + K_L) \leq 0$ *and* $D_1^2K_H(\delta-1) - 6D_2^2K_H + 2D_1D_2(-2K_H + K_L) < 0$, *respectively. We define* $f_1(\delta) = D_1K_H(\delta-1) - 2D_2(K_H+K_L)$ *and* $f_2(\delta) = D_1^2K_H(\delta-1) - 6D_2^2K_H + 2D_1D_2(-2K_H+K_L)$. *Note* $f_1(\delta) \leq 0$ *if* $K_H \leq \frac{2D_1}{D_1+3D_2}K_L$ *and* $f_2(\delta) < 0$ *if* $K_H > \frac{2D_1}{D_1+3D_2}K_L$. *Hence, there are no optimal solutions.*

Boundary solution. $\theta_{1L} = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$.

If $\theta^*_{1L} > 1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}$, *then* $\theta^*_{1L} = 1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}$, $\theta^*_{1H} = \frac{3-3\theta_{1H}}{2} - \frac{K_H+K_L-D_1(1-\theta_L)}{2D_2} = \frac{5}{6}$, $\theta^*_U = 2\theta^*_{1H} - 1 = \frac{2}{3}$, $\theta^*_O = 3\theta^*_{1H} - 2 = \frac{1}{2}$, $\phi^* = \frac{6K_H-2D_1}{3D_2}$, *and*

$\pi^*_J = \frac{(D_1^2+6D_1K_H)\delta-D_1^2-3D_1D_2+6D_1K_H+12D_1K_L-3(D_2-2(K_H+K_L))^2}{12D_1} - F_O$ *provided that* $\frac{D_1}{3} \leq K_H < \min\{\frac{2D_1+3D_2}{6}, \frac{2D_1+D_2}{2} - K_L, \frac{2D_1}{D_1+3D_2}K_L\}$.

Condition 2. $\theta_U = \theta_{1L}$

If $\theta_U = \theta_{1L}$ *when* $\theta_{1L} > 1-\frac{K_H}{D_1}$, *then* $\pi_2 = (1-\frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2})(K_H\delta+K_L-D_1(1-\theta_{1L})\delta)+D_1(\delta-1)(\theta_{1H}\theta_{1L}-\theta_{1L}^2)-F_O$. *The Hessian matrix of the seller's total profit* π_J *is negative definite. By equating the FOCs of* π_J *w.r.t* θ_{1H} *and* θ_{1L} *to zero, we have* $\theta^*_{1H} = \frac{-2K_H\delta-K_L-K_L\delta+4D_1\delta+3D_2\delta+D_2}{4D_1\delta+3D_2\delta+D_2}$, $\theta^*_{1L} = \theta^*_U = \frac{-4K_H\delta-2K_L-2K_L\delta+4D_1\delta+3D_2\delta+D_2}{4D_1\delta+3D_2\delta+D_2}$, $\theta^*_O = \frac{(4D_1D_2-2D_1K_L+3D_2(D_2-K_H-K_L))\delta+D_2^2+2D_1K_L-D_2(K_H+K_L)}{D_2(4D_1\delta+3D_2\delta+D_2)}$,

$\phi^*_J = \frac{3D_2(-\delta D_1+D_1+2D_2)K_H+2D_1((\delta-1)D_1-3D_2)K_L}{3D_2(D_1(K_H-\delta K_H)+2D_2(K_H+K_L))}$, *where* $\phi^*_J = \frac{K_H-D_1(1-\theta^*_{1L})}{K_H+K_L-D_1(1-\theta^*_{1L})}$ *always satifies* $0 \leq \phi^* \leq 1$, *and* $\pi^*_J = \frac{D_2(D_2-K_H-K_L)(3\delta+1)(K_H\delta+K_L)+D_1(K_L^2(\delta-1)^2+4D_2\delta(K_H\delta+K_L))}{D_2(4D_1\delta+3D_2\delta+D_2)} -$

F_O provided that $(3D_2K_H - 2D_1K_L)\delta + D_2K_H - 2D_1K_L > 0$, $(4D_1(D_2 - K_L) + 3D_2(D_2 - 2(K_H + K_L)))\delta + D_2^2 + 4D_1K_L - 2D_2(K_H + K_L \geq 0$, $(4D_1 + 3D_2 - 4K_H - 2K_L)\delta + D_2 - 2K_L \geq 0$, and $(-2D_1K_H + 3D_1K_L + 3D_2K_L)\delta + D_2K_L - D_1K_L \geq 0$. We define $f_3(\delta) = (3D_2K_H - 2D_1K_L)\delta + D_2K_H - 2D_1K_L$, $f_4(\delta) = (4D_1(D_2 - K_L) + 3D_2(D_2 - 2(K_H + K_L)))\delta + D_2^2 + 4D_1K_L - 2D_2(K_H + K_L)$, $f_5(\delta) = (4D_1 + 3D_2 - 4K_H - 2K_L)\delta + D_2 - 2K_L$, and $f_6(\delta) = (-2D_1K_H + 3D_1K_L + 3D_2K_L)\delta + D_2K_L - D_1K_L \geq 0$. Note that $f_3(\delta) > 0$ if $K_H > \frac{D_1}{D_2}K_L$, $f_4(\delta) \geq 0$ if $K_H \leq \min\{\frac{(D_1+D_2)(2D_1+3D_2)^2 - 2(10D_1^2+15D_1D_2+9D_2^2)K_L}{8D_1^2+18D_1D_2+18D_2^2}, \frac{D_1+D_2}{2} - K_L\}$, $f_5(\delta) \geq 0$ if $K_H \leq \min\{D_1 + D_2 - K_L, \frac{(D_1+D_2)(2D_1+3D_2)^2-(4D_1^2+6D_1D_2+6D_2^2)K_L}{4D_1^2+12D_1D_2+12D_2^2}\}$, and $f_6(\delta) \geq 0$ if $K_H \leq \min\{\frac{D_1+2D_2}{D_1}K_L, \frac{2D_1^3+13D_1^2D_2+18D_1D_2^2+9D_2^3}{2D_1^3+6D_1^2D_2+6D_1D_2^2}K_L\}$. Condition $\theta_{1L}^* \geq \frac{\theta_H^*}{2} + \frac{1}{2} - \frac{K_H+K_L-D_1(1-\theta_L^*)}{2D_2}$ reduces to $(-3D_2K_H + 2D_1K_L)\delta + D_2K_H - 2D_1K_L - 2D_2K_L \geq 0$, which does not hold. So, there are no optimal solutions.

(ii) $K_H - D_1(1 - \theta_{1L}) + K_L \leq D_2 < 2(K_H - D_1(1 - \theta_{1L}) + K_L)$.

Condition 1. $\theta_U = \frac{\theta_{1H}}{2} + \frac{D_2}{8(K_H+K_L-D_1(1-\theta_{1L}))}$

If $\frac{D_2}{2D_1} + 1 - \frac{K_H+K_L}{D_1} < \theta_{1L} \leq \frac{D_2}{D_1} + 1 - \frac{K_H+K_L}{D_1}$, then the optimal solutions satisfy the folllowing best response functions if the Hessian matrix is negative definite:

$$\theta_{1H}^* = \frac{2}{3} + \frac{D_2}{12(K_H + K_L - D_1(1 - \theta_{1L}))},$$

$$\frac{-D_2^2 - 2D_2(D_1 - 2(K_H + K_L))(3\theta_{1H}^* - 2) - 8D_1(\delta - 1)(K_H + 2D_1(\theta_{1H}^* - 1))(3\theta_{1H}^* - 2)^3}{2D_2(3\theta_{1H}^* - 2)} = 0.$$

Because $\theta_{1H}^* \neq \theta_{1L}^*$, $\theta_{1H} \neq 1$, $\theta_U^* = 2\theta_{1H}^* - 1$ and $\theta_O^* = \frac{1}{2}$. Hence, the optimal capacity offering can be $(H^P, L^P; H^{U+O}, L^O)$ or $(H^P, L^\varnothing; H^{U+O}, L^O)$. Note that $\frac{\partial \pi_J}{\partial \theta_{1L}} < 0$ when $\theta_{1L} \geq \frac{1}{2}$, or equivalently, $2(K_H + K_L) \leq D_1 + D_2$. So, there are no optimal solutions when $2(K_H + K_L) \leq D_1 + D_2$.

Boundary solution.

The optimal interior solution is a convex combination of the boundary solutions.

(a) $\theta_{1L}^* = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = \frac{5}{6}$, $\theta_U^* = \frac{3}{4}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{4K_H-D_1}{2D_2}$, and $\pi_J^* = \frac{11(\delta-1)D_1^2+36D_1(-D_2+2(\delta+1)K_H+4K_L)-36(D_2-2(K_H+K_L))^2}{144D_1} - F_O$. The effective domain is given by $\max\{\frac{D_1+2D_2}{4} - K_L, \frac{D_1}{4}\} \leq K_H \leq \min\{\frac{2D_1+D_2}{2} - K_L, \frac{D_1+2D_2}{4}\}$.

(b) $\theta_{1L} = 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = \frac{3}{4}$, $\theta_U^* = \frac{1}{2}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{2K_H-D_1}{2D_2}$, and $\pi_J^* = \frac{3(\delta-1)D_1^2+4D_1(-3D_2+(\delta+3)K_H+4K_L)-16(-D_2+K_H+K_L)^2}{16D_1} - F_O$. The effective domain is given by $\max\{\frac{D_1+2D_2}{2} - K_L, \frac{D_1}{2}\} \leq K_H \leq \min\{D_1 + D_2 - K_L, \frac{D_1+2D_2}{2}\}$.

Condition 2. $\theta_U = \theta_{1L}$

The seller's profit of the salvage stage equals $\pi_2 = \frac{D_2(K_H-D_1(1-\theta_{1L}))}{4(K_H+K_L-D_1(1-\theta_{1L}))}(\delta-1) + \frac{D_2}{4} + D_1(\delta-1)(\theta_{1H}\theta_{1L} - \theta_{1L}^2) - F_O$ *if* $\theta_{1L} > 1 - \frac{K_H}{D_1}$, *and the Hessian matrix of* π_J *is negative definite. Consequently, the optimal solutions satisfy*

$$D_1(1-2\theta_{1L}) + \frac{D_1D_2K_L(\delta-1)}{4(K_H+K_L-D_1(1-\theta_{1L}))^2} + D_1(\delta-1)(\theta_{1H}-2\theta_{1L}) = 0, \quad \theta_{1H}^* = \frac{1+\theta_{1L}}{2}.$$

Because $\theta_{1H}^* \neq \theta_{1L}^* = \theta_U^*$, $\theta_{1H} \neq 1$, *and* $\theta_O^* = \frac{1}{2}$. *Hence, the optimal capacity offering is* $(H^P, L^P; H^{U+O}, L^O)$.

Note that $\frac{\partial^2\pi_J}{\partial\theta_{1L}^2} < 0$, *if* $\frac{\partial\pi_J}{\partial\theta_{1L}}\big|_{\theta_{1L}=1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}} < 0$, *or equivalently,* $K_H < \min\{\frac{(D_1+D_2)(2D_1+3D_2)^2-2(10D_1^2+15D_1D_2+9D_2^2)K_L}{8D_1^2+18D_1D_2+18D_2^2}, \frac{D_1+D_2}{2} - K_L\}$, *then* $\frac{\partial\pi_J}{\partial\theta_{1L}} < 0$ *over* $(1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}, 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}]$.

Boundary solution.

The optimal interior solution is a convex combination of the boundary solutions.

(a) $\theta_{1L}^* = \theta_U^* = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = 1 + \frac{D_2}{4D_1} - \frac{K_H+K_L}{2D_1}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{2K_H-2K_L+D_2}{2D_2}$, *and* $\pi_J^* = \frac{4D_1(-\delta D_2+4\delta K_H+2(\delta+1)K_L)-(3\delta+1)(D_2-2(K_H+K_L))^2}{16D_1} - F_O$. *The effective domain is given by* $-\frac{D_2}{2} \leq K_H - K_L \leq \frac{D_2}{2}$ *as well as* $\frac{D_2}{2} \leq K_H + K_L \leq \frac{2D_1+D_2}{2}$.

(b) $\theta_{1L}^* = \theta_U^* = 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{2D_1}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{K_H-K_L+D_2}{2D_2}$, *and* $\pi_J^* = \frac{(-3\delta-1)(-D_2+K_H+K_L)^2+D_1(-3\delta D_2+4\delta K_H+3\delta K_L+K_L)}{4D_1} - F_O$. *The effective domain is given by* $K_L - D_2 \leq K_H \leq D_2$ *as well as* $D_2 \leq K_H + K_L \leq D_1 + D_2$.

(iii) $D_2 < K_H - D_1(1-\theta_{1L}) + K_L$ & $K_H - D_1(1-\theta_U) > \frac{D_2}{2}$.

Condition 1. $\theta_U = \frac{\theta_{1H}}{2} + \frac{1}{8}$

If $\theta_{1L} > 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}$ *and* $\theta_{1H} > \frac{7}{4} + \frac{D_2}{D_1} - \frac{2K_H}{D_1}$, *then the Hessian matrix of* π_J *is negative definite. Hence,* $\theta_{1H}^* = \frac{3}{4}$, $\theta_{1L}^* = \frac{1}{2}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{2K_H-D_1}{2D_2}$, $\theta_U^* = \frac{1}{2}$, and $\pi_J^* = \frac{4(D_1+D_2)+(4K_H+3D_1)(\delta-1)}{16} - F_O$ *provided that* $D_1 < 2D_2$, $K_L > \frac{D_1}{4}$ *and* $K_H + K_L > \frac{D_1}{2} + D_2$.

(iv) $D_2 < K_H - D_1(1-\theta_{1L}) + K_L$ & $K_H - D_1(1-\theta_U) \le \frac{D_2}{2}$.

Condition 1. $\theta_U = \frac{\theta_{1H}}{2}$

If $\theta_{1L} > 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}$ *and* $\theta_{1H} \le 2 + \frac{D_2}{D_1} - \frac{2K_H}{D_1}$, *then* $\theta_{1H}^* = \frac{2}{3}$, $\theta_{1L}^* = \frac{1}{2}$, *and* $\theta_U = \frac{\theta_{1H}^*}{2}$, *while* $\theta_U^* = \frac{1}{3} < \theta_{1L}^* = \frac{1}{2}$, *thus* $\theta_U^* = \max\{\theta_{1L}, 1 - \frac{K_H}{D_1}\}$.

Condition 2. $\theta_U = \theta_{1L}$

If $\theta_{1L} \ge \frac{\theta_{1H}}{2}$ *and* $\theta_{1L} > 1 - \frac{K_H}{D_1}$, *then* $\pi_2 = \frac{D_2}{8}(\delta+1) + D_1(\delta-1)(\theta_{1H}\theta_{1L} - \theta_{1L}^2) - F_O$ *if* $\theta_{1L} > 1 - \frac{K_H}{D_1}$. *The Hessian matrix of* π_J *is negative define, hence,* $\theta_{1H}^* = \frac{2\delta+1}{3\delta+1}$, $\theta_{1L}^* = \theta_U^* = \frac{\delta+1}{3\delta+1}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{1}{2}$, *and* $\pi_J^* = \frac{(8D_1+3D_2)\delta^2+4D_2\delta+D_2}{8+24\delta} - F_O$ *provided that* $\frac{2\delta}{3\delta+1}D_1 < K_H \le \frac{2\delta}{3\delta+1}D_1 + \frac{D_2}{2}$, $K_L > \frac{\delta}{3\delta+1}D_1$ *and* $K_H + K_L > \frac{2\delta}{3\delta+1}D_1 + D_2$. *Conditions* $(3K_H - 2D_1)\delta + K_H > 0$ *and* $(3K_L - D_1)\delta + K_L > 0$ *reduce to* $K_H > \frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$ *and* $K_L > \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$, *respectively. We define* $f_7(\delta) = (4D_1+3D_2-6K_H)\delta+D_2-2K_H$ *and* $f_8(\delta) = (3K_H+3K_L-2D_1-3D_2)\delta+K_H+K_L-D_2$. *Note that* $f_7(\delta) \ge 0$ *if* $K_H \le \frac{D_1+D_2}{2}$, *and* $f_8(\delta) \ge 0$ *if* $K_H > \frac{2D_1^3+10D_1^2D_2+15D_1D_2^2+9D_2^3}{4D_1^2+9D_1D_2+9D_2^2} - K_L$.

So, the effective domain is given by $\max\{\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}, \frac{2D_1^3+10D_1^2D_2+15D_1D_2^2+9D_2^3}{4D_1^2+9D_1D_2+9D_2^2} - K_L\} < K_H < \frac{D_1+D_2}{2}$ *and* $K_L > \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$.

To summarize,

(i) If $\frac{D_1}{3} < K_H < \min\{\frac{2D_1+3D_2}{6}, \frac{2D_1+D_2}{2} - K_L, \frac{2D_1}{D_1+3D_2}K_L\}$ *provided that* $D_1 < 3D_2$, *then* $\theta_{1H}^* = \frac{5}{6}$, $\theta_{1L}^* = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta_U^* = \frac{2}{3}$, *and* $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{6K_H-2D_1}{3D_2}$, *and* $\pi_J^* = \frac{(D_1^2+6D_1K_H)\delta - D_1^2 - 3D_1D_2 + 6D_1K_H + 12D_1K_L - 3(D_2-2(K_H+K_L))^2}{12D_1} - F_O$.

(ii) If $K_H < \frac{D_1}{2}$ *and* $K_L \ge \max\{\frac{D_1-K_H}{2}, \frac{2D_1^2+D_1D_2-7D_1K_H+6K_H^2}{4D_1-6K_H}\}$, *then* $\theta_{1H}^* = 1 - \frac{K_H}{2D_1}$, $\theta_{1L}^* = \frac{1}{2}$, $\theta_U^* = 1 - \frac{K_H}{D_1}$, $\theta_O^* = 1$, *and* $\pi_J^* = \frac{D_1}{4} + K_H(\delta-1)(1 - \frac{3K_H}{4D_1}) - F_O$.

(iii) If $\frac{D_1}{2D_2+D_1}K_L < K_H \le \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2} - K_L\}$, *then*

$\theta_{1H}^* = \frac{-D_1^2\delta+D_1K_H\delta+D_1^2+4D_1D_2+4D_2^2-D_1K_H-2D_2K_H-2D_2K_L}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$, $\theta_O^* = \frac{(2D_1K_H-D_1^2)(\delta-1)+4D_2(D_1+D_2-(K_H+K_L))}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$,

$\theta_{1L}^* = \frac{-D_1^2\delta+D_1K_H\delta+D_1^2+4D_1D_2+4D_2^2-D_1K_H-2D_2K_H-4D_2K_L-2D_2K_H\delta+D_1K_L\delta-D_1K_L}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$,

$\phi^* = \frac{(-D_1-2D_2)K_H+D_1K_L}{(\delta-1)D_1K_H-2D_2(K_H+K_L)}$, *and* $\pi_J^* = \frac{K_L(-D_1^2+4D_2(-D_2+K_H+K_L)+D_1(-4D_2+2K_H+K_L))}{4(D_1+D_2)D_2-D_1^2(\delta-1)} + \frac{\delta^2 D_1K_H(D_1-K_H)+\delta(D_1^2(-K_H+K_L)+4D_2K_H(-D_2+K_H+K_L)-D_1(4D_2K_H-K_H^2+2K_HK_L+K_L^2))}{4(D_1+D_2)D_2-D_1^2(\delta-1)} - F_O$.

(iv) *If* $\max\{\frac{D_1+2D_2}{2} - K_L, \frac{D_1+D_2}{2}\} < K_H \leq \frac{D_1+2D_2}{2}$ *given that* $D_1 < 2D_2$, *and* $K_L > \frac{D_1}{4}$, *then* $\theta_{1H}^* = \frac{3}{4}$, $\theta_{1L}^* = \theta_U^* = \frac{1}{2}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{2K_H-D_1}{2D_2}$ *and* $\pi_J^* = \frac{4(D_1+D_2)+(4K_H+3D_1)(\delta-1)}{16} - F_O$.

(v) *If* $\max\{\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}, \frac{2D_1^3+10D_1^2D_2+15D_1D_2^2+9D_2^3}{4D_1^2+9D_1D_2+9D_2^2} - K_L\} < K_H < \frac{D_1+D_2}{2}$ *and* $K_L > \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$, *then* $\theta_{1H}^* = \frac{2\delta+1}{3\delta+1}$, $\theta_{1L}^* = \theta_U^* = \frac{\delta+1}{3\delta+1}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{1}{2}$, *and* $\pi_J^* = \frac{(8D_1+3D_2)\delta^2+4D_2\delta+D_2}{8+24\delta} - F_O$.

(vi) *If* $\frac{D_1}{4} < K_H < \min\{\frac{8D_1K_L-D_1(D_1+D_2)}{4D_1+12D_2}, \frac{2D_1+D_2}{2} - K_L, \frac{D_1+2D_2}{4}\}$, *then* $\theta_{1H}^* = \theta_U^* = \frac{3}{4}$, $\theta_{1L}^* = 1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{4K_H-D_1}{2D_2}$, *and* $\pi_J^* = \frac{(D_1^2+8D_1K_H)\delta-D_1^2-4D_1D_2+8D_1K_H+16D_1K_L}{16D_1} - \frac{(D_2-2(K_H+K_L))^2}{4D_1} - F_O$.

Overlap Characterization

(a) *Cases (vi) and (i):* $\Delta\pi^{i-vi} > 0$.

(b) *Cases (vi) and (ii):* $\Delta\pi^{ii-vi}$ *increases with* δ *if* $\frac{D_1}{6} < K_H < \frac{D_1}{2}$, *and reaches minimum when* $\delta = 1$: $4(4K_H^2 + (-4D_1 - 4D_2 + 8K_L)K_H + D_1^2 + D_1D_2 + D_2^2 - 4D_1K_L - 4D_2K_L + 4K_L^2)$, *this value is negative when* $\frac{D_1+D_2-\sqrt{D_1D_2}}{2} - K_L < K_H < \frac{D_1+D_2+\sqrt{D_1D_2}}{2} - K_L$. *Note that the intersection between these two cases (the shaded area is* $\frac{D_1}{4} < K_H < \min\{\frac{D_1}{3}, \frac{D_1+2D_2}{4}\}$, $\frac{D_1+2D_2}{4} < K_L < \frac{3D_1+2D_2}{4}$*) satisfies* $\frac{D_1+D_2}{2} - K_L < K_H < \frac{2D_1+D_2}{2} - K_L$. *Because* $\frac{D_1+D_2-\sqrt{D_1D_2}}{2} - K_L < \frac{D_1+D_2}{2} - K_L < \frac{D_1+D_2+\sqrt{D_1D_2}}{2} - K_L < \frac{2D_1+D_2}{2} - K_L$, *so, we conclude that case (ii) dominates case (vi) if* $K_H > \frac{D_1+D_2+\sqrt{D_1D_2}}{2} - K_L$. *If* $\frac{D_1+D_2}{2} - K_L < K_H < \frac{D_1+D_2+\sqrt{D_1D_2}}{2} - K_L$, *then case (ii) dominates case (vi) if* $\delta > \frac{4D_1(D_2-6K_H-4K_L)+4\left(-4D_2(K_H+K_L)+D_2^2+8K_HK_L+7K_H^2+4K_L^2\right)+5D_1^2}{(D_1-6K_H)(D_1-2K_H)}$.

(c) *Cases (vi) and (v): Because* $\frac{\partial^2\Delta\pi}{\partial\delta^2} > 0$, *and* $\frac{\partial\Delta\pi}{\partial\delta}\big|_{\delta=1} > 0$. *So,* $\frac{\partial\Delta\pi}{\partial\delta} > 0$, *note that* $\Delta\pi^{v-vi}|_{\delta=1} > 0$. *So, case (v) dominates case (vi).*

(d) *Cases (ii) and (i): Because* $\frac{\partial\Delta\pi}{\partial\delta} > 0$, $\Delta\pi|_{\delta=1} > 0$ *if* $\frac{D_1+D_2-\sqrt{D_1D_2}}{2} - K_L < K_H < \frac{D_1+D_2+\sqrt{D_1D_2}}{2} - K_L$, $\Delta\pi|_{\delta=1+\frac{3D_1D_2+3D_2^2}{D_1^2}} > 0$ *if*

$\frac{-D_1^2(D_2+4K_L)+\sqrt{D_1^2D_2\left(-2D_1^2(D_2+6K_L)+6D_1\left(-8D_2K_L+D_2^2+6K_L^2\right)+9D_2(D_2-2K_L)^2+5D_1^3\right)}+2D_1^3-3D_2^2D_1}{(D_1-3D_2)(4D_1+3D_2)} <$

$K_H < -\frac{D_1^2(D_2+4K_L)+\sqrt{D_1^2D_2\left(-2D_1^2(D_2+6K_L)+6D_1\left(-8D_2K_L+D_2^2+6K_L^2\right)+9D_2(D_2-2K_L)^2+5D_1^3\right)}-2D_1^3+3D_2^2D_1}{(D_1-3D_2)(4D_1+3D_2)}$.

Note that the region of the shaded area is given by $\frac{D_1}{3} < K_H < \min\{\frac{D_1+3D_2}{6}, \frac{D_1}{2}\}$ *and* $\frac{D_1+3D_2}{6} < K_L < \frac{4D_1+3D_2}{6}$. *If* $3D_2 \leq D_1$*, then the optimal strategy defined in case (ii) does not exist. Otherwise, case (i) dominates case (ii) if* $K_L < \min\{\frac{2D_1+3\sqrt{D_1D_2}}{6}, \frac{D_2+\sqrt{D_1D_2}}{2}\}$. *If* $\min\{\frac{2D_1+3\sqrt{D_1D_2}}{6}, \frac{D_2+\sqrt{D_1D_2}}{2}\} < K_L < \frac{4D_1+3D_2}{6}$*, then* $\Delta\pi^{i-ii} > 0$ *if*

$\delta > \frac{3D_1(D_2-6K_H-4K_L)+3\left(-4D_2(K_H+K_L)+D_2^2+8K_HK_L+7K_H^2+4K_L^2\right)+4D_1^2}{(D_1-3K_H)^2}$.

(e) *Cases (v) and (i): Because* $\frac{\partial^2\Delta\pi}{\partial\delta^2} < 0$, $\frac{\partial\Delta\pi}{\partial\delta}\big|_{\delta=1} < 0$. *So,* $\frac{\partial\Delta\pi}{\partial\delta} < 0$*, note that* $\Delta\pi^{i-v}\big|_{\delta=1} < 0$. *So, case (v) dominates case (i).*

(f) *Cases (ii) and (ii): Because* $\frac{\partial^2\Delta\pi}{\partial\delta^2} < 0$, $\frac{\partial\Delta\pi}{\partial\delta}\big|_{\delta=1} > 0$, $\frac{\partial\Delta\pi}{\partial\delta}\big|_{\delta=1+\frac{3D_1D_2+3D_2^2}{D_1^2}} > 0$ *if* $D_1 > 2D_2$*, and* $\Delta\pi\big|_{\delta=1} < 0$ *if* $\frac{D_1+D_2-\sqrt{D_2(D_1+D_2)}}{2} - K_L < K_H < \frac{D_1+D_2+\sqrt{D_2(D_1+D_2)}}{2} - K_L$. *Note that the range of the shaded area is given by* $\frac{D_1^2}{3D_1+4D_2} < K_H < \frac{D_1^2+2D_1D_2}{4D_1+6D_2}$ *along with* $\frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{3D_1^2+8D_1D_2+6D_2^2} < K_L < \frac{D_1+2D_2}{4}$. *So, we conclude that* $\Delta\pi\big|_{\delta=1} < 0$.

(g) *The right boundary point of condition 1 in case 2 has intersactions with case (iv) and (v), and* $\Delta\pi^{case2-(iv)} > 0$*, and* $\frac{\partial\Delta\pi^{case2-(v)}}{\partial\delta} > 0$*, and* $\Delta\pi^{case2-(v)}\big|_{\delta=1} > 0$. *The left boundary point of condition 1 in case 2 has intersaction with case (vi), and* $\Delta\pi^{case2-(vi)} < 0$.

Proof 37 *Proof of Proposition* 3.4.2. *Recall that*

$$D_{OH} = \begin{cases} \frac{D_2}{4} & if\theta_{1L} > 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}, \theta_{1H} \leq 1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1}, \\ \frac{K_H - D_1(1-\theta_{1H})}{2} & if\theta_{1L} > 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}, \theta_{1H} > 1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1}, \\ \frac{D_2(K_H - D_1(1-\theta_{1H}))}{2(K_H+K_L-D_1(1-\theta_{1L}))} & if\theta_{1L} \in (1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}, 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}], \\ K_H - D_1(1-\theta_{1H}) & if\theta_{1L} \leq 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}. \end{cases}$$

Upgrading mechanism prevails if and only if $K_H - D_1(1-\theta_{1H}) - D_{OH} > 0$. *The optimal solutions satisfy conditions (a) to (d), modified condition (e) (i.e.,* $\theta_U \in [\max\{\theta_{1L}, 1 - \frac{K_H-D_{OH}}{D_1}\}, \theta_{1H}]$*) and newly defined condition (f) (i.e.,* $\theta_{1H} > 1 - \frac{K_H-D_{OH}}{D_1}$*).*

(i) $\theta_{1L} > 1 + \frac{D_2}{D_1} - \frac{K_H + K_L}{D_1}$ *&* $1 + \frac{D_2}{4D_1} - \frac{K_H}{D_1} < \theta_{1H} \leq 1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1}$

The seller's total profit equals

$$\pi_{J1} = D_1(\theta_{1L} - \theta_{1L}^2) + D_1(\delta - 1)(\theta_{1H} - \theta_{1H}^2) + \frac{D_2(\delta + 1)}{8} + \pi_{2U}^* - F_O, \qquad \text{(B.1)}$$

where $\pi_{2U}^* = \frac{D_1(\delta-1)\theta_{1H}^2}{4}$ *if* $\theta_U = \frac{\theta_{1H}}{2}$, $\pi_{2U}^* = D_1(\delta - 1)(\theta_{1H}\theta_L - \theta_{1L}^2)$ *if* $\theta_U = \theta_{1L}$, *or* $\pi_{2U}^* = (\delta - 1)(1 - \frac{K_H - D_{OH}}{D_1})(K_H - D_1(1 - \theta_{1H}) - D_{OH})$ *if* $\theta_U = 1 - \frac{K_H - D_{OH}}{D_1}$. *Moreover,* $\phi^* = \frac{1}{2}$.

(a) $\frac{\theta_{1H}}{2} \geq \max\{\theta_{1L}, 1 - \frac{K_H - \frac{D_2}{4}}{D_1}\}$

Equation B.1 becomes $\pi_{J1} = D_1(\theta_{1L} - \theta_{1L}^2) + D_1(\delta - 1)(\theta_{1H} - \theta_{1H}^2) + \frac{D_1(\delta-1)\theta_{1H}^2}{4} + \frac{D_2(\delta+1)}{8} - F_O$, *the Hessian matrix of* π_{J1} *is negative definite, hence,* $\theta_{1H}^* = \frac{2}{3}$, $\theta_{1L}^* = \theta_O^* = \frac{1}{2}$ *and* $\theta_U^* = \frac{1}{3}$. *While* $\theta_U^* < \theta_{1L}^*$, *which is unreasonable.*

(b) $\frac{\theta_{1H}}{2} < \theta_{1L}$, $\theta_{1L} > 1 - \frac{K_H - \frac{D_2}{4}}{D_1}$

The Hessian matrix of π_{J1} *is negative definite, hence,* $\theta_{1H}^* = \frac{2\delta+1}{3\delta+1}$, $\theta_U^* = \theta_{1L}^* = \frac{\delta+1}{3\delta+1}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{1}{2}$, *and* $\pi_{J1}^* = \frac{(8D_1+3D_2)\delta^2+4D_2\delta+D_2}{8+24\delta} - F_O$ *provided that* $\frac{2\delta D_1}{3\delta+1} + \frac{D_2}{4D_1} < K_H \leq \frac{\delta D_1}{3\delta+1} + \frac{D_2}{2}$, $K_L > \frac{\delta D_1}{3\delta+1}$ *and* $K_H + K_L > \frac{2\delta}{3\delta+1}D_1 + D_2$. *Define* $g_1(\delta) = (3K_H - D_1)\delta + K_H > 0$, $g_2(\delta) = (3K_L - D_1)\delta + K_L > 0$, $g_3(\delta) = (3(K_H + K_L) - 2D_1 - 3D_2)\delta - D_2 + K_H + K_L > 0$, $g_4(\delta) = (2D_1 + 3D_2 - 6K_H)\delta + D_2 - 2K_H \geq 0$, *and* $g_5(\delta) = (12K_H - 8D_1 - 3D_2)\delta + 4K_H - D_2 > 0$, *then above conditions hold if* $K_H > \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$, $K_L > \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$, $K_H > \frac{2D_1^3+10D_1^2D_2+15D_1D_2^2+9D_2^3}{4D_1^2+9D_1D_2+9D_2^2} - K_L$, $K_H \leq \frac{D_1+2D_2}{4}$, *and* $K_H > \frac{8D_1^3+28D_1^2D_2+33D_1D_2^2+9D_2^3}{16D_1^2+36D_1D_2+36D_2^2}$, *respectively. So, the domain is given by* $\max\{\frac{2D_1^3+10D_1^2D_2+15D_1D_2^2+9D_2^3}{4D_1^2+9D_1D_2+9D_2^2} - K_L, \frac{8D_1^3+28D_1^2D_2+33D_1D_2^2+9D_2^3}{16D_1^2+36D_1D_2+36D_2^2}\} < K_H \leq \frac{D_1+2D_2}{4}$ *as well as* $K_L > \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$.

(c) $\frac{\theta_{1H}}{2} < 1 - \frac{K_H - \frac{D_2}{4}}{D_1}$, $1 - \frac{K_H - \frac{D_2}{4}}{D_1} > \theta_{1L}$

The Hessian matrix of π_{J1} *is negative definite, hence,* $\theta_{1H}^* = 1 - \frac{4K_H - D_2}{8D_1}$, $\theta_O^* = \theta_{1L}^* = \frac{1}{2}$, $\phi^* = \frac{1}{2}$, $\pi_{J1}^* = \frac{8D_1((3-\delta)D_2+8(\delta-1)K_H)-3(\delta-1)(D_2-4K_H)^2+16D_1^2}{64D_1} - F_O$ *provided that* $\frac{D_2}{4} \leq K_H \leq \frac{3D_2}{4}$ *and* $K_H + K_L > \frac{D_1}{2} + D_2$.

(ii) $\theta_{1L} > 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}$ *&* $\theta_{1H} > 1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1}$

Equation B.1 becomes $\pi_{J1} = D_1(\theta_{1L}-\theta_{1L}^2)+D_1(\delta-1)(\theta_{1H}-\theta_{1H}^2)+\frac{D_2+(\delta-1)(K_H-D_1(1-\theta_{1H}))}{4}+\pi_{2U}^* - F_O$, $D_{OH} = \frac{K_H-D_1(1-\theta_{1H})}{2}$, *and* $\phi = \frac{K_H-D_1(1-\theta_{1H})}{D_2}$.

(a) $\frac{\theta_{1H}}{2} \geq \max\{\theta_{1L}, 1 - \frac{K_H - \frac{K_H-D_1(1-\theta_{1H})}{2}}{D_1}\}$

The optimal solutions equal $\theta_{1H}^* = \frac{5}{6}$, $\theta_U^* = \frac{5}{12}$, $\theta_{1L}^* = \theta_O^* = \frac{1}{2}$, *and* $\phi^* = \frac{6K_H-D_1}{6D_2}$, *while* $\theta_U^* < \theta_{1L}^*$ *does not hold.*

(b) $\frac{\theta_{1H}}{2} < \theta_{1L}$, $\theta_{1L} > 1 - \frac{K_H - \frac{K_H-D_1(1-\theta_{1H})}{2}}{D_1}$

The Hessian matrix of π_{J1} *is negative definite, hence,* $\theta_{1H}^* = \frac{5\delta+2}{6\delta+2}$, $\theta_U^* = \theta_{1L}^* = \frac{5\delta+3}{12\delta+4}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{(6\delta+2)K_H-\delta D_1}{(6\delta+2)D_2}$, *and* $\pi_{J1}^* = \frac{(13D_1+12K_H)\delta^2+(3D_1+12D_2-8K_H)\delta+4D_2-4K_H}{48\delta+16} - F_O$.

Because conditions $g_6(\delta) = (12K_L - 5D_1)\delta + 4K_L - D_1 > 0$, $g_7(\delta) = (12(K_H + K_L) - 7D_1 - 12D_2)\delta - D_1 - 4D_2 + 4(K_H + K_L) > 0$, $g_8(\delta) = (6K_H - D_1 - 3D_2)\delta + 2K_H - D_2 > 0$, $g_9(\delta) = (6K_H - 6D_1)\delta + 2K_H - D_1 > 0$ *and* $g_{10}(\delta) = (6K_H - 6D_2 - D_1)\delta + 2(K_H - D_2) \leq 0$ *reduce to* $K_L > \frac{6D_1^3+15D_1^2D_2+15D_1D_2^2}{16D_1^2+36D_1D_2+36D_2^2}$, $K_H > \frac{8D_1^3+37D_1^2D_2+57D_1D_2^2+36D_2^3}{16D_1^2+36D_1D_2+36D_2^2} - K_L$, $K_H > \frac{D_1^3+7D_1^2D_2+12D_1D_2^2+9D_2^3}{8D_1^2+18D_1D_2+18D_2^2}$, $K_H > \frac{7D_1^3+18D_1^2D_2+18D_1D_2^2}{8D_1^2+18D_1D_2+18D_2^2}$, *and* $K_H \leq \frac{D_1+8D_2}{8}$ *respectively. So, the effective domain is given by* $\max\{\frac{8D_1^3+37D_1^2D_2+57D_1D_2^2+36D_2^3}{16D_1^2+36D_1D_2+36D_2^2} - K_L, \frac{7D_1^3+18D_1^2D_2+18D_1D_2^2}{8D_1^2+18D_1D_2+18D_2^2}\} < K_H \leq \frac{D_1+8D_2}{8}$ *provided that* $24D_1^3 + 31D_2D_1^2 - 9D_2^2D_1 - 72D_2^3 < 0$.

(c) $\frac{\theta_{1H}}{2} < 1 - \frac{K_H - \frac{K_H-D_1(1-\theta_{1H})}{2}}{D_1}$, $1 - \frac{K_H - \frac{K_H-D_1(1-\theta_{1H})}{2}}{D_1} > \theta_{1L}$

The optimal solutions equal $\theta_{1H}^* = \frac{5}{6}$, $\theta_O^* = \theta_{1L}^* = \frac{1}{2}$, $\theta_U^* = \frac{11}{12} - \frac{K_H}{2D_1}$, $\phi^* = \frac{6K_H-D_1}{6D_2}$, *and* $\pi_{J1}^* = \frac{(D_1^2+36D_1K_H+12K_H^2)(\delta-1)+12D_1^2+12D_1D_2}{48D_1} - F_O$ *provided that* $K_L > \frac{D_1}{3}$ *and* $\max\{\frac{D_1+3D_2}{6}, \frac{D_1+2D_2}{2} - K_L\} < K_H < \min\{\frac{D_1+6D_2}{6}, \frac{5D_1}{6}\}$.

(iii) $1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1} < \theta_{1L} \leq 1 + \frac{D_2}{D_1} - \frac{K_H}{D_1}$

Equation B.1 becomes $\pi_{J1} = (\delta-1)D_1\left(\theta_H - \theta_H^2\right)+\frac{(\delta-1)D_2(K_H-D_1(1-\theta_H))}{4(D_1(\theta_L-1)+K_H+K_L)}+D_1\left(\theta_L - \theta_L^2\right)+\frac{D_2}{4} + \pi_{2U}^*$, *and* $D_{OH} = \frac{D_2(K_H-D_1(1-\theta_{1H}))}{2(K_H+K_L-D_1(1-\theta_{1L}))}$.

(a) $\theta_U = \frac{\theta_{1H}}{2}$

If the Hessian matrix of π_{J1} is negative definite, then the optimal solutions are obtained by solving the FOCs of π_{J1} w.r.t θ_{1H} and θ_{1L}:

$$\frac{\partial \pi_{J1}}{\partial \theta_{1H}} = D_1(\delta-1)(1-2\theta_{1H}+\frac{\theta_{1H}}{2}) + \frac{D_1D_2(\delta-1)}{4(K_H+K_L-D_1(1-\theta_{1L}))} = 0,$$

$$\frac{\partial \pi_{J1}}{\partial \theta_{1L}} = D_1(1-2\theta_{1L}) - \frac{D_1D_2(\delta-1)(K_H-D_1(1-\theta_{1H}))}{4(K_H+K_L-D_1(1-\theta_{1L}))^2} = 0.$$

Because $\theta_{1H}^* \neq \theta_{1L}^*$, $\theta_{1H}^* \neq 1$, $\theta_O^* = \frac{1}{2}$. Note that $\frac{\partial \pi_{J1}}{\partial \theta_{1L}} < 0$ if $D_1+D_2 \geq 2(K_H+K_L)$. Hence, there are no optimal solutions when $D_1+D_2 \geq 2(K_H+K_L)$.

Boundary solution.

The optimal interior solution is a convex combination of the boundary solutions.

(a1) $\theta_{1L}^* = 1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = 1$, $\theta_U^* = \theta_O^* = \frac{1}{2}$, $\phi^* = \frac{2K_H}{D_2}$, *and* $\pi_{J1}^* = \frac{(\delta-1)D_1^2+D_1(-D_2+2(\delta+1)K_H+4K_L)-(D_2-2(K_H+K_L))^2}{4D_1} - F_O$. *The effective domain is given by* $\frac{D_1+D_2}{2} - K_L \leq K_H \leq \min\{\frac{D_2}{2}, \frac{2D_1+D_2}{2} - K_L\}$.

(a2) $\theta_{1L}^* = 1+\frac{D_2}{D_1}-\frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = \frac{5}{6}$, $\theta_U^* = \frac{5}{12}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{6K_H-D_1}{6D_2}$, *and* $\pi_{J1}^* = \frac{13(\delta-1)D_1^2+12D_1(-3D_2+(\delta+3)K_H+4K_L)-48(-D_2+K_H+K_L)^2}{48D_1} - F_O$. *The effective domain is given by* $\max\{\frac{5D_1+12D_2}{12} - K_L, \frac{D_1}{6}\} \leq K_H \leq \min\{\frac{D_1+6D_2}{6}, D_1+D_2-K_L\}$.

(b) $\theta_U = \theta_{1L}$

If the Hessian matrix of π_{J1} is negative definite, then the optimal solutions are obtained by solving the FOCs of π_{J1} w.r.t θ_{1H} and θ_{1L}:

$$\frac{\partial \pi_{J1}}{\partial \theta_{1H}} = D_1(\delta-1)(1-2\theta_{1H}+\theta_{1L}) + \frac{D_1D_2(\delta-1)}{4(K_H+K_L-D_1(1-\theta_{1L}))} = 0,$$

$$\frac{\partial \pi_{J1}}{\partial \theta_{1L}} = D_1(1-2\theta_{1L}) + D_1(\delta-1)(\theta_{1H}-2\theta_{1L}) - \frac{D_1D_2(\delta-1)(K_H-D_1(1-\theta_{1H}))}{4(K_H+K_L-D_1(1-\theta_{1L}))^2} = 0.$$

Because $\theta_{1H}^* \neq \theta_{1L}^*$, $\theta_{1H}^* \neq 1$, $\theta_O^* = \frac{1}{2}$. Note that $\frac{\partial \pi_{J1}}{\partial \theta_{1L}} < 0$ if $D_1+D_2 \geq 2(K_H+K_L)$. Hence, there are no optimal solutions.

Boundary solution.

The optimal interior solution is a convex combination of the boundary solutions.

(b1) $\theta_U^* = \theta_{1L}^* = 1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = \frac{5D_1+D_2-2(K_H+K_L)}{4D_1}$, $\theta_O^* = \frac{1}{2}$, $\phi^* =$

$\frac{D_1+D_2+2K_H-2K_L}{2D_2}$, and $\pi^*_{J1} = \frac{(\delta-1)D_1^2+2D_1((1-3\delta)D_2+2(5\delta-1)K_H+2(3\delta+1)K_L)-(3\delta+1)(D_2-2(K_H+K_L))^2}{16D_1}$ $- F_O$.

(b2) $\theta^*_U = \theta^*_{1L} = 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}$, $\theta^*_{1H} = \frac{9D_1+4D_2-4K_H-4K_L}{8D_1}$, $\theta^*_O = \frac{1}{2}$, $\phi^* = \frac{4(D_2+K_H-K_L)+D_1}{8D_2}$, and $\pi^*_{J1} = \frac{(\delta-1)D_1^2+8D_1((1-7\delta)D_2+(9\delta-1)K_H+(7\delta+1)K_L)-16(3\delta+1)(-D_2+K_H+K_L)^2}{64D_1}$ $- F_O$.

(c) $\theta_U = 1 - \frac{K_H-D_{OH}}{D_1}$

Equation B.1 becomes $\pi_{J1} = D_1(\theta_{1L}-\theta_{1L}^2)+D_1(\delta-1)(\theta_{1H}-\theta_{1H}^2)+\frac{D_2(\delta-1)(K_H-D_1(1-\theta_{1H}))}{4(K_H+K_L-D_1(1-\theta_{1L}))}+\frac{D_2}{4}+(\delta-1)(K_H-D_1(1-\theta_{1H})-D_{OH})(1-\frac{K_H-D_{OH}}{D_1})$, and $D_{OH} = \frac{D_2(K_H-D_1(1-\theta_{1H}))}{2(K_H+K_L-D_1(1-\theta_{1L}))}$.

So, $\pi_{J1} = \frac{1}{4}\left(D_2 - 4(\delta-1)D_1\left(\theta_{1H}-1\right)\theta_{1H}\right) + \frac{(\delta-1)D_2(D_1(\theta_{1H}-1)+K_H)}{4(D_1(\theta_{1L}-1)+K_H+K_L)}$

$+(\delta-1)\left(1-\frac{K_H-\frac{D_2(D_1(\theta_{1H}-1)+K_H)}{2(D_1(\theta_{1L}-1)+K_H+K_L)}}{D_1}\right)\left(D_1\left(\theta_{1H}-1\right)-\frac{D_2(D_1(\theta_{1H}-1)+K_H)}{2(D_1(\theta_{1L}-1)+K_H+K_L)}+K_H\right)-D_1\left(\theta_{1L}-1\right)\theta_{1L}$.

By deriving the FOCs w.r.t θ_{1H} *and* θ_{1L} *and equating them to zero, we obtain the relationship between* θ_{1H} *and* θ_{1L}.

Boundary solution. *The optimal interior solution is a convex combination of the boundary solutions.*

(a1) $\theta^*_U = \theta^*_{1L} = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta^*_{1H} = \theta^*_U = \frac{3}{4}$, $\theta^*_O = \frac{1}{2}$, $\phi^* = \frac{4K_H-D_1}{2D_2}$, and $\pi^*_{J1} = \frac{(\delta-1)D_1^2+D_1(-4D_2+8(\delta+1)K_H+16K_L)-4(D_2-2(K_H+K_L))^2}{16D_1} - F_O$.

(a2) $\theta^*_U = \theta^*_{1L} = 1+\frac{D_2}{D_1}-\frac{K_H+K_L}{D_1}$, $\theta^*_{1H} = \frac{5}{6}$, $\theta^*_U = \frac{11}{12}-\frac{K_H}{2D_1}$, $\theta^*_O = \frac{1}{2}$, $\phi^* = \frac{6K_H-D_1}{6D_2}$, and $\pi^*_{J1} = \frac{(\delta-1)D_1^2+12D_1(-3D_2+3\delta K_H+K_H+4K_L)-12\left(-8D_2(K_H+K_L)+4D_2^2+(\delta+3)K_H^2+8K_HK_L+4K_L^2\right)}{48D_1} - F_O$.

(iv) $\theta_{1L} \le 1 + \frac{D_2}{2D_1} - \frac{K_H}{D_1}$

If $D_{OH} = K_H - D_1(1-\theta_{1H})$, *then* $\pi_{J1} = D_1(\theta_{1L}-\theta_{1L}^2)+D_1(\delta-1)(\theta_{1H}-\theta_{1H}^2)+(1-\frac{K_H+K_L-D_1(1-\theta_{1H})}{D_2})(K_H\delta+K_L-D_1\delta+D_1\theta_{1L}+D_1(\delta-1)\theta_{1H})$. *The optimal outcomes mimick the one in pure opaque selling (i.e., case (i) of Lemma 3.3.3.2).*

To summarize,

(i) *If* $\frac{D_2}{4} \le K_H \le \frac{3D_2}{4}$, *and* $K_H+K_L > \frac{D_1}{2}+D_2$, *then* $\theta^*_{1H} = 1-\frac{4K_H-D_2}{8D_1}$, $\theta^*_O = \theta^*_{1L} = \frac{1}{2}$, $\theta^*_U = 1 - \frac{4K_H-D_2}{4D_1}$, *and* $\pi^*_{J1} = \frac{8D_1((3-\delta)D_2+8(\delta-1)K_H)-3(\delta-1)(D_2-4K_H)^2+16D_1^2}{64D_1} - F_O$.

(ii) *If* $\max\{\frac{8D_1^3+37D_1^2D_2+57D_1D_2^2+36D_2^3}{16D_1^2+36D_1D_2+36D_2^2}-K_L, \frac{7D_1^3+18D_1^2D_2+18D_1D_2^2}{8D_1^2+18D_1D_2+18D_2^2}\} < K_H \leq \frac{D_1+8D_2}{8}$ *provided that* $24D_1^3+31D_2D_1^2-9D_2^2D_1-72D_2^3<0$, *then* $\theta^*_{1H}=\frac{5\delta+2}{6\delta+2}$, $\theta^*_U=\theta^*_{1L}=\frac{5\delta+3}{12\delta+4}$, $\phi^*=\frac{(6\delta+2)K_H-\delta D_1}{(6\delta+2)D_2}$, *and* $\theta^*_O=\frac{1}{2}$, *and* $\pi^*_{J1}=\frac{(13D_1+12K_H)\delta^2+(3D_1+12D_2-8K_H)\delta+4D_2-4K_H}{48\delta+16}-F_O$.

(iii) *If* $\max\{\frac{D_1+3D_2}{6}, \frac{D_1+2D_2}{2}-K_L\} < K_H < \min\{\frac{D_1+6D_2}{6}, \frac{5D_1}{6}\}$ *and* $K_L > \frac{D_1}{3}$, *then* $\theta^*_{1H}=\frac{5}{6}$, $\theta^*_O=\theta^*_{1L}=\frac{1}{2}$, $\theta^*_U=\frac{11}{12}-\frac{K_H}{2D_1}$, $\phi^*=\frac{6K_H-D_1}{6D_2}$, *and* $\pi^*_{J1}=\frac{(D_1^2+36D_1K_H+12K_H^2)(\delta-1)+12D_1^2+12D_1D_2}{48D_1}-F_O$.

(iv) *If* $\frac{D_1}{2D_2+D_1}K_L < K_H \leq \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2}-K_L\}$, *then* $\theta^*_{1H}=\theta^*_U=\frac{-D_1^2\delta+D_1K_H\delta+D_1^2+4D_1D_2+4D_2^2-D_1K_H-2D_2K_H-2D_2K_L}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$, $\theta^*_O=\frac{(2D_1K_H-D_1^2)(\delta-1)+4D_2(D_1+D_2-(K_H+K_L))}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$, $\theta^*_{1L}=\frac{-D_1^2\delta+D_1K_H\delta+D_1^2+4D_1D_2+4D_2^2-D_1K_H-2D_2K_H-4D_2K_L-2D_2K_H\delta+D_1K_L\delta-D_1K_L}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$, *and* $\pi^*_{J1}=$

$$\frac{\delta^2D_1K_H(D_1-K_H)+\delta(D_1^2(-K_H+K_L)+4D_2K_H(-D_2+K_H+K_L)-D_1(4D_2K_H-K_H^2+2K_HK_L+K_L^2))}{4(D_1+D_2)D_2-D_1^2(\delta-1)}$$
$$+\frac{K_L(-D_1^2+4D_2(-D_2+K_H+K_L)+D_1(-4D_2+2K_H+K_L))}{4(D_1+D_2)D_2-D_1^2(\delta-1)}-F_O.$$

(v) *If* $\frac{D_1}{4} < K_H < \min\{\frac{8D_1K_L-D_1(D_1+D_2)}{4D_1+12D_2}, \frac{2D_1+D_2}{2}-K_L, \frac{D_1+2D_2}{4}\}$, *then* $\theta^*_{1H}=\theta^*_U=\frac{3}{4}$, $\theta^*_{1L}=1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}$, $\theta^*_O=\frac{1}{2}$, *and* $\pi^*_{J1}=\frac{(D_1^2+8D_1K_H)\delta-D_1^2-4D_1D_2+8D_1K_H+16D_1K_L-4(D_2-2(K_H+K_L))^2}{16D_1}-F_O$.

Overlap Characterization

Cases (iii) and (i): $\Delta\pi^{i-iii}>0$ *if* $\frac{\left(6D_1+9D_2-\sqrt{3(2D_1-3D_2)^2}\right)}{24} < K_H < \frac{\left(6D_1+9D_2+\sqrt{3(2D_1-3D_2)^2}\right)}{24}$. *Note that the intersection between these two cases is* $\frac{D_1+3D_2}{6} < K_H < \frac{3D_2}{4}$ *provided that* $2D_1 < 3D_2$. *So, case (i) dominates case (iii) if* $\frac{D_1+3D_2}{6} < K_H < \frac{\left(6D_1+9D_2+\sqrt{3(2D_1-3D_2)^2}\right)}{24}$, *otherwise, case (iii) dominates case (i).*

Cases (v) and (i): Because $\frac{\partial\Delta\pi^{i-v}}{\partial\delta}<0$ *if* $K_H<\frac{2D_1+3D_2}{12}$, *and* $\Delta\pi^{i-v}|_{\delta=1}>0$. *So,* $\Delta\pi^{i-v}>0$ *if* $K_H>\frac{2D_1+3D_2}{12}$ *or if* $K_H<\frac{2D_1+3D_2}{12}$ *and* $\delta<\frac{-64K_L(D_1+D_2-2K_H)-96D_1K_H-88D_2K_H+20D_1^2+40D_2D_1+19D_2^2+112K_H^2+64K_L^2}{(3(D_2-4K_H)+2D_1)(2D_1+D_2-4K_H)}$.

Cases (v) and (iii): Because $\frac{\partial\Delta\pi^{iii-v}}{\partial\delta}>0$ *if* $K_H>\frac{(\sqrt{15}-3)D_1}{6}$, $\frac{\partial\Delta\pi^{iii-v}}{\partial\delta}>0$, *and* $\Delta\pi^{iii-v}|_{\delta=1}>0$. *So, case (iii) dominates case (v).*

Proof 38 *Proof of Theorem* 3.4.3.1. *Comparison between the sum of Proposition* 3.4.1.2 *and Lemma* 3.3.1 *and the sum of Lemma* 3.3.2.1, *and Lemma* 3.3.3.2.

(i) $K_H > \frac{D_1+D_2}{2}$.

If $K_H > \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3}{8D_1^2+18D_1D_2+18D_2^2}$, *then* $\pi^J + \pi^P - (\pi^O + \pi^U) = -\frac{D_1(\delta-1)(7\delta-3)}{64(1+3\delta)} < 0$.

(ii) $\frac{D_1+D_2}{2} - K_L < K_H < \frac{D_1+D_2}{2}$.

(a) *If* $\max\{\frac{4D_1+3D_2}{8}, \frac{7D_1+8D_2}{16}\} < K_H < \frac{D_1+D_2}{2}$, *then*

$$\pi^J + \pi^P - (\pi^O + \pi^U)$$
$$= \frac{(\delta-1)\left(D_1^2\left(\left(355\delta^2+210\delta+59\right)D_2 - 64\left(3\delta^2+10\delta+3\right)K_H\right) + 8(3\delta+1)^2 D_2^2\left(D_2-2K_H\right)\right)}{64(3\delta+1)\left(D_1+D_2\right)\left(4(\delta-1)D_1+(3\delta+1)D_2\right)}$$
$$+ \frac{(\delta-1)\left(4\left(37\delta^2+18\delta+9\right)D_1^3 + (3\delta+1)D_1\left(31(3\delta+1)D_2^2 - 16(7\delta+13)D_2K_H - 64(\delta-5)K_H^2\right)\right)}{64(3\delta+1)\left(D_1+D_2\right)\left(4(\delta-1)D_1+(3\delta+1)D_2\right)}.$$

Because $\frac{\partial(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_H}\Big|_{K_H=\frac{D_1+D_2}{2}} < 0$, $\frac{\partial(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_H}\Big|_{K_H=\frac{4D_1+3D_2}{8}} < 0$, *and* $\frac{\partial(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_H}\Big|_{K_H=\frac{7D_1+8D_2}{16}} < 0$ *if* $D_1 < 2D_2$, *then the profit gap decreases with* K_H, $\pi^J + \pi^P - (\pi^O + \pi^U)\big|_{K_H=\frac{4D_1+3D_2}{8}} > 0$ *if* $D_1 < 2D_2$, $\pi^J + \pi^P - (\pi^O + \pi^U)\big|_{K_H=\frac{7D_1+8D_2}{16}} > 0$, *and* $\pi^J + \pi^P - (\pi^O + \pi^U)\big|_{K_H=\frac{D_1+D_2}{2}} < 0$. *Hence,* $\pi^J + \pi^P - (\pi^O + \pi^U) > 0$ *if* $K_H > K_{H1}$, *where* K_{H1} *solves the equation* $\pi^J + \pi^P - (\pi^O + \pi^U) = 0$.

(b) *If* $\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2} < K_H < \frac{4D_1+3D_2}{8}$ *provided that* $\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2} > \frac{7D_1+8D_2}{16}$, *or if* $\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2} < K_H < \frac{7D_1+8D_2}{16}$ *provided that* $\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2} < \frac{7D_1+8D_2}{16}$, *then*

$$\pi^J + \pi^P - (\pi^O + \pi^U) = \frac{1}{64}(\delta-1)\left(D_1\left(\frac{37\delta+7}{3\delta+1} - \frac{64K_H^2}{\left(D_1+D_2\right)\left(4D_1+3D_2\right)}\right) + 8\left(D_2-2K_H\right)\right).$$

Because $\frac{\partial^2(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_H^2} < 0$, $\frac{\partial(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_H}\Big|_{K_H=\frac{2D_1^3+6D_2D_1^2+6D_2^2D_1}{4D_1^2+9D_2D_1+9D_2^2}} < 0$, *and* $\frac{\partial(\pi+\pi^P-(\pi^O+\pi^U))}{\partial K_H}\Big|_{K_H=\frac{4D_1+3D_2}{8}} < 0$. *Therefore, the profit gap decreases with* K_H, *and the minimum value of the profit gap satisfies* $\pi^J + \pi^P - (\pi^O + \pi^U)\big|_{K_H=\frac{4D_1+3D_2}{8}} > 0$ *if* $-D_1^2+2D_1D_2+2D_2^2 > 0$, *under which we have* $\pi^J+\pi^P-(\pi^O+\pi^U) > 0$. *The maximum value is obtained at* $K_H = \frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$. *If the maximum value is positive while the minimum value is negative, then* $\pi^J + \pi^P - (\pi^O + \pi^U) > 0$ *if* $K_H > K_{H2}$, *which solves the equation* $\pi^J + \pi^P - (\pi^O + \pi^U) = 0$.

(c) If $\frac{7D_1+8D_2}{16} < K_H < \frac{4D_1+3D_2}{8}$ provided that $\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2} < \frac{7D_1+8D_2}{16}$, then

$$\pi^J + \pi^P - (\pi^O + \pi^U) = \frac{1}{4}(\delta - 1)D_1 \left(\frac{\delta}{3\delta + 1} - \frac{4K_H^2}{(D_1 + D_2)(4D_1 + 3D_2)} \right).$$

Because $\frac{\partial(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_H} < 0$, $\pi^J + \pi^P - (\pi^O + \pi^U)\big|_{K_H=\frac{4D_1+3D_2}{8}} > 0$ if $15D_1^2 - 16D_2^2 > 0$, and $\pi^J + \pi^P - (\pi^O + \pi^U)\big|_{K_H=\frac{7D_1+8D_2}{16}} > 0$. Hence, if $15D_1^2 > 16D_2^2$, then $\pi^J + \pi^P - (\pi^O + \pi^U) > 0$. Otherwise, $\pi^J + \pi^P - (\pi^O + \pi^U) > 0$ if $K_H > \frac{\sqrt{\delta}\sqrt{D_1+D_2}\sqrt{4D_1+3D_2}}{2\sqrt{3\delta+1}}$.

(d) If $\frac{D_1}{2} < K_H < \min\{\frac{D_1+3D_2}{6}, \frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}\}$ provided that $2D_1 < 3D_2$, then

$$\begin{aligned}
&\pi^J + \pi^P - (\pi^O + \pi^U) \\
&= -\frac{D_2\left((3\delta + 23)D_2^2 - 4D_2\left((3\delta + 17)K_H + 20K_L\right) + 56\left(K_H + K_L\right)^2\right)}{8\left(D_1 + D_2\right)\left(4D_1 + 3D_2\right)} \\
&- \frac{D_1\left((11\delta + 31)D_2^2 - 4D_2\left((7\delta + 15)K_H + 22K_L\right) + 8\left((\delta + 3)K_H^2 + 8K_HK_L + 4K_L^2\right)\right)}{8\left(D_1 + D_2\right)\left(4D_1 + 3D_2\right)} \\
&- \frac{8(2\delta + 1)D_1^4 + D_1^3\left(10(4\delta + 5)D_2 - 48\left((\delta + 1)K_H + 2K_L\right)\right) + 18D_2^2\left(D_2 - 2\left(K_H + K_L\right)\right)^2}{24D_1\left(D_1 + D_2\right)\left(4D_1 + 3D_2\right)}.
\end{aligned}$$

Because $\frac{\partial(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_L} < 0$, and $\pi^J + \pi^P - (\pi^O + \pi^U)\big|_{K_L=\frac{D_1+3D_2}{4}} < 0$, hence, $\pi^J + \pi^P - (\pi^O + \pi^U) < 0$.

(e)

$$\pi^J + \pi^P - (\pi^O + \pi^U) = \frac{1}{48}(\delta - 1)D_1 \left(1 - \frac{48K_H^2}{(D_1 + D_2)(4D_1 + 3D_2)} \right).$$

Note that $\pi^J + \pi^P - (\pi^O + \pi^U) > 0$ if $K_H > \frac{\sqrt{D_1+D_2}\sqrt{4D_1+3D_2}}{4\sqrt{3}}$.

(f)

$$\begin{aligned}
&\pi^J + \pi^P - (\pi^O + \pi^U) \\
&= \frac{3D_2^2\left(-4D_2\left(K_H + K_L\right) + D_2^2 + (7 - 3\delta)K_H^2 + 8K_HK_L + 4K_L^2\right)}{4D_1\left(D_1 + D_2\right)\left(4D_1 + 3D_2\right)} \\
&+ \frac{D_2\left(D_2\left((6\delta - 46)K_H - 40K_L\right) + 10D_2^2 + 7\left((7 - 3\delta)K_H^2 + 8K_HK_L + 4K_L^2\right)\right)}{4\left(D_1 + D_2\right)\left(4D_1 + 3D_2\right)} \\
&+ \frac{D_1^3\left((51 - 7\delta)D_2 + 32\left((\delta - 3)K_H - 2K_L\right)\right) - 4(\delta - 5)D_1^4}{16D_1\left(D_1 + D_2\right)\left(4D_1 + 3D_2\right)} \\
&+ \frac{D_1\left((59 - 3\delta)D_2^2 + 8D_2\left((7\delta - 29)K_H - 22K_L\right) - 64\left((\delta - 2)K_H^2 - 2K_HK_L - K_L^2\right)\right)}{16\left(D_1 + D_2\right)\left(4D_1 + 3D_2\right)}.
\end{aligned}$$

Because $\frac{\partial(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_H}$ decreases with δ if $K_H > \frac{4D_1^3+7D_2D_1^2+3D_2^2D_1}{16D_1^2+21D_2D_1+9D_2^2}$, and $\frac{\partial(\pi^J+\pi^P-(\pi^O+\pi^U))}{\partial K_H}\Big|_{\delta=1} < 0$. Therefore, the profit gap decreases with K_H and *the maximum value is obtained at* $K_H = 0$*:* $\pi^J + \pi^P - (\pi^O + \pi^U)\big|_{K_H=0} = \frac{(5-\delta)D_1^2+4D_1(D_2-4K_L)+4(D_2-2K_L)^2}{16D_1}$.

(iii) $K_H < \frac{D_1+D_2}{2} - K_L$. $\frac{D_1}{D_1+2D_2}K_L < K_H \leq \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2} - K_L\}$. *Profit comparison reduces to comparison between Scenarios U and P, hence,* $\pi^J + \pi^P - (\pi^O + \pi^U) < 0$.

Proof 39 *Proof of Theorem* 3.4.3.2. *By comparing Proposition 3.4.1.2 and Lemmas 3.3.1, 3.3.2.1, and 3.3.3.2, we have the following:*

(i) $\frac{D_1(1+t)}{2} < K_H \leq K_L$.

(a) *If* $\frac{7D_1^3+18D_2D_1^2+18D_2^2D_1}{2\left(4D_1^2+9D_2D_1+9D_2^2\right)} > \frac{D_1+D_2}{2}$*, then*

$$\pi^*_{J1} + \pi^*_P - (\pi^*_O + \pi^*_U) = \frac{(\delta-1)(9\delta+7)D_1}{64(3\delta+1)} > 0.$$

(b) *If* $\max\{\frac{7D_1+8D_2}{16}, \frac{4D_1+3D_2}{8}\} < \frac{7D_1^3+18D_2D_1^2+18D_2^2D_1}{2\left(4D_1^2+9D_2D_1+9D_2^2\right)} < \frac{D_1+D_2}{2}$*, then*

$$\begin{aligned}\pi^*_{J1} + \pi^*_P - (\pi^*_O + \pi^*_U) =& \frac{(\delta-1)D_1\left(4(5\delta+3)^2D_1 + 5(3\delta+1)(5\delta+11)D_2\right)}{64(3\delta+1)\left(4(\delta-1)D_1+(3\delta+1)D_2\right)} \\ &- \frac{(\delta-1)D_1\left(256(3\delta+1)\left(D_1+D_2\right)K_H + 64(\delta-5)(3\delta+1)K_H^2\right)}{64(3\delta+1)\left(D_1+D_2\right)\left(4(\delta-1)D_1+(3\delta+1)D_2\right)}.\end{aligned}$$

Because $\frac{\partial(\pi^*_{J1}+\pi^*_P-(\pi^*_O+\pi^*_U))}{\partial K_H} < 0$*, and* $\pi^*_{J1} + \pi^*_P - (\pi^*_O + \pi^*_U)\big|_{K_H=\frac{D_1+D_2}{2}} > 0$. *Hence,* $\pi^*_{J1} + \pi^*_P - (\pi^*_O + \pi^*_U) > 0$.

(ii) $\frac{D_1(1+t)}{2} - K_L < K_H \leq \min\{K_L, \frac{D_1(1+t)}{2}\}$

(a)

$$\pi^*_{J1} + \pi^*_P - (\pi^*_O + \pi^*_U) = \frac{1}{192}(\delta-1)\left(-\frac{(117\delta+23)D_1}{3\delta+1} - \frac{48K_H^2}{D_1} + 96K_H\right),$$

$\pi^*_{J1} + \pi^*_P - (\pi^*_O + \pi^*_U) > 0$ *if* $-\frac{\sqrt{27\delta+25}D_1}{4\sqrt{9\delta+3}} + D_1 < K_H < \frac{\sqrt{27\delta+25}D_1}{4\sqrt{9\delta+3}} + D_1$.

(b) $\frac{D_1}{2} < K_H < \min\{\frac{7D_1+8D_2}{16}, K_L\}$.

Numerical results show that the profit gap can be negative or positive.

(c) $\frac{D_1}{4} < K_H < \min\{\frac{2D_1+D_2}{2} - K_L, \frac{D_1+2D_2}{4}, \frac{8D_1K_L-D_1(D_1+D_2)}{4D_1+12D_2}\}$. *Comparison reduces to comparison between Scenarios P and U, hence,* $\pi^*_{J1} + \pi^*_P - (\pi^*_O + \pi^*_U) < 0$.

(d) $K_H \leq \min\{K_L, \frac{D_1(1+t)}{2} - K_L\}$. *Comparison reduces to comparison between Scenarios P and U, hence,* $\pi^*_{J1} + \pi^*_P - (\pi^*_O + \pi^*_U) < 0$.

Proof 40 *Proof of Theorem 3.5.1 Capacity satisfies conditions* $K_H \geq 0$, $K_L \geq 0$, *and* $K_H \leq K_L$.

Proof of Pricing.

(i) *If* $K_H > \frac{D_1+D_2}{2}$, *then* $\pi^*_P = \frac{D_1+D_2}{4}\delta$ *is independent of the capacity level.*

(ii) *If* $\frac{D_1+D_2}{2} - K_L < K_H \leq \frac{D_1+D_2}{2}$, *then* $\frac{\partial \pi_P}{\partial K_H} > 0$, *hence,* $K^*_H = \frac{D_1+D_2}{2}$. *Correspondingly,* $\pi^*_P|_{K^*_H = \frac{D_1+D_2}{2}} = \frac{D_1+D_2}{4}\delta$.

(iii) *If* $K_H \leq \frac{D_1+D_2}{2} - K_L$, *then the Hessian matrix of* π_P *is negative definite, by solving the FOCs w.r.t* K_H *and* K_L, *we have* $K^*_H = \frac{D_1+D_2}{2}$ *and* $K^*_L = 0$. *Note that* $K^*_H \leq K^*_L$. *Hence,* $\pi^*_P|_{K^*_H = \frac{D_1+D_2}{4}, K^*_L = \frac{D_1+D_2}{4}} = \frac{D_1+D_2}{16} + \frac{3(D_1+D_2)}{16}\delta$.

Proof of Pure Upgrading.

(i) *If* $K_H > \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3}{8D_1^2+18D_1D_2+18D_2^2}$, $K_L > \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$, *then* $\pi^*_U = \frac{\delta(4\delta D_1+(3\delta+1)D_2)}{12\delta+4}$.

(ii) *If* $\frac{4D_1+3D_2}{8} < K_H \leq \frac{D_1+D_2}{2}$, *then* $\frac{\partial \pi_U}{\partial K_H} > 0$ *over interval* $[\frac{4D_1+3D_2}{8}, \frac{D_1+D_2}{2}]$. *Hence,* $\pi^*_U|_{K^*_H = \frac{D_1+D_2}{2}} = \frac{(\delta-1)(5\delta-1)D_1^2+4\delta(2\delta-1)D_2D_1+\delta(3\delta+1)D_2^2}{16(\delta-1)D_1+4(3\delta+1)D_2}$.

(iii) *If* $\frac{D_1+D_2}{2} - K_L < K_H < \frac{4D_1+3D_2}{8}$, *then* $\frac{\partial \pi_U}{\partial K_H} > 0$. *Hence,* $K^*_H = \frac{4D_1+3D_2}{8}$, $K_L > \frac{D_2}{8}$, *and* $\pi^*_U|_{K^*_H = \frac{4D_1+3D_2}{8}} = \frac{1}{64}\left(4(5\delta-1)D_1 + (15\delta+1)D_2\right)$.

(iv) *If* $K_H < \min\{\frac{D_1+D_2}{2} - K_L, \frac{4D_1D_2+3D_2^2}{2D_1^2+D_1D_2}K_L\}$, *the Hessian matrix of* π_U *is negative definite. Hence,* $K^*_H = \frac{1}{6}(4D_1+3D_2)$, *and* $K^*_L = -\frac{D_1}{6}$, *which are not reasonable. Hence,* $K^*_H = \max\{\frac{D_1+D_2}{4}, \frac{3D_2^2+4D_1D_2}{4D_1+6D_2}\}$, *correspondingly,* $K^*_L = \max\{\frac{D_1+D_2}{4}, \frac{2D_1^2+D_1D_2}{4D_1+6D_2}\}$, $\pi^*_U|_{K^*_H = \frac{3D_2^2+4D_1D_2}{4D_1+6D_2}, K^*_L = \frac{2D_1^2+D_1D_2}{4D_1+6D_2}} = \frac{16\delta D_2D_1^2+3(8\delta-1)D_2^2D_1+9\delta D_2^3+4D_1^3}{4(2D_1+3D_2)^2}$, *and* $\pi^*_U|_{K^*_H = \frac{D_1+D_2}{4}, K^*_L = \frac{D_1+D_2}{4}} = \frac{(D_1+D_2)((13\delta+3)D_1+3(3\delta+1)D_2)}{64D_1+48D_2}$. *Note that condition* $\frac{3D_2^2+4D_1D_2}{4D_1+6D_2} > \frac{D_1+D_2}{4}$ *holds if* $2D_1^2 < 3D_1D_2 + 3D_2^2$, *and condition* $\frac{3D_2^2+4D_1D_2}{4D_1+6D_2} < \frac{2D_1^2+D_1D_2}{4D_1+6D_2}$ *holds if* $2D_1^2 \geq 3D_1D_2 + 3D_2^2$. *While these two inequalities contradict with each other. So,* $\pi^*_U|_{K^*_H = \frac{D_1+D_2}{4}, K^*_L = \frac{D_1+D_2}{4}} = \frac{(D_1+D_2)((13\delta+3)D_1+3(3\delta+1)D_2)}{64D_1+48D_2}$.

Proof of Pure Opaque Selling.

(i) [(i)] If $\frac{D_1}{D_1+2D_2}K_L < K_H \leq \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2} - K_L\}$*, then the Hessian matrix is indefinite which indicates that there are no optimal solutions.*

(ii) If $\frac{D_1}{4} < K_H < \min\{\frac{D_1+2D_2}{2} - K_L, \frac{D_1+2D_2}{4}\}$*, then the Hessian matrix indicates that the optimal solutions are obtained at the boundary points:* $K_L^* = \frac{D_1+2D_2}{4}$. $\frac{\partial \pi_O}{\partial K_H}$ *can be positive or negative for* $\frac{\partial(\frac{\partial \pi_O}{\partial K_H})}{\partial \delta} > 0$, $\left.\frac{\partial \pi_O}{\partial K_H}\right|_{\delta=1} < 0$*, and* $\left.\frac{\partial \pi_O}{\partial K_H}\right|_{\delta=1+\frac{3D_1D_2+3D_2^2}{D_1^2}} > 0$ *if* $-2D_1^2 + 3D_1D_2 + 3D_2^2 > 0$*. So,* $\frac{\partial \pi_O}{\partial K_H} > 0$ *if* $\delta > -\frac{D_1+2D_2-4(K_H+K_L)}{D_1}$*. Then* $K_H^* = \frac{D_1+2D_2}{4}$*. If* $\delta < -\frac{D_1+2D_2-4(K_H+K_L)}{D_1}$*, then* $K_H^* = \frac{D_1}{4}$*. If* $-2D_1^2 + 3D_1D_2 + 3D_2^2 < 0$*, then* $\left.\frac{\partial \pi_O}{\partial K_H}\right|_{\delta=1+\frac{3D_1D_2+3D_2^2}{D_1^2}} > 0$ *if* $K_H < \frac{1}{4}\left(\frac{3D_2^2}{D_1} + 5D_2 + 2D_1 - 4K_L\right)$ *or* $\left.\frac{\partial \pi_O}{\partial K_H}\right|_{\delta=1+\frac{3D_1D_2+3D_2^2}{D_1^2}} < 0$ *otherwise. Hence,* $\pi_O|_{K_H^*=\frac{D_1+2D_2}{4}, K_L^*=\frac{D_1+2D_2}{4}}$
$= \frac{1}{16}\left(4\delta D_2 + (3\delta+1)D_1 - \frac{4D_2^2}{D_1}\right) - F_O.$

(iii) If $\max\{\frac{D_1+2D_2}{2} - K_L, \frac{3D_1+4D_2}{8}\} < K_H \leq \frac{3D_1+8D_2}{8}$*, then* $\frac{\partial \pi_O}{\partial K_H} > 0$*. Hence,* $K_H^* = \frac{3D_1+8D_2}{8}$*,* $K_L \geq \frac{D_1}{8}$*, and* $\pi_O^*|_{K_H^*=\frac{3D_1+8D_2}{8}} = \frac{((15\delta+1)D_1+16\delta D_2)}{64} - F_O$*.*

(iv) The optimal profit equals $\pi_O^* = \frac{(2D_1+D_2)\delta+D_2}{8} - F_O$ *if* $\max\{\frac{D_1+2D_2}{2} - K_L, \frac{D_1}{2}\} < K_H \leq \frac{D_1+D_2}{2}$*.*

Comparison between Scenarios U and P and Scenarios O and P

$$\Delta\pi^{U-P} = \begin{cases} \dfrac{(\delta-1)\delta D_1}{12\delta+4} > 0, \\ \dfrac{(\delta-1)D_1\left((\delta-1)D_1+\delta D_2\right)}{16(\delta-1)D_1+4(3\delta+1)D_2} > 0, \\ \dfrac{1}{64}(\delta-1)\left(4D_1-D_2\right) > 0, \\ \dfrac{(\delta-1)D_1\left(D_1+D_2\right)}{64D_1+48D_2} > 0. \end{cases} \quad \Delta\pi^{O-P} = \begin{cases} -\dfrac{(\delta-1)D_1^2+4D_2^2}{16D_1} - F_O < 0, \\ -\dfrac{1}{64}(\delta-1)D_1 - F_O < 0, \\ -\dfrac{1}{8}(\delta-1)D_2 - F_O < 0. \end{cases}$$

Proof 41 *Proof of Theorem 3.5.2* ***Proof of Joint Adoption with Upgrading Comes First.***

(i) *If* $\frac{D_1}{3} < K_H < \min\{\frac{D_1+3D_2}{6}, \frac{2D_1+D_2}{2} - K_L, \frac{2D_1}{D_1+3D_2}K_L\}$ *provided that* $D_1 < 3D_2$, *then the FOCs can be positive (if* $\delta > \frac{4(K_H+K_L)-(D_1+2D_2)}{D_1}$*) or negative (if* $\delta < \frac{4(K_H+K_L)-(D_1+2D_2)}{D_1}$*). Hence,* $\pi_J|_{K_H^*=\frac{D_1}{3}, K_L^*=\frac{D_1+3D_2}{6}} = \frac{1}{4}(\delta D_1 + D_2) - F_O$, *which is omitted for* $\delta < 1$ *does not hold.* $\pi_J|_{K_H^*=\frac{D_1}{3}, K_L^*=\frac{4D_1+3D_2}{6}} = \frac{1}{4}((\delta-1)D_1 + D_2) - F_O$ *if* $\delta < 3$. $\pi_J|_{K_H^*=\frac{D_1+3D_2}{6}, K_L^*=\frac{(D_1+3D_2)^2}{12D_1}} = \frac{(8\delta+1)D_1^4+12(\delta+1)D_2D_1^3+6D_2^2D_1^2-36D_2^3D_1-27D_2^4}{48D_1^3} - F_O$ *if* $\delta > \frac{2D_1D_2+3D_2^2}{D_1^2}$.

(ii) *If* $D_1 - 2K_L < K_H < \frac{D_1}{2}$, *then* $\frac{\partial \pi_J}{\partial K_H} = (\delta-1)(1-\frac{3K_H}{2D_1}) > 0$, *hence,* $K_H^* = \frac{D_1}{2}$, $K_L > \frac{D_1}{4}$, *and* $\pi^*|_{K_H^*=\frac{D_1}{2}} = \frac{(5\delta-1)D_1}{16} - F_O$.

(iii) *The same as pure opaque selling. The Hessian matrix is indefinite and there are no optimal solutions.*

(iv) *If* $\max\{\frac{D_1+2D_2}{2} - K_L, \frac{D_1+D_2}{2}\} < K_H \leq \frac{D_1+4D_2}{4}$ *provided that* $D_1 < 2D_2$, *then* $\frac{\partial \pi_J}{\partial K_H} = \frac{\delta-1}{4} > 0$, *hence,* $K_H^* = \frac{D_1+4D_2}{4}$, $K_L > \frac{D_1}{4}$, *and* $\pi_J^*|_{K_H^*=\frac{D_1+4D_2}{4}} = \frac{1}{4}\delta(D_1+D_2) - F_O$.

(v) *The optimal profit equals* $\pi_J^* = \frac{8\delta^2D_1+\left(3\delta^2+4\delta+1\right)D_2}{24\delta+8} - F_O$.

(vi) *The same as pure opaque selling:* $\pi_J|_{K_H^*=\frac{D_1+2D_2}{4}, K_L^*=\frac{D_1+2D_2}{4}} = \frac{1}{16}\left(4\delta D_2 + (3\delta+1)D_1 - \frac{4D_2^2}{D_1}\right) - F_O$.

Comparison between Scenarios O, U, P and J

(i) *If* $K_H > \frac{D_1+D_2}{2}$, *then the intersection is given by* $K_H > \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3}{8D_1^2+18D_1D_2+18D_2^2}$, *the profit gap equals* $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*) = \frac{(\delta-1)(13\delta-1)D_1}{64(3\delta+1)} > 0$.

(ii) *If* $\frac{D_1+D_2}{2} - K_L < K_H < \frac{D_1+D_2}{2}$,

(a) *If* $\max\{\frac{4D_1+3D_2}{8}, \frac{7D_1+8D_2}{16}\} < K_H < \frac{D_1+D_2}{2}$, *then the profit gap equals* $\Delta\pi = \pi_J^* + \pi_P^* - (\pi_O^* + \pi_U^*) = \frac{(\delta-1)\left(-4(\delta-1)(7\delta-3)D_1^2+(3\delta+1)(29\delta-33)D_2D_1+8(3\delta+1)^2D_2^2\right)}{64(3\delta+1)(4(\delta-1)D_1+(3\delta+1)D_2)} > 0$ *if* $D_1 < \frac{-87\delta^2D_2-(3\delta+1)\sqrt{1737\delta^2-3194\delta+1473}D_2+70\delta D_2+33D_2}{2(-28\delta^2+40\delta-12)}$.

(b) $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*) = -\frac{(\delta-1)((7\delta-3)D_1-7(3\delta+1)D_2)}{64(3\delta+1)} > 0$ *if* $D_2 > \frac{(7\delta-3)D_1}{3\delta+1}$.

(c) $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*) = -\frac{(\delta-1)(4(\delta-1)D_1+(3\delta+1)D_2)}{64(3\delta+1)} < 0$.

(d) $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*) = \frac{1}{64}\left(4(\delta+3)D_1 + 7(\delta-1)D_2\right) > 0$ *or* $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*) = \frac{1}{192}\left(-3(9\delta+7)D_2 + 4(7\delta-4)D_1 + \frac{108D_2^4}{D_1^3} + \frac{144D_2^3}{D_1^2} - \frac{24D_2^2}{D_1}\right) > 0$.

(e) $\frac{1}{2}(D_1 + D_2) - K_L < K_H < \frac{D_1}{2}$, $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*) = \frac{15}{64}(\delta-1)D_2 - \frac{D_2^2}{4D_1} + \frac{D_1}{4} > 0$, *or* $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*)$
$= \frac{1}{192}\left(-3(\delta+15)D_2 + 4(4\delta-1)D_1 + \frac{108D_2^4}{D_1^3} + \frac{144D_2^3}{D_1^2} - \frac{72D_2^2}{D_1}\right) > 0$.

(f) $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*) = \frac{1}{64}\left((15\delta+1)D_2 - 4(\delta-1)D_1 - \frac{16D_2^2}{D_1}\right) > 0$.

(iii) *If* $K_H < \frac{D_1+D_2}{2} - K_L$, *the intersection is given by* $\frac{D_1}{D_1+2D_2}K_L < K_H \leq \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2} - K_L\}$, *profit gap comparison within four scenarios reduces to profit comparison between Scenario U and P, and* $\Delta\pi = \pi_O^* + \pi_U^* - (\pi_J^* + \pi_P^*) > 0$.

Proof of Joint Adoption with Opaque Selling Comes First.

(i) *If* $\frac{D_2}{4} \leq K_H \leq \frac{3D_2}{4}$, *and* $K_H + K_L > \frac{D_1}{2} + D_2$, *then* $\frac{\partial \pi_{J1}}{\partial K_H} > 0$. *Hence,* $\pi_{J1}^*|_{K_H^* = \frac{3D_2}{4}} = -\frac{3D_2((1-3\delta)D_1 + (\delta-1)D_2)}{16D_1}$.

(ii) *If* $\max\{\frac{8D_1^3+37D_1^2D_2+57D_1D_2^2+36D_2^3}{16D_1^2+36D_1D_2+36D_2^2} - K_L, \frac{7D_1^3+18D_1^2D_2+18D_1D_2^2}{8D_1^2+18D_1D_2+18D_2^2}\} < K_H \leq \frac{D_1+8D_2}{8}$ *provided that* $24D_1^3 + 31D_2D_1^2 - 9D_2^2D_1 - 72D_2^3 < 0$, *then* $\frac{\partial \pi_{J1}}{\partial K_H} > 0$. *Hence,* $\pi_{J1}^*|_{K_H^* = \frac{D_1+8D_2}{8}} = \frac{1}{32}\left(\frac{(\delta(29\delta+4)-1)D_1}{3\delta+1} + 8\delta D_2\right)$.

(iii) *If* $\max\{\frac{D_1+3D_2}{6}, \frac{D_1+2D_2}{2} - K_L\} < K_H < \min\{\frac{D_1+6D_2}{6}, \frac{5D_1}{6}\}$ *and* $K_L > \frac{D_1}{3}$, *then* $\frac{\partial \pi_{J1}}{\partial K_H} > 0$. *Hence,* $\pi_{J1}^*|_{K_H^* = \frac{D_1+6D_2}{6}} = \frac{1}{36}\left(-\frac{9(\delta-1)D_2^2}{D_1} + 3(8\delta-5)D_2 + (5\delta+4)D_1\right)$ *or* $\pi_{J1}^*|_{K_H^* = \frac{5D_1}{6}} = \frac{1}{36}\left((17\delta-8)D_1 + 9D_2\right)$.

(iv) *If* $\frac{D_1}{2D_2+D_1}K_L < K_H \leq \min\{\frac{D_1^2+2D_1D_2}{D_1^2+3D_1D_2+3D_2^2}K_L, \frac{D_1+D_2}{2} - K_L\}$, *then the Hessian matrix is not negative definite. Hence, there are no optimal solutions.*

(v) *If* $\frac{D_1}{4} < K_H < \min\{\frac{8D_1K_L - D_1(D_1+D_2)}{4D_1+12D_2}, \frac{2D_1+D_2}{2} - K_L, \frac{D_1+2D_2}{4}\}$, *then the result is the same as pure opaque selling:* $\pi_J|_{K_H^* = \frac{D_1+2D_2}{4}, K_L^* = \frac{D_1+2D_2}{4}} = \frac{1}{16}\left(4\delta D_2 + (3\delta+1)D_1 - \frac{4D_2^2}{D_1}\right) - F_O$.

Comparison between Scenarios O, U, P and J1

(i) *If* $K_H > \frac{D_1+D_2}{2}$, *then* $\pi^*_{J1}+\pi^*_P-(\pi^*_O+\pi^*_U) = -\frac{3(\delta-1)^2 D_1}{64(3\delta+1)} < 0$ *when* $\frac{7D_1^3+18D_2D_1^2+18D_2^2D_1}{2\left(4D_1^2+9D_2D_1+9D_2^2\right)} > \frac{D_1+D_2}{2}$ *or* $\pi^*_{J1}+\pi^*_P-(\pi^*_O+\pi^*_U) = \frac{(\delta-1)^2D_1(4(\delta-1)D_1-3(3\delta+1)D_2)}{64(3\delta+1)(4(\delta-1)D_1+(3\delta+1)D_2)} < 0$ *otherwise.*

(ii) *If* $\frac{D_1+D_2}{2} - K_L < K_H \le \frac{D_1+D_2}{2}$, *then results show that* $\pi^*_{J1}+\pi^*_P-(\pi^*_O+\pi^*_U)$ *can be positive or negative.*

(iii) *If* $K_H \le \frac{D_1+D_2}{2} - K_L$, *then the profit comparison reduces to comparison between Scenarios P and U, and* $\pi^*_{J1}+\pi^*_P-(\pi^*_O+\pi^*_U) < 0$.

Proof 42 *Proof of Proposition* 3.5.2.

(i) *The salvage stage.*

Case $S^* = K_H - D_H$ *indicates that opaque selling is unavailable. In what follows, we focus on the case that opaque selling is available.*

(a) *If* $D_2 \ge 2(K_H+K_L-D_H-D_L-D_U)$, *then* $\pi_{2C}(S) = (K_H+K_L-D_H-D_L-D_U)(1-\frac{K_H+K_L-D_H-D_L-D_U}{D_2})(\phi\delta+1-\phi)+pS$, *where* $\phi = \frac{K_H-D_H-S}{K_H+K_L-D_H-D_L-D_U}$ *and* $S \in [0, \min\{D_U, K_H-D_H\}]$. *So,* $S^* = 0$ *if* $p-(1-\frac{K_H+K_L-D_H-D_L-D_U}{D_2})(\delta-1) < 0$, $S^* = D_U$ *if* $D_U < K_H - D_H$ *and* $p-(1-\frac{K_H+K_L-D_H-D_L-D_U}{D_2})(\delta-1) > 0$.

(b) *If* $K_H+K_L-D_H-D_L-D_U \le D_2 < 2(K_H+K_L-D_H-D_L-D_U)$, *then* $\pi_{2C}(S) = \frac{D_2}{4}(\phi\delta+1-\phi)+pS$, *where* $\phi = \frac{K_H-D_H-S}{K_H+K_L-D_H-D_L-D_U}$ *and* $S \in [0, \min\{D_U, K_H-D_H\}]$. *So,* $S^* = 0$ *if* $p - \frac{D_2(\delta-1)}{4(K_H+K_L-D_H-D_L-D_U)} < 0$, *or* $S^* = D_U$ *if* $p - \frac{D_2(\delta-1)}{4(K_H+K_L-D_H-D_L-D_U)} > 0$ *and* $D_U < K_H - D_H$.

(c) *If* $D_2 < K_H+K_L-D_H-D_L-D_U$ *and* $D_2 < 2(K_H-D_H-S)$, *then* $\pi_{2C}(S) = \frac{D_2}{4}(\phi\delta+1-\phi)+pS$, *where* $\phi = \frac{K_H-D_H-S}{D_2}$. *So,* $S^* = 0$ *if* $p - \frac{\delta-1}{4} < 0$, *or* $S^* = D_U$ *if* $p - \frac{\delta-1}{4} > 0$ *and* $D_U < K_H - D_H$.

(d) *If* $D_2 < K_H+K_L-D_H-D_L-D_U$ *and* $D_2 \ge 2(K_H-D_H-S)$, *then* $\pi_{2C}(S) = \frac{D_2}{4}(\phi\delta+1-\phi)+pS$, *where* $\phi = \frac{1}{2}$. $\pi_2(S)$ *increases with* S, *so,* $S^* = D_U$ *if* $D_U < K_H - D_H$.

(ii) The regular stage.

Scenario 1. $p_{1L} < \frac{p}{\delta-1}$ ***and*** $\frac{p_{1H}}{\delta} \geq \frac{p_{1L}+\xi p}{\xi\delta+1-\xi}$

In this scenario, $D_{1H} = D_1(1-\theta_{1H})$*,* $D_U = D_1(\theta_{1H}-\theta_U)$*,* $D_{1L} = D_1(\theta_U - \theta_{1L})$*, where* $\theta_{1H} = \frac{p_{1H}-p_{1L}-\xi p}{(1-\xi)(\delta-1)}$*,* $\theta_U = \frac{p}{\delta-1}$ *and* $\theta_{1L} = p_{1L}$*.*

Boundary Solution $S = K_H - D_{1H}$***.***

If $\theta_U \leq 1 - \frac{K_H}{D_1}$*, then* $\xi = \frac{K_H - D_1(1-\theta_{1H})}{D_1(\theta_{1H}-\theta_U)}$*,* $\pi^*_{2C} = \theta_U(\delta-1)(K_H - D_1(1-\theta_{1H}))$*. The seller's total profit equals* $\pi_C = D_1(1-\theta_{1H})(\theta_{1L} + (\delta-1)\xi\theta_U + (1-\xi)(\delta-1)\theta_{1H}) + D_1(\theta_{1H}-\theta_{1L})\theta_{1L} + (\delta-1)\theta_U(K_H - D_1(1-\theta_{1H}))$*, by deriving the FOCs with respect to* θ_{1H}*,* θ_U *and* θ_{1L}*, we have* $\theta^*_{1H} = 1 - \frac{K_H}{D_1}$*,* $\theta^*_U = 1 - \frac{2K_H}{D_1}$*,* $\theta^*_{1L} = \frac{1}{2}$*, and* $\pi^*_C = \frac{D_1}{4} + (\delta-1)(K_H - \frac{K_H^2}{D_1}) - F_O$ *provided that* $K_H \leq \frac{D_1}{4}$ *and* $\frac{D_1}{2} < K_H + K_L$*.*

(a) $D_2 \geq 2(K_H + K_L - D_1(1-\theta_{1L}))$

(a1) If $\theta_U < 1 - \frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2}$*, then* $S^* = 0$*,* $\xi = 0$*, and* $\pi_C = (\theta_{1H}(\delta-1)+\theta_{1L})D_1(1-\theta_{1H}) + \theta_{1L}D_1(\theta_{1H}-\theta_{1L}) + (K_H+K_L-D_1(1-\theta_{1L}))(1-\frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2})(\phi\delta+1-\phi)$*, by solving the FOCs, we find that the optimal solutions are the same as Lemma* 3.3.3.1 *(case (i)). Note that condition* $\frac{p}{\delta-1} < 1 - \frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2}$ *reduces to* $\theta_U = \theta_{1H} < 3\theta_{1H} - 2$*, which does not hold. So, there are no optimal solutions.*

(a2) If $\theta_U > 1 - \frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2}$ *and* $\theta_U > 1 - \frac{K_H}{D_1}$*, then* $S = D_1(\theta_{1H}-\theta_U)$*,* $\xi = 1$*,* $\theta_{1H} = 1$*,* $\phi = \frac{K_H - D_1(1-\theta_U)}{K_H+K_L-D_1(1-\theta_{1L})}$*, and* $\pi_C = \theta_{1L}D_1(1-\theta_{1L}) + (K_H+K_L-D_1(1-\theta_{1L}))(1-\frac{K_H+K_L-D_1(1-\theta_{1L})}{D_2})(\frac{K_H-D_1(1-\theta_U)}{K_H+K_L-D_1(1-\theta_{1L})}(\delta-1)+1) + \theta_U(\delta-1)D_1(1-\theta_U)$*. By solving the FOCs, we find that* θ_U *corresponds to* θ^*_{1H} *under case* $S^* = 0$ *and* θ_{1L} *resembles* θ^*_{1L} *under case* $S^* = 0$*. The effective domain is given as* $\frac{D_1}{D_1+2D_2}K_L < K_H < \min\{\frac{D_1+D_2}{2} - K_L, \frac{4D_1D_2K_L}{D_1^2+6D_2^2+3D_1D_2}, \frac{D_2}{D_1}K_L\}$*.*

(b) $K_H + K_L - D_1(1-\theta_{1L}) \leq D_2 < 2(K_H+K_L-D_1(1-\theta_{1L}))$

Note that cases $S^* = 0$ *and* $S^* = D_1(\theta_{1H}-\theta_U)$ *reduce to the pure opaque selling. Recall that there are no optimal solutions under this case if* $D_1 + D_2 \geq 2(K_H+K_L)$*.*

(c) $D_2 < K_H + K_L - D_1(1-\theta_{1L})$ ***and*** $D_2 < 2(K_H - D_1(1-\theta_{1H}) - S)$

Note that cases $S^* = 0$ *amd* $S^* = D_1(\theta_{1H} - \theta_U)$ *reduce to pure opaque selling. Specifically, if* $p < \frac{\delta-1}{4}$*, then* $S^* = 0$*,* $\xi = 0$*. Hence,* $\theta_{1H}^* = \frac{5}{8}$*,* $\theta_{1L}^* = \frac{1}{2}$*. Note that* $\theta_U < \frac{1}{4}$*, which is not true.*

If $p > \frac{\delta-1}{4}$*, then* $S^* = D_1(\theta_{1H} - \theta_U)$*,* $\xi = 1$*, and* $\theta_{1H} = 1$*. Hence,* $\theta_U^* = \frac{5}{8}$*,* $\theta_{1L}^* = \frac{1}{2}$*,* $S^* = \frac{3D_1}{8}$*, and* $\pi_C^* = \frac{(9D_1+16K_H)\delta-16K_H+7D_1+16D_2}{64} - F_O$ *provided that* $2(K_H+K_L) > D_1 + 2D_2$*,* $K_L \geq \frac{D_1}{2}$ *and* $8K_H \geq 3D_1 + 4D_2$*.*

(d) $D_2 < K_H + K_L - D_1(1-\theta_{1L})$ ***and*** $D_2 \geq 2(K_H - D_1(1-\theta_{1H}) - S)$

Note that case $S^* = D_1(\theta_{1H}-\theta_{1L})$ *reduces to pure opaque selling. Then,* $\xi = 1$*,* $\theta_{1H} = 1$*. Hence,* $\theta_U^* = \frac{1}{2} = \theta_{1L}^* = \theta_O^* = \frac{1}{2}$*,* $S^* = \frac{D_1}{2}$*, and* $\pi_C^* = \frac{2D_1\delta+D_2(\delta+1)}{8} - F_O$ *provided that* $2(K_H+K_L) > D_1 + 2D_2$ *and* $\frac{D_1}{2} \leq K_H \leq \frac{D_1+D_2}{2}$*.*

Scenario 2. $p_{1L} \geq \frac{p}{\delta-1}$ ***and*** $\frac{p_{1H}}{\delta} \geq \frac{p_{1L}+\xi p}{\xi\delta+1-\xi}$

In this scenario, $D_{1H} = D_1(1-\theta_{1H})$*,* $D_U = D_1(\theta_{1H}-\theta_U)$ *and* $D_{1L} = 0$*, where* $\theta_{1H} = \frac{p_{1H}-p_{1L}-\xi p}{(1-\xi)(\delta-1)}$*, and* $\theta_U = \frac{p_{1L}+\xi p}{\xi\delta+1-\xi}$*.*

Boundary Solution $S = K_H - D_1(1-\theta_{1H})$***.***

If $S = K_H - D_1(1-\theta_{1H})$*, then* $\phi = 0$*,* $\xi = \frac{K_H - D_1(1-\theta_{1H})}{D_1(\theta_{1H}-\theta_U)}$*,* $p_{1H} = (\xi\delta+1-\xi)\theta_U + (1-\xi)(\delta-1)\theta_{1H}$*,* $p_{1L} = (\xi\delta+1-\xi)\theta_U - \xi p$*, and* $\pi_C = D_1(1-\theta_{1H})((\xi\delta+1-\xi)\theta_U + (1-\xi)(\delta-1)\theta_{1H}) + D_1(\theta_{1H}-\theta_U)(\xi\delta+1-\xi)\theta_U$*. By deriving the FOCs w.r.t* θ_{1H} *and* θ_U*, we have* $\theta_U^* = 1 - \frac{K_H}{D_1}$ *and* $\theta_{1H}^* = \frac{D_1\delta - K_H - \delta K_H}{D_1(\delta-1)}$*.Note that* $\theta_{1H} \geq \theta_U$ *and* $\theta_{1H} \leq 1$ *reduce to* $K_H \leq \frac{D_1}{2}$ *and* $K_H > \frac{D_1}{2}$*, respectively. So, there are no optimal solutions.*

(i) $D_2 \geq 2(K_H + K_L - D_1(1-\theta_U))$

If $S = D_1(\theta_{1H}-\theta_U)$*, then* $\phi = \frac{K_H - D_1(1-\theta_U)}{K_H+K_L-D_1(1-\theta_U)}$*,* $\xi = 1$*,* $\theta_{1H} = 1$*,* $\theta_U = \frac{p_{1L}+p}{\delta}$*, and* $\pi_C = (K_H+K_L-D_1(1-\theta_U))(1-\frac{K_H+K_L-D_1(1-\theta_U)}{D_2})(1+\frac{K_H-D_1(1-\theta_U)}{K_H+K_L-D_1(1-\theta_U)}(\delta-1)) + \theta_U\delta D_1(1-\theta_U)$*. By solving the FOC, we have* $\theta_U^* = \frac{2D_1\delta+2D_2\delta-2K_H\delta-K_L\delta-K_L}{2(D_1+D_2)\delta}$*,* $\theta_O^* = \frac{\delta(2D_2(D_1+D_2-(K_H+K_L))-D_1K_L)+D_1K_L}{2D_2(D_1+D_2)\delta}$*, and* $\pi_C^* =$
$\frac{-4D_2K_H^2\delta^2+4D_2\delta(D_1\delta+D_2\delta-K_L-K_L\delta)K_H+4D_2\delta(D_2-D_1)K_L+(D_1+D_1\delta^2-4D_2\delta+2D_1\delta)K_L^2}{4D_2\delta(D_1+D_2)}$*. Note that* $\theta_U > 1 - \frac{K_H}{D_1}$ *reduces to* $K_H > \frac{D_1}{D_2}K_L$*, which does not hold, so, there are no optimal solutions.*

(ii) $K_H + K_L - D_1(1-\theta_U) \leq D_2 < 2(K_H + K_L - D_1(1-\theta_U))$

If $S = D_1(\theta_{1H} - \theta_U)$*, then* $\phi = \frac{K_H - D_1(1-\theta_U)}{D_2}$*, and* $\pi_C = \frac{D_2}{4}(1 + \frac{K_H - D_1(1-\theta_U)}{K_H + K_L - D_1(1-\theta_U)}(\delta - 1)) + \theta_U \delta D_1(1-\theta_U)$*, which decreases with* θ_U *if* $K_H < \frac{D_1^3 + 4D_1^2 D_2 + 6D_1 D_2^2 + 3D_2^3 - (5D_1^2 + 9D_1 D_2 + 6D_2^2)K_L}{2D_1^2 + 6D_1 D_2 + 6D_2^3}$.

Boundary solution.

The interior solution is a convex combination of the boundary points.

(a) $\theta_U^* = 1 + \frac{D_2}{2D_1} - \frac{K_H + K_L}{D_1}$, $\theta_{1H}^* = 1$, $S^* = K_H + K_L - \frac{D_2}{2}$, $\phi^* = \frac{D_2 - 2K_L}{2D_2}$.

(b) $\theta_U^* = 1 + \frac{D_2}{D_1} - \frac{K_H + K_L}{D_1}$, $\theta_{1H}^* = 1$, $S^* = K_H + K_L - D_2$, $\phi^* = \frac{D_2 - K_L}{D_2}$.

(iii) $D_2 < K_H + K_L - D_1(1-\theta_U)$ ***and*** $D_2 < 2(K_H - D_1(1-\theta_{1H}) - S)$

If $S^* = D_1(\theta_{1H} - \theta_U)$*, then* $\phi = \frac{K_H - D_1(1-\theta_U)}{D_2}$, $\xi = 1$, $\theta_{1H} = 1$, $\theta_U = \frac{p_{1L} + p}{\delta}$*, and* $\pi_C = \theta_U \delta D_1(1-\theta_U) + \frac{D_2}{4}(1 + (\delta - 1)\frac{K_H - D_1(1-\theta_U)}{D_2})$*. By deriving the FOC, we have* $\theta_U^* = \frac{5\delta - 1}{8\delta}$, $\phi^* = \frac{8\delta K_H - (3\delta+1)D_1}{8\delta D_2}$*, and* $\pi_C^* = \frac{16(\delta^2 - \delta)K_H + 16D_2\delta + D_1(3\delta+1)^2}{64\delta}$ *provided that* $\frac{D_1 + D_2}{2} < K_H \leq \frac{4D_1^3 + 17D_1^2 D_2 + 33D_1 D_2^2 + 24D_2^3}{8D_1^2 + 24D_1 D_2 + 24D_2^2}$, $K_L > \frac{D_1}{2}$*, and* $K_H > \frac{D_1 + 2D_2}{2} - K_L$.

(iv) $D_2 < K_H + K_L - D_1(1-\theta_U)$ ***and*** $D_2 \geq 2(K_H - D_1(1-\theta_{1H}) - S)$

If $S^* = D_1(\theta_{1H} - \theta_U)$*, by deriving the FOC of* $\pi_C^* = \frac{D_2}{8}(\delta + 1) + \theta_U \delta D_1(1-\theta_U)$*, we have* $\theta_U^* = \frac{1}{2}$ *provided that* $K_H \leq \frac{D_1 + D_2}{2}$, $K_H + K_L > \frac{D_1}{2} + D_2$*, and* $K_L \geq \frac{D_1}{2}$.

Scenario 3. $p_{1L} \geq \frac{p}{\delta - 1}$ ***and*** $\frac{p_{1H}}{\delta} < \frac{p_{1L} + \xi p}{\xi\delta + 1 - \xi}$

In this scenario, $D_{1H} = D_1(1 - \theta_{1H})$, $D_U = D_{1L} = 0$*, and* $S = 0$*, where* $\theta_{1H} = \frac{p_{1H}}{\delta}$*. The optimal results mimick the one of* $S^* = D_1(\theta_{1H} - \theta_U)$ *of Scenario 2.*

To summarize,

(i) If $K_H \leq \frac{D_1 + D_2}{2}$, $K_H + K_L > \frac{D_1}{2} + D_2$*, and* $K_L \geq \frac{D_1}{2}$*, then* $\theta_{1H}^* = 1$, $\theta_{1L}^* = \theta_U^* = \theta_O^* = \frac{1}{2}$*, and* $\pi_C^* = \frac{2D_1\delta + D_2(\delta+1)}{8} - F_O$.

(ii) If $\frac{D_1 + D_2}{2} < K_H \leq \frac{4D_1^3 + 17D_1^2 D_2 + 33D_1 D_2^2 + 24D_2^3}{8D_1^2 + 24D_1 D_2 + 24D_2^2}$, $K_L > \frac{D_1}{2}$*, and* $K_H > \frac{D_1 + 2D_2}{2} - K_L$*, then* $\theta_{1H}^* = 1$, $\theta_{1L}^* = \theta_U^* = \frac{5\delta - 1}{8\delta}$, $\theta_O^* = \frac{1}{2}$*, and* $\pi_C^* = \frac{16(\delta^2 - \delta)K_H + 16D_2\delta + D_1(3\delta+1)^2}{64\delta} - F_O$.

(iii) If $K_H \leq \frac{D_1}{4}$ *along with* $\frac{D_1}{2} < K_H + K_L$*, then* $\theta_{1H}^* = 1 - \frac{K_H}{D_1}$, $\theta_U^* = 1 - \frac{2K_H}{D_1}$, $\theta_{1L}^* = \frac{1}{2}$, $\theta_O^* = 1$*, and* $\pi_C^* = \frac{D_1}{4} + (\delta - 1)(K_H - \frac{K_H^2}{D_1}) - F_O$.

(iv) *If* $2(K_H + K_L) > D_1 + 2D_2$, $K_L \geq \frac{D_1}{2}$ *and* $3D_1 + 4D_2 \leq 8K_H \leq 3D_1 + 8D_2$, *then* $\theta^*_{1H} = 1$, $\theta^*_U = \frac{5}{8}$, $\theta^*_{1L} = \theta^*_O = \frac{1}{2}$, *and* $\pi^*_C = \frac{(9D_1+16K_H)\delta-16K_H+7D_1+16D_2}{64} - F_O$.

(v) *If* $\frac{D_1}{D_1+2D_2}K_L < K_H < \min\{\frac{D_1+D_2}{2} - K_L, \frac{4D_1D_2K_L}{D_1^2+6D_2^2+3D_1D_2}, \frac{D_2}{D_1}K_L\}$, *then the optimal solutions are the same as case (i) of Lemma 3.3.3.2.*

Overlap Characterization.

Cases (iv) and (ii): $\Delta^{ii-iv} < 0$. *Cases (iv) and (i):* $\Delta^{iv-i} > 0$ *if* $K_H > \frac{7D_1+8D_2}{16}$. *Cases (iii) and (i): Because* $\frac{\partial\Delta^{i-iii}}{\partial\delta} > 0$, *and* $\left.\frac{\partial\Delta^{i-iii}}{\partial\delta}\right|_{\delta=1} > 0$. *So, case (i) dominates case (iii). Cases (v) and (iii): Because* $\frac{\partial\Delta^{v-iii}}{\partial\delta} > 0$, *and the minimum value* $\left.\frac{\partial\Delta^{v-iii}}{\partial\delta}\right|_{\delta=1} > 0$ *if* $\frac{1}{2}\left(D_1 + D_2 - \sqrt{D_2\left(D_1 + D_2\right)} - 2K_L\right) < K_H < \frac{1}{2}\left(D_1 + D_2 + \sqrt{D_2\left(D_1 + D_2\right)} - 2K_L\right)$. *So, case (v) dominates case (iii).*

Proof 43 *Proof of Theorem* 3.5.2.

(i) *If* $K_H > \frac{D_1+D_2}{2}$, *then* $\pi^*_C + \pi^*_P - (\pi^*_U + \pi^*_O) = -\frac{(\delta-1)\delta D_1}{12\delta+4} < 0$.

(ii) $\frac{D_1+D_2}{2} - K_L < K_H \leq \min\{K_L, \frac{D_1+D_2}{2}\}$. *If* $\frac{7D_1+8D_2}{16} < K_H \leq \min\{K_L, \frac{D_1+D_2}{2}\}$, *then*

$$\pi^*_C+\pi^*_P-(\pi^*_U+\pi^*_O) = \frac{(\delta-1)D_1\left(-16\left(D_1+D_2\right)K_H + 4D_1^2 + 7D_2D_1 + 3D_2^2 - 4(\delta-5)K_H^2\right)}{4\left(D_1+D_2\right)\left(4(\delta-1)D_1 + (3\delta+1)D_2\right)} < 0.$$

If $\frac{D_1+2D_2}{2} - K_L < K_H < \min\{\frac{7D_1+8D_2}{16}, K_L\}$, *and* $\frac{7D_1+8D_2}{16} > \frac{4D_1+3D_2}{8}$, *then*

$$\pi^*_C+\pi^*_P-(\pi^*_U+\pi^*_O) = \frac{(\delta-1)D_1\left(-16\left(D_1+D_2\right)K_H + 4D_1^2 + 7D_2D_1 + 3D_2^2 - 4(\delta-5)K_H^2\right)}{4\left(D_1+D_2\right)\left(4(\delta-1)D_1 + (3\delta+1)D_2\right)} < 0.$$

If $\frac{D_1+D_2}{2} - K_L \leq K_H \leq \frac{D_1+2D_2}{2} - K_L$, *then*

$$\pi^*_C + \pi^*_P - (\pi^*_U + \pi^*_O) = \frac{1}{16}\left(D_1\left(-\delta - \frac{16(\delta-1)K_H^2}{\left(D_1+D_2\right)\left(4D_1+3D_2\right)} + 5\right) + 4\left(D_2 + 2(\delta-3)K_H - 4K_L\right)\right) + \frac{1}{16}\left(\frac{4\left(-4D_2\left(K_H+K_L\right) + D_2^2 - 4(\delta-2)K_H^2 + 8K_HK_L + 4K_L^2\right)}{D_1}\right) < 0.$$

Otherwise, $\pi^*_C + \pi^*_P - (\pi^*_U + \pi^*_O) = -\frac{(\delta-1)D_1K_H^2}{(D_1+D_2)(4D_1+3D_2)} < 0$.

(iii) *If* $K_H \leq \frac{D_1+D_2}{2} - K_L$, *then* $\pi^*_C + \pi^*_P - (\pi^*_U + \pi^*_O) < 0$ *for* $\pi^*_C \leq \pi^*_O$ *and* $\pi^*_P < \pi^*_U$.

B.2 Other Considerations

B.2.1 Platform Intervention in Upgrading

When a third-party platform manages the upgrading mechanism, we make the following assumptions: (i) There is no information asymmetry or deliberate capacity hoarding. (ii) The revenue sharing between the seller and the upgrading platform is captured by a commission rate r_U *and fixed fee* F_U*, where* $r_U = 1$.

The seller's total profit equals $\pi(p_{1H}, p_{1L}) = p_{1H}D_1(1-\theta_{1H}) + p_{1L}D_1(\theta_{1H}-\theta_{1L}) + \pi^*_{2U}(S^*, p^*) + \pi^*_{2O}(p^*_O)$*, where* $\pi_{2U}(S,p) = p\min\{D_1(\theta_{1H}-\theta_U), S\}$*. By computational analysis, we find that the optimal strategy with an upgrading platform is a special case of the one without an upgrading platform.*

Proposition B.2.1 Under joint adoption with an upgrading platform, the seller's optimal product offerings over the whole selling season must be $(H^P, L^P; H^U, L^\varnothing)$*, or* $(H^P, L^\varnothing; H^{U+O}, L^O)$*. In equilibrium,*

(i) The optimal prices charged for high- and low-quality products, the upgrading price and the opaque selling price satisfy $p^*_{1H} > p^*_U > p^*_{1L}$*,* $p^*_{1H} + p^*_{1L} < 2p^*_U$*, and* $p^*_{1L} < p^*_O$*;*

(ii) The number of high-quality products sold in the regular stage and those offered as upgrades satisfy $D_1(1-\theta^*_{1H}) = D_1(\theta^*_{1H} - \theta^*_U)$.

Proposition B.2.1 indicates that the fixed cost gives the upgrading platform an incentive to participate, so the seller always sells through upgrading. And properties regarding the optimal prices and transaction volumes are consistent with those without platform intervention. As for the role of opaque selling and upgrading with an upgrading platform, we find that all results are consistent with Theorem 3.4.3.1 except that opaque selling and upgrading are no longer substitutes when high-quality capacity level is rather low.

Proof 44 *Proof of Proposition B.2.1. In upgrading mechanism, give* S*, the the seller announces* p *to maximize* $\pi_{2U} = \theta_U(\delta-1)\min\{S, D_1(\theta_{1H}-\theta_U)\}$*. If* $S > D_1(\theta_{1H}-\theta_U)$*,* *then* $\pi_{2U}(\theta_U) = D_1(\delta-1)\theta_U(\theta_{1H}-\theta_U)$ *is concave with* θ_U *and yields maximum at* $\theta_U = \frac{\theta_{1H}}{2}$. *Otherwise,* $\pi_{2U}(\theta_U)$ *increases with* θ_U*, so,* $\theta_U = \theta_{1H} - \frac{S}{D_1}$ *for* $\theta_U \leq \theta_{1H} - \frac{S}{D_1}$. *Correspondingly,* $\pi_{2U}(S) = \frac{D_1(\delta-1)\theta_{1H}^2}{4}$ *if* $S > \frac{D_1\theta_{1H}}{2}$ *or* $\pi_{2U}(S) = (\delta-1)(\theta_{1H} - \frac{S}{D_1})S$ *otherwise. Constraints are* $\theta_U^* \geq \theta_{1L}$ *and* $S \leq \min\{K_H - D_1(1-\theta_{1H}), D_1(\theta_{1H}-\theta_{1L})\}$.

By deriving the FOC of $\pi_{2U}(\theta_U)$ *w.r.t* S*, we have* $S^* = \frac{D_1\theta_{1H}}{2}$ *and* $\pi_{2U}^*(\theta_U) = \frac{D_1(\delta-1)\theta_{1H}^2}{4}$ *iff* $\frac{D_1\theta_{1H}}{2} < \min\{K_H - D_1(1-\theta_{1H}), D_1(\theta_{1H}-\theta_{1L})\}$. *Otherwise,* $S^* = D_1(\theta_{1H}-\theta_{1L})$ *and* $\pi_{2U}^* = D_1(\delta-1)(\theta_{1H}\theta_{1L} - \theta_{1L}^2)$ *if* $\theta_{1L} \geq 1 - \frac{K_H}{D_1}$ *or* $S^* = K_H - D_1(1-\theta_{1H})$ *and* $\pi_{2U}^* = (\delta-1)(1-\frac{K_H}{D_1})(K_H - D_1(1-\theta_{1H}))$ *if* $\theta_{1L} < 1 - \frac{K_H}{D_1}$.

(i) $\frac{\theta_{1H}}{2} > \max\{\theta_{1L}, 1-\frac{K_H}{D_1}\}$.

The number of high-quality capacities in the opaque mix equals $K_H - D_1(1-\frac{\theta_{1H}}{2})$.

(a) $K_H + K_L - D_1(1-\theta_{1L}) > D_2$ *&* $K_H - D_1(1-\frac{\theta_{1H}}{2}) \leq \frac{D_2}{2}$.

The Hessian matrix of the seller's profit function π_{J2} *is negative definite. Hence,* $\theta_{1H}^* = \frac{2}{3}$, $\theta_{1L}^* = \theta_O^* = \frac{1}{2}$ *and* $\theta_U^* = \frac{1}{3}$. *While* $\theta_U^* < \theta_{1L}^*$ *is not true.*

(b) $K_H + K_L - D_1(1-\theta_{1L}) > D_2$ *and* $K_H - D_1(1-\frac{\theta_{1H}}{2}) > \frac{D_2}{2}$.

The Hessian matrix of the seller's profit function π_{J2} *is negative definite. Hence,* $\theta_{1H}^* = \frac{3}{4}$, $\theta_{1L}^* = \theta_O^* = \frac{1}{2}$ *and* $\theta_U^* = \frac{3}{8}$. *While* θ_U^* *is not in the effective domain.*

(c) $K_H + K_L - D_1(1-\theta_{1L}) \leq D_2 < 2(K_H + K_L - D_1(1-\theta_{1L}))$.

The seller's profit function $\pi_{J2} = D_1(\delta-1)(\theta_{1H} - \theta_{1H}^2) + D_1(\theta_{1L} - \theta_{1L}^2) + \frac{D_1(\delta-1)\theta_{1H}^2}{4} + \frac{D_2(\delta-1)(K_H - D_1(1-\frac{\theta_{1H}}{2}))}{4(K_H+K_L-D_1(1-\theta_{1L}))} + \frac{D_2}{4} - F_O - F_U$ *is concave with* θ_{1H}*, so,* $\theta_{1H} = \frac{2}{3} + \frac{D_2}{12(K_H+K_L-D_1(1-\theta_{1L}))}$. *Because* $\frac{\partial^3 \pi_{J2}}{\partial \theta_{1L}^3} < 0$, $\frac{\partial^2 \pi_{J2}}{\partial \theta_{1L}^2}\Big|_{\theta_{1L}=1+\frac{D_2}{2D_1}-\frac{K_H+K_L}{D_1}, \theta_{1H}=\frac{5}{6}} < 0$ *if* $K_H < \frac{D_1(7D_1+9D_2)}{12(D_1+D_2)}$. *The seller's profit function decreases with* θ_{1L} *over* $(1+\frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}, 1+\frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}]$ *if the FOC is negative. Note that when* $\theta_{1L} \geq \frac{1}{2}$ *(or equivalently,* $D_1 + D_2 \geq 2(K_H+K_L)$*), Hence, there are no optimal solutions.*

So, the optimal solutions with complicated forms exist if $\frac{D_1+D_2}{2} - K_L < K_H < \frac{D_1(7D_1+9D_2)}{12(D_1+D_2)}$.

Boundary solution.

The optimal interior solution is a convex combination of the boundary solutions.

(c1) $\theta_{1L}^* = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = \frac{5}{6}$, $\theta_U^* = \frac{5}{12}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{12K_H-7D_1}{6D_2}$, *and* $\pi_J^* = \frac{11(\delta-1)D_1^2+12D_1(-D_2+2(\delta+1)K_H+4K_L)-12(D_2-2(K_H+K_L))^2}{48D_1} - F_U - F_O$. *The effective domain is given by* $\max\{\frac{7D_1+6D_2}{12} - K_L, \frac{7D_1}{12}\} \leq K_H \leq \min\{\frac{2D_1+D_2}{2} - K_L, \frac{7D_1+6D_2}{12}\}$.

(c2) $\theta_{1L}^* = 1 + \frac{D_2}{D_1} - \frac{K_H+K_L}{D_1}$, $\theta_{1H}^* = \frac{3}{4}$, $\theta_U^* = \frac{3}{8}$, $\theta_O^* = \frac{1}{2}$, $\phi^* = \frac{8K_H-5D_1}{8D_2}$, *and* $\pi_J^* = \frac{17(\delta-1)D_1^2+16D_1(-3D_2+(\delta+3)K_H+4K_L)-64(-D_2+K_H+K_L)^2}{64D_1} - F_U - F_O$. *The effective domain is given by* $\max\{\frac{5D_1+8D_2}{8} - K_L, \frac{5D_1}{8}\} \leq K_H \leq \min\{D_1 + D_2 - K_L, \frac{5D_1+8D_2}{8}\}$.

(d) $D_2 \geq 2(K_H + K_L - D_1(1 - \theta_{1L}))$.

The Hessian matrix of the seller's profit function π_{J2} *is negative definite. Hence,* $\theta_{1H}^* = \frac{2((-D_1^2+D_1K_H)(\delta-1)-2D_2(3(D_1+D_2)-(K_H+K_L)))}{D_1^2-D_1^2\delta+12D_1D_2+12D_2^2}$, $\theta_{1L}^* = \frac{(-D_1^2+3D_1D_2+D_1K_H-6D_2K_H+D_1K_L)(\delta-1)+12D_2(D_1+D_2-(K_H+K_L))}{D_1^2-D_1^2\delta+12D_1D_2+12D_2^2}$ *provided that* $1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1} - \theta_{1L}^* = \frac{D_2((7D_1^2-12D_1K_H)(\delta-1)-12(D_1+D_2-2(K_H+K_L)))}{2D_1(-D_1^2+D_1^2\delta-12D_1D_2-12D_2^2)} \geq 0$. *Correspondingly,* $\theta_O^* = 3\theta_{1H}^* - 2 = \frac{2((2D_1^2-3D_1K_H)(\delta-1)-6D_2(D_1+D_2-(K_H+K_L)))}{-D_1^2+D_1^2\delta-12D_1D_2-12D_2^2}$, $\theta_U^* = \frac{\theta_{1H}^*}{2} = \frac{(-D_1^2+D_1K_H)(\delta-1)-2D_2(3(D_1+D_2)-(K_H+K_L))}{D_1^2-D_1^2\delta+12D_1D_2+12D_2^2}$, $\phi^* = -\frac{2\left(D_1(3D_2-5K_H+K_L)-6D_2K_H+3D_1^2\right)}{3\left((\delta-1)D_1^2-2(\delta-1)D_1K_H+4D_2(K_H+K_L)\right)}$. *Because condition* $h_1(\delta) = (-D_1^2+3D_1D_2+D_1K_H-6D_2K_H+D_1K_L)(\delta-1)+12D_2(D_1+D_2-(K_H+K_L)) \geq 0$ *holds if* $K_H \leq \min\{\frac{(D_1D_2-3D_1^2)K_L+D_1(3D_1^2+6D_1D_2+3D_2^2)}{3D_1^2+5D_1D_2+6D_2^2}, D_1+D_2-K_L\}$, *condition* $h_2(\delta) = (-2D_1K_H + D_1^2)(\delta-1) + 4D_2(K_H + K_L) \leq 0$ *holds if* $K_H \leq \frac{4D_1K_L+3D_1^2+3D_1D_2}{2D_1+6D_2}$, *condition* $h_3(\delta) = (-7D_1^2 + 12D_1K_H)(\delta - 1) + 12D_2(D_1 + D_2 - 2(K_H + K_L)) \geq 0$ *holds if* $\frac{8D_1K_L+3D_1^2+3D_1D_2}{4D_1+12D_2} \leq K_H \leq \frac{D_1+D_2}{2} - K_L$ *provided that* $D_1 - 6D_2 + 12K_L < 0$, *condition* $h_4(\delta) = (-D_1^3 - D_1^2(3D_2 - K_H) + 6D_1D_2K_H)(\delta - 1) + 8D_1D_2K_H - 4D_1D_2K_L - 12D_2^2K_L < 0$ *holds if* $K_H < \min\{\frac{(4D_1^2+12D_1D_2)K_L+3D_1^3+12D_1^2D_2+9D_1D_2^2}{11D_1^2+21D_1D_2+18D_2^2}, \frac{D_1+3D_2}{2D_1}K_L\}$, *condition* $h_5(\delta) = (-2D_1^2 + 3D_1K_H)(\delta - 1) + 6D_2(D_1 + D_2 - (K_H + K_L)) > 0$ *holds if* $\frac{2D_1}{D_1+3D_2}K_L \leq K_H \leq D_1+D_2-K_L$ *provided that* $K_L < \frac{D_1+3D_2}{3}$, *and condition*

$h_6(\delta) = (-3D_1D_2+6D_2K_H-D_1K_L)(\delta-1)+2D_2(-3(D_1+D_2)+5(K_H+K_L)) > 0$ *holds if* $K_H > \max\{\frac{6D_1^3+15D_1^2D_2+9D_1D_2^2+(-7D_1^2+3D_1D_2)K_L}{10D_1^2+18D_1D_2+18D_2^2}, \frac{3D_1+3D_2}{5} - K_L\}$. *Note that* $\frac{3D_1+3D_2}{5} - K_L > \frac{D_1+D_1}{2} - K_L$, *so, there are no optimal solutions.*

Boundary solution $\theta_{1L} = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$.

The seller's profit function π_{J2} *increases with* θ_{1L} *if* $(-7D_1^2 + 12D_1K_H)(\delta - 1) + 12D_2(D_1 + D_2 - 2(K_H + K_L)) < 0$, *or equivalently,* $\frac{D_1+D_2}{2} - K_L < K_H < \frac{8D_1K_L+3D_1^2+3D_1D_2}{4D_1+12D_2}$ *and* $K_L > \frac{-D_1+6D_2}{12}$. *Hence,* $\theta^*_{1H} = \frac{5}{6}$, $\theta^*_{1L} = 1+\frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta^*_O = \frac{1}{2}$, $\theta^*_U = \frac{5}{12}$, $\phi^* = \frac{12K_H-7D_1}{6D_2}$, *and*

$\pi^*_{J2} = \frac{(D_1^2+24D_1K_H)\delta-D_1^2-12D_1D_2+24D_1K_H+48D_1K_L-12(D_2-2(K_H+K_L))^2}{48D_1} - F_O - F_U$ *provided that* $\max\{\frac{7D_1}{12}, \frac{7D_1+6D_2}{12}-K_L\} < K_H < \min\{\frac{D_1+3D_2}{6}, \frac{2D_1+D_2}{2}-K_L, \frac{8D_1K_L+3D_1^2+3D_1D_2}{4D_1+12D_2}\}$ *and* $5D_1 < 6D_2$.

(ii) $\theta_{1L} > \max\{1 - \frac{K_H}{D_1}, \frac{\theta_{1H}}{2}\}$.

Customers who have purchased low-quality capacity all get upgraded, so, there are K_L *low-quality and* $K_H - D_1(1 - \theta_{1L})$ *high-quality capacities in the opaque mix. The analysis is an analogy to Proposition* 3.4.1.2 *except that condition* $\theta_{1L} > \frac{\theta_{1H}}{2}$ *is verified in each case here. Note that* $\theta_{1L} > \frac{\theta_{1H}}{2}$ *does not change the optimal outcomes of cases (ii), (iii) and (iv), while condition in case (i) reduces to* $(4D_1 + 3D_2 - 6K_H - 3K_L)\delta - 3K_L + D_2 > 0$, *which holds if* $K_H < \min\{\frac{2D_1+2D_2}{3} - K_L, \frac{4D_1^3+16D_1^2D_2+21D_1D_2^2+9D_2^3-(6D_1^2+9D_1D_2+9D_2^2)K_L}{6D_1^2+18D_1D_2+18D_2^2}\}$. *Because condition* $K_H > \frac{D_1}{D_2}K_L$ *does not hold when* $D_2 < D_1$. *So, the optimal solutions under this case are equivalent to case (iii) and (iv) in Proportion* 3.4.1.2.

(iii) $1 - \frac{K_H}{D_1} > \max\{\theta_{1L}, \frac{\theta_{1H}}{2}\}$.

Opaque selling mechanism is not available. By deriving the FOCs of the seller's profit function $\pi_{J2} = D_1(\delta - 1)(\theta_{1H} - \theta_{1H}^2) + D_1(\theta_{1L} - \theta_{1L}^2) + (\delta - 1)(1 - \frac{K_H}{D_1})(K_H - D_1(1 - \theta_{1H})) - F_O - F_U$ *w.r.t* θ_{1H} *and* θ_{1L}, *we have* $\theta^*_{1H} = 1 - \frac{K_H}{2D_1}$ *and* $\theta^*_{1L} = \frac{1}{2}$ *provided that* $K_H < \frac{D_1}{2}$ *and* $\frac{K_H}{2} + K_L > \frac{D_1}{2}$.

To summarize,

(*i*) *If* $\max\{\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}, \frac{D_1+2D_2}{2}-K_L\} < K_H < \frac{D_1+D_2}{2}$ *and* $K_L > \frac{D_1^3+3D_1^2D_2+3D_1D_2^2}{4D_1^2+9D_1D_2+9D_2^2}$, *then* $\theta_{1H}^* = \frac{2\delta+1}{3\delta+1}$, $\theta_{1L}^* = \theta_U^* = \frac{\delta+1}{3\delta+1}$, $\theta_O^* = \frac{1}{2}$, *and* $\pi_{J2}^* = \frac{(8D_1+3D_2)\delta^2+4D_2\delta+D_2}{8+24\delta} - F_O - F_U$.

(*ii*) *If* $K_H < \frac{D_1}{2}$ *and* $\frac{K_H}{2} + K_L > \frac{D_1}{2}$, *then* $\theta_{1H}^* = 1 - \frac{K_H}{2D_1}$, $\theta_{1L}^* = \frac{1}{2}$, $\theta_U^* = 1 - \frac{K_H}{D_1}$, *and* $\pi_{J2}^* = \frac{D_1}{4} + K_H(\delta-1)(1-\frac{3K_H}{4D_1}) - F_O - F_U$.

(*iii*) *If* $\max\{\frac{D_1+2D_2}{2}-K_L, \frac{D_1+D_2}{2}\} < K_H \le \frac{D_1+2D_2}{2}$ *and* $K_L > \frac{D_1}{4}$ *provided that* $D_1 < 2D_2$, *then* $\theta_{1H}^* = \frac{3}{4}$, $\theta_{1L}^* = \theta_U^* = \frac{1}{2}$ *and* $\pi_{J2}^* = \frac{4(D_1+D_2)+(4K_H+3D_1)(\delta-1)}{16} - F_O - F_U$.

(*iv*) *If* $\max\{\frac{7D_1}{12}, \frac{7D_1+6D_2}{12} - K_L\} < K_H < \min\{\frac{D_1+3D_2}{6}, \frac{2D_1+D_2}{2} - K_L, \frac{8D_1K_L+3D_1^2+3D_1D_2}{4D_1+12D_2}\}$ *provided that* $5D_1 < 6D_2$, *then* $\theta_{1H}^* = \frac{5}{6}$, $\theta_{1L}^* = 1 + \frac{D_2}{2D_1} - \frac{K_H+K_L}{D_1}$, $\theta_O^* = \frac{1}{2}$, $\theta_U^* = \frac{5}{12}$ *and* $\pi_{J2}^* = \frac{(D_1^2+24D_1K_H)\delta - D_1^2 - 12D_1D_2 + 24D_1K_H + 48D_1K_L - 12(D_2-2(K_H+K_L))^2}{48D_1} - F_O - F_U$.

Overlap Characterization.

Case (i) and case (iv): Because $\frac{\partial^2\Delta\pi^{iv-i}}{\partial\delta^2} < 0$, *and* $\frac{\partial\Delta\pi^{iv-i}}{\partial\delta}\Big|_{\delta=1} < 0$. *So,* $\frac{\partial\Delta\pi^{iv-i}}{\partial\delta} < 0$. *Note that* $\Delta\pi^{iv-i}|_{\delta=1} < 0$. *So, case (i) dominates case (iv).*

B.2.2 Unavailable Opaque Selling When Opaque Selling Comes First

If only high-quality capacities are left, and last-minute customers are only informed of the opaque selling mechanism, the optimal solutions are summarized as Lemma B.2.2.

Lemma B.2.2 (Optimal Solutions of the Upgrading Mechanism Without Last-minute Customers) *The equilibrium exists if and only if* $K_L \in [\frac{3D_1D_2(D_1+D_2)}{D_1^2+9D_2D_1+9D_2^2}, D_1]$. *In equilibrium,*

(*i*) *If high-quality capacity is large, then the seller uses partial high-quality leftovers to fulfill upgrading demand. Moreover,* $p_{1H}^* + p_{1L}^* = 2p_U^*$, $D_{1H}^* > D_U^*$.

(*ii*) *If high-quality capacity is small, then the seller uses all high-quality leftovers to fulfill upgrading demand. Moreover,* $p_{1H}^* + p_{1L}^* > 2p_U^*$ *if* $K_H > \frac{2\delta D_1 - K_L}{3\delta-1}$, $D_{1H}^* > D_U^*$.

By Lemma B.2.2, if the seller does not target last-minute customers, then high posted price is higher than upgrading price, and the amount of high-quality capacities sold regularly is *more than the amount of high-quality capacities sold as upgrades. This is consistent with the statement that upgrades are complementary capacities, and the additional sales from upgrades are smaller than the original sales* [1].

Proof 45 *Proof of Lemma B.2.2 . In the salvage stage, recall that the seller's profit from upgrading mechanism equals*

$$\pi_{2U} = \begin{cases} \dfrac{D_1(\delta-1)\theta_{1H}^2}{4} & if \dfrac{\theta_{1H}}{2} \geq \max\{\theta_{1L}, 1-\dfrac{K_H}{D_1}\}, \\ D_1(\delta-1)(\theta_{1H}\theta_{1L}-\theta_{1L}^2) & if \theta_{1L} \geq \max\{1-\dfrac{K_H}{D_1}, \dfrac{\theta_{1H}}{2}\}, \\ (\delta-1)(1-\dfrac{K_H}{D_1})(K_H - D_1(1-\theta_{1H})) & if 1-\dfrac{K_H}{D_1} \geq \max\{\theta_{1L}, \dfrac{\theta_{1H}}{2}\}. \end{cases}$$

In the regular stage,

(i) $\theta_U = \frac{\theta_{1H}}{2}$.

The seller's total profit $\pi_U(\theta_{1H}, \theta_{1L}) = (\theta_{1H}(\delta-1)+\theta_{1L})D_1(1-\theta_{1H}) + \theta_{1L}K_L + \frac{D_1(\delta-1)\theta_{1H}^2}{4}$ *is concave with* θ_{1H} *and increases with* θ_{1L}*. Hence,* $\theta_{1H}^* = \frac{2(D_1(\delta-1)+K_L)}{D_1(3\delta-1)}$, $\theta_{1L}^* = \frac{(\delta-1)(2D_1-3K_L)}{D_1(3\delta-1)}$, $\theta_U^* = \frac{D_1(\delta-1)+K_L}{D_1(3\delta-1)}$, *and* $\pi_U^* = \frac{(\delta-1)((1+3\delta^2)D_1^2+(3\delta-5)D_1K_L+3(2-3\delta)K_L^2)}{D_1(1-3\delta)^2}$ *provided that* $K_H > \max\{\frac{2D_1^3+6D_1^2D_2+6D_1D_2^2-D_1^2K_L}{2D_1^2+9D_1D_2+9D_2^2}, D_1-\frac{K_L}{2}\}$ *and* $\frac{3D_1^2D_2+3D_1D_2^2}{D_1^2+9D_1D_2+9D_2^2} < K_L \leq \frac{2D_1}{3}$.

(ii) $\theta_U = \theta_{1L}$.

The Hessian matrix of π_U *is negative definite. Hence,* $\theta_{1H}^* = \frac{2\delta^2D_1+\delta K_L-3\delta D_1-2K_L}{D_1(3\delta-4)\delta}$, *and* $\theta_{1L}^* = \frac{(\delta-1)(\delta D_1+2K_L)}{D_1(3\delta-4)\delta}$*. Because* $\theta_{1L} \geq 0$ *does not always hold when* $\delta \in (1, 1+\frac{3D_1D_2+3D_2^2}{D_1^2})$*. So, there are no optimal solutions.*

(iii) $\theta_U = 1-\frac{K_H}{D_1}$.

By solving the FOCs of π_U *w.r.t* θ_{1H} *and* θ_{1L}*, we find that* π_U *is concave with* θ_{1H} *and increases with* θ_{1L}*. Hence,* $\theta_{1H}^* = \frac{2\delta D_1-\delta K_H+K_H+K_L-2D_1}{D_1(2\delta-1)}$, $\theta_{1L}^* = \frac{2\delta D_1-\delta K_H-2\delta K_L+K_H+2K_L-2D_1}{D_1(2\delta-1)}$, *and* $\pi_U^* = \frac{(\delta-1)(D_1^2-3\delta^2K_H^2+3K_L(K_H+K_L)+D_1((-1-2\delta+4\delta^2)K_H+4(\delta-1)K_L)+2\delta(K_H^2-2K_HK_L-2K_L^2))}{D_1(1-2\delta)^2}$ *provided that* $D_1-K_L < K_H \leq \min\{2D_1-2K_L, \frac{2D_1^3+6D_1^2D_2+6D_1D_2^2-D_1^2K_L}{2D_1^2+9D_1D_2+9D_2^2}\}$.

1. Refer to https://hoteltechreport.com/news/suggestive-selling.