MITIGATING AIRCRAFT MAINTENANCE UNCERTAINTIES IN AIRCRAFT ROUTING PROBLEMS

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Mitigating Aircraft Maintenance Uncertainties in Aircraft Routing Problems

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CERTIFICATE OF ORIGINALITY

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Abstract

Airlines are faced with challenging situations, i.e., narrow profit margins and stochastic environments. Consequently, airlines struggle for operational cost reduction and robust schedules to enhance their profitability and competitiveness. The maintenance cost, due to its significance and expensive nature, has drawn increasing attention. The aircraft maintenance routing problem thus becomes increasingly important for airlines. Hence, this research study focuses on enhancing decision making in aircraft schedules from two aspects, with a view to improve cost-efficiency and robustness.

From the perspective of cost-efficiency enhancement, it can be observed that there is an increasing willingness for airlines to contract-out their aircraft maintenance services, resulting in procurement circumstances in which maintenance providers offer quantity discounts to airlines. However, most of the models in the literature have ignored such advantageous policies. This motivates us to construct a new model, with a piecewise cost function for exploiting the total quantity discount policy. In solving the proposed model, we develop a column generation- based diving heuristic approach, which proves its effectiveness and efficiency through computational experiments. In addition, the computational results also reveal that the new model enables a noteworthy cost reduction compared with the traditional models.

From the perspective of robustness enhancement, this study first concentrates on constructing robust aircraft routes with flexibility. The aircraft maintenance outsourcing creates a new way for airlines to modify the maintenance arrangements. This motivates us to investigate the potential effects of the maintenance distribution structure on the robustness of aircraft routings. Based on this exploration, a robustness strategy encouraging swapping possibilities is additionally integrated into the aircraft

maintenance routing problem. Accordingly, a new robust model is proposed. Our computational results demonstrate that a more concentrated maintenance can create more swapping possibilities for aircraft routes. Furthermore, the results also suggest that the routing solutions resulting from our proposed model enable a further improvement in robustness.

On the other hand, attempts are also made in constructing robust aircraft routes with improved stability. It is acknowledged that a critical challenge in aircraft routings is the stochasticity of maintenance execution, while most of the traditional models of the aircraft maintenance routing considered the sources of disruptions in an aggregated manner. Therefore, we incorporate the uncertainties of heterogenous maintenance tasks into the robust aircraft routing decision framework, while considering other sources of disruptions. Accordingly, a new robust model is constructed, along with a tailored column generation approach. We analyze the impact of distinct degrees of maintenance uncertainties on the robustness of aircraft routes through computational experiments.

In conclusion, the desire for cost and disruption management by airlines motivates the research work conducted in this thesis. Firstly, a new aircraft maintenance routing model integrating the total quantity discount is proposed. Secondly, the effect of the maintenance distribution structure on route robustness is investigated, and a novel robust model considering both the total quantity discount and a robustness strategy encouraging swapping possibilities is constructed. Thirdly, a new robust aircraft maintenance routing model incorporating the maintenance uncertainties of heterogenous maintenance tasks is developed.

Publications Arising from the Thesis

1. **He, Y.**, Chan, F.T.S., Chung, S.H., & Fu, X. (2022). A column generation-based diving heuristic for the piecewise-cost aircraft maintenance routing problem. (Submitted)
2. **He, Y.**, Chan, F.T.S., Chung, S.H., & Fu, X. (2022). Assessing the impact of maintenance distribution structure on the robustness of aircraft routings: an optimization approach. (Submitted)

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List of Abbreviations

AMRP	Aircraft Maintenance Routing Problem
AMR-PC	Aircraft Maintenance Routing with Piecewise Costs
AMRTQD	Aircraft Maintenance Routing with the Total Quantity Discount
ATSP	Asymmetric Traveling Salesman Problem
BTS	Bureau of Transportation Statistics
FAA	Federal Aviation Administration
IAMRP	Integrated Aircraft Maintenance Routing Problem
IATA	International Air Transport Association
LP	Linear Programming
MEL	Minimum Equipment List
MIP	Mixed Integer Programming
MRO	Maintenance, Repair, and Overhaul
TAMRP	Traditional Aircraft Maintenance Routing Problem

Chapter 1. Introduction

1.1 Research Background

1.1.1 The Airline Industry

The air transport sector is an important enabler of the global economy, with airlines facilitating connectivity of city pairs through providing the safest travel means. Statistics from the International Air Transport Association (IATA) reveal that airlines operated more than 22,000 city-pair connections in 2019, safely servicing 4.5 billion passengers (IATA, 2020). Accordingly, air carriers need to upsize and manage their fleet to cope with such operational needs. It is reported by the Bureau of Transportation Statistics (BTS) that, in 2019, the global active fleet size amounted to 210,981 aircraft (BTS, 2020). Notwithstanding such impressive development, there remain tremendous challenges for airlines.

One of the imminent challenges to airlines is that the industry is characterized with surprisingly fierce competition, whether internally or externally generated, and narrow profitability. According to IATA, there has been a steady decrease in the global operating profit margin, from 7.5% of revenue in 2017 to 5.2% in 2019 (IATA, 2021). Furthermore, the COVID-19 pandemic has almost eliminated the chance of revenue enhancement, ultimately leading to financial deterioration. As also reported by IATA, the margin dropped significantly, under the influence of the worldwide pandemic, to -28.2% in the year 2020 (IATA, 2021). As a result, in order to survive in a highly challenging and competitive environment, airlines endeavored to slash expenses. However, among all the expenses, the ownership cost is a fixed cost that is unavoidable over a short period of time, while the fuel cost fluctuates with the market price, and is uncontrollable for airlines.

Furthermore, airline operations are challenged by diverse unexpected disruptions that can lead to significant delays, cancellations and prohibitive recovery costs. According to BTS, 18.87% of flights arrived more than 15 minutes late in 2019, while the cancellation rate reached 2.40% (BTS, 2022a). It was stated by Airlines for America that the annual costs of delays were estimated to be US$28 billion in 2018[1]. The resulting cost loss from delays even accounted for approximately 8% of the industry's revenue (Gershkoff, 2016), and, no less crucial, flight delays generated significant passenger inconvenience and dissatisfaction.

In response to intensifying competition and rapidly changing circumstances, airlines have already placed a high emphasis on strategic and scheduling decisions. Their primary concern should be, due to high operating costs, procuring greater cost efficiencies. Therefore, the first goal of this research study aims at improving the aircraft maintenance routing decisions under the circumstance of maintenance outsourcing, which enables a decrease in operational costs. On the other hand, in face of disruptions, airlines strive to improve the reliability of airline schedules. Hence, this study also concentrates on enhancing the robustness of aircraft schedules, through assessing the impact of the maintenance distribution structure, and integrating multiple uncertainties, especially maintenance stochasticity, into the aircraft maintenance routing framework.

1.1.2 Aircraft Maintenance Routing

Air transport is generally regarded as the safest travel mode of transportation (Chang, 2012). Behind the scenes, the progressive implementation of preventive maintenance, strictly following the Federal Aviation Administration (FAA) regulations, facilitates an enhanced safety performance through detecting hazards before they influence aircraft airworthiness. To accommodate bigger fleets propelled by the expansion of passenger

[1] https://www.airlines.org/dataset/u-s-passenger-carrier-delay-costs/

numbers and the greater need for city connections, the maintenance, repair, and overhaul (MRO) market enjoyed a rapid growth. Statistics from Oliver Wyman further reveal that a rising trend regarding the total MRO, from 68.4 to 117.7 billion USD, will remain in the next decade (Wyman, 2021). In this context, no airline can afford to ignore the expensive and significant nature of the aircraft maintenance cost, which has attracted particularly increasing attention (Haouari et al., 2013; Safaei & Jardine, 2018). The aircraft maintenance routing problem (AMRP), a great, almost striking success of operations research, thereby becomes increasingly important and critical in airline planning. From the operating perspective, airlines design daily flight schedules and determine the exact aircraft type assigned to each flight leg based on market forecasts, network analysis and the available resources. Given the above information, AMRP aims at constructing cost-efficient aircraft routes that satisfy many conditions, including, but not restricted to, maintenance requirements. It is emphasized that different maintenance routing schedules contribute to different maintenance costs (Liang et al., 2011; Orhan et al., 2011; Sriram & Haghani, 2003). There has been an increasing trend to outsource aircraft maintenance services. Under a procurement setting, the maintenance demands of airlines are scheduled and satisfied by purchasing services from multiple independent providers who may offer exclusive discounts on the service volume. The existing research studies regarding aircraft maintenance routing ignored this crucial procurement strategy. Therefore, in Chapter 3 of this thesis, the emphasis is on enhancing the long-term aircraft maintenance routing decisions in terms of the maintenance cost reduction, through integrating the total quantity discount strategy offered by independent maintenance third parties. In the following, we first describe the two variants of aircraft maintenance routing problem in (1). Then, we introduce the background of the quantity discount strategy under the settings of aircraft maintenance outsourcing in detail in (2).

(1) Aircraft maintenance routing: tactical and operational variants

Two variants, i.e., tactical and operational problems, have emerged for the AMRP (Al-Thani et al., 2016; Eltoukhy et al., 2018). Generally, both problems take maintenance requirements into consideration, however, the tactical variant focuses on, for relatively long term scheduling, generating rotations to be repeated (Feo & Bard, 1989; Sriram & Haghani, 2003). The optimization models for tactical aircraft maintenance routing are oriented towards constructing more cost-effective aircraft routes, which however follow diverse cost representations. As expected, the maintenance cost has been extensively considered. The size and complexity of tactical problems are impressively large. The number of possible aircraft routes of these problems increases exponentially with the number of flights. Therefore, it is often impractical to enumerate all the potential routes for tactical AMRP. Accordingly, the column generation technique is usually adopted to handle these large-scale problems. On the other hand, as the day of operation approaches, detailed conditions such as initial position of the aircraft and operational maintenance requirements should be carefully considered to ensure a smooth operation. Therefore, the operational aircraft maintenance routing problem is introduced, permitting customization to meet diverse operational constrictions, while taking the initial conditions of individual aircraft, such as original locations and accumulated flying time, into consideration (Al-Thani et al., 2016).

(2) Aircraft maintenance outsourcing

Aviation maintenance is a highly regulated and crucial issue in the airline industry. It can be discerned that before the deregulation of the market in 1978, airlines completed the majority of maintenance checks relying on self-inspection. In contrast, after this, mounting competition pushed fares so low that, in order to achieve cost savings, maintenance was gradually contracted out to third-party maintenance vendors (Czepiel, 2003). As stated in Tang and Elias (2012), the percentage of outsourced maintenance services was more than 44% in 2011, an increase of over 24% compared with the value in 1990. The popularity of purchasing maintenance services by airlines is expected to continue, not only impelled by the limitation of the qualified technician supply in the labor market, but also on account of its fruitful advantages, e.g., alleviating hard and soft costs associated with maintenance, providing reduced ground time and, due to improved professionalism, achieving better on- time performance. An upside of this procurement situation is that MRO providers usually offer exclusive discounts based on the event amount, known as total quantity discount policy, with various practical reasons for this widespread economic phenomenon. On the one hand, with the increasing reliance on outsourcing, the aviation maintenance industry is saturated with independent maintenance third parties, resulting in a higher level of competition. To cope with tough competition, suppliers tend to charge lower prices (Bağan & Gerede, 2019). In addition, MRO providers normally have invested heavily and may suffer from huge operational costs, and thus are eager to gain business opportunities. In pursuit of stimulating airlines to increase maintenance orders, it is common and operable to provide quantity discounts. On the other hand, airlines usually negotiate a contract with maintenance suppliers under which the price for a certain order quantity is established (Quinlan et al., 2013). Typically, large volume purchasers, by virtue of the larger ordering quantities they bring to suppliers, have relatively stronger negotiation power, and thus are likely to secure exclusive price discounts in contract negotiations (Bugaj et al., 2019; Vega et al., 2016). Airlines have made great attempts to manage their maintenance costs, efforts are thus made on incorporating the discount policy into tactical aircraft routing decisions to activate cost efficiencies, as described in Chapter 3.

1.1.3 Disruption Management

Although airlines try their hardest to create cost-effective schedules, an obstacle to efficient and prompt operations is the disruption effect, which may force airlines to delay their flights and this delay can snowball down the route, hence imposing dire consequences (Clausen et al., 2010). Therefore, there has been increasing interest in managing randomness in operations to mitigate potential delays. In particular, this goal can be achieved through constructing robust airline schedules that possess either of the properties: flexibility and stability (Ahmed et al., 2017a; Aloulou et al., 2013; Liang et al., 2015).

The flexibility within the aircraft routing decisions refers to the number of options provided to recover from disruptions. As soon as there is a disruption, a series of precise adjustments are needed to make it up, and a commonly used, relatively low-cost option is swapping aircraft (Hassan et al., 2021; Liang et al., 2018). To be specific, this strategy allows a disrupted aircraft to cover a later flight leg that in the original schedule is executed by an alternative aircraft. Therefore, a possibility of swapping aircraft enables reallocation of slack time, and if there are not many swapping possibilities inserted in routing plans, the disrupted flight may be seriously delayed or even cancelled. On that account, increasing the number of swapping possibilities in aircraft routing decisions becomes one of the most useful and efficient strategies to ensure the robustness of aircraft routes (Ageeva, 2000; Burke et al., 2010). The maintenance visit is an important procedure for aircraft routes. When implementing routing plans, airlines always expect that their aircraft can be released on time after receiving maintenance. However, a paradox is that aircraft are rarely maintained as planned. One of the common reasons is that maintenance is typically conducted during the night, while the last flight in the day may suffer more significant delays because of delay propagation and, as a result, aircraft cannot arrive at the maintenance station on schedule. Furthermore, while delays by the end of the day may be mitigated during the curfew time, the time is occupied by maintenance and hence the aircraft is more likely to start a new operational day with a late departure. Another trigger could be the extended maintenance duration caused by unanticipated machine/equipment failures. Consequently, the aircraft is unable to fly on time the next scheduled flight early in the following day, further disrupting the routing plan for the rest of the day. Therefore, the robustness in maintenance stations is a key part of the route robustness, which can be enhanced by improving the number of swapping possibilities (Lapp & Cohn, 2012). Aircraft maintenance outsourcing became prevalent after deregulation, making it possible for airlines to intelligently modify the maintenance distribution structure, namely, deciding which maintenance stations should be selected and the number of visits per station. Therefore, efforts are made on improving the robustness performance of aircraft routing decisions, in terms of the number of swapping possibilities in maintenance stations, through investigating the impact of the maintenance distribution structure, as described in Chapter 4.

On the other hand, stability within the aircraft schedules aims at preventing schedules from being influenced by stochastic events. A classical approach is to allocate slacks between connecting flights to absorb disruptions caused by uncertainties and prevent delay propagation throughout the schedule. Actually, airline operations are subject to various sources of disruptions (Choi et al., 2019; Sun et al., 2020). In particular, a number of external uncertainties, such as extreme weather and air traffic control, etc., are out of airlines' control. However, BTS found, through examining the causes of delays in 2020, that 41% of the total delay time is attributable to airline-related disruptions, which include maintenance issues (BTS, 2022b). The airline industry operates under some of the most stringent operational conditions, with high reliance on aircraft maintenance. While the completion of maintenance tasks enables aircraft to leave the ground safely, the dispatch reliability might not be achieved since, in practice, maintenance is inherently uncertain, having the potential to cause remarkable disruptions. This uncertainty lies in the fact that some breakdowns or defects, e.g., electrical wiring discrepancies, may appear during the process of the maintenance check. Consequently, more time will be required to troubleshoot and fix such faults and if, depending on the determined aircraft schedule, these faults cannot be eliminated within the ground time, the flight immediately after the maintenance will be delayed due to aircraft unavailability. More seriously, airline operators may have to completely cancel the flight in a case where the extended maintenance time is too long. Furthermore, the stochasticity of the maintenance duration can be also attributable to the discrepancy in resources at different stations. For instance, a shortage of tools, or spare parts, may prohibit aircraft from undergoing maintenance as planned since the aircraft need to wait until the tools or parts are available. Besides, maintenance staff may be absent from work because of the COVID-19 pandemic (Choi, 2021), leading to higher rates of absence, which significantly affects the maintenance capability. Another example concerns individuals with various levels of proficiency and productivity, which may result in different operating times. In such circumstances, highly skilled staff are able to complete tasks quickly, while for less efficient staff, it might not be possible to complete checks punctually, leading to long delays. The undeniable impacts of maintenance uncertainties on the number and consequence of delays have received special attention. According to Knotts (1999), for Boeing 747s, 20% of delays and cancellations were attributable to technical issues in checking and repairing the aircraft equipment. Specifically, as pointed by Rosenberger (2001), a major airline had to cancel up to 71 flights because of mechanical issues on a single day in June, 2000. Hence, it becomes very important to integrate the inherent uncertainty of maintenance into airline scheduling in order to mitigate the impact of disruptions. In the literature, the sources of disruptions are generally studied in an aggregated manner - known as a non-propagated delay. This approach may underestimate the effects of maintenance uncertainties, resulting in aircraft schedules that are vulnerable to disruptions from maintenance checks. Therefore, the focus in Chapter 5 is on enhancing the robustness performance of aircraft routing decisions with the consideration of maintenance uncertainties.

1.2 Research Gaps

In particular, this thesis focuses on bridging the following research gaps:

1. These recent years have witnessed an upward trend in maintenance outsourcing (Qin et al., 2020), one that plays an increasing important role in common airline practice. This requires airlines to lay great emphasis on procurement-associated decisions. Although the total quantity discount policy is able to bring impressive cost savings and profit enhancement in real procurement problems, it is much less studied in airline planning. Therefore, it is valuable and beneficial to incorporate the consideration of the total quantity discount into the AMRP, with the aim of further reducing the overall maintenance costs. However, the major existing models with respect to AMRP did take maintenance cost into account, but ignored this crucial strategy through depending on a vastly oversimplified assumption-basically, the per unit maintenance cost remains unchangeable with the increasing operation density, leading to a potentially negative impact on the long-term cost management for commercial airlines. Our study tries to bridge the gap in the literature regarding the integration of the total quantity discount problem into AMRP. Obviously, this approach brings challenges in regard to computational complexity since the introduction of the total quantity discount policy brings various constraints and variables into the traditional aircraft maintenance routing model, complicating the decision-making process.

2. Although some research studies have investigated aviation robustness, limited research studies focused on the influence of uncertainties occurring at the maintenance station on the routing plans. From a practical perspective, airlines outsource their maintenance to third-party suppliers and if an aircraft misses the appointed arrival time or unexpected maintenance events prolong the downtime,

the maintenance supplier cannot release the aircraft on time. This will further change the route schedule, causing terrible delays and hence significant costs. Therefore, robustness in maintenance airports is of vital importance to airlines' smooth operations.

It can be seen, from the literature on robust aircraft maintenance routing, that robustness enhancement can be achieved by improving the number of swapping possibilities. Especially, for airlines, it is preferred to have more swapping possibilities in maintenance airports, because swapping aircraft after receiving maintenance will not disrupt the original maintenance plans and thus there is no need, if taking this action, to further adjust the schedule, i.e., rerouting the aircraft to a maintenance airport (Lapp & Cohn, 2012). It is noted that aircraft maintenance outsourcing creates an opportunity to judiciously adjust the maintenance distribution structure, the way in which airlines arrange regular maintenance. However, no studies, to the best of our knowledge, have investigated the impact of maintenance distribution structure on the robustness of aircraft maintenance routes, in terms of swapping possibilities. Therefore, our study attempts to bridge this gap by proposing a framework that analyses the robustness of aircraft routings under different maintenance distribution structures, while further enhancing robustness by encouraging more swapping possibilities at the maintenance stations.

3. It is noted that an increasing number of publications have placed emphasis on the importance of route robustness to alleviate delays, while non-propagated delays, in most of the papers, were normally attributable to an aggregate of all types of stochasticity. However, this aggregated approach did not pay particular attention to specific random sources of disruptions. Thus, some researchers employed an alternative approach that considered particular randomness, such as block-time uncertainties (Sohoni et al., 2011), non-cruise time variability (Duran et al., 2015),

airport congestion (Lee et al., 2020), and the stochasticity of crew availability (Cacchiani & Salazar-González, 2017). However, in the literature, the existing aircraft maintenance routing models made no attempt to incorporate maintenance uncertainties. In practice, maintenance is subject to numerous uncertainties (Dinis et al., 2019; Shahmoradi-Moghadam et al., 2021), whereas, in the previous research on aircraft routings, the duration of a maintenance task was assumed as a deterministic value (Ruther et al., 2017; Safaei & Jardine, 2018). Consequently, the generated aircraft schedules poorly addressed the potential uncertainties in real-world operations and neglected the risks concerning the capacity to satisfy the actual maintenance requirement. In a word, many previous studies set the maintenance capacity per day (Liang et al., 2015) and ignored the impact of delays on the whole maintenance process, which may result in infeasibilities.

Furthermore, most of the traditional aircraft routing models generate generic routes for homogeneous aircraft respecting primary maintenance checks at regular intervals (Lan et al., 2006; Liang et al., 2011). However, they did not consider the states of individual aircraft, e.g., the original location at the beginning of the planning horizon and the associated pending maintenance tasks. Unlike historical practice, nowadays the airlines tend to break letter checks (e.g., A and C checks) into smaller tasks, and thus these tasks have smaller scope and shorter durations than a typical A check and have to be performed more frequently (Ruther et al., 2017; Zhou et al., 2020). In such cases, each maintenance task of each aircraft hence has its own characteristics, i.e., the maximum flight time limit and, as indicated earlier, the execution duration with uncertainty, which need to be respected heterogeneously. In this research, we thus strive for the development of an alternative approach to construct robust routes for individual aircraft with heterogenous maintenance tasks, while taking into account maintenance uncertainties.

1.3 Research Objectives

Motivated by the attempts that airlines made to manage their maintenance costs and the desperate need for more robust schedules to cope with uncertainties from both internal and external environments, this research aims at developing optimization models and corresponding algorithms, with the objective of helping airlines to improve their aircraft maintenance routing decisions. The cost optimization is vital in helping an airline remain ahead in the long run, and therefore this research study focuses on constructing a framework that provides cost-effective tactical decisions. The framework is then extended to provides greater flexibility so as to better react against disruptions. Our last goal aims at scheduling aircraft on a more timely-basis and thus developing an approach for building reliable routes that can withstand the forthcoming uncertainties.

Towards these goals, the concentration is on meeting the following objectives:

1. To investigate the aircraft maintenance routing in the context of maintenance procurement, comprising of formulating a model for AMRP that takes the total quantity discount into account, and developing an efficient solution approach for solving the optimization model.

2. To investigate the potential effects of the maintenance distribution structure on the robustness of aircraft routings, measured by the number of swapping possibilities. In addition, to formulate a robust AMRP model that further incorporates a robustness strategy facilitated by encouraging swapping possibilities.

3. To develop a robust AMRP model that incorporates the uncertainty of heterogenous maintenance tasks, while considering other sources of disruptions. Furthermore, to construct a tailored column generation solution approach for solving the model.

1.4 Research Contributions

The main contributions of this thesis can be stated from two distinct perspectives, i.e., maintenance cost optimization and routing robustness enhancement, as described in the following.

(1) Maintenance cost optimization

In Chapter 3, a novel aircraft maintenance routing model and an efficient solution algorithm for optimizing the maintenance costs of airlines are proposed. The main modelling and methodological contributions are summarized in the following.

On modelling grounds, a mixed integer programming (MIP) model is formulated, with the aim of minimizing a piecewise linear objective function. The proposed model is novel since it is the first one that explicitly captures the total quantity discount policy in aircraft maintenance routing decisions. Even though this policy has been explored in several other transportation problems and is widely applied under real procurement settings, it has not been studied in airline planning, especially the AMRP, in which most of the existing research models, to the best of our knowledge, made an oversimplified assumption, i.e., linear homogeneity in the unit maintenance cost (Eltoukhy et al., 2017; Sriram & Haghani, 2003). Therefore, the incorporation of the total quantity discount into AMRP makes a significant scientific contribution. Computational experiments show that this integrated model has the ability of delivering better cost-effective performance compared to the traditional model. On methodological grounds, we construct a column generation-based heuristic algorithm that exploits the special structure of our optimization model. The framework includes a column generation process for solving the linear programming (LP) relaxation of our proposed problem, in which a two-label shortest path approach with dominance rules is designed to generate promising route candidates. Furthermore, the incorporation of the quantity discount introduces new binary variables to the model and thus further complicates the computational process. To handle this, we propose a two-phase branching scheme, based on which the selected variables are iteratively kept fixed in order to limit the search space. Subsequently, the resulting mixed integer programming sub-problem (i.e., sub-MIP) is solved exactly.

(2) Routing robustness enhancement

In Chapters 4 and 5, investigation and mathematical modelling for enhancing the robustness performance of aircraft routings are described. The major academic and practical contributions are given in the following.

Firstly, this study deals with an important practical question about the effects of the maintenance distribution structure on the robustness of aircraft routing schedules, which is still underexplored in the field. Since maintenance is usually scheduled along with the aircraft routes, through understanding the characteristic of maintenance distribution structure and assessing its possible influence on route robustness, this study is able to deliver some suggestions on the strategy of maintenance distribution design and aircraft routing plans.

Secondly, a robust aircraft maintenance routing model with the consideration of both the total quantity discount policy and the robustness strategy promoting swapping possibilities is proposed. The newly constructed model is novel, which is proven, through intelligent maintenance distribution and solution improvement, to derive cost-efficient aircraft routes with further robustness enhancement.

Thirdly, this research study is the first attempt, to the best of our knowledge, to develop an optimization model that explicitly considers the maintenance uncertainties of heterogenous maintenance tasks in constructing robust aircraft maintenance routings. In such cases, the duration of a heterogenous maintenance task is stochastically modelled using an appropriate probability distribution rather than a deterministic value. Besides maintenance stochasticity, we also incorporate the uncertainties associated with other sources of non-propagated delays to explore the superimposed effects of these two stochastic variables. Our approach can make the resulting routing schedules more robust. Another important contribution is that we improve the maintenance capacity constraints from the maximum quantity each day to maximum quantity per hour, considering that the maintenance tasks consume resources in terms of person-hours, to model the impact of random maintenance duration. Our new approach can circumvent the inaccurate estimation of the maintenance facility occupation due to the assumption of deterministic maintenance times in previous studies.

1.5 Structure of this Thesis

After an introduction of the background, research gaps, objectives, and contributions of this research. This thesis is organized as follows.

Chapter 2 reviews the related literature, while focusing on the aircraft maintenance routing problem and operations research approaches for robustness enhancement. We also provide discussion on the research gaps.

Chapter 3 is devoted to the study on the novel aircraft maintenance routing with the consideration of the total quantity discount strategy. A new mathematical formulation for the proposed problem is presented and discussed. Next, to solve the optimization model, we develop a solution approach based on column generation, and the computational results and discussion are provided.

Chapter 4 examines the impact of the maintenance distribution structure on the robustness of aircraft routes, based on the model proposed in Chapter 3. Then, to further improve the robustness performance, a novel robust aircraft maintenance routing model is proposed, which additionally incorporates a robustness strategy, i.e., encouraging swapping possibilities. Next, computational experiments are conducted to undergo examination and demonstrate the performance of the proposed model.

Chapter 5 focuses on the aircraft maintenance routing closer to the day of operations. Uncertainties of heterogenous maintenance tasks are included in the robust aircraft routing decision framework. Accordingly, a robust aircraft maintenance routing model and a tailored column generation solution approach are proposed. The computational study is carried out to investigate the impact of maintenance uncertainties.

Chapter 6 gives the conclusions to this thesis, points out limitations, and presents potential future directions.

Chapter 2. Literature Review

This chapter presents a review of the relevant literature. First of all, the studies regarding the airline planning process are sequentially introduced (Section 2.1). Then, we focus on the papers with regard to the two variants of the aircraft maintenance routing problem, i.e., tactical models (Section 2.2.1) and operational models (Section 2.2.2). Next, we discuss two distinct approaches to evaluate and enhance the robustness of air transport (Section 2.3), including research on the analytical approach for assessing the robustness of aviation network (Section 2.3.1), and studies on the operations research techniques for improving the robustness of aircraft routings (Section 2.3.2). The applications of quantity discount in the transportation areas are then briefly reviewed (Section 2.4). Finally, we highlight the research gaps identified from the reviewed literature (Section 2.5).

2.1 Four-stage Airline Planning

An airline network is a huge system that consists of masses of flight legs airlines offer, hundreds of aircraft belonging to distinct fleets, and a great number of crews with different levels of experience. Besides this complexity, the airline planning process can be quite challenging, also in the light of demand uncertainties, sets of rules and regulations regarding crews and aircraft, and the unstable environment. Therefore, airline planning, one of the most difficult issues to deal with, is usually decomposed into four stages, i.e., the flight design problem, fleet assignment problem, aircraft maintenance routing problem and crew scheduling problem (Barnhart et al., 1998b; Gao et al., 2009; Jamili, 2017; Rosenberger et al., 2002). However, these sub-problems remain very complex and challenging, with significant computational difficulties. The following provides an overview of the basic concepts and optimization process for each stage as well as integrated problems.

2.1.1 Flight Design Problem

The first stage of airline scheduling aims at designing the flight schedules based on the marketing forecasts, the airline network analysis and the available resources (e.g., airport slots) (Yan & Tseng, 2002). A flight schedule includes, in a particular time horizon (e.g., a day or a week), a set of flight legs with their corresponding flight numbers, origins, destinations, and departure/ arrival times. An example of a flight schedule is presented in **Table 2-1**.

Table 2-1. Example of the flight schedule

Flight number	Origin	Destination	Departure time	Arrival time
N860DN	MSP	SMF	11:20	13:20
…	…	…	…	…
N998DL	SAV	ATL	16:58	18:10

Flight design is fundamental to an airline's profitability and its market share. Therefore, the objective in this stage, typically, is maximizing the profit. This goal was explored by Kim and Barnhart (2007), considering flight schedule design for a charter airline, with passenger demand regarding different fare classes. This problem has its own characteristics, e.g., a weekly fixed demand that fluctuated day by day within a week, and, therefore, it was reasonable to construct a flight schedule that can be repeated weekly. They proposed a MIP model for this problem, and solved it through an exact and a heuristic approach, respectively. Besides the objective of enhancing profits or revenues, some studies also attempted to determine flight schedules, aiming at achieving other objectives, e.g., resource utilization. Abdelghany et al. (2017) considered the impact of competition between airlines on the passenger demand in their flight scheduling model, with the aim of maximizing the passenger revenue, the number of possible rotations and the resource utilization, and constructed a heuristic-based solution approach for solving their proposed model. Kepir et al. (2016) developed a model and a heuristic for addressing the problems of constructing new flight schedules, while considering fleet utilization enhancement and a decrease in the waiting time of passengers.

Considering that airlines do not operate in a deterministic, but an uncertain environment, researchers tried to incorporate uncertainties into the process of flight scheduling decision-making (Jacquillat & Odoni, 2015; Naumann & Suhl, 2013). For example, Lee et al. (2007) improved the schedules of flights, in terms of robustness performance, through retiming the departure times, and, accordingly, a multi-objective optimization model was constructed. Yan et al. (2008) realized that the daily passenger demand in real operations had a significant impact on flight scheduling decisions, and thus took this crucial factor into account when designing schedules, while considering market share and aircraft resources. Accordingly, a stochastic-demand scheduling model was proposed to respond to this randomness. Naumann and Suhl (2013) proposed a stochastic optimization model for the flight design problem, with consideration of both the jet fuel price uncertainties and passenger demand uncertainties.

2.1.2 Fleet Assignment Problem

Once the flight scheduling problems are solved, the generated flight timetables serve as input in the fleet assignment stage. Then, the fleet assignment problem focuses on assigning a particular aircraft type to individual flight legs, considering various factors like the capacity of the aircraft, passenger demand, fleet size, and maintenance (Gu et al., 1994; Hane et al., 1995). Typically, there are three main constraints involved in the basic fleet assignment model: the coverage constraint ensuring that each flight leg can be covered by only one fleet type, the balance constraint requiring that the number of inbound and outbound aircraft are the same, and the aircraft availability constraint, for each fleet, limiting the quantity of aircraft that can be used for operations (Clarke et al., 1996; Sherali et al., 2006).

Generally, the objective of a fleet assignment problem can be in maximizing profit (Barnhart et al., 2002; Belanger et al., 2006; Bélanger et al., 2006; Grothklags, 2003), minimizing the total costs (Hane et al., 1995), or maximizing utilization of the resources (Dožić et al., 2019; Rushmeier & Kontogiorgis, 1997). For example, Abara (1989) proposed an integer linear programming model to formulating the fleet assignment, the objective of which was profit maximization, while in the study of Hane et al. (1995), the fleet assignment was formulated as a multi-commodity flow problem, with the aim of minimizing costs, including the costs for recapturing the "spilled" passengers, and the operational costs. This kind of model was seriously degenerated and, therefore, exact algorithms including the interior-point algorithm were applied to solve the model. Dožić et al. (2019) studied an integrated fleet sizing and fleet assignment problem, for the purpose of ensuring the aircraft utilization, because a spilled number of aircraft results in lower utilization, while insufficient aircraft means a loss of passenger demand. Other works have tried to improve the revenue of flight assignment, taking advantage of the network (Barnhart et al., 2009; Jacobs et al., 2008). For instance, the model proposed in Barnhart et al. (2009) aimed at assigning fleet types to subnetworks, and was more capable of modelling revenue and yielding great profit improvements.

Furthermore, diverse strategies have been investigated to improve the solutions of fleet assignment. For instance, time windows, used to identify how much time is allowed for adjusting the departure time, can be helpful in gaining profitability improvements in fleet assignment, since connecting itineraries are altered to serve the demand (Desaulniers et al., 1997; Jiang & Barnhart, 2009; Sherali et al., 2013). Desaulniers et al. (1997) integrated time windows into the fleet assignment problem and, owing to the resulting larger set of possible connections, the aircraft capacity was better allocated. The experimental results showed that this consideration can improve the total profit by up to 21.9%. Furthermore, it is recognized that, in real operations, passenger demand is updated over time, and the initial aircraft fleet assignment plans are made based on prediction, with low accuracy. Therefore, Sherali et al. (2005) introduced a demand-driven re-fleeting method to dynamically reassign aircraft capacities, and developed a MIP model to formulate this problem, as well as several valid inequalities to tighten the formulation. However, these research studies assumed that the legs were independent from each other. Recognizing the flight dependency on revenue, as a consequence of itineraries, Dumas et al. (2009) tried to iteratively enhance profits through alternately constructing fleet assignments, which were evaluated using a passenger flow model.

2.1.3 Aircraft Maintenance Routing Problem

After constructing the flight schedules and assigning a specific aircraft type (i.e., fleet) to each flight leg, the aircraft maintenance routing stage focuses on, given the aircraft in the same fleet, generating a sequence of flights flown by each aircraft, while satisfying the maintenance restrictions set by the FAA and the airlines' internal regulations (Bulbul & Kasimbeyli, 2021; Haouari et al., 2013; Ma et al., 2022). Therefore, maintenance issues are significantly important to the aircraft maintenance routing decision making.

Specifically, there are two important elements relating to maintenance activities in aircraft routings. One is the maintenance demand, namely, aircraft should perform maintenance before reaching the time limitation, i.e., maximum flying hours, maximum flying days, and maximum number of take-offs (Al-Thani et al., 2016). The other is the maintenance opportunities, namely, aircraft requiring maintenance need to stay sufficiently long in a station that has the capacity to execute this activity. This capacity in the literature is commonly respected by bounding, for each maintenance station, the maximum number of aircraft per night (or per day) (Faust et al., 2017; Khaled et al., 2018; Maher et al., 2014). Typically, there are four types of maintenance for aircraft, called letter checks; however, most of these checks have long intervals and low frequencies, and are out of the scope of aircraft maintenance routing. Instead, the literature considered, in the aircraft routing models, line maintenance, such as daily checks, that can be regularly conducted whilst the aircraft is still in operation and needs to perform a flight mission immediately after maintenance. For example, Haouari et al. (2013) proposed a compact model for a daily aircraft maintenance routing problem, taking into account the maintenance check with 65-hour constraints and the duration for maintenance execution less than 1440 minutes. On the other hand, some studies assumed that airlines operate under a much more stringent maintenance regulation, requiring aircraft to visit a maintenance station every 40 flying hours or 4 days (Gopalan & Talluri, 1998; Talluri, 1998).

A recent trend in the AMRP literature is to focus on individual tasks, instead of maintenance checks with regular intervals. For instance, Ruther et al. (2017) considered distinct maintenance requirements for individual aircraft in an integrated problem. The problem was solved by a branch-and-price framework, in which there were several pricing subproblems to be handled. The authors developed two major approaches, i.e., selecting a subset to handle each iteration, and formulating aggregated pricing subproblems, implicitly solving a single pricing subproblem instead of dealing with several problems. In Safaei and Jardine (2018), each aircraft was assumed to undergo over 50 maintenance tasks, differing in cycle intervals, and therefore they proposed specific constraints, called generalized maintenance constraints, to ensure that there were sufficient maintenance opportunities inserted in the route for completing the tasks of individual aircraft. More recently, Lagos et al. (2020) proposed a framework that simultaneously makes maintenance plans and tail (i.e., individual aircraft) assignment decisions given a set of line-of-flights. The maintenance tasks considered in their approach were disclosed dynamically over time and carried out during the night.

From the perspective of modelling and algorithm development. In particular, much work has been done in modelling approaches, one of which is the compact model that consists of polynomial-sized decision variables and constraints (Haouari et al., 2013). Accordingly, this kind of formulation has the advantage of strong tractability and hence can be tackled by commercial solvers such as Cplex. For example, Liang et al. (2011) illustrated their problem in a time-space network and proposed a new compact aircraft maintenance routing model. In addition, two pre-processing strategies, i.e., node aggregation (node combination) as well as island isolation (arc elimination), were used to reduce the problem size. In contrast, Haouari et al. (2013) proposed a new AMRP formulation base on a connection network, through defining six different types of arcs. Then, a reformulation-linearization technique was used to reconstruct non-linear constraints, and two root-node strategies were developed to improve the model, which could be addressed quickly, while obtaining high-quality solutions and significant savings. Khaled et al. (2018) presented, for the tail assignment problem, a compact model with constraints of polynomial-scale and, therefore, this formulation was capable of significantly reducing the solution space. Computational experiments showed that the largest instance consisting of 1494 flights can be solved (with gaps less than 0.5%) by Cplex in three hours.

In comparison, the other approach, i.e., the set-partitioning based formulation, along with a well-applied methodology, i.e., column generation, allows for an efficient solvability of large scaled models for aircraft maintenance routing (Liang et al., 2015). In particular, these models are typically formulated through the string-based formulation. A string is a flight sequence that starts from and ends at (possibly different) maintenance available airports. This type of model implicitly satisfies the flow balance and maintenance feasibility and, thus, can easily incorporate several operational considerations (Zhou et al., 2020). On the other hand, this formulation usually involves exponential strings and may be solved by some sophisticated methodologies, that is, column generation (Haouari et al., 2013). Column generation is an efficient linear programming technique that can avoid the explicit complete enumeration of all potential columns. In aircraft routing research, column generation is usually used for addressing large-scale problems, by only generating effective aircraft routes (through the labelling approach) that are possible to improve the current solution. Furthermore, to obtain optimal integer solutions, column generation is usually embedded in the branch and bound approach, which is known as branch-and-price. For instance, Sarac et al. (2006) modelled the daily aircraft maintenance routing problem based on set-partitioning formulation and, to solve the model, adopted the branch and price procedure, in which the sub-problem was a constrained shortest path problem, and the branching strategies were modified from the classic branch-on, follow-on branching rule.

2.1.4 Crew Scheduling Problem

As the last stage of airline planning, given the solutions of three previous stages, the crew scheduling problem needs to be solved, with the objective of partitioning a group of flight legs, respecting a set of specific and complex work rules and regulations, so that crew can fly (Barnhart et al., 2003). In light of the complexity of crew scheduling, this problem is typical divided into two steps, i.e., the crew pairing problem and the crew rostering problem (Chung et al., 2017; Gopalakrishnan & Johnson, 2005; Klabjan et al., 2002; Medard & Sawhney, 2007). To be specific, crew pairing aims at providing a set of pairings, with minimal costs, to cover all the pre-generated flight legs so that each leg can be served exactly once. Then the solutions of crew pairing are taken as the inputs of crew rostering, which focuses on creating a schedule, i.e., a roster, for each crew member. In this sub-chapter, we review the research work regarding the two steps in crew scheduling, respectively.

(1) Crew pairing problem

The goal of the crew pairing problem is in generating and selecting the most satisfying pairings, while following specific rules and restrictions regulated by governments, unions, and air carriers (Aydemir-Karadag et al., 2013; Barnhart et al., 1995). Specifically, crew members have to undertake flying duties, while a duty refers to a sequence of flight legs linked by several sits. The sit time is time between two consecutive flight legs in a duty. The elapsed time of one duty is called a duty period. A pairing is thus a sequence of duties undertaken by a crew, starting and ending at the home base where the crew lives (Wen et al., 2020).

The crew pairing problem is normally formulated as a set-partitioning model that can be solved based on the column generation framework. Quesnel et al. (2017) considered the base constraints that limits the total working time at every crew base in crew pairing, which was modelled based on set-partitioning problem and solved by four branch-and-price heuristics. Later, the authors proposed a crew pairing model that incorporated language constraints, i.e., some legs should be covered by crews with specific language qualifications, and a branch-and-price heuristic with a partial pricing strategy was proposed for solving this model (Quesnel et al., 2020a). More recently, in the study of Wen et al. (2022), a novel individual cabin crew pairing framework was constructed, which enabled crew substitution, and a column generation based heuristic was developed for solving the proposed model.

Because of its large scale, metaheuristic methods were also adopted to solve the crew pairing problem. Deveci and Demirel (2018) used two stages to solve the crew pairing problem. The first stage tried to generate pairings while the second stage aimed at choosing the optimal pairings, with the objective of cost minimization. Three evolutionary approaches, including genetic algorithm variants, were developed to address this problem.

As for crew costs, besides the traditional costs adopted by pre-reviewed literature, some efforts have been exerted to capture the diseconomies of scale, which refers to a unit cost increase because of the increment in the operation density, through the approximate piecewise linear function. For instance, in Mercier and Soumis (2007), a piecewise linear waiting cost was included in the total crew costs. More recently, Quesnel et al. (2020b) incorporated base constraints in the crew pairing problem, in an attempt at balancing the workload among different bases. Accordingly, the problem penalized the additional workload following a structure in which the average penalty increased along with more workload, which induced a convex piecewise linear objective function.

(2) Crew rostering problem

After generating pairings, the goal of crew rostering problem aims to assign crew members to pairings, while respecting a series of restrictions, e.g., crew combability, total working days, and requested off-duty period (Cappanera & Gallo, 2004; Caprara et al., 1998). Typically, there are two ways to generate crew rostering. One is called bidline, which requires airlines to construct anonymous schedules first, and these rosters are then assigned to individual crew members according to bids. The other is personalized rostering, in which airlines try to create equal share crew schedules (Kohl & Karisch, 2004).

The crew rostering problem is commonly of large size and computationally challenging, thus motivating several heuristic approaches in previous studies. For instance, Maenhout and Vanhoucke (2010) studied crew rostering with the objective of minimizing the costs and, in addition, penalizing deviational constraints regarding fairness. This problem was solved through their constructed hybrid scatter search heuristic method. More recently, Zhang et al. (2019) proposed a crew rostering model with multiple objectives, such as minimizing crew assignment costs and optimizing individual preferences. They also developed an improved variable neighborhood search approach for solving such a problem, based on construction methods, e.g., crew-by-crew. On the other hand, since the problem can be formulated based on a set-partitioning model, solution algorithms based on column generation also have been developed. The model in Zeren and Özkol (2016) for generating personalized schedules was based on the generalized set partitioning approach. Accordingly, a column generation heuristic was constructed for solving the large- scale rostering problem.

2.1.5 Integrated Airline Planning

A remarkable disadvantage of traditional sequential airline planning is the long lead time between the airline planning solution and the implementation date. Due to the highly competitive environment, airlines seek approaches to integrate their operational subproblems so as to achieve cost reduction, and increased profitability. Here we review the literature that integrates aircraft maintenance routing with some other stages of airline planning.

2.1.5.1 Integrated Fleet Assignment and Aircraft Maintenance Routing

The aircraft maintenance routing and fleet assignment are usually integrated to determine fleet and aircraft routing decisions, with the aim of keeping maintenance feasibility for the aircraft routing when adopting the solutions of fleet assignment.

Barnhart et al. (1998a) proposed a model, along with a solution approach to solve simultaneously the fleet assignment and AMRP. The proposed model was capable of capturing the costs corresponding to aircraft connections and some complicating constraints such as maintenance demand. The problems of generating aircraft routes and of fleet maintenance scheduling were investigated by El Moudani and Mora-Camino (2000). In addition, instead of using methods incorporating artificial intelligence for airline planning to deal with combinatorial optimization problems, a dynamic method to address on-line operation conditions was developed, which was a combination of a dynamic programming for handling the fleet assignment and a heuristic for handling the aircraft maintenance routing. Haouari et al. (2009) also studied a model integrating the pre-mentioned two problems. Accordingly, a two-phase heuristic method was proposed to iteratively solve the minimum-cost flow problem, which can be modified to address other extensions, e.g., flexible flight departure times. The computational results validated the efficiency of the proposed method, which was proven to be able to generate near-optimal solutions (less than 1%) within a very short time. Later, two exact algorithms, i.e., benders decomposition and branch and price, were proposed by Haouari et al. (2011) to solve the integrated fleet assignment and AMRP. For the purpose of reducing the computational time, several acceleration strategies have been proposed, including maximal clique cuts and strong benders cuts. Through experimental studies, it was shown that the former approach performed better in quickly generating near-optimal solutions. In contrast, the latter approach was good at delivering optimal solutions.

2.1.5.2 Integrated Aircraft Maintenance Routing and Crew Paring

The integration of aircraft maintenance routing and crew scheduling aims at incorporating the various regulations and rules, e.g., the minimum turnaround time ensuring that two sequential flights can be connected, and the minimal sit time for making sure the connection for a crew.

Some researchers studied these two problems both in a partly and full integrated way. Díaz-Ramírez et al. (2014) proposed a model for handling both the AMRP and the crew pairing on the condition of only having one fleet with one maintenance and crew base, which was applicable for several low-cost airlines. On the one hand, the problems were tackled in a traditional way, namely, the AMRP was first solved and then, the crew paring was solved by a combined heuristic and column generation method. Lastly, an integrated model was formulated and solved. The study of Parmentier and Meunier

(2020) was similar, in which they firstly solved these two problems separately. Regarding aircraft maintenance, they used a compact integer programming model to obtain the optimal solution by commercial MIP solvers, while column generation was used to address the crew pairing problem. Lastly, the authors solved the integrated problem and obtained near optimal solutions.

On the other hand, several studies have focused on the full integrated problem. To generate minimum-cost routes and crew pairings, Mercier et al. (2005) integrated two stages of airline planning (i.e., aircraft routing and crew pairing), and used linking constraints to connect crew and route decisions. The authors proposed and compared two benders decomposition methods, which had different master problem, i.e., using the aircraft routing or the crew pairing. Furthermore, they generated several Pareto-optimal cuts to improve the convergence, for the purpose of speeding up the computation. Similarly, Mercier (2008) also dealt with the integrated problem with linking constraints, while restricting the minimum connection time for crews when connecting with different aircraft. Furthermore, they proposed a solution method that enabled the generation of feasibility cuts, created by a benders decomposition.

Another extension was in integrating the last three stages of airline planning. Salazar-González (2014) tried to handle an integrated three stages (i.e., fleet assignment, aircraft routing and crew pairing) within a daily planning horizon, similar to a 2-depot vehicle routing problem. A heuristic approach was used to address this integrated model. However, this model did not consider the maintenance operation as they assumed that no maintenance was performed during the daytime. Recently, Shao et al. (2017) also investigated an integrated problem that incorporated the last three steps of the airline planning process. Accordingly, a new model, with consideration of the itinerary-based demands was developed, and solved through a benders decomposition approach.

2.2 Aircraft Maintenance Routing Variants

From the perspective of the problem variants, two main branches have emerged, i.e., tactical and operational problems (Al-Thani et al., 2016). Generally, both problems take maintenance requirements into consideration. The difference is that the tactical aircraft maintenance routing creates generic routes, i.e., sequences of flights, that can be repeated (Feo & Bard, 1989; Sriram & Haghani, 2003), whereas the operational aircraft routing permits customization to meet diverse operational constrictions (Al-Thani et al., 2016; Eltoukhy et al., 2018).

2.2.1 Tactical Aircraft Maintenance Routing

As mentioned earlier, tactical aircraft maintenance routing aims at producing routes for homogenous aircraft. To do so, Clarke et al. (1997) studied an aircraft rotation problem, where the rotation represented a flying sequence for each aircraft, while allowing the execution of maintenance checks. The authors developed a model for this problem and compared it with the asymmetric traveling salesman problem (ATSP). Mak and Boland (2000) treated the aircraft maintenance routing as the ATSP, where the replenishment arc was used to represent the maintenance connection. To solve this problem, the authors constructed a heuristic framework in which upper bounds were obtained by a simulated annealing approach while lower bounds were generated through adopting a subgradient method to address a Lagrangian dual problem. The term "rotation" was also used in Liang and Chaovalitwongse (2013), where a new rotation-tour network model was developed for a weekly AMRP, which enabled a quite tight LP relaxation. Then, based on this formulation, they integrated it with the weekly fleet assignment problem, trying to handle two problems simultaneously using a diving heuristic. Lacasse-Guay et al. (2010) investigated how the three processes, i.e., strings, big cycle, one-day routes, influence the aircraft routing problem, and compared the resulting problem variants from different viewpoints. The results showed that the first variant was the most adaptable among the three processes, but costed more computational time. The second variant performed the worst in most criteria as it had narrow applicability.

The optimization models for the tactical AMRP are oriented towards generating aircraft routes with cost effectiveness. The maintenance cost has been extensively considered. For instance, since Feo and Bard (1989) incorporated maintenance base selection into aircraft routing decisions, there were both the fixed cost regarding base construction and, in their model, the variable costs with respect to each city. It is worthwhile to note that the unit costs stay the same regardless of how much maintenance is performed. Furthermore, the maintenance cost is also considered in conjunction with other expenditure. Sriram and Haghani (2003) formulated a model simultaneously minimizing the total maintenance cost (with stable unit cost for each aircraft in each city) as well as the penalties incurred by the misassignment of aircraft to origin- destination pairs. In Haouari et al. (2013), a comprehensive objective accounting for costs related to aircraft schedules was proposed, which included the maintenance cost, but it was assumed nearly constant for the same aircraft type, and the value for the connections (e.g., negative through values and short connection penalties, etc.). Bazargan (2015) investigated the aircraft dispatch problem, with the objective of minimizing the total maintenance costs. Furthermore, the authors considered that utilization maximization can be achieved by decreasing the number of expected maintenance activities. Finally, computational experiments were conducted, demonstrating that this proposed strategy was able to achieve 2%-5% maintenance cost savings, compared with other strategies. More recently, Eltoukhy et al. (2017) considered, besides the maintenance cost, the impact of labor shortages in the objective function. Furthermore, cost cutting can also be achieved during the solution process, and has been developed by Safaei and Jardine (2018), who provided a framework consisting of a model that minimizes the total route costs, and a new solution approach that avoids maintenance misalignment, thus enabling significant maintenance man-hour reductions.

As mentioned earlier, through values were adapted in the objective function of Liang et al. (2011). Generally, the through value represents the returns that additional passengers are willing to pay when they can stay on the same aircraft instead of changing to another aircraft at a stopover station, and can be defined as negative when appearing in the cost-oriented models. In contrast, in the profit-oriented models, such as Clarke et al. (1997), through values of flight connections are rewarded in the objective function. However, very few studies on aircraft maintenance routing adopted the total profit maximization in their objectives (Desaulniers et al., 1997; Shao et al., 2017).

2.2.2 Operational Aircraft Maintenance Routing

As airlines operate in a dynamic environment, a long- term plan may be not appropriate because it is easily disrupted by unexpected events (Sarac et al., 2006). Furthermore, as the day of operation approaches, detailed conditions such as the initial position of the aircraft and operational maintenance requirements should be carefully considered to ensure the feasibility of aircraft routes in operations. Therefore, the operational aircraft maintenance routing problem is introduced, through incorporating the information on aircraft closer to the day of operations, such as the original locations and accumulated flying time of individual aircraft, into the AMRP decision framework (Başdere & Bilge, 2014; Sarac et al., 2006). Regarding the objectives in the operational AMRP, minimization of the unused flight time (i.e., the difference between the regulated maximum flying hours and the actual flying hours since the latest maintenance) has been widely explored, with the purpose of reducing the amount of maintenance and, as a consequence, cutting the total maintenance costs in the long term (Başdere and Bilge, 2014).

Sarac et al. (2006) were the first to construct routes for each individual aircraft, with a surrogate objective, i.e., minimization of the unused flight hours. A new model, simultaneously considering maintenance person-hours and slots capacity, was developed to formulate the proposed problem. Başdere and Bilge (2014) extended it into a new weekly operational aircraft maintenance routing process, where a modified connection network was constructed to distinguish the before and after maintenance arcs so that the accumulated flight time of each individual aircraft could be tracked. This formulation was then solved by two different approaches to compare the exact and heuristic methods. One was the branch-and-bound with different branch strategies, i.e., selecting variables to branch on first. The other was a heuristic approach derived from compressed annealing. The computational results showed that the adjusted compressed annealing was able to deal with the large-scale problem. Compared to the model in Başdere and Bilge (2014), the compact model developed by Al-Thani et al. (2016) allowed the incorporation of various types of maintenance restrictions, that is, three maintenance constrictions were integrated into operational aircraft maintenance routing decisions. Then, an efficient very large-scale neighborhood search approach was applied to quickly generate (nearly) optimal solutions. Recently, some scholars focused on the multi- objective optimization problem. For instance, Cui et al. (2019) proposed a bi-objective optimization model with the aim of minimizing the number of aircraft used as well as the total unused flight hours. Besides the unused flight time minimization, some other objectives, such as the profit maximization (Eltoukhy et al., 2018), have also been reported in the literature on the operational variant.

2.3 Robustness of Air Transport

Under an extremely stochastic environment, the operation of airlines often encounter various disruptions, which can result in delays and cancellations (Dunbar et al., 2014). The significance of delays has motivated abundant research on the robustness of air transport. To be specific, the literature mainly focused on evaluating and improving robustness, which leads to two types of methods, i.e., the analytical and operations research approaches.

2.3.1 Analytical Approach for Assessing Network Robustness

The analytical approach aims at assessing aviation network robustness based on topological metrics, e.g., betweenness centrality and degree (Roucolle et al., 2020; Zhou et al., 2019).

Lordan et al. (2016) analyzed the impact of different route network configurations (i.e., hub-and-spoke or point-to-point) on the robustness. They tried to assess the robustness of the whole network through the size of giant components and the simulation results showed that the point-to-point network operated by low-cost airlines was more robust. Later, the vulnerability of codesharing networks was examined by Klophaus and Lordan (2018), where the average edge betweenness, used to measure the efficiency of one node (i.e., airport) to connect with others, was extended to assess the robustness. Zhou et al. (2019) constructed a new metric for evaluating the robustness of an air transport network, considering the link weights, which meant the connection strength between two nodes, in terms of specific indicators, e.g., the route quantity and flight frequencies, the authors then adopted the proposed metric to evaluate the robustness of eight domestic networks. Chen et al. (2020) conducted an investigation of the robustness of China's air transport network, to analyze how it was affected by stochastic failures or targeted attacks.

In addition to these internal aviation factors, some research studies also paid attention to other transportation networks and their impacts on air transportation. In Li and Rong (2022), the positive influence of high-speed rail networks on the robustness of airline networks was investigated, based on measurement metrics including the travel time as well as the service frequency between node pairs.

2.3.2 Operations Research Approach for Enhancing Route Robustness

Because of the underlying stochasticity in operations, airline companies and researchers put considerable efforts into making smooth aircraft schedules under uncertainties. The operations research approach is thus applied, striving to improve the robustness of airline schedules through optimization methods, an appropriate application of which is the robust aircraft maintenance routing problem (Ahmed et al., 2017b; Kenan et al., 2018; Maher et al., 2018). This approach is at the route level, and thus needs detailed information on the flight schedule, fleet and maintenance airport availability. Then, taking possible disruptions into consideration, robust aircraft maintenance routing tries to generate aircraft maintenance routes that are less sensitive and more easily recovered from uncertainties. Accordingly, robustness can be generally constructed from two perspectives, i.e., stability preventing schedules from being influenced by stochastic events, and flexibility providing options easily recovered from disruptions (Eltoukhy et al., 2019).

2.3.2.1 Robust Schedules with Flexibility

The options for the former approach, aiming at enhancing route feasibility, include improving the number of aircraft swapping opportunities within the route (Ageeva, 2000; Burke et al., 2010), guaranteeing sufficient short cycles (Rosenberger et al., 2004), and limiting the number of fleets per spoke airport (Smith & Johnson, 2006). Kang (2004) divided a flight schedule into independent layers and, as a result, the unexpected disruptions in a layer cannot influence the sub-schedules, and was able to significantly reduce delays.

In terms of swapping opportunities, it is acknowledged that aircraft swapping is one of the most important and effective recovery policies in the face of disruptions, which means that more swapping possibilities inserted in the original schedule create more flexibilities to alleviate the impacts of uncertainties. This motivated researchers to enhance schedule robustness by taking swapping possibilities into account (Ageeva, 2000; Burke et al., 2010).

In Ageeva (2000), the robustness of aircraft assignment has been improved through maximizing the number of overlaps, which occurred if two flight sequences could meet at some airports. Based on this criterion, the most robust solution was chosen among multiple optimal solutions for the basic aircraft routing model, and the experimental results showed that a remarkable improvement in robustness, by at most 35%, can be observed compared with models without swapping opportunity consideration. Later, Eggenberg (2009) examined three robustness strategies independently, including providing more swapping chances, to explore the corresponding impact on airline recovery. Recognizing the emphasis on cost competitiveness, these authors proposed AMRP models that seek to enhance robustness while maintaining the optimal cost. In contrast, the route cost is ignored in Burke et al. (2010), who attempted to achieve a robustness improvement through deploying multiple robustness strategies, i.e., retiming as well as improving swapping opportunities simultaneously. A hybrid heuristic algorithm was proposed to solve this complex problem.

Since swapping aircraft to cope with disruptions may consequently disrupt the original maintenance plans, special attention therefore should be given to maintenance robustness. Lapp and Cohn (2012) proposed models aiming at making limited changes (i.e., aircraft swaps) so that the possibilities of an aircraft ending its last flight day at a maintenance station (they called it maintenance reachability) can be maximized.

2.3.2.2 Robust Schedules with Stability

Strategies for creating stability normally involve allocating slack time, i.e., the gap between ground time of the flight connection and the minimum turnaround time, such that disruptions can be absorbed (Ahmed et al., 2017a; Liang et al., 2015). In general, delays broadly fit into two categories, i.e., non-propagated and propagated delays (Dunbar et al., 2012; Lan et al., 2006; Liang et al., 2015; Yan & Kung, 2018). Propagated delays occur due to delays in its upstream flight of the same aircraft, while non-propagated delays are caused by all types of uncertainties that are not related to aircraft routes. Lan et al. (2006) proposed a robust aircraft maintenance routing model, with the objective of minimizing the total expected propagated delays, and solved it using a column generation approach. A similar objective can be found in Liang et al. (2015), where they also considered the maintenance constraints and, like many traditional aircraft maintenance routing models, the maintenance capacities in each station per day. These studies assumed the non-propagated delays of flights to be independently distributed. However, Yan and Kung (2018) proposed a robust optimization method that allowed dealing with correlations in flight leg delays, i.e., assuming that non propagated delays of flights lay in a prespecified uncertainty set, with the objective of minimizing the maximal possible total propagated delay. Accordingly, the authors presented an exact method, i.e., column-and-row generation approach, to solve this robust model.

In addition, some studies focused on applying strategies to achieve robust schedules (Burke et al., 2010; Cacchiani & Salazar-González, 2020). For example, retiming the departure time of flights (within a relatively small time window) is a widely-used approach to reallocate slack times and enhance robustness. Ahmed et al. (2017a) applied retiming, after obtaining the maintenance routing solution, to add slack times to absorb delays. Thus, this kind of retiming did not change the maintenance routes and it only impacted on the buffer time and, thereby, improved the on-time performance. In contrast, flight departure retiming adopted by Aloulou et al. (2013) can have an impact on routing schedules. They proposed a model with the aim of creating aircraft routes with robustness, by adjusting and determining the departure times of flights, while respecting the slot capacity in each airport, so that the slack times in the flight connections can be reallocated. On the other hand, cruise speed control was also applied to improve robustness (Gürkan et al., 2016; Şafak et al., 2017). Gürkan et al. (2016) proposed a model that integrated the first three stages of airline planning, while considering cruise speed control, which resulted in a larger number of flight connection alternatives, and was thus able to generate schedules with enhanced robustness performance. Two heuristic methods were presented to address larger size problems.

In recent years, investigation of particular sources of disruptions has been a research hotspot. Sohoni et al. (2011) incorporated block-time uncertainties as random variables in their model, while adopting retiming with the aim of enhancing the robustness of schedules. The authors also carried out computational experiments, which proved that a model considering block-time uncertainties could make a trade-off between service level and profitability. Later, in Duran et al. (2015), non-cruise time was assumed to be uncertain, and modelled through chance constraints, which were reformulated by second-order cone programming constraints, for the purpose of guaranteeing the passenger connection level, while cruise time could be adjusted if necessary. Through a simulation study, it was shown that the schedules resulting from the proposed model, in comparison with the published ones, could enable improved delay performance. Most recently, Lee et al. (2020) divided the sources of non-propagated delays into two classes, i.e., systemic disruptions resulting from airport congestion and contingent disruptions, and then integrated them in the recovery decision framework.

In addition, recognizing that one of the disruptions to aircraft schedules in operations is the availability of crew, i.e., if the crew assigned to an aircraft is late, the aircraft has to wait until the crew is available, and some research studies paid attention to constructing robust aircraft schedules that were less vulnerable to late crews through improving the crew decisions. For example, Weide et al. (2010) studied a problem that integrated the last two steps of airline planning (i.e., aircraft routing and crew pairing), with the aim of minimizing a weighted sum of the costs and improving the robustness of schedules. The latter objective was achieved through penalizing aircraft changes for crews if the connection time was significantly greater than the minimum required ground time. More recently, Cacchiani and Salazar-González (2017) proposed two MIP models for the problem integrating the last three steps of airline scheduling and, in particular, aircraft maintenance was also considered in the problem. To enhance the robustness of the schedules, the goal of the model also aimed at minimizing the total times that crews had to change aircraft. To solve the proposed models (i.e., the path-path and the arc-path model), two exact solution approaches were proposed.

2.4 Quantity Discount

The quantity discount policy is frequently observed in the transportation industry and therefore has been applied in the related research domain. In a situation with multiple third-party suppliers, considering quantity discounts (i.e., lower unit cost for larger purchases) can lead to possible cost savings (Manerba & Perboli, 2019; Nguyen et al., 2014; Podnar et al., 2002; Russell & Krajewski, 1991) and profit improvement (Qiu & Lee, 2019; Yin & Kim, 2012).

Typically, piecewise linear functions are commonly used to formulate this policy. For example, Mansini et al. (2012) proposed an integer programming model with piecewise linear purchasing costs for capturing total quantity discount and truckload shipping costs. Considering that this model was too complex to be solved exactly within a reasonable time, an iterative rounding algorithm was constructed to rapidly generate good solutions. Through computational experiments, it was shown that the total quantity discount had an significant influence on the solution structures. Hanbazazah et al. (2019) considered a freight consolidation problem that adopted the all-units (total) quantity discount policy, reflecting economies of scales, on shipping costs. A MIP model incorporating piecewise shipment costs as well as an exact solution algorithm were proposed accordingly.

However, very limited attention has been paid to the application of quantity discounts in air transportation research. An extremely rare example can be found in Shaban et al. (2021), in which the authors integrated the quantity discount offered by airlines in their model to encourage freight forwarders to order larger quantities in the underutilized routes and thus achieve demand balance. Numerical analysis has proven that the application of this strategy (i.e., quantity discount) can lead to a significant profit improvement, by more than 25%.

2.5 Summary

This chapter presents review works related to airline planning, i.e., flight design, fleet assignment, aircraft maintenance routing, crew scheduling, and the integrated problem, while highlighting the literature regarding the AMRP. Specifically, we present the two branches for the aircraft maintenance routing, i.e., tactical and operational variants, and investigate the costs considered in these two problems. Furthermore, realizing that airline operations can usually be disrupted by diverse uncertainties in the stochastic environment, we thus further investigate the literature concentrating on analyzing or enhancing the robustness of airline network and schedules. In addition, the incorporation of quantity discounts in transportation research areas is comprehensively surveyed, which helps us to understand its importance and significance. From the literature reviewed above, several critical research gaps can be identified.

Firstly, the literature indicates that impressive cost savings can be achieved by incorporating the total quantity discount policy in transportation applications (Hanbazazah et al., 2019; Mansini et al., 2012). However, the traditional optimization models with respect to aircraft maintenance routing have ignored this advantageous strategy through adapting an oversimplified cost structure, i.e., the unit maintenance costs remain fixed.

Secondly, although extensive research efforts have been dedicated to assess the aviation network robustness and improve the robustness of airline plans (Eggenberg, 2009; Li & Rong, 2022; Yan & Kung, 2018; Zou & Hansen, 2012), very limited attention has been paid to the robustness in maintenance stations. In particular, they unfortunately fail to analyze the underlying structure choice, i.e., the maintenance distribution structure, to understand its characteristics and possible impacts on route robustness.

Thirdly, most of the traditional robust aircraft maintenance routing models addressed the sources of disruptions in an aggregated manner, however they ignored the particular influence of maintenance uncertainties (Lan et al., 2006; Liang et al., 2015), which can generate aircraft routes that are vulnerable to disruptions stemming from maintenance tasks.

Chapter 3. The Aircraft Maintenance Routing Problem with Piecewise Maintenance Cost

In this chapter, we investigate a novel aircraft maintenance routing problem that incorporates the impact of the total quantity discount policy, with the aim of constructing aircraft routes at minimal maintenance purchasing cost. This incorporation is primarily motivated by the attempts that airlines made to manage their maintenance costs, and the significant cost savings accrued by taking advantage of the total quantity discount policy in transportation applications, i.e., the price discount relying on the interval in which the total quantity falls (Christensen & Labbé, 2015; Hanbazazah et al., 2019). When it comes to procurement in maintenance operations, by this token, an airline will receive discounts (namely, lower unit costs) if the number of events scheduled to a maintenance provider exceeds the breakpoint quantity. Therefore, the incorporation of a discount policy into aircraft routing decisions may activate significant cost efficiencies.

To integrate the total quantity discount policy into the aircraft routing decision-making framework, an imminent challenge is that the discount policy itself is one of the most technically complicated features of the purchasing problem (Manerba et al., 2014). Furthermore, in achieving the integration of the total quantity discount and aircraft routing, the complexity of the problem can be remarkably increased since diverse factors should be taken into consideration, such as (i) a combinatorial decision framework comprising of the determination of the subset of flight legs on each route, deciding on the total number of maintenance events conducted at each station and, based on this choice, assigning the corresponding unit cost; and (ii) various operational rules, e.g., the flight sequence condition, maintenance requirement, and capacity restrictions, etc. Besides these features, there are a large number of aircraft candidate routes, especially for large- scale networks that the total quantity discount can work on, which are computationally expensive. To overcome these difficulties, our mathematical formulation is based on the set partitioning problem, making it easier to incorporate several operational features and, more notably, avoiding the explicit complete enumeration of all potential aircraft routes. In addition, the total maintenance demand in each station is modelled as an endogenous continuous variable which can be calculated during the process of route selection. The resulting model is strongly NP-hard, and hence needs a sophisticated solution approach. Therefore, we develop a column generation- based diving heuristic framework, which consists of a column generation approach for optimizing the linear programming relaxation of the constructed model, a diving heuristic where the searching space is limited by a novel two-phase branching strategy, and a restricted mixed integer programming problem for generating a feasible integer solution.

The remainder of this chapter is structured in the following way. Section 3.1 is devoted to the overall modelling framework, which includes the basic setting of the proposed problem (Section 3.1.1), connection network construction (Section 3.1.2), and the maintenance cost structure (Section 3.1.3). Then, the new mathematical formulation for the stated problem is presented and discussed in Section 3.2. Next, Section 3.3 illustrates the development of the solution approach. In Section 3.4, we report the computational experiments, including examination of the computational performance of our proposed algorithm (Section 3.4.2), a comparison between our model and the existing modelling approaches which neglect the total quantity discount policy in maintenance operations, i.e., with the unit maintenance cost remaining constant at each station (Section 3.4.3), and a sensitivity analysis on the key parameters of the cost structure (Section 3.4.4). We present the summary of this chapter in Section 3.5.

3.1 Problem Description

In this sub-chapter, we provide a detailed description of the aircraft maintenance routing problem that exploits the total quantity discount policy in maintenance operations, in which the framework is demonstrated in **Figure 3-1**. In the following, we describe the regulations and rules related to aircraft routing, prepare a connection network for generating aircraft routes, and state the maintenance cost structure studied in this chapter.

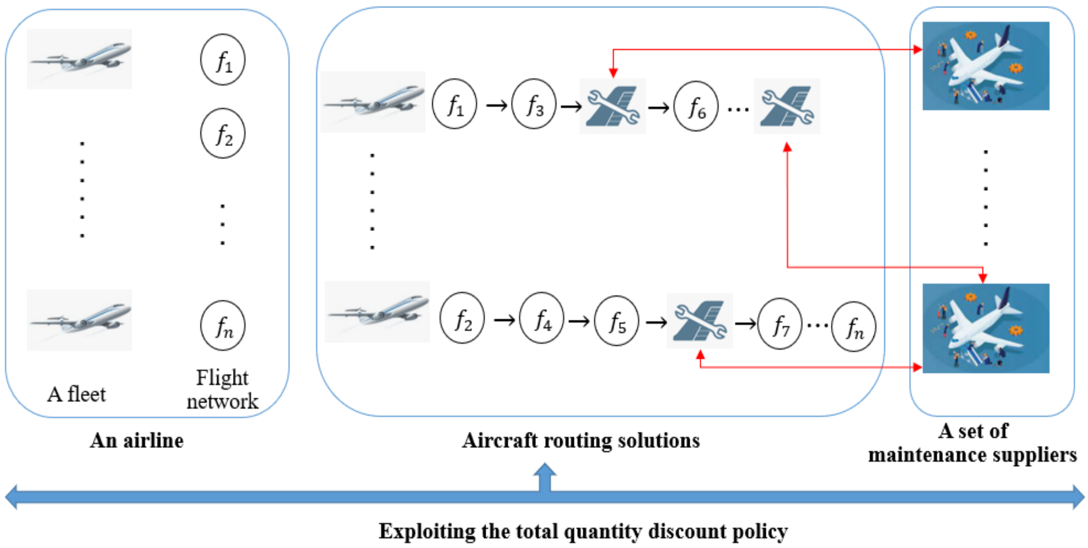

Figure 3-1. The tactical aircraft maintenance routing exploiting total quantity discounts

3.1.1 Basic Setting

Given a flight schedule, a set of homogeneous aircraft which have received maintenance, and coordinating with a few maintenance stations located at specific airports, the problem addressed in this chapter aims to generate effective and efficient aircraft routes with the goal of minimizing the total maintenance cost and, at the same time, accounting for some restrictions and regulations, as discussed in the following.

Our planning horizon is 14 days, as suggested by Feo and Bard (1989), where it is stated that a circulation of an aircraft can be set as multiples of 7.

Consider a pre-defined flight schedule that contains a set of legs, each of which is characterized by a departure and arrival time, while starting from the origin and reaching the destination without any intermediate stop. Following actual practice, we say that two legs can be connected if (i) the arrival airport of the preceding leg happens to be the departure airport of the subsequent leg, and (ii) the minimum turnaround time requirement, i.e., the time spent on necessary operations between two consecutive flying tasks, is met. In addition, considering the improvement in aircraft utilization, the ground time should be no more than 24 hours (Cui et al., 2019). Hence, a sequence of flight connections consecutively covered by the same aircraft constitutes a route. Note that, to maintain the continuity of services, the route should also obey the maintenance requirements as discussed below.

- **Maintenance requirements**

According to the FAA's regulations, aircraft must undergo a series of checks with increasing scope, duration and conversely decreasing frequency. In general, the frequency of maintenance checks is based on the accumulated flight hours, flight cycles or calendar days, whichever happens first. Practically, airlines usually operate a restrictive rule, i.e., every 4 calendar days (Feo & Bard, 1989; Talluri, 1998). We use $maxD$ to represent the maximum calendar days between two successive maintenance activities. It often costs several hours to complete the maintenance check, and usually there is not enough time during the day for conducting maintenance checks; therefore, it should be conducted at night. In order to implement this restriction, each aircraft is forced to visit a maintenance airport, at least once every 4 days, where an overnight maintenance opportunity exists, i.e., allowing the aircraft to stay overnight at the airport for at least 8 hours, while ensuring that the capacity restriction at the station, which limits the number of aircraft undergoing maintenance activities every night, can be satisfied.

3.1.2 Network Structure

We construct a network for generating feasible routes for aircraft. In the literature, there are two common approaches, i.e., time-space and connection network, used to represent the flight network (Zhou et al., 2020). The time-space network, in which each node denotes a departure or arrival event, consists of three types of arcs, i.e., a leg arc representing a scheduled flight, a ground arc standing for aircraft staying on ground, and a wrap-around arc indicating an overnight stay (Liang et al., 2011). In contrast, the flight legs in the connection network are represented by nodes in the network, while the arcs denote feasible connections between two successive flight legs. The time-space network can be used to generate Eulerian tours as routes. However, such a method fails to deliver the flight connections and individual aircraft routes explicitly. In comparison, individual aircraft routes can be constructed from the results of the decision variables by the connection network (Safaei & Jardine, 2018). Therefore, we employ, during the candidate route generation process, the connection network structure and modify it so as to be applicable to our problem (Başdere & Bilge, 2014; Sarac et al., 2006).

In particular, this structure $G = (V, A)$ contains a set of nodes and arcs, where V and A stand for node and arc sets, respectively. Each operational flight leg of the flight set (denoted by F) can be represented by a node in V. Furthermore, dummy nodes, source o and sink t, can be treated as a special type of flight leg with duration 0 and, therefore, V consists of all legs, including source and sink, namely, $V = F \cup \{o\} \cup \{t\}$. In the network G, arc sets can be constructed in advance based on the scheduled flight timetable, so that the precedence relationship can be naturally built. By doing so, redundancy arcs can be eliminated in order to simplify the structure of the connection network and to improve the efficiency of solving the problem. To be specific, the set of arcs A further consists of two types of arcs, i.e., connection arcs (represented by A_f) and maintenance arcs (denoted by A_m). A feasible connection arc (i.e., non-maintenance connection) is included in the network if the conditions identified for the flight connections given in Section 3.1.1 can be satisfied. In addition, if there is a potential maintenance connection (or opportunity) between two successive flight legs, a maintenance arc that belongs to A_m should then be added. We assume that a maintenance check occurs at night, and a maintenance arc starts with an end-day node and connects a node departing from the same airport in the following day. It is worthwhile to mention that since all flight legs are ordered according to their departure time, G is unsurprisingly a directed acyclic graph. For the sake of clarity, an example is shown in **Figure 3-2**, in which there is a maintenance station located at airport B and the ground time between flight legs i and j is longer than the required maintenance execution time.

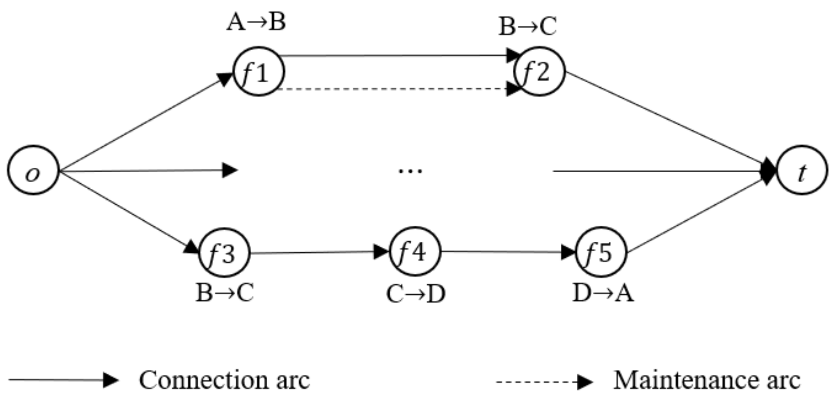

Figure 3-2. An example for the connection network structure.

3.1.3 Maintenance Cost Structure

Before developing the mathematical formulation, we first define the maintenance cost structure, on which the objective function to be studied depends. As mentioned earlier, the cost structure explicitly exploits, under the setting of maintenance outsourcing, the total quantity discount policy, i.e., the unit maintenance cost depends on the discount interval in which the scheduled maintenance activities fall. Not surprisingly, this approach is generally in line with the normal practice since, in the purchasing process, the independent vendors submit quotations (bids) with quantity discounts for larger orders to stimulate demand, while airlines decide on the winners and associated quantities with the aim of minimizing the total maintenance costs as well as meeting the maintenance requirements satisfactorily. Therefore, airlines commonly observe a piecewise maintenance cost, which is non-decreasing in the number of scheduled maintenance operations in each station.

Considering that airlines generally prefer to undertake their maintenance activities at the base airports and, without loss of generality, we suppose that one supplier is chosen from the maintenance station in each base airport. Now, consider a maintenance station (used to represent the third-party service provider at that base hereafter) $s \in S$, the cost function of which has a set of intervals (segments) $L_s = \{1, \cdots, l_s\}$ and each interval $l \in L_s$ is associated with a unique unit cost c_{ls}, which changes as long as the number of maintenance operations exceed a pre-determined breakpoint b_{ls}. Beyond this breakpoint, the total maintenance cost for each station linearly increases with the number of maintenance operations assigned to the station increasing, until the next breakpoint. By the problem definition, this cost structure results in a lower unit cost for a greater amount of maintenance due to the offered discounts. For clarity of illustration, we present an example of the developed cost structure in **Figure 3-3**.

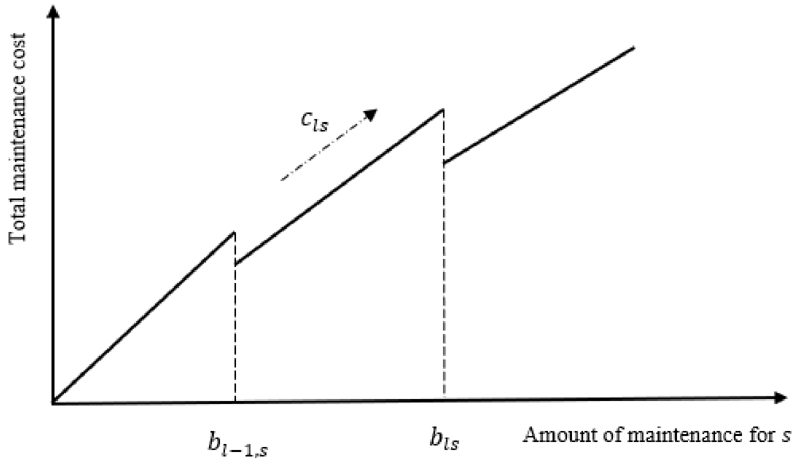

Figure 3-3. Piecewise cost structure.

3.2 Mathematical Formulation

In this sub-chapter, we strive for a mathematical formulation of the problem defined in the previous section. As stated in Section 3.1.3, the maintenance cost in each station follows a piecewise structure. Typically, there are three classic and common formulations for the representation of a discontinuous and piecewise function, i.e., multiple-choice model, incremental model, and convex combination model, whose linear programming relaxations are proven equivalent (Croxton et al., 2003). The multiple-choice model is recommended when the number of intervals, in the transportation problem, is comparatively small (Christensen & Labbé, 2015). Therefore, in this research, we construct the model based on the multiple-choice model to formulate the proposed maintenance cost structure. Hence, the novel aircraft maintenance routing model incorporating piecewise costs (AMR-PC) can be then stated in the following way.

Sets

F	the set of operational flight legs
R	the set of potential routes
D	the set of days
S	the set of maintenance stations
L_s	the set of intervals for station $s \in S$

Parameters

i	index for flight legs
r	index for routes
d	index for days
s	index for maintenance stations
l	index for intervals
b_{ls}	the breakpoint for interval $l \in L_s,\ s \in S$
c_{ls}	the unit maintenance cost allocated to interval $l \in L_s,\ s \in S$
K	the total number of aircraft
Cap_{ds}	the maximum number of aircraft can be maintained at station $s \in S$ on day $d \in D$
θ_{ir}	$= \begin{cases} 1 & \text{if route } r \text{ contains flight leg } i \in F \\ 0 & \text{otherwise} \end{cases}$
φ_r^{ds}	$= \begin{cases} 1 & \text{if maintenance at station } s \in S \text{ on day } d \in D \text{ belongs to route } r \\ 0 & \text{otherwise} \end{cases}$

Variables

y_r	$= \begin{cases} 1 & \text{if route } r \text{ is selected} \\ 0 & \text{otherwise} \end{cases}$
v_{ls}	$= \begin{cases} 1 & \text{if the maintenance demand lies in interval } l \in L_s, s \in S \\ 0 & \text{otherwise} \end{cases}$
u_{ls}	the total maintenance demand in interval $l \in L_s, s \in S$

AMR-PC

$$\min \sum_{s\in S}\sum_{l\in L_s} c_{ls}u_{ls} \tag{3-1}$$

$s.t.$

$$\sum_{r\in R} y_r \le K, \tag{3-2}$$

$$\sum_{r\in R} \theta_{ir} y_r = 1, \qquad \forall\, i \in F \tag{3-3}$$

$$\sum_{r\in R} \varphi_r^{ds} y_r \le Cap_{ds}, \qquad \forall\, d \in D \text{ and } s \in S \tag{3-4}$$

$$\sum_{r\in R}\sum_{d\in D} \varphi_r^{ds} y_r = \sum_{l\in L_s} u_{ls}, \qquad \forall\, s \in S \tag{3-5}$$

$$b_{l-1,s} v_{ls} \le u_{ls}, \qquad \forall\, l-1, l \in L_s \text{ and } s \in S \tag{3-6}$$

$$u_{ls} \le b_{ls} v_{ls}, \qquad \forall\, l \in L_s \text{ and } s \in S \tag{3-7}$$

$$\sum_{l\in L_s} v_{ls} \le 1, \qquad \forall\, s \in S \tag{3-8}$$

$$y_r \epsilon \{0,1\}, \qquad \forall\, r \in R \tag{3-9}$$

$$u_{ls} \ge 0, v_{ls}\ \epsilon \{0,1\}, \qquad \forall\, l \in L_s \text{ and } s \in S \tag{3-10}$$

The objective function (3-1) aims at minimizing the total maintenance cost. This strategy exploits a maintenance cost structure in which the cost per unit is selected depending on the total number of the maintenance operations scheduled at a station, which is calculated using the left side of Constraint (3-5). It is worthwhile to note that this objective function is generic so as to include other cost structures like constant unit cost adopted by most of traditional models for AMRP, through fixing the value of first interval to 1. Furthermore, because the proposed model is constructed based on the multiple-choice model, the cost function can be slightly modified (through adding the intercept-related terms) to formulate the incremental discount type. Hence, the mix of these cost structures can clearly be achieved.

Constraints (3-2)- (3-4) define the important factors or restrictions with which general aircraft scheduling is of concern. To be specific, Constraint (3-2) enforces the required number of aircraft not to exceed the fleet size, while Constraint (3-3) is the coverage constraint, making sure that every flight leg can be covered exactly once. Moreover, the maintenance capacity at each station, limiting the maximum number of aircraft per day (we only count the aircraft receiving maintenance starting on that day), is bounded by Constraint (3-4).

It can be readily observed that the above four constraints are closely related to the aircraft routes, which are followed by Constraint (3-5), which makes sure that the maintenance demand allocated to each station can be fulfilled. Constraints (3-6) and (3-7) warrant that the maintenance allocation (i.e., the total maintenance operations allocated to each station) is in line with the corresponding upper and lower bounds of the interval, respectively. Furthermore, Constraint (3-8) guarantees that at most one interval can be selected for each station. Note that it is possible that none of the intervals will be chosen, which implies that the corresponding station will not be selected by the airline. Lastly, Constraints (3-9) and (3-10) define the domain of the decision variables.

3.3 Solution Approach

There are several challenges in solving the proposed AMR- PC. Firstly, the constructed problem consists of too many potential aircraft routes (or variables in the modelling level) to be explicitly enumerated. Secondly, the problem involving the total quantity discount is strongly NP-hard thus motivating study of effective and efficient heuristic methods for the solution (Manerba & Perboli, 2019). Thirdly, the incorporation of quantity discount introduces new binary variables to the model and thus further complicates the computational process.

In aircraft routing research, column generation is an efficient linear programming algorithm for addressing such large-scale problems, by only generating effective aircraft routes that are possible for improving the current solution. Furthermore, it is noted that matheuristic algorithms, such as primal heuristics, enable generating good, feasible solutions to large- scale optimization problems, through taking advantage of the exact algorithm methods (Sadykov et al., 2019). Therefore, we accordingly construct a suitable column generation-based heuristic algorithm for solving the model. Algorithm 1 provides the pseudo-code of the proposed algorithm.

Algorithm 1 Tailored column generation- based diving heuristic algorithm

Input: Instances

Output: Integer feasible solutions with value of y-, v- and u- variables

Initialize the restricted master problem and solve it via *column generation* (Section 3.3.1 and 3.3.2);

Diving heuristic with sub-MIP (Section 3.3.3)

  for stations with the highest number of intervals l^{max} **do**

    if $v_{l^{max},s}$ is fractional **then**

      create a node according to phase I of the branching scheme;

      conduct column generation process;

    end if

  end for

  if y- variable is fractional **then**

    create a node according to phase II of the branching scheme;

    conduct column generation process;

  end if

  Solve the resulting mixed integer programming problem exactly.

Specifically, our solution algorithm framework is composed of two interdependent modules, one of which is the column generation scheme that endeavors to iteratively solve a restricted master problem as well as a corresponding pricing subproblem, illustrated in Section 3.3.1 and Section 3.3.2, respectively. Secondly, to obtain a good-quality integer solution, this complete column generation process is properly embedded in the diving heuristic with sub-MIP. Considering that the model (i.e., AMR- PC) contains two types of variables, i.e., route variables for selecting the optimal aircraft routes and interval selection variables for identifying which interval will be chosen for the corresponding unit maintenance cost, to handle this, we introduce a two-phase branching scheme, based on which the selected variables are iteratively kept fixed to limit the search space. The details of the extended diving process are demonstrated in Section 3.3.3.

3.3.1 Restricted Master Problem

Model AMR-PC has an exponential number of route / column variables, which makes it impossible to solve directly. Hence, the LP relaxation of the master problem (i.e., AMR-PC) is considered, which we refer to as L- AMR-PC, and in order to apply the column generation procedure, which solves the LP relaxation with a set of route variables $R' \subseteq R$ only, called restricted L- AMR-PC. The restricted L- AMR-PC takes charge of selecting the ideal aircraft routes from the current candidates. To maintain the feasibility in the initial stage, we therefore introduce one artificial variable to each flight coverage constraint. Next, the restricted L- AMR-PC is optimized through the linear programming technique, and the dual information corresponding to route-related constraints is then transferred into the pricing subproblem for finding effective routes (columns) with negative reduced costs, which are again fed into the solution pool of the restricted master problem. Hence, the next iteration begins with the updated restricted master problem. On the other hand, if no such promising route can be found (namely, no improvement on the current solution is expected), the LP relaxation is confirmed to be optimal and the whole column generation process terminates.

3.3.2 Pricing Subproblem

As illustrated earlier, checking for the optimality of the LP relaxation (or the termination criterion for the column generation process), and generating promising routes can be achieved by solving the pricing subproblem, which is a shortest path problem with one resource constraint (i.e., the maintenance requirement in this problem). That is, a route is said to be feasible on condition of meeting the restriction, i.e., the aircraft must receive maintenance at least once every four days. Therefore, we address this problem based on the directed acyclic network with additional maintenance arcs, as described in Section 3.1.2. Now, given current solutions of the restricted L- AMR-PC, we can obtain the following dual variables.

Dual variables	
τ	dual variable of the fleet size constraint (3-2)
α_i	dual variable associated with the coverage constraint (3-3) for flight leg i
β_{ds}	dual variable with respect to the maintenance capacity constraint (3-4) for the maintenance station s on day d
γ_s	dual variable corresponding to the maintenance counting constraint (3-5) for station s

Thus, the reduced cost $\bar{\pi}_r$ of the route r is defined as

$$\bar{\pi}_r = -\tau - \sum_{i \in F} \alpha_i \theta_{ir} - \sum_{s \in S} \sum_{d \in D} (\beta_{ds} + \gamma_s) \varphi_r^{ds} \tag{3-11}$$

where the first term τ is a constant derived from the restricted master problem, and we hence assign $-\tau$ to the cost of the source node. The second and last terms are the costs contributed by the flight selections and maintenance checks along the route, respectively. It is noticeable that our pricing subproblem is different from the previous ones in the AMRP literature due to the cost derived by the maintenance counting (i.e., the existence of γ_s).

The pricing subproblem searches for the optimal route with the most negative reduced cost, simultaneously meeting the maintenance restriction. Therefore, a two-label approach that follows the dynamic programming process is developed. In this approach, a label is comprised of two components where the reduced cost associated with the route checks the optimality and accumulated flying days to ensure the feasibility of the routes in the network. Algorithm 2 provides the pseudo-code of the two-label approach, which works on the connection network $G = (V, A)$ proposed in Section 3.1.2.

Let $e_i = (w_i, \omega_i)$ be a label of node i (which can be a flight leg, source or sink), where w_i tracks the total reduced cost while ω_i records the elapsed calendar days since the last maintenance. We employ a dominance rule to avoid the explicit enumeration of all possible (partial) routes with the purpose of shortening the computational time. To be specific, we consider two labels $e_i' = (w_i', \omega_i')$ and $e_i'' = (w_i'', \omega_i'')$ representing two different partial routes reaching the node i. Then, e_i' is defined to dominate e_i'' on condition that $w_i' \leq w_i''$and $\omega_i' \leq \omega_i''$.

To start the algorithm, we assign a label $(-\tau, 0)$ to the source node and $\emptyset$ to the label set of the remaining nodes. In view of the fact that the network is proven acyclic, the nodes therefore can be checked in topological order, and each time a node is processed, we successively address the labels belonging to this node. Recall that the network contains two arc types: connection arc and maintenance arc, whose costs and flying days are completely distinct. As a result, when extending a label of node i, all the arcs emanating from this node should be successively processed according to the corresponding arc type, which is illustrated in the following.

Algorithm 2 two-label algorithm

Input: fleet size dual τ, flight dual α_i, maintenance capacity dual β_{ds} and counting dual γ_s

Output: route with most negative reduced cost

Initialize the source node's label set as $\{(-\tau, 0)\}$ while the remaining nodes as $\emptyset$;

for each node $i \in V$ **do**

  for each label (w_i, ω_i) of node i **do**

    for each connection arc $(i, j) \in A_f$ **do**

      $\omega_j = \omega_i + (\text{day of flight } j - \text{day of flight } i)$;

      $w_j = w_i - \alpha_j$;

      if $\omega_j \leq maxD$ and (w_j, ω_j) is not dominated by any label tied with j **then**

        delete the label(s) dominated by (w_j, ω_j);

        add (w_j, ω_j) into the label set for node j;

      end if

    end for

    for each maintenance arc $(i, j) \in A_m$ **do**

      $\omega_j = \text{day of flight } j - \text{day of flight } i$;

      $w_j = w_i - \alpha_j - \beta_{ds} - \gamma_s$;

      if $\omega_j \leq maxD$ and (w_j, ω_j) is not dominated by any label tied with j **then**

        delete the label(s) dominated by (w_j, ω_j);

        add (w_j, ω_j) into the label set for node j;

      end if

    end for

  end for

end for

Select the label for the sink node with the minimum reduced cost w_j^*;

if $w_j^* < 0$ **then**

  generate the route;

end if

1. Regarding a connection arc $(i,j) \in A_f$, only the cost with respect to flight selection (i.e., $-\alpha_j$) should be assigned to the arc, and thus contributes to the cost for the subsequent node j (i.e., w_j). Moreover, the accumulated flying days for the subsequent node (i.e., ω_j) is updated based on the label of the predecessor i (i.e., ω_i) and the past days of this arc (i.e., the difference between the scheduled operation day of i and j).

2. Considering a maintenance arc $(i,j) \in A_m$ with the condition that the flight i is operated on day d and the maintenance station s is located in the departure airport of the subsequent flight j, then the cost associated with this arc becomes $-\alpha_j - \beta_{ds} - \gamma_s$. Note that ω_j in this step is set as the past days of this maintenance arc.

After the calculation, we check whether $\omega_j \leq maxD$ and, if yes, the (partial) path is feasible and we can take the next action, namely, checking the dominance conditions. In this regard, two situations should be considered: If the resulting label is not dominated by any other label tied with the subsequent flight (node) j, this label hence can be added into the label list of the node j; otherwise it will be discarded, and ii) if a label, in the label set of the node j, is dominated by the resulting label then it should be deleted. Once all nodes are processed, the algorithm selects the route with the minimum reduced cost and, if all $\bar{\pi}_r \geq 0$, the whole column generation process terminates. Otherwise, this qualified route should be added into the solution pool of the restricted master problem.

3.3.3 Diving Heuristic with Sub-MIP

The column generation is usually combined with the branch and bound approach, which is known as branch-and-price, in order to obtain an optimal but comparatively computationally expensive integer solution. Therefore, with the hope of identifying an acceptable solution in a relatively reasonable time, we construct a diving heuristic in such a way that, at each node of the branch and price tree, column generation should be implemented, and after finding optimal LP solutions, the algorithm creates and processes only one child node. Furthermore, this diving process is then incorporated with sub-MIP which solves a restricted MIP directly (Sadykov et al., 2019).

Considering the proposed model includes two different binary variables, based on the features of each variable type, we propose a two-phase branching scheme that starts from an optimal solution of the restricted master problem. In phase I, we first present some observations regarding the interval selection variable (v_{ls}) which represents whether the corresponding discount interval $l \in L_s$ of the station $s \in S$ is selected or not. Goossens et al. (2007) proposed an optimal solution for LP relaxation of the total quantity discount problem which selects the highest segment for each vendor. That is, services or goods are delivered, in the final LP solution, at the cheapest prices provided by the vendor. Therefore, the value of the v-variable for the highest segment can pose a significant influence on the solution quality, which motivates us to design a branching strategy based on these variables. Let l^{max} be the maximal number of intervals among the maintenance stations and $\bar{S}$ be the set of stations satisfying $l_s = l^{max}$. At each iteration, we select $s \in \bar{S}$ whose $v_{l_s,s}$ is fractional with a value a, denoted by $v'_{l_s,s}$. Then, we have two branching constraints:

$$v'_{l_s,s} + z_t \geq 1 \qquad \forall s \in \bar{S} \tag{3-12}$$

$$v'_{l_s,s} - z_t \leq 0 \qquad \forall s \in \bar{S} \tag{3-13}$$

Since the current restricted master problem with already-generated routes may become infeasible due to the additional constraints during the column generation process, we thus introduce in each iteration (referring to t) an artificial variable z_t, penalized by a "big M" cost, to guarantee feasibility. When implementing this scheme, Constraint (3-12) is added to the restricted master problem if $a \geq 0.5$ or the value of $\sum_{l \in L_s} u_{ls}$ falls in the interval l_s of the station $s \in \bar{S}$, while Constraint (3-13) is employed otherwise. At each iteration, after adding a branching constraint, the restricted master problem is then solved.

Moving to Phase II, in terms of the variables representing the route selection decisions, our branching scheme is thus based on the so-called branch-on follow-on, which is widely applied in the AMRP (Lan et al., 2006; Sarac et al., 2006). In particular, the basic idea is in selecting a pair of flight legs (i, j) through counting $p_{(i,j)} = \sum_{r \in R^{(i,j)}} y_r$, where $R^{(i,j)}$ denotes a line of routes in which the pair (i, j) is consecutively covered. This calculation is done to detect a pair such that $0 < p_{(i,j)} < 1$, reflecting that it appears in several routes with fractional values. Moreover, suppose that more than one such pair can be found, the one closest to an integer, i.e., $(i, j) = \arg \min_{(i,j): 0 < p_{(i,j)} < 1} \{\min[p_{(i,j)}, 1 - p_{(i,j)}]\}$, is selected for branching and for creating a node, where the flight legs i and j should be non-consecutively flown by the same aircraft if $p_{(i,j)}$ is close to 0, otherwise the pair (i, j) is directly connected.

We stop the branch-on follow-on process if no branching leg pair can be detected and thus the integrality of y- variable is guaranteed. However, since not all the v-variables are fixed during the diving heuristic process, to obtain a feasible integer solution of the proposed model, the restricted master problem is thus transferred into a restricted MIP problem and is then solved by a standard branch and bound process.

3.4 Computational Experiments

Here, we describe computational experiments to validate the effective performance of the model as well as the solution framework proposed in this research. The coding implementation used Java programming language while the restricted master problem and restricted MIP were solved through the Concert Technology in CPLEX Studio IDE

12.10. In addition, all the experiments were run on a laptop with Intel(R) Core (TM) i7-9750H CPU@ 2.60GHz and a Windows 10 operational system. Subsequently, we first prepare the data sets and necessary parameters for the experiments. Following these, we first examine the efficiency of the proposed algorithm, and the newly developed two-phase branching scheme. Next, we investigate the advantages of the model incorporating piecewise maintenance costs through comparative experiments. Finally, we conduct sensitivity analysis on the key parameters of the cost structure.

3.4.1 Data Description

In the computational study, we used data from a major U.S. airline's operation schedule during January 2020, which was accessible on the website of the United States Bureau of Transportation Statistics (BTS, 2021). We focused on the Boeing 757 fleet and derived a total of eight scenarios, comprising a broad range of problem sizes over a two-week period. The scenario information is summarized in **Table 3-1**. In particular, each scenario is characterized by the number of flights, aircraft, involved airports and maintenance stations. We record them in an increasingly large manner, in which the first scenario is a small-scale problem with less than 200 flight legs, with the remainder middle and large ones.

With regard to the maintenance cost structure, considering that it was not given in the original data, we assumed in our test that each maintenance station (located at the base airports belonging to the airline) had three cost intervals with two breakpoints (i.e., 10 and 30). In particular, quantity discounts (i.e., 5% and 8%) were assigned the second and third intervals, respectively. Taking the first station as an example, we had the unit variable costs with respect to the three segments: c_{11}, $c_{21} = 0.95 * c_{11}$ and $c_{31} = 0.92 * c_{11}$. In addition, the constant unit cost for each station, prepared for the comparative experiments, was equal to the one for the correspondingly first interval (called the basic price hereafter), which was generated around 500. The resulting scenarios thus have discontinuous, non-decreasing costs. In addition, we then present the general setting for the required operational parameters: the minimum turnaround time between two successive legs is 40 minutes, and the maintenance duration is set as 8 hours.

Table 3-1. Characteristics for scenarios.

Scenario	No. of flights	No. of aircraft	No. of airports	No. of maintenance stations
1	152	6	16	3
2	205	8	16	3
3	263	10	21	4
4	368	14	21	4
5	424	16	22	5
6	512	19	22	6
7	560	20	21	6
8	636	23	21	7

3.4.2 Computational Performance

3.4.2.1 Performance Analysis

Computational experiments were carried out to verify the performance of the algorithm proposed in this chapter through solving the proposed AMR-PC. The numerical results are illustrated in **Table 3-2**, where the "Depth" is depth of the diving heuristic process, i.e., the number of created nodes in the partial branch and price tree, and "No. of routes" gives the total number of existing routes when solving the restricted MIP. Moving to the fourth and fifth columns, "LP objective" represents the lower bound, i.e., the optimal LP solution at the root node, while the "IP objective" stands for the integer solution obtained at the end of the solution process. Given the lower bound and the integer solution (i.e., upper bound), “Gap” means the difference in percentage, which is calculated by (IP objective - LP objective)/ IP objective. Furthermore, the last column records the computational time of each scenario.

Table 3-2. Computational performance of the proposed solution approach when solving AMR-PC.

Scenario	Depth	No. of routes	LP objective	IP objective	Gap (%)	Run time (s)
1	113	483	9025	9125	1.10%	24
2	259	2903	11875	11975	0.84%	230
3	365	3452	14378	14530	1.05%	682
4	790	4764	19833	20010	0.88%	2204
5	997	5717	22235	22700	2.05%	3145
6	1499	9201	26875	27175	1.10%	8394
7	1553	11589	28205.4	28675	1.64%	17109
8	2093	19314	31923.8	32465	1.67%	31224

We first focus particular attention on the run time, where it is clearly observed that, regardless of the size, all scenarios are solved within a reasonable time, which validates the efficiency of our proposed solution approach. To be specific, the small-sized problem, i.e., scenario 1, requires an extraordinarily short time (i.e., only 24 seconds). Another reflective finding is that the computational time generally grows in line with the increasing problem scale. For instance, it only takes 230 seconds to identify a solution for scenario 2; however, this value soars to 3145 seconds with respect to scenario 5, where the flight amount and fleet size are almost doubled. There are two main reasons behind this dramatic growth: i) the increasing number of constraints and route candidates complicates the restricted master problem, and ii) a significant increase in the complexity of dynamic programming since it should traverse all nodes (i.e., flights) and connections. Despite that, our algorithm is able to find a satisfactory solution for the large-scale scenario, comprising 636 flight legs and 23 aircraft, within 9 hours.

From the respective of the solution quality, as can be seen in **Table 3-2**, seven of our test scenarios are shown to be near-optimal with very small optimality gaps (around 1%), while the remaining one (i.e., scenario 5) also generates a relatively high- quality solution (with a gap slightly above 2%). It should be pointed out that the gaps result in large part from the relaxation of the interval selection variables (i.e., the binary variables v) in the restricted master problem. In particular, we can clearly see, from **Table 3-3**, that the customized column generation- based approach is capable of identifying optimal solutions in most cases (five of eight scenarios while the remaining three have minor gaps) when adopted to address the traditional model without a piecewise function.

Table 3-3. Performance comparisons of models with distinct maintenance cost structures.

Scenario	Traditional model			AMR-PC	Cost
	LP objective	IP objective	Gap (%)	IP objective	reduction
1	9500	9500	0.00%	9125	375
2	12500	12500	0.00%	11975	525
3	15080	15080	0.00%	14530	550
4	21540	21540	0.00%	20010	1530
5	24100	24100	0.00%	22700	1400
6	29020	29080	0.21%	27175	1905
7	30580	30660	0.26%	28675	1985
8	34560	34600	0.12%	32465	2135

3.4.2.2 Performance of the Proposed Branching Scheme

In our diving approach, we propose a two-phase branching scheme, the effectiveness of which is validated through comparing it with the other strategy, i.e., only branch-on follow-on is employed (called base scheme hereafter). The computational results are illustrated in **Figure 3-4**, where we present the integrality gaps obtained through solving all scenarios using the solution approach with two distinct branching strategies. In this figure, it can be clearly observed that the integrality gaps yielded by the base scheme are significantly larger than those by the proposed strategy in all scenarios. Special attention is paid to scenario 8, where our strategy reduces the gap by 3.46% (compared to the base one), which shows the advantages of our two-phase branching scheme in terms of effectiveness.

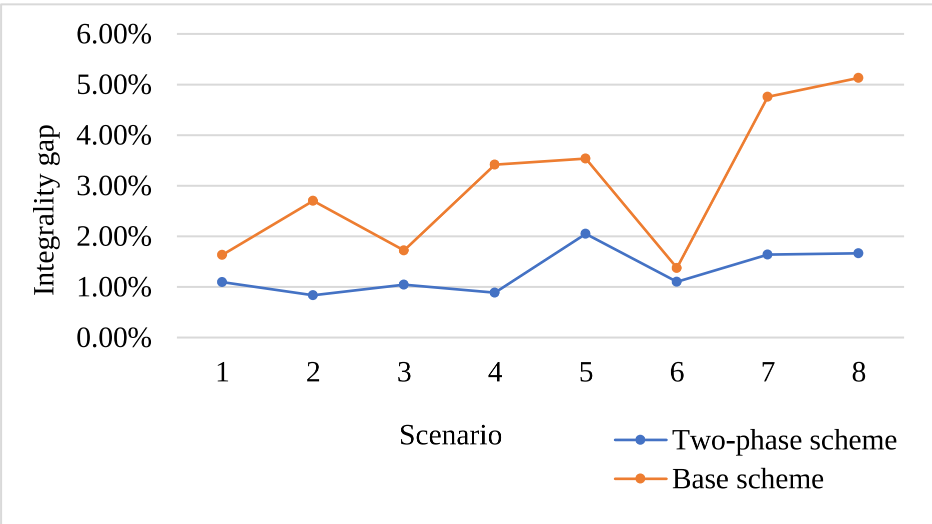

Figure 3-4. Comparison of two branching schemes.

3.4.3 Advantages of the AMR-PC

The model (i.e., AMR-PC) proposed in Section 3.2 exploits the total quantity discount policy through adapting piecewise maintenance costs. To examine the benefits brought about by this novel approach, we carried out a comparative study through further solving the traditional model with constant unit variable costs on all scenarios. The corresponding results are shown in **Table 3-3,** where columns 2 to 4 record the computational performance of the traditional model, while column 5 displays the results of our newly developed model, which is copied from **Table 3-2** for the convenience of comparison. In addition, the difference between the objective values of the two distinct models (i.e., the reduction in the total maintenance cost) can be found in the last column.

Looking at **Table 3-3**, an intuitive observation is that remarkable cost savings can be achieved by the model proposed in this chapter. Taking scenario 4 as an example, we can see a dramatic drop in the objective value, by 1530. Therefore, AMR-PC, taking advantage of the appealing cost structure, shows a clear superiority over the traditional models. This result is consistent with expectations since the average cost stemming from the total quantity discount policy is much less than the constant one and, as a result, significantly drives down the total maintenance cost. We demonstrate the particular impact of this strategy through presenting the solution details of scenario 7, given in **Figure 3-5**, where the maintenance stations are displayed in a nondecreasing order according to the corresponding basic price.

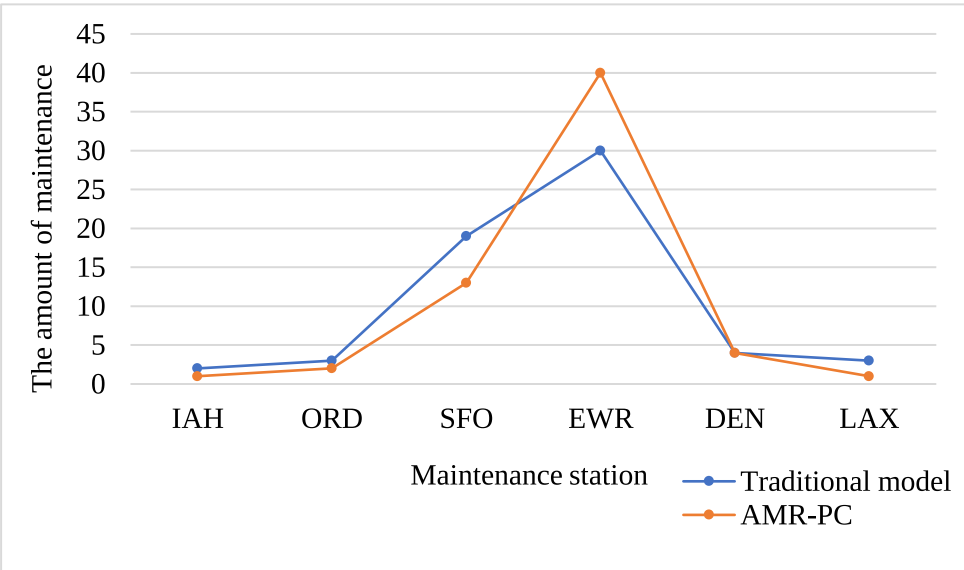

Figure 3-5. Solution details of scenario 7.

It can be clearly seen, in **Figure 3-5**, that the maintenance distribution structure is radically altered, that is, maintenance is more centrally scheduled at the station on EWR (up to 40) after employing our proposed model. The centralization structure reflects that, with the ultimate aim of pursuing a lower unit cost, it is preferrable to concentrate more maintenance activities on the same provider, rather than multiple, individual third parties, in order to accumulate orders, with the aim of pursuing discounts. This is especially true when the basic price offered by different suppliers happens to be identical. Therefore, we can observe that, considering that the basic prices of ORD, SFO and EWR are assumed the same, some of maintenance activities originally scheduled in ORD and SFO in the solution of the traditional model are assigned to EWR if the AMR-PC is applied. In addition, we would do well to pay special attention to the station with a smaller basic price, i.e., IAH. A straightforward finding is that, while selecting it is taken for granted in the case of the traditional model (to minimize total costs), fewer maintenance events are allocated at the station when solving AMR-PC. Going into the reasons, we find that there are only a slight number of maintenance opportunities provided at this station, therefore not sufficient to reach the discounting threshold. In contrast, although the basic price of EWR is a little higher, it can be lowered by exploiting the total quantity discount due to a larger number of scheduled events. In this circumstance, centralized scheduling maintenance (at EWR) may be more beneficial.

Focusing on the performance of different scaled scenarios, we can unsurprisingly observe a generally bigger comparative solution gap along with the growth in fleet size, a direct reason of which is that more cost advantages are gained due to larger scale of operations. As shown in **Table 3-1** and **Table 3-3**, we can witness an impressive improvement in cost reduction, by 1585, when the number of aircraft increases from 10 to 23. However, there is a slight exception: scenario 5 performs slightly worse than scenario 4 in terms of a smaller cost reduction. This might be because (i) the different set of maintenance stations with distinct cost distributions, and (ii) less flexibility in the flight network structure of scenario 5, as a result of which the maintenance distribution is forced to be decentralized in order to retain feasibility of the routing schedules. Despite that, it is worthwhile to note that the total maintenance cost falls significantly by USD 2135 in the scenario of the largest size (i.e., scenario 8). Considering there are 52 weeks over the course of a year, the potential cost reduction on an annual basis can be equivalent to 2135*26 = 55510 USD. More encouragingly, the cost saving is expected to increase for commercial airlines with larger sized fleets.

3.4.4 Impacts of Parameters

Here, we examine the performance of key parameters of the proposed cost structure on the model performance. To this end, we generated more parameter combinations by varying three parameters (i.e., the number of intervals, the breakpoint and the discount rate of the highest interval), and conducted computational experiments on scenario 5 for each combination. The results are illustrated in **Table 3-4**, where the first row presents the different combinations of previously mentioned three parameters, the second row records the corresponding total maintenance costs, and the last five rows show the solution details. It should be pointed out that the setting (1/0/0) represents the case of the traditional model in which the unit cost is fixed without any discounts, while the combination (3/30/8%) stands for the parameter setting in Section 3.4.1.

According to columns 3 and 4, the total maintenance cost reduces notably by 345, as expected, when the discount rate increases slightly from 8% to 10 %. In contrast, it is observed that even though the number of intervals decreases (referring to columns 3 and 5), the results record a very slight variation in total cost. This is because under both settings, the maintenance amount scheduled to EWR (35 and 34, respectively) can fall into the highest discount interval. However, taking a closer look at the last column, since the breakpoint cannot be reached in condition of (2/40/8%), the cost is the same as the solution of traditional model.

Table 3-4. Computational performance of various parameter combinations (scenario 5).

Parameter combinations	(1/0/0)*	(3/30/8%)	(3/30/10%)	(2/30/8%)	(2/40/8%)
Cost (USD)	24100	22700	22355	22740	24100
IAD	0	0	0	0	0
SFO	15	9	11	10	17
EWR	29	35	33	34	27
DEN	3	3	3	3	3
LAX	1	1	3	1	1

* The three parameters in each combination are the number of intervals, the breakpoint and the discount rate of the highest interval, respectively.

3.5 Summary

Operating in an increasingly competitive and volatile environment, major airlines are gradually contracting-out services, including maintenance operations, in order to release fixed assets and therefore enhance cost efficiency. As a result of outsourcing and strong negotiation power, airlines may enjoy quantity discounts from the maintenance providers in the purchase of services. Total quantity discounts can be commonly found in the literature on transportation in general, helping the industry to save significant money. However, in most traditional studies, this phenomenon is not adopted regarding aircraft routing problems. Instead, an oversimplifying assumption (i.e., conforming to a general pattern in which the unit variable costs always remain constant) was applied, which overestimated the total maintenance costs. Recognizing the research gaps and the actual practice of airlines, in this chapter, we consider the tactical aircraft maintenance routing while exploiting the total quantity discount in purchasing maintenance events, which aims to achieve cost efficiencies. A piecewise objective function is central to our proposed model (i.e., AMR-PC). Furthermore, to solve this complex model, we construct a column generation-based heuristic algorithm, which comprises a restricted master problem for optimizing the current solution pool, a dynamic programming process with an appropriate dominance rule for generating a best-quality route candidate and a diving heuristic with Sub-MIP for quickly finding an integer solution.

To verify the performance of our algorithm, computational experiments based on real airline schedules were conducted. The results show that our developed algorithm is capable of obtaining near-optimal solutions, in most cases, with a very small integrality gap in acceptable time limits, which demonstrates the effectiveness and efficiency of our column generation-based algorithm. Specifically, the algorithm can solve the largest scenario consisting of 636 flights in less than 9 hours. In addition, we also carry out computational experiments to examine the efficiency of the newly developed two-phase branching scheme. It is shown that the proposed branching scheme can yield smaller integrality gaps, by at most 3.46%, than a strategy solely employing the branch-on branch-follow. Then, we examine the benefits brought by the model proposed in this study through comparing with the results from the traditional models with constant unit variable costs. It is observed that remarkable cost savings can be achieved by the AMR-PC. Lastly, to examine the impact of these parameters on the computational performance, sensitivity analysis on key parameters of the cost structure, including the number of intervals, the breakpoint and the discount rate of the highest interval, is conducted. It is revealed that when the breakpoints are set smaller, it should be easier to get discounts.

In addition, several managerial implications can be derived from the computational results. On the one hand, total quantity discount enables great cost saving for commercial airlines, arising from the fact that the cost reduction is at most USD 2135 per fortnight compared with traditional models. This can encourage airlines to further outsource maintenance operations and take this policy in procurement negotiation into consideration when making aircraft scheduling decisions. Through further analyzing the detailed solution of the two approaches, interesting findings emerged. That is, the maintenance distribution structure is radically altered, i.e., maintenance is centrally scheduled at the station on EWR (up to 40) after employing our proposed model, which may motivate airlines to place more importance on their maintenance distribution structure. Another interesting finding is that airlines do not always choose the supplier whose original (or basic) price is cheaper, since other suppliers may provide discounts due to larger volume and thus the corresponding price is reduced, maybe less than the original cheapest price. Therefore, airlines are recommended to modify the maintenance distribution structure, i.e., concentrate more services on fewer providers, through optimally restructuring the aircraft routes. The same also applies to airlines that employ an in-house maintenance strategy, where economies of scale, in terms of network size and route structure, can be better exploited through focusing more on fewer stations and thus further cost enhancement can be achieved. On the other hand, larger airlines may benefit more from this attractive cost structure, owing to the potentially more accumulated operations assigned to maintenance providers.

Chapter 4. Assessing the Impact of Maintenance Distribution Structure on Aircraft Routings' Robustness

In this chapter, we focus our attention to two aspects. Firstly, it is acknowledged that, from the computational results of Chapter 3, the introduction of a total quantity discount strategy offered by the third-party maintenance vendors when making aircraft maintenance routing decisions is able to alter the maintenance distribution structure (i.e., which maintenance stations should be selected and the number of visits per station), that is, it is expected that the maintenance distribution will be concentrated in order to get greater discounts and lower maintenance costs. We then investigate how the maintenance distribution structure affects the robustness performance of aircraft maintenance routings, measured by the number of swapping possibilities, and then uncover evidence for the choice of such structure made by airlines. Differing from the traditional literature that assesses network robustness through the analytical approach (e.g., topological metrics) (Roucolle et al., 2020; Zhou et al., 2019), our investigation is based on an optimization approach, i.e., through formulating and solving models, and, considering the significance of delays in maintenance stations, we focus our attention on the maintenance robustness. More specifically, we further introduce the aircraft maintenance routing model incorporating the total quantity discount policy (called AMRTQD) proposed in Chapter 3. Through using a major U.S. airline as a case study in Section 4.3.2, we solve the model and evaluate the robustness of the resulting solution by calculating the number of swapping possibilities in the maintenance stations, and compare it with the solutions using the traditional model without consideration of the total quantity discount policy. Secondly, we make further efforts on robustness enhancement through additionally incorporating a robustness strategy, i.e., encouraging swapping possibilities, into the aircraft routing problem. We hence propose an integrated AMRP model (named as IAMRP) that considers two specific features, i.e., the total quantity discount policy to exploit the impact of the maintenance distribution structure and a robustness strategy facilitated by encouraging swapping possibilities, which is proven capable of constructing aircraft routes that enable a further improvement in robustness.

The rest of this chapter is organized as follows. Section 4.1 presents the background of the problem investigated (Section 4.1.1), and states the situation of swapping possibilities (Section 4.1.2). Then, in Section 4.2, the IAMRP model is proposed (Section 4.2.2), which can be reduced to the AMRTQD model, and a solution algorithm is accordingly constructed (Section 4.2.3). Next, Section 4.3 describes computational studies to examine the robustness of aircraft routings with distinct maintenance distribution structures (Section 4.3.2) and demonstrates the performance of the IAMRP model (Section 4.3.3). We present the summary of this chapter in Section 4.4.

4.1 Problem Statement

Basically, aircraft maintenance routing aims to construct aircraft routes with minimum costs on condition of satisfying the rules and regulations in the industry, while robust aircraft maintenance routing additionally concerns the ability to react to disruptions. Our intention of building a robust aircraft maintenance routing model is to (i) analyze how the maintenance distribution structure affects route robustness and (ii) produce aircraft routing solutions with further robustness improvement, while preserving cost savings. In achieving the first goal, we incorporate the total quantity discount policy into the model in order to create an alternative maintenance distribution structure for the reason that the maintenance orders may be accumulated in pursuit of discounts, resulting in reduced costs and a more centralized maintenance plan. Note that route robustness is assessed through calculating the number of aircraft swapping possibilities.

Furthermore, to seek additional robustness enhancement, i.e., the second goal, we further introduce a robustness strategy that encourages swapping possibilities into the model.

4.1.1 Regulations

A feasible aircraft route is a sequence of flights executed by the same aircraft, which, in general, conforms to two practical rules:

Flight connection: This refers to flight pairs that can be successively covered by the same aircraft. To be specific, given a group of pre-scheduled flight legs F which is indexed by i and j, a flight connection (i, j) can be constructed to be flown by the same aircraft as long as the time gap between the departure of flight j and arrival of flight i exceeds the minimum turnaround time allocated for the completion of ground service while this service is conducted at the same airport.

Maintenance requirement: This denotes the maximum calendar days between two successive maintenance activities, which is regulated by the FAA and the internal rules of the airline. To follow this regulation, aircraft must arrive at an airport in which a qualified maintenance station is located before the maximum allowable days become due, while staying here overnight to permit the completion of maintenance (the duration of maintenance is denoted as $minM$). We remark here that there exists an additional maintenance opportunity (or connection) for a flight connection that satisfies the above conditions.

4.1.2 Aircraft Swapping Possibilities

We adopt the concept of move-up crew for the robust crew pairing from Shebalov and Klabjan (2006) and extended it to apply to the aircraft maintenance routing problem. Precisely, we focus on the robustness in the maintenance airports. Then, the definition of the aircraft swapping possibility for evaluating and improving robustness of aircraft routings is introduced. An example is also demonstrated in **Figure 4-1** for the convenience of presentation.

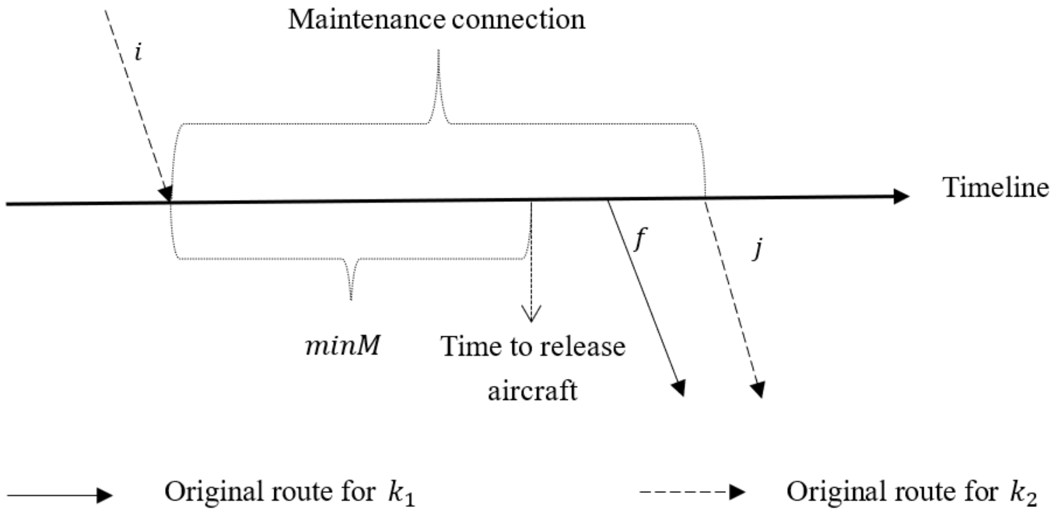

Figure 4-1. An example to demonstrate a swapping possibility for flight f departing from a maintenance airport and originally flown by k_1.

Suppose there are two distinct aircraft denoted by k_1 and k_2, respectively. Let f be the first flight leg to be covered after k_1 receives maintenance, while a maintenance connection (i, j) is prepared for k_2. We further use $arrT_j$ and $depT_j$ to indicate the arrival and departure times of flight leg j, respectively, and $depA_j$ to denote the departure airport of j. Then, a swapping possibility for flight leg f (flown by the aircraft k_1) exists if the following conditions are satisfied.

Airport consistency: $depA_j = depA_f$. That is, the flight legs j and f should depart from the same maintenance airport so that swapping is possible.

Later departure: $depT_j > depT_f$. This inequality means that if the immediately preceding leg of f is delayed or the maintenance duration is prolonged and, as a consequence, k_1 cannot be released punctually from the maintenance station and thus cannot execute f as planned (namely, aircraft k_1 is disrupted), k_1 and k_2 can be swapped so that k_1 is assigned to cover the later leg j. As a result, delay may be avoided.

Maintenance feasibility: $arrT_i + minM \leq depT_f$. This condition is defined to make sure that sufficient downtime is left for aircraft k_2 to complete its maintenance after swapping, i.e., a maintenance connection between i and f can also be constructed for k_2.

4.2 Mathematical Formulation and Solution Framework

Based on the above problem statement, a mixed integer programming model for our integrated problem (i.e., IAMRP) is derived, along with a tailored solution algorithm. While the IAMRP incorporates both the total quantity discount policy and the robustness strategy motivating swapping possibilities, we simplify IAMRP by only considering the discount policy to get AMRTQD, which is the basis for investigating our first goal.

4.2.1 Basic Notation

Before introducing the model for the proposed problem, we summarize the index, parameter and variable definitions of the model. It is noted that, in this chapter, we use the same notations used in Chapter 3, regarding constraints for classic AMRP and the total quantity discount, such as b_{ls}, c_{ls} and Cap_{ds}. In addition to these notations presented in Chapter 3, we further introduce notations used for encouraging aircraft swapping possibilities, which are summarized as follows:

Indices

F_m the set of maintenance flights, $i \in F_m$ if it is feasible to schedule a maintenance before flight $i \in F$;

Parameters

a the award for a swapping possibility

M a considerably large number

μ_{ir} = 1 if a maintenance check scheduled immediately before leg i belongs to r; 0 otherwise

ρ_{ir} = 1 if a swapping possibility for maintenance leg $i \in F_m$ belongs to r; 0 otherwise

Variables

z_i the total number of swapping possibilities for maintenance flight $i \in F_m$

4.2.2 Model Description

Based on the problem statement presented in the previous sub-chapter, we build the mathematical model in order to help understand our analytical work. In our case, the proposed problem is modelled as a set partitioning problem, which captures information carried on the aircraft route ($r \in R$). It should be noted that in the set of routes R, all the constraints (i.e., regulations) with respect to one route (e.g., the precedence relationship of flight connection, and maintenance requirements) can be implicitly satisfied.

Note here that the total quantity discount policy is exploited when constructing aircraft routing schedules, for the purpose of achieving our first goal, i.e., examining the impact of the maintenance distribution structure. Thus, our IAMRP model needs to incorporate this feature, which is also investigated in Chapter 3. Therefore, our IAMRP model includes constraints (3-2) - (3-10) of Chapter 3.

It should be mentioned here that robustness can be further enhanced by motivating more swapping possibilities in maintenance airports in which the maintenance station is located. We thus introduce decision variable z to count the number of swapping possibilities for maintenance flights before which it is feasible to schedule a maintenance activity, and reward the possibilities in the objective function. Then, the following constraints are employed:

$$z_i \leq M \sum_{r \in R} \mu_{ir} y_r, \qquad \forall i \in F_m \tag{4-1}$$

$$z_i \leq \sum_{r \in R} \rho_{ir} y_r, \qquad \forall i \in F_m \tag{4-2}$$

Constraint (4-1) guarantees that only the swapping possibilities for the maintenance flight leg before which a maintenance event is immediately scheduled can be counted. That is, if $\mu_{ir} = 0$, meaning that there is no maintenance visit before leg i within the route r, then $z_i = 0$, otherwise the big M sets the maximum number of swapping possibilities in the stations. Consequently, the model focuses on the swapping possibilities after receiving maintenance. Constraint (4-2) calculates the number of swapping possibilities for each maintenance flight $i \in F_m$. To facilitate the calculation, given the route r, ρ_{ir} is used to record whether there is a swapping possibility prepared for maintenance leg i. Furthermore, in addition to these decision variables bounded in Chapter 3, we further define the bounds on the decision variable z_i using the following constraint.

$$z_i \geq 0, \qquad \forall i \in F_m \tag{4-3}$$

The aim of our model is in encouraging robustness improvement while achieving cost reductions, resulting in a bi-criteria objective function that integrates the minimization of maintenance costs and maximization of the total number of swapping possibilities, as follows.

$$\min \sum_{s \in S} \sum_{l \in L_s} c_{ls} u_{ls} - a \sum_{i \in F_m} z_i \tag{4-4}$$

It is worthwhile to note that the constraints (3-2) - (3-10) of Chapter 3, together with the objective function with only the first component, result in an aircraft maintenance routing model that only considers the total quantity discount policy (i.e., AMRTQD). We seek to use the resulting model to analyze how the maintenance distribution structure affects route robustness, which is described in Section 4.3.2.

4.2.3 Solution Framework

Our optimization model is strongly NP- hard, hence requiring a heuristic algorithm for generating solutions of acceptable accuracy within a reasonable time. Therefore, we construct a column generation based a heuristic algorithm. Column generation is an efficient linear optimization technique to address large-scale problems and, in general, consists of two interdependent parts, i.e., the restricted master problem and the pricing subproblem (Taş, 2021). Here, the restricted master problem is a relaxed version of IAMRP with only a finite number of feasible routes. When implementing column generation, the restricted master problem is solved and the resulting dual information is passed to the pricing subproblem for the generation of promising routes with negative reduced cost, which are in turn fed to the restricted master problem. This process iterates until no such route can be found and so we obtain the optimal linear solution.

4.2.3.1. Pricing Subproblem: Generating Promising Route

The purpose of solving the pricing subproblem is in searching for columns with negative reduced cost, which implies that the current solution is non-optimal. Seeing that the pricing subproblem is constructed based on the dual information from the restricted master problem, we first define the notations on dual variables:

Dual variables	
τ	continuous, derived from the fleet size constraint
α_i	continuous, derived from the coverage constraint for flight leg i
β_{ds}	continuous, derived from the maintenance capacity constraint for the maintenance station s on day d
γ_s	continuous, derived from the maintenance counting constraint for station m
ζ_i	continuous, derived from the swapping possibility bound constraint for maintenance flight i
η_i	continuous, derived from the swapping possibility counting constraint for maintenance flight i

Based on the above definition, the reduced cost $\overline{RC}_r$ of a route r can be formulated as follows.

$$\overline{RC}_r = -\tau - \sum_{i \in L} \theta_{ir}\alpha_i - \sum_{s \in S}\sum_{d \in D} \varphi_r^{ds}(\beta_{ds} + \gamma_s) + M \sum_{i \in F_m} \mu_{ir}\zeta_i + \sum_{f \in F_m} \rho_{fr}\eta_f \quad (4\text{-}5)$$

The pricing subproblem is a shortest path problem with resource constraints (in this chapter the resource is the maximal days between two successive maintenance events), typically addressed by a labelling algorithm. The reader is referred to Irnich and Desaulniers (2005) for a comprehensive description of the solution algorithm. For convenience of illustration, we introduce a connection network $G(V, A)$, where the set of nodes V includes flight legs together with a source and sink, while the set of arcs A is comprised of flight connections and maintenance connections. Next, the reduced cost of a route, i.e., $\overline{RC}_r$ can be demonstrated in this way. It is noted that the first component, i.e., τ, is a constant, while the second component, i.e., $\sum_{i\in L}\alpha_i\theta_{ir}$, is equivalent to the sum of the dual costs corresponding to the flight legs (i.e., nodes in our constructed network) covered by the route. The remaining components are calculated by summing the costs associated with the maintenance connections employed to extend the unprocessed path of route r. Specifically, for a maintenance connection $(i,j)\in A_m$ in the station s on day d, the cost $-\beta_{ds}-\gamma_s$ should be further added to this arc. In addition, considering it is a maintenance arc, therefore, flight j is naturally a maintenance node and $M\zeta_j$ should be recorded. In addition, for each maintenance leg $f\in F_m$, we check if the connection can also serve as a swapping candidate for f, that is, the conditions described in Section 4.1.2 can be satisfied. If yes, $\rho_{fr}=1$, and then an additional cost η_f should be further added to the arc; otherwise, $\rho_{fr}=0$. Suppose that $F_{\overline{m}}$ stands for the set of maintenance legs satisfying $\rho_{fr}=1$ when processing this maintenance arc. Hence, we can conclude that the cost for this connection is thus recorded as $-\alpha_j-\beta_{ds}-\gamma_s+M\zeta_j+\sum_{f\in F_{\overline{m}}}\eta_f$.

4.2.3.2. Procedure of Column Generation-based Heuristic

For the purposing of creating integer solutions, the column generation process is nested in a diving heuristic framework, see Sadykov et al. (2019). A diving heuristic selects and addresses only one branch in the branch-and-bound tree and owing to the existence of two different types of binary variables in our proposed model, distinct branching strategies are accordingly employed. Our solution framework in this chapter is similar with that is proposed in Chapter 3, while the pricing subproblem is a different one. According to the above analysis, the overall algorithm is elaborated as follows:

Step 1: Initialize the restricted master problem by adding artificial variables to the flight coverage constraint and conduct the column generation process, in which the pricing subproblem is solved by the approach proposed in Section 4.2.3.1.

Step 2: Branch the fractional variable v. Note that, to maintain feasibility of the diving process, only variables with respect to the highest number of intervals are addressed. Check whether such variables have fractional values. If yes, round a fractional value and create a node, and then update the route pool through conducting the column generation process; otherwise, go to the next step.

Step 3: Branch the fractional route selection variable y through the branch-on follow-on strategy (Sarac et al., 2006). Select a flight pair with the fractional value closest to an integer and create a node, and then update the route pool through conducting the column generation process. If no fractional pairs can be found, go to the next step.

Step 4: Given the resulting route pool, add the integrality conditions into the restricted master problem and solve the MIP model.

4.3 Computational Experiments

This section demonstrates the computational experiments conducted to examine the potential effects of maintenance distribution structure on robustness of aircraft routings, and to reveal the performance of our proposed model in improving robustness.

4.3.1 Data and Experimental Setup

The experiments were carried out on a laptop with Intel Core i7-9750H CPU 2.60GHz and Windows 10 operation system. The model and solution framework were implemented in Java programming language, which facilitates CPLEX Studio IDE 12.10 as the LP and MIP optimizers.

We collected two weeks of flight data, from a commercial U.S. airline via the Bureau of Transportation Statistics. Then, we established a total of six instances to represent flight networks at different scales, through utilizing a portion of the flight operation data, and we show the detailed information about these instances in **Table**

4-1. After constructing all the test instances, the operational parameters were set as follows.

Table 4-1. Detailed description of instances.

Instances	No. of flights	Fleet size	Maintenance stations
1	189	7	4
2	277	10	4
3	335	12	5
4	427	15	5
5	488	17	5
6	569	20	5

The minimum turnaround time was assumed to be 40 minutes, while the maximum calendar days between two successive maintenance checks should be 4 days, taking 8 hours to complete the whole maintenance check (i.e., $minM$). Parameters concerning the maintenance cost structure, including interval bounds and discount rates, were applied to all stations (service suppliers) that were located in the hub airports (bases). To be specific, unit costs for each interval $l \in L_s$ of each station $s \in S$ were calculated as $c_{ls} = (1 - q_l) * c_{1s}$, where q_l represents the discount rate for interval l and c_{1s} is the unit cost for the first interval of each $s \in S$, which can be regarded as the constant unit cost for traditional models without a piecewise cost structure. The number of intervals was set as 3, with discount rates of 0, 5% and 8%, respectively. Furthermore, data regarding the c_{1s} were randomly generated around 500. As a consequence, each station can observe a discontinuous, non-decreasing cost. Lastly, since higher priority is given to the maintenance costs, for the purpose of preserving costs to the most extent while enhancing robustness, the award for a maintenance possibility (i.e., a) was set as 1, much less than the unit maintenance cost.

4.3.2 The Impact of Maintenance Distribution Structure on Robustness

Here, we investigate the influence of the maintenance distribution structure on the robustness of aircraft routes. To meet this aim, we solve two models which are expected to produce solutions following different maintenance distribution structures. One is the proposed model without elements associated with swapping possibilities (i.e., removing constraints used from Chapter 3, and the second component of the objective function), as demonstrated in Section 4.2.2. By doing so, the model (i.e., AMRTQD) attempts to exert direct effects on the maintenance distribution structure through taking advantage of the total quantity discount policy. The other is the traditional AMRP model (named as TAMRP) with constant unit maintenance costs. Then, based on the aircraft routing solutions, we assess the robustness through counting the number of swapping possibilities at the maintenance stations. **Table 4-2** gives detailed information on the solutions.

The maintenance distribution structure from the aircraft routing solutions, in particular, firstly requires our attention since different structures are the prerequisite for the analysis. We thus keep an eye on columns 4 to 8 of **Table 4-2**, and observe a more concentrated maintenance assignment from solutions of the model exploiting the total quantity discount policy compared with that of TAMRP. To be specific, the resulting aircraft schedule from AMRTQD assigns a higher maintenance amount to the maintenance airport “JFK”. For example, it is interesting to witness, for Instance 5, an increase in the number of maintenance events scheduled at JFK, from 21 to 35, while an opposite trend is observed for the maintenance airports BOS, LAX and FLL. The rationale behind this concentrated structure is that maintenance is accumulated in pursuit of higher discount rates in order to achieve cost savings, as can be seen from column 3 in which the total maintenance costs are reduced for all instances after considering the total quantity discount policy in the model.

Table 4-2. Solution details of AMRTQD and traditional model.

Instance	Model	Maintenance Cost (USD)	The number of planned maintenance activities in each station					No. of SP	SP improvement
			BOS	JFK	LAX	SFO	FLL		
1	AMRTQD	10252	1	16	3	1	-	9	3
	TAMRP	10600	1	10	9	1	-	6	
2	AMRTQD	14659	2	22	5	1	-	18	5
	TAMRP	15160	9	16	4	1	-	13	
3	AMRTQD	17545.2	5	31	1	0	0	29	9
	TAMRP	18600	10	18	4	1	4	20	
4	AMRTQD	21483.6	4	33	6	2	0	51	27
	TAMRP	22670	10	21	10	2	2	24	
5	AMRTQD	24422	6	35	7	3	0	61	34
	TAMRP	25650	10	21	15	2	3	27	
6	AMRTQD	28717.2	8	41	8	2	1	75	28
	TAMRP	30170	12	27	14	2	5	47	

Having acquired two different maintenance distribution structures, we then analyze the corresponding robustness performances. It is intuitive to see, from the last two columns of **Table 4-2**, that AMRTQD creates dramatically more swapping possibilities in maintenance airports than TAMRP. To take Instance 5 as an example again, we note that the number of swapping possibilities from the solutions generated by AMRTQD rises significantly, by 34, in comparison to TAMRP. This proves that the maintenance distribution structure has a major impact on the robustness of aircraft routings. More specifically, a more concentrated structure is shown to be advantageous, in terms of robustness performance, over the unconcentrated one. This outperformance is reasonable because a concentrated maintenance assignment means an increase of aircraft staying at the same maintenance airport on an operating day, which allows an aircraft to have more candidate aircraft to choose for swapping, and as such, making it easier to be recovered from operational disruptions.

4.3.3 The Performance of the Proposed Model

To examine the robustness performance of our proposed model (i.e., IAMRP) which, compared with AMRTQD, further incorporates a robustness enhancement strategy that inspires swapping possibilities, we conducted comparison experiments. It is noticed that Constraint (4-1) of IAMRP includes the big M constant, which imposes an upper bound on the number of swapping possibilities for each maintenance flight. In most mixed integer programming problems, the constraints including big M constants should be improved to get a tighter equivalent formulation, making it faster to solve the problem. For the IAMRP model, in view that the maintenance capacity limits the maximum amount of maintenance assigned to each station per day and, getting rid of the maintenance operation immediately scheduled before the maintenance flight, we thus have $M = \max_{d,s} Cap_{ds} - 1$. The performance comparisons of IAMRP and AMRTQD are shown in **Table 4-3**.

It can be clearly noted, from the last two columns of **Table 4-3**, that our constructed model (i.e., IAMRP) incorporating both strategies (i.e., a more centralized maintenance distribution benefiting from the total quantity discount policy and encouraging swapping possibilities) consistently outperforms AMRTQD which purely exploits the advantage of the maintenance distribution structure, with the number of swapping possibilities of IAMRP increasing for all instances. Of special attention is Instance 3, where IAMRP creates strikingly more swapping possibilities (i.e., 15) in contrast to AMRTQD. This also demonstrates in the robustness strategy facilitated by encouraging swapping possibilities, exhibiting good performance. On the other hand, it should be noted that the total maintenance cost derived by IAMRP turns out to be no greater than that by AMRTQD, which proves that our proposed model is able to retain the advantages in cost reduction resulting from the exploitation of the total quantity discount policy. In some cases, for example, instances 2 and 6, the comparative advantage in maintenance costs is more remarkable because IAMRP has more features, which may motivate the generation of promising routes.

Table 4-3. Comparison of solutions for IAMRP and AMRTQD.

Instance	Model	Maintenance cost (USD)	No. of SP	SP improvement
1	IAMRP	10252	14	5
	AMRTQD	10252	9	
2	IAMRP	14643.5	26	8
	AMRTQD	14659	18	
3	IAMRP	17545.2	44	15
	AMRTQD	17545.2	29	
4	IAMRP	21483.6	59	8
	AMRTQD	21483.6	51	
5	IAMRP	24422	71	10
	AMRTQD	24422	61	
6	IAMRP	28686.4	87	12
	AMRTQD	28717.2	75	

As demonstrated in the previous section, AMRTQD shows a better performance in terms of robustness enhancement and cost reduction compared with TAMRP. Therefore, the degree of solution improvement for the proposed IAMRP can be even more remarkable when we set TAMRP as a benchmark. Based on the above analysis, it is concluded that our constructed model is capable of generating aircraft routing schedules with enhanced robustness through exploiting the more centralized maintenance distribution resulting from taking advantage of the total quantity discount policy and the robustness strategy encouraging swapping possibilities, while achieving the cost savings.

4.4 Summary

In a dynamic environment, airline operations are subject to diverse uncertainties, and thus it is of great importance to improve the robustness of aircraft maintenance routes so as to be less vulnerable to disruptions. One of the most effective and significant strategies is providing flexibility when operating their flights and aircraft. Among all flexibility strategies, improving swapping possibilities is a widely used and efficient approach to achieve robustness of aircraft routings. On the other hand, the trend in growth of contracting-out aircraft maintenance has relieved airlines from fixed costs corresponding to maintenance, which also allows airlines to restructure their maintenance distributions. However, the impact of the maintenance distribution structure on aircraft routing robustness is under-explored in the literature. Acknowledging these research gaps, we investigate a principal question about how distinct maintenance distribution structures affect the robustness of aircraft routes, through solving the models with and without exploiting the total quantity discount. In addition to the attempts to enhance robustness by exploiting the maintenance distribution structure resulting from taking advantage of the total quantity discount policy, this study further considers a robustness strategy, namely, encouraging swapping possibilities, into aircraft maintenance routing, with the aim of pursuing further robustness improvement. To this end, a novel robust aircraft maintenance routing model incorporating both the discount policy and the swapping possibility count is proposed, along with a solution approach.

To examine the effects of maintenance distribution structure on route robustness, computational experiments based on data established from real-world flight operations were carried out to identify the difference, in terms of robustness performance, between divergent maintenance distribution structures. The results demonstrate that the strong impacts of the maintenance distribution structure on robustness can be observed, in view of the outperformance of a more concentrated structure, e.g., the number of swapping possibilities were increased by at most 34 in comparison to unconcentrated maintenance assignment. Furthermore, experiments are also conducted to assess the performance of solutions derived from the proposed model (i.e., IAMRP). It is revealed, from the results, that the newly constructed model with the further consideration of swapping possibilities is able to generate cost- efficient aircraft routes with improved robustness, e.g., a growth in the number of swapping possibilities, from 75 to 87 in the largest-scale instance.

Based on the numerical experiments, this research is favorable for policy makers from several aspects. Firstly, we provide references for the choice of the maintenance distribution structure. The high frequency of the maintenance checks creates significant costs for airlines and thus great efforts are made, through intelligently scheduling aircraft routes, to cut down their costs. At the same time, suffering from delays, airlines also impose great emphasis on the robustness of aircraft routes, especially when aircraft stay at a maintenance airport. A more concentrated maintenance distribution, achieved by considering the total quantity discount policy in aircraft maintenance routing, can deliver aircraft routes with improved robustness and reduced cost, and can achieve what airlines hope for. As a consequence, it is of significant importance for policy makers to understand the effects of the maintenance distribution structure, which guides the policy development. Secondly, it can provide a theoretical framework for airlines to construct their routing schedules with an increased ability to cope with delays. Our proposed robust aircraft maintenance routing model, by both exploiting the impact of maintenance distribution structure benefiting from the total quantity discount policy and awarding the creation of swapping possibilities, is capable of generating aircraft routes with further enhanced robustness that better withstand common disruptions in their day-to-day operations, which can significantly benefit airlines.

Chapter 5. Robust Aircraft Maintenance Routing Problem under Uncertainties Considering Heterogenous Maintenance Tasks

In this chapter, we focus attention on aircraft maintenance routing much closer to the day of operation in order to create routes for individual aircraft, with consideration of the original airport and heterogenous maintenance tasks. Acknowledging the significance of the stochasticity of maintenance execution and the research gaps in the literature, we aim to develop a novel robust aircraft maintenance routing model that minimizes the total propagated delays. A customized column generation-based algorithm is then developed to solve the model. Unlike most of the traditional robust aircraft maintenance routing models in the literature that considered the sources of disruptions in an aggregated manner, our model incorporates the uncertainties of heterogenous maintenance tasks, while considering other uncertainties, and is capable of constructing robust aircraft routes that are less vulnerable to disruptions.

The outline of this chapter is as follows. Firstly, Section 5.1 describes the problem to be studied, while highlighting the modelling of maintenance uncertainties. Next, Section 5.2 presents the proposed robust aircraft maintenance routing model (Section 5.2.1), along with the pricing subproblems (Section 5.2.2), while constructing solution approach for the pricing subproblems (Section 5.2.3). Section 5.3 discusses, based on the flight data derived from real-world schedules, a computational study used to analyze the impact of distinct degrees of maintenance uncertainties on the robustness of aircraft routes, and sensitivity analysis to analyze the value of the maintenance capacity. Lastly, a summary of this chapter is presented in Section 5.4.

5.1 Problem Description

Given a set of scheduled flight legs, the goal of robust aircraft maintenance routing is to generate feasible aircraft routes that are robust to disruptions, and subject to the rules and regulations in the industry. In particular, we first describe the maintenance requirements in the following.

- **Maintenance requirements**

The maintenance restrictions are normally set by the FAA and airlines' internal regulations (Haouari et al., 2013). Typically, a series of maintenance requests are batched into packages that, traditionally, are classified as A, B, C, and D checks, which must be performed regularly (Zhou et al., 2020). The A check is conducted every 400-600 flying hours and takes, for a narrow body aircraft, at most 24 hours, whereas the B check, the next higher-level maintenance, should be conducted every 6-8 months. The C and D checks, which take an aircraft out of flight service for several weeks, are more extensive and complex, and thus carried out much less frequently. Practically, to integrate maintenance into aircraft routings for ensuring airworthiness of flight operations, airline companies commonly carry out minor maintenance checks right before a scheduled flight. Some of these checks are performed every 65 flight hours, the so- called daily check (Zhou et al., 2020). The maintenance durations are usually assumed as fixed parameters, and, in addition, airlines usually need to monitor aircraft instruments and equipment. The inoperative equipment, which still meets airworthiness requirements, is recorded in a Minimum Equipment List (MEL). MEL is tracked by a so-called transit check, which requires aircraft to fly at most 40 hours between two consecutive maintenance checks (Eltoukhy et al., 2017; Talluri, 1998). Furthermore, some airlines also apply a stringent rule, i.e., the 4- day restriction (Talluri, 1998). Only airports with maintenance capability, i.e., maintenance stations, are allowed to carry out maintenance activities.

Recently, in light of the tighter flight schedules, airlines tended to apply a maintenance approach that involves individual tasks, instead of maintenance checks with regular intervals. These individual heterogenous maintenance tasks are generated through breaking the traditional letter checks into smaller packages (Ruther et al., 2017; Zhou et al., 2020). As a result, these heterogenous tasks have shorter durations, compared to traditional checks, and can achieve better flexibility in operation since they can be more easily scheduled and conducted during the time on the ground, and in turn have higher frequencies, with respect to the corresponding remaining flight hours. Accordingly, to reflect the real practice, we model each aircraft at the individual level, considering its initial airport and heterogenous maintenance tasks, which differ in maximum flying time and duration.

Maintenance is typically performed during the night, as seen in most of the traditional studies, and the exact capacity of stations is constrained through limiting the number of aircraft receiving maintenance activities simultaneously each night (or day) in each station (Faust et al., 2017; Khaled et al., 2018; Sriram & Haghani, 2003). We assume that maintenance activities are not necessarily conducted overnight, but any time when the aircraft is on the ground. Furthermore, considering that the maintenance durations of these tasks are highly uncertain (Dinis et al., 2019), and that delays of flights immediately before the maintenance execution will significantly influence the maintenance starting time, we model the capacity per station per hour to accommodate the actual maintenance requirements. Our goal aims at generating and selecting aircraft routes that incur the minimum total expected propagated delays.

The route generation process is carried out based on a connection network $G(V, A)$, similar to the one proposed in Chapter 3. The set of nodes V includes flight legs, together with a source and a sink (represented by s and o, respectively), while the set of arcs A is comprised of connection arcs and maintenance arcs (denoted by A_f and

A_m, respectively). Specifically, given two flight legs i and j, they can be connected (thus resulting in a connection arc) if the time gap between the scheduled ground time (signified by d_{ij}) exceeds the minimum turnaround time allocated for the completion of the ground service, while this service is conducted at the same airport. Furthermore, if this airport happens to be a maintenance station, then there is an additional maintenance arc between i and j.

As aforementioned, our modelling framework provides robust routes that enables disruption absorption. To be specific, the disruptions of any flight can be categorized into two classes, i.e., maintenance and incidental uncertainties. Incidental uncertainties refer to other sources of disruptions, except for maintenance uncertainties. Whereas maintenance arcs may encounter both uncertainties, flight connections only have incidental delays. Thus, the two types of arcs should be treated separately when generating routings. To facilitate the illustration, we define the delays of flight i caused by incidental uncertainty as ID_i, while PD_i refers to the propagated delays of flight i. Based on these definitions, the PD_j occurring in a connection arc, in which flight i is the predecessor flight of j in the route, can be calculated, adapted from Eq. (3) of Lan et al. (2006), by:

$$PD_j = \max\{PD_i + ID_i + MTT - d_{ij}, 0\} \tag{5-1}$$

Note that the incidental delay is stochastic and assumed to follow a specific distribution obtained from historical data, similar to Lan et al. (2006) and Marla et al. (2018). As the number of flights along the route increases, the calculation of the propagated delay can become remarkably difficult because of the high-dimensional complexity resulting from recursion. To simplify the computation, we apply a method from the reference (Dunbar et al., 2012), in which the uncertainty of the previous propagated delay is ignored, and its expectation is used to represent the delay, i.e.,

$$PD_j = \max\{E[PD_i] + ID_i + MTT - d_{ij}, 0\} \tag{5-2}$$

Then, we introduce the maintenance uncertainties in the following.

- **Maintenance duration uncertainty**

To satisfy maintenance requirements, aircraft should visit a maintenance station before the due time of the corresponding task. If the ground time is not sufficient for punctual maintenance execution, the subsequent flight leg will be delayed. A great challenge is due to the maintenance task duration being highly uncertain due to unexpected failures and shortage in repair resources, significantly contributing to flight delays. For the purpose of incorporating the maintenance uncertainty in the aircraft routing decision framework, we introduce D_t to model the duration of task t, which is assumed to follow a truncated lognormal distribution, independent of ID_i (Dinis et al., 2019; Kline, 1984). The parameters of the distribution can be obtained through applying big data and forecasting tools (Chung, 2021; Chung et al., 2020), from historical data that can be monitored by blockchain technology (Choi & Siqin, 2022). Then, supposing that a maintenance task t is conducted through a maintenance arc (i, j), the propagated delay of the flight j in Eq. (5-2) should be adapted by:

$$PD_j = \max\{E[PD_i] + ID_i + D_t - d_{ij}, 0\} \tag{5-3}$$

As stated earlier, the incidental delay and maintenance duration are modelled using specific distributions. To simplify the notation, we further use $f(x)$ and $g(z)$ to represent the probability density functions of stochastic variables ID_i and D_t, respectively, and b to denote all the deterministic items, i.e., $b \triangleq E[PD_i] - d_{ij}$. Subsequently, the expectation of PD_j, i.e., $E[PD_j]$, can be calculated by:

$$
\begin{aligned}
E[PD_j] &= E[\max\{E[PD_i] + ID_i + D_t - d_{ij}, 0\}] \\
&= \int_0^{+\infty}\int_0^{+\infty} \max\{x + z + b, 0\} f(x)g(z)dxdz \\
&= \int_0^{+\infty}\left[\int_{\max(-z-b,0)}^{+\infty} (x + z + b)f(x)dx\right]g(z)dz \qquad (5\text{-}4)
\end{aligned}
$$

5.2 Mathematical Formulation and Column Generation

A column generation-based framework is proposed for formulating and solving the proposed robust aircraft maintenance routing problem. Specifically, our proposed framework involves two interdependent modules, i.e., a master problem for the selection of aircraft routes with the minimum expected propagated delays, and its corresponding pricing subproblems for the generation of promising routes with the most negative reduced cost. Column generation is an efficient linear programming optimization technique that iteratively solves the restricted master problem and the pricing subproblems. To be specific, the restricted master problem is the linear programming relaxation of the master problem, and is initialized by introducing one artificial variable to each flight coverage constraint. Then, after obtaining the optimal solutions of restricted master problem, its dual information is used to construct pricing subproblems, which are solved to generate, for each aircraft, the most promising routes. These routes are then fed into the solution pool of the restricted master problem. This iteration terminates if no routes with negative cost can be generated. Finally, the restricted master problem with routes constructed by the column generation process is thus transferred into an integer programming problem, and solved to obtain the integer solution.

5.2.1 Master Problem

The master problem defines the aircraft count, coverage and maintenance capacity constraints, while maintenance requirements are handled in the pricing subproblems. First, we define the important indices, parameters and variables.

Indices

F	the set of operational flight legs, indexed by i and j.
H	the set of hours, indexed by h.
K	the set of aircraft, indexed by k.
R_k	the set of potential routes of aircraft $k \in K$, indexed by r.
S	the set of maintenance stations, indexed by s.
T_k	the set of heterogenous maintenance tasks for aircraft k, indexed by t.

Parameters

$E[PD_j]$	the expected propagated delays of flight leg $j \in F$.
M_{hs}	the maximum number of aircraft can be maintained in station $s \in S$ at hour $h \in H$.
θ_{ir}	= 1 if flight leg $i \in F$ belongs to $r \in R_k$; 0 otherwise.
φ_r^{hs}	= 1 if a maintenance task for route $r \in R_k$ is scheduled in station $s \in S$ at hour $h \in H$; 0 otherwise.

Variables

y_{kr}	= 1 if potential route r of aircraft $k \in K$ is selected; 0 otherwise.

Then, the master problem can be formulated as follows:

$$\min \sum_{k\in K}\sum_{r\in R_k}\left(\sum_{j\in r} E[PD_j]\right) y_{kr} \tag{5-5}$$

$s.t.$

$$\sum_{r \in R_k} y_{kr} \le 1, \qquad \forall k \in K \tag{5-6}$$

$$\sum_{k \in K} \sum_{r \in R_k} \theta_{ir}\, y_{kr} = 1, \qquad \forall i \in F \tag{5-7}$$

$$\sum_{k \in K} \sum_{r \in R_k} \varphi_r^{hs} y_{kr} \le M_{hs}\,, \qquad \forall\, h \in H \text{ and } s \in S \tag{5-8}$$

The objective function (5-5) minimizes the total expected propagated delays for the selected routes. Constraint (5-6) ensures aircraft count, i.e., an aircraft can fly at most one possible route, while Constraint (5-7) guarantees each flight can be covered by only one aircraft. Lastly, the number of aircraft can be maintained in each station per hour is limited by Constraint (5-8).

5.2.2 Pricing Subproblem

The pricing subproblem is a shortest path problem corresponding to a route of an aircraft (Barnhart et al., 1998a; Liang et al., 2015; Yan & Kung, 2018), with the objective of minimizing the reduced cost under maintenance constraints, i.e., the maximum flight time allowed for each heterogenous task of this aircraft. The shortest path problem is addressed on the proposed connection network $G(V, A)$. Then, given the current solutions of the restricted master problem, we can define the following dual variables.

Dual variables	
τ	dual variable of the aircraft count constraint (5-6)
α_i	dual variable associated with the coverage constraint (5-7) for flight leg i
β_{hs}	dual variable with respect to the maintenance capacity constraint (5-8) for the maintenance station s at hour h

Then, the reduced cost for the route r of aircraft k, denoted by RC_{kr}, is

$$RC_{kr} = \sum_{i \in F} E[PD_i]\theta_{ir} - \sum_{i \in F} \alpha_i \, \theta_{ir} - \sum_{s \in S} \sum_{h \in H} \beta_{hs} \, \varphi_r^{hs} - \tau_k \tag{5-9}$$

Specifically, the first and second terms are the costs incurred by flights comprising the route, where $\sum_{i \in F} E[PD_i]\theta_{ir}$ is equal to the summation of the expected propagated delays and $\sum_{i \in F} \alpha_i \theta_{ir}$ is equivalent to the sum of the dual costs corresponding to the flight legs of the route. The third component, i.e., $\sum_{s \in S} \sum_{h \in H} \beta_{hs} \, \varphi_r^{hs}$, is associated with the maintenance. To be specific, when processing an arc $(i, j) \in A_m$ in which the task $t \in T_k$ is conducted, we have the starting time of this execution $b_t = E[PD_i] + E[ID_i] + Arr_i$, where $E[ID_i]$ is the expected incidental delay of flight i and Arr_i is the scheduled arrival time of flight i, and the ending time $e_t = b_t + E[D_t]$, where $E[D_t]$ is the expected duration of task t. If the time window $[b_t, e_t]$ covers $h \in H$ and the task t is carried out in the station s, then $\varphi_r^{hs} = 1$. Lastly, τ_k is a constant in regard to aircraft k.

5.2.3 Solving the Pricing Subproblem

To solve the pricing subproblem, we propose a multi-label algorithm augmented by dominance rules. The reduced cost is computed using Eq. (5-9), where the $E[PD_i]$ is calculated based on Eqs. (5-2) and (5-4). The nonlinear nature of $E[PD_i]$ hence forces the labels to track both RC and $E[PD_i]$. In the pricing subproblem, the only constraint is the maintenance requirement, which is limited by the feasibility resource, i.e., the maximum flight time of the current processing task (represented by $maxFT$). Therefore, the accumulated flying time (represented by $accFT$) is traced to be no longer than $maxFT$. Note that, in our problem, each task of an individual aircraft has its own $maxFT$; therefore the task being handled should also be dynamically recorded. As a result, our label must track (1) the cost of the (partial) route, (2) the expected propagated delay, (3) the current processing task, and (4) the accumulated flight time. We now describe the proposed multi-label algorithm in detail. For ease of illustration, we specify that the route in the following is prepared for a given aircraft k, with simplified notation, i.e., the index k is omitted.

Let π be a $s-o$ route in G (an ordered collection of nodes $\{s, i_1, \cdots, i_q, o\}$ in V with $(s, i_1), (i_q, o) \in A$ and $(i_l, i_{l+1}) \in A$ for all $l = 1, \cdots, q-1$). For $i \in \pi$, let $\pi(i)$ stand for the ordered collection of nodes in the route π, truncated so that the final node in the list is i. We use $E[PD_{\pi(i)}]$ to denote the expected propagated delay at node i, computed along the route $\pi(i)$, $t_{\pi(i)}$ to represent the current task and $accFT_{\pi(i)}$ to stand for the accumulated flight time at node i along the route $\pi(i)$. Then, as the maintenance duration is enumerated in minutes and the maintenance capacity is controlled hourly, we can define the reduced cost of the truncated route:

$$RC_{\pi(i)} = \sum_{j \in \pi(i)} \left(E\left[PD_j\right] - \alpha_j\right) - \sum_{s \in S} \sum_{h \in H_{\pi(i)}} \beta_{hs} - \tau \tag{5-10}$$

where $H_{\pi(i)}$ refers to the set of hours covered by the maintenance execution time along the route.

It is noted that a node may connect to two types of arcs (i.e., connection and maintenance arcs). When extending two labels $\pi(i)$ and $\eta(i)$ through a maintenance arc $(i,j) \in A_m$ where the arrival airport of i, i.e., $arrA_i$, is a. That is to say, the maintenance station on the arc (i,j) is denoted by a. The costs of the labels of node j are:

$$RC_{\{\pi(i),j\}} = RC_{\pi(i)} + E\left[PD_{\{\pi(i),j\}}\right] - \alpha_j - \sum_{h \in H_{\{\pi(i),j\}}} \beta_{ha} \tag{5-11}$$

and

$$RC_{\{\eta(i),j\}} = RC_{\eta(i)} + E\left[PD_{\{\eta(i),j\}}\right] - \alpha_j - \sum_{h \in H_{\{\eta(i),j\}}} \beta_{ha} \tag{5-12}$$

where $H_{\{\pi(i),j\}}$ and $H_{\{\eta(i),j\}}$ refer to the set of hours covered by the maintenance execution time on the arc (i,j) for two extended routes, which depends, as mentioned earlier, on the expected propagated delay of i and the expected duration of the planned maintenance. Therefore, if the values of $E[PD_{\pi(i)}]$ and $E[PD_{\eta(i)}]$ are different and two labels address distinct maintenance tasks, $H_{\{\pi(i),j\}}$ and $H_{\{\eta(i),j\}}$ are different. Consequently, we are unable to compare the values of $\sum_{h\in H_{\{\pi(i),j\}}}\beta_{ha}$ and $\sum_{h\in H_{\{\eta(i),j\}}}\beta_{ha}$. This leads to the following dominance rule for routes destined for the same node.

Definition 1 (Dominance rule). Let $\pi(i)$, $\eta(i)$ be two different (partial) routes destined for the same node i. Then, we say that $\pi(i)$ dominates $\eta(i)$ if $RC_{\pi(i)} \leq RC_{\eta(i)}$, $E[PD_{\pi(i)}] = E[PD_{\eta(i)}]$, $t_{\pi(i)} = t_{\eta(i)}$ and $accFT_{\pi(i)} \leq accFT_{\eta(i)}$.

The dominance condition originates from the following Lemma 1.

Lemma 1. *Let $j \in V$ such that $(i,j) \in A$. If $\pi(i)$ dominates $\eta(i)$, then $\{\pi(i),j\}$ dominates $\{\eta(i),j\}$.*

Proof. for $(i,j) \in A_f$, from Eq. (5-2) we have

$$E\left[PD_{\{\pi(i),j\}}\right] = \max\left\{E[PD_{\pi(i)}] + E[ID_i] + MTT - d_{ij}, 0\right\} \tag{5-13}$$

and

$$E\left[PD_{\{\eta(i),j\}}\right] = \max\left\{E[PD_{\eta(i)}] + E[ID_i] + MTT - d_{ij}, 0\right\} \tag{5-14}$$

Because $E[PD_{\pi(i)}] = E[PD_{\eta(i)}]$, we have $E\left[PD_{\{\pi(i),j\}}\right] = E[PD_{\{\eta(i),j\}}]$. For the processing task and accumulated flight time

$$t_{\{\pi(i),j\}} = t_{\pi(i)} \text{ and } accFT_{\{\pi(i),j\}} = accFT_{\pi(i)} + Dur_j \tag{5-15}$$

and

$$t_{\{\eta(i),j\}} = t_{\eta(i)} \text{ and } accFT_{\{\eta(i),j\}} = accFT_{\eta(i)} + Dur_j \tag{5-16}$$

Because $t_{\pi(i)} = t_{\eta(i)}$ and $accFT_{\pi(i)} \leq accFT_{\eta(i)}$, we have $t_{\{\pi(i),j\}} = t_{\{\eta(i),j\}}$

and $accFT_{\{\pi(i),j\}} \leq accFT_{\{\eta(i),j\}}$. We also have

$$RC_{\{\pi(i),j\}} = RC_{\pi(i)} + E\left[PD_{\{\pi(i),j\}}\right] - \alpha_j \tag{5-17}$$

and

$$RC_{\{\eta(i),j\}} = RC_{\pi(i)} + E\left[PD_{\{\eta(i),j\}}\right] - \alpha_j \tag{5-18}$$

Because $E\left[PD_{\{\pi(i),j\}}\right] = E[PD_{\{\eta(i),j\}}]$ and $RC_{\pi(i)} \leq RC_{\eta(i)}$, we have $RC_{\{\pi(i),j\}} \leq RC_{\{\eta(i),j\}}$.

On the other hand, for $(i,j) \in A_m$, from Eq. (5-5) we have

$$E\left[PD_{\{\pi(i),j\}}\right] = \max\left\{E[PD_{\pi(i)}] + E[ID_i + D_{t_{\pi(i)}}] - d_{ij}, 0\right\} \tag{5-19}$$

and

$$E\left[PD_{\{\eta(i),j\}}\right] = \max\left\{E[PD_{\eta(i)}] + E[ID_i + D_{t_{\eta(i)}}] - d_{ij}, 0\right\} \tag{5-20}$$

Because $t_{\pi(i)} = t_{\eta(i)}$, then $E[D_{t_{\pi(i)}}] = E[D_{t_{\eta(i)}}]$, and we have $E[PD_{\pi(i)}] = E[PD_{\eta(i)}]$, thus $E\left[PD_{\{\pi(i),j\}}\right] = E[PD_{\{\eta(i),j\}}]$. For the processing task and accumulated flight time

$$t_{\{\pi(i),j\}} = t_{\pi(i)} + 1 \text{ and } accFT_{\{\pi(i),j\}} = accFT_{t_{\{\pi(i),j\}}} + Dur_j \tag{5-21}$$

and

$$t_{\{\eta(i),j\}} = t_{\eta(i)} + 1 \text{ and } accFT_{\{\eta(i),j\}} = accFT_{t_{\{\eta(i),j\}}} + Dur_j \tag{5-22}$$

Because $t_{\pi(i)} = t_{\eta(i)}$ and $accFT_{\pi(i)} \leq accFT_{\eta(i)}$, we have $t_{\{\pi(i),j\}} = t_{\{\eta(i),j\}}$ and $accFT_{\{\pi(i),j\}} \leq accFT_{\{\eta(i),j\}}$. We also have

$$RC_{\{\pi(i),j\}} = RC_{\pi(i)} + E\left[PD_{\{\pi(i),j\}}\right] - \alpha_j - \sum_{s \in S} \sum_{h \in H_{\{\pi(i),j\}}} \beta_{hs} \tag{5-23}$$

and

$$RC_{\{\eta(i),j\}} = RC_{\eta(i)} + E\left[PD_{\{\eta(i),j\}}\right] - \alpha_j - \sum_{s \in S} \sum_{h \in H_{\{\eta(i),j\}}} \beta_{hs} \tag{5-24}$$

Because $E[D_{t_{\pi(i)}}] = E[D_{t_{\eta(i)}}]$ and $t_{\pi(i)} = t_{\eta(i)}$, $b_{t_{\pi(j)}} = b_{t_{\eta(j)}}$ and $e_{t_{\pi(j)}} =$

$e_{t_{\eta(j)}}$. In addition, we have $RC_{\pi(i)} \leq RC_{\eta(i)}$, so $RC_{\{\pi(i),j\}} \leq RC_{\{\eta(i),j\}}$. Thus, the proof is completed.

Particularly, by induction, suppose ω is a route that starts at node j and terminates at sink o, and $(i,j) \in A$, Lemma 1 indicates that $RC_{\{\pi(i),\omega\}} \leq RC_{\{\eta(i),\omega\}}$. Equipped with the dominance rules, we can use the proposed multi-label algorithm to solve (5-9). To be specific, at each node, we only create labels for those routes that are not dominated by any other routes at that node. In addition, for the purpose of saving computational effort, a heuristic pricing algorithm is also adopted before using our proposed dominance rule. That is, considering that the difficulty of comparing two labels lies in the existence of the dual variables $(\beta_{hs})^{h\in H,s\in S}$, these elements are not first allocated to the maintenance arcs and, as a consequence, the dominance rule can be relaxed, which becomes $RC_{\pi(i)} \leq RC_{\eta(i)}$, $E[PD_{\pi(i)}] \leq E[PD_{\eta(i)}]$, $t_{\pi(i)} = t_{\eta(i)}$ and $accFT_{\pi(i)} \leq accFT_{\eta(i)}$. We solve the shortest path problem with the relaxed dominance rule, and then add the corresponding β_{hs} into the reduced cost. If the cost is negative, we add the route to the restricted master problem, and this process is repeated until no negative route can be found. Then, we allocate dual variables $(\beta_{hs})^{h\in H,s\in S}$ to the arcs again, and apply our proposed dominance rule to obtain the optimal solutions for the pricing subproblem.

5.3 Computational Study

Here, we report the results of the computational experiments. The experiments were conducted on a laptop with Intel Core i7-9750H and a Windows 10 operating system. The modelling and solution algorithm framework were implemented in Java programming language, which facilitates IBM ILOG-CPLEX Studio IDE 12.10 as the linear programming and integer programming optimizers. First, we describe the test scenarios constructed for the experiments, along with the parameter settings. Then, we demonstrate how the degree of maintenance uncertainty impacts on the robustness of aircraft routings through comparative experiments. Lastly, the benefits of the maintenance capacity are examined.

5.3.1 Data Description and Experimental Setup

In the experiments, we extracted flight data based on real operational schedules belonging to a major U.S. hub-and-spoke airline available on the BTS. Our planning horizon was set as one week because it is widely used in the literature (Liang et al., 2011; Liang et al., 2015). Furthermore, this horizon is close to the day of operation and, thus, it is possible to construct better schedules by capturing more accurate information. Therefore, we chose scheduled flights on a random week in January 2020 as an example, for conducting the computational experiments.

A total of five scenarios were established from the Boeing 737-800 fleet, for representing the flight networks of different sizes. Detailed information about the scenarios is summarized in **Table 5-1**. Specifically, for each scenario, we give an ID, the number of flight legs, aircraft, airports and maintenance stations. Note that since the maintenance stations are not known from the flight network, we hence assign the airports, with the number of departure/arrival flight legs exceeding a threshold, to be the maintenance stations of each scenario. After constructing all the scenarios, the operational parameters were set as follows. The minimum turnaround time was set as 40 minutes, and the maintenance capacity per hour two thirds of the total number of aircraft in each scenario. The aircraft have heterogenous maintenance tasks, whose durations fit truncated lognormal distributions with values between the lower and upper bounds.

Table 5-1. Summary of scenarios.

Scenario	No. of legs	No. of aircraft	No. of airports	No. of stations
1	79	3	17	1
2	128	5	37	2
3	181	7	53	2
4	221	9	52	2
5	274	11	55	2

5.3.2 The Impact of Maintenance Uncertainties on Robustness

As aforementioned, the maintenance duration follows a truncated distribution defined on a specific interval. The width of the interval indicates the degree of maintenance uncertainty and, specifically, the wider the interval, the greater the uncertainty. It is clear that the idea of airline robust plans stems from attempts to construct schedules that enable the tolerance of a certain degree of uncertainty in operations. Then a question logically arises about how the degree of maintenance duration uncertainty affects the robustness performance of the routing solutions. To find the answer, we conducted comparative experiments through solving the proposed model, given two duration intervals with different widths (i.e., [150, 300] and [120, 480]) that indicate distinct degrees of maintenance uncertainties, and comparing the objective values of the corresponding solutions, i.e., the total expected propagated delays. The computational results are displayed in **Table 5-2**, where the second and third columns report the expected propagated delay in minutes of the solutions when the random maintenance durations are truncated to the intervals, i.e., [150, 300] and [120, 480], respectively. The fourth column indicates the growth in the expected propagated delays in minutes when the width of interval for task durations increases, while the last column records the increased percentage of the expected propagated delays.

Table 5-2. Comparison results under distinct degrees of uncertainty.

Scenario	[150, 300]*	[120,480]	EPD increased (minutes)	% of EPD increased
1	1283	1940	657	51.21%
2	2682	3544	862	32.14%
3	3308	4509	1201	36.31%
4	3696	5542	1846	49.95%
5	3539	4868	1329	37.55%

* [150, 300] represents the duration interval, in which 150 and 300 (minutes) are the lower bound and the upper bound, respectively.

It is intuitive to observe, from **Table 5-2**, that with increase of the interval width, the total expected propagated delays of the resulting aircraft routings increase significantly. Taking scenario 5 as an example, the case with task durations lying between 120 and 480 minutes suffers from much more propagated delays, i.e., 1329 minutes, than the model that applies the duration interval bounded by 150 and 300 minutes, respectively. From the perspective of the increase rate, the duration interval [120,480] worsens delays by 37.55%, in scenario 5, as compared with a narrower interval. This implies that the disruptions stemming from the higher stochasticity in the maintenance tasks provide less planning flexibility and may cause more damage to the flight network. In addition, the variability in the delays in selected routes also highlights the importance of capturing maintenance uncertainties within the aircraft routing framework.

5.3.3 The Benefits of Maintenance Capacity

Airlines tend to purchase maintenance services from suppliers in maintenance stations. The capacity a maintenance station provides to an airline is exactly the maintenance slot quotas, specifying the maximum number of aircraft receiving maintenance per hour, that the airline can and is willing to buy. The maintenance capacity is a scarce resource, and, therefore, it is critical for airlines to decide how many quotas are needed to allow smooth operations. Hence, in this section, we report experiments examining the value of the maintenance capacity. To this end, we assigned the numbers 2 to 6, in turn, to the maintenance capacity and recorded the corresponding propagated delays the resulting routes produce, as shown in **Figure 5-1**.

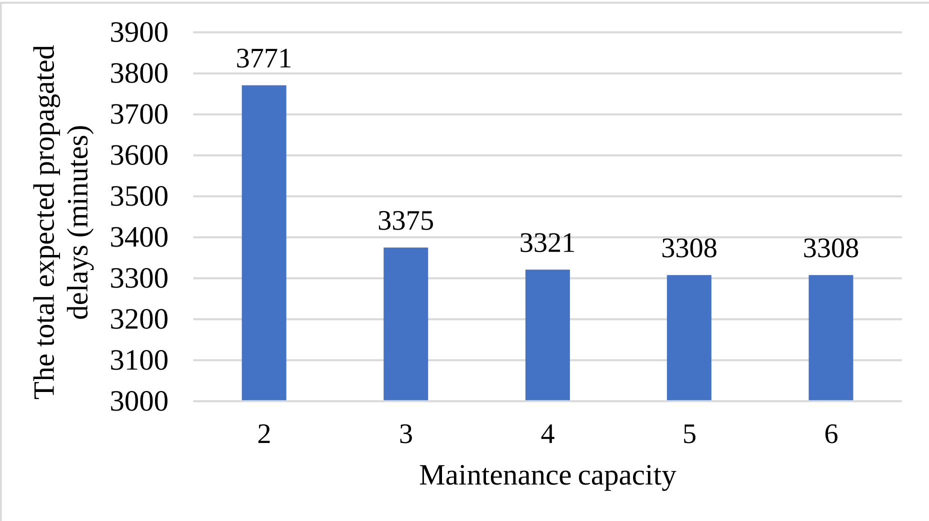

Figure 5-1. The benefits of maintenance capacity.

It can be seen that the total expected propagated delays are reduced when the maintenance capacity is larger. For instance, as the capacity increases from 2 to 3, the propagated delays decrease significantly, by 396 minutes. The underlying reason is that the improvement in capacity provides more opportunities for aircraft to visit the station and thus creates more potential route candidates. Such results demonstrate that when the flight schedules are relatively tight, and when facing maintenance resource supply tension, airlines can benefit from buying maintenance quotas, which are helpful in improving the aircraft schedules.

On the other hand, it is also found that the resulting objective value remains stable, as the value of maintenance quotas increases from 5 to 6. Such results indicate that the improvement in maintenance capacity cannot significantly influence the aircraft routing schedules. This observation may be due to the sufficient supply of maintenance capacity, which implies that, in such cases, it becomes uneconomical to purchase more quotas from suppliers.

5.4 Summary

In an unpredictable and volatile environment especially due to the current Covid-19 pandemic, airline operations are subject to diverse disruptions globally, and thus it is important to improve the robustness of aircraft maintenance routes so as to be less vulnerable to disruptions. The inherent uncertainty of the maintenance duration, one of the most significant disruptions, has received more and more attention from both the aviation industry and academia, which, however, remains under-explored in the literature in regard to the aircraft maintenance routing problem. On the other hand, in light of this uncertainty, the daily planned capacity of maintenance stations, as in many previous studies, may be underutilized or overloaded. To overcome this shortcoming, we propose a novel approach to construct robust aircraft maintenance routings, through taking into account the stochasticity of the durations with regard to heterogenous maintenance tasks. Accordingly, a robust aircraft maintenance routing model is proposed. Because of flight delays and uncertain task durations, the maintenance capacity of each station is modelled hourly to reflect the actual maintenance capacity constrictions, thus complicating the computational process. To address this newly developed model, a tailored column generation-based approach with augmented dominance rules is constructed.

Computational experiments based on the data established from real-world flight operations were carried out to validate managerial implications in terms of robustness performance between distinct degrees of maintenance uncertainties. The results demonstrate that the strong impacts of maintenance duration variability on the robustness of aircraft routes can be observed, seeing that a wider duration interval with more uncertainties produces schedules with worse performance, i.e., the total expected propagate delays significantly increased by at most 51.21%, compared with the solutions generated by a narrower duration interval. Furthermore, experiments were also conducted to evaluate the value of the maintenance capacity, i.e., how many aircraft can be maintained per hour. From the computational results, it is revealed that, under certain circumstances, the improvement in maintenance capacity can significantly reduce the expected propagated delays. For instance, a reduction of 396 minutes can be observed when the maintenance capacity increases from 2 to 3.

One of the derived managerial implications is that airlines should put a high premium on the duration uncertainties of maintenance tasks, and deliberately consider stochastic factors into the decision framework of the aircraft maintenance routing problem. The durations of tasks are inherently stochastic and become a significant source of disruptions in airline operations. As a consequence, ignoring the stochasticity of maintenance may result in aircraft routes lacking robustness. Hence, it is beneficial for airlines to adopt the framework provided in our study, through considering both maintenance and incidental uncertainties, to construct more robust routing schedules.

Secondly, considering that more uncertainties regarding maintenance tasks are able to disrupt aircraft schedules, airline operations can benefit from some useful strategies that may reduce the duration uncertainty of maintenance tasks. For instance, sharing information with maintenance stations (e.g., resource shortages). Taking advantage of these strategies may help airlines to generate routes with less delays.

Thirdly, it is important for airlines to comprehend the potential risks caused by maintenance uncertainties and to determine appropriate maintenance capacities for increasing economic efficiency. To be more specific, if the flight schedules are comparatively tight, it may be beneficial for airlines to purchase more maintenance slot quotas, with the objective of further robustness enhancement. On the other hand, maintenance cost control is also crucial to airlines. As a consequence, there should be a trade-off between the delay cost that airlines can suffer and the procurement cost that airlines are willing to pay. In comparison, when the maintenance resources are over-supplied or the maintenance slot quotas are sufficient, improving the maintenance capacity is not likely to offer significant operational performance improvement to airlines. In such cases, there is a need to seek other strategies to further improve the aircraft maintenance routing solutions.

Chapter 6. Conclusions and Suggestions for Future Research

6.1 Conclusions

Although air transport remains a crucial part of the global economy and development, airlines are faced with dwindling profit margins and stochastic environments. As a result, they endeavor to slash expenses and create robust schedules. In regard to this, operations research plays a significant role in adding value to airline operations, through proposing models and methodologies for effectively addressing airline planning processes, and one of the successful applications is the aircraft maintenance routing problem. While maintenance plays a key part in ensuring safety and airworthiness in the airline industry, there is the significant fact that maintenance checks of aircraft result in considerable costs and uncertainties. Obviously, different maintenance routing schedules contribute to different maintenance costs and reliability performance. To enhance airlines' competitiveness, this research thus concentrates on improving the decision making on aircraft routings from two perspectives: cost-efficiency and robustness. More specifically, in view of the significance nature of maintenance costs and the actual practice of maintenance outsourcing, we propose a new aircraft maintenance routing model with consideration of total quantity discounts, which enables impressive maintenance cost reduction. Furthermore, recognizing the prohibitive consequences of unanticipated disruptions that airlines encounter under the operating environment, two robustness strategies are investigated. One exploits the impacts of the maintenance distribution structure and further integrating the approach encouraging swapping possibilities in the decision-making process. The other is incorporating maintenance uncertainties and other random disruptions into the robust aircraft maintenance routing framework.

To provide an overview of the current studies, Chapter 2 gives a detailed review on the literature with respect to the airline planning process, while concentrating on the tactical and operational aircraft maintenance routing issues. Furthermore, we also draw attention to the contributions, i.e., significant cost savings, made through incorporating quantity discounts in transportation process, which illustrates the importance and benefits of considering the total quantity discounts into the tactical aircraft maintenance routing decision-making process. In addition, the approaches in the literature to evaluate and enhance the robustness are investigated, based on which significant research gaps are identified.

The research gaps regarding the tactical aircraft maintenance routing problem and the procurement strategy existing in common airline practice motivate us to conduct the research work presented in Chapter 3. To be specific, we develop a new aircraft maintenance routing model with a piecewise cost function for capturing the impact of the total quantity discount policy adopted when purchasing maintenance services. The objective of this novel model is in minimizing the total maintenance costs. On the other hand, to solve this complex model and facilitate the computational process, a customized solution algorithm based on column generation is then developed. Actually, from the computational results, the proposed algorithm is proven effective and efficient in solving all the scenarios. Furthermore, owing to the impact of the total quantity discounts, the proposed model demonstrates its advantages in terms of significant cost reduction compared to the existing modelling approaches which neglect the discount strategy in maintenance operations. The results of the comparative experiments also reveal, through analyzing the details of the resulting solutions, the alterability of the maintenance distribution after integrating the total quantity discounts, helping to drive some managerial implications for improving cost efficiencies of airlines, and facilitating the further investigation of the maintenance distribution structure.

Recognizing the approach discovered in Chapter 3 for airlines to modify the maintenance distributions, in Chapter 4, we investigate the potential impact of maintenance distribution structure on the robustness of aircraft routes, measured by the number of swapping possibilities. To be specific, through solving the models with and without consideration of the total quantity discounts, and calculating the number of swapping possibilities of the solutions, the results demonstrate the excellent performance, in terms of robustness, of a more concentrated maintenance distribution. Furthermore, based on the fact that airlines expect their aircraft routings, while maintaining minimum costs, to be ideally as robust as possible, a novel aircraft maintenance routing model is constructed, with consideration of both the total quantity discounts to take advantage of the impact of the maintenance distribution structure and a robustness strategy encouraging swapping possibilities. The proposed robust model shows a clear superiority, through providing more swapping possibilities (i.e., flexibility) to cope with the disruptions, over the traditional models.

Then, observing more accurate information on aircraft, our concern turns to the operational aircraft maintenance routing, and recognizing the research gaps regarding the maintenance process under a stochastic environment, we provide robust solutions from the perspective of schedule stability. Therefore, in Chapter 5, a new robust aircraft maintenance routing model is provided, with consideration of the maintenance uncertainties of the heterogenous maintenance tasks, while taking into account other sources of disruptions. The objective of this proposed model is in minimizing the total expected propagated delays. Accordingly, a tailored column generation algorithm is proposed. With the incorporation of maintenance uncertainties, the constructed robust model demonstrates the significant impacts of the degree of maintenance uncertainty on the aircraft routing solutions' robustness performance. To be specific, the greater uncertainty of maintenance tasks produces more expected propagated delays, which drives managerial implications on managing maintenance uncertainties. Furthermore, the computational experiments on the investigation of maintenance capacity helps us to gain more managerial insights on the maintenance slot quota decisions.

6.2 Suggestions for Future Research

Although this thesis presents new models and algorithms for the aircraft maintenance routing problem, there are some limitations.

1. The model proposed in Chapter 3 (i.e., AMR-PC) considers only the total maintenance cost for the exploitation of the total discount policy. However, there are several other costs to be considered in the aircraft maintenance routing problem, which are also of concern to airlines. Furthermore, in the decision-making framework of Chapter 3, only scheduled maintenance activities are considered. However, unanticipated maintenance, such as repairing malfunctioned components, may occur in regular operations, which should be addressed to ensure safety. This would incur additional costs for airlines and therefore should be carefully taken into account.

2. This study pays special attention to uncertainties in maintenance stations. Furthermore, the improvement of the workforce in maintenance stations may shorten the maintenance duration and mitigate delays. However, the importance of workforce in stations has not been investigated.

3. In Chapter 5, we propose a robust model that considers maintenance uncertainties in the planning stage. However, how maintenance uncertainties influence the recovery stage has not been investigated.

Therefore, based on the research work that has been done in this thesis, several potential research directions worthy of investigating have emerged, which are summarized in the following.

1. It would an interesting direction to integrate more types of operational costs, e.g., through values and penalties for short connections, into the model proposed in Chapter 3, and explore how to make a trade-off between these costs. Moreover, another desirable direction would be further extending this study into the robust aircraft maintenance routing decision-making framework through integrating the unscheduled maintenance demand.

2. It would be valuable to investigate the impact of independent maintenance third parties' ability on the aircraft routing schedules. To be specific, if a service supplier has sufficient resources, e.g., workforce, the normal duration of maintenance can be reduced through assigning more technicians and, as a consequence, the delays resulting from unexpected maintenance or propagated delays can be alleviated, which can help airlines to save significant recovery costs. Additional maintenance resource allocation, however, will trigger increased maintenance costs. It is therefore decisive to achieve a reasonable trade-off between maintenance costs and penalties for delays.

3. In the future, it would be valuable to integrate other robust strategies, such as cruise speed control and time windows, into our robust aircraft routing framework, to further enhance route robustness. On the other hand, it would be an interesting topic to incorporate the maintenance uncertainties into the aircraft recovery problem, which may help airlines to have more savings in recovery costs.

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