INVERSE OPTIMIZATION APPLIED TO OPERATIONS MANAGEMENT IN THE PETROLEUM INDUSTRY

ORISAREMI KAYODE KELVIN

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Department of Industrial and Systems Engineering

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ORISAREMI Kayode Kelvin

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_________________________(Signed)

ORISAREMI Kayode Kelvin_(Student)

Abstract

Routine gas flaring in the petroleum industry poses a serious environmental challenge in most oil-producing nations due to the release of greenhouse gases. In addition to being an environmental threat, gas flaring also leads to a huge economic loss on the scale of billions of dollars when natural resources are depleted. Furthermore, the gas flaring process steals a clean and affordable source of energy from developing nations experiencing energy shortages, such as Nigeria and Venezuela. Considering climate change and a partnership aimed at reducing global gas flaring (GGFR), it is imperative that we estimate potential reductions in global gas flaring to better utilize flare gas within the industry. To this end, this research develops and applies novel strategies for achieving cleaner production of natural gas as a means of improving sustainability measures across the petroleum industry. Three phases of this research are conceptualized, each addressing key research questions concerning energy conversion, economic feasibility of flare gas recovery (FGR) technologies, lean production strategies, and optimal ways to alleviate energy poverty in selected oil-producing nations.

The first phase of this research proposes converting waste gas or flare gas into electricity via gas-to-wire (GTW) process. However, the cost of installing and maintaining a single gas turbine for the GTW process is quite high, particularly for a developing nation like Nigeria. A GTW process must be implemented at an affordable cost by determining the optimal range of turbine units. This phase of the research is therefore highly fundamental, as it involves the development of a novel inverse data envelopment analysis (IDEA) model for the design of a cost-effective GTW process. Based on the developed IDEA model, optimal turbine units were calculated for the deployment of GTW technology in each oil-producing nation. Additionally, the IDEA model was successfully applied to implement policies of the World Bank regarding global gas flaring.

From both an economic and environmental perspective, the second phase of this research examines the dynamics of the petroleum industry with respect to the incremental increases in oil production. A win-win solution is being proposed to achieve both industry and environmental objectives, one that increases oil production while generating less environmental waste in the form of flare gas. In order to achieve this objective, a sustainable lean production framework (SLPF) based on IDEA models was developed. With the introduction of the lean potential

growth (LPG) concept, not only was gas flaring minimized, but it was also possible to determine multiple scenarios with increased oil production and reduced volumes of flared gas. In addition, the lean framework also addressed the limitations of the developed IDEA model in the first phase of this research by minimizing flare gas in efficient oil-producing nations. To meet the energy demand of selected oil-producing nations to some extent, gas flaring reductions were computed and converted to power with the aid of the GT13E2 gas turbine. Nigeria was found to be best positioned to benefit from the energy conversion process, having the highest estimate of gross power output. Further, an energy-based technique was developed for ranking the efficient producers in terms of their net energy production, and the results closely depict the real-world scenario. For the purpose of concluding this second phase, we offer managerial insights for production engineers in this industry. It is through these insights that lean practices can be implemented in the industry, which are far more rewarding than the conventional cost minimization approach.

The final phase of this research seeks to determine the extent to which flare gas can be fully utilized to alleviate energy poverty in some oil-producing nations. This is critical since the gross power outputs computed by the lean framework in the second phase are neither maximum nor minimum. Instead, they are only associated with marginal increases in oil production. In light of this, it is imperative that the maximum amount of power that can be generated by flare gas be estimated. This estimation is especially useful when deciding on an optimal energy mix that includes gas power generation and renewable energy. This objective was accomplished by developing a directional DEA model incorporating both positive and negative data. Based on the findings, Venezuela had a promising energy mix, justifying the use of natural gas as a bridge fuel towards the transition to renewable energy.

Keywords: Gas flaring; climate change; petroleum industry; lean production; inverse DEA; gas-to-wire; optimal energy mix.

Publications

[1] **Orisaremi, K.K.,** Chan, F.T.S., Chung, & N.S.H., 2021. Potential reductions in global gas flaring for determining the optimal sizing of gas-to-wire (GTW) process: An inverse DEA approach. *Journal of Natural Gas Science & Engineering, 93, 103995.*

[2] **Orisaremi, K.K.,** Chan, F.T.S., Chung, S.H., & Fu, X., 2022. A sustainable lean production framework based on inverse DEA for mitigating gas flaring. *Expert Systems with Applications, 206, 117856.*

[3] **Orisaremi, K.K.,** Chan, F.T.S., Fu, X., Chung, N.S.H., 2023. Maximizing flare gas power generation for the design of an optimal energy mix. *Journal of Cleaner Production, 391,136164.*

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Chapter 1- Introduction

1.1 Research Background

A dominant and vital component of the energy sector is the petroleum industry. Crude oil and natural gas, which make up the complex mixture of petroleum, are both raw materials that enable many aspects of modern life. Even though some experts advocate moving away from fossil fuels toward renewable energy due to climate change/global warming, the industry remains highly relevant to humanity for the foreseeable future. In some oil-producing nations, oil remains the mainstay of the economy, providing billions of dollars in revenue annually while preserving industrial civilization. Since the mid-1950s, crude oil has also been the world's most important and reliable source of energy. Energy from its products is essential to modern society, primarily supplying energy to power plants, heating homes, and providing fuel for vehicles and aircraft that transport goods and people around the globe. In addition, refined petroleum products are used in the manufacture of nearly all chemical products, such as plastics, fertilizers, detergents, paints, and even medicines.

The 2015 UK Oil and Gas (UKOG) economic report indicates that the industry employs hundreds of thousands of people and contributes significantly to the British economy in terms of taxes, technologies, and exports. Britain produces about two million tonnes of oil and gas per week. The value of this is about £37 million a day to the British people. The United States relies on oil, natural gas, and coal for 80% of its energy, and the industry is estimated to provide $1.6 trillion in federal and state tax revenue between 2012 and 2025, supporting the maintenance of schools, hospitals, and public infrastructure across the country (Yergin, 2020). An analysis conducted by the U.S Chamber of Commerce indicates that halting oil and gas extraction by 2025 will result in the loss of 19 million jobs (direct and indirect) across seven states in the country. The U.S. trade deficit in 2019 was also estimated to be US$305 billion lower than it would have been without domestic oil and natural gas production (Yergin, 2020).

Yet, it is pertinent to note that, since its inception, the petroleum industry has yet to embody operational sustainability as it generates significant amounts of hazardous wastes throughout its supply chain. As the wastes are generated repeatedly, the associated degradation of the environment is inevitable. Consequently, industry-wide sustainability measures need to be improved urgently. As a means of highlighting the environmental impacts of the petroleum industry, a basic understanding of what a typical petroleum supply chain looks like must be discussed. The petroleum supply chain is often divided into three sectors as shown in Fig. 1.1.

I. Upstream: Exploration and production of oil and gas is the core business of this sector and accounts for the largest portion of overall production costs, including exploration costs, drilling costs, and marginal costs.
II. Midstream: This sector handles the transportation and storage of crude oil as well as associated petroleum gas (APG). Transportation and inventory costs are involved in this sector.
III. Downstream: The sector is primarily responsible for refining crude oil, distributing refined petroleum products, and marketing them.

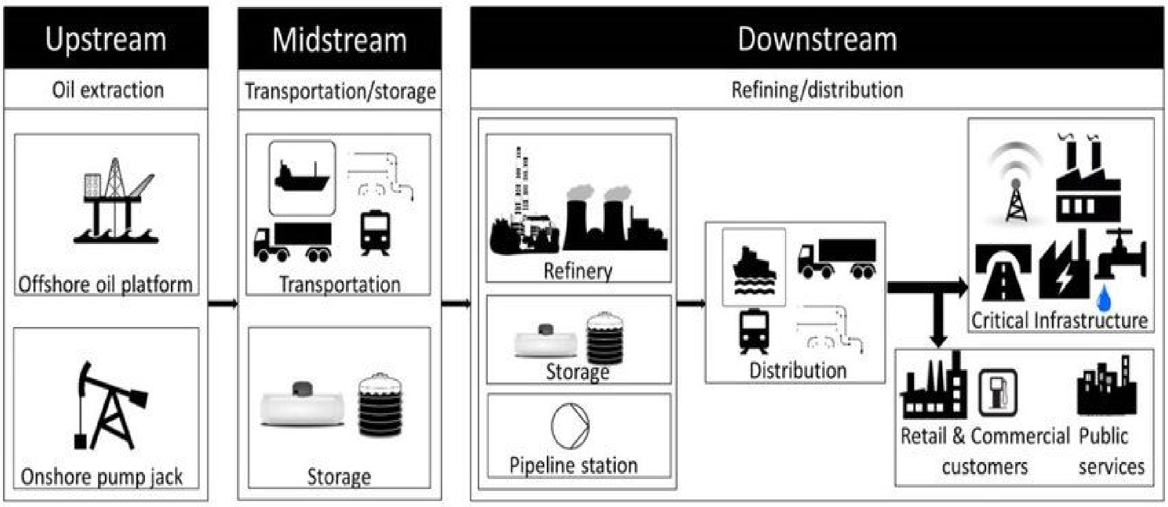

Figure 1.1: Structure of a typical petroleum supply chain (Source: Katopodis and Sfetsos (2019))

Regarding the environmental impacts of the petroleum industry, it is vital to note that the upstream sector is primarily responsible for the generation of hazardous wastes during the oil extraction process. There are two main sources of harmful wastes: oil spills and gas flaring. Pollution of water bodies caused by oil spills makes water unsafe for humans and aquatic mammals. Gas flaring is the controlled burning of associated petroleum gas (APG) using gas flares. APG is technically known as ***flare gas*** when it is burned off in oilfields but is called ***field gas*** when it is converted into electricity for use in homes. It is pertinent to mention that while both wastes have been briefly discussed, gas flaring presents a greater threat because of the release of greenhouse gases (GHGs) that contribute to global warming. Also, gas flaring contributes to air pollution in many oil-producing nations and has been linked to a decrease in life expectancy in some developing countries.

Figure 1.2: A typical gas flaring site in Southern Nigeria (Source: Aggreko)

Gas flaring also results in significant economic losses due to the waste of energy resources on a scale of billions of dollars. Therefore, there is an urgent need to check, minimize, or eliminate gas flaring so that the negative effects can be reduced or eliminated.

Although many authors have examined the issue of gas flaring and its associated environmental degradation in oil-producing nations in the literature (Agboola et al., 2011; Economides et al., 2004; Ishisone, 2004; Odumugbo, 2010; Oni & Oyewo, 2011), it is worth noting that these works were mainly descriptive analyses of the problem, with technology and infrastructure suggestions to reduce gas flaring in the Nigerian oil industry and elsewhere. Therefore, Ojijiagwo et al. (2016) were the first to develop an approach to reduce gas flaring and its impacts in Nigeria based on semi-structured interviews with industry experts. However, gas flaring is a global problem, not unique to a single nation. Further, there is no quantitative approach for dealing with global gas flaring reduction in the literature, in contrast to the qualitative approach developed by Ojijiagwo et al. (2016) that is only applicable to a single oil-producing nation. This study presents for the first time in literature a robust quantitative approach based on novel optimization techniques for managing gas flaring, thereby determining the maximum gas flaring reductions on a global scale.

1.2 Problem Statement

This research involves the development of novel optimization techniques for flare gas management across major oil-producing nations around the world. As such, the proposed research methodologies address three practical problems in the petroleum industry by developing mathematical models for each of these problems within the context of a holistic framework. An OPEC (organization of the petroleum exporting countries) member nation, Nigeria, is chosen as the main case study. The Niger Delta is the oil rich region of southern Nigeria, and it consists of nine oil producing states. Gas flaring at oil production fields in Nigeria is responsible for certain health problems within the Niger Delta region. Respiratory illnesses, adverse skin disorders and heat burns are among other health problems. The first part of this research would involve developing an inverse data envelopment analysis (DEA) model to estimate gas flaring reductions within the country. Gas flaring reductions are vital for converting flare gas into electricity to alleviate the ongoing energy poverty the country has been facing for more than two decades.

This strategy is also expected to reduce air pollution and associated health risks within the oil-producing states. Hence, a crucial objective of this research is, from a purely technical perspective, to determine the maximum amount of flare gas that can be used for power generation within the nation.

The inverse DEA methodology was selected primarily for two reasons. In the first instance, this study examines the global issue of gas flaring, which entails multiple oil-producing countries utilizing multiple inputs to produce multiple outputs. In DEA analysis, this is referred to as homogeneity. The case study, Nigeria, is part of a homogeneous group of oil-producing nations that contribute to global gas flaring. Consequently, the productive efficiency of each oil-producing nation must be evaluated in relation to others. This is a necessary step toward benchmarking and standardization of production processes. DEA is an ideal management tool for carrying out this initial evaluation as its underlying principle is based strictly on the computation of the relative efficiency of each producer. However, a critical inverse problem in the industry arises when it is required to compute the optimal changes in inputs and/or outputs such that the predetermined efficiency score of each producer is preserved or not compromised. Gas flaring reductions at a global scale constitute such an inverse problem, with a higher level of complexity, which requires the use of mathematical theorems and proofs to extend and configure the traditional DEA model. With the advent of inverse DEA, complex inverse problems involving multiple inputs and multiple outputs can be solved in the real world.

Secondly, few works have applied DEA and inverse DEA to the petroleum industry for sustainability development and minimizing GHG emissions (Sueyoshi & Wang, 2014; Wegener & Amin, 2019). Other related works are restricted to estimates of GHG emissions from gas flaring (Anomohanran, 2012; Elvidge et al., 2018; Giwa et al., 2014; Hajilary et al., 2020; Otene et al., 2016) and mitigation strategies for a single region or oil producing nation (Elvidge et al., 2018; Elvidge et al., 2016; Rotty, 1974). Yet, all these studies and innumerable others from the literature have failed to address the collective problem of estimating reductions in gas flaring in accordance with the global gas flaring reduction partnership (GGFR) launched by the World Bank. Therefore, this study closes this critical gap in the literature by developing novel inverse DEA models in compliance with the GGFR initiative. In addition, the first part of this study will conclude by designing an algorithm as a policy-making tool for the implementation of another World Bank policy called the "Zero Routine Flaring (ZRF) by 2030 Initiative". The goal of this initiative is to end global gas flaring at oilfields by 2030.

The lean production philosophy has long been regarded as an effective management technique in many industries, including automobiles, cement, aviation, textiles, healthcare, etc. Utilizing lean tools has achieved satisfactory results within these industries by eliminating waste and non-value-added activities, minimizing input resources, and increasing productivity/profitability. Unsurprisingly, the petroleum industry lags in the adoption of the lean concept. The industry has generated waste products on a continuous basis since its inception, which has had significant consequences during periods of falling oil prices resulting in major revenue losses. The second part of this study focuses on the development of a framework for sustainable lean production based on the inverse DEA. This framework is intended to enable the petroleum industry to reduce production wastes, minimize the use of resources and increase productivity. Experts in the industry have termed this scenario a win-win solution for both the industry and the environment because more revenue could be generated with less environmental waste in the form of flared gas. Benchmarking oil producers also requires the use of the lean production framework to address the perennial problem of ranking efficiency often encountered in DEA analysis. Therefore, it is imperative to establish a hierarchy for efficient producers so that the less efficient producers can improve their production practices to be at par with the most efficient producers.

The final section of this study is solely concerned with determining an optimal energy mix consisting of natural gas power generation and renewable energy for oil-producing developing countries such as Nigeria and Venezuela, which are currently experiencing energy poverty. The goal is to use natural gas as a bridge fuel to help both countries gradually transition to 100 percent renewable energy. Many developed countries see the global push for renewable energy as a positive development. However, the economic crisis in some developing countries, combined with declining purchasing power, suggests that these countries will require more time to completely transition to renewable energy sources. This necessitates a short-term reliance on gas power generation for Nigeria and Venezuela, as both countries are among the top seven gas flaring countries in the world for the ninth consecutive year, creating an energy economy paradox. The motivation for this final part of the study stems from the fact that no previous study has developed a mathematical framework for determining the maximum amount of electricity that can be generated from flare gas. This estimate is required to determine how much of a country's unavailable or lost power capacity can be recovered through flare gas power generation. With this information, the remaining capacity loss in installed power can be allocated to renewable energy sources. To address the last issue, a directional distance DEA model will be developed separately, as this technique is based on determining maximum optimal solutions in DEA analysis.

1.3 Research questions

This study aims to answer the following research questions, which are critical for improving sustainability measures in the petroleum industry in the short and long term:

R1. At current production level of inputs and outputs, and with current technology and workforce, what is the potential reduction in gas flaring for an oil-producing nation?

R2. With better technology and a more highly skilled workforce, what is the maximum potential reduction in gas flaring for an oil-producing nation?

R3. Can the petroleum industry adopt the Zero Routine Flaring (ZRF) Initiative in any given production year?

R4. What is the optimal sizing required for implementing a gas-to-wire (GTW) process for flare gas management?

R5. Can the industry adopt and implement lean production practices without flare gas recovery technology?

R6. How can the industry adopt and implement lean production practices through flare gas recovery technology, such as gas-to-wire (GTW)?

R8. With the current emphasis on transitioning to renewable energy, what is the optimal energy mix for an oil-producing country that combines gas and renewable energy?

1.4 Research objectives

The following objectives will provide the much-needed answers to the research questions posed in the previous section:

1. To develop a novel inverse DEA model for accurately estimating potential reductions in global gas flaring.
2. To develop a new algorithm for effective implementation of the "Zero Routine Flaring by 2030" initiative.
3. To determine the optimal sizing of an economically feasible gas-to-wire (GTW) process.
4. To develop a sustainable lean production framework for improving operational sustainability within the petroleum industry.
5. To develop an optimal energy mix for an oil-producing nation based on flare gas power generation.

1.5 Scope of Research

This research focuses on flare gas management in the petroleum industry as an effective means of alleviating energy poverty in selected OPEC member nations. As opposed to previous studies in the literature that have addressed the issue of gas flaring in such nations using a descriptive and qualitative research approach, this study employs a quantitative method based on linear programming models. There are two different gas flaring policies (the GGFR and ZRF initiatives) published by the World Bank that serve as a guide for the entire research. The GGFR initiative focuses on the reduction of gas flaring, while the ZRF initiative focuses on eliminating gas flaring. In this regard, all the models developed in this study are geared toward the cleaner production of natural gas within OPEC member nations.

Since OPEC member nations experiencing energy poverty are also developing nations, this research also addresses the design of a cost-effective solution scheme for such countries. The solution scheme involves determining the maximum amount of energy that can be produced through gas turbines based on different scenarios. Also, the commercialization of flare gas that is based on the formulated models could serve as a potential source of revenue for OPEC members that are not experiencing any form of energy shortage.

1.6 Research Contributions

The contributions of this study can be summarized as follows:

I. This study is the first to develop a robust quantitative approach to address the persistent issue of gas flaring in the petroleum industry. Thus, the proposed methodologies were developed in strict compliance with the policies of the World Bank regarding gas flaring reductions. In consequence, this study offers both short- and long-term policy-making tools for sustainable oil and gas production across the industry.

II. The present study provides a new theoretical basis that allows the practical application of DEA models in sustainability and waste management, not only within the petroleum industry but also across a wide range of other sectors that require effective management strategies to minimize waste and increase productivity. In general, this study demonstrated that the inverse DEA model can be used as a lean production tool. To combat the increase in gas flaring that is typically associated with increased oil production, as well as to prevent oil price crashes due to an excess supply of crude oil, this study introduced the concept of lean potential growth to the literature for the first time. Managers can also use the lean potential growth formula when optimizing oil inventory.

III. Also, the study was the first in the literature to consider the economic feasibility of deploying flare gas recovery technologies such as gas-to-wire (GTW) to reduce or eliminate gas flaring at oilfields. This was achieved by developing a formula for the optimal sizing of the gas-to-wire (GTW) process. This formula factors in projected reductions in gas flaring and the annual gas consumption of a single turbine unit. It is possible to save capital costs by calculating the maximum number of turbine units. Simply put, any investment beyond the maximum number of turbine units will likely result in a financial loss.

IV. Also, for the first time, an energy transition curve for the petroleum industry is developed in this study. Moreover, the study examines the effect of data type on the transition curve, which leads to a novel concept of optimal power sizing in the industry. Considering this information, the notion of maximum power generation from flare gas becomes more realistic when dealing with positive and negative data in DEA modeling.

V. Finally, as a contribution to the field of operations research, this study successfully introduced new mathematical theorems with proofs and demonstrated that they have profound implications when used in DEA analysis for waste management and circular economy.

1.7 Research Significance

Climate change and global warming are two distinct threats that cannot be overemphasized in the present day. Emissions of greenhouse gases (GHG) are a contributing factor to climate change. GHG emissions from gas flaring have contributed substantially to the push for a transition from fossil fuels to renewable energy sources. The effects of GHG emissions on the atmosphere include air pollution, acid rain, and other harmful consequences. The race toward the elimination of GHG emissions in the short-term may appear haphazard and somewhat challenging, since the policies available to reduce emissions are ineffective or, at best, of limited effectiveness. Therefore, in the short term, the best strategy is to tackle a major source of emissions, which in this case is gas flaring.

By reducing gas flaring, GHG emissions are greatly reduced, and this results in a decline in the number of deaths from air pollution, thus easing pressure on healthcare systems in affected areas. In addition, flare gas commercialization represents a significant economic benefit to oil-producing nations. For instance, this study estimates the maximum revenue that can be generated from flare gas in Nigeria to be US$1.379 billion in 2011. According to a financial report by PricewaterhouseCoopers (PwC), Nigeria lost US$761.6 million (i.e., N233 billion in local currency) to gas flaring in 2018. Fig. 1.3 represents the extent to which this amount can be used to finance government projects, such as housing, road and airport improvements, and healthcare.

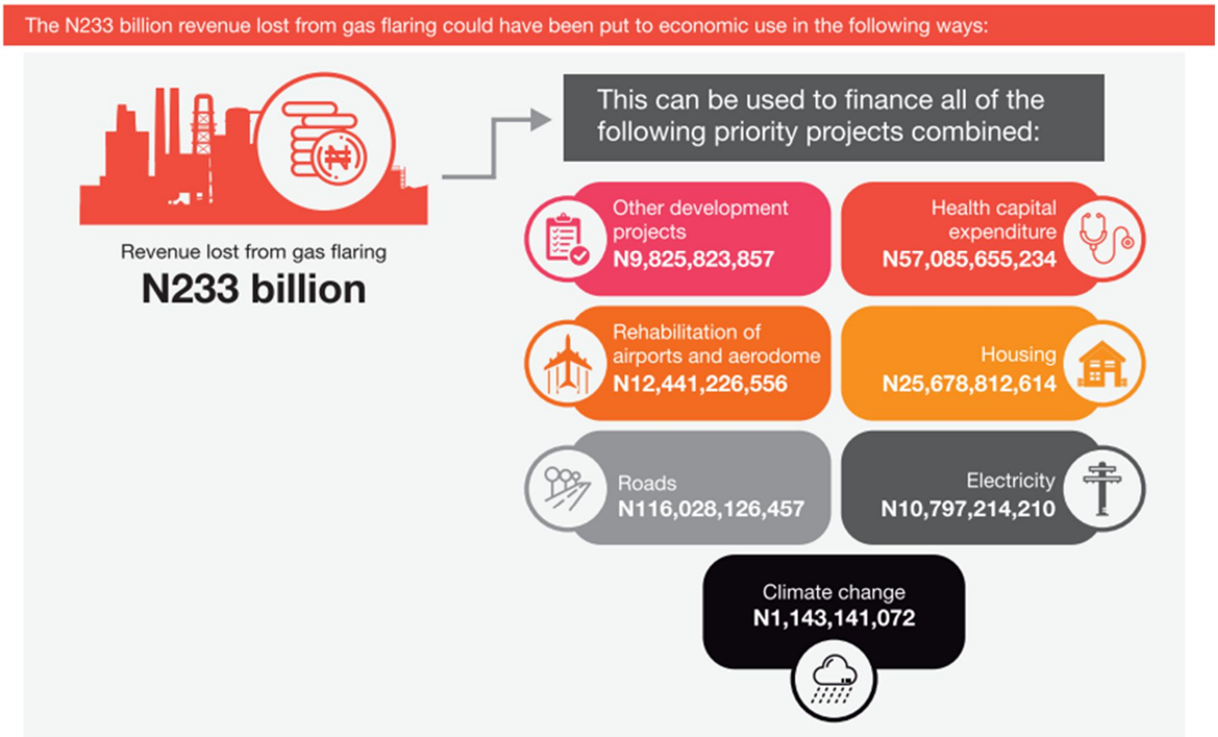

Figure 1.3: Development projects related to flare gas commercialization in Nigeria.

(Source: PwC (2019))

It is often difficult to convince national governments to invest in the technological equipment and infrastructure necessary to reduce gas flaring due to their high cost. In the GTW process, the costs associated with the installation and maintenance of a single turbine unit are quite expensive for a developing nation such as Nigeria. Another important benefit of this study is the provision of a reliable template for computing the optimal number of turbine units based on the availability of gas. With this information, the associated risks of an investment in GTW process can be greatly minimized, resulting in cost savings. The cost savings relate to the excess operating costs of redundant turbines, should a decision-maker neglect to consider safety and maintenance flaring, which is a relatively small fraction of the total volume of flare gas.

1.8 Structure of the Thesis

The remainder of this thesis is organized as follows:

In Chapter 2, a critical assessment of the literature related to the topics of this research is presented. Among the topics addressed in this literature are mitigation strategies for gas flaring, gas-to-wire (GTW) processes, lean production tools, DEA, and inverse DEA models. As well, the application of DEA models to sustainability studies in a variety of industries is covered in detail. Studies involving the use of DEA models in waste management are also critically analyzed. Toward the end of this chapter, research gaps are summarized to give a better understanding of the research problems and subsequent work in the following chapters.

Chapter 3 examines the theoretical basis for the development of a novel inverse DEA that addresses the problem of global gas flaring. Thus, this chapter constitutes the core of this research, as it presents for the first time in the literature a robust mathematical approach that results in the optimal sizing of the gas-to-wire (GTW) process.

Chapter 4 is an extension of chapter 3. By devising a framework for implementing lean production practices in the petroleum industry, this extension addresses the limitations of the inverse DEA developed in chapter 3. In doing so, chapter 4 introduces the concept of lean potential growth to prevent crude oil overproduction and subsequent gas flaring. Furthermore, the managerial implications of the applied models are thoroughly discussed.

Chapter 5 extends both chapters 4 and 3. This chapter presents a novel approach to determining the maximum power that can be generated from flare gas. This chapter focuses primarily on the waste-to-power concept. Additionally, this chapter discusses in detail the energy transition curve and its perturbations due to data type, leading to the notion of optimal power sizing.

In chapter 6, the overall conclusions of the models described in the previous chapters, the limitations of this study, and possible future research directions are presented.

Chapter 2 – Literature Review

2.1 Introduction

This chapter presents a comprehensive review of the pertinent literature, highlighting the importance of this research to the industry and society at large. Previously, studies on gas flaring have been restricted to descriptive and qualitative analyses of single regions, and in some cases, specific oil-producing firms. It is therefore imperative that such studies and those on management approaches proposed for the purpose of reducing gas flaring in the industry be highlighted and critically reviewed. The relevant literature is divided into four sections.

This first section examines previous studies regarding gas flaring estimates and their resulting GHG emissions. An in-depth review of several strategies for mitigating gas flaring as well as technological solutions are presented in the second section. Moreover, since a significant area of this study focuses on the design of a lean production framework as an additional mitigation strategy, it is imperative that a comprehensive review of lean production tools for waste management across several industries also be presented. The third section of this review focuses on the applications of DEA models for sustainability, energy efficiency, and waste reduction. This is further elaborated by illustrating the perennial ranking issues encountered with the use of DEA models for efficiency ranking, and a variety of ranking methods are examined that resolve these issues. A brief discussion of inverse optimization is presented in the fourth section, followed by applications of inverse DEA to real-world problems. This chapter concludes with the identification and discussion of research gaps.

2.2 Estimating Gas Flaring and associated GHG Emissions

Several studies have been conducted to estimate the volumes of flare gas and their constituent greenhouse gas emissions, a problem that has existed in oil-producing nations for decades. A number of these studies investigated causal relationships between oil production and associated gas flaring. Other studies were conducted to estimate the actual volume of GHG emissions during the gas flaring process. These studies utilized satellite imagery, statistical approaches, and algorithms. This section offers a review of related studies.

Rotty (1974) was the first to study the relationship between oil production and gas flaring. Based on data reported or estimated for 1968 to 1971, this was accomplished by fitting a linear regression line as a function of worldwide gas flaring to crude oil production and arriving at flared gas estimates for each year since 1935.

Elvidge et al. (2009) then introduced the use of low light imaging by using data provided by the Defense Meteorological Satellite Program (DMSP) to provide estimates of annual national and global gas flaring over a 15-year period (i.e., 1994-2008). As part of the DMSP estimate, a calibration system has been developed using national gas flaring volumes and data from individual flares. Additionally, the study assessed the global gas flaring efficiency during this period. Flaring efficiency was measured based on the amount of gas flared for every barrel of crude oil produced. During the period of 1994 to 2005, worldwide flaring efficiency was estimated to have been between seven and eight cubic meters per barrel and declined to 5.6 cubic meters per barrel by 2008. It was estimated that 139 billion cubic meters of natural gas were flared in 2008, representing 21% of U.S natural gas consumption with a market value of more than $68 billion. The study also estimated that the 2008 flaring released more than 278 million metric tons of carbon dioxide equivalent (CO_2e). In the DMSP estimates of gas flaring volumes, a 19% decline has occurred since 2005, with the major reductions occurring in Nigeria and Russia, the two countries with the highest level of gas flaring. Both Russia and Nigeria experienced an improvement in their flaring efficiency from 2005 to 2008. Therefore, it might be suggested that the reduction in gas flaring is because of either better utilization of the gas, reinjection, or direct venting of gas into the atmosphere. However, the impact of uncertainties in the satellite data cannot be completely excluded. With the advent of satellite data enabling estimation of gas flaring volumes, it is anticipated that gas that was previously simply burnt as waste will be more effectively utilized this way.

Ismail and Umukoro (2012) have provided an in-depth overview of different approaches utilized by researchers to measure flare gas and its resulting emissions. It includes analytical studies, numerical studies, modeling, computer simulations, etc. All these approaches are aimed at mitigating the consequences of flaring. The study concludes that there does not appear to be a single global method, emission factor, or estimation procedure employed by the oil and gas industry around the world to determine the volume of gas flared and whether its emissions are related to complete or incomplete combustion. Consequently, there is a continuous problem in determining an accurate estimate of the impact of gas flaring on humans and the role it plays in the degradation of the environment on a local and global scale.

Anomohanran (2012) used both descriptive analysis and the reference approach method to determine greenhouse gas (GHG) emissions from gas flaring in Nigeria. Initial findings revealed that the volume of gas produced in Nigeria from 1999 to 2009 was 502 million cubic meters while 237 million cubic meters, or 47%, flared. Moreover, the total amount of gas flared decreased from 23 million cubic meters in 1999 to 14 million cubic meters in 2009. As a result, the total amount of carbon dioxide emissions between 1999 and 2009 was estimated to be 457 million metric tons or 23.1 percent more than the global value of 1979 million metric tons. The obtained results showed that Nigeria flared 47% of total gas produced over a ten-year period (1999 – 2009) and lost an estimated US$11 billion annually.

As a way to monitor gas flaring on a global scale, Casadio et al. (2012) developed a novel active flame detection method that relied on short wavelength infrared measurements (SWIR, 1.6μm), which was tested using measurements of the Along Track Scanning Radiometer (ATSR) family measurements. In addition, a novel algorithm, called ALGO3, was developed based on the confirmed assumption that, at SWIR wavelengths, background contributions to the night-time total radiation measured by ATSR are negligible, whereas flame-emitted radiation from active flames is detectable. Due to their high temperature/small area flames, ALGO3 products can be used to detect gas flares. Flaring sites were categorized based on the persistence of hot spots. Therefore, hot spots present at a frequency greater than four times a year were considered industrial settlements. There has been confirmation of continuity and consistency between the ATSR missions and the results pertaining to the 1991-2009 period have been presented. Visual inspection of high-resolution images of the Earth's surface has been used to validate the flaring site discrimination. There is a high degree of agreement between ALGO3 retrievals and light count data from the DMSP. Elvidge et al. (2015) presented a set of new methods for surveying natural gas flaring globally using data collected by the Visible Infrared Imaging Radiometer Suite (VIIRS). According to the estimates, flared gas volume carries a level of accuracy of

$\pm$9.5%. The VIIRS is especially useful for detecting and measuring the radiant emissions from gas flares based upon the collection of shortwave and near-infrared data at night, as well as recording the peak radiant emissions from flares. The number of flare sites identified in 2012 totaled 7467. According to estimates, the total volume of flared gas is approximately 143($\pm$13.6) billion cubic meters (BCM), which corresponds to 3.5% of worldwide production. Russia leads in terms of volume of flared gas, while the USA has the highest number of flares. 90% of the flared natural gas volume is found in upstream production areas, 8% is found at refineries, and 2% is found at LNG terminals. These findings confirm that natural gas flaring primarily occurs in upstream production areas.

Using Moderate Resolution Imaging Spectroradiometer (MODIS) thermal imagery at night, Anejionu et al. (2015) developed a methodology for locating individual active flaring sites and the volume of flared gas at those sites. As part of its flare detection technique (MODET), MODIS also utilizes a volume estimation technique (MOVET) that combines the absolute and contextual radiometric responses of flare sites. Based on independent observations of flare location and volume, the detection accuracy and estimation error levels were quantified. An archive of MODIS data covering the Niger Delta in Nigeria, a major global hotspot for flaring, was used to apply MODET and MOVET. This study illustrates significant spatial and temporal variability in gas flaring within the region, across states, and across onshore and offshore sites. Thus, while the total volume of gas flared in the region over the study period (350 billion cubic meters) is large, the heterogeneity of flaring indicates that its impact will vary widely between time and space. Combined, MODET and MOVET offer a consistent and objective way to monitor flaring activity across appropriate scales, and the robustness and transferability of these methods need to be tested in other oil-producing regions as well.

A comparative analysis was also provided by Elvidge et al. (2018) based upon satellite-derived gas flaring data of 2015 and the GHG reduction targets indicated by those countries in their nationally determined contributions (NDC), pursuant to the Paris Agreement. This analysis includes three categories of flaring: upstream at oil and gas production sites, downstream at refineries, transportation facilities, and other industrial locations. The results indicate that upstream flaring accounts for 90.6% of all flaring. Flaring worldwide represents less than 2% of the NDC reduction target. Although most gas flaring occurs in a limited set of countries, it is possible that reductions in flaring could make a significant contribution to the NDC targets for specific countries. Additionally, the study revealed that states that could achieve their NDC targets through gas flaring reductions included Yemen (240%), Algeria (197%), and Iraq (136%). The countries that may be able to meet substantial portions of their NDC targets with gas flaring reductions include Gabon (94%), Algeria (48%), Venezuela (47%), Iran (34%), and Sudan (33%). In contrast, several countries with high flaring levels were only able to meet a small portion of their NDC targets by reducing gas flaring, including Russia (2,4%) and the United States (0,1%). The findings of these studies may be useful to guide national efforts to meet greenhouse gas reduction targets under the NDC.

Giwa et al. (2014) used 49-year (i.e., 1965 – 2013) data to determine the relative amounts of black carbon (BC) emissions into the atmosphere via gas flaring in Africa's largest oil producer, Nigeria. They estimated the average annual flare in Nigeria to be 18.27 billion cubic meters. Each year of a 49-year period was assigned an emission factor and a volume of gas flared and they were used to quantify the amount of BC that was released into the atmosphere. During this time, 55% of the gas produced was flared, releasing 4.56×10^5 tons of BC into the environment. From the first decade (1965–1974) of gas flaring to the fourth decade (1995–2004), BC emissions into the environment increased progressively (5.06×10^4 to 1.27×10^5 tons) with a significant decrease (8.74×10^4 tons) in the fifth decade (2005–2013). Emam (2015) also provided estimates of gas flaring in terms of actual volume, composition, and distribution; thereby highlighting flare gas management systems to reduce or recover flare gas.

A study conducted by Umukoro and Ismail (2017) provides material balance equations and presents a prediction technique for non-hydrocarbon emissions such as CO_2, CO, NO, NO_2, and SO_2, from flaring of twelve samples of natural gas representative of natural gas originating worldwide. With the aid of a computer program, the material balance equations for six reaction types and combustion conditions were coded. Based on an average annual global flaring rate of 126 BCM per year (between 2000 and 2011), the expected gaseous emissions from flaring natural gas are 560MMT, 48 MMT, 91 MMT, 93MMT, and 50MMT for CO_2, CO, NO, NO_2 and SO_2, respectively. In their developed model, gaseous emissions are predicted in relation to possible combustion types and conditions. Conrad and Johnson (2017) performed an in-situ field study of BC emission rates and volume-specific BC yields from a variety of flaring systems. In accordance with the results obtained, flare gas volume-specific BC yields were highly correlated with flare gas heating values. Their study concluded by stating that based on the newly derived correlation between current field data as well as previous lab data, the effects of flaring in the energy industry may indeed be underestimated.

2.3 Gas Flaring Mitigation Strategies

In the literature, various proposals have been made to mitigate gas flaring and associated GHG emissions since it is a persistent global problem. In this section, we review these proposals and their significant findings.

In a comparison between three strategies for flare gas recovery at the Asalooye Gas Refinery, Rahimpour et al. (2012) studied GTL production, gas compression, and the production of electricity using gas turbines. As a result of their findings, it was found that GTL had the highest return rate but required the greatest amount of capital investment. Having established a lower rate of return for gas compression, they recommended it for the refinery along with lower capital investment. A study by Davoudi et al. (2013) explained the process of managing gas flares in Iranian gas processing plants and explained their robust strategy for minimizing the associated wastes. To investigate and design the optimal layout of flare gas recovery hybrid systems using liquid ring compressors (LRC) and aqueous amine solvent, Yazdani et al. (2020) constructed three sets of configurations. Specifically, the third configuration showed a lower amine consumption than the first and second configurations by 67% and 44%, respectively. However, compared with the first and second configurations, the third configuration required an increase in power of 58% and 53%, respectively. Abdulrahman et al. (2015) used the triple bottom line approach for analyzing a sustainable gas flare recovery project in Egypt. According to the findings, GHG emission reductions could generate an expected annual revenue of US$ 1.5 million and create job opportunities in the Egyptian petroleum industry. Additionally, it was determined that the removal of energy subsidies will make the study more beneficial to Egypt in the long run, regardless of the revenues from the clean development mechanism (CDM).

Ojijiagwo et al. (2016) investigated the economic feasibility of gas-to-wire technology in Nigerian oil fields. Based on interviews with industry experts across gas and electricity companies in the country, they used a qualitative research methodology. Initial findings suggest that the gas firm flared about 8.33% of the total volume of gas produced on a daily basis, while a gas turbine unit rated for 150MW power output consumes approximately 0.93 million cubic meters of gas per day. According to the main results, 50 turbine units are sufficient to generate 7500MW of electricity for the country based on the total gas consumption of 46.5 million cubic meters. Finally, in the context of the general economic benefits for the Nigerian economy, the project is projected to generate total revenue of £2.68 billion with a rate of return of 16.3% and a payback period of six years. Hajilary et al. (2020) proposed a novel approach for reducing flare gas at the second and third phases of the South Pars refinery. The approach was based on detailed case histories and simulated conditions, which revealed bottlenecks. Results indicate a significant decline in flare gas from 190 to 31 million cubic meters per day. CO_2 emissions decreased by 83.4%, and in total, GHG emissions decreased by over 700% as a result. To effectively extract huge amounts of CO2 from natural gas while generating electricity, Interlenghi et al. (2019) have proposed the use of a combination of gas-to-wire and carbon capture systems. Khalili-Garakani et al. (2020) have provided an in-depth review of the potential of gas flare recovery technologies in Iran.

2.3.1 Technology solutions for gas flaring

With a portable gas compression unit, gas that would otherwise be flared could be compressed, transported, and used elsewhere in the operating area rather than being flared. According to the U.S Environmental Protection Agency (EPA), it was estimated that at least 89% of compressed natural gas (CNG) could be recovered via natural gas compression technique in the western region of North Dakota. Furthermore, General Electric (GE) and Ferus Natural Gas Fuels, Inc. (Ferus NGF) have developed and tested a system called the "*Last Mile Fuelling Solution*" for transporting CNG across the final distance or last mile from the point of supply to the point of use without the use of conveying pipes. Additionally, gas-to-liquids (GTL), liquefied natural gas (LNG), and gas reinjection are viable options that must be discussed separately.

2.3.1.1 Gas-to-liquid (GTL) technology

Gas-to-liquids (GTL) systems have been developed to convert natural gas to liquid products such as high-grade diesel fuel, methanol, and zero sulphur diesel. The GTL produces diesel fuel that provides almost the same amount of energy density as conventional diesel, however the cetane number is higher, so it is possible to design engines that can run better (Stanley, 2009). Furthermore, GTL products not only enhance the value of conventional products, but they also produce less pollution because of their reduced emissions. As well, synthetic liquid fuel (SLF) has been substituted for GTL in Russian literature (Eliseev, 2009). In this process, two technologies are used: direct conversion from gas, and indirect conversion via synthesis gas. By utilizing direct technology, methane can be directly transformed into synthetic gas at a lower cost. By using Fischer-Tropsch (F-T) synthesis or methanol, indirect technology could be applied. Chemical transformation of simple organic compounds into complex hydrocarbons is required by F-T reactors. Cobalt or iron catalysts are typically employed in this process. The F-T synthesis is depicted in Fig. 2.1, which shows a flowchart of the entire process and the final products - LPG, naphtha, and diesel.

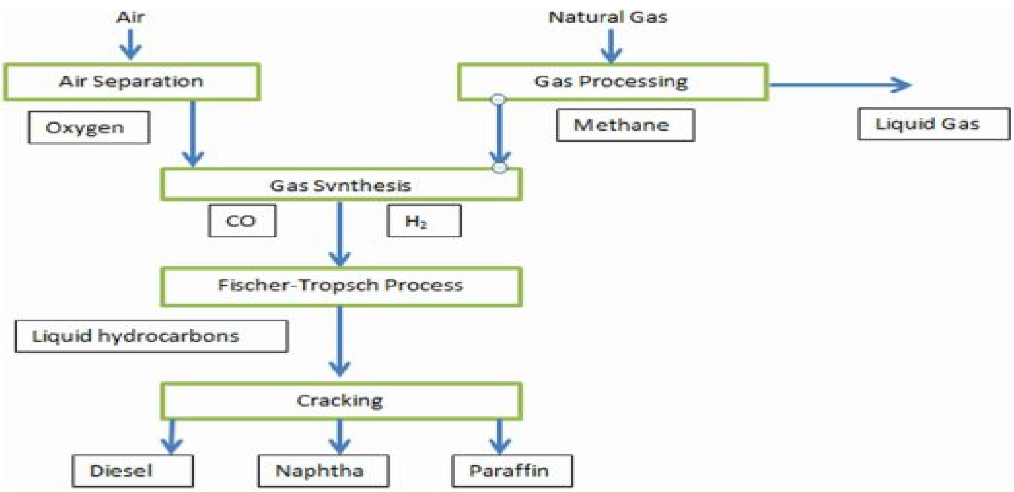

Figure 2.1: Schematic of the F-T process (Source: Ojijiagwo et al. (2016))

With respect to relevant studies, Stanley (2009) evaluated GTL conversion systems as a means of reducing gas flaring in Nigeria. Additionally, the conversion process reduced the overdependence on imported refined petroleum products. Rahimpour et al. (2011) proposed a GTL simulation approach based on reactors for the production of gasoline from synthesis gas. Results showed significant reductions in CO_2 and an increase in gasoline production.

Using a base-case flowsheet synthesis and computer-aided simulation, Bao et al. (2010) carried out a techno-economic analysis of the GTL process. The goal was to find opportunities for cost reductions and energy efficiency without sacrificing the environment. A break-even point and return on investment were also estimated based on an economic analysis. Fei et al. (2014) have also introduced Bio-GTL by providing an overview of bioprocess technologies that use methane as an alternative carbon source. The study further outlined major challenges and research needs for microbial lipid accumulation from methane sources in the future. Upon analysis of raw materials, methane-derived diesel fuel may be more cost effective than petroleum-derived fuel.

2.3.1.2 Liquified natural gas (LNG)

In LNG technology, natural gas is cooled to a temperature of approximately -260 degrees Celsius and at atmospheric pressure. Transport of natural gas over long distances is highly effective with this technology, particularly between countries, and it accounts for about 25% of the world's gas transportation. Since LNG is primarily composed of liquid methane, it presents a viable option for storage and transportation. In most cases, the development of LNG involves large financial investments in liquefaction plants as well as LNG carriers. As a result, this technology presents significant challenges to gas management, in particular in remote stranded gas fields. Due to this, the economic feasibility of developing remote offshore gas by way of LNG is influenced by the cost of transporting the gas to shore.

Fig. 2.2 illustrates the entire process involved in producing LNG from raw natural gas. It begins with the initial processing of natural gas to remove impurities. Thereafter, further purification is accomplished using refrigeration and distillation. The liquefaction process involves the conversion of natural gas into liquid before any other nitrogen compounds/oxides are removed via the nitrogen rejection process. The final product is pure LNG for domestic and international markets.

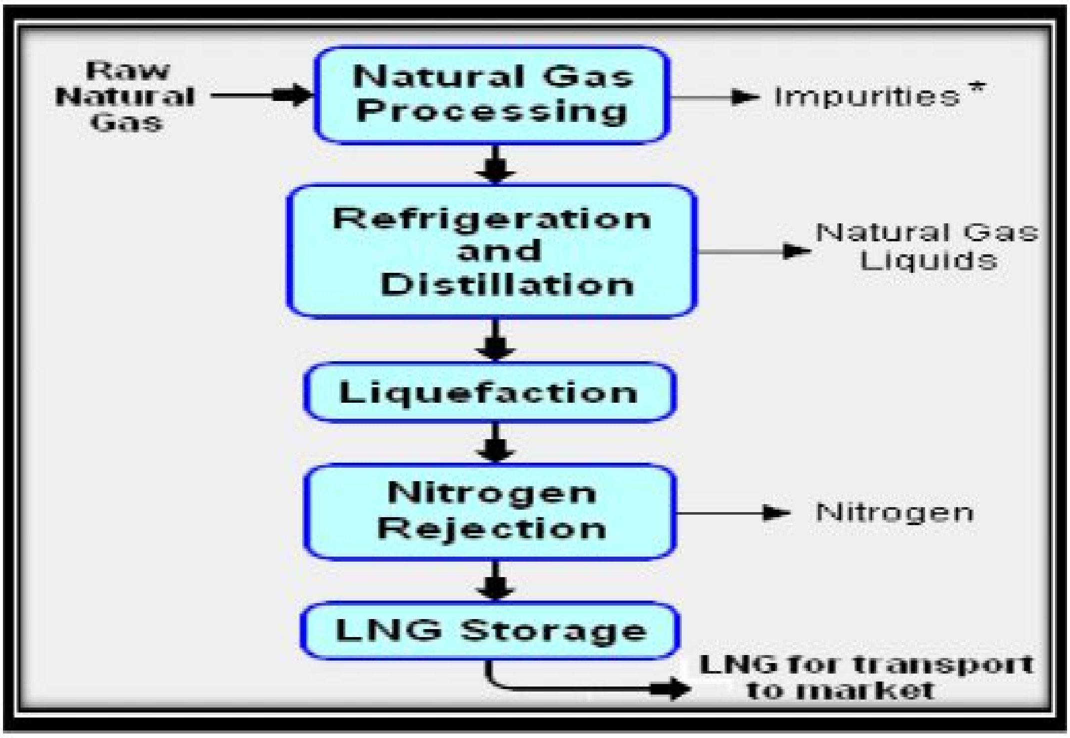

Figure 2.2: Flowchart for production of LNG (Source: Indriani (2005))

2.3.1.3 Gas re-injection process

Natural gas re-injection involves the injection of gas back into an underground reservoir, typically one which already contains each oil and gas. By injecting gas back into the reservoir, the pressure within the reservoir is increased, thereby inducing crude oil flow. A very high pressure of 70 000 kPa is normally required for this process (Jahn et al., 2008). Re-injection of gas is commonly employed in cases where there is no market outlet for the gas. It is possible to inject associated gas that cannot be flared into reservoirs in order to maintain reservoir pressure and increase recovery. There will be an initial liquid separation, and due to the high pressure, it is crucial to dehydrate the gas first. In light of the fact that gas pressures in compressors are much higher than those in gas pipelines, it is imperative that lubricants used in them do not dissolve in high-pressure gas. The Sanha Condensate Project in Angola is an example of a gas re-injection process. By gathering, processing, and re-injecting associated gas, the project is intended to eliminate flaring from existing platforms and to increase oil production.

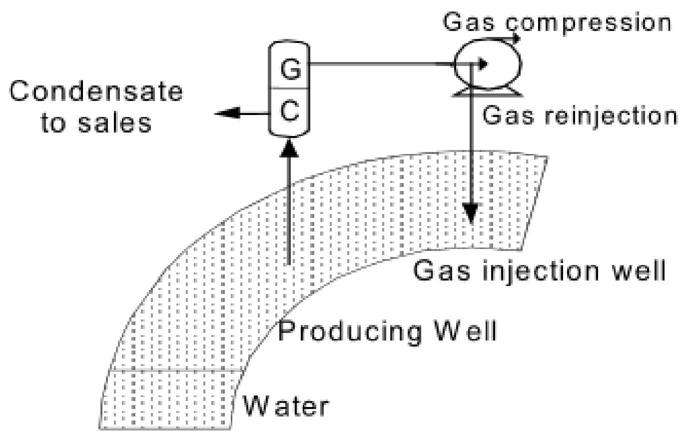

Figure 2.3: Schematic of gas re-injection process (Source: Indriani (2005))

2.3.2 Lean production as a potential mitigation strategy

In the preceding section, recommendations for strategies and solutions to gas flaring were addressed from a technological perspective; however, one major obstacle to the adoption of some technology solutions is the considerable capital expenditure required. Developing countries that produce oil are at a greater disadvantage in this regard. Thus, there is a possibility of adopting an alternative approach to address the persistent problem of gas flaring. In this context, this section introduces the lean concept as an effective managerial approach that aims to minimize waste (in the form of flare gas) while improving the overall productivity of the industry. Hence, this section will provide an overview of lean production as well as a review of related works.

Lean production, which was developed by Toyota Production System (TPS) in the automotive sector, proposes that waste (i.e., '*muda*' in Japanese) can be reduced or eliminated in a work system to increase productivity, profitability, and efficiency. During the past four decades, lean production has become ingrained in many manufacturing companies in the United States (Shah & Ward, 2007). In order to achieve lean operations, considerably emphasis is placed on minimizing the volume of resources utilized while eliminating all waste or non-value-added activities in an organization. In an increasingly competitive environment, lean production provides a competitive advantage. It has even been suggested that lean production may be the dominant manufacturing strategy of the 21st century. Lean production can generate a number of benefits, including, but not limited to, increased customer satisfaction, reduced inventory levels and costs, better utilisation of equipment, and reduced idle time. Other industries which have successfully adopted lean production techniques include healthcare, aviation, construction, cement, textiles, pharmaceutical, and electrical. Thus, it is crucial to give some insight into the extent to which lean tools are being adopted in these industries.

Masmali (2021) implemented Value Stream Mapping (VSM) as a lean framework at a cement company. In the initial investigation, it became apparent that excess inventory between workstations was a source of waste that needed to be reduced, prevented, or eliminated. Also, the Kanban system and CONWIP approach were utilized for inventory control and work-in-process, respectively. According to the main results, non-value time was reduced from 23 days to 4 days in the Kanban approach and 2 days in the CONWIP approach, respectively. It was thus recommended that the company adopt the CONWIP approach. Tortorella et al. (2021) conducted a study that identified potential paths for the implementation of lean automation. By collecting and analysing data based on multivariate techniques, their findings led to three hybrid configurations of lean practices and industry 4.0 (I4.0) technologies. Research findings indicate that companies with higher performance are deploying a more extensive or robust combination of lean practices and I4.0 technologies. During the implementation process of lean manufacturing, Sadiq et al. (2021) incorporated blue ocean manufacturing techniques to achieve manufacturing excellence. After applying both methods to the production/assembly line of an automotive spare parts supplier, it was determined that the reductions in lead-time, value-added time, and GHG emissions were 26%, 39%, and more than 50%, respectively.

According to Agyeman (2021), the implementation of lean principles within the Boeing production system has influenced the production of Boeing aircraft. It is based on prior research that looked at Boeing's pursuit of the lean management approach over the course of many decades and the significant benefits realized across the organization. Based on findings in the study, Boeing showed a robust approach to managing the company's value stream by putting an emphasis on customer demands, industry standards, and novel ways to improve system efficiency. Hao et al. (2021) conducted a study based on the complementarity theory to investigate the effects of lean techniques and servitization of manufacturing on sustainability practice across three different dimensions of practice. Findings indicate that the overall impact of lean practices on sustainability is not always satisfactory. However, the combination of servitization and lean practices led to significant improvements in operational sustainability.

Based upon the use of SciMAT and VOSviewer tools, Furstenau et al. (2021) have provided an in-depth bibliometric analysis of lean production over a 42 year period. According to the study, a total of 4412 articles were reviewed, based on the analysis of various indices, including the overall performance of the publications, the reputation of researchers in the field, and the ranking of universities and their regions. Results indicate that the most relevant themes of the study are related to manufacturing, managerial approaches, and human resources. Galeazzo (2021) conducted a study to investigate the extent to which a firm will be able to adhere to lean production through the application of lean techniques. Furthermore, a hypothesis was tested to see if lean maturity would have any significant impact on the financial performance of firms. Research findings indicate that lean maturity is positively correlated with financial performance and that there is a causal relationship between leanness and financial performance. In a production system characterized by lean production and industry 4.0, Mrugalska and Wyrwicka (2017) studied the coexistence of these concepts. Based on this investigation, they concluded that both approaches can be interconnected without compromising each other. Uzochukwu and Ossai (2016) advocated cellular production as a lean tool for improving overall performance of oil firms in Nigeria. Using the Pearson product moment correlation, their study revealed that there is a positive correlation between cellular production and service delivery.

The petroleum industry, in spite of the lean concept having been adopted by many other industries, has been very slow to adopt and implement it. There are several reasons for this. There has been a lack of sustainability in the petroleum industry from its inception, especially in regards to the practice of waste minimization (Ratnayake & Chaudry, 2017). There are many redundant operations, both onshore and offshore, that result in wasteful activities, which eventually lead to a decline in overall efficiency and productivity (Santamarta et al., 2016). Although this research emphasizes flare gas as a significant waste stream in the industry, there are other hazardous wastes that are generated during exploration and production activities. These include produced waters, drilling fluids, and other associated wastes. Considering that waste minimization is at the core of lean production, it is obvious that the persistent generation of these wastes is an essential justification for adopting and implementing lean practices.

It is also worth noting that the fall in oil prices during 2014 led to a decline in the profit margins of the top oil exploration and production companies (E&P) (Santamarta et al., 2016). Due to the 2014 oil price crash, some E&P companies were forced to reduce their workforces and suspend or cancel a number of scheduled projects. As a result, these actions negatively impacted the upstream sector, resulting in further declines in revenue and profitability. Recent events, culminating most recently in an oil price war and the devastating impact of COVID-19, made it impossible to maintain oil prices at their current levels and caused an unprecedented economic crisis within the industry. Besides posing an environmental risk, gas flaring remains a tremendous waste source, which further degrades the industry's efficiency. Oil price volatility along with the negative impact of gas flaring have shifted the dynamic of the industry in such a way that production managers are forced to consider better strategies other than cost minimization. Under low oil prices, it is imperative that the industry secure its competitiveness by increasing productivity, optimizing resource utilization, and reducing waste. Rats et al. (2015) have suggested that, to improve the overall performance of the industry, the lean concept can be an effective strategy. It thus stands to reason that one of the driving forces behind this research is the need for the petroleum industry to implement lean production practices.

2.4 DEA as a performance evaluation tool

The topic of performance evaluation is one that is of paramount concern to most managers as it can serve as a foundation for decisions made about the allocation of resources, such as budgets and bonus schemes. In conventional terms, performance can be defined both as the inputs and outputs of an organization, or as the relationship between them, usually referred to as efficiency. Across a variety of academic disciplines, the terms 'efficiency' and 'productivity' have been used interchangeably. In essence, both terms are used in the same context, but each has slightly different meanings. Efficiency is a technical measure of the performance of a firm or an organization. In addition, the term can also be defined as the ability of a firm to ensure a low level of resource consumption to achieve a maximum level of output, and this also implies a case of lean production. This is practically possible when, for instance, a firm employs the right people and machine to perform the task correctly (Kvadsheim & Wasamba, 2014). Conversely, the productivity of an inimitable unit is the ratio of its output to the input necessary to produce its output (Haksever & Render, 2013). In general, evaluation characteristics have multidimensional attributes, therefore they cannot be aggregated in a proper way. Thus, it can be said that the main issue of performance measurement is determining the relative efficiency of each business unit within an organization (Tien-Hui, 2011).

Data envelopment analysis is a useful tool for assessing performance and has been used several times in the literature for evaluating the efficiency of decision-making units (DMUs). When discussing a DMU here, we are referring to a producer or service provider. With its nonparametric nature, DEA is an ideal benchmark for relative comparisons of units and can estimate production frontiers. It has also been demonstrated to be an excellent linear programming tool because it can find ways to improve efficiency not apparent in other techniques, particularly when dealing with multiple inputs and multiple outputs. The purpose of this section is to provide a comprehensive review of applications of DEA models for environmental efficiency, waste management and/or reduction, pollution control, and water treatment.

Sueyoshi and Wang (2014) used DEA to evaluate the unified performance of selected petroleum firms in the United States. Based on their results, it was determined that integrated petroleum firms were more efficient than their independent counterparts. Molinos-Senante et al. (2014) conducted the first study to evaluate the efficiency of sixty water treatment plants throughout Spain by incorporating environmental impacts. The benchmarks they identified in their study were the best treatment plants, and the potential reductions of GHG emissions were computed for each plant. Wu et al. (2016c) used DEA to evaluate two subsystems of the Chinese transportation network. By applying a decomposition approach to their evaluation, they were able to determine the maximum efficiency of each subsystem across 30 selected regions in Chinese provinces. Upon detailed analysis, it was found that most regions had low subsystem and overall efficiencies, leading to recommendations for a balance between the largest Chinese territories. Riccardi et al. (2012) used DEA to evaluate the global cement industry considering CO_2 as an undesirable output. According to their findings, countries using unconventional feedstocks have higher efficiency. Wu et al. (2016b) conducted an in-depth examination of a hybrid treatment system using the DEA model, and the results closely reflect the environmental conditions in the eight regions examined. Through a proposed DEA model, Moutinho et al. (2017) assessed the relative impacts of income and other relevant indicators on selected European nations.

Khanna and Kumar (2011) applied the directional distance DEA method to evaluate the ability of environmental management systems (EMS) to decrease toxic emissions and increase environmental efficiency. Twenty nine administrative regions of China were evaluated between 2000 and 2008 using the DEA methodology in order to determine their energy and environmental efficiency (Wang et al., 2013). The DEA and Malmquist productivity index (MPI) were used to measure OECD countries' energy and environmental efficiency (Mavi & Mavi, 2019). Lee et al. (2014) investigated the environmental performance of major port cities in the world using the slacks-based DEA model while taking emissions (i.e., sulphur dioxide, NO_2, and CO_2) into account as bad outputs. Yang et al. (2020) proposed the zero-sum gains DEA model as a template for calculating the optimal CO2 reductions for China's provinces in 2030. In consideration of the technological gap between the cities and the heterogeneity of energy use in

each, DEA models have been used to analyse the reduction of emissions and energy efficiency in selected Chinese cities (Sun et al., 2018). A non-radial DEA model with undesirable outputs was developed to measure the level of environmental efficiency over an eight-year period in EU member states (Vlontzos et al., 2014). An analysis of the environmental performance of dairy farms in four regions of Ireland has been done using DEA and linear regression. There were significant differences in environmental performance among all regions, with Northern Ireland having significantly higher nutrient surpluses than other regions (Adenuga et al., 2018).
A hybrid approach, combining both DEA and stochastic frontier analysis, was proposed by Hoang and Nguyen (2013) to analyse variation in material-based environmental efficiency of selected rice farms in South Korea.

Liu et al. (2017) applied parallel DEA models that consider CO_2 emissions as undesirable outputs to assess the levels of efficiency in the transportation subsectors in China. Chang et al. (2014) developed an SBM-DEA model to assess the environmental performance of 27 global airlines. Asia's airlines are generally more efficient than those in Europe and America, according to the results. L. Chen and Jia (2017) integrated DEA and big data theory to analyse the environmental performance of Chinese provinces over a four-year period. The results indicated that China's industrial sector is inefficient at reducing environmental impact. The SBM DEA model presented by Song et al. (2014) calculated desirable and undesirable outputs separately, effectively resolving the dependency problem of desirable and undesirable outputs.

Halkos and Polemis (2018) proposed a novel power generation efficiency calculation technique for the United States, using window data envelopment analysis (W-DEA). According to the model, the relationship between environmental efficiency and regional economic growth is N-shaped when it comes to global or total pollution. However, the model predicts an inverted N-shaped curve for local pollutants. Geng et al. (2021) integrated the total factor productivity method with the slack-based DEA model to assess the inherent energy structure of 24 countries over an 11-year period.

2.4.1 The perennial problem of efficiency ranking in DEA.

Although DEA methodologies are highly capable of evaluating efficiency, they occasionally fail to separate the computed efficiency of units into well-defined ranks, and this seems to be a recurring issue. The problem of ranking has been dealt with in several works, but not without controversy. Presented here is an overview of the prior ranking methods in the literature that addressed this issue. The cross-efficiency method of ranking is credited to Sexton et al. (1986) who developed a matrix that is used to evaluate each unit two times with regard to its own self-efficiency and its relative efficiency. A unit's self-efficiency is represented by the diagonal of the matrix. As part of this method, an analyst compares the efficiency scores of a selected DMU with those of other homogeneous DMUs n times. There have been numerous developments and demonstrations of this technique (Contreras, 2012; Ramón et al., 2011; Rödder & Reucher, 2011; Wu et al., 2015; Wu et al., 2016a).

DEA and AHP techniques were combined by An et al. (2018) for ranking DMUs based on their cross-efficiency. As part of the super efficiency method, a particular unit is removed and evaluated alongside other units. Several authors have demonstrated the use of this technique (Amirteimoori et al., 2005; Andersen & Petersen, 1993; Balf et al., 2012; Y. Chen et al., 2013; Jahanshahloo et al., 2004; Jahanshahloo et al., 2006; Mehrabian et al., 1999) with further developments. A robust methodology, developed by (Oukil, 2018), combines three techniques for ranking efficient units without bias. Recently, Soleimani-Chamkhorami et al. (2020) developed a novel inverse DEA for ranking units according to their growth potential.

2.5 Inverse Optimization

In linear programming, the standard procedure is to find an optimal solution in the forward direction with respect to a set of well-defined parameters. Generally, the optimal solution is regarded as one that is consistent with the given objective function. Conversely, when some of the parameters of a model are unknown or when a prediction is desired given an already determined optimal solution, a classic case of an inverse problem is presented in reverse order. This is the basis of inverse optimization, which is also called reverse optimization.

Inverse optimization traces its historical roots back to the works of geophysical scientists who attempted to estimate unknown parameters or parameters that, at the time, were impossible to calculate, such as the radius of the earth's core. For example, Tarantola (1987) described the following inverse problem in the geophysical sciences:

"Imagine a physical system denoted by P. Assume there is a set of model parameters Q used to describe the system P. It is possible that there is some element $q \in Q$ that cannot be measured directly such as the radius of the earth's core. A situation such as this might require the definition of another set R of observable parameters that depends on the real value of q. Based on an understanding of the model parameters, predicting the values of the specified observable parameter(s) $r \in R$ is necessary to solve the forward problem. In contrast, the reverse or inverse problem is solved by generating values for model parameters based on predetermined values for the observable parameters".

The observable parameters in the above example correspond to the optimal decision variables given the values of the model parameters, such as the cost coefficients and the right-hand side vector. Still within the realm of geophysical sciences, inverse problems are further demonstrated in the works of Neumann-Denzau and Behrens (1984), Nolet (1987), and Woodhouse and Dziewonski (1984). Predicting the movements of earthquakes is an important application in this area. Modelling earthquake movement can be achieved by considering a network formed by discretizing a geologic zone into several square cells. Each point in the network corresponds to an adjacent cell, and these points are connected by arcs. As arcs are shown as time intervals between earthquake waves, the cost of an arc is determined by the transmission time of particular seismic waves which is not such an accurate measurement. At various observation stations, the arrival times of earthquakes and resulting earthquake perturbations are then observed in relation to earthquakes. Consider the assumption that earthquakes travel along the shortest path between cells. In light of this, geologists face the challenge of re-establishing transmission times between cells using the shortest time waves and prior historical information of the region being investigated. This challenge poses an inverse shortest path problem (Ahuja & Orlin, 2001).

Among mathematicians, the papers by Burton and Toint (1992); (Burton & Toint, 1994) generated a great deal of interest in inverse optimization problems. This led to a variety of inverse optimization problems studied by researchers in the operations research community over the past few years (Ahuja & Orlin, 2001). Other interesting applications of inverse optimization include medical imaging and traffic optimization.

As inverse optimization is a broad field with a variety of real-world inverse problems, it is important to define the nature of inverse problems for this study. In this connection, we provide the following example of a classic inverse problem as part of the DEA methodology for this study:

"Consider a production work system with x and y as its input and output parameters, respectively. Let θ_1 denote the optimal solution of the system at time $t = 1$. Suppose at time $t = 2$, the output parameter, y, changes to $y \pm \Delta y$, to give an optimal solution, θ_2. If Δy is known, we want to determine the corresponding change in input parameter, Δx, that will give a new input, $x \pm \Delta x$, such that $\theta_1 = \theta_2$ (i.e., the optimal solution of the system is retained). This summarizes the evaluation mechanism of the inverse data envelopment analysis (DEA) model which belongs to the family of inverse optimization techniques".

2.5.1 Inverse DEA as an advanced management tool

As an inverse optimization process, inverse DEA is the reverse implementation of conventional DEA. As opposed to DEA, inverse DEA determines the optimal variations of input and output data based upon a predetermined efficiency score of a DMU. In addition, the inverse DEA model provides a greater degree of flexibility. With the advent of practical inverse problems in various work systems, the conventional DEA needed an extension, thereby resulting in the inverse DEA. In the current state of research, there are few works examining the application of inverse DEA to real-world problems. Considering its potential capabilities as an optimization technique, its application to other work systems is of great interest. This notwithstanding, the literature continues to offer some interesting applications of inverse DEA that should be reviewed in this section.

The inverse DEA was first proposed and developed by Wei et al. (2000) in order to address inverse problems, such as: "*Given a group of production firms, suppose we decide to maintain the efficiency of each firm while increasing some inputs, what will be the corresponding increase in outputs?"* As well, the reverse is an inverse problem (i.e., *to estimate the corresponding increase in inputs due to a specified increase in outputs*). They demonstrated the potential of their proposed inverse DEA model using practical examples. The development was closely followed by Yan et al. (2002), who then modified the inverse DEA to allocate resources as efficiently as possible.

Recent years have seen the emergence of new inverse DEA models developed using directional distance DEA, such as the study by Gattoufi et al. (2014), which proposed a novel inverse DEA for bank mergers. Specifically, their proposed model addressed the issue of merging competing banks into a single entity to achieve a significantly improved level of efficiency. The study utilized 42 banks as a case study to demonstrate the practical application of the model, with promising results overall. Lim (2016) proposed an inverse DEA model for use with time series that includes frontier changes. The application of the model to a case study concerning engine development provided valuable insight into how to make optimal decisions regarding the setting of product targets. Amin et al. (2017a) developed an inverse DEA for determining whether formed mergers in a particular industry represent a major or minor consolidation. Specifically, a major consolidation is a combination of at least two decision-making units, where the combined unit perturbs the efficiency frontier relative to a specified set of market or industry standards. Alternatively, if the efficiency frontier shows no perturbation after the formation of a merger or mergers, it is a minor consolidation. Furthermore, all the criteria necessary for a decision-maker to distinguish between the two consolidations were demonstrated using mathematical theorems and real-world applications from the banking industry.

An inverse DEA model has been developed by Amin et al. (2017b) to help managers plan for the possible scenarios that can occur when a number of homogenous units decide to restructure in order to increase their overall efficiency. The model developed was referred to as the generalized inverse DEA because it assists the pre-structuring units to achieve the efficiency target after the entire restructuring phase has been completed. To validate the findings of their study, an application to the banking industry was presented, and overall, the results were satisfactory, illustrating the effectiveness of the inverse DEA as a restructuring tool. Ghiyasi (2017a) developed inverse DEA models for the short, mid, and long-term planning of 19 Iranian provinces based on scenarios that involved pollution-generating and better technologies. CO_2 was the undesirable output that required reduction, whereas GDP was the only desirable output. The results indicated that overall input capital savings were as high as 150%, with the fifth and eighth provinces being the most efficient with respect to all inputs. In addition, with an improved technology, CO_2 reduction required extra inputs to increase desirable output and decrease undesirable output. Ghiyasi (2017b) developed inverse DEA models that can be applied with price data to measure the efficiency of decision-making units regarding revenues and costs. Using real-world examples, the developed models were able to maintain both types of efficiency. Essentially, estimates of input changes were determined in response to perturbations caused by changes in output, while maintaining levels of efficiency for the units.

Hassanzadeh et al. (2018) employed an inverse DEA based on semi-oriented radial measures (SORM) in both input and output formats to allocate resources and develop investment strategies across different European countries. Both positive and negative data can be handled by the developed SORM InvDEA models. To evaluate the sustainability of each country, the outputs and efficiency scores were maintained while determining the optimal changes in input for the input orientation format. In contrast, for the output orientation, optimal changes in output were determined while maintaining the inputs and efficiency of each country. Overall, their findings suggest that their inverse DEA can be an effective sustainability tool. Amin and Al-Muharrami (2018) proposed a new inverse DEA model for merging units when negative data is available, in which the merger has the ability to determine the required inputs and outputs from the initial units to achieve a specific level of efficiency. Emrouznejad et al. (2019) conducted the first study examining the reduction of CO_2 emissions in China with the aid of the inverse DEA method. They accomplished this in their study by developing a robust inverse DEA model for assigning each Chinese region its optimal share of the total emissions in the country. In the first instance, data on the reductions of CO_2 emissions reported by the Chinese government was obtained. Then they applied their developed model to allocate the reductions to several manufacturing industries. The model was also capable of assigning further reductions to the provinces from the industries. Validation of the model was conducted using data from Chinese industries. Wegener and Amin (2019) developed a novel inverse DEA model that addressed the issue of persistent GHG emissions across a set of 23 oil firms in North America. The developed model determines, specifically, for a group of homogenous firms, how much can be increased in the production of the desirable outputs while generating little or no GHG emissions from the group. Initial findings show that 57% of the firms were inefficient, indicating that there is potential to reduce GHG emissions at each of these firms. Their study concluded with recommendations for potential reductions in GHG emissions through effective regulations.

Ghiyasi (2019) developed novel inverse DEA models as a tool to investigate DMUs under perturbation. Comparative analysis was performed to validate the newly developed models against the existing ones. New models outperformed existing ones, solving some persistent difficulties that are characteristic of the existing models. Guijarro et al. (2020) developed a new technique incorporating genetic algorithms and inverse DEA for the purpose of achieving specified levels of efficiency in mergers. With the aid of several assumptions and scenarios, their approach attempts to assess a global efficiency score for mergers with a lesser focus on minimizing input resources. The proposed methodology involves calculating potential input resource savings through inverse DEA, while a genetic algorithm is used to solve the combinatorial issue of mergers. Ghiyasi and Zhu (2020) presented an inverse DEA that considers broader returns-to-scale properties when dealing with negative data. Based on the application of the developed model to a set of China's banks, it was found to be feasible when a combination of positive and negative data was used. In addition, the results were also satisfactory in comparison with the real world.

2.6 Research Gaps

Based on the literature review above, the following research gaps have been identified:

1. There have been several studies ranging from the estimation of gas flaring volumes (Elvidge et al., 2018; Elvidge et al., 2009; Rotty, 1974) to extensive descriptive analyses of GHG emissions from gas flaring (Anomohanran, 2012; Conrad & Johnson, 2017; Giwa et al., 2014; Giwa et al., 2017). Other studies have also shed light on the use of satellite imagery (Elvidge et al., 2009), the use of material balance equations (Ismail & Umukoro, 2012), and other techniques (Anejionu et al., 2015; Casadio et al., 2012; Elvidge et al., 2015; Umukoro & Ismail, 2017) to monitor global gas flaring. Despite this, none of the studies addressed the reduction of global gas flaring in the industry. However, it is important to emphasize that Ojijiagwo et al. (2016) proposed a method to reduce gas flaring, the study remained primarily a qualitative investigation based on semi-structured interviews with field experts. A further limitation is that the study only covered one nation (Nigeria). Therefore, there have been no studies in the literature that have developed a quantitative method for estimating reductions in gas flaring at a global level. As such, this study fills a significant gap in the literature.
2. Regarding mitigation strategies for gas flaring, some studies have utilized cost-benefit analyses as the basis or main evaluation tool of their methodologies (Davoudi et al., 2013; Interlenghi et al., 2019; Ojijiagwo et al., 2016; Rahimpour et al., 2012). In these studies, the final results are usually more or less the estimated rate of return, from which conclusions can be drawn, however, none of them consider the availability of gas for utilization. Based on information from the World Bank archives about routine gas flaring, the availability of gas for use is determined by the difference between the total volume of flared gas and the volume of non-routine flaring. In the context of offshore operations, non-routine flaring is the occurrence of safety and maintenance flaring to minimize or prevent risks to personnel. It is undeniable that this information is crucial to the implementation of all gas flaring mitigation strategies; however, no study has ever considered the effects of non-routine flaring in determining the amount of recoverable gas. Thus, this study fills a critical gap in the literature by integrating a mathematical theorem that is applicable to all forms of non-routine flaring in the industry.

3. Further, technological solutions to gas flaring such as GTL, GTW, and LNG are very expensive or require huge capital. Consequently, the adoption of these technologies in developing nations is considerably more difficult. In a developing nation like Nigeria, for instance, when the currency is devalued often, the capital expenditure for a technology such as GTW will be higher. Thus, there exists another gap in the literature pertaining to the design of a cost-effective solution scheme for the deployment of GTW technology.
4. The advent of lean production by Toyota Production System (TPS) has led to the adoption of lean practices by many manufacturing companies throughout the world. There is also increasing interest among researchers in the application of lean tools to several industries (Agyeman, 2021; Hao et al., 2021; Masmali, 2021; Sadiq et al., 2021; Tortorella et al., 2021). Since lean production places a great emphasis on waste reduction, and that flare gas is hazardous waste, it is rather surprising that no study in the literature has attempted to apply the principles of lean to the issue of gas flaring. Aside from minimizing gas flaring, lean practices can also improve oil production in the industry with the minimum use of resources. In this context, no study has examined the application of any lean tool to operations management in the petroleum industry. This leaves another gap in the literature.
5. One major limitation of all the studies reviewed on gas flaring is that none of them have been able to determine the maximum amount of energy that can be generated from flare gas. Information such as this is highly relevant for policymakers who are seeking to reduce the frequency of blackouts and power outages in some oil-producing nations. Additionally, it facilitates the development of an optimal energy mix of gas power generation and renewable energy in developing nations. Another void in the literature is filled in this study by developing a separate model for such a purpose.

To bridge the identified research gaps, five models are proposed, and their corresponding computational experiments are conducted in this research work. This first model is a novel inverse DEA, which is the most substantial and robust model since it enables the first three research gaps to be filled. Based on the proposed inverse DEA, a mathematical theorem is presented that deals with non-routine flaring in the industry, leading to the emergence of a new algorithm as a policy-making tool, and the resulting development of a new formula for the optimal sizing of the gas-to-wire (GTW) process. In response to the third research gap, which concerns the design of a sustainable lean production framework (SLPF) for the petroleum industry, the second, third, and fourth models are proposed.

A fifth model influenced by both positive and negative data is formulated for bridging the last research gap. Chapters 3-5 provide detailed descriptions, formulations, and solutions available for each model.

Chapter 3 – A Novel Inverse DEA for Estimating Global Gas Flaring Reductions

3.1 Introduction

An overview of routine gas flaring has been presented in the previous chapters. However, before addressing the problem from a managerial perspective, it is important to shed more light on the main causes/reasons for gas flaring in the industry, policies on gas flaring, the current energy situation in Nigeria (i.e., the major case study for this research), as well as further details of the chosen technology, GTW. GTW technology is an ideal choice for this study since it is the only mitigation strategy that converts waste or flare gas into electricity, which contributes to reducing energy poverty in oil-producing nations such as Nigeria and Venezuela.

Gas flaring is usually associated with crude oil extraction. This environmental hazard has arisen because of decades of dependence on crude oil as the engine of global civilization. An increase in oil production due to an increase in oil demand typically causes an increase in flare gas, which illustrates the domino effect in the petroleum industry. Although this domino effect may be deemed as an external factor, it should be noted that there may also be internal factors or underlying causes of gas flaring within the industry. Globally, the flaring of gas is estimated to result in the loss of billions of dollars each year, which could have served as potential revenue for the global economy, and all major oil-producing nations contribute to this loss. Aregbe (2017), for instance, claims that Nigeria flared approximately 12,602,480 cubic feet of gas over a period of 15 years (i.e., 1996 to 2010). This is equivalent to approximately 12,967 × 10^{12} BTU of energy. It is estimated that hundreds of thousands of Nigerian households could benefit from this amount of energy.

3.2 Causes of gas flaring.

The raw form of natural gas is associated petroleum gas (APG) because large quantities are naturally dissolved in crude oil. To extract crude oil, APG must be separated using three different methods. The two common separation methods in developed oil-producing nations are gas reinjection and the conversion of excess gas into electricity for use in oilfields. However, developing nations rely significantly more on the burning of excess gas as it is the cheapest and fastest method of separation. The first two methods are expensive, or they require significant capital investments which developing countries cannot afford. Another cause of gas flaring is the inevitable occurrence of safety and maintenance flaring, which experts claim helps reduce operational risks during offshore operations. Nevertheless, many oil-producing nations still take advantage of this circumstance by continuously flaring beyond the levels permitted by law. Accordingly, routine flaring is intentional and/or beyond safety standards and not to be confused with any form of non-routine flaring. Despite this, there should be a concerted effort to reduce all non-routine flaring.

3.3 World Bank policies on gas flaring

The World Bank launched the Global Gas Flaring Reduction partnership (GGFR) in August 2002 at a summit held in Johannesburg as a means of promoting a global reduction in gas flaring. As a result of climate change, the GGFR was created to serve as a regulatory policy for governments of oil-producing nations to reduce gas flaring over time. According to World Bank archives, over 140 billion cubic meters of natural gas are flared annually, containing more energy than the continent of Africa consumes each year (750kWh). Along with the GGFR, the World Bank and United Nations launched the "Zero Routine Flaring (ZRF) by 2030" initiative as a new industry benchmark to assist in eliminating all forms of routine flaring by 2030. United States-based Occidental was the first oil firm to endorse the ZRF initiative, while in the Middle East, Saudi Arabia's Aramco endorsed the initiative in late 2019. Furthermore, the IEA annual report for 2019 has encouraged the development of novel techniques for offshore practices. These techniques would help in conserving or utilizing excess APG in compliance with the ZRF initiative. Despite these laudable efforts, there was a noticeable rise in gas flares around the world during 2018. This can be attributed to political tensions in the Middle East and Venezuela (Bamji, 2019). Another contributing factor was the increase in shale oil production, which made the U.S. the largest oil producer in the world in 2018.

According to the PwC 2019 report, gas flaring cost the global economy approximately 20 billion U.S. dollars in 2018. Specifically, Nigeria lost US$761.6 million due to gas flaring in the same year (i.e., approximately 3.8% of the total loss). In Fig. 3.1 we can see that Nigeria ranks ninth in the world in terms of gas reserves but is number one on the African continent. However, despite the country's abundant gas reserves along with high volumes of flare gas, Nigeria still faces issues with energy shortages to this day. This can be characterized as a classic case of an energy paradox.

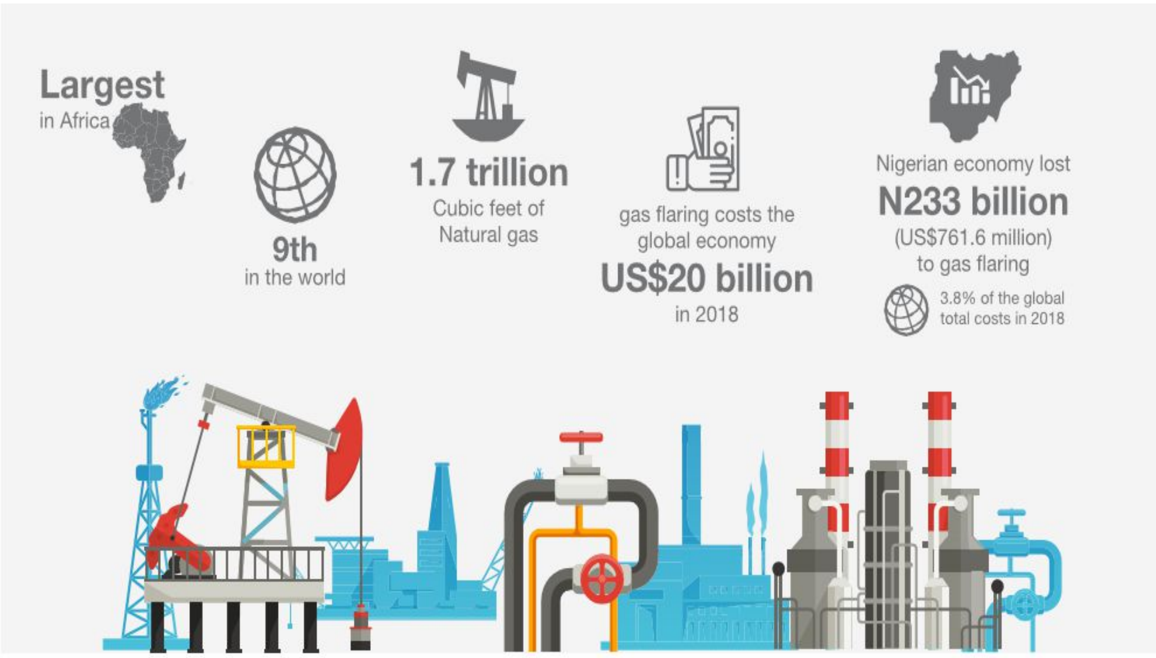

Figure 3.1: Global economic losses resulting from gas flaring in 2018 (Source: PwC (2019))

3.4 The Gas-to-Wire (GTW) process

The gas-to-wire (GTW) process converts natural gas into electricity by means of simple or combined cycle turbines. It is common for the conversion process to involve integration or coupling with a Carbon Capture System (CCS) to further reduce any residual carbon compounds or oxides. In many cases, this type of integration is referred to as a hybrid system, which is more effective in reducing gas flaring. Fig. 3.2 illustrates the mechanism of the hybrid structure, which begins by processing the gas from the wells. A turbine power plant converts the processed gas into electricity, which is then distributed to end-users or the final market. During the conversion process, a small fraction of gas is burned to produce harmful carbon by-products, referred to as exhaust gas or flue gas. The flue gas contains a high carbon content (i.e., CO_2), and this is the point where the CCS would be useful in capturing and injecting this CO_2 back into underground reservoirs to yield more crude oil. In addition, a significant amount of processed gas is purified and dispatched to different segments of the final market based on the type of demand.

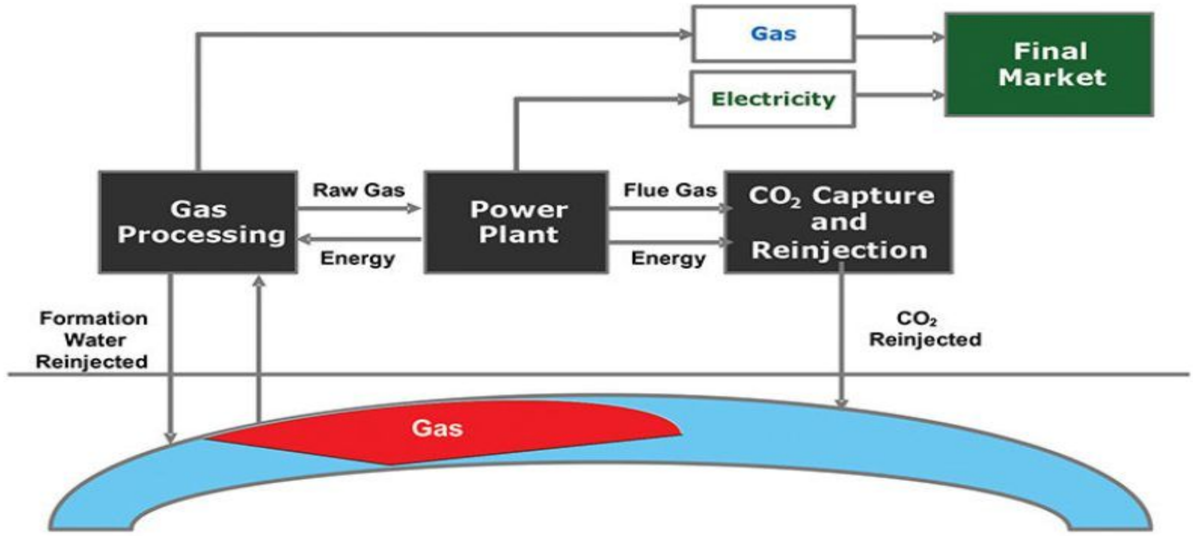

Figure 3.2: Power generation using a hybrid GTW and CCS system (Source: JPT (2018))

In spite of its effectiveness as a mitigation strategy, GTW technology is quite costly. Ojijiagwo et al. (2016) provided information about capital expenditures required for the installation and use of the technology in Nigeria. According to their study, GTW can be adopted and implemented in Nigeria with a 16.3% ROI and a 6-year payback period. A study by Interlenghi et al. (2019) also demonstrated a 20% return on investment for investing in GTW technology. However, both of these studies fail to consider the availability of gas for the GTW process in relation to safety and maintenance flaring. For real-life scenarios, it is highly likely that estimates of the actual volume of gas used for both studies may not be adequate or might be excessive for the GTW process. Further, gas turbines have a moderate demand around the world due to their high capital cost. For example, the popular GT13E2 turbine manufactured by General Electric (GE) has undergone several modifications and improvements in terms of its thermal efficiency and power rating. In spite of this, demand for the GT13E2 is still average at best, and as of 2011, only 146 units had been installed globally for power generation. Thus, a decision-maker is faced with uncertainty when assessing the optimal number of turbine units for the GTW process based on the availability of gas. This can be resolved by calculating the minimum and maximum turbine units necessary for the GTW process. In this way, an investor will be aware of the prudent practice of not investing beyond the maximum number of turbines in order to prevent financial losses. As a practical matter, a decision-maker is responsible for determining the optimal sizing of a cost-effective GTW process, not only for a single oil producer but also for a group of homogenous oil producers.

3.5 The Nigerian energy statistics

At present, all thermal and hydropower plants in Nigeria have a combined installed capacity of 12522 MW. Over the course of time, poor maintenance and ineffective management have led to a loss of around 43%, leaving 7141MW as the utilizable capacity for producing electricity. Other challenges, such as power losses and the uncertain volume of natural gas, reduce capacity to 3879 MW (refer to Fig. 3.3). In addition to political factors, this loss of combined power capacity is one of the major causes of frequent power outages in the country. Consequently, it is anticipated that the implementation of a cost-effective GTW system will compensate for the combined loss in capacity for Nigeria.

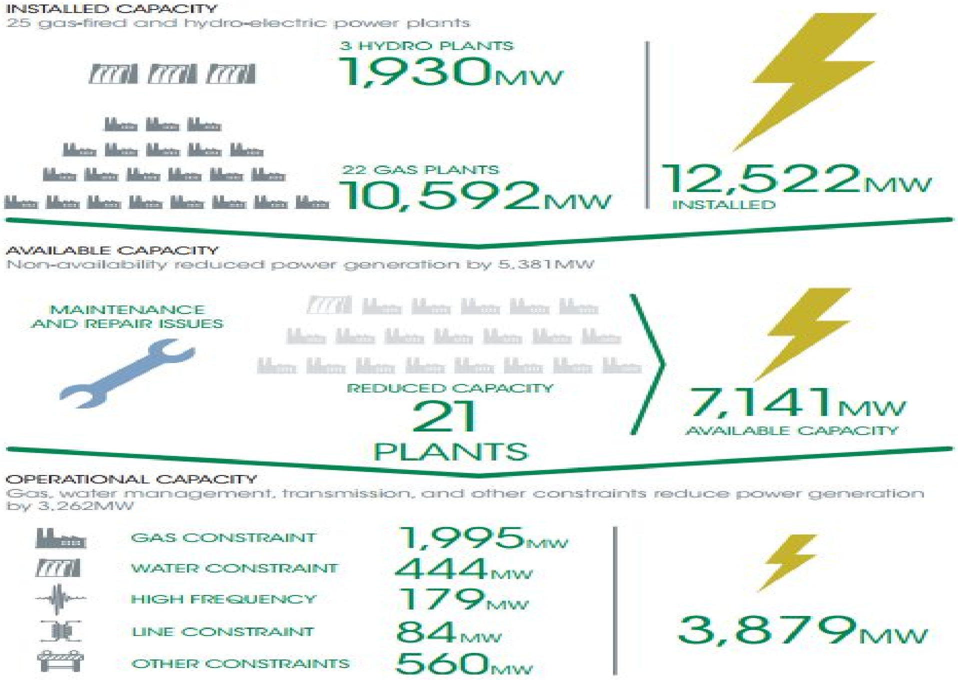

Figure 3.3: Installed power plants capacity in Nigeria (Source: GET.invest)

3.6 Problem statement

As a preface to the development of an inverse DEA for the reduction of gas flaring, it is essential to briefly describe the problem to be addressed within this chapter.

The amount of flared gas that goes to waste each year is difficult to quantify as some oil-producing countries offer plausible explanations for gas flaring, such as safety and maintenance. Therefore, a conundrum arises which is analogous to asking a fundamental research question regarding the most effective way to account for non-routine flaring: *"With successful implementation of GTW technology, what will be the minimum and maximum reduction in waste or flare gas?"*. An accurate response to this question is crucial to the successful implementation of the GGFR. Further, it is imperative to determine the level of commitment that the government of each oil-producing nation has to the ZRF initiative. A second research question may be formulated as follows: *"In real-life scenarios, is the ZRF initiative feasible for an efficient oil-producing nation?"* A feasible GTW process requires the determination of its optimal sizing for cost-effectiveness. This simply refers to the range of turbine units that can be considered economically feasible for the GTW process, since a single turbine unit will require a significant amount of capital. As a result, a more pressing question is: *"How do we determine the minimum and maximum number of turbine units for a group of oil-producing countries in need of GTW technology?"* In general, this chapter proposes a novel methodology to find optimal solutions to these questions.

3.7 Methodology

3.7.1 Nomenclature

Although DEA models have almost similar nomenclatures in several studies in literature, nonetheless, in order to facilitate an accurate representation of all the parameters and variables within this chapter, the following nomenclature applies to all models developed and applied to flare gas management in the industry:

$General\ Parameters$:

n: number of decision-making units (DMUs)

t: number of inefficient decision-making units (DMUs)

m: number of inputs of each DMU

s: number of good outputs of each DMU

q: number of bad outputs of each DMU

$Data\ Parameters$:

x_{ij}: ith input of DMU_j $(j = 1, \dots, n)$

y_{rj}^{g}: rth good output of DMU_j $(j = 1, \dots, n)$

y_{pj}^{b}: pth bad output of DMU_j $(j = 1, \dots, n)$

$\hat{y}_{r}^{g}$: desired production quantity of rth good output by inefficient DMUs

$Decision\ Variables$:

θ_k: inefficiency score of DMU_k $(k = 1, \dots, n)$

λ_j: weight assigned to DMU_j $(j = 1, \dots, n)$

α_{ik}: change in ith input of DMU_k $(k = 1, \dots, t)$

β_{rk}: change in rth good output of DMU_k $(k = 1, \dots, t)$

γ_{pk}: change in pth bad output of DMU_k ($k = 1, \dots, t$)

φ_{uk}: units of turbine u required by DMU_k ($k = 1, \dots, t$)

π_{uk}: annual gas consumption of turbine u required by DMU_k ($k = 1, \dots, t$)

γ_{pk}^{min}: minimum change in pth bad output of DMU_k ($k = 1, \dots, t$)

γ_{pk}^{max}: maximum change in pth bad output of DMU_k ($k = 1, \dots, t$)

3.7.2 Base DEA for efficiency evaluation

To develop an inverse DEA for this study, it is necessary to describe the base model adopted. Every inverse DEA model rely on efficiency scores computed by a base DEA model. The mechanism is as follows: the base DEA model performs forward optimization, while the inverse DEA model performs backward or reverse optimization. The underlying novelty of this chapter is that both models form a ***closed-loop*** optimization process. This refers to an entire loop in which the inefficiency score of each inefficient producer is initially maximized through the base DEA (i.e., reaches a theoretical maximum) but then decreases to zero after the inverse DEA is applied so that an inefficient producer undergoes a complete transition from a state of inefficiency to that of a state of efficiency. Therefore, the following DEA introduced by Chung et al. (1997) will serve as the base model for this chapter, due to its suitability for waste reduction or management:

$$M1: \quad \theta_k^* = \max \theta$$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^g \geq (1+\theta) y_{rk}^g \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^b = (1-\theta) y_{pk}^b \qquad p = 1, \dots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \dots, n$$

There are n decision-making units (DMUs) in model $M1$ that utilize m inputs to produce r good/desirable outputs and q bad/undesirable outputs. The aim of $M1$ is to reduce as much as possible the bad/undesirable outputs of the unit under evaluation (i.e., DMU_k). As a rule of thumb, DMU_k is considered efficient if its inefficiency score, $\theta_k^* = 0$ or if its efficiency score, $\varepsilon_k = 1$. The relationship between these two scores is expressed as follows: $\varepsilon_k = (1 - \theta_k^*)/(1 + \theta_k^*)$. $M1$ has a second benefit in that it increases the good/desirable outputs to the maximum extent.

3.7.3 Inverse DEA for minimizing GHG emissions.

Wegener and Amin (2019) used $M1$ as the basis for an inverse DEA model that minimizes GHG emissions in 23 oil firms located in North America. During the development of their model, they kept the structure of $M1$ by considering changes in inputs that may lead to changes in both good and bad outputs. In mathematical terms, they pondered the following scenario within an inefficient oil firm:

"Suppose an inefficient oil firm denoted by DMU_k increases its ith input by a certain amount α_{ik}; then, corresponding changes will be observed β_{rk} in the rth good output and γ_{pk} in the pth bad output. If we also consider S to be the set of inefficient units or firms (i.e., $k \in S$) that need to manage their GHG emissions, then the collective aim/goal should be to minimize the change γ_{pk} in all of the bad outputs." In order to address this problem mathematically and with respect to $M1$, Wegener and Amin (2019) developed the inverse DEA model shown below:

$$M2: \quad \text{Min}\, \gamma = (\gamma_{11}, \dots, \gamma_{q1}, \dots, \gamma_{1t}, \dots, \gamma_{qt})$$

$$s.t.$$

$$\sum_{j \epsilon F} \lambda_j^k x_{ij} + \sum_{l \epsilon G} \hat{\lambda}_l^k (\alpha_{il} + x_{il}) - (\alpha_{ik} + x_{ik}) \leq 0 \qquad \forall k \in S,\ i = 1, \dots, m$$

$$\sum_{j \epsilon F} \lambda_j^k y_{rj}^g + \sum_{l \epsilon G} \hat{\lambda}_l^k (\beta_{rl} + y_{rl}^g) - (1 + \hat{\theta}_k) \times (\beta_{rk} + y_{rk}^g) \geq 0 \qquad \forall k \in S,\ r = 1, \dots, s$$

$$\sum_{j\epsilon F}\lambda_j^k y_{pj}^b + \sum_{l\epsilon G}\hat{\lambda}_l^k\left(\gamma_{pl} + y_{pl}^b\right) - \left(1-\hat{\theta}_k\right)\times\left(\gamma_{pk} + y_{pk}^b\right) = 0 \qquad \forall k \in S,\ p = 1,\dots,q$$

$$\sum_{j\epsilon F}\lambda_j^k + \sum_{l\epsilon G}\hat{\lambda}_l^k = 1 \qquad \forall k \in S$$

$$\sum_{k\epsilon S}\beta_{rk} = \hat{y}_r^g \qquad r = 1,\dots,s$$

$$\alpha_{ik} \geq 0, \beta_{rk} \geq 0, \gamma_{pk} \geq 0 \ \ \forall k \in S,\ i = 1,\dots,m \ \ r = 1,\dots,s \ \ p = 1,\dots,q$$

$$\lambda_j^k \geq 0, \hat{\lambda}_l^k \geq 0 \ \ \forall k, l \in G,\ \ \forall j \in F$$

The set S, which is convex, consists of two distinct subsets F and G. Subset F contains all efficient firms, whereas subset G contains all inefficient firms. Additionally, there are t DMUs in S, and each DMU_k in S can be represented as a convex combination of the efficient and inefficient units in F and G, respectively. Note that units in F have weights represented as $\lambda_j^k \geq 0 \ j \in F$, while units in G have weights represented as $\hat{\lambda}_l^k \geq 0 \ l \in G$. To ensure that the predetermined efficiency score of each unit does not degrade after producing additional outputs, a decision-maker should set $\hat{\theta}_k \leq \theta_k^*$ as part of the application. Continuing with their model development, Wegener and Amin (2019) used a mathematical theorem to support the conclusion that, after the production of additional outputs, the efficiency frontier created by efficient firms will remain the same. The result was the reduction of the model to a simpler one as follows:

$M3$: $\quad \text{Min}\, \gamma = (\gamma_1 + \gamma_2 +, \dots + \gamma_t)$

$s.t.$

$$\sum_{j\epsilon F}\lambda_j^k x_{ij} - (\alpha_{ik} + x_{ik}) \leq 0 \qquad \forall k \in S,\ \ i = 1,\dots,m$$

$$\sum_{j\epsilon F}\lambda_j^k y_{rj}^g - \left(1+\hat{\theta}_k\right)\times\left(\beta_{rk} + y_{rk}^g\right) \geq 0 \qquad \forall k \in S,\ \ r = 1,\dots,s$$

$$\sum_{j\epsilon F}\lambda_j^k y_{pj}^b - \left(1-\hat{\theta}_k\right)\times\left(\gamma_{pk} + y_{pk}^b\right) = 0 \qquad \forall k \in S,\ p = 1,\dots,q$$

$$\sum_{j \epsilon F} \lambda_j^k = 1 \qquad \forall k \in S$$

$$\sum_{k \epsilon S} \beta_{rk} = \hat{y}_r^g \qquad r = 1, \dots, s$$

$$\alpha_{ik} \geq 0, \beta_{rk} \geq 0, \gamma_{pk} \geq 0 \;\; \forall k \in S,\; i = 1, \dots, m \;\; r = 1, \dots, s \;\; p = 1, \dots, q$$

$$\lambda_j^k \geq 0, \hat{\lambda}_l^k \geq 0 \;\; \forall k, l \in G,\; \forall j \in F$$

Although $M3$ was successfully applied to GHG emissions in oil firms, there is no guarantee that it can also be applied to gas flaring in the same industry. There are two factors at work here. As a first limitation, $M3$ is not equipped to handle negative data, such as the current account balances of oil-producing nations. Current account balances provide information about the import/export status of oil-producing nations. Typically, within the broader field of macroeconomics, a surplus current account balance is regarded as positive data and a deficit current account balance as negative data. $M3$ was developed specifically to handle GHG emissions in oil companies in the United States and Canada, thus information such as the current account balance is not necessary. However, the present study focuses on a global problem involving several oil-exporting nations with current account balances that must be modeled. Therefore, it is of vital importance to test the feasibility of $M3$ when applied to global gas flaring in the industry. This feasibility study will be the first stage in developing a novel inverse DEA for this chapter.

In addition, a major limitation of $M3$, as demonstrated in the practical examples provided by Wegener and Amin (2019), is that it did not reduce the GHG emissions in the selected firms. Instead, it only minimized GHG emissions due to an increase in oil production. Therein lies the greatest challenge associated with the use of $M3$ to estimate gas flaring reductions in the industry. Therefore, $M3$ will need to be further developed and configured to address the research questions contained within this chapter. Such development will be addressed in the second stage of model development discussed in this chapter.

3.7.4 Model development (*stage one*)

To overcome the first limitation of $M3$, it is necessary to incorporate negative data into it, as well as into its base model $M1$. This is imperative since both models must have the same structure. The failure to restructure both models will contradict the fundamental theorems of DEA formulations. Following this, it is necessary to determine whether the source of negative data is input-oriented or output-oriented. For this study, negative data comes from the input variables, thus the models to be developed will only be able to deal with negative inputs.

To deal with negative inputs, we refer to the inputs' constraint of $M1$, and assume a special case of only two DMUs (i.e., $j = 1,2$) with input i having positive value for DMU_1 and negative value for DMU_2

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \quad \dots\dots\dots\dots \text{constraint 1 of } M1$$

The LHS of the above constraint can be rewritten to include both positive and negative inputs as:

$\lambda_1 x_{i1} - \lambda_2 x_{i2} \leq x_{ik}$

By decomposition, this inequality can be represented as the sum of two inequalities:

$$\lambda_1 x_{i1} \leq x_{ik} \quad \dots\dots\dots\dots\dots\dots (3.1)$$

$$-\lambda_2 x_{i2} \leq 0 \quad \dots\dots\dots\dots\dots\dots (3.2)$$

Both can also be represented as:

$$\lambda_1 x_{i1} \leq x_{ik} \quad \dots\dots\dots\dots\dots\dots (3.3)$$

$$\lambda_2 x_{i2} \geq 0 \quad \dots\dots\dots\dots\dots\dots (3.4)$$

Rewriting both types of inputs as $x_{i1} = x_{i1}^{+}$ and $x_{i2} = x_{i2}^{-}$, translates to the following pair:

$$\lambda_1 x_{i1}^{+} \leq x_{ik} \quad \dots\dots\dots\dots\dots\dots (3.5)$$

$$\lambda_1 x_{i2}^{-} \geq 0 \quad \dots\dots\dots\dots\dots (3.6)$$

In general, all positive and negative inputs i for all DMUs including DMU_k can be expressed as:

$$\lambda_j x_{ij}^{+} \leq x_{ik}^{+} \quad \dots\dots\dots\dots\dots\dots (3.7)$$

$$\lambda_j x_{ij}^{-} \geq x_{ik}^{-} \quad \dots\dots\dots\dots\dots (3.8)$$

Constraint 1 of $M1$ can now be expressed as two distinct constraints:

$\sum_{j=1}^{n} \lambda_j x_{ij}^{+} \leq x_{ik}^{+}$ (Positive input i) and $\sum_{j=1}^{n} \lambda_j x_{ij}^{-} \leq x_{ik}^{-}$ (Negative input i)

If DMU_k has no negative input i, then $x_{ik}^{-} = 0$. Similarly, if DMU_k has no positive input i, then $x_{ik}^{+} = 0$. In other words, $x_{ik} = x_{ik}^{+} + x_{ik}^{-}$, such that $x_{ik} = x_{ik}^{+}$ if $x_{ik}^{-} = 0$, and $x_{ik} = x_{ik}^{-}$ if $x_{ik}^{+} = 0$.With this classification of positive and negative inputs, models $M1$ and $M3$ are transformed into models $M2$ and $M4$, respectively.

$M2$: $\theta_k^* = \max \theta$

$$
\begin{aligned}
& s.t. \\
& \sum_{j=1} \lambda_j \, x_{ij}^{+} \le x_{ik}^{+} && i = 1, \dots, m \\
& \sum_{j=1} \lambda_j \, x_{ij}^{-} \le x_{ik}^{-} && i = 1, \dots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^{g} \ge (1+\theta) y_{rk}^{g} && r = 1, \dots, s \\
& \sum_{j=1}^{n} \lambda_j y_{pj}^{b} = (1-\theta) y_{pk}^{b} && p = 1, \dots, q \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0 && j = 1, \dots, n
\end{aligned}
$$

$M4$: $\text{Min}\, \gamma = (\gamma_1 + \gamma_2 +, \dots + \gamma_t)$

$$
\begin{aligned}
& s.t. \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij}^{+} - (\alpha_{ik}^{+} + x_{ik}^{+}) \le 0 && \forall k \in S,\ i = 1, \dots, m \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij}^{-} - (\alpha_{ik}^{-} + x_{ik}^{-}) \ge 0 && \forall k \in S,\ i = 1, \dots, m \\
& \sum_{j \epsilon F} \lambda_j^k y_{rj}^{g} - \left(1 + \hat{\theta}_k\right) \times \left(\beta_{rk} + y_{rk}^{g}\right) \ge 0 && \forall k \in S,\ r = 1, \dots, s \\
& \sum_{j \epsilon F} \lambda_j^k y_{pj}^{b} - \left(1 - \hat{\theta}_k\right) \times \left(\gamma_{pk} + y_{pk}^{b}\right) = 0 && \forall k \in S,\ p = 1, \dots, q \\
& \sum_{j \epsilon F} \lambda_j^k = 1 && \forall k \in S
\end{aligned}
$$

$$\sum_{k \epsilon S} \beta_{rk} = \hat{y}_r^g \qquad r = 1, \dots, s$$

$$\alpha_{ik} \geq 0, \beta_{rk} \geq 0, \gamma_{pk} \geq 0 \ \ \forall k \in S, \ i = 1, \dots, m \ \ r = 1, \dots, s \ \ p = 1, \dots, q$$

$$\lambda_j^k \geq 0, \hat{\lambda}_l^k \geq 0 \ \ \forall k, l \in G, \ \ \forall j \in F$$

It is important to state here that models $M2$ and $M4$ are developed based on the assumption that every input i has both positive and negative values for all DMUs. However, the application of both models might differ due to type of input data in the real world. If only one input takes a negative value for at least one DMU, the application of $M2$ and $M4$ requires partitioning the input set. The semi-oriented radial measure (SORM) introduced by Emrouznejad et al. (2010) can be easily used for this, thereby transforming $M2$ and $M4$ into $M5$ and $M6$, respectively:

$M5$: $\theta_k^* = \max \theta$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, \dots, m \qquad i \, \epsilon \, I_1$$

$$\sum_{j=1} \lambda_j \, x_{ij}^+ \leq x_{ik}^+ \qquad i = 1, \dots, m \qquad i \, \epsilon \, I_2$$

$$\sum_{j=1} \lambda_j \, x_{ij}^- \leq x_{ik}^- \qquad i = 1, \dots, m \qquad i \, \epsilon \, I_2$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^g \geq (1 + \theta) y_{rk}^g \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^b = (1 - \theta) y_{pk}^b \qquad p = 1, \dots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \dots, n$$

Note that $M5$ is the new base model with partitioned input sets. I_1 is the set of inputs that take only positive value for all $DMUs$, while I_2 is the set of inputs that take negative value for at least one DMU.

Similarly, the inverse DEA with partitioned input sets is expressed as follows:

$$
\begin{aligned}
M6:\quad & \text{Min}\, \gamma = (\gamma_1 + \gamma_2 +, \dots + \gamma_t) \\
& s.t. \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij} - (\alpha_{ik} + x_{ik}) \le 0 && \forall k \in S, \quad i = 1, \dots, m \quad i \epsilon I_1 \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij}^+ - (\alpha_{ik}^+ + x_{ik}^+) \le 0 && \forall k \in S, \quad i = 1, \dots, m \quad i \epsilon I_2 \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij}^- - (\alpha_{ik}^- + x_{ik}^-) \ge 0 && \forall k \in S, \quad i = 1, \dots, m \quad i \epsilon I_2 \\
& \sum_{j \epsilon F} \lambda_j^k y_{rj}^g - \left(1 + \hat{\theta}_k\right) \times \left(\beta_{rk} + y_{rk}^g\right) \ge 0 && \forall k \in S, \quad r = 1, \dots, s \\
& \sum_{j \epsilon F} \lambda_j^k y_{pj}^b - \left(1 - \hat{\theta}_k\right) \times \left(\gamma_{pk} + y_{pk}^b\right) = 0 && \forall k \in S, \quad p = 1, \dots, q \\
& \sum_{j \epsilon F} \lambda_j^k = 1 && \forall k \in S \\
& \sum_{k \epsilon S} \beta_{rk} = \hat{y}_r^g && r = 1, \dots, s \\
& \alpha_{ik} \ge 0, \beta_{rk} \ge 0, \gamma_{pk} \ge 0 \;\; \forall k \in S,\; i = 1, \dots, m \;\; r = 1, \dots, s \;\; p = 1, \dots, q \\
& \lambda_j^k \ge 0, \hat{\lambda}_l^k \ge 0 \;\; \forall k, l \in G, \;\; \forall j \in F
\end{aligned}
$$

With their current structure, both models $M5$ and $M6$ are fully capable of handling negative inputs and will be used for conducting a feasibility study. Specifically, the feasibility study is designed to investigate whether the inverse model $M6$ can be utilized to minimize the increase in gas flaring associated with increased oil production.

3.7.5 Model development (*stage two*)

To compute gas flaring reductions, model $M6$ must be further developed in accordance with the GGFR. As it stands, $M6$ is only suitable for preventing an increase in gas flaring associated with increasing oil production. Specifically, the following research questions (i.e., *R1* to *R5* from section 1.3) will aid in the further development of $M6$:

- *At current production level of inputs and outputs, and with current technology and workforce, what is the potential reduction in gas flaring for an oil-producing nation?*
- *With better technology and a more highly skilled workforce, what is the maximum potential reduction in gas flaring for an oil-producing nation?*
- *Could an energy transition curve be developed for the petroleum industry based on investments in flare gas recovery technology and effective management techniques?*
- *Can the petroleum industry adopt the Zero* Routine Flaring (ZRF) Initiative *in any given production year?*
- *Based on the potential reductions in gas flaring, what is the optimal sizing required for implementing a gas-to-wire (GTW) system for flare gas management?*

As a starting point for the first research question, all the changes in the inputs and desirable outputs of $M6$ must be equal to zero, i.e., at current production rates, $\alpha_{ik} = \alpha_{ik}^{+} = \alpha_{ik}^{-} = \beta_{rk} = 0$. Then the anticipated gas flaring reduction necessitates that γ_{pk} must be a negative change in the current level of flare gas. Toward cleaner natural gas production, a new objective function arises in the form of maximizing the reduction in gas flaring for each DMU. As well, there is only one undesirable output for this study (i.e., flare gas), so the index of bad output $p = 1$. The integration of these new changes and features into $M6$ results in a new model $M7$ applicable to same group of DMUs.

$M7$: $\quad \text{Max}\,\gamma = (\gamma_1 + \gamma_2 +, \ldots + \gamma_t)$

$$s.t.$$

$$\sum_{j \epsilon F} \lambda_j^k x_{ij} - x_{ik} \le 0 \qquad \forall k \in S, \quad i = 1, \ldots, m \;\; i \,\epsilon\, I_1$$

$$\sum_{j \epsilon F} \lambda_j^k x_{ij}^{+} - x_{ik}^{+} \le 0 \qquad \forall k \in S, \quad i = 1, \ldots, m \;\; i \,\epsilon\, I_2$$

$$\sum_{j \epsilon F} \lambda_j^k x_{ij}^{-} - x_{ik}^{-} \ge 0 \qquad \forall k \in S, \quad i = 1, \ldots, m \;\; i \,\epsilon\, I_2$$

$$\sum_{j \epsilon F} \lambda_j^k y_{rj}^{g} - \left(1 + \hat{\theta}_k\right) \times y_{rk}^{g} \ge 0 \qquad \forall k \in S, \quad r = 1, \ldots, s$$

$$\sum_{j \epsilon F} \lambda_j^k y_{pj}^b - (1-\hat{\theta}_k) \times (y_{pk}^b - \gamma_{pk}) = 0 \qquad \forall k \in S,\ p = 1, \dots, q$$

$$\sum_{j \epsilon F} \lambda_j^k = 1 \qquad \forall k \in S$$

$$\gamma_{pk} \leq y_{pk}^b$$
$$\alpha_{ik} \geq 0, \beta_{rk} \geq 0, \gamma_{pk} \geq 0 \ \ \forall k \in S,\ i = 1, \dots, m \ \ r = 1, \dots, s \ \ p = 1, \dots, q$$
$$\lambda_j^k \geq 0, \hat{\lambda}_l^k \geq 0 \ \ \forall k, l \in G,\ \ \forall j \in F$$

The new constraint in $M7$ (i.e., $\gamma_{pk} \leq y_{pk}^b$) places an upper bound on the potential reduction in flare gas. It is worth noting that $M7$ still focuses on a group of DMUs or producers, and partially answers the first research question of this study, which refers to gas flaring reduction for each oil-producing nation. Accordingly, $M7$ must be relaxed for a single producer with one bad output to provide the following inverse DEA model:

$M8$: $\quad \text{Max}\, \gamma^* = \gamma_{pk}$

$$s.t.$$
$$\sum_{j \epsilon F} \lambda_j^k x_{ij} \leq x_{ik} \qquad i = 1, \dots, m \quad i \, \epsilon \, I_1$$

$$\sum_{j \epsilon F} \lambda_j^k x_{ij}^+ \leq x_{ik}^+ \qquad i = 1, \dots, m \quad i \, \epsilon \, I_2$$

$$\sum_{j \epsilon F} \lambda_j^k x_{ij}^- \geq x_{ik}^- \qquad i = 1, \dots, m \quad i \, \epsilon \, I_2$$

$$\sum_{j \epsilon F} \lambda_j^k y_{rj}^g \geq (1+\hat{\theta}_k) \times y_{rk}^g \qquad r = 1, \dots, s$$

$$\sum_{j \epsilon F} \lambda_j^k y_{pj}^b = (1-\hat{\theta}_k) \times (y_{pk}^b - \gamma_{pk}) \qquad p = 1, \dots, q$$

$$\sum_{j \epsilon F} \lambda_j^k = 1$$

$$\gamma_{pk} \leq y_{pk}^b$$
$$\alpha_{ik} \geq 0, \beta_{rk} \geq 0, \gamma_{pk} \geq 0$$
$$i = 1, \dots, m \ \ r = 1, \dots, s \ \ p = 1, \dots, q$$

$\lambda_j^k \geq 0, \hat{\lambda}_l^k \geq 0 \quad \forall, l \in G, \ \forall j \in F$

$M8$ is the fully developed inverse DEA model that estimates potential reductions in gas flaring for an inefficient oil-producing nation. It provides the definitive answer to the first research question in this study. To preserve the efficiency score of DMU_k, and to keep $M8$ feasible at all times due to negative data, one must set $\hat{\theta}_k < \theta_k^*$. As an example, but under the full discretion of the decision-maker, one can define $\hat{\theta}_k$ as 1% less than θ_k^*. Where θ_k^* is the optimal solution of base model $M5$. The value of γ^* in $M8$ is the minimum potential reduction in gas flaring (i.e., $\gamma^* = \gamma_{pk}^{min}$). A rare case of $\gamma^* = 0$ implies the producer under evaluation flared the least volume of natural gas relative to other producers. A mathematical relationship does exist between γ^* and $\hat{\theta}_k$, and this leads to a theorem and proof. In this study, all proposed models are solved using the LINGO 18 solver.

THEOREM

The maximum potential reduction occurs at zero inefficiency i.e., when $\hat{\theta}_k = 0, \ y_{pk}^b = \gamma_{pk}^{max}$.

This theorem illustrates the transition phenomenon whereby the directional distance DEA or base model (i.e., $M5$) initially maximizes $\hat{\theta}_k$ for an inefficient unit. This is the preliminary stage of the optimization process. Based on the predetermined value of $\hat{\theta}_k$, the inverse DEA model (i.e., $M8$) calculates the initial potential reduction (i.e., also known as the minimum reduction, γ_{pk}^{min}). Due to the inherent structure of $M8$, this is mathematically correct since inefficiency (i.e., $\hat{\theta}_k$) and potential reductions (i.e., γ_{pk}) are inversely related. It follows that a maximum value of $\hat{\theta}_k$ corresponds to a minimum value of γ_{pk}. Accordingly, the converse must also apply to a case in which a minimum value of $\hat{\theta}_k$ corresponds to a maximum value of γ_{pk}. Therefore, by decreasing the value of $\hat{\theta}_k$ in constant intervals to its minimum value of zero, the corresponding values of γ_{pk} will surely increase to a theoretical maximum. The increase of $\hat{\theta}_k$ to its maximum value by base model $M5$ and subsequent decrease to its minimum value of zero by inverse model $M8$ represent a closed-loop optimization process, like the motion of an object under gravity – one that attains maximum height and returns to its projection point.

Proof

In model $M8$, F denotes the subset of efficient producers that created the efficiency frontier of base model $M5$ and need no further improvement. In constraints 4 and 5 of $M8$, the terms $\sum_{j\epsilon F}\lambda_j^k y_{rj}^g$ and $\sum_{j\epsilon F}\lambda_j^k y_{pj}^b$ only apply to efficient producers in F, while the terms $(1+\hat{\theta}_k)\times y_{rk}^g$ and $(1-\hat{\theta}_k)\times(y_{pk}^b-\gamma_{pk})$ only apply to the inefficient DMU_k that needs improvement. At zero inefficiency, DMU_k is considered efficient and added to set F. This implies from constraint 5 that no further improvement in gas flaring reduction is needed for DMU_k and we have that:

$(1-\hat{\theta}_k)\times(y_{pk}^b-\gamma_{pk})=0$

When $\hat{\theta}_k=0$, $y_{pk}^b-\gamma_{pk}=0$

$y_{pk}^b=\gamma_{pk}=\gamma_{pk}^{max}$ and this completes the proof.

Further evidence for the validity of this theorem is provided in later sections through a sensitivity analysis. Based on this theorem, the second research question in this study is answered, while the sensitivity analysis will provide the energy transition curve for the industry, thereby answering the third research question. Moreover, two corollaries are provided to illustrate the implications of this theorem.

Corollary 1: It is practically possible for the industry to achieve efficiency and reduce gas flaring with investment in better technology, a more highly skilled force, and effective management strategies.

Corollary 2: It is practically difficult for the industry to simultaneously achieve efficiency and zero routine flaring.

The connection between both corollaries is the use of state-of-the-art technology since it allows easier conversion of gases that would be flared to other industrial uses. Nevertheless, non-routine flaring may prevent a producer from achieving an ideal efficiency level with zero flares.

3.7.5.1 The zero routine flaring initiative.

In 2015, the World Bank and United Nations launched the ZRF initiative to eliminate routine gas flaring no later than 2030 by bringing together governments and oil and gas companies. While some industry experts contend that oil-producing nations have sufficient time to standardize their production processes and eliminate routine flaring, the recent spike in global gas flaring suggests that this is not the case. It has been stated that this is partly because the technical means needed to implement this initiative effectively are not available. In short, it is difficult to determine the extent to which oil-producing nations are committed to this goal. The following definitions and algorithm are presented to determine whether this initiative can be adopted by any petroleum industry.

DEFINITION ONE: Through model $M8$, the maximum reduction of bad output γ_{pk}^{max} occurs at $\hat{\theta}_k = 0$ or $\varepsilon_k = 1$

DEFINITION TWO: Zero routine flaring initiative is adoptable, if and only if, $y_{pk}^{b} = \gamma_{pk}^{max}$.

ALGORITHM

Step 1: Through model $M5$ evaluate θ_k^* for each DMU_j $\forall\, j = 1,2,\dots,n$

Step 2: If $\theta_k^* = 0$ add DMU_j to set F and go to step 3; if $\theta_k^* > 0$ add DMU_j to set G and go to step 4

Step 3: For all $\mathrm{DMU}_j \in F$ If $j < n$ go to step 1; if $j = n$ go to step 5

Step 4: For all $\mathrm{DMU}_j \in G$ If $j < n$ go to step 1; if $j = n$ go to step 5

Step 5: Apply $M8$ by combining all $\mathrm{DMU}_j \in F$ with each $\mathrm{DMU}_j \in G$ and go to step 6

Step 6: Through definition one, determine γ_{pk}^{max}

Step 7: Zero routine flaring initiative is adoptable through definition two.

This algorithm provides the answer to the fifth research question because it yields a zero difference if the maximum reduction of bad output is equal to the actual amount of bad output. To provide a more detailed explanation of this algorithm, please refer to Fig. 3.4. As a first step, each producer's inefficiency score is evaluated. The producer with a zero-inefficiency score is considered efficient and is included in the subset F of efficient producers. Otherwise, such a producer will be added to the subset G of inefficient producers. If a producer is inefficient, the maximum reduction in gas flaring, γ^{max}, can be achieved by setting its inefficiency score to zero (i.e., assuming the producer is efficient). If the maximum reduction is equal to the actual volume of gas flared (i.e., y^b) then the ZRF initiative can be adopted by such a producer. Otherwise, we calculate the deviation from a zero flare. Production processes are cleaner when deviations are smaller.

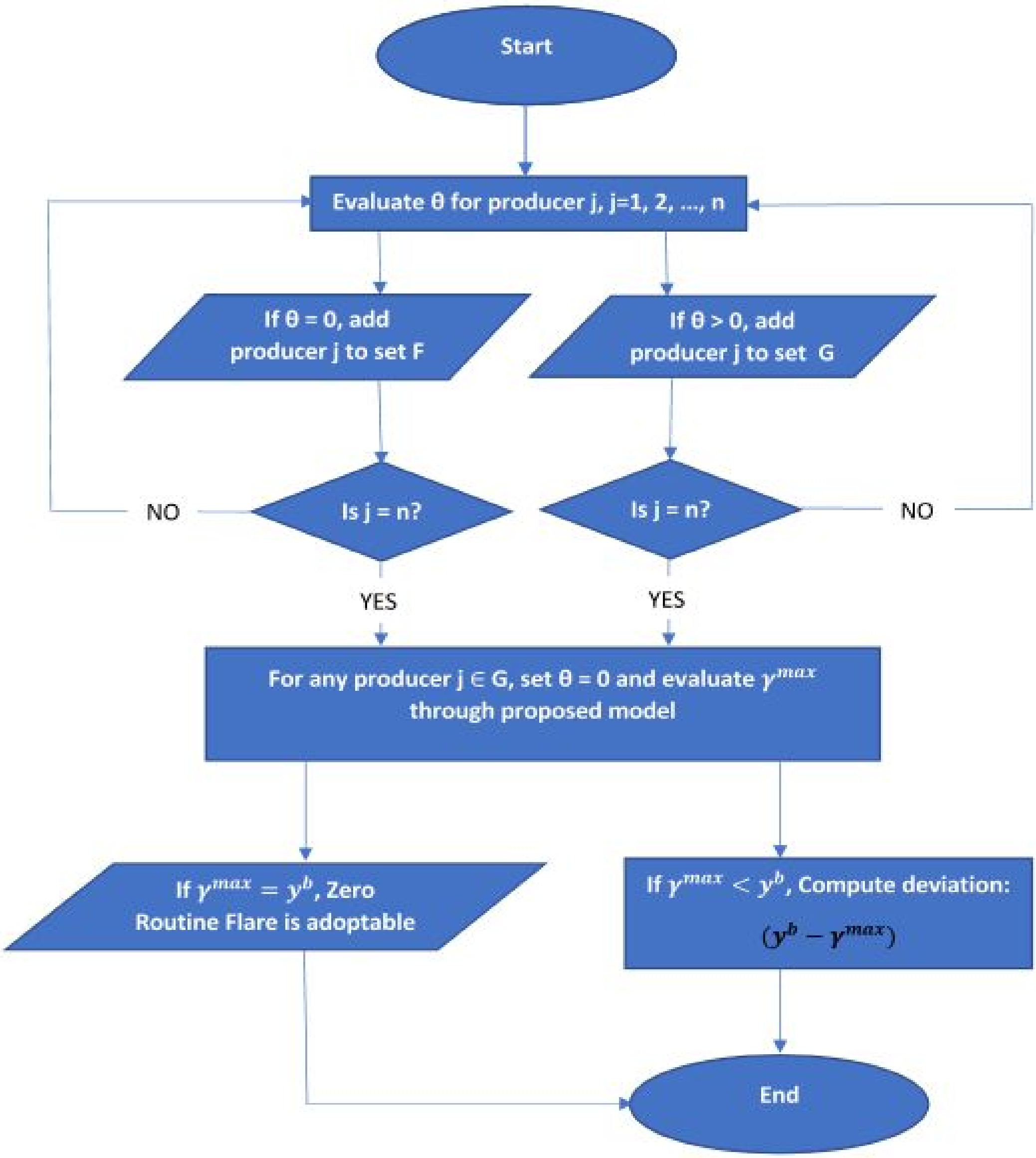

Figure 3.4: Solution algorithm for the Zero Routine Flaring Initiative

3.7.6 Optimal sizing of gas-to-wire (GTW) process

A GTW process that is economically feasible requires a range of turbine units that is optimal. To determine the optimal range, the minimum and maximum turbine units must be computed. With the application of model $M8$, the minimum and maximum units can be determined for any producer k by utilizing the optimal values of its minimum (i.e., γ_{pk}^{min}) and maximum (i.e., γ_{pk}^{max})

potential reductions in gas flaring. Accordingly, the optimal sizing can be expressed mathematically as follows:

$$\frac{\gamma_{pk}^{min}}{\pi_{uk}} \leq \varphi_{uk} \leq \frac{\gamma_{pk}^{max}}{\pi_{uk}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (3.9)$$

Specifically, φ_{uk} is units of turbine u required by DMU_k, while π_{uk} is the annual gas consumption of turbine u required by DMU_k.

In the case of a single bad/undesirable output (i.e., flare gas), we have that the index $p = 1$ and the expression reduces to:

$$\frac{\gamma_{k}^{min}}{\pi_{uk}} \leq \varphi_{uk} \leq \frac{\gamma_{k}^{max}}{\pi_{uk}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (3.10)$$

The process flow diagram (PFD) illustrated in Fig. 3.5 summarizes the steps required to achieve this optimal range. Using the same five inputs in step one of the PFD, all DMUs will produce the same desirable output and undesirable output (flare gas). In the next step, the inefficiency scores will be determined using model $M5$, and this will result in the classification of producers. Through proposed model $M8$, reductions in gas flaring are calculated for each inefficient producer. It is possible to determine the optimal sizing for each producer using equation (3.10). In general, equation (3.10) answers the last research question for this chapter.

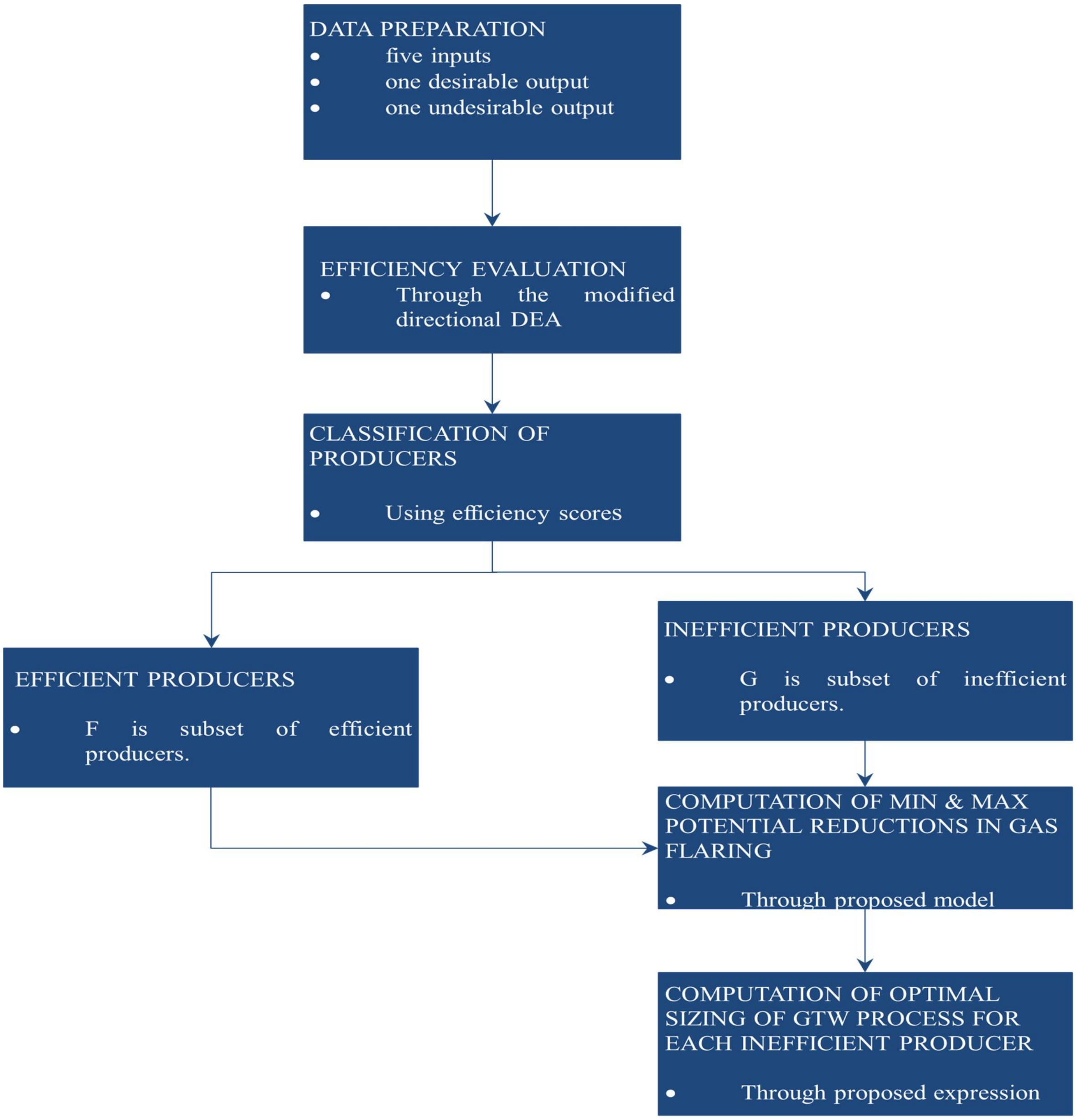

Figure 3.5: Process Flow Diagram for the GTW process

3.7.7 Data collection and description

The OPEC Annual Statistical Bulletin (ASB) for 2016 provides production data for the 13 members of OPEC in 2011. The official OPEC website provides access to open data contained in this bulletin. In this bulletin, OPEC presents data for a five-year period (i.e., from 2011 to 2015). OPEC member nations were chosen for this study since they supply a combined 43.5 percent of global crude oil and hold 81.9 percent of global oil reserves. A second reason for our study is that Nigeria is a major OPEC member nation and is still the largest oil producer in Africa. Considering Nigeria's performance in relation to other OPEC members is imperative.

The rationale for selecting the year 2011 for this study is briefly discussed in this section. To begin with, this is a pioneering study in the literature that develops an optimization model for global gas flaring reductions. In an earlier attempt, Ojijiagwo et al. (2016) used a qualitative approach, consisting of semi-structured interviews with industry experts. It is noteworthy that only Nigeria was included in their analysis, which was based on an average flare report for a period of 49 years (i.e., 1965 to 2013). This time frame is inclusive of the production year (2011 in this case) chosen for this study. As a measure of the performance of the proposed model $M8$, it is pertinent that the results obtained for Nigeria in 2011 be compared with those reported by Ojijiagwo et al. (2016). There is also the issue of OPEC not providing recent flare data. Some OPEC members do not provide flare data on a regular basis. Flare data for all member nations were last published by OPEC in 2015, and other flare data sources conflict with those found in the ASB bulletin. It should be noted, however, that the methodology proposed in this chapter is suitable for a group of oil companies located within one oil-producing nation. Gas flaring is a global problem in this study, so it is interesting to examine it at a local level in a country or region. Additionally, Nigeria had the highest volume of flared gas (14270 million cubic meters) over the five-year period. Consequently, this chapter will analyze results in the context of the 2011 production year.

Throughout this chapter, the same sets of five inputs and two outputs have been selected for all producers to ensure homogeneity in the subsequent analysis. Each producer is required to submit data regarding inputs and outputs annually to OPEC.

Inputs

I. Current account balance

When trying to determine whether an oil-producing nation is a net exporter or importer, it is imperative to know its current account balance. Having a positive balance on the current account establishes the producer as a net exporter, while a negative balance establishes the producer as a net importer. Crude oil production rates are affected by both factors over the long term. In general, countries with a surplus current account tend to have higher GDPs, while those with deficit current accounts usually have lower GDPs. Positive inputs are surpluses, while negative inputs are deficits.

II. Wells completed.

This refers to the number of oil wells developed and completed for oil production.

III. Producing wells

The figure represents the total number of oil wells completed throughout the production year (excluding uneconomical wells).

IV. Active rigs

It is the total number of oil rigs that are currently in operation (including workover rigs) during the course of the production year which is assigned to the production of crude oil. The number of active rigs and wells completed has a significant impact on the amount of refined petroleum produced by a refinery. In other words, as more rigs are engaged and more wells are completed, the refinery capacity will be upgraded to accommodate more crude oil input.

V. Refining capacity

It is the maximum amount of crude oil that can be processed by a refinery to yield petroleum products, usually expressed in barrels per calendar day (1000b/cd). There is a direct relationship between this input and the volume of refined products available for both export and domestic consumption. In addition, countries that have large refining capacities, such as Saudi Arabia and Iran, generate a significant amount of revenue in both the short and long term by exporting refined products to foreign markets. Sadly, this is not the case for an oil giant such as Nigeria with a low refinery capacity and utilization rate. Nigeria currently has four functional refineries - Kaduna Refining and Petrochemicals (KRPC) Limited, Port Harcourt Refining Company (PHRC), Warri Refining and Petrochemicals Company (WRPC), and Niger Delta Petroleum Resources (NDPR). These refineries together have a refining capacity of 446 thousand barrels per day (see Table 3.1), although their combined utilization in 2017 was 8.67 percent. This explains why Nigeria continues to import refined petroleum products from foreign countries despite having a crude oil production capacity of 2.5 million barrels per day. Through a swap deal, crude oil is exported, and refined products are bought back from foreign refineries. Even with this decades-old strategy, a persistent scarcity of refined products such as gasoline, kerosene, and diesel still exist within Nigeria, as the demand for refined products vastly exceeds the supply. It is expected that the proposed Dangote refinery will increase refining capacity by an additional 650 thousand barrels per day, however, it is still in the construction stage and will require more time to be completed. In the meantime, Nigeria will have to continue to import refined products from overseas. This results in the nation losing out on extra revenue that could have been generated if all its refineries were operating at full capacity. Additionally, Nigeria incurs additional costs when purchasing such products from foreign refineries.

Table 3.1: Annual Domestic Refining Capacity Utilization in Nigeria (1000 b/cd)

		Kaduna Refining & Petrochemical Company	Port Harcout Refining Company (New)	Port Harcout Refining Company (Old)	Warri Refining & Petrochemical Company	Niger Delta Petroleum Resources	Total	Capacity Utilization, %
Designed Capacity, BPSD		110,000.00	150,000.00	60,000.00	125,000.00	1000.00	446,000.00	
Crude Oil Processed, BPSD	2010	21,986.72	19,345.38	-	53,345.20	-	94,677.30	21.23
	2011	20,896.79	31,853.02	-	49,731.41	222.03	102,703.25	23.03
	2012	31,981.86	24,530.97	-	34,868.71	557.45	91,938.99	20.61
	2013	32,452.43	44,937.47	-	20,925.04	185.18	98,500.12	22.09
	2014	12,160.39	23,557.15	-	24,049.59	503.71	60,270.84	13.51
	2015	3,297.36	9,274.21	-	8,337.64	714.66	21,623.87	4.85
	2016	10,310.69	32,669.98	-	14,746.13	605.43	58,332.23	13.08
	2017	16,597.23	6,998.94	-	14,775.66	696.90	39,068.73	8.67

(Note: Old Port Harcout Refinery was not operational during the period under review)

Source: 2017 Nigerian Oil and Gas Industry Report by the Department of Petroleum Resources (DPR)

Good Output

I. GDP per capita

In US dollars per person, this is calculated as GDP divided by current oil prices. In general, this economic output is defined as the amount of wealth earned by member states of the Organization of Petroleum Exporting Countries per capita from the exportation of crude oil and refined petroleum products. A case in point is Qatar, which was a member of OPEC from 1961 until the end of 2018 and remains the richest country in the world in terms of gross domestic product per capita. Population is, therefore, an influential factor in this study because the production rates of crude oil could be misleading. For instance, in 2011, Nigeria produced more crude oil than any other African oil-producing nation but had the lowest GDP per capita compared to Algeria, Angola, and Libya. The reason for this can be attributed to Nigeria's large population. Thus, in relation to the size of the Nigerian population, there is a pressing need to improve production efficiency and generate more revenue for the Nigerian economy.

Bad Output

I. Routinely flared gas

In each OPEC member nation, this is the volume of flared gas expressed in million standard cubic meters. The ASB records this information annually, however, some members do not report theirs due to the relatively low levels of flaring observed in a particular year. Two members did not report flare data for the 2011 production year. It is reasonable to assume that the proportion of gas flared in both nations is very low (i.e., negligible). In this case, it would be more appropriate to assume that both nations have had no flares. Thus, we consider two scenarios in this chapter, one excluding both nations, and the other presuming that both nations had no flares. It is pertinent to note that the first scenario involves only 11 members, without any assumptions. In contrast, the second scenario involves all 13 members based on the assumption that both nations adopted zero routine flare. Subsequent analyses will compare the results of both scenarios.

3.8 Application, Results and Analyses

To begin this section, an assessment of the feasibility of the developed base model (i.e., $M5$) and its inverse model (i.e., $M6$) is conducted based on real-life data from the petroleum industry. The original inverse DEA model, $M3$, developed by Wegener and Amin (2019) aimed at preventing increases in GHG emissions and therefore lacked the capability of reducing GHG emissions. The premise here is that if the developed inverse DEA model, $M6$, can also prevent an increase in gas flaring, it is logical to conclude that the further developed and relaxed inverse DEA model, $M8$, will reduce gas flaring.

3.8.1 Results of feasibility study

By using the proposed model $M5$, the inefficiency scores of 11 OPEC members were calculated and presented in Table 3.2. Let F and G represent the subset of efficient and inefficient producers, respectively. This implies $F = \{\mathrm{DMU}_2, \mathrm{DMU}_3, \mathrm{DMU}_6, \mathrm{DMU}_7, \mathrm{DMU}_9\}$, and $G = \{\mathrm{DMU}_1, \mathrm{DMU}_4\ \mathrm{DMU}_5, \mathrm{DMU}_8, \mathrm{DMU}_{10}\mathrm{DMU}_{11}, \}$.

Table 3.2: Production data and inefficiency scores for 11 OPEC members

DMU	Current Account Balance (m US$)	Wells Completed	Producing Wells	Active Rigs	Refining Capacity (1000b/cd)	GDP per Capita (US$/person)	Routinely Flared Gas (M cu m)	Ineff. (θ)
1-Algeria	17770	249	2010	33	592	5453.5	3604	0.83
2-Angola	13085	112	1476	22	65	4666.95	7183	0
3-Ecuador	-402	207	3079	39	188.4	5193.04	539	0
4-Indonesia	1685	838	10423	80	1125	3121	2452	0.78
5-Iraq	26365	76	1695	59	810	5571.55	9612	0.91
6-Kuwait	65743	523	1798	32	936	41672	217	0
7-Libya	3173	76	609	55	380	5858	1302	0
8-Nigeria	10757	124	2116	38	445	2451.75	14270	0.91
9-Qatar	51906	29	517	6	283	97983.27	558	0
10-UAE	50948	266	1592	19	675	40819.31	982	0.55
11-Venezuela	16342	1050	14915	116	1872	10283.2	9284	0.94

Thus, every $DMU \in G$ has more room for minimizing gas flaring compared to the efficient $DMUs \in F$. In this connection, a decision-maker can select any number of inefficient $DMUs$

from set G to form a smaller subset S with a cardinality of t (i.e., $t = |S|$). To ensure the feasibility study is robust, two different cases will be presented in this section. In the first case, we consider the smallest group or set of two random $DMUs$ where $t = 2$, as demonstrated by Wegener and Amin (2019), and gradually increase the cardinality to the total number of inefficient $DMUs$ in G.

Case 1: As an example, let $S = \{DMU_8, DMU_{11}\}$. When real-life scenarios are considered, an increase in the GDP per capita of an oil-producing nation is largely dependent upon an increase in oil production. The reason for this is the fact that the revenues generated from the exportation of crude oil and refined petroleum products take up a substantial portion of their GDP. There is, however, a tendency for such an increase in oil production to be accompanied by an increase in gas flaring as well. Therefore, this feasibility study will aim to determine, based on the application of the proposed inverse DEA, $M6$, an optimal increase in GDP per capita for the group of producers in S that will minimize an increase in gas flaring. Table 3.3 presents three different scenarios for potential growth in GDP per capita for the selected producers. From Table 3.3, it is evident that both producers can achieve a combined GDP per capita growth rate of US$100/person, US$200/person, and US$300/person without increasing their flare gas volumes (i.e., $\gamma_{1,8} = \gamma_{1,11} = 0$). Among the three scenarios in Table 3.3, only DMU_8 (Nigeria) achieved growth in GDP per capita (i.e., $\beta_{1,8} = \hat{y}_1^g$). However, we see that DMU_{11}(Venezuela) has no potential for an increase in GDP per capita (i.e., $\beta_{1,11} = 0$) under the same scenarios. Considering the results of table 3, Nigeria currently dominates Venezuela when both producers are tasked with meeting a combined growth in GDP per capita. Consequently, it is imperative to investigate further by extending the cardinality of the subset S from 2 to 6 $DMUs$. With the addition of DMU_1(Algeria) to the set S, there was no increase in their current levels of flare gas as shown in Table 3.4 (i.e., $\gamma_{1,1} = \gamma_{1,8} = \gamma_{1,11} = 0$). However, like Table 3.3, only DMU_1 could achieve an increase in GDP per capita in each scenario. Accordingly, the other two producers (i.e., DMU_8 and DMU_{11}) cannot boost GDP per capita without a corresponding increase in flare gas. It can be argued that DMU_1 has the greatest potential for GDP growth while maintaining its current flare gas level. The sample size of S was increased to four, five, and six to determine if this dominance of DMU_1 remains true for other combinations with the same increases in GDP per capita. These other three samples had the same results, with DMU_1 being the only producer to increase GDP per capita, although no increase in flare gas levels was observed in any of the scenarios. Therefore, it is imperative to also investigate this dominance in the reverse direction by testing sample sizes of S, beginning with $t,\ t-1,\ t-2,\dots,1$. The findings revealed a consistent pattern of dominance where the inefficient DMU with the lowest value of k is assigned to dominate the other DMUs in S. For instance, at $t = 6$, just like the original set, G, DMU_1 (i.e., having the smallest k value of 1) was the only producer that achieved growth in GDP per capita. When DMU_1 is removed from set S, the smallest k value in the set becomes 4. As a result, DMU_4 becomes the dominant unit. This resulted in DMU_{11} being placed at the bottom of the dominance hierarchy because of this pattern. Therefore, the dominance hierarchy created by the proposed model, $M6$, can be summarized as follows:

$$DMU_1 > DMU_4 > DMU_5 > DMU_8 > DMU_{10} > DMU_{11}$$

Generally, there are many other combinations of $DMUs$ that can form the subset S, but we can see from the examples presented here that only one DMU per combination dominates or has room for growth in GDP per capita. This further explains why, in the first example, only DMU_8 with a lower k value increased GDP per capita in comparison to DMU_{11} with a higher k value. To put it another way, a lower k value indicates a dominant DMU. This can be expressed mathematically as follows:

DEFINITION: Let DMU_p and DMU_q be two inefficient $DMUs$ in S, then through the application of model $M6$, DMU_p is dominant over DMU_q, if and only, if $p < q$. The converse also holds for both $DMUs$.

Based on the above definition, the second case of this feasibility study assesses only one DMU separately to examine its potential for an increase in GDP per capita, flare gas minimization abilities, and input increase requirements.

Table 3.3: Optimal volumes of flare gas for two producers

Increase in GDP per capita	$\beta_{1,8}$	$\beta_{1,11}$	$\gamma_{1,8}$	$\gamma_{1,11}$
$\hat{y}_1^g = 100$	100	0.0000	**0.0000**	**0.0000**
$\hat{y}_1^g = 200$	200	0.0000	**0.0000**	**0.0000**
$\hat{y}_1^g = 300$	300	0.0000	**0.0000**	**0.0000**

Table 3.4: Optimal volumes of flare gas for three producers

Increase in GDP per capita	$\beta_{1,1}$	$\beta_{1,8}$	$\beta_{1,11}$	$\gamma_{1,1}$	$\gamma_{1,8}$	$\gamma_{1,11}$
$\hat{y}_1^g = 100$	100	0.0000	0.0000	**0.0000**	**0.0000**	**0.0000**
$\hat{y}_1^g = 200$	200	0.0000	0.0000	**0.0000**	**0.0000**	**0.0000**
$\hat{y}_1^g = 300$	300	0.0000	0.0000	**0.0000**	**0.0000**	**0.0000**

Case 2: Here a separate analysis is conducted for each $DMU \in G$ or a case where $|S| = 1$. In this second case, every DMU is evaluated for its potential to increase GDP per capita by US$100. The proposed model $M6$ also calculates the change in inputs required by each DMU to increase its GDP per capita. Tables 3.5 through 3.10 present the results obtained. Clearly, the results within Tables 3.5 to 3.10 indicate that each DMU or producer can increase its GDP per capita by US$100/person without causing an increase in flare gas. Interestingly, Table 3.6 revealed that Indonesia could achieve this growth in GDP per capita even without increasing inputs. For Indonesia, this means that all inputs are fully utilized, and to increase its GDP per capita, it would be necessary to improve production efficiency or reduce production losses. An example of a production loss can be attributed to oil spills that occur during the oil extraction process. In turn, this results in a loss of oil revenue, which contributes substantially to the GDP of the nation.

Table 3.5: Optimal changes in inputs and flare gas volume for Algeria

Increase in GDP per capita	$\alpha_{1,1}$	$\alpha_{2,1}$	$\alpha_{3,1}$	$\alpha_{4,1}$	$\alpha_{5,1}$	$\gamma_{1,1}$
$\beta_{1,1} = 100$	25370.47	112.6204	0.0000	7.3036	143.2683	**0.0000**

Table 3.6: Optimal changes in inputs and flare gas volume for Indonesia

Increase in GDP per capita	$\alpha_{1,4}$	$\alpha_{2,4}$	$\alpha_{3,4}$	$\alpha_{4,4}$	$\alpha_{5,4}$	$\gamma_{1,4}$
$\beta_{1,4} = 100$	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**

Table 3.7: Optimal changes in inputs and flare gas volume for Iraq

Increase in GDP per capita	$\alpha_{1,5}$	$\alpha_{2,5}$	$\alpha_{3,5}$	$\alpha_{4,5}$	$\alpha_{5,5}$	$\gamma_{1,5}$
$\beta_{1,5} = 100$	0.0000	175.8459	1107.407	0.0000	0.0000	**0.0000**

Table 3.8: Optimal changes in inputs and flare gas volume for Nigeria

Increase in GDP per capita	$\alpha_{1,8}$	$\alpha_{2,8}$	$\alpha_{3,8}$	$\alpha_{4,8}$	$\alpha_{5,8}$	$\gamma_{1,8}$
$\beta_{1,8} = 100$	0.0000	72.83294	791.4443	0.0000	0.0000	**0.0000**

Table 3.9: Optimal changes in inputs and flare gas volume for UAE

Increase in GDP per capita	$\alpha_{1,10}$	$\alpha_{2,10}$	$\alpha_{3,10}$	$\alpha_{4,10}$	$\alpha_{5,10}$	$\gamma_{1,10}$
$\beta_{1,10} = 100$	16329.65	134.6359	0.0000	7.130176	136.1426	**0.0000**

Table 3.10: Optimal changes in inputs and flare gas volume for Venezuela

Increase in GDP per capita	$\alpha_{1,11}$	$\alpha_{2,11}$	$\alpha_{3,11}$	$\alpha_{4,11}$	$\alpha_{5,11}$	$\gamma_{1,11}$
$\beta_{1,11} = 100$	10913.23	0.0000	0.0000	0.0000	0.0000	**0.0000**

This feasibility study suggests that, since the proposed inverse DEA, $M6$, was successful in preventing an increase in flare gas while increasing GDP per capita for each producer, the further developed inverse DEA, $M8$, will surely reduce flare gas at current GDP per capita. Using the developed model $M8$, the next section provides a detailed analysis of the main findings of this chapter.

3.8.2 Main results

There are two scenarios in which the research findings of this chapter are discussed: one with eleven OPEC members and the other with thirteen OPEC members. As a starting point, the first scenario uses the inefficiency scores of the eleven producers presented in Table 3.2 and their subsets of efficient and inefficient producers.

<u>2011 Production Year (Scenario 1: Eleven OPEC members)</u>

Recall from Table 2, the following subsets:

$F = \{\mathrm{DMU}_2, \mathrm{DMU}_3, \mathrm{DMU}_6, \mathrm{DMU}_7, \mathrm{DMU}_9\}$, and

$G = \{\mathrm{DMU}_1, \mathrm{DMU}_4\ \mathrm{DMU}_5, \mathrm{DMU}_8, \mathrm{DMU}_{10} \mathrm{DMU}_{11}, \}$.

Based on the application of the developed model $M8$, each inefficient DMU in G requires its own analysis.

Taking DMU_8 (Nigeria) as our main case study, $M8$ was applied to determine the potential reduction in gas flaring. From the solution report, the potential reduction in gas flaring for the Nigerian petroleum industry is $\gamma_8 = 382.9901$million cubic metres from a total flare of 14270 million cubic metres in 2011.

This is equivalent to $\gamma_8 = 382.9901 \times 35.3\ million\ cubic\ feet$

$$\gamma_8 = 13519.55 million\ cubic\ feet$$

$$\gamma_8 = 13519550\ thousand\ cubic\ feet$$

At US$3.00 per thousand ft^3 of natural gas, the economic loss equals:

$$\gamma_8 = 13519550\ thousand\ cubic\ feet \times \$3.00 = \text{US}\$40{,}558{,}650$$

3.8.3 Application of Proposed Algorithm

Through steps 1 to 6, we obtain $\gamma_8^{max} = 13020.17$ million cubic meters. This is equivalent to an economic loss of US$1.379 billion. From Table 3.2, the actual volume of routinely flared gas by DMU_8 is $y_8^b = 14270$ million cubic meters. Now, since $y_8^b > \gamma_8^{max}$, we conclude that the Nigerian petroleum industry could not adopt the initiative in 2011, assuming the industry is relatively efficient compared to the other OPEC member nations. However, the deviation from this initiative (i.e., $y_8^b - \gamma_8^{max}$ =1249.83 million cubic metres) suggests that a continual investment in better technology and skilled labor can achieve this goal within the decade.

3.8.4 Sensitivity Analysis

We find from sensitivity analysis that as the inefficiency measure of DMU_8 (i.e., $\theta_8 = 0.91$) decrease in steps of 0.1, the potential reduction, γ_8 increases. When $\theta_8 = 0$, γ_8 achieves a maximum value $\gamma_8^{max} = 13020.17$ million cubic metres from a total flare of 14270 million cubic metres in 2011. This implies an inverse relationship between θ and γ. Fig. 3.6 shows that the measure of inefficiency is a function of the potential reductions. The sensitivity analysis proved to be in accordance with our developed algorithm for the zero routine flaring initiative.

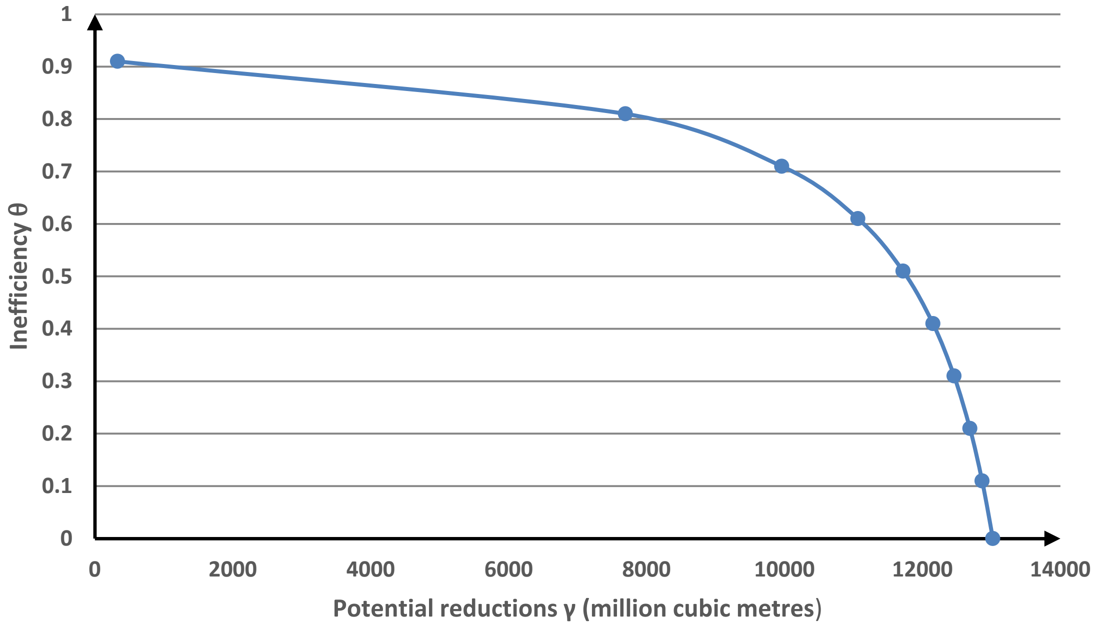

Figure 3.6: Effect of inefficiency on potential reductions in gas flaring for Nigeria

Recall from section 3.7.5, that our proposed theorem states that the maximum potential reduction occurs when the inefficiency score of an inefficient unit approaches zero. This theorem is the underlying principle of step 6 in our algorithm. It is clear from Fig. 3.6 that as the inefficiency score of DMU_8 approaches zero, the potential reduction approaches a maximum value.

3.8.5 Comparative Analysis

We need to apply our extended model and algorithm to $DMU_1, DMU_4, DMU_5, DMU_{10}$ and DMU_{11}, and compare their results with those of DMU_8. The aim is to validate our model and algorithm and to determine which DMU is more committed to the zero-routine initiative.

Table 3.11 presents the obtained results, including the computed deviations. The maximum reductions in gas flaring for all six inefficient producers amounts to 36.1 billion cubic metres, equivalent to US$3.82 billion. It is important to state here that these reductions are computed by our model relative to the inputs and outputs of the efficient producers. Similarly, all five DMUs could not adopt the initiative in 2011 and the deviations can be attributed to non-routine flaring (i.e., safety and maintenance flaring). In terms of least deviation, it is clear from Table 3.11, that DMU_{10}(UAE) is most committed to the zero-routine initiative while

DMU_8(Nigeria) is the least committed because it has the largest deviation. In order of least deviation, we rank them as follows:

$$\mathrm{DMU}_{10} > \mathrm{DMU}_{11} > \mathrm{DMU}_4 > \mathrm{DMU}_1 > \mathrm{DMU}_5 > \mathrm{DMU}_8$$

However, in terms of other yardsticks such as maximum potential revenue, maximum energy savings and maximum potential reductions, our case study, DMU_8 (Nigeria) outperformed the other five DMUs. This implies, to a reasonable extent, that DMU_8 stands a far better chance of improvement with investment in better technology that will place it on par with the other efficient DMUs. In this regard, the efficient DMUs of the subset F serve as the ideal benchmarks for DMU_8. Fig. 3.7 gives a perfect illustration of the scenarios involving maximum savings and makes the strong case that all inefficient DMUs have ample room for reduction in routine gas flaring. The maximum reductions are quite high compared to the other yardsticks. In addition, the low deviations are indicators that the producers are on track to achieving the zero routine initiative provided further investments are made. Fig. 3.8 illustrates the potential improvements with current technology. Here the potential improvements refer to the case where producers are not willing to invest in better technology due to financial constraints or the unpredictability of global oil prices.

Table 3.11: Summary of results for inefficient DMUs (Scenario 1)

DMU	Potential Reduction γ_k^{min} $(m\ cu^3)$	Flare Reduction (%)	Potential Revenue $(m\ \$)$	Maximum Potential Reduction γ_k^{max} $(m\ cu^3)$	Maximum Flare Reduction (%)	Maximum Potential Revenue $(m\ \$)$	Deviation $(y_k^b - \gamma_k^{max})$ $(m\ cu^3)$
1-Algeria	23.69	0.66	2.51	2995.35	83.11	317.20	608.65
4-Indonesia	39.51	1.61	4.18	1921.25	78.35	203.46	530.75
5-Iraq	658.56	6.85	69.74	8806.19	91.61	932.58	805.81
8-Nigeria	382.99	2.68	40.56	13020.20	91.24	1379	1249.83
10-UAE	16.63	1.69	1.76	547.58	55.76	57.99	434.42
11-Venezuela	1229.85	13.25	130.24	8825.04	95.06	934.57	458.96
$Total$	2351.23			36115.61		3824.8	

3.8.6 Optimal Sizing Computations

In section 3.7.6, we introduced the optimal sizing expression for GTW system as:

$$\frac{\gamma_k^{min}}{\pi_{uk}} \leq \varphi_{uk} \leq \frac{\gamma_k^{max}}{\pi_{uk}}$$

Our model computes γ_p^{min} and γ_p^{max} for all inefficient producers. Suppose all producers decide to use same model of turbine for conversion of flared gas to electricity, this implies the annual gas usage, π_{uk}, for each turbine is the same for all producers. For this study, we consider the ALSTOM GT13E2 gas turbine with detailed specifications provided by Ojijiagwo et al. (2016). Its power output is capped at 150MW. The daily gas usage requirement of this turbine was estimated in their study to be 0.93 million cubic metres. This is equivalent to 339.45 million cubic metres per year (i.e., 0.93 x 365 days). Using this value, we present an optimal sizing for all six producers in Table 3.12. Also, the number of turbines, φ_k, can only take integer values. From the results presented in Table 3.12, it is obvious that Iraq, Nigeria, and Venezuela show more potential for power generation via the GTW process. Nigeria and Venezuela are currently experiencing energy supply crises, so this study presents a big opportunity for both nations to solve their energy crisis.

Table 3.12: GTW optimal sizing for inefficient producers (Scenario 1)

DMU	Units of Turbine (φ_k)
1-Algeria	$0 \leq \varphi_k \leq 8$
4-Indonesia	$0 \leq \varphi_k \leq 5$
5-Iraq	$1 \leq \varphi_k \leq 25$
8-Nigeria	$1 \leq \varphi_k \leq 38$
10-UAE	$0 \leq \varphi_k \leq 1$
11- Venezuela	$3 \leq \varphi_k \leq 25$

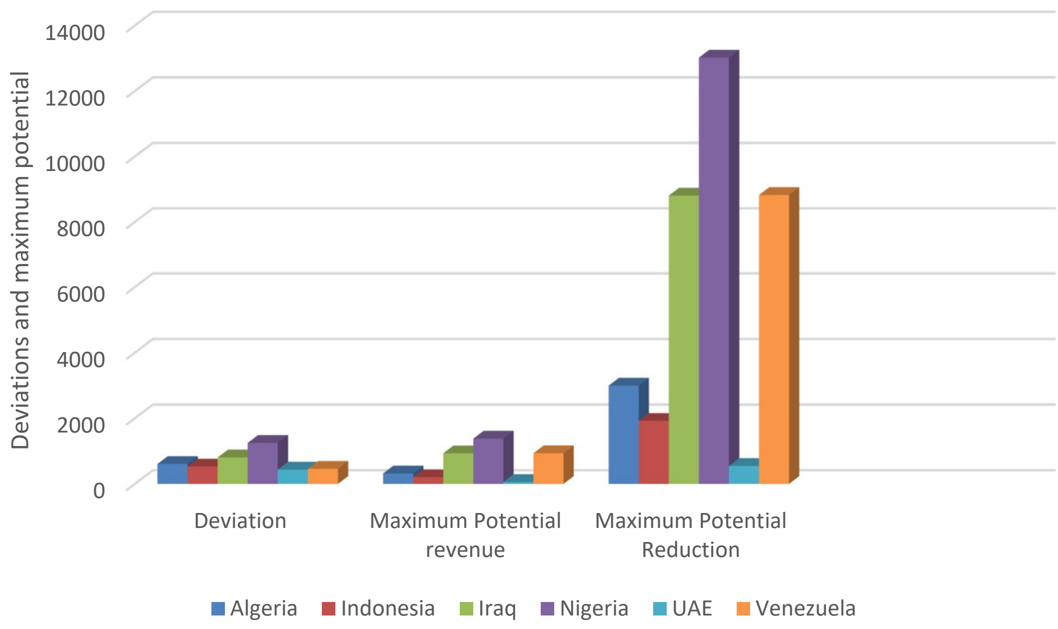

Figure 3.7: Deviations and Maximum Potential Improvements

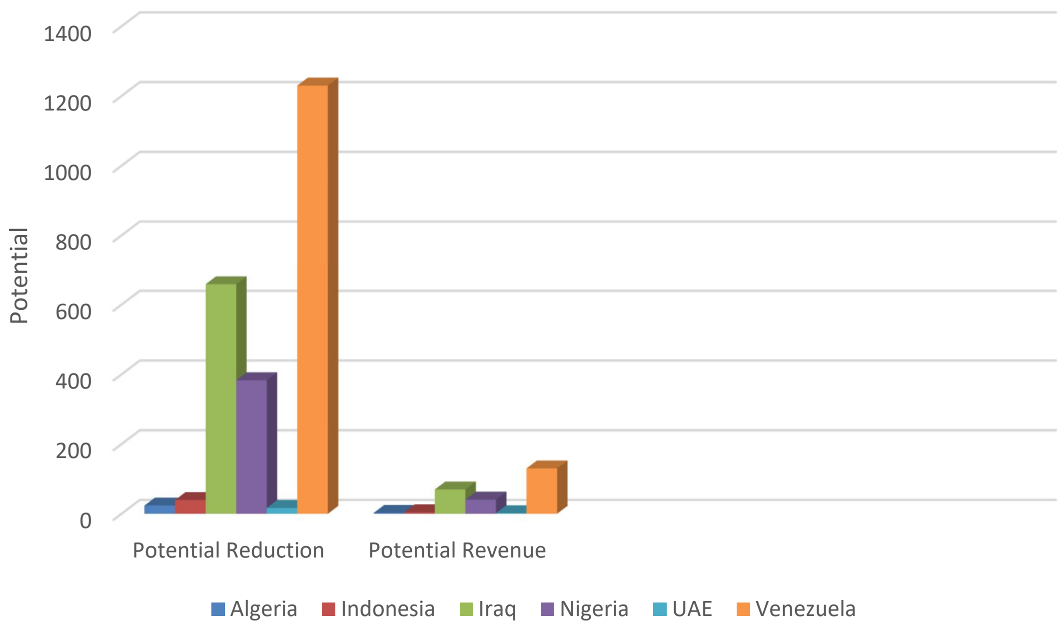

Figure 3.8: Potential Improvements for inefficient producers

To further support our claim that DMU inefficiency is a function of the potential reductions, we extend the sensitivity analysis to the other five inefficient DMUs. The results are presented in Figs. 3.9 to 3.13. In each case, the inefficiency score was decreased in similar steps of 0.1 and the maximum reduction occurred at zero inefficiency or when the DMU became efficient.

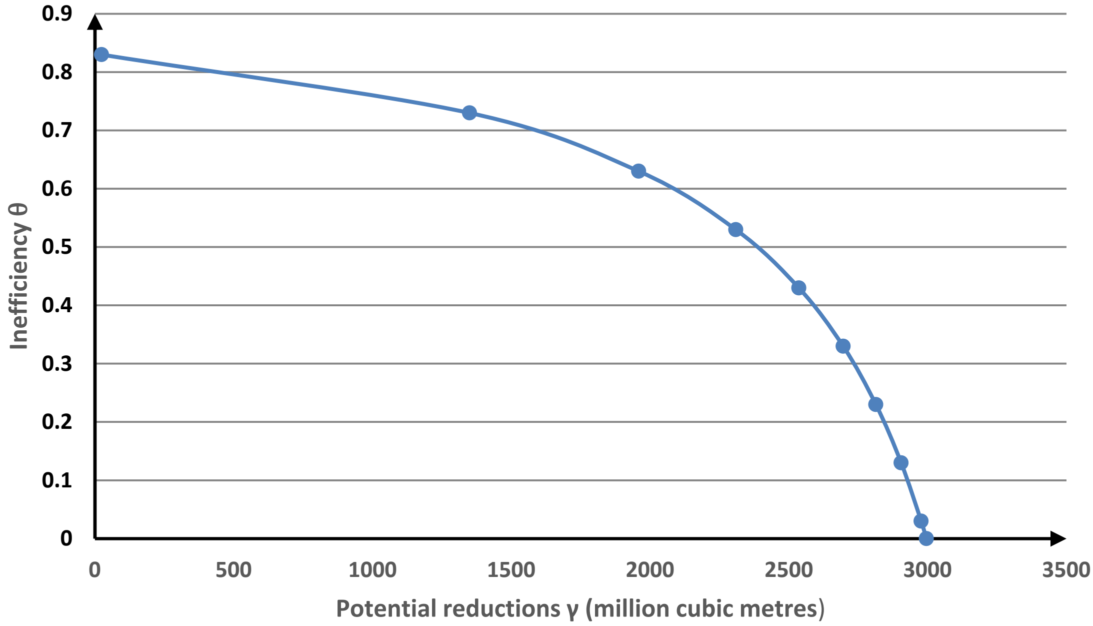

Figure 3.9: Effect of inefficiency on potential reductions in gas flaring for Algeria

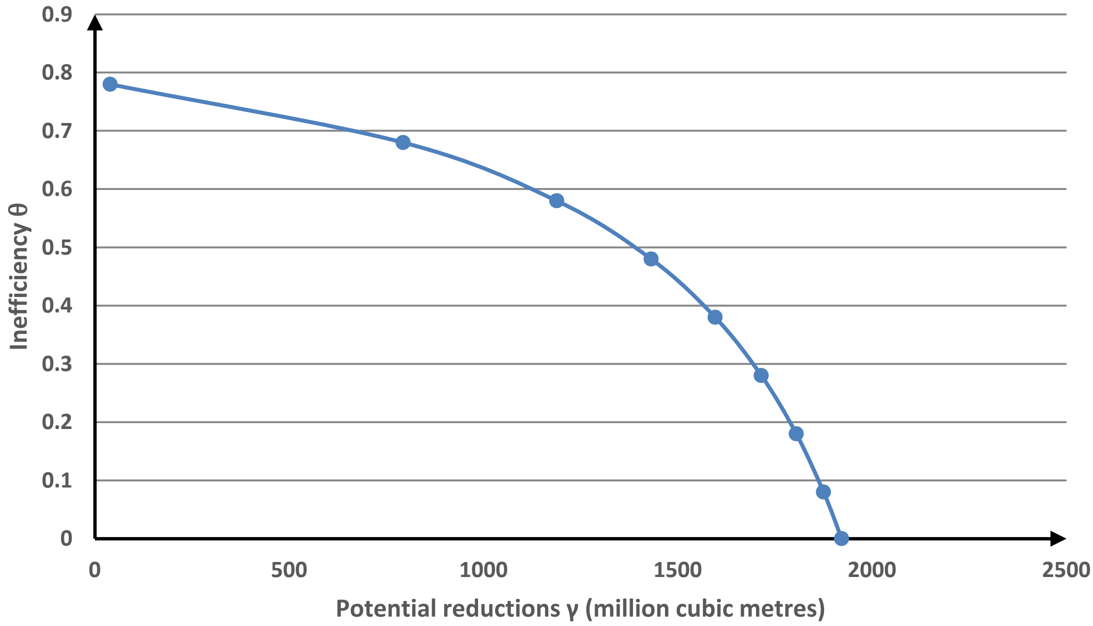

Figure 3.10: Effect of inefficiency on potential reductions in gas flaring for Indonesia

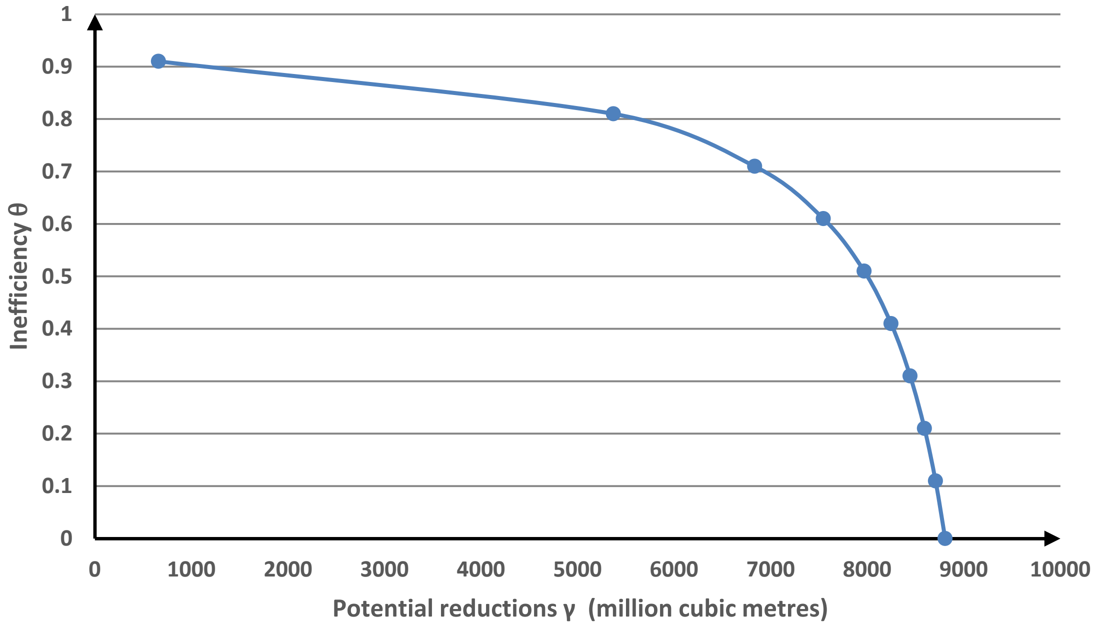

Figure 3.11: Effect of inefficiency on potential reductions in gas flaring for Iraq

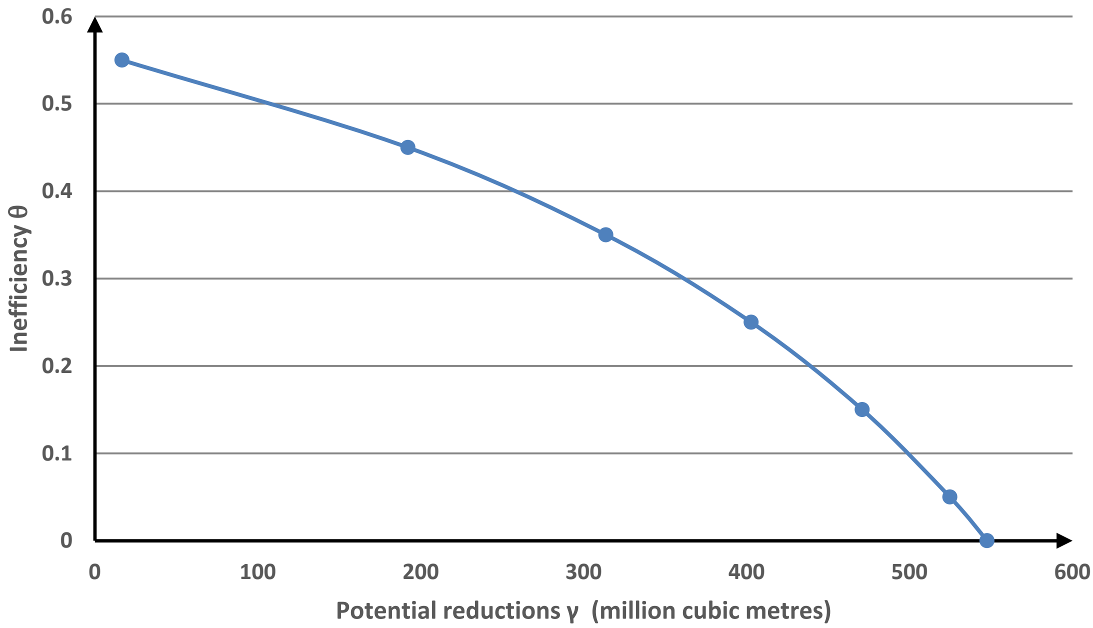

Figure 3.12: Effect of inefficiency on potential reductions in gas flaring for UAE

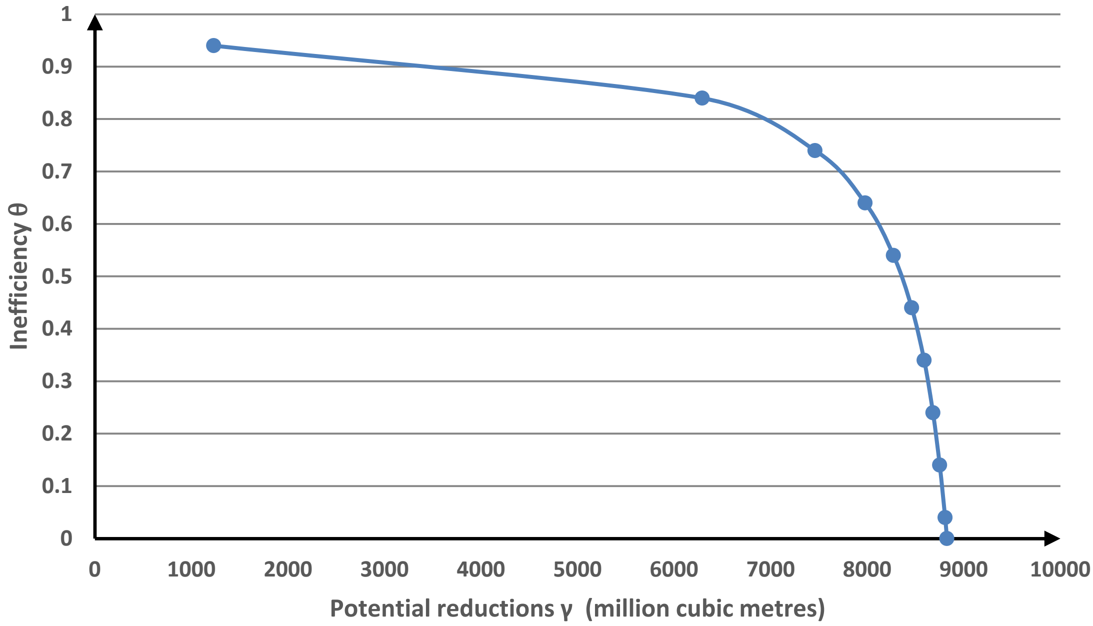

Figure 3.13: Effect of inefficiency on potential reductions in gas flaring for Venezuela

3.8.7 Analysis of Second Scenario

<u>2011 Production Year (Scenario 2: Thirteen OPEC members)</u>

In this section, we consider all the 13 OPEC members by adding Saudi Arabia and Iran to the PPS. We assume that both members had zero routine flaring for the 2011 production year. This makes them ideal benchmarks for the inefficient producers identified by model $M5$. Table 3.13 presents the initial results containing the inefficiency score for each producer. Once again, we have the same number of inefficient DMUs, like Table 3.2, but with two more efficient DMUs taking the number to seven. We define new subsets as follows:

$F = \{\mathrm{DMU}_2, \mathrm{DMU}_3, \mathrm{DMU}_5, \mathrm{DMU}_7, \mathrm{DMU}_8, \mathrm{DMU}_{10}, \mathrm{DMU}_{11}\}$, and

$G = \{\mathrm{DMU}_1, \mathrm{DMU}_4\ \mathrm{DMU}_6, \mathrm{DMU}_9, \mathrm{DMU}_{12}\mathrm{DMU}_{13}, \}$.

Table 3.13: Data and inefficiency scores for 13 OPEC member nations

DMU	Current Account Balance (m US$)	Wells Completed	Producing Wells	Active Rigs	Refining Capacity (1000b/cd)	GDP per Capita (US$/person)	Routinely Flared Gas (M cu m)	Ineff. (θ)
1-Algeria	17770	249	2010	33	592	5453.5	3604	0.83
2-Angola	13085	112	1476	22	65	4666.95	7183	0
3-Ecuador	-402	207	3079	39	188.4	5193.04	539	0
4-Indonesia	1685	838	10423	80	1125	3121	2452	0.78
5-Iran	59364	204	2026	123	1715	7511.1	0	0
6-Iraq	26365	76	1695	59	810	5571.55	9612	0.91
7-Kuwait	65743	523	1798	32	936	41672	217	0
8-Libya	3173	76	609	55	380	5858	1302	0
9-Nigeria	10757	124	2116	38	445	2451.75	14270	0.91
10-Qatar	51906	29	517	6	283	97983.27	558	0
11-Saudi Arabia	158545	312	3245	121	2107	23594.13	0	0
12-UAE	50948	266	1592	19	675	40819.31	982	0.55
13-Venezuela	16342	1050	14915	116	1872	10283.2	9284	0.94

Table 3.14 summarizes the results. Except for DMU_4 (Indonesia) and DMU_{13} (Venezuela), the other four DMUs had same values of reductions and savings, like those of Table 3.11. With the addition of two new members to the PPS, we observe slight improvements in reductions and savings for DMU_4 and DMU_{13}. In order of least deviation, we define a new ranking:

$$\mathrm{DMU}_{13} > \mathrm{DMU}_{12} > \mathrm{DMU}_4 > \mathrm{DMU}_1 > \mathrm{DMU}_6 > \mathrm{DMU}_9$$

DMU_9(Nigeria) is still better than all five in terms of maximum savings and potentials. We also present the optimal sizing for this scenario in Table 3.15. Only Venezuela had changes in sizing compared to Table 3.12. This implies that by expanding the production possibility set (PPS) with the addition of two efficient producers (i.e., Saudi Arabia and Iran), only slight changes were observed in the overall optimal sizing.

Table 3.14: Summary of results for inefficient DMUs (Scenario 2)

DMU	Potential Reduction γ_k^{min} $(m\ cu^3)$	Flare Reduction (%)	Potential Revenue $(m\ \$)$	Maximum Potential Reduction γ_k^{max} $(m\ cu^3)$	Maximum Flare Reduction (%)	Maximum Potential Revenue $(m\ \$)$	Deviation $(y_k^b - \gamma_k^{max})$ $(m\ cu^3)$
1-Algeria	23.69	0.66	2.51	2995.35	83.11	317.20	608.65
4-Indonesia	64.26	2.62	6.81	1928.3	78.63	204.21	523.7
6-Iraq	658.56	6.85	69.74	8806.19	91.61	932.58	805.81
9-Nigeria	382.99	2.68	40.56	13020.20	91.24	1379	1249.83
12-UAE	16.63	1.69	1.76	547.58	55.67	57.99	434.42
13-Venezuela	1502.58	16.18	159.12	8869.33	95.53	939.26	414.67
$Total$	2648.71			36166.95		3830.24	

Table 3.15: GTW optimal sizing for inefficient producers (Scenario 2)

DMU	Units of Turbine (φ_k)
1-Algeria	$0 \leq \varphi_k \leq 8$
4-Indonesia	$0 \leq \varphi_k \leq 5$
5-Iraq	$1 \leq \varphi_k \leq 25$
8-Nigeria	$1 \leq \varphi_k \leq 38$
10-UAE	$0 \leq \varphi_k \leq 1$
11- Venezuela	$4 \leq \varphi_k \leq 26$

3.8.7.1 GTW for Case Study

In this section, we select Nigeria as our case study and carry out a separate analysis for GTW power generation.

Out of 7141MW of power plants' capacity, only 3879MW is currently available for power generation in Nigeria. This leaves a loss in capacity of 3262MW. We obtained the maximum sizing of 38 turbine units for our case study. At 150 MW power output, assuming all turbines are operating at full load, this will give 5700MW for power generation. This is more than enough to cover the loss in capacity. Thus, our analysis makes the case for an investment in

GTW. The economic analysis for such investment is well detailed in Ojijiagwo et al. (2016) and will not be discussed here.

3.8.8 Model Performance

To examine the performance of our proposed model to some extent, we compare our result for Nigeria with previous works, within same time frame. Giwa et al. (2014) estimated the average annual volume of gas flared in Nigeria over a 49-year period (1965 – 2013) to be 18.27 billion cubic metres. Using this estimate, Ojijiagwo et al. (2016) provided an economic analysis with 50 units of the ALSTOM GT13E2 gas turbine resulting in 92.89% flare reduction for Nigeria. Their calculated return on investment was 16.3%, which is greater than the recommended minimum of 15% for any good investment.

The 49-year period highlighted in Giwa et al. (2014) is inclusive of the production year used for this study (i.e., 2011). For the 2011 production year, our proposed model obtained a 91.24% flare reduction for Nigeria in both scenarios. This is a satisfactory performance because the average flare volume stated in Ojijiagwo et al. (2016) is a better representative of flare data and we relaxed some assumptions in the development of our proposed model. It is imperative to state here that our model is well suited for imposing gas flaring reduction on a group of homogenous oil producers. Gas flaring is not confined to a region or a single nation, it is a global problem. If we are to address climate change and global warming, we need the combined effort of oil producing nations to mitigate gas flaring. Our methodology also classifies nations into efficient and inefficient oil producers, so that the inefficient producers can standardize their processes with better technology used by the efficient producers. This is the underlying principle of its evaluation mechanism. Most importantly, our model serves as an important tool for making smart decisions regarding the optimal range of turbine units for implementing the GTW process. For both scenarios, the 2011 reductions in gas flaring for the Nigerian petroleum industry are equal. We conclude our analysis by saying, with the addition of efficient producers (i.e., Saudi Arabia and Iran) to the production possibility set, there are no better results for Nigeria. This implies our proposed model and algorithm yielded the best results for our case study.

3.9 Summary

The future challenges for crude oil make waste management an integral part of corporate long-term planning in the petroleum industry. Routine gas flaring in the industry is an environmental hazard and a multi-billion-dollar waste. In this chapter, we extended the inverse data envelopment analysis (DEA) model for estimating potential reductions in gas flaring. The potential reductions served as a template for computing the optimal range of turbine units required for the gas-to-wire (GTW) process.

We applied our proposed model to member nations of the organization of the petroleum exporting countries (OPEC). We found six nations were inefficient producers. The obtained results showed that the maximum potential reductions in gas flaring for Algeria, Indonesia, Iraq, Nigeria, UAE, and Venezuela are 83.11%, 78.35%, 91.62%, 91.24%, 55.76% and 95.06%, respectively. Based on the obtained reductions, and for all six producers, the turbine units required for the GTW process are 8, 5, 25, 38, 1 and 25, respectively. Iraq, Nigeria, and Venezuela had the highest potential for the conversion of flared gas to electricity using the GTW process. In addition, our proposed model estimated the total potential reduction in global gas flaring across all six oil producing nations to be 36.11 billion cubic meters (BCM). This is equivalent to a potential revenue of US$3.8 billion. In real life scenarios, it is practically difficult for the industry to achieve a zero-routine flare with current technology. In this connection, we developed an algorithm that serves as a tool for computing deviations from a zero-routine flare. The deviations indicate the level of commitment of an oil producing nation to the zero-routine flaring initiative launched by the World Bank. The smaller the deviations, the cleaner the production process. We found that the UAE had the smallest deviation of 0.434 BCM in the first scenario, while Venezuela had the least deviation of 0.414 BCM in the second scenario. However, Nigeria had the largest deviation of 1.250 BCM in both scenarios.

Our extended inverse DEA model provided satisfactory results for a set of homogenous oil producers and is the first to estimate the potential reductions in global gas flaring. However, one limitation is that our model imposes gas flare reductions on inefficient producers but lacks the capability of applying the same technique to efficient producers. On this basis, further research is needed for improving the performance of the efficient producers.

Chapter 4 – A Lean Production Approach to Flare Gas Management

4.1 Introduction

An overview of the gas flaring process, including its main causes, as well as World Bank policies regarding the reduction of global gas flaring was presented in the previous chapter. With a view toward strengthening the developed models and techniques for addressing global gas flaring, this chapter examines the issue more from an economic and environmental perspective. Simply put, there is a need to establish a balance between industry needs (i.e., oil production) and environmental protection. The problem is more dynamic, the kind involving marginal increases in oil production and less environmental waste in the form of flare gas. Thus, it is imperative to investigate any potential trend between oil production and flare gas. Further, the purpose of this chapter is to improve the overall performance of *efficient* oil-producing nations, in order to overcome the limitations of the previous chapter, which focused only on reducing gas flaring in *inefficient* oil-producing nations.

The extraction of oil from reservoirs results in the release of substantial amounts of methane. In oil wells around the world, approximately 140 billion cubic meters of methane is flared annually, creating a significant amount of greenhouse gas (i.e., CO_2) (Elvidge et al., 2009). Furthermore, gas flaring creates heat, air pollution, and noise, which constitute its environmental impact. Flaring of natural gas wastes non-renewable resources. Besides contributing significantly to CO_2 emissions, gas flaring also deprives developing countries of an affordable energy source. Additionally, the volatility of oil prices makes it imperative for gas economies, both mature and developing, to diversify their revenue streams in order to secure a steady stream of income. The United Nations and other international organizations have developed policies aimed at reducing gas flaring, and such policies are crucial to reducing the environmental impact of gas flaring. According to Figure 4.1, gas flaring is correlated with oil production. The uninitiated have sometimes claimed that the volume of gas flared is positively correlated with the volume of oil produced. This claim is not always true because, based on Figure 4.1, gas flaring declined in tandem with an increase in oil production from 2006 to 2010, before rising in 2018. Nevertheless, this chapter will test the validity of this claim. As well, experts in the petroleum industry have considered the feasibility of increasing oil production while reducing environmental waste (i.e., flare gas). It is a win-win situation for both the industry and the environment. The feasibility of such a scenario, however, has not been investigated using any methodology or strategy. To find out whether such a scenario can be realized in the industry, this chapter develops a holistic framework based on lean practices.

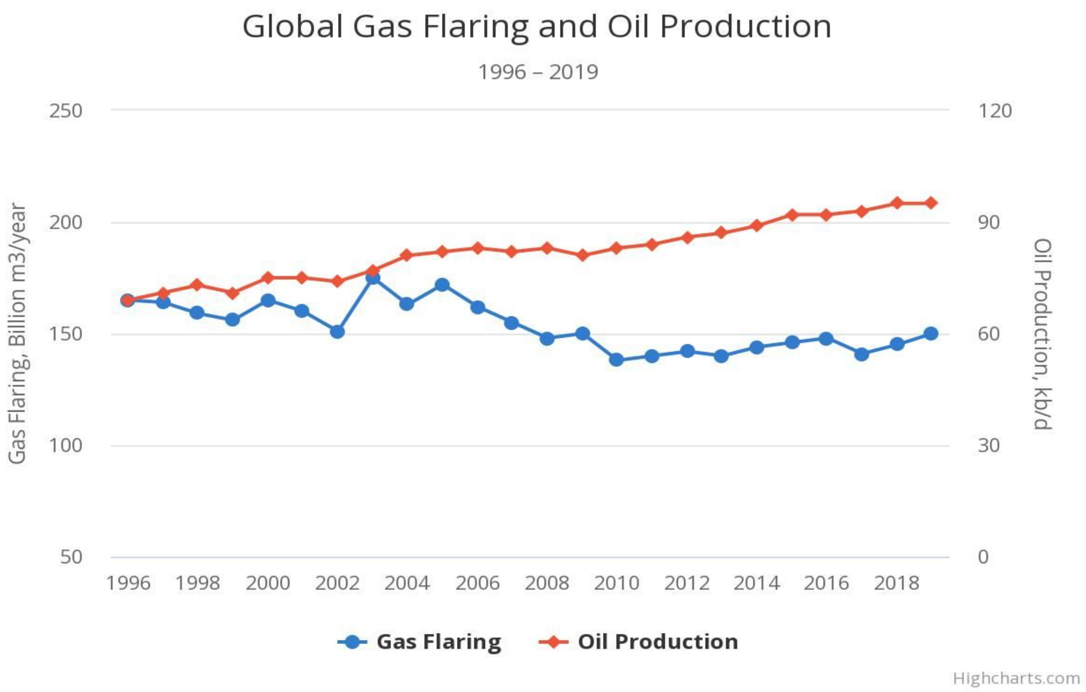

Figure 4.1: Historical trend of gas flaring and oil production

(*Note*: Adopted from *Global Gas Flaring Reduction Partnership*. (https://www.ggfrdata.org/). Copyright 2020 by the World Bank Group)

4.2 Environmental impact of gas flaring

As a result of gas flaring, there are constituents that, when released into the environment, contribute to global warming. This includes carbon dioxide (CO_2). Climate change is a natural phenomenon that is caused by global warming, and it is a phenomenon that is hazardous to humans as well as ecosystems. Due to climate change, wildlife and humans living in arctic environments are vulnerable to problems like melting arctic glaciers and desertification. Among other impacts are flooding and displacement of people and wildlife caused by extreme weather patterns and increased rainfall. People living along ocean coastlines are also threatened by rising coastal waters. The gas flaring process in Nigeria releases approximately 47 million tons of CO_2 and methane into the atmosphere each year. Flooding and other adverse consequences have often resulted from this (Ojijiagwo et al., 2016). GHG emissions from gas flaring consistently destroy the ozone layer that serves as a protective shield from harmful ultraviolet waves (Sekyi, 2017). Moreover, the chemical composition of gas flaring emissions adversely affects the environment. A consequence of gas flaring is acid rain. Snow, fog, or small particles that settle on the ground may be evidence of this type of precipitation, which is primarily composed of sulphuric acid. Forests at higher altitudes are destroyed by acid rain. Plant yields are low, adaptations are poor, and plant production is inhibited by the leaching of essential nutrients (Sekyi, 2017).

A few health complications are caused by gas flaring, such as respiratory illnesses and heat burns. Gas flaring leads to air pollution which produces toxic emissions such as carbon monoxide, hydrogen sulfide, and nitrogen oxide. In large amounts, carbon monoxide can cause blood poisoning and high blood pressure, hydrogen sulfide can irritate soft tissues, and nitrogen oxide can damage tissues. Those who consume locally produced goods, whether they produce it themselves or purchase it at market outlets, are at risk of exposure to harmful health effects associated with air pollution. Benzene, for instance, is a harmful substance that pollutes the air from gas flares that contain widely recognized toxins. Asthma and bronchitis are common respiratory problems that are being complained about by residents within the gas flaring regions. A variety of blood-related disorders have also been linked to benzene exposure in humans, according to the US Environmental Protection Agency (EPA). Approximately 49 premature deaths and 4960 respiratory illnesses among children are likely to be caused by gas flaring in Nigeria annually, according to information from the World Bank. Sulphur species are also formed during the flare of sour gas. It is believed that a number of these chemicals are potentially toxic, such as hydrogen sulfide and carbon disulfide. Smell-free hydrogen sulfide exposure can cause spontaneous abortion at levels below the odor threshold. Thyroid cancer is most caused by radioactivity. In geographical areas with extensive flaring operations, thyroid cancer has an elevated median rate ratio. There is also an association between environmental contaminants and autoimmune rheumatic diseases, endocrine dysfunction, and immune dysfunction. In addition to premature deaths and respiratory illnesses, gas flaring increases cancer risks in surrounding communities. The human body can only tolerate a certain amount of thermal pollution caused by gas flaring. Heat thresholds also apply to habitats and structures nearby.

Despite mitigation measures in some oil-producing nations, gas flaring continues to pose a significant threat to living organisms and the environment. Gas flaring activities are governed by a variety of policies that differ from state to state. Although the United Nations maintains policies such as the GGFR, it has limited capabilities when it comes to implementing them. It is illegal in most countries to flare gas, and such activities are governed by special policies.

4.3 Further mitigation measures for gas flaring

Actions and targets aimed at addressing climate change are incorporated into policies such as the Paris Agreement. A commitment called a Nationally Determined Contribution (NDC) is included in this policy. The Kyoto Protocol's Clean Development Mechanism provides reduction targets for gas flaring reduction policies (Elvidge et al., 2009). A variety of countries set regulations on flaring. In Nigeria and Equatorial Guinea, it is illegal to flare gas, for example. Certain exceptions can be made by governments. Despite reporting requirements at the state level, flaring is legal in the US. Zero Routine Flaring (ZRF) is a global initiative developed by the World Bank that promotes the development of gas infrastructure to a lesser extent. A key recommendation of the GGFR is to reform regulations to reduce the amount of waste or flare gas in the industry. By implementing country-specific flaring reduction programs, GGFR also addresses the major barriers to gas flaring reduction (Calel & Mahdavi, 2020).

Furthermore, oil-producing countries must increase their commitments to end routine flaring and advance measurement and reporting on it, as well as conduct research, raise awareness, and share the best practices in the markets. Government policies relating to gas flaring can be effectively and efficiently achieved through existing regulations and policies on resource management (Oyewunmi & Oyewunmi, 2016). In addition to laws and regulations that govern hydrocarbons, there is the possibility of embedding legal powers regarding gas flaring in secondary legislation, which includes licenses, codes, and guidelines. Natural resource management and oil production conditions are constantly changing, making gas flaring regulations adaptable and flexible. Flaring is assessed from an environmental perspective by government institutions tasked with undertaking regulatory actions and managing natural resources. A production license is revoked if it is not in compliance with state regulations.

4.4 Problem statement

All major oil-producing nations flare natural gas on a regular basis, which creates a global problem. The previous chapter categorized oil-producing nations into efficient and inefficient producers. Inverse DEA models were developed in the previous chapter to enable inefficient producers to deploy a cost-effective gas-to-wire (GTW) process. This leaves room for further research in order to improve the performance of efficient producers. Global gas flaring is therefore a two-sided problem, which leads to the question: *How can we mitigate gas flaring in efficient oil producing nations?*

Lean production has long been considered a promising technique for reducing waste in the petroleum industry. In this chapter, we employ the following lean practices that constitute a subset of lean production:

- Minimizing current level of waste or bad output (i.e., flare gas)
- Minimal use of input resources.
- Increasing productivity by setting targets for good output (i.e., crude oil).
- Waste conversion (i.e., recycling waste for power) as value added.

Based on these practices, this study utilizes inverse DEA as a lean tool to deal with inverse problems such as:

I. For a given producer and in the absence of flare gas recovery (FGR) technology, what changes in inputs are needed to produce the desired level of good output with no increase in associated waste or bad output?

II. With flare gas recovery (FGR) technology such as gas-to-wire (GTW) or gas-to-liquid (GTL), what changes in inputs are needed to produce a desired level of good output with a lower level of associated waste or bad output?

III. With GTW technology, how much power can be generated from recycled waste at the same level of productive efficiency?

4.5 Methodology

4.5.1 Nomenclature

The following notations and indices constitute the entire nomenclature for all the models developed and discussed in this chapter:

$General\ Parameters$:

n: number of decision-making units (DMUs)

m: number of inputs of each DMU

s: number of desirable outputs of each DMU

q: number of undesirable outputs of each DMU

$Data\ Parameters$:

x_{ij}: ith input of DMU_j $(j = 1, \dots, n)$

y_{rj}: rth output of DMU_j $(j = 1, \dots, n)$

y_{rj}^g: rth good output of DMU_j $(j = 1, \dots, n)$

y_{pj}^b: pth bad output of DMU_j $(j = 1, \dots, n)$

$\min\limits_r y_{rj}^g$: minimum rth good output of DMU_j $(j = 1, \dots, n)$

$\min\limits_p y_{pj}^b$: minimum pth bad output of DMU_j $(j = 1, \dots, n)$

$Decision\ Variables$:

θ_o: efficiency score of DMU_o $(o = 1, \dots, n)$

λ_j: weight assigned to DMU_j $(j = 1, \dots, n)$

4.5.2 Preliminaries

Charnes et al. (1978) proposed the following input-oriented CRS DEA model for evaluating the efficiency score of DMU_o, $o \in \{1,2, \dots, n\}$

$M(1)$:
$$
\begin{aligned}
& Min\ \ \theta_o^{CRS} \\
s.t.\ & \sum_{j=1}^{n} x_{ij}\lambda_j \le \theta_o^{CRS} x_{io,} \quad & i = 1,2, \dots, m \\
& \sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{ro,} \quad & r = 1,2, \dots, s \\
& \lambda_j \ge 0, \quad & j = 1,2, \dots, n
\end{aligned}
$$

$M1$ is commonly referred to as the conventional CCR model. However, $M1$ does not account for bad or undesirable outputs. Toward this end, Färe and Grosskopf (2006) proposed the following input oriented measure of efficiency for classifying good and bad outputs of DMU_o:

$M(2)$: $\quad Min\ \theta_o^{CRS}$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o^{CRS} x_{io}, \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^g \geq y_{ro}^g, \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^b = y_{po}^b, \qquad p = 1, \dots, q$$

$$\lambda_j \geq 0, \qquad j = 1, \dots, n$$

Both models can be solved under variable returns to scale (VRS) assumption by adding the constraint $\sum \lambda_j = 1$, and θ_o^{CRS} will be replaced with θ_o^{VRS}.

4.5.3 Inverse DEA for pollution control

Using $M(2)$ as a base model, Ghiyasi (2017a) analyzed a pollution generating work system where it is required to produce more of the good or desirable outputs with less of the bad or undesirable outputs. Mathematically, the inverse problem can be defined as:

If DMU_o changes its output levels from $\left(y_{ro}^g, y_{po}^b\right)$ to $\left(\beta_{ro}^g, \beta_{po}^b\right) = \left(y_{ro}^g + \Delta y_{ro}^g, y_{po}^b + \Delta y_{ro}^b\right)$, what is the required input change required to produce these desired levels of good and bad outputs, such that the efficiency score of DMU_o remains unchanged?

To solve this inverse problem, Ghiyasi (2017a) proposed the following multiple objective linear program:

$M(3)$: $\quad Min\ (\alpha_1, \alpha_2, \dots, \alpha_m) = (x_{1o+}\Delta x_1, x_{2o} + \Delta x_2, \dots, x_{mo} + \Delta x_m)$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o^{CRS} (x_{io} + \Delta x_i), \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^g \geq y_{ro}^g + \Delta y_{ro}^g, \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^b = y_{po}^b + \Delta y_{po}^b, \qquad p = 1, \dots, q$$

$$\lambda_j \geq 0, \qquad j = 1, \dots, n$$

By using the weighted sum technique, the optimal solution to $M(3)$ can be obtained by changing the objective function to $\sum_{i=1}^{m} \alpha_i$. Similarly, $M(3)$ can be solved under variable returns to scale (VRS) assumption by adding the constraint $\sum \lambda_j = 1$, and θ_o^{CRS} will be replaced with θ_o^{VRS}. The study by Ghiyasi (2017a) considered GDP to be the desirable output and CO_2 emissions (i.e., source of environmental pollution) to be the undesirable output, using a set of Iranian provinces as a basis for analysis. Labour and capital were chosen as inputs. Accordingly, the application of $M(3)$ resulted in optimal changes to labour and capital to meet GDP and CO_2 emissions targets.

While $M(3)$ was proposed for pollution control, its application focused only on determining the input changes required for increasing desirable and/or undesirable outputs of a production unit (i.e., DMU). Moreover, there was no defined limit on how much a decision-maker could increase outputs. In real-life scenarios, especially in the petroleum industry, this application of $M(3)$ will cause serious implications for managers or policy makers in the following ways:

- In terms of its outputs, $M(3)$ does not have a safe or maximum production capacity.
- It is permissible to increase the undesirable outputs of $M(3)$ (i.e., Δy_{po}^b), resulting in further pollution of the environment, which is against the core principles of lean production.
- Since the application of $M(3)$ considers undesirable outputs to be parameters rather than variables, it does not support waste conversion or circular economies.
- Furthermore, $M(3)$ can not be used to investigate any interrelationship between desirable and undesirable outputs. Due to the correlation between oil production and gas flaring, this is an important factor in the dynamics of the petroleum industry.

To overcome these four major limitations of $M(3)$, it is imperative to propose new concepts and make slight modifications that will transform $M(3)$ into a robust lean production tool. As a matter of fact, it is possible to overcome the first and most significant limitation of $M(3)$ by designing a production capacity for the petroleum industry that is both economically and environmentally viable.

4.5.4 The lean potential growth

Global oil production correlates with global gas flaring, which is why an increase in oil production is typically associated with an increase in gas flaring. To minimize the environmental impact of associated gas flaring, when applying model $M(3)$, a decision-maker must determine the marginal increase in the desirable output (i.e., crude oil denoted by Δy_{ro}^{g}). For this chapter, we refer to this marginal increase in crude oil production as the lean potential growth (LPG).

The United States Energy Information Administration (EIA) defines an oil well as one with a gas to oil ratio (GOR) of 6000 cubic feet of natural gas per barrel of crude oil. Alternatively, it can be called barrel of oil equivalent (BOE) and is a unit used to convert volumes of natural gas to barrels of crude oil. This value for the BOE was also stated by Wegener and Amin (2019). Using this BOE, we define the mathematical expression for the lean potential growth as follows:

$$L_{pg} \leq min \left\{ \min_{r} y_{rj}^{g}, \quad \frac{\min_{p} y_{pj}^{b}}{BOE}, \quad \left| \min_{r} y_{rj}^{g} - \frac{\min_{p} y_{pj}^{b}}{BOE} \right| \right\} \times 10^{3} \quad \dots\dots\dots\dots\dots (4.1)$$

Where $\min_{r} y_{rj}^{g} > 0$ and $\min_{p} y_{pj}^{b} > 0$ for a given increase in oil production.

Based on a set of n producers such that $j = 1, 2, \dots, n$, we have that $\min_{r} y_{rj}^{g}$ represents the lowest crude oil output of producer j, and $\min_{p} y_{pj}^{b}$ represents the least positive volume of flare gas by producer j.

As a source of reliable data for the application of equation (4.1), we refer to the 2016 Annual Statistical Bulletin (ASB) of the Organization of the Petroleum Exporting Countries (OPEC). It is open-source data that can be downloaded for free. According to the 2016 ASB, Libya was the member nation that produced the least amount of crude oil in 2015, estimated at 403.9 thousand barrels per day, while Saudi Arabia was the member nation that flared the least volume of natural gas, estimated at 40 million cubic meters.

Thus, we have that:

$\min_{r} y_{rj}^{g} = 403.9$, and

$\min_{p} y_{pj}^{b} = 40 \text{ million cubic meters} = 1{,}412{,}586{,}668.9 \text{ cubic feet}$

Using 6000 as BOE in equation 4.5.4.1, we obtain:

$L_{pg} \leq min\{403.9, 235.4, 168.5\} \times 10^3 = \{168.5\} \times 10^3 = 168.5$ thousand barrels

It is worth noting here that another minimum value of L_{pg} could be obtained from the difference between the second and third elements (i.e., 235.4 -168.5 gives 66.9), but this value is too small for scenario analysis. In any case, equation 4.5.4.1 is applied at the discretion of the decision-maker. Furthermore, this proposed concept of lean potential growth (LPG) can be regarded as a ***maximum safe capacity*** for the subsequent increase in oil production when applying model $M3$. Accordingly, this concept overcomes the first and major limitation of model $M3$. As far as economic advantages are concerned, LPG is such that the producer with the lowest crude oil output (e.g., Libya in 2015) could find it economically viable to increase oil production by an amount equal to the value of LPG.

When viewed in the context of environmental protection, the obtained value of LPG will always be less than or equal to the least amount of waste or bad output generated by the producer most committed to waste reduction (i.e., Saudi Arabia in 2015).

4.5.5 Modified models for inverse problems

In this section, we recall the inverse problems for this chapter and describe the modified models for solving them.

Problem one (P1):

For a given producer and in the absence of flare gas recovery (FGR) technology, what changes in inputs are needed to produce the desired level of good output with no increase in associated waste or bad output?

Problem two (P2):

With flare gas recovery (FGR) technology such as gas-to-wire (GTW) or gas-to-liquid (GTL), what changes in inputs are needed to produce a desired level of good output with a lower level of associated waste or bad output?

Problem three (P3):

With GTW technology, how much power can be generated from recycled waste at the same level of productive efficiency?

For P1 and P2, our models consider one good or desirable output which is crude oil, and one bad or undesirable output which is flare gas. As for P3, we need to derive a formula for the conversion of flare gas (i.e., waste) to power. Additionally, we will solve P1 and P2 under two different categories: CRS and VRS assumptions. The purpose is to determine which category will provide the best results for smart decision making. We propose the following models $M(5)^{CRS}$ and $M(6)^{CRS}$ for solving P1 and P2, respectively, under CRS assumption.

$M(5)^{CRS}$: $Min\ (\alpha_1, \alpha_2, \dots, \alpha_m) = (x_{1o+}\Delta x_1, x_{2o} + \Delta x_2, \dots, x_{mo} + \Delta x_m)$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o^{CRS}(x_{io} + \Delta x_i), \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \geq y_{ro}^{g} + \Delta y_{ro}^{g}, \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^{b} = y_{po}^{b} \qquad p = 1, \dots, q$$

$$\lambda_j \geq 0, \qquad j = 1, \dots, n$$

$M(6)^{CRS}$: $Min\ (\alpha_1, \alpha_2, \dots, \alpha_m) = (x_{1o+}\Delta x_1, x_{2o} + \Delta x_2, \dots, x_{mo} + \Delta x_m)$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o^{CRS}(x_{io} + \Delta x_i), \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \geq y_{ro}^{g} + \Delta y_{ro}^{g}, \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^{b} = y_{po}^{b} - \Delta y_{po}^{b} \qquad p = 1, \dots, q$$

$$\Delta y_{po}^{b} < y_{po}^{b}$$

$$\lambda_j \geq 0, \qquad j = 1, \dots, n$$

Under VRS assumption, P1 and P2 can be solved by updating $M(5)^{CRS}$ and $M(6)^{CRS}$ with the additional constraint $\sum \lambda_j = 1$. For the P2 problem, Δy_{po}^{b} is the potential reduction of the bad output (i.e., flare gas), an amount to be converted for power generation or natural gas liquids via GTW or GTL technologies, respectively. This implies the new target levels of gas flaring becomes $(y_{po}^{b} - \Delta y_{po}^{b})$. Compared with the original model $M(3)$, both $M(5)^{CRS}$ and $M(6)^{CRS}$ follow the core principle of lean production (i.e., waste minimization). In other words, $M(5)^{CRS}$ prevents the accumulation of waste or maintain its current level, while $M(6)^{CRS}$ reduces waste.

As both models mitigate waste while increasing desirable outputs, they both overcome the second limitation of $M(3)$ that permits an increase in waste. It should be noted that the additional constraint in $M(6)^{CRS}$ denoted as $\Delta y_{po}^{b} < y_{po}^{b}$ places a limit on the reduction of bad output. For this constraint, a less than sign was chosen rather than a less than or equal sign to eliminate the possibility of infeasibility during the computation process. This additional constraint redefines the bad output as a decision variable, as opposed to a parameter in $M(3)$.

The final limitation of $M(3)$, can be overcome by generating a potential trend that illustrates the interrelationship between oil production and gas flaring. Such a trend is useful when analyzing scenarios that include increased oil production combined with reduced flare gas or waste. It is possible to develop this trend based on the dynamics of the proposed model $M(6)^{CRS}$. For every increase in desirable output, there is a corresponding decrease in undesirable output. Mathematically, the creation of a potential trend is based on the production targets of $M(6)^{CRS}$ denoted as $(y_{ro}^{g} + \Delta y_{ro}^{g})$ and $(y_{po}^{b} - \Delta y_{po}^{b})$. By setting crude oil production targets, Δy_{ro}^{g}, model $M(6)^{CRS}$ seeks to determine the optimal values of Δy_{po}^{b} while minimizing input resources. As a result, $M(6)^{CRS}$ is a robust modification of $M(3)$ that can implement all the lean practices selected in this chapter. Moreover, for both models $M(5)^{CRS}$ and $M(6)^{CRS}$, it is imperative to emphasize that $\Delta y_{ro}^{g} \leq L_{pg}$.

4.5.6 Waste to power generation

A solution to the P2 problem also provides the volume of reduced waste (i.e., Δy_{po}^{b}) that can be utilized by GTW technology. It is necessary to compute the value of Δy_{po}^{b} to solve the P3 problem. To put it another way, the P3 problem is an extension of the P2 problem. With the aid of an industrial gas turbine, the computed values of Δy_{po}^{b} can be converted to power. To do this, we must first define the optimal number of turbine units, φ_o, required for power generation. Suppose ω is the annual volume of gas required by a selected turbine. Then by applying the optimal sizing of GTW proposed in the previous chapter, we have that for any DMU$_o$, the optimal number of turbine units required for power generation can be expressed as:

$$\varphi_o = \frac{\Delta y_{po}^{b}}{\omega} \quad \text{……………...} \quad (4.2)$$

For a typical gas turbine, its actual power output is usually the product of its thermal efficiency and its rated power output. Without considering thermal efficiency or by ignoring heat losses, the rated output is usually regarded as gross power output (Ojijiagwo et al., 2016).

Hence, for each DMU_o, we define the expression for actual power generated via all turbines as follows:

$$P_o = \eta W \varphi_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.3)$$

Where η is the thermal efficiency of the selected turbine, W is the power output of the turbine, and φ_o is an integer representing the optimal number of turbine units.

By substitution, we can rewrite equation 4.5.6.2 as:

$$P_o = \eta W \left(\frac{\Delta y_{po}^b}{\omega}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.4)$$

Then the gross power output can be expressed as:

$$P_{gross} = W \left(\frac{\Delta y_{po}^b}{\omega}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (4.5)$$

As thermal efficiency, η, varies with different types of simple or combined cycle turbines (with a maximum value of about 64% in real life situations), we use only the expression for the gross power output for this study. In addition, equation (4.5) provides the solution to the third inverse problem, P3, referred to in section 4.5.5.

4.5.7 A ranking system based on inverse DEA.

DEA has often failed to distinguish efficient units into well-defined ranks despite its ability to evaluate unit efficiency. Several ranking techniques have been proposed, but none have been based on inverse DEA. As a result, using the CCR DEA (i.e., $M1$ in section 4.5.1) as a base model, Soleimani-Chamkhorami et al. (2020) proposed the following inverse DEA model for evaluating the rank of DMU_o:

$M(7)$: $\quad Min \;\; \alpha_o = \alpha$

$$s.t. \sum_{j \neq o}^{n} \tilde{x}_{ij}\lambda_j \leq \tilde{x}_{io} + \alpha \qquad i = 1,2,\ldots,m$$

$$\sum_{j \neq o}^{n} \tilde{y}_{rj}\lambda_j \geq \tilde{y}_{ro} + k \qquad r = 1,2,\ldots,s$$

$$\tilde{x}_{io} + \alpha \geq x_{io} \qquad i = 1,2,\dots,m$$
$$\lambda_j \geq 0 \qquad j = 1,2,\dots,m$$

$M(7)$ assumes that the change in input is the same for all inputs, and the change in output is also same for all outputs. To avoid dimensional conflict, the changes in input, α, and output, k, are in percentages. Additionally, $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ are normalized inputs and outputs and can be derived from the actual inputs and outputs using the following normalization equations:

$$\tilde{x}_{ij} = \frac{x_{ij}}{\max_i x_{ij}}, \qquad \tilde{y}_{rj} = \frac{y_{rj}}{\max_r y_{rj}}$$

By applying $M(7)$, the following definition for ranking efficient units holds:

DEFINITION 4.1: The rank of DMU_j is better than DMU_k if and only if, $\alpha_j^* \geq \alpha_k^*$

When using $M(7)$, it is imperative to set a limit on the increase in normalized outputs based on our proposed concept of lean potential growth, LPG. A decision-maker should express k as a function of LPG and a specified increase in good or desirable output, denoted by Δy_{ro}^g, to relax the assumed values of k in $M(7)$. This can be done by using the expression: $k = \frac{\Delta y_{ro}^g}{L_{pg}}$.

This expression for k transforms $M(7)$ to a modified version as follows:

$M(8)$: $\quad Min \ \ \alpha_o = \alpha$

$$s.t. \ \sum_{j \neq o}^{n} \tilde{x}_{ij}\lambda_j \leq \tilde{x}_{io} + \alpha \qquad i = 1,2,\dots,m$$
$$\sum_{j \neq o}^{n} \tilde{y}_{rj}^g \lambda_j \geq \tilde{y}_{ro}^g + \frac{\Delta y_{ro}^g}{L_{pg}} \qquad r = 1,2,\dots,s$$
$$\tilde{x}_{io} + \alpha \geq x_{io} \qquad i = 1,2,\dots,m$$
$$\lambda_j \geq 0 \qquad j = 1,2,\dots,m$$

Model $M(8)$ is perfectly capable of ranking units (e.g., oil-producing nations) for a given or specified increase in outputs. Each unit is ranked according to its growth potential, and a higher growth potential indicates a higher ranking. However, it is important to note that $M(8)$ is only appropriate for ranking when only good or desirable outputs are available. Specifically, it does not consider the bad or undesirable output in this chapter, which is waste or flare gas. In this regard, $M(8)$ is not a robust lean tool for this chapter. According to experts in the petroleum industry, input and output cannot change at the same rate for all inputs and outputs under realistic circumstances. It is, therefore, necessary to develop a better method of ranking efficient oil-producing nations based on both their good and bad outputs.

To resolve this ranking problem, it would be ideal to conduct an energy analysis involving both kinds of outputs.

4.5.8 An energy-based ranking technique

First, let us consider each decision-making unit (DMU) or oil-producing nation as an energy system that permits the transfer and conversion of energy. We are dealing with two types of energy sources here, crude oil and flare gas. When refined and processed, crude oil remains the most significant source of energy. The most common products obtained from crude oil are gasoline, kerosine, diesel fuel, and a variety of other products. Accordingly, crude oil can be considered a form of utilized energy for each DMU. Flare gas, however, contributes to environmental degradation through the release of greenhouse gases. In addition, flare gas is also considered to be a significant waste of energy that can be utilized for domestic and industrial purposes. Thus, the dynamics of both sources of energy in this industry closely mimic the first law of thermodynamics, which is a version of the law of conservation of energy. However, there is a significant difference here since each DMU is not a ***closed system***, as stated in the law. The only thing we are interested in is the net energy involved in the extraction of crude oil, which usually involves the burning of associated gas. Since the good output (i.e., crude oil) is utilized energy, it also qualifies as an energy gain, while the bad output (i.e., flare gas) qualifies as an energy loss. Like the first law of thermodynamics, flare gas can be considered as harmful work done by each DMU to the surrounding environment. On the other hand, crude oil is an energy input for each DMU.

Mathematically, the first law of thermodynamics states $\Delta U = Q - W \dots \dots (4.6)$
Now ΔU is the change in internal energy of a closed system, Q is energy supplied to the system, and W is the work done by the system on its surroundings.

Given that each DMU in this study is a production work system, we need to derive a similar expression for the energy dynamics of its outputs in accordance with equation (4.6). For this purpose, both outputs (i.e., crude oil and natural gas) must be expressed in the same energy terms. This can be accomplished easily using the gas-to-oil ratio (GOR) or barrel of oil (BOE) equivalent (as discussed in section 4.5.4). The energy contained in one BOE is equal to the energy contained in 6,000 cubic feet of natural gas, which is approximately 1.7MWh. It is pertinent to recall that this BOE was incorporated into the mathematical expression for lean potential growth in section 4.5.4. This expression is rewritten as follows for analytical purposes:

$$L_{pg} \leq min\left\{\min_{r} y_{rj}^{g}, \quad \frac{\min_{p} y_{pj}^{b}}{BOE}, \quad \left|\min_{r} y_{rj}^{g} - \frac{\min_{p} y_{pj}^{b}}{BOE}\right|\right\} \times 10^{3}$$

This expression for LPG applies to a group of DMUs, but to derive the net energy transfer, ΔU_k, within each DMU_k, we must consider only the third element of the entire set. Next, we rewrite this element using the actual outputs of each DMU_k, and express them in terms of energy transfer as follows:

$$\Delta U_k = 1.7\left(y_{rk}^{g} - \frac{y_{pk}^{b}}{BOE}\right) \dots\dots\dots\dots\dots\dots\dots..(4.7)$$

By expansion, we have: $\Delta U_k = 1.7y_{rk}^{g} - 1.7\left(\frac{y_{pk}^{b}}{BOE}\right) \dots\dots\dots\dots\dots..(4.8)$

Without considering the boundary of both systems in equations (4.6) and (4.8) (i.e., closed or open systems), we can conclude that the energy supplied to each DMU_k in the form of crude oil is expressed as $Q_k = 1.7y_{rk}^{g}$, while the harmful work done by each DMU_k on its surroundings in the form of flare gas is expressed as $W_k = 1.7\left(\frac{y_{pk}^{b}}{BOE}\right)$. Therefore, the energy dynamics or net energy production within each DMU_k can now be expressed in a similar manner to equation (4.6) as follows:

$$\Delta U_k = Q_k - W_k \dots\dots\dots\dots\dots\dots\dots\dots.(4.9)$$

As can be seen from equation (4.9), the lower the value of W_k, the higher the net energy, ΔU_k. The best-case scenario would be if Q_k increases along with a decrease in W_k, which would greatly increase ΔU_k. The worst-case scenario should be when $\Delta U_k = 0$. Negative values of ΔU_k imply significant energy losses for any DMU_k.

In general, net energy production (i.e., ΔU_k) is a derivative of lean potential growth, L_{pg}. Alternatively, the lean potential growth can be viewed as a function of net energy production for a single unit or production work system, expressed as:

$$L_{pg(k)} = f(\Delta U_k) \dots\dots\dots\dots\dots\dots(4.10)$$

Based on this brief energy analysis, a new ranking definition is developed.

DEFINITION 4.2: The rank of DMU_j is better than DMU_k if and only if, $\Delta U_j > \Delta U_k$

In practice, this represents an absolute ranking of efficient oil-producing nations according to their overall net energy production. By utilizing superior technology and effective management strategies, a highly efficient producer will be able to significantly reduce gas flaring. Thus, such producers can produce more crude oil with less flare gas. There is no better example of an extremely efficient producer than Saudi Arabia, which consistently produces the most barrels of crude oil each year while emitting the least amount of flare gas among all OPEC member states. This energy-based ranking technique primarily focuses on increasing energy efficiency in the oil and gas industry, making it a valuable lean tool for this chapter. In subsequent sections, we will use this technique to rank efficient oil producers.

4.5.9 Data collection and classification

In the 2016 OPEC Annual Statistical Bulletin (ASB), available in open source, OPEC provided data of the 13 member nations selected for this chapter. The 2016 ASB publication contains data from a five-year period (i.e., 2011-2015). To conduct an in-depth analysis on an annual basis, the production years 2013 to 2015 were selected. Following is a list of inputs and outputs for the three inverse problems discussed in this chapter:

Table 4.1: Classification of inputs and outputs

Inputs	**Outputs**
1.Wells completed	1.Crude oil (1000b/d)
2.Producing wells	2.Flare gas (million m^3)
3.Active rigs	

4.6 Application, results, and analysis

4.6.1 Application of the base models

Through the application of the CRS and VRS base models, the efficiency scores of all producers are evaluated annually and are presented in Tables 4.2 to 4.4. When comparing the CRS with the VRS base model, it is evident that the VRS model produces more efficient units. As an example, in 2013, the CRS model had seven efficient producers while the VRS model had nine efficient producers. In 2014, there were 8 and 10 efficient producers for the CRS and VRS models, respectively. Under CRS and VRS, there were 7 and 10 efficient producers respectively in 2015. A key objective of this chapter is to mitigate gas flaring in efficient OPEC member nations, thus addressing the limitation of the inverse DEA developed in the previous chapter.

Due to the different number of efficient producers under CRS and VRS, it would be more appropriate to conduct a comparative analysis of truly efficient producers. This chapter defines a truly efficient producer as one who is efficient under both CRS and VRS. This will allow a valid comparison between producers using the CRS and VRS versions of the proposed inverse DEA models. Therefore, let E represent the set of efficient producers across the three tables for the three consecutive production years (i.e., 2013 to 2015) under both CRS and VRS assumptions. Thus, we have that:

2013 production year, $E = \{DMU_2, DMU_6, DMU_7, DMU_9, DMU_{10}, DMU_{11}, DMU_{12}\}$;

2014 production year, $E = \{DMU_2, DMU_5, DMU_6, DMU_8, DMU_9, DMU_{10}, DMU_{11}, DMU_{12}\}$;

2015 production year, $E = \{DMU_2, DMU_5, DMU_6, DMU_8, DMU_9, DMU_{10}, DMU_{11}\}$.

Based on these sets, the models proposed in this chapter, namely $M(5)^{CRS}$ and $M(6)^{CRS}$, and their VRS versions, will be applied to provide solutions to the three inverse problems, P1 through P3. By changing their superscripts to CRS/VRS, we will indicate the application of both models in either CRS or VRS versions for ease of analysis. The subsequent sections will provide an analysis and presentation of the results obtained on an annual basis.

Table 4.2: Efficiency of OPEC member nations (Year 2013)

DMU	θ_o^{CRS}	θ_o^{VRS}
1-Algeria	0.4262	0.5312
2-Angola	1.0000	1.0000
3-Ecuador	0.1297	0.5455
4-Indonesia	0.0886	0.0950
5-Iran	0.5676	1.0000
6-Iraq	1.0000	1.0000
7-Kuwait	1.0000	1.0000
8-Libya	0.5837	0.8966
9-Nigeria	1.0000	1.0000
10-Qatar	1.0000	1.0000
11-Saudi Arabia	1.0000	1.0000
12-UAE	1.0000	1.0000
13-Venezuela	0.4068	1.0000

Table 4.3: Efficiency of OPEC member nations (Year 2014)

DMU	θ_o^{CRS}	θ_o^{VRS}
1-Algeria	0.4568	0.4819
2-Angola	1.0000	1.0000
3-Ecuador	0.1697	0.4510
4-Indonesia	0.1086	0.1225
5-Iran	1.0000	1.0000
6-Iraq	1.0000	1.0000
7-Kuwait	0.8951	1.0000
8-Libya	1.0000	1.0000
9-Nigeria	1.0000	1.0000
10-Qatar	1.0000	1.0000
11-Saudi Arabia	1.0000	1.0000
12-UAE	1.0000	1.0000
13-Venezuela	0.3032	1.0000

Table 4.4: Efficiency of OPEC member nations (Year 2015)

DMU	θ_o^{CRS}	θ_o^{VRS}
1-Algeria	0.3791	0.4118
2-Angola	1.0000	1.0000
3-Ecuador	0.7270	1.0000
4-Indonesia	0.0904	0.1165
5-Iran	1.0000	1.0000
6-Iraq	1.0000	1.0000
7-Kuwait	0.6842	1.0000
8-Libya	1.0000	1.0000
9-Nigeria	1.0000	1.0000
10-Qatar	1.0000	1.0000
11-Saudi Arabia	1.0000	1.0000
12-UAE	0.8380	0.9368
13-Venezuela	0.3028	1.0000

4.6.2 Result presentation format for the proposed models

In sections 4.6.3 to 4.6.5, we will present results obtained from the application of models $M(5)^{CRS/VRS}$ and $M(6)^{CRS/VRS}$ in three stages: stages 1, 2, and 3, which correspond to problems P1, P2, and P3, respectively. Additionally, all results are discussed annually (i.e., 2013 to 2015). There will, however, be a greater focus on the 2015 production year, since it is the most recent year that contains flare data from OPEC nations.

4.6.3 Results analysis for the 2013 production year

Prior to applying the proposed models $M(5)^{CRS/VRS}$ and $M(6)^{CRS/VRS}$ for the 2013 production year, equation 4.5.4.1 must first be applied to determine the value of L_{pg}. For both models, it is crucial that $\Delta y_{ro}^{g} \leq L_{pg}$. Based on the production data for 2013, we determined $L_{pg} \leq 526.4$ thousand barrels. While there is plenty of room for setting target levels for Δy_{ro}^{g}, it is important to keep in mind that the obtained value of L_{pg}(i.e., 526.4) represents the actual amount of crude oil produced by Ecuador in 2013. Over the course of the production year, the nation produced the least amount of crude oil. Therefore, increasing production to levels close to 526.4 thousand barrels may not be feasible for the nation. We thus take into consideration three scenarios with smaller increments of 50 thousand barrels (i.e., $\Delta y_{ro}^{g} = 50, 100, 150$) for each of the producers. Nonetheless, the decision to set target levels is left entirely at the discretion of the decision-maker.

4.6.3.1 Stage one results (2013 production year)

Using the specified target levels of Δy_{ro}^{g}, the P1 problem was solved in this stage under both the CRS and VRS assumptions. By applying model $M(5)^{CRS/VRS}$, it is intended to determine if all the efficient producers can increase production without resulting in an increase in waste (i.e., flare gas). In this stage, we are examining a worst-case scenario of lean production in which producers cannot reduce waste but instead can maintain their current level while increasing production. In fact, it is the same as a feasibility test for waste reduction, since if a producer is efficient enough to avoid a further increase in waste during production, then such a producer may be able to reduce waste through effective waste management strategies and the use of superior technology.

In Tables 4.5 and 4.6, we present the results obtained under CRS and VRS, respectively. As can be seen from Table 4.5, all the efficient producers would be able to increase their production up to 150 thousand barrels without increasing their flare gas levels. However, it was observed that, under VRS as illustrated in Table 4.6, the model was not feasible for Saudi Arabia. A logical explanation for this phenomenon might be the fact that Saudi Arabia produces the highest volume of crude oil among OPEC members. Nevertheless, it is too early to conclude that crude oil production cannot be sustained in this manner. We must therefore perform further experiments not only for the P2 problem but also for the 2014 and 2015 production years. This is crucial for validating results.

Table 4.5: Solution to P1 under CRS (Year 2013)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y^g_{ro} = 50$			Scenario 2: $\Delta y^g_{ro} = 100$			Scenario 3: $\Delta y^g_{ro} = 150$		
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}
2.Angola	0.000	0.000	2.773	0.000	0.000	5.545	0.000	0.000	8.318
6.Iraq	2.226	17.495	0.768	4.452	34.990	1.536	6.677	52.485	2.304
7. Kuwait	0.000	0.000	1.073	0.000	0.000	2.146	0.000	0.000	3.219
9.Nigeria	5.916	0.000	0.879	11.832	0.000	1.758	17.748	0.000	2.637
10.Qatar	0.000	0.000	1.442	0.000	0.000	2.883	0.000	0.000	4.325
11.Saudi Arabia	2.226	17.495	0.768	4.452	34.990	1.536	6.677	52.485	2.304
12. UAE	0.000	0.000	0.956	0.000	0.000	1.912	0.000	0.000	2.867

Table 4.6: Solution to P1 under VRS (Year 2013)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y^g_{ro} = 50$			Scenario 2: $\Delta y^g_{ro} = 100$			Scenario 3: $\Delta y^g_{ro} = 150$		
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}
2.Angola	2.333	0.000	2.733	4.665	0.000	5.467	6.998	0.000	8.200
6.Iraq	11.356	1318.589	10.928	22.712	2637.179	21.855	34.068	3955.768	32.783
7.Kuwait	0.000	8.999	0.916	0.000	17.997	1.832	0.000	26.996	2.748
9.Nigeria	7.228	0.000	0.919	14.457	0.000	1.837	21.685	0.000	2.756
10.Qatar	1.961	16.224	0.817	3.923	32.449	1.634	5.884	48.673	2.450
11.Saudi Arabia	-	-	-	-	-	-	-	-	-
12. UAE	0.000	0.000	1.101	0.000	0.000	2.202	0.000	0.000	3.302

4.6.3.2 Stage two results (2013 production year)

Using the same production targets as in stage one, a reduction in gas flaring was imposed on efficient producers to achieve the needed solutions to the P2 problem. A summary of the results can be found in Tables 4.7 and 4.8. In both tables, the results confirm that each efficient producer can increase production with a reduced level of flare gas designated by

Δy_{po}^{b}. Note that, as stated in the P2 problem, the reductions can be achieved by employing flare gas recovery (FGR) technology such as GTW. Saudi Arabia is excluded from this set of results since the producer reported a zero flare for the 2013 production year.

Upon closer examination of both tables, it becomes clear that none of the scenarios envisioned for Kuwait resulted in any reductions under CRS or VRS. Technically, though, Kuwait is not considered extremely efficient simply because it had zero reductions in flare gas. There are two reasons for this. Firstly, UAE had zero reductions throughout all scenarios, but only in the case of CRS. Under the CRS, it becomes more difficult to identify a more efficient producer between the two. Secondly, Saudi Arabia is also an efficient producer and flared no gas (i.e., reported a zero flare) for the 2013 production year. Regardless of the degree of efficiency, a zero flare is better than a zero reduction in flare gas from a standpoint of environmental protection. In this regard, Kuwait, the UAE, and Saudi Arabia present a conundrum in the context of efficiency ranking. Therefore, it is more appropriate to make a valid conclusion regarding extreme efficiency using the proposed energy-based ranking technique rather than flare gas reductions. If Kuwait dominates the net energy rankings ahead of the other efficient producers at this stage, Kuwait's zero reductions may justify any assumption that the decision-maker may have regarding the country's extreme efficiency.

Table 4.7: Solution to P2 under CRS (Year 2013)

$\mathbf{DMU_o}$	**Scenario 1:** $\Delta y_{ro}^{g} = 50$				**Scenario 2:** $\Delta y_{ro}^{g} = 100$				**Scenario 3:** $\Delta y_{ro}^{g} = 150$			
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}
2.Angola	0.000	0.000	0.000	1090.49	0.000	0.000	0.000	2180.979	0.000	0.000	0.000	3271.469
6.Iraq	0.000	0.000	0.000	314.103	0.000	0.000	0.000	628.205	0.000	0.000	0.000	942.308
7.Kuwait	0.000	0.000	1.073	0.000	0.000	0.000	2.146	0.000	0.000	0.000	3.219	0.000
9.Nigeria	0.000	0.000	0.000	750.265	0.000	0.000	0.000	1500.531	0.000	0.000	0.000	2250.796
10.Qatar	0.000	0.000	1.183	50.278	0.000	0.000	2.366	100.555	0.000	0.000	3.549	150.832
12. UAE	0.000	0.000	0.956	0.000	0.000	0.000	1.912	0.000	0.000	0.000	2.867	0.000

Table 4.8: Solution to P2 under VRS (Year 2013)

$\mathbf{DMU_o}$	**Scenario 1:** $\Delta y_{ro}^{g} = 50$				**Scenario 2:** $\Delta y_{ro}^{g} = 100$				**Scenario 3:** $\Delta y_{ro}^{g} = 150$			
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}
2.Angola	2.142	0.000	0.641	143.716	4.283	0.000	1.282	287.433	6.425	0.000	1.923	431.149
6.Iraq	0.000	0.000	0.000	385.485	0.000	0.000	0.000	770.970	0.000	0.000	0.000	1156.455
7.Kuwait	0.000	8.999	0.916	0.000	0.000	17.997	1.832	0.000	0.000	26.996	2.748	0.000
9.Nigeria	2.070	0.000	0.000	197.556	4.140	0.000	0.000	395.111	6.210	0.000	0.000	592.667
10.Qatar	1.913	16.049	0.802	4.155	3.826	32.099	1.604	8.309	5.739	48.148	2.407	12.464
12. UAE	0.000	0.000	1.042	21.024	0.000	0.000	2.085	42.049	0.000	0.000	3.127	63.073

4.6.3.3 Stage three results (2013 production year)

This stage involves the P3 problem related to the recycling of waste for the generation of electricity. As an extension of the P2 problem, it involves the conversion of the estimated reductions in flare gas into electricity using a gas turbine. As the reductions increase with an increase in oil production, we select only the third scenario for analysis. For this conversion process, we use the turbine specifications from the previous chapter. Further, we employ the equation proposed in section 4.5.6 for gas-to-power (GTP) generation:

$$P_{gross} = W\left(\frac{\Delta y_{po}^{b}}{\omega}\right)$$

Table 4.9 presents the results. We estimate the reductions in million cubic metres and power in megawatts. It should be noted that a value of Δy_{po}^{b} less than the required gas consumption of a single turbine (i.e., ω) is insufficient to power the turbine. We assign a zero value of power generation to such a producer. In Table 4.9, under CRS, Angola gains the most from the conversion process with 1350MW, closely followed by Nigeria with 900MW. VRS benefits Iraq most with 450MW, while Angola and Nigeria each have 150MW. In general, the CRS model produces more power due to larger reductions as opposed to the VRS model. Hence, we recommend GTW technology with CRS properties as a means of achieving cleaner gas production and greater electricity generation for the nations.

Table 4.9: Gas to power generation (Year 2013)

DMU_o	Scenario 3 ($\Delta y_{ro}^{g} = 150$)			
	CRS		VRS	
	Flare reductions (million m^3) (Δy_{po}^{b})	Power (MW) (P)	Flare reductions (million m^3) (Δy_{po}^{b})	Power (MW) (P)
2.Angola	3271.469	1350	431.149	150
6.Iraq	942.308	300	1156.455	450
7.Kuwait	0.000	0	0.000	0
9.Nigeria	2250.796	900	592.667	150
10. Qatar	150.832	0	12.464	0
12. UAE	0.000	0	63.073	0

Finally, to deal with the ranking problem of all efficient producers, including Saudi Arabia, we must apply the energy-based ranking technique discussed in section 4.5.8. The following equation is required to implement this technique:

$$\Delta U_k = 1.7\left(y_{rk}^{g} - \frac{y_{pk}^{b}}{BOE}\right)$$

For DMU_j to be superior to DMU_k, $\Delta U_j > \Delta U_k$, which is the sufficient condition. A comparison of the ranking results based on the net energy of each producer is presented in Table 4.10. The ranking conundrum presented by Kuwait, UAE, and Saudi Arabia at the previous stage has been satisfactorily resolved by our proposed ranking technique, as shown in Table 4.10. The most efficient producer with the highest net energy (i.e., energy gain) is Saudi Arabia, closely followed by Kuwait, which also recorded a net energy gain. However, UAE is ranked third with a net energy loss. The largest net energy loss is attributed to Iraq, which is the least efficient producer. By employing this technique, a decision-maker will be able to draw intelligent and valid conclusions regarding efficiency ranking, particularly when there are multiple efficient producers with zero reductions in flare gas, or another case where two or more efficient producers flared no gas during a particular period.

Table 4.10: Energy-based ranking of efficient OPEC members (Year 2013)

DMU_k	Net energy (MWh) (ΔU_k)	Gain/Loss	Rank
2.Angola	-67159.86	Loss	5
6.Iraq	-119325.06	Loss	7
7.Kuwait	2790.72	Gain	2
9.Nigeria	-118209.23	Loss	6
10.Qatar	-6179.68	Loss	4
11.Saudi Arabia	16382.90	Gain	1
12. UAE	-5461.89	Loss	3

4.6.4 Results analysis for the 2014 production year

We begin our analysis for the 2014 production year by computing the lean potential growth, as we did for the previous production year. Based on production data for 2014, we calculated $L_{pg} \leq 58.9$ thousand barrels. The low value of L_{pg} is a direct result of the fact that the least positive value of flare gas for the current year (i.e., $\min_p y_{pj}^b$) is much lower than what was observed for 2013. Therefore, we propose three scenarios with much smaller increments for this production year, i.e., $\Delta y_{rj}^g = 10, 20, 30$ thousand barrels.

4.6.4.1 Stage one results (2014 production year)

Under CRS, the solutions to the P1 problem are like those of the previous year, in the sense that all efficient producers were able to increase production to 30 thousand barrels without a corresponding increase in flare gas. This category's results are presented in Table 4.11. However, under VRS, as illustrated in Table 4.12, Iran and Saudi Arabia are the two OPEC members who cannot increase their oil production. This may indicate that both nations produced an excessive amount of crude oil in comparison with other members during the production year. Recall that only Saudi Arabia experienced this type of embargo on increasing its oil production in 2013 for this VRS category. Considering this phenomenon repeating over two consecutive years, there is an increasing likelihood that the VRS model could help curtail the excess oil production of OPEC members. Should this phenomenon repeat itself during the analysis of results for the next production year (i.e., 2015), we will draw a conclusion on it.

Table 4.11: Solution to P1 under CRS (Year 2014)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y_{ro}^{g} = 10$			Scenario 2: $\Delta y_{ro}^{g} = 20$			Scenario 3: $\Delta y_{ro}^{g} = 30$		
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}
2.Angola	1.044	0.000	0.149	2.088	0.000	0.298	3.133	0.000	0.447
5.Iran	0.982	7.318	0.433	1.963	14.635	0.866	2.945	21.953	1.299
6. Iraq	0.554	3.506	0.149	1.108	7.012	0.299	1.661	10.518	0.448
8. Libya	0.000	0.862	0.000	0.000	1.725	0.000	0.000	2.587	0.000
9.Nigeria	0.503	0.000	0.147	1.006	0.000	0.294	1.509	0.000	0.442
10.Qatar	0.000	0.000	0.250	0.000	0.000	0.500	0.000	0.000	0.749
11.Saudi Arabia	0.982	7.318	0.433	1.963	14.635	0.866	2.945	21.953	1.299
12. UAE	0.000	0.000	0.204	0.000	0.000	0.408	0.000	0.000	0.612

Table 4.12: Solution to P1 under VRS (Year 2014)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y_{ro}^{g} = 10$			Scenario 2: $\Delta y_{ro}^{g} = 20$			Scenario 3: $\Delta y_{ro}^{g} = 30$		
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}
2.Angola	1.347	0.000	0.127	2.693	0.000	0.255	4.040	0.000	0.382
5.Iran	-	-	-	-	-	-	-	-	-
6. Iraq	5.273	157.164	2.009	10.547	314.328	4.018	15.820	471.492	6.027
8. Libya	0.000	3.301	0.000	0.000	6.603	0.000	0.000	9.904	0.000
9.Nigeria	0.792	0.000	0.148	1.585	0.000	0.296	2.377	0.000	0.444
10.Qatar	0.492	3.271	0.156	0.983	6.542	0.312	1.475	9.813	0.468
11.Saudi Arabia	-	-	-	-	-	-	-	-	-
12. UAE	0.000	0.000	0.212	0.000	0.000	0.424	0.000	0.000	0.636

4.6.4.2 Stage two results (2014 production year)

Tables 4.13 and 4.14 present the solutions to the P2 problem of this stage. Since Iran reported zero flares during this period, it was excluded from the sets of solutions. We see a similar situation in Table 4.14, in which Saudi Arabia is unable to increase oil production under VRS. We obtained much lower reductions across both tables, which will lead to low estimates of converted power for the P3 problem. To achieve the greatest possible reductions, we set the increase in oil production to equal the lean potential growth. Table 4.15 presents the additional reductions, which are later converted to gross power in Table 4.16. Except for Saudi Arabia, the CRS and VRS models have an overall equal performance, imposing greater reductions on three producers each in Table 4.15.

Table 4.13: Solution to P2 under CRS (Year 2014)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y^g_{ro} = 10$				Scenario 2: $\Delta y^g_{ro} = 20$				Scenario 3: $\Delta y^g_{ro} = 30$			
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy^b_{po}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy^b_{po}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy^b_{po}
2.Angola	0.610	0.000	0.000	84.771	1.219	0.000	0.000	169.541	1.829	0.000	0.000	254.312
6. Iraq	0.000	0.000	0.000	75.409	0.000	0.000	0.000	150.818	0.000	0.000	0.000	226.227
8. Libya	0.000	0.000	0.000	341.573	0.000	0.000	0.000	341.573	0.000	0.000	0.000	341.573
9.Nigeria	0.000	0.000	0.000	83.902	0.000	0.000	0.000	167.803	0.000	0.000	0.000	251.705
10.Qatar	0.000	0.000	0.213	9.877	0.000	0.000	0.427	19.755	0.000	0.000	0.640	29.632
11.Saudi Arabia	0.982	7.318	0.433	0.000	1.963	14.635	0.866	0.000	2.945	21.953	1.299	0.000
12. UAE	0.000	0.000	0.200	2.499	0.000	0.000	0.399	4.998	0.000	0.000	0.599	7.497

Table 4.14: Solution to P2 under VRS (Year 2014)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y^g_{ro} = 10$				Scenario 2: $\Delta y^g_{ro} = 20$				Scenario 3: $\Delta y^g_{ro} = 30$			
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy^b_{po}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy^b_{po}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy^b_{po}
2.Angola	0.754	0.000	0.079	29.919	1.507	0.000	0.158	59.838	2.261	0.000	0.237	89.757
6. Iraq	0.000	0.000	0.000	75.436	0.000	0.000	0.000	150.871	0.000	0.000	0.000	226.307
8. Libya	0.000	0.000	0.000	60.072	0.000	0.000	0.000	120.144	0.000	0.000	0.000	180.216
9.Nigeria	0.000	0.000	0.000	78.080	0.000	0.000	0.000	156.160	0.000	0.000	0.000	234.240
10.Qatar	0.488	3.228	0.154	0.777	0.975	6.455	0.309	1.553	1.463	9.683	0.463	2.330
11.Saudi Arabia	-	-	-	-	-	-	-	-	-	-	-	-
12. UAE	0.000	0.000	0.201	3.474	0.000	0.000	0.403	6.949	0.000	0.000	0.604	10.423

Table 4.15: Reductions at LPG (Year 2014)

$\mathrm{DMU_o}$	$\Delta y^{g}_{ro} = LPG$	
	CRS	VRS
	Δy^{b}_{po}	Δy^{b}_{po}
2.Angola	499.299	176.223
6. Iraq	444.159	444.316
8. Libya	341.573	353.825
9.Nigeria	494.180	459.892
10.Qatar	58.178	4.575
11. Saudi Arabia	0.000	-
12. UAE	14.719	20.465

4.6.4.3 Stage three results (2014 production year)

For the P3 problem of this stage, the estimated reductions obtained in Table 4.15 are converted to gross power in a similar manner to the previous year. In Table 4.16, the estimates are presented. The non-zero power estimates in Table 4.16 all have the same value of 150MW. The reason for this is that the flare gas reductions are only sufficient to run a single turbine unit with a rated power output of 150 MW. Moreover, the low LPG value for this production year is the primary contributing factor. As a final point, it is easy to determine which producers are the most and least efficient for this production year by evaluating their net energy production.

The proposed energy-based technique allowed for the segregation of the eight efficient producers, including Iran, into well-defined ranks as shown in Table 4.17. In similar fashion to the previous ranking result (i.e., Table 4.10), only two producers in Table 4.17 had a net energy gain, namely Saudi Arabia and Iran. While Saudi Arabia topped the rankings with the greatest energy gain, Iraq recorded the least net energy production, as was the case in the previous ranking results. Based on these findings, Saudi Arabia has maintained its position as the most efficient oil producer among OPEC member nations for two years in a row.

Table 4.16: Gas to power generation (Year 2014)

DMU_o	$(\Delta y^g_{ro} = LPG = 58.9)$			
	CRS		VRS	
	Flare reductions (million m^3) (Δy^b_{po})	Power (MW) (P)	Flare reductions (million m^3) (Δy^b_{po})	Power (MW) (P)
2.Angola	499.299	150	176.223	0
6. Iraq	444.159	150	444.316	150
8. Libya	341.573	150	353.825	150
9.Nigeria	494.180	150	459.892	150
10.Qatar	58.178	0	4.575	0
11. Saudi Arabia	0.000	0	-	-
12. UAE	14.719	0	20.465	0

Table 4.17: Energy-based ranking of efficient OPEC members (Year 2014)

DMU_k	Net energy (MWh) (ΔU_k)	Gain/Loss	Rank
2.Angola	-67166.43	Loss	6
5.Iran	5299.07	Gain	2
6. Iraq	-123501.09	Loss	8
8. Libya	-44066.28	Loss	5
9.Nigeria	-104358.61	Loss	7
10.Qatar	-5891.49	Loss	4
11. Saudi Arabia	16411.53	Gain	1
12. UAE	-5428.12	Loss	3

4.6.5 Results analysis for the 2015 production year

This production year is of the utmost importance since it is the most recent year in which OPEC published data regarding the annual volumes of gas flared by each member nation. Other sources of flare data are inconsistent with those provided by OPEC. Moreover, there is a need to validate the performance of our proposed models, especially the capability of the VRS versions to regulate oil production. Considering that Saudi Arabia, Iraq, and Iran produced excess crude oil this year, this is extremely important. As well, it is important to consider this production year as an appropriate reference when investigating the relationship between oil production and gas flaring. As a starting point, we computed $L_{pg} \leq 168.5$ thousand barrels. Thus, we specify three scenarios for the subsequent analysis, which are 50, 100, and 150 thousand barrels.

4.6.5.1 Stage one results (2015 production year)

The obtained solution to the P1 problem in Tables 4.18 and 4.19 are in similar manner to the previous years. We observed, however, that the model was infeasible for Iran, Iraq, and Saudi Arabia under the VRS case (see Table 4.19). In other words, all three producers could not increase their current levels of crude output. Under VRS, this is the third consecutive experiment in which a producer (especially Saudi Arabia) has been unable to increase oil production for the P1 problem. The only difference is that the number of producers increased to three during this period. Before drawing any conclusions, it is important to briefly discuss the production levels of these three producers, as 2015 was a significant year for all three.

In 2015, Saudi Arabia produced and exported more crude oil than any other OPEC member country (i.e., over 10 million barrels of crude oil produced and over 7 million barrels exported), according to the 2016 ASB published by OPEC. After Saudi Arabia, Iraq was the second largest producer and exporter of oil within OPEC in 2015. Furthermore, according to Crystol Energy, Iraq was the second largest contributor to global oil supply increases in 2015, with production reaching almost 4 million barrels per day. The Iran Nuclear Deal signed in 2015 resulted in an increase of 500,000 barrels per day in Iranian oil production (Alipour et al., 2017). Based on these facts, it is evident that the VRS model is producing infeasible solutions to curb the relatively excessive crude oil production in all three member countries.

Table 4.18: Solution to P1 under CRS (Year 2015)

$\mathbf{DMU_o}$	**Scenario 1:** $\Delta y_{ro}^{g} = 50$			**Scenario 2:** $\Delta y_{ro}^{g} = 100$			**Scenario 3:** $\Delta y_{ro}^{g} = 150$		
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}
2.Angola	0.000	0.000	1.274	0.000	0.000	2.547	0.000	0.000	3.821
5.Iran	4.569	37.108	2.062	9.138	74.216	4.125	13.707	111.324	6.187
6.Iraq	3.023	17.423	0.711	6.045	34.858	1.422	9.068	52.271	2.133
8.Libya	0.893	13.440	0.000	1.787	26.880	0.000	2.680	40.321	0.000
9.Nigeria	0.000	0.000	1.030	0.000	0.000	2.059	0.000	0.000	3.089
10.Qatar	0.000	0.000	1.100	0.000	0.000	2.200	0.000	0.000	3.300
11.Saudi Arabia	4.569	37.108	2.062	9.138	74.216	4.125	13.707	111.324	6.187

Table 4.19: Solution to P1 under VRS (Year 2015)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y_{ro}^{g} = 50$			Scenario 2: $\Delta y_{ro}^{g} = 100$			Scenario 3: $\Delta y_{ro}^{g} = 150$		
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δx_{1o}	Δx_{2o}	Δx_{3o}
2.Angola	3.302	0.000	0.737	6.603	0.000	1.475	9.905	0.000	2.212
5.Iran	-	-	-	-	-	-	-	-	-
6.Iraq	-	-	-	-	-	-	-	-	-
8.Libya	2.841	17.085	0.578	5.683	34.169	1.156	8.524	51.254	1.735
9.Nigeria	0.000	0.000	1.039	0.000	0.000	2.078	0.000	0.000	3.117
10.Qatar	2.799	16.339	0.756	5.599	32.677	1.513	8.398	49.016	2.269
11.Saudi Arabia	-	-	-	-	-	-	-	-	-

4.6.5.2 Stage two results (2015 production year)

Tables 4.20 and 4.21 present solutions to the P2 problem at this stage. The VRS model was infeasible for only Saudi Arabia, confirming our earlier claim of excess oil production. In this stage, Iran reported zero flares, resulting in its exclusion.

Table 4.20: Solution to P2 under CRS (Year 2015)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y_{ro}^{g} = 50$				Scenario 2: $\Delta y_{ro}^{g} = 100$				Scenario 3: $\Delta y_{ro}^{g} = 150$			
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}
2.Angola	0.000	0.000	0.523	2489.227	0.000	0.000	2.026	1725.767	0.000	0.000	3.530	962.306
6.Iraq	3.907	0.000	0.479	223.267	7.813	0.000	0.959	446.533	11.720	0.000	1.438	669.800
8.Libya	0.000	0.000	0.000	1121.018	0.000	0.000	0.000	1121.018	0.000	0.000	0.000	1121.018
9.Nigeria	0.000	0.000	0.000	3615.890	0.000	0.000	0.000	3248.607	0.000	0.000	0.000	5320.829
10.Qatar	0.000	0.000	0.981	57.647	0.000	0.000	1.962	115.294	0.000	0.000	2.943	172.940
11.Saudi Arabia	4.569	37.108	2.062	0.000	9.138	74.216	4.125	0.000	13.707	111.324	6.187	0.000

Table 4.21: Solution to P2 under VRS (Year 2015)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y_{ro}^{g} = 50$				Scenario 2: $\Delta y_{ro}^{g} = 100$				Scenario 3: $\Delta y_{ro}^{g} = 150$			
	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}	Δx_{1o}	Δx_{2o}	Δx_{3o}	Δy_{po}^{b}
2.Angola	3.302	0.000	0.737	0.000	6.603	0.000	1.475	0.000	9.905	0.000	2.212	0.000
6.Iraq	4.225	0.000	0.437	238.323	8.450	0.000	0.875	476.646	12.675	0.000	1.312	714.969
8.Libya	5.732	0.000	0.000	391.495	11.463	0.000	0.000	782.989	17.195	0.000	0.000	1174.483
9.Nigeria	0.000	0.000	0.000	3876.401	0.000	0.000	0.000	4473.628	0.000	0.000	0.202	4906.535
10.Qatar	2.826	16.190	0.729	3.553	5.652	32.381	1.458	7.105	8.478	48.571	2.186	10.658
11.Saudi Arabia	-	-	-	-	-	-	-	-	-	-	-	-

Table 4.22 presents the estimated reductions for the efficient producers under CRS and VRS cases. In all three scenarios, Libya's constant CRS reductions can be explained by the fact that Libya produced the least amount of crude oil during the production year.

Both CRS and VRS models were unable to impose a reduction in flare gas for Saudi Arabia because the nation flared a relatively low volume of gas despite producing the largest amount of crude oil, and this implies a case of extreme efficiency. We recommend the CRS model for cleaner production based on the overall results of scenario 3 since it imposes larger reductions on more producers. With respect to Angola, the CRS model clearly outperforms the VRS model.

Table 4.22: Comparisons of estimated reductions in undesirable output (Year 2015)

$\mathbf{DMU_o}$	Scenario 1: $\Delta y_{ro}^{g} = 50$		Scenario 2: $\Delta y_{ro}^{g} = 100$		Scenario 3: $\Delta y_{ro}^{g} = 150$	
	Δy_{CRS}^{b}	Δy_{VRS}^{b}	Δy_{CRS}^{b}	Δy_{VRS}^{b}	Δy_{CRS}^{b}	Δy_{VRS}^{b}
2.Angola	2489.23	0.00	1725.77	0.00	962.31	0.00
6.Iraq	223.27	238.32	446.53	476.65	669.80	714.97
8.Libya	1121.02	391.50	1121.02	782.99	1121.02	1174.48
9.Nigeria	3615.89	3876.40	3248.61	4473.63	5320.83	4906.54
10.Qatar	57.65	3.55	115.29	7.11	172.94	10.66
11.Saudi Arabia	0.00	-	0.00	-	0.00	-

4.6.5.3 Stage three results (2015 production year)

In contrast to the 2013 and 2014 production years, we present the 2015 gross power generated for the P3 problem as a chart, as shown in Figure 4.2. The estimates are based on the reductions calculated for scenario 3 in Table 4.22. According to Figure 4.2, Nigeria is the producer with the greatest potential for power generation, with 2250MW under CRS. However, Qatar has the least potential due to its low flare gas reductions. As for Libya, 450MW of electricity is generated equally under CRS and VRS. Angola and Iraq have the potential to generate 300MW of power under the CRS and VRS, respectively. In general, the results obtained are of great benefit to the Nigerian petroleum industry, given the country's current energy supply crisis.

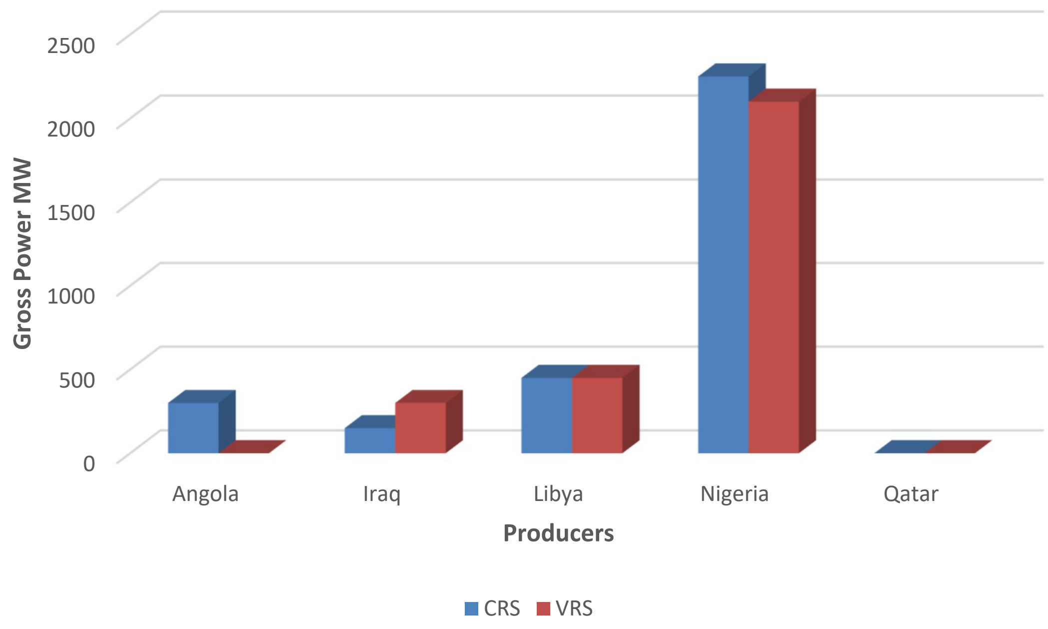

Figure 4.2: Potential power generation by efficient producers (Scenario 3)

To conclude this stage, we present the net energy rankings of all efficient producers in Table 4.23. The results are similar with Saudi Arabia ranked first for the third consecutive year, demonstrating the case for super efficiency. As such, Saudi Arabia serves as the ideal benchmark not only for efficient producers but also for inefficient producers. It stands to reason that Saudi Arabia is the most committed OPEC member to gas flaring reduction. It is therefore possible for other members of OPEC to adopt and implement Saudi Arabia's management strategies for cleaner oil and gas production. The overall ranking results for each producer have been provided in Table 4.24 for an in-depth comparison across the three production years analyzed for this chapter. It is important to note that in Table 4.24, only producers who are efficient under both CRS and VRS are ranked in terms of their net energy, as we have demonstrated throughout this chapter for the solved problems P1 through P3. As an example, Iran was efficient under CRS but inefficient under VRS for the 2013 production year. Therefore, Iran is not included in the ranking results for 2013, as shown in Table 4.24. The same applies to other producers who did not receive a ranking for any of the three years. As can be seen from Table 4.24, there were 7, 8, and 7 ranked producers in 2013, 2014, and 2015, respectively. Saudi Arabia dominates the overall rankings as the most efficient producer, while Iraq is ranked lowest across the rankings, making it the least efficient producer. Qatar and the UAE have maintained their rankings for the first two years, with

Qatar improving slightly in 2015. There is a similar ranking pattern observed between Angola and Nigeria in which the ranking of each nation declined by one step when the number of efficient producers increased. While Iran was excluded from the 2013 rankings, it maintained a decent ranking behind Saudi Arabia in 2014 and 2015.

Table 4.23: Energy-based ranking of efficient OPEC members (Year 2015)

DMU_k	Net energy (MWh) (ΔU_k)	Gain/Loss	Rank
2.Angola	-67006.67	Loss	5
5.Iran	5357.72	Gain	2
6.Iraq	-140194.07	Loss	7
8.Libya	-43746.22	Loss	4
9.Nigeria	-93957.46	Loss	6
10.Qatar	-6064.98	Loss	3
11.Saudi Arabia	16927.19	Gain	1

Table 4.24: Comparison of overall energy rankings

Efficient Producers (CRS & VRS)	2013 Ranking	2014 Ranking	2015 Ranking
Angola	5	6	5
Iran	-	2	2
Iraq	7	8	7
Kuwait	2	-	-
Libya	-	5	4
Nigeria	6	7	6
Qatar	4	4	3
Saudi Arabia	1	1	1
UAE	3	3	-

4.6.5.4 Sensitivity analysis

To investigate the interrelationship between oil production and gas flaring in both the short and long term for stage two producers, we generated more scenarios with successive increases in oil production of 20 thousand barrels. To achieve this, we indicate the new target levels in stage two as follows:

Oil production target: $(y_{ro}^{g} + \Delta y_{ro}^{g})$, Gas flaring target: $(y_{po}^{b} - \Delta y_{po}^{b})$

For a desired increase in oil production, Δy_{ro}^{g}, the applied models compute the value of Δy_{po}^{b} while minimizing the input resources. It is obvious that a successive increase in the computed value of Δy_{po}^{b} for any producer will result in a decrease in its gas flaring target. Under CRS, we find from Table 4.19, that Iraq and Qatar both have increasing values of Δy_{po}^{b} across all three scenarios. Therefore, the relationship between gas flaring and oil production targets will be similar for both producers. As examples, Fig. 4.3 to 4.5 illustrate the relationship between both targets under CRS. This sensitivity analysis was conducted for environmental protection with the aim of decreasing gas flaring levels while increasing oil production.

We observe in Fig. 4.3 that an increase in oil production only has a short-term benefit for Angola due to the initial decrease in gas flaring. However, in the fourth scenario (i.e., at a production target of 1827.1 thousand barrels of crude oil) there is a noticeable increase in gas flaring and continues to rise in tandem with a subsequent increase in oil production. From Fig. 4.4, it is evident that an increase in oil production is perfectly correlated with a linear decrease in gas flaring for Iraq. It is a set of potential win-win scenarios for the Iraqi petroleum industry as well as the environment. The same trend can also be seen in Fig. 4.1, where global gas flaring declined in tandem with an increase in oil production during the 3-year span (2006-2008). Nigeria may initially show potential for a decrease in gas flaring in Fig. 4.5, but in the long run the nation would benefit more from an increase in oil production. The last four scenarios in Fig. 4.5 show a linear decrease in gas flaring for Nigeria, so the long run wins here. We conclude the sensitivity analysis by stating that Fig. 4.3 to 4.5 are clearly integral parts of Fig. 4.1, and our proposed inverse DEA models have made it possible to investigate this salient relationship between oil production and gas flaring.

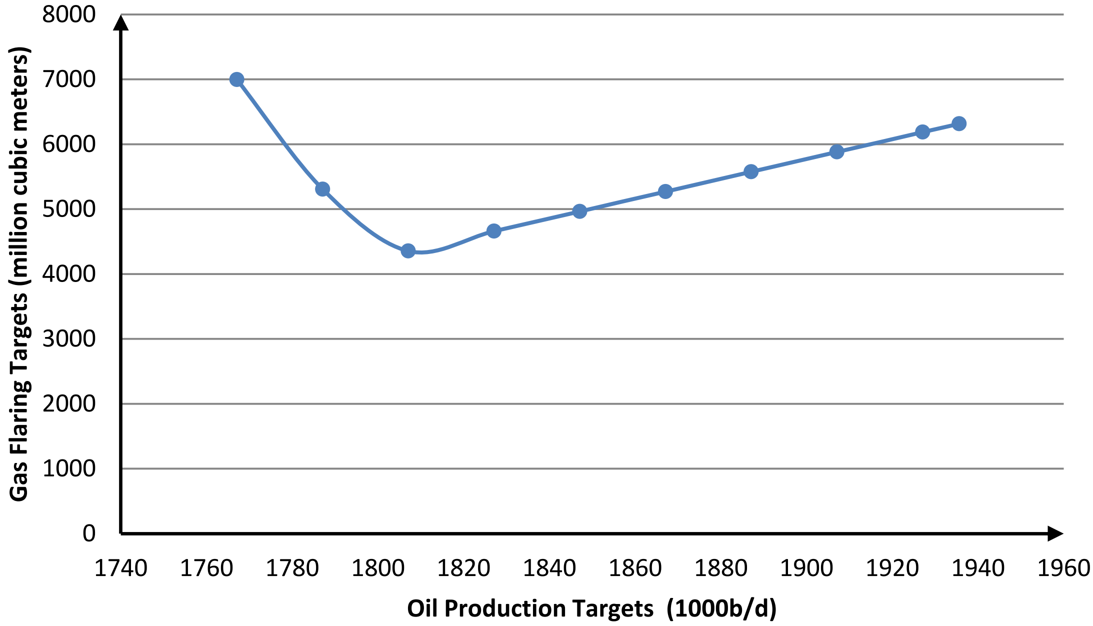

Figure 4.3: Potential trend of gas flaring and oil production for Angola

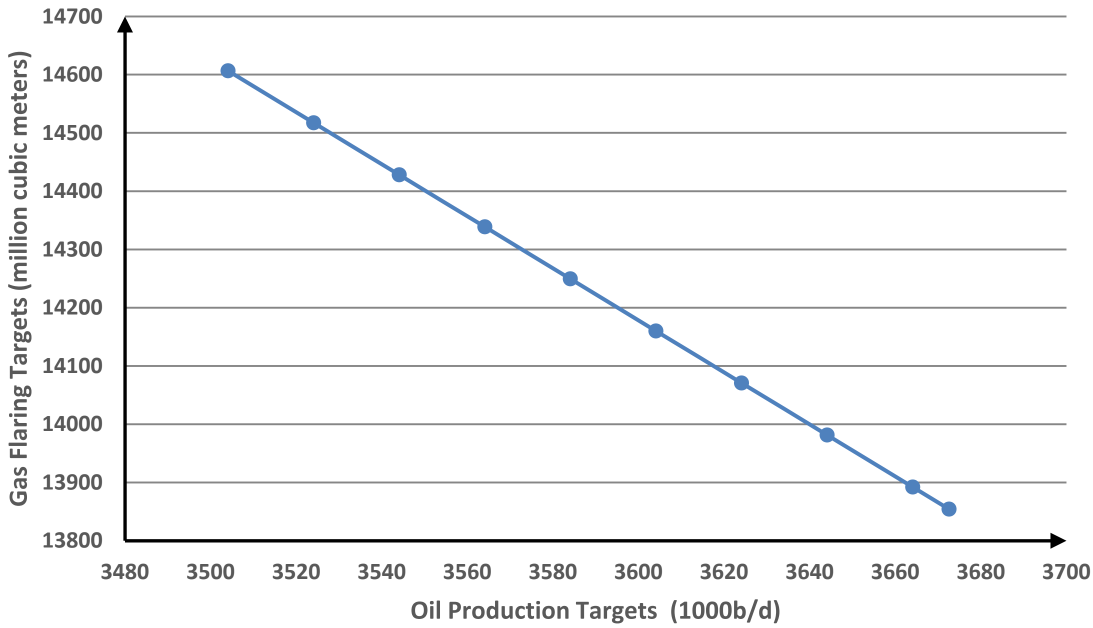

Figure 4.4: Potential trend of gas flaring and oil production for Iraq

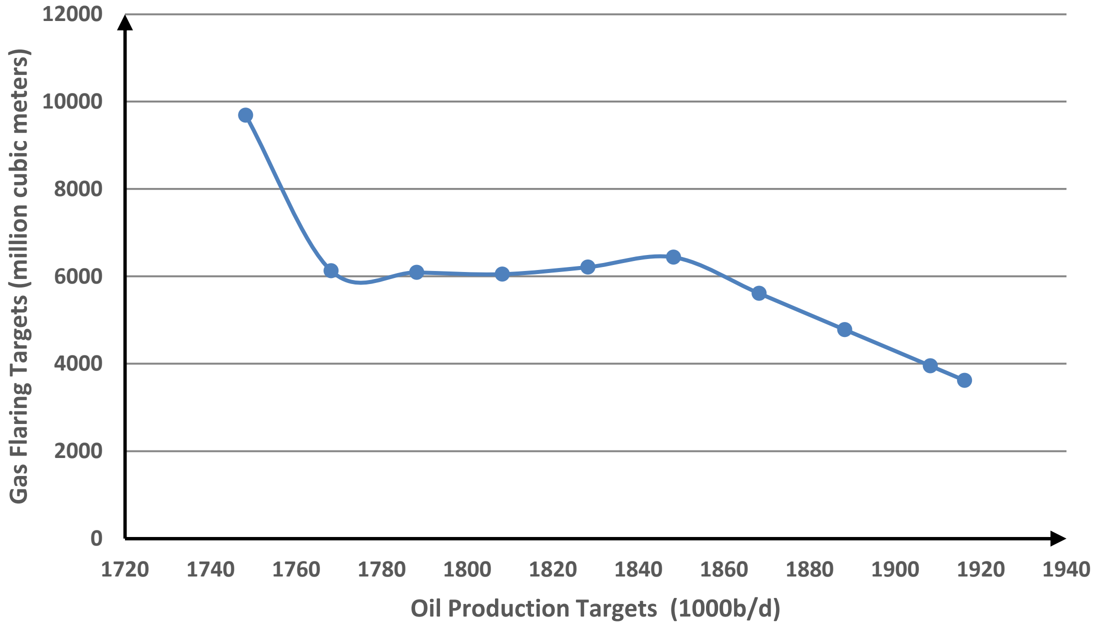

Figure 4.5: Potential trend of gas flaring and oil production for Nigeria

4.6.5.5 Managerial implications

Since the focus of this chapter was the development of a sustainable lean production framework (SLPF) for the petroleum industry, some insights for oil and gas production managers are crucial. The significance of this chapter lies also in the fact that it represents a pioneering study in the literature focused on the implementation of lean production for flare gas management. This further explains why three consecutive production years were considered in this chapter for validation of the results. Therefore, we highlight some general points that must be considered when implementing the proposed models.

Crude oil is more than just a resource. In many ways, it is a global currency because its production and uses have preserved industrial civilization for decades. Most importantly, it contributes significantly to economic development in oil-producing nations. Thus, it is not surprising to see significant increases in oil production in such nations, and, sometimes, OPEC members do not comply with production quotas or cuts leading to oil price wars. The two main consequences or implications of increased global oil production are oil price crashes and increased gas flaring. Environmental degradation continues to be a problem caused by gas flaring. Additionally, since crude oil is a valuable output in the petroleum industry, the application of the inverse DEA can also propose an increase in oil production. In real-life scenarios, such an application poses a serious concern to decision-makers, since they cannot calculate an increase in oil production that would result in relatively low levels of gas flaring. In this chapter, we introduced the concept of lean potential growth for the first time in literature to address these implications. For a set of oil-producing nations, the lean potential growth is the marginal increase in oil production that can result in a minimal volume of flare gas within each nation. With this information, inverse DEA models can be used not only to optimize oil production, but also to improve oil inventory management.

Secondly, we find that it is possible to set new targets for oil production and gas flaring while minimizing input resources. Essentially, our proposed methodology would allow managers to set higher and lower target levels for oil production and gas flaring, respectively, to improve operational sustainability within the petroleum industry. Using sensitivity analysis, this was demonstrated in this chapter as part of the solution to the second inverse problem. The sensitivity analysis helps a decision-maker identify the possible scenarios of increased oil production and a lower level of gas flaring. This is a crucial step towards increasing oil revenue and minimizing the environmental impact of gas flaring. For oil-exporting countries experiencing energy shortages, such as Nigeria, our methodology reveals a reliable comparative analysis of the potential power that may be generated using technologies with CRS and VRS properties. As a result, gas turbine power generation can be handled in a smarter manner.

Furthermore, the conventional ranking system in the petroleum industry has always been based on crude oil output. This method does not give detailed information on the ranks of major oil producers because some factors are ignored. To rank efficient producers based on only desirable or good outputs, we proposed a modified model incorporating lean potential growth. It is important to note, however, that this approach does not fit into the context of lean production, as it does not account for waste or undesirable outputs, such as flare gas. Toward this end, we proposed a new and robust ranking technique based on net energy production of each efficient producer. The new ranking technique is purely scientific and absolute because it is based on the principle of energy conservation.

4.6.5.6 Summary

This chapter proposes inverse DEA models and new concepts to facilitate the implementation of lean practices in the petroleum industry. With lean practices, gas flaring can be reduced, and productivity increased with minimal input resources. Our proposed models were applied to efficient oil-producing nations. Based on our reference year of 2015, we summarize the most significant results across the stages. A first-stage study showed that efficient producers were able to increase oil production up to specified levels in both CRS and VRS cases without increasing flare gas emissions. According to the results of the second stage, the reductions in gas flaring for Angola, Iraq, Libya, and Nigeria were 13.75%, 4.59%, 25.24%, and 54.93 %, respectively, for the same production targets in stage one. Using the GT13E2 turbine, the computed reductions were converted into gross power outputs for all four producers of 300MW, 150MW, 450MW, and 2250MW, respectively. Considering Nigeria's ongoing energy shortage, this is of great benefit to the country. Additionally, a sensitivity analysis revealed that an increase in oil production would benefit Angola in the short term, but Nigeria in the long term. A key finding of the sensitivity analysis was that, for any given scenario, an increase in oil production is accompanied by a reduction in gas flaring in Iraq and Qatar. Therefore, both nations have a high potential for lean production. The solution to the third inverse problem involved developing a new energy-based ranking technique. The most efficient producer was found to be Saudi Arabia, while the least efficient producer was Iraq.

There is no doubt that our proposed models performed reasonably well. This chapter is an insightful and pioneering examination of how lean production can be applied to the management of flare gas. However, the proposed models are limited to only positive data. Considering the possibility of dealing with negative data when using DEA models as demonstrated in chapter three, it is imperative to examine the impact of negative data on the estimated power outputs calculated in this chapter. It is also important to point out that the estimated power outputs are only associated with an increase in oil production. There is a need for further research to maximize the gross power estimates.

Chapter 5 – An Optimal Energy Mix based on Flare Gas Power Generation

5.1 Introduction

Throughout the previous chapters, a solid foundation has been laid for justifying the use of flare gas for power generation. A cost-effective gas-to-wire (GTW) process was developed in chapter three for inefficient producers, whereas a robust lean production framework was developed for efficient producers in chapter four. Both chapters addressed the issue of global gas flaring. Nevertheless, there are some research gaps in both chapters that are addressed in this chapter to provide an overall solution to the global problem. Specifically, this chapter examines the extent to which flare gas can be recycled for alleviating energy poverty in selected countries. Recent literature has seen the use of both positive and negative data when applying DEA models to real-world problems. Thus, an in-depth investigation will be conducted in this chapter to investigate how data type affects estimates of the maximum power that can be generated from flare gas.

5.1.1 A paradoxical view of gas flaring

Energy is recognized as a fundamental resource for advancement, wellbeing, and sustainability in modern society. As such, significant efforts are devoted to harnessing existing energy resources (Pietrosemoli & Rodríguez-Monroy, 2019). Each country is unique when it comes to its energy performance which is based on a combination of resources, policies, and structures. Some are successful in optimizing their energy resources, but others are not (Pietrosemoli & Rodríguez-Monroy, 2019). Nigeria and Venezuela, for instance, are two oil-producing nations with vast reserves of natural gas, which is an affordable and reliable source of energy. There is currently an energy crisis in both nations, which manifests itself in frequent and extended blackouts and service disruptions that affect the entire population. Yet for the ninth year in a row, both countries rank among the top seven gas flaring countries, creating an undesirable paradox in their energy economy. Power outages in Nigeria date back over two decades. In fact, some experts may argue that this has been an ongoing problem since crude oil was discovered in the country. These outages in Nigeria have more serious consequences than those in Venezuela. As an example, multinational companies in Nigeria rely on alternative sources of electricity, such as generating plants. The reason for this is the erratic power supply provided by the national grid. Consequently, such companies factor in the costs involved in operating alternate sources of energy in the final price of their products. In turn, this leads to unwanted inflation, decreasing the purchasing power of consumers in a country regarded as Africa's largest oil producer. Furthermore, power outages in Nigeria on a larger scale have a detrimental effect on the living conditions of rural residents. It is difficult for such people to engage in economic or business activities that sustain their livelihoods. While there have been many plans to increase the installed power capacity of the nation, there is always that tendency to ignore the fact that Nigeria has the largest gas reserves on the African continent and the ninth largest in the world. As most developed nations, such as the United States, rely on natural gas for power generation as their primary energy source, it is surprising that a developing nation like Nigeria, with vast natural gas reserves, would prefer to flare excess gas during oil extraction rather than use it to generate power. In these circumstances, gas flaring in developing nations such as Nigeria and Venezuela might be construed as more than just an energy paradox. It is a shocking and colossal waste of energy. Considering that sustainability refers to the strategic production and use of resources needed for all levels of human life, whether residential, industrial, transportation, commercial or recreational, it is desirable to utilize energy resources in the best possible way.

5.1.2 Energy poverty and statistics

According to the World Economic Forum, energy poverty is defined as the inaccessibility of sustainable energy products and services. The lack of sufficient, affordable, and reliable energy services also contributes to energy poverty. Energy is the engine of civilization, but today access to adequate and affordable sources of energy is not equally distributed around the globe. Often, a lack of energy can impede economic growth in terms of manufacturing and other economic indicators. Energy poverty is delineated as a condition when consumption falls below a certain threshold, which researchers have repeatedly attempted to define in the past. In spite of this, providing universally valid statistics has been unachievable since national needs vary, based on developmental stages and structural characteristics.

Around the world, almost 3.5 billion people lack access to electricity; less than ten percent of their household energy needs are met. The 2018 IEA World Energy Outlook estimates that currently 1 billion people worldwide - 13% of the total population -- lack access to electricity, primarily in Africa and South Asia. There are approximately 600 million people without electricity in Sub-Saharan Africa - 57% of the population. In developing Asia, there are 350 million people without electricity - 9% of the population. Global energy access by 2030 is one of the Goals of the United Nations Agenda for Sustainable Development. The expansion of electricity networks has made a significant difference in the energy accessibility of regions such as East Asia and Latin America since the early 2000s. A great deal of progress has been achieved in many other developing countries as well. Almost 95% of Indonesia's citizens have electricity, up from just 50% in 2000; 80% of people in Bangladesh have electricity, up from just 20% in 2000; 73% of people in Kenya now have electricity, while in Ethiopia, almost half have it compared to 5% in 2000. New programs designed to improve energy efficiency in eastern and central European neighbourhoods have been rolled out by Habitat for Humanity to support such families. It is pertinent to state here that most of the progress made is still far from being environmentally friendly. According to estimates, nearly 2.7 billion people still use solid biomass, coal, or kerosene for cooking, the most polluting of all available energy sources. There is still a need to accelerate efforts to meet the sustainable development goals of the United Nations.

5.1.3 The sustainable energy for all (SE4ALL) initiative

The United Nations General Assembly declared 2012 as the International Year of Sustainable Energy for All (SE4ALL). In a resolution, three global goals were set to be achieved by 2030: ensuring universal access to modern energy services (including electricity and clean, modern cooking solutions), doubling progress in energy efficiency globally, and doubling the share of renewable energy in the global energy mix. The Sustainable Energy for All (SE4ALL) initiative has been formally embraced by 70 countries, while tens of billions of dollars have been pledged to achieve its goals. A decade of sustainable energy for all was later declared by the UN general assembly at the end of 2012, running from 2014-2024. For the SE4ALL objectives to remain on track, global progress will need to be tracked over the years leading up to 2030. Together with 13 other agencies, the World Bank, and the Energy Sector Management Assistance Program (ESMAP) and the International Energy Agency (IEA) have coordinated the process of building the framework. More than 100 stakeholder groups have participated in the public consultation process. In a similar vein, the United Nations collaborated with the World Bank in 2015 by launching the ‘Zero Routine Flaring (ZRF) by 2030’ initiative, which aims to set an industry benchmark for sustainable production of oil and gas. Through the ZRF initiative, oil-producing nations and companies aim to eliminate routine gas flaring by 2030 as a means of combating climate change globally. As demonstrated in the previous chapters, utilizing natural gas to the fullest extent for power generation is an effective means of implementing the ZRF initiative. Thus, we need to examine further the potential of flare gas as a reliable and affordable energy source.

5.1.4 The role of natural gas in power generation

With global warming upon us, natural gas has been deemed a bridge fuel (Hausfather, 2015; Zhang et al., 2016). Due to its versatility for use in industry and the home, natural gas will likely outlast coal and oil. Yet, in regard to the petroleum industry, natural gas is largely present in its true form as associated petroleum gas (APG), since most of it is dissolved in crude oil. We must therefore take APG into account from a dual perspective. APG is technically called field gas when converted into an energy source. In contrast, when excess APG is burned off in the field, it is called flare gas, which contributes to global warming due to the release of greenhouse gases (GHGs). However, it is still necessary to briefly discuss the breakdown of natural gas production in the petroleum industry. According to the OPEC Annual Statistical Bulletin (ASB), gross production of natural gas consists of marketed production, flaring, reinjection, and shrinkage. This is expressed in the following definition:

Gross Production = Marketed production + Flaring + Reinjection + Shrinkage

Marketed production refers to the volume of gas sold to domestic and foreign markets and is a source of revenue. Flaring refers to the total volume of natural gas burned annually in each OPEC member nation. Reinjected gas into underground reservoirs is needed for increasing the yield of crude oil. During the extraction and purification processes, some volume of natural gas is lost due to shrinkage.

Considering the power outages in oil-producing countries with consistently high volumes of flare gas, one may conclude that flare gas represents a tremendous loss in the energy potential of such countries. The potential of natural gas for power generation should thus be highlighted through the relevant literature. Yao et al. (2018) used a multi-objective optimization approach to model a power plant powered by natural gas, yielding a net profit of US$ 3.97 million and a payback period of two years. Man et al. (2018) conducted life cycle assessments for both coal-fired power generation (CPG) and synthetic natural gas (SNG) generation. According to the results, the CPG route had relatively high-water consumption, making the SNG route suitable for water-scarce regions.

A study has been conducted on the environmental and economic impacts of using natural gas for Kuwait's electricity generation. The results showed reductions of 36%, 98%, and 9% in nitrogen oxides, SO_2, and total energy cost, respectively (Alhajeri et al., 2019). Jirutitijaroen et al. (2013) proposed a stochastic programming model to help a power company optimize its natural gas consumption and electricity generation schedule. Two uncertainties were considered in their stochastic model: electricity price and natural gas price. Based on the findings, the power company's expected profits could increase by 25% under price volatility. Xiao et al. (2016) employed system dynamics modelling to examine the development pattern and constraints associated with the development of natural gas power generation in China. The research results indicated that natural gas power generation in China will grow at an average annual rate of 10%, with the natural gas installed capacity expected to reach 235.7GW by 2030. In the context of integrating the planning of electricity and natural gas distribution in the face of uncertain power demand, a probabilistic approach was proposed. This approach revealed a cheaper and more efficient hybrid system than the existing sequential approach (Odetayo et al., 2017).

5.1.4.1 Natural gas as an affordable energy source for Venezuela

In light of the ongoing power outages in Venezuela since late 2009, one cannot help but wonder whether the problem is becoming a conundrum. Although Rosales and Sánchez (2021) have emphasized the political dimensions of the problem, if you take a broader view of the situation, you find out that, according to the World Bank archives, Venezuela owns some of the world's largest gas reserves, but ranks as one of the top seven countries for gas flaring. As the world advocates switching to renewable energy sources, some experts do not realize that oil-exporting nations like Nigeria and Venezuela need time to join the energy transition bandwagon. With an increasing level of economic hardship and the continual devaluation of some national currencies, it will be increasingly difficult for some third-world countries to transition successfully to renewable energy technologies. The cost of certain renewable energy technologies has decreased over time, but if the same cost is converted into devalued currencies, it may remain expensive for some third-world countries. This harsh reality has not been addressed in prior studies, and a gradual approach that makes use of energy sources like natural gas to help alleviate energy poverty in Venezuela is needed.

Pietrosemoli and Rodríguez-Monroy (2019) have provided an in-depth analysis of Venezuela's energy situation, thus highlighting the nation's vast energy resources including natural gas which amounts to approximately 3.1% of the world's proven natural gas reserves. There is therefore an urgent need to explore how flare gas can be used in Venezuela for generating power. Additionally, by reducing its annual gas flaring through power generation, Venezuela will be better equipped to adhere to the Global Gas Flaring Reduction (GGFR) partnership, the Clean Development Mechanism (CDM) of the United Nations, and the ZRF Initiative for 2030. In this context, we first address the gas flaring problem in oil-producing nations, and then narrow it down to Venezuela as our case study. It is also necessary to analyse the installed capacity of power plants in Venezuela, their available capacity, and the total energy demand. In this regard, Table 5.1 presents the breakdown of energy statistics for the nation in 2015.

Table 5.1 shows that out of 34100MW installed capacity (including thermal and hydropower), only 16500MW is available, leaving a deficit of 17600MW. Thermal and hydropower capacity losses total 11100MW and 6500MW, respectively. We must determine how much of this total deficit can be recovered by power generation from flare gas. In order to do this, we need to consider the volume of flare gas within the country in 2015. There is no doubt that this is crucial for addressing the ongoing power outages in the nation. Additionally, this strategy is critical in combating air pollution, or the environmental degradation associated with routine gas flaring within the nation.

Table 5.1: The 2015 Venezuelan energy statistics

Power plant type	Installed capacity (MW)	Available capacity (MW)	Unavailable capacity (MW)
Thermal	17,600	6,500	11,100
Hydropower	16,500	10,000	6,500
Total MW	34,100	16,500	17,600

Source: Adapted from Pietrosemoli and Rodríguez-Monroy (2019)

5.1.5 Problem statement

Studies addressing the issue of gas flaring have yet to develop a robust mathematical framework for determining the maximum amount of electricity that can be produced from flare gas. Such information is vitally important for decision-makers in nations trying to alleviate energy poverty. Essentially, there is a significant gap that necessitates the recycling of flared natural gas for maximum power generation. For this purpose, we will apply the directional distance DEA model. As a second concern, when modelling real-life problems with DEA, negative data may be encountered. Therefore, we will investigate whether negative data has a significant effect on the capability of directional distance DEA to estimate power generation from flare gas.

With regard to both of the above problems, it is pertinent to point out that Chapter 3 of this study only determined the optimal sizing of the gas-to-wire (GTW) process using the inverse DEA, and briefly explained how it can be used for energy analysis. While Chapter 4 took it further to calculate the amount of energy (or gross power) that can be generated from flare gas; the estimates of power are neither maximum nor minimum. The calculations are limited to power generation associated with lean production or a marginal increase in oil production. Generally, neither chapter examined the combined effects of thermal efficiency and negative data on maximum power generation. There is a need for decision makers to pay close attention to this issue, since negative data may also affect the optimality of DEA models, giving rise to the possibility of exploring the concept of optimal power sizing.

Chapter 3 of this study is the first in literature to develop an energy transition curve for the petroleum industry, but no investigation is provided to determine whether negative data impacts the curve. Hence, another gap in the research literature arises: Can the absence or presence of negative data create perturbations in the energy transition curve of oil-producing nations?

To alleviate energy poverty as much as possible in selected oil-producing nations, we propose a novel strategy based on recycling flare gas to generate maximum power. Specifically, this chapter aims to answer the following research questions:

- *To what extent can flare gas be recycled for power generation to address energy shortages in an oil-producing nation?* Due to the fact that our case study, Venezuela, ranks among the top seven nations in the world when it comes to gas flaring, the issue is of utmost importance. Additionally, the nation has one of the largest gas reserves in the world.
- *In the context of the directional distance DEA methodology, what effect does the inclusion of negative data have on the estimates of power generated?* Since the conversion process (i.e., gas-to-wire) is relatively costly, the answer to this question can assist a decision-maker or policymaker in avoiding the error of overestimation.
- *In light of the current push towards the transition from fossil fuels to renewable energy sources, what is the optimal energy mix for an oil-producing nation that combines gas power generation with renewable energy sources?* Identifying an optimal energy mix will involve computing the maximum power that can be generated from flare gas, while considering the nation's unavailable power generation capacity. Should the estimated maximum power not cover the nation's unavailable power generation capacity, the remainder will be allocated to renewable energy generation.

Moreover, this chapter completes a trilogy on novel strategies for gas flaring reduction in the petroleum industry by filling in the research gaps of Chapters 3 and 4. Lastly, in order to assess overall performance, it is necessary to benchmark our proposed methodology against the inverse DEA developed in Chapter 3.

5.2 Methodology

5.2.1 Nomenclature

The nomenclature for this chapter is based on some sets from Chapter 3 of this study with minor additions:

$General\ Parameters$:

n: number of decision-making units (DMUs)

m: number of inputs of each DMU

s: number of good outputs of each DMU

q: number of bad outputs of each DMU

$Data\ Parameters$:

x_{ij}: ith input of DMU_j ($j = 1, \dots, n$)

y^g_{rj}: rth good output of DMU_j ($j = 1, \dots, n$)

y^b_{pj}: pth bad output of DMU_j ($j = 1, \dots, n$)

$Decision\ Variables$:

θ_k: inefficiency score of DMU_k ($k = 1, \dots, n$)

λ_j: weight assigned to DMU_j ($j = 1, \dots, n$)

5.2.2 Preliminaries

As the proposed methodology for this chapter is based on modified versions of the directional distance DEA, we will reintroduce the following DEA proposed by Chung et al. (1997) to reduce bad outputs:

$$M1:\ \theta_k^* = \max \theta$$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik} \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y^g_{rj} \ge (1+\theta) y^g_{rk} \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y^b_{pj} = (1-\theta) y^b_{pk} \qquad p = 1, \dots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \dots, n$$

Where θ_k^* is the inefficiency score of DMU_k, such that DMU_k is regarded as an efficient unit if $\theta_k^* = 0$. For every input i that is positive for some DMUs and negative for others, $M1$ can be modified to give model $M2$ as follows:

$M2$: $\theta^* = \max \theta$

$$s.t.$$

$$\sum_{j=1} \lambda_j \, x_{ij}^{+} \leq x_{ik}^{+} \qquad i = 1, \dots, m$$

$$\sum_{j=1} \lambda_j \, x_{ij}^{-} \leq x_{ik}^{-} \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \geq (1+\theta) y_{rk}^{g} \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^{b} = (1-\theta) y_{pk}^{b} \qquad p = 1, \dots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \dots, n$$

To simplify the model, $M2$ was transformed into its SORM version (i.e., $M5$ in Chapter 3) as follows:

$M3$: $\theta^* = \max \theta$

$$s.t.$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \qquad i = 1, \dots, m \qquad i \in I_1$$

$$\sum_{j=1} \lambda_j \, x_{ij}^{+} \leq x_{ik}^{+} \qquad i = 1, \dots, m \qquad i \in I_2$$

$$\sum_{j=1} \lambda_j \, x_{ij}^{-} \leq x_{ik}^{-} \qquad i = 1, \dots, m \qquad i \in I_2$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \geq (1+\theta) y_{rk}^{g} \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j y_{pj}^b = (1-\theta) y_{pk}^b \qquad p = 1, \dots, q$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0 \qquad j = 1, \dots, n$$

As a result, $M3$ is an applied version of $M2$ based on the data type. Models $M1$ and $M3$ constitute the primary methodology of this chapter since each model is designed to accommodate a specific type of input data. Using both models, we will address the energy supply crisis in our chosen case study. In this regard, we recall the following research questions of this chapter as outlined in section 5.1.5:

- *To what extent can flare gas be recycled for power generation to address energy shortages in an oil-producing nation?*
- *In the context of the directional distance DEA methodology, what effect does the inclusion of negative data have on the estimates of power generated?*
- *In light of the current push towards the transition from fossil fuels to renewable energy sources, what is the optimal energy mix for an oil-producing nation that combines gas power generation with renewable energy sources?*

To answer the first research question, a decision-maker must calculate the maximum amount of power that can be recycled from flare gas. Due to the inevitable occurrence of safety and maintenance flaring, the estimate of maximum power is directly proportional to the maximum reduction of flare gas that can be achieved.

5.2.3 Derivation of maximum power generation from flare gas

Using simple cycle gas turbines, flare gas can be converted into electricity by using the estimated reduction of flare gas, which in this case is the undesirable output in $M1$ and $M3$. Specifically, both models emphasize the reduction of the undesirable output p. To convert such estimated reduction into power, we rewrite the constraint of the undesirable output(s) as follows:

$$(1-\theta)y_{pk}^{b} = y_{pk}^{b} - (\theta y_{pk}^{b})$$

From the above constraint, it is apparent that as θ increases, the reduction of the undesirable output (i.e., θy_{pk}^{b}) will also increase. Let us denote the estimated reduction as $\Delta y = \theta y_{pk}^{b}$. Since the objective is to maximize θ, the maximum reduction of pth undesirable output of DMU_k can be expressed as:

$$\Delta y^{max} = \theta^{*} y_{pk}^{b} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5.1)$$

where θ^{*} is the optimal solution of model $M1$ or $M3$.

As we are dealing with two similar models (i.e., $M1$ and $M3$) but with different structures, it is appropriate to express equation (5.1) separately for each model. In order to accomplish this, we must first recognize that this chapter is concerned only with one undesirable output, namely waste or flare gas. So, at all times, $p = 1$. As a means of distinguishing θ^{*}across both models, let θ_1^{*} and θ_3^{*} be the optimal solutions of models $M1$ and $M3$, respectively. In a similar manner, let Δy_1^{max} and Δy_3^{max}denote the maximum reductions with respect to both models. For ease of analysis, we state the maximum reductions estimated by each model as follows:

$$\Delta y_1^{max} = \theta_1^{*} y_k^{b} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5.2)$$

$$\Delta y_3^{max} = \theta_3^{*} y_k^{b} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5.3)$$

Previously, we demonstrated that when converting gas to electricity, the optimal number of turbine units can be determined based on a turbine's annual gas consumption and gas flaring reductions. Suppose σ is the annual gas consumption of a simple gas turbine, then it follows that the optimal number of turbine units needed is expressed as:

$$\tau = \frac{\Delta y^{max}}{\sigma} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (5.4)$$

Note that τ can only take integer values. As in Chapter 4, let W and φ represent the work output and thermal efficiency of a particular gas turbine. Then power output of the turbine is defined as the product of its thermal efficiency and work output. In mathematical terms, for a single turbine, $Power = \varphi\, W$. Thus for τ turbines, we have:

$$Total\ power\ P = \varphi\, W\tau \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (5.5)$$

Thus, we substitute equation (5.4) into equation (5.5) to obtain actual power generation via all turbines as follows:

$$P = \varphi\, W \left(\frac{\Delta y^{max}}{\sigma}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (5.6)$$

In the same manner as explained in Chapter 4, a turbine's rated output is usually considered its gross power output if we ignore heat losses or thermal efficiency. Hence, we have that:

$$Gross\ maximum\ power, P^{max} = W \left(\frac{\Delta y^{max}}{\sigma}\right) \ldots\ldots\ldots. (5.7)$$

Accordingly, this derived expression for gross maximum power provides the answer to the first research question. In contrast to Chapter 4, where the gross power calculated is neither maximum nor minimum, the gross power in equation (5.7) is a maximum. Clearly, this is evident from the fact that this maximum power is directly proportional to the maximum gas flaring reduction (i.e., Δy^{max}) as indicated on the right-hand side of equation (5.7). In this way, a decision maker will be able to design an energy mix that combines gas and renewable energy sources. A subsequent section will demonstrate this. We can now express the gross maximum power estimated by both models $M1$ and $M3$ as follows:

$$P_1^{max} = W \left(\frac{\Delta y_1^{max}}{\sigma}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (5.8)$$

$$P_3^{max} = W \left(\frac{\Delta y_3^{max}}{\sigma}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.9)$$

Despite the similarities in both expressions, one cannot ignore the influence of negative data when applying the expression for P_3^{max} due to the fact that it is derived for the model $M3$, which accommodates negative input data.

To investigate which model estimates a greater power given the absence or presence of negative data, we propose the following theorem:

Theorem 1 (*Theorem of optimal power sizing*)

The maximum power estimated by $M1$ is greater than or equal to that estimated by $M3$ i.e., $P_1^{max} \geq P_3^{max}$

Proof

For any given DMU_k, recall that the optimal solutions of $M1$ and $M3$ are θ_1^* and θ_3^*, respectively. These optimal solutions will generate maximum reductions of Δy_1^{max} and Δy_3^{max}, respectively. From equation 3.1.7, the gross maximum power P^{max} varies directly as the maximum reduction Δy^{max}. Since $M1$ is a relaxation of $M3$, by omitting both I_2 sets' constraints, the optimal solution of $M3$ will always be a feasible solution to $M1$. Therefore, according to the objective functions of $M1$ and $M3$, we have $\theta_1^* \geq \ \theta_3^*$. Then it follows that $\Delta y_1^{max} \geq \ \Delta y_3^{max}$ and $P_1^{max} \geq P_3^{max}$. This completes the proof.

Consequently, this theorem answers the second research question, and we can deduce three corollaries.

Corollary 1: $P_1^{max} > P_3^{max}$ indicates that negative data reduces the value of maximum power estimated by $M3$. Hence, the optimal power sizing is as follows:

$$P_3^{max} \leq P^{max} \leq P_1^{max}$$

In terms of gas flaring reductions, this optimal power sizing can also be expressed as:

$$W\left(\frac{\Delta y_3^{max}}{\sigma}\right) \leq P^{max} \leq W\left(\frac{\Delta y_1^{max}}{\sigma}\right)$$

Corollary 2: If $P_1^{max} = P_3^{max}$, then negative data has no impact on $M3$

Corollary 3: There is a mathematical possibility that some DMU_k determined to be efficient by $M3$ can also be determined to be inefficient by $M1$, since its inefficiency score θ^* may increase in the absence of negative data.

Corollaries 1 and 2 cannot hold simultaneously for each producer. If the first corollary is satisfied from the obtained results, then we refer to Theorem 1 as the theorem of optimal power sizing. Although we are exploring the effect of negative data on both models, we must take note that $M3$ provides a ***realistic/practical estimate***, whereas $M1$ provides an ***optimistic/theoretical estimate*** (i.e., an upper bound).

As a result of modelling real-world problems with DEA, researchers have discovered that negative data cannot simply be ignored, hence relaxing the prior assumption that all data are positive in the application of DEA models.

5.2.4 Design of an optimal energy mix

A major goal of the Sustainable Energy for All (SE4ALL) initiative is to improve energy efficiency through an energy mix. This goal aims to increase the share of renewable energy in the global energy mix to gradually transition to 100% renewable energy. To aid oil-producing nations in improving energy efficiency, it is imperative that we design an optimal energy mix that includes gas-powered and renewable energy sources. Flare gas is a huge waste of affordable and reliable energy, and to combat its associated environmental degradation, an optimal energy mix must be based on the maximum amount of power that can be recovered from flare gas in conjunction with renewable sources. Additionally, a nation's power generation capacity must also be considered when formulating an optimal energy mix. To begin with, let ω denote the unavailable power generation capacity of a nation. Furthermore, let η_g and η_r represent the proportions (i.e., in percentages) of gas and renewable energy sources that make up the proposed energy mix. Consequently, it follows that:

$$\eta_g + \eta_r = 1 \dots\dots\dots\dots\dots\dots\dots.(5.10)$$

Recall from section 5.2.3, that gross maximum power is expressed as:

$$P^{max} = W\left(\frac{\Delta y^{max}}{\sigma}\right)$$

Hence, from equation (5.10) it can be inferred that the proportions of gas-powered and renewable energy generation are as follows:

$\eta_g = \left(P^{max}/\omega\right)$, and $\eta_r = 1 - \left(P^{max}/\omega\right)$, provided that $P^{max} < \omega$.

Considering models $M1$ and $M3$, the optimal energy mix for each is as follows:

For $M1$: $\eta_g = \left(P_1^{max}/\omega\right)$, and $\eta_r = 1 - \left(P_1^{max}/\omega\right)$, provided that $P_1^{max} < \omega$

For $M3$: $\eta_g = \left(P_3^{max}/\omega\right)$, and $\eta_r = 1 - \left(P_3^{max}/\omega\right)$, provided that $P_3^{max} < \omega$

In the special case where $P^{max} \geq \omega$, gas-powered generation is only recommended, and renewable energy sources may not be required in the short-term, but rather a long-term investment. This means that an energy mix is not necessary if $P^{max} \geq \omega$. However, if an energy mix is still desired for this special case, the proportion of renewable sources should be determined at the discretion of the decision maker. Even so, the most likely scenario in the case of persistent energy shortages is one in which $P^{max} < \omega$. In general, this section addresses the third research question.

5.2.5 Summary of main research methodology

In this section, we summarize the primary methodology of this study. The process flow diagram (PFD) shown in Fig. 5.1 illustrates how this is accomplished. It begins with the classification of the data for our proposed models $M1$ and $M3$. The two models have the same types of two outputs (one desirable output and one undesirable output). In this case, the undesirable output is waste gas or flare gas that must be recycled for power generation. Input classification is the primary difference between the two models. The model $M1$ considers four positive inputs for all DMUs (i.e., oil-producing nations). Alternatively, model $M3$ incorporates another input to make a total of five inputs to examine how negative data affect estimates of power generation. The additional input is positive for some DMUs and negative for others. As previously explained in Chapter 3, a classic example of input with such duality is the current account balance of each oil-producing nation. Current account surpluses are modelled as positive input data, whereas current account deficits are modelled as negative input data.

Following the classification of inputs in Fig. 5.1, the next step is to evaluate the efficiency of each DMU using both models. An efficient DMU has $\theta^* = 0$, whereas an inefficient DMU has $\theta^* > 0$. Next, we calculate the maximum reduction in undesirable output (i.e., Δy^{max}) for each inefficient DMU across both models. Using turbines, the maximum reductions obtained from both models will be converted into maximum power generation (i.e., P^{max}).

In the following step, the estimates of maximum power are compared using our proposed theorem of optimal power sizing. For each DMU, the theorem holds if both power estimates are unequal. The theorem does not hold, however, if the power estimates are equal. Afterward, the decision-maker must compare each power estimate with the unavailable power generation capacity in the case study. An energy mix must be designed if either of the power estimates is less than the unavailable power generation capacity. In the event that the power estimate(s) exceed or equal the unavailable power generation capacity, only gas-powered generation should be considered for the case study, and no mix of energy sources is necessary. This concludes the main methodology for this study.

Since this is a pioneering study on the design of an energy mix based on maximizing power generation from flare gas, it is imperative that our methodology be compared to that of previous research. Furthermore, in this regard, we would like to point out that the proposed methodology of Chapter 3 can also determine the maximum reduction of undesirable outputs, which we have incorporated into our research framework. In order to assess the performance of our proposed models, we will compare the estimated maximum reductions in this chapter with those reported in Chapter 3 for the same set of DMUs or producers. For this purpose, we should examine the inverse DEA model which incorporates all five inputs (including current account balances) developed in Chapter 3 and formulate a corresponding version with only four positive inputs that are perfectly matched to the structure of model $M1$ (i.e., with four positive inputs). The next section will cover the required analysis and related theorems.

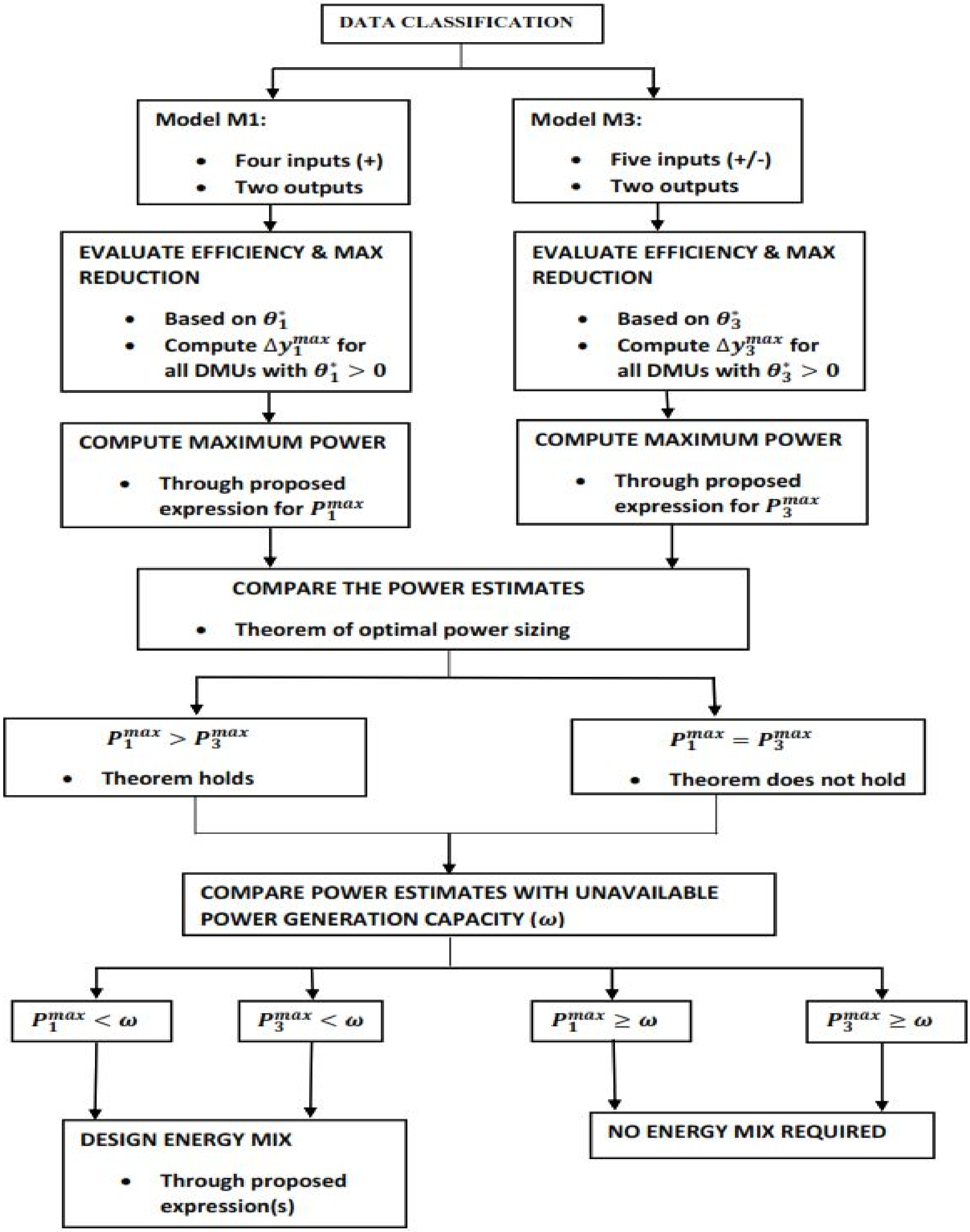

Figure 5.1: Framework of research methodology

5.2.6 A special case of maximum gas flaring reductions

Suppose we are interested in using $M2$ as a base model and need to estimate the minimum and maximum gas flaring reductions for an inefficient DMU_k. The following inverse DEA model is suitable for this application:

$$
\begin{aligned}
M4: \quad & \max \gamma^* = \gamma_{pk} \\
& s.t. \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij}^+ \leq x_{ik}^+ && i = 1, \dots, m \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij}^- \geq x_{ik}^- && i = 1, \dots, m \\
& \sum_{j \epsilon F} \lambda_j^k y_{rj}^g - \left(1 + \hat{\theta}_k\right) \times y_{rk}^g \geq 0 && r = 1, \dots, s \\
& \sum_{j \epsilon F} \lambda_j^k y_{pj}^b - \left(1 - \hat{\theta}_k\right) \times \left(y_{pk}^b - \gamma_{pk}\right) = 0 && p = 1, \dots, q \\
& \sum_{j \epsilon F} \lambda_j^k = 1 \\
& \gamma_{pk} \leq y_{pk}^b \\
& \gamma_{pk} \geq 0 \ \forall k \in G, \ i = 1, \dots, m \ \ r = 1, \dots, s \ \ p = 1, \dots, q \\
& \lambda_j^k \geq 0, \ \forall k \in G, \ \ \forall j \in F
\end{aligned}
$$

To preserve the efficiency score of DMU_k, and to keep $M4$ feasible at all times due to negative data, one must set $\hat{\theta}_k < \theta^*$. As an example, but under the full discretion of the decision-maker, one can define $\hat{\theta}_k$ as 1% less than θ^* . Where θ^* is the optimal solution of $M2$. The application of $M4$ may also require the SORM technique depending on the type of input data. Here, we transform $M4$ into an applied version, $M5$ (i.e., same model $M8$ in Chapter 3), as follows:

$$
\begin{aligned}
M5: \quad & \max \gamma^* = \gamma_{pk} \\
& s.t. \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij} \leq x_{ik} && i = 1,2, \dots, m \quad i \epsilon I_1 \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij}^+ \leq x_{ik}^+ && i = 1,2, \dots, m \quad i \epsilon I_2 \\
& \sum_{j \epsilon F} \lambda_j^k x_{ij}^- \geq x_{ik}^- && i = 1,2, \dots, m \quad i \epsilon I_2 \\
& \sum_{j \epsilon F} \lambda_j^k y_{rj}^g - \left(1 + \hat{\theta}_k\right) \times y_{rk}^g \geq 0 && r = 1, \dots, s
\end{aligned}
$$

$$\sum_{j\epsilon F}\lambda_j^k y_{pj}^b-\left(1-\hat{\theta}_k\right)\times\left(y_{pk}^b-\gamma_{pk}\right)=0 \qquad p=1,\dots,q$$
$$\sum_{j\epsilon F}\lambda_j^k=1$$
$$\gamma_{pk}\le y_{pk}^b$$
$$\gamma_{pk}\ge 0\ \ \forall k\in G,\ i=1,\dots,m\ \ r=1,\dots,s\ \ p=1,\dots,q$$
$$\lambda_j^k\ge 0,\ \forall k\in G,\ \ \forall j\in F$$

It should be noted that $M3$ is the proper base model for $M5$, and both are based on the SORM technique. In the case of a decision-maker wishing to use $M1$ as a base model (i.e., the absence of negative input data), $M5$ must be transformed into $M6$ as follows:

$M6$: $\max \gamma^*=\gamma_{pk}$

$$s.t.$$
$$\sum_{j\epsilon F}\lambda_j^k x_{ij}\ \le\ x_{ik} \qquad i=1,\dots,m$$
$$\sum_{j\epsilon F}\lambda_j^k y_{rj}^g-\left(1+\hat{\theta}_k\right)\times y_{rk}^g\ge 0 \qquad r=1,\dots,s$$
$$\sum_{j\epsilon F}\lambda_j^k y_{pj}^b-\left(1-\hat{\theta}_k\right)\times\left(y_{pk}^b-\gamma_{pk}\right)=0 \qquad p=1,\dots,q$$
$$\sum_{j\epsilon F}\lambda_j^k=1$$
$$\gamma_{pk}\le y_{pk}^b$$
$$\gamma_{pk}\ge 0\ \ \forall k\in G,\ i=1,\dots,m\ \ r=1,\dots,s\ \ p=1,\dots,q$$
$$\lambda_j^k\ge 0,\ \forall k\in G,\ \ \forall j\in F$$

For this case, where there is no negative data, to preserve the efficiency score of DMU_k and to keep $M6$ feasible at all times, one must set $\hat{\theta}_k\le\theta^*$. Where θ^* is the optimal solution of $M1$. When compared to $M2$ and $M4$, which were designed to handle negative input(s), $M6$ is the exact inverse of base model $M1$ and can only be applied to positive data. Moreover, every mathematical relationship that exists between model pair $M2$ and $M4$, will also hold for model pair $M1$ and $M6$. Most importantly, the models $M4, M5$, and $M6$ all have the benefit of a maximum gas flaring reduction when $\hat{\theta}_k=0$.

Toward exploring possible relationships between our proposed models, we pose the following additional questions:

- Are both the models $M5$ and $M6$ capable of estimating equal power?
- Can the base and inverse DEA estimate equal gas flaring reductions?
- Does negative data impact on the energy transition curves of $M5$ and $M6$?

Answers to the additional questions in this section may be found in these theorems and the energy transition curve.

Theorem 2

The maximum power estimated by $M6$ is greater than or equal to that estimated by $M5$

Proof

This proof is similar to that of Theorem 1

Theorem 3 (*Theorem of equivalence*)

The maximum gas flaring reductions estimated by $M2$ and $M4$ (as well as by their SORM versions i.e., $M3$ and $M5$, respectively) are perfectly equal, if and only if, $\theta^* = 1$ in $M2$ and $M3$, and $\hat{\theta}_k = 0$ in $M4$ and $M5$. However, in all other cases, the maximum gas flaring reduction estimated by both models will be approximately equal. (In the presence of only positive input data, this also holds for $M1$ and $M6$).

Proof

Consider the case where an inefficient DMU_k has its undesirable output reduced by $M2$. To obtain an optimal solution to $M2$, one must rewrite the RHS of the equation representing the undesirable output as follows:

$$y_{pk}^b - \left(\theta^* y_{pk}^b\right) = y_{pk}^b - \Delta y^{max} \quad \dots\dots\dots\dots\dots.(5.11)$$

In an ideal or perfect scenario, all waste would be eliminated, or the total undesirable output (i.e., y_{pk}^b) would be reduced to zero if and only if, $\theta^* = 1$ in equation (5.11)

Therefore, when $\theta^* = 1$, $y_{pk}^b - \left(\theta_k^* y_{pk}^b\right) = y_{pk}^b - \Delta y^{max} = 0$

$$y_{pk}^b = \Delta y^{max} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(5.12)$$

If the same inefficient DMU_k is subjected to a reduction in its undesirable output by $M4$, we have already proven in Chapter 3 that the maximum reduction (i.e., γ_{pk}^{max}) occurs when $\hat{\theta}_k = 0$. Mathematically, this can be expressed as follows:

At zero inefficiency (i.e., $\hat{\theta}_k = 0$), $y_{pk}^b = \gamma_{pk}^{max}$ $\dots\dots\dots\dots\dots\dots\dots..(5.13)$

Combining (5.12) and (5.13) for the same DMU_k, we have:

$$\gamma_{pk}^{max} = \Delta y^{max} = \theta^* y_{pk}^b$$

Therefore, for any value of θ^*, we conclude: $\gamma_{pk}^{max} = \theta^* y_{pk}^{b}$ (QED). Since power correlates positively with gas flaring reductions, the maximum power estimated by both models will be the same. This theorem will also be demonstrated in the remaining sections.

5.2.7 The energy transition curve

In Chapter 3, we developed, for the first time in literature, an energy transition curve for the petroleum industry using model $M8$ (i.e., renamed $M5$ in this chapter). This was done through sensitivity analysis. The energy transition curve illustrates our proposed theorem of maximum gas flaring reduction occurring at zero inefficiency, i.e., when an inefficient producer becomes fully efficient. Thus, the curve represents the gradual transition from an inefficient state to an efficient state while reducing gas flaring. The gradual transition is achievable through investment in flare gas recovery technology and better management strategies.

To simplify explanation, we will use models $M3$ and $M5$ for this section in accordance with the model notations of this chapter. Here, we examine the impact of negative data on the projected reductions in gas flaring estimated by $M5$. The original energy transition curve is shown as AB in Fig. 5.2. In contrast to the model $M3$, this curve can locate two extremes, which are the minimum and maximum reductions in gas flaring for an inefficient oil-producing country. The curve AB was designed primarily for optimal sizing of gas-to-wire (GTW) processes. As shown at point A, the producer is significantly inefficient, while at point B (i.e., with zero inefficiency), after completing its potential transition, it becomes fully efficient, thus reducing gas flaring to its lowest levels possible.

Based on Theorem 2, if the maximum power estimated by $M6$ is greater than that estimated by $M5$ for a given producer, the gas flaring reductions estimated by $M6$ will also be greater than those estimated by $M5$. Fig. 5.2 shows the resultant effect of an absence of negative data as lines CD and EF. A rightward perturbation of the energy transition curve AB leads to an increase in both the minimum and maximum reductions estimated by $M6$ in this case, creating a new transition curve $A^{|}B^{|}$. In this regard, it is imperative to note that the increase in both estimated reductions can only be achieved if we have the same number of producers for both curves. If, for example, the number of producers should increase due to the exclusion of negative data (i.e., special case of the third corollary of Theorem 1), then only the maximum reductions would increase. The concept of optimal power sizing is therefore more applicable to maximum reductions.

We must emphasize that the maximum reductions at points B and $B^{|}$ in Fig. 5.2 can also be determined by models $M3$ and $M1$, but with a shorter computation time than their inverse counterparts $M5$ and $M6$, respectively. $M5$ and $M6$, however, have an advantage over $M3$ and $M1$, respectively, in determining the minimum reductions at points A and $A^{|}$ while developing curves AB and $A^{|}B^{|}$. In terms of gas flaring reductions, this is the fundamental difference between the directional distance DEA and the inverse DEA. In summary, recall that Δy_3^{max} and Δy_1^{max} denote the maximum reductions estimated by $M3$ and $M1$, respectively. Hence, we can state here that:

For curve AB, $\gamma_{AB}^{max} = \Delta y_3^{max}$ at point B through the application of $M5$ or $M3$ (i.e., $M5 \equiv M3$). For curve $A^{|}B^{|}$, $\gamma_{A^{|}B^{|}}^{max} = \Delta y_1^{max}$ at point $B^{|}$ through the application of $M6$ or $M1$(i.e., $M6 \equiv M1$).

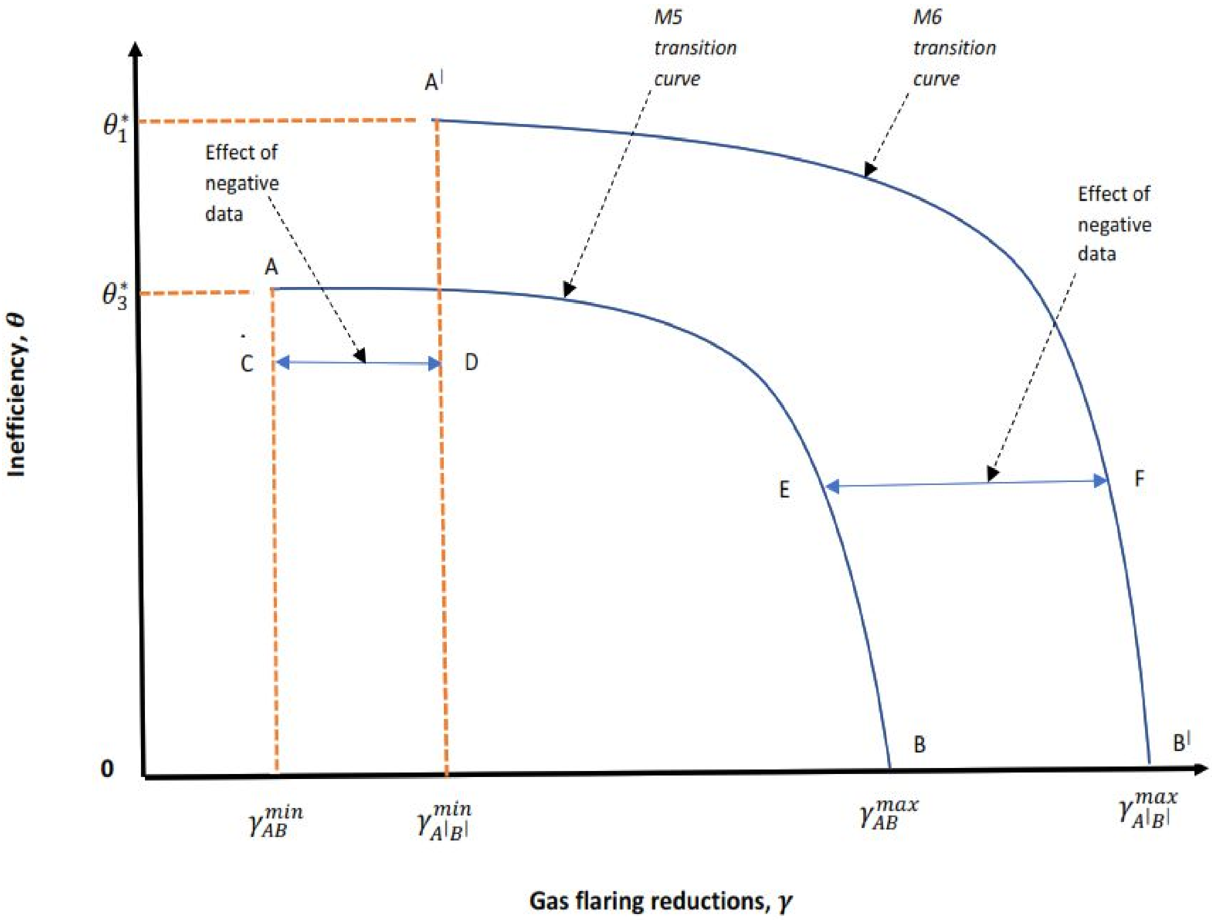

Figure 5.2: Effect of negative data on the energy transition curve

5.3 Application, results, and analysis

5.3.1 Application of the proposed models

We describe the application of our proposed models $M1$ and $M3$ in this section by following the detailed steps in the process flow diagram of Fig. 5.1. Our first step towards benchmarking our proposed models will be to use the oil production data provided in Chapter 3, which is an extract from the ASB database. The data is composed of five inputs: the current account balance (surplus and deficit balances as positive and negative inputs, respectively), wells completed, producing wells, active rigs, and refining capacity; and two outputs: GDP per capita and routinely flared gas. Negative input data comes from deficit current account balances, whereas routinely flared gas is undesirable output or waste that has to be recycled for power generation. By not including the source of negative data (i.e., current account balances), we apply $M1$, which requires using only four inputs while $M3$ requires the use of all inputs. Based on the sated nomenclature, θ_1^* and θ_3^* are the optimal solutions of models $M1$ and $M3$.

Further, it is important to reiterate and justify the rationale for this chapter in terms of maximum gas flaring reductions, which are determined by the optimal solutions of models $M1$ and $M3$. The Zero Routine Flaring (ZRF) initiative is a joint initiative of the World Bank and the United Nations developed with the aim of achieving the greatest possible reduction in gas flaring. This is the underlying principle of our proposed methodology, which is to impose the maximum reduction in gas flaring among OPEC nations.

In Table 5.2, we compare both models' optimal solutions and gas flaring reductions for the six producers discussed in Chapter 3. From Table 5.2, it is clear that, $\theta_1^* > \theta_3^*$ for all six producers. In other words, the optimal solutions, and the flare reductions under $M1$ are greater than those under $M3$. Thus, the first corollary of Theorem 1 is satisfied, leading to the concept of optimal power sizing for the producers. We will need to calculate optimal power sizing for each producer based on the estimated reductions in gas flaring.

Table 5.2: Maximum gas flaring reductions of six OPEC members

DMU_k	Optimal values θ^*		Maximum gas flaring reductions Δy^{max} $(m\ cu^3)$	
	θ_1^*	θ_3^*	Δy_1^{max}	Δy_3^{max}
Algeria	0.8873	0.8311	3197.83	2995.28
Indonesia	0.9115	0.7835	2235.00	1921.14
Iraq	0.9453	0.9162	9086.22	8806.51
Nigeria	0.9655	0.9124	13777.69	13019.95
UAE	0.5984	0.5576	587.63	547.56
Venezuela	0.9766	0.9479	9066.75	8800.30

5.3.2 Comparative analysis

While the overall aim of our proposed methodology is to design an energy mix, it is necessary to compare our obtained results (i.e., the maximum reductions) with those of a prior study. This is intended to provide a means for validating the models. For the same set of producers, this sub-section compares the estimated reductions under $M3$ (i.e., the SORM or applied version of $M2$) in Table 2 with those estimated in Chapter 3. As a point of clarification, we make this initial comparison because only model $M3$ can accommodate negative input data, as with the inverse DEA developed in Chapter 3. A comparative analysis of the estimated reductions is presented in Table 5.3. The reductions under $M3$ are denoted by Δy_3^{max}, while those obtained in Chapter 3 are denoted by γ_5^{max}.

Table 5.3 demonstrates that, except for Venezuela showing a deviation of 0.28% in computed maximum reductions, all other producers have about equal estimates across both models. The reason why there is not a perfect match between the maximum reductions for all the producers in Table 5.3 is because $\theta^* < 1$ for all the producers in Table 5.2. Therefore, it is difficult to eliminate production waste in real-world situations. Likewise, Chapter 3 found that the estimated maximum flare reductions of the same six producers were less than the flare gas volumes they produced. We see an example here where neither of the conditions of Theorem 3 are satisfied due to real-world application of the two models. It is therefore expected that the maximum reductions will be approximately equal. In addition, since the optimal number of turbines, τ, can only take integer values, both models will estimate the same maximum power. The same applies to the models $M1$ and $M6$ as well.

Table 5.3: Comparison of gas flaring reductions in the presence of negative data

Producers	Algeria	Indonesia	Iraq	Nigeria	UAE	Venezuela
Maximum Reduction Δy_3^{max} $(m\ cu^3)$	2995.28	1921.14	8806.51	13019.95	547.56	8800.30
Maximum Reduction $\gamma_5^{max}(m\ cu^3)$	2995.35	1921.25	8806.19	13020.20	547.58	8825.04

Although we did not apply the models $M1$ and $M6$ in Chapter 3, we must compare the reductions under both models in order to further demonstrate our theorem of equivalence. Table 5.4 shows the reductions under $M1$ and $M6$, and here we see a better match of the flaring reductions for all the producers, especially for Venezuela. Hence, we have fully demonstrated our theorem of equivalence, and our proposed models $M1$ and $M3$ demonstrate satisfactory performance.

Table 5.4: Comparison of gas flaring reductions in the absence of negative data

Producers	Algeria	Indonesia	Iraq	Nigeria	UAE	Venezuela
Maximum Reduction Δy_1^{max} $(m\ cu^3)$	3197.83	2235.00	9086.22	13777.69	587.63	9066.75
Maximum Reduction $\gamma_6^{max}(m\ cu^3)$	3197.86	2235.00	9086.44	13777.58	587.60	9067.00

The most noteworthy observation we can make from Tables 5.3 and 5.4 is that for all producers the reductions under $M6$ (i.e., γ_6^{max}) are significantly higher than the reductions under $M5$ (i.e., γ_5^{max}). As a result, the energy transition curve for each producer exhibits a rightward perturbation.

5.3.3 Optimal power sizing computations

In section 5.2.3, we introduced the expression for the optimal power sizing as:

$$P_3^{max} \leq P^{max} \leq P_1^{max}$$

In terms of gas flaring reductions, this expression becomes:

$$W\left(\frac{\Delta y_3^{max}}{\sigma}\right) \leq P^{max} \leq W\left(\frac{\Delta y_1^{max}}{\sigma}\right)$$

As shown in Table 5.2, we have already determined the maximum reductions from each model. Like the analysis in Chapter 3, if all producers use the same type of turbine to convert flare gas into electricity, then the annual gas consumption (i.e., σ) and power output (i.e., W) are constant for each turbine. For computation, we refer to the specifications of the GT13E2 gas turbine highlighted in Chapter 3. For this turbine, $W = 150\text{MW}$ and $\sigma = 339.45$ million cubic metres per year. Based on these values, Table 5.5 presents an optimal power sizing for all six producers. It is also important to note that the number of turbines (i.e., $\tau = {}^{\Delta y^{max}}/_{\sigma}$), can only take integer values.

Limits and bounds in Table 5.5 refer to the two different estimates of maximum power computed by our proposed models. The lower bound (i.e., P_3^{max}) is derived from modelling both positive and negative data, whereas the upper bound (i.e., P_1^{max}) is derived from modelling only positive data. This is done to analyze the impact of negative data in real-world applications of DEA models. It should be noted that the source of negative data in this study is the current account balance of OPEC member nations. The ASB publication released by OPEC records a surplus current account balance as positive data and a deficit account balance as negative data. As can be seen from Table 5.5, except for UAE, the maximum power generated from flare gas varies by input data type across models $M1$ and $M3$. Model $M1$ only considers positive data to establish the upper bound for power generation while $M3$ considers positive and negative data to establish the lower bound. Both upper and lower bounds are equal for UAE because flare reductions calculated by both models are almost equal for UAE, as shown in Table 5.2. Accordingly, negative data does not have a significant impact on power generation for UAE, thus satisfying the second corollary of Theorem 1.

Table 5.5: Optimal power sizing for the producers

DMU_k	Maximum Power (P^{max}) (MW)
Algeria	$1200 \leq P^{max} \leq 1350$
Indonesia	$750 \leq P^{max} \leq 900$
Iraq	$3750 \leq P^{max} \leq 3900$
Nigeria	$5700 \leq P^{max} \leq 6000$
UAE	$150 \leq P^{max} \leq 150$
Venezuela	$3750 \leq P^{max} \leq 3900$

Although sections 5.3.1 to 5.3.3 have utilized the production data contained in Chapter 3, to test the validity of our proposed models, the design of an energy mix for our case study requires more recent data. In particular, we will need to use the 2015 oil production data which corresponds to the 2015 energy statistics provided by Pietrosemoli and Rodríguez-Monroy (2019). Therefore, in the subsequent sections, we will use the 2015 production data to develop an energy mix.

5.3.4 Discussion of further findings

We have already justified using the directional distance DEA model for power generation in previous sections. In this section and for further analysis/validation, we apply both directional distance DEA models to recent production data from the OPEC annual statistical bulletin (ASB). The production data for this section is based on the 2016 ASB publication available as an open source. In the 2016 ASB, production data is provided for all OPEC nations for the period 2011 to 2015. The primary reason for choosing the 2016 ASB for this study is the lack of recent flaring data from OPEC. As previously explained in Chapter 3, the OPEC ASB stopped publishing flaring data for all member nations in 2015, and other sources of flaring data are inconsistent with OPEC's. As such, the 2015 production year includes the most recent flare data among OPEC members. Additionally, it contains production data which can be used together with 2015 energy statistics for our case study (i.e., Venezuela). The 2015 production year data is summarized in Table 5.6. Using the summarized data and models $M1$ and $M3$, we calculated the maximum flare reductions, required turbine units, and power estimates for 13 OPEC nations in 2015. Table 5.7 presents the results.

Table 5.6: Data summary for oil and gas

Variable	Variable name	Mean	Min	Max
Input	Current account balance (US$M)	-7659.62	134	41307
Input	Well completed	301	35	635
Input	Producing wells	3593	540	14685
Input	Active rigs	68	6	182
Input	Refining capacity (1000b/cd)	975.25	65	2907
Desirable output	GDP per capita (US$/person)	15470.16	2645.78	68766.63
Undesirable output	Flare gas (million cu m)	4641.32	0	16539.9

Referring to Table 5.7, let F be the set of producers under model $M1$ that can recycle flare gas for power generation (i.e., producers with $\theta_1^* > 0$). Furthermore, let G be the set of producers under $M3$ that are capable of generating power from flare gas. Therefore, producers in this category have values of $\theta_3^* > 0$. Accordingly, in the context of directional distance DEA, producers with either $\theta_1^* = 0$ or $\theta_3^* = 0$ are already efficient and are unable to reduce their flare gas volumes. According to Table 5.7, we present both sets as follows:

$$F = \{\text{Algeria, Indonesia, Iraq, Nigeria, UAE, Venezuela}\}$$

$$G = \{\text{ Indonesia, Iraq, Nigeria, UAE, Venezuela}\}$$

There is a significant difference between the two sets in terms of Algeria's inclusion or exclusion. The reason for this can be simply traced to the type of data used across both models. Set F, which is based on model $M1$, does not include any negative data. Due to $M1$ producing a non-zero optimal solution (i.e., $\theta_1^* > 0$) for Algeria, within this set F, Algeria can actually reduce its current flare gas levels. In set G, however, which is derived from model $M3$, we observe $\theta_3^* > 0$ for Algeria, which is the result of the inclusion of negative data. This results in Algeria's exclusion from set G.This corresponds to the third corollary of our first theorem, where negative data can flip the efficiency scale for a DMU.

Hence, Algeria is an outlier, since our analysis is based only on the five producers common to both sets F and G.For each producer with $\theta^* > 0$, the respective reductions are computed under both models and converted to optimal power outputs as shown in Table 5.7 and Fig. 5.3. With the exception of Algeria and the UAE, a closer look at Table 5.7 reveals significant increases in the power estimates across models $M3$ and $M1$ for Indonesia, Iraq, Nigeria, and Venezuela. Clearly, $P_1^{max} > P_3^{max}$ for all four producers. Additionally, this applies to their maximum reductions, which will inevitably perturb the energy transition curves of all four producers. Similarly, the required turbine units under both models have also been computed, with $\tau_1^{max} > \tau_3^{max}$ in most cases. However, for UAE, $\tau_1^{max} = \tau_3^{max}$.

In Fig. 5.3, it can be seen that Venezuela stands to gain the most from flare gas power generation, followed closely by Iraq and Nigeria. The UAE generated the least amount of power from flare gas, suggesting a near-efficient state for the producer in comparison to others. Furthermore, unlike Indonesia, Iraq, and Nigeria, our estimation of Venezuela's maximum power is not affected significantly by negative data. Hence, Fig. 5.3 strongly advocates the conversion of flare gas into electricity in Venezuela as a reliable and affordable energy source. Having said that, we must reiterate here that the maximum reduction and power estimated by $M3$ is more realistic due to the existence of negative data in real-world scenarios. Nevertheless, we will design an energy mix for Venezuela based on both power estimates.

Table 5.7: Flare gas power generation for OPEC members

DMU_k	Optimal values θ^*		Maximum gas flaring reductions Δy^{max} $(m\ cu^3)$		Required Turbine units, τ^{max}		Maximum Power P^{max} (MW)		Change in Max Power ΔP^{max} (%)
	θ_1^*	θ_3^*	Δy_1^{max}	Δy_3^{max}	τ_1^{max}	τ_3^{max}	P_1^{max}	P_3^{max}	$\Delta P^{max} = P_1^{max} - P_3^{max}$
Algeria	0.8630	0.0000	3032.75	-	8	-	1200	-	NA
Angola	0.0000	0.0000	-	-	-	-	-	-	-
Ecuador	0.0000	0.0000	-	-	-	-	-	-	-
Indonesia	0.9075	0.6004	1690.67	1118.55	4	3	600	450	33.33
Iran	0.0000	0.0000	-	-	-	-	-	-	-
Iraq	0.9688	0.7917	14150.87	11564.05	41	34	6150	5100	20.58
Kuwait	0.0000	0.0000	-	-	-	-	-	-	-
Libya	0.0000	0.0000	-	-	-	-	-	-	-
Nigeria	0.9350	0.6555	9057.63	6350.03	26	18	3900	2700	44.44
Qatar	0.0000	0.0000	-	-	-	-	-	-	-
Saudi Arabia	0.0000	0.0000	-	-	-	-	-	-	-
UAE	0.4728	0.4728	492.61	492.61	1	1	150	150	0
Venezuela	0.9959	0.9757	16472.09	16137.98	48	47	7200	7050	2.13

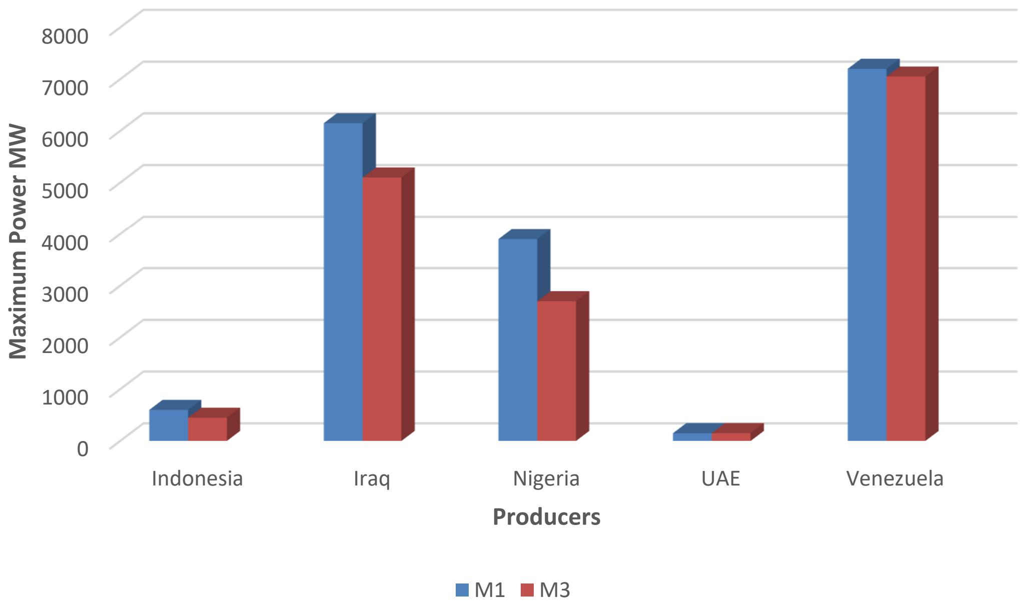

Figure 5.3: Maximum power generation by producers

5.3.5 Proposed energy mix for Venezuela

To determine the optimal energy mix for our case study, we need to restate the energy statistics provided by a previous study and apply our derived formulas.

The total installed capacity of the Venezuelan power plants in 2015 was 34,100MW, leaving a loss in capacity of 17,600MW. In addition to the loss of capacity, the electricity demand shortage in Venezuela for 2016 was 1800MW (Pietrosemoli & Rodríguez-Monroy, 2019).

Recall the following expressions for our proposed energy mix in section 3.2:

For $M1$: $\eta_g = \left(P_1^{max} / \omega \right)$, and $\eta_r = 1 - \left(P_1^{max} / \omega \right)$, provided that $P_1^{max} < \omega$

For $M3$: $\eta_g = \left(P_3^{max} / \omega \right)$, and $\eta_r = 1 - \left(P_3^{max} / \omega \right)$, provided that $P_3^{max} < \omega$

In light of the findings in Table 5.7 and the energy statistics for Venezuela, we have that:

$P_1^{max} = 7200\text{MW}$, $P_3^{max} = 7050\text{MW}$, and $\omega = 17600\text{MW}$.

The necessary conditions for the design of an energy mix for Venezuela have now been met since $P_1^{max} < \omega$ and $P_3^{max} < \omega$.

The resulting energy mix across both models will be as follows:

For $M1$: $\eta_g = 41\%$, and $\eta_r = 59\%$

For $M3$: $\eta_g = 40\%$, and $\eta_r = 60\%$

Although the energy mix from both models is close in proportions, we recommend the $M3$ energy mix for Venezuela due to the presence of negative data in real-world applications of DEA. Thus, flare gas power generation will cover 40% of Venezuela's loss in power capacity, while the country will meet more than three times its 2016 energy deficit. In spite of the fact that it is not a complete solution to Venezuela's energy crisis, our analysis supports the use of natural gas as a bridge fuel. It is possible for the gradual transition to renewable energy sources to compensate for the 60 percent additional loss in power capacity for the nation. Hence, our analysis recommends an energy mix of 40% gas power generation and 60% renewable energy sources for Venezuela in the short term.

5.3.6 Sensitivity analysis

This section conducts a sensitivity analysis using model $M5$ to visualize Venezuela's energy transition curve. As indicated in section 5.3.5, this transition curve represents 40% of the recommended energy mix for Venezuela's gas-powered generation.

Considering $M3$ to be the base model for $M5$, we will investigate the inverse relationship between θ_3^* and γ_5^{max} in our sensitivity analysis. Specifically, we follow the approach discussed in Chapter 3 by reducing θ_3^* to zero to obtain γ_5^{max}. As shown in Table 5.7, for Venezuela, $\theta_3^* = 0.9757$. The resulting curve in Fig. 5.4 shows that when $\theta_3^* = 0$, $\gamma_5^{max} = 16265.30$ million cubic metres. The estimate is close to that presented in Table 5.7 for Venezuela (i.e., $\Delta y_3^{max} = 16137.98$ million cubic metres). The difference between the two estimates (i.e., 127.32 million cubic metres) is less than the annual gas consumption of the GT13E2 turbine (i.e., 339.45 million cubic metres), so both estimates will generate equal amounts of power.

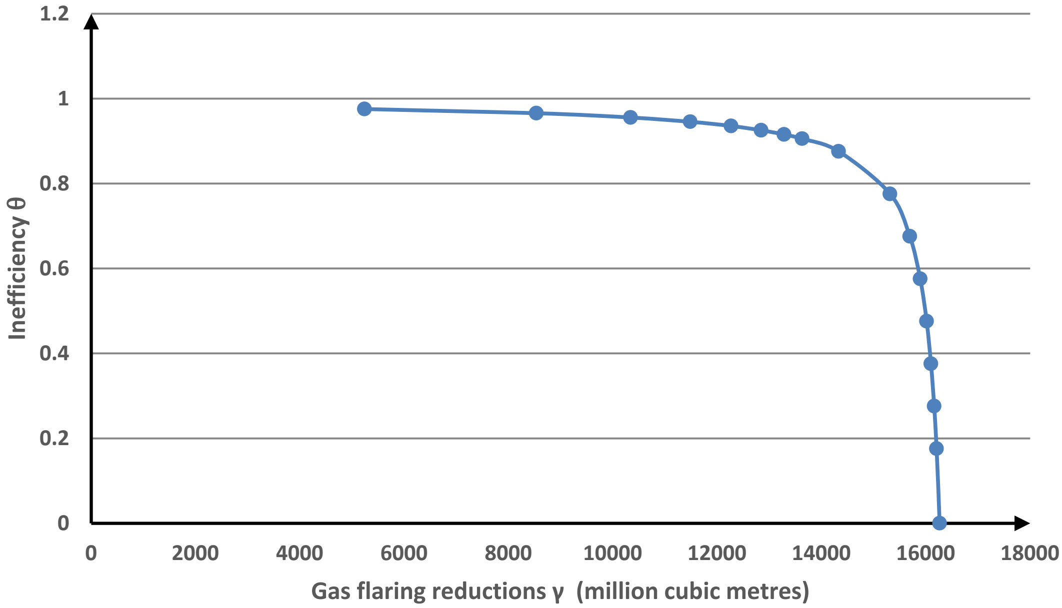

Figure 5.4: Proposed energy transition curve of Venezuela

5.4 Managerial implications

Amidst the increasing push for fossil fuels to be replaced with renewable energy, it is incumbent upon OPEC member nations to consider adopting and implementing energy mixes as a means to improve global energy efficiency. Furthermore, in light of the fact that flare gas contributes significantly to climate change, an adopted energy mix must always incorporate the generation of power from flare gas along with the use of renewable resources. There is no doubt that the DEA methodology, along with its many advanced modifications, is a powerful management tool, not only in business and economics, but also in engineering. In this chapter, from a technical and engineering standpoint, we present an innovative application of DEA to the design of an optimal energy mix for the petroleum industry. However, some implications for oil and gas managers should be highlighted when applying our methodology.

As a starting point, conventional DEA models assume that all inputs and outputs are positive. As can be seen in the current study, this is not always the case when we have an input that contains both positive and negative data for OPEC member nations. We utilized two versions of our proposed model in order to address this issue. In the first version, only positive data was handled, while in the second, both positive and negative data were handled. Based on preliminary results, both models showed significant differences in flare gas reductions. When using either of the proposed models, the reductions computed from both models will help a decision-maker avoid overestimation or underestimation due to data type. Additionally, we found that we can set limits/bounds on the maximum power generated from flare gas based on data type. An upper bound is created by modelling only positive data, while a lower bound is created by modelling both positive and negative data. Generally, the lower bound represents a realistic or practical estimate, while the upper bound represents an optimistic estimate. Using the lower bound estimate of maximum power, a decision-maker can save on costs pertaining to the installation and maintenance of additional turbine(s) to convert flare gas to electricity.

As a final point, the formulation of an energy mix is highly dependent upon the estimate of maximum power obtained by our proposed models. Our study demonstrated that using the lower estimate of maximum power increases the proportion of renewable energy in the energy mix. In accordance with the SE4ALL Initiative, this is of vital importance for the gradual transition to 100% renewable energy in the near future. To this end, it is imperative that decision-makers relax the assumption that all data are positive when using DEA methodology and include relevant negative data whenever available.

5.5 Summary

Climate change is aggravating, and gas flaring is a major contributor. Furthermore, the gas flaring process has paradoxical consequences for the energy economies of some oil-producing nations. There is no doubt that the current shift toward renewable energy sources to fight climate change is a positive development. Yet it is important to note that some developing nations will need more time to jump aboard the energy transition bandwagon than developed nations. Specifically, oil-producing nations experiencing power supply shortages will require a gradual transition based on reliable energy sources in the short term. To help such nations transition gradually to 100 percent renewable energy, this chapter advocates the use of flare gas as a bridge fuel. Therefore, we propose, for the first time in the literature, directional distance DEA models for maximizing flare gas power generation in oil-producing nations.

The application of our proposed models brings useful insights to energy analysts. Our initial findings confirm the impact of negative data on the maximum power estimated by our models. In the absence of negative data, we found that a significant increase in maximum power estimates resulted in a new concept of optimal power sizing. The increases in maximum power outputs for Indonesia, Iraq, Nigeria, and Venezuela were estimated to be 25%, 20.59%, 44.44%, 2.13%, respectively. For all four producers, this will lead to rightward perturbations in their energy transition curves. Another interesting finding of this chapter involves our proposed theorem of equivalence. The theorem found that for every inefficient oil producer, its maximum gas flaring reductions calculated by the directional distance DEA and inverse DEA are approximately equal. This also applies to the equivalent maximum power estimated by both models.

As a final step, we chose Venezuela as our case study. The maximum gas flaring reduction for the nation was estimated to be 97.57% based on our model. With the aid of gas turbines, this reduction was converted into a power output of 7050MW, covering a 40% loss in the installed power plants' capacity of the nation and more than three times the nation's energy shortage in 2016. Based on our findings, an optimal energy mix for Venezuela should be 40% gas power generation and 60% renewable energy. With this, the country would gradually transition to 100% renewable energy. In general, our obtained result for Venezuela confirms that natural gas plays an integral role in the energy economy of the nation.

Chapter 6 – Conclusions and Future Directions

6.1 Introduction

Throughout this chapter, the background to the current research, its objectives, and work done in in previous chapters are briefly summarized. In addition, the findings of the research and its contribution are highlighted. There is also an analysis of the limitations and drawbacks. Finally, it is proposed that future research might address the limitations in order to extend the broader field of gas engineering.

6.2 Conclusions

Gas flaring contributes significantly to climate change due to the release of greenhouse gases. Additionally, it denies oil-producing nations access to affordable energy sources. As concern over climate change grows, experts have repeatedly recommended strategies and/or technologies to address gas flaring in the petroleum industry. Gas compression, gas-to-liquids (GTL), and gas-to-wire (GTW) are the three main technologies recommended. The use of gas compression technology allows gas that would otherwise be flared to be compressed and transported to other locations for use. By using GTL technology, which includes Fischer Tropsch reactors, natural gas is converted into liquid fuels such as methanol, diesel fuel, and zero sulfur diesel. GTW refers to the conversion of natural gas into electricity using a simple or combined cycle gas turbine. This research focuses on the use of GTW technology as a solution to both the problem of gas flaring as well as the ongoing energy crisis in some oil-producing countries.

Previous research indicates that there is no quantitative approach to address the gas flaring issue, since most common methodologies are qualitative or descriptive. Toward this end, the current research develops five models for the first time in the literature to accurately depict real-world scenarios related to gas flaring. Chapter 3 of the current study provides a detailed discussion of the first and second models. The first model is a directional distance DEA integrated with negative data for classifying oil-producing nations into efficient and inefficient producers. For estimating potential reductions in gas flaring, an inverse DEA was developed, which is a robust extension of the first model.

Based on a proposed theorem and sensitivity analysis, it was fully demonstrated that the maximum potential reduction in gas flaring will always occur at zero inefficiency. This led to the development of an energy transition curve for an inefficient producer. Thus, both models form a closed-loop optimization technique, since the first model estimates the inefficiency score of a producer, while the second model reduces it to zero via the energy transition curve. As GTW technology is quite expensive, particularly for developing nations, the inverse DEA played a crucial role in designing a cost-effective GTW process. To achieve this, a formula was developed to determine the optimal range of turbine units for the GTW process. Using this formula, a decision-maker can estimate the maximum number of turbine units required for a GTW process, thereby eliminating excess costs. Any investment that exceeds the maximum number of turbine units will likely result in a financial loss. The inverse DEA developed can therefore be seen as a managerial tool for securing investments in GTW technology and ensuring cleaner production.

In Chapter 4, two inverse DEA models were proposed for implementing lean production practices in efficient oil-producing nations. There were several lean practices implemented, including increasing productivity, minimizing/reducing waste (i.e., flare gas), and recycling waste to generate power. In general, the gross power estimates obtained are of great value to the Nigerian petroleum industry, given the current power outages in the country. Using estimated production targets, a sensitivity analysis was then performed to determine which producers will benefit from both short- and long-term increases in oil production. Short-term benefits accrue to Angola, while long-term benefits accrue to Nigeria. In addition, Iraq and Qatar will benefit both in the short and long term. This chapter also developed an energy-based technique for ranking the efficient oil producers according to their net energy production. Based on three consecutive production years, Saudi Arabia was found to be the most efficient producer, while Iraq was the least efficient.

As the final phase of this research, Chapter 5 examines the maximum amount of power that can be generated from flare gas, thus filling a critical research gap, as estimates of power in chapter four were neither maximum nor minimum. To accomplish this, a directional DEA approach was employed, resulting in the expression of the maximum power that can be produced from waste or flare gas. Estimates of maximum power are required to determine the optimal mix of gas-powered generation and renewable energy sources. There were two different scenarios presented, the first using only positive data, the second including both positive and negative data. According to several computations across multiple experiments, the first scenario generated higher estimates of maximum power than the second scenario. Based on an assessment of the ongoing energy crisis in Venezuela as a case study for this chapter, the maximum power that can be generated from flare gas has been estimated at 7050MW for the country in 2015. This estimate covers approximately 40% of the nation's unavailable power generation capacity. Therefore, in the short term, Venezuela's energy mix was optimized with 40% gas power generation and 60% renewable energy.

6.3 Limitations of the Research

As a result of this research, the following limitations have been identified:

- The focus of this research project is primarily on gas-to-wire (GTW) technology, which offers both the reduction of gas flaring and the generation of electricity. However, there is still a need to explore the other two technologies as well, namely, Gas-to-liquids (GTL) and gas compression, especially in oil producing nations that are not experiencing any energy shortages. It would provide an alternative means or solutions for addressing global gas flaring from an economic perspective, given that experts believe GTL and gas compression techniques are quite expensive.
- In Chapter 3, the models developed are primarily targeted at inefficient producers, causing maximum reduction of their undesirable output (e.g., flare gas). Furthermore, the same pair of models assume that it is not possible to reduce gas flaring for efficient producers. This leaves a gap that has been filled by the proposed models in Chapter 4. Despite this, it is essential to point out that the models in Chapter 4 only imposed reductions in gas flaring on efficient producers due to an increase in oil production. In contrast to chapter three, the computed reductions are not maximum. The global problem can still be further addressed by imposing maximum reductions in gas flaring for efficient producers.

- Chapter 5 has similar limitations to Chapter 3. The maximum power estimates in Chapter 5 were calculated for inefficient producers, but no estimates were possible for efficient producers. It is also due to the underlying assumption of the directional distance DEA which only enables computation of maximum power estimates for inefficient producers. If energy shortages or the need to convert flare gas into other forms of energy arise in efficient oil-producing nations, it is necessary to determine their maximum power generation.
- The ranking method using the inverse DEA remains subject to a final limitation. An inverse DEA was extended in Chapter 4 by incorporating the lean potential growth for ranking oil producers based on their good outputs alone. The current research, however, focuses on both good and bad outputs. As a result, a new energy-based ranking methodology was developed for ranking producers considering both types of outputs. Considering that this energy-based technique only applies to oil and gas production, it would be interesting to develop a ranking technique within the context of inverse DEA to rank oil producers according to good and bad outputs.

6.4 Future Directions

With respect to the identified limitations of this research project, the following suggestions may provide a foundation for future work:

- This research is the first quantitative study on global gas flaring utilizing GTW technology. Therefore, it would be beneficial to develop quantitative approaches or optimization models for Gas-to-Liquids (GTL) and gas compression technologies while considering their economic feasibility. It is also recommended to determine an optimal energy mix consisting of GTL and renewable energy or gas compression and renewable energy, given the push for a transition to renewable energy. For energy analysts, a comparative analysis of both pairs of the energy mix will surely provide valuable information.
- Research in inverse DEA is evolving and is currently attracting the attention of a number of researchers. As the inverse DEA is the primary optimization model in this study, it is essential to develop more robust versions that can impose maximum reductions in the

undesirable outputs of efficient decision-making units (DMUs). This will be highly relevant to other efficient work systems that require waste management techniques.

- A number of techniques are available in the DEA literature for ranking efficient units, however, only the study by Soleimani-Chamkhorami et al. (2020) has developed a ranking method based on inverse DEA. In view of the fact that this technique can only rank units with good outputs when the constant return to scale assumption is applied, it is recommended to develop an inverse DEA that can rank units with both good and bad outputs. Furthermore, it would be interesting to extend such a model to include negative data and to assume variable returns to scale (VRS). As VRS models are well suited to handling negative data, this would be a promising future direction.
- The energy-based ranking technique developed in Chapter 4 considered only crude oil and flare gas as energy sources. The scope of this technique can be extended by considering other sources of energy in the industry, such as LNG and the marketed production of gas, as additional sources of utilized energy. It is also recommended that this technique be applied to other types of decision-making units that use or transfer energy resources.

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