TWO SELECTED TOPICS IN DURABLE GOODS MARKETING STRATEGY FROM THE PERSPECTIVES OF DEPRECIATION AND MAINTENANCE

YANG HONGRU

MPhil

The Hong Kong Polytechnic University

2023

The Hong Kong Polytechnic University

Department of Logistics and Maritime Studies

Two Selected Topics in Durable Goods Marketing Strategy from the Perspectives of Depreciation and Maintenance

Yang Hongru

A thesis submitted in partial fulfilment of the requirements for the degree of Master of Philosophy

May 2023

CERTIFICATE OF ORIGINALITY

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Abstract:

Durable goods are defined as products which can be stored or inventoried for a long time. The degree of product durability is usually measured in terms of its lifespan or expressed in quality, which is a variable as important as price to consumers. Moreover, after-sales maintenance of durable goods has also become a matter of great concern. Therefore, in this thesis we study two selected topics in the marketing strategy of durable goods from the perspectives of consumption depreciation and maintenance.

In the first topic, we study durable goods with consumption depreciation, which are the categories of durable goods, like books, toys, and game cassettes, whose physical properties rarely depreciate but consumption values decline quickly, are especially active in the second-hand and rental platforms. We construct and compare different two-period strategic models of durable goods with a monopolistic manufacturer and obtain some interesting results. First, we focus on the pure leasing and selling strategies, and find that for durable goods the manufacturer's profit is higher for leasing than that for selling. Besides, we show that the manufacturer of durable goods with consumption depreciation will suffer more from second-hand competition than that without consumption depreciation. Then, we consider two hybrid strategies, i.e., selling-leasing strategy and selling-reselling strategy, and find that both hybrid strategies do not necessarily help the manufacturer gain more profit since they cannot subdivide the market. Furthermore, we make some extensions and verify the robustness of our models.

In the second topic, we investigate the maintenance service of durable products. Maintenance has been receiving increasing attention in the manufacturing world, and the development of related technologies has brought about an explosive increase in the performance of equipment maintenance. However, the failure of service providers' operations strategies to keep pace with the technology results in waste of service providers' investment. We develop an analytical and mathematical tool for optimizing operational strategies of competing service providers with imperfect online monitoring. Then, we make sensitivity analyses and explore the meaning behind the influences of parameter changes on the strategies, and the results are robust when extending it to the model of n competing service providers. Besides, we find that the weaker party in the competition could benefit from the deterioration of the market when the stronger party suffers a loss. Finally, we summarize a general pattern of the equilibrium consumer arrival rate in response to the optimal service rate.

Key words: Durable goods; Consumption depreciation; Consumer-intensive service; Preventive maintenance; Competitive strategy.

Publications arising from the thesis

Yang H.(2023). Marketing Strategy for Durable Goods under Consumption Depreciation. Working paper. (co-authored with Cheng, T.C.E., Guo, X., Hua, G. and Ng. C.T.).

Yang H. (2023). Competitive Strategies for Customer-Intensive Service Providers under Imperfect Online Monitoring. Working paper. (co-authored with Cheng, T.C.E. and Ng, C.T.).

Acknowledgements

I am very grateful to my Chief Supervisor, Prof. C.T. Daniel Ng, for his valuable guidance. In the past 2 years, he provided me with lots of help and I have learned a lot from him. For example, in Section 3 of the thesis, I encountered difficulties in solving the model, and Prof. Ng patiently guided me and finally I found a way to solve it. Prof. Ng is a mathematics graduate, and he let me understand and appreciate the importance of the rigor of mathematics and guided me to conduct my research more effectively. In addition, he also took care of me in my life and made me feel at home in the university. I am really grateful to him and proud that I can be his student.

In addition, I would like to thank my Co-Supervisors Prof. T.C. Edwin Cheng and Dr Xiaomeng Guo, who also provided me with lots of advice and comments on my thesis.

Moreover, when I first arrived in Hong Kong, Ms Yuqing Pan provided me with a lot of help in my study and life so that I could adapt to the learning and living environments in Hong Kong faster. In addition, my fellow apprentices Ms Shuen Guo, Ms Yu Du, and Ms Jia Yu were also indispensable partners in my research journey.

My classmates Mr Bin Tian, Mr Shiyi Jiang, Mr Tao Zhang, Mr Zhong Chu, Ms Siyuan Chen, Mr Shucheng Miao, and Ms Xiaoxin Huang also provided a lot of help in my life and study, and it was a great pleasure to be a classmate of them.

Finally, I would like to thank my parents for their kindness in bringing me up and accompanying me along this way, which is the biggest support and motivation for me to embark on this road of research.

I sincerely thank all the people who have helped me.

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Section 1
Introduction

Durable goods are indispensable products in our daily life, and it also attracts many scholars' researches because of its own product characteristics. Mantena et al. (2012) concluded eight primary research issues of durable goods: time inconsistency; pricing; choice of production technology; durability, planned obsolescence and upgrading; leasing versus selling; complementary goods markets; secondary markets channels; and channel design. Among them, durability is one of the most important characteristics that distinguish durable goods from consumable goods and receive scholar’s widespread attention. Thus in this thesis, we study two selected topics in the marketing strategy of durable goods from the perspective of durability, depreciation and maintenance.

The depreciation is known as a reduction in the value of an asset over time, due in particular to wear and tear. And with the emergence of the “collaborative consumption” and the popularization of the Internet, second-hand and rental platforms of durable goods have gradually sprung up, which has led to consumers’ more frequent product transfers. Among them, the categories, like books, toys, and game cassettes, whose physical properties rarely depreciate but the consumption values decline quickly are particularly prominent.

Sheth et al. (1999) stated that when consumers use durable goods with consumption depreciation, different from their physical attributes, the consumption value depreciates rapidly. Coincidentally, Shiller (2013) got a similar conclusion that consumers tired quickly with these products, e.g., games. Unlike other goods which physically depreciate more considerably, Ishihara (2019) held that durable goods with consumption depreciation may face competition from used goods markets almost immediately after the release of a new product.

Based on these, we subdivide the depreciation of durable goods into physical depreciation on the product side and consumption depreciation on the consumer side, similar to Dou et al. (2017). Physical depreciation is defined as the physical attributes of goods or services decreasing gradually after use. For instance, the wear and tear over time, which directly affects the value of a product, is identical to all consumers. In contrast, consumption depreciation only occurs to consumers who have consumed or experienced the goods or services, which refers to that the consumers’ enthusiasm of products will be greatly reduced after achieving a specific goal, e.g., finishing a book and completing a puzzle.

On the other hand, the Industry 4.0 raised by Germany and the intelligent logistics system proposed by China both emphasize the interconnection of people, machines and “everything”, value of data, artificial intelligence, and data-driven operations. Highly automated “intelligent factory” not only brings increased productivity, but also raises the importance and difficulty of equipment maintenance.

In contemporary manufacturing, maintenance has been receiving increasing attention as firms understand that maintenance, when well performed, can be a strategic enabler to achieve corporate goals. Thus, operators of capital-intensive equipment often allocate a large budget to maintenance so as to ensure high equipment reliability. For instance, the operations and maintenance costs of an offshore windmill could contribute a quarter of the life-cycle costs, making it one of the largest cost components (Snyder and Kaiser, 2009), according to a new study by Wood Mackenzie's Power & Renewables Division, global onshore windmill power operations and maintenance costs will reach nearly $15 billion in 2019. Of this, 57% ($8.5 billion) is due to unscheduled maintenance costs due to component failures. Maintenance cost also constitutes around 30-50% of the overall haulage costs of a surface mining operation for overburden and ore removal (Topal and Ramazan, 2010). Furthermore, in the medical imaging equipment industry, a top-of-the-line computer tomography or magnetic resonance imaging device, which typically lasts about ten years, requires annual maintenance expenses amounting to 10% of its purchase price, i.e., the life-time maintenance costs of devices can easily approach the original purchase price (Chan et al., 2019).

The development of emerging technologies, such as the Internet of Things (IoT), big data technology, and data analytics, can facilitate the Condition-based Predictive Maintenance (CPM). In CPM, the condition monitoring is usually realized with equipment-installed sensors, which have the capability of measuring with high frequency a multitude of parameters leading to processing and storage of a huge amount of data. Useful information can be extracted from the available data by data analysis approach, which can be used to predict failures in advance. McKinsey reports that IoT-based predictive maintenance can achieve 10-40% cost reduction, 3-5% longer equipment life, and 50% less downtime (McKinsey, 2015).

Although the development of related technologies has brought about an explosive enhancement in the performance of equipment maintenance, online monitoring is still not perfectly predictable. There are various reasons for this situation, for example, data in the IoT are vulnerable to many risks affecting their quality (Karkouch et al., 2016). In additional, service providers’ investment in online monitoring is limited since they must consider the cost, and the associated data analysis methods can be mismatched. The following is a real-life example. Online condition monitoring is probably the most promising maintenance technique applied to wind turbine components. However, it also faces the probability of making incorrect decisions during continuous condition monitoring (Raza and Ulansky, 2019). The failure of service providers' operations strategies to keep pace with the technologies mentioned above will result in service providers not only failing to take full advantage of relevant emerging technologies, but also wasting of their investment.

The significance of our study of these two topics is that the former can help the manufacturer of a product with consumption value depreciation make better decisions based on the characteristics of the product, and the latter can help a firm make a better maintenance strategy to adapt to modern emerging technology with imperfect online monitoring.

The rest of the thesis is organized as follows: Section 2 researches on the depreciation especially the consumption depreciation of durable goods. In Section 3, we research on maintenance strategies for customer-intensive service providers under imperfect online monitoring. And Section 4 is summary and future research.

Section 2
Marketing Strategy for Durable Goods with Consumption Depreciation

2.1 Introduction

In this thesis, we intend to figure out how the consumption depreciation affects the manufacturer's strategic decisions. We will construct different two-period game-theoretic models with a monopolistic manufacturer. In each period, the manufacturer decides the product quantity and price. In the first period, the consumers decide whether to consume the product. In the second period, the product owners judge whether to resell their products, and the others decide whether and how to adopt these products. Through comparative analysis of the manufacturer's selling, leasing, selling-leasing, and selling-reselling strategies, we obtain some managerial findings. First, consistent with Coase (1972) and Bulow and Jeremy (1982), we show that, for perfectly durable goods, the leasing strategy is always more profitable than the selling strategy.

Second, similar to Dou (2017), we demonstrate that the selling-leasing hybrid strategy will never be a better choice than the pure strategies for the manufacturer. The determining factor in manufacturer's strategic choice is physical depreciation. For products with relatively low physical depreciation, the manufacturer can obtain a higher profit by leasing. With the physical depreciation of product gradually gets higher, the selling strategy will become optimal. This is in line with our common sense. The most essential difference between leasing and selling lies in the division of the use rights and the product ownership. And such division is very difficult and rare for consumables. Therefore, leasing is usually adopted by the durable goods manufacturers in reality.

Third, although the leasing strategy performs better than the selling strategy for durable goods manufacturer, there are few companies implementing pure leasing worldwide. Therefore, an alternative hybrid strategy — selling-reselling is proposed. Interestingly, we find selling-reselling is also not a necessarily better choice even for durable goods manufacturer. The selling-reselling strategy outperforms the pure selling and is worse than the leasing strategy for durable products with relatively high consumption depreciation since the second-hand market scale is quite large. And when the consumption depreciation falls below a threshold, the selling-reselling strategy no longer performs better than the pure selling strategy. Moreover, the optimal second-market share of the strategy decreases in the physical depreciation. We also make some extensions to verify the robustness of our conclusions.

Overall, our thesis contributes to the durable goods literature with both physical depreciation and consumption depreciation. Due to their particularity and importance, durable goods were a considerable hotspot to scholars in academia in the past few decades. Mantena et al. (2012) concluded eight primary research issues of durable goods: time inconsistency; pricing; choice of production technology; durability, planned obsolescence and upgrading; leasing versus selling; complementary goods markets; secondary markets channels; and channel design.

Although there are extensive studies on durable goods in the literature, only few of them are related to consumption depreciation. Dou et al. (2017) divided the depreciation of durable goods into vintage depreciation and individual depreciation, and studied the impact of these two types of depreciation on suppliers. Ishihara and Ching (2019) researched on the depreciation of consumption values for the durable goods. Based on them, we define and explain the phenomenon of consumption depreciation and construct a two-period game-theoretic model incorporating physical and consumption depreciation to comparatively study the marketing strategies of the manufacturer.

We also complement the research on leasing verse selling. Since Coase (1972) proposed that the leasing strategy is preferable to the selling strategy for any durable goods manufacturer affected by time inconsistency, the optimality of the leasing strategy is a common result in the literature (Stokey 1979; Bulow and Jeremy 1982; Hendel and Lizzeri 1999a; Choudhary 2007).

However, there are also research works finding out situations where the leasing strategy may not be optimal. Examples are: (i) when new consumers emerge in each period (Conlisk et al. 1984); (ii) when selling incurs a higher depreciation rate than leasing (Desai and Purohit 1999); (iii) when complementary products are produced by independent manufacturers (Bhaskaran and Gilbert 2005); (iv) when the network effects exist (Chien and Chu 2008); and (v) when the magnitude of consumption depreciation exceeds a certain threshold (Dou et al. 2017). Bhatt (1989) and Biehl (2001) both found selling might dominate leasing with demand uncertainty. Considering the consumption depreciation, our model supports Coase's conclusion that, for a perfectly durable goods manufacturer, leasing can gain higher profits than selling.

Our work also has strong links to the second-hand market, where manufacturers usually concern about the availability of the used products since they will compete with the new products. This competition is widely believed to reduce the sales and profits of the manufacturers. Previous literature found that the second-hand market persistently has a negative effect on the monopolist's profit when the buyer group fails to regenerate over time, which suggests that obsolescence has its benefits (Bulow and Jeremy 1982; Rust 1982; Waldman 1996). Likewise, Iizuka (2007) empirically found that publishers revise editions more frequently when second-hand sales grow. Eliminating the secondary market is a profitable move when old and new commodities become close substitutes (Miller 1974, Liebowitz 1982, Nocke and Peitz 2003). Chen et al. (2013) studied the impact of the used market on the durable goods manufacturer and found that it reduced the net earnings of the manufacturer by 35% by analyzing the US auto market data.

However, there are also articles with contradicting results to the above. Anderson and Ginsburgh (1994) showed that if the quality of second-hand products is controlled, then the cannibalization effect can be eliminated. Hendel and Lizzeri (1999a) pointed out that companies have many approaches to influence the second-hand market, and a reasonable choice could enable manufacturers to gain profits. Hendel and Lizzeri (1999b) found that a monopolist could obtain benefits by actively intervening in the secondary market. Through comparative analysis, Shulman and Coughlan (2007) showed that, compared with the second-hand market closed by the retailers, the manufacturers can obtain more benefits when the market exists. Johnson (2011) found that when consumers' preferences change over time, second-hand market frictions could bring more profits to the monopolists. Lacourbe (2016) showed that the second-hand market is not a threat but an opportunity for a monopolist. Ishihara and Ching (2019) showed that if video game publishers do not adjust their pricing strategies, phasing out the used video game market will reduce the total revenue of new games by an average of 4%.

The rest of the section is organized as follows: Section 2.2 constructs the benchmark model and then compares the leasing and selling strategies. In Section 2.3, we research on the hybrid strategies: selling-leasing and selling-reselling strategies. Section 2.4 discusses some extensions of the model. Finally, Section 2.5 concludes the main findings.

2.2 Selling versus Leasing

In this section, we first introduce the model assumptions and notations. Then we present the benchmark model and manufacturer's two pure strategies. There is a subsection on comparison and analysis of the two pure strategies at the end.

2.2.1 Model assumptions & notations

In our model, a monopolistic manufacturer offers a new product sold in two periods. For simplicity, we assume that the production cost is 0, the whole market size is 1, and the consumers' initial valuation V of the product is uniformly distributed in [0,1]. The manufacturer decides the product quantity sold in each period to maximize its profit. Consumers decide whether to consume the products in each period, and various consumption approaches can be chosen under different marketing strategies adopted by the manufacturer. The consumers face two types of depreciation: the physical depreciation $\gamma_1 \in [0,1]$, which is due to the product itself and the same for all consumers; and the consumption depreciation $\gamma_2 \in [0,1]$, which is due to the product consumption by the consumers. Both values of the depreciation are homogeneous for all consumers in our model. Suppose a product with valuation V is bought by a consumer in period 1. If the consumer does not use the product in period 1, the valuation will depreciate to $(1-\gamma_1)V$. However, if the consumer uses the product in period 1, then the valuation will depreciate to $(1-\gamma_1)(1-\gamma_2)V$. Table 2-1 explains the notations used in Section 2.

Table 2-1 Notations.

Notation	Description
i	The period number, $i = 1, 2$
q_{is}	The quantity of new products sold in period i
q_{il}	The quantity of new products leased in period i
q_{2u}	The quantity of used products sold in period 2
p_{in}	The selling price of a new product in period i
p_{2u}	The price of a used product in period 2
p_{il}	The leasing price of a product in period i
γ_1	Physical depreciation
γ_2	Consumption depreciation
ρ	The discount rate
π	Profit of the manufacturer
λ	The second-market share
$c(\lambda)$	The cost function of the second-market share λ

Let q_{1s} and q_{2s} be the sales quantities of the new products in periods 1 and 2, respectively. We solve the problem by backward induction. Notice that if the consumption depreciation $\gamma_2 =$

0, then everyone will remain interested in the product and keep it in period 2. However, with the increase of the consumption depreciation, the consumers who have used the products in period 1 will lose their interests and decrease the valuation of the product in period 2. Until when $\gamma_2 = 1$, all consumers will lose their interests and resell the product in period 2. Therefore, we follow the setting of Desai and Purohit (1999) and consider that the consumers will resell the product at the rate γ_2.

$$\begin{aligned} p_{2u} &= (1-\gamma_1)(1-q_{1s}-q_{2s}-\gamma_2 q_{1s}), \\ p_{2n} &= 1-(1-\gamma_1+\gamma_2)q_{1s}-q_{2s}. \end{aligned} \qquad (2-1)$$

Given q_{1s} and q_{2s}, according to the consumer utility function, if there is no depreciation, the price of the product in period 2 is $p_2 = 1 - q_{1s} - q_{2s}$. Intuitively, considering the resell quantity, we get $p_2' = p_2 - \gamma_2 q_{1s}$ and then with the physical depreciation, we get $p_{2u} = (1-\gamma_1)p_2'$. However, as to the price of new product p_{2n}, the additional term $\gamma_1 q_{1s}$ and $\gamma_2 q_{1s}$ represent the physical depreciation and consumption depreciation of the sold products. The explanations and corresponding examples are in the next paragraph. Note that the difference between the two prices is $\tilde{p} = p_{2u} - p_{2n} = \gamma_1(q_{1s} - 1) < 0$ when $\gamma_1 > 0$; and $\tilde{p} = 0$ when $\gamma_1 = 0$. This means that only when the physical depreciation equals to 0, the prices of new and used products are identical; otherwise, p_{2u} is strictly less than p_{2n}.

The selling price of new products in period 1 is equal to the market clearing price in period 1 plus the expected value of the product in period 2, i.e.,

$$p_{1s} = 1 - q_{1s} + \rho(1-\gamma_2)p_{2u}, \qquad (2-2)$$

where ρ is the discount rate. To simplify the calculation, we let $\rho = 1$ here. Now, we further explain the term $(1-\gamma_1+\gamma_2)q_{1s}$ which represents the influence of the sales quantity in period 1 on the price of the new products in period 2. We enumerate four special cases and find the corresponding commodity examples to demonstrate the practical significance of our model settings.

Table 2-2 Special cases of γ_1, γ_2.

	Categories	q_{1s}
$\gamma_1 = \gamma_2 = 0$	Electrical appliances	1
$\gamma_1 = 0, \gamma_2 = 1$	Books, Jigsaw	2
$\gamma_1 = 1, \gamma_2 = 0$	Napkins, drinks	0
$\gamma_1 = \gamma_2 = 1$	Failed product	1

i. $\gamma_1 = \gamma_2 = 0$ represents perfectly durable goods with consumers' lasting interests. Therefore, the consumers who buy the products will exit the market with no re-sale. Thus, the coefficient is 1, one unit sale in period 1 and one unit consumer exits the market in period 2.

ii. $\gamma_1 = 0, \gamma_2 = 1$ represents perfectly durable goods, and the consumers buy the products in period 1 but are tired with them and all resell them in period 2. Thus, one unit sale in period 1 will result in two units influence in period 2. Both the original and the new owners will exit the market.

iii. $\gamma_1 = 1, \gamma_2 = 0$ is more like daily consumables. We purchase and use them in period 1, but will not affect our future use. Thus, the coefficient is 0.

iv. $\gamma_1 = \gamma_2 = 1$ represents the situation that consumables, likes textbooks and examination papers, which play a role at the specific times. We consume the product, achieve a specific aim, then we exit the market. Therefore, it is the same as case (i), one unit sale and one unit influence.

2.2.2 Selling strategy

We first establish a benchmark model that does not involve resale, i.e., no c2c transaction will occur. Therefore, the term $\gamma_2 q_{1s}$ is not considered here and the coefficient discussed above will not be greater than 1 as one's consumption behavior will not affect the others. The prices in period 2 can be written as

$$\begin{gathered} p_{2u} = (1-\gamma_1)(1-q_{1s}-q_{2s}), \\ p_{2n} = 1-\min[(1-\gamma_1+\gamma_2),1]\, q_{1s}-q_{2s}. \end{gathered} \tag{2-3}$$

And the price of the sold products in period 1 is

$$p_{1s} = 1-q_{1s}+\rho(1-\gamma_2)p_{2u}. \tag{2-4}$$

The objective of the manufacturer is:

$$max\, \pi = p_{1s}q_{1s}+p_{2n}q_{2s}. \tag{2-5}$$

Solving it, we get when $\gamma_1 > \gamma_2$

$$q_{1s}^* = \frac{\gamma_1\gamma_2-2\gamma_2+2}{(\gamma_1-\gamma_2+1)(-3\gamma_1-\gamma_2+2\gamma_1\gamma_2+5)},$$

$$q_{2s}^* = \frac{3\gamma_1^2(\gamma_2-1)+\gamma_1(-3\gamma_2^2+\gamma_2+4)+3\gamma_2^2-6\gamma_2+3}{2(\gamma_1-\gamma_2+1)(-3\gamma_1-\gamma_2+2\gamma_1\gamma_2+5)},$$

$$\pi^* = \frac{\gamma_1^2(\gamma_2^2+2\gamma_2-3)+2\gamma_1(-3\gamma_2^2+4\gamma_2+1)+5\gamma_2^2-14\gamma_2+9}{4(\gamma_1-\gamma_2+1)(-3\gamma_1-\gamma_2+2\gamma_1\gamma_2+5)}.$$

Similarly, when $\gamma_1 \leq \gamma_2$

$$q_{1s}^* = \frac{\gamma_1\gamma_2-\gamma_1-\gamma_2+2}{-2\gamma_1-2\gamma_2+2\gamma_1\gamma_2+5},$$

$$q_{2s}^{*}=\frac{\gamma_1\gamma_2-\gamma_1-\gamma_2+3}{2(-2\gamma_1-2\gamma_2+2\gamma_1\gamma_2+5)},$$

$$\pi^{*}=\frac{(\gamma_1\gamma_2-\gamma_1-\gamma_2+3)^2}{4(-2\gamma_1-2\gamma_2+2\gamma_1\gamma_2+5)}.$$

However, recent technological advances in online mobile communications have enabled resale among consumers on a massive scale (Jiang and Tian 2018). Therefore, product transfer is adopted in our model.

We assume that there is a third-party platform providing consumers with transaction service. And we let the cost of the transactions equal to 0 for simplicity. The resale quantity is set as $\gamma_2 q_{1s}$, relating to the consumption depreciation. When $\gamma_2=1$, all consumers completely will lose their interests in the products and resell them in period 2; and when $\gamma_2=0$, all consumers will keep the products. Thus, the model formulation is as follows.

$$\begin{aligned} p_{2u}&=(1-\gamma_1)(1-q_{1s}-q_{2s}-\gamma_2 q_{1s}),\\ p_{2n}&=1-(1-\gamma_1+\gamma_2)q_{1s}-q_{2s},\\ p_{1s}&=1-q_{1s}+(1-\gamma_2)p_{2u}.\end{aligned} \qquad (2-6)$$

The objective of manufacturer is:

$$max\,\pi=p_{1s}q_{1s}+p_{2n}q_{2s}. \qquad (2-7)$$

And the result of the model is

$$q_{1s}^{*}=\frac{-2\gamma_2+\gamma_1\gamma_2+2}{2\gamma_1^2\gamma_2-3\gamma_1^2+2\gamma_1\gamma_2^2+2\gamma_1-3\gamma_2^2-2\gamma_2+5},$$

$$q_{2s}^{*}=\frac{3\gamma_1^2(\gamma_2-1)+\gamma_1(\gamma_2^2-3\gamma_2+4)-\gamma_2^2-2\gamma_2+3)}{2(2\gamma_1^2\gamma_2-3\gamma_1^2+2\gamma_1\gamma_2^2+2\gamma_1-3\gamma_2^2-2\gamma_2+5)},$$

$$\pi^{*}=\frac{\begin{array}{c}\gamma_1^2(\gamma_2^2+2\gamma_2-3)+\gamma_1(-2\gamma_2^2+4\gamma_2+2)\\ +\gamma_2^2-10\gamma_2+9)\end{array}}{4(2\gamma_1^2\gamma_2-3\gamma_1^2+2\gamma_1\gamma_2^2+2\gamma_1-3\gamma_2^2-2\gamma_2+5)}.$$

According to the model, in Figure 2-1, we show that regardless of whether the consumer resale exists or not, the manufacturer's profit increases in γ_1 and decreases in γ_2. When the physical depreciation increases, the products gradually change from durable goods to consumables. Thus, the price of the products will decrease since the decrease of product value in period 2. However, the manufacturer can obtain a higher profit since more consumers will purchase the products.

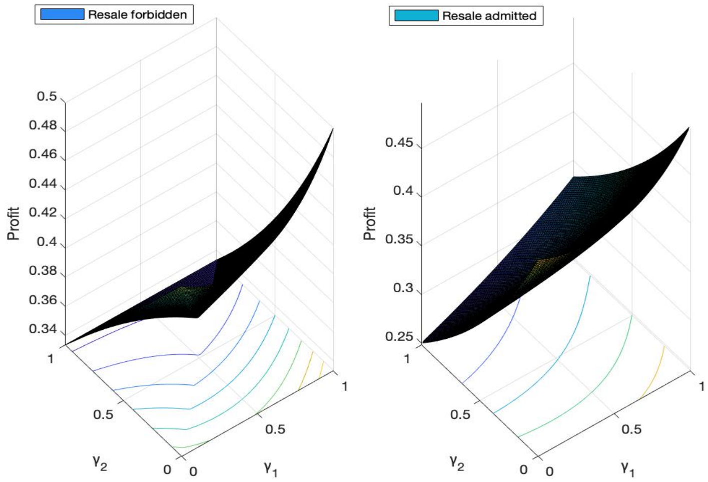


Figure 2-1 Manufacturer's profits with and without resale.

To intuitively represent the profit difference between the two models, we compare the profits and get Figure 2-2. It can be seen that the maximum value is located at $\gamma_1 = 0, \gamma_2 = 1$, and the profit difference increases in γ_2 and decreases in γ_1. Therefore, we find that when it is the perfectly durable product with the consumption value completely depreciated after use, the consumer's resale has the maximum influence on the manufacturer's profit. Since all the consumers will resell the products in period 2 with their physical performance intact, which results in fierce market competition and greatly undermines the profit of the manufacturer. When physical depreciation equals to zero $(\gamma_2 = 0)$, e.g., household appliances like refrigerators and TVs, the consumption value keeps. Therefore, whether consumer resale is admitted will not affect the manufacturer's profit in this situation, just like when $\gamma_2 = 1$ the consumable market case since there will be no used products in the second markets.

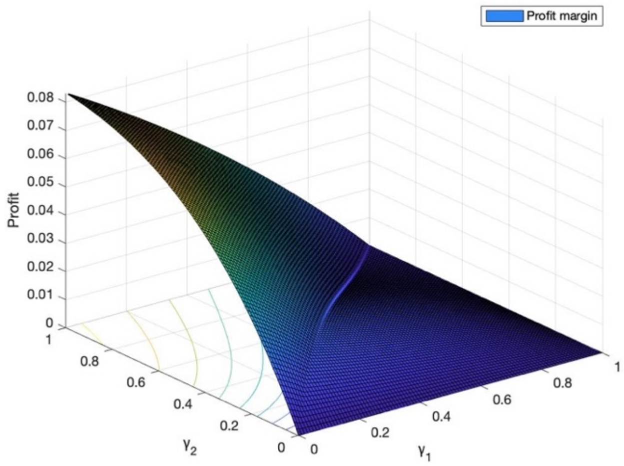


Figure 2-2 Manufacturer profit difference.

2.2.3 Leasing strategy

We find that second-hand trade will harm the profit of the durable goods manufacturer with consumption depreciation. So, what can the manufacturer do to avoid this situation? How can we reduce or even eliminate the second-hand competition? Leasing may be a feasible choice. Leasing versus selling is always a hot topic in durable goods research. Nowadays, many merchants have developed the leasing channel in additional to selling, and leasing has been involved in all aspects of durable goods. Besides, a series of platforms specializing in rental services has also emerged e.g., Furlenco (https://www.furlenco.com/), a popular furniture and appliance rental platform; SabRentKaro (https://www.sabrentkaro.com/), a platform where anyone can list their gadgets, furniture, and bikes for renting; and Cammunity (https://cammunity.co.uk/), a platform offering secure peer-to-peer cameras and equipment rental.

Therefore, we study the manufacturer's leasing strategy next. We assume that the rental period is exactly the length of one period. Noticed that the consumer valuation V in period 1 is uniformly distributed in [0,1]. The valuation of the consumers who have not used the products stay uniform in interval $[0,1-q_{1l}]$ in period 2. For the consumers who leased the products in period 1, their valuations will be uniformly distributed in the interval $[(1-\gamma_2)(1-q_{1l}), 1-\gamma_2]$ due to the consumption depreciation. We only research on the perfectly durable goods here, and the general case will be studied in the next section.

Considering the relationship between γ_2 and q_{1l}, we obtain the probability density function of consumers' second period valuation below

If $\gamma_2 \geq q_{1l}$,

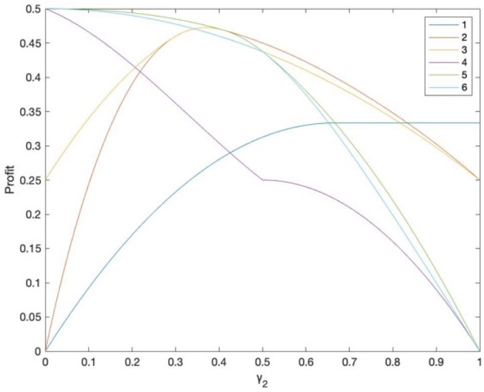
$$g(x)=\begin{cases}1 & 0\leq x\leq(1-\gamma_2)(1-q_{1l})\\ \dfrac{2-\gamma_2}{1-\gamma_2} & (1-\gamma_2)(1-q_{1l})\leq x\leq 1-\gamma_2\\ 1 & 1-\gamma_2\leq x\leq 1-q_{1l}\end{cases}. \qquad (2-8)$$

If $\gamma_2 \leq q_{1l}$,

$$g(x)=\begin{cases}1 & 0\leq x\leq(1-\gamma_2)(1-q_{1l})\\ \dfrac{2-\gamma_2}{1-\gamma_2} & (1-\gamma_2)(1-q_{1l})\leq x\leq 1-q_{1l}\\ \dfrac{1}{1-\gamma_2} & 1-q_{1l}\leq x\leq 1-\gamma_2\end{cases}. \qquad (2-9)$$

According to the different pdf segments the second period leasing quantity (q_2) falls into, we obtain six situations. And considering the constraint of the optimal solution. We subdivide the problem into 15 cases and solve them sequentially, and get the following results and Figure 2-3.

$$\pi^{\mathrm{L}*}=\begin{cases}\dfrac{(\gamma_2-2)^2}{4(\gamma_2^2-2\gamma_2+2)} & 0\le\gamma_2\le\dfrac{2}{3}\\ \dfrac{4-3\gamma_2}{7-3\gamma_2} & \dfrac{2}{3}\le r_2\le\dfrac{5}{6}.\\ \dfrac{1}{3} & \dfrac{5}{6}\le\gamma_2\le 1\end{cases}$$



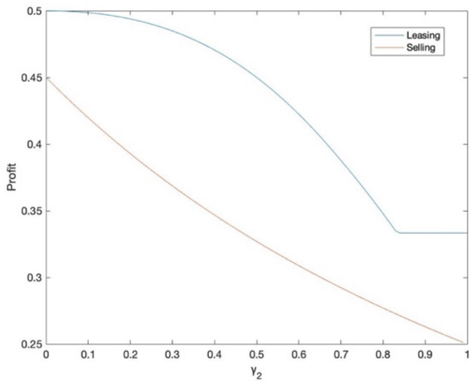


Figure 2-3 Manufacturer's profits in 6 situations (Left).

Figure 2-4 Manufacturers' leasing and selling profits (Right).

2.2.4 Comparison and analysis of the pure strategies

We compare the profits of the leasing and selling strategies in Figure 2-4. It is straightforward to find that the leasing profit first decreases and then is unchanged with the increase of the consumption depreciation. As consumption depreciation increases, the valuations of consumers who have used the product decline in period 2, thus the manufacturer's profit decreases at first. When the valuations of the first period adopters fall below a threshold, it is not worth for the manufacturer to continue the price reduction for these consumers. Then, the increase of the consumption depreciation will have no influence on the manufacturer's profit.

Consistent with Coase (1972), Bulow and Jeremy (1982), and other scholars, we find that the leasing strategy is always better than the selling strategy for a perfectly durable goods manufacturer. Why is this the case? We find that, in period 2, both the price and the quantity under the selling strategy are lower than those under the leasing strategy. It is because the consumers will drop out the market once purchasing the products and resell them in proportion γ_2, resulting in competition with the manufacturer in period 2. With the consumption depreciation increases, this competition will gradually intensify.

Above all, both the manufacturer's selling and leasing profits are decreasing in the consumption depreciation. The former is mainly due to the second-hand competition, and the latter is because of the lower re-leasing rate in period 2.

2.3 Hybrid strategy

We have researched the pure strategies in the previous section. In this section, we would like to explore whether the manufacturer's hybrid strategy can gain a higher profit. In Subsections 3.1 and 3.2, we focus on the leasing-selling and the selling-reselling hybrid strategies, respectively. Subsection 3.3 provides the comparison and analysis of the strategies.

2.3.1 Leasing-selling hybrid strategy

In Subsection 2.3, we find that for a perfectly durable goods manufacturer, the leasing strategy is always better than the selling strategy. Here, we investigate whether the manufacturer's leasing-selling strategy would lead to a higher profit.

Since selling and leasing are carried out at the same time, it is impossible to describe the probability density function of the consumers' valuation like the Subsection 2.3. However, we notice that if a consumer rents the product in both periods, his cost $p_{2u} + p_{1l}$ will be higher than the cost of direct purchase of the product in period 1. Therefore, for all rational consumers, it is not a cost-effective choice. Thus, the consumers who have used the products (no matter through renting or purchasing) will drop out the market in period 2. There will be no difference between leasing and selling in period 2 because our model only lasts for two periods. Here is the model formulation:

$$\begin{gathered} p_{2u} = (1-\gamma_1)(1-q_{1s}-q_{1s}\gamma_2-q_{2s}-q_{1l}-q_{2u}), \\ p_{2n} = 1-(1-\gamma_1+\gamma_2)q_{1s}-q_{2s}-q_{1l}-q_{2u}, \\ p_{1l} = 1-q_{1s}-q_{1l}, \\ p_{1s} = 1-q_{1s}-q_{1l}+(1-\gamma_2)p_{2u}. \end{gathered} \tag{2-10}$$

The objective function and constraints of the model are as follows.

$$\pi_M = p_{1s}q_{1s} + p_{1l}q_{1l} + p_{2n}q_{2s} + p_{2u}q_{2u}. \tag{2-11}$$

s.t.

$$0 \le q \le 1,$$

$$0 \le p \le 1,$$

$$0 \le \sum_{i=1,2} q_i \le 1,$$

$$q_{2u} \le q_{1l} + \gamma_2 q_{1s}.$$

Here, q_{1l} and q_{1s} represent the leasing and selling quantities in period 1, respectively. Moreover, q_{2s} and q_{2u} represent the selling quantities of the new and used products in period 2, respectively. Since the used products in period 2 come from two parts: the manufacturer's leased products in period 1 and the products resold by consumers in period 2, thus, we have $q_{2u} \leq q_{1l} + \gamma_2 q_{1s}$. The p_{2u} and p_{2n} are the prices of the used and new products in period 2, respectively, the same as in the last section. Also, p_{1l} and p_{1s} are the leasing and selling prices in period 1, respectively. Notice that the difference between them is exactly the consumers' valuation in period 2.

Solving the model, we find that the manufacturer will never adopt the selling-leasing hybrid strategy (the same as the result of Dou 2017), and physical depreciation plays a major role in manufacturer's decision-making.

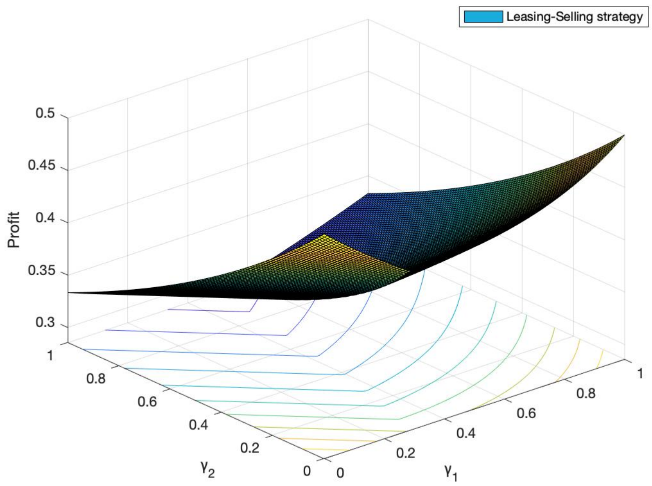


Figure 2-5 Profit of the leasing-selling strategy.

In Figure 2-5, we plot the manufacturer's profit. We find that the profit first decreases and then increases in the physical depreciation, and decreases in the consumption depreciation. When the physical depreciation is relatively small, the manufacturer prefers the leasing strategy. As the physical depreciation increases, the use value of the product in period 2 will gradually decrease, thus decreasing the profit. When the physical depreciation increases and exceeds a threshold, the manufacturer will switch to adopt the selling strategy. Then, the product will face less competition in period 2 with the increase of physical depreciation, thus increasing the manufacturer's profit.

2.3.2 Selling-reselling hybrid strategy

We have got that the pure leasing strategy can basically solve the problem results from resale. However, the companies who adopt the pure leasing strategy in reality are rare. Therefore, we would like to know whether there are any other marketing strategies that can reduce the impact of the collaborative consumption. Controlling the second-hand market seems to be a worth-trying approach. Therefore, we investigate the feasibility of this strategy in this subsection. There are many examples of enterprises who actively intervene in the second-hand market by recycling, remanufacturing, and/or reselling. For instance, Huawei carried out the "Green Action" with Recommerce Solutions and Ateliers du Bocage in 2013. In the first half of 2015, the activity was expanded to include 327 recycling stations worldwide (Huawei News, 2015). Lenovo has attracted much attention since the introduction of the old-for-new service. With the leading quality inspection technology, sophisticated monitoring process, and first-class service, Lenovo has been widely popular among the consumers (World Wide Web, 2020).

We try to find answers to the following questions: What can the manufacturers benefit from the selling-reselling strategy? Is it more profitable for a durable goods manufacturer to adopt the selling-reselling strategy than the pure selling strategy? The model formulation is as follows.

$$\begin{gathered} p_{2u} = (1-\gamma_1)(1-q_{1s}-q_{2s}-\gamma_2 q_{1s}), \\ p_{2n} = 1-(1-\gamma_1+\gamma_2)q_{1s}-q_{2s}, \\ p_{1s} = 1-q_{1s}+(1-\gamma_2)p_{2u}. \end{gathered} \qquad (2-12)$$

The objective function and constraints are:

$$\pi = p_{1s}q_{1s} + p_{2n}q_{2s} + \lambda\gamma_2 p_{2u}q_{1s} - c(\lambda). \qquad (2-13)$$

s.t.

$$0 \le q \le 1,$$

$$0 \le p \le 1,$$

$$0 \le \sum_{i=1,2} q_i \le 1.$$

The parameter λ represents the market share of the manufacturer in the second-hand market. $c(\lambda)$ represents the cost of the manufacturer to control the λ share. It has two properties: When the market share is 0, the cost is 0; and, if the manufacturer would like to have a larger share (λ) of the second-hand market, then its cost will be higher and growing much faster with the increase of λ, i.e., $c(\lambda)' \ge 0$ and $c(\lambda)'' \ge 0$, e.g., $c(\lambda) = (e^{2\lambda}-1)/100$.

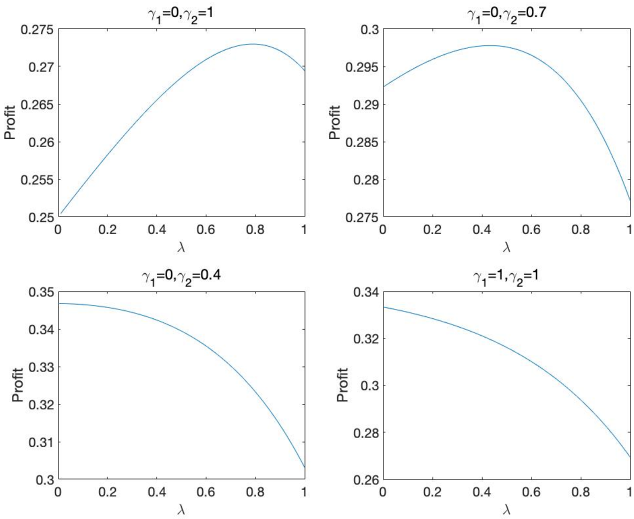


Figure 2-6 The relationship between profit and λ.

We demonstrate the relationship between the manufacturer's profit π and the market share λ in Figure 2-6. When $\gamma_1 = 0, \gamma_2 = 1$, the manufacturer's profit first increases and then decreases with λ, which means that it is indeed profitable to control part of the second-hand market. This is because the product is perfectly durable and all consumers soon tired of the product, the manufacturer faces the largest scale of the second-hand market. However, if the market share is relatively high, then the cost will be greatly increased and thus it is not cost-effective to control such a high market share. The case with $\gamma_1 = 0$ and $\gamma_2 = 0.7$ in the upper right-hand corner of Figure 2-6 also conforms to this trend. The difference is that its turning point slightly shifts to the left. When γ_2 decreases, the number of consumers who are willing to resell the products in period 2 also decreases. Therefore, the manufacturer can obtain profit due to the second-hand market reduction. Furthermore, when γ_2 is reduced to 0.4, it will be unprofitable to adopt the selling-reselling strategy with the specific cost function $c(\lambda)$. Similar to the $\gamma_1 = 1$ case, the product is consumable and no used products exist in period 2, so it is meaningless to involve in the second-hand market. Above all, we find that the selling-reselling strategy is not a necessarily beneficial strategy to the durable goods manufacturer. Figure 2-7 also verifies this conclusion.

Assuming that the manufacturer perfectly decides its optimal market share in every case, we plot the profit difference between the selling-reselling and selling strategies and obtain Figure 2-8. It is shown that the manufacturer's selling-reselling strategy is profitable only when the products have high consumption depreciation and low physical depreciation, and, more fundamentally, when the second-hand market is quite large.

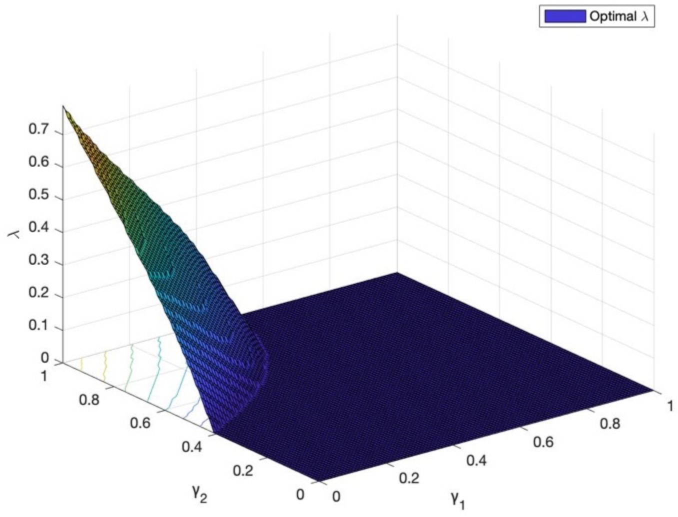


Figure 2-7 The optimal *λ*.

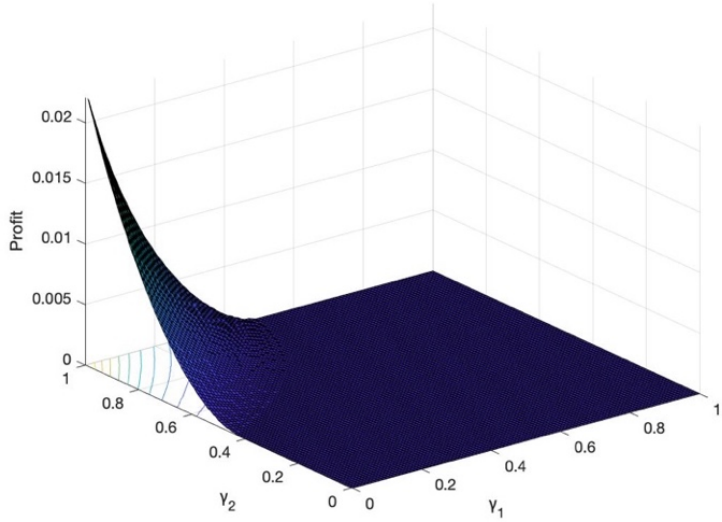


Figure 2-8 Profit margin between the selling-reselling and selling strategies.

2.3.3 Comparison and analysis of the hybrid strategies

In the last section, we find that the selling strategy is less profitable than the leasing strategy for perfectly durable goods. The main reason is that selling will lead to a second-hand market which is uncontrolled by the manufacturer and competes with the manufacturer, thereby hurts its profit. We consider two hybrid strategies in this section and provide some managerial insights.

For the manufacturer, the selling-leasing strategy will never be a better choice compared to the two pure strategies. Since our model only considers two periods, the leasing price must be equal to the selling price in the second period. Thus, if $p_{1s} > p_{1l} + p_{2l}$, no one will choose to purchase the product in period 1; otherwise, no one will rent it for both periods as everyone can resell the product in period 2 if he would only like to use it for one period. Therefore, the hybrid strategy is not helpful to the manufacturer.

However, the selling-reselling strategy may be beneficial to the manufacturer, depending on the cost function $c(\lambda)$. More specifically, for products with a certain degree of durability and high consumption depreciation, the leasing strategy is undoubtedly the best choice, and the selling-reselling strategy is the second-best choice (unless $c(\lambda)$ is high enough). The former can fundamentally eliminate the second-hand competition, and the latter can also alleviate the competition to some extent. This also proves that products with high consumption depreciation will indeed experience stronger second-hand market competition than those with low consumption depreciation, which is consistent with the conclusion of Ishihara (2019).

For consumables manufacturer, the selling strategy is no doubt the optimal since there is no used products in period 2. We conclude the optimal and suboptimal strategies for various types of manufacturers in Table 2-3.

Table 2-3 Strategies with different depreciation rates.

	Durable goods		Consumables
	High γ_2	Low γ_2	
Optimal	Leasing	Leasing	Selling
Suboptimal	Reselling	Selling	/

2.4 Extension

In this section, we would explore some extensions of our model. In Subsection 4.1, we will bring in a competing manufacturer and investigate how the duopoly manufacturers influence each other. We will allow the consumption depreciation to be less than 0 and add in the network effect in Subsection 4.2. In Subsection 4.3, we will extend our two-period model to n periods.

2.4.1 Duopoly manufacturers

In this subsection, we extend our model to duopoly manufacturers with pure strategies. There are three cases to consider: both selling (SS), both leasing (LL), one selling and the other leasing (LS).

To avoid the intractability of the model, we let $\gamma_1 = 0$ in this subsection and show the results of the three cases in Figure 2-9.

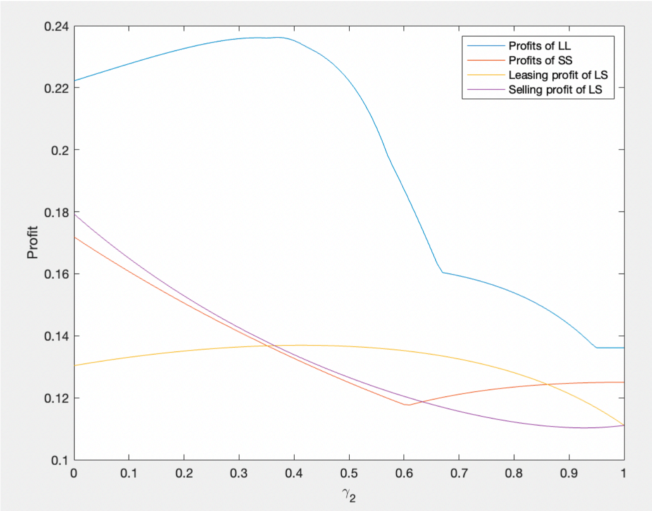


Figure 2-9 Profits of different strategies in duopoly case.

We find that for a perfectly durable goods manufacturer, even if there is a competitor, the optimal strategy is still leasing like Subsection 2.3, which shows our model results are robust.

2.4.2 Negative consumption depreciation

We only consider the case where consumers' interests decay in the above. However, there may be some excellent products which the consumers would not be tired with them and would recommend them to their peers. In this subsection, we incorporate this situation into our model, allowing the consumers' consumption depreciation to be less than 0, i.e., the consumption depreciation will expend the market in period 2. Of course, the term depreciation used here is somehow not so accurate, but we have decided to retain it for consistency.

Note that in Subsection 4.1, we find that the selling-leasing hybrid strategy cannot achieve better results than the pure strategies in any case. If the conclusion also applies here, then we can intuitively show which strategy the manufacturer should adopt and under what circumstances. Therefore, we focus on the selling-leasing strategy directly and find that the conclusion still holds. Following Chau and Desiraju (2017), we use ω_1 and ω_2 to represent the magnitudes of network effects in periods 1 and 2, respectively. We assume that the network benefits for the old and new products in each period are fully compatible. So, the numbers of consumers in the two periods are $q_{1l} + q_{1s}$ and $q_{1s} + q_{2s} + q_{2u}$, respectively, and the network benefits in each period are $b_1 = \omega_1(q_{1l} + q_{1s})$ and $b_2 = \omega_2(q_{1s} + q_{2s} + q_{2u})$. Therefore, the model formulation can be written as follows:

$$p_{2u} = (1-\gamma_1)(1-q_{1s}-q_{1s}\gamma_2-q_{2s}-q_{1l}-q_{2u})+b_2,$$
$$p_{2n} = 1-(1-\gamma_1+\gamma_2)q_{1s}-q_{2s}-q_{1l}-q_{2u}+b_2,$$
$$p_{1l} = 1-q_{1s}-q_{1l}+b_1, \qquad (2-14)$$
$$p_{1s} = 1-q_{1s}-q_{1l}+(1-\gamma_1)(p_{2u}-b_2)+b_2+b_1.$$

We believe that only the value of the product itself will incur consumption depreciation, and the impact of positive network effects is independent of the depreciation. The objective function of the manufacturer is

$$\max \pi_M = p_1q_{1s}+p_{1l}q_{1l}+p_{2n}q_{2s}+p_{2u}q_{2u}\,. \qquad (2-15)$$

Almost the same as conclusions in Subsection 3.1, the manufacturer adopting leasing-selling strategy will never achieve a higher profit compared to the two pure strategies. Moreover, physical depreciation is the major factor in choosing leasing or selling.

As for the consumption depreciation, the manufacturer' profit is decreasing in it. Intuitively, the negative consumption depreciation will further increase the manufacturer's profit.

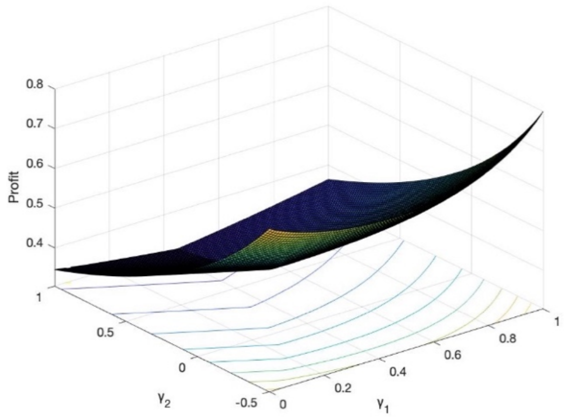


Figure 2-10 Profit with network effect.

2.4.3 Infinite period

In addition, we would also like to find out whether our conclusions still hold when the model is extended to infinite period. Assume that the products will attract q_0 new consumers at the beginning of each period to maintain the market since period 2, and the new consumers' valuations are also uniformly distributed in [0, 1].

Note that a manufacturer will adopt the selling- leasing hybrid strategy only if the profit margin per selling and profit margin per leasing are the same. Thus, in the infinite-period model, the manufacturer can also achieve higher profits through pure strategies. Assuming that each product can only be resold at most once, we construct the equilibrium solution in the N-th period as follows.

Case 1 Leasing only.

$$(1 - q_{1l} - q_{2u})q_0 = q_{1l} + q_{2u},$$
$$q_{1l} = \frac{q_0}{1 + q_0}; q_{2u} = 0.$$

Case 2 Selling only.

$$(1 - q_{1s} - q_{1s}\gamma_2)q_0 = q_{1s} + q_{1s}\gamma_2.$$

Comparing the profits of both cases, we get:

$$\pi_L = \left(\frac{q_0}{1 + q_0}\right)^2 \geq \pi_S = \left[\frac{q_0}{(1 + q_0)(1 + \gamma_2)}\right]^2.$$

Therefore, the conclusion that pure strategy is optimal still holds in the infinite period case. However, different from the above, the manufacturer will only adopt the leasing strategy here, instead of depending on the physical depreciation rate to choose between leasing and selling. It also makes sense, since the difference between leasing and selling is that leasing creates a self-controlled second-hand market, while selling results in a second-hand market out of control and second-hand competition. However, in a monopoly or oligopoly market, even without considering the second-hand competition, leasing is still more profitable than selling since it can fully obtain the value of products through the division of use rights.

2.5 Conclusions

We divide the depreciation of durable goods into consumption depreciation and physical depreciation in Section 2. Based on it, a series of two-period game-theoretic models of monopoly manufacturer is constructed to compare various strategies.

First, we research on the selling versus leasing. we find that although the manufacturer's profits in both strategies decrease in the consumption depreciation. Same as Bulow (1982) and Coase (1972), leasing is more profitable than selling. For durable goods with consumption depreciation $\gamma_2 = 0$, the profit margin is resulted from that leasing can divide the use right of the product. However, when $\gamma_2 > 0$, besides the above reason, a considerable number of consumers will resell the products in period 2, resulting in serious second-hand competition and extra loss of the manufacturer.

Second, we consider two hybrid strategies. We find that the selling-leasing strategy will never achieve better results than the two pure strategies. When the physical depreciation is relatively small, the leasing strategy is optimal. However, if the physical attributes of the products depreciate quickly, the selling strategy will be better. Because the residual value in period 2 of the products with high physical depreciation will be too low to be re-leased to the consumers in the leasing case; however, selling can obtain this value by adding to the selling price in period 1. As to the selling-reselling strategy, it is only profitable to the durable goods manufacturer with high consumption depreciation.

We also make some extensions to verify the robustness of our model. We introduce competing manufacturer in Section 2.4.1 and the conclusion about selling versus leasing still holds. Then, we consider the case where products become more and more popular in consumers, a phenomenon that is especially prevalent in the early stage of the game industry. Same as the conclusions in Section 2.3.1, the leasing-selling strategy will never help the manufacturer gain a higher profit, and physical depreciation determines which pure strategy should be taken. In Section 2.4.3, we extend our 2-period model to n periods and find that leasing is the optimal strategy in any case.

Section 3
Competitive Maintenance Strategies for Customer-Intensive Service Providers under Imperfect Online Monitoring

3.1 Introduction

In Section 3, we focus on the maintenance strategy, for a customer-intensive service, the value provided by the service increases with the time the service-provider spends on the service, so a longer service time usually means more careful inspection and maintenance. However, a longer service time will also result in a longer waiting time for customers (Anand et al., 2011). Many researchers since Anand et al. (2011) have studied this quality-speed tradeoff. Different from the previous research, we consider a competitive model of heterogeneous service providers with IoT-based online monitoring.

We develop analytical models for optimizing the operational strategies of competing service providers with imperfect online monitoring. Our goals are: (i) Derive the optimal competition strategies of the service providers with imperfect online monitoring for the cases of symmetric and asymmetric duopoly service providers, respectively, and extend the analysis to oligopoly service providers. (ii) Study the impacts of the model parameters on the customer arrival rates, service rates, and prices and profits of the service providers in each case. (iii) Strike a balance among online monitoring accuracy, investment cost, and customer waiting cost.

Our main contributions are as follows: First, we derive a complete set of optimal strategies of competitive service providers with imperfect online monitoring, regardless of whether the market is of full coverage or partial coverage, and regardless of the number and strength of the opponents. Second, we explore the impacts of the model parameters on the optimal strategies and discuss their managerial implications. Finally, we find that the optimal customer arrival rate in response to the optimal service rate follows a general pattern.

We organize the rest of the paper as follows: In Section 3.2 we review the related literature. In Section 3.3 we construct symmetric duopoly service provider models with partial market coverage and full market coverage, respectively. We relax the assumption that the service providers are homogeneous and assume that they have different technical levels in Section 3.4.

In Section 3.5 we extend the models from duopoly to n service providers. Finally, in Section 3.6, we conclude Section 3 and suggest some topics for future research. We provide the proofs of all the results in the Appendix.

3.2 Literature Review

There are two main streams of literature related to Section 3, i.e., strategic queuing studies in costumer-intensive services and preventive maintenance optimization.

First, we review studies on customer-intensive services. Anand et al. (2011) pioneer the concept of customer-intensive service, which suggests that the quality or value provided by a service provider increases with the time the service provider spends with the customer. However, a longer service time also results in a longer waiting time for the customer. Anand et al. (2011) consider a monopoly service provider with partial and full market coverage rates, respectively, and then extend the analysis to multiple service providers. They find that customer intensity leads to outcomes different from those of the traditional models. Ni et al. (2013) classify customers into different classes by their intensity levels and investigate the behaviour of each class of customers via a queueing model. Kostami and Rajagopalan (2013) consider dynamic models in the monopoly setting to explore the optimal balance among the speed, price, and waiting time. Li et al. (2016) construct a competing duopoly server model that provides customer-intensive services to boundedly rational customers. Based on the heterogeneity of customer travel burden, Rajan et al. (2019) compare the strategic behaviours of revenue-maximizing and welfare-maximizing experts, and demonstrate that the former would have a lower service rate and a lower customer arrival rate. Sun et al. (2020) focus on the diagnostic service design problem and model a multiple server queue considering imperfect diagnosis with uncertain error cost. Sun et al. (2021) develop a strategic queueing model to investigate a maintenance service provider’s optimal capacity allocation and pricing decisions in the presence of imperfect IoT-based diagnostics. Recently, Sun et al. (2022) examine the effects of spare parts consumption and repairman travel on the service provider’s optimal repair time, price, and number of servers. They find a counter-intuitive result that the service provider will improve the repair quality and decrease the spare parts consumption even if selling the spare parts becomes more profitable. Liu et al. (2022) investigate the participation, competition, and welfare of a platform that focuses on customer-intensive discretionary services with heterogeneous customers. They find that platform price intervention may benefit not only the platform and service providers, but also the customers.

We summarize in Table 3-1 the main features of our work and compare them with the studies reviewed above. It is evident that we consider service providers' competition under partial and full market coverage, respectively. Besides, our study fills the gap in the literature by considering service provider heterogeneity in customer-intensive services.

Table 3-1 Main points of related papers.

	Customer -intensive services	Imperfect online monitoring	Competition		Market share	
			Number of competitors	Asymmetric service providers	Partial coverage	Full coverage
Anand et al. (2011)	√		N		√	√
Ni et al. (2013)	√				√	
Kostami and Rajagopalan (2014)	√				√	
Li et al. (2016)	√		2		√	
Rajan et al. (2019)	√				√	
Sun et al. (2020)	√	√			√	√
Sun et al. (2021)	√	√			√	
Sun et al. (2022)	√		N		√	√
Liu et al. (2022)	√		N		√	√
This thesis	√	√	N	√	√	√

Moreover, preventive maintenance is also highly relevant to our work. Nowadays, preventive maintenance actions, which can better align with the other business functions such as production scheduling and spare parts control, are on the rise (De Jonge and Scarf, 2020). Wang et al. (2014) introduce a two-level inspection policy model based on a three-stage failure process (good, minor defective, and severe defective stages), and derive results corresponding to well-adopted maintenance policies in practice, such as periodic inspections with scheduled maintenance optimization. Ding et al. (2019) design an IoT-based traceability system to realize real-time monitoring of gray market activities, and build three game models to examine how IoT technology affects the gray market and firms' profits. Nguyen et al. (2019) investigate how the adjustment of condition monitoring quality could help reduce the total cost of a CPM programme. Sun et al. (2021) divide the service value into testing (IoT-based diagnostics) and processing (maintenance), and investigate the influence of the accelerating effect on the service provider. Different from the above, we study the optimal decisions of competing service providers with imperfect online monitoring in both homogeneous and heterogeneous cases.

3.3 Model of Symmetric Duopoly Service Providers

We consider a monopoly firm that produces specific devices. The firm has its own sales department and providers of online monitoring, by, which installing condition indicators to collect and analyze product information for early warning of potential failures. Besides, it provides after-sales service by outsourcing two professional maintenance service providers. The firm and its subcontracting service providers share the cost and profit of the after-sales service. In this section, we assume that the maintenance service providers outsourced by the monopoly firm are homogeneous. Gradually relaxing this assumption in the following sections, we consider the case of n heterogeneous service providers.

3.3.1 Model setup

Customers obtain positive utility from both the testing (online monitoring) and processing (maintenance) components (Wang et al., 2014; Levi et al., 2019). In our model, we consider a heavy device firm which installs various types of sensors to detect real-time information such as vibration, temperature, pressure of the product. The firm collects and analyses the information, and returns an error warning signal to the customer in advance when the information is abnormal. Since the online monitoring diagnostic is imperfect, the customer will decide whether to go for the maintenance service according to the reality situation. The processing by maintenance includes inspection, preventive repair, and replacement of defective components (Cui et al., 2004).

However, the monitoring results may produce inaccurate outcomes due to inaccurate information or inappropriate predictive methods. The accuracy of IoT-based diagnostics depends on online monitoring depth l, which measures how many condition indicators of the equipment are installed. Intuitively, the diagnostic accuracy increases with the monitoring depth l, and the marginal increase of the accuracy from an increase in l is diminishing since the difficulty and cost of precision improvement gradually increase. It is worth noting that (1) if we have zero investment, we cannot make predictions since we cannot get any useful information, and (2) the accuracy of the diagnosis will only approach one when the investment tends to infinity. So we have $\theta(0) = 0, \lim_{l \to \infty} \theta(l) = 1$, which is consistent with the reality. Therefore, following Nguyen et al. (2019) and Sun et al. (2021), we assume that the accuracy rate of the prediction signals is

$$\theta(l) = 1 - e^{-rl}, \tag{3-1}$$

where coefficient r reflects the improvement rate of the diagnostic accuracy when l increases.

We use c_t to represent the customer error cost when a misdiagnosis occurs. It can also be regarded as the customer gain c_t, if the diagnosis is correct; and 0, otherwise (Sun et al., 2021). Thus, the expected utility of testing is

$$F(l) = \theta(l)c_t. \tag{3-2}$$

In customer-intensive services, the maintenance quality or value provided to a customer increases with the service time. Thus, following the expression of Anand et al. (2011), we assume

$$Q(\mu) = Q_0 + \alpha(\mu_0 - \mu), \tag{3-3}$$

where Q_0 is the benchmark value when the service provider adopts the benchmark service rate μ_0. We also ensure that $1/\mu_0$ is the minimal service time of a maintenance period by setting $0 \le \mu \le \mu_0$. The parameter $\alpha \ge 0$ determines the sensitivity of the service value to the service speed, and is a descriptor of the "nature" of the customer-intensive service in Anand et al. (2011). However, in our model, we also regard α a descriptor of the technical level of the service provider, and a higher α brings a higher processing value per unit of time to the customer. For example, in the same period, an experienced masseur and a trainee masseur can bring a customer a very different service experience; of course, the difference is also reflected in the price. Therefore, we allow different α for different service providers in the later sections.

In our model, we assume that the customers are homogenous and completely rational who arrive at the service system according to a Poisson process at an exogenous mean rate λ. An arriving customer observes the online monitoring depth l, service rate μ, and price p, and then makes the join-or-balk decision based on the utility function. Notice that when the service provider's price and service rate are determined, the expected probability of a customer choosing either service provider is also determined. Therefore, we approximate the stochastic process of customers arriving at both service providers as Poisson distributions.

$$U(l, \mu, P) = F(l) + Q(\mu) - \frac{c_w}{\mu - \lambda(l, \mu, p)} - p, \qquad (3-4)$$

where $F(l)$ and $Q(\mu)$ represent the values of testing and processing, respectively, as we mention above, and c_w is the waiting cost per unit time, and $\lambda(l, \mu, p)$ is the induced effective demand.

3.3.2 Symmetric model with partial market coverage

We first consider the partial market coverage case. In this case, the potential demand λ is large enough to support two service providers adopting their optimal strategies (μ^*, p^*) independent of each other, i.e., they do not actually have a competitive relationship. A customer procures the service only if his utility is greater than 0; otherwise, he will leave the queue. Using (4), we derive the customer's equilibrium arrival rate as

$$\lambda^e(l, \mu, p) = \mu - \frac{c_w}{F(l) + Q(\mu) - p}. \qquad (3-5)$$

The monopoly firm sets the online monitoring depth l, the long-run decision variable, to maximize its long-run profit, and then each outsourced service provider decides its short-run decision variables. It first makes decisions on the service rate μ, and then sets the corresponding price p based on the service times to maximize its short-run profit. It follows that the profit maximizing function is

$$\max \pi = [p - c_s(1 - \theta(l))]\lambda(l, \mu, p) = (p - c_s e^{-\gamma l})\lambda(l, \mu, p),$$

c_s is the customer error cost. We solve this problem by backward induction. First, we assume that μ is exogenous and find the optimal price $p^*(\mu)$. Then, we replace p with $p^*(\mu)$ and find the optimal service rate μ^*. Accordingly, we derive the following result.

Proposition 1:

The service providers' optimal strategies and profits are

$$\mu^* = \frac{Q_0 + \alpha\mu_0 + c_t - e^{-\gamma l}(c_s + c_t)}{2\alpha} = \lambda^* + \sqrt{\frac{c_w}{\alpha}},$$

$$p^* = \frac{Q_0 + \alpha\mu_0 + c_t + e^{-\gamma l}(c_s - c_t)}{2} - \sqrt{\alpha c_w} = \lambda^* + c_s e^{-\gamma l},$$

$$\pi^* = [p^* - c_s(1-\theta(l))]\lambda^* = \frac{\left(Q_0 + \alpha\mu_0 + c_t - e^{-\gamma l}(c_s + c_t) - 2\sqrt{\alpha c_w}\right)^2}{4\alpha}.$$

The induced equilibrium customer arrival rate is

$$\lambda^* = \frac{Q_0 + \alpha\mu_0 + c_t - e^{-\gamma l}(c_s + c_t) - 2\sqrt{\alpha c_w}}{2\alpha}.$$

The trade-off between quality and speed is at the crux of the service-provider's problem (Anand et al., 2011). When the service rate is low ($\mu < \mu^*$), the customer can gain a higher processing value from the longer service time compared with the μ^* situation, thus the service provider will set a higher price $p > p^*$. However, a longer service time also results in a longer waiting time for the customer. In addition to the higher price, the loss from the low customer arrival rate dominates the profit that the service provider can benefit from the price hike. Thus, the service provider will tend to increase the service rate. And when the service rate is high ($\mu > \mu^*$), the loss in price reduction outweighs the profit from increased customer arrival, and the service provider will increase the service time. Based on Proposition 1, we deduce the following corollaries.

Corollary 1

(i) *The customer arrival rate λ^*, service provider's price p^*, service rate μ^*, and profit π all increase with customers' benchmark service rate μ_0 and benchmark value Q_0.*

(ii) *If the customer error cost c_t increases, then the service provider will increase the service rate μ^* and raise the price p^*, and the customer arrival rate λ^* will also be higher since the customer value increases, and the profit π will increase.*

(iii) *If the service provider error cost c_s increases, then the service provider will decrease the service rate μ^* but increase the price p^*, and the customer arrival rate λ^* will decrease since the customer value decreases, and the profit π will decrease.*

(iv) *If the customer waiting cost c_w increases, then the customer arrival rate λ^* will decrease, and the service provider will lower the price p^*, and the profit π will decrease.*

From (3-3), it is straightforward that if the customer benchmark value Q_0 and benchmark service rate μ_0 increase, then the customer's processing value will increase and the customer will be more willing to join the queue since he can gain more value. Thus, the service provider can increase its service rate, set a higher price, and gain more profit.

If the customer error cost c_t increases, the customer can obtain higher utility when the diagnosis is correct. Therefore, the service provider can set a higher service rate. And if the service provider error cost c_s increases, the service provider will raise the selling price p^* to make up for the loss, and at the same time decrease the service rate to avoid losing too many customers; however, the customer arrival rate will still decrease since the sum of the customer testing value and processing value $F(l) + Q(\mu^*)$ decreases in c_s.

For Corollary 1(iv), the optimal service rate μ^* is independent of the customer waiting cost since it is a seller's market (the customer arrival rate λ is large enough). However, the price p^* and customer arrival rate λ^* both decrease with the waiting cost c_w since the customer is less willing to join the queue.

As for the profit, it obviously increases in c_t and decreases in c_w, and it decreases in c_s since the price increase can only reduce, but cannot offset, the loss caused by the increase in the service provider's error cost.

Corollary 2

(i) *The customer arrival rate λ, service provider's service rate μ^*, and profit π increase with online monitoring depth l.*

(ii) *If $c_t > c_s$, then service providers' optimal price p^* will increase in l; otherwise, decrease in l.*

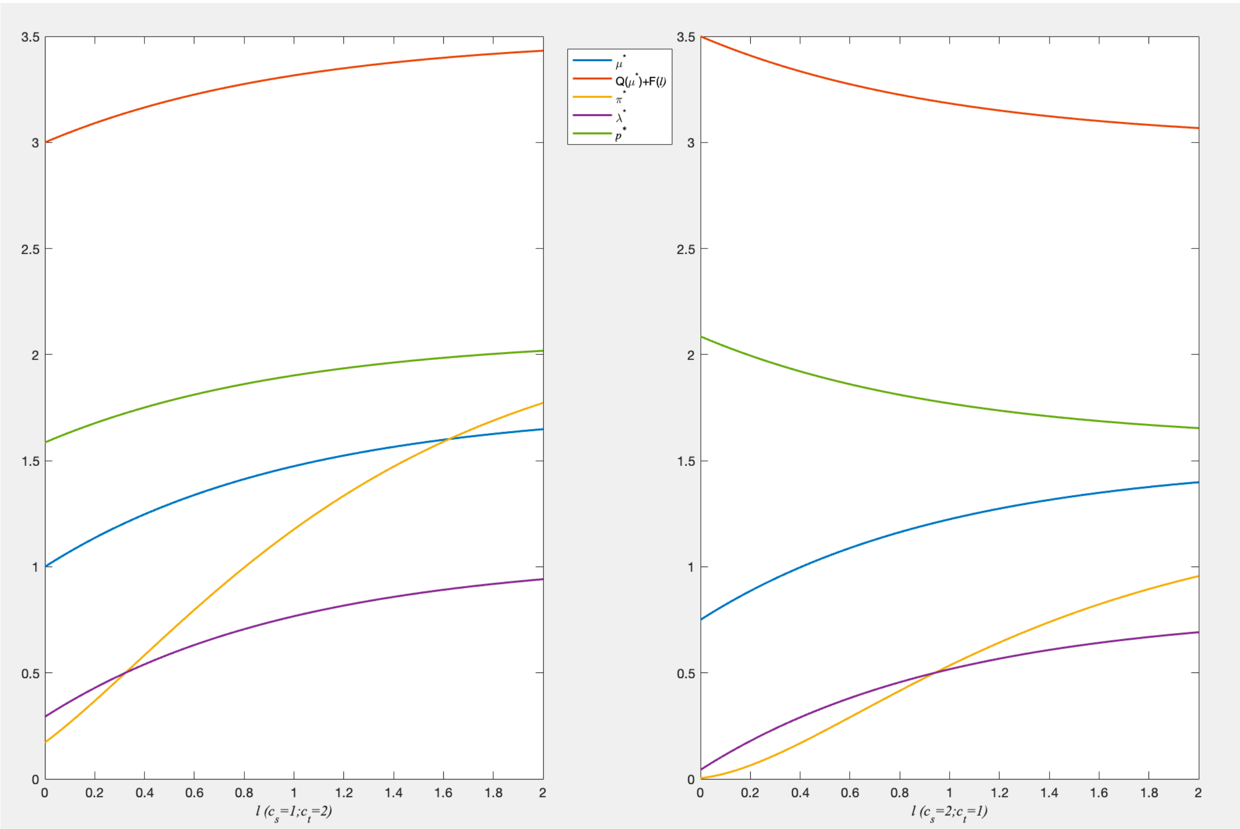


Figure 3-1 Effects of q on the service provider's decisions.

We use numerical examples to make our results more intuitive, and set the parameters used in Figure 3-1 as $\mu_0 = 2, Q_0 = 1, \gamma = 1, \alpha = 2,$ and $c_w = 1$; moreover, we choose $c_s = 1, c_t = 2$ in the left panel, and $c_s = 2, c_t = 1$ in the right panel. Corresponding to Corollary 2(i), Figure 3-1 supports λ^*, μ^*, and π increase in l in both cases (the blue, orange, and purple lines in both panels).

Then, we find that improving the online monitoring depth may damage the customer value, shown as the red line on the right panel, which seems a little bit counterintuitive since the predicted accuracy increases with improved monitoring. How could customer value decrease with the online monitoring depth? The customer value in our model consists of two aspects, testing value and processing value, and as the blue line shows, the service rate μ^* chosen by the service provider always increases in l, which harms customers' processing value. Thus, the relationship between customer value and online monitoring depth depends on whether customers' gain in accuracy raise can make up for the loss of service time reduction. Specifically, if $c_s < c_t$, the customer will suffer more loss when diagnosis is incorrect, so the benefits from increasing the monitoring depth can offset the decrease in the processing value since customers value accuracy more. However, if $c_s > c_t$, the cost of customer error is acceptable, so the decline in processing value dominates variations in customer value, and the service provider needs to cut price to keep its service attractive. In any case, the increasing l improves the service provider's ability, so the profit π will increase accordingly.

Corollary 3

(i) *The customer's optimal service rate* μ^* *increases in* α *when* $Q_0 + F(l) < c_s(1 - \theta(l))$*; otherwise, decreases.*

(ii) *The optimal price* p^* *increases in* α *when* $\alpha > c_w/\mu_0^2$*; otherwise, decreases.*

(iii) *If* $Q_0 + F(l) < c_s\big(1 - \theta(l)\big)$*, the equilibrium arrival rate* λ^* *always increases in* α*; otherwise,* λ^* *first decreases and then increases in* α*.*

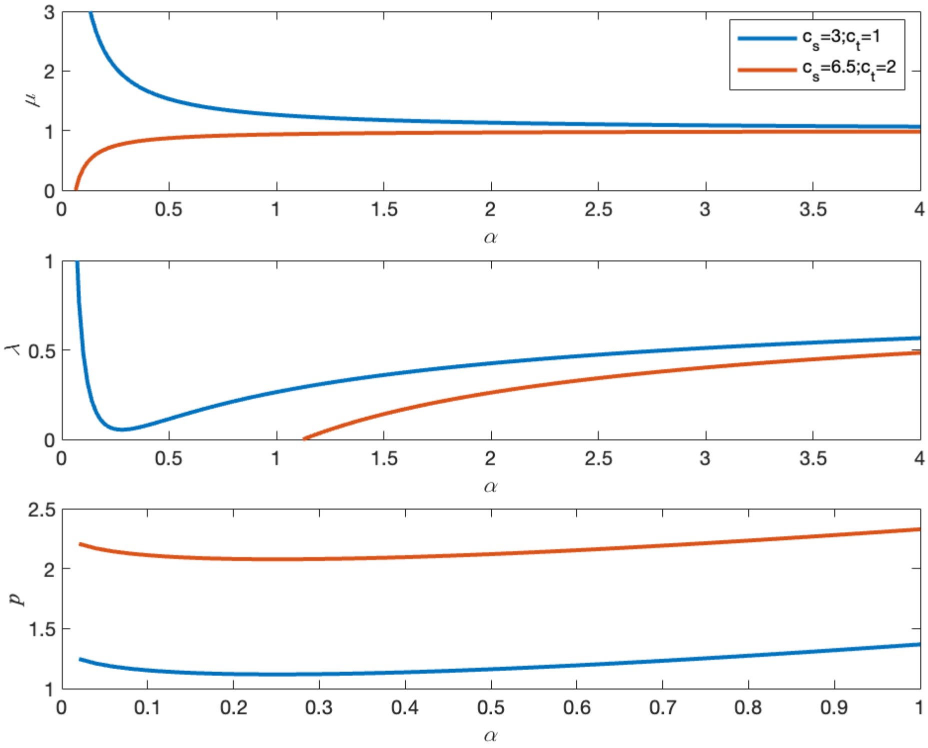


Figure 3-2 Effects of α on the service provider's decisions.

Corollary 3 reveals the relationships between the service providers' strategies, the equilibrium customer arrival rate, and α. We also use numerical examples to make Corollary 3 more intuitive, and set the parameters used in Figure 3-2 as $\mu_0 = 2, Q_0 = 1, \gamma = 1, l = 1,$ and $c_w = 1$. Notice that Q_0 is the benchmark value that customers can at least gain from the processing component. Moreover, $F(l)$ is the exogenous testing value that customers gain from the online monitoring since l is exogenous in the short-run model. Q_0 and $F(l)$ together make up the customer's exogenous surplus and are also the cost of the customer, which is independent of the service rate. In addition, $c_s(1-\theta(l))$ is the service provider's exogenous mis-diagnosis cost when l is given. If $Q_0 + F(l) < c_s(1-\theta(l))$, the customer will keep a low service rate since he will suffer an exogenous loss when he accepts the service, and with increasing α, the service provider can gain profits from processing, so both the service rate and arrival rate increase. However, if $Q_0 + F(l) > c_s(1-\theta(l))$, when α is small, the exogenous surplus is much higher than the endogenous profit $Q(\mu)$, so the service provider chooses a relatively high service rate μ^* to get the exogenous profit as much as possible. With increasing α, $Q(\mu)$ explains more and more of profits, which gradually decreases the service rate until it exceeds the threshold, the loss of the waiting cost dominates the increase in the processing value, then the optimal service rate increases with α instead. And the optimal price first decreases and then increases in α in both cases (shown in the third panel).

Now we consider the long-run model. Notice that in our model the monopoly firm decides on the online monitoring depth l in the first stage, and then each outsourced service provider makes the decision on the service rates μ based on the given l, and sets the corresponding prices p based on μ and l to maximize its profit in the third stage. So the profit maximizing function of the monopoly firm is

$$\max \pi = [p - c_s(1-\theta(l))]\lambda(l,\mu,p) - kl^2 = (p - c_s e^{-\gamma l})\lambda(l,\mu,p) - kl^2.$$

We use the backward induction to solve the long-run model and add a third step to get $l^*(\mu^*, p^*)$ based on Proposition 1. We derive the following result.

Proposition 2:

When $k > \left[Q_0 + \alpha\mu_0 - c_s - 2\sqrt{\alpha c_w}\right]\left(\frac{c_s+c_t}{2\alpha}\right) r$, $l^* = 0$*; otherwise,* l^* *is*

$$\left[Q_0 + \alpha\mu_0 + c_t - e^{-\gamma l^*}(c_s + c_t) - 2\sqrt{\alpha c_w}\right]\left(\frac{c_s + c_t}{2\alpha}\right) r e^{-\gamma l^*} = 2kl^*;$$

and the results of the long-run model are:

$$\mu^* = \frac{Q_0 + \alpha\mu_0 + c_t - e^{-\gamma l^*}(c_s + c_t)}{2\alpha},$$

$$p^* = \frac{Q_0 + \alpha\mu_0 + c_t + e^{-\gamma l^*}(c_s - c_t)}{2} - \sqrt{\alpha c_w},$$

$$\lambda^* = \frac{Q_0 + \alpha\mu_0 + c_t - e^{-\gamma l^*}(c_s + c_t)}{2\alpha} - \sqrt{\alpha c_w},$$

$$\pi^* = [p^* - c_s(1-\theta(l^*))]\lambda^* = \frac{\left(Q_0 + \alpha\mu_0 + c_t - e^{-\gamma l^*}(c_s + c_t) - 2\sqrt{\alpha c_w}\right)^2}{4\alpha} - kl^2.$$

3.3.3 Symmetric model with full market coverage

Then, we consider on the full market coverage case. In this case, the potential demand λ is limited ($\lambda < 2\lambda^*$) and the two service providers compete for customers in a limited pool. Each customer will choose the service provider with higher utility, and if the utilities of the service providers are identical, he will choose at random. Moreover, if the utilities of both service providers are less than 0, he will choose to leave the queue.

Since the strategies and payoffs of the service providers are identical, the strategy space for each service provider is compact and convex ($p \in [0, \overline{p}], \lambda \in [\underline{\lambda}, 2\lambda^e]$), and the payoff function is continuous and quasi-concave with respect to each service provider's own strategy. So there exists at least one symmetric pure strategy NE in this game, and we have the following result.

Proposition 3:

The equilibrium customer arrival rate of each service provider is

$$\lambda^{**} = \frac{\lambda}{2}.$$

And the service provider's optimal strategy and profit are

$$\mu^{**} = \frac{\lambda}{2} + \sqrt{\frac{c_w}{\alpha}} = \lambda^{**} + \sqrt{\frac{c_w}{\alpha}},$$

$$p^{**} = \frac{\alpha\lambda}{2} + c_s e^{-rl} = \alpha\lambda^{**} + c_s e^{-rl},$$

$$\pi^{**} = \frac{\alpha\lambda^2}{2}.$$

The equilibrium utility of customer is

$$U^{**} = c_t + Q_0 + \alpha\mu_0 - \frac{3}{2}\alpha\lambda - 2\sqrt{\alpha c_w} - (c_t + c_s)e^{-rl}$$

From Proposition 3, we derive the following result.

Corollary 4

(i) The customer equilibrium arrival rate λ^{**}, service provider's optimal service rate μ^{**}, optimal price p^{**}, and profit π^{**} are all increasing in the potential demand rate λ.

(ii) The service provider's optimal service rate μ^{**} increases in the customer waiting cost c_w and decreases in the technical level α, and the optimal price p^{**} and profit π^{**} increase in α.

(iii) The customer's equilibrium utility U^{**} increases in $c_t, Q_0, \mu_0, r,$ and l, and decreases in c_w, c_s. When $\lambda > 2\mu_0/3$, U^{**} always decreases in α; otherwise, U^{**} first increases and then decreases in α.

(iv) The optimal price p^{**} increases in the service provider's error cost c_s and decreases in the online monitor depth l.

In this case, the number of potential customers is less than the effective capacity of the service providers. Therefore, it's straightforward that the increase of λ benefits the customers. Moreover, through Proposition 3 and Corollary 4(i), we find that different from the partial coverage case, the decision variables μ and p are independent of the benchmark values

Q_0 and μ_0, and customer error cost c_t. Notice that we have the limited customer quantity $\lambda < 2\lambda^*$. If the service providers still choose the optimal service rate μ^* in the partial coverage case, there will be vacuum periods that nobody is being served, thus resulting in a waste of performance. Therefore, the service providers face customer competition with each other. The exogenous parameters of the customer values have no relationship with the optimal strategy (μ^{**}, p^{**}) anymore, and will only enhance the intensity of competition and increase customer utility instead. Thus, U^{**} increases in $c_t, Q_0,$ and μ_0. And U^{**} increases in l and γ since they will both help improve diagnostic accuracy, leading to stronger competition. When customer error cost and waiting cost increase, the customers are less willing to join the queue, then U^{**} decreases. As for the technical level α, when λ is relatively large, the service providers can gain more profit by raising the price. However, when λ is small, U^{**} increases or decreases in α depending on whether the gains from price raises dominate the losses of the increased waiting times.

Although the number of potential customer declines, we find that the average waiting time of each customer is $\sqrt{\alpha/c_w}$, which is unchanged since $\mu^{**} - \lambda^{**} = \sqrt{c_w/\alpha}$. And μ^{**} increases in c_w for lowering the waiting time when the per minute waiting cost increases and decreases in α for gaining more customer processing value when the technical level rises. Moreover, when service provider's error cost c_s increases, it will increase the price to make up for the misdiagnosis loss.

Proposition 4:

The optimal online monitoring depth l^{**} in the λ limited case is

$$l^{**} = \frac{1}{r} ln \frac{c_t + c_s}{c_t + Q_0 + \alpha\mu_0 - \frac{3}{2}\alpha\lambda - 2\sqrt{\alpha c_w}}.$$

Notice that the profit in the limited case is independent of l^{**}, so the firm will let l be as small as possible in view of the cost kl^2. At the same time, each service provider needs to ensure the non-negativity of the customer's utility. Thus, the optimal l^{**} meets $U(l^{**}) = 0$.

3.4 Model of Asymmetric Duopoly Service Providers

In this section we relax the assumption that the service providers are homogeneous, allowing them to have different technical levels, and the service provider with a high technical level can bring a higher processing value to the customer under the same service time. Without loss of generality, we assume that $\alpha_1 \geq \alpha_2$ and derive

$$U_i(l, \mu, P) = F(l) + Q(\mu_i) - \frac{c_w}{\mu_i - \lambda_i(l_i, \mu_i, p_i)} - P_i, \quad i = 1,2.$$

Similar to Section 3.3, we first consider the full coverage case. If $\Lambda \geq \lambda_1^e + \lambda_2^e = \sum_{i=1,2} \frac{Q_0 + \alpha_i \mu_0 + c_t - e^{-\gamma l}(c_s + c_t) - 2\sqrt{\alpha_i c_w}}{\alpha}$, each service provider does not compete with each other, same as the homogeneous case. So the optimal strategies of the service providers are as follows, $i = 1$ and 2:

$$\mu_i^* = \frac{Q_0 + \alpha_i \mu_0 + c_t - e^{-\gamma l}(c_s + c_t)}{2\alpha_i},$$

$$p_i^* = \frac{Q_0 + \alpha_i \mu_0 + c_t + e^{-\gamma l}(c_s - c_t)}{2} - \sqrt{\alpha_i c_w},$$

$$\lambda_i^* = \frac{Q_0 + \alpha_i \mu_0 + c_t - e^{-\gamma l}(c_s + c_t) - 2\sqrt{\alpha_i c_w}}{2\alpha_i},$$

$$\pi_i^* = [p_i^* - c_s(1 - \theta(l)]\lambda_i^* = \frac{\left(Q_0 + \alpha_i \mu_0 + c_t - e^{-\gamma l}(c_s + c_t) - 2\sqrt{\alpha_i c_w}\right)^2}{4\alpha_i}.$$

Then we consider the full market coverage case. We first prove that $\lambda > \lambda_1' + \lambda_2'$ is not true. Since $\lambda < \lambda_1^e + \lambda_2^e$, if $\lambda_1' + \lambda_2' < \lambda < \lambda_1^e + \lambda_2^e$, then $U_1 = U_2 = 0$; otherwise, the service provider can set a higher price without losing any customer. Since π_i is concave in μ_i and p_i, and (μ_i^*, p_i^*) is the global optimum, service provider i $(i = 1, 2)$ will adjust its strategy (μ_i, p_i) close to (μ_i^*, p_i^*) until $\lambda_1' + \lambda_2' = \lambda$. We also find that $U_1 = U_2$; otherwise, it cannot be an equilibrium strategy because the service provider with larger utility can unilaterally raise its price to increase its revenue. Overall, the model is as follows:

$$Maxmize\ \pi_1 = [p_1 - c_s(1 - \theta(l))]\lambda_1(l, \mu_1, p_1) = (p_1 - c_s e^{rl})\lambda_1(l, \mu_1, p_1)$$

$$Maxmize\ \pi_2 = [p_2 - c_s(1 - \theta(l))]\lambda_2(l, \mu_2, p_2) = (p_2 - c_s e^{rl})\lambda_2(l, \mu_2, p_2)$$

s.t.

$$Q_1(\mu_1) - \frac{c_w}{\mu_1 - \lambda_1} - P_1 = Q_1(\mu_2) - \frac{c_w}{\mu_2 - \lambda_2} - P_2,$$

$$Q_i(\mu) = Q_0 + \alpha_i(\mu_0 - \mu_i),$$

$$\lambda_1 + \lambda_2 = \lambda.$$

Solving the model, we derive the optimal decisions of the two service providers in the following.

Proposition 5:

The equilibrium customer arrival rates of service providers 1 and 2 are

$$\lambda_1^* = \frac{\mu_0(\alpha_1 - \alpha_2) - 2\sqrt{c_w}(\sqrt{\alpha_1} - \sqrt{\alpha_2}) + \lambda(\alpha_1 + 2\alpha_2)}{3(\alpha_1 + \alpha_2)},$$

$$\lambda_2^* = \frac{\mu_0(\alpha_2 - \alpha_1) - 2\sqrt{c_w}(\sqrt{\alpha_2} - \sqrt{\alpha_1}) + \lambda(2\alpha_1 + \alpha_2)}{3(\alpha_1 + \alpha_2)}.$$

The optimal strategies and profits of service providers 1 and 2 are

$$\mu_1^* = \frac{\mu_0(\alpha_1 - \alpha_2) - 2\sqrt{c_w}(\sqrt{\alpha_1} - \sqrt{\alpha_2}) + \lambda(\alpha_1 + 2\alpha_2)}{3(\alpha_1 + \alpha_2)} + \sqrt{c_w/\alpha_1},$$

$$\mu_2^* = \frac{\mu_0(\alpha_2 - \alpha_1) - 2\sqrt{c_w}(\sqrt{\alpha_2} - \sqrt{\alpha_1}) + \lambda(2\alpha_1 + \alpha_2)}{3(\alpha_1 + \alpha_2)} + \sqrt{c_w/\alpha_2},$$

$$p_1^* = \frac{\mu_0(\alpha_1 - \alpha_2) - 2\sqrt{c_w}(\sqrt{\alpha_1} - \sqrt{\alpha_2}) + \lambda(\alpha_1 + 2\alpha_2)}{3} + c_s e^{-rl},$$

$$p_2^* = \frac{\mu_0(\alpha_2 - \alpha_1) - 2\sqrt{c_w}(\sqrt{\alpha_2} - \sqrt{\alpha_1}) + \lambda(2\alpha_1 + \alpha_2)}{3} + c_s e^{-rl},$$

$$\pi_1^* = {\lambda_1^*}^2(\alpha_1 + \alpha_2),$$

$$\pi_2^* = {\lambda_2^*}^2(\alpha_1 + \alpha_2).$$

From Proposition 5, we deduce the following results.

Corollary 5

(i) *The optimal price p_1^* and customer arrival rate of service provider 1 are increasing in μ_0 and decreasing in c_w. Moreover, p_2^* and λ_2^* are decreasing in μ_0 and increasing in c_w.*

(ii) *For $i = 1$ and 2, the optimal price p_i^* of service provider i increases in its own technical level α_i and decreases in the technical level α_{3-i} of service provider $3 - i$ if $\lambda < (\mu_0 - \sqrt{c_w/\alpha_{3-i}})/2$; otherwise, increases in α_i.*

(iii) *The equilibrium arrival rate λ_1^* (λ_2^*) decreases (increases) in α_2. If $\lambda > 3\lambda_1^* + \sqrt{c_w/\alpha_1} - \mu_0$, then λ_1^* increases in α_1, and λ_2^* and μ_2^* decrease in α_1; otherwise, λ_1^* decreases in α_1, and λ_2^* and μ_2^* increase in α_1. Specifically, if $\alpha_1 = \alpha_2$, λ_i^* increases in α_i and decreases in α_{3-i}, $i = 1$ and 2.*

(iv) *If $\lambda > \frac{(5\alpha_1+3\alpha_2)}{2\alpha_1}\sqrt{\frac{c_w}{\alpha_1}} + 3\lambda_1^* - \mu_0$, then the optimal service rate of service provider i, i.e., μ_i^*, increases in α_i; otherwise, decreases in α_i, $i = 1$ and 2.*

In this case, the increase of μ_0 and c_w will have different changes according to the technical levels of the two service providers. Specifically, since we specify $\alpha_1 > \alpha_2$, we have the arrival rate, price, and profit of service provider 1 increase in μ_0 and decrease in c_w. The former will strengthen service provider 1's advantage in terms of its technical level, thus encroaching more on service provider 2's profit. And if c_w raises, the customers are less willing to wait in the queue, service provider 1 will suffer more losses due to its higher customer arrival rate. On the contrary, service provider 2 can acquire customers who are not willing to wait for a long time at service provider 1. Thus, he could achieve an increase in customer arrival rate and profit. From this, we find that, to some extent, the deterioration of the market may be an opportunity for the weaker party in the competition, as he may be able to gain profit from it.

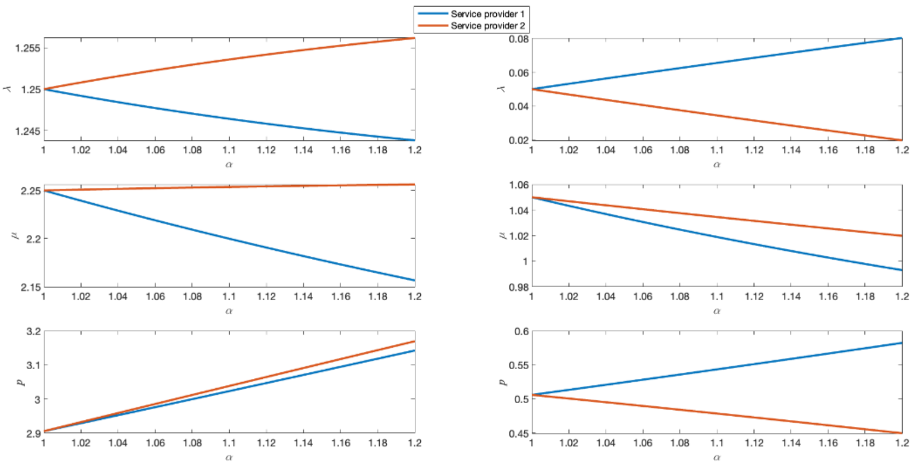


Figure 3-3 Effects of α_1 on the service provider's decisions.

The relationships between the decision variables of the service providers and technical levels are pretty complicated, and we analyze here one by one in Figure 3-3. It is quite straightforward that service provider i's optimal price p_i^* increases in its technical level α_i, like the blue lines in the two bottom panels of Figure 3-3. And when λ is small i.e., $\lambda < (\mu_0 - \sqrt{c_w/\alpha_{3-i}})/2$, the competition is extremely intensified, the raise of opponent's technical level α_{3-i} will further increase the competition intensity, which will cause p_i^* to drop (the red line in the bottom left panel). However, if $\lambda > (\mu_0 - \sqrt{c_w/\alpha_{3-i}})/2$, as the red line in the bottom left panel, the service provider 2's price p_2^* can benefit from α_1 since its opponent's price p_1 also increases with α_1.

Noting that $\alpha_1 > \alpha_2$ in our setting, we have $\lambda_1^* > \lambda/2 > \lambda_2^*$ (the proof is given in the Appendix). If α_2 increases, service provider 2's arrival rate will increase since customers can gain more value from processing. Thus, service provider 1's customer arrival rate and service rate will decrease in α_2 since $\lambda_1 + \lambda_2 = \lambda$. For α_1, $\lambda < 3\lambda_1^* + \sqrt{c_w/\alpha_1} - \mu_0$ represents that service provider 1 occupying the vast majority of the market at this time, and if α_1 increases, service provider 1 can gain more profit from price hike and decreasing the customer arrival rate (the blue line in the top left panel). And service provider 2's optimal customer arrival rate and service rate increase in α_1 as the red lines in the top left and middle left panels in Figure 3. However, if $\lambda > 3\lambda_1^* + \sqrt{c_w/\alpha_1} - \mu_0$, the changes in λ_1^* and λ_2^* are shown in the top right panel. Similarly, if $\lambda < \frac{(5\alpha_1+3\alpha_2)}{2\alpha_1}\sqrt{\frac{c_w}{\alpha_1}} + 3\lambda_1^* - \mu_0$, then μ_1^* will decreases in α_1; otherwise, increases in α_1.

3.5 Extension to Oligopoly Service Providers

In the above, we study the case of two subcontracting firms. Now we extend our model to n subcontracting firms and allow them to have various technical levels α_i. In the partial market coverage case, the decisions of the service providers are the same as those in Proposition 3. In the full coverage case ($\lambda < \sum_{k=1}^{n} \lambda_k^*$), the service providers compete with one another and seek to maximize their own profits. Similarly, the equilibrium customer utility from each service provider is identical $U_1 = U_2 = \cdots = U_n$.

Solving the model, we derive the following result.

Proposition 6:

For $k = 1, \ldots, n$, the optimal strategy and profit of service provider k are

$$\mu_k^* = \lambda_k^* + \sqrt{\frac{c_w}{\alpha_k}}, p_k^* = \lambda_k^* \alpha_k \left(1 + \frac{1}{\alpha_k A - 1}\right) + c_s e^{-rl},$$

$$\pi_k^* = {\lambda_k^*}^2 \alpha_k \left(1 + \frac{1}{\alpha_k A - 1}\right).$$

The corresponding equilibrium customer arrival rate λ_k^* is

$$\lambda_k^* = \frac{\lambda + \sum_{i=1}^{n} \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}}{A_k \sum_{i=1}^{n} \frac{1}{A_i}},$$

where $A = \sum_{j=1}^{n} \frac{1}{\alpha_j}; A_i = \alpha_i \left(2 + \frac{1}{\alpha_i A - 1}\right) = \left(2\alpha_i + \frac{1}{A - 1/\alpha_i}\right).$

We find that even in the n service providers case, the waiting time is still $\sqrt{\alpha_i/c_w}$, which is only related to customer's waiting cost and service provider's technical level, increasing in the former and decreasing in the latter. This finding will help service providers adjust their service rates more easily and quicker based on the observed changes in the customer waiting cost and service providers' own technical levels. Specifically, if the service provider's technical level grows over time or if the customer's waiting cost decreases, then the optimal service rate chosen by the service provider increases based on the observed customer arrival rate; otherwise, the optimal service rate chosen by the service provider decreases.

We find that the results in Corollary 5 are robust and hold when extending our model to n service providers. For example, same as Corollary 5(i), increases in μ_0 and c_w have different impacts on the service providers depending on their technical levels. If one's technical level α_k is relatively high compared with the others ($\sum_{i=1}^{n}\frac{\alpha_k-\alpha_i}{A_i}>0$), then μ_0's raise will benefit its customer arrival rate and profit. However, the high technical level service provider ($\sum_{i=1}^{n}\frac{\sqrt{\alpha_k}-\sqrt{\alpha_i}}{A_i\sqrt{c_w}}>0$) will suffer more loss when the customer waiting cost increases, and the service provider with a relatively low α_k can capture the customers lost from the high technology level service providers and gain more profit. In addition, we find that the service providers' prices, customer arrival rates, and profits all increase with the potential customer quantity λ.

Moreover, the impact of α_k is more complex: when $H_n(\alpha_k)\left(\sum_{i=1}^{n}\frac{A_k}{A_i}\right)^2 = B_{\mu_0}\mu_0 + \sum_{i=1,i\neq k}^{n} B_{c_w}\left(\sqrt{c_w}(\sqrt{\alpha_k}-\sqrt{\alpha_i})\right) - B_\lambda\lambda - B_{\alpha_k}\frac{\sqrt{c_w}}{\sqrt{\alpha_k}} > 0$, service provider k's customer arrival rate increases in its technical level α_k; otherwise, decreases; when the number of service providers $n=2$, $H_n(\alpha_k)\left(\sum_{i=1}^{n}\frac{A_k}{A_i}\right)^2 = H_2(\alpha_1)3(\alpha_1+\alpha_2)^2 = 2\alpha_2\mu_0 + \sqrt{c_w}(\sqrt{\alpha_1} - 2\sqrt{\alpha_2}) - \alpha_2\lambda - \alpha_2\sqrt{\frac{c_w}{\alpha_1}} > 0$, which further illustrates the robustness of our model results when the number of service providers increases to n.

3.6 Conclusions

We study the competitive strategies of customer-intensive service providers in several scenarios with imperfect online monitoring. Specifically, we first study the optimal strategies of 2 homogeneous service providers in the partial and full market coverage cases, respectively. And then allow service providers to have different technical levels to study the impact of technical levels on decision making and finally expand our model to n heterogeneous service providers in Section 3.5. We get the following findings:

From Proposition 1, we find that when there are enough potential customers in the market ($\lambda \geq 2\lambda^*$), the customers will choose to join the service as long as their utility is greater than 0. Thus, there is no competition between the two service providers at this case. However, Proposition 3 indicates that when the market only has a small number of potential customers ($\lambda < 2\lambda^*$), the two service providers compete for the limited customers, and Corollary 4 (iv) reveals that customer utility and the intensity increase in the parameters, which can enhance the service providers' service quality and decreases in the number of potential customers. And in the short-run model, we find that service providers' profit is always increasing in online monitoring quality l since it can help improve the monitoring accuracy, while Proposition 2 shows that in the long-run model, the company cannot infinitely improve the quality of monitoring because of the continuous growth of its cost.

In Section 3.4 we allow the service providers to have different technical levels, the optimal strategies of service providers are almost the same as Proposition 1 in the partial market coverage case. However, in the case of full market coverage, the level of technology will have an impact on the competitor's profit. Corollary 5(i) finds that when one provider has the advantage of technical level, he can gain profit from better market conditions (μ_0 increases), or suffer greater losses because of the worse market conditions. The relatively weaker side, on the other hand, has the potential to reap profits when market conditions deteriorate.

Finally, we extend the model to n heterogeneous service providers in Section 3.5 and find that, regardless of the case (the number of potential customers in the market /the number of service providers), the differences between the optimal service rates of the service providers and the equilibrium customer arrival rates are always $\sqrt{\alpha_i/c_w}$, this suggests that the customer waiting time is only related to the service provider's service level and the customer waiting cost, and this pattern can help service providers adjust their strategies more quickly.

Section 4
Conclusions and Future Research

In this thesis, we study two selected topics in the marketing strategy of durable goods providers from the perspectives of depreciation and maintenance. In Section 2, we divide the depreciation of durable goods into consumption depreciation and physical depreciation and construct a series of two-period game-theoretic models of monopoly manufacturer to compare various strategies. We find that pure leasing strategy is more profitable than pure selling strategy since it can divide the use right of the product and at the same time avoid second hands competition. Moreover, we consider two hybrid strategies, the selling-leasing hybrid strategy and selling-reselling hybrid strategy and find that the two strategies cannot achieve better results than the pure strategies. And we make some extensions in Section 2.4 such as competing manufacturer, positive network effect, and n period model, the results of the above verify the robustness of our model.

In Section 3, we focus on the maintenance strategies for customer-intensive service providers under imperfect online monitoring. Maintenance has been receiving increasing attention in the manufacturing world, and the development of related technologies has brought about an explosive increase in the performance of equipment maintenance. To keep pace with the advanced technologies, we develop analytical and mathematical models for optimizing operational strategies of competing service providers under imperfect online monitoring. We find that when there are enough potential customers in the market, no competition exists between the two service providers at this case. However, when the market only has a small number of potential customers, the two service providers will compete for the limited customers. And we reveal that customer utility and the competition intensity increase in the parameters, which can enhance the service providers' service quality and decreases in the number of potential customers. Moreover, we allow the service providers to have different technical levels in Section 3.4 and get that the weaker party in the competition has the potential to reap profits from the deterioration of the market when the stronger party suffers a loss. Finally, we summarize a general pattern of the equilibrium consumer arrival rate in response to the optimal service rate, the differences between the optimal service rates of the service providers and the equilibrium customer arrival rates are always $\sqrt{\alpha_i/c_w}$.

There are still several issues that can be further studied in our model. For example, in Section 2, an important assumption is that the consumption depreciation is homogeneous among the consumers. Therefore, in future studies, we may relax this assumption and study the case of heterogeneous consumption depreciation. And in Section 3, all the customers are homogeneous in the model. However, in reality, customers' service valuation and time cost are heterogeneous, so this can be our future research direction. In addition, service providers' service rates and prices can be changed, and in our subsequent research, we can also consider constructing a multi-period model, in which service providers are free to change their strategies in each period. In addition, the quality of online monitoring can be different for different companies, and in this case, the decision of company monitoring quality also needs to take into account the quality levels and strategies of the rival companies.

Appendix

Proposition 1:

We first use the objective function to find the second derivatives of μ and p, respectively, and obtain

$$\frac{\partial^2\pi}{\partial\mu^2} = -\frac{2\alpha^2 c_w\big(p - c_s(1-\theta(l)\big)}{(c_t\theta(l)+Q_0+\alpha(\mu_0-\mu)-p)^3} = -\frac{2\alpha^2 c_w\big(p - c_s(1-\theta(l)\big)}{(F(l)+Q(\mu)-p)^3} < 0$$

$$\frac{\partial^2\pi}{\partial p^2} = -\frac{2c_w(Q(\mu)-p+F(l)+p-e^{-\gamma l}c_s)}{\big(Q(\mu)-p+F(l)\big)^3} < 0$$

We have $p - c_s(1-\theta(l)) > 0$ since the selling price must greater than error cost. And note that $c_t\theta(l)+Q_0+\alpha(\mu_0-\mu)-p = F(l)+Q(\mu)-p > U_i(l,\mu,p) \geq 0$. Thus, we get that profit π concave in service rate μ and price p.

Since the service provider decides on its service rate at first, and then determines the price according to the service rate to maximize its own profit, we solve the price stage $p^*(\mu)$ first following the reverse induction method.

$$p^*(\mu) = c_t\theta(l) + Q(\mu) + \sqrt{\frac{c_w\big(Q(\mu)-c_s+(c_s+c_t)\theta(l)\big)}{\mu}}$$

And then we turn to the service rate stage, using $\mu^*(p^*)$, and here is the result:

$$\mu^* = \frac{Q_0+\alpha\mu_0+c_t-e^{-\gamma l}(c_s+c_t)}{2\alpha}$$

$$p^* = \frac{Q_0+\alpha\mu_0+c_t+e^{-\gamma l}(c_s-c_t)}{2} - \sqrt{\alpha c_w}$$

$$\lambda^* = \frac{Q_0+\alpha\mu_0+c_t-e^{-\gamma l}(c_s+c_t)}{2\alpha} - \sqrt{\frac{c_w}{a}}$$

$$\pi^* = [p^* - c_s(1-\theta(l))]\lambda^* = \frac{\big(Q_0+\alpha\mu_0+c_t-e^{-\gamma l}(c_s+c_t)-2\sqrt{\alpha c_w}\big)^2}{4\alpha}$$

Corollary 1

We make the sensitivity analysis by taking partial derivatives of the optimal μ^*, p^*, λ^* and π^* with respect to parameters.

$$\frac{\partial\mu^*}{\partial\mu_0}, \frac{\partial\lambda^*}{\partial\mu_0}, \frac{\partial p^*}{\partial\mu_0}, \frac{\partial\pi^*}{\partial\mu_0} > 0$$

$$\frac{\partial\mu^*}{\partial Q_0}, \frac{\partial\lambda^*}{\partial Q_0}, \frac{\partial p^*}{\partial Q_0}, \frac{\partial\pi^*}{\partial Q_0} > 0$$

$$\frac{\partial\mu^*}{\partial c_t}=\frac{\theta(l)}{2\alpha}>0\ \frac{\partial\mu^*}{\partial c_s}=\frac{-e^{-\gamma l}}{2\alpha}<0\ \frac{\partial\mu^*}{\partial c_w}=0$$

$$\frac{\partial\lambda^*}{\partial c_t}=\frac{\theta(l)}{2\alpha}>0\ \frac{\partial\lambda^*}{\partial c_s}=\frac{-e^{-\gamma l}}{2\alpha}<0\ \frac{\partial\lambda^*}{\partial c_w}=-\frac{1}{2\sqrt{\alpha c_w}}<0$$

$$\frac{\partial p^*}{\partial c_t}=\frac{\theta(l)}{2\alpha}>0\ \frac{\partial p^*}{\partial c_s}=\frac{e^{-\gamma l}}{2\alpha}>0\ \frac{\partial p^*}{\partial c_w}=-\frac{\alpha}{2\sqrt{c_w}}<0$$

$$\pi^*=[p^*-c_s(1-\theta(l))]\lambda^*=\frac{\left(Q_0+\alpha\mu_0+c_t-e^{-\gamma l}(c_s+c_t)-2\sqrt{\alpha c_w}\right)^2}{4\alpha}$$

$$\frac{\partial\pi^*}{\partial c_s}=-2[p^*-c_s(1-\theta(l))]\lambda^* e^{-\gamma l}<0$$

Corollary 2

Same as the corollary 1, we get

$$\frac{\partial\mu^*}{\partial l}=\frac{\partial\lambda^*}{\partial l}=\frac{e^{-\gamma l}(c_s+c_t)}{2\alpha}>0$$

$$\frac{\partial p^*}{\partial l}=\frac{e^{-\gamma l}(c_t-c_s)}{2}$$

$$\frac{\partial\pi^*}{\partial l}=\frac{\left(Q_0+\alpha\mu_0+c_t-e^{-\gamma l}(c_s+c_t)-2\sqrt{\alpha c_w}\right)\gamma e^{-\gamma l}(c_s+c_t)}{2\alpha}>0$$

Corollary 3

Same as the corollary 1, we get

$$\frac{\partial\mu^*}{\partial\alpha}=-\frac{Q_0+c_t-e^{-\gamma l}(c_s+c_t)}{2\alpha^2}=-\frac{Q_0+(c_s+c_t)\theta(l)-c_s}{2\alpha^2}=\frac{c_s\left(1-\theta(l)\right)-Q_0-F(l)}{2\alpha^2}$$

$$\frac{\partial p^*}{\partial\alpha}=\frac{\mu_0}{2}-\frac{\sqrt{\alpha c_w}}{2\alpha}=\frac{\sqrt{\alpha}\mu_0-c_w}{2\sqrt{\alpha}}$$

$$\frac{\partial\lambda^*}{\partial\alpha}=-\frac{Q_0+c_t-e^{-\gamma l}(c_s+c_t)}{2\alpha^2}+\frac{\sqrt{\alpha c_w}}{2\alpha^2}=\frac{\sqrt{\alpha c_w}+c_s\left(1-\theta(l)\right)-Q_0-F(l)}{2\alpha^2}$$

Proposition 2:

We solve the long-run model here, we use objective function to find the derivatives of l at first.

$$\frac{\partial\pi^*}{\partial l}=\frac{\left(Q_0+\alpha\mu_0+c_t-e^{-\gamma l}(c_s+c_t)-2\sqrt{\alpha c_w}\right)\gamma e^{-\gamma l}(c_s+c_t)}{2\alpha}-2kl$$

$$\frac{\partial^2\pi^*}{\partial l^2} = -2k - \frac{\gamma^2 e^{-l\gamma}(c_s + c_t)\left(Q_0 + \alpha\mu_0 + c_t - 2e^{-l\gamma}(c_s + c_t) - 2\sqrt{\alpha c_w}\right)}{2\alpha}$$

$$\frac{\partial^3\pi^*}{\partial l^3} = \gamma^3 e^{-l\gamma}(c_s + c_t)\frac{\left(Q_0 + \alpha\mu_0 + c_t - 4e^{-l\gamma}(c_s + c_t) - 2\sqrt{\alpha c_w}\right)}{2\alpha}$$

To simplify, we make $H_1(l) = Q_0 + \alpha\mu_0 + c_t - 4e^{-l\gamma}(c_s + c_t) - 2\sqrt{\alpha c_w}$. Since $p > 0$, we have $Q_0 + \alpha\mu_0 + c_t - e^{-\gamma l}(c_s + c_t) - 2\sqrt{\alpha c_w} > 0$. Thus, $\lim_{l\to\infty} H_1(l) = Q_0 + \alpha\mu_0 + c_t - 2\sqrt{\alpha c_w} > p > 0$.

If $H_1(0) = Q_0 + \alpha\mu_0 + c_t - 4(c_s + c_t) - 2\sqrt{\alpha c_w} > 0$, since $\partial H_1(l)/\partial l > 0$, we will have $\partial^3\pi^*/\partial l^3 > 0$. Thus, $\partial^2\pi^*/\partial l^2$ increases in l and $\lim_{l\to\infty}\partial^2\pi^*/\partial l^2 = -2k < 0$. Thus $\pi(l)$ is a concave function with a global maximize point $l^* = \{l|\partial\pi^*/\partial l = 0\}$.

However, if $H_1(0) = Q_0 + \alpha\mu_0 + c_t - 4(c_s + c_t) - 2\sqrt{\alpha c_w} < 0$, which means $\partial^2\pi^*/\partial l^2$ first decreases and then increases in l. Similarly, we have $\lim_{l\to\infty}\partial^2\pi^*/\partial l^2 = -2k < 0$. Thus, when $l = 0$, if $\partial^2\pi^*/\partial l^2 = -2k - \gamma^2(c_s + c_t)\left(Q_0 + \alpha\mu_0 + c_t - 2(c_s + c_t) - 2\sqrt{\alpha c_w}\right)/2\alpha \leq 0$, $\pi(l)$ is also a concave function with the global maximize point l^*. Otherwise, $\partial\pi^*/\partial l$ first increases and then decreases in l and notice that $\lim_{l\to\infty}\partial\pi^*/\partial l = -2kl < 0$ and when $l = 0$, $\partial\pi^*/\partial l = p\gamma e^{-\gamma l}(c_s + c_t)/2\alpha > 0$. Therefore, function $\pi(l)$ has a local maximize point $l^* = \{l|\partial\pi^*/\partial l = 0\}$.

After all, if $\pi(l^*) > \left(Q_0 + \alpha\mu_0 - 2\sqrt{\alpha c_w}\right)^2/4\alpha$, i.e., the optimal profit of service provider with online monitoring is greater than the profit without it, the optimal online monitor quality $l = l^*$, otherwise, $l = 0$.

Proposition 3:

Since we have $\lambda^{**} = \lambda/2$. And the consumer utility is

$$U_i\left(l,\mu,P,\frac{\lambda}{2}\right) = F(l_i) + Q(\mu_i) - \frac{c_w}{\mu_i - \Lambda/2} - P_i,$$

if $\mu > \lambda/2$, $U_i(l,\mu,P,\lambda/2)$ is concave in μ_i since

$$\frac{\partial^2 U_i\left(l,\mu,P,\frac{\lambda}{2}\right)}{\partial\mu_i^2} = \frac{2c_w}{\left(\frac{\Lambda}{2} - \mu\right)^3} < 0.$$

Let the first order derivative condition equals to 0,

$$\frac{\partial U_i\left(l,\mu,P,\frac{\lambda}{2}\right)}{\partial \mu_i} = \frac{c_w}{\left(\frac{\lambda}{2}-\mu\right)^2} - \alpha = 0,$$

we have

$$\mu^{**} = \frac{\lambda}{2} + \sqrt{\frac{c_w}{\alpha}}.$$

Since the limit of the market size, service provider can't obtain sufficient consumer when chooses the global optimal service rate i.e., if $\lambda^{**} < \lambda^e$, the service provider will decrease its service rate to obtain higher profit, thus, we will have $\mu^{**} < \mu^*$.

Now we focus on the price, notice that $\pi = [p - c_s(1-\theta(l))]\lambda$ infers that the service providers can set a price as high as possible if they can cooperate with each other, then the selling price $p = [Q_0 + \alpha\mu_0 + c_t - (c_t + c_s)e^{-\gamma l} - \frac{\alpha\lambda}{2} - 2\sqrt{\alpha c_w}$, which is the conclusion of Anand et al. (2011). However, service providers will deviate from the equilibrium in normal case, for example, we assume that the service provider 1 will deviate the equilibrium and set price p_1, and the corresponding arrival rates are $\lambda_1, 1-\lambda_1$. Differentiate both sides of $U_1 = U_2$ with respect to p_1, we have

$$\frac{\partial \lambda_1}{\partial p_1} = -\frac{1}{\alpha_1 + \alpha_2} < 0$$

$$\frac{\partial \pi_1}{\partial p_1} = \lambda_1 - \frac{p_1 - c_s e^{-rl}}{\alpha_1 + \alpha_2}$$

We get a local optimum $p^{**} = \alpha\lambda^{**} + c_s e^{-rl}$

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{\partial \lambda_1}{\partial p_1} - \frac{1}{\alpha_1 + \alpha_2} < 0$$

Thus $p^{**} = \alpha\lambda^{**} + c_s e^{-rl}$ is the optimal price service provider will choose, and the equilibrium consumer utility is,

$$U^{**} = F(l) + Q(\mu) - \frac{c_w}{\mu - \lambda} - p = c_t + Q_0 + \alpha\mu_0 - \frac{3}{2}\alpha\lambda - 2\sqrt{\alpha c_w} - (c_t + c_s)e^{-rl}$$

$$\pi^{**} = \frac{\alpha\lambda^2}{2}$$

Overall,

$$\lambda^{**} = \frac{\lambda}{2}$$

$$\mu^{**} = \frac{\lambda}{2} + \sqrt{\frac{c_w}{\alpha}} = \lambda^{**} + \sqrt{\frac{c_w}{\alpha}}$$

$$p^{**} = \frac{\alpha\lambda}{2} + c_s e^{-rl} = \alpha\lambda^{**} + c_s e^{-rl}$$

$$\pi^{**} = \frac{\alpha\lambda^2}{2}$$

$$U^{**} = c_t + Q_0 + \alpha\mu_0 - \frac{3}{2}\alpha\lambda - 2\sqrt{\alpha c_w} - (c_t + c_s)e^{-rl}$$

Corollary 4

Similarly, we use the optimal strategy to find the partial derivatives of each parameter

(i)

$$\frac{\partial\mu^{**}}{\partial\lambda} = \frac{\partial\lambda^{**}}{\partial\lambda} = \frac{\partial p^{**}}{\alpha\partial\lambda} = \frac{\partial\pi^{**}}{2\alpha\partial\lambda} = \frac{1}{2} > 0$$

(ii)

$$\frac{\partial\mu^{**}}{\partial c_w} = \frac{1}{2\sqrt{\alpha c_w}} > 0; \frac{\partial\mu^{**}}{\partial\alpha} = -\frac{\sqrt{c_w}}{2\alpha^{\frac{3}{2}}} < 0$$

$$\frac{\partial\pi^{**}}{\partial\alpha} = \lambda\frac{\partial p^{**}}{\partial\alpha} = \lambda > 0$$

(iii)

$$\frac{\partial p^{**}}{\partial l} = -c_s\gamma e^{-\gamma l} < 0;$$

$$\frac{\partial p^{**}}{\partial c_s} = e^{-\gamma l} > 0$$

(iv)

$$\frac{\partial U^{**}}{\partial c_t} = 1 - e^{-rl} > 0; \frac{\partial U^{**}}{\partial Q_0} = 1 > 0, \frac{\partial U^{**}}{\partial\mu_0} = \alpha > 0$$

$$\frac{\partial U^{**}}{\gamma\partial l} = \frac{\partial U^{**}}{l\partial\gamma} = (c_t + c_s)e^{-rl} > 0$$

$$\frac{\partial U^{**}}{\partial c_w} = -\sqrt{\frac{\alpha}{c_w}} < 0, \frac{\partial U^{**}}{\partial c_s} = -e^{-rl} < 0$$

$$\frac{\partial U}{\partial\alpha} = \mu_0 - \frac{3}{2}\lambda - \sqrt{\frac{c_w}{\alpha}}$$

$\frac{\partial U}{\partial\alpha} > 0$ if $\mu_0 > \frac{3}{2}\lambda + \sqrt{\frac{c_w}{\alpha}}$, otherwise, $\frac{\partial U}{\partial\alpha} < 0$.

Proposition 5:

Note that

$$U_i(l,\mu,P) = F(l) + Q(\mu_i) - \frac{c_w}{\mu_i - \lambda_i(l,\mu_i,p_i)} - p_i$$

Assume $t_i = F(l_i) + Q(\mu_i) - P_i$, then

$$t_1 - \frac{c_w}{\mu_1 - \lambda_1} = t_2 - \frac{c_w}{\mu_2 - \lambda_2}$$

$$t_1 - t_2 = \frac{c_w}{\mu_1 - \lambda_1} - \frac{c_w}{\mu_2 - \lambda_2} = \frac{c_w}{\mu_1 - \lambda_1} - \frac{c_w}{\mu_2 - \lambda + \lambda_1} \qquad (A1)$$

Differentiate both sides of Equation (A1) with respect to μ_1

$$\frac{c_w}{\mu_1 - \lambda_1} - \frac{c_w}{\mu_2 - \lambda + \lambda_1}$$

$$-\alpha_1 = -\frac{c_w}{(\mu_1 - \lambda_1)^2}\left(1 - \frac{\partial\lambda_1}{\partial\mu_1}\right) + \frac{c_w}{(\mu_2 - \lambda + \lambda_1)^2}\frac{\partial\lambda_1}{\partial\mu_1}$$

$$\frac{c_w}{(\mu_1 - \lambda_1)^2} - \alpha_1 = \left[\frac{c_w}{(\mu_2 - \lambda + \lambda_1)^2} + \frac{c_w}{(\mu_1 - \lambda_1)^2}\right]^{-1}\frac{\partial\lambda_1}{\partial\mu_1}$$

$$\frac{\partial\lambda_1}{\partial\mu_1} = \left[\frac{1}{(\mu_1 - \lambda_1)^2} - \frac{\alpha_1}{c_w}\right]\left[\frac{1}{(\mu_2 - \lambda + \lambda_1)^2} + \frac{1}{(\mu_1 - \lambda_1)^2}\right]^{-1}$$

Find the point of first derivative equals to 0

$$\mu_1 = \lambda_1 + \sqrt{c_w/\alpha_1}$$

Differentiate both sides of Equation (A1) with respect to p_1

$$\frac{\partial\lambda_1}{\partial p_1} = -\frac{1}{c_w}\left[\frac{1}{(\mu_2 - \lambda_2)^2} + \frac{1}{(\mu_1 - \lambda_1)^2}\right]^{-1} < 0$$

Similarly, find the point of first derivative equals to 0

$$\pi_1 = \lambda_1(p_1 - c_s e^{-rl})$$

Differentiate both sides with respect to μ_1, p_1

$$\frac{\partial\pi_1}{\partial\mu_1} = \frac{\partial\lambda_1}{\partial\mu_1}(p_1 - c_s e^{-rl})$$

$$\frac{\partial\pi_1}{\partial p_1} = \frac{\partial\lambda_1}{\partial p_1}(p_1 - c_s e^{-rl}) + \lambda_1 = \lambda_1 - \frac{1}{c_w}\left[\frac{1}{(\mu_2 - \lambda_2)^2} + \frac{1}{(\mu_1 - \lambda_1)^2}\right]^{-1}(p_1 - c_s e^{-rl})$$

And we get the point of first derivative equals to 0 is

$$\mu_1 = \lambda_1 + \sqrt{c_w/\alpha_1}$$

$$p_1 = c_w\lambda_1\left[\frac{1}{(\mu_2 - \lambda_2)^2} + \frac{1}{(\mu_1 - \lambda_1)^2}\right] + c_s e^{-rl} = \lambda_1(\alpha_1 + \alpha_2) + c_s e^{-rl}$$

Similarly, we get

$$\mu_2 = \lambda_2 + \sqrt{c_w/\alpha_2}$$

$$p_2 = \lambda_2(\alpha_1 + \alpha_2) + c_s e^{-rl}$$

Find the possible equilibrium point μ_i^*, p_i^* and λ_i^*

$$U_i = F(l) + Q(\mu_i) - \frac{c_w}{\mu_i - \lambda_i(l, \mu_i, p_i)} - p_i$$

$$U_1 = F(l) + Q_0 + \alpha_1\mu_0 - 2\sqrt{\alpha_2 c_w} - c_S e^{-rl} - \lambda_1(2\alpha_1 + \alpha_2)$$

$$U_2 = F(l) + Q_0 + \alpha_2\mu_0 - 2\sqrt{\alpha_1 c_w} - c_S e^{-rl} - \lambda_2(2\alpha_2 + \alpha_1)$$

Since $U_1 = U_2$

$$\alpha_1\left(\mu_0 - \lambda_1 - \sqrt{\frac{c_w}{\alpha_1}}\right) - \sqrt{\alpha_1 c_w} - \lambda_1(\alpha_1 + \alpha_2)$$

$$= \alpha_2\left(\mu_0 - \lambda_2 - \sqrt{\frac{c_w}{\alpha_2}}\right) - \sqrt{\alpha_2 c_w} - \lambda_2(\alpha_1 + \alpha_2)$$

$$\alpha_1\mu_0 - 2\sqrt{\alpha_1 c_w} - \lambda_1(2\alpha_1 + \alpha_2) = \alpha_2\mu_0 - 2\sqrt{\alpha_2 c_w} - (\lambda - \lambda_1)(\alpha_1 + 2\alpha_2)$$

$$3\lambda_1(\alpha_1 + \alpha_2) = \mu_0(\alpha_1 - \alpha_2) - 2\sqrt{c_w}\left(\sqrt{\alpha_1} - \sqrt{\alpha_2}\right) - \lambda(\alpha_1 + 2\alpha_2)$$

we get

$$\lambda_1^* = \frac{\mu_0(\alpha_1 - \alpha_2) - 2\sqrt{c_w}(\sqrt{\alpha_1} - \sqrt{\alpha_2}) - \lambda(\alpha_1 + 2\alpha_2)}{3(\alpha_1 + \alpha_2)}$$

$$\lambda_2^* = \frac{\mu_0(\alpha_2 - \alpha_1) - 2\sqrt{c_w}(\sqrt{\alpha_2} - \sqrt{\alpha_1}) - \lambda(2\alpha_1 + \alpha_2)}{3(\alpha_1 + \alpha_2)}$$

Now, we prove that the possible equilibrium point μ_i^*, p_i^* is the maximum strategy, we focus on the second derivative of μ_1^* at first.

$$\frac{\partial^2\lambda_1}{\partial\mu_1^2} = -\frac{2}{(\mu_1 - \lambda_1)^3}\left(1 - \frac{\partial\lambda_1}{\partial\mu_1}\right)\left[\frac{1}{(\mu_2 - \lambda + \lambda_1)^2} + \frac{1}{(\mu_1 - \lambda_1)^2}\right]^{-1}$$

$$\frac{\partial^2\lambda_1^*}{\partial{\mu_1^*}^2} = -\frac{2c_w}{\alpha_1 + \alpha_2}\left(\frac{\alpha_1}{c_w}\right)^{\frac{3}{2}} < 0$$

And

$$H_{11} = \frac{\partial^2\pi_1^*}{\partial{\mu_1^*}^2} = \frac{\partial^2\lambda_1^*}{\partial{\mu_1^*}^2}(p_1 - c_s e^{-rl}) < 0$$

Then, we calculate the second derivative of p_1^*

$$\frac{\partial^2\lambda_1}{\partial p_1^2} = \frac{2}{c_w}\left[\frac{1}{(\mu_1 - \lambda_1)^2} + \frac{1}{(\mu_2 - \lambda_2)^2}\right]^{-2}\left[\frac{1}{(\mu_1 - \lambda_1)^3} - \frac{1}{(\mu_2 - \lambda_2)^3}\right]\frac{\partial\lambda_1}{\partial p_1}$$

$$\frac{\partial^2\lambda_1^*}{\partial{p_1^*}^2} = \frac{2\left(\alpha_1^{\frac{3}{2}} - \alpha_2^{\frac{3}{2}}\right)}{\sqrt{c_w}(\alpha_1 + \alpha_2)^2}\frac{\partial\lambda_1^*}{\partial p_1^*}$$

$$H_{22} = \frac{\partial^2 \pi_1^*}{\partial p_1^{*2}} = \frac{\partial \lambda_1^*}{\partial p_1^*}\left[2 + \frac{2\left(\alpha_1^{\frac{3}{2}} - \alpha_2^{\frac{3}{2}}\right)}{\sqrt{c_w}(\alpha_1 + \alpha_2)^2}(p_1^* - c_s e^{-rl})\right] = \frac{\partial^2 \pi_1^*}{\partial p_1^{*2}}$$

$$= \frac{\partial \lambda_1^*}{\partial p_1^*}\left[2 + \frac{2\left(\alpha_1^{\frac{3}{2}} - \alpha_2^{\frac{3}{2}}\right)\lambda_1^*}{\sqrt{c_w}(\alpha_1 + \alpha_2)}\right]$$

Next, calculate the second partial derivatives of π_1^* with respect to p_1^* and μ_1^*

$$\frac{\partial^2 \lambda_1}{\partial \mu_1 \partial p_1} = -\frac{2}{(\mu_1 - \lambda_1)^3}\left(-\frac{\partial \lambda_1}{\partial p_1}\right)\left[\frac{1}{(\mu_2 - \lambda + \lambda_1)^2} + \frac{1}{(\mu_1 - \lambda_1)^2}\right]^{-1}$$

$$\frac{\partial^2 \lambda_1^*}{\partial \mu_1^* \partial p_1^*} = \frac{-2\alpha_1^{\frac{3}{2}}}{\sqrt{c_w}(\alpha_1 + \alpha_2)^2}$$

$$H_{12} = H_{21} = \frac{\partial^2 \pi_1^*}{\partial \mu_1^* \partial p_1^*} = \frac{\partial^2 \lambda_1^*}{\partial \mu_1^* \partial p_1^*}(p_1^* - c_s e^{-rl}) = \frac{-2\lambda^* \alpha_1^{\frac{3}{2}}}{\sqrt{c_w}(\alpha_1 + \alpha_2)}$$

And we get the Hessian matrix of (μ_1, p_1^*) is

$$H_1 = \begin{vmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{vmatrix}$$

Note that we have proved that $H_{11} < 0$, then we need to prove $H_{11}H_{22} - H_{12}H_{21} > 0$

$$H_{11}H_{22} - H_{12}H_{21} = \frac{4\alpha_1^{\frac{3}{2}}\lambda_1^*[\sqrt{c_w}(\alpha_1 + \alpha_2) - \alpha_2^{\frac{3}{2}}\lambda_1^*]}{c_w(\alpha_1 + \alpha_2)^2}$$

Therefore, if $\sqrt{c_w}(\alpha_1 + \alpha_2) - \alpha_2^{\frac{3}{2}}\lambda_i^* > 0$,i.e.,

$$\lambda_1^* < \frac{\sqrt{c_w}(\alpha_1 + \alpha_2)}{\alpha_2^{\frac{3}{2}}}$$

$$\lambda_2^* < \frac{\sqrt{c_w}(\alpha_1 + \alpha_2)}{\alpha_1^{\frac{3}{2}}}$$

Furthermore, since $\alpha_1 > \alpha_2$ we can

$$\lambda < \left[\frac{3\sqrt{c_w}(\alpha_1 + \alpha_2)^2}{\alpha_1^{\frac{3}{2}}} - \mu_0(\alpha_2 - \alpha_1) + 2\sqrt{c_w}\left(\sqrt{\alpha_2} - \sqrt{\alpha_1}\right)\right](2\alpha_1 + \alpha_2)^{-1}$$

Overall, we find that (μ_1^*, p_1^*), (μ_2^*, p_2^*) are the optimal decision. Therefore,

$$\lambda_1^* = \frac{\mu_0(\alpha_1 - \alpha_2) - 2\sqrt{c_w}(\sqrt{\alpha_1} - \sqrt{\alpha_2}) + \lambda(\alpha_1 + 2\alpha_2)}{3(\alpha_1 + \alpha_2)}$$

$$\lambda_2^* = \frac{\mu_0(\alpha_2 - \alpha_1) - 2\sqrt{c_w}(\sqrt{\alpha_2} - \sqrt{\alpha_1}) + \lambda(2\alpha_1 + \alpha_2)}{3(\alpha_1 + \alpha_2)}$$

$$\mu_1^* = \frac{\mu_0(\alpha_1 - \alpha_2) - 2\sqrt{c_w}(\sqrt{\alpha_1} - \sqrt{\alpha_2}) + \lambda(\alpha_1 + 2\alpha_2)}{3(\alpha_1 + \alpha_2)} + \sqrt{c_w/\alpha_1}$$

$$\mu_2^* = \frac{\mu_0(\alpha_2 - \alpha_1) - 2\sqrt{c_w}(\sqrt{\alpha_2} - \sqrt{\alpha_1}) + \lambda(2\alpha_1 + \alpha_2)}{3(\alpha_1 + \alpha_2)} + \sqrt{c_w/\alpha_2}$$

$$p_1^* = \frac{\mu_0(\alpha_1 - \alpha_2) - 2\sqrt{c_w}(\sqrt{\alpha_1} - \sqrt{\alpha_2}) + \lambda(\alpha_1 + 2\alpha_2)}{3} + c_s e^{-rl}$$

$$p_2^* = \frac{\mu_0(\alpha_2 - \alpha_1) - 2\sqrt{c_w}(\sqrt{\alpha_2} - \sqrt{\alpha_1}) + \lambda(2\alpha_1 + \alpha_2)}{3} + c_s e^{-rl}$$

$$\pi_1^* = {\lambda_1^*}^2(\alpha_1 + \alpha_2)$$

$$U = F(l) + Q(\mu) - \frac{c_w}{\mu - \lambda} - p$$

$$= c_t + Q_0$$

$$+ \frac{\mu_0(\alpha_1^2 + 4\alpha_1\alpha_2 + \alpha_2^2) - 2\sqrt{c_w}[(\alpha_1 + 2\alpha_2)\sqrt{\alpha_1} + (2\alpha_1 + \alpha_2)\sqrt{\alpha_2}] - \lambda(2\alpha_1^2 + 5\alpha_1\alpha_2 + 2\alpha_2^2)}{3(\alpha_1 + \alpha_2)}$$

$$- (c_t + c_s)e^{-rl}$$

Corollary 5

Similarly, we get

$$\frac{\partial \lambda_1^*}{\partial \mu_0} = \frac{\partial \mu_1^*}{\partial \mu_0} = (\alpha_1 + \alpha_2)\frac{\partial p_1^*}{\partial \mu_0} = \frac{\alpha_1 - \alpha_2}{3(\alpha_1 + \alpha_2)} > 0$$

$$\frac{\partial \lambda_2^*}{\partial \mu_0} = \frac{\partial \mu_2^*}{\partial \mu_0} = (\alpha_1 + \alpha_2)\frac{\partial p_2^*}{\partial \mu_0} = \frac{\alpha_2 - \alpha_1}{3(\alpha_1 + \alpha_2)} < 0$$

$$\frac{\partial \lambda_1^*}{\partial c_w} = \frac{\partial p_1^*}{\partial c_w}(\alpha_1 + \alpha_2)^{-1} = -\frac{\sqrt{\alpha_1} - \sqrt{\alpha_2}}{3\sqrt{c_w}(\alpha_1 + \alpha_2)} < 0$$

$$\frac{\partial \mu_1^*}{\partial c_w} = -\frac{\sqrt{\alpha_1} - \sqrt{\alpha_2}}{3\sqrt{c_w}(\alpha_1 + \alpha_2)} + \frac{1}{2\sqrt{c_w \alpha_1}}$$

$$\frac{\partial \lambda_2^*}{\partial c_w} = \frac{\partial \mu_2^*}{\partial c_w} - \frac{1}{2\sqrt{c_w \alpha_2}} = \frac{\partial p_2^*}{\partial c_w}(\alpha_1 + \alpha_2)^{-1} = -\frac{\sqrt{\alpha_2} - \sqrt{\alpha_1}}{3\sqrt{c_w}(\alpha_1 + \alpha_2)} > 0$$

The relationship between price of α

$$3\frac{\partial p_1^*}{\partial \alpha_1} = \mu_0 - \sqrt{\frac{c_w}{\alpha_1}} + \lambda > \mu_1^* - \sqrt{\frac{c_w}{\alpha_1}} + \lambda = \lambda_1^* + \lambda > 0$$

$$3\frac{\partial p_1^*}{\partial \alpha_2} = -\mu_0 + \sqrt{\frac{c_w}{\alpha_2}} + 2\lambda$$

And,

$$3\frac{\partial p_2^*}{\partial \alpha_1} = -\mu_0 + \sqrt{\frac{c_w}{\alpha_1}} + 2\lambda$$

$$3\frac{\partial p_2^*}{\partial \alpha_2} = \mu_0 - \sqrt{\frac{c_w}{\alpha_2}} + \lambda > 0$$

The relationship between arrival rate of α

$$p_1^* = \lambda_1^*(\alpha_1 + \alpha_2) + c_s e^{-rl}$$

$$\frac{\partial p_1^*}{\partial \alpha_1} = \frac{\partial \lambda_1^*}{\partial \alpha_1}(\alpha_1 + \alpha_2) + \lambda_1^* \Leftrightarrow \frac{\partial \lambda_1^*}{\partial \alpha_1} = (\frac{\partial p_1^*}{\partial \alpha_1} - \lambda_1^*)(\alpha_1 + \alpha_2)^{-1}$$

$$\frac{\partial \lambda_1^*}{\partial \alpha_1} = \left(\frac{\partial p_1^*}{\partial \alpha_1} - \lambda_1^*\right)(\alpha_1 + \alpha_2)^{-1} = \frac{1}{3(\alpha_1 + \alpha_2)}\left(\mu_0 - \sqrt{\frac{c_w}{\alpha_1}} + \lambda - 3\lambda_1^*\right)$$

Notice that if $\alpha_1 = \alpha_2$, then $\lambda_1^* = \lambda_2^* = \lambda/2$. And,

$$\mu_0 - \sqrt{\frac{c_w}{\alpha_1}} + \lambda - 3\lambda_1^* = \mu_0 - \sqrt{\frac{c_w}{\alpha_1}} - \lambda_1^* \geq \mu_1^* - \sqrt{\frac{c_w}{\alpha_1}} - \lambda_1^* = 0$$

$$\mu_0 - \sqrt{\frac{c_w}{\alpha_2}} + \lambda - 3\lambda_2^*$$

We have $\lambda_1^* > \lambda_2^*$ since $\alpha_1 > \alpha_2$, thus, we have if $\mu_0 < 3\lambda_1^* + \sqrt{c_w/\alpha_1} - \lambda$, $\partial\lambda_1^*/\partial\alpha_1 < 0$, and $\partial\lambda_1^*/\partial\alpha_1 > 0$ otherwise. Moreover,

$$3(\alpha_1 + \alpha_2)\frac{\partial \lambda_2^*}{\partial \alpha_2} = \mu_0 - \sqrt{\frac{c_w}{\alpha_2}} + \lambda - 3\lambda_2^* > \mu_2^* - \sqrt{\frac{c_w}{\alpha_2}} - \lambda_2^* = 0$$

Since $\lambda = \lambda_1^* + \lambda_2^*$

$$\frac{\partial \mu_1^*}{\partial \alpha_2} = \frac{\partial \lambda_1^*}{\partial \alpha_2} = -\frac{\partial \lambda_2^*}{\partial \alpha_2} < 0$$

$$\frac{\partial \mu_2^*}{\partial \alpha_1} = \frac{\partial \lambda_2^*}{\partial \alpha_1} = -\frac{\partial \lambda_1^*}{\partial \alpha_1}$$

The relationship between service rate of α

$$\frac{\partial \mu_1^*}{\partial \alpha_1} = \frac{\partial \lambda_1^*}{\partial \alpha_1} - \frac{1}{2\alpha_1}\sqrt{\frac{c_w}{\alpha_1}} = \frac{2\alpha_1\left(\mu_0 - \sqrt{\frac{c_w}{\alpha_1}} + \lambda - 3\lambda_1^*\right) - 3(\alpha_1+\alpha_2)\sqrt{\frac{c_w}{\alpha_1}}}{6\alpha_1(\alpha_1+\alpha_2)}$$

$\partial \mu_1^*/\partial \alpha_1 < 0$ if

$$\lambda < \frac{(5\alpha_1 + 3\alpha_2)}{2\alpha_1}\sqrt{\frac{c_w}{\alpha_1}} + 3\lambda_1^* - \mu_0,$$

otherwise, $\partial \mu_1^*/\partial \alpha_1 > 0$. And similarly, $\partial \mu_2^*/\partial \alpha_2 < 0$ if

$$\lambda < \frac{(5\alpha_2 + 3\alpha_1)}{2\alpha_2}\sqrt{\frac{c_w}{\alpha_2}} + 3\lambda_2^* - \mu_0.$$

Proposition 6:

Same as the proposition 4, we have equation (A2) and (A3) here.

$$U_1 = U_2 = \cdots = U_n \tag{A2}$$

$$\lambda_1 + \lambda_2 + \cdots + \lambda_n = \lambda \tag{A3}$$

And $U_i = F(l) + Q_i(\mu) - c_w/(\mu_i - \lambda_i) - p_i$.

From (A2), we have

$$\alpha_1(\mu_0 - \mu_1) - \frac{c_w}{\mu_1 - \lambda_1} - p_1 = \cdots = \alpha_n(\mu_0 - \mu_n) - \frac{c_w}{\mu_n - \lambda_n} - p_n \tag{A4}$$

Use equation (A4) derivative w.r.t p_1

$$-\frac{c_w}{(\mu_1-\lambda_1)^2}\frac{\partial \lambda_1}{\partial p_1} - 1 = -\frac{c_w}{(\mu_2-\lambda_2)^2}\frac{\partial \lambda_2}{\partial p_1} = \cdots = -\frac{c_w}{(\mu_n-\lambda_n)^2}\frac{\partial \lambda_n}{\partial p_1}$$

$$\frac{\partial \lambda_n}{\partial p_1} = \frac{\partial \lambda_2}{\partial p_1}\frac{(\mu_n-\lambda_n)^2}{(\mu_2-\lambda_2)^2} \tag{A5}$$

And from (A5) we have

$$-\frac{\partial \lambda_1}{\partial p_1} = \frac{\partial \lambda_2}{\partial p_1} + \cdots + \frac{\partial \lambda_n}{\partial p_1} = \frac{\partial \lambda_2}{\partial p_1} + \frac{\partial \lambda_3}{\partial p_1}\frac{(\mu_3-\lambda_3)^2}{(\mu_2-\lambda_2)^2} + \cdots + \frac{\partial \lambda_n}{\partial p_1}\frac{(\mu_n-\lambda_n)^2}{(\mu_2-\lambda_2)^2}$$

$$= \frac{\partial \lambda_2}{\partial p_1}\left[\sum_{i=2}^{n}\frac{(\mu_i-\lambda_i)^2}{(\mu_2-\lambda_2)^2}\right]$$

$$-\frac{\partial \lambda_2}{\partial p_1} = \frac{\partial \lambda_1}{\partial p_1}\frac{(\mu_2-\lambda_2)^2}{\sum_{i=2}^{n}(\mu_i-\lambda_i)^2}$$

Substitute the value of $\partial \lambda_2/\partial p_1$ into (A4)

$$\frac{\partial \lambda_1}{\partial p_1} = -\frac{1}{c_w}\left[\frac{1}{(\mu_1-\lambda_1)^2} + \frac{1}{\sum_{i=2}^{n}(\mu_i-\lambda_i)^2}\right]^{-1} < 0$$

Notice that $\pi_1 = \lambda_1(p_1 - c_s e^{-rl})$, and

$$\frac{\partial \pi_1}{\partial p_1} = \frac{\partial \lambda_1}{\partial p_1}(p_1 - c_s e^{-rl}) + \lambda_1 = -\frac{1}{c_w}\left[\frac{1}{(\mu_1-\lambda_1)^2} + \frac{1}{\sum_{i=2}^{n}(\mu_i-\lambda_i)^2}\right]^{-1}(p_1 - c_s e^{-rl}) + \lambda_1$$

Let $\partial\pi_1/\partial\pi_1 = 0$, we have

$$p_1 = \lambda_1 c_w\left[\frac{1}{(\mu_1-\lambda_1)^2} + \frac{1}{\sum_{i=2}^{n}(\mu_i-\lambda_i)^2}\right] + c_s e^{-rl},$$

similarly,

$$p_i = \lambda_i c_w\left[\frac{1}{(\mu_i-\lambda_i)^2} + \frac{1}{\sum_{j=1}^{n}(\mu_j-\lambda_j)^2 - (\mu_i-\lambda_i)^2}\right] + c_s e^{-rl}$$

Now, use equation (A4) derivative w.r.t μ_1, we have

$$-\alpha_1 + \frac{c_w}{(\mu_1-\lambda_1)^2}\left(1 - \frac{\partial\lambda_1}{\partial\mu_1}\right) = -\frac{c_w}{(\mu_2-\lambda_2)^2}\frac{\partial\lambda_2}{\partial\mu_1} = \cdots = -\frac{c_w}{(\mu_n-\lambda_n)^2}\frac{\partial\lambda_n}{\partial\mu_1}$$

$$\frac{\partial\lambda_n}{\partial\mu_1} = \frac{\partial\lambda_2}{\partial\mu_1}\frac{(\mu_n-\lambda_n)^2}{(\mu_2-\lambda_2)^2}$$

Same to equation (A5), we can get

$$-\frac{\partial\lambda_2}{\partial\mu_1} = \frac{\partial\lambda_1}{\partial\mu_1}\frac{(\mu_2-\lambda_2)^2}{\sum_{i=2}^{n}(\mu_i-\lambda_i)^2},$$

substitute the value into (A4),

$$\frac{\partial\lambda_1}{\partial\mu_1} = \left[\frac{1}{(\mu_1-\lambda_1)^2} - \frac{\alpha_1}{c_w}\right]\left[\frac{1}{(\mu_1-\lambda_1)^2} + \frac{1}{\sum_{i=2}^{n}(\mu_i-\lambda_i)^2}\right]^{-1},$$

$$\frac{\partial\lambda_i}{\partial\mu_i} = \left[\frac{1}{(\mu_i-\lambda_i)^2} - \frac{\alpha_i}{c_w}\right]\left[\frac{1}{(\mu_i-\lambda_i)^2} + \frac{1}{\sum_{j=1}^{n}(\mu_j-\lambda_j) - (\mu_i-\lambda_i)^2}\right]^{-1}.$$

We take the partial derivative of the profit π with respect to μ_1 and obtain

$$\frac{\partial\pi_1}{\partial\mu_1} = \frac{\partial\lambda_1}{\partial\mu_1}(p_1 - c_s e^{-rl})$$

Let $\partial\pi_1/\partial\mu_1 = 0$, we get $\mu_1 = \lambda_1 + \sqrt{c_w/\alpha_1}$. Similarly, $\mu_i = \lambda_i + \sqrt{c_w/\alpha_i}$. Substitute the value of $\mu_1, \mu_2, \dots, \mu_i, p_1, p_2, \dots p_i$ into (6), we get

$$\alpha_1\mu_0 - 2\sqrt{c_w\alpha_1} - \lambda_1 c_w\left(\frac{2\alpha_1}{c_w} + \frac{1}{\sum_{i=2}^{n}\frac{1}{\alpha_1}}\right)$$

$$= \alpha_i\mu_0 - 2\sqrt{c_w\alpha_i} - \lambda_i c_w\left(\frac{2\alpha_i}{c_w} + \frac{1}{\sum_{j=1}^{n}\frac{1}{\alpha_j} - \frac{1}{\alpha_i}}\right),$$

Let $h = \alpha_1\mu_0 - 2\sqrt{c_w\alpha_1} - \lambda_1 c_w\left(\frac{2\alpha_1}{c_w} + \frac{1}{\sum_{i=2}^{n}\frac{c_w}{\alpha_i}}\right)$, we have

$$\lambda_i = \frac{\alpha_i\mu_0 - 2\sqrt{c_w\alpha_i} - h}{\alpha_i\left(2 + \frac{1}{\alpha_i A - 1}\right)}, \qquad (A6)$$

$A = \sum_{j=1}^{n}\frac{1}{\alpha_j}$.

And through equation (A3), we have

$$\lambda = \sum_{i=1}^{n}\lambda_i = \sum_{i=1}^{n}\frac{\alpha_i\mu_0 - 2\sqrt{c_w\alpha_i} - h}{\alpha_i\left(2 + \frac{1}{\alpha_i A - 1}\right)} = \sum_{i=1}^{n}\frac{\alpha_i\mu_0 - 2\sqrt{c_w\alpha_i}}{\alpha_i\left(2 + \frac{1}{\alpha_i A - 1}\right)} - h\sum_{i=1}^{n}\frac{1}{\alpha_i\left(2 + \frac{1}{\alpha_i A - 1}\right)}$$

$$h = \frac{\sum_{i=1}^{n}\frac{\alpha_i\mu_0 - 2\sqrt{c_w\alpha_i}}{\alpha_i\left(2 + \frac{1}{\alpha_i A - 1}\right)} - \lambda}{\sum_{i=1}^{n}\frac{1}{\alpha_i\left(2 + \frac{1}{\alpha_i A - 1}\right)}} = \frac{\sum_{i=1}^{n}\frac{\alpha_i\mu_0 - 2\sqrt{c_w\alpha_i}}{A_i} - \lambda}{\sum_{i=1}^{n}\frac{1}{A_i}}$$

$$= \sum_{i=1}^{n}\frac{(\alpha_i\mu_0 - 2\sqrt{c_w\alpha_i})(1/A_i) - \lambda}{1/A_i}$$

$A_i = \alpha_i\left(2 + \frac{1}{\alpha_i A - 1}\right)$.

And substitute the value of h into (8), we have

$$\lambda_k = \frac{\lambda + \sum_{i=1}^{n}\frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}}{A_k\sum_{i=1}^{n}\frac{1}{A_i}},$$

and then

$$\mu_k = \lambda_k + \sqrt{\frac{c_w}{\alpha_k}}$$

$$p_k = \lambda_k\alpha_k\left(1 + \frac{1}{\alpha_k A - 1}\right) + c_s e^{-rl}.$$

After obtaining these values, we need to prove that they are the optimal decisions for the service providers, and as before, we also use the hessian matrix here. We have

$$H_1 = \begin{vmatrix} \frac{\partial^2\pi_1^*}{\partial\mu_1^{*2}} & \frac{\partial^2\pi_1^*}{\partial\mu_1^*\partial p_1^*} \\ \frac{\partial^2\pi_1^*}{\partial p_1^*\partial\mu_1^*} & \frac{\partial^2\pi_1^*}{\partial p_1^{*2}} \end{vmatrix},$$

since $\frac{\partial^2\pi_1^*}{\partial\mu_1^{*2}} = -2\lambda_1^*\left(\frac{\alpha_1}{c_w}\right)^{\frac{3}{2}} c_w^2 < 0$, we need to prove $\frac{\partial^2\pi_1^*}{\partial\mu_1^{*2}}\frac{\partial^2\pi_1^*}{\partial p_1^{*2}} - \left(\frac{\partial^2\pi_1^*}{\partial p_1^*\partial\mu_1^*}\right)^2 > 0$.

$$\frac{\partial^2\pi_1^*}{\partial\mu_1^{*2}}\frac{\partial^2\pi_1^*}{\partial p_1^{*2}} - \left(\frac{\partial^2\pi_1^*}{\partial p_1^*\partial\mu_1^*}\right)^2 = \sqrt{c_w}\left(\alpha_1 + \frac{1}{\sum_{j=1}^n \frac{1}{\alpha_j} - \alpha_1}\right) - \lambda_1^* \frac{\sum_{j=2}^n \left(\frac{1}{\alpha_j}\right)^{\frac{3}{2}}}{\left(\sum_{j=1}^n \frac{1}{\alpha_j} - \alpha_1\right)^3} > 0$$

$$\lambda_1^* < \sqrt{c_w}\left(\alpha_1 + \frac{1}{\sum_{j=1}^n \frac{1}{\alpha_j} - \alpha_1}\right)\frac{\left(\sum_{j=1}^n \frac{1}{\alpha_j} - \alpha_1\right)^3}{\sum_{j=1}^n \left(\frac{1}{\alpha_j}\right)^{\frac{3}{2}} - \left(\frac{1}{\alpha_1}\right)^{\frac{3}{2}}}$$

Similarly, if all λ_k satisfy

$$\lambda_k^* < \sqrt{c_w}\left(\alpha_k + \frac{1}{\sum_{j=1}^n \frac{1}{\alpha_j} - \alpha_k}\right)\frac{\left(\sum_{j=1}^n \frac{1}{\alpha_j} - \alpha_k\right)^3}{\sum_{j=1}^n \left(\frac{1}{\alpha_j}\right)^{\frac{3}{2}} - \left(\frac{1}{\alpha_k}\right)^{\frac{3}{2}}},$$

then, (μ_k^*, p_k^*) are the optimal strategies for all service providers. Moreover, notice that each λ_k needs to be within the interval $[0, \lambda]$. Thus, we have the constrain that $\lambda_k \geq 0$.

$$\lambda_k = \frac{\lambda + \sum_{i=1}^n \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}}{A_k \sum_{i=1}^n \frac{1}{A_i}}$$

$$A_k \sum_{i=1}^n \frac{1}{A_i}\lambda_k = \lambda + \sum_{i=1}^n \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}$$

Let $x_k = \sqrt{\alpha_k}, y_k = A_k \sum_{i=1}^n \frac{1}{A_i}\lambda_k$, so we have

$$y_k = \left(\sum_{i=1}^n \frac{1}{A_i}\right)\mu_0 x_k^2 - 2\sqrt{c_w}\left(\sum_{i=1}^n \frac{1}{A_i}\right)x_k + 2\sqrt{c_w}\sum_{i=1}^n \frac{\sqrt{\alpha_i}}{A_i} - \mu_0\sum_{i=1}^n \frac{\alpha_i}{A_i} + \lambda$$

Notice that the minimum of y_k locate at $x_k = \sqrt{c_w}/\mu_0$. Therefore, if $y_{k_{min}} \geq 0$, all $\lambda_k \geq 0$.

$$y_{k_{min}} = -\frac{c_w}{\mu_0}\left(\sum_{i=1}^n \frac{1}{A_i}\right) + 2\sqrt{c_w}\sum_{i=1}^n \frac{\sqrt{\alpha_i}}{A_i} - \mu_0\sum_{i=1}^n \frac{\alpha_i}{A_i} + \lambda \geq 0$$

$$\lambda \geq \frac{c_w}{\mu_0}\left(\sum_{i=1}^n \frac{1}{A_i}\right) - 2\sqrt{c_w}\sum_{i=1}^n \frac{\sqrt{\alpha_i}}{A_i} + \mu_0\sum_{i=1}^n \frac{\alpha_i}{A_i} = \mu_0\sum_{i=1}^n \frac{1}{A_i}\left(\sqrt{\alpha_i} - \frac{\sqrt{c_w}}{\mu_0}\right)^2$$

$$\mu_k^* = \lambda_k^* + \sqrt{\frac{c_w}{\alpha_k}}, p_k^* = \lambda_k^* \left(\alpha_k + \frac{1}{A - \frac{1}{\alpha_k}} \right) + c_s e^{-rl},$$

$$\pi_k^* = \lambda_k^{*2} \alpha_k \left(1 + \frac{1}{\alpha_k A - 1}\right)$$

$$\lambda_k^* = \frac{\lambda + \sum_{i=1}^{n} \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}}{A_k \sum_{i=1}^{n} \frac{1}{A_i}}.$$

And $A = \sum_{j=1}^{n} \frac{1}{\alpha_j}; A_i = \alpha_i \left(2 + \frac{1}{\alpha_i A - 1}\right) = \left(2\alpha_i + \frac{1}{A - 1/\alpha_i}\right)$

Corollary 6

Corollary 6 is similar to Corollary 5, except that the service provider is changed from 2 to n

$$\frac{\partial p_k^*}{\partial \lambda} \left[\alpha_k \left(1 + \frac{1}{\alpha_k A - 1}\right)\right]^{-1} = \frac{\partial \mu_k^*}{\partial \lambda} = \frac{\partial \lambda_k^*}{\partial \lambda} = \frac{1}{A_k \sum_{i=1}^{n} \frac{1}{A_i}} > 0$$

$$\frac{\partial p_k^*}{\partial \mu_0} \left[\alpha_k \left(1 + \frac{1}{\alpha_k A - 1}\right)\right]^{-1} = \frac{\partial \mu_k^*}{\partial \mu_0} = \frac{\partial \lambda_k^*}{\partial \mu_0} = \frac{\sum_{i=1}^{n} \frac{\alpha_k - \alpha_i}{A_i}}{A_k \sum_{i=1}^{n} \frac{1}{A_i}}$$

$$\frac{\partial p_k^*}{\partial c_w} \left[\alpha_k \left(1 + \frac{1}{\alpha_k A - 1}\right)\right]^{-1} = \frac{\partial \lambda_k^*}{\partial c_w} = -\frac{\sum_{i=1}^{n} \frac{\sqrt{\alpha_k} - \sqrt{\alpha_i}}{A_i \sqrt{c_w}}}{A_k \sum_{i=1}^{n} \frac{1}{A_i}}$$

The change of α is more complex.

$$\frac{\partial A}{\partial \alpha_k} = -\frac{1}{\alpha_k^2} < 0; \frac{\partial A_k}{\partial \alpha_k} = 2; \frac{\partial A_{i,i\neq k}}{\partial \alpha_k} = \frac{1}{\alpha_k^2 \left(A - \frac{1}{\alpha_i}\right)^2} > 0$$

$$\frac{\partial \lambda_k^*}{\partial \alpha_k} = \frac{\sum_{i=1,i\neq k}^{n}(\frac{\mu_0 - \frac{\sqrt{c_w}}{\sqrt{\alpha_k}}}{A_i} - \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i^2}\frac{\partial A_i}{\partial \alpha_k})}{A_k \sum_{i=1}^{n}\frac{1}{A_i}}$$

$$- \frac{\lambda + \sum_{i=1}^{n}\frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}}{\left(\sum_{i=1}^{n}\frac{A_k}{A_i}\right)^2}(2\sum_{i=1}^{n}\frac{1}{A_i} - \frac{2}{A_k}$$

$$- A_k \sum_{i=1,i\neq k}^{n}\frac{1}{A_i^2}\frac{\partial A_i}{\partial \alpha_k})$$

$$2\sum_{i=1}^{n}\frac{1}{A_i} - \frac{2}{A_k} - A_k\sum_{i=1,i\neq k}^{n}\frac{1}{A_i^2}\frac{\partial A_i}{\partial \alpha_k} = 2\sum_{i=1}^{n}\frac{1}{A_i} - A_k\sum_{i=1}^{n}\frac{1}{A_i^2}\frac{\partial A_i}{\partial \alpha_k}$$

Since $\left(A_k \sum_{i=1}^{n}\frac{1}{A_i}\right)^2 > 0$,

$$\frac{\partial \lambda_k^*}{\partial \alpha_k}\left(\sum_{i=1}^{n}\frac{A_k}{A_i}\right)^2$$

$$= A_k \sum_{i=1}^{n}\frac{1}{A_i}\sum_{i=1,i\neq k}^{n}(\frac{\mu_0 - \frac{\sqrt{c_w}}{\sqrt{\alpha_k}}}{A_i} - \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i^2}\frac{\partial A_i}{\partial \alpha_k}) - (\lambda$$

$$+ \sum_{i=1}^{n}\frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i})(\sum_{i=1,i\neq k}^{n}\frac{2}{A_i} - \frac{A_k}{A_i^2}\frac{\partial A_i}{\partial \alpha_k})$$

$$= A_k(\sum_{i=1,i\neq k}^{n}\frac{1}{A_i} + \frac{1}{A_k})\sum_{i=1,i\neq k}^{n}(\frac{\mu_0 - \frac{\sqrt{c_w}}{\sqrt{\alpha_k}}}{A_i} - \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i^2}\frac{\partial A_i}{\partial \alpha_k})$$

$$- \lambda\left(\sum_{i=1,i\neq k}^{n}\frac{2}{A_i} - \frac{A_k}{A_i^2\alpha_k^2\left(A - \frac{1}{\alpha_i}\right)^2}\right)$$

$$- \sum_{i=1,i\neq k}^{n}\frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}(\sum_{i=1,i\neq k}^{n}\frac{2}{A_i} - \frac{A_k}{A_i^2}\frac{\partial A_i}{\partial \alpha_k})$$

$$= (\sum_{i=1,i\neq k}^{n} \frac{A_k}{A_i} + 1) \sum_{i=1,i\neq k}^{n} (\frac{\mu_0 - \frac{\sqrt{c_w}}{\sqrt{\alpha_k}}}{A_i} - \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i^2} \frac{\partial A_i}{\partial \alpha_k})$$

$$- \lambda \left(\sum_{i=1,i\neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2 \alpha_k^2 \left(A - \frac{1}{\alpha_i}\right)^2} \right)$$

$$- \sum_{i=1,i\neq k}^{n} \frac{(\alpha_k - \alpha_i)\mu_0 - 2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i} (\sum_{i=1,i\neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2} \frac{\partial A_i}{\partial \alpha_k})$$

$$= -\lambda \left(\sum_{i=1,i\neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2 \alpha_k^2 \left(A - \frac{1}{\alpha_i}\right)^2} \right)$$

$$+ \left(\sum_{i=1,i\neq k}^{n} \frac{A_k}{A_i} \right.$$

$$\left. + 1 \right) \sum_{i=1,i\neq k}^{n} \left(\frac{\mu_0}{A_i} - \frac{(\alpha_k - \alpha_i)\mu_0}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} + \frac{2\sqrt{c_w}\left(\sqrt{\alpha_k} - \sqrt{\alpha_i}\right)}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} - \frac{\sqrt{c_w}}{A_i \sqrt{\alpha_k}} \right)$$

$$- \sum_{i=1,i\neq k}^{n} [\frac{(\alpha_k - \alpha_i)\mu_0}{A_i} - \frac{2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}](\sum_{i=1,i\neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2} \frac{\partial A_i}{\partial \alpha_k})$$

$$= -B_\lambda \lambda + \left(\sum_{i=1,i\neq k}^{n} \frac{A_k}{A_i} + 1 \right) [\sum_{i=1,i\neq k}^{n} \left(\frac{\mu_0}{A_i} - \frac{\mu_0(\alpha_k - \alpha_i)}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} \right)$$

$$+ \sum_{i=1,i\neq k}^{n} \left(\frac{2\sqrt{c_w}\left(\sqrt{\alpha_k} - \sqrt{\alpha_i}\right)}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} - \frac{\sqrt{c_w}}{A_i \sqrt{\alpha_k}} \right)] - [\sum_{i=1,i\neq k}^{n} \frac{(\alpha_k - \alpha_i)\mu_0}{A_i}$$

$$- \sum_{i=1,i\neq k}^{n} \frac{2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i}](\sum_{i=1,i\neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2} \frac{\partial A_i}{\partial \alpha_k})$$

$$= -B_\lambda \lambda + \mu_0 \left(\sum_{i=1, i \neq k}^{n} \frac{A_k}{A_i} + 1 \right) \sum_{i=1, i \neq k}^{n} \left(\frac{1}{A_i} - \frac{(\alpha_k - \alpha_i)}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} \right)$$

$$+ \left(\sum_{i=1, i \neq k}^{n} \frac{A_k}{A_i} + 1 \right) \sum_{i=1, i \neq k}^{n} \left(\frac{2\sqrt{c_w}\left(\sqrt{\alpha_k} - \sqrt{\alpha_i}\right)}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} - \frac{\sqrt{c_w}}{A_i \sqrt{\alpha_k}} \right)$$

$$- \mu_0 \sum_{i=1, i \neq k}^{n} \frac{(\alpha_k - \alpha_i)}{A_i} \left(\sum_{i=1, i \neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} \right)$$

$$+ \sum_{i=1, i \neq k}^{n} \frac{2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i} \left(\sum_{i=1, i \neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} \right)$$

$$= -B_\lambda \lambda + B_{\mu_0} \mu_0 + \left(\sum_{i=1, i \neq k}^{n} \frac{A_k}{A_i} + 1 \right) \sum_{i=1, i \neq k}^{n} \left(\frac{2\sqrt{c_w}\left(\sqrt{\alpha_k} - \sqrt{\alpha_i}\right)}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} - \frac{\sqrt{c_w}}{A_i \sqrt{\alpha_k}} \right)$$

$$+ \sum_{i=1, i \neq k}^{n} \frac{2\sqrt{c_w}(\sqrt{\alpha_k} - \sqrt{\alpha_i})}{A_i} \left(\sum_{i=1, i \neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} \right)$$

$$= -B_\lambda \lambda + B_{\mu_0} \mu_0 + C_1 \sum_{i=1, i \neq k}^{n} C_3 \left(\sqrt{c_w}\left(\sqrt{\alpha_k} - \sqrt{\alpha_i}\right) \right) - C_1 \sum_{i=1, i \neq k}^{n} \left(\frac{\sqrt{c_w}}{A_i \sqrt{\alpha_k}} \right)$$

$$+ C_2 \sum_{i=1, i \neq k}^{n} \frac{2\sqrt{c_w}\left(\sqrt{\alpha_k} - \sqrt{\alpha_i}\right)}{A_i}$$

$$= B_{\mu_0} \mu_0 + \sum_{i=1, i \neq k}^{n} B_{c_w} \left(\sqrt{c_w}\left(\sqrt{\alpha_k} - \sqrt{\alpha_i}\right) \right) - B_\lambda \lambda - B_{\alpha_k} \frac{\sqrt{c_w}}{\sqrt{\alpha_k}}$$

$$B_\lambda = \left(\sum_{i=1, i \neq k}^{n} \frac{2}{A_i} - \frac{A_k}{A_i^2 \alpha_k^2 \left(A - \frac{1}{\alpha_i} \right)^2} \right); B_{c_w} = C_1 C_3 + C_2 C_4; B_{\alpha_k} = C_1 \sum_{i=1, i \neq k}^{n} \frac{1}{A_i}$$

$$C_1 = \left(\sum_{j=1, j \neq k}^{n} \frac{A_k}{A_j} + 1 \right); C_2 = \left(\sum_{j=1, j \neq k}^{n} \frac{2}{A_j} - \frac{A_k}{A_j^2} \frac{\partial A_j}{\partial \alpha_k} \right); C_3 = \frac{2}{A_i^2} \frac{\partial A_i}{\partial \alpha_k}; C_4 = \frac{2}{A_i}$$

$$B_{\mu_0} = C_1 \sum_{i=1, i \neq k}^{n} \left(\frac{1}{A_i} - \frac{(\alpha_k - \alpha_i)}{A_i^2} \frac{\partial A_i}{\partial \alpha_k} \right) - \sum_{i=1, i \neq k}^{n} \frac{(\alpha_k - \alpha_i)}{A_i} C_2$$

$$A = \sum_{j=1}^{n} \frac{1}{\alpha_j}; A_i = \alpha_i \left(2 + \frac{1}{\alpha_i A - 1} \right) = \left(2\alpha_i + \frac{1}{A - 1/\alpha_i} \right)$$

If $n = 2$

$$\frac{\partial \lambda_1^*}{\partial \alpha_1} = \frac{2\mu_0\alpha_2 + \sqrt{c_w}(\sqrt{\alpha_1} - 2\sqrt{\alpha_2}) - \alpha_2\lambda - \alpha_2\sqrt{\frac{c_w}{\alpha_1}}}{3(\alpha_1 + \alpha_2)^2},$$

which is same to the result in Corollary 5.

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