SELECTED TOPICS IN SUSTAINABILITY MANAGEMENT: EMERGENCY RELIEF ALLOCATION, VACCINATION, AND BLOCKCHAIN TECHNOLOGY

YUQING PAN

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Selected Topics in Sustainability Management: Emergency Relief Allocation, Vaccination, and Blockchain Technology

Yuqing Pan

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CERTIFICATE OF ORIGINALITY

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Yuqing PAN (Name of student)

Abstract

Both healthcare management and environmental management are vital components of sustainability management, with the common goal of striking a balance between addressing present healthcare needs and safeguarding the environment for future generations. Sustainability management acknowledges the importance of healthcare and environmental factors, striving to find sustainable solutions that ensure long-term health and well-being. Today's society places significant emphasis on addressing healthcare and environmental management issues. Despite researchers having focused on these matters for over a decade, numerous aspects remain unexplored due to the complexity of the social environment and technological advancements. In this thesis, we conduct three studies on healthcare and environmental management, considering different aspects in terms of emergency relief allocation, vaccination, and blockchain technology.

In the first study, we study medical resources allocation during epidemic outbreaks. While some reports show that the existing real-life medical resources allocations during epidemic outbreaks are myopic, some experts claim that medical resources allocations based on foresighted future allocations might enable a better balance of supply and demand. To investigate this claim, we develop a foresighted medical resources allocation model to help governments manage large-scale epidemic outbreaks. We formulate a demand forecasting model with a general demand forecasting function based on the last-period demands, extra demand caused by the last-period unfulfilled demand, and uncertain demand. In the foresighted allocation model, the government decides the current-period allocation based on the foresighted demand, which considers the last-period area demand and uncertain demand from the current period to the end of a planning horizon, using a stochastic dynamic

program. We find that the optimal allocation is a function of the allocation capacity in each period. The optimal foresighted allocation is always higher than the optimal static (one-period) allocation and decreases with allocation capacity. When the allocation capacity is sufficiently large, the foresighted demand is close to the static demand. Besides, if the cost of oversupply is close to zero, the optimal allocations for both the foresighted allocation and one-period models are the allocation capacity. Our results provide useful managerial implications for a government contemplating medical resources allocation in response to an epidemic outbreak.

In the second study, we discuss the coordination between public and private resources for vaccination. Vaccination is a well-known method to protect the public against an epidemic outbreak, e.g., COVID-19. To this end, the government of a country or region would strive to achieve its target of vaccination coverage. Limited by the total vaccine capacity of public hospitals, the government may need to cooperate with private hospitals or clinics for more vaccination. Exploring in this study government coordination of public and private resources for vaccination, we model a vaccine system consisting of a public hospital, a profit-maximizing private clinic, and self-interested individuals, under three scenarios: (i) without information sharing (concerning vaccine inventory and vaccine price), (ii) with information sharing and subsidy, and (iii) with information sharing and allocation. We find that, under scenario (i), the vaccine demand is fully satisfied by the public hospital and the private clinic cannot make any profit. Under scenario (ii), the private clinic is willing to enter the vaccine market with a positive profit-maximizing vaccination coverage. Under scenario (iii), the socially optimal vaccination coverage may be lower than that under scenario (i). Moreover, we conduct a sensitivity analysis to generate practical implications of the research findings for vaccination policy-making. Our results provide both theoretical and managerial insights on vaccine supply decision, government intervention, and vaccination coverage.

In the third study, we examine how a sustainable firm should communicate its environmental quality to consumers. Environmental labels are commonly used in practice, but their proliferation leads to label confusion among consumers. Blockchain-based transparency can solve the above dilemma. As such, whether and when to adopt environmental labels or blockchain technology to reveal its environmental efforts are critical questions faced by firms investing in environmental quality. To answer these questions, we develop a game-theoretic model with a sustainable firm and a non-sustainable firm. The sustainable firm needs to communicate its environmental quality to consumers via either environmental labels or blockchain-based transparency. In the case of environmental labels, a fraction of consumers are confused about the label standards and may underestimate or overestimate the sustainable firm's environmental quality; in the case of blockchain-based transparency, all consumers have full information. We highlight several main findings. First, under environmental labels, as the fraction of confused consumers increases, the sustainable firm may either switch from a high-tier label to a low-tier one, or counterintuitively switch from a low-tier label to a high-tier one to differentiate itself from the competitor. Second, blockchain-based transparency is not always preferred by the sustainable firm. That is, full information is not necessarily better than partial information for the sustainable firm. Third, when the sustainable firm prefers blockchain-based transparency to environmental labels, the sustainable firm may improve or reduce its environmental quality and the non-sustainable firm may be better or worse off.

Publications Arising from the Thesis

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Table of Contents

List of Figures

List of Tables

Chapter 1

Introduction

Sustainability management emphasizes the comprehensive consideration of environmental, social, and economic factors while striving for balanced development in all aspects. Healthcare and environmental management are closely associated with the goals of sustainable development. The Sustainable Development Goals (SDGs), also known as the Global Goals, were adopted by the United Nations in 2015 as a universal call to action to end poverty, protect the planet, and ensure that all people enjoy peace and prosperity by 2030. The 17 SDGs are integrated; they recognize that action in one area will affect outcomes in others, and that development must balance social, economic, and environmental sustainability. Goal 3, "GOOD HEALTH AND WELL-BEING," is related to healthcare management, while Goal 11, "SUSTAINABLE CITIES AND COMMUNITIES," is related to environmental management. Healthcare pertains to the domain of human health and well-being, while environmental management focuses on the protection of natural resources, reduction of environmental pollution, and addressing climate change issues. Achieving these goals requires innovation and collaboration, including exploring new technologies, methods, and models to improve the sustainability of healthcare and environmental management. In conclusion, healthcare and environmental management are integral components of sustainability management. We must strive to achieve their sustainability through comprehensive management, innovation, and collaboration.

Operations management can be used to identify and address inefficiencies in healthcare and environmental systems. Developing strategies by an operations management approach can help healthcare organizations optimize the processes of resources allocation, reduce costs, and increase social benefits. Environmental management can also benefit from the principles of operations management. By improving environmental performance, firms can reduce their adverse impact on the environment while also improving their profits.

With the rapid development of society, health and environmental management has been attracting the attention of researchers. On the one hand, the emergence of COVID-19 has posed many challenges to healthcare systems. The virus, which first appeared in Wuhan, China, has spread rapidly and has affected millions of people. The healthcare systems have had to adapt to the new demands placed on them by COVID-19. On the other hand, the emergence of new technologies, such as blockchain technology, has brought changes to people's lives. Blockchain has given rise to diverse applications, from cryptocurrencies, such as Bitcoin and Ethereum, to smart contracts and supply chain management systems. The emergence of blockchain technology has opened up new possibilities and opportunities for environmental management.

Considering the current medical situation and new technologies, we conduct three studies on healthcare and environmental management, regarding different aspects in terms of emergency relief allocation, vaccination, and blockchain technology. Our aim is to improve the medical resources allocation decisions of governments during large-scale epidemic outbreaks, the coordination between public and private resources for vaccination, and the way of firms to communicate its environmental quality to consumers.

First, COVID-19 has exposed the vulnerability of healthcare systems of countries across the world (Jain et al. 2020). In February 2020, all resources arriving in Hubei Province, China, were allocated to cities or hospitals (areas) by the provincial health commission as follows. Based on the reported number of patients in each area on the day of arrival of the resources, the provincial health commission calculated the current demand in each area and then allocated the medical resources accordingly (EEO 2020). Experts in public administration suggested that a more reasonable way was to forecast the future demand in each area and then distribute the scarce resources (Beijing News 2020). They claimed that allocation based on foresighted prior allocation would enable a better balance of supply and demand.

In Chapter 2, we develop a foresighted allocation model for medical resources allocation by governments. In this model, the government considers deterministic and random demands from the current period to the future. The foresighted allocation attempts to minimize the expected cost of each area from the current period to the end of a planning horizon, which results in a stochastic dynamic programming problem. The government then makes a proportional allocation based on the available medical resources in the current period. Our foresighted allocation model captures the unique characteristics of medical resources demand and results in the following contributions to the literature: (1) We construct a time-dependent medical resources demand model that considers both random demand and forecasted additional demand arising from resources shortages. This new formulation resembles real logistical practices during epidemic outbreaks. (2) We formulate the foresighted allocation model for medical resources as a stochastic dynamic program based on the infection spread of the epidemic. This is a difficult problem in general, but we exploit the special demand forecasting structure of medical resources during an epidemic outbreak, derive some important properties of the solution, and propose the resulting policy implementation. (3) We develop the foresighted allocation model from the humanitarian perspective. We consider random demand, limited allocation capacity, time-varying supply, and proportional allocation to reflect the reality.

Second, vaccination is effective in preventing seasonal flu and the government aims to increase vaccination coverage in Hong Kong. To achieve this goal, public hospitals and private clinics need to cooperate. The Centre for Health Protection of Hong Kong has introduced the Vaccination Programme for more than ten years. The Vaccination Programme provides free vaccinations to priority groups, but it is not sufficient to cover the entire population. A few years ago, the Hong Kong government launched the "Vaccination Subsidy Scheme", under which the government provides subsidy to private clinics for the vaccines they have administered to qualified citizens. In addition, in 2020, people were afraid of the double infections of COVID-19 and influenza. The private clinics in Hong Kong faced an unprecedented flu vaccine shortage (On.cc 2020). According to the Medical Association, the Secretary for Food and Health approved an agreement with vaccine manufacturers to distribute part of the vaccine supply to private doctors (OnNews 2020). Given the vaccination subsidy scheme and government allocation scheme, it is unclear as to which scheme is better, in terms of private clinics' profitability and vaccination coverage.

In Chapter 3, to study the effectiveness of the two schemes, we first model a vaccine system without information sharing as the benchmark. We model the "Vaccination Programme" and the "Vaccination Subsidy Scheme" as a vaccine system with information sharing and subsidy, which resembles the vaccine market in Hong Kong. The public hospital only provides free vaccines to the priority group and decides the subsidy for the private clinic for the vaccines they have administered to qualified citizens. Observing the vaccine inventory in the public hospital and vaccine subsidy, the private clinic decides its vaccine inventory and vaccine price. We find that, in the vaccine system with information sharing and subsidy, the private clinic is willing to order vaccines and enter the vaccine market. Besides, as the range of the priority group decreases, the socially optimal subsidy decreases and the vaccine demand of the non-priority group increases. Moreover, we model the vaccine system with information sharing and allocation to study the effectiveness of this cooperation scheme. Under this scheme, the public hospital provides free vaccines to the priority group and the private clinic. The private clinic makes profit from administering the vaccines to qualified citizens, but the vaccine inventory and vaccine price of the private clinic are decided by the public hospital. In this vaccine system, the public hospital can increase the vaccine inventory in the private clinic in order to increase the supply and decrease the vaccine price to induce more demand, so increasing the vaccination coverage. The vaccination coverage is not affected by the range of the priority group because all the vaccines are ordered by the public hospital. As the vaccine cost increases, the socially optimal vaccine inventory decreases and the vaccine price increases.

Third, with consumers growing more aware of the environmental impact of products and services, firms are making efforts to improve their sustainable performance. Such efforts often involve higher costs but allow firms to meet the needs of environmentally conscious consumers and build competitive advantages. For example, IKEA used recycled material in 10% of its products in 2018 (Ringstrom 2018). Unlike conventional quality attributes, a product's environmental attributes cannot be observed or experienced by consumers (Baksi and Bose 2007). Indeed, environmental attributes are usually firms' private information. For this reason, environmental labels (also called ecolabels or green labels) have emerged. While environmental labels have the potential to disclose important information about firms' sustainable efforts, consumers may be unfamiliar with or confused by them, especially given the presence of numerous labels with different standards. With the increasing use of blockchain technology, blockchain-based transparency is attracting the attention of researchers as a potential solution to the dilemma caused by label confusion. Blockchain technology improves the information transparency within supply chains and is able to reliably reveal firms' environmental efforts to consumers (see Shen et al. 2022 for evidence on the reliability of such disclosure).

In Chapter 4, we explore how sustainable firms should communicate their environmental quality to consumers and how this affects their environmental quality in a competitive market. We use a game-theoretic model involving a sustainable firm and a non-sustainable firm. The sustainable firm offers eco-friendly products, but communication is challenging as environmental quality cannot be directly observed. Communication can be done through environmental labels or blockchain-based transparency. With environmental labels, some consumers may be confused about label standards, whereas blockchain-based transparency reveals the actual quality to all consumers. We compare these two means of communication and provide insights into the challenges faced by sustainable firms. We highlight several main findings. First, under environmental labels, as the fraction of consumers who are confused about label standards increases, the sustainable firm may switch from a high-tier label to a low-tier label when the fraction of confused consumers is sufficiently high; but may counterintuitively switch from a low-tier label to a high-tier label when the fraction of confused consumers is moderate or low. Second, the sustainable firm does not always prefer blockchain-based transparency to environmental labels, which indicates that full information is not necessarily more beneficial than partial information for the sustainable firm. Third, when the sustainable firm prefers blockchain-based transparency to environmental labels, the sustainable firm may improve or reduce its environmental quality, and the non-sustainable firm may be better or worse off.

Chapter 2

Foresighted Medical Resources Allocation during an Epidemic Outbreak

2.1 Introduction

Recent outbreaks of epidemics, such as SARS, MERS, and COVID-19, have caused physical and psychological pain to millions of people. Especially, COVID-19 has exposed the vulnerability of healthcare systems of countries across the world (Jain et al. 2020). At the beginning of the COVID-19 outbreak, almost every affected region experienced a medical resources shortage.

In February 2020, all resources arriving in Hubei Province, China, were allocated to cities or hospitals (areas) by the provincial health commission as follows. Based on the reported number of patients in each area on the day of arrival of the resources, the provincial health commission calculated the current demand in each area and then allocated the medical resources accordingly (EEO 2020). Experts in public administration suggested that a more reasonable way was to forecast the future demand in each area and then distribute the scarce resources (Beijing News 2020). They claimed that allocation based on foresighted prior allocation would enable a better balance of supply and demand.

To investigate this claim, we develop a foresighted allocation model for medical resources allocation by governments. In this model, the government considers deterministic and random demands from the current period to the future. The foresighted allocation attempts to minimize the expected cost of each area from the current period to the end of a planning horizon, which results in a stochastic dynamic programming problem. The government then makes a proportional allocation based on the available medical resources in the current period.

In contrast to business logistics and other general emergency logistics of allocating non-medical resources (such as food), our foresighted allocation model captures the unique characteristics of medical resources demand as listed below.

1. The probability distribution of the medical resources demand varies with time because both the spread and control of the disease are time-dependent and vary with regions/areas. Because of the urgency and imperfect substitutability of medical resources, delayed deliveries of medical resources are unacceptable. For example, mask shortages contribute to contact infection, and shortages of goggles and other protective gear increase infection among the health care workers. Besides, unpredictable epidemics break out in unexpected areas.

2. Demand is not only time-dependent and random but also dynamic. Specifically, the allocation decision in the current period affects the future demand distributions and decisions. So the foresighted allocation needs to be considered.

3. Because medical resources allocation is a societal problem, profit is not the main consideration. For example, in the real case of medical resources allocation in Hubei province, proportional allocation rather than profit-maximizing allocation was adopted.

Our analysis of the medical resources allocation problem during an epidemic outbreak results in the following contributions to the literature.

1. We construct a time-dependent medical resources demand model that considers both random demand and forecasted additional demand arising from resources shortages. This new formulation resembles real logistical practices during epidemic outbreaks.

2. We formulate the foresighted allocation model for medical resources as a stochastic dynamic program based on the infection spread of the epidemic. This is a difficult problem in general, but we exploit the special demand forecasting structure

of medical resources during an epidemic outbreak, derive some important properties of the solution, and propose the resulting policy implementation.

3. We develop the foresighted allocation model from the humanitarian perspective. We consider random demand, limited allocation capacity, time-varying supply, and proportional allocation to reflect the reality.

The rest of the study is organized as follows: In Section 2.2, we review the related literature. In Section 2.3, we develop a foresighted allocation model. In Section 2.4, we present the analytical and numerical studies of the foresighted allocation problem formulated as a stochastic dynamic program. In Section 2.5, we compare the unsatisfied demands under both models. In Section 2.6, we present and discuss the results of the numerical studies to generate managerial insights. Finally, in Section 2.7, we conclude the study and suggest topics for future research. We present all proofs in the Appendix A.

2.2 Literature Review

COVID-19 has challenged supply chain viability under severe uncertainty (Choi 2021, Ivanov and Dolgui 2020, Ivanov 2021). Many studies have attempted to combine medical services with emergency logistics (Jia et al. 2007, Berman and Gavious 2007, Mete and Zabinsky 2010, Sheu and Pan 2014, Barz and Rajaram 2015, Ramirez-Nafarrate et al. 2015). These studies have made remarkable advances in deriving the optimal decisions on the locations of medical facilities and the distribution of medical resources. However, most of them focus on emergency logistics after a large-scale natural disaster and regard medical resources as common resources such as food or tents. In our model, we consider the unique characteristics of epidemic diseases. We construct a time-dependent medical resources demand model that considers the random demand as well as the forecasted additional demand arising from resources shortages. This new formulation resembles real logistics practices during epidemic outbreaks.

Meanwhile, considerable efforts have been made to evaluate various proposals for the logistics and distribution of antibiotics and providing hospital care after a bioterror attack (Kaplan et al. 2003, Craft et al. 2005, Miller et al. 2006, Zaric et al. 2008, Hu and Zhao 2011, Hansen and Day 2011, Gralla et al. 2014, Paciarotti and Valiakhmetova 2021, Corominas 2021). Almost all recent terrorist attacks involve only two biological agents – smallpox and anthrax – whose infection spread can be accurately forecasted or simulated. There are also studies on vaccine allocation (Chick et al. 2008, Pan et al. 2021). But they do not consider the strategies for the time-varying allocation of medical resources, whereas we consider epidemics that may break out in any area unexpectedly, leading to time-varying medical resources demand that requires time-varying allocation strategies to deal with.

Resource allocation is an important part of healthcare operations management (Dai and Tayur, 2020). Concerning the emergency response to epidemics, some studies consider the time evolution and dynamic nature of the demand for medical resources. Zhong et al. (2020) constructed a risk-averse optimization model to deliver disaster supplies under stochastic demand. Liu et al. (2019) determined the optimal temporary facility locations and allocation plans for post-disaster humanitarian medical service. Zaric and Brandeau (2001) and Zaric and Brandeau (2002) presented dynamic models for epidemic resources allocation and developed heuristics for solving the models. In addition, these studies suggest that allowing the reallocation of funds may generate additional health benefits. Mylius et al. (2008) studied the relationships between optimal vaccine allocation, age, risk, and timing. Patel et al. (2005) used stochastic epidemic simulation, genetic algorithm, and random mutation hill climbing to find near-optimal vaccine distributions. Ekici et al. (2014) considered food distribution during an epidemic outbreak. Wang et al. (2009) built a multi-objective stochastic programming model to help select logistics hubs and distribute medical resources and used a genetic algorithm based on Monte Carlo simulation to solve the model. Baghalian et al. (2013) studied a stochastic model for designing a network of multi-product supply chains. Salmerón and Apte (2010) developed a two-stage stochastic optimization model in pre-disaster planning. Alanis et al. (2013) analyzed a two-dimensional Markov chain model for an emergency medical services system to reposition ambulances. Rachaniotis et al. (2012) proposed a deterministic medical resources scheduling model for epidemic control. The model works for large populations in which random interactions can be averaged. He and Liu (2015) developed and compared three emergency medical logistics models, consisting of two recursive mechanisms, namely time-varying demand forecasting for medical resources and distribution of the medical resources. Liu and Xiao (2015) presented a discrete time-space network model for a dynamic resources allocation problem following an epidemic outbreak in a region. They developed a genetic algorithm to solve the model. In addition, several similar models have been built for this problem (Liu and Liang 2013, Liu et al. 2015).

While the extant studies provide insights into medical supply allocation and epidemic control, they often overlook some critical aspects of the problem: (1) Some previous works consider only deterministic problems. In our model, the demand is stochastic as is the case in the real world. (2) Although the analysis and exact solution of the medical supply allocation problem is desired, many previous studies only solve their problems using heuristic methods. We provide general analytical framework for the multi-period medical resources allocation problem under consideration. (3) Although aware of the multi-period issue, most previous studies essentially apply the repetitive one-period model and do not consider a foresighted allocation. Our model is set to optimize the decisions for the whole horizon. To the best of our knowledge, no study has developed a foresighted medical resources allocation model considering the characteristics of epidemic diseases.

2.3 Model Development

Following the reported allocation process we mentioned in the Introduction, we make the following assumptions for our model development:

(1) The demand in the current period is calculated based on information about the last period, and all the information is accurate. We do not consider the misreports in this study.

(2) All the allocation periods are set in advance, and the lead time for distributing the medical resources is less than the length of one period.

The government undertakes prior allocation for each area based on foresighted allocation underpinned by demand in the last period. Then, based on the total available medical resources in the current period, the posterior (actual) allocation for each area is proportional to its prior allocation. The proportional allocation is based on the status quo, which we can see from the following reports on the allocation of medical resources in the aftermath of the COVID-19 outbreak in Hubei in 2020.

"The Hubei Provincial Health Commission (HPHC) compiles the number of inpatients reported the day before by each city. According to this list combined with the number of medical workers in each city, HPHC calculates the total demand for various materials for each city to obtain the corresponding distribution coefficient of medical materials in each city. Then, according to the donation reported by the Material Security Group of the Provincial Command Materials, the amount of medical materials that should be allocated to each city is obtained." (EEO 2020).

"For example, if the demand of the province is 1 million masks and a certain city needs 10,000 masks, then it accounts for 1% of the total. Currently, if 10,000 masks could be distributed, then 10,000 times 1%, that is, 100 masks, will be distributed to the city." (EEO 2020).

Table 2.1 lists all notations used throughout the study.

We use q_{jt} to denote the inventory of medical resources in epidemic area j at the beginning of period t. x_{jt} and a_{jt} are the prior and posterior (actual) allocations for epidemic area j in period t, respectively. Foresighted allocation considers deterministic demand and random variation. The forecast demand in epidemic area j in period t is

$$d_{jt} = M(d_{j,t-1}) + r(d_{j,t-1} - a_{j,t-1} - q_{j,t-1})^+ + \sigma_{jt}, \quad 0 < t \leq n, \quad \forall j. \tag{2.1}$$

In Equation (2.1), $M(d_{j,t-1})$ is a demand forecasting function that is non-negative and increasing in $d_{j,t-1}$. Further, $(d_{j,t-1} - a_{j,t-1} - q_{j,t-1})^+$ is the amount of unfulfilled demand in area j in period t-1, and r is the extra demand in a period caused by one unit of unfulfilled demand in the previous period. Besides, we also consider random variation σ_{jt}. Random disturbances arise from other inadvertent events. We will discuss the stochastic demand and its probability distribution later.

Table 2.1: Notations used throughout this study

x_{jt}^F	Amount of foresighted prior allocation for area j in period t (a decision variable)
x_{jt}^S	Amount of prior allocation of the static model for area j in period t
a_{jt}^F	Actual allocation for epidemic area j in period t
n	Number of periods
J	Number of affected areas
q_{jt}	Inventory of medical resources in epidemic area j at the beginning of period t
d_{jt}	Demand for medical resources in epidemic area j in period t (a random variable)
S_t	Total available medical resources (i.e., total supply) for all areas in period t
C_j	Allocation capacity in epidemic area j in each period
$M(d_{jt})$	Function to forecast the medical resources demand in epidemic area j in period $t+1$, according to the demand in period t
r	Extra demand in a period caused by per unit unfulfilled demand in the previous period
σ_{jt}	Random variation in demand in epidemic area j in period t
$w(.)$	Probability density function of σ_{jt}
$W(.)$	Cumulative distribution function of σ_{jt}
λ	Mean of σ_{jt}
$g_{jt}(.)$	Probability density function of d_{jt}
$G_{jt}(.)$	Cumulative distribution function of d_{jt}
α	Storage cost per unit per period of oversupplied medical resources
β	Shortage cost per unit per period of unfulfilled demand for medical resources
$f_{jt}(q_{jt})$	Sum of expected penalties in epidemic area j from period t to period n, given that initial inventory of epidemic area j from period t is q_{jt}
$Y_{jt}(q_{jt}, d_{jt}, x_{jt}^F)$	Sum of penalties of epidemic area j at period t
$L_t(x)$	Expected unsatisfied demand based on available resources x
x_{jt}^{F*}	Optimal foresighted prior allocation of the foresighted allocation model for area j in period t
x_{jt}^{S*}	Optimal prior allocation of the static model for area j in period t
μ_{jt}	Mean of d_{jt}

The foresighted allocation problem can be divided into n periods with the policy decisions required in each period. In each period, the over-supply causes a storage cost; and if the supply cannot satisfy the demand, there is also a shortage cost including the cost of substitutes, the loss of goodwill for hospitals, and so on. The objective function for the prior allocation for every area x_{jt}^F is to minimize the expected total cost from the current period to the end. The transition functions are as follows:

$$d_{jt} = M(d_{j,t-1}) + r(d_{j,t-1} - x_{j,t-1}^F - q_{j,t-1})^+ + \sigma_{jt}, \quad 0 < t \leq n, \quad \forall j; \tag{2.2}$$

$$q_{jt} = (q_{j,t-1} + x_{j,t-1}^F - d_{j,t-1})^+, \quad 0 < t \leq n, \quad \forall j. \tag{2.3}$$

The prior allocation x_{jt}^F is determined using Equation (2.4).

$$\begin{cases} f_{j,n+1}(q_{j,n+1}) = 0; \\ f_{jt}(q_{jt}) = \min\limits_{x_{jt}^F} E_{d_{jt}}\{Y_{jt}(q_{jt}, d_{jt}, x_{jt}^F) + f_{j,t+1}(q_{j,t+1})\} \end{cases} \tag{2.4}$$

$$(0 \leq x_{jt}^F \leq C_j, \quad t = 1, 2, \ldots, n),$$

where C_j is the allocation capacity in epidemic area j, which is restricted by road capacity and storage capacity in this area, and $Y_{jt}(q_{jt}, d_{jt}, x_{jt}^F)$ is the cost contribution of period t to the objective function, that is,

$$Y_{jt}(q_{jt}, d_{jt}, x_{jt}^F) = \alpha(x_{jt}^F + q_{jt} - d_{jt})^+ + \beta(d_{jt} - x_{jt}^F - q_{jt})^+. \tag{2.5}$$

In Equation (2.5), α is the storage cost per unit per period of over-supplied medical resources, β is the shortage cost per unit per period of unfulfilled demand for medical resources, $(x_{jt}^F + q_{jt} - d_{jt})^+$ is the excess amount of medical resources in area j at the end of period t, and $(d_{jt} - x_{jt}^F - q_{jt})^+$ is the amount of unfulfilled demand in period t. For each area, we aim to (1) fulfill the demand and (2) avoid over-supply. This results in allocating medical resources in such a way that the allocation is close to the demand in each area. Following the classical inventory problem in the beer game, we assume that $\alpha < \beta$.

Each area considers not only the demand in the current period but also future demands. If the predicted future demand is high and the supply is limited, each area probably maintains some inventory to meet the anticipated future demands. Therefore, each area needs to solve a stochastic dynamic programming problem, as discussed in Subsection 2.4.

The sequence of events in each period is as follows:

Stage 1: Given $d_{j,t-1}$, it is easy to derive d_{jt} using Equation (2.1). The foresighted allocation x_{jt}^F is obtained by solving the objective function in (2.4).

Stage 2: Based on the available resources, the actual allocation is

$$a_{jt}^F = \min\{1, \frac{S_t}{\sum_{j=1}^J x_{jt}^F}\} x_{jt}^F. \tag{2.6}$$

2.4 Analysis of the Prior Allocation

In this section, we solve the stochastic dynamic programming problem for stage 1. In Section 2.4.1, we formulate the probability distribution of the demand. In Section 2.4.2, we analyze the optimal foresighted prior allocation x_{jt}^{F*}. In Section 2.4.3, we discuss some properties of the foresighted prior allocation.

2.4.1 Probability Distribution of Demand

In the optimization models above, d_{jt} is a random variable characterized by the epidemic under study. In this subsection, we discuss the probability distribution of d_{jt}, which is strongly related to the status in the previous period.

As shown in Figure 2.1 (Dong et al. 2020), the confirmed cases of COVID-19 in the United States and India (two countries with the most confirmed cases around the world) from January to November 2020, the probability distribution of the demand for medical resources varies with time because both the spread and control of the disease are dynamic and time-dependent.

We formulate a time-dependent probability distribution of the medical resources demand as follows:

$$d_{j,t+1} = M(d_{jt}) + r(d_{jt} - a_{jt} - q_{jt})^+ + \sigma_{j,t+1}, \quad 0 < t \leq n-1, \quad \forall j. \tag{2.7}$$

$M(d_{jt})$ is a function to forecast the demand in epidemic area j in period $t+1$, given the demand d_{jt} in period t. If d_{jt} increases, $M(d_{jt})$ will also increase. However,

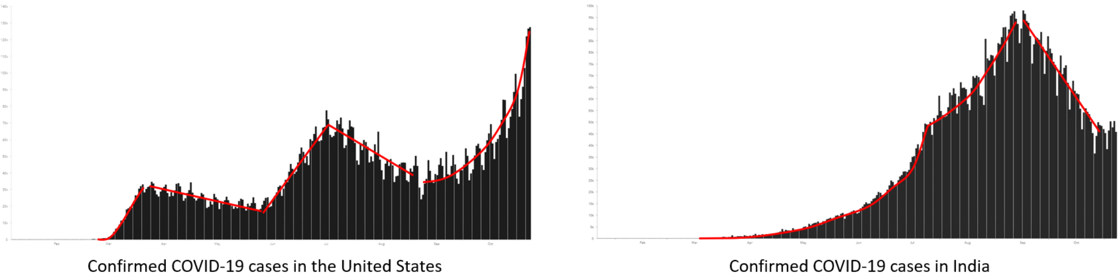

Figure 2.1: Distribution of the number of confirmed cases of COVID-19 over time

this does not mean that $M(d_{jt})$ is always greater than d_{jt}. At the beginning of the disease outbreak, $M(d_{jt})$ is greater than d_{jt} as the epidemic breaks out. When the disease is under control, $M(d_{jt})$ may be smaller than d_{jt}.

During the outbreak of COVID-19, medical resources shortages were prevalent, which could lead to more widespread infections. For example, the mask shortage accelerated contact infection, while the shortage of goggles or protective coveralls caused infection cases in healthcare workers. Therefore, medical resources shortages can lead to increased demand. Let r denote the extra demand in a period caused by per unit unfulfilled demand in the previous period. For different diseases, $r \geq 0$ and have different values. If the unfulfilled demand in the previous period does not cause any extra demand, then r is equal to 0.

In addition to the main spread of the epidemic in a closed area, there are cases beyond expectations. We use σ_{ij} to denote the demands from cases without a clear contact history (The Economic Times 2020) and other unexpected cases. We assume that $\sigma_{j,t+1}$ follows an exponential distribution with mean λ, which can vary among the areas. $w(.)$ and $W(.)$ are the probability density function and cumulative distribution function of σ_{jt}, respectively.

$$w(\sigma_{jt}) = \begin{cases} 0 & , \quad \sigma_{jt} < 0 \\ \lambda e^{-\lambda \sigma_{jt}} & , \quad \sigma_{jt} > 0 \end{cases} \quad \textit{and} \quad W(\sigma_{jt}) = \begin{cases} 0 & , \quad \sigma_{jt} < 0 \\ 1 - e^{-\lambda \sigma_{jt}} & , \quad \sigma_{jt} \geq 0 \end{cases}. \tag{2.8}$$

These two functions describe the natural characteristics of an epidemic disease. We define $g_{jt}(d_{jt})$ and $G_{jt}(d_{jt})$ as the probability density function and cumulative probability function of d_{jt}, respectively. Then we have

$$G_{j,t+1}(y)|_{d_{jt}} = \begin{cases} 0 & , \quad y \leq M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \\ 1 - e^{-\lambda[y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+]} & , \quad y > M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \end{cases}$$

and

$$g_{j,t+1}(y)|_{d_{jt}} = \begin{cases} 0 & , \quad y < M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \\ \lambda e^{-\lambda[y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+]} & , \quad y > M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \end{cases} .$$

Also, $G_{jt}(d_{jt})$ and $g_{jt}(d_{jt})$ have the following properties.

Proposition 1. *For any given d_{jt}, we have $\frac{\partial G_{j,t+1}(y)|_{d_{jt}}}{\partial x_{jt}} \geq 0$, $\frac{\partial^2 G_{j,t+1}(y)|_{d_{jt}}}{\partial^2 x_{jt}} \leq 0$,*

$$\frac{\partial G_{j,t+1}(y)|_{d_{jt}}}{\partial x_{jt}} = \begin{cases} 0 & , \quad y < M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \\ & \quad \text{or } d_{jt} < x_{jt} + q_{jt} \\ \lambda r e^{-\lambda[y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})]} & , \quad y > M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \\ & \quad \text{and } d_{jt} > x_{jt} + q_{jt} \end{cases}$$

and

$$\frac{\partial^2 G_{j,t+1}(y)|_{d_{jt}}}{\partial^2 x_{jt}} = \begin{cases} 0 & , \quad y < M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \\ & \quad \text{or } d_{jt} < x_{jt} + q_{jt} \\ -\lambda^2 r^2 e^{-\lambda[y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})]} & , \quad y > M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \\ & \quad \text{and } d_{jt} > x_{jt} + q_{jt} \end{cases} .$$

Proposition 1 shows that for any given demand in the last period, the expected satisfied demand in this period increases concavely in the amount of allocated medical resources in the last period.

2.4.2 Analysis of the Stochastic Dynamic Program

Because the analysis of the foresighted prior allocation decision problem in each area is similar, we remove the subscript j in this section to simplify the notation. After obtaining the results, we can add back the subscript j.

We solve the above model by first solving the problem from period n-1 to period n and then finding a general form of the optimal policy in each period in the n-period stochastic dynamic programming model.

To facilitate solving of the model, we define the function

$$L_t(x) = \int_x^\infty (d_t - x) g_t(d_t)\, d(d_t), \quad x \geq 0, \tag{2.9}$$

$L_t(x)$ is decreasing and convex in x as $\frac{dL_t(x)}{dx} = G_t(x) - 1 \leq 0$. Indeed, $L_t(x)$ is the expected unsatisfied demand based on the available resources x. Thus, the expected unsatisfied demand decreases with x, and the decreasing rate decreases with x.

In Equations (2.4) and (2.5), we can write $E_{d_t}Y_t(q_t, d_t, x_t^F)$ as

$$\begin{aligned} E_{d_t}Y_t(q_t, d_t, x_t^F) &= E_{d_t}\{\alpha(x_t^F + q_t - d_t)^+ + \beta(d_t - x_t^F - q_t)^+\}, \\ &= \alpha \int_0^{x_t^F+q_t} (x_t^F + q_t - d_t) g_t(d_t)\, d(d_t) + \beta \int_{x_t^F+q_t}^{\infty} (d_t - x_t^F - q_t) g_t(d_t)\, d(d_t) \\ &= \alpha(x_t^F + q_t - \mu_t) + (\alpha+\beta)L_t(x_t^F + q_t), \end{aligned} \tag{2.10}$$

where μ_t is the mean of d_t.

By solving the function in Equation (2.4) for period n, we derive the optimal solution for period n, which is also the optimal solution for the static (one-period) model in Proposition 2. We use x_t^{S*} to denote the optimal solution for the static model.

Proposition 2. *The optimal solution in period n is $x_t^{S*} = x_n^{F*} = \min\left\{C, \left(G_n^{-1}\left(\frac{\beta}{\alpha+\beta}\right) - q_n\right)^+\right\}$, i.e.,*

$$x_t^{S*} = x_n^{F*} = \begin{cases} C & , \quad q_n \le G_n^{-1}(\frac{\beta}{\alpha+\beta}) - C \\ G_n^{-1}(\frac{\beta}{\alpha+\beta}) - q_n & , \quad G_n^{-1}(\frac{\beta}{\alpha+\beta}) - C \le q_n \le G_n^{-1}(\frac{\beta}{\alpha+\beta}) \\ 0 & , \quad q_n \ge G_n^{-1}(\frac{\beta}{\alpha+\beta}) \end{cases} .$$

Proposition 2 states that the areas will retain the available resources within $G_n^{-1}(\frac{\beta}{\alpha+\beta})$, which is also the optimal solution for the static (one-period) model. In Section 2.5, we compare the optimal solutions between the dynamic and static models. When the initial inventory q_n is small, all possible medical resources are allocated. When the inventory is moderate, the optimal allocation amount is a linearly decreasing function of the inventory, which is affected by storage and shortage costs. Finally, when the initial inventory is sufficiently large, no resources are allocated.

Lemma 1. $\frac{\partial G_n^{-1}(\frac{\beta}{\alpha+\beta})}{\partial x_{n-1}^F} \le 0$ *and* $\frac{\partial G_n^{-1}(\frac{\beta}{\alpha+\beta})}{\partial d_{n-1}} \ge 0$, *except for point $d_{n-1} = x_{n-1}^F + q_{n-1}$.*

Lemma 1 states that the available resources that the areas would like to have will decrease with the allocation in the last period and increase with the demand in the last period. Note that both $G_n^{-1}(\frac{\beta}{\alpha+\beta})$ and q_n are functions of $x_{n-1}^F + q_{n-1}$ and d_{n-1} (from Equations (2.2) and (2.3), and Proposition 1). Furthermore, $G_n^{-1}(\frac{\beta}{\alpha+\beta}) - q_n$ has the properties given in Lemma 2.

Lemma 2. $G_n^{-1}(\frac{\beta}{\alpha+\beta}) - q_n$ *strictly increases in* d_{n-1}*. There exist unique* d'_{n-1} *and* d''_{n-1} *such that (1) if* $d_{n-1} = d'_{n-1}$*, then* $G_n^{-1}(\frac{\beta}{\alpha+\beta}) - q_n = 0$*; (2) if* $d_{n-1} = d''_{n-1}$*, then* $G_n^{-1}(\frac{\beta}{\alpha+\beta}) - q_n = C$*; and (3)* $d'_{n-1} < x^F_{n-1} + q_{n-1}$*.*

Increasing d_{n-1} forecasts more demand in period n and leaves less inventory in period n, so we obtain Lemma 2. The existence of d'_{n-1} and d''_{n-1} can be explained as follows: There must exist a d'_{n-1} that makes the government not desire any allocation, which only happens when the demand in period $n-1$ has been fully satisfied, i.e., $d'_{n-1} < x^F_{n-1} + q_{n-1}$. There is also a d''_{n-1} that makes the government desire C to be the allocation.

Given Lemma 2, we can write x_n^{F*} as

$$x_n^{F*} = \begin{cases} 0 & , \quad d_{n-1} \le d'_{n-1} \\ G_n^{-1}(\frac{\beta}{\alpha+\beta})|_{d_{n-1}} - (x^F_{n-1} + q_{n-1} - d_{n-1})^+ & , \quad d'_{n-1} < d_{n-1} \le d''_{n-1} \\ C_j & , \quad d_{n-1} > d''_{n-1} \end{cases} ,$$

where d'_{n-1} and d''_{n-1} are defined in Lemma 2.

Thus,

$$\begin{aligned} & f_{n-1}(q_{n-1}) \\ =& \min_{x^F_{n-1}} E_{d_{n-1}}\{Y_{n-1}(q_{n-1}, d_{n-1}, x^F_{n-1}) + f_n(q_n)\} \\ =& \min_{x^F_{n-1}}\{\alpha(q_{n-1} + x^F_{n-1} - \mu_{n-1}) + (\alpha+\beta)L_{n-1}(q_{n-1} + x^F_{n-1}) \\ & + \int_0^{d'_{n-1}} E_{d_n}\{\alpha(x^F_{n-1} + q_{n-1} - d_{n-1} - d_n)^+ \\ & + \beta(d_n - q_{n-1} - x^F_{n-1} + d_{n-1})^+\} g_{n-1}(d_{n-1})\, d(d_{n-1}) \\ & + \int_{d'_{n-1}}^{d''_{n-1}} E_{d_n}\{\alpha(G_n^{-1}(\frac{\beta}{\alpha+\beta}) - d_n)^+ + \beta(d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta}))^+\} g_{n-1}(d_{n-1})\, d(d_{n-1}) \\ & + \int_{d''_{n-1}}^{\infty} E_{d_n}\{\alpha[C + (q_{n-1} + x^F_{n-1} - d_{n-1})^+ - d_n]^+ \\ & + \beta[d_n - (q_{n-1} + x^F_{n-1} - d_{n-1})^+ - C]^+\} g_{n-1}(d_{n-1})\, d(d_{n-1}). \end{aligned} \tag{2.11}$$

The objective function in the (n-1)-th period $E_{d_{n-1}}\{Y_{n-1}(q_{n-1}, d_{n-1}, x^F_{n-1}) + f_n(q_n)\}$, which is the expectation of the total cost in the (n-1)-th and n-th periods, is convex in x^F_{n-1} for any given C, q_{n-1}, $G_{n-1}(d_{n-1})$, and $G_n(d_n)$. This indicates that there exists an optimal foresighted prior allocation. Set $I_t = \frac{dE_{d_t}\{Y_t(q_t,d_t,x^F_t)+f_{t+1}(q_{t+1})\}}{dx^F_t}$. I_{n-1} has the properties given in Lemma 3.

Lemma 3. *(1) I_{n-1} is an increasing function of q_{n-1} and C. (2)* $\lim_{q_{n-1}\to\infty} I_{n-1} > 0$.

Lemma 3 (1) states that the rate of change in the expected cost increases with q_{n-1} and C. Set $I_t^0 = I_t|_{x_t^F=0}$. For any given C, there exists a unique $h_{n-1}(C) > 0$ such that $I_{n-1}^0|_{q_{n-1}<h_{n-1}(C)} < 0$, $I_{n-1}^0|_{q_{n-1}=h_{n-1}(C)} = 0$, and $I_{n-1}^0|_{q_{n-1}>h_{n-1}(C)} > 0$. This shows that although the foresighted prior allocation is zero, there exists an inventory level that achieves the minimum expected cost, and it changes with C. Thus, the optimal solution can be derived.

Proposition 3. *For any given C, q_{n-1}, and $G_{n-1}(d_{n-1})$, the optimal solution of the two-period subproblem is*

$$x_{n-1}^{F*} = \begin{cases} 0 & , \quad q_{n-1} \geq h_{n-1}(C) \\ h_{n-1}(C) - q_{n-1} & , \quad h_{n-1}(C) - C \leq q_{n-1} < h_{n-1}(C) \\ C & , \quad q_{n-1} < h_{n-1}(C) - C \end{cases}$$

and $x_n^{F*} = \min\left\{C, \left(G_n^{-1}\left(\frac{\beta}{\alpha+\beta}\right) - \left(q_{n-1} + x_{n-1}^{F*} - d_{n-1}\right)\right)^+\right\}$*, where $h_{n-1}(C)$ is a function of C defined by* $I_{n-1}^0|_{q_{n-1}=h_{n-1}(C)} = 0$.

Proposition 3 gives the optimal solutions for periods n and $n-1$. When the initial inventory q_{n-1} is small, many medical resources are allocated as much as possible. When the initial inventory is sufficient, no resources are allocated. When the inventory is moderate, the optimal allocation amount is a linearly decreasing function of the inventory. In addition, x_n^{F*} is not only related to the on-hand inventory but is also affected by x_{n-1}^{F*}.

We provide a general form of the optimal solution in each period as follows:

In the t-th period,

$$\begin{aligned} f_t(q_t) &= \min_{x_t^F} E_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1})\} \\ &= \min_{x_t^F}\{\alpha(q_t + x_t^F - \mu_t) + (\alpha+\beta)L_t(x_t^F + q_t) + E_{d_t} f_{t+1}(q_{t+1})\} \\ s.t. \quad 0 \leq & x_t^F \leq C. \end{aligned}$$

Given an optimal policy for the $(t+1)$-th period, the objective function in the t-th period $E_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1})\}$, which is the expectation of the total cost from period t to period n, is convex in x_t^F for any given C, q_t, and $G_t(d_t)$. Lemma 4 shows that I_t has properties similar to those of I_{n-1}.

Lemma 4. *I_t is an increasing function of q_t and C, where I_t is defined as*

$$I_t = \frac{dE_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1})\}}{dx_t^F} \ (t = 1, 2, \ldots, n).$$

For any given C, there exists $h_t(C) > 0$ such that $I_t^0|_{q_t<h_t(C)} < 0$, $I_t^0|_{q_t=h_t(C)} = 0$, and $I_t^0|_{q_t>h_t(C)} > 0$. In addition, $h_t(C)$ decreases in C. Based on the above propositions and lemmas, we derive the optimal solution in period t, as specified in Proposition 4 and illustrated in Figure 2.2.

Proposition 4. *For any given C, q_t, and $g_t(d_t)$ in period t, the general form of the optimal solution is*

$$x_t^{F*} = \begin{cases} 0 & , \quad q_t \geq h_t(C) \\ h_t(C) - q_t & , \quad h_t(C) - C \leq q_t < h_t(C) \\ C & , \quad q_t < h_t(C) - C \end{cases} .$$

In other words, $x_t^{F*} = min\{C, (h_t(C) - q_t)^+\}$, where $h_t(C)$ is a function of C defined by $I_t^0|_{q_t=h_t(C)} = 0$ and $h_t(C) > 0$.

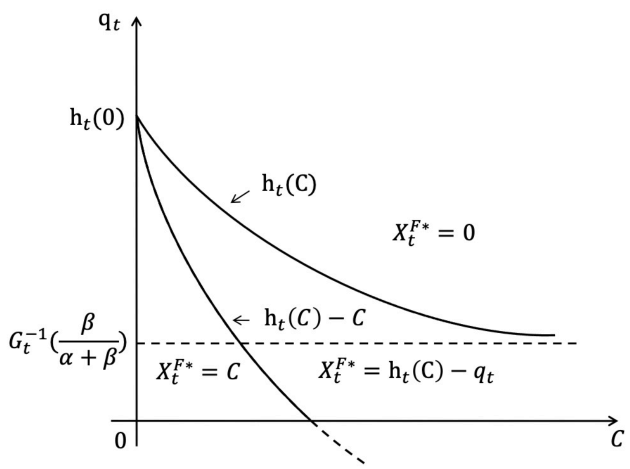

Figure 2.2: General form of the optimal solution

In Figure 2.2, for a given q_t, when C increases, x_t^{F*} decreases.

According to Propositions 2.2 and 4, the optimal solution in each period is a piecewise function of the quantities allocated in the previous periods, the maximum supply, the demands in the previous periods, and the distribution of the demands in the following periods. We resolve the stochastic dynamic programming problem, which is difficult to solve in general, by using a special demand forecasting structure of medical resources during an epidemic outbreak. Figure 2.2 shows the plane of C and q_t. The plane is segmented into three parts by functions $h_t(C)$ and $h_t(C) - C$. The different combinations of C and q_t determine the different forms of the optimal solution.

2.4.3 Properties of the Optimal Solution

Based on the solution of the proposed stochastic dynamic programming model, we provide several further properties of the optimal solution in this section.

First, we provide the properties of $h_t(C)$, an important function for solving the proposed problem, in Proposition 5.

Proposition 5.

(1) $\frac{\partial h_t(C)}{\partial C} \leq 0$;

(2) $h_t(C) \geq G_t^{-1}\left(\frac{\beta}{\alpha+\beta}\right)$;

(3) $\frac{\partial h_{t+1}(C)}{\partial d_t} \geq 0$ $(d_t \neq x_t^F + q_t)$.

Proposition 5 (1) states that $h_t(C)$ is decreasing in C. This is because when supply increases, the probability of demand unfulfillment in the following periods will decline. Thus, the allocated amount can be reduced. Proposition 5 (2) shows that, in any period, the value of $h_t(C)$ is greater than or equal to $G_t^{-1}(\frac{\beta}{\alpha+\beta})$, which is the optimal solution to the static model. As C increases, $h_t(C)$ is close to $G_t^{-1}(\frac{\beta}{\alpha+\beta})$, which shows that when the allocation capacity is sufficiently large, the foresighted demand is close to the static demand. Proposition 5 (3) states that if the demand in a specific period increases, then, in the next period, the value of $h_t(C)$ will also increase because the increased demand indicates more patients in the following periods.

According to Proposition 4, we have

$$x_{t+1}^{F*} = \begin{cases} 0 & , \quad d_t \leq q_t + x_t^F - h_{t+1}(C) \\ h_{t+1}(C) - (q_t + x_t^F - d_t)^+ & , \quad q_t + x_t^F - h_{t+1}(C) < d_t \\ & \qquad \leq q_t + x_t^F - (h_{t+1}(C) - C)^+ \\ C - (C - h_{t+1}(C))^+ & , \quad d_t \geq q_t + x_t^F - (h_{t+1}(C) - C)^+ \end{cases} .$$

Based on the above results and Proposition 4, we can determine $\frac{\partial x_t^{F*}}{\partial q_t}$, $\frac{\partial x_{t+1}^{F*}}{\partial q_t}$, $\frac{\partial x_{t+1}^{F*}}{\partial x_t^F}$, $\frac{\partial q_{t+1}}{\partial x_t^F}$, $\frac{\partial q_{t+1}}{\partial q_t}$, $\frac{\partial x_t^{F*}}{\partial C}$, and $\frac{\partial x_{t+1}^{F*}}{\partial d_t}$ as follows:

Proposition 6.

(1) $\frac{\partial x_t^{F*}}{\partial q_t} = \begin{cases} 0 & , \quad q_t < (h_t(C)-C)^+ \\ -1 & , \quad (h_t(C)-C)^+ < q_t < h_t(C) \\ 0 & , \quad q_t > h_t(C) \end{cases}$;

$\frac{\partial^2 x_t^{F*}}{\partial q_t^2} = 0$ *($q_t \neq h_t(C)$ and $q_t \neq (h_t(C)-C)^+$), respectively.*

(2) $\frac{\partial q_{t+1}}{\partial x_t^F} = \frac{\partial q_{t+1}}{\partial q_t} = \begin{cases} 1 & , \quad d_t < q_t + x_t^F \\ 0 & , \quad d_t > q_t + x_t^F \end{cases}$;

$\frac{\partial^2 q_{t+1}}{\partial x_t^{F2}} = \frac{\partial^2 q_{t+1}}{\partial q_t^2} = 0$ *($d_t \neq q_t + x_t^F$).*

(3) $\frac{\partial x_{t+1}^{F*}}{\partial x_t} = \frac{\partial x_{t+1}^{F*}}{\partial q_t} = \begin{cases} 0 & , \quad d_t < q_t + x_t^F - h_{t+1}(C) \\ -1 & , \quad q_t + x_t^F - h_{t+1}(C) < d_t \\ & \quad\; < q_t + x_t^F - (h_{t+1}(C)-C)^+ \\ 0 & , \quad d_t < q_t + x_t^F - (h_{t+1}(C)-C)^+ \end{cases}$;

$\frac{\partial^2 x_{t+1}^{F*}}{\partial x_t^{F2}} = \frac{\partial^2 x_{t+1}^{F*}}{\partial q_t^2} = 0$ *($d_t \neq q_t + x_t^F - h_{t+1}(C)$ and $d_t \neq q_t + x_t^F - (h_{t+1}(C)-C)^+$).*

(4) $\frac{\partial x_t^{F*}}{\partial C} = \begin{cases} 1 & , \quad q_t < (h_t(C)-C)^+ \\ \frac{dh_t(C)}{dC} \leq 0 & , \quad (h_t(C)-C)^+ < q_t < h_t(C) \\ 0 & , \quad q_t > h_t(C) \end{cases}$.

(5) $\frac{\partial x_{t+1}^{F*}}{\partial d_t} = \begin{cases} 1 & , \quad d_t < q_t + x_t^F - h_{t+1}(C) \\ \frac{dh_{t+1}(C)}{d(d_t)} + 1 \geq 1 & , \quad q_t + x_t^F - h_{t+1}(C) < d_t \\ & \quad\; < q_t + x_t^F - (h_{t+1}(C)-C)^+ \\ 0 & , \quad d_t < q_t + x_t^F - (h_{t+1}(C)-C)^+ \end{cases}$.

Proposition 6 shows the relationships between the optimal solution x_t^{F*} and inventory q_t, demand d_t, and allocation capacity C. Specifically, Proposition 6(1) provides the relationship between the optimal solution and the initial inventory in a period. When the inventory is large or small enough, the solution is not affected by the inventory; however, when the inventory is moderate, the inventory decreases linearly.

Proposition 6 (2) shows how the inventory and allocated amount in a period affect the inventory in the next period. Note that $q_{t+1} = (q_t + x_t^F - d_t)^+$. When the demand is relatively small, if the inventory and allocated amount increase by one unit, then the inventory in the next period also increases by one unit.

Proposition 6 (3) gives the relationship between the optimal solution in a period and the inventory and solution in the previous period. This relationship is similar to that in Equation (1).

Proposition 6 (4) provides the relationship between the optimal solution and supply. When the inventory is relatively small, a unit increase in supply will lead to a unit increase in the solution. With more inventory, the solution will decline as the supply increases, but their relationship is not linear.

Proposition 6 (5) shows the relationship between the optimal solution in a period and the demand in the previous period. Indeed, increasing demand leads to an increase in the optimal solution. In addition, when the demand is small, that is, $d_t < q_t + x_t^F - h_{t+1}(C)$, the optimal solution will increase by one unit if the demand in the previous period increases by one unit. However, when the demand exceeds a critical value, that is, $q_t + x_t^F - h_{t+1}(C) < d_t < q_t + x_t^F - (h_{t+1}(C) - C)^+$, a unit increase in demand will lead to a unit increase in the optimal solution in the next period. When the demand is greater than $q_t + x_t^F - (h_{t+1}(C) - C)^+$, the optimal solution is not affected in the next period.

From the humanitarian perspective, the government might care so much about the unsatisfied demand and ignore the storage cost for oversupplied medical resources. In this case, α is close to 0, we get the properties for the static allocation model and foresighted allocation model in Proposition 7.

Proposition 7. *When α is close to 0, the optimal allocation for both the static allocation model and foresighted allocation model is the allocation capacity, i.e.,*

$$x_t^{S*} = x_n^{F*} = C.$$

Proposition 7 states that when the storage cost for oversupplied medical resources is extremely small, the optimal foresighted allocation is equal to the optimal allocation in the static model. In this case, the optimal allocations in both models are as large as possible.

2.4.4 Linear forecasting model

In this section, we use a linear forecasting function. It should be noted that our allocation model is not restricted to the linear model, as the model works with other forms of the forecasting function.

Set $M(d_{jt})$ as a linear function of d_{jt}, that is, $M(d_{jt}) = A_j d_{jt}$. Then, we express

$d_{j,t+1}$ as:

$$d_{j,t+1} = A_j d_{jt} + r(d_{jt} - x_{jt} - q_{jt})^+ + \sigma_{j,t+1},$$

where A_j is a positive growth coefficient. When $A_j > 1$, demand increases.

Proposition 8. *If the medical resources in the epidemic area j are sufficient, then*

$$G_{jt}(y) = \begin{cases} 0 & , \quad y < A_j^t D_{j0} \\ 1 - \frac{1}{\prod_{l=1}^{t-1}(1-A_j^l)} \sum_{i=0}^{t-1} B_j^i(t-1, A_j) e^{\frac{-\lambda(y - A_j^t D_{j0})}{A_j^i}} & , \quad y \geq A_j^t D_{j0} \end{cases}$$

and

$$g_{jt}(y) = \begin{cases} 0 & , \quad y < A_j^t D_{j0} \\ \frac{\lambda}{\prod_{l=1}^{t-1}(1-A_j^l)} \sum_{i=0}^{t-1} B_j^i(t-1, A_j) e^{\frac{-\lambda(y - A_j^t D_{j0})}{A_j^i}} \frac{1}{A_j^i} & , \quad y \leq A_j^t D_{j0} \end{cases},$$

where $B_j^i(t-1, A_j)$ is the coefficient of x^i in the expression of $(1 - A_j x)(1 - A_j^2 x) \ldots (1 - A_j^{t-1} x)$.

Proposition 8 gives the probability density function of demand for the linear demand forecasting model.

2.5 Numerical Studies

In this section, we use the linear forecasting function to do some numerical analysis of the optimal policies. We find the probability and cumulative distribution functions according to Proposition 1.

Study 1 This study compares the optimal solution of the dynamic programming model with the corresponding $G_t^{-1}\left(\frac{\beta}{\alpha+\beta}\right)$.

We designed this numerical study as a simplification of the real case of the severe acute respiratory syndrome (SARS) outbreak in China in the first quarter of 2003. The parameters were set according to the situation on February 9, 2003, in Guangzhou, a city in southern China as follows: $\lambda = 0.25$, $P_0 = 226$, $A = 1.08$, $C = 200$, $n = 3$, $\alpha = 0.3$, $\beta = 0.7$, and $D_t = \mu_t$ $(t = 1, 2, 3)$. We obtain these data by consulting the relevant parties. For the values of A, 1.08 is the parameter determined by fitting the actual growth, and 0.9 is a random experimental value. In addition, for generality, we test an experimental situation as follows: the number of patients is decreasing, that is, $0 < A < 1$. α and β reflect the weightings of the oversupply and unfulfilled demand, respectively.

Table 2.2 reports the solutions of $h_t(C)$ $(t = 1, 2, 3)$, which can be compared with the corresponding $G_t^{-1}\left(\frac{\beta}{\alpha+\beta}\right)$ and the expectation of the demand.

Table 2.2: Policy comparison between the dynamic and static models

Period	$A = 1.08$			$A = 0.9$		
	$h_t(C)$	$G_t^{-1}\left(\frac{\beta}{\alpha+\beta}\right)$	$E\{D_t\}$	$h_t(C)$	$G_t^{-1}\left(\frac{\beta}{\alpha+\beta}\right)$	$E\{D_t\}$
$t = 1$	247.2	200.1	198.5	167.5	167.5	165.9
$t = 2$	256.4	221.0	217.5	155.7	155.7	152.5
$t = 3$	243.4	243.4	238.1	144.9	144.9	140.5

$h_t(C)$ is the optimal policy for stochastic dynamic programming; $G_t^{-1}\left(\frac{\beta}{\alpha+\beta}\right)$ is the optimal policy of the stochastic static programming; and $E\{D_t\}$ is the expectation of D_t.

We allocate once in each period. Regardless of the model being adopted, the optimal allocation of medical resources in each period is greater than the expectation of the corresponding demand because the cost of unfulfilled demand is greater than that of over-supply, i.e., $\alpha < \beta$. In addition, when the epidemic continues to spread, that is, $A > 1$, the optimal allocation obtained by the foresighted model is greater than that of the static model. This is because to meet the increasing demand, greater quantities of medical resources are allocated to earlier periods in advance. However, when the epidemic is under control and the number of patients is declining (i.e., $0 < A < 1$), the solutions of the two models are the same. Referring to Proposition 5, when C tends to infinity, $h_t(C)$ is close to $G_t^{-1}(\frac{\beta}{\alpha+\beta})$. It shows that when the allocation capacity is sufficiently large, the foresighted demand is close to static demand. In Table 2.2, $C = 200$ is sufficiently large to make $h_t(C) = G_t^{-1}(\frac{\beta}{\alpha+\beta})$. However, this is not always the case. When C is small, $h_t(C) > G_t^{-1}(\frac{\beta}{\alpha+\beta})$, as shown in Figure 2.2. In this study, we have used the results of Propositions 2 and 4, and the result of the numerical study is consistent with Proposition 5 (2).

In the following numerical studies, we consider only the first situation, that is,

$A = 1.08$.

Study 2 This study tests how the optimal policy changes with λ, C, and α. We set the other parameters to be the same as those in Study 1.

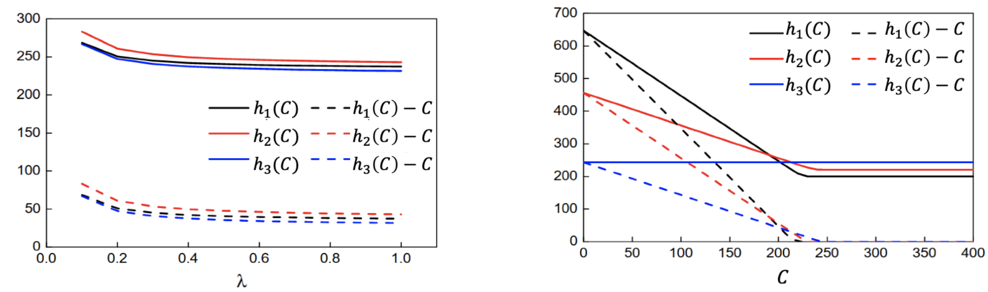

Figure 2.3: Sensitivity analysis of λ and C

The left-hand side of Figure 2.3 illustrates that with a given allocation capacity, as λ (i.e., mean of random demand) increases, both $h_t(C)$ and $h_t(C) - C$ $(t = 1, 2, 3)$ decrease, and the rates of decrease decline. It shows that, as the mean of the unexpected cases increases, the foresighted demand in each period decreases. Smaller values of λ lead to higher values of expectation and variance in demand. Thus, more medical resources are allocated.

The right side of Figure 2.3 shows how the optimal policy is affected by the storage and transport capacity. Both $h_t(C)$ $(t = 1, 2)$ and $h_t(C) - C$ $(t = 1, 2, 3)$ decrease as the allocation capacity increases when the allocation capacity is less than the corresponding boundary points, whereas they remain unchanged when the capacity is adequate. In addition, the rates of their decrease are the fastest in the beginning $(t = 1)$. We show that the relationship between the optimal policy and the allocation capacity is consistent with our result in Proposition 5 (1). These results provide valuable references for long-term decisions on capacity investment.

The left side of Figure 2.4 presents the sensitivity analysis for the storage cost. In this analysis, $\beta = 1 - \alpha$. As the storage cost (i.e., α) increases, $h_t(C)$ and $h_t(C) - C$ $(t = 1, 2, 3)$ decrease. However, the increase is very slow when the storage cost is greater than 0.6. In addition, a stepped growth trend is found, which benefits the application. The shortage cost can be specified within certain intervals, which is more realistic than stipulating that the shortage cost has a precise value.

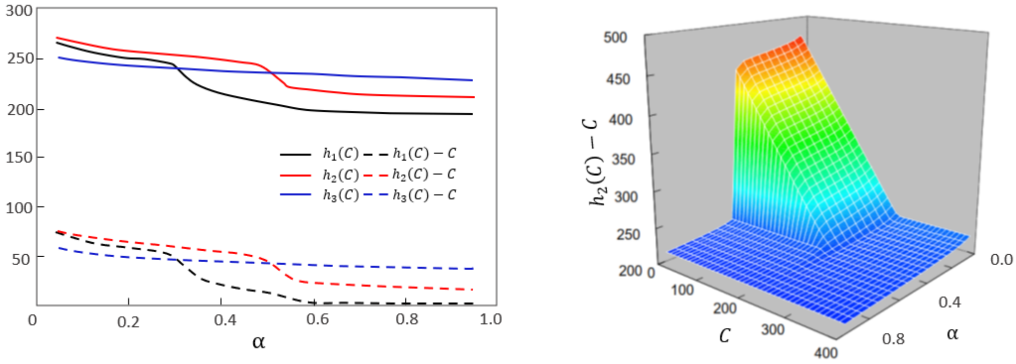

Figure 2.4: Sensitivity analysis of α and combinations of C and α

Study 3 This study considers the relationships between the optimal policy, warehouse capacity, and storage cost. The right-hand side of Figure 2.4 reports how the allocation capacity (i.e, C) and the storage cost (i.e., α) affect $h_2(C)$. In addition, testing $h_1(C)$, $h_1(C)-C$, and $h_2(C)-C$, we find that they have similar properties.

When the storage cost for the oversupplied resources is not small, the effect of the allocation capacity on $h_2(C)$ is negligible. Only when the storage cost is smaller than (approximately) 0.55, do the allocation capacity and $h_2(C)$ show relations resembling those shown on the right-hand side of Figure 2.3 in Study 2. However, when the allocation capacity is large (greater than approximately 260), $h_2(C)$ shows a slight increase as the storage cost decreases. However, $h_2(C)$ grows step wisely when the allocation capacity is less than approximately 260, and the smaller the allocation capacity is, the greater the $h_2(C)$ is.

Study 4 By the report we cited at the beginning of model development, the Hubei Provincial Health Commission makes a proportional allocation by the demand in the last period. After this, we call the existing allocation model a myopic allocation model (MAM). In this numerical study, we compare our foresighted allocation model (FAM) with MAM. We evaluate the total unsatisfied demands under both models involving three areas over a three-period horizon, i.e., $J=2$ and $n=3$. We define UD_{jt} as the unsatisfied demand of area j in period t as $(d_{jt}-a_{jt}-q_{jt})^+$. The total UD denotes the unsatisfied demand for all the allocated areas over the planning horizon. We use the superscript "M" to denote MAM. We set $S_t=300$ for $t=1,...,n$,

$\lambda = 0.25$, $\alpha = 0.3$, and $\beta = 0.7$. On the left side of Figure 2.5, $A = 1.08$, and on the right side of Figure 2.5, $A = 0.9$.

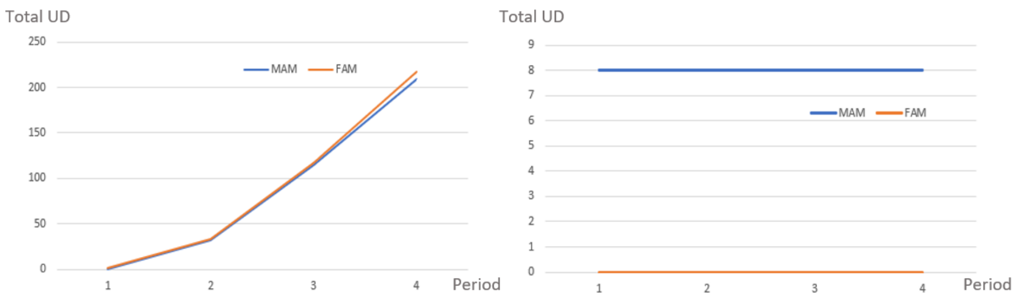

Figure 2.5: Total unsatisfied demands under MAM and FAM

The left side of Figure 2.5 illustrates the situation where $\frac{S_t}{\sum_{j=1}^{J} x_{jt}^M} \leq 1$. In each period, the total unsatisfied demand under MAM is less than that under FAM. As time goes by, the difference in the total unsatisfied demands of the two models increases. The right side of Figure 2.5 shows the case where $\frac{S_t}{\sum_{j=1}^{J} x_{jt}^M} \leq 1$. The total unsatisfied demand under MAM is always greater than that under FAM, because MAM only considers the deterministic demand, while FAM considers both the deterministic demands and random demands over the planning horizon.

2.6 Conclusions

In this study, we develop a time-dependent demand forecasting model that includes demand forecasting, extra demand from unsatisfied demand, and some unexpected cases as a basis for building the foresighted model to optimize medical resources allocation in response to epidemic outbreaks. We derive some important properties of the corresponding dynamic programming problem and obtain a general form of the foresighted allocation model in each period.

We build the foresighted allocation model as a stochastic dynamic programming problem. We divide the problem into several finite periods with the policy decision required in each period. The inventory of the medical resources and the probability distributions of the demands in the epidemic areas change among the periods, and the demand in each period is regarded as a stochastic variable. To solve the model, we provide a general form of the optimal allocation policy in each period to minimize the expected overall cost. The results of the numerical studies reveal several properties of the optimal policy. These results corroborate and supplement the analytical findings.

In addition, analytical and numerical analyses provide managerial implications for improving decisions on medical resources allocation in response to epidemic outbreaks. The relationship between the optimal policy and each parameter reflects the sensitivity analysis, that is, changes in the optimal policy in response to changes in each parameter, which shows the significant effect of the allocation capacity. It is affected by the local production, storage, and transport capacity. While medicines usually require strict conditions of storage and transport, storage, transport, and production capacity can hardly be expanded on short notice. When the capacity is insufficient, the optimal amount of medical resources allocated in each period decreases as the capacity increases, and the amount in the previous periods decreases faster. However, these changes are not significant if the cost of the unfulfilled demand is relatively small. Furthermore, in a situation where the epidemic disease continues to spread and the demand for medical resources continues to increase, the optimal amounts derived from the proposed dynamic stochastic model are not always less than those derived from the static model. The differences in the early periods are larger than those in the later periods.

Finally, there are some directions for future research. First, we consider the unexpected cases to follow an exponential distribution. In future research, we can consider it follows other kinds of distributions, like uniform or normal distribution. Second, in our model, we simplify that the allocation capacity is independent between different areas. Network-restricted capacity can be considered in future research. Third, we do not consider the private sector in our models. It is conceivable that governments may cooperate with the private sector to provide emergency medical supplies. Therefore, this work can be extended to include the private sector.

Chapter 3

Information Sharing and Coordination in a Vaccine Supply Chain

3.1 Introduction

Vaccination is one of the most cost-effective medical interventions to prevent seasonal influenza infection (CDC 2019). Unfortunately, the vaccination coverage is always undesirably low in real practice. On the supply side, production uncertainty is considered as the main cause of the low coverage (Deo and Corbett 2009). On the demand side, the positive externalities of vaccination, i.e., the indirect benefits accruing to other individuals, affect individuals' vaccination decisions (Galvani et al. 2007).

To safeguard the public against flu infection, the government is committed to increasing the overall flu vaccination coverage and enhancing the disease's prevention (Hong Kong Government News 2019). The government is usually one of the key parties in the vaccine supply chain. A typical vaccine supply chain consists of a manufacturer, a government that makes vaccine orders, and self-interested individuals (Adida et al. 2013, Arifoğlu et al. 2012). Meanwhile, in a vaccine market (e.g., Hong Kong), there exist not only public hospitals, but also private clinics, which the government cannot make orders for. Our interview with a public health researcher reveals that, in the vaccine market of Hong Kong, the government and private clinics make vaccine orders to manufacturers before the flu season and the orders are always satisfied. But vaccine shortage still happens in private clinics (On.cc 2020). To improve vaccination coverage, the government needs to cooperate with private clinics to eliminate supply shortage and stimulate vaccine demand.

For example, the Hong Kong government has implemented policies to improve vaccination coverage. The Centre for Health Protection of Hong Kong has introduced the Vaccination Programme for more than ten years. Under this programme, the priority group (e.g., all the citizens aged 65 or above) can take vaccination for free in public hospitals (Center of Health Protection 2020). However, limited by the total vaccine capacity of public hospitals, the vaccination coverage is not high enough to keep the whole population in a safe status (Capital 2019). Therefore, it is necessary for the government to encourage residents to take vaccination in private clinics. A few years ago, the Hong Kong government launched the "Vaccination Subsidy Scheme", under which the government provides subsidy to private clinics for the vaccines they have administered to qualified citizens. As a result, the private clinics ordered more vaccines and the vaccine price decreased, which stimulated people to take vaccination (Hong Kong Government News 2020a). In addition, in 2020, people were afraid of the double infections of COVID-19 and influenza. The private clinics in Hong Kong faced an unprecedented flu vaccine shortage (On.cc 2020). Some experts suggested that the government collaborated with private doctors and allocated vaccines to private clinics. According to the Medical Association, the Secretary for Food and Health approved an agreement with vaccine manufacturers to distribute part of the vaccine supply to private doctors (OnNews 2020). Given the vaccination subsidy scheme and government allocation scheme, it is unclear as to which scheme is better, in terms of private clinics' profitability and vaccination coverage. For example, the Hong Kong government has implemented policies to improve vaccination coverage. The Centre for Health Protection of Hong Kong has introduced the Vaccination Programme for more than ten years. Under this programme, the priority group (e.g., all the citizens aged 65 or above) can take vaccination for free in public hospitals (Center of Health Protection 2020). However, limited by the total vaccine capacity of public hospitals, the vaccination coverage is not high enough to keep the

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To study the effectiveness of the two schemes, we first model a vaccine system without information sharing as the benchmark. In this vaccine system, there are a profit-maximizing private clinic, a social-cost-minimizing public hospital, and self-interested individuals. Each of the public hospital and private clinic decides its vaccine inventory and vaccine price independently with no knowledge of the other's inventory and price information. We show that restricted by limited information and insufficient public health care resources, some problems emerge from the vaccine system without information sharing as follows: (1) The vaccine demand is fully satisfied by the public hospital and the private clinic cannot make any profit. As such, the private clinic has no incentive to order vaccines, which is adverse to the vaccine market's development. (2) The public hospital allocates too many medical resources to the vaccination programme, which might undermine the other parts of the public health care system.

We model the "Vaccination Programme" and the "Vaccination Subsidy Scheme" as a vaccine system with information sharing and subsidy, which resembles the vaccine market in Hong Kong. The public hospital only provides free vaccines to the priority group and decides the subsidy for the private clinic for the vaccines they have administered to qualified citizens. Observing the vaccine inventory in the public hospital and vaccine subsidy, the private clinic decides its vaccine inventory and vaccine price. We find that, in the vaccine system with information sharing and subsidy, the private clinic is willing to order vaccines and enter the vaccine market. When the vaccine subsidy is less than or equal to the vaccine cost, the profit-maximizing vaccine inventory cannot satisfy half of the demand of the non-priority group. As the range of the priority group decreases, the profit-maximizing vaccination coverage decreases and the profit-maximizing price increases. This is because when the range of the priority group decreases, some customers with high infection disutility that cannot get the vaccine in the public hospital are willing to pay a high price for the vaccine in the private clinic. The private clinic increases its vaccine price to maximize its profit and does not serve the customers with low infection disutility any more. Moreover, as the vaccine subsidy increases, the profit-maximizing inventory increases and the profit-maximizing price decreases. This implies that vaccine subsidy can stimulate vaccine supply and demand simultaneously. Besides, as the range of the priority group decreases, the socially optimal subsidy decreases and the vaccine demand of the non-priority group increases.

Moreover, we model the vaccine system with information sharing and allocation to study the effectiveness of this cooperation scheme. Under this scheme, the public hospital provides free vaccines to the priority group and the private clinic. The private clinic makes profit from administering the vaccines to qualified citizens, but the vaccine inventory and vaccine price of the private clinic are decided by the public hospital. In this vaccine system, the public hospital can increase the vaccine inventory in the private clinic in order to increase the supply and decrease the vaccine price to induce more demand, so increasing the vaccination coverage. The vaccination coverage is not affected by the range of the priority group because all the vaccines are ordered by the public hospital. As the vaccine cost increases, the socially optimal vaccine inventory decreases and the vaccine price increases.

Furthermore, we conduct a sensitivity analysis to study the effects of the vaccine cost. In the vaccine system without information sharing, both the socially optimal coverage of the public hospital and the profit-maximizing coverage of the private clinic decrease with the vaccine cost. This is because as the vaccine cost increases, the social cost increases and the profit of the private clinic decreases. The socially optimal coverage is always higher than the profit-maximizing coverage because the public hospital considers not only the profit from selling vaccines, but also the infection cost of the non-vaccinated individuals. In both vaccine systems with information sharing, the socially optimal coverage decreases with the vaccine cost, where the decreasing rate in the vaccine system with allocation is higher than that in the vaccine system with subsidy. This is because in the vaccine system with allocation, all the vaccines are ordered by the public hospital, while in the vaccine system with subsidy, the public hospital only orders vaccines for itself. So the socially optimal coverage in the vaccine system with allocation is more affected by the vaccine cost. When the vaccine cost is low, the socially optimal coverage in the vaccine system with allocation is higher than that in the vaccine system with subsidy. But when the vaccine cost is high, the socially optimal coverage in the vaccine system with allocation is lower than that in the vaccine system with subsidy.

We organize the rest of this study as follows: In Section 3.2 we review the related literature to position our study. In Section 3.3 we introduce the model and discuss the assumptions. In Section 3.4 we analyze the vaccine system without information sharing. In Section 3.5 we study the vaccine system with information sharing and subsidy. In Section 3.6 we model the vaccine system with information sharing and allocation. Finally, in Section 3.7, we conclude the study and suggest topics for future research. We provide the proofs of all the results in the Appendix B.

3.2 Literature Review

Vaccination is an important measure in public health policy through which the government seeks to achieve a high immunization level. However, due to supply uncertainties and insufficient incentives of taking vaccination, the vaccination coverage is often below the socially optimal level (Duijzer et al. 2018). Many studies consider government coordination in the vaccine market. To improve vaccination coverage, the manufacturer needs to produce adequate quantities of the vaccine (Deng et al. 2008). Chick et al. (2008) studied several types of contracts with the objective of maximizing the benefits of the government and the manufacturer simultaneously. Dai (2015) and Dai et al. (2016) indicated that the existing contracts do not consider the supply inefficiency resulting from late delivery. They proposed a new contract to coordinate vaccine supply with on-time delivery. Arifoğlu et al. (2012) studied the vaccine supply chain with rational consumer behavior. Self-interested individuals make vaccination decisions considering infection risks and vaccine prices. In view of the fact that giving subsidies to vaccinated individuals and taxing non-vaccinated individuals can induce vaccine demand (Brito et al. 1991). Demirci and Erkip (2020) adopted the bilevel programming approach to study the intervention problem for a vaccine market. Extending coordination to affect both the supply and demand sides, Adida et al. (2013) proposed a two-side subsidy mechanism depending on the vaccination coverage to achieve the socially optimal coverage. Arifoğlu and Tang (2022) studied the vaccine supply chain as a sequential game. They developed a two-sided incentive programme to eliminate the inefficiencies on both the supply and demand sides. However, there are few studies considering the private retailer in the vaccine market. We exclude production uncertainty, which is not a serious problem in some vaccine markets (e.g., Hong Kong), and consider the not-for-profit public hospital and for-profit private clinic as the vaccine retailers.

Vaccines are examples of public interest goods, whose demands are influenced by the related externalities and prices. The *positive externality effect*, i.e., vaccination not only protects the vaccinated people, but also decreases the infection probability of the non-vaccinated people by decreasing their contacts with the infected people, impacts consumers' vaccination decisions (Brito et al. 1991). Consumers are utility maximizing and forward-looking (Aviv and Pazgal 2008, Su and Zhang 2008). Self-interested individuals will compare the vaccine price with the expected infection cost and make vaccination decisions (Mamani et al. 2012). Pan et al. (2021) studied the effect of the free-riding behaviour on vaccination coverage, considering customer regret. Xie et al. (2021) analyzed the government subsidy on the R&D of vaccine products with a risk-averse buyer. Governments in developing countries often dictate the retail prices of subsidized food and drugs (Tuck and Lindert 1996), which is commonly assumed in the related studies on the vaccine market (see, e.g., Adida et al. 2013 and Arifoğlu et al. 2012). But in some vaccine markets (e.g., Hong Kong), the government cannot control the vaccine prices charged by private clinics. Therefore, similar to Erhun et al. (2008) and Cho and Tang (2013), we consider the case where the private clinic decides the vaccine price and faces a price-sensitive demand.

Research on public-private partnership has enabled a better understanding of the relationship between the public and private sectors. Public-private partnership refers to the cooperative relationship between the public and private sectors for efficient provision of public goods. Besley and Ghatak (2001) studied the ownership structure of public products between the public and private sectors, and proposed public-private cooperation. Kivleniece and Quelin (2012) determined the value creation based on a theoretical model of two conceptual public-private structural alternatives. Iossa and Martimort [2015] (2015) compared several existing incentives for public-private partnership and derived the optimal contract. They presented a basic model of procurement in a multi-task environment, in which a risk-averse firm makes non-contractible efforts on cost reduction and quality improvement. Berenguer et al. (2017) studied the effects of subsidy on for-profit and not-for-profit organizations in a vaccine market with uncertain demand. Lin et al. (2022) considered influenza vaccine supply chain coordination in a centralized system and a decentralized system. Besides subsidy, the government might also allocate the vaccine to private clinics to improve vaccine supply. Differing from the above papers, we study and compare both the subsidy and allocation mechanisms.

3.3 Modelling

We consider a population of N individuals. Infected individuals incur an expected infection disutility δ (Meltzer et al. 1999, Galvani et al. 2007), with probability density function $g(.)$ and cumulative probability function $G(.)$. Similar to Arifoğlu and Tang (2022), we consider δ follows an uniform distribution in $[0, \bar{\delta}]$. We assume that the vaccine is perfectly effective, i.e., all the vaccinated individuals are immunized against the infection (Brito et al. 1991, Arifoğlu et al. 2012). The non-vaccinated individuals may be infected with probability $P(f)$, which is continuous and non-increasing in $f \in [0, 1]$, the vaccinated fraction of the population. Similar to Brito et al. (1991), we assume that $P(.)$ is common knowledge. In the literature, the expected number of infected people is usually derived as $P(f) = \max\{1 - \frac{1}{R_0} - f, 0\}$[3.1] (Mamani et al. 2012). Referring to this estimation, we consider $\frac{d^2P(f)}{df^2} = 0$. Then the expected number of infected people in the population, i.e., $N(1-f)P(f)$, is a convex decreasing function of f. Individuals take vaccination in the public hospital or private clinic at different vaccine prices. We use r_h and r_c to represent the vaccine prices in the public hospital and private clinic, respectively. To exclude the case where no one is willing to pay for the vaccine in the public hospital or private clinic, we assume that the vaccine price in any case is less than $\bar{\delta}P(0)$. When people can take vaccination in either the public hospital or the private clinic, we assume that they will choose the vaccine that has a lower price. The public hospital operates for public health, which aligns with the government's objective, while the private clinic operates for profit. Thus, we consider in this study that the objective of the public hospital is to minimize the social cost, whereas the objective of the private clinic is to maximize its profit.

Governments in some countries or areas (e.g., Hong Kong) have launched different programmes to improve the vaccination coverage. We model and compare such programmes to derive management insights for vaccine market coordination. To explore the effectiveness of different measures to promote vaccination, we model a non-cooperative vaccine market shown in Figure 3.1 as the benchmark. The manufacturer charges the vaccine cost (per vaccine) c to the public hospital and private clinic. The public hospital (private clinic) makes decisions independently without sharing any information about its vaccine inventory and vaccine price to the private clinic (public hospital). Specifically, the sequence of events in the vaccine system without information sharing is as follows:

- Stage 1: The public hospital decides its vaccine inventory q_h and vaccine price r_h to minimize the social cost, without any information on q_c and r_c. Meanwhile, the private clinic decides its vaccine inventory q_c and vaccine price r_c to maximize its profit, without any information on q_h and r_h.

[3.1] R_0 represents the basic reproduction number and is a measure of the infectiousness of a disease (Anderson and May 1992, Murray 1993).

- Stage 2: Observing the vaccine prices in the public hospital and private clinic, individuals decide whether or not and where to take vaccination.

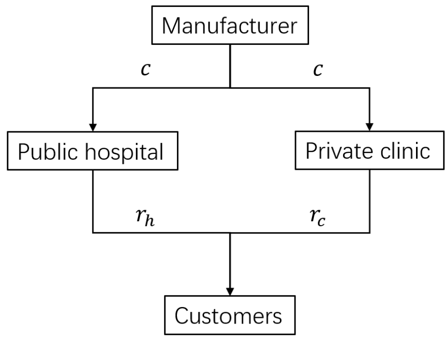

Figure 3.1: Non-cooperative vaccine market

Restricted by limited information sharing and insufficient public health care resources, the vaccine system without information sharing is hard to achieve the socially optimal vaccination coverage. The government needs to implement some policies to improve the vaccination coverage. For example, the Centre for Health Protection of Hong Kong has launched the Vaccination Programme for more than ten years, under which public hospitals provide free vaccines to the priority group rather than the non-priority group (Center of Health Protection 2020). It provides social benefits to the priority group while keeping the public hospitals from being overloaded. The priority group is characterized with a higher mortality and morbidity risk than the non-priority group (Meltzer et al. 1999, Galvani et al. 2007). To ensure analytical tractability, we prioritize individuals based on their infection disutility (Arifoğlu et al. 2012). We assume the individuals whose infection disutility is higher than β (i.e., $\delta \in [\beta, \bar{\delta}]$) are in the priority group. Otherwise, they are in the non-priority group. The public hospital orders $N[\bar{G}(\beta)]$ vaccines for the priority group. Individuals always prefer a vaccine that has a lower price. So all the individuals in the priority group, i.e., $\delta \in [\beta, \bar{\delta}]$, take free vaccination in the public hospital. The range of the priority group is restricted by the public hospital's capacity planning and the government's fiscal policy. Thus we do not discuss the decision on β in this study.

In addition to the policy for the priority group, the government proposes several cooperation schemes with the private clinic to improve the vaccination coverage for the non-priority group. The private clinic operates for profit and does not consider the social benefits. For example, in the last few years, the Hong Kong government has run the "Vaccination Subsidy Scheme" to stimulate vaccine supply and demand at the same time (Hong Kong Government News 2020b). Under the Vaccination Subsidy Scheme, individuals in the non-priority group can take vaccination in the private clinic at the vaccine price r_s and the private clinic receives $r_s + s$ per vaccine sold, where s is the subsidy per vaccine from the government. Figure 3.2 shows the vaccine system with information sharing and subsidy. The sequence of events is as follows:

- Stage 1: Given that the vaccine inventory in the public hospital is $N\bar{G}(\beta)$, the government decides the subsidy s per vaccinated person for the private clinic.
- Stage 2: With the information on the vaccine inventory in the public hospital and the subsidy, i.e., $N\bar{G}(\beta)$ and s, the private clinic decides the vaccine inventory q_s and vaccine price r_s.
- Stage 3: Given r_s, individuals make vaccination decisions. When the vaccine inventory in the private clinic is less than the vaccine demand, every individual in the non-priority group has the same probability of being vaccinated.

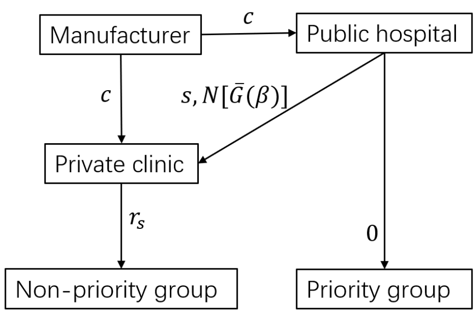

Figure 3.2: Vaccination subsidy scheme

In the past two years, people were afraid of the double infections of COVID-19 and influenza. Private clinics in Hong Kong faced an unprecedented flu vaccination shortage (On.cc 2020). Some experts suggested that the government worked with private doctors and allocated part of the vaccines to private clinics. According to the Medical Association in Hong Kong, the Secretary for Food and Health approved an agreement with vaccine manufacturers to distribute part of the vaccines to private clinics, but the clinics must comply with government regulations and must not increase the price of the vaccine (OnNews 2020). In this way, the non-priority group can take vaccination in cooperating private clinics at the price r_g set by the government. This scheme has not been proposed officially. We study this vaccine system with information sharing and allocation as depicted in Figure 3.3. The sequence of events is as follows:

- Stage 1: Given that the vaccine inventory in the public hospital is $N\bar{G}(\beta)$, the public hospital decides the vaccine inventory q_g and vaccine price r_g for the private clinic. The private clinic receives r_g per vaccinated person.
- Stage 2: With the information of vaccine price r_g, individuals make vaccination decisions. When the vaccine inventory in the private clinic is less than the vaccine demand, every individual in the non-priority group has the same probability of getting vaccinated.

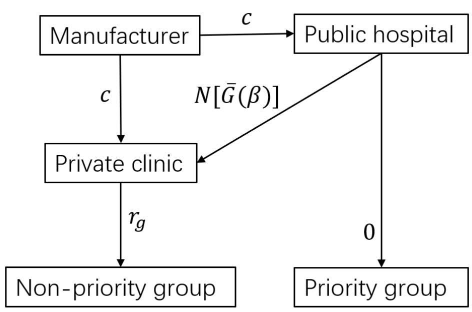

Figure 3.3: Government allocation scheme

Table 3.1 summarizes the notation in this study.

3.4 Vaccine System without Information Sharing

We model a vaccine system consisting of a public hospital that operates for public benefits, a profit-maximizing private clinic, and self-interest individuals that make

Table 3.1: Notations used throughout this study

δ	An individual's infection disutility.
$g(.)$	Probability density function of δ.
$G(.)$	Cumulative probability function of δ.
f	Vaccination coverage for the population.
$P(f)$	Infection probability for the non-vaccinated group with vaccination coverage f.
N	Number of people in the population.
c	Vaccine cost (per vaccine) charged by the manufacturer to the public hospital and private clinic.
r_i	Vaccine price in the non-cooperative vaccine market, where $i = h, c$ representing the public hospital and the private clinic, repectively.
q_i	Vaccine inventory in the non-cooperative vaccine market, where $i = h, c$.
s	The subsidy per vaccinated person from the government to the private clinic.
ϕ_j	The probability of being vaccinated in the vaccine system j, where $j = w, s, g$ referring the vaccine system without information sharing, with information sharing and subsidy, and with information sharing and allocation, respectively.
r_j	Vaccine price in the vaccine system j, where $j = s, g$.
q_j	Vaccine inventory in the vaccine system j, where $j = s, g$.
δ_i	The infection disutility of the marginal customer with respect to r_i, where $i = h, c$.
f_j	The vaccination coverage in the vaccine system j, where $j = w, s, g$.
SC_j	The social cost in the vaccine system j, where $j = w, s, g$.
π_j	The profit of the private clinic in the vaccine system j, where $j = w, s, g$.
δ_j	The infection disutility of the marginal customer with respect to r_j, where $j = s, g$.

their own vaccination decisions. There is no cooperation or information sharing between the public hospital and private clinic, and they make decisions without knowledge of the other's inventory and price information. First, each of the public hospital and private clinic independently decides its own vaccine inventory and vaccine price. Second, individuals make their own vaccination decisions. We use the subscript "w" to denote the situation of the non-cooperative vaccine market.

3.4.1 Individuals' Problem

In the second stage of the game, observing the vaccine prices of the private clinic and public hospital, each individual decides whether and where to take vaccination. An individual with infection disutility δ that decides not to take vaccination will be healthy with probability $1 - P(f)$. He will be infected with probability $P(f)$ and will cause infection disutility δ. His expected cost of not taking vaccination is $\delta P(f)$. So, an individual with infection disutility δ is willing to pay for the vaccine in the public hospital when

$$\delta P(f) \geq r_h. \tag{3.1}$$

An individual with infection disutility δ is willing to pay for the vaccine in the private clinic when

$$\delta P(f) \geq r_c. \tag{3.2}$$

Clearly, in equilibrium, if an individual with infection disutility $\hat{\delta}$ is not willing to pay for the vaccine in the public hospital (or in the private clinic), then none of the individuals with $\delta < \hat{\delta}$ is willing to pay for the vaccine in the public hospital (or in the private clinic). Therefore, the marginal customer that is indifferent to taking vaccination in the public hospital (or in the private clinic) satisfies the following condition

$$\delta_h P(\bar{G}(\delta_h)) = r_h \; (\delta_c P(\bar{G}(\delta_c)) = r_c), \tag{3.3}$$

where $\bar{G}(.) = 1 - G(.)$.

Given $P(.)$ and $G(.)$, r_h and r_c can be decided by δ_h and δ_c, respectively. To facilitate the presentation, we set δ_h and δ_c as decision variables of the public hospital and private clinic, respectively. Once δ_h and δ_c are settled, we can derive the vaccine prices.

Lemma 5. *In equilibrium, there is a unique δ_h (δ_c) at which all the individuals with $\delta > \delta_h$ ($\delta > \delta_c$) are willing to pay for the vaccine in the public hospital (or in the private clinic) and all the individuals with $\delta < \delta_h$ ($\delta < \delta_c$) are not willing to pay for the vaccine in the public hospital (or in the private clinic).*

Lemma 5 implies the existence and uniqueness of the marginal customer in the public hospital (or in the private clinic). The public hospital and private clinic make their decisions independently, so we cannot directly characterize the relationship between δ_h and δ_c. For clarity, we use i (j) to denote the public hospital or the private clinic with the lower (higher) vaccine price. Then we have

$$r_i < r_j,$$

and

$$\delta_i < \delta_j.$$

For an individual with infection disutility δ, his probability of being vaccinated is

$$\phi_w(\delta) = \begin{cases} \min\{1, \frac{q_i}{N[\bar{G}(\delta_i)]} + \frac{q_j}{N[\bar{G}(\delta_j)]}\} & \delta \geq \delta_j, \\ \min\{1, \frac{q_i}{N[\bar{G}(\delta_i)]}\} & \delta_i \leq \delta < \delta_j, \\ 0 & \delta \leq \delta_i. \end{cases} \tag{3.4}$$

For an individual with $\delta P(\bar{G}(\delta)) \geq r_j$, he first tries to take the vaccine at price r_i. If he does not get the vaccine at a lower price, he will try to take the vaccine at a higher price r_j. All the individuals that are willing to pay for the same type of vaccine have the same probability of being vaccinated. So the vaccination coverage is

$$f_w = \bar{G}(\delta_j)\min\{1, \frac{q_i}{N[\bar{G}(\delta_i)]} + \frac{q_j}{N[\bar{G}(\delta_j)]}\} + (G(\delta_j) - G(\delta_i))\min\{1, \frac{q_i}{N[\bar{G}(\delta_i)]}\}, \tag{3.5}$$

where the first term is the vaccination coverage from the customers that are willing to pay r_j for the vaccine and the second term is the vaccination coverage from the customers that are only willing to pay r_i for the vaccine.

3.4.2 Public Hospital's Problem

In the first stage of the non-cooperative market, the public hospital and private clinic make decisions before the flu season without information sharing. So both make decisions based on their own information and do not consider the inventory and price of the other. The public hospital decides the vaccine price, determined by δ_h, and the inventory q_h to minimize the social cost, which includes the profit of selling the vaccine and individuals' utility. So the public hospital's problem is as follows:

$$\begin{aligned} min\ SC_w(q_h, \delta_h) =& cq_h - r_h \min\{N\bar{G}(\delta_h), q_h\} + \int_0^{\delta_h} \delta N P(f_w) dG(\delta) \\ &+ \int_{\delta_h}^{\bar{\delta}} \delta(1-\phi_w) N P(f_w) dG(\delta), \end{aligned} \tag{3.6}$$

where the first term is the cost of the vaccine, the second term is the revenue of the sold vaccines, and the third term and the fourth term are individuals' infection disutility. The public hospital tries to minimize the social cost. Solving the optimization problem in Equation (3.6), we obtain the following result that characterizes the socially optimal decision for the public hospital.

Proposition 9. *The socially optimal inventory for the public hospital q_h^* is $N\bar{G}(\delta_h^*)$ and the socially optimal price r_h^* is $\delta_h^* P(\bar{G}(\delta_h^*))$, where $\delta_h^* \geq \frac{\bar{\delta}}{3}$ and satisfies* $g(\delta_h^*)P(\bar{G}(\delta_h^*))(3\delta_h^* - \bar{\delta}) + \frac{dP(\bar{G}(\delta_h^*))}{d\delta_h^*}\left(\int_0^{\delta_h^*} \delta dG(\delta) - \delta_h g(\delta_h^*)(\bar{\delta} - \delta_h^*)\right) - cg(\delta_h^*) = 0.$

Proposition 9 states the socially optimal inventory and price for the public hospital. It suggests that the socially optimal vaccine inventory would make at most two thirds of the population being vaccinated. The vaccine price affects the vaccine demand and makes it equal to the vaccine inventory. When the vaccine price is higher than r_h^*, as vaccine price increases, vaccine demand decreases and individuals' utility decreases. When vaccine price is lower than r_h^*, as the vaccine price decreases, the vaccine demand increases, the probability of being vaccinated for individuals with high infection disutility decreases, and the social cost increases. Besides, we obtain that δ_h^* increases in c. Clearly, as the vaccine cost increases, the socially optimal inventory decreases and the socially optimal vaccine price increases.

It is worth noting that, in real practice, the public hospitals in some cities provide free medical service to residents. So we consider the special case where the public hospital provides free vaccines and the corresponding problem is as follows:

$$min\ SC_w^{'}(q_h^{'}, \delta_h^{'}) = cN(\bar{G}(\delta_h^{'})) + \int_0^{\bar{\delta}} \delta(1-\phi)NP(f_w^{'})dG(\delta). \tag{3.7}$$

The following result characterizes the property of the socially optimal decision for free vaccines.

Lemma 6. *The socially optimal inventory for free vaccines is* $N[\bar{G}(\delta_h^{'*})]$, *where* $\delta_h^{'*}$ *satisfies* $c - \delta_h^{'*}P(\bar{G}(\delta_h^{'*})) < 0$.

Lemma 6 shows that free vaccines should be allocated to individuals with high infection disutility. Besides, the lowest expected infection cost for the vaccinated people should be higher than the vaccine cost.

3.4.3 Private Clinic's Problem

In the first stage of the game, the private clinic decides the vaccine inventory q_c and vaccine price δ_c to maximize its profit. The private clinic's problem is as follows:

$$\max \pi_w(q_c, \delta_c) = \delta_c P(\bar{G}(\delta_c)) \min\{q_c, N[\bar{G}(\delta_c)]\} - cq_c. \tag{3.8}$$

Solving the optimization problem (3.8), we characterize the profit-maximizing decision for the private clinic in the following result.

Proposition 10. *The profit-maximizing inventory for the private clinic* q_c^* *is* $N[\bar{G}(\delta_c^*)]$ *and the profit-maximizing price* r_c^* *is* $\delta_c^* P(\bar{G}(\delta_c^*))$, *where* $\delta_c^* \geq \frac{\bar{\delta}}{2}$ *and satisfies* $Ng(\delta_c)\left(P(\bar{G}(\delta_c))(\bar{\delta} - 2\delta_c) + \delta_c(\bar{\delta} - \delta_c)\frac{dP(\bar{G}(\delta_c))}{d\delta_c} + c\right) = 0.$

The profit-maximizing decision for the private clinic suggests that the vaccine inventory would make at most half of the population being vaccinated. The vaccine price affects the vaccine demand and makes it equal to the vaccine inventory. When the vaccine price is higher than r_c^*, as the vaccine price increases, the vaccine demand decreases and the profit decreases. When the vaccine price is lower than r_c^*, as the vaccine price decreases, the profit decreases. Besides, as the vaccine cost increases, the profit-maximizing inventory decreases and the profit-maximizing vaccine price increases. Comparing Proposition 9 with Proposition 10, we derive the following result.

Corollary 1. *In the non-cooperative market, the socially optimal vaccine inventory for the public hospital is higher than the profit-maximizing vaccine inventory for the private clinic, i.e.,* $q_h^* > q_c^*$*, and the socially optimal vaccine price is lower than the profit-maximizing vaccine price, i.e.,* $\delta_h^* < \delta_c^*$.

Corollary 1 provides the relationship between the socially optimal decision for the public hospital and the profit-maximizing decision for the private clinic. Clearly, the public hospital considers not only profit but also individuals' utility. So the socially optimal vaccination coverage is higher than the profit-maximizing coverage. It implies that in the vaccine market without the public hospital, the vaccination coverage would not be socially optimal. By Equation (3.5), Propositions 9 and 10, and Corollary 1, we derive the vaccination coverage in the non-cooperative vaccine market as

$$f_w = \bar{G}(\delta_h^*).$$

From the above analyses, we find several problems in the vaccine system without information sharing as follows:

(1) In the vaccine system without information sharing, all the vaccine demand will be satisfied by the public hospital and the private clinic cannot make any profit. So the private clinic has no incentive to order vaccines, which is adverse to the vaccine market's development.

(2) The public hospital allocates too many medical resources to the vaccination programme, which might undermine the other parts of the public health care system.

To deal with these problems, we study several types of cooperation schemes in the vaccine market in the following.

3.5 Vaccine System with Information Sharing and Subsidy

As discussed in Section 3.3, some countries or cities (e.g., Hong Kong) now implement the vaccination programme under the vaccination subsidy scheme. Because of their limited capacity, the public hospitals in Hong Kong only order a small quantity of vaccines and provide free vaccines to the high-risk group, i.e., the priority group. The quantity of free vaccines is limited by the public hospitals' capacity planning and the government's fiscal policy, which does not change every year. So we do not discuss the decision on the range of the priority group in this study. Besides, the government provides subsidy to private clinics as an incentive for them to order vaccines and serve the vaccine market. We use the subscript "s" to denote the vaccine system with information sharing and subsidy. We model the vaccination subsidy scheme in the following steps: First, given the priority group $N[\bar{G}(\beta)]$, the public hospital decides the subsidy per vaccine s for the private clinic. Second, the private clinic decides the vaccine inventory and vaccine price. Third, individuals make their own vaccination decisions. We use backward induction to characterize the equilibrium of this three-stage game.

3.5.1 Individuals' Problem

In the third stage of the game, all the individuals in the priority group, i.e., $\delta \geq \beta$, take free vaccination in the public hospital. Observing the vaccine price, the non-priority individuals decide whether or not to take vaccination in the private clinic. By Lemma 5, we find that the infection disutility of the marginal customer δ_s under the vaccination subsidy scheme satisfies the following condition

$$\delta_s P(\bar{G}(\delta_s)) = r_s. \tag{3.9}$$

For an individual with infection disutility δ, his probability of being vaccinated is

$$\phi_s(\delta) = \begin{cases} 1 & \beta \leqslant \delta, \\ \min\{1, \frac{q_s}{N[\bar{G}(\delta_s)-\bar{G}(\beta)]}\} & \delta_s \leq \delta < \beta. \end{cases} \tag{3.10}$$

A non-priority individual with $\delta_s \leq \delta < \beta$ is willing to pay r_s for the vaccine in the private clinic. So the vaccination coverage is

$$f_s = \min\{\bar{G}(\beta) + \frac{q_s}{N}, \bar{G}(\delta_s)\}, \tag{3.11}$$

where the first term is the normalized total vaccine supply in the public hospital and private clinic, and the second term is the normalized total vaccine demand.

3.5.2 Private Clinic's Problem

In the second stage under the vaccination subsidy scheme, observing the public hospital's vaccine subsidy, the private clinic decides its vaccine inventory q_s and vaccine price, determined by δ_s, to maximize its profit. The public hospital gives the private clinic s per vaccine sold. So the private clinic gets $r_s + s$ per vaccine sold. It follows that the private clinic's problem is as follows:

$$\max \pi_s(q_s, \delta_s) = (r_s + s) \min\{q_s, (N[\bar{G}(\delta_s) - \bar{G}(\beta)])^+\} - cq_s, \tag{3.12}$$

where the first term is the total revenue of the sold vaccines and the second term is the total vaccine cost.

Solving the optimization problem (3.12), we obtain the following result.

Lemma 7. *The profit-maximizing decision for the private clinic satisfies the condition* $\delta_s^* < \beta$*.*

Lemma 7 implies that, under the vaccination subsidy scheme, the private clinic is willing to order vaccines and enter the vaccine market. The marginal customer is in the non-priority group. The following result characterizes the profit-maximizing decision for the private clinic under the vaccination subsidy scheme.

Proposition 11. *The profit-maximizing vaccine inventory* q_s^* *is* $N[\bar{G}(\delta_s^*) - \bar{G}(\beta)]$ *and the profit-maximizing vaccine price* r_s^* *is* $\delta_s^* P(\bar{G}(\delta_s^*))$*. When* $c - s \geq 0$, $\frac{\beta}{2} \leq \delta_s^* \leq \beta$ *and satisfies* $P(\bar{G}(\delta_s))(\beta - 2\delta_s) + \delta_s(\beta - \delta_s)\frac{dP(\bar{G}(\delta_s))}{d\delta_s} + c - s = 0$*; when* $c - s < 0$, δ_s^* *might be smaller than* $\frac{\beta}{2}$*.*

Proposition 11 illustrates that the profit-maximizing decision achieves the equilibrium between vaccine inventory and vaccine demand. In this case, the profit-maximizing vaccination coverage achieves $\bar{G}(\delta_s^*)$. When the vaccine subsidy is less than or equal to the vaccine cost, i.e., $c - s \geq 0$, the profit-maximizing vaccine inventory cannot satisfy half of the demand of the non-priority group. As β increases, δ_s^* increases. This implies that as the range of the priority group decreases, the profit-maximizing vaccination coverage decreases and the profit-maximizing price increases. This is because when the range of the priority group decreases, some customers with high infection disutility that cannot get the vaccine in the public hospital are willing to pay a high price for the vaccine in the private clinic. The private clinic increases its vaccine price to maximize its profit and does not serve customers with low infection disutility any more. When the vaccine subsidy is larger than the vaccine cost, i.e., $c - s < 0$, the profit-maximizing vaccine inventory can satisfy more than half of the demand of the non-priority group. As the vaccine subsidy increases, the profit-maximizing inventory increases and the profit-maximizing price decreases. This shows that vaccine subsidy can stimulate vaccine supply and demand simultaneously.

Corollary 2. *Comparing Proposition 10 and Proposition 11, we have*

$$\delta_s^* \leq \delta_c^*. \tag{3.13}$$

Clearly, under the vaccination subsidy scheme, the infection disutility of the marginal customer in the private clinic is lower than that under the vaccine system without information sharing, and the vaccine price in the private clinic will be lower than or equal to that in the vaccine system without information sharing, i.e., $r_s^* \leq r_c^*$.

3.5.3 Public Hospital's Problem

By Proposition 11, we see that the vaccine subsidy affects the profit-maximizing decision. The public hospital decides the vaccine subsidy s to minimize the social cost. So the public hospital's problem is

$$min\ SC_s(s) = sV_c + \int_0^{\delta_s} \delta NP(f)dG(\delta) + \int_{\delta_s}^{\beta} \delta(1-\phi_s)NP(f)dG(\delta), \tag{3.14}$$

where the first term is the total subsidy for the private clinic, and the second and third terms are individuals' infection costs.

Given q_s^* in Proposition 11, the objective function is changed to

$$min\ SC_s(s) = sq_s^* + \int_0^{\delta_s^*} \delta NP(f)dG(\delta). \tag{3.15}$$

The following result characterizes the socially optimal decision.

Proposition 12. *The socially optimal subsidy for the government s^* satisfies the condition*

$$N[\bar{G}(\delta_s^*) - \bar{G}(\beta)] + N\frac{d\delta_s^*}{ds}\left(\frac{dP(f)}{d\delta_s^*}\int_0^{\delta_s^*} \delta dG(\delta) + P(f)\delta_s^* g(\delta_s^*)\right) = 0. \tag{3.16}$$

Proposition 12 shows the socially optimal subsidy. As β increases, s^* decreases. This indicates that as the range of the priority group decreases, the vaccine demand in the non-priority group increases, so the socially optimal subsidy decreases. In this case, the public hospital provides the subsidy $s(N[\bar{G}(\delta_s) - \bar{G}(\beta)])$ and charges the vaccine cost $c(N[\bar{G}(\beta)])$ to achieve the vaccination coverage $\bar{G}(\delta_s)$.

Comparing Proposition 9 with Proposition 12, we derive the following result.

Corollary 3. *If $N[\bar{G}(\beta)-\bar{G}(\delta_s^*)]\frac{ds}{d\delta_s^*}-Ng(\delta_s^*)\left(P(\bar{G}(\delta_s^*))(\bar{\delta} - 2\delta_s^*) + \delta_s^*(\bar{\delta} - \delta_s^*)\frac{dP(\bar{G}(\delta_s^*))}{d\delta_s^*} + c\right) \leq 0$, $\delta_h^* \geq \delta_s^*$; otherwise, $\delta_h^* < \delta_s^*$.*

Corollary 3 compares the socially optimal vaccine price in the non-cooperative vaccine market with that in the vaccine system with information sharing and subsidy. The result is affected by the range of the priority group and the vaccine cost.

3.6 Vaccine System with Information Sharing and Allocation

In this section we consider the case where the public hospital orders vaccines for the high-risk group, i.e., the priority group. Besides, the public hospital also orders and allocates vaccines to the private clinic. The private clinic does not order any vaccine, but it can make profit by providing vaccinations to individuals at the vaccine price decided by the public hospital. We use the subscript "g" to denote the vaccine system with information sharing and allocation. In the first stage, the public hospital decides the vaccine inventory and vaccine price for the private clinic. In the second stage, individuals make vaccination decisions.

3.6.1 Individuals' Problem

In the second stage, observing the vaccine price, non-priority individuals decide whether or not to take vaccination in the private clinic. Similar to Equation (3.9), the infection disutility of the marginal customer under the government allocation scheme satisfies the following condition

$$\delta_g P(\bar{G}(\delta_g)) = r_g. \tag{3.17}$$

Individuals in the priority group, i.e., $\delta \geq \beta$, take vaccination for free in the public hospital. Non-priority individuals with $\delta_g \leq \delta < \beta$ are willing to pay r_g for vaccination in the private clinic. For an individual with infection disutility δ, his probability of being vaccinated is

$$\phi_g(\delta) = \begin{cases} 1 & \beta \leqslant \delta, \\ \min\{1, \frac{q_g}{N[\bar{G}(\delta_g) - \bar{G}(\beta)]}\} & \delta_g \leq \delta < \beta. \end{cases} \tag{3.18}$$

The vaccination coverage is

$$f_g = \min\{\bar{G}(\beta) + \frac{q_g}{N}, \bar{G}(\delta_g)\}. \tag{3.19}$$

The profit of the private clinic is

$$\pi_g = r_g \min\{q_g, (N[\bar{G}(\delta_g) - \bar{G}(\beta)])^+\}. \tag{3.20}$$

3.6.2 Public Hospital's Problem

In the first stage of the game, the public hospital decides q_g and δ_g for the private clinic to minimize the social cost. So the public hospital's problem is

$$min\ SC_g(q_g, \delta_g) = c(N[\bar{G}(\beta)] + q_g) + \int_0^{\delta_g} \delta NP(f_g)dG(\delta) + \int_{\delta_g}^{\beta} \delta(1 - \phi_g)NP(f_g)dG(\delta), \tag{3.21}$$

where the first term is the total vaccine cost in the public hospital and private clinic, and the second and third terms are individuals' infection costs.

Solving the optimization problem (3.21), we characterize the socially optimal decision for the vaccine system with information sharing and allocation in the following result.

Proposition 13. *The socially optimal inventory is* $N[\bar{G}(\delta_g^*)-\bar{G}(\beta)]$ *and the socially optimal price is* $P(\bar{G}(\delta_g^*))\delta_g^*$*, where* δ_g^* *satisfies* $c+\left(\frac{dP(\bar{G}(\delta_g^*))}{d\bar{G}(\delta_g^*)}\int_0^{\delta_g^*}\delta dG(\delta) - P(\bar{G}(\delta_g^*))\delta_g^*\right) = 0$.

Proposition 13 implies that the vaccination coverage under the government allocation scheme is $\bar{G}(\delta_g)$. The vaccination coverage is not affected by β. This is because all the vaccines are ordered by the public hospital. As c increases, δ_g increases. This indicates that as the vaccine cost increases, the socially optimal vaccine inventory decreases and the vaccine price increases. In total, the public hospital spends $cN[\bar{G}(\delta_g)]$ and achieves vaccination coverage $\bar{G}(\delta_g)$. Comparing Proposition 9 with Proposition 13, we derive the following result.

Corollary 4. *If* $P(\bar{G}(\delta_g^*))(\bar{\delta}-2\delta_g^*)+\delta_g^*(\bar{\delta}-\delta_g^*)\frac{dP(\bar{G}(\delta_g^*))}{d\delta_g^*}\leq 0$*,* $\delta_g^*\leq\delta_h^*$*; otherwise,* $\delta_g^*>\delta_h^*$.

Under the vaccination subsidy scheme, the public hospital provides the subsidy $s(N[\bar{G}(\delta_s)-\bar{G}(\beta)])$ and charges the vaccine cost $c(N[\bar{G}(\beta)])$ to achieve the vaccination coverage $\bar{G}(\delta_s)$. Under the government allocation scheme, the public hospital spends $cN[\bar{G}(\delta_g)]$ and achieves the vaccination coverage $\bar{G}(\delta_g)$. Comparing δ_s^* in Proposition 12 with δ_g^* in Proposition 13, we derive the following result.

Corollary 5. *If* $\frac{ds}{d\delta_s^*}(\bar{G}(\beta)-\bar{G}(\delta_s^*))\leq cNg(\delta)$*,* $\delta_s^*\leq\delta_g^*$*; otherwise,* $\delta_s^*>\delta_g^*$*, where* $\frac{ds}{d\delta_s}=-2P(\bar{G}(\delta_s))+2(\beta-2\delta_s)\frac{dP(\bar{G}(\delta_s))}{d\delta_s}+\delta_s(\beta-\delta_s)\frac{d^2P(\bar{G}(\delta_s))}{d(\delta_s)^2}$.

Corollary 5 compares the socially optimal vaccine price in the vaccine system with information sharing and subsidy with that in the vaccine system with information sharing and allocation. The result is affected by the range of the priority group and the vaccine cost. Given that $\frac{ds}{d\delta_s}\leq 0$, as the vaccine subsidy increases, the vaccination coverage increases.

3.6.3 Sensitivity Analysis

In this subsection we test the effects of the vaccine cost on the socially optimal coverage and the profit-maximizing coverage in different vaccine systems. Following Mamani et al. (2012) and Adida et al. (2013), we assume that $g(.)$ follows a uniform distribution and $P(f) = 1 - f - \frac{1}{R_0}$, where $R_0 = 2$ (Wikipedia 2018). Following Arifoğlu et al. (2012), we assume $\bar{\delta} = 100$. We set $\beta = 0.6\bar{\delta}$ and vary c. The result of the vaccine system without information sharing is shown in Figure 3.4 and the result of the vaccine systems with information sharing is shown in Figure 3.5.

From Figure 3.4, we see that both the socially optimal coverage of the public hospital and the profit-maximizing coverage of the private clinic decrease with the vaccine cost. As the vaccine cost increases, the social cost increases and the profit of the private clinic decreases. The socially optimal coverage is always higher than the profit-maximizing coverage. The public hospital considers not only the profit from selling vaccines, but also the infection cost of the non-vaccinated individuals.

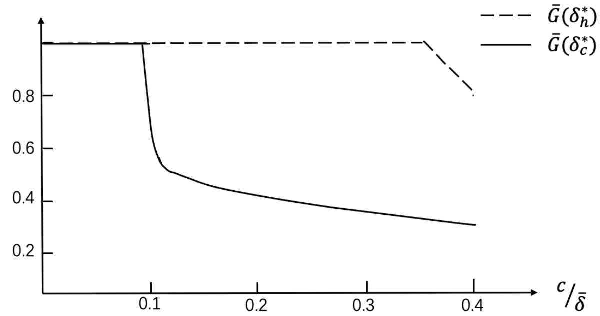

Figure 3.4: Sensitivity analysis of the vaccine system without information sharing

Figure 3.5 shows the effects of the vaccine cost on the vaccine system with information sharing and subsidy, and on the vaccine system with information sharing and allocation. In both vaccine systems, the socially optimal coverage decreases with the vaccine cost, where the decreasing rate in the vaccine system with allocation is higher than that in the vaccine system with subsidy. This is because in the vaccine system with allocation all the vaccines are ordered by the public hospital, whereas in the vaccine system with subsidy the public hospital only orders vaccines for itself. So the socially optimal coverage in the vaccine system with allocation is affected more by the vaccine cost. When the vaccine cost is low, the socially optimal coverage in the vaccine system with allocation is higher than that in the vaccine system with subsidy. But when the vaccine cost is high, the socially optimal coverage in the vaccine system with allocation is lower than that in the vaccine system with subsidy.

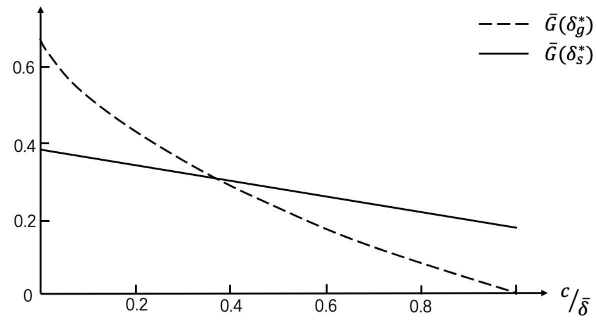

Figure 3.5: Sensitivity analysis of the vaccine systems with information sharing

3.7 Conclusions

In this study we build models of a vaccine system to study the cooperation between the government and a private clinic for vaccination. In the vaccine system, there are a profit-maximizing private clinic, a public hospital that seeks to minimize the social cost, and self-interested individuals. We construct three models including a vaccine system without information sharing, a vaccine system with information sharing and subsidy, and a vaccine system with information sharing and allocation.

In the vaccine system without information sharing, the public hospital and private clinic decide the vaccine inventories and vaccine prices independently. Restricted by limited information sharing and insufficient public health care resources, some problems emerge in the vaccine system without information sharing. First, all the vaccine demand will be satisfied by the public hospital and the private clinic cannot make any profit. So the private clinic has no incentive to order vaccines, which is adverse to the vaccine market's development. Second, the public hospital allocates too many medical resources to vaccination, which might undermine the other parts of the public health care system.

In the vaccine system with information sharing and subsidy, we consider the

"Vaccination Programme" and the "Vaccination Subsidy Scheme" based on real practice. The public hospital only provides free vaccines to the priority group. The public hospital decides the subsidy for the sold vaccines for the private clinic. Observing the vaccine inventory in the public hospital and vaccine subsidy, the private clinic decides its own vaccine inventory and vaccine price.

We find that, in the vaccine system with information sharing and subsidy, the private clinic is willing to order vaccines and enter the vaccine market. When the vaccine subsidy is less than or equal to the vaccine cost, the profit-maximizing vaccine inventory cannot satisfy half of the demand of the non-priority group. As the range of the priority group decreases, the profit-maximizing vaccination coverage decreases and the profit-maximizing price increases. This is because when the range of the priority group decreases, some customers with high infection disutility that cannot get the vaccine in the public hospital are willing to pay a high price for the vaccine in the private clinic. The private clinic increases its vaccine price to maximize its profit and does not serve customers with low infection disutility any more. As the vaccine subsidy increases, the profit-maximizing inventory increases and the profit-maximizing price decreases. This implies that vaccine subsidy can stimulate vaccine supply and demand simultaneously. Besides, as the range of the priority group decreases, the vaccine demand of the non-priority group increases and the socially optimal subsidy decreases.

Following real practices, we further model the vaccine system with information sharing and allocation. The public hospital provides free vaccines to the priority group and private clinic. The private clinic makes profit from selling vaccines, where the vaccine inventory and vaccine price of the private clinic are decided by the public hospital. In this vaccine system, the public hospital can increase the vaccine inventory of the private clinic to increase the supply and decrease the vaccine price to increase the demand, thereby increasing the vaccination coverage. The vaccination coverage is not affected by the range of the priority group because all the vaccines are ordered by the public hospital. As the vaccine cost increases, the socially optimal vaccine inventory decreases and the vaccine price increases.

We also conduct a sensitivity analysis of the effects of the vaccine cost. In the vaccine system without information sharing, both the socially optimal coverage of the public hospital and the profit-maximizing coverage of the private clinic decrease with the vaccine cost. As the vaccine cost increases, the social cost increases and the profit of the private clinic decreases. The socially optimal coverage is always higher than the profit-maximizing coverage because the public hospital considers not only the profit from selling vaccines, but also the infection cost of the non-vaccinated individuals. In both vaccine systems with information sharing, the socially optimal coverage decreases with vaccine cost, where the decreasing rate in the vaccine system with allocation is higher than that in the vaccine system with subsidy. This is because in the vaccine system with allocation, all the vaccines are ordered by the public hospital, while in the vaccine system with subsidy, the public hospital only orders vaccines for itself. So the socially optimal coverage in the vaccine system with allocation is more affected by the vaccine cost. When the vaccine cost is low, the socially optimal coverage in the vaccine system with allocation is higher than that in the vaccine system with subsidy. But when the vaccine cost is high, the socially optimal coverage in the vaccine system with allocation is lower than that in the vaccine system with subsidy.

In this study we assume that the public hospital and private clinic provide the same product and service. For future research, it would be interesting to consider the differences in service quality between the public hospital and private clinic. Besides, future research should also consider the cost of searching for vaccines in the vaccine system.

Chapter 4

Communicating Environmental Quality to Consumers: Impacts of Label Confusion and Blockchain-Based Transparency

4.1 Introduction

Consumers are increasingly aware of sustainability. According to a survey conducted by Accenture, 72% of respondents buy more environmentally sustainable products today than before, and 81% of respondents expect to buy more green products in the future (Accenture 2019). Consumers' sustainability awareness may translate into higher willingness-to-pay for environmentally friendly products. Indeed, some consumers are willing to buy green products at a premium (Miremadi et al. 2012, Nielsen 2015). The survey by Accenture (2019) also shows that 36% of consumers are willing to pay extra for a product made from recycled material.

With consumers growing more aware of the environmental impact of products and services, firms are making efforts to improve their sustainable performance. Such efforts often involve higher costs but allow firms to meet the needs of environmentally conscious consumers and build a competitive advantage. For example, Coca-Cola uses recycled material in its packaging: its plastic bottles, cans, and glass bottles contain 25% recycled plastic, 49% recycled aluminum, and 56% recycled glass, respectively (Coca-Cola Journey 2017). Adidas has partnered with the non-profit organization Parley for the Oceans to launch sportswear made from recycled ocean waste since 2015 (Aziz 2018). IKEA used recycled material in 10% of its products in 2018 (Ringstrom 2018).

Unlike conventional quality attributes, a product's environmental attributes cannot be observed or experienced by consumers (Baksi and Bose 2007). Indeed, environmental attributes are usually firms' private information. For this reason, environmental labels (also called ecolabels or green labels) have emerged. Environmental labels provide sustainability information about a product or service, such as its material, recyclability, packaging, or level of energy consumption (ISO 2019). They function as an important marketing tool for firms to communicate their sustainable efforts to consumers, and they also help consumers make more environmentally conscious purchasing decisions. In practice, green labels can help consumers who care about the environment but lack expertise evaluate a product's sustainability. Consumers want green labels on products and are willing to pay extra for labeled products (PEFC News 2014, UL Environment UL Environment).

However, the proliferation of labels leads to label confusion among consumers. Ecolabel Index, a global directory of ecolabels, currently identifies 455 ecolabels in 25 industry sectors (Ecolabel Index 2022). Through analyzing a dataset between 1970 and 2012 covering 197 countries, Gruère (2013) states that the proliferation of environmental labels and information schemes causes consumer misperceptions. Indeed, 91% of Europeans believe that product labels do not provide enough information (59%) or provide unclear information (32%) about the environmental impact of products (European Commission 2013). Moreover, according to the French Institute of Public Opinion, 91% of French people say that labels are useful in guiding their purchases, but half find it difficult to distinguish between various labels with different standards (de Malleray 2022). Thus, while environmental labels have the potential to disclose improtant information about firms' sustainable efforts, consumers may be unfamiliar with or confused by them, especially given the presence of numerous labels with different standards.

With the increasing use of blockchain technology, blockchain-based transparency is attracting attention as a potential solution to the dilemma caused by label confu-

sion. Blockchain technology improves the information transparency within supply chains and is able to reliably reveal firms' environmental efforts to consumers (see Shen et al. 2022 for evidence on the reliability of such disclosure). Several companies have adopted this approach. For example, Patagonia, a well-known outdoor clothing and gear company, has a strong commitment to sustainability and has been a pioneer in the outdoor industry, focusing on environmentally-friendly practices and promoting responsible consumption. Patagonia uses environmental labels on their products to inform consumers about their sustainability initiatives. These labels may include information about the materials used, certifications obtained (such as Fair Trade, Bluesign, or Organic certifications), and the carbon footprint of the product. Nowadays, to communicate their sustainability efforts transparently and effectively to consumers, they have implemented blockchain technology in their supply chain. By using blockchain, Patagonia can track and verify the entire lifecycle of their products, from sourcing raw materials to manufacturing and distribution, thereby improving the quality transparency (Forbes 2018, Patagonia 2023). Beauty brand Tropic Skincare uses blockchain technology to improve shoppers' understanding of its social and environmental impact (Provenance 2022). Blockchain-based transparency has also been used to indicate whether a fish sold at a fish market comes from a sustainable fisherman or whether a bag of coffee comes from a fair trade producer (Futurethinkers 2022). Additionally, blockchain-based transparency provides accurate information about environmental impacts. For instance, two Japanese companies, Teijin Ltd. and Fujitsu Ltd., have launched a project that uses blockchain technology and a life cycle assessment calculation method to provide accurate information about recycled materials' environmental quality, e.g., data on greenhouse gas emissions (Recycling Today 2022). Similarly, TRACKCYCLE (a joint project launched by TotalEnergies, Circulor, Innovate UK, and Recycling Technologies) uses blockchain technology to provide a traceable and accurate record of recycled materials, thus ensuring full visibility on the provenance and quality of recycled materials (Plastics Today 2021).

Motivated by the above observations, this study examines how a sustainable firm should communicate its environmental quality to consumers in a competitive market and how the means of communication affects the firm's environmental quality when there exists label confusion among consumers. The research questions are as follows: First, given the existence of label confusion, how should a sustainable firm decide its level of environmental quality when using labels to communicate its environmental efforts? Second, can blockchain-based transparency benefit a sustainable firm more than environmental labels, and if so, under which conditions? Third, what are the effects of blockchain-based transparency on a sustainable firm's environmental quality and a non-sustainable firm's profit?

To answer the above research questions, we develop a game-theoretic model with a sustainable firm and a non-sustainable firm. The sustainable firm offers an eco-friendly product with some level of environmental quality, while the non-sustainable firm sells a regular product without any environmental quality. Given that environmental quality is a credence attribute and cannot be directly observed or experienced by consumers, the sustainable firm needs to communicate its environmental quality to consumers via either environmental labels or blockchain-based transparency. In the case of environmental labels, a fraction of consumers are confused about label standards and may underestimate or overestimate the sustainable firm's environmental quality; whereas the actual quality is revealed to all consumers under blockchain-based transparency. By comparing the performance of these two means of communication when some consumers are confused about label standards, our model provides novel insights into the operational issues faced by sustainable firms.

We highlight several main findings. *First, under environmental labels, as the fraction of consumers who are confused about label standards increases, the sustainable firm may switch from a high-tier label to a low-tier label when the fraction of confused consumers is sufficiently high, but may counterintuitively switch from a low-tier label to a high-tier label when the fraction of confused consumers is moderate or low.* When there are enough confused consumers in the market, the sustainable firm prefers to serve both informed and confused consumers, regardless of the prevailing label. In this case, the firm is damaged by confused consumers' underestimation of its environmental quality when there is a high-tier label on the product, but benefits from their overestimation when there is a low-tier label on the product. Thus, the sustainable firm prefers a low-tier label as the fraction of confused consumers increases. By contrast, when there is a moderate or small number of confused consumers in the market, the sustainable firm may serve a single type of consumers (either confused or informed). In this case, a high-tier label helps soften market competition by leading the two competing firms to target different types of consumers, whereas a low-tier label may intensify competition by inducing both firms to rely on confused consumers. Consequently, the sustainable firm may counterintuitively switch from a low-tier label to a high-tier label as the fraction of confused consumers increases.

Second, the sustainable firm does not always prefer blockchain-based transparency over environmental labels. In particular, if blockchain adoption is free, the cost of environmental quality is low, and there is a great number of confused consumers, then the sustainable firm prefers blockchain-based transparency when the fraction of confused consumers is relatively small and prefers environmental labels otherwise. This is because, when the fraction of confused consumers is large, the sustainable firm prefers a low-tier label when using ecolabels to communicate its environmental quality, in which case it benefits from confused consumers' overestimation. However, such benefit is impractical under blockchain-based transparency. Thus, the sustainable firm may or may not make a higher profit under blockchain-based transparency than under environmental labels. Given that environmental labels provide partial information about the sustainable firm's environmental quality for confused consumers, while blockchain-based transparency allows all consumers to gain full information, the second result indicates that *full information is not necessarily more beneficial than partial information for the sustainable firm.*

Third, when the sustainable firm prefers blockchain-based transparency to environmental labels, the sustainable firm may improve or reduce environmental quality, and the non-sustainable firm may be better or worse off. Blockchain-based transparency allows the sustainable firm to flexibly choose the desired environmental quality. However, such flexibility may lead the firm to reduce its quality level, especially when environmental quality is costly. Moreover, while the non-sustainable firm may benefit from the sustainable firm's adoption of blockchain when the cost of environmental quality is low, it may suffer negative consequences when environmental quality is costly, in which case, the sustainable firm reduces its quality, causing the difference between the two products to shrink and the non-sustainable firm's competitive advantage stemming from the absence of environmental costs to decrease. Lastly, we identify the conditions under which blockchain adoption can lead to a win-win-win situation, wherein both firms make higher profits and the sustainable firm provides a higher quality level than those under environmental labels.

The rest of this study is organized as follows. In Section 4.2, we review the relevant literature. Section 4.3 introduces the model. Then, we solve the model and derive the main results in Section 4.4. Section 4.5 concludes the study. All proofs and technical details are presented in the Appendix C.

4.2 Literature Review

This study is closely related to two literature streams: certification/labeling and blockchain technology adoption.

This study contributes to the literature on voluntary environmental certification/labeling in economics and operations areas (e.g., Heyes and Maxwell 2004, Bottega and De Freitas 2009, Yenipazarli 2015, Plambeck and Taylor 2019, Lim et al. 2019). Many works in this stream consider competitive settings. For example, Ben Youssef and Lahmandi-Ayed (2008) consider a certifier who decides a label's criterion and offers labeling services for competing firms. Fischer and Lyon (2014) study the competition between an NGO label and an industry label, and find that the label competition may reduce environmental benefits. Heyes and Martin (2016) focus on NGOs providing competing labels. They examine how the number of NGOs affects labels' standards and firms' socially responsible behavior. In addition, some papers consider a multi-tier label design (e.g., Li and van't Veld 2015, Fischer and Lyon 2019, Nadar and Ertürk 2020). Nevertheless, the above papers mainly consider settings where label information is credible and certain.

Some other papers take label credibility into account. For example, Hamilton and Zilberman (2006) study the performance of ecolabels in a market with some extent of fraud. Mason (2011) pays attention to certification error and treats labeling as a noisy test. Murali et al. (2019) study the effects of competing firms' credibility asymmetry on the firms' labeling decisions (i.e., adopting self-labels or external certifications) and environmental quality decisions. In particular, some works consider ecolabel proliferation and the resulting label confusion faced by consumers. For example, Brécard (2014) examines the performance of private labels, NGOs' labels, and public labels when there exists label confusion among consumers. Similarly, Brécard (2017) considers a setting where an unlabeled product and two labeled products of medium and high quality are in competition, but focuses on the effects of consumer misperceptions of ecolabels on firms' pricing decisions, the market structure, and social welfare when the firms' labels are exogenously given. Harbaugh et al. (2011) study the impacts of consumer confusion on the adoption and effective use of voluntary labels. Heyes et al. (2020) extend this research stream by considering a situation where consumers can pay a cost to acquire the information about green labels. In contrast to the above papers that focus on the performance of ecolabels when label information is uncertain or there exists label confusion among consumers, we compare the performance of ecolabels and blockchain-based transparency when some consumers are confused about label standards. We also contribute to the literature on cerification/labeling by considering blockchain-based transparency as a new means of communication.

Our study is also related to the literature on blockchain technology adoption in operations areas. Recent papers in this stream discuss a variety of practical issues, including data quality problems (Choi and Luo 2019), supply chain finance (Dong et al. 2021, Dong et al. 2022b), food and/or pharmaceuticals supply chains (Dong et al. 2022a, Lu et al. 2022), and combating copycats (Shen et al. 2022). Particularly, some works incorporate blockchain-based transparency into environmental issues. For example, Benjaafar et al. (2018) examine whether blockchain technology can facilitate green sourcing. Guo et al. (2020) investigate the role of blockchain-based transparency in disclosing a manufacturer's environmental efforts to a retailer. Wu et al. (2020) analyze the interactions between information transparency and greenwashing. In contrast to the above papers, which usually assume away the strategic role of consumers, we apply blockchain-based transparency to the situation in which consumers are confused about label standards. We compare the performance of ecolabels and blockchain-based transparency and examine their impacts on environmental quality. Our results surprisingly show that a firm's environmental quality may decrease when switching from environmental labels to blockchain-based transparency, even when blockchain adoption is free.

4.3 Model Setup

This study investigates how a sustainable firm should communicate its environmental quality to consumers in a competitive market when consumers face label confusion and how the means of communication affects the sustainable firm's environmental quality. To that end, we first describe the players and then present the game sequence. For convenience, the key notations are summarized in Table 4.1.

Firms: We consider a sustainable firm (denoted as "firm S") and a non-sustainable firm (i.e., a conventional firm, denoted as "firm N"). Each firm sells a single product at price p_i, where $i \in \{S, N\}$. The sustainable firm's product is associated with some level of environmental quality (denoted by $q \geq 0$), while the non-sustainable firm offers a product without any environmental quality. We normalize both firms' unit production costs to zero, but firm S needs to incur a marginal cost $\frac{1}{2}cq^2$ to offer the environmental quality q, where $c > 0$ refers to the marginal cost factor of environmental quality. This assumption suggests that the total quality cost of the product is a convex increasing function of its quality q, with marginal cost increases as the quality improves. It is because a unit increase in q for higher quality products often requires the use of more expensive materials or components. In practice, offering some level of environmental quality usually incurs additional marginal and/or fixed costs. This study focuses on situations in which environmental efforts are associated with marginal costs. For example, the usage of recycled or organic materials, ethical sourcing, and energy consumption affect a firm's marginal costs. In this study, the sustainable firm chooses q along with p_S, and the non-sustainable firm sets p_N, both aiming to maximize their own profits. Let D_i and π_i denote the demand and profit of firm $i \in \{S, N\}$, respectively.

The retail prices p_S and p_N are known to consumers, while the environmental quality q is the sustainable firm S's private information and unknown to consumers. In practice, unlike conventional quality attributes, a product's environmental attributes cannot be observed or experienced by consumers (Baksi and Bose 2007) and thus are usually a firm's private information. Nevertheless, the NGOs that provide labeling services can verify and certify a product's environmental quality. As such, firm S can use an *environmental label* offered by NGOs to communicate its environmental quality to consumers. Indeed, environmental labels provide credible sustainability information and hence function as an important marketing tool for firms to communicate their sustainable efforts to consumers while also helping consumers make environmentally conscious purchasing decisions (ISO 2019). In addition to environmental labels, firm S can adopt blockchain technology and make use of *blockchain-based transparency* to share its environmental quality with consumers.

If firm S adopts *environmental labels*, then it needs to further choose the desired label standard. In practice, an NGO may set several levels of standards and provide a distinct logo for each level. For example, the Forest Stewardship Council allows a forest product to bear an "FSC Recycled" logo if the product is made from 100% recycled material and an "FSC Mix" logo if it is made from a mixture of recycled and non-recycled materials. This study considers that there are two distinct tiers of environmental labels with criteria Q_L and Q_H, respectively, where $0 < Q_L < Q_H$. The label standard reveals the minimum environmental quality level of a labeled product. Specifically, a high-tier label certification is granted if firm S's environmental quality satisfies $q \geq Q_H$, and a low-tier label certification is granted if $Q_L \leq q < Q_H$; otherwise, firm S cannot obtain a label.

If firm S chooses to adopt *blockchain-based transparency* to communicate its

Table 4.1: Notations used throughout this study

Notation	Description
p_i	Selling price of firm i's product, where $i = S, N$.
q	Environmental quality of firm S's product.
c	Marginal cost factor of environmental quality relative to firm S's environmental quality.
D_i	Demand of firm i, where $i = S, N$.
π_i	Profit of firm i, where $i = S, N$.
Q_j	Environmental quality criteria of j-tier label, where $j = H, L$.
f_B	Fixed implementation cost of blockchain technology.
v	Consumers' homogeneous base value for the two products.
θ	A consumer's heterogeneous taste for environmental quality, which follows a uniform distribution in $[0, 1]$.
ϕ_q	A consumer's belief about the environmental quality q. ϕ_q depends on firm S's environmental quality q, its means of communication, and the consumer's type. $\phi_q = \tilde{q}$ for a confused consumer under environmental labels.
α	Fraction of confused consumers.
$\tilde{q}$	A confused consumer's belief about the environmental quality of a labeled product, where $\tilde{q} = \rho Q_H + (1 - \rho) Q_L$.
ρ	The proportion of the high standard Q_H in a confused consumer's belief in the case of environmental labels.

environmental quality to consumers, then it can set its environmental quality q at any level, and the quality will be accurately and reliably revealed to all consumers. Nevertheless, firm S needs to pay a fixed implementation cost (denoted by $f_B \geq 0$) for the blockchain technology. As such, firm S can use either environmental labels or blockchain-based transparency to communicate its environmental quality to consumers. We will characterize firm S's optimal means of communication when facing competition from the non-sustainable firm N.

Consumers: There is a mass of infinitesimal consumers such that the strategic interaction among consumers can be reasonably ignored. The market size is deterministic and normalized to 1. Each consumer purchases at most one unit of the products from the two firms. Consumers have a homogeneous base value (denoted by $v > 0$) for the two products. Meanwhile, they value firm S's environmental quality but have heterogeneous taste for it. A consumer's taste for environmental quality usually depends on her environmental consciousness and level of wealth. Let θ denote the taste, where θ follows a uniform distribution in $[0, 1]$. Additionally, each consumer's utility of purchasing from firm N is $v - p_N$, while that of purchasing from firm S is affected by her belief about the environmental quality, which is denoted by ϕ_q. Note that ϕ_q depends on firm S's environmental quality q, firm S's means of communication, and the consumer's type, as described below.

In the case of environmental labels, motivated by the observation that some consumers are confused due to the proliferation of labels, we consider two types of consumers: A fraction $\alpha \in [0, 1]$ of consumers (referred to as "*confused consumers*") value environmental labels but are confused about the label standards, whereas the remaining $1 - \alpha$ (referred to as "*informed consumers*") value labels and exactly know the relevant label standards. This study models label confusion as follows: First, confused consumers value ecolabels; that is, they prefer a labeled product to an unlabeled product if the two products are charged at the same price. Second, confused consumers cannot distinguish different types of labels or identify the quality level required by various labels. Following the literature on label confusion (e.g., Brécard 2014 and Brécard 2017), we assume that the label standard is the same for confused consumers, regardless of the specific labels. To be specific, a confused consumer's belief about the environmental quality of a labeled product is given by $\phi_q = \tilde{q}$ and $\tilde{q} = \rho Q_H + (1 - \rho)Q_L$, where ρ is the proportion of the high standard Q_H and $1 - \rho$ is the proportion of the low standard Q_L in a confused consumer's belief for $0 \leq \rho \leq 1$. In this case, a confused consumer's utility of buying one unit of the labeled product from firm S is given by: $v + \theta\tilde{q} - p_S$. By contrast, an informed consumer's belief about the environmental quality of the labeled product exactly matches the label standards, that is, $\phi_q = Q_L$ if the product has a low-tier label, and $\phi_q = Q_H$ if it has a high-tier label. As such, given there is a label on the product, an informed consumer's utility of buying from firm S is given by: $v + \theta Q_j - p_S$ for $j \in \{H, L\}$.

In the case of blockchain-based transparency, since firm S can accurately and reliably declare its environmental quality q to all consumers, each consumer's belief about the quality exactly matches firm S's environmental quality, i.e., $\phi_q = q$. As such, given firm S adopts blockchain technology, a consumer's utility of purchasing from firm S is given by: $v + \theta q - p_S$.

We assume that consumers have no other outside options and their reserved utility is zero. Each consumer makes purchasing decisions to maximize her own non-negative utility.

Sequence of events: In practice, firms that invest in environmental quality usually engage in marketing efforts to establish their environmental reputation or image. As such, we assume that firm S needs to choose the means of communication and its environmental quality before both firms' pricing decisions. Specifically, the game sequence is as follows:

Stage 1. Firm S chooses between environmental labels and blockchain-based transparency to communicate its environmental quality to consumers. If firm S selects environmental labels, then it needs to further set its environmental quality level q and choose either a high-tier label (with criterion Q_H) or a low-tier label (with criterion Q_L). If blockchain-based transparency is adopted, then firm S only needs to decide its environmental quality q.

Stage 2. Given firm S's means of communication and environmental quality q, both firm S and firm N determine their own prices p_S and p_N simultaneously.

Stage 3. Given both firms' decisions, consumers form their beliefs about firm S's environmental quality and decide which product to purchase, if any.

Lastly, we end this section by introducing some additional assumptions that help eliminate some trivial analyses. Throughout this study, we assume that the difference between the two labels' standards is relatively small, i.e., $Q_H < 2Q_L$, thus inducing label confusion among some consumers. Moreover, this study focuses on pure-strategy equilibria. In addition, we assume that $c \leq \min\{\frac{4-2\alpha}{(1-\alpha)Q_H}, \frac{2(1+\alpha)(\rho Q_H+(1-\rho)Q_L)}{\alpha Q_L^2}\}$ to ensure that both firms have positive demand in the equilibrium. Furthermore, we assume that $v > \underline{v}$ such that the market is fully covered, where $\underline{v} = \max\{\frac{1}{-6+6\alpha}(-2Q_H - cQ_H^2 - 2Q_H\alpha + cQ_H^2\alpha), \frac{1}{6\alpha}(4Q_L - 2Q_L\alpha + cQ_L^2\alpha + 4\rho(Q_H - Q_L) - 2\alpha\rho(Q_H - Q_L))\}$.

4.4 Analysis

In this section, we first analyze the case in which the sustainable firm uses environmental labels to communicate its environmental quality to consumers in Subsection 4.4.1. Subsection 4.4.5 then studies the case in which the sustainable firm adopts blockchain-based transparency to share its environmental quality. In each subsection, we derive the sustainable firm's optimal environmental quality and both firms' optimal pricing decisions. Lastly, through a comparative analysis, Subsection 4.4.6 compares these two means of communication and discusses how the sustainable firm's environmental quality and the non-sustainable firm's profit are affected.

Label confusion plays a critical role only in the case of environmental labels. Specifically, under environmental labels, informed consumers have full information about firm S's environmental quality, but confused consumers have partial information. As such, firm S can segment the market and choose to serve *both informed and confused consumers* or *a single type of consumers*. In addition, when serving a single type of consumers, firm S may adopt either *a differentiated targeting strategy* or *a uniform targeting strategy* to compete with firm N. By contrast, in the case of blockchain-based transparency, all consumers have full information about firm S's environmental quality, in which case, the two firms always adopt a uniform targeting strategy and serve both informed and confused consumers.

4.4.1 Environmental Labels

When firm S adopts environmental labels to disclose its environmental quality, it needs to set its environmental quality level q and choose either a high-tier label with criterion Q_H or a low-tier label with criterion Q_L, where $Q_H > Q_L > 0$. In what follows, we study first the high-tier label scenario and then the low-tier label scenario. After that, we compare these two scenarios to derive the sustainable firm's optimal choice of environmental label when both labels are available.

Note that when firm S chooses *a high-tier (a low-tier)* label, confused consumers tend to *underestimate (overestimate)* the environmental quality of the labeled product, as their belief about the environmental quality of a labeled product is given by:

$\phi_q = \tilde{q} = \rho Q_H + (1-\rho)Q_L$ for $0 \leq \rho \leq 1$.

4.4.2 A High-Tier Label with Underestimation

We first analyze the scenario of a high-tier label. We solve the game by backward induction and start with consumers' purchasing decisions at Stage 3, which help derive each firm's demand and profit function. Given that there is a high-tier label on the product, an informed consumer's belief about firm S's environmental quality is given by $\phi_q = Q_H$, whereas that of a confused consumer is given by $\phi_q = \tilde{q}$. In this case, the utility of purchasing from firm N is $v - p_N$, and the utility of purchasing from firm S is $v + \theta Q_H - p_S$ for an informed consumer and $v + \theta\tilde{q} - p_S$ for a confused consumer.

At Stage 2, both firm S and firm N set their prices p_S and p_N simultaneously to maximize their individual profits, which are given as follows:

$$\max_{p_S \geq 0} \pi_S = \begin{cases} (p_S - \frac{1}{2}cq^2)((1-\alpha)(1-\frac{p_S-p_N}{Q_H}) + \alpha(1-\frac{p_S-p_N}{\tilde{q}})), & \frac{p_S-p_N}{\tilde{q}} < 1 \\ (p_S - \frac{1}{2}cq^2)(1-\alpha)(1-\frac{p_S-p_N}{Q_H}), & \frac{p_S-p_N}{Q_H} < 1 < \frac{p_S-p_N}{\tilde{q}} \end{cases}, \text{ and}$$

$$\max_{p_N \geq 0} \pi_N = \begin{cases} p_N((1-\alpha)\frac{p_S-p_N}{Q_H} + \alpha\ (\frac{p_S-p_N}{\tilde{q}})), & \frac{p_S-p_N}{\tilde{q}} < 1 \\ p_N((1-\alpha)\frac{p_S-p_N}{Q_H} + \alpha), & \frac{p_S-p_N}{Q_H} < 1 < \frac{p_S-p_N}{\tilde{q}} \end{cases}, \tag{4.1}$$

where $q \geq Q_H$ and $\tilde{q} = \rho Q_H + (1-\rho)Q_L$. Solving the two firms' pricing decisions simultaneously, we can obtain two pure-strategy equilibria, depending on whether firm S serves the confused consumers in the equilibrium. We refer to the equilibrium where firm S serves both types of consumers as "HA" and use the superscript "HA" to denote the equilibrium outcome therein. We refer to the equilibrium where firm S serves a proportion of consumers (i.e., only the informed consumers) as "HP" and use the superscript "HP" to denote the equilibrium outcome therein. ("A" and "P" are the abbreviations for serving "all" and a "proportion" of consumers, respectively.)

At Stage 1, in anticipation of both firms' prices, firm S chooses q to maximize its own profit. Since environmental quality incurs an additional marginal cost and consumers would not expect a labeled product's environmental quality to be higher than the label standard, firm S has no incentive to provide a quality level beyond the corresponding label standard. As such, firm S's optimal quality is $q = Q_H$ in the scenario of a high-tier label. The equilibrium outcomes under a high-tier label are summarized in Lemma 8.

Lemma 8. *In the scenario of a high-tier label, there exist thresholds $\alpha_{HA} \geq \alpha_{HP}$ such that the following statements hold:*[4.1]

(i) If $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$, then both firms serve both informed and confused consumers. In this HA equilibrium, $q^{HA} = Q_H$, $p_S^{HA} = \frac{cQ_H^3\alpha + Q_H\tilde{q}(2+c(Q_H - Q_H\alpha))}{3(\tilde{q}+Q_H\alpha-\tilde{q}\alpha)}$, $p_N^{HA} = \frac{cQ_H^3\alpha + Q_H\tilde{q}(2+c(Q_H - Q_H\alpha))}{6(\tilde{q}+Q_H\alpha-\tilde{q}\alpha)}$, $\pi_S^{HA} = \frac{Q_H(cQ_H^2\alpha+\tilde{q}(-4+c(Q_H-Q_H\alpha)))^2}{36\tilde{q}(Q_H\alpha-\tilde{q}(-1+\alpha))}$, *and* $\pi_N^{HA} = \frac{Q_H(cQ_H^2\alpha+\tilde{q}(2+c(Q_H-Q_H\alpha)))^2}{36\tilde{q}(Q_H\alpha-\tilde{q}(-1+\alpha))}$.

(ii) If $\alpha < \alpha_{HP}$, then firm S serves only informed consumers while firm N serves both. In this HP equilibrium, $q^{HP} = Q_H$, $p_S^{HP} = \frac{Q_H(-2+cQ_H(-1+\alpha)+\alpha)}{3(-1+\alpha)}$, $p_N^{HP} = \frac{1}{6}Q_H(cQ_H - \frac{2(1+\alpha)}{-1+\alpha})$, $\pi_S^{HP} = \frac{Q_H(4+cQ_H(-1+\alpha)-2\alpha)^2}{36(1-\alpha)}$, *and* $\pi_N^{HP} = \frac{Q_H(cQ_H(-1+\alpha)-2(1+\alpha))^2}{36(1-\alpha)}$.

In the scenario of a high-tier label, the confused consumers underestimate firm S's environmental quality and have a lower willingness-to-pay for firm S's product than the informed consumers. In this case, if the marginal cost factor of environmental quality is low and there exist enough confused consumers (i.e., $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$), then firm S can set a low price to serve both informed and confused consumers (i.e., the HA equilibrium), as shown in Lemma 8 (i). This HA equilibrium becomes suboptimal when the marginal cost factor is high, in which case, firm S needs to charge a high price to compensate for the high environmental cost (i.e., $\frac{1}{2}cQ_H^2$), resulting in the loss of confused consumers. Similarly, when there exist few confused consumers in the market, the increase in demand achieved by serving both types of consumers at a low price is small and outweighed by the loss in profit caused by the low price. Thus, when few confused consumers are present in the market (i.e., $\alpha < \alpha_{HP}$), firm S will quit the confused consumers and serve only the informed

[4.1] Either when $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha_{HP} < \alpha < \alpha_{HA}$, or when $c \geq \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HP}$, we will not have any pure-strategy equilibrium and mix-strategy equilibrium arises, because firm N always has an incentive to lower its price to poach all confused consumers whenever firm S sets a price to serve all consumers. This study rules out this case and focuses on pure-strategy equilibria in order to eliminate trivial analysis.

consumers to charge a high price (i.e., the HP equilibrium), as shown in Lemma 8 (ii).

Next, we examine the effects of the fraction of confused consumers (i.e., α) and the proportion of the high standard Q_H in a confused consumer's belief about firm S's environmental quality (i.e., ρ) on both firms' profits when a high-tier label is adopted.[4.2] The results are presented in Lemma 9 and depicted in Figures 4.1 and 4.2.

Lemma 9. *In the scenario of a high-tier label, the following statements hold:*

(i) [The fraction of confused consumers] (a) In the HA equilibrium (i.e., when $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$), π_S^{HA} always decreases in α, and π_N^{HA} decreases in α iff $\alpha < \frac{(-2+cQ_H)\tilde{q}}{cQ_H(-Q_H+\tilde{q})}$. (b) In the HP equilibrium (i.e., when $\alpha < \alpha_{HP}$), both π_S^{HP} and π_N^{HP} always increase in α.

(ii) [The proportion of the high standard in a confused consumer's belief] (a) In the HA equilibrium (i.e., when $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$), π_S^{HA} always increases in ρ, and π_N^{HA} decreases in ρ iff $\rho < \frac{cQ_H^2\alpha+Q_L(-2+c(Q_H-Q_H\alpha))}{(Q_H-Q_L)(2+cQ_H(-1+\alpha))}$. (b) In the HP equilibrium (i.e., when $\alpha < \alpha_{HP}$), both π_S^{HP} and π_N^{HP} are independent of ρ.

One might intuit that both firms' profits should decrease in the fraction of confused consumers (i.e., α). The reason for this intuition is twofold. On the one hand, an increase in the fraction of confused consumers means that more consumers tend to underestimate the sustainable firm's environmental quality and reduce their willingness-to-pay for its product, which hurts the sustainable firm. On the other hand, with more consumers underestimating firm S's quality, the difference between the two products shrinks, resulting in fierce price competition that damages both firms.

The above intuition is true for firm S when both firms serve both types of consumers (i.e., the HA equilibrium), as shown in Lemma 9 (i)(a) and Figure 4.1 (a). However, in the HA equilibrium, the non-sustainable firm's profit (i.e., π_N) may increase in the fraction of confused consumers when $\alpha > \frac{(-2+cQ_H)\tilde{q}}{cQ_H(-Q_H+\tilde{q})}$ (see Lemma

[4.2] The effects of α and ρ on all equilibrium outcome parameters are presented and proved in the Appendix.

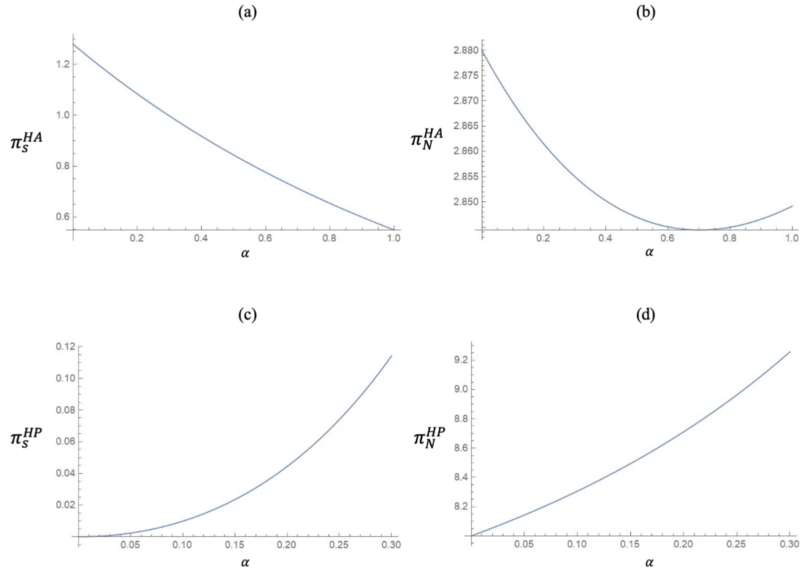

Figure 4.1: Effects of α on each firm's profit under a high-tier label

Note: In Figures 4.1 (a) and (b), $c = 0.2$, $\rho = 0.3$, $Q_H = 8$, and $Q_L = 5$; in Figures 4.1 (c) and (d) $c = 0.2$, $\rho = 0.5$, $Q_H = 8$, and $Q_L = 5$.

9 (i)(b) and Figure 4.1 (b)). The underlying reason for this non-monotone effect is as follows. When price competition is fierce enough (which is the case when α is large enough in the HA equilibrium), the non-sustainable firm has a competitive advantage, due to the absence of the marginal cost caused by environmental quality. As such, the non-sustainable firm can set a lower price than the sustainable firm and benefit from the competition. Thus, firm N's profit π_N may increase in α in the HA equilibrium.

Moreover, when the sustainable firm prefers to serve only the informed consumers (i.e., the HP equilibrium), both firms' profits increase in the fraction of confused consumers. The reason for this is as follows. In the HP equilibrium, the two firms compete only over informed consumers (i.e., $1 - \alpha$). In this case, as α increases, the non-sustainable firm faces more confused consumers, inducing it to raise its price p_N and profit mainly from the confused consumers rather than compete with the sustainable firm over the informed consumers. As a result, an increase in α leads the two firms to take different targeting strategies and focus on different types of consumers, which softens competition and benefits both firms in the HP equilibrium.

With regard to the effects of the proportion of the high standard in a confused

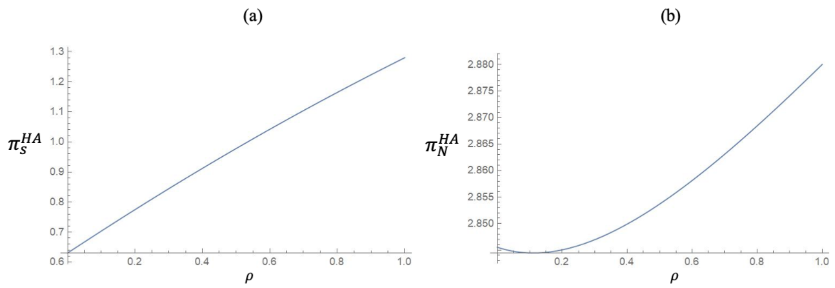

Figure 4.2: Effects of ρ on each firm's profit in the HA equilibrium under a high-tier label

Note: In Figure 4.2, $c = 0.5$, $\alpha = 0.5$, $Q_H = 8$, and $Q_L = 5$.

consumer's belief (i.e., ρ), following the above intuition, one might expect an increase in ρ to benefit both firms in a weak sense, because it leads the confused consumers to form a more accurate belief about firm S's environmental quality and, meanwhile, expands the difference between the two products, thus softening market competition. This is true for most cases, except when $\rho < \frac{cQ_H^2\alpha+Q_L(-2+c(Q_H-Q_H\alpha))}{(Q_H-Q_L)(2+cQ_H(-1+\alpha))}$ in the HA equilibrium, where the non-sustainable firm's profit (i.e., π_N) decreases in ρ, as shown in Lemma 9 (ii). One possible explanation is as follows. From the perspective of firm N, an increase in ρ has both positive and negative effects. The positive effect is that an increase in ρ weakens market competition and causes both prices p_S and p_N to increase; while the negative effect is that firm N suffers a competitive disadvantage because the confused consumers have a higher willingness-to-pay for firm S's product than firm N's; this situation raises firm S's demand D_S but reduces firm N's demand D_N. The negative effect dominates the positive effect when ρ is small and is dominated by the positive one when ρ is large. Thus, in the HA equilibrium, firm N's profit first decreases and then increases as ρ increases. Note that changes in ρ will not affect either firm's profit in the HP equilibrium, because ρ plays a role only when the confused consumers need to evaluate firm S's product; such effort is unnecessary in the HP equilibrium, wherein firm S serves only the informed consumers.

Lastly, we compare the two firms' profits in the scenario of a high-tier label and summarize the results in Corollary 6.

Corollary 6. *In the scenario of a high-tier label, firm S's profit is higher than* *firm N's profit iff the cost of environmental quality is low and meanwhile there is a moderate number of confused consumers in the market. Specifically, (i) in the HA equilibrium (i.e., when* $c < \frac{4\tilde{q}}{Q_H^2}$ *and* $\alpha > \alpha_{HA}$*),* $\pi_S^{HA} > \pi_N^{HA}$ *iff* $\alpha < \frac{\tilde{q}(1-cQ_H)}{cQ_H(Q_H-\tilde{q})}$*. (ii) In the HP equilibrium (i.e., when* $\alpha < \alpha_{HP}$*),* $\pi_S^{HP} < \pi_N^{HP}$ *always holds.*

One might expect the sustainable firm to gain more profit than the non-sustainable firm, as providing a product of high environmental quality is expected to build a competitive advantage. However, Corollary 6 states that firm S's profit is higher than that of firm N if the cost of environmental quality is low (i.e., $c < \frac{4\tilde{q}}{Q_H^2}$) and meanwhile there is a moderate number of confused consumers in the market (i.e., $\alpha_{HA} < \alpha < \frac{\tilde{q}(1-cQ_H)}{cQ_H(Q_H-\tilde{q})}$), in which case, firm S can set a moderately low price to serve both types of consumers. Otherwise, firm S make a less profit than firm N due to the loss of confused consumers (when α is small) or the extremely low price required (when α is large). Nevertheless, without the environmental quality, the two firms would engage in Bertrand competition with homogeneous products, and both of them would make zero profit.

4.4.3 A Low-Tier Label with Overestimation

Next, we analyze the scenario of a low-tier label by following a similar logic to that in Section 4.4.2. With a low-tier label on the product, at Stage 3, an informed consumer's belief about firm S's environmental quality is given by $\phi_q = Q_L$, whereas a confused consumer's belief is given by $\phi_q = \tilde{q}$. At Stage 2, both firms determine their prices simultaneously to maximize their own profits, which are given by:

$$\max_{p_S \geq 0} \pi_S = \begin{cases} (p_S - \frac{1}{2}cq^2)((1-\alpha)(1-\frac{p_S-p_N}{Q_L}) + \alpha(1-\frac{p_S-p_N}{\tilde{q}})), & \frac{p_S-p_N}{Q_L} < 1 \\ (p_S - \frac{1}{2}cq^2)\alpha(1-\frac{p_S-p_N}{\tilde{q}}), & \frac{p_S-p_N}{\tilde{q}} < 1 < \frac{p_S-p_N}{Q_L} \end{cases}, \text{ and}$$
$$\max_{p_N \geq 0} \pi_N = \begin{cases} p_N((1-\alpha)\frac{p_S-p_N}{Q_L} + \alpha\, \frac{p_S-p_N}{\tilde{q}}), & \frac{p_S-p_N}{Q_L} < 1 \\ p_N((1-\alpha) + \alpha\, \frac{p_S-p_N}{\tilde{q}}), & \frac{p_S-p_N}{\tilde{q}} < 1 < \frac{p_S-p_N}{Q_L} \end{cases}, \tag{4.2}$$

where $Q_L \leq q < Q_H$ and $\tilde{q} = \rho Q_H + (1-\rho)Q_L$. Similar to the high-tier label scenario, in the scenario of a low-tier label, solving the two firms' pricing decisions simultaneously leads to two pure-strategy equilibria. We refer to the equilibrium where firm S serves both types of consumers as "LA" and use the superscript "LA"

to denote the equilibrium therein. We refer to the equilibrium where firm S serves a proportion of consumers as "LP" and use the superscript "LP" to denote the equilibrium therein.

At Stage 1, anticipating the prices at Stage 2, firm N needs to decide q. Firm S has no incentive to provide a quality level beyond the corresponding label standard, and the optimal quality is $q = Q_L$. The equilibrium outcomes for a low-tier label are summarized in Lemma 10.

Lemma 10. *In the scenario of a low-tier label, there exist thresholds $\alpha_{LA} \leq \alpha_{LP}$ such that the following statements hold:*[4.3]

(i) If $c < \frac{4}{Q_L}$ and $\alpha < \alpha_{LA}$, then both firms serve both informed and confused consumers. In this LA equilibrium, $q^{LA} = Q_L$, $p_S^{LA} = \frac{cQ_L^3\alpha + Q_L\tilde{q}(2+c(Q_L - Q_L\alpha))}{3(\tilde{q}+Q_L\alpha-\tilde{q}\alpha)}$, $p_N^{LA} = \frac{cQ_L^3\alpha + Q_L\tilde{q}(2+c(Q_L - Q_L\alpha))}{6(\tilde{q}+Q_L\alpha-\tilde{q}\alpha)}$, $\pi_S^{LA} = \frac{Q_L(cQ_L^2\alpha+\tilde{q}(-4+c(Q_L-Q_L\alpha)))^2}{36\tilde{q}(Q_L\alpha-\tilde{q}(-1+\alpha))}$, and $\pi_N^{LA} = \frac{Q_L(cQ_L^2\alpha+\tilde{q}(2+c(Q_L-Q_L\alpha)))^2}{36\tilde{q}(Q_L\alpha-\tilde{q}(-1+\alpha))}$.

(ii) If $\alpha > \alpha_{LP}$, then firm S serves only confused consumers while firm N serves both. In this LP equilibrium, $q^{LP} = Q_L$, $p_S^{LP} = \frac{\tilde{q}+cQ_L^2\alpha+\tilde{q}\alpha}{3\alpha}$, $p_N^{LP} = \frac{4\tilde{q}+cQ_L^2\alpha-2\tilde{q}\alpha}{6\alpha}$, $\pi_S^{LP} = \frac{(cQ_L^2\alpha-2\tilde{q}(1+\alpha))^2}{36\tilde{q}\alpha}$, and $\pi_N^{LP} = \frac{(-2\tilde{q}(-2+\alpha)+cQ_L^2\alpha)^2}{36\tilde{q}\alpha}$.

Lemma 10 states that, similar to the high-tier label scenario, there exist two pure-strategy equilibria in the low-tier label scenario, depending on the marginal cost factor of environmental quality (i.e., c) and the fraction of confused consumers (i.e., α). However, different from the scenario of a high-tier label, with a low-tier label, firm S serves both informed and confused consumers when the marginal cost factor is low and there exist few confused consumers (i.e., $c < \frac{4}{Q_L}$ and $\alpha < \alpha_{LA}$), and it serves only confused consumers when many confused consumers are present in the market (i.e., $\alpha > \alpha_{LP}$). The underlying reason for this difference is as follows. Compared to the informed consumers, the confused consumers always underestimate firm S's environmental quality (and have a lower willingness-to-pay for firm S's product) in the high-tier label scenario, but always overestimate it (and have a

[4.3]Note that a mixed strategy equilibrium arises either when $c < \frac{4}{Q_L}$ and $\alpha_{LA} < \alpha < \alpha_{LP}$ or when $c \geq \frac{4}{Q_L}$ and $\alpha < \alpha_{LP}$. In this study, we rule out this case to focus on pure-strategy equilibria.

higher willingness-to-pay for firm S's product) in the low-tier label scenario. As such, firm S prefers informed consumers if a high-tier label is adopted, but that preference changes to confused consumers if a low-tier label is adopted.

Having established the equilibria for the scenario of a low-tier label, we now turn to the impacts of the fraction of confused consumers (i.e., α) and the proportion of the high standard in a confused consumer's belief about firm S's environmental quality (i.e., ρ) on both firms' profits. The results are summarized in Lemma 11 and illustrated in Figures 4.3 and 4.4.

Lemma 11. *In the scenario of a low-tier label, the following statements hold:*

(i) [The fraction of confused consumers] (a) In the LA equilibrium (i.e., when $c < \frac{4}{Q_L}$ and $\alpha < \alpha_{LA}$), π_S^{LA} always increases in α, and π_N^{LA} decreases in α iff $\alpha < \frac{-2\tilde{q}+cQ_L\tilde{q}}{-cQ_L^2+cQ_L\tilde{q}}$. (b) In the LP equilibrium (i.e., when $\alpha > \alpha_{LP}$), both π_S^{LP} and π_N^{LP} always decrease in α.

(ii) [The proportion of the high standard in a confused consumer's belief] (a) In the LA equilibrium (i.e., when $c < \frac{4}{Q_L}$ and $\alpha < \alpha_{LA}$), π_S^{LA} always increases in ρ, and π_N^{LA} decreases in ρ iff $\rho < \frac{Q_L(-2+cQ_L)}{(Q_H-Q_L)(2+cQ_L(-1+\alpha))}$. (b) In the LP equilibrium (i.e., when $\alpha > \alpha_{LP}$), π_S^{LP} always increases in ρ, and π_N^{LP} decreases in ρ iff $\rho < \frac{\alpha cQ_L^2+2\alpha Q_L-4Q_L}{-2\alpha Q_H+4Q_H+2\alpha Q_L-4Q_L}$.

In the scenario of a low-tier label, the confused consumers always overestimate the environmental quality and have a higher willingness-to-pay for firm S's product. As such, one might intuit that an increase in the fraction of confused consumers always benefits firm S, but may or may not benefit firm N, depending on the positive effect of weakened price competition and the negative effect of some lost demand, as aforementioned in the discussion of Lemma 9. This intuition is true when both firms serve both types of consumers (i.e., in the LA equilibrium). However, when firm S serves only the confused consumers (in the LP equilibrium), both firms' profits decrease as the fraction of confused consumers increases. The reason for this is as follows. In the LP equilibrium, both firms compete over confused consumers (i.e., α). In this case, an increase in α will not expand the difference between the two

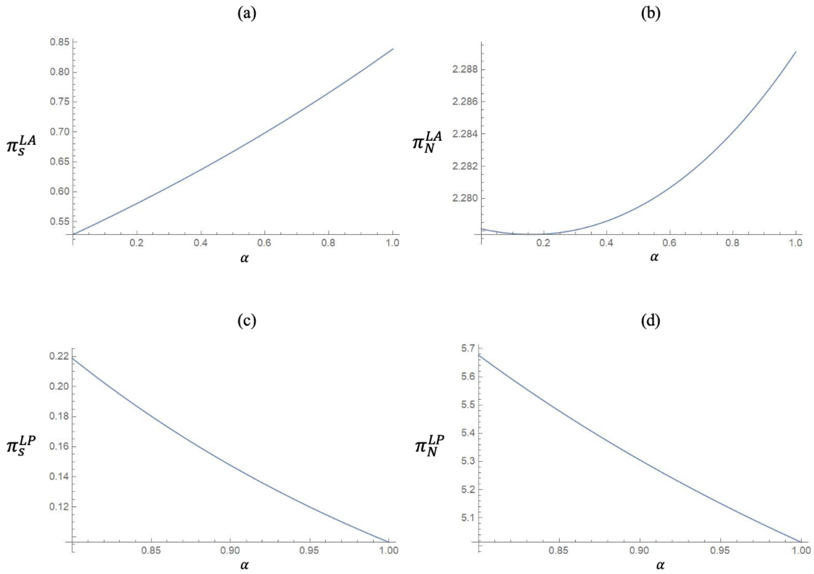

Figure 4.3: Effects of α on each firm's profit under a low-tier label

Note: In Figures 4.3 (a) and (b), $c = 0.41$, $\rho = 0.3$, $Q_H = 8$, and $Q_L = 5$; in Figures 4.3 (c) and (d) $c = 0.8$, $\rho = 0.5$, $Q_H = 8$, and $Q_L = 5$.

products for a given ρ, but will intensify the competition between the two firms as the confused consumers play an increasingly crucial role in both firms' profits. This increase in competition hurts both of them. (Indeed, both firms' prices decrease in α.) Thus, both π_S^{LP} and π_N^{LP} decrease in α when firm S serves a proportion of consumers in the equilibrium.

Regarding the effects of the proportion of the high standard in a confused consumer's belief (i.e., ρ), Lemma 11 (ii) states that when both firms serve both informed and confused consumers, firm S's profit increases in ρ, while firm N's profit may first decrease and then increases in ρ. This is is similar to the scenario of a high-tier label, and the underlying reason is similar. However, when firm S serves only a proportion of consumers, different from the high-tier label scenario where both firms' profits are independent of ρ, in the low-tier label scenario, both firms' profits are affected by ρ. This difference is driven by the fact that firm S prefers to serve only the confused consumers (whose belief about the environmental quality relies on ρ) if a low-tier label is adopted, and prefers to serve only the informed consumers (whose belief is independent of ρ) if a high-tier label is adopted. In the scenario of a low-tier label, an increase in ρ raises the confused consumers' perception of firm S's environmental

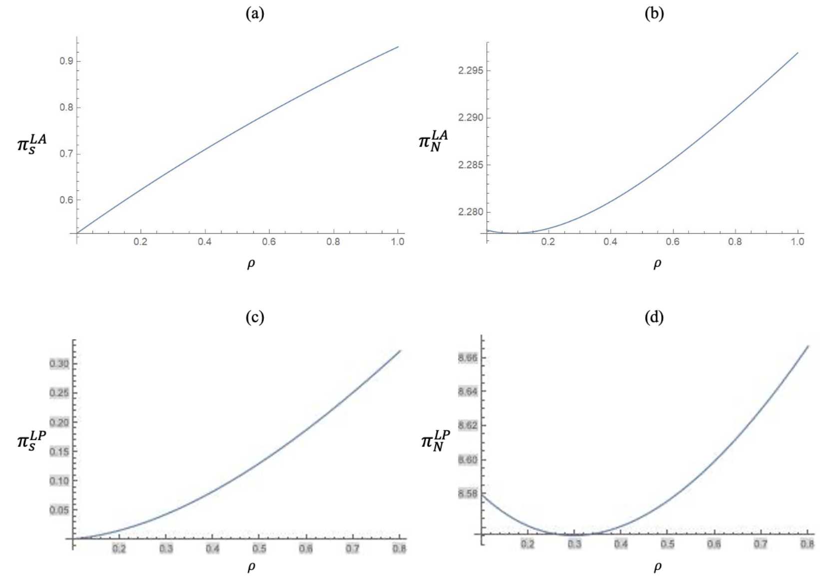

Figure 4.4: Effects of ρ on each firm's profit under a low-tier label

Note: In Figures 4.4 (a) and (b), $c = 0.41$, $\alpha = 0.5$, $Q_H = 8$, and $Q_L = 5$; in Figures 4.4 (c) and (d) $c = 1.1$, $\alpha = 0.6$, $Q_H = 8$, and $Q_L = 5$.

quality and expands the difference between the two products, benefiting firm S. For firm N, however, this situation has both a positive effect via weakened competition and a detrimental effect via the loss of some demand. Thus, firm S's profit increases in ρ, while firm N's profit may first decrease and then increase in ρ.

Before ending this part, we compare the two firms' profits when a low-tier label is adopted and summarize the results in Corollary 7.

Corollary 7. *In the scenario of a low-tier label, firm S's profit is higher than firm N's profit iff the cost of environmental quality is low and meanwhile there is a moderate number of confused consumers in the market. Specifically, (i) in the LA equilibrium (i.e., when* $c < \frac{4}{Q_L}$ *and* $\alpha < \alpha_{LA}$*),* $\pi_S^{LA} > \pi_N^{LA}$ *iff* $\alpha > \frac{\tilde{q}(1-cQ_L)}{cQ_L(Q_L-\tilde{q})}$*. (ii) In the LP equilibrium (i.e., when* $\alpha > \alpha_{LP}$*),* $\pi_S^{LP} < \pi_N^{LP}$ *always holds.*

Corollary 7 states that in the low-tier label scenario, the sustainable firm may earn a higher profit than the non-sustainable firm if the cost of environmental quality is low and the fraction of confused consumers is moderate, which is similar to the high-tier label scenario. Nevertheless, the sustainable firm is more likely to make a higher profit than the non-sustainable firm in the low-tier label scenario, compared to that in the high-tier label scenario (i.e., $c < \frac{4\tilde{q}}{Q_H^2} \cap \alpha_{HA} < \alpha < \frac{\tilde{q}(1-cQ_H)}{cQ_H(Q_H-\tilde{q})}$ is a subset of $c < \frac{4}{Q_L} \cap \frac{\tilde{q}(1-cQ_L)}{cQ_L(Q_L-\tilde{q})} < \alpha < \alpha_{LA}$). The reason for this is that confused consumers always underestimate firm S's environmental quality when a high-tier label is adopted, but always overestimate its quality when a low-tier label is adopted. Firm S is hurt by the underestimation but benefits from the overestimation.

4.4.4 Choice of Environmental Label

So far, we have analyzed the scenarios of a high-tier label and a low-tier label, and identified four equilibrium outcomes (i.e., HA, HP, LA, and LP). From the perspective of the sustainable firm S, a high-tier label has the detrimental effect of being underestimated by the confused consumers in the HA equilibrium, and the beneficial effect of softened price competition arising from a differentiated targeting strategy in the HP equilibrium. Moreover, if a low-tier label is adopted, firm S may benefit from being overestimated by the confused consumers in the LA equilibrium, but may be hurt by the intensified price competition as a result of a uniform targeting strategy in the LP equilibrium.

Next, we study firm S's preference for environmental labels when both labels are available. We are also interested in how the fraction of confused consumers (i.e., α) affects firm S's environmental quality decision. We first compare firm S's profit in the scenario of a high-tier label with that in the scenario of a low-tier label when the pure-strategy equilibria exist in both scenarios, in order to derive its label preferences. The results are presented in Proposition 14.

Proposition 14. *Given that the sustainable firm adopts environmental labels and needs to choose between a high- and a low-tier labels, as α increases, it may switch from a high-tier label to a low-tier one, or switch from a low-tier label to a high-tier one. Specifically, there exist thresholds α_1, α_2, and α_3, such that the following statements hold:*[4.4]

(i) When the HA equilibrium and the LA equilibrium coexist (i.e., $c < \frac{4\tilde{q}}{Q_H^2}$ and

[4.4]Note that $\alpha_{HA} \geq \alpha_{HP}$, $\alpha_{LA} \leq \alpha_{LP}$, and $\frac{4\tilde{q}}{Q_H^2} < \frac{4}{Q_L}$. Moreover, the HA equilibrium and the LP equilibrium cannot coexist. In addition, there exist situations where a single pure-strategy equilibrium occurs and the choice between a high- or a low-tier label is irrelevant. For example, the LA equilibrium is the single pure-strategy equilibrium under environmental labels either when $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha_{HP} < \alpha < \alpha_{HA}$, or when $\frac{4\tilde{q}}{Q_H^2} < c < \frac{4}{Q_L}$ and $\alpha_{HP} < \alpha < \alpha_{LA}$.

$\alpha > \alpha_{HA}$), firm S prefers a high-tier label (i.e., $\pi_S^{HA} > \pi_S^{LA}$) if $\alpha < \alpha_1$, and a low-tier label (i.e., $\pi_S^{HA} \leq \pi_S^{LA}$) otherwise.

(ii) When the HP equilibrium and the LP equilibrium coexist (i.e., $\alpha_{LP} < \alpha < \alpha_{HP}$), firm S prefers a low-tier label (i.e., $\pi_S^{HP} < \pi_S^{LP}$) if $\alpha < \alpha_2$, and a high-tier label (i.e., $\pi_S^{HP} \geq \pi_S^{LP}$) otherwise.

(iii) When the HP equilibrium and the LA equilibrium coexist (i.e., $c < \frac{4}{Q_L}$ and $\alpha < \min\{\alpha_{HP}, \alpha_{LA}\}$), firm S prefers a low-tier label (i.e., $\pi_S^{HP} < \pi_S^{LA}$) if $\alpha < \alpha_3$, and a high-tier label (i.e., $\pi_S^{HP} \geq \pi_S^{LA}$) otherwise.

Intuitively, one might expect the sustainable firm to prefer a high-tier label when there are few confused consumers (i.e., α is small), and a low-tier label when there are many confused consumers. The reason for this intuition is that the confused consumers underestimate firm S's environmental quality in the high-tier label scenario, but overestimate it in the low-tier label scenario. Thus, firm S should adopt a high-tier label when α is small, but a low-tier label when α is large to take advantage of the overestimation. This is true when the marginal cost factor of environmental quality is low and many confused consumers are present in the market (i.e., $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$), such that firm S serves both informed and confused consumers, regardless of the type of labels adopted, as shown in Proposition 14 (i) and Figure 4.5 (a).

However, when the number of confused consumers in the market is moderate or small, firm S may switch from a low-tier label to a high-tier label, as the fraction of confused consumers (i.e., α) increases. Specifically, when the fraction of confused consumers is moderate (i.e., $\alpha_{LP} < \alpha < \alpha_{HP}$), such that the sustainable firm always prefers to charge a high price and serve only a proportion of consumers, regardless of the environmental label in force, firm S may switch from LP to HP when more consumers are confused about the label standards (i.e., when α increases). As mentioned above, firm S benefits from an increase in α in the HP equilibrium, because the increase leads to differentiated targeting strategies by the two firms and helps soften market competition between them. By contrast, an increase in α may hurt firm S in the LP equilibrium by intensifying market competition, as both firms target the confused consumers. (Actually, both firms in the LP equilibrium increasingly rely on the confused consumers as α increases.) As such, firm S may first prefer LP, but then switch to HP as α increases.

Moreover, when the marginal cost is low and few confused consumers are present in the market (i.e., $c < \frac{4}{Q_L}$ and $\alpha < \min\{\alpha_{HP}, \alpha_{LA}\}$), as α increases, firm S first prefers a low-tier label and serves both informed and confused consumers (i.e., LA), but then switches to a high-tier label and serves only the informed consumers (i.e., HP). One possible explanation is as follows. Firm S benefits from an increasing α in both the HP and LA equilibria, as discussed in Lemmas 9 (i) and 11 (i). Nevertheless, there are some differences between these two trends. In the HP equilibrium, the sustainable firm can take a differentiated targeting strategy by serving only informed consumers. By contrast, in the LA equilibrium, firm S benefits from the confused consumers' overestimation, causing its profit to increase in α, but firm S takes a uniform targeting strategy to serve both types of consumers, which actually intensifies the competition between the two firms. Thus, the sustainable firm may prefer HP to LA as α increases.

Next, we examine how the fraction of confused consumers (i.e., α) affects the sustainable firm's environmental quality under firm S's optimal choice of environmental label. Corollary 8 summarizes the result, which is a byproduct of Proposition 14.

Corollary 8. *Given that the sustainable firm uses labels to reveal its environmental quality q, q may increase or decrease as the fraction of confused consumers (i.e., α) increases. Specifically, as α increases, firm S's optimal environmental quality q decreases when $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$, and increases either when $\alpha_{LP} < \alpha < \alpha_{HP}$ or when $c < \frac{4}{Q_L}$ and $\alpha < \min\{\alpha_{HP}, \alpha_{LA}\}$.*

Corollary 8 states that the sustainable firm may counterintuitively prefer high quality to low quality as more confused consumers are present in the market, when the number of confused consumers in the market is moderate or small (i.e., either when $\alpha_{LP} < \alpha < \alpha_{HP}$, or when $c < \frac{4}{Q_L}$ and $\alpha < \min\{\alpha_{HP}, \alpha_{LA}\}$). This is because a high-tier label associated with high quality helps firm S to differentiate its product from the competitor's product. That is, firm S may prefer the HP equilibrium to either the LA or LP equilibrium when α is small, as previously mentioned in the discussion of Proposition 14.

4.4.5 Blockchain-Based Transparency

In this section, we analyze the case in which the sustainable firm S uses blockchain-based transparency to communicate its environmental quality. With blockchain-based transparency, firm S can accurately and reliably share its environmental quality q to all consumers. In this case, each consumer's belief about firm S's environmental quality is given by $\phi_q = q$. We solve this case by backward induction and start with consumers' behavior at Stage 3, which defines both firms' demand. At Stage 2, both firms S and N set their prices simultaneously to maximize their own profits, which are given as follows:

$$\max_{p_S \geq 0} \pi_S = (p_S - \frac{1}{2}cq^2)(1 - \frac{p_S - p_N}{q}) - f_B, \text{ and } \max_{p_N \geq 0} \pi_N = p_N \frac{p_S - p_N}{q}, \tag{4.3}$$

where f_B is the fixed implementation cost of blockchain technology. At Stage 1, in anticipation of both firms' prices at Stage 2, firm S chooses q to maximize its own profit. We refer to the case of blockchain-based transparency as "B" and use the superscript "B" to denote the equilibrium outcome therein. The equilibrium is summarized in Lemma 12.

Lemma 12. *Given that the sustainable firm adopts blockchain-based transparency, in the equilibrium,* $q^B = \frac{4}{3c}$, $p_S^B = \frac{40}{27c}$, $p_N^B = \frac{20}{27c}$, $\pi_S^B = \frac{64}{243c} - f_B$, *and* $\pi_N^B = \frac{100}{243c}$.

Recall that, firm S's profit is higher than firm N's profit when the marginal cost factor of environmental quality is low (i.e., c is small), and is lower otherwise (see Corollaries 6 and 7). However, Lemma 12 states that when firm S adopts blockchain-based transparency, it always sets a higher price (i.e., $p_S^B > p_N^B$) but meanwhile earns a lower profit (i.e., $\pi_S^B < \pi_N^B$) than firm N, an outcome mainly driven by the cost of environmental quality (i.e., $\frac{1}{2}cq^2$).

4.4.6 Comparisons

Next, we compare environmental labels with blockchain-based transparency in order to derive the optimal means of communication. We are also interested in how the optimal means of communication affects firm S's environmental quality and firm N's profit when there exists label confusion among consumers. For expositional convenience, in what follows, we focus on the case where the HA equilibrium and the LA equilibrium coexist (i.e., $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$, see Proposition 14 (i)). We note that the main insights hold qualitatively when this assumption is relaxed. The details are available from the authors upon request.

To begin with, through comparing firm S's optimal profit under environmental labels with that under blockchain-based transparency, we characterize the conditions under which blockchain-based transparency is more profitable for firm S. The result is presented in Proposition 15 and depicted in Figure 4.5 (b).

Proposition 15. *The sustainable firm S would not prefer blockchain-based transparency in all cases, even when blockchain adoption is free (i.e., $f_B = 0$). Particularly, when $f_B = 0$, $c < \frac{4\tilde{q}}{Q_H^2}$, and $\alpha > \alpha_{HA}$, there exists a threshold α_B such that firm S prefers blockchain-based transparency if $\alpha < \alpha_B$ and prefers environmental labels otherwise, where $\alpha_B > \alpha_1$ and α_1 is defined in Proposition 14 (i).*

One might intuit that firm S will always prefer blockchain-based transparency to environmental labels when some consumers are confused about the label standards, because blockchain-based transparency provides consumers with full information about firm S's environmental efforts. However, Proposition 15 states that, under certain conditions (i.e., $f_B = 0$, $c < \frac{4\tilde{q}}{Q_H^2}$, and $\alpha > \alpha_{HA}$), firm S prefers blockchain-based transparency when there is a small number of confused consumers (i.e., $\alpha < \alpha_B$); and prefers environmental labels otherwise. When $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$, firm S under environmental labels prefers a high-tier label and serves both informed and confused consumers (i.e., HA) when $\alpha < \alpha_1$; but it prefers a low-tier label and serves both consumers (i.e., LA) otherwise (see Proposition 14 (i)). Under HA, the confused consumers tend to underestimate firm S's environmental quality. Moreover, the uniform targeting strategy taken by firm S and firm N intensifies the market

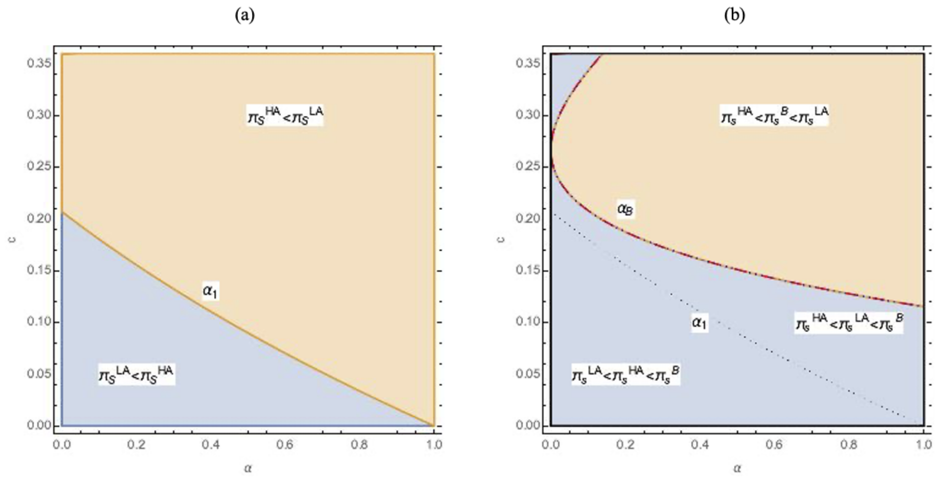

Figure 4.5: Environmental labels or blockchain-based transparency

Note: In Figures 4.5 (a) and (b), $\rho = 0.5$, $Q_H = 8$, and $Q_L = 5$. Moreover, Figure 4.5 (a) is an illustration of Proposition 14 (i), and Figure 4.5 (b) is an illustration of Proposition 15.

competition between them. As such, firm S always prefers to adopt blockchain in the HA region (i.e., when $\alpha < \alpha_1$). By contrast, under LA, environmental labels enable overestimation, which increases in α. Therefore, in the LA region (i.e., when $\alpha > \alpha_1$), firm S prefers blockchain-based transparency when α is small (i.e., when $\alpha_1 < \alpha < \alpha_B$), and prefers to implement a low-tier label and serve both types of consumers to benefit from confused consumers' overestimation (i.e., LA) otherwise.

We then analyze how blockchain-based transparency affects firm S's environmental quality. The result is presented in Proposition 16 and illustrated in Figure 4.6 (a).

Proposition 16. *Blockchain-based transparency may improve or reduce firm S's environmental quality. Specifically, given that blockchain adoption is free and firm S prefers blockchain-based transparency to environmental labels (i.e., when $f_B = 0$, $c < \frac{4\tilde{q}}{Q_H^2}$, and $\alpha_{HA} < \alpha < \alpha_B$), firm S's environmental quality increases with blockchain adoption either if $\alpha < \alpha_1$ and $c < \frac{4}{3Q_H}$, or if $\alpha > \alpha_1$ and $c < \frac{4}{3Q_L}$; and decreases otherwise.*

Proposition 16 states that blockchain adoption improves environmental quality if the marginal cost factor is small, but reduces environmental quality if such quality is costly. When serving both informed and confused consumers (which is the case

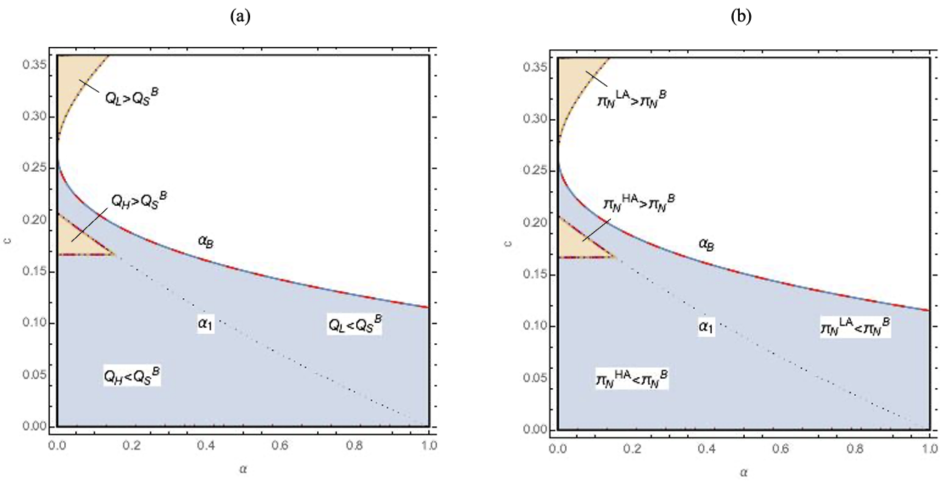

Figure 4.6: Effects of blockchain adoption on the market

Note: In Figures 4.6 (a) and (b), $\rho = 0.5$, $Q_H = 8$, and $Q_L = 5$. Moreover, Figure 4.6 (a) compares the sustainable firm's environmental quality, and Figure 4.6 (b) compares the non-sustainable firm's profits.

when $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha_{HA} < \alpha < \alpha_B$, regardless of the means of communication), providing some level of environmental quality enables firm S to differentiate its product from the competitor's. Compared with environmental labels, blockchain-based transparency improves the efficiency of communication by eliminating the underestimation that occurs under a high-tier label and the constraints of label standards on the quality level. That is, blockchain-based transparency allows firm S to flexibly choose the desired environmental quality. However, a drawback of such flexibility is that firm S may reduce its environmental quality, especially when the environmental cost is large, as shown in Proposition 16.

Lastly, we study the effects of blockchain adoption from the perspective of the non-sustainable firm N. We find that, although firm N does not make decision on blockchain adoption, it may benefit from firm S's adoption of blockchain, as stated in Corollary 9 and depicted in Figure 4.6 (b).

Corollary 9. *The non-sustainable firm N may benefit from or be hurt by firm S's adoption of blockchain. Specifically, given that blockchain adoption is free and firm S prefers blockchain-based transparency to environmental labels (i.e., when $f_B = 0$, $c < \frac{4\tilde{q}}{Q_H^2}$, and $\alpha_{HA} < \alpha < \alpha_B$), there exist thresholds $c_{B1}(\alpha)$ and $c_{B2}(\alpha)$, such that firm N is more profitable with blockchain adoption either if $\alpha < \alpha_1$ and $c < c_{B1}(\alpha)$,* *or if* $\alpha > \alpha_1$ *and* $c < c_{B2}(\alpha)$*; and is less profitable otherwise, where* $c_{B1}(\alpha) > \frac{4}{3Q_H}$ *and* $c_{B2}(\alpha) < \frac{4}{3Q_L}$.

Corollary 9 states that blockchain-based transparency may lead to a win-win situation in which both firms benefit from blockchain adoption. The underlying rationale is as follows. When $f_B = 0$, $c < \frac{4\tilde{q}}{Q_H^2}$, and $\alpha_{HA} < \alpha < \alpha_B$, firm S under environmental labels prefers a high-tier label and serves both informed and confused consumers (i.e., HA) when $\alpha < \alpha_1$; but it prefers a low-tier label and serves both consumers (i.e., LA) otherwise (see Proposition 14 (i)). Under HA, blockchain-based transparency improves firm S's quality when $c < \frac{4}{3Q_H}$ (compared with that under environmental labels, see Proposition 16), which efficiently differentiates firm S from the competitor, thus softening market competition and meanwhile benefiting both firms. This trend continues to hold even when firm S's quality is slightly lower. This is because firm S's quality is underestimated under environmental labels, allowing the two means of communication to be different in a larger space. Consequently, firm N is more profitable with blockchain adoption if $\alpha < \alpha_1$ and $c < c_{B1}(\alpha)$, where $c_{B1}(\alpha) > \frac{4}{3Q_H}$. Under LA, blockchain-based transparency improves firm S's quality when $c < \frac{4}{3Q_L}$ (compared with that under environmental labels). Nevertheless, the higher quality under blockchain adoption does not necessarily benefit firm N, as firm S's quality is overestimated under environmental labels, allowing the two means of communication to be different in a smaller space. Thus, firm N benefits from blockchain adoption if $\alpha > \alpha_1$ and $c < c_{B2}(\alpha)$, where $c_{B2}(\alpha) < \frac{4}{3Q_L}$.

It is noteworthy that, either if $c < \frac{4}{3Q_H}$ when $\alpha_{HA} < \alpha < \alpha_1$, or if $c < c_{B2}(\alpha)$ when $\alpha_1 < \alpha < \alpha_B$, blockchain-based transparency leads to a win-win-win situation, where both firms make higher profits, and firm S provides a higher quality level than those seen under environmental labels. Nevertheless, we caution that blockchain-based transparency is not always beneficial. We have so far characterized the conditions under which blockchain-based transparency is beneficial from the perspectives of firm S, firm N, and environmental quality, as shown in Propositions 15 and 16, and Corollary 9.

4.5 Conclusions

This study examines how a sustainable firm should communicate its environmental quality to consumers in a competitive market, and how the means of communication affects the sustainable firm's environmental quality, when there exists label confusion among consumers. We develop a game-theoretic model with a sustainable firm and a non-sustainable firm. The sustainable firm offers an eco-friendly product with some level of environmental quality, while the non-sustainable firm sells a regular product without any environmental quality. The sustainable firm needs to communicate its environmental quality to consumers via either environmental labels or blockchain-based transparency. In the case of environmental labels, a fraction of consumers are confused about the label standards and may underestimate or overestimate the sustainable firm's environmental quality; whereas the actual quality is revealed to all consumers under blockchain-based transparency.

We highlight several main findings. First, under environmental labels, as the number of consumers who are confused about label standards increases, the sustainable firm may switch from a high-tier label to a low-tier label when the number of confused consumers in the market is sufficiently high, but may counterintuitively switch from a low-tier label to a high-tier label when the prevailing fraction of confused consumers is moderate or low. Second, blockchain-based transparency does not always benefit the sustainable firm more than environmental labels. That is, from the perspective of the sustainable firm, full information is not necessarily better than partial information. In particular, under certain conditions (e.g., when blockchain adoption is free, the cost of environmental quality is low, and there is a great number of confused consumers), the sustainable firm prefers blockchain-based transparency if the fraction of confused consumers is small and prefers environmental labels otherwise, which allows it to benefit from confused consumers' overestimation. Third, when the sustainable firm prefers blockchain-based transparency to environmental labels, the sustainable firm may improve or reduce its environmental quality, and the non-sustainable firm may be better off or worse off. Moreover, we characterize the conditions under which blockchain adoption leads to a win-win-win situation, where both firms make higher profits, and the sustainable firm provides a higher quality level than those seen under environmental labels.

Chapter 5

Conclusions and Future Research

In this thesis, we conduct three studies on healthcare and environmental management, considering different medical and technology aspects in terms of emergency relief allocation, vaccination, and blockchain technology. The three studies conducted in this thesis hold significant value in addressing critical challenges faced by society in the realms of healthcare management, environmental management, and sustainability. We first point out some future research directions for each study respectively, and then summarize the overall contributions of the three studies.

In the first study, we develop a time-dependent demand forecasting model that includes demand forecasting, extra demand from unsatisfied demand, and some unexpected cases as a basis for building the foresighted model to optimize medical resources allocation in response to epidemic outbreaks. We derive some important properties of the corresponding dynamic programming problem and obtain a general form of the foresighted allocation model in each period. This study provides valuable insights for policymakers and healthcare administrators to optimize the allocation of medical resources during emergencies. A more foresighted approach can lead to better preparedness and response, ultimately saving lives and reducing the burden on healthcare systems during crises.

This study provides managerial implications for improving decisions on medical resources allocation in response to epidemic outbreaks. There are some directions for future research. First, we consider the unexpected cases to follow an exponential distribution. In future research, we can consider that it follows other kinds of distributions, like uniform or normal distribution. Second, in our model, we simplify that the allocation capacity is independent between different areas. Network-restricted capacity can be considered in future research. Third, we do not consider the private sector in our models. It is conceivable that governments may cooperate with the private sector to provide emergency medical supplies. Therefore, this work can be extended to include the private sector.

In the second study, we build models of a vaccine system to study the cooperation between the government and a private clinic for vaccination. In the vaccine system, there are a profit-maximizing private clinic, a public hospital that seeks to minimize the social cost, and self-interested individuals. We construct three models including a vaccine system without information sharing, a vaccine system with information sharing and subsidy, and a vaccine system with information sharing and allocation. The effectiveness of these schemes has been investigated. By understanding the dynamics of public-private collaboration in vaccination efforts, this study informs policymakers on strategies to enhance vaccination coverage. Coordinating resources effectively can help ensure equitable vaccine distribution and expedite the containment of infectious diseases.

This study provides both theoretical and managerial insights on vaccine supply decision, government intervention, and vaccination coverage. In this study we assume that the public hospital and private clinic provide the same product and service. For future research, it would be interesting to consider the differences in service quality between the public hospital and private clinic. Besides, future research should also consider the cost of searching for vaccines in the vaccine system.

In the third study, we examine how a sustainable firm should communicate its environmental quality to consumers in a competitive market, and how the means of communication affects the sustainable firm's environmental quality, when there exists label confusion among consumers. Interestingly, blockchain-based transparency does not always benefit the sustainable firm more than environmental labels. Futhermore, when the sustainable firm prefers blockchain-based transparency to environmental labels, the sustainable firm may improve or reduce its environmental quality, and the non-sustainable firm may be better off or worse off. The study provides valuable guidance to firms on how to effectively communicate their environmental efforts to consumers. By adopting blockchain-based transparency, firms can build trust and credibility, fostering sustainable practices throughout the supply chain. Clear communication empowers consumers to make informed choices, thus driving demand for environmentally friendly products and encouraging more businesses to adopt sustainable practices.

This study provides some managerial insights for the sustainable firm. We develop a game-theoretic model with a sustainable firm and a non-sustainable firm. The sustainable firm offers an eco-friendly product with some level of environmental quality, while the non-sustainable firm sells a regular product without any environmental quality. For future research, we can consider two sustainable firms competing in the market. Additionally, we can consider that there are more than two firms in the market.

In summary, the three studies capture the most recent issues in healthcare and environmental management. These studies will contribute to the decision making on emergency logistics, resource allocation, vaccination policy, public-private cooperation, pricing policy, and blockchain technology adoption in operations areas.

Appendix A

Proofs for Chapter 2

Proof of Proposition 1.

From Equations (2.7) and (2.8), the following equations are obtained:

$$G_{j,t+1}(y) = \int_0^\infty W(y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+)|_{d_{jt}} g_{jt}(d_{jt})\, d(d_{jt})$$

$$g_{j,t+1}(y) = \int_0^\infty w(y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+)|_{d_{jt}} g_{jt}(d_{jt})\, d(d_{jt})$$

where

$$\begin{aligned} &W\left(y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+\right)|_{d_{jt}} \\ &= \begin{cases} 0 & , \quad y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+ < 0 \\ 1 - e^{-\lambda\left(y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+\right)} & , \quad y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+ \geq 0 \end{cases} \end{aligned}$$

and

$$\begin{aligned} &w\left(y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+\right)|_{d_{jt}} \\ &= \begin{cases} 0 & , \quad y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+ < 0 \\ \lambda e^{-\lambda\left(y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+\right)} & , \quad y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+ > 0 \end{cases} \end{aligned}$$

Therefore,

$$G_{j,t+1}(y) = \begin{cases} 0 & , \quad y \leq M\left(D_{jt}\right) + r\left(D_{jt} - X_{jt} - Q_{jt}\right)^+ \\ 1 - e^{-\lambda\left[y - M(D_{jt}) - r(D_{jt} - X_{jt} - Q_{jt})^+\right]} & , \quad y > M\left(D_{jt}\right) + r\left(D_{jt} - X_{jt} - Q_{jt}\right)^+ \end{cases}$$

$$g_{j,t+1}(y) = \begin{cases} 0 & , \quad y < M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \\ \lambda e^{-\lambda[y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+]} & , \quad y > M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ \end{cases}$$

Note that $G_{j,t+1}(y)|_{d_{jt}} = Pr\{d_{j,t+1} \leq y\} = Pr\{M(d_{jt}) + r(d_{jt} - x_{jt} - q_{jt})^+ + \sigma_{jt} \leq y\} = Pr\{\sigma_{jt} \leq y - M(d_{jt}) - r(d_{jt} - x_{jt} - q_{jt})^+\}$. Then with the distribution of σ, we can obtain the results.

Proof of Proposition 2.

$$
\begin{aligned}
&f_n(q_n) = \min_{x_n^F}\{\alpha(x_n^F + q_n - \mu_n) + (\alpha+\beta)L_n(x_n^F + q_n) + E_{d_n} f_{n+1}(q_{n+1})\}\\
&f_{n+1}(q_{n+1}) = 0\\
&\frac{d[\alpha(x_n^F + q_n - \mu_n) + (\alpha+\beta)L_n(x_n^F + q_n)]}{dx_n^F} = (\alpha+\beta)G_n(x_n^F + q_n) - \beta\\
&\frac{d^2[\alpha(x_n^F + q_n - \mu_n) + (\alpha+\beta)L_n(x_n^F + q_n)]}{dx_n^{F2}} = (\alpha+\beta)g_n(x_n^F + q_n) \geq 0
\end{aligned}
$$

The first order condition is $x_n^F = G_n^{-1}\left(\frac{\beta}{\alpha+\beta}\right) - q_n$. Recall $0 \leq x_n^F \leq C$, the optimal solution is $x_n^{F*} = \min\left\{C, \left(G_n^{-1}\left(\frac{\beta}{\alpha+\beta}\right) - q_n\right)^+\right\}$.

Proof of Lemma 1.

One can also calculate that

$$
\frac{\partial G_n^{-1}(\frac{\beta}{\alpha+\beta})}{\partial x_{n-1}^F} = \begin{cases} -r & , \quad x_{n-1}^F < d_{n-1} - q_{n-1} \\ 0 & , \quad x_{n-1}^F > d_{n-1} - q_{n-1} \end{cases}
$$

and

$$
\frac{\partial G_n^{-1}(\frac{\beta}{\alpha+\beta})}{\partial d_{n-1}} = \begin{cases} \frac{dM(d_{n-1})}{d(d_{n-1})} & , \quad d_{n-1} < x_{n-1}^F + q_{n-1} \\ \frac{dM(d_{n-1})}{d(d_{n-1})} + r & , \quad d_{n-1} > x_{n-1}^F + q_{n-1} \end{cases}
$$

Recall that $\frac{dM(d_{n-1})}{d(d_{n-1})} \geq 0$. One can obtain $\frac{\partial G_n^{-1}(\frac{\beta}{\alpha+\beta})}{\partial x_{n-1}^F} \leq 0$ and $\frac{\partial G_n^{-1}(\frac{\beta}{\alpha+\beta})}{\partial d_{n-1}} \geq 0$ (except the point $d_{n-1} = x_{n-1}^F + q_{n-1}$).

Proof of Lemma 2.

Set $y = G_n^{-1}(\frac{\beta}{\alpha+\beta})$, that is, $G_n(y) = \frac{\beta}{\alpha+\beta}$. With a given d_{n-1}, we have

$$
\begin{aligned}
&\begin{cases} y > M(d_{n-1}) + r(d_{n-1} - x_{n-1}^F - q_{n-1})^+ \\ 1 - e^{-\lambda[y - M(d_{n-1}) - r(d_{n-1} - x_{n-1}^F - q_{n-1})^+]} = \frac{\beta}{\alpha+\beta} \end{cases}\\
&\Rightarrow e^{-\lambda[y - M(d_{n-1}) - r(d_{n-1} - x_{n-1}^F - q_{n-1})^+]} = \frac{\alpha}{\alpha+\beta}\\
&\Rightarrow y = -\frac{1}{\lambda}ln(\frac{\alpha}{\alpha+\beta}) + M(d_{n-1}) + r(d_{n-1} - x_{n-1}^F - q_{n-1})^+
\end{aligned}
$$

Thus,

$$\frac{\partial}{\partial d_{n-1}}\left[G_n^{-1}(\frac{\beta}{\alpha+\beta})-q_n\right]$$
$$=\frac{\partial}{\partial d_{n-1}}\left[-\frac{1}{\lambda}ln(\frac{\alpha}{\alpha+\beta})+M(d_{n-1})+r(d_{n-1}-x_{n-1}^F-q_{n-1})^+-(x_{n-1}^F+q_{n-1}-d_{n-1})^+\right]$$
$$=\begin{cases}\frac{dM(d_{n-1})}{d(d_{n-1})}+1 & , \quad d_{n-1}<x_{n-1}^F+q_{n-1}\\ \frac{dM(d_{n-1})}{d(d_{n-1})}+r & , \quad d_{n-1}>x_{n-1}^F+q_{n-1}\end{cases}$$
$$>0$$

That is, $G_n^{-1}(\frac{\beta}{\alpha+\beta})-q_n$ is strictly increasing in d_{n-1}.

When $d_{n-1}\to-\infty$, $G_n^{-1}(\frac{\beta}{\alpha+\beta})-q_n\to-\infty$; when $d_{n-1}\to+\infty$, $G_n^{-1}(\frac{\beta}{\alpha+\beta})-q_n\to+\infty$. So there exists unique d'_{n-1} and d''_{n-1} defined as Lemma 2.

Proof of Lemma 3.

We first prove the objective function in the (*n-1*)-th period $E_{d_{n-1}}\{Y_{n-1}(q_{n-1},d_{n-1},x_{n-1}^F)+f_n(q_n)\}$, which is the expectation of the total cost in the (*n-1*)-th and n-th periods, is convex in x_{n-1}^F for any given C, q_{n-1}, $G_{n-1}(d_{n-1})$, and $G_n(d_n)$.

We will first get the first-order derivative of the objective function, followed by the second-order derivative. Then prove that the second-order derivation is not less than 0.

Note that when $d_{n-1}=d'_{n-1}$, $(\alpha+\beta)G_n(q_{n-1}+x_{n-1}^F-d_{n-1})-\beta=0$; when $d_{n-1}=d''_{n-1}$, $(\alpha+\beta)G_n(C+q_{n-1}+x_{n-1}^F-d_{n-1})-\beta=0$.

$$\frac{d}{dx_{n-1}^F}\int_0^{d'_{n-1}}E_{d_n}\{\alpha(x_{n-1}^F+q_{n-1}-d_{n-1}-d_n)^+$$
$$+\beta(d_n-q_{n-1}-x_{n-1}^F+d_{n-1})^+\}g_{n-1}(d_{n-1})\,d(d_{n-1})$$
$$=\frac{d}{dx_{n-1}^F}\int_0^{d'_{n-1}}\left[\alpha\int_0^{x_{n-1}^F+q_{n-1}-d_{n-1}}(x_{n-1}^F+q_{n-1}-d_{n-1}-d_n)G_n(d_n)\,d(d_n)\right.$$
$$\left.+\beta\int_{x_{n-1}^F+q_{n-1}-d_{n-1}}^{\infty}(d_n-q_{n-1}-x_{n-1}^F+d_{n-1})G_n(d_n)\,d(d_n)\right]g_{n-1}(d_{n-1})\,d(d_{n-1})$$
$$=\int_0^{d'_{n-1}}\left[\alpha\int_0^{x_{n-1}^F+q_{n-1}-d_{n-1}}G_n(d_n)\,d(d_n)\right.$$
$$\left.+\beta\int_{x_{n-1}^F+q_{n-1}-d_{n-1}}^{\infty}(-1)G_n(d_n)\,d(d_n)\right]g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$=\int_0^{d'_{n-1}} [(\alpha+\beta)G_n(x^F_{n-1}+q_{n-1}-d_{n-1})-\beta]g_{n-1}(d_{n-1})\,d(d_{n-1})$$

Let $\psi = \frac{dG_n(d_n)|_{d_{n-1}}}{dx^F_{n-1}}$. When $d_{n-1} > q_{n-1} + x^F_{n-1}$ and $d_n > M(d_{n-1}) + r(d_{n-1} - q_{n-1} - x^F_{n-1})$, $\psi = -\lambda^2 r e^{-\lambda[d_n - M(d_{n-1}) - r(d_{n-1}-q_{n-1}-x^F_{n-1})]}$; otherwise, $\psi = 0$.

If $d''_{n-1} \le q_{n-1} + x^F_{n-1}$, then

$$\frac{d}{dx^F_{n-1}}\int_{d'_{n-1}}^{d''_{n-1}} E_{d_n}\{\alpha(G_n^{-1}(\frac{\beta}{\alpha+\beta})-d_n)^+ + \beta(d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta}))^+\}g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$=\frac{d}{dx^F_{n-1}}\int_{d'_{n-1}}^{d''_{n-1}}\left[\alpha\int_0^{G_n^{-1}(\frac{\beta}{\alpha+\beta})}(G_n^{-1}(\frac{\beta}{\alpha+\beta})-d_n)G_n(d_n)\,d(d_n)\right.$$

$$\left.+\beta\int_{G_n^{-1}(\frac{\beta}{\alpha+\beta})}^{\infty}(d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta}))G_n(d_n)\,d(d_n)\right]g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$=0$$

and

$$\frac{d}{dx^F_{n-1}}\int_{d''_{n-1}}^{\infty} E_{d_n}\{\alpha[C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+-d_n]^+$$

$$+\beta[d_n-(q_{n-1}+x^F_{n-1}-d_{n-1})^+-C]^+\}g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$=\frac{d}{dx^F_{n-1}}\int_{d''_{n-1}}^{q_{n-1}+x^F_{n-1}}\{\alpha\int_0^{C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+}[C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+-d_n]G_n(d_n)\,d(d_n)$$

$$+\beta\int_{C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+}^{\infty}[d_n-(q_{n-1}+x^F_{n-1}-d_{n-1})^+-C]G_n(d_n)\,d(d_n)\}g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$+\frac{d}{dx^F_{n-1}}\int_{q_{n-1}+x^F_{n-1}}^{\infty}\{\alpha\int_0^{C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+}[C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+-d_n]G_n(d_n)\,d(d_n)$$

$$+\beta\int_{C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+}^{\infty}[d_n-(q_{n-1}+x^F_{n-1}-d_{n-1})^+-C]G_n(d_n)\,d(d_n)\}g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$=\int_{d''_{n-1}}^{q_{n-1}+x^F_{n-1}}[\alpha\int_0^{C+q_{n-1}+x^F_{n-1}-d_{n-1}}G_n(d_n)\,d(d_n)$$

$$+\beta\int_{C+q_{n-1}+x^F_{n-1}-d_{n-1}}^{\infty}(-1)G_n(d_n)\,d(d_n)]g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$+\int_{q_{n-1}+x^F_{n-1}}^{\infty}[\alpha\int_0^C(C-d_n)\psi\,d(d_n)+\beta\int_C^{\infty}(d_n-C)\psi\,d(d_n)]g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$=\int_{d''_{n-1}}^{q_{n-1}+x^F_{n-1}}[(\alpha+\beta)G_n(C+q_{n-1}+x^F_{n-1}-d_{n-1})-\beta]g_{n-1}(d_{n-1})\,d(d_{n-1})$$

$$+\int_{q_{n-1}+x^F_{n-1}}^{\infty}[\alpha\int_0^C(C-d_n)\psi\,d(d_n)+\beta\int_C^{\infty}(d_n-C)\psi\,d(d_n)]g_{n-1}(d_{n-1})\,d(d_{n-1})$$

If $d''_{n-1} > q_{n-1} + x^F_{n-1}$, then

$$\begin{aligned}
&\frac{d}{dx^F_{n-1}} \int_{d'_{n-1}}^{d''_{n-1}} E_{d_n}\{\alpha(G_n^{-1}(\frac{\beta}{\alpha+\beta}) - d_n)^+ + \beta(d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta}))^+\} g_{n-1}(d_{n-1})\, d(d_{n-1})\\
=&\frac{d}{dx^F_{n-1}} \int_{q_{n-1}+x^F_{n-1}}^{d''_{n-1}} \left[\alpha \int_0^{G_n^{-1}(\frac{\beta}{\alpha+\beta})} (G_n^{-1}(\frac{\beta}{\alpha+\beta}) - d_n) G_n(d_n)\, d(d_n)\right.\\
&\left. +\beta \int_{G_n^{-1}(\frac{\beta}{\alpha+\beta})}^{\infty} (d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta})) G_n(d_n)\, d(d_n)\right] g_{n-1}(d_{n-1})\, d(d_{n-1})\\
=&\int_{q_{n-1}+x^F_{n-1}}^{d''_{n-1}} \{\alpha \int_0^{G_n^{-1}(\frac{\beta}{\alpha+\beta})} [-rG_n(d_n) + (G_n^{-1}(\frac{\beta}{\alpha+\beta}) - d_n)\psi]\, d(d_n)\\
&+ \beta \int_{G_n^{-1}(\frac{\beta}{\alpha+\beta})}^{\infty} [rG_n(d_n) + (d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta}))\psi]\, d(d_n)\} g_{n-1}(d_{n-1})\, d(d_{n-1})\\
=&\int_{q_{n-1}+x^F_{n-1}}^{d''_{n-1}} \{\alpha \int_0^{G_n^{-1}(\frac{\beta}{\alpha+\beta})} [G_n^{-1}(\frac{\beta}{\alpha+\beta}) - d_n]\psi\, d(d_n)\\
&+ \beta \int_{G_n^{-1}(\frac{\beta}{\alpha+\beta})}^{\infty} [d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta})]\psi\, d(d_n)\} g_{n-1}(d_{n-1})\, d(d_{n-1})
\end{aligned}$$

and

$$\begin{aligned}
&\frac{d}{dx^F_{n-1}} \int_{d''_{n-1}}^{\infty} E_{d_n}\{\alpha[C + (q_{n-1} + x^F_{n-1} - d_{n-1})^+ - d_n]^+\\
&+ \beta[d_n - (q_{n-1} + x^F_{n-1} - d_{n-1})^+ - C]^+\} g_{n-1}(d_{n-1})\, d(d_{n-1})\\
=&\frac{d}{dx^F_{n-1}} \int_{d''_{n-1}}^{\infty} \{\alpha \int_0^{C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+} [C + (q_{n-1} + x^F_{n-1} - d_{n-1})^+ - d_n] G_n(d_n)\, d(d_n)\\
&+ \beta \int_{C+(q_{n-1}+x^F_{n-1}-d_{n-1})^+}^{\infty} [d_n - (q_{n-1} + x^F_{n-1} - d_{n-1})^+ - C] G_n(d_n)\, d(d_n)\} g_{n-1}(d_{n-1})\, d(d_{n-1})\\
=&\int_{d''_{n-1}}^{\infty} \left[\alpha \int_0^C (C - d_n)\psi\, d(d_n) + \beta \int_C^{\infty} (d_n - C)\psi\, d(d_n)\right] g_{n-1}(d_{n-1})\, d(d_{n-1})
\end{aligned}$$

So the first-order derivative of the objective function is as follows.

If $d''_{n-1} \le q_{n-1} + x^F_{n-1}$, then

$$\begin{aligned}
&\frac{\partial}{\partial x^F_{n-1}} E_{d_{n-1}}\{Y_{n-1}(d_{n-1}, q_{n-1}, x^F_{n-1}) + f_n(q_n)\}\\
=&(\alpha+\beta) G_{n-1}(x^F_{n-1} + q_{n-1}) - \beta\\
&+ \int_0^{d'_{n-1}} [(\alpha+\beta) G_n(x^F_{n-1} + q_{n-1} - d_{n-1}) - \beta] g_{n-1}(d_{n-1})\, d(d_{n-1})\\
&+ \int_{d''_{n-1}}^{q_{n-1}+x^F_{n-1}} [(\alpha+\beta) G_n(C + q_{n-1} + x^F_{n-1} - d_{n-1}) - \beta] g_{n-1}(d_{n-1})\, d(d_{n-1})
\end{aligned}$$

$$+\int_{q_{n-1}+x_{n-1}^F}^{\infty}[\alpha\int_0^C (C-d_n)\psi\, d(d_n)+\beta\int_C^{\infty}(d_n-S)\psi\, d(d_n)]g_{n-1}(d_{n-1})\, d(d_{n-1})$$

If $d''_{n-1} > q_{n-1}+x_{n-1}^F$, then

$$\begin{aligned}
&\frac{\partial}{\partial x_{n-1}^F}E_{d_{n-1}}\{Y_{n-1}(d_{n-1},q_{n-1},x_{n-1}^F)+f_n(q_n)\}\\
=&(\alpha+\beta)G_{n-1}(x_{n-1}^F+q_{n-1})-\beta\\
&+\int_0^{d'_{n-1}}[(\alpha+\beta)G_n(x_{n-1}^F+q_{n-1}-d_{n-1})-\beta]g_{n-1}(d_{n-1})\, d(d_{n-1})\\
&+\int_{q_{n-1}+x_{n-1}^F}^{d''_{n-1}}\{\alpha\int_0^{G_n^{-1}(\frac{\beta}{\alpha+\beta})}[G_n^{-1}(\frac{\beta}{\alpha+\beta})-d_n]\psi\, d(d_n)\\
&+\beta\int_{G_n^{-1}(\frac{\beta}{\alpha+\beta})}^{\infty}[d_n-G_n^{-1}(\frac{\beta}{\alpha+\beta})]\psi\, d(d_n)\}g_{n-1}(d_{n-1})\, d(d_{n-1})\\
&+\int_{d''_{n-1}}^{\infty}\left[\alpha\int_0^C (C-d_n)\psi\, d(d_n)+\beta\int_C^{\infty}(d_n-C)\psi\, d(d_n)\right]g_{n-1}(d_{n-1})\, d(d_{n-1})
\end{aligned}$$

Note that when $x_{n-1}^F = d_{n-1}-q_{n-1}$, $\psi = 0$. Then we will get the second-order derivative of the objective function and prove it is not less than 0.

If $d''_{n-1} \le q_{n-1}+x_{n-1}^F$, then

$$\begin{aligned}
&\frac{\partial^2}{\partial^2 x_{n-1}^F}E_{d_{n-1}}\{Y_{n-1}(d_{n-1},q_{n-1},x_{n-1}^F)+f_n(q_n)\}\\
=&\alpha g_{n-1}(x_{n-1}^F+q_{n-1})+(\alpha+\beta)G_n(C)|_{d_{n-1}=q_{n-1}+x_{n-1}^F}g_{n-1}(q_{n-1}+x_{n-1}^F)\\
&+\int_0^{d'_{n-1}}(\alpha+\beta)G_n(x_{n-1}^F+q_{n-1}-d_{n-1})g_{n-1}(d_{n-1})\, d(d_{n-1})\\
&+\int_{d''_{n-1}}^{q_{n-1}+x_{n-1}^F}(\alpha+\beta)G_n(C+q_{n-1}+x_{n-1}^F-d_{n-1})g_{n-1}(d_{n-1})\, d(d_{n-1})\\
&+\int_{q_{n-1}+x_{n-1}^F}^{\infty}\left[\alpha\int_0^C (C-d_n)\frac{\partial\psi}{\partial x_{n-1}^F}\, d(d_n)+\beta\int_C^{\infty}(d_n-C)\frac{\partial\psi}{\partial x_{n-1}^F}\, d(d_n)\right]g_{n-1}(d_{n-1})\, d(d_{n-1})
\end{aligned}$$

If $d''_{n-1} > q_{n-1}+x_{n-1}^F$, then

$$\begin{aligned}
&\frac{\partial^2}{\partial^2 x_{n-1}^F}E_{d_{n-1}}\{Y_{n-1}(d_{n-1},q_{n-1},x_{n-1}^F)+f_n(q_n)\}\\
=&(\alpha+\beta)g_{n-1}(x_{n-1}^F+q_{n-1})\\
&+\int_0^{d'_{n-1}}(\alpha+\beta)G_n(x_{n-1}^F+q_{n-1}-d_{n-1})g_{n-1}(d_{n-1})\, d(d_{n-1})\\
&+\int_{q_{n-1}+x_{n-1}^F}^{d''_{n-1}}\left[\alpha\int_0^{G_n^{-1}(\frac{\beta}{\alpha+\beta})}\left(G_n^{-1}(\frac{\beta}{\alpha+\beta})-d_n\right)\frac{\partial\psi}{\partial x_{n-1}^F}\, d(d_n)\right.
\end{aligned}$$

$$+\beta \int_{G_n^{-1}(\frac{\beta}{\alpha+\beta})}^{\infty} \left(d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta})\right) \frac{\partial \psi}{\partial x_{n-1}^F}\, d(d_n)\Bigg]\, g_{n-1}(d_{n-1})\, d(d_{n-1})$$

$$+\int_{d''_{n-1}}^{\infty} \left[\alpha \int_0^C (C-d_n)\frac{\partial \psi}{\partial x_{n-1}^F}\, d(d_n) + \beta \int_C^{\infty} (d_n - C)\frac{\partial \psi}{\partial x_{n-1}^F}\, d(d_n)\right] g_{n-1}(d_{n-1})\, d(d_{n-1})$$

where

$$\frac{\partial \psi}{\partial x_{n-1}^F} = \begin{cases} 0 & , \quad d_n < M(d_{n-1}) + r(d_{n-1} - q_{n-1} - x_{n-1}^F) \\ \lambda^3 r^2 e^{-\lambda[d_n - M(d_{n-1}) - r(d_{n-1} - q_{n-1} - x_{n-1}^F)]} & , \quad d_n > M(d_{n-1}) + r(d_{n-1} - q_{n-1} - x_{n-1}^F) \end{cases}$$

Obviously, in both situations, $\frac{\partial^2}{\partial^2 x_{n-1}^F} E_{d_{n-1}}\{Y_{n-1}(d_{n-1}, q_{n-1}, x_{n-1}^F) + f_n(q_n)\} > 0$.

And since both first-order and second-order derivatives are continuous at the point $x_{n-1}^F = d''_{n-1} - q_{n-1}$, we can obtain that $E_{d_{n-1}}\{Y_{n-1}(d_{n-1}, q_{n-1}, x_{n-1}^F) + f_n(q_n)\}$ is convex in x_{n-1}^F, for any C, $G_{n-1}(d_{n-1})$, and $G_n(d_n)$.

Then we prove (1) I_{n-1} is an increasing function of q_{n-1} and C. (2) $\lim_{q_{n-1}\to\infty} I_{n-1} > 0$.

(1) For Q, note that $E_{d_{n-1}}\{Y_{n-1}(d_{n-1}, q_{n-1}, x_{n-1}^F) + f_n(q_n)\}$ is a function of $x_{n-1}^F + q_{n-1}$.

$$\frac{\partial I_{n-1}}{\partial q_{n-1}} = \frac{\partial I_{n-1}}{\partial x_{n-1}^F} = \frac{\partial^2}{\partial^2 x_{n-1}^F} E_{d_{n-1}}\{Y_{n-1}(d_{n-1}, q_{n-1}, x_{n-1}^F) + f_n(q_n)\} > 0$$

That is, I_{n-1} is increasing in q_{n-1}.

For C, if $C < G_n^{-1}(\frac{\beta}{\alpha+\beta})$, then

$$\begin{aligned}\frac{\partial I_{n-1}}{\partial C} =& (\alpha+\beta) \int_{x_{n-1}^F + q_{n-1} - \left(G_n^{-1}(\frac{\beta}{\alpha+\beta}) - C\right)}^{x_{n-1}^F + q_{n-1}} G_n(C + x_{n-1}^F + q_{n-1} - d_{n-1}) g_{n-1}(d_{n-1})\, d(d_{n-1}) \\ &\geq 0\end{aligned}$$

If $C \geq G_n^{-1}(\frac{\beta}{\alpha+\beta})$, then

$$\frac{\partial I_{n-1}}{\partial C} = 0$$

(2) According to the definition of d''_{n-1}, when $q_{n-1} \to \infty$, $d''_{n-1} \to \infty$. Therefore,

$$\lim_{q_{n-1}\to\infty} I_{n-1} = \alpha + \int_0^{d'_{n-1}} \alpha g_{n-1}(d_{n-1})\, d(d_{n-1}) > 0$$

For any given C, there exists a unique $h_{n-1}(C) > 0$ such that $I^0_{n-1}|_{q_{n-1}<h_{n-1}(C)} < 0$, $I^0_{n-1}|_{q_{n-1}=h_{n-1}(C)} = 0$, and $I^0_{n-1}|_{q_{n-1}>h_{n-1}(C)} > 0$.

According to the definition of I^0_{n-1},

If $d''_{n-1} \le q_{n-1}$, then

$$\begin{aligned} I^0_{n-1} =& (\alpha+\beta)G_{n-1}(q_{n-1}) - \beta \\ &+ \int_0^{d'_{n-1}} [(\alpha+\beta)G_n(q_{n-1}-d_{n-1}) - \beta] g_{n-1}(d_{n-1})\, d(d_{n-1}) \\ &+ \int_{d''_{n-1}}^{q_{n-1}} [(\alpha+\beta)G_n(C+q_{n-1}-d_{n-1}) - \beta] g_{n-1}(d_{n-1})\, d(d_{n-1}) \\ &+ \int_{q_{n-1}}^{\infty} [\alpha \int_0^C (C-d_n)\psi\, d(d_n) + \beta \int_C^\infty (d_n - C)\psi\, d(d_n)] g_{n-1}(d_{n-1})\, d(d_{n-1}) \end{aligned}$$

If $d''_{n-1} > q_{n-1}$, then

$$\begin{aligned} I^0_{n-1} =& (\alpha+\beta)G_{n-1}(q_{n-1}) - \beta \\ &+ \int_0^{d'_{n-1}} [(\alpha+\beta)G_n(q_{n-1}-d_{n-1}) - \beta] g_{n-1}(d_{n-1})\, d(d_{n-1}) \\ &+ \int_{q_{n-1}}^{d''_{n-1}} \{\alpha \int_0^{G_n^{-1}(\frac{\beta}{\alpha+\beta})} [G_n^{-1}(\frac{\beta}{\alpha+\beta}) - d_n]\psi\, d(d_n) \\ &+ \beta \int_{G_n^{-1}(\frac{\beta}{\alpha+\beta})}^{\infty} [d_n - G_n^{-1}(\frac{\beta}{\alpha+\beta})]\psi\, d(d_n)\} g_{n-1}(d_{n-1})\, d(d_{n-1}) \\ &+ \int_{d''_{n-1}}^{\infty} \left[\alpha \int_0^C (C-d_n)\psi\, d(d_n) + \beta \int_C^\infty (d_n - C)\psi\, d(d_n)\right] g_{n-1}(d_{n-1})\, d(d_{n-1}) \end{aligned}$$

We can obtain

$$\begin{aligned} I^0_{n-1}|_{q_{n-1}=0} =& -\beta + \int_0^\infty \left[\alpha \int_0^C (C-d_n)\psi\, d(d_n) + \beta \int_C^\infty (d_n - C)\psi\, d(d_n)\right] g_{n-1}(d_{n-1})\, d(d_{n-1}) \\ &< 0 \quad (Recall \quad \psi \le 0) \end{aligned}$$

According to Lemma 3, $\lim\limits_{q_{n-1}\to\infty} I_{n-1} > 0$ and $\frac{\partial I^0_{n-1}}{\partial q_{n-1}} > 0$. So we can obtain: for any given C, there exists a unique $h_{n-1}(C) > 0$, such that $I^0_{n-1}|_{q_{n-1}<h_{n-1}(C)} < 0$, $I^0_{n-1}|_{q_{n-1}=h_{n-1}(C)} = 0$ and $I^0_{n-1}|_{q_{n-1}>h_{n-1}(C)} > 0$.

Proof of Proposition 3.

According to the definition of d''_{n-1}, when $x^F_{n-1} \to \infty$, $d''_{n-1} \to \infty$. Therefore,

$$\lim_{x^F_{n-1}\to\infty} I_{n-1} = \alpha + \int_0^{d'_{n-1}} \alpha g_{n-1}(d_{n-1})\, d(d_{n-1}) > 0$$

And since Lemma 3 and $E_{d_{n-1}}\{Y_{n-1}(q_{n-1}, d_{n-1}+f_n(q_n), x^F_{n-1}\}$ is convex in x^F_{n-1}, we can obtain: (1) If $q_{n-1} < h_{n-1}$, then there exists $\bar{x}_{n-1} > 0$, such that $I_{n-1}|_{x^F_{n-1}<\bar{x}_{n-1}} < 0$, $I_{n-1}|_{x^F_{n-1}=\bar{x}_{n-1}} = 0$ and $I_{n-1}|_{x^F_{n-1}>\bar{x}_{n-1}} > 0$. Thus, $X^*_{n-1} = min\{C, \bar{x}_{n-1}\}$. (2) If $q_{n-1} \geq h_{n-1}(C)$, then $X^*_{n-1} = 0$;

In addition, note that $I_{n-1}|_{x^F_{n-1}=x, q_{n-1}=q} = I_{n-1}|_{x^F_{n-1}=x+\Delta, q_{n-1}=q-\Delta}$ (x, q, $x+\Delta$, $q-\Delta \geq 0$). So $\bar{x}_{n-1} = h_{n-1}(C) - q_{n-1}$. Therefore, the optimal solution is

$$x^{F*}_{n-1} = \begin{cases} 0 & , \quad q_{n-1} \geq h_{n-1}(C) \\ h_{n-1}(C) - q_{n-1} & , \quad h_{n-1}(C) - C \leq q_{n-1} < h_{n-1}(C) \\ C & , \quad q_{n-1} < h_{n-1}(C) - C \end{cases}$$

and $x^{F*}_n = \min\left\{C, \left(G^{-1}_n\left(\frac{\beta}{\alpha+\beta}\right) - \left(q_{n-1} + x^{F*}_{n-1} - d_{n-1}\right)\right)^+\right\}$.

Proof of Lemma 4.

We use induction to prove these propositions and lemmas. First, assume these propositions and lemmas hold for period $t+1 (t = 1, 2, \ldots, n-1)$, that is:

(1) Given an optimal policy of the (*i+2*)-th period, the objective function in the (*t+1*)-th period $E_{d_{t+1}}\{Y_{t+1}(q_{t+1}, d_{t+1}, x^F_{t+1}) + f_{t+2}(q_{t+2})\}$, which represents the expectation of the total cost from the (*t+1*)-th to the n-th period) is convex in x^F_{t+1} for any given C, q_{t+1} and $G_{t+1}(d_{t+1})$;

(2) The optimal solution at period *t+1* is

$$x^{F*}_{t+1} = \begin{cases} 0 & , \quad q_{t+1} \geq h_{t+1}(C) \\ h_{t+1}(C) - q_{t+1} & , \quad h_{t+1}(C) - C \leq q_{t+1} < h_{t+1}(C) \\ C & , \quad q_{t+1} < h_{t+1}(C) - C \end{cases}$$

where $h_{t+1}(C)$ is a function of C defined as $I^0_{t+1}|_{q_{t+1}=h_{t+1}(C)} = 0$.

We have proved that these propositions and lemmas hold for periods *n-1* and *n*. Then we prove below that they hold for $t = 1, 2, \ldots, n-2$.

We first show that given an optimal policy in period $t+1$, the objective function $E_{d_t}\{Y_t(q_t, d_t, x^F_t) + f_{t+1}(q_{t+1}, x^{F*}_{t+1})\}$ in period t, namely the expectation of the total cost from period t to period n, is convex in x^F_t for any given C, q_t, and $G_t(d_t)$.

According to the assumptions of induction,

$$x_{t+1}^{F*} = \begin{cases} 0 & , \quad q_{t+1} \geq h_{t+1}(C) \\ h_{t+1}(C) - q_{t+1} & , \quad h_{t+1}(C) - C \leq q_{t+1} < h_{t+1}(C) \\ C & , \quad q_{t+1} < h_{t+1}(C) - C \end{cases}$$

And since $\frac{\partial q_{t+1}}{\partial x_t^F} = \begin{cases} 1 & , \quad d_t < q_t + x_t^F \\ 0 & , \quad d_t > q_t + x_t^F \end{cases}$, we get $\frac{\partial^2 (q_{t+1} + x_{t+1}^F)}{\partial x_t^{F^2}} = 0$, almost everywhere. Thus, almost everywhere,

$$\begin{aligned} &\frac{d^2 f_{t+1}(q_{t+1}, x_{t+1}^{F*})}{dx_t^{F^2}} \\ =&\frac{d^2}{d\left(q_{t+1} + x_{t+1}^{F*}\right)^2} E_{d_{t+1}}\{Y_{t+1}(q_{t+1}, d_{t+1}, x_{t+1}^F) + f_{t+2}(q_{t+2}, x_{t+2}^{F*})\} \left[\frac{d(q_{t+1} + x_{t+1}^{F*})}{dx_t^F}\right]^2 \\ =&\frac{\partial^2}{\partial x_{t+1}^{L*2}} E_{d_{t+1}}\{Y_{t+1}(q_{t+1}, d_{t+1}, x_{t+1}^F) + f_{t+2}(q_{t+2}, x_{t+2}^{F*})\} \left[\frac{d(q_{t+1} + x_{t+1}^{F*})}{dx_t^F}\right]^2 \end{aligned}$$

Since $E_{d_{t+1}}\{Y_{t+1}(q_{t+1}, d_{t+1}, x_{t+1}^F) + f_{t+2}(q_{t+2}, x_{t+2}^{F*})\}$ is convex in x_{t+1}^F (assumption of induction), $\frac{d^2 E_{d_{t+1}}\{Y_{t+1}(q_{t+1}, d_{t+1}, x_{t+1}^F) + f_{t+2}(q_{t+2}, x_{t+2}^{F*})\}}{dx_{t+1}^{F2}} \geq 0$. Thus, $\frac{d^2 f_{t+1}(q_{t+1}, x_{t+1}^{F*})}{dx_t^{F^2}} \geq 0$.

Therefore, $\frac{d^2 f_{t+1}(q_{t+1}, x_{t+1}^{F*})}{dx_t^{F^2}} \geq 0$, almost everywhere.

Also,

$$\begin{aligned} &\frac{d^2 E_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1}, x_{t+1}^{F*})\}}{dx_t^{F^2}} \\ =&\frac{d^2}{dx_t^{F^2}} \left[\alpha(q_t + x_t^F - \mu_t) + (\alpha + \beta) L_t(x_t^F + q_t) + E_{d_t} f_{t+1}(q_{t+1}, x_{t+1}^{F*})\right] \\ =&(\alpha + \beta) g_t(x_t^F + q_t) + \int_0^\infty \frac{d^2 f_{t+1}(q_{t+1}, x_{t+1}^{F*})}{dx_t^{F^2}} g_t(d_t)\, d(d_t) \\ \geq& 0 \end{aligned}$$

And since $\frac{d^2 E_{d_{n-1}}\{Y_{n-1}(x_{n-1}^F, d_{n-1}) + f_n(q_n, x_n^{F*})\}}{dx_{n-1}^{F2}} \geq 0$ (we can obtain that for any $i = 1, 2, \ldots, n-1$, $\frac{d^2 E_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1}, x_{t+1}^{F*})\}}{dx_t^{F^2}} \geq 0$. That is, $E_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1}, x_{t+1}^{F*})\}$ is convex in x_t^F.

$$\frac{\partial I_t}{\partial q_t} = \frac{\partial I_t}{\partial x_t^F} = \frac{d^2 E_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1}, x_{t+1}^{F*})\}}{dx_t^{F^2}} \geq 0$$

$$
\begin{aligned}
\frac{\partial I_t}{\partial C} =& \frac{\partial}{\partial C}\int_0^{q_t+x_t^F-h_{t+1}(C)} \frac{\partial f_{t+1}(q_{t+1},0)}{\partial x_t^F} g_t(d_t)\, d(d_t) \\
&+ \frac{\partial}{\partial C}\int_{q_t+x_t^F-(h_{t+1}(C)-C)^+}^{q_t+x_t^F} \frac{\partial f_{t+1}(q_{t+1}, C-(C-h_{t+1}(C))^+)}{\partial x_t^F} g_t(d_t)\, d(d_t) \\
=& \frac{\partial}{\partial C}\int_0^{q_t+x_t^F-h_{t+1}(C)} \frac{\partial f_{t+1}(q_{t+1},0)}{\partial q_{t+1}} \frac{\partial q_{t+1}}{\partial x_t^F} g_t(d_t)\, d(d_t) \\
&+ \frac{\partial}{\partial C}\int_{q_t+x_t^F-(h_{t+1}(C)-C)^+}^{q_t+x_t^F} \frac{\partial f_{t+1}(q_{t+1}, C-(C-h_{t+1}(C))^+)}{\partial q_{t+1}} \frac{\partial q_{t+1}}{\partial x_t^F} g_t(d_t)\, d(d_t) \\
=& \frac{\partial}{\partial C}\int_0^{q_t+x_t^F-h_{t+1}(C)} \frac{\partial f_{t+1}(q_{t+1},0)}{\partial q_{t+1}} g_t(d_t)\, d(d_t) \\
&+ \frac{\partial}{\partial C}\int_{q_t+x_t^F-(h_{t+1}(C)-C)^+}^{q_t+x_t^F} \frac{\partial f_{t+1}(q_{t+1}, C-(C-h_{t+1}(C))^+)}{\partial q_{t+1}} g_t(d_t)\, d(d_t) \\
=& \int_0^{q_t+x_t^F-h_{t+1}(C)} \frac{\partial^2 f_{t+1}(q_{t+1},0)}{\partial C\partial q_{t+1}} g_t(d_t)\, d(d_t) \\
&+ \frac{\partial f_{t+1}(q_{t+1},0)}{\partial q_{t+1}}\Big|_{d_t=q_t+x_t^F-h_{t+1}(C)} g_t\left(q_t+x_t^F-h_{t+1}(C)\right) \frac{d\left(q_t+x_t^F-h_{t+1}(C)\right)}{dC} \\
&+ \int_{q_t+x_t^F-(h_{t+1}(C)-C)^+}^{q_t+x_t^F} \frac{\partial^2 f_{t+1}(q_{t+1}, C-(C-h_{t+1}(C))^+)}{\partial C\partial q_{t+1}} g_t(d_t)\, d(d_t) \\
&- \frac{\partial f_{t+1}(q_{t+1}, C-(C-h_{t+1}(C))^+)}{\partial q_{t+1}}\Big|_{d_t=q_t+x_t^F-(h_{t+1}(C)-C)^+} \\
&\quad g_t\left(q_t+x_t^F-(h_{t+1}(C)-C)^+\right) \frac{d\left(q_t+x_t^F-(h_{t+1}(C)-C)^+\right)}{dC}
\end{aligned}
$$

And

$$
\begin{aligned}
\because\quad & \frac{\partial^2 f_{t+1}(q_{t+1},0)}{\partial C\partial q_{t+1}} = 0 \\
& \frac{\partial f_{t+1}(q_{t+1},0)}{\partial q_{t+1}}\Big|_{d_t=q_t+x_t^F-h_{t+1}(C)} = \frac{\partial f_{t+1}(q_{t+1},0)}{\partial q_{t+1}}\Big|_{q_{t+1}=h_{t+1}(C)} = 0 \\
& \frac{\partial f_{t+1}(q_{t+1}, C-(C-h_{t+1}(C))^+)}{\partial q_{t+1}}\Big|_{d_t=q_t+x_t^F-(h_{t+1}(C)-C)^+} \\
& = \begin{cases} \frac{\partial f_{t+1}(q_{t+1},C)}{\partial q_{t+1}}\big|_{q_{t+1}=h_{t+1}^M(C)} = 0 \quad , & C \le h_{t+1}(C) \\ \frac{\partial f_{t+1}(q_{t+1},h_{t+1}(C))}{\partial q_{t+1}}\big|_{q_{t+1}=0} = 0 \quad , & C > h_{t+1}(C) \end{cases} \\
\therefore\quad & \frac{\partial I_t}{\partial C} = \int_{q_t+x_t^F-(h_{t+1}(C)-C)^+}^{q_t+x_t^F} \frac{\partial^2 f_{t+1}(q_{t+1}, C-(C-h_{t+1}(C))^+)}{\partial C\partial q_{t+1}} g_t(d_t)\, d(d_t)
\end{aligned}
$$

Assume $\frac{\partial^2 f_{t+2}(q_{t+2},x_{t+2}^{F*})}{\partial C\partial q_{t+2}} \ge 0$, except finite number of points,

$$
\begin{aligned}
\because\quad & f_{t+1}(q_{t+1}, x_{t+1}^{F*}) \\
& = \alpha(q_{t+1}+x_{t+1}^{F*}-\mu_{t+1}) + (\alpha+\beta)L_{t+1}(q_{t+1}+x_{t+1}^{F*})
\end{aligned}
$$

$$+\int_0^{\infty} f_{t+2}(q_{t+2}, x_{t+2}^{F*}) g_{t+1}(d_{t+1})\, dd_{t+1}$$

$$\therefore \quad \frac{\partial f_{t+1}(q_{t+1}, x_{t+1}^{F*})}{\partial q_{t+1}}$$

$$= (\alpha+\beta) G_{t+1}(q_{t+1} + x_{t+1}^{F*}) - \beta + \int_0^{\infty} \frac{\partial f_{t+2}(q_{t+2}, x_{t+2}^{F*})}{\partial q_{t+2}} \frac{\partial q_{t+2}}{\partial q_{t+1}} g_{t+1}(d_{t+1})\, dd_{t+1}$$

$$= (\alpha+\beta) G_{t+1}(q_{t+1} + x_{t+1}^{F*}) - \beta + \int_0^{q_{t+1}+x_{t+1}^{F*}} \frac{\partial f_{t+2}(q_{t+2}, x_{t+2}^{F*})}{\partial q_{t+2}} g_{t+1}(d_{t+1})\, dd_{t+1}$$

$$\therefore \quad \frac{\partial^2 f_{t+1}(q_{t+1}, x_{t+1}^{F*})}{\partial C \partial q_{t+1}}$$

$$= \int_0^{q_{t+1}+x_{t+1}^{F*}} \frac{\partial^2 f_{t+2}(q_{t+2}, x_{t+2}^{F*})}{\partial C \partial q_{t+2}} g_{t+1}(d_{t+1})\, dd_{t+1} \geq 0$$

And since $\frac{\partial^2 f_n(q_n, x_n^{F*})}{\partial C \partial q_n} = \begin{cases} 0 & , \quad q_n > G_n^{-1}(\frac{\beta}{\alpha+\beta}) \\ (\alpha+\beta) G_n(S+q_n) & , \quad q_n < G_n^{-1}(\frac{\beta}{\alpha+\beta}) \end{cases} \geq 0$, $\forall q_{t+1}$ (t=1, 2, ..., $n-1$), $\frac{\partial^2 f_{t+1}(q_{t+1}, x_{t+1}^{F*})}{\partial C \partial q_{t+1}} \geq 0$ (except finite number of points).

Therefore, $\frac{\partial I_t}{\partial C} = \int_{q_t+x_t^F-(h_{t+1}(C)-C)^+}^{q_t+x_t^F} \frac{\partial^2 f_{t+1}(q_{t+1}, C-(C-h_{t+1}(C))^+)}{\partial C \partial q_{t+1}} g_t(d_t)\, d(d_t) \geq 0$. I_t is an increasing function of q_t and C.

Then we prove that, for any given C, there exists $h_t(C) > 0$ such that $I_t^0|_{q_t<h_t(C)} < 0$, $I_t^0|_{q_t=h_t(C)} = 0$ and $I_t^0|_{q_t>h_t(C)} > 0$. In addition, $h_t(C)$ is decreasing in C. Based on the assumption of induction, for any period i ($i = t+1, t+2, ..., n$),

$$x_i^{F*} = \begin{cases} 0 & , \quad q_i \geq h_i(C) \\ h_i(C) - q_i & , \quad h_i(C) - C \leq q_i < h_i(C) \\ C & , \quad q_i < h_i(C) - C \end{cases}$$

and

$$\frac{df_i(q_i, x_i^{F*})}{d(q_i + x_i^{F*})} \begin{cases} > 0 & , \quad q_i > h_i(C) \\ = 0 & , \quad h_i(C) - C < q_i < h_i(C) \\ < 0 & , \quad q_i < h_i(C) - C \end{cases}$$

According to the definition of I_t,

$$I_t = \frac{dE_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1}, x_{t+1}^{F*})\}}{dx_t^F}$$

$$= \frac{dE_{d_t}\{Y_t(q_t, d_t, x_t^F) + f_{t+1}(q_{t+1}, x_{t+1}^{F*})\}}{d(q_t + x_t^F)}$$

$$= (\alpha+\beta) g_t(q_t + x_t^F) - \beta + \int_0^{\infty} \frac{df_{t+1}(q_{t+1}, x_{t+1}^{F*})}{dx_t^F} g_t(d_t)\, d(d_t)$$

$$= (\alpha+\beta) g_t(q_t + x_t^F) - \beta + \int_0^{\infty} \frac{df_{t+1}(q_{t+1}, x_{t+1}^{F*})}{d(q_{t+1} + x_{t+1}^{F*})} \frac{d(q_{t+1} + x_{t+1}^{F*})}{dx_t^F} g_t(d_t)\, d(d_t)$$

When $q_t + x_t^F \to +\infty$, we have $\frac{d(q_{t+1}+x_{t+1}^{F*})}{dx_t^F} = 1$ and $\frac{df_{t+1}(q_{t+1},x_{t+1}^{F*})}{d(q_{t+1}+x_{t+1}^{F*})} > 0$. So

$$I_t|_{q_t+x_t^F\to+\infty} = \alpha + \int_0^\infty \frac{df_{t+1}(q_{t+1}, x_{t+1}^{F*})}{d(q_{t+1}+x_{t+1}^{F*})}|_{q_t+x_t^F\to+\infty} g_t(d_t)\, d(d_t) > \alpha > 0$$

When $q_t + x_t^F = 0$, we have: (1) if $h_{t+1} < C$, then $\frac{df_{t+1}(q_{t+1},x_{t+1}^{F*})}{d(q_{t+1}+x_{t+1}^{F*})} = 0$; (2) if $h_{t+1} > C$, then $\frac{d(q_{t+1}+x_{t+1}^{F*})}{dx_t^F} = 0$. So

$$I_t|_{q_t+x_t^F=0} = -\beta < 0$$

Recall that $q_t + x_t^F \geq 0$ and $\frac{dI_t}{d(q_t+x_t^F)} = \frac{\partial I_t}{\partial q_t} = \frac{\partial I_t}{\partial x_t^F} \geq 0$ (Lemma 4), we can obtain: there exists $h_t(C) > 0$, such that $I_t|_{q_t+x_t^F<h_t(C)} \leq 0$, $I_t|_{q_t+x_t^F=h_t(C)} = 0$ and $I_t|_{q_t+x_t^F>h_t(C)} \leq 0$. That is, there exists $h_t(C) > 0$, such that $I_t^0|_{q_t<h_t(C)} < 0$, $I_t^0|_{q_t=h_t(C)} = 0$ and $I_t^0|_{q_t>h_t(C)} > 0$. And according to Lemma 4, $h_t(C)$ is decreasing in C.

Proof of Proposition 4.

From Lemmas 3 and 4, we can obtain: (1) If $q_t < h_t$, then there exists $\bar{x}_t > 0$, such that $I_t|_{x_t^F} < \bar{x}_t < 0$, $I_t|_{x_t^F=\bar{x}_t} = 0$ and $I_t|_{x_t^F>\bar{x}_t} > 0$. Thus, $x_t^* = min\{C, \bar{x}_t\}$. In addition, recall the proof of Lemma 3. So $\bar{x}_t = h_t(C) - q_t$. (2) If $q_t \geq h_t(C)$, then $x_t^* = 0$.

Therefore, the optimal solution is

$$x_t^{F^*} = \begin{cases} 0 & , \quad q_t \geq h_t(C) \\ h_t(C) - q_t & , \quad h_t(C) - C \leq q_t < h_t(C) \\ S & , \quad q_t < h_t(C) - C \end{cases}$$

where $h_t(C)$ is defined as $I_t^0|_{q_t=h_t(C)} = 0$.

When $t = n$, $h_n(C) = G_n^{-1}(\frac{\beta}{\alpha+\beta})$. So Proposition 4 is proved for any period t.

Proof of Proposition 5.

(1) According to the proof of Lemma 3, $\frac{\partial h_t(C)}{\partial C} \leq 0$.

(2) Since $\frac{df_{t+1}(q_t+x_t^F-d_t,0)}{dx_{t+1}^{F*}} \geq 0$, $\frac{df_{t+1}(q_t+x_t^F-d_t,C-(C-h_{t+1}(C))^+)}{dx_{t+1}^{F*}} \leq 0$ and Proposition 4, we can get $\frac{df_{t+1}(q_t+x_t^F-d_t,0)}{dx_t^F} \geq 0$ and $\frac{df_{t+1}(q_t+x_t^F-d_t,C-(C-h_{t+1}(C))^+)}{dx_t^F} \leq 0$.

Thus,

$$\begin{aligned}
I_t^0 =&(\alpha+\beta)g_t(q_t)-\beta+\int_0^{q_t-h_{t+1}(C)}\frac{df_{t+1}(q_t+x_t^F-d_t,0)}{dx_t^F}|_{x_t^F=0}g_t(d_t)d(d_t)\\
&+\int_{q_t-(h_{t+1}(C)-C)^+}^{q_t}\frac{df_{t+1}(q_t+x_t^F-d_t,C-(C-h_{t+1}(C))^+)}{dx_t^F}|_{x_t^F=0}g_t(d_t)d(d_t)\\
\geq&(\alpha+\beta)g_t(q_t)-\beta+\frac{df_{t+1}(x_t^F+h_{t+1}(C),0)}{dx_t^F}|_{x_t^F=0}g_t\left(q_t-h_{t+1}(C)\right)\\
&+\frac{df_{t+1}\left(x_t^F-(h_{t+1}(C)-C)^+,C-(C-h_{t+1}(C))^+\right)}{dx_t^F}|_{x_t^F=0}g_t\left(q_t-(h_{t+1}(C)-C)^+\right)\\
=&(\alpha+\beta)g_t(q_t)-\beta
\end{aligned}$$

$\therefore I_t^0|_{q_t=g_t^{-1}(\frac{\beta}{\alpha+\beta})}\geq(\alpha+\beta)g_t^{-1}(\frac{\beta}{\alpha+\beta})=0$

$\therefore h_t(C)\geq g_t^{-1}(\frac{\beta}{\alpha+\beta})$

(3) $\frac{\partial q_{t+1}}{\partial d_t}=\begin{cases}-1 & , \quad d_t<x_t^F+q_t\\ 0 & , \quad d_t>x_t^F+q_t\end{cases}$ and $\frac{\partial I_{t+1}^0}{\partial q_{t+1}^0}\geq 0$

$\therefore \frac{\partial I_{t+1}^0}{\partial d_t}\geq 0 \ (d_t\neq x_t^F+q_t)$

Note that $h_{t+1}(C)$ is defined as $I_{t+1}^0|_{q_{t+1}=h_{t+1}(C)}=0$, we can obtain $\frac{\partial h_{t+1}(C)}{\partial d_t}\geq 0$ $(d_t\neq x_t^F+q_t)$.

Proof of Proposition 8.

According to the definition of $B_j^i(t-1,A_j)$, it has the following properties:

(1) $B_j^0(t-1,A_j)=1$;

(2) $B_j^i(t-1,A_j)-B_j^{i-1}(t-1,A_j)A_j^t=B_j^i(t,A_j)$;

(3) $B_j^i(t-1,A_j)=(-1)^tA_j^{\frac{t(t+1)}{2}}$.

And we can calculate out $B_j^i(t-1,A_j)=(-1)^i\frac{A_j^{\frac{i(i+1)}{2}}(1-A_j^{t-1})(1-A_j^{t-2})...(1-A_j^{t-i})}{(1-A_j^i)(1-A_j^{i-1})...(1-A_j)}$.

Therefore

$$G_{j1}(y)=Pr\{A_jd_{j0}+\sigma_{j1}\leq y\}=Pr\{\sigma_{j1}\leq y-A_jd_{j0}\}=\begin{cases}0 & , \quad y<A_jd_{j0}\\ 1-e^{-\lambda(y-A_jd_{j0})} & , \quad y\geq A_jd_{j0}\end{cases}$$

$$g_{j1}(y)=\frac{dG_{j1}(y)}{dy}=\begin{cases}0 & , \quad y<A_jd_{j0}\\ \lambda e^{-\lambda(y-A_jd_{j0})} & , \quad y>A_jd_{j0}\end{cases}$$

Assume $G_{jt}(y)=\begin{cases}0 & , \quad y<A_j^td_{j0}\\ 1-\frac{1}{\prod_{l=1}^{t-1}(1-A_j^F)}\sum_{i=0}^{t-1}B_j^i(t-1,A_j)e^{\frac{-\lambda(y-A_j^td_{j0})}{A_j^i}} & , \quad y\geq A_j^td_{j0}\end{cases}$.

Then,

$$
\begin{aligned}
G_{j,t+1}(y) =& \int_0^{\infty} G_{jt}(y)|_{d_{jt}} g_{jt}(d_{jt})\, d(d_{jt}) \\
=& \int_{A_j^t d_{j0}}^{\frac{y}{A_j}} \left(1 - e^{-\lambda(y - A_j d_{jt})}\right) \frac{\lambda}{\prod_{l=1}^{t-1}(1 - A_j^F)} \sum_{i=0}^{t-1} B_j^i(t-1, A_j) e^{\frac{-\lambda(d_{jt} - A_j^t d_{j0})}{A_j^i}} \frac{1}{A_j^i}\, d(d_{jt}) \\
=& \frac{\lambda}{\prod_{l=1}^{t-1}\left(1 - A_j^F\right)} \int_{A_j^t d_{j0}}^{\frac{y}{A_j}} \left[\sum_{i=0}^{t-1} B_j^i(t-1, A_j) e^{\frac{-\lambda(d_{jt} - A_j^t d_{j0})}{A_j^i}} \frac{1}{A_j^i}\right. \\
& \left. - \sum_{i=0}^{t-1} B_j^i(t-1, A_j) e^{\lambda(\frac{d_{jt} - A_j^t d_{j0}}{A_j^i} + y - A_j d_{jt})} \frac{1}{A_j^i}\right] d(d_{jt}) \\
=& - \frac{\lambda}{\prod_{l=1}^{t-1}\left(1 - A_j^F\right)} \left[- \sum_{i=0}^{t-1} B_j^i(t-1, A_j) \frac{A_j^{k+1}}{1 - A_j^{k+1}} e^{\frac{-\lambda(y - A_j^{t+1} d_{j0})}{A_j^{i+1}}}\right. \\
& \left. - \sum_{i=0}^{t-1} B_j^i(t-1, A_j) + \sum_{i=0}^{t-1} B_j^i(t-1, A_j) \frac{1}{1 - A_j^{k+1}} e^{-\lambda(y - A_j^{t+1} d_{j0})}\right] \\
=& 1 - \frac{1}{\prod_{l=1}^{t-1}\left(1 - A_j^F\right)} \left[\sum_{i=1}^{t} \frac{B_j^i(t, A_j)}{1 - A_j^t} e^{\frac{-\lambda(y - A_j^{t+1} d_{j0})}{A_j^{i+1}}} + \frac{1}{1 - A_j^t} e^{-\lambda(y - A_j^{t+1} d_{j0})}\right] \\
=& 1 - \frac{1}{\prod_{l=1}^{t}\left(1 - A_j^F\right)} \sum_{i=0}^{t} B_j^i(t, A_j) e^{\frac{-\lambda(y - A_j^{t+1} d_{j0})}{A_j^i}} (y \geq A_j^{t+1} d_{j0})
\end{aligned}
$$

And easily get

$$
g_{j,t+1}(y) = \begin{cases} 0 & , \quad y < A_j^{t+1} d_{j0} \\ \frac{\lambda}{\prod_{l=1}^{t}\left(1 - A_j^F\right)} \sum_{i=0}^{t} B_j^i(t-1, A_j) e^{\frac{-\lambda(y - A_j^{t+1} d_{j0})}{A_j^i}} \frac{1}{A_j^i} & , \quad y > A_j^{t+1} d_{j0} \end{cases}
$$

Appendix B

Proofs for Chapter 3

Proof of Lemma 5.

$\frac{d\delta P(\bar{G}(\delta))}{d\delta} = P(\bar{G}(\delta)) - \delta g(\delta)\frac{dP(\bar{G}(\delta))}{d\bar{G}(\delta)} > 0$. When $\delta = 0$, $\delta P(\bar{G}(\delta)) = 0$; when $\delta = \bar{\delta}$, $\delta P(\bar{G}(\delta)) = \bar{\delta}P(0) > \max\{r_h, r_c\}$. So there exists a unique δ_h (δ_c) satisfying $\delta_h P(\bar{G}(\delta_h)) = r_h$ ($\delta_c P(\bar{G}(\delta_c)) = r_c$).

Proof of Proposition 9.

Both the public hospital and private clinic make decisions based on their own information and do not consider the inventory and price of the other. In this case, they consider the other party's inventory as zero. By δ following an uniform distribution, we have $\frac{dg(\delta)}{d\delta} = 0$. Then we have $\frac{dP(\bar{G}(\delta))}{d\delta} = -g(\delta)\frac{dP(\bar{G}(\delta))}{d\bar{G}(\delta)} \geqslant 0$,
$\frac{d^2P(\bar{G}(\delta))}{d\delta^2} = -\frac{dg(\delta)}{d\delta}\frac{dP(\bar{G}(\delta))}{d\bar{G}(\delta)} - g(\delta)\frac{d^2P(\bar{G}(\delta))}{d(\bar{G}(\delta))^2}\frac{d\bar{G}(\delta)}{d\delta} = 0$,
$\frac{dr_h}{d\delta_h} = \frac{d(\delta_h P(\bar{G}(\delta_h)))}{d\delta_h} = P(\bar{G}(\delta_h)) - \delta_h g(\delta_h)\frac{dP(\bar{G}(\delta_h))}{d\bar{G}(\delta_h)} \geqslant 0$,
$\frac{d\int_0^{\delta_h}\delta dG(\delta)}{d\delta_h} = G(\delta_h) + \delta_h g(\delta_h) - G(\delta_h) = \delta_h g(\delta_h) \geqslant 0$, $\frac{d(P(\bar{G}(\delta_h))\int_0^{\delta_h}\delta dG(\delta))}{d\delta_h} \geqslant 0$,
$\frac{d(\frac{q_h}{N[\bar{G}(\delta_h)-\bar{G}(\beta)]})}{d\delta_h} = \frac{q_h}{Ng(\delta)(\beta-\delta_h)^2} \geqslant 0$, and $\frac{d^2(\frac{q_h}{N[\bar{G}(\delta_h)-\bar{G}(\beta)]})}{d(\delta_h)^2} = \frac{2q_h}{Ng(\delta)(\beta-\delta_h)^3} \geqslant 0$.

For $q_h \leq N\bar{G}(\delta_h)$, $\frac{\partial SC}{\partial q_h} = c - r_h + \int_0^{\delta_h}\delta dG(\delta)\frac{dP(f)}{df} + \int_{\delta_h}^{\bar{\delta}}\delta dG(\delta)\left((1 - \frac{q_h}{N\bar{G}(\delta_h)})\frac{dP(f)}{df} - \frac{P(f)}{\bar{G}(\delta_h)}\right) \leq 0$; for $q_h > N\bar{G}(\delta_h)$, $\frac{\partial SC}{\partial q_h} = c > 0$. So $q_h^* = N\bar{G}(\delta_h)$. Given q_h^*, the function changes to

$$SC(\delta_h \mid_{q_h^*}) = (c - \delta_h P(\bar{G}(\delta_h)))N\bar{G}(\delta_h) + \int_0^{\delta_h}\delta NP(\bar{G}(\delta_h))dG(\delta).$$

It is easy to get that $\frac{\partial SC}{\partial \delta_h} = -Ng(\delta_h)\left(P(\bar{G}(\delta_h))(\bar{\delta} - 2\delta_h) + \delta_h(\bar{\delta} - \delta_h)\frac{dP(\bar{G}(\delta_h))}{d\delta_h} + c\right)$
$+N\left(\frac{dP(\bar{G}(\delta_h))}{d\delta_h}\int_0^{\delta_h}\delta dG(\delta) + P(\bar{G}(\delta_h))\delta_h g(\delta_h)\right)$
$= Ng(\delta_h)P(\bar{G}(\delta_h))(3\delta_h - \bar{\delta}) + N\frac{dP(\bar{G}(\delta_h))}{d\delta_h}\left(\int_0^{\delta_h}\delta dG(\delta) - \delta_h g(\delta_h)(\bar{\delta} - \delta_h)\right) - cNg(\delta_h)$,

where $\int_0^{\delta_h} \delta dG(\delta) = \int_0^{\delta_h} \delta g(\delta) d\delta = g(.) \int_0^{\delta_h} \delta d\delta = \frac{1}{2}\delta_h^2 g(\delta_h)$. $\int_0^{\delta_h} \delta dG(\delta) - \delta_h g(\delta_h)(\bar{\delta} - \delta_h) = \delta_h g(.)(\frac{3\delta_h}{2} - \bar{\delta})$.

When $3\delta_h - \bar{\delta} < 0$, we have $N g(\delta_h) P(\bar{G}(\delta_h))(3\delta_h - \bar{\delta}) < 0$ and $N \frac{dP(\bar{G}(\delta_h))}{d\delta_h} \left(\int_0^{\delta_h} \delta dG(\delta) - \delta_h g(\delta_h)(\bar{\delta} - \delta_h) \right) < 0$. Then $\frac{\partial SC}{\partial \delta_h} < 0$, so $\delta_h^* \geq \frac{\bar{\delta}}{3}$.

Moreover, $\frac{\partial^2 SC}{\partial \delta_h^2} = -N g(\delta_c) \left(-2P(\bar{G}(\delta_c)) + 2(\bar{\delta} - 2\delta_c) \frac{dP(\bar{G}(\delta_c))}{d\delta_c} + \delta_c(\bar{\delta} - \delta_c) \frac{d^2 P(\bar{G}(\delta_c))}{d(\delta_c)^2} \right)$
$+N \left(\frac{d^2 P(\bar{G}(\delta_h))}{d\delta_h^2} \int_0^{\delta_h} \delta dG(\delta) + \frac{dP(\bar{G}(\delta_h))}{d\delta_h} \delta_h g(\delta_h) + \frac{dP(\bar{G}(\delta_h))}{d\delta_h} \delta_h g(\delta_h) + P(\bar{G}(\delta_h)) g(\delta_h) \right)$
$= 2N g(\delta_c) P(\bar{G}(\delta_c)) + 2N g(\delta) \frac{dP(\bar{G}(\delta_c))}{d\delta_c}(3\delta_h - \bar{\delta}) + N P(\bar{G}(\delta_h)) g(\delta_h)$.

When $3\delta_h - \bar{\delta} \geq 0$, $\frac{\partial^2 SC}{\partial \delta_h^2} \geq 0$. So δ_h^* satisfies $\frac{\partial SC}{\partial \delta_h} = 0$.

Proof of Lemma 6.

Similar to the proof of Proposition 9, it is easy to derive $q_h^{'*} = N[\bar{G}(\delta_h^{'*})]$.

Then the problem becomes $min\ SC(\delta_h^{'} \mid_{q_h^{'*}}) = cN(\bar{G}(\delta_h^{'})) + \int_0^{\delta_h^{'}} \delta N P(f^{'}) dG(\delta)$.

It is easy to get $\frac{\partial SC}{\partial \delta_h} = N g(\delta)(c - \delta_h^{'} P(\bar{G}(\delta_h^{'}))) + N \frac{dP(\bar{G}(\delta_h^{'}))}{d\delta_h^{'}} \int_0^{\delta_h^{'}} \delta dG(\delta)$. So $\delta_h^{'*}$ satisfies $c - \delta_h^{'*} P(\bar{G}(\delta_h^{'*})) < 0$.

Proof of Proposition 10.

$$\frac{\partial \pi}{\partial q_c} = \begin{cases} r_c - c & q_c \leq N[\bar{G}(\delta_c)], \\ \\ -c & q_c > N[\bar{G}(\delta_c)]. \end{cases}$$

So $q_c^* = N[\bar{G}(\delta_c)]$. Given q_c^*, $\pi(\delta_c \mid_{q_c^*}) = (\delta_c P(\bar{G}(\delta_c)) - c) N[\bar{G}(\delta_c)]$.
$\frac{\partial \pi}{\partial \delta_c} = N g(\delta_c) \left(P(\bar{G}(\delta_c))(\bar{\delta} - 2\delta_c) + \delta_c(\bar{\delta} - \delta_c) \frac{dP(\bar{G}(\delta_c))}{d\delta_c} + c \right)$.
It is easy to derive $\frac{\partial \pi}{\partial \delta_c} \geq 0$ when $\bar{\delta} - 2\delta_c \geq 0$.
$\frac{\partial^2 \pi}{\partial (\delta_c)^2} = N g(\delta_c) \left(-2P(\bar{G}(\delta_c)) + 2(\bar{\delta} - 2\delta_c) \frac{dP(\bar{G}(\delta_c))}{d\delta_c} + \delta_c(\bar{\delta} - \delta_c) \frac{d^2 P(\bar{G}(\delta_c))}{d(\delta_c)^2} \right)$.
When $\bar{\delta} - 2\delta_c < 0$, $\frac{\partial^2 \pi}{\partial (\delta_c)^2} \leq 0$. So δ_c^* satisfies $\frac{\partial \pi}{\partial \delta_c} = 0$.

Proof of Lemma 7.

$$\frac{\partial \pi}{\partial q_s} = \begin{cases} r_s + s - c & q_s < (N[\bar{G}(\delta_s) - \bar{G}(\beta)])^+, \\ \\ -c & q_s \geq (N[\bar{G}(\delta_s) - \bar{G}(\beta)])^+. \end{cases}$$

Then we find the optimal inventory

$$q_s^* = \begin{cases} 0 & \beta \leqslant \delta_s, \\ \\ N[\bar{G}(\delta_s) - \bar{G}(\beta)] & \delta_s < \beta. \end{cases}$$

The total profit is

$$\pi(\delta_s \mid_{q_s^*}) = \begin{cases} 0 & \beta \leqslant \delta_s, \\ \\ (\delta_s P(\bar{G}(\delta_s)) + s - c)(N[\bar{G}(\delta_s) - \bar{G}(\beta)]) & \delta_s < \beta. \end{cases}$$

It is easy to derive $(\delta_s P(\bar{G}(\delta_s)) + s - c)(N[\bar{G}(\delta_s) - \bar{G}(\beta)]) > 0$. So we have $\delta_s^* < \beta$.

Proof of Proposition 11.

$$\frac{\partial \pi}{\partial \delta_s} = \frac{N}{\bar{\delta}} \left(P(\bar{G}(\delta_s))(\beta - 2\delta_s) + \delta_s(\beta - \delta_s) \frac{dP(\bar{G}(\delta_s))}{d\delta_s} + c - s \right),$$

$$\frac{\partial^2 \pi}{\partial (\delta_s)^2} = \frac{N}{\bar{\delta}} \left(-2P(\bar{G}(\delta_s)) + 2(\beta - 2\delta_s) \frac{dP(\bar{G}(\delta_s))}{d\delta_s} + \delta_s(\beta - \delta_s) \frac{d^2 P(\bar{G}(\delta_s))}{d(\delta_s)^2} \right).$$

When $c - s \geq 0$, $\frac{\partial \pi}{\partial \delta_s} \geq 0$ for $\delta_s \leq \frac{\beta}{2}$. When $\delta_s \geq \frac{\beta}{2}$, $\frac{\partial^2 \pi}{\partial (\delta_s)^2} \leq 0$. So, for $c - s \geq 0$, δ_s^* satisfies $\frac{\partial \pi}{\partial \delta_s} = 0$. Referring to $\frac{\partial^2 \pi}{\partial (\delta_s)^2} \leq 0$ and $\frac{\partial \pi}{\partial \delta_s}$ increases with β, we can get that as β increases, δ_s^* increases.

$s = P(\bar{G}(\delta_s))(\beta - 2\delta_s) + \delta_s(\beta - \delta_s) \frac{dP(\bar{G}(\delta_s))}{d\delta_s} + c$ for $s \leq c$, so $\frac{ds}{d\delta_s} \leq 0$.

When $c - s < 0$, δ_s^* might be smaller than $\frac{\beta}{2}$.

Proof of Proposition 12.

$\frac{d \int_{\delta_c}^{\beta} \delta dG(\delta)}{d\delta_c} = G(\delta_c) + \delta_c g(\delta_c) - G(\delta_c) = -\delta_c g(\delta_c) \leqslant 0$. $\frac{d^2 \int_{\delta_c}^{\beta} \delta dG(\delta)}{d(\delta_c)^2} = -g(\delta_c) \leqslant 0$.

By Proposition 11, $h = \bar{G}(\delta_s^*) = \bar{G}(\beta) + \frac{q_s^*}{N}$ and $s = P(\bar{G}(\delta_s^*))(\beta - 2\delta_s^*) + \delta_s^*(\beta - \delta_s^*) \frac{dP(\bar{G}(\delta_s^*))}{d\delta_s^*} + c$.

$\frac{dSC}{ds} = q_s^* + N \frac{d\delta_s^*}{ds} \left(\frac{dP(f)}{d\delta_s^*} \int_0^{\delta_s^*} \delta dG(\delta) + P(f) \delta_s^* g(\delta_s^*) \right)$.

$\frac{d^2 SC}{ds^2} = N(\frac{d\delta_s^*}{ds})^2 \left(\frac{d^2 P(f)}{d(\delta_s^*)^2} \int_0^{\delta_s^*} \delta dG(\delta) + 2 \frac{dP(f)}{d\delta_s^*} \delta_s^* g(\delta_s^*) + P(f) g(\delta_s^*) \right) \geq 0$.

So s^* satisfies $N[\bar{G}(\delta_s^*) - \bar{G}(\beta)] + N \frac{d\delta_s^*}{ds} \left(\frac{dP(f)}{d\delta_s^*} \int_0^{\delta_s^*} \delta dG(\delta) + P(f) \delta_s^* g(\delta_s^*) \right) = 0$.

Proof of Proposition 13.

$SC(q_g,\delta_g) = c(N[\bar{G}(\beta)]+q_g)+NP(f)\int_0^{\delta_g}\delta dG(\delta)+(1-\frac{q_g}{N[\bar{G}(\delta_g)-\bar{G}(\beta)]})^{+}NP(f)\int_{\delta_g}^{\beta}\delta d(G(\delta)).$

$$\frac{\partial SC(q_g,\delta_g)}{\partial\delta_g} = \begin{cases} N\frac{dP(\bar{G}(\delta_g))}{d\delta_g}\int_0^{\delta_g}\delta dG(\delta) + NP(\bar{G}(\delta_g))\delta_g g(\delta_g) & \delta_g \geq \beta - \frac{Q_g}{Ng(\delta)}, \\ \frac{NP(f)q_g g(\delta_g)}{N[\bar{G}(\delta_g)-\bar{G}(\beta)]}\left(\delta_g - \frac{1}{\bar{G}(\delta_g)-\bar{G}(\beta)}\int_{\delta_g}^{\beta}\delta d(G(\delta))\right) & \delta_g < \beta - \frac{Q_g}{Ng(\delta)}. \end{cases}$$

It is easy to derive $\frac{dP(\bar{G}(\delta_g))}{d\delta_g}\int_0^{\delta_g}\delta dG(\delta)+P(\bar{G}(\delta_g))\delta_g g(\delta_g) \geq 0$ and $\delta_g-\frac{1}{\bar{G}(\delta_g)-\bar{G}(\beta)}\int_{\delta_g}^{\beta}\delta d(G(\delta)) < 0$. So $\delta_g^* = \beta - \frac{q_g}{Ng(\delta)}$.

$h = \bar{G}(\beta)+\frac{q_g}{N} = \bar{G}(\delta_g^*)$, then we calculate q_g^*, $\frac{\partial SC(q_g|_{\delta_g^*})}{\partial q_g} = c+\left(\frac{dP(f)}{df}\int_0^{\delta_g^*}\delta dG(\delta) - P(f)\delta_g^*\right)$.

$\frac{\partial^2 SC(q_g|_{\delta_g^*})}{\partial q_g^2} = \frac{1}{N}\left(\frac{d^2P(f)}{df^2}\int_0^{\delta_g^*}\delta dG(\delta) - 2\delta_g^*\frac{dP(f)}{df} + \frac{P(f)}{g(\delta_g)}\right) \geq 0.$

Then we find q_g^* satisfying $c + \left(\frac{dP(f)}{df}\int_0^{\delta_g^*}\delta dG(\delta) - P(f)\delta_g^*\right) = 0.$

Proof of Corollary 4.

By Proposition 9, we have

$-g(\delta_h^*)\left(P(\bar{G}(\delta_h^*))(\bar{\delta}-2\delta_h^*) + \delta_h^*(\bar{\delta}-\delta_h^*)\frac{dP(\bar{G}(\delta_h^*))}{d\delta_h^*} + c\right)+\left(\frac{dP(\bar{G}(\delta_h^*))}{d\delta_h^*}\int_0^{\delta_h^*}\delta dG(\delta) + P(\bar{G}(\delta_h^*))\delta_h^* g(\delta_h^*)\right) = 0,$

$-g(\delta_h^*)\left(P(\bar{G}(\delta_h^*))(\bar{\delta}-2\delta_h^*) + \delta_h^*(\bar{\delta}-\delta_h^*)\frac{dP(\bar{G}(\delta_h^*))}{d\delta_h^*} + c\right)-g(\delta_h^*)\left(\frac{dP(\bar{G}(\delta_h^*))}{df}\int_0^{\delta_h^*}\delta dG(\delta) - P(\bar{G}(\delta_h^*))\delta_h^*\right) = 0,$

$P(\bar{G}(\delta_h^*))(\bar{\delta}-2\delta_h^*) + \delta_h^*(\bar{\delta}-\delta_h^*)\frac{dP(\bar{G}(\delta_h^*))}{d\delta_h^*} + c + \frac{dP(\bar{G}(\delta_h^*))}{df}\int_0^{\delta_h^*}\delta dG(\delta) - P(\bar{G}(\delta_h^*))\delta_h^* = 0.$

By Proposition 13, we can easily derive Corollary 4.

Proof of Corollary 5.

Under the vaccination subsidy scheme, we have

$N[\bar{G}(\delta_s^*) - \bar{G}(\beta)] + N\frac{d\delta_s^*}{ds}\left(\frac{dP(\bar{G}(\delta_s^*))}{d\delta_s^*}\int_0^{\delta_s^*}\delta dG(\delta) + P(\bar{G}(\delta_s^*))\delta_s^* g(\delta)\right) = 0.$

$\frac{dP(\bar{G}(\delta_s^*))}{d\delta_s^*}\int_0^{\delta_s^*}\delta dG(\delta) + P(\bar{G}(\delta_s^*))\delta_s^* g(\delta) = -\frac{ds}{d\delta_s^*}(\bar{G}(\delta_s^*) - \bar{G}(\beta)).$

Under the government allocation scheme, we have

$c + \frac{1}{N}\left(\frac{dP(\bar{G}(\delta_g^*))}{d\bar{G}(\delta_g^*)}\int_0^{\delta_g^*}\delta dG(\delta) - P(\bar{G}(\delta_g^*))\delta_g^*\right) = 0.$

$c - \frac{1}{Ng(\delta)}\left(\frac{dP(\bar{G}(\delta_g^*))}{d\delta_g^*}\int_0^{\delta_g^*}\delta dG(\delta) + P(\bar{G}(\delta_g^*))\delta_g^* g(\delta_g^*)\right) = 0.$

$\frac{dP(\bar{G}(\delta_g^*))}{d\delta_g^*}\int_0^{\delta_g^*}\delta dG(\delta) + P(\bar{G}(\delta_g^*))\delta_g^* g(\delta) = cNg(\delta).$

Setting $K(\hat{\delta}) = \frac{dP(\bar{G}(\hat{\delta}))}{d\hat{\delta}} \int_0^{\hat{\delta}} \delta dG(\delta) + P(\bar{G}(\hat{\delta}))\hat{\delta}g(\delta)$, we have $\frac{dK(\hat{\delta})}{d\hat{\delta}} \geq 0$. Then it is easy to derive Corollary 5.

Appendix C

Proofs for Chapter 4

Proof of Lemma 8.

In the high-tier label scenario, at Stage 2, each firm $i \in \{S, N\}$ sets price p_i to maximize its profit π_i, which is given by Equation (1) in Section 4.4.2. One can verify that $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$ when $\frac{p_S - p_N}{\tilde{q}} < 1$ and also when $\frac{p_S - p_N}{Q_H} < 1 < \frac{p_S - p_N}{\tilde{q}}$. Moreover, it can be shown that π_S is bimodal with respect to p_S, while π_N is unimodal with respect to p_N. At Stage 1, in anticipation of the equilibrium prices at Stage 2, firm S decides the quality level q to maximize its own profit. One can verify that there are two types of pure-strategy equilibrium as follows.

Equilibrium 1: Firm S serves both types of consumers.

Firm S's profit is equal to $(p_S - \frac{1}{2}cq^2)((1-\alpha)(1 - \frac{p_S - p_N}{Q_H}) + \alpha(1 - \frac{p_S - p_N}{\tilde{q}}))$, and firm N's profit is equal to $p_N((1-\alpha)\frac{p_S - p_N}{Q_H} + \alpha\ (\frac{p_S - p_N}{\tilde{q}}))$.

At Stage 2, given the environmental quality q, the two firms' optimal prices and profits are $p_S^{*a}(q) = \frac{1}{3}(cq^2 + \frac{2Q_H(\rho Q_H - \rho Q_L + Q_L)}{Q_H(\alpha(-\rho)+\alpha+\rho)+(\alpha-1)(\rho-1)Q_L})$,
$p_N^{*a}(q) = \frac{1}{6}(cq^2 + \frac{2Q_H(\rho Q_H - \rho Q_L + Q_L)}{Q_H(\alpha(-\rho)+\alpha+\rho)+(\alpha-1)(\rho-1)Q_L})$,
$\pi_S^{*a}(q) = -\frac{(cq^2(Q_H(\alpha(-\rho)+\alpha+\rho)+(\alpha-1)(\rho-1)Q_L)-4Q_H(\rho Q_H - \rho Q_L + Q_L))^2}{36 Q_H(\rho Q_H - \rho Q_L + Q_L)(\alpha\rho Q_H - Q_H(\alpha+\rho)+Q_L(\alpha(-\rho)+\alpha+\rho-1))}$, and
$\pi_N^{*a}(q) = -\frac{(cq^2(Q_H(\alpha(-\rho)+\alpha+\rho)+(\alpha-1)(\rho-1)Q_L)+2Q_H(\rho Q_H - \rho Q_L + Q_L))^2}{36 Q_H(\rho Q_H - \rho Q_L + Q_L)(\alpha\rho Q_H - Q_H(\alpha+\rho)+Q_L(\alpha(-\rho)+\alpha+\rho-1))}$.

At Stage 1, firm S decides q to maximize $\pi_S^{*a}(q)$. One can verify that $\frac{\partial \pi_S^{*a}(q)}{\partial q} < 0$ and the optimal quality level is $q^{HA} = Q_H$. Then, we can derive the HA equilibrium as summarized in Lemma 8 (i). In this HA equilibrium, $D_S^{HA} = \frac{\tilde{q}(4+cQ_H(\alpha-1))-cQ_H^2\alpha}{6\tilde{q}}$ and $D_N^{HA} = \frac{cQ_H^2\alpha+\tilde{q}(2+c(Q_H - Q_H\alpha))}{6\tilde{q}}$.

The HA equilibrium can exist if $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$, where $\alpha_{HA} = \frac{1}{2}(3\sqrt{W_1} +$

W_2) for
$W_1 = -\frac{(\rho Q_H-\rho Q_L+Q_L)^2(4cQ_H^2+(9-25\rho)Q_H+25(\rho-1)Q_L)}{(\rho-1)(Q_H-Q_L)(c^2Q_H^3-3\rho(2cQ_H-3)(Q_H-Q_L)-6cQ_HQ_L+9Q_L)^2}$ and
$W_2 = \frac{(\rho Q_H-\rho Q_L+Q_L)(-3\rho(4cQ_H-11)(Q_H-Q_L)+Q_H(2c(Q_H(cQ_H-1)-6Q_L)-9)+33Q_L)}{(\rho-1)(Q_H-Q_L)(c^2Q_H^3-3\rho(2cQ_H-3)(Q_H-Q_L)-6cQ_HQ_L+9Q_L)}$. Otherwise, firm S has an incentive to deviate to serve only informed consumers.

Equilibrium 2: Firm S serves only informed consumers. Firm S's profit is equal to $(p_S - \frac{1}{2}cQ_H^2)((1-\alpha)(1-\frac{p_S-p_N}{Q_H}))$, and firm N's profit is equal to $p_N((1-\alpha)\frac{p_S-p_N}{Q_H}+\alpha)$.

At Stage 2, given the environmental quality q, the two firms' optimal prices and profits are $p_S^{*b}(q) = \frac{1}{3}(cq^2 - \frac{Q_H}{\alpha-1} + Q_H)$, $p_N^{*b}(q) = \frac{1}{6}(cq^2 - \frac{2(\alpha+1)Q_H}{\alpha-1})$, $\pi_S^{*b}(q) = -\frac{((\alpha-1)cq^2-2(\alpha-2)Q_H)^2}{36(\alpha-1)Q_H}$, and $\pi_N^{*b}(q) = -\frac{((\alpha-1)cq^2-2(\alpha+1)Q_H)^2}{36(\alpha-1)Q_H}$.

At Stage 1, firm S decides q to maximize $\pi_S^{*b}(q)$. One can verify that $\frac{\partial\pi_S^{*b}(q)}{\partial q} < 0$ and $q^{HP} = Q_L$. Given this optimal quality level, we can get the HP equilibrium as shown in Lemma 8 (ii). In this HP equilibrium, $D_S^{HP} = \frac{1}{6}(4 + cQ_H(\alpha-1) - 2\alpha)$, and $D_N^{HP} = \frac{1}{6}(2 + cQ_H + 2\alpha - cQ_H\alpha)$.

The HP equilibrium can exist if $\alpha < \alpha_{HP}$, where α_{HP} is the unique solution (under the condition that $Q_L < Q_H < 2Q_L$) of $-3\sqrt{\frac{(\alpha-1)(\rho-1)^2(Q_H-Q_L)^2(\rho Q_H-\rho Q_L+Q_L)}{Q_H^4(\alpha(\rho-1)Q_H-\rho Q_H+Q_L(\alpha(-\rho)+\alpha+\rho-1))}} + \frac{3(\rho Q_H-\rho Q_L+Q_L)}{Q_H^2} + \frac{\alpha+1}{Q_H-\alpha Q_H} = c$.

Otherwise, firm S has an incentive to deviate to serve both types of consumers.

Proof of Lemma 9.

Given the pure-strategy equilibria summarized in Lemma 8, we characterize the effects of α and ρ by taking the derivatives of the equilibrium outcome parameters with respect to α and ρ. The details are as follows.

In the HA equilibrium: (a) $\frac{\partial p_S^{HA}}{\partial\alpha} = \frac{2Q_H\tilde{q}(\tilde{q}-Q_H)}{3(\tilde{q}(\alpha-1)-Q_H\alpha)^2} < 0$, $\frac{\partial D_S^{HA}}{\partial\alpha} = \frac{cQ_H(\tilde{q}-Q_H)}{6\tilde{q}} < 0$, and
$\frac{\partial\pi_S^{HA}}{\partial\alpha} = \frac{Q_H(Q_H-\tilde{q})\left(cQ_H^2\alpha+\tilde{q}(-4+c(Q_H-Q_H\alpha))\right)\left(cQ_H^2\alpha+\tilde{q}(4+c(Q_H-Q_H\alpha))\right)}{36\tilde{q}(\tilde{q}(-1+\alpha)-Q_H\alpha)^2} < 0$.

(b) $\frac{\partial p_N^{HA}}{\partial\alpha} = \frac{Q_H\tilde{q}(\tilde{q}-Q_H)}{3(\tilde{q}(\alpha-1)-Q_H\alpha)^2} < 0$ and $\frac{\partial D_N^{HA}}{\partial\alpha} = \frac{cQ_H(Q_H-\tilde{q})}{6\tilde{q}} > 0$. $\frac{\partial^2\pi_N^{HA}}{\partial\alpha^2} = -\frac{2Q_H(Q_H-\tilde{q})^2\tilde{q}}{9(\tilde{q}(\alpha-1)-Q_H\alpha)^3} > 0$, $\frac{\partial\pi_N^{HA}}{\partial\alpha} = \frac{Q_H(Q_H-\tilde{q})\left(cQ_H^2\alpha+\tilde{q}(-2+c(Q_H-Q_H\alpha))\right)\left(cQ_H^2\alpha+\tilde{q}(2+c(Q_H-Q_H\alpha))\right)}{36\tilde{q}(\tilde{q}(-1+\alpha)-Q_H\alpha)^2}$, and $\frac{\partial\pi_N^{HA}}{\partial\alpha} < 0$ iff $\alpha < \frac{(-2+cQ_H)\tilde{q}}{cQ_H(-Q_H+\tilde{q})}$.

(c) $\frac{\partial p_S^{HA}}{\partial\rho} = \frac{2Q_H^2(Q_H-Q_L)\alpha}{3(Q_L(\alpha-1)(\rho-1)+Q_H(\alpha+\rho-\alpha\rho))^2} > 0$, $\frac{\partial D_S^{HA}}{\partial\rho} = \frac{cQ_H^2(Q_H-Q_L)\alpha}{6(Q_L(\rho-1)-Q_H\rho)^2} > 0$, and

$\frac{\partial \pi_S^{HA}}{\partial \rho} = \frac{W_3}{36(Q_L(-1+\rho)-Q_H\rho)^2(Q_L(-1+\alpha)(-1+\rho)+Q_H(\alpha+\rho-\alpha\rho))^2} > 0$, where $W_3 = (Q_H^2(Q_H - Q_L)\alpha(-Q_L(4+cQ_H(-1+\alpha))(-1+\rho)+Q_H(cQ_H(\alpha(-1+\rho)-\rho)+4\rho))(Q_L(-4+cQ_H(-1+\alpha))(-1+\rho)+Q_H(4\rho+cQ_H(\alpha+\rho-\alpha\rho))))$.

(d) $\frac{\partial p_N^{HA}}{\partial \rho} = \frac{Q_H^2(Q_H-Q_L)\alpha}{3(Q_L(\alpha-1)(\rho-1)+Q_H(\alpha+\rho-\alpha\rho))^2} > 0$ and $\frac{\partial D_N^{HA}}{\partial \rho} = -\frac{cQ_H^2(Q_H-Q_L)\alpha}{6(Q_L(\rho-1)-Q_H\rho)^2} < 0$. $\frac{\partial \pi_N^{HA}}{\partial \rho} = \frac{1}{(36(Q_L(-1+\rho)-Q_H\rho)^2(Q_L(-1+\alpha)(-1+\rho)+Q_H(\alpha+\rho-\alpha\rho))^2))}(-((Q_H^2(Q_H-Q_L)\alpha(Q_H(cQ_H(\alpha(-1+\rho)-\rho)-2\rho)-Q_L(-2+cQ_H(-1+\alpha))(-1+\rho))(-Q_L(2+cQ_H(-1+\alpha))(-1+\rho)+Q_H(cQ_H(\alpha(-1+\rho)-\rho)+2\rho))))$ and $\frac{\partial \pi_N^{HA}}{\partial \rho} > 0$ iff $\rho > \frac{cQ_H^2\alpha+Q_L(-2+c(Q_H-Q_H\alpha))}{(Q_H-Q_L)(2+cQ_H(-1+\alpha))}$.

In the HP equilibrium: (a) $\frac{\partial p_S^{HP}}{\partial \alpha} = \frac{Q_H}{3(\alpha-1)^2} > 0$. $\frac{\partial D_S^{HP}}{\partial \alpha} = \frac{1}{6}(cQ_H - 2)$ and $\frac{\partial D_S^{HP}}{\partial \alpha} > 0$ iff $cQ_H > 2$. $\frac{\partial \pi_S^{HP}}{\partial \alpha} = -\frac{Q_H(cQ_H(\alpha-1)-2\alpha)(4+cQ_H(\alpha-1)-2\alpha)}{36(\alpha-1)^2} > 0$.

(b) $\frac{\partial p_N^{HP}}{\partial \alpha} = \frac{2Q_H}{3(\alpha-1)^2} > 0$. $\frac{\partial D_N^{HP}}{\partial \alpha} = \frac{1}{6}(2 - cQ_H)$ and $\frac{\partial D_N^{HP}}{\partial \alpha} > 0$ iff $cQ_H < 2$. $\frac{\partial \pi_N^{HP}}{\partial \alpha} = -\frac{Q_H(6+cQ_H(\alpha-1)-2\alpha)(cQ_H(\alpha-1)-2(1=\alpha))}{36(\alpha-1)^2} > 0$.

Proof of Corollary 6.

In the high-tier label scenario, there are two types of pure-strategy equilibria. (i) In the HA equilibrium (i.e., when $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$), $\pi_S^{HA} = \frac{Q_H(cQ_H^2\alpha+\tilde{q}(-4+c(Q_H-Q_H\alpha)))^2}{36\tilde{q}(Q_H\alpha-\tilde{q}(-1+\alpha))}$ and $\pi_N^{HA} = \frac{Q_H(cQ_H^2\alpha+\tilde{q}(2+c(Q_H-Q_H\alpha)))^2}{36\tilde{q}(Q_H\alpha-\tilde{q}(-1+\alpha))}$. Through straightforward algebra, one can show that $\pi_S^{HA} > \pi_N^{HA}$ iff $\alpha < \frac{\tilde{q}(1-cQ_H)}{cQ_H(Q_H-\tilde{q})}$. (ii) In the HP equilibrium (i.e., when $\alpha < \alpha_{HP}$), $\pi_S^{HP} = \frac{Q_H(4+cQ_H(-1+\alpha)-2\alpha)^2}{36(1-\alpha)}$ and $\pi_N^{HP} = \frac{Q_H(cQ_H(-1+\alpha)-2(1+\alpha))^2}{36(1-\alpha)}$. By straightforward algebraic analysis, one can verify that $\pi_S^{HP} < \pi_N^{HP}$ always holds. Then we have the results summarized in Corollary 6.

Proof of Lemma 10.

In the low-tier label scenario, at Stage 2, each firm $i \in \{S, N\}$ sets price p_i to maximize its profit π_i, which is given by Equation (2) in Section 4.4.3. One can verify that $\frac{\partial^2 \pi_i}{\partial p_i^2} < 0$ when $\frac{p_S-p_N}{Q_L} < 1$ and also when $\frac{p_S-p_N}{\tilde{q}} < 1 < \frac{p_S-p_N}{Q_L}$. Moreover, it can be shown that π_S is bimodal with respect to p_S, while π_N is unimodal with respect to p_N. At Stage 1, anticipating the equilibrium prices at Stage 2, firm S decides the quality level q to maximize its own profit. One can verify that there are the following two types of pure-strategy equilibrium.

Equilibrium 1: Firm S serves both types of consumers. Firm S's profit is equal to $(p_S - \frac{1}{2}cq^2)((1-\alpha)(1-\frac{p_S-p_N}{Q_L}) + \alpha(1-\frac{p_S-p_N}{\tilde{q}}))$, and firm N's profit is equal to $p_N((1-\alpha)\frac{p_S-p_N}{Q_L}+\alpha\ \frac{p_S-p_N}{\tilde{q}})$. Following a similar logic to that in Proof of Lemma 8, one can derive the LA equilibrium as summarized in Lemma 10 (i). The details are available from the authors upon request. In this LA equilibrium, $D_S^{LA}=\frac{\tilde{q}(4+cQ_L(\alpha-1))-cQ_L^2\alpha}{6\tilde{q}}$ and $D_N^{LA}=\frac{cQ_L^2\alpha+\tilde{q}(2+c(Q_L-Q_L\alpha))}{6\tilde{q}}$.

The LA equilibrium can exist if $c<\frac{4}{Q_L}$ and $\alpha<\alpha_{LA}$, where $\alpha_{LA}=\frac{1}{2}(\frac{(-14cQ_L^2+2c^2Q_L^3+Q_L(24-9\rho)+9Q_H\rho)(Q_L+Q_H\rho-Q_L\rho)}{(Q_H-Q_L)\rho(-6cQ_L^2+c^2Q_L^3-9Q_L(\rho-1)+9Q_H\rho)}-3\sqrt{\frac{(Q_L+Q_H\rho-Q_L\rho)^2(4cQ_L^2+9Q_H\rho-Q_L(16+9\rho))}{(Q_H-Q_L)\rho(-6cQ_L^2+c^2Q_L^3-9Q_L(-1+\rho)+9Q_H\rho)^2}})$. Otherwise, firm S has an incentive to deviate to serve only confused consumers.

Equilibrium 2: Firm S serves only confused consumers. Firm S's profit is equal to $(p_S-\frac{1}{2}cq^2)\alpha(1-\frac{p_S-p_N}{\tilde{q}})$, and firm N's profit is equal to $p_N((1-\alpha)+\alpha\ \frac{p_S-p_N}{\tilde{q}})$. Following a similar logic to that in Proof of Lemma 8, one can derive the LP equilibrium as summarized in Lemma 10 (ii). In this LP equilibrium, $D_S^{LP}=-\frac{cQ_L^2\alpha}{6\tilde{q}}+\frac{1+\alpha}{3}$ and $D_N^{LP}=\frac{4\tilde{q}+cQ_L^2\alpha-2\tilde{q}\alpha}{6\tilde{q}}$.

The LP equilibrium can exist if $\alpha>\alpha_{LP}$, where α_{LP} is the unique solution (under the condition that $Q_L<Q_H<2Q_L$) of $\frac{-\alpha\rho Q_H+2\rho Q_H-3\alpha Q_L^2\sqrt{Z}+\alpha\rho Q_L+2\alpha Q_L-2\rho Q_L+2Q_L}{\alpha Q_L^2}=c$ for

$Z=\frac{\alpha\rho^2(Q_L-Q_H)^2}{Q_L^3(-\alpha\rho Q_H+\rho Q_H+\alpha\rho Q_L-\rho Q_L+Q_L)}$. Otherwise, firm S has an incentive to deviate to serve both types of consumers.

Proof of Lemma 11.

Given the pure-strategy equilibria summarized in Lemma 10, we characterize the effects of α and ρ by taking the derivatives of the equilibrium outcome parameters with respect to α and ρ. The details are as follows.

In the LA equilibrium: (a) $\frac{\partial p_S^{LA}}{\partial\alpha}=\frac{2Q_L\tilde{q}(\tilde{q}-Q_L)}{3(\tilde{q}(\alpha-1)-Q_L\alpha)^2}>0$, $\frac{\partial D_S^{LA}}{\partial\alpha}=\frac{cQ_L(\tilde{q}-Q_L)}{6\tilde{q}}>0$, and $\frac{\partial\pi_S^{LA}}{\partial\alpha}=\frac{Q_L(Q_L-\tilde{q})\left(cQ_L^2\alpha+\tilde{q}(-4+c(Q_L-Q_L\alpha))\right)\left(cQ_L^2\alpha+\tilde{q}(4+c(Q_L-Q_L\alpha))\right)}{36\tilde{q}(\tilde{q}(-1+\alpha)-Q_L\alpha)^2}>0$.

(b) $\frac{\partial p_N^{LA}}{\partial\alpha}=\frac{Q_L\tilde{q}(\tilde{q}-Q_L)}{3(\tilde{q}(\alpha-1)-Q_L\alpha)^2}>0$, and $\frac{\partial D_N^{LA}}{\partial\alpha}=\frac{cQ_L(Q_L-\tilde{q})}{6\tilde{q}}<0$. $\frac{\partial^2\pi_N^{LA}}{\partial\alpha^2}=-\frac{2Q_L(Q_L-\tilde{q})^2\tilde{q}}{9(\tilde{q}(\alpha-1)-Q_L\alpha)^3}>0$ and $\frac{\partial\pi_N^{LA}}{\partial\alpha}<0$ iff $\alpha<\frac{-2\tilde{q}+cQ_L\tilde{q}}{-cQ_L^2+cQ_L\tilde{q}}$.

(c) $\frac{\partial p_S^{LA}}{\partial\rho}=\frac{2(Q_H-Q_L)Q_L^2\alpha}{3(Q_L-Q_H(\alpha-1)\rho+Q_L(\alpha-1)\rho)^2}>0$, $\frac{\partial D_S^{LA}}{\partial\rho}=\frac{c(Q_H-Q_L)Q_L^2\alpha}{6(Q_L(\rho-1)-Q_H\rho)^2}>0$, and $\frac{\partial\pi_S^{LA}}{\partial\rho}=\frac{W_4}{(36(Q_L(-1+\rho)-Q_H\rho)^2(Q_L-Q_H(-1+\alpha)\rho+Q_L(-1+\alpha)\rho)^2)}>0$, where $W_4=(((Q_H-Q_L)Q_L^2\alpha(4Q_H\rho+Q_L(4+(-4+cQ_H(-1+\alpha))\rho)-Q_L^2(c+c(-1+\alpha)\rho))(4Q_H\rho+Q_L^2(c+c(-1+\alpha)\rho)+Q_L(4+(-4+cQ_H-cQ_H\alpha)\rho))))$.

(d) $\frac{\partial p_N^{LA}}{\partial \rho} = \frac{(Q_H-Q_L)Q_L^2\alpha}{3(Q_L-Q_H(\alpha-1)\rho+Q_L(\alpha-1)\rho)^2} > 0$ and $\frac{\partial D_N^{LA}}{\partial \rho} = -\frac{c(Q_H-Q_L)Q_L^2\alpha}{6(Q_L(\rho-1)-Q_H\rho)^2} < 0$. $\frac{\partial \pi_N^{LA}}{\partial \rho} = -\frac{1}{(36(Q_L(-1+\rho)-Q_H\rho)^2(Q_L-Q_H(-1+\alpha)\rho+Q_L(-1+\alpha)\rho)^2))}$ $((((Q_H-Q_L)Q_L^2\alpha(2Q_H\rho+Q_L(2+(-2+cQ_H(-1+\alpha))\rho)-Q_L^2(c+c(-1+\alpha)\rho))(-2Q_H\rho+Q_L(-2+(2+cQ_H(-1+\alpha))\rho)-Q_L^2(c+c(-1+\alpha)\rho))))$ and $\frac{\partial \pi_N^{LA}}{\partial \rho} > 0$ iff $\rho > \frac{Q_L(-2+cQ_L)}{(Q_H-Q_L)(2+cQ_L(-1+\alpha))}$.

In the LP equilibrium: (a) $\frac{\partial p_S^{LP}}{\partial \alpha} = -\frac{\tilde{q}}{3\alpha^2} < 0$. $\frac{\partial D_S^{LP}}{\partial \alpha} = \frac{1}{3} - \frac{cQ_L^2}{6\tilde{q}}$ and $\frac{\partial D_S^{LP}}{\partial \alpha} > 0$ iff $c < \frac{2\tilde{q}}{Q_L^2}$.

$\frac{\partial \pi_S^{LP}}{\partial \alpha} = \frac{1}{36}(-4cQ_L^2 + \frac{c^2Q_L^4}{\tilde{q}} + \tilde{q}(4 - \frac{4}{\alpha^2})) < 0$.

(b) $\frac{\partial p_N^{LP}}{\partial \alpha} = -\frac{2\tilde{q}}{3\alpha^2} < 0$. $\frac{\partial D_N^{LP}}{\partial \alpha} = \frac{1}{6}(-2 + \frac{cQ_L^2}{\tilde{q}})$ and $\frac{\partial D_N^{LP}}{\partial \alpha} > 0$ iff $c > \frac{2\tilde{q}}{Q_L^2}$.

$\frac{\partial \pi_N^{LP}}{\partial \alpha} = \frac{1}{36}(-4cQ_L^2 + \frac{c^2Q_L^4}{\tilde{q}} + 4\tilde{q}(1 - \frac{4}{\alpha^2})) < 0$.

(c) $\frac{\partial p_S^{LP}}{\partial \rho} = \frac{(Q_H-Q_L)(1+\alpha)}{3\alpha} > 0$, $\frac{\partial D_S^{LP}}{\partial \rho} = \frac{c(Q_H-Q_L)Q_L^2\alpha}{6(Q_L+Q_H\rho-Q_L\rho)^2} > 0$, and $\frac{\partial \pi_S^{LP}}{\partial \rho} = \frac{(Q_H-Q_L)(-cQ_L^2\alpha-2Q_L(1+\alpha)(\rho-1)+2Q_H(1+\alpha)\rho)(cQ_L^2\alpha-2Q_L(1+\alpha)(\rho-1)+2Q_H(1+\alpha)\rho)}{36\alpha(Q_L(-1+\rho)-Q_H\rho)^2} > 0$.

(d) $\frac{\partial p_N^{LP}}{\partial \rho} = -\frac{(Q_H-Q_L)(-2+\alpha)}{3\alpha} > 0$, $\frac{\partial D_N^{LP}}{\partial \rho} = \frac{cQ_L^2(Q_L-Q_H)\alpha}{6(Q_L(-1+\rho)-Q_H\rho)^2} < 0$, and $\frac{\partial^2 \pi_N^{LP}}{\partial \rho^2} = -\frac{c^2(Q_H-Q_L)^2Q_L^4\alpha}{18(Q_L(-1+\rho)-Q_H\rho)^3}$. $\frac{\partial \pi_N^{LP}}{\partial \rho} = \frac{1}{(36\alpha(Q_L(-1+\rho)-Q_H\rho)^2)}$ and $\frac{\partial \pi_N^{LP}}{\partial \rho}((Q_H-Q_L)(-cQ_L^2\alpha - 2Q_L(-2+\alpha)(-1+\rho)+2Q_H(-2+\alpha)\rho)(cQ_L^2\alpha-2Q_L(-2+\alpha)(-1+\rho)+2Q_H(-2+\alpha)\rho)) > 0$ iff $\rho > -\frac{Q_L(-4+(2+cQ_L)\alpha)}{2(Q_H-Q_L)(-2+\alpha)}$.

Proof of Corollary 7.

In the low-tier label scenario, there are two types of pure-strategy equilibria. (i) In the LA equilibrium (i.e., when $c < \frac{4}{Q_L}$ and $\alpha < \alpha_{LA}$), $\pi_S^{LA} = \frac{Q_L(cQ_L^2\alpha+\tilde{q}(-4+c(Q_L-Q_L\alpha)))^2}{36\tilde{q}(Q_L\alpha-\tilde{q}(-1+\alpha))}$ and $\pi_N^{LA} = \frac{Q_L(cQ_L^2\alpha+\tilde{q}(2+c(Q_L-Q_L\alpha)))^2}{36\tilde{q}(Q_L\alpha-\tilde{q}(-1+\alpha))}$. By straightforward algebra, one can verify that $\pi_S^{LA} > \pi_N^{LA}$ iff $\alpha > \frac{\tilde{q}(1-cQ_L)}{cQ_L(Q_L-\tilde{q})}$. (ii) In the LP equilibrium (i.e., when $\alpha > \alpha_{LP}$), $\pi_S^{LP} = \frac{(cQ_L^2\alpha-2\tilde{q}(1+\alpha))^2}{36\tilde{q}\alpha}$ and $\pi_N^{LP} = \frac{(-2\tilde{q}(-2+\alpha)+cQ_L^2\alpha)^2}{36\tilde{q}\alpha}$. By straightforward algebra, one can show that $\pi_S^{LP} < \pi_N^{LP}$ always holds. Then we can get the results presented in Corollary 7.

Proof of Proposition 14.

So far, we have identified four types of pure-strategy equilibrium, that is HA (when $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$), HP (when $\alpha < \alpha_{HP}$), LA (when $c < \frac{4}{Q_L}$ and $\alpha < \alpha_{LA}$), and LP (when $\alpha > \alpha_{LP}$). We next compare the high-tier label with the low-tier label. One can verify that there are three intersections, in which pure-strategy equilibria exist in both the high-tier label scenario and the low-tier label scenario. We compare firm S's profits in these intersections to derive its label preferences. The details are as follows.

(i) When $c < \frac{4\tilde{q}}{Q_H^2}$ and $\alpha > \alpha_{HA}$, HA and LA coexist. Given π_S^{HA} convexly decreases in α (see Lemma 9 and its proof) and π_S^{LA} convexly increases in α (see Lemma 11 and its proof), there exists a threshold α_1 such that $\pi_S^{HA} > \pi_S^{LA}$ iff $\alpha < \alpha_1$.

(ii) When $\alpha_{LP} < \alpha < \alpha_{HP}$, HP and LP coexist. Given π_S^{HP} convexly increases in α (see Lemma 9 and its proof) and π_S^{LA} convexly decreases in α (see Lemma 11 and its proof), there exists a threshold α_2 such that $\pi_S^{LP} > \pi_S^{HP}$ iff $\alpha < \alpha_2$.

(iii) When $c < \frac{4}{Q_L}$ and $\alpha < \min\{\alpha_{HP}, \alpha_{LA}\}$, HP and LA coexist. One can show that π_S^{HP} and π_S^{LA} convexly increase in α (see Lemmas 9 and 11 and their proofs). By straightforward algebraic calculation, one can verify that $\pi_S^{HP} - \pi_S^{LA}$ is convex in α. Moreover, it can be verified that solving $\pi_S^{HP} - \pi_S^{LA} = 0$ yields at most one root with respect to α. Let α^{HPvsLA} denote this root. One can further show that if $(\pi_S^{HP} - \pi_S^{LA})|_{\alpha=0} > 0$, then $\frac{\partial(\pi_S^{HP} - \pi_S^{LA})}{\partial\alpha}|_{\alpha=0} > 0$ and $\pi_S^{HP} - \pi_S^{LA} > 0$ for $0 < \alpha < 1$. Then we can conclude that there exists a threshold $\alpha_3 = \max\{0, \min\{\alpha^{HPvsLA}, 1\}\}$ such that $\pi_S^{LA} > \pi_S^{HP}$ iff $\alpha < \alpha_3$.

Proof of Corollary 8.

Given Proposition 14 and its proof, Corollary 8 can be easily verified.

Proof of Lemma 12.

When firm S chooses to adopt blockchain technology, at Stage 2, given q, both firms S and N set their prices simultaneously to maximize their own profits, which are given by Equation (3) in Section 4.4.5. One can verify that $\frac{\partial^2\pi_i}{\partial p_i^2} < 0$ for $i \in \{S, N\}$. Solving the two firms' pricing decisions simultaneously, we have $p_S^{*c}(q) = \frac{1}{3}q(2+cq))$, $p_N^{*c}(q) = \frac{1}{6}q(2+cq))$, $\pi_S^{*c}(q) = \frac{1}{36}q(-4+cq)^2$, and $\pi_N^{*c}(q) = \frac{1}{36}q(2+cq)^2$.

At Stage 1, firm S decides q to maximize $\pi_S^{*c}(q)$. It can be verified that that the optimal quality level is $q^B = \frac{4}{3c}$. With this quality level, we can derive the equilibrium summarized in Lemma 12.

Proof of Proposition 15.

Given that $f_B = 0$, $c < \frac{4\tilde{q}}{Q_H^2}$, and $\alpha_{HA} < \alpha < \alpha_B$, we need to compare π_S^B (as given in Lemma 12) with π_S^{HA} (as given in Lemma 8) when $\alpha < \alpha_1$, and with π_S^{LA} (as given in Lemma 10) when $\alpha > \alpha_1$. The details are as follows.

(i) When $\alpha < \alpha_1$, by straightforward algebraic calculation, one can verify that $\pi_S^{HA} < \pi_S^B$.

(ii) When $\alpha > \alpha_1$, it can be shown that π_S^{LA} increases in α (see Lemma 11 and its proof) while π_S^B is independent of α. Thus, there exists a threshold α_B such that $\pi_S^{LA} < \pi_S^B$ iff $\alpha < \alpha_B$, where α_B is the solution of $\pi_S^{LA} - \pi_S^B = 0$.

Then we have the results summarized in Proposition 15.

Proof of Proposition 16.

We prove Proposition 16 by following a similar logic to that in Proof of Proposition 15. Given that $f_B = 0$, $c < \frac{4\tilde{q}}{Q_H^2}$, and $\alpha_{HA} < \alpha < \alpha_B$, we need to compare $q^B = \frac{4}{3c}$ (as given in Lemma 12) with $q^{HA} = Q_H$ (as given in Lemma 8) when $\alpha < \alpha_1$, and with $q^{LA} = Q_L$ (as given in Lemma 10) when $\alpha > \alpha_1$. One can verify that $q^B > q^{HA} = Q_H$ iff $c < \frac{4}{3Q_H}$, and $q^B > q^{LA} = Q_L$ iff $c < \frac{4}{3Q_L}$. Then we can get Proposition 16.

Proof of Corollary 9.

We prove Proposition 9 by following a similar logic to that in Proof of Proposition 15. Given that $f_B = 0$, $c < \frac{4\tilde{q}}{Q_H^2}$, and $\alpha_{HA} < \alpha < \alpha_B$, we need to compare π_N^B (as given in Lemma 12) with π_N^{HA} (as given in Lemma 8) when $\alpha < \alpha_1$, and with π_N^{LA} (as given in Lemma 10) when $\alpha > \alpha_1$. The details are as follows.

(i) When $\alpha < \alpha_1$, by straightforward algebra, one can verify that $\frac{\partial \pi_N^{HA}}{\partial c} = \frac{cQ_H^4\alpha + Q_H^2\tilde{q}(2+c(Q_H - Q_H\alpha))}{18\tilde{q}} > 0$ and $\frac{\partial \pi_N^B}{\partial c} = 0$. That is, π_N^{HA} increases in c, while π_N^B is independent of c. Thus, we can conclude that there exists a threshold $c_{B1}(\alpha)$ such that $\pi_N^{HA} < \pi_N^B$ iff $c < c_{B1}(\alpha)$.

(ii) Similarly, when $\alpha > \alpha_1$, it can be shown that $\frac{\partial \pi_N^{LA}}{\partial c} = \frac{cQ_L^4\alpha + Q_L^2\tilde{q}(2+c(Q_L - Q_L\alpha))}{18\tilde{q}} > 0$. That is π_N^{LA} increases in c. Thus, there exists a threshold $c_{B2}(\alpha)$ such that

$\pi_N^{LA} < \pi_N^B$ iff $c < c_{B2}(\alpha)$.

Then Corollary 9 is proved.

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