DATA ENVELOPMENT ANALYSIS, INVERSE DATA ENVELOPMENT ANALYSIS, AND MACHINE LEARNING: A NOVEL FRAMEWORK FOR STOCK PORTFOLIO MANAGEMENT

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Data Envelopment Analysis, Inverse Data Envelopment Analysis, and Machine Learning: A Novel Framework for Stock Portfolio Management

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__________________________(Signed)

KEHINDE Temitope Olubanjo_(Student)

Dedication

This thesis is dedicated to the Almighty God, whose unwavering presence and guidance have been my constant companion and source of strength during the most turbulent times of my PhD journey.

Abstract

The stock market is a vital component of the financial sector, facilitating the accumulation of money, spreading risk, setting prices, and communicating information. Given the substantial growth of the global stock market, investors, traders, analysts, and researchers are facing considerable difficulties in accurately selecting stocks and predicting their prices. Traditional statistical and econometric methods have shortcomings when dealing with the inherent uncertainty, noise, nonlinearity, and high dimensionality of stock markets. This thesis presents an innovative framework that combines data envelopment analysis (DEA), inverse DEA (IDEA), and machine learning techniques to improve stock market analysis and decision-making.

The first phase of this research presents a novel approach for assessing and ranking equities operating on the Taiwan Stock Exchange (TWSE). This phase addresses the issue of bias and subjectivity in current ranking approaches by combining the strengths of the Shannon Entropy Technique (SET), DEA, and IDEA, which allows a more objective evaluation. The SET method was utilized to identify the most significant financial indicators from a collection of 13 financial ratios. These indicators were selected based on their importance from several financial perspectives, such as liquidity, asset usage, leverage, profitability, and valuation. This approach substantially reduced the complexity of the analysis by lowering the number of dimensions. For each equity under consideration, the DEA conducted further analysis to obtain efficiency scores. The DMUs that demonstrated high efficiency, indicated by a score of 1, were subjected to a ranking process using an IDEA model. The outcome of this phase establishes a standard for the evaluation of DEA models, aiming to improve the level of objectivity and accuracy.

In the second phase, a novel DEA methodology framework was proposed, and it deviates from the traditional approach that considers risk as an input and returns as an output. Instead, this concept is reframed by considering return and risk as outputs that arise from a financial production process. However, applying DEA in stock market settings encounters limitations due to its inherent difficulty in efficiently handling negative inputs and outputs. Also, no existing studies have been conducted on applying DEA to reduce potential equity risk using an inverse optimization approach. To effectively address these problems, the directional distance function DEA (DDF-DEA) and IDEA were integrated to estimate inefficiency and potential reductions in non-performing stocks. Using this novel methodology, an analysis was conducted on stocks belonging to the food industry listed on the TWSE as well as consumer staples within

the Standard & Poor's 500 (S&P 500) index. This phase addresses a significant gap in the current body of literature, providing a foundation for enhanced decision-making in managing equity risk.

Going forward, a study similar to phase 2 was conducted in phase 3, but in a portfolio-related context. This phase introduces a novel approach to portfolio optimization using IDEA. The study uses a combination of a DDF-DEA and a novel IDEA method to assess the efficiency and volatility reduction of industry-based portfolios, where each industry is a combination of several related firms listed on the TWSE. The empirical analysis shows that only 7 of 20 industry-based portfolios were underperforming. The methodology calculates the potential and maximum reduction in the volatility of all underperforming portfolios. Additionally, for the first time in the literature, the phase proposed a net-zero volatility risk initiative for investors and analysts to keep track of their portfolios efficiently at all times. This work holds potential utility for investors and fund managers seeking to enhance the performance of their investment portfolios.

The last phase explores the application of a Transformer model, enhanced with a Tree-structured Parzen Estimator (TPE) for hyperparameter optimization, in predicting stock market indices. The phase focuses on the prediction of three major global stock indices: the S&P 500, Financial Times Stock Exchange 100 (FTSE 100), and Hang Seng Index (HSI), using deep learning techniques. This is a notable advancement from conventional forecasting techniques by incorporating a cutting-edge Transformer model, which is widely recognized for its achievements in natural language processing (NLP), into the field of financial forecasting. The work relies on the innovative use of the Transformer model, which employs self-attention mechanisms to effectively deal with complex and non-linear financial time series features, surpassing traditional recurrent neural network (RNN) and its variants, long short-term memory (LSTM) and gated recurrent units (GRU). The study showcases the model's proficiency in extracting relevant features from structured financial data, in contrast to prior research that predominantly concentrated on unstructured data such as social media sentiment. An essential aspect of this phase is applying a straightforward trading strategy that relies on the model's predictions, demonstrating the approach's practical monetary implications and possible investment gains.

To summarize, this thesis introduces a thorough and original framework that integrates DEA, IDEA, and machine learning methods to improve stock portfolio management. The proposed

frameworks offer resilient and adaptable strategies for investors and analysts, considering the uncertain and external disturbances encountered by stock markets, such as economic recessions, pandemics, and political instability. This framework provides significant insights for investors, analysts, and decision-makers in navigating the complicated and constantly changing stock market landscape by overcoming the limits of traditional methodologies and utilizing data-driven algorithms. This research holds great value for investors, managers, regulators, and scholars interested in assessing the efficiency of financial markets.

Keywords: DEA, IDEA, stock, stock market forecasting, Transformer

Publications

[1] **Kehinde, T. O**., Felix TS Chan, and S. H. Chung. “Scientometric review and analysis of recent approaches to stock market forecasting: Two decades survey.” *Expert Systems with Applications* 213 (2023):119299.

[2] **Kehinde, T.O.**, Waqar Ahmed Khan, Chung, S.H. (2023). A Novel Inverse DEA-based Portfolio Optimization: Evidence from the Taiwan Stock Exchange, *“International Review of Economics & Finance (IREF),”*(Under Review).

[3] **Kehinde, T.O.**, Chung, S.H., Chan, F.T.S. (2023). Benchmarking TPU and GPU for Stock Price Forecasting Using LSTM Model Development. In: Arai, K. (eds) Intelligent Computing. SAI 2023. Lecture Notes in Networks and Systems, vol 711. Springer, Cham. https://doi.org/10.1007/978-3-031-37717-4_20.

[4] **Kehinde, T.**, Khan, W., & Chung, S. (2023, October). Financial Market Forecasting using RNN, LSTM, BiLSTM, GRU and Transformer-Based Deep Learning Algorithms. In 1st International Conference on Smart Mobility and Vehicle Electrification, https://doi.org/10.46254/EV01.20230037.

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List of Abbreviations

AE: Auto Encoder

ABC: Artificial Bee Colony

AHP: Analytic Hierarchy Process

AI: Artificial Intelligence

AIC: Akaike Information Criterion

ANN: Artificial Neural Network

ANP: Analytic Network Process

ARCH: Autoregressive Conditional Heteroskedasticity

ARIMA: Autoregressive Integrated Moving Average

ARMA: Autoregressive Moving Average

Cd-DLA: Cross-domain Deep Learning Approach

CES: Comprehensive Efficiency Score

CNN: Convolutional Neural Network

CPU: Central Processing Unit

CRS: Constant Returns To Scale

CSI 300: China Securities Index 300

DDF: Directional Distance Function

DDF-DEA: Directional Distance Function-Data Envelopment Analysis

DEA: Data Envelopment Analysis

DEA-SET: Data Envelopment Analysis- Shannon Entropy Technique

DMU: Decision Making Unit

DT: Decision Tree

EMH: Efficient Market Hypothesis

ETF: Exchange-Traded Fund

FMCG: Fast Moving Consumer Goods

FTSE 100: Financial Times Stock Exchange 100 Index

Fuzzy AHP: Fuzzy Analytic Hierarchy Process

GARCH: Generalized AutoRegressive Conditional Heteroskedasticity

GBDT: Gradient Boosted Decision Tree

GDP: Gross Domestic Product

GP: Genetic Programming

GPU: Graphics Processing Unit

GRU: Gated Recurrent Unit

HKEX: Hong Kong Exchanges and Clearing Limited

HSI: Hang Seng Index

IDEA: Inverse Data Envelopment Analysis

IT: Information Technology

KELM: Kernel Extreme Learning Machine

KNN: K-Nearest Neighbors

LSTM: Long Short-Term Memory

M&A: Mergers And Acquisitions

M-V: Mean-Variance

MAE: Mean Absolute Error

MAPE: Mean Absolute Percentage Error

MCDM: Multi-Criteria Decision-Making

MODM: Multi-Objective Decision-Making

MSE: Mean Squared Error

NASDAQ: National Association of Securities Dealers Automated Quotations

NLP: Natural Language Processing

NSE: National Stock Exchange of India

NYSE: New York Stock Exchange

PCA: Principal Component Analysis

PROMETHEE: Preference Ranking Organization Method for Enrichment Evaluation

RF: Random Forest

RNN: Recurrent Neural Network

RMSE: Root Mean Squared Error

S&P 500: Standard and Poor's 500 Index

SDTP: Series Decomposition Transformer

SEE: South Eastern Europe

SET: Shannon Entropy Technique

SOCP: Second-Order Cone Programming

SMBO: Sequential Model-Based Optimization

SVM: Support Vector Machine

TEJ: Taiwan Economic Journal

TPE: Tree-structured Parzen Estimator

TPU: Tensor Processing Unit

TSE: Tehran Stock Exchange

TWSE: Taiwan Stock Exchange

UK: United Kingdom

UNWTO: United Nations World Tourism Organisation

US: United States

VIKOR: VIekriterijumsko KOmpromisno Rangiranje

WRDS: Wharton Research Data Services

Nomenclature

General Parameters:

$n = number\ of\ decision\ making\ units\ (DMUs)$

$t = number\ of\ inefficient\ decision\ making\ units\ (DMUs)$

$m = number\ of\ inputs\ of\ each\ DMU$

$s = number\ of\ good\ outputs\ of\ each\ DMU$

$q = number\ of\ bad\ outputs\ of\ each\ DMU$

$b = bad\ output$

$g = good\ output$

$A = a\ production\ possibility\ set\ of\ efficienct\ DMUs$

$B = a\ production\ possibility\ set\ of\ inefficient\ DMUs$

Data Parameters:

$x_{ij} = i^{th}\ input\ of\ DMU_j\ (j = 1,...,n)$

$y_{rj}^{g} = r^{th}\ good\ output\ of\ DMU_j\ (j = 1,...,n)$

$y_{pj}^{b} = p^{th}\ output\ of\ DMU_j\ (j = 1,...,n)$

Decision variables:

$\phi_k = inefficiency\ score\ of\ DMU_k (k = 1,...,n)$

$\psi_k = efficiency\ score\ of\ DMU_k (k = 1,...,n)$

$\lambda_j = weight\ assigned\ to\ DMU_j (j = 1,...,n)$

$\alpha_{ik} = change\ \text{in}\ i^{th}\ input\ of\ DMU_k (k = 1,...,t)$

$\beta_{rk} = change\ \text{in}\ r^{th}\ good\ output\ of\ DMU_k (k = 1,...,t)$

$\gamma_{pk} = change\ \text{in}\ p^{th}\ bad\ output\ of\ DMU_k (k = 1,...,t)$

$\gamma_{pk}^{\max} = \max\ change\ \text{in}\ p^{th}\ bad\ output\ of\ DMU_k (k = 1,...,t)$

$\lambda_j = weight\ assigned\ to\ DMU_j (j = 1,...,n)$

Chapter 1 – Introduction

1.1 Research background

The stock market plays a crucial role in the financial sector, facilitating the trading of corporation shares to investors. It serves as a medium for the accumulation of capital, the spreading of risk, the determination of prices, and the distribution of information. In addition to reflecting the expectations and attitudes of investors, the stock market serves as an indicator of a country's economic success and prospects. Traditionally, firms have utilized the issuance of stocks as a means to raise capital, while individuals view it as a pathway to amass riches. Investing in stocks offers the potential for huge financial profits, as seen by the recent notable expansion of the global domestic stock market. The market's valuation increased from 65.04 trillion US dollars in 2013 to a remarkable 98.5 trillion U.S. dollars by 2022. By July 2023, the combined value of domestic enterprises registered on stock exchanges worldwide had reached an impressive 112 trillion U.S. dollars (Statista, 2023b). The impressive upward trend demonstrates the growing significance and prospects of investments in equities. Significantly, the United States (US) has established itself as the leading participant in this field, exerting control over the largest portion of global stock holdings as of 2023, thereby solidifying its status as a financial hub in the global equities market.

Several prominent stock markets in the world include New York Stock Exchange (NYSE), National Association of Securities Dealers Automated Quotations (NASDAQ), Hong Kong Exchange (HKEX), Taiwan Stock Exchange (TWSE), National Stock Exchange (NSE), and Tehran Stock Exchange (TSE). The stock market is characterized by great dynamism and complexity, which is subject to the effects of multiple factors, including macroeconomic conditions, corporate earnings, news events, investor psychology, and market trends. Hence, the accurate prediction of future stock price fluctuations is a formidable and significant undertaking for investors, traders, analysts, and researchers. The ability to efficiently select and accurately predict stock prices can significantly assist investors in making well-informed decisions, optimizing their portfolio allocation, and maximizing their financial gains. However, the selection and prediction of stock prices are prone to various challenges, such as uncertainty, noise, nonlinearity, and high dimensionality. These factors limit conventional statistical and econometric approaches to stock selection and predictions.

Data Envelopment Analysis (DEA) is a non-parametric technique that plays a vital role in operations research and economics. It is used to evaluate the efficiency of decision making units (DMUs). As introduced by Charnes et al. (1978), the application of DEA enables the ranking and screening of stocks by evaluating their efficiency scores. The wide-ranging use of this technique in several industries, such as healthcare, banking, and particularly the stock market, is due to its flexibility to adjust and strength. Within the domain of stock market analysis, individual stocks can be seen as discrete units involved in decision-making. When these stocks are aggregated, they collectively represent a homogeneous set of DMUs. This viewpoint is consistent with the ideas of DEA, a very suitable technique for evaluating performance. Through the utilization of DEA, one can accurately distinguish between equities performing well and those performing poorly. As we advance, Wei et al. (2000) introduce the idea of inverse DEA (IDEA) as a sensitivity technique for performing the reverse process of DEA. IDEA is a notable progression in this domain. In contrast to standard DEA, IDEA aims to modify the input-output relationship to estimate variation in input-output data while keeping the efficiency constant within an existing DEA model. This unique approach has crucial implications in financial markets, notably helping in risk management and investment strategies for improved performance.

In recent years, researchers have shifted to machine learning and deep learning techniques to address the issues associated with stock selection and price prediction. Machine learning and deep learning are artificial intelligence (AI) subfields that employ data-driven algorithms to acquire knowledge from data and generate predictions or choices. Machine learning and deep learning techniques have the capability to process extensive datasets characterized by high dimensionality and nonlinearity effectively. These methodologies possess the ability to discern complex patterns and establish correlations. Several machines and deep learning techniques are frequently employed to predict stock prices, including linear regression, support vector machine (SVM), artificial neural network (ANN), decision tree (DT), random forest (RF), k-nearest neighbors (KNN), recurrent neural network (RNN), long short-term memory (LSTM), gated recurrent unit (GRU), convolutional neural network (CNN), and transformer model. The integration of machine learning and financial analysis represents a significant advancement in predictive analytics and decision-making.

Given the growing complexity and data-driven nature of stock markets, these approaches hold considerable promise for investors and analysts. It is worth noting that stock markets frequently encounter unpredictability and exogenous disruptions, such as the economic downturn of 2008 (Apergis & Dastidar, 2024), the COVID-19 pandemic (Gao et al., 2022), and the political instability of 2023 (Israel-Hamas War, 2023). These factors substantially influence the market's stability, which poses difficulties for making well-informed decisions. Using DEA, IDEA, and deep learning techniques offers a more robust and flexible strategy, which is essential for effectively navigating these unpredictable circumstances. The suggested framework seeks to address the gaps left by existing methodologies in the changing financial landscape characterized by growing data abundance. It intends to utilize the strengths of DEA, IDEA, and deep learning to enhance stock market analysis and decision-making. This study presents a robust and novel approach integrating DEA and IDEA to select stocks and analyze portfolio risks. Furthermore, this study introduces the utilization of a state-of-the-art transformer model, using sophisticated hyperparameter tuning approaches, to attain accurate forecasts of stock indexes. This novel methodology represents a notable progression in the domain of financial analysis.

1.2 Problem statement

The design and management of financial investment are among the most researched areas in finance, operation research, and computer science. It involves the complex task of maintaining and creating investment strategies. The core of this domain revolves around purchasing, holding, or selling diverse assets, such as stocks, bonds, commodities, and currencies, with the ultimate goal of achieving a positive return. Investing in stock markets is important since it provides significant financial benefits for companies and society. This concept is underscored by the substantial fortunes many investors accumulate through their stock investments. As an illustration, the Standard & Poor’s 500 (S&P 500), a prominent market index, experienced a remarkable surge of 24,708% from 1965 to 2022. In comparison, Warren Buffet's firm achieved an astronomical growth of 3,787,464% during the same time frame. This serves as evidence supporting Bill Gates’ quote: “If you are born poor, it is not your fault, but if you die poor, blame yourself.” Equities symbolize partial ownership in corporations, while stock portfolios consist of carefully selected equities aimed at maximizing profit and minimizing risk. Investment objectives and risk aversion are crucial for investors and management in stock analysis. Most investors prefer avoiding risk and are actively looking for effective methods to invest in stocks with a strong track record of performance while still having risks that can be easily managed. Nevertheless, it is crucial to acknowledge that risk is an intrinsic component of existence.

The study is additionally inspired by a biblical viewpoint, particularly Ecclesiastes 11:1-2 (GNT version), which promotes diversifying investments in order to reduce risk, including engaging in overseas trade, to achieve financial gain. The vast quantity of stocks (exceeding 65,000) and prominent stock exchanges (about 60) globally pose a difficulty in choosing equities for investment purposes as of 2023. Portfolio management encompasses several steps, namely stock selection, portfolio optimization, and price forecasting (Solares et al., 2022). This study investigates novel methodologies for making decisions in the areas of stock selection, portfolio optimization, and price forecasting in the financial markets. The initial phase of the study utilizes multi-criteria decision-making (MCDM) methods to choose equities and identify investment-worthy stocks that are not influenced by expert opinions. In addition, the study incorporates Shannon Entropy Technique (SET), DEA, and IDEA to evaluate, rank, and prioritize stocks through their input-output relationship with DMUs. The next phase involves applying a novel method of IDEA to theoretically estimate the possible reduction and maximum reduction in equity risk. In order to address this issue, the strength of the Directional Distance Function DEA (DDF-DEA) and a novel IDEA were suggested to accomplish this objective. The aim is to minimize the potential risk associated with non-performing stock investments to obtain an optimal and efficient state. In addition, the third phase, like the preceding phase, focuses on the issue of portfolio optimization using TWSE as a case study. The aim is to categorize assets based on sectors and employ inverse optimization to reduce portfolio volatility while maintaining a consistent return.

The complex, chaotic, non-linear, and unpredictable nature of the global stock market poses considerable limitations to conventional financial models. Existing models face difficulties in effectively capturing market trends, dynamics, and non-linear characteristics. The last section of this study is on price forecasting, emphasizing the significance of advanced machine learning techniques such as RNN, LSTM, GRU, and Transformer architecture. Previous studies have used the Transformer model in stock market forecasting by analyzing unstructured data like social media sentiment; however, its implementation on structured datasets like stock market technical indicators is still in its early stages. This study uses the Transformer model, which has shown success in natural language processing, language translation, and speech processing, to forecast the performance of three global stock indices: S&P 500, Financial Times Stock Exchange 100 (FTSE 100), and Hang Seng Index (HSI). The study assesses the predictive capabilities of the Transformer model using structured data and compares its performance to RNN and variants of RNN. Hyperparameter optimization using a Tree-structured Parzen Estimator (TPE) is also examined. The study also evaluates the computation demand using Google Colab hardware accelerators for model training. It implements a simple trading strategy to emphasize the investment reward of each model under evaluation in real-life trading scenarios.

1.3 Research questions

The investigation of more integrated and sophisticated ways is essential due to the complex and ever-changing nature of the stock market and the limitations of current analytical tools. This thesis is driven by the following research questions:

RQ1: How can DEA, SET, and IDEA methodologies be effectively utilized in stock analysis to assess performance, operational efficiency, and optimize risk management?

RQ2: What are the novel methodologies and contributions to enhancing stock efficiency evaluation and portfolio optimization, considering the complexities of financial data?

RQ3: How does the Transformer model, enhanced with Tree-structured Parzen Estimator (TPE) optimization, perform in financial time series prediction compared to existing models?

RQ4: What are the practical implications and computational demands of employing advanced machine learning models like the Transformer in financial market forecasting?

1.4 Research objectives

The following objectives will offer the essential responses to the research inquiries presented in the preceding section:

1. To develop a novel integrated methodology for assessing and ranking stocks.
2. To develop a novel IDEA model to estimate possible and maximum reduction in equity risk.
3. To develop a novel IDEA model to estimate the possible and maximum volatility reductions in portfolio risk.
4. To develop an algorithm to achieve net zero portfolio risk.
5. To advance stock prediction using a Transformer model, enhanced with a TPE for hyperparameter optimization.
6. To explore the practical implications of using a Transformer model in trading for investors and analysts

1.5 Scope of research

This research investigates the three phases of stock portfolio management: stock selection, portfolio optimization, and price forecasting. The study focuses on industries and sectors that hold significant global importance and are vulnerable to external disruptions. Also, this study examines the implementation of the proposed models by analyzing case studies of stock markets from both developed and developing economies. The research employs DEA and IDEA as the principal methodologies in the first three sections of the work. At the same time, sophisticated deep learning algorithms were adopted in the last section of this study for analyzing time-series data and identifying patterns to make accurate time-series predictions.

The study utilizes both technical and fundamental market data. Technical indicators include historical stock market data, including prices, volumes, and other pertinent variables; meanwhile, fundamental indicators include financial ratios such as asset turnover, inventory turnover, receivables turnover, current ratio, quick ratio, price-to-sales ratio, and a host of others. The data range encompasses various market conditions, including stability, expansion, pandemics, and volatility periods. The research concentrates on certain stock exchanges that provide a rich dataset and demonstrate varied market behaviours; however, the approaches used can be applied generally, whether they are developed markets or emerging markets. The analysis covers a specific historical era, which is crucial for comprehending market trends and validating the proposed framework.

Last, the study applies sophisticated computational methods and software tools like LINGO, R, and Google Colab to implement DEA, IDEA, and deep learning algorithms. The work provides advantages to many stakeholders in the financial sector, such as investors, financial analysts, portfolio managers, and policymakers. The results offer valuable perspectives for effective and efficient stock investment and market analysis decision-making.

1.6 Research contributions

This work achieves significant progress in stock portfolio management by exploring the strengths of DEA, IDEA, and deep learning techniques. The main contributions of this work are as follows:

1. This study is the first in the literature to present a novel methodology to address the issue of bias and subjectivity in current ranking approaches by integrating the strengths of SET, DEA, and IDEA in the assessment and ranking of stocks.

2. This work makes a scholarly contribution to the current body of research on DEA by proposing new models that address the inclusion of negative data in DEA models.
3. This research is the first in the literature to apply an IDEA model to equity evaluation using modern axioms that perceive the return-risk relationship as a financial production process.
4. This research is the first in the literature to apply an IDEA model to portfolio optimization using modern axioms that perceive the return-volatility relationship as a financial production process.
5. This study introduces a novel approach by demonstrating the robustness of TPE hyperparameter tuning in optimizing a Transformer model. It is the first study to showcase how this technique can significantly enhance prediction accuracy and trading strategies, leading to superior returns compared to other deep learning models.

1.7 Research significance

The research holds great importance due to its capacity to revolutionize the methodology of stock portfolio management and the decision-making process. The significance of this study is manifold, spanning theoretical, practical, technological, and educational aspects.

First, this study greatly adds to the theoretical foundations of financial analysis approaches. This work offers novel approaches to comprehending and examining stock efficiency and dynamics complexities by integrating DEA and IDEA. The research expands the theoretical scope by applying these approaches to stock market analysis. Consequently, this creates opportunities for future investigations and studies into implementing these sophisticated methods in different financial scenarios, perhaps resulting in more perceptive and efficient financial analysis procedures. Practically, the methodology developed in this study has substantial usefulness, especially in improving investment decision-making processes. This framework enhances the ability of investors and financial experts to evaluate stock portfolio efficiency, estimate risk reduction, and predict market changes using proposed methods. This progress results in better-informed, innovative, and perhaps more lucrative investment choices. Moreover, the capacity to adjust quickly and make informed choices is crucial in the current dynamic and unpredictable financial markets. Incorporating deep learning into this system is remarkable, as it allows for real-time analysis capabilities. This feature provides a substantial advantage in market analysis and investment strategy by helping to navigate and respond to unpredictable and changeable market conditions.

Also, this research holds technological relevance. At first, it represents significant progress in financial research by employing sophisticated data analytics techniques. This study utilizes deep learning algorithms to use the potential of big data, marking a significant advancement in the technical progression of financial analysis. Further, this research has far-reaching implications for education and policy. The results can significantly impact the content of academic courses in finance, operations research, and data science. The knowledge obtained from this research is essential when it comes to policy and regulatory frameworks. They can provide valuable insights for policy-making and contribute to the domain of investment-related legislation.

Last, the global market significance of this framework is particularly remarkable. The fact that it may be used in many financial markets, regardless of whether they are emerging or developed, emphasizes its universal significance in the finance industry. This research demonstrates that the approaches and tools created may be successfully utilized worldwide, providing valuable insights and advantages beyond geographical and economic limitations.

1.8 Structure of the Thesis

The remaining chapters of this thesis are structured as follows:

Chapter 2 of this thesis covers a comprehensive review of the existing literature pertaining to the subject matter. This chapter comprehensively examines the pertinent literature on the background, concepts, methods, and methodologies employed in stock portfolio management, including stock selection, portfolio optimization, and price prediction, by utilizing DEA, IDEA, and deep learning models. This chapter additionally identifies the research gaps in the current body of literature and the potential areas for further investigation.

Chapter 3 introduces a novel framework for stock selection and ranking by integrating the techniques of SET, DEA, and IDEA. The concept is implemented using the TWSE tourist sector as a case study. The utilization of SET can potentially decrease the dimensionality and redundancy of data while simultaneously enhancing the discrimination strength and robustness of DEA. The IDEA technique is utilized to conduct an inverse optimization by maintaining a constant efficiency score for DMUs. Additionally, sensitivity analysis was conducted to validate the models and results.

Chapter 4 delves into a novel conceptualization of reframing return and risk as outputs of a financial production process. Implementing this concept involves utilizing DDF-DEA and

IDEA methodologies to estimate the inefficiency levels and potential reduction in equity risk. The chapter tested the developed models using two similar sectors from different markets. Additionally, the chapter provides an overview of the data sources, data preprocessing, model formulation, model implementation, and model evaluation.

Chapter 5 of this study focuses on extending the developed models in the preceding chapter to optimize industry-based portfolios. The empirical evidence presented in this chapter is tested using datasets from various sectors belonging to TWSE. In addition, the chapter developed a framework for net zero portfolio risk initiatives. Also, it provides an overview of the data sources, data preprocessing, model formulation, model implementation, and model evaluation.

Chapter 6 concentrates on improving the prediction of stock market indices by utilizing a Transformer model combined with a TPE for tuning hyperparameters. The chapter introduced sophisticated deep learning techniques to forecast three notable stock indices: the S&P 500, FTSE 100, and HSI. It highlights the model's practical relevance and ability to generate substantial investment benefits.

Chapter 7 presents the overall summary of the models discussed in the preceding chapters, along with the limits of this study and potential avenues for further research.

Chapter 2 – Literature Review

This chapter provides an in-depth investigation of existing studies on DEA, IDEA, and deep learning techniques in the domain of stock portfolio management. The chapter tackles the problem of stock management using DEA, a non-parametric technique in operations research, to evaluate stock performance, optimize portfolios, and assess the efficiency of investment strategies. The study also investigates the role of IDEA in conducting reverse optimization to estimate variations in input-output relationships without altering the efficiency state of DMUs. In addition, the chapter explores the use of sophisticated deep learning techniques, highlighting their capacity to analyze vast amounts of data to forecast stock patterns and execute investment strategies. This study highlights the major contributions made by scholars in these fields, demonstrating the development of techniques and their practical implications in financial settings. Nevertheless, in addition to these achievements, the chapter thoroughly examines the limitations of past studies, which justifies the need for current research.

2.1 Shannon Entropy Technique (SET)

In MCDM, determining the relative weights of indicators is a fundamental and essential step in the problem-solving process. Several methods are prominent and generally recognized for determining these weights, including expert opinion-based approaches, the least squares method, the special vector technique, and SET. Notably, the SET stands out as one of the most crucial methods for determining criteria weights (Peykani et al., 2022). Qualitative approaches such as Analytic Hierarchy Process (AHP) (Teixeira et al., 2023), Fuzzy AHP (Faisal et al., 2022; Yilmaz et al., 2022), Analytic Network Process (ANP) (Percin, 2008), PROMETHEE (Agrawal, 2022), and VIekriterijumsko KOmpromisno Rangiranje (VIKOR) (Sahu et al., 2018; Sahu & Raut, 2023), are common methods that require expert opinions in their evaluation processes, often criticized for creating bias and subjectivity. Principal Component Analysis (PCA) does not possess the inherent characteristic of being a tool for MCDM, yet the statistical technique is typically used for dimensionality reduction, particularly in DEA (Sarkar, 2016). Nevertheless, the utilization of PCA in this situation has its limitations. Applying PCA for dimension reduction in DEA models may lead to a reduction in the amount of original data, perhaps resulting in an oversimplification of the DEA model. The possible loss discussed above has the capacity to not only impact the assessment of DMU efficiency but also impose constraints on subsequent investigations, such as inverse optimisation, as a result of the altered data. Moreover, the effectiveness of PCA is greatly impacted by the scaling of variables, rendering it susceptible to variations.

The concept of SET originated from information theory and was initially presented by Shannon in 1948 (Shannon, 1948). SET quantifies the level of uncertainty or unpredictability inherent in a given system or variable. The level of entropy increases proportionally with the degree of uncertainty or unpredictability exhibited by the system or variable. SET has several applications across various disciplines, including cryptography, communication theory, physics, biology, ecology, economics, and statistics. One of the practical applications of SET is in the domain of feature selection. This process involves the careful selection of a subset of relevant features or variables from a vast pool of accessible options tailored to address a given task or problem. The feature selection process can decrease the dimensionality and intricacy of the data while also enhancing the effectiveness and precision of the models or techniques that rely on the chosen features or variables. One notable benefit of employing SET for feature selection lies in its independence from prior knowledge or expert judgments. Instead, it leverages the information content associated with each feature or variable to ascertain their significance and pertinence. Another benefit of utilizing SET for feature selection is its ability to effectively handle many features or variables, including numerical and categorical ones. Furthermore, it is capable of accommodating linear as well as nonlinear associations among these features.

Few studies have employed SET in portfolio stocks. Xie et al. (2014) suggest a novel approach that integrates the efficiency of several variable subsets. These subsets are then weighted based on their respective degrees of relevance, which are determined using SET. The process yields a Comprehensive Efficiency Score (CES), enabling a more precise and consistent ranking of all DMUs. The authors implement their methodology on several datasets from existing literature and conduct a comparative analysis with alternative DEA models. The authors demonstrate that their methodology can potentially enhance the discriminatory power of DEA while retaining valuable variable information. Furthermore, their versatile approach can be applied across various DEA assumptions and orientations. A similar approach was adopted by Gupta et al. (2020) to rank four distinguished DEA models. Peykani et al. (2022) introduce a hybrid DEA–Shannon entropy (DEASE) method in their study. This approach addresses the issue of selecting appropriate input and output indicators within DEA models. The strategy employs SET to ascertain the weights of indicators and select the most significant ones from each cluster of comparable indicators. The approach considers negative data and values in some indicators, employing the range directional measure DEA as the fundamental model. The authors showcase the practicality of the DEASE strategy through its implementation in a case study involving 15 equities from the food sector of TSE. The findings indicate that the DEASE technique can potentially enhance the DEA model's discriminatory capability and achieve a more efficient stock ranking compared to a traditional DEA model. In a recent investigation, Karagiannis and Karagiannis (2023) present a non-parametric model utilizing DEA to quantify the degree of overpricing for each brand in relation to its number of features. The authors additionally present a novel use of SET to aggregate vitamin and mineral items, thereby mitigating the curse of dimensionality.

2.2 Efficiency evaluation using DEA

DEA is a non-parametric method utilized to assess the relative efficiency of DMUs by considering their weighted inputs and outputs. The DEA methodology has the capability to offer benchmarks and targets that can be utilized to enhance the performance of inefficient DMUs. Asides stock selection, DEA has been extensively applied for performance improvement in other sectors such as pension management (Demirtaş & Keçeci, 2020), energy assessment (Liu et al., 2022), transport assessment (Li et al., 2023), bank assessment (W. Zhu et al., 2023), port assessment (Q. Wang et al., 2022), safety management (Ye et al., 2023), city assessment (Wu et al., 2023), loan assessment (Partovi & Matousek, 2019), outsourcing firms assessment (Valiyattoor & Bhandari, 2020), sports (Adhikari et al., 2020), value chain (Pourbabagol et al., 2023), R&D (X. Chen et al., 2021), water services (Tourinho et al., 2022), fintech (Wang et al., 2021), production and manufacturing (Oukil et al., 2022), blockchain technology (Zhou et al., 2022), healthcare (Wu et al., 2022), logistics (Ho et al., 2022), fishery (Pratama et al., 2023), farm and forest resources (Liu et al., 2023), equity market (Gyan et al., 2017), insurance (Alshammari et al., 2019), power (Zhu et al., 2022), pulp and paper (Jauhar et al., 2022), online customer services (Park, 2023), warehousing (AlAlawin et al., 2022), and education (Guo & Chen, 2023).

The notion of the DEA was originally proposed by Charnes et al. (1978), drawing upon Pareto efficiency and linear programming principles. DEA has the capability to discern the most effective DMUs as efficient entities while categorizing the remaining ones as inefficient. The DEA methodology is capable of offering benchmarks and targets that can be utilized to enhance the performance of inefficient DMUs. The technique of DEA offers numerous advantages in comparison to alternative methods when it comes to measuring efficiency and evaluating performance. One notable feature of this approach is its ability to generate an empirical frontier that encompasses all DMUs without relying on any assumptions regarding the functional form or distribution of the data. Instead, it utilizes the data itself to derive this frontier. An additional benefit is its ability to effectively manage many inputs and outputs that possess distinct units and scales without necessitating any pre-existing weights or preferences. A significant issue in implementing the DEA approach across different applications is the absence of consensus about selecting and determining input and output variables. Another crucial aspect when employing the DEA methodology is its discriminatory strength. Despite the absence of a universally accepted scientific norm or agreement in the existing literature about the optimal number of DMUs for a robust efficiency estimation, several researchers suggest employing Eq. (2.1) as a means to establish the minimal number of DMUs needed by any standard DEA model to discern effectively (da Silva et al., 2023). In Eq. (2.1), a represents the minimum count of DMUs, b represents the total count of inputs, and c represents the total count of outputs.

$$a \geq \max\{3*(b+c), b*c\} \tag{2.1}$$

Few studies have used DEA models and methodologies to determine the performance of stock portfolios. Although some investigations have yielded valuable insights and implications for enhancing the efficiency of stocks, none of these works has employed integrated DEA, SET, and IDEA to perform efficiency evaluation and ranking.

2.3 Application of DEA to various sectors

Various studies have employed diverse DEA models and methodologies to measure the performance of portfolio stocks belonging to various sectors, including capital markets, power generation, telecommunications, tourism, health care, banking, sea ports, airlines, and many more. Chen (2008) conducted a study that analyzed portfolios managed by DEA in eight prominent industries. The findings indicated that these portfolios demonstrated incredible performance compared to industry averages and portfolios of small-size enterprises. The methodology employed encompasses stock screening, portfolio selection, and capital allocation, demonstrating superior performance compared to benchmark indices regarding both return rate and Sharpe ratio across many testing intervals. In related research by Gardijan and Škrinjarić (2015), using the Croatian market as a case study, the same findings were achieved when a DEA-selected portfolio was compared with a market capitalization-based portfolio. This superiority was observed in terms of higher return rates and risk-adjusted returns.

Hsu (2014) introduced a novel methodology for portfolio optimization by integrating DEA with Artificial Bee Colony (ABC) and Genetic Programming (GP). The effectiveness of this approach was demonstrated through a case study conducted in the semiconductor sector of TWSE, where it successfully generated significant returns and mitigated investment risks. In addition, Huang et al. (2015) introduced an integrated methodology combining DEA with multi-objective decision making (MODM) methodologies to optimize stock investments in Taiwan. The methodology employed encompasses stock screening, portfolio selection, and capital allocation, demonstrating superior performance compared to benchmark indices regarding both return rate and Sharpe ratio across many testing intervals. Other domains where researchers have explored the applications of DEA include healthcare (Chiu et al., 2022), education (Lin & Yu, 2023), manufacturing (Lu et al., 2021), agriculture (Wei et al., 2023), tourism (Yu & Chen, 2020), port management (Hsu et al., 2023), banking (Li et al., 2019), power and energy (Mariano et al., 2021).

2.4 Extended versions of DEA

The DEA methodology has the capability to integrate various assumptions on returns to scale and restrictions on variable weights. This allows for a more precise representation of the features and preferences of the DMUs. Nevertheless, the use of DEA in the stock market necessitates the identification and resolution of certain constraints and problems in order to ensure its successful and accurate implementation. Several researchers have made significant contributions to the discipline by addressing this limitation by including negative data handling in DEA models. For instance, Portela et al. (2004) outline strategies for handling negative data DEA using a radial distance measure approach. An extended dynamic radial distance function strategy that allows for negative data has also been developed by Tavana et al. (2018). Additional adjustments to various versions of DEA models to account for negative values have been proposed by other researchers, such as Allahyar and Rostamy-Malkhalifeh (2015), Lin and Chen (2017), Lin and Liu (2019), Tone et al. (2020), and Omrani et al. (2022). Another drawback of the DEA method is the assumption that all outputs are inherently desirable or valuable, which may not necessarily hold true in some cases. For instance, specific outcomes such as waste, pollution, and risk may be deemed undesirable or pose potential harm in a production process. To address these shortcomings, many improvements and adjustments to DEA have been proposed. According to Chung et al. (1997), DDF-DEA is one such extension, as it accounts for bad output in DEA models.

As defined by DDF-DEA, efficiency is the maximization of the proportional increase in good outputs and the minimization of bad outputs along a given direction vector while maintaining constant inputs. In addition, other researchers such as Ghiyasi (2017), Wegener and Amin (2019), Yang et al. (2020), Orisaremi et al. (2021), and Zeng et al. (2022) have made notable advancements in addressing undesirable outputs such as pollution Ghiyasi (2017), gas emissions, and waste in the energy industry using the DEA framework. The developments above have significantly expanded the applicability of DEA, enabling it to effectively capture and analyze situations involving negative data or undesirable outcomes. However, none of these works has addressed the inverse process of DEA to assess stock using new axioms of financial production technology where wealth investment is viewed as a by-product with risk-return values.

Kuo et al. (2021) authored a novel hybrid approach for assessing the effectiveness of Chinese-listed companies in the stock market. The authors adhered to the principle established by Tarnaud and Leleu (2018), which demonstrated that within a financial production framework, risk should be regarded as an outcome rather than a factor of production. The proposed model employs DEA, AHP, and Second-Order Cone Programming (SOCP) methodologies. Also, the study employs a combination of fundamental and technical ratio methodologies to evaluate the performance of companies throughout two distinct stages: production and finance production. Furthermore, the study employs trend analysis to forecast prospective efficiency by leveraging past data. The study concludes that the production stage holds greater significance than the finance production stage in determining total efficiency. Nevertheless, Kuo et al. (2021) did not engage in inverse optimization to ascertain the minimum adjustments necessary to enhance the efficiency of the selected portfolios.

2.5 Selection of inputs and outputs in a stock portfolio problem

The assessment of firm performance is crucial when making investment decisions. The selection of appropriate metrics is crucial for evaluating performance, as it ensures that the chosen indicators accurately represent the comprehensive financial well-being of a company. These measures have the potential to be either fundamental, technical, or a combination of both. The primary sources of a company's financial information typically consist of its balance sheet and income statement, which can be analyzed in either absolute values or in the form of ratios. The process of selecting inputs and outputs for DEA involves careful consideration and deliberation. This step is crucial, as it directly impacts the accuracy and reliability of the analysis. The inputs and outputs chosen must accurately represent the underlying factors and outcomes being evaluated. Therefore, a thoughtful and systematic approach is necessary. The selected inputs and outputs should align with the production process and objectives of the DMUs while also considering the characteristics and preferences of the investors or stakeholders involved.

Fundamental and technical indicators are widely employed methodologies for the appraisal of stocks and forecasting equity returns (Kumbure et al., 2022; Nazareth & Reddy, 2023; Zhou et al., 2021). The existing body of literature indicates that fundamental indicators have consistently been preferred as the primary approach for assessing and choosing equities with the intention of long-term investing. The evaluation of firm well-being is often indicated in publicly available financial statements. Fundamental indicators serve not only to assess a firm's current performance relative to its past but also facilitate inter-firm comparisons. Consequently, it gives investors significant insights to inform their long-term investment decisions. However, it is important to acknowledge that one potential limitation of this particular methodology lies in the fact that financial statements can present erroneous data due to many factors, such as accounting errors or deliberate fraudulent activities. In addition, the process of gathering, organizing, and preparing data from financial reports might prove to be a time-intensive endeavour for investors. In contrast to fundamental indicators, technical indicators assert that an asset's historical trading records and price fluctuations frequently serve as insightful indications for predicting its future price movements. The operations of these entities are predicated upon the underlying assumptions that the market efficiently incorporates all available information, price movements exhibit discernible trends, and historical patterns tend to recur (Jiang, 2021).

According to Murphy (1999), several scholars contend that fundamental analysis is unnecessary, as they believe all relevant information on a stock is already incorporated into its market price. However, one limitation of this methodology is the potential impact of investor sentiments on market data. Despite its limitations, the majority of portfolio selection strategies focus exclusively on technical indicators, operating under the assumption that the stock market is efficient and resistant to the influence of investor activity. Recent research indicates that a combination of fundamental and technical elements should be employed in the process of selecting stocks (Chen et al., 2016). Bettman et al. (2009) introduced a theoretical framework integrating fundamental and technical assessments of stock valuation. An empirical examination of this model substantiated the notion that these two approaches are mutually reinforcing and should not be seen as interchangeable alternatives. In support of these experimental findings, a study conducted by Gardijan and Škrinjarić (2015) provides empirical evidence to support the assertion that integrating technical and financial indicators enhances the effectiveness and performance of portfolio selection, particularly in the context of small and illiquid markets. Even though most studies have presented significant evidence regarding the effectiveness of utilizing fundamental and technical indicators for stock selection, a few have explored the synergistic relationship between these variables in the context of stock selection (Ejaz et al., 2017). While each analysis demonstrates satisfactory performance when considered independently, their integration yields a more comprehensive explanatory capacity. According to the study conducted by Chen et al. (2016), it was uncovered that the accounting ratio has the potential to serve as a supplementary tool to the technical ratio. Furthermore, the researchers discovered that the combination approach, incorporating accounting and technical ratios, exhibited superior performance compared to the individual method.

To solve this dispute, Arasu et al. (2021) conducted a comparative analysis of three distinct categories of variables employed in stock appraisal: fundamental indicators, technical indicators, and a combination of both, using a dataset of 69 stocks from NSE. The study uncovers that all three sets of variables possess the capability to produce a set of efficient stocks that produce high returns. Even though it is evident in their results that technical indicators yield higher returns (aggressive portfolio) than the other two, the authors posit that the condition is not sufficient to say that the technical indicator is the most proficient, as some stocks that are missing in the second distinct category but listed in the third category have higher annualized returns with dividends (conservative portfolio). The study proposes that the integration of fundamental and technical indicators has the potential to enhance stock selection and optimize portfolio performance. In this regard, Chapters 4 and 5 of this research explore a combination of financial and technical indicators in implementing the proposed DEA framework.

Despite the work of Arasu et al. (2021), there is still a discrepancy between the traditional and modern description of an investment production process, where equity risk is measured as input and return is measured as output (Branda, 2013, 2015). Devaney et al. (2016) utilize a DDF-DEA to evaluate the performance of 188 mutual funds in the US. The authors established an optimal frontier that considers both risk and return while accounting for the transaction costs involved with portfolio management. Risk is an unwanted product of return; therefore, risk aversion is presumed to be the desire of any investor (Tarnaud & Leleu, 2018). In contemporary times, the assessment of portfolio efficiency has consistently emerged as a prominent area of research within finance. The estimation of stock efficiency has been a subject of interest among numerous scholars. One of its key advantages is that DEA evaluation does not require the specification of a production function. This technique not only mitigates the potential error in setting the production function but also serves as a reference point for decision-makers (Xiao et al., 2022). The classic DEA and diversification DEA models are often employed in this context. As mentioned earlier, the evaluation does not necessitate assumptions on the effectiveness of the financial market. Instead, they rely solely on utilizing multi-dimensional technical indicators, such as return and risk, to conduct a comparative evaluation of stock portfolios. Based on the review conducted, previous studies commonly employed the aforementioned technical indicators to construct the portfolio production potential set.

Nevertheless, some scholars have raised concerns regarding the validity of the input-output process, arguing that these financial metrics alone may not provide a comprehensive depiction of portfolio performance (Ebrahimi et al., 2021; Kuo et al., 2021; Ren et al., 2021; Tarnaud & Leleu, 2018; Zhou et al., 2019). The input-output attribute between return and risk is usually viewed from two distinct angles in the literature. The first perspective takes equity risk as an input indicator of initial wealth, while the expected return is regarded as an output indicator of a terminal wealth investment. In contrast, the second perspective holds that real investment is one where both assessment indicators (risk and return) are generated from the terminal wealth viewed as either good or bad; hence, this present work is in alignment with the second perspective since a stock should always be seen as a financial production process where return and risk are its by-products. The work of Tarnaud and Leleu (2018) and Ebrahimi et al. (2021) gave valid and convincing proofs to support this claim in linear programming and non-linear programming contexts, respectively.

In this thesis, it is imperative to ensure that the choice of inputs and outputs aligns with the established ideas and practices, and so, we align with the second perspective view of return and risk as an output of a financial production process, as proved by Tarnaud and Leleu (2018).

2.6 Inverse DEA (IDEA)

The IDEA technique is widely utilized in conducting inverse optimization. This involves maintaining the efficiency score of DMUs at a constant level while systematically adjusting the input or output variables by a certain percentage. This adjustment allows for examining the corresponding changes in the output or input variables. The notion of IDEA was initially introduced by Wei et al. (2000). This approach is rooted in inverse linear programming and sensitivity analysis principles. The IDEA framework offers a distinctive approach to rating efficient DMUs by evaluating their capacity for improvement or decrease. Recently, researchers have developed and studied inverse issues. Two major categories of models exist in this area, with the first focusing on resource allocation and the second on investment analysis (Emrouznejad & Amin, 2023). The IDEA concept has become the state-of-the-art analytical technique in energy and environmental studies. Prominent scholarly contributions that employ the IDEA framework to mitigate environmental pollution and address the climatic impact of greenhouse gases in production include Ghiyasi (2017), Wegener and Amin (2019), Emrouznejad et al. (2019), Orisaremi et al. (2022), Lu et al. (2022), and Yang and Lu (2023).

Further, the utilization of the IDEA has been established in the banking industry, particularly in mergers and acquisitions (M&A) scenarios. M&A in the banking industry often arises from the objective of attaining enhanced operational effectiveness, enlarging market share, and strengthening financial performance. The IDEA framework provides a method for assessing the potential operational efficiency of a merged company before the merger and acquisition process is completed. Several notable works have employed the IDEA framework to assess the effectiveness of bank consolidations. These works include Amin and Al-Muharrami (2018), Amin et al. (2019), Amin and Oukil (2019), Amin and Ibn Boamah (2020), Amin and Ibn Boamah (2021), and Soltanifar et al. (2023). In recent studies, scholars have investigated other industries using the IDEA technique. These sectors encompass healthcare (Ghiyasi et al., 2022; Jahani Sayyad Noveiri & Kordrostami, 2023), education (Foladi et al., 2020), transportation (L. Chen et al., 2021), supply chain (Aslani Khiavi et al., 2023; Moghaddas et al., 2022), textiles (Mahla et al., 2023), and automobiles (Lim, 2016). The works of Emrouznejad et al. (2023) and Emrouznejad and Amin (2023) provide in-depth reviews on the advances and state-of-the-art applications of IDEA in various domains.

In stock selection, IDEA is rarely covered in the literature, making it a hotspot for investigators. The two closely related articles in this regard include those of Çakır (2017) and Goyal et al. (2023). Çakır (2017) presents a two-phase approach to address resource allocation issues within a fuzzy context. The initial stage involves selecting input and output variables for DEA using an imprecise SET. This method can accommodate fuzzy and interval data while also facilitating the calculation of criteria weights. In the second phase, an interval IDEA model is utilized to estimate the optimal input values for DMUs in case of changes in some output values. This estimation is done while maintaining the efficiency score of the DMU and enhancing the efficiency scores of other DMUs. Their results showcase the practicality and effectiveness of the suggested technique by examining a genuine case study involving 16 cement companies in Turkey. However, the findings indicate that the hybrid model only addresses challenges related to input-output selection and resource allocation in situations characterized by fuzzy conditions. Another limitation is that the interval IDEA formulated could only be used for ranking purposes of input and output data but is unsuitable for ranking efficient DMUs. Also, the interval IDEA only preserves the efficiency score of the DMU under evaluation while enhancing the efficiency scores of other DMUs. Similarly, Goyal et al. (2023) put forth an IDEA approach by introducing a ranking system based on a super-efficiency IDEA model. The proposed model assesses and prioritizes 52 bus depots by considering their inputs and outputs. The study compares the suggested and conventional super-efficiency DEA models, revealing high consistency between their respective outcomes. The limitation of this work is that the number of inputs and outputs selected does not compromise the discerning power of DEA. Also, it is worth noting that super-efficiency DEA and super-efficiency IDEA are sensitive to outliers and sometimes encounter infeasibility issues.

A noticeable gap is observed in the existing body of literature regarding the utilization of IDEA in the context of stock market or stock portfolio analysis. Based on the available literature, no prior work has been conducted in this field, emphasizing the need for this research. This present study can enhance comprehension of stock dynamics and portfolio performance while promoting the development of more efficient and successful investing techniques.

2.7 Portfolio optimization using DEA

Portfolio optimization is widely utilized in asset allocation, garnering significant attention from investors and portfolio managers (Trichilli et al., 2020). The application of portfolio optimization as a tool of diversity (Singh et al., 2023) has conventionally relied on the M-V paradigm proposed by Markowitz (1952). Nevertheless, researchers have endeavoured to explore new techniques that can effectively tackle the limits of the existing approach and offer a more comprehensive examination of the trade-off between risk and return. Therefore, investors have access to several strategies to mitigate risk while effectively maximizing returns. Regrettably, the Markowitz mean-variance portfolio optimization technique has encountered numerous drawbacks that have resulted in identifying a limited number of top assets. First, it is imperative to acknowledge that this process exhibits high sensitivity towards even minor input alterations. Second, this approach relies on previous pricing data, limiting investors from formally incorporating their expertise into the market. Bauder et al. (2021) have indicated that the M-V analysis is susceptible to the traditional weaknesses and limitations inherently linked to extreme portfolio weights that often arise when building a set of efficient portfolios. The M-V optimization technique serves as a fundamental component of contemporary portfolio theory. However, in the context of practical application, it is essential to acknowledge that estimating errors have the potential to result in portfolios that exhibit extreme characteristics.

Basso and Funari (2001) are one of the initial studies utilizing DEA for portfolio selection. The researchers put forth a model based on DEA to select a subset of mutual funds. This model demonstrated superior performance compared to conventional approaches in the context of the Italian market. Portfolio diversification is a fundamental principle in the field of investing theory. It proposes that by maintaining a combination of different assets, investors can reduce the potential risks connected with specific securities. The fundamental premise might be straightforward: "Avoid concentrating all of your resources or investments in a single entity or venture." The scholarly discussion surrounding diversification has undergone significant changes over several decades, exerting a profound influence on contemporary portfolio management methodologies. Whether local or international, diversification is critical in portfolio selection (Yadav et al., 2023). Solnik (1974) emerged as a prominent advocate of international diversification. The author suggested that diversification investments between countries may yield more benefits than diversifying across industries, primarily due to fewer correlations between international markets. To support this claim, Oloko (2018) explores the possible advantages of portfolio diversification for US and UK investors who invest in Nigerian companies. His analysis reveals that both US and UK investors can experience advantageous outcomes by incorporating Nigerian companies into their investment portfolios. Furthermore, the inclusion of Nigerian stocks can serve as an effective hedging technique against financial crises.

Similarly, Pirgaip et al. (2021) investigate the effects of the South Eastern Europe (SEE) Link trading platform on the potential for portfolio diversification among investors in the region. The SEE Link project serves as a platform for interconnecting 7 stock exchanges located in South Eastern Europe. Its primary objective is to streamline cross-border investments and bolster regional market liquidity. The authors employ a range of correlation and regression models to examine the daily returns of the Zagreb Stock Exchange and the Bulgarian Stock Exchange, both prominent stock exchanges, throughout the period spanning from 2005 to 2019. The researchers observed a decline in the correlation and spillover effects between the two markets after introducing the SEE Link in 2016, which suggests that investors have the potential to gain advantages through diversification in the regional markets.

The closest study to the current study is the work of Yu et al. (2023). The authors present a novel approach that utilizes inverse optimization through online learning to acquire insights into the risk preferences of investors based on their investment portfolios. The methodology assumes that investors exhibit rational behaviour and engage in portfolio optimization using the mean-variance paradigm. By examining portfolios and analyzing market prices over time, their methodology enables the estimation of a risk tolerance factor that captures the balance between anticipated returns and associated risks. The authors illustrate their methodology using both simulated and real-world data and establish its capability to effectively capture the dynamic character of risk preferences.

2.8 Predictability of stock markets

The pursuit of predicting stock prices has gained significant attention in recent decades, mainly because of the crucial role that financial markets play in national economies and their widespread impact on different aspects of daily life. Accurate forecasting of stock prices is an ongoing process that is continuously developing. The methodology selection mainly relies on the distinct attributes of the stock data, the prevailing market conditions, and the desired objectives of the forecasting task. The theory of Efficient Market Hypothesis (EMH), as proposed by Fama (1970), posits that financial markets are efficient and that all available information is already reflected in the prices of assets; as a result, making a super profit is almost infeasible. The hypothesis has played a fundamental role in financial theory, influencing investment strategies and regulatory policies. However, the validity of it has been a topic of vigorous discussion, as empirical research has produced conflicting outcomes. Although EMH disproves the existence of abnormal returns, this hypothesis has faced both critical opposition and support throughout its existence, as many researchers have reported their models to generate tangible profits (Grudniewicz & Ślepaczuk, 2023; C. Wang et al., 2022).

Stock market evaluation is divided into two main approaches: fundamental analysis and technical analysis (Lohrmann & Luukka, 2019). Fundamental analysis evaluates a company's financial well-being, management, industry standing, and economic indicators to determine its inherent value. In contrast, technical analysis examines price fluctuations, chart formations, and trading volume to anticipate future patterns. Although there are supporters for both sides, the question of whether they can continuously outperform the market is still a subject of debate.

Traditional financial time series analysis was based on the assumptions of linearity and stationarity, often utilizing the linear regression model. However, non-linear connections and other complications in financial time series have rendered classic linear models insufficient. Accurate stock market prediction is arduous due to the immense amount of financial data and the complex dynamics of stock markets. The complexity is additionally expanded by variables such as market volatility, economic data, investor mood, and global events, all contributing to the unpredictability of stock movements. Stock price prediction is essentially the task of forecasting time-series data, aiming to estimate future values based on past data. To tackle this task, one might utilize three main classifications of forecasting approaches: classical, machine, and deep learning.

2.9 Classical approaches to stock market forecasting.

This approach comprises statistical models, such as moving average, autoregressive moving average (ARMA), autoregressive integrated moving average (ARIMA), autoregressive conditional heteroskedasticity (ARCH), and generalized ARCH (GARCH), which have long been fundamental in financial forecasting (Rounaghi & Zadeh, 2016; Zhang et al., 2016). These models are based on statistical theory and are efficient in certain market scenarios, especially when the market exhibits linearly predicted patterns or trends. Generally, classical models marked a notable progression in quantitative finance. Based on statistical theory, these models have been extensively utilized for predicting time series data, such as stock prices. Their popularity stems from their simplicity and interpretability; however, they frequently fail to capture the non-linear nature and complexities of the financial market.

Mondal et al. (2014) employ the Akaike Information Criterion (AIC) to pick the optimal ARIMA model for individual stocks and quantify prediction accuracy using mean absolute error (MAE). In addition, they assess the predictive accuracy across various training data set sizes and employ t-tests to evaluate the statistical significance of any observed differences. The ARIMA model used in their study demonstrates high predictive accuracy for some industries, particularly fast moving consumer goods (FMCG) and information technology (IT) while exhibiting lower effectiveness in the banking and automobile sectors. Additionally, they uncovered that the magnitude of the training data set does not substantially influence the prediction's accuracy. Singh et al. (2021) put forth a novel method that integrates wavelet decomposition, wavelet denoising, ARMA, and ARIMA models to produce precise forecasts using historical data from the Bombay Stock Exchange (BSE) 100 S&P index. The authors compare their hybrid models against the baseline models, demonstrating that their models exhibit reduced predicting errors and improved accuracy. The proposed model by the authors effectively mitigates risk and uncertainty for investors in stock market investments.

However, the limitation of using statistical models is their dependence on linear assumptions regarding market behaviour. The stock market is a complex system that is affected by various elements such as noise, policy changes, and human manipulation, which can substantially impact market dynamics. Due to their reliance on past data and patterns, these elements introduce a degree of uncertainty and chance that these models frequently find difficult to consider. Statistical models offer significant insights, but their ability to capture the complex character of stock markets is restricted in practice. The existence of this gap has resulted in an increasing trend towards the use of more advanced methods that can effectively handle the intricacies and uncertainties that are inherent in financial markets.

2.10 Machine learning approach to stock market forecasting

Driven by the rapid progress in AI, stock market forecasting has experienced a notable transition towards machine learning techniques due to their capacity to better adapt to data distributions and consequently achieve improved performance compared to conventional statistical methods. These models have gained significance because of their capacity to manage non-linear interactions and flexibility in adapting to various market conditions. Machine learning models can reveal complex patterns in data that may not be immediately obvious, providing a more sophisticated comprehension of market dynamics compared to conventional statistical models. In this realization, some researchers have already employed machine learning approaches to study stock market prediction. For instance, Yunneng (2020) proposed an improved KNN algorithm that integrates the historical stock price data from the past N days with the overall pattern observed over the initial N days to predict the price trend for the following day. The study evaluates the effectiveness of the suggested approach by benchmarking it with the conventional KNN algorithm and the regression prediction method. The results demonstrate that the proposed method achieves superior accuracy and exhibits a reduced standard error. Hindrayani et al. (2020) utilize historical fundamental data from four firms to train and evaluate four regression models (Multiple Linear Regression, Support Vector Regression, Decision Tree Regression, and K-Nearest Regression). The results indicate that the Decision Tree Regression method yields the most favourable outcomes in terms of correlation coefficient and MAPE.

Due to its promising results, SVM has emerged as the most widely used machine learning algorithm for stock prediction. SVM provides more adaptability to acquire knowledge from data, resulting in greater forecast accuracy. Nevertheless, it encountered difficulties managing the complex nature and interference in financial data due to its high dimensionality and noise. Lin et al. (2013) present an SVM-based market trend prediction. The method consists of a feature selection and a prediction model. The authors use a correlation-based SVM filter to rank and select financial indicators that are highly correlated with the stock market trend. The prediction model uses a quasi-linear SVM to avoid over-fitting and improve generalization. The article by Khaidem et al. (2016) suggests a method for predicting stock price trends using RF. The authors use historical trading data from four US-listed companies and apply exponential smoothing to reduce noise and volatility. The authors calculate ten technical indicators as features and use random search to optimize parameters. The optimized model achieves higher accuracy, precision, recall, and F1 score, making it more stable and reliable. Similarly, Basak et al. (2019) applied RF and GBDT algorithms to classify the direction of stock prices throughout a trading period spanning 3 to 90 days. The study's results suggest that the models are suitable for trading over longer durations, as accuracy improved as the trading period rose. Other notable works on the application of machine learning to stock predictions include Wang et al. (2020), Yun et al. (2021), Nti et al. (2020), Rath et al. (2022) and Koukaras et al. (2022),

While the approaches may yield satisfactory results in many cases of stock predictions, their application to extensive datasets is hindered by their limited capacity to extract relevant features (Vadlamudi, 2017). To fill this gap, Yun et al. (2021) developed a three-stage predictive model utilizing a hybrid GA-XGBoost, that specifically aimed to improve feature expansion, data preprocessing, and optimal feature selection. Similarly, Mohanty et al. (2021) devised a hybrid model that integrates Auto Encoder (AE) and KELM to boost the robustness of stock predictions. Although machine learning methods show proficiency in modelling non-linear connections and extracting insights from complex datasets, they still encounter difficulties in adequately addressing the complexities and vastness of massive data inherent in stock market forecasting. The complex and intricate dynamics of financial markets, which are influenced by numerous quantitative and qualitative elements, necessitate models that not only accurately represent non-linear connections but also possess the ability to adjust to quick market changes and incorporate a more comprehensive array of market indicators. Consequently, investors and researchers increasingly emphasize new approaches, such as deep learning models. These models are expected to provide more thorough comprehension and predictive abilities in the constantly changing stock market.

2.11 Deep learning approach to stock market forecasting

The exponential increase in computational capacity in recent years has accelerated the emergence of deep learning methodologies, fundamentally transforming diverse financial domains, such as stock market analysis. Deep learning, a kind of machine learning distinguished by its utilization of multi-layered neural networks, has significantly transformed various domains, such as finance. The emergence of deep learning models, such as ANN, CNN, RNN, LSTM, GRU, and CNN, signifies the most recent frontier in forecasting stock prices, as they stand out in capturing temporal dependencies and non-linear correlations. These models utilize intricate structures to acquire knowledge from extensive datasets, capturing hidden and intricate patterns that conventional machine learning models may overlook. The ability of deep learning to analyze sequential data and its proficiency in managing extensive datasets make it very suitable for evaluating the sequential characteristics of stock prices.

2.11.1 Application of CNN in time series forecasting

CNNs, RNNs, LSTMSs, GRU, and Transformers play a novel role in enhancing stock prediction approaches. For example, Gunduz et al. (2017) introduced a CNN-based stock market prediction using diverse variables. The research explores CNN architectures, including convolutional, pooling, and fully connected layers, and introduces two models, 2D-CNNpred and 3D-CNNpred, for improved prediction accuracy. The study compares these models against baseline algorithms and discusses their practical applicability in real-world trading scenarios. Nevertheless, the proposed CNN performance was surpassed by the one developed by Hoseinzade and Haratizadeh (2019). Hoseinzade and Haratizadeh (2019) present a CNN structure to forecast the intraday movement of Borsa Istanbul 100 equities. The distinguishing characteristic of this CNN model is that it utilizes a meticulously arranged feature set obtained by using diverse indicators, pricing data, and temporal information. The study emphasizes the significance of arranging features according to their correlations before feeding them into the CNN. This approach is contrasted with a CNN that employs randomly ordered features and logistic regression. The results indicate that by using ordered features, the suggested classifier surpasses the randomly ordered CNN and logistic regression. This emphasizes the effectiveness of feature selection in improving prediction accuracy, as well as lowering model complexity and training time.

2.11.2 Application of RNN in time series forecasting

RNNs, renowned for their effectiveness in managing time-series data using past information, have also found extensive use in stock prediction. For instance, Jiang et al. (2018) introduce a new deep learning technique called Cross-domain Deep Learning Approach (Cd-DLA) to address the challenge of predicting financial markets. This approach aims to tackle the growing globalization and cross-border financial flows by examining three types of correlations: intra-domain correlations between similar markets in different countries, inter-domain correlations between different types of markets, and temporal correlations between current and past market data. The model adopts RNN and integrates an attention mechanism to examine correlations inside and across different domains. The research showcases the robustness of Cd-DLA by conducting experiments on a 10-year dataset involving currency and stock markets from three nations. The results highlight its superior performance in comparison to alternative baseline methodologies. In Kaczmarek et al. (2022), the authors find safe haven assets that effectively boost portfolio performance in out-of-sample scenarios. The authors adopt RNN volatility forecasts to determine market conditions and develop an investment approach that adaptively blends stocks, cash, and low-risk assets. The work uses the S&P 500 index in conjunction with 13 prospective safe haven assets. Based on a rigorous 20-year study, it is determined that out of all the assets examined, only long-term Treasury bonds serve as a genuine safe haven and improve the effectiveness of investment strategies, whereas other assets do not exhibit similar potential. Unfortunately, RNNs have a notable drawback: their vulnerability to the vanishing and ballooning gradient problem, especially when dealing with long sequence dependencies (Noh, 2021).

2.11.3 Application of LSTM in time series forecasting

The emergence of LSTMs directly responded to RNN limitations, as they incorporated gate logic units into RNNs to tackle these problems effectively. LSTMs have been prominent in stock forecasting. For instance, Rokhsatyazdi et al. (2020) introduced a neural network model using LSTM and differential evolution (DE) to predict future stock prices. The model optimizes ten hyperparameters, including time window size, batch size, LSTM units, and dropout coefficient. The optimized model outperforms state-of-the-art models and reduces training duration, resulting in a shallower, quicker, and more precise network. Moghar and Hamiche (2020) explore the difficulties associated with investing in financial markets, which arise from their intrinsic intricacy and the limited ability of fundamental models to forecast future asset values reliably. Addressing this, the researchers apply RNN and LSTM to forecast future stock market values. Similarly, Kijewski and Ślepaczuk (2020) applied algorithmic investment strategies utilizing classical techniques and LSTM to forecast buy/sell signals for the S&P500 index. The research assesses the performance of these algorithms by using a dataset that covers 20 years. The sensitivity study revealed that classical approaches exhibited more adaptability to parameter changes than the LSTM model, which relied on heuristically determined hyperparameters. More recently, LSTM has been successively applied in sentiment analysis of the impacts of social media and news in making accurate stock predictions, which is evident in the works of Guo (2020), Jin et al. (2020), and Swathi et al. (2022). Notwithstanding these progressions, LSTM models still encounter the problem of information decay during forward propagation, impeding their capacity to comprehend the iterative shift patterns and trends. In this realization, an increasing number of scholars now explore the Transformer model to address challenges in predicting financial time series.

2.11.4 Application of GRU in time series forecasting

In 2014, Cho et al. (2014) presented a novel technique, which is a gating technique for RNN. Like LSTM with a gating system, GRU has often outperformed LSTM or exhibited identical performances in making accurate stock predictions. For example, Gao et al. (2021) present a novel approach to enhance stock prediction by incorporating a diverse range of technical and investor mood indicators and financial data. The method applies deep learning algorithms, notably LASSO and PCA, to reduce the dimensions of these components. The study compares the LSTM and GRU models across different parameters, demonstrating that both models exhibit identical robustness in predicting stock prices. In another research, Gupta et al. (2022) present the StockNet model, which leverages GRU and consists of two modules: an Injection module to mitigate overfitting and an Investigation module for predicting stock index movements. This strategy tackles the prevalent problem of overfitting in deep learning models. The effectiveness of the StockNet model was validated using data from CNX-Nifty. The results imply that the StockNet model substantially reduces test loss. Specifically, it reduces the RMSE, MAE, and MAPE by 65.59%, 27.30%, and 14.89%, respectively, compared to the TargetNet model. Although GRU has demonstrated impressive results in applications like sequence modelling and natural NLP, it has some limitations. While GRU is specifically designed to address the issue of vanishing gradients, it may still face difficulties in capturing dependencies that extend over a substantial number of time steps. This limitation can impact the model's robustness when handling unusually lengthy sequences over prolonged durations.

2.11.5 Application of Transformer in time series forecasting

The Transformer model, initially designed for NLP, has shown remarkable potential in the field of financial forecasting in recent years. The introduction of the Transformer model by Vaswani et al. (2017), featuring its self-attention mechanism, has significantly transformed time series prediction. Following the publication of its novel discovery, Transformer-based designs have emerged as the leading approach in various domains, including image classification (Zamir et al., 2022), audio processing (Lin et al., 2022), language translation (Di Gangi et al., 2019), machinery diagnosis (S. Zhu et al., 2023), computer vision (Han et al., 2022), asset allocation (Ma et al., 2023), retrosynthesis (Karpov et al., 2019), water prediction (Xu et al., 2023) and so on. Transformers, which can simultaneously process information from all nodes, arose as a way to address the issue of information loss that is inherent in iterative training models.

The application of Transformers in stock market prediction started with the work of Liu et al. (2019). Liu et al. (2019) present the CapTE model, a novel method for forecasting market trends by leveraging social media data. The CapTE model applies a Transformer Encoder to extract profound semantic characteristics from social media information and employs a capsule network to capture the structural connections within the text. The results prove that the CapTE model dramatically increases the accuracy of stock movement forecasts compared to existing models. Due to the success of Liu et al. (2019) in predicting stock movement, Ding et al. (2020) advanced this path by offering a novel strategy using the Transformer model to address the difficulty of predicting stock price changes. Their results emphasize that this approach is exceptionally skilled at extracting long-term relationships in financial time series. The experimental findings indicate that the suggested models surpass many other competing techniques, especially in stock price prediction using the NASDAQ and the China A-shares market as case studies. Since the publication of Liu et al. (2019), researchers have dedicated their efforts to exploring novel applications of transformers in transforming the prediction of stock markets. Specifically, they have focused on harnessing sentiments derived from diverse social media platforms, including news headlines, Twitter, Facebook, and more (Devlin et al., 2018; Köksal & Özgür, 2021; Sonkiya et al., 2021; Zhang et al., 2022). However, these methods do not increase the capacity to extract features from the historical series. Instead, they utilize the Transformer model to analyze social data and acquire sentiment information. However, acknowledging social information from several sources is challenging, as it is usually unstructured, energy-consuming, and prone to significant uncertainty and bias. As a result, the methodologies used to analyze this information may exhibit inconsistent performance across various stock markets.

Unlike previous studies on the application of Transformers to stock prediction using unstructured data such as social media information, C. Wang et al. (2022) conducted a back-testing experiment on major global stock market indices using the structured datasets of China Securities Index 300 (CSI 300), S&P 500, HSI, and Nikkei 225. The results from these experiments indicate that the Transformer model significantly outperforms traditional methods in prediction accuracy, highlighting its potential to generate excess earnings for investors. However, it is noteworthy that the analysis only encompasses datasets from two continents. Chapter 6 of the present work expands on this by including datasets across three continents, offering a more comprehensive global perspective. Additionally, the data collected spans a longer timeframe, covering the post-COVID era and its impact on stock markets, which is particularly relevant for stocks like HSI. This broader and more current dataset provides a more detailed view of market dynamics and enhances the relevance and applicability of research findings in today's rapidly evolving financial landscape.

Recently, Tao et al. (2024) conducted a similar study to C. Wang et al. (2022) using structured data. Tao et al. (2024) introduced an SDTP with period correlation, an innovative method to improve the accuracy of stock price predictions. The SDTP model addresses the shortcomings of deep learning models using RNNs and LSTMs, which have faced challenges in dealing with the unpredictable nature of stock prices and the loss of relevant information from past data. The SDTP model stands out by incorporating a period-correlation mechanism and series decomposition layers, enabling a more comprehensive understanding of historical data relationships and the detection of evolving trends in the stock market using structured datasets, as opposed to depending on unreliable social media data. However, the drawback for this study lies in its failure to execute a trading strategy to showcase the model's effectiveness in generating superior financial returns compared to other benchmarking models.

The present work aims to fill these gaps by employing the Transformer model to forecast structured data from three global stock indices across three continents (North America, Europe, and Asia): S&P 500, FTSE 100, and HSI. In light of the necessity to use practical models in financial markets, Chapter 6 of this thesis takes another step by executing a straightforward trading strategy. This approach evaluates the economic significance of developed models and their practicality in trading and investment situations. To assess the robustness of the proposed models in Chapter 6, a comparison of the trading performance of the Transformer model with other competing models, such as RNN, LSTM, and GRU, was carried out. This complete methodology guarantees not only the improvement of predictive precision in stock market prediction but also offers a clearer understanding of the performance of these models in real-life trading settings, providing valuable insights for investors and market analysts.

2.12 Hyperparameter optimization

Hyperparameters play a vital role in determining the behaviour of machine learning algorithms and substantially influence the performance of the generated models. Based on the Bayesian theorem, Bayesian optimization is a notable technique for performing hyperparameter tuning. This approach converts the task of tuning hyperparameters into an optimization problem by utilizing Gaussian processes to establish a connection between the performance of machine learning models and their corresponding hyperparameters. Bayesian optimization is a highly efficient approach for identifying optimal hyperparameters using smaller samples, unlike other methods, such as random search, grid search, and stopping epochs, that need an explicit statement of the function (Rawi et al., 2023). Wu et al. (2019) employ this methodology across multiple machine learning models, encompassing RF methods, deep neural networks, and deep forests. Using Bayesian optimization across these varied models showcases its versatility and capacity to improve machine learning efficiency significantly. TPE is a widely used Sequential Model-Based Optimization (SMBO) commonly applied in optimizing machine learning algorithms (Lima et al., 2021). Its algorithm aims to find the optimal set of hyperparameters in any developed machine learning model. Methods such as TPE have become increasingly popular due to their effectiveness in exploring the hyperparameter space. This approach fills a gap in prior research by applying a novel perspective on TPE hyperparameter optimization on the Transformer model for stock market forecasting.

Integrating TPE into the optimization process not only corresponds to the ongoing development in the area but also provides a valuable assessment of the effectiveness of various strategies. This progress enhances the field of hyperparameter optimization, demonstrating the continuous improvement and fine-tuning of techniques in this crucial domain of machine learning. When optimizing a Transformer model using TPE, you can define a search space that includes the hyperparameters of the model, such as the number of layers, hidden dimensions, attention heads, learning rate, dropout rate, and batch size. TPE will then explore this search space by evaluating different combinations of hyperparameters and progressively refining its search based on the observed results. By optimizing parameters, models can improve generalization and performance.

2.13 Research gaps and challenges

This section concisely overviews the research gaps identified in the state-of-the-art review on DEA, IDEA, and deep learning approaches in stock analysis. These approaches are significant techniques used in stock portfolio management, which consists of three fundamental stages: stock selection, portfolio optimization, and price forecasting. The review conducted has found the following research gaps:

1. The current body of literature lacks robust research that integrates SET, DEA, and IDEA methodologies to evaluate and rank stocks. Furthermore, there is a research gap in previous studies using the methodology to analyze the tourist industry. This sector is crucial since it is recognized as one of the most vital and rapidly evolving industries globally.
2. Also, previous studies have limited applications of DEA for negative input and output variables. Also, there is no research on the inverse optimization of DEA for equity risk evaluation.
3. Another gap in the literature is the absence of research on portfolio optimization using IDEA. None of the existing studies in the literature has conducted portfolio optimization using a reverse process of DEA to estimate possible volatility reductions.
4. None of the previous studies in the literature has taken up an initiative to pursue the goal of the net zero portfolio risk initiative.
5. Although Transformer models have been applied innovatively in different sectors, their use in predicting the stock market is still limited. Most existing research employing these financial market models concentrates on unstructured data sources such as social media content. Moreover, there is a significant lack of research on using TPE for hyperparameter tuning in combination with Transformer models to improve stock market predictions.

To address the stated research gaps, this thesis offers robust models and conducts extensive experiments in the succeeding chapters of this work. The purpose of these proposed models and their empirical evaluations is to help in stock portfolio management. The succeeding chapters address the existing literature gaps and contribute value to advancing stock portfolio management.

Chapter 3 – An Integrated Approach to Stock Selection and Ranking: Combining SET, DEA, and IDEA

3.1 Introduction

The appraisal and ranking of stocks play a crucial role for investors, managers, and policymakers seeking to make well-informed and logical decisions within the stock market. Nevertheless, evaluating and ranking stocks is not simple, as it encompasses various criteria, views, and uncertainties. The emergence of the COVID-19 epidemic posed unprecedented obstacles to the worldwide financial environment (Verma, 2023). For instance, between January and March 2020, the weighted index of TWSE had a devastating decline of 29% (Lee & Lu, 2021). One of the areas that experienced significant repercussions was the tourist business (Sánchez-Sánchez & Sánchez-Sánchez, 2023), which is inherently interconnected with the unrestricted mobility of individuals across national boundaries.

The tourism and hospitality industry plays a significant role in stimulating economic growth on a global scale. According to estimations from the United Nations World Tourism Organisation (UNWTO), its impact is comparable to global oil, food, or vehicle trade (Nurmatov et al., 2021). Before the onset of the COVID-19 pandemic, the industry consistently exceeded the global Gross Domestic Product (GDP) growth rate over 9 years. During this time, it significantly contributed USD 8.9 trillion to the world's GDP, accounting for 10.3% of the total GDP (Tewari & Arya, 2023). The remarkable expansion mentioned above has been driven mainly by emerging economies in Asia rather than the Western world. The trend above is projected to persist until 2030, with UNWTO anticipating a higher growth rate in travel to emerging nations than developed economies. Taiwan, well-known for its flourishing tourism sector (Dai & Fang, 2023), encountered a number of challenges. The TWSE tourism sector faced significant hurdles in the financial year 2021 due to the country's decision to restrict international tourists from entering its borders in response to the ongoing epidemic. The said period has examined the resilience of tourism stocks and offered a distinctive perspective for comprehending the flexibility of stocks in the face of unforeseen and highly disruptive events. Due to the intricate nature of these unexpected events, conventional approaches to assessing stock efficiency, which mostly rely on qualitative methodologies or expert judgments, may prove inadequate. Although these methodologies have previously yielded significant insights in relatively stable circumstances, they may possess inherent biases, particularly in times of disruption.

DEA is a non-parametric method that enables the estimation of the comparative efficiency of DMUs by considering multiple inputs and outputs (Pimentel & Mora-Monge, 2023). The DEA methodology identifies efficient DMUs based on their superior performance while categorizing the remaining DMUs as inefficient. Nevertheless, DEA technique does possess certain drawbacks. These include challenges in selecting suitable inputs and outputs to avoid the curse of dimensionality and, more so, the absence of a definitive ranking system for efficient DMUs. The extensive range of inputs and outputs utilized to evaluate efficiency exacerbates this challenge, bringing the possible risk of the "curse of dimensionality" (Ünsal et al., 2022). This phenomenon entails evaluations becoming excessively complex and perhaps less dependable. Given the aforementioned issues, it becomes apparent that there is a need for a new and unbiased method for assessing the value of stocks. This technique should effectively explain the complexities of a sector experiencing significant pressure and offer complete insights free from subjective biases.

The central proposition of this study posits that the proposed methodology holds the potential to provide an alternative and novel assessment of ranking stocks compared to prevailing techniques that rely on qualitative methodologies or expert judgments in their ranking models. Inherent biases and subjectivity constrain these conventional approaches. SET is a quantitative metric for assessing the amount of information contained within a given dataset (Feutrill & Roughan, 2021). It serves as a valuable tool for identifying and selecting the most pertinent and representative inputs and outputs from a vast array of variables. One approach to assessing the efficiency of the tourism sector is using DEA methodology (Chaabouni, 2019). This method allows for ranking tourism stocks based on their efficiency levels. The SET is chosen as a dimensionality reduction technique due to its method of weighted relativity in variable selection. This approach contrasts with other strategies that rely on subjective expert judgments, which may introduce bias (Peykani et al., 2022). The SET measure assigns weights to individual variables depending on their information richness, quantifying their contribution to the overall data variance. By identifying the variables with the most significant weights from each perspective, one may effectively gather the most significant elements of each dimension while minimizing the loss of information or the introduction of extraneous factors. The utilization of SET can potentially decrease the dimensionality and redundancy of data while simultaneously enhancing the discrimination strength and robustness of DEA. The IDEA technique is utilized to conduct inverse optimization by maintaining a constant efficiency score for DMUs. This involves adjusting the input or output variables by a specific percentage to observe changes in the corresponding output or input variables. The utilization of IDEA enables the establishment of a distinct hierarchy for efficient DMUs by evaluating their potential for expansion or reduction.

The existing studies have a research gap where these techniques were integrated into ranking stocks. This study aims to present a novel methodology by integrating SET, DEA and IDEA in the assessment and ranking of stocks using the tourist sector of the TWSE as a case study. This approach incorporates SET in making dimensionality reduction, traditional DEA in estimating efficiency scores of selected stocks, and IDEA technique in checking out the input variation due to constant output variation.

3.2 Rationale for choosing the tourism sector of TWSE

The rationale behind selecting the tourism sector of TWSE as a case study stems from the country's status as the 20th most visited country in Asia in 2022, as published by Yahoo Finance (Yahoo_Finance, 2023), with considerable prospects for expansion and growth. It is noteworthy that TWSE possesses a resilient and transparent securities market, and is recognized as one of the major and dynamic stock exchanges in the Asian region. According to Statista's report in 2023, TWSE holds the position of the eighth largest equities market in Asia and the seventeenth largest equity market operator globally, as measured by market capitalization. (Statista, 2023a). Furthermore, Taiwan has garnered numerous accolades for its commendable tourist quality and commitment to sustainability. Nevertheless, Taiwan has had significant repercussions from the COVID-19 pandemic, resulting in a substantial decrease in both international tourists and financial gains. The COVID-19 pandemic has significantly influenced tourism, affecting economies, livelihoods, public services, and opportunities across all continents (Soliku et al., 2021).

Based on data provided by UNWTO, there was a significant decline of over 74% in international tourist arrivals worldwide during 2020. This decline had substantial economic implications, leading to an estimated loss of 1.3 trillion USD in export revenues from the tourism sector (UNWTO, 2021). The global pandemic has significantly threatened employment within the tourist sector, potentially jeopardizing over 100 million jobs (UNWTO, 2021). This impact is most pronounced among small enterprises operating within the industry, which tend to have a substantial representation of women and young individuals in their workforce. The durability and adaptability of Taiwan's tourism business and its capacity to navigate unparalleled obstacles and uncertainties have been put to the test by the COVID-19 pandemic. The decision to concentrate data analysis on the financial year 2021 stems from the fact that during this period, the industry experienced the detrimental effects of the COVID-19 pandemic, leading to the closure of the country's borders to foreign tourists for almost two years. The current period presents a crucial examination of the performance and effectiveness of tourism stocks, given the substantial disruptions and losses they have encountered due to the epidemic. Through a comprehensive analysis of this time frame, it becomes possible to evaluate these companies' resilience and recuperative abilities in the face of the crisis. The selection of this specific industry and time frame has significant value for this research as it enables one to assess and prioritize tourist stocks amidst unprecedented circumstances, potentially highlighting their inherent strengths and shortcomings more prominently than in typical situations.

Sensitivity analysis is a fundamental method employed to ascertain the strengths and weaknesses of a model (Aghakarimi et al., 2023). The inclusion of a "what-if" analysis is crucial in light of the intrinsic uncertainty and variability included in empirical data and parameters. Sensitivity analysis contributes to the validation, reliability, confidence level, and internal consistency of decision-making processes (Tao et al., 2023). This is achieved by emphasizing the impact of variations in parameters and the resulting outcomes, hence ensuring the resilience and adaptability of models under various circumstances. In the context of the present study, which assesses stock efficiency through the utilization of SET, DEA, and IDEA, sensitivity analysis holds particular relevance. Financial ratios and stock measurements can be influenced by a range of factors, leading to potential swings. Through the implementation of sensitivity analysis, one may ascertain the reliability and robustness of the findings, validate the novel approach that integrates SET, DEA, and IDEA, and provide valuable strategic implications by examining the model's performance under different scenarios.

3.3 Methodology

3.3.1 Data sources and collection

The primary data utilized in this study comprises the financial ratios of 16 stocks belonging to the tourist sector of the TWSE, specifically for the fiscal year 2021. The data utilized in this study was sourced from the Taiwan Economic Journal (TEJ) database, a reputable and comprehensive source of financial information on publicly traded firms in Taiwan. Sixteen companies from the tourist sector were chosen for analysis, taking into consideration their market capitalization and the availability of relevant data. The tourism industry encompasses a range of establishments and enterprises, such as hotels, restaurants, travel agents, airlines, and other interconnected entities. The study incorporates a collection of 13 financial ratios that assess different aspects of a firm's economic performance, including liquidity, asset usage, leverage, profitability, and valuation. These ratios are frequently employed in financial analysis and stock appraisal (Çakır, 2017; Edirisinghe & Zhang, 2008; Peykani et al., 2022). The definitions and formulas pertaining to these ratios are shown in Table 3.1.

Table 3.1: Description of the financial variables

S/N	Ratio	Perspective	Formula
1	Current ratio (CR)	Liquidity ratio	Current assets / Current liabilities
2	Quick ratio (QR)	Liquidity ratio	(Current assets - Inventories) / Current liabilities
3	Cash ratio (CAR)	Liquidity ratio	Cash and cash equivalents / Current liabilities
4	Asset turnover (AT)	Asset utilization	Revenue / Total assets
5	Receivables turnover (RT)	Asset utilization	Revenue / Average accounts receivable
6	Solvency ratio I (SOL I)	Leverage ratio	Total debt / Total assets
7	Solvency ratio II (SOL II)	Leverage ratio	Total debt / Total equity
8	Return on equity (ROE)	Profitability ratio	Net Income / Average Equity
9	Net profit margin (NPM)	Profitability ratio	Net Income / Revenue
10	Earnings per share (EPS)	Profitability ratio	Net income / Weighted average number of common shares outstanding
11	Price to sales ratio (PSR)	Valuation ratio	Stock price / Revenue per share
12	Price to book ratio (PBR)	Valuation ratio	Stock price / Book value per share
13	Price to earnings ratio (PER)	Valuation ratio	Stock price / Earnings per share

3.3.2 Dimensionality reduction using SET

SET quantifies the amount of information contained within a given set of variables. It serves as a valuable tool for identifying and selecting the most pertinent and representative inputs and outputs from a vast array of options. The utilization of SET can potentially decrease the dimensionality and redundancy of the variables while simultaneously enhancing the discrimination power and resilience of DEA. The process of employing SET for input and output variable selection involves the following procedures:

Step 1: First, Eq. (3.1) is set up to create a decision matrix with P alternatives and Q criteria, where p^{th} alternative and q^{th} criterion denotes the value of individual n_{pq}.

$$N = \left[n_{pq} \right]_{PXQ} = \begin{pmatrix} n_{11} & \cdots & n_{1Q} \\ \vdots & \ddots & \vdots \\ n_{P1} & \cdots & n_{PQ} \end{pmatrix} \tag{3.1}$$

Step 2: The second step involves applying the min-max normalization procedure to the data in order to mitigate the influence of varying units and scales. Eq. (3.2) is used to standardize the constructed matrix. The normalized estimate, V_{pq}, is calculated by expressing individual n_{pq} on each column as a ratio of its sum.

$$V_{pq} = \frac{n_{pq}}{\left(\sum_{p=1}^{P} n_{pq} \right)}, \forall p, q \tag{3.2}$$

Step 3: In the third step of the process, the entropy of each variable is determined by employing Eq. (3.3). The entropy, e_q, can be calculated by solving Eq. (3.3). The parameter is kept between 0 and 1 by having a fixed value.

$$e_q = -(\ln(P))^{-1} \sum_{p}^{P} V_{pq} \ln(V_{pq}), \forall q \tag{3.3}$$

Step 4: In this step, the degree of deviation d_q using Eq. (3.4) is estimated. The magnitude of the deviation can be used to infer how much insight the relevant criteria provide into the decision.

$$d_q = 1 - e_q, \forall q \tag{3.4}$$

Step 5: In the fifth step, the variables with the highest weight from each perspective are chosen as the inputs and outputs of the efficiency model. Since a low weight indicates that all the options perform equally, the one with the highest weight is chosen. Using Eq. (3.5), division d_q by the total of d_q is carried out; the result gives the weight w_q.

$$w_q = \frac{d_q}{\sum_{q}^{Q} d_q}, \forall q \tag{3.5}$$

In the present study, 13 variables were grouped into five distinct views: liquidity, asset usage, leverage, profitability, and valuation. The initial viewpoint to consider is liquidity, which encompasses the assessment of various financial ratios, such as the current, quick, and cash ratios. The subsequent viewpoint pertains to asset usage, wherein the available variables encompass asset and receivables turnover. The third perspective pertains to the concept of leverage, which involves considering the factors of solvency ratio I and solvency ratio II ratio when making choices. The fourth aspect pertains to profitability, encompassing a range of factors such as return on equity, net profit margin, and earnings per share. Valuation constitutes the fifth perspective, wherein the available variables for consideration encompass the price-to-sales ratio, price-to-book ratio, and price-to-earnings-per-share. In choosing the appropriate indicators for DEA, the selection of variables for input or output is determined by their respective weights, which are calculated using SET.

3.3.3 Integrated DEA-SET

One of the shortcomings of SET is its limitation in considering the objective or purpose of decision-making. To handle this limitation, SET is integrated with DEA to create an objective function. The selection of SET as a method for reducing dimensionality is notably significant. DEA-SET is a hybrid model for evaluating the performance level of stocks using a combination of DEA and SET to increase the model's discerning power when used to assess the performance of the DMUs. The required steps are presented as follows:

Step 1: The first step involves the establishment of clear definitions for the inputs and outputs of the DEA method. Variables are classified based on financial perspectives. The first perspective is Liquidity, and the indicators in this category include QR, CR, and CAR. The second perspective is Asset Usage, and the indicators in this category include AT and RT. The third perspective is Leverage, and the indicators in this category include SOL I and SOL II. The fourth perspective is Profitability, and the indicators in this category include ROE, NPM, and EPS. The last perspective is Valuation, and the indicators in this category include PSR, PBR, and PER. It is worth noting that the first three perspectives are considered inputs because they relate to operational plans, while the last two perspectives relate to operational outcomes. The most weighted indicator in each perspective using SET represents all the variables in such perspective.

Step 2: In the second step, selecting a suitable DEA model is necessary by considering the orientation and assumptions related to returns to scale. For this study, an input-oriented DEA model that incorporates the assumption of constant returns to scale (CRS) is considered. The input-oriented DEA model posits that DMUs can decrease their inputs while maintaining a consistent output level. The assumption of CRS posits that DMUs function at an optimal scale and that any proportional alteration in inputs will yield a proportional alteration in outputs.

Step 3: The third step involves formulating the chosen DEA model as a linear programming problem. In this work, in a similar manner to Soleimani-Chamkhorami et al. (2020) and Orisaremi et al. (2022), the classical input-oriented DEA model with CRS assumption is adopted as a base model, which is expressed as model (M1).

$$
\begin{aligned}
& Min\ \psi_o^{CCR} \\
& s.t. \\
& \sum_{k=1}^{n} x_{jk}\lambda_k \le \psi_o^{CCR} x_{jo} \qquad j = 1,2,...,m \\
& \sum_{k=1}^{n} y_{lk}\lambda_k \ge y_{lo} \qquad l = 1,2,...,s \\
& \lambda_k \ge 0 \qquad k = 1,2,...,n
\end{aligned}
\tag{M1}
$$

ψ_o^{CCR} represents the Efficiency Score of DMU_o
x_{jk} represents the jth input of DMU_k
y_{lk} represents the lth output of DMU_k
λ_k represents the assigned weight of DMU_k
n = number of DMUs
m = number of inputs
s = number of outputs
o=*DMU under consideration for optimization*

Step 4: In the fourth step, the DEA model is solved for each DMU by employing a linear programming solver. This work uses the deaR package of R software to evaluate the efficiency score of individual DMUs. The efficiency score of each DMU varies between 0 and 1. A DMU is deemed efficient if its efficiency score is equal to one, but a DMU is regarded inefficient if its efficiency score is less than one.

Step 5: In the fifth step of the analysis, the results are interpreted by comparing the efficiency scores of various DMUs. This process involves identifying the efficient DMUs and those that exhibit inefficiencies.

3.3.4 IDEA for ranking evaluation

The IDEA technique is utilized to conduct inverse optimization by maintaining the efficiency score of DMUs at a constant level. IDEA has proven in recent years that it can handle difficult decision-making situations. This involves adjusting the input or output variables by a specific percentage to observe the resulting changes in the related output or input variables. The IDEA methodology offers a distinctive approach to rate efficient DMUs by assessing their capacity for progress or decline. If a DMU output is perturbed, the classical IDEA problem estimates the corresponding input change while the efficiency score is constant. The IDEA model seeks to determine the slight change needed to reach a new level $x_o + \Delta x_o = \varphi_o$ that maintains DMU's efficiency score after the output is increased to $y_o + \Delta y_o = \beta_o$. The following MOLP model (M2) proposed by Soleimani-Chamkhorami et al. (2020) can be used to achieve this goal:

$$
\begin{aligned}
& Min(\varphi_{1o}, \varphi_{2o}, \ldots, \varphi_{mo}) \\
& s.t. \\
& \sum_{k \neq o}^{n} \lambda_k x_{jk} \leq \psi^{CCR} \varphi_{jo}, \quad j = 1, 2, \ldots, m \\
& \sum_{k \neq o}^{n} \lambda_k y_{lk} \geq \beta_{lo}, \quad l = 1, 2, \ldots, s \\
& \varphi_{jo} \geq x_{jo} \\
& \lambda_k \geq 0, \quad k = 1, 2, \ldots, n
\end{aligned}
\tag{M2}
$$

Soleimani-Chamkhorami et al. (2020) created the first IDEA ranking model to rank Iranian banks. The IDEA implementation phase is carried out using Lingo software. The process for implementing the IDEA method to rank efficient DMUs consists of the following procedures:

Step 1: Normalization of inputs and outputs is required to put all variables on a standard scale. By so doing, the effect of noise and outliers are managed without compromising stability and accuracy. Eq. (3.6) is applied to normalize both the input and output variables.

$$
\begin{aligned}
\bar{x}_{jk} &= \frac{x_{jk}}{\max x_{jk}} \\
\bar{y}_{lk} &= \frac{y_{lk}}{\max y_{lk}}
\end{aligned}
\tag{3.6}
$$

Step 2: The second step involves creating sets A and B to categorize DMUs into a pool of efficient and inefficient stocks. In this study, we classify the DMUs with an efficiency score of

1 as efficient, while the remaining DMUs with an efficiency score of less than 1 are categorized as inefficient.

Step 3: Step 3 involves formulating the IDEA model as a linear programming problem, incorporating a constant percentage change denoted as C. Suppose all outputs of efficient DMUs are increased by C % (where β=C) to give Y+C%. Then, the corresponding increase in the input values is calculated using the inverse optimization model (M3). The value of C% is initially set as a very small percentage change, specifically within the range of 0-10%. This decision is grounded in financial principles, acknowledging that dramatic changes in financial values between successive years are uncommon, especially for investments considered to be "safe havens". The rationale behind this choice is to reflect realistic and practical financial scenarios, ensuring that the model remains relevant and applicable to actual financial data and investment practices. By selecting a conservative percentage change, the model aims to simulate incremental adjustments that are feasible in the real-world financial context. For mathematical proof, kindly refer to the work of Soleimani-Chamkhorami et al. (2020).

$$
\begin{aligned}
&\varphi_0 = \min \varphi \\
&S.T, \\
&\sum_{k \neq o}^{n} \bar{x}_{jk} \lambda_k \leq \bar{x}_{jo} + \varphi, \qquad \text{where } \mathrm{j} = 1, 2, \ldots, m \\
&\sum_{k \neq o}^{n} \bar{y}_{lk} \lambda_k \geq \bar{y}_{lo} + C, \qquad \text{where } \mathrm{l} = 1, 2, \ldots, s \\
&\bar{x}_{jo} + \varphi \geq \bar{x}_{jo} \\
&\lambda_k \geq 0, \qquad \text{where } k = 1, 2, \ldots, n
\end{aligned}
\tag{M3}
$$

Step 4: The ranking order of DMU_o where $\mathrm{o} \in setA$. Set A is the production possibility set that encompasses all efficient stocks under consideration, computed using model (M3), and sorted in descending order of φ. Fig. 3.1 details the summary of steps and methods used in this study.

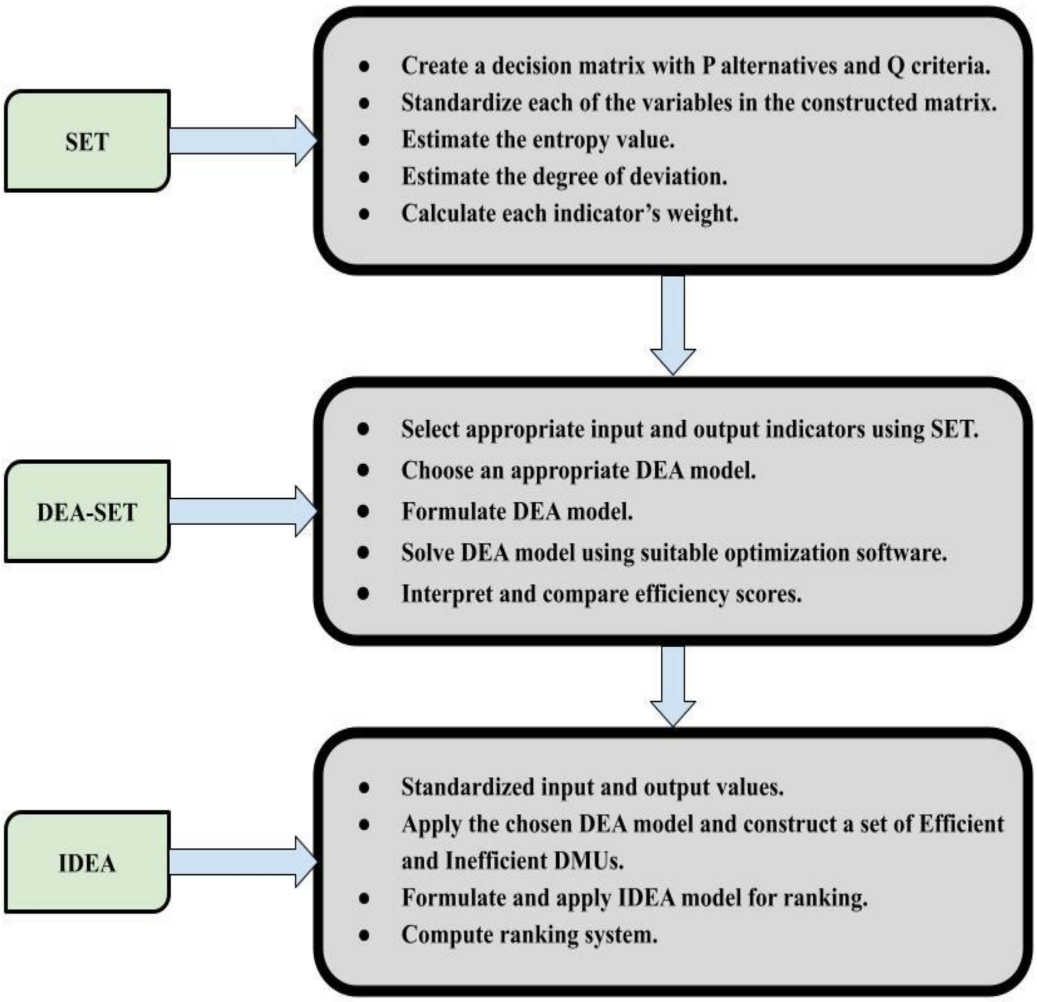

Fig. 3.1: Summary of steps and methods.

3.4 Application of SET

SET methodology is applied to the datasets, and the results are then analyzed and discussed. The entropy values and corresponding weights for each financial ratio, categorized by their financial perspective, are displayed in Table 3.2.

Table 3.2: Indicator selection using SET

S/N	Financial ratio	Perspective	Entropy (E_j)	Weight (W_j)
1	Current ratio (CR)	Liquidity	0.7681	0.4074
2	Quick ratio (QR)	Liquidity	0.7365	0.4629
3	Cash ratio (CAR)	Liquidity	0.9262	0.1297
4	Asset turnover (AT)	Asset utilization	0.8898	0.1108
5	Receivables turnover (RT)	Asset utilization	0.1163	0.8892
6	Solvency ratio I (SOL I)	Leverage	0.9712	0.3060
7	Solvency ratio II (SOL II)	Leverage	0.9347	0.6940
8	Return on equity (ROE)	Profitability	0.7772	0.3356
9	Net profit margin (NPM)	Profitability	0.7826	0.3275
10	Earnings per share (EPS)	Profitability	0.7763	0.3370
11	Price to sales ratio (PSR)	Valuation	0.7413	0.5952
12	Price to book ratio (PBR)	Valuation	0.9567	0.0995
13	Price to earnings ratio (PER)	Valuation	0.8673	0.3052

Liquidity is a crucial aspect in financial analysis, and among the several liquidity ratios, QR holds particular significance with a weight of 0.4629, rendering it the most prominent variable within this category. While CR carries a significant weight of 0.4074, it is surpassed by QR. The CAR is assigned a weight of 0.1297, suggesting that it holds relatively less importance in capturing the liquidity of the analyzed equities. Asset utilization is a crucial aspect to consider, and the variable that holds the most significance in this regard is RT, which carries a weight of 0.8892. This weight underscores the importance of RT in accurately reflecting asset utilization. AT exhibits a significantly reduced weight of 0.1108. SOL II holds greater significance in the context of leverage, as indicated by its weight of 0.694, compared to SOL I, which carries a weight of 0.306. In terms of profitability, the weight assigned to EPS is the greatest at 0.337, closely followed by ROE and NPM, with weights of 0.3356 and 0.3275, respectively. In the context of valuation, PSR holds significant importance as the primary variable, carrying a weight of 0.5952. PER and PBR carry weights of 0.3052 and 0.0995, respectively, signifying their relatively lower relevance when compared to PSR.

Using SET, the variables inside each perspective have been appropriately sorted based on their weights. The weights assigned to each variable offer an impartial assessment of their significance, promoting a data-centric approach to variable selection that is free from subjective biases. By employing a strategy of picking the variable with the highest weight from each perspective, it is possible to efficiently decrease the number of dimensions and eliminate redundancy in the dataset. This approach guarantees that the most indicative variables are picked for the study using DEA. This technique improves the efficiency of the evaluation process and strengthens the ability to differentiate and withstand the challenges of DEA. Based on the weights derived from Table 3.2, the variables chosen for the subsequent study are QR, RT, SOL II, EPS, and PSR. The liquidity metric known as the QR is utilized to assess the ability of a company to meet its short-term obligations. Asset utilization is evaluated through the RT ratio, which measures the efficiency of a company in collecting its accounts receivable. Leverage is analyzed using SOL II, which provides insight into the proportion of total liability and equity financing employed by a company. Profitability is evaluated through the EPS metric, which indicates the amount of earnings generated per outstanding share of common stock. Lastly, the valuation metric known as PSR is employed to assess the market value of a company relative to its sales revenue.

Table 3.3 displays the final financial ratios chosen for each DMU (stock) using SET, with the most significant weights assigned to them.

Table 3.3: Dataset using SET

DMU	STOCK_ID	QR	RT	SOL II	EPS	PSR
1	1259	116.2400	49.6600	152.7900	3.0700	0.4400
2	1268	111.7100	18.5300	133.5300	2.0100	1.6300
3	2701	1131.2200	13329.7800	16.2500	0.1900	22.8800
4	2706	1835.7500	319.9000	16.2000	0.1600	28.4700
5	2707	161.2200	25.9900	113.3100	17.0900	3.9000
6	2722	117.9100	43.6100	33.8400	0.1800	6.0700
7	2723	115.4600	60.1400	90.6100	6.0700	1.0200
8	2729	83.5100	14.8100	167.8500	7.0400	1.2100
9	2732	103.9600	8.5600	99.4400	3.0800	1.0100
10	2752	210.9000	10.3700	83.0400	6.6800	1.9500
11	2754	68.7900	27.6400	215.0400	0.4900	1.3700
12	2755	129.9800	53.5700	110.5800	3.6500	0.7000
13	3252	51.5100	12.3200	102.9800	0.7100	1.8900
14	5704	186.3600	32.2300	30.2900	1.5100	3.2200
15	5706	169.8700	11.2800	86.2000	1.1700	19.1100
16	9943	145.8200	70.6000	52.8800	0.4300	5.2900

Liquidity, as measured by QR: The stocks demonstrate a diverse range of QR values, whereas DMU 4 (STOCK_ID 2706) stands out with an extraordinarily high QR of 1835.75. This observation indicates a robust short-term liquidity position. On the other hand, DMU 13 (STOCK_ID 3252) exhibits the lowest QR of 51.51, suggesting possible difficulties in fulfilling immediate financial obligations. Asset utilization, specifically RT, is a metric used to assess the efficiency with which a company manages its accounts receivable. The DMU 3 (STOCK_ID 2701) exhibits a notable RT of 13329.78, indicating proficient handling of receivables. Conversely, DMU 9 (STOCK_ID 2732) has a minimum value of 8.56 for the variable RT. The concept of leverage, specifically SOL II, is a significant metric in financial analysis. DMU 11, identified explicitly by its STOCK_ID 2754, exhibits the highest SOL II of 215.04, indicating substantial financial leverage. In contrast, DMU 3 (STOCK_ID 2701) and DMU 4 (STOCK_ID 2706) exhibit significantly low SOL II values of 16.25 and 16.20, respectively, suggesting a diminished dependence on external financing. Profitability, measured explicitly by EPS, is a key metric to assess a company's financial performance. The company with STOCK_ID 2707, known as DMU 5, demonstrates a notable level of profitability, as seen by its EPS of 17.09. On the other hand, DMU 6 (STOCK_ID 2722) and DMU 4 (STOCK_ID 2706) exhibit the lowest EPS values, precisely 0.18 and 0.16, respectively. The valuation metric is represented as PSR. DMU 4, identified by its STOCK_ID 2706, exhibits the most significant PSR of 28.47, indicating a comparatively elevated valuation or market anticipation. The DMU 1 (STOCK_ID 1259) exhibits a PSR value of 0.44, which suggests a comparatively lower valuation in relation to its sales.

The variables that were chosen for analysis were picked based on their relevance as defined by SET. This selection process ensures that the variables chosen provide a concise yet complete depiction of the performance of each stock based on a financial perspective. Table 3.4 provides the statistical description of the dataset.

Table 3.4: Statistical representation of datasets

Index	**QR**	**RT**	**SOL II**	**EPS**	**PSR**
count	16.000	16.000	16.000	16.000	16.000
mean	296.263	880.562	94.052	3.346	6.260
std	482.762	3320.627	56.297	4.359	8.865
min	51.510	8.560	16.200	0.160	0.440
25%	109.773	14.188	48.120	0.475	1.163
50%	123.945	29.935	95.025	1.760	1.920
75%	173.993	55.213	118.365	4.255	5.485
max	1835.750	13329.780	215.040	17.090	28.470
variance	233059.583	11026570.000	3169.384	18.997	78.583

3.5 DEA-SET

The individual DMU efficiency score (ES) is displayed in Table 3.5, and it was obtained using the CCR-DEA as the base efficiency evaluation model.

Table 3.5: Base model efficiency evaluation

DMU	STOCK_ID	Stock Name	ES
1	1259	An-Shin	0.24915
2	1268	Hi-Lai Foods	0.25812
3	2701	Wan Hwa	1.00000
4	2706	First Hotel	1.00000
5	2707	Formosa Intl Hotels	1.00000
6	2722	Chateau	0.73862
7	2723	Gourmet	0.49595
8	2729	TTFB	0.79526
9	2732	La Kaffa	0.56521
10	2752	TOFU	1.00000
11	2754	Kura Sushi Asia	0.22137
12	2755	YoungQin	0.26491
13	3252	Haiwan	0.41280
14	5704	Chihpen Royal	0.57776
15	5706	PHX Tour	1.00000
16	9943	Holiday	0.44165

The DMUs with the highest level of efficiency, namely DMUs 3 (STOCK_ID 2701), 4 (STOCK_ID 2706), 5 (STOCK_ID 2707), 10 (STOCK_ID 2752), and 15 (STOCK_ID 5706), all exhibit ES of 1. This observation suggests that the equities above are positioned on the efficiency frontier, signifying their superior efficiency compared to other stocks in the sample. These stocks function as reference points for the remaining equities within the dataset. The DMUs with moderate efficiency ratings include DMU 6 (STOCK_ID 2722), DMU 7 (STOCK_ID 2723), DMU 8 (STOCK_ID 2729), DMU 9 (STOCK_ID 2732), DMU 13 (STOCK_ID 3252), DMU 14 (STOCK_ID 5704), and DMU 16 (STOCK_ID 9943). These DMUs have efficiency values that fall between the range of 0.4 to 0.8. The efficiency of these stocks can be characterized as modest, with identifiable opportunities for enhancement to attain the efficiency frontier. The DMUs with the lowest efficiency ratings, specifically DMUs 1 (STOCK_ID 1259), 2 (STOCK_ID 1268), 11 (STOCK_ID 2754), and 12 (STOCK_ID 2755), exhibit efficiency levels below 0.3. The stocks in the sample exhibit a lower level of efficiency and possess considerable potential for enhancing their operational efficiency.

Table 3.5 presents a comprehensive overview of the comparative efficiency of the selected tourism stocks within TWSE during the financial year 2021. The efficiency scores obtained through the DEA analysis provide crucial insights into the operational performance of each stock compared to its counterparts. Stocks that possess an efficiency score of 1 are operating at their maximum potential, taking into account the inputs and outputs involved. These equities have the potential to function as benchmarks for other firms within the same sector, offering valuable insights into optimal strategies and streamlined operations. Conversely, equities exhibiting efficiency scores below 1 possess potential opportunities for enhancement. Through the examination of the disparities between these companies and the benchmark stocks, stakeholders can discern potential opportunities for operational improvements and strategic modifications. The efficiency scores serve as a basis for conducting additional analysis, such as IDEA, which allows for ranking efficient DMUs and provides more detailed insights into their performance. Fig. 3.2-3.5 display the graphical results of DMU efficiency analysis using the deaR package of R software. Fig. 3.2 and 3.3 depict an efficiency graph and a network graph of efficiency evaluation. In Fig. 3.3, the green circles symbolize the efficient DMUs, while the red circles represent the inefficient ones. Fig. 3.3 was produced with R software, using all the input and output data of individual DMUs in Table 3.3 to create network graphs illustrating the relationships between efficient and inefficient DMUs. The arcs in the graph represent benchmarking relationships where inefficient DMUs are compared against those on the efficiency frontier (efficient DMUs). The decision to draw an arc between a red node (inefficient DMU) and a green one (efficient DMU) is based on the benchmarking analysis where inefficient DMUs are projected onto the efficient frontier, identifying which efficient DMUs they reference for potential improvement. Fig. 3.4 illustrates the frequency with which an efficient DMU is included in the reference set of inefficient DMUs, whereas Fig. 3.5 illustrates the count plot of DMU categorization based on efficiency scores.

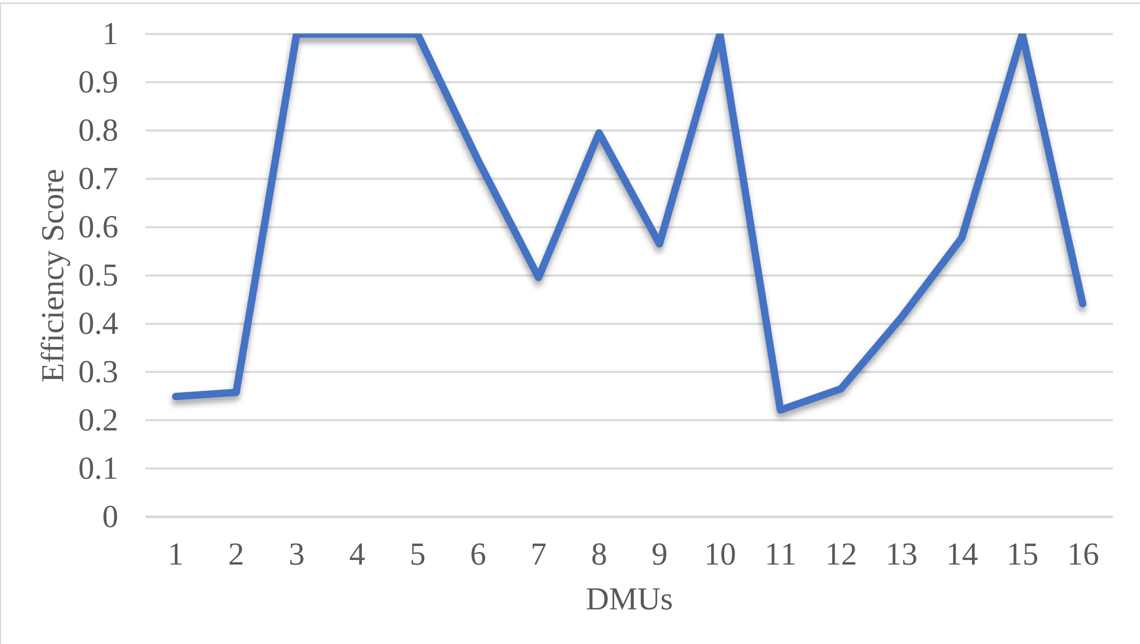

Fig. 3.2: Efficiency plot of DMUs.

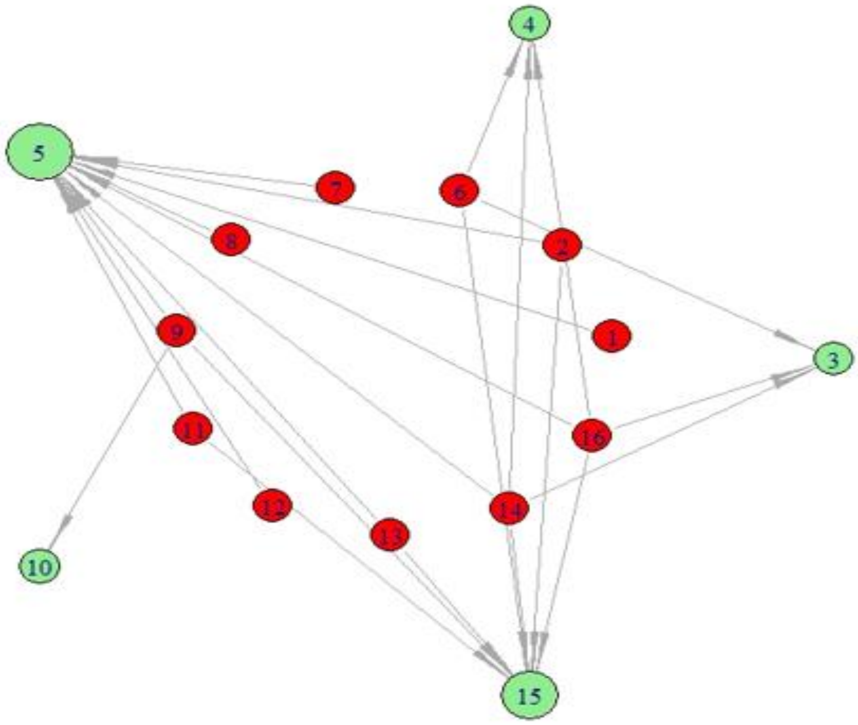

Fig. 3.3: Network graph of DMUs.

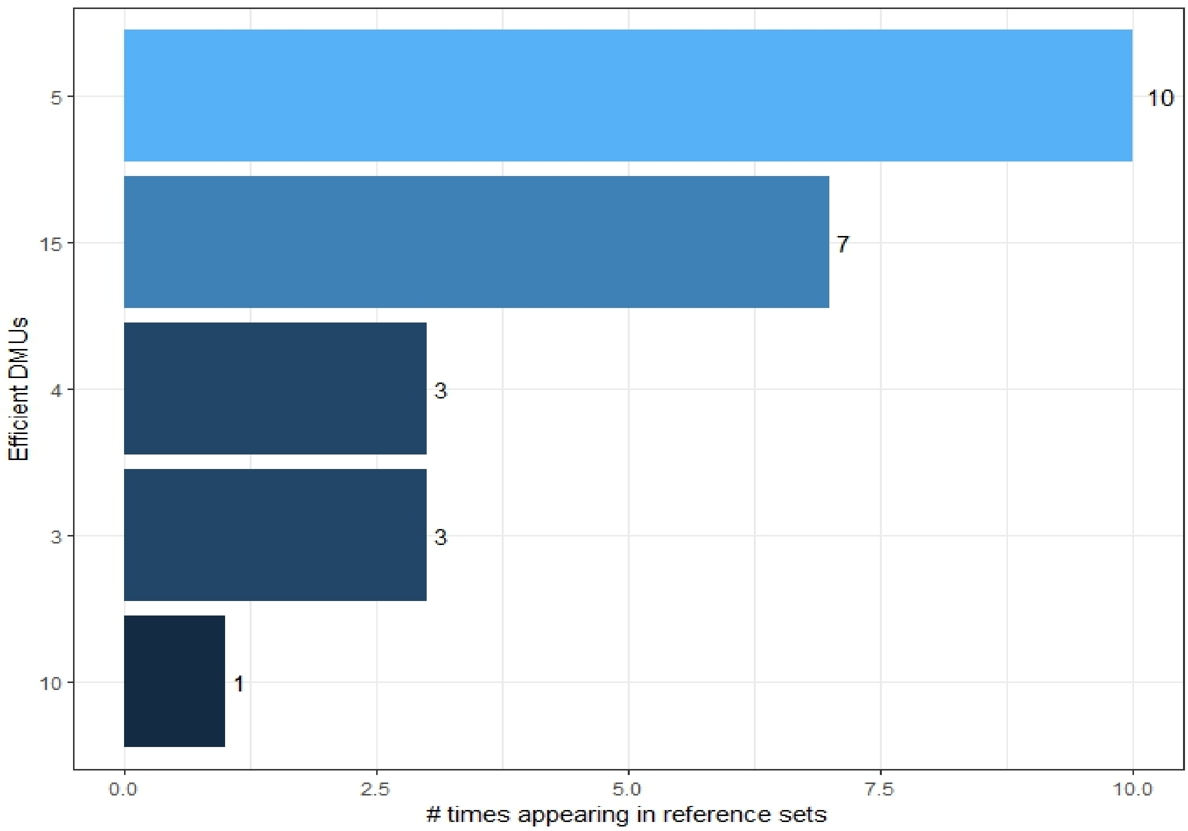

Fig. 3.4: Reference sets evaluation.

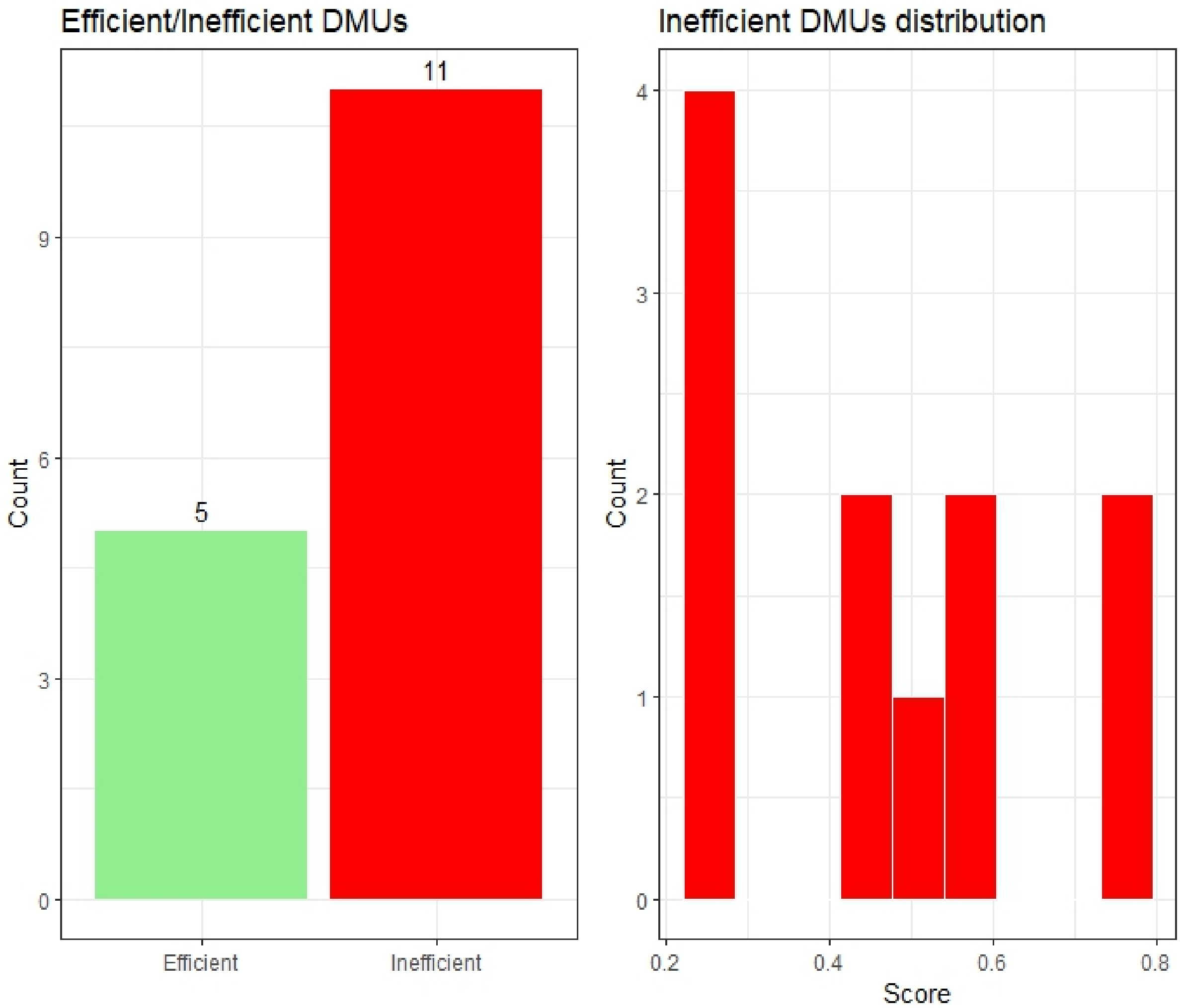

Fig. 3.5: DMUs count plot and distribution.

3.6 Application of IDEA for ranking

Table 3.6 displays the findings of the IDEA analysis, where the outputs of efficient DMUs were increased by varied percentages (C=1% to 10%), and the associated input growth rates were estimated.

Table 3.6: Ranking order of stocks using IDEA

Output Increment percentage	DMU 3 (STOCK_ID 2701)	DMU 4 (STOCK_ID 2706)	DMU 5 (STOCK_ID 2707)	DMU 10 (STOCK_ID 2752)	DMU 15 (STOCK_ID 5706)
C=1%	0.0656	0.3693	0.4710	0.0000	0.1175
Ranking	4	2	1	5	3
C=2%	0.0725	0.3735	0.4808	0.0000	0.1244
Ranking	4	2	1	5	3
C=3%	0.0797	0.3778	0.4907	0.0000	0.1313
Ranking	4	2	1	5	3
C=4%	0.0868	0.3820	0.5006	0.0000	0.1381
Ranking	4	2	1	5	3
C=5%	0.0940	0.3863	0.5107	0.0000	0.1450
Ranking	4	2	1	5	3
C=6%	0.1012	0.3905	0.5211	0.0001	0.1519
Ranking	4	2	1	5	3
C=7%	0.1084	0.3948	0.5315	0.0002	0.1587
Ranking	4	2	1	5	3
C=8%	0.1155	0.4020	0.5419	0.0002	0.1656
Ranking	4	2	1	5	3
C=9%	0.1227	0.4094	0.5524	0.0002	0.1725
Ranking	4	2	1	5	3
C=10%	0.1299	0.4167	0.5628	0.0002	0.1793
Ranking	4	2	1	5	3

Consistent Rankings: The rankings of the DMUs stay consistent throughout all percentages of output increments. DMU 5, identified by STOCK_ID 2707, routinely attains the top position in rankings, signifying the highest rate of input growth. This observation implies that when the level of outputs is augmented, DMU 5 exhibits the highest degree of input augmentation necessary to uphold its efficiency score of 1. In contrast, DMU 10 (STOCK_ID 2752) continuously occupies the lowest position in the rankings, suggesting the lowest rate of input increase. The order of ranking from highest to lowest is seen as DMU 5 > DMU 4 > DMU 15 > DMU 3 > DMU 10.

Growth Patterns: A positive correlation is observed between the percentage rise in output and the corresponding growth rate in input for all DMUs. This phenomenon is anticipated, as a larger increase in output typically necessitates a proportionately greater augmentation in inputs in order to sustain the same level of efficiency.

Variability in Input Growth: The analysis of DMU 10 (STOCK_ID 2752) reveals a lack of input growth, as seen by the negligible or non-existent increase in output for increments of up to 5%. This implies that DMU 10 has the capacity to accommodate higher levels of production without necessitating extra resources up to a specific threshold. Nevertheless, it is important to acknowledge that even when the output increments are increased to 6%, 7%, 8%, 9%, and 10%, the input growth for DMU 10 continues to exhibit little change.

Table 3.6 provides significant information on the robustness and adaptation of the effective DMUs. The IDEA analysis is a method that estimates the necessary output growth to achieve a specific rise in input growth. This approach provides valuable insights into how each DMU might react to output variations. DMU 5, identified by STOCK_ID 2707, exhibits a consistent pattern of leading input growth. The need for a proportional increase in outputs to achieve an increase in inputs renders it less flexible to changes than other DMUs. In contrast, DMU 10 (STOCK_ID 2752) exhibits notable resilience. The capacity to manage higher outputs without substantial increases in inputs implies the presence of untapped potential or resources, hence enhancing its adaptability to changes. The robustness of the IDEA method in assessing the adaptation and resilience of the DMUs is highlighted by the constant rankings seen across different output increments. In summary, it can be shown that although all DMUs listed in Table 3.6 exhibit an initial efficiency score of 1, their capacity to respond to increased output capacity differs. IDEA analysis uncovers the adaptability of the system, which offers supplementary insights to aid decision-makers in strategic planning and resource allocation.

3.7 Sensitivity analysis

Sensitivity analysis is a reliable method for evaluating the robustness and flexibility of a model (Azizi et al., 2023). The introduction of additional input indicators from 3 to 7 increases complexity and possibly redundancy, potentially affecting the efficiency scores of the DMUs. This methodology will facilitate comprehension of the extent to which variations influence the efficiency scores and rankings in the inputs of the model. Additionally, the input values were augmented by 20% while reducing the output values by 20%. The augmentation of input variables results in heightened intricacy by subjecting the model to unfavourable conditions. This experiment is subjected to a sensitivity test to determine its impact on efficiency scores and rankings. It assesses the model's ability to rank and evaluate DMUs rigorously. Table 3.7 gives the new dataset based on the above conditions, while Table 3.8 details the efficiency scores of individual DMUs using the new dataset.

Table 3.7: Dataset for sensitivity analysis

DMU	CAR	CR	QR	SOL I	SOL II	AT	RT	EPS	PSR
1	0.771	146.124	139.488	72.528	183.348	1.212	59.592	2.456	0.352
2	0.964	186.372	134.052	68.616	160.236	0.768	22.236	1.608	1.304
3	1.396	1358.904	1357.464	16.776	19.500	0.036	15995.74	0.152	18.304
4	2.584	2209.584	2202.900	16.728	19.440	0.024	383.880	0.128	22.776
5	0.190	197.052	193.464	63.744	135.972	0.540	31.188	13.672	3.120
6	0.336	148.404	141.492	30.348	40.608	0.264	52.332	0.144	4.856
7	0.974	161.028	138.552	57.048	108.732	1.104	72.168	4.856	0.816
8	0.698	115.596	100.212	75.204	201.420	0.924	17.772	5.632	0.968
9	0.715	154.644	124.752	59.832	119.328	1.152	10.272	2.464	0.808
10	1.940	263.688	253.080	54.444	99.648	1.404	12.444	5.344	1.560
11	0.641	90.000	82.548	81.912	258.048	0.852	33.168	0.392	1.096
12	1.449	172.212	155.976	63.012	132.696	1.572	64.284	2.920	0.560
13	0.358	176.940	61.812	60.876	123.576	0.132	14.784	0.568	1.512
14	1.269	238.944	223.632	27.900	36.348	0.636	38.676	1.208	2.576
15	0.291	209.520	203.844	55.560	103.440	0.072	13.536	0.936	15.288
16	0.576	183.660	174.984	41.508	63.456	0.348	84.720	0.344	4.232

Table 3.8: Efficiency evaluation using new dataset

DMU	STOCK_ID	Stock Name	ES
1	1259	An-Shin	0.24915
2	1268	Hi-Lai Foods	0.25812
3	2701	Wan Hwa	1.00000
4	2706	First Hotel	1.00000
5	2707	Formosa Intl Hotels	1.00000
6	2722	Chateau	0.73862
7	2723	Gourmet	0.49595
8	2729	TTFB	0.79526
9	2732	La Kaffa	0.56521
10	2752	TOFU	1.00000
11	2754	Kura Sushi Asia	0.22137
12	2755	YoungQin	0.26491
13	3252	Haiwan	0.41280
14	5704	Chihpen Royal	0.57776
15	5706	PHX Tour	1.00000
16	9943	Holiday	0.44165

The findings presented in Table 3.8 offer strong evidence in favour of the feature reduction methodology utilized in this investigation. The persistent retention of efficiency scores, even when faced with more unfavourable conditions, serves as a strong testament to the resilience of the DEA-SET model and the appropriateness of employing SET for dimensionality reduction. The findings indicate that the initial selection effectively captured the essential elements of the DMUs' performances; meanwhile, the new variables or value revisions did not substantially impact the efficiency scores. The consistency seen in this study enhances the credibility of the research findings and underscores the reliability of the utilized DEA-SET model. Table 3.9 displays the ranking analysis of efficient stocks using the IDEA model, where the outputs of efficient DMUs were increased by varied percentages (C = 1% to 10%), and the associated input growth rates were estimated.

Table 3.9: Ranking order of stocks using IDEA (Sensitivity data)

Output Increment percentage	**DMU 3 (STOCK_ID 2701)**	**DMU 4 (STOCK_ID 2706)**	**DMU 5 (STOCK_ID 2707)**	**DMU 10 (STOCK_ID 2752)**	**DMU 15 (STOCK_ID 5706)**
C=1%	0.115	0.4991	1.3326	0.0000	0.3175
Ranking	4	2	1	5	3
C=2%	0.123	0.5054	1.3528	0.0000	0.3238
Ranking	4	2	1	5	3
C=3%	0.1324	0.5117	1.373	0.0000	0.3301
Ranking	4	2	1	5	3
C=4%	0.1418	0.518	1.3932	0.0000	0.3364
Ranking	4	2	1	5	3
C=5%	0.1512	0.5243	1.4134	0.0000	0.3428
Ranking	4	2	1	5	3
C=6%	0.1607	0.5308	1.4336	0.0001	0.3491
Ranking	4	2	1	5	3
C=7%	0.1701	0.541	1.4538	0.0002	0.3554
Ranking	4	2	1	5	3
C=8%	0.1795	0.5512	1.474	0.0002	0.3617
Ranking	4	2	1	5	3
C=9%	0.1889	0.5614	1.4942	0.0002	0.368
Ranking	4	2	1	5	3
C=10%	0.1984	0.5716	1.5144	0.0002	0.3744
Ranking	4	2	1	5	3

Table 3.9 provides significant insights into the durability and adaptation of the efficient DMUs in light of the revised conditions where consistent rankings were maintained as with the original dataset and conditions. The observed rankings consistently hold over various increments in output, indicating that the relative performance of these DMUs stays constant, even when exposed to multiple unfavourable conditions. In summary, the findings in Table 3.9 enhance the comprehension of the ranking of efficient DMUs across different scenarios, reinforcing the reliability and robustness of the model utilized.

3.8 Summary

This study undertook a novel investigation to assess and rank efficient stocks within the tourist sector of the TWSE using an integrated approach of SET, DEA, and IDEA techniques. The analysis notably concentrated on the financial year 2021, characterized by the exceptional difficulties posed by the COVID-19 pandemic. The year was marked by the implementation of border closures and a notable decline in international tourism, rendering it a pivotal moment for evaluating the resilience of stocks within this industry. The main aim of this chapter was to develop a methodology that may address the inherent constraints of bias and subjectivity for estimating the efficiency ratings and ranking of equities. In order to accomplish this objective, the valuable attributes of SET, DEA, and IDEA were integrated. SET, a method grounded in information theory, was implemented to decrease dimensionality and choose 5 most appropriate financial ratios from 13 available financial metrics. The utilization of weighted relativity in this method renders it more favourable compared to alternative dimensionality reduction techniques, as it obviates the requirement for potentially expert views that may be biased. The outcomes derived from SET facilitated the refinement of variables, enabling the identification of the most influential indicators across many dimensions, such as liquidity, asset usage, leverage, profitability, and valuation. The decrease in dimensionality was of utmost importance to mitigate the curse of dimensionality, particularly due to the constrained number of DMUs being analyzed, consisting of only 16 stocks. The efficiency scores for each stock were obtained by subsequent analysis conducted by the integrated DEA-SET, utilizing the selected inputs and outputs. A total of 5 stocks exhibiting an efficiency score of 1 were classified as efficient, whereas 11 stocks displaying scores below 1 were classified as inefficient. In order to enhance the strength of the findings and establish a hierarchical order of efficient DMUs, the IDEA model was applied.

The IDEA methodology employed in this study entailed a progressive augmentation of the output units associated with high-performing equities while simultaneously monitoring the resultant alterations in the input units. The conducted sensitivity analysis strengthened initial findings by incorporating an expansion in the number of input variables from 3 to 7 and adjustments to their respective values by simultaneously increasing the input values by 20% and reducing the output values by 20%. Notably, despite the imposition of stricter restrictions, the efficiency scores of the five efficient stocks exhibited a constant pattern, thereby highlighting the resilience of the proposed model and the reliability of the proposed dimensionality reduction technique. This study has effectively showcased a unique methodology for evaluating the performance and ranking of stocks by integrating SET, DEA, and IDEA. The methodology effectively mitigates the limits associated with prejudice and subjectivity and demonstrates resilience in many scenarios. The results provide significant implications for investors and stakeholders in screening stocks, particularly in comprehending the capacity of equities to withstand adverse circumstances.

Chapter 4 – A Novel Inverse DEA Approach for Estimating Potential Reduction in Equity Risk

4.1 Introduction

The stock market is a significant and complex financial system since it reflects the success and worth of several firms, industries, and public sectors. The importance of stock market volatility cannot be overstated in risk management, portfolio selection, asset pricing, and other domains (He et al., 2023). Market trends, investor behaviour, economic conditions, and political events are some of the many sources of uncertainty and volatility in the stock market. Because of this, it is crucial to constantly appraise the performance of the stock market while also controlling and lowering its risk exposure. DEA is one of the most popular tools for evaluating DMUs efficiency and performance across various disciplines. To estimate the performance of a set of DMUs, DEA employs linear programming to create an efficient frontier and then compares each DMU to the frontier. In addition to pinpointing the causes and extents of inefficiency across all DMUs, DEA can set improvement goals and standards for those that fall short of the efficient frontier.

However, some limitations and challenges with DEA need to be worked out before it can be reliably applied to the stock market. As an example of its shortcomings, DEA, as a linear programming technique, makes the unrealistic and sometimes non-practical assumption that all inputs and outputs are non-negative. Negative values may be assigned to inputs or outputs due to measurement errors, imprecise data, or unintended consequences. Variables in finance often have negative values, representing declines or losses in some contexts. Returns on investments are one such instance, with negative returns indicating a decline in the value of the investment. Also, earnings per share growth may become negative under certain circumstances (Karimi & Barati, 2018). Furthermore, interest rates might be negative in some economic conditions, meaning borrowers are charged for holding money. Negative profits are common in financial statements, signaling a company's overall financial performance downturn. Assessing risk, weighing investing opportunities, and comprehending the ever-changing financial world rely heavily on these unfavourable factors. In any investment, decision-makers seek to maximize their returns while minimizing the associated risks in their investment choices (Hosseinzadeh et al., 2023). Assuming all outputs are desirable or beneficial is another limitation of standard DEA, which is not always the case. Some outputs, like risk, may be detrimental or unwelcome. Thus, mishandling these constraints may result in inaccurate or misleading results or interpretations of DMU efficiency.

To address these shortcomings, many extensions and modifications to the DEA have been proposed. According to Chung et al. (1997), the DDF-DEA is one such extension, as it permits both good and bad outputs in DEA models. The DDF-DEA defines efficiency as the maximization of the proportional increase in good outputs and the minimization of bad outputs along a given direction vector while maintaining constant inputs. An additional extension in the field of DEA is the IDEA model, as introduced by Wei et al. (2000). The IDEA is a computational approach that entails solving a mathematical optimization problem in order to ascertain the appropriate change in input or output levels that results in the desired efficiency. The IDEA methodology considers the pre-existing efficiency scores of DMUs, as well as their input-output relationships. By examining the IDEA results, analysts can acquire valuable knowledge on the most effective approaches for resource allocation, pinpoint areas that require enhancement, and make well-informed choices to optimize efficiency and productivity.

IDEA has been extensively utilized in various sectors, including energy, environment, supply chain management, banking, healthcare, and education (Emrouznejad et al., 2023). This widespread application showcases the robustness and reliability of IDEA in facilitating well-informed decision-making. Nevertheless, a noticeable gap is observed in the existing body of literature regarding the utilization of IDEA in the context of stock selection or stock portfolio analysis. Based on the available literature, no prior research has been conducted in this field, emphasizing the need for more investigation. There are multiple compelling justifications for pioneering research in the utilization of IDEA for stock market analysis. First, the concept of decision support refers to using tools and techniques to assist individuals or organizations in making informed and effective decisions. This can be particularly advantageous in situations involving MCDM, which are frequently encountered in the field of stock selection (Emamat et al., 2023); hence, IDEA can be a valuable tool for facilitating intricate investment choices by integrating crucial elements, such as return, and risk, which are hitherto considered as good and bad outputs, that is, by-products of an investment decision. This research enhances comprehension of stock dynamics and equity performance while promoting the development of more efficient and successful investing techniques. Considering the existing knowledge gap and the promising potential, it is imperative to explore the utilization of IDEA in the investigation of stock selection. In this context, a novel approach is proposed whereby stocks are evaluated as financial production technologies, with risk being regarded as a bad output and return being considered a good output. Ji et al. (2020) posit that investors incorporate safe-haven assets into their investment portfolios to mitigate potential losses during crises. In this connection, this study presents a novel model that integrates the strengths of DDF-DEA and IDEA methodologies and applies it to analyze:

(i) 28 stocks belonging to the food industry in the TWSE
(ii) 20 stocks belonging to the consumer staples in the S&P 500 index

In case study 1, a total of 28 stocks that are categorized under the food industry and are listed on TWSE were examined. The concept of regional specificity refers to the unique characteristics and attributes that are specific to a certain region or geographical area. As a prominent participant in the Asia-Pacific region, Taiwan presents a distinctive economic milieu and investor sentiment that may diverge from those observed in Western countries. The food business holds significant importance on a global scale and is commonly regarded as a defensive sector, particularly in times of economic recession. Although TWSE has a strong presence, it works within a context that is sometimes characterized as an emerging market. Case 2 considers a selection of 20 stocks from the consumer staples sector within the S&P 500 index. The S&P 500 index encompasses a collection of prominent multinational corporations holding significant global market sway. These companies operate within an established and mature economic environment. Examining consumer staples in this context can provide valuable insights into the potential outcomes of equity risk mitigation methods in a stable market setting. Like the food business, consumer staples are commonly regarded as defensive in character. These products encompass necessary goods that individuals purchase irrespective of prevailing economic circumstances.

The proposed models, which incorporate the ability to handle negative data in inputs and output variables, are extensions of works by Wegener and Amin (2019) and Orisaremi et al. (2021). Additionally, the new model involves extending an IDEA approach to enable inverse optimization on inefficient DMUs, utilizing the DDF-DEA methodology as a base model. This research adds to the existing body of knowledge in the following ways: Foremost, this study is the first to apply DDF-DEA to stock selection using modern axioms that perceive the return-risk relationship as a financial production process (Tarnaud & Leleu, 2018). Second, it stands out as one of the limited numbers of research that considers combining both technical and financial indicators in choosing the variables of its financial production process. Third, it modifies the work of Wegener and Amin (2019) and Orisaremi et al. (2021) to create a DDF-DEA that handles negative input and output data. Next is a new IDEA for determining a potential reduction in bad output (risk). Further, based on the developed model, the maximum possible reduction in bad output (risk) is estimated for all inefficient stocks.

4.2 Methodology

4.2.1 Directional Distance Function DEA (DDF-DEA)

DDF-DEA is a novel methodology for measuring productivity that considers both good and bad output, frequently overlooked in conventional productivity metrics. The DDF was developed as part of a productivity indicator. In numerous sectors, manufacturing sought-after commodities often leads to the simultaneous generation of undesirable output as by-products, such as pollution or waste. This work chooses to use the DDF version of DEA due to its dual benefit of simultaneously reducing undesired output and expanding desirable output (Shetty et al., 2012). In contrast to conventional axioms that perceive risk as an input, this study adopted Tarnaud and Leleu (2018) theoretical proof to demonstrate that within a financial production framework, risk is a secondary unwanted output that arises from the primary output: return. Wegener and Amin (2019) applied the DDF-DEA model in model M-1 to estimate the inefficiency score of $DMU_k \ \forall \ k = 1,...,n$.

$$
\begin{aligned}
&\phi_k^* = \max \phi_k \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik} \quad where \quad i = 1,....,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \ge (1+\phi) y_{rk}^{g} \quad where \quad \mathrm{r} = 1,....,s \\
&\sum_{j=1}^{n} \lambda_j y_{pj}^{b} = (1-\phi) y_{pk}^{b} \quad where \quad \mathrm{p} = 1,....,q \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \quad where \quad j = 1,....,n
\end{aligned}
\tag{M-1}
$$

When the value of $\phi_k^* = 0$, the concept of DDF-DEA considers DMU_k to be an efficient unit, characterized by an efficiency score (ψ) of 1 (Toloo et al., 2018). Hence, the connection between inefficiency and efficiency might be articulated in Eq. (4.1).

$$
Efficiency = \psi_k = \frac{(1-\phi_k^*)}{(1+\phi_k^*)} \tag{4.1}
$$

It should be noted that the value k pertains explicitly to the DMU being evaluated, but the index j serves as a general index for any particular DMU.

4.2.2 Inverse DEA (IDEA)

Orisaremi et al. (2021) extend the work of the IDEA developed by Wegener and Amin (2019) to reduce the waste of gas flaring in the petroleum industry. Wegener and Amin (2019) developed a model that increases the production rate, a good output, while decreasing gas emission, a bad output. However, Orisaremi et al. (2021) performed some modifications to generate a new model that could help keep the production rate (good output) while lowering the waste of gas flaring (bad output). In addition, the authors were able to estimate the potential reduction in bad output and the maximum potential reduction in bad output without altering the production rate. The breakdown of Wegener and Amin (2019) and Orisaremi et al. (2021) models are expressed as follows:

Let α_{ik} represents the variation in the i^{th} input resulting in a matching variation β_{rk} in the r^{th} desirable output, accompanied by a change γ_{pk} in the p^{th} undesirable output. The goal is to minimize the changes γ_{pk} in all the undesirable output. The IDEA model developed by Wegener and Amin (2019), together with their established notations and meanings for clarity, is presented as model M-2;

$$
\begin{aligned}
&\min \gamma = (\gamma_{11},\dots,\gamma_{q1},\dots,\gamma_{1t},\dots,\gamma_{qt})\\
&s.t.\\
&\sum_{j\in A}\lambda_j^k x_{ij} + \sum_{l\in B}\bar{\lambda}_l^k(\alpha_{il}+x_{il}) - (\alpha_{ik}+x_{ik}) \le 0\\
&\forall k\in S,\quad i=1,\dots,m\\
&\sum_{j\in A}\lambda_j^k y_{rj}^g + \sum_{l\in B}\bar{\lambda}_l^k(\beta_{rl}+y_{rl}^g) - (1+\bar{\phi}_k)\mathrm{x}(\beta_{rk}+y_{rk}^g) \ge 0\\
&\forall k\in S,\quad r=1,\dots,s\\
&\sum_{j\in A}\lambda_j^k y_{pj}^b + \sum_{l\in B}\bar{\lambda}_l^k(\gamma_{pl}+y_{pl}^b) - (1-\bar{\phi}_k)\mathrm{x}(\gamma_{pk}+y_{pk}^b) = 0\\
&\forall k\in S,\quad p=1,\dots,q\\
&\sum_{j\in A}\lambda_j^k + \sum_{l\in B}\bar{\lambda}_l^k = 1 \quad \forall k\in S\\
&\sum_{k\in S}\beta_{rk} = \bar{y}_r^g \quad r=1,\dots,s\\
&\alpha_{ik}\ge 0,\quad \beta_{rk}\ge 0,\quad \gamma_{pk}\ge 0 \quad \forall k\in S,\quad i=1,\dots,m\\
&\lambda_j^k \ge 0,\quad \bar{\lambda}_l^k \ge 0,\quad \forall k,l\in B,\quad \forall j\in A
\end{aligned}
\tag{M-2}
$$

The MOLP in M-2 was solved using the Weighted Sum Technique, which allows multiple objectives to be considered by assigning weights to each objective function and combining them into a single scalar objective function. The set S is a production possibility set and can be divided into sets A and B. The set A comprises all the stocks considered efficient, while the set B comprises all stocks considered inefficient. It is worth noting that the units in set A are assigned weights denoted as $\lambda_j^k \geq 0$ for each unit j in set A, whereas $\bar{\lambda}_l^k \geq 0$ for each unit l in set B. In order to maintain the specified efficiency score of each unit without degradation following the production of new outputs, it is advisable for a decision-maker to establish a value of $\bar{\phi}_k$ that is less than or equal to ϕ_k^* , that is, $\bar{\phi}_k \leq \phi_k^*$ as a component of the application. Thus, Wegener and Amin (2019) theoretical framework, model M-2, was simplified to get model M-3.

$$
\begin{aligned}
&\min \gamma = (\gamma_1 + \gamma_2 +, \ldots, +\gamma_t) \\
&s.t. \\
&\sum_{j \in A} \lambda_j^k x_{ij} \leq (\alpha_{ik} + x_{ik}) \quad \forall k \in S, \\
&\sum_{j \in A} \lambda_j^k y_{rj}^g \geq (1 + \bar{\phi}_k) \mathrm{x} (\beta_{rk} + y_{rk}^g) \quad \forall k \in S, \\
&\sum_{j \in A} \lambda_j^k y_{pj}^b = (1 - \bar{\phi}_k) \mathrm{x} (\gamma_{pk} + y_{pk}^b) \quad \forall k \in S, \\
&\sum_{j \in A} \lambda_j^k = 1 \quad \forall k \in S \\
&\sum_{k \in S} \beta_{rk} = \bar{y}_r^g \\
&\alpha_{ik} \geq 0, \quad \beta_{rk} \geq 0, \quad \gamma_{pk} \geq 0 \quad \forall k \in S, \\
&\lambda_j^k \geq 0, \quad \bar{\lambda}_l^k \geq 0, \forall k, l \in B, \quad \forall j \in A
\end{aligned}
\tag{M-3}
$$

While the application of M-3 to reduce bad output in oil and gas production, there exist some limitations. First, M-3 is unable to process negative data, specifically pertaining to the current account balances of oil nations. Second, another limitation of M-3 is that it did not reduce bad output but only minimize bad output due to an increase in both good and bad outputs. In this realization, Orisaremi et al. (2022) extend the Wegener and Amin (2019) model to handle the first limitation by transforming models M-1 and M-3 to produce M-4 and M-5.

Model M-4:

$$
\begin{aligned}
&\phi_k^* = \max \phi \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^+ \le x_{ik}^+ \quad where \quad i = 1,....,m \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^- \ge x_{ik}^- \quad where \quad i = 1,....,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^g \ge (1+\phi) y_{rk}^g \quad where \quad \mathrm{r} = 1,....,s \\
&\sum_{j=1}^{n} \lambda_j y_{pj}^b = (1-\phi) y_{pk}^b \quad where \quad \mathrm{p} = 1,....,q \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \quad where \quad j = 1,....,n
\end{aligned}
\tag{M-4}
$$

Model M-5:

$$
\begin{aligned}
&\min \gamma = (\gamma_1 + \gamma_2 +,...,+\gamma_t) \\
&s.t. \\
&\sum_{j \in F} \lambda_j^k x_{ij}^+ - (\alpha_{ik}^+ + x_{ik}^+) \le 0 \quad \forall k \in S, \\
&\sum_{j \in F} \lambda_j^k x_{ij}^- - (\alpha_{ik}^- + x_{ik}^-) \ge 0 \quad \forall k \in S, \\
&\sum_{j \in F} \lambda_j^k y_{rj}^g - (1+\overline{\theta}_k) \mathrm{x} (\beta_{rk} + y_{rk}^g) \ge 0 \quad \forall k \in S, \\
&\sum_{j \in F} \lambda_j^k y_{pj}^b - (1-\overline{\theta}_k) \mathrm{x} (\gamma_{pk} + y_{pk}^b) = 0 \quad \forall k \in S, \\
&\sum_{j \in F} \lambda_j^k = 1 \quad \forall k \in S \\
&\sum_{k \in S} \beta_{rk} = \overline{y}_r^g \\
&\alpha_{ik} \ge 0, \quad \beta_{rk} \ge 0, \quad \gamma_{pk} \ge 0 \quad \forall k \in S, \\
&\lambda_j^k \ge 0, \quad \overline{\lambda}_l^k \ge 0 \quad \forall k, l \in B, \quad \forall j \in A
\end{aligned}
\tag{M-5}
$$

Given the existing levels of inputs and outputs, as well as the current state of financial production technology, what is the potential for reducing risk in any equity? To handle this, one must ensure that all the changes in inputs and good outputs are equal to zero. This implies that, in the current financial production process, the values $\beta_{rk} = \alpha_{ik} = \alpha_{ik}^- = \alpha_{ik}^+$ must be equal to zero. The expected decrease in bad output implies that the parameter γ_{pk} must exhibit a negative variation from its present level. In the pursuit of sustainability, a new objective function emerges, which aims to maximize the reduction in bad output for each DMU. Since there is only one bad output in the case study, the index of undesirable output, denoted as $p=1$. The larger the risk reduction, the closer the equity is to efficiency. The incorporation of these novel modifications leads to model M-6.

$$
\begin{aligned}
& \max \gamma = (\gamma_1 + \gamma_2 +, \ldots, +\gamma_t) \\
& s.t. \\
& \sum_{j \in A} \lambda_j^k x_{ij}^{+} \le x_{ik}^{+} \quad \forall k \in S, \quad i = 1, \ldots, m \\
& \sum_{j \in A} \lambda_j^k x_{ij}^{-} \ge x_{ik}^{-} \quad \forall k \in S, \quad i = 1, \ldots, m \\
& \sum_{j \in A} \lambda_j^k y_{rj}^{g} \ge (1+\overline{\phi}_k) \text{ x } y_{rk}^{g} \quad \forall k \in S, \quad r = 1, \ldots, s \\
& \sum_{j \in A} \lambda_j^k y_{pj}^{b} = (1-\overline{\phi}_k) \text{ x } (y_{pk}^{b} - \gamma_{pk}) \quad \forall k \in S, \quad p = 1, \ldots, q \\
& \sum_{j \in A} \lambda_j^k = 1 \quad \forall k \in S \\
& \gamma_{pk} \le y_{pk}^{b} \\
& \overline{\theta}_k \ge 0, \quad \gamma_{pk} \ge 0 \quad \forall k \in S, \quad i = 1, \ldots, m \\
& \lambda_j^k \ge 0, \quad \forall k, l \in B, \quad \forall j \in A
\end{aligned}
\tag{M-6}
$$

The M-6 model introduces a novel restriction that imposes a maximum limit on the potential decrease in bad output (gas flare). Notably, the concept of model M-6 continues to center around a group of DMUs. In order to achieve a reduction in individual DMU, it is therefore necessary to modify the model M-6 model to accommodate a single DMU with one bad output, resulting in the IDEA model M-7.

$$
\begin{aligned}
& \max \gamma^{*} = \gamma_{pk} \\
& s.t. \\
& \sum_{j \in A} \lambda_j^k x_{ij}^{+} \le x_{ik}^{+} \quad i = 1, \ldots, m \\
& \sum_{j \in A} \lambda_j^k x_{ij}^{-} \ge x_{ik}^{-} \quad i = 1, \ldots, m \\
& \sum_{j \in A} \lambda_j^k y_{rj}^{g} \ge (1+\overline{\phi}_k) \text{ x } y_{rk}^{g} \quad r = 1, \ldots, s \\
& \sum_{j \in A} \lambda_j^k y_{pj}^{b} = (1-\overline{\phi}_k) \text{ x } (y_{pk}^{b} - \gamma_{pk}) \quad p = 1, \ldots, q \\
& \sum_{j \in A} \lambda_j^k = 1 \\
& \gamma_{pk} \le y_{pk}^{b} \\
& \gamma_{pk} \ge 0 \\
& \lambda_j^k \ge 0, \quad \forall l \in B, \quad \forall j \in A
\end{aligned}
\tag{M-7}
$$

4.3 Proposed DDF-DEA and IDEA model

Since standard DEA and traditional IDEA models are non-applicable in the presence of negative data (Amin & Al-Muharrami, 2018), it is essential to revise the base model and IDEA model to suit this study. The extended model accommodates negative data inclusion within the framework of the DDF-DEA model. As defined by the DDF-DEA framework, efficiency refers to the optimal achievable increase in good outputs and decrease in bad outputs relative to a specified direction vector while maintaining constant levels of inputs. The applicability of the models presented by Orisaremi et al. (2022) is non-applicable in a scenario where negative data is absent in input variables but present in the output variables or negative data is present in both input and output variables. In case study 1, the input variables do not contain any negative data; however, it is worth noting that certain negative data are present in the output (good) variables. Also, in case study 2, negative data exists in both input and output (good) variables. Therefore, the proposed model extends Orisaremi et al. (2022) work to create new DDF-DEA and IDEA models that could handle these limitations.

4.3.1 Case study 1: Modelling negative data in output variable

In case study 1, negative data exists in the good output (return). Considering a case of two stocks (DMU_1 *and* DMU_2) and one good output (R). The good output (R) constraint is expressed in Eq. (4.2).

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{g} \geq (1+\theta) y_{rk}^{g} \tag{4.2}$$

Let us examine a specific scenario involving two DMU_s, denoted as $j=1$ and $j=2$, this leads to Eq. (4.3).

$$\lambda_1 y_{r1}^{g} + \lambda_2 y_{r2}^{g} \geq (1+\theta) y_{rk}^{g} \tag{4.3}$$

Return is a good output; thus, it is classified as a good output into positive and negative variables. In this situation, the good output exhibits a positive value for DMU_1 and a negative value for DMU_2. Therefore, Eq. (4.3) becomes Eq. (4.4).

$$\lambda_1 y_{r1}^{g} - \lambda_2 y_{r2}^{g} \geq (1+\theta) y_{rk}^{g} \tag{4.4}$$

The inequality can be expressed as a composite of two distinct inequalities, as shown in Eq. (4.5) and Eq. (4.6).

$$\lambda_1 y_{r1}^{g} \geq (1+\theta) y_{rk}^{g} \tag{4.5}$$

$$-\lambda_2 y_{r2}^{g} \geq 0 \tag{4.6}$$

Multiplying Eq. (4.6) by (-1) gives Eq. (4.7)

$$\lambda_2 y_{r2}^{g} \leq 0 \tag{4.7}$$

By transformation, $y_{r1}^{g} = y_{r1}^{g+}$ and $y_{r2}^{g} = y_{r2}^{g-}$, Eq. (4.5) and Eq. (4.7) give Eq. (4.8) and Eq. (4.9).

$$\lambda_1 y_{r1}^{g+} \geq (1+\theta) y_{rk}^{g+} \tag{4.8}$$

$$\lambda_2 y_{r2}^{g-} \leq 0 \tag{4.9}$$

Extending this classification of LHS of Eq. (4.8) and Eq. (4.9) to the RHS to ensure uniformity. Hence, the model can be generalized to give Eq. (4.10) and Eq. (4.11).

$$\lambda_1 y_{r1}^{g+} \geq (1+\theta) y_{rk}^{g+} \tag{4.10}$$

$$\lambda_2 y_{r2}^{g-} \leq (1+\theta) y_{rk}^{g-} \tag{4.11}$$

If DMU_k where $k \in setA$ under consideration has no negative good output, then $(1+\theta) y_{rk}^{g-} = 0$. Similarly, if DMU_k has no positive good output, then $(1+\theta) y_{rk}^{g+} = 0$. In a scenario where only one good output, that is, return (R) exhibits a negative value, the utilization of M-1 and M-7 necessitates the division of the input set. The semi-oriented radial measure (SORM) was introduced by Emrouznejad et al. (2010) as a technique for transforming M-1 and M-7 into M-8 and M-9, respectively. As a result, a new base model M-8 is derived and expressed as:

$$\begin{aligned}
&\phi_k^{*} = \max \phi \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{g+} \geq (1+\phi) y_{rk}^{g+} \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{g-} \leq (1+\phi) y_{rk}^{g-} \\
&\sum_{j=1}^{n} \lambda_j y_{pj}^{b} = (1-\phi) y_{pk}^{b} \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0 \quad where \quad j = 1, \ldots, n
\end{aligned} \tag{M-8}$$

Also, as a result of the transformation, a new IDEA model M-9 is created and expressed as:

$$
\begin{aligned}
&\max \gamma^{*} = \gamma_{pk} \\
&s.t. \\
&\sum_{j \in F} \lambda_j^k x_{ij} \le x_{ik} \\
&\sum_{j \in F} \lambda_j^k y_{rj}^{g+} \ge (1+\overline{\phi}_k) \text{ x } y_{rk}^{g+} \\
&\sum_{j \in F} \lambda_j^k y_{rj}^{g-} \le (1+\overline{\phi}_k) \text{ x } y_{rk}^{g-} \\
&\sum_{j \in F} \lambda_j^k y_{pj}^{b} - (1-\overline{\phi}_k) \text{ x } (y_{pk}^{b} - \gamma_{pk}) = 0 \\
&\sum_{j \in F} \lambda_j^k = 1 \\
&\gamma_{pk} \le y_{pk}^{b} \\
&\overline{\theta}_k \ge 0, \quad \gamma_{pk} \ge 0, \quad i = 1,\ldots,m \quad r = 1,\ldots.,s \quad p = 1,\ldots.,q \\
&\lambda_j^k \ge 0, \quad \forall l \in B, \quad \forall j \in A
\end{aligned}
\tag{M-9}
$$

4.3.2 Case study 2: Modelling negative data in input and output (good) variables

In case study 2, negative data exists in the inputs and output (good) variables. Therefore, a new base model is created to handle this scenario by integrating model M-4 with model M-8 to create a new base model M-10. Similarly, for this case study, a new IDEA model, M-11, is created by integrating model M-7 and model M-9. Thus, model M-10 and model M-11 are hitherto expressed as follows;

Model M-10:

$$
\begin{aligned}
&\phi_k^{*} = \max \phi \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^{+} \le x_{ik}^{+} \quad where \quad i = 1,\ldots.,m \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^{-} \ge x_{ik}^{-} \quad where \quad i = 1,\ldots.,m \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{g+} \ge (1+\phi) y_{rk}^{g+} \quad where \quad \mathrm{r} = 1,\ldots.,s \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^{g-} \le (1+\phi) y_{rk}^{g-} \quad where \quad \mathrm{r} = 1,\ldots.,s \\
&\sum_{j=1}^{n} \lambda_j y_{pj}^{b} = (1-\phi) y_{pk}^{b} \quad where \quad \mathrm{p} = 1,\ldots.,q
\end{aligned}
\tag{M-10}
$$

$$\sum_{j=1}^{n} \lambda_j = 1$$
$$\lambda_j \geq 0 \quad where \quad j = 1, \ldots, n$$

Model M-11:

$$\begin{aligned}
&\max \gamma^* = \gamma_{pk} \\
&s.t. \\
&\sum_{j \in A} \lambda_j^k x_{ij}^+ \leq x_{ik}^+ \quad i = 1, \ldots, m \\
&\sum_{j \in A} \lambda_j^k x_{ij}^- \geq x_{ik}^- \quad i = 1, \ldots, m \\
&\sum_{j \in F} \lambda_j^k y_{rj}^{g+} \geq (1 + \bar{\phi}_k) \text{ x } y_{rk}^{g+} \quad r = 1, \ldots, s \\
&\sum_{j \in F} \lambda_j^k y_{rj}^{g-} \leq (1 + \bar{\phi}_k) \text{ x } y_{rk}^{g-} \quad r = 1, \ldots, s \\
&\sum_{j \in A} \lambda_j^k y_{pj}^b = (1 - \bar{\phi}_k) \text{ x } (y_{pk}^b - \gamma_{pk}) \quad p = 1, \ldots, q \\
&\sum_{j \in A} \lambda_j^k = 1 \\
&\gamma_{pk} \leq y_{pk}^b \\
&\gamma_{pk} \geq 0 \\
&\lambda_j^k \geq 0, \quad \forall l \in B, \quad \forall j \in A
\end{aligned} \tag{M-11}$$

M-8 and M-9 models represent the new base model and IDEA framework designed to assess the possible reduction in the bad output (σ) for case study 1, whereas M-10 and M-11 models represent the new base model and IDEA framework designed to assess the possible reduction in the bad output (σ) for case study 2. In any case study, the inefficiency score is represented as ϕ for any DMU_k. Trying to choose $\bar{\phi}_k$ such that it $\bar{\phi}_k \prec \phi_k^*$. Decisively, $\bar{\phi}_k$ is set to be less that ϕ_k^* at a value with two decimal places without any approximation. The γ^* is the minimum potential reduction in risk (σ), denoted as $\gamma_{pk}^{\min}$ for a specific financial year. This value serves as the solution to one of the study objectives. When γ^* equals zero, it signifies that the present amount of risk is at its minimal level and cannot be further decreased. A relationship can be observed between the variable γ^* and the estimated parameter $\bar{\phi}_k$, which subsequently gives rise to a proposition and its accompanying proof. The optimization software used in solving all models in this study is LINGO 20 solver.

Theorem: The highest possible level of reduction is achieved when DMU is efficient, that is zero inefficiency, specifically when the estimated value of $\overline{\phi}_k$ is equal to zero, resulting in $y_{pk}^{b} = \gamma_{pk}^{\max}$

Proof: set A represents the group of efficient DMUs that have established the efficiency frontier and do not require any additional improvement. Accordingly, the constraints $\sum_{j \in A} \lambda_j^k y_{rj}^{g+}$, $\sum_{j \in A} \lambda_j^k y_{rj}^{g-}$ and $\sum_{j \in A} \lambda_j^k y_{pj}^{b}$ apply to the efficient stocks in A while $(1+\overline{\phi}_k)$ x y_{rk}^{g+}, $(1+\overline{\phi}_k)$ x y_{rk}^{g-} and $(1-\overline{\phi}_k)$ x $(y_{pk}^{b} - \gamma_{pk})$ only applies to inefficient DMU_k. This proof is expressed in Eq. (4.12), Eq. (4.13) and Eq. (4.14)

$$(1-\overline{\phi}_k) \text{ x } (y_{pk}^{b} - \gamma_{pk}) = 0 \tag{4.12}$$

$$\text{When } \overline{\phi}_k = 0,\; y_{pk}^{b} - \gamma_{pk} = 0 \tag{4.13}$$

$$y_{pk}^{b} = \gamma_{pk} = \gamma_{pk}^{\max} \tag{4.14}$$

Eq. (4.14) serves to finalize the proof.

4.4 Data collection and sources

This study applies the developed DDF-DEA and IDEA models to datasets using two case studies. In case study 1, a comprehensive analysis is conducted on a sample of 28 stocks belonging to the food industry and listed on TWSE. As a significant actor in the Asia-Pacific region, Taiwan exhibits a unique economic environment and investor attitude that may deviate from the patterns observed in Western nations. The food industry is of great significance globally and is widely recognized as a defensive sector, particularly during periods of economic downturn. Case 2 considers a subset of 20 equities from the consumer staples sector within the S&P 500 index. The S&P 500 index comprises a selection of famous multinational firms that substantially influence the worldwide market. Analyzing consumer staples within this framework offers valuable insights into the potential results of equity risk mitigation strategies in a stable market environment. Like the food industry, consumer staples are widely recognized for their defensive nature. For this research, data for Case Study 1 were sourced from the TEJ, while data for Case Study 2 were obtained from the Wharton Research Data Services (WRDS) for the financial year 2020. This study analyzes the consumer staples sector in the S&P 500 and compares its findings with the food business in the TWSE. This comparative analysis can reveal both universal trends and market-specific nuances. In either of the case studies, the data set consists of three inputs and two outputs for each DMU.

4.5 Data description and statistical analysis

This study considers three inputs, including the current ratio, asset turnover, and solvency ratio, which are deemed significant since they provide insight into a corporation's plans and operating strategies. The current ratio (X1), the asset turnover (X2), the solvency ratio I (X3), and the solvency ratio II (X4) represent each of the perspectives mentioned, respectively. For the outputs, this study considered both return and risk to be the outputs of a financial production process. The performance and volatility of the DMUs are reflected in the outputs, that is, return (R) and risk (σ). In this work, return is classified as a good output, while risk is regarded as a bad output. These outputs are of particular interest as they pertain to the amount and strength of earnings investors derive from the stock market. Moreover, the selection of risk as an output variable is grounded in the theorem proposed and proved by Tarnaud and Leleu (2018), which emphasizes the necessity of accounting for the distinction between conventional production processes and financial production processes when employing DEA in the context of stocks or any other financial assets. The notations and definitions of variables used are illustrated below:

1. Current ratio (XI): The current ratio is a metric applied to assess a business organization's capacity to settle its current obligations, such as debts and payables, by utilizing its short-term assets, including cash, inventory, and receivables.
2. Asset turnover (X2): The asset turnover ratio is a metric employed to assess a firm's capacity to produce revenue from its assets. It accomplishes this by comparing the net sales of the company with the average total assets it possesses (Jing et al., 2023).
3. Solvency ratio I (X3): The solvency ratio is a significant financial indicator that evaluates the extent of a firm's total liabilities in relation to its total assets.
4. Solvency ratio II (X4): The solvency ratio is a significant financial indicator that evaluates the extent of a firm's total liabilities in relation to its shareholder equity.
5. Return (R): The term "return" pertains to the financial outcome, either positive or negative, resulting from an investment. In the context of financial markets, stock returns refer to the financial outcome resulting from the ownership of shares in a specific company within a designated timeframe. In this work, the annual stock return is measured using Eq. (4.15) to estimate Eq. (4.16):

$$\text{Daily Return} = \frac{(\text{Closing price at day t} - \text{Closing price at day t-1})}{\text{Closing price at day t} - 1} \quad (4.15)$$

$$\text{Annual Return=Sum of daily return over a year} \quad (4.16)$$

6. Risk (σ): Risk is quantified by calculating the standard deviation of daily returns.

In order to make the data consistent and comparable throughout the DMUs, the data was processed and cleaned to remove any missing or erroneous values. Table 4.1 presents the descriptive statistics for the dataset of Case Study 1, while Table 4.2 provides similar statistics for Case Study 2.

Table 4.1: Descriptive statistics of datasets for 28 food stocks listed on TWSE

index	X1	X2	X3	R	σ
count	28	28	28	28	28
mean	231.88786	0.86214	40.33607	0.11328	0.01764
std	152.85275	0.57060	18.19267	0.15518	0.00801
min	48.51000	0.02000	17.82000	-0.30120	0.00900
25%	102.08000	0.41000	23.27750	0.01785	0.01243
50%	184.82500	0.87000	40.01500	0.11665	0.01575
75%	319.31500	1.10000	54.27000	0.19348	0.01900
max	525.63000	2.87000	77.73000	0.38810	0.03900
variance	23363.96397	0.32559	330.97310	0.02408	0.00006

Table 4.2: Descriptive statistics of datasets for 20 consumer staples listed on S&P 500

index	X1	X2	X4	R	σ
count	20	20	20	20	20
mean	1.3760	1.0318	13.8571	0.1490	0.0247
std	0.8817	0.6936	35.8507	0.1367	0.0063
min	0.3293	0.2324	-4.1079	-0.2124	0.0194
25%	0.8134	0.6082	1.3322	0.0947	0.0201
50%	1.0287	0.8197	2.9558	0.1617	0.0232
75%	1.6269	1.2097	5.2937	0.2006	0.0253
max	3.7517	2.7768	154.4318	0.4370	0.0438
variance	0.7774	0.4811	1285.2741	0.0187	0.0000

4.6 Model application for case study 1

This section reports the outcomes and analysis derived from implementing the proposed model and methodology on the 28 food stocks traded on the TWSE. The inefficiency ratings of the DMUs derived using model M-8 model are presented in Table 4.3. Next, the results of the inverse optimization of the inefficient DMUs are provided by applying model M-9.

Table 4.3: Datasets and efficiency analysis

DMU	STOCK_ID	X1	X2	X3	R	σ	Inefficiency (∅)
1	1201	96.2300	1.0000	61.9800	-0.0777	0.0186	0.1883
2	1203	272.6000	0.6400	32.3500	0.3098	0.0116	0.0000
3	1210	111.2100	1.6500	46.0500	0.1933	0.0160	0.0116
4	1213	48.5100	0.3200	57.2900	0.3582	0.0316	0.0000
5	1215	97.4100	1.1600	55.3200	0.1189	0.0155	0.0000
6	1216	103.5200	0.9200	63.5400	-0.0717	0.0137	0.0000
7	1217	63.8800	0.3500	48.5900	0.3881	0.0213	0.0000
8	1218	234.0400	0.9000	23.9000	0.2706	0.0209	0.0808
9	1219	206.6300	1.4100	48.0300	0.1892	0.0171	0.2788
10	1220	520.8200	0.8900	17.8200	0.1937	0.0114	0.0000
11	1225	144.0200	1.3200	51.4800	0.1144	0.0181	0.2560
12	1227	235.8900	1.2900	35.2500	-0.0808	0.0153	0.3238
13	1229	82.2000	0.1900	20.6100	0.1778	0.0178	0.0000
14	1231	163.0200	1.0800	53.9200	0.1328	0.0127	0.0000
15	1232	359.6200	2.8700	26.9000	0.1813	0.0146	0.2556
16	1233	132.1100	0.7700	38.1900	-0.0800	0.0116	0.0000
17	1234	419.0300	0.4100	18.6500	0.0177	0.0090	0.0000
18	1235	282.8000	0.0200	28.0800	0.1979	0.0164	0.0000
19	1236	127.1800	0.6000	41.8400	0.0588	0.0147	0.0000
20	1256	222.2200	1.0400	24.9300	0.3223	0.0202	0.0000
21	1258	86.1300	0.9700	77.7300	0.1013	0.0375	0.5412
22	1264	305.8800	1.1600	21.4100	0.1077	0.0095	0.0000
23	1702	97.7600	0.7200	72.5500	-0.0488	0.0163	0.0910
24	1737	417.4000	0.4100	18.3800	0.0179	0.0097	0.0000
25	1796	462.3200	0.3700	59.6800	0.0827	0.0311	0.6307
26	4205	517.7600	0.8500	17.9600	0.1043	0.0099	0.0000
27	4207	157.0400	0.7600	47.6800	0.1934	0.0129	0.0000
28	4712	525.6300	0.0700	19.3000	-0.3012	0.0390	0.0000

In case study 1, it was observed that 18 out of 28 DMUs (64% of the total sample) exhibited optimal efficiency, as evidenced by a lack of inefficiency represented by a score of zero. The selected stocks, representing different firms, demonstrated exemplary performance by efficiently utilizing their resources to achieve the highest possible desirable outcomes. The list of efficient stocks comprises the following STOCK_IDs: 1203, 1213, 1215, 1216, 1217, 1220, 1229, 1231, 1233, 1234, 1235, 1236, 1256, 1264, 1737, 4205, 4207, and 4712. In contrast, the study has identified a total of 10 equities, which account for around 36% of the sample, as being less efficient. These stocks have been assigned inefficiency scores that exceed zero. The inefficient stocks, identified explicitly by their respective STOCK_ID: 1201, 1210, 1218, 1219, 1225, 1227, 1232, 1258, 1702, and 1796, demonstrated varied levels of inefficiency. Of all the options considered, STOCK_ID 1796 had notable inefficiency, as indicated by its highest inefficiency score of 0.6307. This number implies that there is significant potential for enhancing its operational performance. On the opposite side of the continuum, STOCK_ID 1210, despite being categorized as inefficient, exhibited the lowest inefficiency score of 0.0116 among the inefficient stocks, suggesting that it is comparatively closer to attaining efficiency in relation to its counterparts within this classification.

The findings above provide significant insights into the operational efficiency of the selected equities, which may have possible implications for various stakeholders such as investors, regulators, and corporate management. The findings highlight the adaptability and novelty of the revised DDF-DEA model to handle negative data in output variables. Accordingly, two sets are created to represent the collection of the two types of DMU classification. Set A is a set of all the efficient stocks, while Set B is a collection of all the inefficient stocks.

$$Set\ A = DMUs\ (2,\ 4,\ 5,\ 6,\ 7,\ 10,\ 13,\ 14,\ 16,\ 17,\ 18,\ 19,\ 20,\ 22,\ 24,\ 26,\ 27,\ 28) \quad (4.17)$$

$$Set\ B = DMUs\ (1,\ 3,\ 8,\ 9,\ 11,\ 12,\ 15,\ 21,\ 23,\ 25) \quad (4.18)$$

While inefficient stocks may be seen as appalling, it is crucial to additionally contemplate the potential for growth and enhancement within the realm of reducing undesirable output and maximizing desirable output. Thus, a novel IDEA model is applied to estimate the potential risk reduction and maximum potential reduction in stocks. The potential risk reduction and maximum risk reduction for each DMU in Set B may be observed in Table 4.4. The potential risk reduction refers to the immediate and attainable decrease in risk that can be accomplished, whereas the maximum risk reduction indicates the greatest possible reduction in risk. The inefficient stocks are ranked in ascending order based on the deviation from the lowest to the highest. The order of rankings for DMUs can be represented as follows:

$$DMUs\ (15 \le 12 \le 25 \le 9 \le 11 \le 23 \le 1 \le 3 \le 21 \le 8) \quad (4.19)$$

The aforementioned sequence suggests that DMU 15 exhibits the lowest deviation after implementing maximal risk reduction measures, with DMU 12 and DMU 25 closely following suit. In contrast, it can be observed that DMU 8 exhibits the most significant deviation after the implementation of the maximum risk mitigation measures, with DMU 21 and DMU 3 preceding it in terms of deviation from initial risk. Table 4.4 gives the results of inverse optimization on inefficient stocks, while Fig. 4.1 and Fig. 4.2 show the bar plot of maximum risk reduction and risk deviation.

Table 4.4: Inefficient DMUs risk reduction analysis

DMU	Risk (σ)	Potential risk reduction	Max. Risk Reduction	Deviation
1	0.0186	0.000188916	0.00350232	0.01509768
3	0.0160	0.000030190	0.00021573	0.01578427
8	0.0209	0.000068620	0.00342289	0.01747711
9	0.0171	0.000223730	0.00515445	0.01194555
11	0.0181	0.000143070	0.0046323	0.01346770
12	0.0153	0.000085550	0.00495418	0.01034582
15	0.0146	0.000131470	0.00433523	0.01026477
21	0.0375	0.000096260	0.0202943	0.01720570
23	0.0163	0.000018630	0.00140788	0.01489212
25	0.0311	0.000064940	0.0205278	0.01057220

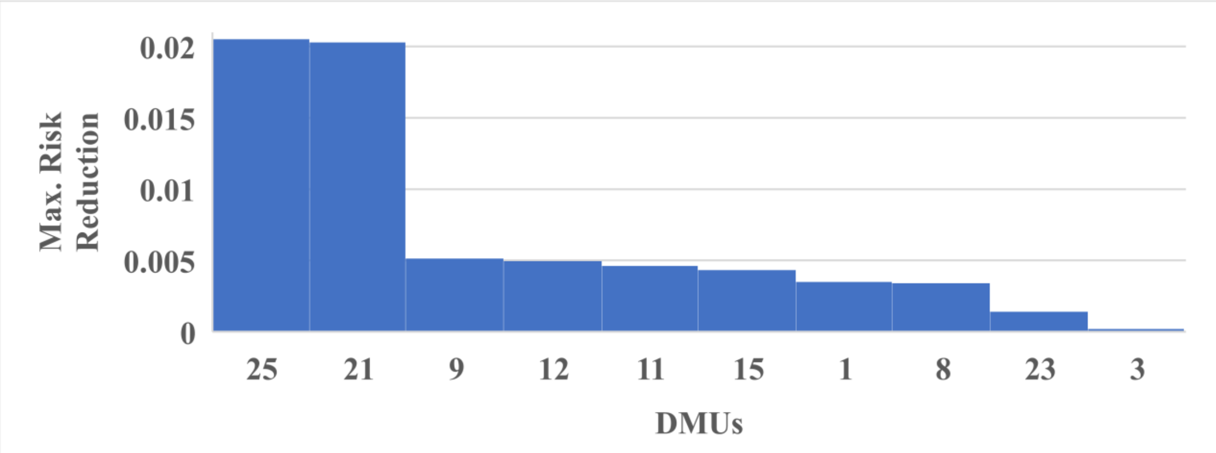

Fig. 4.1: Maximum risk reduction plot.

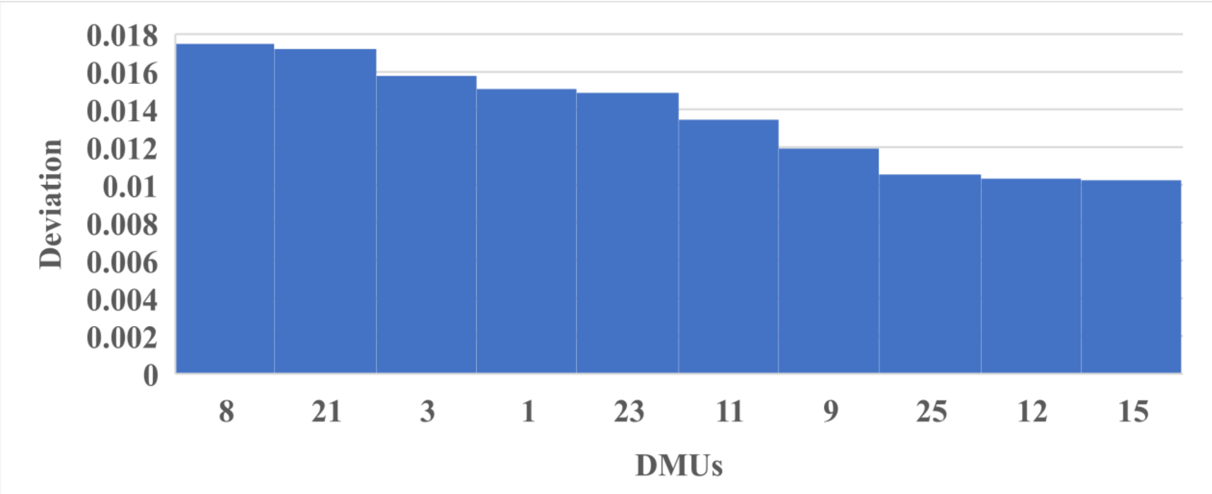

Fig. 4.2: Risk deviation plot.

The examination of the given DMUs in Table 4.4 indicates a variety of risk profiles, with DMU 15 displaying the lowest risk level of 0.0146 and DMU 21 demonstrating the greatest risk level of 0.0375. Specifically, DMU 21 is notable for its heightened level of volatility. When evaluating possible risk reductions, it is observed that DMU 23 exhibits the lowest potential reduction. However, it is noteworthy that DMU 23 also has a substantial maximum risk reduction capability, suggesting a discrepancy between both measurements. In contrast, DMU 25 demonstrates the greatest potential for mitigating risks. Based on a careful analysis of deviations, it can be shown that DMU 15 exhibits the closest proximity to its efficient frontier, suggesting a high level of efficiency. Conversely, DMU 21 has significant potential for improvement, indicating ample room for enhancement. It is evident that DMUs, such as DMU 23, which exhibit the smallest disparities between possible and maximum risk reductions, are positioned in closer proximity to their efficiency thresholds. DMU 21 is identified as a prominent contender for risk mitigation endeavours owing to its existing state of heightened risk. On the other hand, DMU 15 exhibits a reasonably high level of efficiency, as it is positioned in close proximity to its efficiency frontier. Fig. 4.3-4.12 illustrates the relationship between the measure of inefficiency and the potential reductions. The risk reduction curve is derived from the algorithm utilized in the zero portfolio risk initiative. In each instance, the inefficiency scores were divided into equal ranges, and it was apparent that the most considerable reduction in inefficiency occurred at a score of zero.

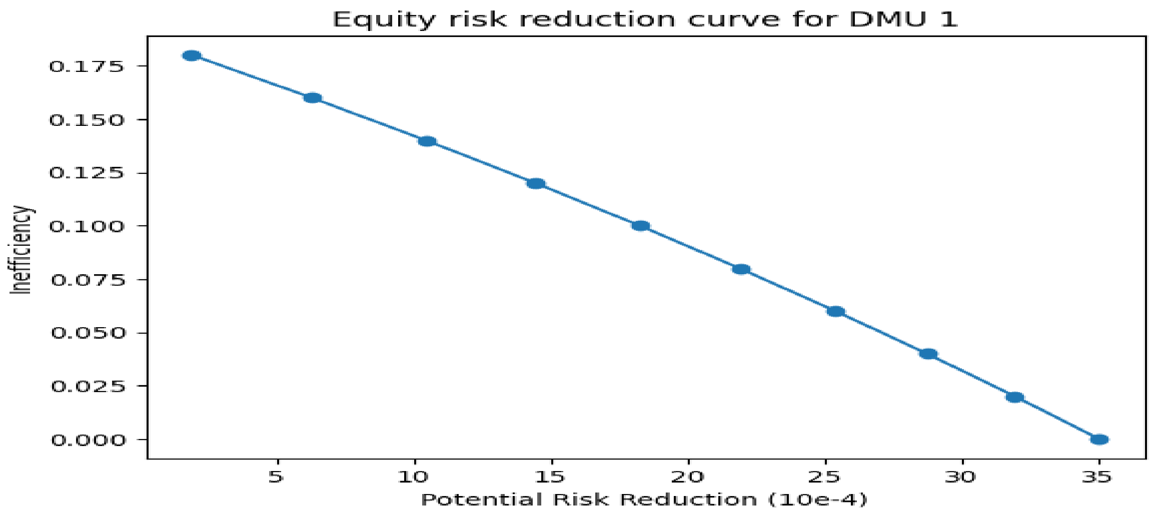

Fig. 4.3: Risk Reduction curve for DMU 1.

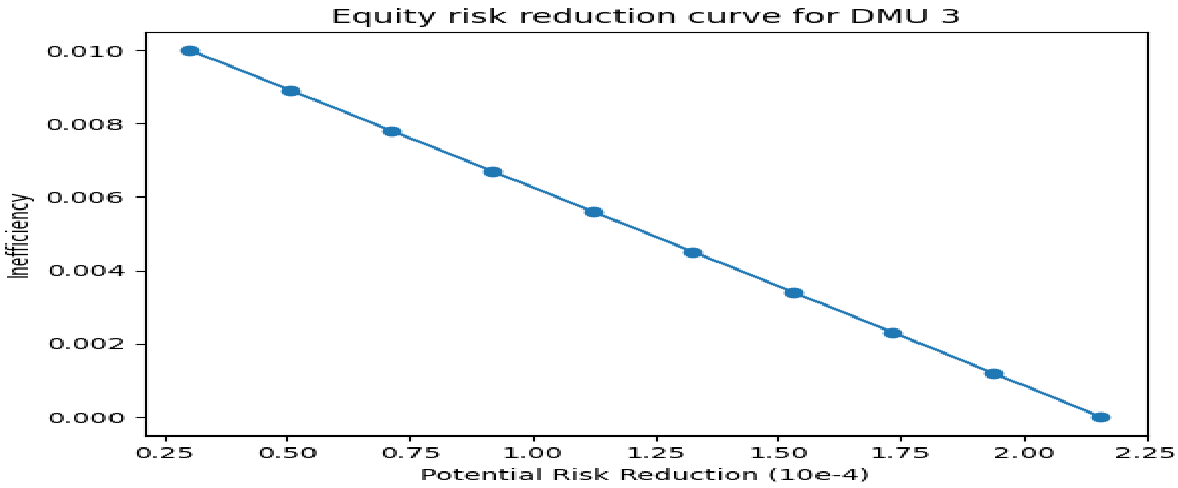

Fig. 4.4: Risk Reduction curve for DMU 3.

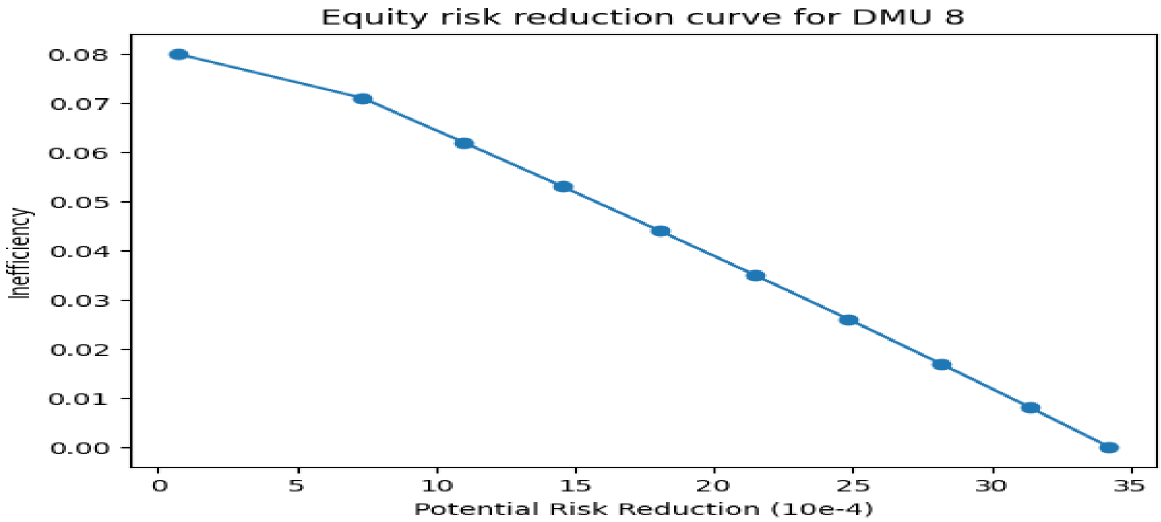

Fig. 4.5: Risk Reduction curve for DMU 8.

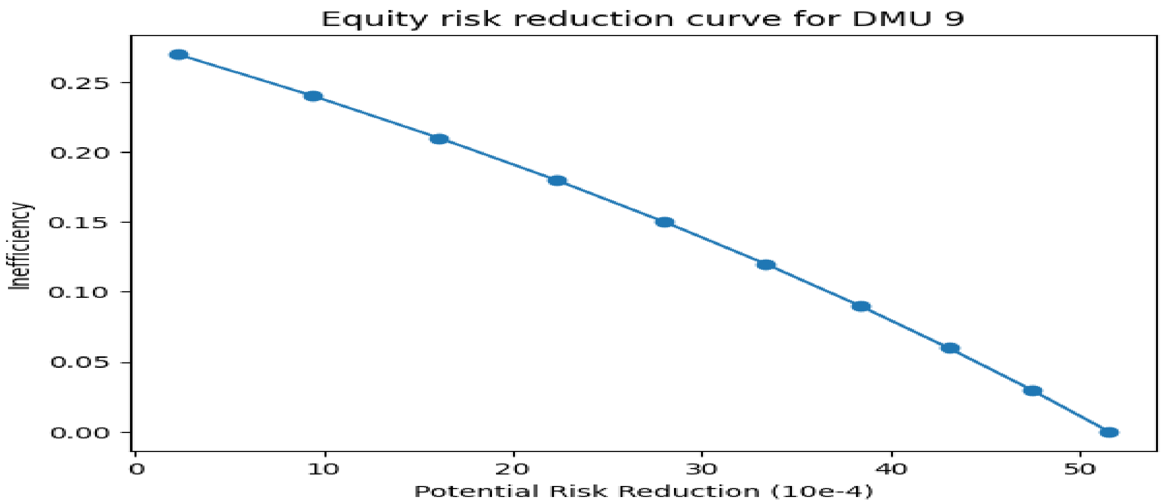

Fig. 4.6: Risk Reduction curve for DMU 9.

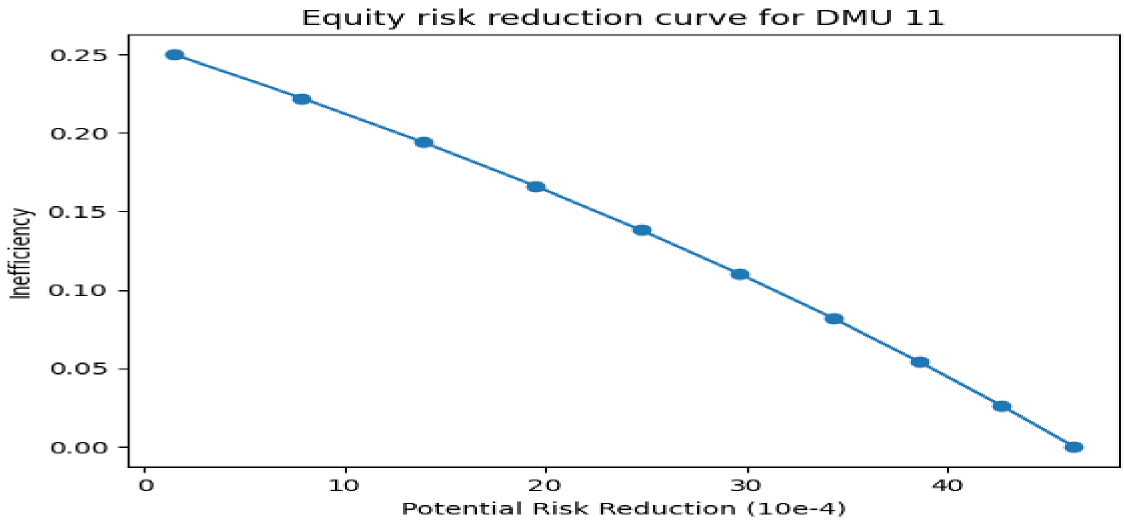

Fig. 4.7: Risk Reduction curve for DMU 11.

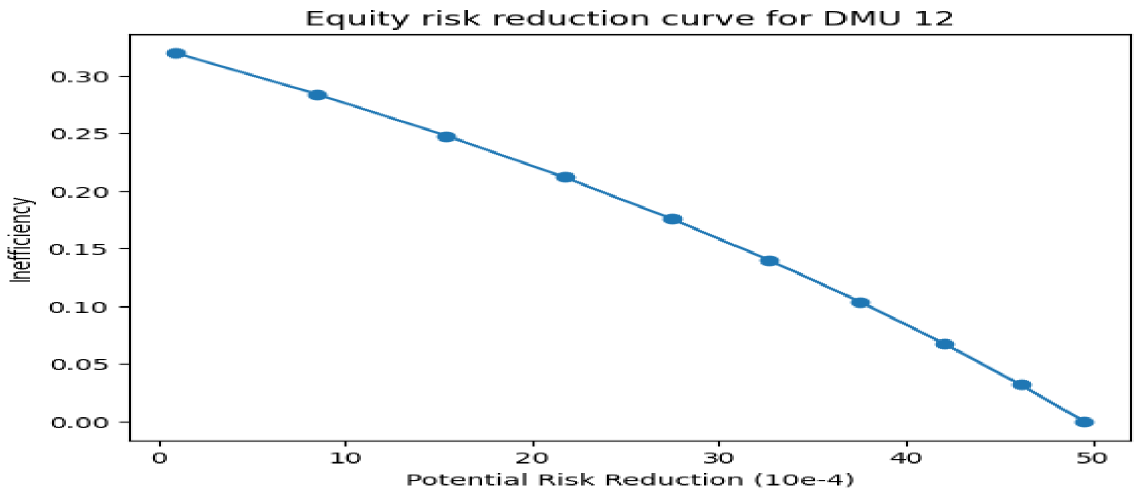

Fig. 4.8: Risk Reduction curve for DMU 12.

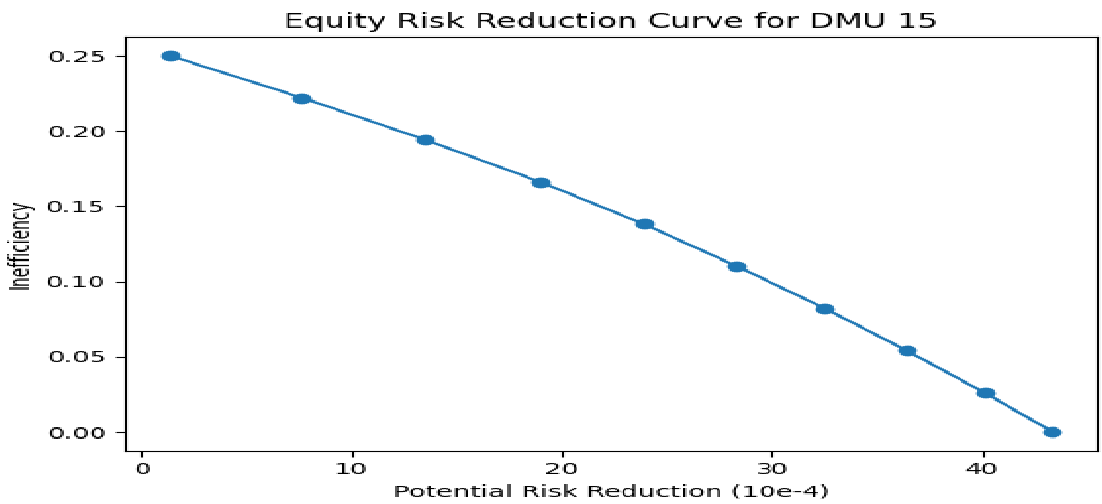

Fig. 4.9: Risk Reduction curve for DMU 15.

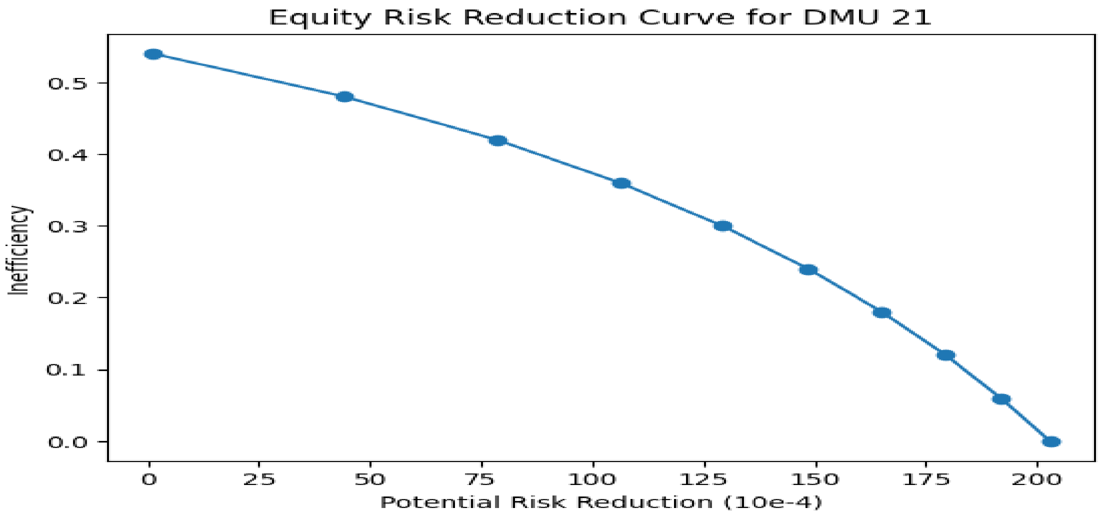

Fig. 4.10: Risk Reduction curve for DMU 21.

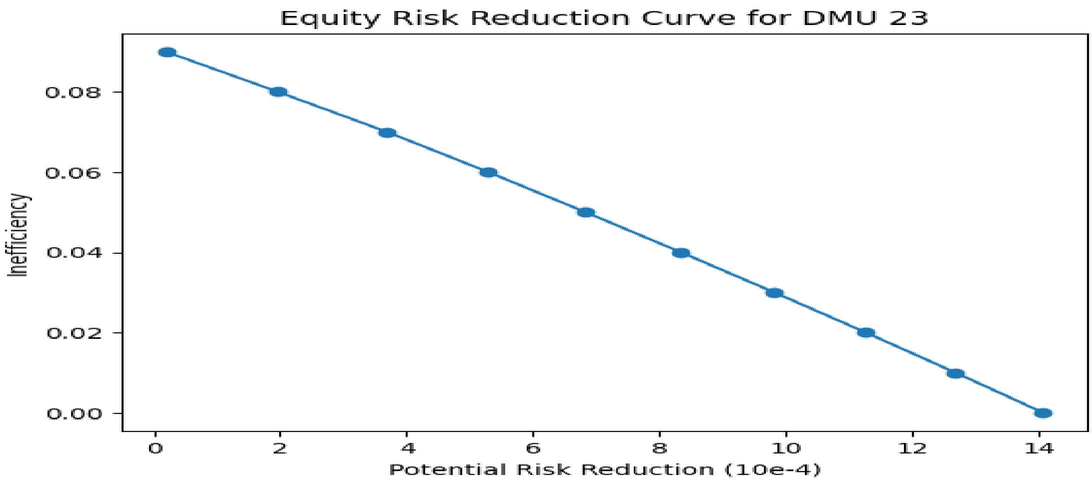

Fig. 4.11: Risk Reduction curve for DMU 23.

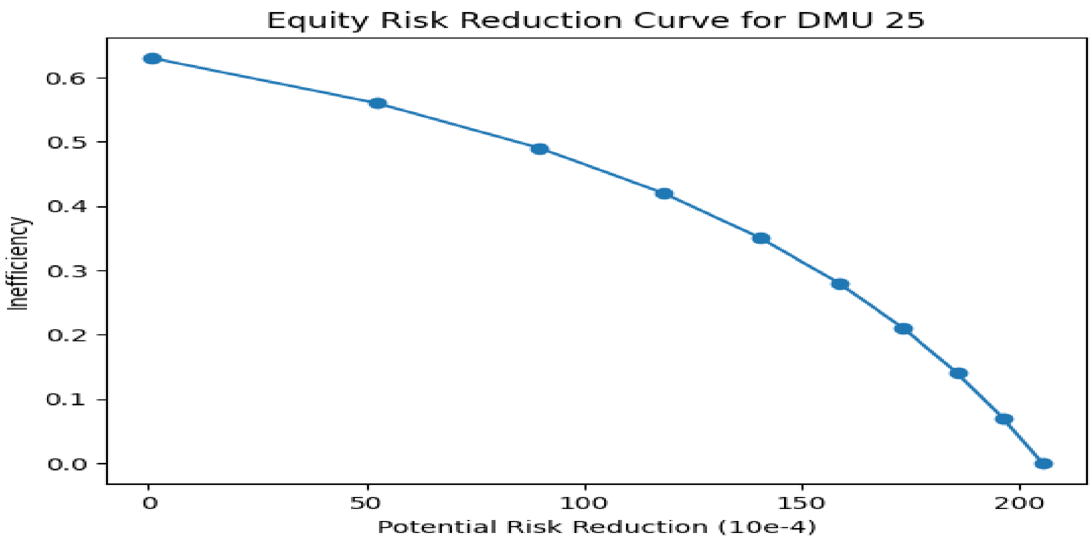

Fig. 4.12: Risk Reduction curve for DMU 25.

4.7 Model application for case study 2

This section discusses the outcomes and analysis derived from implementing the proposed model and methodology on the 20 consumer staples stocks listed on the S&P 500. The inefficiency ratings of the DMUs derived using the M-10 model are presented in Table 4.5. Next, the results of the inverse optimization of the inefficient DMUs are provided by applying model M-11.

Table 4.5: Datasets and efficiency analysis

DMU	TICKER	X1	X2	X4	R	σ	Inefficiency (∅)
1	ADM	1.5973	1.3817	1.3595	0.1697	0.0252	0.1797
2	BF.B	3.7517	0.5618	1.7921	0.2484	0.0250	0.1112
3	CL	0.9694	1.0648	62.7103	0.2774	0.0200	0.0000
4	EL	1.7156	0.7758	3.1569	0.3397	0.0257	0.0000
5	GIS	0.6605	0.5861	2.7546	0.168	0.0194	0.0000
6	HSY	1.1859	0.9138	3.8614	0.1276	0.0234	0.1709
7	K	0.7621	0.7385	5.2721	-0.0339	0.0197	0.0152
8	KDP	0.3293	0.2324	1.1361	0.179	0.0240	0.0000
9	KHC	1.2248	0.2566	0.9984	0.187	0.0275	0.0000
10	KMB	0.8725	1.1867	154.4318	0.0418	0.0197	0.0152
11	KO	0.9586	0.3596	3.9851	0.0575	0.0217	0.0000
12	KR	0.8185	2.7768	3.9756	0.154	0.0201	0.0348
13	LW	2.2549	0.9264	14.2064	0.1071	0.0367	0.4714
14	MNST	3.4014	0.8449	0.2386	0.437	0.0225	0.0000
15	PEP	0.9652	0.7945	5.3585	0.1553	0.0230	0.1539
16	PG	0.7939	0.6156	1.5264	0.1724	0.0204	0.0000
17	PM	1.088	0.7302	-4.1079	0.0512	0.0249	0.0000
18	SYY	1.5778	2.3184	11.1226	0.1126	0.0438	0.5562
19	TSN	1.7945	1.2786	1.2502	-0.2124	0.0312	0.3317
20	WMT	0.7979	2.2921	2.1134	0.2414	0.0198	0.0000

In Table 4.5, it was observed that 10 out of 20 DMUs (50% of the total sample) exhibited optimal efficiency, as evidenced by a lack of inefficiency represented by a score of zero. The selected stocks, representing different firms, demonstrated exemplary performance by efficiently utilizing their resources to achieve the highest possible desirable outcomes. The list of efficient stocks comprises the following stocks represented by TICKER: CL, EL, GIS, KDP, KHC, KO, MNST, PG, PM, and WMT. In contrast, the study has identified a total of 10 equities, which accounts for around 50% of the sample, as being less efficient. These stocks have been assigned inefficiency scores that exceed zero. The inefficient stocks, identified explicitly by their respective TICKER: ADM, BF.B, HSY, K, KMB, KR, LW, PEP, SYY, and TSN, demonstrated varied levels of inefficiency. SYY had notable inefficiency of all the options considered, as indicated by its highest inefficiency score of 0.5562. This number implies that there is significant potential for enhancing its operational performance. On the opposite side of the continuum, K and KMB exhibited the lowest inefficiency score of 0.0152 among the inefficient stocks, suggesting that it is comparatively closer to attaining efficiency in relation to their counterparts within this classification.

The findings above provide significant insights into the operational efficiency of the selected equities, which may have possible implications for various stakeholders such as investors, regulators, and corporate management. The findings highlight the adaptability and novelty of the revised DDF-DEA model to handle negative data in both input and output variables. Accordingly, two sets are created to represent the collection of the two types of DMU classification. Set A is a set of all the efficient stocks, while Set B is a collection of all the inefficient stocks.

$$Set\ A = DMUs\ (3,\ 4,\ 5,\ 8,\ 9,\ 11,\ 14,\ 16,\ 17,\ 20) \qquad (4.20)$$

$$Set\ B = DMUs\ (1,\ 2,\ 6,\ 7,\ 10,\ 12,\ 13,\ 15,\ 18,\ 19) \qquad (4.21)$$

While inefficient stocks may be seen as appalling, it is crucial to additionally contemplate the potential for growth and enhancement within the realm of reducing undesirable output and maximizing desirable output. Thus, novel IDEA model is applied to estimate the potential risk reduction and maximum potential reduction in stocks. The potential risk reduction and maximum risk reduction for each DMU in Set B may be observed in Table 4.6. The potential risk reduction refers to the immediate and attainable decrease in risk that can be accomplished, whereas the maximum risk reduction indicates the greatest possible reduction in risk. The inefficient stocks are ranked in ascending order based on the deviation from the lowest to the highest. The order of rankings of DMUs can be represented as follows:

$$DMUs\ (6 \le 7 \le 10 \le 12 \le 13 \le 15 \le 18 \le 1 \le 19 \le 2) \qquad (4.22)$$

The sequence above suggests that DMU 6, 7, 10, 12, 13, 15, and 18 exhibit the lowest and similar deviation after implementing maximal risk reduction measures. In contrast, it can be observed that DMU 2 exhibits the most significant deviation after the implementation of the maximum risk mitigation measures, with DMU 19 and DMU 1 preceding it in terms of deviation from initial risk. Table 4.6 gives the results of inverse optimization on inefficient stocks, while Fig. 4.13 and Fig. 4.14 show the bar plot of maximum risk reduction and risk deviation.

Table 4.6: Inefficient DMUs risk reduction analysis

DMU	Risk (σ)	Potential risk reduction	Max. Risk Reduction	Deviation
1	0.0252	0.000293781	0.00452784	0.02067216
2	0.0250	0.000042171	0.00342522	0.02157478
6	0.0234	0.000026506	0.00400000	0.01940000
7	0.0197	0.000104040	0.00030000	0.01940000
10	0.0197	0.000104040	0.00030000	0.01940000
12	0.0201	0.000100000	0.00070000	0.01940000
13	0.0367	0.000096226	0.01730000	0.01940000
15	0.0230	0.000108461	0.00360000	0.01940000
18	0.0438	0.000609809	0.02440000	0.01940000
19	0.0312	0.000080059	0.01034960	0.02085040

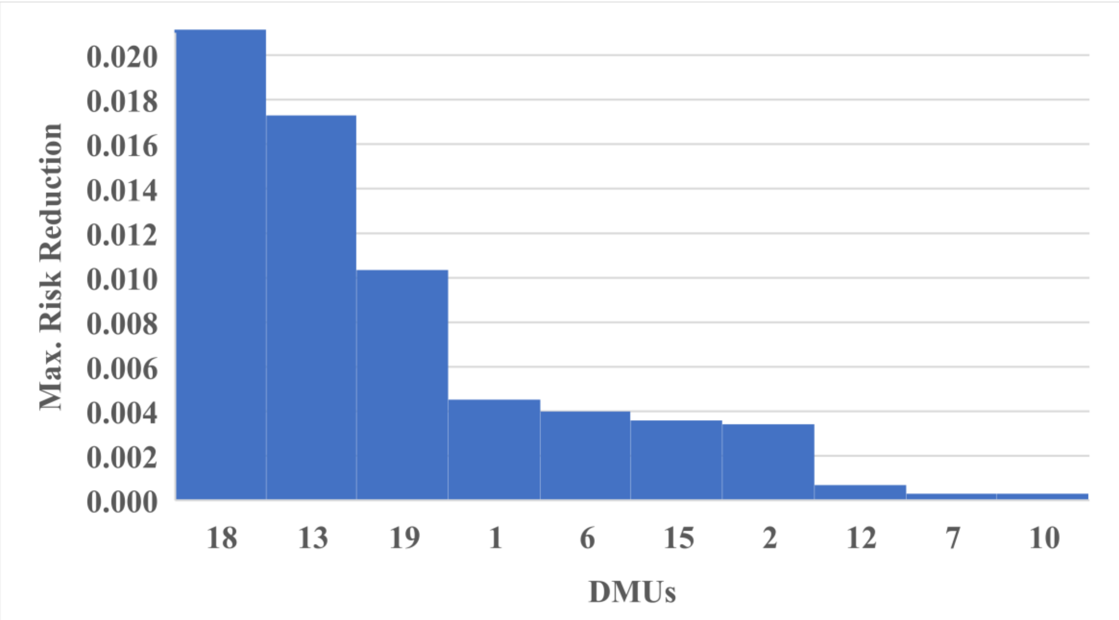

Fig. 4.13: Maximum risk reduction plot.

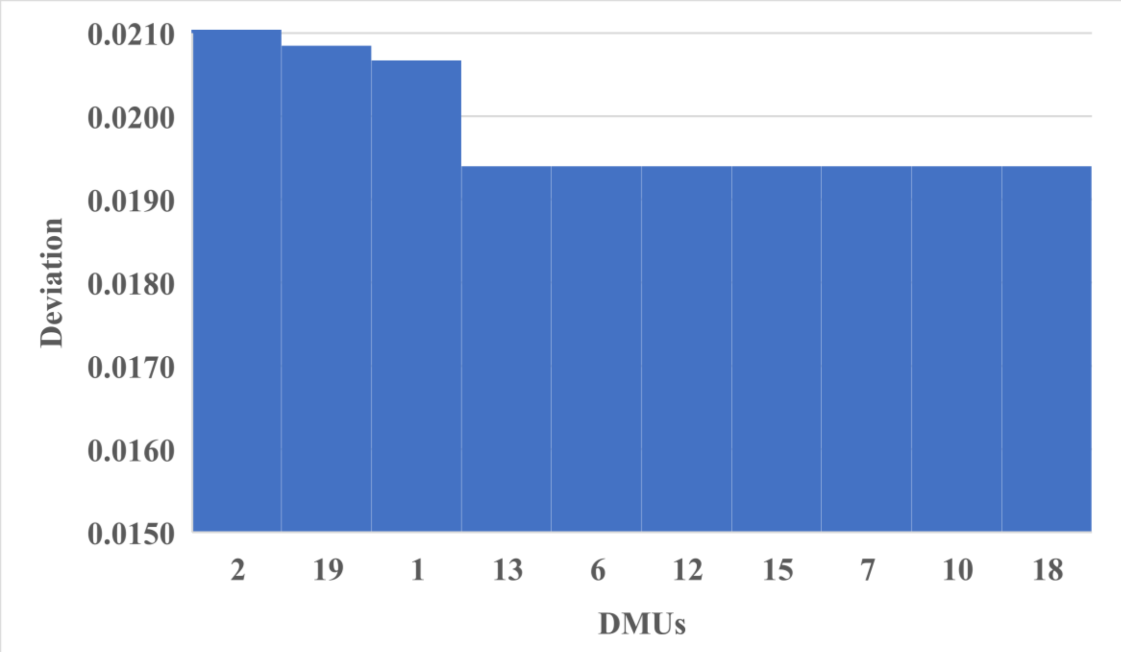

Fig. 4.14: Risk deviation plot.

Table 4.6 presents the risk assessments for various inefficient DMUs. The findings indicate that DMU 18 displays a marked level of risk, which is counterbalanced by its considerable capacity for risk mitigation. Conversely, DMUs 7 and 10 demonstrate the lowest levels of risk, suggesting that they may be functioning at or near optimal efficiency. However, the significant deviation seen in DMU 2 indicates the presence of potential areas for enhancement, requiring a more comprehensive examination of its inefficiencies. Simultaneously, it is observed that DMUs 6, 7, 10, 12, 13, 15, and 18 exhibit slight deviation, making them possible benchmarks for other units. Fig. 4.15-4.24 illustrates the relationship between the measure of inefficiency and the potential reductions. The risk reduction curve is derived from the algorithm utilized in the zero portfolio risk initiative. In each instance, the inefficiency scores were divided into equal ranges, and it was apparent that the most considerable reduction in inefficiency occurred at a score of zero.

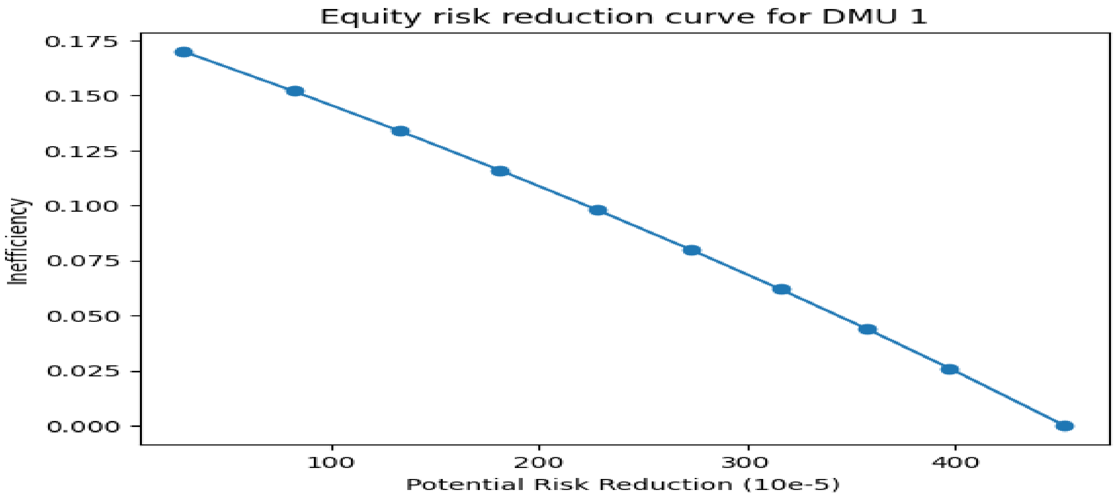

Fig. 4.15: Risk Reduction curve for DMU 1.

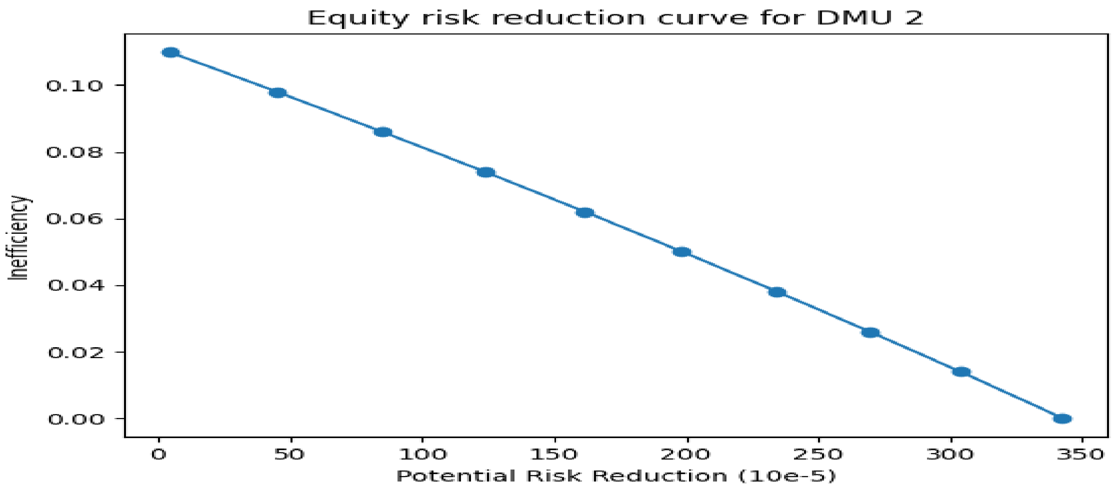

Fig. 4.16: Risk Reduction curve for DMU 2.

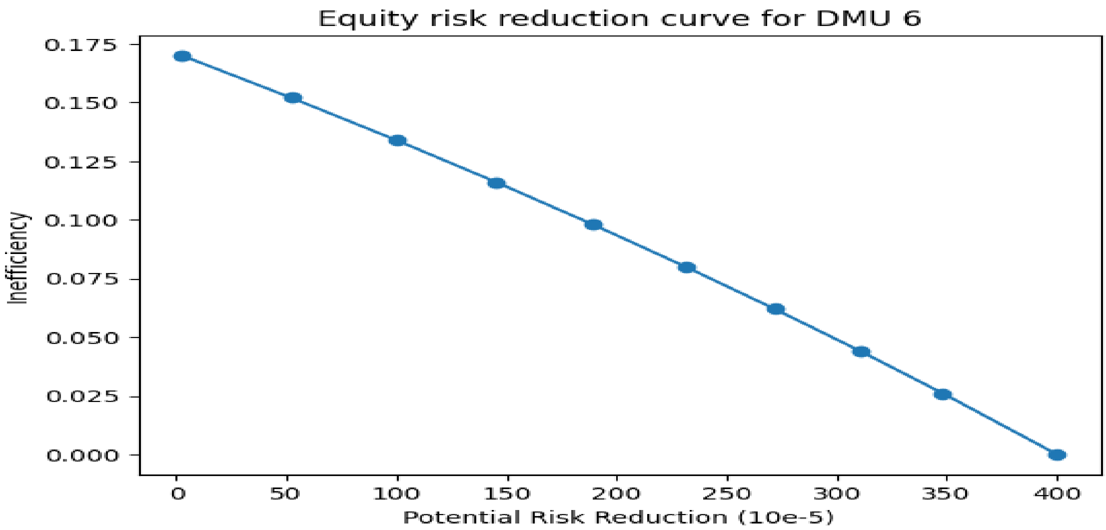

Fig. 4.17: Risk Reduction curve for DMU 6.

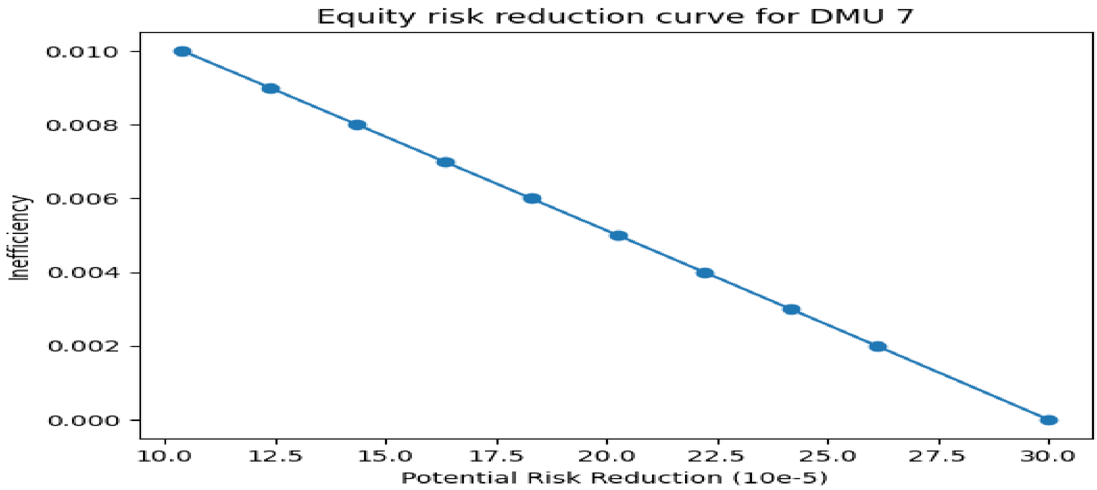

Fig. 4.18: Risk Reduction curve for DMU 7.

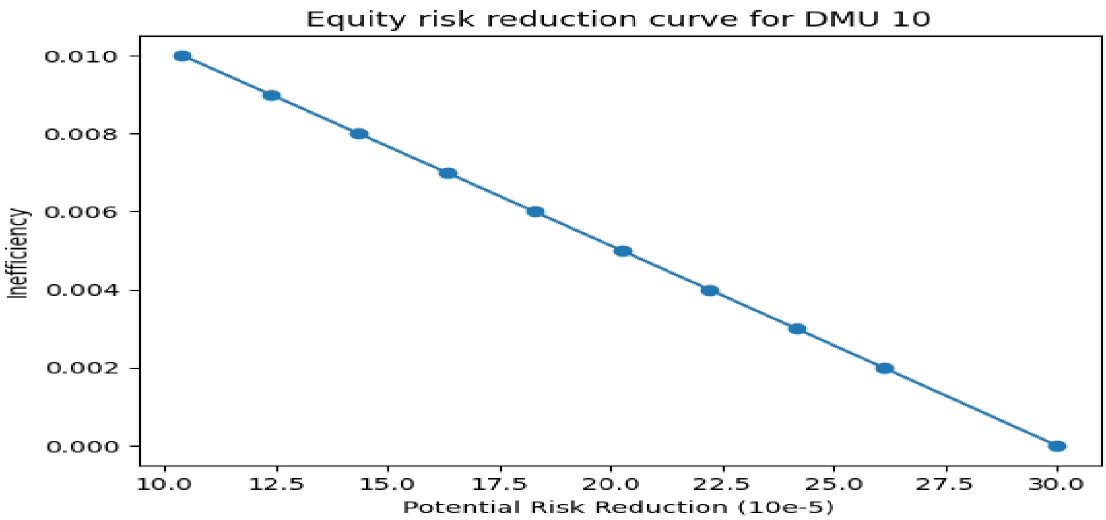

Fig. 4.19: Risk Reduction curve for DMU 10.

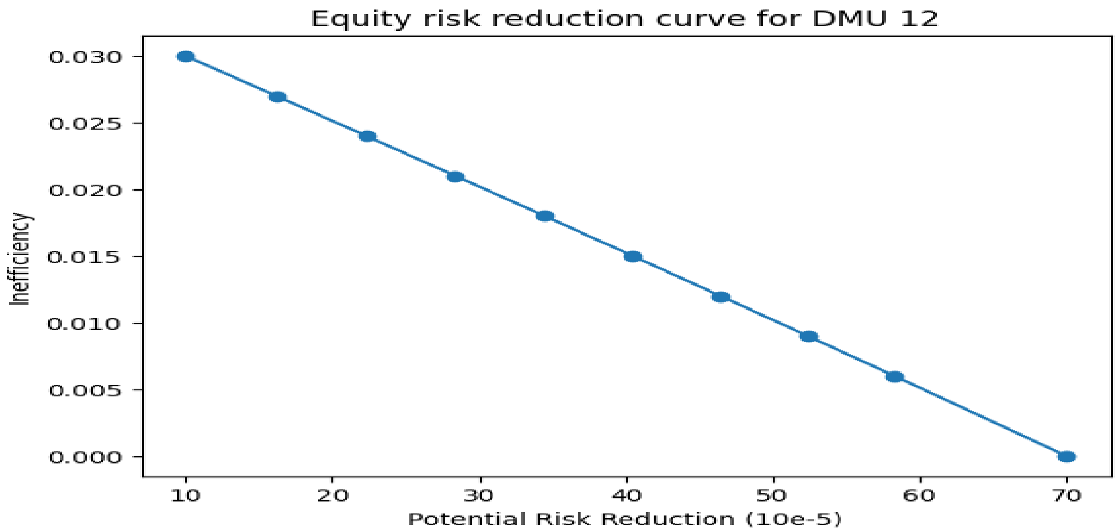

Fig. 4.20: Risk Reduction curve for DMU 12.

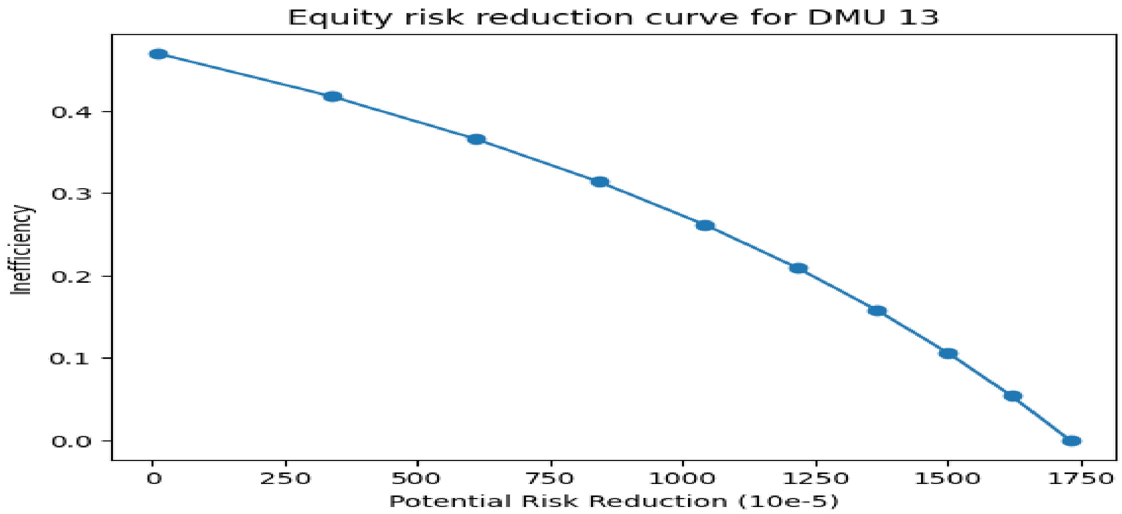

Fig. 4.21: Risk Reduction curve for DMU 13.

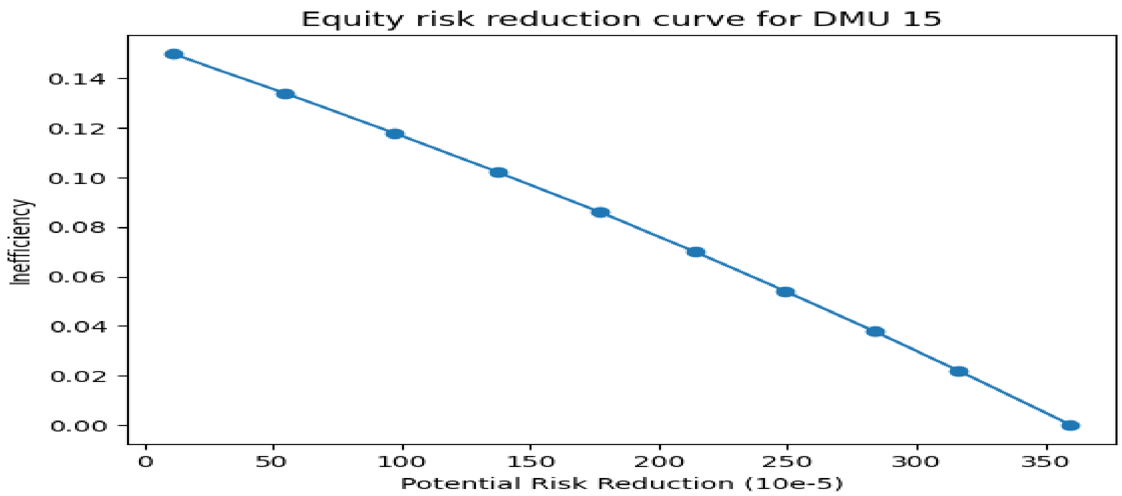

Fig. 4.22: Risk Reduction curve for DMU 15.

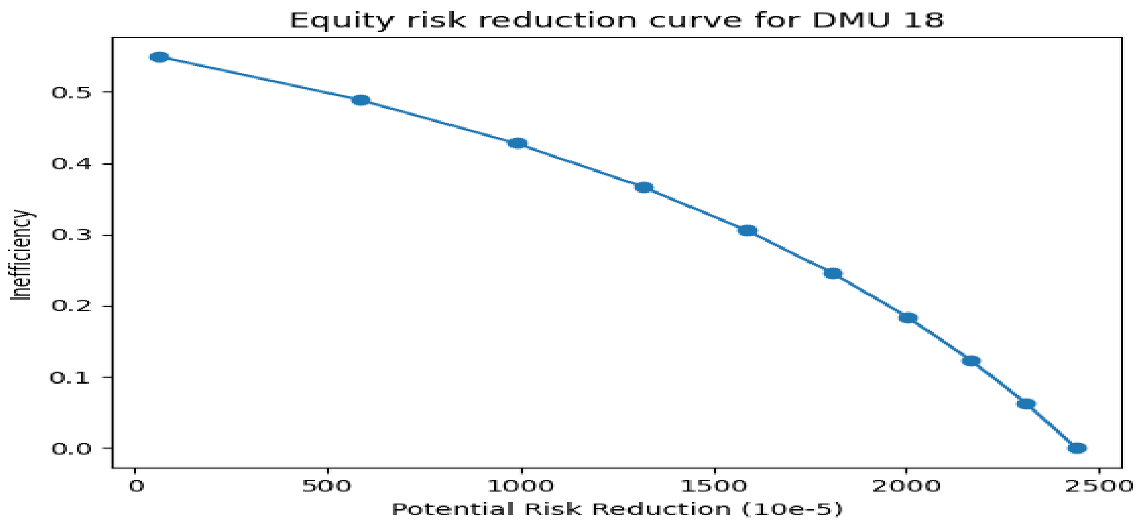

Fig. 4.23: Risk Reduction curve for DMU 18.

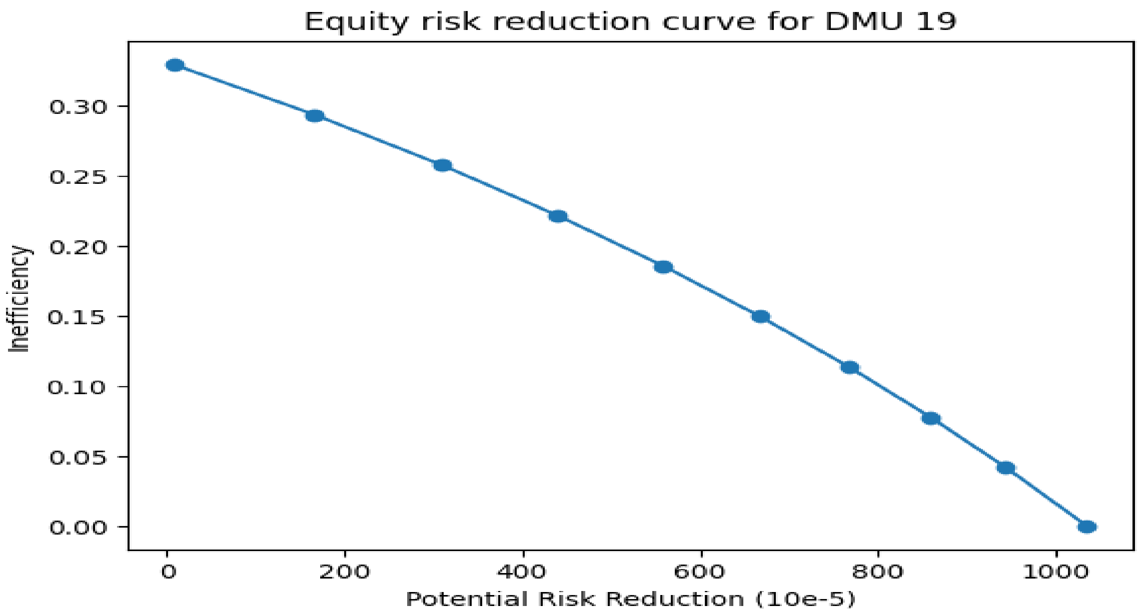

Fig. 4.24: Risk Reduction curve for DMU 19.

4.8 Verification

In order to validate the new IDEA model and technique, a sensitivity analysis was performed on the inefficient stocks within Set B. The initial risk values were substituted with updated ones obtained by subtracting the maximum risk reduction from the original risk. Following this substitution, the stocks underwent re-evaluation for inefficiency scores utilizing the proposed IDEA model. The findings are briefly presented in Table 4.7 and Table 4.8, representing verification studies for Case 1 and Case 2 studies.

Table 4.7: Case 1 sensitivity analysis

DMU	Initial Risk	New Risk	Initial Inefficiency Score	New Inefficiency Score
1	0.0186	0.01509768	0.18830000	0
3	0.016	0.01578427	0.00021573	0
8	0.0209	0.01747711	0.00342289	0
9	0.0171	0.01194555	0.00515445	0
11	0.0181	0.01346770	0.00463230	0
12	0.0153	0.01034582	0.00495418	0
15	0.0146	0.01026477	0.00433523	0
21	0.0375	0.01720570	0.02029430	0
23	0.0163	0.01489212	0.00140788	0
25	0.0311	0.01057220	0.02052780	0

Table 4.8: Case 2 sensitivity analysis

DMU	Initial Risk	New Risk	Initial Inefficiency Score	New Inefficiency Score
1	0.0252	0.02067216	0.1797	0
2	0.0250	0.02157478	0.1112	0
6	0.0234	0.01940000	0.1709	0
7	0.0197	0.01940000	0.0152	0
10	0.0197	0.01940000	0.0152	0
12	0.0201	0.01940000	0.0348	0
13	0.0367	0.01940000	0.4714	0
15	0.0230	0.01940000	0.1539	0
18	0.0438	0.01940000	0.5562	0
19	0.0312	0.02085040	0.3317	0

Based on the data presented in Table 4.7 and Table 4.8, it is evident that upon replacing the initial risk with the updated risk value, the inefficiency scores of $DMU_j, \forall j \in setB$ have decreased to zero. The observed results provide a favourable indication of the robustness of the proposed model. This study confirms that the maximum risk reductions predicted by the proposed model can potentially eliminate inefficiencies in these equities and reach optimal levels. The verification process instills assurance and confidence in adopting the proposed models to find potential and maximum risk reductions in underperforming firms. Furthermore, it ensures the resilience of the proposed models in accurately forecasting the efficiency improvements that can be attained through these reductions in risk (bad output). When analyzing data-driven insights, it is crucial to consider external factors and overall strategies to choose the most appropriate course of action. Also, it is essential to acknowledge that the effective implementation of these improvements will be contingent upon the specific strategies employed to attain the stated reductions in risk. Hence, the implementation of these findings necessitates the utilization of strategic decision plans and the adoption of appropriate risk practices by management. The findings of this study offer valuable insights for all inefficient DMUs in set B to minimize risk (bad output) in a financial production process and improve their efficiency. The provided framework presents a theoretical approach for mitigating risk to achieve an optimal state, thus informing strategic choices to enhance efficiency.

4.9 Summary

This chapter depicts a novel methodology for assessing the efficiency of stocks using two case studies, including an emerging market (TWSE) and a developed market (S&P 500), taking into account negative data in input and output variables. An extended version of the DDF-DEA methodology was developed to calculate the inefficiency scores of stocks. Additionally, a novel IDEA was proposed to estimate the potential and maximum risk reduction while maintaining a constant level of inputs and good output (return). The suggested model and approach were implemented using datasets of selected stocks belonging to the food sector of TWSE and the consumer staples sector of the S&P 500. The dataset spans the financial year 2020. In either case, only 10 stocks were found to be inefficient. The inefficient stocks were further subjected to an inverse optimization approach to analyze the potential reduction in bad output (risk) to reach an optimal target. Specific risk reduction targets are identified for each inefficient stock, offering a distinct trajectory toward enhancing efficiency. After applying maximum risk reductions, all re-evaluated stocks demonstrated zero inefficiency ratings. The findings indicate that inefficient equities would only require a little risk reduction to achieve efficiency. The robustness of the proposed model is validated through a thorough sensitivity analysis, enhancing its reliability for practical implementation. The implementation of these models depends on the management strategies employed using several means such as jumbo sales, promotions, advertisements, and the development of new and innovative products that could promote the company's image and consequently establish the company as a leading firm among its contemporaries, to maintain an efficient frontier at all times. This work makes a scholarly contribution to the current body of research on DEA by expanding and modifying established models and methodologies to effectively address the inclusion of negative data in DEA models. Unlike conventional methods, this study adopts a novel approach where stocks are evaluated as financial production technologies, with return and risk as key output variables.

It is posited that the innovative methodology being proposed has the potential to generate significant advancements in the respective domain and establish a foundation for the development of more sophisticated, intelligent, and streamlined decision-making processes. In sum, this chapter has expanded the potential applications of DEA models in assessing equity risk, offering a helpful means for strategic decision-making.

Chapter 5 – A Novel Inverse DEA-based Portfolio Optimization: Evidence from the Taiwan Stock Exchange

Chapter 4 of this thesis focuses on optimizing risk in relation to individual equities, examining both favorable and unfavorable factors in input and output data. This provides a detailed understanding of risk optimization at the individual securities level, establishing a strong basis for exploring increasingly complex investing situations. Chapter 5 extends this methodology by shifting from examining individual stocks to thoroughly investigating portfolio stocks. The shift is based on the characteristics of the data used for portfolios, which are completely positive, representing the combined good qualities of stocks associated with the industry inside each DMU. Chapter 5 shifts the focus from individual stocks to portfolio stocks, modifying the analytical framework to focus primarily on positive data models. This sophisticated technique exemplifies the pragmatic aspects of portfolio management, where investors and fund managers often encounter and employ favorable facts in their decision-making procedures. This chapter offers valuable insights that can be applied to the wider financial management and investment strategy optimization domains.

5.1 Introduction

Financial markets such as stock markets are complex systems that exhibit the complicated interplay of actions, emotions, and decisions made by numerous individuals. These markets are influenced by two significant factors: uncertainty and randomness (Li & Teo, 2021). Although there is often an overlap between these notions, they are diverse in nature and have unique impacts on market dynamics. Uncertainty emerges when the result of an occurrence or the magnitude of a variable is not known. The phenomenon in the realm of finance might be likened to a state of obscurity, where the existence of some factors is acknowledged, although their precise outcomes remain uncertain and difficult to ascertain with absolute certainty. The concept of randomness is closely associated with the notion of unpredictability. The proposition posits that irrespective of the extent of one's knowledge, certain occurrences within financial markets are inherently stochastic in nature and impervious to accurate prediction (Fama, 1965).

The importance of stock market volatility cannot be overstated in risk management, portfolio selection, asset pricing, and other domains (He et al., 2023). Portfolio selection problem holds significant importance within the domain of computational finance. The perpetual endeavour to achieve a harmonious equilibrium between risk and return has proven to be a persistent and formidable obstacle. The emergence of this difficulty has led to the development of the notion of portfolio optimization, a highly advanced technique that has become essential for investors and financial institutions on a global scale. The concept of portfolio optimization can be traced back to the pioneering research conducted by Markowitz (1952), which serves as a witness to the progression of financial theory. Portfolio optimization is a discipline that involves the strategic selection of a combination of assets with the aim of attaining the highest attainable return for a given degree of risk (Abolmakarem et al., 2023; Cao et al., 2023). Ji et al. (2020) posit that investors incorporate safe-haven assets into their investment portfolios as a means of mitigating potential losses during times of crisis. This is not merely a hypothetical endeavour; within the dynamic scope of international finance, effective portfolio optimization can determine the disparity between significant gains and notable losses. The increasing complexity of markets has necessitated the evolution of portfolio optimization tools and methodologies. These adaptations are crucial to effectively navigate the shifting dynamics of global economies and their inherent uncertainties.

5.2 Rationale for selecting the Taiwan Stock Exchange (TWSE)

The rationale behind selecting the TWSE as the focus of this research is based on several strategic reasons and justifications. First, Taiwan is characterized as a well-developed free-market economy. It is worth noting that Taiwan possesses a robust and transparent securities market, which stands as one of the most prominent and dynamic stock exchanges in the Asian region. In 2023, TWSE was ranked the eighth biggest equity market in Asia and the seventeenth biggest equity market operator in the world by market capitalization (Statista, 2023a). Second, Taiwan holds a prominent position as a major manufacturer and distributor of semiconductors, electronics, and various technological goods. These products have experienced a surge in worldwide demand as a result of the COVID-19 pandemic and the prevailing trend of digital transformation. In 2021, despite the ravaging COVID-19 pandemic, the semiconductor industry in Taiwan achieved a notable output value of around 145.8 billion US dollars, thereby solidifying its global dominance as the foremost producer in this sector, which, in turn, made the US government allied in response to Taiwan's significant role as a notable manufacturer and distributor of semiconductors, electronics, and numerous technology items (Zhou, 2023). The 'Chip4 Alliance' was founded on 9th August 2022, with the participation of the United States, Taiwan, Japan, and South Korea. Data pertaining to the fiscal year 2020 was gathered with the objective of capturing the latest and pertinent performance of the TWSE and its industry-based portfolios. In the year 2020, Taiwan's economy exhibited noteworthy performance by surpassing China's economic growth for the first time in three decades. Taiwan achieved a growth rate of 2.98%, while China recorded a growth rate of 2.3% (Bloomberg, 2021). Also, in 2020, despite the global pandemic of COVID-19, Taiwan's technology sector demonstrated notable resilience and competitiveness, contributing to substantial export growth and achieving record-high levels of the TWSE index. Hence, the dataset pertaining to the fiscal year 2020 possesses the capacity to offer a thorough and representative depiction of the TWSE and its inherent attributes.

The present work aims to evaluate the efficiency of different industry-based portfolios, introducing a novel IDEA approach to estimate the potential and maximum potential reductions in portfolio volatility. The research holds a more profound importance that extends beyond its immediate conclusions by presenting an alternative theoretical framework for portfolio management. By doing so, it can establish a new standard for attaining optimal efficiency in a financial landscape that is becoming more unpredictable. In this connection, this study applies a novel inverse optimization technique to analyze 20 industry-based portfolios selected from the TWSE. This study improves the existing literature in the following manners: First, this work is the first attempt at portfolio optimization using the IDEA framework. Second, while integrating both technical and financial analysis, this study appears to be one of the few works employing modern axioms that conceptualise the link between return and volatility as a terminal output of a financial production process. Third, the proposed model in this work is capable of estimating a range from minimum to maximum reduction in risk to generate volatility reduction curves for all inefficient portfolios. Last, it proposes potential mitigation strategies to attain optimal stock portfolios.

5.3 Base model development

The evaluation of return and risk serves as crucial metrics in portfolio selection and optimization. In financial assets, the term "return" pertains to the valuation of an asset, typically denoting either the wealth value or the rate of return. In a modern financial production setting, it has been theoretically proven that return and risk should be classified as production outputs, which can later be viewed as either desirable or undesirable (Tarnaud & Leleu, 2018). The DDF-DEA methodology presents an approach to assessing productivity by considering both good and bad outputs, a factor often neglected in traditional productivity metrics. The development of DDF was undertaken as an integral component of a productivity indicator. In several industries, producing highly demanded goods frequently results in the concurrent production of undesired outputs. Every return comes alongside its volatility, and since risk aversion is the desire of most investors, it is essential to ensure that a bad output like portfolio volatility is minimized at all times. The base model (M1) helps estimate the inefficiency (ϕ) score in individual DMUs, as similarly applied by Wegener and Amin (2019), Orisaremi et al. (2021) and (Lu et al., 2022) for any $DMU_k \;\forall\; k = 1, \ldots, n$.

$$
\begin{aligned}
&\phi_k^* = \max \phi_k \\
&s.t. \\
&\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ik} \quad where \quad i = 1, \ldots., m \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^g \ge (1+\phi) y_{rk}^g \quad where \quad \mathrm{r} = 1, \ldots., s \\
&\sum_{j=1}^{n} \lambda_j y_{pj}^b = (1-\phi) y_{pk}^b \quad where \quad \mathrm{p} = 1, \ldots., q \\
&\sum_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \quad where \quad j = 1, \ldots., n
\end{aligned}
\tag{M1}
$$

When the value of $\phi_k^* = 0$, the concept of DDF-DEA considers DMU_k to be an efficient.

When the value of $\phi_k^* \succ 0$, the concept of DDF-DEA considers DMU_k to be an inefficient.

When a DMU has $\phi_k^* = 0$, it signifies that the DMU in question operates with optimal efficiency, reflected by an efficiency score of 1.0 (Toloo et al., 2018). The efficiency score is demonstrated in the subsequent Eq. (5.1).

$$Efficiency = \psi_k = \frac{(1-\phi_k^*)}{(1+\phi_k^*)} \tag{5.1}$$

It is essential to acknowledge that the index k relates explicitly to the assessed DMU, while the index j acts as a general index for any given DMU.

5.4 Inverse DEA (IDEA)

Orisaremi et al. (2021) modified the research conducted by Wegener and Amin (2019) by expanding the use of the IDEA framework to address the minimization of greenhouse gases in the oil and gas sector. The models proposed by Wegener and Amin (2019) and Orisaremi et al. (2021) can be described as follows:

Let α_{ik} denote the fluctuation in the i^{th} input, which leads to a corresponding change β_{rk} in the r^{th} desired output, accompanied by a modification γ_{pk} in the p^{th} undesired output. The intent is to minimize the variations in all the undesired outputs. The model (M2) is formulated by Wegener and Amin (2019) to model this problem using an inverse optimization approach.

$$
\begin{aligned}
&\min \gamma = (\gamma_{11},\dots,\gamma_{q1},\dots,\gamma_{1t},\dots,\gamma_{qt})\\
&s.t.\\
&\sum_{j\in A}\lambda_j^k x_{ij} + \sum_{l\in B}\bar{\lambda}_l^k(\alpha_{il}+x_{il}) - (\alpha_{ik}+x_{ik}) \le 0\\
&\forall k\in S,\quad i=1,\dots,m\\
&\sum_{j\in A}\lambda_j^k y_{rj}^g + \sum_{l\in B}\bar{\lambda}_l^k(\beta_{rl}+y_{rl}^g) - (1+\bar{\phi}_k)\mathrm{x}(\beta_{rk}+y_{rk}^g) \ge 0\\
&\forall k\in S,\quad r=1,\dots,s\\
&\sum_{j\in A}\lambda_j^k y_{pj}^b + \sum_{l\in B}\bar{\lambda}_l^k(\gamma_{pl}+y_{pl}^b) - (1-\bar{\phi}_k)\mathrm{x}(\gamma_{pk}+y_{pk}^b) = 0\\
&\forall k\in S,\quad p=1,\dots,q\\
&\sum_{j\in A}\lambda_j^k + \sum_{l\in B}\bar{\lambda}_l^k = 1 \quad \forall k\in S\\
&\sum_{k\in S}\beta_{rk} = \bar{y}_r^g \quad r=1,\dots,s\\
&\alpha_{ik}\ge 0,\quad \beta_{rk}\ge 0,\quad \gamma_{pk}\ge 0 \quad \forall k\in S,\quad i=1,\dots,m\\
&\lambda_j^k\ge 0,\quad \bar{\lambda}_l^k\ge 0,\quad \forall k,l\in B,\quad \forall j\in A
\end{aligned}
\tag{M2}
$$

In the context of portfolio evaluation, set C is introduced as a production possibility set encapsulating t DMUs. This set can be dichotomized into sets A and B. Subset A aggregates portfolios deemed efficient, while set B collates the inefficient ones. Importantly, it is worth noting that within set A, weights of efficient units are assigned $\lambda_j^k \ge 0,\ \forall j\in A$. Conversely, units in set B are assigned weights $\bar{\lambda}_l^k \ge 0,\ \forall l\in B$. To preserve the intrinsic ϕ_k^* of DMUs post-output production, decision-makers are guided to ensure $\bar{\phi}_k \le \phi_k^*$ within the application framework. By simplification, model (M2) approach produced a streamlined model, termed model (M3).

$$
\begin{aligned}
&\min \gamma = (\gamma_1 + \gamma_2 +,\ldots,+\gamma_t) \\
&s.t. \\
&\sum_{j\in A} \lambda_j^k x_{ij} \le (\alpha_{ik} + x_{ik}) \quad \forall k \in S, \\
&\sum_{j\in A} \lambda_j^k y_{rj}^g \ge (1+\bar{\phi}_k)\mathrm{x}(\beta_{rk} + y_{rk}^g) \quad \forall k \in S, \\
&\sum_{j\in A} \lambda_j^k y_{pj}^b = (1-\bar{\phi}_k)\mathrm{x}(\gamma_{pk} + y_{pk}^b) \quad \forall k \in S, \\
&\sum_{j\in A} \lambda_j^k = 1 \quad \forall k \in S \\
&\sum_{k\in S} \beta_{rk} = \bar{y}_r^g \\
&\alpha_{ik} \ge 0, \quad \beta_{rk} \ge 0, \quad \gamma_{pk} \ge 0 \quad \forall k \in S, \\
&\lambda_j^k \ge 0, \quad \bar{\lambda}_l^k \ge 0, \forall k, l \in B, \quad \forall j \in A
\end{aligned}
\tag{M3}
$$

Although implementing model (M3) to mitigate bad output in production processes is effective, it is important to acknowledge that certain limitations are associated with its application. One shortcoming of model (M3) is its inability to reduce the occurrence of undesirable output without initially increasing both the undesirable and desirable outputs. It solely focuses on minimizing the occurrence of undesirable output by increasing the occurrence of desirable output. In this study, the focus lies on developing a model to address this particular limitation.

5.5 Proposed IDEA model

Given the existing levels of inputs and outputs, as well as the current state of financial production technology, what is the amount of volatility reduction required to make an underperforming portfolio efficient? To handle this, all the variations in the desirable output and inputs are assumed to equal zero. This simply means that, in the present financial production process, the values $\beta_{rk} = \alpha_{ik}$ must be equal to zero. The anticipated decrease in volatility suggests that γ_{pk} must exhibit a reduction from its current level. In pursuing net zero-risk portfolios, a new objective function arises, which aims to maximize the reduction in volatility of individual portfolios. In this work, there exists only a single undesirable output, that is, volatility; therefore, set $p = 1$ which serves as the index of undesirable output. The larger the volatility reduction, the closer the portfolio is to efficiency. The integration of these novel adjustments results in the formulation of model (M4).

$$
\begin{aligned}
&\max \gamma = (\gamma_1 + \gamma_2 +, \ldots, +\gamma_t) \\
&s.t. \\
&\sum_{j \in A} \lambda_j^k x_{ij} \le x_{ik} \quad \forall k \in S, \quad i = 1, \ldots, m \\
&\sum_{j \in A} \lambda_j^k y_{rj}^g \ge (1 + \bar{\phi}_k) \text{ x } y_{rk}^g \quad \forall k \in S, \quad r = 1, \ldots, s \\
&\sum_{j \in A} \lambda_j^k y_{pj}^b = (1 - \bar{\phi}_k) \text{ x } (y_{pk}^b - \gamma_{pk}) \quad \forall k \in S, \quad p = 1, \ldots, q \\
&\sum_{j \in A} \lambda_j^k = 1 \quad \forall k \in S \\
&\gamma_{pk} \le y_{pk}^b \\
&\bar{\theta}_k \ge 0, \quad \gamma_{pk} \ge 0 \quad \forall k \in S, \quad i = 1, \ldots, m \\
&\lambda_j^k \ge 0, \quad \forall k, l \in B, \quad \forall j \in A
\end{aligned}
\tag{M4}
$$

Model (M4) presents a constraint, setting an upper threshold on the feasible reduction in volatility. While the underlying premise of model (M4) predominantly revolves around a collection of DMUs, adjustments are required to target a reduction in a singular DMU. By refining model (M4) to cater for a solitary DMU with just one undesirable output, the IDEA model, termed model (M5) is derived.

$$
\begin{aligned}
&\max \gamma^* = \gamma_{pk} \\
&s.t. \\
&\sum_{j \in A} \lambda_j^k x_{ij} \le x_{ik} \quad i = 1, \ldots, m \\
&\sum_{j \in A} \lambda_j^k y_{rj}^g \ge (1 + \bar{\phi}_k) \text{ x } y_{rk}^g \quad r = 1, \ldots, s \\
&\sum_{j \in A} \lambda_j^k y_{pj}^b = (1 - \bar{\phi}_k) \text{ x } (y_{pk}^b - \gamma_{pk}) \quad p = 1, \ldots, q \\
&\sum_{j \in A} \lambda_j^k = 1 \\
&\gamma_{pk} \le y_{pk}^b \\
&\gamma_{pk} \ge 0 \\
&\lambda_j^k \ge 0, \quad \forall l \in B, \quad \forall j \in A
\end{aligned}
\tag{M5}
$$

Model (M5) introduces an IDEA model tailored to evaluate potential volatility reductions. Using the value of ϕ_k^* derived from model (M1), strategically, $\bar{\phi}_k$ is determined to ensure its value aligns with specific criteria. Crucially, $\bar{\phi}_k$ is established to be slightly below ϕ_k^*, that is $\bar{\phi}_k \le \phi_k^*$, exacted to three decimal points without rounding. The parameter γ^* in model (M5) stands for the least possible decrease in volatility, represented as $\gamma_{pk}^{\min}$. This numeric becomes pivotal in achieving one of the study goals. When a value of $\gamma^* = 0$, it indicates the existing volatility is at its lowest point, making further reductions infeasible. A discernible link materializes between the variable γ^* and the estimated parameter $\bar{\phi}_k$, leading to the formulation of a specific proposition and its subsequent validation. For computational purposes, the LINGO 20 solver was employed across all model evaluations in this research.

Theorem: This optimal state emerges precisely when the projected value of $\bar{\phi}_k = 0$, leading to the condition $y^b_{pk} = \gamma^{\max}_{pk}$.

Proof: In model (M5), set A encompasses a group of DMUs that are efficient, having delineated the efficiency frontier in model (M1) and thereby necessitating no further enhancement.

Thus, the constraints $\sum_{j\in A} \lambda^k_j y^g_{rj}$, and $\sum_{j\in A} \lambda^k_j y^b_{pj}$ in model (M5) apply to the DMUs that are efficient while $(1+\bar{\phi}_k)$ x y^g_{rk}, and $(1-\bar{\phi}_k)$ x $(y^b_{pk} - \gamma_{pk})$ applies to inefficient DMU_k. This proof is articulated through Eq. (5.2-5.4).

$$(1-\bar{\phi}_k) \text{ x } (y^b_{pk} - \gamma_{pk}) = 0 \tag{5.2}$$

$$\text{At } \bar{\phi}_k = 0,\ y^b_{pk} - \gamma_{pk} = 0 \tag{5.3}$$

$$y^b_{pk} = \gamma_{pk} = \gamma^{\max}_{pk} \tag{5.4}$$

Eq. (5.4) culminates and validates the proof of this theorem with a test in a later section. As mentioned earlier, the theorem addresses and resolves one of the research objectives and finalizes the proof.

Efficiency in multi-stock portfolios can often be achieved through diversification strategies. Distributing investments across assets with minimal correlation can potentially boost returns while mitigating volatility. Balancing heightened returns with controlled volatility in a stock portfolio remains a formidable task for fund managers. In this realization, a framework is developed to see the possibility of attaining efficiency in a portfolio with net zero volatility.

5.6 Net-zero portfolio risk initiative

Participating in stock market investments inherently involves an apparent degree of risk. The notion of achieving "net zero volatility" in an equity portfolio is often perceived as elusive, given the inherent risks present even in seemingly secure investments like treasury bills and government bonds. In some cases, the uncertainty may include the risks connected with inflation and interest rates, contributing to the total volatility observed in these assets. Nonetheless, alternative strategies can be utilized to reduce risk when engaging in stock portfolio investments. These strategies encompass diversification, asset allocation, dollar-cost averaging, hedging, and low-volatility asset investments. The present study only aims to evaluate the theoretical method of incorporating the concept of "net zero volatility" into stock portfolio investment. To achieve this objective, a comprehensive overview of the definitions and methods employed in this analysis are stated as follows.

Definition 1: Model (M5) pinpoints the juncture of optimal reduction in volatility (bad output). $y_{pk}^{\max}$ only occurs when $\overline{\phi}_k = 0$.

Definition 2: The feasibility of a net zero portfolio volatility centers on the scenario where $y_{pk}^{b} = \gamma_{pk}^{\max}$

The provided definitions address the goal of net zero volatility by underscoring that no discrepancy exists when the maximum reduction in volatility aligns with the actual volatility value. Fig. 5.1 provides a detailed representation of the methodological structure. The process commences with the evaluation of the inefficiency score for every DMU. Portfolios are deemed efficient and sorted into subset A if their $\phi_k^* = 0$. Conversely, portfolios are grouped into subset B if found to be inefficient, that is, their $\phi_k^* \succ 0$. For any inefficient portfolio, the utmost reduction in volatility, symbolized as $\gamma^{\max}$ is reached when the target $\overline{\phi}_k = 0$, thus nullifying its ϕ score. If the deduced maximum reduction mirrors the actual volatility value, it implies no divergence, suggesting a net zero portfolio risk for any inefficient portfolio to become efficient.

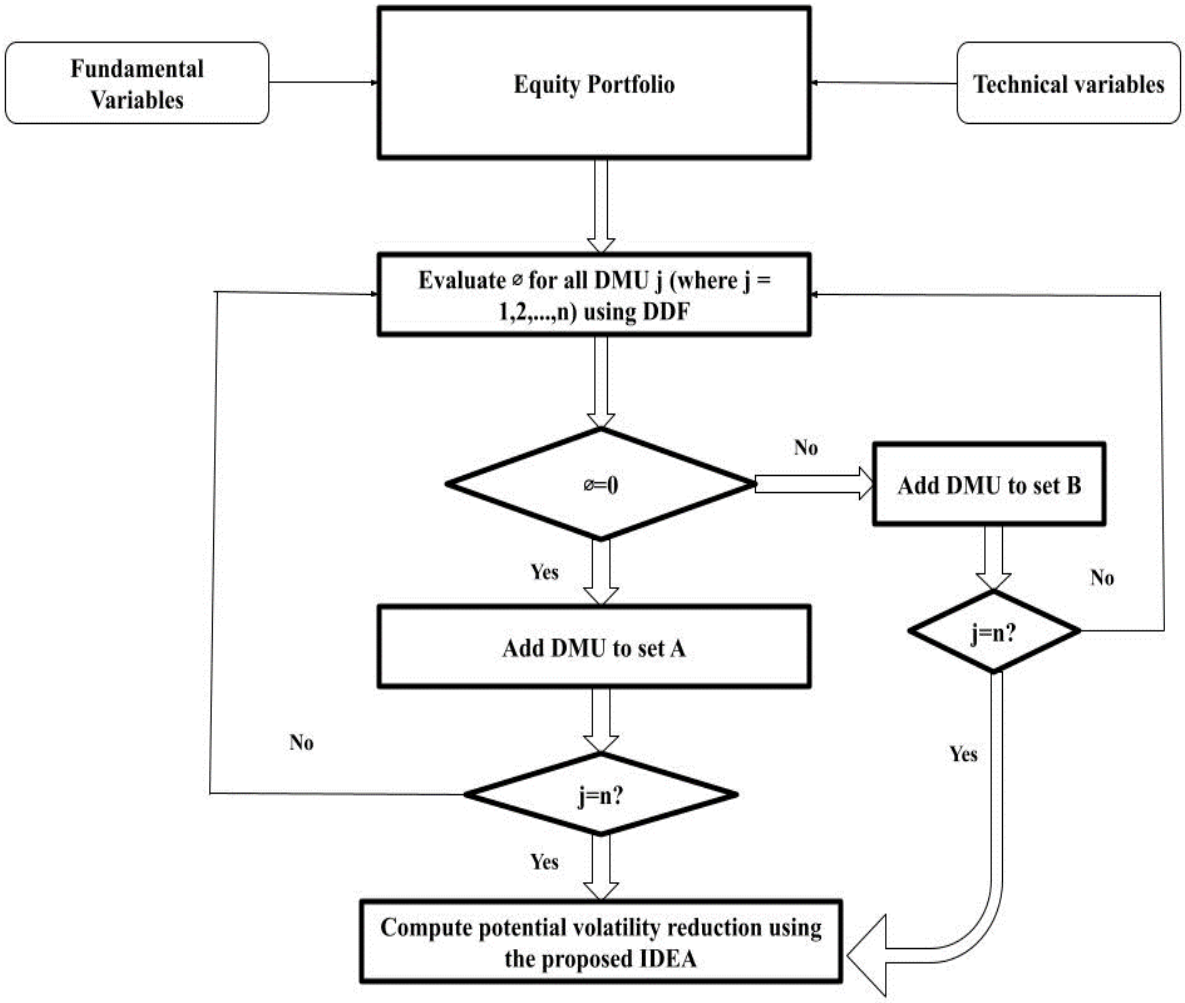

Fig. 5.1: Framework for net zero portfolio risk initiative.

5.7 Data collection and description

The data utilized in this research was acquired from the TEJ and contains a range of fundamental and momentum data for the fiscal year 2020. The said year was deliberately chosen due to its significance as part of the time frame encompassing the worldwide COVID-19 pandemic. This unprecedented pandemic tested the resilience of businesses and the performance of equities in TWSE. The dataset encompasses a comprehensive collection of data for 1,365 stocks, all categorized into 20 distinct industries listed on TWSE, where each industry is considered an individual portfolio. The breakdown of the industry is as follows: (1) Automobile = AU = 31 firms (2) Biotech. & Medical = BM = 121 firms (3) Building Material = BU = 78 firms (4) Chemical = CH = 41 firms (5) Comm. & Internet = CI = 86 firms (6) Computer & Peripherals = CP = 109 firms (7) Cultural & Creative = CC = 25 firms (8) Elec. Parts & Comp. = EC = 198 firms (9) Elec. Products Dist. = EP = 33 firms (10) Electric &

Machinery = EM = 91 firms (11) Foods = FO = 28 firms (12) Information Service = IF = 35 firms (13) Iron & Steel = IS = 45 firms (14) Optoelectronic = OE = 113 firms (15) Plastics = PL = 25 firms (16) Semiconductor = SC = 151 firms (17) Shipping & Trans. = ST = 28 firms (18) Textiles = TX = 53 firms (19) Tourism = TO = 40 firms (20) Trading & Cons. = TC = 34 firms. It is important to acknowledge that, in the process of compiling data, firms that exhibited gaps or missing data points between the commencement and conclusion of the fiscal year were deliberately removed.

Furthermore, the exclusion of financial-related enterprises, such as banks, from the original dataset is due to the presence of accounting standards and regulatory issues associated with such firms. The purpose of this exclusion is to maintain consistency with existing studies where firms with varying industry standards were eliminated (Jothimani et al., 2017; Kuo et al., 2021). Additionally, it is imperative to mention that the dataset employed in this work has three inputs and two outputs for each portfolio, which function as distinct DMUs. Three key views are employed to assess the DMUs, including liquidity, asset usage, and leverage. The current ratio, asset turnover, and solvency ratio are important as they offer valuable insights into the operational strategies and plans a firm implements. The viewpoints discussed are represented by the current ratio (K1), the asset turnover (K2), and the solvency ratio (K3), respectively. For the outputs, return (R) and volatility (σ) serve as indicators of the performance and risk of the DMUs. In the context of the present study, the measure of return is considered a desirable output, but risk, commonly referred to as volatility, is perceived as an undesirable output (Tarnaud & Leleu, 2018). These outputs are of specific significance as they relate to customer motivations for investment choices. Table 5.1 briefly describes variables, while Table 5.2 presents the descriptive statistics of the datasets.

Table 5.1: Variable description of datasets

Variables	Symbols	Description	Perception
Inputs	K1	The current ratio is expressed as a fraction of current assets to its current liability.	Liquidity
	K2	Asset turnover is expressed as a fraction of net sales to its mean total assets.	Asset Utilization
	K3	The solvency ratio is expressed as a fraction of total liability to its total assets.	Leverage
Outputs	R	The annual stock return is calculated as the cumulative daily stock returns.	Profitability
	σ	Volatility is calculated as a measure of the dispersion of daily stock returns.	Uncertainty

Table 5.2: Statistical representation of datasets

index	K1	K2	K3	R	σ
count	20	20	20	20	20
mean	275.7521	0.7639	44.9438	0.2268	0.0264
std	85.4976	0.3653	8.5394	0.1066	0.0057
min	164.6368	0.3308	32.1940	0.0946	0.0176
25%	215.8115	0.5801	39.5355	0.1354	0.0227
50%	252.6442	0.6721	42.4166	0.2131	0.0259
75%	341.4549	0.8527	52.3626	0.2712	0.0287
max	482.6248	2.1173	58.7947	0.4763	0.0431
variance	7309.8323	0.1334	72.9208	0.0114	0.0000

5.8 Model application

This section examines the results of applying the proposed model and approach to create a new dataset of 20 industry-based portfolios in TWSE. Table 5.3 displays the inefficiency scores of each DMU (industry) obtained through model (M1) utilization.

Table 5.3: Results from using the base model (M1) to estimate efficiency scores.

DMU	Industry	K1	K2	K3	R	σ	Inefficiency (∅)
1	AU	165.3803	0.5971	52.0781	0.1398	0.02676	0.0000
2	BM	482.6248	0.5057	33.4804	0.2457	0.03058	0.0000
3	BU	226.3188	0.3308	58.7947	0.1223	0.02075	0.0000
4	CH	285.4622	0.6998	39.1417	0.1431	0.02115	0.0000
5	CI	342.7023	0.7821	40.1926	0.1954	0.02789	0.2287
6	CP	218.7953	1.0585	46.865	0.2614	0.02693	0.1924
7	CC	356.1352	0.512	38.8936	0.297	0.04306	0.0000
8	EC	247.6624	0.7547	43.3909	0.2474	0.02943	0.1870
9	EP	192.1785	2.1173	56.5897	0.189	0.02169	0.0381
10	EM	235.1505	0.6158	44.8115	0.1186	0.02369	0.0000
11	FO	231.8879	0.8621	40.3361	0.1133	0.01764	0.0000
12	IF	280.8194	0.9726	40.648	0.2171	0.02508	0.1344
13	IS	206.8602	0.8496	48.3002	0.314	0.02127	0.0000
14	OE	257.6259	0.6272	41.4422	0.3795	0.03234	0.0000
15	PL	414.0608	0.632	33.5508	0.2626	0.02422	0.0000
16	SC	353.3933	0.7707	32.194	0.4763	0.03329	0.0000
17	ST	164.6368	0.6443	58.7457	0.4008	0.0231	0.0000
18	TX	320.1915	0.5289	39.6668	0.209	0.02675	0.0000
19	TO	192.116	0.518	56.5375	0.1093	0.02842	0.1995
20	TC	341.0391	0.8982	53.2159	0.0946	0.02378	0.2541

From Table 5.3, it is evident that using the DDF-DEA model (M1), the proportion of efficient portfolios is 65%, indicating that 13 out of 20 portfolios meet the criteria for efficiency. This observation is indicative of a positive trend, as it suggests that the majority of portfolios are demonstrating strong performance and have effectively optimized their input and output variables. The proportion of inefficient portfolios is 35%, indicating that 7 out of 20 portfolios exhibit inefficiency. This issue raises concerns as it suggests that specific portfolios are inefficiently utilizing resources or failing to capitalize on opportunities for enhancing their performance. The set of efficient portfolios includes AU, BM, BU, CH, CC, EM, FO, IS, OE, PL, SC, ST, and TX, corresponding to DMUs (1, 2, 3, 4, 7, 10, 11, 13, 14, 15, 16, 17, 18). More so, the set of inefficient portfolios includes CI, CP, EC, EP, IF, TO, and TC, corresponding to DMUs (5, 6, 8, 9, 12, 19, 20). The ranking of inefficient portfolios, arranged in ascending order of inefficiency, is as follows: TC, CI, TO, CP, EC, IF, and EP, corresponding to DMUs (20, 5, 19, 6, 8, 12, 9). The portfolios exhibit a range of inefficiency scores, spanning from 0.2541 to 0.0381. The findings indicate that the portfolios characterized by inefficiency exhibit varying degrees of inefficiency. The IDEA model can be employed to identify the potential reduction in undesirable output to enhance efficiency. A potential decrease in the undesirable output of any inefficient portfolios or DMUs may result in the portfolio becoming efficient. Hence, specific portfolios may exhibit a greater potential for improvement when compared to others. The utilization of the IDEA model facilitates the determination of the optimal direction to estimate the possible and maximum possible decrease in the volatility level of non-efficient portfolios. Accordingly, two distinct sets are established to symbolize the assemblage of the two categorizations of chosen portfolios. Set A comprises all the efficient DMUs, whereas Set B encompasses all the inefficient DMUs, as shown in Eq. 5.5-5.8.

$$Set\ A\ =\ DMUs\ \left(AU,\ BM,\ BU,\ CH,\ CC,\ EM,\ FO,\ IS,\ OE,\ PL,\ SC,\ ST,\ TX\right) \tag{5.5}$$

$$Set\ A\ =\ DMUs\ \left(1,\ 2,\ 3,\ 4,\ 7,\ 10,\ 11,\ 13,\ 14,\ 15,\ 16,\ 17,\ 18\right) \tag{5.6}$$

$$Set\ B\ =\ DMUs\ \left(CI,\ CP,\ EC,\ EP,\ IF,\ TO,\ TC\right) \tag{5.7}$$

$$Set\ B\ =\ DMUs\ \left(5,\ 6,\ 8,\ 9,\ 12,\ 19,\ 20\right) \tag{5.8}$$

The proposed IDEA model (M5) is adopted to assess the potential and maximum volatility reduction in individual portfolios. The potential reduction of volatility is the immediate and feasible decrease that can be achieved, while the maximum reduction of volatility is the ultimate and optimal decrease that can be attained. Table 5.4 shows the potential and maximum volatility reduction for each DMU in Set B. The portfolios are ordered from the lowest to the highest deviation. The ranking of the portfolios is DMUs ($20 \leq 5 \leq 6 \leq 12 \leq 9 \leq 8 \leq 19$). The ranking of the portfolios shows that DMU 20 has the most negligible residual volatility after applying the maximum volatility reduction measures, followed by DMU 5 and DMU 6. On the other hand, DMU 19 has the most considerable residual volatility after applying the maximum volatility reduction measures, followed by DMU 8 and DMU 9. Table 5.4 presents the IDEA results due to the application of model (M5) on all the inefficient portfolios or DMUs. Fig. 5.2 and Fig. 5.3 display the graphical representation of volatility reduction and deviations.

Table 5.4: Volatility reduction analysis

DMU	Volatility (σ)	Potential Volatility reduction	Max. Volatility Reduction	Deviation
5	0.02789	0.00002911	0.007610	0.020280
6	0.02693	0.00001658	0.006612	0.020318
8	0.02943	0.00004598	0.007267	0.022163
9	0.02169	0.00000192	0.000827	0.020863
12	0.02508	0.00001329	0.004401	0.020679
19	0.02842	0.00001948	0.005672	0.022748
20	0.02378	0.00000470	0.006140	0.017640

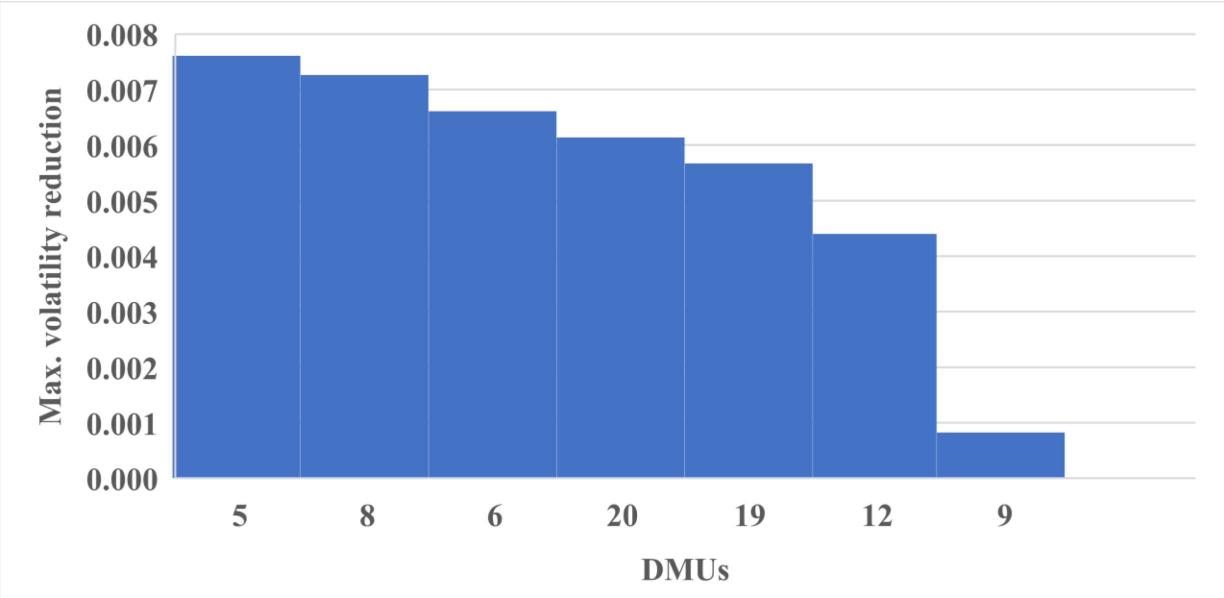

Fig. 5.2: Volatility reduction bar graph.

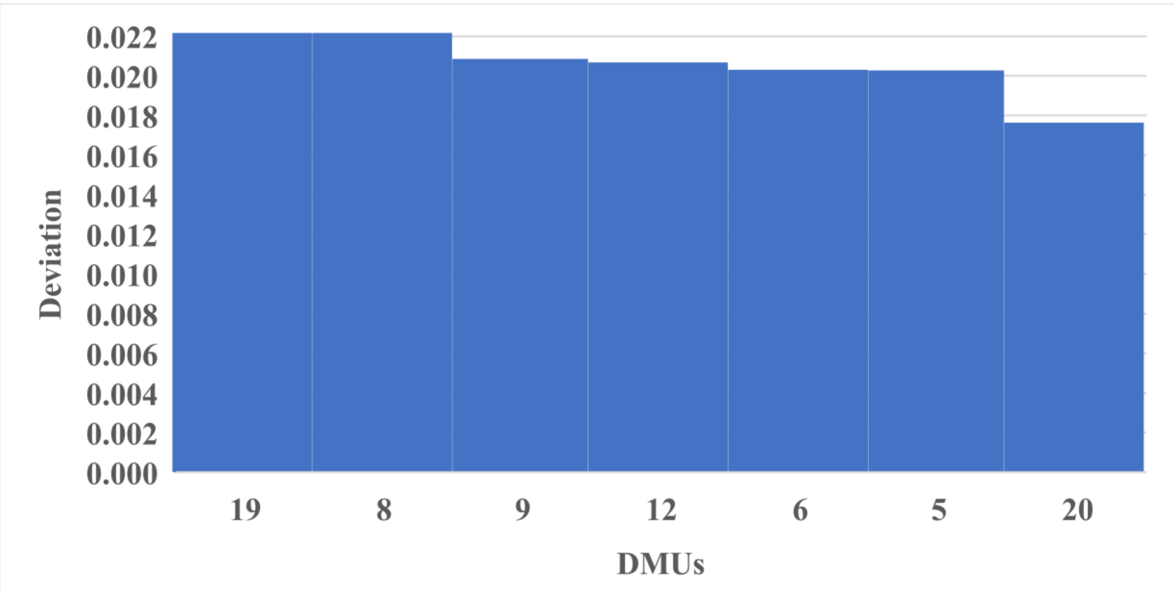

Fig. 5.3: Volatility deviation bar graph.

The present analysis showcases the successful implementation of the proposed IDEA technique in mitigating volatility for inefficient portfolios. However, it is essential to consider that the proposed reduction is a theoretical maximum, and the actual achievable decrease may be smaller due to several factors, such as market conditions and uncertainties associated with stock data. The relationship between the variations of ϕ and the possible volatility reduction is depicted in Fig. 5.4-5.10. The volatility curve is obtained by implementing the net zero volatility algorithm. In every case, the inefficiency scores were partitioned into equivalent intervals to estimate the corresponding reduction in volatility. The plots depict that a maximum reduction will always occur at $\phi = 0$.

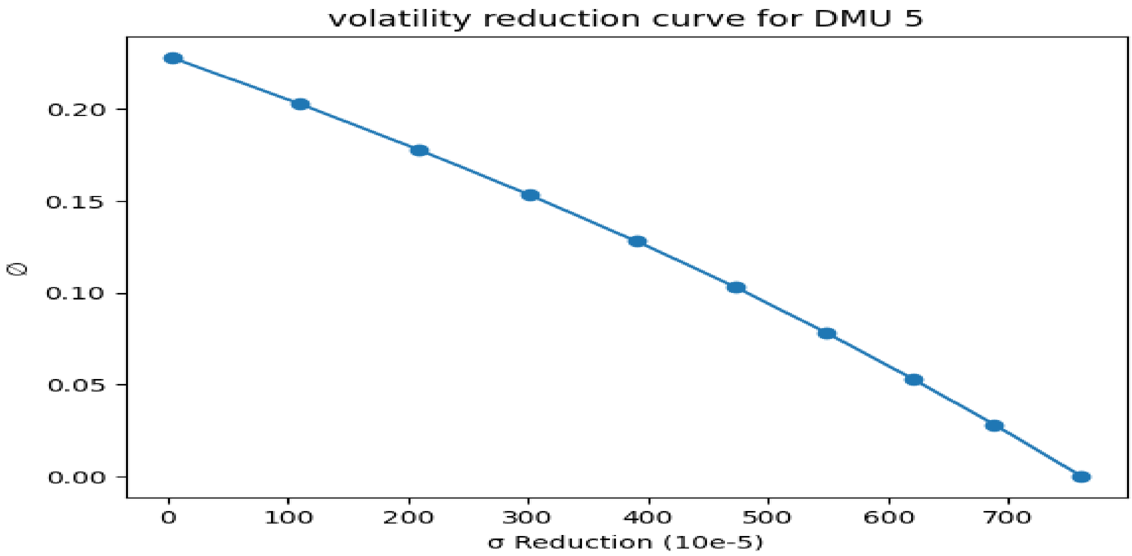

Fig. 5.4: Volatility curve for DMU 5.

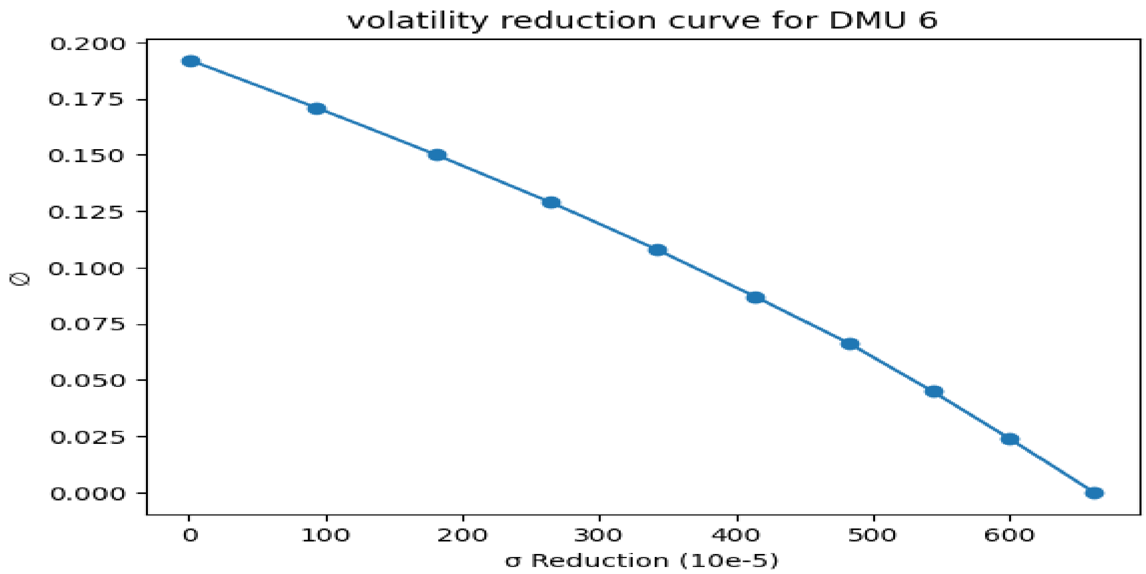

Fig. 5.5: Volatility curve for DMU 6.

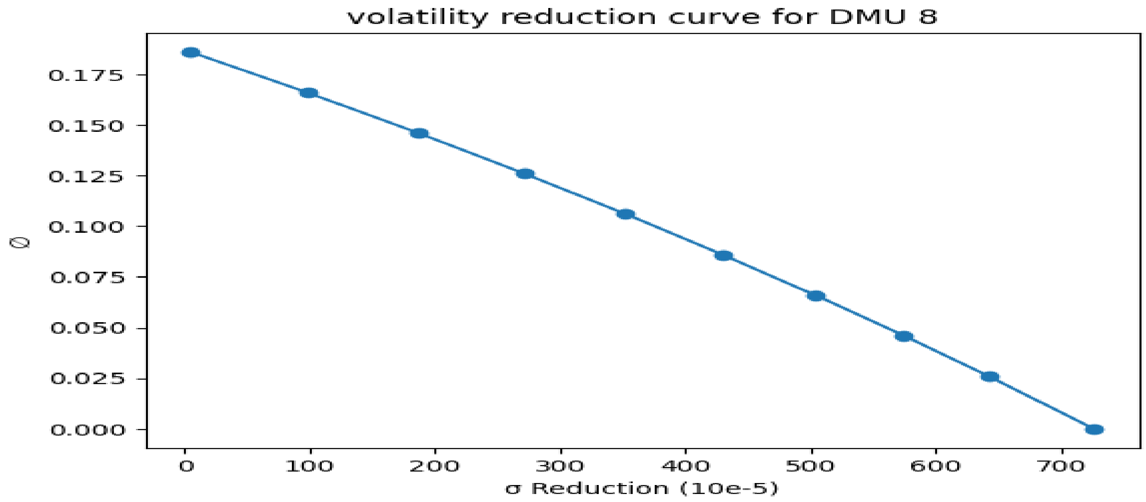

Fig. 5.6: Volatility curve for DMU 8.

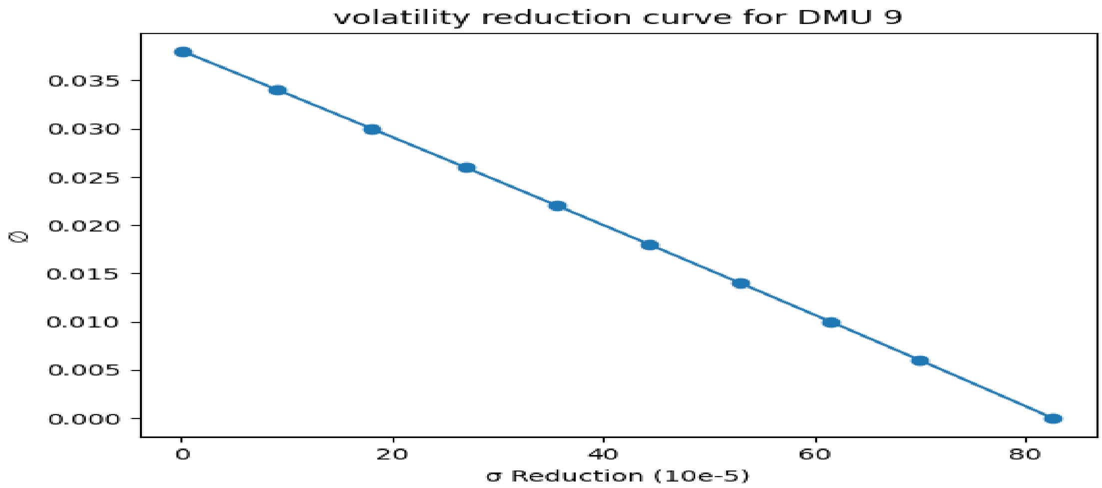

Fig. 5.7: Volatility curve for DMU 9.

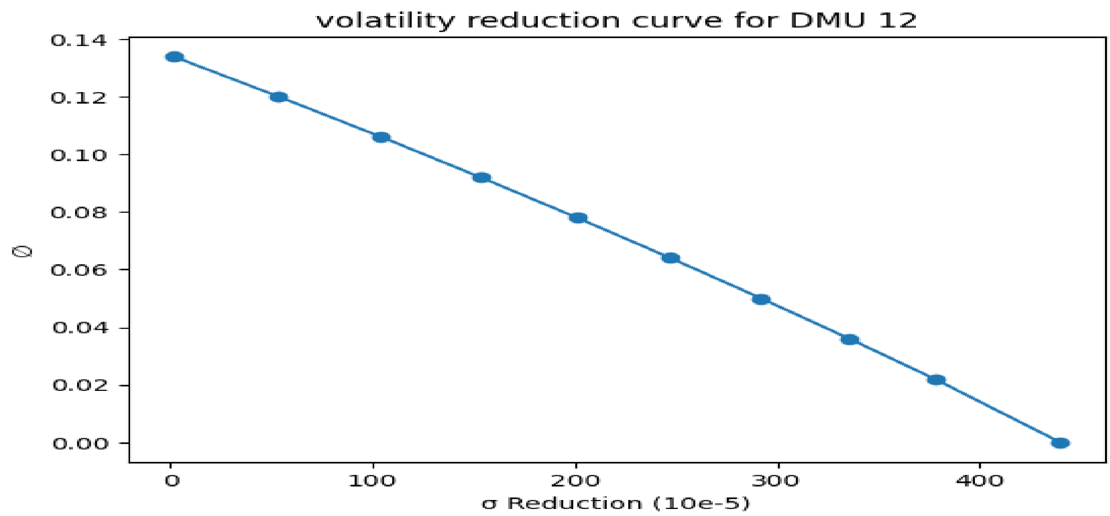

Fig. 5.8: Volatility curve for DMU 12.

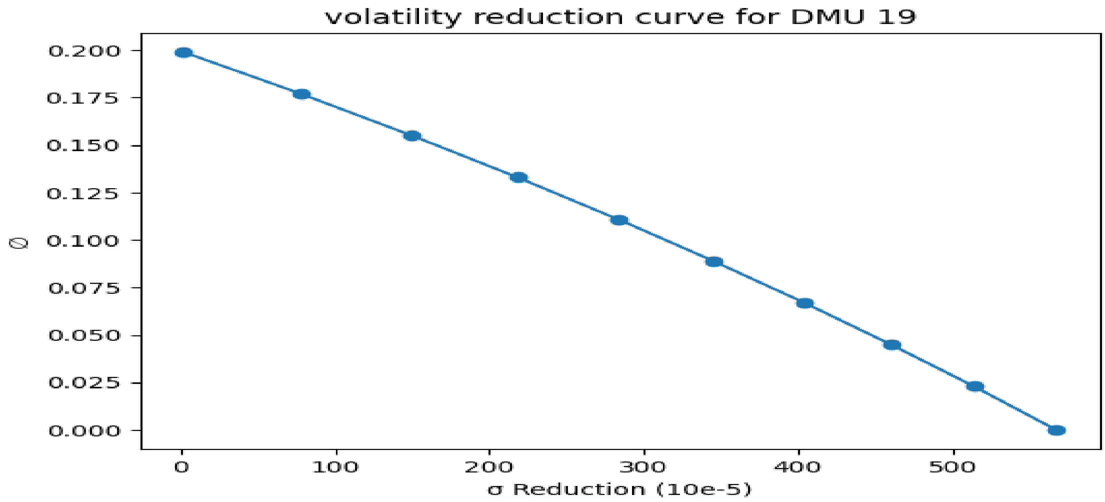

Fig. 5.9: Volatility curve for DMU 19.

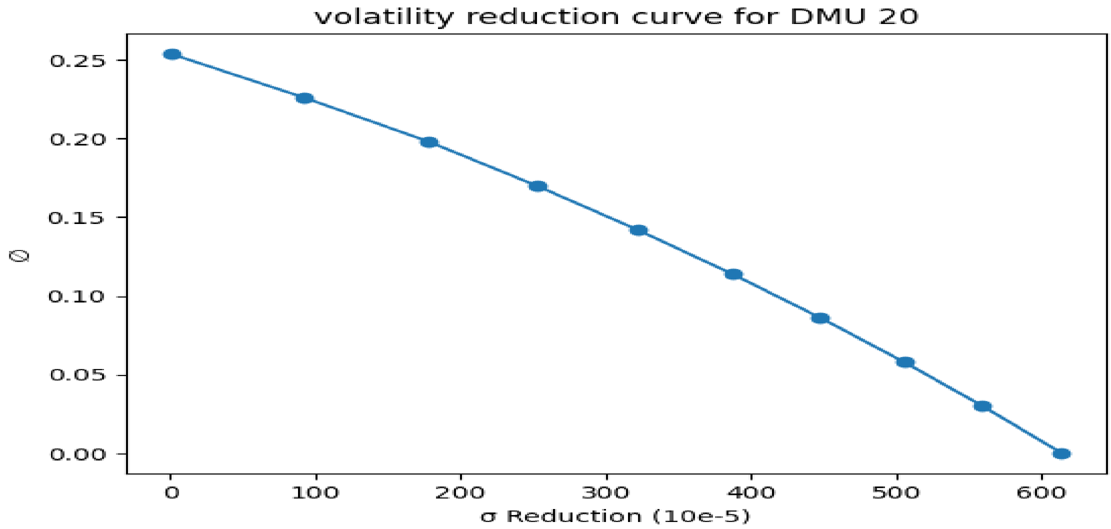

Fig. 5.10: Volatility curve for DMU 20.

This study introduces a theoretical method to estimate the reduction in volatility (undesirable output) so that all inefficient portfolios in set B become efficient. The proposed framework offers a systematic method for identifying the theoretical and optimal possibilities for lowering volatility, thus guiding strategic decisions to improve efficiency. The real-life application of the proposed model is possible through effective management strategies. Habib and Hasan (2017) analyze the impact of managerial competence on investment efficiency and the risk of stock price crashes. The researchers employ a metric of management aptitude DEA and ascertain that managers with higher levels of ability have a tendency to engage in excessive investment relative to their less capable counterparts, even when accounting for variables such as financial reporting quality and other relevant aspects. Their findings provide empirical evidence for the rent extraction theory, a theoretical framework that posits that managers with higher levels of ability may prioritize their personal interests over the interests of shareholders. With appropriate strategies by competent managers, the practical implementation of the proposed strategy is feasible.

5.9 Verification

To determine the robustness of the proposed model in achieving study objective, a verification test was conducted on all the inefficient DMUs in Set B using the base model (M1). The original volatility values were replaced with revised volatility values in the model (M1). The new volatility is derived by estimating the difference in maximum volatility reduction away from the initial risk. After implementing this substitution, the ϕ values of all affected DMUs were re-evaluated using the base model (M1) to determine their new ϕ values. The results are briefly displayed in Table 5.5.

Table 5.5: Verification analysis on portfolios

DMU	Initial σ	Final σ	Initial ∅	Final ∅
5	0.02789	0.020280	0.2287	0
6	0.02693	0.020318	0.1924	0
8	0.02943	0.022163	0.1870	0
9	0.02169	0.020863	0.0381	0
12	0.02508	0.020679	0.1344	0
19	0.02842	0.022748	0.1995	0
20	0.02378	0.017640	0.2541	0

The verification analysis on DMUs in subset B is presented in Table 5.5. Based on the available data, it is evident that substituting the initial risk with an updated risk value has a discernible impact on the ϕ ratings of the DMUs. The initial and final volatility comparison seems to pertain to a portfolio risk assessment method. The Final volatility is seen to be lower than the Initial volatility for each DMU listed. This observation indicates a general decrease in the risk measure across all the affected DMUs. Also, there was a noticeable distinction between the initial ϕ and the final ϕ. Upon verification, it has been seen that all the DMUs have experienced ϕ reduction to 0, suggesting that they have achieved a state of efficiency. After revising the risk measurements, a discernible enhancement in the efficiency of the portfolios has been seen. All the previously inefficient ϕ scores have decreased to zero. This suggests that the initial inefficiencies were closely associated with the volatility values (expressed by σ), and by modifying these volatility values, the portfolios have achieved efficiency. The said result provides a positive indicator of the robustness of the proposed method. The findings of this study confirm that the developed models accurately predict the extent to which volatility can be reduced in portfolios. This reduction in volatility can potentially address inefficiencies in the portfolios under consideration to become efficient. The verification method imbues a sense of confidence in applying the proposed model to identify possible and potential reductions in volatility within portfolios. Moreover, it guarantees the proposed model's robustness in accurately predicting efficiency enhancements that can be achieved through IDEA.

5.10 Summary

Taiwan, frequently recognized as an exemplary economic development model in Asia, has consistently exhibited robust economic expansion, notable technological expertise, and a vibrant stock market. Moreover, the fiscal year 2020 encompassed a time of unparalleled difficulties and disturbances within worldwide financial markets due to the COVID-19 pandemic. The said year presents a unique perspective as companies and industries have confronted the negative consequences of the pandemic; the year presented a formidable challenge to corporations, testing their durability, resilience, agility, and financial stability. In unfolding this novel research methodology, 20 industry-based portfolios were assessed as a financial production process, explicitly examining their performance in terms of returns (desirable output) and volatility (undesirable output) to gain insights. This study proposes an inverse optimization approach, referred to as IDEA. This work combines technical and fundamental indicators to enhance the performance of industry-based portfolios in the TWSE. The inefficiency scores for these portfolios were determined using the DDF-DEA. The findings assessed 1365 equities to create 20 industry-based portfolios in TWSE, indicating that just 7 of the 20 industry portfolios exhibited underperformance. Nevertheless, our model posits that by making slight reductions in volatility, these portfolios have the potential to achieve optimal efficiency.

Additionally, this chapter proposed the net zero portfolio risk initiative for the first time in the literature. The essence of this goal is to optimize a portfolio towards becoming a "safe-haven" asset for investors and analysts. The real-world applicability of our approach is strengthened by a verification test, enhancing its robustness. Through a focused examination of the year 2020, the study delves into a distinct economic era, intensifying the complexity of the optimization quandary and enhancing the significance and reliability of the proposed models.

Chapter 6 – Advancing Stock Market Index Prediction with Transformer Model: Leveraging TPE for the Optimization of Hyperparameters

6.1 Introduction

Forecasting financial market trends, particularly regarding the stock market, is a topic that attracts considerable attention and holds great significance. This is primarily due to the volatile and unpredictable nature of stock prices. Stocks are financial assets that represent fractional ownership in a firm, providing the potential for financial gains as the company's value rises. The impressive upward trend demonstrates the growing significance and prospects of investments in equities. Significantly, the US has established itself as the leading participant in this field, exerting control over the largest portion of global stock holdings as of 2023, thereby solidifying its status as a financial hub in the global equities market. However, it is crucial to recognize that this undertaking carries inherent risks, especially due to the volatile nature of stock price movements.

Significant fluctuations undermine the stability of global financial systems (Anagnostidis et al., 2016), as demonstrated by the occurrences during the 2008 financial crisis (Apergis & Dastidar, 2024) and the 2020 COVID-19 pandemic (Gao et al., 2022). Historically, technical and fundamental analysis approaches have been the foundation for predicting stock market trends (Krishnapriya & James, 2023). Based on the historical development of methods for predicting stock market behaviour, it is evident that the sector has experienced substantial changes, particularly in terms of tools for analyzing data. Initially, statistical techniques such as moving average, ARMA, ARIMA, ARCH, and GARCH were employed (Kehinde, Chan, & Chung, 2023; Kumbure et al., 2022). Although these methods are efficient in terms of time and are efficient when certain assumptions are met, they generally fail to uncover non-linear relationships in large market data. As a result of this limitation, researchers have investigated several machine learning methods, such as KNN, DT, SVM, and RF (Kehinde, Chan, & Chung, 2023). Even though these techniques showed significant progress in predicting stock market trends, they faced difficulties identifying and analyzing complex features and patterns in more complex stock market situations (Tao et al., 2024).

Nonetheless, the emergence of deep learning algorithms has expanded the novelty of stock market prediction using neural networks. Models like ANN, CNN, RNN, LSTM, and GRU have shown exceptional abilities to uncover hidden patterns in large sets of historical stock market data, effectively dealing with the intricacies present in unpredictable financial settings

(C. Wang et al., 2022). The works of Jiang (2021), Kumbure et al. (2022), Nazareth and Reddy (2023), and Kehinde, Chan and Chung (2023) highlight the increasing dependence on these sophisticated models to offer a more profound understanding of market dynamics. Nevertheless, the difficulty remains in effectively handling the significant fluctuations in stock prices and the potential deterioration of insights obtained from past data. These problems emphasize the need for continuous demands for models that could comprehend complex patterns and adjust to the chaotic nature of stock markets, taking into account abrupt shifts and the non-linear interaction of different market components. The changing environment emphasizes the need for ongoing innovation in developing and optimizing models to improve the accuracy and reliability of stock market predictions.

Going forward, these challenges have been successfully resolved by the implementation of the Transformer architecture by Vaswani et al. (2017), representing a noteworthy achievement in deep learning. Transformer is a neural network that has proven increasingly popular. OpenAI has recently employed transformers in their language models, while DeepMind has also recently utilized them in AlphaStar. The attention mechanism, in theory, has the ability to reference an unlimited window of information, provided there are sufficient computational resources. The Transformer model, which includes a self-attention mechanism, significantly benefits its ability to learn in parallel. This model employs an attention mechanism to capture global dependencies more efficiently than conventional ones relying on sequential processing. Inspired by the significant achievements of Transformer models in other domains, researchers progressively embrace this methodology in predicting financial time-series data.

The utilization of Transformers in stock market forecasting has been pioneered by Liu et al. (2019), Ding et al. (2020), Yoo et al. (2021) and Zhang et al. (2022). However, a significant pattern observed in these approaches is the utilization of transformers primarily for examining social media data, which is unstructured, and generating sentiment data as supplementary inputs rather than primarily concentrating on improving the direct extraction of features from past financial data (structured). This rationale to deal with historical data only aligns with the fundamental ideology of technical analysis, that the price of a stock reflects all factors. Social media poses a notable difficulty, as social data from many sources is undoubtedly filled with uncertainties (Tao et al., 2024). The factors that make social media full of uncertainties include the techniques used to gather data, the range and reliability of data sources, and the bias of commentators, which contribute to a significant degree of uncertainty. As a result, these uncertainties can lead to an uneven and unstable performance of these models in various stock market conditions. This issue highlights the need for a more reliable implementation of Transformer models that can effectively process and extract significant patterns from past stock data, reducing the need for less reliable external social data. This approach has the potential to provide a more robust and more dependable framework for predicting stocks, capable of effectively managing its complexities and unpredictable nature of markets. The utilization of the Transformer model for analyzing unstructured data has gained significant popularity. However, its implementation on structured datasets, such as stock market technical indicators, is still in its early stage (Tao et al., 2024; C. Wang et al., 2022).

In this study, applying the Transformer model, which has demonstrated success in NLP, language translation, and speech processing, is a remarkable progression in predicting the stock market. This study uses state-of-the-art deep learning techniques to forecast the performance of three prominent global stock indices: S&P 500, FTSE 100, and HSI. A wide range of explanatory variables is considered to predict the index values for the following day. However, to simplify the complexity, the closing price is selected as the central focus due to its strong correlation with other variables and its importance in stock market assessment. This study aims to comprehensively assess the predictive capabilities of the Transformer model by employing structured data and comparing its performance to RNN and variants of RNN such as LSTM and GRU. Furthermore, this study examines the impact of hyperparameter optimization using a Tree-structured Parzen Estimator (TPE) to improve prediction accuracy. Evaluation is conducted by utilizing dissimilarity-based metrics such as RMSE, MAPE, and MAE and a similarity-based statistic called R squared. Additionally, this study evaluates the computation demand in terms of runtime performance using Google Colab hardware accelerators for model training. Last, this study implemented a simple trading strategy to emphasize the investment reward of each model under evaluation when subjected to real-life trading scenarios.

6.2 Model configurations

6.2.1 RNN configuration

The most challenging part of the time series prediction problem is how to model all the interdependent data. RNN is one of the earliest attempts, and it solves this issue by inserting a memory cell, an internal state that stores historical data. Although RNN can accurately characterize the contextual relationship between sequential data, this relationship weakens as the gap between them grows. Back-propagation issues, including disappearing gradients and gradient explosions, have been linked to RNNs' propensity for long-term reliance (Huang et al., 2019). RNNs are notable among neural networks for their distinctive capacity to handle sequential input, making them especially well-suited for analyzing time-series data such as stock market predictions. RNNs, in contrast to CNNs, are specifically built to preserve a "memory" of previous inputs within their internal state and use earlier outputs as successive inputs, which allows them to demonstrate temporal dynamics and contextual awareness (Goodfellow et al., 2016). It is critical in financial contexts to consider the sequence of past data points as it greatly influences the ability to forecast future trends. RNNs accomplish this by iterating through sequential steps, modifying their internal state at each step based on both the current input and the preceding state. RNNs utilize this technique to preserve and acquire knowledge from sequential patterns in the data, which is crucial for comprehending and predicting stock market fluctuations by leveraging historical trends. The mathematical formulation of this configuration is expressed in Eq. 6.1.

$$\begin{aligned} h_t &= \tanh(W^x \mathrm{x}_t + W^h h_{t-1} + b^h) \\ y_t &= W^y h_t + b^y \end{aligned} \tag{6.1}$$

Where the weight matrix W^x is applied to the input, the weight matrix W^h is used to map values from the previous hidden layer to the next one. The weight matrix W^y is responsible for mapping the values of the current hidden layer to the output layer. In Eq. 6.1, h_t represents the hidden state at time step t, h_{t-1} represents the hidden state at previous time step t-1, x_t represents the input vector at time step t, y_t represents the output vector at time step t. In addition, the variable, b^y, represents the bias vector for the output, while the variable, b^h, represents the bias vector for the hidden layer. Tan h is the activation function used to maintain stability in gradients during training.

Optimizing the performance of an RNN for stock market index prediction requires several crucial considerations. The model often begins with an input layer specifically built to receive a sequence of historical data, such as a window of past closing prices of a stock index. Next are the hidden layers, where the processing takes place. These layers are structured with a specific quantity of neurons, and these layers are accountable for modifying the internal state of the RNN according to the input sequence. The output layer is responsible for generating the predictive output, representing the closing price of the stock market for the following day. While configuring an RNN, important factors include selecting appropriate activation functions, such as tanh or ReLU, which impact the model's capacity to capture non-linear patterns, and designing the network architecture by determining the number of hidden layers and neurons. A loss function directs the learning process of the model, often MSE for regression tasks, and an optimizer like Adam or RMSprop, which reduces the loss. In order to improve the model's ability to generalize and avoid overfitting, regularisation approaches such as dropout procedures are expected. The cumulative effect of these variables is crucial in determining the accuracy of the RNN's stock market trend forecasting. Fig. 6.1 illustrates a typical structure of an RNN model.

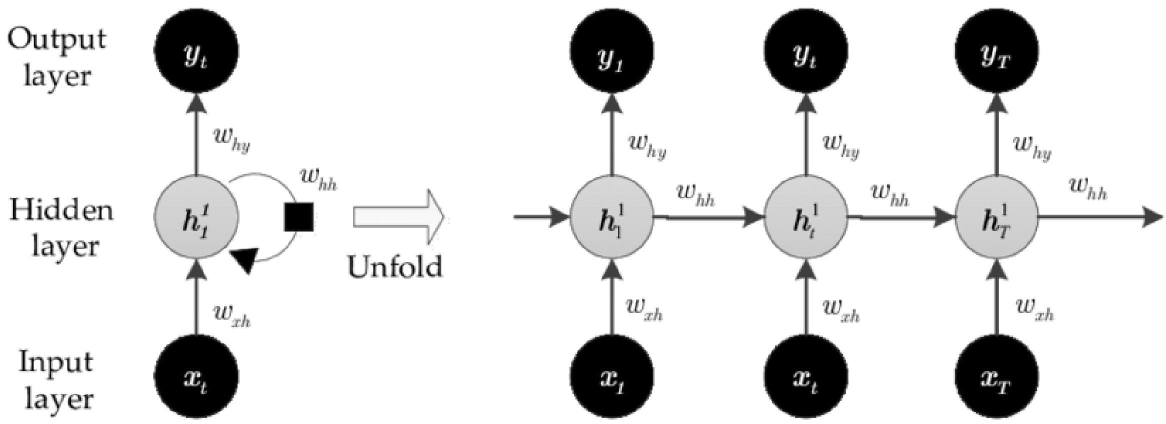

Fig. 6.1: RNN model configuration.

6.2.2 LSTM configuration

LSTM is a type of RNN that can enhance models with a specific gate structure. LSTM was designed by Hochreiter and Schmidhuber (1997) to overcome the difficulty of learning long-term dependencies, a common problem in ordinary RNNs. The fundamental concept of LSTM is to retain information for long durations, making it exceptionally efficient for analyzing sequential data in which the significance of information extends over a substantial time frame, especially in domains like financial time series forecasting. LSTMs accomplish this by employing a distinctive internal configuration of three gates: the forget, input, and output. The forget gate selects which information should be eliminated from the cell state, the input gate modifies it with fresh information, and the output gate controls the flow of information from the memory cell to the output of the LSTM unit. The gating mechanism efficiently controls the information flow in the LSTM, enabling it to store or reject input across extended periods selectively. This helps address problems such as vanishing or exploding gradients frequently encountered with conventional RNNs. The mathematical formulation of this configuration is expressed in Eq. 6.2.

$$
\begin{aligned}
f_t &= \sigma(U^f \mathrm{x}_t + W^f h_{t-1} + b^f) \\
i_t &= \sigma(U^i \mathrm{x}_t + W^i h_{t-1} + b^i) \\
o_t &= \sigma(U^o \mathrm{x}_t + W^o h_{t-1} + b^o) \\
g_t &= \tanh(U^g \mathrm{x}_t + W^g h_{t-1} + b^g) \\
c_t &= f_t \odot c_{t-1} + i_t \odot g_t \\
y_t &= h_t = o_t \odot \tanh(c_t)
\end{aligned}
\tag{6.2}
$$

In Eq. 6.2, the symbol $\odot$ represents the Hadamard product. The symbols f, i, o, g, c, and h subsequently indicate the forget gate, the input gate, the output gate, the classic RNN gate, the cell state, and the hidden state. Also, U, W, and b represent the input weight matrices, recurrent weight matrices, and bias vectors of different gates within the model.

When setting up an LSTM model to predict stock indices, specific components and parameters are carefully adjusted to maximize the model's effectiveness. The input layer is specifically designed to process sequential data, sometimes including previous stock values within a defined time frame. The core of the LSTM model is in its hidden layers, comprising a sequence of LSTM units or cells, each equipped with the aforementioned gating mechanisms. The quantity of hidden layers and the quantity of LSTM units within these layers are critical characteristics that require meticulous adjustment since they profoundly influence the model's capacity to acquire and comprehend intricate patterns in the input. The LSTM model commonly utilizes a fully connected layer for the output layer, which generates the prediction. In the case of stock market indices, this prediction corresponds to the anticipated future price. The selection of activation functions, such as tanh or ReLU, used in the LSTM gates is crucial for improving the model's ability to learn and process non-linear patterns. The training procedure of the model entails the selection of a suitable loss function, such as MSE for regression tasks, and an optimizer, typically Adam or RMSprop, to minimize the loss. Furthermore, in order to improve the model's resilience and avoid overfitting, it is possible to employ regularisation methods like dropout. Dropout layers in neural networks selectively deactivate a fraction of neurons at random during the training process, which helps create a more generalized model. A typical LSTM architecture illustrated in Fig. 6.2 is designed to capture the underlying temporal dynamics and complexities in stock market data, making it an invaluable tool for forecasting financial time series.

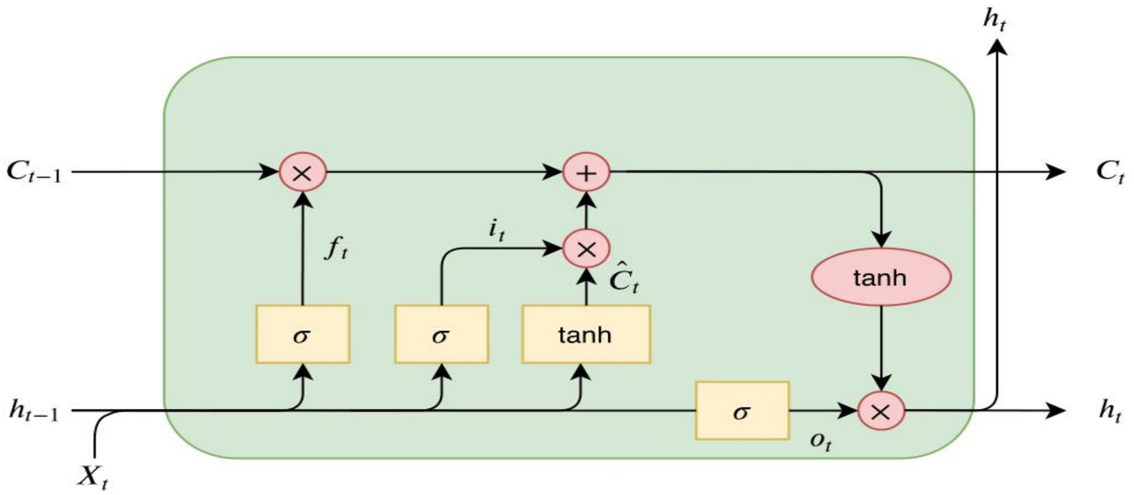

Fig. 6.2: LSTM model configuration.

6.2.3 GRU configuration

Cho et al. (2014) presented GRU as a variant of RNN. Similar to LSTM with a forget gate, the GRU lacks an output gate and has fewer parameters than an LSTM. GRU networks are a variant of RNNs designed explicitly for handling sequential data. They are comparable to LSTM networks in their ability to process time series data, but they have a more straightforward architectural structure. GRUs were developed as a substitute for LSTMs to tackle the difficulty of capturing long-term relationships in time series data. While LSTM incorporates distinct forget, input, and output gates, GRU employs an update gate and a reset gate to regulate the flow of information. This makes it well-suited for tasks like stock market prediction, where the impact of past data can span several periods. In addition, they combine the cell state and hidden state, leading to a more concise and computationally optimized structure. GRUs utilize a streamlined gating mechanism to regulate the information flow, eliminating the requirement for a distinct memory unit. This enables them to effectively preserve important information over extended periods and overcome the issue of disappearing gradients commonly encountered in conventional RNNs. The properties of GRUs frequently result in accelerated training times and a reduced need for extensive data to achieve generalization, rendering them an attractive option for sequence modelling problems. The mathematical formulation of GRU configuration is expressed in Eq. 6.3.

$$
\begin{aligned}
h_t &= (1 - z_t) \odot h_{t-1} + z_t \odot \overline{h}_t \\
\overline{h}_t &= g(U^h (r_t \odot h_{t-1}) + W^h x_t + b^h) \\
z_t &= \sigma(U^z h_{t-1} + W^z x_t + b^z) \\
r_t &= \sigma(U^r h_{t-1} + W^r x_t + b^r)
\end{aligned}
\tag{6.3}
$$

The variables h_t and h_{t-1} represent the output of the current and prior states, respectively, while r_t and z_t represent the reset and update gates, respectively.

When setting up a GRU model to forecast stock market indices, various elements and parameters are taken into account to customize the network's structure for the best possible performance. The model commences with an input layer that receives sequential data, usually historical stock prices, spanning a specified time interval. The central component of the GRU model consists of its hidden layers, which comprise several GRU units. The responsibility of these units is to process the input data and the information from previous time steps. This is accomplished through the management of the update and reset gates. The quantity of GRU units and the extent of hidden layers are important variables that greatly impact the model's learning capacity and capability to capture intricate temporal patterns in the stock market data. The output layer of the GRU model, often implemented as a dense layer, is specifically designed to generate the ultimate prediction output, such as the anticipated future price of a stock index. Important factors to consider in the setup of the GRU also involve selecting activation functions, such as tanh or ReLU, which control the gate operations and data translation inside the network. Training the GRU model entails carefully selecting a suitable loss function, such as MSE for regression tasks, and an optimizer, typically Adam or SGD, to progressively minimize the loss via iterations. Regularisation methods such as dropout can mitigate overfitting and boost the model's generalization ability. This involves randomly deactivating a portion of the neurons during the training process. A typical GRU architecture illustrated in Fig. 6.3.

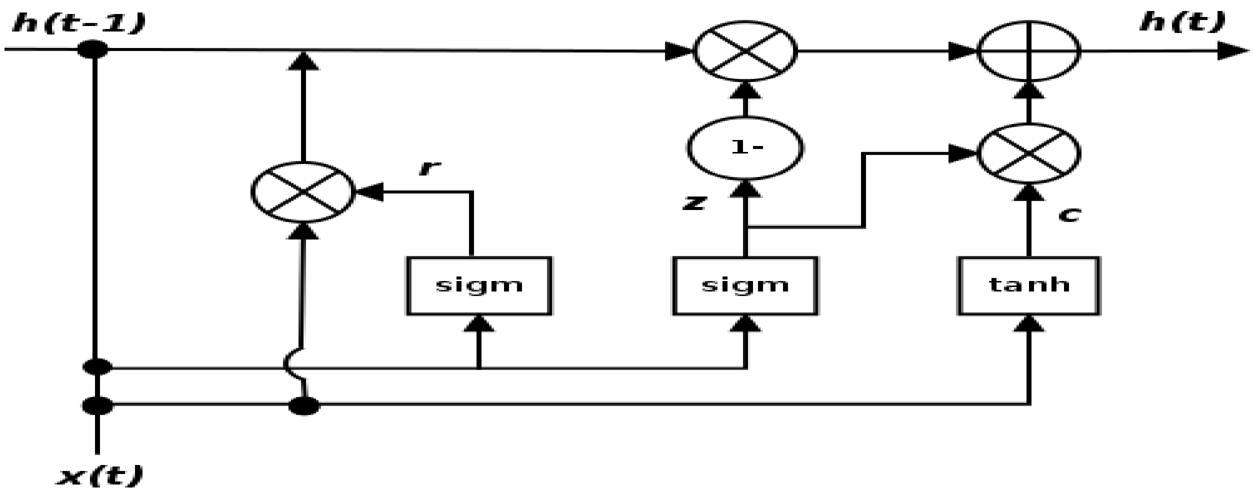

Fig. 6.3: GRU model configuration.

6.2.4 Transformer configuration

Vaswani et al. (2017) proposed the technique of Transformer model, which marks a notable departure from the conventional recurrent architectures commonly employed in sequence modelling tasks such as stock market prediction. Unlike RNN and its variants, the Transformer model does not handle data sequentially. Instead, it employs a technique known as 'attention', which enables it to process large data sequences concurrently. The capacity to handle tasks simultaneously dramatically improves the efficiency and scalability of the model. The fundamental concept behind the Transformer is to represent the connections between various elements in a sequence, irrespective of their relative positions. This allows the model to comprehend complex interdependencies that could be disregarded by models focusing only on immediate or nearby contexts. The Transformer excels at managing distant relationships in data, which is vital when working with extensive time-series data such as stock market indexes, where previous events might have prevailing effects.

A typical Transformer model architecture, as illustrated in Fig. 6.4, usually consists of two primary elements: an encoder and a decoder. The encoder is usually on the left side and comprises multiple layers, including a multi-head self-attention mechanism and a position-wise feed-forward network, with some residual connections and normalizations. The encoder generates an encoded representation of the source sequence and applies it in conjunction with the decoder to forecast the subsequent target output (Chollet, 2021). The majority of the sub-layers exhibit similarities in both the encoder and decoder. The self-attention method enables the model to assign weights to distinct segments of the input sequence, allowing it to concentrate on pertinent data sections while making predictions. This functionality is valuable in analyzing time series data, such as stock market indices, as specific past data points may significantly impact predicting future patterns.

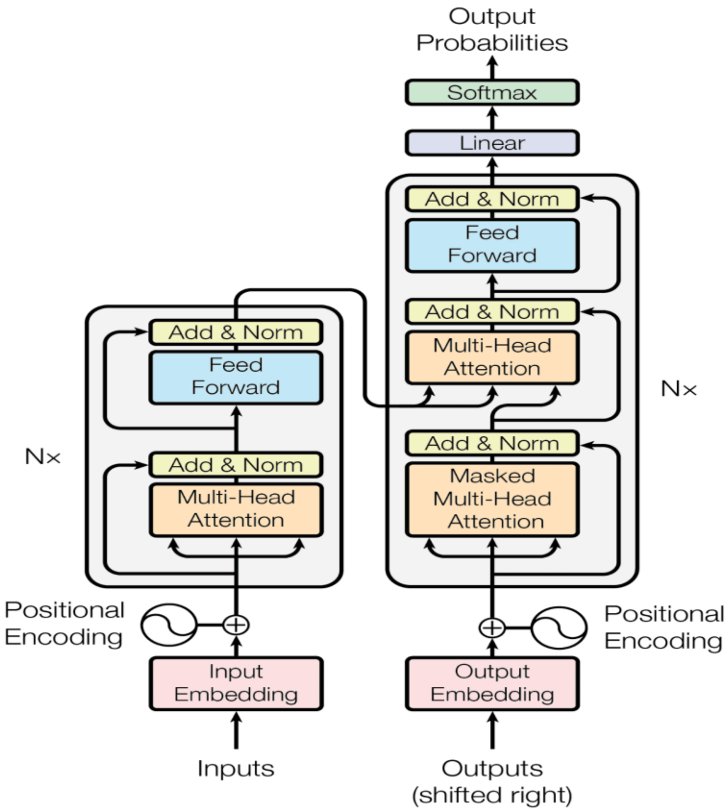

Fig. 6.4: Transformer model configuration (Vaswani et al., 2017).

Generally, a self-attention configuration of the Transformer model is defined using Eq. 6.4.

$$Attention(Q,K,V) = soft\max(\frac{QK^T}{\sqrt{d}})V \tag{6.4}$$

Where d is the hidden dimension of the keys. The matrices $Q,K,V \in \mathbb{R}^{T*d}$ represent the query, key, and value matrices, respectively. These matrices are the outputs of three distinct linear layers that share the same input.

The self-attention mechanism offers a novel approach to concentrate on crucial local information. Nonetheless, using several self-attention mechanisms, referred to as multi-head attention, can lead to improved performance. Each attention function is performed simultaneously within the multi-head attention mechanism with the corresponding projected versions of the query, key, and value matrices. Subsequently, the results of all attention functions are combined by concatenation to generate the outcome using a linear layer. The formula for multi-head attention is represented using Eq. 6.5.

$$\begin{aligned} MultiHead(Q,K,V) &= Concat(head_1, head_2, ..., head_h)W^O \\ head_i &= Attention(QW_i^Q, KW_i^K, VW_i^V) \end{aligned} \tag{6.5}$$

Where, $i = 1,...,h$ and W_i^Q, W_i^K, W_i^V are weights of networks. Fig. 6.5 illustrates the architecture of the self-attention mechanism.

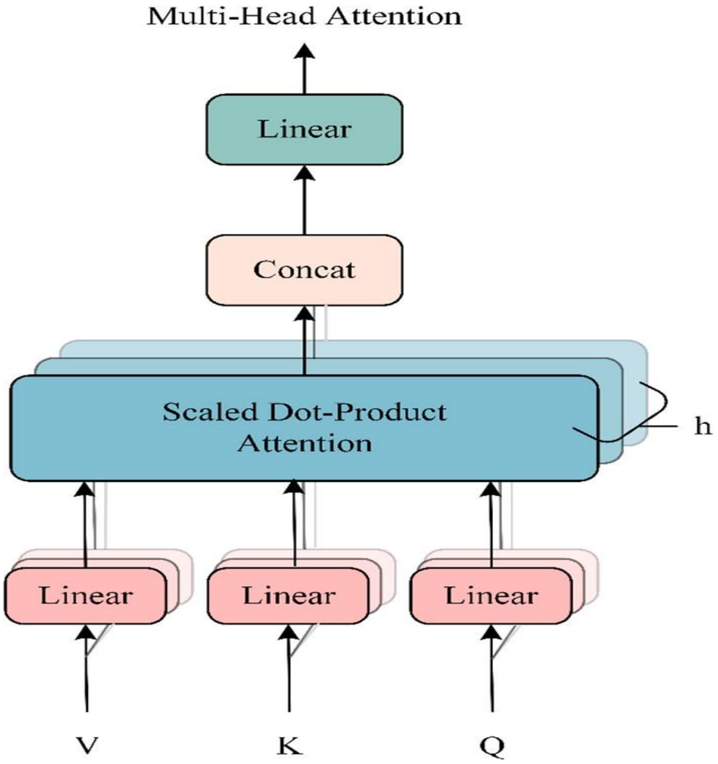

Fig. 6.5: Model structure of multi-head attention (Ma et al., 2023).

6.2.5 Proposed model

Vaswani et al. (2017) paper, "Attention is All You Need," presents the Transformer model, which consists of separate encoder and decoder components. The encoder is responsible for handling the input sequence, whilst the decoder is responsible for generating the equivalent output sequence. The dual-component design is well-suited for applications like language translation, where the model analyses a sequence in one language and generates a corresponding sequence in another. In contrast, the Transformer model employed in the current study is primarily designed for time series forecasting, particularly for predicting stock values. The aim is not to translate or alter sequences but to predict a singular future value using past data. For this purpose, the proposed model revised the existing Transformer model using only the encoder component, as shown in Fig. 6.6.

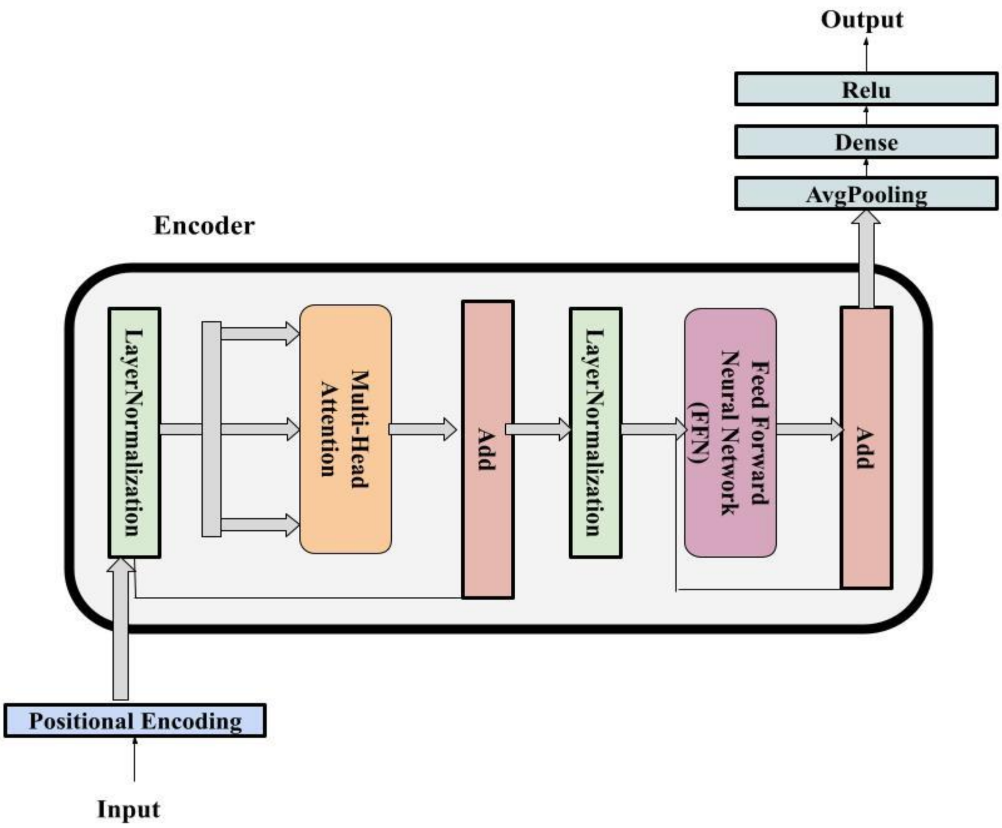

Fig. 6.6: Proposed Transformer model configuration.

This study effectively utilized a Transformer model to predict stock index values, employing a systematic and multi-layered technique. The method commences with an input layer, wherein each node represents a distinct time interval in the stock daily price data. Above this layer lies the positional encoding, a vital element in Transformers that accounts for their non-recurrent nature by offering essential information on the sequence order of the data. The core of this model consists of multiple encoder layers, each consisting of two main components: the Multi-Head Attention mechanism and a Feed-Forward Neural Network. Parallel lines or nodes represent the Multi-Head Attention and effectively focus on different parts of the input sequence, simultaneously catching multiple aspects of the data. The Feed-Forward Network applies the ReLU activation function to perform transformations on the attention-enhanced input. The layers in the model are coupled by skip connections and surrounded by layer normalisation, enhancing the model's ability to learn and store information during training. After undergoing the complex operations of the encoder, the resulting output is a modified version of the input sequence enriched with highly acquired temporal connections. The output is subsequently subjected to Global Average Pooling, which condenses the encoder's output into a more understandable form while retaining its essential features. The aggregated data is thereafter transmitted through a sequence of compact layers, responsible for additional analysis and improvement of the data, ultimately leading to the final prediction layer. Here, the model presents its prediction, including the expected stock price for the next day, based on a thorough review of past stock market trends and patterns. The Adam optimizer is employed in this method because it effectively manages extensive datasets and intricate structures. The performance and accuracy of the model are thoroughly assessed by employing the MSE as the loss function, guaranteeing accurate and reliable predictions of stock indices.

6.3 Model development

This section provides a detailed description of the methodological framework. The primary objective of Phase 1 is to establish a solid foundation by concentrating on data sourcing, preprocessing, and analysis. This ensures the data is robust and adequately prepared for the upcoming modelling process. During Phase 2, predictive models were developed. This stage is crucial as it entails developing and improving both base and optimized models utilizing four sophisticated deep learning techniques: RNN, LSTM, GRU, and the Transformer model. Phase 3 is centered on thoroughly assessing the models' performance, employing a comprehensive set of measures. This stage is essential in evaluating the accuracy and reliability of predictive models. Subsequently, Phase 4 focuses on the implementation of these models in real-world scenarios, specifically examining the trading strategy that exploits the forecasting skills of the produced models. Phase 5 concludes by interpreting the models, delivering valuable insights into their decision-making processes and underlying mechanisms. The method visualized in Fig. 6.7 guarantees the current study's thorough and systematic methodology.

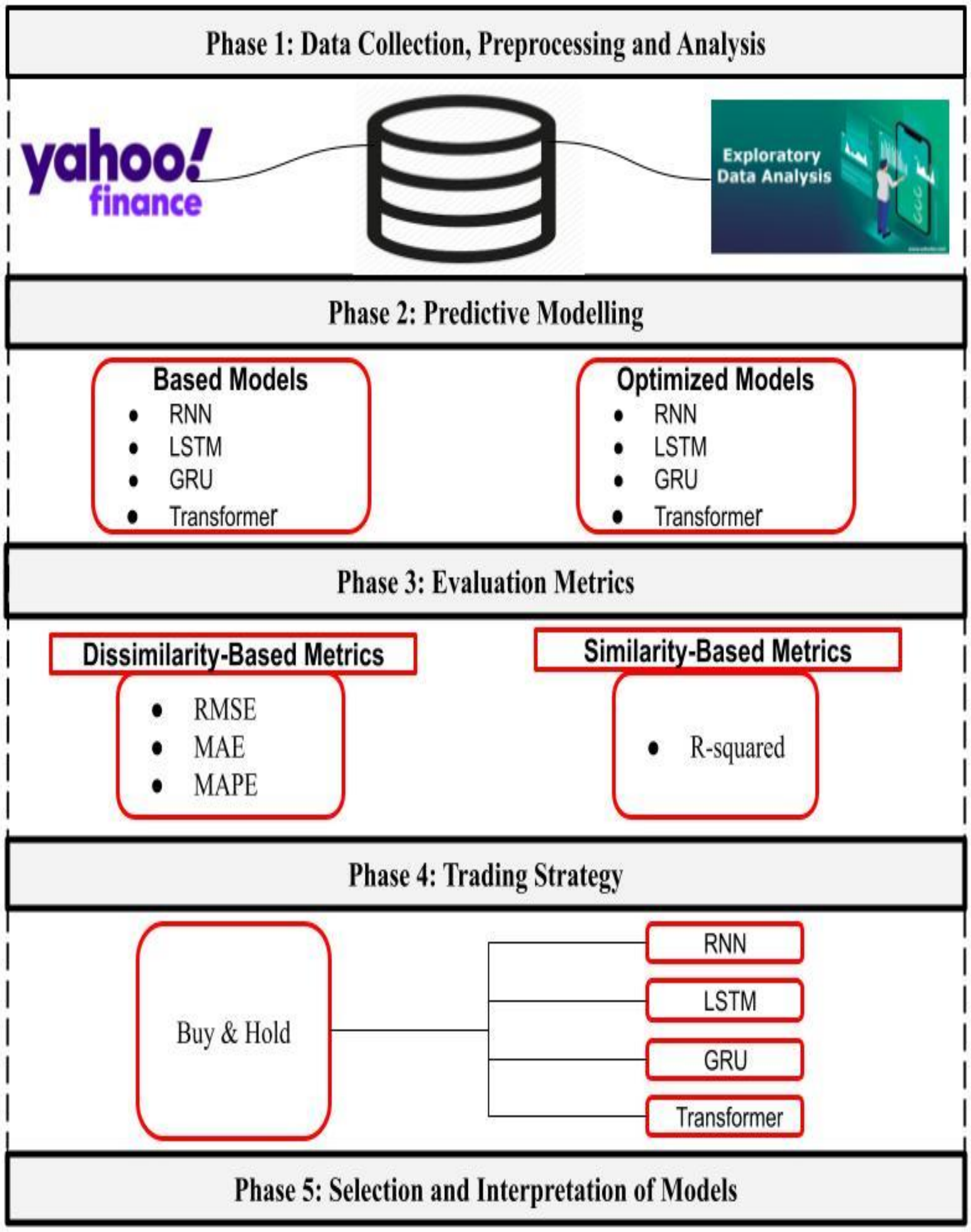

Fig. 6.7: The proposed framework for forecasting stock indices.

6.4 Data sourcing

The datasets used in this study were sourced from Yahoo Finance, a publicly available database recognized for its vast financial information. This work specifically examined three prominent global stock indices - the S&P 500, FTSE 100, and HSI - which reflect significant economic regions spanning three continents. The dataset, as shown in Table 6.1, covers a period of 13 years, starting from January 1, 2010, and ending on December 31, 2022. This extended duration was selected to offer a thorough and all-encompassing understanding of seasonality and trends throughout the years.

Table 6.1: Descriptive statistics of stock indices

S/N	Index	Samples	Start date	End date	Mean	Std. Dev.
1	S&P 500	3272	1-Jan-10	31-Dec-22	2401.4860	988.7889
2	FTSE 100	3281	1-Jan-10	31-Dec-22	6597.5617	707.1860
3	HSI	3203	1-Jan-10	31-Dec-22	23871.7701	3272.6246

The first dataset, S&P 500 Index, is a prominent indicator of the US stock market, comprising 500 of the biggest firms from various industries. It serves as a crucial gauge of the overall health and patterns of the US economy. The S&P500 index considers market capitalization, which assigns greater importance to more giant corporations, thus acknowledging their substantial influence on the market. The performance of the S&P 500 serves as a crucial benchmark for investors, analysts, and financial professionals. Additionally, it serves as the foundation for other investment instruments, such as index funds, mutual funds, and Exchange-Traded Funds (ETFs). Although the S&P 500 has shown consistent growth over the years, it is nonetheless susceptible to changes and instability. The second dataset, FTSE 100 Index, sometimes known as "Footsie," consists of the 100 biggest firms listed on the London Stock Exchange. It is a significant measure of the success of the UK stock market. Similar to the S&P 500, it follows a market capitalization-weighted methodology and is commonly employed as a benchmark for evaluating investment portfolios. The third dataset, HSI measures the performance of the Hong Kong stock market. It includes the 66 largest firms listed on the Hong Kong Stock Exchange, which accounts for about 58% of the entire Hong Kong market. HSI considers the market capitalization of freely tradable shares, resulting in a complete measure of the Hong Kong economy.

All the indices are decisive for investors and financial experts to assess market conditions and investment success in their respective regions. The dataset for each index had multiple parameters, namely, opening price, closing price, high price, low price, adjusted closing price, and volume. However, only the closing price was used in this research, as it is a single data point that accurately reflects market patterns and investor emotion. The decision to consider only the closing price is based on the strong correlation between the various price levels and the fact that closing prices effectively summarise the day's market operations and investor behaviour.

6.5 Data preprocessing

Upon data collection, the datasets were subjected to cleaning. This step was essential to guarantee the integrity and quality of the data, which serves as the basis for robust model predictions. This phase carefully examined the datasets to identify missing variables and outliers. Fortunately, no absent data were identified, and there were no occurrences of outliers, signifying the dependability and uniformity of the data obtained from Yahoo Finance. After undergoing the cleaning process, the data for each index was divided into training and testing sets, with a ratio of 75:25. Consequently, 75% of the data was utilized for training the models, specifically RNN, LSTM, GRU, and Transformer, and the remaining 25% was set aside for assessing their performance. The split was meticulously selected to ensure the models could access sufficient data to learn and capture the underlying market patterns. It is worth noting that the rationale behind partitioning data into training and testing sets is a customary procedure in machine learning, guaranteeing that models are trained on a comprehensive dataset and also assessed on unfamiliar data to determine their ability to generalize. In order to account for the inherent noise in the stock market data and create strong models, $\overline{x}_t$ is denoted as the normalized price at any time t, while x_t is the original price at any time t. Normalization using the Min-Max Scaler technique was performed as shown in Eq. 6.6.

$$\overline{x}_t = \frac{x_t - \min(x_t)}{\max(x_t) - \min(x_t)} \tag{6.6}$$

Eq.6.6 applies the normalization algorithm, where min and max represent the minimum and maximum values in the dataset, respectively. By normalizing the data to a range of [0,1], the models can handle the data more effectively, minimizing the risk of bias caused by different scales. In addition, a look-back of 60 days was employed to make each forecast. Each prediction in this method relied on a collection of the preceding 60 days' closing prices. This enabled the models to utilize recent historical data to anticipate the closing price for the following day. By carefully performing these preprocesses, the study guaranteed that the data for each stock index was adequately prepared for model training.

6.6 The Tree-structured Parzen Estimator (TPE).

6.6.1 Overview of TPE

The Tree-structured Parzen Estimator (TPE) is a Bayesian optimization method family member used for hyperparameter tuning. It is a state-of-the-art technique for the optimization of algorithms due to its notable improvement compared to conventional grid or random search methods in exploring the hyperparameter space (Rawi et al., 2023). Given the objective function, this technique constructs a model representing the hyperparameter's likelihood distribution. It then selects new hyperparameters to evaluate based on this model. This methodology enables TPE to determine the most suitable combination of hyperparameters more accurately and productively than alternative search techniques, particularly in spaces with many dimensions. It is worth noting that the fundamental mechanism of TPE entails partitioning the hyperparameter space into distinct regions according to their probability of improving the model's performance. At first, it randomly selects hyperparameters and assesses the model's performance. TPE constructs a probabilistic model based on increasing data, which predicts the probability of a hyperparameter set improving the model's performance. TPE subsequently prioritizes sampling hyperparameters from areas with a greater likelihood of success. The term "tree-structured" in TPE suggests its utilization of hierarchical partitioning, enabling it to effectively explore continuous and discrete hyperparameters. TPE becomes essential when optimizing deep learning models because it can effectively manage intricate and high-dimensional hyperparameter spaces inherent in these models. When optimizing an RNN, LSTM, GRU, or Transformer model, hyperparameters such as layer count, unit count per layer, learning rate, dropout rate, activation functions, and more can significantly influence the model's performance. The TPE algorithm systematically investigates these hyperparameters to identify the combination that produces the highest prediction accuracy. Fig. 6.8 illustrates the Bayesian optimization framework of models used in thee current study.

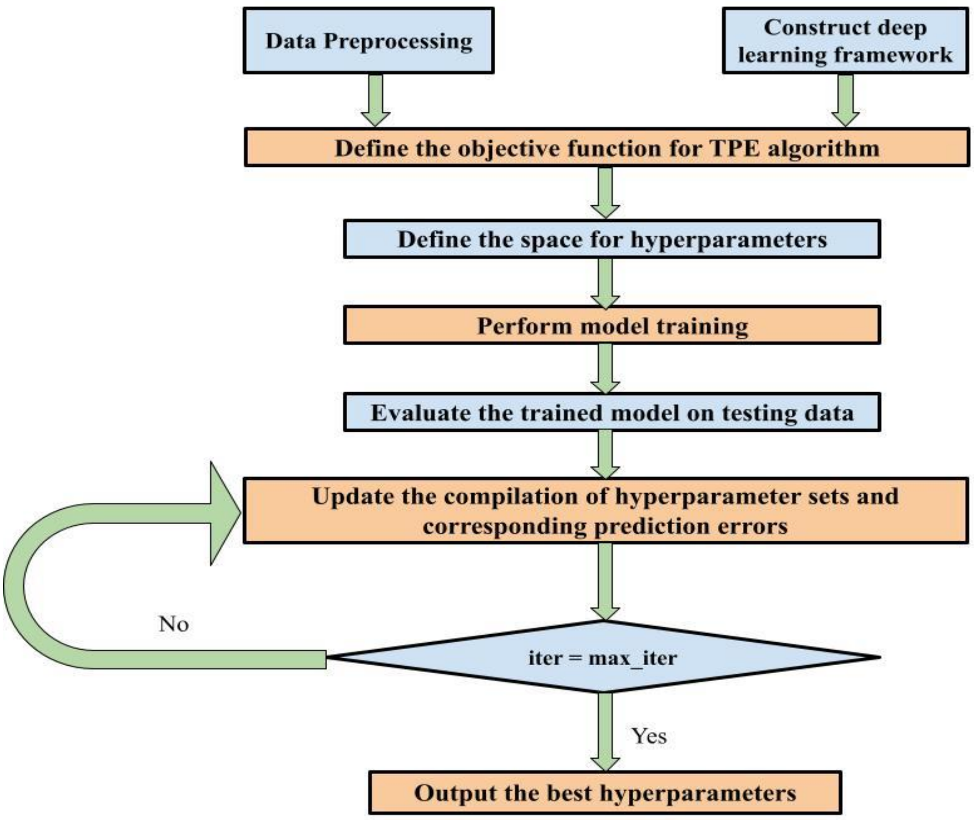

Fig. 6.8: TPE optimization framework.

6.6.2 Application of TPE in the present study

The TPE is a Bayesian optimization approach that applies a sequence model to estimate the loss function. It employs a probabilistic model and makes informed assumptions about the number of repetitions. The TPE technique surpasses other sequential-based model optimization (SBMO), such as random search, grid search, and stopping epochs strategies in optimizing numerous hyperparameters, especially for deep learning models like RNN, LSTM, GRU, and Transformer, with more hyperparameters than traditional machine learning models (Rawi et al., 2023). TPE models P(x|y) and P(y) instead of P(y|x) as observed in other SMBO methods, such that x specifies the hyperparameters while y represents the corresponding loss. The TPE method functions by creating a probabilistic model that associates hyperparameters with the likelihood of achieving a specific score on the goal function. The algorithm continuously improves this model by collecting additional data and utilizing the model to identify the most favourable hyperparameters to evaluate the objective function. The TPE approach employs two mathematical distributions: l(x) for the optimal observed hyperparameters and g(x) for the other hyperparameters. Eq. 6.7 provides the Expected Improvement (EI) criterion, which the method utilizes to choose the subsequent set of hyperparameters for evaluating the objective function (Wu et al., 2019).

$$EI(x) = E[\max(f(x) - f(x^*), 0)] \tag{6.7}$$

Where x represents the assessed location, x* represents the current optimal position, f(x) represents the anticipated value of the function at x, and E[.] represents the expectation. Table 6.2 presents the hyperparameters of the four deep learning algorithms considered in the current study. It includes information on their type (continuous, discrete, categorical) and the range within which they were adjusted. Also, It gives a thorough overview of the suggested hyperparameter optimization parameters, outlining the parameters taken into account and the extensive range of their investigation to find the best model configurations. The utilization of TPE in this work played a crucial role in improving the models' capacity to forecast stock market trends precisely, showcasing the effectiveness of this sophisticated optimization method in financial time-series prediction.

Table 6.2: TPE hyperparameters of various model configurations

Model	Hyperparameter	Type	Range
RNN	Units	Integer	(25, 100, 5)
	Epochs	Integer	(20, 100, 5)
	Learning Rate	Continuous	(0.0001, 0.01)
	Dropout	Continuous	(0, 0.3)
	Batch Size	Integer	[16, 32, 64]
LSTM	Units	Integer	(25, 100, 5)
	Epochs	Integer	(20, 100, 5)
	Learning Rate	Continuous	(0.0001, 0.01)
	Dropout	Continuous	(0, 0.3)
	Batch Size	Integer	[16, 32, 64]
GRU	Units	Integer	(25, 100, 5)
	Epochs	Integer	(20, 100, 5)
	Learning Rate	Continuous	(0.0001, 0.01)
	Dropout	Continuous	(0, 0.3)
	Batch Size	Integer	[16, 32, 64]
Transformer	Head_Size	Integer	[32, 64, 128]
	Num_Heads	Integer	[4, 8, 16]
	ff_dim	Integer	[4, 8, 16]
	Num_Transformer_Blocks	Integer	[2, 4, 6]
	MLP_Units	Integer	[64, 128, 256]
	Dropout	Continuous	(0, 0.3)
	MLP_Dropout	Continuous	(0, 0.3)
	Batch Size	Integer	[16, 32, 64]
	Epochs	Integer	(20, 100, 5)
	Learning Rate	Continuous	(0.0001, 0.01)

6.7 Evaluation metrics

It is essential to assess the performance of models during training, especially when dealing with time-series data such as stock market indices. To achieve robust prediction, choosing suitable evaluation measures becomes imperative. This study examined four essential metrics: RMSE, MAPE, MAE, and R-squared. The performance of the RNN, LSTM, GRU, and Transformer models was assessed using these measures on the S&P 500, FTSE 100, and HSI datasets.

RMSE is a statistical measure used to assess the accuracy of a prediction or model by calculating the square root of the average of the squared differences between the predicted values and the actual values. In the present study, RMSE served as a suitable metric to assess the accuracy of the model's predictions in relation to actual stock values. Smaller RMSE values indicate superior model performance, representing reduced discrepancies between the anticipated and actual values. Eq. 6.8 presents the formula for RMSE.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\overline{x}_i - x_i)^2} \tag{6.8}$$

MAPE is a metric used to measure the accuracy of a forecast by calculating the average absolute percentage difference between the forecasted and actual values. MAPE is highly beneficial in the current study for comprehending the models' performance in relation to the magnitude of the stock prices. This metric facilitates the comparison of model accuracy across datasets with varying scales or price ranges. Eq. 6.9 presents the formula for MAPE.

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\overline{x}_i - x_i}{x_i}\right| * 100\% \tag{6.9}$$

MAE is an evaluation metric that quantifies the average absolute value of errors in a given set of predictions, regardless of their direction. The metric represents the mean absolute deviation between the predicted values and the actual observations. MAE offered a user-friendly metric to assess any model's average error size. Similar to RMSE, a lower MAE signifies superior model performance. Eq. 6.10 presents the formula for MAE.

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|\overline{x}_i - x_i\right| \tag{6.10}$$

R-squared, or the coefficient of determination quantifies the fraction of the variance in the dependent variable that the independent variables can predict. It quantifies the extent to which the model accurately reproduces observed results. In the current study, R-squared is used to evaluate how well the models could accurately represent and account for the fluctuations in stock prices. A greater R-squared value signifies a superior model alignment with the data. Eq. 6.11 presents the formula for R-squared.

$$R^2 = 1 - \frac{\sum_{i-1}^{N}(\overline{x}_i - x_i)^2}{\sum_{i-1}^{N}(\hat{x}_i - x_i)^2} \tag{6.11}$$

Where, x_i is the actual value, $\overline{x}_i$ is the predicted value, $\hat{x}_i$ is the mean value, and N is the length of the dataset.

6.8 Hardware and software requirements

All models are executed with Python 3.10.12 using Google Colab. The decision to use Google Colab for all models presented in this chapter was motivated by a strategic need to achieve computing efficiency with accessibility. Colab provided an optimal platform because of its robust and expandable cloud-based infrastructure, which is essential for managing the extensive computational requirements of deep-learning models. Using in-built cutting-edge hardware accelerators, such as T4 GPU, V100, and A100 GPUs, ensured the requisite computing capacity and speed for handling extensive datasets and intricate model architectures. By utilizing the powerful computational capabilities of Colab, models were trained, tested, and assessed in an environment that provided excellent performance without the need for physical hardware setup and maintenance. This significantly improved the efficiency and practicality of the current work. Generally, Colab offers user-friendly environments with external hardware accelerators and compute units to reduce computational workloads. The Keras library operates on Tensorflow 2.14.0 and includes vast deep-learning models already installed as ready-made libraries. Most visualizations and data processing in this study are conducted using Matplotlib and Seaborn libraries in Python.

Google Colab provides five distinct types of runtime hardware accelerators, each designed to meet specific computational requirements:

1. Central Processing Unit (CPU): it is a standard computing choice appropriate for activities with lower intensity levels.
2. Tensor Processing Unit (TPU): it is a specialized hardware component specifically built to handle machine learning tasks, with a particular emphasis on its compatibility with TensorFlow.
3. T4 GPU: it is a better alternative to the CPU, specifically designed for handling moderate machine learning applications.
4. V100: it is a premium GPU that provides improved performance and faster processing capabilities. It is exclusively available in the paid version.
5. A100: The most sophisticated and state-of-the-art GPU currently offered in Colab, only available in its premium version. It is renowned for its outstanding computational power and rapid processing capabilities.

According to the research conducted by Kehinde, Chung and Chan (2023), it was evident that GPUs exhibit superior speed compared to TPUs. Therefore, preference was given to GPUs over TPUs and CPUs. This decision is appropriate for quick processing speeds and effective management of extensive datasets and intricate model structures. Due to the high computational demands of developed models and their hyperparameter tunings, a premium version of Google Colab, which is an A100 GPU, was chosen. The choice was motivated by the requirement for premium computing capacity, speed, and supplementary computing units offered by this sophisticated GPU. The NVIDIA A100 GPU employed in this study provides many characteristics that render it suitable for high-performance computing jobs. Presented in Table 6.3 is a tabular depiction of the features of the chosen hardware accelerator.

Table 6.3: Hardware accelerator configuration

Property	Description
Architecture	Ampere
Tensor Cores	Third-generation Tensor Cores
CUDA Cores	6,912
Memory Capacity	Up to 40 GB or 80 GB of HBM2
Memory Bandwidth	Up to 1.6 TB/s
Peak Floating-Point Performance	19.5 teraflops (FP32), 156 teraflops (Tensor TF32)
AI Performance (FP16)	312 teraflops
AI Performance (INT8)	624 teraflops
Power Consumption	Up to 400W

6.9 Trading strategy

All developed models are subjected to a simple trading strategy to estimate the net values in making a portfolio investment using the developed models. The strategy is to capitalize on the algorithm's capacity to predict future stock prices correctly and get monetary rewards. Its design aimed to evaluate the effectiveness of translating predictive models into practical trading choices in real-life situations. The trading technique was implemented on the S&P 500, FTSE 100, and HSI datasets to assess its effectiveness under varying models. A trading strategy was formulated using total return, volatility, maximum drawdown, and Sharpe ratio. If the forecasted value $\overline{x}_{t+1}$ for the next day exceeds the most recent observed value x_t, the strategy would initiate a long one position in the index. Alternatively, if $\overline{x}_{t+1}$ is lesser than x_t, it would initiate a short one position index. Perhaps there is no difference; then no position is held. The calculation of the return at any particular time $t+1$ is determined according to Eq. 6.12:

$$R_{t+1} = \ln\frac{x_{t+1}}{x_t} * sign(\overline{x}_{t+1} - x_t) \tag{6.12}$$

The sign (.) represents the sign function. The net value (NV) of the strategy, which represents the total return, is calculated using Eq. 6.13, where $NV_1 = 1$ and $t > 1$.

$$NV_t = 1 + \sum_{i=2}^{t} R_t \tag{6.13}$$

Volatility is a term that quantifies the degree of change in the value of a security, index, or market across a given period. It plays a crucial role as a tool for investors and traders to evaluate risk and make well-informed decisions. Eq. 6.14 is commonly used in computing volatility.

$$Volatility = \sigma(R_t) \tag{6.14}$$

Where σ represents the standard deviation of returns.

Maximum drawdown is a risk indicator that quantifies the most significant decline in the value of a portfolio or investment from its highest point to its lowest point before reaching a new high. It is frequently employed to assess the risk associated with a particular investment or compare various asset risk levels. Eq. 6.15 is commonly used in computing maximum drawdown.

$$Max\_drawdown = \max_{i<j} \frac{NV_j - NV_i}{NV_i} \tag{6.15}$$

The Sharpe Ratio is a financial metric that quantifies an investment's performance to its level of risk. The Sharpe ratio measures the additional return gained per unit of risk assumed in an investment. The Sharpe Ratio can be calculated using Eq. 6.16.

$$Sharpe\_ratio = \frac{R_t - R_f}{\sigma} \tag{6.16}$$

R_f represents risk free interest rate. In this stuudy, R_f is assumed to be 1%. Also, the number of trading days per year is set at 252.

6.10 TPE-optimized model implementation and discussion

6.10.1 S&P 500 – Optimized model analysis and discussion

Implementing TPE hyperparameter adjustment has significantly enhanced model accuracy when forecasting the S&P 500 index. The Transformer model currently exhibits superior performance in terms of RMSE (59.1469) and MAE (43.4004), suggesting that its predictions have been much enhanced and are now more precise and closely aligned with the actual values. The GRU model exhibits significant enhancement, particularly in RMSE (61.0501) and R-squared (0.9885), demonstrating its proficient predictive capacity and substantial ability to account for variations in the data. Remarkably, the RNN and LSTM models demonstrate nearly identical performances in terms of RMSE and R-squared, with minor discrepancies in MAE. This indicates that hyperparameter tuning has improved their prediction skills, but not as significantly as the Transformer model. The Transformer model exhibits the greatest R-squared value (0.9892), closely trailed by the GRU model. In general, the utilization of TPE hyperparameter tuning has dramatically enhanced the performance of all models, with the

Transformer model exhibiting the most notable improvement in its predictive ability. The results of the evaluation metrics are presented in Table 6.4.

Table 6.4: Evaluation metrics on testing datasets – S&P 500 (optimized model)

Metrics	RNN	LSTM	GRU	Transformer
RMSE	77.7684	77.8831	61.0501	59.1469
MAPE	17.47%	17.51%	17.66%	17.77%
MAE	63.0126	65.6568	48.3526	43.4004
R-Squared	0.9813	0.9812	0.9885	0.9892

6.10.2 FTSE 100 – Optimized model analysis and discussion

The optimization of TPE hyperparameters has resulted in improved performances for all models used to predict the FTSE 100 index. The Transformer model now has superior performance in terms of RMSE (91.2344) and MAE (62.5082), suggesting a greater level of accuracy in its predictions when compared to other models. The LSTM model exhibits notable enhancement, particularly in terms of RMSE and R-squared (0.9749), indicating a superior fit and predictive capacity after fine-tuning. Although the RNN and GRU models exhibit enhancements, they fail to surpass the Transformer model in any measure. This represents a departure from the previous analysis (without TPE adjustment) in which the GRU model demonstrated superiority in multiple criteria. All models have similar MAPE values, with the LSTM model having a slightly higher MAPE (9.68%) than the rest. The Transformer model exhibits the highest R-squared value (0.9758), demonstrating its superior capacity to explain the variability of the FTSE 100 index closing prices after tuning. In general, optimizing TPE hyperparameters has enhanced the performance of all models. Among them, the Transformer model has exhibited the most substantial improvement in terms of both error magnitude and explanatory power. The results of the evaluation metrics are presented in Table 6.5.

Table 6.5: Evaluation metrics on testing datasets – FTSE 100 (optimized model)

Metrics	RNN	LSTM	GRU	Transformer
RMSE	98.0826	92.9279	94.0607	91.2344
MAPE	9.52%	9.68%	9.52%	9.65%
MAE	78.0497	70.2576	73.1244	62.5082
R-Squared	0.9721	0.9749	0.9743	0.9758

6.10.3 HSI – Optimized model analysis and discussion

Applying TPE hyperparameter tuning to the HSI index has led to diverse improvements across the models. The Transformer model has superior performance in terms of RMSE (412.0157) and MAE (320.8853), showing a substantial improvement in prediction accuracy. The LSTM model demonstrates significant enhancement, particularly in terms of R-squared (0.9846), indicating that it has become more proficient in elucidating the fluctuations in the closing prices of the HSI index after fine-tuning. Although the RNN model does not have the lowest RMSE or MAE, it demonstrates a high R-squared value (0.9823), suggesting its ability to explain the data. The MAPE values for all models are pretty similar, with the LSTM and GRU models exhibiting somewhat higher MAPEs (16.75% and 16.72%, respectively) in comparison to the Transformer and RNN models. The Transformer model exhibits the greatest R-squared value (0.9855), indicating its exceptional ability to account for the fluctuations in the closing prices of the HSI index after fine-tuning. In general, tweaking the TPE hyperparameters has significantly enhanced the performance of the Transformer and LSTM models for the HSI index. The Transformer model has shown the most significant improvement in both the number of errors and its ability to explain the data. The results of the evaluation metrics are presented in Table 6.6.

Table 6.6: Evaluation metrics on testing datasets – HSI (optimized model)

Metric	**RNN**	**LSTM**	**GRU**	**Transformer**
RMSE	455.7988	425.2154	492.5732	412.0157
MAPE	16.37%	16.75%	16.72%	16.52%
MAE	345.3747	328.1241	400.4278	320.8853
R-Squared	0.9823	0.9846	0.9793	0.9855

The following bar graphs, Fig. 6.9-6.11, depict the evaluation measures of the selected prominent stock indices: the S&P 500, FTSE 100, and HSI. The figures present the findings from Table 6.4-6.6, facilitating a straightforward and concise comparison of the performance measures among four distinct deep learning models: RNN, LSTM, GRU, and Transformer. Each bar graph depicts the results of each model's particular evaluation metrics (RMSE, MAPE, MAE, and R-squared). These graphs provide a clear visual picture of the predictive accuracy and efficiency of the models after implementing TPE hyperparameter optimization.

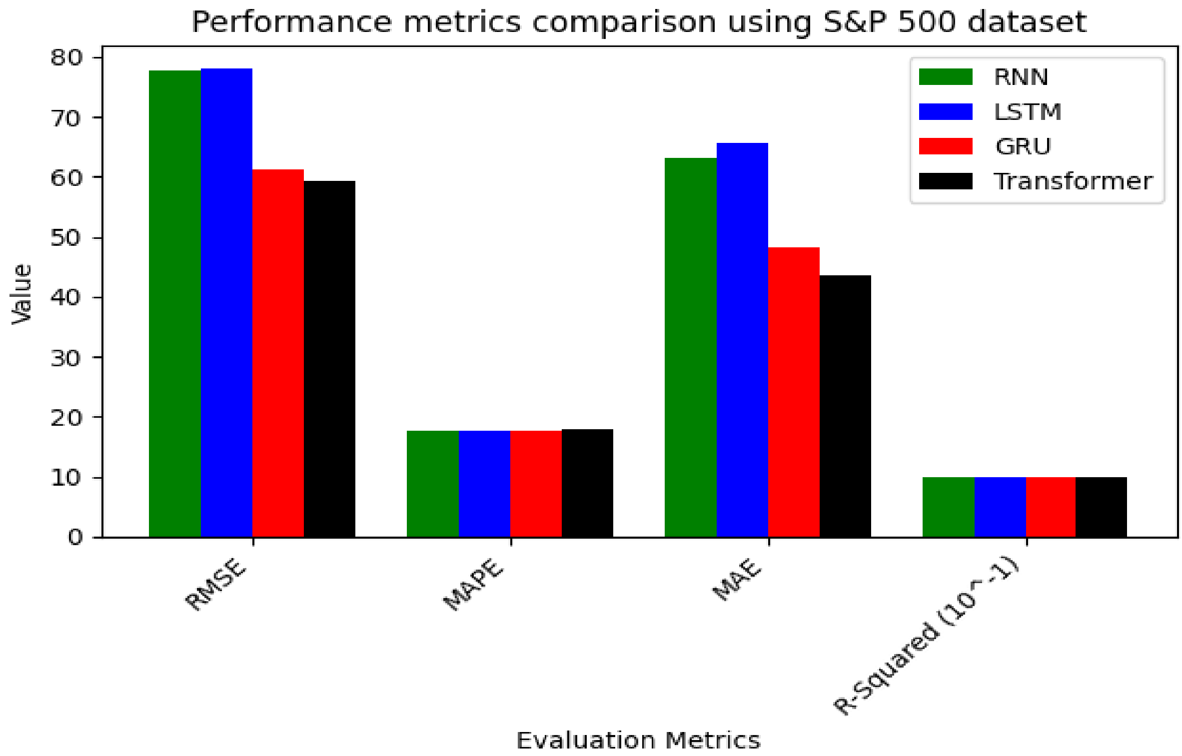

Fig. 6.9: Performance metrics comparison using the S&P 500 dataset.

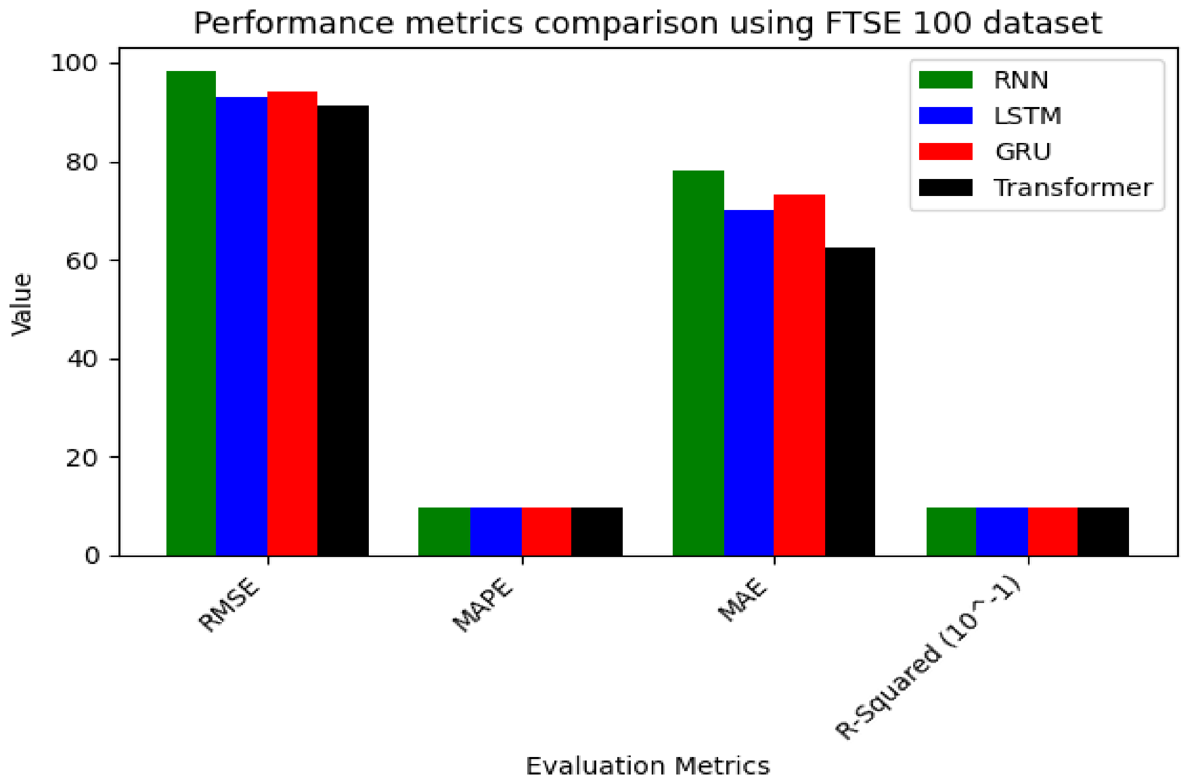

Fig. 6.10: Performance metrics comparison using the FTSE 100 dataset.

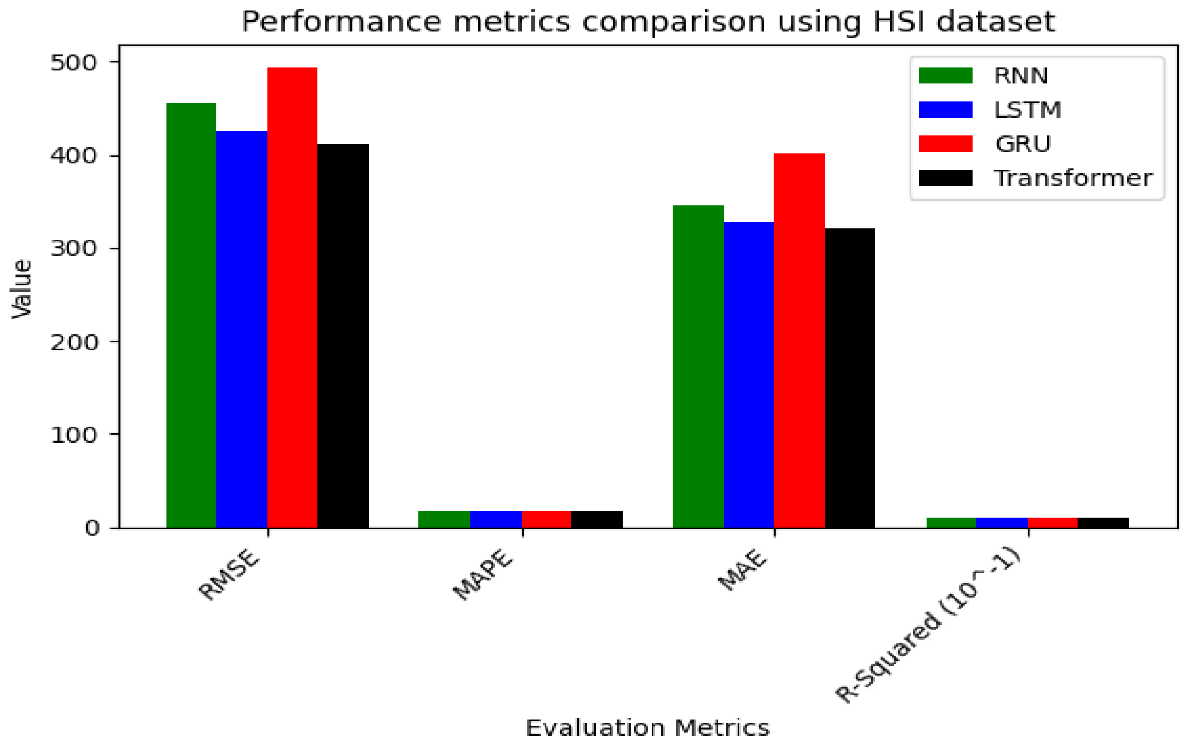

Fig. 6.11: Performance metrics comparison using the HSI dataset.

Fig. 6.12-6.14 depicts the loss functions as they change with the number of epochs for the three stock indices analyzed in this study: the S&P 500, FTSE 100, and HSI, respectively. The graphical depiction offers a detailed view of these variations. Each figure depicts the evolving learning process of the model throughout multiple epochs, showcasing the reduction and stabilization of the loss as the RNN, LSTM, GRU, and Transformer models converge toward optimal performance. The instability and poor convergence of the RNN on the HSI dataset, as observed in Fig. 6.14, could be attributed to several factors, including political instability, market volatility, national laws, and pandemic effect. The plotted graphs provide valuable insights into each model's training efficiency and learning rate, showcasing their behaviour and reactivity during the training phase for each stock index.

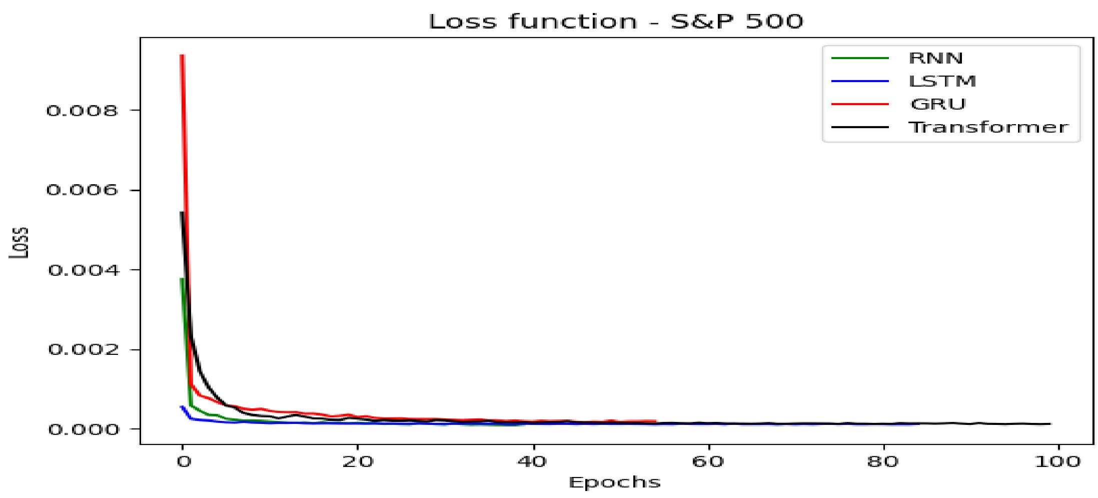

Fig. 6.12: Loss function – S&P 500.

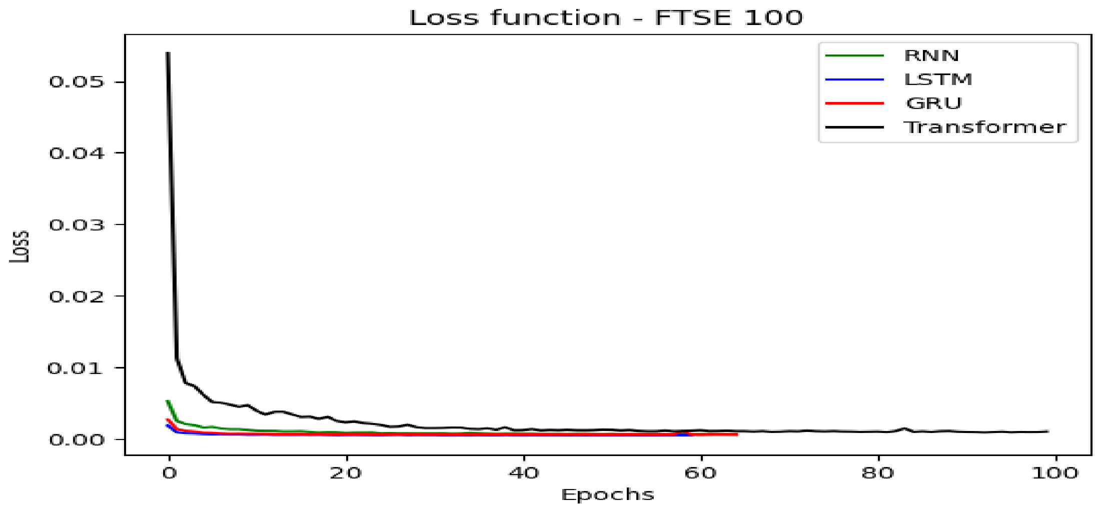

Fig. 6.13: Loss function – FTSE 100.

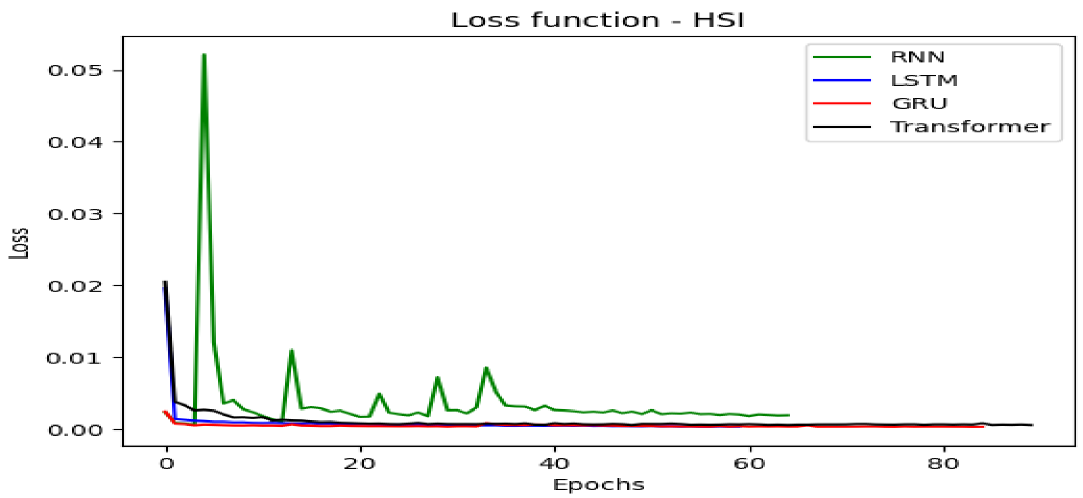

Fig. 6.14: Loss function – HSI.

6.11 Model interpretation with predicted curves

Fig. 6.15-6.18 illustrates the predictions of RNN, LSTM, GRU, and Transformer models for the S&P 500 index. It compares the true values (represented by the blue line) with the model's performance throughout the training period (depicted by the orange line) and the testing period (represented by the green line) from 2010 to 2022. The general trajectory is ascending, indicating the expansion of the market, with a notable decline observed around the year 2020, presumably aligning with the market collapse caused by the COVID-19 pandemic. The model's training forecasts accurately track the true values, including the recessionary drop. Similarly, the testing predictions capture this downturn, indicating the model's ability to effectively capture the fluctuating influence of the pandemic on the market.

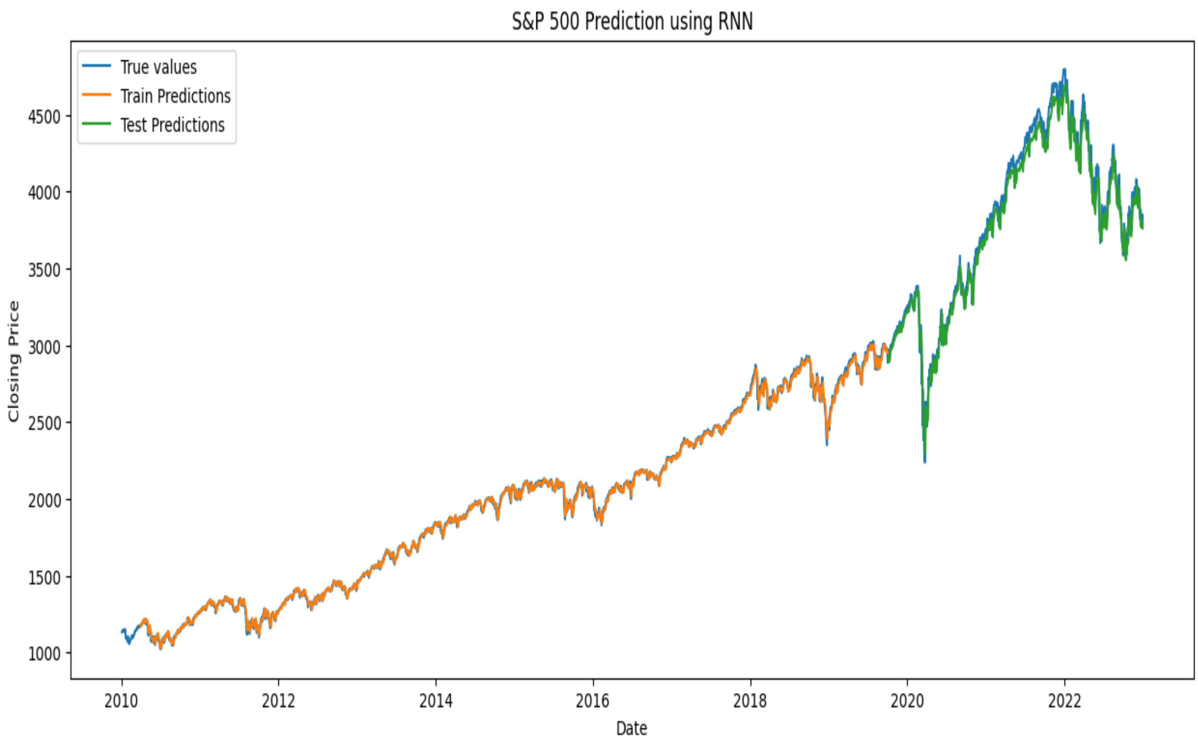

Fig. 6.15: Prediction curves on dataset using RNN model – S&P 500.

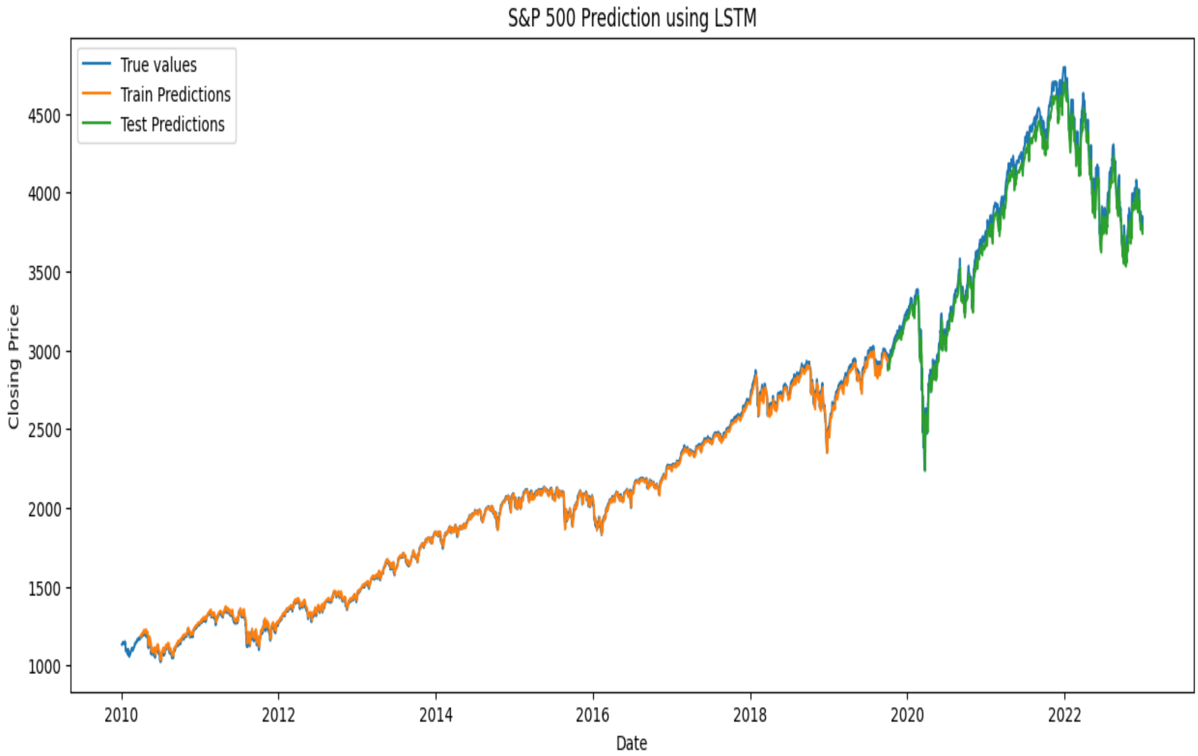

Fig. 6.16: Prediction curves on dataset using LSTM model – S&P 500.

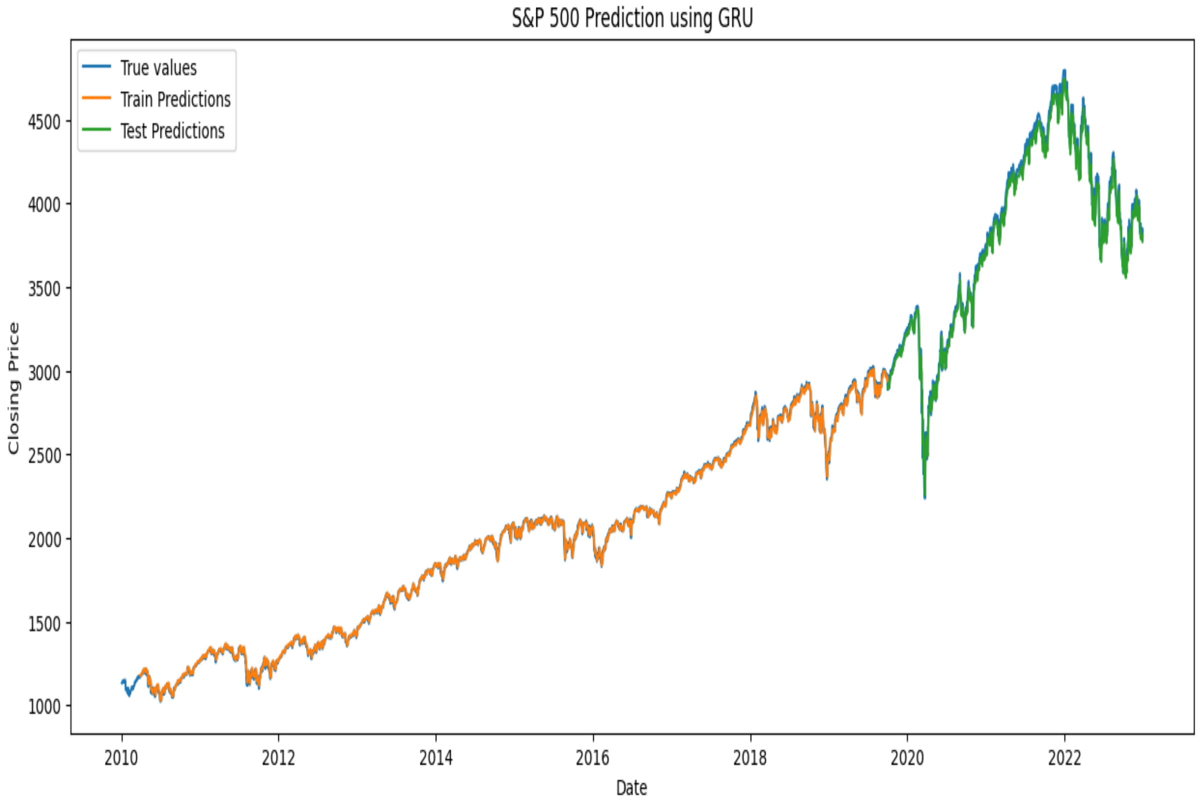

Fig. 6.17: Prediction curves on dataset using GRU model – S&P 500.

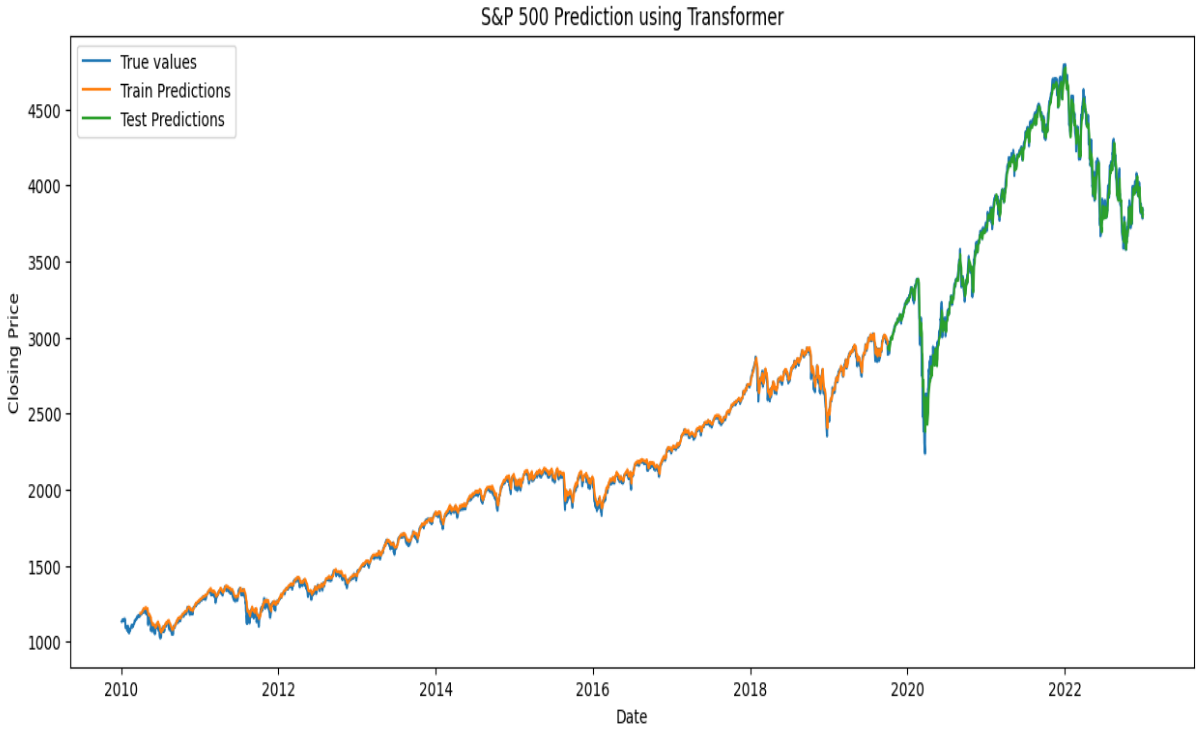

Fig. 6.18: Prediction curves on dataset using Transformer model – S&P 500.

Similarly, Fig. 6.19-6.22 illustrates the predictions of RNN, LSTM, GRU and Transformer models for the FTSE 100 index spanning 2010 to 2022. The blue line depicts the true values, signifying the real-life performance of the market. The model's predictions are illustrated in two distinct stages: the training phase (represented by the orange line) and the testing phase (represented by the green line). The training predictions utilize historical data to discern patterns, whereas the testing predictions strive to anticipate future market fluctuations. The index demonstrates a recurrent pattern of fluctuation, characterized by crests and troughs throughout the years. Notably, there is a significant and sudden decrease around 2020, likely attributable to the economic repercussions of the COVID-19 pandemic. After 2020, the market seems to be experiencing a rebound, and the test predictions are trying to imitate this recovery phase. The proximity of the training and testing predictions to the true values indicates the model's accuracy and resilience.

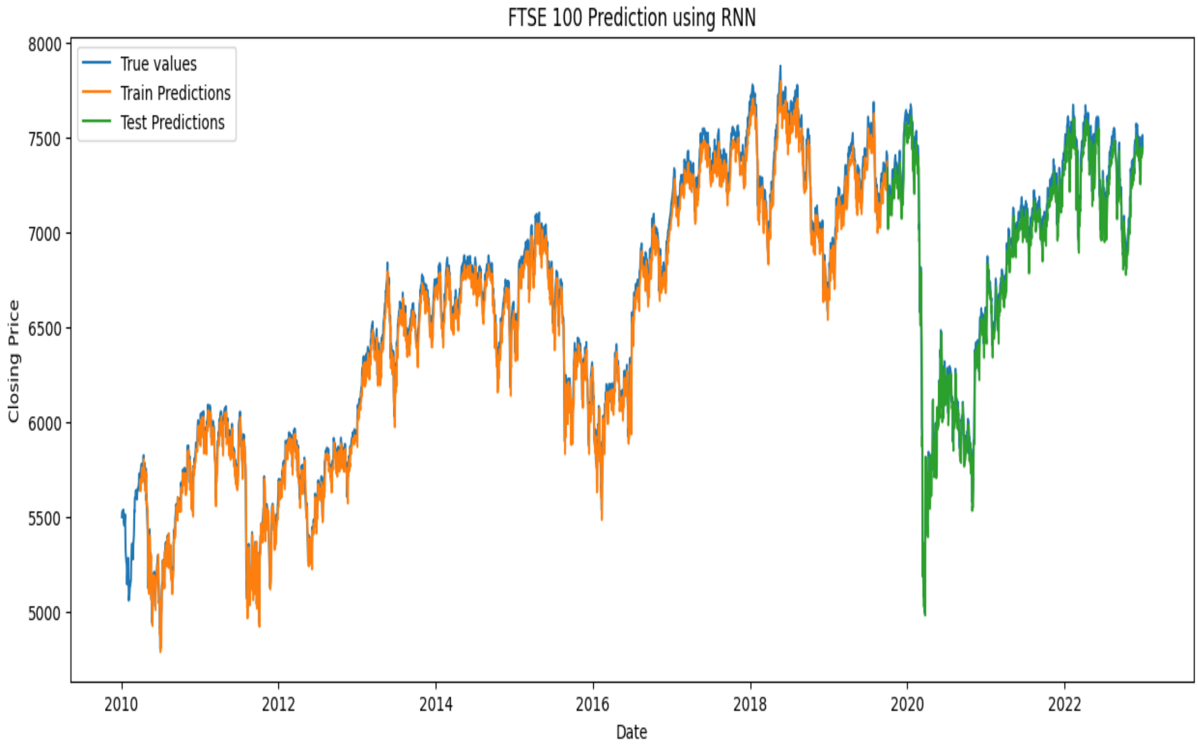

Fig. 6.19: Prediction curves on dataset using RNN model – FTSE 100.

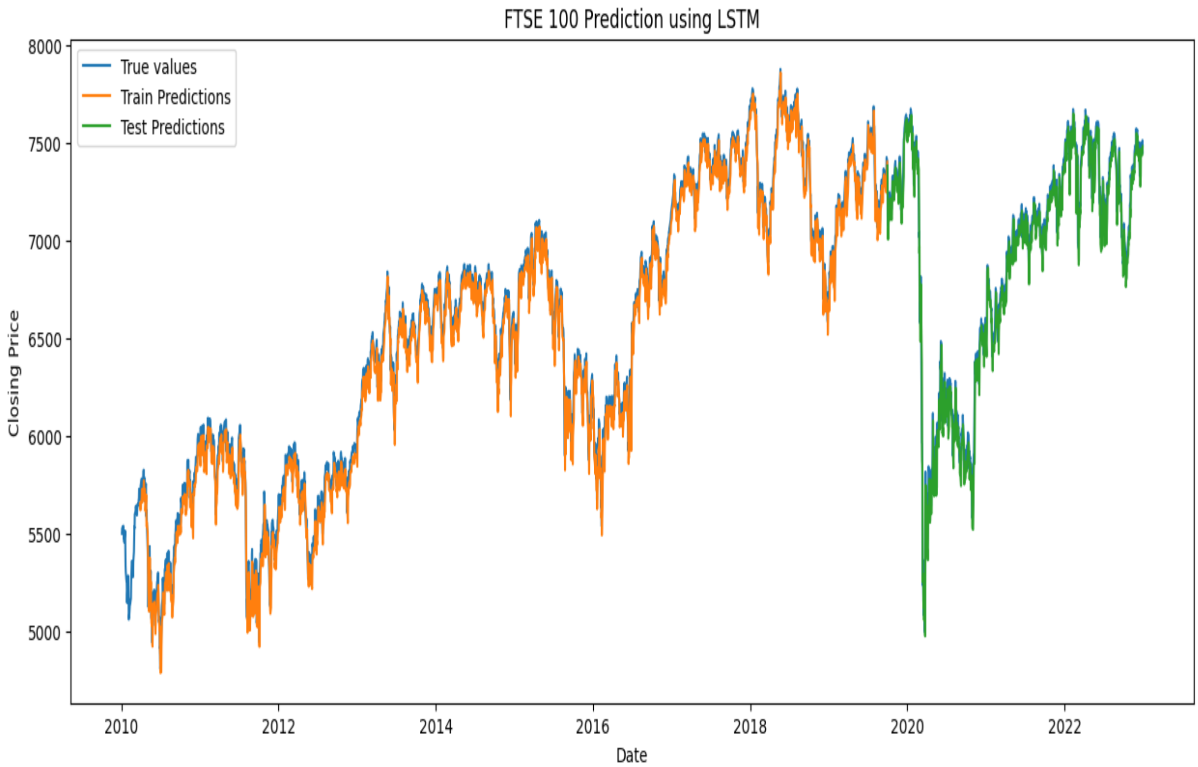

Fig. 6.20: Prediction curves on dataset using LSTM model – FTSE 100.

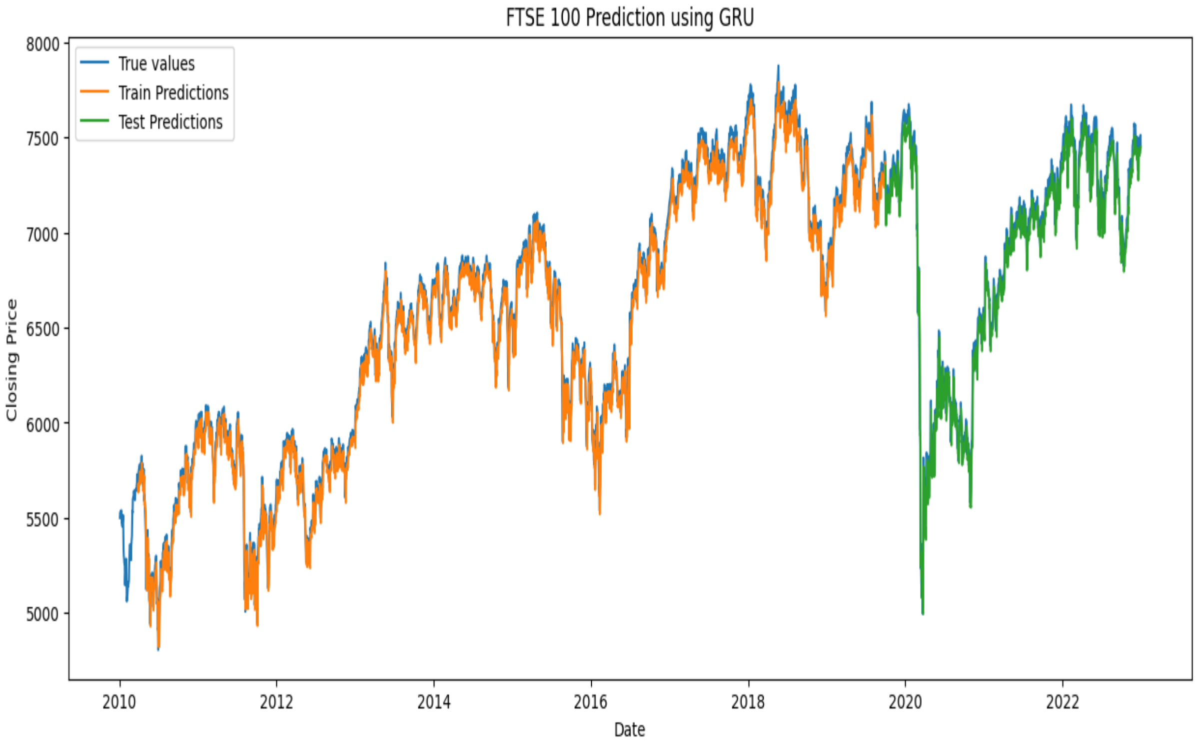

Fig. 6.21: Prediction curves on dataset using GRU model – FTSE 100.

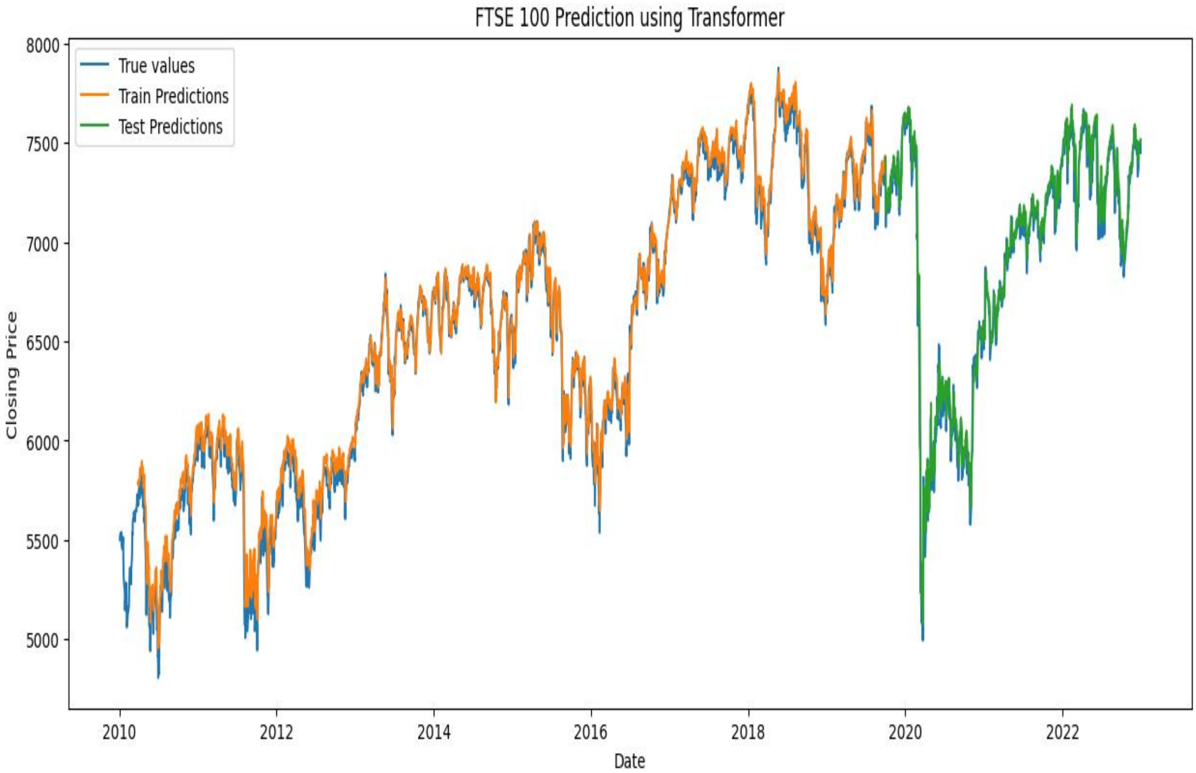

Fig. 6.22: Prediction curves on dataset using Transformer model – FTSE 100.

Lastly, Fig. 6.23-6.26 illustrates the predictions of RNN, LSTM, GRU, and Transformer models for the HSI spanning 2010 to 2022. The blue line represents the actual HSI values, displaying the historical performance of the index with typical market volatility. The model's forecasts are observed in two distinct stages: the training period (shown by the orange line), during which the model learns from past data, and the testing period (represented by the green line), during which the model endeavours to predict future trends based on its training. Throughout the period, the index exhibits volatility with many fluctuations. Contrary to the S&P 500 and FTSE 100, the HSI saw a comparatively shorter decline during the COVID-19 pandemic period in 2020. However, it is essential to highlight that the stock index's performance since 2021 has been unimpressive and is currently facing difficulties recovering from its previous decline.

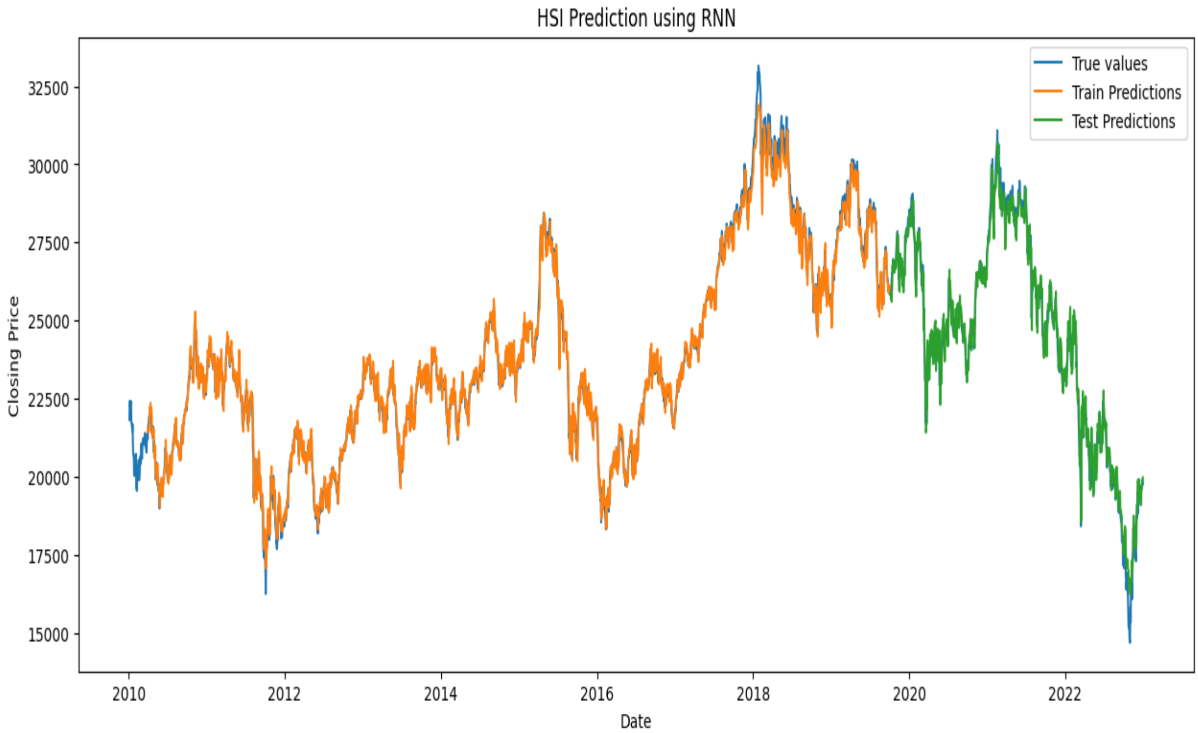

Fig. 6.23: Prediction curves on dataset using RNN model – HSI.

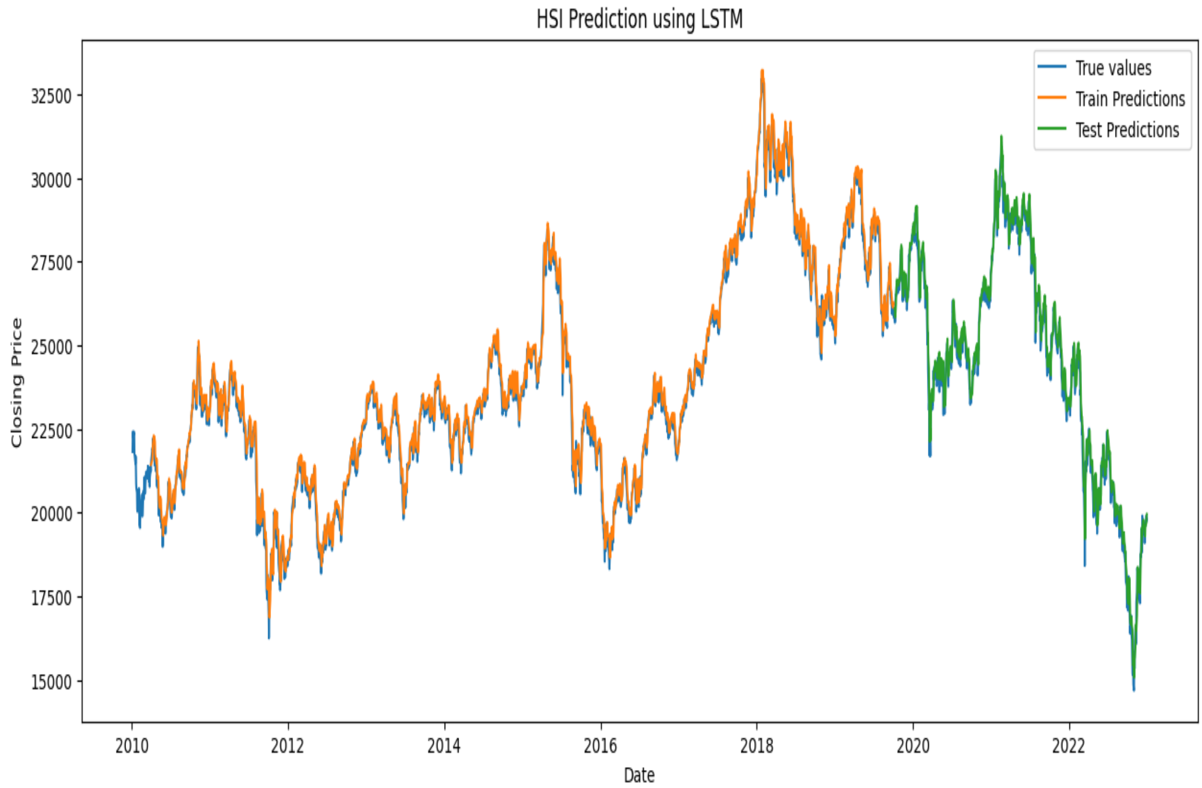

Fig. 6.24: Prediction curves on dataset using LSTM model – HSI.

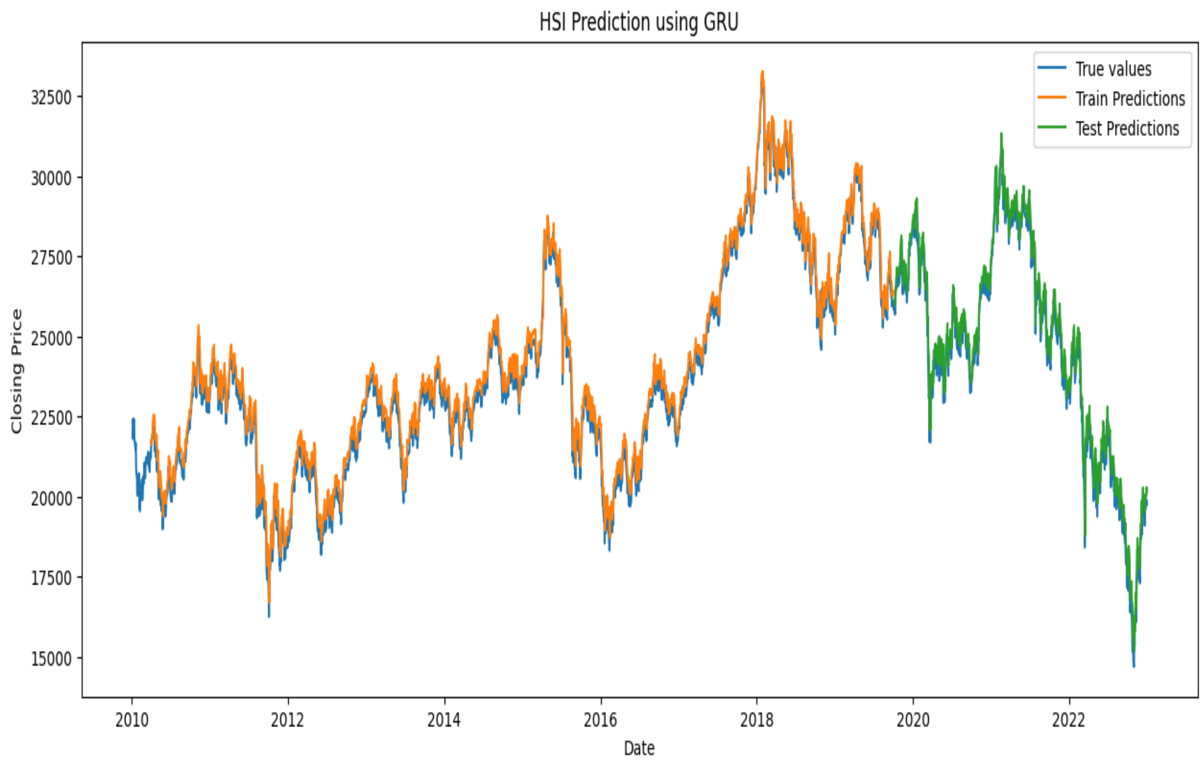

Fig. 6.25: Prediction curves on dataset using GRU model – HSI.

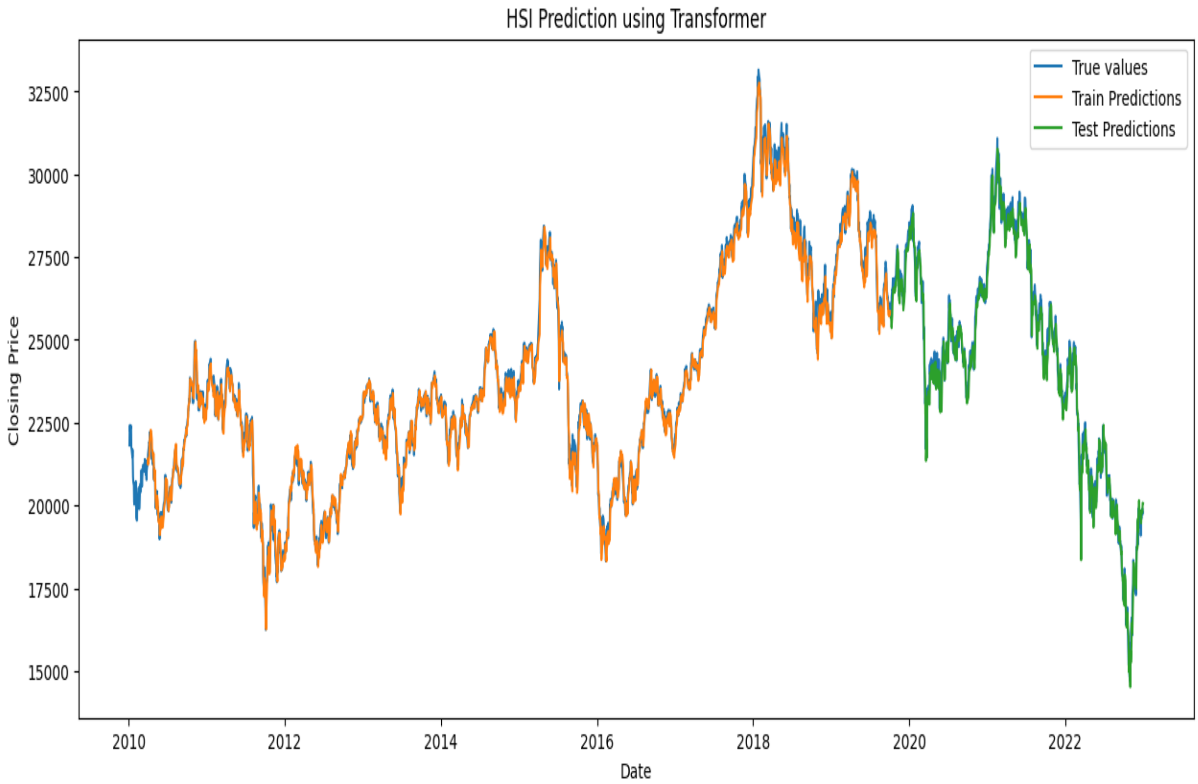

Fig. 6.26: Prediction curves on dataset using Transformer model – HSI.

6.12 Model runtime performance and analysis

Runtime performance in the context of training deep learning models refers to the efficiency and effectiveness of the model training process. It entails considering multiple factors and employing diverse methodologies to guarantee that the models are trained within a shorter duration without compromising their accuracy or effectiveness. It is worth noting that distinct trends become apparent when examining the runtime performance of several deep learning models on the S&P 500, FTSE 100, and HSI datasets. Judging by the training runtimes recorded in Table 6.7, the GRU model consistently demonstrates the highest level of efficiency in terms of training time across all three indices, with durations of 2419 seconds for S&P 500, 2275 seconds for FTSE 100, and 2969 seconds for HSI. The LSTM model exhibits enhanced efficiency compared to the RNN, with significantly shorter training times across all datasets (2870 seconds for S&P 500, 2356 seconds for FTSE 100, and 3198 seconds for HSI). On the other hand, the RNN model, quicker than the Transformer, exhibits comparatively lengthier training durations (7078 seconds for S&P 500, 6873 seconds for FTSE 100, and 7830 seconds for HSI), suggesting considerable computing resource utilization. Although the Transformer model exhibits superior prediction accuracy in previous evaluations, it necessitates a more extended training duration on each dataset (11854 seconds for S&P 500, 9463 seconds for FTSE 100, and 10209 seconds for HSI). This emphasizes a significant compromise between the number of computational resources required and the level of accuracy achieved, establishing the Transformer as the most demanding model in terms of resources. The GRU model is notable for its optimal training efficiency and competitive predictive performance.

Table 6.7: Runtime performance analysis

Runtime (Seconds)	RNN (s)	LSTM (s)	GRU (s)	Transformer (s)
S&P 500	7078	2870	2419	11854
FTSE 100	6873	2356	2275	9463
HSI	7830	3198	2969	10209

The bar graph in Fig. 6.27 represents the runtime performance of different deep learning models - RNN, LSTM, GRU, and Transformer - when applied to three selected stock indices: S&P 500, FTSE 100, and HSI. It depicts the duration of training for each model on each dataset, offering a concise and rapid way to compare their computational efficiencies. Across all three indices, the GRU model consistently exhibits the highest level of training efficiency, with a much smaller duration compared to the RNN, LSTM, and Transformer models.

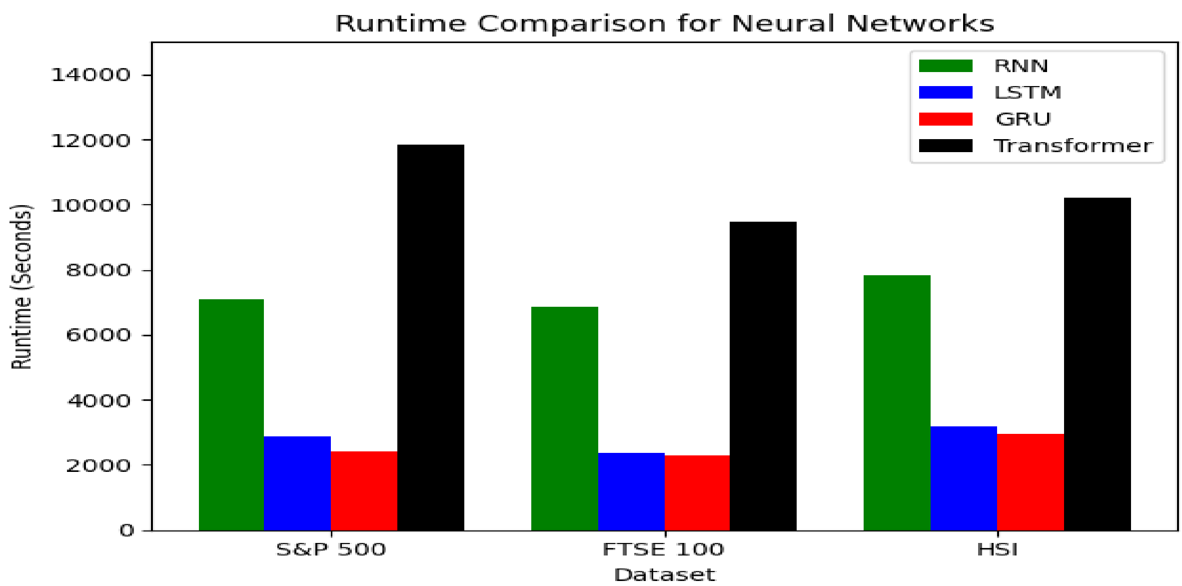

Fig. 6.27: Runtime performance of models.

6.13 Implementation of trading strategy

In addition to assessing prediction accuracy and runtime performance, model performance based on net values derived from the provided trading strategy was evaluated. The performance analysis based on trading strategy is discussed in the subsections below.

6.13.1 S&P 500 – Trading strategy analysis

Table 6.8 shows the performance of each trading strategy in the S&P 500 index, which is illustrated explicitly by the Sharpe ratio, maximum drawdown, annualized volatility, and total return. The Transformer model-based strategy surpasses other strategies with a Sharpe ratio of 0.5245, indicating greater risk-adjusted returns. In addition, it exhibits a comparatively least maximum drawdown (-0.2718) compared to other models. This suggests a reduced risk of experiencing significant losses from the highest to the lowest point. It is essential to mention that the Transformer model demonstrates the highest annualized volatility (0.2350), indicating more significant variations in returns. The Transformer method exhibits a significantly better total return of 40.83%, outperforming the passive buy-and-hold (B&H) technique that generates a total return of 31.91%. The RNN and LSTM models have Sharpe ratios of 0.4473 and 0.3552, respectively, suggesting lesser risk-adjusted returns than the Transformer model. The LSTM model exhibits the highest maximum drawdown of -0.3684, indicating a likelihood of substantial drops and a comparatively modest total return of 22.53%. The GRU model exhibits a competitive maximum drawdown of -0.2875 but falls behind in the Sharpe ratio and total return. The passive B&H strategy, often regarded as a standard in trading, exhibits a reasonable Sharpe ratio of 0.4304 and a robust total return of 31.91%. However, it is surpassed by the Transformer approach in terms of total return, risk-adjusted returns, maximum drawdown, and volatility. Ultimately, when considering the S&P 500 index, the Transformer-based trading method is the most efficient, yielding the most overall profit, superior risk-adjusted returns, and the least maximum drawdown despite its heightened volatility. Although the passive B&H approach is not as practical as the Transformer model, it still demonstrates impressive performance by surpassing the RNN, LSTM, and GRU models in terms of total return, maximum Drawdown, and Sharpe ratio.

Table 6.8: Trading Strategy – S&P 500

Models	RNN	LSTM	GRU	Transformer	Buy & Hold
Sharpe Ratio	0.4473	0.3552	0.3483	0.5245	0.4304
Max Drawdown	-0.3059	-0.3684	-0.2875	-0.2718	-0.3392
Annualized Volatility	0.2130	0.2090	0.2218	0.2350	0.2459
Total Return	30.82%	22.53%	22.63%	40.83%	31.91%

6.13.2 FTSE 100 – Trading strategy analysis

Table 6.9 details the trading strategies implemented using the FTSE 100 index. The Transformer model-based strategy exhibits the highest Sharpe ratio of 0.3460, signifying the most favourable risk-adjusted returns compared to other models. Furthermore, it attains an impressive overall return of 22.68%, surpassing both the other models and the passive B&H strategy by a substantial margin. Nevertheless, it is crucial to acknowledge that the Transformer approach demonstrates the most significant maximum drawdown (-0.4159) and annualized volatility (0.2307), indicating a greater level of possible risk and uncertainty in returns. The GRU model demonstrates a favourable equilibrium with a respectable Sharpe ratio of 0.2309, the lowest maximum drawdown of -0.2421, and a commendable total return of 11.98%. The RNN and LSTM models exhibit lower Sharpe ratios (0.0351 and 0.0917, respectively), suggesting inferior risk-adjusted returns in comparison to the Transformer and GRU algorithms. The LSTM model exhibits a modest total return of 3.62% and a slightly higher maximum drawdown of -0.2990 compared to the RNN model. Although commonly used as a benchmark, the passive B&H strategy exhibits a poor Sharpe ratio of 0.0713 and a minimal total return of 1.24%, significantly underperforming the Transformer and GRU techniques. In short, the Transformer model-based trading strategy proves to be the most efficient for the FTSE 100 index, resulting in the highest overall return and the most robust risk-adjusted returns.

Table 6.9: Trading Strategy – FTSE 100

Models	RNN	LSTM	GRU	Transformer	Buy & Hold
Sharpe Ratio	0.0351	0.0917	0.2309	0.3460	0.0713
Max Drawdown	-0.2901	-0.2990	-0.2421	-0.4159	-0.3493
Annualized Volatility	0.1798	0.1744	0.1670	0.2307	0.2025
Total Return	0.09%	3.62%	11.98%	22.68%	1.24%

6.13.3 HSI – Trading strategy analysis

Table 6.10 records the data for the HSI performance using the proposed trading techniques. The Transformer model-based approach is notable for its remarkably high Sharpe ratio of 0.9225, which indicates exceptionally favourable risk-adjusted returns. This method also attains a remarkable cumulative return of 75.66%, far outperforming other models and the passive B&H technique. Significantly, this strategy has the smallest maximum drawdown (-0.2449) and the lowest annualized volatility (0.2044) compared to all other strategies, indicating reduced risk and more consistent returns. The RNN, LSTM, and GRU models and the B&H strategy have negative Sharpe ratios, suggesting poor risk-adjusted performance. The LSTM model had the most loss (-40.54%) among these models and the passive strategy, resulting in considerable total returns losses. The RNN, LSTM, GRU, and B&H strategies exhibited negative Sharpe ratios (-0.2441, -0.3858, -0.1688, and -0.2488, respectively), indicating that these approaches not only had lower effectiveness but also had a negative impact on risk-adjusted returns for the HSI index. The maximum drawdowns for these models are significantly greater, with the RNN model exhibiting the greatest value (-0.6814), suggesting the possibility of considerable drops. The annualized volatility for these strategies is higher in comparison to the Transformer model, with the RNN model exhibiting the highest volatility (0.3511). In sum, the Transformer model-based trading strategy proves to be very effective and remarkably superior for the HSI index. It demonstrates exceptionally high total returns, favourable risk-adjusted returns, and the lowest volatility and max drawdown levels. This emphasizes the robustness of the Transformer model within the framework of the HSI index trading technique.

Table 6.10: Trading Strategy – HSI

Models	RNN	LSTM	GRU	Transformer	Buy & Hold
Sharpe Ratio	-0.2441	-0.3858	-0.1688	0.9225	-0.2488
Max Drawdown	-0.6814	-0.6223	-0.6339	-0.2449	-0.5275
Annualized Volatility	0.3511	0.3197	0.3288	0.2044	0.2532
Total Return	-35.17%	-40.54%	-26.92%	75.66%	-23.60%

As net value analysis using testing data was performed on all the datasets, Fig. 6.28-6.30, representing the algorithm trading on S&P 500, FTSE 100, and HSI, were plotted. The models encompass RNN, LSTM, GRU, and Transformer. In addition, the B&H strategy is added to establish a benchmark for other models. The y-axis represents the net value in relation to the initial investment, which is standardized to 1.0 for all the strategies, while the x-axis shows the passage of time. The curves represent the changes in the net value of an investment portfolio that is managed based on the signals produced by each model. These changes can indicate either growth or decline.

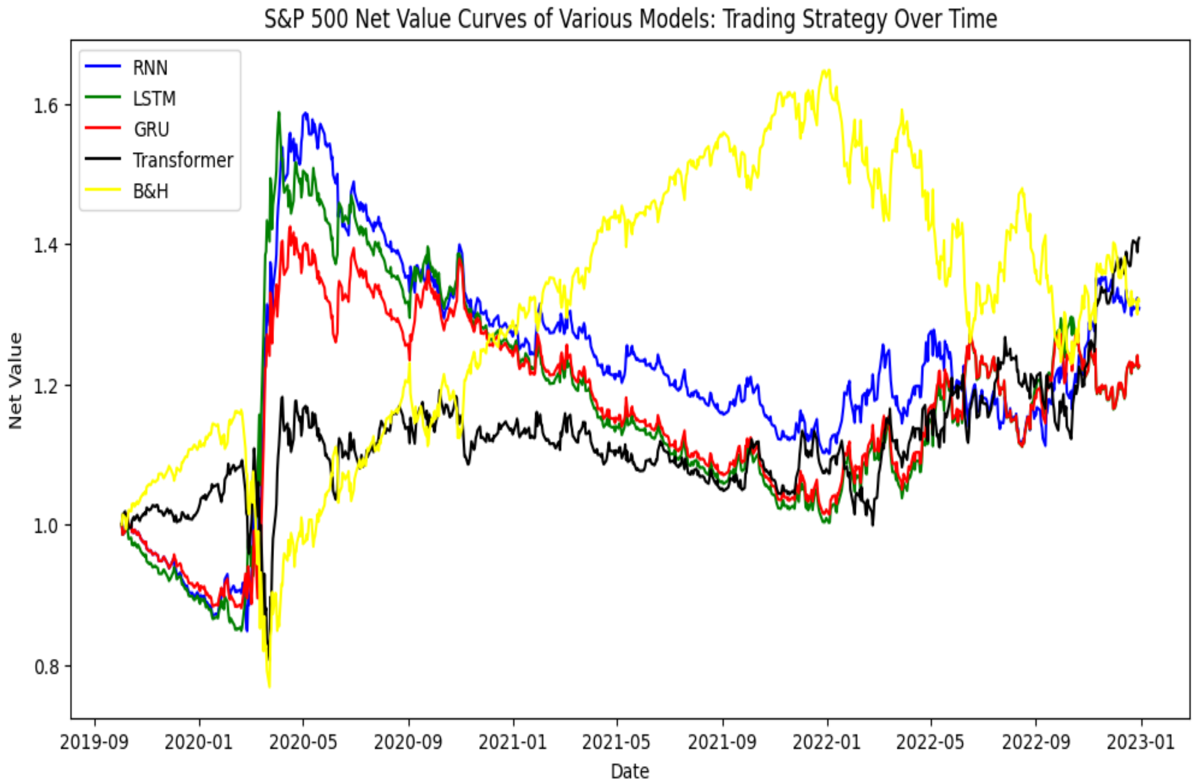

Fig. 6.28: Net value curves – S&P 500.

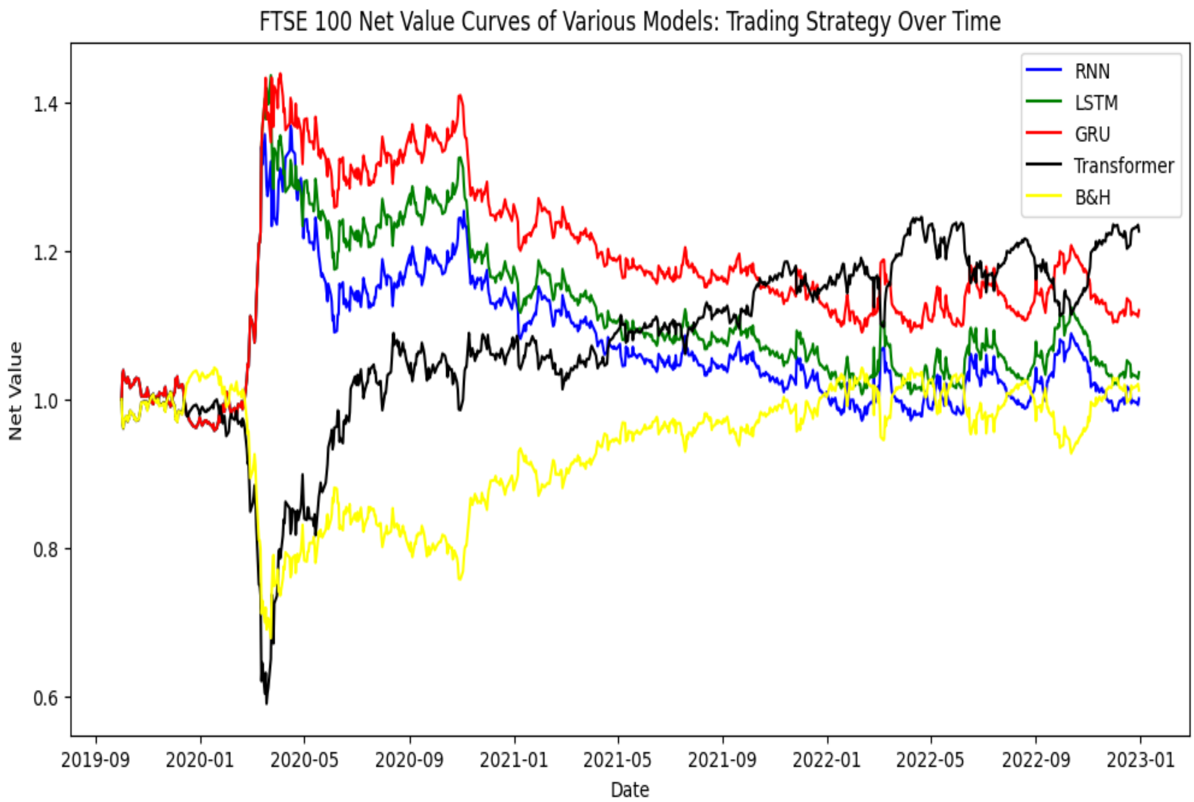

Fig. 6.29: Net value curves – FTSE 100.

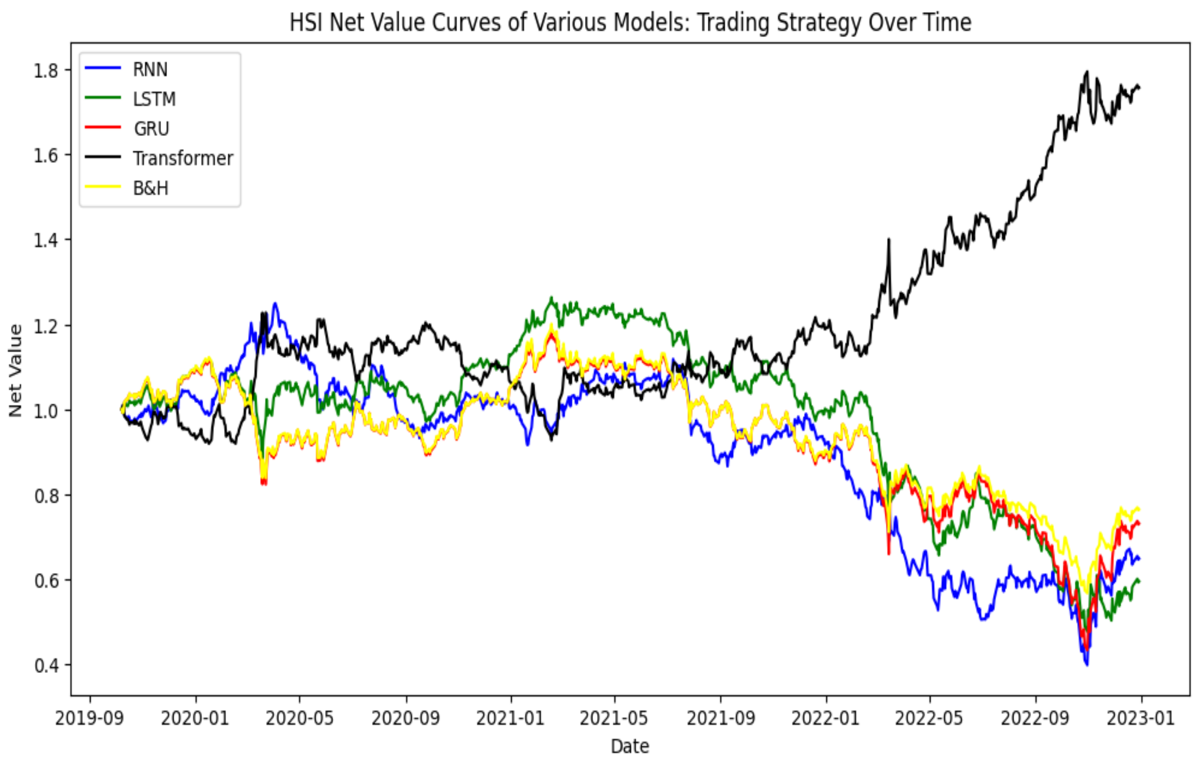

Fig. 6.30: Net value curves – HSI.

6.14 Summary

This study seeks to question the conventional EMH by providing evidence that trained deep learning models possess the capacity to transform stock prediction and yield substantial monetary gains. This chapter reveals favourable outcomes and substantiates the potential that comes from utilizing deep learning models, especially the Transformer model's state-of-the-art technique, to accurately and profitably predict stock performance, achieved through intensive training and assessment. This chapter utilized TPE for hyperparameter optimization, encompassing four models: RNN, LSTM, GRU, and Transformer. These models were applied to the S&P 500, FTSE 100, and HSI datasets. The objective was to improve the architecture and learning process of each model to suit the individual features of each stock index dataset. After fine-tuning, the Transformer model demonstrated exceptional performance in forecasting all three selected stock indices, S&P 500, FTSE 100, and HSI, exhibiting the lowest RMSE, MAE, and the highest R-squared values, in so doing, emphasizing the effectiveness of hyperparameter optimization in enhancing prediction accuracy.

A vital aspect of the study involved analyzing the runtime performance. The GRU model consistently demonstrated superior runtimes across all datasets, balancing computational efficiency and predictive performance. While exhibiting more robust accuracy in the hyperparameter settings, the Transformer model requires much greater processing resources, highlighting the trade-off between accuracy and training runtime. Going beyond the measurement of prediction accuracy, another stage was set to assess net values yielded by trading strategies. The utilization of the Transformer model-based approach, specifically for the HSI index, yielded outstanding results with the most elevated overall returns and Sharpe ratios, signifying great returns adjusted for risk. This contrasts the other models and the passive B&H technique, which exhibited diverse but less remarkable performances.

This thorough investigation highlights the importance of hyperparameter tuning in prediction modelling and trading strategies. The findings emphasize that although models like the Transformer can provide high accuracy, this compromises higher computational resource utilization. On the other hand, models like the GRU give a more balanced choice in terms of efficiency and performance. This study significantly contributes to using deep learning models in forecasting financial markets and investment returns. On a final note, while the Transformer has been a prominent result in this area of research, the complex, chaotic, and non-linear nature of the financial market necessitates a regular appraisal to understand and simulate the ever-growing dynamic characteristics.

Chapter 7 – Conclusion and Future Works

7.1 Conclusion

Stock market analysis remains a hotspot for scholars, investors, and managers since it offers the possibility of substantial profits. However, this field is inherently risky and complex, characterized by chaotic, nonlinear, and unpredictable elements. EMH posits that it is practically impossible to consistently surpass the average returns of the market. However, many studies have periodically been conducted to challenge this idea and attain significant profits. Investors desire portfolios that achieve a harmonious combination of minimal risk and significant returns, leading to the subject of stock portfolio management. The process consists of three fundamental phases: stock screening, portfolio optimization, and price prediction. This thesis primarily centers around the efficient management of stock portfolios, explicitly addressing the concerns of investors and analysts. The preceding chapters have presented novel approaches for accurate stock selection, mitigation of risk in equities, optimization of portfolios, and accurate forecasting of prominent global market indexes.

The tourism and hospitality industry plays a significant role in stimulating economic growth on a global scale. The remarkable expansion in this sector has been driven mainly by emerging economies in Asia rather than the Western world. Taiwan, well-known for its flourishing tourism sector, encountered a number of challenges during the pandemic-ravaged period of 2020-2021. The TWSE tourism sector faced significant hurdles in the financial year 2021 due to the country's decision to restrict international tourists from entering its borders in response to the ongoing epidemic. The said period has examined the resilience of tourism stocks and offered a distinctive perspective for comprehending the flexibility of stocks in the face of unforeseen and highly disruptive events. Chapter 3 of this thesis study assessed and ranked efficient stocks in the tourist sector of the TWSE using an integrated approach of SET, DEA, and IDEA techniques. The analysis focused on the financial year 2021, characterized by the COVID-19 pandemic and a decline in international tourism. The study developed a methodology that addresses bias and subjectivity in estimating efficiency ratings and ranking equities. SET, a technique based on information theory, was used to remove the curse of dimensionality associated with our dataset and select the most appropriate financial ratios from 13 available metrics. The IDEA model was applied to establish a hierarchical order of efficient DMUs. During sensitivity analysis, the efficiency scores of efficient stocks remained consistent, demonstrating the resilience and robustness of the proposed models and the reliability of the dimensionality reduction technique.

The work presented in Chapter 4 proposes a new method for evaluating the efficiency of stocks using the hybrid strength of DDF-DEA. It uses two case studies, the TWSE and the S&P 500, to consider negative data in input and output variables. Using the proposed methodology, the model calculates inefficiency scores. The study proposed a model to estimate the possible and maximum reductions in equity risk. The research contributes to the existing literature on stock market analysis by addressing negative data problems in DEA models. Taiwan, frequently recognized as an exemplary economic development model in Asia, has consistently exhibited robust economic expansion, notable technological expertise, and a vibrant stock market. Chapter 5 of this work assessed 20 industry-based portfolios as a financial production process to optimize their volatility using an inverse optimization approach, IDEA, and combining technical and fundamental indicators to improve performance. The study found that 7 of the 20 portfolios exhibited underperformance, and by reducing volatility through the proposed IDEA model, they could achieve optimal efficiency. A validation test strengthened the real-world applicability of the approach. Furthermore, the chapter proposed a model for pursuing a net zero portfolio risk initiative, which aims to achieve the least bearable risk to create an efficient portfolio of stocks.

Chapter 6 challenges EMH theory by demonstrating that deep learning models can transform stock prediction and yield substantial financial gains. The Transformer model, along with other models like RNN, LSTM, and GRU, was applied to forecast the closing prices of three major stock indices, including S&P 500, FTSE 100, and HSI. TPE hyperparameter optimization was employed to optimize the models, resulting in the Transformer model showing exceptional performance in forecasting all three indices. Even though the GRU model exhibited superior runtimes across all datasets, balancing computational efficiency and predictive performance, the Transformer model gave the highest investment returns and Sharpe ratio. The study emphasizes the importance of hyperparameter tuning in prediction modelling and trading strategies, highlighting the trade-off between accuracy and training runtime. In addition, the study demonstrates robustness as it generates satisfactory performances despite using datasets spanning three geographical nations.

To summarize, this thesis has made significant contributions to the field of stock portfolio management, encompassing stock selection, portfolio optimization, and price forecasting, thereby providing a crucial tool for informed financial decision-making. The research introduced a new approach to stock selection and ranking, offering an objective technique that surpasses many existing methods reliant on expert opinions. It also presented a new method for estimating potential and maximum potential volatility, which helps construct efficient portfolios. Furthermore, this work introduced the net zero volatility goal, a management framework designed to continuously optimize operations, ensuring that firms remain on the efficient frontier among their peers. Additionally, this thesis has expanded our understanding of how asset prices, like stocks, can be predicted using state-of-the-art techniques such as the transformer model, which has demonstrated impressive results in various domains. Beyond accuracy, the research illustrated the practical implications of applying such cutting-edge models to trading, revealing that the outcomes, in terms of returns, tend to be more favorable compared to results from existing deep learning techniques.

In conclusion, the findings and methodologies of this thesis have broader implications for sustainable development. By providing tools and frameworks that enhance financial market efficiency and decision-making, this research indirectly supports Sustainable Development Goal (SDG) 8, which aims to promote sustained, inclusive, and sustainable economic growth. The improved investment strategies and market insights derived from this study contribute to more stable and resilient financial markets, which are crucial for economic stability and growth, aligning with sustainable development's broader objectives.

7.2 Future works

In financial research, it is worth noting that stock market data can occasionally be inaccurate or contain inherent uncertainty. In light of this understanding, it would be advantageous for future research to investigate similar studies as Chapter 3 under conditions that can accommodate these uncertainties. This may involve exploring fuzzy, stochastic, or interval-based scenarios, yielding a more sophisticated comprehension of financial information amidst different levels of uncertainty and unpredictability. This method has the potential to provide a more flexible framework for analyzing and predicting the stock market.

Chapter 5 of this work evaluates portfolio optimization using datasets of 1365 equities from TWSE. However, due to the annual surge in the debut of new stocks, it may be challenging and time-consuming to repeat a similar model formulation and execution process to cover up-to-date datasets of equities. The possible integration of IDEA and deep learning or blockchain technologies to derive financial intelligence and insights is an opportunity for future research, offering potential breakthroughs in informed business decision-making.

In Chapter 6, future research can explore and conduct similar experiments in alternative financial markets like cryptocurrency, bonds, currency exchange, and forex markets. This can involve testing various versions of the deep learning models, incorporating external economic or geopolitical factors, or employing ensemble techniques to leverage the strengths of multiple models. These potential avenues are worth considering. Furthermore, the application of hybrid deep learning models has the capacity to improve forecasting accuracy, perhaps attaining higher levels of accuracy.

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