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# ADVANCED TOPOLOGY OPTIMIZATION IN<br>TYBER-PHYSICAL DISTRIBUTION SYSTEMS<br>FOR MULTI-SECURITY AND OPERATIONAL ADVANCED TOPOLOGY OPTIMIZATION IN<br>CYBER-PHYSICAL DISTRIBUTION SYSTEMS<br>FOR MULTI-SECURITY AND OPERATIONAL<br>FLEXIBILITY ENHANCEMENT ADVANCED TOPOLOGY OPTIMIZATION IN<br>CYBER-PHYSICAL DISTRIBUTION SYSTEMS<br>FOR MULTI-SECURITY AND OPERATIONAL<br>FLEXIBILITY ENHANCEMENT NCED TOPOLOGY OPTIMIZATION IN<br>:-PHYSICAL DISTRIBUTION SYSTEMS<br>|ULTI-SECURITY AND OPERATIONAL<br>FLEXIBILITY ENHANCEMENT OLOGY OPTIMIZATION IN<br>L DISTRIBUTION SYSTEMS<br>URITY AND OPERATIONAL<br>ITY ENHANCEMENT<br>LEI CHAO

# PhD

LEI CHAO<br>PhD<br>The Hong Kong Polytechnic University

2024

# The Hong Kong Polytechnic University

# The Hong Kong Polytechnic University<br>Department of Electrical and Electronic Engineering

# The Hong Kong Polytechnic University<br>
Department of Electrical and Electronic Engineering<br>
Advanced Topology Optimization in Cyber-Physical<br>
Distribution Systems for Multi-Security and<br>
Operational Flexibility Enhancement The Hong Kong Polytechnic University<br>
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Distribution Systems for Multi-Security and<br>Operational Flexibility Enhancement<br>Lei Chao<br>A thesis submitted in partial fulfillment of the requirements for<br>the degree of Doctor of Philosophy rational Flexibility Enhancement<br>Lei Chao<br>itted in partial fulfillment of the requirements for<br>the degree of Doctor of Philosophy ial fulfillment of the requirements for<br>f Doctor of Philosophy<br>April 2024

# CERTIFICATE OF ORIGINALITY

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## Abstract

**Abstract**<br>The distribution grids evolve from the passive network with having the goal of<br>supplying reliably and efficiently the end users, gradually to active networks with<br>integrating distributed energy resources (DERs). **Abstract**<br>The distribution grids evolve from the passive network with having the goal of<br>supplying reliably and efficiently the end users, gradually to active networks with<br>integrating distributed energy resources (DERs). **integration**<br>**integration** and efficiently the end users, gradually to active networks with<br>integrating distributed energy resources (DERs). With the extensive use of operational<br>technologies (OT) and information and comm **Abstract**<br>The distribution grids evolve from the passive network with having the goal of<br>supplying reliably and efficiently the end users, gradually to active networks with<br>integrating distributed energy resources (DERs). **Abstract**<br>The distribution grids evolve from the passive network with having the goal of<br>supplying reliably and efficiently the end users, gradually to active networks with<br>integrating distributed energy resources (DERs). **Abstract**<br>The distribution grids evolve from the passive network with having the goal of<br>supplying reliably and efficiently the end users, gradually to active networks with<br>integrating distributed energy resources (DERs). The distribution grids evolve from the passive network with having the goal of<br>supplying reliably and efficiently the end users, gradually to active networks with<br>integrating distributed energy resources (DERs). With the e The distribution grids evolve from the passive network with having the goal of supplying reliably and efficiently the end users, gradually to active networks with integrating distributed energy resources (DERs). With the e The distribution grids evolve from the passive network with having the goal of supplying reliably and efficiently the end users, gradually to active networks with integrating distributed energy resources (DERs). With the e supplying reliably and efficiently the end users, gradually to active networks with<br>integrating distributed energy resources (DERs). With the extensive use of operational<br>technologies (OT) and information and communication stakeholders. Ethiologics (OT) and information and communication technologies (ICT) networks,<br>
Ethion to cyber-physical distribution systems enables the complete<br>
servability enhancement of measurements and smartization of control compo the transition to cyber-physical distribution systems enables the complete<br>observability enhancement of measurements and smartization of control components.<br>Under this background, an effective distribution network reconfig observability enhancement of measurements and smartization of control components.<br>
Under this background, an effective distribution network reconfiguration (DNR)<br>
scheme plays a key role in smart energy management of today

Under this background, an effective distribution network reconfiguration (DNR)<br>scheme plays a key role in smart energy management of today's active distribution<br>networks (ADNs) for substantial cost reductions and operation scheme plays a key role in smart energy management of today's active distribution<br>networks (ADNs) for substantial cost reductions and operational flexibility<br>enhancements subject to system observability and privacy concern networks (ADNs) for substantial cost reductions and operational flexibility<br>enhancements subject to system observability and privacy concerns of different<br>stakeholders.<br>Firstly, we propose a disjunctive convex hull relaxat hancements subject to system observability and privacy concerns of different<br>keholders.<br>Firstly, we propose a disjunctive convex hull relaxation (DCHR) to tackle with the<br>assical DNR problem. This classic DNR problem is a stakeholders.<br>
Firstly, we propose a disjunctive convex hull relaxation (DCHR) to tackle with the<br>
classical DNR problem. This classic DNR problem is a mixed integer second order<br>
conic programming (MISOCP) problem which i

system observability as a disjunctive relaxed connected dominating set problem for<br>reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained D reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained DNR model has been proposed.<br>Thirdly, the topology switch for the loss minimization system observability as a disjunctive relaxed connected dominating set problem for<br>reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained D stem observability as a disjunctive relaxed connected dominating set problem for<br>configurable ADNs with the least defense cost in theory. For the benefits of system<br>servability, an observability defense-constrained DNR mod

system observability as a disjunctive relaxed connected dominating set problem for<br>reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained D system observability as a disjunctive relaxed connected dominating set problem for<br>reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained D system observability as a disjunctive relaxed connected dominating set problem for<br>reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained D system observability as a disjunctive relaxed connected dominating set problem for<br>reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained D system observability as a disjunctive relaxed connected dominating set problem for<br>reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained D reconfigurable ADNs with the least defense cost in theory. For the benefits of system<br>observability, an observability defense-constrained DNR model has been proposed.<br>Thirdly, the topology switch for the loss minimization observability, an observability defense-constrained DNR model has been proposed.<br>
Thirdly, the topology switch for the loss minimization may expose the private load<br>
change information of an agent, e.g., transition from a Thirdly, the topology switch for the loss minimization may expose the private load<br>change information of an agent, e.g., transition from a light load to a heavy load, in<br>interconnected ADNs managed by multiple agents. To a change information of an agent, e.g., transition from a light load to a heavy load, in<br>interconnected ADNs managed by multiple agents. To address this issue, this paper<br>proposes a differentially private distribution networ interconnected ADNs managed by multiple agents. To address this issue, this paper<br>proposes a differentially private distribution network reconfigu-ration (DP-DNR)<br>mechanism based on a consensus alternating direction method proposes a differentially private distribution network reconfigu-ration (DP-DNR)<br>mcchanism based on a consensus alternating direction method of multipliers<br>(C-ADMM) algorithm. This can tackle privacy leakage challenges on mechanism based on a consensus alternating direction method of multipliers<br>(C-ADMM) algorithm. This can tackle privacy leakage challenges on the agent's and<br>eustomer's levels. To suppress private load change leakage as an -ADMM) algorithm. This can tackle privacy leakage challenges on the agent's and<br>stomer's levels. To suppress private load change leakage as an agent's concern, this<br>P-DNR mechanism provides a mixture output of realisticall enter's levels. To suppress private load change leakage as an agent's concern, this<br>DP-DNR mechanism provides a mixture output of realistically optimal topology<br>switch status and corresponding obfuscated-but-feasible load DP-DNR mechanism provides a mixture output of realistically optimal topology<br>switch status and corresponding obfuscated-but-feasible load flows, part of which<br>may have reverse load flow directions. On the customer's level,

switch status and corresponding obfuscated-but-feasible load flows, part of which<br>may have reverse load flow directions. On the eustomer's level, the C-ADMM-based<br>decentralized DP-DNR approach can seek the optimal topology may have reverse load flow directions. On the customer's level, the C-ADMM-based<br>decentralized DP-DNR approach can seek the optimal topology switch without<br>customer's load datasets of agents, whilst exchanged communication decentralized DP-DNR approach can seek the optimal topology switch without<br>customer's load datasets of agents, whilst exchanged communication signals in<br>C-ADMM algorithm are also synthetic based on the proposed DP-DNR mech developed. This proposed look-ahead economic dispatch model is cast as a MISOCP<br>problem. For this established MISOCP-based model, it is highly desirable to combine<br>the Multi-cut Benders Decomposition (MBD) and Generalized developed. This proposed look-ahead economic dispatch model is cast as a MISOCP<br>problem. For this established MISOCP-based model, it is highly desirable to combine<br>the Multi-cut Benders Decomposition (MBD) and Generalized developed. This proposed look-ahead economic dispatch model is cast as a MISOCP<br>problem. For this established MISOCP-based model, it is highly desirable to combine<br>the Multi-cut Benders Decomposition (MBD) and Generalized developed. This proposed look-ahead economic dispatch model is cast as a MISOCP<br>problem. For this established MISOCP-based model, it is highly desirable to combine<br>the Multi-cut Benders Decomposition (MBD) and Generalized developed. This proposed look-ahead economic dispatch model is cast as a MISOCP<br>problem. For this established MISOCP-based model, it is highly desirable to combine<br>the Multi-cut Benders Decomposition (MBD) and Generalized developed. This proposed look-ahead economic dispatch model is east as a MISOCP<br>problem. For this established MISOCP-based model, it is highly desirable to combine<br>the Multi-cut Benders Decomposition (MBD) and Generalized

# List of Publications

- **List of Publications**<br> *Journal papers*<br>
1. C. Lei, S. Bu, Q. Wang, N. Zhou, L. Yang, and X. Xiong, "Load<br>
optimization considering hot-spot and top-oil temperature limits of trans **1. C. Lei, S. Bu, Q. Wang, N. Zhou, L. Yang, and X. Xiong, "Load transfer**<br>1. **C. Lei, S. Bu, Q. Wang, N. Zhou, L. Yang, and X. Xiong, "Load transfer**<br>optimization considering hot-spot and top-oil temperature limits of tr optimization considering hot-spot and top-oil temperature limits of transformers,"<br>C. Lei, S. Bu, Q. Wang, N. Zhou, L. Yang, and X. Xiong, "Load transformers,"<br>*IEEE Transactions on Power Delivery*, vol. 37, no. 3, pp. 219 **St of Publications**<br> **C. Lei,** S. Bu, Q. Wang, N. Zhou, L. Yang, and X. Xiong, "Load transfer<br>
optimization considering hot-spot and top-oil temperature limits of transformers,"<br> *IEEE Transactions on Power Delivery*, vol **st of Publications**<br> **C. Lei, S. Bu, Q. Wang, N. Zhou, L. Yang, and X. Xiong, "Load transfer**<br> **C. Lei, S. Bu, Q. Wang, N. Zhou, L. Yang, and X. Xiong, "Load transfer**<br> *IEEE Transactions on Power Delivery*, vol. 37, no. **List of Publications**<br>
2. C. Lei, S. Bu, Q. Wang, N. Zhou, L. Yang, and X. Xiong, "Load transfer<br>
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1. C. Lei, S. Bu, Q. Wang and Q. Chen, "Dynamic Ramping of Retrofitted Coal-Fired<br>Power Plants: Basic Formulation and Tightened Approximation," in the 2023<br>IEEE Power & Energy Society General Meeting (PESGM), Orlando, Flor P. Lei, S. Bu, Q. Wang and Q. Chen, "Dynamic Ramping of Retrofitted Coal-Fired<br>Power Plants: Basic Formulation and Tightened Approximation," in the 2023<br>JEEE Power & Energy Society General Meeting (PESGM), Orlando, Florida E. Lei, S. Bu, Q. Wang and Q. Chen, "Dynamic Ramping of Retrofitted Coal-Fired<br>Power Plants: Basic Formulation and Tightened Approximation," in the 2023<br>IEEE Power & Energy Society General Meeting (PESGM), Orlando, Florida E. Lei, S. Bu, Q. Wang and Q. Chen, "Dynamic Ramping of Retrofitted Coal-Fired<br>Power Plants: Basic Formulation and Tightened Approximation," in the 2023<br>IEEE Power & Energy Society General Meeting (PESGM), Orlando, Florida 1. **C. Lei**, S. Bu, Q. Wang and Q. Chen, "Dynamic Ramping of Retrofitted Coal-Fired<br>
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## Patents

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1. S. Bu, G. Lu, E. Yim and C. Lei, "A Health Index System and Method<br>
Predicting Health Con 1. C. Lei, Q. Wang and N. Zhou, "Load Transfer Optimization in Smart<br>Discrete Delta Section 2003.2023.10253043.<br>
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1. C. Lei, Q. Wang and N. Zhou, "Load Transfer Optimization in Smart<br>
Distribution Networks," Publisher: China Electric Power Press, 2024. (Accepted)<br>
2. C. Lei and Q. Wang, "Look-ahead Rolli C. Lei, Q. Wang and N. Zhou, "Load Transfer Optimization in Smart<br>Distribution Networks," Publisher: China Electric Power Press, 2024. (Accepted)<br>C. Lei and Q. Wang, "Look-ahead Rolling Economic Dispatch Approach for<br>Wind-
- (Accepted)

## Acknowledgments

**Example 12 Example 12**<br>First and foremost, I would like to express my most sincere gratitude to my chief<br>ervisor Dr. Siqi. Bu. He is one of the top-tier scholars in the field of power system<br>ility and control. It is truly **Acknowledgments**<br>First and foremost, I would like to express my most sincere gratitude to my chief<br>supervisor Dr. Siqi. Bu. He is one of the top-tier scholars in the field of power system<br>stability and control. It is trul **Acknowledgments**<br>First and foremost, I would like to express my most sincere gratitude to my chief<br>supervisor Dr. Siqi. Bu. He is one of the top-tier scholars in the field of power system<br>stability and control. It is trul Acknowledgments<br>First and foremost, I would like to express my most sincere gratitude to my chief<br>supervisor Dr. Siqi. Bu. He is one of the top-tier scholars in the field of power system<br>stability and control. It is truly Acknowledgments<br>First and foremost, I would like to express my most sincere gratitude to my chief<br>supervisor Dr. Siqi. Bu. He is one of the top-tier scholars in the field of power system<br>stability and control. It is truly Acknowledgments<br>
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guidance. His attitude and enthusiasm for scientific research has always been a role<br>model for me to follow. From academic research to life value, he has given me<br>countless suggestions for thoughts shaping and paper writin model for me to follow. From academic research to life value, he has given me<br>countless suggestions for thoughts shaping and paper writing. His research integrity,<br>academic guidance and great support in my research is real countless suggestions for thoughts shaping and paper writing. His research integrity,<br>academic guidance and great support in my research is really a valuable treasure in<br>my life.<br>Morcover, I am cternally grateful to my coacademic guidance and great support in my research is really a valuable treasure in<br>my life.<br>Moreover, I am eternally grateful to my co-supervisors, Professor C. Y. Chung,<br>Professor Edward Chung from The Hong Kong Polytech my life.<br>
Moreover, I am eternally grateful to my co-supervisors, Professor C. Y. Chung,<br>
Professor Edward Chung from The Hong Kong Polytechnic University and Professor<br>
Dipti Srinivasan from National University of Singapo Moreover, I am eternally grateful to my co-supervisors, Professor C. Y. Chung,<br>Professor Edward Chung from The Hong Kong Polytechnic University and Professor<br>Dipti Srinivasan from National University of Singapore, for thei fessor Edward Chung from The Hong Kong Polytechnic University and Professor<br>ti Srinivasan from National University of Singapore, for their insightful<br>gestions on my research and advice in polishing my journal papers. Besid Dipti Srinivasan from National University of Singapore, for their insightful<br>suggestions on my research and advice in polishing my journal papers. Besides, I owe<br>my special thanks to my thesis committee members for their p suggestions on my research and advice in polishing my journal papers. Besides, I owe<br>my special thanks to my thesis committee members for their precious time in reading<br>my thesis and giving me valuable suggestions for impr

I will never forget this marvelous research journey in Hong Kong.<br>Last but not least, I would like to thank the support from The Hong Kong<br>Polytechnic University. Ill never forget this marvelous research journey in Hong Kong.<br>Last but not least, I would like to thank the support from The Hong Kong<br>ytechnic University. I will never forget this marvelous research journey in Hong Kong.<br>Last but not least, I would like to thank the support from The Hong<br>Polytechnic University.

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# List of Abbreviations





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System operational actions, which plays a key role in smart energy management of<br>the system operator (DSO) point of view, a high penetration of<br>energy 12.<br>Fig. 1.3 Cyber-physical systems on the decarbonization of energy 12 THE SPUT TIME STATES OF SUBSERVIEWS CONTRACT THE SPUT TIME STATES CONTRACT THE SUBSERVIEWS CONTRACT TIME STATES THE UNIVERSITY OF SUBSTRIAL REALM TIME STATES THE UNIVERSITY OF SUBSERVIEWS FOR SUBSTRIAL PREALM THE SPUT OF S Toward Content of measurements and smartization of control components via the computation of energy and the distribution systems operator (DSO) point of view, a high penetration of active resources and an extensive use of **EVALUAT STANA REALM**<br>
Fig. 1.3 Cyber-physical systems on the decarbonization of energy [2].<br>
From the distribution system operator (DSO) point of view, a high penetration of<br>
active resources and an extensive use of intel Fig. 1.3 Cyber-physical systems on the decarbonization of energy [2].<br>
From the distribution system operator (DSO) point of view, a high penetration of<br>
active resources and an extensive use of intelligent devices will clo From the distribution system operator (DSO) point of view, a high penetration of active resources and an extensive use of intelligent devices will closely interact with system operational actions, which plays a key role in From the distribution system operator (DSO) point of view, a high penetration of<br>active resources and an extensive use of intelligent devices will elosely interact with<br>system operational actions, which plays a key role in active resources and an extensive use of intelligent devices will closely interact with<br>system operational actions, which plays a key role in smart energy management of<br>today's DNs for substantial cost reductions [5]. Unde system operational actions, which plays a key role in smart energy management of today's DNs for substantial cost reductions [5]. Under this background, the evolution toward cyber-physical distribution systems enables the today's DNs for substantial cost reductions [5]. Under this background, the evolution<br>toward cyber-physical distribution systems enables the complete observability<br>enhancement of measurements and smartization of control co toward cyber-physical distribution systems enables the complete observability<br>cnhancement of measurements and smartization of control components via optic fiber<br>communications systems. Plus, circuit breakers (CBs) or reclo enhancement of measurements and smartization of control components via optic fiber<br>communications systems. Plus, circuit breakers (CBs) or reclosers, instead of usual<br>sectionalizers, are deployed for remote control and fre communications systems. Plus, circuit breakers (CBs) or reclosers, instead of usual<br>sectionalizers, are deployed for remote control and frequent operational switching in<br>recent years. Thus, it is clear that real-time topol



Problem of DNs over decades. It is a specific topology optimization about choosing<br>the optimal variable dual associated benefits<br>Decades. It is a specific topology optimization about choosing<br>the optimal switch status of s Sensitive data **Privacy Level** Sensitive data  $\begin{bmatrix} 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 &$ Fig. 1.4 Three concerned levels of cyber-physical distribution systems and<br>associated benefits<br>1.2 Research Motivations<br>1.2 Research Motivations<br>5.<br>Distribution network reconfiguration (DNR) is a classical optimal operatio associated benefits<br>associated benefits<br>associated benefits<br>associated benefits<br>associated benefits<br>points are associated benefits<br>points are achievable optimal operation<br>problem of DNs over decades. It is a specific topol (i) maintaining real-time load balancing (DNR) is a classical optimal operation<br>problem of DNs over decades. It is a specific topology optimization about choosing<br>the optimal switch status of sectionalizing switches (norma 1.2 Research Motivations<br>Distribution network reconfiguration (DNR) is a classical optimal operation<br>problem of DNs over decades. It is a specific topology optimization about choosing<br>the optimal switch status of sectional 1.2 Research Motivations<br>
Distribution network reconfiguration (DNR) is a classical optimal operation<br>
problem of DNs over decades. It is a specific topology optimization about choosing<br>
the optimal switch status of sectio Distribution network reconfiguration (DNR) is a classical optimal operation<br>problem of DNs over decades. It is a specific topology optimization about choosing<br>the optimal switch status of sectionalizing switches (normally problem of DNs over decades. It is a specific topology optimization about choosing<br>the optimal switch status of sectionalizing switches (normally closed), tie-switches<br>(normally open), and/or controllable power flows by so the optimal switch status of sectionalizing switches (normally closed), tie-switches<br>(normally open), and/or controllable power flows by soft open points (SOPs) [8]. In<br>contrast to passive DNs, two objectives are achievabl (normally open), and/or controllable power flows by soft open points (SOPs) [8]. In contrast to passive DNs, two objectives are achievable from performing DNR actions:<br>(i) maintaining real-time load balancing and loss redu

1.2.1 Primary Approaches for Topology Optimization of ADNs<br>To address the quick solvability of DNR problems, heuristic methods and convex<br>relaxations are two primary effective solving approaches for ADNs. To address the quick solvability of DNR problems, heuristic methods and convex<br>To address the quick solvability of DNR problems, heuristic methods and convex<br>axations are two primary effective solving approaches for ADNs.<br> Finary Approaches for Topology Optimization of ADNs<br>To address the quick solvability of DNR problems, heuristic methods and convex<br>relaxations are two primary effective solving approaches for ADNs.<br>In terms of heuristic me

2.1 Primary Approaches for Topology Optimization of ADNs<br>To address the quick solvability of DNR problems, heuristic methods and convex<br>axations are two primary effective solving approaches for ADNs.<br>In terms of heuristic **Polynomy Approaches for Topology Optimization of ADNs**<br> **Polynomy Solution** To address the quick solvability of DNR problems, heuristic methods and convex<br>
relaxations are two primary effective solving approaches for ADNs non-lineary *Approaches for Topology Optimization of ADNs*<br>To address the quick solvability of DNR problems, heuristic methods and convex<br>relaxations are two primary effective solving approaches for ADNs.<br>In terms of heuri *Primary Approaches for Topology Optimization of ADNs*<br>To address the quick solvability of DNR problems, heuristic methods and convex<br>relaxations are two primary effective solving approaches for ADNs.<br>In terms of heuristic 1.2.1 Primary Approaches for Topology Optimization of ADNs<br>
To address the quick solvability of DNR problems, heuristic methods and convex<br>
relaxations are two primary effective solving approaches for ADNs.<br>
In terms of h To address the quick solvability of DNR problems, heuristic methods and convex<br>relaxations are two primary effective solving approaches for ADNs.<br>In terms of heuristic methods, there is a rich set of optimization approache relaxations are two primary effective solving approaches for ADNs.<br>
In terms of heuristic methods, there is a rich set of optimization approaches in the<br>
power systems literature that tries to circumvent these problems. In In terms of heuristic methods, there is a rich set of optimization approaches in the<br>power systems literature that tries to circumvent these problems. In fact, the<br>non-linearity of AC power flow complicate solving a DNR pr power systems literature that tries to circumvent these problems. In fact, the<br>non-linearity of AC power flow complicate solving a DNR problem. To handle this<br>problem, researchers used DC power flow, a *DistFlow* model [10 non-linearity of AC power flow complicate solving a DNR problem. To handle this<br>problem, researchers used DC power flow, a *DistFlow* model [10] that is a<br>second-order mathematical programming model, and full AC power flow problem, researchers used DC power flow, a *DistFlow* model [10] that is a<br>second-order mathematical programming model, and full AC power flow [11].<br>However, the DC power flow model is inaccurate while the *DistFlow* and t second-order mathematical programming model, and full AC power flow [11].<br>However, the DC power flow model is inaccurate while the *DistFlow* and the full AC<br>power flow models are accurate but very time-consuming for a lar wever, the DC power flow model is inaccurate while the *DistFlow* and the full AC<br>wer flow models are accurate but very time-consuming for a large system.<br>ack-box heuristic methods, which push power flow calculations outsi power flow models are accurate but very time-consuming for a large system.<br>Black-box heuristic methods, which push power flow calculations outside the<br>optimization solver, have become very popular, owing to their broad app Black-box heuristic methods, which push power flow calculations outside the<br>optimization solver, have become very popular, owing to their broad applicability [12].<br>In summary, heuristic methods perform well in small system

optimization solver, have become very popular, owing to their broad applicability [12].<br>In summary, heuristic methods perform well in small systems but might converge<br>slowly, especially in large-scale systems [13]. And the In summary, heuristic methods perform well in small systems but might converge<br>slowly, especially in large-scale systems [13]. And the results obtained by heuristic<br>methods for different runs might not be the same, which slowly, especially in large-scale systems [13]. And the results obtained by heuristic<br>methods for different runs might not be the same, which prevents them from being<br>widely used in power system applications.<br>Regarding con computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina computational time. Moreover, a DNR problem is cast as a combinatorial explosion in<br>the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a bina the number of total possible solutions as the number of branches increases, if the<br>open/closed status of each branch is regarded as a binary variable. For instance, the<br>number of total possible solutions is 2136 for a DN h open/closed status of each branch is regarded as a binary variable. For instance, the<br>number of total possible solutions is 2136 for a DN has 136 circuit breakers. This<br>induces that a DNR problem becomes high-dimensional, number of total possible solutions is 2136 for a DN has 136 circuit breakers. This<br>induces that a DNR problem becomes high-dimensional, thus spending significant<br>amounts of computational time [18]. To reduce the computatio induces that a DNR problem becomes high-dimensional, thus spending significant<br>amounts of computational time [18]. To reduce the computational time caused by<br>high-dimension binary variables, it is essential to exploit the amounts of computational time [18]. To reduce the computational time caused by<br>high-dimension binary variables, it is essential to exploit the DNR model<br>characterization that can be used to accelerate this entire computat high-dimension binary variables, it is essential to exploit the D<br>characterization that can be used to accelerate this entire computation. On<br>with a high penetration of DERs into DNs, the fluctuated power injection<br>trigger aracterization that can be used to accelerate this entire computation. On top of that,<br>th a high penetration of DERs into DNs, the fluctuated power injections intricately<br>gger a complicated DNR decision-making process due with a high penetration of DERs into DNs, the fluctuated power injections intricately<br>trigger a complicated DNR decision-making process due to bi-directional power flows<br>[19], [20]. This suggests that today's active DNs in trigger a complicated DNR decision-making process due to bi-directional power flows<br>[19], [20]. This suggests that today's active DNs increases the difficulty level of quick<br>solvability. In the light of loosened relaxation

[19], [20]. This suggests that today's active DNs increases the difficulty level of quick<br>solvability. In the light of loosened relaxation bounds, high-dimensional space of<br>binary variables and integrations of enrichable solvability. In the light of loosened relaxation bounds, high-dimensional space of<br>binary variables and integrations of enrichable DERs, there is a research gap to<br>explore the efficient and tight relaxation approach for DN binary variables and integrations of enrichable DERs, there is a research gap to<br>explore the efficient and tight relaxation approach for DNR problems in large-scale<br>active DNs.<br>Based on the convex hull (CH) of *DistFlow*

1.2.2 Cyber-Physical Security Enhancement for Topology Optimization of ADNs<br>Apart from the fast computation methods for DNR problems, we have to consider<br>the cyber-physical security enhancement for topology optimization of 2.2 Cyber-Physical Security Enhancement for Topology Optimization of ADNs<br>Apart from the fast computation methods for DNR problems, we have to consider<br>2 cyber-physical security enhancement for topology optimization of ADN The cyber-Physical Security Enhancement for Topology Optimization of ADNs.<br>
Apart from the fast computation methods for DNR problems, we have to consider<br>
the cyber-physical security enhancement for topology optimization o *I.2.2 Cyber-Physical Security Enhancement for Topology Optimization of ADNs*<br>Apart from the fast computation methods for DNR problems, we have to consider<br>the cyber-physical security enhancement for topology optimization *Chart The Physical Security Enhancement for Topology Optimization of ADNs*<br>Apart from the fast computation methods for DNR problems, we have to consider<br>the cyber-physical security enhancement for topology optimization of Fact Cyber-Physical Security Enhancement for Topology Optimization of ADNs<br>Apart from the fast computation methods for DNR problems, we have to consider<br>the cyber-physical security enhancement for topology optimization of *I.2.2 Cyber-Physical Security Enhancement for Topology Optimization of ADNs*<br>Apart from the fast computation methods for DNR problems, we have to consider<br>the cyber-physical security enhancement for topology optimization 1.2.2 Cyber-Physical Security Enhancement for Topology Optimization of ADNs<br>Apart from the fast computation methods for DNR problems, we have to consider<br>the cyber-physical security enhancement for topology optimization o *I.2.2 Cyber-Physical Security Enhancement for Topology Optimization of ADNs*<br>Apart from the fast computation methods for DNR problems, we have to consider<br>the cyber-physical security enhancement for topology optimization Apart from the fast computation methods for DNR problems, we have to consider<br>the cyber-physical security enhancement for topology optimization of ADNs. The<br>deployment of cyber-physical systems with ADNs has led to an inc the cyber-physical security enhancement for topology optimization of ADNs. The<br>deployment of cyber-physical systems with ADNs has led to an increase in efficiency,<br>observability, and flexibility to facilitate the real-time deployment of cyber-physical systems with ADNs has led to an increase in efficiency,<br>observability, and flexibility to facilitate the real-time operation of ADNs. However,<br>some security threats from the inter dependency of servability, and flexibility to facilitate the real-time operation of ADNs. However,<br>me security threats from the inter dependency of the cyber and physical components<br>CPDS cannot be sufficiently tackled only with the simp some security threats from the inter dependency of the cyber and physical components<br>of CPDS cannot be sufficiently tackled only with the simplest protection measures<br>such as data encryption [23]. [24]. Protecting DNs agai of CPDS cannot be sufficiently tackled only with the simplest protection measures<br>such as data eneryption [23], [24]. Protecting DNs against cyber physical threats<br>typically is simply to eliminate the threat of false data

such as data eneryption [23], [24]. Protecting DNs against cyber-physical threats<br>typically is simply to climinate the threat of false data injection attacks (FDIAs) on<br>state estimation [25]-[28], where the data integrity typically is simply to eliminate the threat of false data injection attacks (FDIAs) on<br>state estimation [25]-[28], where the data integrity of state estimation is greatly<br>relevant to limited security resources, e.g., distr state estimation [25]–[28], where the data integrity of state estimation is greatly<br>relevant to limited security resources, e.g., distribution-level phasor measurement<br>units (D-PMUs) [29] and communication networks.<br>Agains relevant to limited security resources, e.g., distribution-level phasor measurement<br>units (D-PMUs) [29] and communication networks.<br>Against these possible cyber-physical threats, the defense level of cyber-physical<br>distrib units (D-PMUs) [29] and communication networks.<br>
Against these possible cyber-physical threats, the defense level of eyber-physical<br>
distribution system security for the real-time DNR has not been widely concerned to<br>
date Against these possible cyber-physical threats, the defense level of cyber-physical<br>distribution system security for the real-time DNR has not been widely concerned to<br>date. In existing studies, K. C. Sou [30], [31] constr distribution system security for the real-time DNR has not been widely concerned to<br>date. In existing studies, K. C. Sou [30], [31] constructs a minimum cost placement of<br>PMUs such that no FDIA is possible. However, this w

security issues that cannot be observed. To migrate this issue, we are focused on the full system observability of DNs that is crucial to understand the physical system states [35]. Full system observability of DNs that is crucial to understand the physical system<br>states [35].<br>With the full system observability of ADNs, the various grid operations depending security issues that cannot be observed. To migrate this issue, we are focused on the full system observability of DNs that is crucial to understand the physical system states [35].<br>With the full system observability of AD

security issues that cannot be observed. To migrate this issue, we are focused on the<br>full system observability of DNs that is crucial to understand the physical system<br>states [35].<br>With the full system's behaviors, e.g., security issues that cannot be observed. To migrate this issue, we are focused on the<br>full system observability of DNs that is crucial to understand the physical system<br>states [35].<br>With the full system observability of AD security issues that cannot be observed. To migrate this issue, we are focused on the full system observability of DNs that is crucial to understand the physical system states [35].<br>With the full system observability of AD security issues that cannot be observed. To migrate this issue, we are focused on the<br>full system observability of DNs that is crucial to understand the physical system<br>states [35].<br>With the full system observability of AD security issues that cannot be observed. To migrate this issue, we are focused on the<br>full system observability of DNs that is crucial to understand the physical system<br>states [35].<br>With the full system observability of AD full system observability of DNs that is crucial to understand the physical system<br>states [35].<br>With the full system observability of ADNs, the various grid operations depending<br>on the physical system's behaviors, e.g., ge states [35].<br>With the full system observability of ADNs, the various grid operations depending<br>on the physical system's behaviors, e.g., generator redispatch, fault location, can be<br>under monitoring and control. For exampl on the physical system's behaviors, e.g., generator redispatch, fault location, can be<br>under monitoring and control. For example, the DSOs can observe the voltage<br>excursion and overloading power flows as soon as early, and er monitoring and control. For example, the DSOs can observe the voltage<br>ursion and overloading power flows as soon as early, and then can remove these<br>curity problems in time [36]. Accordingly, the effective cyber-physica exeursion and overloading power flows as soon as early, and then ean remove these<br>insecurity problems in time [36]. Accordingly, the effective cyber-physical system<br>security defense, e.g., the full system observability gua

insecurity problems in time [36]. Accordingly, the effective cyber-physical system<br>security defense, e.g., the full system observability guarantees, cannot be neglected<br>[37]. At present, few studies deal with this defense security defense, e.g., the full system observability guarantees, cannot be neglected<br>
[37]. At present, few studies deal with this defense issue at the lowest expense of<br>
different topology schemes from the perspective of [37]. At present, few studies deal with this defense issue at the lowest expense of different topology schemes from the perspective of full system observability.<br>
1.2.3 Privacy-Preserving Enhancement for Topology Optimiza different topology schemes from the perspective of full system observability.<br>
1.2.3 Privacy-Preserving Enhancement for Topology Optimization of ADNs<br>
Even though cyber-physical security of ADNs can be guaranteed,<br>
privacy *the use of both resources to meet the load demands, and also helps to reduce new also errowser-physical security of ADNs can be guaranteed, privacy-preserving data sharing should be also crucial for distribution-level age* 1.2.3 Privacy-Preserving Enhancement for Topology Optimization of ADNs<br>
Even though cyber-physical security of ADNs can be guaranteed,<br>
privacy-preserving data sharing should be also crucial for distribution-level agents,<br> Even though cyber-physical security of ADNs can be guaranteed,<br>privacy-preserving data sharing should be also crucial for distribution-level agents,<br>especially for those with conflicting interests. At the operation level,

stakeholders, e.g., load aggregators. And their network connections are tie-lines<br>across different agents.<br>For interconnected multi-agent ADNs, tie-line load flow information and topology

stakeholders, e.g., load aggregators. And their network connections are tie-line<br>across different agents.<br>For interconnected multi-agent ADNs, tie-line load flow information and topolo<br>switch status are generally shared wi Keholders, e.g., load aggregators. And their network connections are tie-lines<br>For interconnected multi-agent ADNs, tie-line load flow information and topology<br>For interconnected multi-agent ADNs, tie-line load flow inform stakeholders, e.g., load aggregators. And their network connections are tie-lines<br>across different agents.<br>For interconnected multi-agent ADNs, tie-line load flow information and topology<br>switch status are generally shared stakeholders, e.g., load aggregators. And their network connections are tie-lines<br>across different agents.<br>For interconnected multi-agent ADNs, tie-line load flow information and topology<br>switch status are generally shared stakeholders, e.g., load aggregators. And their network connections are tie-lines<br>across different agents.<br>For interconnected multi-agent ADNs, tie-line load flow information and topology<br>switch status are generally shared stakeholders, e.g., load aggregators. And their network connections are tie-lines<br>across different agents.<br>For interconnected multi-agent ADNs, tie-line load flow information and topology<br>switch status are generally shared stakeholders, e.g., load aggregators. And their network connections are tie-lines<br>across different agents.<br>For interconnected multi-agent ADNs, tie-line load flow information and topology<br>switch status are generally shared across different agents.<br>
For interconnected multi-agent ADNs, tie-line load flow information and topology<br>
switch status are generally shared with different agents for interconnected operational<br>
and/or marketing purposes For interconnected multi-agent ADNs, tie-line load flow information and topology<br>switch status are generally shared with different agents for interconnected operational<br>and/or marketing purposes, which energy data-sharing switch status are generally shared with different agents for interconnected operational<br>and/or marketing purposes, which energy data-sharing may evoke privacy-related<br>complications, i.e., inference of sensitive information and/or marketing purposes, which energy data-sharing may evoke privacy-related<br>complications, i.e., inference of sensitive information [39]. In the future energy data<br>asset market, the energy data-sharing mechanisms has dr complications, i.e., inference of sensitive information [39]. In the future energy data<br>asset market, the energy data-sharing mechanisms has drawn extensive attention [40],<br>especially for the auction market with fair comme asset market, the energy data-sharing mechanisms has drawn extensive attention [40],<br>especially for the auction market with fair commercial competition [41]. Under this<br>background, we specifically focus on the privacy-rela especially for the auction market with fair commercial competition [41]. Under this<br>background, we specifically focus on the privacy-related information leakage issue of<br>loads caused by the DNR operation on two load levels background, we specifically focus on the privacy-related information leakage issue of<br>loads caused by the DNR operation on two load levels, i.e., agent's and customer's<br>levels. For agent's privacy concerns, sharing tic-lin loads caused by the DNR operation on two load levels, i.e., agent's and customer's<br>levels. For agent's privacy concerns, sharing tic-line load flow information may suffer<br>from leaking the private load change information of il evels. For agent's privacy concerns, sharing tie-line load flow information may suffer<br>from leaking the private load change information of an agent, i.e., transition from a<br>light load to a heavy load. This information c light load to a heavy load. This information can be acknowledged by other agents who<br>are stakeholders with conflicting interests, e.g., bidding for grid services in energy<br>market [41]. On the customer's load privacy level, are stakeholders with conflicting interests, e.g., bidding for grid services in energy<br>market [41]. On the customer's load privacy level, all customer's load datasets from<br>smart meters are obliged to be uploaded to the dis

and decryption operation by a trusted third party who should own a large amount of available computational resources in [45] and [47]. However, finding an authorized and high-performance computing third party for this job and decryption operation by a trusted third party who should own a large amount of<br>available computational resources in [45] and [47]. However, finding an authorized<br>and high-performance computing third party for this job and decryption operation by a trusted third party who should own a large amount of<br>available computational resources in [45] and [47]. However, finding an authorized<br>and high-performance computing third party for this job and decryption operation by a trusted third party who should own a large amount of<br>available computational resources in [45] and [47]. However, finding an authorized<br>and high-performance computing third party for this job and decryption operation by a trusted third party who should own a large amount of<br>available computational resources in [45] and [47]. However, finding an authorized<br>and high-performance computing third party for this job and decryption operation by a trusted third party who should own a large amount of<br>available computational resources in [45] and [47]. However, finding an authorized<br>and high-performance computing third party for this job and decryption operation by a trusted third party who should own a large amount of<br>available computational resources in [45] and [47]. However, finding an authorized<br>and high-performance computing third party for this job and decryption operation by a trusted third party who should own a large amount of available computational resources in [45] and [47]. However, finding an authorized and high-performance computing third party for this job and decryption operation by a trusted third party who should own a large amount of<br>available computational resources in [45] and [47]. However, finding an authorized<br>and high-performance computing third party for this job available computational resources in [45] and [47]. However, finding an authorized<br>and high-performance computing third party for this job is also very costly for<br>real-time DNR operations. Therefore, we concentrate on a di and high-performance computing third party for this job is also very costly for<br>real-time DNR operations. Therefore, we concentrate on a differential privacy<br>mechanism to increase the data privacy and it can be used to sha real-time DNR operations. Therefore, we concentrate on a differential privacy<br>mechanism to increase the data privacy and it can be used to share sensitive data<br>without a trusted third party. Regarding differential privacy mechanism to increase the data privacy and it can be used to share sensitive data<br>without a trusted third party. Regarding differential privacy mechanism in power<br>systems, it can quantify and bound privacy risks through th without a trusted third party. Regarding differential privacy mechanism in power<br>systems, it can quantify and bound privacy risks through the randomization of<br>sensitive datasets, e.g., leveraging a carefully calibrated noi sensitive datasets, e.g., leveraging a carefully calibrated noise to solve the<br>private-preserving optimal power flow (OPF) problems in ADNs [48] and<br>transmission systems [49], or obfuscating power grid parameters for netwo private-preserving optimal power flow (OPF) problems in ADNs [48] and<br>transmission systems [49], or obfuscating power grid parameters for network privacy<br>preservation [50]. Recently, the program perturbation strategy [51], transmission systems [49], or obfuscating power grid parameters for network privacy<br>prescrvation [50]. Recently, the program perturbation strategy [51], [52] is created to<br>ensure the feasibility of privacy-preserving optim preservation [50]. Recently, the program perturbation strategy [51], [52] is ereated to<br>ensure the feasibility of privacy-preserving optimal solutions with the high probability<br>via a stochastic chance-constrained optimizat ensure the feasibility of privacy-preserving optimal solutions with the high probability<br>via a stochastic chance-constrained optimization reformulation. This is superior to the<br>bi-level optimization based on the output/obj solutions.

Regarding the customer's privacy, since this privacy leakage may be caused by<br>loading sensitive load datasets to the distribution dispatch center, a decentralized<br>mework is well suited for the DSO to deal with this DNR pro Regarding the customer's privacy, since this privacy leakage may be caused by uploading sensitive load datasets to the distribution dispatch center, a decentralized framework is well suited for the DSO to deal with this DN Regarding the customer's privacy, since this privacy leakage may be caused by<br>uploading sensitive load datasets to the distribution dispatch center, a decentralized<br>framework is well suited for the DSO to deal with this DN Regarding the customer's privacy, since this privacy leakage may be caused by<br>uploading sensitive load datasets to the distribution dispatch center, a decentralized<br>framework is well suited for the DSO to deal with this DN Regarding the customer's privacy, since this privacy leakage may be caused by<br>uploading sensitive load datasets to the distribution dispatch center, a decentralized<br>framework is well suited for the DSO to deal with this DN Regarding the customer's privacy, since this privacy leakage may be caused by<br>uploading sensitive load datasets to the distribution dispatch center, a decentralized<br>framework is well suited for the DSO to deal with this DN Regarding the customer's privacy, since this privacy leakage may be caused by uploading sensitive load datasets to the distribution dispatch center, a decentralized framework is well suited for the DSO to deal with this DN Regarding the customer's privacy, since this privacy leakage may be caused by uploading sensitive load datasets to the distribution dispatch center, a decentralized framework is well suited for the DSO to deal with this DN Regarding the customer's privacy, since this privacy leakage may be caused by<br>uploading sensitive load datasets to the distribution dispatch center, a decentralized<br>framework is well suited for the DSO to deal with this DN uploading sensitive load datasets to the distribution dispatch center, a decentralized<br>framework is well suited for the DSO to deal with this DNR problem [53] and the<br>resilience enhancement of ADNs in recent years [54], [5 framework is well suited for the DSO to deal with this DNR problem [53] and the<br>resilience enhancement of ADNs in recent years [54], [55], [56], [57]. This also<br>contributes to relieving the communication burden and preserv resilience enhancement of ADNs in recent years [54], [55], [56], [57]. This also<br>contributes to relieving the communication burden and preserving the privacy of<br>customers' load datasets. In terms of decentralized framework contributes to relieving the communication burden and preserving the privacy of customers' load datasets. In terms of decentralized frameworks, the alternating direction method of multipliers (ADMM) approach and its varian customers' load datasets. In terms of decentralized frameworks, the alternating<br>direction method of multipliers (ADMM) approach and its variants [58], are typical<br>decentralized solutions for such privacy-preserving concern direction method of multipliers (ADMM) approach and its variants [58], are typical<br>decentralized solutions for such privacy-preserving concerns [59]. Recently, it can be<br>used in mixed-integer quadratic programming (MIQP) p decentralized solutions for such privacy-preserving concerns [59]. Recently, it can be<br>used in mixed-integer quadratic programming (MIQP) problems with good<br>performance as reported in [56], [57], [60]. To be specific, the used in mixed-integer quadratic programming (MIQP) problems with good<br>performance as reported in [56], [57], [60]. To be specific, the DNR problem can be<br>approximated as a MIQP problem. Thus, the consensus ADMM (C-ADMM)<br>ap performance as reported in [56], [57], [60]. To be specific, the DNR problem can be<br>approximated as a MIQP problem. Thus, the consensus ADMM (C-ADMM)<br>approach can be adopted to deal with this DNR problem by breaking the co approximated as a MIQP problem. Thus, the consensus ADMM (C-ADMM)<br>approach can be adopted to deal with this DNR problem by breaking the complex<br>computational DNR tasks into much smaller ones. Each smaller computational tas approach can be adopted to deal with this DNR problem by breaking the complex<br>computational DNR tasks into much smaller ones. Each smaller computational task is<br>performed by an individual agent who only communicates and to computational DNR tasks into much smaller ones. Each smaller computational task is<br>performed by an individual agent who only communicates and works collectively<br>with the DSO by exchanging their tie-switch states and tie-li performed by an individual agent who only communicates and works collectively<br>with the DSO by exchanging their tie-switch states and tie-line load flows. However,<br>the explicit communication exchanging signals of realistic

1.2.4 Operational Flexibility Enhancement by Topology Optimization of HVDNs<br>
On top of that, growing penetration of renewable energy in power generation areas<br>
provides a green solution to the decarbonization of power syst 2.4 *Operational Flexibility Enhancement by Topology Optimization of HVDNs*<br>On top of that, growing penetration of renewable energy in power generation areas<br>ovides a green solution to the decarbonization of power systems *Provides a green solution to the decarbonization of HVDNs*<br> **provides a green solution to the decarbonization of power systems [61]-[64].** To<br>
accommodate more renewable energy integration, the wind-thermal-bundled power<br> 1.2.4 Operational Flexibility Enhancement by Topology Optimization of HVDNs<br>
On top of that, growing penetration of renewable energy in power generation areas<br>
provides a green solution to the decarbonization of power syst 1.2.4 Operational Flexibility Enhancement by Topology Optimization of HVDNs<br>
On top of that, growing penetration of renewable energy in power generation areas<br>
provides a green solution to the decarbonization of power syst 1.2.4 Operational Flexibility Enhancement by Topology Optimization of HYDNs<br>On top of that, growing penetration of renewable energy in power generation areas<br>provides a green solution to the decarbonization of power system 1.2.4 Operational Flexibility Enhancement by Topology Optimization of HYDNs<br>
On top of that, growing penetration of renewable energy in power generation areas<br>
provides a green solution to the decarbonization of power syst *I.2.4 Operational Flexibility Enhancement by Topology Optimization of HVDNs*<br>
On top of that, growing penetration of renewable energy in power generation areas<br>
provides a green solution to the decarbonization of power sy *I.2.4 Operational Flexibility Enhancement by Topology Optimization of I*<br>On top of that, growing penetration of renewable energy in power gen<br>provides a green solution to the decarbonization of power systems [<br>accommodate On top of that, growing penetration of renewable energy in power generation areas<br>avides a green solution to the decarbonization of power systems [61]-[64]. To<br>commodate more renewable energy integration, the wind-thermalprovides a green solution to the decarbonization of power systems [61]-[64]. To<br>accommodate more renewable energy integration, the wind-thermal-bundled power<br>system (WTBPS) is a suitable option to increase power system fle accommodate more renewable energy integration, the wind-thermal-bundled power<br>system (WTBPS) is a suitable option to increase power system flexibility [65]. To<br>explore an efficient economic dispatch of WTBPS, distribution-

system (WTBPS) is a suitable option to increase power system flexibility [65]. To<br>explore an efficient economic dispatch of WTBPS, distribution-level topology<br>optimization [66] and energy storage [67] can be used to increa explore an efficient economic dispatch of WTBPS, distribution-level topology<br>optimization [66] and energy storage [67] can be used to increase the flexibility of<br>WTBPS in which this generation system instantly accommodates optimization [66] and energy storage [67] can be used to increase the flexibility of<br>WTBPS in which this generation system instantly accommodates the rapid growth of<br>wind farms.<br>Since WTBPS can mitigate the uncertainty and WTBPS in which this generation system instantly accommodates the rapid growth of<br>wind farms.<br>Since WTBPS can mitigate the uncertainty and variability of renewable resources,<br>the transition of optimal generation dispatch is wind farms.<br>
Since WTBPS can mitigate the uncertainty and variability of renewable resources,<br>
the transition of optimal generation dispatch is underway to multi-energy generation<br>
systems. Previous work has investigated t Since WTBPS can mitigate the uncertainty and variability of renewable resources,<br>the transition of optimal generation dispatch is underway to multi-energy generation<br>systems. Previous work has investigated the economic di the transition of optimal generation dispatch is underway to multi-energy generation<br>systems. Previous work has investigated the economic dispatch methods in bulk<br>AC/DC hybrid WTBPS [68], combined generation system of mult systems. Previous work has investigated the economic dispatch methods in bulk<br>AC/DC hybrid WTBPS [68], combined generation system of multiple renewable<br>energy resources and energy storage [69], multi-fuel [70], and integra

On the one hand, retrofitted coal-fired plants are worldwide concerned, since<br>al-fired plants are not phased out, especially in middle-income countries [75].<br>ithout retrofits, coal power plants can run at the minimum level Coal-fired plants are not phased out, especially in middle-income countries [75].<br>Without retrofits, coal power plants can run at the minimum level of 50%, and the<br>tramp rate for an inflexible unit is 0.6–2% per minute of on the one hand, retrofitted coal-fired plants are worldwide concerned, since<br>coal-fired plants are not phased out, especially in middle-income countries [75].<br>Without retrofits, coal power plants can run at the minimum le on the one hand, retrofitted coal-fired plants are worldwide concerned, since<br>coal-fired plants are not phased out, especially in middle-income countries [75].<br>Without retrofits, coal power plants can run at the minimum le On the one hand, retrofitted coal-fired plants are worldwide concerned, since<br>coal-fired plants are not phased out, especially in middle-income countries [75].<br>Without retrofits, coal power plants can run at the minimum l On the one hand, retrofitted coal-fired plants are worldwide concerned, since<br>coal-fired plants are not phased out, especially in middle-income countries [75].<br>Without retrofits, coal power plants can run at the minimum le On the one hand, retrofitted coal-fired plants are worldwide concerned, since<br>coal-fired plants are not phased out, especially in middle-income countries [75].<br>Without retrofits, coal power plants can run at the minimum l On the one hand, retrofitted coal-fired plants are worldwide concerned, since<br>coal-fired plants are not phased out, especially in middle-income countries [75].<br>Without retrofits, coal power plants can run at the minimum l coal-fired plants are not phased out, especially in middle-income countries [75].<br>Without retrofits, coal power plants can run at the minimum level of 50%, and the<br>ramp rate for an inflexible unit is 0.6-2% per minute of r Without retrofits, coal power plants can run at the minimum level of 50%, and the<br>ramp rate for an inflexible unit is 0.6 2% per minute of rated power [76]. After<br>retrofits, retrofitted power plants can enable sufficient ramp rate for an inflexible unit is 0.6-2% per minute of rated power [76]. After<br>retrofits, retrofitted power plants can enable sufficient flexibility [73], [77], the most<br>beneficial advances of which are the reduction of retrofits, retrofitted power plants can enable sufficient flexibility [73], [77], the most<br>beneficial advances of which are the reduction of minimum load levels to 15%-30%<br>of rated capacities and the increase of ramp rate beneficial advances of which are the reduction of minimum load levels to 15%-30%<br>of rated capacities and the increase of ramp rate to 2-6% per minute of rated power.<br>For example, the ramp rates of retrofitted power plants of rated capacities and the increase of ramp rate to 2-6% per minute of rated power.<br>For example, the ramp rates of retrofitted power plants are generally raised to 2 6%<br>in China, 2-6% in Poland, and 3-6% in Germany. The m For example, the ramp rates of retrofitted power plants are generally raised to  $2-6\%$ <br>in China, 2 6% in Poland, and 3-6% in Germany. The minimal load of coal-fired<br>plant Bexbach has been reduced by 11%, and coal-fired p in China, 2–6% in Poland, and 3-6% in Germany. The minimal load of coal-fired<br>plant Bexbach has been reduced by 11%, and coal-fired plant Wes Weiler has<br>increased the ramp rate by 10MW/min in Germany [77]. As reported in [ plant Bexbach has been reduced by 11%, and coal-fired plant Wes Weiler has<br>increased the ramp rate by 10MW/min in Germany [77]. As reported in [78], ramping<br>limits in practical applications should be a function of the unit increased the ramp rate by 10MW/min in Germany [77]. As reported in [78], ramping<br>limits in practical applications should be a function of the unit's generating output.<br>Thus, ramp rates are dynamic at different output powe limits in practical applications should be a function of the unit's generating output.<br>Thus, ramp rates are dynamic at different output power levels. Existing pieces of<br>literature regarding dynamic ramp rates mainly fit in Thus, ramp rates are dynamic at different output power levels. Existing pieces of literature regarding dynamic ramp rates mainly fit into two categories, i.e., piecewise linear functions and stepwise linear representations
On the other hand, storage energy response [81] or distribution-side load transfer<br>tions [82] will be an alternative to offset the insufficient ramping margins of<br>al-fired units in WTBPS. Reference [83] adopts an adjustabl on the other hand, storage energy response [81] or distribution-side load transfer<br>actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjus On the other hand, storage energy response [81] or distribution-side load transfer<br>actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjus on the other hand, storage energy response [81] or distribution-side load transfer<br>actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjus The other hand, storage energy response [81] or distribution-side load transfer<br>actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjustab On the other hand, storage energy response [81] or distribution-side load transfer<br>actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjus On the other hand, storage energy response [81] or distribution-side load transfer<br>actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjus On the other hand, storage energy response [81] or distribution-side load transfer<br>actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjus On the other hand, storage energy response [81] or distribution-side load transfer<br>actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjus actions [82] will be an alternative to offset the insufficient ramping margins of<br>coal-fired units in WTBPS. Reference [83] adopts an adjustable heat storage strategy<br>from solar power stations to shift excess wind power in coal-fired units in WTBPS. Reference [83] adopts an adjustable heat storage strategy<br>from solar power stations to shift excess wind power in combined wind-thermal<br>generation systems. However, compared with solar power reso from solar power stations to shift excess wind power in combined wind-thermal<br>generation systems. However, compared with solar power resources, the operation<br>cost of stored energy is too expensive to be widely used in WTBP generation systems. However, compared with solar power resources, the operation<br>cost of stored energy is too expensive to be widely used in WTBPS. Load resources in<br>distribution networks have instinctive flexibility, which cost of stored energy is too expensive to be widely used in WTBPS. Load resources in<br>distribution networks have instinctive flexibility, which has great potential to install<br>with WTBPS in order to maximally avoid wind curt distribution networks have instinctive flexibility, which has great potential to install<br>with WTBPS in order to maximally avoid wind curtailment. As the desirable load<br>provider, the distribution-level topology actions have with WTBPS in order to maximally avoid wind curtailment. As the desirable load<br>provider, the distribution-level topology actions have been performed via<br>reconfigurable HVDNs integrated with renewable resources under<br>stabil provider, the distribution-level topology actions have been performed via<br>reconfigurable HVDNs integrated with renewable resources under<br>stability-constrained conditions [84]. HVDNs are sub-transmissions on 110kV voltage<br>l reconfigurable HVDNs integrated with renewable resources under<br>stability-constrained conditions [84]. HVDNs are sub-transmissions on 110kV voltage<br>level [82] constructed in meshed topology (closed loop) but operated in rad stability-constrained conditions [84]. HVDNs are sub-transmissions on 110kV voltage<br>level [82] constructed in meshed topology (closed loop) but operated in radial<br>structures (open loop), which network can be found in China level [82] constructed in meshed topology (closed loop) but operated in radial<br>structures (open loop), which network can be found in China [82], Spain [84] and<br>Finland [85]. The HVDNs are composed of specific topological u structures (open loop), which network can be found in China [82], Spain [84] and Finland [85]. The HVDNs are composed of specific topological units, which can<br>reduce the computational complexity of distribution-level topol

1.3 Primary Contributions<br>Severe issues in facets of physical-oriented, cyber-oriented, and privacy-oriented<br>security can be emerged in the topology reconfiguration of smart DNs, which she Primary Contributions<br>Severe issues in facets of physical-oriented, cyber-oriented, and privacy-oriented<br>arity can be emerged in the topology reconfiguration of smart DNs, which should<br>considered for real-time operational Subsecurity can be emerged in the topology reconfiguration of smart DNs, which should<br>be considered for real-time operational actions. This thesis aims to enhance different<br>facets of security using the advanced topology op 1.3 Primary Contributions<br>Severe issues in facets of physical-oriented, cyber-oriented, and privacy-oriented<br>security can be emerged in the topology reconfiguration of smart DNs, which should<br>be considered for real-time op Facetive issues in facets of physical-oriented, cyber-oriented, and privacy-oriented security can be emerged in the topology reconfiguration of smart DNs, which should be considered for real-time operational actions. This 1.3 Primary Contributions<br>Severe issues in facets of physical-oriented, cyber-oriented, and privacy-oriented<br>security can be emerged in the topology reconfiguration of smart DNs, which should<br>be considered for real-time op 1.3 Primary Contributions<br>Severe issues in facets of physical-oriented, cyber-oriented, and privacy-oriented<br>security can be emerged in the topology reconfiguration of smart DNs, which should<br>be considered for real-time op 1) To avoid worldwides the DNR model with enrichable DERs using the theoretical vectors of smart phase climate climate climate change distinct climate change distinct climate change distinct climate change distinct climate 1.3 Primary Contributions<br>Severe issues in facets of physical-oriented, cyber-oriented, and privacy-oriented<br>security can be emerged in the topology reconfiguration of smart DNs, which should<br>be considered for real-time op Severe issues in facets of physical-oriented, cyber-oriented, and privacy-oriented<br>security can be emerged in the topology reconfiguration of smart DNs, which should<br>be considered for real-time operational actions. This th

security can be emerged in the topology reconfiguration of smart DNs, which should<br>be considered for real-time operational actions. This thesis aims to enhance different<br>facets of security using the advanced topology optim be considered for real-time operational actions. This thesis aims to enhance different<br>faccts of sccurity using the advanced topology optimization, and then explores a<br>distribution-level topology optimization for flexibili facets of security using the advanced topology optimization, and then explores a<br>distribution-level topology optimization for flexibility enhancement in economic<br>dispatch of wind-thermal-bundled power system.<br>1) To avoid w distribution-level topology optimization for flexibility enhancement in economic<br>dispatch of wind-thermal-bundled power system.<br>1) To avoid worldwide climate change effects, decarbonization initiatives transit<br>the conventi dispatch of wind-thermal-bundled power system.<br>
1) To avoid worldwide climate change effects, decarbonization initiatives transit<br>
the conventional DNs to be smart DNs mixed with a high penetration of DERs. This<br>
paper the 1) To avoid worldwide climate change effects, decarbonization initiatives transit<br>the conventional DNs to be smart DNs mixed with a high penetration of DERs. This<br>paper theoretically reformulates the DNR model with enricha the conventional DNs to be smart DNs mixed with a high penetration of DERs. This<br>paper theoretically reformulates the DNR model with enrichable DERs using the<br>disjunctive convex hull approach. Continuous parent-child relat per theoretically reformulates the DNR model with enrichable DERs using the<br>sjunctive convex hull approach. Continuous parent-child relationship variables in<br>anning tree constraints can be regarded as disjunctive variables disjunctive convex hull approach. Continuous parent-child relationship variables in<br>spanning tree constraints can be regarded as disjunctive variables to represent<br>disjunctive convex hull of *DistFlow* equations. And this spanning tree constraints can be regarded as disjunctive variables to represent<br>disjunctive convex hull of *DistFlow* equations. And this disjunctive convex hull<br>relaxation (DCHR) is proven as a tighter relaxation than the disjunctive convex hull of *DistFlow* equations. And this disjunctive convex hull<br>relaxation (DCHR) is proven as a tighter relaxation than the existing relaxation<br>techniques for DNR problems, such as the Big-M and McCormic

relaxation (DCHR) is proven as a tighter relaxation than the existing relaxation<br>techniques for DNR problems, such as the Big-M and McCormick linearization<br>methods. Case studies also demonstrate that the DCHR's computing p

model is then converted to a mixed integer second-order conic programming problem,<br>which can be solved with commercial solvers easily.<br>3) The topology switch for the loss minimization may expose the private which can be solved with commercial solvers easily.<br>
The topology switch for the loss minimization may expose the private<br>
Shown the solved with commercial solvers easily.<br>
The topology switch for the loss minimization may

3) The topology switch for the loss minimization may expose the private command of light and heavy loads for interconnected DNs owned by different exponention of light and heavy loads for interconnected DNs owned by differ information of light and wave integer second-order conic programming problem,<br>which can be solved with commercial solvers easily.<br>3) The topology switch for the loss minimization may expose the private<br>information of light stakeholders with connucted to a mixed integer second-order conic programming problem,<br>which can be solved with commercial solvers easily.<br>3) The topology switch for the loss minimization may expose the private<br>information model is then converted to a mixed integer second-order conic programming problem,<br>which can be solved with commercial solvers easily.<br>3) The topology switch for the loss minimization may expose the private<br>information of model is then converted to a mixed integer second-order conic programming problem,<br>which can be solved with commercial solvers easily.<br>3) The topology switch for the loss minimization may expose the private<br>information of model is then converted to a mixed integer second-order conic programming problem,<br>which can be solved with commercial solvers easily.<br>3) The topology switch for the loss minimization may expose the private<br>information of model is then converted to a mixed integer second-order conic programming problem,<br>which can be solved with commercial solvers easily.<br>3) The topology switch for the loss minimization may expose the private<br>information of which can be solved with commercial solvers easily.<br>
3) The topology switch for the loss minimization may expose the private<br>
information of light and heavy loads for interconnected DNs owned by different<br>
stakeholders wit 3) The topology switch for the loss minimization may expose the private information of light and heavy loads for interconnected DNs owned by different stakeholders with conflicting interests. For agent's privacy concerns, information of light and heavy loads for interconnected DNs owned by different<br>stakeholders with conflicting interests. For agent's privacy concerns, we propose the<br>DP-DNR mechanism in this chapter. This DP-DNR mechanism p stakeholders with conflicting interests. For agent's privacy concerns, we propose the<br>DP-DNR mechanism in this chapter. This DP-DNR mechanism provides a mixture<br>output of realistically optimal tie-switch status and corresp DP-DNR mechanism in this chapter. This DP-DNR mechanism provides a mixture<br>output of realistically optimal tie-switch status and corresponding<br>obfuscated-but-feasible load flows, part of which may have reverse load flow<br>di output of realistically optimal tie-switch status and corresponding<br>obfuscated-but-feasible load flows, part of which may have reverse load flow<br>directions. This privacy-preserving mechanism is used to mitigate agent's pri obfuscated-but-feasible load flows, part of which may have reverse load flow directions. This privacy-preserving mechanism is used to mitigate agent's privacy concerns against private load change leakage from DNR operation directions. This privacy-preserving mechanism is used to mitigate agent's privacy<br>concerns against private load change leakage from DNR operations, which has not<br>been concerned yet. On the customer's privacy-preserving lev concerns against private load change leakage from DNR operations<br>been concerned yet. On the customer's privacy-preserving level, the C<br>decentralized DP-DNR approach can seek the optimal DNR s<br>customer's load datasets of ag en concerned yet. On the customer's privacy-preserving level, the C-ADMM-based<br>centralized DP-DNR approach can seek the optimal DNR solution without<br>stomer's load datasets of agents. The exchanged communication signals are decentralized DP-DNR approach can seek the optimal DNR solution without<br>customer's load datascts of agents. The exchanged communication signals are also<br>synthetic based on the proposed DP-DNR mechanism, which perfectly pro customer's load datasets of agents. The exchanged communication signals are also<br>synthetic based on the proposed DP-DNR mechanism, which perfectly protects the<br>realistic communication messages between agents and the DSO. T synthetic based on the proposed DP-DNR mechanism, which perfectly protects the<br>realistic communication messages between agents and the DSO. Thus, this proposed<br>decentralized reconfiguration approach is applicable for inter

HVDNs is summarized, and then the simplified voltage-constrained load transfer<br>strategy via topological structures can be developed. Moreover, this proposed<br>look-ahead economic dispatch model is cast as a mixed-integer sec HVDNs is summarized, and then the simplified voltage-constrained load transfer<br>strategy via topological structures can be developed. Moreover, this proposed<br>look-ahead economic dispatch model is cast as a mixed-integer sec HVDNs is summarized, and then the simplified voltage-constrained load transfer<br>strategy via topological structures can be developed. Moreover, this proposed<br>look-ahead economic dispatch model is cast as a mixed-integer sec HVDNs is summarized, and then the simplified voltage-constrained load transfer<br>strategy via topological structures can be developed. Moreover, this proposed<br>look-ahead economic dispatch model is cast as a mixed-integer sec HVDNs is summarized, and then the simplified voltage-constrained load transfer<br>strategy via topological structures can be developed. Moreover, this proposed<br>look-ahead economic dispatch model is cast as a mixed-integer sec HVDNs is summarized, and then the simplified voltage-constrained load transfer<br>strategy via topological structures can be developed. Moreover, this proposed<br>look-ahead economic dispatch model is cast as a mixed-integer sec HVDNs is summarized, and then the simplified voltage-constrained load transfer<br>strategy via topological structures can be developed. Moreover, this proposed<br>look-ahead economic dispatch model is cast as a mixed-integer sec HVDNs is summarized, and then the simplified voltage-constrained load transfer<br>strategy via topological structures can be developed. Moreover, this proposed<br>look-ahead economic dispatch model is cast as a mixed-integer sec Example 1.1 Highlights of contributions in this thesis<br>
The Mixed-integral Control of Control of the Multi-cut Benders Occupation<br>
1.1 Highlights of contributions in this thesis<br>
Table 1.1 Highlights of contributions in th Items<br>
International and the model of the state of the Multi-cut Bender, this proposed<br>
Items (MISOCP) problem. For this established MISOCP-based model, it is<br>
Items (MISOCP) problem. For this established MISOCP-based mode dispatch model is cast as a mixed-integer second-order cone<br>
P) problem. For this established MISOCP-based model, it is<br>
mbine the Multi-cut Benders Decomposition (MBD) [86] and<br>
Decomposition (GBD) [87] as the devised Mul beak. Moreover, this proposed<br>
ixed-integer second-order cone<br>
ed MISOCP-based model, it is<br>
ecomposition (MBD) [86] and<br>
s the devised Multi-cut GBD<br>
enhance overall computational<br>
lispatch.<br>
in this thesis<br> **Weakness of** 

	programming (MISOCP) problem. For this established MISOCP-based model, it is		
	highly desirable to combine the Multi-cut Benders Decomposition (MBD) [86] and		
	Generalized Benders Decomposition (GBD) [87] as the devised Multi-cut GBD		
	(MGBD) to tackle this MISOCP problem, which can enhance overall computational		
	efficiency and be suitable for online rolling economic dispatch.		
	Table 1.1 Highlights of contributions in this thesis		
<b>Items</b>	Key characteristics of research problems	<b>Weakness of existing</b> references	<b>Contributions</b>
Computational challenges of <b>DNR</b> (Chapter 3)	Mixed-integer $\bullet$ second-order conic optimization problem Computationally $\bullet$ challenging, especially for the large-scale networks	Big-M method $[14]$ McCormick linearization $\bullet$ method [15], [16] Loosened relaxation $\bullet$ techniques	<b>DCHR</b> with provably tightened relaxation
Cyber-physical security of DNR (Chapter 4)	System observability cost can be optimized w.t.r. reconfiguration	<b>Conventional DNR</b> model $\bullet$ System observability $\bullet$ cannot be guaranteed[37] <b>RCDS</b> formulation $\bullet$ $[30]$ , $[31]$	Observability defense-constrain ed DNR
Privacy-preservi ng DNR (Chapter 5)	Topology switch for the loss minimization may expose the private information of light and heavy loads	<b>Conventional DNR</b> model ● Private information for agents cannot be guaranteed $[39]$ , $[40]$	DP-DNR mechanism
Flexibility enhancement of economic dispatch (Chapter 6)	Look-ahead rolling $\bullet$ economic dispatch of <b>WTBPS</b> Dispatchable load $\bullet$ resources Mixed-integer $\bullet$	Conventional ramping $\bullet$ model of thermal power units $[80]$ Conventional dispatch model of thermal-wind-bundled	Decentralized MGBD-based for approach rolling economic dispatch of <b>WTBPS</b>
	17		



second-order conic<br>
optimization problem<br>
computationally<br>
computationally<br>
challenging, especially<br>
for the large-scale<br>
networks<br>
1.4 Thesis Layout<br>
The rest of this thesis consists of seven Chapters. Chapter 2 reviews t **•** Computationally<br>
of the large-scale<br>
for the large-scale<br>
networks<br>
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fundamentals of *DistFlow* equations and its convex rel for the large-scale<br>
networks<br>
The rest of this thesis consists of seven Chapters. Chapter 2 reviews the<br>
fundamentals of *DistFlow* equations and its convex relaxation formulations. Chapter 3<br>
firstly proposes the disjunc 1.4 Thesis Layout<br>The rest of this thesis consists of seven Chapters. Chapter 2 reviews the<br>fundamentals of *DistFlow* equations and its convex relaxation formulations. Chapter 3<br>firstly proposes the disjunctive convex hul 1.4 Thesis Layout<br>The rest of this thesis consists of seven Chapters. Chapter 2 reviews the<br>fundamentals of *DistFlow* equations and its convex relaxation formulations. Chapter 3<br>firstly proposes the disjunctive convex hul 1.4 Thesis Layout<br>The rest of this thesis consists of seven Chapters. Chapter 2 reviews the<br>fundamentals of *DistFlow* equations and its convex relaxation formulations. Chapter 3<br>firstly proposes the disjunctive convex hul The rest of this thesis consists of seven Chapters. Chapter 2 reviews the fundamentals of *DistFlow* equations and its convex relaxation formulations. Chapter 3 firstly proposes the disjunctive convex hull approach to deal fundamentals of *DistFlow* equations and its convex relaxation formulations. Chapter 3<br>firstly proposes the disjunctive convex hull approach to deal with the reconfiguration<br>of DNs. This approach is theoretically tighter t firstly proposes the disjunctive convex hull approach to deal with the reconfiguration<br>of DNs. This approach is theoretically tighter than the McCormick linearization<br>method and the Big-M method, and it is especially suita of DNs. This approach is theoretically tighter than the McCormick linearization<br>method and the Big-M method, and it is especially suitable for smart DNs with<br>directional power flows. Chapter 4 develops the observability de method and the Big-M method, and it is especially suitable for smart DNs with<br>directional power flows. Chapter 4 develops the observability defense-constrained<br>topology optimization of DNs, which perfectly enables an obser directional power flows. Chapter 4 develops the observability defense-constrained<br>topology optimization of DNs, which perfectly enables an observable DNR solution<br>just with the cyber-physical security enhancement. Chapter topology optimization of DNs, which perfectly enables an observable DNR solution<br>just with the cyber-physical security enhancement. Chapter 5 presents a differentially<br>private topology optimization of ADNs, which provides iust with the cyber–physical security enhancement. Chapter 5 presents a differentially private topology optimization of ADNs, which provides a pair of realistic optimal topology variables and obfuscated-but-feasible power



## **Chapter 2**<br>Fundamentals of *DistFlow* Equations and its Fundamentals of *DistFlow* Equations and its<br>Fundamentals of *DistFlow* Equations and its<br>Convex Relaxation Formulations Chapter 2<br>Fundamentals of *DistFlow* Equations and its<br>Convex Relaxation Formulations<br>In view of distribution networks (DNs) highly penetrated by enrichable distributed

Chapter 2<br> **Fundamentals of DistFlow Equations and its**<br> **Convex Relaxation Formulations**<br>
In view of distribution networks (DNs) highly penetrated by enrichable distributed<br>
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Convex Relaxation Formulations**<br>
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energy resources (DERs) and inverter-based resources (IBR **Fundamentals of DistFlow Equations and its**<br> **Convex Relaxation Formulations**<br>
In view of distribution networks (DNs) highly penetrated by enrichable distributed<br>
try resources (DERs) and inverter-based resources (IBRs), **CONVEX Relaxation Formulations**<br>In view of distribution networks (DNs) highly penetrated by enrichable distributed<br>energy resources (DERs) and inverter-based resources (IBRs), a fast power flow<br>calculation method is esse In view of distribution networks (DNs) highly penetrated by enrichable distributed<br>energy resources (DERs) and inverter-based resources (IBRs), a fast power flow<br>calculation method is essential not only for the load balan

In view of distribution networks (DNs) highly penetrated by enrichable distributed<br>energy resources (DERs) and inverter-based resources (IBRs), a fast power flow<br>calculation method is essential not only for the load balanc energy resources (DERs) and inverter-based resources (IBRs), a fast power flow<br>calculation method is essential not only for the load balancing and loss reduction at<br>the voltage security-constrained level, but also for real calculation method is essential not only for the load balancing and loss reduction at<br>the voltage security-constrained level, but also for real-time transactive dispatch tasks<br>between supply and demand at the market level the voltage security-constrained level, but also for real-time transactive dispatch tasks<br>between supply and demand at the market level of DNs.<br>This chapter lays the theoretical foundation of *DistFlow* equations and its c between supply and demand at the market level of DNs.<br>
This chapter lays the theoretical foundation of *DistFlow* equations and its convex<br>
relaxation formulations. Initially, *DistFlow* equations can be derived from the b relaxation formulations. Initially, *DistFlow* equations can be derived from the branch<br>flow model, and then a linearized *DistFlow* equations can be obtained if the<br>non-convex terms are negligible. Subsequently, the SOCP w model, and then a linearized *DistFlow* equations can be obtained if the<br>n-convex terms are negligible. Subsequently, the SOCP and SDP form of *DistFlow*<br>uations are formulated according to SOCP and SDP convex relaxatio mon-convex terms are negligible. Subsequently, the SOCP and SDP form of *DistFlow*<br>equations are formulated according to SOCP and SDP convex relaxation techniques.<br>Additionally, the polyhedral approximation formulation is networks are formulated according to SOCP and SDP convex relaxation techniques.<br>Additionally, the polyhedral approximation formulation is included for linearizing the<br>SOC constraints in SOCP-based reactive power optimizat

networks with a tree topology) is illustrated in Fig. 2.1. For this branch l,  $P_n^d$  and  $Q_n^d$ 

are fixed active and reactive power demands with  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{mn}^l$  refers to the<br>impedance with  $(z_{mn}^l)^2 = (r_{mn}^l)^2 + (x_{mn}^l)^2$ ;  $P_n^s$  and  $Q_n^g$  are active and reactive power<br>generation at node *n* with  $\$  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{mn}^l$  refers to the  $z_{mn}^l$  refers to the<br>nd reactive power are fixed active and reactive power demands with  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{mn}^l$  refers<br>impedance with  $(z_{mn}^l)^2 = (r_{mn}^l)^2 + (x_{mn}^l)^2$ ;  $P_n^g$  and  $Q_n^g$  are active and reactive<br>generation at node *n* with  $\tilde{S}_n^g = P_n^g$ impedance with  $(z_{mn}^l)^2 = (r_{mn}^l)^2 + (x_{mn}^l)^2$ ;  $P_n^g$  and  $Q_n^g$  are active and reactive power mds with  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{mn}^l$  refers to the<br>  $P_n^g$  and  $Q_n^g$  are active and reactive power<br>
, respectively;  $P_{mn}^l$  and  $Q_{mn}^l$  are active and are fixed active and reactive power demands with  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{mn}^l$  refers to impedance with  $(z_{mn}^l)^2 = (r_{mn}^l)^2 + (x_{mn}^l)^2$ ;  $P_n^g$  and  $Q_n^g$  are active and reactive pogeneration at node *n* with  $\tilde{S}_n^g$ ve power demands with  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{mn}^l$  refers to the<br>  $(z_{mn}^l)^2 + (x_{mn}^l)^2$ ;  $P_n^g$  and  $Q_n^g$  are active and reactive power<br>  $\tilde{S}_n^g = P_n^g + jQ_n^g$ , respectively;  $P_{mn}^l$  and  $Q_{mn}^l$  are active and<br>  $\tilde$ are fixed active and reactive power demands with  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{nn}^l$  refers to the impedance with  $(z_{mn}^l)^2 = (r_{mn}^l)^2 + (x_{mn}^l)^2$ ;  $P_n^g$  and  $Q_n^g$  are active and reactive power generation at node *n* with  $\$ re power demands with  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{nm}^l$  refers to the<br>  $\frac{d}{dm}y^2 + (x_{mn}^l)^2$ ;  $P_n^g$  and  $Q_n^g$  are active and reactive power<br>  $\tilde{S}_n^g = P_n^g + jQ_n^g$ , respectively;  $P_m^l$  and  $Q_{mn}^l$  are active and<br>  $\tilde$  $\frac{Q_n^d + jQ_n^d : z_{mn}^l$  refers to the<br>
e active and reactive power<br>
n and  $Q_{mn}^l$  are active and<br>  $P_{nk}^l + jQ_{nk}^l$  $P_n^g$  and  $Q_n^g$  are active and reactive power<br>  $Q_n^g$ , respectively;  $P_{mn}^f$  and  $Q_{mn}^f$  are active and<br>  $jQ_{mn}^f$ , respectively.<br>  $P_n^g + jQ_n^g$ <br>  $P_n^g + jQ_n^g$ <br>  $P_n^g + jQ_n^g$ <br>  $P_n^f + jQ_n^f$ <br>  $P_n^f + jQ_n^f$ d reactive power demands with  $\tilde{S}_n^d = P_n^d + jQ_n^d$ ;  $z_{nm}^d$  refers to the<br>  $m_m^d = (r_{nm}^d)^2 + (x_{nm}^d)^2$ ;  $P_n^s$  and  $Q_n^s$  are active and reactive power<br>  $n$  with  $\tilde{S}_n^s = P_n^s + jQ_n^s$ , respectively;  $P_m^l$  and  $Q_{nm}^l$ 



 $\begin{array}{ccc}\nP_n^u + jQ_n^u & \cdots & P_m^u + jQ_m^d \\
\hline\nS_m^u = P^u + jQ^u & \cdots & P_m^d + jQ_m^d\n\end{array}$ Fig. 2.1 Typical connection of a branch in DNs.<br>
In practice, we have one assumption that the shunt elements in DNs are assumed<br>
zero, namely  $b_{nm$  $\begin{array}{ccc}\nP_1^k & +jQ_2^k & \cdots & P_m^k & +jQ_m^j \\
\hline\nS_m^i = P^i + jQ^j & & P_m^k & +jQ_m^j\n\end{array}$ <br>
Fig. 2.1 Typical connection of a branch in DNs.<br>
In practice, we have one assumption that the shunt elements in DNs are assumed<br>
zero, namely  $\frac{U_m}{S_m^i = P^i + jQ^i}$ <br>  $\frac{V_m}{V_m^i + jQ^i}$ <br>
Fig. 2.1 Typical connection of a branch in DNs.<br>
In practice, we have one assumption that the shunt elements in DNs are assumed<br>
zero, namely  $b_{mn}/2$ –0. This assumption is reaso  $\tilde{S}_{mn}^l = \dot{U}_m (I_{mn}^l)^*$ ,  $P_{nk}^{j} + jQ_{nk}^{j}$ <br>  $P_{nk}^{j} + jQ_{nk}^{j}$ <br>
the shunt elements in DNs are assumed<br>
easonable for realistic DNs due to short<br>
i, we can express the apparent power flow<br>  $T_{m}$  refers to the voltage phasor at node *m*<br>
urrent p and  $(I_{mn}^l)^*$  refers to the conjuga  $U_n$ <br>  $\overrightarrow{S}_{\text{new}}^i = P^i + jQ^i$ <br>
Fig. 2.1 Typical connection of a branch in DNs.<br>
In practice, we have one assumption that the shunt elements in DNs are assumed<br>
zero, namely  $b_{\text{new}}/2=0$ . This assumption is reasonable fo **Fig. 2.1 Typical connection of a branch in DNs.**<br>
In practice, we have one assumption that the shunt elements in DNs are assumed<br>
zero, namely  $b_{nm}/2=0$ . This assumption is reasonable for realistic DNs due to short<br>
dis In practice, we have one assumption that the shunt elements in DNs are assumed<br>zero, namely  $b_{mw}/2=0$ . This assumption is reasonable for realistic DNs due to short<br>distances of branches. Under this assumption, we can expr as  $\tilde{S}_{mn}^l - j \frac{b_{mn} U_m^2}{2} = U_m (I_{mn}^l)^*$ . 2  $l_{mn}^l - j\frac{b_{mn}C_m}{2} = U_m(I_{mn}^l)$ In practice, we have one assumption that the shunt elements in DNs are assumed<br>
co, namely  $b_{nm}/2-0$ . This assumption is reasonable for realistic DNs due to short<br>
tances of branches. Under this assumption, we can express distances of branches. Under this assumption, we can express the apparent power flow<br>for this branch *l* as  $\tilde{S}_{mn}^i = U_m(I_{mn}^i)^*$ , where  $U_m$  refers to the voltage phasor at node *m*<br>and  $(I_{mn}^i)^*$  refers to the conjuga this assumption, we can express the apparent power flow<br>  $(L_m^f)$ , where  $U_m$  refers to the voltage phasor at node m<br>
conjugate of current phasor between nodes m and n.<br>
odeled by a series admittance  $y_{mn}$  with shunt elem

formulating distribution power flow equations in phasor form, we can achieve the<br>BFM-based power flow equality, yielding formulating distribution power flow equations in phasor form, we can achieve the<br>BFM-based power flow equality, yielding<br> $\begin{cases} \dot{U}_n = \dot{U}_m - z_{mn}^l \cdot I_{mn}^l \\ \tilde{S}_{mn}^l = \dot{U}_m (I_{mn}^l)^* \end{cases}$ ,  $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.1

$$
\begin{cases}\n\dot{U}_n = \dot{U}_m - z_{mn}^l \cdot I_{mn}^l \\
\tilde{S}_{mn}^l = \dot{U}_m (I_{mn}^l)^* \\
\tilde{S}_{mn}^l + \tilde{S}_n^g - z_{mn}^l |I_{mn}^l|^2 = \sum_{k \in \pi(n)} \tilde{S}_{nk}^l + \tilde{S}_n^d\n\end{cases}
$$
\n
$$
(2.1)
$$

formulating distribution power flow equations in phasor form, we can achieve the<br>
BFM-based power flow equality, yielding<br>  $\begin{cases} \dot{U}_n = \dot{U}_m - z_{mn}^t \cdot I_{mn}^t \\ \dot{S}_{nm}^t = \dot{U}_m (I_{mn}^t)^* \\ \dot{S}_{nm}^t + \tilde{S}_n^s - z_{mn}^t |I_{mn}^t|^2 = \$ formulating distribution power flow equations in phasor form, we can achiev<br>
BFM-based power flow equality, yielding<br>  $\left\{\n\begin{aligned}\n\dot{U}_n &= \dot{U}_m - z_{mn}^i \cdot I_{mn}^i \\
\dot{S}_{mn}^i &= \dot{U}_m (I_{mn}^i)^* \\
\dot{S}_{mn}^i &= \dot{U}_m (I_{mn}^i)^* \\
\dot{S}_{mn}^i +$  $\dot{\cal{U}}_m({\cal I}^l_{mn})^*$  . tion power flow equations in phasor form, we can achieve the<br>
ow equality, yielding<br>  $\int_{m} = U_m - z'_{nm} \cdot I'_{nm}$ <br>  $\int_{m} = U_m (I'_{m})^*$ ,  $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.1)<br>  $\int_{m} + \tilde{S}_n^s - z'_{mn} |I'_{m}|^2 = \sum_{k \in \mathcal{K}(n)} \tilde{S}'_{nk} + \til$  $\overline{(n)}$  $\int_{mn}^{kl} + S_n^g - z_{mn}^l |I_{mn}^l|^2 = \sum \tilde{S}_{nk}^l + S_n^d$  $k \in \pi(n)$  $\tilde{S}_{mn}^l + S_n^g - z_{mn}^l |I_{mn}^l|^2 = \sum \tilde{S}_{nk}^l + S_n^d$  $\in \pi$ equations in phasor form, we can achieve the<br>
ding<br>  $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.1)<br>  $=\sum_{k \in \pi(n)} \tilde{S}_{nk}^i + \tilde{S}_n^d$ <br>
thes that connect the node *n*.<br>
1 power flow equations are non-convex due to<br>  $\tilde{S}_{mn}^i + S_n^g - z_{mn}^$ BFM-based power flow equality, yielding<br>  $\left\{\begin{aligned}\n\dot{U}_n &= \dot{U}_m - z'_{mn} \cdot I'_m \\
\dot{S}'_{mn} &= \dot{U}_m (I'_m)^* \\
\dot{S}'_{mn} &= \dot{Z}'_{mn} |I'_m\|^2 = \sum_{k \in \pi(n)} \tilde{S}'_{nk} + \tilde{S}^d_n\n\end{aligned}\right.$ (2.1<br>
where  $\pi(n)$  refers to the set of branches that conn equation caused by  $|I_{mn}^l|^2$ . In other words, BFM-based power flow equality is not w equality, yielding<br>  $= U_m - z'_{mn} \cdot I'_{mn}$ <br>  $= U_m (I'_{mn})^*$ ,  $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.1)<br>  $+ \tilde{S}_n^g - z'_{mn} |I'_{mn}|^2 = \sum_{k \in \pi(n)} \tilde{S}_{nk}^l + \tilde{S}_n^d$ <br>
the set of branches that connect the node *n*.<br>
1), distribution power  $\begin{cases} \n\dot{U}_n = \dot{U}_m - \frac{J^2}{2m} \cdot \dot{I}^T_{nn} \\ \n\tilde{S}^T_{nn} = \dot{U}_m (I^T_{nn})^* \\ \n\tilde{S}^T_{nn} + \tilde{S}^g_n - \frac{J^2}{2m} |I^T_{nn}|^2 = \sum_{k \neq i} \tilde{S}^i_{nk} + \tilde{S}^d_n \n\end{cases}$ where  $\pi(n)$  refers to the set of branches that connect the nod  $\begin{aligned}\nU_n &= U_m - z_{mn}^*, I_{mn}^* \\
\hat{S}_{mn}^i &= U_m (I_{mn}^i)^* \\
\hat{S}_{mn}^i &= \hat{S}_m^* - z_{mn}^i |I_{mn}^i|^2 = \sum_{k \neq i(n)} \tilde{S}_{nk}^i + \tilde{S}_n^d\n\end{aligned}$ where  $\pi(n)$  refers to the set of branches that connect the node *n*.<br>
As observed in (2.1), distrib As observed in (2.1), distribution power flow equations are non-convex due to<br>
n-convex terms  $U_m(I'_{mn})^*$ . And  $\tilde{S}'_{mn} + S''_n = z'_{mn}|I'_{mn}|^2 = \sum_{k \in \mathcal{K}(s)} \tilde{S}'_{nk} + S''_n$  is a nonlinear<br>
uation caused by  $|I'_{mn}|^2$ . In oth The boost of the  $U_m(I_{mn}^i)^*$ . And  $\tilde{S}_{mn}^i + S_n^g - z_{mn}^i |I_{mn}^i|^2 = \sum_{k \in \mathcal{I}(n)} \tilde{S}_{nk}^i + S_n^g$  is a nonlinear equation caused by  $|I_{mn}^i|^2$ . In other words, BFM-based power flow equality is not suitable for tractable . And  $\tilde{S}_{mn}^i + S_n^g - z_{mn}^i |I_{mn}^i|^2 = \sum_{k \in \pi(n)} \tilde{S}_{nk}^i + S_n^d$  is a nonlinear<br>in other words, BFM-based power flow equality is not<br>ation, and thus we need to reformulate this BFM-based<br>equation, and thus we need to ref non-convex terms  $U_m(I_{mn}^r)$ . And  $S_{mn}^r + S_n^s - z_{md}^r I_{nm}^r] = \sum_{k=r(v)} S_{nk} + S_n^s$  is a nonlinear<br>equation caused by  $|I_{mn}^t|^2$ . In other words, BFM-based power flow equality is not<br>suitable for tractable computation, and th

## 2.2 DistFlow Equations

suitable for tractable computation, and thus we need to reformulate this BFM-based<br>power flow equality.<br>2.2 *DistFlow* Equations<br>Due to this voltage drop equation in phasor form, we solve this equation with<br>squares on bot

$$
|\dot{U}_{m} - z_{mn}^{l} \cdot I_{mn}^{l}|^{2} = (\dot{U}_{m} - z_{mn}^{l} \cdot I_{mn}^{l})(\dot{U}_{m} - z_{mn}^{l} \cdot I_{mn}^{l})^{*}
$$
  
\n
$$
= |U_{m}|^{2} + |z_{mn}^{l} \cdot I_{mn}^{l}|^{2} - 2 \operatorname{Re}(\dot{U}_{m}(z_{mn}^{l} \cdot I_{mn}^{l})^{*})
$$
  
\n
$$
= |U_{m}|^{2} + |z_{mn}^{l}|^{2} \cdot |I_{mn}^{l}|^{2} - 2 \operatorname{Re}(\dot{U}_{m} \cdot z_{mn}^{l*} \cdot I_{mn}^{l*})
$$
  
\n
$$
= |U_{m}|^{2} + |z_{mn}^{l}|^{2} \cdot |I_{mn}^{l}|^{2} - 2 \operatorname{Re}(z_{mn}^{l*} \cdot \tilde{S}_{mn}^{l})
$$
  
\n
$$
= |U_{m}|^{2} + |z_{mn}^{l}|^{2} \cdot |I_{mn}^{l}|^{2} - 2r_{mn}^{l} \cdot P_{mn}^{l} - 2x_{mn}^{l} \cdot Q_{mn}^{l}
$$
  
\n(2.2)

where  $r_{mn}^l$  and  $x_{mn}^l$  indicate the resistance and  $x'_{mn}$  indicate the resistance and reactance of branch *l.*  $|U_m|$  and voltage magnitude of voltage phasor  $U_m$  and current modulus of  $U'_m$ , respectively. Note that for any vector *x*, Re(*x*) refers to the real  $|I_{mn}^l|$  denote the voltage magnitude of voltage phasor  $\dot{U}_m$  and current modulus of where  $r_{mn}^l$  and  $x_{mn}^l$  indicate the resistance and reactance of branch *l.*  $|U_m|$  and  $I_{mn}^l|$  denote the voltage magnitude of voltage phasor  $U_m$  and current modulus of urrent phasor  $I_{mn}^l$ , respectively. Note t  $\begin{aligned}\n\text{Let } \text{a} \text{ be a constant, } \text{a} \text{ is a constant, } \text{a} \text{ and } \text{a} \text{ is a constant, } \text{a} \text{ and } \text{a} \text{ and } \text{a} \text{ is a constant, } \text{a} \text{ and } \text{a} \text{ is a constant, } \text{a} \text{ is a constant.}\n\end{aligned}$ where  $r_{mn}^l$  and  $x_{mn}^l$  indicate the resistance and reactance of branch *l*. |
| $I_{mn}^l$ | denote the voltage magnitude of voltage phasor  $U_m$  and current n<br>current phasor  $I_{mn}^l$ , respectively. Note that for any vect  $x'_{mn}$  indicate the resistance and reactance of branch *l.*  $|U_m|$  and<br>voltage magnitude of voltage phasor  $U_m$  and current modulus of<br> $I'_{mn}$ , respectively. Note that for any vector x, Re(x) refers to the real<br>noted tha where  $r'_{ms}$  and  $x'_{ms}$  indicate the resistance and reactance of branch *l.*  $|U_m|$  and  $|I'_{ms}|$  denote the voltage magnitude of voltage phasor  $U_m$  and current modulus of current phasor  $I'_{ms}$ , respectively. Note that example and reactance of branch *l.*  $|U_m|$  and<br>magnitude of voltage phasor  $U_m$  and current modulus of<br>pectively. Note that for any vector *x*, Re(*x*) refers to the real<br>nat (2.1) is actually a phase angle free equation

variables  $|U_m|$ ,  $|I_{mn}^l|$ ,  $P_{mn}^l$  and  $Q_{mn}^l$  are where  $r'_{mn}$  and  $x'_{mn}$  indicate the resistance and reactance of branch *l.*  $|U_m|$  and<br>  $|I'_{mn}|$  denote the voltage magnitude of voltage phasor  $U_n$  and current modulus of<br>
current phasor  $I'_{mn}$ , respectively. Note th  $(|U_m|, |I_{mn}^l|, P_{mn}^l, Q_{mn}^l)$ -space as a projection of complex-valued  $(\dot{U}_m, I_{mn}^l, \tilde{S}_{mn}^l)$ -space. *l<sub>mn</sub>* indicate the resistance and reactance of branch *l.*  $|U_m|$  and coltage magnitude of voltage phasor  $U_m$  and current modulus of  $U_m$ , respectively. Note that for any vector *x*, Re(*x*) refers to the real oted tha where  $r_{ms}^l$  and  $x_{ms}^l$  indicate the resistance and reactance of branch *l.*  $|U_m|$  and  $|I_{nm}^l|$  denote the voltage magnitude of voltage phasor  $U_m$  and current modulus of current phasor  $I_{ms}^l$ , respectively. Note resistance and reactance of branch *l.*  $|U_m|$  and<br>de of voltage phasor  $U_m$  and current modulus of<br>Note that for any vector *x*, Re(*x*) refers to the real<br>is actually a phase angle free equation, since all<br> $Q'_{mn}$  are r  $|I'_{nn}|$  denote the voltage magnitude of voltage phasor  $U_m$  and eurrent modulus of<br>current phasor  $I'_m$ , respectively. Note that for any vector x, Re(x) refers to the real<br>part of x.<br>It should be noted that (2.1) is actu current phasor  $I'_{mn}$ , respectively. Note that for any vector x, Re(x) refers to the real<br>part of x.<br>It should be noted that (2.1) is actually a phase angle free equation, since all<br>variables  $|U_m|$ ,  $|I'_{mn}|$ ,  $P'_{mn}$  an is, Re(x) refers to the real<br>free equation, since all<br>the mathematics, we call<br>consider the real-valued<br>ed  $(\dot{U}_m, I_{mn}^I, \tilde{S}_{mn}^I)$ -space.<br>a point in the complex<br>bind from the origin. The<br> $\dot{U}_m = |U_m|e^{i\theta}$  as shown i part of x.<br>
It should be noted that (2.1) is actually a phase angle free equation, since all<br>
variables  $|U_m|$ ,  $|I'_{m\pi}|$ ,  $P'_m$  and  $Q'_m$  are real-valued numbers. In mathematics, we call<br>
this equation as the angle rela hat (2.1) is actually a phase angle free equation, since all  $P'_{nm}$  and  $Q'_{nm}$  are real-valued numbers. In mathematics, we call angle relaxation. Indeed, we can consider the real-valued pace as a projection of complex-v It should be noted that (2.1) is actually a phase angle free equation, since all<br>variables  $|U_m|$ ,  $|I'_{mn}|$ ,  $P'_m$  and  $Q'_m$  are real-valued numbers. In mathematics, we call<br>this equation as the angle relaxation. Indeed, bles  $|U_m|$ ,  $|I'_{mn}|$ ,  $P'_{mn}$  and  $Q'_{mn}$  are real-valued numbers. In mathematics, we call<br>equation as the angle relaxation. Indeed, we can consider the real-valued<br>equation as the angle relaxation. Indeed, we can consid distance of the point from the origin. The<br>
n can be taken  $U_m = |U_m|e^{i\theta}$  as shown in<br>
coordinate space is a point. Then, a circle<br>
tions for any real-valued variable  $|U_m|$ .<br>
b a circle.<br>  $\vec{U}_j = |U_j|e^{i\theta}$ <br>  $\overline{U}_j = |$ 



Similarly, we take the modulus with squares on both side of  $\tilde{S}_{mn}^i = U_m (I_{mn}^i)^*$ ,<br>elding<br> $|\vec{U}(I^i)|^2 = |\tilde{S}^i|^2 \Leftrightarrow |U|^{2} \cdot |I^i|^2 = (P^i)^2 + (O^i)^2$  (2.3)  $\tilde{S}_{mn}^l = \dot{U}_m (I_{mn}^l)^*$ , yielding

$$
|\dot{U}_m(I_{mn}^l)^*|^2 = |\tilde{S}_{mn}^l|^2 \iff |U_m|^2 \cdot |I_{mn}^l|^2 = (P_{mn}^l)^2 + (Q_{mn}^l)^2 \tag{2.3}
$$

Similarly, we take the modulus with squares on both side of  $\tilde{S}_{mn}^i = U_m (I_{mn}^i)^*$ ,<br>
Edding<br>  $|\tilde{U}_m (I_m^i)^*|^2 = |\tilde{S}_{mn}^i|^2 \iff |U_m|^2 \cdot |I_{mn}^i|^2 = (P_m^i)^2 + (Q_m^i)^2$  (2.3)<br>
As mentioned above, if a shunt admittance  $b_{mn}/2$  is  $|\tilde{S}_{mn}^l - j\frac{b_{mn}U_m^2}{2}|^2 = |\dot{U}_m(I_{mn}^l)^*|^2$ 2  $\int_{mm}^{l} -j\frac{D_{mn}U_m}{2}|^2 = |\dot{U}_m(I_{mn}^l)|^2$ Similarly, we take the modulus with squares on both side of  $S_{mn}^i = U_m (I_{mn}^i)^*$ .<br>
viciding<br>  $|\dot{U}_m (I_{mn}^i)^*|^2 = |\dot{S}_{mn}^i|^2 \iff |U_m|^2 \cdot |I_{mn}^i|^2 = (P_{mn}^i)^2 + (Q_{mn}^i)^2$  (2.3)<br>
As mentioned above, if a shunt admittance  $b_{mn}/2$  i Similarly, we take the modulus with squares on both side of  $\tilde{S}_{\text{ave}}^i = U_{\text{av}}(t_{\text{inv}}^i)^2$ ,<br>
yielding<br>  $|\tilde{U}_{\text{av}}(t_{\text{inv}}^i)^2|^2 = |\tilde{S}_{\text{row}}^i|^2 \iff |U_{\text{av}}|^2 \cdot |I_{\text{inv}}^i|^2 = (P_{\text{inv}}^i)^2 + (Q_{\text{inv}}^i)^2$  (2.3)<br>
As mention Similarly, we take the modulus with squares on both side of  $\tilde{S}_{\text{max}}^i = U_{\text{max}}(I_{\text{max}}^i)^*$ ,<br>yielding<br> $|U_{\text{max}}(I_{\text{max}}^i)^*|^2 = |\tilde{S}_{\text{max}}^i|^2 \iff |U_{\text{max}}^i|^2 + (I_{\text{max}}^i)^2 + (Q_{\text{max}}^i)^2$  (2.3)<br>As mentioned above, if a shu Similarly, we take the modulus with squares on both side of  $\tilde{S}_{\text{ave}}^i = U_{\text{in}}(I_{\text{in}}^i)^*$ ,<br>
yielding<br>  $|U_{\text{in}}(I_{\text{in}}^i)^*|^2 = |\tilde{S}_{\text{out}}^i|^2 \iff |U_{\text{in}}|^2 \cdot |I_{\text{in}}^i|^2 = (P_{\text{in}}^i)^2 + (Q_{\text{out}}^i)^2$  (2.3)<br>
As mentioned yielding<br>  $|U_m (I'_{mn})^2|^2 - |\tilde{S}^2_{mn}|^2 \iff |U_m|^2 \cdot |I'_{m\ell}|^2 = (P'_m)^2 + (Q'_m)^2$  (2.3)<br>
As mentioned above, if a shunt admittance  $b_{mn}/2$  is included, then<br>  $|\tilde{S}^d_{mn} - J \frac{b_{mn} U_m^2}{2}|^2 = |U_m (I'_{mn})^2|^2$  is a complicated quadratic  $|U_{m}(t'_{mn})^{\dagger}|^{2} = |\tilde{S}_{mn}^{l}|^{2} \iff |U_{m}|^{2} + |I_{mn}^{l}|^{2} = (P_{mn}^{l})^{2} + (Q_{mn}^{l})^{2}$  (2.3)<br>
As mentioned above, if a shunt admittance  $b_{mn}/2$  is included, then<br>  $|\tilde{S}_{mn}^{l} - f \frac{b_{mn}U_{m}^{l}}{2}|^{2} = |U_{m}(I_{mn}^{l})^{2}|^{2}$  is a com  $(|U_m|, |I'_{mn}|, P^l_{mn}, Q^l_{mn})$  based on n  $\left[\hat{U}_m(I'_{mn})^*\right]^2$  is a complicated quadratic equation that cannot be<br>  $\left[\hat{U}_m(I'_{mn})^*\right]^2$  is a complicated quadratic equation that cannot be<br>
SOC form. In summary, it can be seen that (2.1) and (2.2) are two<br>
tions af  $|\hat{S}_{nm}^{U} - j\frac{b_m U_m^2}{2}|^2 = |U_m(I_m^U)^2|^2$  is a complicated quadratic equation that cannot be<br>converted to this SOC form. In summary, it can be seen that (2.1) and (2.2) are two<br>angle free equations after applying angle rel converted to this SOC form. In summary, it can be seen that (2.1) and (2.2) are two<br>angle free equations after applying angle relaxation. According to the reference<br>direction of power flow shown in Fig. 2.1, the left-hand angle free equations after applying angle relaxation. According to the reference<br>direction of power flow shown in Fig. 2.1, the left-hand (right-hand) side gathers total<br>active and reactive power injected (withdrawn) in ( direction of power flow shown in Fig. 2.1, the left-hand (right-hand) side gathers total<br>active and reactive power injected (withdrawn) in (from) node *n*. We can express the<br>corresponding *DistFlow* equations with respec

$$
P_{mn}^l + P_n^g - r_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} P_{nk}^l + P_n^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.4)

$$
Q_{mn}^l + Q_n^g - x_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} Q_{nk}^l + Q_n^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.5)

$$
v_n = v_m - 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) + |z_{mn}^l|^2 \ell_{mn}^l \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.6)

$$
\ell_{mn}^l \cdot \nu_m = (P_{mn}^l)^2 + (Q_{mn}^l)^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.7)

where  $\ell_{mn}^l = |I_{mn}^l|^2$ ,  $v_m = |U_m|^2$  and  $v_n = |U_n|^2$ .

In this set of *DistFlow* equations,  $(2.4)-(2.5)$  are nodal active power and reactive<br>wer balancing conditions.  $(2.6)$  describes forward voltage drop on each branch<br>tained by  $(2.2)$  and  $(2.7)$  is derived from  $(2.3)$  wh In this set of **DistFlow** equations,  $(2.4)-(2.5)$  are nodal active power and reactive<br>power balancing conditions.  $(2.6)$  describes forward voltage drop on each branch<br>obtained by  $(2.2)$  and  $(2.7)$  is derived from  $(2.3)$ In this set of **DistFlow** equations, (2.4)-(2.5) are nodal active power and reactive<br>power balancing conditions. (2.6) describes forward voltage drop on each branch<br>obtained by (2.2) and (2.7) is derived from (2.3) which In this set of **DistFlow** equations, (2.4)-(2.5) are nodal active power and reactive<br>power balancing conditions. (2.6) describes forward voltage drop on each branch<br>obtained by (2.2) and (2.7) is derived from (2.3) which  $v_0 = |U_0|^2$  at the PCC is a constant. In this regard, we can define a vector of variables In this set of **DistFlow** equations, (2.4)-(2.5) are nodal active power and reactive<br>power balancing conditions. (2.6) describes forward voltage drop on each branch<br>obtained by (2.2) and (2.7) is derived from (2.3) which as  $x:=(\nu, \ell^l, P^l, Q^l)$  for  $\forall l \in \mathcal{E}$ , **istFlow** equations, (2.4)-(2.5) are nodal active power and reactive<br>conditions. (2.6) describes forward voltage drop on each branch<br>and (2.7) is derived from (2.3) which defines apparent power flow<br>ead node m of each bra In this set of **DistFlow** equations,  $(2.4)-(2.5)$  are nodal active power and reactive<br>power balancing conditions.  $(2.6)$  describes forward voltage drop on each branch<br>obtained by  $(2.2)$  and  $(2.7)$  is derived from  $(2.3)$ In this set of *DistFlow* equations,  $(2.4)-(2.5)$  are nodal active power and reactive<br>power balancing conditions.  $(2.6)$  describes forward voltage drop on each branch<br>obtained by  $(2.2)$  and  $(2.7)$  is derived from  $(2.3)$ In this set of *DistFlow* equations,  $(2.4)-(2.5)$  are nodal active power and reactive<br>power balancing conditions.  $(2.6)$  describes forward voltage drop on each branch<br>obtained by  $(2.2)$  and  $(2.7)$  is derived from  $(2.3)$ power balancing conditions. (2.6) describes forward voltage drop on each branch<br>obtained by (2.2) and (2.7) is derived from (2.3) which defines apparent power flow<br>injection at the head node m of each branch. The squared tained by (2.2) and (2.7) is derived from (2.3) which defines apparent power flow<br>ection at the head node *m* of each branch. The squared voltage magnitude<br> $= |U_0|^2$  at the PCC is a constant. In this regard, we can define Solution at the head node *m* of each branch. The squared voltage magnitude  $V_0|^2$  at the PCC is a constant. In this regard, we can define a vector of variables  $(\mathbf{v} \cdot \ell^i, \ell^i, \mathbf{P}^i, \mathbf{Q}^i)$  for  $\forall l \in \mathcal{E}$ ,  $\vert \exists U_0 \vert^2$  at the PCC is a constant. In this regard, we can define a vector of variables<br>  $x:=(v_-, t', P', Q')$  for  $\forall l \in \mathcal{E}$ , the key feature of which is that x does not involve<br>
gles of voltage and current phasors. Mos as  $x:=(v, e', P', Q')$  for  $\forall l \in \mathcal{E}$ , the key feature of which is that x does not involve<br>angles of voltage and current phasors. Most importantly,  $(2.4)-(2.7)$  in BFM are linear,<br>and non-convexity only appears in branch flow

angles of voltage and current phasors. Most importantly,  $(2.4)-(2.7)$  in BFM are linear,<br>and non-convexity only appears in branch flow equality  $(2.7)$ . This BFM-based power<br>flow equation is much different from bus injecti and non-convexity only appears in branch flow equality (2.7). This BFM-based power<br>flow equation is much different from bus injection model in which power balancing<br>conditions render non-convex quadratic equatities.<br>**It s** flow equation is much different from bus injection model in which power balancing<br>conditions render non-convex quadratic equalities.<br> **It should be also noted that:**<br>
• **Uniqueness of solutions for** *DistFlow* **equations<br>
T** solutions, but for radial distribution networks with  $|U| \approx 1$  p.u. and small  $r_{mn}^l$  and  $x'_{mn}$ , the solution of (2.4)-(2.7) is unique and same to the one produced by the conditions render non-convex quadratic equalities.<br> **U** should be also noted that:<br> **U** Uniqueness of solutions for *DistFlow* equations<br>
There are  $2|\mathcal{E}|+2|\mathcal{E}|=4|\mathcal{N}|-4$  equations in  $|\mathcal{N}|-1+3|\mathcal{E}|=4|\mathcal{N}|-4$  rea It should be also noted that:<br>
• Uniqueness of solutions for *DistFlow* equations<br>
There are  $2|\mathcal{E}|+2|\mathcal{E}|=4|\mathcal{N}|-4$  equations in  $|\mathcal{N}|-1+3|\mathcal{E}|=4|\mathcal{N}|-4$  real<br>
variables. It is clear that the number of variables • Uniqueness of solutions for *DistFlow* equations<br>
There are  $2|\mathcal{E}|+2|\mathcal{E}|=4|\mathcal{N}|-4$  equations in  $|\mathcal{N}|-1+3|\mathcal{E}|=4|\mathcal{N}|-4$  real<br>
variables. It is clear that the number of variables is equal to the number of equat There are  $2|\mathcal{E}|+2|\mathcal{E}|=4|\mathcal{N}|-4$  equations in  $|\mathcal{N}|-1+3|\mathcal{E}|=4|\mathcal{N}|-4$  real<br>variables. It is clear that the number of variables is equal to the number of equations,<br>which means that this *DistFlow* equations woul variables. It is clear that the number of variables is equal to the number of equations,<br>which means that this **DistFlow** equations would have a unique solution or multiple<br>solution. It is shown in [88], [89] that this se

These solutions may be spurious, i.e., they do not correspond to a solution of the original branch flow equations.<br>
• Application to general networks (loop networks) These solutions may be spurious, i.e., they do not correspond to a solution of the original branch flow equations.<br>
• Application to general networks (loop networks)<br>
The *DistFlow* equations can be extended to be used for

Solutions may be spurious, i.e., they do not correspond to a solution of the<br>
al branch flow equations.<br>
• **Application to general networks (loop networks)**<br> **• DistFlow equations can be extended to be used for general net** The Sections may be spurious, i.e., they do not correspond to a solution of the extended to a solution of the **Application to general networks (loop networks)**<br>The *DistFlow* **equations can be extended to be used for gener** these solutions may be spurious, i.e., they do not correspond to a solution of the<br>original branch flow equations.<br>
• Application to general networks (loop networks)<br>
The *DistFlow* equations can be extended to be used fo these solutions may be spurious, i.e., they do not correspond to a s<br>original branch flow equations.<br>
• Application to general networks (loop networks)<br>
The *DistFlow* equations can be extended to be used for general net<br> **Example 10** is space solutions may be spurious, i.e., they do not correspond to a solution of the giginal branch flow equations.<br> **• Application to general networks (loop networks)**<br>
The *DistFlow* equations can be ext these solutions may be spurious, i.e., they do not correspond to a solution of the<br>original branch flow equations.<br>
• **Application to general networks (loop networks)**<br>
The *DistFlow* equations can be extended to be used **• Application to general networks (loop networks)**<br>
The *DistFlow* equations can be extended to be used for general networks that may<br>
contain cycles by introducing a cycle condition. We define the angle difference acros

$$
\beta_{mn}(x) := \angle(\dot{U}_m - z_{mn}^l \cdot I_{mn}^l)
$$
\n(2.8)

$$
(2.4) - (2.7), \exists \theta \in \mathbb{R}, \text{ s.t. } \beta(x) = A \cdot \theta \tag{2.9}
$$

The *DistFlow* equations can be extended to be used for general networks that may<br>contain cycles by introducing a cycle condition. We define the angle difference across<br>for branch / as<br> $\beta_{nn}(x) := \angle(\hat{U}_m - z_{nn}^i \cdot I_{nn}^i)$  ( contain cycles by introducing a cycle condition. We define the angle difference across<br>for branch l as<br> $\beta_{nm}(x) := \angle(\vec{U}_m - z'_{mn} \cdot \vec{I}_{mn})$  (2.8)<br>For convenience, we let  $\beta(x) := (\beta_{mn}(x), l = (m, n), \forall l \in \mathcal{E})$ . Thus, the **DistFlow** for branch *l* as<br>  $\beta_{nn}(x) := \angle (\tilde{U}_m - z_m^{\ell} \cdot I_{nn}^{\ell})$  (2.8)<br>
For convenience, we let  $\beta(x) = (\beta_{nn}(x), l = (m, n), \forall l \in \mathcal{E})$ . Thus, the *DistFlow*<br>
equations are extended to general networks as:<br>  $(2.4) - (2.7), \exists \theta \in \mathbb{R}, s.t. \beta(x$  $\beta_{n\omega}(x) := \angle (\vec{U}_m - z'_{n\omega} \cdot I'_{n\omega})$  (2.8)<br>
For convenience, we let  $\beta(x) = (\beta_{n\omega}(x), l = (m, n), \forall l \in \mathcal{E})$ . Thus, the **DistFlow**<br>
equations are extended to general networks as:<br>  $(2.4) - (2.7), \exists \theta \in \mathbb{R}, s.t. \beta(x) = A \cdot \theta$  (2.9)<br> For convenience, we let  $\beta(x) := (\beta_{mn}(x), l) = (m, n), \forall l \in \mathcal{E})$ . Thus, the *DistFlow*<br>equations are extended to general networks as:<br> $(2.4) - (2.7), \exists \theta \in \mathbb{R}, s.t. \beta(x) = A \cdot \theta$  (2.9)<br>where A is a  $|\mathcal{E}|$  by  $|\mathcal{N}|$  branch-node inc cquations are extended to general networks as:<br>  $(2.4) - (2.7), \exists \theta \in \mathbb{R}, s.t. \beta(x) = A \cdot \theta$  (2.9)<br>
where *A* is a |ξ| by |*N*| branch-node incidence matrix in tree graph *G* with *A*<sub>*m*<sup>--1</sup></sub> if *I* = *m* → *n* for some *n* Example 1 as a  $|\mathcal{E}|$  by  $|\mathcal{N}|$  branch-node incidence matrix in tree graph  $\mathcal{G}$  with  $A_m=1$ <br>  $l = m \rightarrow n$  for some  $n$ ,  $A_m=-1$  if  $l = n \rightarrow m$  for some  $m$ , and 0 otherwise. This<br>
anch-node incidence matrix  $A$  will be di if  $l = m \rightarrow n$  for some  $n$ ,  $A_mr-1$  if  $l = n \rightarrow m$  for some  $m$ , and 0 otherwise. This<br>branch-node incidence matrix A will be discussed later. We refer to the condition<br> $\beta(x) = A \cdot \theta$  on x in (2.9) as the cycle condition, which branch-node incidence matrix A will be discussed later. We refer to the condition  $\beta(x) = A \cdot \theta$  on x in (2.9) as the cycle condition, which can be enforced by introducing  $\theta$  as additional variables. For general networks  $\beta$  (x) =  $\Lambda \cdot \theta$  on x in (2.9) as the cycle condition, which can be enforced by<br>introducing  $\theta$  as additional variables. For general networks (2.4)-(2.7) can thus be<br>interpreted as a relaxation of (2.9) where the cycl

 $l \cdot v = v_m - v_n$  holds

where A is a  $|\mathcal{E}|$  by  $|\mathcal{N}|$  branch-node incidence matrix in tree graph  $\mathcal{G}$  and  $\mathcal{A}^l$ <br>refers to the *l*-th row. Expressing with  $\mathcal{A} \cdot \mathcal{V}$  is generally formulated in the<br>matrix-vector form. To avoid h where *A* is a  $|\mathcal{E}|$  by  $|\mathcal{N}|$  branch-node incidence matrix in tree graph  $\mathcal{G}$  and  $\mathcal{A}^l$ <br>refers to the *I*-th row. Expressing with  $\mathcal{A} \cdot v$  is generally formulated in the<br>matrix-vector form. To avoid heav where *A* is a  $|\mathcal{E}|$  by  $|\mathcal{N}|$  branch-node incidence matrix in tree graph *G* and *A<sup>t</sup>*<br>refers to the *I*-th row. Expressing with *A* · *v* is generally formulated in the<br>matrix-vector form. To avoid heavy notations where *A* is a  $|\mathcal{E}|$  by  $|N|$  branch-node incidence matrix in tree graph *G* and *A<sup>t</sup>*<br>refers to the *I*-th row. Expressing with  $A \cdot v$  is generally formulated in the<br>matrix-vector form. To avoid heavy notations, the where *A* is a  $|\mathcal{E}|$  by  $|\mathcal{N}|$  branch-node incidence matrix in tree graph *G* and *A<sup>t</sup>*<br>refers to the *l*-th row. Expressing with *A* · *v* is generally formulated in the<br>matrix-vector form. To avoid heavy notations b the *l*-th row. Expressing with  $\Lambda \cdot v$  is generally formulated in the<br>
ector form. To avoid heavy notations, the following steady-state network<br>
low equality (2.10)-(2.13) can be derived by the real-valued **DistFlow**<br>

$$
-P^g + P^d = A^T P^l - D_r \ell^l \tag{2.10}
$$

$$
-\mathbf{Q}^g + \mathbf{Q}^d = A^T \mathbf{Q}^l - \mathbf{D}_x \ell^l \tag{2.11}
$$

$$
A v - 2D_r P^l - 2D_x Q^l + D_z \ell^l = 0
$$
 (2.12)

$$
\ell^l \cdot \mathbf{D}_v = |\mathbf{P}^l|^2 + |\mathbf{Q}^l|^2 \tag{2.13}
$$

where  $P<sup>i</sup>$  and  $Q<sup>i</sup>$  refer to the vectors of sending-end active and reactive power flows matrix-vector form. To avoid heavy notations, the following steady-state network<br>power flow equality (2.10)-(2.13) can be derived by the real-valued **DistFlow**<br>equations for  $\forall l \in \mathcal{E}$  in the compact matrix vector form |and |Q||. Ps, Qs and P<sup>d</sup>, Q<sup>d</sup> indicate the vectors of given bid heavy notations, the following steady-state network<br>  $(\mathbf{P}^{\mu})$ -(2.13) can be derived by the real-valued **DistFlow**<br>
compact matrix vector form.<br>  $\mathbf{P}^{\mu} + \mathbf{P}^{\mu} = A^T \mathbf{P}^{\mu} - \mathbf{D}_{\mu} \ell^{\mu}$  (2.10)<br>  $\mathbf{Q}$ Nowing steady-state network<br>
y the real-valued *DistFlow*<br>
(2.10)<br>
(2.11)<br>
(2.12)<br>
(2.13)<br>
ve and reactive power flows<br>
indicate the vectors of given<br>
d reactive loads at nodes.  $Q^{cr}$ <br>
is the vector of squared power flow equality (2.10)-(2.13) can be derived by the real-valued *DistFlow*<br>equations for  $\forall l \in \mathcal{E}$  in the compact matrix vector form.<br> $-P^s + P^{\ell} = A^T P^{\ell} - D_r \ell^{\ell}$  (2.10)<br> $- Q^s + Q^{\ell} = A^T Q^{\ell} - D_s \ell^{\ell}$  (2.11)<br> $A v$ cquations for  $\forall l \in \mathcal{E}$  in the compact matrix vector form.<br>  $-P^s + P^l = A^t P^l - D_r \ell^l$  (2.10)<br>  $-Q^s + Q^l = A^T Q^l - D_s \ell^l$  (2.11)<br>  $A v - 2D_r P^l - 2D_s Q^l + D_s \ell^l = 0$  (2.12)<br>  $\ell^l \cdot D_r = |P^l|^2 + |Q^l|^2$  (2.13)<br>
where  $P^l$  and  $Q^l$ (2.10)<br>
(2.11)<br>
(2.12)<br>
(2.13)<br>
ive and reactive power flows<br>
indicate the vectors of given<br>
d reactive loads at nodes.  $Q^{cr}$ <br>  $\ell^l$  is the vector of squared<br>
al matrices whose diagonal<br>
ector, respectively.  $D_z$  is the  $-P^x + P^d = A^T P^t - D_r \ell^t$  (2.10)<br>  $-Q^x + Q^d = A^T Q^t - D_x \ell^t$  (2.11)<br>  $Av - 2D_r P^t - 2D_x Q^t + D_z \ell^t = 0$  (2.12)<br>  $\ell^t \cdot D_r = |P^t|^2 + |Q^t|^2$  (2.13)<br>
where  $P^s$  and  $Q^t$  refer to the vectors of sending-end active and reactive power fl  $-\mathbf{Q}^{\varepsilon} + \mathbf{Q}^{\varepsilon} = A^T \mathbf{Q}^t - \mathbf{D}_x \mathbf{C}^t$  (2.11)<br>  $A\mathbf{v} - 2\mathbf{D}_x \mathbf{P}^t - 2\mathbf{D}_x \mathbf{Q}^t + \mathbf{D}_z \ell^t = 0$  (2.12)<br>  $\ell^t \cdot \mathbf{D}_x = |\mathbf{P}^t|^2 + |\mathbf{Q}^t|^2$  (2.13)<br>
where  $P^t$  and  $Q^t$  refer to the vectors  $A\nu \cdot 2D_r P' \cdot 2D_s Q' + D_z \ell' = 0$  (2.12)<br>  $\ell' \cdot D_r = |P'|^2 + |Q'|^2$  (2.13)<br>
where P' and Q' refer to the vectors of sending-end active and reactive power flows<br>
with the moduli equal to  $|P|$  and  $|Q|$ . P<sup>g</sup>, Q<sup>g</sup> and P<sup>f</sup>, Q<sup>f</sup>  $\ell = |r'|^2 + |x'|^2$ .  $D_{\nu}$  is the diagonal matrix (2.12)<br>
(2.13)<br>
e and reactive power flows<br>
dicate the vectors of given<br>
reactive loads at nodes.  $Q^{cr}$ <br>
is the vector of squared<br>
matrices whose diagonal<br>
tor, respectively.  $D_z$  is the<br>
.  $D_v$  is the diagonal matrix<br>
t  $t^l \cdot \mathbf{D}_r = |\mathbf{P}'|^2 + |\mathbf{Q}'|^2$  (2.13)<br>where  $P^l$  and  $Q^l$  refer to the vectors of sending-end active and reactive power flows<br>with the moduli equal to  $|P^l|$  and  $|Q^l|$ .  $P^g$ ,  $Q^g$  and  $P^g$ ,  $Q^l$  indicate the ve branches. th the moduli equal to  $|P|$  and  $|Q|$ .  $P^g$ ,  $Q^g$  and  $P^d$ ,  $Q^d$  indicate the vectors of given<br>dal active and reactive power injections and active and reactive loads at nodes.  $Q^{gr}$ <br>the vector of nodal reactive powe nodal active and reactive power injections and active and reactive loads at nodes.  $Q^{\pi}$ <br>is the vector of nodal reactive power compensation.  $\ell'$  is the vector of squared<br>current on branches.  $D_r$  and  $D_x$  indicate the is the vector of nodal reactive power compensation.  $\ell^i$  is the vector of squared<br>current on branches.  $D_r$  and  $D_x$  indicate the diagonal matrices whose diagonal<br>elements are the resistance vector and the reactance vec current on branches. **D**, and **D**<sub>x</sub> indicate the diagonal matrices whose diagonal<br>elements are the resistance vector and the reactance vector, respectively. **D**<sub>z</sub> is the<br>diagonal matrix whose diagonal elements are  $|z^{\$ 









where  $\tilde{A}$  refers to the reduced branch-node incidence matrix. Thus, (2.13) can be<br>further rearranged as<br> $-\tilde{A}v - 2D_rP^t - 2D_xQ^t + D_z\ell^t = a_0v_0$  (2.15) where  $\tilde{A}$  refers to the reduced branch-node incidence matrix. Thus, (2.13) can be<br>further rearranged as<br> $-\tilde{A}v - 2D_rP^l - 2D_xQ^l + D_z\ell^l = a_0v_0$  (2.15)<br>According to Ohm's law, when each node is injected by the current

$$
-\tilde{A}v - 2D_r P^l - 2D_x Q^l + D_z \ell^l = a_0 v_0 \tag{2.15}
$$

here  $\tilde{A}$  refers to the reduced branch-node incidence matrix. Thus, (2.13) can be<br>
ther rearranged as<br>  $-\tilde{A}v - 2D_rP^t - 2D_sQ^t + D_z\ell^t = a_0v_0$  (2.15)<br>
According to Ohm's law, when each node is injected by the current where  $\tilde{A}$  refers to the reduced branch-node incidence matrix. Thus, (2.<br>
further rearranged as<br>  $-\tilde{A}v - 2D_rP^i - 2D_xQ^i + D_z\ell^i = a_0v_0$ <br>
According to Ohm's law, when each node is injected by the current in<br>
can obtai

$$
A\cdot\boldsymbol{1}_{|\varepsilon|+1}=0\Longrightarrow a_0+\tilde{A}\cdot\boldsymbol{1}_{|\varepsilon|}=0\Longrightarrow \boldsymbol{1}_{|\varepsilon|}=-a_0\cdot\tilde{A}^{-1}
$$

where  $-a_0 \cdot \tilde{A}^{-1}$  can be defined as  $\tilde{A}$  refers to the reduced branch-node incidence matrix. Thus, (2.13) can be<br>rearranged as<br> $-\tilde{A}v - 2D_xP^i - 2D_xQ^i + D_z\ell^i = a_0v_0$  (2.15)<br>rding to Ohm's law, when each node is injected by the current in one unit, we<br>a where  $\tilde{A}$  refers to the reduced branch-node incidence matrix. Thus, (2.13) can be<br>further rearranged as<br> $-\tilde{A}v-2D_rP^i-2D_sQ^i+D_s\ell^i=a_0v_0$  (2.15)<br>According to Ohm's law, when each node is injected by the current in where  $\tilde{A}$  refers to the reduced branch-node incidence matrix. Thus, (2.13) can be<br>further rearranged as<br> $-\tilde{A}v - 2D_zD' + 2D_zD' + D_zC' = a_0v_0$  (2.15)<br>According to Ohm's law, when each node is injected by the current in further rearranged as<br>  $-\tilde{A}v - 2D_rP' - 2D_xQ' + D_z\ell' = a_0v_0$ <br>
According to Ohm's law, when each node is injected by the current in one u<br>
can obtain<br>  $A \cdot 1_{|a|+1} = 0 \Rightarrow a_0 + \tilde{A} \cdot 1_{|a|} = 0 \Rightarrow 1_{|a|} = -a_0 \cdot \tilde{A}^{-1}$ <br>
wher incidence matrix  $-a_0 \cdot \tilde{A}^{-1}$  can be equal to as<br>  $-\tilde{A}v - 2D_rP^i - 2D_sQ^i + D_z\ell^i = a_0v_0$  (2.15)<br>
hm's law, when each node is injected by the current in one unit, we<br>  $A^{-1}I_{g(+)} = 0 \Rightarrow a_0 + \tilde{A}^{-1}I_{g(+)} = 0 \Rightarrow 1_{g(+)} = -a_0 \cdot \tilde{A}^{-1}$ <br>
can be defined as the branch-branc



where  $a_0=1$ .

brin#5:1 1 1 1 (2.10)<br>
brin#6:1 1 1 1 1<br>
brin#6:1 1 1 1 1<br>
brin#9:1 1 1 1 1<br>
brin#9:1 1 1 1 1<br>
where  $a_0$ =1.<br>
2.3 Linearized DistFlow Equations<br>
The compact matrix-vector form of **DistFlow** equations can be simplified if

$$
-P^g + P^d = A^T \tilde{P}^l \tag{2.17}
$$

notation using the LDF method  
\n
$$
-P^{g} + P^{d} = A^{T} \tilde{P}^{l}
$$
\n(2.17)\n
$$
-Q^{g} + Q^{d} = A^{T} \tilde{Q}^{l}
$$
\n(2.18)

$$
-A\tilde{\mathbf{v}} - 2D_r\tilde{P}^{\prime} - 2D_x\tilde{Q}^{\prime} = 0
$$
 (2.19)

notation using the LDF method<br>  $-P^g + P^d = A^T \tilde{P}^l$  (2.17)<br>  $-Q^g + Q^d = A^T \tilde{Q}^l$  (2.18)<br>  $-A\tilde{v} - 2D_x \tilde{P}^l - 2D_x \tilde{Q}^l = 0$  (2.19)<br>
where  $\tilde{P}^l$  and  $\tilde{Q}^l$  refer to the vector of sending-end active and react From the LDF method<br>  $-P^s + P^d = A^T \tilde{P}^t$  (2.17)<br>  $-Q^s + Q^d = A^T \tilde{Q}^t$  (2.18)<br>  $- A\tilde{v} - 2D_x \tilde{P}^t - 2D_x \tilde{Q}^t = 0$  (2.19)<br>
where  $\tilde{P}^t$  and  $\tilde{Q}^t$  refer to the vector of sending-end active and reactive powe tation using the LDF method<br>  $-P^s + P^d = A^T \tilde{P}^t$  (2.17)<br>  $-Q^s + Q^d = A^T \tilde{Q}^t$  (2.18)<br>  $-A\tilde{v}-2D_r \tilde{P}^t - 2D_s \tilde{Q}^t = 0$  (2.19)<br>
here  $\tilde{P}^t$  and  $\tilde{Q}^t$  refer to the vector of sending-end active and reactive  $-2\mathbf{P} \cdot \mathbf{P} \cdot \mathbf{P} = A^T \mathbf{Q} \cdot \mathbf{P}$  (2.18)<br>  $- A\vec{v} - 2D_x \mathbf{P} \cdot \mathbf{P} - 2D_x \mathbf{Q} \cdot \mathbf{P} = 0$  (2.19)<br>
here  $\tilde{P}'$  and  $\tilde{Q}'$  refer to the vector of sending-end active and reactive power<br>
ws;  $\tilde{v}$  denotes

$$
\tilde{P}^{\prime} = A(-P^s + P^d) \tag{2.20}
$$

$$
\tilde{\mathbf{Q}}^{\prime} = A(\mathbf{-Q}^s + \mathbf{Q}^d) \tag{2.21}
$$

$$
-A\tilde{v} - 2D_rA(-P^s + P^d) - 2D_xA(-Q^s + Q^d) = 0
$$
 (2.22)

equations

(i) All equations are linear and are only with respect to a vector of<br>  $\tilde{p}^i$  blenotes the vector of squared voltage profiles.<br>
(ii) all equations as<br>  $\tilde{p}^i = A(-P^g + P^d)$  (2.20)<br>  $\tilde{Q}^i = A(-Q^g + Q^d)$  (2.21)<br>  $-A\tilde{$ variables  $\tilde{\bm{x}} {:=} (\tilde{\bm{\nu}}$  ,  $\tilde{\bm{P}}^{t}$  ,  $\tilde{\bm{Q}}^{t}$  ), which is sn e vector of squared voltage profiles.<br>
earrange this set of LDF equations as<br>  $\tilde{P}' = A(-P^s + P^4)$  (2.20)<br>  $\tilde{Q}' = A(-Q^s + Q^t)$  (2.21)<br>  $-A\tilde{v} - 2D_rA(-P^s + P^d) - 2D_xA(-Q^s + Q^d) = 0$  (2.22)<br>
noted that there are several charact (2.20)<br>  $\vec{P}' = A(-P^g + P^d)$  (2.20)<br>  $\vec{Q}' = A(-Q^g + Q^d)$  (2.21)<br>  $-A\tilde{v} - 2D_xA(-P^g + P^d) - 2D_xA(-Q^g + Q^d) = 0$  (2.22)<br>
should also be noted that there are several characteristies for this set of LDF<br>
tions<br>
(i) All equations ar  $\tilde{P}^i = A(-P^g + P^d)$ <br>  $\tilde{Q}^i = A(-Q^g + Q^d)$ <br>  $- A\tilde{v} - 2D_r A(-P^g + P^d) - 2D_x A(-Q^g + Q^d) = 0$ <br>
It should also be noted that there are several characteristics for this<br>
equations<br>
(*i*) All equations are linear and are only wi  $\vec{Q}' = A(-Q^x + Q^d)$  (2.21)<br>  $-A\vec{v} - 2D_rA(-P^s + P^d) - 2D_rA(-Q^s + Q^d) = 0$  (2.22)<br>
should also be noted that there are several characteristics for this set of LDF<br>
tions<br>
(i) All equations are linear and are only with respect t  $-A\tilde{v}-2D_rA(-P^s + P^d) - 2D_sA(-Q^s + Q^d) = 0$  (2.22)<br>
It should also be noted that there are several characteristics for this set of LDF<br>
equations<br>
(*i*) All equations are linear and are only with respect to a vector of<br>
vari

riables  $\tilde{\mathbf{x}}:=(\tilde{v}, \tilde{P}', \tilde{Q}')$ , which is smaller than the size of x in *DistFlow* equations<br>
(*ii*) Voltage drop and line power flows are approximately linearly relate<br>
wer injections.<br>
(*iii*) LDF gives an over-e than the size of x in *DistFlow* equations.<br>
ws are approximately linearly related to<br>
quared voltage magnitudes.<br>
te<br>  $Q' + D_{\xi} \ell' = a_0 v_0$  (2.23)<br>  $2D_x Q' + (\tilde{A})^{-1} D_{\xi} \ell'$  (2.24)<br>  $\tilde{A}^{-1} \cdot [a_0 v_0 + 2D_{\xi} P' + 2D_{x} Q' ]$ 

$$
-\tilde{A}v - 2D_rP^{\prime} - 2D_xQ^{\prime} + D_z\ell^{\prime} = a_0v_0
$$
 (2.23)

Rearranging (2.23) as

$$
\mathbf{v} = -(\tilde{A})^{-1} \cdot [a_0 v_0 + 2D_r \mathbf{P}' + 2D_x \mathbf{Q}'] + (\tilde{A})^{-1} D_z \ell'
$$
 (2.24)

 $(\tilde{A})^{-1} D_z \ell^l < 0$  and  $-(\tilde{A})^{-1} \cdot [a_0 v_0 + 2D_r P' + 2D_x Q'] > 0$ ,

obtain this inequality as  
\n
$$
\mathbf{v} < -(\tilde{A})^{-1} \cdot [a_0 v_0 + 2\mathbf{D}_r \mathbf{P}^T + 2\mathbf{D}_x \mathbf{Q}^T]
$$
\nThe RHS of (2.25) is equal to  $\tilde{\mathbf{v}} = -(\tilde{A})^{-1} \cdot [a_0 v_0 + 2\mathbf{D}_r \mathbf{P}^T + 2\mathbf{D}_x \mathbf{Q}^T]$ , which can be

tain this inequality as<br>  $v < -(\tilde{A})^{-1} \cdot [a_0 v_0 + 2D_r P^T + 2D_x Q^T]$  (2.25)<br>
The RHS of (2.25) is equal to  $\tilde{v} = -(\tilde{A})^{-1} \cdot [a_0 v_0 + 2D_r P^T + 2D_x Q^T]$ , which can be<br>
hieved by LDF. Thus, we prove that  $\tilde{v} > v$ .<br>
(*iv*) L  $= -(\tilde{A})^{-1} \cdot [a_0 v_0 + 2D_r P^{\prime} + 2D_x Q^{\prime}],$  $v_0 + 2D_r P' + 2D_x Q'$  (2.25)<br>  $\tilde{v} = -(\tilde{A})^{-1} \cdot [a_0 v_0 + 2D_r P' + 2D_x Q']$ , which can be<br>
aat  $\tilde{v} > v$ .<br>
ator for line flows. obtain this inequality as<br>  $v < -(\tilde{A})^{-1} \cdot [a_0v_0 + 2D_r P' + 2D_x Q']$  (2.25)<br>
The RHS of (2.25) is equal to  $\tilde{v} = -(\tilde{A})^{-1} \cdot [a_0v_0 + 2D_r P' + 2D_x Q']$ , which can be<br>
achieved by LDF. Thus, we prove that  $\tilde{v} > v$ .<br>
(iv) LDF g obtain this inequality as<br>  $v < -(\tilde{A})^{-1} \cdot [a_0v_0 + 2D_r P' + 2D_x Q']$  (2.25)<br>
The RHS of (2.25) is equal to  $\tilde{v} = -(\tilde{A})^{-1} \cdot [a_0v_0 + 2D_r P' + 2D_x Q']$ , which can be<br>
achieved by LDF. Thus, we prove that  $\tilde{v} > v$ .<br>
(iv) LDF g obtain this inequality as<br>  $v < -(\tilde{A})^{-1} \cdot [a_0v_0 + 2D_rP' + 2D_sQ']$  (2.2<br>
The RHS of (2.25) is equal to  $\tilde{v} = -(\tilde{A})^{-1} \cdot [a_0v_0 + 2D_rP' + 2D_sQ']$ , which can<br>
achieved by LDF. Thus, we prove that  $\tilde{v} > v$ .<br>
(*iv*) LDF give  $v < -(\tilde{A})^{-1} \cdot [a_0v_0 + 2D_r P' + 2D_x Q']$  (2.25)<br>
(c) RHS of (2.25) is equal to  $\tilde{v} = -(\tilde{A})^{-1} \cdot [a_0v_0 + 2D_r P' + 2D_x Q']$ , which can be<br>
veed by LDF. Thus, we prove that  $\tilde{v} > v$ .<br>
(*iv*) LDF gives an under-estimator for l The RHS of (2.25) is equal to  $\tilde{v} = -(\tilde{A})^{-1} \cdot [a_0 v_0 + 2D_r P' + 2D_s Q']$ , which can be<br>achieved by LDF. Thus, we prove that  $\tilde{v} > v$ .<br>(iv) LDF gives an under-estimator for line flows.<br>**Proof:** According to (2.22), we ca

$$
P^{l} = A(-P^{g} + P^{d} + D_{r}\ell^{l}) > A(-P^{g} + P^{d}) = \tilde{P}^{l}
$$
 (2.26)

$$
\mathbf{Q}^l = A(-\mathbf{Q}^s + \mathbf{Q}^d + \mathbf{D}_x l^l) > A(-\mathbf{Q}^s + \mathbf{Q}^d) = \tilde{\mathbf{Q}}^l
$$
 (2.27)

 $\boldsymbol{P}^{\prime} > \tilde{\boldsymbol{P}}^{\prime}$  and  $\boldsymbol{Q}^{\prime} > \tilde{\boldsymbol{Q}}^{\prime}$ .

ved by LDF. Thus, we prove that  $\vec{v} > v$ .<br>
it) LDF gives an under-estimator for line flows.<br>
it According to (2.22), we can directly derive the following inequality<br>  $P' = A(-P^s + P^d + D_y c^t) > A(-P^s + P^d) = \vec{P}'$  (2.26)<br>  $Q' = A(-Q$ 

- 
- 
- 

(*i*) LDF gives an under-estimator for line flows.<br>  $\mathbf{P}^i = A(\mathbf{P}^g + \mathbf{P}^d + \mathbf{D}_x t^i) > A(\mathbf{P}^g + \mathbf{P}^d) = \mathbf{P}^i$  (2.26)<br>  $\mathbf{P}^i = A(\mathbf{P}^g + \mathbf{P}^d + \mathbf{D}_x t^i) > A(\mathbf{P}^g + \mathbf{P}^d) = \mathbf{P}^i$  (2.27)<br>
we prove th  $P' = A(-P^s + P^d + D_x t^d) > A(-P^s + P^d) = \tilde{P}'$  (2.26)<br>  $Q' = A(-Q^s + Q^d + D_x t^d) > A(-Q^s + Q^d) = \tilde{Q}'$  (2.27)<br>
uus, we prove that  $P' > \tilde{P}'$  and  $Q' > \tilde{Q}'$ .<br>
(v) Approximation accuracy by LDF depends on loading conditions. The<br>
proxim  $Q' = A(-Q^s + Q^d + D_x c') > A(-Q^s + Q^d) = \tilde{Q}'$  (2.27)<br>
Thus, we prove that  $P' > \tilde{P}'$  and  $Q' > \tilde{Q}'$ .<br>
(v) Approximation accuracy by LDF depends on loading conditions. The<br>
approximation error of this LDF can be acceptable if<br>
• Thus, we prove that  $P' > \tilde{P}'$  and  $Q' > \tilde{Q}'$ .<br>
(v) Approximation error of this LDF can be acceptable if<br> **e** Voltage magnitudes close to unity, namely  $|\nu| = 1 + \varepsilon$  with  $|\varepsilon| \approx 0$ ;<br> **e** Voltage angle differences acro (v) Approximation accuracy by LDF depends on loading conditions. The<br>approximation error of this LDF can be acceptable if<br>
• Voltage magnitudes close to unity, namely  $|v| = 1 + \varepsilon$  with  $|\varepsilon| = 0$ ;<br>
• Voltage angle differ approximation error of this LDF can be acceptable if<br>
• Voltage magnitudes close to unity, namely  $|\mathbf{v}| = 1 + \varepsilon$  with  $|\varepsilon| = 0$ ;<br>
• Voltage angle differences across lines close to zero, i.e.,  $\theta_{\text{ave}} = \theta_n - \theta_m \approx 0$ <br>
• expansion around the normal voltage profile  $|U_0| = 1$ .

# 2.4 Convex Relaxation Formulation of *DistFlow* Equations<br>2.4.1 Second-order Conic Programming Formulation<br>In the sake of realizing minimum nower losses, we discuss the reactive nower

2.4 Convex Relaxation Formulation of *DistFlow* Equations<br>2.4.1 Second-order Conic Programming Formulation<br>In the sake of realizing minimum power losses, we discuss the reactive power<br>optimization problem for the fixed typ In the sake of realization Formulation of *DistFlow* Equations<br>
In the sake of realizing minimum power losses, we discuss the reactive power<br>
timization problem for the fixed typology of DNs. The two convex relaxations of<br> 2.4 Convex Relaxation Formulation of *DistFlow* Equations<br>2.4.1 Second-order Conic Programming Formulation<br>In the sake of realizing minimum power losses, we discuss the reactive power<br>optimization problem for the fixed typ 2.4 Convex Relaxation Formulation of *DistFlow* Equations<br>2.4.1 Second-order Conic Programming Formulation<br>In the sake of realizing minimum power losses, we discuss the reactive power<br>optimization problem for the fixed typ 2.4 Convex Relaxation Formulation of *DistFlow* Equations<br>
2.4.1 Second-order Conic Programming Formulation<br>
In the sake of realizing minimum power losses, we discuss the reactive power<br>
optimization problem for the fixed DNs:

relaxations, we at first discuss the conventional reactive power optimization model for  
\nDNs:  
\n
$$
\lim_{\tilde{S}',I',U,\mathcal{Q}''=C} \text{Re}(\tilde{S}'_0)
$$
\n
$$
\begin{cases}\n\dot{U}_n = \dot{U}_m - z_{mn}^l \cdot I_m^l, & \forall l \in \mathcal{E}, \forall m,n \in \mathcal{N} \\
\dot{S}^l_{mn} = \dot{U}_m(I_{mn}^l)^s, & \forall l \in \mathcal{E}, \forall m,n \in \mathcal{N} \\
\dot{S}^l_{mn} = \dot{U}_m(I_{mn}^l)^s, & \forall l \in \mathcal{E}, \forall m,n \in \mathcal{N}\n\end{cases}
$$
\n
$$
s.t.\n\begin{cases}\n\dot{S}^l_{mn} = \dot{U}_m(I_{mn}^l)^s, & \forall l \in \mathcal{E}, \forall m,n \in \mathcal{N} \\
\dot{S}^l_{mn} + \tilde{S}^s_n - z_{mn}^l |I_{mn}^l|^2 = \sum_{k \in \{n\}} \tilde{S}^l_{nk} + \tilde{S}^d_n, & \forall l \in \mathcal{E}, \forall m,n \in \mathcal{N}\n\end{cases}
$$
\n(2.28)  
\n
$$
\begin{cases}\nQ^l_{mj} \leq \text{Im}(\tilde{S}^l_{mj}) \leq \overline{Q}^l, & \text{if } \tilde{Q}^l_{mj} \leq \overline{Q}^l.\n\end{cases}
$$
\nwhere  $\tilde{S}^l_0$  refers to the complex power on the first branch between the PCC and a PQ  
\nbus; Re( $\tilde{S}^l_0$ ) and Im( $\tilde{S}^l_0$ ) refer to the real part and imaginary part of  $\tilde{S}^l_0$ , respectively.  
\nOther symbols are illustrated in Section 1.  
\nRecall that the conventional reactive power optimization model (2.28) is  
\nnon-convex, we can further modify this conventional model based on *DistFlow*  
\nequations for the preparation of convex relaxations:

where  $\tilde{S}_0^l$  refers to the complex p bus; Re( $\tilde{S}_0^l$ ) and Im( $\tilde{S}_0^l$ ) refer to the real part and imaginary part of  $\tilde{S}_0^l$  $\begin{aligned}\n\mathcal{E}_{\text{tot}} & \int_{\text{S}_{\text{int}}} \mathcal{S}_{\text{int}} + \mathcal{S}_{\text{n}}^{\circ} - \mathcal{E}_{\text{int}} |V_{\text{int}}|^{2} = \sum_{k \neq i(\epsilon)} \mathcal{S}_{ik} + \mathcal{S}_{\text{n}}^{\circ}, \quad \forall i \in \mathcal{E}, \forall m, n \in \mathcal{N} \end{aligned}$ (2.26)<br>  $\begin{aligned}\n\mathcal{E}_{\text{tot}}^{N} &\leq \text{Im}(\tilde{S}_{\text{tot}}^{i}) \leq \tilde{Q}_{\text{tot}}^{i} \\
\$ 

$$
\min_{x' \in \mathbb{R}} P_0^l
$$
\n
$$
\sum_{k \in \mathbb{N}} P_0^l
$$
\n
$$
\int_{R_{nn}} P_0^l = \sum_{k \in \mathbb{N}(n)} P_{nn}^l e^k = \sum_{k \in \mathbb{N}(n)} P_{nn}^l + P_n^g \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
\int_{Q_{nn}} Q_{nn}^l + Q_n^g - \sum_{k \in \mathbb{N}(n)} Q_{nn}^l e^k = \sum_{k \in \mathbb{N}(n)} Q_n^l + Q_n^g \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
\sum_{k=1}^l (p_{nn}^l - 2(p_{nn}^l)^2 + (Q_{nn}^l)^2) + |\sum_{n}^l e^m| \sum_{k=1}^l (p_{nn}^l, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
\sum_{k=1}^l (q_{ni} \cdot \nabla_{m} = (P_{nn}^l)^2 + (Q_{nn}^l)^2, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
\sum_{k=1}^l (q_{n}^l \leq Q_{nn}^l \leq \overline{Q}_{nn}^l)
$$
\n
$$
\sum_{k=1}^l (q_{n}^l \leq Q_{nn}^l \leq \overline{Q}_{nn}^l)
$$
\nwhere  $P_0^l$  refers to the active power injection of the first branch between the PCC and a PQ bus.  
\nFor (2.29), all constraints are linear only except for  $\ell_{mn}^l \cdot \nu_m = (P_{nm}^l)^2 + (Q_{mn}^l)^2$ . Here,  
\nwe equivalently slack this quadratic equalities to a pair of two inequalities below:  
\n
$$
\ell_{mn}^l \cdot \nu_m \geq (P_{nn}^l)^2 + (Q_{nn}^l)^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
\sum_{k=1}^l (q_{nm}^l \cdot \nu_m \leq (P_{nn}^l)^2 + (Q_{nn}^l)^2) \quad \forall l \in \mathcal{E}, \forall m, n \in \math
$$

where  $P_0^l$  refers to the active power injection of the first branch between the PCC where  $P_0^l$  refers to the active power injection of the first branch between the l<br>and a PQ bus.<br>
For (2.29), all constraints are linear only except for  $\ell_{mn}^l \cdot v_m = (P_{mn}^l)^2 + (Q_{mn}^l)^2$ <br>
we equivalently slack this quad

 $v_{mn}^l \cdot v_m = (P_{mn}^l)^2 + (Q_{mn}^l)$ 

$$
\ell_{mn}^l \cdot v_m \ge (P_{mn}^l)^2 + (Q_{mn}^l)^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.30)

$$
\ell_{mn}^l \cdot \nu_m \le (P_{mn}^l)^2 + (Q_{mn}^l)^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.31)

d a PQ bus.<br>
For (2.29), all constraints are linear only except for  $t_{nn}^t \cdot v_m = (P_{nn}^t)^2 + (Q_{nn}^t)^2$ . Here,<br>
equivalently slack this quadratic equalities to a pair of two inequalities below:<br>  $t_{nn}^t \cdot v_m \ge (P_{nn}^t)^2 + (Q_{nn}^$ be equivalently slack this quadratic equalities to a pair of two inequalities be<br>  $\ell'_{mn} \cdot v_m \ge (P'_{mn})^2 + (Q'_{mn})^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$ <br>  $\ell'_{mn} \cdot v_m \le (P'_{mn})^2 + (Q'_{mn})^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$ <br>
After dropping (2.31) lack this quadratic equalities to a pair of two inequalities below:<br>  $w_m \cdot v_m \ge (P'_{mn})^2 + (Q'_{mn})^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.30)<br>  $\ell'_{mn} \cdot v_m \le (P'_{mn})^2 + (Q'_{mn})^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.31)<br>
(2.31), we can relax the  $t_{mn}^l \cdot v_m \le (P_{mn}^l)^2 + (Q_{mn}^l)^2$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.31)<br>
After dropping (2.31), we can relax the non-convex quadratic equalities as an<br>
equality below:<br>  $t_{mn}^l \cdot v_n \ge (P_{mn}^l)^2 + (Q_{mn}^l)^2$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal$ 

$$
\ell_{mn}^l \cdot \nu_m \ge (P_{mn}^l)^2 + (Q_{mn}^l)^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.32)

$$
(2Pmnl)2 + (2Qmnl)2 - 2\ellmnl \cdot vm \le 2\ellmnl \cdot vm \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.33)

 $(\ell_{mn}^l)^2 + (\nu_m)^2$ 

$$
(2P_{mn}^l)^2 + (2Q_{mn}^l)^2 + (\ell_{mn}^l - \nu_m)^2 \le (\ell_{mn}^l + \nu_m)^2 \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.34)

$$
\left\| \begin{array}{c} 2P_{mn}^l \\ 2Q_{mn}^l \\ \hline \epsilon_{mn}^l - v_m \end{array} \right\|_{2} \leq (\ell_{mn}^l + v_m) \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n(2.35)  
Note that (2.32) also refers to a rotated SOC inequality. The standard SOCP-based formulation of *DistFlow* equations is  

$$
P_{mn}^l + P_n^g - r_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} P_{nk}^l + P_n^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n(2.36)

$$
P_{mn}^l + P_n^g - r_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} P_{nk}^l + P_n^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.36)

$$
Q_{mn}^l + Q_n^g - x_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} Q_{nk}^l + Q_n^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.37)

$$
v_n = v_m - 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) + |z_{mn}^l|^2 \ell_{mn}^l \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.38)

$$
\begin{vmatrix} 2P_{mn}^{l} \\ 2Q_{mn}^{l} \\ \ell_{mn}^{l} - \nu_{m} \end{vmatrix}_{2} \leq (\ell_{mn}^{l} + \nu_{m}) \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.39)

Note that (2.32) also refers to a rotated SOC inequality. The standard SOCP-based<br>
rmulation of DistFlow equations is<br>  $P_{nn}^l + P_n^u - r_{nn}^l e_{nn}^l = \sum_{k \in \mathbb{N}(n)} P_{nk}^l + P_n^s \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.36)<br>  $Q_{nn}^l + Q_n^e - x_{$ formulation of *DistFlow* equations is<br>  $P_{mn}^l + P_n^g - r_{mn}^l e_{mn}^l = \sum_{k \in \pi(n)} P_m^l + P_n^g \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (3)<br>  $Q_{mn}^l + Q_n^g - x_{mn}^l e_{mn}^l = \sum_{k \in \pi(n)} Q_m^l + Q_n^g \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (3)<br>  $v_n = v_m - 2(r_{mn}^l P_m^l + x_{mn}^$ equations is<br>  $\sum_{k \in \mathcal{I}(n)} P'_{nk} + P''_{n} \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$ (2.36)<br>  $\chi'_{mn} e'_{mn} = \sum_{k \in \mathcal{I}(n)} Q'_{nk} + Q''_{n} \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$ (2.37)<br>  $\chi'_{mn} P'_{mn} + \chi'_{mn} Q'_{mn} + |\zeta'_{mn}|^2 e'_{mn} \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$ (2.3  $P_{mn}^i + P_{\alpha}^g - r_{mn}^i \ell_{mn}^i = \sum_{k \in \{1, 6\}} P_{nk}^i + P_{n}^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$ (2.36)<br>  $Q_{mn}^i + Q_n^g - x_{mn}^i \ell_{mn}^i = \sum_{k \in \{1, 6\}} Q_{nk}^i + Q_n^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$ (2.37)<br>  $v_n = v_m - 2(r_{mn}^i P_{mn}^i + x_{mn}^i Q_{mn}^i) + |z_{mn$  $Q'_{nn} + Q''_{n} = \sum_{k=r(s)} Q'_{ni} + Q''_{n}$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.37)<br>  $v_x = v_m - 2(r'_{nn} P'_{mn} + x'_{mn} Q'_{nn}) + |\varepsilon'_{nn}|^2 t'_{nn}$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.38)<br>  $\begin{vmatrix}\n2P'_{mn} \\
2Q'_{mn} \\
\epsilon'_{mn} - v_m\n\end{vmatrix} \leq (\epsilon'_{mn} + v_m)$   $\forall l \in \mathcal{E}, \forall m, n$ optimization variables involves a set of state variables  $[P^l, Q^l, \ell^l, \nu_{PQ}]^T$  $l, n \in \mathcal{N}$  (2.37)<br>  $\forall m, n \in \mathcal{N}$  (2.38)<br>  $n \in \mathcal{N}$  (2.39)<br>
e define the vector of<br>
er loss. This is also called<br>
logy of DNs. The set of<br>  $\left[P^l, Q^l, \ell^l, \nu_{PQ}\right]^T$  and a<br>
refers to squared voltage<br>
ltage profile a  $\mathcal{L}_{mn} + \mathcal{L}_n - \lambda_{mn} \epsilon_{mn} = \sum_{k \neq n(m)} \mathcal{L}_{nk} + \mathcal{Q}_n$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.38)<br>  $v_n = v_m - 2(r_m^l P_m^l + x_m^l Q_m^l) + |\epsilon_{mn}^l|^2 \epsilon_{mn}^l$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.38)<br>  $\begin{bmatrix} 2P_m^l \\ 2Q_m^l \\ \epsilon_{mn}^l - v_n \end{bmatrix} \leq (\epsilon_{mn}^l +$  $_{cr}$  $\neg$ <sup>T</sup>  $\sum_{m\in\{n\}} C_{nk} + Q_n$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.38)<br>  $\leq (\ell'_{mn} + \nu_m)$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.38)<br>  $\exists \ell'_{mn} + \nu_m$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.39)<br>  $\exists \ell'_{mn} + \nu_m$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.39)<br>  $\exists \ell'_{mn} + \nu_m$  $v_n = v_m - 2(r'_{mn}P'_{mn} + x'_{mn}Q'_{mn}) + |z'_{mn}|^2 t'_{mn}$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.38)<br>  $\begin{vmatrix}\n2P'_m \\
2Q'_m \\
t'_{mn} - v_m\n\end{vmatrix} \leq (t'_{mn} + v_m) \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.39)<br>
According to SOCP-based *DistFlow* equations, we define th **a**<br>  $\begin{vmatrix}\n2P'_{mn} \\
2P'_{mn} \\
e'_{nm}-v_m\n\end{vmatrix} \leq (e'_{mn}+v_n) \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.39)<br>
According to SOCP-based *DistFlow* equations, we define the vector of<br>
optimization variables  $x^l$  for the minimization of real po **c**  $\left| \begin{matrix} \ell_{m}^{H} & -\gamma_{m} \end{matrix} \right|_{2}$ <br> **According to SOCP-based** *DistFlow* equations, we define the vector of optimization variables  $\mathbf{x}^{t}$  for the minimization of real power loss. This is also called reactive pow active power optimization problem for the fixed typology of DNs. The set of<br>timization variables involves a set of state variables  $[P', Q', \ell', v_{reg}]^T$  and a<br>ctor of controllable variables  $[v_{\text{ACC}}, Q^{e\tau}]^T$ . Here,  $v_{\text{FQ}}$  r typology of DNs. The set of<br>
les  $[P^l, Q^l, \ell^l, v_{PQ}]^T$  and a<br>  $v_{PQ}$  refers to squared voltage<br>
d voltage profile at PCC node;<br>
sation sources. In addition, as<br>
pporated to represent:<br>
(2.40)<br>
d' can be defined as<br>
(2.4

$$
\begin{cases}\n w^l = \ell_{mn}^l - v_m \\
 m^l = \ell_{mn}^l + v_m\n\end{cases}
$$
\n(2.40)

$$
\mathbf{x}^{\prime} := [P^{\prime}, Q^{\prime}, \ell^{\prime}, \nu, Q^{\prime\prime}, w^{\prime}, m^{\prime}]^{T}
$$
 (2.41)

The constraints are composed of *Distflow* equations and reactive power injection<br>nstraint of the first branch between the PCC and a PQ bus, reactive power capacity<br>mpensation constraints for all reactive power compensatio The constraints are composed of *Distflow* equations and reactive power injection<br>constraint of the first branch between the PCC and a PQ bus, reactive power capacity<br>compensation constraints for all reactive power compens The constraints are composed of *Distflow* equations and reactive power injection<br>constraint of the first branch between the PCC and a PQ bus, reactive power capacity<br>compensation constraints for all reactive power compen The constraints are composed of *Distflow* equations and reactive power injection<br>constraint of the first branch between the PCC and a PQ bus, reactive power capacity<br>compensation constraints for all reactive power compen The constraints are composed of *Distflow* equations and reactive power injection<br>constraint of the first branch between the PCC and a PQ bus, reactive power capacity<br>compensation constraints for all reactive power compen The constraints are composed of *Distflow* equations and reactive power injection<br>constraint of the first branch between the PCC and a PQ bus, reactive power capacity<br>compensation constraints for all reactive power compen The constraints are composed of *Distylow* equations and reactive power injordination of the first branch between the PCC and a PQ bus, reactive power care<br>compensation constraints for all reactive power compensation sour metraint of the first branch between the PCC and a PQ bus, reactive power capacity<br>mensation constraints for all reactive power compensation sources as well as<br>lage security constraints for all nodes.<br> $Q^{\prime\prime} \leq Q^{\prime\prime} \le$ 

$$
Q^{cr} \le Q^{cr} \le \overline{Q}^{cr} \tag{2.42}
$$

$$
\underline{v} \le v \le \overline{v} \tag{2.43}
$$

compensation constraints for all reactive power compensation sources as well as<br>voltage security constraints for all nodes.<br> $Q'' \leq Q'' \leq \overline{Q}''$  (2.42)<br> $y \leq y \leq \overline{y}$  (2.43)<br>where  $Q''$  and  $\overline{Q}''$  indicate the lower an voltage security constraints for all nodes.<br>  $Q^{\sigma} \leq Q^{\sigma} \leq \overline{Q}^{\sigma}$  (2.42)<br>  $\underline{\nu} \leq \nu \leq \overline{\nu}$  (2.43)<br>
where  $Q^{\sigma}$  and  $\overline{Q}^{\sigma}$  indicate the lower and upper limits of reactive power<br>
compensation sources.

$$
\min_{x' \in \mathbb{R}} P_0^l
$$
\n
$$
\left\{\n\begin{aligned}\nP_m^l + P_n^g - r_m^l v_m^l &= \sum_{k \in \pi(n)} P_m^l + P_n^d, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N} \\
Q_{mn}^l + Q_n^g - x_{mn}^l v_{mn}^l &= \sum_{k \in \pi(n)} Q_m^l + Q_n^d, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}\n\end{aligned}\n\right.
$$
\n
$$
s.t.\n\left\{\n\begin{aligned}\n&v_m = v_m - 2(r_m^l P_m^l + x_{mn}^l Q_{mn}^l) + |z_m^l|^2 (r_m^l, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}\n\end{aligned}\n\right.
$$
\n
$$
s.t.\n\left\{\n\begin{aligned}\n&2P_m^l \\
&2P_m^l \\
&2Q_m^l \\
&w^l\n\end{aligned}\n\right\} \leq m', \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}\n\right.
$$
\n
$$
\left\{\n\begin{aligned}\n&2P_m^l \\
&2Q_m^l \\
&w^l\n\end{aligned}\n\right\} \leq m', \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}\n\end{aligned}
$$
\nwhere  $P_0^l$  is the active power injection at the root node 0.\n\n**Exactness of SOCP convex relaxation**\n
$$
35
$$

where  $P_0^l$  is the active power injection at the root node 0.

However, is the optimal solution of (2.44) the same as the one solved by (2.1)? If<br>d only if the convex relaxation techniques are exact, the obtained solution is the<br>me as the optimal solution of original non-convex nonlin However, is the optimal solution of (2.44) the same as the one solved by (2.1)? If<br>and only if the convex relaxation techniques are exact, the obtained solution is the<br>same as the optimal solution of original non-convex no However, is the optimal solution of (2.44) the same as the one solved by (2.1)? If<br>and only if the convex relaxation techniques are exact, the obtained solution is the<br>same as the optimal solution of original non-convex no However, is the optimal solution of (2.44) the same as the one solved by (2.1)? If<br>and only if the convex relaxation techniques are exact, the obtained solution is the<br>same as the optimal solution of original non-convex no However, is the optimal solution of (2.44) the same as the one solved by (2.1)? If<br>and only if the convex relaxation techniques are exact, the obtained solution is the<br>same as the optimal solution of original non-convex no However, is the optimal solution of  $(2.44)$  the same as the one solved by  $(2.1)$ ? If and only if the convex relaxation techniques are exact, the obtained solution is the same as the optimal solution of original non-conv However, is the optimal solution of (2.44) the same as the one solved by (2.1)? If<br>and only if the convex relaxation techniques are exact, the obtained solution is the<br>same as the optimal solution of original non-convex no However, is the optimal solution of  $(2.44)$  the same as the one solved by  $(2.1)$ ? If and only if the convex relaxation techniques are exact, the obtained solution is the same as the optimal solution of original non-conv and only if the convex relaxation techniques are exact, the obtained solution is the<br>same as the optimal solution of original non-convex nonlinear optimization. This<br>section we discuss whether SOCP relaxations for reactiv same as the optimal solution of original non-convex nonlinear optimization. This<br>section we discuss whether SOCP relaxations for reactive power optimization<br>problems are exact or not. Looking from case studies in previous

as  $\min \sum r_{mn}^l \ell_{mn}^l$ . To prov l n l  $r_{mn}^l$ l $_{mn}^l$  $\sum_{l \in \mathcal{E}} r_{mn}^l \ell^i$ the relaxation is exactly the relaxation in provides in previous subsections, it seems<br>plutions are satisfied with SOC equality. This suggests that the SOC<br>r. Now, let us give the exactness of SOCP convex relaxation and i relaxation is exact. Now, let us give the exactness of SOCP convex relaxation and its<br>associated proof by contradiction [88], [89].<br> **Theorem:** The SOCP-based reactive power optimization formulation (2.44) is<br>
convex. Mor  $l_{mn}^{l} \cdot v_{m}^{l} = (P_{mn}^{l})^{2} + (Q_{mn}^{l})^{2}$ n n is so the solution of solution and its<br>
were optimization formulation (2.44) is<br>
al solution of (2.44) is also optimal for the<br>
m (2.1).<br>
we rewrite the objective function  $\min P_{inj}^{l}$ <br>
tion is exact, it suffices to s associated proof by contradiction [88], [89].<br> **Theorem:** The SOCP-based reactive power optimization formulation (2.44) is<br>
convex. Moreover, it is exact, i.e., an optimal solution of (2.44) is also optimal for the<br>
origi  ${x^{\prime}}^{\star}:=(P_{ij}^{l^{\bigstar}},Q_{ij}^{l^{\bigstar}},\ell^{l^{\bigstar}},\nu^{\bigstar},Q^{cr})$ on [88], [89].<br>
d reactive power optimization formulation (2.44) is<br>
i.e., an optimal solution of (2.44) is also optimal for the<br>
zation problem (2.1).<br>
hodel (2.44), we rewrite the objective function min  $P_{mj}^l$ <br>
ant t **Theorem:** The SOCP-based reactive power optimization formulation (2<br>convex. Moreover, it is exact, i.e., an optimal solution of (2.44) is also optimal<br>original reactive power optimization problem (2.1).<br>**Proof:** For this inal reactive power optimization problem (2.1).<br>
of: For this SOCP-based model (2.44), we rewrite the objective function  $\min P'_{mj}$ <br>  $\min \sum_{i \in \mathcal{S}} \gamma'_{mn} e^{i}_{mn}$ . To prove that the relaxation is exact, it suffices to show bjective function min  $P_{inj}^l$ <br>suffices to show that any<br> $+(\underline{Q}_{mn}^l)^2$ . Assume for the<br>is optimal but has strict<br>(2.45)<br> $(\tilde{\ell}^l, \tilde{v}, \tilde{Q}^{cr})$ , which is **Proof:** For this SOCP-based model (2.44), we rewrite the objective functio<br>as  $\min \sum_{h \in \mathcal{E}} r_{mn}^l \ell_{mn}^l$ . To prove that the relaxation is exact, it suffices to show<br>optimal solution of (2.1) has equality in  $\ell_{mn}^l \cdot$ 

$$
\ell_{mn}^{l^{\star}} \cdot v_m^{\star} > (P_{mn}^{l^{\star}})^2 + (Q_{mn}^{l^{\star}})^2, \forall l \in \mathcal{E}
$$
 (2.45)

 $\tilde{x}^l \coloneqq (\tilde{P}_{ij}^l, \tilde{Q}_{ij}^l, \tilde{\ell}^l, \tilde{v}, \tilde{Q}^{cr})$ 

$$
\begin{aligned}\n\tilde{v} &= v^*, \tilde{Q}^{cr} = Q^{cr*} \\
\tilde{P}_{mn}^i &= P_{mn}^{i*} - r_{mn}^{j*} \epsilon/2, \quad \tilde{P}_{-mn}^i = P_{-mn}^{j*}\n\end{aligned}
$$
\n
$$
\begin{aligned}\n\tilde{Q}_{mn}^i &= Q_{mn}^{i*} - r_{mn}^{j*} \epsilon/2, \quad \tilde{Q}_{-mn}^i = Q_{-mn}^{i*}\n\end{aligned}
$$
\n
$$
\begin{aligned}\n\tilde{Q}_{mn}^i &= Q_{mn}^{i*} - \epsilon_n, \quad \tilde{\epsilon}_{-mn}^i = \epsilon_n^{i*}\n\end{aligned}
$$
\n
$$
\begin{aligned}\n\tilde{P}_{mn}^i &= e^{i*}\n\end{aligned}
$$
\n
$$
\begin{aligned}\n\tilde{P}_{mn}^i &= P_{n}^{i*} + r_{mn}^{i*} \epsilon/2, \quad \tilde{P}_{mn}^{i*} = P_{-mn}^{i*}\n\end{aligned}
$$
\n
$$
\begin{aligned}\n\tilde{Q}_{m}^i &= Q_{m}^{i*} + x_{mn}^{i*} \epsilon/2, \quad \tilde{Q}_{m}^i = Q_{m}^{i*} + x_{mn}^{i*} \epsilon/2, \\
\tilde{P}_{m}^i &= P_{m}^i, \tilde{P}_{m}^i = P_{m}^i, \tilde{Q}_{m}^i = Q_{m}^i, \tilde{Q}_{-m}^i = Q_{-m}^i\n\end{aligned}
$$
\nwhere the negative indices mean excluding elements from a vector.\n\nWhen  $\tilde{\epsilon}_{mn}^i$ ,  $\tilde{v}_m = (\tilde{P}_{mn}^i)^2 + (\tilde{Q}_{mn}^i)^2$  holds for certain  $\varepsilon > 0$ , it can be verified that  $\tilde{\chi}^i$  can satisfy\n
$$
\tilde{v}_n = \tilde{v}_m - 2(r_{mn}^i \tilde{P}_{mn}^i + x_{mn}^i \tilde{Q}_{mn}^i) + |z_{mn}^i|^2 \tilde{\epsilon}_{mn}^i + (r_{mn}^i)^2 \epsilon + (x_{mn}^i)^2 \epsilon - |z_{mn}^i|^2 \epsilon \quad (2.47)\n= \tilde{v}_m - 2(r_{mn}^i \tilde{P
$$

When  $\tilde{\ell}_{mn}^l \cdot \tilde{v}_m = (\tilde{P}_{mn}^l)^2 + (\tilde{Q}_{mn}^l)^2$  holds  $v_{mn}^l \cdot \tilde{v}_m = (\tilde{P}_{mn}^l)^2 + (\tilde{Q}_{mn}^l)$ 

$$
\begin{bmatrix}\n\tilde{P}_{-n}^{d} = P_{-n}^{d}, \ \tilde{P}_{-m}^{d} = P_{-m}^{d}, \ \tilde{Q}_{-n}^{d} = Q_{-n}^{d}, \ \tilde{Q}_{-m}^{d} = Q_{-m}^{d}\n\end{bmatrix}
$$
\nref: the negative indices mean excluding elements from a vector.  
\nThen  $\tilde{\ell}_{mn}^{l} \cdot \tilde{v}_m = (\tilde{P}_{mn}^{j})^2 + (\tilde{Q}_{mn}^{j})^2$  holds for certain  $\varepsilon > 0$ , it can be verified that  $\tilde{x}^{l}$   
\nsatisfy  
\n
$$
\tilde{v}_n = \tilde{v}_m - 2(r_{mn}^{l} \tilde{P}_{mn}^{l} + x_{mn}^{l} \tilde{Q}_{mn}^{l}) + |z_{mn}^{l}|^2 \tilde{\ell}_{mn}^{l}
$$
\n
$$
= \tilde{v}_m - 2(r_{mn}^{l} P_{mn}^{l*} + x_{mn}^{l} Q_{mn}^{l*}) + |z_{mn}^{l}|^2 (\tilde{\ell}_{mn}^{l*} + (r_{mn}^{l})^2 \varepsilon + (x_{mn}^{l})^2 \varepsilon - |z_{mn}^{l}|^2 \varepsilon \quad (2.47)
$$
\n
$$
= \tilde{v}_m - 2(r_{mn}^{l} P_{mn}^{l*} + x_{mn}^{l} Q_{mn}^{l*}) + |z_{mn}^{l}|^2 (\tilde{\ell}_{mn}^{l*} + (r_{mn}^{l})^2 \varepsilon + (x_{mn}^{l})^2 \varepsilon - |z_{mn}^{l}|^2 \varepsilon \quad (2.47)
$$
\n
$$
= \tilde{v}_m - 2(r_{mn}^{l} P_{mn}^{l*} + x_{mn}^{l} Q_{mn}^{l*}) + |z_{mn}^{l}|^2 (\tilde{\ell}_{mn}^{l*} + (r_{mn}^{l})^2 \varepsilon + (x_{mn}^{l})^2 \varepsilon - |z_{mn}^{l}|^2 \varepsilon \quad (2.47)
$$
\n
$$
= \tilde{v}_m - 2(r_{mn}^{l} P_{mn}^{l*} + x_{mn}^{l} Q_{mn}^{l*}) + |z_{mn}^{l}|^2 (\tilde{\ell}_{mn}^{l*} + (x_{mn}^{l})^
$$

For  $\tilde{P}_{mn}^l$ 

$$
\tilde{P}_{mn}^l + P_n^g - r_{mn}^l \tilde{\ell}_{mn}^l = \sum_{k \in \pi(n)} \tilde{P}_{nk}^l + \tilde{P}_n^d \qquad , \qquad \text{we} \qquad \text{can}
$$

find 
$$
P_{mn}^{\prime\star} - r_{mn}^l \varepsilon / 2 + P_n^g - r_{mn}^l \ell_{mn}^l + r_{mn}^l \varepsilon = \sum_{k \in \pi(n)} P_{nk}^l + P_n^d + r_{mn}^l \varepsilon / 2
$$
 holds and for

$$
Q_{mn}^l + Q_n^g - x_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} Q_{nk}^l + Q_n^d
$$
, this point  $\tilde{\chi}^l$  still holds under this over-satis  
factor

can satisfy<br>  $\tilde{v}_{e} = \tilde{v}_{m} - 2(r'_{m} \overline{P'_{m}} + x'_{m} \overline{Q'_{m}}) + |\tilde{\tau}'_{m}|^{2} \tilde{\ell}'_{m}$ <br>  $= \tilde{v}_{m} - 2(r'_{m} P''_{m\pi} + x'_{m} Q''_{m\pi}) + |\tilde{\tau}'_{m}|^{2} \tilde{\ell}'_{m} + (r'_{m\pi})^{2} \varepsilon + (x'_{m\pi})^{2} \varepsilon - |\tilde{\tau}'_{m}|^{2} \varepsilon$  (2.47)<br>  $= \tilde{v}_{m} - 2$ 1.1.  $\tilde{v}_n = \tilde{v}_m - 2(r_{mn}^l \tilde{P}_{mn}^l + x_{mn}^l \tilde{Q}_{mn}^l) + |z_{mn}^l|^2 \tilde{t}_{mn}^l$ <br>  $= \tilde{v}_m - 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) + |z_{mn}^l|^2 \tilde{t}_{mn}^l + (r_{mn}^l)^2 \varepsilon + (x_{mn}^l)^2 \varepsilon - |z_{mn}^l|^2 \varepsilon$  (2.<br>  $= \tilde{v}_m - 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{$ increased the loads  $\tilde{P}_n^d$  and  $\tilde{Q}_n^d$  on buses and to obtain the alternative feasible  $\begin{aligned}\n& -x_{mn}^j \tilde{Q}_{mn}^l + |z_{mn}^l|^2 \tilde{\epsilon}_{mn}^l \\
& +x_{mn}^j Q_{mn}^l + |z_{mn}^l|^2 \epsilon_{mn}^{l*} + (r_{mn}^l)^2 \varepsilon + (x_{mn}^l)^2 \varepsilon - |z_{mn}^l|^2 \varepsilon \quad (2.47) \\
& + x_{mp}^l Q_{mn}^l + |z_{mn}^l|^2 \epsilon_{mn}^{l*} \\
& + x_{mp}^l Q_{mn}^l + |z_{mn}^l|^2 \epsilon_{mn}^{l*} \\
& = \sum_{k \in \pi(n)} P_{nk}^{l$ solution  $\tilde{x}^i$ .

 $= v_m - 2(r_{mn}^r r_{mn}^r + x_{mn}^r C_{mn}) + |z_{mn}^r|^r C_{mn}$ <br>
For  $\tilde{P}_m^i + P_n^g - r_{mn}^l \tilde{\ell}_{mn}^l = \sum_{k \in \pi(n)} \tilde{P}_m^k + \tilde{P}_n^d$ , we can be  $\tilde{P}_m^i + r_{mn}^k \varepsilon / 2 + P_n^g - r_{mn}^l \ell_{mn}^{i*} + r_{mn}^l \varepsilon = \sum_{k \in \pi(n)} P_m^{i*} + P_n^d + r_{mn}^l \varepsilon / 2$  h  $-2(V_{na}F_{ma} + x_{ma}Q_{ma}) + |\xi_{mn}|^2 V_{ma}$ <br>  $P_n^g - r_{mn}^h \tilde{\ell}_{mn}^l = \sum_{k \in \pi(\alpha)} \tilde{P}_{nk}^i + \tilde{P}_n^d$ , we can<br>  $P_n^g - r_{mn}^l \ell_{mn}^k + r_{mn}^l \sigma = \sum_{k \in \pi(\alpha)} P_{nk}^i + P_n^d + r_{mn}^l \sigma / 2$  holds and for<br>  $\sum_{k \in \pi(\alpha)} Q_{nk}^l + Q_n^d$ , this point  $\tilde$ Since  $\tilde{\ell}_{mn}^l = {\ell}_{mn}^{\prime\star} - \varepsilon$ , the  $\tilde{x}^l$  has a strictly smaller objective value  $\sum r_m^l$  $\frac{d}{dm}\mathcal{E}/2 + P_n^g - r_m^f \frac{e^{i\pi}}{mn} + r_m^f \mathcal{E} = \sum_{k \in \pi(n)} P_n^{i\pi} + P_n^d + r_m^f \mathcal{E}/2$  holds and for<br>  $m e^{i\pi} m \mathcal{E} = \sum_{k \in \pi(n)} Q_m^i + Q_n^d$ , this point  $\tilde{\chi}^j$  still holds under this over-satisfaction<br>
se note that this ov  $r_{mn}^{m} \int_{k=\pi(n)}^{k} r_{mn}^{n} \mathcal{E} = \sum_{k=\pi(n)} P_{nk}^{i*} + P_{n}^{d} + r_{mn}^{d} \mathcal{E}/2$  holds and for  $Q_{nk}^{j} + Q_{n}^{d}$ , this point  $\tilde{\chi}^{j}$  still holds under this over-satisfaction that this over-satisfaction of load is needed beca mn l l  $r_{mn}^l$ l $_{mn}^l$  $\sum_{l\in\mathcal{E}} r_{mn}^l \ell^u$ than  $x^{i^*}$ . that  $I_{mn}^{\mu} - I_{mn}^{\mu} \epsilon^{i} / 2 + I_{n}^{\mu} - I_{mn}^{\mu} \epsilon_{mn} + I_{mn}^{\mu} \epsilon^{i} = \sum_{k \geq \epsilon(n)} I_{nk}^{\mu} + I_{n}^{\mu} + I_{m}^{\mu} \epsilon^{i} / 2$  holds and for<br>  $Q_{mn}^{i} + Q_{n}^{\mu} - X_{mn}^{\mu} \ell_{mn}^{i} = \sum_{k \geq \epsilon(n)} Q_{nk}^{i} + Q_{n}^{\mu}$ , this point  $\vec{x}^{i}$  stil This contradicts the optimality of  $x'^{\star}$ . This theorem indicates that the optimal solution  ${}_{k}g_{k} = \sum_{k \in R(n)} P_{nk}^{k} + P_{n}^{m} + r_{mn}^{m}g/2$  holds and for<br>this point  $\tilde{\chi}^{j}$  still holds under this over-satisfaction<br>r-satisfaction of load is needed because we have<br>on buses and to obtain the alternative feasible  $Q'_{nn} + Q''_{n} - x'_{mn} \ell'_{nn} = \sum_{k \in \{r(s)\}} Q'_{nk} + Q''_{n}$ , this point  $\tilde{\chi}^j$  still holds under this over-satisfaction<br>of load. Please note that this over-satisfaction of load is needed because we have<br>increased the loads  $\bar{$  strictly increasing in the power injections. Once there is an inverse power flow in DNs,<br>i.e. DGs to generate power to grids, this relaxation may be inexact. So this exact<br>SOCP relaxation condition holds under a load overs strictly increasing in the power injections. Once there is an inverse power flow in DNs,<br>i.e. DGs to generate power to grids, this relaxation may be inexact. So this exact<br>SOCP relaxation condition holds under a load over strictly increasing in the power injections. Once there is an inverse power flow in DNs,<br>i.e. DGs to generate power to grids, this relaxation may be inexact. So this exact<br>SOCP relaxation condition holds under a load over node *n*,  $\tilde{P}_n^d - P_n^d \ge 0$  and radial reasing in the power injections. Once there is an inverse power flow in DNs,<br>o generate power to grids, this relaxation may be inexact. So this exact<br>xation condition holds under a load oversatisaction assumption (i.e., f strictly increasing in the power injections. Once there is an inverse power flow in DNs,<br>i.e. DGs to generate power to grids, this relaxation may be inexact. So this exact<br>SOCP relaxation condition holds under a load over ■ Production of the Contract of **Example 12.4.2**<br>
2.4.2 Semi-Definite formulation. Once there is an inverse power flow in DNs,<br>
i.e. DGs to generate power to grids, this relaxation may be inexact. So this exact<br>
SOCP relaxation condition holds under a l ictly increasing in the power injections. Once there is an inverse power flow in DNs,<br>DGs to generate power to grids, this relaxation may be inexact. So this exact<br>OCP relaxation condition holds under a load oversatisacti strictly increasing in the power injections. Once there is an inverse power flow in DNs,<br>
i.e. DGs to generate power to grids, this relaxation may be inexact. So this exact<br>
SOCP relaxation condition holds under a load ov i.e. DGs to generate power to grids, this relaxation may be inexact. So this exa<br>
SOCP relaxation condition holds under a load oversatisaction assumption (i.e., for<br>
node *n*,  $\tilde{P}_n^d - P_n^d \ge 0$ ) and radial networks, wh

2.4.2 Semi-Definite Programming Formulation  
\nWe firstly recall the general mathematic conversion between inequality constraints  
\nand semi-definite formulation. For example, we can find the following two  
\nformulations are equivalent:  
\n
$$
\begin{cases}\n(x, y) \in \mathbb{R}^2 : X = \begin{bmatrix} x & 0 & y \\ 0 & 1 & -x \\ y & -x & 1 \end{bmatrix} \succeq 0, \text{rank}(X) = 1\n\end{cases}\n\Leftrightarrow \begin{cases}\n0 \le x \le 1 \\
(x, y) \in \mathbb{R}^2 : x \ge y^2 \\
x - x^3 - y^2 \ge 0\n\end{cases}
$$
\n(2.4  
\n8)  
\nSimilarly, we can apply this equivalent conversion for the quadratic constraints in  
\nDistFlow equations. The alternative formulation of non-convex quadratic equalities  
\n1.7) can be expressed in SDP form. We establish

2.4.2 Semi-Definite Programming Formulation<br>
We firstly recall the general mathematic conversion between inequality constraints<br>
and semi-definite formulation. For example, we can find the following two<br>
formulations are We firstly recall the general mathematic conversion between inequality constraints<br>and semi-definite formulation. For example, we can find the following two<br>formulations are equivalent:<br> $\begin{cases} (x, y) \in \mathbb{R}^2 : X = \begin{bmatrix} x & 0 &$ \*  $(I^l)^*$  $\left[\begin{matrix} \dot U_m^* & \left( I_{mn}^l \right)^* \end{matrix} \right] = \begin{matrix} V_m & \tilde S_{mn}^l \ \tilde S_{mn}^l & \rho^l \end{matrix}$  $m \left\{ \begin{array}{c} \left\{ \boldsymbol{I} \right\} \\ \left\{ \boldsymbol{I} \right\} \end{array} \right\}$ l  $\begin{array}{ccc} \n\begin{array}{ccc} \n\end{array} & \n\end{array} & \n\begin{array}{ccc} \n\end{array} & \n\end{array}$  $mn \perp$   $\qquad \qquad \qquad \Box m n \qquad \sim mn$  $\left[\begin{matrix} U_m \\ \vdots \end{matrix}\right] \left[\begin{matrix} U_m^* & (I_m^l)^* \end{matrix}\right] = \left[\begin{matrix} V_m & \tilde{S}_{mn}^l \end{matrix}\right]$  $\bar{S_i}$ I I  $\begin{bmatrix} \dot{U}_m \end{bmatrix}$  $\begin{bmatrix} \dot{V}_m & \dot{V}_m \end{bmatrix}$  $\boldsymbol{X}^{\boldsymbol{l}}=\begin{bmatrix} \boldsymbol{U}_m \ I_m^{\boldsymbol{l}} \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{U}}_m^* & (\boldsymbol{I}_{mn}^{\boldsymbol{l}})^* \end{bmatrix}=\begin{bmatrix} \boldsymbol{\nu}_m & \boldsymbol{S}_{mn} \\ \tilde{S}_{mn}^{\boldsymbol{l}*} & \ell_{mn}^{\boldsymbol{l}} \end{bmatrix}$  1  $\ell'$  $\mathcal{L}_{m}^{j}$   $\left[\begin{matrix} \dot{U}_{m}^{*} & (I_{mn}^{l})^{*} \end{matrix}\right] = \begin{bmatrix} v_{m} & \tilde{S}_{mn}^{l} \\ \tilde{S}_{l}^{l*} & e^{l} \end{bmatrix}$  that satisfies  $\det(\mathbf{X}^{l}) = \ell_{mn}^{l} \cdot v_{m} - \tilde{S}_{mn}^{l}$ tion. For example, we can find the following two<br>  $\begin{aligned}\ny \\
x \\
y \\
y \\
z\n\end{aligned} \ge 0$ , rank $(X) = 1$   $\Leftrightarrow$   $\begin{cases}\n0 \le x \le 1 \\
(x, y) \in \mathbb{R}^2 : x \ge y^2 \\
x - x^3 - y^2 \ge 0\n\end{cases}$  (2.4<br>
S)<br>
his equivalent conversion for the quadratic constraints an find the following two<br>  $0 \le x \le 1$ <br>  $x-y^2 \ge 0$ <br>  $(2.4$ <br>  $x-x^3-y^2 \ge 0$ <br>
8)<br>
or the quadratic constraints in<br>
n-convex quadratic equalities<br>
form. We establish<br>  $X' = \ell_{mn}^l \cdot v_m - \tilde{S}_{mn}^l \tilde{S}_{mn}^{l*}$ . If  $X'$  is<br>  $w \tilde{$ is Similarly, we can apply this equivalent conversion for the quadratic constraint<br>
Similarly, we can apply this equivalent conversion for the quadratic constraint<br>
Distribute equations. The alternative formulation of non-co Let  $\begin{cases} x & 0 \quad y \\ 0 & 1 \quad -x \end{cases} \ge 0$ , rank $(X) = 1$ ,  $\Leftrightarrow \begin{cases} (x, y) \in \mathbb{R}^2 : x \ge y^2 \\ x - x^3 - y^2 \ge 0 \end{cases}$  (2.4<br>
can apply this equivalent conversion for the quadratic constraints in<br>
ons. The alternative formulation of no  $(x, y) \in \mathbb{R}^2 : X = \begin{bmatrix} 0 & 1 & -x \\ y & -x & 1 \end{bmatrix} \implies 0, \text{rank}(X) = 1 \Rightarrow \begin{cases} (x, y) \in \mathbb{R}^2 : & x \ge y^2 \\ (x - x^2 - y^2 \ge 0) \end{cases}$  (2.4<br>
Similarly, we can apply this equivalent conversion for the quadratic constraints in<br>
strilow equatio

 $l \tilde{C}l^*$  $\sum_{mn}^{l} \cdot \mathcal{V}_m - \tilde{S}_{mn}^{l}$ 

$$
X^{l} = \begin{bmatrix} v_m & \tilde{S}_{mn}^l \\ \tilde{S}_{mn}^{l*} & \ell_{mn}^l \end{bmatrix} \succeq 0 \tag{2.49}
$$

$$
rank(X^l)=1
$$
 (2.50)

However,  $rank(X')=l$  is a non-convex constraint, and thus the standard SDP-based<br>rmulation omits this constraint. Therefore, the standard SDP-based formulation of<br>stFlow equations [89] is However, rank $(X^t)$ =1 is a non-convex constraint, and thus the standard SDP-based<br>formulation omits this constraint. Therefore, the standard SDP-based formulation of<br>*DistFlow* equations [89] is<br> $P^t \leftarrow P^s$ ,  $e^{t} e^{t}$   $\$ Dist $Flow$  equations [89] is mulation omits this constraint. Therefore, the standard SDP-based formulation of<br>
stFlow equations [89] is<br>  $P'_{ma} + P''_n - r'_{ma}t''_{mn} = \sum_{k \in \pi(x)} P'_{nk} + P''_n \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.51)<br>  $Q'_{mn} + Q''_n - x'_{mn}t''_{mn} = \sum_{k \in \pi(x)} Q$ b, the standard SDP-based formulation of<br>  $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.51)<br>  $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.52)<br>  $\tilde{S}_{mn}^{l^*}$  + $|z_{mn}^l|^2 \ell_{mn}^l$   $\forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.53)<br>  $\geq 0$  (2.54)<br>  $\Rightarrow$  we can decompose

$$
P_{mn}^l + P_n^g - r_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} P_{nk}^l + P_n^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.51)

$$
Q_{mn}^l + Q_n^g - x_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} Q_{nk}^l + Q_n^d \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
 (2.52)

$$
v_n = v_m - 2(r_{mn}^l \frac{\tilde{S}_{mn}^l + \tilde{S}_{mn}^{l*}}{2} + x_{mn}^l \frac{\tilde{S}_{mn}^l - \tilde{S}_{mn}^{l*}}{2}) + |z_{mn}^l|^2 \ell_{mn}^l \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N} \quad (2.53)
$$

$$
X^{\prime} = \begin{bmatrix} v_m & \tilde{S}^{\prime} \\ \tilde{S}^{\prime*} & \ell^{\prime} \\ m_m & \ell^{\prime} \end{bmatrix} \succeq 0 \tag{2.54}
$$

For the 2  $\times$  2 complex-valued matrix  $X<sup>t</sup>$ , we can decompose it as a linear matrix DistFlow equations [89] is<br>  $P'_{mn} + P''_n - r'_{mn} \ell'_{mn} = \sum_{k \in \pi(n)} P'_{nk} + P''_n \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.51<br>  $Q'_{mn} + Q''_n - \chi''_{mn} \ell'_{mn} = \sum_{k \in \pi(n)} Q'_{nk} + Q''_n \quad \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}$  (2.52<br>  $v_n = v_m - 2(r'_{mn} \frac{\tilde{S}'_{mn} + \tilde{S}''_{$ 

$$
X' = \begin{bmatrix} v_m & \tilde{S}'_{mn} \\ \tilde{S}^{i*}_{mn} & \ell'_{mn} \end{bmatrix} \succeq 0 \tag{2.54}
$$
\nFor the 2 × 2 complex-valued matrix  $X'$ , we can decompose it as a linear matrix

\nequality (LMI) constraint

\n
$$
X' = \begin{bmatrix} v_m & \tilde{S}'_{mn} \\ \tilde{S}^{i*}_{mn} & \ell'_{mn} \end{bmatrix}
$$
\n
$$
= v_m \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + (P'_{mn} + jQ'_{mn}) \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + (P'_{mn} - jQ'_{mn}) \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \ell'_{mn} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.55}
$$
\n
$$
= v_m \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + P'_{mn} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + Q'_{mn} \begin{bmatrix} 0 & j \\ -j & 0 \end{bmatrix} + \ell'_{mn} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \succeq 0
$$
\nAccording to SDP-based *DistFlow* equations, we define the vector of optimization

\nriables  $x'$  for the minimization of real power loss. By applying LMI constraint, the

\nimplex-valued matrix  $X'$  is rearranged to a constraint with respect to

\nriables  $P'_{mn}$ ,  $Q'_{mn}$ ,  $\ell'_{mn}$ , and  $v_m$ . Therefore, the vector of optimization variables  $x'$  can

variables  $x<sup>1</sup>$  for the minimization of real power loss. By applying LMI constraint, the For the 2 × 2 complex-valued matrix  $X^i$ , we can decompose it as a linear matrix<br>
inequality (LMI) constraint<br>  $X^i = \begin{bmatrix} v_m & \tilde{S}^i_m \\ \tilde{S}^m_m & \ell^i_m \end{bmatrix}$ <br>  $= v_m \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + (P^j_m + jQ^j_m) \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{b$  $\begin{aligned}\n\mathbf{E}_{mn}^{j} & \begin{bmatrix}\n0 & 1 \\
0 & 0\n\end{bmatrix} + (P_{mn}^{j} - jQ_{mn}^{j}) \begin{bmatrix}\n0 & 0 \\
1 & 0\n\end{bmatrix} + \mathcal{E}_{mn}^{j} \begin{bmatrix}\n0 & 0 \\
0 & 1\n\end{bmatrix} + \mathcal{E}_{mn}^{j} \begin{bmatrix}\n0 & 0 \\
0 & 1\n\end{bmatrix} + \mathcal{E}_{mn}^{j} \begin{bmatrix}\n0 & j \\
-j & 0\n\end{bmatrix} + \mathcal{E}_{mn}^{j$ variables  $P_{mn}^l$ ,  $Q_{mn}^l$ ,  $\ell_{mn}^l$ , and  $v_m$ . Therefore, the vector of optimization variables  $x^l$  can  $\begin{aligned} &\nu_{\text{m}} \left[\sum_{S_{mn}}^{V_m} \frac{\tilde{S}_{mn}^i}{\tilde{S}_{mn}}\right] \\ &= \left[\sum_{S_{mn}}^{V_m} \frac{\tilde{S}_{mn}^i}{\ell_{mn}^i}\right] \\ &\nu_m \left[\begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array}\right] + \left(P_{mn}^j + jQ_{mn}^j\right) \left[\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array}\right] + \left(P_{mn}^j - jQ_{mn}^j\right) \left[\begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array}\right] + \ell_{mn$  $X' = \begin{bmatrix} v_m & \tilde{S}_{mn}^t \\ \tilde{S}_{mn}^s & e_{mn}^t \end{bmatrix}$ <br>  $= v_m \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + (P_{mn}^i + jQ_{mn}^i) \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + (P_{mn}^i - jQ_{mn}^i) \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + e_{mn}^i \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ <br>  $= v_m \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + P_{mn$  $\begin{bmatrix} 1 & 0 \ 0 & 1 \end{bmatrix}$  (b)  $\begin{bmatrix} 0 & 0 \ 0 & 0 \end{bmatrix}$  (consequently  $\begin{bmatrix} 0 & 0 \ 0 & 0 \end{bmatrix} + P'_{\text{max}} \begin{bmatrix} 0 & 1 \ 1 & 0 \end{bmatrix} + Q'_{\text{max}} \begin{bmatrix} 0 & 0 \ -j & 0 \end{bmatrix} + e''_{\text{max}} \begin{bmatrix} 0 & 0 \ 0 & 1 \end{bmatrix} \ge 0$ <br>According to SDP-based *D* 

$$
\mathbf{x}^{\prime} := [\boldsymbol{P}^{\prime}, \boldsymbol{Q}^{\prime}, \ell^{\prime}, \nu, \boldsymbol{Q}^{\mathrm{cr}}]^T
$$
 (2.56)

$$
\min_{x' \in \mathbb{R}} P_0^l
$$
\n
$$
\int_{x' \in \mathbb{R}} P_m^l + P_n^g - r_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} P_{nk}^l + P_n^d, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
Q_{mn}^l + Q_n^g - x_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} Q_m^l + Q_n^d, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
s.t. \begin{cases} v_n = v_m - 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) + |z_{mn}^l|^2 \ell_{mn}, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N} \\ v_n = v_m - 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) + |z_{mn}^l|^2 \ell_{mn}, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N} \end{cases}
$$
\n
$$
\begin{cases} 1 & 0 \\ v_m \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + P_{mn}^l \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + Q_{mn}^l \begin{bmatrix} 0 & j \\ -j & 0 \end{bmatrix} + \ell_{mn}^l \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \geq 0, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
\begin{cases} 2^{cr} \leq Q^{cr} \leq \overline{Q}^{cr}, \underline{v} \leq v \leq \overline{v} \end{cases}
$$
\nNote that there is another formulation of this model with respect to complex-valued vector  $x^l := [\tilde{S}^l, \ell^l, v, Q^{cr}]^T$ . In this vein, we formulate this complex-valued reactive power optimization model below:  
\n
$$
\min_{x' \in \mathcal{C}} \frac{\text{Tr}(S_0^l + S_0^r)}{2}
$$

vector  $\mathbf{x}^l := [\tilde{S}^l, \ell^l, \nu, Q^{cr}]^T$ .

$$
\min_{x' \in \mathbb{C}} \frac{\text{Tr}(S_0^l + S_0^{l^*})}{2}
$$
\n
$$
\int_{x' \in \mathbb{C}} \frac{\int_{x''} (S_0^l + S_0^{l^*})}{2}
$$
\n
$$
\int_{x''} \left| P_{mn}^l + P_n^g - r_{mn}^l e_{mn}^l = \sum_{k \in \pi(n)} P_{nk}^l + P_n^g, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
S_0^l = \sum_{k \in \pi(n)} \frac{S_m^l + (S_m^l)}{2} + \sum_{k \in \pi(n)} \frac{S_m^l + (S_m^l)}{2} + \sum_{k \in \mathcal{N}} \frac{S_m^l - (S_m^l)}{2} + \sum_{k \in \mathcal{N}} \frac{S_m^l}{2} + \sum_{k \in \mathcal{N}} \frac{S_m^l - (S_m^l)}{2} + \sum_{k \in \mathcal{N}} \frac
$$

where  $Tr(S_0^l + S_0^{l*})$  refers to the trace of  $S_0^l + S_0^{l*}$ 

how can a SOC cone can be approximated into a set of linearizations. Mathematically,<br>the second-order cone is also called a Lorentz cone or an ice-cream cone. The general<br>mathematical definition is cast as how can a SOC cone can be approximated into a set of linearizations. Mathematically,<br>the second-order cone is also called a Lorentz cone or an ice-cream cone. The general<br>mathematical definition is cast as<br> $L^n := \{(x, t) \in \mathbb$ how can a SOC cone can be approximated into a set of linearizations. Mathematically,<br>the second-order cone is also called a Lorentz cone or an ice-cream cone. The general<br>mathematical definition is cast as<br> $L^n := \{(x, t) \in \mathbb$ w can a SOC cone can be approximated into a set of linearizations. Mathematically,<br>
second-order cone is also called a Lorentz cone or an ice-ercam cone. The general<br>
athematical definition is cast as<br>  $L^n = \{(x, t) \in \mathbb{R}^$ Exations. Mathematically,<br>ream cone. The general<br>(2.59)<br>is displayed in Fig. 2.4.

$$
L^n := \{(x, t) \in \mathbb{R}^n \times \mathbb{R} \, \| \, |x|_2 \le t\}
$$
\n(2.59)

For instance, if  $n=2$ , then the geometry of a Lorentz cone  $L^2$  is displayed in Fig. 2.4.



Fig. 2.4 Geometry of a Lorentz cone  $L^2$  is displayed in Fig. 2.4.<br>
Fig. 2.4 Geometry of a Lorentz cone  $L^2$ .<br>
The feasible region of a second-order cone can be well-approximated by a nedral cone, as presented in Fig. 2 **Polyhedral cone, as presented in Fig. 2.5.** This polyhedral approximated by a polyhedral cone, as presented in Fig. 2.5. This polyhedral approximation makes a SOC constraint become a series of linear constraints. The app Solution and the constraints. The feasible region of a second-order cone can be well-approximated by a polyhedral cone, as presented in Fig. 2.5. This polyhedral approximation makes a solution become a series of linear co Fig. 2.4 Geometry of a Lorentz cone L<sup>2</sup>.<br>
The feasible region of a second-order cone can be well-approximated by a<br>
polyhedral cone, as presented in Fig. 2.5. This polyhedral approximation makes a<br>
SOC constraint become a Example region of a second-order cone can be well-approximated by a<br>
cone, as presented in Fig. 2.5. This polyhedral approximation makes a<br>
raint become a series of linear constraints. The approximation accuracy<br>
the numb Iyhedral cone, as presented in Fig. 2.5. This polyhedral approximation makes a<br>DC constraint become a series of linear constraints. The approximation accuracy<br>pends on the number of outer polyhedral linearizations.<br><br>Fig.



polyhedral cone *P* is an ε-approximation of  $\mathcal{F}$  if  $\mathcal{F} \subseteq P \subseteq \mathcal{F}_\varepsilon$ .<br>
Next, we seek to find an ε-approximation of  $\mathcal{F}$ . Recall that a 3-dimensional<br>
Lorentz cone  $L^2 = \{(x, t) \in \mathbb{R}^2 \times \mathbb{R} : ||x|| \le t\}$ . G polyhedral cone *P* is an *e*-approximation of  $\mathcal{F}$  if  $\mathcal{F} \subseteq P \subseteq \mathcal{F}_z$ .<br>
Next, we seek to find an *e*-approximation of  $\mathcal{F}$ . Recall that a 3-dimensional<br>
Lorentz cone  $L^2 = \{(x, t) \in \mathbb{R}^2 \times \mathbb{R} : ||x|| \le t\}$ .  $\mathcal{L} \subseteq \mathcal{F}_{\varepsilon}.$ <br>
Recall that a 3-dimensional<br>  $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \overline{t}) \in \mathbb{R}^3$ , how can<br>  $= 1, \tilde{x} \ge 0$ . Ideally, rotating the<br>
clear that  $\tilde{x} \in I^2$  iff  $\tilde{x} \le 1$ . If polyhedral cone *P* is an *e*-approximation of  $\mathcal{F}$  if  $\mathcal{F} \subseteq P \subseteq \mathcal{F}_z$ .<br>
Next, we seek to find an *e*-approximation of  $\mathcal{F}$ . Recall that a 3-dimensional<br>
Lorentz cone  $L^2 = \{(x, t) \in \mathbb{R}^2 \times \mathbb{R} : ||x|| \le t\}$ . determine this any point lies in  $L^2$ . Assume w.l.o.g.  $\bar{t} = 1$ ,  $\tilde{x} \ge 0$ . Ideally, rotating the mation of  $\mathcal{F}$  if  $\mathcal{F} \subseteq P \subseteq \mathcal{F}_\varepsilon$ .<br>
pproximation of  $\mathcal{F}$ . Recall that a 3-dimensional<br>  $\mathbb{R}: ||x|| \le t$ , Given  $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \overline{t}) \in \mathbb{R}^3$ , how can<br>
Assume w.l.o.g.  $\overline{t} = 1$ ,  $\tilde{x} \ge 0$ . Ideal point  $(\tilde{x}_1, \tilde{x}_2, 1)$  to  $(\tilde{x}_1', 0, 1)$  is shown in Fig. 2.6. It is clear that  $\tilde{x} \in L^2$  iff  $\tilde{x}_1' \le 1$ . If an *ε*-approximation of  $\mathcal{F}$  if  $\mathcal{F} \subseteq P \subseteq \mathcal{F}_\varepsilon$ .<br>
( find an *ε*-approximation of  $\mathcal{F}$ . Recall that a 3-dimensional  $\{(x,t) \in \mathbb{R}^2 \times \mathbb{R} : ||x|| \le t\}$ . Given  $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{t}) \in \mathbb{R}^3$ , how can The interaction of the same state<br>  $\tilde{x}_1 \leq 1$ . If  $x_1' \in L^2$  for polyhedral cone *P* is an *e*-approximation of  $\mathcal{F}$  if  $\mathcal{F} \subseteq P \subseteq \mathcal{F}_z$ .<br>
Next, we seek to find an *e*-approximation of  $\mathcal{F}$ . Recall that a 3-dimensional<br>
Lorentz cone  $L^2 = \{(x,t) \in \mathbb{R}^3 \times \mathbb{R} : ||x|| \le t\}$ . G  $\tilde{x}_1 \in L^2_{\varepsilon}$  for polyhedral cone *P* is an *e*-approximation of  $\mathcal{F}$  if  $\mathcal{F} \subseteq P \subseteq \mathcal{F}$ .<br>
Next, we seek to find an *e*-approximation of  $\mathcal{F}$ . Recall that a 3-dimensional<br>
Lorentz cone  $L^2 = \{(x,t) \in \mathbb{R}^2 \times \mathbb{R} : ||x|| \le t\}$ . Gi First  $\mathcal{F} \subseteq P \subseteq \mathcal{F}_\varepsilon$ .<br>  $\text{in } \mathcal{F} \subseteq P \subseteq \mathcal{F}_\varepsilon$ .<br>  $\text{Given } \tilde{x} = (\tilde{x}_1, \tilde{x}_2, \overline{t}) \in \mathbb{R}^3$ , how can<br>  $\text{in } \mathcal{F} \subseteq \{1, \tilde{x}_2\} \cup \{1, \tilde{x}_1\} \cup \{1, \tilde{x}_1\}$ .<br>  $\text{in } \mathcal{F} \subseteq \mathcal{F}_\varepsilon$  and  $\tilde{x}_1 \in \mathcal{F}_\$ of  $\mathcal{F}$ . Recall that a 3-dimensional<br>
iiven  $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \overline{t}) \in \mathbb{R}^3$ , how can<br>
1.g.  $\overline{t} = 1, \tilde{x} \ge 0$ . Ideally, rotating the<br>
6. It is clear that  $\tilde{x} \in L^2$  iff  $\tilde{x}_1 \le 1$ . If<br>
whether the rotate ven  $x = (x_1, x_2, t) \in \mathbb{R}^{\infty}$ , now can<br>  $\overline{t} = 1, \tilde{x} \ge 0$ . Ideally, rotating the<br>
It is clear that  $\tilde{x} \in L^2$  iff  $\tilde{x}_1 \le 1$ . If<br>
whether the rotated point  $\tilde{x}_1 \in L^2$  for<br>
very close to 0.<br>  $\tilde{x}_1, \tilde{x}_2$ )<br>



2 a set of  $\sim$  3 a set of  $\sim$ 

appropriate small  $\varepsilon$ , since the component  $\hat{x}_i$  is very close to 0.<br>  $\begin{pmatrix} \hat{x}_i, \hat{x}_2 \\ \hat{x}_i, \hat{x}_2 \end{pmatrix}$ <br>  $\begin{pmatrix} \hat{x}_i, \hat{x}_2 \\ \hat{x}_i, 0 \end{pmatrix}$ <br>  $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ <br>  $\begin{pmatrix} x_1 \\ y_2 \end{pmatrix}$ <br>  $\begin{pmatrix} x_1 \\ y_2 \end{pmatrix}$ <br>  $\begin$ [91]. For  $\sqrt{x_1^2 + x_2^2} \le t$  can be approximated by a system of linear homogeneous <sup>22</sup><br>
1  $(\tilde{x}_1, \tilde{x}_2)$ <br>
1  $(\tilde{x}_1, 0)$ <br>
and  $\alpha t = 1$ <br>
Fig. 2.6 *c*-approximation of a Lorentz cone *L*<sup>2</sup>.<br>
and approximation method is proposed by Ben-Tal and Nemirovski<br>  $x_1^2 + x_2^2 \le t$  can be approximated by a syste equalities and inequalities in terms of  $x_1$ ,  $x_2$ , , and 2( $v+1$ ) variables  $c^2$ ,  $\pi^2 + x_2^2 \le t$  can be approximated by a system of linear homogeneous equalities and inequalities in terms of  $x_1$ ,  $x_2$ ,  $t$ , and 2(  $\begin{array}{c}\n x_1, x_2\n\end{array}$ <br>  $\begin{array}{c}\n x_1 \\
 \hline\n x_2\n\end{array}$ <br>  $\begin{array}{c}\n x_2 \\
 \hline\n x_3\n\end{array}$ <br>  $\begin{array}{c}\n x_1 \\
 \hline\n x_2\n\end{array}$ <br>
This polyhedral approximation method is proposed by Ben-Tal and Nemirovski<br>
91]. For  $\sqrt{x_1^2 + x_2^2} \le t$  can **a**<br>
Fig. 2.6  $\varepsilon$ -approximation of a Lorentz cone  $L^2$ .<br>
This polyhedral approximation method is proposed by Ben-Tal and Nemirovski<br>
[91]. For  $\sqrt{x_1^2 + x_2^2} \le t$  can be approximated by a system of linear homogeneous<br> Fig. 2.6 e-approximation of a Lorentz cone  $L^2$ .<br>
This polyhedral approximation method is proposed by Ben-Tal and Ne<br>
1]. For  $\sqrt{x_1^2 + x_2^2} \le t$  can be approximated by a system of linear homeological<br>  $= 0,1,2,...$  where v g. 2.6 e-approximation of a Lorentz cone  $L^2$ .<br>
broximation method is proposed by Ben-Tal and Nemirovski<br>
t can be approximated by a system of linear homogeneous<br>
ities in terms of  $x_1, x_2, t$ , and  $2(v+1)$  variables  $\varsigma$ This polyhedral approximation method is proposed by Ben-Tal and Nemirovski<br>
[91]. For  $\sqrt{x_1^2 + x_2^2} \le t$  can be approximated by a system of linear homogeneous<br>
equalities and inequalities in terms of  $x_1, x_2, t$ , and  $2(\$ method is proposed by Ben-Tal and Nemirovski<br>proximated by a system of linear homogeneous<br>
of  $x_1, x_2, t$ , and  $2(v+1)$  variables  $\varsigma^j$ ,  $\eta^j$  for<br>
parameter of the polyhedral  $\varepsilon(v)$  relaxed<br>  $(v) = \frac{1}{\cos(\frac{\pi}{2^{v+1}})} - 1$ 

$$
\varepsilon(v) = \frac{1}{\cos(\frac{\pi}{2^{v+1}})} - 1
$$
\n(2.60)

This gives  $\varepsilon(v) \approx 3 \times 10^{-7}$  when  $v=11$ ; the relaxed approximation in  $(1+\varepsilon)t \ge \sqrt{x_1^2 + x_2^2}$  will have  $(1+\varepsilon(v))^2 - 1 \approx 6 \times 10^{-7}$ . The system of linear

homogeneous equations is given by the following polyhedron *T*.  
\n
$$
T := \{(x_1, x_2, \varsigma^0, \eta^0, ..., \varsigma^v, \eta^v, t) : \varsigma^0 \ge |x_1| \eta^0 \ge |x_2| \}
$$
\n
$$
\frac{\tau^j}{\varsigma^j} = \cos(\frac{\pi}{2^{j+1}}) \varsigma^{j-1} + \sin(\frac{\pi}{2^{j+1}}) \eta^{j-1}, \quad j = 1, ..., v
$$
\n
$$
\eta^j \ge -\sin(\frac{\pi}{2^{j+1}}) \varsigma^{j-1} + \cos(\frac{\pi}{2^{j+1}}) \eta^{j-1}, \quad j = 1, ..., v
$$
\n
$$
\eta^j \ge \sin(\frac{\pi}{2^{j+1}}) \varsigma^{j-1} - \cos(\frac{\pi}{2^{j+1}}) \eta^{j-1}, \quad j = 1, ..., v
$$
\n
$$
\varsigma^v \le t \eta^v \le \tan(\frac{\pi}{2^{v+1}}) \varsigma^v
$$
\nwhere  $(\xi_0, \eta_0)$  rotates  $(x_1, x_2)$  so that  $x_1, x_2 \ge 0$ , and we have two additional variables  $\xi_j$ ,  
\n $\eta_j$  for each iteration  $j = 1, ..., v$ .  
\nThe polyhedral approximation given by (2.61) can be reduced by using the linear equality constraints in  $\varsigma^j = \cos(\frac{\pi}{2^{j+1}}) \varsigma^{j-1} + \sin(\frac{\pi}{2^{j+1}}) \eta^{j-1}, \quad j = 1, ..., v$  to solve for  $\varsigma^j$ ,  $j = 1, ..., v$  in terms of  $\varsigma^0$ ,  $\eta^{j-1}$ ,  $j = 1, ..., v$  and then substitute  $\varsigma^j$  out of the system (2.61) by this linear equality constraint. The resulting system will only have linear

 $\cos(\frac{\pi}{2^{j+1}})\zeta^{j-1} + \sin(\frac{\pi}{2^{j+1}})\eta^{j}$  $j = \cos(\frac{\pi}{2}) e^{j-1} + \sin(\frac{\pi}{2}) n^{j-1}$  $\frac{1}{j+1}$  /  $\frac{1}{j+1}$  +  $\frac{1}{2j+1}$  $\varsigma^{j} = \cos(\frac{\pi}{2^{j+1}}) \varsigma^{j-1} + \sin(\frac{\pi}{2^{j+1}}) \eta^{j-1}$  $j = 1,...,v$  in terms of  $\zeta^0, \eta^{j-1}, j=1,...,v$  and  $\eta^j \ge -\sin(\frac{\pi}{2^{j+1}}) \xi^{j+1} + \cos(\frac{\pi}{2^{j+1}}) \eta^{j+1}, \quad j = 1,..., v$  (2.61)<br>  $\eta^j \ge \sin(\frac{\pi}{2^{j+1}}) \xi^{j-1} - \cos(\frac{\pi}{2^{j+1}}) \eta^{j-1}, \quad j = 1,..., v$ <br>  $\xi^* \le t$ <br>  $\eta^* \le \tan(\frac{\pi}{2^{j+1}}) \xi^*$ <br>
where  $(\xi_0, \eta_0)$  rotates  $(x_1, x_2)$  so that  $x_1$  $\int_{0}^{1} \frac{z}{2^{j+1}} e^{-j\frac{\pi}{2}} + \cos(\frac{\pi}{2^{j+1}})\eta^{j-1}$ ,  $j = 1,...,v$  (2.61)<br>  $\int_{0}^{1} \frac{z}{2^{j+1}} e^{-j\frac{\pi}{2}} - \cos(\frac{\pi}{2^{j+1}})\eta^{j-1}$ ,  $j = 1,...,v$ <br>  $\int_{0}^{v} \frac{z}{2^{j+1}} e^{-j\frac{\pi}{2}} e^{-j\frac{\pi}{2}}$ <br>  $\int_{0}^{v} \frac{z}{2^{j+1}} e^{-j\frac{\pi}{2}} e^{-j\frac{\pi}{2}}$ <sup>1</sup>,  $j = 1,...,v$  (2.61)<br>  $j = 1,...,v$  (2.61)<br>
ditional variables  $\xi_j$ ,<br>
discussed by using the linear<br>  $1,...,v$  to solve for<br>  $\zeta^j$  out of the system<br>
vill only have linear<br>
the  $(v+1)$  variables  $\eta' \ge \sin(\frac{\pi}{2^{j+1}})z^{j+1} - \cos(\frac{\pi}{2^{j+1}})\eta^{j+1}, \quad j = 1,..., \nu$ <br>  $\zeta' \le t$ <br>  $\eta' \le \tan(\frac{\pi}{2^{j+1}})z^{\nu}$ <br>
where  $(\zeta_0, \eta_0)$  rotates  $(x_1, x_2)$  so that  $x_1, x_2 \ge 0$ , and we have two additional variables  $\zeta_j$ ,<br>  $\eta_j$  for each  $\zeta^* \le t$ <br>  $\eta^* \le \tan(\frac{\pi}{2^{s+1}}) \zeta^*$ <br>
where  $(\zeta_0, \eta_0)$  rotates  $(x_1, x_2)$  so that  $x_1, x_2 \ge 0$ , and we have two additional variables  $\zeta_j$ ,<br>  $\eta_j$  for each iteration  $j=1, \dots, v$ .<br>
The polyhedral approximation given 1 we have two additional variables  $\xi_j$ ,<br>
can be reduced by using the linear<br>  $\frac{\pi}{2^{j+1}}$ ,  $\eta^{j-1}$ ,  $j=1,..., \nu$  to solve for<br>
then substitute  $\zeta^j$  out of the system<br>
ulting system will only have linear<br>  $x_1, x_2, x_3$ where  $(\xi_0, \eta_0)$  rotates  $(x_1, x_2)$  so that  $x_1, x_2 \ge 0$ , and we have two additional variables  $\xi_j$ ,<br>  $\eta_j$  for each iteration  $j=1, \dots, v$ .<br>
The polyhedral approximation given by  $(2.61)$  can be reduced by using the li *n<sub>b</sub>* for each iteration *j*=1, ..., v.<br>
The polyhedral approximation given by (2.61) can be reduced by using the linear<br>
equality constraints in  $\zeta^j = \cos(\frac{\pi}{2^{j+1}})\zeta^{j-1} + \sin(\frac{\pi}{2^{j+1}})\eta^{j-1}$ ,  $j=1,...,v$  to solve for<br> be reduced by using the linear<br>  $|\eta^{j-1} \rangle$ ,  $j=1,...,v$  to solve for<br>
substitute  $\zeta^j$  out of the system<br>
ng system will only have linear<br>  $x_1, x_3, \zeta^0$  and the  $(v+1)$  variables<br>
inequalities (2.61) is known as<br>
. Inspi The polyhedral approximation given by (2.61) can be reduced by using the linear<br>equality constraints in  $\zeta^j = \cos(\frac{\pi}{2^{j+1}}) \zeta^{j-1} + \sin(\frac{\pi}{2^{j+1}}) \eta^{j-1}$ ,  $j=1,...,v$  to solve for<br> $\zeta^j$ ,  $j = 1,...,v$  in terms of  $\zeta^0$ , using the linear<br>
., v to solve for<br>
only have linear<br>
( $v+1$ ) variables<br>
61) is known as<br>
is idea, we can<br>
. Assume w.l.o.g<br>
ulation for<br>
(2.62) equality constraints in  $\varsigma^{i} = \cos(\frac{\pi}{2^{j+1}})s^{i-1} + \sin(\frac{\pi}{2^{j+1}})\eta^{i-1}$ ,  $j=1,...,v$  to solve for  $\varsigma^{i}$ ,  $j = 1,...,v$  in terms of  $\varsigma^{0}$ ,  $\eta^{i-1}$ ,  $j=1,...,v$  and then substitute  $\varsigma^{i}$  out of the system (2.61) by this

the polyhedral approximation for a Lorentz cone  $L^2$ . Inspired by this idea, we can extend this polyhedral approximation for an arbitrary Lorentz cone  $L^n$ . Assume w.l.o.g  $n = 2^k$  for some  $k \in \mathbb{Z}$ . We show that we can give an extended formulation for 61) by this linear equality constraint. The resulting system will only have linear<br>equality constraints in terms of the variables  $x_1, x_2, x_3, \zeta^0$  and the  $(\nu+1)$  variables<br> $(\pi)^i$  for  $j=0,..., \nu$ .<br>The system of linear ho inequality constraints in terms of the variables  $x_1, x_2, x_3, c^0$  and the  $(v+1)$  variables<br>for  $\eta^j$  for  $j=0,...,v$ .<br>The system of linear homogeneous equalities and inequalities (2.61) is known as<br>the polyhedral approxima

$$
L^n := \left\{ (x_{0,1}, \dots, x_{0,n}, t) \in \mathbb{R}^n \times \mathbb{R} \middle\| \left( x_{0,1}, \dots, x_{0,n} \right) \right\|_2 \le t \right\}
$$
\n(2.62)

 $||(x_{0,2j-1}, x_{0,2j})||_2 \leq x_{1,j}$ , where  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})$ .  $||(x_{0,2j-1}, x_{0,2j})||_2 \le x_{1,j}$ , where  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})$ .<br>Then, introducing "2nd" level variables  $x_{2,j}$  is subject to  $||(x_{1,2j-1}, x_{1,2j})||_2 \le x_{2,j}$  in a similar way. We the  $\text{rule for a pair } (x_{0,2j-1}, x_{0,2j}) \,.$ <br>  $||(x_{1,2j-1}, x_{1,2j})||_2 \leq x_{2,j} \quad \text{in a}$ <br>  $\text{rule} \text{1 in } i\text{-th level variable has as}$ <br>
preserved up to the level  $||(x_{0,2j-1}, x_{0,2j})||_2 \le x_{1,j}$ , where  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})$ .<br>Then, introducing "2nd" level variables  $x_{2,j}$  is subject to  $||(x_{1,2j-1}, x_{1,2j})||_2 \le x_{2,j}$  in a<br>similar way. We the  $||(x_{0,2j+1}, x_{0,2j})||_2 \le x_{1,j}$ , where  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})$ .<br>Then, introducing "2nd" level variables  $x_{2,j}$  is subject to  $||(x_{1,2j+1}, x_{1,2j})||_2 \le x_{2,j}$  in a<br>similar way. We the  $\|(x_{0,2j+1}, x_{0,2j})\|_2 \le x_{1,j}$ , where  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j+1}, x_{0,2j})$ .<br>Then, introducing "2nd" level variables  $x_{2,j}$  is subject to  $\|(x_{1,2j+1}, x_{1,2j})\|_2 \le x_{2,j}$  in a similar way. We  $log_2(n)$ , which has a single node  $x_{log_2(n)} = t$ . At each level of the tree, we have an a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})$ .<br>
ables  $x_{2,j}$  is subject to  $||(x_{1,2j-1}, x_{1,2j})||_2 \le x_{2,j}$  in a<br>
a binary tree, where each *i*-th level variable has as<br>
s. The same structure is preserved up to th  $||(x_{0,2j+1}, x_{0,2j})||_2 \le x_{1,j}$ , where  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})$ .<br>Then, introducing "2nd" level variables  $x_{2,j}$  is subject to  $||(x_{1,2j-1}, x_{1,2j})||_2 \le x_{2,j}$  in a similar way. We the  $\epsilon$ -approximation of the  $L^2$  cones with the construction from the previous section. This e  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})$ .<br>
evel variables  $x_{2,j}$  is subject to  $||(x_{1,2j-1}, x_{1,2j})||_2 \le x_{2,j}$  in a<br>
construct a binary tree, where each *i*-th level variable has as<br>
variables.  $\|(x_{0,2j-1}, x_{0,2j})\|_2 \le x_{1,j}$ , where  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})\|_2 \le x_{1,j}$ .<br>
Then, introducing "2nd" level variables  $x_{2,j}$  is subject to  $\|(x_{1,2j-1}, x_{1,2j})\|_2 \le x_{2,j}$ <br>
similar  $\sum_{i,j}$ , where  $x_{i,j}$  is a new introduced variable for a pair  $(x_{0,2j+1}, x_{0,2j})$ .<br>
19 "2nd" level variables  $x_{2,j}$  is subject to  $||(x_{1,2j+1}, x_{1,2j})||_2 \le x_{2,j}$  in a<br>
therefore construct a binary tree, where each *i*-th  $\mathfrak n$  $||x_1, x_{0,2}||_2 \le x_{1,j}$ , where  $x_{1,j}$  is a new introduced variable for a pair  $(x_{0,2j-1}, x_{0,2j})$ .<br>  $||x_1||_2 \le x_{1,j}$ , introducing "2nd" level variables  $x_{2,j}$  is subject to  $||(x_{1,2j-1}, x_{1,2j})||_2 \le x_{2,j}$  in a<br>  $||x_1||_2 \le x$ 1, introducing "2nd" level variables  $x_{2,j}$  is subject to  $||(x_{1,2j-1}, x_{1,2j})||_2 \le x_{2,j}$  in a<br>
ar way. We therefore construct a binary tree, where each *i*-th level variable has as<br>
ren two (*i*-1)-th level variables. The ables. The same structure is preserved up to the level<br>
de  $x_{\log_2(n)} = t$ . At each level of the tree, we have an<br>
s with the construction from the previous section. This<br>
-approximation for  $L^n$ .<br>
mplify this idea by<br>  $x_{0,$ ariables. The same structure is preserved up to the level<br>
node  $x_{\text{log}_2(n)} = t$ . At each level of the tree, we have an<br>
nes with the construction from the previous section. This<br>
-1)-approximation for  $L^n$ .<br>
xemplify this ame structure is preserved up to the level<br>  $t$ . At each level of the tree, we have an<br>
anstruction from the previous section. This<br>
tion for  $L^n$ .<br>
dea by<br>  $\mathbb{R} \left\| \left( x_{0,1}, x_{0,2}, x_{0,3}, x_{0,4} \right) \right\|_2 \le t \right\}$  (2.63)<br>

$$
L^4 := \left\{ (x_{0,1}, x_{0,2}, x_{0,3}, x_{0,4}, t) \in \mathbb{R}^4 \times \mathbb{R} \middle\| | (x_{0,1}, x_{0,2}, x_{0,3}, x_{0,4}) ||_2 \le t \right\}
$$
 (2.63)

approximation of the 
$$
L^2
$$
 cones with the construction from the previous section. This  
\nves in total a  $( (1 + \varepsilon)^{\log_2(n)} - 1)$ -approximation for  $L^n$ .

\nFor explanation, let us exemplify this idea by

\n
$$
L^4 := \left\{ (x_{0,1}, x_{0,2}, x_{0,3}, x_{0,4}, t) \in \mathbb{R}^4 \times \mathbb{R} \middle\| |(x_{0,1}, x_{0,2}, x_{0,3}, x_{0,4})| \geq \varepsilon \right\}
$$
\n(2.63)

\nThe approximation process can be summarized in Fig. 2.7.

\n1<sup>n</sup> level

\n
$$
\frac{x_{0,1}}{\left(x_{0,1}^2 + x_{0,2}^2\right)^{1/2} \leq x_{1,1}} \quad \frac{x_{0,2}}{\left(x_{0,3}^2 + x_{0,4}^2\right)^{1/2} \leq x_{1,2}} \times \frac{x_{0,3}}{x_{1,1}} \quad \frac{x_{0,4}}{\left(x_{1,1}^2 + x_{1,2}^2\right)^{1/2} \leq x_{1,3}} \times \frac{x_{0,4}}{x_{1,2}}
$$
\n2<sup>n4</sup> level

\n
$$
\frac{x_{1,1}}{\left(x_{1,1}^2 + x_{1,2}^2\right)^{1/2} \leq t} \times \frac{x_{1,2}}{x_{2,1} - t}
$$
\nFig. 2.7  $\varepsilon$ -approximation flow chart of a Lorentz cone L<sup>2</sup>.

\nConsider the following SOCP problem by the polyhedral approximation method.

\n
$$
\min x_1 + x_2 + 3x_3
$$
\n
$$
x_1, x_2 + 2x_3 = 3
$$
\n(2.64)

$$
2^{nd} \text{ level}
$$
\n
$$
2^{nd} \text{ level}
$$
\n
$$
2^{n} \left( \frac{(x_{1,1}^2 + x_{1,2}^2)^{1/2} \le t}{x_{2,1} - t} \right)
$$
\n
$$
x_{3,1} = t
$$
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$$
x_{4,1} = t
$$
\n
$$
x_{5,1} = t
$$
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x_{6,1} = t
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x_{7,1} = t
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x_{8,1} = t
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x_{9,1} = t
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x_{11} = t
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\n<

SOC constraint satisfies with equality as observed by the gap error 3.1666×10<sup>-7</sup>.<br>Subsequently, we linearize this SOC constraint with respect to  $x_1, x_2, x_3$ <br>and 2(3+1)=8 variables  $\varsigma^0$ ,  $\varsigma^1$ ,  $\varsigma^2$ ,  $\varsigma^3$ ,  $\$ SOC constraint satisfies with equality as observed by the gap error 3.1666×10<sup>-7</sup>.<br>
Subsequently, we linearize this SOC constraint with respect to  $x_1, x_2, x_3$ <br>
and 2(3+1)=8 variables  $\zeta^0, \zeta^1, \zeta^2, \zeta^3, \eta^0, \eta^1, \$ SOC constraint satisfies with equality as observed by the gap error 3.1666×10<sup>-7</sup>.<br>
Subsequently, we linearize this SOC constraint with respect to  $x_1, x_2, x_3$ <br>
and 2(3+1)=8 variables  $\zeta^0, \zeta^1, \zeta^2, \zeta^3, \eta^0, \eta^1, \$ 

$$
\begin{cases}\n-g^0 \le x_1 \le g^0 \\
-\eta^0 \le x_2 \le \eta^0 \\
g^1 - \cos(\frac{\pi}{2^2})g^0 - \sin(\frac{\pi}{2^2})\eta^0 = 0 \\
g^2 - \cos(\frac{\pi}{2^3})g^1 - \sin(\frac{\pi}{2^3})\eta^1 = 0 \\
g^3 - \cos(\frac{\pi}{2^4})g^2 - \sin(\frac{\pi}{2^4})\eta^2 = 0\n\end{cases}
$$

$$
\begin{vmatrix}\n\zeta^3 - \cos(\frac{\pi}{2})\zeta^2 - \sin(\frac{\pi}{2})\eta^2 = 0 \\
\sin(\frac{\pi}{2^2})\zeta^0 + \cos(\frac{\pi}{2^2})\eta^0 - \eta^1 \le 0 \\
\sin(\frac{\pi}{2^2})\zeta^0 - \cos(\frac{\pi}{2^2})\eta^0 - \eta^1 \le 0 \\
-\sin(\frac{\pi}{2^3})\zeta^1 + \cos(\frac{\pi}{2^3})\eta^1 - \eta^2 \le 0 \\
\sin(\frac{\pi}{2^3})\zeta^1 - \cos(\frac{\pi}{2^3})\eta^1 - \eta^2 \le 0\n\end{vmatrix} = \begin{cases}\n\zeta^3 \le x_3 \\
\zeta^3 \le \tan(\frac{\pi}{2^4})\zeta^3 \end{cases}
$$
\n
$$
\begin{vmatrix}\n\sin(\frac{\pi}{2^3})\zeta^1 - \cos(\frac{\pi}{2^3})\eta^1 - \eta^2 \le 0 \\
-\sin(\frac{\pi}{2^4})\zeta^2 + \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \le 0\n\end{vmatrix} = \begin{cases}\n\eta^3 \le \tan(\frac{\pi}{2^4})\zeta^3 \end{cases}
$$
\n
$$
\begin{cases}\n\sin(\frac{\pi}{2^4})\zeta^2 - \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \le 0 \\
\sin(\frac{\pi}{2^4})\zeta^2 - \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \le 0\n\end{cases}
$$
\nThe vector of optimization variables is  $x = [x_1, x_2, x_3, \zeta^0, \zeta^1, \zeta^2, \zeta^3, \eta^0, \eta^1, \eta^2, \eta^3]^T$ , the total of which is  $3+2(3+1)=11$ . By solving this linear programming model, the minimum objective value is  $3.8787$  and the optimal vector  $x=(0.6213, 0.6213, 0.8787)^T$  is achieved. The SOC constraint satisfies inequality by the gap error -0.0301.

The vector of optimization variables is  $x=[x_1, x_2, x_3, \zeta^0, \zeta^1, \zeta^2, \zeta^3, \eta^0, \eta^1, \eta^2, \eta^3]^T$ ,  $\sin(\frac{\pi}{2})s$ ,  $+\cos(\frac{\pi}{2})\eta^0 - \eta^1 \le 0$ <br>  $\sin(\frac{\pi}{2})\zeta^1 + \cos(\frac{\pi}{2})\eta^1 - \eta^2 \le 0$ <br>  $-\sin(\frac{\pi}{2})\zeta^1 + \cos(\frac{\pi}{2})\eta^1 - \eta^2 \le 0$ <br>  $\sin(\frac{\pi}{2})\zeta^1 - \cos(\frac{\pi}{2})\eta^1 - \eta^2 \le 0$ <br>  $\sin(\frac{\pi}{2})\zeta^2 - \cos(\frac{\pi}{2})\eta^2 - \eta^3 \le 0$ <br>  $-\sin(\frac{\pi}{2})\zeta^2 + \$  $(0.8787)^T$  is achieved. The SOC constraint satisfies inequality by the gap error -0.0301.  $\begin{aligned}\n\left\{\n\begin{aligned}\n\sin(\frac{\pi}{2})s & -\cos(\frac{\pi}{2})\eta - \eta \leq 0 \\
-\sin(\frac{\pi}{2})s^2 + \cos(\frac{\pi}{2})\eta - \eta^2 \leq 0\n\end{aligned}\n\right.\n\left\{\n\begin{aligned}\n\int_0^s & \leq x, \\
\sin(\frac{\pi}{2})s^2 - \cos(\frac{\pi}{2})\eta - \eta^2 \leq 0\n\end{aligned}\n\right.\n\left\{\n\begin{aligned}\n\int_0^s & \leq \tan(\frac{\pi}{2})s^3 \cdot (2.65) \\
\int_0^s & = \sin(\frac{\pi}{$  $\int \sin(\frac{\pi}{2^3})\zeta^2 - \cos(\frac{\pi}{2^3})\eta - \eta \le 0$ <br>  $\int \sin(\frac{\pi}{2^4})\zeta^3 - \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \le 0$ <br>  $\int \sin(\frac{\pi}{2^4})\zeta^2 + \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \le 0$ <br>
The vector of optimization variables is  $x = [x_1, x_2, x_3, \zeta^0, \zeta^1, \zeta^2, \zeta^3, \eta^0$  $\left\{\n\begin{aligned}\n\sin(\frac{\pi}{2^3})s^2 - \cos(\frac{\pi}{2^3})\eta^2 - \eta^3 \leq 0 \\
-\sin(\frac{\pi}{2^4})s^2 + \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \leq 0\n\end{aligned}\n\right.\n\left.\n\left.\n\begin{aligned}\n\sin(\frac{\pi}{2^4})s^2 + \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \leq 0 \\
\sin(\frac{\pi}{2^4})s^2 - \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \leq 0\n\end{aligned}\n\right.\n\left.\n\text{The vector of optimization$  $\begin{aligned}\n&\begin{aligned}\n&-\sin(\frac{\pi}{2^4})\zeta^2 + \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \leq 0 \\
&\sin(\frac{\pi}{2^4})\zeta^2 - \cos(\frac{\pi}{2^4})\eta^2 - \eta^3 \leq 0\n\end{aligned}\n\end{aligned}$ The vector of optimization variables is  $x = [x_1, x_2, x_3, c^0, c^1, c^2, c^3, \eta, c^4, c^2, c^3, \eta, c^4, c^2, c^3, \eta, c$ 

2.6 Case Study<br>2.6.1 Simple 6-node DN<br>The following simple 6-node DN is used to exemplify this SOC 2.6.1 Simple 6-node DN<br>
2.6.1 Simple 6-node DN<br>
The following simple 6-node DN is used to exemplify this SOCP-based<br>
formulation of *DistFlow* equations. The network topology is shown in Fig. 2.8. The 5.1 Simple 6-node DN<br>The following simple 6-node DN is used to exemplify this SOCP-based<br>mulation of DistFlow equations. The network topology is shown in Fig. 2.8. The<br>de 1 is the PCC bus, while nodes 2-6 are PQ buses. In 2.6 Case Study<br>2.6.1 Simple 6-node DN<br>The following simple 6-node DN is used to exemplify this SOCP-based<br>formulation of *DistFlow* equations. The network topology is shown in Fig. 2.8. The<br>node 1 is the PCC bus, while no 2.6 Case Study<br>
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2.6.1 Simple 6-node DN<br>
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1.0.00de 1. is the PCC bus, while n 2.6.1 Simple 6-node DN<br>
The following simple 6-node DN is used to exemplify this SOCP-based<br>
formulation of *DistFlow* equations. The network topology is shown in Fig. 2.8. The<br>
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2.6.1 Simple 6-node DN<br>
1. The following simple 6-node DN is used to exemplify this SOCP-based<br>
formulation of *DistFlow* equations. The network topology is shown in Fig. 2.8. The<br>
node 1 is the PCC bus, w 2.6.1 Simple 6-node DN<br>
The following simple 6-node DN is used to exemplify this SOCP-based<br>
formulation of *DistFlow* equations. The network topology is shown in Fig. 2.8. The<br>
node 1 is the PCC bus, while nodes 2-6 are 2.6.1 Simple 6-node DN<br>The following simple 6-node DN is used to exemplify this SOCP-based<br>formulation of *DistFlow* equations. The network topology is shown in Fig. 2.8. The<br>node 1 is the PCC bus, while nodes 2-6 are PQ The following simple 6-node DN is used to exemplify this SOCP-based<br>formulation of *DistFlow* equations. The network topology is shown in Fig. 2.8. The<br>node 1 is the PCC bus, while nodes 2-6 are PQ buses. In other words,<br>



 $\frac{1}{\sqrt{5}}\sqrt{2}$ <br>  $\frac{3}{2}$ <br>  $\frac{$ 

in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerci in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerci in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerci in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerci in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerci in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerci in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerci in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerc in our designed MATDNR Toolbox v1.0 [92] to run this SOCP-based reactive power<br>optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commerc optimization model (2.18) with commercial solvers MOSEK [93], Baron [94] and<br>SDP-based reactive power optimization model (2.57) by the commercial solver<br>SeduMi [95]. Please note that no overlapping variables are allowed fo SDP-based reactive power optimization model (2.57) by the commercial solver<br>SeduMi [95]. Please note that no overlapping variables are allowed for all SOC<br>constraints, and thus more new variables are included for the solve SeduMi [95]. Please note that no overlapping variables are allowed for all SOC constraints, and thus more new variables are included for the solver MOSEK. More information can be found at the handbook of MOSEK online. We a constraints, and thus more new variables are included for the solver MOSEK. More<br>information can be found at the handbook of MOSEK online. We also employ the<br>reactive power optimization solver based on conventional power f information can be found at the handbook of MOSEK online. We also employ the<br>reactive power optimization solver based on conventional power flow equations in<br>polar coordinates (Varopt) [96], and the solving algorithm is in reactive power optimization solver based on conventional power flow equations in<br>polar coordinates (Varopt) [96], and the solving algorithm is interior point algorithm.<br>To compare the accuracy of the polyhedral method, we figure. edually of the polyhedral method, we solve this feachive power<br>
all with polyhedral approximations with 32 segments of<br>
1. The nodal voltage profiles by SOCP-based and polyhedral<br>
1. The nodal voltage profiles by SOCP-base arizations by commercial SOCP solvers MOSEK and Baron. The solution can be<br>
d in Table 2.1. The nodal voltage profiles by SOCP-based and polyhedral<br>
oximation formulations of reactive power optimization models can be prese in<br>
Society MOSEK and Baron. The solution can be<br>
dal voltage profiles by SOCP-based and polyhedral<br>
reactive power optimization models can be presented<br>
re x-axis refers to branches. For instance, node (1,2)<br>
refers to b CP solvers MOSEK and Baron. The solution can be<br>
1 voltage profiles by SOCP-based and polyhedral<br>
eactive power optimization models can be presented<br>
2 x-axis refers to branches. For instance, node (1,2)<br>
fers to branch 2 al voltage profiles by SOCP-based and polyhedral<br>reactive power optimization models can be presented<br>ex-axis refers to branches. For instance, node (1,2)<br>fers to branch 2, node (3,4) refers to branch 3 in this<br>point of th FILM SOCP-based and polyhedral<br>
Solution can be presented<br>
Solution models can be presented<br>
Solution models can be presented<br>
Solution models can be presented<br>
Solution Connect 1,2)<br>
2, node (3,4) refers to branch 3 in t OSEK and Baron. The solution can be<br>
files by SOCP-based and polyhedral<br>
optimization models can be presented<br>
to branches. For instance, node (1,2)<br>
2, node (3,4) refers to branch 3 in this<br>
sof the 6-node DN.<br>
Real Power ution can be<br>
1 polyhedral<br>
be presented<br>  $\therefore$ , node (1,2)<br>
nch 3 in this<br>
<br>
Computational<br>
Time (seconds)<br>
<br>
0.2030<br>
0.1895<br>
0.0469

		pproximation formulations of reactive power optimization models can be presented			
		in a tree-shaped Fig. 2.9, where x-axis refers to branches. For instance, node $(1,2)$			
		efers to branch 1, node $(2,3)$ refers to branch 2, node $(3,4)$ refers to branch 3 in this			
igure.					
		Table 2.1 Optimal solutions of the 6-node DN.			
Models	Solvers	Minimal injected real power at PCC node	Real Power Loss $(p.u.)$	Algorithm Iterations	Computational Time (seconds)
	<b>MOSEK</b>	(p.u.) 1.501	0.101453	10	0.2030
<b>SOCP</b>	Baron	1.501	0.101453	10	0.1895
	Varopt	1.501	0.101453	7	0.0469
<b>SDP</b>	SeduMi	1.501	0.101453	16	0.6406
Polyhedral	<b>MOSEK</b>	1.5005	0.1005	16	0.3280
		$47\,$			



 $\frac{2}{3}$ <br>  $\frac{2}{3}$ <br> **EXECUTE:** 1<br>  $\frac{56}{5}$ <br>  $\frac{9.96}{1}$ <br>  $\frac{1}{1.5}$ <br>  $\frac{2}{2.5}$ <br>  $\frac{3}{3.5}$ <br>  $\frac{4}{4}$ <br>
Fig. 2.9 Tree-shaped voltage profile of this simple 6-node DN.<br>
Observing Table 2.1 tells us that the commercial solvers MOSEK, Bar **Power Fig. 2.9 Tree-shaped voltage profile of this simple 6-node DN.**<br> **Powering Table 2.1 tells us that the commercial solvers MOSEK, Baron and ScduMi can be effective to solve the SOCP-based/semi-definite-based reactiv** Social and Fig. 2.9 Tree-shaped voltage profile of this simple 6-node DN.<br>
Social and Fig. 2.9 Tree-shaped voltage profile of this simple 6-node DN.<br>
Observing Table 2.1 tells us that the commercial solvers MOSEK, Baron a Fig. 2.9 Tree-shaped voltage profile of this simple 6-node DN.<br>
Observing Table 2.1 tells us that the commercial solvers MOSEK, Baron and<br>
SeduMi can be effective to solve the SOCP-based/semi-definite-based reactive power Observing Table 2.1 tells us that the commercial solvers MOSEK, Baron and SeduMi can be effective to solve the SOCP-based/semi-definite-based reactive power optimization model, since the optimal solutions including power Observing Table 2.1 tells us that the commercial solvers MOSEK, Baron and<br>ScduMi can be effective to solve the SOCP-based/semi-definite-based reactive power<br>optimization model, since the optimal solutions including power l SeduMi can be effective to solve the SOCP-based/semi-definite-based reactive power<br>optimization model, since the optimal solutions including power loss and voltage<br>profiles are the same with ones by Varopt that is based on timization model, since the optimal solutions including power loss and voltage<br>offiles are the same with ones by Varopt that is based on the conventional nonlinear<br>wer flow equations. Moreover, the optimal solution is conv profiles are the same with ones by Varopt that is based on the conventional nonlinear<br>power flow equations. Moreover, the optimal solution is converged at the equality of<br>SOC constraints. This means that this optimal solut

16-node, 33-node, 123-node and 1060-node DNs with DERs that are used for tests.<br>The switch-off circuit breakers for these benchmark systems are shown in Table 2.2,<br>while other circuit breakers are switched on. 16-node, 33-node, 123-node and 1060-node DNs with DERs that are used for tests.<br>The switch-off circuit breakers for these benchmark systems are shown in Table 2.2,<br>while other circuit breakers are switched on.<br>Table 2.2 Sw 33-node, 123-node and 1060-node DNs with DERs that are used for tests.<br>
h-off circuit breakers for these benchmark systems are shown in Table 2.2,<br>
r circuit breakers are switched on.<br>
Table 2.2 Switch-off circuit breakers

				16-node, 33-node, 123-node and 1060-node DNs with DERs that are used for tests.			
				The switch-off circuit breakers for these benchmark systems are shown in Table 2.2,			
				while other circuit breakers are switched on.			
				Table 2.2 Switch-off circuit breakers for these benchmark systems			
	<b>DNs</b>				switch-off circuit breakers		
				start node		end node	
	16-node			6		12 15	
				11 8		17	
				9		8	
				32		31	
33-node			28	29			
				15		14	
				8		21	
				76		72	
		123-node		105		101	
				80		102	
				17		111	
				54 117 143			
		1060-node		45			
				73	134		
				97		76	
				28		56	
				The computational performance in terms of CPU time in seconds and algorithm iterations are given in Table 2.3. The nodal voltage profiles by SOCP-based and polyhedral approximation formulations of reactive power optimization models can be presented in a tree-shaped is presented in Fig. 2.10 (a) $-(d)$ . Table 2.3 Optimal solutions of different scalability of DNs			Computat
<b>DNs</b>	Models		Solvers	Minimal injected real power at PCC node(p.u.)	Real Power Loss(p.u.)	Algorithm Iterations	ional Time (seconds)

The computational performance in terms of CPU time in seconds and algorithm<br>
The computational performance in terms of CPU time in seconds and algorithm<br>
rations are given in Table 2.3. The nodal voltage profiles by SOCP-<sup>13</sup><br>
<sup>134</sup><br>
97 <sup>76</sup><br>
28 56 <br>
<br>
nee in terms of CPU time in seconds and algorithm<br>
3. The nodal voltage profiles by SOCP-based and<br>
lations of reactive power optimization models can be<br>
sented in Fig. 2.10 (a) –(d).<br>
<br>
so 97<br>
97<br>
97<br>
28 56<br>
29<br>
2.10 (a) – (d)<br>
2.10 (a) – (d).<br>
2.10 (a 28 56<br>
in terms of CPU time in seconds and algorithm<br>
The nodal voltage profiles by SOCP-based and<br>
ions of reactive power optimization models can be<br>
ted in Fig. 2.10 (a) –(d).<br>
utions of different scalability of DNs<br>
ni U time in seconds and algorithm<br>
gge profiles by SOCP-based and<br>
ower optimization models can be<br>
(a)–(d).<br>
It scalability of DNs<br>
Real<br>
Power Algorithm Computat<br>
Loss (p.u.) Iterations Time<br>
(seconds)<br>
0.042 17 0.219<br>
0.

<b>DNs</b>	Models	Solvers	Minimal injected real power at PCC node(p.u.)	Real Power Loss(p.u.)	<b>Algorithm</b> Iterations	Computat 10 <sub>nal</sub> Time (seconds)
16	<b>SOCP</b>	<b>MOSEK</b>	2.972	0.042	17	0.219
node		Baron	2.972	0.042		0.201






Fig. 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b)  $\frac{1}{2}$ <br>
Fig. 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b)  $\frac{1}{2}$ <br>
Fig. 2.10 Tree-shaped voltage profiles of different Example 18<br>
Fig. 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node;<br>
(c) (d)<br>
Fig. 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node;<br>
(c) 123-node and (d) 1060-node.<br> Fig. 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node;<br>
(c) (d)<br>
Fig. 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node;<br>
(c) 123-node and (d) 1060-node.<br>
Table 2.2 s IFER 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node;<br>
(c) 123-node and (d) 1060-node.<br>
(c) 123-node and (d) 1060-node.<br>
Table 2.2 shows that the above-mentioned solvers can be effective to so (c) (d)<br>
Fig. 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node;<br>
(c) 123-node and (d) 1060-node.<br>
Table 2.2 shows that the above-mentioned solvers can be effective to solve the<br>
reactive power Fig. 2.10 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node;<br>
(c) 123-node and (d) 1060-node.<br>
Table 2.2 shows that the above-mentioned solvers can be effective to solve the<br>
reactive power optimizat Table 2.2 shows that the above-mentioned solvers can be effective to solve the reactive power optimization problem formulated in the SOCP-based and SDP-based *DistFlow* form. Fig.2.10 (a)-(d) display that the nodal voltag Table 2.2 shows that the above-mentioned solvers can be effective to solve the reactive power optimization problem formulated in the SOCP-based and SDP-based *DistFlow* form. Fig.2.10 (a)-(d) display that the nodal voltage DistFlow form. Fig.2.10 (a)-(d) display that the nodal voltage profiles by<br>and polyhedral approximation formulations are very close. The maximident approximation formulations are very close. The maximident and 4.211%, resp d polyhedral approximation formulations are very close. The maximum errors of<br> *EE* 16-node, 33-node, 123-node and 1060-node DNs are 0.286%, 0.510%, 1.005%<br>
d 4.211%, respectively. This demonstrates that *DistFlow* equatio IEEE 16-node, 33-node, 123-node and 1060-node DNs are 0.286%, 0.510%, 1.005%<br>and 4.211%, respectively. This demonstrates that *DistFlow* equations have<br>advantageous properties that ean be reformulated in the SOCP and SDP f and 4.211%, respectively. This demonstrates that *DistFlow* equations have<br>advantageous properties that can be reformulated in the SOCP and SDP form, and the<br>polyhedral approximation formulation of SOC constraints can be

of SOC constraints can be also accurate and effective for quick convergence. This<br> *DistFlow* equations and its convex relaxation formulations lays the theoretical<br>
foundation for the following DNR problems with convex opt The SOC constraints can be also accurate and effective for quick convergence. This DistFlow equations and its convex relaxation formulations lays the theoretical foundation for the following DNR problems with convex optimi foundation for the following DNR problems with convergence. This *DistFlow* equations and its convex relaxation formulations lays the theoretical foundation for the following DNR problems with convex optimization solvers.

# **Chapter 3<br>Topology Optimization of Active Distribution** Chapter 3<br>Topology Optimization of Active Distribution<br>Network based on Disjunctive Convex Hull Chapter 3<br>Topology Optimization of Active Distribution<br>Network based on Disjunctive Convex Hull<br>Approach for Operational Security Enhancement Chapter 3<br>
Topology Optimization of Active Distribution<br>
Network based on Disjunctive Convex Hull<br>
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Distribution network reconfiguration (DNR) is a classical optimal operation Chapter 3<br>
Topology Optimization of Active Distribution<br>
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**Chapter 3**<br> **Topology Optimization of Active Distribution**<br> **Network based on Disjunctive Convex Hull**<br> **Approach for Operational Security Enhancement**<br>
Distribution network reconfiguration (DNR) is a classical optimal op Chapter 3<br>Topology Optimization of Active Distribution<br>Network based on Disjunctive Convex Hull<br>Approach for Operational Security Enhancement<br>Distribution network reconfiguration (DNR) is a classical optimal operation<br>prob Topology Optimization of Active Distribution<br>Network based on Disjunctive Convex Hull<br>Approach for Operational Security Enhancement<br>Distribution network reconfiguration (DNR) is a classical optimal operation<br>problem over d **Topology Optimization of Active Distribution**<br> **Network based on Disjunctive Convex Hull**<br> **Approach for Operational Security Enhancement**<br>
Distribution network reconfiguration (DNR) is a classical optimal operation<br>
pro **Approach for Operational Security Enhancement**<br>
Distribution network reconfiguration (DNR) is a classical optimal operation<br>
problem over decades. It aims to maintain load balancing and loss reduction at the<br>
voltage sceu **Approach for Operational Security Enhancement**<br>Distribution network reconfiguration (DNR) is a classical optimal operation<br>problem over decades. It aims to maintain load balancing and loss reduction at the<br>voltage securit The Distribution network reconfiguration (DNR) is a classical optimal operation<br>blem over decades. It aims to maintain load balancing and loss reduction at the<br>age security-constrained operation level, and to coordinate re Distribution network reconfiguration (DNR) is a classical optimal operation<br>problem over decades. It aims to maintain load balancing and loss reduction at the<br>voltage security-constrained operation level, and to coordinate problem over decades. It aims to maintain load balancing and loss reduction at the voltage sceurity-constrained operation level, and to coordinate real-time transactive dispatch tasks between supply and demand at the marke

voltage security-constrained operation level, and to coordinate real-time transactive<br>dispatch tasks between supply and demand at the market level of DNs. However, the<br>computing performance of existing methods in terms of dispatch tasks between supply and demand at the market level of DNs. However, the<br>computing performance of existing methods in terms of running time and iterations is<br>not satisfactory for a large-scale network. This chapte computing performance of existing methods in terms of running time and iterations is<br>not satisfactory for a large-scale network. This chapter investigates the convex hull<br>(CH) of *DistFlow* equations for the superior numer not satisfactory for a large-scale network. This chapter investigates the convex hull (CH) of *DistFlow* equations for the superior numerical performance.<br>This chapter proposes the disjunctive convex hull relaxation (DCHR)

3.1 Radiality Constraints<br>
In this thesis, we suppose that each branch of DNs has a sectionalizing switch or<br>
switch Fach bus in DNs is connected by a branch with a switch and one tie switch I Radiality Constraints<br>
In this thesis, we suppose that each branch of DNs has a sectionalizing switch or tie<br>
itch. Each bus in DNs is connected by a branch with a switch, and one tie switch<br>
in form a loop with other se 3.1 Radiality Constraints<br>In this thesis, we suppose that each branch of DNs has a sectionalizing switch or tie<br>switch. Each bus in DNs is connected by a branch with a switch, and one tie switch<br>can form a loop with other 3.1 Radiality Constraints<br>
In this thesis, we suppose that each branch of DNs has a sectionalizing switch or tie<br>
switch. Each bus in DNs is connected by a branch with a switch, and one tie switch<br>
can form a loop with oth 3.1 Radiality Constraints<br>
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switch. Each bus in DNs is connected by a branch with a switch, and one tie switch<br>
can form a loop with oth 3.1 Radiality Constraints<br>In this thesis, we suppose that each branch of DNs has a sectionalizing switch or tie<br>switch. Each bus in DNs is connected by a branch with a switch, and one tie switch<br>can form a loop with other 3.1 Radiality Constraints<br>
In this thesis, we suppose that each branch of DNs has a sectionalizing switch or tie<br>
switch. Each bus in DNs is connected by a branch with a switch, and one tie switch<br>
can form a loop with oth 3.1 Radiality Constraints<br>In this thesis, we suppose that each branch of DNs has a sectionalizing switch or tic<br>switch. Each bus in DNs is connected by a branch with a switch, and one tic switch<br>can form a loop with other 3.1 Radiality Constraints<br>
In this thesis, we suppose that each branch of DNs has a sectionalizing switch or tie<br>
switch. Each bus in DNs is connected by a branch with a switch, and one tie switch<br>
can form a loop with ot DistFlow equations. can form a loop with other sectionalizing switches. The DNR problem can be deemed<br>as a combinatorial issue about choosing optimal switch status of sectionalizing<br>switches (normally closed) and tie-switches (normally open). a combinatorial issue about choosing optimal switch status of sectionalizing<br>
itches (normally closed) and tie-switches (normally open). The DNR provides an<br>
timal network structure to realize minimum power losses and achi switches (normally closed) and tie-switches (normally open). The DNR provides an<br>optimal network structure to realize minimum power losses and achieve better load<br>balancing; or to be used for post-outage restoration and pl

imal network structure to realize minimum power losses and achieve better load<br>balancing; or to be used for post-outage restoration and planned maintenance, etc. In<br>this section, we discuss the relaxation techniques for DN balancing; or to be used for post-outage restoration and planned maintenance, etc. In<br>this section, we discuss the relaxation techniques for DNR problems based on<br>DistFlow equations.<br>3.1.1 Virtual Commodity Flow Constrain this section, we discuss the relaxation techniques for DNR problems based on<br> *DistFlow* equations.<br>
3.1.1 Virtual Commodity Flow Constraints<br>
In the traditional DNs, DNR is not a frequent operation. However, with the<br>
ap ques for DNR problems based on<br>
uent operation. However, with the<br>
DNS, the DNR is developing toward<br>
imal operation condition of DNs. The<br>
ables a class of radial grids subject to<br>
<sup>1</sup> be the binary state vector of circui *BistFlow* equations.<br>
3.1.1 Virtual Commodity Flow Constraints<br>
In the traditional DNs, DNR is not a frequent operation. However, with the<br>
application of high-speed switching devices in DNs, the DNR is developing toward  $u' \in \{0,1\}$ ,  $u' \in \mathbb{Z}$ . *l.1 Virtual Commodity Flow Constraints*<br>
In the traditional DNs, DNR is not a frequent operation. However, with the<br>
plication of high-speed switching devices in DNs, the DNR is developing toward<br>
al-time reconfiguration In the traditional DNs, DNR is not a frequent operation. However, with the application of high-spccd switching devices in DNs, the DNR is developing toward real-time reconfiguration for maintaining the optimal operation c application of high-speed switching devices in DNs, the DNR is developing toward<br>real-time reconfiguration for maintaining the optimal operation condition of DNs. The<br>ability to switch between different topologies enables

also been recently. In this section, we advance upon the commodity flow approach<br>and propose a more succinct model with fewer variables and constraints. We consider<br>a virtual single commodity flowing on the network graph also been recently. In this section, we advance upon the commodity flow approach<br>and propose a more succinct model with fewer variables and constraints. We consider<br>a virtual single commodity flowing on the network graph also been recently. In this section, we advance upon the commodity flow approach<br>and propose a more succinct model with fewer variables and constraints. We consider<br>a virtual single commodity flowing on the network graph also been recently. In this section, we advance upon the commodity flow approach<br>and propose a more succinet model with fewer variables and constraints. We consider<br>a virtual single commodity flowing on the network graph also been recently. In this section, we advance upon the commodity flow approach<br>and propose a more succinet model with fewer variables and constraints. We consider<br>a virtual single commodity flowing on the network graph also been recently. In this section, we advance upon the commodity flow approach<br>and propose a more succinct model with fewer variables and constraints. We consider<br>a virtual single commodity flowing on the network graph also been recently. In this section, we advance upon the commodity flow approach<br>and propose a more succinct model with fewer variables and constraints. We consider<br>a virtual single commodity flowing on the network graph also been recently. In this section, we advance upon the commodity flow approach<br>and propose a more succinct model with fewer variables and constraints. We consider<br>a virtual single commodity flowing on the network graph incidence matrix A, by forcing the virtual flows in  $f$  to be zero for open lines: on the commodity flow approach<br>bles and constraints. We consider<br>graph  $G$  and set a demand of one<br>oad nodes to have nonzero power<br>ently, all demands supplied by<br>on ensuring connectivity. Virtual<br>with respect to the branc all non-substation buses. This is to say, all load nodes to have nonzero power<br>
n (no transfer nodes in DNs). Consequently, all demands supplied by<br>
ion and all nodes have a path to substation ensuring connectivity. Virtu ection (no transfer nodes in DNs). Consequently, all demands supplied by<br>bstation and all nodes have a path to substation ensuring connectivity. Virtual<br>mmodity flow constraints can be expressed with respect to the branch

$$
A^T f = 1 \tag{3.1}
$$

$$
-N \cdot u^l \le f^l \le N \cdot u^l, \quad \forall l \in \mathcal{E}
$$
\n
$$
(3.2)
$$

$$
1^T u^l = |\mathcal{N}| - 1 \tag{3.3}
$$

where  $f$  refers to the vector of virtual flows on each branch.

In this set of virtual commodity flow constraints, the virtual flow variable  $f$  does substation and all nodes have a path to substation ensuring connectivity. Virtual<br>commodity flow constraints can be expressed with respect to the branch-bus<br>incidence matrix **A**, by forcing the virtual flows in f to be ze commodity flow constraints can be expressed with respect to the branch-bus<br>incidence matrix **A**, by forcing the virtual flows in  $f$  to be zero for open lines:<br> $A^T f = 1$  (3.1)<br> $- N \cdot u' \le f' \le N \cdot u'$ ,  $\forall l \in \mathcal{E}$  (3.2)<br> $1^T$ incidence matrix **A**, by forcing the virtual flows in  $f$  to be zero for open lines:<br>  $A^T f = 1$  (3.1)<br>  $- N \cdot u^i \le f^i \le N \cdot u^i$ ,  $\forall l \in \mathcal{E}$  (3.2)<br>  $1^T u^i = |\mathcal{N}| - 1$  (3.3)<br>
where  $f$  refers to the vector of virtual flows or open lines:<br>
(3.1)<br>
(3.2)<br>
(3.3)<br>
(3.3)<br>
flow variable  $f$  does<br>
enforce connectivity.<br>
w balance (KCL) on a<br>  $A^T f = 1$ . Moreover,<br>
by linking  $f$  and  $u^l$ .<br>
on solution will be an  $A^T f = 1$  (3.1)<br>  $-N \cdot u' \le f' \le N \cdot u'$ ,  $\forall l \in \mathcal{E}$  (3.2)<br>  $1^T u' = |N| - 1$  (3.3)<br>
where  $f$  refers to the vector of virtual flows on each branch.<br>
In this set of virtual commodity flow constraints, the virtual flow variable  $-N \cdot u' \le f' \le N \cdot u'$  can be used to maintain radiality of DNs by linking  $f$  and  $u'$ .  $l$ .  $-N \cdot u' \le f' \le N \cdot u'$ ,  $\forall l \in \mathcal{E}$  (3.2)<br>  $1^T u' = |\mathcal{N}| - 1$  (3.3)<br>
Where  $f'$  refers to the vector of virtual flows on each branch.<br>
In this set of virtual commodity flow constraints, the virtual flow variable  $f'$  does<br>
n  $1^T u' = |N| - 1$  (3.3)<br>
where  $f'$  refers to the vector of virtual flows on each branch.<br>
In this set of virtual commodity flow constraints, the virtual flow variable  $f$  does<br>
not relate to the actual line flows and is int where  $f$  refers to the vector of virtual flows on each branch.<br>
In this set of virtual commodity flow constraints, the virtual flow variable  $f$  does<br>
not relate to the actual line flows and is introduced only to enforce In this set of virtual commodity flow constraints, the virtual flow variable  $f$  does<br>not relate to the actual line flows and is introduced only to enforce connectivity.<br>Given that flows are allowed only on active lines,

3.1.2 Spanning Tree Constraints<br>The spanning tree (ST) constraints are used to formulate a linear program of the<br>minimum spanning tree problem [98]. For a DNR problem, we can encode every The spanning tree Constraints<br>The spanning tree (ST) constraints are used to formulate a linear program of the<br>inimum spanning tree problem [98]. For a DNR problem, we can encode every<br>anch as a directed are with respect t 3.1.2 Spanning Tree Constraints<br>The spanning tree (ST) constraints are used to formulate a linear program of the<br>minimum spanning tree problem [98]. For a DNR problem, we can encode every<br>branch as a directed are with resp 3.1.2 Spanning Tree Constraints<br>The spanning tree (ST) constraints are used to formulate a linear program of the<br>minimum spanning tree problem [98]. For a DNR problem, we can encode every<br>branch as a directed are with res 3.1.2 Spanning Tree Constraints<br>The spanning tree (ST) constraints are used to formulate a linear program of the<br>minimum spanning tree problem [98]. For a DNR problem, we can encode every<br>branch as a directed are with res 3.1.2 Spanning Tree Constraints<br>
The spanning tree (ST) constraints are used to formulate a linear program of the<br>
minimum spanning tree problem [98]. For a DNR problem, we can encode every<br>
branch as a directed are with 3.1.2 Spanning Tree Constraints<br>
The spanning tree (ST) constraints are used to formulate a linear program of the<br>
minimum spanning tree problem [98]. For a DNR problem, we can encode every<br>
branch as a directed are with instance,  $\beta_{mn}^l = 1$  and  $\beta_{nm}^l = 0$  means that *n* is the traints<br>
(constraints are used to formulate a linear program of the<br>
problem [98]. For a DNR problem, we can encode every<br>
with respect to an acyclic network rooted at node, which ST<br>
for network reconfiguration. To enfor  $\beta_{mn}^l = 0$  and  $\beta_{nm}^l = 1$  implies that *m* is the pa *l i nee Constraints*<br>
ig tree (ST) constraints are used to formulate a linear program of the<br>
ining tree problem [98]. For a DNR problem, we can encode every<br>
rected are with respect to an acyclic network rooted at n parent-child relationship between nodes *m* and *n* for the radiality of networks. For<br>instance,  $\beta'_{mn} = 1$  and  $\beta'_{mn} = 0$  means that *n* is the parent node of *m*, otherwise<br> $\beta'_{mn} = 0$  and  $\beta'_{nm} = 1$  implies that *m* 

$$
\beta_{mn}^l + \beta_{nm}^l = u^l \tag{3.4}
$$

$$
\beta_{mn}^l = 0, \quad \text{if } m = S \tag{3.5}
$$

$$
\sum_{m(m,n)\in E} \beta_{mn}^l = 1, \quad \forall \ m \in N \setminus S \tag{3.6}
$$

$$
\beta_{mn}^l \in [0,1] \tag{3.7}
$$

stance,  $\beta_{nn}^i = 1$  and  $\beta_{nn}^i = 0$  means that *n* is the parent node of *m*, otherwise<br>  $\beta_{nn}^i = 0$  and  $\beta_{nn}^i = 1$  implies that *m* is the parent node of *n*.<br>  $\beta_{nn}^i = \nu_i^i$  (3.4)<br>  $\beta_{nn}^i = 0$ , if  $m = S$  (3.5)<br>  $\$  $\beta'_{mn} = 0$  and  $\beta'_{mn} = 1$  implies that m is the parent-node of n.<br>  $\beta'_{mn} = 0$ ,  $i f \, m = S$  (3.4)<br>  $\beta'_{mn} = 0$ ,  $i f \, m = S$  (3.5)<br>  $\sum_{\alpha(n,\mu) \in E} \beta'_{mn} = 1$ ,  $\forall m \in N \setminus S$  (3.6)<br>  $\beta'_{mn} = [0,1]$  (3.7)<br>
where *S* is the set of  $\beta_{nm}^i + \beta_{nm}^i = u^i$  (3.4)<br>  $\beta_{nm}^i = 0$ , if  $m = S$  (3.5)<br>  $\sum_{a(n,n)\in E} \beta_{nm}^i = 1$ ,  $\forall m \in N \setminus S$  (3.6)<br>  $\beta_{nm}^i \in [0,1]$  (3.7)<br>
where *S* is the set of source nodes.<br>
Flowing on the branch with nodes  $(m,n)$  can only in one  $\beta_{nm}^i = 0$ ,  $if \ m = S$  (3.5)<br>  $\sum_{a(m,n)\in E} \beta_{nm}^i = 1$ ,  $\forall m \in N \setminus S$  (3.6)<br>  $\beta_{mn}^i \in [0,1]$  (3.7)<br>
where *S* is the set of source nodes.<br>
Flowing on the branch with nodes  $(m,n)$  can only in one direction, which explicitly<br>
i  $\sum_{n=m+2}^{\infty} \beta_{nm}^{l} = 1, \forall m \in \mathbb{N} \setminus S$  (3.6)<br>  $\beta_{nm}^{l} \in [0,1]$  (3.7)<br>
Where *S* is the set of source nodes.<br>
Flowing on the branch with nodes  $(m,n)$  can only in one direction, which explicitly<br>
is set to a combination  $\frac{1}{\sigma(m,n)=R}$  (3.0)<br>
where *S* is the set of source nodes.<br>
Flowing on the branch with nodes  $(m,n)$  can only in one direction, which explicitly<br>
is set to a combination of variables  $\beta_{nm}^i$  and  $\beta_{nm}^i$ . The parent-chi  $\beta_{\text{me}}^j \in [0,1]$  (3.7)<br>
Flowing on the branch with nodes  $(m,n)$  can only in one direction, which explicitly<br>
set to a combination of variables  $\beta_{\text{me}}^i$  and  $\beta_{\text{em}}^i$ . The parent-child relationship<br>
riables  $\beta^i$ where *S* is the set of source nodes.<br>
Flowing on the branch with nodes  $(m,n)$  can only in one direction, which explicitly<br>
is set to a combination of variables  $\beta_{na}^i$  and  $\beta_{ma}^i$ . The parent-child relationship<br>
varia

example from reference [99] where we suppose that node 1 is a point of common coupling (PCC) node. example from reference [99] where we suppose that node 1 is a point of comme<br>coupling (PCC) node.<br>PCC bus Q



			8	
			Fig. 3.1 An example of inadequate ST constraints.	
				By ST constraints, we give the following $\beta^l$ values for this figure in Table. 3.1.
			Table 3.1 Values of parent-child relationship variable $\beta$ .	
	<b>Connected Branches</b>			Parent-child
Node $m$	Node $n$	$\beta^{l}_{\scriptscriptstyle{mn}}$	$\beta_{\scriptscriptstyle \!\!nm}^{\scriptscriptstyle l}$	relationship
6			$\boldsymbol{0}$	1-parent, 6-child
7	6		$\boldsymbol{0}$	6-parent, 7-child
3	$\boldsymbol{2}$	$\boldsymbol{0}$		3-parent, 2-child
4	3	$\boldsymbol{0}$		4-parent, 3-child
5	$\overline{\mathbf{4}}$	$\boldsymbol{0}$		5-parent, 4-child
5	$\boldsymbol{2}$		$\boldsymbol{0}$	2-parent, 5-child
9	5		$\boldsymbol{0}$	5-parent, 9-child
8	4		$\boldsymbol{0}$	4-parent, 8-child
		It can be inferred from Table. 3.1 that all $\beta^l$		values satisfy ST constraints (2.25)-(2.28). Unfortunately, there is a cycle topology that is not a radial network. For a DNR problem, power flow equations are hard constraints in this DNR model. Clearly, this disconnected network in Fig. 3.1 leads to an infeasible power flow
		57		

 $\mathcal{U}$ .

solution. In other words, the above-mentioned example cannot satisfy the nodal power<br>flow balance at node 7 under assumed power flow directions in Fig. 3.1.<br>But from our perspective, the power flow equations are necessary

flow balance at node 7 under assumed power flow directions in Fig. 3.1.<br>flow balance at node 7 under assumed power flow directions in Fig. 3.1.<br>But from our perspective, the power flow equations are necessary but insuffici solution. In other words, the above-mentioned example cannot satisfy the nodal power<br>flow balance at node 7 under assumed power flow directions in Fig. 3.1.<br>But from our perspective, the power flow equations are necessary solution. In other words, the above-mentioned example cannot satisfy the nodal power<br>flow balance at node 7 under assumed power flow directions in Fig. 3.1.<br>But from our perspective, the power flow equations are necessary



Fig. 3.2 An example of inadequate ST and power flow constraints.<br>
In this case, we can find that ST and power flow constraints.<br>
In this case, we can find that ST and power flow equations constraints cannot<br>
guarantee a r s<br> $\begin{array}{r} 3 \ 3 \ 3 \ 4 \ 6 \ 7 \ 8 \end{array}$ <br>Fig. 3.2 An example of inadequate ST and power flow constraints.<br>In this case, we can find that ST and power flow equations constraints cannot<br>guarantee a radial topology when there is <sup>3</sup><br><sup>3</sup><br><sup>5</sup><br><sup>9</sup><br><sup>8</sup><br>Fig. 3.2 An example of inadequate ST and power flow constraints.<br>In this case, we can find that ST and power flow equations constraints cannot<br>guarantee a radial topology when there is a DG source at n Fig. 3.2 An example of inadequate ST and power flow constraints.<br>In this case, we can find that ST and power flow equations constraints cannot<br>guarantec a radial topology when there is a DG source at node 2 in this distrib Fig. 3.2 An example of inadequate ST and power flow constraints.<br>
In this case, we can find that ST and power flow equations constraints cannot<br>
guarantee a radial topology when there is a DG source at node 2 in this distr

3.2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>programming (SOCP) or a semi-definite programming (SDP) formulation for 2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>ogramming (SOCP) or a semi-definite programming (SDP) formulation for<br>*stFlow* equations. By these convex relaxati 3.2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>programming (SOCP) or a semi-definite programming (SDP) formulation for<br>*DistFlow* equations. By these convex re 3.2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>programming (SOCP) or a semi-definite programming (SDP) formulation for<br>*DistFlow* equations. By these convex re 3.2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>programming (SOCP) or a semi-definite programming (SDP) formulation for<br>*DistFlow* equations. By these convex re 3.2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>programming (SOCP) or a semi-definite programming (SDP) formulation for<br>*DistFlow* equations. By these convex re 3.2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>programming (SOCP) or a semi-definite programming (SDP) formulation for<br>*DistFlow* equations. By these convex re 3.2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>programming (SOCP) or a semi-definite programming (SDP) formulation for<br>*DistFlow* equations. By these convex re 3.2 Conventional DNR Models<br>By the angle relaxation, *DistFlow* equations can be cast as a second-ord-<br>programming (SOCP) or a semi-definite programming (SDP) formulat<br>*DistFlow* equations. By these convex relaxations, the By the angle relaxation, *DistFlow* equations can be cast as a second-order conic<br>programming (SOCP) or a semi-definite programming (SDP) formulation for<br>*DistFlow* equations. By these convex relaxations, the DNR model ca gramming (SOCP) or a semi-definite programming (SDP) formulation for<br>sthe sake of the sake of the model of the minimal power loss in the model of as<br>mixed-integer second-order conic programming (MISOCP) or a mixed integer *DistFlow* equations. By these convex relaxations, the DNR model can be modeled as<br>a mixed-integer second-order conic programming (MISOCP) or a mixed integer<br>semi-definite programming (MISDP) problem in the following. The a mixed-integer second-order conic programming (MISOCP) or a mixed integer<br>semi-definite programming (MISDP) problem in the following. These convex models<br>exploit the optimal distribution topology with reduced computation ic programming (MISOCP) or a mixed integer<br>
P) problem in the following. These convex models<br>
ology with reduced computational complexity. In<br>
OCP-based model of DNR problems with Big-M<br>
vith Big-M Relaxation Method<br>
powe

 $\left[P^{\prime}, Q^{\prime}, \ell^{\prime}, \nu_{PQ}\right]^{T}$  and  $\left[ Q^{cr},f^{l},\pmb{\beta}^{l}_{mn},\ \pmb{\beta}^{l}_{nm},\pmb{u}^{l}\ \right] ^{T}$  $\nu_{mn}$ ,  $\nu_{nm}$ semi-definite programming (MISDP) problem in the following. These convex models<br>exploit the optimal distribution topology with reduced computational complexity. In<br>this subsection, we only discuss SOCP-based model of DNR exploit the optimal distribution topology with reduced computational complexity. In<br>this subsection, we only discuss SOCP-based model of DNR problems with Big-M<br>relaxation method.<br>3.2.1 MISOCP-based DNR Model with Big-M R this subsection, we only discuss SOCP-based model of DNR problems with Big-M<br>relaxation method.<br>3.2.1 MISOCP-based DNR Model with Big-M Relaxation Method<br>For the sake of the minimal power loss in reconfigurable DNs, the s relaxation method.<br>
3.2.1 MISOCP-based DNR Model with Big-M Relaxation Method<br>
For the sake of the minimal power loss in reconfigurable DNs, the set of<br>
optimization variables involves a set of operational variables  $[p^i,$ with Big-M Relaxation Method<br>
power loss in reconfigurable DNs, the set of<br>
et of operational variables  $[p', Q', \ell', v_{pQ}]^T$  and<br>  $[v_0, Q^\sigma, f', \beta'_{mn}, \beta'_{mn}, u'^T]^T$ . Here,  $v_0$  refers to the<br>
node 0. Accordingly, once controllabl 3.2.1 MISOCP-based DNR Model with Big-M Relaxation Method<br>
For the sake of the minimal power loss in reconfigurable DNs, the set of<br>
optimization variables involves a set of operational variables  $\left[P, Q', \ell', v_{rQ}\right]^T$  and<br> pptimization variables involves a set of operational variables  $\left[P', Q', \ell', v_{\ell} \right]^T$  and<br>a vector of controllable variables  $\left[v_0, Q^{\sigma}, f', \beta'_{mn}, \beta'_{mn}, u'\right]^T$ . Here,  $v_0$  refers to the<br>squared voltage profile at the root no a vector of controllable variables  $[v_0, Q^\sigma, f^j, \beta_m^j, \beta_m^j, \mu_j^{\sigma}]$ . Here,  $v_0$  refers to the<br>squared voltage profile at the root node 0. Accordingly, once controllable variables<br>are provided, operational variables can

 $2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) - |z_{mn}^l|^2 \ell_{mn}^l = 0$  $n_{n} - \nu_{m} + 2(r_{mn}^{l}P_{mn}^{l} + x_{mn}^{l}Q_{mn}^{l}) - |z_{mn}^{l}|^{2}\ell_{m}^{l}$ 

$$
x' := [P^l, Q^l, \ell^l, v, Q^{cr}, w^l, m^l, f^l, \beta^l_{mn}, \beta^l_{nm}, u^l]^T, \forall l \in \mathcal{E}
$$
. Thus, the entire network  
reconfiguration model with the loss minimization is cast as  

$$
\min_{x'} P_0^l
$$

$$
x' := [P', Q', t', v, Q'', v', m', f', \beta'_m, \beta'_{m}, \beta'_{m}, u']^T, \forall l \in \mathcal{E} \text{ . Thus, the entire networkreconfiguration model with the loss minimization is cast as\n
$$
\min_{m} P'_{0}
$$
\n
$$
\begin{cases}\nP'_{m} + P_{s}^{0} - r'_{m}t'_{m} = \sum_{s=r(s)} P'_{m} + P'_{s}, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N} \\
Q'_{m} + Q_{n}^{0} - x'_{m}t'_{m} = \sum_{s=r(s)} Q'_{m} + Q'_{s}, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}\n\end{cases}
$$
\n
$$
v_{n} - v_{n} + 2(r'_{m}P'_{m} + x'_{m}Q'_{m}) - |x'_{m}|^{2}t'_{m} - (1 - u')M \leq 0, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}\n\end{cases}
$$
\n
$$
y_{n} - y_{n} + 2(r'_{m}P'_{m} + x'_{m}Q'_{m}) - |x'_{m}|^{2}t'_{m} + (1 - u')M \geq 0, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}\n\end{cases}
$$
\n
$$
2Q'_{m}
$$
\n
$$
\begin{vmatrix}\n2P'_{m} \\
2Q'_{m} \\
2Q'_{m} \\
2Q'_{m}\n\end{vmatrix} \leq m' \sqrt{v} \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
Q' = Q'' \leq Q''
$$
\n
$$
Q' = Q''
$$
\n
$$
Q' = Q''
$$
\n
$$
Q'' = Q'''
$$
\n
$$
Q''' = 1, \forall m \in \mathcal{N}, S
$$
\
$$

where  $P_0^l$  is the active power injection at the root node S.

 $\mathbf{v}_l = (\mathbf{v}_l - \mathbf{v}_m) \cdot \mathbf{u}^l$  $y' = (v_n - v_m) \cdot u'$ . Then, the McCormick linearization method is utilized to relax<br>equality  $v_n - v_m + 2(r'_{mn} P'_{mn} + x'_{mn} Q'_{mn}) - |z'_{mn}|^2 t'_{mn} = 0$  as<br> $y' + 2(r'_{mn} P'_{mn} + x'_{mn} Q'_{mn}) - |z'_{mn}|^2 t'_{mn} = 0$  (3.9) equality  $v_n - v_m + 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) - |z_{mn}^l|^2 \ell_{mn}^l = 0$  $v_n - v_m + 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) - |z_{mn}^l|^2 \ell_{mn}^l = 0$  as

$$
y^{l} + 2(r_{mn}^{l}P_{mn}^{l} + x_{mn}^{l}Q_{mn}^{l}) - |z_{mn}^{l}|^{2}\ell_{mn}^{l} = 0
$$
\n(3.9)

$$
(\underline{v} - \overline{v}) \cdot u^l \leqslant y^l \leqslant (\overline{v} - \underline{v}) \cdot u^l \tag{3.10}
$$

$$
v_n - v_m - (\overline{v} - \underline{v}) \cdot (u^l - 1) \leqslant y^l \leqslant v_n - v_m - (\underline{v} - \overline{v}) \cdot (u^l - 1) \tag{3.11}
$$

=  $(v_n - v_m) \cdot u'$ . Then, the McCormick linearization method is utilized to relax<br>
uality  $v_n - v_m + 2(r'_{mn}P'_{mn} + x'_{mn}Q'_{m}) - |z'_{mn}|^2 \ell^1_{mn} = 0$  as<br>  $y' + 2(r'_{mn}P'_{mn} + x'_{mn}Q'_{mn}) - |z'_{mn}|^2 \ell^1_{mn} = 0$  (3.9)<br>  $(\underline{v} - \overline{v}) \cdot u' \leq y' \leq$ as  $\mathbf{x}' \coloneqq \left[ \boldsymbol{P}^{\prime}, \boldsymbol{Q}^{\prime}, \ell^{\prime}, \nu, \boldsymbol{Q}^{cr}, \boldsymbol{w}^{\prime}, \boldsymbol{m}^{\prime}, \boldsymbol{y}^{\prime}, \boldsymbol{f}^{\prime}, \boldsymbol{\beta}_{mn}^{\prime}, \boldsymbol{\beta}_{nm}^{\prime}, \boldsymbol{u}^{\prime} \right]^{T}, \forall l \in \mathcal{E}$  $(v_n - v_m) \cdot u'$ . Then, the McCormick linearization method is utilized to relax<br>
dity  $v_n - v_m + 2(r_m' P_m' + x_m' Q_m') - |z_m'|^2 (r_m' = 0$  as<br>  $y' + 2(r_m' P_m' + x_m' Q_m') - |z_m'|^2 (r_m' = 0)$  (3.9)<br>  $(v - \overline{v}) \cdot u' \le y' \le (\overline{v} - \underline{v}) \cdot u'$  (3.10)<br>  $v_n - v_m - (\over$  $y' = (v_n - v_m) \cdot u'$ . Then, the McCormick linearization method is utilized to relax<br>equality  $v_n - v_m + 2(v_{mn}^i P_{mn}^i + x_{mn}^i Q_{mn}^i) - |z_{mn}^i|^2 t_{mn}^i = 0$  as<br> $y' + 2(v_{mn}^i P_{mn}^i + x_{mn}^i Q_{mn}^i) - |z_{mn}^i|^2 t_{mn}^i = 0$  (3.9)<br> $(y - \overline{v}) \cdot u^i$ 

$$
\min_{s'} P'_n
$$
\n
$$
\int_{\infty}^{m} P'_n + P''_n - r'_m e'_m = \sum_{k \in \mathcal{K}(s)} P'_m + P''_n, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
Q'_{mn} + Q^s_n - x'_{mn} e'_m = \sum_{k \in \mathcal{K}(s)} Q^s_{mk} + Q^d_n, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
y' + 2(r'_m P'_m + x'_m Q'_m) - |x'_m| e'_m = 0, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
\sum_{r} v_m - \sum_{u'} (\sum_{v} - \sum_{v'} u', \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
v_p - v_m - (\nabla - \sum_{v'} u', \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
P''_{mn} = (\nabla - \sum_{v'} u', \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
2Q_m^{lm} = \sum_{m'} \sum_{m'} m' = e'_{mn} + v_m, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
w' = e'_{mn} - v_m, m' = e'_{mn} + v_m, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
Q'' = \sum_{u'} \sum_{v'} Q' \leq Q'_v
$$
\n
$$
Q'' \leq Q'' \leq \overline{Q''}
$$
\n
$$
Q''' \leq Q'' \leq \overline{Q''}
$$
\n
$$
Q''' \leq Q'' \leq \overline{Q''}
$$
\n
$$
Q'''' = 0, \text{ if } m = 0
$$
\n
$$
\sum_{m,m,\nu \in \mathcal{N}} \sum_{v'} \sum_{m'} \sum_{m'} \text{ and } M \subset \text{Cormick Linearization Methods}
$$
\n
$$
\text{With LDF-based equations, the Big-M method is utilized to relax equality}
$$
\n
$$
v_n - v_m + 2(v'_{mn} P'_m + x'_m
$$

 $2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) = 0$  $\begin{aligned}\n\beta'_{mn} &= a' \\
\beta'_{mn} &= 0, \text{ if } m = 0 \\
\sum_{m,n\geq 0} \beta'_{mn} &= 1, \forall m \in \mathcal{N} \cup \{0\} \\
\beta'_{mn} &\geq 0\n\end{aligned}$ <br>  $\begin{aligned}\nA''_n &= 0 \\
A''_n &= 2\n\end{aligned}$ <br>  $A''_n &= 1\n\end{aligned}$ <br>
3.2.3 Quadratic DNR Model with Big-M and McCormick Li  $\begin{cases} \n\frac{\mu_{m0}}{m_{m0,mP}} = 0, \quad \text{if } m = 0 \\ \n\frac{\mu_{m1}}{m_{m2}} \geq 0 \\ \n\frac{\mu_{m2}}{m_{m2}} \geq 0 \n\end{cases}$ <br>  $\begin{cases} \n\frac{\mu_{m2}}{m_{m2}} = 1, \forall m \in \mathcal{N}0 \\ \n-\frac{\lambda}{2} \left( \frac{\mu_{m2}}{2} \right) \left( \frac{\mu_{m1}}{2} \right) \left( \frac{\mu_{m2}}{2} \right) \left( \frac{\mu_{m2}}{2} \right) \left( \frac{\mu_{m2}}{$  $\boldsymbol{d}^l := \Bigr[\!\!\left[ \boldsymbol{P}^l,\boldsymbol{Q}^l,\nu,\boldsymbol{Q}^{cr},\boldsymbol{f}^l,\boldsymbol{\beta}^l_{mn},\boldsymbol{\beta}^l_{nm},\boldsymbol{u}^l \right]\!\!\!\right] ^T, \,\, \forall l \in \mathcal{E}$  $\int_{m+n/2}^{m+n/2} \int_{m}^{d} \int_{m}^{d} \xi \ge 0$ <br>  $\int_{1}^{d} f = 1$ <br>  $- N \cdot u' \le f' \le N \cdot u', \quad \forall l \in \mathcal{E}$ <br>  $\int_{1}^{d} I' = [N-l], \quad u' \in \{0,1\}$ <br>
S.2.3 Quadratic DNR Model with Big-M and McCormick Linearization Methods<br>
With LDF-based equations,  $\begin{aligned}\nA^T f &= 1 \\
-A^T u^T \le f^T \le N \cdot u^T, \quad \forall I \in \mathcal{E} \\
-\frac{1}{2} N \cdot u^T \le f^T \le N \cdot u^T, \quad u^T \in \{0,1\}.\n\end{aligned}$ 3.2.3 Quadratic DNR Model with Big-M and McCormick Linearization Methods<br>
With LDF-based equations, the Big-M method is

 $\sum_{l \in \mathcal{E}} r_{mn}^l ((P_{mn}^l)^2 + (Q_{mn}^l)^2)$  $r_{mn}^l((P_{mn}^l)^2+(Q_{mn}^l)$  $\sum_{l\in\mathcal{E}}r_{mn}^l((P_{mn}^l)^2 +$ 

where voltage profiles are all assumed as one unit. Due to this<br>notion, the entire linearized DNR model with the loss  $\sum_{l \in \mathcal{E}} r_{nn}^l \left( (P_{nn}^l)^2 + (Q_{nn}^l)^2 \right)$  where voltage profiles are all assumed as one unit. Due to this quadratic objective function, the entire linearized DNR model with the loss minimization is reformulated as a m  $\sum_{h \in \mathcal{E}} r_{mn}^l((P_{mn}^l)^2 + (Q_{mn}^l)^2)$  where voltage profiles are all assumed as one unit. Due to this quadratic objective function, the entire linearized DNR model with the loss minimization is reformulated as a mixed

2 2 ( ) ( ) , , , , (1 ) 0, , , min (( ( ( , ) ) 1 ) 0 , , ( ) ) , , 2( 2 ) . . l l l mn mn mn l l g l d mn n nk n k n l g l d mn n nk n k n l l l l n m mn mn mn mn l l l l n m mn m l n mn m l n l m n l m n u M l m n u M l m n r P Q P P P P Q Q Q Q v v r P x Q v v r P x Q s t l x 0 max max ( 0 , ) 0 Γ , Γ 0, if 1, \ 0 1 , 1 | | 1 {0,1} l l l l l l l l mn l l l cr cr cr n nm l m l mn n m n l mn l l l T l l T l u P Q u u m S Q Q Q Q Q Q v v v u m S l f N u f u u N A : (3.13) 3.2.4 Polyhedral Approximation of DNR Model using Big-M Relaxation Method For SOC constraints in the DNR model, the polyhedral approximation can be used to simplify the SOC constraints to be a family of linear constraints. We can express

 $(2P_{mn}^l \quad 2Q_{mn}^l \quad w^l)^T\Big\|_2 \leq m^l, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{E}$  $\boldsymbol{T}$ m  $\begin{aligned}\n&\begin{bmatrix}\n\theta_{na}^{l}_{na} = 0, \text{ if } m = S \\
&\sum_{m,l,m \in \mathcal{S}} \theta_{ma}^{l}_{m,l} = 1, \forall m \in \mathcal{N} \setminus S\n\end{bmatrix} \\
&\begin{bmatrix}\n\theta_{na} > 0 \\
A^{T}f = 1 \\
-I & -N \cdot u^{l} \leq f^{l} \leq N \cdot u^{l}, \quad \forall l \in \mathcal{E} \\
I^{T}u^{l} = |N| - 1\n\end{bmatrix} \\
A \cdot Polyhedral Approximation of DNR Model using Big-M Relation Method\n\end{aligned} \\
A \cdot Polyhedral Approximation of DNR Model$  $\int_{m_m/2}^{m_m/2} \frac{\int_{m_m/2}^{m_m/2} e^{2\pi i m}}{m^3} dx$ <br>  $\int_{m'}^{N'} = \int_{N'}^{N'} = \$  $\overline{A}^T f = 1$ <br>  $-N \cdot u' \le f' \le N \cdot u', \quad \forall l \in \mathcal{E}$ <br>  $l^T u' = |N| - 1$ <br>  $u' \in \{0,1\}$ <br>
3.2.4 Polyhedral Approximation of DNR Model using Big-M Relaxation Method<br>
For SOC constraints in the DNR model, the polyhedral approximation



 $2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) - |z_{mn}^l|^2 \ell_{mn}^l = 0$  $n_{n} - \nu_{m} + 2(r_{mn}^{l}P_{mn}^{l} + x_{mn}^{l}Q_{mn}^{l}) - |z_{mn}^{l}|^{2}\ell_{m}^{l}$ Fig. 3.3 Polyhedral cone for SOC constraint.<br>
Moreover, the Big-M method is utilized to relax equality<br>  $v_n - v_m + 2(v_m^{\prime}P_m^{\prime} + x_m^{\prime}Q_m^{\prime}) - |z_m^{\prime}|^2(v_m = 0$  as two inequalities with a large-enough<br>
positive scalar M. Theref Fig. 3.3 Polyhedral cone for SOC constraint.<br>
Moreover, the Big-M method is utilized to relax equality<br>  $v_n - v_m + 2(v_m^t m_m^t + x_m^t \mathcal{Q}_m^t) - |z_m^t|^2 (v_m^t = 0)$  as two inequalities with a large-enough<br>
positive scalar M. Theref Fig. 3.3 Polyhedral cone for SOC constraint.<br>
Moreover, the Big-M method is utilized to relax equality<br>  $v_n - v_m + 2(r_m^t P_m^t + x_m^t Q_m^t) - |z_m^t|^2 t_m^t = 0$  as two inequalities with a large-enough<br>
positive scalar *M*. Therefore,  $\bm{M}^{l} := \left[\bm{P}^{l},\bm{Q}^{l},\bm{v},\bm{Q}^{cr},\bm{w}^{l},\bm{m}^{l},\bm{S}^{l},\varsigma,\eta,\omega,\kappa,\bm{f}^{l},\bm{\beta}^{l}_{mn},\bm{\beta}^{l}_{nm},\bm{u}^{l}\right]^{T},\ \ \forall l\in\mathcal{E}^{l}$ mn nm  $w^l$ <br>  $\sqrt{(2P_{mn}^l)^2 + (2Q_{mn}^l)^2}$ <br>
Fig. 3.3 Polyhedral cone for SOC constraint.<br>
Moreover, the Big-M method is utilized to relax equality<br>  $v_n - v_n + 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) - |z_{mn}^l|^2 e_{mn}^l = 0$  as two inequalities Fig. 3.3 Polyhedral cone for SOC constraint.<br>
Moreover, the Big-M method is utilized to relax equality<br>  $v_n - v_m + 2(r_m' P_m' + x_m' Q_m') - |z_m'|^2 (t_m' = 0)$  as two inequalities with a large-enough<br>
positive scalar *M*. Therefore, a pol Moreover, the Big-M method is utilized to relax equality<br>  $v_n - v_m + 2(v_m^l v_m^l + x_m^l Q_m) - |z_m^l|^2 (v_m^l = 0)$  as two inequalities with a large-enough<br>
positive scalar *M*. Therefore, a polyhedral approximation DNR model with Big-

$$
\min_{s'} P_0^l
$$
\n
$$
\int P_{mn}^l + P_n^g - r_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} P_m^l + P_n^d, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
Q_{mn}^l + Q_n^g - x_{mn}^l \ell_{mn}^l = \sum_{k \in \pi(n)} Q_m^l + Q_n^d, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
v_n - v_m + 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) - (1 - u^l)M \le 0, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
v_n - v_m + 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) + (1 - u^l)M \ge 0, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
v' = \ell_{mn}^l - v_n, m' = \ell_{mn}^l + v_n, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}
$$
\n
$$
\forall^2 \ge 22r_{mn}^l, \forall l \in \mathcal{E}
$$
\n
$$
T^0 \ge 2Q_{mn}^l, \forall l \in \mathcal{E}
$$
\n
$$
T^0 \ge -\cos(\frac{\pi}{2^{j+1}})S^{j-1} + \sin(\frac{\pi}{2^{j+1}})\eta^{j-1}, \quad j = 1, ..., D
$$
\n
$$
T^0 \ge -\sin(\frac{\pi}{2^{j+1}})S^{j-1} + \cos(\frac{\pi}{2^{j+1}})\eta^{j-1}, \quad j = 1, ..., D
$$
\n
$$
C^0 \le S_{mn}^l, \forall l \in \mathcal{E}
$$
\n
$$
\eta^0 \ge |S_{mn}^l|, \forall l \in \mathcal{E}
$$
\n
$$
\eta^0 \ge |S_{mn}^l|, \forall l \in \mathcal{E}
$$
\n
$$
K^0 \ge |w'|, \forall l \in \mathcal{E}
$$
\n
$$
K^0 \ge |w'|, \forall l \in \mathcal{E}
$$
\n<math display="block</math>

where  $\upsilon$  is the parameter of the polyhedral  $\varepsilon(\upsilon)$  relaxed approximation in (2.60).<br>3.3 Disjunctive Convex Hull Approach for DNR Formulation<br>4.8 stated the above, convex relaxation approaches in the conventional DNR The above, convex relay in the polyhedral  $\varepsilon(\nu)$  relaxed approximation in (2.60).<br>
As stated the above, convex relaxation approaches in the conventional DNR models<br>
clude Big-M method and McCormick linearization method. where v is the parameter of the polyhedral  $\varepsilon(\nu)$  relaxed approximation in (2.60).<br>
3.3 Disjunctive Convex Hull Approach for DNR Formulation<br>
As stated the above, convex relaxation approaches in the conventional DNR mod where  $\nu$  is the parameter of the polyhedral  $\varepsilon(\nu)$  relaxed approximation in (2.60).<br>3.3 Disjunctive Convex Hull Approach for DNR Formulation<br>As stated the above, convex relaxation approaches in the conventional DNR mo status indicators, or construct relaxation approaches in the conventionism in (2.60).<br>
3.3 Disjunctive Convex Hull Approach for DNR Formulation<br>
4.8 stated the above, convex relaxation approaches in the conventional DNR m satisfied for a disconnected line. Since the schemes, the disconnection in (2.60).<br>
3.3 Disjunctive Convex Hull Approach for DNR Formulation<br>
4.5 stated the above, convex relaxation approaches in the conventional DNR mode where v is the parameter of the polyhedral  $\varepsilon(\nu)$  relaxed approximation in (2.60).<br>
3.3 Disjunctive Convex Hull Approach for DNR Formulation<br>
As stated the above, convex relaxation approaches in the conventional DNR mod 3.3 Disjunctive Convex Hull Approach for DNR Formulation<br>
As stated the above, convex relaxation approaches in the conventional DNR models<br>
include Big-M method and McCormick linearization method. These two existing<br>
conv **3** Disjunctive Convex Hull Approach for DNR Formulation<br>As stated the above, convex relaxation approaches in the conventional DNR models<br>thude Big-M method and McCormick linearization method. These two existing<br>nvex rela As stated the above, convex relaxation approaches in the conventional DNR models<br>include Big-M method and McCormick linearization method. These two existing<br>convex relaxation approaches are used to slack power flow constr inventional DNR models<br>
d. These two existing<br>
constraints by switch<br>
variables automatically<br>
on methods suffer from<br>
e the efficient and tight<br>
able DNs, the set of<br>  $:= [\mathbf{P}', \mathbf{Q}', \ell', \nu, \mathbf{Q}^{cr}]^T$ ,<br>  $\in \mathbb{R}^{2\varepsilon}$  i clude Big-M method and McCormick linearization method. These two existing<br>novex relaxation approaches are used to slack power flow constraints by switch<br>tus indicators, or construct relaxation constraints with variables a od. These two existing<br>v constraints by switch<br>variables automatically<br>on methods suffer from<br>e the efficient and tight<br>rable DNs, the set of<br> $:= [\mathbf{P}', \mathbf{Q}', \ell', \nu, \mathbf{Q}^{cr}]^T$ ,<br> $\in \mathbb{R}^{2\varepsilon}$  in spanning tree<br> $1$ ,  $\mathbf{u$ 

 $\mathcal{U} := [\boldsymbol{P^\prime},\ \boldsymbol{Q^\prime},\ \ \ell^{\, \prime}\, \ , \ \boldsymbol{v},\ \boldsymbol{Q^{cr}}]^T,$ ,  $x' \in \mathbb{R}^n$ , the continuous parent-child relationship variable  $\beta' \in \mathbb{R}^{2\varepsilon}$  in spannin convex relaxation approaches are used to slack power flow constraints by switch<br>status indicators, or construct relaxation constraints with variables automatically<br>satisfied for a disconnected line. Since these convex rel  $l \in \{0, 1\}$   $u^{l} \in \mathbb{Z}^{\varepsilon}$  where  $u^{l}$  is flow constraints by switch<br>with variables automatically<br>elaxation methods suffer from<br>explore the efficient and tight<br>onfigurable DNs, the set of<br>ples  $x' := [P', Q', \ell', v, Q^{cr}]^T$ ,<br>ble  $\beta' \in \mathbb{R}^{2\epsilon}$  in spanning tree<br> $\in \{0,$ straints by switch<br>bles automatically<br>ethods suffer from<br>efficient and tight<br>DNs, the set of<br> $\mathbf{v}^l$ ,  $\mathbf{Q}^l$ ,  $\ell^l$ ,  $\mathbf{v}$ ,  $\mathbf{Q}^{cr}$ ]<sup>T</sup>,<br> $\ell^s$  in spanning tree<br> $\ell \in \mathbb{Z}^s$ , where  $\mathbf{u}^l$  is<br>s to seek is status indicators, or construct relaxation constraints with variables automatically<br>satisfied for a disconnected line. Since these convex relaxation methods suffer from<br>loosened relaxation bounds, there is a research gap satisfied for a disconnected line. Since these convex relaxation methods suffer from<br>loosened relaxation bounds, there is a research gap to explore the efficient and tight<br>relaxation approach for DNR problems.<br>For the sak loosened relaxation bounds, there is a research gap to explore the eff-<br>relaxation approach for DNR problems.<br>
For the sake of the minimal power loss in reconfigurable D?<br>
optimization variables involves a set of operatio m l  $\binom{l}{mn}$  -  $|z^l_{mn}|^2 \ell^l_{mn}$ n bounds, there is a research gap to explore the efficient and tight<br>
if the minimal power loss in reconfigurable DNs, the set of<br>
les involves a set of operation variables  $x^l := [P, Q^l, \ell^l, v, Q^{\alpha}]^T$ ,<br>
muous parent-child  $P_{mn}^l = Q_{mn}^l = 0$ ,  $\ell^l = 0$  and  $\underline{v} \leq$ roach for DNR problems.<br>
ke of the minimal power loss in reconfigurable DNs, the set of<br>
ariables involves a set of operation variables  $x' := [P', Q', \ell', v, Q^{cr}]^T$ ,<br>
continuous parent-child relationship variable  $\beta' \in \mathbb{R}^{2\$ m l  $v_n - v_m + 2(r_{mn}^l P_{mn}^l + x_{mn}^l Q_{mn}^l) - |z_{mn}^l|^2 \ell_{mn}^l = 0$  may For the sake of the minimal power loss in reconfigurable DNs, the set of<br>optimization variables involves a set of operation variables  $x' := [P', Q', \ell', v, Q'']$ ,<br> $x' \in \mathbb{R}^n$ , the continuous parent-child relationship variable optimization variables involves a set of operation variables  $x' := [P', Q', \ell', v, Q^{\alpha}]^T$ ,<br>  $x' \in \mathbb{R}^n$ , the continuous parent-child relationship variable  $\beta' \in \mathbb{R}^{2\varepsilon}$  in spanning tree<br>
constraints, and binary state v  $x^l \in \mathbb{R}^n$ , the continuous parent-child relationship variable  $\beta^l \in \mathbb{R}^{2\epsilon}$  in spanning tree<br>constraints, and binary state vector of circuit breakers  $u^l \in \{0, 1\}$ ,  $u^l \in \mathbb{Z}^{\epsilon}$ , where  $u^l$  is<br>zero if constraints, and binary state vector of circuit breakers  $u^l \in \{0, 1\}$ ,  $u^l \in \mathbb{Z}^{\ell}$ , where  $u^l$  is<br>zero if the switch is open and one if closed. The DNR model is to seek the loss<br>minimization over *DistFlow* equa

According to [14], continuous parent-child relationship variables  $\beta^l$  are proved to the converged as binary solutions from real-valued continuous numbers. Inspired by a characteristic, we can ideally link disjunctive p <sup>l</sup> are proved to<br>
ers. Inspired by<br>
hip variables  $\beta_l$ According to [14], continuous parent-child relationship variables  $\beta^l$  are proved to get converged as binary solutions from real-valued continuous numbers. Inspired by this characteristic, we can ideally link disjunctiv According to [14], continuous parent-child relationship variables  $\beta^i$  are proved to<br>get converged as binary solutions from real-valued continuous numbers. Inspired by<br>this characteristic, we can ideally link disjunctiv According to [14], continuous parent-child relationship variables  $\beta^l$  are proved to<br>get converged as binary solutions from real-valued continuous numbers. Inspired by<br>this characteristic, we can ideally link disjunctiv (According to [14], continuous parent-child relationship variables  $\beta^i$  are proved to get converged as binary solutions from real-valued continuous numbers. Inspired by this characteristic, we can ideally link disjuncti According to [14], continuous parent-child relationship variables  $\beta$  are proved to<br>get converged as binary solutions from real-valued continuous numbers. Inspired by<br>this characteristic, we can ideally link disjunctive According to [14], continuous parent-child relationship variables  $\beta^i$  are proved to<br>get converged as binary solutions from real-valued continuous numbers. Inspired by<br>this characteristic, we can ideally link disjunctiv  $l = 1$  and  $l$ ild relationship variables  $\beta^l$  are proved to<br>1-valued continuous numbers. Inspired by<br>netive parent-child relationship variables  $\beta_l$ <br>a the disjunctive convex hull relxations<br>an arbitrary branch  $l := (m, n)$  with<br>ding on mumbers  $\beta^l$  are proved to<br>ontinuous numbers. Inspired by<br>nnt-child relationship variables  $\beta_l$ <br>unctive convex hull relxations<br>ary branch  $l := (m, n)$  with<br>nrent-child relationship variables<br> $\frac{l}{mn} = 0$ , or  $v_m - v_n < 0$  is with  $\beta_{mn}^{\ell}$ =1 and  $\beta_{nm}^{\ell}$ =0. Mathematically, ording to [14], continuous parent-child relationship variables  $β'$  are proverged as binary solutions from real-valued continuous numbers. Insparance and a binary solutions from real-valued continuous numbers. Insparance [14], continuous parent-child relationship variables  $\beta^i$  are proved to<br>s binary solutions from real-valued continuous numbers. Inspired by<br>c, we can ideally link disjunctive parent-child relationship variables  $\beta_i$ <br>qu According to [14], continuous parent-child relationship variables  $\beta'$  are proved to<br>t converged as binary solutions from real-valued continuous numbers. Inspired by<br>scharacteristic, we can ideally link disjunctive paren get converged as binary solutions from real-valued continuous numbers. Inspired by<br>this characteristic, we can idcally link disjunctive parent-child relationship variables  $\beta_i$ <br>with "on/off" equality (1.6) and (1.7) via get converged as binary solutions from real-valued continuous numbers. Inspired by<br>this characteristic, we can ideally link disjunctive parent-child relationship variables  $\beta_i$ <br>with "on/oft" equality (1.6) and (1.7) via i ideally link disjunctive parent-child relationship variables *βi*<br>
(1.6) and (1.7) via the disjunctive convex hull relxations<br>
other words, if an arbitrary branch *l* := (*m*, *n*) with<br>
w directions depending on pare with "on/off" equality (1.6) and (1.7) via the disjunctive convex hull relxations<br>
(DCHR) approach. In other words, if an arbitrary branch  $l := (m, n)$  with<br>
active/reactive power flow directions depending on parent-child r

<sup>l</sup>,  $\beta$ <sup>l</sup>,  $u$ <sup>l</sup>). For an arbitrary branch  $l := (n)$  $\mathcal{Q}_{mn}^{l}$  and  $\mathcal{Q}_{mn}^{l}$  simultaneously depend on the combination of  $\beta_{mn}^{l}$  and  $\beta_{mn}^{l}$ , **Theorem:** Let  $\Omega'$  be the feasible set of the DNR problem with respect to<br>optimization variables  $(x', \beta', u')$ . For an arbitrary branch  $l := (m, n)$  for  $\forall (m, n) \in \mathcal{N}$ ,<br>if the signs of  $P'_{nn}$  and  $Q'_{nn}$  simultaneously dep poptimization variables  $(x', \beta', u')$ . For an arbitrary branch  $l := (m, n)$  for  $\forall (m, n) \in \mathcal{N}$ ,<br>
if the signs of  $P_m^l$  and  $Q_m^l$  simultaneously depend on the combination of  $\beta_m^l$  and  $\beta_m^l$ ,<br>
then  $\Omega^l$  can be expressed

$$
\Omega^{l} = \left\{ (\mathbf{x}^{l}, \mathbf{\beta}^{l}, \mathbf{u}^{l}) \in \mathbb{R}^{n+2|\varepsilon|} \times \mathbb{Z}^{2|\varepsilon|} \middle| \begin{matrix} (2.4) - (2.7) \\ (3.4) - (3.7) \\ (v_{m} - v_{n}) \cdot \beta_{nm}^{l} > 0 \\ \forall (v_{m} - v_{n}) \cdot \beta_{nm}^{l} < 0 \\ \forall (v_{m} - v_{n}) \cdot u_{mn}^{l} = 0 \end{matrix} \right\}
$$
(3.15)

if the signs of  $P'_{\text{nn}}$  and  $Q'_{\text{nn}}$  simultaneously depend on the combination of  $β'_{\text{nn}}$  and  $β'_{\text{nn}}$ ,<br>
then  $Ω'$  can be expressed as:<br>  $\Omega' = \begin{cases} (x', β', u') \in \mathbb{R}^{n-2[d]} \times \mathbb{Z}^{2[d]} \begin{pmatrix} (2.4) - (2.7) \\ (3.4) - (3.7) \\$  $l-1$  and  $l = 0$  Due to  $l^l$ by depend on the combination of  $\beta'_{mn}$  and  $\beta'_{mn}$ ,<br>  $\begin{vmatrix} (2.4) - (2.7) \\ (3.4) - (3.7) \\ (3.4) - (3.7) \\ \n\sqrt{(v_m - v_n) \cdot \beta'_{mn}} > 0 \\ \n\sqrt{(v_m - v_n) \cdot u_{mn}^l} = 0 \end{vmatrix}$  (3.15)<br>
disjunction.<br>
ional branch power flow from parent node *n*<br> nm=0. Due to <sup>ℓ</sup> ation of  $\beta_{nm}^i$  and  $\beta_{mn}^i$ ,<br>  $\begin{cases}\n0 \\
\vdots \\
0\n\end{cases}$ (3.15)<br>  $\begin{cases}\n(3.15) \\
\vdots \\
0\n\end{cases}$ (3.15)<br>
from parent node *n*<br>  $\begin{cases}\n\frac{i}{mn} > 0, \text{ then we have} \\
\frac{m}{mn} - \left|\frac{z}{mn}\right| \cdot \frac{2t}{mn} < 0\n\end{cases}$  $-|z_{mn}|^2 \ell_{mn}^1 < 0$ . And directional branch power flow from *n* to node *m* means that  $P_{mn}^1 < 0$ mn < 0. And directional branch power flow from <sup>n</sup> to node <sup>m</sup> means that <sup>P</sup> and  $Q_{mn}^l$ <0. Subsequently, it is derived that  $v_m - v_n = 2R_{mn}P_{mn}^l + 2X_{mn}Q_{mn}^l - |z_{mn}^l|^2 \ell_{mn}^l$  < 0  $\Omega' = \begin{cases} (2.4)-(2.7) \\ (3.4)-(3.7) \\ (x', \beta', \mu') \in \mathbb{R}^{n+2|d|} \times \mathbb{Z}^{2|d|} \begin{cases} (2.4)-(2.7) \\ (y_m-y_n) \cdot \beta'_{nn} > 0 \\ (y_m-y_n) \cdot \beta'_{nn} < 0 \end{cases} \end{cases}$  (3.15)<br>  $\forall$  denotes the logic operator for disjunction.<br>
Suppose that there exists a dir (3.15)<br>
mode *n*<br>
e have<br>  $P_{mn}^l < 0$ <br>  $\frac{l}{mn} < 0$ <br>
es  $v_m$  –  $\Omega' = \begin{cases} (x^i, \beta^i, u^i) \in \mathbb{R}^{n+2[d]} \times \mathbb{Z}^{2[d]} \quad (3.4) - (3.7) \\ (y_m - v_n) \cdot \beta^i_{mn} < 0 \\ \sqrt{(y_m - v_n) \cdot \beta^i_{mn}} = 0 \end{cases}$  (3.15)<br>
where  $\vee$  denotes the logic operator for disjunction.<br> *Proof:* Suppose that there exists a dire vn contradicts vm − v<sub>n</sub>>0 as the initial assumption. Similarly, this theorem v<sub>n</sub> − 0, which contradicts v<sub>m</sub> − v<sub>n</sub>>0 as the initial assumption.<br>
Simples variable values of the initial assumption.<br>
Simples that there

can be also proven for which  $v_m - v_n < 0$  is consistent with  $\beta_{mn}^i = 1$  and  $\beta_{nm}^i = 0$ . If  $u_{mn}^i = 0$ ,<br>then  $\beta_{nm}^i = \beta_{mn}^i = 0$  by (3.15) and  $v_m - v_n$  can be free.  $\int_{mn}^{l} = 1$  and  $\beta_{nm}^{l} = 0$ . If  $u_{mn}^{l} = 0$ ,  $m_{mn} = 1$  and  $\beta_{nm}^{l} = 0$ . If  $u_{mn}^{l} = 0$ ,  $u'_{nm}=0.$  If  $u'_{mn}=0$ , then  $\beta_{nm}^l = \beta_{mn}^l = 0$  by (3.15) and  $v_m - v_n$  can be free. Let also proven for which  $v_m - v_n < 0$  is consistent with  $\beta_{mn}^t = 1$  and  $\beta_{nm}^t = 0$ . If  $u_{mn}^t = 0$ ,<br>  $\beta_{nm}^l = \beta_{mn}^l = 0$  by (3.15) and  $v_m - v_n$  can be free. ■<br>
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At this theorem's framework, it is found that no ad

Theorem's framework  $v_m - v_n < 0$  is consistent with  $\beta'_{mn} = 1$  and  $\beta'_{nm} = 0$ . If  $u'_{mn} = 0$ ,<br>  $\beta'_{nm} = \beta'_{mn} = 0$  by (3.15) and  $v_m - v_n$  can be free.<br>
With this theorem's framework, it is found that no additional disjuncti can be also proven for which  $v_m - v_n < 0$  is consistent with  $\beta'_{mn} = 1$  and  $\beta'_{mn} = 0$ . If  $u'_{mn} = 0$ ,<br>then  $\beta'_{nm} = \beta'_{nm} = 0$  by (3.15) and  $v_m - v_n$  can be free.<br>With this theorem's framework, it is found that no addition can be also proven for which  $v_m - v_n < 0$  is consistent with  $\beta'_{mn} = 1$  and  $\beta'_{nm} = 0$ . If  $u'_{mm} = 0$ ,<br>then  $\beta'_{nm} = \beta'_{mn} = 0$  by (3.15) and  $v_{m} - v_{n}$  can be free.<br>With this theorem's framework, it is found that no addi can be also proven for which  $v_m - v_n < 0$  is consistent with  $\beta'_{nn} = 1$  and  $\beta'_{nn} = 0$ . If  $u^1_{nn} = 0$ ,<br>then  $\beta'_{nn} = \beta^1_{nn} = 0$  by (3.15) and  $v_m - v_n$  can be free.<br>With this theorem's framework, it is found that no additi can be also proven for which  $v_m - v_n < 0$  is consistent with  $\beta'_{nn} = 1$  and  $\beta'_{nn} = 0$ . If  $u'_{nn} = 0$ ,<br>then  $\beta'_{nm} = \beta'_{nm} = 0$  by (3.15) and  $v_{m} - v_{n}$  can be free.<br>With this theorem's framework, it is found that no addi can be also proven for which  $v_m - v_n < 0$  is consistent with  $\beta'_{nn} = 1$  and  $\beta'_{nn} = 0$ . If  $u'_{nm} = 0$ ,<br>then  $\beta'_{nm} = \beta'_{nm} = 0$  by (3.15) and  $v_{m} - v_{n}$  can be free.<br>With this theorem's framework, it is found that no addi can be also proven for which  $v_m - v_n < 0$  is consistent with  $\beta'_{m} = 1$  and  $\beta'_{m} = 0$ . If  $u'_{mn} = 0$ ,<br>then  $\beta'_{nm} = \beta'_{nm} = 0$  by (3.15) and  $v_m - v_n$  can be free.<br>With this theorem's framework, it is found that no addition  $\beta_{mn}^{\ell} = 1$ ,  $\beta_{nm}^{\ell} = 0$ . It is known that leading  $β'_{nm} = β'_{mn} = 0$  by (3.15) and  $ν_m − ν_n$  can be free.<br>
With this theorem's framework, it is found that no additional disjunct<br>
re incorporated for tighter relaxations, but it can guarantee a more rapid<br>
nan using  $= \beta_{\text{nw}}^i = 0$  by (3.15) and  $v_m-v_n$  can be free. <br>
is theorem's framework, it is found that no additional disjunctive variables<br>
orated for tighter relaxations, but it can guarantee a more rapid convergence<br>
<sup>2</sup> Big-M With this theorem's framework, it is found that no additional disjunctive vari<br>are incorporated for tighter relaxations, but it can guarantee a more rapid converg<br>than using Big-M and McCormick linearization method. Indee  $(v_m - v_n) \cdot u_{mn}^l \neq 0$ ,  $(v_m - v_n) \cdot \beta_{mn}^l = 0$  and  $(v_m - v_n) \cdot \beta_{nm}^l < 0$  disobeys are incorporated for tighter relaxations, but it can guarantee a more rapid convergence<br>than using Big-M and McCormick linearization method. Indeed, if there exists a DN<br>without any reactive compensation or DERs injection than using Big-M and McCormick linearization method. Indeed, if there exists a DN<br>without any reactive compensation or DERs injection, the enforced disjunctive<br>constraint naturally holds for each branch since the voltage without any reactive compensation or DERs injection, the enforced disjunctive<br>constraint naturally holds for cach branch since the voltage profile at the root node is<br>the only highest. For a branch *l*:=  $(m, n)$  with addit constraint naturally holds for each branch since the voltage profile at the root node is<br>the only highest. For a branch  $l := (m, n)$  with additional reactive compensation or<br>DERs injection at the ending node n, this theorem the only highest. For a branch  $l := (m, n)$  with additional reactive compensation or<br>
DERs injection at the ending node *n*, this theorem may not hold due to  $v_m - v_n < 0$  and<br>  $\theta_m^{\mu} = 1$ ,  $\theta_m^{\mu} = 0$ . It is known that this DERs injection at the ending node *n*, this theorem may not hold due to  $v_w - v_w < 0$  and  $\beta_{\text{new}}^L = 1$ ,  $\beta_{\text{new}}^L = 0$ . It is known that this *l* is a connected branch with parent node m and child node *n*, but  $(v_m - v_n) \cdot$  $\beta_{\text{inc}}^{\mu} = 1$ ,  $\beta_{\text{inc}}^{\mu} = 0$ . It is known that this *l* is a connected branch with parent node m and child node *n*, but  $(v_m - v_n) \cdot u_{\text{min}}^{\mu} \neq 0$ ,  $(v_m - v_n) \cdot \beta_{\text{min}}^{\mu} = 0$  and  $(v_m - v_n) \cdot \beta_{\text{min}}^{\mu} < 0$  disobeys child node *n*, but  $(v_m - v_n) \cdot u'_{mn} \neq 0$ ,  $(v_m - v_n) \cdot \beta'_{mn} = 0$  and  $(v_m - v_n) \cdot \beta'_{mn} < 0$  disobeys<br>this **Theorem**. Therefore, if a branch has a bi-directional power flow caused by<br>reactive compensation or DERs injection, any this **Theorem**. Therefore, if a branch has a bi-directional power flow caused by<br>reactive compensation or DERs injection, any enforced disjunctive constraints<br>between parent-child relationship variables and voltage drops reactive compensation or DERs injection, any enforced disjunctive constraints<br>between parent-child relationship variables and voltage drops in  $(3.15)$  should be<br>relaxed. Otherwise, additional disjunctive variables should between parent-child relationship variables and voltage drops in (3.15) should be<br>relaxed. Otherwise, additional disjunctive variables should be incorporated to replace<br>parent-child relationship variables in (3.15). Exclud

corresponding disjunctive constraints between parent-child relationship variables and<br>voltage drops in (1.6).<br>For (1.6), we define  $\Omega_{1, mn}$  and  $\Omega_{1, mn}$  as the disjunctive convex sets for (3.15)

Corresponding disjunctive constraints between parent-child relationship variables<br>voltage drops in (1.6).<br>For (1.6), we define  $\Omega_1$ , mn and  $\Omega_1$ , mn as the disjunctive convex sets for (3)<br>integrated with  $β'_{mn} = 1$  a Fresponding disjunctive constraints between parent-child relationship variables and<br>
Itage drops in (1.6).<br>
For (1.6), we define  $\Omega_{1, \text{ mm}}$  and  $\Omega_{1, \text{ mm}}$  as the disjunctive convex sets for (3.15)<br>
egrated with  $\beta_{nn}$ corresponding disjunctive constraints between parent-child relationship varial<br>voltage drops in (1.6).<br>For (1.6), we define  $\Omega_1$ , <sub>nm</sub> and  $\Omega_1$ , <sub>nm</sub> as the disjunctive convex sets fo<br>integrated with  $\beta'_{mn} = 1$  and  $l = 1$  and  $l^l$ sjunctive constraints between parent-child relationship variables and<br>
1.6).<br>
define  $\Omega_1$ ,  $_{mn}$  and  $\Omega_1$ ,  $_{nm}$  as the disjunctive convex sets for (3.15)<br>  $_{mn}^l = 1$  and  $\beta_{mn}^l = 1$  for branch  $l := (m, n)$ , respectivel mstraints between parent-child relationship variables and<br>
mn and  $\Omega_1$ ,  $nm$  as the disjunctive convex sets for (3.15)<br>  $\frac{l}{nm} = 1$  for branch  $l := (m, n)$ , respectively. Subsequently,<br>
on of two convex sets  $\Omega_{nm}^{0,1}$  an corresponding disjunctive constraints between parent-child relationship variables and<br>voltage drops in (1.6).<br>
For (1.6), we define Ω<sub>1, mn</sub> and Ω<sub>1, mn</sub> as the disjunctive convex sets for (3.15)<br>
integrated with  $\beta_m^t$ t-child relationship variables and<br>
isjunctive convex sets for (3.15)<br> *m*, *n*), respectively. Subsequently,<br>
<sup>01,</sup> and  $\Omega_{nm}^{11}$  corresponding to  $\beta_{nm}^l =$ <br>  $\frac{1}{2} \beta_{nm}^{l} = 0$ .<br>  $\frac{1}{2} \left( \Omega_{1nm}^{0} \cup \Omega_{1nm}^{1} \right)$  elationship variables and<br>
e convex sets for (3.15)<br>
spectively. Subsequently,<br>  $\lim_{n=1}^{1,1}$  corresponding to  $\beta_{nm}^l =$ <br>  $\cup \Omega_{1,nm}^l$  (3.16)  $\Omega_{1,mn}$  can be written as a union of two convex sets  $\Omega_{mn}^{0,1}$  and  $\Omega_{nm}^{1,1}$  corresponding to  $\beta_{nm}^l$  $\frac{n}{nm}$  = corresponding disjunctive constraints between parent-child relationsh<br>
voltage drops in (1.6).<br>
For (1.6), we define  $\Omega_1$ , mm and  $\Omega_1$ , nm as the disjunctive convex<br>
integrated with  $β'_m = 1$  and  $β'_m = 1$  for branch  $l_{mn}^l = 0$ . This is similar to  $\Omega_{1, nm}$  under  $\beta_{nm}^l = 1$  and  $\beta_{nm}^l = 0$ . onding disjunctive constraints between parent-child relationship variables and<br>drops in (1.6).<br>(1.6), we define  $\Omega_{1, mm}$  and  $\Omega_{1, mm}$  as the disjunctive convex sets for (3.15)<br>led with  $\beta_{nn}^{\ell} = 1$  and  $\beta_{nn}^{\ell} = 1$  orial parametrical diversion by variables and<br>
s the disjunctive convex sets for (3.15)<br>
ch *l* := (*m*, *n*), respectively. Subsequently,<br>
x sets Ω<sup>0,1,</sup> and Ω<sup>1,1,</sup> corresponding to β<sup>*l<sub>um</sub>*</sup> =<br> *l*<sub>um</sub> = 1 and β<sup>*l<sub></sup>*</sub> Itage drops in (1.6).<br>
For (1.6), we define Ω<sub>1, mn</sub> and Ω<sub>1</sub>, <sub>mn</sub> as the disjunctive convex sets for (3.15)<br>
egrated with  $\beta'_{m} = 1$  and  $\beta'_{m} = 1$  for branch  $l := (m, n)$ , respectively. Subsequently,<br>
<sub>*m*n</sub> can be wr For (1.6), we define  $\Omega_{1, mm}$  and  $\Omega_{1, mm}$  as the disjunctive convex sets fitted with  $\beta_{nm}^i = 1$  and  $\beta_{nm}^i = 1$  for branch  $l := (m, n)$ , respectively. Subs<br>  $\Omega_{1, mn}$  can be written as a union of two convex sets  $\Omega_{nm$ define  $\Omega_1$ ,  $_{mn}$  and  $\Omega_1$ ,  $_{mn}$  as the disjunctive convex sets for (3.15)<br>  $\Omega_m^{-1} = 1$  and  $\beta_m^{i} = 1$  for branch  $l := (m, n)$ , respectively. Subsequently,<br>  $\Omega_m^{-1}$  on as a union of two convex sets  $\Omega_m^{0,i}$  and  $\Omega_m$ sets for (3.15)<br>
Subsequently,<br>
onding to  $\beta_{nm}^i =$ <br>
(3.16)<br>
(3.17)<br>
(3.18)<br>
h a continuous<br>
"')-space in Fig.<br>
edral sets with<br>
Note that the

$$
\Omega_{1,mn} = \text{Conv}(\Omega_{1,mn}^0 \cup \Omega_{1,mn}^1), \ \Omega_{1,mn} = \text{Conv}(\Omega_{1,mn}^0 \cup \Omega_{1,mn}^1) \tag{3.16}
$$

$$
\Omega_{1,mn}^0, \Omega_{1,nm}^0 = \{ \boldsymbol{x}^l \in \mathbb{R}^n \mid \underline{\boldsymbol{\nu}} \leq \boldsymbol{\nu} \leq \overline{\boldsymbol{\nu}}, \ell_{mn}^l, P_{mn}^l, Q_{mn}^l = 0 \}
$$
(3.17)

$$
\Omega_{1,mn}^1, \Omega_{1,nm}^1 = \{ \mathbf{x}^l \in \mathbb{R}^n \mid (1.6), \mathbf{y} \le \mathbf{v} \le \overline{\mathbf{v}}, 0 \le \ell_{mn}^l \le \overline{\ell}^l \}
$$
(3.18)

 $\beta_{mn}^l \in [0, 1]$  on  $(v_m - v_n, 2R_{mn}P_{mn}^l + 2X_{mn}Q_{mn}^l - |z_{mn}^l|^2 \ell_{mn}^l, \beta^l)$ -space in Fig. integrated with  $\beta'_{mn} = 1$  and  $\beta'_{mn} = 1$  for branch  $l := (m, n)$ , respectively. Subsequently,<br>  $\Omega_{1, mn}$  can be written as a union of two convex sets  $\Omega_{nm}^{s1}$  and  $\Omega_{nm}^{11}$  corresponding to  $\beta'_{nm} =$ <br>  $1$  and  $\beta'_{mn}$  $\Omega_{1,mn}$  can be written as a union of two convex sets  $\Omega_{1,m}^{2+}$  and  $\Omega_{1,m}^{1+}$  corresponding to  $\beta_{m}^{l} =$ <br>
1 and  $\beta_{m}^{l} = 0$ . This is similar to  $\Omega_{1,nm}$  under  $\beta_{m}^{l} = 1$  and  $\beta_{m}^{l} = 0$ .<br>  $\Omega_{1,mn} = \text{Conv}$ 1 and  $β'_{nn} = 0$ . This is similar to Ω<sub>1,an</sub> under  $β'_{n}=1$  and  $β'_{n}=0$ .<br>  $Ω_{1,mn} = Conv(Ω^{0}_{1,m})$ ,  $Ω_{1,mn} = Conv(Ω^{0}_{1,m} ∪ Ω^{1}_{1,mn})$  (3.16)<br>  $Ω^{0}_{1,mn}, Ω^{1}_{1,mn} = {x' ∈ ℝ<sup>n</sup> | ⊻ ≤ ν ≤ Γ, θ'_{mn}, Π''_{mn}, Q''_{mn} = 0}$  (3.17)<br>  $Ω'_{1,mn}, Ω^{1}_{1$ former vertical coordinate axis is upward  $\beta_{nm}^{\prime}$  while the latter vertical axis is downward  $\beta_{nm}^{l} = 1$  and  $\beta_{nm}^{l} = 0$ .<br>  $\sum_{1, mn} = \text{Conv}(\Omega_{1, mn}^{0} \cup \Omega_{1, mn}^{l})$  (3.16)<br>  $\sum_{i} \overline{v}_{i} e_{mn}^{l}, P_{mn}^{l}, Q_{mn}^{l} = 0$ } (3.17)<br>  $\sum_{i} v_{i} \leq \overline{v}_{i} \in \overline{v}_{i}$ ,  $P_{mn}^{l}, Q_{mn}^{l} \leq \overline{c}^{l}$ } (3.18)<br>
Soure of  $\Omega_{1, mn}$  a  $\beta_{mn}^{\mu}$ . Clearly, the hyperplane  $\psi_1$  with  $\Omega_{1,mn} = \text{Conv}(\Omega_{1,mn}^0 \cup \Omega_{1,mn}^1)$ ,  $\Omega_{1,mn} = \text{Conv}(\Omega_{1,mn}^0 \cup \Omega_{1,mn}^1)$  (3.16)<br>  $\Omega_{1,mn}^0, \Omega_{1,mn}^0 = \{x' \in \mathbb{R}^n \mid \underline{v} \le v \le \overline{v}, \ell'_{mn}, P'_m, Q'_m = 0\}$  (3.17)<br>  $\Omega_{1,mn}^1, \Omega_{1,mn}^1 = \{x' \in \mathbb{R}^n \mid (1.6), \underline{v} \le v \le \overline{v$  $Ω_{1,mn}^{0}$ ,  $Ω_{1,nm}^{0} = {x' ∈ ℝ<sup>n</sup> | y ≤ ν ≤ Ψ,  $\ell_{mn}^{0}P_{mn}^{0}/P_{mn}^{0} ≤ \ell_{n}^{0}} = 0$  (3.17)<br>  $Ω_{1,nm}^{1}$ ,  $Ω_{1,nm}^{1} = {x' ∈ ℝ<sup>n</sup> | (1.6), y ≤ ν ≤ ν, θ ≤  $\ell_{mn}^{0} ≤ \ell^{0}$ ) (3.18)<br>
In this vein, we sketch the geometry closure of  $Ω_{1,$$$  $\Omega'_{1,m}, \Omega^1_{1,nm} = \{x^i \in \mathbb{R}^n \mid (1.6), y \le v \le \overline{v}, 0 \le \ell'_{1,m} \le \overline{\ell}'\}$  (3.18)<br>
In this vein, we sketch the geometry closure of  $\Omega_{1,mn}$  and  $\Omega_{1,mn}$  with a continuous<br>
variation of  $\beta'_{nn}$ ,  $\beta'_{nn} \in [0, 1]$  on  $(v$ In this vein, we sketch the geometry closure of  $\Omega_{1,mm}$  and  $\Omega_{1,mm}$  with a continuous<br>variation of  $\beta_{\alpha\alpha}^i, \beta_{\alpha\alpha}^i \in [0, 1]$  on  $(v_m - v_n, 2R_{mn}P_{\alpha\alpha}^i + 2X_{mn}Q_{\alpha\alpha}^i - |z_{\alpha\alpha}^i|^2|^2|^2_{\alpha\alpha\gamma} \beta^i$ )-space in Fig. variation of  $\beta_{\text{inv}}^i \beta_{\text{inv}}^i$   $\in [0, 1]$  on  $(v_m^- v_n, 2R_{\text{inv}} P_{\text{inv}}^i + 2X_{\text{inv}} Q_{\text{inv}}^i - |z_{\text{inv}}^i|^2 \ell_{\text{inv}}^i \beta^i$ )-space in Fig.<br>
3.4 (a). The geometry closure of  $\Omega_{1, \text{inv}}$  and  $\Omega_{1, \text{inv}}$  are disjunctive pol 3.4 (a). The geometry closure of  $\Omega_{1,,mn}$  and  $\Omega_{1,amn}$  are disjunctive polyhedral sets with<br>vertexes  $V_1 - V_4$  and  $V_1$ ,  $V_5 - V_7$  in blue and green CHs, respectively. Note that the<br>former vertical coordinate axis is vertexes  $V_1 - V_4$  and  $V_1$ ,  $V_5 - V_7$  in blue and green CHs, respectively. Note that the<br>former vertical coordinate axis is upward  $\beta_{\text{on}}^l$  while the latter vertical axis is downward<br>former vertical coordinate axis former vertical coordinate axis is upward  $\beta'_{\text{inv}}$  while the latter vertical axis is downward  $\beta'_{\text{inv}}$ . Clearly, the hyperplane  $\psi_1$  with vertexes  $V_1$ ,  $V_2$  and  $V_3$  is at the bottom of the blue polyhedral hul

Subsequently, the third constraint in (3.19) using  $u_{mn}^l = \beta_{mn}^l + \beta_{nm}^l$  can represent the top hyperplane with vertexes  $V_1$ ,  $V_2$ ,  $V_3$  or  $V_5$ ,  $V_6$ ,  $V_7$ . These linear constraints from equations of the above-m  $u_{mn}^l = \beta_{mn}^l + \beta_{nm}^l$  can represent the<br>V<sub>7</sub>. These linear constraints from Subsequently, the third constraint in (3.19) using  $u'_{mn} = \beta'_{mn} + \beta'_{nm}$  can represent the<br>top hyperplane with vertexes V<sub>1</sub>, V<sub>2</sub>, V<sub>3</sub> or V<sub>5</sub>, V<sub>6</sub>, V<sub>7</sub>. These linear constraints from<br>equations of the above-mentioned Subsequently, the third constraint in (3.19) using  $u_{mn}^l = \beta_{mn}^{l} + \beta_{nm}^{l}$  can represent the<br>top hyperplane with vertexes V<sub>1</sub>, V<sub>2</sub>, V<sub>3</sub> or V<sub>5</sub>, V<sub>6</sub>, V<sub>7</sub>. These linear constraints from<br>equations of the above-menti

Subsequently, the third constraint in (3.19) using 
$$
u'_{mn} = \beta_{mn}^l + \beta_{nm}^l
$$
 can represent the  
\ntop hyperplane with vertices V<sub>1</sub>, V<sub>2</sub>, V<sub>3</sub> or V<sub>5</sub>, V<sub>6</sub>, V<sub>7</sub>. These linear constraints from  
\nequations of the above-mentioned hyperplane:  
\n
$$
\begin{vmatrix}\n\frac{1}{\overline{v}-\underline{v}} \cdot A \cdot \nu - \frac{1}{\overline{v}-\underline{v}} \cdot (2D_r P^l + 2D_x Q^l - |z^l|^2 \ell^l) - \beta_{nm}^l \ge -1 \\
\frac{1}{\overline{v}-\underline{v}} \cdot A \cdot \nu - \frac{1}{\overline{v}-\underline{v}} \cdot (2D_r P^l + 2D_x Q^l - |z^l|^2 \ell^l) - \beta_{nm}^l \ge -1\n\end{vmatrix}
$$
\n(3.19)  
\nAs a DCHR formation via  $u^l = \beta_{mn}^{l-1} + \beta_{mn}^{l}$ , the disjunctive CH  $\Omega_1 = \text{Conv}(\Omega_1, nm \cup \Omega_1, mn)$   
\nof (1.6) and the enforced disjunctive constraint in (3.15) yields  
\n
$$
\Omega_1 = \{(x^l, \beta^l, u^l) \in \mathbb{R}^{n+2l} \times \mathbb{Z}^{|\alpha|} | (3.19) \text{ and } (3.21) - (3.23) \}
$$
\n(3.20)  
\nwhere corresponding perspective linear cuts are imposed to improve bounds of v and  
\n
$$
\ell^l
$$
 by  
\n
$$
(1 - \beta_{mn}^l)(v - \overline{v}) \le v_m - v_n \le \beta_{mn}^l(\overline{v} - v)
$$
\n(3.21)

 $l = \beta_{mn}^l + \beta_{nm}^l$ , the disjunctive CH  $\Omega_1 =$ Conv

$$
\Omega_1 = \{ (\mathbf{x}^t, \mathbf{\beta}^t, \mathbf{u}^t) \in \mathbb{R}^{n+2|\varepsilon|} \times \mathbb{Z}^{|\varepsilon|} \mid (3.19) \text{ and } (3.21) - (3.23) \}
$$
(3.20)

 $\ell^l$  by

$$
(1 - \beta_{nm}^l)(\underline{v} - \overline{v}) \le v_m - v_n \le \beta_{nm}^l(\overline{v} - \underline{v})
$$
\n(3.21)

$$
\beta_{mn}^l(\underline{v}-\overline{v}) \le v_m - v_n \le (1-\beta_{mn}^l)(\overline{v}-\underline{v})
$$
\n(3.22)

$$
\ell^l = \ell_+^l + \ell_-^l, \ 0 \le \ell_+^l \le \beta_{mn}^l \overline{\ell}^l, 0 \le \ell_-^l \le \beta_{nm}^l \overline{\ell}^l \tag{3.23}
$$



For (1.7) of the branch  $l := (m, n)$ , the possible CH of both (1.7) and the enforced sjunctive constraint in (3.15) can be expressed as the union of two points  $(x^l, u^l) = (0, 0, \nu, 0, 0)$  and  $(0, 0, 0, \overline{\nu}, 0, 0)$  and a con For (1.7) of the branch  $l := (m, n)$ , the possible CH of both (1.7) and the enforced<br>disjunctive constraint in (3.15) can be expressed as the union of two points  $(x^l, u^l) = (0, 0, 0, \nu, 0, 0)$  and  $(0, 0, 0, \overline{\nu}, 0, 0)$  and  $\ell^{l}$ , $u^{l}$  $\equiv$   $(0, 0)$ For (1.7) of the branch  $l := (m, n)$ , the possible CH of both (1.7) and the enforced disjunctive constraint in (3.15) can be expressed as the union of two points  $(x^l, u^l) = (0, 0, 0, v_1, 0, 0)$  and  $(0, 0, 0, \overline{v}, 0, 0)$  and For (1.7) of the branch  $l := (m, n)$ , the possible CH of both (1.7) and the enforced<br>disjunctive constraint in (3.15) can be expressed as the union of two points  $(x', u') = (0, 0, 0, \nu, 0, 0)$  and  $(0, 0, 0, \overline{\nu}, 0, 0)$  and a co For (1.7) of the branch  $l := (m, n)$ , the possible CH of both (1.7) and the enformulation<br>disjunctive constraint in (3.15) can be expressed as the union of two points  $(x', u')$ <br>0, 0,  $\frac{v}{v}$ , 0, 0) and (0, 0, 0,  $\overline{v}$ , 0, For (1.7) of the branch  $l := (m, n)$ , the possible CH of both (1.7) and the enforced<br>sjunctive constraint in (3.15) can be expressed as the union of two points  $(x^t, u^t) = (0,$ <br>0,  $y$ , 0, 0) and  $(0, 0, 0, \overline{v}, 0, 0)$  and a c For (1.7) of the branch  $l := (m, n)$ , the possible CH of both (1.7) and the enforced<br>disjunctive constraint in (3.15) can be expressed as the union of two points  $(x', u')=(0,$ <br>0, 0,  $v_x$ , 0, 0) and (0, 0, 0,  $\overline{v}$ , 0, 0) and (1.7) and the enforced<br>
order conic (SOC) set,<br>
ation of the quadratic<br>  $N$  (3.24)<br>
metry closure of this<br>  $\frac{1}{2}$ , 20 $\frac{1}{mn}$ ,  $\beta$ )-space as<br>
the second-order cone<br>  $\ell_{mn}^i \geq 0$  or  $\nu_m - \nu_n \leq 0$ Let enforced<br>  $\sum_{i=1}^{\infty} (x^i, u^i) = (0,$ <br>  $\sum_{i=1}^{\infty} (3.24)$ <br>
(3.24)<br>
The space as order cone<br>  $\sum_{i=1}^{\infty} (x^i - y^i)$   $\leq 0$ For (1.7) of the branch  $l := (m, n)$ , the possible CH of both (1.7) and the enforced<br>disjunctive constraint in (3.15) can be expressed as the union of two points  $(x^l u^l) = (0,$ <br>0, 0,  $v_1$ , 0, 0) and (0, 0, 0,  $\overline{v}$ , 0, 0)

$$
\| (2P_{mn}^l, 2Q_{mn}^l, v_m - \ell_{mn}^l)^T \|_2 \le v_m + \ell_{mn}^l, \forall m, n \in \mathcal{N}
$$
 (3.24)

 $\sum_{mn}^{l}$ , 2Q  $_{mn}^{l}$ ,  $\beta$ <sup>l</sup>)-space as disjunctive constraint in (3.15) can be expressed as the union of two points  $(x^t,u^t)=(0,$ <br>
0, 0,  $v_2$ , 0, 0) and (0, 0, 0,  $\overline{v}$ , 0, 0) and a continuous second-order conic (SOC) set,<br>
where this SOC set is drawn from a  $\sum_{mn}^{l} + 2Q_{mn}^{l} - |z_{mn}^{l}|^2 \ell_{mn}^{l} \ge 0$  or  $v_m - v_n \le 0$ d as the union of two points  $(x^l, u^l) = (0,$ <br>tinuous second-order conic (SOC) set,<br>ous SOC relaxation of the quadratic<br> $\int_{m}^{m} + \ell_{mn}^l, \forall m, n \in \mathcal{N}$  (3.24)<br>sualize the geometry closure of this<br>ive CH on  $(2P_{mn}^{l}, 2Q_{mn}$ From two points  $(x^l, u^l) = (0,$ <br>
order conic (SOC) set,<br>
tion of the quadratic<br>  $\sqrt{2}$ <br>
or  $(3.24)$ <br>
metry closure of this<br>  $\sqrt{2}$ ,  $2Q_{mn}^l$ ,  $\beta^l$ )-space as<br>
the second-order cone<br>
the second-order cone<br>  $\frac{l}{mn} \geq 0$ 0, 0,  $y_1$ , 0, 0) and (0, 0, 0,  $\overline{v}$ , 0, 0) and a continuous sccond-order conic (SOC) set,<br>where this SOC set is drawn from a continuous SOC relaxation of the quadratic<br>equality (1.7), yielding<br> $\|(2P_{\text{ms}}^i, 2Q_{\text{ms$ where this SOC set is drawn from a continuous SOC relaxation of the quadratic<br>equality (1.7), yielding<br> $\|(2P'_{ms}, 2Q'_{ms}, v_n - t'_{ms})^T\|_2 \le v_m + t'_{ms}, \forall m, n \in \mathcal{N}$  (3.24)<br>With the above-mentioned theorem, we visualize the geome equality (1.7), yielding<br>  $||(2P'_{mn}, 2Q'_{mn}, v_m - \ell'_{mn})^T||_2 \le v_m + \ell'_{mn}, \forall m, n \in \mathcal{N}$ <br>
With the above-mentioned theorem, we visualize the geometry close<br>
SOC-representable set by (3.24) as a disjunctive CH on  $(2P'_{mn}, 2Q'_{mn}, \beta$  $\beta_{nm}^{l}$  and  $\beta_{nm}^{l}$ , which is expressed as ding<br>  $\|(2P'_{mn}, 2Q'_{mn}, v_m - \ell'_{mn})^T\|_2 \le v_m + \ell'_{mn}, \forall m, n \in \mathcal{N}$  (3.24)<br>
-mentioned theorem, we visualize the geometry closure of this<br>
set by (3.24) as a disjunctive CH on  $(2P'_{mn}, 2Q'_{mn}, \beta')$ -space as<br>
(b). In this figure, With the above-mentioned theorem, we visualize the geometry elosure of this<br>
SOC-representable set by (3.24) as a disjunctive CH on  $(2P_w^i, 2Q_w^i, \beta^i)$ -space as<br>
shown in Fig. 3.4 (b). In this figure, the intersection b d a valid cutting plane by either  $v_m - v_n = 2P'_{\text{tot}} + 2Q'_{\text{tot}} - |z'_{\text{tot}}|^2 \ell'_{\text{tot}} > 0$  or  $v_m - v_n < 0$ <br>ntributes to forming a nearly half second-order cone. Evidently, this disjunctive<br>sure is tighter than a full secondcontributes to forming a nearly half second-order cone. Evidently, this disjunctive<br>closure is tighter than a full second-order cone. Let  $\Omega_2$  be a DCHR formation with<br>respect to  $\beta_{aa}^{\mu}$  and  $\beta_{aa}^{\mu}$ , which is ex

$$
\Omega_2 = \{ (\mathbf{x}^1, \mathbf{\beta}^1, \mathbf{u}^1) \in \mathbb{R}^{n+2|\varepsilon|} \times \mathbb{Z}^{|\varepsilon|} \mid (3.24) \text{ and } (3.26) \cdot (3.27) \}
$$
(3.25)

$$
0 \le 2D_r P^l + 2D_x Q^l - D_z \ell^l \le (\bar{v} - \underline{v}) \cdot \beta_{nm}^l \tag{3.26}
$$

$$
(\underline{v} - \overline{v}) \cdot \boldsymbol{\beta}_{mn}^l \le 2D_r \boldsymbol{P}^l + 2D_x \boldsymbol{Q}^l - D_z \ell^l \le 0
$$
 (3.27)

closure is tighter than a full second-order cone. Let  $\Omega_2$  be a DCHR formation with<br>respect to  $\beta_m^i$  and  $\beta_m^i$ , which is expressed as<br> $\Omega_2 = \{ (x^i, \beta^i, u^i) \in \mathbb{R}^{n+2d} \times \mathbb{Z}^d | (3.24) \text{ and } (3.26) - (3.27) \}$  (3.25) respect to  $\beta_{\text{max}}^{\mu}$  and  $\beta_{\text{max}}^{\mu}$  which is expressed as<br>  $\Omega_2 = \{ (x', \beta', u') \in \mathbb{R}^{n+2d} \times \mathbb{Z}^{d} \mid (3.24) \text{ and } (3.26) - (3.27) \}$ (3.25)<br>
where corresponding perspective linear cuts are given as<br>  $0 \le 2D_p P^l + 2D_s$ (3.25)<br>
(3.26)<br>
(3.27)<br>
ch is tighter than the<br>
or any branch *l* under<br>  $y' = u^l \cdot A \cdot v$  and then<br>
for connected and/or  $\Omega_2 = \{ (x', \beta', \mu') \in \mathbb{R}^{n+2\kappa} \times \mathbb{Z}^{|\kappa|} | (3.24) \text{ and } (3.26) - (3.27) \}$  (3.25)<br>
where corresponding perspective linear cuts are given as<br>  $0 \le 2D_r P' + 2D_s Q' - D_{\epsilon} \ell' \le (\bar{v} - \underline{v}) \cdot \beta'_{nm}$  (3.26)<br>  $(\underline{v} - \bar{v}) \cdot \beta'_{mn} \le$ 

 $y' - 2D_r P^{\prime} - 2D_x Q^{\prime} + D_z \ell^{\prime} = 0$ 

unconnected branches subject to  $A \cdot v + (\overline{v} - \underline{v}) \cdot (u' - 1) \le y' \le A \cdot v - (\overline{v} - \underline{v}) \cdot (u' - 1)$ .<br>It is clear that, due to the large-enough positive scalar number  $M \ge (\overline{v} - \underline{v})$ ,<br>McCormick linearization method has tighter method. connected branches subject to  $A \cdot v + (\overline{v} - \underline{v}) \cdot (u^t - 1) \le y^t \le A \cdot v - (\overline{v} - \underline{v}) \cdot (u^t - 1)$ .<br>
is clear that, due to the large-enough positive scalar number  $M \ge (\overline{v} - \underline{v})$ ,<br>
eCormick linearization method has tighter r unconnected branches subject to  $A \cdot v + (\overline{v} - \underline{v}) \cdot (u' - 1) \leq v' \leq A \cdot v - (\overline{v} - \underline{v}) \cdot (u' - 1)$ .<br>
It is clear that, due to the large-enough positive scalar number  $M \geq (\overline{v} - \underline{v})$ ,<br>
McCormick linearization method has tig subject to  $A \cdot v + (\overline{v} - \underline{v}) \cdot (u' - 1) \le y' \le A \cdot v - (\overline{v} - \underline{v}) \cdot (u' - 1)$ .<br>
to the large-enough positive scalar number  $M \ge (\overline{v} - \underline{v})$ ,<br>
ion method has tighter relaxation bounds of (1.6) than Big-M<br>
anch *l*, the lower bo connected branches subject to  $A \cdot v + (\overline{v} - \underline{v}) \cdot (u' - 1) \le y' \le A \cdot v - (\overline{v} - \underline{v}) \cdot (u' - 1)$ ,<br>is clear that, due to the large-enough positive scalar number  $M \ge (\overline{v} - \underline{v})$ ,<br>cCormick linearization method has tighter relaxa anconnected branches subject to  $A \cdot v + (\overline{v} - \underline{v}) \cdot (u' - 1) \le y' \le A \cdot v - (\overline{v} - \underline{v}) \cdot (u' - 1)$ .<br>
It is clear that, due to the large-enough positive scalar number  $M \ge (\overline{v} - \underline{v})$ ,<br>
McCormick linearization method has tighte It is clear that, due to the large-enough positive scalar number  $\Lambda$ <br>McCormick linearization method has tighter relaxation bounds of (1.6)<br>method.<br>For an arbitrary branch *l*, the lower bound of (1.6) by McCormick<br>method

$$
L_1^{lower} = A \cdot \mathbf{v} + (\overline{\mathbf{v}} - \underline{\mathbf{v}}) \cdot (\mathbf{u}^l - 1) - 2D_r \mathbf{P}^l - 2D_x \mathbf{Q}^l + \mathbf{D}_z \ell^l
$$
 (3.28)

 $L_2^{lower}$  is derived from the intersection set of first two inequality in (3.29), which is

$$
L_2^{lower} = A \cdot \mathbf{v} - 2D_r \mathbf{P}^1 - 2D_x \mathbf{Q}^1 + \mathbf{D}_z \ell^1
$$
 (3.29)

end as the difference in algebra between  $L_i^{lower}$  and  $L_i^{lower}$  and  $L_i^{lower}$  and  $L_i^{lower} = A \cdot v + (\overline{v} - \underline{v}) \cdot (u^l - 1) - 2D_z P^l - 2D_z Q^l + D_z \ell^l$  (3.28)<br>
However, in terms of DCHR, the lower bound of (1.6) for branch *l* denoted as  $L_1^{lower}$  and  $L_2^{lower}$  by  $L_1^{lower} - L_2^{lower} = (\overline{v} - \underline{v}) \cdot (\mathbf{u}^1 - 1)$ . Since  $u^1 \in \{0,1\}$ , this clearly renders  $L_1^{lower} - L_2^{lower} \le 0$ . nch *l*, the lower bound of (1.6) by McCormick linearization<br>  $A \cdot v + (\overline{v} - \underline{v}) \cdot (u' - 1) - 2D$ ,  $P' - 2D$ ,  $Q' + D$ <sub>s</sub> $\ell'$  (3.28)<br>
of DCHR, the lower bound of (1.6) for branch *l* denoted as<br>
the intersection set of first t method is denoted as  $L_1^{lower}$ , yielding<br>  $L_2^{lower} = A \cdot v + (\overline{v} - \underline{v}) \cdot (u^t - 1) - 2D_r P^t - 2D_x Q^t + D_z \ell^t$  (3.28)<br>
However, in terms of DCHR, the lower bound of (1.6) for branch *l* denoted as<br>  $L_2^{lower}$  is derived from the inte  $I_2^{lower} = A \cdot v + (\overline{v} - \underline{v}) \cdot (u^l - 1) - 2D_z P^l - 2D_x Q^l + D_z \ell^l$  (3.28)<br>
However, in terms of DCHR, the lower bound of (1.6) for branch *l* denoted as<br>
were is derived from the intersection set of first two inequality in (3.29 However, in terms of DCHR, the lower bound of (1.6) for branch *l* denoted as<br>  $I_2^{lower}$  is derived from the intersection set of first two inequality in (3.29), which is<br>
given by<br>  $I_2^{lower} = A \cdot \mathbf{v} - 2D_x \mathbf{P}' - 2D_x \mathbf{Q}' + \$  $L_2^{\text{lower}}$  is derived from the intersection set of first two inequality in (3.29), which is<br>given by<br>given by<br> $L_2^{\text{lower}} = A \cdot \nu - 2D_r P' - 2D_s Q' + D_z \ell'$  (3.29)<br>Hence, we can achieve the difference in algebra between  $L_1^{\text{lower$  $I_2^{lower} = A \cdot v - 2D_r P' - 2D_s Q' + D_s e'$  (3.29)<br>
Hence, we can achieve the difference in algebra between  $L_i^{lower}$  and  $L_i^{lower}$  by<br>  $I_2^{lower} = ( \overline{v} - \underline{v}) \cdot (\underline{u}' - 1)$ . Since  $\underline{u}' \in \{0,1\}$ , this clearly renders  $L_i^{lower} - L_i^{lower} \le 0$ .<br>

$$
\Phi_2 = {\Phi_1 \cap (v_m - v_n > 0)} \cup {\Phi_1 \cap (v_m - v_n < 0)} \cup {\Phi_1 \cap (v_m - v_n = 0)} \quad (3.30)
$$

Hence, we can achieve the difference in algebra between  $L_1^{\text{lower}}$  and  $L_2^{\text{lower}}$  by<br>  $L_2^{\text{lower}} - L_2^{\text{lower}} = (\bar{v} - \underline{v}) \cdot (\mathbf{u}^t - 1)$ . Since  $\mathbf{u}^t \in \{0,1\}$ , this clearly renders  $L_1^{\text{lower}} - L_2^{\text{lower}} \le 0$ .<br>
Similarl  $L_1^{\text{lower}} - L_2^{\text{lower}} = (\overline{v} - \underline{v}) \cdot (\mathbf{u}' - 1)$ . Since  $\mathbf{u}' \in \{0,1\}$ , this clearly renders  $L_1^{\text{lower}} - L_2^{\text{lower}} \le 0$ .<br>
Similarly, we can also achieve the upper bound of (1.6) by two methods are equal.<br>
Moreover, this M Similarly, we can also achieve the upper bound of (1.6) by two methods are equal.<br>
Morcover, this McCormick linearization method for (1.7) adopts a Lorentz cone  $L$ <br>
defined from (3.24). Let  $\Phi_1$  be the feasible set of

With tighter bounds, this DCHR approach can also converge at the optimal point<br>th achieving SOC equality for a load oversatisaction assumption and radial<br>tworks as proved in Subsection 2.4. Making use of the above, the los With tighter bounds, this DCHR approach can also converge at the optimal point<br>with achieving SOC equality for a load oversatisaction assumption and radial<br>networks as proved in Subsection 2.4. Making use of the above, the With tighter bounds, this DCHR approach can also converge at the optimal point<br>with achieving SOC equality for a load oversatisaction assumption and radial<br>networks as proved in Subsection 2.4. Making use of the above, the With tighter bounds, this DCHR approach can also converge at the optimal point<br>with achieving SOC equality for a load oversatisaction assumption and radial<br>networks as proved in Subsection 2.4. Making use of the above, the With tighter bounds, this DCHR approach can also converge at the optimal point<br>with achieving SOC equality for a load oversatisaction assumption and radial<br>networks as proved in Subsection 2.4. Making use of the above, th With tighter bounds, this DCHR approach can also converge at the optimal point<br>with achieving SOC equality for a load oversatisaction assumption and radial<br>networks as proved in Subsection 2.4. Making use of the above, th  $\bm{d}^l := \left[\bm{P}^l,\bm{Q}^l,\ell^l,\nu,\bm{Q}^{cr},\bm{w}^l,\bm{m}^l,\bm{f}^l,\bm{\beta}^l_{mn},\bm{\beta}^l_{nm},\bm{u}^l\right]^T, \,\,\forall l \in \mathcal{E}^l$ With tighter bounds, this DCHR approach can also converge at the optimal point<br>with achieving SOC equality for a load oversatisaction assumption and radial<br>networks as proved in Subsection 2.4. Making use of the above, th With tighter bounds, this DCHR approach can also converge at the optimal point<br>with achieving SOC equality for a load oversatisaction assumption and radial<br>networks as proved in Subsection 2.4. Making use of the above, th

$$
\begin{aligned}\n\min \quad P_0^t &\quad P_m^t + P_m^x - r_m^t e^t_{mn} = \sum_{k \leq x(n)} P_m^t + P_m^t, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N} \\
\frac{Q_m^t + Q_n^x - r_m^t e^t_{mn}}{\omega_m + \omega_m^x - \omega_m^x e^t_{mn}} &\quad \sum_{k \leq x(n)} Q_m^t + Q_m^t, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N} \\
\frac{1}{\overline{v} - \underline{v}} \cdot A \cdot \nu - \frac{1}{\overline{v} - \underline{v}} \cdot (2D_r P^t + 2D_x Q^t - |\underline{z}^t|^2 e^t) - \beta_m^t \geq -1 \\
\frac{1}{\overline{v} - \underline{v}} \cdot A \cdot \nu - \frac{1}{\overline{v} - \underline{v}} \cdot (2D_r P^t + 2D_x Q^t - |\underline{z}^t|^2 e^t) + u_m^t \geq 1 \\
\frac{1}{\overline{v} - \underline{v}} \cdot A \cdot \nu - \frac{1}{\overline{v} - \underline{v}} \cdot (2D_r P^t + 2D_x Q^t - |\underline{z}^t|^2 e^t) + u_m^t \geq 1 \\
(1 - \beta_m^t) (\underline{v} - \overline{v}) \leq v_m - v_n \leq \beta_m^t (\overline{v} - \underline{v}) \\
\beta_m^t (\underline{v} - \overline{v}) \leq \omega_m - v_n \leq (1 - \beta_m^t) (\overline{v} - \underline{v}) \\
e^t = e^t + e^t, 0 \leq e^t, \leq \beta_m^t \overline{v}, 0 \leq e^t \leq \beta_m^t \overline{v}^t \\
\frac{2Q_m^t}{\omega_m^t} &\quad \text{sn}^t, \forall l \in \mathcal{E}, \forall m, n \in \mathcal{N}\n\end{aligned}\n\tag{3.31}
$$
\n
$$
s.t. \begin{aligned}\n\frac{1}{\sqrt{v}} &\frac{1}{\sqrt{v}} &\frac{1}{\sqrt{v}} &\frac{1}{\sqrt{v}} &\frac{1}{\sqrt{v}} &\frac{1}{\sqrt{v}} &\frac{1}{\sqrt{v}} &\frac{1}{\sqrt{v}}
$$

where  $P_0^l$  is the active power injection at the root node 0.

3.4 Case Study<br>3.4.1 Simple 6-node DN<br>The following simple 6-node DN is used to exemplify this pol 3.4 Case Study<br>3.4.1 Simple 6-node DN<br>The following simple 6-node DN is used to exemplify this polyhedral<br>approximation of DNR model using Big-M relaxation method. The network is shown 4 Case Study<br>
1. *I Simple 6-node DN*<br>
The following simple 6-node DN is used to exemplify this polyhedral<br>
proximation of DNR model using Big-M relaxation method. The network is shown<br>
Fig. 3.5. The node 1 is the PCC bus, 3.4 Case Study<br>3.4.1 Simple 6-node DN<br>The following simple 6-node DN is used to exemplify this polyhedral<br>approximation of DNR model using Big-M relaxation method. The network is shown<br>in Fig. 3.5. The node 1 is the PCC b 3.4. *I Simple 6-node DN*<br>
The following simple 6-node DN is used to exemplify this polyhedral<br>
approximation of DNR model using Big-M relaxation method. The network is shown<br>
in Fig. 3.5. The node 1 is the PCC bus, while 3.4 Case Study<br>3.4.1 Simple 6-node DN<br>3.4.1 Simple 6-node DN is used to exemplify this polyhedral<br>approximation of DNR model using Big-M relaxation method. The network is shown<br>1. Fig. 3.5. The node 1 is the PCC bus, whil 3.4. *I Simple 6-node DN*<br>3.4. *I Simple 6-node DN*<br>3.4. *I Simple 6-node DN*<br>3.4. *I Simple 6-node 1* is the PCC bus, while nodes 2-6 are PQ buses. In other words,<br> $\mathcal{N} = \{1,2,3,4,5,6\}$  and  $\mathcal{E} = \{1,2,3,4,5\}$ , whe 3.4 Case Study<br>3.4.1 Simple 6-node DN<br>3.4.1 Simple 6-node DN<br>3.4.1 Simple 6-node DN<br>3.4.1 Simple 6-node DN<br>3.4.1 Simple 6-node DN<br>3.5. The nodel using Big-M relaxation method. The network is shown<br>in Fig. 3.5. The node 1 3.4.1 Simple 6-node DN<br>
3.4.1 Simple 6-node DN<br>
The following simple 6-node DN is used to exemplify this polyhedral<br>
approximation of DNR model using Big-M relaxation method. The network is shown<br>
in Fig. 3.5. The node 1 3.4.1 Simple 6-node DN<br>The following simple 6-node DN is used to exemplify this polyhedral<br>approximation of DNR model using Big-M relaxation method. The network is shown<br>in Fig. 3.5. The node 1 is the PCC bus, while nodes The following simple 6-node DN is used to exemplify this polyhedral<br>approximation of DNR model using Big-M relaxation method. The network is shown<br>in Fig. 3.5. The node 1 is the PCC bus, while nodes 2-6 are PQ buses. In o approximation of DNR model using Big-M relaxation method. The network is shown<br>in Fig. 3.5. The node 1 is the PCC bus, while nodes 2-6 are PQ buses. In other words,<br> $\mathcal{N} = \{1,2,3,4,5,6\}$  and  $\mathcal{E} = \{1,2,3,4,5\}$ , whe live power compensation capacity of installed capacitors are labeled<br>
e voltage allowance band of each node is set to 0.97-1.07 p.u. The<br>
breakers are marked with " $\Box$ " and the rest are switch-on marked by<br>
topology, we



3.6=18. The remaining  $u^l$  is a vector of  $u^l = [u_{12}, u_{23}, u_{34}, u_{25}, u_{36}, u_{56}]$ . Thus, the total

number of variables is 25+24+18+6=73. The SCF constraints and spanning tree<br>constraints are the same as the MISOCP-based DNR model with Big-M relaxation<br>method. The tree-shaped voltage profiles before and after DNR operati number of variables is 25+24+18+6=73. The SCF constraints and spanning tree<br>constraints are the same as the MISOCP-based DNR model with Big-M relaxation<br>method. The tree-shaped voltage profiles before and after DNR operati mumber of variables is 25+24+18+6=73. The SCF constraints and spanning tree<br>constraints are the same as the MISOCP-based DNR model with Big-M relaxation<br>method. The tree-shaped voltage profiles before and after DNR operati number of variables is 25+24+18+6=73. The SCF constraints and spanning tree<br>constraints are the same as the MISOCP-based DNR model with Big-M relaxation<br>method. The tree-shaped voltage profiles before and after DNR operati number of variables is 25+24+18+6=73. The SCF constraints and spanning tree<br>constraints are the same as the MISOCP-based DNR model with Big-M relaxation<br>method. The tree-shaped voltage profiles before and after DNR operati



	$\frac{1}{2}$ .04 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0.98 0.96 1.5 $\overline{2}$	2.5 $\overline{3}$ branch	3.5 4					
	Fig. 3.6 Tree-shaped voltage profiles of simple 6-node DN.							
	Table 3.2 Optimal DNR objective of the 6-node DN.							
Solvers	real power at PCC	Minimal injected <b>Real Power</b> Algorithm Loss (p.u.) Iterations node (p.u.) 0.1035 0.031 1.504 $\tau$		Computational Time (seconds)				
<b>MOSEK</b>								
Baron	1.504 0.1035		6	0.030				
Table 3.3 Optimal DNR solutions of the 6-node DN. Circuit breakers Solvers Swicth-on status of Branch No. Swicth-off status of Branch No.								
<b>MOSEK</b>	5		1, 2, 3, 4, 6					
Baron	5		1, 2, 3, 4, 6					
		76						





3.4.2 *Large-scale DNs*<br>To validate the effectiveness of DCHR, we show the voltage profiles before and<br>after DNR operations by this proposed DCHR approach. The tree-shaped voltage *A.2 Large-scale DNs*<br>To validate the effectiveness of DCHR, we show the voltage profiles before and<br>er DNR operations by this proposed DCHR approach. The tree-shaped voltage<br>ofiles are presented in Fig. 3.7 (a)-(d) for di 3.4.2 *Large-scale DNs*<br>To validate the effectiveness of DCHR, we show the voltage profiles before and<br>after DNR operations by this proposed DCHR approach. The tree-shaped voltage<br>profiles are presented in Fig. 3.7 (a)-(d) 3.4.2 Large-scale DNs<br>To validate the effectiveness of DCHR, we show the voltage profiles before and<br>after DNR operations by this proposed DCHR approach. The tree-shaped voltage<br>profiles are presented in Fig. 3.7 (a)-(d) f 3.4.2 Large-scale DNs<br>
To validate the effectiveness of DCHR, we show the voltage profiles before and<br>
after DNR operations by this proposed DCHR approach. The tree-shaped voltage<br>
profiles are presented in Fig. 3.7 (a)-( 3.4.2 Large-scale DNs<br>To validate the effectiveness of DCHR, we show the voltage profiles before and<br>after DNR operations by this proposed DCHR approach. The tree-shaped voltage<br>profiles are presented in Fig. 3.7 (a)-(d) f 3.4.2 Large-scale DNs<br>To validate the effectiveness of DCHR, we show the voltage profiles before and<br>after DNR operations by this proposed DCHR approach. The tree-shaped voltage<br>profiles are presented in Fig. 3.7 (a)-(d) 3.4.2 Large-scale DNs<br>
To validate the effectiveness of DCHR, we show the voltage profiles before and<br>
after DNR operations by this proposed DCHR approach. The tree-shaped voltage<br>
profiles are presented in Fig. 3.7 (a)-( 3.4.2 Large-scale DNs<br>
To validate the effectiveness of DCHR, we show the voltage profiles before and<br>
after DNR operations by this proposed DCHR approach. The tree-shaped voltage<br>
profiles are presented in Fig. 3.7 (a)-(





Fig. 3.7 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node; (c) 123-node and (d) 1060-node. ndependent DNs: (a) 16-node; (b) 33-node; (c)<br>123-node and (d) 1060-node.<br>11 tightness of DCHR, we employ the DNR models using

Fig. 3.7 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node; (c) 123-node and (d) 1060-node.<br>To compare the relaxation tightness of DCHR, we employ the DNR models using<br>are methods: (*i*) the Big-M met Fig. 3.7 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node; (c)<br>
123-node and (d) 1060-node.<br>
To compare the relaxation tightness of DCHR, we employ the DNR models using<br>
four methods: (*i*) the Big-Fig. 3.7 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node; (c)<br>123-node and (d) 1060-node.<br>To compare the relaxation tightness of DCHR, we employ the DNR models using<br>four methods: (*i*) the Big-M me Fig. 3.7 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node; (c)  $123$ -node and (d) 1060-node.<br>
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123-node and (d) 1060-node.<br>
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f Fig. 3.7 Tree-shaped voltage profiles of different DNs: (a) 16-node; (b) 33-node; (c) 123-node and (d) 1060-node.<br>
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To compare the relaxation tightness of DCHR, we employ the DNR models using<br>
four methods: (i) the Big-M To compare the relaxation tightness of DCHR, we employ the DNR models using<br>four methods: (*i*) the Big-M method for (1.6) and continuous SOC relaxation for (1.7)<br>(M1) [14], (*ii*) the McComick linearization method for (1 To compare the relaxation tightness of DCHR, we employ the DNR models using<br>four methods: (*i*) the Big-M method for (1.6) and continuous SOC relaxation for (1.7)<br>(M1) [14], (*ii*) the McComnick linearization method for ( four methods: (*i*) the Big-M method for  $(1.6)$  and continuous SOC relaxation for  $(1.7)$ <br>(M1) [14], (*ii*) the McCormick linearization method for  $(1.6)$  and continuous SOC<br>relaxation for  $(1.7)$  (M2) [15], and (*iii*) 1.7) (M2) [15], and (iii) the polyhedral approximation method with 32<br>eaerizations (M3) [17], and (iv) the proposed DCHR approach (M4).<br>ble M1-M4 are implemented for the low and the high penetrations of<br>r capacitors and D Example M1-M4 are implemented for the low and the high pene<br>
hese comparable M1-M4 are implemented for the low and the high pene<br>
active power capacitors and DERs for cases I and II, respective<br>
mputational performance in

are given in Table 3.3 for M1-M4. The convergence performance of duality gaps is									
presented in Fig. 3.8 by M1-M4.									
		Table 3.3 Computational Performance Among M1-M4.							
		CPU Time(s)				Iterations			
Cases Syst.		Convex		Relaxations	Approx.	Convex	Relaxations		Approx.
		M1	M <sub>2</sub>	M4	M <sub>3</sub>	M1	M <sub>2</sub>	M4	M3
$\mathbf I$	16-node	0.281	0.250	0.234	0.297	202	172	113	99
	33-node	2.906	1.985	1.187	1.781	3452	2259	949	1290
	$123$ -node	1.613	1.488	1.171	1.875	237	329	135	275
	$1060$ -node	329.437	56.922	18.187	20.328	116663	16114	3764	4510
$\mathbf{I}$	16-node	0.309	0.297	0.228	0.275	175	151	133	109
	33-node	3.703	2.141	1.432	1.890	2021	1432	635	1136
	$123$ -node	1.703	1.641	1.218	1.918	237	329	179	152
	1060-node	686.813	97.969	39.094	46.938	127315	13929	5567	5936



1060-node.

Fig. 3.8 Convergence performance: (a) 16-node; (b) 33-node; (c) 123-node; (d)<br>
Fig. 3.8 Convergence performance: (a) 16-node; (b) 33-node; (c) 123-node; (d)<br>
Table 3.3 shows that M4 significantly outperforms M1 and M2 wit Fig. 3.8 Convergence performance: (a) 16-node; (b) 33-node; (c) 123-node; (d)<br>
The 3.3 shows that M4 significantly outperforms M1 and M2 with the less<br>
CPU running time and M4's solutions are more accurate than M3 in theo **Example 19** Interation windered<br>
Fig. 3.8 Convergence performance: (a) 16-node; (b) 33-node; (c) 123-node; (d)<br>
1060-node.<br>
Table 3.3 shows that M4 significantly outperforms M1 and M2 with the less<br>
CPU running time and Fig. 3.8 Convergence performance: (a) 16-node; (b) 33-node; (c) 123-node; (d) 1060-node.<br>
1060-node.<br>
Table 3.3 shows that M4 significantly outperforms M1 and M2 with the less<br>
CPU running time and M4's solutions are more 1060-node.<br>
Table 3.3 shows that M4 significantly outperforms M1 and M2 with the less<br>
CPU running time and M4's solutions are more accurate than M3 in theory for all<br>
systems of two cases. Regarding the number of iterati Table 3.3 shows that M4 significantly outperforms M1 and M2 with the less<br>CPU running time and M4's solutions are more accurate than M3 in theory for all<br>systems of two cases. Regarding the number of iterations, it sugges branches that may have bi-directional power flows in case II. These conclusions are<br>also justified by the fast convergence in two test systems in Fig 3.8 (a)-(d).<br>Moreover, we examine the relaxation bounds by M1, M2 and M4 branches that may have bi-directional power flows in case II. These conclusions are<br>also justified by the fast convergence in two test systems in Fig 3.8 (a)-(d).<br>Moreover, we examine the relaxation bounds by M1, M2 and M4

Thes that may have bi-directional power flows in case II. These conclusions are<br>intified by the fast convergence in two test systems in Fig 3.8 (a)-(d).<br>Moreover, we examine the relaxation bounds by M1, M2 and M4 to showca branches that may have bi-directional power flows in case II. These conclusions are<br>also justified by the fast convergence in two test systems in Fig 3.8 (a)-(d).<br>Moreover, we examine the relaxation bounds by M1, M2 and M4 branches that may have bi-directional power flows in case II. These conclusions are<br>also justified by the fast convergence in two test systems in Fig 3.8 (a)-(d).<br>Moreover, we examine the relaxation bounds by M1, M2 and M4 branches that may have bi-directional power flows in case II. These conclusions are<br>also justified by the fast convergence in two test systems in Fig 3.8 (a)-(d).<br>Moreover, we examine the relaxation bounds by M1, M2 and M branches that may have bi-directional power flows in case II. These conclusions are<br>also justified by the fast convergence in two test systems in Fig 3.8 (a)-(d).<br>Moreover, we examine the relaxation bounds by M1, M2 and M **EXECUTE:** Both the relaxation bounds the relaxation bounds by M1, M2 and M4 to showcase<br>the limitations of M1 and M2. For an arbitrary branch *l*, the bounds by M1 and by M4<br>are denoted as (3.28) and (3.29), respectively



Clearly, the relaxation bounds by M1, M2 and M4 are significantly different, in<br>ich M4 has the tightest relaxation bounds as indicated in Fig. 3.9(a)-(d). This is<br>cause of the least numbers of relaxations in the B&B iterat Clearly, the relaxation bounds by M1, M2 and M4 are significantly different, in which M4 has the tightest relaxation bounds as indicated in Fig. 3.9(a)-(d). This is because of the least numbers of relaxations in the B&B i Clearly, the relaxation bounds by M1, M2 and M4 are significantly different, in which M4 has the tightest relaxation bounds as indicated in Fig. 3.9(a)-(d). This is because of the least numbers of relaxations in the B&B i Clearly, the relaxation bounds by M1, M2 and M4 are significantly different, in which M4 has the tightest relaxation bounds as indicated in Fig. 3.9(a)-(d). This is because of the least numbers of relaxations in the B&B i tighter feasibility space for this DNR optimization due to  $L_2^{lower} > L_1^{lower}$  in (3.28) and Clearly, the relaxation bounds by M1, M2 and M4 are significantly different, in which M4 has the tightest relaxation bounds as indicated in Fig. 3.9(a)-(d). This is because of the least numbers of relaxations in the B&B i Clearly, the relaxation bounds by M1, M2 and M4 are significantly different, in which M4 has the tightest relaxation bounds as indicated in Fig. 3.9(a)-(d). This is because of the least numbers of relaxations in the B&B i Clearly, the relaxation bounds by M1, M2 and M4 are significantly<br>which M4 has the tightest relaxation bounds as indicated in Fig. 3.9<br>because of the least numbers of relaxations in the B&B iterations. In the<br>tighter feas Clearly, the relaxation bounds by M1, M2 and M4 are significantly different, in<br>
inch M4 has the tightest relaxation bounds as indicated in Fig. 3.9(a)-(d). This is<br>
cause of the least numbers of relaxations in the B&B it which M4 has the tightest relaxation bounds as indicated in Fig. 3.9(a)-(d). This is<br>because of the least numbers of relaxations in the B&B iterations. In theory, M4 has a<br>tighter feasibility space for this DNR optimizati

because of the least numbers of relaxations in the B&B iterations. In theory, M4 has a<br>tighter feasibility space for this DNR optimization due to  $L_2^{lower} > L_1^{lower}$  in (3.28) and<br>(3.29) and a tighter Lorentz cone L by (3.30) tighter feasibility space for this DNR optimization due to  $L_2^{lower} > L_1^{lower}$  in (3.28) and (3.29) and a tighter Lorentz cone *L* by (3.30). Based on these two aspects, M4 has tighter relaxation bounds than the M1 and M2.<br>3. (3.29) and a tighter Lorentz cone  $L$  by (3.30). Based on these two aspects, M4 has<br>tighter relaxation bounds than the M1 and M2.<br>3.5 Summary<br>This chapter proposes a DCHR approach to tackle the disjunctive nature of DNR<br>p runder relaxation bounds than the M1 and M2.<br>
3.5 Summary<br>
This chapter proposes a DCHR approach to tackle the disjunctive nature of DNR<br>
problems. With continuous parent-child relationship variables as disjunctive variabl 3.5 Summary<br>This chapter proposes a DCHR approach to tackle the disjunctive nature of DNR<br>problems. With continuous parent-child relationship variables as disjunctive variables,<br>this DCHR approach is theoretically tighter

# **Chapter 4**<br> **Observability Defense-Constrained Topology** Chapter 4<br>Observability Defense-Constrained Topology<br>Optimization of Active Distribution Networks for Chapter 4<br>
Observability Defense-Constrained Topology<br>
Optimization of Active Distribution Networks for<br>
Cyber–Physical System Security Enhancement Chapter 4<br>
Observability Defense-Constrained Topology<br>
Optimization of Active Distribution Networks for<br>
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The system observability is crucial for a sufficient level of controllabi Chapter 4<br>
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**Chapter 4**<br> **Observability Defense-Constrained Topology**<br> **Optimization of Active Distribution Networks for**<br> **Cyber–Physical System Security Enhancement**<br>
The system observability is encial for a sufficient level of cont **Chapter 4**<br> **Observability Defense-Constrained Topology**<br> **Optimization of Active Distribution Networks for**<br> **Cyber–Physical System Security Enhancement**<br>
The system observability is crucial for a sufficient level of con **Observability Defense-Constrained Topology**<br> **Optimization of Active Distribution Networks for**<br> **Cyber-Physical System Security Enhancement**<br>
The system observability is crucial for a sufficient level of controllability **Observability Defense-Constrained Topology**<br> **Optimization of Active Distribution Networks for**<br> **Cyber–Physical System Security Enhancement**<br>
The system observability is crucial for a sufficient level of controllability **Optimization of Active Distribution Networks for**<br>Cyber–Physical System Security Enhancement<br>The system observability is crucial for a sufficient level of controllability on ADNs,<br>which provides the ability to understand **Cyber–Physical System Security Enhancement**<br>The system observability is crucial for a sufficient level of controllability on ADNs,<br>which provides the ability to understand the physical system states. This has proved to<br>be The system observability is crucial for a sufficient level of controllability on ADNs, which provides the ability to understand the physical system states. This has proved to be extremely powerful, especially with the vari The system observability is crucial for a sufficient level of controllability on ADNs,<br>nich provides the ability to understand the physical system states. This has proved to<br>extremely powerful, especially with the various which provides the ability to understand the physical system states. This has proved to<br>be extremely powerful, especially with the various grid operations that depend on the<br>physical system's behavior, e.g., generator redi be extremely powerful, especially with the various grid operations that depend on the<br>physical system's behavior, e.g., generator redispatch, fault location. Indeed, a huge<br>number of feeders and nodes with limited metering

physical system's behavior, e.g., generator redispatch, fault location. Indeed, a huge<br>number of feeders and nodes with limited metering points such as D-PMU units are<br>essential to achieve this merit of observability. With number of feeders and nodes with limited metering points such as D-PMU units are<br>essential to achieve this merit of observability. With D-PMU units, the full system<br>observability for system-wide security operation in ADNs essential to achieve this merit of observability. With D-PMU units, the full system<br>observability for system-wide security operation in ADNs can be available with a<br>good level of service continuity.<br>This chapter is focused observability for system-wide security operation in ADNs can be available with a good level of service continuity.<br>
This chapter is focused on the defense level of eyber physical security in the DNR<br>
model. Motivated by th

4.1 System Observability for Cyber–Physical Security Enhancement<br>D-PMUs are advanced grid measurements for security and economic operation,<br>which measurement data are uploaded to dispatch centers via wireless/wired 1 System Observability for Cyber–Physical Security Enhancement<br>
D-PMUs are advanced grid measurements for security and economic operation,<br>
inch measurement data are uploaded to dispatch centers via wireless/wired<br>
mmunica 4.1 System Observability for Cyber-Physical Security Enhancement<br>D-PMUs are advanced grid measurements for security and economic operation,<br>which measurement data are uploaded to dispatch centers via wireless/wired<br>communi 4.1 System Observability for Cyber–Physical Security Enhancement<br>
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D-PMUs are advanced grid measurements for security and economic operation,<br>
which measurement data are uploaded to dispatch centers via wireless/wired<br>
comm D-PMUs are advanced grid measurements for security and economic operation,<br>which measurement data are uploaded to dispatch centers via wireless/wired<br>communication layers. From the perspective of cybersecurity, it is cruci in cyber-physical DN entities against cyber attacks [35], where<br>ysical DNs, cyber-physical DNs and cyber DNs is displayed in<br>parts of DNs cannot be observed, then cyber attacks for these<br>vy not be casily detected. As such, hysical DNs, cyber-physical DNs and cyber DNs is displayed in<br>parts of DNs cannot be observed, then cyber attacks for these<br>agy not be easily detected. As such, we propose the term<br>se" that can be used to describe the defe observability defense" that can be used to describe the defense cost against<br>ber-physical threats subject to the full observability of DNs.<br> **Physical DN**<br> **Physical DN**<br> **Physical DN**<br> **Physical DN**<br> **Physical DN**<br> **Physi** 



very essential to prevent false data injection attacks (FDIAs) as common<br>exper-physical to prevent false data injection attacks (FDIAs) as common<br>exper-physical to prevent false data injection attacks (FDIAs) as common<br>exp cyber–physical threats subject to the full observability of DNs.<br> **Cyber–Physical diverse of D-PMUs are attack**<br> **Cyber DN**<br>
Fig. 4.1 Cyber-physical interactions in DNs<br>
The observability defense cost is on the D-PMU measu **Cyber-Physical Cyber-Physical Cyber-Physical Cyber DN**<br>
Fig. 4.1 Cyber-physical interactions in DNs<br>
Fig. 4.1 Cyber-physical interactions in DNs<br>
The observability defense cost is on the D-PMU measurement protection, whic **Physical DN**<br> **Physical DN**<br> **Probably defense cost is on the D-PMU measurement protection, which is<br>
very essential to prevent false data injection attacks (FDIAs) as common<br>
eyber-physical threats. If any measurements o** Eq. 4.1 Cyber-physical interactions in DNs Incoming Signals<br>
Fig. 4.1 Cyber-physical interactions in DNs<br>
The observability defense cost is on the D-PMU measurement protection, which is<br>
very essential to prevent false dat Fig. 4.1 Cyber-physical interactions in DNs<br>The observability defense cost is on the D-PMU measurement protection, which is<br>very essential to prevent false data injection attacks (FDIAs) as common<br>over-physical threats. If

strategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other word strategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other word strategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other word strategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other word strategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other word strategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other word ategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>hall number of D-PMU devices can actually cover the full observability of DNs,<br>nece some nodes have zero power injection. In other words, th strategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other word strategies focus on the full protection of D-PMU data for the entire DNs. However, a<br>small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other word

small number of D-PMU devices can actually cover the full observability of DNs,<br>since some nodes have zero power injection. In other words, this observability<br>defense strategy varies as the DNs change, which can be benefic since some nodes have zero power injection. In other words, this observability<br>defense strategy varies as the DNs change, which can be beneficial for reducing the<br>defense cost. From this perspective, we study the defense i defense strategy varies as the DNs change, which can be beneficial for reducing the<br>defense cost. From this perspective, we study the defense issue of system<br>observability in the DNR model.<br>Given that deployed D-PMUs and z defense cost. From this perspective, we study the defense issue of system<br>observability in the DNR model.<br>Given that deployed D-PMUs and zero injection nodes are available in ADNs,<br>power flow and nodal voltage phasor can b observability in the DNR model.<br>
Given that deployed D-PMUs and zero injection nodes are available in ADNs,<br>
power flow and nodal voltage phasor can be observable just based on a proper use<br>
combination of them. In terms o Given that deployed D-PMUs and zero injection nodes are available in ADNs,<br>power flow and nodal voltage phasor can be observable just based on a proper use<br>combination of them. In terms of FDIAs during DNR operations, both prover flow and nodal voltage phasor can be observable just based on a proper use<br>combination of them. In terms of FDIAs during DNR operations, both zero injection<br>nodes without having generation or loads and substation me combination of them. In terms of FDIAs during DNR operations, both zero injection<br>nodes without having generation or loads and substation measurements cannot be<br>attacked due to physical property and private communication n



**Example 19 Follows**<br> **Example 2** Control Center<br> **Protected Devices**<br> **Fig. 4.2 Illustration of the system observability defense.**<br> **4.2 RCDS Formulation For Fixed ADNs**<br> **To begin with, we introduce the RCDS problem for** Fig. 4.2 Illustration of the system observability defense.<br>
4.2 RCDS Formulation For Fixed ADNs<br>
To begin with, we introduce the RCDS problem for fixed ADNs. Generally, ADNs<br>
are considered as a connected undirected tree Control Center<br>
Protected Devices<br>
Fig. 4.2 Illustration of the system observability defense.<br>
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are consi Fig. 4.2 Illustration of the system observability defense.<br>
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are considered as a connected undirected tree 4.2 RCDS Formulation For Fixed ADNs<br>
To begin with, we introduce the RCDS problem for fixed ADNs. Generally, ADNs<br>
are considered as a connected undirected tree  $G = (N, \mathcal{E})$ , where  $N := [1, 2, ..., |N|]$ <br>
is the set of nodes an yielding arbitrary branch  $l := (i, j)$ ,  $l \in \mathcal{E}$  is between nodes  $(i, j)$ . For any RCDS solution <br>  $\subseteq \mathcal{N}$  under a fixed radial topology, we denote  $\mathcal{E}[D] := \{l \in \mathcal{E}(i, j) \cap \mathcal{D} \neq \emptyset\}$  as the<br>
of cdges connected to  $\mathcal{D}$ 

$$
\min_{\mathcal{D}\subseteq\mathcal{N}} \quad F = |\mathcal{D}| \tag{4.1a}
$$

s.t.  $(N, \mathcal{E}[\mathcal{D}])$  connected (4.1b)
protection set in [30] and [31]. Mathematically, we can explain the constraint (4.1b)<br>that (*i*) every node in  $N$  either belongs to  $\mathcal{D}^*$  or is adjacent to a node in  $\mathcal{D}^*$ ; and<br>(*ii*) any node in  $\mathcal{D}^*$  can protection set in [30] and [31]. Mathematically, we can explain the constraint (4.1b)<br>that (*i*) every node in  $N$  either belongs to  $D^*$  or is adjacent to a node in  $D^*$ ; and<br>(*ii*) any node in  $D^*$  can reach any othe protection set in [30] and [31]. Mathematically, we can explain the constraint (4.1b) that (*i*) every node in  $N$  either belongs to  $D^*$  or is adjacent to a node in  $D^*$ ; and (*ii*) any node in  $D^*$  can reach any othe protection set in [30] and [31]. Mathematically, we can explain the constraint (4.1b)<br>that (*i*) every node in  $N$  either belongs to  $\mathcal{D}^*$  or is adjacent to a node in  $\mathcal{D}^*$ ; and<br>(*ii*) any node in  $\mathcal{D}^*$  can protection set in [30] and [31]. Mathematically, we can explain the constraint (4.1b)<br>that (*i*) every node in  $N$  either belongs to  $\mathcal{D}^*$  or is adjacent to a node in  $\mathcal{D}^*$ ; and<br>(*ii*) any node in  $\mathcal{D}^*$  can Solution set in [30] and [31]. Mathematically, we can explain the constraint (4.1b) at (*i*) every node in  $N$  either belongs to  $D^*$  or is adjacent to a node in  $D^*$ ; and any node in  $D^*$  can reach any other node in protection set in [30] and [31]. Mathematically, we can explain the constraint (4.1b)<br>that (*i*) every node in  $N$  either belongs to  $D^*$  or is adjacent to a node in  $D^*$ ; and<br>(*ii*) any node in  $D^*$  can reach any othe

protection set in [30] and [31]. Mathematically, we can explain the constraint (4.1b)<br>that (i) every node in  $N$  either belongs to  $\mathcal{D}^*$  or is adjacent to a node in  $\mathcal{D}^*$ ; and<br>(ii) any node in  $\mathcal{D}^*$  can rea  $\psi(\mathbf{u}^l) = 0$  denotes the spanning tree constraints for radiality and  $h(\mathbf{u}^l) \leq 0$  denotes that (*i*) every node in  $N$  either belongs to  $\mathcal{D}^*$  or is adjacent to a node in  $\mathcal{D}^*$ ; and<br>
(*ii*) any node in  $\mathcal{D}^*$  can reach any other node in  $\mathcal{D}^*$  by a path that stays entirely<br>
within  $\mathcal{D}^*$  (*ii*) any node in  $\mathcal{D}^*$  can reach any other node in  $\mathcal{D}^*$  by a path that stays entirely<br>within  $\mathcal{D}^*$  or a "relaxed path" that there exists one node not in  $\mathcal{D}^*$  along this path.<br>4.3 Disjunctive RCDS F within  $\mathcal{D}^*$  or a "relaxed path" that there exists one node not in  $\mathcal{D}^*$  along th<br>within  $\mathcal{D}^*$  or a "relaxed path" that there exists one node not in  $\mathcal{D}^*$  along th<br>4.3 Disjunctive RCDS Formulation For un *D* can clear any once node in *D* oy a pain that stays chirecty<br>a "relaxed path" that there exists one node not in *D*" along this path.<br>ive RCDS Formulation For Reconfigurable ADNs<br>berational perspective, ADNs are modified as  $\mathcal{E}^u[D] := \left\{l \in \mathcal{E}^u | (i, j) \cap \mathcal{D} \neq \phi \right\}$ , where  $\mathcal{E}^u := \left\{l \in \mathcal{E} | \psi(u^l) = 0, u^l \in \mathbb{Z} \right\}$ . Within  $D$  or a "relaxed path" that there exists one node not in  $D$  along this path.<br>
4.3 Disjunctive RCDS Formulation For Reconfigurable ADNs<br>
From an operational perspective, ADNs are generally reconfigurable. Thereby, 4.3 Disjunctive RCDS Formulation For Reconfigurable ADNs<br>
From an operational perspective, ADNs are generally reconfigurable. Thereby, the<br>
RCDS solution D is dependent on a binary state vector of circuit breakers  $u'$  fo graph  $G$ . Here,  $u'$  is zero if the switch is open and one if closed, which<br>  $(u')=0$  denotes the spanning tree constraints for radiality and  $h(u')\leq 0$  denotes<br>
stem-wide operational constraints. The subgraph induced by R  $\psi(u') = 0$  denotes the spanning tree constraints for radiality and  $h(u') \le 0$  denotes<br>system-wide operational constraints. The subgraph induced by RCDS is then<br>modified as  $\mathcal{E}^u[D] := \{l \in \mathcal{E}^u(i,j) \cap \mathcal{D} \neq \emptyset\}$ , wher

$$
\min_{\mathcal{D}\subseteq\mathcal{N},\ \mathbf{u}'\in\mathbb{Z}}\quad F=\big|\mathcal{D}\big|\tag{4.2a}
$$

s.t. 
$$
(\mathcal{N}, \mathcal{E}^u[\mathcal{D}])
$$
 connected (4.2b)

$$
\psi(\mathbf{u}^l) = 0 \quad \text{and} \quad h(\mathbf{u}^l) \leq 0 \tag{4.2c}
$$

system-wide operational constraints. The subgraph induced by RCDS is then<br>modified as  $\mathcal{E}^{\alpha}[D] := \{l \in \mathcal{E}^{\alpha}|(i,j) \cap \mathcal{D} \neq \emptyset\}$ , where  $\mathcal{E}^{\alpha} := \{l \in \mathcal{E} | \psi(\mathbf{u}') = 0, \mathbf{u}' \in \mathbb{Z}\}$ .<br>Due to the disjunctive modified as  $\mathcal{E}^{\alpha}[\mathcal{D}] := \{l \in \mathcal{E}^{\alpha}|(i,j) \cap \mathcal{D} \neq \emptyset\}$ , where  $\mathcal{E}^{\alpha} := \{l \in \mathcal{E} | \psi(\mathbf{u}') = 0, \mathbf{u}' \in \mathbb{Z}\}$ .<br>
Due to the disjunctive nature of DNR problems [9], we propose a disjunctive RCDS<br>
formulatio Due to the disjunctive nature of DNR problems [9], we propose a disjunctive RC<br>formulation for reconfigurable  $\mathcal{E}^u$  in theory based on the existing RCDS model:<br> $\lim_{D \to \mathcal{N} : u^2 \in \mathcal{Z}} F = |\mathcal{D}|$  (4.<br> $s.t. \quad (\mathcal{N}, \mathcal{E$ 



The topology distinction is obvious since Fig. 4.2 (b) has four leaves but Fig. 4.2 (a)<br>
The topology  $\varepsilon^u$  with two leaves; (b) topology  $\varepsilon^v$  with five leaves.<br>
By the observation, it is easily found that Fig. 4.2 ( witch-on<br>
root node 0<br>
Tig. 4.3. (a) Topology  $g^2$  with two leaves; (b) topology  $g^2$  with five leaves.<br>
By the observation, it is easily found that Fig. 4.2 (b) only employs one D-PMU at<br>
node 1, whereas Fig. 4.2 (a) m Fig. 4.3. (a) Topology  $\varepsilon^u$  with two leaves; (b) topology  $\varepsilon^u$  with fiv<br>By the observation, it is easily found that Fig. 4.2 (b) only employs one<br>node 1, whereas Fig. 4.2 (a) must deploy two D-PMUs at nodes 2 and 4, 2 (a) (a) (b)<br>
Fig. 4.3. (a) Topology  $g^u$  with two leaves; (b) topology  $g^u$  with five leaves.<br>
By the observation, it is easily found that Fig. 4.2 (b) only employs one D-PMU at<br>
de 1, whereas Fig. 4.2 (a) must deploy Fig. 4.3. (a) Topology  $\varepsilon^e$  with two leaves; (b) topology  $\varepsilon^e$  with five leaves.<br>By the observation, it is easily found that Fig. 4.2 (b) only employs one D-PMU at<br>node 1, whereas Fig. 4.2 (a) must deploy two D-PMUs By the observation, it is easily found that Fig. 4.2 (b) only employs one D-PMU at <br>
le 1, whereas Fig. 4.2 (a) must deploy two D-PMUs at nodes 2 and 4, respectively.<br>
topology distinction is obvious since Fig. 4.2 (b) ha

 $\psi(\mathbf{u}^l) = 0$  in (4.2c). Without loss of generosity, if all non-leaf nodes have deployed By the observation, it is easily found that Fig. 4.2 (b) only employs one D-PMU at node 1, whereas Fig. 4.2 (a) must deploy two D-PMUs at nodes 2 and 4, respectively.<br>The topology distinction is obvious since Fig. 4.2 (b) node 1, whereas Fig. 4.2 (a) must deploy two D-PMUs at nodes 2 and 4, respectively.<br>The topology distinction is obvious since Fig. 4.2 (b) has four leaves but Fig. 4.2 (a)<br>has one leaf, where a leaf refers to a terminal n The topology distinction is obvious since Fig. 4.2 (b) has four leaves but Fig. 4.2 (a)<br>has onc leaf, where a leaf refers to a terminal node with degree one for these two<br>rooted trees.<br>This indicates that the topology det has onc leaf, where a leaf refers to a terminal node with degree one for these two<br>rooted trees.<br>This indicates that the topology determined by the optimal RCDS solution  $D^{n^*}$  is<br>equivalent to deal with a maximum leaf rooted trees.<br>
This indicates that the topology determined by the optimal RCDS solution  $\mathcal{D}^*$  is<br>
equivalent to deal with a maximum leaf spanning tree problem (MLSTP) subject to<br>  $\psi(u') = 0$  in (4.2c). Without loss of This indicates that the topology determined by the optimal RCDS solution  $\mathcal{D}^{\circ\circ}$  is<br>equivalent to deal with a maximum leaf spanning tree problem (MLSTP) subject to<br> $\psi(u') = 0$  in (4.2c). Without loss of generosity, i DS solution  $\mathcal{D}^{t*}$  is<br>
(MLSTP) subject to<br>
nodes have deployed<br>
y  $\mathcal{D}^{*}$  and by the<br>
below:<br>
g a spanning tree  $\tilde{\mathcal{G}}$ <br>
uch that  $s \ge 1$  and<br>
the optimal RCDS<br>
. By the Handshake<br>
be expressed as

equivalent to deal with a maximum leaf spanning tree problem (MLSTP) subject to  $\psi(u') = 0$  in (4.2c). Without loss of generosity, if all non-leaf nodes have deployed D-PMU units, then the maximum leaf numbers determined by  $w(u') = 0$  in (4.2c). Without loss of generosity, if all non-leaf nodes have deployed<br>
D-PMU units, then the maximum leaf numbers determined by  $\mathcal{D}^*$  and by the<br>
optimal MLSTP solution  $\tilde{\mathcal{D}}$  are equivalent, which

$$
\sum_{\nu=1}^{n} \deg \nu = \sum_{\nu=n-s+1}^{n} \deg \nu + s = \sum_{\nu=n-m+1}^{n} \deg \nu + m \qquad \qquad \text{This} \qquad \text{further} \qquad \text{induces}
$$

 $v=n-m+1$  $\sum_{n=1}^{\infty}$  deg  $v > \sum_{n=1}^{\infty}$  deg  $v = n - s + 1$   $v = n - m +$  $v > \sum_{n=1}^{\infty} \deg v$  $= n-s+1$   $v=n-m+1$  $\sum_{y=n-m+1}^{n} \text{deg } v > \sum_{y=n-m+1}^{n} \text{deg } v$  since  $s < m$ . Recall that the total degree of non-leaf nodes as endpoints. Thus,  $\sum_{y=n-s+1}^{n} \deg v > \sum_{y=n-m+1}^{n} \deg v$  since  $s < m$ . Recall that the total degree of non-leaf nodes<br>represents the total number of edges that have these non-leaf nodes as endpoints. Thus,<br>we have  $\sum_{y=n-s+1}^{n} \deg v = 2(n-(n-s+1)) = 2s-$ 

we have 
$$
\sum_{v=n-s+1}^{n} \deg v = 2(n-(n-s+1)) = 2s-2
$$
 and

$$
\sum_{\substack{v=n-s+1\\v=n-m+1}}^{n} \text{deg } v > \sum_{\substack{v=n-m+1\\v=n-s+1}}^{n} \text{deg } v \quad \text{since } s < m. \text{ Recall that the total degree of non-leaf nodes}
$$
\nrepresents the total number of edges that have these non-leaf nodes as endpoints. Thus,\n
$$
\text{we have } \sum_{\substack{v=n-s+1\\v=n-s+1}}^{n} \text{deg } v = 2(n - (n - s + 1)) = 2s - 2 \quad \text{and}
$$
\n
$$
\sum_{\substack{v=n-m+1\\v=n-m+1}}^{n} \text{deg } v = 2(n - (n - m + 1)) = 2m - 2. \text{ By substitution, it is clear that } 2s - 2 > 2m - 2
$$
\ncan hold, i.e.,  $s > m$  stands, which contradicts the assumption  $s < m$ . This demonstrates

 $\sum_{n=n+1}^{n} \deg v > \sum_{n=n+1}^{\infty} \deg v$  since s<m. Recall that the total degree of non-leaf nodes<br>represents the total number of edges that have these non-leaf nodes as endpoints. Thus,<br>we have  $\sum_{n=n+1}^{n} \deg v = 2(n - (n - s + 1)) = 2s \sum_{n=m+1}^{\infty} \deg v > \sum_{n=m+1}^{\infty} \deg v$  since s<*m*. Recall that the total degree of non-leaf nodes<br>represents the total number of edges that have these non-leaf nodes as endpoints. Thus,<br>we have  $\sum_{n=m+1}^{\infty} \deg v = 2(n - (n - s + 1$ 

 $\sum_{y=x,y=1}^{n} \deg v > \sum_{y=x,y=1}^{n} \deg v$  since  $s \le m$ . Recall that the total degree of non-leaf nodes<br>presents the total number of edges that have these non-leaf nodes as endpoints. Thus,<br> $\sum_{y=x,y=1}^{n} \deg v = 2(n - (n - s + 1)) = 2s - 2$  an  $\sum_{n=n+1}^{\infty} \frac{\log V}{2} \sum_{n=n+2}^{\infty} \frac{\log V}{2}$  since *s*-*m*. Recall that the othat degree of non-leaf nodes<br>represents the total number of edges that have these non-leaf nodes as endpoints. Thus,<br>we have  $\sum_{n=n+1}^{\infty} \deg v =$ represents the total number of edges that have these non-leaf nodes as endpoints. Thus,<br>we have  $\sum_{m=n+1}^{n} \deg v = 2(n - (n - s + 1)) = 2s - 2$  and<br> $\sum_{n=m+1}^{n} \deg v = 2(n - (n - m + 1)) = 2m - 2$ . By substitution, it is clear that 2s-2>2m-2<br>ca represents the total number of edges that have these non-leaf nodes as endpoints. Thus,<br>we have  $\sum_{m=s+1}^{n} \deg v = 2(n - (n - s + 1)) = 2s - 2$  and<br> $\sum_{n=s+1}^{n} \deg v = 2(n - (n - m + 1)) = 2m - 2$ . By substitution, it is clear that 2s-2>2m-2<br>ca have  $\sum_{v=\pi+i}^n \deg v = 2(n-(n-s+1)) = 2s-2$  and<br>  $\sum_{v=1}^n \deg v = 2(n-(n-m+1)) = 2m-2$ . By substitution, it is clear that  $2s-2>2m-2$ <br>
hold, i.e.,  $s>m$  stands, which contradicts the assumption  $s < m$ . This demonstrates<br>
ts should be equ  $\sum_{n=m+1}^{\infty}$  deg  $v = 2(n - (n - m + 1)) = 2m - 2$ . By substitution, it is clear that 2s-2>2m-2<br>can hold, i.e., s>m stands, which contradicts the assumption s<m. This demonstrates<br>that s should be equal to m.<br>It is assumed that  $\sum_{\text{even}} \deg v = 2(n - (n - m + 1)) = 2m - 2$ . By substitution, it is clear that 2s-2>2m-2<br>can hold, i.e., s>m stands, which contradicts the assumption s<m. This demonstrates<br>that s should be equal to m.<br>It is assumed that  $M^e$  repr  $\int_{P}$ <sup> $\frac{2}{i}$ </sup> i z  $\sum_{\in {\cal M}^P}$  $\mathcal{M}^r$ **Example 12** and to *m*.<br> **Example 12** and the reference of the lowest of D-PMUs other than the root node 0<br>  $l^p \in \mathcal{N}$ . Then, the decision vector is encoded by binary variables as z for<br>
s in the dimension  $|\mathcal{N}| \times 1$ It is assumed that  $M^P$  represents all nodes of D-PMUs other than the root node 0<br>and  $M^P \in \mathcal{N}$ . Then, the decision vector is encoded by binary variables as z for<br>D-PMUs in the dimension  $|\mathcal{N}| \times 1$ , i.e.,  $z_i \in \{0,$ proposents all nodes of D-PMUs other than the root node 0<br>
independent of the decision vector is encoded by binary variables as z for<br>
ion  $|\mathcal{N}| \times 1$ , i.e.,  $z_i \in \{0,1\}, \forall i \in \mathcal{M}^p$ ; otherwise,  $z_i = 0$ <br>  $\phi$  and the p for  $i \notin \mathcal{M}^{\nu}$ . With  $\mathcal{M}^{\nu} \neq \emptyset$  and the physical property of MLSTP, we can observe the<br>minimum cardinality  $|\mathcal{D}|$  has the maximum leaf spanning tree if  $h(u') \le 0$  in (4.2c)<br>is relaxed. This is essential for

$$
\sum_{i \in \mathcal{M}^P} z_i \ge z^g \tag{4.3}
$$

where  $z^s$  refers to the lower bound by solving (4.2a)-(4.2c) with relaxed  $h(u^i) \le 0$ .  $\sum_{i \in \mathcal{M}^P} z_i \geq 1$  $\mathcal{M}^r$ For the sets of zero injection nodes for non-leaf nodes and leaf the sets of *M*<sup>2</sup> in the sets of zero in the sets of zero injection of  $\sum_{n \in \mathcal{N}^e} z_i$  and  $\sum_{n \in \mathcal{N}^e} z_i \ge z^e$  (4.3) there  $z^e$  refers to the l is relaxed. This is essential for strengthening the lower bound of  $\sum_{i \in M'} z_i$ <br>  $\sum_{k \in M''} z_i \ge z^g$ <br>
where  $z^g$  refers to the lower bound by solving (4.2a)-(4.2c) with relaxes<br>
Since  $M'' \neq \phi$ , then  $z^g \ge 1$  holds, whic 2d. This is essential for strengthening the lower bound of  $\sum_{i \in \mathcal{M}^e} z_i$  (4.3)<br>  $\sum_{i \in \mathcal{M}^e} z_i \ge z^e$  (4.3)<br>
2<sup>g</sup> rcfers to the lower bound by solving (4.2a)-(4.2e) with relaxed  $h(u') \le 0$ .<br>  $\mathcal{M}^e \neq \phi$ , then  $\sum_{w,n'} z_i \ge z^{\epsilon}$  (4.3)<br>
where  $z^s$  refers to the lower bound by solving (4.2a)-(4.2c) with relaxed  $h(u') \le 0$ .<br>
Since  $\mathcal{M}'' \ne \phi$ , then  $z^s \ge 1$  holds, which indicates (4.3) should be tighter than<br>  $\sum_{v \in \mathcal{M}'} z_i \ge$ 

Namely,  $y_i \in \{0,1\}$ ,  $\forall i \in \mathcal{M}^I$  and  $y_k = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for I We denote  $I(w) = \int i \in \Lambda/[w - 1]$  or  $\exists i \in \Lambda/[w - 1] \forall l \in \mathcal{C}^u$ Namely,  $y_i \in \{0,1\}$ ,  $\forall i \in \mathcal{M}^i$  and  $y_k = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for  $i \notin \mathcal{M}^i$ . We denote  $I(y) := \{i \in \mathcal{N} | y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, \forall l \in \mathcal{E}^u\}$  for all zero injection nodes and associ  $\in \mathcal{M}^l$  and  $y_k = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for <br>  $I(y) := \{ i \in \mathcal{N} | y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, \forall l \in \mathcal{E}^u \}$  for all zero<br>
sociated dominated nodes, where an associated dominated node<br>
minated non Namely,  $y_i \in \{0,1\}$ ,  $\forall i \in \mathcal{M}^l$  and  $y_k = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for  $i \notin \mathcal{M}^l$ . We denote  $I(\mathbf{y}) := \{i \in \mathcal{N} | y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, \forall l \in \mathcal{E}^u\}$  for all zero injection nodes and as Namely,  $y_i \in \{0,1\}$ ,  $\forall i \in \mathcal{M}^i$  and  $y_k = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for  $i \notin \mathcal{M}^i$ . We denote  $I(\mathbf{y}) = \{i \in \mathcal{N} | y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, \forall l \in \mathcal{E}^u\}$  for all zero injection nodes and as Namely,  $y_i \in \{0,1\}$ ,  $\forall i \in \mathcal{M}^i$  and  $y_k = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for  $i \notin \mathcal{M}^i$ . We denote  $I(y) := \{i \in \mathcal{N} | y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, \forall l \in \mathcal{E}^* \}$  for all zero injection nodes and associ  $P(z) := \{ i \in \mathcal{N} | z_i = 1 \text{ or } \exists j \in \mathcal{N} : z_j = 1, \forall l \in \mathcal{E}^u \} \text{ for all } D\text{-PMU nodes and }$ Namely,  $y_i \in \{0,1\}$ ,  $\forall i \in \mathcal{M}'$  and  $y_k = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for  $i \notin \mathcal{M}'$ . We denote  $I(y) := \{i \in \mathcal{N} | y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, \forall l \in \mathcal{E}^v\}$  for all zero injection nodes and associated Namely,  $y_i \in \{0,1\}$ ,  $\forall i \in \mathcal{M}^i$  and  $y_k = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for  $i \notin \mathcal{M}^i$ . We denote  $I(y) = \{i \in \mathcal{N} | y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, \forall l \in \mathcal{E}^u\}$  for all zero injection nodes and associ Namely,  $y_i \in \{0,1\}$ ,  $\forall i \in \mathcal{M}'$  and  $y_i = 1$  and  $\forall k \in \mathcal{M}^k$ ; otherwise,  $y_i = 0$  for  $i \in \mathcal{M}'$ . We denote  $I(y) := \{i \in \mathcal{N} | y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, \forall l \in \mathcal{E}^u\}$  for all zero injection nodes and associated  $i \notin M'$ . We denote  $I(y) = {i \in N | y_i = 1 \text{ or } \exists j \in N : y_j = 1, \forall l \in \mathcal{E}^v}$  for all zero<br>injection nodes and associated dominated nodes, where an associated dominated node<br>for node *i* refer to a dominated non-zero injection nod injection nodes and associated dominated nodes, where an associated dominated node<br>for node *i* refer to a dominated non-zero injection node on a branch  $l := (i,j)$  whose the<br>other side node *j* is a zero injection node with for node *i* refer to a dominated non-zero injection node on a branch  $l := (i,j)$  whose the<br>
other side node *j* is a zero injection node with  $y_i = 1$ . And<br>  $P(z) := \{i \in \mathcal{N} | z_i = 1 \text{ or } \exists j \in \mathcal{N} : z_j = 1, \forall l \in \mathcal{E}^o\}$  for a other side node *j* is a zero injection node with  $y_i = 1$ . And<br>  $P(z) := \{ i \in \mathcal{N} \mid z_i = 1 \text{ or } \exists j \in \mathcal{N} : z_j = 1, \forall l \in \mathcal{E}^* \}$  for all D-PMU nodes and<br>
associated dominated nodes, where an associated dominated node *f* in

Iominated node on a branch  $l := (i_j j)$  whose the other side node *j* is a D-PMU node<br>th  $z_j = 1$ .<br>In order to understand  $P(z)$  and  $I(y)$ , the decision vector is encoded by binary<br>riables as z for D-PMUs in the dimension  $|\mathcal$ riables as z for D-PMUs in the dimension  $|\mathcal{N}| \times 1$ . The vector y is the auxiliary<br>
vary-based decision variable vector for zero injection nodes in the dimension<br>  $\mathcal{I}|\times 1$ . Mathematically, we express as<br>  $z_i = 0$ ,  $\$ 

$$
z_i = 0, \quad \forall i \notin \mathcal{M}^P, \quad z_j \in \{0,1\} \quad \forall j \in \mathcal{M}^P \tag{4.4}
$$

$$
y_i = 0, \quad \forall i \notin \mathcal{M}^I, \quad y_k = 1, \quad \forall k \in \mathcal{M}^k, \quad y_j \in \{0, 1\} \quad \forall j \in \mathcal{M}^I
$$
 (4.5)

$$
P(z) = \{i \in \mathcal{N} \mid |z_i = 1 \text{ or } \exists k \in \mathcal{N} : z_k = 1, (i, k) \in \mathcal{E}\}\tag{4.6}
$$

$$
I(y) = \{i \in \mathcal{N} \mid y_i = 1 \text{ or } \exists j \in \mathcal{N} : y_j = 1, (i, j) \in \mathcal{E}\}\tag{4.7}
$$



node  $\left\{\n\begin{array}{c}\n\text{node} \\
\text{node } i \text{ node } j\n\end{array}\n\right.\n\left.\n\left.\n\begin{array}{c}\n\text{node} \\
\text{node } i \text{ node } j\n\end{array}\n\right.\n\left.\n\left.\n\begin{array}{c}\n\text{node } i \text{ node } j\n\end{array}\n\right.\n\left.\n\left.\n\begin{array}{c}\n\text{node } i \text{ node } j\n\end{array}\n\right.\n\left.\n\left.\n\begin{array}{c}\n\text{node } i \text{ node } j\n\end{array}\n\right.\n\right.\n\left.\n\left.\$ **IFFORT ACTS ASSET ASSET ASSET ASSET ASSET ASSET ASSET ASSET ASSET AND INDUSTRELLATION OF SET ALL INSTED AND INTERENT ACTS AND FOR Fig. 4.4(a),**  $i \in P(z)$  **holds according to (4.6) if a D-PMU unit is active at node<br>i.e., z\_i** 

 $y_i = 1$  and  $i \in I(y)$ <br>
(b) Illustration of sets  $I(y)$ <br>
Fig. 4.4 Illustration of sets  $P(z)$  and  $I(y)$ <br>
For Fig. 4.4(a),  $i \in P(z)$  holds according to (4.6) if a D-PMU unit is active at node<br>  $i$  (i.e.,  $z_i = 1$ ) or a D-PMU uni (b) Illustration of set  $I(y)$ <br>
Fig. 4.4 Illustration of sets  $P(z)$  and  $I(y)$ <br>
For Fig. 4.4(a),  $i \in P(z)$  holds according to (4.6) if a D-PMU unit is active at node<br>  $i$  (i.e.,  $z_i = 1$ ) or a D-PMU unit is active at adjacent Fig. 4.4 Illustration of sets  $P(z)$  and  $I(y)$ <br>
For Fig. 4.4(a),  $i \in P(z)$  holds according to (4.6) if a D-PMU unit is active at node<br>  $i$  (i.e.,  $z_i = 1$ ) or a D-PMU unit is active at adjacent node  $k$  (i.e.,  $z_k = 1$ ). For For Fig. 4.4(a),  $i \in P(z)$  holds according to (4.6) if a D-PMU unit is active at node  $i$  (i.e.,  $z_i = 1$ ) or a D-PMU unit is active at adjacent node  $k$  (i.e.,  $z_k = 1$ ). For Fig. 4.4(b), if node *i* has zero injection with



**IF THIS EXECUTE THE SURVEY OF A ZET ON THE SURVEY OF A ZET ON THE SEXAMPLE SURVEY AND INTEREST ASSOCIATED ASSESS AND A SURVEY AND A SURVEY AND A SURVEY OF A ZET OUTLET AND A SURVEY AND A SURVEY AND A SURVEY AND A SURVEY** P z I y ( ) ( ) {0,1,2,3,4,5} . Under this condition P z I y ( ) ( ) , we Fig. 4.5 Illustration of  $P(z) \cup I(y) = N$ <br>
In this example, it is clear that  $P(z) = \{0, 1, 2, 4, 5\}$  and  $I(y) = \{1, 2, 3\}$ , and then  $P(z) \cup I(y) = \{0, 1, 2, 3, 4, 5\} = N$ . Under this condition  $P(z) \cup I(y) = N$ , we incorporate an auxi <sup>l</sup> <sup>w</sup>ij for the branch l := (i,j) if the adjacent node <sup>j</sup> is dominated by the zero injection node <sup>i</sup>; otherwise <sup>0</sup> <sup>l</sup> <sup>w</sup>ij . Following [31], we express the connectivity constraints to formulate (4.2b): Fig. 4.5 Illustration of  $P(z) \cup I(y) = \mathcal{N}$ <br>
In this example, it is clear that  $P(z) = \{0, 1, 2, 4, 5\}$  and  $I(y) = \{1, 2, 3\}$ , and then  $P(z) \cup I(y) = \{0, 1, 2, 3, 4, 5\} = \mathcal{N}$ . Under this condition  $P(z) \cup I(y) = \mathcal{N}$ , we<br>
inc In this example, it is clear that  $P(z) = \{0, 1, 2, 4, 5\}$  and  $I(y) = \{1, 2, 3\}$ , and then  $P(z) \cup I(y) = \{0, 1, 2, 3, 4, 5\} = \mathcal{N}$ . Under this condition  $P(z) \cup I(y) = \mathcal{N}$ , we more provide an auxiliary binary variable  $w'_0 =$  $P(z) \cup I(y) = \{0, 1, 2, 3, 4, 5\} = \mathcal{N}$ . Under this condition  $P(z) \cup I(y) = \mathcal{N}$ , we<br>incorporate an auxiliary binary variable  $w'_0 = 1$  for the branch  $1 = (i,j)$  if the adjacent<br>node *j* is dominated by the zero injection node

$$
z_i + \sum_{k:(i,k)\in\mathcal{E}^u} z_k + \sum_{j\in\mathcal{N}} \left( w_{ij}^l \cdot u_{ij}^l \right) \ge 1, \quad \forall i \in\mathcal{M}^P
$$
\n(4.8a)

$$
\sum_{i \in \mathcal{N}} \left( w_{ij}^l \cdot u_{ij}^l \right) \le y_j, \quad \forall j \in \mathcal{M}^l \tag{4.8b}
$$

incorporate an auxiliary binary variable  $w'_y = 1$  for the branch  $1 := (i, j)$  if the adjacent<br>node *j* is dominated by the zero injection node *i*; otherwise  $w'_y = 0$ . Following [31],<br>we ex<br> $\sum_{x_i + \sum_{k \neq k} x_k = \sum_{j \neq k} (w'_y \cdot$ 

and also have nonlinear terms  $w_g^i - u_g^i$ , we equivalently cluster  $\mathcal{L}_i$  and  $\mathcal{L}_k$  with  $\mathcal{L}_k$  and  $\sum_{k \in \mathcal{N}} (w_g^i \cdot u_g^i) \leq 1$ ,  $\forall i \in \mathcal{M}^n$  (4.8a)<br>
where (4.8a) represents the connectivity constraint of Let union the set of them as a set of linear<br>
let union them as a set of linear<br>  $\sum_{j \in \mathcal{N}} (w_{ij}^f \cdot u_{ij}^f) \ge 1$ ,  $\forall i \in \mathcal{M}^P$  (4.8a)<br>
dependicivity constraint of the graph  $G$  such that<br>
that among all active branc press the connectivity constraints to formulate (4.20).<br>  $z_i + \sum_{k \in A} z_k e^{z_k} + \sum_{k \in A} (w'_g \cdot u'_g) \ge 1$ ,  $\forall i \in \mathcal{M}^{\rho}$  (4.8a)<br>  $\sum_{k \in A} (w'_g \cdot u'_g) \ge y_j$ ,  $\forall j \in \mathcal{M}^{\prime}$  (4.8b)<br>
where (4.8a) represents the connectivity constraints with the auxiliary variables  $m_{ij}^l = w_{ij}^l \cdot u_{ij}^l$  and  $e_k = z_k \cdot u_{ik}^l$  using Where (4.8a) represents the connectivity constraint of the graph  $G$  such that  $P(z) \cup I(y) = N$ ; and (4.8b) states that among all active branches adjacent to an arbitrary zero injection node *i*, there is at most one adjacent

$$
m_{ij}^l \leq w_{ij}^l, \quad m_{ij}^l \leq u_{ij}^l, \quad e_k \leq z_k, \quad e_k \leq u_{ik}^l \tag{4.9a}
$$

$$
m_{ij}^l \ge w_{ij}^l + u_{ij}^l - 1, \quad e_k \ge z_k + u_{ik}^l - 1 \tag{4.9b}
$$

Therefore, the linear formulation of disjunctive RCDS constraint consists of:  
\n
$$
\sum_{i\in\mathcal{N}} P_{ij} \leq 1, \quad \forall i \in \mathcal{M}^P
$$
\n
$$
\sum_{i\in\mathcal{N}} m'_{ij} \leq y_j, \quad \forall j \in \mathcal{M}^I
$$
\n
$$
\sum_{i\in\mathcal{N}^P} \sum_{i\in\mathcal{N}^P} \sum_{j\in\mathcal{N}^P} \sum
$$

Therefore, the linear formulation of disjunctive RCDS constraint consists of:<br>  $z_i + \sum_{j \in \mathcal{N}} a_j \leq 1$ ,  $\forall i \in \mathcal{M}^P$ <br>  $\sum_{k \in \mathcal{N}^P} a_k = \sum_{j \in \mathcal{N}^P} a_j \leq y_j$ ,  $\forall j \in \mathcal{M}^P$ <br>  $\sum_{k \in \mathcal{M}^P} a_k = \sum_{j \in \mathcal{M}^P} a_j \le$  $:=\left[ \left. \boldsymbol{P}^{l} \boldsymbol{,} \boldsymbol{\mathcal{Q}}^{l} \boldsymbol{,} \ell^{l} \boldsymbol{,} \nu \boldsymbol{,} \boldsymbol{\mathcal{Q}}^{cr} \boldsymbol{,} \boldsymbol{\beta}^{l} \right. \right]^{T} \boldsymbol{,} \boldsymbol{x}_{c}^{l}$  $\begin{cases} z_i + \sum_{j \in \mathcal{N}} e_i + \sum_{j \in \mathcal{N}} v'_j \ge 1, & \forall i \in \mathcal{M}^p \\ \sum_{i \in \mathcal{N}} m'_i \le y_j, & \forall j \in \mathcal{M}^l \\ \sum_{i \in \mathcal{N}} x_i^r \ge 2^{-g} \end{cases}$  (4.10)<br>  $m'_0 \le w'_0, m'_0 \le u'_0, e_i \le z_z, e_k \le u'_k$ <br>  $m'_0 \ge w'_0 + u'_0 -1, e_k \ge z_z + u'_k -1$ <br>
4.5 Observability Defe  $\begin{cases} \sum_{k \leq N} m_{ij} \leq y_j, & \forall j \in \mathcal{M}' \\ \sum_{k \leq N} z_i \geq z^k & (4.10) \\ m_{ij}^j \leq w_{ij}^j, & m_{ij}^j \leq u_{ij}^j, & e_k \leq z_i, & e_k \leq u_k^j \\ m_{ij}^j \geq w_{ij}^j + u_{ij}^j - 1, & e_k \geq z_k + u_k^j - 1 \end{cases}$ <br>4.5 Observability Defense-Constrained DNR Formulation<br>Fo (4.10)<br>  $\iota_{ij}^l, e_k \leq z_k, e_k \leq u_{ik}^l$ <br>  $e_k \geq z_k + u_{ik}^l - 1$ <br>
NNR Formulation<br>
in reconfigurable ADNs, the set of<br>
set of operation variables<br>  $\iota^l$  and  $Q^l$  refer to the vectors of<br>  $\ell^l$  is the vector of squared curr **branches;**  $\begin{aligned}\n\int_{\theta_0}^{\theta_0} &\leq w'_\theta, & m'_\theta \leq u'_\theta, & m'_\theta \leq u'_\theta, & e_\xi \leq z_\xi, & e_\xi \leq u'_\alpha\n\end{aligned}$ <br> **4.5 Observability Defense-Constrained DNR Formulation**<br>
For the sake of the minimal power loss in reconfigurable ADNs, t **For all and the continuous parent-child**<br> **For the sake of the minimal power loss in reconfigurable ADNs, the set of**<br>
optimization variables involves a set of operation variables<br>  $x'_n := [P', Q', \ell', v, Q'', \rho']^T, x'_n \in \mathbb{R}$ , 4.5 Observability Defense-Constrained DNR Formulation<br>
For the sake of the minimal power loss in reconfigurable ADNs, the set of<br>
optimization variables involves a set of operation variables<br>  $x'_c := [P', Q', \ell', v, Q'', \rho^{i'}]^T, x'_c \$ 4.5 Observability Defense-Constrained DNR Formulation<br>
For the sake of the minimal power loss in reconfigurable ADNs, the optimization variables involves a set of operation<br>  $x'_c := [P^i, Q^i, \ell^i, v, Q^\sigma, \beta^\mu]^T, x'_c \in \mathbb{R}$ , w variable vector  $\mathbf{x}_d^l := \left[\mathbf{u}^l, z, \mathbf{y}, \mathbf{w}^l, \mathbf{m}^l, \mathbf{e}\right]^T$ ,  $\mathbf{x}_d^l \in \mathbb{Z}$ , is also included. Now, we express ity Defense-Constrained DNR Formulation<br>of the minimal power loss in reconfigurable ADNs, the set of<br>variables involves a set of operation variables<br> $v, Q^{\alpha}, \beta^{\beta}]^T$ ,  $x_c^{\gamma} \in \mathbb{R}$ , where  $P^{\gamma}$  and  $Q^{\gamma}$  refer to For the sake of the minimal power loss in reconfigurable ADNs, the set of<br>optimization variables involves a set of operation variables<br> $x_c^t := [P^t, Q^t, \ell^t, y, Q^\infty, \beta^t]^T, x_c^t \in \mathbb{R}$ , where  $P^t$  and  $Q^t$  refer to the ve optimization variables involves a set of operation variables<br>  $x_c' = [P', Q', \ell', v, Q'', \beta']^T, x_c' \in \mathbb{R}$ , where  $P'$  and  $Q'$  refer to the vectors of<br>
sending-end active and reactive power flows;  $\ell'$  is the vector of squared c  $x_c' := [P', Q', P', P', Z' \in \mathbb{R}$ , where  $P'$  and  $Q'$  refer to the vectors of<br>sending-end active and reactive power flows;  $\ell'$  is the vector of squared current on<br>branches;  $v$  is the vector of squared voltage profiles;  $Q^v$  sending-end active and reactive power flows;  $\ell'$  is the vector of squared current on<br>branches;  $\mathbf{v}$  is the vector of squared voltage profiles;  $\mathbf{Q}^{\circ}$  is the vector of nodal<br>reactive power compensation; and  $\bold$ branches; **v** is the vector of squared voltage profiles;  $Q^{\sigma}$  is the vector of nodal<br>reactive power compensation; and  $\beta'$  denotes the continuous parent-child<br>relationship variable. For the proposed defense-constraine active power compensation; and  $\beta'$  denotes the continuous parent-child<br>lationship variable. For the proposed defense-constrained DNR model, a binary<br>riable vector  $x'_a := [u', z, y, w', m', e', z, z, z$  is also included. Now, we expre relationship variable. For the proposed defense-constrained DNR model, a binary<br>variable vector  $x'_a = [u', z, y, w', m', e]^T, x'_a \in \mathbb{Z}$ , is also included. Now, we express<br>this DNR problem using the DCHR approach in the Chapter 3

proportional to the consumption of decryption service and shared channel resources,<br>this number of D-PMUs equivalently represents the minimum defense cost function.<br>From the perspective of cyber-physical security, the mini proportional to the consumption of decryption service and shared channel resources,<br>this number of D-PMUs equivalently represents the minimum defense cost function.<br>From the perspective of cyber-physical security, the min proportional to the consumption of decryption service and shared channel resources,<br>this number of D-PMUs equivalently represents the minimum defense cost function.<br>From the perspective of cyber-physical security, the min proportional to the consumption of decryption service and shared channel resources,<br>this number of D-PMUs equivalently represents the minimum defense cost function.<br>From the perspective of cyber-physical security, the min proportional to the consumption of decryption service and shared channel resources,<br>this number of D-PMUs equivalently represents the minimum defense cost function.<br>From the perspective of cyber-physical security, the min proportional to the consumption of decryption service and shared channel rest<br>this number of D-PMUs equivalently represents the minimum defense cost fu<br>From the perspective of cyber-physical security, the minimum power lo i z  $\sum_{i\in\mathcal{N}}$ umption of decryption service and shared channel resources,<br>
sequivalently represents the minimum defense cost function.<br>
cyber-physical security, the minimum power loss  $P_{loss}$  can be<br>
on topology with the minimum cost of proportional to the consumption of decryption service and shared channel resources,<br>this number of D-PMUs equivalently represents the minimum defense cost function.<br>From the perspective of cyber-physical security, the min i z  $\sum_{i\in\mathcal{N}}$ d channel resources,<br>efense cost function.<br>wer loss  $P_{loss}$  can be<br>bservability defense.<br>ith units \$ and \$/p.u.<br>study, we select  $c_1$  =<br>for positive integers<br>chieve the minimum proportional to the consumption of decryption service and shared channel resources,<br>this number of D-PMUs equivalently represents the minimum defense cost function.<br>From the perspective of cyber-physical security, the min i i z  $\sum_{i\in\mathcal{N}}$ in priority, and then obtain the minimum defense cost function.<br>
the perspective of cyber-physical security, the minimum power loss  $P_{loss}$  can be<br>
to the distribution topology with the minimum cost of observability defens subject to the distribution topology with the minimum cost of observability defense.<br>
To scale the physical units, we define  $c_1$  and  $c_2$  as coefficients with units S and S/p.u.<br>
for defense cost  $\sum_{n \in \mathbb{N}} z_i$  and p To scale the physical units, we define  $c_1$  and  $c_2$  as coefficients with units \$ a<br>for defense cost  $\sum_{i \in \mathcal{N}} z_i$  and power loss  $P_{loss}$ , respectively. In this study, we se<br> $c_2 = 1$ , since  $P_{loss}$  per unit is generall

$$
\min_{x_c^l \in \mathbb{R}, x_d^l \in \mathbb{Z}} \quad P_{\text{loss}} + \sum_{i \in \mathcal{N}} z_i \tag{4.11a}
$$

$$
\mathbf{s}.\mathbf{t}.\mathbf{-P}^{\mathcal{B}} + \mathbf{P}^d = \mathbf{A}^T \mathbf{P}^l - \mathbf{D}_r \ell^l \tag{4.11b}
$$

$$
-\mathbf{Q}^g - \mathbf{Q}^{cr} + \mathbf{Q}^d = A^T \mathbf{Q}^l - \mathbf{D}_x \ell^l
$$
 (4.11c)

$$
\frac{1}{\overline{v}-\underline{v}}\cdot A\cdot \underline{v}-\frac{1}{\overline{v}-\underline{v}}\cdot (2D_rP^{\prime}+2D_xQ^{\prime}-|z^{\prime}|^2\ell^{\prime})-\beta_{ji}^{\prime}\geq -1
$$
\n(4.11d)

$$
\frac{1}{\overline{v}-\underline{v}}\cdot A\cdot \underline{v}-\frac{1}{\overline{v}-\underline{v}}\cdot (2D_rP^{\prime}+2D_xQ^{\prime}-|z^{\prime}|^2\ell^{\prime})-\beta_{ij}^{\prime}\geq -1
$$
\n(4.11e)

$$
\frac{1}{\overline{v}-\underline{v}}\cdot A\cdot \nu - \frac{1}{\overline{v}-\underline{v}}\cdot (2D_rP^{\prime} + 2D_xQ^{\prime} - |z^{\prime}|^2 \ell^{\prime}) + \mathbf{u}_{ij}^{\prime} \le 1
$$
\n(4.11f)

$$
(1-\boldsymbol{\beta}_{ji}^l)(\underline{v}-\overline{v}) \leq A \cdot v \leq \boldsymbol{\beta}_{ji}^l(\overline{v}-\underline{v}), \boldsymbol{\beta}_{ij}^l(\underline{v}-\overline{v}) \leq A \cdot v \leq (1-\boldsymbol{\beta}_{ij}^l)(\overline{v}-\underline{v})^{(4.11g)}
$$

$$
\ell^{l} = \ell^{l}_{+} + \ell^{l}_{-}, \ 0 \leq \ell^{l}_{+} \leq \beta^{l}_{ij} \overline{\ell}^{l}, 0 \leq \ell^{l}_{-} \leq \beta^{l}_{ji} \overline{\ell}^{l}
$$
 (4.11h)

$$
0 \le 2D_r P^l + 2D_x Q^l - |z^l|^2 \ell^l \le (\bar{v} - \underline{v}) \cdot \beta_{ji}^l \tag{4.11i}
$$

$$
(\underline{v} - \overline{v}) \cdot \boldsymbol{\beta}_{ij}^l \le 2 \boldsymbol{D}_r \boldsymbol{P}^l + 2 \boldsymbol{D}_x \boldsymbol{Q}^l - |\boldsymbol{z}^l|^2 \ell^l \le 0 \tag{4.11j}
$$

$$
\left|\boldsymbol{P}^{l}\right|^{2}+\left|\boldsymbol{Q}^{l}\right|^{2}\le\boldsymbol{D}_{v}\cdot\ell^{l},\quad-B\cdot\boldsymbol{u}^{l}\le\boldsymbol{P}^{l},\boldsymbol{Q}^{l}\le B\cdot\boldsymbol{u}^{l}
$$
\n(4.11k)

$$
\underline{\mathbf{v}} \le \overline{\mathbf{v}}, \quad 0 \le \ell^l \le \overline{\ell}^l, \quad \underline{\mathbf{Q}}_{cr} \le \underline{\mathbf{Q}}^{cr} \le \overline{\mathbf{Q}}_{cr} \tag{4.111}
$$

$$
z_i + \sum_{k:(i,k)\in\mathcal{E}} e_k + \sum_{j\in\mathcal{N}} m_{ij}^l \ge 1, \quad \forall i \in \mathcal{M}^P
$$
\n(4.11m)

$$
\sum_{i \in \mathcal{N}} m_{ij}^l \le \mathcal{Y}_j, \quad \forall j \in \mathcal{M}^l \tag{4.11n}
$$

$$
\sum_{i \in \mathcal{M}^P} z_i \ge z^g \tag{4.11o}
$$

$$
m_{ij}^{l} \leq w_{ij}^{l}, \quad m_{ij}^{l} \leq u_{ij}^{l}, \quad e_{k} \leq z_{k}, \quad e_{k} \leq u_{ik}^{l}
$$
 (4.11p)

$$
m_{ij}^l \geq w_{ij}^l + u_{ij}^l - 1, \quad e_k \geq z_k + u_{ik}^l - 1 \tag{4.11q}
$$

$$
\beta_{ij}^l + \beta_{ji}^l = u_{ij}^l, \quad \beta_{ij}^l = 0, \quad \text{if} \quad i = 0 \tag{4.11r}
$$

$$
\sum_{j:(i,j)\in\mathcal{E}}\beta_{ij}^l=1,\forall i\in\mathcal{N}\setminus 0,\quad 0\leq\beta_{ij}^l\leq 1,\forall l\in\mathcal{E}
$$
\n(4.11s)

 $y \le y \le \bar{y}, \quad 0 \le \ell' \le \bar{\ell}', \quad Q_{\omega'} \le Q_{\omega}$  (4.111)<br>  $z_i + \sum_{k \in \mathcal{N}'} e_k + \sum_{j \in \mathcal{N}'} m'_{ij} \ge 1, \quad \forall i \in \mathcal{M}^{\ell}$  (4.11m)<br>  $\sum_{i \in \mathcal{N}'} m'_{ij} \le y_j, \quad \forall j \in \mathcal{M}^{\ell}$  (4.11m)<br>  $\sum_{i \in \mathcal{N}'} z_i \ge z^s$  (4.11n)<br>  $m'_{ij} \le w'_{ij},$  $z_i + \sum_{k(l,k) \in \mathcal{E}} e_k + \sum_{j \in \mathcal{N}} m'_{ij} \ge 1, \quad \forall i \in \mathcal{M}^P$ <br>  $\sum_{i \in \mathcal{N}'} w'_{ij} \le y_j, \quad \forall j \in \mathcal{M}^I$ <br>  $\sum_{i \in \mathcal{N}'} z_i \ge z^g$ <br>  $m'_{ij} \le w'_{ij}, \quad m'_{ij} \le u'_{ij}, \quad e_k \le z_k, \quad e_k \le u'_{ik}$ <br>  $m'_{ij} \ge w'_{ij} + u'_{ij} - 1, \quad e_k \ge z_k + u'_{ik} - 1$ <br> positive number.  $P^g$ ,  $Q^g$  and  $P^d$ ,  $Q^d$  indicate the vectors of given nodal active and  $\sum_{n} m_{ij}^{l} \ge 1, \forall i \in M^{P}$  (4.11m)<br>  $\sum_{n} \sum_{n} x_{ij}^{l} \le 1, \forall i \in M^{I}$  (4.11n)<br>  $\sum_{n} \sum_{n} x_{ij}^{l} \le 1, \forall i \in M^{I}$  (4.11n)<br>  $\sum_{n} \sum_{n} x_{ij}^{l} \le 1, \forall i \in M^{I}$  (4.11p)<br>  $\sum_{n} \sum_{n} x_{ij}^{l} \le 1, \forall i \in M^{I}$  (4.11q)<br>  $\sum_{n} \sum_{n}$  $\sum_{n \in \mathcal{N}} n'_0 \leq y_j$ ,  $\forall j \in \mathcal{M}'$  (4.11n)<br>  $\sum_{n \in \mathcal{N}'} z_n \geq z^{\epsilon}$  (4.11o)<br>  $m'_0 \leq w'_0, m'_0 \leq w'_0, e_k \leq z_k, e_k \leq u'_k$  (4.11p)<br>  $m'_0 \geq w'_0 + u'_0 - 1, e_k \geq z_k + u'_k - 1$  (4.11q)<br>  $m'_0 \geq w'_0 + u'_0 - 1, e_k \geq z_k + u'_k - 1$  (4.11q)<br>  $\beta'_0$  $\sum_{k=M'}^{T} z_k \ge \varepsilon^s$  (4.11o)<br>  $m'_0 \le w'_0, m'_0 \le u'_0, e_k \le z_k, e_k \le u'_k$  (4.11p)<br>  $m'_0 \ge w'_0 + u'_0 - 1, e_k \ge z_k + u'_k - 1$  (4.11q)<br>  $\beta'_0 + \beta'_n = u'_0, \beta'_0 = 0, \text{ if } t = 0$  (4.11r)<br>  $\sum_{j(i,j)\in\mathcal{J}_0} \beta'_j = 1, \forall i \in \mathcal{N} \setminus 0, 0 \le \beta'_0 \le 1, \forall l \in \math$  $m'_{ij} \le w'_{ij}, \quad m'_{ij} \le w'_{ij}, \quad e_k \le z_k, \quad e_k \le u'_k$  (4.11p)<br>  $m'_i \ge w'_{ij} + u'_{ij} - 1, \quad e_k \ge z_k + u'_k - 1$  (4.11q)<br>  $\beta'_{ij} + \beta'_{ji} = u'_{ij}, \quad \beta'_{ij} = 0, \quad \text{if} \quad i = 0$  (4.11r)<br>  $\sum_{j(i,j)\in\mathcal{S}'_{ij}} \beta'_{ij} = 1, \forall i \in \mathcal{N} \setminus 0, \quad 0 \le \beta'_{ij} \le 1,$  $m_g^j \ge w_g^j + u_g^j - 1$ ,  $\varepsilon_k \ge \varepsilon_k + u'_k - 1$  (4.11q)<br>  $\beta_g^i + \beta_g^i = u_g^i$ ,  $\beta_g^i = 0$ , if  $i = 0$  (4.11r)<br>  $\sum_{j(i,j) \in \beta_g^i} \beta_g^i = 1, \forall i \in \mathcal{N} \setminus 0$ ,  $0 \le \beta_g^i \le 1, \forall i \in \mathcal{E}$  (4.11s)<br>
where  $P_{\text{loss}}$  is the active power lo  $\beta'_u + \beta'_u = u'_v$ ,  $\beta'_u = 0$ , if  $i = 0$  (4.11r)<br>  $\sum_{j(i,j)=s} \beta'_y = 1, \forall i \in \mathcal{N} \setminus 0, \quad 0 \leq \beta'_y \leq 1, \forall l \in \mathcal{E}$  (4.11s)<br>
where  $P_{\text{iso}}$  is the active power loss of ADNs and the root node is 0. M is the big<br>
positive numb  $\sum_{j\in J, j\in J} \beta'_{ij} = 1, \forall i \in \mathcal{N} \setminus 0, \quad 0 \leq \beta'_{ij} \leq 1, \forall l \in \mathcal{E}$  (4.11s)<br>where  $P_{\text{loss}}$  is the active power loss of ADNs and the root node is 0. M is the big<br>positive number.  $P^s$ ,  $Q^s$  and  $P^d$ ,  $Q^d$  indicat  $\underline{\boldsymbol{\mathcal{Q}}}_{cr}, \overline{\boldsymbol{\mathcal{Q}}}_{cr}, \underline{\boldsymbol{\mathcal{v}}} \ , \ \overline{\boldsymbol{\mathcal{v}}} \ , \ \overline{\boldsymbol{\mathcal{Q}}}^\prime$  represents th value  $l \in \mathcal{E}$  (4.11s)<br>
e root node is 0. *M* is the big<br>
tors of given nodal active and<br>
e loads at nodes. *A* is a<br>
indicate the diagonal matrices<br>
the reactance vector and the<br>
is the diagonal matrix whose<br>
sending  $\mathbf{Q}^{cr}$ , v and  $\ell^l$ , respectively. is the active power loss of ADNs and the root node is 0. M is the inder.  $P^s$ ,  $Q^s$  and  $P^d$ ,  $Q^d$  indicate the vectors of given nodal active  $\epsilon$  wer injections and active and reactive loads at nodes. A is anch-node

4.6 Case Study<br>The *IEEE* 16-node, 33-node, 123-node, and 1060-node distribution systems<br>few DERs are used for tests. We have highlighted our RCDs solutions for the The *IEEE* 16-node, 33-node, 123-node, and 1060-node distribution systems with a<br>w DERs are used for tests. We have highlighted our RCDs solutions for the *IEEE*<br>-node and 123-node DNs. With given zero injection nodes {3, 4.6 Case Study<br>The *IEEE* 16-node, 33-node, 123-node, and 1060-node distribution systems with a<br>few DERs are used for tests. We have highlighted our RCDs solutions for the *IEEE*<br>33-node and 123-node DNs. With given zero 4.6 Case Study<br>
The *IEEE* 16-node, 33-node, 123-node, and 1060-node distribution systems with a<br>
few DERs are used for tests. We have highlighted our RCDs solutions for the *IEEE*<br>
33-node and 123-node DNs. With given ze 4.6 Case Study<br>The *IEEE* 16-node, 33-node, 123-node, and 1060-node distribution systems with a<br>few DERs are used for tests. We have highlighted our RCDs solutions for the *IEEE*<br>33-node and 123-node DNs. With given zero 4.6 Case Study<br>
The *IEEE* 16-node, 33-node, 123-node, and 1060-node distribution systems with a<br>
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33-node and 123-node DNs. With given zer 4.6 Case Study<br>
The *IEEE* 16-node, 33-node, 123-node, and 1060-node distribution systems with a<br>
few DERs are used for tests. We have highlighted our RCDs solutions for the *IEEE*<br>
33-node and 123-node DNs. With given ze



108, 110, 115, 116, 117, 119, 120, 121, 122} and the minimum RCDS solution<br>  $D^* = \{5, 8, 13, 14, 15, 19, 21, 25, 29, 31, 35, 38, 42, 47, 50, 52, 54, 55, 58, 60, 63, 65, 68, 70, 72, 74, 76, 78, 82, 84, 87, 89, 94, 95, 97,$ = switch-on<br>
DER<br>
DER<br>
TOT EXECUTE TRANSITY AND THE TRANSITY OF THE CONDITION OF THE TRANSITY OF THE CONDITION OF THE TRANSITY OF THE TRANSIT **18. 10. 11. 12. Proof bus**  $\frac{1}{2}$  **present in Fig. 4.6.** Minimal RCDS solution  $p^*$  of *IEEE* 33-node system<br>In terms of the *IEEE* 123-node ADN, the zero injection nodes are given as {3, 8, 1<br>15,18, 21, 23, 25, 26, 27, 44, 40, 54,



locations of defensed D-PMU units, which means every branch flow can be **Example 1.**  $\frac{3}{5}$  **b**  $\frac{1}{5}$  **c**  $\frac{6}{3}$  **c**  $\frac{4}{3}$  **c**  $\frac{5}{3}$  **c**  $\frac{5}{3}$  **c**  $\frac{5}{81}$  **c**  $\frac{5}{81}$  **c**  $\frac{3}{81}$  **c**  $\frac{3}{81}$  **c**  $\frac{3}{81}$  **c**  $\frac{3}{81}$  **c**  $\frac{3}{81}$  **c**  $\frac{3}{81}$  **c**  $\frac{$ **Example 1998 87** 86<br> **Proposed D-PMU • Zero injection nodes**<br> **Proposed observable of the Color-coded solution**  $D^{\nu}$  of *IEEE* 123-node system.<br>
In Fig. 4.6, the color-coded solid edges of  $\mathcal{E}^{\nu}[D^{\nu}]$  indicate **Example 10** PMU **C** Exercuises Eig. 4.7 Minimal RCDS solution  $D^{\circ}$  of *IEEE* 123-node system.<br>
In Fig. 4.7 Minimal RCDS solution  $D^{\circ}$  of *IEEE* 123-node system.<br>
In Fig. 4.6, the color-coded solid edges of  $\mathcal{E}^$ Fig. 4.7 Minimal RCDS solution  $\mathcal{D}^*$  of *IEEE* 123-node system.<br>
In Fig. 4.6, the color-coded solid edges of  $\mathcal{E}^*[\mathcal{D}^*]$  indicate that the graph<br>  $(\mathcal{N}, \mathcal{E}^*[\mathcal{D}^*])$  is connected. Similarly to Fig. 4.6, In Fig. 4.6, the color-coded solid edges of  $\mathcal{E}^n[D^*]$  indicate that the graph  $N, \mathcal{E}^n[D^*]$  ) is connected. Similarly to Fig. 4.6, Fig. 4.7 presents the optimal rations of defensed D-PMU units, which means every b In Fig. 4.6, the color-coded solid edges of  $\mathcal{E}^*[D^*]$  indicate that the graph  $(\mathcal{N}, \mathcal{E}^*[D^*])$  is connected. Similarly to Fig. 4.6, Fig. 4.7 presents the optimal locations of defensed D-PMU units, which means eve  $(\mathcal{N}, \mathcal{E}^{\circ}[\mathcal{D}^{\circ}])$  is connected. Similarly to Fig. 4.6, Fig. 4.7 presents the optimal locations of defensed D-PMU units, which means every branch flow can be observable under defensed D-PMUs at dominated node s

the corresponding power loss cost (p.u.), and CPU time (in seconds) for two<br>penetration rates of D-PMUs. The corresponding power loss cost (p.u.), and CPU time (in seconds) for two<br>penetration rates of D-PMUs.<br>Table 4.1 Optimal observability defense-constrained DNR solutions<br>Cost of Low D-PMUs

the corresponding power loss cost (p.u.), and CPU time (in seconds) for two									
penetration rates of D-PMUs.									
					Table 4.1 Optimal observability defense-constrained DNR solutions				
	<b>DNR</b>			Cost of Low D-PMUs			Cost of High D-PMUs		
<b>DNs</b>	Loss	$\mathbb{Z}^g$	Defens e	Loss	Time	$\mathbb{Z}^g$	Defense	Loss	Time
16-node	0.0302	5	6	0.0323	0.521	5	5	0.0318	0.500
33-node	0.0542	9	11	0.0594	2.368	9	9	0.0586	1.719
123-node	0.0995	$\mathfrak{Z}$	41	0.1029	175.542	35	35	0.0995	151.024
1060-node	0.1404	5	59	0.1823	225.032	45	45	0.1409	179.078
					For these exhibited cases, our proposed observability defense-constrained DNR				
solution with the low and the high penetrations of D-PMUs successfully achieves the									
complete system observability status in four test systems. The distinction between									
them is the different total cost. The defense cost and power loss cost for the high									
					penetration of D-PMUs is much lower than the one for the low penetration of				

DNs  $\xrightarrow{L \text{res}}$   $z^8$  Defense Loss Time  $z^8$  Defense Loss Time<br>
16-node 0.0302 5 6 0.0323 0.521 5 5 0.0318 0.500<br>
33-node 0.0542 9 11 0.0594 2.368 9 9 0.0586 1.719<br>
123-node 0.0995  $\frac{3}{5}$  41 0.1029 175.542 35 35 0.099 16-node  $0.0302 \t 5$  6  $0.0323$  0.521 5 5  $0.0318$  0.500<br>
33-node  $0.0542$  9 11  $0.0594$  2.368 9 9  $0.0886$  1.719<br>
123-node  $0.1404$   $\frac{4}{5}$  59  $0.1823$  225.032 45 45 0.1409 179.078<br>
For these exhibited cases, our propo 123-node 0.0995  $\frac{3}{5}$  41 0.1029 175.542 35 35 0.0995 151.024<br>1060-node 0.1404  $\frac{4}{5}$  59 0.1823 225.032 45 45 0.1409 179.078<br>For these exhibited cases, our proposed observability defense-constrained DNR<br>solution wit 1060-node 0.1404  $\frac{4}{5}$  59 0.1823 225.032 45 45 0.1409 179.078<br>
For these exhibited cases, our proposed observability defense-constrained DNR<br>
solution with the low and the high penetrations of D-PMUs successfully achi 0.1409 179.078<br>
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for test systems<br>
proposed DNR<br>
es the minimal For these exhibited cases, our proposed observability defense-constrained DNR solution with the low and the high penetrations of D-PMUs successfully achieves the complete system observability status in four test systems. For these exhibited cases, our proposed observability defense-constrained DNR solution with the low and the high penetrations of D-PMUs successfully achieves the complete system observability status in four test systems. solution with the low and the high penetrations of D-PMUs successfully achieves the<br>complete system observability status in four test systems. The distinction between<br>them is the different total cost. The defense cost and complete system observability status in four test systems. The distinction between<br>them is the different total cost. The defense cost and power loss cost for the high<br>penetration of D-PMUs is much lower than the one for th them is the different total cost. The defense cost and power loss cost for the high<br>penctration of D-PMUs is much lower than the one for the low penctration of<br>D-PMUs. This is because more D-PMUs locations lead to a large penetration of D-PMUs is much lower than the one for the low penetration of D-PMUs. This is because more D-PMUs locations lead to a larger feasible space for the DNR optimization. For this reason, the defense cost is equal

which induces the desirable computational efficiency in terms of CPU time for two<br>case studies. which induces the desirable computational efficiency in terms of CPU<br>case studies.

which induces the desirable computational efficiency in terms of CPU<br>case studies.<br>4.7 Summary<br>This chapter proposes a disjunctive RCDS formulation for reconfigure<br>with the least defense cost in theory. With this formulati This chapter proposes a disjunctive RCDS formulation for reconfigurable networks<br>This chapter proposes a disjunctive RCDS formulation for reconfigurable networks<br>the the least defense cost in theory. With this formulation, which induces the desirable computational efficiency in terms of CPU time for two<br>case studies.<br>4.7 Summary<br>This chapter proposes a disjunctive RCDS formulation for reconfigurable networks<br>with the least defense cost in th which induces the desirable computational efficiency in terms of CPU time for two<br>case studies.<br>4.7 Summary<br>This chapter proposes a disjunctive RCDS formulation for reconfigurable networks<br>with the least defense cost in th which induces the desirable computational efficiency in terms of CPU time for two<br>case studies.<br>4.7 Summary<br>This chapter proposes a disjunctive RCDS formulation for reconfigurable networks<br>with the least defense-cost in th which induces the desirable computational efficiency in terms of CPU time for two<br>case studies.<br>4.7 Summary<br>This chapter proposes a disjunctive RCDS formulation for reconfigurable networks<br>with the least defense cost in th 4.7 Summary<br>This chapter proposes a disjunctive RCDS formulation for reconfigurable networks<br>with the least defense cost in theory. With this formulation, an observability<br>defense-constrained DNR model can be constructed a

# Chapter 5<br>A Consensus ADMM-based Differentially Private Chapter 5<br>A Consensus ADMM-based Differentially Private<br>Topology Optimization Approach for Privacy Chapter 5<br>
A Consensus ADMM-based Differentially Private<br>
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A Consensus ADMM-based Differentially Private<br>
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Preservation Enhancement of Multi-Agent Active<br>
Distribution Networks<br>
The topology switch for the loss minimization may **Chapter 5**<br> **A Consensus ADMM-based Differentially Private**<br> **Topology Optimization Approach for Privacy**<br> **Preservation Enhancement of Multi-Agent Active**<br> **Distribution Networks**<br>
The topology switch for the loss minimi **Chapter 5**<br> **A Consensus ADMM-based Differentially Private**<br> **Topology Optimization Approach for Privacy**<br> **Preservation Enhancement of Multi-Agent Active**<br> **Distribution Networks**<br>
The topology switch for the loss minimi

**interpoonly ADMM-based Differentially Private**<br> **Topology Optimization Approach for Privacy**<br> **Preservation Enhancement of Multi-Agent Active**<br> **Distribution Networks**<br>
The topology switch for the loss minimization may ex **A Consensus ADMM-based Differentially Private**<br> **Topology Optimization Approach for Privacy**<br> **Preservation Enhancement of Multi-Agent Active**<br> **Distribution Networks**<br>
The topology switch for the loss minimization may ex **Topology Optimization Approach for Privacy**<br>**Preservation Enhancement of Multi-Agent Active**<br>**Distribution Networks**<br>The topology switch for the loss minimization may expose the private load change<br>information of an agent **Preservation Enhancement of Multi-Agent Active**<br> **Distribution Networks**<br>
The topology switch for the loss minimization may expose the private load change<br>
information of an agent, e.g., transition from a light load to a **Distribution Networks**<br>The topology switch for the loss minimization may expose the private load change<br>information of an agent, e.g., transition from a light load to a heavy load, in<br>interconnected ADNs managed by multip The topology switch for the loss minimization may expose the private load change information of an agent, e.g., transition from a light load to a heavy load, in interconnected ADNs managed by multiple agents. To address th The topology switch for the loss minimization may expose the private load change<br>information of an agent, e.g., transition from a light load to a heavy load, in<br>interconnected ADNs managed by multiple agents. To address th information of an agent, e.g., transition from a light load to a heavy load, in<br>interconnected ADNs managed by multiple agents. To address this issue, this paper<br>proposes a DP-DNR mechanism based on the C-ADMM algorithm. T interconnected ADNs managed by multiple agents. To address this issue, this paper<br>proposes a DP-DNR mechanism based on the C-ADMM algorithm. This can tackle<br>privacy leakage challenges on the agent's and customer's levels. proposes a DP-DNR mechanism based on the C-ADMM algorithm. This can tackle<br>privacy leakage challenges on the agent's and customer's levels. To suppress private<br>load change leakage as an agent's concern, this DP-DNR mechani foad change leakage as an agent s concern, this DP-DNK mechanism provides a<br>mixture output of realistically optimal topology switch status and corresponding<br>obfuscated-but-feasible load flows, part of which may have revers metry weper of remaining spinnal septency of this mass and consequently discussed doen fluxed but-feasible load flows, part of which may have reverse load flow<br>rections. On the customer's level, the C-ADMM-based decentrali

of noise along with maintaining a healthy trade-off between privacy and accuracy.<br>This process can be illustrated in Fig. 5.1. The protection of complete data from<br>database can be achieved by DP; otherwise unprotected data This process can be illustrated in Fig. 5.1. The protection of complete data from<br>database can be achieved by DP; otherwise unprotected data can be analyzed or<br>inferred by doing analyst query attacks. of noise along with maintaining a healthy trade-off between privacy and accuracy.<br>This process can be illustrated in Fig. 5.1. The protection of complete data from<br>database can be achieved by DP; otherwise unprotected data inferred by doing analyst query attacks.



**EXAMPLE ADVIDENTIFY CONTROLLED INTERNATIONAL CONTROLLED CONTROLLED DIFFERENCES**<br>
Total business of the control of t **Example 2018**<br> **Example 2019**<br> **Example 2019 Example 12**<br> **Example 12**<br> **Contentional Data District Contention**<br>
Fig. 5.1 Data output with DP preservation and without DP preservation.<br>
<br>
Accoding to DP theory, DP guarantees for optimization datasets are achieved<br>
t Database  $\frac{\text{Original Data}}{\text{Unprotected}}$   $\sum_{\text{practical}}^{\text{practical}} \sum_{\text{pmodel}}^{\text{practical}}$  Analyst<br>
Fig. 5.1 Data output with DP preservation and without DP preservation.<br>
Analyst<br>
Fig. 5.1 Data output with DP preservation and without DP preservatio Fig. 5.1 Data output with DP preservation and without DP preservation.<br>
Accoding to DP theory, DP guarantees for optimization datasets are achieved<br>
through randomization. Thus, when answering optimization queries, the Accoding to DP theory, DP guarantees for optimization datasets are achieval<br>through randomization. Thus, when answering optimization queries, the DP-bi-<br>mechnism is to make adjacent optimization datasets statistically sim mechnism is to make adjacent optimization datasets statistically similar. Suppose  $\tilde{x}(d)$ <br>be a randomized counterpart of optimization map  $x(d)$ , and two datasets  $d'$ ,  $d$  are<br>dijacent with the Euclidean distance  $||d-d'||{\$ 

$$
Pr[\tilde{x}(d) = \hat{x}] \leq Pr[\tilde{x}(d') = \hat{x}] exp(\varepsilon) + \delta
$$
\n(5.1)

According to this definition, the probabilities of observing the same optimization<br>sult on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $> 0$ , termed *probability of failure*. Accord According to this definition, the probabilities of observing the same optimization<br>result on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $\delta > 0$ , termed *probability of failure*. Ac According to this definition, the probabilities of observing the same optimization<br>result on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $\delta > 0$ , termed *probability of failure*. Ac According to this definition, the probabilities of observing the same optimization<br>result on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $\delta > 0$ , termed *probability of failure*. Ac According to this definition, the probabilities of observing the same optimization<br>result on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $\delta > 0$ , termed *probability of failure*. Ac According to this definition, the probabilities of observing the same optimization<br>sult on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $> 0$ , termed *probability of failure*. Accord optimization<br>
acy loss, and<br>  $\pm$  statistically<br>
parameters  $\varepsilon$ <br>  $\vdots$ , the output<br>  $\frac{1}{2}$ -sensitivity According to this definition, the probabilities of observing the same optimization<br>result on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $\delta > 0$ , termed *probability of failure*. Ac efinition, the probabilities of observing the same optimization<br>sets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br>*ility of failure*. Accordingly,  $\tilde{x}(d)$  and  $\tilde{x}(d')$  are statistically<br>and  $\delta$  ta Lities of observing the same optimization<br>parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br>rdingly,  $\tilde{x}(d)$  and  $\tilde{x}(d')$  are statistically<br>lues. In other words, smaller parameters  $\varepsilon$ <br>tion.<br>pr dataset universe  $\mathbb{D}$ According to this definition, the probabilities of observing the same optimization<br>result on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $\delta > 0$ , termed *probability of failure*. Ac According to this definition, the probabilities of observing the same optimization<br>sult on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br> $> 0$ , termed *probability of failure*. Accord poptimization<br>  $ucy loss$ , and<br>
statistically<br>
parameters  $\varepsilon$ <br>
the output<br>  $t$ -sensitivity<br>  $t$ -sensitivity<br>
where  $\Delta_1$  is<br>
followed by result on adjacent datasets are similar up to parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br>  $\delta > 0$ , termed *probability of failure*. Accordingly,  $\tilde{x}(d)$  and  $\tilde{x}(d')$  are statistically<br>
similar if parameters  $\varepsilon$  parameters  $\varepsilon > 0$ , termed *privacy loss*, and<br>ordingly,  $\tilde{x}$  (*d*) and  $\tilde{x}$  (*d*') are statistically<br>alues. In other words, smaller parameters  $\varepsilon$ <br>ation.<br><sup>3</sup> or dataset universe  $\mathbb{D} \subset \mathbb{R}^k$ , the output<br> $\$ 

perturbation is  $x(d) + \hat{\zeta}$  with perturbation  $\hat{\zeta}$ , where  $\Delta_1$  is the worst-case  $\ell_1$ -sensitivity

 $\delta > 0$ , termed *probability of failure*. Accordingly,  $\tilde{x}(d)$  and  $\tilde{x}(d')$  are statistically<br>similar if parameters  $\varepsilon$  and  $\delta$  take smaller values. In other words, smaller parameters  $\varepsilon$ <br>and  $\delta$  can have the s similar if parameters  $\varepsilon$  and  $\delta$  take smaller values. In other words, smaller and  $\delta$  can have the stronger privacy preservation.<br> **Definition 5.2** (Output perturbation). For dataset universe  $\mathbb{D} \subset \mathbb{D}$  perturb the map  $x(\tilde{d})$ . d  $\delta$  can have the stronger privacy preservation.<br>
Definition 5.2 (Output perturbation). For dataset universe  $\mathbb{B} \subset \mathbb{R}^k$ , the output<br>
rturbation is  $x(d) + \zeta$  with perturbation  $\zeta$ , where  $\Delta_1$  is the worst-cas **Definition 5.2** (Output perturbation). For dataset universe  $\mathbb{D} \subset \mathbb{R}^k$ , the output<br>perturbation is  $x(d) + \hat{\zeta}$  with perturbation  $\hat{\zeta}$ , where  $\Delta_1$  is the worst-case  $\ell_1$ -sensitivity<br>of the map to adjacent perturbation is  $x(d) + \zeta$  with perturbation  $\zeta$ , where  $\Delta_1$  is the worst-case  $\ell_1$ -sensitivity<br>of the map to adjacent datasets.<br>**Definition 5.3** (Input perturbation). For dataset universe  $\mathbb{D} \subset \mathbb{R}^k$ , the i **Definition 5.3** (Input perturbation). For dataset universe  $\mathbb{D} \subset \mathbb{R}^k$ , the input perturbation is twofold: data perturbation  $\tilde{d} = d + \zeta$  with perturbation  $\zeta$ , where  $\Delta_1$  is the worst-case  $\ell_1$ -sensitivit et the DNs are considered as a connected undirected tree  $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ , where  $\Delta_1$  is<br>e worst-case  $\ell_1$ -sensitivity of an identity query to adjacent datasets, is followed by<br>emap  $x(\tilde{d})$ .<br>**Proposition 5.1** T

the worst-case  $\ell_1$ -sensitivity of an identity query to adjacent datasets, is followed by<br>the map  $x(\bar{d})$ .<br>**Proposition 5.1** The output and input perturbation strategies are  $(\varepsilon, 0)$ -differentially<br>private. Both stra the map  $x(\bar{d})$ .<br> **Proposition 5.1** The output and input perturbation strategies are  $(\epsilon, 0)$ -differentially<br>
private. Both strategies directly extend to  $(\epsilon, \delta)$ -differential privacy by choosing the<br>
Laplace distributio **Proposition 5.1** The output and input perturbation strategies are  $(\varepsilon, 0)$ -differentially<br>private. Both strategies directly extend to  $(\varepsilon, \delta)$ -differential privacy by choosing the<br>Laplace distribution of random pertu private. Both strategies directly extend to  $(\varepsilon, \delta)$ -differential privacy by choosing the<br>Laplace distribution of random perturbations calibrated to  $\ell_2$ -sensitivities.<br>5.2 Non-Private DNR Formulation<br>The DNs are cons

The set of optimization variables involves a set of operational variables  $x' := [P', Q', Q^{cr}, \beta^t]^T$ , switch status indicator variables  $u' \in \mathbb{Z}^{|\mathcal{E}|}$ . In this vein,  $P' \in \mathbb{R}^{|\mathcal{E}|}$  and  $Q' \in \mathbb{R}^{|\mathcal{E}|}$  refer to  $:=\bigr\lceil\!\!\bigl[P^l,\boldsymbol{Q}^l,\boldsymbol{Q}^{cr},\boldsymbol{\beta}^l\bigr\rceil\!\!\bigl]^T$ The set of optimization variables involves a set of operational variables<br>  $\mathbf{x}^l := [\mathbf{P}^l, \mathbf{Q}^l, \mathbf{Q}^{\alpha}, \mathbf{\beta}^l]^T$ , switch status indicator variables  $\mathbf{u}^l \in \mathbb{Z}^{|\mathcal{E}|}$ . In this vein,<br>  $\mathbf{P}^l \in \mathbb{R}^{|\$ erational variables<br>  $\mathbb{Z}^{|\mathcal{E}|}$ . In this vein,<br>
and reactive power<br>
tion and  $n_{cr}$  is the  $\boldsymbol{P}' \in \mathbb{R}^{|\mathcal{E}|}$  and  $\boldsymbol{Q}' \in \mathbb{R}^{|\mathcal{E}|}$  refer to the vectors of ization variables involves a set of operational variables<br>  $\mathbf{u}^l \in \mathbb{Z}^{|\mathcal{E}|}$ . In this vein,<br>  $\mathbb{R}^{|\mathcal{E}|}$  refer to the vectors of sending-end active and reactive power<br>
sthe vector of nodal reactive power com The set of optimization variables involves a set of operational variables  $x' := [P^i, Q^i, Q^{cr}, \beta^r]^T$ , switch status indicator variables  $u' \in \mathbb{Z}^{|\mathcal{E}|}$ . In this vein,  $P^i \in \mathbb{R}^{|\mathcal{E}|}$  and  $Q^i \in \mathbb{R}^{|\mathcal{E}|}$  ref The set of optimization variables involves a set of operational ve<br>  $x' = [P', Q', Q'', \beta']^T$ , switch status indicator variables  $u' \in \mathbb{Z}^{|\mathcal{E}|}$ . In this<br>  $P' \in \mathbb{R}^{|\mathcal{E}|}$  and  $Q' \in \mathbb{R}^{|\mathcal{E}|}$  refer to the vectors of variables involves a set of operational variables<br>tch status indicator variables  $u^l \in \mathbb{Z}^{|\mathcal{E}|}$ . In this vein,<br>per to the vectors of sending-end active and reactive power<br>tor of nodal reactive power compensation an The set of optimization variables involves a set of operational variables  $x' = [P', Q', Q'', B']^\top$ , switch status indicator variables  $u' \in \mathbb{Z}^{\beta}$ . In this vein,  $P' \in \mathbb{R}^{\beta}$  and  $Q' \in \mathbb{R}^{\beta}$  refer to the vectors of s The set of optimization variables involves a set of operational variables<br>  $x' := [P', Q', Q'', P']$ , switch status indicator variables  $u' \in \mathbb{Z}^{\beta_1}$ . In this vein,<br>  $P' \in \mathbb{R}^{\beta_1}$  and  $Q' \in \mathbb{R}^{\beta_2}$  refer to the vector

$$
\min_{x' \in \mathbb{R}, u' \in \mathbb{Z}} \quad \left[\begin{array}{c} \boldsymbol{P}' \\ \boldsymbol{Q}' \end{array}\right]^T \left[\begin{array}{cc} \boldsymbol{D}_r & \boldsymbol{0}_{|\mathcal{E}| \neq |\mathcal{E}|} \\ \boldsymbol{0}_{|\mathcal{E}| \neq |\mathcal{E}|} & \boldsymbol{D}_r \end{array}\right] \left[\begin{array}{c} \boldsymbol{P}' \\ \boldsymbol{Q}' \end{array}\right] \tag{5.3a}
$$

$$
\text{s.t.} \begin{bmatrix} A^T & \mathbf{0}_{|\varepsilon| \times |\varepsilon|} & \mathbf{0}_{|\varepsilon| \times n_{cr}} \\ \mathbf{0}_{|\varepsilon| \times |\varepsilon|} & A^T & A_{cr} \end{bmatrix} \begin{bmatrix} P^l \\ Q^l \\ Q^{cr} \end{bmatrix} = \begin{bmatrix} -P^{\varepsilon} + P^d \\ Q^{\varepsilon} + Q^d \end{bmatrix} \tag{5.3b}
$$

$$
2D_{r}P' + 2D_{x}Q' - (1 - u')M \le Av \le 2D_{r}P' + 2D_{x}Q' + (1 - u')M \quad (5.3c)
$$

$$
-u^{t}M \leq P^{t}, Q^{t} \leq u^{t}M, \quad \underline{v} \leq \overline{v} \leq \overline{Q}^{cr} \leq Q^{cr} \leq \overline{Q}^{cr} \tag{5.3d}
$$

$$
\beta_{mn}^l + \beta_{nm}^l = u_{mn}^l, \quad \beta_{mn}^l = 0, \text{ if } m = S \tag{5.3e}
$$

$$
\sum_{m(m,n)\in\mathcal{E}}\beta_{mn}^l=1,\quad\forall m\in\mathcal{N}\setminus S\tag{5.3f}
$$

$$
0 \leqslant \beta_{mn}^l \leqslant 1, \quad \forall l \in \mathcal{E} \tag{5.3g}
$$

 $\lim_{x \to a} \left[ \frac{P}{Q'} \right] \begin{bmatrix} \frac{1}{\omega} & \frac{1}{\omega} \\ \frac{1}{\omega} & \frac{1}{\omega} \end{bmatrix} \begin{bmatrix} \frac{1}{\omega} \\ \frac{1}{\omega} \end{bmatrix}$  (5.3a)<br>
S.t.  $\left[ \frac{A^T}{\omega_{\alpha+1}} \frac{\omega_{\beta+\alpha_{\alpha}}}{A^T} \right] \begin{bmatrix} \frac{1}{\omega'} \\ \frac{1}{\omega'} \end{bmatrix} = \left[ \frac{-P^S + P^S}{Q^S + Q^S} \right]$  (5.3b)<br>  $2D$ **EXAMPLE 10**  $\mathbf{L} = \begin{bmatrix} \mathbf{A}^T & \mathbf{0}_{[k]k|T} & \mathbf{0}_{[k]k|T} & \mathbf{0}_{[k]} & \mathbf{0}_{[k]} & \mathbf{0}_{[k]} & \mathbf{0}_{[k]} \end{bmatrix}$ <br> **S.f.**  $\begin{bmatrix} \mathbf{A}^T & \mathbf{0}_{[k]k|T} & \mathbf{0}_{[k]k|T} & \mathbf{0}_{[k]} & \mathbf{0}_{[k]} & \mathbf{0}_{[k]} & \mathbf{0}_{[k]} & \mathbf{0}_{[k]} & \mathbf{0}_{[k$ s.t.  $\begin{bmatrix} A^T & 0_{|\xi| \leq |\xi|} & 0_{|\xi| \leq |\xi|} & P' \\ 0_{|\xi| \leq |\xi|} & A^T & A_{cr} \end{bmatrix} \begin{bmatrix} P' \\ Q' \\ Q'' \end{bmatrix} = \begin{bmatrix} -P^{\xi} + P^{\xi} \\ Q^{\xi} + Q^{\xi} \end{bmatrix}$  (5.3b)<br>  $2D_r P' + 2D_s Q' - (1 - u')M \leq Av \leq 2D_r P' + 2D_s Q' + (1 - u')M$  (5.3c)<br>  $-u'M \leq P', Q' \leq u'M, \quad \underline{v$ denotes the branch capacity.  $P^s, Q^s$  and  $P^d, Q^d$  indicate the vectors of given  $\begin{aligned}\n\begin{bmatrix}\ne^{\eta_v} \\
\mathbf{Q}^t \\
\mathbf{Q}^r\n\end{bmatrix} & = \begin{bmatrix}\n-P^g + P^d \\
\mathbf{Q}^s + \mathbf{Q}^d\n\end{bmatrix}$ (5.3b)<br>  $v \le 2D_r P^t + 2D_x Q^t + (1 - u^t)M$  (5.3c)<br>  $\le v \le \overline{v}, \quad \mathbf{Q}^{\sigma} \le \mathbf{Q}^{\sigma} \le \overline{\mathbf{Q}}^{\sigma}$  (5.3d)<br>  $= 0, \text{ if } m = S$  (5.3e)<br>  $\forall m$ s.t.  $\left[\begin{array}{cc} \mathbf{s} \cdot \mathbf{t} \\ \mathbf{0} \cdot \mathbf{s} \cdot \mathbf{t} \end{array}\right] \cdot \left[\begin{array}{c} \mathbf{0}^T \\ \mathbf{0}^T \end{array}\right] = \left[\begin{array}{c} \mathbf{0}^T \cdot \mathbf{s} \cdot \mathbf{0} \end{array}\right]$  (5.30)<br>  $2\mathbf{D}_r \mathbf{P}' + 2\mathbf{D}_x \mathbf{Q}' - (1 - \mathbf{u}')M \le Av \le 2\mathbf{D}_r \mathbf{P}' + 2\mathbf{D}_x \mathbf{Q$  $2D_x P^i + 2D_x Q^i - (1 - u^i)M \le Av \le 2D_x P^i + 2D_x Q^i + (1 - u^i)M$  (5.3c)<br>  $-u^i M \le P^i, Q^i \le u^i M, \quad 1 \le v \le \overline{v}, \quad Q^{iv} \le Q^{iv} \le \overline{Q}^{iv}$  (5.3d)<br>  $\beta^i_{m\pi} + \beta^i_{m\pi} = u^i_{m\pi}, \quad \beta^i_{m\pi} = 0, \text{ if } m = S$  (5.3e)<br>  $\sum_{\substack{n \in [m,n] \le n}} \beta^i_{m\pi}$  $2D_rF + 2D_sQ - (1 - u^r)M \le Av \le 2D_rF + 2D_sQ + (1 - u^r)M$  (3.3c)<br>  $-u^tM \le P^t,Q^t \le u^tM$ ,  $\underline{v} \le v \le \overline{v}$ ,  $Q^{\circ} \le Q^{\sigma} \le \overline{Q}^{\circ}$  (5.3d)<br>  $\beta^t_{m\pi} + \beta^t_{m\pi} = u^t_{mn}$ ,  $\beta^t_{m\pi} = 0$ , if  $m = S$  (5.3e)<br>  $\sum_{n \in m, n \neq 0} \beta^t_{m\pi}$  $-u^t M \le P^t$ ,  $Q^t \le u^t M$ ,  $\underline{v} \le \nu \le \overline{v}$ ,  $Q^{\alpha} \le Q^{\alpha} \le Q^{\alpha}$  (5.3d)<br>  $\beta_{nn}^t + \beta_{nn}^t = u_{nn}^t$ ,  $\beta_{nn}^t = 0$ , if  $m = S$  (5.3e)<br>  $\sum_{\substack{n \in \mathbb{N} \text{ s} \\ n \in \mathbb{N}}} \beta_{nn}^t = 1$ ,  $\forall m \in \mathbb{N} \setminus S$  (5.3f)<br>  $0 \le \beta_{nn}^t \le \overline$  $\beta'_{mn} + \beta'_{mn} = u'_{mn}, \quad \beta'_{mn} = 0, \text{ if } m = S$  (5.3e)<br>  $\sum_{\alpha(m,\alpha) \in S} \beta'_{mn} = 1, \quad \forall m \in \mathcal{N} \setminus S$  (5.3f)<br>  $0 \le \beta'_{mn} \le 1, \quad \forall l \in \mathcal{E}$  (5.3g)<br>
where (5.3a) states the quadratic active power loss of DNs under the assumption of  $\sum_{n(m,n)\in\mathcal{E}} \beta_{nm}^{l'} = 1, \forall m \in \mathcal{N} \setminus S$  (5.3f)<br>  $0 \leq \beta_{mn}^{l'} \leq 1, \forall l \in \mathcal{E}$  (5.3g)<br>
where (5.3a) states the quadratic active power loss of DNs under the assumption of<br>
flat voltage profiles for all nodes. *M* refe

respectively.

spectively.<br>Therefore, we can summarize  $c, A, G_v, d, G_c, b_v, b_{cr}, K$  and h from (5.3e)-(5.3g). To<br>void heavy notions, we express the general mathematical formulation of a<br>pn-private DNR model in the MIOP form with respect to op  $b_{cr}$ , **K** and **h** from (5.3e)-(5.3g). To<br>mathematical formulation of a<br>espect to operational variables  $x^l$ respectively.<br>
Therefore, we can summarize  $c, A, G_v, d, G_v, b_v, b_{cr}, K$  and h from (5.3c)-(5.3g). To<br>
avoid heavy notions, we express the general mathematical formulation of a<br>
non-private DNR model in the MIQP form with respec respectively.<br>
Therefore, we can summarize  $c, A, G_v, d, G_o, b_v, b_{cv}$ , *K* and *h* from (5.3e)-(5.3g). To<br>
avoid heavy notions, we express the general mathematical formulation of a<br>
non-private DNR model in the MIQP form with including actively.<br>
Therefore, we can summarize  $c, A, G_c, d, G_c, b_c, K$  and h from (5.3c)-(5.3g). To<br>
avoid heavy notions, we express the general mathematical formulation of a<br>
non-private DNR model in the MIQP form with resp respectively.<br>
Therefore, we can summarize  $c, A, G_v, d, G_v, b_v, b_{\sigma}, K$  and h from (5.3e)-(5.3g). To<br>
avoid heavy notions, we express the general mathematical formulation of a<br>
non-private DNR model in the MIQP form with respe respectively.<br>
Therefore, we can summarize  $c, A, G_c, d, G_c, b_c, b_c, K$  and h from (5.3c)-(5.3g). To<br>
avoid heavy notions, we express the general mathematical formulation of a<br>
non-private DNR model in the MIQP form with respect below. pectively.<br>
herefore, we can summarize  $c, A, G_c, d, G_c, b_v, b_{cv}, K$  and h from (5.3e)-(5.3g). The<br>
heavy notions, we express the general mathematical formulation of<br>
h-private DNR model in the MIQP form with respect to operatio non-private DNR model in the MIQP form with respect to operational variables  $x^i$ <br>including active/reactive power flow variables and reactive power compensation<br>variables, continuous parent-child relationship variables including active/reactive power flow variables and reactive power compensation<br>variables, continuous parent-child relationship variables  $\beta'$  and switch status<br>indicator variables  $u'$ . We summarize this MIQP-based non-p

$$
\textbf{Non-Private DNA:} \qquad \min_{x' \in \mathbb{R}, u' \in \mathbb{Z}} F_0 = (x')^T c(x')
$$
\n(5.4a)

$$
\text{s.t.} \mathcal{X} := \left\{ (\mathbf{x}^l, \mathbf{u}^l) \; \begin{array}{l} \left| \tilde{A} \mathbf{x}^l = \tilde{d}, \left[ \begin{matrix} \mathbf{G}_v \\ \mathbf{G}_u \end{matrix} \right] \left[ \begin{matrix} \mathbf{x}^l \\ \mathbf{u}^l \end{matrix} \right] \leq \left[ \begin{matrix} \mathbf{b}_v \\ \mathbf{b}_u \end{matrix} \right], K \left[ \begin{matrix} \mathbf{x}^l \\ \mathbf{u}^l \end{matrix} \right] = \mathbf{h} \right\} \quad (5.4b)
$$

variables, continuous parent-child relationship variables  $\beta'$  and switch status<br>indicator variables  $u'$ . We summarize this MIQP-based non-private DNR model<br>below.<br>Non-Private DNR:  $\min_{x \in \mathbb{R}, u' \in \mathbb{R}} F_0 = (x')^T c(x')$  ( indicator variables  $u'$ . We summarize this MIQP-based non-private DNR model<br>below.<br>
Non-Private DNR:<br>  $\lim_{x'\to x} \int_{u'\in K} F_0 = (x')^T c(x')$  (5.4a)<br>  $s.t. \mathcal{X} := \begin{cases} (x', u') & \begin{bmatrix} \lambda u' = \tilde{d} \end{bmatrix} \begin{bmatrix} \alpha_v \\ \alpha_u \end{bmatrix} \begin{bmatrix} x' \\ h_u \end{b$ below.<br> **Subscripts DNR:**  $\min_{x \in \mathbb{R}, d \in \mathbb{Z}} F_0 = (\mathbf{x}')^T \mathbf{c}(\mathbf{x}')$  (5.4a)<br>  $\text{s.t.} \mathcal{X} := \begin{cases} \mathbf{x}' \cdot \mathbf{a}' & \text{if } \mathbf{x}' = \mathbf{a} \cdot \begin{bmatrix} \mathbf{a}' \\ \mathbf{b}' \end{bmatrix} \leq \begin{bmatrix} \mathbf{a}' \\ \mathbf{b}' \end{bmatrix} \times \begin{bmatrix} \mathbf{a}' \\ \mathbf{b}' \end{bmatrix} \times \begin{bmatrix} \$ **Non-Private DNR:**  $\lim_{x \to 0} x \in G(x')^T c(x')$  (5.4a)<br>  $s.t. \mathcal{X} = \begin{cases} x^4 \in \mathbf{A} \cdot \mathbf{B}^T \mathbf{B} = (x')^T c(x') \end{cases}$  (5.4b)<br>  $s.t. \mathcal{X} = \begin{cases} x^4 \in \mathbf{A} \cdot \mathbf{B}^T \mathbf{B} = \mathbf{A} \cdot \mathbf{B} \end{cases}$   $\mathbf{A} \cdot \mathbf{A}^T = \mathbf{A} \cdot \mathbf{B} \cdot \mathbf{B$ S.L.A.  $-\begin{bmatrix} 1 & .0 & .0 \\ 0 & .1 & .0 \end{bmatrix}$   $\begin{bmatrix} 0 & .0 & .0 \\ 0 & .0 & .0 \end{bmatrix}$ <br>
Where X refers to the non-empty feasibility space and f denotes the approximate<br>
system power loss with a fixed diagonal matrix e. The inequality in where  $X$  refers to the non-empty feasibility space and  $f$  denotes the approximate<br>system power loss with a fixed diagonal matrix  $c$ . The inequality in (5.4b) represents<br>the voltage security constraints, physical ranges stem power loss with a fixed diagonal matrix  $c$ . The inequality in (5.4b) represents<br>
c voltage security constraints, physical ranges of reactive power compensation<br>
pacitors and topology-linked branch capacity constrain The inequality in (5.4b) represents<br>s of reactive power compensation<br>onstraints, which are marked by the<br>rst and second equality denotes the<br>nstraints.<br> $N$ R Model<br>and corresponding tie-line load<br>ared with neighbor agents age security constraints, physical ranges of reactive power compensation<br>rs and topology-linked branch capacity constraints, which are marked by the<br>ts v, cr and u for G and b. The first and second equality denotes the<br>wi capacitors and topology-linked branch capacity constraints, which are marked by the<br>subscripts v, cr and u for G and b. The first and second equality denotes the<br>system-wide load balance of DNs and radiality constraints.<br>

flows  $x^{l_t}$  for an arbitrary tie-line  $l_t$  have to be shared with neighbor agents by a joint

interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tie-lines. For agent's privacy concerns, sharing raw data o interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tie-lines. For agent's privacy concerns, sharing raw data o interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tie-lines. For agent's privacy concerns, sharing raw data o interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tie-lines. For agent's privacy concerns, sharing raw data o interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tie-lines. For agent's privacy concerns, sharing raw data o interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tie-lines. For agent's privacy concerns, sharing raw data o interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tie-lines. For agent's privacy concerns, sharing raw data o interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tie-lines. For agent's privacy concerns, sharing raw data o interests, different agents as stakeholders may bid for grid services such as demand<br>response, which is dependent on operational topology as well as load flows of<br>tic-lines. For agent's privacy concerns, sharing raw data o response, which is dependent on operational topology as well as load flows of<br>tic-lines. For agent's privacy concerns, sharing raw data of tic-lines between agents<br>may cause privacy challenges, i.e., the private load chang -lines. For agent's privacy concerns, sharing raw data of tie-lines between agents<br>ty cause privacy challenges, i.e., the private load change information may be<br>posed. On the customer's privacy-preserving level, since a jo may cause privacy challenges, i.e., the private load change information may be<br>exposed. On the customer's privacy-preserving level, since a joint DSO performs the<br>DNR operation for the loss minimization of the entire inter exposed. On the customer's privacy-preserving level, since a joint DSO performs the<br>DNR operation for the loss minimization of the entire interconnected ADNs, all load<br>datasets from different agents are obliged to upload t

DNR operation for the loss minimization of the entire interconnected ADNs, all load<br>datasets from different agents are obliged to upload to the distribution dispatch center.<br>This may increase the possibility of exposing se datasets from different agents are obliged to upload to the distribution dispatch center.<br>This may increase the possibility of exposing sensitive load consumption of<br>customers managed by an agent to adversaries who can fur This may increase the possibility of exposing sensitive load consumption of<br>customers managed by an agent to adversaries who can further infer commercial<br>behaviors or perform cyber-physical attacks.<br>We illustrate these two customers managed by an agent to adversaries who can further infer commercial<br>behaviors or perform cyber-physical attacks.<br>We illustrate these two categories of privacy concerns in one example. For the<br>agent's concerns, as behaviors or perform cyber-physical attacks.<br>
We illustrate these two eategories of privacy concerns in one example. For the agent's concerns, as shown in Fig. 5.2(a), four agents A-D with three normally-open tie-switches We illustrate these two eategories of privacy concerns in one example. For the agent's concerns, as shown in Fig. 5.2(a), four agents A-D with three normally-open tie-switches where each agent manages a specific ADN with agent's concerns, as shown in Fig. 5.2(a), four agents A-D with three normally-open<br>tie-switches where each agent manages a specific ADN with one substation. If agent<br>A has an heavily loading event while others have light tie-switches where each agent manages a specific ADN with one substation. If agent<br>A has an heavily loading event while others have light loads displayed in the top layer<br>of Fig. 5.2(a), an optimal DNR decision with the mi A has an heavily loading event while others have light loads displayed in the top layer<br>of Fig. 5.2(a), an optimal DNR decision with the minimal power loss objective can be<br>made to close on all tie-switches between agent A have been transferred to other agents. Then, agents B, C and D can bid high prices for grid services, while agent A cannot win the contracts at the low price. Because they know that agent A has insufficient loads to respon have been transferred to other agents. Then, agents B, C and D can bid high prices for grid services, while agent A cannot win the contracts at the low price. Because they know that agent A has insufficient loads to respon have been transferred to other agents. Then, agents B, C and D can bid high prices for<br>grid services, while agent A cannot win the contracts at the low price. Because they<br>know that agent A has insufficient loads to respon have been transferred to other agents. Then, agents B, C and D can bid high prices for<br>grid services, while agent A cannot win the contracts at the low price. Because they<br>know that agent A has insufficient loads to respon Interaction of agents. Then, agents B, C and D can bid high prices for grid services, while agent A cannot win the contracts at the low price. Because they know that agent A has insufficient loads to respond this request. have been transferred to other agents. Then, agents B, C and D can bid high prices for grid services, while agent A cannot win the contracts at the low price. Because they know that agent A has insufficient loads to respon have been transferred to other agents. Then, agents B, C and D can bid high prices for grid services, while agent A cannot win the contracts at the low price. Because they know that agent A has insufficient loads to respon have been transferred to other agents. Then, agents B, C and D can bid high prices for<br>grid services, while agent A cannot win the contracts at the low price. Because they<br>know that agent A has insufficient loads to respon have been transferred to other agents. Then, agents B, C and D can bid high prices for<br>grid services, while agent A cannot win the contracts at the low price. Because they<br>know that agent A has insufficient loads to respon grid services, while agent A cannot win the contracts at the low price. Because they<br>know that agent  $\Lambda$  has insufficient loads to respond this request. As such, a synthetic<br>DNR solution with realistic tie-switch status know that agent A has insufficient loads to respond this request. As such, a synthetic<br>DNR solution with realistic tie-switch status can be against this load change<br>information of agent A leaked by realistic DNR operations DNR solution with realistic tie-switch status can be against this load change<br>information of agent A leaked by realistic DNR operations. As indicated in the third<br>DP-DNR layer, as long as tie-line load flows and directions



**EXECUTE:**<br> **EXEC**  $x^{l_t}$  from  $x^{l}$  and realistically optimal topology variables  $u^{l}$ . To do this, we ain **Example 19 AVE CONTAINE (b)**<br> **(b)**<br>
Fig. 5.2. (a) An exposure event of agent's privacy; (b) leakage of customer's privacy<br>
by centralized DNR model.<br>
.3.2 Privacy-Preserving Criteria For DP-DNR Mechanism<br>
The well-known Final Transformation of the set of customer's privacy<br>
to make queries over<br>  $\tilde{M}$  for  $(x', u')$ . This<br>
sible tie-line load flows<br>  $\tilde{M}$ :<br>  $\tilde{M}$ :<br>
(5.5) Fig. 5.2. (a) An exposure event of agent's privacy; (b) leakage of customer's privacy<br>by centralized DNR model.<br>
5.3.2 Privacy-Preserving Criteria For DP-DNR Mechanism<br>
The well-known Laplace mechanism can be used to make

$$
\min_{x' \in \mathbb{R}, u' \in \mathbb{Z}} F_1 = \left(x'\right)^T \mathbf{c} x' \tag{5.5}
$$

s.t. optimality of topology variables  $\mathbf{u}^l$  (5.6)

 $l_t$  (5.7)

s.t. optimality of topology variables  $u^{l}$  (5.6)<br>easibility of operational variables for  $x^{l_t}$  (5.7)<br>ction (5.6) represents the system loss of the entire s.t. optimality of topology variables  $u^l$  (5.6)<br>feasibility of operational variables for  $x^{l_t}$  (5.7)<br>notion (5.6) represents the system loss of the entire<br>notion (5.7) enforces  $\tilde{M}$  to output realistically optimal s.t. optimality of topology variables  $u^l$  (5.6)<br>feasibility of operational variables for  $x^l$  (5.7)<br>where the objective function (5.6) represents the system loss of the entire<br>interconnected ADNs. Constraint (5.7) enfo S.t. optimality of topology variables  $u^l$  (5.6)<br>feasibility of operational variables for  $x^l$  (5.7)<br>where the objective function (5.6) represents the system loss of the entire<br>interconnected ADNs. Constraint (5.7) enfo s.t. optimality of topology variables  $u^l$ <br>feasibility of operational variables for  $x^{l_t}$ <br>where the objective function (5.6) represents the system loss of th<br>interconnected ADNs. Constraint (5.7) enforces  $\tilde{M}$  to o topology solutions  $u^{l*}$  equal to the ones by the non-private DNR model. Constraint s.t. optimality of topology variables  $u^l$  (5.6)<br>feasibility of operational variables for  $x^{l_t}$  (5.7)<br>where  $(5.6)$  represents the system loss of the entire<br>Constraint (5.7) enforces  $\tilde{\mathcal{M}}$  to output realistically S.t. optimality of topology variables  $u^i$  (5.6)<br>feasibility of operational variables for  $x^{l_c}$  (5.7)<br>where the objective function (5.6) represents the system loss of the entire<br>interconnected ADNs. Constraint (5.7) en s.t. optimality of topology variables  $u'$  (5.6)<br>feasibility of operational variables for  $x^{i}$  (5.7)<br>where the objective function (5.6) represents the system loss of the entire<br>interconnected ADNs. Constraint (5.7) enfo enable  $\tilde{M}$  to provide obfuscated-but-feasible tie-line load flows  $x^{i^*}$ . s.t. optimality of topology variables  $u^l$  (5.6)<br>feasibility of operational variables for  $x^{l_i}$  (5.7)<br>enere the objective function (5.6) represents the system loss of the entire<br>erconnected ADNs. Constraint (5.7) enfor ε. optimality of topology variables  $\mathbf{r}$  (3.6)<br>feasibility of operational variables for  $\mathbf{x}^{i}$  (5.7)<br>where the objective function (5.6) represents the system loss of the entire<br>interconnected ADNs. Constraint (5. here the objective function (5.6) represents the system loss of the entire<br>erconnected ADNs. Constraint (5.7) enforces  $\tilde{\mathcal{M}}$  to output realistically optimal<br>ology solutions  $\mathbf{u}^{\mu}$  equal to the ones by the nonincomostraint (5.7) enforces  $\tilde{\mathcal{M}}$  to output realistically optimal<br>
lutions  $u^{i*}$  equal to the ones by the non-private DNR model. Constraint<br>
si the operational constraints involving random perturbations, which ca Let realistically optimal<br>
MR model. Constraint<br>
rturbations, which can<br>
s  $x^{i'}$ .<br>
m  $\tilde{M}$  can be an<br>
:<br>
<br>
irbations obey Laplace<br>  $l_i \in T$ . And  $(\hat{x}^i, u^{i*}) \in$ <br>
<br>  $\ge$  DP-DNR mechanism<br>
onal variable solution

distribution  $\xi^{l_i} \sim \mathbb{P}_{\xi}$ ,  $\mathbb{P}_{\xi} = \text{Lap}(\Delta_{\rho} / \varepsilon)$ , for an arbitrary tie-line  $l_i \in \mathcal{T}$ . And  $(\hat{\mathbf{x}}^l, \mathbf{u}^{l*}) \in$ is the operational constraints involving random perturbations, which can<br>stable  $\tilde{M}$  to provide obfuscated-but-feasible tie-line load flows  $x^{\zeta}$ .<br>According to this framework, this DP-DNR mechanism  $\tilde{M}$  can be (5.8) denotes the operational constraints involving random perturbations, which can<br>rable  $\mathcal M$  to provide obfuscated-but-feasible tie-line load flows  $x^{\zeta}$ .<br>According to this framework, this DP-DNR mechanism  $\mathcal M$  c  $\vec{r}_t$  and realistically optimal topology solution  $\vec{u}^{l*}$  such that  $\hat{x}^{l_t} = \vec{r}^{l^*} + \vec{a}^{l^*} \xi^{l_t}$  for enable  $\hat{M}$  to provide obfuscated-but-feasible tie-line load flows  $x^{\zeta}$ .<br>
According to this framework, this DP-DNR mechanism  $\hat{M}$  can be an<br>
z-differential private algorithm based on the following Theorem:<br> **The** ine load flows  $x^{i}$ .<br>
R mechanism  $\tilde{M}$  can be an<br>
ing Theorem:<br>
andom perturbations obey Laplace<br>
ary tie-line  $l_i \in T$ . And  $(\hat{x}^i, u^{i*}) \in$ <br>
lel. Then, the DP-DNR mechanism<br>
sible operational variable solution<br>
, s x<sup>i</sup>.<br>
m  $\tilde{\mathcal{M}}$  can be an<br>
:<br>
<br>
x bations obey Laplace<br>  $l_i \in \mathcal{T}$ . And  $(\hat{x}^i, u^{i*}) \in \mathcal{D}P$ -DNR mechanism<br>
<br>
onal variable solution<br>  $\hat{x}^{l_i} = x^{l_i^*} + \alpha^{l_i^*} \xi^{l_i}$  for this<br>
<br>
x roof is as follows: tie-line  $l_t$ . This can be an  $\epsilon$ -differential private algorithm. The proof is as follows: ding to this framework, this DP-DNR mechanism  $\tilde{M}$  can be an trial private algorithm based on the following Theorem:<br> **em 5.1** (*DP-DNR Mechanism*): Suppose random perturbations obey Laplace<br>  $\text{tan } \tilde{\xi}^b - \mathbb{P}_{\xi}$ e-differential private algorithm based on the following Theorem:<br> **Theorem 5.1** (*DP-DNR Mechanism*): Suppose random perturbations obey Laplace<br>
distribution  $\xi^b \sim \mathbb{P}_z$ ,  $\mathbb{P}_z = Lap(\Lambda_p/s)$ , for an arbitrary tie-line  $l$ **Theorem 5.1** (*DP-DNR Mechanism*): Suppose random perturbations obey<br>distribution  $\xi^{l_i} \sim \mathbb{P}_{\xi}$ ,  $\mathbb{P}_{\xi} = Lap(\Delta_{\rho}/\varepsilon)$ , for an arbitrary tic-line  $l_i \in \mathcal{T}$ . And (<br> $\mathcal{X}$  is the optimal solution of non-privat **5.1** (*DP-DNR Mechanism*): Suppose random perturbations obey Laplace<br>  $\psi \sim \mathbb{P}_{\xi}$ ,  $\mathbb{P}_{\xi} = Lap(\Delta_{\rho} / \varepsilon)$ , for an arbitrary tie-line  $l_{\xi} \in T$ . And  $(\hat{x}', u'') \in$ <br>
mal solution of non-private DNR model. Then, the DP **EXECUTE:** LET DAN *INCRETION*, Explorational polarizations over Lagridiative distribution  $\xi^{i_k} \sim \mathbb{P}_{\xi}$ .  $\mathbb{P}_{\xi} = Lap(\Delta_{\rho}/\varepsilon)$ , for an arbitrary tie-line  $l_i \in \mathcal{T}$ . And  $(\hat{x}', u' \mathcal{X})$  is the optimal solution o *for* can output a mixture of obfuscated-but-feasible operational variable solution  $x^6$  and realistically optimal topology solution  $u^6$ , such that  $\dot{x}^6 = x^6 + a^6 \dot{x}^6$  for this tite-line *l<sub>i</sub>*. This can be an *c*-d

answers be  $Q = x^{l_t}$ , and we alternatively rewrite this theorem in the definition of

$$
\mathbb{P}_{\xi}[\tilde{\mathcal{M}}(\boldsymbol{d}) = \mathcal{O}] \leq \mathbb{P}_{\xi}[\tilde{\mathcal{M}}(\boldsymbol{d}') = \mathcal{O}]e^{\varepsilon} \tag{5.8}
$$

For convenience, we define  $\xi^l = [\xi^{l_1}, \dots, \xi^{l_n}]$ ,  $\{\alpha^{l_i}\} = [\alpha^{l_1}, \dots, \alpha^{l_n}]$ ,<br>  $\{\alpha^{l_i}\} = [\alpha^{l_1}, \dots, \alpha^{l_n}]$ , where *n* refers to the total number of tie-lines. Thus, the query<br>
tput  $\{\mathcal{O}^{l_i}\}\$  for all tie-lines  $\forall l$  $\bm{\xi}^l = \begin{bmatrix} \bm{\xi}^{l_1}, \ldots, \bm{\xi}^{l_n} \end{bmatrix} \quad , \hspace{5mm} \left\{ \bm{a}^{l_t} \right\} \!=\!\! \begin{bmatrix} \alpha^{l_1}, \ldots, \alpha^{l_n} \end{bmatrix} \quad ,$  $\left\{ \mathbf{x}^{l_i} \right\} = \left\lceil x^{l_1}, \ldots, x^{l_n} \right\rceil$ , where *n* refers For convenience, we define  $\xi^l = [\xi^{l_1}, \dots, \xi^{l_n}]$ ,  $\{\alpha^{l_i}\} = [\alpha^{l_1}, \dots, \alpha^{l_n}]$ ,<br>  $x^{l_i}\} = [x^{l_1}, \dots, x^{l_n}]$ , where *n* refers to the total number of tie-lines. Thus, the query<br>
utput  $\{O^{l_i}\}$  for all tie-lines  $\forall l_i \in T$ , output  $\{\mathcal{O}^{l_i}\}\$  for all tie-lines  $\forall l_i \in \mathcal{T}$ ,  $\forall t = 1,...,n$ , with the vectors of  $\xi^l$ , can be convenience, we define  $\xi^{t} = \left[\xi^{t_1}, \dots, \xi^{t_n}\right]$ ,  $\left\{\alpha^{t_i}\right\} = \left[\alpha^{t_1}, \dots, \alpha^{t_n}\right]$ ,<br>  $x^{t_1}, \dots, x^{t_n}$ , where *n* refers to the total number of tie-lines. Thus, the query<br>  $\mathcal{O}^{t_i}$  for all tie-lines  $\forall l_i \in \mathcal{T}$ For convenience, we define  $\xi^{l} = [\xi^{l_1}, \dots, \xi^{l_n}]$ ,  $\{\alpha^{l_i}\} = [\alpha^{l_1}, \dots, \alpha^{l_n}]$ ,<br>  $\{x^{l_i}\} = [x^{l_1}, \dots, x^{l_n}]$ , where *n* refers to the total number of tie-lines. Thus,<br>
output  $\{O^{l_i}\}$  for all tie-lines  $\forall l_i \in \mathcal{T}$ , For convenience, we define  $\zeta' = [\zeta^{t_1},...,\zeta^{t_n}]$ ,  $\{a^{t_i}\} = [\alpha^t, ..., \alpha^{t_n}]$ ,<br>  $\kappa^t_i = [x^{t_1},...,x^{t_n}]$ , where *n* refers to the total number of tie-lines. Thus, the query<br>
tput  $\{\emptyset^{t_i}\}$  for all tie-lines  $\forall l_i \in \mathcal{T}$ , For convenience, we define  $\xi^{i} = [\xi^{i_1}, ..., \xi^{i_n}]$ ,  $\{\alpha^{i_i}\} = [\alpha^{i_1}, ..., \alpha^{i_n}]$ ,<br>  $\{\mathbf{x}^{i_j}\} = [\mathbf{x}^{i_1}, ..., \mathbf{x}^{i_n}]$ , where *n* refers to the total number of tie-lines. Thus, the query<br>
output  $\{\mathcal{O}^{i_j}\}$  for all tie-line

$$
\mathcal{O}^{l_t} = \mathbf{x}^{l_t^*} = \hat{\mathbf{x}}^{l_t} - \mathbf{\alpha}^{l_t^*} \xi^{l_t}, \quad \forall t \in [1, n] \tag{5.9}
$$

$$
\{x^i\} = [x^i, \dots, x^i] \text{ , where } n \text{ refers to the total number of ic-lines. Thus, the query output  $\{\emptyset^i\}$  for all tic-lines  $\forall l, \in \mathcal{T}$ ,  $\forall l = 1, \dots, n$ , with the vectors of  $\xi^i$ , can be written as\n
$$
\mathcal{O}^k = x^{\hat{i} \hat{i}} = \hat{x}^{\hat{i} \hat{j}} = \hat{x}^{\hat{i
$$
$$

 $\hat{\mathbf{x}}^{l_t}(\bm{d}) \!-\! \hat{\mathbf{x}}^{l_t}\left(\bm{d'}\right)\!\!\Big\|_2$ 

 $\hat{\mathbf{x}}^{l}(\boldsymbol{d}) - \hat{\mathbf{x}}^{l}(\boldsymbol{d}')\|_{2} \leq \Delta \rho$ . Accordingly  $\hat{x}^{i}(d) - \hat{x}^{i}(d')\|_{2} \leq \Delta \rho$ . Accordingly, it is clear that (5.8) holds based on (5.10), which proves this Theorem.  $\left\|\hat{x}^{i}(d) - \hat{x}^{i}(d')\right\|_{2} \leq \Delta \rho$ . Accordingly, it is clear that (5.8) holds based on (5.10),<br>which proves this Theorem.

 $\left| \mathbf{F}^{(d)}(\mathbf{d}') \right|_{2} \leq \Delta \rho$ . Accordingly, it is clear that (5.8) holds based on (5.10),<br>
inch proves this Theorem.<br>
Please note that  $\alpha'$  represents a vector of recourse variables by the DP-DNR<br>
schanism  $\tilde{\mathcal{M$  $\|\hat{x}'(d) - \hat{x}'(d')\|_2 \leq \Delta \rho$ . Accordingly, it is clear that (5.8) holds based on (5.10),<br>which proves this Theorem.<br>Please note that  $\alpha'$  represents a vector of recourse variables by the DP-DNR<br>mechanism  $\tilde{M}$ . For t  $\boldsymbol{\alpha}^{l_i} \in [-\overline{\boldsymbol{\alpha}}^l, \overline{\boldsymbol{\alpha}}^l]$  and  $\boldsymbol{\alpha}^{l_b} = 0$  for other internal (a s) holds based on (5.10),<br>
■<br>
variables by the DP-DNR<br>  $α^{l_b} = 0$  for other internal<br>
Therefore, we can express<br>  $l \in \varepsilon$ . The recourse function  $\|\hat{x}'(d) - \hat{x}'(d')\|_2 \leq \Delta \rho$ . Accordingly, it is clear that (5.8) holds based on (5.10),<br>which proves this Theorem.<br>Please note that  $\alpha'$  represents a vector of recourse variables by the DP-DNR<br>mechanism  $\tilde{M}$ . For t  $\left|\hat{x}'(d) - \hat{x}'(d')\right|_{2} \leq \Delta \rho$ . Accordingly, it is clear that (5.8) holds based on (5.10),<br>
which proves this Theorem.<br>
Please note that  $\alpha'$  represents a vector of recourse variables by the DP-DNR<br>
mechanism  $\hat{M}$ .  $\|\hat{x}^{i}(d) - \hat{x}^{i}(d^{i})\|_{2} \leq \Delta \rho$ . Accordingly, it is clear that (5.8) holds based on (5.10),<br>which proves this Theorem.<br>Please note that  $\alpha^{i}$  represents a vector of recourse variables by the DP-DNR<br>mechanism  $\tilde{M$  $\|\hat{x}^{i}(d) - \hat{x}^{i}(d')\|_{2} \leq \Delta \rho$ . Accordingly, it is clear that (5.8) holds based on (5.10),<br>which proves this Theorem.<br>Please note that  $\alpha^{i}$  represents a vector of recourse variables by the DP-DNR<br>mechanism  $\tilde{\mathcal{$ Solutions, we further propose two privacy-preserving criteria for shared tie-line load<br>flow data, i.e., both and there is the propose variables by the DP-DNR<br>mechanism  $\bar{M}$ . For tie-lines,  $a^k \in [-\bar{a}^i, \bar{a}^i]$  and Please note that  $\alpha^i$  represents a vector of recourse variables by the DP-DNR<br>mechanism  $\bar{M}$ . For tie-lines,  $\alpha^i \in [-\bar{\alpha}^i, \bar{\alpha}^i]$  and  $\alpha^k = 0$  for other internal<br>branches  $\{l_b\}$  in each agent, where  $\epsilon = \{l_b\$ chanism  $\hat{M}$ . For tic-lines,  $\alpha^i \in [-\overline{\alpha}', \overline{\alpha}']$  and  $\alpha^i = 0$  for other internal<br>anches  $\{b_i\}$  in each agent, where  $\varepsilon = \{b_i\} \cup T$  holds. Therefore, we can express<br> $+\alpha' \xi'$  to represent the load flows of an arbi anches  $\{l_b\}$  in each agent, where  $\varepsilon = \{l_b\} \cup \mathcal{T}$  holds. Therefore, we<br>  $+\alpha' \xi'$  to represent the load flows of an arbitrary line  $l \in \varepsilon$ . The recou<br>  $\alpha'$  is to scale the random perpetuation  $\xi'$  in the program each agent, where  $\varepsilon = \{l_b\} \cup T$  holds. Therefore, we can express<br>sesent the load flows of an arbitrary line  $l \in \varepsilon$ . The recourse function<br>cale the random perpetuation  $\xi^l$  in the program of DP-DNR<br>ch will be discus  $x' + \alpha' \xi'$  to represent the load flows of an arbitrary line  $l \in \varepsilon$ . The recourse function<br>of  $\alpha'$  is to scale the random perpetuation  $\xi'$  in the program of DP-DNR<br>mechanism, which will be discussed in the later con

of  $\alpha'$  is to scale the random perpetuation  $\xi'$  in the program of DP-DNR<br>mechanism, which will be discussed in the later context. Regarding synthetic<br>solutions, we further propose two privacy-preserving criteria for sh innechanism, which will be discussed in the later context. Regarding synthetic<br>solutions, we further propose two privacy-preserving criteria for shared tie-line load<br>flow data, i.e., load flow quantity and direction obfus solutions, we further propose two privacy-preserving criteria for shared tie-line load<br>flow data, i.e., load flow quantity and direction obfuscations.<br>(1) Load flow quantity obfuscation<br>To obfuscate  $x^i$  with quantities, flow data, i.e., load flow quantity and direction obfuscations.<br>
(1) Load flow quantity obfuscation<br>
To obfuscate  $x^l$  with quantities, we incorporate a virtual power injection variable<br>
vector  $g, g \in \mathbb{R}$ , at boundar supposed to be  $m \rightarrow n$ , i.e.,  $\hat{x}^{l_i} = d_n > 0$ , and the obfuscated load flow directions are obthis and direction obfuscations.<br> **Subsection**<br>
antities, we incorporate a virtual power injection variable<br>
nodes of tie-lines, which aims to balance the random<br>
and flows. Let  $g_m$  and  $g_n$  represent the virtual power (1) Load flow quantity obfuscation<br>
To obfuscate  $x^i$  with quantities, we incorporate a virtual power injection variable<br>
vector  $g, g \in \mathbb{R}$ , at boundary nodes of tie-lines, which aims to balance the random<br>
perturbati To obfuscate  $x^6$  with quantities, we incorporate a virtual power injection variable<br>vector  $g, g \in \mathbb{R}$ , at boundary nodes of tie-lines, which aims to balance the random<br>perturbations in tie-line load flows. Let  $g_m$  a

$$
A_m x^l = d_m - g_m, \qquad A_n x^l = d_n - g_n \tag{5.11}
$$

$$
\boldsymbol{g}_m = -\boldsymbol{\alpha}^{l_i} \boldsymbol{\xi}^{l_i}, \ \ \boldsymbol{g}_n = \boldsymbol{\alpha}^{l_i} \boldsymbol{\xi}^{l_i}, \ \ \boldsymbol{g} \in \left[-\overline{\boldsymbol{g}}, \overline{\boldsymbol{g}}\right]
$$
 (5.12)

$$
-\boldsymbol{u}^{l_i}\overline{\boldsymbol{\alpha}}^l \leq \boldsymbol{\alpha}^{l_i} \leq \boldsymbol{u}^{l_i}\overline{\boldsymbol{\alpha}}^l
$$
\n
$$
(5.13)
$$

 $-u^{i} \overline{\alpha}^{i} \le \alpha^{i} \le u^{i} \overline{\alpha}^{i}$  (5.13)<br>where  $A_{m}$  and  $A_{n}$  refer to the *m*-th and *n*-th row of matrix *A*, respectively. And  $-\overline{g}$ <br>and  $\overline{g}$  are the lower and upper boundaries of *g*. By constraint (5.13  $-u^{i} \overline{\alpha}^{i} \le \alpha^{i} \le u^{i} \overline{\alpha}^{i}$  (5.13)<br>where  $A_{m}$  and  $A_{n}$  refer to the *m*-th and *n*-th row of matrix *A*, respectively. And  $-\overline{g}$ <br>and  $\overline{g}$  are the lower and upper boundaries of *g*. By constraint (5.13 variable  $\alpha^{l_i}$  is enforced to a zero if this tie-line  $l_i$  is switch-off by  $\mathbf{u}^{l_i} = 0$ ;  $-u^{i} \overline{\alpha}^{i} \le \alpha^{i} \le u^{i} \overline{\alpha}^{i}$  (5.13)<br>
and  $A_{n}$  refer to the *m*-th and *n*-th row of matrix *A*, respectively. And  $-\overline{g}$ <br>
re the lower and upper boundaries of *g*. By constraint (5.13), the recourse<br>  $\alpha^{i}$  otherwise,  $\alpha^{l_1}$  can be in the range of  $[-\overline{\alpha}^l, \overline{\alpha}^l]$ . For a switch-on tie-line  $l_1$ ,  $\alpha^{l_1}$  is  $-u^k \overline{\alpha}^l \le \alpha^k \le u^k \overline{\alpha}^l$  (5.13)<br>
and  $A_n$  refer to the *m*-th and *n*-th row of matrix *A*, respectively. And  $-\overline{g}$ <br>
ce the lower and upper boundaries of *g*. By constraint (5.13), the recourse<br>  $x^k$  is enforc  $\leq \alpha^{l_i} \leq u^{l_i} \overline{\alpha}^{l_i}$  (5.13)<br>
th row of matrix *A*, respectively. And  $-\overline{g}$ <br>
ries of *g*. By constraint (5.13), the recourse<br>
this tie-line *l<sub>t</sub>* is switch-off by  $u^{l_i} = 0$ ;<br>  $-\overline{\alpha}^{l_i}, \overline{\alpha}^{l_i}$ ]. For a sw  $-i\ell^2 \vec{\alpha} \le \alpha^k \le \mu^k \vec{\alpha}$  (5.13)<br>
where  $A_m$  and  $A_a$  refer to the *m*-th and *n*-th row of matrix  $A$ , respectively. And  $-\vec{R}$ <br>
and  $\vec{R}$  are the lower and upper boundaries of  $g$ . By constraint (5.13), the recou incorporated to scale the random perpetuation  $\xi^{l_i}$  in constraints (5.12).  $\leq u^h \overline{a}^l$  (5.13)<br>
w of matrix *A*, respectively. And  $-\overline{g}$ <br>
of *g*. By constraint (5.13), the recourse<br>
tie-line *l<sub>t</sub>* is switch-off by  $u^h = 0$ ;<br>  $\overline{a}^l$ ]. For a switch-on tie-line *l<sub>t</sub>*,  $\alpha^h$  is<br>  $\xi^h$ 



 $l_t^*$  $l_t^*$ Fig.5.3(a), it maybe smaller than the realistically optimal load flow <sup>ˆ</sup> Fig. 5.3(a), it maybe smaller than the realistically optimal load flow  $\hat{x}^{l_i}$  $\hat{\mathbf{x}}^{l_t}$  if  $0 \le a^{l_i} \xi^{l_i} \le d_n$  with the direction  $m \to n$ , or larger than  $\hat{x}^{l_i}$  if  $a^{l_i} \xi^{l_i} \le 0$  with the the dum state of the direction of  $\alpha^{l_1^* g l_1} > d_n$  d<sub>n</sub><br>
atted flow of<br>  $\alpha^{l_2^* g l_2} > d_n$ <br>
b)<br>  $\rightarrow m$ .<br>
njections  $g_m$  and  $g_n$ <br>
ad flow  $x^{l_1^*}$ . In<br>
aad flow  $\hat{x}^{l_1^*}$  if<br>  $\alpha^{l_1^* g l_1^*} < 0$  with the<br>
nen  $\alpha^{l_1^* g l_1^*} > d_n$  in<br>
obfuscation. defined the most of the flow of  $g_m$  obfuscated flow of  $g_m$  (a) (b)<br>Fig. 5.3. (a) Realistic flow  $m\rightarrow n$ ; (b) reversed flow  $n\rightarrow m$ .<br>Indeed, if  $u^c = 1$  holds for  $a^{l_i} \xi^{l_i} > d_n$  in Fig.5.3(b), which will be discussed in the load flow direction obfuscation. Even (a)<br>
Fig. 5.3. (a) Realistic flow  $m\rightarrow n$ ; (b) reversed flow  $n\rightarrow m$ .<br>
Indeed, if  $u^c = 1$  holds for tie-line *l<sub>0</sub>*, then non-zero virtual injections  $g_w$  and  $g_w$ <br>
through constraints (5.11)-(5.13) can obfuscate the tie-l Indeed, if  $u^c = 1$  holds for tie-line *l<sub>i</sub>*, then non-zero virtual injections  $g_w$  and  $g_w$ <br>through constraints (5.11)-(5.13) can obfuscate the tie-line load flow  $x^6$ . In<br>Fig.5.3(a), it maybe smaller than the realisti Indeed, if  $u^c = 1$  holds for tie-line  $l_i$ , then non-zero virtual injections  $\mathbf{g}_m$  and  $\mathbf{g}_n$ <br>through constraints (5.11)-(5.13) can obfuscate the tie-line load flow  $x^c$ . In<br>Fig.5.3(a), it maybe smaller than the  $u^{\prime\prime}$ . Therefore, we propose the load flow quantity obfuscation criteria for  $x^{\prime\prime}$  which through constraints (5.11)-(5.13) can obfuscate the tie-line load flow  $x^6$ . In Fig.5.3(a), it maybe smaller than the realistically optimal load flow  $\hat{x}^k$  if  $0 < \alpha^c \xi^k < d_n$  with the direction  $m \to n$ , or larger than

**Criteria 1**(*Quantity Obfuscation of Tie-lines*): With virtual power injection **g** at undary nodes of tie-lines, the proposed DP-DNR mechanism  $\tilde{M}$  with the odified objective function  $F_1$ **Criteria 1**(*Quantity Obfuscation of Tie-lines*): With virtual power injection **g** at boundary nodes of tie-lines, the proposed DP-DNR mechanism  $\tilde{M}$  with the modified objective function  $F_1$ <br> $F_1 = (x^t + \alpha^t \xi^t)^T \mathbf$ **Criteria 1**(*Quantity Obfuscation of Tie-lines*): With virtual power injection *g* at<br>boundary nodes of tie-lines, the proposed DP-DNR mechanism  $\tilde{M}$  with the<br>modified objective function  $F_1$ <br> $F_1 = (x^l + a^l \xi^l)^T c (x^$ **Criteria 1**(*Quantity Obfuscation of Tie-lines*): With virtual power injection **g** at boundary nodes of tie-lines, the proposed DP-DNR mechanism  $\hat{\mathcal{M}}$  with the modified objective function  $F_1$ <br>  $F_1 = (x^i + \alpha^i \xi^i)^T$ If Tie-lines): With virtual power injection g at<br>
oposed DP-DNR mechanism  $\tilde{\mathcal{M}}$  with the<br>  $+\alpha' \xi^{l} \int^{T} c(x^{l} + \alpha' \xi^{l})$  (5.14)<br>  $\exists \alpha^{l'} \neq 0$  for then switch-on tie-line  $l_i$  is true.<br>
can be achieved as a non-zero **Criteria 1**(*Quantity Obfuscation of Tie-lines*): With virtual power injection *g* at undary nodes of tie-lines, the proposed DP-DNR mechanism  $\tilde{M}$  with the odified objective function *F<sub>1</sub>*<br> $F_i = (x^i + \alpha^i \xi^i)^T c (x^i +$ l α cap achieved DP-DNR mechanism  $\tilde{\mathcal{M}}$  with the<br>
c proposed DP-DNR mechanism  $\tilde{\mathcal{M}}$  with the<br>
c  $(\mathbf{x}' + \alpha' \xi')^T c(\mathbf{x}' + \alpha' \xi')$  (5.14)<br>
ere  $\exists \alpha' \neq 0$  for then switch-on tie-line *l<sub>i</sub>* is true.<br>
α<sup>*i*</sup> can **Criteria 1**(*Quantity Obfuscation of Tie-lines*): With virtual power injection *g* at boundary nodes of tie-lines, the proposed DP-DNR mechanism  $\tilde{\mathcal{M}}$  with the modified objective function  $F_1$ <br>  $F_i = (x^i + \alpha^i \xi^i)^T$ **Criteria 1**(*Quantity Obfuscation of Tie-lines*): With virtual power inj<br>boundary nodes of tie-lines, the proposed DP-DNR mechanism  $\tilde{M}$ <br>modified objective function  $F_1$ <br> $F_1 = (\mathbf{x}' + \mathbf{\alpha}' \xi')^T \mathbf{c} (\mathbf{x}' + \mathbf{\alpha}' \xi')$ <br> mitiy Obfuscation of Tie-lines): With virtual power injection **g** at<br>
of tie-lines, the proposed DP-DNR mechanism  $\hat{M}$  with the<br>
function  $F_1$ <br>  $F_1 = (x^t + a^t \xi^t)^T c (x^t + a^t \xi^t)$  (5.14)<br>
t  $(x^{\mu}, u^{\mu})$ , if there  $\exists a^$ boundary nodes of tie-lines, the proposed DP-DNR mechanism  $\hat{\mathcal{M}}$  with the<br>modified objective function  $F_1$ <br> $F_i = (x^i + a^i \xi^i)^T e(x^i + a^i \xi^i)$  (5.14)<br>should converge at  $(x^i, a^{i'})$ , if there  $\exists a^{i'} \neq 0$  for then switc

$$
F_1 = \left(\mathbf{x}^l + \mathbf{\alpha}^l \xi^l\right)^T \mathbf{c} \left(\mathbf{x}^l + \mathbf{\alpha}^l \xi^l\right) \tag{5.14}
$$

 $t \neq 0$ 

\* t

*Proof*: Suppose  $(\hat{x}^i, u^{i*}) \in \mathcal{X}$  is the optimal solution of non-private DNR model. It is solution in the same of  $(x^i, u^{i*})$ , if there  $\exists \alpha^{\xi} \neq 0$  for then switch-on tie-line  $l_i$  is true.<br>
solution-on tie-line  $l_i$ ,  $\alpha^{i}$  can be achieved as a non-zero number, which<br>
proof is as follows:<br>
ppose  $(\hat{x}^i,$ c proof is as follows:<br>
suppose  $(\hat{x}', u'') \in \mathcal{X}$  is the optimal solution of non-private DNR model. It is<br>
tthe following constraints can stand<br>  $G_v^{i'}\begin{bmatrix} \hat{x}' \\ u'' \end{bmatrix} \leq \phi_v^{i}$ ,  $\forall l_i \in \mathcal{T}$  (5.15)<br>  $\hat{x}^k = x^k + a^k \hat$ 

$$
\boldsymbol{G}_{v}^{l_{t}}\left[\begin{matrix} \hat{\boldsymbol{x}}^{l} \\ \boldsymbol{u}^{l^{*}} \end{matrix}\right] \leqslant \boldsymbol{b}_{v}^{l_{t}}, \quad \forall l_{t} \in \mathcal{T} \tag{5.15}
$$

Since  $\hat{x}^{l_t} = x^{l_t^*} + \alpha^{l_t^*} \xi^{l_t}$  holds accor

$$
\boldsymbol{G}_{v}^{l_{t},*}\boldsymbol{\alpha}^{l_{t}}\boldsymbol{\xi}^{l_{t}}\leqslant \boldsymbol{b}_{v}^{l_{t}}-\boldsymbol{G}_{v}^{l_{t}}\begin{bmatrix}\boldsymbol{x}^{l_{t}^{*}}\\ \boldsymbol{u}^{l^{*}}\end{bmatrix},\quad \forall l_{t}\in\mathcal{T}
$$
\n(5.16)

Where  $G_v^{l_{t,*}}$  is a real number on the v

oof: Suppose  $(\hat{x}^i, u^i) \in \mathcal{X}$  is the optimal solution of non-private DNR model. It is<br>
car that the following constraints can stand<br>  $G_{\psi}^{l} \begin{bmatrix} \hat{x}^{l} \\ u^{i*} \end{bmatrix} \le b_{\psi}^{l}$ ,  $\forall l_{l} \in \mathcal{T}$  (5.15)<br>
Since  $\hat{x}^{$ clear that the following constraints can stand<br>  $G_s^{\xi} \left[ \hat{x}^i \atop u^{\alpha} \right] \leq \hat{b}_s^{\xi}, \quad \forall l_s \in \mathcal{T}$  (5.15)<br>
Since  $\hat{x}^k = x^{\xi^k} + \alpha^{\xi^k} \hat{c}^{\xi^k}$  holds according to Theorem 2, we can find<br>  $G_s^{\xi, \alpha} \alpha^{\xi} \hat{c}^{\xi$  $G_v^{\dagger} \begin{bmatrix} \ddot{x} \\ \ddot{x} \\ \vdots \\ \ddot{x} \end{bmatrix} \leq b_v^{\dagger}, \quad \forall l_i \in \mathcal{T}$  (5.15)<br>
Since  $\dot{x}^{\dagger} = x^{\dagger} + a^{\dagger} \dot{\xi}^{\dagger}$  holds according to Theorem 2, we can find<br>  $G_v^{\dagger}{}^{\dagger} a^{\dagger} \dot{\xi}^{\dagger} \leq b_v^{\dagger} - G_v^{\dagger} \begin{bmatrix} x^{\dagger} \\ u$  $l_t$  $\exists T$  (5.15)<br>
m 2, we can find<br>  $\forall l_i \in T$  (5.16)<br>
e row vector  $G_v^{l_i}$ .<br>
(5.16)<br>
(5.16)<br>
(5.16)<br>
(5.16)<br>
(5.16)<br>
(5.16)<br>
(5.16)<br>
(5.16)<br>
(5.17)<br>
(5.17)

$$
\boldsymbol{\xi}^{l_i} \boldsymbol{G}_v^{l_i,*} \boldsymbol{\alpha}^{l_i} \leqslant \boldsymbol{b}_v^{l_i} - \boldsymbol{G}_v^{l_i} \begin{bmatrix} \hat{\boldsymbol{x}}^{l_i} \\ \boldsymbol{u}^{l*} \end{bmatrix}, \quad \forall l_i \in \mathcal{T}
$$
\n(5.17)

If  $\mathbf{G}_{\nu}^{l_i} \left| \begin{array}{c} \mathbf{x} \\ \mathbf{y} \end{array} \right| \leq b_{\nu}^{l_i}$  hold  $\left\Vert \hat{\mathbf{x}}^{l_{t}}\right\Vert _{<\boldsymbol{h}^{l_{t}}}$  $\mathbf{u}_l \begin{bmatrix} \hat{\mathbf{x}}^{l_t} \end{bmatrix}$   $\mathbf{L}^{l_t}$  $\left| \begin{array}{c} \nabla \nu \\ \nabla \nu \end{array} \right|$   $\left| \begin{array}{c} \nabla \nu \\ \nabla \nu \end{array} \right|$  $\left\lceil \hat{\mathbf{x}}^{l_t} \right\rceil$  $\bm{G}^{l_t}_v\left[\begin{matrix} \hat{\bm{x}}^{l_t} \ \bm{u}^{l*} \end{matrix}\right]\!\!\!<\!\!\bm{b}^{l_t}_v$  $\boldsymbol{u}^{\prime}$  $\langle \mathbf{b}_{\nu}^{l_i} \rangle$  holds, then  $\mathbf{b}_{\nu}^{l_i} - \mathbf{G}_{\nu}^{l_i} \begin{bmatrix} \hat{\mathbf{x}}^{l_i} \\ \mathbf{u}^{l^*} \end{bmatrix} \neq 0$  can be achieved. This means<br>
17). Otherwise,  $\mathbf{\alpha}^{l_i} = 0$  if and only if  $\mathbf{G}_{\nu}^{l_i} \begin{bmatrix} \hat{\mathbf{x}}^{l_i} \\ \mathbf{u}^{l^*}$  $\left\Vert \boldsymbol{\mathcal{G}}_{v}^{l_{t}}\right\Vert \boldsymbol{\hat{\mathcal{X}}}^{l_{t}}\left\Vert \boldsymbol{\neq}\boldsymbol{0}\right\Vert$  $\begin{bmatrix} l_i & \mathbf{C}^{l_i} \end{bmatrix} \hat{\mathbf{x}}^{l_i}$  $\mathbf{v}_{\nu}^{\mu}$   $-\mathbf{G}_{\nu}^{\mu}$   $\mathbf{u}_{\nu}^{\mu}$  $\lceil \, \hat{\mathbf{x}}^{l_t} \, \rceil$  $\boldsymbol{b}_v^{l_t} - \boldsymbol{G}_v^{l_t} \begin{bmatrix} \hat{\boldsymbol{x}}^{l_t} \\ \boldsymbol{u}^{l^*} \end{bmatrix} \neq 0$  $\boldsymbol{u}^{\prime}$ can be achieved. This means<br>  $\mathbf{G}_{v}^{l_{t}}\left[\hat{\mathbf{x}}^{l_{t}}_{t}\right]=\mathbf{b}_{v}^{l_{t}}$ . But as voltage  $l_i^* \neq 0$ If  $\mathbf{G}_{v}^{l} \left[ \hat{\mathbf{x}}_{v}^{l_{l}} \right] < \mathbf{b}_{v}^{l_{l}}$  holds, then  $\mathbf{b}_{v}^{l_{l}} - \mathbf{G}_{v}^{l_{l}} \left[ \hat{\mathbf{x}}_{v}^{l_{l}} \right] \neq 0$  can be achieved. This means  $\mathbf{a}^{l_{l}} \neq 0$  by (5.17). Otherwise,  $\mathbf{a}^{l_{l}} = 0$  if and only if  $\mathbf{I}_{i}^{t} = 0$  if and only if  $\mathbf{G}_{i}^{l_{t}} \begin{bmatrix} \hat{\mathbf{x}}^{l_{t}} \\ = \mathbf{b}_{i}^{l_{t}} \end{bmatrix}$ **a**  $\mathbf{b}_{v}^{l_i} - \mathbf{G}_{v}^{l_i} \left[ \hat{\mathbf{x}}_{v}^{l_i} \right] \neq 0$  can be achieved. This means<br>  $\alpha^{l_i^*} = 0$  if and only if  $\mathbf{G}_{v}^{l_i} \left[ \hat{\mathbf{x}}_{v}^{l_i} \right] = \mathbf{b}_{v}^{l_i}$ . But as voltage<br>
traints,  $\alpha^{l_i^*} = 0$  can be avoided as  $\mathbf{u}_l \begin{bmatrix} \hat{\mathbf{x}}^{l_t} \end{bmatrix}$   $\mathbf{h}_l$  $\begin{array}{c|c} \begin{array}{c} \n\ddots \\ \n\end{array} & \n\end{array}$  $\lceil \hat{x}^{l_t} \rceil$  $\boldsymbol{G}_{v}^{l_{t}}\left[\begin{array}{c} \hat{\boldsymbol{\mathcal{X}}}^{l_{t}} \\ \boldsymbol{u}^{l^{*}} \end{array}\right] = \boldsymbol{b}_{v}^{l_{t}}$  .  $\boldsymbol{u}^{\prime}$ hieved. This means<br> $= b_v^{l_i}$ . But as voltage<br>as long as the range If  $G_v^{l} \left[ \hat{x}^{l} \atop u^{l*} \right] < b_v^{l}$  holds, then  $b_v^{l} - G_v^{l} \left[ \hat{x}^{l} \atop u^{l*} \right] \neq 0$  can be achieved. This means  $a^{l'} \neq 0$  by (5.17). Otherwise,  $a^{l'} = 0$  if and only if  $G_v^{l} \left[ \hat{x}^{l'} \atop u^{l*} \right] = b_v^{l}$ . But as v  $l_t^*$  $\vec{r}_{v}^{l_{t}}\left[\hat{\vec{x}}^{l_{t}}\right] \neq 0$  can be achieved. This means<br>if and only if  $G_{v}^{l_{t}}\left[\hat{\vec{x}}^{l_{t}}\right] = b_{v}^{l_{t}}$ . But as voltage<br> $\alpha^{l_{t}} = 0$  can be avoided as long as the range<br>little bit. This proof also demonstrate If  $G_v^l \left[\frac{\hat{x}^l}{u^l}\right] < b_v^l$  holds, then  $b_v^l - G_v^l \left[\frac{\hat{x}^l}{u^l}\right] \neq 0$  can be achieved. This means  $\alpha^r \neq 0$  by (5.17). Otherwise,  $\alpha^r = 0$  if and only if  $G_v^l \left[\frac{\hat{x}^l}{u^r}\right] = b_v^l$ . But as voltage security co  $l_t^*$ If  $G_v^{l} \left[ \frac{\hat{\mathbf{x}}^{l_i}}{u^n} \right] < \delta_v^{l_i}$  holds, then  $b_v^{l_i} - G_v^{l_i} \left[ \frac{\hat{\mathbf{x}}^{l_i}}{u^n} \right] \neq 0$  can be achieved. This means  $\alpha^c \neq 0$  by (5.17). Otherwise,  $\alpha^c = 0$  if and only if  $G_v^{l_i} \left[ \frac{\hat{\mathbf{x}}^{l_i}}{u^n} \right] = b_v^{l_i$  $\boldsymbol{u}^{l_t^*}$  . ■ Production of the Contract of If  $G_v^{i} \begin{bmatrix} \hat{x}^{i} \\ \hat{u}^{i} \end{bmatrix} < b_v^{i'}$  holds, then  $b_v^{i} - G_v^{i} \begin{bmatrix} \hat{x}^{i} \\ \hat{u}^{i'} \end{bmatrix} \neq 0$  can be achieved. This  
\n $\vec{v} \neq 0$  by (5.17). Otherwise,  $a^{\vec{v}} = 0$  if and only if  $G_v^{i} \begin{bmatrix} \hat{x}^{i} \\ \hat{u}^{i'} \end{bmatrix} = b_v^{i'}$ . But as  
\n*unity* constraints are soft constraints,  $a^{\vec{v}} = 0$ b holds, then  $b_v^i - G_v^i \begin{bmatrix} \hat{x}^i \\ a^n \end{bmatrix} \neq 0$  can be achieved. This means<br>
. Otherwise,  $\alpha^i = 0$  if and only if  $G_v^i \begin{bmatrix} \hat{x}^i \\ a^n \end{bmatrix} = b_v^i$ . But as voltage<br>
sare soft constraints,  $\alpha^i = 0$  can be avoided as l If  $G_v^k \begin{bmatrix} a_v^{k-1} \end{bmatrix} \le b_v^k$  holds, then  $b_v^k - G_v^k \begin{bmatrix} a_v^{k-1} \end{bmatrix} \ne 0$  can be achieved. This means  $a^k \ne 0$  by (5.17). Otherwise,  $a^k = 0$  if and only if  $G_v^k \begin{bmatrix} \dot{x}_v^{k-1} \end{bmatrix} = b_v^k$ . But as voltage secur  $\alpha^c \neq 0$  by (5.17). Otherwise,  $\alpha^c = 0$  if and only if  $G_i^{\xi} \begin{bmatrix} \hat{\mathbf{x}}^k \\ \hat{\mathbf{u}}^n \end{bmatrix} = \mathbf{b}_i^{\xi}$ . But as voltage<br>security constraints are soft constraints,  $\alpha^c = 0$  can be avoided as long as the range<br>of v

 $l_t^*$ **EVALUATE:** Fig.5.3(a) shows the realistically optimal load flow direction of  $\mathbf{g}^{\mathcal{F}}$  and  $\mathbf{g}^{\mathcal{F}}$  and  $\mathbf{g}^{\mathcal{F}}$  cannot change the optimality of topology variables at  $\mathbf{g}^{\mathcal{F}}$ .<br>
<br> **E**<br>
With a p  $\alpha^{l_i^*} \xi^{l_i} > 0$  and  $g_m$  ejects  $-\alpha^{l_i^*} \xi^{l_i} < 0$ , then  $x^{l_i^*} + \alpha^{l_i^*} \xi^{l_i} = \hat{x}^{l_i}$ . This m the set of the minimal  $\kappa^e + \alpha^e \xi^h = \xi^h$  and  $\kappa^e + \alpha^e \xi^h = \xi^h$  and  $\kappa^e$  are the set of the virtual power injection vector **g** in this maintains the optimality of topology solutions and the virtual power injecti is proof also demonstrates that<br>
blogy variables at  $u^r$ .<br>
wever injection vector g in this<br>
stimality of topology solutions<br>
his criteria from the physical<br>
cond flow direction  $m \rightarrow n$ .<br>  $x^{r^r} + \alpha^{r^r} \xi^{t_i} = \hat{x}^{t_i}$ . minimization of  $F_1$ , denoted as  $F_1^*$ , is equal to the minimal  $F_0$  over  $\chi^C = \chi^C$ .<br> **The minimal is equal to the minimal is to the minimal to the minimal external in the proposed DP-DNR mechanism**  $\tilde{M}$  **maintain** if  $g_n = \alpha^{l_i^*} \xi^{l_i} < 0$  and  $g_m = -\alpha^{l_i^*} \xi^{l_i} > 0$ , then  $x^{l_i^*} + \alpha^{l_i^*} \xi^{l_i} = \hat{x}^{l_i}$ the separation of the virtual power injection vector **g** in this<br>
anism  $\tilde{M}$  maintains the optimality of topology solutions<br>
understanding, we discuss this criteria from the physical<br>
ows the realistically optimal loa x and power injection vector g in this<br>the optimality of topology solutions<br>uss this criteria from the physical<br>mal load flow direction  $m\rightarrow n$ .<br>hen  $x^{l_i} + \alpha^{l_i} \xi^{l_i} = \hat{x}^{l_i}$ . This means the<br>e minimal  $F_0^*$  over  $\mathcal$ **i**<br>
With a proper  $\alpha^E$ , we elaborate how the virtual power injection vector g in this<br>
proposed DP-DNR mechanism  $\hat{M}$  maintains the optimality of topology solutions<br>  $\alpha^V$ . With an intuitive understanding, we discu With a proper  $\alpha^{\mathcal{E}}$ , we elaborate how the virtual power injection vector  $g$  in this<br>proposed DP-DNR mechanism  $\hat{M}$  maintains the optimality of topology solutions<br> $u^*$ . With an intuitive understanding, we discus proposed DP-DNR mechanism  $\tilde{M}$  maintains the optimality of topology solutions  $u^*$ . With an intuitive understanding, we discuss this criteria from the physical perspective. Fig.5.3(a) shows the realistically optimal If  $g_n$  injects  $a^k \xi^k > 0$  and  $g_n$  cjects  $-a^k \xi^k = 0$ , then  $x^k + a^k \xi^k = x^b$ . This means the<br>minimization of  $F_1$ , denoted as  $F_1^*$ , is equal to the minimal  $F_0^*$  over  $\mathcal X$ . Conversely,<br>if  $g_n = a^k \xi^k < 0$  and

$$
\min_{x',a',g \in \mathbb{R}, u' \in \mathbb{Z}} F_1 \tag{5.18}
$$

$$
\text{s.t.} \mathcal{X}_1 := \left\{ (\mathbf{x}^l, \mathbf{\alpha}^l, \mathbf{g}, \mathbf{u}^l) \; \begin{vmatrix} \tilde{\mathbf{\Lambda}} \mathbf{x}^l = \tilde{\mathbf{d}}, \begin{bmatrix} \mathbf{G}_v \\ \mathbf{G}_u \end{bmatrix} \begin{bmatrix} \mathbf{x}^l \\ \mathbf{u}^l \end{bmatrix} \leq \begin{bmatrix} \mathbf{b}_v \\ \mathbf{b}_u \end{bmatrix}, \mathbf{K} \begin{bmatrix} \mathbf{x}^l \\ \mathbf{u}^l \end{bmatrix} = \mathbf{h} \right\}
$$
(5.19)

s.t. $\mathcal{X}_1 := \left\{ (\mathbf{x}', \mathbf{\alpha}', \mathbf{g}, \mathbf{u}') \middle| \begin{aligned} \tilde{\mathbf{\mathcal{A}}} \mathbf{x}' &= \tilde{\mathbf{d}} \cdot \begin{bmatrix} \mathbf{G}_v \\ \mathbf{G}_u \end{bmatrix} \begin{bmatrix} \mathbf{x}' \\ \mathbf{u}' \end{bmatrix} \leq \begin{bmatrix} \mathbf{b}_v \\ \mathbf{b}_u \end{bmatrix}, \mathbf{K} \begin{bmatrix} \mathbf{x}' \\ \mathbf{u}' \end{bmatrix} = \mathbf{h} \right\}$  (5.19)<br>where the s.t. $\mathcal{X}_i = \left\{ (\mathbf{x}', \mathbf{\alpha}', \mathbf{g}, \mathbf{u}') \middle| \begin{aligned} \tilde{A}\mathbf{x}' &= \tilde{d}, \begin{bmatrix} \mathbf{G}_v \\ \mathbf{G}_s \end{bmatrix} \begin{bmatrix} \mathbf{x}' \\ \mathbf{b}' \end{bmatrix} \leq \begin{bmatrix} \mathbf{b}_v \\ \mathbf{b}_s \end{bmatrix}, \mathbf{K} \begin{bmatrix} \mathbf{x}' \\ \mathbf{u}' \end{bmatrix} = \mathbf{h} \right\}$  (5.19)<br>where the superscript sym model. s.t. $\mathcal{X}_i := \left\{ (x', \alpha', g, u') \middle| \begin{aligned} \tilde{A}x' &= \tilde{d}_s \begin{bmatrix} G_v \\ G_u \end{bmatrix} \begin{bmatrix} x' \\ u' \end{bmatrix} \leq \begin{bmatrix} b_v \\ b_u \end{bmatrix}, K \begin{bmatrix} x' \\ u' \end{bmatrix} = h \right\}$  (5.19)<br>where the superscript symbol  $\sim$  of *A* and *A* represent the matrices and vectors  $\begin{aligned}\n(x^l, \alpha^l, g, u^l) & \left| \tilde{A}x^l = \tilde{d} \left[ \begin{matrix} G_v \\ G_u \end{matrix} \right] \begin{bmatrix} x^l \\ u^l \end{matrix} \right] \le \begin{bmatrix} b_v \\ b_u \end{bmatrix}, K \begin{bmatrix} x^l \\ u^l \end{bmatrix} = h$ (5.19)
(5.11)–(5.13),  $G_v, x^l \le B_v, x^l \le x^l \le \overline{x}^l$ (5.19)
<br>
secript symbol ~ of *A* and *d*

Obfuscating  $x^{l_i}$  with reversed directions can avoid the exposure event of private s.t. $\mathcal{X}_i = \begin{cases} (x', a', g, u') & \mathcal{A}x' = \vec{d} \cdot \begin{bmatrix} \mathbf{G}_v \\ \mathbf{G}_s \end{bmatrix} u' \le \begin{bmatrix} \mathbf{B}_v \\ \mathbf{B}_s \end{bmatrix}, K \begin{bmatrix} \mathbf{K} \\ \mathbf{K}' \end{bmatrix} = \vec{h} \end{cases}$  (5.19)<br>
where the superscript symbol  $\sim$  of  $A$  and  $d$  represent the matrices (5.11) – (5.13),  $G_{\alpha}x' \leq b_{\alpha}$ ,  $x' \leq x' \leq x' \leq \overline{x'}$ <br>where the superscript symbol ~ of *A* and *A* represent the matrices and vectors<br>excluding boundary nodes, and  $X_1$  refers to the non-empty feasibility space of where the superscript symbol  $\sim$  of *A* and *d* represent the matrices and vectors<br>excluding boundary nodes, and  $X_i$  refers to the non-empty feasibility space of this<br>model.<br>(2) Load flow direction obfuscation<br>Obfuscati  $u_i^*$  equals to  $\hat{x}^{l_i} - \alpha^{l_i^*} \xi^{l_i} < 0$  if resent the matrices and vectors<br>
in-empty feasibility space of this<br>
in Fig.5.3(a). Thus, the load flow<br>
selection of  $\alpha^l$ . For instance, as<br>  $x^{l_i^*}$  equals to  $\hat{x}^{l_i} - \alpha^{l_i^*} \xi^{l_i} < 0$  if<br>
d load flow direction is  $\alpha^{l_i^*} \xi^{l_i} > d_n$  and  $\hat{x}^{l_i} = d_n$ . Consequently, the obf lary nodes, and  $X_1$  refers to the non-empty feasibility space of this<br>irection obfuscation<br> $x^i$  with reversed directions can avoid the exposure event of private<br>by the obtacted load is reversed in Fig.5.3(a). Thus, the to n and the express the criteria of tie-line load flow direction obfuscating  $x^k$  with reversed directions can avoid the exposure event of private load change information as previously exemplified in Fig.5.3(a). Thus, t yielding Obfuscating  $x^{\xi}$  with reversed directions can avoid the exposure event of private<br>ad change information as previously exemplified in Fig.5.3(a). Thus, the load flow<br>rection obfuscation can be realized with the proper s to the image information as previously exemplified in Fig.5.3(a). Thus, the load flow<br>direction obfuscation can be realized with the proper selection of  $\alpha'$ . For instance, as<br>displayed in Fig.5.3(b), the obfuscated load istantial differential as previously extemptives in rigional (i.e., rians, the base now<br>direction obfuscation can be realized with the proper selection of  $\alpha^i$ . For instance, as<br>displayed in Fig.5.3(b), the obfuscated l

displayed in Fig.5.3(b), the obfuscated load flow  $x^{\xi}$  equals to  $\dot{x}^{\xi} - a^{\xi} \xi^{\xi} < 0$  if  $a^{\xi} \xi^{\xi} > d_n$  and  $\dot{x}^{\xi} = d_n$ . Consequently, the obfuscated load flow direction is reversed to  $n \to m$ . Now, we express  $\alpha^{i} \xi^{i} > d_{\alpha}$  and  $\hat{x}^{i} = d_{\alpha}$ . Consequently, the obfuscated load flow direction is<br>to  $n \rightarrow m$ . Now, we express the criteria of tie-line load flow direction ob<br>yielding<br>**Criteria 2**(*Direction Obfuscation of Tie-li*  $l_t^*$ x s<sub>hould</sub> be realistical to the obfuscated load flow direction is reversed<br>w, we express the criteria of tie-line load flow direction obfuscation,<br>We express the criteria of tie-line load flow direction obfuscation,<br>Dire  $\mu$ ,  $\mu$ , and  $\mu$  =  $\mu$ , consequently, we obtained to the dow direction is revessed to  $n \rightarrow m$ . Now, we express the criteria of tie-line load flow direction obfuscation, yielding<br> **Criteria 2**(*Direction Obfuscation of* For the load flow direction of the lines of the lines of the lines of the lines connected to the *i*-th agent where  $\forall i \in A$  and  $A$  is the set of agents. The *i*-th agent connects at least two switch-on tie-lines with th

$$
(f_i-1)M \leq \tau_i \sum_{l_i \in \mathcal{I}_i} \boldsymbol{u}^{l_i} - \mathcal{S} \leq f_i M \tag{5.20}
$$

$$
(f_i-1)M \leqslant \tau_i \sum_{l_i \in \mathcal{I}_i} \mathbf{x}^{l_i} \leqslant (1-f_i)M
$$
\n
$$
(5.21)
$$

 $(f_i-1)M \le \tau_i \sum_{l_i \in T_i} \mathbf{u}^{l_i} - \theta \le f_i M$  (5.20)<br>  $(f_i-1)M \le \tau_i \sum_{l_i \in T_i} x^{l_i} \le (1-f_i)M$  (5.21)<br>
where *M* is a big positive scalar. *9* is the given constant and we select  $\theta$ =1.5 in this<br>
chapter. And  $\tau_i$  is a given pa  $(f_i-1)M \le \tau_i \sum_{i_i \in \mathcal{I}_i} \mathbf{u}^{i_i} - 9 \le f_i M$  (5.20)<br>  $(f_i-1)M \le \tau_i \sum_{i_i \in \mathcal{I}_i} x^i \le (1-f_i)M$  (5.21)<br>
where *M* is a big positive scalar. *9* is the given constant and we select *9*=1.5 in this<br>
chapter. And  $\tau_i$  is a g  $(f_i-1)M \leq \tau_i \sum_{\xi_i \in \mathcal{I}_i} u^i - \theta \leq f_i M$  (5.20)<br>  $(f_i-1)M \leq \tau_i \sum_{\xi_i \in \mathcal{I}_i} x^i \leq (1-f_i)M$  (5.21)<br>
where *M* is a big positive scalar.  $\theta$  is the given constant and we select  $\theta = 1.5$  in this<br>
chapter. And  $\tau_i$  is a  $(f_i-1)M \leq \tau_i \sum_{\xi_i \in \mathcal{I}_i} x^{\xi_i} \leq (1-f_i)M$  (5.20)<br>  $(f_i-1)M \leq \tau_i \sum_{\xi_i \in \mathcal{I}_i} x^{\xi_i} \leq (1-f_i)M$  (5.21)<br>
here *M* is a big positive scalar. *9* is the given constant and we select *9*–1.5 in this<br>
apter. And  $\tau_i$  is a

 $(f_i-1)M \leq \tau_i \sum_{\xi \in \mathcal{I}_i} u^{\xi} - \theta \leq f_i M$  (5.20)<br>  $(f_i-1)M \leq \tau_i \sum_{\xi \in \mathcal{I}_i} x^{\xi} \leq (1-f_i)M$  (5.21)<br>
where *M* is a big positive scalar. *9* is the given constant and we select  $\theta = 1.5$  in this<br>
chapter. And  $\tau_i$  is  $(f_i-1)M \le \tau_i \sum_{k \in \mathbb{Z}_i} \mu^k - \Re \le f_i M$  (5.20)<br>  $(f_i-1)M \le \tau_i \sum_{k \in \mathbb{Z}_i} \chi^k \le (1-f_i)M$  (5.21)<br>
where *M* is a big positive scalar. *9* is the given constant and we select  $\theta = 1.5$  in this<br>
chapter. And  $\tau_i$  is a given  $(f_i-1)M \le \tau_i \sum_{k \in \mathcal{I}_i} x^k \le (1-f_i)M$  (5.21)<br>where *M* is a big positive scalar. *9* is the given constant and we select *9*–1.5 in this<br>chapter. And  $\tau_i$  is a given parameter such that  $\tau_i=1$  if the *i*-th agent with where *M* is a big positive scalar. *9* is the given constant and we select  $\theta$ –1.5 in this<br>chapter. And  $\tau_i$  is a given parameter such that  $\tau_i$ –1 if the *i*-th agent with heavy loads<br>have a request of tie-line load f t  $t \in T_i$  $\sum_{l_t \in \mathcal{T}_i} \mathbf{x}^{l_t} =$ be select  $9=1.5$  in this<br>
ent with heavy loads<br>
wise  $\tau_i=0$ .<br>
explore the *i*-th agent<br>
iable  $f_i=1$ . Otherwise,<br>
e. Then, for this *i*-th<br>
ming into all optimal<br>  $\tau_i$   $\mathbf{x}^{l_i} = 0$  for this *i*-th<br>  $\sum_{l_i \in T_i} \mathbf{x}^{l$ where *m* is a  $\omega_{\rm g}$  positive setalation in the given constant and we seted of the in-<br>chapter. And  $\tau_i$  is a given parameter such that  $\tau_i$ =1 if the *i*-th agent with heavy lo<br>have a request of tie-line load flows  $u_i^* \xi^{l_i} > d_n$  as indicated in Fig.5.3 (b). Hence,  $\sum_{l_i \in \mathcal{I}_i} x^{l_i}$ a given parameter such that  $\tau_i$ =1 if the *i*-th agent with heavy loads<br>e-line load flows direction obfuscations; otherwise  $\tau_i$ =0.<br>h be elaborated here. Constraint (5.20) can explore the *i*-th agent<br>o switch-on tie-li  $t \in T_i$  $\sum_{l_i \in \mathcal{T}_i} \mathbf{x}^{l_i} =$ vith heavy loads<br>  $\tau_i=0$ .<br>
e the *i*-th agent<br>  $\tau_i=1$ . Otherwise,<br>
en, for this *i*-th<br>
into all optimal<br>
= 0 for this *i*-th<br>  $\tau_i$   $\mathbf{x}^{i_i} = 0$  can be<br>
e automatically<br>
m Fig.5.4(a) and Exampled with  $f_i=1$ . Instead,  $f_i=0$  induces that  $\sum_{i\in\mathcal{I}_i} x^{i}=0$  can be another in the set of the line between the elaborated here. Constraint (5.20) can explore the *i*-th agent who has at least two switch-on tie t  $t \in I_i$  $\sum_{l_i \in \mathcal{T}_i} \mathbf{x}^{l_i} =$ ions; otherwise  $\tau_i=0$ .<br>
5.20) can explore the *i*-th agent<br>
y binary variable  $f_i=1$ . Otherwise,<br>
n-on tie-line. Then, for this *i*-th<br>
s, when running into all optimal<br>
rcing  $\sum_{l_i \in \mathcal{T}_i} x^{l_i} = 0$  for this *i*-th<br>
b This criteria can be elaborated here. Constraint (5.20) can explore the *i*-th agent<br>who has at least two switch-on tie-lines, if an auxiliary binary variable *f*-1. Otherwise,<br>*f*-0 refers to other agents with less than This criteria can be elaborated here. Constraint (5.20) can explore the *i*-th agent<br>who has at least two switch-on tic-lines, if an auxiliary binary variable  $f_f$ =1. Otherwise,<br> $f_f$ =0 refers to other agents with less tha  $t=1$ i  $\sum_{t \in \mathcal{T}_i} u^{l_i} =$ has at least two switch-on tie-lines, if an auxiliary binary variable *f*<sub>i</sub>=1. Otherwise,<br>refers to other agents with less than one switch-on tie-line. Then, for this *i*-th<br>t, we consider reversing one of tie-line load  $f_i=0$  refers to other agents with less than one switch-on tie-line. Then, for this *i*-th agent, we consider reversing one of tie-line load flows, when running into all optimal tie-lines' loads flow towards the same dire agent, we consider reversing one of tie-line load flows, when running into all optimal<br>tie-lines' loads flow towards the same direction. Enforcing  $\sum_{i_i \in \mathcal{I}_i} x^{i_i} = 0$  for this *i*-th<br>agent can lead to  $a^{i_i} \xi^{i_j} > d$  $t = 2$  $t \in T_i$  $\sum_{l_i \in \mathcal{I}_i} u^{l_i} = 2$  and  $f_i = 1$ , and then  $\sum_{l_i \in \mathcal{I}_i} x^{l_i}$ nes' loads flow towards the same direction. Enforcing  $\sum_{\ell \in \mathcal{I}_{\ell}} x^{\ell} = 0$  for this *i*-th<br>t can lead to  $a^{\ell} \xi^{\ell} > d_{n}$  as indicated in Fig.5.3 (b). Hence,  $\sum_{\ell \in \mathcal{I}_{\ell}} x^{\ell} = 0$  can be<br>led with  $f_i$ =1. Inst  $t \in T_i$  $\sum_{l_t \in \mathcal{T}_i} \mathbf{x}^{l_t} =$ etion. Enforcing  $\sum_{l_i \in T_i} x^{l_i} = 0$  for this *i*-th<br>
1 Fig.5.3 (b). Hence,  $\sum_{l_i \in T_i} x^{l_i} = 0$  can be<br>
that  $\sum_{l_i \in T_i} x^{l_i} = 0$  can be automatically<br>
We exemplify two cases in Fig.5.4(a) and<br>
witch-on tie-line for the agent can lead to  $\alpha^r \xi^i > \delta_\epsilon$  as indicated in Fig.5.3 (b). Hence,  $\sum_{i_i \in \mathcal{I}} x^i = 0$  can be coupled with  $f_i$ –1. Instead,  $f_i$ –0 induces that  $\sum_{i_i \in \mathcal{I}} x^i = 0$  can be automatically relaxed, which forms the cons



direction obfuscation subject to the optimality of topology variables at language objection obfuscation subject to the optimality of topology variables at language objection obfuscation subject to the optimality of topolo **Example 12** For a gent to the objective function (b) two active tie-lines.<br>
In light of the above, this proposed DP-DNR mechanism  $\tilde{M}$  can involve two<br>
privacy-preserving criteria, i.e., load flow quantity obfuscatio

$$
P(X|X|Y|X|Z) = P(X|X|X) + P(X|X|Z) + P
$$

s.t. $\mathcal{X}_u := \left\{ (x', \alpha', f, g, u') \mid \begin{aligned} & \lambda x' = \overline{d} \cdot \begin{bmatrix} G_v \\ G_u \end{bmatrix} \begin{bmatrix} x' \\ u' \end{bmatrix} \leq \begin{bmatrix} b_v \\ b_u \end{bmatrix}, K \begin{bmatrix} x' \\ u' \end{bmatrix} = h \\ & (5.11) - (5.13), (5.20) - (5.21), G_{av} x' \leq b_{av}, x' \leq x' \leq \overline{x'} \end{aligned} \right\}$ <br>(5.23)<br>where  $\mathcal{X}_n$  refers  $\mathbf{u}^l \cdot \mathbf{u}^l \cdot \mathbf{f}^T$  $\begin{aligned}\n\mathbf{b}_v \mathbf{b}_v \mathbf{b}_v &\mathbf{K} \begin{bmatrix} \mathbf{x}' \\ \mathbf{u}' \end{bmatrix} = \mathbf{h} \\
\text{(5.21)}, \mathbf{G}_c, \mathbf{x}' \leq \mathbf{b}_c, \mathbf{x}' \leq \mathbf{x}' \leq \mathbf{\overline{x}}'\n\end{aligned}$ (5.23)<br>
f this DP-DNR model.<br>
and the i-th  $\mathbf{X}_i = [\mathbf{x}'_i, \mathbf{a}'_i, \mathbf{u}'_i, \mathbf{f}_i]^T$  s.t. $\mathcal{X}_{\pi} := \left\{ (x', \alpha', f, g, u') \mid \begin{aligned} Ax' &= d \cdot \begin{bmatrix} x \\ G_{\mu} \end{bmatrix} \begin{bmatrix} x \\ u' \end{bmatrix} \leq \begin{bmatrix} x \\ b_{\mu} \end{bmatrix} \cdot K \begin{bmatrix} x \\ u' \end{bmatrix} = h \right\}$ <br>
(5.11) – (5.13), (5.20) – (5.21),  $G_{\alpha}x' \leq b_{\alpha}, \underline{x}' \leq x' \leq \overline{x}'$ <br>
(5.23)<br>
where  $\mathcal{$  $\boldsymbol{a}_{i}^{l_{i}},\boldsymbol{a}_{i,b}^{l},\boldsymbol{u}_{i,b}^{l},\boldsymbol{f}_{i,b}^{l}\big]^{T}$  $Ax' = d$ ,  $\begin{bmatrix} c \\ c \\ d \end{bmatrix} \begin{bmatrix} u' \\ u' \end{bmatrix} \leq \begin{bmatrix} \sum_{b} b \\ b' \end{bmatrix}$ ,  $K \begin{bmatrix} u' \\ u' \end{bmatrix} = h$ <br>
(5.11) – (5.13), (5.20) – (5.21),  $G_{cr}x' \leq b_{cr}$ ,  $x' \leq x' \leq x'$ <br>
(5.23)<br>
apty feasibility space of this DP-DNR model.<br>
gorith (5.11)–(5.13),(5.20)–(5.21), $G_{\alpha} x' \leq b_{\alpha}, \underline{v} \leq x' \leq \overline{x}$ "<br>
(5.23)<br>
where  $X_{\mathbb{I}}$  refers to the non-empty feasibility space of this DP-DNR model.<br>
5.4 Consensus ADMM Algorithm<br>
In this section, we reformulate thi consensus variables for tie-line  $l_t$  can be composed of  $\mathbf{Z}_x = [\mathbf{x}_z^t, \mathbf{\alpha}_z^t, \mathbf{u}_z^t, \mathbf{f}_z]^T$  and<br>squared voltage profiles  $W_y$ . Accordingly,  $X_{i,b} - \mathbf{Z}_x = 0$  can hold for either  $\mathbf{u}^{l_t} = 0$ <br>or  $\mathbf{u}^{l$  $\boldsymbol{Z}_{\scriptscriptstyle \mathcal{X}} = \bigr[ \begin{matrix} \boldsymbol{x}^{\scriptscriptstyle I}_{\scriptscriptstyle \mathcal{Z}} , \boldsymbol{a}^{\scriptscriptstyle I}_{\scriptscriptstyle \mathcal{Z}} , \boldsymbol{u}^{\scriptscriptstyle I}_{\scriptscriptstyle \mathcal{Z}} , \boldsymbol{f}_{\scriptscriptstyle \mathcal{Z}} \end{matrix} \bigr]^T$  and consensus variables for tie-line  $l_t$  can be composed of  $\mathbf{Z}_x = [\mathbf{x}'_z, \mathbf{\alpha}'_z, \mathbf{\mu}'_z, \mathbf{f}_z]^T$  and<br>squared voltage profiles  $W_v$ . Accordingly,  $X_{i,b} - \mathbf{Z}_x = 0$  can hold for either  $\mathbf{\mu}'_t = 0$ <br>or  $\mathbf{\mu}'_t = 1$ .  $\boldsymbol{u}^{l_t} =$ or  $\mathbf{u}^{l_i} = 1$ . Please note sensus variables for tie-line  $l_i$  can be composed of  $\mathbf{Z}_x = [\mathbf{x}'_z, \mathbf{\alpha}'_z, \mathbf{\mu}'_z, \mathbf{f}_z]^T$  and<br>ared voltage profiles  $W_y$ . Accordingly,  $X_{i,b} - \mathbf{Z}_x = 0$  can hold for either  $\mathbf{\mu}^l_i = 0$ <br> $\mathbf{\mu}^l = 1$ . Please thus it would be not regarded as consensus variables. And  $\mathbf{Z}_x = [\mathbf{x}'_z, \mathbf{\alpha}'_z, \mathbf{\alpha}'_z, \mathbf{\alpha}'_z, \mathbf{\beta}'_z]^T$  and squared voltage profiles  $W_y$ . Accordingly,  $X_{i,b} - Z_x = 0$  can hold for either  $\mathbf{\alpha}' = 0$  or  $\mathbf{\alpha}' =$ if  $\mathbf{u}^{l_i} = 1$ ; otherwise it should be relaxed when  $\mathbf{u}^{l_i}$ SECTING INTERT IN EXECT:<br>
IN INSTERT IN EXECT INTERT INTERT INTERT INTERT INTERT INTERTATION INTERTATION INTERTATION INTERTATION INTERT<br>  $u^L = 1$ . Please note that g can be achieved by sampled  $\xi^t$  and variables  $\alpha^t$ , ISSONSING INTERT IN EXECT UP:  $l_i$  can be composed of  $\mathbf{Z}_x = [\mathbf{x}'_z, \mathbf{a}'_z, \mathbf{u}'_z, \mathbf{f}'_z]^T$  and<br>uared voltage profiles  $W_v$ . Accordingly,  $X_{i,b} - Z_x = 0$  can hold for either  $\mathbf{u}^k = 0$ <br> $\mathbf{u}^k = 1$ . Please note consensus variables for tie-line *l<sub>i</sub>* can be composed of  $\mathbf{Z}_s = [\mathbf{x}'_s, \mathbf{a}'_s, \mathbf{u}'_s, \mathbf{f}_s]^T$  and<br>squared voltage profiles  $W_x$ . Accordingly,  $X_{i,b} - \mathbf{Z}_s = 0$  can hold for either  $\mathbf{u}^i = 0$ <br>or  $\mathbf{u}^i = 1$ consensus variables for tie-line  $l_t$  can be composed of  $\mathbf{Z}_x = [\mathbf{x}_s^t, \mathbf{a}_s^t, \mathbf{u}_s^t, \mathbf{f}_z]^T$  and<br>squared voltage profiles  $\mathbf{W}_v$ . Accordingly,  $\mathbf{X}_{i,b} - \mathbf{Z}_x = 0$  can hold for either  $\mathbf{u}^t = 0$ <br>or  $\$ or tie-line  $l_i$  can be composed of  $\mathbf{Z}_s = [\mathbf{x}_z, \mathbf{\alpha}_z, \mathbf{u}_z, \mathbf{y}_z]$  and<br>
les  $\mathbf{W}_v$ . Accordingly,  $\mathbf{X}_{i,b} - \mathbf{Z}_x = 0$  can hold for either  $\mathbf{u}^{l_i} = 0$ <br>
c that  $\mathbf{g}$  can be achieved by sampled  $\xi^i$   $u^k = 1$ . Please note that g can be achieved by sampled  $\xi'$  and variables  $\alpha'$ , and<br>
as it would be not regarded as consensus variables. And  $v_{i,b} - W_v = 0$  can hold only<br>  $u^k = 1$ ; otherwise it should be relaxed when  $u$ 

$$
\mathcal{L}_{\lambda} = \sum_{i=1}^{n_{\mathcal{A}}} \Big( F_{i} \big( \bm{X}_{i} \big) + \lambda / 2 \big\| \bm{X}_{i,b} - \bm{Z}_{x} + \bm{\mu}_{i,b} \big\|_{2}^{2} + \lambda / 2 \big\| \bm{V}_{i,b} \big\|_{2}^{2} \Big)
$$
(5.24)

thus it would be not regarded as consensus variables. And  $v_{i,b} - W_v = 0$  can hold only<br>
if  $u^i = 1$ ; otherwise it should be relaxed when  $u^i = 0$ .<br>
Since the DP-DNR model can be represented by the block variables  $X_i$  and<br> if  $u^i = 1$ ; otherwise it should be relaxed when  $u^i = 0$ .<br>
Since the DP-DNR model can be represented by the block variables  $X_i$  and<br>
associated separable objective function  $F_{1,i}(X_i)$  for the *i*-th agent, the augmented  $\psi_{i,b}$  –  $Wv + \gamma_{i,b}$ ) and id be relaxed when  $u^i = 0$ .<br>
del can be represented by the block variables  $X_i$  and<br>
tive function  $F_{1,i}(X_i)$  for the *i*-th agent, the augmented<br>
n be expressed in the scaled form:<br>  $Y_i + \lambda/2 ||X_{i,b} - Z_x + \mu_{i,b}||_2^2 + \lambda/2 ||V_{$ Since the DP-DNR model can be represented by the block variables  $X_i$  and<br>associated separable objective function  $F_{1,i}(X_i)$  for the *i*-th agent, the augmented<br>lagrangian function  $\mathcal{L}_z$  can be expressed in the scaled grangian function  $\mathcal{L}_\lambda$  can be expressed in the scaled form:<br>  $\mathcal{L}_\lambda = \sum_{i=1}^{n} \left[ F_i(X_i) + \lambda / 2 \| X_{i,b} - \mathbf{Z}_x + \boldsymbol{\mu}_{i,b} \|_2^2 + \lambda / 2 \| V_{i,b} \|_2^2 \right]$  (5.24)<br>
where  $n_A$  is the number of agents, and  $\boldsymbol{\mu}_{i,b}$  is the *i* be expressed in the scaled form:<br>  $\langle j \rangle + \lambda / 2 \| X_{i,b} - Z_x + \mu_{i,b} \|_2^2 + \lambda / 2 \| V_{i,b} \|_2^2$  (5.24)<br>
of agents, and  $\mu_{i,b}$  is the *i*-th agent's dual variables for<br>
And  $\lambda$  is a given positive scalar. And  $V_{i,b}$  is an<br>  $\langle i,$  $\mathcal{L}_2 = \sum_{i=1}^{n_2} \left( F_i(X_i) + \lambda/2 \|X_{i,b} - Z_x + \mu_{i,b}\|_2^2 + \lambda/2 \|V_{i,b}\|_2^2 \right)$  (5.24)<br>
where  $n_A$  is the number of agents, and  $\mu_{i,b}$  is the *i*-th agent's dual variables for<br>
the equality  $X_{i,b} - Z_x = 0$ . And  $\lambda$  is a give where  $n_A$  is the number of agents, and  $\boldsymbol{\mu}_{i,b}$  is the *i*-th agent's dual variables for<br>the equality  $X_{i,b} - Z_x = 0$ . And  $\lambda$  is a given positive scalar. And  $V_{i,b}$  is an<br>incorporated variable for  $V_{i,b} = u^k (v_{i,b} - Wv$ mber of agents, and  $H_{i,b}$  is the *i*-th agent's dual variables for<br>  $= 0$ . And  $\lambda$  is a given positive scalar. And  $V_{i,b}$  is an<br>
for  $V_{i,b} = u^i (v_{i,b} - Wv + \gamma_{i,b})$  and  $\gamma_{i,b}$  is the associated dual<br>
can be further equi e equality  $X_{i,b} - Z_x = 0$ . And  $\lambda$  is a given positive scalar. And  $V_{i,b}$  is an corporated variable for  $V_{i,b} = u^k (v_{i,b} - W_V + \gamma_{i,b})$  and  $\gamma_{i,b}$  is the associated dual<br>riable. This equality can be further equivalent to<br>

$$
\boldsymbol{v}_{i,b} - \boldsymbol{W}_{v} + \boldsymbol{\gamma}_{i,b} - (1 - \boldsymbol{u}^{l_{t}}) M \leqslant \boldsymbol{V}_{i,b} \leqslant \boldsymbol{v}_{i,b} - \boldsymbol{W}_{v} + \boldsymbol{\gamma}_{i,b} + (1 - \boldsymbol{u}^{l_{t}}) M \qquad (5.25)
$$

Subsequently, we define  $V_i = \{V_{i,b}, W_v, v_{i,b} \in \mathbb{R}, u^{l_i} \in \mathbb{Z} \mid (5.26)\}$  for the *i*-th agent. As the for  $V_{i,b} = u^h (v_{i,b} - Wv + \gamma_{i,b})$  and  $\gamma_{i,b}$  is the associated dual<br>
can be further equivalent to<br>  $v_{i,b} - (1 - u^k) M \le V_{i,b} \le v_{i,b} - W_v + \gamma_{i,b} + (1 - u^k) M$  (5.25)<br>
fine  $V_i = \{V_{i,b}, W_v, v_{i,b} \in \mathbb{R}, u^h \in \mathbb{Z} \mid (5.26)\}$  for the *i*  $\mathbf{u}_{i,b} = \mathbf{u}^{l_i} (\mathbf{v}_{i,b} - \mathbf{W}v + \mathbf{y}_{i,b})$  and  $\mathbf{y}_{i,b}$  is the associated dual<br>further equivalent to<br> $-\mathbf{u}^{l_i} \Big) M \le V_{i,b} \le \mathbf{v}_{i,b} - \mathbf{W}_v + \mathbf{y}_{i,b} + (1 - \mathbf{u}^{l_i}) M$  (5.25)<br> $V_i = \{V_{i,b}, W_v, v_{i,b} \in \mathbb{R}, \mathbf{u}^{l$ 

$$
\left(\boldsymbol{X}_{i,b}^{k+1},\boldsymbol{V}_{i,b}^{k+1}\right)=\underset{\boldsymbol{X}_{i}\in\mathcal{X}_{\mathrm{II},i},\boldsymbol{V}_{i,b}\in\mathcal{V}_{i}(\boldsymbol{W}_{v}^{k})}{\operatorname{argmin}}F_{i}\left(\boldsymbol{X}_{i}\right)+\lambda/2\left\|\boldsymbol{X}_{i,b}-\boldsymbol{Z}_{x}^{k}+\boldsymbol{\mu}_{i}^{k}\right\|_{2}^{2}+\lambda/2\left\|\boldsymbol{V}_{i,b}\right\|_{2}^{2},\quad\forall i=1,2...,n_{A}
$$

(5.26)

$$
\left(\mathbf{Z}_{x}^{k+1}, \mathbf{W}_{v}^{k+1}\right) = \Pi_{c} \left( \begin{bmatrix} \mathbf{X}_{i,b}^{k+1} + \mathbf{\mu}_{i}^{k} \\ \mathbf{v}_{i,b}^{k+1} + \mathbf{y}_{i}^{k} \end{bmatrix} \right)
$$
\n
$$
\mathbf{X}_{i,b}^{k+1} - \mathbf{Z}_{x}^{k+1}, \mathbf{y}_{i,b}^{k+1} = \mathbf{y}_{i,b}^{k} + \mathbf{Z}_{x,u}^{k+1} \left( \mathbf{V}_{i,b}^{k+1} - \mathbf{W}_{v}^{k+1} \right)
$$
\n(5.28)

$$
\boldsymbol{\mu}_{i,b}^{k+1} = \boldsymbol{\mu}_{i,b}^k + \boldsymbol{X}_{i,b}^{k+1} - \boldsymbol{Z}_x^{k+1}, \boldsymbol{\gamma}_{i,b}^{k+1} = \boldsymbol{\gamma}_{i,b}^k + \boldsymbol{Z}_{x,u}^{k+1} (\boldsymbol{V}_{i,b}^{k+1} - \boldsymbol{W}_v^{k+1})
$$
(5.28)

 $\left(Z_s^{k+1}, W_s^{k+1}\right) = \Pi_c\left[\begin{bmatrix} X_{j,b}^{k+1} + \mu_i^k \ \nu_{j,b}^{k+1} + \gamma_i^k \end{bmatrix}\right]$  (5.27)<br>  $\mu_{i,b}^{k+1} = \mu_{i,b}^k + X_{i,b}^{k+1} - Z_s^{k+1}, \gamma_{i,b}^{k+1} = \gamma_{i,b}^k + Z_{s,u}^{k+1} \left(V_{i,b}^{k+1} - W_s^{k+1}\right)$  (5.28)<br>
where *C* is the Cartesian product **projection**<br> **projection**  $(Z_s^{k+1}, W_s^{k+1}) = \Pi_c \left[ \begin{bmatrix} X_{j,s}^{k+1} + \mu_i^k \\ v_{j,s}^{k+1} + \gamma_i^k \end{bmatrix} \right]$  (5.27)<br>  $\mu_{i,b}^{k+1} = \mu_{i,b}^k + X_{i,b}^{k+1} - Z_s^{k+1}, \gamma_{i,b}^{k+1} = \gamma_{i,b}^k + Z_{s,a}^{k+1} \left( V_{i,b}^{k+1} - W_s^{k+1} \right)$  (5.28)<br>
where *C* ,  $x_{x,u}^{k+1}$  in the vector  $\mathbf{Z}_x^{k+1}$ ,  $\Pi_c$  $\begin{pmatrix} \mathbf{I}_{i}^{k} \\ \mathbf{I}_{i}^{k} \end{pmatrix}$  (5.27)<br>  $(\mathbf{V}_{i,b}^{k+1} - \mathbf{W}_{v}^{k+1})$  (5.28)<br>  $\ldots$ , *n* and  $\Pi_{c}$  stands for the<br>  $\mathbf{Z}_{x,u}^{k+1}$  in the vector  $\mathbf{Z}_{x}^{k+1}$ ,  $\Pi_{c}$ <br>
by [60], and  $C_{i} \in \mathbb{R}$  for the<br>
ati  $\left(\mathbf{Z}_{s}^{k+1}, \mathbf{W}_{s}^{k+1}\right) = \Pi_{\mathcal{C}}\left[\begin{bmatrix} \mathbf{X}_{j,s}^{k+1} + \mathbf{\mu}_{s}^{k} \\ \mathbf{v}_{j,s}^{k+1} + \mathbf{\gamma}_{s}^{k} \end{bmatrix}\right]$  (5.27)<br>  $\mathbf{\mu}_{l,b}^{k+1} = \mathbf{\mu}_{l,b}^{k} + \mathbf{X}_{l,b}^{k+1} - \mathbf{Z}_{s}^{k+1}, \mathbf{\gamma}_{l,b}^{k+1} = \mathbf{\gamma}_{l,b}^{k} + \mathbf{Z}_{s,a}^{k+1} \left$  $(Z_{\zeta}^{k+1}, W_{\zeta}^{k+1}) = \Pi_{\zeta} \left( \begin{bmatrix} X_{\zeta s}^{k+1} + \mu_{\zeta}^k \\ v_{\zeta s}^{k+1} + \mu_{\zeta}^k \end{bmatrix} \right)$  (5.27)<br>  $\mu_{\zeta s}^{k+1} = \mu_{\zeta s}^k + X_{\zeta s}^{k+1} - Z_{\zeta s}^{k+1}, \gamma_{\zeta s}^{k+1} = \gamma_{\zeta s}^k + Z_{\zeta s}^{k+1} (V_{\zeta s}^{k+1} - W_{\zeta}^{k+1})$  (5  $\left( \mathbf{Z}_{s}^{k+1}, \mathbf{W}_{s}^{k+1} \right) = \Pi_{\mathcal{C}} \left[ \begin{bmatrix} \mathbf{X}_{i,k}^{k+1} + \mathbf{\mu}_{i}^{k} \\ \mathbf{v}_{i,k}^{k+1} + \mathbf{y}_{i}^{k} \end{bmatrix} \right]$  (5.27)<br>  $\mathbf{\mu}_{i,s}^{k+1} = \mathbf{\mu}_{i,s}^{k} + \mathbf{X}_{i,s}^{k+1} - \mathbf{Z}_{s}^{k+1}, \mathbf{y}_{i,s}^{k+1} = \mathbf{y}_{i,s}^{k+1} + \mathbf{Z}_{s,s}^{$ , , , from two  $(Z_s^{k+1}, W_s^{k+1}) = \Pi_{\mathcal{C}} \left[ \begin{pmatrix} X_{s,b}^{k+1} + \mu_s^k \ Y_{l,b}^{k+1} + \eta_s^k \end{pmatrix} \right]$  (5.27)<br>  $\mu_{i,b}^{k+1} = \mu_{i,b}^k + X_{i,b}^{k+1} - Z_s^{k+1}, \gamma_{i,b}^{k+1} = \gamma_{i,b}^k + Z_{s,a}^{k+1} \left( V_{i,b}^{k+1} - W_s^{k+1} \right)$  (5.28)<br>
where *C* is the Cartesian pro



 $x_{i,b}^{k+1} = 0$  can be achieved by (5.26) no matter if  $u^{l_i} = 0$  or  $\mathbf{V}_{i}^{l} = 1$ . This is because if  $\mathbf{u}^{l_i} = 0$ , then  $V_{i,b}^{k+1} = 0$ 2  $\left|V_{i,b}\right|_{2}^{2}$  equals  $u^{l_i} = 1$ , then  $V_{i,b} = v_{i,b} - W_v^k + \gamma_{i,b}^k$  stands v ( $\chi_{\alpha}^{(k+1)} \mu_{\alpha}^{(k+1)}$ )<br>
( $\chi_{\alpha}^{(k+1)} \mu_{\alpha}^{(k+1)}$ )<br>
( $\chi_{\alpha}^{(k+1)} \mu_{\alpha}^{(k+1)}$ )<br>
( $\chi_{\alpha}^{(k)} \mu_{\alpha}^{(k)}$ )<br>
( $\chi_{\alpha}^{(k)} \mu_{\alpha}^{(k)}$ )<br>
( $\chi_{\alpha}^{(k)} \mu_{\alpha}^{(k)}$ )<br>
( $\chi_{\alpha}^{(k)} \mu_{\alpha}^{(k)}$ )<br>
(Fig. 5.5. Relationship bet optimization variables<br>
(*X<sub>b</sub>*  $X_{j,b}V_{j,b}$ )<br> **Agent** *i*<br>
Fig. 5.5. Relationship between consensus variables and optimization variables.<br>
Note that the optimal  $V_{i,b}^{k+1} = 0$  can be achieved by (5.26) no matter if  $u^i$  $W_{\nu}^{k+1} = 0$ , then  $W_{\nu}^{k+1}$  can be free in (5.27)  $\gamma_{i,b}^{k+1}$ **e-switch**<br>  $\frac{u^{l_i}}{u^{l_i}}$ <br>  $\frac{dx^{l_i}}{(X_j, X_{l,b}, V_{l,b})}$ <br> **tie-line**  $l_i$ <br> **Agent** *j*<br> **C**<br> potimization variables<br>  $(X_j, X_{i,b}, V_{i,b})$ <br>
Agent *j*<br>
s and optimization variables.<br>
y (5.26) no matter if  $u^{l_i} = 0$  or<br>
ds as minimizing  $||V_{i,b}||_2^2$  equals<br>  $W_v^k + \gamma_{i,b}^k$  stands which enables<br>
an function  $\mathcal{L}_\lambda$  i ( $X_i$ ,  $X_i$ ,  $V_i$ ,  $Y_i$ )<br> **Can be kept in the kept unchanged by (5.26)** and optimization variables.<br>
Note that the optimal  $V_{i,b}^{k+1} = 0$  can be achieved by (5.26) no matter if  $u^i = 0$  or  $u^i = 1$ . This is because if  $u$ 

(5.28) are performed by each individual agent simultaneously and (5.26) is calculated<br>by the proximal operator by the DSO. The convergence of C-ADMM is characterized<br>in terms of the primal residual and the dual residual wi  $(5.28)$  are performed by each individual agent simultaneously and  $(5.26)$  is calculated<br>by the proximal operator by the DSO. The convergence of C-ADMM is characterized<br>in terms of the primal residual and the dual residu (5.28) are performed by each individual agent simultaneously and (5.26) is calculated<br>by the proximal operator by the DSO. The convergence of C-ADMM is characterized<br>in terms of the primal residual and the dual residual w

(5.28) are performed by each individual agent simultaneously and (5.26) is calculated<br>by the proximal operator by the DSO. The convergence of C-ADMM is characterized<br>in terms of the primal residual and the dual residual w (5.28) are performed by each individual agent simultaneously and (5.26) is calce<br>by the proximal operator by the DSO. The convergence of C-ADMM is charact<br>in terms of the primal residual and the dual residual with predefi  $l_t^*$ If the DSO. The convergence of C-ADMM is characterized<br>attact by the DSO. The convergence of C-ADMM is characterized<br>al residual and the dual residual with predefined thresholds [60].<br>exhibit the algorithmic pseudo-code f (5.28) are performed by each individual agent simultaneously and (5.26) is calculated<br>by the proximal operator by the DSO. The convergence of C-ADMM is characterized<br>in terms of the primal residual and the dual residual w by the proximal operator by the DSO. The convergence of C-ADMM is characterin terms of the primal residual and the dual residual with predefined thresholds [6]<br>Next, we will exhibit the algorithmic pseudo-code for this pr Algorithm 1 C-ADMM-based DP-DNR Mechanism  $\tilde{\mathcal{M}}$ <br>
1: Initialization with a random perpetuation vector  $\xi^i$ .<br>
As stated previously, the output of  $\tilde{\mathcal{M}}$  can be a mixture of obfuse<br>ated-but-feasible tic-line loa Next, we will exhibit the algorithmic pseudo-code for this<br>
LDMM-based DP-DNR mechanism  $\tilde{M}$  with a random perpetuation ve<br>
stated previously, the output of  $\tilde{M}$  can be a mixture of obfuscated-bu<br>
line load flows Next, we will exhibit the algorithmic pseudo-code for this proposed<br>
C-ADMM-based DP-DNR mechanism  $\tilde{\mathcal{M}}$  with a random perpetuation vector  $\zeta^i$ .<br>
As stated previously, the output of  $\tilde{\mathcal{M}}$  can be a mixture o C-ADMIN-based DP-DINK Incedianism  $\mathcal{M}$  with a random perpetuation vector  $\zeta$ .<br>
As stated previously, the output of  $\mathcal{M}$  can be a mixture of obfuscated-but-feasible<br>
tie-line load flows  $x^c$  and realistically op For a control of the entire of obfuscated-but-feasible<br>ty solution  $u^{l^*}$  of the entire<br>then we can summarize this<br> $\tilde{M}$ <br>wer  $n_A$  agents and<br> $P_{\xi}$ ;<br> $Z_x^k$ ,  $W_y^k$  by (5.26) and sends<br> $(Z_x^{k+1}, W_y^{k+1})$ <br>27) and sends ated previously, the output of  $\tilde{M}$  can be a mixture of obfuscated-but-feasible<br>
ne load flows  $x^{\ell}$  and realistically optimal topology solution  $u^{i*}$  of the entire<br>
is. The maximum iteration number is set to  $k_{max$ tie-line load flows  $x^{\xi}$  and realistically optimal topology solution  $u^{\prime\prime}$  of the<br>ADNs. The maximum iteration number is set to  $k_{max}$ , and then we can summari<br>algorithm as below.<br>Algorithm 1 C-ADMM-based DP-DNR Mec and realistically optimal topology solution  $u^{i*}$  of the entire<br>iteration number is set to  $k_{max}$ , and then we can summarize this<br>M-based DP-DNR Mechanism  $\tilde{\mathcal{M}}$ <br>mput  $c, A, G_{i}, G_{\varphi}, b_{\varphi}, b_{\varphi}, b_{\varphi}, K, h$  over  $n_A$ Ns. The maximum iteration number is set to  $k_{max}$ , and then we can summ<br>
orithm as below.<br>
<br>
<br> **Initialization with input c**,  $A, G_v, G_v, b_v, b_v, b_u, K, h$  over  $n_A$  agents and<br>
<br>
Initialization with input c,  $A, G_v, G_v, b_v, b_u, K, h$ 

- $\epsilon, \vartheta, \Delta_{\rho}, \tau, \overline{\bm{g}}, \overline{\bm{a}}^{l}\,;$
- 
- 3: while  $k \leq k_{\text{max}}$  do
- $1 \mathbf{L}^{k+1}$  $(X_{i,b}^{k+1}, V_{i,b}^{k+1}) \leftarrow (Z_x^k, W_y^k)$  by (5.26) and sends  $(X_{i,b}^{k+1}, V_{i,b}^{k+1})$  to the DSO;  $_{i,b}^{k+1},V_{i,b}^{k+1}$  $_{i,b}$ ,  $\boldsymbol{V}_{i,b}$
- $\left\{ \mathbf{u}^{k+1} \right\}$ ,  $\left( \mathbf{v}^{k+1} \mathbf{v}^{k+1} \right)$  $(x_{k}^{k+1}, W_{\nu}^{k+1}) \leftarrow (X_i^{k+1}, V_{i,b}^{k+1})$  by (5.27) and sends  $(Z_x^{k+1}, W_{\nu}^{k+1})$  $\boldsymbol{Z}_x^{k+1}, \boldsymbol{W}_v^{k+1}$

Altrid. The internal netation number is set to state, that there is a summarize this<br>algorithm as below.<br>
Algorithm 1 C-ADMM-based DP-DNR Mechanism  $\tilde{\mathcal{M}}$ <br>
1: Initialization with input  $\mathbf{c}, \mathbf{A}, \mathbf{G}_{\mathbf{y}}, \mathbf{G}_{$  $1 \bullet^{k+1}$  $(\mu_{i,b}^{k+1}, \gamma_{i,b}^{k+1}) \leftarrow (\mu_{i,b}^{k}, \gamma_{i,b}^{k})$  by (5.29) and in the call sammarize and<br>
ightharpoonup is the same of  $W_v^k$  by (5.26) and sends<br>
ightharpoonup is the sends  $(Z_x^{k+1}, W_y^{k+1})$ <br>  $\mu_{i,b}^k, \gamma_{i,b}^k$  by (5.29) and  $\mathcal{V}_i\big(\pmb{W}_v^k\big) \!\leftarrow\! \mathcal{V}_i\big(\pmb{W}_v^{k+\!1}\big)\!$ Algorithm 1 C-ADMM-based DP-DNR Mechanism  $\mathcal{M}$ <br>
1: Initialization with input  $c, A, G_{\gamma}, b_{\gamma}, b_{\alpha}, b_{\beta}, K, h$  over  $n_A$  agents and<br>
input parameters  $\varepsilon, \vartheta, \Delta_{\rho}, \tau, \overline{g}, \overline{a}$ ;<br>
2: Sample a random perturbation vecto 1: Initialization with input  $c, A, G_{\varphi}, G_{\varphi}, b_i, b_{\varphi}, b_k, K, h$  over  $n_A$  agents and<br>
input parameters  $\varepsilon, \vartheta, \Delta_{\varphi}, \tau, \overline{g}, \overline{\alpha}'$ ;<br>
2: Sample a random perturbation vector  $\xi^i$ , i.i.d.  $\xi^i \sim \mathbb{P}_{\xi}$ ;<br>
3: **while**  $\mathcal{F}_{v}$ ,  $\mathcal{G}_{\alpha}$ ,  $b_{v}$ ,  $b_{\alpha}$ ,  $b_{u}$ ,  $K$ ,  $h$  over  $n_{A}$  agents and<br>  $\overline{\alpha}^{l}$ ;<br>
vector  $\xi^{l}$ , i.i.d.  $\xi^{l} \sim \mathbb{P}_{\xi}$ ;<br>
dates  $(X_{i,b}^{k+1}, V_{i,b}^{k+1}) \leftarrow (\mathbf{Z}_{x}^{k}, \mathbf{W}_{v}^{k})$  by (5.26) and sends<br>  $\leftarrow ($ 3: while  $k \le k_{\text{max}}$  do<br>
4: Each agent distributively updates  $(X_{i,b}^{k+1}, V_{i,b}^{k+1}) \leftarrow (Z_x^k, W_y^k)$  by (5.26) an<br>  $(X_{i,b}^{k+1}, V_{i,b}^{k+1})$  to the DSO;<br>
5: DSO updates  $(Z_x^{k+1}, W_y^{k+1}) \leftarrow (X_i^{k+1}, V_{i,b}^{k+1})$  by (5.27) and send 4: Each agent distributively updates  $(X_{i,b}^{k+1}, V_{i,b}^{k+1}) \leftarrow (Z_s^k, W_s^k)$  by (5.26)<br>  $(X_{i,b}^{k+1}, V_{i,b}^{k+1})$  to the DSO;<br>
5: DSO updates  $(Z_s^{k+1}, W_s^{k+1}) \leftarrow (X_i^{k+1}, V_{i,b}^{k+1})$  by (5.27) and sends (*i*<br>
to all agents;<br>
6: Ea  $(X_{i,b}^{k+1}, V_{i,b}^{k+1})$  to the DSO;<br>
5: DSO updates  $(Z_x^{k+1}, W_x^{k+1}) \leftarrow (X_i^{k+1}, V_{i,b}^{k+1})$  by (5.27) and sends  $(Z_x^{k+1}, W_y^{k+1})$ <br>
to all agents;<br>
6: Each agent distributively updates  $(\mu_{i,b}^{k+1}, \gamma_{i,b}^{k+1}) \leftarrow (\mu_{i,b}^k, \gamma_{i,b}$  $\left( \mathbf{Z}_{x}^{k+1} \right)$  by (5.27) and sends  $\left( \mathbf{Z}_{x}^{k+1}, \mathbf{W}_{y}^{k+1} \right)$ <br>  $\left( \mathbf{\mu}_{i,b}^{k}, \mathbf{y}_{i,b}^{k+1} \right) \leftarrow \left( \mathbf{\mu}_{i,b}^{k}, \mathbf{y}_{i,b}^{k} \right)$  by (5.29) and<br>
en<br>
or the entire ADNs;<br>  $\mathbf{x}^{\ell}$  and realistically optim

 $^*, u^{l^*})$ 

9: else

```
10: k \leftarrow k + 1
```
- 
- 

13: Release both obfuscated-but-feasible  $x^{l_i}$ 

variables  $u^{l_i^*}$  for  $\forall l_i \in \mathcal{T}$ .

variables  $u^{\ell_i}$  for  $\forall l_i \in \mathcal{T}$ .<br>
5.5 Case Study<br>
To validate this C-ADMM-based DP-DNR mechanism  $\tilde{\mathcal{M}}$ , we have consimulation experiments on the *IFFF* RBTS-Rus 4 system with 68 nodes [10] variables  $u^{\zeta}$  for  $\forall l_i \in T$ .<br>
5 Case Study<br>
To validate this C-ADMM-based DP-DNR mechanism  $\tilde{M}$ , we have conducted<br>
mulation experiments on the *IEEE* RBTS-Bus 4 system with 68 nodes [101] and a<br>
ddified real lar variables  $u^{\zeta}$  for  $\forall l_i \in \mathcal{T}$ .<br>
5.5 Case Study<br>
To validate this C-ADMM-based DP-DNR mechanism  $\bar{\mathcal{M}}$ , we have conducted<br>
simulation experiments on the *IEEE* RBTS-Bus 4 system with 68 nodes [101] and a<br>
modifi variables  $u^6$  for  $\forall l_i \in T$ .<br>
5.5 Case Study<br>
To validate this C-ADMM-based DP-DNR mechanism  $\bar{M}$ , we have conducted<br>
simulation experiments on the *IEEE* RBTS-Bus 4 system with 68 nodes [101] and a<br>
modified real la variables  $u^{\delta}$  for  $\forall l_i \in T$ .<br>
5.5 Case Study<br>
To validate this C-ADMM-based DP-DNR mechanism  $\hat{M}$ , we have conducted<br>
simulation experiments on the *IEEE* RBTS-Bus 4 system with 68 nodes [101] and a<br>
modified real Variations  $u^2$  for  $v_i \in I$ .<br>
To validate this C-ADMM-based DP-DNR mechanism  $\hat{M}$ , we have conducted<br>
mulation experiments on the *IEEE* RBTS-Bus 4 system with 68 nodes [101] and a<br>
podified real large-scale European 5.5 Case Study<br>
To validate this C-ADMM-based DP-DNR mechanism  $\tilde{\mathcal{M}}$ , we have conducted<br>
simulation experiments on the *IEEE* RBTS-Bus 4 system with 68 nodes [101] and a<br>
modified real large-scale European distribut

To validate this C-ADMM-based DP-DNR mechanism  $\hat{M}$ , we have conducted<br>simulation experiments on the *IEEE* RBTS-Bus 4 system with 68 nodes [101] and a<br>modified real large-scale European distribution networks with 906 To validate unis C HDMH oased DF D-IW incellation  $\mathcal{M}_1$ , we have conducted<br>simulation experiments on the IEEE RBTS-Bus 4 system with 68 nodes [102]. The<br>C-ADMM-based DP-DNR mechanism is calculated with MOSEK package [ Example 1 and a large-scale European distribution networks with 906 nodes [102]. The<br>C-ADMM-based DP-DNR mechanism is calculated with MOSEK package [103].<br>5.5.1 IEEE RBTS-Bus 4 System<br>To validate this DP-DNR mechanism  $\bar$ 20 C-ADMM-based DP-DNR mechanism is calculated with MOSEK package [103].<br>
20 foctor levels (103).<br>
20 foctor levels and 6 on, we have<br>with 68 load<br>with 68 load<br>ed ADNs are<br> $5\}$ , {28, 67}.<br>2 and 3, and<br>We display<br>ith load flow 5.5.1 IEEE RBTS-Bus 4 System<br>
To validate this DP-DNR mechanism  $\mathcal{M}$  about privacy preservation, we have<br>
conducted simulation experiments on the IEEE RBTS-Bus 4 system with 68 load<br>
points, 7 feeders and 6 distribute To validate this DP-DNR mechanism  $\bar{M}$  about privacy preservation, we have conducted simulation experiments on the IEEE RBTS-Bus 4 system with 68 load points, 7 feeders and 6 distributed generators (DGs). This intercon To validate this DP-DNR mechanism  $\bar{M}$  about privacy preservation, we have<br>conducted simulation experiments on the IEEE RBTS-Bus 4 system with 68 load<br>points, 7 feeders and 6 distributed generators (DGs). This intercon conducted simulation experiments on the IEEE RBTS-Bus 4 system with 68 load<br>points, 7 feeders and 6 distributed generators (DGs). This interconnected ADNs are<br>managed by agents 1, 2 and 3 with 4 tie-lines {8, 10}, {28, 29 points, 7 feeders and 6 distributed generators (DGs). This interconnected ADNs are<br>managed by agents 1, 2 and 3 with 4 tie-lines {8, 10}, {28, 29}, {47, 45}, {28, 67}.<br>We assume  $\varepsilon = 1, \rho = 0.01p.u.\tau = 1$  for agent 1 and  $\$ managed by agents 1, 2 and 3 with 4 tie-lines {8, 10}, {28, 29}, {47, 45}, {28, 67}.<br>We assume  $\varepsilon = 1, \rho = 0.01p.u, \tau = 1$  for agent 1 and  $\tau = 0$  for agents 2 and 3, and  $\overline{\alpha}' = 20$ . The sampled vector  $\xi'$  is [0.3252, -We assume  $\varepsilon = 1, \rho = 0.01p.u., \tau = 1$  for agent 1 and  $\tau = 0$  for agents 2 and 3, and  $\overline{\alpha}^t = 20$ . The sampled vector  $\xi^t$  is  $[0.3252, -0.7549, 1.3703, -1.7115]$ <sup>r</sup>. We display the heat map of load distribution and asso <sup>*l*</sup>,  $Q^l$ ,  $v_f$  and  $v_t$ , respectively.



# (a)



(b)

No.	<b>Nodes</b>	Obfuscated Values [p.u.]	Realistic Values [p.u.]

$\boldsymbol{Q}^l$ $\int$ $\boldsymbol{Q}^l$ ${\boldsymbol P}^l$ $\boldsymbol{t}$ $\boldsymbol{P}^l$ $v_f$ $v_f$ $v_{t}$ $\boldsymbol{v}_t$ 1.026 0.165 1.014 45 $-0.217$ $-0.134$ 1.031 0.615 1.011 47 -1 1.013 $\,8\,$ 0.336 0.223 1.027 1.007 0.095 0.225 1.014 10 $\overline{2}$ 28 29 $-0.118$ $-0.089$ 1.021 1.026 0.205 0.090 1.020 1.018 3 28 67 1.026 1.032 $\boldsymbol{0}$ $\boldsymbol{0}$ 1.020 1.032 $\boldsymbol{0}$ $\boldsymbol{0}$ 4 0.0506 0.0506 Loss [p.u.]								
Fig. $5.6(a)$ exhibits some of load nodes in agent 1 are transferred to neighborhood								
agents 2 and 3, where the topology switch-off status is highlighted in red dashed lines.								
This is also the same optimal topology solution by solving the non-private DNR								
model. By observation, the tie-lines $\{47, 45\}$ and $\{28, 29\}$ have reversed the load								

 $f$   $t$   $p'$   $Q'$   $v_f$   $v_i$   $p'$   $Q'$   $v_f$   $v_i$ <br>  $\frac{1}{2}$   $\frac{1}{47}$   $\frac{45}{328}$   $\frac{0.217}{328}$   $\frac{0.336}{0.223}$   $\frac{0.223}{1.027}$   $\frac{1.026}{1.020}$   $\frac{0.0595}{0.0225}$   $\frac{0.225}{0.104}$   $\frac{1.011}{1.013}$ <br>  $\frac{28}{4}$   $f$   $f$   $p'$   $Q'$   $v_f$   $v_t$   $p'$   $Q'$   $v_f$   $v_t$ <br>  $\frac{1}{2}$   $47$   $45$   $-0.217$   $-0.134$   $1.026$   $1.031$   $0.615$   $0.165$   $1.014$   $1.011$ <br>  $\frac{2}{3}$   $108$   $0.336$   $0.223$   $1.027$   $1.007$   $0.095$   $0.225$   $1.014$   $1.013$ <br>  $\frac{$  $f$   $t$   $p'$   $Q'$   $v_j$   $v_j$   $p'$   $Q'$   $v_j$   $v_j$ <br>  $\frac{1}{2}$   $\frac{47}{2}$   $\frac{45}{2}$   $\frac{-0.173}{2}$   $\frac{-0.134}{2}$   $\frac{1.026}{2}$   $\frac{1.031}{2}$   $\frac{-0.655}{2}$   $\frac{-0.165}{2}$   $\frac{-1.014}{2}$   $\frac{-1.013}{2}$   $\frac{-0.089}{2}$   $\frac{-0.183}{2}$  1 47 45 - 0.217 - 0.134 1.026 1.031 0.615 0.165 1.014 1.011<br>
2 10 8 0.336 0.223 1.027 1.007 0.099 0.225 1.014 1.013<br>
3 28 29 -0.118 -0.089 1.021 1.026 0.205 0.090 1.020 1.018<br>
4 28 67 0 0 1.026 1.032 0 0 1.020 1.032<br>
Loss 3 28 29 -0.118 -0.089 1.021 1.026 -0.205 -0.090 1.020 1.018<br>  $\frac{4}{\sqrt{188}}$   $\frac{67}{\sqrt{100}}$  -0 -1.026 1.032 -0 -0.1032<br>  $\frac{0.0506}{\sqrt{1000}}$ <br>  $\frac{0.0506}{\sqrt{1000}}$ <br>
Fig. 5.6(a) exhibits some of load nodes in agent 1 are tr 21 1.026 0.205 0.090 1.020 1.018<br>
26 1.032 0 0 1.020 1.032<br>
0.0506<br>
<br>
les in agent 1 are transferred to neighborhood<br>
ch-off status is highlighted in red dashed lines.<br>
y solution by solving the non-private DNR<br>
7, 45} an Loss [p.u.] 0.0506 0.0506 0.0506<br>
Fig. 5.6(a) exhibits some of load nodes in agent 1 are transferred to neighborhood<br>
agents 2 and 3, where the topology switch-off status is highlighted in red dashed lines.<br>
This is also Fig. 5.6(a) exhibits some of load nodes in agent 1 are transferred to neighborhood<br>agents 2 and 3, where the topology switch-off status is highlighted in red dashed lines.<br>This is also the same optimal topology solution b Eas 2 and 3, where are optimal topology solution by solving the non-private DNR<br>del. By observation, the tie-lines  $\{47, 45\}$  and  $\{28, 29\}$  have reversed the load<br>w directions. And the tie-line  $\{8, 10\}$  provides t significantly by the tie-lines (47, 45) and (28, 29) have reversed the load<br>flow directions. And the tie-line  $\{8, 10\}$  provides the obfuscated-but-feasible load<br>flow in the realistic direction. The algebra sums of tie-However, as indicated in the third diagram of Fig. 5.6(b), this load proportions maintains quite similarly to the original load privacy of agent 1 equal to zero, as shown in the  $P'$  and  $Q'$  columns of Tab.5.1. This prot

mow anceasions. Final the therme [6, 10] provides the contradicate ordered from the proportions of the original load privacy of agent 1 and corresponding boundary voltage profiles are slightly obfuseated due to the feasib Evantual to zero, as shown in the  $P'$  and  $Q'$  columns of Tab.5.1. This protects the<br>load privacy of agent 1 and corresponding boundary voltage profiles are slightly<br>obfuscated due to the feasibility of power flow equati if there is no internal topology information available. In summary, this DP-DNR solution not only guarantees the DSO's operation available. In summary, this DP-DNR solution not only guarantees the DSO's operation available solution in Fig. 5.6(b), the proportions of three agent's loads have varied significantly by the DNR operation from the observation of the first two pie charts.<br>However, as indicated in the third diagram of Fig. 5.6(b), th As shown in Fig. 5.6(b), the proportions of three agent's loads have varied significantly by the DNR operation from the observation of the first two pie charts.<br>However, as indicated in the third diagram of Fig. 5.6(b), t As shown in Fig. 5.6(b), the proportions of three agent's loads have varied<br>significantly by the DNR operation from the observation of the first two pie charts.<br>However, as indicated in the third diagram of Fig. 5.6(b), t
Moreover, we compare the feasibility and optimality of tie-line load flow<br>fuscations with the proposed DP-DNR mechanism  $\tilde{M}$  and the Laplace<br>echanism with the output perturbation (OP) strategy [50] and program perturb Moreover, we compare the feasibility and optimality of tie-line load flow<br>obfuscations with the proposed DP-DNR mechanism  $\tilde{M}$  and the Laplace<br>mechanism with the output perturbation (OP) strategy [50] and program pert Moreover, we compare the feasibility and optimality of tie-line load flow<br>obfuscations with the proposed DP-DNR mechanism  $\tilde{M}$  and the Laplace<br>mechanism with the output perturbation (OP) strategy [50] and program pert Moreover, we compare the feasibility and optimality of tie-line load flow<br>obfuscations with the proposed DP-DNR mechanism  $\tilde{M}$  and the Laplace<br>mechanism with the output perturbation (OP) strategy [50] and program pert Morcover, we compare the feasibility and optimality of tie-line load flow<br>obfuscations with the proposed DP-DNR mechanism  $\tilde{M}$  and the Laplace<br>mechanism with the output perturbation (OP) strategy [50] and program pert Moreover, we compare the feasibility and optimality of tie-line load flow<br>obfuscations with the proposed DP-DNR mechanism  $\tilde{M}$  and the Laplace<br>mechanism with the output perturbation (OP) strategy [50] and program pert Moreover, we compare the feasibility and optimality of tie-line load flow<br>obfuscations with the proposed DP-DNR mechanism  $\tilde{\mathcal{M}}$  and the Laplace<br>mechanism with the output perturbation (OP) strategy [50] and program p Moreover, we compare the feasibility and optimality of tie-line load flow<br>obfuscations with the proposed DP-DNR mechanism  $\mathcal{M}$  and the Laplace<br>mechanism with the output perturbation (OP) strategy [50] and program pert Moreover, we compare the feasibility and optimality of tie-line load flow<br>obfuscations with the proposed DP-DNR mechanism  $\tilde{M}$  and the Laplace<br>mechanism with the output perturbation (OP) strategy [50] and program pert with the output perturbation (OP) strategy [50] and program perturbation<br>gy [49], respectively. We run 1000 random perturbations  $\xi^l$  for<br>as shown in Tab. 5.2. The first column refers to the number of tie-lines.<br>It colu strategy [49], respectively. We run 1000 random perturbations  $\xi'$  for<br>titions as shown in Tab. 5.2. The first column refers to the number of tie-lines.<br>econd column denotes the mean of active load flows of tie-lines, an 2 10 8 0.095 0.103 1.625 0.0 98.124 0.0 able to the mean of active load flows of tie-lines, and the third<br>
in indicates the constraint violation percentage of the non-private DNR model.<br>
ie last column, we examine the optimality loss of output topology variable

Table 5.2 Syntneic Tie-line Load Flows under 1000 Samples<br>
No.  $\frac{\text{Nodes}}{f}$   $\frac{\text{Mean of } P'}{\text{OP [50]}}$   $\frac{\text{Constant } \sqrt{9}}{\text{OP [50]}}$   $\frac{\text{Optimality Loss } (\%)}{\text{OP [49]}}$ <br>  $\frac{1}{\sqrt{4}}$   $\frac{47}{1}$   $\frac{45}{1}$   $\frac{60.15}{10.0}$   $\frac{0.244}{10.651}$  No.  $\frac{\text{Nodes}}{f}$   $\frac{f}{f}$   $\frac{\text{Mean of } P'}{\text{OP [50]}}$   $\frac{\sqrt{1}}{\sqrt{1}}$   $\frac{\text{Opi (190)}}{\text{OP [50]}}$   $\frac{\sqrt{1}}{\sqrt{1}}$   $\frac{\text{Pp [49]}}{\text{P [49]}}$   $\frac{\sqrt{1}}{\sqrt{1}}$ <br>1 47 45 0.615 0.244 1.651 0.0 86.251 0.0<br>2 10 8 0.095 0.103 1.625 0.0 98.124 0 mechanism and  $\hat{M}$  is obvious that the PP-based Laplace mechanism and  $\hat{M}$ , it is obvious that the PP-based Laplace mechanism cannot<br>mechanism and  $\hat{M}$ , it is obvious that the PO of  $\hat{M}$ ,  $\hat{M}$  and  $\hat{M}$  a 1 47 45 0.615 0.244 1.651 0.0 86.251 0.0<br>
2 10 8 0.095 0.13 1.625 0.0 98.124 0.0<br>
3 28 29 0.090 0.154 1.647 0.0 97.632 0.0<br>
4 28 67 0 0 1.664 0.0 95.121 0.0<br>
As shown in Tab.5.2, the OP-based Laplace mechanism returns pri  $3 \times 29$  0.090 0.154 1.647 0.0 97.632 0.0<br>  $4 \times 28$  67 0 0 1.664 0.0 95.121 0.0<br>
As shown in Tab.5.2, the OP-based Laplace mechanism returns private solutions<br>
with a large number of violated constraints due to imbalances

comparison verifies that  $\tilde{M}$  can achieve the optimality of topology variables  $u^l$  and<br>the feasibility of operational variables  $x^l$  with a satisfactory operational and<br>privacy-preserving performance. comparison verifies that  $\tilde{M}$  can achieve the optimality of topology variables  $u^l$  and comparison verifies that  $\tilde{M}$  can achieve the optimality of topology variables  $u^l$  and<br>the feasibility of operational variables  $x^l$  with a satisfactory operational and<br>privacy-preserving performance.<br>5.5.2 Large-s Let us optimality of topology variables  $u^l$  and  $u^l$  with a satisfactory operational and  $u$ <br>variables  $u^l$  and  $u$ <br>variables  $u^l$  and  $u$ <br>variables  $u^l$  and  $u$ <br>variables  $u^l$  and  $u^l$  and  $u^l$  are  $u^l$  and  $u^$ comparison verifies that  $\tilde{M}$  can achieve the optimality of topology variables  $u^i$  and<br>the feasibility of operational variables  $x^i$  with a satisfactory operational and<br>privacy-preserving performance.<br>5.5.2 Large-s comparison verifies that  $\bar{M}$  can achieve the optimality of topology variables  $u^i$  and<br>the feasibility of operational variables  $x^i$  with a satisfactory operational and<br>privacy-preserving performance.<br>5.5.2 Large-sc mparison verifies that  $\hat{M}$  can achieve the optimality of topology variables  $u^i$  and<br>
2 feasibility of operational variables  $x^i$  with a satisfactory operational and<br>
yacy-preserving performance.<br>
5.2 Large-scale Pr

comparison verifies that  $\tilde{M}$  can achieve the optimality of topology variables  $u^i$  and<br>the feasibility of operational variables  $x^i$  with a satisfactory operational and<br>privacy-preserving performance.<br>5.5.2 Large-s comparison verifies that  $\hat{M}$  can achieve the optimality of topology variables  $u'$  and<br>the feasibility of operational variables  $x'$  with a satisfactory operational and<br>privacy-preserving performance.<br>5.5.2 Large-scal comparison verifies that  $\hat{M}$  can achieve the optimality of topology variables  $n'$  and<br>the feasibility of operational variables  $x'$  with a satisfactory operational and<br>privacy-preserving performance.<br>5.5.2 Large-scal the feasibility of operational variables  $x^i$  with a satisfactory operational and<br>privacy-preserving performance.<br>5.5.2 Large-scale Practical European Distribution Networks<br>For the scalability analysis, the large-scale p The presenting of operational variables  $\lambda$  with a statistically operational and<br>privacy-preserving performance.<br>
S.5.2 Large-scale Practical European Distribution Networks<br>
To structure the interconnected ADNs, we have 5.5.2 Large-scale Practical European Distribution Networks<br>
For the scalability analysis, the large-scale practical European distribution networks<br>
are adopted from reference [102], which operates at 0.416 kV and includes For the scalability analysis, the large-scale practical European distribution networks<br>are adopted from reference [102], which operates at 0.416 kV and includes 906 nodes.<br>To structure the interconnected ADNs, we have add For the scalability analysis, the large-scale practical European distribution networks<br>are adopted from reference [102], which operates at 0.416 kV and includes 906 nodes.<br>To structure the interconnected ADNs, we have add are adopted from reference [102], which operates at 0.416 kV and includes 906 nodes.<br>To structure the interconnected ADNs, we have added 10 DGs and 9 points of<br>common coupling (PCC) in this distribution networks, and then To structure the interconnected ADNs, we have added 10 DGs and 9 common coupling (PCC) in this distribution networks, and then the 1<br>tie-lines is 8. The parameter  $\tau = 1$  for agents 2 and 5, and  $\tau = 0$  for other a<br>visual  $\mathbf{Q}^l$ ,  $\mathbf{V}_f$  and  $\mathbf{v}_t$ , respectiv interconnected ADNs, we have added 10 DGs and 9 points of<br>ng (PCC) in this distribution networks, and then the number of<br>e parameter  $\tau = 1$  for agents 2 and 5, and  $\tau = 0$  for other agents. We<br>t map of load distribution Eines is 8. The parameter  $\tau$  = 1 for agents 2 and 5, and  $\tau$  = 0 for other agents. We sualize the heat map of load distribution and associated optimal DNR solution with Freent agents in Fig. 5.7. To be a clear exhibiti by observing the switch-off branches marked in red dashed lines. The tie-line numbers marked in red in red inc. The tie-line numbers matched the marked in red in red in red in the time spin of tie-lines and associated bou Fig. 5.7. To be a clear exhibition, we display the realistic load<br>flows of tic-lines and associated boundary voltage profiles and the mean of these<br>obfuscated values under 1000 samples of random perturbations  $\xi^i$  in Ta

altective agents in Fig. 3.7. For the architection, we display the relation band flows of tic-lines and associated boundary voltage profiles and the mean of these obfuseated values under 1000 samples of random perturbatio revers of the lines and associated boundary voltage profites dual to the linear of the loop<br>obfuscated values under 1000 samples of random perturbations  $\xi^i$  in Tab.5.3. In this<br>table, f and t refers to "from agent" and table, f and t refers to "from agent" and "to agent" in the column of agents, and this is applied for  $P'$ ,  $Q'$ ,  $v_f$  and  $v_b$  respectively.<br>Fig. 5.7 exhibits some of load nodes in agent 5 are switched to neighborhood ag this optimal DNR operation. Agents 1 and 3 inject the power via tie-line numbers 5 and 2, but agent 1 still supplies several loads in agent 5 via tie-line number 6. Since the agent 5 is heavily loaded, neighborhood agents This optimal DNR operation. Agents 1 and 3 inject the power via tie-line numbers 5 and 2, but agent 1 still supplies several loads in agent 5 via tie-line number 6. Since the agent 5 is heavily loaded, neighborhood agents this optimal DNR operation. Agents 1 and 3 inject the power via tie-line numbers 5 and 2, but agent 1 still supplies several loads in agent 5 via tie-line number 6. Since the agent 5 is heavily loaded, neighborhood agents this optimal DNR operation. Agents 1 and 3 inject the power via tie-line numbers 5 and 2, but agent 1 still supplies several loads in agent 5 via tie-line number 6. Since the agent 5 is heavily loaded, neighborhood agents In this optimal DNR operation. Agents 1 and 3 inject the power via tie-line numbers 5 and 2, but agent 1 still supplies several loads in agent 5 via tie-line number 6. Since the agent 5 is heavily loaded, neighborhood agen this optimal DNR operation. Agents 1 and 3 inject the power via tie-line numbers 5 and 2, but agent 1 still supplies several loads in agent 5 via tie-line number 6. Since the agent 5 is heavily loaded, neighborhood agents This optimal DNR operation. Agents 1 and 3 inject the power via tie-line numbers 5 and 2, but agent 1 still supplies several loads in agent 5 via tie-line number 6. Since the agent 5 is heavily loaded, neighborhood agents this optimal DNR operation. Agents 1 and 3 inject the power via tie-line number of and 2, but agent 1 still supplies several loads in agent 5 via tie-line number 6.<br>the agent 5 is heavily loaded, neighborhood agents 1, 4,









2 3  $-0.117$   $-0.125$  1.000  $-0.034$   $-0.094$  1.005 1.018<br>
5 4  $-0.109$   $-0.056$  1.021 1.017  $-0.381$   $-0.264$  1.024 1.021<br>
5 7 0.168 0.018 1.018  $-0.23$   $-0.216$  1.012 1.014<br>
2 1  $-0.016$   $-0.035$  1.029  $-0.04$   $-0.09$  because of the load flow direction obfuscation criteria, while the possible ranges of the load flow direction obfuscation criteria, while the possible ranges of the load flow direction obfuscation criteria, while the pos 2 1 -0.010 -0.033 -1.029 -1.033 -1.044 -0.034 -0.004 -1.080 -1.031 -1.011<br>
6 8 0.335 0.160 1.026 1.014 0.213 0.089 1.026 1.015<br>
5 6 0.192 0.122 1.067 1.061 -0.013 -0.001 1.065 1.062<br>
1.065 1.062 -0.2036 0.2036 0.2036 0.20  $56$  8 0.335 0.160 1.026 1.014 0.213 0.089 1.026 1.014<br>  $56$  0.192 0.122 1.067 1.061 -0.013 -0.0011 1.065 1.062<br>
Loss [p.u.]<br>
We compare the load proportion of each agent before and after this DNR operation<br>
as highlighte  $\frac{0.2036}{2}$  0.2036<br>
We compare the load proportion of each agent before and after this DNR operation<br>
as highlighted by blue and red radar forms in Fig. 5.8. With 1000 samples of random<br>
perturbations  $\xi^i$ , this figu



Moreover, this C-ADMM algorithm, we suppose that the parameter  $\lambda$  is set to 100.<br>
ie initial dual variables  $\mu_{i,b}$  and  $\gamma_{i,b}$  are zeros vectors and a good warm start  $Z_x$  and  $\mu$  is predefined for the iteration  $k =$ Moreover, this C-ADMM algorithm, we suppose that the parameter  $\lambda$  is set to 100.<br>The initial dual variables  $\mu_{i,b}$  and  $\gamma_{i,b}$  are zeros vectors and a good warm start  $\mathbb{Z}_x$  and  $W_y$  is predefined for the iteratio Moreover, this C-ADMM algorithm, we suppose that the parameter  $\lambda$  is set to 100.<br>The initial dual variables  $\mu_{i,b}$  and  $\gamma_{i,b}$  are zeros vectors and a good warm start  $\mathbb{Z}_x$  and  $\mathbb{W}_y$  is predefined for the ite Moreover, this C-ADMM algorithm, we suppose that the parameter  $\lambda$  is set to 100.<br>The initial dual variables  $H_{i,b}$  and  $T_{i,b}$  are zeros vectors and a good warm start  $Z_x$  and  $W_y$  is predefined for the iteration  $k = 1$ Morcover, this C-ADMM algorithm, we suppose that the parameter  $\lambda$  is set to 100.<br>The initial dual variables  $\mu_{i,b}$  and  $\gamma_{i,b}$  are zeros vectors and a good warm start  $\mathbb{Z}_x$  and  $\mathbb{W}_v$  is predefined for the ite Moreover, this C-ADMM algorithm, we suppose that the parameter  $\lambda$  is set to 100.<br>The initial dual variables  $\mu_{\lambda}$  and  $\gamma_{\lambda}$  are zeros vectors and a good warm start  $\mathbb{Z}_x$  and  $\mathbb{W}_v$  is predefined for the ite Moreover, this C-ADMM algorithm, we suppose that the parameter  $\lambda$  is set to 100.<br>The initial dual variables  $\mu_{i,\delta}$  and  $\gamma_{i,\delta}$  are zeros vectors and a good warm start  $Z_x$  and  $W_y$  is predefined for the iteration Moreover, this C-ADMM algorithm, we suppose that the parameter  $\lambda$  is set to 100.<br>The initial dual variables  $\mu_{i,b}$  and  $\gamma_{i,b}$  are zeros vectors and a good warm start  $\mathbb{Z}_k$  and  $\mathbb{W}_r$  is predefined for the ite



Fig. 5.9. Convergence performance of C-ADMM algorithm: (a) norms of primal<br>residuals; (b) norms of dual residuals; (c) norms of consensus residuals 5.9. Convergence performance of C-ADMM algorithm: (a) norms of primal<br>residuals; (b) norms of dual residuals; (c) norms of consensus residuals<br>iscuss the privacy-preserving performance for our proposed C-ADMM-based

Fig. 5.9. Convergence performance of C-ADMM algorithm: (a) norms of primal<br>residuals; (b) norms of dual residuals; (c) norms of consensus residuals<br>We discuss the privacy-preserving performance for our proposed C-ADMM-bas Fig. 5.9. Convergence performance of C-ADMM algorithm: (a) norms of primal<br>residuals; (b) norms of dual residuals; (c) norms of consensus residuals<br>We discuss the privacy-preserving performance for our proposed C-ADMM-bas Fig. 5.9. Convergence performance of C-ADMM algorithm: (a) norms of primal<br>residuals; (b) norms of dual residuals; (c) norms of consensus residuals<br>We discuss the privacy-preserving performance for our proposed C-ADMM-bas Fig. 5.9. Convergence performance of C-ADMM algorithm: (a) norms of primal<br>residuals; (b) norms of dual residuals; (c) norms of consensus residuals<br>We discuss the privacy-preserving performance for our proposed C-ADMM-bas norms of primal<br>
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non-private DNR<br>
e can observe the<br>  $P^l$  as boundary<br>
models per twenty<br>
sesents the tie-line's Fig. 5.9. Convergence performance of C-ADMM algorithm: (a) norms of pri<br>residuals; (b) norms of dual residuals; (c) norms of consensus residuals<br>We discuss the privacy-preserving performance for our proposed C-ADMM<br>DP-DNR Let be performance of C-ADMM algorithm: (a) norms of primal<br>orms of dual residuals; (c) norms of consensus residuals<br>acy-preserving performance for our proposed C-ADMM-based<br> $\hat{M}$  as compared to the C-ADMM-based non-pri Fig. 5.9. Convergence performance of C-ADMM algorithm: (a) norms of primal<br>residuals; (b) norms of dual residuals; (c) norms of consensus residuals<br>We discuss the privacy-preserving performance for our proposed C-ADMM-bas ractive load flow. It is clear that the C-ADMM algorithm can exchange privaty-preserving performance for our proposed C-ADMM-based DP-DNR mechanism  $\hat{M}$  as compared to the C-ADMM-based non-private DNR approach. To exem Francularis, (b) infinite of data resindants, (b) infinite of connectional resindants.<br>
We discuss the privacy-preserving performance for our proposed C-ADMM-based<br>
DP-DNR mechanism  $\tilde{M}$  as compared to the C-ADMM-base privacy-preserving information  $x^{l_i^*} = \hat{x}^{l_i} - \alpha^{l_i^*} \xi^{l_i}$  based on Theorem 2 in Fig 5.10 (a), **Executes**, (c) norms of each reasonals, (c) norms of ecnsesses restorates<br>
DP-DNR mechanism  $\hat{M}$  as compared to the C-ADMM-based non-private DNR<br>
approach. To exemplify this privacy-preserving performance, we can obs instead of realistic boundary continuous variables  $\hat{x}^{l_1}$  from each agent as shown in for our proposed C-ADMM-based<br>ADMM-based non-private DNR<br>erformance, we can observe the<br>2 load flow  $P^l$  as boundary<br>-private DNR models per twenty<br>oded line represents the tie-line's<br>9MM algorithm can exchange<br>ed on The DP-DNR mechanism  $\hat{M}$  as compared to the C-ADMM-based non-private DNR<br>approach. To exemplify this privacy-preserving performance, we can observe the<br>different convergence performances of the active load flow  $P^t$  as b approach. To exemplify this privacy-preserving performance, we can observe the<br>different convergence performances of the active load flow  $P^k$  as boundary<br>continuous variables  $x^k$  of the DP-DNR and non-private DNR mode  $\mathbf{x}^{l^*_t}$  .





 $l_t^*$  $\hat{\mathbf{x}}^{l_t}$ 

Fig. 5.10. Convergence performance of C-ADMM algorithm: (a)  $x^6$  by DP-DNR; (b)<br>Fig. 5.10. Convergence performance of C-ADMM algorithm: (a)  $x^6$  by DP-DNR; (b)<br> $\dot{x}^6$  by non-private DNR<br>5.6 Summary<br>This chapter propos  $\frac{1}{20}$   $\frac{1}{40}$   $\frac{1}{60}$   $\frac{1}{80}$   $\frac{1}{100}$   $\frac{1}{120}$   $\frac{1}{140}$   $\frac{1}{160}$   $\frac{1}{180}$ <br>
Iterations (b)<br>
Fig. 5.10. Convergence performance of C-ADMM algorithm: (a)  $x^6$  by DP-DNR; (b)<br>  $\hat{x}^6$  by non-pr **Example 1998** (b)<br>
(b)<br>
(b)<br>  $\hat{x}^k$  by non-private DNR<br>
5.6 Summary<br>
This chapter proposes a DP-DNR mechanism based on a C-ADMM approach for<br>
interconnected multi-agent ADNs. This query mechanism provides a mixture out Fig. 5.10. Convergence performance of C-ADMM algorithm: (a)  $x^6$  by DP-DNR; (b)<br>  $\hat{x}^6$  by non-private DNR<br>
5.6 Summary<br>
This chapter proposes a DP-DNR mechanism based on a C-ADMM approach for<br>
interconnected multi-age  $\hat{x}^i$  by non-private DNR<br>
5.6 Summary<br>
This chapter proposes a DP-DNR mechanism based on a C-ADMM approach for<br>
interconnected multi-agent ADNs. This query mechanism provides a mixture output<br>
of both realistically opt 5.6 Summary<br>This chapter proposes a DP-DNR mechanism based on a C-ADMM approach for<br>interconnected multi-agent ADNs. This query mechanism provides a mixture output<br>of both realistically optimal tie-switch status and corres 5.6 Summary<br>This chapter proposes a DP-DNR mechanism based on a C-ADMM approach for<br>interconnected multi-agent ADNs. This query mechanism provides a mixture output<br>of both realistically optimal tic-switch status and corres This chapter proposes a DP-DNR mechanism based on a C-ADMM approach for<br>interconnected multi-agent ADNs. This query mechanism provides a mixture output<br>of both realistically optimal tie-switch status and corresponding obfu interconnected multi-agent ADNs. This query mechanism provides a mixture output<br>of both realistically optimal tie-switch status and corresponding obfuscated-<br>but-feasible tie-line load flows, part of which may have reverse

## Chapter 6<br>Distribution-Level Topology Optimization in Chapter 6<br>Distribution-Level Topology Optimization in<br>Economic Dispatch of Wind-Thermal-Bundled Chapter 6<br>
Distribution-Level Topology Optimization in<br>
Economic Dispatch of Wind-Thermal-Bundled<br>
Power System for Operational Flexibility Chapter 6<br>Distribution-Level Topology Optimization in<br>Economic Dispatch of Wind-Thermal-Bundled<br>Power System for Operational Flexibility<br>Enhancement Enhancement Chapter 6<br>
Distribution-Level Topology Optimization in<br>
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Enhancement<br>
With a wind-thermal-bundled power system (WTBPS) under high wind<br>
et **Chapter 6**<br> **Distribution-Level Topology Optimization in**<br> **Economic Dispatch of Wind-Thermal-Bundled**<br> **Power System for Operational Flexibility**<br> **Enhancement**<br>
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**Distribution-Level Topology Optimization in**<br> **Economic Dispatch of Wind-Thermal-Bundled**<br> **Power System for Operational Flexibility**<br> **Enhancement**<br>
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penetration levels, the sharp power fluctuations of ti **Economic Dispatch of Wind-Thermal-**<br> **Power System for Operational F**<br> **Enhancement**<br>
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ger a signif **Enhancement**<br>With a wind-thermal-bundled power system (WTBPS) under high wind<br>penetration levels, the sharp power fluctuations of tie-lines for interconnected grids<br>trigger a significant challenge of security-constrained With a wind-thermal-bundled power system (WTBPS) under high wind<br>penetration levels, the sharp power fluctuations of tie-lines for interconnected grids<br>trigger a significant challenge of security-constrained power system o

With a wind-thermal-bundled power system (WTBPS) under high wind<br>penctration levels, the sharp power fluctuations of tic-lines for interconnected grids<br>trigger a significant challenge of security-constrained power system o penetration levels, the sharp power fluctuations of tie-lines for interconnected grids<br>trigger a significant challenge of security-constrained power system operation.<br>Smoothing power fluctuations with economic dispatch is trigger a significant challenge of security-constrained power system operation.<br>Smoothing power fluctuations with economic dispatch is widely concerned against<br>this challenge.<br>This chapter proposes a distribution-level top Smoothing power fluctuations with economic dispatch is widely concerned against<br>this challenge.<br>This chapter proposes a distribution-level topology optimization contributing to<br>the flexibility enhancement of a look-ahead r this challenge.<br>
This chapter proposes a distribution-level topology optimization contributing to<br>
the flexibility enhancement of a look-ahead rolling economic dispatch of WTBPS.<br>
The contributions of this paper are three-This chapter proposes a distribution-level topology optimization contributing to<br>the flexibility enhancement of a look-ahead rolling economic dispatch of WTBPS.<br>The contributions of this paper are three-fold: 1) This study the flexibility enhancement of a look-ahead rolling economic dispatch of WTBPS.<br>The contributions of this paper are three-fold: 1) This study derives a new family of<br>tightened ramping constraints of retrofitted coal-fired

LTS-based strategy is then reformulated as a mixed-integer second-order cone<br>programming (MISOCP) problem for a long look-ahead period, which has not been<br>studied to date. 3) For this established MISOCP-based model, it is LTS-based strategy is then reformulated as a mixed-integer second-order cone<br>programming (MISOCP) problem for a long look-ahead period, which has not been<br>studied to date. 3) For this established MISOCP-based model, it is LTS-based strategy is then reformulated as a mixed-integer second-order cone<br>programming (MISOCP) problem for a long look-ahead period, which has not been<br>studied to date. 3) For this established MISOCP-based model, it is LTS-based strategy is then reformulated as a mixed-integer second-order cone<br>programming (MISOCP) problem for a long look-ahead period, which has not been<br>studied to date. 3) For this established MISOCP-based model, it is LTS-based strategy is then reformulated as a mixed-integer second-order cone<br>programming (MISOCP) problem for a long look-ahead period, which has not been<br>studied to date. 3) For this established MISOCP-based model, it is LTS-based strategy is then reformulated as a mixed-integer second-order cone<br>programming (MISOCP) problem for a long look-ahead period, which has not been<br>studied to date. 3) For this established MISOCP-based model, it is LTS-based strategy is then reformulated as a mixed-integer second-order cone<br>programming (MISOCP) problem for a long look-ahead period, which has not been<br>studied to date. 3) For this established MISOCP-based model, it is LTS-based strategy is then reformulated as a mixed-integer second-order cone<br>programming (MISOCP) problem for a long look-ahead period, which has not been<br>studied to date. 3) For this established MISOCP-based model, it is by a by a stablished MISOCP based period, which has not been<br>idid to date. 3) For this established MISOCP-based model, it is highly desirable to<br>mbine the Multi-cut Benders Decomposition (MBD) and Generalized Benders<br>recom studied to date. 3) For this established MISOCP-based model, it is highly desirable to<br>combine the Multi-cut Benders Decomposition (MBD) and Generalized Benders<br>Decomposition (GBD) as the devised Multi-cut GBD (MGBD) to ta

combine the Multi-cut Benders Decomposition (MBD) and Generalized Benders<br>Decomposition (GBD) as the devised Multi-cut GBD (MGBD) to tackle this<br>MISOCP problem, which can enhance overall computational efficiency and be<br>sui Decomposition (GBD) as the devised Multi-cut GBD (MGBD) to tackle this<br>MISOCP problem, which can enhance overall computational efficiency and be<br>suitable for online rolling economic dispatch.<br>6.1 Modeling of WTBPS and Asso MISOCP problem, which can enhance overall computational efficiency and be<br>suitable for online rolling economic dispatch.<br>(5.1 Modeling of WTBPS and Associated Constraints<br>For the rapid growth of wind farms only with a fixe suitable for online rolling economic dispatch.<br>
6.1 Modeling of WTBPS and Associated Constraints<br>
For the rapid growth of wind farms only with a fixed capacity of coal-fired plants,<br>
there are two ways to avoid suffering f for the rapid growth of wind farms only with a fixed capacity of coal-fired plants,<br>there are two ways to avoid suffering from wind curtailment. *i*) The one option is to<br>upgrade coal-fired units, e.g. retrofitted coal-fir 6.1 Modeling of WTBPS and Associated Constraints<br>For the rapid growth of wind farms only with a fixed capacity of coal-fired plants,<br>there are two ways to avoid suffering from wind curtailment. *i*) The one option is to<br>up For the rapid growth of wind farms only with a fixed capacity of coal-fired plants,<br>there are two ways to avoid suffering from wind curtailment.  $i$ ) The one option is to<br>upgrade coal-fired units, e.g. retrofitted coal-fi there are two ways to avoid suffering from wind curtailment. *i*) The one option is to upgrade coal-fired units, e.g. retrofitted coal-fired units have faster ramp rates. Herein, they can be used to rapidly track changes w upgrade coal-fired units, e.g. retrofitted coal-fired units have faster ramp rates. Herein,<br>they can be used to rapidly track changes with unexpected ramp down of wind power.<br>ii) Various alternatives such as distribution-s In this study, integrating wind-thermal-bundled transmission system wind power.<br>
In Various alternatives such as distribution-side load transfer and energy storage can<br>
be used to increase the flexibility of wind-thermal-b ii) Various alternatives such as distribution-side load transfer and energy storage can<br>be used to increase the flexibility of wind-thermal-bundled transmission system<br>further. In this paper, we discuss the flexible load t

There are quite a few coastal cities in the world suitable for constructing this WTBPS<br>for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting There are quite a few coastal cities in the world suitable for constructing this WTBPS<br>for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting There are quite a few coastal cities in the world suitable for constructing this WTBPS<br>for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting There are quite a few coastal cities in the world suitable for constructing this WTBPS<br>for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting There are quite a few coastal cities in the world suitable for constructing this WTBPS<br>for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting There are quite a few coastal cities in the world suitable for constructing this WTBPS<br>for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting There are quite a few coastal cities in the world suitable for constructing this WTBPS<br>for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting Frame are quite a few coastal cities in the world suitable for constructing this WTBPS<br>of the countries, where these places accommodate abundant dispatchable loads,<br>d also are equipped with long-distance transporting coals There are quite a few coastal cities in the world suitable for constructing this WTBPS<br>for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting for other countries, where these places accommodate abundant dispatchable loads,<br>and also are equipped with long-distance transporting coals by sea as well as off-shore<br>wind resources. Towards this flexible control objecti

and also are equipped with long-distance transporting coals by sea as well as off-shore<br>wind resources. Towards this flexible control objective, we concentrate on its<br>look-ahead economic dispatch problem in a rolling windo wind resources. Towards this flexible control objective, we concentrate on its<br>look-ahead economic dispatch problem in a rolling window, considering the dynamic<br>ramping of retrofitted coal-fired units and the LTS via HVDNs look-ahead economic dispatch problem in a rolling window, considering the dynamic<br>ramping of retrofitted coal-fired units and the LTS via HVDNs.<br>6.1.1 Static Modeling of WTBPS<br>The 110kV dispatchable load resources from sub ramping of retrofitted coal-fired units and the LTS via HVDNs.<br>6.1.1 Static Modeling of WTBPS<br>The 110kV dispatchable load resources from substations in HVDNs are generally<br>in the range of 0 and 480 MW or even larger, acco 6.1.1 Static Modeling of WTBPS<br>The 110kV dispatchable load resources from substations in HVDNs are generally<br>in the range of 0 and 480 MW or even larger, accounting for nearly 5%-30% capacity<br>of WTBPS. This flexibility in m substations in HVDNs are generally<br>accounting for nearly 5%-30% capacity<br>r quick-response capacity than that of<br>HVDNs to a wind-thermal generation<br>other grids forms a WTBPS, which<br>this flexibility, multiple independent<br> in the range of 0 and 480 MW or even larger, accounting for nearly 5%-30% eapacity<br>of WTBPS. This flexibility indicates the better quick-response capacity than that of<br>any coal-fired units. Consequently, integrating HVDNs for nearly 5%-30% capacity<br>ponse capacity than that of<br>o a wind-thermal generation<br>s forms a WTBPS, which<br>bility, multiple independent<br>alle HVDNs, each of which<br>deled with several retrofitted<br> $=\sum_{i=1}^{N_G} r_{i,u}^t P_{Gi,u,max}$ , w any coal-fired units. Consequently, integrating HVDNs to a wind-thermal generation<br>system with connected AC/DC tie-lines to other grids forms a WTBPS, which<br>produces more steady output power. Due to this flexibility, mult

G  $C_{\text{G},u} = \sum_{i=1}^{n} r_{i,u} P_{\text{G}i,u,\text{max}}$  $t = \sum_{t=1}^{N_{\rm G}} t^{t}$  $u = \sum_{i=1}^r r_{i,u} F_{Gi,u,n}$  $P_{\rm G, u}^t = \sum_{l}^{\rm o} r_{l, u}^t P_{\rm G, u}^t$ Ξ  $t_{u}^{t} = P_{Gi, u}^{t} / P_{Gi, u, max}$ system with connected AC/DC tie-lines to other grids forms a WTBPS, which<br>produces more steady output power. Due to this flexibility, multiple independent<br>agents of WTBPS can integrate with the same large-scale HVDNs, eac C tie-lines to other grids forms a WTBPS, which<br>power. Due to this flexibility, multiple independent<br>e with the same large-scale HVDNs, each of which<br>as displayed in Fig. 6.1.<br>wind power  $P'_{w,u}$  is bundled with several r loads  $P_{D,u}^t$  with reconfigurable HVDNs. For every  $t \in T_s$ , constraint (6.1) can be as more steady output power. Due to this flexibility, multiple independent<br>of WTBPS can integrate with the same large-scale HVDNs, each of which<br>stic-lines to other grids, as displayed in Fig. 6.1.<br>the u-th agent of WTBPS xibility, multiple independent<br>scale HVDNs, each of which<br>.<br>.<br>undled with several retrofitted<br> $\int_{G_{\alpha}u}^{u} = \sum_{i=1}^{N_G} r_{i,u}^t P_{G_{i,u,\text{max}}}$ , where<br>tted coal-fired units in the *u*-th<br>y coordinate with dispatchable<br> $t \in T_s$ agents of WTBPS can integrate with the same large-scale HVDNs, each<br>connects tie-lines to other grids, as displayed in Fig. 6.1.<br>For the *u*-th agent of WTBPS, wind power  $P'_{W,u}$  is bundled with severa<br>coal-fired units o TBPS can integrate with the same large-scale HVDNs, each of which<br>lines to other grids, as displayed in Fig. 6.1.<br>th agent of WTBPS, wind power  $P'_{w,x}$  is bundled with several retrofitted<br>units of total output power  $P'_{$  $(1 - \delta\%) P_{\text{base}}$ ,  $(1 + \delta\%)$ .  $P_{\text{T},u} \in [(1-\delta\%)P_{\text{base}} , (1+\delta\%)P_{\text{base}} ]\frac{1}{2}$  on tie-lines.

$$
P_{W,u}^t + P_{G,u}^t - P_{D,u}^t = P_{T,u}^t, (1 - \delta^0/0)P_{base} \le P_{T,u}^t \le (1 + \delta^0/0)P_{base}
$$
 (6.1)



RETRON THE TRANSFORM COMPARED CONDITION TO THE CONSIDERATION CONDUCT THE CONDUCT SURVISION CONDUCT THE CONDUCT SURVISION CONDUCT THE METRON MANUS (SYSTEM) THE MOREOVER (footherm), a medi-herman-bundled layer (medium) and a

THE SET AND THE STAND TO THE SURFACT THE SURFACT THE SURFACT THE SURFACT THE SURFACT THAN THE SURFACT THAN THE SURFACT ON THE SURFACT ON THE SURFACT ON THE SURFACT ON THE DURING THE SURFACT ON THE DURING THE SURFACT ON THE Fig. 6.1 An illustration of WTBPS consisting of the HVDNs layer (bottom), a<br>wind-thermal-bundled layer (medium) and a control layer (bottom). HVDNs layer, load flows are indicated with arrows. The vertical dashed lines co wind-thermal-bundled layer (medium) and a control layer (top). In the bottom<br>HVDNs layer, load flows are indicated with arrows. The vertical dashed lines connect<br>he nodes in the bottom and medium layers, and to nodes in th the nodes in the bottom and medium layers, and to nodes in the top layer, which<br>aggregate at bus *i<sub>8</sub>* by different agents. Also, bus *i<sub>8</sub>* in each agent is a starting node of<br>tie-lines to connect other grids.<br> $6.1.2$  O **6.1.2 Operational Constraints of Retrofitted Coal-fired Units**<br>**Retrofitting flexibility measures consist of upgrading the control system, reducing**<br>the wall thickness of key components, auxiliary firing with dried lignit 6.1.2 Operational Constraints of Retrofitted Coal-fired Units<br>Retrofitting flexibility measures consist of upgrading the control system, reducing<br>the wall thickness of key components, auxiliary firing with dried lignite i 6.1.2 Operational Constraints of Retrofitted Coal-fired Units<br>Retrofitting flexibility measures consist of upgrading the control system, reducing<br>the wall thickness of key components, auxiliary firing with dried lignite i Retrofitting flexibility measures consist of upgrading the control system, reducing<br>the wall thickness of key components, auxiliary firing with dried lignite ignition<br>burner in booster operation, and so forth [77]. These

on a dispatch period ΔT level.<br>As clearly shown in Table 6.1, conventional coal-fired plants without retrofits are<br>inherently less flexible than retrofitted coal-fired plants in the minimum load level, a dispatch period  $\Delta T$  level.<br>As clearly shown in Table 6.1, conventional coal-fired plants without retrofits are<br>nerently less flexible than retrofitted coal-fired plants in the minimum load level,<br>np rate utilization, in a dispatch period  $\Delta T$  level.<br>As clearly shown in Table 6.1, conventional coal-fired plants without retrofits are<br>inherently less flexible than retrofitted coal-fired plants in the minimum load level,<br>ramp rate utiliz on a dispatch period  $\Delta T$  level.<br>As clearly shown in Table 6.1, conventional coal-fired plants without retrofits are<br>inherently less flexible than retrofitted coal-fired plants in the minimum load level,<br>ramp rate utiliza on a dispatch period  $\Delta T$  level.<br>
As clearly shown in Table 6.1, conventional coal-fired plants without retrofits are<br>
inherently less flexible than retrofitted coal-fired plants in the minimum load level,<br>
ramp rate uti on a dispatch period  $\Delta T$  level.<br>As clearly shown in Table 6.1, conventional coal-fired plants without retrofits are inherently less flexible than retrofitted coal-fired plants in the minimum load level,<br>ramp rate utiliz on a dispatch period  $\Delta T$  level.<br>As clearly shown in Table 6.1, conventional coal-fired plants without retrofits are<br>inherently less flexible than retrofitted coal-fired plants in the minimum load level,<br>ramp rate utiliz on a dispatch period  $\Delta T$  level.<br>
As clearly shown in Table 6.1, conventional coal-fired plants without retrofits are<br>
inherently less flexible than retrofitted coal-fired plants in the minimum load level,<br>
ramp rate util Items<br>
Conventional coal-fired plants without retrofits are<br>
less flexible than retrofitted coal-fired plants in the minimum load level,<br>
utilization, and hot and cold start-up time. As merits, the minimum load<br>
contritte fitted coal-fired plants in the minimum load level,<br>
1 cold start-up time. As merits, the minimum load<br>
s can be reduced to 15–35% of the rated power, and<br>
0 2–6% per minute of rated power, any of which<br>
lispatch of WTBPS nts without retrofits are<br>
the minimum load level,<br>
erits, the minimum load<br>
of the rated power, and<br>
d power, any of which<br>
amping limit is a linear<br>
ed coal-fired plants<br>
Retrofitted coal-fired<br>
power plants<br>
15-35<br>
2-6<br> minimum load level,<br>the minimum load<br>the rated power, and<br>ower, any of which<br>ing limit is a linear<br>oal-fired plants<br>ofited coal-fired<br>power plants<br>15-35<br>2-6<br>1.5-4<br>5-6 Minimum load level (%)<br>
Minimum condenties the minimum load level,<br>
amp rate utilization, and hot and cold start-up time. As merits, the minimum load<br>
vel of retrofitted coal-fired plants can be reduced to 15-35% of the r p rate utilization, and hot and cold start-up time. As merits, the minimum load<br>
of retrofitted coal-fired plants can be reduced to 15–35% of the rated power, and<br>
ramp rate can be enhanced to 2–6% per minute of rated pow matrix in the start-up time. As merits, the minimum load<br>
el of retrofitted coal-fired plants can be reduced to 15–35% of the rated power, and<br>
ramp rate can be enhanced to 2.6% per minute of rated power, any of which<br>
tr vel of retrofitted coal-fired plants can be reduced to 15-35% of the rated power, and<br>
ramp rate can be enhanced to 2-6% per minute of rated power, any of which<br>
ntributes to a flexible rolling dispatch of WTBPS. The ramp

	$180^\circ$ of retrofficed coal-fired plants can be reduced to $10-30\%$ of the rated power, and		
	the ramp rate can be enhanced to $2-6\%$ per minute of rated power, any of which		
	contributes to a flexible rolling dispatch of WTBPS. The ramping limit is a linear		
function of the unit's generating output [78].			
	Table 6.1. Differences between conventional and retrofitted coal-fired plants		
Items	Conventional power plants (without retrofits)	Retrofitted coal-fired power plants	
Minimum load level $(\%)$	50-60	$15 - 35$	
Ramp rate $(min\%)$	$0.6 - 2$	$2 - 6$	
Hot start-up time $(h)$	$3 - 5$	$1.5 - 4$	
Cold start-up time (h)	$5 - 8$	$5-6$	
	However, experimentally measuring ramping limits with respect to each output		
	power point is very tough in practice. In this regard, we adopt the upper envelope to		
	approximate the ramping limits per minute so as to capture the dynamic ramp rates		

point is very tough in practice. In this regard, we adopt the upper envelope to a physical spherime of the unit's generating output [78].<br>
Table 6.1. Differences between conventional and retrofitted coal-fired plants<br>
Icm function of the unit's generating output [78].<br>
Table 6.1. Differences between conventional and retrofitted coal-fired plants<br>
Items<br>
Conventional power plants<br>
Retrofitted coal-fired<br>
Minimum load level (%)<br>
Minimum rate Table 6.1. Differences between conventional and retrofitted coal-fired plants<br>
Items<br>
Items<br>
Conventional power plants<br>
Minimum load level (%)<br>
(without retrofits) power plants<br>
Minimum load level (%)<br>
50-60 15-35<br>
Ramp r Table 6.1. Differences between conventional and retrofitted coal-fired plants<br>
Items<br>
Conventional power plants<br>
(without retrofits)<br>
Minimum load level (%)<br>
5-060 15-55<br>
Ramp rate (min.<sup>9%</sup>)<br>
60.6-2<br>
Hot start-up time (h The sum of the china. The china. The x-axis refers to the percentage of the output power plants<br>
Hinimum load level (%) 50-60 power plants<br>
Hot start-up time (h) 3-5 0.5-4<br>
Cold start-up time (h) 3-5<br>
Cold start-up time ( Mummum load tevel (%)<br>
Ramp rate (min.<sup>96</sup>) 0.6-2<br>
Hot start-up time (h) 3-5<br>
Hot start-up time (h) 3-5<br>
Hot start-up time (h) 3-5<br>
However, experimentally measuring ramping limits with respect to each output<br>
power point First start-up time (h)  $5-8$   $5-6$ However, experimentally measuring ramping limits with respect to each output<br>power point is very tough in practice. In this regard, we adopt the upper envelope to<br>approximate the ramping limits per minute so as to capture



output power points, where the minimum load level is reduced to 30% and the ramp<br>and the minimum load level is reduced to 30% and the ramp<br>of  $\frac{1}{2}$ . Fig. 6.2. Historical ramp rates for the retrofitted coal-fired unit. Fig. 6.2 shows 2, 500 samples of operational ramp rates. It is observed that the operational ramp rate is freed to 2% when the other than the red points. Although the red line of the sampled points, a linear function in t see Fig. 6.2 shows 2, 500 samples of operational ramp rates. It is observed that the operational ramp rate is freedom as the dynamic ramp rates with respect to different output power points, where the minimum load level i  $\frac{1}{20}$  0.5% and  $\frac{1}{20\%}$  when the output active power (%)<br>
Fig. 6.2 shows 2, 500 samples of operational ramp rates with respect to different<br>
Fig. 6.2 shows 2, 500 samples of operational ramp rates with respect to 30% and 60%, the ramping limit is an approximated linear function of the unit's<br>generating of output active power (%)<br>Fig. 6.2 shows 2, 500 samples of operational ramp rates with respect to different<br>output power points, Fig. 6.2. Historical ramp rates for the retrofitted coal-fired unit.<br>
Fig. 6.2 shows 2, 500 samples of operational ramp rates with respect to different<br>
output power points, where the minimum load level is reduced to 30% generating output. Therefore, the appropriate dynamic ramp rates  $v_{i,u}^t$  (%/min) at unit.<br>
9% and the ramp<br>
1 in the red line<br>
bserved that the<br>
s 60%. Between<br>
on of the unit's<br>  $v_{i,u}^t$  (%/min) at<br>
eneration power, Fig. 6.2 shows 2, 500 samples of operational ramp rates with respect to different<br>output power points, where the minimum load level is reduced to 30% and the ramp<br>rate is improved to 2%. For these sampled points, a linear Fig. 6.2 shows 2, 500 samples of operational ramp rates with respect<br>output power points, where the minimum load level is reduced to 30% as<br>rate is improved to 2%. For these sampled points, a linear function in<br>segment is segment is the upper envelope as the dynamic ramp rates. It is observed that the<br>operational ramp rate is fixed to 2% when the output power exceeds 60%. Between<br>30% and 60%, the ramping limit is an approximated linear fun operational ramp rate is fixed to 2% when the output power exceeds 60%. Between<br>30% and 60%, the ramping limit is an approximated linear function of the unit's<br>generating output. Therefore, the appropriate dynamic ramp ra 20% and 60%, the ramping limit is an approximated linear function of the unit's<br>generating output. Therefore, the appropriate dynamic ramp rates  $v'_{i,n}$  (%/min) at<br>time t can be reasonably treated as a linear function of imated linear function of the unit's<br>ynamic ramp rates  $v'_{i,u}$  (%/min) at<br>unction of output generation power,<br> $r'_{i,u} \le d$  (6.2)<br>ed ramp rate of retrofitted coal-fired<br>nated from historical ramping data,<br>refers to the per

$$
v_{i,u}^t = \begin{cases} a \cdot r_{i,u}^t - b, & r_{i,u}^t \le d \\ c, & r_{i,u}^t > d \end{cases}
$$
 (6.2)

minimum load level and  $d=(c+b)/a$ ; and  $r_{i,u}^t$ 

 $P_{\mathrm{G}i,\mathrm{u}}^{t}$  /  $P_{\mathrm{G}i,\mathrm{u},\mathrm{max}}$  .

 $P_{Gi,u}^{t} / P_{Gi,u,max}$ .<br>Three kinds of models can be used to describe dynamic ramp rates: approximated<br>linear model by Eq. (6.2), piecewise linear model by approximated thresholds [78],<br>and stepwise linear model by given thres  $P_{G,i,n}^{\epsilon}$  /  $P_{G,i,n,max}$ .<br>Three kinds of models can be used to describe dynamic ramp rates: approximated linear model by Eq. (6.2), piecewise linear model by approximated thresholds [78], and stepwise linear model by give  $P_{\text{G}_{i,d},\text{max}}^{\ell}$ .<br>Three kinds of models can be used to describe dynamic ramp rates: approximated<br>linear model by Eq. (6.2), piecewise linear model by approximated thresholds [78],<br>and stepwise linear model by given t F<sub>Gist</sub> /  $P_{Gis,max}$ .<br>Three kinds of models can be used to describe dynamic ramp rates: approximated<br>linear model by Eq. (6.2), piecewise linear model by approximated thresholds [78],<br>and stepwise linear model by given thre  $P_{\text{G}_{\text{off}}}/P_{\text{G}_{\text{off,off}}}$ .<br>Three kinds of models can be used to describe dynamic ramp rates: approximated<br>linear model by Eq. (6.2), piccewise linear model by approximated thresholds [78],<br>and stepwise linear model by  $P_{\text{Ga},\mu}^{c}/P_{\text{Ga},\text{mm}}$ .<br>Three kinds of models can be used to describe dynamic ramp rates: approximated<br>linear model by Eq. (6.2), piecewise linear model by approximated thresholds [78],<br>and stepwise linear model by g  $P_{\text{Gia},\text{u,max}}^c$ .<br>Three kinds of models can be used to describe dynamic ramp rates: approximated<br>linear model by Fq. (6.2), piccewise linear model by approximated thresholds [78],<br>and stepwise linear model by given thr  $P_{\text{G}ij,n}$ / $P_{\text{G}ij,n}$ .<br>Three kinds of models can be used to describe dynamic ramp rates: approximated<br>linear model by Eq. (6.2), piecewise linear model by approximated thresholds [78],<br>and stepwise linear model by giv Three kinds of models can be used to describe dynamic ramp rates: approximated<br>linear model by Eq. (6.2), piecewise linear model by approximated thresholds [78],<br>and stepwise linear model by given thresholds [78]. For the linear model by Eq. (6.2), piecewise linear model by approximated thresholds [78],<br>and stepwise linear model by given thresholds [78]. For their main differences, the<br>first kind of ramp-rate expressions is a continuous fu Fig. 6.3(a) reveals the ramp rate  $v_{i}^{t}$  in a solid blue line using Eq. (6.2) with is linear model by approximated thresholds [78],<br>thresholds [78]. For their main differences, the<br>a continuous function, whereas the rest two are a<br>local exercutive coal-fired units can output more<br>es, our proposed linear and stepwise linear model by given thresholds [78]. For their main differences, the<br>first kind of ramp-rate expressions is a continuous function, whereas the rest two are a<br>discrete function of thresholds. Since retrofitt first kind of ramp-rate expressions is a continuous function, whereas the rest two are a<br>discrete function of thresholds. Since retrofitted coal-fired units can output more<br>stable and accurate power in minutes, our propos discrete function of thresholds. Since retrofitted coal-fired units can output more<br>stable and accurate power in minutes, our proposed linear model of dynamic ramp<br>rates can capture available operational ramp rates. Howev stable and accurate power in minutes, our proposed linear model of dynamic ramp<br>rates can capture available operational ramp rates. However, the other two models of<br>dynamic ramp rates cannot provide fast-tracking ability rates can capture available operational ramp rates. However, the other two models of<br>dynamic ramp rates cannot provide fast-tracking ability because of their fixed<br>thresholds. For example, to distinguish between proposed periods.



Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute, and (b) percentage of maximum active power variations for fifteen minutes using the proposed and piecewise linear models of dynamic ramp Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute, and (b) percentage of maximum active power variations for fifteen minutes using the proposed and piecewise linear models of dynamic ramp Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute,<br>and (b) percentage of maximum active power variations for fifteen minutes using the<br>proposed and piecewise linear models of dynamic ramp

Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute,<br>
d (b) percentage of maximum active power variations for fifteen minutes using the<br>
posed and piecewise linear models of dynamic ramp ra Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute,<br>and (b) percentage of maximum active power variations for fifteen minutes using the<br>proposed and piecewise linear models of dynamic ramp Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute,<br>and (b) precentage of maximum active power variations for fifteen minutes using the<br>proposed and piecewise linear models of dynamic ramp ,  $y_{i,u}^{t-1}$ minic ramp rates per minute,<br>or fifteen minutes using the<br>es.<br>ns of a retrofitted coal-fired<br>poposed and piecewise linear<br> $\frac{1}{u}$  and y-axis indicates the<br>time t. In this figure,  $\Delta y'_{i,u}$ <br>egments, which correspond Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute,<br>and (b) percentage of maximum active power variations for fifteen minutes using the<br>proposed and piecewise linear models of dynamic ramp t of dynamic ramp rates per minute,<br>ations for fifteen minutes using the<br>amp rates.<br>variations of a retrofitted coal-fired<br>y the proposed and piecewise linear<br>s to  $y_{i,u}^{t-1}$  and y-axis indicates the<br> $y_{i,u}^{t}$  at time t. ,  $\Delta y^t_{i,u}$ Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute,<br>and (b) percentage of maximum active power variations for fifteen minutes using the<br>proposed and piecewise linear models of dynamic ramp Fig. 6.3. (a) Proposed and piecewise linear models of dynamic ramp rates per minute,<br>and (b) percentage of maximum active power variations for fifteen minutes using the<br>proposed and piecewise linear models of dynamic ramp proposed and piecewise linear models of dynamic ramp rates.<br>
Fig. 6.3(b) presents the maximum active power variations of a retrofitted coal-fired<br>
unit under a dispatchable period  $\Delta T=15$  minutes by the proposed and piec Fig. 6.3(b) presents the maximum active power variations of a retrofitted coal-fired<br>unit under a dispatchable period  $\Delta T = 15$  minutes by the proposed and piecewise linear<br>models of dynamic ramp rates, where x-axis refer when  $r_{i,u}^t \leq 60\%$ . In other words, I 5.3(b) presents the maximum active power variations of a retrofitted coal-fired<br>ler a dispatchable period  $\Delta T=15$  minutes by the proposed and piecewise linear<br>of dynamic ramp rates, where x-axis refers to  $y_{i,u}^{t-1}$  an unit under a dispatchable period  $\Delta T=15$  minutes by the proposed and piecewise linear<br>models of dynamic ramp rates, where x-axis refers to  $y_{i,s}^{t-1}$  and y-axis indicates the<br>percentage of maximum active power variatio models of dynamic ramp rates, where *x*-axis refers to  $y_{i,n}^{t-1}$  and *y*-axis indicates the<br>percentage of maximum active power variations  $y_{i,n}^{t}$  at time *t*. In this figure,  $Ay_{i,n}^{t}$ <br>can be sketched out by blue c percentage of maximum active power variations  $y'_{i,n}$  at time *t*. In this figure,  $\Delta y'_{i,n}$ <br>can be sketched out by blue curves and several green line segments, which correspond<br>to the proposed and piecewise linear mode can be sketched out by blue curves and several green line segments, which correspond<br>to the proposed and piecewise linear models shown in Fig. 6.3(a), respectively. The<br>maximum active power variations in blue line segment to the proposed and piecewise linear models shown in Fig. 6.3(a), respectively. The<br>maximum active power variations in blue line segments surround the area enclosed by<br>green line segments, showing that its area is smaller aximum active power variations in blue line segments surround the area enclosed by<br>
seen line segments, showing that its area is smaller than the blue one, especially<br>
ten  $r'_{i,\mu} \le 60\%$ . In other words, Eq. (6.2) is a is in blue line segments surround the area enclosed by<br>that its area is smaller than the blue one, especially<br>ords, Eq. (6.2) is a more adaptable solution to the<br>tion challenges, especially during peak shaving stages.<br>see green line segments, showing that its area is smaller than the blue one, especially<br>when  $r'_{i\omega} \le 60\%$ . In other words, Eq. (6.2) is a more adaptable solution to the<br>tremendous wind power fluctuation challenges, especi when  $r'_{i,n} \le 60\%$ . In other words, Eq. (6.2) is a more adaptable solution to the tremendous wind power fluctuation challenges, especially during peak shaving stages.<br>Moreover, the gaps between these two areas in Fig. 6 table solution to the<br>
g peak shaving stages.<br>
cly indicate that using<br>
utions. Thus, it can be<br>
output ramp-up/down<br>  $T = 15$  min accurately.<br>
d as  $(6.3a)$ - $(6.3d)$  for<br>
lower boundaries as<br>  $r_{i,u}^t$ )-space. Therefore, tremendous wind power fluctuation challenges, especially during peak shaving stages.<br>Moreover, the gaps between these two areas in Fig. 6.3(b) clearly indicate that using<br>the piecewise linear model can result in suboptima

, t ,  $r_{i,u}^{t-1}$  ,  $r_{i,u}^t$  )-space. Ther t

$$
r_{i,u}^t \le r_{i,u}^{t-1} + c \cdot (\Delta T - 1) \quad \text{if } r_{i,u}^{t-1} \ge d \tag{6.3a}
$$

$$
r_{i,u}^t \le (r_{i,u}^{t-1} - b/a)(a+1)^{\Delta T - 1} + b/a \quad \text{if } r_{i,u}^{t-1} \le \xi_{i,u}
$$
 (6.3b)

$$
r_{i,u}^t \le 1 \tag{6.3c}
$$

$$
r_{i,u}^t \le (r_{i,u}^{t-1} - b/a)(a+1)^{\eta-1} + b/a + c \cdot (\Delta T - \eta) \text{ if } \xi_{i,u} < r_{i,u}^{t-1} < d \quad (6.3d)
$$

$$
r_{i,u}^t \ge r_{i,u}^{t-1} - c \cdot (\Delta T - 1) \quad \text{if } r_{i,u}^{t-1} \ge \chi_{i,u} \tag{6.4a}
$$

$$
r_{i,u}^t \ge (r_{i,u}^{t-1} - b/a)(1-a)^{\Delta T - 1} + b/a \quad \text{if } r_{i,u}^{t-1} \le d \tag{6.4b}
$$

$$
r_{i,u}^t \ge 0.3\tag{6.4c}
$$

$$
r_{i,u}^{t} \ge (r_{i,u}^{t-1} - c \cdot (\omega_{i,u} - 1) - b/a)(1 - a)^{\Delta T - \omega_{i,u}} + b/a \text{ if } d < r_{i,u}^{t-1} < \mathcal{X}_{i,u} \tag{6.4d}
$$

where 
$$
\xi_{i,u} = \frac{d-b/a}{(a+1)^{\Delta T-1}} + \frac{b}{a}
$$
,  $\chi_{i,u} = d + c(\Delta T - 1)$ , and  $d = (c+b)/a$ ,

$$
r_{i,u}^{t} \le (r_{i,u}^{t-1} - b/a)(a+1)^{\Delta T-1} + b/a \quad \text{if } r_{i,u}^{t-1} \le \xi_{i,u} \tag{6.3b}
$$
\n
$$
r_{i,u}^{t} \le 1 \tag{6.3c}
$$
\n
$$
r_{i,u}^{t} \le (r_{i,u}^{t-1} - b/a)(a+1)^{\eta-1} + b/a + c \cdot (\Delta T - \eta) \quad \text{if } \xi_{i,u} < r_{i,u}^{t-1} < d \tag{6.3d}
$$
\n
$$
r_{i,u}^{t} \ge r_{i,u}^{t-1} - c \cdot (\Delta T - 1) \quad \text{if } r_{i,u}^{t-1} \ge \chi_{i,u} \tag{6.4a}
$$
\n
$$
r_{i,u}^{t} \ge (r_{i,u}^{t-1} - b/a)(1-a)^{\Delta T-1} + b/a \quad \text{if } r_{i,u}^{t-1} \le d \tag{6.4b}
$$
\n
$$
r_{i,u}^{t} \ge 0.3 \tag{6.4c}
$$
\n
$$
r_{i,u}^{t} \ge (r_{i,u}^{t-1} - c \cdot (\omega_{i,u} - 1) - b/a)(1-a)^{\Delta T - \omega_{i,u}} + b/a \quad \text{if } d < r_{i,u}^{t-1} < \chi_{i,u} \tag{6.4d}
$$
\n
$$
\text{where } \xi_{i,u} = \frac{d - b/a}{(a+1)^{\Delta T-1}} + \frac{b}{a} \quad \chi_{i,u} = d + c(\Delta T - 1) \quad \text{and} \quad d = (c+b)/a \quad \text{,}
$$
\n
$$
\eta_{i,u} = round \quad (\ln(c/(a \cdot r_{i,u}^{t-1} - b)) / \ln(a+1) + 1) \quad \text{and} \quad \omega_{i,u} = round((r_{i,u}^{t-1} - d)/c + 1), \text{ in}
$$
\n
$$
\text{which } round(\cdot) \text{ refers to a function of round towards negative infinity.}
$$
\n
$$
\text{An intuitive explanation for ramping margins is given under different dispatch}
$$

(6.3c)<br>  $r'_{i,n} \le (r_{i,n}^{i-1} - b/a)(a+1)^{n+1} + b/a + c \cdot (\Delta T - \eta)$  if  $r'_{i,n} < r'_{i,n}^{i-1} < d$  (6.3d)<br>  $r'_{i,n} \ge r'_{i,n}^{i-1} - c \cdot (\Delta T - 1)$  if  $r'_{i,n}^{i-1} \ge \chi_{i,n}$  (6.4a)<br>  $r'_{i,n} \ge (r'_{i,n}^{i-1} - b/a)(1-a)^{n+1} + b/a$  if  $r'_{i,n}^{i-1} \le d$  (6.4b)  $F_{i,s} \le (r_{i,s} - b/a)(a+1)^{y} + b/a + c \cdot (\Delta I - \eta) \int f_{s,u} < d$  (6.3d)<br>  $r'_{i,s} \ge r_{i,s}^{d-1} - c \cdot (\Delta T - 1) \quad \text{if } r_{i,s}^{d-1} \ge \chi_{i,s}$  (6.4a)<br>  $r'_{i,s} \ge (r_{i,s}^{d-1} - b/a)(1-a)^{3^{T-1}} + b/a \quad \text{if } r_{i,s}^{d-1} \le d$  (6.4b)<br>  $r'_{i,s} \ge (r_{i,s}^{d-1} - c \cdot (\omega_{i,s$  $r'_{i,n} \ge r'^{-1}_{i,n} - c \cdot (\Delta T - 1)$  if  $r'^{-1}_{i,n} \ge \chi_{i,n}$  (6.4a)<br>  $r'_{i,n} \ge (r'^{-1}_{i,n} - b/a)(1-a)^{N-1} + b/a$  if  $r'^{-1}_{i,n} \le d$  (6.4b)<br>  $r'_{i,n} \ge 0.3$  (6.4c)<br>  $r'_{i,n} \ge (r'^{-1}_{i,n} - c \cdot (\omega_{i,n} - 1) - b/a)(1-a)^{M-\omega_{i,n}} + b/a$  if  $d < r'^{-1}_{i,n} < \chi_{i,n}$  (6  $r_{i,s}^{\ell} \ge (r_{i,s}^{\ell-1} - b/a)(1-a)^{\Delta T-1} + b/a$  if  $r_{i,s}^{\ell-1} \le d$  (6.4b)<br>  $r_{i,s}^{\ell} \ge (r_{i,s}^{\ell-1} - c \cdot (a_{i,s} - 1) - b/a)(1-a)^{\Delta T-\alpha_{i,s}} + b/a$  if  $d < r_{i,s}^{\ell-1} < \chi_{i,s}$  (6.4d)<br>
where  $\xi_{i,s} = \frac{d-b/a}{(a+1)^{\Delta T-1}} + \frac{b}{a}$ ,  $\chi_{i,s} = d + c(\Delta T$  $r'_{i,n} \ge 0.3$  (6.4c)<br>  $r'_{i,n} \ge (r'^{-1}_{i,n} - c \cdot (\omega_{i,n} - 1) - b/a)(1-a)^{\sqrt{1-\omega_{i,n}}} + b/a$  if  $d < r'^{-1}_{i,n} < \chi_{i,n}$  (6.4d)<br>
where  $\xi_{i,n} = \frac{d - b/a}{(a+1)^{N-1}} + \frac{b}{a}$ ,  $\chi_{i,n} = d + c(\Delta T - 1)$ , and  $d = (c + b)/a$ ,<br>  $\eta_{i,n} = round(\ln(c/(a \cdot r'^{-1}_{i,n} - b))/\ln(a$  $r'_{\alpha} = (r'_{\alpha})^4 - c \cdot (\omega_{\alpha} - 1) - b / a (1 - a)^{3T - \omega_{\alpha}} + b / a \text{ if } d < r'^{-1}_{\alpha} < \chi_{\alpha}$  (6.4d)<br>
where  $\zeta_{\alpha} = \frac{d - b/a}{(a + 1)^{N-1}} + \frac{b}{a}$ ,  $\chi_{\alpha} = d + c(\Delta T - 1)$ , and  $d = (c + b) / a$ ,<br>  $\eta_{\alpha} = round \left( \ln(c/(a \cdot r'^{-1}_{\alpha} - b)) / \ln(a + 1) + 1 \right)$ , and  $r_{i,u}^{t-1} = 60\%$  $-a)^{\Delta T - \omega_{i,x}} + b/a$  if  $d < r_{i,u}^{t-1} < \chi_{i,u}$  (6.4d)<br>  $-c(\Delta T - 1)$ , and  $d = (c+b)/a$ ,<br>
(b), and  $\omega_{i,x} = round((r_{i,x}^{t-1} - d)/c+1)$ , in<br>
cowards negative infinity.<br>
argins is given under different dispatch<br>
inutes, 15 minutes, and 5 min where  $\xi_{i,x} = \frac{d-b/a}{(a+1)^{\Delta i-1}} + \frac{b}{a}$ ,  $\chi_{i,x} = d + c(\Delta T - 1)$ , and  $d = (c+b)/a$ ,<br>  $\eta_{i,x} = round(\ln(c/(a \cdot r_{i,x}^{(d)} - b))/\ln(a+1)+1)$ , and  $\omega_{i,x} = round((r_{i,x}^{(d)} - d)/c+1)$ , in<br>
which round(·) refers to a function of round towards negative inf  $\eta_{i,a}$  = round (ln( $c/(a \cdot r_{i,a}^{e^{-1}} - b)$ )/ln( $a+1$ )+1), and  $\omega_{i,a}$  = round(( $r_{i,a}^{e^{-1}} - d/c+1$ ), in<br>which round( $\cdot$ ) refers to a function of round towards negative infinity.<br>An intuitive explanation for ramping margins ,  $r_{i,u}^{t-1}$ orthomorphical in the dispatch<br>
induction in the shows that a<br>
rter dispatch<br>
lies in [60%,<br>
in  $\Delta T=60$  min,<br>
in  $\Delta T=60$  min,<br>
in  $\Delta T=6$  min,<br>
in also<br>
ivel changes<br>
margins, are which *round*( $\cdot$ ) refers to a function of round towards negative infinity.<br>
An intuitive explanation for ramping margins is given under different dispateh<br>
periods  $\Delta T$  scaled in 60 minutes, 30 minutes, 15 minutes, and MECH FORMIC J FICENS to a function of Found towards negative infinity.<br>
An intuitive explanation for ramping margins is given under different dispatch<br>
periods  $\Delta T$  scaled in 60 minutes, 30 minutes, 15 minutes, and 5 min An matrive expranation for tamping magnits is given under unfecting to applied periods  $\Delta T$  scaled in 60 minutes, 30 minutes, 15 minutes, and 5 minutes in 2-dimensional Fig. 6.4 with the above-mentioned parameters. This Since some retrofitted coal-fired plants have more than 2% ramp rate, we depict<br>fired on a different some retrofit of  $\Delta T$  has larger ramp-up/down margins than a shorter dispatch<br>riod. For instance, when  $\Delta T$ =15min and **Example 12** 6.4 which diated parameters. This ngute shows that a<br>bonger dispatch period  $\Delta T$  has larger ramp-up/down margins than a shorter dispatch<br>period. For instance, when  $\Delta T$ =15min and  $r_{i,n}^{t-1}$  = 60%, the ra begave the period of  $\Delta T$  and  $\Delta T = 15$  min and  $r_{i,s}^{t-1} = 60\%$ , the ramp-up band lies in [60%,<br>88%], while the ramp-down range varies from 60% to 37%; whereas when  $\Delta T$ =60min,<br>88%], while the ramp-down range varies

suggests that larger parameter c results in larger ramp-up/down margins under  $\Delta T$ <br>=15min for the better quick-response capacity of coal-fired plants.<br>With the loss of generality, we depict (6.3a)-(6.4d) for boundaries a

suggests that larger parameter c results in larger ramp-up/down margins under  $\Delta T$ <br>=15min for the better quick-response capacity of coal-fired plants.<br>With the loss of generality, we depict (6.3a)-(6.4d) for boundaries a ggests that larger parameter c results in larger ramp-up/down margins under  $\Delta T$ <br>
5min for the better quick-response capacity of coal-fired plants.<br>
With the loss of generality, we depict (6.3a)-(6.4d) for boundaries as suggests that larger parameter c results in larger ramp-up/down margins under  $\Delta T$ <br>=15min for the better quick-response capacity of coal-fired plants.<br>With the loss of generality, we depict (6.3a)-(6.4d) for boundaries a suggests that larger parameter c results in larger ramp-up/down margins under  $\Delta T$ <br>=15min for the better quick-response capacity of coal-fired plants.<br>With the loss of generality, we depict (6.3a)-(6.4d) for boundaries a suggests that larger parameter c results in larger ramp-up/down margins under  $\Delta T$ <br>-1 Smin for the better quick-response capacity of coal-fired plants.<br>With the loss of generality, we depict (6.3a)-(6.4d) for boundaries and (6.4d).





Eq. 40%<br>  $20\%$ <br>  $20\$  $f(x) = (x - b/a)(a+1)^{n-1} + b/a + c \cdot (\Delta T - \eta)$  where  $x = r_{i,u}^{t-1}$ , we can deduce  $x = r_{i,u}^{t-1}$  $\frac{1}{\pi}$ <br>  $\frac{1$ 20% 40% 60% 80% 100%<br>
Percentage of active power at time *t*-1 (%)<br>
Fig. 6.5 Ramping boundaries under  $\Delta T=15$  minutes with different sets of para<br>
and *b*.<br>
We can prove these two nonlinear constraints as a convex functi nction, respectively.<br>  $\omega \sigma f$ : For (6.3d) and (6.4d), it is assumed that the  $\overline{BC}$  and  $\overline{HG}$  functions are<br>
ntinuous and  $\eta$  and  $\omega$  are continuous variables. In terms of (6.3d), let<br>  $(x) = (x - b/a)(a + 1)^{\eta-1} + b/a + c \cdot (\$ ntinuous and  $\eta$  and  $\omega$  are continuous variables. In terms of (6.3d), let<br>  $(x) = (x - b/a)(a + 1)^{\gamma - 1} + b/a + c \cdot (\Delta T - \eta)$  where  $x = r_{i,a}^{i-1}$ , we can deduce the<br>
llowing equations:<br>  $\int f' = (a + 1)^{\gamma - 1} + (x - b/a)[(a + 1)^{\gamma - 1}] - c \cdot \eta'$ 

$$
\int f' = (a+1)^{\eta-1} + (x-b/a)[(a+1)^{\eta-1}] - c \cdot \eta'
$$
  
\n
$$
[(a+1)^{\eta-1}]' = (a+1)^{\eta-1} \ln(a+1) \cdot \eta'
$$
  
\n
$$
[(a+1)^{\eta-1}]'' = (a+1)^{\eta-1} \ln(a+1)(\ln(a+1) \cdot (\eta')^2 + \eta'')
$$
  
\n
$$
\eta' = -a/[(ax-b)\ln(a+1)], \quad \eta'' = a^2/[(ax-b)^2 \ln(a+1)]
$$
\n(6.5)

$$
f'' = 2[(a+1)^{n-1}]^{1} + (x-b/a)[(a+1)^{n-1}]^{n} - c \cdot \eta''
$$
\n(6.6)

$$
[(a+1)^{\eta-1}]<0, \ \ [(a+1)^{\eta-1}]^n>0
$$
\n(6.7)

From Eq.(6.6), we calculate 
$$
\Delta h = [(a+1)^{n-1}]^+ \varepsilon \cdot [(a+1)^{n-1}]^n
$$
, where  $\varepsilon = x - b/a > 0$ .  
\n
$$
\Delta h = (a+1)^{n-1} \ln(a+1) \{ \eta' - \varepsilon \ln(a+1) (\eta')^2 - \varepsilon \eta'' \} \qquad (6.8)
$$
\nWe substitute  $\eta'' = c \ln(a+1)(\eta')^2$  from Eq. (6.5) into Eq. (6.8), and obtain

From Eq.(6.6), we calculate 
$$
\Delta h = [(a+1)^{\eta-1}]^+ \varepsilon \cdot [(a+1)^{\eta-1}]^n
$$
, where  $\varepsilon = x - b/a > 0$ .  
\n
$$
\Delta h = (a+1)^{\eta-1} \ln(a+1) \{ \eta' - \varepsilon \ln(a+1) (\eta')^2 - \varepsilon \eta'' \} \qquad (6.8)
$$
\nWe substitute  $\eta'' = c \ln(a+1)(\eta')^2$  from Eq. (6.5) into Eq. (6.8), and obtain  
\n
$$
\Delta h = \eta'(a+1)^{\eta-1} \ln(a+1) \{ 1 - \varepsilon \ln(a+1) \eta'(1-c) \} \qquad (6.9)
$$
\nIt is clear that  $\Delta h < 0$  due to  $\eta' < 0$ ,  $\{ 1 - \varepsilon \ln(a+1) \eta'(1-c) \} > 0$ . Consider  $\Delta h < 0$  and

IF IF IF  $\mathbb{E}(\mathcal{A}(\epsilon_0)$ , we calculate  $\Lambda h = [(a+1)^{n-1}]^2 + \varepsilon \cdot [(a+1)^{n-1}]^n$ , where  $\varepsilon = x \cdot b/a > 0$ .<br>  $\Delta h = (a+1)^{n-1} \ln(a+1) \{ \eta' - \varepsilon \ln(a+1)(\eta' \}^2 - \varepsilon \eta'' \}$  (6.8)<br>
We substitute  $\eta'' = c \ln(a+1)(\eta')^2$  from Eq. (6.5) int From Eq.(6.6), we calculate Δh =  $[(a+1)^{n-1}] + \varepsilon \cdot [(a+1)^{n-1}]^n$ , where  $\varepsilon = x - b/a > 0$ .<br>  $\Delta h = (a+1)^{n-1} \ln(a+1) \{ \eta \} - \varepsilon \ln(a+1) (\eta \}^2 - \varepsilon \eta^n \}$  (6.8)<br>
We substitute  $\eta^n = c \ln(a+1)(\eta \}^2$  from Eq. (6.5) into Eq. (6.8), and From Eq.(6.6), we calculate  $\Delta h = [(a+1)^{\gamma-1}]^+ + \varepsilon \cdot [(a+1)^{\gamma-1}]^n$ , where  $\varepsilon = x \cdot b/a > 0$ .<br>  $\Delta h = (a+1)^{\gamma-1} \ln(a+1) \{ \eta \}^{\prime} - \varepsilon \ln(a+1) (\eta \}^2 - \varepsilon \eta^{\prime \prime} ) \}$  (6.8)<br>
We substitute  $\eta'' = c \ln(a+1) (\eta')^2$  from Eq. (6.5) into Eq. ( =  $x - b/a > 0$ .<br>
(6.8)<br>
btain<br>
(6.9)<br>
sider  $\Delta h < 0$  and<br>
that  $f(x)$  is a<br>
=  $y_{i,u}^{t-1}$ , and we From Eq.(6.6), we calculate  $\Delta h = [(a+1)^{\gamma-1}] + \varepsilon \cdot [(a+1)^{\gamma-1}]^n$ , where  $\varepsilon = x - b/a > 0$ .<br>  $\Delta h = (a+1)^{\gamma-1} \ln(a+1) \{ \eta \} - \varepsilon \ln(a+1) (\eta \}^2 - \varepsilon \eta \eta \}$  (6.8)<br>
We substitute  $\eta'' = \varepsilon \ln(a+1)(\eta')^2$  from Eq. (6.5) into Eq. (6.8), and It is clear that  $\Delta h < 0$  due to  $\eta' < 0$ ,  $\{1 - \varepsilon \ln(a + 1)\eta'(1 - \varepsilon)\} > 0$ . Consider  $\Delta h < 0$  and<br>  $\pi+1)^{n+1} < 0$ , thus causing  $f'' = \Delta h + [(a+1)^{n+1}]' - \varepsilon \eta'' < 0$ . This proves that  $f(x)$  is a<br>
nvex function for  $\forall x \in (b/a,d)$ .

For (6.4d), let 
$$
g(x) = (x - c \cdot (\omega - 1) - b/a)(1 - a)^{\Delta T - \omega} + b/a
$$
 where  $x = y_{i,u}^{t-1}$ , and we

$$
\begin{cases}\ng' = (1-a)^{\Delta T - \omega} (1-c \cdot \omega') + (x-c \cdot (\omega - 1) - b/a) [(1-a)^{\Delta T - \omega}]' \\
[(1-a)^{\Delta T - \omega}]' = -(1-a)^{\Delta T - \omega} \ln(1-a) \cdot \omega' \\
[(1-a)^{\Delta T - \omega}]'' = -(1-a)^{\Delta T - \omega} \ln(1-a) \cdot (\omega'' - \ln(1-a) \cdot (\omega')^2) \\
\omega' = 1/c, \quad \omega'' = 0\n\end{cases} (6.10)
$$

$$
g'' = -\omega'(1-a)^{\Delta T - \omega} \ln(1-a)(1-c \cdot \omega') + (x-c \cdot (\omega-1)-b/a) \left[(1-a)^{\Delta T - \omega}\right]^{n}
$$
  
+ 
$$
[(1-a)^{\Delta T - \omega}]^{n}
$$
 (6.11)

For  $(6.4d)$ , let  $g(x) = (x - c \cdot (\omega - 1) - b/a)(1 - a)^{3/2-\omega} + b/a$  where  $x = y_{i,a}^{i-1}$ , and we<br>
milarly yield the following equations:<br>  $g' = (1 - a)^{3/2-\omega}[(1 - c \cdot \omega') + (x - c \cdot (\omega - 1) - b/a)[(1 - a)^{3/2-\omega}]$ <br>  $[(1 - a)^{3/2-\omega}]^* = -(1 - a)^{3/2-\omega} \ln(1 - a) \cdot (\omega$ similarly yield the following equations:<br>  $\begin{cases} g' = (1-a)^{\Delta T-\omega}(1-c \cdot \omega^r) + (x-c \cdot (\omega-1) - b/a)[(1-a)^{\Delta T-\omega}] \\ [(1-a)^{\Delta T-\omega}]^n = -(1-a)^{\Delta T-\omega} \ln(1-a) \cdot (\omega^n - \ln(1-a) \cdot (\omega^n)^2) \\ [(1-a)^{\Delta T-\omega}]^n = -(1-a)^{\Delta T-\omega} \ln(1-a) \cdot (\omega^n - \ln(1-a) \cdot (\omega^n)^2) \end{cases}$ (6.10)<br>
Based on the Example  $\begin{cases} g' = (1-a)^{x\gamma - \omega}(1-c \cdot \omega') + (x-c \cdot (\omega - 1) - b/a)[(1-a)^{x-\omega}] \\ [(1-a)^{x\gamma - \omega}]^* = -(1-a)^{x\gamma - \omega} \ln(1-a) \cdot \omega' \\ [(1-a)^{x\gamma - \omega}]^* = -(1-a)^{x\gamma - \omega} \ln(1-a) \cdot (\omega^* - \ln(1-a) \cdot (\omega^*)^*) \\ (\omega^* - 1/c, \omega^* = 0 \end{cases}$  (6.10)<br>
Based on the above, we further ach



Fig. 6.6. General ramping boundaries of retrofitted coal-fired units.<br>
Fig. 6.6. General ramping boundaries of retrofitted coal-fired units.<br>
Moreover, the spinning reserve requirement of WTBPS should be considered<br>
befor

$$
\sum_{i=1}^{N_G} (1 - r_{i,u}^t) P_{Gi,u,\text{max}} \ge P_u^{\text{spin}} \tag{6.12}
$$

**Example 10 Example 10** Moreover, the spinning reserve requirement of WTBPS should be considered<br>before performing an economic dispatch. The spinning reserve capacity provided by<br>retrofitted coal-fired units should not exceed the system spinning Moreover, the spinning reserve requirement of WTBPS should be cons<br>before performing an economic dispatch. The spinning reserve capacity provident<br>retrofitted coal-fired units should not exceed the system spinning reserve before performing an economic dispatch. The spinning reserve capacity provided by<br>retrofitted coal-fired units should not exceed the system spinning reserve capacity:<br> $\sum_{i=1}^{N_0} (1 - r'_{i\mu}) P_{0i\mu\mu\mu\alpha} \ge P_{\sigma}^{\text{spin}}$  (6. profitted coal-fired units should not exceed the system spinning reserve capacity:<br>  $\sum_{i=1}^{N_0} (1 - r_{i,a}^i) P_{Gi,a,max} \ge P_a^{\text{spin}}$  (6.12)<br>
here  $P_a^{\text{spin}}$  refers to system spinning reserve capacity for the *u*-th agent of WTBP  $\sum_{i=1}^{N_0} (1 - r'_{i,n}) P_{G,i,n;\text{max}} \ge P_n^{\text{spin}}$  (6.12)<br>where  $P_n^{\text{spin}}$  refers to system spinning reserve capacity for the *u*-th agent of WTBPS<br>and  $u=1,2,...,N_n$ . Considering wind turbines have no contribution to the spinning<br>  $\sum_{c=1}^{n} (1-r_{i\pi})V_{G,i\pi,\text{max}} \ge P_{i\pi}^{even}$  (6.12)<br>
where  $P_{i}^{sym}$  refers to system spinning reserve capacity for the *u*-th agent of WTBPS<br>
and  $u=1,2,...,N_u$ . Considering wind turbines have no contribution to the spinning<br>

(SSC) are typical grid-scale structures for 110kV HVDNs as shown in Fig. 6.7, where  $P_c^t$  and  $P_d^t$  are active loads at 110kV substations C and D, respectively;  $P_{A1}^t$  and  $P_{A2}^t$  refer to the total active dis-patc (SSC) are typical grid-scale structures for 110kV HVDNs as shown in Fig. 6.7, where  $P_e^t$  and  $P_d^t$  are active loads at 110kV substations C and D, respectively;  $P_{A1}^t$  and  $P_{A2}^t$  refer to the total active dis-patc  $P_c^t$  and  $P_d^t$  are active loads at 110kV substations C and D, respectively;  $P_{A1}^t$  and  $P_{\scriptscriptstyle{A2}}^t$ (SSC) are typical grid-scale structures for 110kV HVDNs as shown in Fig. 6.7, where  $P_c^t$  and  $P_a^t$  are active loads at 110kV substations C and D, respectively;  $P_{A1}^t$  and  $P_{A2}^t$  refer to the total active dis-patc respectively. HVDNs as shown in Fig. 6.7, where<br>ons C and D, respectively;  $P'_{\lambda 1}$  and<br>dds in 220kV stations A1 and A2,<br>ubstation C<br> $P_c, Q_c$ <br>tation D



Station Allen Substation C<br>
Station Allen Substation C<br>
Substation C<br>
Substation C<br>
Substation D<br>
Station Allen S<sub>i</sub>,  $\overline{S}_j$ <br>  $\overline{S}_k$ <br>
Substation D<br>
Station Allen S<sub>i</sub>,  $\overline{S}_j$ <br>  $P_c$ ,  $Q_c$ <br>  $P_d$ ,  $Q_d$ <br>
Fig. 6.7. Tw Saction A2<br>
Simulation C Substation D<br>
Studien A1  $S_i$   $\blacktriangleright S_j$   $\blacktriangleright S_k$  Station A2<br>  $P_c, Q_c$   $P_d, Q_d$ <br>
Fig. 6.7. Two typical HVDNs of DSC and SSC.<br>
Thanks to simple topological units of HVDNs just with one or two buses, SSC Substation C Substation D<br>
Station A1  $S_i$   $\overline{S_j}$   $\overline{S_j}$   $\overline{S_k}$  Station A2<br>  $P_c, Q_c$   $P_d, Q_d$ <br>
Fig. 6.7. Two typical HVDNs of DSC and SSC.<br>
Thanks to simple topological units of HVDNs just with one or two buses Station A1  $S_i$   $\overrightarrow{S_j}$   $\overrightarrow{S_j}$   $\overrightarrow{S_k}$  Station A2<br>  $P_c, Q_c$   $P_d, Q_d$ <br>
Fig. 6.7. Two typical HVDNs of DSC and SSC.<br>
Thanks to simple topological units of HVDNs just with one or two buses, the<br>
well-known simplified *D* Fig. 6.7. Two typical HVDNs of DSC and SSC.<br>
Fig. 6.7. Two typical HVDNs of DSC and SSC.<br>
Thanks to simple topological units of HVDNs just with one or two buses, the<br>
well-known simplified *DistFlow* equations with neglig Fig. 6.7. Two typical HVDNs of DSC and SSC.<br>
Thanks to simple topological units of HVDNs just with one or two buses, the<br>
well-known simplified *DistFlow* equations with negligible power loss can achieve<br>
acceptable accur Thanks to simple topological units of HVDNs just with one or two buses, the well-known simplified *DistFlow* equations with negligible power loss can achieve acceptable accuracy. As reference [90] reported, nonlinear term Thanks to simple topological units of HVDNs just with one or two buses, the well-known simplified  $DistFlow$  equations with negligible power loss can achieve acceptable accuracy. As reference [90] reported, nonlinear terms are

As indicated in Fig. 6.8, the general form of (6.13a) and (6.13b) can be<br>formulated as  $P'_{s,j} = L'_j S' + b'_j$ ,  $\forall j \in N_T, \forall t \in T_s$ , where  $P'_{s,j}$  denotes total active<br>wer on transformers in the *j*-th station and  $L'_i$  and  $b'_$ As indicated in Fig. 6.8, the general form of (6.13a) and (6.13b) can<br>reformulated as  $P'_{s,j} = L'_j S' + b'_j$ ,  $\forall j \in N_T, \forall t \in T_s$ , where  $P'_{s,j}$  denotes total as<br>power on transformers in the *j*-th station and  $L'_j$  and  $b'_j$  reformulated as  $P_{s,j}^t = L_j S' + b_j'$ ,  $\forall j \in N_{\text{T}}$ ,  $\forall t \in T_s$ , where  $P_{s,j}^t$  denotes total active ightharpoontal in Fig. 6.8, the general form of  $(6.13a)$  and  $(6.13b)$  can be  $P_{S,j}^t = L_j' S' + b_j'$ ,  $\forall j \in N_{\tau}, \forall t \in T_s$ , where  $P_{S,j}^t$  denotes total active<br>mers in the *j*-th station and  $L_j'$  and  $b_j'$  refer to vectors  $\mathbf{P}_{\mathbf{S},j}^{t}$  denotes total active<br>refer to vectors of load<br>tration Specifically when As indicated in Fig. 6.8, the general form of (6.13a) and (6.13b) can be<br>reformulated as  $P'_{s,j} = L'_j S' + b'_j$ ,  $\forall j \in N_{\tau}, \forall t \in T_s$ , where  $P'_{s,j}$  denotes total active<br>power on transformers in the *j*-th station and  $L'_j$  a As indicated in Fig. 6.8, the general form of (6.13a) and (6.13b) can be<br>reformulated as  $P'_{s,j} = L'_{j}S' + b'_{j}$ ,  $\forall j \in N_{\tau}$ ,  $\forall t \in T_{s}$ , where  $P'_{s,j}$  denotes total active<br>power on transformers in the *j*-th station a As indicated in Fig. 6.8, the general form of (6.13a) and (6.13b) can be<br>reformulated as  $P_{s,j}^{\ell} = L_j S^{\ell} + b_j^{\ell}$ ,  $\forall j \in N_T, \forall t \in T_s$ , where  $P_{s,j}^{\ell}$  denotes total active<br>power on transformers in the *j*-th station an  $P_{D,u}^t = P_{S,j}^t$  holds at time t; for other stations,  $P_{S,j}^t$  should satisfy (6.13a). In addition, As indicated in Fig. 6.8, the general form of (6.13a) and (6.13b) can be<br>reformulated as  $P'_{s,j} = L'_{j}S' + b'_{j'}$ ,  $\forall j \in N_{\tau}, \forall t \in T_{s}$ , where  $P'_{s,j}$  denotes total active<br>power on transformers in the *j*-th station and form of (6.13a) and (6.13b) can be<br>  $T_s$ , where  $P_{s,j}^t$  denotes total active<br>  $L_j^t$  and  $b_j^t$  refer to vectors of load<br>
ates the *j*-th station. Specifically, when<br>
BPS belongs to 220kV station *j*, then<br>  $P_{s,j}^t$  sh As indicated in Fig. 6.8, the general form of (6.13a) and (6.13b) can be<br>reformulated as  $P'_{s,j} = L'_j S' + b'_j$ ,  $\forall j \in N_{\tau}, \forall t \in T_s$ , where  $P'_{s,j}$  denotes total active<br>power on transformers in the *j*-th station and  $L'_j$  a As indicated in Fig. 6.8, the general form of (6.13a) and (6.13b) can be<br>reformulated as  $P'_{s,j} = L'_j S' + b'_j$ ,  $\forall j \in N_T$ ,  $\forall t \in T_s$ , where  $P'_{s,j}$  denotes total active<br>power on transformers in the *j*-th station and  $L'_j$ 

$$
\begin{bmatrix} P_{\mathrm{A1}}^t \\ P_{\mathrm{A2}}^t \end{bmatrix} = \boldsymbol{L}^t \begin{bmatrix} S_1^t \\ S_2^t \end{bmatrix} + \boldsymbol{b}^t = \begin{bmatrix} P_{\mathrm{c}}^t & 0 \\ 0 & P_{\mathrm{c}}^t \end{bmatrix} \begin{bmatrix} S_1^t \\ S_2^t \end{bmatrix} + \boldsymbol{0}
$$
 (6.13a)

$$
\begin{bmatrix} P'_{A1} \\ P'_{A2} \end{bmatrix} = \mathbf{L}^{t} \begin{bmatrix} S_{3}^{t} \\ S_{4}^{t} \\ S_{5}^{t} \end{bmatrix} + \mathbf{b}^{t} = \begin{bmatrix} P_{c}^{t} + P_{d}^{t} & P_{d}^{t} & 0 \\ 0 & P_{c}^{t} & P_{c}^{t} + P_{d}^{t} \end{bmatrix} \begin{bmatrix} S_{3}^{t} \\ S_{4}^{t} \\ S_{5}^{t} \end{bmatrix} + \begin{bmatrix} -P_{d}^{t} \\ -P_{c}^{t} \end{bmatrix}
$$
(6.13b)

$$
0 \le P_{S,j}^t \le (n_{s,j} \cdot S_{N,j}), \forall j \in N_{\text{T}} \tag{6.14a}
$$

$$
0 \le P_{\mathbf{S},j}^t \le S_{\mathbf{L},j}, \forall j \in N_T
$$
\n
$$
(6.14b)
$$

F<sub>na</sub> -  $s_{x,j}$  node a clinic, for other stations,  $r_{s,j}$  should station formerly (0.1.04). In addition,<br>
branch capacity constraints (6.14b) for cables and overhead lines in HVDNs are also<br>
considered in LTS model.<br>  $\begin$ singlered in LTS model.<br>  $\begin{bmatrix} P'_{N1} \\ P'_{N2} \end{bmatrix} = E \begin{bmatrix} S'_1 \\ S'_2 \end{bmatrix} + b' = \begin{bmatrix} P'_c & 0 \\ 0 & P'_c \end{bmatrix} \begin{bmatrix} S'_1 \\ S'_2 \end{bmatrix} + \theta$  (6.13a)<br>  $\begin{bmatrix} P'_{N2} \\ P'_{N2} \end{bmatrix} = E \begin{bmatrix} S'_1 \\ S'_2 \\ S'_3 \end{bmatrix} + b' = \begin{bmatrix} P'_c + P'_d & P'_d & 0 \\ 0 & P'_c & P'_c +$  $\begin{bmatrix} P'_{s1} \\ P'_{s2} \end{bmatrix} = U \begin{bmatrix} S'_1 \\ S'_2 \end{bmatrix} + b' = \begin{bmatrix} P'_c & 0 \\ 0 & P'_c \end{bmatrix} \begin{bmatrix} S'_1 \\ S'_2 \end{bmatrix} + \theta$  (6.13a)<br>  $\begin{bmatrix} P'_{s1} \\ P'_{s2} \end{bmatrix} = U \begin{bmatrix} S'_1 \\ S'_1 \\ S'_2 \end{bmatrix} + b' = \begin{bmatrix} P'_c + P'_d & P'_d & 0 \\ 0 & P'_c & P'_c + P'_d \end{bmatrix} \begin{bmatrix} S'_1 \\ S'_2 \\ S'_$  $\begin{bmatrix} r_{N_1} \\ p'_{N_2} \end{bmatrix} = U \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + b' = \begin{bmatrix} r_c & 0 \\ 0 & P'_c \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + b$  (6.13a)<br>  $\begin{bmatrix} p'_{N_1} \\ p'_{N_2} \end{bmatrix} = U \begin{bmatrix} S'_1 \\ S'_2 \\ S'_3 \end{bmatrix} + b' = \begin{bmatrix} P'_c + P'_d & P_d & 0 \\ 0 & P'_c & P'_c + P'_d \end{bmatrix} \begin{bmatrix} s'_1 \\ s'_2 \\ s'_$ 

 $\begin{bmatrix} P'_{\alpha} \\ P'_{\beta} \end{bmatrix} = E \begin{bmatrix} S'_{3} \\ S'_{4} \\ S'_{5} \end{bmatrix} + b' = \begin{bmatrix} P''_{c} + P''_{d} & P'_{d} & 0 \\ 0 & P''_{c} & P''_{c} + P''_{d} \end{bmatrix} \begin{bmatrix} S'_{3} \\ S'_{4} \end{bmatrix} + \begin{bmatrix} -P'_{d} \\ -P''_{c} \end{bmatrix}$  (6.13b)<br>  $0 \le P'_{5,j} \le (n_{s,j} \cdot S_{5,j})$ ,  $\forall j \in N_{\tau}$  (6.14a  $v_m^2 - v_k^2 = 2(R_{mk}P_{mk}^t + X_{mk}Q_{mk}^t)$  in simplified *DistFlow* equations, we rewrite this  $\left[\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\right] - E \left[\begin{matrix} S_1^t \\ S_2^t \end{matrix}\right] + b' - \left[\begin{matrix} x_1 + x_3 & 0 \\ 0 & P_2^t & P_2^t + P_3^t \end{matrix}\right] \left[\begin{matrix} S_2^t \\ S_3^t \end{matrix}\right] + \left[\begin{matrix} -t_2 \\ -P_2^t \end{matrix}\right]$  (6.13b)<br>  $0 \le P_{S,j}^t \le (n_{s,j} \cdot S_{N,j})$ ,  $\forall j \in N_T$  (6.14a)<br>  $0 \le P_{S,j}$ equation:

$$
R_{mk}P_{mk}^{t} + X_{mk}Q_{mk}^{t} = (v_{m}^{2} - v_{n}^{2})/2 = \frac{(v_{m} - v_{n})^{2}}{2} + v_{n}(v_{m} - v_{n})
$$
  
= 
$$
\frac{(\Delta v_{mk}^{t})^{2}}{2} + v_{n}\Delta v_{mk}^{t} \le \frac{(\Delta \bar{v})^{2}}{2} + \bar{v} \cdot \Delta \bar{v}
$$
 (6.15)

where  $R_{mk}$  and  $X_{mk}$  refer to resistance, reactance of branch  $(m,k)$ , and  $P_{mk}^{i}$  and  $Q_{mk}^{i}$  denote active and reactive power flow on branch  $(m,k)$ ,  $\overline{v}$  is the maximum vectors are file and  $A_{mk}^{i}$  are for to where  $R_{mk}$  and  $X_{mk}$  refer to resistance, reactance of branch  $(m,k)$ , and  $P'_{mk}$  and  $Q'_{mk}$  denote active and reactive power flow on branch  $(m,k)$ ,  $\overline{v}$  is the maximum voltage profile, and  $\overline{\Delta v}$  refers to the a where  $R_{mk}$  and  $X_{mk}$  refer to resistance, reactance of branch  $(m,k)$ , and  $P'_{mk}$  and  $Q'_{mk}$  denote active and reactive power flow on branch  $(m,k)$ ,  $\overline{v}$  is the maximum voltage profile, and  $\Delta \overline{v}$  refers to the where  $R_{nk}$  and  $X_{mk}$  refer to resistance, reactance of branch  $(m,k)$ , and  $P'_{nk}$  and  $Q'_{nk}$  denote active and reactive power flow on branch  $(m,k)$ ,  $\overline{v}$  is the maximum voltage profile, and  $\Delta \overline{v}$  refers to the reactance of branch  $(m,k)$ , and  $P'_{mk}$  and<br>we on branch  $(m,k)$ ,  $\overline{v}$  is the maximum<br>wable voltage drop, e.g.  $\Delta v = 0.1$  p.u. for<br> $\Delta \overline{V} = \frac{(\Delta v)^2}{2} + \overline{v} \cdot \Delta v$ , and since DSC has a<br>uct linear voltage security const where  $R_{\text{int}}$  and  $X_{\text{int}}$  refer to resistance, reactance of branch  $(m,k)$ , and  $P'_{\text{int}}$  and  $Q'_{\text{int}}$  denote active and reactive power flow on branch  $(m,k)$ ,  $\bar{v}$  is the maximum voltage profile, and  $\Delta \bar{v}$  refe where  $R_{\text{int}}$  and  $X_{\text{int}}$  refer to resistance, reactance of branch  $(m,k)$ , and  $P_{\text{int}}^i$  and  $Q_{\text{int}}^i$  denote active and reactive power flow on branch  $(m,k)$ ,  $\overline{v}$  is the maximum voltage profile, and  $\Delta \overline{v}$  Let allowable squared voltage drop be  $\Delta \overline{V} = \frac{(\Delta \overline{V})^2}{2} + \overline{V} \cdot \Delta \overline{V}$  and since DSC has a<br>Ns [82].<br>Let allowable squared voltage drop be  $\Delta \overline{V} = \frac{(\Delta \overline{V})^2}{2} + \overline{V} \cdot \Delta \overline{V}$ , and since DSC has a<br>red d

 $\left(\overline{\Delta v}\right)^2$ 2  $\overline{V} = \frac{(\Delta v)^2}{2} + \overline{v} \cdot \Delta \overline{v}$ ,

$$
\left[R_{A1\text{-}C}P_c^t + X_{A1\text{-}C}Q_c^t \quad R_{A2\text{-}C}P_c^t + X_{A2\text{-}C}Q_c^t\right]\left[\begin{matrix} S_1^t\\ S_2^t \end{matrix}\right] \leq \Delta \overline{V}
$$
\n(6.16)

voltage profile, and  $\Delta \bar{v}$  refers to the allowable voltage drop, e.g.  $\Delta \bar{v} = 0.1$  p.u. for<br>
DNs [82].<br>
Let allowable squared voltage drop be  $\Delta \overline{V} = \frac{(\Delta \bar{v})^2}{2} + \bar{v} \cdot \Delta \bar{v}$ , and since DSC has a<br>
fixed dir  $S_3^t$ ,  $S_4^t$  and  $S_5^t$ . Given that this g.  $\Delta v = 0.1$  p.u. for<br>and since DSC has a<br>urity constraints with<br> $\Delta \overline{V}$  (6.16)<br>pends on the circuit<br> $S_5^t$ . Given that this<br>les among  $S_3^t$ ,  $S_4^t$ <br>constraints: bi-directional power flow we can construct linear voltage security constraints with<br>the allowable ranges of voltage profiles:<br> $\left[R_{A1c}P_c^e + X_{A1c}Q_c^i - R_{A2c}P_c^e + X_{A2c}Q_c^i\right]\left[\begin{array}{c} S_1^i \ S_2^i \end{array}\right] \leq \Delta \overline{V}$  (6.16)<br>  $S_3^t$  ,  $S_4^t$ and  $S_5^t$ , we can establish two linear equivalent voltage security constraints: t allowable squared voltage drop be  $\Delta \overline{V} = \frac{(\overline{\Lambda v})^2}{2} + \overline{v} \cdot \Delta \overline{v}$ , and since DSC has a<br>directional power flow, we can construct linear voltage security constraints with<br>lowable ranges of voltage profiles:<br> $\$  $\left[R_{A i, C}P_{e}^{i} + X_{A i, C}Q_{e}^{i} - R_{A \lambda c}Q_{e}^{i}\right] \begin{bmatrix} S_{1}^{i} \\ S_{2}^{i} \end{bmatrix} \leq \Delta \overline{V}$  (6.16)<br>
The power flow between substations C and D in SSC depends on the circuit<br>
switching status of circuit breakers among  $S_{1}^{i$ 

$$
\begin{bmatrix} \lambda_1 & 0 & R_{\text{c-D}} P_d^t + X_{\text{c-D}} Q_d^t \\ 0 & \lambda_2 & R_{\text{c-D}} P_c^t + X_{\text{c-D}} Q_c^t \end{bmatrix} \begin{bmatrix} S_3^t \\ S_4^t \\ S_5^t \end{bmatrix} - \begin{bmatrix} R_{\text{c-D}} P_d^t + X_{\text{c-D}} Q_d^t \\ R_{\text{c-D}} P_c^t + X_{\text{c-D}} Q_c^t \end{bmatrix} \le \begin{bmatrix} \Delta \overline{V} \\ \Delta \overline{V} \end{bmatrix} \tag{6.17}
$$

Where  $\lambda_1 = R_{A1\text{-}C}P_c^t + X_{A1\text{-}C}Q_c^t + R_{C\text{-}D}P_d^t + X_{C\text{-}D}Q_d^t$ ,  $\lambda_2 = R_{A2\text{-}D}P_d^t + X_{A2\text{-}D}Q_d^t + R_{C\text{-}D}P_c^t + X_{C\text{-}D}Q_c^t$ ;

switching status of circuit breakers among  $S'_3$ ,  $S'_4$  and  $S'_5$ . Given that this<br>bi-directional power flow for different circuit switching variables among  $S'_3$ ,  $S'_4$ <br>and  $S'_7$ , we can establish two linear equivalent v bi-directional power flow for different circuit switching variables among  $S'_3$ ,  $S'_4$ <br>and  $S'_7$ , we can establish two linear equivalent voltage security constraints:<br> $\begin{bmatrix} \lambda_1 & 0 & R_{\text{CD}}P'_4 + X_{\text{CD}}Q'_6 \\ 0 & \lambda_2 & R_{\text{CD}}P'_$ and  $S'_1$ , we can establish two linear equivalent voltage security constraints:<br>  $\begin{bmatrix} \lambda_1 & 0 & R_{C:D}P'_6 + X_{C:D}Q'_6 \\ 0 & \lambda_2 & R_{C:D}P'_6 + X_{C:D}Q'_c \end{bmatrix} \begin{bmatrix} S'_1 \\ S'_2 \\ S'_3 \end{bmatrix} - \begin{bmatrix} R_{C:D}P'_6 + X_{C:D}Q'_6 \\ R_{C:D}P'_6 + X_{C:D}Q'_6 \end{bmatrix} \le \begin{$ Solution  $\begin{bmatrix} \lambda_1 & 0 & R_{\text{CD}}P_4' + X_{\text{CD}}Q_4' \\ 0 & \lambda_2 & R_{\text{CD}}P_6' + X_{\text{CD}}Q_6' \end{bmatrix} \begin{bmatrix} S_1^* \\ S_2^* \\ S_3^* \end{bmatrix} - \begin{bmatrix} R_{\text{CD}}P_4' + X_{\text{CD}}Q_6' \\ R_{\text{CD}}P_6' + X_{\text{CD}}Q_6' \end{bmatrix} \leq \begin{bmatrix} \Delta \overline{V} \\ \Delta \overline{V} \end{bmatrix}$  (6.17)<br>Where  $\begin{bmatrix} \lambda_1 & 0 & R_{c,D}P'_d + X_{c,D}Q'_s \\ 0 & \lambda_2 & R_{c,D}P'_c + X_{c,D}Q'_s \end{bmatrix} \begin{bmatrix} S'_1 \\ S'_2 \\ S'_3 \end{bmatrix} = \begin{bmatrix} R_{c,D}P'_c + X_{c,D}Q'_s \\ R_{c,D}P'_c + X_{c,D}Q'_d \end{bmatrix} \le \begin{bmatrix} \Delta F \\ \Delta F \end{bmatrix}$  (6.17)<br>
Where  $\lambda_1 = R_{\text{ALC}}P'_c + X_{\text{ALC}}Q'_c + R_{\text{C-D}}P'_d + X_{c,D}Q'_$ 

Accordingly, this constraint is bounded by  
\n
$$
\sum_{t=1}^{N_{Tx}} (S_j^t - S_j^{t-1})^2 = \sum_{t=1}^{N_{Tx}} ((S_j^t)^2 + (S_j^{t-1})^2 - 2 \cdot S_j^t S_j^{t-1}) \le N_w
$$
\n(6.18)

where  $N_{T_s}$  refers to the number of time-horizons in a rolling window and  $N_{w}$ Accordingly, this constraint is bounded by<br>  $\sum_{i=1}^{N_n} (S'_i - S_i^{i-1})^2 = \sum_{i=1}^{N_n} ((S'_i)^2 + (S_i^{i-1})^2 - 2 \cdot S_i^i S_i^{i-1}) \le N_w$  (6.18)<br>
where  $N_{TS}$  refers to the number of time-horizons in a rolling window and  $N_w$ <br>
denotes the

cordingly, this constraint is bounded by<br>  $\sum_{i=1}^{N_m} (S'_j - S'^{-1}_j)^2 = \sum_{i=1}^{N_m} ((S'_i)^2 + (S'^{-1}_j)^2 - 2 \cdot S'_j S'^{-1}_j) \le N_w$  (6.18)<br>
here  $N_{T_S}$  refers to the number of time-horizons in a rolling window and  $N_w$ <br>
notes the number  $\le N_w$  (6.18)<br>ing window and  $N_w$ <br> $(S_j')^2 = S_j'$ , and let<br>mick envelopes [100],  $t_i = S_i^t + S_i^{t-1}$  that satisfy  $z_i^t$ Accordingly, this constraint is bounded by<br>  $\sum_{i=1}^{N_x} (S_j^t - S_j^{t-1})^2 = \sum_{i=1}^{N_x} ((S_j^t)^2 + (S_j^{t-1})^2 - 2 \cdot S_j^t S_j^{t-1}) \le N_w$  (6.18)<br>
where  $N_{TS}$  refers to the number of time-horizons in a rolling window and  $N_w$ <br>
denotes t Accordingly, this constraint is bounded by<br>  $\sum_{i=1}^{N_x} (S'_i - S''_j)^2 = \sum_{i=1}^{N_x} ((S'_i)^2 + (S'^{+}_i)^2 - 2 \cdot S'_j S'^{+}_j) \le N_w$  (6.18)<br>
where  $N_{TS}$  refers to the number of time-horizons in a rolling window and  $N_w$ <br>
denotes the numb  $z_j^t = S_j^t + S_j^{t-1}$  with  $\sum_{i=1}^{N_n} (S_j^t - S_j^{t-1})^2 = \sum_{i=1}^{N_n} ((S_j^t)^2 + (S_j^{t-1})^2 - 2 \cdot S_j^t S_j^{t-1}) \le N_w$ <br>
nere  $N_{TS}$  refers to the number of time-horizons in a rolling window<br>
notes the number of allowable switching actions.<br>
Consider the propert  $\sum_{i=1}^{N_{th}} (S'_{i} - S'^{-1}_{i})^{2} = \sum_{i=1}^{N_{th}} ((S'_{i})^{2} + (S'^{-1}_{i})^{2} - 2 \cdot S'_{i} S'^{-1}_{i}) \le N_{w}$  (6.18)<br>  $r_{s}$  refers to the number of time-horizons in a rolling window and  $N_{w}$ <br>
number of allowable switching actions.<br>  $r_{t}$  As<br>
Lonsider the property of binary number operations  $(S'_j)^2 = S'_j$ , and let<br>  $= S'_j + S'^{-1}_j$  that satisfy  $z'_j \in \{0,1\}$ . By using piecewise McCormick envelopes [100],<br>
e exactly replace  $z'_j = S'_j + S'^{-1}_j$  with<br>  $z'_j \geq S'_j + S'^{-$ 

$$
z_j^t \ge S_j^t + S_j^{t-1} - 1, \ z_j^t \le S_j^t, \ z_j^t \le S_j^{t-1}
$$
\n
$$
(6.19)
$$

Due to  $z_j^t = S_j^t + S_j^{t-1}$ , constraint (6.18) can be subsequently rearranged as

$$
\sum_{t=1}^{N_{T_s}} (S_j^t + S_j^{t-1} - 2 \cdot z_j^t) \le \tau \tag{6.20}
$$

denotes the number of allowable switching actions.<br>
Consider the property of binary number operations  $(S'_j)^2 = S'_j$ , and let  $z'_j = S'_j + S'^{-1}_j$  that satisfy  $z'_j \in \{0,1\}$ . By using piccewise McCormick envelopes [100], we exa Consider the property of binary number operations  $(S'_j)^2 = S'_j$ , and let  $z'_j = S'_j + S'^{-1}_j$  that satisfy  $z'_j \in \{0,1\}$ . By using piecewise McCormick envelopes [100], we exactly replace  $z'_j = S'_j + S'^{-1}_j$  with<br>  $z'_j \geq S'_j + S'^{-1}_$  $z'_j = S'_j + S'^{-1}_j$  that satisfy  $z'_j \in \{0,1\}$ . By using piecewise McCormick envelopes [100],<br>we exactly replace  $z'_j = S'_j + S'^{-1}_j$  with<br> $z'_j \geq S'_j + S'^{-1}_j - 1$ ,  $z'_j \leq S'_j$ ,  $z'_j \leq S'^{-1}_j$  (6.19)<br>Due to  $z'_j = S'_j + S'^{-1}_j$ , const we exactly replace  $z'_j = S'_j + S'^{-1}_j$  with<br>  $z'_j \geq S'_j + S'^{-1}_j - 1, z'_j \leq S'_j, z'_j \leq S'^{-1}_j$  (6.19)<br>
Due to  $z'_j = S'_j + S'^{-1}_j$ , constraint (6.18) can be subsequently rearranged as<br>  $\sum_{i=1}^{N_s} (S'_j + S'^{-1}_j - 2 \cdot z'_j) \leq r$  (6.20)<br>
As  $z'_j \geq S'_j + S''_j^{-1} - 1$ ,  $z'_j \leq S'_j$ ,  $z'_j \leq S''_j^{-1}$  (6.19)<br>
Due to  $z'_j = S'_j + S''_j^{-1}$ , constraint (6.18) can be subsequently rearranged as<br>  $\sum_{i=1}^{S_0} (S'_j + S''_j^{-1} - 2 \cdot z'_i) \leq \tau$  (6.20)<br>
As stated in Subsection 6.1.1, t Due to  $z_j' = S_j' + S_j'^{-1}$ , constraint (6.18) can be subsequently rearranged as<br>  $\sum_{i=1}^{N_E} (S_j' + S_j'^{-1} - 2 \cdot z_j') \le \tau$  (6.20)<br>
As stated in Subsection 6.1.1, the maximum amount of dis-patchable loads only<br>
accounts for nearly As stated in Subsection 6.1.1, the maximum amount of dis-patchable<br>accounts for nearly 5%-30% capacity of WTBPS, which directly indicates<br>coal-fired units play a major role and load transfer operations only play  $\varepsilon$ <br>rol As sacted in stassection 6.1.1, the maximum amount or uss-pactable toats omy<br>counts for nearly 5%-30% capacity of WTBPS, which directly indicates retrofitted<br>al-fired units play a major role and load transfer operations o coal-fired units play a major role and load transfer operations only play a secondary<br>role in tracking wind power of WTBPS. Thus, the substantial switching times of<br>eircuit breakers cannot be realistic for economic dispat

DSC: 
$$
S_1^t + S_2^t = 1
$$
 SSC:  $S_3^t + S_4^t + S_5^t = 2$ . (6.21)

6.2 Tightened Ramping Constraints<br>6.2.1 Linear Ramping Constraints<br>As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we 6.2 Tightened Ramping Constraints<br>6.2.1 Linear Ramping Constraints<br>8.5 As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we<br>19 As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d 2 Tightened Ramping Constraints<br>
2.1 Linear Ramping Constraints<br>
As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we<br>
place arcs  $\widehat{BC}$  and  $\widehat{HG}$  by connecting a line between B and C and Fightened Ramping Constraints<br>
6.2.*I Linear Ramping Constraints*<br>
As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we<br>
replace arcs  $\widehat{BC}$  and  $\widehat{HG}$  by connecting a line between B and C 6.2.*I* Linear Ramping Constraints<br>6.2.*I* Linear Ramping Constraints<br>As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we<br>replace arcs  $\overline{BC}$  and  $\overline{HG}$  by connecting a line between B and 6.2 Tightened Ramping Constraints<br>6.2.*1 Linear Ramping Constraints*<br>4s displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we<br>replace ares  $\overline{BC}$  and  $\overline{HG}$  by connecting a line between *B* an 6.2 Tightened Ramping Constraints<br>
6.2.1 Linear Ramping Constraints<br>
As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we<br>
replace arcs  $\widehat{BC}$  and  $\widehat{HG}$  by connecting a line between B and 6.2 Tightened Ramping Constraints<br>
6.2.1 Linear Ramping Constraints<br>
As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we<br>
replace ares  $\widehat{BC}$  and  $\widehat{HG}$  by connecting a line between B and As displayed in Fig. 6.7(a), to simplify convex constraints (6.3d) and (6.4d), we<br>replace arcs  $\widehat{BC}$  and  $\widehat{HG}$  by connecting a line between B and C and a line from H<br>to G. Clearly,  $\widehat{BC}$  and  $\widehat{HG}$  are strictly replace ares  $\widehat{BC}$  and  $\widehat{HG}$  by connecting a line between B and C and a line from H<br>to G. Clearly,  $\widehat{BC}$  and  $\widehat{HG}$  are strictly tighter than ares  $\widehat{BC}$  and  $\widehat{HG}$ , which<br>ramping area is a decagon whose ve

$$
\begin{bmatrix} 1 & -A_{\text{upper}} \\ -1 & A_{\text{lower}} \end{bmatrix} \begin{bmatrix} r_{i,u}^t \\ r_{i,u}^{t-1} \end{bmatrix} + \begin{bmatrix} -B_{\text{upper}} \\ B_{\text{lower}} \end{bmatrix} \le 0
$$
 (6.22)

$$
\begin{cases}\nA_{\text{upper}} = \frac{a(\Delta T - 1) (a + 1)^{\Delta T - 1}}{(a + 1)^{\Delta T - 1} - 1}, & \begin{cases}\nB_{\text{upper}} = d - \frac{\Delta T - 1}{(a + 1)^{\Delta T - 1} - 1} \cdot (c + b(a + 1)^{\Delta T - 1}) \\
\end{cases} \\
A_{\text{lower}} = \frac{1 - (1 - a)^{\Delta T - 1}}{a(\Delta T - 1)} & \begin{cases}\nB_{\text{lower}} = d - \frac{(1 - (1 - a)^{\Delta T - 1})d}{a(\Delta T - 1)} - \frac{c(1 - (1 - a)^{\Delta T - 1})}{a}\n\end{cases}\n\end{cases}
$$
\n(6.2





Fig. 6.8. (a) Ramping boundaries of a retrofitted coal-fired unit; (b) quadratic<br>functions for  $f(x)$  and  $g(x)$ .<br>Fig. 6.8. (a) Ramping boundaries of a retrofitted coal-fired unit; (b) quadratic<br>functions for  $f(x)$  and  $g(x)$ (6.22) under  $\Delta T = 15$  minutes, respectively. As suggested in Fig. 6.8(a), the tightened increase of a retrofitted coal-fired unit; (b) quadratic functions for  $f(x)$  and  $g(x)$ .<br>Fig. 6.8(a) compares the ramping boundaries **Example of a retrofitted** coal-fired unit; (b) quadratic functions for  $f(x)$  and  $g(x)$ .<br>
Fig. 6.8(a) compares the ramping boundaries using the piecewise and proposed<br>
linear models, and tightened ramp-up/down margins by (b)<br>
Fig. 6.8. (a) Ramping boundaries of a retrofitted coal-fired unit; (b) quadratic<br>
functions for  $f(x)$  and  $g(x)$ .<br>
Fig. 6.8(a) compares the ramping boundaries using the piecewise and proposed<br>
linear models, and tight Fig. 6.8. (a) Ramping boundaries of a retrofitted coal-fired unit; (b) quadratic<br>functions for  $f(x)$  and  $g(x)$ .<br>Fig. 6.8(a) compares the ramping boundaries using the piecewise and proposed<br>linear models, and tightened ram functions for  $f(x)$  and  $g(x)$ .<br>
Fig. 6.8(a) compares the ramping boundaries using the piecewise and<br>
linear models, and tightened ramp-up/down margins by linear ramping (<br>
(6.22) under  $\Delta T$ =15 minutes, respectively. As s Fig. 6.8(a) compares the ramping boundaries using the piecewise and proposed<br>ear models, and tightened ramp-up/down margins by linear ramping constraints<br>22) under  $\Delta T=15$  minutes, respectively. As suggested in Fig. 6.8( Fig. 6.8(a) compares the ramping boundaries using the piecewise and proposed<br>linear models, and tightened ramp-up/down margins by linear ramping constraints<br>(6.22) under  $\Delta T = 15$  minutes, respectively. As suggested in Fi linear models, and tightened ramp-up/down margins by linear ramping constraints<br>
(6.22) under  $\Delta T$ =15 minutes, respectively. As suggested in Fig. 6.8(a), the tightened<br>
ramping boundaries in pink line are very close to r ramping boundaries in pink line are very close to ramping boundaries using the<br>proposed linear model in blue line, and also are more accurate than ramping<br>boundaries using the piecewise linear model in green line, especia

$$
\Delta f_{upper}(x) = f(x) - A_{upper}x - B_{upper}
$$
\n(6.24)

$$
-c \cdot \eta' - A_{\text{upper}} = 0 \tag{6.25}
$$

Herein,  $d\Delta f_{upper}(x)/dx=0$ , yielding<br> $-c \cdot \eta' - A_{upper} = 0$  (6.25)<br>Substituting  $\eta' = -a/[(ax-b)\ln(a+1)]$  from Eq. (6.26), we can obtain the Herein,  $d\Delta f_{upper}(x)/dx=0$ , yielding<br>  $-c \cdot \eta' - A_{upper} = 0$  (6.25)<br>
Substituting  $\eta' = -a/[(ax - b)\ln(a + 1)]$  from Eq. (6.26), we can obtain the<br>
timal x<sup>\*</sup> as optimal  $x^*$  as as Herein,  $d\Delta f_{upper}(x)/dx=0$ , yielding<br>  $-c \cdot \eta' = A_{upper} = 0$  (6.25)<br>
Substituting  $\eta' = -a/[(ax-b)\ln(a+1)]$  from Eq. (6.26), we can obtain the<br>
timal x<sup>\*</sup> as<br>  $x' = \frac{c}{A_{upper} \ln(a+1)} + \frac{b}{a}$  (6.26)<br>
Consequently, the maximum error between ti Herein,  $d\Delta f_{upper}(x)/dx=0$ , yielding<br>  $-c \cdot \eta - A_{upper} = 0$  (6.25)<br>
Substituting  $\eta' = -a/[(ax-b)\ln(a+1)]$  from Eq. (6.26), we can obtain the<br>
optimal x<sup>\*</sup> as<br>  $x^* = \frac{c}{A_{upper}} \ln(a+1) + \frac{b}{a}$  (6.26)<br>
Consequently, the maximum error between

$$
x^* = \frac{c}{A_{\text{upper}} \ln(a+1)} + \frac{b}{a}
$$
 (6.26)

Substituting 
$$
\eta' = -a/[(ax - b)\ln(a + 1)]
$$
 from Eq. (6.26), we can obtain the  
\noptimal  $x^*$  as  
\n
$$
x^* = \frac{c}{A_{upper} \ln(a + 1)} + \frac{b}{a}
$$
\n(6.26)  
\nConsequently, the maximum error between tightened and untightened ramp-up  
\nmargins can be calculated in  
\n
$$
\Delta f_{upper}(x^*) = f(x^*) - A_{upper}x^* - B_{upper}
$$
\n
$$
= (\frac{c}{A_{upper} \ln(a + 1)}) (a + 1)^{n^2 + 1} + \frac{b}{a} (1 - A_{upper}) + c \cdot (\Delta T - \eta^*) - \frac{c}{\ln(a + 1)} - B_{upper}
$$
\n(6.27)  
\nwhere  $\eta^* = round(-\ln(\frac{a}{A_{upper} \ln(a + 1)}) / \ln(a + 1) + 1)$ .  
\nMoreover, the maximum error of the linear ramp-down constraint (6.22) can be  
\nsummarized as  
\n
$$
\Delta f_{lower}(x) = A_{lower}x + B_{lower} - g(x)
$$
\n(6.28)  
\nwhere  $A_{lower}$ ,  $B_{lower}$  can be found in (6.23).  
\nAccording to  $d\Delta f_{lower}(x)/dx = 0$  and  $c(\omega - 1) = (x - d)$ , we express  
\n
$$
A_{lower} - (1 - a)^{\Delta T - \omega} - \frac{\ln(1 - a)}{a} = 0
$$
\n(6.29)

where  $\eta^* = round(-\ln(\frac{a}{A_{\text{upper}}\ln(a+1)})/\ln(a+1)+1)$ .<br>
Moreover, the maximum error of the linear ramp-down constraint (6.22) can be<br>
mmarized as<br>  $A_{\text{lower}}(x) = A_{\text{lower}}x + B_{\text{lower}} - g(x)$  (6.28)<br>
where  $A_{\text{lower}}$ ,  $B_{\text{lower}}$  can be found in

$$
\Delta f_{lower}(x) = A_{lower}x + B_{lower} - g(x) \tag{6.28}
$$

$$
A_{\text{lower}} - (1 - a)^{\Delta T - \omega} \frac{-\ln(1 - a)}{a} = 0 \tag{6.29}
$$

as

$$
x^* = c(\Delta T - \frac{\ln(-\frac{aA_{\text{lower}}}{\ln(1-a)})}{\ln(1-a)} - 1) + d
$$
 (6.30)

 $\Delta f_{\text{source}}(x) = A_{\text{lower}} + B_{\text{lower}} - g(x)$  (6.28)<br>
where  $A_{\text{lower}}$ ,  $B_{\text{lower}}(x)dx-0$  and  $c(\omega-1) = (x-d)$ , we express<br>  $A_{\text{lower}} = (1-a)^{\delta T-\omega} \frac{-\ln(1-a)}{a} = 0$  (6.29)<br>
As a result, we can obtain the optimal  $x^*$  as<br>  $x^* = c(\Delta T - \frac{\ln(-\frac{aA_{$ where  $A_{\text{lower}}$ ,  $B_{\text{lower}}$  can be found in (6.23).<br>
According to d $\Delta f_{lower}(x)/dx=0$  and  $c(\omega-1)=(x-d)$ , we express<br>  $A_{\text{lower}}-(1-a)^{\Delta T-\omega}\frac{-\ln(1-a)}{a}=0$ <br>
As a result, we can obtain the optimal  $x^*$  as<br>  $x^* = c(\Delta T - \frac{\ln(-\frac{aA_{\text{lower}}}{\ln($ 

$$
\Delta f_{lower}(x^*) = A_{lower}x^* + B_{lower} - g(x^*)
$$
(6.31)  
where  $\omega^* = round(\Delta T - \ln\left(-\frac{aA_{lower}}{\ln(1-a)}\right)/\ln(1-a))$ .  
6.2.2 SOC Ramping Constraints  
As mentioned above, we simplify convex constraints (6.3d) and (6.4d) as linear  
constraints; However, it may induce approximations, which confine actual

 $A_{lower}(x^*) = A_{lower}x^* + B_{lower} - g(x^*)$  (6.31)<br>
where  $\omega^* = round(\Delta T - \ln\left(-\frac{aA_{lower}}{\ln(1-a)}\right)/\ln(1-a))$ .<br>
2.2 SOC Ramping Constraints<br>
As mentioned above, we simplify convex constraints (6.3d) and (6.4d) as linear<br>
nstraints; However, it may  $\Delta_{lower}(x^*) = A_{lower}x^* + B_{lower} - g(x^*)$  (6.31)<br>
where  $\omega^* = round(\Delta T - \ln\left(-\frac{aA_{lower}}{\ln(1-a)}\right)/\ln(1-a))$ .<br>
6.2.2 SOC Ramping Constraints<br>
As mentioned above, we simplify convex constraints (6.3d) and (6.4d) as linear<br>
constraints; However, it  $\Delta f_{\text{tree}}(x^*) = A_{\text{tree}}x^* + B_{\text{tree}} = g(x^*)$  (6.31)<br>
where  $\omega^* = round(\Delta T - \ln\left(-\frac{aA_{\text{tree}}}{\ln(1-a)}\right) / \ln(1-a))$ .<br>
6.2.2 SOC Ramping Constraints<br>
As mentioned above, we simplify convex constraints (6.3d) and (6.4d) as linear<br>
constrai  $\Delta f_{source}(x') = A_{true}x' + B_{true} - g(x')$  (6.31)<br>
where  $\omega' = round(\Delta T - \ln\left(-\frac{aA_{base}}{\ln(1-a)}\right)/\ln(1-a))$ .<br>
6.2.2 SOC Ramping Constraints<br>
As mentioned above, we simplify convex constraints (6.3d) and (6.4d) as linear<br>
constraints; However, it ma 1 .<br>و  $x = r_{i,u}^{t-1}$  $A_{lower}(x^*) = A_{lower}x^* + B_{lower} - g(x^*)$  (6.31)<br>
where  $\omega^* = round(\Delta T - \ln\left(-\frac{aA_{lower}}{\ln(1-a)}\right)/\ln(1-a))$ .<br>
2.2 SOC Ramping Constraints<br>
As mentioned above, we simplify convex constraints (6.3d) and (6.4d) as linear<br>
mstraints; However, it may  $\widehat{HG}$  are As mentioned above, we simplify convex constraints (6.3d) and (6.4d) as linear<br>constraints; However, it may induce approximations, which confine actual<br>ramp-up/down margins of retrofitted coal-fired units. We consider dra

$$
\begin{cases}\nf(x) = (x - b/a)(a+1)^{n-1} + b/a + c \cdot (\Delta T - \eta) \\
g(x) = (x - c \cdot (\omega - 1) - b/a)(1 - a)^{\Delta T - \omega} + b/a\n\end{cases}
$$
\n(6.32)

 $f_U(x) = a_{f,upper} \cdot x^2 + b_{f,upper} \cdot x + c_{f,upper}$  and  $g_L(x) = a_{f,lower} \cdot x^2 + b_{f,lower} \cdot x + c_{f,lower}$ rann-many contributions of retrofitted coal-fired units. We consider drawing a<br>tightened quadratic curve to replace ares  $\widehat{BC}$  and  $\widehat{HG}$ . For convenience, we define<br> $x = r_{i\omega}^{(-1)}$  and then ramp-up/down margin functi represent the quadratic functions to approximate  $f(x)$  and  $g(x)$ , where  $a_{f,upper}$ ,  $b_{f,upper}$ ,  $c_{f,\text{upper}}$  and  $a_{f,\text{lower}}$ ,  $b_{f,\text{lower}}$ ,  $c_{f,\text{lower}}$  are quadratic coefficients of  $f_U(x)$  and for any set of  $\overline{BC}$  and  $\overline{HG}$ . For convenience, we define<br>margin functions  $f(x)$  and  $g(x)$  for arcs  $\overline{BC}$  and<br> $\frac{b}{a}(a+1)^{n-1}+b/a+c\cdot(\Delta T-\eta)$  (6.32)<br> $c\cdot(\omega-1)-b/a(1-a)^{\Delta T-\omega}+b/a$ <br>defined as Lipschitz continuous funct  $x = r_{i,s}^{t-1}$  and then ramp-up/down margin functions  $f(x)$  and  $g(x)$  for arcs<br>  $\overline{HG}$  are<br>  $\begin{cases} f(x) = (x - b/a)(a + 1)^{n-1} + b/a + c \cdot (\Delta T - \eta) \\ g(x) = (x - c \cdot (\omega - 1) - b/a)(1 - a)^{\Delta T - \omega} + b/a \end{cases}$ <br>
where  $f(x)$  and  $g(x)$  are defined as Lipschi As displayed in Fig. 6.8(b), let us prove the quadratic constraint  $f_U(x)$  in red<br>
As displayed in Fig. 6.8(b), let us prove the quadratic continuous functions. Let<br>  $f(x) = a_{f, wpgor} \cdot x^2 + b_{f, wpgor} \cdot x + c_{f, wpgor}$  and  $g_L(x) = a_{f, w$ (6.32)<br>
unctions. Let<br>  $\begin{aligned}\n &\text{var} \cdot x + c_{f,\text{lower}} \\
 &\text{var} \cdot y + c_{f,\text{lower}}\n \end{aligned}$ <br>  $f_U(x)$  and<br>  $f_U(x)$  in red<br>
ximated error<br>
is an arbitrary  $\int f(x) = (x - b/a)(a + 1)^{n-1} + b/a + c \cdot (\Delta T - \eta)$  (6.32)<br>  $\int g(x) = (x - c \cdot (\omega - 1) - b/a)(1 - a)^{\Delta T - \omega} + b/a$  (6.32)<br>
where  $f(x)$  and  $g(x)$  are defined as Lipschitz continuous functions. Let<br>  $f_v(x) = a_{f,upper} \cdot x^2 + b_{f,upper} \cdot x + c_{f,upper}$  and  $g_x(x) = a_{f,lower} \$  $\int g(x) = (x - c \cdot (\omega - 1) - b/a)(1 - a)^{\Delta t - \omega} + b/a$  (6.32)<br>
where  $f(x)$  and  $g(x)$  are defined as Lipschitz continuous functions. Let<br>  $f_0(x) = a_{f,upper} \cdot x^2 + b_{f,upper} \cdot x + c_{f,upper}$  and  $g_L(x) = a_{f,lower} \cdot x^2 + b_{f,lower} \cdot x + c_{f,lower}$ <br>
represent the quadrati

where  $f(x)$  and  $g(x)$  are defined as Lipschitz continuous functions. Let<br>  $f_U(x) = a_{j,upper} \cdot x^2 + b_{j,upper} \cdot x + c_{j,upper}$  and  $g_L(x) = a_{j,lower} \cdot x^2 + b_{j,lower} \cdot x + c_{j,lower}$ <br>
represent the quadratic functions to approximate  $f(x)$  and  $g(x)$ , wher  $f_U(x) = a_{f, \text{super}} \cdot x^2 + b_{f, \text{upper}} \cdot x + c_{f, \text{upper}}$  and  $g_L(x) = a_{f, \text{lower}} \cdot x^2 + b_{f, \text{lower}}$ ,  $b_{f, \text{upper}}$ ,  $b_{f, \text{upper}}$ ,  $c_{f, \text{upper$ represent the quadratic functions to approximate  $f(x)$  and  $g(x)$ , where  $a_{f,upper}$ ,  $b_{f,upper}$ ,  $b_{f,upper}$ ,  $c_{f,lower}$ ,  $c_{f,lower}$ ,  $c_{f,lower}$  are quadratic coefficients of  $f_U(x)$  and  $g_L(x)$ , respectively.<br>As displayed in Fig. 6.8(b)

$$
\begin{bmatrix} x_{\rm B}^2 & x_{\rm B} & 1 \\ x_{\rm C}^2 & x_{\rm C} & 1 \\ x_{\rm U}^2 & x_{\rm U} & 1 \end{bmatrix} \begin{bmatrix} a_{f,\text{upper}} \\ b_{f,\text{upper}} \\ c_{f,\text{upper}} \end{bmatrix} = \Gamma \begin{bmatrix} y_{\rm B} \\ y_{\rm C} \\ y_{\rm D} + h_{\rm U} \end{bmatrix}
$$
(6.33)

 $\begin{bmatrix} x_{\rm B}^2 & x_{\rm B} & 1 \\ x_{\rm C}^2 & x_{\rm C} & 1 \\ x_{\rm C}^2 & x_{\rm U} & 1 \end{bmatrix} \begin{bmatrix} a_{f, \text{upper}} \\ b_{f, \text{upper}} \\ c_{f, \text{upper}} \end{bmatrix} = \Gamma \begin{bmatrix} y_{\rm B} \\ y_{\rm C} \\ y_{\rm D} + h_{\rm U} \end{bmatrix}$  (6.33)<br>
where  $\Gamma$  is the coefficient matrix of  $f_U(x)$ , and  $x_{\rm B}$  $f_{\text{y, upper}}$ <br>  $\left[\begin{array}{c} y_B \\ y_C \\ y_D + h_U \end{array}\right]$ <br>  $f_{\text{y, x}}$ ,  $x_C$ ,  $x_D$  and  $x_U$  refer to<br>  $x_D$ ,  $y_D$ ,  $y_C$ ,  $y_D$  and  $y_U$  refer to  $y$ -coordinates  $\begin{bmatrix} x_8^2 & x_{B} & 1 \ x_0^2 & x_{C} & 1 \ x_1^2 & x_{U} & 1 \end{bmatrix} \begin{bmatrix} a_{f, \text{upper}} \\ b_{f, \text{upper}} \\ c_{f, \text{upper}} \end{bmatrix} = \begin{bmatrix} y_B \\ y_C \\ y_D + h_U \end{bmatrix}$  (6.33)<br>
where  $\Gamma$  is the coefficient matrix of  $f_U(x)$ , and  $x_B$ ,  $x_C$ ,  $x_D$  and  $x_U$  refer to<br>
x-co  $\begin{bmatrix} x_{B}^2 & x_{B} & 1 \ x_{C}^2 & x_{C} & 1 \ x_{C}^2 & x_{C} & 1 \end{bmatrix} \begin{bmatrix} a_{f, \text{upper}} \\ b_{f, \text{upper}} \\ c_{f, \text{upper}} \end{bmatrix} = \begin{bmatrix} y_{B} \\ y_{C} \\ y_{D} + h_{U} \end{bmatrix}$  (6.33)<br>where  $\Gamma$  is the coefficient matrix of  $f_{U}(x)$ , and  $x_{B}$ ,  $x_{C}$ ,  $x_{D}$  and  $\begin{bmatrix} x_{\rm B}^2 & x_{\rm B} & 1 \ x_{\rm C}^2 & x_{\rm C} & 1 \ x_{\rm C}^2 & x_{\rm C} & 1 \end{bmatrix} \begin{bmatrix} a_{f,\text{upper}} \\ b_{f,\text{upper}} \\ c_{f,\text{upper}} \end{bmatrix} = \Gamma \begin{bmatrix} a_{f,\text{upper}} \\ b_{f,\text{upper}} \\ c_{f,\text{upper}} \end{bmatrix} = \begin{bmatrix} y_{\rm B} \\ y_{\rm C} \\ y_{\rm D} + h_{\rm U} \end{bmatrix}$  (6.33)<br>
nere  $\Gamma$  is the coeffi  $\begin{bmatrix} x_{\rm B}^2 & x_{\rm B} & 1 \\ x_{\rm C}^2 & x_{\rm C} & 1 \\ x_{\rm C}^2 & x_{\rm C} & 1 \end{bmatrix} \begin{bmatrix} a_{f,\text{upper}} \\ b_{f,\text{upper}} \\ c_{f,\text{upper}} \end{bmatrix} = \begin{bmatrix} a_{f,\text{upper}} \\ b_{f,\text{upper}} \\ c_{f,\text{upper}} \end{bmatrix} = \begin{bmatrix} y_{\rm B} \\ y_{\rm C} \\ y_{\rm D} + h_{\rm U} \end{bmatrix}$  (where  $\Gamma$  is the coefficient ma where  $\left[\begin{array}{cc} x_0^2 & x_0 & 1 \end{array}\right] \left[\begin{array}{c} x_{\text{comp}} & e_{\text{comp}} \\ \cos \theta & \cos \theta \end{array}\right] \left[\begin{array}{c} y_{\text{p}} + h_0 \end{array}\right]$ <br>where  $\Gamma$  is the coefficient matrix of  $f_U(x)$ , and  $x_B$ ,  $x_C$ ,  $x_D$  and  $x_U$  refer to  $x$ -coordinates of points B, where  $\Gamma$  is the coefficient matrix of  $f_U(x)$ , and  $x_B$ ,  $x_C$ ,  $x_D$  and  $x_U$  refer to<br>  $x$ -coordinates of points B, C, D and U and  $y_B$ ,  $y_C$ ,  $y_D$  and  $y_U$  refer to  $y$ -coordinates of<br>
points B, C, D and U, respectively

Since  $\Gamma$  is invertible, the quadratic coefficients  $a_{f,\text{lower}}$ ,  $b_{f,\text{lower}}$  and  $c_{f,\text{lower}}$  can

$$
\begin{bmatrix} a_{f,\text{upper}} \\ b_{f,\text{upper}} \\ c_{f,\text{upper}} \end{bmatrix} = \Gamma^{-1} \begin{bmatrix} y_B \\ y_C \\ y_D \end{bmatrix} + \Gamma^{-1} \begin{bmatrix} 0 \\ 0 \\ h_U \end{bmatrix} = \Gamma^{-1} \begin{bmatrix} y_B \\ y_C \\ y_D \end{bmatrix} - \frac{1}{|\Gamma|} \begin{bmatrix} x_C - x_B \\ x_B^2 - x_C^2 \\ x_C^2 x_B - x_B^2 x_C \end{bmatrix} \cdot h_U \qquad (6.34)
$$

points B, C, and U. By (6.34), we express  $f_U(x)$  as

where 
$$
|\Gamma|
$$
 is the moduli of  $\Gamma$  and  $\Gamma$  is only with respect to the *x*-coordinates of  
points B, C, and U. By (6.34), we express  $f_U(x)$  as  

$$
f_U(x) = \begin{bmatrix} x^2 & x & 1 \end{bmatrix} \begin{bmatrix} a_{f,upper} \\ b_{f,upper} \end{bmatrix} = \begin{bmatrix} x^2 & x & 1 \end{bmatrix} (\Gamma^{-1} \begin{bmatrix} y_B \\ y_C \end{bmatrix} - \frac{x_C - x_B}{|\Gamma|} \begin{bmatrix} 1 \\ -x_C - x_B \end{bmatrix} \cdot h_U)
$$

$$
= \begin{bmatrix} \Gamma^{-1} \begin{bmatrix} y_B \\ y_C \end{bmatrix} \end{bmatrix} \begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix} + h_U \cdot \gamma^T \cdot \begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix}
$$
where  $\Gamma = -\frac{x_C - x_B}{|\Gamma|} \begin{bmatrix} 1 & -x_C - x_B & x_C x_B \end{bmatrix}^T$ .  
Observeing Eq. (6.35),  $[x^2 x 1] \cdot \Gamma^{-1} \cdot [y_B y_C y_D]^T$  is the quadratic function for arc *BCD*.  
If  $h_U=0$ , arc *BCD* overlaps arc *BCD*. Moreover, for any  $\forall x \ge x_C$ , we  
rearrange  $\gamma^T \cdot [x^2 \quad x \quad 1]^T$  as

where  $\Upsilon = -\frac{x_c - x_B}{|\Gamma|} \left[ 1 - x_c - x_B \right] x_c x_B$ .  $\Upsilon = -\frac{x_{\rm C} - x_{\rm B}}{|\Gamma|} \Big[ 1 \quad -x_{\rm C} - x_{\rm B} \quad x_{\rm C} x_{\rm B} \Big]^T.$ 

Observing Eq. (6.35),  $[x^2 \times 1] \cdot \Gamma^{-1} \cdot [y_B y_C y_D]^T$  is the quadratic function for arc  $\widehat{BCD}$ . rearrange  $\Upsilon^T \cdot \begin{bmatrix} x^2 & x & 1 \end{bmatrix}^T$  as  $\mathbf{r} = \left[\mathbf{\Gamma}^{-1}\begin{bmatrix} y_B \\ y_C \\ y_D \end{bmatrix}\right] \begin{bmatrix} x^2 \\ x \\ y_D \end{bmatrix} + h_U \cdot \mathbf{\Gamma}^{-1} \begin{bmatrix} x^2 \\ x \\ 1 \end{bmatrix}$ <br>
where  $\mathbf{\Upsilon} = -\frac{x_C - x_B}{|\mathbf{\Gamma}|} \begin{bmatrix} 1 & -x_C - x_B & x_C x_B \end{bmatrix}^T$ .<br>
Observing Eq. (6.35),  $[x^2 \times 1] \cdot \mathbf{\Gamma}^{-1} \cdot [y_B y_C y_D]^T$  is the

$$
\Upsilon^T \cdot \left[ x^2 \quad x \quad 1 \right]^T = -\frac{x_{\rm C} - x_{\rm B}}{|\Gamma|} (x^2 - (x_{\rm C} + x_{\rm B})x + x_{\rm C}x_{\rm B}) \tag{6.36}
$$

It is clear that  $x^2 - (x_c + x_B)x + x_cx_B$  is a quadratic function where the parameters upward. The minimization achieves at  $x^* = (x_B + x_C)/2$ , but  $x > x_c > x^*$ , so in minimization must be at  $x_c$ . The minimizal value of Eq. (6.36) v  $x^2 - (x_C + x_B)x + x_Cx_B$  is a quadratic function where the parabola<br>
are minimization achieves at  $x^* = (x_B + x_C)/2$ , but  $x > x_C > x^*$ , so the<br>
st be at x. The minimal value of Eq. (6.36) vields It is clear that  $x^2 - (x_c + x_B)x + x_cx_B$  is a quadratic function where the parabola<br>opens upward. The minimization achieves at  $x^*=(x_B+x_C)/2$ , but  $x>x_C>x^*$ , so the<br>minimization must be at  $x_C$ . The minimal value of Eq. (6.36) yi parabola<br>
, so the<br>
) yields It is clear that  $x^2 - (x_c + x_b)x + x_cx_b$  is a quadratic function where the parabola<br>opens upward. The minimization achieves at  $x^* = (x_b + x_c)/2$ , but  $x > x_c > x^*$ , so the<br>minimization must be at  $x_c$ . The minimal value of Eq. (6.36 ear that  $x^2 - (x_c + x_h)x + x_cx_h$  is a quadratic function where the parabola<br>ward. The minimization achieves at  $x^* = (x_B + x_c)/2$ , but  $x > x_c > x^*$ , so the<br>ion must be at  $x_c$ . The minimal value of Eq. (6.36) yields<br> $(x_c^2 - (x_c + x_h)x_c + x$ dratic function where the parabola<br>
\*= $(x_B+x_C)/2$ , but  $x>x_C>x^*$ , so the<br>
d value of Eq. (6.36) yields<br>  $Y^T \cdot \begin{bmatrix} x^2 & x & 1 \end{bmatrix}^T \ge 0$  is proven.<br>
point U is higher than point D, then Lear that  $x^2 - (x_c + x_B)x + x_cx_B$  is a quadratic function where the parabola<br>
sward. The minimization achieves at  $x^* = (x_B + x_C)/2$ , but  $x > x_c > x^*$ , so the<br>
tion must be at  $x_C$ . The minimal value of Eq. (6.36) yields<br>  $(x_c^2 - (x_c +$ 

$$
-\frac{x_{\rm C} - x_{\rm B}}{|\Gamma|} (x_{\rm C}^2 - (x_{\rm C} + x_{\rm B})x_{\rm C} + x_{\rm C}x_{\rm B}) = 0.
$$
 Therefore,  $\Upsilon^T \cdot \left[x^2 - x \quad 1\right]^T \ge 0$  is proven.

With  $\Upsilon^T \cdot \begin{bmatrix} x^2 & x & 1 \end{bmatrix}^T \ge 0$ , if  $h_U \ne 0$  r It is clear that  $x^2 - (x_c + x_0)x + x_cx_0$  is a quadratic function where the parabola<br>opens upward. The minimization achieves at  $x^* - (x_B + x_c)/2$ , but  $x > x_c > x^*$ , so the<br>minimization must be at  $x_c$ . The minimal value of Eq. (6. It is clear that  $x^2 - (x_c + x_n)x + x_cx_n$  is a quadratic function where the parabola<br>opens upward. The minimization achieves at  $x^* = (x_0 + x_c)/2$ , but  $x > x_c > x^*$ , so the<br>minimization must be at  $x_c$ . The minimal value of Eq. (6.3 It is clear that  $x^2 - (x_c + x_n)x + x_cx_n$  is a quadratic function where the parabola<br>opens upward. The minimization achieves at  $x^* = (x_0 + x_0)/2$ , but  $x > x_c > x^*$ , so the<br>minimization must be at  $x_c$ . The minimal value of Eq. (6. opens upward. The minimization achieves at  $x^*=(x_0+x_C)/2$ , but  $x>x_Cx^*$ , so the<br>minimization must be at  $x_C$ . The minimal value of Eq. (6.36) yields<br> $-\frac{x_C-x_B}{|\Gamma|}(x_C^2-(x_C+x_R)x_C+x_Cx_R)=0$ . Therefore,  $\Gamma^T \cdot [x^2 \times 1]^T \ge 0$  is prov minimization must be at  $x_c$ . The minimal value of Eq. (6.<br>  $-\frac{x_c - x_b}{|\Gamma|}(x_c^2 - (x_c + x_b)x_c + x_cx_b) = 0$ . Therefore,  $\Upsilon^T \cdot [x^2 \times 1]^T \ge 0$  is<br>
With  $\Upsilon^T \cdot [x^2 \times 1]^T \ge 0$ , if  $h_U \ne 0$  means that point U is higher than po<br>
we c must be at  $x_c$ . The minimal value of Eq. (6.36) yields<br>  $-(x_c + x_n)x_c + x_cx_n) = 0$ . Therefore,  $\Upsilon^T \cdot [x^2 \times 1]^T \ge 0$  is proven.<br>  $\begin{bmatrix} x^2 \times 1 \end{bmatrix}^T \ge 0$ , if  $h_0 \ne 0$  means that point U is higher than point D, then<br>
sive  $-\frac{x_C - x_B}{|\Gamma|} (x_C^2 - (x_C + x_B)x_C + x_Cx_B) = 0$ . Therefore,  $\Upsilon^T \begin{bmatrix} x^2 & x & 1 \end{bmatrix}^T \ge 0$  is proven.<br>
With  $\Upsilon^T \begin{bmatrix} x^2 & x & 1 \end{bmatrix}^T \ge 0$ , if  $h_V \neq 0$  means that point U is higher than point D, then<br>
we can perceive that  $f$  $X_c = (x_c + x_s)x_c + x_cx_s = 0$ . Therefore,  $1 + [A - A - 1] \ge 0$  is proven.<br>
With  $Y^T \cdot [x^2 \times 1]^T \ge 0$ , if  $h_{U} = 0$  means that point U is higher than point D, then<br>
we can perceive that  $f_L(x)$  can be a monotonically increasing functi With  $Y' \cdot [x^2 \cdot x \cdot 1]^T \ge 0$ , if  $h_V \ne 0$  means that point U is higher than point D, then<br>
can perceive that  $f_l(x)$  can be a monotonically increasing function with respect to<br>
for any fixed x from (6.35). If and only if we can perecive that  $f_k(x)$  can be a monotonically increasing function with respect to<br>  $h_U$  for any fixed x from (6.35). If and only if  $h_U=0$ , then the minimization of  $f(x) \cdot f_k(x)$ <br>
can be achieved, which indicates that by for any fixed x from (6.35). If and only if  $h_V$ =0, then the minimization of  $f(x)$ - $f_x(x)$ <br>can be achieved, which indicates that the quadratic constraints  $\overline{BCD}$  has the<br>minimal inner-approximated error for arc  $\overline{$ 

Analogously, the quadratic constraint  $f_L(x)$  can be justified as are<br>nvelop the point F.<br>nner-approximated errors  $\Delta s_U$  and  $\Delta s_L$  between arcs  $\widehat{BCD}$  and<br>arcs  $\widehat{HRG}$  and  $\widehat{HG}$  can be obtained by establishing  $df$ FHG, which can envelop the point F.<br>
The maximum inner-approximated errors  $\Delta s_U$  and  $\Delta s_L$  between arcs  $\widehat{BC}$  and  $\widehat{BC}$  and between arcs  $\widehat{HG}$  and  $\widehat{HG}$  can be obtained by establishing  $df(x)dx - df_L(x)dx$  and  $dg(x)dx$  $\overline{BC}$  and between arcs  $\overline{HG}$  and  $\overline{HG}$  can be obtained by establishing  $df(x)/dx = df_c(x)/dx$  and  $dg(x)/dx = dg_c(x)/dx$ , respectively. The solutions yield:<br>  $\frac{a \cdot c}{(a \cdot x_i - b) \ln(a + 1)} = 2a_{f, \text{supc}} x_i^+ + b_{f, \text{supc}}$  (6.37a)<br>  $2a_{f, \text{sup$ 

$$
\frac{a \cdot c}{(a \cdot x_{+}^{*} - b) \ln(a + 1)} = 2a_{f, \text{upper}} x_{+}^{*} + b_{f, \text{upper}}
$$
(6.37a)

$$
2a_{f,lower}x_{-}^{*} + b_{f,lower} = (1-a)^{\Delta T - \frac{x_{-}^{*}-d}{c} - 1} \cdot \frac{-\ln(1-a)}{a}
$$
 (6.37b)

where  $x^*$  and  $x^*$  denotes the x-coordinates of

 $x^*_{+}$  and  $x^*_{-}$ ,

the maximum inner-approximated errors  $\Delta s_U$  and  $\Delta s_L$  between quadratic and<br>untightened ramp-up/down margins can be calculated in<br> $\Delta s_U = f(x^*) - f_U(x^*)$ ,  $\Delta s_L = g_L(x^*) - g(x^*)$  (6.38) the maximum inner-approximated errors  $\Delta s_U$  and  $\Delta s_L$  between quadratic and<br>untightened ramp-up/down margins can be calculated in<br> $\Delta s_U = f(x^*) - f_U(x^*)$ ,  $\Delta s_L = g_L(x^*) - g(x^*)$  (6.38)<br>Based on the above, the quadratic ramping con Examinum inner-approximated errors  $\Delta s_U$  and  $\Delta s_L$  between quadratic and<br>tightened ramp-up/down margins can be calculated in<br> $\Delta s_U = f(x^*) - f_U(x^*)$ ,  $\Delta s_L = g_L(x^*) - g(x^*)$  (6.38)<br>Based on the above, the quadratic ramping constrai the maximum inner-approximated errors  $\Delta s_U$  and  $\Delta s_L$  between quadratic and<br>untightened ramp-up/down margins can be calculated in<br> $\Delta s_U = f(x_i^*) - f_U(x_i^*)$ ,  $\Delta s_L = g_L(x^*) - g(x^*)$  (6.38)<br>Based on the above, the quadratic ramping c where af,upper,  $\log_{10}$  and  $\cos_{10}$  and  $\cos_{10}$  and  $\cos_{10}$  between quadratic and untightened ramp-up/down margins can be calculated in<br>  $\Delta s_{\nu} = f(x_{\nu}^{*}) - f_{\nu}(x_{\nu}^{*})$ ,  $\Delta s_{\lambda} = g_{\lambda}(x_{\nu}^{*}) - g(x_{\nu}^{*})$  (6.38)<br>
Bas

$$
\Delta s_U = f(x_+^*) - f_U(x_+^*) \quad , \quad \Delta s_L = g_L(x_-^*) - g(x_-^*) \tag{6.38}
$$

$$
a_{f,\text{upper}} \cdot (r_{i,u}^{t-1})^2 + b_{f,\text{upper}} \cdot r_{i,u}^{t-1} + c_{f,\text{upper}} \ge r_{i,u}^t \tag{6.39a}
$$

$$
a_{f,\text{lower}} \cdot (r_{i,u}^{t-1})^2 + b_{f,\text{lower}} \cdot r_{i,u}^{t-1} + c_{f,\text{lower}} \le r_{i,u}^t \tag{6.39b}
$$

untightened ramp-up/down margins can be calculated in<br>  $\Delta s_o = f(x^*) - f_o(x^*)$   $\Delta s_t = g_z(x^*) - g(x^*)$  (6.38)<br>
Based on the above, the quadratic ramping constraints with minimal<br>
inner-approximated errors are given as<br>  $a_{f, \text{upper}} \cdot (r$ 

$$
\Delta s_{U} = f(x') - f_{U}(x') , \Delta s_{L} = g_{L}(x') - g(x')
$$
 (6.38)  
\nBased on the above, the quadratic ramping constraints with minimal  
\ninner-approximated errors are given as  
\n
$$
a_{f, \text{upper}} \cdot (r_{i,s}^{i-1})^{2} + b_{f, \text{upper}} \cdot r_{i,s}^{i-1} + c_{f, \text{upper}} \ge r_{i,s}'
$$
 (6.39a)  
\n
$$
a_{f, \text{lower}} \cdot (r_{i,s}'^{i-1})^{2} + b_{f, \text{lower}} \cdot r_{i,s}^{i-1} + c_{f, \text{lower}} \le r_{i,s}'
$$
 (6.39b)  
\nwhere  $a_{f, \text{upper}}$ ,  $b_{f, \text{upper}}$ ,  $c_{f, \text{upper}}$ ,  $a_{f, \text{lower}}$ , and  $c_{f, \text{lower}}$  can be obtained by arcs  $\widehat{BCD}$   
\nand  $\widehat{HRG}$ .  
\nTo convert this quadratic constraint (6.39a) and (6.39b) in the rotated SOC form,  
\nwe have  
\n
$$
\begin{cases}\n2 \cdot m_{\text{upper}} - 1/2 \geq ||J_{\text{upper}}||^{2}, m_{\text{upper}} = -r_{i,s}' - \frac{b_{f, \text{upper}}^{2}}{4a_{f, \text{upper}}} + c_{f, \text{upper}} \\
\frac{2}{u_{\text{upper}}} - \sqrt{-a_{f, \text{upper}}}\cdot r_{i,s}'^{-1} - \frac{1}{2}b_{f, \text{upper}}\sqrt{-1/a_{f, \text{upper}}}}{4a_{f, \text{upper}}} - c_{f, \text{lower}}\n\end{cases}
$$
 (6.40a)  
\n
$$
\begin{cases}\n2 \cdot m_{\text{lower}} - 1/2 \geq ||J_{\text{lower}}||^{2}, m_{\text{lower}} = r_{i,s}' - \frac{b_{f, \text{upper}}^{2}}{4a_{f, \text{upper}}} - c_{f, \text{lower}} \\
\frac{2}{u_{\text{lower}}} = \sqrt{a_{f, \text{lower}}r_{i,s}^{i-1} + \frac{1}{2}b_{f, \text{lower}}\sqrt{1/a_{f, \text{lower}}}}{1/a_{f, \text{lower}}}\n\end{cases}
$$
 (6.40b)  
\

# Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>Financement in Look-ahead Rolling Economic Dispatch Approach<br>Financement in Look-ahead Rolling Economic Dispatch Approach<br>Finance Dispatch Model Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>Figure 2.1 Rolling Economic Dispatch Model<br>With the day-ahead unit commitment scheme and forecasted

4.3. Distribution-Level Topology Optimization for Flexibility<br>
Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>
4.3.1 Rolling Economic Dispatch Model<br>
With the day-ahead unit commitment scheme and forecasted w 3 Distribution-Level Topology Optimization for Flexibility<br>nhancement in Look-ahead Rolling Economic Dispatch Approach<br>3.1 Rolling Economic Dispatch Model<br>With the day-ahead unit commitment scheme and forecasted wind power **6.3 Distribution-Level Topology Optimization for Flexibility**<br> **Enhancement in Look-ahead Rolling Economic Dispatch Approach**<br> **6.3.1 Rolling Economic Dispatch Model**<br>
With the day-ahead unit commitment scheme and forecas 6.3 Distribution-Level Topology Optimization for Flexibility<br>Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>6.3.1 Rolling Economic Dispatch Model<br>With the day-ahead unit commitment scheme and forecasted wind **involves Several Took-ahead Rolling Economic Dispatch Approach**<br> **6.3.1 Rolling Economic Dispatch Model**<br> **6.3.1 Rolling Economic Dispatch Model**<br>
With the day-ahead anti commitment scheme and forecasted wind power data, 6.3 Distribution-Level Topology Optimization for Flexibility<br>Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>6.3.1 Rolling Economic Dispatch Model<br>With the day-ahead unit commitment scheme and forecasted wind 6.3 Distribution-Level Topology Optimization for Flexibility<br>Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>6.3.1 Rolling Economic Dispatch Model<br>With the day-ahead unit commitment scheme and forceasted wind 6.3 Distribution-Level Topology Optimization for Flexibility<br>
Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>
6.3.1 Rolling Economic Dispatch Model<br>
With the day-ahead unit commitment scheme and forceasted w Enhancement in Look-ahead Rolling Economic Dispatch Approach<br>
6.3.1 Rolling Economic Dispatch Model<br>
With the day-ahead unit commitment scheme and forecasted wind power data, we<br>
can establish the coonomic dispatch model 6.3.*I* Rolling *Economic Dispatch Model*<br>With the day-ahead unit commitment scheme and forecasted wind power data, we<br>can establish the economic dispatch model over a rolling window, during which<br>WTBPS maintains steady t With the day-ahead unit commitment scheme and forecasted wind power data, we<br>can establish the economic dispatch model over a rolling window, during which<br>WTBPS maintains steady transmission power within specified bands. can establish the economic dispatch model over a rolling window, during which<br>WTBPS maintains steady transmission power within specified bands. Each dispatch<br>involves several look-ahead hours. In general, the wind power f WTBPS maintains steady transmission power within specified bands. Each dispatch<br>involves several look-ahead hours. In general, the wind power forecast for 4–6 hours<br>has relatively low forecast errors [73]. In this study, process:



(a)



$$
\min_{r'_{i,u} \in \mathbb{R}} F_{o1} = \sum_{t=1}^{N_{T_s}} \sum_{u=1}^{N_u} \sum_{i=1}^{N_{G,u}} \left[ c_{2i,u} (r'_{i,u} P_{Gi,u,\text{max}})^2 + c_{1i,u} r'_{i,u} P_{Gi,u,\text{max}} + c_{0i,u} \right] \tag{6.41}
$$

(a)  $t = 96$  (b)<br>
Fig. 6.9. (a) Look-ahead rolling economic dispatch framework; (b) rolling process.<br>
Since different  $y_i^t$  have similar production cost coefficients, we approximately<br>
express the coal-fired generation co (b)<br>
Fig. 6.9. (a) Look-ahead rolling economic dispatch framework; (b) rolling process.<br>
Since different  $y_i'$  have similar production cost coefficients, we approximately<br>
express the coal-fired generation cost  $F_0$  as<br> respectively. Since different  $y'_i$  have similar production cost coefficients, we approximately<br>press the coal-fired generation cost  $F_{o1}$  as<br> $\min_{\theta_i \in \mathbb{R}} F_{o1} = \sum_{i=1}^{N_c} \sum_{i=1}^{N_c} [c_{\theta_i \mu} G_{\theta_i \mu_i \mu_i \mu_i}^* + c_{\theta_i \mu_i}^*] + c_{\theta_i \mu_i$ 

Since different  $y'_i$  have similar production cost coefficients, we approximately<br>express the coal-fired generation cost  $F_{01}$  as<br> $\min_{f_0 \in \mathbb{R}} F_{01} = \sum_{i=1}^{N_E} \sum_{i=1}^{N_E} \sum_{i=1}^{N_E} [c_{2i,n}(r'_i, P_{G,n,\text{max}})^2 + c_{1i,n}r'_i, P_{$ express the coal-fired generation cost  $F_{01}$  as<br>  $\min_{r_1 \in \Omega} F_{01} = \sum_{i=1}^{N_1} \sum_{i=1}^{N_i} \sum_{i=1}^{N_i} \sum_{i=1}^{N_i} [c_{2i,a} (r'_{i,a} P_{0,i,a,max})^2 + c_{1i,a} r'_{i,a} P_{0,i,a,max} + c_{0i,a}]$  (6.41)<br>
where  $c_{2i,u}$ ,  $c_{1i,u}$  and  $c_{0i,u}$  refer t  $\min_{C_1 \text{ cell}} E_2 \sum_{k=1}^{N_{C_1} N_{C_2}} \sum_{l=1}^{N_{C_2}} [c_{2l,a}(r'_{l,a}P_{G_{l,a,\text{max}}})^2 + c_{l,a}r'_{l,a}P_{G_{l,a,\text{max}}} + c_{0l,a}]$  (6.41)<br>
where  $c_{2l,a}$ ,  $c_{l,a}$  and  $c_{0l,a}$  refer to production cost coefficients for the *i*-th retrofitted<br>
co where  $c_{2i,w}$ ,  $c_{1i,w}$  and  $c_{0j,w}$  refer to production cost coefficients for the *i*-th retrofitted<br>coal-fired unit in the *u*-th agent of WTBPS;  $N_{G,w}$  and  $N_u$  indicate the total number of<br>retrofitted coal-fired uni  $_{,u}\cdot (P_{D,u}^t)^2$ ost coefficients for the *i*-th retrofitted<br>  $B_{i,u}$  and  $N_u$  indicate the total number of<br>
t and the total number of agents,<br>
nimization issue in an LTS task is to<br>
ing operation times and the minimum<br>
window. This chapt it in the *u*-th agent of WTBPS;  $N_{G,u}$  and  $N_u$  indicate the total number of<br>oal-fired units in the *u*-th agent and the total number of agents,<br>addressing the operational cost minimization issue in an LTS task is to<br>dl respectively.<br>
Moreover, addressing the operational cost minimization issue in an LTS task is to<br>
seek the smallest number of total circuit switching operation times and the minimum<br>
amount of dispatchable loads during a

$$
F_{o2} = \sum_{t=1}^{N_{T_s}} \sum_{u=1}^{N_u} \left\{ c_{D,u} \cdot (P_{D,u}^t)^2 + c_{H,u} \sum_{j=1}^{N_s} (S_j^t - S_j^{t-1})^2 \right\}.
$$
 After applying  $z_j^t = S_j^t S_j^{t-1}$  to  $F_{o2}$ ,  $F_{o2}$ 

$$
F_{o2} = \sum_{t=1}^{N_{T_s}} \sum_{u=1}^{N_u} \left\{ c_{D,u} \cdot (P_{D,u}^t)^2 + c_{H,u} \sum_{j=1}^{N_s} (S_j^t - S_j^{t-1})^2 \right\}
$$
(6.42)

 $F_{o2} = \sum_{t=1}^{N_{T_s}} \sum_{u=1}^{N_u} \left\{ c_{D,u} \cdot (P'_{D,u})^2 + c_{H,u} \sum_{j=1}^{N_u} (S'_j - S'^{-1}_j)^2 \right\}$  (6.42)<br>In light of all these facts, we consider that two objectives are assumed to have equal<br>iorities and weighting coefficients are  $F_{\alpha 2} = \sum_{t=1}^{N_{\alpha}} \sum_{u=1}^{N_u} \left\{ c_{D,u} \cdot (P'_{D,u})^2 + c_{H,u} \sum_{j=1}^{N_u} (S'_j - S'^{-1}_j)^2 \right\}$  (6.42)<br>In light of all these facts, we consider that two objectives are assumed to have equal<br>priorities and weighting coefficients  $F_{\text{o}2} = \sum_{i=1}^{N_h} \sum_{u=1}^{N_e} \left\{ c_{\text{D},u} \cdot (P_{\text{D},u}^u)^2 + c_{\text{H},u} \sum_{j=1}^{N_e} (S_j^t - S_j^{t-1})^2 \right\}$  (6.42)<br>In light of all these facts, we consider that two objectives are assumed to have equal<br>priorities and weighting  $F_{\text{o,2}} = \sum_{r=1}^{N_{\text{f}}} \sum_{u=1}^{N_{\text{e}}} \left\{ c_{0,u} \cdot (P_{0,u}^{\prime})^2 + c_{\text{H},u} \sum_{j=1}^{N_{\text{e}}} (S_j^{\prime} - S_j^{\prime -1})^2 \right\}$  (6.42)<br>
In light of all these facts, we consider that two objectives are assumed to have equal<br>
priorities a  $F_{e2} = \sum_{i=1}^{N_E} \sum_{u=1}^{N_E} \left\{ c_{\text{D},u} \cdot (P_{\text{D},u}^i)^2 + c_{\text{H},u} \sum_{j=1}^{N_E} (S_j^i - S_j^i)^2 \right\}$  (6.42)<br>In light of all these facts, we consider that two objectives are assumed to have equal<br>priorities and weighting coeff  $F_{\text{c},3} = \sum_{r=1}^{N_{\text{c}}}\sum_{\text{e}}^{N_{\text{c}}} \left\{c_{\text{D},\text{u}} \cdot (P_{\text{D},\text{u}}^{\text{f}})^2 + c_{\text{R},\text{a}} \sum_{\text{e}}^{N_{\text{c}}} (S_i^{\text{f}} - S_i^{\text{f-1}})^2 \right\}$  (6.42)<br>
In light of all these facts, we consider that two objectives are assumed to iorities and weighting coefficients are selected as  $\mu_1 = \mu_2 = 1/2$ . Thus, a look-ahead<br>
Iling economic dispatch model is the minimization of  $F_o$  subject to the power<br>
lance of tic-lines (6.1), operational limits of ret

$$
\min_{r_{i,u}^t, P_{D,u}^t, P_{T,u}^t \in \mathbb{R}, S_i^t, z_j^t \in \mathbb{N}} F_o = \mu_1 F_{o1} + \mu_2 F_{o2}
$$
\n  
s.t. (6.1), (6.3*a*)-(6.3*c*),(6.4*a*)-(6.4*c*),(6.12)-(6.23),  $\forall t \in T_s$  (6.43)

$$
\min_{r'_{i,u}, P'_{D,u}, P'_{1,u}, m_{\text{upper}}, m_{\text{lower}}, J_{\text{upper}}, J_{\text{lower}} \in \mathbb{R}, S'_i, z'_j \in \mathbb{N}} F_o = \mu_1 F_{o1} + \mu_2 F_{o2}
$$
\ns.t. (6.1), (6.3a)-(6.3c), (6.4a)-(6.4c), (6.5)-(6.14), (6.25a), (6.25b)  $\forall t \in T_s$  (6.44)

4a)-(6.4c) and (6.12), and linear ramping constraints of retrofitted coal-fired units<br>
22)-(6.23) or SOC ramping constraints of retrofitted coal-fired units (6.41a) and<br>
41b), plus simplified voltage-constrained LTS via H (6.22)-(6.23) or SOC ramping constraints of retrofitted coal-fired units (6.41a) and<br>
(6.41b), plus simplified voltage-constrained LTS via HVDNs (6.13)-(6.21).<br> **P1**: Rolling cconomic dispatch model with linear ramping co (6.41b), plus simplified voltage-constrained LTS via HVDNs (6.13)-(6.21).<br> **P1:** Rolling economic dispatch model with linear ramping constraints<br>  $\frac{\sin n}{\sqrt{2\pi\sigma_{\text{tot}}R_{\text{tot}}R_{\text{tot}}}}$   $F_a = \mu_i F_{a1} + \mu_2 F_{a2}$ <br> **c.** (6.1), as  $y_{1,u} := \{ (r_u^t, P_{D,u}^t, P_{T,u}^t) \in \mathbb{R}, t \in T_s \}$  and  $Y_1 = \{ y_{1,u} \}$ ,  $u=1,2,..., N_u$  for (6.43) and  $y_{2,u} := \{ (r_u^t, P_{D,u}^t, P_{T,u}^t, m_{\text{upper}}, m_{\text{lower}}, J_{\text{upper}}, J_{\text{lower}}) \in \mathbb{R}, t \in T_s \}$  and  $Y_2 = \{ y_{2,u} \}$ ,  $u=1$ , **P2**: Rolling economic dispatch model with SOC ramping constraints<br>  $\lim_{t'_a,t''_{b,a},t''_{b,a},m_{\text{max}},t'_{\text{max}},t'_{\text{max}},t'_{\text{max}},t'_{\text{max}},t'_{\text{max}}$  is  $\in$   $\mu_1F_{s_0}$  +  $\mu_2F_{s_2}$  (6.44)<br>
s.t. (6.1), (6.3a)-(6.3c), (6.4a)-(6.4c), other category of discrete variables refers to <sup>s</sup>  $\mu_2 F_{o2}$  (6.44)<br>
(6.14), (6.25a), (6.25b)  $\forall t \in T_s$  (6.44)<br>
ch model (6.43) or (6.44) has different<br>
(6.44) in a more compact model form.<br>
f continuous variables is defined<br>  $\{ \alpha_i =: \{ y_{i,u} \}$ ,  $u=1,2,..., N_u$  for (6.43)<br> of generality, we further modify (6.43) and (6.43) and (6.44) in more general forms as<br>
of generality, we further model  $(6.43)$  or  $(6.44)$  has different<br>
groups of variables, we can express  $(6.43)$  or  $(6.44)$  in a mor Since this proposed rolling economic dispatch model (6.43) or (6.44) has different<br>groups of variables, we can express (6.43) or (6.44) in a more compact model form.<br>To avoid heavy notion, one group of continuous variable

Compact P1 subject to linear constraints:

\n
$$
\min_{X \in \mathbb{N}, Y_i \in \mathbb{R}} \mu_1 (Y_1^T D_{q_1}^y Y_1 + (D_{l_1}^y)^T Y_1) + \mu_2 (X^T D_q^x X + (D_l^x)^T X)
$$
\n(6.45a)

\n
$$
s.t. \mathbf{A} \cdot \mathbf{X} + \mathbf{B} \cdot \mathbf{Y} < \mathbf{b}
$$
\n(6.45b)

$$
s.t. \quad A_l \cdot X + B_l \cdot Y_1 \le b_l \tag{6.45b}
$$

$$
E_t \cdot Y_1 \le h_t \tag{6.45c}
$$

$$
\boldsymbol{F}_l \cdot \boldsymbol{X} \le \boldsymbol{r}_{le} \tag{6.45d}
$$

$$
G_l \cdot X = r_k \tag{6.45e}
$$

where 
$$
Y_1^T D_{q_1}^{\gamma} Y_1 + (D_{l_1}^{\gamma})^T Y_1 = \sum_{t=1}^{N_{T_s}} \sum_{u=1}^{N_u} \sum_{i=1}^{N_{G,u}} [c_{2i,u} (r_{i,u}^t P_{Gi,u,\max})^2 + c_{1i,u} r_{i,u}^t P_{Gi,u,\max}] \text{ and } D_{q_1}^{\gamma}, D_{q}^{\gamma},
$$

 $D_l^y$  and  $D_l^x$  in (6.45a) are constant matrices summarizing from (6.41) and (6.42). Compact P1 subject to linear constraints:<br>  $\lim_{x \to i, 1 \to \infty} H_i (Y_i^T D_s^y Y_i + (D_i^x)^T Y_i) + \mu_2 (X^T D_s^x X + (D_i^x)^T X)$  (6.45a)<br>  $s.t. A_i \cdot X + B_i \cdot Y_i \leq b_i$  (6.45b)<br>  $E_i \cdot Y_i \leq h_i$  (6.45c)<br>  $F_i \cdot X \leq r_i$  (6.45c)<br>  $G_i \cdot X - r_k$  (6.45d)<br>
where  $\begin{array}{lll} \displaystyle \min_{X\in\mathbb{R}\setminus I_f\in\mathbb{R}} \mu_1(Y_i^T D_u^y Y_i + (D_i^y)^T Y_i) + \mu_2(X^T D_u^y X + (D_i^y)^T X) & (6.45a) \\[2mm] & \text{s.t.} & A_i \cdot X + B_i \cdot Y_i \leq b_i & (6.45b) \\[2mm] & E_i \cdot Y_i \leq h_i & (6.45c) \\[2mm] & F_i \cdot X \leq r_k & (6.45d) \\[2mm] & G_i \cdot X = r_k & (6.45d) \\[2mm] & G_i \cdot X = r_k & (6.45$ *st.*  $A_i \cdot X + B_i \cdot Y_1 \le b_i$  (6.45b)<br>  $E_i \cdot Y_1 \le h_i$  (6.45c)<br>  $F_i \cdot X \le r_k$  (6.45d)<br>  $G_i \cdot X = r_b$  (6.45d)<br>
Where  $Y_1^T D_{ij}^N Y_1 + (D_{ij}^N)^T Y_1 = \sum_{i=1}^{N_c} \sum_{i=1}^{N_c} [c_{2i,a} (r_{i,a}^i P_{(i,a,max})^2 + c_{i,i,a} r_{i,a}^i P_{(i,a,max)}]$  and  $D_{ij}^N$ ,  $D_{$  $E_i \cdot Y_i \le h_i$  (6.45c)<br>  $F_i \cdot X \le r_k$  (6.45d)<br>  $G_i \cdot X - r_k$  (6.45d)<br>
Where  $Y_i^T D_n^y Y_i + (D_i^y)^T Y_i = \sum_{i=1}^{N_E} \sum_{u=1}^{N_D} [c_{2i,u} (r_{i,u}^t P_{G_i,u,max})^2 + c_{1i,u} r_{i,u}^t P_{G_i(u,max})$  and  $D_n^y$ ,  $D_q^x$ ,<br>  $D_{i_1}^y$  and  $D_i^t$  in (6.45a) are c  $E_i \cdot X \le r_k$  (6.45d)<br>  $G_i \cdot X = r_k$  (6.45d)<br>  $G_i \cdot X = r_k$  (6.45e)<br>
where  $Y_i^T D_{ij}^{\alpha} Y_i + (D_{i_i}^{\gamma})^T Y_i = \sum_{i=1}^{N_k} \sum_{k=1}^{N_k} \sum_{l=1}^{N_k} (E_{2j_{il}} (r_{i_{il}}^{\alpha} P_{0j_{il}, \text{max}})^2 + c_{1l,kl} r_{i_{il}}^{\alpha} P_{0j_{il}, \text{max}}$  and  $D_{ij}^{\gamma}$ ,  $D_{ij}^{\gamma}$  $F_1 \cdot X \leq r_k$  (6.450)<br>  $G_i \cdot X \to r_k$  (6.45e)<br>
where  $Y_i^T D_{ij}^N Y_1 + (D_{i}^N)^T Y_1 = \sum_{i=1}^{N_s} \sum_{i=1}^{N_s} \sum_{i=1}^{N_s} [c_{2j,a} (r_{is}^i P_{(3j,a,max)})^2 + c_{ijs} r_{is}^i P_{(3j,a,max)}]$  and  $D_{ij}^N$ ,  $D_{ij}^S$ ,  $D_{ij}^N$  and  $D_i^S$  in (6.45a) are cons  $G_t \cdot X = r_h$  (6.45e)<br>
where  $Y_1^T D_{q_1}^N Y_1 + (D_{l_1}^N)^T Y_1 = \sum_{i=1}^{N_c} \sum_{i=1}^{N_c} [c_{2i,a} (r_{i,a}^i P_{0i,a,\text{max}})^2 + c_{1i,a} r_{i,a}^i P_{0i,a,\text{max}}]$  and  $D_q^N$ ,  $D_q^N$  and  $D_i^T$  in (6.45a) are constant matrices summarizing from (6.41)  $D_i^y$  and  $D_i^z$  in (6.45a) are constant matrices summarizing from (6.41) and (6.42).<br>Constraint (6.45b) refers to constraint (6.1) which  $A_i$ ,  $B_i$  and  $b_i$  are constant vectors<br>drawn from constraint (6.1). Constraint (  $y_i^2$  and  $D_i^*$  in (6.45a) are constant matrices summarizing from (6.41) and (6.42).<br>
constraint (6.45b) refers to constraint (6.1) which  $A_i$ ,  $B_i$  and  $b_i$  are constant vectors<br>
awn from constraint (6.1). Constraint (

$$
\min_{X \in \mathbb{N}, Y_2 \in \mathbb{R}} \mu_1(Y_2^T D_{q_2}^y Y_2 + (D_{l_2}^y)^T Y_2) + \mu_2(X^T D_q^x X + (D_l^x)^T X) \tag{6.46a}
$$

$$
s.t. \quad A_l \cdot X + B_l \cdot Y_2 \le b_l \tag{6.46b}
$$

$$
E_t \cdot Y_2 \le h_t \tag{6.46c}
$$

$$
\boldsymbol{F}_l \cdot \boldsymbol{X} \le \boldsymbol{r}_{le} \tag{6.46d}
$$

$$
G_l \cdot X = r_k \tag{6.46e}
$$

$$
\boldsymbol{Y}_2^T \boldsymbol{Q}_r \boldsymbol{Y}_2 + \boldsymbol{l}_r^T \boldsymbol{Y}_2 \leq \boldsymbol{g}_r \tag{6.46f}
$$

where  $Y_2^T D_y^y Y_2 + (D_y^y)^T Y_2 = \sum_{i=1}^{N_{T_s}} \sum_{j=1}^{N_{G_i}} \sum_{j=1}^{N_{G_i}}$  $2^{\sim}2^{\sim}$   $\sqrt{-1}2^{\sim}$  $T_{\mathbf{D}}^y \mathbf{V} + (\mathbf{D}^y)^T \mathbf{V} = \sum_{\alpha} \sum_{\alpha} \sum_{\alpha} \sum_{\alpha} \mathbf{V}^{\alpha} \mathbf{V}^{\alpha} \mathbf{V}^{\alpha}$  $2 \left[ D_{q_2}^r I_2 + (D_{l_2}^r) I_2 \right] = \sum_{i} \sum_{i} [C_{2i,u} (V_{i,u}^r C_{i,u,max}) + C_{1i,u} V_{i,u}^r C_{i,u,max}]$  $\frac{1}{1}$   $\frac{1}{u-1}$   $\frac{1}{i-1}$  $\left( \begin{array}{l l} \sum\limits_{i=1}^N Y_1 + (D_k^\nu)^T Y_2 = \sum\limits_{i=1}^{N_{T_{\rm s}}} \sum\limits_{i=1}^{N_{u}} \sum\limits_{i=1}^{N_{G,u}} [c_{2i,u} (r_{i,u}^\mu P_{{\rm G}i,u,{\rm max}})^2 + c_{1i,u} r_{i,u}^\mu P_{{\rm G}i,u,{\rm max}} ] \end{array} \right) \; ,$  $\mathcal{L}_{q_2}^{\mathbf{r}} \mathbf{I}_2 + (\mathcal{D}_{l_2}^{\mathbf{r}})$   $\mathbf{I}_2 = \sum_{i} \sum_{i} \sum_{i} [C_{2i,u} (V_{i,u}^{\mathbf{r}} C_{i,u}^{\mathbf{r}} + C_{1i,u}^{\mathbf{r}} C$  $\overline{t=1}$   $\overline{u=1}$   $\overline{i=1}$  $D_a^{\gamma} Y_2 + (D_b^{\gamma})^T Y_2 = \sum_{i} \sum_{j} \sum_{j} \sum_{l} \left[ c_{2i,u} (r_{i,u}^t P_{Gi,u, \text{max}})^2 + c_{1i,u} r_{i,u}^t P_{Gi,u} \right]$  $\sum_{i=1}^{n} \sum_{u=1}^{n} \sum_{i=1}^{n} [c_{2i,u} (r_{i,u}^t P_{Gi,u,\text{max}})^2 + c_{1i,u} r_{i,u}^t P_{Gi,u,\text{max}}]$ , and  $D_{q_2}^y$  $Y_2^T Q_r Y_2 + l_r^T Y_2 \leq g_r$  (6.46f)<br>  $Y_2^T D_{q_2}^y Y_2 + (D_{l_2}^y)^T Y_2 = \sum_{t=1}^{N_{l_2}} \sum_{u=1}^{N_{u}} \sum_{i=1}^{N_{u}} [c_{2i,u} (r_{i,u}^t P_{Gi,u,mux})^2 + c_{1i,u} r_{i,u}^t P_{Gi,u,mux}]$ , and  $D_{q_2}^y$ <br>
in (6.46a) are constant matrices. Quadratic constra

and  $D_{l_2}^{\gamma}$  in (6.46a) are constant ma  $Y_1^T Q_r Y_2 + I_r^T Y_2 \leq g_r$  (6.46f)<br>
re  $Y_2^T D_{q_2}^V Y_2 + (D_{l_2}^V)^T Y_2 = \sum_{r=1}^{N_{q_2}} \sum_{u=1}^{N_{r}} \sum_{i=1}^{N_{r}N_{r}N_{r}m} \left[ C_{2l,u} (r_{i,u}^t P_{Gi,u,max})^2 + C_{1l,u} r_{i,u}^t P_{Gi,u,max} \right]$ , and  $D_{q_2}^V$ <br>  $D_{l_2}^V$  in (6.46a) are constant **can be view**  $\mathbf{Y}_2^T \mathbf{Q}_r \mathbf{Y}_2 + \mathbf{I}_r^T \mathbf{Y}_2 \leq \mathbf{g}_r$ **.** (6.46f)<br>
where  $\mathbf{Y}_2^T D_{q_2}^V \mathbf{Y}_2 + (D_{l_2}^V)^T \mathbf{Y}_2 = \sum_{i=1}^{N_{l_2}} \sum_{u=1}^{N_{l_2}} \sum_{i=1}^{N_{l_2}} [C_{2i,u} (r'_{i,u} P_{Gi,u,\text{max}})^2 + C_{1i,u} r'_{i,u} P_{Gi,u,\text{max}}]$  $Y_1^T Q_r Y_2 + I_r^T Y_2 \leq g_r$  (6.46f)<br>
where  $Y_2^T D_0^Y Y_2 + (D_0^Y)^T Y_2 = \sum_{i=1}^{N_E} \sum_{i=1}^{N_E} \sum_{i=1}^{N_E} [C_{2i,u} (Y_{i,u}^T P_{Gi,u,\text{max}})^2 + C_{i,u} Y_{i,u}^T P_{Gi,u,\text{max}}]$ , and  $D_{\ell_2}^Y$ <br>
and  $D_{\ell_2}^Y$  in (6.46a) are constant matrices.  $Y_i^T Q_i Y_1 + I_i^T Y_2 \leq g_i$ , (6.461)<br>
where  $Y_2^T D_{ij}^V Y_2 + (D_{ij}^V)^T Y_2 = \sum_{i=1}^{N_L} \sum_{i=1}^{N_C} [c_{2i,a} (r_{ij}^L P_{Gix,max})^2 + c_{1i,a} r_{ix}^L P_{Gix,max}]$ , and  $D_{ij}^V$ <br>
and  $D_{ij}^V$  in (6.46a) are constant matrices. Quadratic constraints  $Y_2^T Q_1 Y_2 + I_1^T Y_2 \leq g$ , (6.46f)<br>
here  $Y_2^T D_{ij}^N Y_2 + (D_{ij}^N)^T Y_2 = \sum_{i=1}^{N_L} \sum_{u=1}^{N_u} \sum_{l=1}^{N_u} [C_{2j,u} (Y_{i,u}^I P_{G(i,u,mux})^2 + C_{1i,u} Y_{i,u}^I P_{G(i,u,mux})]$ , and  $D_{ij}^N$ <br>
id  $D_{ij}^N$  in (6.46a) are constant matrices. Quadra  $Y_2 Q_y Y_1 + l_y Y_2 \leq g$ , (6.461)<br>
where  $Y_2^T D_{\varphi_1}^y Y_2 + (D_{l_1}^y)^T Y_2 = \sum_{i=1}^{N_2} \sum_{i=1}^{N_i} [c_{2j,a} (r_{i,a}^i P_{G,i,a,\text{max}})^2 + c_{1i,a} r_{i,a}^i P_{G,i,a,\text{max}}]$ , and  $D_{\varphi_2}^y$ <br>
and  $D_{l_2}^y$  in (6.46a) are constant matrices. Quad

where  $Y_2^T D_{y_2}^V Y_1 + (D_{\xi}^V)^T Y_2 = \sum_{i=1}^{N_E} \sum_{i=1}^{N_E} [C_{2j,n} (V'_{i,n} P_{G,i,n,\text{max}})^2 + C_{1i,n} V'_{i,n} P_{G,i,n,\text{max}}]$ , and  $D_{y_2}^V$  and  $D_{y_2}^V$  in (6.466) are constant matrices. Quadratic constraints in constraint (6.46f) and  $D_{l_0}^{r}$  in (6.46a) are constant matrices. Quadratic constraints in constraint (6.46f)<br>can automatically convert to standard rotated SOC constraints with the auxiliary<br>variables ( $J_{\text{upppers}}$ ,  $m_{\text{upppers}}$ ,  $J_{\text{downers}}$ and  $D_{\xi}^{x}$  in (6.46a) are constant matrices. Quadratic constraints in constraint (6.461)<br>can automatically convert to standard rotated SOC constraints with the auxiliary<br>variables ( $J_{\text{upper}}$ ,  $m_{\text{upper}}$ ,  $J_{\text{lower}}$ ,  $m_{$ can automatically convert to standard rotated SOC constraints with the auxiliary<br>variables ( $J_{\text{hyperper}}$ ,  $J_{\text{lower}}$ ,  $m_{\text{lower}}$ ,  $m_{\text{lower}}$ ,  $m_{\text{lower}}$ ). The constant matrices  $Q$ ,  $L$  and  $g$ , are symbols of<br> $a_{\text{AugPer}}$ ,  $b_{\text$ variables (J<sub>upper</sub>, *B<sub>L</sub>*op<sub>per</sub>, *B<sub>L</sub>*opper, *B*<sub>L</sub>opper, *B*<sub>L</sub>opper, *B*  $a_{\text{Supper}}$ ,  $b_{\text{Supper}}$ ,  $a_{\text{Supper}}$ ,  $a_{\text{Supper}}$ ,  $a_{\text{Supper}}$  and  $c_{\text{Supper}}$ .<br>
6.3.2 Multi-cut Generalized Benders Decomposition<br>
As observed in compact **P1 and P2**, this look-ahead economic dispatch model is a<br>
MISOCP probl framework. As observed in compact **P1 and P2**, this look-ahead economic dispatch model is a<br>ISOCP problem. This MISOCP-based optimization problem is established on<br>rge-scale HVDNs and multiple agents of WTBPS, which contains a subst MISOCP problem. This MISOCP-based optimization problem is established on<br>large-scale HVDNs and multiple agents of WTBPS, which contains a substantial<br>number of continuous and integer variables during the look-ahead period large-scale HVDNs and multiple agents of WTBPS, which contains a substantial<br>number of continuous and integer variables during the look-ahead period. Running<br>this large-scale optimization model mixed with the LTS-based ne

number of continuous and integer variables during the look-ahead period. Running<br>this large-scale optimization model mixed with the LTS-based network operation<br>model is inevitably time-consuming. Fortunately, the compact this large-scale optimization model mixed with the LTS-based network operation<br>model is inevitably time-consuming. Fortunately, the compact **P1** and **P2** can be<br>decomposed into a relaxed master problem (*MP*) with respect model is inevitably time-consuming. Fortunately, the compact **P1** and **P2** can be decomposed into a relaxed master problem  $(MP)$  with respect to X and many sub-problems with respect to Y<sub>1</sub> or Y<sub>2</sub>, which perfectly suits decomposed into a relaxed master problem (*MP*) with respect to *X* and many<br>sub-problems with respect to *Y*<sub>1</sub> or *Y*<sub>2</sub>, which perfectly suits GBD decomposition<br>framework.<br>Given disercte variables  $\hat{X}$  from *MP*, we
1 <sup>2</sup> ˆ Agent 1 Agent 2 Agent <sup>u</sup> <sup>N</sup><sup>u</sup> N <sup>u</sup> u u l,1 l,1 l,1 l,2 l,2 l,2 l, l,N N l,N B y b A B y b A X B y b A (6.47) We summarize this special ′′block-angular′′ decomposable structure in (6.47) with three characteristics as convexity, linear separability and linear independence. <sup>i</sup>) Convexity: P2 is all convex on continuous variables yu with given the discrete ii) Linear separability: constraints (6.45b)-(6.45e) and (6.46b)-(6.46e) are all linear on discrete variables <sup>X</sup> with given the continuous variables <sup>ˆ</sup>

variables  $\hat{X}$ ;

linear on discrete variables X with given the continuous variables  $\hat{y}_u$ ;

Agent 1<br>
Agent  $N_e$ <br>  $\begin{bmatrix}\n\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots\n\end{bmatrix}\n\begin{bmatrix}\ny_i \\
y_i \\
z_i \\
y_{i}\n\end{bmatrix}\n=\n\begin{bmatrix}\n\ddots \\
\ddots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots\n\end{bmatrix}\n\begin{bmatrix}\n\ddots \\
\ddots \\
\vdots \\
\vdots \\
\vdots \\
\vdots\n\end{bmatrix}\n\begin{bmatrix}\n\ddots \\
\ddots \\
\vdots \\
\vdots \\
\vdots \\
\vdots\n\end{bmatrix}\n\begin$ continuous variables yu are linearly independent. Under these three characteristics, it **Example 19**  $\left[\cos\theta + \frac{1}{2}\cos\theta + \frac$ We summarize this special "block-angular" decomposable structure in (6.47) with<br>three characteristies as convexity, linear separability and linear independence.<br> *i*) **Convexity: P2** is all convex on continuous variables three characteristics as convextly, incer separability and linear independence.<br> *i*) **Convextly:** P2 is all convex on continuous variables  $y_n$  with given the disercte variables  $\hat{\chi}$ :<br> *ii*) **Linear separability**: con *a*) **Convextiy:** P2 is all convex on continuous variables  $y_u$  with given the discrete variables  $\hat{X}$ ;<br> *ii*) **Linear separability**: constraints (6.45b)-(6.45e) and (6.46b)-(6.46e) are all linear on discrete variables variables  $\chi$ :<br>
ii) Linear separability: constraints (6.45b)-(6.45e) and (6.46b)-(6.46e) are all<br>
linear on discrete variables  $X$  with given the continuous variables  $\hat{y}_a$ :<br>
iii) Linear independence: with given discr *ii*) Linear separability: constraints (6.45b)-(6.45c) and (6.46b)-(6.46c) are all<br>linear on discrete variables *X* with given the continuous variables  $\hat{y}_s$ ;<br>*iii*) Linear independence: with given discrete variables linear on diserte variables X with given the continuous variables  $\hat{y}_n$ ;<br> *iii*) **Linear independence**: with given discrete variables  $\hat{y}$  different groups of<br>
continuous variables  $y_u$  are linearly independent. Und *iii*) Linear independence: with given discrete variables  $\hat{X}$ , different groups of continuous variables  $y_u$  are linearly independent. Under these three characteristics, it is highly desirable to utilize the GBD metho Intinuous variables  $y_u$  are linearly independent. Under these three characteristics, it highly desirable to utilize the GBD method with multiple cuts as MGBD. MGBD and decompose the sub-problem of GBD into multiple indep is highly desirable to utilize the GBD method with multiple cuts as MGBD. MGBD<br>can decompose the sub-problem of GBD into multiple independent smaller<br>optimization models by (6.47), which sub-problems engender multiple fea

variables  $\hat{X}^{k-1}$ :

$$
\min_{\mathbf{y}_{2,u}\in\mathbb{R}} \mu_1(\mathbf{y}_{2,u}^T D_{q_2}^{\mathbf{y}} \mathbf{y}_{2,u} + (D_{l_2}^{\mathbf{y}})^T \mathbf{y}_{2,u}) + \mu_2((\hat{\mathbf{X}}^{k-1})^T D_q^{\mathbf{x}} \hat{\mathbf{X}}^{k-1} + (D_l^{\mathbf{x}})^T \hat{\mathbf{X}}^{k-1}) \tag{6.48a}
$$

s.t. 
$$
\mathbf{B}_{l,u} \cdot \mathbf{y}_{2,u} \le \mathbf{b}_{l,u} - A_{l,u} \cdot \hat{\mathbf{X}}^{k-1}
$$
 (6.48b)

$$
E_{t,u} \cdot y_{2,u} \le h_{t,u} \tag{6.48c}
$$

$$
\mathbf{y}_{2,u}^T \mathbf{Q}_{r,u} \mathbf{y}_{2,u} + \mathbf{l}_{r,u}^T \mathbf{y}_{2,u} \leq \mathbf{g}_{r,u}
$$
 (6.48d)

where  $A_i = \{A_{i,u}\}\;$ ,  $I_r = \{I_{r,u}\}\;$ ,  $B_i = \{B_{i,u}\}\;$ ,  $b_i = \{b_{i,u}\}\;$ ,  $E_i = \{E_{i,u}\}\;$ ,  $h_i = \{h_{i,u}\}\;$ ,  ${E}_{ia} \cdot y_{2,n} \le h_{ta}$  (6.48c)<br>  $y_{2,n}^T Q_{ca} y_{2,n} + t_{ca}^T y_{2,n} \le g_{ca}$  (6.48d)<br>
where  $A_i = \{A_{ta}\}, \quad I_i = \{I_{ca}\}, \quad B_i = \{B_{ta}\}, \quad b_i = \{b_{ta}\}, \quad E_i = \{E_{ta}\}, \quad h_i = \{h_{ta}\},$ <br>  $Q_i = \{Q_{ra}\}$  and  $g_i = \{g_{ra}\}$  for  $u=1, 2, ..., N_u$ .<br>
If this  $SP_u$  pro (6.48c)<br>
(6.48d)<br>  $\}, E_t = \{E_{t,u}\}, h_t = \{h_{t,u}\},$ <br>  $\hat{v}_{2,u}^k$ , the upper bound

this  $SP_u$  problem is optimal with  $\hat{y}_{2,u}^k$ , the upper is

$$
E_{t,u} \cdot y_{2,u} \le h_{t,u} \qquad (6.48c)
$$
\n
$$
y_{2,u}^T Q_{r,u} y_{2,u} + I_{r,u}^T y_{2,u} \le g_{r,u} \qquad (6.48d)
$$
\nwhere  $A_t = \{A_{t,u}\}, I_r = \{I_{r,u}\}, B_t = \{B_{t,u}\}, b_t = \{b_{t,u}\}, E_t = \{E_{t,u}\}, h_t = \{h_{t,u}\},$ \n
$$
Q_r = \{Q_{r,u}\} \text{ and } g_r = \{g_{r,u}\} \text{ for } u=1, 2, ..., N_u.
$$
\nIf this  $SP_u$  problem is optimal with  $\hat{y}_{2,u}^k$ , the upper bound is\n
$$
UB_k = \min \{UB_{k-1}, \mu_1 \sum_{u=1}^{N_u} ((\hat{y}_{2,u}^k)^T D_{q_1}^v \hat{y}_{2,u}^k) + (\mu_2 ((\hat{X}^{k-1})^T D_q^v \hat{X}^{k-1} + (D_l^v)^T \hat{X}^{k-1}))\}
$$
\n
$$
\vdots \text{ otherwise it is infeasible, we turn to solve an } l_1\text{-minimization feasibility check\nproblem with the relaxed variable  $\sigma_u \ge 0$  where for  $u=1, 2, ..., N_u$  as follows:\n
$$
\min_{y_u, \sigma_u \in \mathbb{R}} \sigma_u \qquad (6.49a)
$$
$$

$$
\min_{\mathbf{y}_u, \sigma_u \in \mathbb{R}} \sigma_u \tag{6.49a}
$$

s.t. 
$$
\mathbf{B}_{l,u} \cdot \mathbf{y}_{2,u} - \sigma_u \le \mathbf{b}_{l,u} - A_{l,u} \cdot \hat{\mathbf{X}}^{k-1}
$$
 (6.49b)

$$
E_{t,u} \cdot y_{2,u} - \sigma_u \le h_{t,u} \tag{6.49c}
$$

$$
y_{2,u}^T Q_{r,u} y_{2,u} + l_{r,u}^T y_{2,u} - \sigma_u \leq g_{r,u}
$$
 (6.49d)

 $B_k = \min \{UB_{k-1}, \mu_1 \sum_{u=1}^{N_1} ((\hat{J}_{2,u}^{k})^T D_{k}^{v} \hat{J}_{2,u}^{k} + (D_{k}^{v})^T \hat{J}_{2,u}^{k}) + \mu_2 ((\hat{X}^{k-1})^T D_{q}^{v} \hat{X}^{k-1} + (D_{l}^{v})^T \hat{X}^{k-1})\}$ <br>
otherwise it is infeasible, we turn to solve an *l*<sub>1</sub>-minimization feasibil  $UB_k = \min\{UB_{k+1}, \mu_1 \sum_{n=1}^{\infty} ((\hat{y}_{2,n}^{k})^T D_{xy}^{v_k} \hat{y}_{2,n}^{k} + (D_{ij}^{v_k})^T \hat{y}_{2,n}^{k+1}) + \mu_2((\hat{X}^{k+1})^T D_{ij}^{v_k} \hat{X}^{k+1} + (D_{ij}^{v_k})^T \hat{X}^{k+1})\}$ ; otherwise it is infeasible, we turn to solve an *l<sub>1</sub>*-minimization fe (6.48a)-(6.48d), we substitute the obtained continuous variables  $\hat{y}_{2,u}^k$  and Lagrange  $\hat{\mathbf{x}}^{k-1} + (D_i^x)^T \hat{\mathbf{x}}^{k-1}$ )}<br>
n feasibility check<br>
follows:<br>
(6.49a)<br>
(6.49b)<br>
(6.49c)<br>
(6.49d)<br>
ion as indicated in<br>  $\hat{\mathbf{y}}_{2,\mu}^k$  and Lagrange<br>
(8.49d)<br>
(8.49d)<br>
(8.49d)<br>
(8.49d)<br>
(8.49d)<br>
(8.49d)<br>
(8.49d)<br> problem with the relaxed variable  $\sigma_u \ge 0$  where for  $u=1, 2,..., N_u$  as follows:<br>  $\min_{y_u, \sigma_u \in \mathbb{R}} \sigma_u$ <br>
s.t.  $B_{l,u} \cdot y_{2,u} - \sigma_u \le b_{l,u} - A_{l,u} \cdot \hat{X}^{k-1}$ <br>  $E_{l,u} \cdot y_{2,u} - \sigma_u \le b_{l,u} - A_{l,u} \cdot \hat{X}^{k-1}$ <br>  $E_{l,u} \cdot y_{2,u} - \sigma_u \le b_{l$  $\hat{\lambda}_{u,1}^k$ ,  $\hat{\lambda}_{u,2}^k$ , and  $\hat{\lambda}_{u,3}^k$ ble, we turn to sorve an  $t_1$ -imminization reasonity clieck<br>variable  $\sigma_u \ge 0$  where for  $u=1, 2, ..., N_u$  as follows:<br><br> $\min_{y_u, \sigma_u \in \mathbb{R}} \sigma_u$  (6.49a)<br> $\sum_{y_u, \sigma_u \in \mathbb{R}} \sigma_u \le b_{Lu} - A_{Lu} \cdot \hat{X}^{k-1}$  (6.49b)<br><br> $\sum_{l,u} y_{2,u} - \sigma_u \le b$  $\[\sigma_u \ge 0\]$  where for  $u=1, 2, ..., N_u$  as follows:<br>  $\[\sigma_u \le b_{\mu\mu} - A_{\mu\mu} \cdot \hat{X}^{k-1}\]$  (6.49a)<br>  $\[\sigma_u \le b_{\mu\mu} - A_{\mu\mu} \cdot \hat{X}^{k-1}\]$  (6.49b)<br>  $\[\sigma_u \le h_{\mu\mu}\]$  (6.49c)<br>  $\[\sigma_u \le h_{\mu\mu}\]$  (6.49c)<br>
oblem is feasible at *k*-th (6.49a)<br>
S.t.  $B_{\mu\nu} \cdot y_{z,\mu} - \sigma_{\mu} \leq b_{\mu\nu} - A_{\mu\nu} \cdot \hat{X}^{\lambda+1}$  (6.49b)<br>  $E_{\mu\nu} \cdot y_{2,\mu} - \sigma_{\mu} \leq b_{\mu\nu} - A_{\mu\nu} \cdot \hat{X}^{\lambda+1}$  (6.49b)<br>  $F_{\mu\nu} \cdot y_{2,\mu} - \sigma_{\mu} \leq b_{\mu\nu}$  (6.49d)<br>
On the one hand, if the  $SP_{\mu}$  p  $E_{1a} \cdot y_{2a} - \sigma_a \ge \mathbf{h}_{1a}$ <br>  $y_{2a}^T Q_{2a} y_{2a} + t_{1a}^T y_{2a} - \sigma_a \le g_{2a}$  (6.49d)<br>
On the one hand, if the *SPu* problem is feasible at *k*-th iteration as indicated in<br>
48a)-(6.48d), we substitute the obtained continuou On the one hand, if the *SP<sub>u</sub>* problem is feasible at *k*-th iteration as indicated in  $(6.48a)$ -(6.48d), we substitute the obtained continuous variables  $\hat{y}_{2,n}^k$  and Lagrange multiplier vectors  $\hat{\lambda}_{n,1}^k$ ,  $\hat{\lambda}_{$ 

$$
(\hat{\lambda}_{u,1}^{k})^{T} A_{l,u} X + (\hat{\lambda}_{u,1}^{k})^{T} (B_{l,u} \cdot \hat{y}_{2,u}^{k} - b_{l,u}) + (\hat{\lambda}_{u,2}^{k})^{T} (E_{l,u} \cdot \hat{y}_{2,u}^{k} - h_{l,u}) +
$$
  

$$
(\hat{\lambda}_{u,3}^{k})^{T} ((\hat{y}_{2,u}^{k})^{T} \mathbf{Q}_{r,u} \hat{y}_{2,u}^{k} + \mathbf{I}_{r,u}^{T} \hat{y}_{2,u}^{k} - \mathbf{g}_{r,u}) + \mu_{1} \sum_{u=1}^{N_{u}} ((\hat{y}_{2,u}^{k})^{T} D_{q_{2}}^{y} \hat{y}_{2,u}^{k} + (D_{l_{2}}^{y})^{T} \hat{y}_{2,u}^{k}) \leq q_{u}
$$
(6.50)

and then we substitute continuous variables  $\hat{y}_u^{f,k}$  and Lagrange multiplier vectors  $\hat{\gamma}_{u,1}^k$ ,<br>  $\hat{\gamma}_{u,2}^k$ , and  $\hat{\gamma}_{u,3}^k$  for constraints (6.49b)-(6.49d) to form the feasibility cut:  $f^k_i$  and Lagrange multiplier vectors  $\hat{\gamma}^k_{u,1}$ ,  $\hat{y}_{u}^{f,k}$  and Lagrange multiplier vectors  $\hat{y}_{u,1}^{k}$ ,<br>form the feasibility cut:  $\hat{\gamma}_{u,2}^k$ , and  $\hat{\gamma}_{u,3}^k$  for constraints (6.49b)-(6.49 we substitute continuous variables  $\hat{y}_u^{f,k}$  and Lagrange multiplier vectors  $\hat{\gamma}_{u,1}^k$ ,<br>  $\hat{\gamma}_{u,3}^k$  for constraints  $(6.49b)$ - $(6.49d)$  to form the feasibility cut:<br>  $(\hat{\gamma}_{u,1}^k)^T A_{i,u} X + (\hat{\gamma}_{u,1}^k)^T (\boldsymbol{B}_{i,u} \cdot$ d then we substitute continuous variables  $\hat{y}_a^{fA}$  and Lagrange multiplier vectors  $\hat{\gamma}_{n,1}^k$ ,<br>  $\hat{y}_{n,2}^k$ , and  $\hat{\gamma}_{n,3}^k$  for constraints (6.49b)-(6.49d) to form the feasibility cut:<br>  $(\hat{y}_{n,1}^k)^T A_{n,2} X + (\$ and then we substitute continuous variables  $\hat{y}_u^{tA}$  and Lagrange multiplier vectors  $\hat{j}_{u,1}^k$ ,<br>  $\hat{j}_{u,2}^k$  and  $\hat{j}_{u,3}^k$  for constraints  $(6.49b)$ - $(6.49d)$  to form the feasibility cut:<br>  $(\hat{y}_{u,3}^k)^T A_{i,u} X +$ straints (6.49b)-(6.49d) to form the feasibility cut:<br>
straints (6.49b)-(6.49d) to form the feasibility cut:<br>  ${}^{T}A_{i,x}X + (\hat{\tau}_{u,i}^{k})^{T}(\boldsymbol{B}_{i,u} \cdot \hat{\tau}_{2,u}^{th} - \boldsymbol{b}_{i,u}) + (\hat{\tau}_{u,2}^{k})^{T}(\boldsymbol{E}_{i,u} \cdot \hat{\tau}_{2,u}^{th} - \boldsymbol{h}_{i,u}) +$ <br>

or constraints (6.49b)-(6.49d) to form the feasibility cut:  
\n
$$
(\hat{y}_{u,1}^k)^T A_{l,u} X + (\hat{y}_{u,1}^k)^T (B_{l,u} \cdot \hat{y}_{2,u}^{f,k} - b_{l,u}) + (\hat{y}_{u,2}^k)^T (E_{l,u} \cdot \hat{y}_{2,u}^{f,k} - h_{l,u}) +
$$
\n
$$
(\hat{y}_{u,3}^k)^T ((\hat{y}_{2,u}^{f,k})^T Q_{r,u} \hat{y}_{2,u}^{f,k} + I_{r,u}^T \hat{y}_{2,u}^{f,k} - g_{r,u}) \le 0
$$
\ning the optimality cuts (6.50) and the feasibility cuts (6.51) for  $u=1$ ,

\nstablish the relaxed *MP* model at *k*-th iteration for all agents:

\n
$$
\min_{X, q_u \in \mathbb{R}} \sum_{u=1}^{N_u} q_u + \mu_2 (X^T D_q^x X + (D_l^x)^T X) \qquad (6.52a)
$$
\ns.t.

\n
$$
F_l \cdot X \le r_{l,e}, \quad G_l \cdot X = r_{l,s}, \text{ and } (6.50) \cdot (6.51) \qquad (6.52b)
$$
\nsound is obtained as

\n
$$
LB_k = \mu_2 ((X^k)^T D_q^x X^k + (D_l^x)^T X^k) + \sum_{u=1}^{N_u} q_u^k, \text{ where}
$$
\nare optimal solutions from (6.52a) - (6.52b). Until 
$$
||LB_k - UB_k||
$$
 is less

 $\int_{a}^{L} \frac{d\mathbf{r}}{dt} d\mathbf{r} dt$ <br>  $\int_{a}^{L} \frac{d\mathbf{r}}{dt} d\mathbf{r} dt$ , for constraints  $(6.49b) - (6.49d)$  to form the feasibility cut:<br>  $(\hat{y}_{a,i}^k)^T (A_{i,a}X + (\hat{y}_{a,i}^k)^T (B_{i,a} \cdot \hat{y}_{i,a}^k - \mathbf{b}_{ia}) + (\hat{y}_{a,i}^k)^T (E_{i,a} \cdot \hat{y}_{i,a}^k$ 

$$
\min_{X, q_u \in \mathbb{R}} \sum_{u=1}^{N_u} q_u + \mu_2 (X^T D_q^* X + (D_l^*)^T X) \tag{6.52a}
$$

s.t. 
$$
F_i \cdot X \le r_{le}
$$
,  $G_i \cdot X = r_{ls}$ , and (6.50)-(6.51) 
$$
(6.52b)
$$

1  $((X^k)^T D_a^x X^k + (D_l^x)^T X^k) + \sum_{k=1}^{N_u} q_u^k$  $_{k} = \mu_2((X \cup D_q X + (D_l) X) + \sum_{u=1} q_u$  $LB_{k} = \mu_2((X^k)^T D_a^X X^k + (D_i^X)^T X^k) + \sum_{k=1}^{n} q_{k}^k$  $=$ 

 $X^k$  and  $q_{\mu}^k$  are optimal solutions from  $(\hat{\jmath}_{u_1}^k)^T A_{u_n} X + (\hat{\jmath}_{u_1}^k)^T (B_{u_n} \cdot \hat{\jmath}_{u_n}^{tk} - B_{u_n}) + (\hat{\jmath}_{u_n}^k)^T (E_{u_n} \cdot \hat{\jmath}_{u_n}^{tk} - B_{u_n}) +$ <br>  $(\hat{\jmath}_{u_n}^k)^T ((\hat{\jmath}_{u_n}^{tk})^T Q_{uu} \hat{\jmath}_{u_n}^{tk} + I_{uu}^T \hat{\jmath}_{u_n}^{tk} - B_{uu}) \le 0$ <br>
(mposing the optimality cuts (6.50) and  $(\hat{y}_{n,s}^{k})^T (\hat{y}_{2,s}^{ck})^T Q_{en}\hat{y}_{2,s}^{ck} + I_{en}^T \hat{y}_{2,s}^{ck} - g_{en}) \leq 0$  (6.51)<br>
After imposing the optimality cuts (6.50) and the feasibility cuts (6.51) for  $u=1$ ,<br>
2,...,  $N_u$ , we establish the relaxed MP model at k-th i

After imposing the optimality cuts (6.50) and the feasibility cuts (6.51) for  $u=1$ ,<br>
...,  $N_b$ , we establish the relaxed **MP** model at *k*-th iteration for all agents:<br>  $\min_{X_{\alpha_0}\in\mathbb{R}} \sum_{i=1}^{N_c} q_x + \mu_2 (X^T D_i^* X + (D_i^*)$ 2,...,  $N_u$ , we establish the relaxed *MP* model at *k*-th iteration for all agents:<br>  $\min_{X_{A_u} \in \mathbb{R}} \sum_{n=1}^{N_u} q_u + \mu_2(X^T D_u^* X + (D_i^*)^T X)$  (6.52a)<br>
s.t.  $F_i \cdot X \le r_{i_n}$ ,  $G_i \cdot X = r_{i_n}$ , and (6.50)-(6.51) (6.52b)<br>
The lo  $\lim_{x_{ch}=n} \sum_{e=1}^{N_x} q_e + \mu_2 (X^T D_e^* X + (D_i^*)^T X)$  (6.52a)<br>
s.t.  $F_i \cdot X \le r_{le}$ ,  $G_i \cdot X = r_{li}$ , and  $(6.50)-(6.51)$  (6.52b)<br>
The lower bound is obtained as  $LB_k = \mu_2 ((X^k)^T D_e^* X^* + (D_i^*)^T X^k) + \sum_{e=1}^{N_x} q_e^k$ , where<br>  $X^*$  and  $q$ data to the dispatch center, such as power outputs, power capacities, utilization levels, s.t.  $F_i \cdot X \le r_{i_s}$ ,  $G_i \cdot X = r_{i_s}$ , and  $(6.50)(6.51)$  (6.52b)<br>
The lower bound is obtained as  $LB_k = \mu_2((X^k)^T D_i^* X^k + (D_i^*)^T X^k) + \sum_{s=1}^{N_i} q_{i_s}^*$ , where<br>  $X^k$  and  $q_s^k$  are optimal solutions from  $(6.52a)(6.52b)$ . Unti The lower bound is obtained as  $LB_k = \mu_2((X^k)^T D_q^* X^k + (D_i^*)^T X^k) + \sum_{n=1}^{N} q_k^k$ , where  $X^k$  and  $q_n^k$  are optimal solutions from  $(6.52a)-(6.52b)$ . Until  $||LB_k - UB_k||$  is less than the given tolerance, this MGBD algorithm ca  $X^*$  and  $q'_s$  are optimal solutions from  $(6.52a)$ - $(6.52b)$ . Until  $||LB_s -UB_s||$  is less<br>than the given tolerance, this MGBD algorithm can be converged.<br>This MGBD can generally be solved by servers at the dispatch enter, w  $X^*$  and  $q_u^*$  are optimal solutions from (6.52a)-(6.52b). Until  $||LB_k - UB_k||$  is less<br>than the given tolerance, this MGBD algorithm can be converged.<br>This MGBD can generally be solved by servers at the dispatch center, wh 6.10.



Convergence N<br>
Fig. 6.10. Computation procedure of MGBD for P1 and P2.<br>
6.4 Case Studies<br>
A real modified HVDN system with 220kV and 110kV voltage levels in central<br>
China as shown in Fig. 6.11 [82]. Four agents A<sub>1</sub>-A<sub>4</sub> Fig. 6.10. Computation procedure of MGBD for **P1** and **P2**.<br>
6.4 Case Studies<br>
A real modified HVDN system with 220kV and 110kV voltage levels in central<br>
China as shown in Fig. 6.11 [82]. Four agents A<sub>1</sub>-A<sub>4</sub> of WTBPS ar **PT1-PT4.** This integrated practical system is used for case studies to validate the proposed look-ahead rolling economic dispatch approach.<br> **PT1-PT4.** This integrated practical system is used for case studies to validat **Fig. 6.10. Computation procedure of MGBD for P1 and P2.**<br>6.4 Case Studies<br>A real modified HVDN system with 220kV and 110kV voltage levels in central<br>China as shown in Fig. 6.11 [82]. Four agents A<sub>1</sub>-A<sub>4</sub> of WTBPS are int Fig. 6.10. Computation procedure of MGBD for **P1** and **P2**.<br>
4 Case Studies<br>
A real modified HVDN system with 220kV and 110kV voltage levels in central<br>
hina as shown in Fig. 6.11 [82]. Four agents A<sub>1</sub>-A<sub>4</sub> of WTBPS are 6.4 Case Studies<br>  $\Lambda$  real modified HVDN system with 220kV and 110kV voltage levels in central<br>
China as shown in Fig. 6.11 [82]. Four agents A<sub>1</sub>-A<sub>4</sub> of WTBPS are integrated in four<br>
different 220kV stations, namely st 6.4 Case Studies<br>A real modified HVDN system with 220kV and 110kV voltage levels in central<br>China as shown in Fig. 6.11 [82]. Four agents A<sub>1</sub>-A<sub>4</sub> of WTBPS are integrated in four<br>different 220kV stations, namely station δ.4 Case Studies<br>
A real modified HVDN system with 220kV and 110kV voltage levels in central<br>
China as shown in Fig. 6.11 [82]. Four agents A<sub>1</sub>-A<sub>4</sub> of WTBPS are integrated in four<br>
different 220kV stations, namely stat

(\$/MW<sup>2</sup>h), 18.23 (\$/MWh) and 35.54 (\$). The  $c_{D,u}$  and  $c_{H,u}$  are set to 4.17 h), 18.23 (\$/MWh) and 35.54 (\$). The  $c_{D,\mu}$  and  $c_{H,\mu}$  are set to 4.17<br>
(a) and 10 (\$/p.u.) respectively for all agents. This HVDN system has 17 units<br>
V stations labeled with long rectangle boxes and rated capacities (\$/MW<sup>2</sup>h), 18.23 (\$/MWh) and 35.54 (\$). The  $c_{D,\mu}$  and  $c_{H,\mu}$  are set to 4.17<br>(\$/MWh) and 10 (\$/p.u.) respectively for all agents. This HVDN system has 17 units<br>of 220kV stations labeled with long rectangle boxes and (\$/MW<sup>2</sup>h), 18.23 (\$/MWh) and 35.54 (\$). The  $c_{D,w}$  and  $c_{H,w}$  are set to 4.17 (\$/MWh) and 10 (\$/p.u.) respectively for all agents. This HVDN system has 17 units of 220kV stations labeled with long rectangle boxes and r (\$/MW<sup>2</sup>h), 18.23 (\$/MWh) and 35.54 (\$). The  $c_{D,x}$  and  $c_{H,x}$  are set to 4.17<br>(\$/MWh) and 10 (\$/p.u.) respectively for all agents. This HVDN system has 17 units<br>of 220kV stations labeled with long rectangle boxes and r (S/MW<sup>2</sup>h), 18.23 (S/MWh) and 35.54 (\$). The  $c_{D,n}$  and  $c_{H,n}$  are set to 4.17 (S/MWh) and 10 (S/p.u.) respectively for all agents. This HVDN system has 17 units of 220kV stations labeled with long rectangle boxes and r (\$/MW<sup>2</sup>h), 18.23 (\$/MWh) and 35.54 (\$). The  $c_{b,w}$  and  $c_{B,w}$  are set to 4.17 (\$/MWh) and 10 (\$/p.u.) respectively for all agents. This HVDN system has 17 units of 220kV stations labeled with long rectangle boxes and r (S/MW<sup>2</sup>h), 18.23 (\$/MWh) and 35.54 (\$). The  $c_{D,n}$  and  $c_{N,n}$  are set to 4.17 (\$/MWh) and 10 (\$/p.u.) respectively for all agents. This HVDN system has 17 units of 220kV stations labeled with long rectangle boxes and r (S/MW<sup>2</sup>h), 18.23 (S/MWh) and 35.54 (\$). The  $c_{\rho,w}$  and  $c_{\pi,w}$  are set to 4.17 (S/MWh) and 10 (S/p.u.) respectively for all agents. This HVDN system has 17 units of 220kV stations labeled with long rectangle boxes and (S/MW<sup>2</sup>h), 18.23 (S/MWh) and 35.54 (\$). The  $c_{n,\nu}$  and  $c_{n,\nu}$  are set to 4.17 (S/MWh) and 10 (S/p.u.) respectively for all agents. This HVDN system has 17 units of 220kV stations labeled with long rectangle boxes and (S/MW<sup>2</sup>h), 18.23 (S/MWh) and 35.54 (\$). The  $c_{0,\nu}$  and  $c_{n,\nu}$  are set to 4.17 (S/MWh) and 10 (S/p.u.) respectively for all agents. This HVDN system has 17 units of 220kV stations labeled with long rectangle boxes and (\$/MWh) and 10 (\$/p.u.) respectively for all agents. This HVDN system has 17 u<br>of 220kV stations labeled with long rectangle boxes and rated capacities in li<br>yellow color and 57 units of 110kV substations marked with blac  $\hat{b}/p$ .u.) respectively for all agents. This HVDN system has 17 units<br>labeled with long rectangle boxes and rated capacities in light<br>7 units of 110kV substations marked with black-filled circles. The<br>eakers is shown in of 220kV stations labeled with long rectangle boxes and rated capacities in light yellow color and 57 units of 110kV substations marked with black-filled circles. The status of switch breakers is shown in Fig. 6.11 for th yellow color and 57 units of 110kV substations marked with black-filled circles. The<br>status of switch breakers is shown in Fig. 6.11 for the initial period. The number of<br>allowable switch actions during a rolling window i status of switch breakcrs is shown in Fig. 6.11 for the initial period. The number of<br>allowable switch actions during a rolling window is set to 6 times [103]. Load data for<br>each substation and wind power data for four ag



		Fig. 6.11. A real modified HVDNs in central China.
	Table 6.2. Comparative coal-fired plants	
Types of coal-fired plants	Minimum load level $(\% )$	Ramp rate $(min\%)$
Conventional coal-fired units with normal ramp rate	60	1.1%
Retrofitted coal-fired units with piecewise linear model	30	$v_{i,u}^t = \begin{cases} 0.35\%, & r_{i,u}^t \in [0.3, 0.4] \\ 1.0\%, & r_{i,u}^t \in [0.4, 0.5] \\ 1.5\%, & r_{i,u}^t \in [0.5, 0.6] \end{cases}$
of dynamic ramp rates [78]		$\left[2\%, r_{i,u}^t \in [0.6,1.0]\right]$
Retrofitted coal-fired units with proposed dynamic ramp	30	$v_{i,u}^t = \begin{cases} a \cdot r_{i,u}^t - b, & r_{i,u}^t \le d \\ c, & r_{i,u}^t > d \end{cases}$ where
rates Eq. $(6.2)$		$a=0.055, b=0.013, c=0.02, d=0.6.$
		In this study, we adopt the following comparative rolling economic dispatch (ED)
		models for agent $A_1$ in Table 6.3. We mainly divide two categories of factors for
		economic dispatch models: dispatchable loads and ramp rate. For dispatchable loads,
		reference [77] adopts a load-shedding variable in the look-ahead rolling economic
		dispatch model instead of a network-based model, namely unrestricted dispatchable

Retrofitted coal-fired units<br>
with piccewise linear model<br>
of dynamic ramp rates [78]<br>
of dynamic ramp rates [78]<br>  $v'_{i,x} = \begin{cases} 0.35\%, & r'_{i,x} \in [0.4, 0.5] \\ 1.5\%, & r'_{i,x} \in [0.5, 0.6] \\ 2\%, & r'_{i,x} \in [0.6, 1.0] \end{cases}$ <br>
Retrof Netrofitted coal-fired units<br>
with piccewise linear model<br>
of dynamic ramp rates [78]<br>
of dynamic ramp rates [78]<br>  $2\%$ ,  $r'_{i,a} \in [0.6, 1.0]$ <br>
Retrofitted coal-fired units<br>
with proposed dynamic ramp<br>  $\gamma'_{i,a} = \begin{cases} a \cdot r'$ or dynamic ramp rates [78]<br>
Retrofitted coal-fired units<br>
with proposed dynamic ramp<br>
rates Eq.(6.2)<br>
The second column refers to unrestricted dispatchable loads,<br>
In this study, we adopt the following comparative rolling Retrofitted coal-fired units<br>
with proposed dynamic ramp<br>
rates Eq.(6.2)<br>
The this study, we adopt the following comparative rolling economic dispatch (ED)<br>
In this study, we adopt the following comparative rolling econom with proposed dynamic ramp<br>
are interesting and  $\frac{a=0.055, b=0.013, c=0.02, d=0.6$ .<br>
In this study, we adopt the following comparative rolling economic dispatch (ED)<br>
models for agent A<sub>1</sub> in Table 6.3. We mainly divide tw In this study, we adopt the following comparative rolling economic dispatch (ED) models for agent A<sub>1</sub> in Table 6.3. We mainly divide two categories of factors for economic dispatch models: dispatchable loads and ramp rate In this study, we adopt the following comparative rolling economic dispatch (ED) models for agent A<sub>1</sub> in Table 6.3. We mainly divide two categories of factors for economic dispatch models: dispatchable loads and ramp rate models for agent A<sub>1</sub> in Table 6.3. We mainly divide two categories of factors for<br>economic dispatch models: dispatchable loads and ramp rate. For dispatchable loads,<br>reference [77] adopts a load-shedding variable in the l economic dispatch models: dispatchable loads and ramp rate. For dispatchable loads, reference [77] adopts a load-shedding variable in the look-ahead rolling economic dispatch model instead of a network-based model, namely reference [77] adopts a load-shedding variable in the look-ahead rolling economic<br>dispatch model instead of a network-based model, namely unrestricted dispatchable<br>loads. The second column refers to unrestricted dispatchab dispatch model instead of a network-based model, namely unrestricted dispatchable<br>loads. The second column refers to unrestricted dispatchable loads. In contrast, the<br>LTS-based HVDN operation model only outputs disertee di loads. The second column refers to unrestricted dispatchable loads. In LTS-based HVDN operation model only outputs discrete dispatchable are indicated in the third column. For ramp rates, we consider converterofitted coal-

	Table 6.3 Comparative rolling economic dispatch models for agent A1					
ED	Dispatchable loads		Normal		Dynamic ramp rates	
models	unrestricted	<b>HVDNs</b>	ramp rate	piecewise	linear	<b>SOC</b>
ED1						
ED <sub>2</sub>						
ED3 ED4						
ED <sub>5</sub>						
ED <sub>6</sub>						
ED7						
ED <sub>8</sub>						
	6.4.1 Maximum Inner-Approximated Errors of Tightened Ramping Constraints We present accuracy discussions for maximum inner-approximated errors between					
	the untightened and linear and SOC ramping constraints. Fig. $6.13(a)-(c)$ present the					
	maximum inner-approximated errors between the untightened and linear ramping					
				constraints, i.e. $\Delta e_U$ and $\Delta e_L$ , and between the untightened and SOC ramping		
					constraints, i.e. $\Delta s_U$ and $\Delta s_L$ , with respect to different ranges of experimental	

constraints, i.e. ΔeU and ΔeL, and between the untightened and SOC ramping ED3  $\sqrt$ <br>
ED5  $\sqrt$ <br>
ED5  $\sqrt$ <br>
ED5  $\sqrt$ <br>
ED7  $\sqrt$ <br>
ED7  $\sqrt$ <br>
6.4.1 Maximum Inner-Approximated Errors of Tightened Ramping Constraints<br>
We present accuracy discussions for maximum inner-approximated errors between<br>
the unt ED6  $\gamma$ <br>
ED6  $\gamma$ <br>
ED7  $\gamma$ <br>
ED8  $\gamma$ <br>
EQ10 FOLDED and c where  $d=(b+c)/a$ . A.I Maximum Inner-Approximated Errors of Tightened Ramping Constraints<br>We present accuracy discussions for maximum inner-approximated errors between<br>entingled and linear and SOC ramping constraints. Fig. 6.13(a)-(c) prese Maximum Inner-Approximated Errors of Tightened Ramping Constraints<br>present accuracy discussions for maximum inner-approximated errors between<br>tightened and linear and SOC ramping constraints. Fig. 6.13(a)-(c) present the<br> We present accuracy discussions for maximum inner-approximated errors between<br>e untightened and linear and SOC ramping constraints. Fig. 6.13(a)-(c) present the<br>aximum inner-approximated errors between the untightened and tightened and linear and SOC ramping constraints. Fig. 6.13(a)-(c) present the<br>num inner-approximated errors between the untightened and linear ramping<br>aints, i.e.  $\Delta e_U$  and  $\Delta e_L$ , and between the untightened and SOC ra aximum inner-approximated errors between the untightened and linear ramping<br>
anstraints, i.e.  $\Delta \epsilon_U$  and  $\Delta \epsilon_L$ , and between the untightened and SOC ramping<br>
anstraints, i.e.  $\Delta s_U$  and  $\Delta s_L$ , with respect to different aints, i.e.  $\Delta e_U$  and  $\Delta e_L$ , and between the untightened and SOC ramping<br>aints, i.e.  $\Delta s_U$  and  $\Delta s_L$ , with respect to different ranges of experimental<br>eters a, b, c. In this experiment, we define the following sets of

- 
- 
- 







(b)



With different sets of parameters. Following  $\Delta \epsilon_U$  and  $\Delta \epsilon_U$  Examples the SOC ramping constraints are more accurate than corresponding linear ramping<br>constraints with different sets of parameters<br>From Fig. 6.12(a)-(c), it should be noted that the maximum inner-approximated<br>errors Fig. 6.12 Maximum inner-approximated errors using linear and SOC ramping<br>constraints with different sets of parameters<br>From Fig. 6.12(a)-(c), it should be noted that the maximum inner-approximated<br>errors  $\Delta e_U$ ,  $\Delta e_L$ , (c)<br>
Fig. 6.12 Maximum inner-approximated errors using linear and SOC ramping<br>
constraints with different sets of parameters<br>
From Fig. 6.12(a)-(c), it should be noted that the maximum inner-approximated<br>
errors  $\Delta ev_i$ , Fig. 6.12 Maximum inner-approximated errors using linear and SOC ramping<br>constraints with different sets of parameters<br>from Fig. 6.12(a)-(c), it should be noted that the maximum inner-approximated<br>crrors  $\Delta e_{U_2}$   $\Delta e_{U$ constraints with different sets of parameters<br>
From Fig. 6.12(a)-(e), it should be noted that the maximum inner-approximated<br>
errors  $\Delta e_t$ ,  $\Delta e_t$ ,  $\Delta v_t$  and  $\Delta s_t$  are quantified below 3.21%, 2.45%, 2.84%, and 1.36%<br>
w From Fig. 6.12(a)-(c), it should be noted that the maximum inner-approximated<br>rors  $\Delta e_i$ ,  $\Delta e_i$ ,  $\Delta s_i$  and  $\Delta s_i$  are quantified below 3.21%, 2.45%, 2.84%, and 1.36%<br>th different sets of parameters. Following  $\Delta e_i > \Delta s$ From Fig. 6.12(a)-(c), it should be noted that the maximum inner-approximated<br>errors  $\Delta e_U$ ,  $\Delta e_U$  and  $\Delta s_U$  are quantified below 3.21%, 2.45%, 2.84%, and 1.36%<br>with different sets of parameters. Following  $\Delta e_U \ge \Delta s_U$  errors  $\Delta e_U$ ,  $\Delta e_L$  and  $\Delta s_L$  are quantified below 3.21%, 2.45%, 2.84%, and 1.36%<br>with different sets of parameters. Following  $\Delta e_L > \Delta s_L$  and  $\Delta e_L > \Delta s_L$ , this suggests that<br>the SOC ramping constraints are more accura with different sets of parameters. Following  $\Delta \epsilon_{U} > \Delta S_{U}$  and  $\Delta \epsilon_{U} > \Delta S_{Z}$ , this suggests that<br>the SOC ramping constraints are more accurate than corresponding linear ramping<br>constraints. For SOC ramping constrain

black, pink and blue lines in Fig. 6.13. The ramping boundary of this installed wind power capacity is depicted in green lines, and three forecasted sets of wind power data are also included in Fig. 6.13. black, pink and blue lines in Fig. 6.13. The ramping boundary of this installed wind<br>power capacity is depicted in green lines, and three forecasted sets of wind power data<br>are also included in Fig. 6.13. black, pink and blue lines in Fig. 6.13. The ramping boundary of this installed wind<br>power capacity is depicted in green lines, and three forecasted sets of wind power data<br>are also included in Fig. 6.13.



in black lines, blue lines, and pink lines. Moreover, the theoretical ramping area in Fig. 6.13 Different allowable ranping margins of WPTs and be enlarged as<br>
lines is determined by installed wind power at time  $F1$  (%)<br>
Fig. 6.13 Different allowable ranping margins of WPTs.<br>
Fig. 6.13 suggests that the a Fig. 6.13 Different allowable ramping margins of WPTs.<br>Fig. 6.13 Different allowable ramping margins of WPTs.<br>Fig. 6.13 Suggests that the allowable ramping margin of WPTs can be enlarged as<br>long as coal-fired plants are r **Example 1996 Example 1996 Example 1997 Example 1998 Example 1998 Example 1998 Example 1998 Example 1998 Example 19** Fig. 6.13 Different allowable ramping margins of WPTs.<br>
Fig. 6.13 suggests that the allowable ramping margin of WPTs can be enlarged as<br>
long as coal-fired plants are retrofitted, as justified by ramping areas with bounda Fig. 6.13 Different allowable ramping margins of WPTs.<br>
Fig. 6.13 suggests that the allowable ramping margin of WPTs can be enlarged as<br>
long as coal-fired plants are retrofitted, as justified by ramping areas with boundar Fig. 6.13 suggests that the allowable ramping margin of WPTs can be enlarged as<br>long as coal-fired plants are retrofitted, as justified by ramping areas with boundaries<br>in black lines, blue lines, and pink lines. Moreover Fig. 6.13 suggests that the allowable ramping margin of WPTs can be enlarged as<br>mg as coal-fired plants are retrofitted, as justified by ramping areas with boundaries<br>black lines, blue lines, and pink lines. Moreover, the long as coal-fired plants are retrofitted, as justified by ramping areas with boundaries<br>in black lines, blue lines, and pink lines. Moreover, the theoretical ramping area in<br>green lines is determined by installed wind po

proposed operational ramping constraints. The red and black lines refer to tightened<br>ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forecasted wind power dataset 2 rema proposed operational ramping constraints. The red and black lines refer to tightened<br>ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forecasted wind power dataset 2 rema proposed operational ramping constraints. The red and black lines refer to tightened<br>ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forecasted wind power dataset 2 rema proposed operational ramping constraints. The red and black lines refer to tightened<br>ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forecasted wind power dataset 2 rema proposed operational ramping constraints. The red and black lines refer to tightened ramping boundary of WPTs caused by linear and SOC ramping constraints. It is shown that one point of forecasted wind power dataset 2 rema proposed operational ramping constraints. The red and black lines refer to tightened<br>ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forecasted wind power dataset 2 rema proposed operational ramping constraints. The red and black lines refer to tightened<br>ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forecasted wind power dataset 2 rema proposed operational ramping constraints. The red and black lines refer to tightened<br>ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forecasted wind power dataset 2 rema oposed operational ramping constraints. The red and black lines refer to tightened<br>mping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>own that one point of forceasted wind power dataset 2 remarked i proposed operational ramping constraints. The red and black lines refer to tightened<br>ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forceasted wind power dataset 2 rema ramping boundary of WPTs caused by linear and SOC ramping constraints. It is<br>shown that one point of forceasted wind power dataset 2 remarked in the blue-filled<br>square box lies on the SOC ramping constraint. The linear con

shown that one point of forecasted wind power dataset 2 remarked in the blue-filled<br>square box lies on the SOC ramping constraint. The linear constraints lead to<br>purchasing dispatchable loads via a few switch-over operatio square box lies on the SOC ramping constraint. The linear constraints lead to<br>purchasing dispatchable loads via a few switch-over operations of HVDNs if linear<br>ramping constraints are involved. Instead, adopting tightened purchasing dispatchable loads via a few switch-over operations of HVDNs if linear<br>ramping constraints are involved. Instead, adopting tightened SOC ramping<br>constraints can accommodate this problem.<br>6.4.3 Linear Versus SOC



(a)



**EXECUTE CONSUMIDED SOLUTION CONSUMIDED**<br>
SOLUTION CONSUMPTED SOLUTION CONSUMPTION CONSUM solution is the embedded graph, since the optimal solutions<br>of ED1 shown in the embedded graph, since the optimal solutions<br>of ED1 shown in the embedded graph, since the optimal solution of ED2 is<br>older to the untightened Soon  $\frac{600}{400}$  600 600 1000 1200 1400<br>
(b)<br>
Fig. 6.14. (a) Untightened and tightened ramping margins of WPTs under  $\Delta T$  =15min;<br>
(b) Optimal rolling ED solutions between ED1 and ED2.<br>
Remarkable differences can be in **Example 19.** The power at time  $t-1/MW$  (b)<br>
Fig. 6.14. (a) Untightened and tightened ramping margins of WPTs under  $\Delta T = 15$ min;<br>
(b) Optimal rolling ED solutions between ED1 and ED2.<br>
Remarkable differences can be infer (b)<br>
Fig. 6.14. (a) Untightened and tightened ramping margins of WPTs under  $\Delta T$  =15min;<br>
(b) Optimal rolling ED solutions between ED1 and ED2.<br>
Remarkable differences can be inferred from Fig. 6.15(b) between optimal so Fig. 6.14. (a) Untightened and tightened ramping margins of WPTs under  $\Delta T = 15$ min;<br>
(b) Optimal rolling ED solutions between ED1 and ED2.<br>
Remarkable differences can be inferred from Fig. 6.15(b) between optimal solutio (b) Optimal rolling ED solutions between ED1 and ED2.<br>
Remarkable differences can be inferred from Fig. 6.15(b) between optimal solutions<br>
of ED1 and ED2. The optimal solutions of ED2 is more accurate than the ones<br>
obtai solutions. Remarkable differences can be inferred from Fig. 6.15(b) between optimal solutions<br>of ED1 and ED2. The optimal solutions of ED2 is more accurate than the ones<br>obtained by ED1 shown in the embedded graph, since the optimal FD1and ED2. The optimal solutions of ED2 is more accurate than the ones<br>vained by ED1 shown in the embedded graph, since the optimal solution of ED2 is<br>oser to the untightened ramping margin in blue line. With acceptable obtained by ED1 shown in the embedded graph, since the optimal solution of ED2 is<br>eloser to the untightened ramping margin in blue line. With acceptable errors<br>indicated by maximum error analysis in Fig. 6.15(a)-(e), the o closer to the untightened ramping margin in blue line. With acceptable errors<br>indicated by maximum error analysis in Fig. 6.15(a)-(e), the optimal rolling economic<br>dispatch solution of ED2 can be decmed as accurate, regard

solutions of ED3, ED4 and ED5 are compared under the typical load demands and the<br>mean of wind power data in 100 random scenarios in Fig. 6.15(a), and optimal<br>dispatchable loads are collected under 100 random wind power sc solutions of ED3, ED4 and ED5 are compared under the typical load demands and the<br>mean of wind power data in 100 random scenarios in Fig. 6.15(a), and optimal<br>dispatchable loads are collected under 100 random wind power sc solutions of ED3, ED4 and ED5 are compared under the typical load demands and the<br>mean of wind power data in 100 random scenarios in Fig. 6.15(a), and optimal<br>dispatchable loads are collected under 100 random wind power s solutions of ED3, ED4 and ED5 are compared under the typical load demands and the<br>mean of wind power data in 100 random scenarios in Fig. 6.15(a), and optimal<br>dispatchable loads are collected under 100 random wind power s solutions of ED3, ED4 and ED5 are compared under the typical load demands and the<br>mean of wind power data in 100 random scenarios in Fig. 6.15(a), and optimal<br>dispatchable loads are collected under 100 random wind power s solutions of ED3, ED4 and ED5 are compared under the typical load demands and the<br>mean of wind power data in 100 random scenarios in Fig. 6.15(a), and optimal<br>dispatchable loads are collected under 100 random wind power s solutions of ED3, ED4 and ED5 are compared under the typical load demands and the<br>mean of wind power data in 100 random scenarios in Fig. 6.15(a), and optimal<br>dispatchable loads are collected under 100 random wind power s



(a)



(b)<br>Fig. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>power data; (b) optimal dispatchable loads in 100 wind power scenarios. (b)<br>
Fig. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>
power data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>
Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind

(b)

(b)<br>
g. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>
wer data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>
Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind po (b)<br>Fig. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>power data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind po (b)<br>Fig. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>power data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind po (b)<br>
Fig. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>
power data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>
Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind (b)<br>
Fig. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>
power data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>
Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind (b)<br>
Fig. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>
power data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>
Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to win Fig. 6.15. (a) optimal ED solutions under typical load demands and mean of wind<br>power data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind power power data; (b) optimal dispatchable loads in 100 wind power scenarios.<br>
Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind power fluctuations<br>
with multiple output power below 600MW, while ED3 is incapable o Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind power fluctuations<br>with multiple output power below 600MW, while ED3 is incapable of<br>accommodating these wind power variations. This suggests that retrofitte Fig. 6.15(a) reveals that ED4 and ED5 can quickly react to wind power fluctuations<br>with multiple output power below 600MW, while ED3 is incapable of<br>accommodating these wind power variations. This suggests that retrofitte these wind power variations. This suggests that retrofitted coal-fired<br>exible than conventional coal-fired plants. To compare ED4 and ED5,<br>to ED4 since ED5 can seek many optimal output solutions that are<br>9. MW whereas ED4 perior to ED4 since ED5 can seek many optimal output solutions that are<br>
aan 600MW whereas ED4 cannot do so. Moreover, Fig. 6.15(b) presents that<br>
s the least dispatchable loads from HVDNs than the quantities obtained by<br> EDS secks the least dispatchable loads from HVDNs than the quantities obtained by<br>
ED3 and ED4 in each dispatch period. These results also validate EDS's superiority in<br>
the quick-response to unexpected wind power fluctua Both and ED4 in each dispatch period. These results also validate ED5's superiority in<br>
equick-response to unexpected wind power fluctuations, as also reflected in the<br>
nallest total cost  $F_{e1}$ ,  $F_{e2}$ , and  $F_{n}$  for

ED Models	$(x10^5\$ 'ol ∶	$({\times}10^{5}$ \$) $\frac{1}{2}$ 02	$1 \times 10^{5}$ ് റ
ED3	0.40	82.420	46.41
ED4	8.67	5.400	7.01
ED5	8.49	3.031	5.75

the quick-response to unexpected wind power fluctuations, as also reflected in the<br>
smallest total cost  $F_{01}$ ,  $F_{02}$ , and  $F_{0}$  for EDS in Table 6.4.<br>
Table 6.4. Average costs of 100 wind power scenarios<br>
ED Models smallest total cost  $F_{01}$ ,  $F_{02}$ , and  $F_{0}$  for ED5 in Table 6.4.<br>
Table 6.4. Average costs of 100 wind power scenarios<br>
ED Models  $F_{01}$  ( $\times 10^{5}$ S)  $F_{02}$  ( $\times 10^{5}$ S)  $F_{03}$  ( $\times 10^{5}$ S)  $\overline{10.40}$ <br>
ED3 1 Table 6.4. Average costs of 100 wind power scenarios<br>
ED Models  $F_{\text{el}}$  (×10<sup>x</sup>\$)  $F_{\text{el}}$  (×10<sup>x</sup> Table 6.4. Average costs of 100 wind power scenarios<br>
ED Models  $F_{\text{ol}}$  (×10<sup>5</sup>\$)  $F_{\text{ol}}$  (×10<sup>5</sup>\$)  $F_{\text{ol}}$  (×10<sup>5</sup>\$)<br>
ED3 10.40 82.420 46.41<br>
ED5 8.49 5.031 7.01<br>
ED5 8.49 5.031 7.57<br>
6.4.5 Unrestricted Versus LTS-ED Models  $F_{cd}$  (×10<sup>5</sup>\$)  $F_{c2}$  (×10<sup>5</sup>\$)  $F_{c3}$  (×10<sup>5</sup>\$) ED3 10.44) 8.647 6.440 6.41<br>ED5 8.49 3.031 5.75<br>Com ED5 8.49 3.031 5.75<br>6.4.5 Unrestricted Versus LTS-based HVDNs<br>Fig. 6.16(a) and (b) show the rolling ED dif

MW is essential at 09:30, since the wind power varies by 614.72 MW, which is larger<br>than the maximum ramp-down margin 251.67 MW of retrofitted coal-fired units at<br>that time. Otherwise, it has to conduct load transfer opera MW is essential at 09:30, since the wind power varies by 614.72 MW, which is larger<br>than the maximum ramp-down margin 251.67 MW of retrofitted coal-fired units at<br>that time. Otherwise, it has to conduct load transfer opera that is essential at 09:30, since the wind power varies by 614.72 MW, which is larger<br>than the maximum ramp-down margin 251.67 MW of retrofitted coal-fired units at<br>that time. Otherwise, it has to conduct load transfer ope MW is essential at 09:30, since the wind power varies by 614.72 MW, which is larger<br>than the maximum ramp-down margin 251.67 MW of retrofitted coal-fired units at<br>that time. Otherwise, it has to conduct load transfer opera MW is essential at 09:30, since the wind power varies by 614.72 MW, which is larger<br>than the maximum ramp-down margin 251.67 MW of retrofitted coal-fired units at<br>that time. Otherwise, it has to conduct load transfer opera MW is essential at 09:30, since the wind power varies by 614.72 MW, which is larger than the maximum ramp-down margin 251.67 MW of retrofitted coal-fired units at that time. Otherwise, it has to conduct load transfer opera variable.



(a)



(b)

172



(c)

We conduct simulation experiments to validate the computational efficiency<br>
We content power by EDT and EDT: (a) set of forecasted wind<br>
wer 2; (b) set of forecasted wind power 3; and (c) legends for (a) and (b).<br>
4.6 Cen **between the centralized MICP-** and MISOCP-based methods and the proposed decentralized MGD-<br> **exists** centralized MILP- and MISOCP-based methods and the proposed of and ED7: (a) set of forecasted wind<br>
power 2; (b) set o decentralized Magnetian Show TDF and ED7: (a) set of forecasted wind<br>
The active output power by ED6  $\rightarrow$  output power of WTBPS by ED7<br>
output power of WTBPS by ED6  $\rightarrow$  output power of WTBPS by ED7<br>
(c)<br>
Fig. 6.16. Optim **Example 19 Control Centralized Centralized MICP**<br>
Fig. 6.16. Optimal ED solutions between ED6 and ED7: (a) set of forecasted wind<br>
power 2; (b) set of forecasted wind power 3; and (c) legends for (a) and (b).<br>
6.4.6 Centr (c)<br>
Fig. 6.16. Optimal ED solutions between ED6 and ED7: (a) set of forceasted wind<br>
power 2; (b) set of forceasted wind power 3; and (c) legends for (a) and (b).<br>
6.4.6 Centralized Versus Decentralized MBGD Methods<br>
We c Fig. 6.16. Optimal ED solutions between ED6 and ED7: (a) set of forecasted wind<br>power 2; (b) set of forecasted wind power 3; and (c) legends for (a) and (b).<br>6.4.6 Centralized Versus Decentralized MBGD Methods<br>We conduct s power 2; (b) set of forecasted wind power 3; and (c) legends for (a) and (b).<br>
6.4.6 Centralized Versus Decentralized MBGD Methods<br>
We conduct simulation experiments to validate the computational efficiency<br>
between the ce 4.6 Centralized Versus Decentralized MBGD Methods<br>We conduct simulation experiments to validate the computational efficiency<br>tween the centralized MILP- and MISOCP-based methods and the proposed<br>centralized MGBD-based meth 6.4.6 Centralized Versus Decentralized MBGD Methods<br>We conduct simulation experiments to validate the computational efficiency<br>between the centralized MILP- and MISOCP-based methods and the proposed<br>decentralized MGBD-base We conduct simulation experiments to validate the computational efficiency<br>between the centralized MILP- and MISOCP-based methods and the proposed<br>decentralized MGBD-based method for ED7 and ED8 in case studies. Four meth between the centralized MILP- and MISOCP-based methods and the proposed<br>decentralized MGBD-based method for ED7 and ED8 in case studies. Four methods<br>are compared: Centralized MILP programming solver for ED8 (M1); Central

decentralized MGBD-based method for ED7 and ED8 in ease studies. Four methods<br>are compared: Centralized MILP programming solver for ED8 (M1); Centralized<br>MISOCP programming solver for ED7 (M2); Centralized MISOCP programm are compared: Centralized MILP programming solver for ED8 (M1); Centralized MISOCP programming solver for ED7 (M2); Centralized MISOCP programming solver embedded in a GBD framework for ED7 (M3); Decentralized MISOCP prog MISOCP programming solver for ED7 (M2); Centralized MISOCP pro<br>solver embedded in a GBD framework for ED7 (M3); Decentralized<br>programming solver embedded an MGBD framework for ED7(M4).<br>Three four methods are implemented b ver embedded an MGBD framework for ED7(**M4**).<br>
Athods are implemented by using CPLEX tools and MOSEK in the<br>
nment with an AMD Ryzen 75800X 8-core CPU 3.80GHz processor.<br>
toechastic wind power scenarios for four agents ar These four methods are implemented by using CPLEX tools and MOSEK in the<br>These four methods are implemented by using CPLEX tools and MOSEK in the<br>ATLAB environment with an AMD Ryzen 75800X 8-core CPU 3.80GHz processor.<br>Th using CPLEX tools and MOSEK in the<br>
175800X 8-core CPU 3.80GHz processor.<br>
<br>
rios for four agents are generated where<br>
(for wind power rated capacity between<br>
1.5, and 0.8. Note that all forecasted wind<br>
in of WPTs under

Cases	Methods	Iterations	Entire CPU	CPU Time(s)	
			Time(s)	<b>TMP</b>	TCD ـ ⊔ت⊥
	M1		5.34		
$\Delta P = 0.3$	M <sub>2</sub>		1911 14. I 1		
			173		



 $\frac{N_1-8}{N_1-8}$  M3 8 18.42 14.22 4.20<br>
II M1 *1* 5.6 5.54 2.02<br>
II M1 *1* 5.6 5.54 2.02<br>  $\Delta P=0.5$  M3 9 20.08 *15.75* 4.33<br>  $\frac{N_1-8}{N_1-8}$  M4 5 20.08 15.75 4.33<br>
II M1 *1* 5.02 *1* 7<br>  $\Delta P=0.8$  M3 15 21.73 15.24 6.49 II M1 / 5.41 / 1<br>  $\Delta P = 0.5$  M3 / 16.36 / / 16.35<br>  $N_i = 8$  M4 5 7.47 5.12 2.35<br>
II M1 / 5.02 / / /<br>
II M1 / 5.02 / / /<br>  $\Delta P = 0.8$  M3 15 21.73 15.24 6.49<br>
M4 5 7.51 5.40 2.11<br>
Table 6.5 displays the numerical results for  $\frac{N}{N_e}$ =0. M3 9 20.08 15.75 4.33<br>  $\frac{M4}{N_e}$  5 7.47 5.12 2.25<br>
II M1 / 5.02 / 16.81<br>  $\frac{M^2}{N_e}$  -0.8 M3 15 21.73 15.24 6.49<br>  $\frac{M^2}{N_e}$  M4 5 7.51 15.24 6.49<br>
Table 6.5 displays the numerical results for case studi II M1 / 5.02 / 16.81 / 16.81 / 16.81 / 16.81 / 16.81 / 16.81 / 16.81 / 16.81 / 16.81 / 16.81 / 16.81 / 16.91 / 16.81 / 16.91 / 16.49 M4 5 7.51 5.40 2.11 Table 6.5 displays the numerical results for case studies in differe  $\frac{\Delta P - 0.8}{N_A - 8}$  M3 15 21.73 15.24 6.49<br>Table 6.5 displays the numerical results for case studies in different wind power<br>secnarios. For case II, we consider involving eight agents of WTBPS connecting to<br>220kV stations Table 6.5 displays the numerical results for case studies in different wind power<br>secnarios. For case II, we consider involving eight agents of WTBPS connecting to<br>220kV stations FZ, RD, XEC, and SY. The number of constrai Table 6.5 displays the numerical results for case studies in different wind pot<br>scenarios. For case II, we consider involving eight agents of WTBPS connecting<br>220kV stations FZ, RD, XEC, and SY. The number of constraints a enarios. For ease II, we consider involving eight agents of WTBPS connecting to  $20kV$  stations FZ, RD, XEC, and SY. The number of constraints and variables is the as many as case I. The first column indicates case numbers 220kV stations FZ, RD, XEC, and SY. The number of constraints and variables is<br>twice as many as case I. The first column indicates case numbers with different ranges<br>of wind power fluctuations. The second column denotes fo twice as many as case I. The first column indicates case numbers with different ranges<br>of wind power fluctuations. The second column denotes four methods. The third<br>column shows the total number of iterations required to r

stems from the MILP-based ED8, whereas M2 is applied to address the<br>MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less stems from the MILP-based ED8, whereas M2 is applied to address the<br>MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number Externs from the MILP-based ED8, whereas M2 is applied to address the<br>MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>numb stems from the MILP-based ED8, whereas M2 is applied to address the<br>MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number stems from the MILP-based ED8, whereas M2 is applied to address the<br>MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number stems from the MILP-based ED8, whereas M2 is applied to address the MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number stems from the MILP-based ED8, whereas M2 is applied to address the<br>MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number ems from the MILP-based ED8, whereas M2 is applied to address the ISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>
sompared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>
umber o stems from the MILP-based ED8, whereas M2 is applied to address the<br>MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number stems from the MILP-based ED8, whereas M2 is applied to address the<br>MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number MISOCP-based ED7 with a more accurate solution than the MILP-based ED8.<br>Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number of iterations in two cases. And the CPU processing time for M3 subs

Compared to M3, M4 shows a remarkably smaller amount of CPU time and a less<br>number of iterations in two cases. And the CPU processing time for M3 substantially<br>increases with larger system sizes. However, the effectiveness number of iterations in two cases. And the CPU processing time for M3 substant increases with larger system sizes. However, the effectiveness of the proposed M<br>be ensured within less than 8 seconds of CPU processing time, be ensured within less than 8 seconds of CPU processing time, which sa<br>65% of computation time compared with M3.<br>Based on the above, M4 outperforms M2-M3 in the least Cl<br>outperforms M1 in the accuracy of the solution. Prov Based on the above, M4 outperforms M2-M3 in the least CPU time and<br>tperforms M1 in the accuracy of the solution. Provided that M4 is performed on<br>altiple distributed computing servers, it can be certainly expected that the outperforms M1 in the accuracy of the solution. Provided that M4 is performed on<br>multiple distributed computing servers, it can be certainly expected that the<br>computing performance of M4 can bring better computational eff

multiple distributed computing servers, it can be certainly expected that the<br>computing performance of M4 can bring better computational efficiency to this<br>decomposition-coordination computing framework, especially for a computing performance of M4 can bring better computational efficiency to this<br>decomposition-coordination computing framework, especially for a large-scale<br>optimization problem.<br>6.5 Summary<br>The chapter proposes a look-ahea decomposition-coordination computing framework, especially for a large-scale<br>optimization problem.<br>
The chapter proposes a look-ahead rolling economic dispatch approach of WTBPS<br>
considering the variable ramp rate of retr optimization problem.<br>
6.5 Summary<br>
The chapter proposes a look-ahead rolling economic dispatch approach of WTBPS<br>
considering the variable ramp rate of retrofitted coal-fired units and flexible<br>
voltage-constrained LTS v 6.5 Summary<br>The chapter proposes a look-ahead rolling economic dispatch approach of WTBPS<br>considering the variable ramp rate of retrofitted coal-fired units and flexible<br>voltage-constrained LTS via HVDNs. The SOC ramping 6.5 Summary<br>The chapter proposes a look-ahead rolling economic dispatch approach of WTBPS<br>considering the variable ramp rate of retrofitted coal-fired units and flexible<br>voltage-constrained LTS via HVDNs. The SOC ramping The chapter proposes a look-ahead rolling economic dispatch approach of WTBPS<br>considering the variable ramp rate of retrofitted coal-fired units and flexible<br>voltage-constrained LTS via HVDNs. The SOC ramping constraints large-scale rolling economic dispatch model to be quickly solved, saves around 65%<br>computational time and releases more computational resources. Therefore, the<br>computing performance for the proposed MGBD-based method is pr alarge-scale rolling economic dispatch model to be quickly solved, saves around 65% computational time and releases more computational resources. Therefore, the computing performance for the proposed MGBD-based method is p large-scale rolling economic dispatch model to be quickly solved, saves around 65%<br>computational time and releases more computational resources. Therefore, the<br>computing performance for the proposed MGBD-based method is pr large-scale rolling economic dispatch model to be quickly solved, saves around 65% computational time and releases more computational resources. Therefore, the computing performance for the proposed MGBD-based method is pr

# **Chapter 7<br>Conclusions and Future Work** Chapter 7<br>Conclusions and Future Work<br>Conclusions

Chapter 7<br>
Conclusions and Future Work<br>
7.1 Conclusions<br>
DNs can adaptively maintain load balancing and loss reduction<br>
security-constrained operation level, and to coordinate real-time transa Chapter 7<br>
Conclusions and Future Work<br>
Conclusions<br>
DNs can adaptively maintain load balancing and loss reduction at the voltage<br>
urity-constrained operation level, and to coordinate real-time transactive dispatch<br>
as bet **Chapter 7**<br>Conclusions and Future Work<br>T.1 Conclusions<br>DNs can adaptively maintain load balancing and loss reduction at the voltage<br>security-constrained operation level, and to coordinate real-time transactive dispatch<br>ta **Conclusions and Future Work**<br>T.1 Conclusions<br>T.1 Conclusions<br>DNs can adaptively maintain load balancing and loss reduction at the voltage<br>security-constrained operation level, and to coordinate real-time transactive dispa **Conclusions and Future Work**<br>
Conclusions<br>
DNs can adaptively maintain load balancing and loss reduction at the voltage<br>
urity-constrained operation level, and to coordinate real-time transactive dispateh<br>
shetween supply

(e.g. renewable generation, electric vehicles and storage) into small the voltage security-constrained operation level, and to coordinate real-time transactive dispatch tasks between supply and demand at the market level o **Conclusions and Future Work**<br>
7.1 Conclusions<br>
DNs can adaptively maintain load balancing and loss reduction at the voltage<br>
security-constrained operation level, and to coordinate real-time transactive dispatch<br>
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DNs can adaptively maintain load balancing and loss reduction at the voltage<br>
security-constrained operation level, and to coordinate real-time transactive dispatch<br>
tasks between supply and demand at the 7.1 Conclusions<br>
DNs can adaptively maintain load balancing and loss reduction at the voltage<br>
security-constrained operation level, and to coordinate real-time transactive dispatch<br>
tasks between supply and demand at the DNs can adaptively maintain load balancing and loss reduction at the voltage<br>security-constrained operation level, and to coordinate real-time transactive dispatch<br>tasks between supply and demand at the market level of DNs security-constrained operation level, and to coordinate real-time transactive dispatch<br>tasks between supply and demand at the market level of DNs.<br>On system-wide operation level, with the proliferation of diverse power ent tasks between supply and demand at the market level of DNs.<br>
(e.g. renewable generation, electric vehicles and storage) into smart DNs, adaptive<br>
reconfiguration of the electrical topology of DNs may run into physical secu On system-wide operation level, with the proliferation of diverse power entities (e.g. renewable generation, electric vehicles and storage) into smart DNs, adaptive reconfiguration of the electrical topology of DNs may run (e.g. renewable generation, electric vehicles and storage) into smart DNs, adaptive<br>reconfiguration of the electrical topology of DNs may run into physical security issues,<br>i.e., over-voltage and under-voltage excursions. reconfiguration of the electrical topology of DNs may run into physical security issues,<br>i.e., over-voltage and under-voltage excursions. Moreover, on cyber-physical system<br>security level, massive D-PMU devices are connect i.e., over-voltage and under-voltage excursions. Moreover, on cyber-physical system<br>security level, massive D-PMU devices are connected into smart DNs for the full<br>system observability in recent years. The cyber-physical n security level, massive D-PMU devices are connected into smart DNs for the full<br>system observability in recent years. The cyber-physical nature of DNs facilitates the<br>exchange of erafted D-PMU signals amongst actuating and system observability in recent years. The cyber-physical nature of DNs facilitates the exchange of crafted D-PMU signals amongst actuating and monitoring power entities in DNs. This renders that an effective defense of obs

Thereby, in order to proactively mitigate these problems, this thesis proposes<br>eral advanced topology optimization methods to achieve a DNR solution for<br>er-physical security, privacy-preserving and dispatch flexibility enh Thereby, in order to proactively mitigate these problems, this thesis proposes<br>several advanced topology optimization methods to achieve a DNR solution for<br>cyber-physical security, privacy-preserving and dispatch flexibili Thereby, in order to proactively mitigate these problems, this thesis proposes<br>several advanced topology optimization methods to achieve a DNR solution for<br>cyber-physical security, privacy-preserving and dispatch flexibili Thereby, in order to proactively mitigate these problems, this thesis proposes<br>several advanced topology optimization methods to achieve a DNR solution for<br>eyber-physical security, privacy-preserving and dispatch flexibili Thereby, in order to proactively mitigate these problems, this thesis proposes<br>veral advanced topology optimization methods to achieve a DNR solution for<br>ber-physical security, privacy-preserving and dispatch flexibility e Thereby, in order to proactively mitigate these problems, this thesis proposes<br>several advanced topology optimization methods to achieve a DNR solution for<br>eyber-physical security, privacy-preserving and dispatch flexibili

Thereby, in order to proactively mitigate these problems, this thesis proposes<br>several advanced topology optimization methods to achieve a DNR solution for<br>eyber-physical security, privacy-preserving and dispatch flexibili Thereby, in order to proactively mitigate these problems, this thesis proposes<br>several advanced topology optimization methods to achieve a DNR solution for<br>cyber-physical security, privacy-preserving and dispatch flexibili Thereby, in order to proactively mitigate these problems, this thesis proposes<br>several advanced topology optimization methods to achieve a DNR solution for<br>eyber-physical security, privacy-preserving and dispatch flexibili Thereby, in order to proactively mitigate these problems, this thesis proposes<br>several advanced topology optimization methods to achieve a DNR solution for<br>eyber-physical security, privacy-preserving and dispatch flexibili several advanced topology optimization methods to achieve a DNR solution for<br>eyber-physical security, privacy-preserving and dispatch flexibility enhancement. The<br>primary conclusions and contributions of this thesis are su (2) The proposed disjunctive RCDS formulation can be applicable for the proposed political state of DNR approach can tackle the disjunctive nature of DNR blems. With continuous parent-child relationship variables as disjun primary conclusions and contributions of this thesis are summarized as follows:<br>
(1) The proposed DCHR approach can tackle the disjunctive nature of DNR<br>
problems. With continuous parent-child relationship variables as dis an observability defense-constrained DNR model can be constructed as a MISOCP

this DCHR approach is theoretically tighter than the McCormick linearization method<br>and the Big-M method, and it is especially suitable for DNs with directional power<br>flows. As demonstrated in case studies, the computing p and the Big-M method, and it is especially suitable for DNs with directional power<br>flows. As demonstrated in case studies, the computing performance in terms of<br>running time and iterations using a DCHR approach yields supe vs. As demonstrated in case studies, the computing performance in terms of<br>ining time and iterations using a DCHR approach yields superior numerical<br>formance than prior relaxation methods.<br>(2) The proposed disjunctive RCDS running time and iterations using a DCHR approach yields superior numerical<br>performance than prior relaxation methods.<br>(2) The proposed disjunctive RCDS formulation can be applicable for<br>reconnection extractions with the l performance than prior relaxation methods.<br>
(2) The proposed disjunctive RCDS formulation can be applicable for<br>
reconfigurable networks with the least defense cost in theory. With this formulation,<br>
an observability defen

(2) The proposed disjunctive RCDS formulation can be applicable for<br>reconfigurable networks with the least defense cost in theory. With this formulation,<br>an observability defense-constrained DNR model can be constructed as reconfigurable networks with the least defense cost in theory. With this formulation,<br>an observability defense-constrained DNR model can be constructed as a MISOCP<br>problem, which perfectly enables an observable DNR solutio an observability defense-constrained DNR model can be constructed as a MISOCP<br>problem, which perfectly enables an observable DNR solution just with the minimal<br>defense cost and active power loss for cyber-physical security problem, which perfectly enables an observable DNR solution just with the minimal<br>defense cost and active power loss for cyber-physical security enhancement.<br>(3) For multi-agent ADNs, we proposes a DP-DNR mechanism based o

In the future energy-sharing market with mutual trust, this well-designed<br>C-ADMM-based DP-DNR management will be much applicable for<br>privacy-preserving grid operation of multi-agent ADNs, especially for agents with In the future energy-sharing market with mutual trust, this well-designed<br>C-ADMM-based DP-DNR management will be much applicable for<br>privacy-preserving grid operation of multi-agent ADNs, especially for agents with<br>conflic In the future energy-sharing market with mutual trust, this well-designed<br>C-ADMM-based DP-DNR management will be much applicable for<br>privacy-preserving grid operation of multi-agent ADNs, especially for agents with<br>conflic In the future energy-sharing market with mutual trust, this well-de<br>C-ADMM-based DP-DNR management will be much applicable<br>privacy-preserving grid operation of multi-agent ADNs, especially for agen<br>conflicting interests.<br>( (4) The proposed distribution-level topology optimization contributes to the proposed distribution-level topology optimization contributes to the proposed distribution-level topology optimization contributes to the initial In the future energy-sharing market with mutual trust, this well-designed<br>C-ADMM-based DP-DNR management will be much applicable for<br>privacy-preserving grid operation of multi-agent ADNs, especially for agents with<br>conflic

In the future energy-sharing market with mutual trust, this well-designed<br>C-ADMM-based DP-DNR management will be much applicable for<br>privacy-preserving grid operation of multi-agent ADNs, especially for agents with<br>confli In the future energy-sharing market with mutual trust, this well-designed<br>C-ADMM-based DP-DNR management will be much applicable for<br>privacy-preserving grid operation of multi-agent ADNs, especially for agents with<br>confli In the future energy-sharing market with mutual trust, this well-designed<br>C-ADMM-based DP-DNR management will be much applicable for<br>privacy-preserving grid operation of multi-agent ADNs, especially for agents with<br>confli C-ADMM-based DP-DNR management will be much applicable for<br>privacy-preserving grid operation of multi-agent ADNs, especially for agents with<br>conflicting interests.<br>(4) The proposed distribution-level topology optimization privacy-preserving grid operation of multi-agent ADNs, especially for agents with<br>conflicting interests.<br>(4) The proposed distribution-level topology optimization contributes to the<br>flexibility enhancement of a look-ahead conflicting interests.<br>
(4) The proposed distribution-level topology optimization contributes to the<br>
flexibility enhancement of a look-ahead rolling economic dispatch of WTBPS, which<br>
offsets the insufficient ramping mar (4) The proposed distribution-level topology optimization contributes to the<br>flexibility enhancement of a look-ahead rolling economic dispatch of WTBPS, which<br>offsets the insufficient ramping margins of retrofitted coal-f flexibility enhancement of a look-ahead rolling economic dispatch of WTBPS, which<br>offsets the insufficient ramping margins of retrofitted coal-fired units. The proposed<br>SOC ramping constraints of retrofitted coal-fired un economic dispatch model to be quickly solved, saves around the proposed to the proposed SOC ramping constraints of retrofitted coal-fired units are validated with acceptable inner-approximated error (at most 2.84%) when SOC ramping constraints of retrofitted coal-fired units are validated with acceptable<br>inner-approximated error (at most 2.84%) when  $\Delta T$  =15 min. Results from the case<br>studies demonstrate that this proposed rolling econo inner-approximated error (at most 2.84%) when  $\Delta T = 15$  min. Results from the case<br>studies demonstrate that this proposed rolling economic dispatch approach is<br>applicable for multiple WTBPS agents to accommodate wind powe applicable for multiple WTBPS agents to accommodate wind power fluctuation<br>the minimization of production cost, purchasing cost, and switch-over operation<br>Moreover, the proposed MGBD-based decentralized method enables<br>subp minimization of production cost, purchasing cost, and switch-over operation cost.<br>
reover, the proposed MGBD-based decentralized method enables many<br>
problems to be solved in parallel, which facilitates this large-scale ro Moreover, the proposed MGBD-based decentralized method enables many<br>subproblems to be solved in parallel, which facilitates this large-scale rolling<br>economic dispatch model to be quickly solved, saves around 65% computatio subproblems to be solved in parallel, which facilitates this large-scale rolling<br>economic dispatch model to be quickly solved, saves around 65% computational time<br>and releases more computational resources. Therefore, the c

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circuit breakers are gradually upgraded with high-speed switching properties. Thanks<br>
to smart (remotely controlled) circu 1) Under the digital transformation of the energy industry, power electronic-based<br>circuit breakers are gradually upgraded with high-speed switching properties. Thanks<br>to smart (remotely controlled) circuit breakers, the r circuit breakers are gradually upgraded with high-speed switching properties. Thanks<br>to smart (rcmotcly controlled) circuit breakers, the real-time topology optimization<br>technology can be a promising load transition event to smart (remotely controlled) circuit breakers, the real-time topology optimization<br>technology can be a promising load transition event via network reconfiguration,<br>which can be used to relieve stress on a primary energy technology can be a promising load transition event via network reconfiguration,<br>which can be used to relieve stress on a primary energy sources when demand for<br>electric is greater than the primary power source can supply. which can be used to relieve stress on a primary energy sources when demand for<br>electric is greater than the primary power source can supply. At the operation level,<br>DSOs perform the topology optimization strategy for load electric is greater than the primary power source can supply. At the operation level,<br>DSOs perform the topology optimization strategy for load balancing and/or loss<br>reduction by the means of choosing optimal status of sect DSOs perform the topology optimization strategy for load balancing and/or loss<br>reduction by the means of choosing optimal status of sectionalizing switches and<br>tie-switches in energy-intensive ADNs on different voltage lev reduction by the means of choosing optimal status of sectionalizing switches and<br>tie-switches in energy-intensive ADNs on different voltage levels, i.e., HVDNs,<br>medium-voltage DNs (MVDNs) and low-voltage DNs (LVDNs). This tie-switches in energy-intensive ADNs on different voltage levels, i.e., HVDNs,<br>medium-voltage DNs (MVDNs) and low-voltage DNs (LVDNs). This also facilitates<br>interacting with transmission system operators with the provisio medium-voltage DNs (MVDNs) and low-voltage DNs (LVDNs). This also facilitates<br>interacting with transmission system operators with the provision of grid services at<br>the transmission-distribution interface. In other words, d racting with transmission system operators with the provision of grid services at<br>transmission-distribution interface. In other words, doing this topology<br>mization task on multi-voltage level ADNs actually responds to mult the transmission-distribution interface. In other words, doing this topology<br>optimization task on multi-voltage level ADNs actually responds to multiple<br>operational requests by DSOs, e.g., quick load balance, system loss m optimization task on multi-voltage level ADNs actually responds to multiple<br>operational requests by DSOs, e.g., quick load balance, system loss minimization<br>and/or virtual power plants, as a load transition event towards o

voltages from 10kV up to 35kV, and HVDNs are operated on 110kV as developed in<br>Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>characterizations. As reported, LVDNs or MVDNs generally have Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>characterizations. As reported, LVDNs or MVDNs generally h voltages from 10kV up to 35kV, and HVDNs are operated on 110kV as developed in<br>Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>characterizations. As reported, LVDNs or MVDNs generally have voltages from 10kV up to 35kV, and HVDNs are operated on 110kV as developed in<br>Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>characterizations. As reported, LVDNs or MVDNs generally have voltages from 10kV up to 35kV, and HVDNs are operated on 110kV as developed in<br>Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>characterizations. As reported, LVDNs or MVDNs generally have voltages from 10kV up to 35kV, and HVDNs are operated on 110kV as developed in<br>Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>characterizations. As reported, LVDNs or MVDNs generally have voltages from 10kV up to 35kV, and HVDNs are operated on 110kV as developed in Finland, China, and Spain, etc. Different categories of ADNs have different graph characterizations. As reported, LVDNs or MVDNs generally have voltages from 10kV up to 35kV, and HVDNs are operated on 110kV as developed in Finland, China, and Spain, etc. Different categories of ADNs have different graph characterizations. As reported, LVDNs or MVDNs generally have voltages from 10kV up to 35kV, and HVDNs are operated on 110kV as developed in<br>Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>characterizations. As reported, LVDNs or MVDNs generally have Finland, China, and Spain, etc. Different categories of ADNs have different graph<br>characterizations. As reported, LVDNs or MVDNs generally have single-meshed<br>network structures with depth but non-width properties. In contr characterizations. As reported, LVDNs or MVDNs generally have single-meshed<br>nctwork structures with depth but non-width properties. In contrast to LVDNs or<br>MVDNs, HVDNs are sub-transmissions constructed in multi-meshed top network structures with depth but non-width properties. In contrast to LVDNs or<br>MVDNs, HVDNs are sub-transmissions constructed in multi-meshed topology<br>(closed loop) but operated in radial structures (open loop), where net MVDNs, HVDNs are sub-transmissions constructed in multi-meshed topology<br>(closed loop) but operated in radial structures (open loop), where network structures<br>normally develop wide but non-deep around each station. It is cl (closed loop) but operated in radial structures (open loop), where network structures<br>normally develop wide but non-deep around each station. It is clear that the physical<br>properties of graph characterizations between HVDN normally develop wide but non-deep around each station. It is clear that the physical<br>properties of graph characterizations between HVDNs, LVDNs and MVDNs can be<br>significant distinct, i.e, topology structures of HVDNs are properties of graph characterizations between HVDNs, LVDNs and MVDNs can be significant distinet, i.e, topology structures of HVDNs are more simplified than general DNs. In the future, we wonder how physical network proper significant distinct, i.e, topology structures of HVDNs are more singeneral DNs. In the future, we wonder how physical network properti<br>ADNs can be utilized to reduce the computational complexity<br>optimization. This is very

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