RECYCLED LABEL DESIGN AND GREEN COMPETITION

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Recycled Label Design and Green Competition

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Abstract

Facing markets with growing green awareness, an increasing number of firms are using recycled materials in their products. The most effective way to convey such green efforts may be getting labeled by a third-party certifier, i.e., NGO or government agency. Across industries, we observe two types of recycled labels: The *continuous* label precisely displays the percentage of recycled materials in a product; the *binary* label sets a minimum-percentage standard and is issued to a product if the standard is met. In this paper, we build a game-theoretic model to investigate how a certifier should design the recycled label to boost an industry's environmental performance. The industry is captured by a duopoly where firms determine product composition through recycled-technology investment and thereby compete for market share. We consider three key metrics characterizing the industry: competition intensity, average recycled-investment efficiency, and symmetricity of recycled-investment efficiency. We figure out a *sandwich principle* for label selection: For any of the three metrics, the certifier should design the recycled label as continuous if the metric is intermediate, and as binary if the metric is low or high. This principle is rationalized by contrasting between the continuous label's *transparency* effect—firms are spontaneously motivated by being able to entirely transfer their recycled investment into market competitiveness—and the binary label's *enforcement* effect—the certifier can control firms to deviate from their self-motivated investment. We further demonstrate that the cerifier's label preference cannot align with both firms, but can align with the industry as a whole. Comparative statics reveal that environmental performance and industry profitability change nontrivially in the three metrics. In particular, intenser (milder) competition or more efficient recycled investment does not necessarily improve environmental performance (industry profitability).

Keywords: recycled label; market competition; ESG operations; operations-marketing interface.

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Chapter 1

Introduction

Consumers are increasingly concerned with the utilization of recycled materials in products they purchase. Studies have demonstrated that female millennials are willing to pay 5–10% more for athleisure apparel made from recycled polyester (Chi et al. 2021); that students are willing to pay a price premium for mineral water bottled by recycled plastic (Galati et al. 2022); and that eco-friendly consumers are willing to pay around 7% more for recycled paper towels (Srinivasan and Blomquist 2009). Such green awareness of consumers is changing the market. For example, merely for recycled plastic, the global market size reached 47.60 billion USD in 2022 and will exhibit a compound annual growth rate of 4.9% between 2023 and 2030 (Grand View Research 2023). As a result, it becomes essential for many firms to gain competitiveness by building recycled elements into their operational and marketing strategies, which can be generally attributed to "green competition"—a specific form of competition that is focused on the ecological side of the companies' strategies and that refers to a green promotion strategy of the companies' goods and services (Popescu 2020).

However, utilizing recycled materials is not trivial. While firms compete on the market shares of recycled products, they face varying challenges to invest in the relevant technologies. First, the type of material matters. For example, paper can be much more easily recycled than plastic (Miller 2021), which requires advanced technologies to change the chemical structures (PlasticsEurope 2023). Second, even for the same type of material, brands can have distinct efficiencies of recycled-technology investment, which depends on product design, existing technological base, and supply chain environment. For example, in the apparel industry, product design plays a crucial role. Napapijri, an outdoor brand, can easily employ recycled materials because of its mono-material product design (Napapijri 2023); in contrast, some other brands have to invest heavily in near-infrared (NIR) sorting technology because of their multi-fabric product designs (STADLER Anlagenbau GmbH 2023).

When shaping demand for recycled products, firms need not only the above efforts in utilizing recycled materials, but also a credible channel to convey these efforts. The most effective way may be getting labeled by a third-party certifier, e.g., NGOs or the environmental departments of governments. According to a survey by PEFC (Programme for the Endorsement of Forest Certification), over half of consumers consider certification labels as the most reassuring proof of sustainable materials (PEFC 2014). A multi-national survey conducted by GlobalScan further reveals that 74% of global consumers believe strongly in the importance of independent certification for paper products; they check product information to make informed choices, and a certified label motivates them to buy the products (Forest Stewardship Council 2021). In sum, these labels help firms communicate their green efforts to consumers in a trustworthy manner and enable consumers to assess products' recycled contents without being an expert.

Interestingly, we observe two approaches to designing recycled labels across various industries: *continuous* and *binary*. A continuous label directly displays the percentage of recycled content in a product. As is shown in Table 1.1, in the industries of recycled plastic, bio-circular material, textile, and general post-consumer resin (PCR) and post-industrial resin (PIR), there are various certifiers that issue continuous labels. A binary label, by contrast, does not display the actual percentage of recycled content. Rather, the certifier sets a minimum standard for the percentage, and a product is issued the label as long as it satisfies the standard. Table 1.1 also displays several instances of binary labels across industries of recycled forest-based

Table 1.1: Instances of Recycled Labels

	Continuous			
Label				
Certifier	International Sustainability & Carbon Certifier	RecyClass	Textile Exchange	Green Circle
Industry	Plastic & bio -circular material	Plastic	Textile	PCR & PIR
	Binary			
Label				
Certifier	Forest Stewardship Council	Program for the Endorsement of Forest Certification	The Association of Plastic Recyclers	OceanCycle
Industry	Forest-based material	Forest-based material	PCR	Ocean-bound plastic
Standard	≥100%	≥70%	≥90%	≥30%

material, PCR, and ocean-bound plastic. The minimum standards for these labels range from 30% to 100% of recycled content.

The above context implies the multifacetedness of recycled-product industries. Several considerations play significant roles in the formation of a recycled economy: the design of labels by certifiers (the regulation side), the competition between firms under varying technological efficiency (the operations side), and the consumers' willingness to pay for recycled products (the marketing side). In this paper, we seek to understand the dynamics of this system. We ask the following research questions: First, should the certifier design the label as continuous or binary to motivate competing firms' usage of recycled materials? Second, do competing firms benefit from a continuous label or a binary label? How do their preferences align with the certifier's selection? Third, how does the industrial condition, such as competition intensity and maturity of recycled technology, influence the label design?

To address these questions, we build a game-theoretic model consisting of one certifier, two competing firms in an industry, and a mass of consumers with green awareness. The certifier optimizes the environmental impact of the firms' productions. It can design the recycled label to be either continuous (i.e., fully reveal a firm's recycled-content percentage) or binary (i.e., issue the label as long as a firm's recycled-content percentage exceeds the minimum standard) coupled with a minimum-standard decision. The firms are profit maximizers. Each firm determines product constitution as well as product price. More recycled content (and less virgin content) in the product leads to a higher production cost and, more importantly, a heavier investment in recycled technology. The two firms' efficiencies of such investment differ, which reflects their distinct product designs, existing technology bases, or business environments. Consumers are located along a Hotelling line and their utilities comprise products' *green value*, which depends on the recycled label. In particular, if the label is continuous, consumers can perceive the exact recycled-content percentage in products; by contrast, if the label is binary, consumers can only perceive whether the minimum standard is met. In this game, the certifier first designs the label, the two firms second determine product constitution simultaneously, the pricing decisions follow, and finally consumers purchase. We start with the analysis of each subgame under the continuous/binary label and then move on to the two labels' comparison regarding environmental performance and firm profits.

Our analysis on recycled label design surrounds three key metrics of the industry. First, the competition intensity between firms. Second, the average recycled investment efficiency of the industry. As we mentioned earlier, depending on the type of material, the maturity of recycled technology differs largely. Therefore, the corresponding investment efficiency of the industry also varies and should be considered as a key factor in recycled label design. Third, the symmetricity of investment efficiency between the two firms. Symmetricity is not only a matter of theoretical interest in competition, but also an important feature of recycled investment—again, as mentioned earlier, even for the same type of material, firms may face distinct recycled investment efficiencies due to multifaceted causes.

Recycled label design principle. We figure out a *sandwich* principle for recycled label design: To maximize environmental performance, the certifier should design the label as continuous if the three above-stated metrics are intermediate, whereas as binary if these metrics are low or high. The rationale of this principle stems from distinct effects that the two labels impose on the industry. The continuous label has a *transparency effect* that allows firms to entirely convert their green efforts into public visibility and market competitiveness; this may provide them with sufficient motivation for recycled investment. The binary label has an *enforcement effect* that pushes firms to deviate from their own willingness-to-invest because any effort beyond the standard is invisible; the label's standard may be targeted to both firms, evening out their efficiency advantages/disadvantages, or targeted to the more efficient firm only, leaving the less efficient firm fully virgin.

When the competition intensity is low, both firms utilize a low percentage of recycled materials under the continuous label and it is better for the certifier to enforce both to invest more with a common standard of the binary label. When the competition intensity is intermediate, the common standard limits the market expansion of the more efficient firm, so the continuous label, which allows the more efficient firm to spontaneously utilize a higher percentage of recycled materials and obtain a larger market share, is better. When the competition intensity is high, the certifier should adopt the binary label that targets only to the more efficient firm. As a result, fully-recycled product from the more efficient firm dominates the market, and fully-virgin product from the less efficient firm gains only a tiny market share.

Similar rationales apply for the average and symmetricity of recycled investment efficiency. Along the path that the industry averagely grows more efficient (or becomes more symmetric) in recycled investment, the certifier should first focus on motivating the relatively more efficient firm with the binary label, then leverage the spontaneous competition between firms with the continuous label, and finally enforce both firms to increase the recycled-material percentage with the binary label. Notably, the binary label always outperforms when the firms are completely symmetric because a single standard can achieve the most effective control over them.

Other implications. By examining the firms' label preferences, we conclude that the certifier's choice cannot simultaneously align with both firms, but can align with the industry as a whole. Two situations may emerge: The two firms align but conflict with the certifier, or the certifier aligns with one firm but conflicts with the other. In the former situation, both firms prefer spontaneous competition that leads to adequate investment levels and market share allocation for them. However, the certifier selects the binary label that distorts their competition, leading to investment burden or market share shrinkage. In the latter situation, the certifier's choice of either label may turn out helping the more efficient firm further expand the market, or protecting the less efficient firm from her rival's market erosion. One firm's loss may exceed the other firm's gain such that the industry benefits overall.

By sensitivity analysis, we find high non-monotonicity of environmental performance and industry profit in the three key metrics. In particular, more intense green competition may not benefit the environment. When the certifier selects the binary label and sets a standard targeting both firms, more intense competition requires a lower standard because otherwise the less efficient firm will quit due to the investment burden. Given the continuous label, more efficient recycled investment may hurt the environment as well, although the certifier would select the binary label instead in this circumstance. With a milder competition or a higher investment efficiency, the industry do not necessarily become more profitable because the certifier may extract the industry surplus and turn it into environmental surplus with its label-selection power.

Chapter 2

Literature Review

Our paper is most related to two streams of literature: labeling/certification and green competition. In the following, we review each stream of literature and position our paper within.

Labeling or certification is considered as a tool to convey product quality information from sellers to buyers. We classify the labeling/certification literature into three branches. The first branch seeks to understand and explain the information mechanism underlying certification in a general setting. Biglaiser (1993) shows that a certifier with expertise can help mitigate information asymmetry and improve welfare. Lizzeri (1999) argues that a certifier may reveal only partial information to uninformed parties so as to extract surplus from the seller and buyers. Harbaugh et al. (2011) demonstrate that, when buyers are unsure of a label's standard, a small extent of such uncertainty may reduce or even eliminate the label's value to sellers. Farhi et al. (2013) rationalize why a certifier may not publicize rejected applications. Stahl and Strausz (2017) identify the distinct roles certification play for sellers (as a signaling device) and for buyers (as an inspection device). The second branch focuses on certification in specific contexts. In the context of intellectual property (IP) promotion, Lerner and Tirole (2006) study the IP owner's "forum shopping" behavior—strategically selecting the most favorable certifier to endorse their work. In the context of supply chain responsibility, Chen and Lee (2017) demonstrate that certification is a more effective mechanism than process audit and contigent payment for buyers to deal with suppliers' responsibility risk. In the context of social-good labeling, Heyes and Martin (2017) study the competition between NGOs, whose objectives are a mixture of their own label coverage and social benefit. In the context of green product development, Murali et al. (2019) find that, under competition, the less credible firm always seeks for external certification, whereas the more credible firm may self-certificate. In the context of electronic waste recovery, Esenduran et al. (2020) study the competition between two collector-recycler channels that can choose from either a high- or a low-standard certification. In the context of agricultural products, Agrawal and Zhang (2024) compare flexible premium vs. fixed premium that a firm should pay farmers to obtain an NGO's certification. The third branch empirically examines the effects of labeling/certification. With a data set on an environmental certification program, Rysman et al. (2020) find that increasing the number of certification tiers (information granularity) may reduce some firms' quality investment but promote the overall quality investment. Furthermore, Hui et al. (2023) conduct a field experiment on an e-commerce platform and find that introducing a new certification tier may raise or reduce a seller's quality effort, depending on whether the seller is young or established. Houde (2022) uses the data set of an energy-certification program to document that firms strategically use certification to extract consumer surplus.

We position our paper within the second branch discussed above because our research is motivated by a specific context of recycled label design. We capture three salient features of this context: First, the environmental quality of products has a clear measurement, the percentage of recycled materials, with a 100% ceiling. Second, two particular labeling formats, binary label and continuous label, are observed across various industries. Third, the effort required to obtain certification highly depends on material category and industry characteristics. These features differ our paper from the previous literature. Moreover, our paper also contributes insights to the first branch and the third branch, because the continuous label and the binary label represent distinct information mechanisms—the former has the highest information granularity whereas the latter has the lowest information granularity.

Green competition, as a general concept, can be related to various dimensions of firm competition. We classify this stream of literature into two branches: without or with a third-party policy (government or NGO) maker's intervention. In the first branch, green competition can manifest in various forms. Ferguson and Toktay (2006) show that a manufacturer's profit can increase in the presence of competition with a remanufacturer. Örsdemir et al. (2014) further demonstrate that a manufacturer can either use quality or quantity as a lever to deal with a remanufacturer's competition; moreover, encouraging independent remanufacturing can worsen the environmental performance. Raz and Souza (2018) study manufacturers' competition motivated by the metal cutting tools industry and show that recycling can serve as a strategic supply source that improves profitability. Tian et al. (2019) employs cooperative game theory to study the stability of recycling cooperation between competing firms. Fatehi et al. (2023) consider competition between an intrinsically responsible firm and a greenwasher who only makes observable environmental investments and ignores unobservable ones; they show that greenwashing may improve social welfare and a higher degree of transparency may reduce social welfare. In the second branch, a third party (government or NGO) with environmental or social-welfare objective can control green competition with various formats of policy intervention. Plambeck and Wang (2009) study e-waste regulation on competing firms when they introduce new products. Cohen et al. (2015) investigate a government's consumer subsidy on competing firms' green technology products. Park et al. (2015) find that, by imposing carbon penalty to competing retailers, a policy maker can improve social welfare especially when the competition is intense. Anand and Giraud-Carrier (2020) compare two approaches of pollution regulation to oligopoly: cap-and-trade and taxes. Wang et al. (2021) employ the global game framework to show that industry regulation should take into account competing firms' voluntary adoption level. Mohammadi et al. (2024) investigate how a budget-constrained regulator should adopt the penalty-and-subsidy combo to motivate competing firms' clean technology investment.

Our paper belongs to the second branch as we consider a third-party policy maker who controls green competition by labeling. The green competition in our paper has a specific format of recycled-material usage and investment, which straightforwardly connects the environmental aspect—the amount of recycled material consumed in the market—with the competition aspect—the market share of firms. Such a connection brings interesting and novel trade-offs. Regarding policy intervention, different from direct penalty or subsidy, labeling is an indirect approach that utilizes marketing motive to encourage firms' green effort. The third-party certifier's primary decision is on how to reveal firms' green effort to the market; that is, full transparency (continuous label) or limited transparency (binary label). In sum, our contribution to the green competition literature is twofold: introducing a new problem context coupled with new trade-offs and deepening the understanding of indirect, informational approaches for third-party intervention.

Chapter 3

Model Setup

We consider a game-theoretic model consisting of one certifier ("it"), two competing firms ("she"), and a mass of consumers ("he"). The certifier determines how to design the recycled label; each firm determines her product constitution and product price; consumers, with green awareness, make purchasing decisions based on their personal tastes, product prices, and the recycled labels attached to products. In the following, we elaborate on the details of the game, with a summary of notation presented in Appendix C.

Firms. We capture the competition between the two firms, firm 1 and firm 2, following the Hotelling fashion. They face a unit market interval $[0, 1]$ and each sells one product. Firm 1 is located at the left-hand side of the interval (point 0) and firm 2 is located at the right-hand side (point 1).

Each firm's product is processed using a combination of two types of materials: recycled and virgin (non-recycled). A key decision of firm i ($i \in \{1, 2\}$) is her product composition; that is, the percentage of recycled content, $\phi_i \in [0, 1]$, in her product, product i. Then $1 - \phi_i$ is the percentage of virgin content. In practice, following ISO 14021, the recycled content is measured by the weight ratio of recycled material to the whole product. For expositional convenience, we refer to a product with recycled-content percentage ϕ_i as a ϕ_i-recycled product.

To prepare for the production of ϕ_i-recycled products, firm i needs to first invest in the corresponding technology and facility that deal with the recycled materials, which incurs a fixed cost of $k_i\phi_i^2$. The quadratic functional form captures the decreasing marginal return of investment. Without loss of generality, we let the investment coefficients $k_1 \leq k_2$; i.e., firm 1 is more cost-efficient than firm 2 in recycled investment. As mentioned in the Introduction section, the type of material determines the recycled technological investment within a certain industry; e.g., utilizing recycled paper is easier than utilizing recycled textiles. We then employ $\kappa = (k_1 + k_2)/2$ to measure the *average investment coefficient* of the industry. Moreover, within the same industry, firms also face heterogeneous recycled investment efficiencies due to their distinct product designs, existing technological bases, and business environments. We then define $\gamma = k_1/k_2 \in [0, 1]$ to measure the *investment symmetricity* between the two firms. With κ and γ defined, we can rewrite $k_1 = 2\gamma\kappa/(1 + \gamma)$ and $k_2 = 2\kappa/(1 + \gamma)$. As we will elaborate later, both κ and γ will play important roles in the subsequent analysis.

Besides technology investment, the firms incur variable costs of materials for production. The variable cost of a ϕ_i-recycled product is $c_i = \phi_i \cdot c_r + (1 - \phi_i) \cdot c_v$, the weighted average of the two types of materials' unit costs c_r (subscript $_r$ stands for "recycled") and c_v (subscript $_v$ stands for "virgin"). Recycled materials are usually more expensive because they are labor and energy-intensive (Green and Grumpy 2022). Therefore, we assume $c_r > c_v$ and let $\Delta c = c_r - c_v$ denote the unit cost differential. Although there are also situations where recycled materials are cheaper, we only consider the more-expensive situations to rule out any potential cost-reduction incentive of recycled investment for firms and focus on the standard-compliance and consumer-satisfaction incentives. Our analysis can easily carry over to cases of $c_r \leq c_v$. Besides materials costs, we normalize any other source of production cost to zero, which does not affect the qualitative trade-offs.

After ϕ_i is set, each firm needs to determine her product price p_i. The two firms' products are horizontally differentiated and share a common base value v, which helps us concentrate on the recycled-content aspect of products, the theme of this paper. The demand for firm i, d_i, is characterized jointly by p_i, p_j ($j \neq i$), ϕ_i, and

ϕ_j. The detailed demand formulations will be derived in the "consumers" part. The payoff for firm i is thus formulated as:

$$\pi_i = \Big(p_i - [\phi_i \cdot c_r + (1 - \phi_i) \cdot c_v]\Big) \cdot d_i - k_i \phi_i^2. \tag{3.1}$$

Certifier. The certifier is a third party (NGO or government agency) that provides a voluntary and credible label of recycled content for products. It is able to verify the recycled-content percentage ϕ_i in firm i's product and, correspondingly, issue a recycled label. The certifier can design the recycled label in two ways: The *continuous* label exactly presents the value of ϕ_i. In contrast, the *binary* label sets a pass/fail standard $\underline{\phi} \in [0, 1]$, and product i obtains the label only if $\phi_i \geq \underline{\phi}$; i.e., the standard is met. The certifier's decision is on which type of label to provide for the industry and, if the label is binary, the standard $\underline{\phi}$.

The certifier is an environmental proactivist who only cares about the environmental impact of products. Let β_r (β_v) be the environmental impact of one unit of recycled (virgin) content. β_r and β_v can be either positive (meaning environmental benefit) or negative (meaning environmental damage). Apparently, $\beta_r > \beta_v$. We denote the certifier as player 0 in this game. Its payoff, the *environmental performance* of the industry, is then formulated as:

$$\pi_0 = \sum_{i \in 1,2} [\beta_r \phi_i + \beta_v (1 - \phi_i)] \cdot d_i. \tag{3.2}$$

The linear formation of environmental performance is widely used in the literature, e.g, Atasu and Souza (2013), Huang et al. (2019) and Long and Gui (2024). To encourage the firms' usage of recycled materials, the certifier does not charge them any fee for adopting the label. That is, the firms have no costs for adopting the recycled label, as long as they can meet the standard. Including the certification fee only adds a trivial fixed-cost trade-off, which will not change the main insights.

Consumers. The total number of consumers is normalized to one, and they are uniformly distributed over the Hotelling interval $[0, 1]$. A consumer located at position x experiences a mismatch disutility of $t \cdot |x - (i-1)|$ (tx if $i = 1$ and $t(1-x)$ if $i = 2$) if purchasing from firm i, where $|x-(i-1)|$ measures the extent of consumer-product mismatch and t is the unit cost of mismatch. Notably, according to the literature (e.g., Villas-Boas and Schmidt-Mohr 1999, Boone 2001, and Bijlsma et al. 2018), a higher t also means that the two firms are more differentiated, and thus their competition is less intense. We then define $\tau = 1/t$ to measure the *competition intensity* between firms, which is a crucial metric of the industry. When purchasing product i, a consumer also obtains its base value v and incurs disutility from its price p_i.

The final segment of a consumer's utility is the green value he obtains from purchasing the products. That is, they care about how much materials are recycled in the products. However, consumers are unable to directly learn the recycled-content percentage in a product; instead, they have to perceive this number through the label issued by the certifier. If the label is designed as continuous, a consumer observes ϕ_i and obtains utility

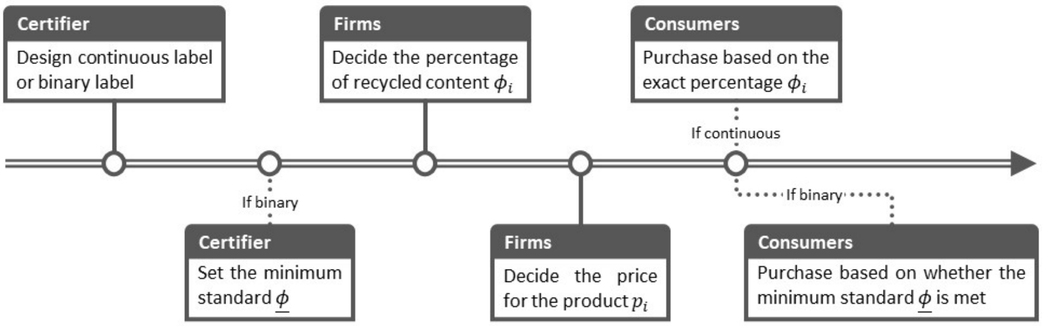
$$u_i = v + \theta \cdot \phi_i - \frac{1}{\tau} \cdot |x - (i-1)| - p_i \tag{3.3}$$

by purchasing product i. Here, we let θ denote the unit value obtained from recycled content, which also measures the *green awareness* of consumers. We focus on the situations of $\theta > \Delta c$; otherwise the model collapses to a trivial one that no firm uses any recycled content. If the label is designed as binary, a consumer only observes whether product i is issued the label or not; i.e., it passes or fails the standard $\underline{\phi}$. By purchasing product i, his utility is

$$u_i = v + \theta \cdot \underline{\phi} \cdot \mathbb{1}_{\{\phi_i \geq \underline{\phi}\}} - \frac{1}{\tau} \cdot |x - (i-1)| - p_i. \tag{3.4}$$

In other words, a consumer believes that product i has a recycled-content percentage of $\underline{\phi}$ if the product is issued the label, and the belief is zero recycled content if the label is absent. We note that this belief is rational because, besides demand boosting, investing in recycled content does not bring the firms any other benefit.

Figure 3.1: Timeline of the Game



Hence, lacking an effective information channel to convey any additional usage of recycled materials, firm i has no incentive to make ϕ_i deviate from 0 (if she does not intend to be labeled) or $\underline{\phi}$ (if she intends to be labeled). This fact will be validated later by equilibrium characterization.

Demand d_i is shaped by consumers' utility maximization. To ensure that the two firms are in an competitive setting instead of isolated monopolies, we assume that the base value v is sufficiently large ($v \geq 3/(2\tau) + c_v$) such that the market is fully covered; i.e., a consumer either purchases product 1 or product 2 and does not seek the outside option. This full-coverage assumption is a norm in the literature that study competition within the Hotelling framework (see, e.g., the seminal textbook Tirole 1988). Under this assumption, the total amount of materials used by the two firms is constant and so, in the whole industry, one more unit of recycled material used means one less unit of virgin material used. Moreover, to make sure that neither firm is forced out of the market, we assume the competition intensity between the two firms is not too high ($\tau < 3/(\theta - \Delta c)$).

Game sequence. The timeline of this game is depicted in Figure 3.1. The certifier first designs the recycled label as continuous or binary. If the label is binary, the certifier also sets the minimum standard $\underline{\phi}$ for a product to obtain the label. Then the two firms simultaneously determine ϕ_1 and ϕ_2, the recycled-content percentages in their products; the labeling follows. After observing each other's product composition, the firms further simultaneously determine p_1 and p_2, their product prices. Finally, consumers make purchasing decisions. As aforementioned, if the label is continuous, consumers purchase according to the exact recycled-content percentages; if the label is binary, consumers purchase according to whether the minimum standard is met.

Throughout the paper, we used accent ($\tilde{}$)/($\ddot{}$) to mark certain variables in the subgame where the recycled label is designed as continuous/binary. We also make the following tie-breaking rules: When a firm is indifferent between two recycled-content percentages, she adopts the higher one to benefit the certifier. When the certifier is indifferent between the two labels, it chooses the binary label. For the binary label, when the certifier is indifferent between two standards, it sets the high one. The latter two rules have no impact on our results as they only occur in boundary situations.

Finally, we elaborate on several issues regarding the scope of this model. First, we focus on the continuous label and the binary label instead of other labels in more complex formats. This is consistent with our industry observation that the focal two label types are most prevalent. Theoretically speaking, the continuous label and the binary label respectively represents the highest and the lowest information granularity of labeling, and therefore understanding them can also shed light on other labels, which has intermediary information granularity. Second, the usage of recycled content does not affect product quality. This is claimed by firms across industries, such as the usage of recycled aluminum in Apple (Apple 2023), wood in IKEA (IKEA 2022), plastic in Logitech (Logitech 2023), and textiles in Patagonia (Bastone 2022). We refer readers to Gao and Souza (2022) for a similar assumption. Third, we focus on a non-profit-driven (instead of profit-driven) certifier issuing voluntary (instead of mandatory) recycled labels. This captures the majority of the cases in practice. Most recycled-label issuers are government agencies or NGOs that do not seek profitability. While there are initiatives for mandatory usage of recycled content, the current and near-future practices are still voluntary (Taylor 2022). Fourth, we consider a monopolistic certifier. In practice, it is prevalent that a certain label dominates a specific domain. For example, according to the official statistics in FSC (2023) and PEFC (2023), for recycled forest-based materials in the

US, the FSC (Forest Stewardship Council) label is issued eight times as the PEFC (Program for the Endorsement of Forest Certification) label. For recycled textiles, the GRS (Global Recycled Standard) label is the most common and is issued more than three times the second-most common (Textile Exchange 2022). Moreover, as most certifiers share a common goal of promoting environmental well-being, the monopoly can also be regarded as a union of multiple certifiers.

Chapter 4

Analysis

4.1 Continuous Label

In this section, we characterize the equilibrium of the subgame when the certifier selects the continuous label. We also conduct some primary sensitivity analysis to show how firms' competition dynamics affect environmental performance.

4.1.1 Equilibrium

We solve for the equilibrium following backward induction. Recall that consumers are uniformly located in the interval $[0, 1]$ and that the market is fully covered by the two firms. Given utility Formulation (3.3), we suffice to find the location of the consumer who is indifferent between products 1 and 2, then derive the two firms' demand functions accordingly. Combining the demand functions with Equation (3.1), we formulate firm i's profit as:

$$\pi_i = \frac{1}{2} \cdot \Big(p_i - [\phi_i \cdot c_r + (1 - \phi_i) \cdot c_v]\Big) \cdot \Big(1 + \tau \cdot [\theta(\phi_i - \phi_j) - p_i + p_j]\Big) - k_i \phi_i^2, \quad (4.1)$$

where $(i, j) \in \{(1, 2), (2, 1)\}$, $k_1 = 2\gamma\kappa/(1 + \gamma)$, and $k_2 = 2\kappa/(1 + \gamma)$.

The firms make two-stage simultaneous decisions, first on product composition ϕ_i

and then price p_i. The sequence naturally follows the practice that firms normally first develop products and then determine prices when the selling season starts. Throughout the paper, we use the prime symbol ($'$) to denote interim equilibrium characterizations. We have:

Lemma 1. *Under the continuous label and given product compositions ϕ_1 and ϕ_2, the firms' pricing decisions are:*

$$(\tilde{p}_1', \tilde{p}_2') = \Big(\frac{1}{\tau} + \frac{\theta(\phi_1 - \phi_2) + 2c_1 + c_2}{3}, \frac{1}{\tau} + \frac{\theta(\phi_2 - \phi_1) + 2c_2 + c_1}{3}\Big),$$

where $c_i = \phi_i \cdot c_r + (1 - \phi_i) \cdot c_v$.

Some direct observations can be obtained from Lemma 1. A firm prices higher when: The competition is less intense (a lower τ); the firm's product contains more recycled content than her competitor (a higher $\phi_i - \phi_j$); the material cost for either firm is higher (a higher c_i or c_j, with c_i more impactful). Note that the product costs also depend on recycled-content percentages. Substituting $\tilde{p}_1'$ and $\tilde{p}_2'$ back into Equation (4.1), we can reformulate the firms' profits that only depend on ϕ_1 and ϕ_2 and then solve for the corresponding equilibrium decisions.

Lemma 2. *Under the continuous label, the firms' equilibrium recycled-content decisions are:*

$$(\tilde{\phi}_1, \tilde{\phi}_2) = \begin{cases} \big(1, 1\big) & \text{if } (\tau, \kappa, \gamma) \in \tilde{\Omega}_{11} \\ \big(1, \frac{(\gamma+1)(\theta-\Delta c)(3-\tau\theta+\tau\Delta c)}{36\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}\big) & \text{if } (\tau, \kappa, \gamma) \in \tilde{\Omega}_{1\phi} \\ \big(\frac{(\gamma+1)(\theta-\Delta c)[18\kappa-\tau(\gamma+1)(\theta-\Delta c)^2]}{6\kappa[36\gamma\kappa-\tau(\gamma+1)^2(\theta-\Delta c)^2]}, \frac{(\gamma+1)(\theta-\Delta c)[18\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2]}{6\kappa[36\gamma\kappa-\tau(\gamma+1)^2(\theta-\Delta c)^2]}\big) & \text{if } (\tau, \kappa, \gamma) \in \tilde{\Omega}_{\phi\phi} \end{cases}, \tag{4.2}$$

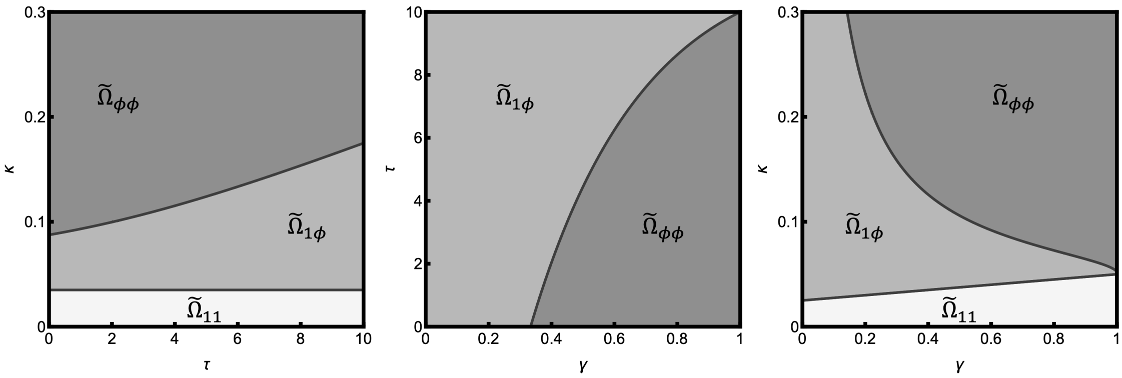
where parameter regions $\tilde{\Omega}_{11}$, $\tilde{\Omega}_{1\phi}$, and $\tilde{\Omega}_{\phi\phi}$ are characterized in Appendix A[1].

Throughout this paper, we focus on three key metrics (parameters) that represent important characteristics of the industry: first, the competition intensity τ; second,

[1] In a small sub-region of $\tilde{\Omega}_{1\phi}$, multiple equilibria may arise. We refine the equilibrium following the plausible criterion that the more efficient firm 1 invests more in recycled-content usage. This results in the uniqueness of equilibrium. In Appendix B, we provide the results in alternative criteria and show the robustness of our results.

Figure 4.1: Equilibrium Outcome Under Continuous Label

Parameter Region	Equilibrium Outcome Pattern
$\tilde{\Omega}_{11}$	Both firms conduct fully-recycled production
$\tilde{\Omega}_{1\phi}$	Firm 1 conducts fully-recycled production & firm 2 conducts partially-recycled production
$\tilde{\Omega}_{\phi\phi}$	Both firms conduct partially-recycled production



(a) $\gamma = 0.4$ (b) $\kappa = 0.1$ (c) $\tau = 5.2$

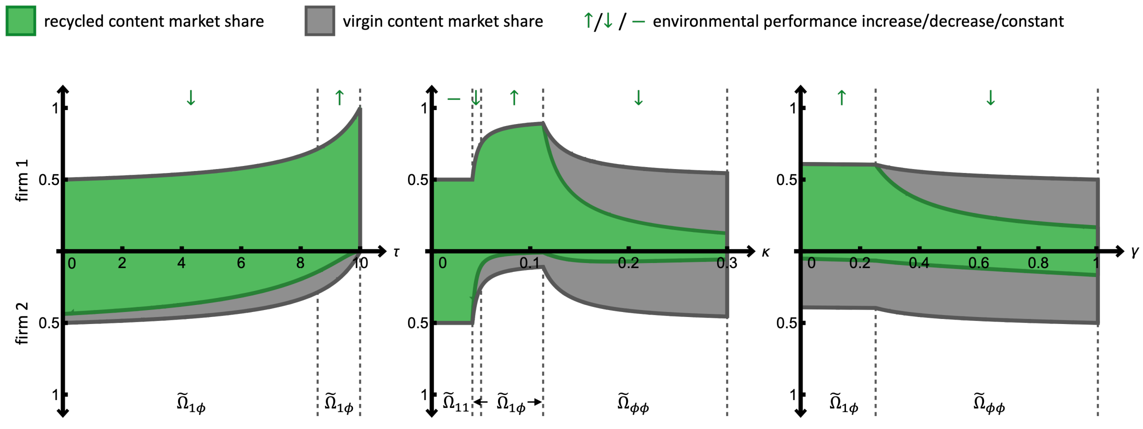
Note. $\theta = 0.4$ and $\Delta c = 0.1$.

the average recycled investment coefficient κ; third, the investment symmetricity γ. Lemma 2 then characterizes the equilibrium of the continuous-label subgame based on (τ, κ, γ). The whole parameter space can be segmented into three regions. Figure 4.1 illustrates these regions by depicting two-dimensional slices of the (τ, κ, γ) space. As shown in Figure 4.1(a), when κ is low (region $\tilde{\Omega}_{11}$), both firms enjoy efficient recycled investments and are willing to make their products with 100% recycled materials. When κ is not low: If the competition is not intense (τ is low), both firms use recycled materials partially to save the investment cost (region $\tilde{\Omega}_{\phi\phi}$); if the competition is intense (τ is high), the more efficient firm 1 can maintain 100% recycled content to grasp more market share whereas the less efficient firm 2 cannot (region $\tilde{\Omega}_{1\phi}$). The former situation tends to occur more if the firms' investment efficiencies are more symmetric (γ is high), as is shown in Figure 4.1(b) and (c).

4.1.2 Environmental Performance

To further understand the dynamics in the equilibrium, we examine how $\tilde{\pi}_0$, the environmental performance as well as the certifier's payoff, changes in the three

Figure 4.2: Environmental Impact of (τ, κ, γ) Under the Continuous Label (Proposition 1 Illustration)



(a) Impact of competition intensity $\kappa = 0.04, \gamma = 0.4$

(b) Impact of average investment coefficient $\tau = 8.5, \gamma = 0.65$

(c) Impact of investment symmetricity $\tau = 2.5, \kappa = 0.15$

Note. $\beta_r = 1$, $\beta_v = 0$, $\theta = 0.4$, and $\Delta c = 0.1$.

key parameters τ, κ, and γ. We note that, since we are focusing on the situation of full market coverage, the two firms' equilibrium demands $\tilde{d}_1$ and $\tilde{d}_2$ sum up to one and therefore also represent the corresponding market shares. Moreover, $\tilde{d}_i$ ($i \in \{1, 2\}$) can be further divided into $\tilde{\phi}_i\tilde{d}_i$ and $(1 - \tilde{\phi}_i)\tilde{d}_i$, the recycled content and virgin content respectively. The whole size-one market is therefore separated into four segments: firms 1 and 2's recycled and virgin market shares. The total recycled-content market share then equals $\tilde{\phi}_1\tilde{d}_1 + \tilde{\phi}_2\tilde{d}_2$. By Equation (3.2):

$$\tilde{\pi}_0 = \sum_{i \in 1,2} [\beta_r\tilde{\phi}_i + \beta_v(1 - \tilde{\phi}_i)] \cdot \tilde{d}_i = \beta_r + (\beta_r - \beta_v) \cdot (\tilde{\phi}_1\tilde{d}_1 + \tilde{\phi}_2\tilde{d}_2). \tag{4.3}$$

Since $\beta_r > \beta_v$ (recycled content has a more positive environmental impact than virgin content), the monotonicity of $\tilde{\pi}_0$ aligns with that of the total recycled-content market share. It then suffices to examine the latter. Throughout this paper, we use a sequence of "↑" and "↓" to represent monotonicity. For example, "↑↓" means "first increases and then decreases". Following this norm, we have:

Proposition 1. *Under the continuous label, environmental performance $\tilde{\pi}_0$: (i) ↑, ↓↑, or ↑↓↑ in τ; (ii) ↓ or ↓↑↓ in κ; (iii) ↑, ↓, ↑↓, ↓↑, or ↓↑↓ in γ. In particular, more intense competition or a lower average investment coefficient may lead to worse* *environmental performance.*

Proposition 1 shows various patterns of environmental performance's monotonicity, and we discuss several representative ones: ↓↑ in τ, ↓↑↓ in κ, and ↑↓ in γ; the other ones are delegated to Appendix A. We illustrate these representative patterns in Figure 4.2. In this figure, without loss of generality, we let $\beta_r = 1$ and $\beta_v = 0$ to *normalize* the environmental performance such that it numerically equals the total recycled-content market share. We depict how the market shares of each firm's recycled/virgin content vary in τ, κ, and γ, with firm 1 represented above the x-axis and firm 2 below. Green (grey) fills represent recycled (virgin) content. Given any τ, κ, or γ, the thickness of the green area then represents the total recycled-content market share (environmental performance).

While the first intuition suggests that more intense green competition pushes the industry to employ more recycled contents, our results demonstrate that it is not that trivial. Figure 4.2(a) examines a situation in which the industry is overall efficient in recycled investment (κ is low) such that firm 1 always keeps 100% recycled content in her product as τ varies. When τ is low, increasing τ can worsen the environmental performance. This is because firm 2 chooses cost saving (a lower $\tilde{\phi}_2$) coupled with market shrinkage (a lower $\tilde{d}_2$) to avoid competing with the aggressive firm 1; here, firm 2 still maintains a considerable market share ($\tilde{d}_2$) such that her reduction of recycled investment is sufficiently influential to lower the total recycled content in the market ($\tilde{\phi}_1\tilde{d}_1 + \tilde{\phi}_2\tilde{d}_2$, as well as $\tilde{\pi}_0$ under normalization). Competition intensity benefits the environment only when τ becomes high. Now firm 2 loses too much market share such that her decision is no longer that influential to the industry. The 100%-recycled firm 1 dominates to a greater extent as τ increases, so the industry-wise recycled content grows.

Moreover, even when the whole industry becomes more efficient in recycled investment (a lower κ, or k_1 and k_2 are both lower with their ratio fixed), the environment performance does not necessarily improve. In Figure 4.2(b), $\tilde{\pi}_0$ may locally increase as κ increases; i.e., a less efficient industry leads to better environmental performance. The dynamics is as follows. When κ is low, both firms make 100%-recycled products. When κ is moderate, firms 1 maintains 100%-recycled because k_1 is still low; we note that the 100%-recycled ceiling plays a key role here because it locally fixes firm 1's product composition as κ varies. Firm 2 makes her product partially recycled because k_2 is high now. Her recycled content percentage ($\tilde{\phi}_2$) and market share ($\tilde{d}_2$) both decrease in κ, which makes $\tilde{\pi}_0$ first decrease (when firm 2 still mains a considerable market share and is influential) and then increase (when firm 2's market share becomes too small and firm 1 dominates). As κ becomes high, the more efficient firm 1 also turns into partial-recycled production. The whole industry thus reduces recycled-content usage as κ further increases, which follows intuition.

The increase of γ means that the two firms are getting closer in the investment of recycled technology, while the whole industry's investment efficiency is fixed. In particular, firm 1 becomes less efficient (k_1 becomes larger) and firm 2 becomes more efficient (k_2 becomes smaller). This, as demonstrated in Proposition 1, can lead to a variety of monotonicity patterns for $\tilde{\pi}_0$ in γ, and Figure 4.2(c) captures one of them. Here, when γ is low, firm 1 conducts 100%-recycled production, and firm 2 conducts partial-recycled production. An increasing γ makes firm 2 raise $\tilde{\phi}_2$ such that $\tilde{\pi}_0$ also increases. When γ is high, both firms conduct partial-recycled production. An increasing γ makes firm 1 reduce $\tilde{\phi}_1$ and firm 2 raises $\tilde{\phi}_2$; the former prevails the latter due to firm 1's larger market share, such that $\tilde{\pi}_0$ also decreases.

4.2 Binary Label

In this section, we move on to characterize the equilibrium of the binary-label subgame, as well as the sensitivity analysis regarding environmental performance.

4.2.1 Equilibrium

Different from the case of continuous label, the binary-label subgame has an additional stage of decision: the minimum standard set by the certifier (See Figure 3.1). Again, we solve for the equilibrium of this three-stage subgame following backward induction. Firm i's profit, following Equation (3.1) and utility Formulation (3.4), is formulated as:

$$\pi_i = \frac{1}{2}\cdot\Big(p_i - [\phi_i\cdot c_r + (1-\phi_i)\cdot c_v]\Big)\cdot\Big(1+\tau\cdot[\theta\underline{\phi}(\mathbb{1}_{\{\phi_i\geq\underline{\phi}\}} - \mathbb{1}_{\{\phi_j\geq\underline{\phi}\}}) - p_i + p_j]\Big) - k_i\phi_i^2, \quad (4.4)$$

where $(i,j) \in \{(1,2),(2,1)\}$, $k_1 = 2\gamma\kappa/(1+\gamma)$, and $k_2 = 2\kappa/(1+\gamma)$. This is analogous to the characterization in the continuous-label case, except that now consumers cannot observe each firm's exact product composition; they can only observe the minimum standard and whether this standard is satisfied. The demand is therefore shaped by ϕ_i and ϕ_j in a discrete way, as represented by the indicator functions ($\mathbb{1}_{\{\cdot\}}$). We can obtain the final-stage pricing decisions as:

Lemma 3. *Under the binary label, given standard $\underline{\phi}$ and recycled contents ϕ_1 and ϕ_2, the firms' pricing decisions are:*

$$(\ddot{p}_1', \ddot{p}_2') = \Big(\frac{1}{\tau} + \frac{\theta\underline{\phi}(\mathbb{1}_{\{\phi_1\geq\underline{\phi}\}} - \mathbb{1}_{\{\phi_2\geq\underline{\phi}\}}) + 2c_1 + c_2}{3}, \frac{1}{\tau} + \frac{\theta\underline{\phi}(\mathbb{1}_{\{\phi_2\geq\underline{\phi}\}} - \mathbb{1}_{\{\phi_1\geq\underline{\phi}\}}) + 2c_2 + c_1}{3}\Big),$$

where $c_i = \phi_i \cdot c_r + (1-\phi_i)\cdot c_v$.

In the next, we directly present the equilibrium of the whole subgame, which combines the characterizations of the certifier's minimum standard and the firms' product-composition decisions.

Lemma 4. *Under the binary label, the certifier's equilibrium minimum-standard decision and the firms' equilibrium recycled-content decisions are:*

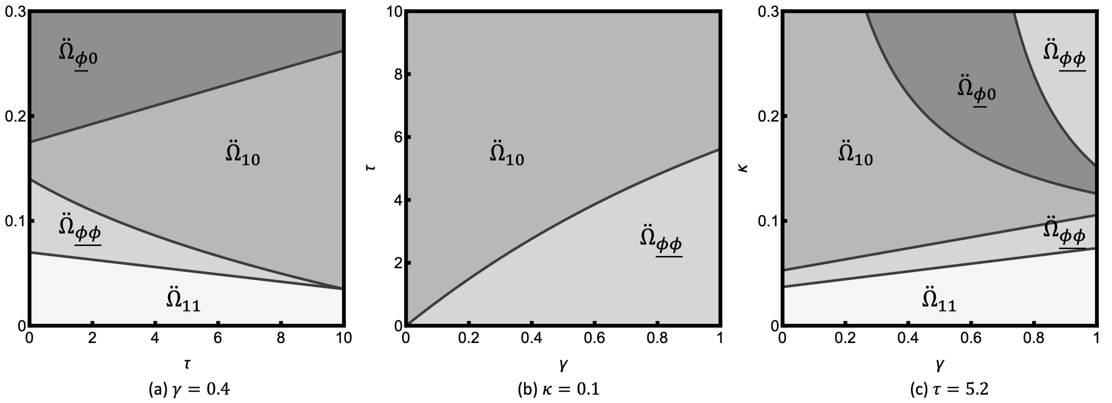
$$(\ddot{\underline{\phi}}, \ddot{\phi}_1, \ddot{\phi}_2) = \begin{cases} \big(1,1,1\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}_{11} \\ \big(\frac{6(\gamma+1)(\theta-\Delta c)}{36\kappa+\tau(\gamma+1)(\theta-\Delta c)^2}, \frac{6(\gamma+1)(\theta-\Delta c)}{36\kappa+\tau(\gamma+1)(\theta-\Delta c)^2}, \frac{6(\gamma+1)(\theta-\Delta c)}{36\kappa+\tau(\gamma+1)(\theta-\Delta c)^2}\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}_{\underline{\phi}\underline{\phi}} \\ \big(1,1,0\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}_{10} \\ \big(\frac{6(\gamma+1)(\theta-\Delta c)}{36\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}, \frac{6(\gamma+1)(\theta-\Delta c)}{36\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}, 0\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}_{\underline{\phi}0} \end{cases},$$

where parameter regions $\ddot{\Omega}_{11}$, $\ddot{\Omega}_{\underline{\phi}\underline{\phi}}$, $\ddot{\Omega}_{10}$, and $\ddot{\Omega}_{\underline{\phi}0}$ are characterized in Appendix A[2]

[2]Again, multiple equilibria may arise in a small sub-region of $\ddot{\Omega}_{10}$. We refine the equilibrium following the plausible rule that the more efficient firm 1 invests more in recycled technology. We show the robustness of our results under other rules in Appendix B.

Figure 4.3: Equilibrium Outcome Under Binary Label

Parameter Region	**Equilibrium Outcome Pattern**
$\ddot{\Omega}_{11}$	Standard is fully-recycled and met by both firms
$\ddot{\Omega}_{\underline{\phi}\underline{\phi}}$	Standard is partially-recycled and met by both firms
$\ddot{\Omega}_{10}$	Standard is full-recycled and met by firm 1 only; firm 2 conducts fully-virgin production
$\ddot{\Omega}_{\underline{\phi}0}$	Standard is partially-recycled and met by firm 1 only; firm 2 conducts fully-virgin production



Note. $\theta = 0.4$ and $\Delta c = 0.1$.

An initial observation from Lemma 4 is that firms only choose from two levels of product composition: exactly the minimum standard ($\ddot{\phi}_i = \ddot{\underline{\phi}}$) or zero ($\ddot{\phi}_i = 0$). This fact echos consumer-utility Formulation (3.4) that consumers do not believe any additional recycled content usage beyond what the label tells. Therefore, the certifier essentially sets a target product composition, only leaving firms to adopt or deny it. Moreover, the certifier can adjust the standard to induce different extents of adoption in the industry. On one hand, it can set a *both-targeted standard*; that is, the standard is sufficiently low[3] such that both firms are willing to adopt it. On the other hand, it can also set a *single-targeted standard*; that is, the standard is relatively high such that only the more efficient firm 1 is willing to adopt it.

Figure 4.3 depicts the parameter regions characterized in Lemma 4. Figure 4.3(a) shows that, when the industry is in general very efficient (κ is very low), the certifier sets a both-targeted standard that requires 100% recycled content. When κ is medium, competition intensity τ matters. If the competition is not intense (τ is low), the certifier simply lowers the both-targeted standard. If the competition is intense

[3]The definition of being "low" varies in the industry's investment efficiency; when κ is very small, 100% can be considered as "low."

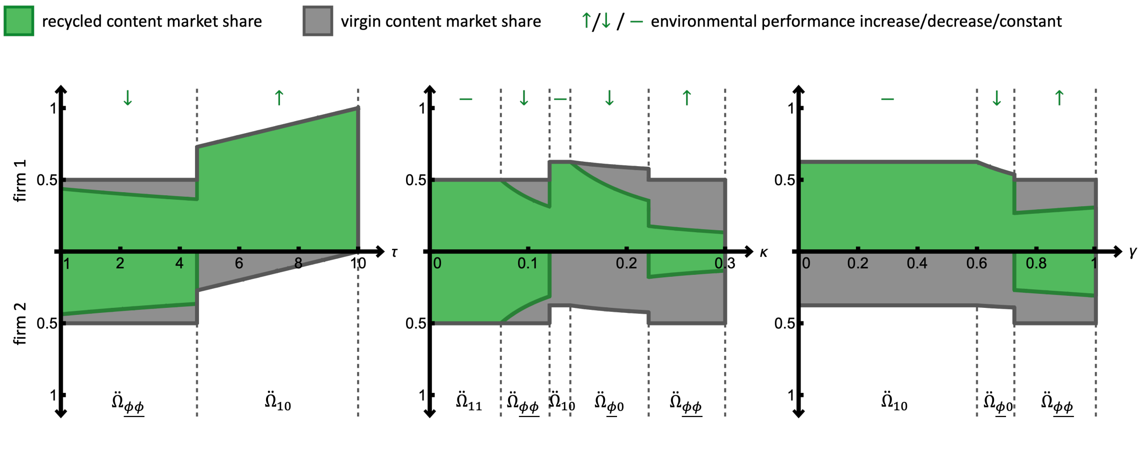
(τ is high), the certifier intentionally induces an extreme strategic differentiation between the two firms—one focusing on market expansion while the other focusing on cost reduction. That is, it insists on the 100% recycled standard such that firm 1 conducts 100%-recycled production and firm 2 completely gives up recycled content. We note that such strategic differentiation may occur even if the two firms are completely symmetric ($\gamma = 1$), as is shown in Figure 4.3(b) and Figure 4.3(c). This occurs when the competition is intense (a high τ) such that both firms suffer if they employ symmetric strategies; for one firm, the best response to the other firm's 100%-recycled production is not to adopt the standard. When κ is high, even the one-adopted standard cannot be 100%. Based on competition intensity τ and investment symmetricity γ, the certifier selects between a low, both-targeted standard and a relatively high, single-targeted standard. A related observation is drawn in Figure 4.3(b) and Figure 4.3(c): As the two firms are more symmetric (γ is higher), the standard is more likely to be both-targeted than single-targeted (in the figures, the height of region $\ddot{\Omega}_{\underline{\phi\phi}}$ becomes greater). Moreover, as shown in Figure 4.3(c), when γ is high, region $\ddot{\Omega}_{\underline{\phi\phi}}$ first arises when κ is relatively low and then arises again when it is high enough. This is because when κ is relatively low, both firms are willing to invest in recycled content, and thus the certifier is willing to target both firms. As κ increases within a moderate region, the certifier prefers to raise the standard to target a single firm. However, as κ becomes high enough, given that γ is also high, both firms are very inefficient, which makes a single firm's effort insufficient. As a result, the certifier switches back to a both-targeted standard.

4.2.2 Environment performance

We next conduct sensitivity analysis on $\ddot{\pi}_0$ following a similar fashion as in Section 4.2. Again, we normalize $\beta_r = 1$ and $\beta_v = 0$ such that environmental performance is equivalent to the total market share of recycled content. A sequence of $\uparrow$ and $\downarrow$ are used to represent various patterns of monotonicity. We have:

Proposition 2. *Under the binary label, environmental performance $\ddot{\pi}_0$: (i) $\uparrow$, $\downarrow$, or $\downarrow\uparrow$ in τ; (ii) $\downarrow$ in κ; (iii) $\uparrow$, $\downarrow$, or $\downarrow\uparrow$ in γ. In particular, more intense competition*

Figure 4.4: Environmental Impact of (τ, κ, γ) Under the Binary Label (Proposition 2 Illustration)



(a) Impact of competition intensity $\kappa = 0.08, \gamma = 0.4$

(b) Impact of average investment coefficient $\tau = 2.5, \gamma = 0.65$

(c) Impact of investment symmetricity $\tau = 2.5, \kappa = 0.15$

Note. $\beta_r = 1$, $\beta_v = 0$, $\theta = 0.4$, and $\Delta c = 0.1$.

may lead to worse environmental performance, whereas a lower average investment coefficient always leads to better environmental performance.

We select several representative monotonicity patterns for τ (↓↑), κ (↓), and γ (↓↑) and illustrate them in Figure 4.4. The other possible patterns are presented in Appendix A. Figure 4.4 not only presents the market share of each firm's each type of content, but also implies the minimum standard set by the certifier. Namely, in a both-targeted situation, the two firms' common product composition, which equals the total recycled market share, reflects the standard; in a single-targeted situation, firm 1's product composition reflects the standard.

As competition intensity τ increases, $\ddot{\pi}_0$ may first decrease and then increase, as is depicted in Figure 4.4 (a). When τ is low, the certifier gradually adjusts down the standard as τ increases; the environmental performance also gradually decreases. Here, given a mild competition, both firms have the incentive to utilize a certain portion of recycled content. The certifier thus sets a low standard to comply with their common interest. A more intense competition requires more compliance (a lower standard) from the certifier to maintain both firms' willingness-to-adopt. When τ becomes high, such a compliance becomes unacceptable to the certifier, who thus switches to a high, single-targeted standard. Now the firms employ differentiated strategies—firm 1 focuses on market expansion and firm 2 focuses on cost saving. Therefore, a more intense competition makes firm 1 willing to adopt a higher standard set by the certifier, which helps firm 1 further dominate the market. The environmental performance then improves.

In contrast to the continuous label, under the binary label, more efficient recycled investment in the industry (a lower κ) always benefits the environment. In Figure 4.4 (b), we observe that the certifier changes the standard scheme from both-targeted to single-targeted and back to both-targeted as κ increases. Yet, under these changes, $\ddot{\pi}_0$ always decreases in κ. The rationale can be understood by examining how the minimum standard $\underline{\ddot{\phi}}$ changes in κ. When κ is low, the both-targeted standard can be 100%. As κ increases, the both-targeted standard decreases. Here, $\underline{\ddot{\phi}} = \ddot{\phi}_1 = \ddot{\phi}_2$ so the total recycled market share as well as the environmental performance also decreases. When κ is medium, the certifier switches to a single-targeted standard. Note that, at the switching point, the environmental performance must be the same under the optimal both-targeted standard and the optimal single-targeted standard; otherwise the certifier would switch at a smaller or a larger κ. In this single-targeted range, an increased κ requires the certifier to tune down the standard such that firm 1 is willing to compromise. When κ becomes high, a single firm's effort is insufficient, so the certifier switches back to a both-targeted standard and the decrease of $\ddot{\pi}_0$ continues. In summary, under the binary label, the power of setting the minimum standard always enables the certifier to take advantage of the industry-wise recycled investment efficiency by manipulating the recycled-content market share.

The investment symmetricity γ also plays a key role in the certifier's standard decision. In Figure 4.4 (c), as γ increases (κ is fixed, k_1 becomes higher and k_2 becomes lower), $\ddot{\pi}_0$ first decreases (coupled with a single-targeted standard) and then increases (coupled with a both-targeted standard). When γ is low, the two firms' recycled investment efficiencies are distinct such that the ceitifier focuses on inducing firm 1's compliance. Increasing γ means that firm 1 becomes less efficient (k_1 is higher) so the standard has to be lowered. When γ is high, the two firms have close investment efficiencies such that the certifier focuses on making both comply. The standard has to be tailored for the less efficient firm 2. Hence, increasing γ means that firm 2 becomes more efficient (k_2 is lower) so the standard can be higher, which improves the environmental performance.

4.3 Label Selection

After understanding the dynamics under each type of label, we are now able to make a comparison between them. In this game and in practice, it is the certifier's entitlement to determine the label design. Therefore, we first examine which type of label the certifier should choose. Nevertheless, it is also important for social planners to understand how label selection influences industry profitability. Hence, we also examine the firms' preferences and whether they align with the certifier's selection. We finally re-conduct sensitivity analysis with respect to τ, κ, and γ under the certifier's label choice.

4.3.1 The Certifier's Choice

The certifier compares the environmental performances under the two types of labels, $\tilde{\pi}_0$ and $\ddot{\pi}_0$. Then, we have the core result of this paper:

Proposition 3. *There exist parameter regions $\mathcal{C}$, $\mathcal{B}$, and $\mathcal{I}$ (characterized in Appendix A) such that the certifier chooses the continuous label if $(\tau, \kappa, \gamma) \in \mathcal{C}$, chooses the binary label if $(\tau, \kappa, \gamma) \in \mathcal{B}$, and is indifferent if $(\tau, \kappa, \gamma) \in \mathcal{I}$. In particular:*
(i) When τ increases, the choice changes following the pattern $\mathcal{B} \to \mathcal{C} \to \mathcal{B}$, $\mathcal{C} \to \mathcal{B}$, $\mathcal{B}$, or $\mathcal{I}$.
(ii) When κ increases, the choice changes following the pattern $\mathcal{I} \to \mathcal{B} \to \mathcal{C} \to \mathcal{B}$ or $\mathcal{I} \to \mathcal{B}$.
(iii) When γ increases, the choice changes following the pattern $\mathcal{B} \to \mathcal{C} \to \mathcal{B}$ or $\mathcal{C} \to \mathcal{B}$ or $\mathcal{B} \to \mathcal{I}$.

The most notable result from Proposition 3 as well as the corresponding Figure 4.5 is the "sandwich" principle of label selection: With respect to competition intensity

τ, average recycled investment coefficient κ, and investment symmetricity γ, the certifier should choose the continuous label if these metrics fall within an intermediate range, whereas the binary label if these metrics are low or high.

Before diving into the detailed comparison result between the two labels, we qualitatively discuss the pros and cons of each label based on the implications obtained in Sections 4.1 and 4.2.

The advantage of the continuous label is the *transparency effect*. That is, it endows firms with full information transparency to convey their recycled effort to consumers, which turns into market competitiveness. Competition under transparency may drive firms to spontaneously intensify recycled investment. This is a disadvantage for the binary label—once the minimum standard is set, no firm is willing to invest beyond the standard because such effort is unobservable to consumers and does not bring additional market share. Such a disadvantage is especially significant when firms' investment efficiencies are distinct. To cope with the less efficient firm, the certifier has to tune down the standard, which loses potential effort boost from the more efficient firm.

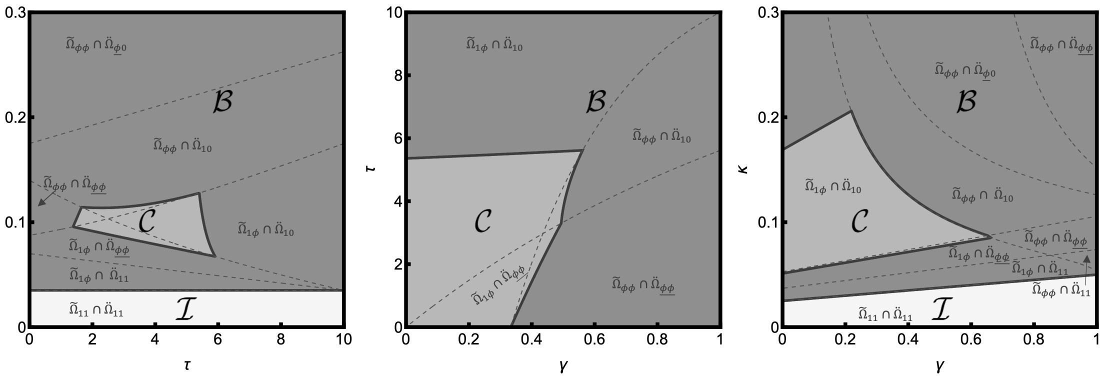
Conversely, the advantage of the binary label is the *enforcement effect*. That is, the certifier can set a standard that pushes the firms to deviate from their own willingness to invest. Under the binary label, a firm only has two options: to adopt or not to adopt. To induce a firm's adoption, the certifier only needs to make sure that the firm prefers to adopt than not to, instead of offering the most favorable standard for the firm. The certifier then has the space to enforce a standard that is higher or lower than firms' spontaneous choices, by which controlling their competition. This is a disadvantage for the continuous label—the certifier essentially has no control over the firms' product composition decisions, which is purely driven by market conditions and firm costs.

Next we provide the rationales behind the sandwich principle by discussing when one label scheme's advantageous effect can outweigh the other. We preface the discussion with a general principle: The continuous label induces a moderate level of strategic differentiation between firms with their spontaneous competition, whereas the binary label enforces either no differentiation (with a both-targeted standard) or an extreme differentiation (with a single-targeted standard). When the firms have little incentive to be differentiated (a low τ), the whole industry is efficient in recycled investment (a low κ), or the firms are relatively symmetric (a high γ), the enforcement effect of the binary label is particularly effectual, since the certifier can set a both-targeted standard to enforce both firms to invest "a bit more". Besides, when the firms have great incentive to be differentiated (a high τ), the industry overall is inefficient in recycled investment (a high κ), or the firms are relatively asymmetric (a low γ), the enforcement effect of the binary label is also effectual, since the certifier can set a single-targeted standard to greatly leverage the investment capability of the more efficient firm by enforcing extreme differentiation between the firms. However, when these three metrics are within the intermediate range, the disadvantage of the binary label becomes prominent. To cope with the investment capability of the less efficient firm, the certifier has to knock down the standard, which loses potential additional effort from the more efficient firm. Alternatively, to exploit the investment capability of the more efficient firm, the certifier has to build up the standard, which deters the less efficient firm from participating in the green competition. Under these conditions, the continuous label then stands out, since the transparency effect enables each firm to sustain adequate recycled investment under differentiation. Thus, the sandwich principle is intuitively explained.

Next, recall the parameter regions characterized in Lemmas 2 and 4. We elaborate the detailed dynamics behind the sandwich principle by going through these parameter regions with the help of Figure 4.5.

We highlight several direct observations across Figure 4.5(a), (b), and (c). First, the certifier is indifferent between the two label types when κ is very low. The corresponding parameter regions is $\tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, where both firms conduct 100%-recycled production under each label. Second, the binary label always outperforms in $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{11}$ and $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{11}$. These regions are where the binary label's enforcement effect can push both firms to conduct 100%-recycled production, whereas at least one firm spontaneously conducts partial-recycled production under the continuous label. Third, the binary label always outperforms in $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi} 0}$ as well. In this region, κ is high such that, under the continuous label, both firms utilize low percentages of recycled materials in their products. With the binary label, however, the certifier can enforce their differentiation with a single-targeted standard: The more efficient firm 1 is pushed to utilize more recycled materials and grasp more market share whereas the less efficient firm 2 is pushed out of the recycled strategy and shrink her market share. Letting firm 1 dominate the market turns out more beneficial for the environmental performance.

Figure 4.5: Certifier's Label Preference



(a) $\gamma = 0.4$ (b) $\kappa = 0.1$ (c) $\tau = 5.2$

Note. $\theta = 0.4$ and $\Delta c = 0.1$.

With the discussions above, the continuous label may be selected only in four scenarios (region intersections): $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$, and $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$. We will then focus on these four scenarios in the subsequent discussions.

Competition intensity. Let us first pay attention to the left side of Figure 4.5(a), region $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$. Here, under either label, both firms partially utilize recycled materials in their products. As competition intensity τ increases, the label selection changes from $\mathcal{B}$ to $\mathcal{C}$. Under a very mild competition, the firms have very weak motivations to differentiate. Then the binary label's enforcement effect can push both firms invest more in recycled technology. When the competition becomes a bit more intense, the firms gain higher incentive to differentiate, such that a common minimum standard is insufficient to serve as an effective control. Instead, the transparency effect of the continuous label allows them to maintain sufficient recycled investment under differentiation, and is therefore selected.

Moving to the right side of Figure 4.5(a), let us then pay attention to region $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$. Here, under either label, firm 1 conducts 100%-recycled production. As competition intensity τ increases, the label selection changes from $\mathcal{C}$ to $\mathcal{B}$. When the competition is not too high, spontaneous competition under the continuous label results in firm 1 100%-recycled and firm 2 partially-recycled. Under the binary label, due to lack of transparency, the certifier has to set a 100% standard, making firm 2 give up recycled materials. The continuous label is then selected. However, when the competition becomes very high, the two firms are too differentiated such that, given her small market share, firm 2's partial-recycled product under the continuous label benefits the environment insignificantly. The certifier would rather select the binary label to enforce firm 2 out of the recycled strategy, shrink her market size, and therefore help the 100%-recycled firm 1 seize an even larger market share.

In sum, along the increasing path of competition intensity τ, the certifier's strategy evolves according to the following pattern: enforcing both firms to adopt the minimum standard with the binary label $\rightarrow$ leveraging the firms' spontaneous competition with the continuous label $\rightarrow$ enforcing the more efficient, 100%-recycled firm to dominate the market with the binary label. The $\mathcal{B} \rightarrow \mathcal{C} \rightarrow \mathcal{B}$ sandwich principle is thus rationalized. Besides Figure 4.5(a), such a rational is also reflected by Figure 4.5(b), which illustrates parameter regions in (γ, τ)-coordinates.

Average recycled-investment coefficient. Let us first pay attention to the upper side of Figure 4.5(a), $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$. Here, the firms act distinctly under the two labels. Under the continuous label, both firms produce partially-recycled products; under the binary label, the certifier adopts a single-targeted standard such that firm 1 conducts 100%-recycled production whereas firm 2 conducts 100%-virgin production. Locally, when κ is high, both firms are not so efficient and their recycled-content percentages would be low under the continuous label. It is better for the certifier to enforce one firm's 100%-recycled production with the binary label. On the contrary, when κ is locally low, both firms are relatively efficient such that the continuous label, which allows both firms to exert recycled efforts, outperforms.

Moving to the bottom side of Figure 4.5(a), let us then pay attention to region

$\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$. Here, under the continuous label, firm 1 conducts 100%-recycled production and firm 2 conducts partially-recycled production; under the binary label, the certifier sets a both-targeted standard to motivate partially-recycled production. Locally, a relatively high κ means that the binary label's standard has to be lowered and letting 100%-recycled firm 1 dominate the market is a better choice. In contrast, a relatively low κ means that the binary label's standard can be set higher and enforcing both firms adopt this standard leads to a better environmental performance.

In sum, along the decreasing path of average recycled-investment coefficient (increasing path of average recycled-investment efficiency), the certifier's strategy evolves according to the following pattern: enforcing the more efficient, 100%-recycled firm to dominate the market with the binary label $\rightarrow$ leveraging the firms' spontaneous competition with the continuous label $\rightarrow$ enforcing both firms to adopt the minimum standard with the binary label. The $\mathcal{B} \rightarrow \mathcal{C} \rightarrow \mathcal{B}$ sandwich principle is thus rationalized. Besides Figure 4.5(a), such a rational is also reflected by Figure 4.5(c), which illustrates parameter regions in (γ, κ)-coordinates.

Recycled-investment symmetricity. Like competition intensity τ, the two firms' symmetricity γ also influences their differentiation strategies. They are less inclined to differentiate as γ increases. As a result, in Figure 4.5(b) and (c), we observe that increasing γ locally leads to a switch from $\mathcal{B}$ to $\mathcal{C}$ within $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$ and a reversed switch from $\mathcal{C}$ to $\mathcal{B}$ within $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$. The sandwich principle for γ is therefore established.

An accompanied implication is that, where the firms are completely symmetric ($\gamma = 1$), the binary label always outperforms the continuous label. This is because one standard can uniformly control both firms when they share a common investment efficiency. Hence, the continuous label is meaningful only when the firms have a sufficient degree of differentiation.

4.3.2 Preference Alignment with Firms

Although the recycled label is selected by the certifier, it also matters for social planners to understand how label selection affects industry profitability. In the next, we examine which label is preferred by the firms, and how their preferences align with the certifier's label selection. We obtain:

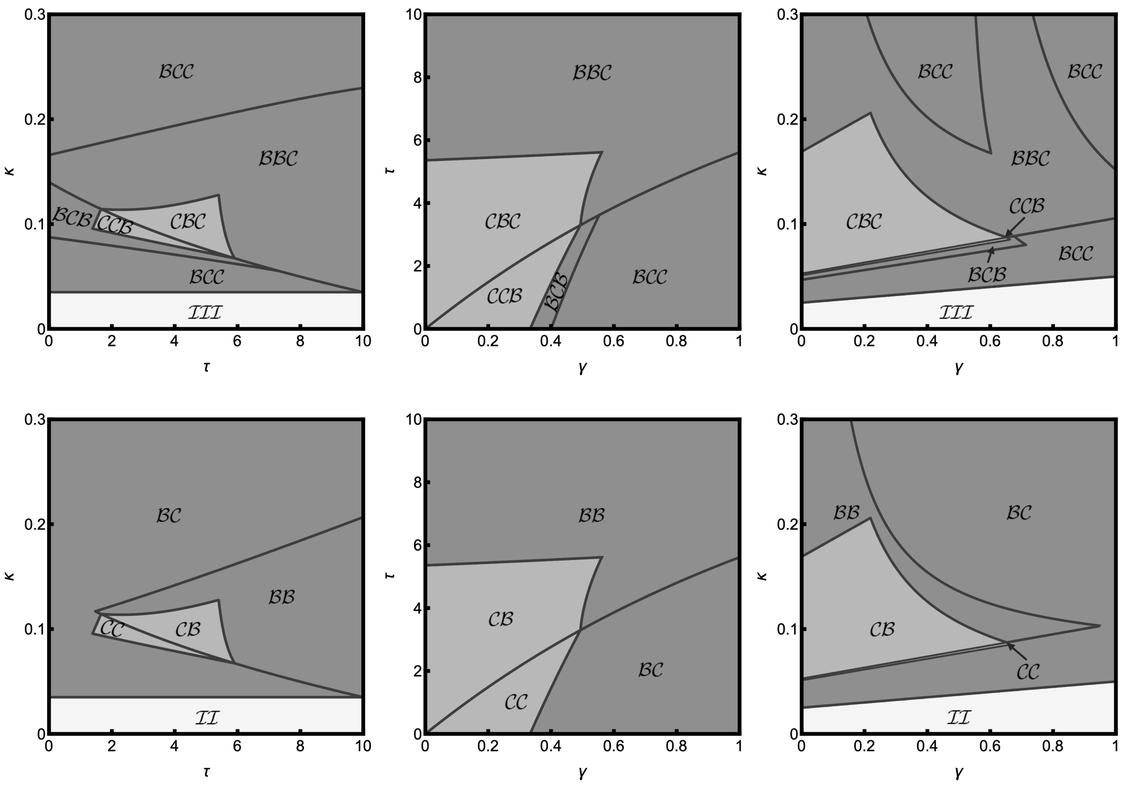
Proposition 4. *The certifier's label selection never aligns with both firms' preferences (except for the cases that the two labels are indifferent), but may align with the industry's (two firms' total) preference. Formally: (i)* $\tilde{\pi}_0 > \ddot{\pi}_0$*,* $\tilde{\pi}_1 > \ddot{\pi}_1$*, and* $\tilde{\pi}_2 > \ddot{\pi}_2$ *(*$\tilde{\pi}_0 < \ddot{\pi}_0$*,* $\tilde{\pi}_1 < \ddot{\pi}_1$*, and* $\tilde{\pi}_2 < \ddot{\pi}_2$*) cannot simultaneously hold; (ii)* $\tilde{\pi}_0 > \ddot{\pi}_0$ *and* $\tilde{\pi}_1 + \tilde{\pi}_2 > \ddot{\pi}_1 + \ddot{\pi}_2$ *(*$\tilde{\pi}_0 < \ddot{\pi}_0$ *and* $\tilde{\pi}_1 + \tilde{\pi}_2 < \ddot{\pi}_1 + \ddot{\pi}_2$*) can simultaneous hold for certain parameters.*

Figure 4.6 illustrates the preferences of the certifier, firm 1, and firm 2. Despite the many combinations that may arise, we prove in Proposition 4 that the $\mathcal{BBB}$ pattern or the $\mathcal{CCC}$ pattern may never arise. There may be two situations: the firms' preferences align, but conflict with the certifier; the certifier's preference aligns with one firm but conflicts with the other firm.

The former situation specifically refers to the $\mathcal{BCC}$ pattern (the $\mathcal{CBB}$ pattern never arises); i.e, the firms both prefer the continuous label whereas the certifier selects the binary label. Here, both firms hope they can enjoy free competition, under which the market share allocation and investment levels are adequate for them. However, the certifier's binary-label standard may distort their competition in two ways. In the first way, the standard is both-targeted, which is too low for firm 1 to fully exploit her efficiency advantage, but too high for firm 2 to escape from investment burden. In the second way, the standard is single-targeted, which creates investment burden even for firm 1 but makes firm 2 lose too much market share.

The latter situation contains various patterns, including $\mathcal{BBC}$, $\mathcal{BCB}$, $\mathcal{CBC}$, and $\mathcal{CCB}$. That is, the certifier's either label selection may align with either firm and conflicts with the other. When aligning with firm 1, the certifier may select a binary label ($\mathcal{BBC}$) with single-targeted standard to enhance her market share, or select a con-

Figure 4.6: Label Preference of Certifier and Firms



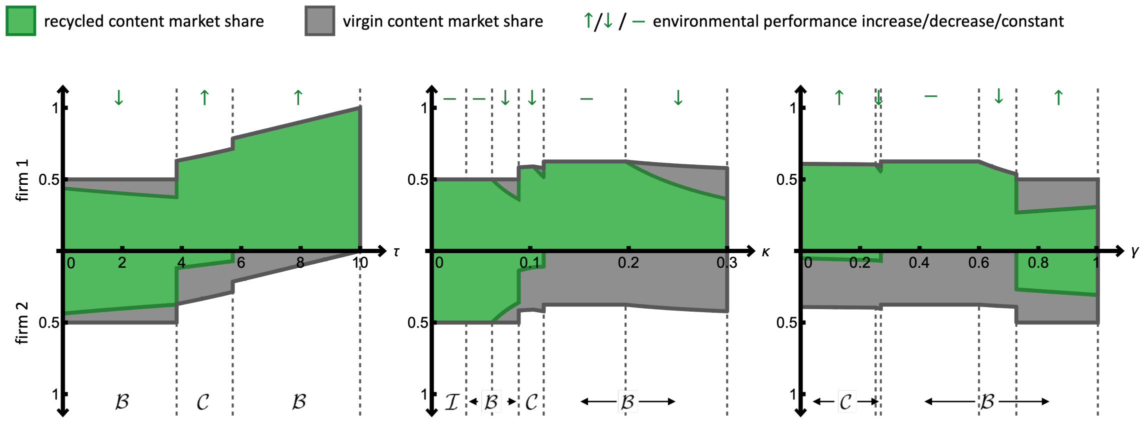
(a) $\gamma = 0.4$ (b) $\kappa = 0.1$ (c) $\tau = 5.2$

Note. $\theta = 0.4$ and $\Delta c = 0.1$. For example, $\mathcal{BCB}$ means that the certifier prefers the binary label, firm 1 prefers the continuous label, and firm 2 prefers the binary label. $\mathcal{BC}$ means that the certifier prefers the binary label whereas the industry prefers the continuous label.

tinuous label ($\mathcal{CCB}$) that otherwise the binary label's both-targeted standard would undermine firm 1's efficiency advantage. When aligning with firm 2, the certifier may select a binary label ($\mathcal{BCB}$) with both-targeted standard that protects firm 2 from the aggressive market share erosion of firm 1, or select a continuous label ($\mathcal{CBC}$) that otherwise the binary label's single-targeted standard would enforce firm 2 to be fully virgin and lose too much market share.

When the certifier's label selection aligns with one firm, the firm's total gain in market share expansion, price increase, or investment savings may exceed the other firm's loss in these aspects. Consequently, the certifier and the industry as a whole reach an agreement. As is shown in Figure 4.6, the break-even boundaries of such alignment can present many patterns and, for the ease of exposition, we no longer go through them one by one.

Figure 4.7: Environmental Impact of (τ, κ, γ) Under the Certifier's Choice (Proposition 5 Illustration)



(a) Impact of competition intensity $\kappa = 0.08, \gamma = 0.4$

(b) Impact of average investment coefficient $\tau = 2.5, \gamma = 0.4$

(c) Impact of investment symmetricity $\tau = 2.5, \kappa = 0.15$

Note. $\beta_r = 1$, $\beta_v = 0$, $\theta = 0.4$, and $\Delta c = 0.1$.

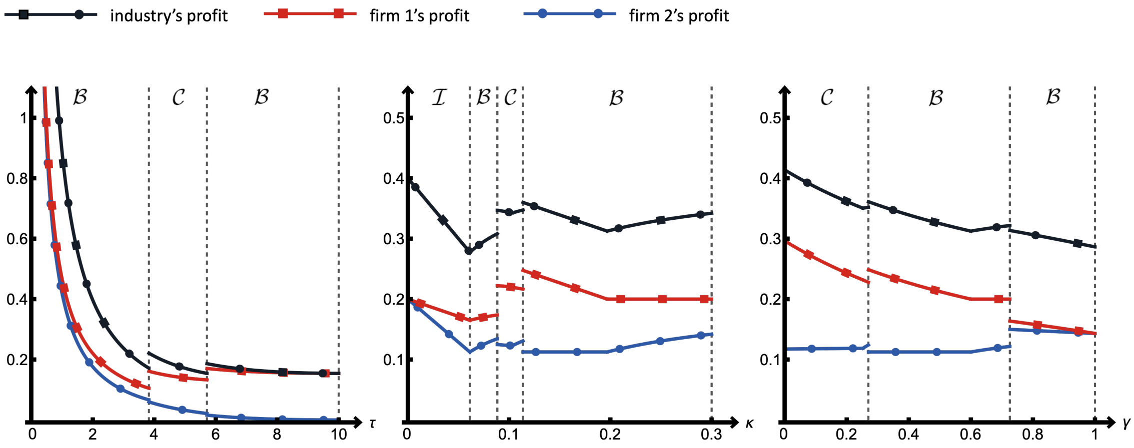
4.3.3 Sensitivity Analysis

We finally examine how the environmental performance and industry profit change in the three key parameters τ, κ, and γ.

Proposition 5. *Under the certifier's label choice, environmental performance* $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$*: (i)* $\uparrow$*,* $\downarrow$*, or* $\downarrow\uparrow$ *in* τ*; (i)* $\downarrow$ *in* κ*; (iii)* $\uparrow$*,* $\downarrow$*,* $\uparrow\downarrow$*,* $\downarrow\uparrow$*, or* $\uparrow\downarrow\uparrow$ *in* γ*.*

Proposition 5 is a result of combining Propositions 1 and 2 under the certifier's label selection. For τ, as is illustrated in Figure 4.7, the decreasing monotonicity occurs when the certifier selects the binary label. This is because the certifier has to lower the both-targeted standard when the firms compete more intensely and the less efficient firm has a stronger incentive to quit recycled-content utilization. The increasing monotonicity occurs when the certifier switches to the continuous label, such that more intense competition leads to higher recycled-content percentage and larger market share for the more efficient firm 1. Then the certifier switches to the binary label with single-targeted binary to further help the more efficient firm enlarge market share such that the increasing monotonicity patterns continues. For κ, since the certifier has an even stronger control over the industry than the scenario of binary label only, it can also take more advantage of a more efficient industry. For

Figure 4.8: Industrial Impact of (τ, κ, γ) Under the Certifier's Choice (Proposition 6 Illustration)



(a) Impact of competition intensity $\kappa = 0.08, \gamma = 0.4$

(b) Impact of average investment coefficient $\tau = 2.5, \gamma = 0.4$

(c) Impact of investment symmetricity $\tau = 2.5, \kappa = 0.15$

Note. $\beta_r = 1$, $\beta_v = 0$, $\theta = 0.4$, and $\Delta c = 0.1$.

γ, we observe a first-increase-then-decrease pattern when the certifier selects the continuous label, whereas a first-decrease-then-increase pattern when the certifier selects the binary label. The combination gives rise to the increase-decrease-increase pattern.

Proposition 6. *Under the certifier's label choice, industry profit* $(\tilde{\pi}_1+\tilde{\pi}_2)\cdot\mathbb{1}_{\{\tilde{\pi}_0>\ddot{\pi}_0\}}+(\ddot{\pi}_1+\ddot{\pi}_2)\cdot\mathbb{1}_{\{\tilde{\pi}_0\leq\ddot{\pi}_0\}}$ *is non-monotonic in* τ, κ, *and* γ, *and may increase and decrease multiple times.*

The monotonicity of industry profit is complex because the firms do not own the power to select the label, and therefore have to face the many distortions brought by the certifier's decision. We highlight two observations. First, milder competition does not necessarily lead to higher industry profit. As is shown in Figure 4.8, although the general trend is decreasing, local upward jumps occur when the certifier switches from one label to the other. Second, higher efficiency does not always benefit the industry as well. Here, even without the certifier's label switch, a lower κ (higher industry efficiency) may still lead to lower industry profit. Under the continuous label, a general high efficiency helps the less efficient firm 2 but hinders the relative competitiveness of firm 1, making the total effect ambiguous. Under the binary label, the certifier can manipulate the minimum standard such that the profit surplus brought by higher efficiency is extracted and transformed to environmental surplus.

Chapter 5

Conclusion

Firms who proactively invest in the usage of recycled materials should be recognized and rewarded by the market. In this process, government agencies or NGOs can serve as a credible information channel by recycled-label certification. Across various recycled industries–paper, plastic, and textile, two types of labels are most commonly adopted by certifiers: The continuous label precisely reports a product's recycled-content percentage, while the binary label sets a minimum standard and is issued only when the standard is met. This research addresses a fundamental question in this context: Which label should the certifier select to maximize environmental performance?

The answer depends on three key metrics of the industry: firms' competition intensity, the average efficiency of recycled technology investment, and firms' symmetricity of investment efficiency. When the competition is fierce, the recycled investment is inefficient, or the investment symmetricity is low, the certifier should adopt the binary label associated with a standard that targets only at advanced (relatively higher investment efficiency) firms. When the competition is moderate, the recycled investment is mediumly efficient, or the investment symmetricity is intermediate, the certifier should adopt the continuous label that allows firms to compete spontaneously on the green market. When the competition is mild, the recycled investment is very efficient, or the investment symmetricity is high, the certifier should adopt the binary label associated with a common standard that enforces all firms in the industry. In summary, along the increasing/decreasing paths of the three key metrics, the certifier should follow the binary→continuous→binary "sandwich" principle to determine the recycled label selection.

Policy makers and stakeholders should keep in mind that the more environmental-friendly label may not favor industry profitability. In particular, when the binary label is selected by the certifier, all the firms in the industry may prefer the continuous label. It can also be that the certifier's label preference, either continuous or binary, aligns with the more (less) advanced firms but conflicts with the less (more) advanced firms. In this situation, the winners' gain may exceed the losers' loss, making the whole industry better off. Policy makers and stakeholders should also be aware that fiercer green competition or higher efficiency of recycled investment may not benefit the environment, due to the distortion brought by the label mechanisms. Milder competition or higher efficiency may not benefit the industry as well, as the economic benefit may be extracted by the certifier with sophisticated label design, and converted into environmental benefit.

The study in this thesis can be extended in several possible directions for future research. One potential research direction is to consider the existence of multiple recycled labels. Although a single label often predominates in a specific area, exploring the coexistence of multiple recycled labels can introduce an additional competitive dimension—the competition among labels. This might provide valuable insights and a deeper understanding of label dynamics. Moreover, it is also interesting to investigate the impact of different recycled labels on the overall social welfare and explore appropriate mechanisms to align the incentives of various parties.

Appendix A

Proofs of Results

Proof of Lemma 1: Under the continuous label, each firm chooses a price and the percentage of recycled material to maximize her profit. Given the firms' decisions, consumers located at $\tilde{x}(p_1, p_2, \phi_1, \phi_2) = \frac{1}{2} + \frac{\tau[\theta(\phi_1-\phi_2)-p_1+p_2]}{2}$ are indifferent between products 1 and 2. Then, the demand of firm 1 is $\tilde{d}_1(p_1, p_2, \phi_1, \phi_2) = \tilde{x}(p_1, p_2, \phi_1, \phi_2)$ and of firm 2 is $\tilde{d}_2(p_1, p_2, \phi_1, \phi_2) = 1 - \tilde{x}(p_1, p_2, \phi_1, \phi_2)$. Combining the demand functions with Equation (3.1), each firm's profit maximization problem is shown in Equation (4.1). The profit of firm i is concave in her price p_i. Thus, solving the first-order condition $\frac{\partial \tilde{\pi}_i(p_1,p_2,\phi_1,\phi_2)}{\partial p_i} = 0$ yields the maximizer $\tilde{p}'_i(\phi_1, \phi_2) = \frac{1}{\tau} + \frac{\theta(\phi_i-\phi_j)+2c_i+c_j}{3}$, where $c_i = \phi_i \cdot c_r + (1 - \phi_i) \cdot c_v$. □

Proof of Lemma 2: When the prices are as shown in Lemma 1, the corresponding profit for firm i can be reformulated as $\tilde{\pi}_i(\tilde{p}'_1(\phi_1, \phi_2), \tilde{p}'_2(\phi_1, \phi_2), \phi_1, \phi_2) = \frac{[3+\tau(\theta-\Delta c)(\phi_i-\phi_j)]^2}{18\tau} - k_i\phi_i^2$ by substituting $\tilde{p}'_1(\phi_1, \phi_2)$ and $\tilde{p}'_2(\phi_1, \phi_2)$ back into Equation (4.1). Then given ϕ_j, we discuss the best response recycled content of firm i (i.e, $\tilde{\phi}_i(\phi_j)$) by cases.

1. If $0 \leq k_i \leq \frac{\theta-\Delta c}{6}$, for any given $\phi_j \in [0, 1]$, firm i's profits always increases in ϕ_i when $\phi_i \in [0, 1]$. Thus, in this case, we have:

$$\tilde{\phi}_i(\phi_j) = 1.$$

2. If $\frac{\theta-\Delta c}{6} < k_i \leq \frac{3(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{18}$, there are two subcases: 1) Given $\phi_j \in [0, \frac{-18k_i+3(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{\tau(\theta-\Delta c)^2}]$, firm i's profit increases in ϕ_i when $\phi_i \in [0,1]$; 2) Given $\phi_j \in (\frac{-18k_i+3\tau(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{\tau(\theta-\Delta c)^2}, 1]$, firm i's profit is concave in ϕ_i when $\phi_i \in [0,1]$. Specifically, it increases in ϕ_i when $\phi_i \in [0, \frac{3(\theta-\Delta c)-\phi_j\tau(\theta-\Delta c)^2}{18k_i-\tau(\theta-\Delta c)^2}]$ and decreases when $\phi_i \in (\frac{3(\theta-\Delta c)-\phi_j\tau(\theta-\Delta c)^2}{18k_i-\tau(\theta-\Delta c)^2}, 1]$. Thus, in this case, we have:

$$\tilde{\phi}_i(\phi_j) = \begin{cases} 1, & if \phi_j \in [0, \frac{-18k_i+3(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{\tau(\theta-\Delta c)^2}] \\ \frac{3(\theta-\Delta c)-\phi_j\tau(\theta-\Delta c)^2}{18k_i-\tau(\theta-\Delta c)^2}, & if \phi_j \in (\frac{-18k_i+3(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{\tau(\theta-\Delta c)^2}, 1] \end{cases}.$$

3. If $k_i > \frac{3(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{18}$, for any given $\phi_j \in [0,1]$, firm i's profit is concave in ϕ_i when $\phi_i \in [0,1]$, increasing in ϕ_i when $\phi_i \in [0, \frac{3(\theta-\Delta c)-\phi_j\tau(\theta-\Delta c)^2}{18k_i-\tau(\theta-\Delta c)^2}]$ and decreasing when $\phi_i \in (\frac{3(\theta-\Delta c)-\phi_j\tau(\theta-\Delta c)^2}{18k_i-\tau(\theta-\Delta c)^2}, 1]$. Thus, in this case, we have:

$$\tilde{\phi}_i(\phi_j) = \frac{3(\theta-\Delta c)-\phi_j\tau(\theta-\Delta c)^2}{18k_i-\tau(\theta-\Delta c)^2}.$$

Denote $\tilde{k}_L = \frac{\theta-\Delta c}{6}$ and $\tilde{k}_H = \frac{3(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{18}$. Then, there are six cases regarding the relationships between $\tilde{k}_L$, $\tilde{k}_H$ and the two firms' fixed recycled cost k_i. By solving the firms' best response recycled content simultaneously under these cases (i.e., $\tilde{\phi}_i = \tilde{\phi}_i(\tilde{\phi}_j)$), we derive the recycled content equilibria $(\tilde{\phi}_1, \tilde{\phi}_2)$ in each case. The conditions are collected in sets $\tilde{\Omega}_i$, $\forall i \in \{1,2,...,9\}$. Moreover, due to the focus of our study, all sets are described with focus on (τ, κ, γ).

1. If $0 \leq k_1 \leq k_2 \leq \tilde{k}_L$: $(\tilde{\phi}_1, \tilde{\phi}_2) = (1,1)$ in set $\tilde{\Omega}_1$, where $\tilde{\Omega}_1 \doteq \{(\tau,\kappa,\gamma) : 0 \leq \kappa \leq \frac{(\gamma+1)\tilde{k}_L}{2}\}$.

2. If $0 \leq k_1 \leq \tilde{k}_L < k_2 \leq \tilde{k}_H$: $(\tilde{\phi}_1, \tilde{\phi}_2) = (1, \tilde{\phi}_{1\phi})$ in set $\tilde{\Omega}_2$, where $\tilde{\Omega}_2 \doteq \{(\tau,\kappa,\gamma) : (\gamma = 0, \frac{\tilde{k}_L}{2} < \kappa \leq \frac{\tilde{k}_H}{2}) || (0 < \gamma < 1, \frac{(\gamma+1)\tilde{k}_L}{2} < \kappa \leq \min\{\frac{(\gamma+1)\tilde{k}_L}{2\gamma}, \frac{(\gamma+1)\tilde{k}_H}{2}\})\}$ and $\tilde{\phi}_{1\phi} = \frac{(\gamma+1)(\theta-\Delta c)[3-\tau(\theta-\Delta c)]}{36\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}$.

3. If $0 \leq k_1 \leq \tilde{k}_L < \tilde{k}_H < k_2$: $(\tilde{\phi}_1, \tilde{\phi}_2) = (1, \tilde{\phi}_{1\phi})$ in set $\tilde{\Omega}_3$, where $\tilde{\Omega}_3 \doteq \{(\tau,\kappa,\gamma) : (\gamma = 0, \kappa > \frac{\tilde{k}_H}{2}) || (0 < \gamma < 1, \frac{(\gamma+1)\tilde{k}_H}{2} < \kappa \leq \frac{(\gamma+1)\tilde{k}_L}{2\gamma})\}$. Note this case only exists when $0 \leq \gamma \leq \frac{\tilde{k}_L}{\tilde{k}_H} < 1$, that is when $0 \leq \gamma \leq \frac{1}{2}$ or when $\frac{1}{2} < \gamma < 1$ and

$\tau < \frac{3(1-\gamma)}{\gamma(\theta-\Delta c)}$.

4. If $\tilde{k}_L < k_1 \leq k_2 \leq \tilde{k}_H$, there are three subcases:
   - $(\tilde{\phi}_1, \tilde{\phi}_2) = (1, \tilde{\phi}_{1\phi})$ in set $\tilde{\Omega}_4$;
   - $(\tilde{\phi}_1, \tilde{\phi}_2) = (\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$ in set $\tilde{\Omega}_5$;
   - $(\tilde{\phi}_1, \tilde{\phi}_2) = (1, \tilde{\phi}_{1\phi})$, $(\tilde{\phi}_{\phi 1}, 1)$ and $(\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$ in set $\tilde{\Omega}_6$;

   Where $\tilde{\phi}_{\phi\phi 1} = \frac{(\gamma+1)(\theta-\Delta c)[18\kappa-\tau(\gamma+1)(\theta-\Delta c)^2]}{6\kappa[36\gamma\kappa-\tau(\gamma+1)^2(\theta-\Delta c)^2]}$, $\tilde{\phi}_{\phi\phi 2} = \frac{(\gamma+1)(\theta-\Delta c)[18\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2]}{6\kappa[36\gamma\kappa-\tau(\gamma+1)^2(\theta-\Delta c)^2]}$), $\tilde{\phi}_{\phi 1} = \frac{(\gamma+1)(\theta-\Delta c)[3-\tau(\theta-\Delta c)]}{36\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}$, $\tilde{\Omega}_4 \doteq \{(\tau,\kappa,\gamma) : \frac{1}{2} < \gamma \leq 1, (\tilde{\tau}_1 \leq \tau < \frac{3}{\theta-\Delta c}, (\frac{(\gamma+1)\tilde{k}_L}{2\gamma} < \kappa < \tilde{\kappa}_1) || (\tilde{\kappa}_2 < \kappa \leq \min\{\frac{(\gamma+1)\tilde{k}_H}{2}, \tilde{\kappa}_3\})) || (\frac{3(1-\gamma)}{\gamma(\theta-\Delta c)} \leq \tau < \tilde{\tau}_1, (\frac{(\gamma+1)\tilde{k}_L}{2\gamma} < \kappa \leq \min\{\frac{(\gamma+1)\tilde{k}_H}{2}, \tilde{\kappa}_3\}))\}$, $\tilde{\Omega}_5 \doteq \{(\tau,\kappa,\gamma) : \frac{1}{2} < \gamma \leq 1, \frac{3(1-\gamma)}{\gamma(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}, \tilde{\kappa}_3 < \kappa \leq \frac{(\gamma+1)\tilde{k}_H}{2}\}$, $\tilde{\Omega}_6 \doteq \{(\tau,\kappa,\gamma) : \frac{1}{2} < \gamma \leq 1, \tilde{\tau}_1 \leq \tau < \frac{3}{\theta-\Delta c}, \tilde{\kappa}_1 \leq \kappa \leq \tilde{\kappa}_2\}$,
   $\tilde{\tau}_1 = \frac{3\gamma}{[3-\gamma-2\sqrt{2(1-\gamma)}](\theta-\Delta c)}$,
   $\tilde{\kappa}_1 = \frac{(\gamma+1)(\theta-\Delta c)[3\gamma+\tau(\gamma+1)(\theta-\Delta c)+\sqrt{\gamma^2(3+\tau\theta-\tau\Delta c)^2+\tau^2(2\gamma+1)(\theta-\Delta c)^2-18\tau\gamma(\theta-\Delta c)}]}{72\gamma}$,
   $\tilde{\kappa}_2 = \frac{(\gamma+1)(\theta-\Delta c)[3\gamma+\tau(\gamma+1)(\theta-\Delta c)+-\sqrt{\gamma^2(3+\tau\theta-\tau\Delta c)^2+\tau^2(2\gamma+1)(\theta-\Delta c)^2-18\tau\gamma(\theta-\Delta c)}]}{72\gamma}$,
   $\tilde{\kappa}_3 = \frac{(\gamma+1)(\theta-\Delta c)[3+\tau(\gamma+1)(\theta-\Delta c)+\sqrt{\tau^2(\gamma+1)^2(\theta-\Delta c)^2+6\tau(1-3\gamma)(\theta-\Delta c)+9}]}{72\gamma}$. Note this case only exists when $\frac{\tilde{k}_L}{\tilde{k}_H} < \gamma \leq 1$, that is when $\frac{1}{2} < \gamma \leq 1$ and $\frac{3(1-\gamma)}{\gamma(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}$.

5. If $\tilde{k}_L < k_1 \leq \tilde{k}_H < k_2$, there are two subcases:
   - $(\tilde{\phi}_1, \tilde{\phi}_2) = (1, \tilde{\phi}_{1\phi})$ in set $\tilde{\Omega}_7$;
   - $(\tilde{\phi}_1, \tilde{\phi}_2) = (\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$ in set $\tilde{\Omega}_8$;

   Where $\tilde{\Omega}_7 \doteq \{(\tau,\kappa,\gamma) : \max\{\frac{(\gamma+1)\tilde{k}_L}{2\gamma}, \frac{(\gamma+1)\tilde{k}_H}{2}\} < \kappa \leq \tilde{\kappa}_3\}$, $\tilde{\Omega}_8 \doteq \{(\tau,\kappa,\gamma) : \max\{\frac{(\gamma+1)\tilde{k}_L}{2\gamma}, \frac{(\gamma+1)\tilde{k}_H}{2}, \tilde{\kappa}_3\} < \kappa \leq \frac{(\gamma+1)\tilde{k}_H}{2\gamma}\}$. Note $\frac{(\gamma+1)\tilde{k}_L}{2\gamma} < \tilde{\kappa}_3$ always holds.

6. If $\tilde{k}_H < k_1 \leq k_2$, $(\tilde{\phi}_1, \tilde{\phi}_2) = (\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$ in $\tilde{\Omega}_9$, where $\tilde{\Omega}_9 \doteq \{(\tau,\kappa,\gamma) : \kappa > \frac{(\gamma+1)\tilde{k}_H}{2\gamma}\}$.

The results show that there exist multiple equilibria in set $\tilde{\Omega}_6$. We implement a strategic policy assigning a higher percentage of recycled content to the more efficient firm in the equilibrium, refining these equilibria. This is based on the rationale that greater efficiency provides stronger incentives for increased utilization of recycled materials. Note in set $\tilde{\Omega}_6$, we have, $\tilde{\phi}_{1\phi} < 1$, $\tilde{\phi}_{\phi 1} < 1$ and $\tilde{\phi}_{\phi\phi 1} \leq \tilde{\phi}_{\phi\phi 2}$. Thus after the refinement, there remains a unique equilibrium $(\tilde{\phi}_1, \tilde{\phi}_2) = (1, \tilde{\phi}_{1\phi})$ in set $\tilde{\Omega}_6$. Then, there are three possible equilibrium candidates under the continuous label, namely

$(1,1)$, $(1,\tilde{\phi}_{1\phi})$, $(\tilde{\phi}_{\phi\phi1},\tilde{\phi}_{\phi\phi2})$, and we combine the condition sets for each equilibrium in $\tilde{\Omega}_{11}$, $\tilde{\Omega}_{1\phi}$ and $\tilde{\Omega}_{\phi\phi}$, respectively, where $\tilde{\Omega}_{11} \doteq \tilde{\Omega}_1 \doteq \{(\tau,\kappa,\gamma) : 0 \leq \kappa \leq \frac{(\gamma+1)\tilde{k}_L}{2}\}$, $\tilde{\Omega}_{1\phi} \doteq \tilde{\Omega}_2 \cup \tilde{\Omega}_3 \cup \tilde{\Omega}_4 \cup \tilde{\Omega}_6 \cup \tilde{\Omega}_7 \doteq \{(\tau,\kappa,\gamma) : (\gamma = 0, \kappa > \frac{(\gamma+1)\tilde{k}_L}{2})||(0 < \gamma \leq 1, \frac{(\gamma+1)\tilde{k}_L}{2} < \kappa \leq \tilde{\kappa}_3)\}$, and $\tilde{\Omega}_{\phi\phi} \doteq \tilde{\Omega}_5 \cup \tilde{\Omega}_8 \cup \tilde{\Omega}_9 \doteq \{(\tau,\kappa,\gamma) : 0 < \gamma \leq 1, \kappa > \tilde{\kappa}_3\}$.

Given the recycled content equilibrium, the corresponding pricing equilibrium $(\tilde{p}_1,\tilde{p}_2)$ is given by:

$$(\tilde{p}_1,\tilde{p}_2) = \Big(\frac{1}{\tau} + \frac{\theta(\tilde{\phi}_1 - \tilde{\phi}_2) + 2\tilde{c}_1 + \tilde{c}_2}{3}, \frac{1}{\tau} + \frac{\theta(\tilde{\phi}_2 - \tilde{\phi}_1) + 2\tilde{c}_2 + \tilde{c}_1}{3}\Big),$$

where $\tilde{c}_i = \tilde{\phi}_i \cdot c_r + (1-\tilde{\phi}_i)\cdot c_v$.

Then, given the recycled content and pricing equilibrium, the corresponding firms' equilibrium demand $(\tilde{d}_1,\tilde{d}_2)$, profits $(\tilde{\pi}_1,\tilde{\pi}_2)$ and the certifier's equilibrium payoff $\tilde{\pi}_0$ are respectively given by:

$$(\tilde{d}_1,\tilde{d}_2) = \Big(\frac{1}{2} + \frac{\tau[\theta(\tilde{\phi}_1 - \tilde{\phi}_2) - \tilde{c}_1 + \tilde{c}_2]}{6}, \frac{1}{2} + \frac{\tau[\theta(\tilde{\phi}_2 - \tilde{\phi}_1) - \tilde{c}_2 + \tilde{c}_1]}{6}\Big),$$

$$(\tilde{\pi}_1,\tilde{\pi}_2) = \Big(\frac{[3+\tau(\theta-\Delta c)(\tilde{\phi}_1 - \tilde{\phi}_2)]^2}{18\tau} - \frac{2\gamma\kappa\tilde{\phi}_1^2}{1+\gamma}, \frac{[3+\tau(\theta-\Delta c)(\tilde{\phi}_2 - \tilde{\phi}_1)]^2}{18\tau} - \frac{2\kappa\tilde{\phi}_2^2}{1+\gamma}\Big),$$

$$\tilde{\pi}_0 = \beta_r + (\beta_r - \beta_v)\frac{3(\tilde{\phi}_1 + \tilde{\phi}_2) + \tau(\theta-\Delta c)(\tilde{\phi}_1 - \tilde{\phi}_2)^2}{6}. \tag{A.1}$$

□

Proof of Proposition 1: Given Equation (A.1), the monotonicities of $\tilde{\pi}_0$ with respect to τ, κ, γ within region $\tilde{\Omega}_{ij}$, for $i,j \in \{1,\phi\}$, can be easily proved via checking the first-order conditions. Thus, to prove this proposition, we first analyze the monotonicities within each region, and then combine the results in these regions.

1. With respect to τ, we can easily check: (1) Within $\tilde{\Omega}_{11}$, $\frac{\partial\tilde{\pi}_0}{\partial\tau} = 0$; (2) Within $\tilde{\Omega}_{1\phi}$, $\frac{\partial\tilde{\pi}_0}{\partial\tau} < 0$, if $\kappa \leq \min\{\frac{(\gamma+1)(\theta-\Delta c)}{6}, \frac{(\gamma+1)(\theta-\Delta c)}{3(2\gamma+1)}\}$ and $\tau < \frac{6[(\gamma+1)(\theta-\Delta c)-6\kappa]}{(\gamma+1)(\theta-\Delta c)^2}$;

Otherwise $\frac{\partial \tilde{\pi}_0}{\partial \tau} \geq 0$; (3) Within $\tilde{\Omega}_{\phi\phi}$, $\frac{\partial \tilde{\pi}_0}{\partial \tau} > 0$.

Then we combine the results in these regions, which are shown in Table A.1. Note $\frac{(\gamma+1)(\theta-\Delta c)}{12}$ is the upper bound κ of region $\tilde{\Omega}_{11}$ and the lower bound κ of region $\tilde{\Omega}_{1\phi}$. $\frac{(\gamma+1)(\theta-\Delta c)}{12\gamma}$ is the lower bound κ of region $\tilde{\Omega}_{\phi\phi}$. $\frac{(\gamma+1)(\theta-\Delta c)}{6\gamma}$ is the upper bound κ of region $\tilde{\Omega}_{1\phi}$.

Table A.1: The Monotonicities of $\tilde{\pi}_0$ with Respect to τ by Cases

Cases	$\tilde{\Omega}_{\phi\phi}$	$\tilde{\Omega}_{1\phi}$	$\tilde{\Omega}_{11}$
$0 \leq \kappa \leq \frac{(\gamma+1)(\theta-\Delta c)}{12}$	$\diagup$	$\diagup$	–
$\frac{(\gamma+1)(\theta-\Delta c)}{12} < \kappa \leq \min\{\frac{(\gamma+1)(\theta-\Delta c)}{6}, \frac{(\gamma+1)(\theta-\Delta c)}{12\gamma}\}$	$\diagup$	$\downarrow\uparrow$	$\diagup$
$\frac{(\gamma+1)(\theta-\Delta c)}{6} < \kappa \leq \frac{(\gamma+1)(\theta-\Delta c)}{12\gamma}$	$\diagup$	$\uparrow$	$\diagup$
$\frac{(\gamma+1)(\theta-\Delta c)}{12\gamma} < \kappa \leq \frac{2(\gamma+1)(\theta-\Delta c)}{6(2\gamma+1)}$	$\uparrow$	$\downarrow\uparrow$	$\diagup$
$\max\{\frac{(\gamma+1)(\theta-\Delta c)}{12\gamma}, \frac{2(\gamma+1)(\theta-\Delta c)}{6(2\gamma+1)}\} < \kappa \leq \frac{(\gamma+1)(\theta-\Delta c)}{6\gamma}$	$\uparrow$	$\uparrow$	$\diagup$
$\kappa > \frac{(\gamma+1)(\theta-\Delta c)}{6\gamma}$	$\uparrow$	$\diagup$	$\diagup$

2. With respect to κ, we can easily check: (1) Within $\tilde{\Omega}_{11}$, $\frac{\partial \tilde{\pi}_0}{\partial \kappa} = 0$; (2) Within $\tilde{\Omega}_{1\phi}$, $\frac{\partial \tilde{\pi}_0}{\partial \kappa} > 0$ if $\frac{9}{(-2\gamma+6)(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}$ and $\kappa > \frac{\tau(\gamma+1)(\theta-\Delta c)^2}{12[2\tau(\theta-\Delta c)-3]}$; Otherwise, $\tilde{\Omega}_{1\phi}$, $\frac{\partial \tilde{\pi}_0}{\partial \kappa} \leq 0$. (3) Within $\tilde{\Omega}_{\phi\phi}$, $\frac{\partial \tilde{\pi}_0}{\partial \kappa} < 0$.

   Thus, there are two cases, i.e, $\tau < \frac{9}{(-2\gamma+6)(\theta-\Delta c)}$ and $\frac{9}{(-2\gamma+6)(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}$. When $\tau < \frac{9}{(-2\gamma+6)(\theta-\Delta c)}$, $\tilde{\pi}_0$ always decreases in κ. When $\frac{9}{(-2\gamma+6)(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}$, $\tilde{\pi}_0$ firstly remains constant in κ (i.e, in region $\tilde{\Omega}_{11}$), then experiences a decreasing-then-increasing trend (i.e, in region $\tilde{\Omega}_{1\phi}$), and may finally consistently decreases when $\tilde{\Omega}_{\phi\phi}$ exists.

3. With respect to γ, we can easily check: (1) Within $\tilde{\Omega}_{11}$, $\frac{\partial \tilde{\pi}_0}{\partial \gamma} = 0$; (2) Within $\tilde{\Omega}_{1\phi}$, $\frac{\partial \tilde{\pi}_0}{\partial \gamma} \leq 0$, if $0 \leq \gamma \leq \tilde{\gamma}_{\gamma 1}$, where $\tilde{\gamma}_{\gamma 1} = \frac{12\kappa[-3+2\tau(\theta-\Delta c)]}{\tau(\theta-\Delta c)^2} - 1$; Otherwise, $\frac{\partial \tilde{\pi}_0}{\partial \gamma} > 0$. (3) Within $\tilde{\Omega}_{\phi\phi}$, $\frac{\partial \tilde{\pi}_0}{\partial \gamma} < 0$.

   Then we combine the results in these regions, which are shown in Table A.2 by cases. Note $\frac{\theta-\Delta c}{12}$ is the lower bound κ of region $\tilde{\Omega}_{1\phi}$, and $\frac{\theta-\Delta c}{6}$ is the upper bound κ of region $\tilde{\Omega}_{11}$. In particular, $\frac{\theta-\Delta c}{6}$ is also the lower bound κ of region $\tilde{\Omega}_{\phi\phi}$ when $\tau < \frac{3}{2(\theta-\Delta c)}$, while when $\frac{3}{2(\theta-\Delta c)} \leq \tau < \frac{3}{(\theta-\Delta c)}$, the lower bound κ of region $\tilde{\Omega}_{\phi\phi}$ is $\frac{\tau(\theta-\Delta c)^2}{9}$. Thus, we discuss the results in two subcases, i.e., $\frac{3}{2(\theta-\Delta c)} \leq \tau < \frac{3}{(\theta-\Delta c)}$, and $\tau < \frac{3}{2(\theta-\Delta c)}$. When $\frac{3}{2(\theta-\Delta c)} \leq \tau < \frac{3}{(\theta-\Delta c)}$, $\tilde{\gamma}_{\gamma 1}$ is smaller than the lower bound γ of region $\tilde{\Omega}_{1\phi}$ if $\kappa < \tilde{\kappa}_{\gamma 1}$. This implies when

$\kappa < \tilde{\kappa}_{\gamma 1}$, $\gamma > \tilde{\gamma}_{\gamma 1}$ always satisfies in $\tilde{\Omega}_{1\phi}$, and thereby $\frac{\partial \tilde{\pi}_0}{\partial \gamma} > 0$ holds. Moreover, $\tilde{\gamma}_{\gamma 1}$ is larger than the upper bound γ of region $\tilde{\Omega}_{1\phi}$ if $\kappa > \tilde{\kappa}_{\gamma 2}$ or $\kappa > \tilde{\kappa}_{\gamma 3}$. This means when $\kappa > \tilde{\kappa}_{\gamma 2}$ or $\kappa > \tilde{\kappa}_{\gamma 3}$, $\gamma < \tilde{\gamma}_{\gamma 1}$ always satisfies in $\tilde{\Omega}_{1\phi}$, and thereby $\frac{\partial \tilde{\pi}_0}{\partial \gamma} < 0$. When $\tau < \frac{3}{2(\theta - \Delta c)}$, we find $\tilde{\gamma}_{\gamma 1} < 0$, which means $\gamma > \tilde{\gamma}_{\gamma 1}$ always satisfies in $\tilde{\Omega}_{1\phi}$. Hence, $\frac{\partial \tilde{\pi}_0}{\partial \gamma} > 0$ holds.

Table A.2: The Monotonicities of $\tilde{\pi}_0$ with Respect to γ by Cases

Cases		$\tilde{\Omega}_{1\phi}$	$\tilde{\Omega}_{11}$	$\tilde{\Omega}_{\phi\phi}$
$0 \le \kappa \le \frac{\theta - \Delta c}{12}$		$\nearrow$	$-$	$\nearrow$
$\frac{3}{2(\theta - \Delta c)} \le \tau < \frac{3}{(\theta - \Delta c)}$	$\frac{\theta - \Delta c}{12} < \kappa \le \min\{\frac{\theta - \Delta c}{6}, \tilde{\kappa}_{\gamma 1}\}$	$\uparrow$	$-$	$\nearrow$
	$\tilde{\kappa}_{\gamma 1} < \kappa \le \frac{\theta - \Delta c}{6}$	$\downarrow\uparrow$	$-$	$\nearrow$
	$\frac{\theta - \Delta c}{6} < \kappa \le \min\{\tilde{\kappa}_{\gamma 1}, \frac{\tau(\theta - \Delta c)^2}{9}\}$	$\uparrow$	$\nearrow$	$\nearrow$
	$\max\{\frac{\theta - \Delta c}{6}, \tilde{\kappa}_{\gamma 1}\} < \kappa \le \min\{\tilde{\kappa}_{\gamma 2}, \frac{\tau(\theta - \Delta c)^2}{9}\}$	$\downarrow\uparrow$	$\nearrow$	$\nearrow$
	$\tilde{\kappa}_{\gamma 2} < \kappa \le \frac{\tau(\theta - \Delta c)^2}{9}$	$\downarrow$	$\nearrow$	$\nearrow$
	$\frac{\tau(\theta - \Delta c)^2}{9} < \kappa \le \tilde{\kappa}_{\gamma 1}$	$\uparrow$	$\nearrow$	$\downarrow$
	$\max\{\tilde{\kappa}_{\gamma 1}, \frac{\tau(\theta - \Delta c)^2}{9}\} < \kappa \le \tilde{\kappa}_{\gamma 3}$	$\downarrow\uparrow$	$\nearrow$	$\downarrow$
	$\kappa > \max\{\frac{\tau(\theta - \Delta c)^2}{9}, \tilde{\kappa}_{\gamma 3}\}$	$\downarrow$	$\nearrow$	$\downarrow$
$\tau < \frac{3}{2(\theta - \Delta c)}$	$\frac{\theta - \Delta c}{12} < \kappa \le \frac{\theta - \Delta c}{6}$	$\uparrow$	$-$	$\nearrow$
	$\kappa > \frac{\theta - \Delta c}{6}$	$\uparrow$	$\nearrow$	$\downarrow$

Note: $\tilde{\kappa}_{\gamma 1} = \frac{\tau(\theta - \Delta c)^2}{12[-3 + 2\tau(\theta - \Delta c)]}$, $\tilde{\kappa}_{\gamma 2} = \frac{\tau(\theta - \Delta c)^2}{6[-3 + 2\tau(\theta - \Delta c)]}$, $\tilde{\kappa}_{\gamma 3} = \frac{(\theta - \Delta c)[9 - 8\tau(\theta - \Delta c)]}{24[3 - 2\tau(\theta - \Delta c)]}$.

□

Proof of Lemma 3: Under the binary label, given the label standard $\underline{\phi}$ and the firms' decisions (p_1, p_2) and (ϕ_1, ϕ_2), marginal consumers who are indifferent between products 1 and 2 are located at $\ddot{x}(p_1, p_2, \phi_1, \phi_2, \underline{\phi}) = \frac{1}{2} + \frac{\tau[\theta \underline{\phi}(\mathbb{1}_{\{\phi_1 \ge \underline{\phi}\}} - \mathbb{1}_{\{\phi_2 \ge \underline{\phi}\}}) - p_1 + p_2]}{2}$. Thus, the demand of firm 1 is $\ddot{d}_1(p_1, p_2, \phi_1, \phi_2, \underline{\phi}) = \ddot{x}(p_1, p_2, \phi_1, \phi_2, \underline{\phi})$ and of firm 2 is $\ddot{d}_2(p_1, p_2, \phi_1, \phi_2, \underline{\phi}) = 1 - \ddot{x}(p_1, p_2, \phi_1, \phi_2, \underline{\phi})$. Then, given the label standard $\underline{\phi}$, each firm's profit maximization profit is as shown in Equation (4.4). The profit of firm i is concave in her price p_i. Solving the first-order condition $\frac{\partial \ddot{\pi}_i(p_1, p_2, \phi_1, \phi_2, \underline{\phi})}{\partial p_i} = 0$ yields the maximizer $\ddot{p}_i(\phi_1, \phi_2, \underline{\phi}) = \frac{1}{\tau} + \frac{\theta \underline{\phi}(\mathbb{1}_{\{\phi_i \ge \underline{\phi}\}} - \mathbb{1}_{\{\phi_j \ge \underline{\phi}\}}) + 2c_i + c_j}{3}$, where $c_i = \phi_i \cdot c_r + (1 - \phi_i) \cdot c_v$. Substituting the prices $\ddot{p}_i(\phi_1, \phi_2, \underline{\phi})$ back into Equation (4.4), the profit for firm i can be reformulated as $\ddot{\pi}_i(\ddot{p}_1(\phi_1, \phi_2, \underline{\phi}), \ddot{p}_2(\phi_1, \phi_2, \underline{\phi}), \phi_1, \phi_2, \underline{\phi}) = \frac{[3 + \tau(\theta - \Delta c)(\mathbb{1}_{\{\phi_i \ge \underline{\phi}\}} - \mathbb{1}_{\{\phi_j \ge \underline{\phi}\}})]^2}{18\tau} - k_i \phi_i^2$. □

Proof of Lemma 4: We prove Lemma 4 by firstly showing given binary label standard $\underline{\phi}$ and $\phi_i \in [0, 1]$, any $\phi_j \in (0, \underline{\phi})$ and $\phi_j \in (\underline{\phi}, 1]$ cannot be an equilibrium.

We firstly show given $\underline{\phi}$ and $\phi_i \in [0,1]$, any $\phi_j \in (0,\underline{\phi})$ cannot be an equilibrium. For $\phi_i \in [0,1]$, we have $\mathbb{1}_{\{\phi_i \geq \underline{\phi}\}} = 0$ if $\phi_i \in [0,\underline{\phi})$, and $\mathbb{1}_{\{\phi_i \geq \underline{\phi}\}} = 1$ if $\phi_i \in [\underline{\phi},1]$. Then we proceed to analyze two cases, i.e., $\phi_i \in [0,\underline{\phi})$ and $\phi_i \in [\underline{\phi},1]$. Firstly, given $\underline{\phi}$ and $\phi_i \in [0,\underline{\phi})$, $\ddot{\pi}_j(\phi_j|\phi_i,\underline{\phi}) = \frac{1}{2\tau} - k_j\phi_j^2$ when $\phi_j \in [0,\underline{\phi})$. Secondly, given $\underline{\phi}$ and $\phi_i \in [\underline{\phi},1]$, $\ddot{\pi}_j(\phi_j|\phi_i,\underline{\phi}) = \frac{[3-\tau(\theta-\Delta c)\underline{\phi}]^2}{18\tau} - k_j\phi_j^2$ when $\phi_j \in [0,\underline{\phi})$. Thus, we can verify $\ddot{\pi}_j(0|\phi_i,\underline{\phi}) > \max\limits_{\phi_j \in (0,\underline{\phi})} \ddot{\pi}_j(\phi_j|\phi_i,\underline{\phi})$, since $\ddot{\pi}_j(\phi_j|\phi_i,\underline{\phi})$ decreases in ϕ_j when $\phi_j \in [0,\underline{\phi})$. This implies given $\underline{\phi}$ and $\phi_i \in [0,1]$, for any $\phi_j = (0,\underline{\phi})$, firm j would at least deviate to $\phi_j = 0$. Therefore, we can conclude given $\underline{\phi}$ and $\phi_i \in [0,1]$, any $\phi_j \in (0,\underline{\phi})$ cannot be an equilibrium.

Next, we show given $\underline{\phi}$ and $\phi_i \in [0,1]$, $\phi_j \in (\underline{\phi},1]$ cannot be an equilibrium. For $\phi_i \in [0,1]$, as earlier mentioned, there are two cases and we proceed to analyze these two cases, respectively. Given $\underline{\phi}$ and $\phi_i \in [0,\underline{\phi})$, we have $\ddot{\pi}_j(\phi_j|\phi_i,\underline{\phi}) = \frac{[3+\tau(\theta-\Delta c)\underline{\phi}]^2}{18\tau} - k_j\phi_j^2$ when $\phi_j \in [\underline{\phi},1]$. Given $\underline{\phi}$ and $\phi_i \in [\underline{\phi},1]$, we have $\ddot{\pi}_j(\phi_j|\phi_i,\underline{\phi}) = \frac{1}{2\tau} - k_j\phi_j^2$ when $\phi_j \in [\underline{\phi},1]$. Thus, we can verify $\ddot{\pi}_j(\underline{\phi}|\phi_i,\underline{\phi}) > \max\limits_{\phi_j \in (\underline{\phi},1]} \ddot{\pi}_j(\phi_j|\phi_i,\underline{\phi})$, since $\ddot{\pi}_j(\phi_j|\phi_i,\underline{\phi})$ decreases in ϕ_j when $\phi_j \in [\underline{\phi},1]$. This implies given $\underline{\phi}$ and $\phi_i \in [0,1]$, for any $\phi_j = (\underline{\phi},1)$, firm j would at least deviate to $\phi_j = \underline{\phi}$. Therefore, we can conclude given $\underline{\phi}$ and $\phi_i \in [0,1]$, any $\phi_j \in (\underline{\phi},1]$ cannot be an equilibrium.

Then, given $\underline{\phi}$ and $\phi_i \in [0,1]$, by ruling out the equilibrium of any $\phi_j \in (0,\underline{\phi})$ and $\phi_j \in (\underline{\phi},1]$, we can show there remain four possible equilibrium candidates, in which $(\ddot{\phi}_1,\ddot{\phi}_2|\underline{\phi}) = (0,0)$, $(\underline{\phi},0)$, $(0,\underline{\phi})$ and $(\underline{\phi},\underline{\phi})$.

Next, to verify when each candidate is an equilibrium, we need to solve the corresponding no-deviation requirements. Note when firms are indifferent from obtaining or not obtaining the label, we apply the tie-breaking rules that the firms will obtain the label so as to benefit the certifier.

1. $(\ddot{\phi}_1,\ddot{\phi}_2|\underline{\phi}) = (\underline{\phi},\underline{\phi})$ is an equilibrium when both firms cannot be better off by deviating from $\ddot{\phi}_i = \underline{\phi}$ to any other recycled content ϕ_i, given $\phi_j = \underline{\phi}$ and the binary label with standard $\underline{\phi}$, i.e., $\ddot{\pi}_i(\underline{\phi}|\underline{\phi},\underline{\phi}) \geq \max\limits_{\phi_i \in [0,1]} \ddot{\pi}_i(\phi_i|\underline{\phi},\underline{\phi})$, for $i \in \{1,2\}$. Firstly, we observe that $\ddot{\pi}_i(\underline{\phi}|\underline{\phi},\underline{\phi}) \geq \max\limits_{\phi_i \in [\underline{\phi},1]} \ddot{\pi}_i(\phi_i|\underline{\phi},\underline{\phi})$ holds trivially. This means to verify the no-deviation requirements, we only need to check

$\ddot{\pi}_i(\underline{\phi}|\underline{\phi},\underline{\phi}) \geq \max\limits_{\phi_i\in[0,\underline{\phi})} \ddot{\pi}_i(\phi_i|\underline{\phi},\underline{\phi})$. Then due to the fact that $\max\limits_{\phi_i\in[0,\underline{\phi})} \ddot{\pi}_i(\phi_i|\underline{\phi},\underline{\phi}) = \ddot{\pi}_i(0|\underline{\phi},\underline{\phi})$, we can rewrite the above requirements as $\ddot{\pi}_i(\underline{\phi}|\underline{\phi},\underline{\phi}) \geq \ddot{\pi}_i(0|\underline{\phi},\underline{\phi})$, for both $i \in \{1,2\}$. By comparing the profits, we find these inequalities holds when in set $\ddot{Q}_{\underline{\phi}\underline{\phi}}$, where $\ddot{Q}_{\underline{\phi}\underline{\phi}} \doteq \{(\tau,\kappa,\gamma,\underline{\phi}) : (0 \leq \kappa \leq \ddot{\kappa}_{L1})||(\kappa > \ddot{\kappa}_{L1}, 0 \leq \underline{\phi} \leq \underline{\phi}_{L1})\}$, $\ddot{\kappa}_{L1} = \frac{(\gamma+1)[6(\theta-\Delta c)-\tau(\theta-\Delta c)^2]}{36}$ and $\underline{\phi}_{L1} = \frac{6(\gamma+1)(\theta-\Delta c)}{36\kappa+\tau(\gamma+1)(\theta-\Delta c)^2}$.

2. $(\ddot{\phi}_1, \ddot{\phi}_2|\underline{\phi}) = (\underline{\phi}, 0)$ is an equilibrium when firm 1 cannot be better off by deviating from $\ddot{\phi}_1 = \underline{\phi}$ to any other recycled content ϕ_1, given $\phi_2 = 0$ and the label standard $\underline{\phi}$, i.e., $\ddot{\pi}_1(\underline{\phi}|0,\underline{\phi}) \geq \max\limits_{\phi_1\in[0,1]} \ddot{\pi}_1(\phi_1|0,\underline{\phi})$, and firm 2 cannot be better off by deviating from $\ddot{\phi}_2 = 0$ to any other recycled content ϕ_2, given $\phi_1 = \underline{\phi}$ and the label standard $\underline{\phi}$, i.e., $\ddot{\pi}_2(0|\underline{\phi},\underline{\phi}) > \max\limits_{\phi_2\in(0,1]} \ddot{\pi}_2(\phi_2|\underline{\phi},\underline{\phi})$. For the first no-deviation requirement, we find $\ddot{\pi}_1(\underline{\phi}|0,\underline{\phi}) \geq \max\limits_{\phi_1\in[\underline{\phi},1]} \ddot{\pi}_1(\phi_1|0,\underline{\phi})$ holds trivially. This means to verify the first no-deviation requirement, we only need to check $\ddot{\pi}_1(\underline{\phi}|0,\underline{\phi}) \geq \max\limits_{\phi_1\in[0,\underline{\phi})} \ddot{\pi}_1(\phi_1|0,\underline{\phi})$. Due to the fact that $\max\limits_{\phi_1\in[0,\underline{\phi})} \ddot{\pi}_1(\phi_1|0,\underline{\phi}) = \ddot{\pi}_1(0|0,\underline{\phi})$, the first requirement can be rewritten as $\ddot{\pi}_1(\underline{\phi}|0,\underline{\phi}) \geq \ddot{\pi}_1(0|0,\underline{\phi})$. For the second no-deviation requirement, it is easy to verify $\ddot{\pi}_2(0|\underline{\phi},\underline{\phi}) > \max\limits_{\phi_2\in(0,\underline{\phi})} \ddot{\pi}_2(\phi_2|\underline{\phi},\underline{\phi})$. This means for the second no-deviation requirement, we only need to check $\ddot{\pi}_2(0|\underline{\phi},\underline{\phi}) > \max\limits_{\phi_2\in[\underline{\phi},1]} \ddot{\pi}_2(\phi_2|\underline{\phi},\underline{\phi})$. Then by using the fact that $\max\limits_{\phi_2\in[\underline{\phi},1]} \ddot{\pi}_2(\phi_2|\underline{\phi},\underline{\phi}) = \ddot{\pi}_2(\underline{\phi}|\underline{\phi},\underline{\phi})$, we can rewrite the second requirement as $\ddot{\pi}_2(0|\underline{\phi},\underline{\phi}) > \ddot{\pi}_2(\underline{\phi}|\underline{\phi},\underline{\phi})$. By comparing the profits, we find these two inequalities hold in $\ddot{Q}_{\underline{\phi}0}$, where $\ddot{Q}_{\underline{\phi}0} \doteq \{(\tau,\kappa,\gamma,\underline{\phi}) : \kappa > \ddot{\kappa}_{L1}, \underline{\phi}_{L1} < \underline{\phi} \leq \min\{\underline{\phi}_{H1}, 1\}\}$, and $\underline{\phi}_{H1} = \frac{6(\gamma+1)(\theta-\Delta c)}{36\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}$.

3. $(\ddot{\phi}_1, \ddot{\phi}_2|\underline{\phi}) = (0, \underline{\phi})$ is an equilibrium when firm 1 cannot be better off by deviating from $\ddot{\phi}_1 = 0$ to any other recycled content ϕ_1, given $\phi_2 = \underline{\phi}$ and the label standard $\underline{\phi}$, i.e., $\ddot{\pi}_1(0|\underline{\phi},\underline{\phi}) > \max\limits_{\phi_1\in(0,1]} \ddot{\pi}_1(\phi_1|\underline{\phi},\underline{\phi})$, and firm 2 cannot be better off by deviating from $\ddot{\phi}_2 = \underline{\phi}$ to any other recycled content ϕ_2, given $\phi_1 = 0$ and the label standard $\underline{\phi}$, i.e., $\ddot{\pi}_2(\underline{\phi}|0,\underline{\phi}) \geq \max\limits_{\phi_2\in[0,1]} \ddot{\pi}_2(\phi_2|\underline{\phi},\underline{\phi})$. For the first no-deviation requirement, we find $\ddot{\pi}_1(0|\underline{\phi},\underline{\phi}) > \max\limits_{\phi_1\in(0,\underline{\phi})} \ddot{\pi}_1(\phi_1|\underline{\phi},\underline{\phi})$ holds trivially. This means to verify the first no-deviation requirement, we only need to check $\ddot{\pi}_1(0|\underline{\phi},\underline{\phi}) > \max\limits_{\phi_1\in[\underline{\phi},1]} \ddot{\pi}_1(\phi_1|\underline{\phi},\underline{\phi})$. Due to the fact that $\max\limits_{\phi_1\in[\underline{\phi},1]} \ddot{\pi}_1(\phi_1|\underline{\phi},\underline{\phi}) = \ddot{\pi}_1(\underline{\phi}|\underline{\phi},\underline{\phi})$, the first requirement can be rewritten as $\ddot{\pi}_1(0|\underline{\phi},\underline{\phi}) > \ddot{\pi}_1(\underline{\phi}|\underline{\phi},\underline{\phi})$.

For the second no-deviation requirement, it is easy to verify $\ddot{\pi}_2(\underline{\phi}|0,\underline{\phi}) \geq \max_{\phi_2\in[\underline{\phi},1]} \ddot{\pi}_2(\phi_2|0,\underline{\phi})$. Then we only need to check $\ddot{\pi}_2(\underline{\phi}|0,\underline{\phi}) \geq \max_{\phi_2\in[0,\underline{\phi})} \ddot{\pi}_2(\phi_2|0,\underline{\phi})$. By using the fact that $\max_{\phi_2\in[0,\underline{\phi})} \ddot{\pi}_2(\phi_2|0,\underline{\phi}) = \ddot{\pi}_2(0|0,\underline{\phi})$, we can rewrite the second requirement as $\ddot{\pi}_2(\underline{\phi}|0,\underline{\phi}) \geq \ddot{\pi}_2(0|0,\underline{\phi})$. By comparing the profits, we find these two requirements hold when $(\tau,\kappa,\gamma,\underline{\phi}) \in \ddot{Q}_{0\underline{\phi}}$, where $\ddot{Q}_{0\underline{\phi}} \doteq \{(\tau,\kappa,\gamma,\underline{\phi}) : 0 < \gamma \leq 1, \kappa > \ddot{\kappa}_{L2}, \underline{\phi}_{L2} < \underline{\phi} \leq \min\{\underline{\phi}_{H2}, 1\}\}$, $\ddot{\kappa}_{L2} = \frac{(\gamma+1)[6(\theta-\Delta c)-\tau(\theta-\Delta c)^2]}{36\gamma}$, $\underline{\phi}_{L2} = \frac{6(\gamma+1)(\theta-\Delta c)}{36\gamma\kappa+\tau(\gamma+1)(\theta-\Delta c)^2}$, and $\underline{\phi}_{H2} = \frac{6(\gamma+1)(\theta-\Delta c)}{36\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}$.

4. $(\ddot{\phi}_1, \ddot{\phi}_2|\underline{\phi}) = (0,0)$ is an equilibrium when both firms cannot be better off by deviating from $\ddot{\phi}_i = 0$ to any other recycled content ϕ_i, given $\phi_j = 0$ and the binary label with standard $\underline{\phi}$, i.e., $\ddot{\pi}_i(0|0,\underline{\phi}) > \max_{\phi_i\in(0,1]} \ddot{\pi}_i(\phi_i|0,\underline{\phi})$, for $i \in \{1,2\}$. Firstly, we observe that $\ddot{\pi}_i(0|0,\underline{\phi}) > \max_{\phi_i\in(0,\underline{\phi})} \ddot{\pi}_i(\phi_i|0,\underline{\phi})$ holds trivially. This means to verify the no-deviation requirements, we only need to check $\ddot{\pi}_i(0|0,\underline{\phi}) > \max_{\phi_i\in[\underline{\phi},1]} \ddot{\pi}_i(\phi_i|0,\underline{\phi})$. Then due to the fact that $\max_{\phi_i\in[\underline{\phi},1]} \ddot{\pi}_i(\phi_i|0,\underline{\phi}) = \ddot{\pi}_i(\underline{\phi}|0,\underline{\phi})$, we can rewrite the above requirements as $\ddot{\pi}_i(0|0,\underline{\phi}) > \ddot{\pi}_i(\underline{\phi}|0,\underline{\phi})$, for both $i \in \{1,2\}$. By comparing the profits, we find these inequalities hold when in $\ddot{Q}_{00}$, where $\ddot{Q}_{00} \doteq \{(\tau,\kappa,\gamma,\underline{\phi}) : \kappa > \ddot{\kappa}_{H1}, \underline{\phi}_{H1} < \underline{\phi} \leq 1\}$, and $\ddot{\kappa}_{H1} = \frac{(\gamma+1)[6(\theta-\Delta c)+\tau(\theta-\Delta c)^2]}{36\gamma}$.

From the above discussions, we find $\ddot{\kappa}_{L1} \leq \ddot{\kappa}_{L2}$, $\underline{\phi}_{L1} \leq \underline{\phi}_{L2}$, and $\underline{\phi}_{H1} \geq \underline{\phi}_{H2}$, which means $\ddot{Q}_{0\underline{\phi}} \in \ddot{Q}_{\underline{\phi}0}$. Thus, there are multiple equilibria (i.e., $(\underline{\phi},0)$ and $(0,\underline{\phi})$) in $\ddot{Q}_{0\underline{\phi}}$. Nevertheless, after implementing the refinement policy as mentioned earlier, the only left equilibrium in $\ddot{Q}_{0\underline{\phi}}$ is $(\underline{\phi},0)$. Moreover, sets $\ddot{Q}_{00}$, $\ddot{Q}_{\underline{\phi}0}$ and $\ddot{Q}_{\underline{\phi}\underline{\phi}}$ are exclusive and complementary. Therefore, given the binary label standard $\underline{\phi}$, we conclude the recycled content equilibrium under the binary label is $(\ddot{\phi}_1, \ddot{\phi}_2|\underline{\phi}) = (0,0)$ in $\ddot{Q}_{00}$, $(\ddot{\phi}_1, \ddot{\phi}_2|\underline{\phi}) = (\underline{\phi},0)$ in $\ddot{Q}_{\underline{\phi}0}$, and $(\ddot{\phi}_1, \ddot{\phi}_2|\underline{\phi}) = (\underline{\phi},\underline{\phi})$ in $\ddot{Q}_{\underline{\phi}\underline{\phi}}$.

Finally, the certifier chooses the binary label standard $\underline{\phi}$ to maximize its payoff $\ddot{\pi}_0(\underline{\phi})$. For a clear discussion, we next discuss the certifier's decision by cases in Table A.3, where $(\tau,\kappa,\gamma,\underline{\phi})$ space is divided into three components. Each part could correspond to various subcases, requiring additional comparison of the certifier's associated payoffs. Note in Table A.3, $\underline{\phi}_{L1}$ intersects $\underline{\phi} = 1$ at $\ddot{\kappa}_{L1}$, and $\underline{\phi}_{H1}$ intersects $\underline{\phi} = 1$ at $\ddot{\kappa}_{H1}$. Moreover, $\ddot{\Omega}_1 \doteq \{(\tau,\kappa,\gamma) : (0 \leq \gamma \leq \frac{1}{2}, \ddot{\kappa}_{L1} < \kappa < \ddot{\kappa}_1)||(\frac{1}{2} < \gamma \leq 1, (\ddot{\tau}_1 \leq$

$\tau < \frac{3}{\theta-\Delta c}, \ddot{\kappa}_{L1} < \kappa < \ddot{\kappa}_1)||(\tau < \ddot{\tau}_1, \ddot{\kappa}_{L1} < \kappa \le \ddot{\kappa}_{H1}))\}$, $\ddot{\Omega}_2 \doteq \{(\tau,\kappa,\gamma) : (\gamma = 0, \kappa \ge \ddot{\kappa}_1)||(0 < \gamma \le \frac{1}{2}, \ddot{\kappa}_1 \le \kappa \le \ddot{\kappa}_{H1})||(\frac{1}{2} < \gamma \le 1, (\ddot{\tau}_1 \le \tau < \frac{3}{\theta-\Delta c}, \ddot{\kappa}_1 \le \kappa \le \ddot{\kappa}_{H1})\}$, $\ddot{\Omega}_3 \doteq \{(\tau,\kappa,\gamma) : \frac{1}{2} < \gamma \le 1, (\ddot{\tau}_1 \le \tau < \frac{3}{\theta-\Delta c}, \kappa > \ddot{\kappa}_2)||(\tau < \ddot{\tau}_1, \kappa > \ddot{\kappa}_{H1})\}$, $\ddot{\Omega}_4 \doteq \{(\tau,\kappa,\gamma) : (0 < \gamma \le \frac{1}{2}, \kappa > \ddot{\kappa}_{H1})||(\frac{1}{2} < \gamma \le 1, \ddot{\tau}_1 \le \tau < \frac{3}{\theta-\Delta c}, \ddot{\kappa}_{H1} < \kappa \le \ddot{\kappa}_2)\}$, where $\ddot{\kappa}_1 = \frac{(\gamma+1)(\theta-\Delta c)[36-3\tau(\theta-\Delta c)-\tau^2(\theta-\Delta c)^2]}{36[3+\tau(\theta-\Delta c)]}$, $\ddot{\kappa}_2 = \frac{\tau(\gamma+1)(5\gamma+1+\sqrt{17\gamma^2+14\gamma+1})(\theta-\Delta c)^2}{72\gamma(2\gamma-1)}$, and $\ddot{\tau}_1 = \frac{12(2\gamma-1)}{(\gamma+3+\sqrt{17\gamma^2+14\gamma+1})(\theta-\Delta c)}$.

Table A.3: The Certifier's Minimum-standard Decision Under Binary Label

Cases	Subcases	$(\ddot{\phi}_1, \ddot{\phi}_2 \vert \underline{\phi})$	$\underline{\phi}$	$\ddot{\pi}_0(\underline{\phi})$	$\ddot{\underline{\phi}}$
$0 \le \kappa \le \ddot{\kappa}_{L1}$	$\ddot{Q}_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0,1]$	β_r	1
$\ddot{\kappa}_{L1} < \kappa \le \ddot{\kappa}_{H1}$	$\ddot{Q}_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, \underline{\phi}_{L1}]$	$\beta_v + (\beta_r - \beta_v)\underline{\phi}_{L1}$	$\underline{\phi}_{L1}$ if $(\tau,\kappa,\gamma) \in \ddot{\Omega}_1$; 1 if $(\tau,\kappa,\gamma) \in \ddot{\Omega}_2$
	$\ddot{Q}_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{L1}, 1]$	$\beta_v + (\beta_r - \beta_v)\frac{3+\tau(\theta-\Delta c)}{6}$	
$\kappa > \ddot{\kappa}_{H1}$	$\ddot{Q}_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, \underline{\phi}_{L1}]$	$\beta_v + (\beta_r - \beta_v)\underline{\phi}_{L1}$	$\underline{\phi}_{L1}$ if $(\tau,\kappa,\gamma) \in \ddot{\Omega}_3$; $\underline{\phi}_{H1}$ if $(\tau,\kappa,\gamma) \in \ddot{\Omega}_4$
	$\ddot{Q}_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{L1}, \underline{\phi}_{H1}]$	$\beta_v + (\beta_r - \beta_v)\frac{3\underline{\phi}_{H1}+\tau(\theta-\Delta c)\underline{\phi}_{H1}^2}{6}$	
	$\ddot{Q}_{00}$	$(0,0)$	$(\underline{\phi}_{H1}, 1]$	β_v	

Table A.3 shows there are four possible recycled content equilibria for firms, namely $(\ddot{\phi}_1, \ddot{\phi}_2) = (1,1)$, $(\underline{\phi}_{L1}, \underline{\phi}_{L1})$, $(1,0)$ and $(\underline{\phi}_{H1}, 0)$. Then we combine the conditions for each equilibrium in sets $\ddot{\Omega}_{11}$, $\ddot{\Omega}_{\underline{\phi}\underline{\phi}}$, $\ddot{\Omega}_{10}$ and $\ddot{\Omega}_{\underline{\phi}0}$, respectively, where $\ddot{\Omega}_{11} \doteq \{(\tau,\kappa,\gamma) : 0 \le k \le \ddot{\kappa}_{L1}\}$, $\ddot{\Omega}_{\underline{\phi}\underline{\phi}} \doteq \ddot{\Omega}_1 \cup \ddot{\Omega}_3 \doteq \{(\tau,\kappa,\gamma) : (0 \le \gamma \le \frac{1}{2}, \ddot{\kappa}_{L1} < \kappa < \ddot{\kappa}_1)||(\frac{1}{2} < \gamma \le 1, (\ddot{\tau}_1 \le \tau < \frac{3}{\theta-\Delta c}, (\ddot{\kappa}_{L1} < \kappa < \ddot{\kappa}_1)||(\kappa > \ddot{\kappa}_2))||(\tau < \ddot{\tau}_1, \kappa > \ddot{\kappa}_{L1}))\}$, $\ddot{\Omega}_{10} \doteq \ddot{\Omega}_2 \doteq \{(\tau,\kappa,\gamma) : (\gamma = 0, \kappa \ge \ddot{\kappa}_1)||(0 < \gamma \le \frac{1}{2}, \ddot{\kappa}_1 \le \kappa \le \ddot{\kappa}_{H1})||(\frac{1}{2} < \gamma \le 1, \ddot{\tau}_1 \le \tau < \frac{3}{\theta-\Delta c}, \ddot{\kappa}_1 \le \kappa \le \ddot{\kappa}_{H1})\}$,$\ddot{\Omega}_{\underline{\phi}0} \doteq \ddot{\Omega}_4 \doteq \{(\tau,\kappa,\gamma) : (0 < \gamma \le \frac{1}{2}, \kappa > \ddot{\kappa}_{H1})||(\frac{1}{2} < \gamma \le 1, \ddot{\tau}_1 \le \tau < \frac{3}{\theta-\Delta c}, \ddot{\kappa}_{H1} < \kappa \le \ddot{\kappa}_2)\}$. The corresponding label decision (i.e., $\ddot{\underline{\phi}}$) are 1, $\underline{\phi}_{L1}$, 1 and $\underline{\phi}_{H1}$, respectively.

Given the certifier's optimal label standard and the firms' recycled content equilibrium, the corresponding pricing equilibrium $(\ddot{p}_1, \ddot{p}_2)$ is given by:

$$(\ddot{p}_1, \ddot{p}_2) = \Big(\frac{1}{\tau} + \frac{\theta(\ddot{\phi}_1 - \ddot{\phi}_2) + 2\ddot{c}_1 + \ddot{c}_2}{3}, \frac{1}{\tau} + \frac{\theta(\ddot{\phi}_2 - \ddot{\phi}_1) + 2\ddot{c}_2 + \ddot{c}_1}{3}\Big),$$

where $\ddot{c}_i = \ddot{\phi}_i \cdot c_r + (1 - \ddot{\phi}_i) \cdot c_v$.

Then, given the recycled content and pricing equilibrium, the corresponding firms' equilibrium demand $(\ddot{d}_1, \ddot{d}_2)$, profits $(\ddot{\pi}_1, \ddot{\pi}_2)$ and the certifier's equilibrium payoff $\ddot{\pi}_0$

are respectively given by:

$$(\ddot{d}_1, \ddot{d}_2) = (\frac{1}{2} + \frac{\tau[\theta(\ddot{\phi}_1 - \ddot{\phi}_2) - \ddot{c}_1 + \ddot{c}_2]}{6}, \frac{1}{2} + \frac{\tau[\theta(\ddot{\phi}_2 - \ddot{\phi}_1) - \ddot{c}_2 + \ddot{c}_1]}{6}),$$

$$(\ddot{\pi}_1, \ddot{\pi}_2) = (\frac{[3 + \tau(\theta - \Delta c)(\ddot{\phi}_1 - \ddot{\phi}_2)]^2}{18\tau} - \frac{2\gamma\kappa\ddot{\phi}_1^2}{1+\gamma}, \frac{[3 + \tau(\theta - \Delta c)(\ddot{\phi}_2 - \ddot{\phi}_1)]^2}{18\tau} - \frac{2\kappa\ddot{\phi}_2^2}{1+\gamma}),$$

$$\ddot{\pi}_0 = \beta_r + (\beta_r - \beta_v)\frac{3(\ddot{\phi}_1 + \ddot{\phi}_2) + \tau(\theta - \Delta c)(\ddot{\phi}_1 - \ddot{\phi}_2)^2}{6}. \quad \text{(A.2)}$$

□

Proof of Proposition 2: Given Equation (A.2), we first check the monotonicities of $\ddot{\pi}_0$ with respect to τ, κ, γ within each region $\ddot{\Omega}_{ij}$, for $i, j \in \{1, \underline{\phi}, 0\}$, via solving the first-order conditions. Then we combine the results in these regions.

1. With respect to τ, we can easily check: (1) Within $\ddot{\Omega}_{11}$, $\frac{\partial \ddot{\pi}_0}{\partial \tau} = 0$; (2)Within $\ddot{\Omega}_{\underline{\phi}\underline{\phi}}$, $\frac{\partial \ddot{\pi}_0}{\partial \tau} < 0$; (3) Within $\ddot{\Omega}_{10}$, $\frac{\partial \ddot{\pi}_0}{\partial \tau} > 0$; (4) Within $\ddot{\Omega}_{\underline{\phi}0}$, $\frac{\partial \ddot{\pi}_0}{\partial \tau} > 0$.

   Then we combine the results in these regions, which are shown in Table A.4 by cases. Note $\frac{(\gamma+1)(\theta-\Delta c)}{12}$ is the lower bound κ of region $\ddot{\Omega}_{\underline{\phi}\underline{\phi}}$, $\frac{(\gamma+1)(\theta-\Delta c)}{6}$ is the upper bound κ of region $\ddot{\Omega}_{11}$, and $\frac{(\gamma+1)(\theta-\Delta c)}{4\gamma}$ is the upper bound κ of region $\ddot{\Omega}_{10}$. When $0 \le \gamma \le \frac{1}{2}$, $\frac{(\gamma+1)(\theta-\Delta c)}{3}$ is the upper bound κ of region $\ddot{\Omega}_{\underline{\phi}\underline{\phi}}$, $\frac{(\gamma+1)(\theta-\Delta c)}{6\gamma}$ is the lower bound κ of region $\ddot{\Omega}_{\underline{\phi}0}$. Moreover, when $\gamma > \frac{1}{2}$, $\ddot{\kappa}_{\tau 1}$ is the lower bound κ of region $\ddot{\Omega}_{\underline{\phi}0}$, $\ddot{\kappa}_{\tau 2}$ is the upper bound κ of region $\ddot{\Omega}_{\underline{\phi}0}$.

2. With respect to κ, we can easily check: (1) Within $\ddot{\Omega}_{11}$, $\frac{\partial \ddot{\pi}_0}{\partial \kappa} = 0$; (2)Within $\ddot{\Omega}_{\underline{\phi}\underline{\phi}}$, $\frac{\partial \ddot{\pi}_0}{\partial \kappa} < 0$; (3) Within $\ddot{\Omega}_{10}$, $\frac{\partial \ddot{\pi}_0}{\partial \kappa} = 0$; (4) Within $\ddot{\Omega}_{\underline{\phi}0}$, $\frac{\partial \ddot{\pi}_0}{\partial \kappa} < 0$.
   Thus, we can summarize $\ddot{\pi}_0$ always (weakly) decreases in κ.

3. With respect to γ, we can easily check: (1) Within $\ddot{\Omega}_{11}$, $\frac{\partial \ddot{\pi}_0}{\partial \gamma} = 0$; (2)Within $\ddot{\Omega}_{\underline{\phi}\underline{\phi}}$, $\frac{\partial \ddot{\pi}_0}{\partial \gamma} > 0$; (3) Within $\ddot{\Omega}_{10}$, $\frac{\partial \ddot{\pi}_0}{\partial \gamma} = 0$; (4) Within $\ddot{\Omega}_{\underline{\phi}0}$, $\frac{\partial \ddot{\pi}_0}{\partial \gamma} < 0$.

   Then we combine the results in these regions, which are shown in Table A.5

Table A.4: The Monotonicities of $\ddot{\pi}_0$ with Respect to τ by Cases

Cases		$\ddot{\Omega}_{11}$	$\ddot{\Omega}_{\underline{\phi}\phi}$	$\ddot{\Omega}_{\underline{\phi}0}$	$\ddot{\Omega}_{10}$
$0 \leq \kappa \leq \frac{(\gamma+1)(\theta-\Delta c)}{12}$		–	/	/	/
$\frac{(\gamma+1)(\theta-\Delta c)}{12} \leq \kappa < \frac{(\gamma+1)(\theta-\Delta c)}{6}$		–	↓	/	↑
$0 \leq \gamma \leq \frac{1}{2}$	$\frac{(\gamma+1)(\theta-\Delta c)}{6} < \kappa \leq \frac{(\gamma+1)(\theta-\Delta c)}{3}$	/	↓	/	↑
	$\frac{(\gamma+1)(\theta-\Delta c)}{3} < \kappa \leq \frac{(\gamma+1)(\theta-\Delta c)}{6\gamma}$	/	/	/	↑
	$\frac{(\gamma+1)(\theta-\Delta c)}{6\gamma} < \kappa \leq \frac{(\gamma+1)(\theta-\Delta c)}{4\gamma}$	/	/	↑	↑
	$\kappa > \frac{(\gamma+1)(\theta-\Delta c)}{4\gamma}$	/	/	↑	/
$\gamma > \frac{1}{2}$	$\frac{(\gamma+1)(\theta-\Delta c)}{6} < \kappa \leq \ddot{\kappa}_{\tau 1}$	/	↓	/	↑
	$\ddot{\kappa}_{\tau 1} < \kappa \leq \frac{(\gamma+1)(\theta-\Delta c)}{4\gamma}$	/	↓	↑	↑
	$\frac{(\gamma+1)(\theta-\Delta c)}{4\gamma} < \kappa \leq \ddot{\kappa}_{\tau 2}$	/	↓	↑	/
	$\kappa > \ddot{\kappa}_{\tau 2}$	/	↓	/	/

Note: $\ddot{\kappa}_{\tau 1} = \frac{(\gamma+1)(5\gamma+1+\sqrt{17\gamma^2+14\gamma+1})(\theta-\Delta c)}{6\gamma(\gamma+3+\sqrt{17\gamma^2+14\gamma+1})}$, $\ddot{\kappa}_{\tau 2} = \frac{(\gamma+1)(5\gamma+1+\sqrt{17\gamma^2+14\gamma+1})(\theta-\Delta c)}{24\gamma(2\gamma-1)}$.

by cases. Note $\frac{6(\theta-\Delta c)-\tau(\theta-\Delta c)^2}{36}$ is the lower bound κ of region $\ddot{\Omega}_{\underline{\phi}\phi}$, $\ddot{\kappa}_{\gamma 1}$ is the lower bound κ of region $\ddot{\Omega}_{10}$, $\frac{6(\theta-\Delta c)-\tau(\theta-\Delta c)^2}{18}$ is the upper bound κ of region $\ddot{\Omega}_{11}$. When $\frac{3}{(1+\sqrt{2})(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}$, there emerges two separate regions within $\ddot{\Omega}_{\underline{\phi}\phi}$, $2\ddot{\kappa}_{\gamma 1}$ is the upper bound κ of one, and $\frac{(6+4\sqrt{2})\tau(\theta-\Delta c)^2}{36}$ is the lower bound κ of the other. Moreover, $\frac{6(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{18}$ is the lower bound κ of region $\ddot{\Omega}_{\underline{\phi}0}$ when $\frac{3}{(1+\sqrt{2})(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}$. Otherwise, $\frac{(\theta-\Delta c)[9+\tau(\theta-\Delta c)]}{6[3+\tau(\theta-\Delta c)]}$ serves as the lower bound κ.

Table A.5: The Monotonicities of $\ddot{\pi}_0$ with Respect to γ by Cases

Cases		$\ddot{\Omega}_{10}$	$\ddot{\Omega}_{\underline{\phi}0}$	$\ddot{\Omega}_{\underline{\phi}\phi}$	$\ddot{\Omega}_{11}$
$0 \leq \kappa \leq \frac{6(\theta-\Delta c)-\tau(\theta-\Delta c)^2}{36}$		/	/	/	–
$\frac{6(\theta-\Delta c)-\tau(\theta-\Delta c)^2}{36} < \kappa \leq \ddot{\kappa}_{\gamma 1}$		/	/	↑	–
$\ddot{\kappa}_{\gamma 1} < \kappa \leq \frac{6(\theta-\Delta c)-\tau(\theta-\Delta c)^2}{18}$		–	/	↑	–
$\frac{3}{(1+\sqrt{2})(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}$	$\frac{6(\theta-\Delta c)-\tau(\theta-\Delta c)^2}{18} < \kappa \leq 2\ddot{\kappa}_{\gamma 1}$	–	/	↑	/
	$2\ddot{\kappa}_{\gamma 1} < \kappa \leq \frac{6(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{18}$	–	/	/	/
	$\frac{6(\theta-\Delta c)+\tau(\theta-\Delta c)^2}{18} < \kappa \leq \frac{(6+4\sqrt{2})\tau(\theta-\Delta c)^2}{36}$	–	↓	/	/
	$\kappa > \frac{(6+4\sqrt{2})\tau(\theta-\Delta c)^2}{36}$	–	↓	↑	/
$\tau < \frac{3}{(1+\sqrt{2})(\theta-\Delta c)}$	$\frac{6(\theta-\Delta c)-\tau(\theta-\Delta c)^2}{18} < \kappa \leq \frac{(\theta-\Delta c)[9+\tau(\theta-\Delta c)]}{6[3+\tau(\theta-\Delta c)]}$	–	/	↑	/
	$\kappa > \frac{(\theta-\Delta c)[9+\tau(\theta-\Delta c)]}{6[3+\tau(\theta-\Delta c)]}$	–	↓	↑	/

Note: $\ddot{\kappa}_{\gamma 1} = \frac{(\theta-\Delta c)[36-3\tau(\theta-\Delta c)-\tau^2(\theta-\Delta c)]}{36[3+\tau(\theta-\Delta c)]}$.

Proof of Proposition 3: Following Lemma 2 and 4, we compare the certifier's payoff under the continuous and binary label. The comparison is valid in the following non-empty regions: (1) When $\gamma = 0$, there are $\tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi}\phi}$, and

$\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$; (2) When $0 < \gamma \leq \frac{1}{4}$, there are $\tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$ and $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi}0}$; (3) When $\frac{1}{4} < \gamma \leq \frac{1}{2}$, there are $\tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi}0}$ and $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$; (4) When $\frac{1}{2} < \gamma \leq 1$, there are $\tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi}0}$, $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{11}$ and $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$. Otherwise, the intersection is empty. Then, the comparison is given below by cases.

1. When $\gamma = 0$: (1) $\tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\tilde{\pi}_0 = \ddot{\pi}_0$ holds trivially. (2) $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{11}$, $\tilde{\pi}_0 < \ddot{\pi}_0$ holds trivially. (3) $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, $\tilde{\pi}_0 - \ddot{\pi}_0$ increases in κ. The solution to $\tilde{\pi}_0 = \ddot{\pi}_0$ is the third root to this equation, which we denote as κ_L. Thus $\tilde{\pi}_0 \leq \ddot{\pi}_0$ when $\kappa \leq \kappa_L$ and $(\tau, \kappa, \gamma) \in \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, otherwise, $\tilde{\pi}_0 > \ddot{\pi}_0$. (4) $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\pi}_0 \leq \ddot{\pi}_0$ when $\frac{3}{2(\theta - \Delta c)} \leq \tau < \frac{3}{\theta - \Delta c}$, $\kappa \geq \kappa_{H1}$ and $(\tau, \kappa, \gamma) \in \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, otherwise, $\tilde{\pi}_0 > \ddot{\pi}_0$, where $\kappa_{H1} = \frac{(\gamma+1)\tau^2(\theta-\Delta c)^3}{36[-3+2\tau(\theta-\Delta c)]}$.

2. When $0 < \gamma \leq \frac{1}{4}$: The first three cases are the same. For $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\pi}_0 \leq \ddot{\pi}_0$ when $\tau_1 \leq \tau < \frac{3}{\theta - \Delta c}$, $\kappa \geq \kappa_{H1}$ and $(\tau, \kappa, \gamma) \in \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, otherwise, $\tilde{\pi}_0 > \ddot{\pi}_0$, where $\tau_1 = \frac{12}{5(\theta-\Delta c)+\sqrt{(9-8\gamma)(\theta-\Delta c)^2}}$. For $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\pi}_0 - \ddot{\pi}_0$ decreases in κ. Thus, the solution to $\tilde{\pi}_0 = \ddot{\pi}_0$ is unique, which can be either the first or the third root to this equation, specifically depending on the symmetricity of the two firms' recycled efficiency (i.e., γ) and the competition intensity (i.e., τ). Note that regardless of the specific root, it is the upper bound κ in $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$ that the certifier prefers the continuous label, which we denote as κ_{H2}. For $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi}0}$, $\tilde{\pi}_0 < \ddot{\pi}_0$.

3. When $\frac{1}{4} < \gamma \leq \frac{1}{2}$: The first six cases are the same. For $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, There are two subcases, i.e., $\frac{1}{4} < \gamma \leq \frac{1}{3}$ and $\frac{1}{3} < \gamma \leq \frac{1}{2}$. When $\frac{1}{4} < \gamma \leq \frac{1}{3}$, $\tilde{\pi}_0 > \ddot{\pi}_0$. When $\frac{1}{3} < \gamma \leq \frac{1}{2}$, there is a unique solution to $\tilde{\pi}_0 = \ddot{\pi}_0$, which is identified as the first root to this equation and denoted as κ_{H3}. Moreover, $\tilde{\pi}_0 \leq \ddot{\pi}_0$ when $\kappa \geq \kappa_{H3}$, and $\tilde{\pi}_0 > \ddot{\pi}_0$ when $\kappa < \kappa_{H3}$.

4. When $\frac{1}{2} < \gamma \leq 1$: The first seven cases are the same. For $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{11}$, $\tilde{\pi}_0 < \ddot{\pi}_0$ holds trivially.

Note κ_L and κ_{H1} intersect at $\tau = \frac{24}{5+\sqrt{73}(\theta-\Delta c)}$ which we denote as τ_2; κ_{H1}, κ_{H2} and $\tilde{\kappa}_3$ intersect at τ_1; κ_{H2}, κ_{H3} and $\ddot{\kappa}_1$ intersect at τ_3 which is the second root to the equation $\kappa_{H2} = \ddot{\kappa}_1$. κ_L, κ_{H3} and $\tilde{\kappa}_3$ intersect at τ_4 which is the first root to the equation $\kappa_L = \tilde{\kappa}_3$. Moreover, when $\gamma = \frac{5\sqrt{73}-31}{16}$, κ_L, κ_{H1}, κ_{H2} κ_{H3}, $\ddot{\kappa}_1$ and $\tilde{\kappa}_3$ all intersect at the same point τ_2, which means κ_L, κ_{H1}, κ_{H2} and κ_{H3} become a point. Recall, κ_L is lower bound κ that the certifier prefers the continuous label, and κ_{H1}, κ_{H2} and κ_{H3} are the upper bounds κ that the certifier prefers the continuous label. Thus, when $\frac{5\sqrt{73}-31}{16} < \gamma \leq 1$, the certifier would always (weakly) prefers the binary label, since $\tilde{\pi}_0 > \ddot{\pi}_0$ no longer exists. Then we summarize the comparison results and combine the conditions in space (τ, κ, γ) in sets $\mathcal{I}$, $\mathcal{C}$ and $\mathcal{B}$ as follows. $\mathcal{I} \doteq \tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\mathcal{C} \doteq \{(\tau, \kappa, \gamma) : (\gamma = 0, (\tau_1 \leq \tau < \tau_2, \kappa_L < \kappa < \kappa_{H1})||(\tau < \tau_1, \kappa > \kappa_L))||(0 < \gamma \leq \frac{1}{3}, (\tau_1 \leq \tau < \tau_2, \kappa_L < \kappa < \kappa_{H1})||(\tau < \tau_1, \kappa_L < k < \kappa_{H2}))||(\frac{1}{3} < \gamma \leq \frac{5\sqrt{73}-31}{16}, (\tau_1 \leq \tau < \tau_2, \kappa_L < \kappa < \kappa_{H1})||(\tau_3 \leq \tau < \tau_1, \kappa_L < \kappa < \kappa_{H2})||(\tau < \tau_3, \kappa_L < \kappa < \kappa_{H3}))\}$. Otherwise $(\tau, \kappa, \gamma) \in \mathcal{B}$. By the definition of each region, we can easily find the certifier is indifferent between the two labels (i.e., $\tilde{\pi}_0 = \ddot{\pi}_0$) when $(\tau, \kappa, \gamma) \in \mathcal{I}$, prefers the continuous label (i.e., $\tilde{\pi}_0 > \ddot{\pi}_0$) when $(\tau, \kappa, \gamma) \in \mathcal{C}$ and prefers the binary label (i.e., $\tilde{\pi}_0 \leq \ddot{\pi}_0$) when $(\tau, \kappa, \gamma) \in \mathcal{B}$. □

Proof of Proposition 4: Let $\tilde{\pi}_I = \tilde{\pi}_1 + \tilde{\pi}_2$ and $\ddot{\pi}_I = \ddot{\pi}_1 + \ddot{\pi}_2$ denote the overall industry profit under the continuous and binary label, respectively. Then following Lemma 2 and 4, we also compare the firms' and the whole industry's profits under the continuous and binary label in the non-empty intersections. To simplify the analysis, we'll stick to strict inequalities for now and address equality in the final results. Note we use the tie-breaking rule that when the firms and the industry are indifferent between the two labels, they prefer the binary label. This rules has no impact on our results as it only occurs in boundary situation. Then, the comparison is given below.

1. When $\gamma = 0$: (1) $\tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\tilde{\pi}_i = \ddot{\pi}_i$ holds trivially for $i \in \{1, 2, I\}$. (2) $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{11}$, $\tilde{\pi}_i > \ddot{\pi}_i$ holds for $i \in \{1, 2, I\}$. (3) $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, for firm 1, we obtain $\tilde{\pi}_1 > \ddot{\pi}_1$. For firm 2, we have $\tilde{\pi}_2 - \ddot{\pi}_2$ decreases in κ, and we also find $\tilde{\pi}_2 > \ddot{\pi}_2$ at the lower bound κ of $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$. At the upper bound κ of $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, there

are two subcases: $\tilde{\pi}_2 > \ddot{\pi}_2$ if $\hat{\tau}_{f1} < \tau < \frac{3}{\theta-\Delta c}$, and $\tilde{\pi}_2 < \ddot{\pi}_2$ if $\tau < \hat{\tau}_{f1}$, where $\hat{\tau}_{f1} = \frac{6}{(\sqrt{3}+1)(\theta-\Delta c)}$. Thus, when $\hat{\tau}_{f1} < \tau < \frac{3}{\theta-\Delta c}$, $\tilde{\pi}_2 > \ddot{\pi}_2$ always holds within this set. When $\tau < \hat{\tau}_{f1}$, a unique solution will emerge for $\tilde{\pi}_2 = \ddot{\pi}_2$, which is the first root of this equation and we denote as $\hat{\kappa}_{f1}$. Thus, in the latter subcase, we obtain $\tilde{\pi}_2 > \ddot{\pi}_2$ when $\kappa < \hat{\kappa}_{f1}$ and $\tilde{\pi}_2 < \ddot{\pi}_2$ when $\kappa > \hat{\kappa}_{f1}$. Additionally, we can derive $\hat{\kappa}_{f1}$ intersects with the upper bound of this intersection at $\tau = \hat{\tau}_{f1}$. For the industry, we have $\tilde{\pi}_I > \ddot{\pi}_I$. (4) $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, we have $\tilde{\pi}_1 < \ddot{\pi}_1$, $\tilde{\pi}_2 > \ddot{\pi}_2$, and $\tilde{\pi}_I < \ddot{\pi}_I$.

2. When $0 < \gamma \leq \frac{1}{4}$: The first four cases are the same as above. Within $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$, there is a unique solution, denoted as $\check{\kappa}_{f1}$, to the equation $\tilde{\pi}_1 = \ddot{\pi}_1$. This solution corresponds to the forth root of the equation. Consequently, for firm 1, we have $\tilde{\pi}_1 < \ddot{\pi}_1$ when $\kappa < \check{\kappa}_{f1}$ and $\tilde{\pi}_1 > \ddot{\pi}_1$ when $\kappa > \check{\kappa}_{f1}$. For firm 2, we have $\tilde{\pi}_2 > \ddot{\pi}_2$. For the industry, a unique solution exists for the equation $\tilde{\pi}_I = \ddot{\pi}_I$, identified as the third root to this equation and denoted as κ_I. As a result, for the industry, we have $\tilde{\pi}_I < \ddot{\pi}_I$ when $\kappa < \kappa_I$ and $\tilde{\pi}_I > \ddot{\pi}_I$ when $\kappa > \kappa_I$. Within $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi}0}$, $\tilde{\pi}_i > \ddot{\pi}_i$, for $i \in \{1, 2, I\}$.

3. When $\frac{1}{4} < \gamma \leq \frac{1}{2}$: The first sixth cases are the same as above, except for $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$. In this specific set, both $\tilde{\pi}_1 > \ddot{\pi}_1$ for firm 1 and $\tilde{\pi}_I > \ddot{\pi}_I$ for the industry persist. For firm 2, we discuss the outcomes in two subcases, depending on the symmetricity of firms' recycled efficiency (i.e., γ). In the first case where $\frac{1}{4} < \gamma \leq \frac{2}{5}$, the results are the same as above. In the second case where $\frac{2}{5} < \gamma \leq \frac{1}{2}$, $\hat{\kappa}_{f1}$ intersects with the upper bound of this intersection not only at $\tau = \hat{\tau}_{f1}$ but also at $\tau = \hat{\tau}_{f2}$. $\hat{\tau}_{f2}$ is the third root to the equation $\tilde{\kappa}_3 = \hat{\kappa}_{f1}$ and is established as greater than $\hat{\tau}_{f1}$. Recall that $\tilde{\pi}_2 - \ddot{\pi}_2$ decreases in κ and $\hat{\kappa}_{f1}$ stands as the unique solution to $\tilde{\pi}_2 = \ddot{\pi}_2$ within this set. Thus in this subcase, we can derive $\tilde{\pi}_2 > \ddot{\pi}_2$ when $\hat{\tau}_{f1} < \tau < \frac{3}{\theta-\Delta c}$ and $\tau < \hat{\tau}_{f2}$. Additionally, when $\hat{\tau}_{f2} < \tau < \hat{\tau}_{f1}$, we have $\tilde{\pi}_2 > \ddot{\pi}_2$ if $\kappa < \hat{\kappa}_{f1}$ and $\tilde{\pi}_2 < \ddot{\pi}_2$ if $\kappa > \hat{\kappa}_{f1}$. For $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, $\tilde{\pi}_1 > \ddot{\pi}_1$ holds for firm 1. For firm 2, similarly, we analyze the results in two subcases. In the first case where $\frac{1}{4} < \gamma \leq \frac{2}{5}$, we have $\tilde{\pi}_2 < \ddot{\pi}_2$. In the second case where $\frac{2}{5} < \gamma \leq \frac{1}{2}$, a unique solution, denoted

as $\hat{\kappa}_{f2}$, exists for the equation $\tilde{\pi}_2 = \ddot{\pi}_2$, where $\hat{\kappa}_{f2}$ is the third root to this equation. Furthermore, within this intersection set, we obtain $\tilde{\pi}_2 < \ddot{\pi}_2$ when $\kappa < \hat{\kappa}_{f2}$, and $\tilde{\pi}_2 > \ddot{\pi}_2$ when $\kappa > \hat{\kappa}_{f2}$. For the industry, we have $\tilde{\pi}_I > \ddot{\pi}_I$.

4. When $\frac{1}{2} < \gamma \leq 1$, the first seven cases are the same as above for the industry. For the firms, the outcomes in cases $\tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{11}$ and $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$ are the same as the above, and the outcomes in cases $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$ and $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$ are the same as those when $\frac{2}{5} < \gamma \leq \frac{1}{2}$. Within $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$ and $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi}0}$, $\tilde{\pi}_2 > \ddot{\pi}_2$ still holds for firm 2. While for firm 1, we need to discuss the results in two subcases, i.e., $\frac{1}{2} < \gamma \leq \frac{6-\sqrt{6}}{5}$ and $\frac{6-\sqrt{6}}{5} < \gamma \leq 1$. In the first case where $\frac{1}{2} < \gamma \leq \frac{6-\sqrt{6}}{5}$, the outcome within $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$ is the same as the above. While within $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi}0}$, a unique solution, denoted as $\check{\kappa}_{f2}$, emerges from the equation $\tilde{\pi}_1 = \ddot{\pi}_1$, where $\check{\kappa}_{f2} = \frac{(\gamma+1)[(\gamma+1)(3\gamma-1)+(1-\gamma)\sqrt{9\gamma^2-2\gamma+1}]\tau(\theta-\Delta c)^2}{72\gamma(2\gamma-1)}$. Furthermore, within this set, we obtain $\tilde{\pi}_1 > \ddot{\pi}_1$ when $\kappa < \check{\kappa}_{f2}$, and $\tilde{\pi}_1 < \ddot{\pi}_1$ when $\kappa > \check{\kappa}_{f2}$. In the second case where $\frac{6-\sqrt{6}}{5} < \gamma \leq 1$, $\tilde{\pi}_1 < \ddot{\pi}_1$ consistently holds in $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$ and $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi}0}$. Within $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{11}$, $\tilde{\pi}_1 > \ddot{\pi}_1$, $\tilde{\pi}_2 > \ddot{\pi}_2$, and $\tilde{\pi}_I > \ddot{\pi}_I$.

Note $\check{\kappa}_{f1}$, $\check{\kappa}_{f2}$ and $\ddot{\kappa}_{H1}$ intersect at $\check{\tau}_{f1}$, where $\check{\tau}_{f1} = \frac{12(2\gamma-1)}{[(\gamma+1)(3\gamma-1)+(1-\gamma)\sqrt{9\gamma^2-2\gamma+1}](\theta-\Delta c)}$. Moreover, $\hat{\kappa}_{f1}$, $\hat{\kappa}_{f2}$ and $\tilde{\kappa}_3$ intersect at $\hat{\tau}_{f2}$. $\hat{\kappa}_{f2}$ intersects $\ddot{\kappa}_1$ at $\hat{\tau}_{f3}$, where $\hat{\tau}_{f3}$ is the sixth root to $\hat{\kappa}_{f2} = \ddot{\kappa}_1$. Particularly, when $\gamma = \frac{9-3\sqrt{3}}{4}$, $\hat{\kappa}_{f1}$, $\hat{\kappa}_{f2}$, $\tilde{\kappa}_3$, $\ddot{\kappa}_1$ all intersect at the same point $\hat{\tau}_{f1}$, which means $\hat{\kappa}_{f1}$ and $\hat{\kappa}_{f2}$ become a point. Recall that $\hat{\kappa}_{f1}$ is the lower bound κ and $\hat{\kappa}_{f2}$ is the upper bound κ that firm 2 prefers the binary label. Thus, when $\frac{9-3\sqrt{3}}{4} < \gamma \leq 1$, firm 2 would always (weakly) prefers the continuous label, as $\tilde{\pi}_2 < \ddot{\pi}_2$ no longer exists. Then we summarize the comparison results for each firm and combine the conditions in space (τ, κ, γ) in sets $\check{\mathcal{I}}$, $\check{\mathcal{B}}$ and $\check{\mathcal{C}}$ for firm 1 and $\hat{\mathcal{I}}$, $\hat{\mathcal{B}}$ and $\hat{\mathcal{C}}$ for firm 2. Specifically, $\check{\mathcal{I}} \doteq \tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\check{\mathcal{B}} \doteq \{(\tau, \kappa, \gamma) : (\gamma = 0, \kappa \geq \ddot{\kappa}_1)||(0 < \gamma \leq \frac{1}{2}, \ddot{\kappa}_1 \leq \kappa \leq \check{\kappa}_{f1})||(\frac{1}{2} < \gamma \leq \frac{6-\sqrt{6}}{5}, \check{\tau}_{f1} \leq \tau < \frac{3}{\theta-\Delta c}, (\ddot{\kappa}_1 \leq \kappa \leq \check{\kappa}_{f1})||(\check{\kappa}_{f2} \leq \kappa \leq \ddot{\kappa}_2))||(\ddot{\tau}_1 \leq \tau < \check{\tau}_{f1}, \ddot{\kappa}_1 \leq \kappa \leq \ddot{\kappa}_2))||(\frac{6-\sqrt{6}}{5} < \gamma \leq 1, \ddot{\kappa}_1 \leq \kappa \leq \ddot{\kappa}_2)\}$. Otherwise, $(\tau, \kappa, \gamma) \in \check{\mathcal{C}}$. Additionally, $\hat{\mathcal{I}} \doteq \tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$, $\hat{\mathcal{B}} \doteq \{(\tau, \kappa, \gamma) : (0 \leq \gamma \leq \frac{2}{5}, \hat{\kappa}_{f1} \leq \kappa \leq \ddot{\kappa}_1)||(\frac{2}{5} < \gamma \leq \frac{9-3\sqrt{3}}{4}, (\hat{\tau}_{f3} \leq \tau < \hat{\tau}_{f1}, \hat{\kappa}_{f1} \leq \kappa \leq \ddot{\kappa}_1))||(\hat{\tau}_{f2} \leq \tau < \hat{\tau}_{f3}, \hat{\kappa}_{f1} \leq k \leq \hat{\kappa}_{f2}))\}$. Otherwise, $(\tau, \kappa, \gamma) \in \hat{\mathcal{C}}$. By the definition of the regions, we can easily find firm i is indifferent between the two labels (i.e., $\tilde{\pi}_i = \ddot{\pi}_i$) when

$(\tau,\kappa,\gamma) \in \check{\mathcal{I}}/\hat{\mathcal{I}}$, prefers the continuous label (i.e., $\tilde{\pi}_i > \ddot{\pi}_i$) when $(\tau,\kappa,\gamma) \in \check{\mathcal{C}}/\hat{\mathcal{C}}$, and prefers the binary label (i.e., $\tilde{\pi}_i \leq \ddot{\pi}_i$) when $(\tau,\kappa,\gamma) \in \check{\mathcal{B}}/\hat{\mathcal{B}}$.

For the industry, we find κ_I will intersect with the lower bound of set $\tilde{\Omega}_{LL} \cap \ddot{\Omega}_{10}$ (i.e., $\ddot{\kappa}_1$) at $\tau = \tau_I$ when $\frac{1+\sqrt{65}}{32} < \gamma \leq 1$, where τ_I is the sixth root to the equation $\kappa_I = \ddot{\kappa}_1$. Then, we also summarize the comparison results for the industry and combine the conditions in space (τ,κ,γ) in sets $\mathring{\mathcal{I}}$, $\mathring{\mathcal{B}}$, and $\mathring{\mathcal{C}}$. Specifically, $\mathring{\mathcal{I}} \doteq \tilde{\Omega}_{11} \cap \ddot{\Omega}_{11}$ $\mathring{\mathcal{B}} \doteq \{(\tau,\kappa,\gamma) : (\gamma = 0, \kappa \geq \ddot{\kappa}_1)||(0 < \gamma \leq \frac{1+\sqrt{65}}{32}, \ddot{\kappa}_1 \leq \kappa \leq \kappa_I)||(\frac{1+\sqrt{65}}{32} < \gamma \leq 1, \tau_I \leq \tau < \frac{3}{\theta-\Delta c}, \ddot{\kappa}_1 \leq \kappa \leq \kappa_I)\}$, otherwise, $(\tau,\kappa,\gamma) \in \mathring{\mathcal{C}}$. By the definition of these regions, we can easily find the industry is indifferent between the two labels (i.e., $\tilde{\pi}_I = \ddot{\pi}_I$) when $(\tau,\kappa,\gamma) \in \mathring{\mathcal{I}}$, prefers the continuous label (i.e., $\tilde{\pi}_I > \ddot{\pi}_I$) when $(\tau,\kappa,\gamma) \in \mathring{\mathcal{C}}$, and prefers the binary label (i.e., $\tilde{\pi}_I \leq \ddot{\pi}_I$) when $(\tau,\kappa,\gamma) \in \mathring{\mathcal{B}}$.

After showing the label preferences of the certifier, the firms, and the entire industry, we proceed to compare these preferences to demonstrate the alignment among them.

Next, we show there is no win-win-win label design for the certifier and the two firms. To elaborate, we begin by demonstrating that they cannot all be better off under the binary label, and then we illustrate that the same holds true when under the continuous label. Firstly, when contrasting the label preferences of the two firms, we find both firms cannot simultaneously benefit from the binary label. This is because firm 2 would prefer the binary label only within $(\tilde{\Omega}_{1\phi} \cup \tilde{\Omega}_{\phi\phi}) \cap \ddot{\Omega}_{\underline{\phi\phi}}$. In contrast, within region $\ddot{\Omega}_{\underline{\phi\phi}}$, firm 1 always prefers the continuous label. Consequently, we can straightforwardly infer that the certifier and the two firms cannot collectively achieve a better outcome under the binary label. Secondly, we establish that they cannot collectively be better off under the continuous label. This is demonstrated by proving that when the certifier prefers the continuous label, either firm 1 or firm 2 would prefer the binary label. Recall the proof of Proposition 3, the certifier would prefer the continuous label exclusively in the intersection sets $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$, $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, and $\tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$. This means we can divide $\mathcal{C}$ into four subsets: $\mathcal{C} \cap \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$, $\mathcal{C} \cap \tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$, $\mathcal{C} \cap \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, and $\mathcal{C} \cap \tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$. Then we proceed to compare the firms' label preference in these four subsets. Within $\mathcal{C} \cap \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$ and $\mathcal{C} \cap \tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$, we obtain $\tilde{\pi}_1 < \ddot{\pi}_1$, indicating firm 1 prefers the binary label within these two subsets.

Similarly, within $\mathcal{C} \cap \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$ and $\mathcal{C} \cap \tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$, we find $\tilde{\pi}_2 < \ddot{\pi}_2$, indicating firm 2 prefers the binary label within these two subsets. As a result, we establish that in the subsets where the certifier prefers the continuous label, either firm 1 or firm 2 prefers the binary label.

Finally, we show there is a win-win label design for the certifier and the industry either under the binary or continuous label. Firstly, we establish that they can be both better off under the binary label. This is because the certifier prefers the binary label within a subset of the intersection $\tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$ and the industry prefers the binary label across this whole intersection. Thus, there must be an overlap where both the certifier and the industry prefer the binary label. Secondly, we show they can be both better off under the continuous label. This is because, the certifier may prefer the continuous label in the intersection $(\tilde{\Omega}_{1\phi} \cup \tilde{\Omega}_{\phi\phi}) \cap \ddot{\Omega}_{\underline{\phi\phi}}$ and the industry prefers the continuous label across this whole intersection. Hence, there must be an overlap where both the certifier and the industry prefer the continuous label. $\square$

Proof of Proposition 5: To assess the monotonicities of the certifier's final payoff $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ with respect to τ, κ, and γ, we can process as follows: Firstly, recall the monotonicities of the certifier's payoff under the binary label within each region $\ddot{\Omega}_{ij}$, where $i, j \in \{1, \underline{\phi}, 0\}$; Secondly, recall the regions where the certifier would prefer the continuous label in the final equilibrium and check the monotonicities of the certifier's payoff under the continuous label in these identified regions; Thirdly, combine and analyze the results under the two labels in the final equilibrium by enumerating all the possibilities and verifying their existence.

We first show the monotonicity of $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ with respect to τ. Under the binary label, $\ddot{\pi}_0$ decreases in τ within $\ddot{\Omega}_{\underline{\phi\phi}}$, increases in τ within $\ddot{\Omega}_{10}$ and $\ddot{\Omega}_{\underline{\phi}0}$, and remains unchanged within $\ddot{\Omega}_{11}$. In the final equilibrium, we note the certifier may prefer the continuous label within $\ddot{\Omega}_{\underline{\phi\phi}}$ and $\ddot{\Omega}_{10}$. Then, we check the monotonicities of $\tilde{\pi}_0$ in these potential continuous label preference regions. The result turns out that $\tilde{\pi}_0$ always increases in τ in these regions. Considering both labels, if the certifier prefers the continuous label in region $\ddot{\Omega}_{10}$, $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ continuous to increase in τ, as both $\tilde{\pi}_0$ and $\ddot{\pi}_0$ increase in τ in this region. In contrast, if the certifier prefers the continuous label in region $\ddot{\Omega}_{\underline{\phi\phi}}$, $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ may decrease, increase or firstly decrease and then decrease in τ, as $\tilde{\pi}_0$ increases in τ, $\ddot{\pi}_0$ decreases in τ and the certifier chooses the label offering higher payoff. Combing these results with the provided Table A.4, we can conclude the final payoff $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ may "↑" or "↓↑" in τ. These outcomes are feasible because when γ is large (i.e., $\gamma > \frac{5\sqrt{73}-31}{16}$), the binary label dominates over the continuous label (i.e, $\max\{\tilde{\pi}_0, \ddot{\pi}_0\} = \ddot{\pi}_0$). In this circumstances, $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$(i.e, $\ddot{\pi}_0$) can either "↑" or "↓↑" in τ.

Next, we show the monotonicity of $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ with respect to κ. Under the binary label, $\ddot{\pi}_0$ always weakly decreases in κ. Under the continuous label, $\tilde{\pi}_0$ can increase in κ in region $\tilde{\Omega}_{1\phi}$. Specifically, within this region, $\tilde{\pi}_0$ increases in κ when $\frac{9}{(-2\gamma+6)(\theta-\Delta c)} \leq \tau < \frac{3}{\theta-\Delta c}$ and $\kappa > \frac{\tau(\gamma+1)(\theta-\Delta c)^2}{12[2\tau(\theta-\Delta c)-3]}$. Referring to the proof of Proposition 3, we find $\frac{9}{(-2\gamma+6)(\theta-\Delta c)} > \tau_2$, where τ_2 is the upper bound τ that the certifier prefers the continuous label. Thus when the certifier finally chooses the continuous label, its payoff cannot increase in κ. Considering both labels, we can conclude the final payoff $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ always decreases in κ, as both $\tilde{\pi}_0$ and $\ddot{\pi}_0$ decrease in κ.

Finally, we show the monotonicity of $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ with respect to γ. Under the binary label, $\ddot{\pi}_0$ increases in γ within $\ddot{\Omega}_{\underline{\phi\phi}}$, decreases in γ within $\ddot{\Omega}_{\underline{\phi}0}$, and remains unchanged within $\ddot{\Omega}_{10}$ and $\ddot{\Omega}_{11}$. In the final equilibrium, the certifier may prefer the continuous label only within $\ddot{\Omega}_{\underline{\phi\phi}}$ and $\ddot{\Omega}_{10}$. Then, we check the monotonicities of $\tilde{\pi}_0$ in these potential continuous label preference regions. The result turns out that $\tilde{\pi}_0$ increases in γ when $\tilde{\Omega}_{1\phi}$ arises in the final equilibrium, and decreases in γ when $\tilde{\Omega}_{\phi\phi}$ arises in the final equilibrium. Apparently, as γ increases, the region may switch from $\tilde{\Omega}_{1\phi}$ to $\tilde{\Omega}_{\phi\phi}$. Thus, in the final equilibrium, $\tilde{\pi}_0$ may increase, decrease or first increase and then decrease in γ. Considering both labels, if the certifer finally prefers the continuous label in region $\ddot{\Omega}_{10}$, the overall trend of $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ follows that of $\tilde{\pi}_0$, as $\ddot{\pi}_0$ is not affected by γ. That is $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ may increase, decrease or first increase and then decrease in γ. If the certifier prefers the continuous label in region $\ddot{\Omega}_{\underline{\phi\phi}}$ and $\ddot{\Omega}_{10} \rightarrow \ddot{\Omega}_{\underline{\phi\phi}}$, $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ may increase, decrease, first increase and then decrease, first decrease and then increase, or first increase then decrease finally increase in γ. Here, $\ddot{\Omega}_{10} \rightarrow \ddot{\Omega}_{\underline{\phi\phi}}$ refers to combination of $\ddot{\Omega}_{10}$ and $\ddot{\Omega}_{\underline{\phi\phi}}$ when the region switches from $\ddot{\Omega}_{10}$ to $\ddot{\Omega}_{\underline{\phi\phi}}$ under the binary label. This is because, in these regions, $\ddot{\pi}_0$ weakly increases in γ, $\tilde{\pi}_0$ is non-monotonic (i.e, "↑", "↓" or "↑↓") in γ and the certifier selects the label that yields higher payoff. Next, we combine these results in the provided Table A.5. We note the certifier would not choose the continuous label in the specific case of $\ddot{\Omega}_{\underline{\phi\phi}}$ that follows $\ddot{\Omega}_{\underline{\phi}0}$. Recall that region $\ddot{\Omega}_{\underline{\phi}0}$ only intersects with region $\tilde{\Omega}_{\phi\phi}$, and the certifier always prefers the binary label in region $\ddot{\Omega}_{\underline{\phi}0}$. This preference implies $\ddot{\pi}_0$ in region $\ddot{\Omega}_{\underline{\phi}0}$ is greater than $\tilde{\pi}_0$ in $\tilde{\Omega}_{\phi\phi}$. As γ increases, the region under the binary label switches from $\ddot{\Omega}_{\underline{\phi}0}$ to $\ddot{\Omega}_{\underline{\phi\phi}}$, and the region under the continuous label remains in $\tilde{\Omega}_{\phi\phi}$. Consequently, $\tilde{\pi}_0$ continuous to decrease in γ in region $\tilde{\Omega}_{\phi\phi}$, while $\ddot{\pi}_0$ begins to increase in γ in region $\ddot{\Omega}_{\underline{\phi\phi}}$. Thus, we can easily infer $\ddot{\pi}_0$ in region $\ddot{\Omega}_{\underline{\phi\phi}}$ that follows $\ddot{\Omega}_{\underline{\phi}0}$ is greater than $\tilde{\pi}_0$ in $\tilde{\Omega}_{\phi\phi}$. Then, based on this label preference analysis and the outcomes detailed in Table A.5, we can conclude the final payoff $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ may "↑", "↓", "↓↑", "↑↓",and "↑↓↑" in γ. The first three cases are feasible because when τ is large (i.e., $\tau \geq \frac{24}{5+\sqrt{73}(\theta-\Delta c)}$), the binary label dominates over the continuous label (i.e, $\max\{\tilde{\pi}_0, \ddot{\pi}_0\} = \ddot{\pi}_0$). In this circumstances, $\ddot{\pi}_0$ can either "↑", "↓" or "↓↑" in γ. Next, we prove patterns "↑↓" and "↑↓↑" are also feasible. When $\gamma = 0$, as shown in Proposition 3, the certifier prefers the continuous label (i.e., $\max\{\tilde{\pi}_0, \ddot{\pi}_0\} = \tilde{\pi}_0$) under the conditions when $\tau < \tau_1$ and $\kappa > \kappa_L$. This preference corresponds to the region $\tilde{\Omega}_{1\phi}$ under the continuous label, which indicates that $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ (i.e., $\tilde{\pi}_0$) would firstly increase in γ in these conditions. Moreover, when $\gamma = 0$, we find $\tau_1 > \frac{3}{(1+\sqrt{2})(\theta-\Delta c)}$ and $\kappa_L < 2\ddot{\kappa}_{\gamma 1}$. Therefore, referring to Table A.5, we can conclude the fifth and the last case in the table satisfy the conditions when $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ would first increase with γ. Combining this result with that of the last case, we can easily establish the pattern "↑↓↑". For the fifth case in the table, after the initial increase, we can infer the certifier's final payoff would experience a subsequent decrease. This is because when γ reaches to 1, the certifier payoff under the continuous label should reduce to be lower than that under the binary label, since the certifier always prefers the binary label when the two firms are symmetric. Hence, this observation explains the "↑↓" pattern. □

Proof of Proposition 6: We define $(\tilde{\pi}_1 + \tilde{\pi}_2) \cdot \mathbb{1}_{\{\tilde{\pi}_0 > \ddot{\pi}_0\}} + (\ddot{\pi}_1 + \ddot{\pi}_2) \cdot \mathbb{1}_{\{\tilde{\pi}_0 \leq \ddot{\pi}_0\}}$ as the industry's final profit. To show the monotonicities of the industry's final profit with respect to τ, κ, and γ, we firstly analyze those monotonicities separately by solving the first-order condition within each region under both the binary and continuous label. Then we integrate the findings to reveal the combined results in the final equilibrium.

We begin by analyzing the monotonicity of the final industry profit with respect to τ. To elaborate on this, we first check the monotonicity of $(\tilde{\pi}_1+\tilde{\pi}_2)\cdot \mathbb{1}_{\{\tilde{\pi}_0>\ddot{\pi}_0\}}+(\ddot{\pi}_1+\ddot{\pi}_2)\cdot \mathbb{1}_{\{\tilde{\pi}_0\leq\ddot{\pi}_0\}}$ when the certifier chooses the continuous label in the final equilibrium (i.e., $\tilde{\pi}_0 > \ddot{\pi}_0$), and then examine this profit expression when the certifier chooses the binary label in the final equilibrium(i.e., $\tilde{\pi}_0 \leq \ddot{\pi}_0$). Our analysis reveals that within each region, both $\tilde{\pi}_1+\tilde{\pi}_2$ and $\ddot{\pi}_1+\ddot{\pi}_2$ decrease in τ, indicating a decrease in the final industry profit concerning τ within each region. However, when $\tilde{\pi}_0 \leq \ddot{\pi}_0$, the final profit can increase in τ when the region switches from $\ddot{\Omega}_{\underline{\phi\phi}}$ to $\ddot{\Omega}_{\underline{\phi}0}$ or $\ddot{\Omega}_{10}$. Moreover, when the certifier's label preference switches from the binary label to continuous label, or from the continuous label to binary label, the final profit may also increase in τ. Specifically, when $\tau = \kappa_L^{-1}(\kappa)$, where $\kappa_L^{-1}(\kappa)$ is the inverse function of $\kappa = \kappa_L(\tau)$, the label preference switches from the binary label to continuous label. We find at this boundary, $\tilde{\pi}_1+\tilde{\pi}_2$ may surpass $\ddot{\pi}_1+\ddot{\pi}_2$. Similarly, when $\tau = \kappa_{H1}^{-1}(\kappa)$, where $\kappa_{H1}^{-1}(\kappa)$ is the inverse function of $\kappa = \kappa_{H1}(\tau)$, the label preference switches from the continuous label to binary label. At this boundary, we also find $\ddot{\pi}_1+\ddot{\pi}_2$ may be greater than $\tilde{\pi}_1+\tilde{\pi}_2$.

We next show the monotonicity of the final industry profit with respect to κ. When $\tilde{\pi}_0 > \ddot{\pi}_0$, it is noticed that $\tilde{\pi}_1+\tilde{\pi}_2$ is non-monotonic in κ. Specifically, within region $\tilde{\Omega}_{1\phi}$, it increases in κ when κ is greater than a threshold which is the third root to the equation $\frac{\partial(\tilde{\pi}_1+\tilde{\pi}_2)}{\partial\kappa} = 0$. Otherwise, it decreases in κ. Additionally, within region $\tilde{\Omega}_{\phi\phi}$, it increases in κ when κ is smaller than a threshold which is the first root to the equation $\frac{\partial(\tilde{\pi}_1+\tilde{\pi}_2)}{\partial\kappa} = 0$. Otherwise, it decreases in κ. When $\tilde{\pi}_0 \leq \ddot{\pi}_0$, $\ddot{\pi}_1+\ddot{\pi}_2$ increases in κ within regions $\ddot{\Omega}_{\underline{\phi\phi}}$ and $\ddot{\Omega}_{\underline{\phi}0}$, decreases in κ within regions $\ddot{\Omega}_{11}$ and $\ddot{\Omega}_{10}$. Therefore, even when the binary label dominates the continuous label in the final equilibrium (e.g, $\gamma > \frac{5\sqrt{73}-31}{16}$), the final industry profit will experience a "↓↑↓↑↓↑"

trend as the region switches from $\ddot{\Omega}_{11} \rightarrow \ddot{\Omega}_{\underline{\phi\phi}} \rightarrow \ddot{\Omega}_{10} \rightarrow \ddot{\Omega}_{\underline{\phi}0} \rightarrow \ddot{\Omega}_{\underline{\phi\phi}}$. Notably, as the region switches from $\ddot{\Omega}_{\underline{\phi}0}$ to $\ddot{\Omega}_{\underline{\phi\phi}}$, there is a sharp decrease in industry's profit due to the intensified competition between the firms.

Finally, we examine the monotonicity of the final industry profit with respect to γ. When $\tilde{\pi}_0 > \ddot{\pi}_0$, it is noticed that $\tilde{\pi}_1 + \tilde{\pi}_2$ is non-monotonic in γ. Specifically, within region $\tilde{\Omega}_{1\phi}$, it decreases in γ. Additionally, within region $\tilde{\Omega}_{\phi\phi}$, it increases in γ when $\kappa > \frac{2\tau(\theta-\Delta c)}{9}$ and $\gamma > \frac{9\kappa[3\tau(\theta-\Delta c)^2+\sqrt{8\tau^2(\theta-\Delta c)^2+162\kappa^2-108\kappa\tau(\theta-\Delta c)^2}]-81\kappa^2-2\tau^2(\theta-\Delta c)^2}{[2\tau^2(\theta-\Delta c)^2-27\kappa\tau](\theta-\Delta c)^2}$. Otherwise, it decreases in γ. When $\tilde{\pi}_0 \leq \ddot{\pi}_0$, $\ddot{\pi}_1 + \ddot{\pi}_2$ increases in γ within region $\ddot{\Omega}_{\underline{\phi}0}$, decreases in γ within regions $\ddot{\Omega}_{\underline{\phi\phi}}$ and $\ddot{\Omega}_{10}$, and remains unchanged within region $\ddot{\Omega}_{11}$. Thus, it is obvious when combining the results of both labels in the final equilibrium, the industry profit exhibits a non-monotonic trend in γ. Specifically, as show in Figure 4.8, it may initially undergo a "↓↑" pattern when the certitifer prefers the continuous label, and then transition to a"↓↑↓" pattern when the certifer switches to prefer the binary label. □

Appendix B

Alternative Refinement Criterion

In the main text, when there are multiple recycled content equilibria, we follow the refinement policy that the more efficient firm adopts a higher percentage of recycled content. In this section, we employ an alternative policy by allocating a higher percentage of recycled content to the less efficient firm. The subsequent results validate the robustness of our earlier analytical findings.

Next, under this new refinement policy, we first analyze the case of continuous label in Section B.1, and then study the binary label in Section B.2. The comparison regarding the certifier's payoff and the industry's profit under the two labels are given in Section B.3.

B.1 Continuous Label

Here, we extend Lemma 2 to scenarios where the less efficient firm 2 invests more in recycled-content usage under the continuous label.

Lemma 5. *Under the continuous label, two cases emerge regarding firms' equilibrium recycled-content decisions:*

Case a:

$$(\tilde{\phi}_1, \tilde{\phi}_2) = \begin{cases} (1,1) & \textit{if } (\tau,\kappa,\gamma) \in \tilde{\Omega}^{ra}_{11} \\ \left(1, \frac{(\gamma+1)(\theta-\Delta c)(3-\tau\theta+\tau\Delta c)}{36\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}\right) & \textit{if } (\tau,\kappa,\gamma) \in \tilde{\Omega}^{ra}_{1\phi} \\ \left(\frac{(\gamma+1)(\theta-\Delta c)(3-\tau\theta+\tau\Delta c)}{36\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}, 1\right) & \textit{if } (\tau,\kappa,\gamma) \in \tilde{\Omega}^{ra}_{\phi 1} \\ \left(\frac{(\gamma+1)(\theta-\Delta c)[18\kappa-\tau(\gamma+1)(\theta-\Delta c)^2]}{6\kappa[36\gamma\kappa-\tau(\gamma+1)^2(\theta-\Delta c)^2]}, \frac{(\gamma+1)(\theta-\Delta c)[18\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2]}{6\kappa[36\gamma\kappa-\tau(\gamma+1)^2(\theta-\Delta c)^2]}\right) & \textit{if } (\tau,\kappa,\gamma) \in \tilde{\Omega}^{ra}_{\phi\phi} \end{cases}, \tag{B.1}$$

Case b:

$$(\tilde{\phi}_1, \tilde{\phi}_2) = \begin{cases} (1,1) & \textit{if } (\tau,\kappa,\gamma) \in \tilde{\Omega}^{rb}_{11} \\ \left(1, \frac{(\gamma+1)(\theta-\Delta c)(3-\tau\theta+\tau\Delta c)}{36\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}\right) & \textit{if } (\tau,\kappa,\gamma) \in \tilde{\Omega}^{rb}_{1\phi} \\ \left(\frac{(\gamma+1)(\theta-\Delta c)[18\kappa-\tau(\gamma+1)(\theta-\Delta c)^2]}{6\kappa[36\gamma\kappa-\tau(\gamma+1)^2(\theta-\Delta c)^2]}, \frac{(\gamma+1)(\theta-\Delta c)[18\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2]}{6\kappa[36\gamma\kappa-\tau(\gamma+1)^2(\theta-\Delta c)^2]}\right) & \textit{if } (\tau,\kappa,\gamma) \in \tilde{\Omega}^{rb}_{\phi\phi} \end{cases}, \tag{B.2}$$

where parameter regions $\tilde{\Omega}^{ra}_{11}$, $\tilde{\Omega}^{ra}_{1\phi}$, $\tilde{\Omega}^{ra}_{\phi 1}$, $\tilde{\Omega}^{ra}_{\phi\phi}$ and $\tilde{\Omega}^{rb}_{11}$, $\tilde{\Omega}^{rb}_{1\phi}$, $\tilde{\Omega}^{rb}_{\phi\phi}$ are characterized in the following proof.

Proof: By the proof of Lemma 2, we establish that multiple recycled content equilibria exist exclusively in set $\tilde{\Omega}_6$. Therefore, within the other remaining sets, the results remain unaffected. Specifically, in set $\tilde{\Omega}_6$, there are multiple equilibria: $(1, \tilde{\phi}_{1\phi})$, $(\tilde{\phi}_{\phi 1}, 1)$ and $(\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$, where $\tilde{\phi}_{1\phi} < 1$, $\tilde{\phi}_{\phi 1} < 1$ and $\tilde{\phi}_{\phi\phi 1} \leq \tilde{\phi}_{\phi\phi 2}$. Thus, under the new refinement policy, the recycled content equilibrium within this set can be either $(\tilde{\phi}_{\phi 1}, 1)$ or $(\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$. Our following analysis will cover both cases. Specifically, we refer to **Case a** when considering $(\tilde{\phi}_{\phi 1}, 1)$ as the equilibrium, and **Case b** when considering $(\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$ as the equilibrium.

Case a: There are four possible equilibrium candidates, namely $(1,1)$, $(1, \tilde{\phi}_{1\phi})$, $(\tilde{\phi}_{\phi 1}, 1)$, $(\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$, and we combine the condition sets for each equilibrium in $\tilde{\Omega}^{ra}_{11}$, $\tilde{\Omega}^{ra}_{1\phi}$, $\tilde{\Omega}^{ra}_{\phi 1}$ and $\tilde{\Omega}^{ra}_{\phi\phi}$, respectively, where $\tilde{\Omega}^{ra}_{11} \doteq \tilde{\Omega}_1 \doteq \{(\tau,\kappa,\gamma) : 0 \leq \kappa \leq \frac{(\gamma+1)\tilde{k}_L}{2}\}$, $\tilde{\Omega}^{ra}_{1\phi} \doteq \tilde{\Omega}_2 \cup \tilde{\Omega}_3 \cup \tilde{\Omega}_4 \cup \tilde{\Omega}_7 \doteq \{(\tau,\kappa,\gamma) : (\gamma = 0, \kappa > \frac{(\gamma+1)\tilde{k}_L}{2}) || (0 < \gamma \leq \frac{1}{2}, \frac{(\gamma+1)\tilde{k}_L}{2} < \kappa \leq \tilde{\kappa}_3) || (\frac{1}{2} < \gamma \leq 1, (\tilde{\tau}_1 \leq \tau < \frac{3}{\theta-\Delta c}, (\frac{(\gamma+1)\tilde{k}_L}{2} < \kappa < \tilde{\kappa}_1) || (\tilde{\kappa}_2 < \kappa \leq \tilde{\kappa}_3)) || (\tau < \tilde{\tau}_1, \frac{(\gamma+1)\tilde{k}_L}{2} < \kappa \leq \tilde{\kappa}_3)))\}$, $\tilde{\Omega}^{ra}_{\phi 1} \doteq \tilde{\Omega}_6 \doteq \{(\tau,\kappa,\gamma) : \frac{1}{2} < \gamma \leq 1, \tilde{\tau}_1 \leq \tau < \frac{3}{\theta-\Delta c}, \tilde{\kappa}_1 \leq \kappa \leq \tilde{\kappa}_2\}$, and $\tilde{\Omega}^{ra}_{\phi\phi} \doteq \tilde{\Omega}_5 \cup \tilde{\Omega}_8 \cup \tilde{\Omega}_9 \doteq \{(\tau,\kappa,\gamma) : 0 < \gamma \leq 1, \kappa > \tilde{\kappa}_3\}$.

Case b: There are three possible equilibrium candidates, namely namely $(1,1)$, $(1,\tilde{\phi}_{1\phi})$, $(\tilde{\phi}_{\phi\phi 1},\tilde{\phi}_{\phi\phi 2})$, and we combine the condition sets for each equilibrium in $\tilde{\Omega}^{rb}_{11}$, $\tilde{\Omega}^{rb}_{1\phi}$, $\tilde{\Omega}^{rb}_{\phi\phi}$, respectively, where $\tilde{\Omega}^{rb}_{11} \doteq \tilde{\Omega}_1 \doteq \{(\tau,\kappa,\gamma) : 0 \leq \kappa \leq \frac{(\gamma+1)\tilde{k}_L}{2}\}$, $\tilde{\Omega}^{rb}_{1\phi} \doteq \tilde{\Omega}_2 \cup \tilde{\Omega}_3 \cup \tilde{\Omega}_4 \cup \tilde{\Omega}_7 \doteq \{(\tau,\kappa,\gamma) : (\gamma = 0, \kappa > \frac{(\gamma+1)\tilde{k}_L}{2})||(0 < \gamma \leq \frac{1}{2}, \frac{(\gamma+1)\tilde{k}_L}{2} < \kappa \leq \tilde{\kappa}_3)||(\frac{1}{2} < \gamma \leq 1, (\tilde{\tau}_1 \leq \tau < \frac{3}{\theta-\Delta c}, (\frac{(\gamma+1)\tilde{k}_L}{2} < \kappa < \tilde{\kappa}_1)||(\tilde{\kappa}_1 < \kappa \leq \tilde{\kappa}_3))||(\tau < \tilde{\tau}_1, \frac{(\gamma+1)\tilde{k}_L}{2} < \kappa \leq \tilde{\kappa}_3)))\}$, and $\tilde{\Omega}^{rb}_{\phi\phi} \doteq \tilde{\Omega}_5 \cup \tilde{\Omega}_6 \cup \tilde{\Omega}_8 \cup \tilde{\Omega}_9 \doteq \{(\tau,\kappa,\gamma) : (0 < \gamma \leq 1, \kappa > \tilde{\kappa}_3)||(\frac{1}{2} < \gamma \leq 1, \tilde{\tau}_1 \leq \tau < \frac{3}{\theta-\Delta c}, \tilde{\kappa}_1 \leq \kappa \leq \tilde{\kappa}_2)\}$. □

Proposition 7. *Under the continuous label, environmental performance $\tilde{\pi}_0$ is non-monotonic in τ, κ, and γ. In particular, more intense competition or a lower average investment coefficient may lead to worse environmental performance.*

Proof: By the proof of Proposition 1, $\tilde{\pi}_0$ can be non-monotonic in the three key parameters (i.e., τ, κ, and γ) within region $\tilde{\Omega}_{1\phi}$. Under the new refinement policy, we recheck the monotonicities of $\tilde{\pi}_0$ within regions $\tilde{\Omega}^{ra}_{1\phi}$ and $\tilde{\Omega}^{rb}_{1\phi}$. The results confirm that $\tilde{\pi}_0$ continuous to be non-monotonic in these parameters, with the threshold remaining consistent with that established in Proposition 1. □

B.2 Binary Label

Lemma 6. *Under the binary label, the certifier's equilibrium minimum-standard decision and the firms' equilibrium recycled-content decisions are:*

$$(\ddot{\underline{\phi}}, \ddot{\phi}_1, \ddot{\phi}_2) = \begin{cases} \big(1,1,1\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}^r_{11} \\ \big(\frac{6(\gamma+1)(\theta-\Delta c)}{36\kappa+\tau(\gamma+1)(\theta-\Delta c)^2}, \frac{6(\gamma+1)(\theta-\Delta c)}{36\kappa+\tau(\gamma+1)(\theta-\Delta c)^2}, \frac{6(\gamma+1)(\theta-\Delta c)}{36\kappa+\tau(\gamma+1)(\theta-\Delta c)^2}\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}^r_{\underline{\phi\phi}} \\ \big(1,1,0\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}^r_{10} \\ \big(1,0,1\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}^r_{10} \\ \big(\frac{6(\gamma+1)(\theta-\Delta c)}{36\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}, \frac{6(\gamma+1)(\theta-\Delta c)}{36\gamma\kappa-\tau(\gamma+1)(\theta-\Delta c)^2}, 0\big) & \text{if } (\tau,\kappa,\gamma) \in \ddot{\Omega}^r_{\underline{\phi}0} \end{cases},$$

where parameter regions $\ddot{\Omega}^r_{11}$, $\ddot{\Omega}^r_{\underline{\phi\phi}}$, $\ddot{\Omega}^r_{10}$, $\ddot{\Omega}^r_{01}$, and $\ddot{\Omega}^r_{\underline{\phi}0}$ are characterized in the following proof.

Proof: By the proof of Lemma 4, given the binary label standard $\underline{\phi}$, we find there are two recycled content equilibria, namely $(\underline{\phi}, 0)$ and $(0, \underline{\phi})$ in set $\ddot{Q}_{0\underline{\phi}}$. Thus, under the new refinement policy, the unique recycled content equilibrium in set $\ddot{Q}_{0\underline{\phi}}$ becomes $(0, \underline{\phi})$. Then we combine the conditions sets for each equilibrium (i.e., $(0,0)$, $(\underline{\phi}, 0)$, $(0, \underline{\phi})$, and $(\underline{\phi}, \underline{\phi})$) in $\ddot{Q}^r_{00}$, $\ddot{Q}^r_{\underline{\phi}0}$, $\ddot{Q}^r_{0\underline{\phi}}$, and $\ddot{Q}^r_{\underline{\phi}\underline{\phi}}$. Specifically, $\ddot{Q}^r_{00} = \ddot{Q}_{00}$, $\ddot{Q}^r_{\underline{\phi}0} = \ddot{Q}_{\underline{\phi}0} - \ddot{Q}_{0\underline{\phi}}$, $\ddot{Q}^r_{0\underline{\phi}} = \ddot{Q}_{0\underline{\phi}}$, and $\ddot{Q}^r_{\underline{\phi}\underline{\phi}} = \ddot{Q}_{\underline{\phi}\underline{\phi}}$.

Next, the certifier chooses the binary label standard $\underline{\phi}$ to maximize its payoff $\ddot{\pi}_0(\underline{\phi})$. For a clear discussion, we discuss the certifier's decision by cases in Table B.1, where (τ, κ, γ) space is divided into seven components. Note in Table B.1, $\underline{\phi}_{L2}$ intersects $\underline{\phi} = 1$ at $\ddot{\kappa}_{L2}$, $\underline{\phi}_{H2}$ intersects $\underline{\phi} = 1$ at $\ddot{\kappa}_{H2}$, and $\underline{\phi}_{L2}$ intersects $\underline{\phi}_{H2}$ at $\ddot{\kappa}_{I2}$, where $\ddot{\kappa}_{L2} = \frac{(\gamma+1)[6(\theta-\Delta c)-\tau(\theta-\Delta c)^2]}{36\gamma}$, $\ddot{\kappa}_{H2} = \frac{(\gamma+1)[6(\theta-\Delta c)+\tau(\theta-\Delta c)^2]}{36}$, and $\ddot{\kappa}_{I2} = \frac{(\gamma+1)\tau(\theta-\Delta c)^2}{18(1-\gamma)}$.

Recalling the impact of the new refinement policy, the only change occurs in region $\ddot{Q}^r_{0\underline{\phi}}$, where firms' recycled content equilibrium shifts from $(\underline{\phi}, 0)$ to $(0, \underline{\phi})$. Due to the equivalence in the certifier's payoff between $(\underline{\phi}, 0)$ and $(0, \underline{\phi})$, we can easily find the certifier's binary label standard decision, as shown in Table B.1, remains unchanged from Table A.3. Nevertheless, given the label standard $\underline{\phi}$, the recycled content equilibrium for cases 3, 4, and 6 in Table B.1 may undergo adjustments, considering the emergence of $\ddot{Q}^r_{0\underline{\phi}}$ in these cases. A noteworthy finding is that $(0, \underline{\phi})$ cannot materialize in cases 4 and 6. This is because under these two cases, the certifier, instead of allowing firm 2 to obtain the label (i.e., $(0, \underline{\phi})$), at least benefits from setting a higher label standard to only encourage firm 1 to obtain the label (i.e., $(\underline{\phi}, 0)$). In contrast, in case 3, the certifier may benefit from only inducing recycled content usage from firm 2. By further comparing the certifier's payoff in case 3, we find the certifier would set $\ddot{\underline{\phi}} = 1$ when $\ddot{\Omega}^r_{01} \doteq \{(\tau, \kappa, \gamma) : \gamma > \frac{1}{3}, \max\{\ddot{\kappa}_1, \ddot{\kappa}_{L2}\} < \kappa < \ddot{\kappa}_{H2}\}$, resulting in the firms' recycled content equilibrium being $(0, 1)$ within this set. Specifically, $\ddot{\Omega}^r_{01} \in \ddot{\Omega}_2$. Thus, there are five possible recycled content equilibria for firms, namely $(1,1)$, $(\underline{\phi}_{L1}, \underline{\phi}_{L1})$, $(1,0)$, $(0,1)$ and $(\underline{\phi}_H, 0)$. Then we combine the conditions for each equilibrium in set $\ddot{\Omega}^r_{11}$, $\ddot{\Omega}^r_{\underline{\phi}\underline{\phi}}$, $\ddot{\Omega}^r_{10}$, $\ddot{\Omega}^r_{01}$ and $\ddot{\Omega}^r_{\underline{\phi}0}$, respectively, where $\ddot{\Omega}^r_{11} \doteq \{(\tau, \kappa, \gamma) : 0 \leq \kappa \leq \ddot{\kappa}_{L1}\}$, $\ddot{\Omega}^r_{\underline{\phi}\underline{\phi}} \doteq \ddot{\Omega}_1 \cup \ddot{\Omega}_3 \doteq \{(\tau, \kappa, \gamma) : (0 \leq \gamma \leq \frac{1}{2}, \ddot{\kappa}_{L1} < \kappa < \ddot{\kappa}_1) || (\frac{1}{2} < \gamma \leq 1, (\ddot{\tau}_1 \leq \tau < \frac{3}{\theta-\Delta c}, (\ddot{\kappa}_{L1} < \kappa < \ddot{\kappa}_1) || (\kappa > \ddot{\kappa}_2)) || (\tau < \ddot{\tau}_1, \kappa > \ddot{\kappa}_{L1}))\}$,

Table B.1: Alternative Results of the Certifier's Minimum-standard Decision Under Binary Label

Cases	Subcases	$(\ddot{\phi}_1, \ddot{\phi}_2 \vert \underline{\phi})$	$\underline{\phi}$	$\ddot{\pi}_0(\underline{\phi})$	$\ddot{\underline{\phi}}$
$0 \le \kappa \le \ddot{\kappa}_{L1}$	$\ddot{Q}^r_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, 1]$	β_r	1
$\ddot{\kappa}_{L1} < \kappa \le \ddot{\kappa}_{L2}$	$\ddot{Q}^r_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, \underline{\phi}_{L1}]$	$\beta_v + (\beta_r - \beta_v)\underline{\phi}_{L1}$	$\underline{\phi}_{L1}$ if $(\tau, \kappa, \gamma) \in \ddot{\Omega}_1$; 1 if $(\tau, \kappa, \gamma) \in \ddot{\Omega}_2$
	$\ddot{Q}^r_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{L1}, 1]$	$\beta_v + (\beta_r - \beta_v)\frac{3+\tau(\theta-\Delta c)}{6}$	
$\ddot{\kappa}_{L2} < \kappa \le \ddot{\kappa}_{H2}$	$\ddot{Q}^r_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, \underline{\phi}_{L1}]$	$\beta_v + (\beta_r - \beta_v)\underline{\phi}_{L1}$	
	$\ddot{Q}^r_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{L1}, \underline{\phi}_{L2}]$	$\beta_v + (\beta_r - \beta_v)\frac{3\underline{\phi}_{L2}+\tau(\theta-\Delta c)\underline{\phi}^2_{L2}}{6}$	
	$\ddot{Q}^r_{0\underline{\phi}}$	$(0, \underline{\phi})$	$(\underline{\phi}_{L2}, 1]$	$\beta_v + (\beta_r - \beta_v)\frac{3+\tau(\theta-\Delta c)}{6}$	
$\ddot{\kappa}_{H2} < \kappa \le \min\{\ddot{\kappa}_{I2}, \ddot{\kappa}_{H1}\}$	$\ddot{Q}^r_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, \underline{\phi}_{L1}]$	$\beta_v + (\beta_r - \beta_v)\underline{\phi}_{L1}$	
	$\ddot{Q}^r_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{L1}, \underline{\phi}_{L2}]$	$\beta_v + (\beta_r - \beta_v)\frac{3\underline{\phi}_{L2}+\tau(\theta-\Delta c)\underline{\phi}^2_{L2}}{6}$	
	$\ddot{Q}^r_{0\underline{\phi}}$	$(0, \underline{\phi})$	$(\underline{\phi}_{L2}, \underline{\phi}_{H2}]$	$\beta_v + (\beta_r - \beta_v)\frac{3\underline{\phi}_{H2}+\tau(\theta-\Delta c)\underline{\phi}^2_{H2}}{6}$	
	$\ddot{Q}^r_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{H2}, 1]$	$\beta_v + (\beta_r - \beta_v)\frac{3+\tau(\theta-\Delta c)}{6}$	
$\ddot{\kappa}_{I2} < \kappa \le \ddot{\kappa}_{H1}$	$\ddot{Q}^r_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, \underline{\phi}_{L1}]$	$\beta_v + (\beta_r - \beta_v)\underline{\phi}_{L1}$	
	$\ddot{Q}^r_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{L1}, 1]$	$\beta_v + (\beta_r - \beta_v)\frac{3+\tau(\theta-\Delta c)}{6}$	
$\ddot{\kappa}_{H1} < \kappa < \ddot{\kappa}_{I2}$	$\ddot{Q}^r_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, \underline{\phi}_{L1}]$	$\beta_v + (\beta_r - \beta_v)\underline{\phi}_{L1}$	$\underline{\phi}_{L1}$ if $(\tau, \kappa, \gamma) \in \ddot{\Omega}_3$; $\underline{\phi}_{H1}$ if $(\tau, \kappa, \gamma) \in \ddot{\Omega}_4$
	$\ddot{Q}^r_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{L1}, \underline{\phi}_{L2}]$	$\beta_v + (\beta_r - \beta_v)\frac{3\underline{\phi}_{L2}+\tau(\theta-\Delta c)\underline{\phi}^2_{L2}}{6}$	
	$\ddot{Q}^r_{0\underline{\phi}}$	$(0, \underline{\phi})$	$(\underline{\phi}_{L2}, \underline{\phi}_{H2}]$	$\beta_v + (\beta_r - \beta_v)\frac{3\underline{\phi}_{H2}+\tau(\theta-\Delta c)\underline{\phi}^2_{H2}}{6}$	
	$\ddot{Q}^r_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{H2}, \underline{\phi}_{H1}]$	$\beta_v + (\beta_r - \beta_v)\frac{3\underline{\phi}_{H1}+\tau(\theta-\Delta c)\underline{\phi}^2_{H1}}{6}$	
	$\ddot{Q}^r_{00}$	$(0, 0)$	$(\underline{\phi}_{H1}, 1]$	β_v	
$\kappa > \max\{\ddot{\kappa}_{I2}, \ddot{\kappa}_{H1}\}$	$\ddot{Q}^r_{\underline{\phi}\underline{\phi}}$	$(\underline{\phi}, \underline{\phi})$	$[0, \underline{\phi}_{L1}]$	$\beta_v + (\beta_r - \beta_v)\underline{\phi}_{L1}$	
	$\ddot{Q}^r_{\underline{\phi}0}$	$(\underline{\phi}, 0)$	$(\underline{\phi}_{L1}, \underline{\phi}_{H1}]$	$\beta_v + (\beta_r - \beta_v)\frac{3\underline{\phi}_{H1}+\tau(\theta-\Delta c)\underline{\phi}^2_{H1}}{6}$	
	$\ddot{Q}^r_{00}$	$(0, 0)$	$(\underline{\phi}_{H1}, 1]$	β_v	

$\ddot{\Omega}^r_{10} \doteq \ddot{\Omega}_2 - \ddot{\Omega}^r_{01} \doteq \{(\tau, \kappa, \gamma) : (\gamma = 0, \kappa \ge \ddot{\kappa}_1) || (0 < \gamma \le \frac{1}{3}, \ddot{\kappa}_1 \le \kappa \le \ddot{\kappa}_{H1}) || (\frac{1}{3} < \gamma \le \frac{1}{2}, ((\ddot{\kappa}_1 \le \kappa \le \max\{\ddot{\kappa}_1, \ddot{\kappa}_{L2}\}) || (\ddot{\kappa}_{H2} \le \kappa \le \ddot{\kappa}_{H1}))) || (\frac{1}{2} < \gamma \le 1, \ddot{\tau}_1 \le \tau < \frac{3}{\theta - \Delta c}, ((\ddot{\kappa}_1 \le \kappa \le \max\{\ddot{\kappa}_1, \ddot{\kappa}_{L2}\}) || (\ddot{\kappa}_{H2} \le \kappa \le \ddot{\kappa}_{H1})))\}$, $\ddot{\Omega}^r_{01} \doteq \{(\tau, \kappa, \gamma) : \gamma > \frac{1}{3}, \max\{\ddot{\kappa}_1, \ddot{\kappa}_{L2}\} < \kappa < \ddot{\kappa}_{H2}\}$, and $\ddot{\Omega}^r_{\underline{\phi}0} \doteq \ddot{\Omega}_4 \doteq \{(\tau, \kappa, \gamma) : (0 < \gamma \le \frac{1}{2}, \kappa > \ddot{\kappa}_{H1}) || (\frac{1}{2} < \gamma \le 1, \ddot{\tau}_1 \le \tau < \frac{3}{\theta - \Delta c}, \ddot{\kappa}_{H1} < \kappa \le \ddot{\kappa}_2)\}$. □

Proposition 8. *Under the binary label, environmental performance $\ddot{\pi}_0$: (i)↑, ↓, or ↓↑ in τ; (ii) ↓ in κ; (iii) ↑, ↓, or ↓↑. In particular, more intense competition* *may lead to worse environmental performance, whereas a lower average investment coefficient always leads to better environmental performance.*

Proof: By the proof of Lemma 6, we observe that the region $\ddot{\Omega}^r_{01}$ under the new refinement criterion is a sub-region of $\ddot{\Omega}_{10}$ under the previous refinement criterion. As the certifier's payoff remains equivalent in both cases, we can conclude that the certifier is indifferent to the change in firms' recycled content equilibrium under the binary label with the alternative refinement criterion, indicating that the monotonicity of the environmental performance is also consistent. □

B.3 Label Selection

Proposition 9. *The certifier's label selection remains consistent: It chooses the continuous label if $(\tau, \kappa, \gamma) \in \mathcal{C}$, chooses the binary label if $(\tau, \kappa, \gamma) \in \mathcal{B}$, and is indifferent if $(\tau, \kappa, \gamma) \in \mathcal{I}$.*

Proof: Recalling the impact of the new refinement criterion, the change under the continuous label arises in region $\tilde{\Omega}_6$, where firms' recycled content equilibrium shifts from $(1, \tilde{\phi}_{1\phi})$ to either $(\tilde{\phi}_{\phi 1}, 1)$ (i.e., **Case a**) or $(\tilde{\phi}_{\phi\phi 1}, \tilde{\phi}_{\phi\phi 2})$ (i.e., **Case b**). Meanwhile, under the binary label, the change arises in region $\ddot{\Omega}^r_{01}$, where firms' recycled content equilibrium shifts from $(1, 0)$ to $(0, 1)$. Thus, alongside the non-empty intersections outlined in Proposition 3, we identify additional intersections: $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{01}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{\phi\phi}$/ $\ddot{\Omega}^r_{01} \cap (\tilde{\Omega}^{rb}_{\phi\phi} - \tilde{\Omega}_6)$, and $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{1\phi}$/ $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{rb}_{1\phi}$. Note that region $\tilde{\Omega}^{ra}_{\phi\phi}$ in **Case a** is equivalent to the region $\tilde{\Omega}^{rb}_{\phi\phi} - \tilde{\Omega}_6$ in **Case b**. Similarly, region $\tilde{\Omega}^{ra}_{1\phi}$ in **Case a** is equivalent to the region $\tilde{\Omega}^{rb}_{1\phi}$ in **Case b**. Then, to validate the robustness, our focus lies on examining the certifier's label selection in these new intersections, as the results drawn from the earlier intersections remain unchanged.

Next, we compare the certifier's payoff under the continuous and binary label (i.e., $\tilde{\pi}_0$ and $\ddot{\pi}_0$) in these intersections. Note that the certifier is indifferent to the change in firms' recycled content under the binary label, as its payoff is consistent in regions

$\ddot{\Omega}^r_{10}$ and $\ddot{\Omega}^r_{01}$. As a result, our focus is directed towards the new intersections arising from the continuous label: $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{01}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$ and $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$. The payoff comparison in these intersections is given in two cases, depending on the firms' recycled content equilibrium under the continuous label.

Case a: (1) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\tilde{\pi}_0 < \ddot{\pi}_0$. (2) $\tilde{\Omega}_6 \cap (\ddot{\Omega}^r_{10} \cap \ddot{\Omega}^r_{01})$, $\tilde{\pi}_0 < \ddot{\pi}_0$. (3) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$, $\tilde{\pi}_0 < \ddot{\pi}_0$.

Case b: (1) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\tilde{\pi}_0 < \ddot{\pi}_0$. (2) $\tilde{\Omega}_6 \cap (\ddot{\Omega}^r_{10} \cap \ddot{\Omega}^r_{01})$, $\tilde{\pi}_0 < \ddot{\pi}_0$. (3) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$, $\tilde{\pi}_0 < \ddot{\pi}_0$.

Therefore, regardless of firms' recycled content equilibrium in set $\tilde{\Omega}_6$ under the continuous label, the certifier always prefers the binary label in region $\tilde{\Omega}_6$. Note when $\gamma < \frac{5\sqrt{73}-31}{16}$, $\tau_1 < \tilde{\tau}_1$, which means $\tilde{\Omega}_6$ falls within region $\mathcal{B}$—the region where the certifier prefers the binary label in the prior refinement policy. This observation establishes a robustness pattern in the certifier's label preference across distinct refinement criteria. □

Proposition 10. *The certifier's label selection never aligns with both firms' preferences, but may align with the industry's (two firms' total) preference.*

Proof: Then we compare the firms' and the industry's profits under the continuous and the binary label in the new intersections. The profits comparison is given in two cases, depending on the firms' recycled content equilibrium under the continuous label.

Case a: (1) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\tilde{\pi}_i > \ddot{\pi}_i$ holds for $i \in \{1, 2, I\}$. (2) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\pi}_1 < \ddot{\pi}_1$, $\tilde{\pi}_2 > \ddot{\pi}_2$, and $\tilde{\pi}_I < \ddot{\pi}_I$. (3) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{01}$, $\tilde{\pi}_1 > \ddot{\pi}_1$, $\tilde{\pi}_2 < \ddot{\pi}_2$ and $\tilde{\pi}_I < \ddot{\pi}_I$. (4) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$, for firm 1, a unique solution exists for $\tilde{\pi}_1 = \ddot{\pi}_1$, which is the first root to this equation and we denoted it as $\breve{\kappa}^{ra}_f$. Moreover, in this subcase, $\tilde{\pi}_1 > \ddot{\pi}_1$ if $\kappa > \breve{\kappa}^{ra}_f$ and $\tilde{\pi}_1 < \ddot{\pi}_1$ if $\kappa < \breve{\kappa}^{ra}_f$. For firm 2, $\tilde{\pi}_2 > \ddot{\pi}_2$. For the industry, $\tilde{\pi}_I > \ddot{\pi}_I$. (5) $\tilde{\Omega}^{ra}_{1\phi} \cap \ddot{\Omega}^r_{01}$, $\tilde{\pi}_1 > \ddot{\pi}_1$, $\tilde{\pi}_2 < \ddot{\pi}_2$. For the industry, a unique solution exists for the equation $\tilde{\pi}_I = \ddot{\pi}_I$, which is the third root to this equation and we denote it as $\mathring{\kappa}^{ra}_{I1}$. In this subcase, when $\kappa > \mathring{\kappa}^{ra}_{I1}$, $\tilde{\pi}_I > \ddot{\pi}_I$, otherwise, $\tilde{\pi}_I < \ddot{\pi}_I$. (6) $\tilde{\Omega}^{ra}_{\phi\phi} \cap \ddot{\Omega}^r_{01}$, $\tilde{\pi}_1 > \ddot{\pi}_1$, $\tilde{\pi}_2 < \ddot{\pi}_2$. For the industry, two solutions, denoted as $\mathring{\kappa}^{ra}_{I2}$ and $\mathring{\kappa}^{ra}_{I3}$, exist for the the equation $\tilde{\pi}_I = \ddot{\pi}_I$. These solutions correspond to the second and third roots of this equation, respectively. More specifically, when $\kappa < \mathring{\kappa}^{ra}_{I2}$ or $\kappa > \mathring{\kappa}^{ra}_{I3}$, $\tilde{\pi}_I > \ddot{\pi}_I$, otherwise, $\tilde{\pi}_I < \ddot{\pi}_I$.

Case b: (1) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\tilde{\pi}_i > \ddot{\pi}_i$ holds for $i \in \{1, 2, I\}$. (2) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\pi}_1 < \ddot{\pi}_1$, $\tilde{\pi}_2 > \ddot{\pi}_2$, and and $\tilde{\pi}_I < \ddot{\pi}_I$. (3) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{01}$, $\tilde{\pi}_1 > \ddot{\pi}_1$, $\tilde{\pi}_2 < \ddot{\pi}_2$, and $\tilde{\pi}_I < \ddot{\pi}_I$. (4) $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$, for firm 1, two solutions exist for $\tilde{\pi}_1 = \ddot{\pi}_1$, which is the second and the third root to this equation and we denoted them as $\breve{\kappa}^{rb}_{f1}$ and $\breve{\kappa}^{rb}_{f2}$, respectively. Moreover, in this subcase, $\tilde{\pi}_1 > \ddot{\pi}_1$ if $\kappa < \breve{\kappa}^{rb}_{f1}$ or $\kappa > \breve{\kappa}^{rb}_{f2}$, and $\tilde{\pi}_1 < \ddot{\pi}_1$ if $\breve{\kappa}^{rb}_{f1} < \kappa < \breve{\kappa}^{rb}_{f2}$. For firm 2, $\tilde{\pi}_2 > \ddot{\pi}_2$. For the industry, $\tilde{\pi}_I > \ddot{\pi}_I$. (5) $\tilde{\Omega}^{rb}_{1\phi} \cap \ddot{\Omega}^r_{01}$, $\tilde{\pi}_1 > \ddot{\pi}_1$, $\tilde{\pi}_2 < \ddot{\pi}_2$. For the industry, a unique solution exists for the equation $\tilde{\pi}_I = \ddot{\pi}_I$, which is the third root to this equation and we denote it as $\mathring{\kappa}^{ra}_{I1}$. In this subcase, when $\kappa > \mathring{\kappa}^{ra}_{I1}$, $\tilde{\pi}_I > \ddot{\pi}_I$, otherwise, $\tilde{\pi}_I < \ddot{\pi}_I$. (6) $(\tilde{\Omega}^{rb}_{\phi\phi} - \tilde{\Omega}_6) \cap \ddot{\Omega}^r_{01}$, $\tilde{\pi}_1 > \ddot{\pi}_1$, $\tilde{\pi}_2 < \ddot{\pi}_2$. For the industry, two solutions, denoted as $\mathring{\kappa}^{ra}_{I2}$ and $\mathring{\kappa}^{ra}_{I3}$, exist for the the equation $\tilde{\pi}_I = \ddot{\pi}_I$. These solutions correspond to the second and third roots of this equation, respectively. More specifically, when $\kappa < \mathring{\kappa}^{ra}_{I2}$ or $\kappa > \mathring{\kappa}^{ra}_{I3}$, $\tilde{\pi}_I > \ddot{\pi}_I$, otherwise, $\tilde{\pi}_I < \ddot{\pi}_I$.

Then we summarize the comparison results in these new intersections for each firm. In **Case a**, firm 1 prefers the continuous label in $\ddot{\Omega}^r_{01}$ (i.e., $\ddot{\Omega}^r_{01} \cap (\tilde{\Omega}_6 \cup \tilde{\Omega}^{ra}_{1\phi} \cup \tilde{\Omega}^{ra}_{\phi\phi})$), $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap (\kappa > \breve{\kappa}^{ra}_f)$, while prefers the binary label in $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap (\kappa < \breve{\kappa}^{ra}_f)$. In **Case b**, firm 1 prefers the continuous label in $\ddot{\Omega}^r_{01}$ (i.e., $\ddot{\Omega}^r_{01} \cap (\tilde{\Omega}_6 \cup \tilde{\Omega}^{rb}_{1\phi} \cup \tilde{\Omega}^{rb}_{\phi\phi})$), $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap ((\kappa < \breve{\kappa}^{rb}_{f1}) \cup (\kappa > \breve{\kappa}^{rb}_{f2}))$, while prefers the binary label in $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap (\breve{\kappa}^{rb}_{f1} < \kappa < \breve{\kappa}^{rb}_{f2})$. In these two cases, firm 2 prefers the continuous label in $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$, while prefers the binary label in $\ddot{\Omega}^r_{01}$. Notably, we observe $\ddot{\Omega}^r_{01}$ and $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$ fall within region $\check{\mathcal{B}}$-the region where firm 1 prefers the binary label in the prior refinement policy. While $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$ and $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$ fall within region $\check{\mathcal{C}}$-the region where firm 1 prefers the continuous label in the prior refinement policy. This observation establishes that in $\ddot{\Omega}^r_{01}$, firm 1's label preference shifts from binary to continuous. In **Case a**, the preference shifts from continuous label to binary label in $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap (\kappa < \breve{\kappa}^{ra}_f)$, while in **Case b**, this shift occurs in $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap (\breve{\kappa}^{rb}_{f1} < \kappa < \breve{\kappa}^{rb}_{f2})$. Moreover, we observe all these new intersections fall within $\hat{\mathcal{C}}$-the region where firm 2 prefers the continuous label. This observation establishes that firm 2's label preference shifts from continuous label to binary label in $\ddot{\Omega}^r_{01}$. After the preference adjustment under the new refinement policy, we show the firms' label preference as below. Specifically, firm 1/firm 2 prefers the binary label if $(\tau, \kappa, \gamma) \in \check{\mathcal{B}}^{ra}/\hat{\mathcal{B}}^{ra}$, prefers the continuous label if $(\tau, \kappa, \gamma) \in \check{\mathcal{C}}^{ra}/\hat{\mathcal{C}}^{ra}$, and is indifferent if $(\tau, \kappa, \gamma) \in \check{\mathcal{I}}^{ra}/\hat{\mathcal{I}}^{ra}$.

Case a: $\check{\mathcal{I}}^{ra} \doteq \hat{\mathcal{I}}^{ra} \doteq \check{\mathcal{I}} \doteq \check{\mathcal{I}}$, $\check{\mathcal{B}}^{ra} \doteq \check{\mathcal{B}} \cup (\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap (\kappa \leq \check{\kappa}^{ra}_f)) - \ddot{\Omega}^r_{01}$, $\check{\mathcal{C}}^{ra} \doteq \check{\mathcal{C}} \cup \ddot{\Omega}^r_{01} - (\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap (\kappa > \check{\kappa}^{ra}_f))$, $\hat{\mathcal{B}}^{ra} \doteq \hat{\mathcal{B}} \cup \ddot{\Omega}^r_{01}$, and $\hat{\mathcal{C}}^{ra} \doteq \hat{\mathcal{C}} - \ddot{\Omega}^r_{01}$.

Case b: $\check{\mathcal{I}}^{rb} \doteq \hat{\mathcal{I}}^{rb} \doteq \check{\mathcal{I}} \doteq \check{\mathcal{I}}$, $\check{\mathcal{B}}^{rb} \doteq \check{\mathcal{B}} \cup (\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap (\check{\kappa}^{rb}_{f1} \leq \kappa \leq \check{\kappa}^{rb}_{f2})) - \ddot{\Omega}^r_{01}$, $\check{\mathcal{C}}^{rb} \doteq \check{\mathcal{C}} \cup \ddot{\Omega}^r_{01} - (\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}} \cap ((\kappa < \check{\kappa}^{rb}_{f1}) \cup (\kappa > \check{\kappa}^{rb}_{f2})))$, $\hat{\mathcal{B}}^{rb} \doteq \hat{\mathcal{B}} \cup \ddot{\Omega}^r_{01}$, and $\hat{\mathcal{C}}^{rb} \doteq \hat{\mathcal{C}} - \ddot{\Omega}^r_{01}$.

Next, we summarize the comparison results in these new intersections for the industry. We observe regardless of the firms' recycled content equilibrium in set $\tilde{\Omega}_6$ under the continuous label, the industry prefers the continuous label in $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$, $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{1\phi} \cap (\kappa > \mathring{\kappa}^{ra}_{I1})$, and $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{\phi\phi} \cap ((\kappa < \mathring{\kappa}^{ra}_{I2}) \cup (\kappa > \mathring{\kappa}^{ra}_{I3}))$, while prefers the binary label in $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{01}$, $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{1\phi} \cap (\kappa < \mathring{\kappa}^{ra}_{I1})$, and $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{\phi\phi} \cap (\mathring{\kappa}^{ra}_{I2} < \kappa < \mathring{\kappa}^{ra}_{I3})$. Notably, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{\underline{\phi\phi}}$ and $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{11}$ fall into the region $\mathring{\mathcal{C}}$–the region where the industry prefers the continuous label in the previous refinement. $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{10}$, $\tilde{\Omega}_6 \cap \ddot{\Omega}^r_{01}$, $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{1\phi} \cap (\kappa < \mathring{\kappa}^{ra}_{I1})$, and $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{\phi\phi} \cap (\mathring{\kappa}^{ra}_{I2} < \kappa < \mathring{\kappa}^{ra}_{I3})$ fall into the region $\mathring{\mathcal{B}}$–the region where the industry prefers the binary label in the previous refinement. Thus, within these sets, the industry's label preference maintain robust. However, we find $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{1\phi} \cap (\kappa > \mathring{\kappa}^{ra}_{I1})$ and $\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{\phi\phi} \cap ((\kappa < \mathring{\kappa}^{ra}_{I2}) \cup (\kappa > \mathring{\kappa}^{ra}_{I3}))$ may fall into region $\mathring{\mathcal{B}}$. This observation establishes that in these sets, the industry's label preference may shift from binary to continuous. Then after the label preference adjustment under the new refinement policy, we conclude the industry is indifferent to the two labels when $(\tau, \kappa, \gamma) \in \mathring{\mathcal{I}}^r$, prefers the binary label when $(\tau, \kappa, \gamma) \in \mathring{\mathcal{B}}^r$, while prefers the continuous label when $(\tau, \kappa, \gamma) \in \mathring{\mathcal{C}}^r$, where $\mathring{\mathcal{I}}^r \doteq \mathring{\mathcal{I}}$, $\mathring{\mathcal{B}}^r \doteq \mathring{\mathcal{B}} - (\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{1\phi} \cap (\kappa \geq \mathring{\kappa}^{ra}_{I1})) - \ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{\phi\phi} \cap ((\kappa \leq \mathring{\kappa}^{ra}_{I2}) \cup (\kappa \geq \mathring{\kappa}^{ra}_{I3})))$, $\mathring{\mathcal{C}}^r \doteq \mathring{\mathcal{C}} \cup (\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{1\phi} \cap (\kappa > \mathring{\kappa}^{ra}_{I1})) \cup (\ddot{\Omega}^r_{01} \cap \tilde{\Omega}^{ra}_{\phi\phi} \cap ((\kappa < \mathring{\kappa}^{ra}_{I2}) \cup (\kappa > \mathring{\kappa}^{ra}_{I3})))$.

By showing the label preferences of the certifier, the firms, and the entire industry under the new refinement policy, we proceed to analyze the robustness of label preference alignment between them.

We first show the conclusion that there is no win-win-win label design for the certifier and the two firms is consistent. This conclusion is supported by the following rationale. Firstly, given both firms cannot be better off under the binary label with the previous refinement alignment–implying the absence of any intersection between $\check{\mathcal{B}}$ and $\hat{\mathcal{B}}$, it follows that they cannot both be better off under the binary label with the new refinement alignment, as indicated by the absence of any intersection between $\check{\mathcal{B}}^{ra}$ and $\hat{\mathcal{B}}^{ra}$, and between $\check{\mathcal{B}}^{rb}$ and $\hat{\mathcal{B}}^{rb}$. Thus, we can also readily conclude the certifier and two firms cannot achieve a better outcome under the binary label simultaneously. Secondly, similar to the proof of Proposition 4, we show that when the certifier prefers the continuous label, one of the firms would prefer the binary label. Note the certifier's label preference remain consistent under different refinement policies. Within the four subsets where the certifier prefers the continuous label, we observe $\mathcal{C} \cap \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$ and $\mathcal{C} \cap \tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$ fall into $\check{\mathcal{B}}$, while $\mathcal{C} \cap \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$ and $\mathcal{C} \cap \tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$ fall into $\hat{\mathcal{B}}$. Under the new refinement policy, recall the definition of $\check{\mathcal{C}}^{ra}$ in **Case a** and $\check{\mathcal{C}}^{rb}$ in **Case b**, we find firm 1's label preference may shift from binary to continuous within $\ddot{\Omega}^{r}_{01}$. Nevertheless, the intersection of $\ddot{\Omega}^{r}_{01}$ and $\mathcal{C}$ is empty. Hence, we can deduce $\mathcal{C} \cap \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{10}$ and $\mathcal{C} \cap \tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{10}$ also fall into $\check{\mathcal{B}}^{ra}$ or $\check{\mathcal{B}}^{rb}$. Moreover, recalling the definition of $\hat{\mathcal{B}}^{ra}$ in **Case a** and $\hat{\mathcal{B}}^{rb}$ in **Case b**, we find $\hat{\mathcal{B}} \in \hat{\mathcal{B}}^{ra} = \hat{\mathcal{B}}^{rb}$. Thus, we can get $\mathcal{C} \cap \tilde{\Omega}_{1\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$ and $\mathcal{C} \cap \tilde{\Omega}_{\phi\phi} \cap \ddot{\Omega}_{\underline{\phi\phi}}$ also fall into $\hat{\mathcal{B}}^{ra}$ or $\hat{\mathcal{B}}^{rb}$. Finally we conclude when the certifier prefers the continuous label, it is evident that either firm 1 or firm 2 would prefer the binary label.

Finally, we show the conclusion that there is a win-win label design for the certifier and whole industry is consistent. Recalling the definition of $\mathring{\mathcal{C}}^{r}$, we observe $\mathring{\mathcal{C}} \in \mathring{\mathcal{C}}^{r}$. Therefore, since there is an intersection between $\mathcal{C}$ and $\mathring{\mathcal{C}}$, it logically follows that there must still be a intersection between $\mathcal{C}$ and $\mathring{\mathcal{C}}^{r}$. This implies that both the certifier and industry can potentially benefit under the continuous label with the new refinement policy. □

Proposition 11. *Under the certifier's label choice: Environmental performance* $\max\{\tilde{\pi}_0, \ddot{\pi}_0\}$ *remain non-monotonic in* τ *and* γ*, while always decreases in* κ*; Industry profit* $(\tilde{\pi}_1 + \tilde{\pi}_2) \cdot \mathbb{1}_{\{\tilde{\pi}_0 > \ddot{\pi}_0\}} + (\ddot{\pi}_1 + \ddot{\pi}_2) \cdot \mathbb{1}_{\{\tilde{\pi}_0 \leq \ddot{\pi}_0\}}$ *remain non-monotonic in* τ*,* κ*, and* γ*.*

Proof: In addition to the original equilibrium regions, a new region $\ddot{\Omega}^{r}_{01}$ may emerge within the existing region $\ddot{\Omega}_{10}$ in the final equilibrium under the new refinement policy. The certifier's payoff is consistent across these two regions, ensuring that the monotonic trends in environment performance remain unaffected. Moreover, industry profit displays non-monotonic behavior even without this new region under the previous refinement policy. Thus, the introduction of this additional region further solidifies the non-monotonic nature of industry profit. □

Appendix C

Summary of Notations

Table C.1: Notation

Endogenous variables	
ϕ_i	the percentage of recycled content in product i (firm $i's$ product)
c_i	variable cost of a ϕ_i-recycled product
p_i	the price of product i
d_i	demand for product i
$\underline{\phi}$	the binary recycling label's standard (lower bar for recycled-content percentage)
π_i	firm i's payoff
π_0	the environmental performance, the certifier's payoff
Exogenous variables	
k_i	firm i's cost factor of recycling investment
$\kappa = (k_1 + k_2)/2$	average recycled investment coefficient of the industry
$\gamma = k_1/k_2$	investment symmetricity of the industry
c_r (c_v)	unit cost of recycled (virgin) material
$\Delta c = c_r - c_v$	the unit cost differential between recycled and virgin materials
β_r (β_v)	unit environmental impact of recycled (virgin) material
v	the base value of products
x	location of a consumer in the Hotelling interval
t	unit mismatch cost in the Hotelling framework
$\tau = 1/t$	competition intensity
θ	consumers' green awareness (unit willingness-to-pay for recycled content)
Accents	
$\tilde{}$	the accent for variables when continuous label is adopted
$\ddot{}$	the accent for variables when binary label is adopted

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