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Modelling and Analysis of Strategic Supply Chain Management Systems

by
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A thesis submitted to
The Hong Kong Polytechnic University
for the degree of
Doctor of Philosophy

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Abstract of the thesis entitled ‘Modelling and Analysis of Strategic Supply Chain Management Systems’
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This thesis is a research study on modelling and analysis of strategic supply chain management systems. In a comprehensive survey of the literature on supply chain modelling and analysis, we present and discuss the results according to three issues, namely supply chain network, supply chain uncertainties, and supply chain coordination. Although there is a considerable body of literature on these three issues of supply chain management, we observe that rich opportunities for further research still exist in these issues. This thesis focuses on a portion of these potential research areas, which are incorporating logical constraints in supply chain design models, modelling supplier's uncertainty in a two-stage supply chain, and modelling the benefits of information sharing-based supply chain partnerships. Specifically, we study and examine the following problems.

First, in a strategic supply chain design model, we introduce logical constraints to include the constraints related to bills of materials (BOM). Reflecting intimate material supply relationships with vendors, the bottom level of BOM can be described as a list of specified quantities of materials or components supplied by different vendors. We can express BOM as logical rules such as “if product i will be produced in at least s of the proposed plants, then at least t of the candidate vendors must be selected”. This formulation of BOM constraints provides a reasonable way to capture the role of vendors in the strategic supply chain design model. We will show how BOM constraints are formulated in our proposed mixed integer programming (MIP) model to assist in the selection of vendors and how to develop the linear representation of logical constraints. Logical constraints can also be used to represent the relationships among the main entities of a supply chain such as vendors, plants, and distribution centers. We will also show how these relationships are formulated as logical constraints.

Second, we study an optimal inventory policy for a two-stage supply chain, which includes a supplier and a retailer. The supplier orders from an unstable outer source, which means the supplier can’t receive the placed order on time and the supplier’s lead time is variable. We use the supplier’s variable lead time to formulate the uncertainty from the supplier. The supplier’s variable lead time is assumed to be independently and identically distributed with mean and variance. The retailer faces customer demands which form a Poisson process with intensity λ.
Each customer demand arrival brings a batch of $Q$ units. Both of the supplier and retailer adopt periodic-review batch-ordering policies. We present a modelling framework to demonstrate the modelling and analysis of supplier's lead time variability in this two-stage supply chain. We identify the relation between inventory costs and lead time variability. Comparing the results to the model with constant lead time, we can see that this variability can cause excess inventory. We also suggest further approaches in this topic about the reduction of the deficiency of lead time variability and the possible coordination mechanisms to reduce the variability.

Third, in a two-level decentralized supply chain, we present a study aimed at quantifying the benefits of information sharing-based supply chain partnerships. Based on a modelling framework, we introduce three levels of information integration for modelling the partnerships in a two-stage supply chain consisting of a single retailer and a single manufacturer, namely decentralized control, coordinated control and centralized control. We derive the optimal inventory policies for the manufacturer and the retailer under these three different information sharing scenarios. We show that increasing information sharing among the members in a decentralized supply chain will lead to Pareto improvement in the performance of the entire chain. Specifically, both of the manufacturer and the retailer can obtain benefits in terms of reductions in inventory levels and cost savings.

These three problems are very typical issues in modelling and analysis of strategic supply chain management systems. For each of them, we give the illustration of the specific settings of the supply chain structure. The relative modelling frameworks are formulated for quantitative analysis on each problem. We apply some traditional operations research techniques to carry out our modelling and analysis on these issues. Some practical cases are also included for testing and validating our mathematical modelling results.
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Contents

1 Introduction ......................................................... 1
   1.1 BOM logical constraints ........................................ 3
   1.2 Supplier's uncertainties ........................................ 3
   1.3 Information sharing-based supply chain partnerships ........... 4

2 A survey of supply chain modelling and analysis .................. 7
   2.1 Introduction .................................................. 7
   2.2 Modelling and analysis of strategic supply chain design .......... 9
   2.3 Modelling and analysis of supply chain uncertainty ............. 18
   2.4 Modelling and analysis of supply chain coordination .......... 22
   2.5 Potential research opportunities ................................ 29

3 Incorporating logical constraints in supply chain design models 31
   3.1 Introduction .................................................. 31
   3.2 Model formulation ............................................. 33
      3.2.1 Parameter notations and definitions ...................... 34
      3.2.2 The objective function ................................... 35

3.2.3 The constraints ................................................. 36
3.3 BOM logical constraints ........................................ 38
3.4 Linear representation of logical constraints ............... 42
  3.4.1 Logical consistency constraints ......................... 43
  3.4.2 BOM logical constraints ................................. 44
  3.4.3 Reduction of linear inequalities ......................... 46
3.5 An illustration problem ........................................ 48
3.6 Conclusions ..................................................... 53

4 Modelling and analysis of supplier's uncertainty in a two-stage supply chain 54
  4.1 Introduction ....................................................... 54
  4.2 Model formulation .............................................. 56
  4.3 One-for-one replenishment system ......................... 59
    4.3.1 Performance evaluation at the supplier ................. 61
    4.3.2 One-for-one replenishment at the retailer ............. 63
    4.3.3 Customer waiting time at the retailer ................ 64
    4.3.4 The whole system ......................................... 64
  4.4 Analysis of the inventory cost under the one-for-one system .... 65
  4.5 Conclusions ..................................................... 67

5 Benefits of information sharing-based supply chain partnerships 68
  5.1 Introduction ....................................................... 68
  5.2 Three information integration levels and the normative model .... 71
    5.2.1 Three information integration levels .................... 72
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2.2</td>
<td>Normative inventory model</td>
<td>73</td>
</tr>
<tr>
<td>5.3</td>
<td>Inventory control policies under different information sharing-Based partnerships</td>
<td>78</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Optimal policies under decentralized control</td>
<td>78</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Optimal policies under coordinated control</td>
<td>80</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Optimal policies under centralized control</td>
<td>81</td>
</tr>
<tr>
<td>5.4</td>
<td>The benefits of information sharing-based partnership</td>
<td>82</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Inventory levels under different situations</td>
<td>84</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Cost reduction based on the information sharing-based partnership</td>
<td>85</td>
</tr>
<tr>
<td>5.4.3</td>
<td>The retailer's benefits</td>
<td>86</td>
</tr>
<tr>
<td>5.4.4</td>
<td>Pareto improvement</td>
<td>87</td>
</tr>
<tr>
<td>5.5</td>
<td>Conclusion</td>
<td>87</td>
</tr>
<tr>
<td>6</td>
<td>Conclusions and future research</td>
<td>89</td>
</tr>
<tr>
<td>6.1</td>
<td>Conclusions</td>
<td>89</td>
</tr>
<tr>
<td>6.2</td>
<td>Future research</td>
<td>91</td>
</tr>
</tbody>
</table>

References                                                                 | 94   |

Appendix                                                                 | 102  |
Chapter 1

Introduction

Globalization of business has been accelerating in pace in the last two decades due to rapid developments in technology for manufacturing and information, increased cost pressure, and more aggressive demand from customers. Traditional production-distribution schemes have been dramatically changed. Most companies have been compelled to redesign or restructure their manufacturing network in different countries. New partnership relationships among suppliers, manufacturers, retailers and other parties have replaced the conventional free market structure. With these evolutions, research on supply chain management has gained increasing attention in both academics and industrial practices in recent years. Supply chain management has also been an active research area in the field of management science and operations management.

A supply chain is a multi-echelon network of suppliers, manufacturers, distribution centers, and retailers through which raw materials are acquired, transformed, and delivered to customers. In a global market environment, companies have to compete based on flexibility and responsiveness. A number of companies attempt to improve the agility level by distributed enterprises. The concept of supply chain management helps the companies in this environment to achieve effective enterprise integration. In the literature, there are numerous papers dealing with the concepts and strategies of supply chain management. However, there is a need for modelling and analysis of supply chain management systems with the objective of measuring the performance of various strategies, methods and technologies. Application of operations research models or techniques can be extensively used in solving the problems of supply chain management. The essential motivation behind the modelling and analysis is to effectively design and control supply chain management systems for improving competitiveness by producing quality
products and services. These operations research models and techniques can be applied to solve the problems of supply chain management that include the strategic supply chain design, supplier selection, economic ordering management, and costing system evaluation.

There is a considerable body of literature on modelling and analysis of supply chain management systems. These research works discuss many aspects of supply chain management. The existing traditional models for strategic supply chain design are developed to determine the number, location, capacity, and type of production plants and distribution centers so as to minimize the total cost, or maximize the after-tax profit, of the supply chain. For the entire supply chain design and planning, it is also necessary to select the set of suppliers (or vendors), hence to determine the number of vendors supply contracts. The modelling and analysis of supply chain management systems has evolved from the work on multi-echelon network design and inventory control to recent studies of various mechanisms of coordinating supply chain decisions. There is a clear shift from studies on centralized planning models, in which a central planner makes all decisions to optimize a systemwide objective, to focusing on decentralized system with independent decision makers. From the general trend towards global markets and growing customer orientation, one key requirement of the challenges is to incorporate supply chain partnerships in supply chain management systems, which contribute to reallocating stocking decisions right, sharing information and coordinating decisions.

The papers appearing in current literature deal with various modelling and analysis methodologies of supply chain management systems. There are three fundamental issues associated with a supply chain: (a) the supply chain network; (b) the nature of supply chain uncertainty; (c) the relationships between different stages of the supply chain. These three issues follow a logical hierarchical structure. The analysis of the supply chain network design focuses on strategic supply chain design, which is the application of traditional logistics research on production-distribution systems with the explicit consideration of the whole network. Within a deterministic supply chain, the performance of the chain’s operations often encounters supply chain uncertainty including demand uncertainty, lead time variability and other uncertain information between the supply chain entities. In order to mitigate the deficiency of supply chain uncertainty, researchers suggest addressing the issue of supply chain partnerships between different stages of the supply chain, which involves information sharing, ordering coordination, vendor managed inventory, etc. This thesis will follow this logical framework to discuss the findings in all these three featured areas of supply chain modelling research: modelling and analysis on supply chain design, modelling and analysis on supply chain uncertainty, and modelling and analysis on supply chain partnerships.
1.1 BOM logical constraints

This a joint work with Hong Yan and T.C.Edwin Cheng.

In a mixed integer programming (MIP) model of a strategic supply chain design, we describe a reasonable way to include bills of materials (BOM) as logical constraints, which reflect intimate material supply relationships with vendors. We show how these relationships are formulated as logical constraints in a MIP model.

The bottom level of BOM can be described as a list of specified quantities of materials or components supplied by different vendors. We suggest that BOM be formulated as logical constraints in supply chain design models. This formulation is based on recent developments in logical inference and integer programming. We decompose the BOM as logical rule “if at least \( p \) of the proposed plants are open, then at least \( q \) of the candidate vendors should be chosen”. The logical rule can be represented as:

\[
(\tilde{y}_1 \lor \tilde{y}_2 \lor \ldots \lor \tilde{y}_K)_p \Rightarrow (\tilde{x}_1 \lor \tilde{x}_2 \lor \ldots \lor \tilde{x}_J)_q,
\]

(1.1)

where \( \tilde{x}_j \) is the 0-1 indicator for candidate vendor \( j \) and \( \tilde{y}_k \) as the 0-1 indicator for potential plant \( k \). \( J \) is the set of candidate vendors and \( K \) is the set of potential plants. The constraint (1.1) is a cardinality rule. The convex hull representation of this rule can be given by the following inequalities as the main facets:

\[
-qe\tilde{y} + (1 + K - p)e\tilde{x} \geq q(1 - p),
\]

(1.2)

where \( e \) is a vector of 1s with the corresponding dimension. The convex hull representation (1.2) consists of many linear inequalities. These inequalities can be achieved via a simple recursive procedure (see details in Yan and Hooker (1999)).

The formulation captures the role of BOM in the selection of vendors in the strategic design of a supply chain. We give a test problem to illustrate the model formulation and relative solutions. BOM logical constraints in MIP model will not only achieve substantial savings in solution time, but also enable adequate descriptions of relevant relationships among the entities involved in the supply chain design model.

1.2 Supplier’s uncertainties

This a joint work with Hong Yan and T.C.Edwin Cheng.
Normally, there are five lead time components: procurement, set-up, processing, handling, and transportation. All of these components contain uncertainty. A possible approach is to accumulate all the uncertainty and formulate the variability of lead time as independently identical distribution with mean and variance. This modelling approach is quite straightforward. We can formulate a modelling framework to demonstrate the modelling and analysis of lead time variability. In a given two-stage supply chain which includes a supplier and a retailer, we assume the customer demand process is compound Poisson process with intensity $\lambda$. The retailer’s lead time $L_t$ is defined by

$$L_t = (1 - \varepsilon)b + \varepsilon \alpha_t,$$  \hspace{1cm} (1.3)

where $b$ is a constant and $\alpha_t$ is an error term which is identically distributed with mean 0 and variance $\sigma$. In Eq.(1.3), parameter $\varepsilon$ is the measure of lead time variability. The variance of lead time decreases when $\varepsilon$ decreases. We assume that the customer demand during the lead time $L$ is distributed with density $f(\cdot)$, distribution $F(\cdot)$, mean $\mu(\varepsilon)$ and variance $\varphi(\varepsilon)$. The retailer’s inventory policy is a popular two-bin $(R, Q)$ policy which operates by placing an order of size $Q$ whenever the inventory level (stock on hand) drops to or below the reorder level $R$. The notation is:

$h$: holding cost per unit per time period;
$g$: backorder cost per unit per time period.

The model, which is to minimize the average inventory cost at each time period, can be given by

$$\min Z(R, Q, \varepsilon) = h \int_0^{R(\varepsilon)} (R(\varepsilon) - x)f(x)dx + g \int_{R(\varepsilon)}^{\infty} (x - R(\varepsilon))f(x)dx.$$ \hspace{1cm} (1.4)

The objective is to find the inherit relation among $Q, R$ and inventory costs with respect to lead time variability measure $\varepsilon$. Comparing the results to the model without lead time variability ($\varepsilon = 0$), we expect to see that this variability can cause excess inventory. Further studies in this field can be the reduction of the deficiency of lead time variability and the formulation of coordination policies to reduce the variability.

### 1.3 Information sharing-based supply chain partnerships

This a joint work with Hong Yan and T.C.Edwin Cheng.
To identify the benefits of supply chain partnerships, we give a modelling analysis framework to formulate the partnerships between two supply chain members: manufacturer and retailer. We assume the coordination between the two members are ordering information sharing, i.e., sharing customer demand information. To capture the different situations of information sharing and ordering coordination, we formulate the information sharing-based relationship between the retailer and the manufacturer as three information integration levels: Level 1—“decentralized control” is described as both the retailer and the manufacturer make their inventory decisions according to their own forecasting without any information sharing: Level 2—“coordinated control” is referred to as the two neighboring inventories are coordinated with sharing of the customer ordering information: Level 3—“centralized control” is defined as both the retailer and the manufacturer can retrieve the customer’s demand information in a synchronized manner by using of Electronic Data Interchange (EDI) and the two partners can establish a strategic partnership by adopting a Vendor Managed Inventory (VMI) strategy.

The basic cost-minimization model is presented for the two supply chain members. It is assumed that the retailer orders a single item from the manufacturer at each time period. Both of them adopt the order-up-to inventory policy with a periodic review procedure and any excess demand is backlogged. At the beginning of each period, the retailer and manufacturer should decide their order-up-to levels. For each of them, the decision is made based on such a basic inventory model. Mathematically, the inventory problem is given by

\[
\text{Minimize } Z = \sum_{t=1}^{\infty} \beta^{t-1} \left[ cR_t + \beta^L G(S_t, \sum_{l=t}^{t+L} D_l) \right],
\]

where

\[
G(S_t, \sum_{l=t}^{t+L} D_l) = h \left( S_t - \sum_{l=t}^{t+L} D_l \right)^+ + g \left( \sum_{l=t}^{t+L} D_l - S_t \right)^+
\]

with \( x^+ = \max\{0, x\} \) and \( S_t = W_t + R_t \). The notation is:

- \( R_t \): quantity ordered in period \( t \);
- \( W_t \): quantity on hand plus on order in period \( t \);
- \( L \): lead time in terms of number of periods;
- \( c \): unit cost of ordering;
- \( h \): unit holding cost;
- \( g \): unit shortage cost;
- \( \beta \): discount factor for each period \((0 < \beta < 1)\).
This model is used to find the optimal order-up-to level $S_t$ of each supply chain member. Based on the optimal inventory control policies of the two members, the average inventory levels and expected inventory costs can be achieved. From the comparison of inventory reductions and cost savings among the three information integration levels, Yu et al. (2000) show that "Pareto improvement" will be achieved, i.e., both the retailer and the manufacturer are at least as well off and at least one of them is better off. Therefore, this modelling framework shows that the supply chain partnerships based on information sharing can improve the overall performance of a decentralized supply chain.

The rest of this thesis is organized as follows. In Chapter 2, a comprehensive literature review is given to identify the existing research opportunities in modelling and analysis of strategic supply chain management systems. We focus our attention on the three featured research fields, which are specified in the following three chapters. In Chapter 3, we introduce the research work on BOM logical constraints in strategic supply chain design. Supplier's uncertainty in a two-stage supply chain is discussed in Chapter 4. We introduce the research about information sharing-based supply chain partnerships in Chapter 5. Conclusions and future research directions are presented in Chapter 6.
Chapter 2

A survey of supply chain modelling and analysis

2.1 Introduction

A large stream of literature on supply chain management has been published over the last two decades. These research works discuss many aspects of supply chain management, including organizational structures, financial arrangements, accounting management, strategic approaches, operations design and implementation. Although the supply chain is frequently referred to as a logistics network in the literature, the integration of supply networks, distribution networks, manufacturing networks and assembly networks from different parties in different countries, at different times has become tremendously important and complex. It is vastly different from traditional logistics management or purchasing management where minimization of the operation cost to a single owner is often the major concern. Due to the high complexity of a supply chain network, it would be very difficult, if not impossible, to reveal and to describe the inter-relationships existing between the partners in a production-distribution chain. The modelling and analysis of supply chain problems will significantly improve our understanding of global logistics planning systems, and provide important decision support tools for supply chain design. In this survey, we set out to review the status of research on the modelling and analysis of supply chain design and management.

There are previous reviews on supply chain management research and development. The advances of computers have made the optimization of large-scale models for distribution systems
design technically feasible. In a review paper, Geoffrion and Powers (1995) report the evolution of strategic distribution systems design over the previous twenty years. They stress that the corporate status of logistics has changed dramatically during the previous decades. Thomas and Griffin (1996) emphasize coordination planning among the three fundamental stages of a supply chain: procurement, production, and distribution. Specifically, they classify research on supply chain coordination into three specific areas of study: buyer-vendor coordination, production-distribution coordination, and inventory-distribution coordination. It is pointed out that managing facilities in a supply chain without cooperation, usually implies poor overall behavior, and coordinating the planning and operation of the different stages of a supply chain could lead to significant savings in logistics costs. Vidal and Goetschalckx (1997) give an extensive literature review of strategic production-distribution models. They focus on formulations of relevant factors, characteristics of solution methods and computational experiences in mixed integer programming models. The review shows that inadequate consideration of some stochastic aspects such as service level and lead time, and lack of representation of bills of materials (BOM) are the major drawbacks of existing strategic supply chain design models. Maloni and Benton (1997) review both conceptual logistics literature and supply chain modelling literature. They show that the conceptually based supply chain literature has been well aware of the promise of win-win supply chain partnerships while the application of operations research tools has not met the requirements of the extensive coordination needs of supply chain management. Cohen and Mallik (1997) review the state of knowledge and practice of supply chain globalization. They characterize global supply chain management strategy planning by two modelling approaches: network flow and option valuation models. The paper points out that the reported global supply chain planning models lack practicality and ignore price and demand uncertainty in international markets. In a recent paper, Beamon (1998) provides a review of performance measures of supply chain modelling. Beamon indicates that the majority of the models use inventory level as a decision variable and cost as a performance measure. He suggests that research is required to incorporate appropriate performance measurement systems with critical supply chain decision variables. In a review of production-distribution planning approaches, Erenbüç et al. (1999) give general models of different production-distribution issues at supplier, plant, and distribution stages.

For a long time, the various processes of the supply chain have been investigated individually. Recently, increasing attention is being paid to investigating the design, performance, and analysis of the supply chain as a whole. In this work, we review the supply chain modelling and analysis literature, which addresses the issues of coordinated planning between two or more stages of the supply chain, especially modelling and analysis based on a total supply chain model. Our aim is to present a logical framework which can be used to generalize the
key issues of contemporary supply chain research. We first give a classification of the supply chain literature. There are three fundamental issues associated with a supply chain: (a) the supply chain network; (b) the nature of supply chain uncertainty; (c) the relationships between different stages of the supply chain. These three issues follow a logical hierarchical structure. The analysis of the supply chain network focuses on strategic supply chain design, which is the application of traditional logistics research on production-distribution systems with the explicit consideration of the whole network. Within a deterministic supply chain, the performance of the chain's operations often encounters supply chain uncertainty including demand uncertainty, lead time variability and other uncertain information between the supply chain entities. In order to mitigate the deficiency of supply chain uncertainty, researchers suggest addressing the issue of supply chain coordination between different stages of the supply chain, which involves information sharing, ordering coordination, supply chain partnerships, etc. Our review follows this logical framework to discuss the findings in existing supply chain literature.

2.2 Modelling and analysis of strategic supply chain design

A supply chain is an integrated manufacturing process in which raw materials are converted into final products, then delivered to customers. This process is composed of two basic processes: (i) the production planning and inventory control process, and (ii) the distribution and logistics process. The two processes interact with each other to form an integrated supply chain. The formulation of strategic production-distribution models for supply chain design and management has received increasing research interest. Managing a large-scale supply chain or production-distribution network is often concerned with optimizing the locations of manufacturing sites, allocations of product mix at each site, times, routes, and volumes of product delivery, under various constraints and requirements. In other words, strategic supply chain design determines the extent to which the supply chain works as a unit to meet the required performance objectives. Using optimization models to make such decisions is a key feature in this modelling research area. The optimization issues can be considered under two scenarios:

1. Supply chain facility optimization. Under this scenario, the objective is to minimize the total cost incurred by supply chain facilities while designing the structure of the supply chain based on the candidate entities. We call this scenario supply chain structure design.

2. Supply chain performance optimization. This scenario optimizes the performance of each supply chain entity under a deterministic network, while deciding the supply chain entities'
optimal control policies, especially the inventory control policies. We name this supply chain performance design.

Generally, the objective of the above optimization problems is either cost minimizing or profit maximizing, subject to some constraints from the supply chain entities. The total costs of the two scenarios do not differ significantly. However, their optimization models and decision variables are very different. We discuss the two scenarios as follows.

Supply chain structure design

The supply chain structure design problem commonly occurs in the following case: A number of candidate supply chain entities (such as vendors, production plants, distribution centers (DCs) and retailers) and some customer zones with specified demand quantities of the different products are given. The objective is to determine the number, locations, capacities, and types of supply chain entities so as to minimize the total cost or maximize the after-tax profit of the supply chain, subject to supply chain entity capacities, material balance and service level constraints. The decision variable associated with each candidate entity is a 0-1 variable to indicate if the entity is selected (the variable equals 1) or not (the variable equals 0). We call these binary variables structural variables. Most of the structural design models are therefore mixed integer programming (MIP) models. Geoffrion and Graves (1974) give probably the first modelling work that presents a comprehensive MIP model for strategic design of supply chains. Cohen et al. (1989) formulate a modelling framework for the strategic design of a global supply chain. We summarize the two modelling works in the following cost minimization model to demonstrate the general model of multi-echelon multi-commodity supply chain structure design.

Minimize

\[
Z = \sum_{i,j} c_{ij} x_{ij} + \sum_j g_j y_j + \sum_k f_k z_k + \sum_{m,i,j} C_{mij} X_{mij} + \sum_{p,j,k,l} P_{pj} Y_{pjk} + \sum_{p,j,k,l} u_{pjk} Y_{pjk} Y_{pjk} + \sum_{p,k,l} v_{pkl} D_{pl} \tilde{z}_{kl}
\]  

(2.1)
subject to

\[ \sum_{p, k, l} R_{pj} Y_{pjk} \leq W_j y_j, \quad \forall j \]  
(2.2)

\[ \sum_{p, j, l} U_{pk} Y_{pjk} \leq V_k z_k, \quad \forall k \]  
(2.3)

\[ \sum_{i, j} X_{mij} = \sum_{p, j, k, l} M_{mp} Y_{pjk} \]  
(2.4)

\[ \frac{\sum_{p, j, k, l} t_{pkl} D_{pl} z_{kl}}{\sum_{l} D_{pl}} \leq T_p \]  
(2.5)

\[ \sum_{j, k} Y_{pjk} \tilde{y}_{jk} = D_{pl} \tilde{z}_{kl} \]  
(2.6)

\[ x, y, z, \tilde{y}, \tilde{z} \in \{0, 1\}, \quad \forall i, j, k, l \]  
(2.7)

\[ X, Y \geq 0, \quad \forall p, i, j, k, l. \]  
(2.8)

In this model, \( m, p, i, j, k \) and \( l \) represent the indices for materials, products, vendors, plants, distribution center sites, and customer zones within given finite sets, respectively. Customer demand is denoted by \( D_{pl} \). The decision variables are \( x, y, z, \tilde{y}, \tilde{z}, X \) and \( Y \). \( X_{mij} \) is the amount of material \( m \) purchased from vendor \( i \) by plant \( j \). \( x_{ij} \) is 1 if the supply channel from vendor \( i \) to plant \( j \) is chosen. \( Y_{pjk} \) denotes the amount of product \( p \) shipped from plant \( j \) through DC \( k \) to customer zone \( l \). \( M_{mp} \) represents the material balance factor. \( \tilde{y}_{jk} \) is 1 if the distribution channel from plant \( j \) to DC \( k \) is chosen. The objective function (2.1) specifies the fixed cost including fixed ordering cost \((c_{ij})\), plant opening cost \((g_i)\), and DC opening cost \((f_k)\). The unit costs include: material purchasing cost \((C_{mij})\), production cost \((P_{pj})\), transportation cost \((u_{pjk})\) and distribution cost \((v_{pkl})\). Constraints (2.2) and (2.3) are related to capacity limits for each plant \((W_j)\) and DC \((V_k)\). Service level is ensured by constraint (2.5), where \( t_{pkl} \) is the average time to make a delivery of product \( p \) to customer zone \( l \) after receiving an order at DC \( k \), and \( T_p \) is a desired bound on the average delivery delay for product \( p \).

The above model describes a typical multi-commodity production-distribution problem, and can be solved by Benders Decomposition as suggested by Geoffrion and Graves (1974). In their paper, they formulate a model to represent the design of a production-distribution system with several plants with known capacities, distribution centers, and a number of customer zones. Fixed and linear costs of DC, production costs, and linear transportation costs are considered in the objective function. The constraints include plant capacities, throughput limits of DC, customer demand satisfaction, and binary variable configuration constraints (see Eqs. (2.2).
(2.3), (2.5), and (2.6), respectively). Subsequently, Geoffrion et al. (1978) present a status report on research of strategic distribution system planning. In this report, they introduce some new ideas such as nonlinear facility throughput constraints and tradeoffs between distribution and customer service based on the work of Geoffrion and Graves (1974).

The objective function of a strategic supply chain design problem is either a total cost minimization or profit maximization. Many past studies on this problem differ in the formulation of some constraints and the consideration of miscellaneous environmental elements of the real world, as summarized in Table 2.1.

<table>
<thead>
<tr>
<th>Authors and References</th>
<th>Objective Function</th>
<th>Model Type</th>
<th>BOM</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Service level</td>
</tr>
<tr>
<td>Geoffrion &amp; Graves</td>
<td>Minimize cost</td>
<td>Single period, multi-product</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>(1974)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohen et al.</td>
<td>Maximize profit</td>
<td>Multi period, multi-product</td>
<td>Material Balance</td>
<td>No</td>
</tr>
<tr>
<td>(1989)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohen &amp; Lee</td>
<td>Maximize profit</td>
<td>Single period, multi-product</td>
<td>Material Balance</td>
<td>Yes</td>
</tr>
<tr>
<td>(1989)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cohen &amp; Moon</td>
<td>Minimize cost</td>
<td>Single period, multi-product</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>(1991)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arntzen et al.</td>
<td>Minimize cost</td>
<td>Multi period, multi-product</td>
<td>Rooted tree</td>
<td>No</td>
</tr>
<tr>
<td>(1995)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cole (1995)</td>
<td>Minimize cost</td>
<td>Single period, multi-product</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Most modelling work follows the general formulation Eqs. (2.2) and (2.3) as the facility capacity constraints. The consideration of bills of materials (BOM) can be treated as a source of production data for the development of consistency constraints which balance the material flows among vendors, plants, and DCs like Eq. (2.4) (Geoffrion and Graves (1974). Cohen and Lee (1989). and Cohen et al. (1989)). Another BOM formulation is given by Arntzen et al. (1995). They consider BOM as a collection of rooted arborescence, on which each vertex represents a product and the fabrication facility. The vertices are numbered in preorder. This order can be formulated as the index of the relevant parameters to demonstrate the arboreal structure.
of BOM. The customer service level constraint (2.5) is given by Geoffrion and Graves (1974). Cohen and Lee (1989) suggest an alternative way to formulate the service level constraint (in contrast to Eq. (2.5)), as follows

$$\beta_{gh} \left[ 1 - \prod_{f} \prod_{l \in k} (1 - y_{fg}) \right] \leq \text{TOT}_h / \text{TOT}. \tag{2.9}$$

Here, $\beta_{gh}$ is the minimum content percentage required by country $h$ to sell to market segment $g$ and $y_{f,g}$ is a 0-1 variable representing whether product $f$ is to be sold to market segment $g$ of market region $l$. $\text{TOT}_h / \text{TOT}$ is the value-added ratio, with $\text{TOT}_h$ being the total value of sourcing and manufacturing activities in country $h$, and $\text{TOT}$ being the global sum of the $\text{TOT}_h$'s.

In practical problems, some environmental factors should be considered in supply chain design modelling. For a supply chain crossing international boarders, duty is an important element which should be included in the objective function or constraints (see Cohen and Lee (1989) and Arntzen et al. (1995)). Cohen et al. (1989) give a comprehensive modelling framework of strategic supply chain design, which considers exchange rate factors, feasible flow constraints, and capacity of suppliers. Cohen and Lee (1989) present a deterministic, mixed integer, non-linear mathematical programming model that analyzes resource deployment decisions for multi-national firm. They include government offset requirements, tariffs, duties, and transfer pricing in the model. The model by Cohen and Lee (1989) is a simplified version of the model presented by Cohen et al. (1989). An algorithm to solve production-distribution problems with piecewise linear concave costs of production is given by Cohen and Moon (1991). The consideration and determination of stochastic factors play an active role in supply chain design modelling in recent years. To optimize a strategic production-distribution system, Cole (1995) presents a single-period mixed integer model, which considers normal demands and stochastic customer service by carrying safety stocks, together with warehouse location, customer allocation, and channel selection. The contribution of the work is the consideration of normal demands and safety stocks in an MIP supply chain design model.

Supply chain performance design

Within a deterministic supply chain network, each organization should design its performance variables such as batch size, reorder point, and order-up-to level. A typical supply chain performance design model is to determine the optimal inventory control policies for each level of a multi-echelon production-distribution system with the fixed or variable costs associated
with the single level. Since a supply chain consists of various levels or echelons, an important question that needs to be addressed is how supply chain entities at these various levels interact with one another. There are two major but contrasting philosophies in the literature: the push and pull system. In a production-distribution system, the push philosophy means that there is a central decision maker, say the manufacturer, who can access information about inventory levels at all the concerned facilities. All inventory decisions are made centrally based on this information. This approach is called centralized optimization. In the pull system, the inventory decisions are made by local managers based on their local conditions. This approach is called decentralized optimization. In both push and pull systems, the decision variables (order quantity, reorder point, etc.) should be determined in order that the overall system costs are minimized.

Based on the work of Pyke and Cohen (1993, 1994), Graves et al. (1998) and Ganeshan (1999), we give a general framework for the modelling study of the multi-stage multi-commodity centralized control problem. It is assumed that the lead times between adjacent stages are zero and the model can be given as follows.

Minimize

\[
Z(Q_{ipt}) = \sum_{i,p,t} \{c_{ipt} X_{ipt} + C_{ipt} Q_{ipt} + H_{ipt} S_{ipt}\}
\]  (2.10)

subject to

\[
S_{ipt-1} + Q_{ipt} - S_{ipt} = Q_{i-1pt} \quad \forall i, p, t,
\]  (2.11)

\[
X_{ipt} = \begin{cases} 
1 & \text{if } Q_{ipt} > 0, \\
0 & \text{if } Q_{ipt} \leq 0,
\end{cases} \quad \forall i, p, t,
\]  (2.12)

\[
Q_{ipt} \geq 0, \quad \forall i, p, t,
\]  (2.13)

where \(i, p, \text{ and } t\) are the indices for stage levels, products, and time periods, respectively. \(Q_{ipt}\) is the amount of product \(p\) ordered by level \(i\) at time period \(t\), while \(S_{ipt}\) is the inventory of product \(p\) at level \(i\). The objective function (2.10) describes the summation of the ordering costs (\(c_{ipt}\) is the fixed ordering cost and \(C_{ipt}\) is the unit ordering cost) and holding costs (\(H_{ipt}\) is the unit holding cost) of all supply chain stages. In this general model, if we need to include lead times of adjacent supply chain stages, we just define the demand faced by a stage in a particular time period as an aggregate quantity of the demands for its downstream stage lagged by the lead time. Two types of control policies: echelon inventory and installation inventory, can be designed by using this model. The echelon inventory position at a particular stage is the inventory position of the subsystem consisting of the stage itself as well as all the downstream
stages. The installation inventory of a stage is its local inventory level. These two stock policies are studied by Chen (1998).

Pyke and Cohen (1993) present a model of a simple production-distribution network and examine its performance characteristics. The system consists of a factory, a finished goods stockpile, and a single retailer. The authors present near-optimal algorithms for determining the expedite batch size, the normal replenishment batch size, the expedite reorder point, the normal reorder point, and the order-up-to level at the retailer. In a follow-up paper, Pyke and Cohen (1994) present multi-product extensions to their previous work. Graves et al. (1998) describe how to provide the analysis for determining performance measures on production smoothness, production stability, and inventory requirements in a single production-inventory stage. The optimization problem minimizes production smoothing, which is given by the standard deviation of the production output, subject to a constraint on the standard deviation of the inventory and the requirement that the weights sum to one. This model can be used to determine the inventory placement across a multi-stage supply chain. Ganeshan (1999) presents a near-optimal (s, Q) type inventory policy for a production-distribution network with multiple suppliers replenishing a central warehouse, which in turn distributes to a large number of identical retailers. The model synthesizes three components: the inventory analysis at the retailers, the demand process at the warehouse, and the inventory analysis at the warehouse. A case study by Escudero et al. (1999) presents a modelling framework for the optimization of a manufacturing, assembly and distribution supply chain planning problem under uncertainty in product demand and component supplying cost and delivery time. The framework identifies four key aspects of this problem as time, uncertainty, cost and customer service level.

For a decentralized control system, each supply chain echelon designs its performance variables based on its own knowledge. The models are usually formulated as dynamic programming models that seek to minimize (maximize) finite or infinite horizon discounted expected costs (profits) (see models by Clark and Scarf (1960) and Federgruen and Zipkin (1984a, 1984b)). According to these previous works, the general model of the decentralized supply chain at each single stage can be formulated as the following problem. It is assumed that the demand from the downstream level is independently and identically distributed (i.i.d).

Minimize

\[ f_t(y_t) = \{C(z_t) + L(z_t + y_t) + \alpha E[f_{t+1}(y_t + z_t - D_t)] \} \]  \hspace{1cm} (2.14)
subject to

\[
C(x) = \begin{cases} 
K + c \cdot x : & x > 0 \\
0 : & x = 0 
\end{cases}
\]  \quad (2.15)

\[y_{t+1} = y_t + z_t - D_t, \quad (2.16)\]

\[z_t \geq 0 \quad \forall t. \quad (2.17)\]

In this model, \(f_i(\cdot)\) is the discounted cost function and \(C(\cdot)\) is the order cost function including the fixed cost \((K)\) and unit cost \((c)\). \(D_t\) is the demand from the downstream level and \(\alpha\) is the discount factor. The lost function \(L(\cdot)\) is defined as

\[
L(Y_t) = hE(Y_t - D_t)^+ + gE(D_t - Y_t)^+, 
\]

where \(h\) = unit holding cost,

\(g\) = unit shortage cost,

\(Y_t = z_t + y_t \quad (Y_t\) denotes the amount available),

and \(x^+ = \max(x, 0)\).

Originally, model (2.14) comes from a multi-echelon inventory system. Clark and Scarf (1960) provide one of the earliest efforts in decentralized optimization of a two-echelon system. Their paper forms the basis of subsequent work. They present a recursive decomposition approach to determine optimal policies for serial multi-echelon systems. Under periodic review inventory control with no setup costs, they show that an order-up-to policy at each node is optimal. Most research in this area is based on this classic work. Federgruen and Zipkin (1984a) extend Clark and Scarf's result to the infinite-horizon case (for both discounted and average costs). Furthermore, Federgruen and Zipkin (1984b) are able to obtain tractable approximation results for the optimal centralized control policy when the network structure is of the multi-echelon type. It is assumed that each node in the multi-echelon network can have at most one supplying mode. They show that the model can be systematically approximated by a single-location inventory problem. This is the basic work for obtaining the optimal centralized control policy by decentralized optimization.

Cohen and Lee (1988) develop a comprehensive stochastic model framework for linking material management activities throughout production-distribution systems. The framework is decomposed into four stochastic sub-models which are material control, production, stockpile inventory and distribution. Another modelling framework in managing material flows in decentralized supply chains by Lee and Billington (1993) describes a pull-type, periodic, order-up-to inventory system in which the review period and the order-up-to quantity are decision variables.
Table 2.2: A summary of some approaches for supply chain performance design

<table>
<thead>
<tr>
<th>Authors and References</th>
<th>Model Type</th>
<th>Echelons</th>
<th>Decision variables</th>
<th>Inventory policy</th>
<th>Solution techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pyke &amp; Cohen (1993)</td>
<td>Single product, finite period</td>
<td>Three levels</td>
<td>Reorder point, order-up-to level</td>
<td>Local inventory, centralized control</td>
<td>Approximation</td>
</tr>
<tr>
<td>Pyke &amp; Cohen (1994)</td>
<td>Multi product, finite period</td>
<td>Three levels</td>
<td>Reorder point, order-up-to level</td>
<td>Local inventory, centralized control</td>
<td>Approximation</td>
</tr>
<tr>
<td>Graves et al. (1998)</td>
<td>Single product, finite period</td>
<td>Single stage</td>
<td>Performance measures on production and inventory</td>
<td>Safety stock</td>
<td>Lagrangian relaxation</td>
</tr>
<tr>
<td>Ganeshan (1999)</td>
<td>Single product, finite period</td>
<td>Three levels</td>
<td>Reorder point, Order quantity</td>
<td>(s, Q) type, centralized control</td>
<td>Simulation</td>
</tr>
<tr>
<td>Clark &amp; Scarf (1960)</td>
<td>Single product, finite period</td>
<td>Two echelons</td>
<td>Order size</td>
<td>Echelon inventory, decentralized control</td>
<td>Recursive decomposition</td>
</tr>
<tr>
<td>Federgruen &amp; Zipkin (1984a)</td>
<td>Single product, infinite horizon</td>
<td>Two echelons</td>
<td>Order size</td>
<td>Echelon inventory, decentralized control</td>
<td>Approximation</td>
</tr>
<tr>
<td>Federgruen &amp; Zipkin (1984b)</td>
<td>Single product, finite horizon</td>
<td>Two echelons</td>
<td>Order quantity allocation</td>
<td>Local inventory, decentralized control</td>
<td>Recursive decomposition</td>
</tr>
<tr>
<td>Cohen &amp; Lee (1988)</td>
<td>Multi Product, finite horizon</td>
<td>More than three echelons</td>
<td>Reorder point, Order quantity</td>
<td>Local inventory, decentralized control</td>
<td>Decomposition</td>
</tr>
<tr>
<td>Lee &amp; Billington (1993)</td>
<td>Multi Product, finite horizon</td>
<td>More than three echelons</td>
<td>Order up to level or service level</td>
<td>Local inventory, decentralized control</td>
<td>Approximation</td>
</tr>
</tbody>
</table>

The model is used to determine the material ordering policy by calculating the required stock levels to achieve a target service level, or to determine the service level for a given material ordering policy. Lee et al. (1993) present a stochastic, periodic review, order-up-to inventory model to develop a procedure for process localization in the supply chain. The objective of this research is to design the product and production processes that are suitable for different markets in order to achieve the lowest cost and highest customer service levels. A summary of literature on supply chain performance design can be seen in Table 2.2.
2.3 Modelling and analysis of supply chain uncertainty

In almost any industry, inventories are needed to protect against uncertainty in the real world. Thus, to establish a proper inventory control policy, we should identify the variability in the system. Generally speaking, there are three distinct sources of uncertainty that affect a supply chain: suppliers, manufacturing, and customers (see Davis (1993)). Uncertainty can propagate through a supply chain. The typical uncertainty in supply chain operations is presented as demand uncertainty and information distortion along the chain. The traditional way of coping with uncertainty, caused by supplier unreliability and unpredictable customer demand in each stage of the supply chain, is to build inventories or to provide excess capacity. Much literature discusses the issues of supply chain uncertainty based on uncertainty from customer demand. We review the literature focusing on two aspects: identifying the sources of supply chain uncertainty and examining approaches to cope with them.

The bullwhip effect and supply chain uncertainty

A very well addressed phenomenon in many supply chains is that the variability of buyers’ demand increases as one moves up a supply chain, which is largely an effect of the ordering policies. This phenomenon, known as the “bullwhip effect”, makes supply chain planning difficult. The bullwhip effect is a major concern for many manufacturers, distributors and retailers because it usually leads to excess safety stock, increased logistics costs and inefficient use of resources. The modelling and analysis of the bullwhip effect often follow the following framework: given the timely demand of the downstream stage, a cost minimizing or profit maximizing objective function is formulated to decide the optimal control policy of the stage studied, and then the order process is deduced to demonstrate the variance amplification.

To illustrate such a general modelling framework, we introduce a formulation based on a two-stage supply chain with one manufacturer and \( N \) identical retailers (see details in Baganha and Cohen (1998)). The managerial objective is to determine the order quantities for each period and location so as to minimize the total expected discounted cost. There are two cost components in the objective function: retailers—\( RC_i \) and manufacturer—\( MC \).

\[
\text{Minimize } E \left\{ \sum_{i=1}^{N} RC_i + MC \right\}.
\]  

(2.18)

In order to deduce the ordering policies of the retailers and manufacturer, we should treat the cost-minimization problems of retailers and manufacturer separately. The sub-problems of the
retailers and manufacturer can be the general model formulated for a decentralized system in Section 2.2 (see Eq.(2.14)), or a discounted inventory model proposed by Heyman and Sobel (1984). The customer demand at retailer $i$ is defined as an AR($p$) process as follows:

$$D_{i,t} = d + \sum_{j=1}^{p} \phi_j D_{i,t-j} + \alpha_t,$$  \hspace{1cm} (2.19)

where $d > 0$, $-1 < \phi_j < 1$ are constant, and $\alpha_t$ is the random disturbance term which is independent and identically distributed with mean 0 and variance $\sigma$. It is also assumed that $\sigma$ is significantly smaller than $d$, so that the probability of a negative demand is negligible. Each retailer orders a single item from the manufacturer each period. All the retailers and manufacturer adopt the order-up-to policies with a periodic review procedure. The excess demand is backlogged. The order quantity of the retailer $i$ ($Q_{i,t}$) and the order quantity of the manufacturer ($R_t$) can be derived from the optimal inventory sub-problems. The "bullwhip effect" arises when $\text{Var}(R_t) \geq \text{Var}(\sum_{i=1}^{N} Q_{i,t})$ and $\text{Var}(Q_{i,t}) \geq \text{Var}(D_{i,t})$ (note that this phenomenon only arises under some conditions).

Kahn (1987) firstly presents a model of production decisions with demand uncertainty to show that the variance of production exceeds the variance of sales under the optimal behavior by the firm, either if demand is positively correlated, or if the excess demand can be backlogged. This phenomenon is now known as the "bullwhip effect", pointed out by Lee et al. (1997a. b). Kahn defines customer demand as an AR(1) process (see Eq.(2.19) when $p = 1$) and uses a maximizing discounted profit model to determine the manufacturer’s optimal policy. Lee et al. (1997a, b) also use similar approaches to demonstrate how demand forecasting can cause the bullwhip effect. In addition, they identify the causes of the bullwhip effect: the rationing game, order batching, and price variations. Chen et al. (1998, 2000a, and 2000b) demonstrate how a moving average forecast or an exponential forecast can induce a bullwhip effect in a two-stage supply chain, and quantify the bullwhip effect by the size of the increase in customer demand variability. The causes of the bullwhip effect are identified as demand forecasting, lead time, batch ordering, supply shortage and price variation. In their models, they consider two forecasting techniques: exponential smoothing and moving average forecast, under two types of demand processes: a correlated demand (Eq.(2.19) when $p = 1$) and a linear trend demand (see Eq.(2.20)).

$$D_t = d + bt + \alpha_t,$$  \hspace{1cm} (2.20)

where $d$ represents the constant demand level at time $t = 0$, $b$ is the linear trend factor, and $\alpha_t$ is the random error term which is i.i.d with mean 0 and variance $\sigma$. Their research results suggest that the increase in variability does exist for both demand processes, and exponential
smoothing forecasts lead to a larger increase in variability than moving average forecasts for certain demand processes, i.e., i.i.d. demands or demand with a linear trend. Schultz (1983) gives computational formulas for the mean, variance, and autocorrelation function of the demand process at a warehouse supplying \( N \) independent stores in parallel. The warehouse employs the \((s, S)\) inventory replenishment policy with backlogging to satisfy its i.i.d. demand. For a similar production-distribution system, Baganha and Cohen (1998) present a hierarchical model framework to analyze the stabilizing effect of inventories at different echelons. By comparing the variance of demand to the variance of replenishment orders at different echelons of the system, the analysis suggests that even though the bottom echelon (retailer or wholesaler) faces the normally distributed demand, the order quantity of the upper echelons at each time period would autocorrelate with a certain lag of the following consecutive time series. Furthermore, if the order quantities at different time periods are positively correlated \((\phi_j \geq 0 \text{ in (2.19)})\), there will be a variance amplification, and there will not be amplification if negatively correlated \((\phi_j < 0 \text{ in (2.19)})\). Graves (1999) demonstrates demand amplification for a single-item two-stage inventory system with an integrated moving average demand process of order \((0, 1, 1)\).

An organization of supply chain is linked with an uncertain external environment by customer demand from one side and a raw material supplier from another side. Traditionally, attention has been focused on the uncertainty in customer demand. However, uncertainty is inherent in the market at the supplier side also, and the quantity and quality of raw material delivered from an external supplier may differ from those requested. Petrovic et al. (1998) use fuzzy sets to represent these two sources of uncertainty. Their paper describes supply chain fuzzy models and supply chain simulation under an uncertain environment. Van Der Vorst et al. (1998) identify three clusters of sources of uncertainty: order forecast horizon, input data, and administrative and decision processes, and some inherent uncertainties of operational processes. They suggest that the real time information systems will be a requirement for achieving efficient and effective supply chain management.

*Dealing with uncertainty*

The identification of supply chain uncertainty helps researchers go further to explore the strategies to deal with uncertainty. Lee et al. (1997a, b) and Chen et al. (1998) give some suggestions to eliminate the impact of the bullwhip effect. The key aspect of their suggestions is information sharing. To alleviate the detrimental impact of the bullwhip effect, they
propose some strategies with the requirements of information sharing and ordering coordination. Especially, Chen et al. (1998) stress that strategic partnerships can change the practice of information sharing and inventory management within the supply chain, and therefore can reduce or eliminate the impact of the “bullwhip effect”. The modelling and analysis of the benefits of information sharing and order coordination often follow such a framework: giving the cost-minimization or profit-maximization problems of all supply chain members, defining different scenarios by applying these strategies, finding the optimal policies under each scenario, and comparing the optimal policies.

Many parameters of a production-distribution system may have direct or indirect influence on the cause of bullwhip effect. By comparing the optimal production policies under different operational settings, some researchers investigate the ways to cope with the bullwhip effect in their modeling work. Metters (1997) utilizes a dynamic programming model to estimate the excess costs of the bullwhip effect. His paper indicates that the importance of reducing the bullwhip effect to a firm differs greatly depending on the specific business environment. The profits gained by eliminating the bullwhip effect is affected by the relative capacity and the structure of lost sales penalty cost. Furthermore, Kelle and Milne (1999) investigate how the parameters of a $(s, S)$ policy influence the variabilities of the retailer’s individual purchase order and aggregate order. The analysis suggests that small, frequent orders can reduce the effect of variability and uncertainty. This study is ratified by Cachon (1999). The research shows there are two strategies to reduce the supplier’s demand variance. The first, balancing retailer order intervals, is effective over a broad range of conditions. The second is to reduce the supplier’s demand variance by lengthening the retailer’s order interval and increasing batch size. This strategy is effective when there are few retailers and consumer demand variability is low. The information flow between a supplier and a retailer in a two-echelon supply chain is incorporated by Gavirneni et al. (1999) in a model that captures the supply chain’s capacitated setting. D’Amours et al. (1999) address the impact of information sharing on networked manufacturing. Chen (1998) quantifies the value of centralized demand information by considering two inventory policy settings with different informational requirement in a serial inventory system: echelon inventory policy and installation inventory policy. The computational study suggests that the value of centralized demand information increase with increasing number of stages, lead time, or batch sizes. Bourland et al. (1996) show the customer could reduce inventories if its supplier can offer more accurate demand information, or improve the reliability of its deliveries. Their results also show that inventory-related benefits are particularly sensitive to the variability of demand and the uncertainty of the supplier.
2.4 Modelling and analysis of supply chain coordination

Most manufacturing enterprises are organized as networks of manufacturing and distribution sites that procure raw materials, transform them into intermediate and finished products, and distribute the finished products to customers. The fundamental stages of these networks, procurement, production and distribution, have been managed independently, buffered by inventories. Historically, the resource management policy of each independent stage is the key issue studied by logistics research, of which the objective is to seek more effective and efficient management of resources to increase overall productivity in a resource-constrained environment. However, managing a supply chain is very different from managing an individual company. The inventory stockpiles at various sites, including both incoming materials and finished products, have complex interrelationships. Thus, coordination is the core mechanism to manage the supply chain as a whole.

Multi-echelon inventory control policies

Within operations management research, the supply chain concept grows largely out of two-echelon inventory models. Most of the research in the design and analysis of two echelon systems is based on the classic work of Clark and Scarf (1960), which discusses the optimal policies for a multi-echelon inventory problem. They introduce the concept of echelon stock. A firm's local inventory is its on-hand inventory, and its echelon inventory is its local inventory plus all inventory held lower in the supply chain. Clark and Scarf show that echelon base-stock policies are optimal in a periodic-review, finite-horizon setting. The result is extended to periodic-review, infinite-horizon models by Federgruen and Zipkin (1984).

Centralized stock information can be utilized through echelon stock policies. However, most research has focused on installation stock policies that use only local stock information. To deploy the two inventory control policies under different settings, some approaches are proposed in the recent literature. Let us introduce the formulations of echelon stock and local stock policies in a serial inventory systems with $N$ stages (the details can be seen in Chen (1998)). Stage 1 orders from 2, 2 from 3, ..., and stage $N$ orders from an outside supplier (denoted by $N + 1$) with unlimited stock. Each stage $i$ replenishes a stage-specific inventory level according to a stage-specific ($R_i$, $nQ_i$) policy: when the inventory level falls to or below a reorder point $R_i$, a minimum integer multiple of $Q_i$ (base quantity of stage $i$) is ordered to replenish the stock. Based on different stock policies, the reorder point can be the echelon

22
reorder point or local reorder point. If we assume that the base quantity \( Q_i \) is fixed, the decision variable should be the reorder point \( R_i \) at each stage. The objective function can be written as the following infinite-horizon discounted cost problem.

\[
\text{Minimize } C(R_1, \ldots, R_N) = E \left\{ \sum_{t=1}^{\infty} \beta^{t-1} \left[ \sum_{i=1}^{N} h_i IL_i(t) + (p + H_i)B(t) \right] \right\}.
\]  

(2.21)

In (2.21), \( IL_i(t) \) is the echelon inventory level at stage \( i \) (except the backorder) and \( B(t) \) is the backorder level at stage 1. \( p \) is backorder cost at stage 1. \( H_i \) is the local holding cost at stage \( i \), and \( h_i \) is the echelon holding cost with \( h_i = H_i - H_{i+1} \) (\( H_{N+1} = 0 \)). If we define the echelon stock \( ES_i(t) \) as echelon inventory position \( IL_i(t) \) plus orders in transit and outstanding backlogged orders at stage \( i \), the local stock \( IS_i(t) \) should be \( ES_i(t) - ES_{i-1}(t) \) for \( i \geq 2 \) while \( IS_1(t) = ES_1(t) \). The approaches to determine the optimal echelon reorder point and local reorder point are discussed in Chen and Zheng (1997) and Chen (1998).

In a supply chain, each entity is the division of the same firm. If the manager does not have perfect knowledge about the demand distribution or the firm faces a fluctuating demand environment, decentralized decision making is beneficial (see details in Chen(1999)). In a decentralized supply chain, each site manager optimizes his own performance while incentive misalignment arises. Lee and Whang (1999) discuss a set of corporate rules to mitigate the general problems of incentive misalignment in a decentralized supply chain. Three of these corporate rules, namely cost conservation, incentive compatibility and informational decentralizability are specified as properties of a performance measurement scheme in their paper. Most research suggests centralized information can improve the decentralized supply chain performance. One important mechanism for coordination in a supply chain is the information flows among members of the supply chain.

The coordination mechanisms can be drawn from comparing the optimal policies under different operational scenarios in supply chain modelling and analysis. Axsäter and Zhang (1999) investigate the retailer's joint replenishment policy in a two-level inventory system with a central warehouse and a number of identical retailers. The joint replenishment policy is compared to an installation stock and to an echelon stock policy. The result indicates that only small improvements can be achieved, and the installation stock policy or the echelon stock policy is a better alternative for the joint replenishment policy. Inventory “balancing” policy is examined by Mcgavin et al. (1997). In a one-warehouse \( N \)-retailer distribution system facing stochastic demand, they show that balancing is optimal if the retailers are identical, and balancing is generally not optimal for the nonidentical-retailer case. Chan and Simchi-Levi (1998) investigate cross-docking strategy in a distribution system consisting of a vendor. a fixed
number of warehouses and many retailers. With consideration of ordering and holding cost, Tzafestas and Kapsiotis (1994) give multi-level models to study the coordinated control of the manufacturer and suppliers under three different scenarios. Scenario I is that the manufacturer optimizes his own costs and imposes the resulting policy on the suppliers. In scenario II, all levels of the chain are assumed to cooperate in order to minimize the overall operational cost. Scenario III is that each level minimizes a local cost function in a decentralized way. The optimal policies of the production and inventory problem of "style goods", which have a short selling season, are studied by Kouvelis and Gutierrez (1997). They develop both the optimal centralized control policies and decentralized control policies, and demonstrate that the decentralized control policies with purchasing coordination are equally effective in terms of overall profitability as the centralized control policies.

Supply chain partnerships and restructuring

Supply chain management emphasizes the overall and long-time benefit of all parties in the chain through cooperation and information sharing. It is therefore different from traditional logistics management or purchasing management where to minimize the operation cost of a single owner is often the major concern. By coordinating different parties along the logistics network, or establishing business partnerships, supply chain management creates win-win situations for all players. Information sharing on a supply chain brings about a great advance in business connections, such as vendor managed inventory, cross-docking and postponement of products. The growing inter-company coordination makes logistics a corporate function. The result of this coordination is a mutually beneficial logistics partnership or strategic alliance (see Geoffrion and Powers (1995)). Companies are increasingly using strategic partnerships to create competitive advantages, and partnerships can also provide firms with the opportunity to improve their conduct of business by means of cooperation.

There are some modelling works which study the benefits of supply chain partnerships under some certain cooperation framework. Anupindi and Bassok (1999) consider a multi-decision maker system, and discuss if the interests of the manufacturers and retailers coincide under an inventory centralization policy, where the inventory at a central location is owned jointly by the retailers. Their analysis shows that centralization, while always beneficial to the retailers, does not always benefit the manufacturer. The benefit of coordination between the departments of manufacturing and supply within a firm is illustrated by Parlar and Weng (1997). If all demand for a product should be satisfied and the product has very short life cycles, there will
be a second run to meet unsatisfied demand. The coordination between manufacturing and supplying is beneficial when the material-related cost and unit production cost in the second run are higher than those in the first run. Li and O'Brien (1999) give a modelling work on assessing potential partners in a supply chain, which aims to improve supply chain efficiency and effectiveness under four criteria: profit, lead time, delivery and waste elimination.

With short product life cycles and uncertain, rapidly changing markets and technology, Quick Response (QR) has gained more and more attention. Fundamentally, QR refers to speed-to-market of products which move rapidly through the production and delivery channels. The impact of QR on the fashion apparel industry is given by Iyer and Bergen (1997). They build formal models to address who wins and who loses under QR. Their result shows that both the manufacturer and the retailer can obtain benefits from QR, i.e., Pareto improving. A case study by Perry et al. (1999) describes the QR program in the Australian textiles, clothing and footwear industry. They stress that multi-directional information sharing between all partners is very important for implementing a QR program.

To meet requirements of business competition, nowadays, many companies begin reconstructing their supply chains involving reducing transit times, centralizing inventories, and postponing product differentiation. A field research project was conducted by Kopczak (1998) to investigate how computer companies use logistics partnerships and supply chain reconstructing. The research shows that restructurers achieve greater cost and service benefits from the combination of restructuring and partnership formation than do nonrestructurers from partnership formation alone. The author indicates that supply chain restructuring will be the future trend for computer companies in forming supply chain partnerships.

In order to increase the flexibility and service level of the manufacturing system, product/process redesign is proposed to achieve the benefits. Delayed product differentiation offers such an opportunity. Lee and Tang (1997) formalize three basic approaches to make product differentiation delayed: standardization, modular design, and process restructuring. They develop a model to evaluate the costs and benefits of each basic approach. The benefits of the differentiation delayed include simplifying the manufacturing process, increasing the flexibility of buffer inventories, and improving the service level. Furthermore, Lee and Tang (1998) use a simple model to measure the variability of production under reversing the sequence of two consecutive stages in a supply chain provided that they are reversible, which is considered as a way of process restructuring. They suggest that operation reversal is a powerful tool to reengineer a supply chain. Replacing several products by a single common product is another approach to streamline the production process and reduce the required safety stock levels. Hillier (1999)
studies the concept of product commonality. In his paper, the effectiveness of product commonality is investigated by a general multiple-period model, of which the objective is to minimize total costs under two different scenarios—one that utilizes a common product and one that does not. The results show that commonality is beneficial under some circumstances, such as when there are more products replaced, when holding costs are high, and when the discount factor is low.

The lack, as pointed out by Maloni and Benton (1997), still exists in the area of modelling research of supply chain partnerships. Only a few features of supply chain partnerships have been studied in modelling research like QR systems and supplier-buyer (two-phase) relationships. Recent efforts of supply chain restructuring focus mainly on redesigning the processes of product design and production. The restructuring of other processes should also be considered, such as the processes involving a supply chain's transportation and distribution channel. Especially, the recent partnerships formed with logistics service providers present a great opportunity for modelling and analysis of process redesign.

*Supply chain game*

The optimal solution of the supply chain may not minimize an individual member's own costs. The manufacturer may care more than a retailer about consumer backorders for the manufacturer's product, or the retailer's cost to hold inventory may be higher than the manufacturer's. Under a framework of supply chain coordination, each member should cooperate with one another while each may face the temptation to deviate from any agreement to reduce its own costs. Game theory is applied to study such supply chain coordination issues in some literature. Generally, the approaches for this analysis would follow such a modelling framework: formulating the objective functions of each member as cost-minimization or profit-maximization problems, designing games under different cooperation scenarios, finding the Nash equilibrium in each game, comparing the equilibrium to each other, and identifying the gain or loss from different cooperation scenarios.

To investigate the preferences of echelon inventory and installation inventory control policies for supply chain members, Cachon and Zipkin (1999) conduct a game study in a two-stage serial supply chain with stationary stochastic demand and fixed transportation times. The result shows that the supplier prefers installation inventory, but the retailer's preference depends on the parameters of the game. If the supplier in a supply chain has limited capacity, retailers will compete for the scarce capacity. Cachon and Lariviere (1999a, 1999b) consider the allocation
game in such a supply chain. Some capacity allocation mechanisms are discussed in the two papers. They report that retailers will order more than they need to gain a more favorable allocation for a broad class mechanism. Van Mieghem (1999) values the option of subcontracting and outsourcing to improve financial performance and system coordination by analyzing a two-player, two-market stochastic game. The author shows that the higher complexity of subcontracting makes coordination more difficult compared to traditional outsourcing models.

In the real world, supply chain coordination often encounters supply chain partners' agreement deviation. The modelling research involving game theory can be a powerful tool for coordination mechanism testing in supply chain modelling and analysis.
<table>
<thead>
<tr>
<th>Authors and References</th>
<th>Model Types</th>
<th>Performance Measures</th>
<th>Coordinated Issues</th>
<th>Decision Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axsi&amp; Ko Zhang (1999)</td>
<td>Single period, one warehouse with N retailers</td>
<td>Total costs</td>
<td>Joint replenishment policy</td>
<td>Joint retailer reorder point and batch size</td>
</tr>
<tr>
<td>Cachon &amp; Larivere (1999a, 1999b)</td>
<td>Single period, one supplier with N retailers</td>
<td>Expected profit</td>
<td>Supplier’s capacity allocation</td>
<td>Retailer’s allocation</td>
</tr>
<tr>
<td>Cachon &amp; Zipkin (1995)</td>
<td>Multi-period, one supplier with one retailer</td>
<td>Total average cost per period</td>
<td>Echelon inventory vs. local inventory</td>
<td>Retailer’s base stock level</td>
</tr>
<tr>
<td>Chen (1999)</td>
<td>Multi-period, a serial chain with N divisions</td>
<td>Long-run average total costs</td>
<td>Decentralized local inventory policy</td>
<td>Order-up-to level of the base stock</td>
</tr>
<tr>
<td>Chen &amp; Zheng (1994)</td>
<td>Multi-period, one warehouse with N retailers</td>
<td>Average holding and backorder costs</td>
<td>Centralized echelon inventory policy</td>
<td>Retailer’s reorder point and batch size</td>
</tr>
<tr>
<td>Chen &amp; Zheng (1997)</td>
<td>Multi-period, a serial chain with N stages</td>
<td>Average holding and backorder costs</td>
<td>Centralized echelon inventory policy</td>
<td>Retailer’s reorder point and batch size</td>
</tr>
<tr>
<td>Hillier (1999)</td>
<td>Multi-period, one make-to-order system</td>
<td>Expected discounted costs</td>
<td>Product commonality</td>
<td>Order-up-to level</td>
</tr>
<tr>
<td>Iyer &amp; Bergen (1997)</td>
<td>Single period, one manufacturer with one retailer</td>
<td>Expected profit</td>
<td>Quick response policy with lead time reduction</td>
<td>Retailer’s order quantity</td>
</tr>
<tr>
<td>Kouvelis &amp; Gutierrez (1997)</td>
<td>Newsvendor model in a two-market system</td>
<td>Total costs</td>
<td>Centralized vs. decentralized control policies</td>
<td>Quantity shipped and produced for the secondary market</td>
</tr>
<tr>
<td>Lee &amp; Tang (1997)</td>
<td>Single period, N stages of the production system</td>
<td>Total costs</td>
<td>Delayed product differentiation</td>
<td>The point of product differentiation</td>
</tr>
<tr>
<td>Lee &amp; Tang (1998)</td>
<td>Single period, two consecutive stages of the production system</td>
<td>Variability of production volumes</td>
<td>Product/process redesign</td>
<td>N/A</td>
</tr>
<tr>
<td>McGavin et al. (1997)</td>
<td>Multi-period, one warehouse with N retailers</td>
<td>Total costs</td>
<td>Balancing retailers’ inventories</td>
<td>Stocks shipped from warehouse to retailer</td>
</tr>
<tr>
<td>Parlar &amp; Weng (1997)</td>
<td>Newsvendor model, one supplier with one manufacturer</td>
<td>Expected profit</td>
<td>Joint information coordination</td>
<td>Order and production quantity</td>
</tr>
</tbody>
</table>
2.5 Potential research opportunities

According to this review, there are rich opportunities in all three featured areas of supply chain modelling research. We will focus on these three key issues on modelling and analysis of supply chain systems.

Modelling and analysis on supply chain design

For supply chain network, the stochastic factors should be concerned in the design model. Exploring multi-period MIP models with normally distributed customer demand, like Cole’s work (1995), is a good practice in this field. The lack of complementary work on supply chain performance design is that the approaches only consider single stage or at most two stage models. Integrated approaches to make supply chain performance design decisions at all stages of the supply chain need to be developed. If some stochastic factors (like timely customer demand and inventory level) are introduced to the traditional MIP model, some constraints such as service level and facility capacity will change their roles in supply chain structure design models. Such models will also become the integration of supply chain structure design and performance design. The development of mixed-integer stochastic programming models will be helpful to explore more comprehensive approaches for both supply chain structure and performance design. The consideration of BOM constraints is another interesting topic. The logical interactions among the three basic supply chain operation processes, say, procurement, production and distribution should be formulated as constraints in the current mixed integer programming models. To reflect the logical consistent relationships between the supplier and manufacturer, BOM can play a useful role in supply chain design models.

Modelling and analysis on supply chain uncertainty

Second, the novelty of modelling supply chain uncertainty requires analysis of the uncertainty from the supply side. Most of the previous research focuses on demand uncertainty. The modelling study on supply chain uncertainty from the supplier’s side is required, such as the variability of lead time and the supplier’s uncertainty. Demand uncertainty from the customer’s side may prompt the manufacturer and supplier to hold excess inventory, while uncertainty from the supplier’s side may lead to an increase in downstream supply chain members’ inventory. However, there is a lack of attention to the modelling and analysis of supply chain uncertainty.
from the supplier.

Modelling and analysis on supply chain coordination

Third, the use of information sharing appears to be a viable strategy to deal with supply chain uncertainty. However, effective modelling of supply chain partnership issues are required to measure the effects and benefits of information sharing and supply chain partnerships. From an individual partner’s perspective, he/she may hesitate to share certain information since this could lead to competitive advantage loss for the partner. There is a need to develop effective mechanisms to motivate the partner’s willingness to share essential information. Even though the supply chain partnership is a promising way to establish these mechanisms, rigorous testing of different coordination mechanisms under supply chain partnerships is urgently needed in supply chain modelling.
Chapter 3

Incorporating logical constraints in supply chain design models

3.1 Introduction

The formulation of strategic production-distribution models for supply chain design has been a popular research topic in the field of Supply Chain Management for two decades. Most formulations are in the form of mixed integer programming (MIP) models. The problem commonly arises in the following scenario: a number of production plants supply a collection of distribution centers (DCs) with multiple products, which, in turn, supply customers with specified demand quantities of the different products. The challenge is to determine the number, location, capacity, and type of production plants and DCs to use so as to minimize the total cost or maximize the after-tax profit of the supply chain. For the entire supply chain, it is also necessary to select the set of suppliers (or vendors), hence to determine the number of vendor supply contracts. A diversity of mathematical programming models dealing with different issues of production and distribution can be found in literature.

In their pioneering paper, Geoffrion and Graves (1974) describe a multi-commodity single-period production-distribution problem and solve it by Benders Decomposition. This is probably the first paper that presents a comprehensive MIP model for the strategic design of supply chains. The main contribution of this paper is the proposed method of solution to the problem under study. Subsequently, Geoffrion et al. (1978) present a status report on research into strategic distribution system planning based on decomposition techniques. In both papers, the
vendor factors are not considered.

Cohen and Lee (1988) develop a comprehensive modelling framework for linking material management activities throughout the material production-distribution supply chain. The framework consists of four stochastic sub-models. Under some assumptions, they are able to optimize each sub-model. However, the optimization of all the sub-models collectively is intractable. A heuristic procedure is presented and demonstrated to be able to yield quality approximate optimal policies. A dynamic, nonlinear MIP model presented by Cohen et al. (1989) is concerned with the operation of a network of vendors, plants, and markets. The main contribution of this model is the explicit inclusion of vendor supply contracts in the model. Bills of materials are mentioned, but they are simply treated as material requirement balance constraints in the model. Cohen and Lee (1989) present a single-period multi-commodity model that analyzes resource deployment decisions for an international firm. This model is a simplified version of the model presented by Cohen et al. (1989). It considers plant production capacity, material requirements at each plant (major components and sub-assemblies based on BOM), balance constraints at plants and DCs, demand limits, feasible flow constraints, and capacity of suppliers. BOM are treated as a source of production data for the development of consistency constraints that balanced the material flows among vendors, plants, and DCs. Arntzen et al. (1995) consider BOM as a collection of rooted arborescence, on which each vertex represents a product and the fabrication facility. The contribution of this paper is the inclusion of BOM constraints and duty considerations in an international supply chain. But it is not clear how the vendors and DCs are considered in the formulation. In a recent review, Vidal and Goetschalckx (1997) highlight that there is a lack of models with the inclusion of BOM constraints. They argue that BOM should be considered as constraints in a strategic production-distribution model while designing a complete supply chain, but they concede that it is difficult to formulate such constraints in a mathematical model.

Most past efforts consider the coordination between supply and production-distribution activities separately. As a result, BOM just act as some consistency constraints and many of the complex supply chain inter-relationships are ignored, despite that some researchers such as Arntzen et al. (1995) and Cohen and Lee (1989) suggest BOM be exploited to coordinate the behavior of suppliers with the production and distribution activities. The set of vendors was seldom included under the assumption they were less involved in the coordination with production and distribution. But in reality it is necessary to consider vendors in the comprehensive design of a supply chain. BOM provide key information for coordinating the activities between material procurement and production planning. The introduction of BOM as constraints in MIP-based supply chain design models will be a reasonable way to consider the selection of
vendors, especially for assembly systems, where different parts making up a finished product may come from different locations or even countries. The challenge is to find a way to effectively include BOM in the strategic supply chain design model. The early researchers often focus their attention on formulating the material balance constraints between vendors and plants. However, this is far from an effective way to truthfully reflect the close relationships between vendors and manufacturers.

Recently, with new developments in computational logic and integer programming, logical rules and propositions play an increasingly important role in mathematical programming models. Such simple logical constraints as “if A is true, then either B or C must be true” have been explicitly considered in mathematical programming problems (see Yan and Hooker 1999). Introducing logical constraints in an MIP model will not only achieve substantial savings in solution time (see Hooker et al. 1994 and Raman and Grossmann 1993), but also enable adequate descriptions of relevant relationships among the entities involved in the model.

Reflecting intimate material supply relationships with vendors, the bottom level of BOM can be described as a list of specified quantities of materials or components supplied by different vendors. According to the types and capacity limits of vendors and plants, we can see that each candidate vendor can provide a subset of materials (or components) in a finite amount and each proposed plant can produce a subset of products with throughput limit. To fulfill the customer demand, we can incorporate BOM into the logical rules, which express the logical relationships between the vendors and plants. One of these logical rules can be presented as “if product $i$ will be produced in at least $s$ of the proposed plants, then at least $t$ of the candidate vendors must be selected”. This formulation of BOM makes it possible to capture the role of vendors in the strategic supply chain design model. We will show how BOM logical rules are formulated as constraints in our proposed MIP model to assist in the selection of vendors, and how linear representation of logical constraints is developed. A numerical example is solved by LINDO to illustrate the effectiveness of including BOM constraints for vendor selection.

### 3.2 Model formulation

The MIP model developed in this section aims to select suppliers from a candidate set of material (or component) vendors, as well as to locate a given number of production plants and DCs, subject to plant and DC capacity restrictions. We assume that the customer zone locations and their specific demand estimates for multiple products are given in advance. The potential plant and DC locations as well as their capacities are also known. We present a multi-commodity.
multi-echelon, and single-period MIP model with the objective of minimizing the total cost. with consideration of BOM logical constraints for vendor selection.

3.2.1 Parameter notations and definitions

Before formulating the model, we introduce the basic parameter notations and definitions. In this study, we use the following indices: \( j \in J \), a set of candidate vendors; \( k \in K \), a set of potential plants; \( l \in L \), a set of possible distribution centers; \( n \in N \), a set of customer zones; \( m \in M \), a set of materials (or components) needed for production; and \( i \in I \), a set of products. We define the problem parameters and decision variables as follows.

I. Parameters:

\[
\begin{align*}
FO_j & \quad \text{fixed ordering cost of vendor } j. \\
g_k & \quad \text{fixed cost to open and operate plant } k. \\
f_l & \quad \text{fixed cost to open and operate DC } l, \\
OC_{mjk} & \quad \text{unit ordering cost of material (or component) } m \text{ from vendor } j \text{ to plant } k, \\
PC_{ik} & \quad \text{unit production cost of product } i \text{ in plant } k, \\
TC_{ikl} & \quad \text{unit transportation cost of product } i \text{ from plant } k \text{ to DC } l, \\
DC_{iln} & \quad \text{unit distribution cost of product } i \text{ from DC } l \text{ to customer zone } n, \\
CV_{mj} & \quad \text{capacity limit of material (or component) } m \text{ of vendor } j, \\
CP_k & \quad \text{capacity limit of plant } k, \\
LC_{ik}, UC_{ik} & \quad \text{lower and upper production capacity limits of product } i \text{ in plant } k. \\
W_l & \quad \text{throughput limit of DC } l, \\
D_{in} & \quad \text{demand of product } i \text{ in customer zone } n.
\end{align*}
\]

II. Decision variables:

\[
\begin{align*}
G_{mjk} & \quad \text{total units of material (or components) } m \text{ purchased from vendor } j \text{ for plant } k, \\
H_{iln} & \quad \text{total units of product } i \text{ distributed from DC } l \text{ to customer zone } n, \\
F_{ikl} & \quad \text{total units of product } i \text{ shipped from plant } k \text{ to DC } l, \\
\tilde{x}_j & \quad \text{a 0-1 variable indicating whether vendor } j \text{ is selected } (\tilde{x}_j = 1) \text{ or not } (\tilde{x}_j = 0), \\
\hat{y}_k & \quad \text{a 0-1 variable indicating whether plant } k \text{ is open } (\hat{y}_k = 1) \text{ or not } (\hat{y}_k = 0).
\end{align*}
\]
\( \tilde{z}_l \) a 0-1 variable indicating whether DC \( l \) is open (\( \tilde{z}_l = 1 \)) or not (\( \tilde{z}_l = 0 \)).

\( x_{mj} \) a 0-1 variable indicating whether a contract for material (or component) \( m \) is entered into with vendor \( j \) (\( x_{mj} = 1 \)) or not (\( x_{mj} = 0 \)).

\( y_{ik} \) a 0-1 variable indicating whether any amount of product \( i \) is produced in plant \( k \) (\( y_{ik} = 1 \)) or not (\( y_{ik} = 0 \)).

\( z_{il} \) a 0-1 variable indicating whether any amount of product \( i \) is delivered to DC \( l \) (\( z_{il} = 1 \)) or not (\( z_{il} = 0 \)).

The problem is to select a subset of vendors according to the BOM for supplying some materials (or components) required, and to choose a subset of plants and DCs to open for producing and distributing some products to satisfy the demand requirements at the customer zones in order to minimize the overall supply chain management cost. For each open plant and DC, a decision must be made on the total units of products that need to be transported from the open plant to the open DC and the total units of products that need to be distributed from the open DC based on the given service level.

### 3.2.2 The objective function

The total cost of the supply chain includes purchasing cost, production cost, transportation and distribution cost, and fixed costs such as the fixed ordering cost, the fixed cost to open and operate a plant, and the fixed cost to open and operate a DC. Therefore, the objective function to be minimized is given by:

\[
\text{Min } Z = \sum_{m,j,k} OC_{mjk}G_{mjk} + \sum_{i,k,l} PC_{ik}F_{ikl} + \sum_{i,k,l} TC_{ikl}F_{ikl} + \sum_{i,l,n} DC_{iln}H_{iln} + \sum_{j} FO_{j} \tilde{x}_{j} + \sum_{k} g_{k} \tilde{y}_{k} + \sum_{l} f_{l} \tilde{z}_{l} \tag{3.1}
\]

The first term in the objective function is the total purchasing cost of materials and components from all vendors. The second term is the total production cost in all plants. The third and fourth terms are the total transportation cost (from plants to DCs) and distribution cost (from DCs to customer zones). The remaining three terms are the various fixed costs including the fixed ordering cost, the fixed cost to open and operate plants, and the fixed cost to open and operate DCs.
3.2.3 The constraints

Like any other MIP supply chain design model, many constraints need to be considered, including the balance constraints of materials (or components) and products, the capacity limit constraints, the throughput limit constraints, and the service level constraints. They will be discussed under constraints I to V below. We also introduce some new constraints concerning the logical rules involving BOM and ensuring logical consistency. These will be discussed under constraints VI and VII below.

I. Materials requirements:

\[ R_{mi} \]  
units of material (or component) \( m \) required to produce one unit of product \( i \) according to the product BOM.

The materials (or components) balance constraint is:

\[ \sum_{i,l} F_{ikl} R_{mi} \leq \sum_j G_{mjk}, \quad \text{for all } k, \ m. \quad (3.2) \]

II. Vendor’s capacity limits:

The capacity limits, for vendor \( j \), can be formulated as:

\[ \sum_k G_{mjk} \leq CV_{mj} x_{mj}, \quad \text{for all } m. \quad (3.3) \]

III. Production capacity limits of plants:

There are lower and upper production capacity limits of product \( i \) in plant \( k \). So, for each plant \( k \), the production capacity limits constraints are:

\[ LC_{ik} y_{ik} \leq \sum_l F_{ikl} \leq UC_{ik} y_{ik}, \quad \text{for all } i. \quad (3.4) \]

IV. The throughput limit constraints:

For DC \( l \), the throughput limit is:

\[ \sum_n H_{itn} \leq W_{it} z_{i,t}, \quad \text{for all } i. \quad (3.5) \]

For plant \( k \), the throughput limit is:

\[ \sum_{i,l} F_{ikl} \leq CP_k. \quad (3.6) \]

(Note: In each candidate plant site, we must have \( \sum_i UC_{ik} = CP_k \).)
V. Service level constraints:

\[ \alpha: \quad \text{percentage of customer demands met.} \]

Under the circumstances of deterministic customer demands, the customer service level can be defined as a percentage of the total demand met. We can formulate the service level constraints as:

\[ \sum_{l} H_{l in} \geq \frac{\alpha}{100} D_{in}, \quad \text{for all } i, \ n, \quad (3.7) \]

where

\[ \sum_{k} F_{ikl} \geq \sum_{n} H_{iln}, \quad \text{for all } i, \ l, \quad (3.8) \]

VI. BOM logical constraints:

Taking into consideration the types and capacity limits of vendors and plants, we find that the logical relationships between the candidate vendors and proposed plants can be connected by BOM. Each candidate vendor can provide a subset of materials (or components) in a finite amount and each proposed plant can produce a subset of products with throughput limit, while generally the bottom level of BOM can be described as a list of specified quantities of materials or components needed by the set of products. Considering the customer demands, we can incorporate BOM into such logical rules as “if at least \( p \) of the proposed plants are open, then at least \( q \) of the candidate vendors should be chosen” and “if product \( i \) will be produced in at least \( s \) of the proposed plants, then at least \( t \) of the candidate vendors should be selected to supply the material \( m \) according to the BOM”. These two types of logical rules are represented in the following forms, known as cardinality rules (see McKinnon and Williams 1989):

\[ (\bar{y}_1 \lor \bar{y}_2 \lor \ldots \lor \bar{y}_K)_p \Rightarrow (\bar{x}_1 \lor \bar{x}_2 \lor \ldots \lor \bar{x}_J)_q \quad (3.9) \]

and

\[ (\bar{y}_1 \lor \bar{y}_2 \lor \ldots \lor \bar{y}_K)_s \Rightarrow (\bar{x}_m \lor \bar{x}_m \lor \ldots \lor \bar{x}_m)_t. \quad (3.10) \]

The number of open plants is subject to the plant type and the throughput limit of each plant. These logical rules will be discussed in more detail and their formulation illustrated in Section 3.3.

VII. Logical consistency constraints:

Another important set of constraints represents the fact that if a product is to be produced, then the corresponding type and amount of parts or materials must be supplied. This
set of constraints is called logical consistency constraint. For example, for vendor \( j \), the variables \( x_{mj} \) should be consistent with \( \tilde{x}_j \) in the MIP model. The logical rule representing this consistency relationship is “if at least one \( x_{mj} \) is true (\( x_{mj} = 1 \)), then \( \tilde{x}_j \) is true”. Such logical consistency also exists for the binary variables related to plants and DCs. We list the relevant logical consistency constraints in the following:

1. Whether a vendor is selected is subject to if any amount of materials (or components) is purchased from it. The material purchasing indicators \( x_{mj} \) should be consistent with the vendor selection indicators \( \tilde{x}_j \). So, we have

\[
(x_{1j} \lor x_{2j} \lor \ldots \lor x_{Mj}) \Rightarrow \tilde{x}_j.
\] (3.11)

It means if any of the indicators \( x_{mj} \) is true (some amount of material \( m \) is ordered from vendor \( j \)), then \( \tilde{x}_j \) is true (vendor \( j \) is selected).

2. Whether a plant should be open is subject to if any product is produced in that plant. This can be formulated into logical consistency constraints as follows.

\[
(y_{1k} \lor y_{2k} \lor \ldots \lor y_{Ik}) \Rightarrow \tilde{y}_k.
\] (3.12)

3. The same logic can be applied to DCs. If any product is distributed from a DC, then it is open. Thus,

\[
(z_{1l} \lor z_{2l} \lor \ldots \lor z_{ll}) \Rightarrow \tilde{z}_l.
\] (3.13)

The logical rules (3.11), (3.12), and (3.13) above are also cardinality rules.

### 3.3 BOM logical constraints

As basic input to manufacturing resources planning, BOM play a pivotal role in production planning and scheduling. In supply chain management, BOM also play a fundamental role in the chain design. However, there does not exist a formal and consistent way to represent BOM constraints in strategic supply chain design models (see Vidal and Goetschalckx 1997), although some researchers (such as Arntzen et al. 1995 and Cohen and Lee 1989) proposed that BOM information be exploited to coordinate the behavior of suppliers with the production and distribution activities of a manufacturer. In fact, many important interactions and their logical consistency among the entities of a supply chain can be captured via expressing them as BOM logical constraints.
Based on recent advances in computational logical integer programming (Williams and Brailsford 1996, Yan and Hooker 1999), we propose to capture BOM information as logical constraints and introduce them in a strategic supply chain design model. The introduction of BOM as constraints in MIP models will enable the inclusion of vendor selection in the strategic design of supply chains.

In a supply chain design model, the limits of the vendor’s capacity are commonly expressed as constraints for vendor selection. Generally, BOM can be described as a hierarchical product structure that specifies the quantity and lead time of each item, ingredient, or material needed to assemble, mix, or produce the end product. The bottom level of BOM consists a set of materials (or components) supplied by vendors. Considering other related important constraints such as vendor’s type, plant’s type, and plant’s throughput limits, we identify relevant logical relationships between vendors and production plants. Such relationships can be expressed as cardinality rules which say that “if at least \( p \) of the proposed plants are open, then at least \( q \) of the candidate vendors should be chosen”, or “if product \( i \) will be produced in at least \( s \) different plants, there are at least \( t \) vendors to be chosen for supplying the materials (or components) of product \( i \)”. In the following, we discuss these two types of logical rules, respectively.

A. Consider all products with all vendors and plants involved.

For a multi-commodity supply chain model as discussed in Section 2, a set of products is to be produced to meet the demands of customer zones. From the BOM of these products, we obtain information about a set of materials (or components) supplied by vendors. Each vendor can only supply a subset of these materials (or components) within its capacity limit. Meanwhile, each plant only produces a subset of products demanded within its throughput limit. Considering the demands in customer zones, we can state the logical rule “if at least \( p \) of the proposed plants are open, then at least \( q \) of the candidate vendors should be chosen”, and formulate it as
\[
(y_1 \lor y_2 \lor \ldots \lor y_K)_p \Rightarrow (x_1 \lor x_2 \lor \ldots \lor x_J)_q \quad \text{(see constraint VI)}.
\]

In the model presented in Section 2, we use \( x_j \) as the 0-1 indicator for candidate vendors and \( y_k \) as the 0-1 indicator for potential plants.

B. Consider one product with certain vendors and plants involved.

Each product has its own set of materials (or components) as input for production. According to plant type, certain types of products can be produced in each plant. And, at the same time, each vendor can only supply certain materials (or components). Therefore, for a particular product, it can be produced only by a subset of proposed plants, and its
materials (or components) can only be supplied by a subset of candidate vendors. We can formulate this constraint as a logical rule “if product $i$ will be produced in at least $s$ different plants, there are at least $t$ vendors to be chosen for supplying the materials (or components) of product $i$” represented by

$$(y_{i1} \lor y_{i2} \lor \ldots \lor y_{ik})_s \Rightarrow (x_{m1} \lor x_{m2} \lor \ldots \lor x_{mJ})_t \text{ (see constraint VI).}$$

We can see, from the above, that applying logical rules to formulate BOM constraints is a natural way to take BOM into account in a supply chain design model. They precisely reflect the inherent logical relationships among products, vendors, and plants. In addition, some related factors can also be stated as logical rules formulated via BOM constraints such as plant type, vendor’s capacity, throughput limit, and customer demand. To illustrate how to formulate BOM as logical constraints, an example is given at following. The BOM data of this example are from the illustration problem in Section 3.5.

**An Example of Formulating BOM Logical Constraints**

Consider a simplified supply chain operated by an international computer company located in Southeast Asia. The supply chain includes four candidate vendors, three proposed plants, three potential DCs, and four customer zones (please see the solution of this example in Section 3.5). The total demands for two products of the four customer zones are: A (personal computer) 300 units and B (server) 250 units. The BOM of the two products are illustrated in Figure 3.1.

![Figure 3.1: The BOM of products A and B](image)

Table 3.1 gives the vendor type and vendors’ capacity limits.

---

40
Table 3.1: Vendor type and their capacity limits for the example

<table>
<thead>
<tr>
<th>Vendors</th>
<th>Components supplied (capacity limits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>$a_2 (300), b_2 (250)$</td>
</tr>
<tr>
<td>$V_2$</td>
<td>$a_1 (300), a_1 (300), b_1 (250), b_1 (250)$</td>
</tr>
<tr>
<td>$V_3$</td>
<td>$a_1 (300), a_1 (300), b_1 (250), b_1 (250)$</td>
</tr>
<tr>
<td>$V_4$</td>
<td>$a_1 (300), b_1 (250)$</td>
</tr>
</tbody>
</table>

Table 3.2: Plant type and their throughput limits for the example

<table>
<thead>
<tr>
<th>Plants</th>
<th>Plant type (throughput limits)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>Product A (300 units)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Product A and B (600 units)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>Product B (300 units)</td>
</tr>
</tbody>
</table>

Information about the plant type and plants’ throughput limits is given in Table 3.2.

From the data in Table 3.1 and Table 3.2, together with demands of customer zones, we can formulate the following logical constraints based on BOM considerations.

- For both products A and B, at least one plant ($P_2$) should be open in order to satisfy customer demands. And, according to the BOM, at least two vendors should be selected for supplying the components needed. This can be expressed as a logical rule “if at least 1 of the proposed plants is open, then at least 2 of the candidate vendors should be chosen”, formulated as

  $$(\bar{y}_1 \lor \bar{y}_2 \lor \bar{y}_3) \Rightarrow (\bar{x}_1 \lor \bar{x}_2 \lor \bar{x}_3 \lor \bar{x}_4)_2.$$  \hspace{1cm} (3.14)

- For each product A or B, if it is produced in a particular plant, the necessary components should be ordered from the corresponding vendors. We use $y_{A,P_1}$ to indicate if product A is produced in plant $P_1$ and $x_{a_1,V_2}$ to indicate if component $a_1$ is ordered from vendor $V_2$. Based on the BOM of product A and vendor type information given in Table 3.3, we have the following logical constraints

  $$(y_{A,P_1} \lor y_{A,P_2}) \Rightarrow (x_{a_1,V_2} \lor x_{a_1,V_4}),$$

  $$y_{A,P_1} \lor y_{A,P_2} \Rightarrow (x_{a_2,V_1} \lor x_{a_2,V_3}),$$

  $$y_{A,P_1} \lor y_{A,P_2} \Rightarrow (x_{a_3,V_1} \lor x_{a_3,V_2}).$$
and

\((yA, P_1 \lor yA, P_2) \Rightarrow (x_{a_3}, v_2 \lor x_{a_3}, v_3)\).

Similar logical constraints can be formulated for product B.

### 3.4 Linear representation of logical constraints

The supply chain design model we presented in Section 3.2 is a mixed integer programming (MIP) model which, excluding the constraints VI and VII, can be solved by Benders Decomposition (Geoffrion and Graves 1974) or factorization methods (Brown and Olson 1994). The sets of logical constraints VI and VII can be incorporated into the model involving 0-1 variables. In this section, we briefly introduce some theoretical background about logical constraints.

One conventional method for representing logical relations in an MIP model involves two stages. A logical rule is first rewritten as a conjunction of logical "clauses", i.e., in the Conjunctive Normal Form (CNF) (Williams and Brailsford 1996). A clause is a disjunction of literals, such as

\[ x_1 \lor \neg x_2 \lor x_3, \]

where \( \neg \) means logic "not," and \( \lor \) means logic "or". Each clause is then written as an inequality in 0-1 variables, which for this example is

\[ x_1 + (1 - x_2) + x_3 \geq 1, \]

where \( x \) is interpreted as true when \( x = 1 \) and false when \( x = 0 \).

Generally speaking, the specific logical relationships among the units in the process can be explicitly converted into the CNF form:

\[ \big[ \forall i \in P_1(x_i) \lor \forall i \in \bar{P}_1(\neg x_i) \big] \land \big[ \forall i \in P_2(x_i) \lor \forall i \in \bar{P}_2(\neg x_i) \big] \land \ldots \land \big[ \forall i \in P_j(x_i) \lor \forall i \in \bar{P}_j(\neg x_i) \big], \]

where \( P_j \) is the subset of positive literals and \( \bar{P}_j \) is the subset of negative literals in the \( j \)th clause (for details, see for example, Hooker and Chandru 1999). Each conjunction in the CNF can be written as an inequality in 0-1 variables like the example above. A logical relationship is thus converted into the CNF, and then the CNF is transformed as linear inequalities in binary variables. However, the linear inequalities so obtained do not usually give a "tight" representation of the original logic rule. In their work, Yan and Hooker (1999) discussed the cardinality logic rules, and provided a convex hull representation via a simple recursive procedure.
There are two types of logical constraints formulated. One includes the logical constraints from the BOM requirements (constraint VI), while the other includes some logical consistency constraints related to some 0-1 variables for each entity of the supply chain (constraint VII). Each of these logical constraints can thus be transformed into the corresponding convex hull representation, which can then be embedded in the mixed integer programming model. In the following, we will briefly exhibit these two types of logical constraints.

3.4.1 Logical consistency constraints

We discuss the logical consistency constraints first, because the logical consistency constraints in the model follow a special form of the cardinality rules. The form of logical consistency constraints in Section 3.2 is \((x_1 \lor x_2 \lor \ldots \lor x_n) \Rightarrow y\). This can be converted into the CNF:

\[
(\neg x_1 \lor y) \land (\neg x_2 \lor y) \land \ldots \land (\neg x_n \lor y).
\]

And, they can be represented by the linear inequalities:

\[
-x_i + y \geq 0 \quad x_i, y \in \{0, 1\} \quad i = 1, 2, \ldots, n. \tag{3.15}
\]

This representation adds \(n\) linear inequalities in the MIP model. According to the result by Yan and Hooker (1999), the logical rule \((x_1 \lor x_2 \lor \ldots \lor x_n) \Rightarrow y\) can be alternatively represented by

\[
-(x_1 + x_2 + \ldots + x_n) + ny \geq 0. \tag{3.16}
\]

Here, we introduce a theorem.

**Theorem 3.1** If \(x_i, y \in \{0, 1\}, i = 1, 2, \ldots, n\), then the inequality

\[
-(x_1 + x_2 + \ldots + x_n) + ny \geq 0.
\]

is equivalent to the inequalities

\[
-x_i + y \geq 0 \quad i = 1, 2, \ldots, n.
\]

**Proof of Theorem 3.1**: If \(-x_i + y \geq 0\), for all \(i = 1, 2, \ldots, n\), then

\[
-(x_1 + x_2 + \ldots + x_n) + ny \geq 0.
\]
On the other hand, if
\[ (-x_1 + y) + (-x_2 + y) + \ldots + (-x_n + y) \geq 0, \]
then there is at least one \( i \), such that
\[ -x_i + y \geq 0. \]

We consider two cases:

i. if \( y = 1 \), then for any \( i \in \{1, 2, \ldots, n\}, x_i \in \{0, 1\} \), we must have
\[ -x_i + y \geq 0; \]

ii. if \( y = 0 \), then from (16), we have \( x_i = 0 \) for all \( i \); therefore, \( -x_i + y \geq 0 \) for \( i = 1, 2, \ldots, n \).

So, we have
\[ -(x_1 + x_2 + \ldots + x_n) + ny \geq 0 \]
equivalent to
\[ -x_i + y \geq 0, \quad x_i, y \in \{0, 1\}, \quad i = 1, 2, \ldots, n. \]

Theorem 3.1 shows that inequality (3.16) is equivalent to all the equalities (3.15) in the sense that they have the same satisfiability set. In the work of Yan and Hooker (1999), (3.16) is called a “main facet” of the convex hull representation. In this simple case, it is more compact than (3.15). Thus, the logical consistency constraints (3.11), (3.12), and (3.13)
\[
(x_{1j} \lor x_{2j} \lor \ldots \lor x_{Mj}) \Rightarrow \hat{x}_j,
\]
\[
(y_{1k} \lor y_{2k} \lor \ldots \lor y_{lk}) \Rightarrow \hat{y}_k,
\]
\[
(z_{1l} \lor z_{2l} \lor \ldots \lor z_{ll}) \Rightarrow \hat{z}_l,
\]
can be represented as linear inequalities respectively:
\[
-(x_{1j} + x_{2j} + \ldots + x_{Mj}) + M\hat{x}_j \geq 0,
\]
\[
-(y_{1k} + y_{2k} + \ldots + y_{lk}) + I\hat{y}_k \geq 0,
\]
\[
-(z_{1l} + z_{2l} + \ldots + z_{ll}) + I\hat{z}_l \geq 0.
\]

3.4.2 BOM logical constraints

According to special considerations of all products and one product in Section 3.3. the logical constraints (in Section 3.2) derived from BOM are expressed in two different logical rules
\[
(\hat{y}_1 \lor \hat{y}_2 \lor \ldots \lor \hat{y}_K)_p \Rightarrow (\hat{x}_1 \lor \hat{x}_2 \lor \ldots \lor \hat{x}_j)_q
\]
and

$$(y_1 \lor y_2 \lor \ldots \lor y_K)_s \Rightarrow (x_{m1} \lor x_{m2} \lor \ldots \lor x_{mj})_t.$$  

Both of them are cardinality rules, and in the same form. Thus, we only consider the former type of BOM logical constraints above:

$$(y_1 \lor y_2 \lor \ldots \lor y_K)_p \Rightarrow (x_1 \lor x_2 \lor \ldots \lor x_J)_q.$$  

The convex hull of this rule is given by Theorem 3.1 of Yan and Hooker (1999). The convex hull representation has the following inequality as the main facet:

$$-qey + (1 + K - p)ex \geq q(1 - p),$$  \hspace{1cm} (3.17)

where $e$ is a vector of 1s with corresponding dimension. In general, the convex hull representation consists of many linear inequalities. In order to illustrate the convex hull description of this rule, we consider a simple example which states that “at least 2 vendors will be chosen from 3 candidate vendors, and at least 2 plants will be open among 3 potential plant sites”. This logical rule can be written as

$$(y_1 \lor y_2 \lor y_3) \Rightarrow (x_1 \lor x_2 \lor x_3)_2,$$  \hspace{1cm} (3.18)

where $x_i$ is a 0-1 indicator for vendors and $y_j$ is a 0-1 indicator for plants. Then the convex hull representation is given by:

$$-2(y_1 + y_2 + y_3) + 2(x_1 + x_2 + x_3) \geq -2$$
$$-2(y_1 + y_2) + x_1 + x_2 + x_3 \geq -2$$
$$-2(y_1 + y_3) + x_1 + x_2 + x_3 \geq -2$$
$$-2(y_2 + y_3) + x_1 + x_2 + x_3 \geq -2$$
$$-(y_1 + y_2) + 2(x_1 + x_2) \geq -1$$
$$-(y_1 + y_2 + y_3) + 2(x_1 + x_3) \geq -1$$
$$-(y_1 + y_2 + y_3) + 2(x_2 + x_3) \geq -1$$
$$-y_1 - y_2 + x_1 + x_2 \geq -1$$
$$-y_1 - y_2 + x_1 + x_3 \geq -1$$
$$-y_1 - y_2 + x_2 + x_3 \geq -1$$
$$-y_1 - y_3 + x_1 + x_2 \geq -1$$
$$-y_1 - y_3 + x_1 + x_3 \geq -1$$
$$-y_1 - y_3 + x_2 + x_3 \geq -1$$
$$-y_2 - y_3 + x_1 + x_2 \geq -1$$
$$-y_2 - y_3 + x_1 + x_3 \geq -1$$
$$-y_2 - y_3 + x_2 + x_3 \geq -1.$$
3.4.3 Reduction of linear inequalities

The convex hull of a cardinality rule provides a tight representation for that rule. However, such a tight representation often involves a large number (it is exponential in the number of variables) of linear inequalities. When several cardinality rules are considered in a mixed integer programming model simultaneously, it would sometimes render the computation impractical. As shown earlier in this section, for logical consistency constraints, only the main facet is needed to represent the rule since it is equivalent to the CNF form in terms of integer feasibility. For the general BOM constraints, we may need to use some heuristics to speed up the efficiency of computation. Yan (1998) discussed the computational strategies for this problem. A set of symbolic constraints, which are the logic rules separated from the MIP model, are used as cutting rules to prune further branching on solutions whenever these solutions potentially violate the rules. In his work, the following three rules are tested and compared: 1) logic rules are represented by the linear inequalities which fully describe the convex hull; 2) logic rules are separated from the model and used as symbolic constraints; and 3) logic rules are used as symbolic constraints, but the main facet of each logic rule is added into the linear constraints to tighten the feasible region of the LP relaxation. It is shown that the third strategy is preferable to speed up computation. In this study, we propose a new strategy where the “sense of management” is used.

Consider the example given above. The convex hull description of logical rules (3.18) contains 16 linear inequalities. If some specific information about vendors is provided, e.g. one of the three vendors is considered an “important vendor” to the manufacturer, we can remove some of the facet representing inequalities provided that they are not the main facet inequality. An “important vendor” may imply that the vendor is designated to supply some essential material (or component), or has some strategic relationship with the manufacturer. The important vendor should have a high priority to be chosen as a supplier, although it is possible that it may still be excluded due to high ordering cost or other related restrictions. The common way to deal with this kind of information is to assign a weight to the important vendor artificially in the model.

A more natural way to give priority to an important vendor in the model is to remove some inequalities defining the logical constraints. In this example, the inequality

$$-y_1 - y_2 + x_2 + x_3 \geq -1$$  \hspace{1cm} (3.19)

says that “if $y_1$ and $y_2$ are true, then at least one of $x_2$ and $x_3$ must be true”. Thus, withdrawing this constraint implies that the opportunity for vendor 2 and vendor 3 to be selected is reduced.
Therefore higher priority is shifted to vendor 1. Inequality (3.19) is not the main facet defining inequality. It is easy to show that to remove such inequalities will not change the optimal solution (see Yan and Hooker (1999)). Clearly, this is another way to balance the problem size and the tightness of representation.

Now, assume that vendor 1 is an important vendor in the above example. We can remove the following inequalities from the original linear descriptions of the logical constraints, since \( x_1 \) is not involved.

\[
\begin{align*}
-(y_1 + y_2 + y_3) + 2(x_2 + x_3) & \geq -1 \\
-y_1 - y_2 + x_2 + x_3 & \geq -1 \\
-y_1 - y_3 + x_2 + x_3 & \geq -1 \\
-y_2 - y_3 + x_2 + x_3 & \geq -1.
\end{align*}
\]

This heuristic leads to a great reduction in the number of inequalities of the linear representation. Generally, we suppose that there are \( K \) potential plants and \( J \) candidate vendors. According to BOM considerations, we have such a logical rule as “if at least \( s \) plants are open, there are at least \( t \) vendors to be chosen for supplying materials (or components)”. This logical rule can be formulated as

\[(y_1 \lor y_2 \lor \ldots \lor y_K) \Rightarrow (x_1 \lor x_2 \lor \ldots \lor x_J).\]  

(3.20)

The number of linear inequalities generated from the BOM logical rule (3.20) can be expressed as a number function

\[F(K, J, s, t) = (C^K_K + C^K_{K-1} + C^K_{K-2} + \ldots + C^K_s)(C^J_J + C^J_{J-1} + C^J_{J-2} + \ldots + C^J_s).\]  

(3.21)

If there are \( n \) (\( n < s \)) vendors which have good relationships with the manufacturer, for example, they have some priority to be chosen as suppliers. We should exclude the logical cuts of these \( n \) vendors. Then, the number function \( F(K, J, s, t) \) will become

\[F_n(K, J, s, t) = (C^K_K + C^K_{K-1} + \ldots + C^K_s)(C^J_J - n + C^J_{J-n} - 1 + \ldots + C^J_{J-n}).\]  

(3.22)

So, the reduction in the number of inequalities is \( F(K, J, s, t) - F_n(K, J, s, t) \). To illustrate how efficient this reduction is, we consider a supply chain in which there are 4 potential plants and 6 candidate vendors. The logical rule based on BOM is “if at least 2 plants is open, then at least 2 vendors should be chosen”. From equation (3.21), we see that the total number of linear inequalities from the logical rule is

\[F(4, 6, 2, 2) = (C^4_4 + C^4_3 + C^4_2 + C^4_1)(C^6_6 + C^6_5 + C^6_4) = 627.\]
If one vendor has priority to be chosen, then the number of linear inequalities for such a case from equation (3.22) is

$$F_1(4, 6, 2, 2) = (C_5^5 + C_5^4 + C_5^3 + C_5^2 + C_5^1)(C_4^4 + C_4^3 + C_4^2) = 341,$$

after removing some inequalities—those that include logical cuts of the important vendor. The resulting number of linear inequalities is $F - F_1 = 627 - 341 = 286$, amounting to a reduction of 46 per cent.

Therefore, to exclude some inequalities of logical rule for the important vendors is not only a natural way to give priority to these vendors in the model, but also enables a great reduction in the solution time of the MIP model. Having the logical constraints expressed as integer programming constraints, the model we formulate in Section 2 becomes a tractable production-distribution MIP model. In the next section, we present a supply chain design test problem and solve it by our model.

### 3.5 An illustration problem

In this section, we present a small-scale supply chain design problem adapted from a real-life situation. We illustrate how the problem can be formulated following our model. Although to solve the real problem is often seen as the ultimate goal of the problem modelling, the more important purpose of model construction is to reveal insight of the problem structure. Based on the model and some test computation, we further discuss the results obtained, on issues of the viability, validity, and sensibility of the proposed model.

The potential design of a supply chain being considered by an international computer company in Southeast Asia is illustrated in Figure 3.2, which includes four vendors, three plants, three DCs, and four customer zones. Based on historical marketing data provided by the company, we acquire some basic information about costs and demands for use as input to our general model. However, full and exact commercial data are modified numerically to preserve business privacy. We adopt some assumptions for the information in order to generate adequate data needed to formulate the model. The unit ordering costs are generated from 20 to 100 according to different components. The unit transportation and distribution costs are in the range between 10 and 20 per cent of the unit product prices. The fixed costs related to vendors, plants, and DCs are generated over a range of values to provide a realistic scenario for the supply chain design problem. The BOM of the two products and the demands of customer zones are discussed in the example of Section 3.3.
Based on our general model, we formulate this supply chain design problem as an MIP model which is composed of 80 variables (including 26 binary variables) and 260 rows. The representation of the BOM logical constraints and logical consistency constraints follows the formulation presented for the example in Section 3.3. It is solved by a general integer programming problem solver. The resulting supply chain design is also shown by the arrow line connections in Figure 3.2.

![Figure 3.2: Illustration of a supply chain design problem](image)

To observe the sensitivity of the model to different operating conditions, we first increase the unit ordering cost, unit transportation cost, and unit distribution cost, and then increase the fixed ordering cost and the fixed costs of plants and DCs in order to detect their effects on the total cost. The computational results and discussion are given as follows.

1. Increase unit ordering, transportation, and distribution costs by a step increment of 10 per cent. The resulting minimum total cost and percentage change in total cost are shown in Table 3.3.
Table 3.3: Change in total cost against increasing unit ordering, transportation, and distribution costs

<table>
<thead>
<tr>
<th>Percentage change (%)</th>
<th>Increasing unit ordering cost</th>
<th>Increasing unit transportation cost</th>
<th>Increasing unit distribution cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum total cost</td>
<td>Change in total cost (%)</td>
<td>Minimum total cost</td>
</tr>
<tr>
<td>10</td>
<td>99810.0</td>
<td>5.776</td>
<td>94762.0</td>
</tr>
<tr>
<td>20</td>
<td>105260.0</td>
<td>11.552</td>
<td>95140.0</td>
</tr>
<tr>
<td>30</td>
<td>110710.0</td>
<td>17.327</td>
<td>95490.0</td>
</tr>
<tr>
<td>40</td>
<td>114580.0</td>
<td>21.429</td>
<td>95840.0</td>
</tr>
<tr>
<td>50</td>
<td>121610.0</td>
<td>28.879</td>
<td>96160.0</td>
</tr>
</tbody>
</table>

|                       | Change in total cost (%)      |                                   |                                  |
| 10                    | 0.426                         |                                   |                                  |
| 20                    | 0.827                         |                                   |                                  |
| 30                    | 1.198                         |                                   |                                  |
| 40                    | 1.568                         |                                   |                                  |
| 50                    | 1.908                         |                                   |                                  |

Table 3.4: Change in total cost against increasing the three fixed costs

<table>
<thead>
<tr>
<th>Percentage change (%)</th>
<th>Increasing fixed order cost</th>
<th>Increasing fixed cost of plants</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum total cost</td>
<td>Change in total cost (%)</td>
</tr>
<tr>
<td>10</td>
<td>94434.0</td>
<td>0.078</td>
</tr>
<tr>
<td>20</td>
<td>94508.0</td>
<td>0.157</td>
</tr>
<tr>
<td>30</td>
<td>94582.0</td>
<td>0.235</td>
</tr>
<tr>
<td>40</td>
<td>94656.0</td>
<td>0.314</td>
</tr>
<tr>
<td>50</td>
<td>94730.0</td>
<td>0.392</td>
</tr>
</tbody>
</table>

|                       | Change in total cost (%)    |                                  |                                  |
| 10                    | 0.127                       |                                  |                                  |
| 20                    | 0.254                       |                                  |                                  |
| 30                    | 0.382                       |                                  |                                  |
| 40                    | 0.509                       |                                  |                                  |
| 50                    | 0.625                       |                                  |                                  |

2. Increase fixed ordering cost, fixed cost of each plant, and fixed cost of each DC by a step increment of 10 per cent. The resulting minimum total cost and percentage change in total cost are given in Table 3.4.

A comparison of the effect on the total cost by increasing the three unit costs and increasing the three fixed costs is presented in Figure 3.3. It can be seen that the total cost is relatively sensitive to changes in unit ordering cost. Increasing unit ordering cost by 10 per cent will cause a more than 5 per cent increase in total cost. Changes in any of the other five costs only have minor effects on total cost—a 10 per cent increase in any of the five costs will cause a no more than 0.5 per cent increase in total cost. In Figure 3.3, we also see that all unit costs (unit ordering cost, unit transportation cost, and unit distribution cost) have greater impact on total cost than fixed costs (fixed order cost, plants' fixed cost, and DCs' fixed cost). Therefore,
ordering cost is an important cost component in the company's cost structure, and the main element of the ordering cost is related to vendor selection. So we can draw the conclusion that vendor selection is a controlling factor for supply chain design.

From a practical perspective, the results obtained from our model are also seen to be reasonable. Generally, logistics costs (including transportation and distribution costs) are very high in Asian companies which account for about 15 to 20 per cent of total cost, particularly for consumer products. But, for equipment or industrial products, these costs are very low, about 3 per cent of total cost. Ordering cost is a main component in the cost structure for any company in Asia. A manager of the computer company, from which we acquired the data for our test problem, admitted that, in this company, logistics costs accounted for no more than 10 per cent of total manufacturing cost and increasing logistics costs by 10 per cent will roughly cause about a one per cent increase in total cost.

From the solution of this numerical example, we see that formulating BOM as logical constraints provides a new and viable approach to incorporate BOM constraints in supply chain design models. It also enables the explicit inclusion of vendor selection in the strategic design of supply chains.
Figure 3.3: Example
3.6 Conclusions

Although there is a wealth of literature and research on modelling of strategic supply chain design, there is an apparent lack of theoretical consideration of BOM constraints. We formulate a strategic supply chain design model which includes BOM in the form of logical constraints. We express the relationships among products, vendors, and plants as logical constraints via BOM considerations. These logical constraints include two types: BOM logical constraints and logical consistency constraints. Through these two types of logical constraints, we can capture the role of BOM in the selection of vendors in an MIP-based model for the strategic design of supply chains.

In order to make the MIP model with logical constraints to be tractable, we extensively discuss the representation of logical constraints. We present logical constraints as linear inequalities based on the CNF representation of logical rules. But the difficulty with CNF representation is not how to represent individual logical rules. The large number of clauses in the CNF will result in a great number of inequalities embedded in the original model. This can lead to a significant increase in the solution time of the model. We show how to achieve simplified representation of the logical consistency constraints and obtain an efficient set of inequalities with the help of additional information about vendors of the supply chain.

We formulate a test problem and solve it to demonstrate that the model is valid and viable as a tool for realistic design of strategic supply chains. The solution also reveals that making use of logical constraints not only provides a new and effective means to incorporate BOM constraints, but also enables an explicit consideration of vendor selection in supply chain design.
Chapter 4

Modelling and analysis of supplier’s uncertainty in a two-stage supply chain

4.1 Introduction

Coordination is a very important mechanism to manage supply chain as a whole. This mechanism is a main result from the acknowledgment of supply chain uncertainty. A supply chain member is often linked with a downstream customer from one side and a upstream supplier from the other side. Due to the imperfect information from each side, uncertainty happens. The uncertainty from customer demand is well addressed in past literature, and many coordination mechanisms are suggested to mitigate the deficiency. Therefore, the current research stream of supply chain coordination mainly focuses on ordering information coordination. Based on a well addressed phenomenon, “bullwhip effect”, many researchers propose central ordering information to deal with it. However, uncertainty may also be inherent in the market at the supplier side, and the quantity and quality of material delivered from an external supplier may differ from those requested. The traditional way of coping with uncertainty is to build inventories or to provide excess capacity. The deficiency caused by unpredictable customer demand and the relative coordination mechanism has been well studied by recent literature. The modelling study on supply chain uncertainty from the supplier’s side is required, such as the supplier’s uncertainty and the ways of coordinating supply chain partners to cope with such uncertainty.
To capture the quantitative characteristics of supplier’s uncertainty, we assume supplier’s uncertainty is only indicated by the lead time of its downstream customer, which means the customer can receive the exact quantity it orders but the customer doesn’t know the exact arriving time of the ordered item. We commence our study in a two-stage supply chain which includes a supplier and a retailer. The retailer faces an independent and identically distributed (i.i.d.) customer demand process. The supplier’s uncertainty is from the outer source to which it places order. The supplier can’t receive its order in time. The order quantity may arrive earlier or later than the time is supposed to be. To ensure the retailer’s order fulfillment, the supplier may wait until its order quantity arrives. Therefore, the retailer will get a delayed replenishment. On the other hand, if the supplier gets its order quantity earlier, it will dispatch the retailer’s replenishing quantity as soon as possible to reduce its own holding cost. Under these circumstances, the uncertainty of the supplier’s delivery to the retailer is decided by the variability of the supplier’s lead time. Our modelling framework forms a multi-echelon inventory system. We give assumption the base-stock with batch-order policy to both of the supplier and retailer’s inventory control. We will discuss the optimal settings of the ordering policies and inventory costs in this system. Our focus is not simply to find an optimal solution of a inventory control model. What we are trying to do is to discuss and outline the coordination mechanisms between the supplier and the retailer in order to obtain an improvement of the overall supply chain system performance.

The similar multi-echelon inventory system has been studied by other researchers for years. We give a review of relative literature before the introduction of our modelling work. In a multi-item base-stock inventory system, Song (1998) investigates the order fill rate with assumptions of compound Poisson demand and constant lead time. Furthermore, Song et al. (1999) analyze the order-fulfillment performance of an assemble-to-order system. This modelling framework is also a two-echelon inventory system with base-stock policy. They assume that the demand is a Poisson process and lead time is exponentially distributed. In a multi-item spares inventory model, Cheung and Hausman (1995) consider i.i.d. replenishment lead time and Poisson demand model. Optimal stationary policies in multi-echelon inventory systems can be seen in Chen (1998 and 2000). Chen (1998) presents a stationary policy for a serial system with N stages. The transportation lead times at all stages are assumed to be zero. For the constant lead times (transportation times are greater than zero), Chen (2000) derives optimal policies for a multi-echelon serial system with batch ordering. The evaluation of \((R, Q)\) policies of a two-stage inventory system is studied by Axståter (1998). He also adopts the assumptions of constant lead times and Poisson demand processes. Nahmias (1979) gives a first step on the approximations of the optimal policies under variable lead times. Federgruen (1993) indicates there is a little consideration of stochastic lead time within the study of multi-echelon inventory
systems. A continuous-review inventory system with lost sales and variable lead times is studied by Mohebbi and Posner (1998). They assume the demand process is compound Poisson process and lead times are i.i.d. random variables following Erland and hyperexponential distributions. Zhang (1999) assumes a multivariate Poisson demand process, and each item is supplied by a dedicated facility with general i.i.d. processing times. The same kind of supply system is studied by Glasserman and Wang (1998). Ettl et al. (2000) develop a supply network model featuring both performance evaluation and optimization. Each site in the network is modeled as an inventory queue and the model generates the base-stock level at each site.

4.2 Model formulation

In this section, we study an optimal inventory policy for a two-stage supply chain, which includes a supplier and a retailer. The retailer faces customer demands which form a Poisson process with intensity \( \lambda \). Assume that the demands only take integer values. Each customer demand arrival brings a batch of \( Q \) units. The retailer deploys a periodic-review batch-ordering \((R_r, Q_r)\) inventory policy, which means a batch of size \( Q_r \) is ordered when retailer’s on-hand inventory level declines to or below the reorder level \( R_r \). Let \( Q \) denote the base batch for the whole system. Both of \( R_r \) and \( Q_r \) are in multiple units of base batch size \( Q \). When customer demand exceeds the retailer’s on-hand inventory, the excess is backlogged. The inventory position is the on-hand inventory plus outstanding orders minus backorders. All backorders are delivered on a first-come-first-serve basis. The supplier also adopts a batch-ordering policy \((R_s, Q_s)\). It is assumed that the inventory level at supplier is always an integer multiple of order batch placed by retailer. We define \( R_s \) and \( Q_s \) are multiple units of retailer’s batch size \( Q_r \). Considering this two-stage inventory control system, a replenishment cycle starts when the retailer places an order batch \( Q_r \), which is triggered by a customer demand batch \( Q \), and completes when an order batch is received by the retailer. The supplier’s replenishment order batch \( Q_s \) is triggered by an order batch from the retailer and is filled by when the corresponding order is received after an uncertain lead time. The system incurs holding and backorder costs and the objective is to minimize the long-run average total cost in the system.
Figure 4.1: The general two-stage supply chain with \((R, Q)\) policies

We introduce several notations:

\(L_s\): the supplier's lead time (current order will arrive after \(L_s\) periods);
\(L_r\): the retailer's lead time;
\(I_r\): transportation time from the supplier to the retailer;
\(W_r\): random delay at the retailer of the customer demand;
\(W_s\): random delay at the supplier of replenishment orders from the retailer;
\(h_s\): the supplier’s holding cost per unit per time period;
\(h_r\): the retailer’s holding cost per unit per time period;
\(g_s\): the supplier’s backorder cost per unit per time period;
\(g_r\): the retailer’s backorder cost per unit per time period;
\(I_s\): the inventory level of the supplier in the one-for-one replenishment;
\(I_r\): the inventory level of the retailer in the one-for-one replenishment.

To capture the supplier’s uncertainty, we assume the supplier orders units from an unstable outer source. We assume the supplier can not receive the placed order on time and the supplier’s lead time is variable. Generally, there are several lead time constituents: the time of procurement, set-up, processing, handling, and transportation. All of these sub-parts of the lead time contain uncertainty. The possible approach is to accumulate all the uncertainty and formulate the variability of lead time as an independent and identically distributed variable with mean and variance. Therefore, we formulate the supplier’s lead time \(L_s\) is i.i.d. and the probability distribution function is \(G(\cdot)\). We assume the mean of the lead time is \(L_s\).

Axsäter (1993) used a one-for-one replenishment policy to analyze the continuous review \((R, Q)\) policies in two echelon inventory systems with compound Poisson demand. We also
adopt the same procedure here to carry out our study in the two-stage supply chain system. A one-for-one replenishment system is defined as: when a customer demand occurs at the retailer, a unit is immediately ordered from the supplier to satisfy the demand and the supplier also orders a unit to replenish the retailer’s replenishment order. If customer demands occur while the supplier’s inventory is empty, the retailer’s replenishment order is backlogged. When units are available at the supplier’s inventory, the retailer’s backlogged orders are served according to a first come, first served policy.

The one-for-one system can be a building block for the analysis of periodic review \((R, Q)\) policies in our two-stage supply chain system. Let \(C(I_s, I_r)\) denote the total holding and backorder costs per time unit when applying one-for-one replenishment policy. \(I_s\) and \(I_r\) are inventory positions of the supplier and the retailer. Based on the assumption of one-for-one replenishment, we have \(R_s = I_s - 1, R_r = I_r - 1\) and \(Q_s = Q_r = 1\). The one-for-one replenishment cost \(C(I_s, I_r)\) can be used to evaluate different batch-ordering policies of two-level inventory system. Recalling the multiple unit assumption of order batches of the supplier and the retailer, we classify our system into following two different scenarios with the different settings of \(Q_r\).

A. One-for-one replenishment at the retailer (\(Q_s\) is multiple units of \(Q_r\) and \(Q_r = Q\))

We assume the retailer’s order batch is just same as the customer’s demand. Each customer demand triggers a separate order. In this scenarios, the customer demand process is a Poisson process, and so does the retailer’s ordering process. When the retailer applies one-for-one replenishment, the demand process at the supplier is still the Poisson process. The cost is \(C(R_s + 1, I_r)\). An order placed by the supplier will be used to fill the \((R_s + 1)\)th demand at the supplier following this order. Similarly, an order placed by the retailer will be used to fill the \(I_r\)th demand at the retailer following this order. Now, a batch of \(Q_s\) units is ordered by the supplier. The first unit in this batch (the one which is delivered first to the retailer) will fill \((R_s + 1)\)th demand at the supplier and the \(I_r\)th demand at the retailer. The cost incurred is \(C(R_s + 1, I_r)\). The second unit in the batch will be used to fill the \((R_s + 2)\)th demand at the supplier and will incur cost \(C(R_s + 2, I_r)\). etc. Then, the average cost per time unit is:

\[
C = \frac{1}{Q_s} \sum_{j=R_s+1}^{R_s+Q_s} C(j, I_r).
\] (4.1)


B. General serial system (\(Q_s\) is multiple units of \(Q_r\) and \(Q_r\) is multiple units of \(Q\))
The similar approach above can be used to deduce a general batch-ordering policy for our two-echelon serial system. Recall that \( R_s \) and \( Q_s \) are in the multiple units of retailer batch \( Q_r \). A batch ordered by the supplier consists of \( Q_s \) subbatches of size \( Q_r \). The first subbatch will be used to fill the \( (R_s + 1) \)th demand for a retailer batch at the supplier, which means that it is released from the supplier when \( (R_s + 1)Q_r \) demands occur after the order at the retailer. The next subbatch will be used to fill the \( (R_r + 2) \)th demand at the supplier, etc. The first unit in a subbatch will fill the \( (R_r + 1) \)th customer demand; the second unit will fill the \( (R_r + 2) \)th customer demand, etc. The average cost per time unit by averaging over the individual unit in a supplier batch can be determined by:

\[
C = (1/Q_s Q_r) \sum_{j=R_r+1}^{R_r+Q_s} \sum_{k=R_r+1}^{R_r+Q_r} C(jQ_r, k) .
\] (4.2)

The average cost per time unit of the two-stage system can be presented as the summation of the average costs of one-for-one replenishment system under different inventory level settings. If we can give the exact evaluation of one-for-one replenishment policy, the two different scenarios above can be deduced straightforward. Now, we consider a one-for-one replenishment system with uncertain supplier’s lead time.

4.3 One-for-one replenishment system

In a two-stage supply chain which includes one supplier and one retailer, the supplier replenish units from the outer source using a one-for-one replenishment policy. The retailer also adopts the same policy to order units from the supplier. The replenishment lead times for the supplier are independent and identically distributed. Under the one-for-one replenishment policy, a customer demand arrival at the instantaneously translates into a retailer’s replenishment order to the supplier. The retailer’s replenishment order is either satisfied immediately or backlogged at the retailer, depending on if the retailer has inventory on hand. Then, each replenishment order placed by the retailer instantaneously triggers a supplier’s replenishment order, which means replenishment order is received instantaneously by the supplier. The supplier’s lead time is defined as the time between the receipt of the retailer’s order and the receipt of the supplier replenishment order shipped by the outer source. Under this policy, the inventory position, which is the on-hand inventory plus outstanding orders minus backorders, is kept at a constant base-stock level by replenishing one unit immediately upon receiving a demand. We assume that the transportation time from the supplier to the retailer is constant. The retailer’s lead time include the transportation time and the random delay at the supplier.
Specifically, in this one-for-one replenishment system, the retailer faces a stationary and independent Poisson customer demand with rate $\lambda$. Each arrival customer demand triggers a retailer order. Therefore, the retailer replenishment orders at the supplier also form a Poisson process with rate $\lambda$. The control policies of the supplier and the retailer become base-stock policies with order-up-to levels $I_s$ and $I_r$ respectively. The supplier’s lead time $L_s$ is defined as an i.i.d. and the distribution function is $G(\cdot)$. The retailer’s lead time is summation of the transportation time $l_r$ and random delay $W_r$. The retailer faces a Poisson demand with rate $\lambda$. With the one-for-one replenishment policy, each customer demand triggers a retailer order to the supplier and each retailer’s replenishment order triggers a supplier’s order to the outer source. Therefore, the retailer’s replenishment order also forms a Poisson process with the same rate $\lambda$ and so does the supplier’s replenishment ordering process. Let $W_r$ denote the time elapsed from when the customer order is placed to when its associated demand arrives. Actually, this distribution of $W_r$ is an Erland distribution with parameters $\lambda$ and $I_r$. Let $f_{I_r}^{\tau} (\cdot)$ denote the density function of $W_r$. We have

$$f_{I_r}^{\tau}(t) = \frac{\lambda^t r (r-1)e^{-\lambda t}}{(I_r - 1)!}. \quad (4.3)$$

The corresponding cumulative distribution function $F_{I_r}^{\tau}(t)$ is

$$F_{I_r}^{\tau}(t) = \sum_{k=I_r}^{-\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t}. \quad (4.4)$$

The same approach can also be applied to find out the distribution of the retailer’s random delay at the supplier.
4.3.1 Performance evaluation at the supplier

The retailer’s replenishment order arrivals at the supplier are assumed to be sequential, because the supplier follows the first-come-first-serve rule and the transportation times are constant. A retailer’s replenishment order arrives after \( L_s \) time periods. \( L_s \) is i.i.d. with distribution function \( G(\cdot) \). If the ordered unit arrives before its assigned retailer’s order, it is kept in stock and incurs holding cost; if it arrives after its assigned retailer’s order, this assigned replenishment order is backlogged and shortage cost occurs until the ordered unit arrives. The time elapsed from when the retailer’s order is placed to when its associated order arrives is defined as \( W_s \). The distribution of \( W_s \) is an Erlang distribution with parameters \( \lambda \) and \( I_s \). Let \( f^{I_s}_s(t) \) denote the density function of \( W_s \). We have

\[
f^{I_s}_s(t) = \frac{\lambda^{I_s} t^{I_s-1} e^{-\lambda t}}{(I_s - 1)!}.
\]  

(4.5)

The corresponding cumulative distribution function \( F^{I_s}_s(t) \) is

\[
F^{I_s}_s(t) = \sum_{k=1}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t}.
\]  

(4.6)

We use \( \psi(I_s) \) to denote the expected supplier holding and shortage costs incurred to fill a unit of the retailer’s replenishment order. Let \( \psi(I_s) \) denote the expected holding and shortage costs per time unit. We evaluate this value by conditioning on \( L_s \). The conditional expected cost \( \psi(I_s|L_s) \) is given by

\[
\psi(I_s|L_s) = g_s \int_0^{L_s} f^{I_s}_s(x)(L_s - x)dx + h_s \int_{L_s}^{\infty} f^{I_s}_s(x)(x - L_s)dx,
\]  

(4.7)

while

\[
\psi(0|L_s) = g_s(L_s).
\]

Thus,

\[
\Psi(I_s) = \int_0^{\infty} \psi(I_s|L_s)dG(L_s).
\]  

(4.8)

We will derive the probability distribution of the random delay of the retailer’s replenishment order at supplier \( W_s \). Both of the timely results and steady-state results of the system properties will be given.

If we assume that the supplier starts with \( I_s \) units on hand at the beginning. The retailer places a typical replenishment order at time \( t \). Let \( W_s(t) \) denote the random delay experienced
by the order. For $\tau > 0$, we want to get $\Pr W_s(t) \leq \tau$. Let $S(t, t+\tau)$ be the supplier’s inventory level at time $t + \tau$ plus the demands arrived in $[t, t+\tau)$. Then, we have (Wang, et al., 2000)

$$\{W_s(t) \leq \tau\} \Leftrightarrow \{S(t, t+\tau) \geq 1\}.$$  \hspace{1cm} (4.9)

Each retailer’s replenishment order received before $t$ reduces $S(t, t+\tau)$ by one unit upon their arrival, while each replenishment order filled by the outer source by time $t + \tau$. We use definitions of $M_t(\tau)$ and $N_t(\tau)$ as follows (see detail in Wang, et al., 2000):

$M_t(\tau)$ = the number of orders arrived during $(0, t)$ from the retailer and whose corresponding supplier’s replenishment order have not been filled by $t + \tau$;

$N_t(\tau)$ = the number of orders arrived during $(t, t+\tau)$ from the retailer and whose corresponding supplier’s replenishment order have been filled by $t + \tau$:

$$\eta(\tau) = \begin{cases} 1, & \text{if the supplier’s order corresponding to the specific retailer order at } t \text{ is filled by } t + \tau; \\ 0, & \text{otherwise.} \end{cases}$$

Then, we have the following balance equation:

$$S(t, t+\tau) = I_s - M_t(\tau) + N_t(\tau) + \eta(\tau),$$  \hspace{1cm} (4.10)

where

$i$. $M_t(\tau)$, $N_t(\tau)$ are independent Poisson random variables with mean of $\lambda m_t(\tau)$ and $\lambda n_t(\tau)$, where

$$m_t(\tau) = \int_0^t [1 - G(t + \tau - s)] ds = \int_0^{t+\tau} [1 - G(s)] ds$$  \hspace{1cm} (4.11)

$$n_t(\tau) = \int_0^\tau G(\tau - s) ds = \int_0^\tau G(s) ds$$  \hspace{1cm} (4.12)

$ii.$

$$\eta(\tau) = \begin{cases} 1, & \text{with the probability of } G(\tau), \\ 0, & \text{with the probability of } 1 - G(\tau). \end{cases}$$

The steady state properties of the systems can be obtained by letting $t \to \infty$. We then have:

$$S(\tau) = I_s - M(\tau) + N(\tau) + \eta(\tau).$$  \hspace{1cm} (4.13)

The distribution of the delay experienced by the retailer’s replenishment orders is:
\[ H(\tau) = \Pr\{W_s(t) \leq \tau\} = \Pr\{S(\tau) \geq 1\}: \]
\[ = \Pr\{I_s - M(\tau) + N(\tau) + \eta(\tau) \geq 1\}: \]
\[ = \Pr\{I_s - M(\tau) + N(\tau) \geq 1\} + G(\tau) \Pr\{I_s - M(\tau) + N(\tau) = 0\} \quad (4.14) \]

The random delay experienced by replenishment orders placed by the retailer is stochastically decreasing in the supplier’s base-stock level \(I_s\). Therefore, we have the retailer’s lead time, which is \(L_r = l_r + W_s\). Considering the transportation time \(l_r\) is a constant, the probability distribution of \(L_r\) can be denoted by \(H(\cdot)\).

### 4.3.2 One-for-one replenishment at the retailer

An order placed by the retailer will arrive after \(l_r + W_s\) time periods, where \(W_s\) is the random delay encountered at the supplier (\(0 \leq W_s \leq L_s\) conditioning on \(L_s\)). The probability distribution of \(W_s\) is \(H(\cdot)\). At the retailer, if the ordered unit from the supplier arrives before its assigned customer demand, it is kept in stock and incurs holding cost; if it arrives after its assigned demand, this assigned customer demand is backlogged and shortage cost occurs until the ordered unit arrives. We use \(\Theta_r^{l_r}(I_s)\) to denote the expected retailer holding and shortage costs incurred to fill a unit of demand at the retailer. We evaluate this value by conditioning on \(W_s = t\). The conditional expected cost \(\theta_r^{l_r}(t)\) is independent of \(I_s\) and is given by

\[ \theta_r^{l_r}(t) = g_r \int_0^{l_r + t} f_s^{l_r}(x)(l_r + t - x)dx + h_r \int_{l_r + t}^{\infty} f_s^{l_r}(x)(x - l_r - t)dx, \quad (4.15) \]

while

\[ \theta_r^0(t) = g_r(l_r + t). \]

Note that the supplier also faces a Poisson demand process with rate \(\lambda\). The expected retailer holding and shortage cost \(\Theta_r^{l_r}(I_s)\) can be obtained by

\[ \Theta_r^{l_r}(I_s) = \int_0^{L_s} f_s^{l_r}(\bar{L}_s - t)\theta_r^{l_r}(t)dt + (1 - F_s^{l_r}(\bar{L}_s))\theta_r^{l_r}(0). \quad (4.16) \]

For \(I_s = 0\) and \(\Delta = \bar{L}_s\), \(\Theta_r^{l_r}(0) = \theta_r^{l_r}(L_0)\). The long-run average retailer holding and shortage costs are given by \(\lambda \Theta_r^{l_r}(I_s)\).
4.3.3 Customer waiting time at the retailer

When a demand occurs at the retailer, a unit is immediately ordered from the supplier and the supplier orders a new unit at the same time. If demands occur while the retailer’s inventory is empty, the replenishment to customer is delayed. When units are available at the retailer, the backordered customer demands are served according to a first come, first serve policy. We define the time elapsed from when the customer demand occurs to when its associated demand unit arrives as the customer waiting time at the retailer, denoted by $W_r$.

The distribution of $W_r$ can be easily obtained. See,

$$\Pr(W_r = 0) = \sum_{k=0}^{I_r-1} \frac{\lambda^k r^k}{k!} e^{-\lambda L_r} = 1 - F_r^I(L_r)$$  \hspace{1cm} (4.17)

and the density function $\xi(t)$ for $0 \leq W_r \leq L_r$ is:

$$\xi(t) = f_r^I(L_r - t) = \frac{\lambda^I_r (L_r - t)^{I_r-1}}{(I_r-1)!} e^{-\lambda(L_r-t)}.$$  \hspace{1cm} (4.18)

4.3.4 The whole system

In many cases, the assumption that the delayed retailer demands at the supplier does not cause shortage cost, which means only the retailer takes the shortage cost. We will discuss the two scenarios. We give definitions of the average supplier holding and shortage cost per unit as following.

i. The retailer takes backorder cost. No backorder cost happens at the supplier. The holding cost per time unit of the supplier (4.7) is

$$\psi(I_s|L_s) = h_s \int_{L_s}^{\infty} f_s^I(x)(x - L_s)dx,$$  \hspace{1cm} (4.19)

and the holding and shortage cost per time unit of the retailer is still equation (4.15).

ii. Backorder cost happens at the supplier. We assume the backorder cost only happens with one member of this serial two-stage supply chain. If the supplier takes the backorder order cost associated with the retailer replenishment orders, there is no backorder happening at the retailer. The holding cost per time unit of the supplier is the same formula as equation (4.7). The holding cost per time unit of the retailer is

$$\theta_r^I(t) = h_r \int_{I_r+t}^{\infty} f_r^I(x)(x - I_r - t)dx$$  \hspace{1cm} (4.20)
According to the assumptions of the one-for-one replenishment system, we conclude the long-run system inventory costs:

\[ C(I_s, I_r) = \lambda [\Theta^{I_r}(I_s) + \Psi(I_s)] \]  

(4.21)

4.4 Analysis of the inventory cost under the one-for-one system

The backorder cost part in (4.7) and (4.15) are in the same form of the formula. Let \( \omega_r^{I_r} \) denote the part of the backorder cost in the retailer's holding and backorder costs per unit time (4.15).

\[ \omega_r^{I_r}(t) = g_r \int_0^{I_r+t} f_r^{I_r}(x)(l_r + t - x)dx \]  

(4.22)

Since the function (4.3) has following property,

\[ \int_0^t x \cdot f_r^{I_r}(x)dx = \int_0^t f_r^{I_r+1}(x) \frac{F_r}{\lambda}dx = F_r^{I_r+1}(t) \frac{F_r}{\lambda}, \]

we can obtain

\[
\omega_r^{I_r}(t) = g_r \int_0^{I_r+t} f_r^{I_r}(x)(l_r + t - x)dx
\]

\[= \frac{g_rF_r}{\lambda} \left[ 1 - F_r^{I_r+1}(l_r + t) \right] - g_r(l_r + t) \left[ 1 - F_r^{I_r}(l_r + t) \right]
\]

\[+ g_r(l_r + t - \frac{I_r}{\lambda}) \]

\[= \frac{g_r}{\lambda} e^{-\lambda(l_r+t)} \sum_{k=0}^{I_r-1} \binom{I_r-k}{k} \lambda^k + g_r(l_r + t - \frac{I_r}{\lambda}) \]  

(4.23)

We derive the differential equation for \( \omega_r^{I_r}(t) \) from (4.23). It is easy to obtain the derivative of \( \omega_r^{I_r}(t) \).

\[
\frac{d\omega_r^{I_r}(t)}{dt} = \lambda [\omega_r^{I_r-1}(t) - \omega_r^{I_r}(t)]
\]

\[= \frac{g_r}{\lambda} \frac{e^{-\lambda(l_r+t)}(I_r + t)^{I_r-1}}{(I_r - 1)!} - \lambda^{(I_r-1)} \]
\[
\frac{g_r}{\lambda} \left[ 1 - \frac{\lambda^{I_r}(l_r + t)^{I_r-1}}{\lambda(I_r - 1)!} e^{-\lambda(l_r+t)} \right] \\
= \frac{g_r}{\lambda} \left[ 1 - \frac{f_r^I(l_r + t)}{\lambda} \right]
\]

(4.24)

Then, we have following proposition:

**Proposition 4.1** For \( \lambda \geq 1 \), we have: \( \frac{d\omega_r^I(t)}{dt} > 0 \); the retailer’s backorder cost is increasing with \( t \).

The second derivative of \( \omega_r^I(t) \) can be given by:

\[
\frac{d^2\omega_r^I(t)}{dt^2} = \frac{d}{dt} \left\{ \frac{\lambda \omega_r^I(t-1) - \omega_r^I(t)}{\lambda^2} \right\} = g_r e^{-\lambda(l_r+t)} \frac{(l_r + t)^{I_r} \lambda^{I_r+1}}{I_r!} > 0.
\]

(4.25)

Thus, we can see that:

**Proposition 4.2** The retailer’s backorder cost \( \omega_r^I(t) \) is convex with \( t \).

From the two propositions above, we can see that the retailer’s backorder cost is subject to the random delay experienced by the replenishment orders at the supplier. The similar approach can be applied in the suppliers. The distribution of supplier’s lead time can influence the distribution of the retailer’s random delay at the supplier (4.14). The distribution types of the retailer’s random delay is not only subject to the customer demand process but also the different distribution of the supplier’s lead time. The retailer’s random delay at the supplier can be a significant part of the overall replenishment lead time of the retailer. Therefore, the differences in the random delay can be represented by the difference in the service performance at the retailer.

The supplier’s uncertainty affects the system’s cost structure. We give the analysis of the two scenarios with the different backorder cost bearers. Generally, when supply chain goes to downstream level, the average unit backorder cost will be increasing. We give the possible scenarios of a one-for-one system, which are either the supplier or the retailer is the whole system backorder cost bearer. With the variable supplier’s lead time, if the backorder cost happens with the retailer, there will be an increase on the total system inventory cost. To reduce the extra cost which is caused by the variability of the supplier’s lead time. We should choose the supplier to be the backorder cost bearer. In fact, in a decentralized supply chain, it is impractical to let the supplier be the single bearer of the system shortage cost. The possible
coordination mechanisms may be discussed under these two situations: one is the retailer affords the additional cost to the supplier in order to let the supplier maintain the retailer's lead time stability; the other is the two members share the extra cost in order to obtain an optimal inventory cost of the whole system. There will be an optimal proportion allocation of the additional cost between the supplier and the retailer to minimize the whole system's inventory cost. Therefore, we can discuss the coordination mechanisms in this two-echelon system to deal with the uncertainty from the outer source, which makes the supplier's replenishment unstable.

4.5 Conclusions

This Chapter presents a modeling study on supplier's uncertainty in a two-stage supply chain. The supplier's uncertainty can be indicated by the variability of downstream customer's lead time. We present a modeling framework to demonstrate the modeling analysis of lead time variability. In our modelling framework, the supplier's lead time is formulated to be i.i.d. with distribution $G(\cdot)$ and a known mean. A one-for-one replenishment system is used to help us give an exact evaluation of the periodic review $(R, Q)$ policies of the supplier and the retailer. We find the variability of the supplier's lead time can be a significant part of the retailer's random delay at the supplier, which is an important part of the retailer's lead time. The variability of the retailer's lead time can also cause extra inventory cost, especially when the retailer is the shortage cost bearer. In many practical cases, the retailer is more likely to be the shortage cost bearer. We discuss the different situations with either the retailer or the supplier being the shortage cost bearer. The related coordination mechanisms can be deduced based on the different allocations of the extra cost caused by the variability of the supplier's lead time. Our modelling analysis leads to an exact performance evaluation procedure of the two-stage $(R, Q)$, which can be easily used in system optimization models. We suggest further approaches in this field about the reduction of the deficiency of lead time variability and the relevant coordination mechanisms to reduce the extra cost caused by the variable supplier's lead time.
Chapter 5

Benefits of information sharing-based supply chain partnerships

5.1 Introduction

The motivation of this research is a recent successful application of Electronic Data Interchange (EDI) by P&S Company, one of the two largest supermarket chains in Hong Kong, and its important supplier J&J, a famous local beauty care product distributor. Before the adoption of EDI, P&S retail stores needed to hold certain amount of inventory in the storeroom of each store in addition to the shelf inventory of the products on product shelves. The distribution center of J&J replenished the beauty care products for a particular P&S store when the inventory level of the storeroom dropped to the reorder point and an order was placed by the store. Each order from a retail store triggered a single replenishment. With EDI, P&S and J&J determined to establish a long term partnership based on information sharing. Under the new scheme, a retailing store of P&S can retrieve its product inventory by bar code scanning at each POS (point of sales). This real time information is simultaneously sent to J&J’s distribution center, which then replenishes the product inventory of the retail store according to a pre-determined shelf inventory reorder point.

As a result, the P&S store no longer needs to keep the inventory in its storeroom. neither does it need to place the order to J&J. The real time vendor-managed-inventory (VMI) is realized.
and P&S thus can make a significant saving on its daily administration costs. Meanwhile, with real-time information about its product inventory at all retail stores of P&S, J&J's distribution center can make joint replenishments for P&S's retailing network periodically. J&J can therefore arrange its inventory and delivery planning at its distribution center. It is found that both its inventory cost and delivering cost have been greatly reduced. Information sharing leads P&S and J&J to forming a stable partnership. Both of them can improve their performance and obtain economic benefits for the long run.

This research aims to study the benefit of information sharing among members in a supply chain through mathematical programming models. As a member in a supply chain continues to implement information sharing with others, the partnerships among the supply chain members have become more and more important to practitioners. The increasing attention paid to supply chain partnerships in industry provides a wealth of research opportunities concerning the modelling and quantitative study of supply chain partnerships. Research on quantifying the benefits of information sharing and partnerships is valuable to practitioners as it provides a solid analysis of this issue. There is a body of research focused on the conceptual design and analysis of decentralized supply chains, most of which is qualitatively based. In the following, we present a brief review of the related literature.

An important phenomenon observed in supply chain management is that demand variability increases as one moves up a supply chain. Most past research efforts focus on the nature of this phenomenon and how to deal with it. Kahn (1987) first uses a serially correlated demand model to show the variance of production exceeds the variance of sales even if the firm can backlog. This phenomenon is now known as the "bullwhip effect", a term popularized by Lee et al. (1997a and 1997b). Bagamah and Cohen (1998) present a hierarchical model as an analytical framework to examine the stabilizing effect of inventories in supply chains. Their analysis suggests that even though the bottom echelon, i.e., the retailer or wholesaler, faces a normally distributed demand, the order quantity of the upper echelons at each time period would autocorrelate with a certain lag of the following consecutive time series: furthermore, if they are positively correlated, there will be a variance amplification, while there will be stabilization if they are negatively correlated.

On identifying the causes of the bullwhip effect, Chen et al. (1999) propose three options to reduce its impact, namely reducing uncertainty, reducing lead time and forming strategic partnerships. The expanding importance of supply chain integration also prompts increasing attention being paid to the study of integrated supply chain partnerships. Srinivasan et al. (1994) investigate the degree to which increasing vertical information integration using Elec-
tronic Data Interchange (EDI) technology enhances shipment performance of suppliers in a just-in-time (JIT) environment. Their analytical study reveals that vertical information integration along the value chain appears to greatly facilitate coordination in a JIT assembly environment and improve the performance of the logistical system. Gavirneni et al. (1999) incorporate information flow between a supplier and a retailer in a two-echelon supply chain model. They evaluate the benefit of information by capturing different supply chain's capacitated settings. The results show that if the end-item demand variance is moderate, and the order-up-to value is not extreme, the benefits of information are great: if the supplier's capacity is high, the information link is to be expected to be most beneficial with EDI implementation.

There exists a large amount of literature on the concepts of supply chain partnerships projecting extremely optimistic views about their promise as win-win partnerships without rigorous analysis to support the cause of optimism. Maloni and Benton (1997) point out that while there is a plenty of qualitative, conceptual literature on supply chain partnerships, very few researchers have attempted a rigorous analytical approach to examine the supply chain partnership issues.

Among the existing quantitative studies, Iyer and Bergen (1997) examine the impact of Quick Response (QR) on the fashion apparel industry. They introduce formal inventory models with the assumption of normally distributed demand. The manufacturer-expected profit of the old system and that of the QR system are compared, revealing that Pareto improvement—all partners are at least as well off, and some partners are better off—is achieved (1997). Under QR, they show that the manufacturer will obtain more expected profit by shortening the lead time for fashionable merchandise. Kouvelis and Gutierrez (1997) investigate the optimal policies of the production and inventory problem of “style goods”, which have a short selling season. A producer of a “style good” sells it to two markets (a primary and a secondary market) with nonoverlapping selling seasons. The results demonstrate that decentralized control policies with purchasing coordination are equally effective in terms of overall profitability as centralized control policies. Decentralized control policies suggest that the third party deliver units from one market to another. It is well known in the literature that enhancing information integration will reduce the impact of the bullwhip effect. Lee et al. (2000) first derive the benefits of information sharing in a two-level supply chain based on analytical models. Information sharing is modeled in such a way that the manufacturer could know the customer demand information at the present time period. With this kind of one-time information sharing, they show that the manufacturer will achieve inventory reduction and cost savings. The authors quantify the benefits of information sharing and stress the benefits of ordering coordination. However, information sharing is just the first step for supply chain integration. In decentralized supply
chains, supply chain partnerships could be introduced via applying a vendor managed inventory (VMI) program in order to obtain the centralized control optimum. In Lee et al. (2000), no quantitative study on the coordination mechanism of supply chain partnerships is presented although the VMI program is mentioned.

This research is an extension of the study by Lee et al. (2000), under the assumption that the coefficient of correlation of orders $\phi$ satisfies $-1 < \phi < 1$. Some important results are obtained by further restricting that $0 \leq \phi < 1$. That is, the orders are positively correlated. We present a comprehensive model on information sharing-based supply chain partnerships. We formulate a decentralized supply chain under different scenarios of information sharing and ordering coordination with a view to quantifying the benefits of information sharing-based partnerships for different supply chain partners. The results obtained for the first and the second information sharing levels in our work are similar to Lee et al. (2000). The difference is we illustrate the coordination mechanism under information sharing-based partnerships and quantify its benefits. Especially, we show the benefits under the third information sharing level by introducing the VMI policy in the manufacturer and retailer.

To better understand the benefits of information sharing-based partnerships in a decentralized supply chain, three levels of information integration are introduced for modelling the partnerships in a two-stage supply chain consisting of a single retailer and a single manufacturer. From the comparisons of inventory reductions and cost savings among the three information integration levels, we show that Pareto improvement will be achieved, i.e., both the retailer and the manufacturer are at least as well off, and one of them—the manufacturer—is better off. Therefore, the information sharing-based partnership can improve the overall performance of the decentralized supply chain. Consequently, we draw the conclusion that supply chain management should choose strategic partnerships based on information sharing via adopting VMI using EDI to achieve overall optimum in decentralized supply chains.

5.2 Three information integration levels and the normative model

In this study, we consider a two-stage supply chain involving one manufacturer and one retailer. This is a decentralized supply chain in which the two entities belong to different organizations. In this section, we introduce the three information integration levels and the basic models based on this assumption.
5.2.1 Three information integration levels

According to different situations of information sharing and ordering coordination, we formulate the information sharing-based relationship between the retailer and the manufacturer as three information integration levels:

*Level 1:* 
- Manufacturer
- Retailer
- Customer
- Order

*Level 2:* 
- Manufacturer
- Retailer
- Customer
- Order
- Demand

*Level 3:* 
- Manufacturer
- Retailer
- Customer
- Order
- VMI with EDI

Figure 5.1: Three integration levels of information sharing-based relationship

The three levels of information integration can be described as follows. *Level 1* is described as “decentralized control”. At this level, the inventory of two members of the supply chain is controlled independently. Both the retailer and the manufacturer make their inventory decisions according to their own forecasting. We suppose all the stock control policies involved are of the order-up-to type with periodic review procedures. *Level 2* is referred to as “coordinated control”. The two neighboring inventories are coordinated with the sharing of the customer ordering information. In this situation, the manufacturer will obtain the customer demand information, together with the retailer’s ordering information, and then make its inventory decision. *Level 3* is named as “centralized control”. In this situation, the decentralized supply chain can achieve optimal performance achievable by a supply chain under centralized control. Based on EDI, both the retailer and the manufacturer can retrieve the customer’s demand information in a synchronized manner. The two partners can establish a strategic partnership by adopting a
vendor managed inventory (VMI) strategy based on this synchronized information sharing. This means the manufacturer takes the initiative to make major inventory replenishment decisions for the retailer in parallel with its own inventory decision. In this case, the manufacturer will not depend on the retailer’s ordering information, but on the customer’s demand directly.

At Level 1 of information integration, we assume that there is neither information sharing nor any ordering coordination between the retailer and the manufacturer. We present the cost-minimization models for this two-stage supply chain and their normative forms. Next, we formulate the mathematical model for Level 2 of information integration with customer ordering information coordination. Finally, we consider information integration at Level 3 with full information sharing via VMI based on EDI.

5.2.2 Normative inventory model

We assume that the retailer orders a single item from the manufacturer each period. Both of them adopt the order-up-to policy with a periodic review procedure and any excess demand is backlogged. We assume that the manufacturer can make an expedite delivery from an outer source to fulfill the retailer’s replenishing requirement when stock-outs occur at the manufacturer. The manufacturer absorbs the expedite cost solely, which is in terms of the manufacturer’s shortage cost. At the beginning of each period, the retailer and manufacturer should decide their order-up-to levels. We formulate the two partners’ inventory decisions under three different scenarios characterized by the three information integration levels. We consider the inventory problems of the retailer and the manufacturer as single-item multi-period problems (Heyman and Sobel (1984)). First, we introduce some notation and assumptions for our models. It is suggested that there is no fixed cost of ordering and all other costs (unit costs of ordering, holding cost and shortage cost) are constant. The time period is indexed by \( t \). We define

- \( L \): lead time;
- \( c \): unit cost of ordering;
- \( h \): unit holding cost;
- \( g \): unit shortage cost;
- \( \beta \): discount factor for each period \( (0 < \beta < 1) \);
- \( R_t \): quantity ordered in period \( t \);
- \( W_t \): quantity on hand plus on order (i.e., plus \( R_{t-L} + R_{t-L+1} + \ldots + R_{t-1} \)) in period \( t \);
- \( S_t \): quantity on hand plus on order (includes those in transit) after the decision \( R_t \) has
been made in period $t$, i.e., $W_t + R_t$.

In our decentralized supply chain models, we define customer demand in period $t$ as $D_t$. To model the demand process, we assume that the retailer faces autoregressive customer demands according to the $AR(1)$ process

$$D_t = d + \phi D_{t-1} + \alpha_t,$$  \hspace{1cm} (5.1)

where $d > 0$ and $-1 < \phi < 1$ are constant, and $\alpha_t$ is the random disturbance term, which is independent and identically normally distributed with a mean $0$ and a variance $\sigma$. It is also assumed that $\sigma$ is significantly smaller than $d$, so that the probability of a negative demand is negligible. When $-1 < \phi < 0$, the demand process has negative auto-correlation; when $0 < \phi < 1$, the demand process has positive auto-correlation; when $\phi = 0$, the customer demand is $i.i.d.$ normally distributed over time. The demand model (5.1) is adopted on the assumptions that the retailer faces nonstationary demand over time and demand forecasts are updated based on observed demand, which is less restrictive than the assumption of $i.i.d.$ demand. Examples of such demand models can be seen in previous research by Kahn (1987), Urban (2000), and Lee et al. (1997b and 2000). The demand process parameters ($d$, $\alpha$ and $\sigma$) are known. Similar assumptions are also made in the work by Lee et al. (1997b and 2000). However, in practical situations, these parameters need to be estimated. The assumption with known parameters can technically be considered as the “large-sample” case, in which the parameter estimation can be made on a one-time basis from an initial set of data. In many practical situations, a large set of observed data at one time is not available, i.e., the “small-sample” case. To obtain more accurate estimates, the data set should be updated timely by some pre-defined recording procedures. The estimates of our demand model (5.1), for example, can be updated through time as each new demand information is received.

At the beginning of period $t$, an order quantity $R_{t-L-1}$ of the item arrives after demand $D_t$ is realized. Then, a decision to order a quantity $R_t$ is made. This time point is the “decision point” for period $t$. The items ordered, $R_t$, will arrive $L$ periods later—the beginning of time period $t + L + 1$. Suppose, the partner expects to operate the business for a long time. The managerial objective is to determine the order-up-to levels for each time period $t$ while minimizing the expected discounted inventory cost over an infinite horizon. Let $x^+$ denote $\max(0, x)$. The cost-minimization problems for both the retailer and the manufacturer have a normative form as follows:

$$\min \sum_{t=1}^{\infty} \beta^{t-1} \left[ c R_t + \beta^L G(S_t, \sum_{l=t+1}^{t+L+1} D_l) \right],$$  \hspace{1cm} (5.2)
where
\[
G(S_t, \sum_{i=t+1}^{t+L+1} D_i) = h \left( S_t - \sum_{i=t+1}^{t+L+1} D_i \right)^+ + g \left( \sum_{i=t+1}^{t+L+1} D_i - S_t \right)^+ .
\]

The discount factor \( \beta \), \( 0 < \beta < 1 \), embodies the partners’ time preference for money. The meaning of the discount factor, \( \beta \), can be interpreted as the most amount of money which can be invested to earn a riskless reward at a specified interest rate (see the detail interpretation and calculation of \( \beta \) in the book by Heyman and Sobel (1984)). It is assumed that each partner’s attitude to risk is consistent with a linear utility function, which means that the partners are risk neutral. For each time period \( t \), the cost function (5.2) represents the expected cost discounted to the decision point of time period \( t \). If we assume that the future consequences of the present decision can be safely ignored, the myopic (or “greedy”) behavior will be optimal (see detail in Heyman and Sobel (1984)). For the normative inventory model in this chapter, the myopic policy means the ordering decision made at time period \( t \) will be optimal if the expected inventory cost discounted to the decision point of time period \( t \) is minimized. Similar models are adopted by Baganha and Cohen (1998) to study the stabilizing effect of inventory in supply chains. By using this myopic policy, Heyman and Sobel show that
\[
S_t^* = Q^{-1}_{L+1} \left( \frac{g - c(1-\beta)/\beta^L}{h + g} \right), \tag{5.3}
\]
where \( Q_{L+1}(\cdot) \) denotes the distribution function of \( \sum_{i=t+1}^{L+1} D_i \) (conditional on the demand record up to period \( t \)).

Now note that, in period \( t \), repeating the demand process (5.1) for the next \( i \) consecutive periods yields
\[
D_{t+i} = d \frac{1-\phi^i}{1-\phi} + \phi^i D_t + \sum_{j=1}^{i} \phi^{j-1} \alpha_{t+i-j+1}. \tag{5.4}
\]

It follows that the demand over the lead time \( L \) is given by
\[
\sum_{i=1}^{L+1} D_{t+i} = d \sum_{k=1}^{L+1} \frac{1-\phi^k}{1-\phi} + \phi(1-\phi^{L+1}) \frac{1-\phi}{1-\phi} D_t
\]
\[
+ \frac{1}{1-\phi} \sum_{j=1}^{L+1} (1-\phi^j) \alpha_{t+L+2-j} .
\]

Thus, \( \sum_{i=t+1}^{t+L+1} D_i \) at the decision point in period \( t \) is a normally distributed random variable \( N(M_t, V_t) \), where \( M_t \) and \( V_t \) are the conditional expectation (conditioned on \( D_t \)) and the conditional variance of demand (conditioned on \( D_t \)) over the lead time \( L \). We have
\[
M_t = E(\sum_{i=1}^{L+1} D_{t+i} | D_t) = d \sum_{k=1}^{L+1} \frac{1-\phi^k}{1-\phi} + \phi(1-\phi^{L+1}) \frac{1-\phi}{1-\phi} D_t,
\]

75
and
\[ V_t = \text{Var}\left( \sum_{i=1}^{L+1} D_{t-i} \mid D_t \right) = \frac{\sigma^2}{(1-\phi)^2} \sum_{j=1}^{L+1} (1-\phi^j)^2. \]

Then, from (5.3), the optimal order-up-to level \( S_t^* \) is
\[ S_t^* = M_t + \Phi^{-1}\left( \frac{g - c(1-\beta)/\beta^L}{h + g} \right) \sqrt{V_t}, \tag{5.5} \]
where \( \Phi^{-1} \) is the inverse function of the standard normal distribution function \( \Phi \).

From the expressions of \( D_t \) and \( S_t \), the order quantity \( R_t \) can be written as
\[ R_t = S_t^* - S_{t-1}^* + D_t = \frac{\phi(1-\phi^L)}{1-\phi} (D_t - D_{t-1}) + D_t. \]

We should note here that \( R_t \) may be negative, which means that excess inventory can be returned. Define the constant function \( z_L(\phi) = \frac{\phi^{(1-\phi^L)}}{1-\phi} \). Generally, we have
\[ R_t = D_t + z_L(\phi)(D_t - D_{t-1}). \tag{5.6} \]

Note that
\[ \text{Var}(R_t) = \text{Var}(D_t) + [z_L(\phi)]^2 \text{Var}(D_t - D_{t-1}) + 2z_L(\phi)\text{Cov}(D_t - D_{t-1}, D_t). \]

Using the fact that \( \text{Cov}(D_t, D_{t-1}) = \phi \text{Var}(D_t) \), we have \( \text{Cov}(D_t - D_{t-1}, D_t) = (1-\phi)\text{Var}(D_t) \) and
\[ \text{Var}(R_t) = \text{Var}(D_t) \left\{ 1 + 2(1-\phi) \left[ z_L(\phi) + z_L(\phi)^2 \right] \right\}. \]

The details are given in Proof 1.

**Proof 1:**
We have
\[ \text{Var}(R_t) = \text{Var}(D_t) + [z_L(\phi)]^2 \text{Var}(D_t - D_{t-1}) + 2z_L(\phi)\text{Cov}(D_t - D_{t-1}, D_t). \]

The key issue here is to express \( \text{Var}(D_t - D_{t-1}) \) and \( \text{Cov}(D_t - D_{t-1}, D_t) \) in terms of \( \text{Var}(D_t) \). For the \( AR(1) \) process, we have \( \text{Var}(D_t) = \text{Var}(D_{t-1}) = \frac{\sigma^2}{1-\phi^2} \) (detail can be seen in the book
by Box et al (1994)). We also have

\[ \text{Cov}(D_t, D_{t-1}) = E[(D_t - E(D_t))(D_{t-1} - E(D_{t-1}))] \]
\[ = E[(d + \phi D_{t-1} + \alpha_t - E(d + \phi D_{t-1} + \alpha_t))(D_{t-1} - E(D_{t-1}))] \]
\[ = \phi E[(D_{t-1} - E(D_{t-1}))(D_{t-1} - E(D_{t-1})) + \alpha_t(D_{t-1} - E(D_{t-1}))] \]
\[ = \phi \text{Var}(D_{t-1}) = \phi \text{Var}(D_t). \]

Then, we can see

\[ \text{Var}(D_t - D_{t-1}) = \text{Var}(D_t) + \text{Var}(-D_{t-1}) + 2\text{Cov}(D_t, -D_{t-1}) \]
\[ = 2\text{Var}(D_t) - 2\text{Cov}(D_t, D_{t-1}) = 2(1 - \phi)\text{Var}(D_t). \]

Using the fact that \( \text{Cov}(D_t, D_{t-1}) = \phi \text{Var}(D_t) \), we have

\[ \text{Cov}(D_t - D_{t-1}, D_t) = E[(D_t - D_{t-1} - E(D_t - D_{t-1}))(D_t - E(D_t))] \]
\[ = E[(D_t - E(D_t) - D_{t-1} + E(D_{t-1}))(D_t - E(D_t))] \]
\[ = E[(D_t - E(D_t))[D_t - E(D_t)] - E(D_{t-1} - E(D_{t-1}))[D_t - E(D_t)]] \]
\[ = \text{Var}(D_t) - \text{Cov}(D_t, D_{t-1}) = (1 - \phi)\text{Var}(D_t). \]

With the fact that \( \text{Cov}(D_t, D_{t-1}) = \phi \text{Var}(D_t) \) and \( \text{Cov}(D_t - D_{t-1}, D_t) = (1 - \phi)\text{Var}(D_t) \), it is easy to show

\[ \text{Var}(R_t) = \text{Var}(D_t) \left\{ 1 + 2(1 - \phi) \left[ z_L(\phi) + z_L(\phi)^2 \right] \right\}. \]

\[ \square \]

It is easy to show that \( \text{Var}(R_t) > \text{Var}(D_t) \) for \( 0 < \phi < 1 \) and \( \text{Var}(R_t) < \text{Var}(D_t) \) for \( -1 < \phi < 0 \). This indicates that, in a multi-stage supply chain, there will be a variance amplification if \( 0 < \phi < 1 \), while there will be stabilization if \( -1 < \phi < 0 \). The same results are also obtained in Baganha and Cohen (1998). Under the condition of \( 0 < \phi < 1 \), the variance of order quantity exceeds the variance of demand. This is the so-called “bullwhip effect”, first pointed out by Lee et al. (1997a and 1997b).
5.3 Inventory control policies under different information sharing-based partnerships

We will investigate the ordering decisions and inventory control policies of the two partners at the three levels of information integration. There are two important indices in this section: the retailer is indexed by \( r \), while the manufacturer is indexed by \( m \).

5.3.1 Optimal policies under decentralized control

Under Level 1—decentralized control, both the retailer and the manufacturer make their inventory decisions independently according to their own forecasting. The retailer uses the customer demand information and the manufacturer uses the retailer’s ordering information.

I. Retailer’s inventory policy

The retailer faces the customer demand process (5.1). According to the normative model and its optimal policy (5.5) in Section 2.2, we have the conditional expectation (conditioned on \( D_t \)) and the conditional variance of \( \sum_{i=1}^{L_r+1} D_{t+i} \) (conditioned on \( D_t \)) over the lead time \( L_r \) as follows:

\[
M_t^r = d \sum_{k=1}^{L_r+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_r+1})}{1 - \phi} D_t, \\
V_t^r = \frac{\sigma^2}{(1 - \phi)^2} \sum_{j=1}^{L_r+1} (1 - \phi^j)^2,
\]

and

\[
S_t^r = M_t^r + K_r \sqrt{V_t^r},
\]

where \( K_r = \Phi^{-1}\left(\frac{\sigma - \sigma(1 - \phi) / \phi}{\sigma + \phi}\right) \).

According to (5.1) and (5.6), the retailer’s order quantity \( R_{t+1}^r \) is

\[
R_{t+1}^r = d + \phi R_t^r + \left[1 + z_{L_r}(\phi)\right]\alpha_{t+1} - z_{L_r}(\phi)\alpha_t
= d + \phi R_t^r + \frac{1 - \phi^{L_r+2}}{1 - \phi} \alpha_{t+1} - \frac{\phi(1 - \phi^{L_r+1})}{1 - \phi} \alpha_t.
\]

Here, we introduce a lemma useful for subsequent analysis.
Lemma 5.1 For the retailer ordering process (5.7), for \(-1 < \phi < 1\) and \(t \to \infty\), \(E(R_t^r) = \frac{d}{1-\phi}\) and \(\text{Var}(R_t^r) = \frac{\sigma^2}{1-\phi^2} \left\{ 1 + \frac{2\phi(1-\phi^{L_r+1})(1-\phi^{L_r+2})}{1-\phi} \right\} \).

Proof of Lemma 5.1: From (5.6), and using the fact that \(E(D_t) = \frac{d}{1-\phi}\) for \(t \to \infty\). it is easy to obtain \(E(R_t^r) = \frac{d}{1-\phi}\).

With customer demand information sharing, the manufacturer follows the retailer’s ordering process (5.7), which can be expressed as

\[
\tilde{R}_{t+1} - \phi \tilde{R}_t = a_{t+1} - \theta a_t,
\]

where \(\tilde{R}_t = R_t^r - \frac{d}{1-\phi}\), \(a_t = (1 + z_{L_r}(\phi))\alpha_t\), and \(\theta = \frac{z_{L_r}(\phi)}{1+z_{L_r}(\phi)}\). Process (5.8) is an ARMA(1, 1) process, and it has

\[
\text{Var}(\tilde{R}_t) = \frac{1 + \phi^2 - 2\phi \theta}{1 - \phi^2} \text{Var}(a_t).
\]

We have \(\text{Var}(R_t^r) = \text{Var}(\tilde{R}_t)\), so

\[
\text{Var}(R_t^r) = \frac{1 + \phi^2 - 2\phi \theta}{1 - \phi^2} \text{Var}(a_t) = \frac{\sigma^2}{1-\phi^2} \left\{ 1 + \frac{2\phi(1-\phi^{L_r+1})(1-\phi^{L_r+2})}{(1-\phi)} \right\}.
\]

The retailer’s demand over the manufacturer’s lead time \(L_m\) is

\[
\sum_{i=1}^{L_m+1} R_{t+i}^r = d \sum_{k=1}^{L_m+1} \frac{1-\phi^k}{1-\phi} R_t^r + \frac{\phi(1-\phi^{L_m+1})}{1-\phi} R_t^r
\]

\[
+ \frac{1}{1-\phi} \sum_{i=1}^{L_m+1} (1-\phi^{L_r+i}) \alpha_{t+L_m+2-i} - \frac{z_{L_r}(\phi)(1-\phi^{L_m+1})}{(1-\phi)} \alpha_t.
\]

II. Manufacturer’s inventory policy

We suppose the manufacturer has enough data to conduct statistical analysis to formulate a mathematical model for the retailer’s ordering process. Under such an ideal situation, the manufacturer can obtain the conditional expectation \(E(\sum_{i=1}^{L_m+1} R_{t+i}^r \mid R_t^r)\) over the manufacturer’s lead time \(L_m\) as

\[
M_t^m = E(\sum_{i=1}^{L_m+1} R_{t+i}^r \mid R_t^r) = d \sum_{k=1}^{L_m+1} \frac{1-\phi^k}{1-\phi} + \frac{\phi(1-\phi^{L_m+1})}{1-\phi} R_t^r,
\]

and the conditional variance \(\text{Var}(\sum_{i=1}^{L_m+1} R_{t+i}^r \mid R_t^r)\) as

\[
V_t^m = \text{Var}(\sum_{i=1}^{L_m+1} R_{t+i}^r \mid R_t^r) = \frac{\sigma^2}{(1-\phi)^2} \left\{ \sum_{i=1}^{L_m+1} (1-\phi^{L_r+i})^2 + \frac{\phi^2(1-\phi^{L_m+1})^2(1-\phi^{L_r+1})^2}{(1-\phi)^2} \right\}.
\]
According to the normative model (5.2), the manufacturer's optimal policy \( S_t^{m*} \) is

\[
S_t^{m*} = M_t^m + K_m \sqrt{V_t^m},
\]

where \( K_m = \Phi^{-1} \left( \frac{g_m - c_m(1 - \beta_m)/\beta_m^m}{g_m + \lambda_m} \right) \).

5.3.2 Optimal policies under coordinated control

At Level 2—coordinated control, the manufacturer can obtain the ordering information about the customer's demand process as well as the retailer's ordering information in each time period \( t \), and then make its own inventory decision with this information.

I. The retailer

We assume that the retailer has full information about the customer's demand process. Under coordinated control, there is no change of retailer's inventory decision and no substantial improvement in its inventory level, even though the retailer offers the customer demand information in time period \( t \) to the manufacturer. However, the manufacturer can offer the retailer some incentives to induce the retailer to take the initiative for information sharing.

The retailer's optimal order-up-to level under coordinated control \( S_t^{r*}|C_0 \) is equal to \( S_t^{r*} \): that is

\[
S_t^{r*}|C_0 = S_t^{r*} = M_t^r + K_r \sqrt{V_t^r}.
\]

II. The manufacturer

With information about the customer's demand in period \( t \), the manufacturer will appreciate the error term \( \alpha_t \) and the retailer's ordering process (5.7). Under this condition, the retailer's demand over the manufacturer's lead time \( L_m \) is

\[
\sum_{i=1}^{L_m+1} R_{t+i}^r = d \sum_{k=1}^{L_m+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_m+1})}{1 - \phi} R_t^r - z_L(\phi)(1 - \phi^{L_m+1}) \alpha_t \\
+ \frac{1}{1 - \phi} \sum_{i=1}^{L_m+1} (1 - \phi^{L_r+1+i}) \alpha_{t+L_m+2-i}.
\]

The conditional expectation (conditioned on \( R_t^r \)) and the conditional variance of the retailer's demand (conditioned on \( R_t^r \)) over the manufacturer's lead time \( L_m \) are \( M_t^m \) and
\[ V_t^m, \text{ respectively, as follows} \]
\[ M_t^m\mid_{C_o} = d \sum_{k=1}^{L_m+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_m+1})}{1 - \phi} R_t^r - z_{L_t}(\phi)(1 - \phi^{L_m+1}) \alpha_t, \]

and
\[ V_t^m\mid_{C_o} = \frac{\sigma^2}{(1 - \phi)^2} \left\{ \sum_{i=1}^{L_m+1} (1 - \phi^{L_r+1+i})^2 \right\}. \]

The manufacturer’s optimal make-up-to level is
\[ S_t^{m*}\mid_{C_o} = M_t^m\mid_{C_o} + K_m \sqrt{V_t^m\mid_{C_o}}. \]

### 5.3.3 Optimal policies under centralized control

Under Level 3—centralized control, both the retailer and the manufacturer know the customer’s demand process in each period. This can be achieved by EDI between them. The manufacturer can help the retailer to establish its inventory policy and decide its own inventory via the VMI policy. This means that the inventory policies of the two partners are related only to the demand process of the customer. The VMI policy suggests that the manufacturer help the retailer to make its replenishment decision, so there is no need for the retailer to place orders for replenishing. Under this centralized information sharing scenario, the manufacturer makes its own and the retailer’s inventory control decisions based on customer’s demand information.

For consistency, we still call the inventory level of the retailer after the manufacturer replenishes the its inventory as the retailer’s order-up-to level. Thus, the optimal order-up-to level under centralized control \( S_t^{r*}\mid_{C_r} \) is equal to \( S_t^{r*} \), that is
\[ S_t^{r*}\mid_{C_r} = S_t^{r*} = M_t^r + K_r \sqrt{V_t^r}. \]

The manufacturer makes its own inventory policy by using the customer demand information. In period \( t \), the manufacturer delivers \( R_t^{r} \) units of the item as the retailer’s replenishing quantity. However, this \( R_t^{r} \) should be subject to customer demand \( D_t \) (see (5.6)), but not retailer’s ordering information (see (5.7)). According to (5.6), the retailer’s replenishing quantity over the manufacturer’s lead time \( L_m \) should be

\[ \sum_{i=1}^{L_m+1} R_{t+i}^{r} = \sum_{k=1}^{L_m+1} D_{t+i} + \frac{\phi(1 - \phi^{L_r+1})}{1 - \phi} (D_{t+L_m+1} - D_t) \]
\[ = d \left[ \sum_{k=1}^{L_m+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_r+1})(1 - \phi^{L_m+1})}{(1 - \phi)^2} \right] + \frac{\phi^{L_r+2}(1 - \phi^{L_m+1})}{1 - \phi} D_t \]

81
\[ + \frac{1}{1 - \phi} \sum_{i=1}^{L_m + 1} (1 - \phi L_r + 1 + i) \alpha_{t + L_m + 2 - i}. \]

Therefore, the manufacturer’s conditional expectation (conditioned on \( D_t \)) and the conditional demand (conditioned on \( D_t \)) over the lead time \( L_m \) are

\[
M_t^m|_{Ce} = d \left[ \sum_{k=1}^{L_m + 1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_r + 1})(1 - \phi^{L_m + 1})}{(1 - \phi)^2} \right] + \frac{\phi^{L_r + 2}(1 - \phi^{L_m + 1})}{1 - \phi} D_t
\]

and

\[
V_t^m|_{Ce} = \frac{\sigma^2}{(1 - \phi)^2} \left\{ \sum_{i=1}^{L_m + 1} (1 - \phi^{L_r + 1 + i})^2 \right\}.
\]

The manufacturer’s optimal order-up-to level \( S_t^{m*}|_{Ce} \) is

\[
S_t^{m*}|_{Ce} = M_t^m|_{Ce} + K_m \sqrt{V_t^m|_{Ce}}.
\]

### 5.4 The benefits of information sharing-based partnership

Based on the optimal policies of the three information integration levels, the benefits of the information sharing-based partnership will be derived in two ways: (1) inventory reduction, and (2) expected cost reduction.

First, we introduce two propositions based on Lemma 5.1.

**Proposition 5.1** For \( 0 \leq \phi < 1 \), we have: (i) \( V_t^m \geq V_t^m|_{Co} = V_t^m|_{Ce} \); (ii) \( V_t^m \). \( V_t^m|_{Co} \), and \( V_t^m|_{Ce} \) are increasing with \( L_m \).

**Proof of Proposition 5.1:**

(i) Note that

\[
V_t^m = \frac{\sigma^2}{(1 - \phi)^2} \left\{ \sum_{i=1}^{L_m + 1} (1 - \phi^{L_r + 1 + i})^2 + \frac{\phi^2(1 - \phi^{L_m + 1})^2(1 - \phi^{L_r + 1})^2}{(1 - \phi)^2} \right\}
\]

\[\geq V_t^m|_{Co} = \frac{\sigma^2}{(1 - \phi)^2} \left\{ \sum_{i=1}^{L_m + 1} (1 - \phi^{L_r + 1 + i})^2 \right\}.
\]

So, we have \( V_t^m \geq V_t^m|_{Co} = V_t^m|_{Ce} \).
(ii) For \(0 \leq \phi < 1\), we see that the partial differentiations \((\partial V_t^m / \partial L_m), (\partial V_t^m|_{Co}/\partial L_m), \) and \((\partial V_t^m|_{Ce}/\partial L_m) \geq 0\). Then, we have that \(V_t^m, V_t^m|_{Co},\) and \(V_t^m|_{Ce}\) are non-decreasing with \(L_m\).

**Proposition 5.2** For \(0 \leq \phi < 1\) and \(t \to \infty\), we have:

(i) \(E(M_t^m) = E(M_t^m|_{Co}) = E(M_t^m|_{Ce})\);
(ii) \(Var(M_t^m) = Var(M_t^m|_{Co}) \geq Var(M_t^m|_{Ce})\).

**Proof of Proposition 5.2:**

(i) Based on Lemma 5.1, we have \(E(R_t^r) = E(D_t) = d \frac{1}{1-\phi}\). For \(0 \leq \phi < 1\) and \(t \to \infty\),

\[
E(M_t^m) = E(M_t^m|_{Co})
= d \left[ \sum_{k=1}^{L_m+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_m+1})}{1 - \phi} \right]
= d \left[ \sum_{k=1}^{L_m+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_m+1})}{(1 - \phi)^2} \right],
\]

and

\[
E(M_t^m|_{Ce}) = d \left[ \sum_{k=1}^{L_m+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_m+1})(1 - \phi^{L_{m+1}})}{(1 - \phi)^2} \right] + \frac{\phi^{L_r+2}(1 - \phi^{L_m+1})}{1 - \phi} E(D_t)
= d \left[ \sum_{k=1}^{L_m+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_m+1})(1 - \phi^{L_{m+1}})}{(1 - \phi)^2} \right] + \frac{\phi^{L_r+2}(1 - \phi^{L_m+1})}{(1 - \phi)^2}
= d \left[ \sum_{k=1}^{L_m+1} \frac{1 - \phi^k}{1 - \phi} + \frac{\phi(1 - \phi^{L_{m+1}})}{(1 - \phi)^2} \right].
\]

Therefore, \(E(M_t^m) = E(M_t^m|_{Co}) = E(M_t^m|_{Ce})\).

(ii) For \(0 \leq \phi < 1\) and \(t \to \infty\), based on Lemma 5.1, we have

\[
Var(M_t^m) = Var(M_t^m|_{Co}) = z_{L_m}^2(\phi) Var(R_t^r) = \frac{\sigma^2 z_{L_m}^2(\phi)}{1 - \phi^2} \left\{ [1 + z_{L_r}(\phi)^2] + z_{L_r}(\phi)^2 \right\},
\]

and

\[
Var(M_t^m|_{Ce}) = \phi^{2(L_r+1)} z_{L_m}^2(\phi) Var(D_t) = \frac{\sigma^2 \phi^{2(L_r+1)} z_{L_m}^2(\phi)}{1 - \phi^2}.
\]

It is obvious that \(Var(M_t^m) = Var(M_t^m|_{Co}) \geq Var(M_t^m|_{Ce})\).
From the expressions of $S_t^{m*}$, $S_t^{m*}|C_0$ and $S_t^{m*}|C_e$, we see that they are random variables which are conditioned on $R_t^*$ or $D_t$. We see that $\text{Var}(S_t^{m*}) = \text{Var}(M_t^{m*})$, $\text{Var}(S_t^{m*}|C_0) = \text{Var}(M_t^{m}|C_0)$ and $\text{Var}(S_t^{m*}|C_e) = \text{Var}(M_t^{m}|C_e)$. According to Proposition 5.2, for $0 \leq \phi < 1$ and $t \to \infty$, it is obvious that $\text{Var}(S_t^{m*}) = \text{Var}(S_t^{m*}|C_0) \geq \text{Var}(S_t^{m*}|C_e)$. This result tells us that variability of the manufacturer's optimal order-up-to level is decreasing with increasing level of information integration, i.e., the manufacturer's inventory will be more stabilized under the partnership relationship.

Based on the above two propositions, we have the following theorem.

**Theorem 5.1** The manufacturer's optimal order-up-to level is decreasing with increasing level of information integration, i.e., for $0 \leq \phi < 1$,

$$\lim_{t \to \infty} E(S_t^{m*}) \geq \lim_{t \to \infty} E(S_t^{m*}|C_0) = \lim_{t \to \infty} E(S_t^{m*}|C_e).$$

**Proof of Theorem 5.1:**

For $0 \leq \phi < 1$, we have $\lim_{t \to \infty} E(S_t^{m*}) = E(M_t^{m}) + K_m \sqrt{V_t^{m}}$, $\lim_{t \to \infty} E(S_t^{m*}|C_0) = E(M_t^{m}|C_0) + K_m \sqrt{V_t^{m}|C_0}$ and $\lim_{t \to \infty} E(S_t^{m*}|C_e) = E(M_t^{m}|C_e) + K_m \sqrt{V_t^{m}|C_e}$. According to Proposition 5.1 and Proposition 5.2, it is easy to show that $\lim_{t \to \infty} E(S_t^{m*}) \geq \lim_{t \to \infty} E(S_t^{m*}|C_0) = \lim_{t \to \infty} E(S_t^{m*}|C_e)$. \hfill \Box

From the results obtained, we note that for each partner's inventory control policy, the retailer will not see any change in its order-up-to level whereas the manufacturer will achieve a reduction in its order-up-to level under an information sharing-based partnership. In the following discussion of the benefits of the partnership, we will focus on quantifying the manufacturer's reductions in inventory level and expected cost.

### 5.4.1 Inventory levels under different situations

It should be noted that the decision variable $S_t$ is not the inventory level. The inventory level of the manufacturer at the end of period $t - L_m$ (possibly negative if there is excess demand) is $S_{t-L_m}^{m*} - \sum_{i=1}^{L_m+1} R_{t+2-i}$ (conditioned on $R_{t-L_m}^*$). We consider the negative value of inventory level as zero. It can also be represented by $S_{t-L_m}^{m*} - \sum_{i=1}^{L_m+1} R_{t-i}$ (conditioned on $R_t^*$) because $\sum_{i=1}^{L_m+1} R_{t+2-i}$ (conditioned on $R_{t-L_m}^*$) has the same distribution as $\sum_{i=1}^{L_m+1} R_{t+i}$ (conditioned on $R_t^*$) (see Heyman and Sobel (1984)).

To obtain the manufacturer's average inventory level at the end of period $t$, we give the
expectation of the inventory level, denoted as \( I_m \), as follows

\[
I_m = E \left( S_t^{m^*} - \sum_{i=1}^{L+1} R_{t+i} \right) = E(S_t^{m^*}) - E \left( \sum_{i=1}^{L+1} R_{t+i} \right) = K_m \sqrt{V_t^m}.
\]

The inventory levels under different information sharing scenarios are \( I_m = K_m \sqrt{V_t^m} \), \( I_m|_{C_o} = K_m \sqrt{V_t^m|_{C_o}} \) and \( I_m|_{C_e} = K_m \sqrt{V_t^m|_{C_e}} \), based on the expressions of the manufacturer’s order-up-to levels. Then, from Proposition 5.1, we have \( I_m \geq I_m|_{C_o} = I_m|_{C_e} \). This observation shows that the manufacturer’s inventory level decreases with an increase in information sharing level and the manufacturer can obtain a reduction in its inventory level.

### 5.4.2 Cost reduction based on the information sharing-based partnership

According to Heyman and Sobel (1984), the cost-minimization problem (5.2) can be formulated as

\[
\min \sum_{t=1}^{\infty} \beta^{t-1} E[G(S_t)],
\]

where

\[
G(S_t) = c(1-\beta)S_t + \beta^L E \left[ G(S_{t+L}, \sum_{l=1}^{L+1} D_l) \right] + \beta^L E \left[ G(S_{t+L+1}) \right].
\]

The expectation operator \( E \) denotes the expectation taken at the decision point of period \( t \).

Let \( q_{L+1}(\cdot) \) denote the density function for \( \sum_{i=1}^{L+1} D_i \) (given by the \( L+1 \)-fold convolution of \( q \)), and \( r \) denote the fixed retail price of the item. With \( E(D_t) = \frac{d}{1-\sigma^2} \), the cost function \( G(S_t) \) can be expressed as

\[
G(S_t) = c(1-\beta)S_t + \beta^L \left[ h \int_0^{S_t} (S_t - x) q_{L+1}(x) \ dx \right]
\]

\[
+ g \int_{S_t}^{\infty} (x - S_t) q_{L+1}(x) \ dx - \frac{r(L+1)d}{1-\phi}.
\]

We define the function \( F(S_t) \) as

\[
F(S_t) = h \int_0^{S_t} (S_t - x) q_{L+1}(x) \ dx + g \int_{S_t}^{\infty} (x - S_t) q_{L+1}(x) \ dx.
\]

Let \( L(x) \) be the right loss function of the standard normal distribution, where

\[
L(x) = \int_x^{\infty} (z - x) \ d\Phi(z).
\]
and $\Phi(x)$ is the standard normal probability distribution function.

In Section 2, we deduce that the optimal order-up-to level is $S_t^* = Q_{L+1}^{-1}\left(\frac{g-c(1-\beta)/\beta^L}{h+g}\right) = M_t + K\sqrt{V_t}$ and assume that $Q_{L+1}(\cdot)$ is normally distributed, where $K = \Phi^{-1}\left(\frac{g-c(1-\beta)/\beta^L}{h+g}\right)$. Hence, the function $\mathcal{F}(S_t)$ can be represented as

$$\mathcal{F}(S_t^*) = \sqrt{V_t}[(h + g)L(K) + hK].$$

Next, the expectation of the cost function $G(S_t^*)$ can be the period average cost when $\beta \to 1$. $t \to \infty$, which is

$$E(G(S_t^*)) = c(1-\beta)E(S_t^*) + \sqrt{V_t}[(h + g)L(K) + hK] - \frac{r(L+1)d}{1-\varphi}.$$

For the manufacturer, the expected costs under the three information integration levels are $E(G(S_t^{m*}))$, $E(G(S_t^{m*}|C_o))$ and $E(G(S_t^{m*}|C_e))$, respectively. According to Proposition 5.1 and Theorem 5.1, the manufacturer’s expected cost under the three scenarios have the relationship: $E(G(S_t^{m*})) \geq E(G(S_t^{m*}|C_o)) = E(G(S_t^{m*}|C_e))$. Hence, the manufacturer can also achieve a reduction in expected cost under the information sharing-based supply chain partnership.

### 5.4.3 The retailer’s benefits

Under the three proposed information sharing scenarios, the retailer’s order-up-to level remains the same and so the retailer will not gain any improvement in its inventory level and expected cost. There are two reasons for this outcome. One is that we assume the retailer has perfect information about the customer’s demand and there is no further customer demand information that the retailer can obtain when it shares the information with the manufacturer. The other is that we assume the retailer’s lead time $L_r$ is fixed, which is subject to the manufacturer’s supplying reliability. Without any information sharing between the supply chain members, the retailer’s lead time is an estimation of the manufacturer’s time for order processing, manufacturing and delivering. The existing supply chain partnerships can let the manufacturer to share the lead time information with the retailer, or even make the manufacturer shorten the retailer’s lead time $L_r$. The retailer can also achieve more accurate lead time information with information sharing-based partnerships. From Proposition 5.1, we see that $V_t^{m}$, $V_t^{m}|C_o$ and $V_t^{m}|C_e$ are increasing with the manufacturer’s lead time $L_m$. The same approach can be applied to $V_t^{r}$, $V_t^{r}|C_o$ and $V_t^{r}|C_e$ and it is easy to show $V_t^{r}$, $V_t^{r}|C_o$ and $V_t^{r}|C_e$ are increasing with the retailer’s lead time $L_r$. If the retailer’s lead time can be shortened, applying our approaches to analyze the manufacturer’s benefits, we will reach the conclusions that the retailer can achieve benefits in both inventory level and expected cost from information sharing-based partnerships.
5.4.4 Pareto improvement

From the comparisons of inventory reductions and cost savings of the manufacturer among the three relationship integration levels, we see that the manufacturer can obtain benefits from increasing the level of information integration when the customer demand is positively correlated over time. Under the information sharing-based supply chain partnership both the retailer and the manufacturer are at least as well off, and at least one of them—the manufacturer—is better off. The information sharing-based partnership can improve the overall performance of the decentralized supply chain. Therefore, we conclude that with the information sharing-based supply chain partnership, Pareto improvement will be achieved in respect of the overall performance of the decentralized supply chain.

5.5 Conclusion

We attempt to quantify the benefits of information sharing-based supply chain partnerships in this chapter. Based on a two-stage decentralized supply chain including a retailer and a manufacturer, a comprehensive study of modelling and analysis has been performed to investigate the partners’ optimal order-up-to policies under three levels of information sharing-based relationship. From the comparisons of inventory reductions and cost savings of the two partners, Pareto improvement is achieved in respect of the entire supply chain performance. Specifically, the manufacturer can obtain considerable performance improvement from increasing the level of information sharing, while there is no change in the retailer's inventory level and cost (based on our assumptions). The retailer can also gain benefits from lead time information sharing.

The “bullwhip effect” exists in the decentralized supply chain in our model because the manufacturer uses the retailer's ordering information to determine its inventory policy without any information about the customer demand process. With access to the customer ordering information, the manufacturer can eliminate the amplified customer’s demand variance in its replenishment process. If the manufacturer and the retailer can retrieve the customer demand information in a synchronized manner using EDI, a VMI strategy can be adopted in the decentralized supply chain to obtain optimal performance under centralized control. In the VMI case, the manufacturer makes the major inventory replenishment decision depending on the customer demand process directly, without regard to the retailer's ordering information.

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of the two partners, *Pareto improvement* is achieved in respect of the entire supply chain performance. Specifically, the manufacturer can obtain considerable performance improvement with increasing level of information sharing, while there is no change in the retailer's inventory level and cost (based on our assumptions). The retailer can also get benefits from lead time information sharing. We achieve the conclusion that the information sharing-based partnership can improve the overall performance of the decentralized supply chain, and the supply chain management should choose strategic partnerships based on information sharing via adopting VMI using EDI to achieve overall optimum in decentralized supply chains.

According to our analysis, we see that the supply chain partnership can not only help the entities of a decentralized supply chain to eliminate the "bullwhip effect", but also improve the overall performance of the supply chain. The supply chain partners can use EDI to adopt a VMI strategy for inventory decision making. In our modelling study, the manufacturer obtains more benefits than the retailer does. Therefore, we suggest that the manufacturer should take the initiative to establish information sharing-based partnerships and also give the retailer some incentives to induce the retailer's cooperation.
Chapter 6

Conclusions and future research

6.1 Conclusions

Modelling and analysis of strategic supply chain systems have been important topics in the field of Management Science and Operations Management. In the past two decades, a considerable body of literature on the topics was published, which covered many aspects of supply chain management. The existing studies show a clear evolution that the modelling study of supply chain management has evolved from the work on multi-echelon network design and inventory control to recent studies of various mechanisms of coordinating supply chain decisions. From the trend towards global markets and growing customer orientation, one key requirement of these challenges is to incorporate supply chain partnerships in supply chain management systems, which contribute to reallocation of stocking decisions right, sharing information and coordinating decisions. There still exist rich opportunities for researchers to further the study in the area of modelling and analysis of strategic supply chain systems. This research aims to extend these studies.

In Chapter 2, we give a comprehensive review of the the papers appearing in current literature deal with various modelling and analysis methodologies of supply chain management systems. The review shows that there is a need for modelling and analysis of supply chain management systems with the objective of measuring the performance of various strategies, methods and technologies. We identify three fundamental issues associated with a supply chain, namely, the supply chain network design, the nature of supply chain uncertainty and the relationships between different stages of the supply chain. The literature is reviewed based on the classifi-
cation of these three issues of supply chain management. This thesis discusses the findings in the existing opportunities of all these three featured areas of supply chain modelling research. We name the three parts of the thesis as: modelling and analysis on supply chain design, modelling and analysis on supply chain uncertainty, and modelling and analysis on supply chain partnerships under information sharing and coordination.

In Chapter 3, we introduce logical constraints in a strategic supply chain design model to include the constraints related to bills of materials (BOM). The consideration of BOM constraints is an interesting topic in the strategic supply chain design. The logical interactions among the three basic supply chain operation processes, say, procurement, production and distribution should be formulated as constraints in the current mixed integer programming models. To reflect the logical consistent relationships between the supplier and manufacturer, BOM can play an active role in supply chain design models. We present the bottom level of BOM can be described as a list of specified quantities of materials or components supplied by different vendors. BOM can be formulated as logical rules such as “if product $i$ will be produced in at least $s$ of the proposed plants, then at least $t$ of the candidate vendors must be selected”. This formulation of BOM constraints provides a reasonable way to capture the role of vendors in the strategic supply chain design model. We show how BOM constraints are formulated in our proposed MIP model to help the selection of vendors. and how linear representation of logical constraints is developed. In order to make the MIP model with logical constraints to be tractable, we extensively discuss the representation of logical constraints. Based on the CNF representation of logical rules, the large number of clauses in the CNF will result in a great number of inequalities embedded in the original model. This can lead to a significant increase in the solution time of the model. We show how to achieve simplified representation of the logical consistency constraints and obtain an efficient set of inequalities with the help of additional information about vendors of the supply chain.

In Chapter 4, we study a periodic review $(R, Q)$ inventory policy for a two-stage supply chain, which includes a supplier and a retailer. The supplier orders from an unstable outer source, which means the supplier can’t receive the placed order on time and its lead time is variable. We formulate this uncertain lead time to be i.i.d. with distribution function $G(\cdot)$. and the mean of the supplier’s lead time is also known. The retailer faces customer demands which form a Poisson process with intensity $\lambda$. Each customer demand arrival brings a batch of $Q$ units. Both of the supplier and retailer adopt periodic-review batch-ordering policies. We present a modelling framework to demonstrate the modelling analysis of supplier’s lead time variability in this two-stage supply chain. Specifically, a one-for-one replenishment system is adopted to give an performance measure of our serial two-stage supply chain. We show the
relationships between inventory costs and lead time variability. Comparing the results to the model with constant lead time, we can see that this variability can cause excess inventory. We also suggest further approaches in this topic about the reduction of the deficiency of lead time variability and the possible coordination mechanisms to reduce the variability.

In Chapter 5, we present a modelling framework to quantify the benefits of information sharing-based supply chain partnerships. In a two-stage supply chain consisting of a single retailer and a single manufacturer, we introduce three levels of information integration for modelling the partnerships, namely decentralized control, coordinated control and centralized control. We derive the optimal inventory policies for the manufacturer and the retailer under these three different information sharing scenarios. According to the comparative analysis of inventory levels and expected inventory costs of the manufacturer and the retailer, we show that increasing information sharing among the members in a decentralized supply chain will lead to Pareto improvement in the performance of the entire chain. We see that the supply chain partnership can not only help the entities of a decentralized supply chain to eliminate the "bullwhip effect", but also improve the overall performance of the supply chain. Specifically, both of the manufacturer and the retailer can obtain benefits in terms of reductions in inventory levels and cost savings by using EDI to adopt a VMI strategy for inventory decision making.

6.2 Future research

We discuss three major issues of strategic supply chain systems in this thesis. There are still many research opportunities to further our study on the three particular topics. To extend our research further, we suggest some directions for future work.

Strategic supply chain design

In order to incorporate BOM constraints more effectively in a strategic supply chain design model, we introduce logical constraints to formulate BOM as logical rules and give the linear representation of the logical rules. In fact, similar approaches can also be applied to formulate more complicated relationships among the supply chain entities, such as the transportation connections between manufacturers and distribution centers. Meanwhile, testing the effectiveness of logical constraints in the strategic supply chain network design models from the real world is needed in order to give more guidelines for enterprise applications of conducting a
comprehensive supply chain design.

Furthermore, to give a more effective formulation of BOM in supply chain design models is only a particular topic of the potential research opportunities in the area of supply chain design. There is still a need to develop integrated approaches to make supply chain performance design decisions at all stages of the supply chain. If some stochastic factors (like timely customer demand and inventory level) are introduced to the traditional MIP model, some constraints such as service level and facility capacity will change their roles in supply chain structure design models. The development of mixed-integer stochastic programming models will be helpful to explore more comprehensive approaches for both supply chain structure and performance design.

Supply chain uncertainties

To formulate the supplier’s uncertainties in a two-stage supply chain, we give the assumption that the supplier’s lead time is i.i.d. with the distribution function $G(\cdot)$. We use one-for-one replenishment system to give a exact approximation of our supply chain. Research is still needed to consider the multiple retailer with shared units. Under the circumstances of perfect and imperfect information between the supplier and the retailers, the value of reducing lead time variability in a two-stage system should be identified and more accurately quantified. With different distributions of the supplier’s lead time, the influence of the supplier’s lead time may be different on the inventory cost settings of the whole system. Same questions also exist with the different types of customer demand. The research on the performance evaluations of the supply chain system with the combination of both the uncertainties from the upper stream suppliers and downstream customers will be an very interesting and promising research topic in the area of the modelling and analysis of strategic supply chain management systems.

Supply chain partnerships

For the customer demand process (5.1), most of the results are obtained for the case of $0 \leq \phi < 1$, which means the orders are positively correlated. The reason is that we want to capture the benefits of the supply chain partnerships based on information sharing, especially the ordering information. However, what happens when $-1 < \phi < 0$? We know that there is no variance amplification along the supply chain when $-1 < \phi < 0$. We also know that
the partnerships can lead to supply chain performance improvement. But, how to model the benefits of partnerships under this condition is an interesting research question. We also see that the benefits of supply chain partnerships can be captured by modelling the supplying reliability of the supplier, especially the lead time reduction.

In our modelling analysis on the information sharing-based supply chain partnerships, we focus on the partnerships between the manufacturer and the retailer, although our results still hold for the wholesaler and distributor or manufacturer and assembler for the case of single product or multiple independent products (the demand process of each product is independent of those of other products). Our model can be extended to the case in which one manufacturer supplies a single item or independent items to multiple retailers. When there are more than two levels in a supply chain, for example, which includes a manufacturer, a distributor and a retailer, our analysis is applicable to the manufacturer and distributor or the distributor and the retailer. However, modelling analysis of information sharing and partnerships among the three partners is a quite promising research area. It is also very interesting to consider the situation where there are a number of manufacturers in the supply chain, especially manufacturers with capacity constraints.
References


94


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97


Appendix

Publications

Refereed Journal Articles:


Conference Presentation:


Other Publications: