PRICE COMMITMENT VS. PRICE FLEXIBILITY: EFFECTS ON NEW PRODUCT INTRODUCTION WITH QUALITY IMITATION AND SPILLOVER

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Price Commitment vs. Price Flexibility: Effects on New Product Introduction with Quality Imitation and Spillover

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Abstract

Introducing new products into the marketplace presents significant challenges, notably the free-riding behavior of copycats that affects both large corporations and small business innovators. Developing effective strategies to mitigate this issue is crucial for achieving success in competitive markets. Additionally, when competing firms embark on exploring new markets, they often face dual pressures: the need to enhance product quality by accumulating experience and progressing along learning curves, and the inadvertent expansion of brand awareness that benefits all competing products. These dynamics underscore the complex interplay between innovation, competition, and market entry strategies that are central to overcoming obstacles in new product introduction and market exploration.

In the first study, we examine the role of price commitment and quality management as strategies to tackle the threat of imitation. We develop a parsimonious game-theoretic model involving an innovator and a copycat to analyze the impact of committed pricing and flexible pricing strategies on imitation deterrence and their effects on welfare. Our findings reveal several key insights. First, when quality is exogenous, flexible pricing is more effective in blockading copycats, whereas committed pricing excels in deterring them. However, when quality is endogenously managed, flexible pricing may outperform committed pricing in deterring copycats. Second, innovators should pursue price leadership through committed pricing when quality investment is inexpensive. By contrast, when quality investment is moderately affordable, the

flexibility provided by flexible pricing consistently makes the firm better off. Third, the threat of imitation incentivizes innovators to strategically limit both price and quality under either pricing strategy. In this context, limit quality and limit pricing function as strategic complements, and the presence of replica products can lead to a more aggressive approach in both dimensions. Last, a reduction in quality investment costs or an increase in market competition does not necessarily benefit consumers or society. Their potential benefits can be outweighed by the strategic reduction in product quality, resulting in adverse outcomes for both consumer surplus and social welfare. We further validate the robustness of our findings in the presence of strategic consumers. This study underscores the intricate interplay between pricing strategies, quality management, and imitation, highlighting the nuanced challenges innovators face in safeguarding their market position.

In the second study, we study a two-period duopoly price competition where firms can improve their quality based on the accumulated demand (learn-by-doing effect) and their potential market size is positively affected by both firms' quality levels (quality spillover effect). In addition, we investigate two pricing schemes, namely, committed pricing and dynamic pricing, and their impact on the equilibrium outcomes. Assuming the two firms are symmetric in every aspect, our main findings include the following. First, we establish the existence and uniqueness of the pure Nash equilibrium for the dynamic game under either pricing scheme, and show that firms always set a low price in the first period to leverage quality improvement. As the quality spillover effect gets stronger, firms tend to raise their first-period price, leading to a lower individual quality improvement and a non-monotonic impact on firms' profit. Moreover, we find that committed pricing scheme benefits the duopoly when the spillover effect is strong, otherwise dynamic pricing scheme brings more profits. Finally, we examine two asymmetric cases where the firms are different in certain attributes pertaining to their learning speed and the quality spillover strength. Our analysis shows that the findings in the symmetric case still hold qualitatively.

Useful managerial insights are derived from these studies.

Publications Arising from the Thesis

1. Chen, Z., Guo, X., Wang, Y., Xiao, G., Pricing Strategies Against Quality Imitation. To be submitted.

2. Geng X., Chen Z., Guo X., Xiao G. (2022) Duopoly Price Competition with Quality Improvement Spillover. ***Naval Resaerch Logistics*** 69(7): 958-973.

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The past five years have indeed been extensive. I distinctly recall arriving in Hong Kong in September 2020, prior to the major outbreak of the pandemic. Even then, pursuing academic studies in Hong Kong necessitated a 14-day home quarantine period. Upon formal enrollment, all courses were conducted online to maintain social distancing protocols. Consequently, I initially did not have a strong sense of pursuing doctoral studies. Professor Guang Xiao provided me with a detailed research direction, which marked the beginning of my somewhat challenging research journey. In retrospect, the initial progress and work were modest, and I remain grateful for the professor's meticulous guidance and patience during that period.

The pandemic escalated shortly after my enrollment, with the most significant impact being the severe restriction on mainland travel. I thus spent two Lunar New Years in Hong Kong, and despite the geographical proximity to my hometown, the experience was analogous to studying in a distant foreign country. Returning to mainland China required a minimum quarantine period of fourteen to twenty-one days. In contemporary society, temporal barriers prove more formidable than spatial ones. It wasn't until 2022, following the completion of my MPhil coursework and thesis, that I was able to visit my hometown, necessitating a fourteen-day quarantine period in Zhuhai.

The initial two years of my academic pursuit progressed satisfactorily, characterized by both promising research developments and positive feedback in theoretical coursework. This performance led me to conclude that I possessed sufficient intellectual capacity to pursue a research-oriented career trajectory. However, this assumption proved significantly miscalibrated, as independent research presented challenges far exceeding my initial expectations. Subsequently, it required approximately one year of persistent effort before I successfully completed my first independent manuscript. During this period, I encountered numerous obstacles that frequently prompted considerations of discontinuation.

While deriving research ideas from practical phenomena is relatively straightforward, given the expansive and engaging nature of the business domain, the primary challenge lies in the theoretical modeling of these ideas. This process demands both comprehensive mastery of existing theoretical frameworks and an exceptional understanding of the specific research topic that surpasses conventional knowledge levels. The construction of this bridge between theoretical frameworks and empirical reality remains a challenging endeavor, one that I cannot yet claim to have mastered. Nevertheless, this developmental process has gradually crystallized my professional aspirations, leading me to recognize this as my desired career trajectory.

Throughout my academic journey, I am profoundly grateful to my chief supervisor, Professor Xiao, whose guidance and mentorship were instrumental in facilitating my entry into the research domain. I extend my sincere appreciation to my co-supervisors, Professor Yulan Wang and Professor Xiaomeng Guo, for their consistent support and assistance throughout my doctoral studies. My research capabilities and manuscript development skills were significantly enhanced through collaboration with and mentorship from Professor Fasheng Xu and Professor Xin Geng.

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Chapter 1

Introduction

In today's intensely competitive markets, firms are not only challenged by rivals but also face the paradox where their own quality improvements can become a source of competitive advantage for others. Competitors can gain an edge directly from the enhancements one company makes, intensifying the race to stay ahead. For example, imitation has become a pervasive issue; copycat firms are ubiquitous, exploiting their late-mover advantage to replicate successful products. The more successful and innovative a product is, the more these imitators benefit, which further undermines the original creator's market position. Moreover, as companies strive to improve product quality—often by integrating customer feedback—they must contend with the reality of quality improvement spillovers. In many industries, these spillover effects mean that competitors can capitalize on each other's advancements without bearing the same costs or efforts. This interconnectedness allows rivals to benefit from the innovations of others, potentially eroding the original firm's competitive edge. These dual challenges of rampant imitation and quality spillovers imply that firms need to exercise caution with their quality improvement.

A common thread linking these challenges is the critical role of pricing strategies. Firms must choose between price commitment and pricing flexibility - while price commitment may help alleviate market competition (e.g., through pre-announced prices or price matching guarantees), pricing flexibility allows firms to build competitive advantages by responding to market changes (e.g., through dynamic pricing or promotional adjustments). Both strategies are prevalent in practice, as seen in retailers' different approaches to holiday sales or airlines' contrasting pricing models. Moreover, since quality improvements may benefit competitors through imitation or spillover effects, firms must carefully evaluate their quality investment decisions. Understanding how these pricing and quality decisions interact in competitive environments is essential for developing effective strategies that balance competition, imitation risks, and market growth.

The first study addresses the pressing issue of copycat products and their detrimental impact on innovation. In today's digital era, opportunistic imitators have unprecedented access to innovative ideas and popular products, posing significant threats to both small-scale entrepreneurs and large corporations. For instance, crowdfunding projects like the KAISR air lounge and the Fidget Cube have quickly encountered copycats, undermining the original creators' ability to secure capital and invest in patent protection. Even major industry players like Apple face imitation, as seen with Xiaomi producing phones that closely resemble the iPhone.

To combat copycats, the study explores alternatives to traditional intellectual property enforcement, which can be costly and ineffective due to blurred lines between imitation and originality. It examines the use of marketing and operational strategies, specifically pricing strategies and quality management, as tools for innovators. By analyzing flexible (contingent) pricing, where innovators adjust prices in response to competition, versus committed pricing, where prices remain consistent, the research investigates the optimal approach for innovators to deter copycat entry. Furthermore, it considers how quality investment affects these strategies and the potential benefits to consumer surplus and social welfare.

The second study delves into the dynamics of emerging "blue ocean" industries, where start-up firms face the dual challenges of improving product quality and expanding market awareness. Innovative products often lack extensive trials and may not fully meet customer needs, making it vital for firms to climb a learning curve through experience-based quality improvements. The more these firms engage with customers, the more they can refine their offerings. Concurrently, expanding brand awareness is crucial, often achieved through word-of-mouth that can lead to a quality spillover effect benefiting all firms in the industry.

In this context, pricing is the central decision variable linking learn-by-doing, quality improvement, and profitability. The study investigates two common pricing schemes: committed pricing, where firms set and maintain prices over time, and dynamic pricing, where prices are adjusted periodically. By building a two-period duopoly model incorporating experience-based quality improvement and positive spillover effects, the research examines how these pricing strategies affect competition and firms' profits. It also explores how differences between firms, such as varying abilities to improve quality or capitalize on spillover effects, influence the competitive landscape.

By addressing these interconnected challenges through the lenses of pricing strategies and quality improvement, both studies contribute valuable insights into strategic decision-making in competitive environments. They highlight the importance of selecting appropriate pricing schemes and managing quality improvements to deter imitation, enhance competitiveness, and ultimately promote consumer welfare and industry advancement.

Chapter 2

Pricing Strategies Against Quality Imitation

2.1 Introduction

Copycat products are a pressing threat to innovation. In today's digital era, innovative ideas and popular products are much more readily accessible than ever–not just to consumers but also to opportunistic imitators. The consequences of replication are far-reaching, impacting both small-scale entrepreneurs and large-scale corporations. For instance, crowdfunding platforms have become hotbeds for copycats. Successful crowdfunding projects like the KAISR air lounge on Indiegogo (Guzman 2017) and the Fidget Cube on Kickstarter (Lee 2017) quickly spawn imitation and encounter copycats. Startups who initiate crowdfunding projects have limited resources and need to secure capital, leaving them difficult to invest in patent protection. Copycats do not just target small business innovators; even major industry players fall victim to imitation. For example, Xiaomi, a leading Chinese electronics manufacturer, has gained significant market share with producing phones that closely resemble Apple's iPhone, recently surpassing Apple as one of the top five smartphone vendors (Chiang

2024).

Copycat strategies greatly reshape competitive dynamics in industries worldwide. Replica products, though often of inferior quality compared to the originals, are typically sold at significantly lower prices. For example, the Fidget Cube, originally launched on Kickstarter for $20, faces competition from replicas priced as low as $4 on Amazon and $1.50 on Taobao (Lee 2017). In another instance, Xiaomi provides remarkably affordable prices to its budget-conscious consumers who cannot afford $1,000 for a iPhone (Russell 2018). This aggressive pricing strategy gives replica products a distinct advantage, as many consumers prioritize cost savings over the authenticity or origin of innovation. Such consumer behavior poses a serious threat to innovators, as it undermines their ability to recoup research and development investments, thereby reducing incentives to pursue innovation. The prevailing availability of low-priced replicas not only erodes the market share of original products but also hurts their profitability, ultimately stifling the overall progress within the industry. As a result, innovators face increasing challenges in sustaining their competitive edge and securing the financial resources necessary for continued innovation and growth.

A prevalent approach to counteract copycats involves the implementation of intellectual property (IP) strategies. However, enforcing patent rights through legal proceedings can be prohibitively expensive, rendering it unaffordable for many small businesses and startups (Key 2017). Although large industrial firms can absorb these costs, they often find that pursuing legal action against copycats is a lengthy and resource-draining process. Furthermore, the lines between imitation and originality are often blurred, and patents typically provide only limited protection for innovations. For instance, Thatchers attempted to assert that Aldi had deliberately imitated its Cloudy Lemon Cider. However, their claim was rejected in the High Court ruling, where judge acknowledged the similarity between the appearance of the two products, albeit to a low degree (Farrell 2024). This highlights the need to identify alternative and more effective mechanisms to tackle the challenge posed by copycats.

In this paper, we explore the use of common marketing and operational levers, including pricing strategies and quality investment, as tools to combat copycatting. Our model examines two pricing strategies: flexible pricing, also known as contingent pricing, where the innovator adjusts pricing decisions contingently, and committed pricing, where firms maintain fixed prices and offer price commitments to consumers. For example, Tesla implemented aggressive price reductions in response to increasing competition (He 2024), while Apple maintains notably consistent pricing throughout a specific iPhone model's lifecycle. The pricing flexibility offered by flexible pricing allows innovators to significantly erode the profits of copycats. Anticipating intense competition, copycats may choose not to enter the market. In contrast, committed pricing allows innovators to establish pricing leadership, which can be an effective strategy for countering replica products. Consequently, understanding how imitation impacts an innovator's choice of pricing strategies has become increasingly important.

In addition to selecting an appropriate pricing strategy, innovators must first consider the extent of their investment in quality. While high-quality products are more likely to be favored by consumers, they are also more susceptible to imitation by copycats. Previous research has shown that as a follower, the quality of a copycat's product is heavily influenced by the quality of the innovator's original creation, which we refer to as the "free-riding" behavior (Qian et al. 2015). This free-riding behavior clearly undermines innovators' incentives to invest in quality. In such cases, appropriate quality investment becomes a vital tool for innovators to counteract the threats posed by copycats.

Building on the preceding discussions, this paper aims to examine the role of price commitment and quality investment as strategic approaches to address the threat posed by copycats. Specifically, we seek to investigate the following research questions that have not been adequately addressed in the literature: (i). In scenarios with exogenous quality, what constitutes the optimal pricing strategy for the innovator to effectively compete with copycats? Can such strategy deter potential market entry?

(ii). How does the capability to manage product quality affect the optimal pricing strategy, and what are the resulting effects on entry deterrence? and (iii). Can the adoption of optimal pricing strategy and quality investment improve consumer surplus and social welfare? To address these questions, we build a parsimonious game-theoretic model involving an innovator and a copycat, and examine the implications of committed pricing and flexible pricing strategies in deterring copycats, as well as their impacts on welfare. Among other results, we highlight the following key findings.

First, we identify the optimal pricing strategy when product quality is exogenous. Under flexible pricing, the innovator is compelled to accommodate copycats and cannot effectively deter their entry as product quality increases. By contrast, committed pricing allows the innovator to use price commitments as a deterrent against copycat entry. As a result, committed pricing outperforms flexible pricing for the innovator when its product quality is high. However, flexible pricing may be optimal when product quality is moderate, as the flexibility it offers enables the innovator to better blockade copycat entry. Notably, flexible pricing can lead to higher consumer surplus and greater social welfare, but only when the original product quality is high.

Second, when product quality is endogenously determined, we show that the innovator should adopt committed pricing when the cost of quality investment is low, but switch to flexible pricing if the cost is moderate. Moreover, the key results derived above, assuming exogenous product quality, remain valid in this context. Here, committed pricing does not always outperform flexible pricing in deterring entry. We find that under both pricing strategies, the innovator tends to adopt a “limit quality” strategy by deliberately curtailing product quality to mitigate the threat of copycats. The limit quality strategy together with flexible pricing enables the innovator to deter copycat entry. Furthermore, the capability to manage quality enhances the effectiveness of flexible pricing.

Finally, we extend our main model by considering two additional scenarios. First, when the copycat can freely determine its level of imitation efficiency, we find that flexible pricing always weakly outperforms committed pricing by leading to a less efficient copycat. Second, when the innovator adopts uniform pricing instead of committed pricing, flexible pricing emerges as the optimal pricing strategy for the case with endogenized product quality decision. The uniform pricing constraint significantly limits the effectiveness of price leadership endowed by price commitment.

The rest of the paper is organized as follows. Section 2.2 reviews the existing literature and positions our work. Section 2.3 introduces the model setup. We study the optimal pricing strategy and its social impact under both exogenous and endogenous product quality in Sections 2.4 and 2.5, respectively. Section 2.6 discusses two extensions. Concluding remarks are provided in Section 2.7. All proofs and supporting results are relegated to the appendices.

2.2 Literature Review

Entry deterrence has been extensively explored in the literature. Various strategies have been proposed by economists to deter entry, including pricing (Salop 1979), advertising (Thomas 1999), capacity (Basu and Singh 1985), product proliferation (Bonanno 1987), and channel management (Liu et al. 2006). Ofek and Turut (2008) examine how firms should respond to entrants possessing both innovation and imitation capabilities. Li (2019) shows that incumbents can launch line extensions to diminish competitors' profits and deter entry. In this line of research, our work is closely related to the studies by Jost (2023) and Wang et al. (2016). Jost (2023) considers endogenous price leadership, focusing on its effect when introducing a fighter brand. Wang et al. (2016) compare Stackelberg and Nash price competition in the presence of an entrant with limited capacity and zero entry cost. Our study contributes to this stream of research by examining a special type of entrant whose replica product quality depends on the quality of the original product. The entrant's imitation behavior incentivizes the incumbent to limit quality. We highlight the distinct roles of pricing strategies and quality investment, as well as their strategic interactions, in combating copycats.

Our work contributes to the emerging literature exploring the issue of copycats and their impact on brand management (Qian 2014), global supply chain (Cho et al. 2015), luxury management (Gao et al. 2017), and crowdfunding (Chen et al. 2023). These studies examine the impact of copycats in various business environments. Other research study various strategies to counteract copycats. For example, Sun et al. (2010) propose barrier-erecting and market-grabbing strategies to deter copycat entry in the presence of technology transfer. Pun and DeYong (2017) investigate optimal advertising and pricing policies in the presence of strategic customers, finding that lower product quality may increase profits. Yi et al. (2022) explore the impact of counterfeiting from a global supply chain perspective, addressing how supply chain members combat counterfeiting. Jin et al. (2023) discuss the advantages of dynamic and committed contracts in a supply chain with network externalities to combat copycats. A common assumption in the copycat literature is that innovators frequently adjust retail prices to counter copycats. However, these studies largely overlook price commitments and quality investment. In contrast, we explore different pricing strategies and derive novel insights in this study. For example, we show that under exogenous quality, flexible pricing is more effective in blockading copycats, whereas committed pricing is better in deterring them. Moreover, the ability to adjust product quality further influence the effectiveness of pricing strategy in combating copycats.

Last, our work is also related to the stream of literature on pricing strategies, which primarily focuses on the trade-off between price flexibility and price commitment. Aviv and Pazgal (2008) demonstrate that announced pricing can be more advantageous for the seller than contingent pricing in the presence of strategic consumer behavior. Özer and Zheng (2016) reinstate the profitability of a markdown strategy compared to an everyday-low-price strategy in the presence of consumers with anticipated regret. Liu and Zhang (2013) examine duopoly price competition with strategic customers and show that static pricing can generally improve profits. Wang and Hu (2014) investigate whether competing firms should commit to a fixed price ex ante or adopt contingent pricing ex post under demand uncertainty. They show that flexible pricing can intensify competition compared to committed pricing. Selcuk and Gokpinar (2018) compare fixed and flexible pricing when consumers have bargaining power. Kabul and Parlaktürk (2019) study the value of price and quantity commitments from both retailer and supplier perspectives. Chen and Jiang (2021) compare the effects of flexible pricing and price commitment on new experience goods selling where consumers learn product quality from informed consumers. Geng et al. (2022) show that committed pricing benefits the duopoly if the spillover effect is strong. Dong et al. (2023) investigate the interactions between price commitment and mutlti-sourcing in mitigating supply yield risk. Wu et al. (2023) discuss manufacturers' commitment strategies in the presence of supply disruption risk. However, none of the aforementioned studies examine the role of pricing strategies in combating copycats. We complement their results by analyzing optimal quality and pricing strategies in the presence of potential copycats.

2.3 Model Setup

In this section, we develop the framework for the strategic interaction between an innovator and a potential copycat. First, we outline the objectives and decisions of each player in Section 2.3.1. Then, in Section 2.3.2, we introduce two specific pricing strategies that will be examined in detail.

2.3.1 Players

Our model consists of two key players: an innovator and a potential imitator. The innovator markets an innovative product to consumers over two periods, facing the possibility of an imitator (i.e., the copycat) entering the market in the second period to compete. In what follows, we detail the role and decisions of each player.

Innovator. Consider an innovator introducing a new product to the market. The innovator incurs costs when investing in product quality. That is, the innovator incurs a fixed cost of $\frac{1}{2}\gamma q^2$ to achieve a product quality level q, where γ represents the quality cost coefficient. Such quadratic form of the quality cost function has been widely utilized in the economics, marketing, and operations management literature, as it captures the increasing costs associated with achieving higher quality levels (Purohit 1994, Li 2019). The selling season spans two periods. In each period $j \in \{1, 2\}$, the innovator sells the product at a price denoted by p_{ij}, where subscript i represents the innovator.

Copycat. The introduction of a new product often attracts opportunistic firms seeking to free-ride on the innovator's efforts. Following the incumbent and entrant framework, we consider a copycat in the second period, reflecting the fact that the copycat acts as a follower of the innovation. After the first period, the copycat evaluates the potential profitability of imitation and will enter the market if the expected profit is sufficient to cover the entry cost, denoted as K.

When the copycat chooses to enter the market, it produces a replica product, sets its selling price p_c, and competes with the innovator's original product in the second period. We denote the quality of the replica product as δq, where $\delta \in (0, 1)$ represents the copycat's imitation efficiency. It is assumed that $0 < \delta < 1$, i.e., the copycat can only replicate part of the original product's quality (Gao et al. 2017, Pun and DeYong 2017). The parameter δ can also be interpreted as the "competitiveness" of the replica product compared to the original (Jin et al. 2023). We further remark that when the quality level of the replica product and the original product are very close, the copycat's equilibrium profit may decrease as its imitation efficiency increases, due to the intensified price competition. To avoid this extreme scenario, which is also uncommon in practice, we require that $\delta \in (0, \frac{4}{7})$. This indicates that the imitation efficiency cannot be too high, or perfect imitation is prohibitively costly, ensuring that the copycat always benefits from its improved imitation efficiency. Later, we relax this assumption by endogenizing the copycat's imitation efficiency δ in Section 2.6.1.

Customers. Following Anand et al. (2008) and Jin et al. (2023), we assume that one unit mass of customers arrives in each period, with each customer demanding at most one unit of a product. Consumers are heterogeneous in their willingness-to-pay for quality, denoted by θ, which is distributed uniformly on $[0, 1]$; that is, $\theta \sim U[0, 1]$. Consumers buy the product if and only if they receive non-negative utility, i.e., $\theta q - p \geq 0$, where q and p are the quality and the price. In our base model, we focus on the case where customers are short-lived and myopic, meaning their demands must be satisfied within the period of their arrival; otherwise, their demands are lost.[1]

We remark that customers in our model are able to distinguish between the original product and the replica product. Importantly, the replica product is distinct from a counterfeit product, which can sometimes be mistaken for the original by customers (Gao et al. 2017, Cho et al. 2015). This distinction highlights the challenge of addressing copycats through patent protection, as most copycats offer a lower-quality alternative at a reduced price rather than misrepresenting their products as the original. For example, Raspberry Pi can be used in a wide range of industry and business. However, due to its high price, consumers often consider more affordable alternatives, such as Orange Pi (Osborne 2024).

[1] Furthermore, we stand ready to expand our analysis to encompass considerations of strategic consumer behavior, if warranted.

2.3.2 Pricing Strategies

There are various approaches to addressing the challenges posed by copycats. An innovator can seek patent protection prior to a product launch or initiate legal proceedings against the copycat. However, legal actions are often time-consuming and costly, potentially exceeding the innovator's financial capacity. In this paper, we explore the role of pricing strategies in mitigating the impact of copycats. Unlike other methods, pricing strategies are relatively easy to implement and have an immediate effect. Specifically, we analyze and compare two distinct pricing strategies, flexible pricing and committed pricing.

Flexible Pricing. Flexible pricing, also known as responsive pricing in the literature, enables innovators to adjust their pricing decisions based on the most up-to-date information available. This approach offers the innovator a strategic advantage referred to as *price flexibility*. Price flexibility allows the innovator to determine optimal pricing in a competitive market environment. In practice, many firms adjust the prices of their new products in response to the entry of copycats. For example, Tesla announced substantial price reductions in response to increased competition (He 2024).

Under flexible pricing, the sequence of events is as follows. First, prior to the selling period, the innovator determines the quality level of its original product, q. Next, during selling period 1, the innovator determines the price p_{i1}, and sells the original product to the market. Following this, the copycat decides whether to enter the market, incurring the entry cost K if it chooses to do so. Finally, during selling period 2, both firms compete by making their respective pricing decisions, p_{i2} and p_c.

Committed Pricing. Under committed pricing, the innovator sets prices at the beginning of each period and commits to them. Compared to flexible pricing, committed pricing may be less effective in a dynamic market since the innovator cannot adjust prices in response to changing market conditions. However, it can offer the innovator a strategic advantage known as *price leadership*, which is similar to a first-mover advantage. This pricing strategy is particularly applicable to many seasonal products, such as fashion items and electronic goods (Aviv and Pazgal 2008, Aviv et al. 2019, Arifoğlu et al. 2020).

The sequence of events under committed pricing is as follows. First, prior to the selling period, the innovator determines the quality level of its original product, q. The innovator then sets the price p_{i1} for selling period 1. During selling period 1, only the innovator sells the original product to the market. At the onset of selling period 2, the innovator decides the price p_{i2} and commits to it. Following this, the copycat decides whether to enter the market, incurring the entry cost K if it chooses to do so, and sets its price, p_c. Finally, demand in selling period 2 is realized through consumers' purchasing decisions.

Note that a specific variant of committed pricing is uniform pricing, where the innovator commits to a single price for the entire selling horizon. This pricing strategy is commonly adopted in practice. For instance, Apple maintains notably consistent pricing throughout a specific iPhone model's lifecycle. In our main model, we focus on comparing flexible pricing with committed pricing. We further examine uniform pricing as an extension in Section 2.6.2 and find that our key findings remain valid. Key notations are summarized in Table 2.1.

2.4 Exogenous Quality

In this section, we examine the innovator's optimal pricing strategy for countering copycats by considering the case where the product quality is exogenously determined. The exogenous quality case applies to scenarios where engaging in product quality management is either impractical–such as when production is outsourced–or prohibitively expensive due to high research and development costs. This case serves as a benchmark for understanding the effectiveness of pricing strategies in combating

Table 2.1: Summary of Model Notations

Parameters	
γ	coefficient of quality cost
K	copycat's entry cost
δ	copycat's imitation efficiency
Decisions	
q	innovator's quality decision
p_{ij}	innovator's pricing decisions in period j, $j \in \{1, 2\}$
p_c	copycat's pricing decision
Superscripts	
FB (CB)	copycat blockade under flexible (committed) pricing
FD (CD)	copycat deterrence under flexible (committed) pricing
FA (CA)	copycat accommodation under flexible (committed) pricing

copycats when adjusting quality is costly. By assuming exogenous quality, we isolate the impact of pricing strategies, removing any confounding effects of quality investment. The solution concept used is the subgame perfect Nash equilibrium, obtained through backward induction.

2.4.1 Equilibrium Outcomes

We begin by examining the optimal pricing decisions under different pricing strategies. For each given strategy, we derive the innovator's pricing decisions and the copycat's entry decision. For clarity, we use the superscripts F and C to denote notations under flexible and committed pricing, respectively. Under flexible pricing, both firms simultaneously choose p_{i2}^F and p_c^F in period 2 to maximize their profits:

$$\begin{cases} \max\limits_{p_{i2}^F} \quad \Pi_{i2}^F = p_{i2}^F(1 - \theta_{ic}), \\ \max\limits_{p_c^F} \quad \Pi_c^F = p_c^F \left(\theta_{ic} - \dfrac{p_c^F}{\delta q} \right), \end{cases}$$

where $\theta_{ic} := \frac{p_{i2}-p_c}{(1-\delta)q}$ is the the willingness-to-pay for quality at which consumers are indifferent between the original product and the replica product. The following lemma provides the equilibrium pricing decisions in the context of flexible pricing. For the sake of brevity, all proofs are relegated to the online appendices.

Lemma 1. *When the innovator adopts a flexible pricing strategy, there exists a quality threshold* $q^F := \frac{(4-\delta)^2}{\delta(1-\delta)}K$ *such that:*

(i) If the product quality $q \leq q^F$*, the innovator sets prices* $p_{i1}^{F*} = p_{i2}^{F*} = \frac{q}{2}$*, and the copycat does not enter the market.*

(ii) If $q > q^F$*, the innovator sets* $p_{i1}^{F*} = \frac{q}{2}$ *and* $p_{i2}^{F*} = \frac{2q(1-\delta)}{4-\delta}$*. The copycat enters the market and sets its own price* $p_c^{F*} = \frac{\delta q(1-\delta)}{4-\delta}$*.*

As shown by Lemma 1(i), the copycat will refrain from entering the market if the original product's quality is low, as it would be unable to recoup its entry costs through imitation. We refer to the scenario described in Lemma 1(i) as the *copycat blockade* region. Conversely, when the quality of the original product is high ($q > q^F$), the copycat can achieve a positive profit through imitation. This scenario incentivizes the innovator to reduce the price in the second period, employing a strategy known as *limit pricing*, a term widely used in the economic literature on entry deterrence (see, e.g., Salop 1979, Milgrom and Roberts 1982). By setting prices below the first-best level, firms can deter or weaken rivals. We refer to the scenario described in Lemma 1(ii) as the *copycat accommodation* region, where the copycat enters the market, and the innovator strategically sets a price below the first-best level to counteract copycat competition. Throughout the paper, we use the superscripts B and A to denote equilibrium outcomes for the regions of copycat blockade and copycat accommodation, respectively.

Next, we proceed to examine the equilibrium outcomes under committed pricing. In contrast to flexible pricing, committed pricing endows the innovator with price leadership. Consequently, the copycat assumes a follower role, making pricing decisions after observing the innovator's price commitments. Under committed pricing, the innovator chooses p_{i2}^C in period 2 to maximize

$$\Pi_i^C = \begin{cases} \Pi_i^{CA} = \frac{q}{4} + p_{i2}^C(1-\theta_{ic}), & \text{if } p_{i2}^C > p_{i2}^{CD}; \\ \Pi_i^{CB} = \frac{q}{4} + p_{i2}^C(1-\frac{p_{i2}^C}{q}), & \text{if } p_{i2}^C \leq p_{i2}^{CD}; \end{cases}$$

$$\text{s.t.} \quad \Pi_c^{C*}\left(p_{i2}^C\right)\Big|_{p_{i2}^C = p_{i2}^{CD}} = 0.$$

The following lemma characterizes equilibrium pricing and entry decisions under committed pricing.

Lemma 2. *When the innovator adopts a committed pricing strategy, there exists quality thresholds,* $q_1^C := \frac{16(1-\delta)}{\delta}K$ *and* $q_2^C := \frac{8(2-\delta)\left(1+\sqrt{\delta(2-\delta)}\right)}{\delta(1-\delta)}K$, *where* $q_1^C < q_2^C$, *such that:*

(i) If the product quality $q \leq q_1^C$, *the innovator sets prices* $p_{i1}^{C*} = p_{i2}^{C*} = \frac{q}{2}$, *and the copycat does not enter the market.*

(ii) If $q_1^C < q \leq q_2^C$, *the innovator sets* $p_{i1}^{C*} = \frac{q}{2}$ *and* $p_{i2}^{C*} = \frac{2\sqrt{Kq(1-\delta)}}{\sqrt{\delta}}$. *The copycat does not enter the market.*

(iii) If $q > q_2^C$, *the innovator sets* $p_{i1}^{C*} = \frac{q}{2}$ *and* $p_{i2}^{C*} = \frac{q(1-\delta)}{2-\delta}$. *The copycat enters the market and sets its price* $p_c^{C*} = \frac{\delta}{2}p_{i2}^{C*}$.

Similar to that under flexible pricing, the first and third statements of Lemma 2 show that under committed pricing, imitation is blockaded when the original product's quality is low ($q \leq q_1^C$) and accommodated when it is high ($q > q_2^C$). However, Lemma 2(ii) reveals a nuanced outcome for original products with moderate quality, i.e., $q_1^C < q \leq q_2^C$. In this situation, the innovator commits to a lower-than-first-best price in the second period, but the price reduction is more aggressive compared to the case of copycat accommodation, i.e., $p_{i2}^{CD} < p_{i2}^{CA}$. Here, price leadership acts as a strategic barrier to imitation, discouraging the copycat from entering the market. We refer to this scenario, described in Lemma 2(ii), as the *copycat deterrence* region, where the innovator deliberately deviates from the first-best pricing to deter entry. Throughout the paper, we use the superscript D to denote equilibrium outcomes for the regions of copycat deterrence.

A close look at Lemmas 1 and 2 reveals that committed pricing grants the innovator price leadership, thereby deterring entry–a capability that flexible pricing lacks. Intuitively, when product quality is exogenously determined, committed pricing is expected to outperform flexible pricing in effectiveness. However, the following proposition shows that this conjecture is not necessarily true.

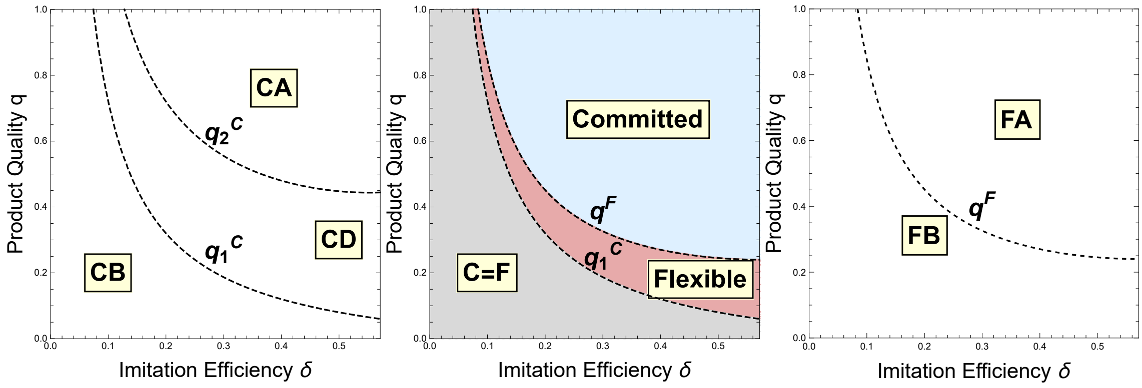
Proposition 1. *Given that $q_2^C > q^F > q_1^C$ always holds, flexible pricing is more effective in blockading copycats than committed pricing, while committed pricing is more effective in deterring copycats than flexible pricing. Moreover, we have:*

(i) If the product quality $q \leq q_1^C$, the two pricing strategies yield the same profit for the innovator, i.e., $\Pi_i^{F*} = \Pi_i^{C*}$.

(ii) If $q_1^C < q \leq q^F$, flexible pricing leads to a higher profit for the innovator compared to committed pricing, i.e., $\Pi_i^{F*} > \Pi_i^{C*}$.

(iii) If $q > q^F$, committed pricing leads to a higher profit for the innovator compared to flexible pricing, i.e., $\Pi_i^{F*} < \Pi_i^{C*}$.

It can be shown that the quality threshold q^F falls into the region (q_1^C, q_2^C); see the proof of Proposition 1. This implies that the region of copycat blockade under flexible pricing is larger than that under committed pricing, while the region of copycat deterrence under committed pricing is larger than that under flexible pricing. The underlying reason is that in a competitive environment, the copycat gains greater profit under committed pricing and finds it easier to enter the market. Yet, by committing to a lower price, committed pricing can deter entry over a broader range. See Figure 1 for an illustration.



(a) Committed pricing (b) Optimal Pricing (c) Flexible pricing

Figure 2.1: The Equilibrium with Exogenous Quality

Proposition 1 further identifies the optimal pricing strategy when product quality is exogenously determined. Specifically, when the original product's quality is low (i.e., $q \leq q_1^C$), the innovator is indifferent between the two pricing strategies. In this scenario, the copycat cannot cover its entry costs, and imitation is blockaded under both flexible and committed pricing. When the original product's quality is sufficiently high (i.e., $q > q^F$), committed pricing proves more advantageous for the innovator compared to flexible pricing. This is driven by the following underlying reasons: (1) when $q^F < q \leq q_2^C$, the innovator can deter copycats under committed pricing but must share the market with the copycat under flexible pricing; and (2) when $q > q_2^C$, the copycat enters the market regardless of the pricing strategy, and the innovator benefits from taking price leadership.

Somewhat surprisingly, flexible pricing outperforms committed pricing for the innovator when the original product's quality is moderate and falls within the intermediate range $(q_1^C, q^F]$. Note that in this range, the copycat does not enter the market under either pricing strategy, and flexible pricing is more effective in blockading copycats than committed pricing. This is because pricing flexibility creates a highly competitive environment, which helps deter imitation. In contrast, price leadership associated with committed pricing, while enhancing the innovator's profit, also improves the copycat's potential profit. Hence, the barrier to imitation is lower under committed pricing due to a less competitive environment. As a result, flexible pricing allows the innovator to achieve the first-best outcome, while committed pricing requires a sacrifice–a price lower than the first-best–to deter market entry.

2.4.2 Impact of Pricing Strategy

In this section, we investigate the innovator's limit pricing strategy by analyzing the second-period price, p_{i2}^*, as it adequately captures the trade-off between copycat deterrence and copycat accommodation. As previously discussed, the innovator sets a price lower than the first-best to either deter entry or accommodate imitation. While limit pricing has been studied in the literature on industrial organization and marketing, how the innovator employs limit pricing under different pricing strategies in the presence of copycats remains unclear. In particular, it is unclear whether the innovator will adopt a more aggressive limit pricing strategy when the original product has lower quality or when the copycat exhibits greater efficiency in imitation. The following proposition provides an answer:

Proposition 2. *(Limit Pricing) Under committed pricing, the second-period price increases with the original product's quality (q) and is non-monotonic in the copycat's imitation efficiency (δ). By contrast, it is non-monotonic in q and decreases with δ under flexible pricing. Moreover,* $p_{i2}^{F*} \geq p_{i2}^{C*}$ *if and only if (iff)* $q \leq q_2^C$*; otherwise,* $p_{i2}^{F*} < p_{i2}^{C*}$.

Some sensitivity results are monotonic in a straightforward way. The second-period price p_{i2}^{C*} under committed pricing increases with the original product quality q; the innovator will charge a higher price for a better product, regardless of copycat entry. Under flexible pricing, the second-period price p_{i2}^{F*} decreases with imitation efficiency δ since the competitive environment forces the innovator to cut prices. However, the proposition shows that p_{i2}^{C*} may increase in δ, whereas p_{i2}^{F*} may decrease in q.

Under committed pricing, the case shifts from CD to CA as δ increases, meaning the innovator is compelled to accommodate the copycat once δ becomes sufficiently large. In both cases, p_{i2}^{C*} decreases with δ. Nonetheless, in case CD, the innovator uses more aggressive limit pricing to deter copycats compared to case CA. When δ crosses the threshold between the two cases, such aggressive limit pricing becomes more costly when facing a more efficient copycat, and there is no need to lower the price to such an extent after entry. In the case of flexible pricing, p_{i2}^{F*} may decrease in q when the copycat enters the market, i.e., the innovator cannot deter copycats and is forced to accommodate imitation through lower prices.

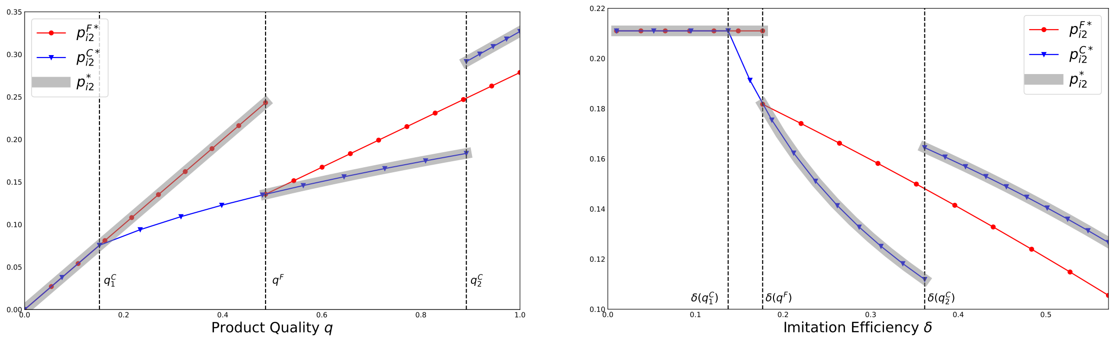


Figure 2.2: Optimal Limit Pricing

We next examine whether the innovator would choose a lower second-period price under the optimal pricing strategy. Proposition 2 indicates that the reduction in the second-period price under the optimal pricing strategy does not necessarily align with the more aggressive pricing approach. Figure 2.2 compares the second-period prices under committed and flexible pricing strategies alongside the optimal strategy, revealing three interesting findings. First, when the product quality falls into an intermediate-low range ($q \in (q_1^C, q^F]$), limit pricing is more aggressive under committed pricing, i.e., $p_{i2}^{C*} \leq p_{i2}^{F*}$. However, the optimal pricing strategy in this range is flexible pricing, which directly blockades copycats. Committed pricing, by contrast, can result in excessive price reduction that is not in the innovator's best interest. Second, when $q^F < q \leq q_2^C$, the more aggressive limit pricing appears under the optimal pricing strategy, which is committed pricing. Under flexible pricing, improved product quality attracts imitation, making it impossible for the innovator to deter entry. Instead, the innovator must accommodate the copycat, leading to fierce price competition. Committed pricing, on the other hand, enables the innovator to deter copycats through aggressive limit pricing. Notably, the gap between second-period prices under committed and flexible pricing varnishes at the quality level $q = q^F$, meaning the copycat enjoys the same profit under both strategies, and it increases in q for $q \in [q^F, q_2^C]$. Third, when $q > q_2^C$, flexible pricing results in excessive price reduction to compete with the copycat. In contrast, committed pricing, which is the optimal pricing strategy, mitigates the ensuing price competition and allows the innovator to charge a higher second-period price despite the entry of the copycat.

Finally, we discuss the impacts of imitation efficiency δ and product quality q on both the innovator and the copycat. First, irrespective of the pricing strategy, a more efficient imitator always benefits itself while harming the innovator. Second, due to free-riding behavior, the imitator gains from improvements in the original product's quality under both pricing strategies. This holds even when the optimal pricing strategy is adopted. As the original product's quality q increases, the optimal pricing strategy switches from flexible pricing to committed pricing. This strategy switching reduces competitive pressures and further boosts the imitator's profitability. We thus hypothesize that the innovator will also benefit from an increase in product quality. This holds true under committed pricing; however, it does not apply under flexible pricing, as outlined in the following corollary.

Corollary 1. *Under flexible pricing, the innovator's profit* Π_i^{F*} *is non-monotonic in the original product's quality (*q*).*

Interestingly, Corollary 1 shows that under flexible pricing, an increase in the original product's quality does not necessarily enhance the innovator's profitability. This counterintuitive outcome occurs because higher-quality products are more likely to attract imitation. Once the product quality reaches the imitation threshold ($q = q^F$), the imitator finds market entry profitable. Consequently, further increases in product quality beyond this threshold may ultimately harm the innovator.

2.4.3 Social Welfare Implication

In this section, we investigate the social impacts of the optimal pricing strategy. Recall that under flexible pricing, the second-period price (p_{i2}^{F*}) exhibits a non-monotonic relationship with respect to the original product's quality q, while under committed pricing, the second-period price (p_{i2}^{C*}) is non-monotonic in the copycat's imitation efficiency δ. These non-monotonic effects on selling prices suggest that changes in market conditions may lead to nuanced impacts on consumer surplus and social welfare. We explore these impacts in the subsequent analysis.

Lemma 3. *Under flexible pricing, consumer surplus and social welfare both increase with the original product's quality q and the imitator's imitation efficiency δ; while they are non-monotonic in q and δ under committed pricing.*

Lemma 3 shows that despite that p_{i2}^{F*} is non-monotonic in q, consumer surplus and social welfare under flexible pricing increase with both the original product's quality (q) and the imitator's imitation efficiency (δ). This is because, in general, a better product and a more efficient copycat provide greater benefits to consumers. Moreover, when the innovator accommodates the copycat under flexible pricing, the price reduction resulting from imitation also benefits consumers. By contrast, under committed pricing, the opposite patterns emerge: consumer surplus and social welfare may decrease with q and δ. Unlike flexible pricing, an innovator using committed pricing tends to raise prices when accommodating imitation. This increase in prices diminishes the advantages typically associated with competitive markets. Consequently, both consumers and society may experience adverse effects from quality improvements and imitation under committed pricing. This raises the question of which pricing strategy is socially optimal. To facilitate the analysis, define $q^{CF} \in (q^F, q_2^C)$

such that $\left(CS^{CD*} - CS^{FA*}\right)\big|_{q=q^{CF}} = 0$.

Proposition 3. *There exists a quality threshold q^{CF} such that:*

(i) Consumers are better off under flexible pricing if the product quality $q \in (q^F, q^{CF}) \bigcup (q_2^C, \infty)$, under committed pricing if $q \in (q_1^C, q^F) \bigcup (q^{CF}, q_2^C)$, and indifferent between the two pricing strategies otherwise.

(ii) Flexible pricing is socially optimal if $q > q_2^C$, while committed pricing is socially optimal if $q_1^C < q < q_2^C$. Otherwise, both pricing strategies lead to the same level of social welfare.

Proposition 3 has the following implications. First, when the product quality satisfies $q_1^C < q < q^F$, both consumers and society are better off under committed pricing. This is because the innovator sets a price lower than the first-best level to deter the copycat's potential entry. Second, flexible pricing fosters a more competitive environment, enhancing both consumer surplus and social welfare when $q > q_2^C$. However, under these circumstances, the innovator's preference is misaligned with those of consumers and society, as the innovator does not favor the pricing option that maximizes consumer surplus and social welfare.

When the original product's quality is moderate ($q^F < q < q_2^C$), the effects of price reduction and competition are intertwined. In this range, flexible pricing accommodates imitation while committed pricing does not. Despite this, consumers benefit from a lower second-period price under committed pricing. However, the competition effect outweighs the price reduction effect when $q^F < q < q^{CF}$, making consumers better off under flexible pricing. In contrast, committed pricing always benefits the innovator and improves the social welfare. Thus, a "win-win-win" outcome for the innovator, consumers, and society occurs when $q^{CF} < q < q_2^C$, where the innovator chooses committed pricing and the copycat refrains from entering the market. Finally, we summarize the equilibrium outcome regarding the copycat's potential entry and the preferences of all parties towards the two pricing strategies, depending on the magnitude of the original product's quality q, in Table 2.2.

Table 2.2: Preference of Different Parties with Exogenous Quality

Product	Pricing		Preference		
Quality	Committed	Flexible	Innovator	Consumer	Society
$q < q_1^C$	CB	FB	$C = F$	$C = F$	$C = F$
$q_1^C < q < q^F$	CD	FB	F	C	C
$q^F < q < q^{CF}$	CD	FA	C	F	C
$q^{CF} < q < q_2^C$	CD	FA	C	C	C
$q > q_2^C$	CA	FA	C	F	F

2.5 Endogenous Quality

In this section, we endogenize the quality decision by allowing the innovator to decide the product's quality level prior to the selling period. This is motivated by the business practice of using effective quality investment as a means to deter imitation. Our analysis aims to provide valuable insights into how innovators can effectively integrate quality investment with pricing strategies to combat copycat.

2.5.1 Equilibrium Outcomes

Before the selling period, the innovator selects the pricing strategy and decides the level of quality investment. It is worth noting that the innovator's profit is concave with respect to product quality, q, given a pricing strategy. Therefore, we can analyze the problem in a sequential manner via backward induction. Specifically, we first solve for the optimal quality decision under a given pricing strategy. Then, by comparing the equilibrium outcomes associated with two pricing strategies, we identify the optimal pricing strategy. For the flexible pricing strategy, Lemma 4 follows directly from Lemma 1:

Lemma 4. *Under flexible pricing, there exist two quality cost coefficient thresholds,*

$$\gamma_1^F := \frac{\delta(1-\delta)}{2K(4-\delta)^2} - \frac{\delta(1-\delta)\sqrt{\delta(8+\delta)(3\delta^2-40\delta+64)}}{4K(4-\delta)^4} \quad \textit{and} \quad \gamma_2^F := \frac{\delta(1-\delta)}{2K(4-\delta)^2},$$

such that:

(i) If the innovator's quality cost coefficient $\gamma < \gamma_1^F$, the innovator sets its quality level $q^{F*} = \frac{32-24\delta+\delta^2}{4\gamma(4-\delta)^2}$ and prices $p_{i1}^{F*} = \frac{q^{F*}}{2}$ and $p_{i2}^{F*} = \frac{2q^{F*}(1-\delta)}{4-\delta}$. The copycat enters the market and sets the price $p_c^{F*} = \frac{\delta q^{F*}(1-\delta)}{4-\delta}$.

(ii) If $\gamma_1^F \leq \gamma < \gamma_2^F$, the innovator sets its quality level $q^{F*} = \frac{K(4-\delta)^2}{\delta(1-\delta)}$ and prices $p_{i1}^{F*} = p_{i2}^{F} = \frac{q^{F*}}{2}$; the copycat does not enter the market.

(iii) If $\gamma \geq \gamma_2^F$, the innovator sets its quality level $q^{F*} = \frac{1}{2\gamma}$ and prices $p_{i1}^{F*} = p_{i2}^{F} = \frac{q^{F*}}{2}$; the copycat does not enter the market.

A close look at Lemmas 1 and 4 reveals that the optimal flexible pricing strategy with the endogenous quality decision resembles that with the exogenous quality. Specifically, the first and third statements of Lemma 4 indicate that imitation is effectively blockaded when the quality cost coefficient is high ($\gamma \geq \gamma_2^F$) and that imitation is accommodated when $\gamma < \gamma_1^F$. However, when the quality cost coefficient is intermediate ($\gamma_1^F \leq \gamma < \gamma_2^F$), the innovator can successfully deter imitators under flexible pricing due to its capability of managing quality. Moreover, the innovator can also choose a first-best price in the second period. In contrast, an innovator lacking this capability fails to achieve such deterrence, as shown in Lemma 1. This result arises because an innovator with quality management capabilities can strategically invest less in quality, thereby reducing the profitability of imitation for potential copycats. Analogous to the concept of limit pricing in the exogenous quality scenario, we refer to this strategy as *limit quality*.

Next, we analyze the optimal committed pricing. To facilitate the analysis, define γ_1^C as the threshold that satisfies $\Pi_i^{CD*}\big|_{\gamma=\gamma_1^C} = \Pi_i^{CA*}\big|_{\gamma=\gamma_1^C}$ and

$$\gamma_2^C = \frac{\delta}{32K(1-\delta)}, \quad q^{CD} := \left(\frac{1}{4} - \gamma q^{CD} + \frac{\sqrt{q^{CD}K(1-\delta)}}{\sqrt{\delta}q^{CD}} = 0\right).$$

We then have the following:

Lemma 5. *Under committed pricing, the following statements hold:*

(i) If the quality cost coefficient $\gamma < \gamma_1^C$, the innovator sets its quality level $q^{C*} = \frac{4-3\delta}{4\gamma(2-\delta)}$ and prices $p_{i1}^{C*} = \frac{q^{C*}}{2}$ and $p_{i2}^{C*} = \frac{q^{C*}(1-\delta)}{2-\delta}$. The copycat enters the market and sets the price $p_c^{C*} = \frac{\delta}{2}p_{i2}^{C*}$.

(ii) If $\gamma_1^C \leq \gamma < \gamma_2^C$, the innovator sets its quality level $q^{C*} = q^{CD}$ and prices $p_{i1}^{C*} = \frac{q^{C*}}{2}$ and $p_{i2}^F = \frac{2\sqrt{Kq^{C*}(1-\delta)}}{\sqrt{\delta}}$; the copycat does not enter the market.

(iii) If $\gamma \geq \gamma_2^C$, the innovator sets its quality level $q^{C*} = \frac{1}{2\gamma}$ and prices $p_{i1}^{C*} = p_{i2}^{C*} = \frac{q^{C*}}{2}$; the copycat does not enter the market.

Lemma 5 shows that the qualitative findings obtained under the exogenous quality, as stated in Lemma 2, continue to hold when the quality level is endogenized. The key difference is that the regions of copycat accommodation, deterrence, and blockade now depend on the magnitude of the quality cost coefficient γ. Having analyzed both pricing strategies, we now derive the innovator's optimal pricing strategy in the context of endogenous quality. Define γ_1^{CF} as the threshold that satisfies $\Pi_i^{CD*}\big|_{\gamma=\gamma_1^{CF}} = \Pi_i^{FD*}\big|_{\gamma=\gamma_1^{CF}}$. We then obtain the following results.

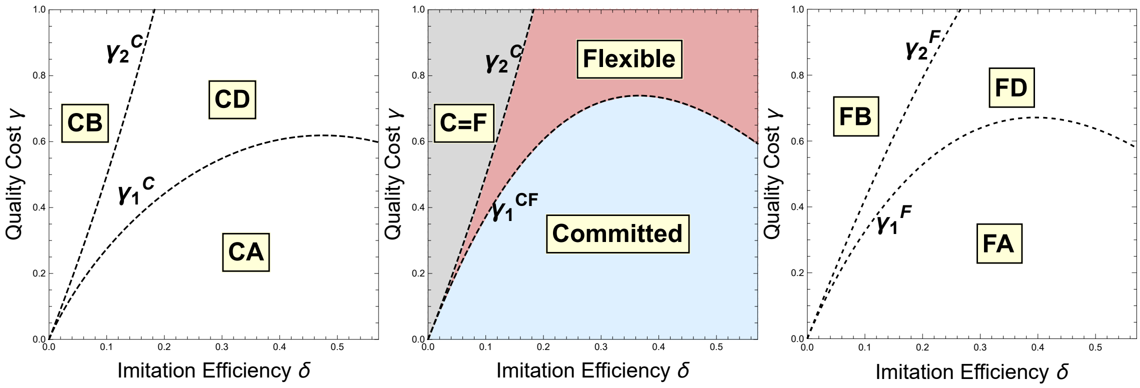
Proposition 4. *Flexible pricing is more effective in blockading copycats than committed pricing (since $\gamma_2^F < \gamma_2^C$). Regarding deterring entry, the relative performance of each strategy depends on the level of imitation efficiency, δ.*[2] *Moreover,*

[2]For detailed analysis and comparisons, please refer to Lemma A.2.3 in the appendix.

(i) if the quality cost coefficient $\gamma \geq \gamma_2^C$*, the two pricing strategies yield the same profit for the innovator, i.e.,* $\Pi_i^{F*} = \Pi_i^{C*}$*;*

(ii) if $\gamma_1^{CF} \leq \gamma < \gamma_2^C$*, flexible pricing leads to a higher profit for the innovator compared to committed pricing, i.e.,* $\Pi_i^{F*} > \Pi_i^{C*}$*;*

(iii) if $\gamma < \gamma_1^{CF}$*, committed pricing leads to a higher profit for the innovator compared to flexible pricing, i.e.,* $\Pi_i^{F*} < \Pi_i^{C*}$*.*



(a) Committed pricing (b) Optimal Pricing (c) Flexible pricing

Figure 2.3: The Equilibrium with Endogenous Quality

Proposition 4 indicates that flexible pricing still outperforms committed pricing in blockading copycats even when the quality level is endogenized. However, committed pricing now does not always outperform flexible pricing in deterring entry. We can further show that committed pricing is more effective in deterring entry only when the copycat's imitation efficiency δ is small.[3] This finding highlights an important implication: the ability to manage quality enhances the effectiveness of flexible pricing. While copycat deterrence may not be achievable under committed pricing when the innovator faces a highly efficient imitator, it can potentially be achieved within a flexible pricing framework; see Figure 2.3 for an illustration.

[3]See Lemma A.2.3 in the appendix for the details.

Proposition 4 further reveals that the innovator's equilibrium pricing strategy preference under endogenous quality is qualitatively similar to that under exogenous quality (see Proposition 1). However, the former depends on the magnitude of the quality cost coefficient, whereas the latter depends on the magnitude of the exogenous quality level. Specifically, there exists a threshold γ_1^{CF} regarding quality cost coefficient, below which the innovator prefers committed pricing. Moreover, we can show that γ_1^{CF} is greater than both γ_1^C and γ_1^F, where $\gamma_1^C < \gamma_1^F$ when the imitation efficiency δ is low and $\gamma_1^C \geq \gamma_1^F$ otherwise[4]. We then obtain the following two interesting cases. First, when $\gamma_1^F < \gamma < \gamma_1^C$, the innovator can deter the copycat and thereby becomes a monopolist under flexible pricing, whereas imitation must be accommodated under committed pricing. However, the innovator ultimately prefers to accommodate the copycat through committed pricing. This preference arises because, compared to the exogenous quality scenario, there is a substantial reduction in quality investment when the innovator seeks to deter copycats under flexible pricing. This distortion in quality investment significantly diminishes monopoly profits, making flexible pricing less favorable for the innovator.

Second, when $\max(\gamma_1^C, \gamma_1^F) < \gamma < \gamma_2^F$, the innovator prioritizes copycat deterrence under both pricing strategies. Interestingly, the profit gap between committed pricing and flexible pricing narrows as γ increases.[5] Specifically, committed pricing is preferred by the innovator when quality investment is relatively inexpensive, whereas flexible pricing is preferred when quality investment becomes costly. To better explain the underlying reasons, we conduct a comprehensive analysis of the interaction between limit quality and limit pricing in the next section.

Endogenous quality also yields additional new results. When quality is exogenous, the optimal pricing strategy shifts from flexible pricing to committed pricing as the imitation efficiency (δ) increases, as illustrated in Figure 1. However, when quality

[4]See Lemma A.2.3 for the detail.

[5]Detailed expressions are provided in Lemma A.2.3.

is endogenous, the optimal pricing strategy first transitions from flexible pricing to committed pricing and may subsequently revert from committed pricing to flexible pricing as δ increases. With exogenous quality, flexible pricing can lead to intense competition when facing highly efficient copycats, which can be detrimental. Conversely, with endogenous quality, the innovator can deter copycats via flexible pricing too, thereby avoiding intense competition.

2.5.2 Impact of Pricing Strategy

In this section, we examine the innovator's limit quality strategy, which emerges as a response to the copycat's free-riding behavior that disincentivizes the innovator from investing in quality. We first analyze how the quality cost coefficient, γ, and the copycat's imitation efficiency, δ, affect the innovator's optimal quality decision. Let $\gamma_2^{CF} := \frac{\delta(1-\delta)(8-5\delta)}{4K(4-\delta)^3}$. We have the following:

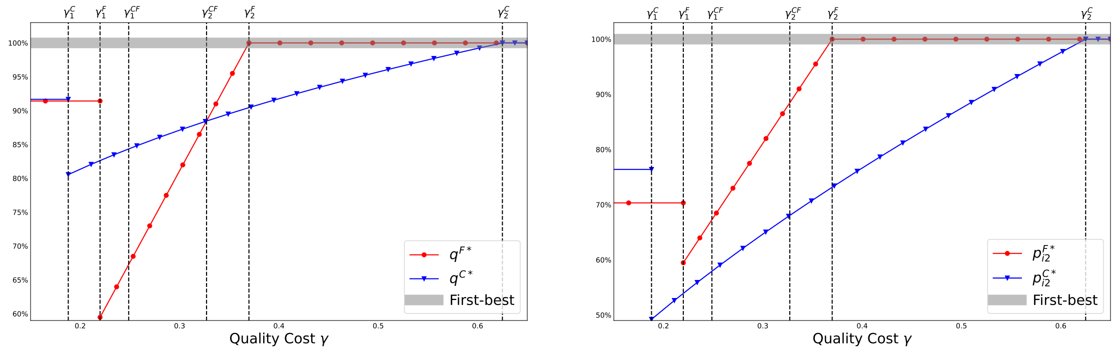
Proposition 5. *(Limit Quality) Under both flexible and committed pricing, the innovator's optimal quality level q^{i*} ($i \in \{C, D\}$) decreases with γ and is non-monotonic in δ. Moreover, $q^{F*} \geq q^{C*}$ if either $\gamma_1^C < \gamma < \gamma_1^F$ or $\gamma \geq \gamma_2^{CF}$; otherwise, $q^{F*} < q^{C*}$.*

Although the innovator's pricing decision under flexible pricing is non-monotonic with the original product quality q in Proposition 2, Proposition 5 demonstrates that, regardless of the pricing strategy, the innovator reduces its quality level q^{i*} as quality investment becomes more costly (i.e., with a larger γ). This is because, without the ability to manage quality, the innovator can only passively accommodate imitation by reducing prices when quality exceeds a certain threshold. In contrast, the innovator with the ability to manage quality can choose to deter the copycat rather than passively accommodate imitation. The transitional case of copycat deterrence ensures the monotonicity of quality decisions with respect to γ. Meanwhile, the relationship between quality decisions and δ exhibits non-monotonicity: an increase in the copycat's imitation efficiency (δ) can push the innovator to increase its quality level q^{i*}.

This stems from the aforementioned effects: A more efficient copycat pressurizes the innovator to accommodate imitation through a more moderate limit quality strategy.

We are now ready to explain why the profit gap between committed pricing and flexible pricing decreases with the quality cost coefficient γ for $\max(\gamma_1^C, \gamma_1^F) < \gamma < \gamma_2^F$. Within this range of the quality cost coefficient, the innovator's objective in employing either pricing strategy is to deter copycats and monopolize the market. Limit quality and limit pricing serve as strategic complements for the innovator in combating copycats, as both can effectively undermine and deter their entry. Under committed pricing, limit quality and limit pricing can be employed together, whereas only limit quality can be employed under flexible pricing, as previously discussed. Compared to committed pricing, the limit quality strategy under flexible pricing is more sensitive to changes in γ; that is, the innovator adopting flexible pricing makes more significant quality adjustments as γ varies. As shown in Figure 2.4a, when quality investment becomes more expensive (i.e., γ increases), the innovator has less incentive to limit quality, as product imitation becomes less attractive to copycats. Furthermore, the equilibrium quality under flexible pricing is lower than that under committed pricing when quality investment is inexpensive (with a small γ), but the opposite holds true when it is costly.

The innovator's quality investment capabilities influence both its limit quality and limit pricing strategies. Under committed pricing, the innovator can combine limit quality and limit pricing strategies to deter copycat entry. However, under flexible pricing, copycat deterrence relies exclusively on limit quality strategies, but this also implies that the innovator can implement monopoly pricing. As illustrated in Figure 2.4b, in the region of copycat deterrence, innovators employing flexible pricing consistently chooses a lower level of limit pricing compared to those utilizing committed pricing. Consequently, when quality investment is expensive, the innovator sacrifices less quality and pricing power under flexible pricing compared to that under committed pricing, resulting in a more favorable position in the copycat deterrence region. In this case, the ability of committed pricing to combine limit quality and limit pricing strategies may yield contrasting outcomes.



(a) Limit Quality (b) Limit Pricing

Figure 2.4: Equilibrium Quality and Pricing Strategy with Endogenous Quality

Note: The ratio on the vertical axis represents the proportion of the equilibrium decision to the first-best decision.

Next, we examine which pricing strategy results in a lower quality investment level, referred to as a more aggressive implementation of the limit quality strategy. Recall from Proposition 2 that when quality is exogenously given, limit pricing under committed pricing is more aggressive than under flexible pricing when the original product's quality is low ($q \leq q_2^C$). This leads us to conjecture that, when the quality decision is endogenous, limit quality under committed pricing should also be more aggressive than under flexible pricing when quality investment is costly (i.e., $\gamma \geq \gamma_1^C$). However, this argument does not hold when $\gamma_1^F < \gamma < \gamma_2^{CF}$. To understand why, note that two factors contribute to the outcome $p_{i2}^{F*} \geq p_{i2}^{C*}$ under exogenous quality. First, when $q_1^C < q < q^F$, $p_{i2}^{F*} > p_{i2}^{C*}$ occurs because flexible pricing achieves the first-best outcome. This corresponds to the case where $\gamma_2^F < \gamma < \gamma_2^C$. Second, when $q^F < q < q_2^C$, $p_{i2}^{F*} > p_{i2}^{C*}$ occurs because committed pricing requires undercutting prices to deter entry. This corresponds to the case where $\gamma_1^C < \gamma < \gamma_1^F$.

The remaining case, $\gamma_1^F < \gamma < \gamma_2^F$, does not correspond to any of the above discussed scenarios under exogenous quality. This case arises due to the effects described earlier: the ability to manage quality enables the innovator using flexible pricing to deter copycats. When quality investment is moderate, i.e., $\gamma_1^F < \gamma < \gamma_2^{CF}$, the innovator employing flexible pricing has a stronger incentive to limit quality, as this is the only means of deterring copycats. In contrast, the innovator adopting committed pricing can deter copycats by combining limit quality and limit pricing; therefore, there is no need to offer a low-quality product. When γ exceeds γ_2^{CF}, the situation reverses. As previously discussed, flexible pricing is more effective in deterring imitation, and the innovator's decisions are more sensitive to changes in quality investment. Consequently, innovators offering lower-quality products are those employing committed pricing. Next, we examine the profit impact of these strategies.

Corollary 2. *Under both flexible and committed pricing, the innovator's profit Π_i^* decreases in both the quality cost coefficient γ and the copycat's imitation efficiency δ, while the copycat's profit Π_c^* decreases in γ and is non-monotonic in δ.*

Corollary 2 further confirms our earlier findings that increased efficiency in imitation adversely affects the innovator's profitability. Unlike the conclusions drawn in Corollary 1, the innovator's profit declines as the quality investment becomes more costly (i.e., with a larger γ), regardless of the pricing strategy employed. The underlying reason is that as the cost of quality investment decreases, the innovator can strategically leverage quality management to deter imitation through flexible pricing. Moreover, quality management introduces a nuanced effect on the copycat's profitability. The value of imitation efficiency diminishes for the copycat, as the innovator may strategically curtail quality investment to weaken the copycat. This outcome is in contrast to the model with exogenous quality, where the copycat always benefits from improved imitation efficiency.

2.5.3 Social Welfare Implication

We now investigate how system parameters, including the quality cost coefficient γ and the copycat's imitation efficiency, affect the equilibrium consumer surplus and social welfare under the two pricing strategies. The results are summarized in Proposition 6.

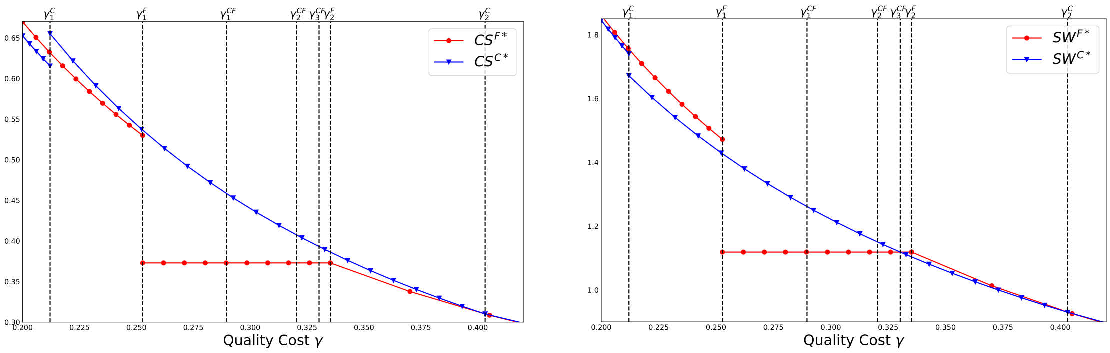
Lemma 6. *The following statements hold:*

(i) CS^{F*}*,* SW^{F*} *and* SW^{C*} *decrease in* γ*, while* CS^{C*} *is non-monotonic in* γ*.*

(ii) Both consumer surplus and social welfare under flexible and committed pricing are non-monotonic in δ*.*

Lemma 6(i) shows that consumer surplus under flexible pricing (CS^{F*}) and social welfare under both pricing strategies decrease in the quality cost coefficient γ, as shown by Figure 2.5. To understand why, two key points should be considered. First, as the cost of quality investment decreases, the innovator can offer a higher-quality product in the market, thereby enhancing both consumer surplus and social welfare. Second, as the quality cost decreases and the equilibrium transitions from copycat deterrence to copycat accommodation, the innovator tends to adopt more moderate levels of limit quality and limit pricing strategies. However, this intuition does not apply under committed pricing. Specifically, consumer surplus under committed pricing may decrease even when a higher-quality product is offered. This can be explained by the dynamics outlined in Proposition 5: the innovator employing committed pricing strategically combines limit quality and limit pricing to deter imitation. Notably, compared to the innovator employing flexible pricing, the one employing committed pricing offers a higher-quality product at a lower price. Consequently, consumers may benefit more from committed pricing when quality investment is expensive.

As the copycat's imitation efficiency δ increases, a more efficient copycat could intensify price competition and provide an alternative for consumers with a low willingness to pay for quality, potentially increasing consumer surplus and social welfare. However, Lemma 6(ii) indicates that this may not always hold. With the ability to manage quality, a higher-efficiency copycat impels the innovator to reduce quality investment,



(a) Consumer Surplus (b) Social Welfare

Figure 2.5: The Social Impact of Quality Cost

either to deter or accommodate the copycat. This reduction in quality investment can lead to lower consumer surplus and social welfare. In what follows, we identify the pricing strategy that maximizes consumer surplus and social welfare. To facilitate the analysis, define K_1 as the entry cost threshold satisfying $\Pi_i^{CD*}\big|_{K=K_1} = \Pi_i^{CA*}\big|_{K=K_1}$, K_2 as the one satisfying $CS^{CD*}\big|_{K=K_2} = CS^{FA*}\big|_{K=K_2}$, and

$$K_3 = \frac{(1-\delta)\delta\left(2\delta^2 - \sqrt{\delta\left(3\delta^3 - 16\delta^2 - 256\delta + 512\right)} - 16\delta + 32\right)}{4\gamma(\delta-4)^4},$$

$$\gamma_3^{CF} := \frac{7(1-\delta)\delta\left(11\delta^2 - 2\sqrt{14}(\delta-1)\sqrt{11\delta^2 - 40\delta + 56} - 40\delta + 56\right)}{8\left(11\delta^2 - 40\delta + 56\right)^2 K}.$$

Proposition 6. *Depending on the magnitudes of entry cost K and quality cost coefficient γ, the following statements hold:*

(i) If $\gamma > \gamma_2^C$, flexible and committed pricing yield the same consumer surplus and social welfare.

(ii) Flexible pricing makes consumers better off if (1) $\gamma < \gamma_1^C$ or (2) $K_2 < K < K_3$; otherwise, committed pricing makes consumers better off.

(iii) Flexible pricing is socially optimal if (1) $\gamma < \gamma_1^F$ or (2) $\gamma_3^{CF} < \gamma < \gamma_2^C$; otherwise, committed pricing is socially optimal.

Proposition 6 shows that flexible pricing benefits consumers only when either the quality investment is inexpensive ($\gamma < \gamma_1^C$) or the entry cost is moderate ($K_2 < K < K_3$). Under the former, the innovator offers a product of similar quality under both pricing strategies; however, flexible pricing results in a significantly lower price, thereby benefiting consumers. Under the latter, copycats are deterred by committed pricing but are accommodated under flexible pricing, where the resulting competition helps enhance consumer surplus. Interestingly, when both pricing strategies successfully deter copycats, committed pricing proves to be more beneficial for consumers. This is because, in this situation, flexible pricing deploys monopoly pricing, which benefits the innovator at the expense of consumers.

Proposition 6 further reveals that committed pricing is socially optimal when the quality investment cost is intermediate ($\gamma \in (\gamma_1^F, \gamma_3^{CF})$); otherwise, flexible pricing is socially optimal. Two points are particularly noteworthy. First, when $\gamma_1^C < \gamma < \gamma_1^F$, the innovator will choose committed pricing, and consumers may also prefer it; however, flexible pricing remains the socially optimal one. This is because copycats enter the market under flexible pricing but are deterred under committed pricing. Second, when the quality investment cost is intermediate ($\gamma_3^{CF} < \gamma < \gamma_2^C$), flexible pricing is socially optimal. Under committed pricing, the innovator tends to excessively rely on both limit quality and limit pricing, which ultimately reduces social welfare. Finally, we summarize the equilibrium outcome regarding the copycat's potential entry and the preferences of all parties towards the two pricing strategies, depending on the quality cost coefficient, in Table 2.3. However, since the relationships among certain thresholds may vary, we present the most representative outcome.

2.6 Extensions

In this section, we extend our main model to two alternative settings to check the robustness of our key results and to derive additional insights. First, in Section

Table 2.3: Preference of Different Parties with Endogenous Quality

Quality	Pricing		Preference		
Cost	Committed	Flexible	Innovator	Consumer	Society
$\gamma < \gamma_1^C$	CA	FA	C	F	F
$\gamma_1^C < \gamma < \gamma_1^F$	CD	FA	C	C or F	F
$\gamma_1^F < \gamma < \gamma_1^{CF}$	CD	FD	C	C	C
$\gamma_1^{CF} < \gamma < \gamma_2^{CF}$	CD	FD	F	C	C
$\gamma_2^{CF} < \gamma < \gamma_3^{CF}$	CD	FD	F	C	C
$\gamma_3^{CF} < \gamma < \gamma_2^F$	CD	FD	F	C	F
$\gamma_2^F < \gamma < \gamma_2^C$	CD	FB	F	C	F
$\gamma > \gamma_2^C$	CB	FB	$C = F$	$C = F$	$C = F$

Note: When $\gamma_1^C < \gamma < \gamma_1^F$, consumers prefer flexible pricing iff $K_2 < K < K_3$ and committed pricing otherwise.

2.6.1, we relax our assumption regarding exogenous imitation efficiency and allow the copycat to decide its level of imitation. Next, our main model focuses on short-lived customers whose demands must be satisfied in their arrival period. In other words, consumers do not delay purchases even in anticipation of a lower price in selling period 2. We relax these assumptions and consider a setting in which the innovator commits to a uniform price for both selling periods in Section 2.6.2.

2.6.1 Strategic Copycat

In our main model, we consider that the copycat's imitation efficiency is not too high. This applies to replica products with quality considerably lower than that of the original. For instance, Lululemon faces competition from firms claiming to offer leggings with softness comparable to Lululemon's (Maruf 2024). Nonetheless, Lululemon is often considered the preferred choice due to its distinctive fabric (Lindeman 2024). Here, we explore an alternative setting in which the copycat's imitation efficiency can be sufficiently high, i.e., $\delta \in [0, 1]$, enabling the copycat's product to match the quality of the original. Furthermore, we endogenize the copycat's imitation decision, allowing it to optimally select δ.

In what follows, we replicate the analysis in Sections 2.4 and 2.5 with a strategic copycat. We begin by studying the copycat's imitation decisions under two pricing strategies and summarize the key findings in the following lemma.

Lemma 7. *The following statements hold:*

(i) When imitation efficiency is endogenized, the copycat chooses a lower imitation efficiency under flexible pricing than under committed pricing, specifically, $\delta^{F*} = \frac{4}{7} < \delta^{C*} = \frac{2}{3}$.

(ii) When quality is exogenous, flexible pricing is more effective in blockading copycats than committed pricing (since $q^F > q_1^C$), while committed pricing is more effective in deterring entry than flexible pricing (since $q_2^C > q^F$).

(iii) When quality is endogenous, flexible pricing is more effective in both blockading copycats and deterring entry than committed pricing (since $\gamma_2^F < \gamma_2^C$ and $\gamma_1^F < \gamma_1^C$).

Lemma 7(i) first show that when the copycat makes its imitation efficiency decision, it adopts a strategy of partial imitation efficiency, i.e., $\delta^* \in (0, 1)$. Specifically, the copycat chooses a partial imitation efficiency of $\delta^{F*} = \frac{4}{7}$ under flexible pricing and $\delta^{C*} = \frac{2}{3}$ under committed pricing. This indicates that the price leadership endowed by committed pricing incentivizes more efficient imitation. When quality is exogenous, part (ii) reveals that the conclusion of the Proposition 1 remains robust, flexible pricing is more effective in blockading copycats than committed pricing, and committed pricing is more effective in deterring copycats than flexible pricing. However, when quality is endogenous, part (iii) shows that flexible pricing is more effective in both blockading copycats and deterring entry than committed pricing. Here, both endogenous quality and endogenous imitation efficiency enhances the effectiveness of flexible pricing, as the copycat is incentivized to select a lower imitation efficiency under flexible pricing.

With the optimal imitation decision established, we then examine the optimal pricing strategy under both exogenous and endogenous quality, as outlined in the following proposition.

Proposition 7. *Regardless of whether quality is exogenous or endogenous, flexible pricing leads to a weakly higher profit for the innovator compared to committed pricing.*

Proposition 7 shows that flexible pricing always weakly outperforms committed pricing for the innovator, irrespective of whether the innovator has the quality management capability. In our main model, price leadership reduces competition and benefits the innovator in the region of copycat accommodation. However, when imitation efficiency is endogenous, the benefits of reduced competition are offset by a more efficient copycat. Consequently, flexible pricing weakly dominates committed pricing.

2.6.2 Uniform Pricing

In our main model with a committed pricing strategy, we consider the scenario where the innovator commits to two distinct prices prior to each selling periods. This setup reflects business environments where the innovator adjusts prices specifically in response to potential copycat entry. Practical situations may also arise where the innovator commits to a uniform price for the entire selling horizon.

We now examine the scenario where the innovator adopts a uniform pricing strategy, an alternative approach to establish price leadership. A notable example of this strategy is Apple's practice of maintaining stable prices throughout the iPhone product cycle, which aims to discourage consumers from postponing their purchases strategically. For ease of notation, We use the superscript U to denote variables and outcomes under uniform pricing, where the innovator commits to a uniform price p_i^U for the entire selling horizon. The equilibrium outcome under uniform pricing with exogenous product quality is presented in Lemma A.2.4 of the appendix, which are similar to those presented in Lemma 2 under committed pricing. Specifically, imitation is blockaded when the original product's quality is low and accommodated when it is high. It is worth noting that uniform pricing is a specialized form of committed pricing, constrained by a single uniform price, and is always weakly dominated by the more general committed pricing strategy. Below, we compare the performances associated with uniform and flexible pricing in the context of exogenous quality. Define

$$q_1^U := \frac{16(1-\delta)}{\delta}K,\ q_2^U = \frac{2\left(-\delta+\sqrt{\delta}\sqrt{4-3\delta}+2\right)(4-3\delta)K}{(1-\delta)\delta},$$

and

$$q^{FU} := \frac{32K(1-\delta)(4-\delta)^2\left(\sqrt{\delta(8+\delta)}-\sqrt{2}\delta+4\sqrt{2}\right)^2}{\delta(\delta^2-24\delta+32)^2}.$$

Lemma 8. *Under exogenous product quality, flexible pricing is more effective in blockading copycats than uniform pricing (since $q^F > q_1^U$), while uniform pricing is more effective in deterring entry than flexible pricing (since $q_2^U > q^F$). Moreover, we have:*

(i) If the product quality $q \leq q_1^U$, the two pricing strategies yield the same profit for the innovator, i.e., $\Pi_i^{F*} = \Pi_i^{U*}$.

(ii) If $q^F < q < q^{FU}$, uniform pricing leads to a higher profit for the innovator compared to flexible pricing, i.e., $\Pi_i^{F*} < \Pi_i^{U*}$.

(iii) Otherwise, flexible pricing leads to a higher profit for the innovator compared to uniform pricing, i.e., $\Pi_i^{F*} > \Pi_i^{U*}$.

Lemma 8 shows that when the original product's quality is low ($q \leq q_1^U$), both flexible and uniform pricing yield the same profit for the innovator. However, uniform pricing makes the innovator better off when the quality falls within an intermediate level. This is because the single uniform price constraint significantly reduces the effectiveness of price leadership. Consequently, in competitive environments, uniform pricing is strictly dominated by flexible pricing. In the quality range $q^F < q \leq q^{FU}$, uniform pricing enables the innovator to deter imitators, whereas flexible pricing fails to do so. Nonetheless, flexible pricing is always weakly optimal when the innovator faces an efficient imitator. Therefore, uniform pricing become a viable strategy only when the imitator is less capable. In the following proposition, we examine how quality investment affects the innovator's optimal pricing strategy selection.

Proposition 8. *Under endogenous product quality, flexible pricing is more effective in blockading entry than uniform pricing. Regarding deterring entry, the relative performance of each strategy depends on the level of imitation efficiency, δ. Moreover, flexible pricing leads to a weakly higher profit for the innovator compared to uniform pricing, i.e., $\Pi_i^{U*} \leq \Pi_i^{F*}$.*

When product quality is endogenous, flexible pricing is weakly preferred by the innovator over uniform pricing. Flexible pricing outperforms uniform pricing in both the copycat deterrence and copycat accommodation regions. This is because quality management enhances the advantages of price flexibility inherent in flexible pricing, while the single uniform price diminishes the effectiveness of price leadership under uniform pricing. To summarize, an innovator with the ability to manage quality should not adopt uniform pricing when facing the threat of imitation.

2.7 Summary

Copycats contribute to substantial societal losses each year. Like large corporations, small business innovators are also susceptible to imitation. Despite the significant time and resources dedicated to mitigating this risk, their efforts often fail. Unlike corporate giants, small businesses encounter greater difficulties in absorbing the financial burdens associated with imitation. Pricing and quality investment are two fundamental levers for firms, serving as strategic tools in a competitive environment. In this paper, we develop a parsimonious game-theoretic model involving an innovator and a copycat to explore the effectiveness of price commitment and quality investment in addressing copycat threats. Specifically, we investigate the roles of committed and flexible pricing strategies in deterring copycats and analyze their implications on consumers and social welfare.

We have obtained the following several key findings. First, under exogenous product quality, flexible pricing is more effective in blockading copycats, while committed pricing is more effective in deterring entry. However, when product quality can be managed, flexible pricing may outperform committed pricing in deterring copycats. Second, the innovator should pursue price leadership through committed pricing when quality investment is inexpensive. Conversely, when quality investment is intermediately affordable, the pricing flexibility under flexible pricing consistently results in a higher profit for the innovator. Third, the presence of imitation impels the innovator to limit both price and quality under either pricing strategy. In this context, limit quality and limit pricing act as strategic complements: the threat of replica products can drives the innovator towards more aggressive limit pricing and limit quality. Finally, lower quality investment costs or a more competitive environment does not necessarily improve consumer surplus or social welfare. The strategic reduction in product quality can offset the benefits brought by decreased quality costs and increased competition, leading to worse outcomes for both consumers and society.

Chapter 3

Duopoly Price Competition with Quality Improvement Spillover

3.1 Introduction

In recent years, business innovations of various kinds have been constantly introducing new and emerging industries, opening up unfilled markets and attracting start-up entrepreneurs. In these so-called blue ocean industries, competing firms, especially at their starting stages, will typically face two major challenges when trying to survive. First, they need to improve their own quality level in terms of providing the customers with better products or services. Indeed, an innovative new product often lacks extensive trials and is likely to suffer from mismatch between the product features and the customers' needs. Hence, it is of vital importance for the firms to climb up a learning curve and improve the quality as perceived by the customers. The way to achieve this goal in the emerging industries is through accumulating serving experiences and the learn-by-doing effect.[1] Specifically, the more demand a firm

[1]Note that this paper particularly focuses on the quality that can be improved via the learn-by-doing effect and neglects other quality determinants (e.g., at the beginning the firms are already

obtains, the more it can learn from the past experiences, and the higher quality it may achieve in the future. Such an experience-based quality improvement has been widely seen in practice and well-documented in relevant literature (Pinker and Shumsky 2000, Misra et al. 2004, Ryder et al. 2008).

The second major challenge facing the start-up firms in new and emerging industries is the expansion of brand awareness in potential markets. Although an innovative product or service does uncover a new market and create new demands, the size of the demand is usually not very profitable due to the limited product awareness. Hence, enlarging the potential market with more informed customers should be a high-priority goal to the firms that materialize innovative ideas into business. On the one hand, the market growth surely depends on the quality level of the firms. On the other hand, more importantly, it depends on the awareness expansion among customers, which is often seen in the form of word-of-mouth spread (Godes 2017). Specifically, there exist interpersonal communications among customers who have used the product or service and the potential customers who have not. Interestingly, these communications could be either within-brand or cross-brand (Libai et al. 2009), meaning that the total potential market may be expanded for the benefit of all competing firms, rendering a quality spillover effect. Therefore, with respect to the product awareness expansion, firms will not necessarily take competitive actions; rather, they may leverage the spillover effect to their own advantage.

Illustrating examples combining both experience-based quality improvement and quality spillover effect abound in practice. For example, in the Internet age, many innovative ways of entertainment have emerged. Among others, playing the multiplayer online battle arena (MOBA) games (such as Heroes Evolved and DOTA 2) and reading online serial novels (updated actively and regularly on websites such as Qidian.com and WuxiaWorld.com) are the most popular categories, and the game and novel pub-

doing the best within their capabilities, but will be able to further improve quality by improving their capabilities through learning).

lishing industries are still growing. One major feature of these industries is that the publishers must constantly enhance the quality of the contents by correcting bugs and revising plots according to the feed-backs from the existing users. This means that their quality improvement is based on the number of the accumulated players/readers. Another important feature of these industries is that similar products are constantly reviewed, rated, and discussed altogether on forums. Hence, abundant social interactions, mostly in the form of word-of-mouth, are present in the users communities (e.g., game players and novel readers). Since the word-of-mouth communications among users may not be product-specific, quality spillover may transpire, for example, when the potential users of all similar products increase due to the word-of-mouth recommendation from a user of one particular product. In fact, similar effect has also been observed in fiction books sales, where the bestsellers appear to increases sales for both bestsellers and non-bestsellers in similar genres (Sorensen 2007). As another example, when introducing ground-breaking smartwatch, competing firms, such Apple and Samsung, frequently issue upgrades via incremental innovations to incorporate additional features selected from user feedback. Moreover, high product quality and good user experience of one firm can foster the growth of the entire market significantly, benefiting other smartwatch producers.[2]

The central decision variable that links the learn-by-doing process, the quality improvement, and eventually the profit of the competing firms is the price. Since the firms are climbing up the learning curve over times, they may decide different prices for the product over multiple periods to reflect the quality improvement. Depending on the specific setting, two pricing schemes are often observed in practice. First, firms may announce the prices for all time periods at the beginning of their market entry and keep them unchanged, which is referred to as *committed* pricing scheme. The benefit of such a pricing scheme is easy implementation for management and

[2]Supporting evidences can be found from three sources: `https://tinyurl.com/applesw1`; `https://tinyurl.com/applesw2`; and `https://tinyurl.com/applesw3`, respectively. Accessed on Jan. 10th, 2022.

less risk in irritating customers with changing prices in short notice. Second, without any commitment device, it is not uncommon that the prices are inter-temporal variables decided dynamically in each time period, which is referred to as *dynamic* pricing scheme. Obviously, this pricing scheme endows firms with a level of flexibility that may be valued in a dynamic and competitive environment. In addition to being commonly observed in practice, both pricing schemes are also widely studied in the related literature (see, e.g., Liu and Zhang 2013, Wang and Hu 2014, Shang et al. 2021). In our setting, it is expected that different pricing schemes may have distinct impacts on competing firms' quality improvement and spillover.

Motivated by the above discussion, the primary objective of our paper is to investigate the following research questions in a duopoly dynamic competition setting.

(1) What does the equilibrium outcome, if one exists, of the duopoly price competition look like, and how is it affected by the experience-based quality improvement and spillover under different pricing schemes?

(2) How do the two pricing schemes compare to each other in terms of their impacts on the duopoly competition and the firms' profits?

(3) How would the above results change if the duopoly is different in certain attributes related to quality improvement and its spillover?

To answer these questions, we build a two-period duopoly pricing model where firms set prices, under either committed or dynamic pricing schemes, and compete with each other as their quality levels not only increase according to a linear learning curve, but also have positive spillover effect on the rival's potential demand. Moreover, we first consider the case where the two firms are symmetric in every aspect, and then examine the possible asymmetric case where the firms differ in certain important attributes pertaining to learning and quality improvement. In the following, we highlight our major findings.

First, for a symmetric duopoly, regardless of the pricing scheme, we establish the existence and uniqueness of the Nash equilibrium under some mild condition. In the equilibrium, firms always set a lower price in the first period and raise it in the second period. The lower first-period price is charged with the purposes to both attract more demand as a source for self quality improvement and prevent the competitor from learning too fast. As the quality spillover effect gets stronger, firms tend to raise their first-period price, leading to a lower individual quality improvement and a non-monotonic impact on firms' profit. This is because stronger quality spillover alleviates competition intensity and reduces firms' reliance on quality improvement. In addition, compared to the benchmark where no quality learning exists, both firms choose to sacrifice some short-term profit by setting a lower price, in exchange for climbing up their learning curve fast and the corresponding long-run benefit.

Second, we further compare symmetric firms' equilibrium prices and profits under both committed and dynamic pricing schemes. When the strength of the spillover effect is large compared to the competition intensity level, committed pricing can help alleviate the price competition, and benefit the firms more than dynamic pricing does, which is consistent with the existing literature (Liu and Zhang 2013, Wang and Hu 2014, Shang et al. 2021). By contrast, when the quality spillover is weak, dynamic pricing scheme brings more profits for the duopoly, as firms can take advantage of the pricing flexibility and earn a high second-period profit without sacrificing the first-period profit for self quality improvement.

Finally, we extend our model to two asymmetric cases with respect to quality learning speed and quality spillover effect, respectively. For the first case, the firm with the learning advantage dominates the market by setting prices aggressively, which, surprisingly, may even benefit the firm without the learning advantage, especially when the quality spillover effect is strong. This is true under both pricing schemes. However, dynamic pricing scheme further intensifies firms' asymmetry, and benefits the firm with the learning advantage more than the committed pricing. For the second case, by contrast, the firm without the spillover advantage tends to price more aggressively, regardless of the pricing scheme, as it must rely on its own quality improvement more than the spillover from the competitor. Moreover, we find that dynamic pricing further provides pricing flexibility for the firm with the spillover disadvantage and helps it better leverage the individual quality learning and gain a higher profit.

The reminder of the paper is organized as follow. In Section 3.2, we give a review on related literature. Section 3.3 formulates the model and presents a benchmark case without quality learning. Section 3.4 studies the symmetric two-period duopoly game to develop the conditions for the existence and uniqueness of the equilibrium and answer our research questions in the symmetric case. Moreover, we extend this duopoly game to two asymmetric cases: asymmetric learning speed and quality spillover in Section 3.5. Finally, Section 3.6 concludes the paper with discussions. All proofs are relegated to the appendices.

3.2 Related Literature

Broadly speaking, this paper contributes to the literature on oligopoly price and quality competition; (see, e.g., Banker et al. 1998, Chambers et al. 2006, Liu and Zhang 2013, Gallego and Hu 2014, Geng et al. 2021). Although this stream of literature is vast, there are three important features that distinguish our paper from all previous works. These three differences are related to three streams of literature, respectively.

First, this paper is built upon the literature on learn-by-doing and experience-based quality improvement. Here, we focus on the perceived quality that cannot be directly controlled by firms, and thus is not a decision variable. Instead, the firms' pricing decision indirectly affects their quality improvements according to a learning curve; see Yelle (1979) for a comprehensive survey on learning curve. Quality improvement and learning was first studied by Fine (1986) in the context of productive system, and later introduced to service systems (Pinker and Shumsky 2000, Misra et al. 2004). In particular, price competition with the presence of learning, which is similar to our setting, has been studied by prior works such as Spence (1981), Dasgupta and Stiglitz (1988), and Cabral and Riordan (1994). These papers show that the high-quality firm has an incentive to set low price (predatory pricing) in order to prevent the low-quality entrant from learning and improving. Apparently, the quality spillover effect is absent in those works. Hence, by considering the beneficial spillover of quality improvement between firms, we manage to derive more insights in this novel setting.

Second, our paper is also related to the literature on spillover effect in operations management. For example, Agrawal et al. (2016) investigate the investment in shared suppliers in the presence of spillover effect and competition, the interplay of which is shown to have different impact on investment decision; Hu et al. (2020) study the outsourcing decision where the outsourcing may lead to knowledge spillover, and show that the innovator may strategically outsource the product out of technical and non-technical motivations. Different from these papers, our paper focuses on a duopoly game and identifies the impact on pricing decisions with quality improvement and its spillover effect. Moreover, the extant studies towards quality spillover are largely empirical and covers a variety of topics: inventory management (e.g., Yao et al. 2012), supply chain management (e.g., Muthulingam and Agrawal 2016) and platform strategy (e.g., Haviv et al. 2020). To the best of our knowledge, we are the first to build and solve a theoretical model of duopoly pricing in the presence of quality spillover. Hence, our work provides important and practical insights into the fundamental impact of quality improvement in many business settings.

Third, this paper joins the group of prior works that study the comparison between committed and dynamic pricing schemes. Liu and Zhang (2013) examine the duopoly price competition with strategic customers and show that static pricing can generally improves profits. Wang and Hu (2014) discuss whether competing firms should commit to a fixed price ex ante or elect to price contingently ex post in the presence of demand uncertainty. They conclude that, compared to committed pricing, dynamic pricing may intensify the competition. Shang et al. (2021) consider firms' positioning and pricing choices in the presence of variety-seeking and strategic consumers, and show that price commitment softens horizontal competition and increases profits. There are several fundamental differences between our paper and theirs. First, the quality spillover effect can mitigate competition and this impact may be magnified if prices are pre-determined. Second, the ability of quality improvement may intensify price competition and thus influence the results under each pricing scheme.

Lastly, the quality spillover effect captured by our model could call for certain collaborative actions from the competing firms; e.g., they may allow the other firm to learn and improve quality so that the total market may be expanded. In this sense, our work is also related to the notion of co-opetition (Brandenburger and Nalebuff 1996). Indeed, as discussed by many previous papers (see, e.g., Arya et al. 2007, Hu et al. 2017, Niu et al. 2019), certain competitive strategies may result in mutual benefit for competing firms in many practical settings. Hence, our paper simply identifies and studies another example setting in this regard.

To sum up, our primary contribution to the literature is as follows. We are among the first to consider possible quality spillover of firms that engage in price and quality competition, and the impacts of different pricing schemes on the equilibrium outcomes. This not only advances the relevant literature, but also captures many of the real-life situations and therefore has a considerable practical value. Moreover, by introducing the learning curve to firms' quality improvement in a competitive environment, our model links the firms' quality levels to their pricing decisions, which has important managerial implications but is largely overlooked in the literature that studies quality competition.

3.3 Model Setup and Preliminary Analysis

In this section, we first formulate our model by laying out the basic assumptions and describing the key model elements, and then analyze a static model without quality improvement to provide some benchmark results.

3.3.1 Model Setup

Consider a duopoly price competition in a two-period setting. The two firms (A and B) provide similar products to a market of customers and, as a result, the demand of one firm is interdependent with the demand of the other firm via their prices and quality levels. Moreover, we assume that both firms are on the learning curve in terms of their quality improvement. That is, each firm's quality level may increase in the second period due to improvement made based on the accumulated experience, which is represented by the market size gained by the firm in the first period. The main distinguishing feature of our model is the assumption that the rival firm's quality level can expand the total market size and positively affect a firm's demand. As discussed in Section 1, such a spillover phenomenon can be observed in industries wherein the competing firms are at the startup stage or the service is relatively new to customers. As such, the pricing schemes considered in our model is of particular importance, because the firms' pricing decisions will jointly affect their demands and thus influence the quality improvement, which in turn has an impact on the competition. Hence, the duopoly pricing schemes and the quality improvement spillover are intertwined in a non-intuitive way. In the following, we detail the key elements to complete the model setup.

Pricing Schemes

In a dynamic setting such as ours, two types of pricing schemes are commonly studied, i.e., committed pricing and dynamic pricing. In the former case, the prices for all periods are decided at the beginning and committed throughout the horizon, whereas in the latter case, the price for one period is announced in a dynamic fashion, when that period starts. These two pricing schemes are both employed by firms in practice and well-documented in literature (e.g., Wang and Hu 2014, Geng et al. 2021). In particular, the equilibrium under committed pricing is often referred to as "open-loop" equilibrium, and that under dynamic pricing as "closed-loop" equilibrium. Although dynamic pricing seems to have the obvious benefit of pricing flexibility, which allows the firm to always price according to the most updated information, it may still be outperformed by committed pricing in a competitive environment. Indeed, when considering firms that are climbing learning curves, dynamic pricing may excessively intensify the competition and beneficial quality improvement spillover cannot be fostered. In other words, the combined effect of pricing flexibility and quality improvement spillover is not immediate in our setting. Therefore, one of the primary objectives of our analysis is to compare the two pricing schemes and to characterize the condition under which one can outperform the other.

Quality Improvement and Learning Process

Quality improvement and learning is first studied in the context of production system, and the underlying assumption is that the unit production cost is reduced according to a learning curve that is based on production volume (e.g., Fine 1986, Spence 1981). In service industries, similar learning process can be observed (e.g., Misra et al. 2004, Ryder et al. 2008). In this paper, we focus on the experience-based quality improvement; i.e., the learning is based on the total past sales. Indeed, as a firm serves more customers, it has to apply a correspondingly large amount of care and scrutiny to the process, and as a result the quality can be enhanced.

The learning curve (as a function of the accumulated sales) is often increasing and concave, and has an upper bound. Although the log-linear functional form can be used to describe the learning curve, one approach commonly seen in the related literature to reduce the analytical difficulty is to employ linear learning curve as an approximation (see Jin et al. 2004). Hence, in this paper, we use a linear approximation to describe the firms' learning process. Let q_t be the quality level in period t and $q_{max} = 1$ be the maximal quality level. Then, the quality improvement from period t to $t+1$ can be written as

$$q_{t+1} = \min\{1, q_t + \delta D_t\},$$

where $\delta \geq 0$ represents the learning speed of the firm and D_t is the firm's demand in period t. To focus on the effect of learning and quality improvement spillover, we assume that the learning occurred in the first period is not sufficient for either firm to achieve the maximal quality level in the second period. Thus, the firms' decisions will not be restricted by the ceiling of quality improvement. It is straightforward to see that this assumption can be satisfied as long as the total market size and the learning speed are finite. Therefore, in our model, the learning process is further simplified to be

$$q_{t+1} = q_t + \delta D_t. \tag{3.1}$$

Demand and Profit Functions

Consistent with the most commonly seen demand model in the literature, we focus on linear demand function, which is based on the quadratic utility function maximization derived in Shubik and Levitan (1980). Suppose that the total market size is Λ. In period t ($t = 1, 2$), firm i's ($i = A, B$) demand when using pricing scheme k ($k = c, d$)

is a function of both firms' prices (p^k_{At} and p^k_{Bt}) and their quality levels (q^k_{At} and q^k_{Bt}):

$$D^k_{it} = \frac{1}{2}\left[\Lambda\left(\left(1-\theta_i\right)q^k_{it} + \theta_i q^k_{jt}\right) - p^k_{it} + \alpha\left(p^k_{jt} - p^k_{it}\right)\right] \quad i,j \in \{A,B\}, i \neq j, k \in \{c,d\}. \tag{3.2}$$

In the above, the superscript "c" means committed pricing scheme and "d" means dynamic pricing scheme. Here, $\alpha \geq 0$ is the substitution parameter, which indicates the intensity of the price competition; e.g., larger α means that firms compete more fiercely. On the other hand, the parameter θ_i measures the effect of firm j's quality on firm i's demand. Deviating from the traditional economic and operations management models, where a firm's market share and its rival's quality level are negatively correlated, we assume that $\theta_i \geq 0$ in our setting. That is, a firm can benefit from a higher quality provided by the rival firm. This assumption aims to capture the situations discussed in Section 3.1, where one firm's higher quality level can help expand the entire market and improve consumers' perceived quality of both firms (Libai et al. 2009). Moreover, we only consider $\theta_i < \frac{1}{2}$, meaning that, each firm's own quality has a stronger impact on its own demand than the other firm's quality. Note that, in our model, the first-period demand improves firms' quality levels via experience-based learn-by-doing, which in turn affects each firm's demand in the second period. Hence, our model shares a similar nature with those studying the market expansion via social interactions (e.g., Geng et al. 2021).

The duopoly independently and simultaneously decide the prices to maximize their respective profits. Without loss of generality, we normalize the firms' unit cost to zero. Thus, given firm j's prices, firm i's problem is formulated as below:

$$\max_{p^k_{i1}, p^k_{i2}} \sum_{t=1,2} p^k_{it} D^k_{it}.$$

Under committed pricing, the two firms decide the price in each period at the beginning of the game; however, if dynamic pricing scheme is in effect, firms will decide only the first-period price at the beginning of the game and announce the second-period price after they know their sales and profits of the first period. Regardless of the pricing scheme, the firms' quality level will be improved according to (3.1) at the end of the first period.

With the price competition on the one hand, and the quality improvement spillover on the other, the duopoly faces a trade-off between the degree of the competition intensity and the amount of quality spillover. In our model formulation, α measures the competition intensity and θ_i and δ are closely related to the quality improvement spillover, these parameters are at the core of our analysis on the firms' optimal pricing decisions and they shed lights on the firms' profits in equilibrium.

3.3.2 Benchmark: Static Duopoly Game

We first investigate the benchmark model where quality improvement is absent. Specifically, consider a single-period duopoly pricing game. Without learning or quality change, the game is degenerated to a simple price competition with exogenous quality levels. Therefore, the demands of firm A and B are given by:

$$\begin{cases} D_A = \frac{1}{2}\left[\Lambda\left(\left(1-\theta_A\right)q_A + \theta_A q_B\right) - p_A + \alpha\left(p_B - p_A\right)\right] \\ D_B = \frac{1}{2}\left[\Lambda\left(\left(1-\theta_B\right)q_B + \theta_B q_A\right) - p_B + \alpha\left(p_A - p_B\right)\right] \end{cases} \tag{3.3}$$

Then, each firm will simultaneously set price to maximize its own profit.

We focus on the pure Nash equilibrium prices. It is straightforward to solve the first order conditions and find the unique equilibrium prices to be

$$\begin{cases} p_A^* &= \frac{2(1+\alpha)((1-\theta_A)q_A+\theta_A q_B)+\alpha((1-\theta_B)q_B+\theta_B q_A)}{(\alpha+2)(3\alpha+2)}\Lambda \\ p_B^* &= \frac{2(1+\alpha)((1-\theta_B)q_B+\theta_B q_A)+\alpha((1-\theta_A)q_A+\theta_A q_B)}{(\alpha+2)(3\alpha+2)}\Lambda \end{cases} \tag{3.4}$$

Moreover, the equilibrium market share and revenue are, respectively,

$$D_A^* = \frac{1+\alpha}{2}p_A^*, \quad D_B^* = \frac{1+\alpha}{2}p_B^*;$$

and

$$\pi_A^* = \frac{1+\alpha}{2}(p_A^*)^2, \quad \pi_B^* = \frac{1+\alpha}{2}(p_B^*)^2.$$

Note that, if $q_A > q_B$, then $p_A^* > p_B^*$, $D_A^* > D_B^*$ and $\pi_A^* > \pi_B^*$. That is, the firm with higher quality would price in a more aggressive way and dominate the low-quality firm in both market share and profit.

This simple static model serves as a natural benchmark for our study on the two-period model. In particular, by comparing the equilibrium outcomes of the two-period model against the benchmark, we can examine the role of the quality improvement in the duopoly game, as well as the impact of its possible spillover to the opponent. Next, in Section 3.4, we study the two-period model under the assumption that the two firms are symmetric in all aspects; and then, we repeat the analysis in Section 3.5 for the case where the two firms are asymmetric in certain key attributes.

3.4 The Two-Period Model with Symmetric Firms

In this section, we focus on the two-period duopoly price competition model with symmetric firms to study how the experience-based quality improvement and its spillover influence firms' equilibrium prices and profits, and how the quality learning process interact with the pricing schemes. It is noteworthy that the two-period setting can both capture the essential characteristics of the quality improvement in our model and yield tractable analysis by avoiding the unnecessary technical difficulties.

Our primary goal is to investigate the impacts of the learning process, the quality improvement spillover, and the pricing schemes on the equilibrium outcomes of the duopoly game. To that end, we develop our research from three perspectives. First, we solve the two-period game to examine firms' prices change in different periods and their quality improvement across periods. Second, we investigate the role of the quality improvement process in the duopoly game. In particular, we compare the first-period price and profit in the two-period model against the benchmark results, which reveals how much the firms are willing to cut prices and/or sacrifice profit in order to improve faster and benefit in the long run. Third, we scrutinize the performances of different pricing schemes, committed and dynamic, and derive insights into the interaction between the quality improvement spillover and the pricing flexibility.

To further facilitate the model analysis and the derivation of useful insights, we assume that both firms are identical in all aspects. Specifically, all the relevant attributes of the firms are the same; i.e., their initial quality satisfies $0 < q_{A1} = q_{B1} = q_1 < 1$, their learning speed $\delta_A = \delta_B = \delta$, and their quality spillover coefficients $\theta_A = \theta_B = \theta$. Such a symmetric setting depicts the situation where firms are roughly at the same stage of their development. All our following discussions are therefore based on this unique equilibrium of the game. Before proceeding to formal analysis, we first confirm the existence and uniqueness of pure symmetric Nash equilibrium for the duopoly pricing game.

Theorem 1. *For any pricing scheme $k \in \{c, d\}$, when $\delta\Lambda \leq 1$, there exists a unique pure symmetric Nash equilibrium (p_1^{k*}, p_2^{k*}), which is an interior point of the feasible region.*

3.4.1 Equilibrium Prices, Quality Improvement, and Profits

In this subsection, we solve the duopoly pricing game and fully characterize the equilibrium outcomes for both firms, including the prices, the quality levels, and the profits. Then, for different pricing schemes, we examine the price change over periods and the quality improvement due to learning. Moreover, we conduct sensitivity analysis with respect to the competition intensity and the quality spillover strength to illustrate their impacts.

We start with the comparison between the prices across the two periods, and the result is summarized by the following proposition.

Proposition 9. *For any $\alpha \geq 0$, $\delta\Lambda \leq 1$, $\theta \in [0, 1/2]$ and pricing scheme $k \in \{c, d\}$,*

$p_1^{k*} < p_2^{k*}$ *always holds.*

This proposition shows that firms would set a low price in the first period and raise it in the second period. In our particular setting, there are two different reasons for such a price increase. First, firms want to improve quality faster so that they may collect higher total profit in the long run, because higher quality improvement means a larger market in the second period. Consequently, setting a lower first-period price helps the firms attract more demand as a source of higher quality improvement, which allows them to charge a higher price in the second period. The second reason why firms offer lower prices in the first period is purely strategic — to prevent the opponent from learning too fast. When the competition intensity is strong compared to the strength of the quality spillover effect, firms choose to set a low price in the beginning to deter the opponent's gaining competitive advantage through quality improvement; this result also echoes with that in Cabral and Riordan (1994).

It is noteworthy that $p_1^{k*} < p_2^{k*}$ holds largely because of the condition $\theta < \frac{1}{2}$, $k \in \{c, d\}$. If, however, the opponent's quality affects the firm's demand more than its own quality does (i.e., $\theta > \frac{1}{2}$), then the firm could find it more beneficial to set a high price in the first period to help the opponent learn and improve quickly, hoping that this could booster its second period demand increment

How would the system parameters affect the equilibrium prices of the firms? The following proposition reports the sensitivity results in this regard. Throughout the paper, we use "decrease" and "increase" in the weak sense, unless otherwise specified.

Proposition 10. *Suppose* $\alpha \geq 0$, $\delta\Lambda \leq 1$, *and* $\theta \in [0, 1/2]$. *Then, for pricing scheme* $k \in \{c, d\}$, p_1^{k*} *increases in* θ, *decreases in* δ, *and decreases in* α, *whereas* p_2^{k*} *decreases in* θ, *increases in* δ, *and decreases in* α.

This proposition shows that firms tend to raise their price in the first period when θ increases or when δ and/or α decreases. When the quality spillover effect is strong (i.e., θ is large), firms will set a higher price in the first period to avoid fierce competition. In this case, the benefit of quality spillover effect outperforms the benefit of quality improvement. The quality learning speed affects the prices in an intuitive way in our model. To be specific, when learning can improve quality efficiently (i.e., δ is large), there is an incentive for firms to lower prices in the first period for higher quality level in the second period, which allows them to compete with a higher prices. Lastly, in a more competitive market (i.e., α is large), firms tend to set a lower price in the first period in order to make themselves more competitive in the second period. Indeed, the firm with greater quality improvement can lower the price in the second period to attract more customers and gain a higher profit. In this case, maintaining competitive advantage is more important than enjoying the benefit of quality improvement, and thus the second-period prices would be lower despite the greater quality improvement.

Next, we turn to analyze the firms' quality improvement after the first period, and its dependence on the competition level and the quality spillover effect. Since the firms are symmetric, the quality improvement for both firms are identical. Therefore, we focus on the quality improvement defined by $QI = \delta D_1^k$, where D_1^k is the demand in the first period. In the equilibrium of the duopoly game under pricing scheme $k \in \{c, d\}$, we have

$$QI^{k*} = \delta \frac{1}{2}(\Lambda q_1 - p_1^{k*}).$$

The equilibrium prices in the two pricing schemes are solved from the first order conditions, as can be seen in the proof of Proposition 9. Consequently, the monotonicity of the quality improvement can be directly link to that of the first period equilibrium price p_1^{k*}. Specifically, the next proposition provides us a view on how price competition and quality spillover effect influence the firms' quality improvement at equilibrium.

Proposition 11. *Suppose $\alpha \geq 0$, $\delta\Lambda \leq 1$, and $\theta \in [0, 1/2]$. Then, for pricing scheme $k \in \{c, d\}$, QI^{k*} decreases in θ and increases in δ and α, respectively.*

We can see from Proposition 11 that the competition intensity (α) acts as a positive driving force on firms' quality improvement; i.e., more intense price competition between the firms induces larger quality improvement. On the other hand, however, we observe a surprising result that the quality spillover effect (θ) has a negative impact on the quality improvement. Indeed, as Proposition 11 shows, when the firm depends more on the opponent's quality to induce the demand, the quality improvement actually diminishes. These results can be explained by the following fact: High quality spillover effect counteracts the competition intensity and therefore firms do not have to rely on quality improvement too much for competition reason. Hence, this finding reveals an important managerial insight that a competitive environment helps the firms improve quality faster, whereas the quality spillover effect delays the learning process.

Lastly, we look into the equilibrium profits of the firms and study how they are affected by the system parameters.

Proposition 12. *For any $\alpha \geq 0$, $\delta\Lambda \leq 1$, $\theta \in [0, 1/2]$ and pricing scheme $k \in \{c, d\}$, the following statements hold:*

(i) π_1^{k*} increases in θ, decreases in δ and α;

(ii) π_2^{k*} decreases in θ, increases in δ, and first increases and then decreases in α;

(iii) π^{k*} first increases and then decreases in θ, increases in δ, and first increases and then decreases in α.

The above proposition shows that the learning speed always plays a positive role on firms' equilibrium profits. However, it is not the case for the competition level and the quality spillover effect. A small quality spillover effect benefits firms while a strong quality spillover effect may hurt firms. Note that with quality spillover, firms tend to avoid fierce competition and enjoy higher profits in the first period; but in the second period, they have to compete with each other at a lower prices and can only collect lower profits. Moreover, when the competition is intense enough, the gain in the first period is greater than the loss in the second period. In a less competitive market, however, there exists an intermediate quality spillover level that is optimal for firms. At that point, the impacts of θ on the first- and the second-profit are balanced out.

Regardless of the pricing schemes, the main trade-off for the firms is to balance the first-period profit and the second-period profit. Note that there are two critical factors that determine the second-period profit, namely, the quality spillover and the quality learning. We show that they are substitutes in terms of the contribution to the second-period profit.

3.4.2 The Impact of Quality Improvement on Firms' First-Period Decisions and Performances

In this subsection, we compare the static benchmark and the dynamic models, seeking to investigate the role quality improvement plays in the duopoly game. For example, in the presence of learning, firms may leverage on pricing to affect the opponent's market share, which in turn affects its quality improvement and the second-period demand. Hence, we will examine the difference between the static price/profit in the benchmark model and the first-period price/profit in the dynamic model. The second-period prices, although important to the total profit, do not reflect the role of learning in the firms' interaction. Moreover, we will analyze how the price and profit changes depend on the firms' learning speed and quality spillover strength.

In our symmetric setting, the two firms are identical, and thus we focus on firm A and compare its price and profit in the static model and its first-period price in the two-period dynamic model. Specifically, we examine the percentage price and profit changes[3], defined by:

$$P^k(\delta,\theta) := \frac{p^* - p_1^{k*}}{p^*} \quad \text{and} \quad R^k(\delta,\theta) := \frac{\pi^* - \pi_1^{k*}}{\pi^*},$$

where the benchmark equilibrium p^* and π^* are given by Equation (3.4). The above function P^k and R^k represent how the learning and quality improvement across periods affect the firm's pricing decision and profit in terms of both the changing direction (increase or decrease) and the magnitude (by how much compared to the benchmark case). Then, we have the following proposition, which details the comparison and characterizes the sensitivity of the price and profit changes with respect to firms' quality learning and quality spillover.

Proposition 13. *For any $\alpha \geq 0$, $\delta\Lambda \leq 1$, $\theta \in [0, 1/2]$ and pricing scheme $k \in \{c, d\}$, the following statements hold:*

(i) $P^k(\delta,\theta) > 0$ *and* $R^k(\delta,\theta) > 0$.

(ii) $\frac{\partial P^k(\delta,\theta)}{\partial \theta} < 0$ *and* $\frac{\partial P^k(\delta,\theta)}{\partial \delta} > 0$; $\frac{\partial R^k(\delta,\theta)}{\partial \theta} < 0$ *and* $\frac{\partial R^k(\delta,\theta)}{\partial \delta} > 0$.

(iii) $\frac{\partial^2 P^k(\delta,\theta)}{\partial \theta \partial \delta} < 0$ *and* $\frac{\partial^2 R^k(\delta,\theta)}{\partial \theta \partial \delta} < 0$.

The foremost result shown in Proposition 13, given by part (i), is that the price and profit changes are always positive. This means that, compared to the situation where learning does not exist, the symmetric firms in the two-period model will set a lower price and receive less profit regardless of the pricing schemes. To understand this result, note that both firms are still climbing the learning curve. Hence, the benefit of learning and quality improvement prevails for both firms, and as a result they both choose to sacrifice some short-term profit in return for the long-run benefit. Therefore, for any δ and θ, we have $p^* > p_1^{k*}$ and $\pi^* > \pi_1^{k*}$. It is worth mentioning that the presence of quality spillover would refrain the firms from lowering the price

[3]Note that our results hold true for the case of absolute change as well. We take the percentage change in this paper because it represents a relative and more objective comparison.

in the first period too much, but this factor is not strong enough to justify increasing the first-period price.

Although the percentage change functions are always positive, the learning speed and spillover strength do have an impact on the magnitude of the price and profit changes, as given in part (ii). To be specific, with all other parameters fixed, a stronger quality spillover effect or a lower quality learning speed would yield smaller price and profit decreases. That is, stronger quality spillover effect helps firms avoid low price and fierce competition, and therefore gain greater profit. On the other hand, faster quality learning induces the firms to cut prices deeper in order to enlarge sales, which hurts their short-run profits.

Finally, part (iii) reveals an interesting result regarding the cross effect of δ and θ on the price and profit changes. Specifically, we can see that $P^k(\delta, \theta)$ and $R^k(\delta, \theta)$ are both submodular functions, indicating that the learning and the quality spillover are strategic substitutes. With a faster quality learning, the effect of quality spillover in diminishing the strategic price drop and profit loss becomes more manifested. Similarly, if the quality spillover becomes stronger, then the learning speed would only lead to a smaller price and profit decrease. Therefore, there is an internal conflict between the quality learning and the quality spillover in terms of affecting the price and profit changes between the static and dynamic models.

3.4.3 Comparison Between the Two Pricing Schemes

In our model, two pricing schemes are considered. Both commonly seen in practice, the committed pricing and the dynamic pricing schemes have their respective advantages and disadvantages. For example, dynamic pricing allows firms to make inter-temporal changes to the posted price so that it is contingent to the evolving environment; committed pricing, on the other hand, could generally relax the competition and is relatively easy to implement. Depending on the practical situation, one strategy may outperform the other, as shown by several prior works in economics and operation management literature (Liu and Zhang 2013, Wang and Hu 2014, Shang et al. 2021). Here, we compare the duopoly pricing game under these two pricing schemes in the presence of the learning and quality improvement spillover. When the firms decide the prices of both periods at the beginning of the game and commit to them, the game outcome is given by the open-loop equilibrium; however, when the firms set the prices of each period dynamically, the resulting prices and profits are called closed-loop equilibrium. In the following, we compare the open-loop and closed-loop equilibrium prices, quality improvement, and profits, and then conduct sensitivity analysis with respect to the quality learning and quality spillover parameters. We start with the proposition below.

Proposition 14. *For any $\alpha \geq 0$, $\delta\Lambda \leq 1$ and $\theta \in [0, 1/2]$, the following statement holds,*

(i) when $\theta < \frac{\alpha(1+\alpha)}{(2\alpha+1)(\alpha+2)}$, $p_1^{c*} < p_1^{d*}$, $p_2^{c*} > p_2^{d*}$, $QI^{c*} > QI^{d*}$, and $\pi^{c*} < \pi^{d*}$.

(ii) when $\theta > \frac{\alpha(1+\alpha)}{(2\alpha+1)(\alpha+2)}$, $p_1^{c*} > p_1^{d*}$, $p_2^{c*} < p_2^{d*}$ and $QI^{c*} < QI^{d*}$. Moreover, $\pi^{c*} > \pi^{d*}$ if $\alpha > 0.51$.

One of the main contributions of this paper is to characterize the conditions under which the open-loop, and respectively the closed-loop, equilibrium profits of the firms are higher, which is summarized by Proposition 14. From the proposition, we derive three interesting insights regarding the pricing schemes comparison.

First, quality improvement spillover plays an important moderating role in the comparison. If the strength of the spillover effect is large compared to the competition intensity level (i.e. $\theta > \frac{\alpha(1+\alpha)}{(2\alpha+1)(\alpha+2)}$), then committed pricing can benefit the firms more than dynamic pricing does. Indeed, stronger quality spillover effect means that firms do not have to aggressively set low prices in the first period for quality improvement, and therefore the competition is softened. Conversely, if the quality spillover is relatively weak in strength (i.e. $\theta < \frac{\alpha(1+\alpha)}{(2\alpha+1)(\alpha+2)}$), then dynamic pricing scheme brings more profits than committed pricing scheme. In this case, firms can take advantage of the pricing flexibility and earn a high second-period profit without sacrificing the first-period profit for quality improvement. Hence, in a sense, the quality spillover and the pricing flexibility are substitutes when it comes to the firms' total profits.

Second, the quality improvement is smaller under the pricing scheme that results in higher profit in each case; moreover, under that strategy, the first-period price is higher and the second-period price is lower. This means that the quality spillover and the pricing flexibility can both reduce a firm's dependence on its own quality improvement. However, they function differently. Specifically, when the quality spillover effect is weak, dynamic pricing can use its pricing flexibility to hedge against the insufficient quality increase (that is, the advantage of contingent pricing overcomes the lack of quality improvement). On the other hand, stronger quality spillover effect can alleviate the competition and help both firms expand the market without setting too low prices in the first period. Thus, the firm's own quality improvement does not have to increase a lot.

Third, the firms should be cautioned against overdoing the learning and the quality improvement. While it is usually tempting to increase its quality and rely on it to gain more long run benefit, decreasing the first-period prices too much may hurt the firm under certain pricing scheme. In Proposition 14 (i) and (ii), we observe that, under the pricing scheme that results in lower profit, firms' lowering the prices in the first period is an overdoing of quality improvement by inducing too much demand. In fact, although doing so could increase their second-period profits, it hurts their first-period profits even more. Hence, in the presence of quality improvement, firms should take the pricing scheme into consideration when evaluating the benefit of lowering the prices to induce large demand for learning.

Finally, we remark that the above results hold under the assumption $\alpha > 0.51$, i.e., the duopoly competition is not too soft. In a market with weak competition intensity (i.e., $\alpha < 0.51$), part (i) still holds, while the profit of dynamic pricing may outperform the profit of committed pricing even under the condition of part (ii), if the quality spillover is strong enough. Note that in this case, in contrast to the relationship of substitution, quality spillover complements quality learning. Hence the dynamic pricing scheme, which contributes greater quality improvement, yields more profit.

Next, we further investigate how the comparison between the pricing schemes depends on the learning speed and the quality spillover. Under certain conditions on competition intensity, the following proposition shows the results of the sensitivity analysis regarding the relevant parameters.

Proposition 15. *For any $\alpha \geq 0$, $\delta\Lambda \leq 1$ and $\theta \in [0, 1/2]$, the following statement holds.*

(i) $p_1^{d*} - p_1^{c*}$ decreases in θ, $p_2^{d*} - p_2^{c*}$ increases in θ, and $QI^{d*} - QI^{c*}$ increases in θ. Moreover, $\pi^{d*} - \pi^{c*}$ decreases in θ if $\alpha > 0.97$.

(ii) Suppose $\theta < \frac{\alpha(1+\alpha)}{(2\alpha+1)(\alpha+2)}$. Then, $p_1^{d*} - p_1^{c*}$ increases in δ, $p_2^{d*} - p_2^{c*}$ decreases in δ, $QI^{d*} - QI^{c*}$ decreases in δ, and $\pi^{d*} - \pi^{c*}$ increases in δ.

(iii) Suppose $\theta > \frac{\alpha(1+\alpha)}{(2\alpha+1)(\alpha+2)}$. Then, $p_1^{d*} - p_1^{c*}$ decreases in δ, $p_2^{d*} - p_2^{c*}$ increases in δ, and $QI^{d*} - QI^{c*}$ increases in δ. Moreover, $\pi^{d*} - \pi^{c*}$ decreases in δ if $\alpha > 1.46$.

Recall that Proposition 14 shows that $\pi^{d*} - \pi^{c*}$ is positive when θ is small and negative when θ is large. Thus, Proposition 15(i) further reveals that the profit difference between the two pricing schemes is in fact decreasing in θ when $\alpha > 0.97$. Note that the comparison result holds when $\alpha > 0.51$, whereas the monotonicity result requires the competition to be even more intense. As previously discussed, the flexibility endowed by dynamic pricing scheme becomes less beneficial when the quality spillover effect gets stronger; therefore, with intensified competition (from $\alpha > 0.51$ to $\alpha > 0.97$), the substitutability of pricing flexibility and the quality spillover is strengthened, as implied by the monotonicity of $\pi^{d*} - \pi^{c*}$ in θ.

Proposition 15(ii) and (iii) indicate that the profit difference between dynamic pricing and committed pricing could be either increasing or decreasing in δ. However, the seemingly opposite impacts actually consistently show the positive influence of the learning speed on the consequences brought by the pricing schemes. Specifically, the conditions in part (ii) imply $\pi^{d*} > \pi^{c*}$, and faster learning can further improve the advantage of dynamic pricing; on the other hand, the conditions in part (iii) ensures $\pi^{d*} < \pi^{c*}$, and therefore committed pricing is even more advantageous with faster learning speed. It is worth noting that the underlying rationale behind the results in the two cases are different. When the spillover effect is weak, the dynamic pricing provides flexibility to firms so they can better improve their own quality; thus, faster learning leads to more advantage. When the spillover effect is strong, committed pricing along with faster learning let firms spill over more quality improvement to each other, and the total market size is expanded more, which results in more advantage of committed pricing.

Similar to Proposition 15, our results continue to hold with a small α, as long as the quality spillover θ is not so strong. When competition is weak, $\pi^{d*} - \pi^{c*}$ may increase in θ if quality spillover and quality learning are complements rather than substitutes, i.e., θ is large enough in part (i). The impact of quality learning on the profits gap may also change under this condition, since the complementary relationship between quality spillover and quality learning appeals for greater quality improvement; and the dynamic pricing scheme meets this requirement by overdoing the quality learning and improvement.

3.5 The Two-Period Model with Asymmetric Firms

Previously, we studied the two-period model in a symmetric duopoly setting. How would our results alter if the two firms are asymmetric in some key attribute? In this section, we attempt to answer this question. Particularly, since firms' learning and the quality spillover effect are pivotal factors in our model, we focus on firms' asymmetry in these two attributes. It is noteworthy that such asymmetries are common in reality; for example, the quality improvement of an incumbent firm may induce more demand to an entrant firm than the other way around (i.e., asymmetric quality spillover strength), while an entrant firm can typically learn and improve quality faster than an incumbent (i.e., asymmetric quality learning speed).

To isolate the sole impact of the firms' asymmetry in each of the above attributes, we separately study the following two cases: First, we consider asymmetric learning speed by letting $\delta_A = \delta$ and $\delta_B = 0$. Second, we consider asymmetric quality spillover by assuming $\theta_A = \theta$ and $\theta_B = 0$. In each case, with all other parameters being the same as in the symmetric model, we characterize the equilibrium results, examine how equilibrium prices and profits depend on θ and δ, and determine the impact of firms' asymmetry on the differences between the two pricing schemes.

3.5.1 Asymmetric Learning Speed

In this subsection, we study the case where the duopoly is asymmetric in the learning speed with firm A learning and firm B not. That is, $\delta_A = \delta > 0$ and $\delta_B = 0$. We repeat the equilibrium analysis as conducted in Section 4, starting with the characterization of the equilibrium outcome. Compared to the symmetric case, here firms will set prices differently in both periods, as shown by the following proposition.

Proposition 16. *Under pricing scheme $k \in \{c, d\}$, when $\delta\Lambda \leq 1$, there exists a unique pure Nash equilibrium $(p^{k*}_{A1}, p^{k*}_{A2}, p^{k*}_{B1}, p^{k*}_{B2})$, which is an interior point of the feasible region. Moreover, the following statements hold.*

(i) $p^{k*}_{A1} < p^{k*}_{A2}$ *and* $p^{k*}_{B1} < p^{k*}_{B2}$.

(ii) $p^{k*}_{A1} < p^{k*}_{B1}$ *and* $p^{k*}_{A2} > p^{k*}_{B2}$.

As shown by Proposition 16, the existence and uniqueness of the Nash equilibrium for the asymmetric duopoly game can be guaranteed by the same condition as in the symmetric case. Note that, while δ is the learning speed of both firms in Theorem 1, firm B does not learn in this case here and δ is only firm A's learning speed.

Since firm B is unable to improve its quality through learning, one may intuit that it does not have the incentive to cut the first-period price in exchange for quality improvement. However, Proposition 16(i) reveals an intriguing finding that both firms would lower prices in the first period. Firm B's behavior can be explained by the quality spillover effect. Indeed, firm A's asymmetric learning speed can also benefit firm B by expanding the total market in the long run, allowing firm B to charge a higher price to a larger market in the second period. Furthermore, part (ii) of the above proposition demonstrates that firm A sets a lower price in the first period and a higher price in the second period than firm B does. That is, the initiative in quality learning drives firm A to sacrifice more short-run profit. Conversely, knowing that the opponent's quality improvement may be beneficial, firm B may voluntarily set a higher price (than firm A) in the first period so that more customers are sent to firm A to facilitate a even faster quality improvement.

How would the learning speed and the quality improvement spillover influence the equilibrium prices and profits? The next Proposition demonstrates the impact of these system parameters in the asymmetric setting.

Proposition 17. *Suppose $\alpha \geq 0$, $\delta\Lambda \leq 1$ and $\theta \in [0, 1/2]$. Under pricing scheme $k \in \{c, d\}$, the equilibrium outcomes have the following properties.*

(i) Prices: p_{A1}^{k*} increases in θ and decreases in δ, p_{A2}^{k*} decreases in θ and increases in δ; p_{B1}^{k*} increases in θ, and decreases in δ when θ is small and increases in δ otherwise, and p_{B2}^{k*} increases in θ and δ.

(ii) Quality improvement: QI_A^{k*} decreases in θ and increases in δ.

(iii) Profits: π_A^{k*} *first decreases and then increases in* θ*, and always increases in* δ*;* π_B^{k*} *increases in* θ*. As for* δ*,* π_B^{c*} *increases in* δ*, while* π_B^{d*} *decreases in* δ *when* θ *is small and increases otherwise.*

Parts (i) and (ii) of Proposition 17 indicate that the impact of δ and θ remains the same as in the symmetric case for firm A, i.e., the one with the learning ability. However, for firm B, we have different results. Specifically, in the first period, stronger quality spillover incentivizes firm B to raise price, which shows again that firm B would voluntarily assist the opponent in quality improvement. Moreover, firm A's faster learning speed could either positively or negatively influence the price, depending on the strength of the quality spillover: When the quality spillover effect is not very significant, larger δ will result in lower equilibrium prices, implying an intensified duopoly price competition. In the second period, interestingly, the quality spillover and the opponent's asymmetric quality learning positively affect firm B's price, because they both increase firm B's potential market size.

Part (iii) of the above proposition presents a series of interesting results regarding the effects of quality learning and quality spillover on firms' profits. First, firm A's profit increases in the learning speed δ and firm B's profit increases in the quality spillover strength. Here, faster learning enhances firm A's advantage in quality improvement, whereas a stronger quality spillover effect always benefits firm B, the free rider. Second, the impact of the quality spillover effect on firm A's profit is more complicated. Specifically, when the quality spillover is strong enough, firm A's profit is positively influenced; however, firm A can be hurt by the free-rider effect induced by the quality spillover. Thus, larger θ could make the learning firm worse off because the free riding firm may take most of the benefit. Third, the impact of the learning speed on firm B's profit depends on the pricing schemes. Under committed pricing, an increase in firm A's learning speed always benefits firm B, a consequence of the free-rider effect. Under dynamic pricing, however, larger δ may hurt firm B when θ is small. This finding indicates that the pricing flexibility could reduce the benefit enjoyed by the free rider, especially in the presence of a weak quality spillover.

Lastly, we compare the equilibrium results under different pricing schemes and study how the setting of the asymmetric learning speed alter our findings from Section 4.3. The following proposition is presented in contrast with Proposition 14.

Proposition 18. *For any $\alpha \geq 0$, $\delta\Lambda \leq 1$ and $\theta \in [0, 1/2]$, the following statements hold.*

(i) $p_{A1}^{d*} - p_{A1}^{c*} < 0$, $p_{A2}^{d*} - p_{A2}^{c*} > 0$, $p_{B1}^{d*} - p_{B1}^{c*} > 0$ *and* $p_{B2}^{d*} - p_{B2}^{c*} > 0$.

(ii) $QI_A^{d*} - QI_A^{c*} > 0$.

(iii) $\pi_A^{d*} - \pi_A^{c*} > 0$, *and* $\pi_B^{d*} - \pi_B^{c*}$ *is first negative and then positive in* δ.

As shown by parts (i) and (ii) of Proposition 18, when implementing dynamic pricing scheme, firm A tends to pursue greater quality improvement by lower first-period price. By contrast, firm B without quality learning chooses higher prices when both firms adjust prices sequentially, since firm B can only enlarge second-period sales through quality spillover and thus has an incentive to assist firm A in quality improvement.

The most contrasting result here is Proposition 18(iii). Unlike the symmetric case, where either of the two pricing schemes may outperform the other depending on the parameter θ, firm A with the asymmetric learning speed always achieve higher profit under dynamic pricing. Indeed, being the only firm that is able to improve quality by learning, firm A could achieve greater quality improvement by setting contingent prices across periods. As such, the pricing flexibility always facilitates the quality learning and they two can be seen as complements in increasing firm A's profit. In the symmetric setting, by contrast, the pricing flexibility and fast learning speed may together harm the learning firm.

On the other hand, the comparison between firm B's profit under the two pricing schemes depends on firm A's asymmetric learning speed δ. Based on Proposition 18, dynamic pricing can yield a higher profit for firm B when the learning speed of firm A is large enough. In fact, while using dynamic pricing helps firm B leverage on the contingent pricing flexibility, firm B also voluntarily gives in some profits in the first period to facilitate firm A's quality improvement. As a result, firm B's loss in the first period can only be offset if the gain in the second period is substantial, which can be guaranteed by a strong free-rider effect induced by the fast learning speed.

3.5.2 Asymmetric Quality Spillover Effect

In this subsection, we study the case where the duopoly is asymmetric in the quality spillover effect with $\theta_A = \theta > 0 = \theta_B$. That is, firm A enjoys the quality spillover whereas firm B is rather independent with respect to the impact of quality on demand. As did in Section 5.1, we investigate this case from three aspects, namely, the equilibrium structural results, the moderating effect of quality learning and quality spillover parameters, and the impact on the comparison between pricing schemes, respectively. We start with the following proposition.

Proposition 19. *Under pricing scheme $k \in \{c, d\}$, when $\delta\Lambda \leq 1$, there exists a unique pure Nash equilibrium $(p_{A1}^{k*}, p_{A2}^{k*}, p_{B1}^{k*}, p_{B2}^{k*})$, which is an interior point of the feasible region. Moreover, the following statements hold.*

(i) $p_{A1}^{k*} < p_{A2}^{k*}$ *and* $p_{B1}^{k*} < p_{B2}^{k*}$.

(ii) $p_{A1}^{k*} > p_{B1}^{k*}$ *and* $p_{A2}^{k*} < p_{B2}^{k*}$.

Similar to the previous case, Proposition 19 first confirms the existence and uniqueness of equilibrium under the assumption of asymmetric quality spillover. Moreover, Proposition 19(i) reveals that both firms cut prices in the first period, which is consistent with the intuition that they have the incentive to sacrifice the short-run profit in exchange for the benefit of quality improvement in the long run, even though only firm A enjoys the quality spillover. Furthermore, part (ii) of the above proposition shows that firm B, not being affected by the quality spillover, sets the prices more aggressively. To be specific, compared to firm A, firm B sets a lower price in the first period and a higher price in the second. This observation can be explained in the following way. On the one hand, since firm A can take advantage of the quality spillover effect, it takes firm B's quality improvement into consideration and thus does not set too low a first-period price. On the other hand, being independent from quality spillover, firm B must count on its own quality improvement, and therefore $p_{B1}^{k*} < p_{A1}^{k*}$ holds.

Next, under the assumption of asymmetric quality spillover, we conduct sensitivity analysis of equilibrium prices, quality improvement and profits with respect to the learning speed and quality spillover strength. The main findings are summarized in the following proposition.

Proposition 20. *Suppose $\alpha \geq 0$, $\delta\Lambda \leq 1$ and $\theta \in [0, 1/2]$. Under pricing scheme $k \in \{c, d\}$, the equilibrium outcomes have the following properties.*

(i) Prices: p_{A1}^{k*} increases in θ and decreases in δ, p_{A2}^{k*} first decreases and then increases in θ, and increases in δ; p_{B1}^{k*} increases in θ and decreases in δ, and p_{B2}^{k*} increases in θ and δ.

(ii) Quality improvement: QI_A^{k*} decreases in θ, and QI_A^{c*} increases in δ, while QI_A^{d*} increases in δ when θ is small and decreases otherwise; QI_B^{k*} increases in θ and δ.

(iii) Profits: π_A^{c*} increases in θ, while π_A^{d*} first decreases and then increases in θ, and π_A^{k*} increases in δ; π_B^{k*} increases in θ, and increases in δ.

Part (i) of Proposition 20 demonstrates that in the presence of asymmetric quality spillover, firms' equilibrium prices depend on the system parameters in a different way than before. Specifically, in the first period, both firms raise prices with a larger quality spillover or a lower learning speed. By contrast, depending on the quality spillover strength, θ may play a positive or negative role in firm A's second period pricing decision. Part (ii) confirms that the impacts of both quality spillover and learning speed on firm A's quality improvement largely remain robust, with an exception that firm A may be hurt by a faster learning speed under dynamic pricing when the asymmetric spillover effect is strong. By contrast, firm B always enjoys a larger quality improvement when firm A counts more on the quality spillover to take advantage of firm B's effort.

Furthermore, part (iii) of Proposition 20 elaborates the impact of quality learning and quality spillover on both firms' profits. It is noteworthy that stronger quality spillover always benefit the independent firm, firm B. Hence, when firm A relies more on firm B's quality improvement through the spillover effect, firm B has a greater advantage over firm A. As for firm A, the quality learning speed is always beneficial, whereas the impact of quality spillover depends on the pricing schemes. Under committed pricing, firm A earns more profit from stronger quality spillover effect. However, under dynamic pricing, the impact of quality spillover on firm A's profit would be positive only when the spillover is strong enough. Here, the pricing flexibility endowed by dynamic pricing intensifies the duopoly competition. With weak quality spillover, firm A would lose much market to firm B, which is not affected by quality spillover and therefore focuses solely on improving its own quality to become more competitive. As such, relying too much on the spillover effect may not always be beneficial in a competitive environment, especially when such spillover is not strong.

Finally, we compare the two pricing schemes in the asymmetric quality spillover setting. The following proposition provides insights into how the asymmetry in θ influences the differences of equilibrium outcomes under the two commonly observed pricing schemes.

Proposition 21. *Suppose that* $\alpha \geq 0$ *and* $\delta\Lambda \leq 1$. *As* θ *increases in the interval* $[0, 1/2]$*, the following statements hold:*

(i) $p_{A1}^{d*} - p_{A1}^{c*}$ *is first positive and then negative,* $p_{A2}^{d*} - p_{A2}^{c*}$ *is first negative and then positive,* $p_{B1}^{d*} - p_{B1}^{c*}$ *is first positive and then negative, and* $p_{B2}^{d*} - p_{B2}^{c*}$ *is first negative and then positive;*

(ii) $\pi_A^{d*} - \pi_A^{c*}$ *can be positive or negative, and* $\pi_B^{d*} - \pi_B^{c*}$ *is always positive.*

In part (i) of Proposition 21, both firms exhibit similar patterns when comparing different pricing schemes. Under dynamic pricing, both firms can set contingent prices and therefore focus more on the quality improvement; as a result, when the quality spillover is strong, they would offer lower first-period prices compared to those under committed pricing. Moreover, in the second period, firms under dynamic pricing are more capable to harvest the sales by contingently setting higher prices than those under committed pricing.

Proposition 20(ii) states that firm B always obtains higher profit under dynamic pricing. This result is comparable to that in Proposition 18, where firm A, the only learning firm, always gets higher profit under dynamic pricing. Like the previous case, since firm B is independent of the quality spillover, it has to rely on the pricing flexibility brought by the dynamic pricing as the competitive advantage. On the other hand, firm A must consider the quality spillover and may even need to help firm B learn in order to enjoy the benefit of the spillover. Hence, firm B is more competitive under dynamic pricing and can gain a higher profit compared to that obtained under committed pricing.

3.6 Summary

Focusing on a dynamic duopoly pricing game with quality considerations, this paper incorporates two new features into the setting. First, the quality can be improved over times via a learning process that is based on the previous sales volume (experience-based learn-by-doing). Second, there exists a quality spillover effect in the sense that both firms' quality levels are positively correlated with the total market size so that one firm may benefit from the rival's quality improvement. Indeed, both quality learning and quality spillover are salient features for situations where, for example, competing startup firms sell similar innovative products/services to customers who are not familiar them. Together, the two features considered in our model introduce new and interesting dynamics to the duopoly pricing game.

We concentrate on a two-period dynamic model and assume a linear learning curve of both firms; i.e., the firm's quality increases linearly with its previous sales volume. Besides, we respectively consider two pricing schemes, namely, committed pricing and dynamic pricing. Assuming symmetric firms, we first establish the existence and uniqueness of Nash equilibrium, and then derive useful insights into the effect of quality learning and quality spillover on the equilibrium outcomes such as prices, quality improvement, and profits. We find that, for either firm, the first-period price is always lower than the second-period price. Such a discount price in the first period is offered because the firm wants to attract more demand to improve its quality level on the one hand, and to slow down its opponent's learning on the other. Moreover, our results indicate that stronger quality spillover relaxes the duopoly competition and reduces the need for competing for the quality improvement opportunity. In addition, the existence of the experience-based learning and quality improvement is shown to be the main driving force for firms to sacrifice some short-term profit in exchange for the long-run benefit.

Another major finding in our paper concerns the comparison between the pricing schemes with respect to the firms' equilibrium outcomes. We find that the comparison result hinges on the strength of the quality spillover effect. When the spillover effect is strong relative to the competition intensity, committed pricing can help soften the price competition, which is more beneficial to firms than dynamic pricing scheme. On the contrary, if the spillover effect is relatively weak, the the pricing flexibility endowed by the dynamic pricing scheme can grant higher second-period profit without losing too much first-period profit, thereby leading to higher total profit for the firms.

Finally, we relax the assumption regarding firms' symmetry in every aspect and investigate two asymmetric cases. First, we assume that one firm has the learning ability whereas the other does not. We find that the firm with the learning ability would price lower and gain more market share; but this surprisingly may also benefit the firm that does not learn, especially when the quality spillover effect is strong. Such an advantage of learning is amplified under the dynamic pricing scheme compared to the committed pricing. Second, we assume that one firm can enjoy the quality spillover from the opponent while the other firm cannot. In this case, the firm without the spillover advantage tends to price lower, because, rather than leveraging on the spillover from the other firm, it must rely on its own quality improvement. As such, the lower price is for accumulating more demand as the source of learning. Finally, the pricing flexibility endowed by the dynamic pricing can further help the firm disadvantageous in terms of quality spillover gain more second-period, as well as total, profit.

We conclude by pointing out the caveats of our model and suggesting some directions for further research. First, the linear functional form of the learning curve assumed in our paper certainly facilitates producing tractable model and yielding useful insights. However, a general log-linear function will accommodate more realistic settings, although incorporating it in our model requires much more computational efforts. Secondly, this paper considers a complete information game. If, however, the firms cannot observe the opponents' learning speed (δ), which is possible in reality, then the analysis and result will be changed. Thirdly, because the quality improvement is based on the generated demand, extending the deterministic demand model to account for stochastic environment would be interesting and useful. All the aforementioned directions are left for future research.

Chapter 4

Conclusions

Our research provides comprehensive insights into how firms should strategically manage their pricing and quality decisions in competitive markets characterized by imitation threats and quality spillovers. The analysis spans two critical market scenarios: one where copycat firms pose direct imitation threats, and another where quality improvements generate spillover effects between competing firms.

In the first study, we provide comprehensive insights into how innovators should strategically manage their pricing decisions when facing copycat threats. With exogenous product quality, we find that the optimal choice between committed and flexible pricing critically depends on the product's quality level. Committed pricing proves superior for high-quality products by enabling effective entry deterrence through price commitments, while flexible pricing becomes optimal for moderate-quality products by providing the necessary adaptability to blockade copycat entry. When extending to endogenous quality decisions, we discover that the cost of quality investment significantly influences this choice - firms should adopt committed pricing under low investment costs but switch to flexible pricing under moderate costs. Notably, firms often employ a "limit quality" strategy, deliberately restricting quality improvements to manage copycat threats.

These findings remain robust across various extensions, including scenarios with endogenous imitation efficiency, uniform pricing constraints, and strategic consumers. When copycats can freely determine their imitation efficiency, flexible pricing weakly dominates committed pricing by inducing less efficient imitation. Under uniform pricing constraints, the effectiveness of price commitment diminishes, making flexible pricing optimal for endogenous quality decisions. Even with strategic consumers who may postpone purchases, our core results hold: committed pricing remains optimal for high-quality products, while flexible pricing proves optimal for moderate-quality products. These insights challenge the conventional wisdom that price commitment always provides stronger deterrence against market entry, highlighting how the interplay between quality levels, investment costs, and pricing flexibility shapes competitive outcomes.

In the second study, we reveal how quality spillovers and learning effects shape firms' pricing strategies in duopolistic competition. In symmetric markets, we establish the existence and uniqueness of Nash equilibrium where firms consistently adopt a low-price-then-high strategy across periods, regardless of their pricing scheme. This initial price reduction serves dual purposes: attracting demand for self-improvement while limiting competitors' learning opportunities. Interestingly, stronger quality spillovers lead firms to raise first-period prices, resulting in reduced individual quality improvements and non-monotonic profit impacts. This occurs because enhanced spillovers moderate competition intensity and decrease firms' dependence on self-improvement. Compared to scenarios without quality learning, firms willingly sacrifice short-term profits through lower initial prices to accelerate their learning trajectory and secure long-term benefits.

The effectiveness of pricing strategies varies significantly with market conditions and firm characteristics. In symmetric markets with strong spillover effects relative to competition intensity, committed pricing outperforms dynamic pricing by alleviating price competition. Conversely, weak spillovers favor dynamic pricing, allowing firms to leverage pricing flexibility for higher second-period profits without compromising first-period gains. In asymmetric markets, the impacts become more nuanced. When firms differ in learning speeds, dynamic pricing amplifies market asymmetries, particularly benefiting firms with learning advantages. However, when firms face asymmetric spillover effects, those at a spillover disadvantage price more aggressively and benefit more from dynamic pricing's flexibility, as it enables them to better capitalize on individual quality improvements. These findings highlight how market asymmetries and spillover effects critically influence the optimal choice between committed and dynamic pricing strategies.

Collectively, our research demonstrates that successful competition in modern markets requires careful coordination of pricing strategies and quality improvements. The choice between price commitment and flexibility must balance multiple factors: the threat of imitation or the strength of quality spillovers, and the costs of quality improvements. These insights contribute to both theoretical understanding and practical management of competitive strategy in innovation-driven markets.

Appendix A

Proofs for Chapter 2

This appendix consists of two parts: Appendix A.1 gives the proofs of the statements in the main paper. In appendix A.2, we provide detailed supporting results for Appendix A.1.

A.1 Proofs

Proof of Lemma 1 In the second period, both firms choose p_{i2}^F and p_c^F simultaneously to maximize their profits:

$$\begin{cases} \max\limits_{p_{i2}^F} \quad \Pi_{i2}^F = p_{i2}^F(1-\theta_{ic}), \\ \max\limits_{p_c^F} \quad \Pi_c^F = p_c^F\left(\theta_{ic} - \dfrac{p_c^F}{\delta q}\right), \end{cases}$$

where $\theta_{ic} := \frac{p_{i2}-p_c}{(1-\delta)q}$ is the the willingness-to-pay for quality at which consumers are indifferent between purchasing from the innovator and purchasing from the copycat. Using first-order conditions, we derive the equilibrium prices

$$p_{i2}^F = \frac{2(1-\delta)}{4-\delta}q \quad \text{and} \quad p_c^F = \frac{\delta(1-\delta)}{4-\delta}q.$$

There exists a threshold $q^F := \frac{(4-\delta)^2}{\delta(1-\delta)}K$ such that if $q \leq q^F$, the copycat cannot derive a positive profit and does not enter the market. Hence, the copycat enters the market if $q > q^F$ and stays out of the market otherwise. The first-period pricing decision would not affect the later decisions and is thus always $p_{i1}^F = \frac{q}{2}$ on the basis of first-order conditions.

In equilibrium, the innovator enjoys

$$\Pi_i^{F*} = \begin{cases} \Pi_i^{FA} = \frac{q}{4} + \frac{4(1-\delta)}{(4-\delta)^2}q, & \text{if } q > q^F; \\ \Pi_i^{FB} = \frac{q}{2}, & \text{if } q \leq q^F. \end{cases}$$

□

Proof of Lemma 2 We solve the game backwards. In the second period, the copycat decides p_c^C after observing p_{i2}^C, i.e.,

$$\max_{p_c^C} \Pi_c^C = p_c^C \left(\theta_{ic} - \frac{p_c^C}{\delta q} \right).$$

Using first-order conditions, the optimal decision and profit are

$$p_c^{C*} = \frac{\delta}{2} p_{i2}^C \quad \text{and} \quad \Pi_c^{C*} = \frac{\delta}{4q(1-\delta)}(p_{i2}^C)^2.$$

Under committed pricing, the innovator can commit a price p_{i2}^{CD} to deter entry, where

$$p_{i2}^{CD} := \frac{2\sqrt{(1-\delta)qK}}{\sqrt{\delta}}.$$

Since $p_{i1}^{C*} = \frac{q}{2}$ and the innovator's objective is thus

$$\Pi_i^C = \begin{cases} \Pi_i^{CA} = \frac{q}{4} + p_{i2}^C(1-\theta_{ic}), & \text{if } p_{i2}^C > p_{i2}^{CD}; \\ \Pi_i^{CB} = \frac{q}{4} + p_{i2}^C(1-\frac{p_{i2}^C}{q}), & \text{if } p_{i2}^C \leq p_{i2}^{CD}. \end{cases}$$

Let

$$p_{i2}^{CB} \left(:= \frac{q}{2} \right) \quad > \quad p_{i2}^{CA} \left(:= \frac{1-\delta}{2-\delta} q \right)$$

denote two solutions derived by first-order conditions, respectively. We next study the optimal decision globally.

Case $p_{i2}^{CB} \leq p_{i2}^{CD}$: Since $p_{i2}^{CA} < p_{i2}^{CB}$, we have $p_{i2}^{CA} < p_{i2}^{CD}$ too. Π_i^C increases in $p_{i2}^C \in [0, p_{i2}^{CB}]$, decreases in $p_{i2}^C \in (p_{i2}^{CB}, p_{i2}^{CD}]$ and decreases in $p_{i2}^C \in (p_{i2}^{CD}, q]$. Thus, the optimal decision is $p_{i2}^{C*} = p_{i2}^{CB}$ and the entry is blockaded.

Case $p_{i2}^{CA} \leq p_{i2}^{CD} < p_{i2}^{CB}$: Π_i^C increases in $p_{i2}^C \in [0, p_{i2}^{CD}]$ and decreases in $p_{i2}^C \in (p_{i2}^{CD}, q]$. Thus, the innovator should set $p_{i2}^{C*} = p_{i2}^{CD}$ to deter entry.

Case $p_{i2}^{CA} > p_{i2}^{CD}$: Π_i^C increases in $p_{i2}^C \in [0, p_{i2}^{CD}]$, increases in $p_{i2}^C \in (p_{i2}^{CD}, p_{i2}^{CA}]$ and decreases in $p_{i2}^C \in (p_{i2}^{CA}, q]$. Thus, the optimal decision is $p_{i2}^{C*} = p_{i2}^{CA}$ if $\Pi_i^{CA*} > \Pi_i^{CD*}$ and $p_{i2}^{C*} = p_{i2}^{CD}$ otherwise.

Let

$$q_1^C := \frac{16(1-\delta)}{\delta}K \quad \text{and} \quad q_2^C := \frac{8\left(2-\delta+(2-\delta)\sqrt{\delta(2-\delta)}\right)}{\delta(1-\delta)}K,$$

we can rewrite the equilibrium as follows.

(i) If $q \leq q_1^C$, the innovator sets $p_{i1}^{C*} = p_{i2}^{C*} = \frac{q}{2}$, and the copycat would not enter the market.

(ii) If $q_1^C < q \leq q_2^C$, the innovator sets $p_{i1}^{C*} = \frac{q}{2}$ and $p_{i2}^{C*} = \frac{2\sqrt{(1-\delta)qK}}{\sqrt{\delta}}$, and the copycat would not enter the market.

(iii) If $q > q_2^C$, the innovator sets $p_{i1}^{C*} = \frac{q}{2}$ and $p_{i2}^{C*} = \frac{q(1-\delta)}{2-\delta}$, and the copycat enters the market and sets $p_c^{C*} = \frac{\delta}{2}p_{i2}^{C*}$.

□

Proof of Proposition 1 On the basis of Lemma A.2.1, we have following cases.

Case $q < q_1^C$: The entry is blockaded under both pricing strategies, and thus the innovator enjoys the same profit.

Case $q_1^C < q < q^F$: Under committed pricing, the innovator has to set a lower-than-first-best price to deter entry. Under flexible pricing, the innovator achieves the first-best outcome. Obviously, flexible pricing is the dominant one.

Case $q^F < q < q_2^C$: Under committed pricing, the innovator sets a lower-than-first-best price to deter entry. Under flexible pricing, the innovator accommodates imitation. We have

$$\Pi_i^{CD*} - \Pi_i^{FA*} = \frac{2\sqrt{Kq(1-\delta)}}{\sqrt{\delta}} - \frac{4K(1-\delta)}{\delta} - \frac{4q(1-\delta)}{(4-\delta)^2}.$$

Note that the gap is concave in q. To show that the gap is non-negative, it is enough to show that the gap is non-negative at $q = q^F$ and $q = q_2^C$:

$$\Pi_i^{CD*} - \Pi_i^{FA*}\big|_{q=q^F} = 2K > 0,$$

$$\Pi_i^{CD*} - \Pi_i^{FA*}\big|_{q=q_2^C} = \frac{4\delta\left(1+\sqrt{\delta(2-\delta)}\right)}{(4-\delta)^2} > 0.$$

Case $q > q_2^C$: The innovator accommodates imitation under both pricing strategies. We have

$$\Pi_i^{CA*} - \Pi_i^{FA*} = \frac{\delta^2 q(1-\delta)}{2(2-\delta)(4-\delta)^2} > 0.$$

□

Proof of Proposition 2 We aim to prove:

(i) p_{i2}^{F*} decreases in δ, while is non-monotonic in q;

(ii) p_{i2}^{C*} is non-monotonic in δ, while increases in q;

(iii) p_{i2}^{*} is non-monotonic in δ and q.

Using Lemma A.2.1, we can easily derive the sequences of cases switch. We thus focus on the sensitivity analysis for each cases.

Case FB and CB:

$$\frac{\partial p_{i2}^{FB*}}{\partial q} = \frac{\partial p_{i2}^{CB*}}{\partial q} = \frac{1}{2}, \quad \frac{\partial p_{i2}^{FB*}}{\partial \delta} = \frac{\partial p_{i2}^{CB*}}{\partial \delta} = 0.$$

Case FA:

$$\frac{\partial p_{i2}^{FA*}}{\partial q} = \frac{\delta(1-\delta)}{4-\delta} > 0, \quad \frac{\partial p_{i2}^{FA*}}{\partial \delta} = -\frac{6q}{(4-\delta)^2} < 0.$$

Moreover,

$$p_{i2}^{FB*} - p_{i2}^{FA*}\big|_{q=q^F} = \frac{3(\delta-4)K}{2(\delta-1)} > 0,$$

which explains why p_{i2}^{F*} is non-monotonic in q even though p_{i2}^{F*} increases in $q \in (0, q^F]$ and $q \in (q^F, \infty)$, respectively.

Case CA:

$$\frac{\partial p_{i2}^{CA*}}{\partial q} = \frac{\delta-1}{\delta-2} > 0, \quad \frac{\partial p_{i2}^{CA*}}{\partial \delta} = -\frac{q}{(\delta-2)^2} < 0.$$

Case CD:

$$\frac{\partial p_{i2}^{CD*}}{\partial q} = \frac{\sqrt{qK(1-\delta)}}{\sqrt{\delta}q} > 0, \quad \frac{\partial p_{i2}^{CD*}}{\partial \delta} = -\frac{Kq}{\delta^{3/2}\sqrt{qK(1-\delta)}} < 0.$$

Moreover,

$$p_{i2}^{CA*} - p_{i2}^{CD*}\big|_{q=q_2^C} = 4\left(\frac{\sqrt{2-\delta}}{\sqrt{\delta}} + 1\right)K > 0, \quad p_{i2}^{CB*} - p_{i2}^{CD*}\big|_{q=q_1^C} = 0,$$

which explains why p_{i2}^{C*} is non-monotonic in δ and increases in q.

Then we focus on the comparison under different pairs of cases.

Case $q \leq q_1^C$: In this case, $p_{i2}^{C*} = p_{i2}^{F*}$.

Case $q_1^C < q < q^F$: In this case, $p_{i2}^{C*} = p_{i2}^{CD*} \leq p_{i2}^{CB*} = p_{i2}^{FB*} = p_{i2}^{F*}$.

Case $q^F \leq q < q_2^C$: In this case,

$$\begin{aligned} p_{i2}^{F*} - p_{i2}^{C*} &= p_{i2}^{FA*} - p_{i2}^{CD*} \\ &= \frac{2(\delta-1)q}{\delta-4} - \frac{2\sqrt{qK(1-\delta)}}{\sqrt{\delta}}. \end{aligned}$$

Note that

$$\frac{\partial^2\left(p_{i2}^{F*} - p_{i2}^{C*}\right)}{\partial q^2} = \frac{(\delta-1)^2K^2}{2\sqrt{\delta}(qK(1-\delta))^{3/2}} > 0,$$

it is enough to show that

$$\left.\frac{\partial\left(p_{i2}^{F*}-p_{i2}^{C*}\right)}{\partial q}\right|_{q=q^F}=\frac{\delta-1}{\delta-4}>0.$$

Thus, $p_{i2}^{F*}-p_{i2}^{C*}$ increases in q and

$$p_{i2}^{F*}-p_{i2}^{C*}\geq\left.\left(p_{i2}^{F*}-p_{i2}^{C*}\right)\right|_{q=q^F}=0.$$

Case $q\geq q_2^C$: In this case,

$$p_{i2}^{C*}-p_{i2}^{F*}=p_{i2}^{CA*}-p_{i2}^{FA*}=\frac{(1-\delta)\delta q}{(\delta-4)(\delta-2)}>0.$$

□

Proof of Corollary 1 We aim to show:

(i) Π_i^{C*} increases in q, while Π_i^{F*} and Π_i^* is non-monotonic in q;

(ii) Π_c^{C*}, Π_c^{F*} and Π_c^* increases in q;

(iii) Π_i^{C*}, Π_i^{F*} and Π_i^* decreases in δ;

(iv) Π_c^{C*}, Π_c^{F*} and Π_c^* increases in δ.

Similarly, we display the sensitivity analysis for each cases.

Case FB and CB:

$$\frac{\partial\Pi_i^{FB*}}{\partial q}=\frac{\partial\Pi_i^{CB*}}{\partial q}=\frac{1}{2},\quad\frac{\partial\Pi_i^{FB*}}{\partial\delta}=\frac{\partial\Pi_i^{CB*}}{\partial\delta}=0.$$

Case FA:

$$\frac{\partial\Pi_i^{FA*}}{\partial q}=\frac{\delta^2-24\delta+32}{4(\delta-4)^2}>0,\quad\frac{\partial\Pi_i^{FA*}}{\partial\delta}=-\frac{4(\delta+2)q}{(4-\delta)^3}<0,$$
$$\frac{\partial\Pi_c^{F*}}{\partial q}=\frac{(1-\delta)\delta}{(\delta-4)^2}>0,\quad\frac{\partial\Pi_c^{F*}}{\partial\delta}=\frac{(4-7\delta)q}{(4-\delta)^3}>0.$$

Table A.1: Sensitivity Analysis Results of CS

Derivative	CS^{FB*} (CS^{CB*})	CS^{FA*}	CS^{CA*}	CS^{CD*}
q	$\frac{1}{4}$	$\frac{\delta^2+12\delta+32}{8(\delta-4)^2}(>0)$	$\frac{8-3\delta}{8(\delta-2)^2}(>0)$	$\frac{5}{8}-\frac{\sqrt{qK(1-\delta)}}{\sqrt{\delta}q}$ $(>\frac{3}{8})$
δ	0	$\frac{(5\delta+28)q}{2(4-\delta)^3}(>0)$	$\frac{(3\delta-10)q}{8(\delta-2)^3}(>0)$	$\frac{K\left(\frac{\sqrt{\delta}q}{\sqrt{qK(1-\delta)}}-2\right)}{\delta^2}$ $(>\frac{2K}{\delta^2})$

Table A.2: Sensitivity Analysis Results of SW

Derivative	SW^{FB*} (SW^{CB*})	SW^{FA*}	SW^{CA*}	SW^{CD*}
q	$\frac{3}{4}$	$\frac{-5\delta^2-28\delta+96}{8(\delta-4)^2}(>0)$	$\frac{4\delta^2-21\delta+24}{8(\delta-2)^2}(>0)$	$\frac{7}{8}$
δ	0	$\frac{(17\delta-20)q}{2(\delta-4)^3}(>0)$	$\frac{(5\delta-6)q}{8(\delta-2)^3}(>0)$	$\frac{2K}{\delta^2}$

Case CA:

$$\frac{\partial\Pi_i^{CA*}}{\partial q}=\frac{4-3\delta}{8-4\delta}>0,\quad \frac{\partial\Pi_i^{CA*}}{\partial\delta}=-\frac{q}{2(\delta-2)^2}<0,$$
$$\frac{\partial\Pi_c^{C*}}{\partial q}=\frac{(1-\delta)\delta}{4(\delta-2)^2}>0,\quad \frac{\partial\Pi_c^{C*}}{\partial\delta}=\frac{(3\delta-2)q}{4(\delta-2)^3}>0.$$

Case CD:

$$\frac{\partial\Pi_i^{CD*}}{\partial q}=\frac{\sqrt{(\delta-1)(-K)q}}{\sqrt{\delta}q}+\frac{1}{4}>0,\quad \frac{\partial\Pi_i^{CD*}}{\partial\delta}=-\frac{2K\left(\delta+\sqrt{(2-\delta)\delta}\right)}{\delta^2(1-\delta)}<0.$$

Moreover, Π_i is continuous, this completes the proof. □

Proof of Lemma 3 We aim to show that:

- CS^{F*} increases in q and δ, while CS^{C*} and CS^* are non-monotonic in q and δ;
- SW^{F*} increases in q and δ, while SW^{C*} and SW^* are non-monotonic in q and δ.

The sensitivity analysis results are summarized in Table A.1 and Table A.2.

□

Proof of Proposition 3 We focus on the comparison under different pairs of cases.

Case $q \leq q_1^C$: In this case, $CS^{C*} = CS^{F*}$ and $SW^{C*} = SW^{F*}$.

Case $q_1^C < q < q^F$: In this case,

$$CS^{C*} - CS^{F*} = CS^{CD*} - CS^{FB*} = \left(\frac{2}{\delta} - 2\right) K - \frac{2\sqrt{qK(1-\delta)}}{\sqrt{\delta}} + \frac{3q}{8},$$

$$\frac{\partial^2 \left(CS^{C*} - CS^{F*}\right)}{\partial q^2} = \frac{(\delta-1)^2 K^2}{2\sqrt{\delta}(qK(1-\delta))^{3/2}} > 0 \quad \text{and} \quad \left.\frac{\partial \left(CS^{C*} - CS^{F*}\right)}{\partial q}\right|_{q=q^F} = \frac{1}{8} > 0.$$

Therefore,

$$CS^{C*} - CS^{F*} \geq \left(CS^{C*} - CS^{F*}\right)\Big|_{q=q^F} = 0.$$

And

$$SW^{C*} - SW^{F*} = SW^{CD*} - SW^{FB*} = \frac{2(\delta-1)K}{\delta} + \frac{q}{8} \geq \left(\frac{2(\delta-1)K}{\delta} + \frac{q}{8}\right)\Bigg|_{q=q_1^C} = 0.$$

Case $q^F \leq q < q_2^C$: In this case,

$$CS^{C*} - CS^{F*} = CS^{CD*} - CS^{FA*} = \left(\frac{2}{\delta} - 2\right) K - \frac{2\sqrt{(\delta-1)(-K)q}}{\sqrt{\delta}} - \frac{\left(\delta^2 + 12\delta + 32\right) q}{8(\delta-4)^2} + \frac{5q}{8},$$

which is convex in q. And

$$\left.\frac{\partial \left(CS^{CD*} - CS^{FA*}\right)}{\partial q}\right|_{q=q^F} = \frac{-\delta^2 - 3\delta + 4}{2(\delta-4)^2} > 0,$$

which means that $CS^{CD*} - CS^{FA*}$ increases in $q \in (q^F, q_2^C)$. Moreover,

$$\left(CS^{CD*} - CS^{FA*}\right)\Big|_{q=q^F} < 0, \left(CS^{CD*} - CS^{FA*}\right)\Big|_{q=q_2^C} > 0.$$

Let

$$q^{CF} := \left(CS^{CD*} - CS^{FA*}\right)\Big|_{q=q^{CF}} = 0,$$

$CS^{CD*} < CS^{FA*}$ iff $q^F < q < q^{CF}$. Moreover,

$$SW^{C*} - SW^{F*} = SW^{CD*} - SW^{FA*} = \frac{2(\delta-1)K}{\delta} + \frac{\left(5\delta^2 + 28\delta - 96\right) q}{8(\delta-4)^2} + \frac{7q}{8}$$

increases in q and

$$SW^{CD*} - SW^{FA*} \geq \left(SW^{CD*} - SW^{FA*}\right)\big|_{q=q^F} = \frac{K}{2} > 0.$$

Case $q > q_2^C$: In this case,

$$CS^{F*} - CS^{C*} = CS^{FA*} - CS^{CA*} = \frac{\delta\left(\delta^3 + 11\delta^2 - 44\delta + 32\right)q}{8(\delta-4)^2(\delta-2)^2} > 0,$$

$$SW^{F*} - SW^{C*} = SW^{FA*} - SW^{CA*} = \frac{\delta\left(-9\delta^3 + 45\delta^2 - 68\delta + 32\right)q}{8(\delta-4)^2(\delta-2)^2} > 0.$$

□

Proof of Lemma 4 When the quality is endogenized, the innovator chooses q to maximize

$$\Pi_i^F = \begin{cases} \Pi_i^{FA} = \frac{q}{4} + \frac{4q(1-\delta)}{(4-\delta)^2} - \frac{1}{2}\gamma q^2, & \text{if } q > q^F; \\ \Pi_i^{FB} = \frac{q}{2} - \frac{1}{2}\gamma q^2, & \text{if } q \leq q^F. \end{cases}$$

It is easy to see that both Π_i^{FA} and Π_i^{FB} are concave in q. Using first-order conditions, we derive

$$q^{FA} = \frac{32 - 24\delta + \delta^2}{4\gamma(4-\delta)^2} \quad \text{and} \quad q^{FB} = \frac{1}{2\gamma}.$$

Let $q^{FD} := q^F$, we need to discuss the global optimum. Specifically, there are three cases.

- If $q^{FB} \leq q^{FD}$, then $q^{FA} < q^{FD}$ too. Π_i^F increases in $q \in [0, q^{FB}]$ and decreases in $q \in (q^{FB}, q^{FD}]$ and decreases in $q \in (q^{FD}, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{FB}$.
- If $q^{FA} \leq q^{FD} < q^{FB}$, i.e., , Π_i^F increases in $q \in [0, q^{FD}]$ and decreases in $q \in (q^{FD}, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{FD}$.
- If $q^{FD} < q^{FB}$ and $q^{FA} > q^{FD}$, Π_i^F increases in $q \in [0, q^{FD}]$, increases in $q \in (q^{FD}, q^{FA}]$ and decreases in $q \in (q^{FA}, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{FD}$ if $\Pi_i^{FA*} \leq \Pi_i^{FB*}$ and $q^* = q^{FA}$ otherwise.

We next rewrite these conditions for different equilibrium cases.

Case $q^{FB} \leq q^{FD}$: The conditions can be rewritten as

$$\gamma \geq \gamma_2^F \left(:= \frac{\delta(1-\delta)}{2K(4-\delta)^2} \right).$$

Case $q^{FA} \leq q^{FD} < q^{FB}$: The conditions can be rewritten as

$$\frac{\delta(1-\delta)(\delta^2 - 24\delta + 32)}{4K(4-\delta)^4} \leq \gamma < \frac{\delta(1-\delta)}{2K(4-\delta)^2}.$$

Case $q^{FD} < q^{FB}$ and $q^{FA} > q^{FD}$ and $\Pi_i^{FA*} \leq \Pi_i^{FB*}$: Note that $q^{FD} < q^{FB}$ is redundant, and $q^{FA} > q^{FD}$ implies $\frac{\delta(1-\delta)(\delta^2-24\delta+32)}{4K(4-\delta)^4} > \gamma$. For the third condition, we can see that

$$\Pi_i^{FA*} - \Pi_i^{FB*} = \frac{1}{32\gamma\delta^2(1-\delta)^2(4-\delta)^4} \left(16K^2(4-\delta)^8\gamma^2 - 16K\delta(1-\delta)(4-\delta)^6\gamma + \delta^2(\delta^3 - 25\delta^2 + 56\delta - 32)^2\right)$$

and the molecular is exactly a quadratic function of γ. Let $\gamma_1 < \gamma_2$ denote two roots to the quadratic equation, and it is easy to see that only γ_1 satisfy the condition of $q^{FA} > q^{FD}$. Thus, the conditions can be rewritten as

$$\gamma_1^F \left(:= \frac{\delta(1-\delta)}{2K(4-\delta)^2} - \frac{\delta(1-\delta)\sqrt{\delta(8+\delta)(3\delta^2 - 40\delta + 64)}}{4K(4-\delta)^4} \right) \leq \gamma < \gamma_2^F.$$

Case $q^{FD} < q^{FB}$ and $q^{FA} > q^{FD}$ and $\Pi_i^{FA*} > \Pi_i^{FB*}$: The conditions can be rewritten as $\gamma < \gamma_1^F$. □

<u>Proof of Lemma 5</u> When the quality is endogenized, the innovator chooses q to maximize

$$\Pi_i^C = \begin{cases} \Pi_i^{CA} = \frac{q}{4} + \frac{q(1-\delta)}{2(2-\delta)} - \frac{1}{2}\gamma q^2, & \text{if } q > q_2^C; \\ \Pi_i^{CD} = \frac{q}{4} - \frac{1}{2}\gamma q^2 - \frac{4K(1-\delta)}{\delta} + \frac{2\sqrt{qK(1-\delta)}}{\sqrt{\delta}}, & \text{if } q_1^C < q \leq q_2^C; \\ \Pi_i^{CB} = \frac{q}{2} - \frac{1}{2}\gamma q^2, & \text{if } q \leq q_1^C. \end{cases}$$

Note that Π_i^C is continuous on q. It is easy to see that all objective functions are concave in q. Using first-order conditions, we derive

$$q^{CA} = \frac{4-3\delta}{4\gamma(2-\delta)} \quad \text{and} \quad q^{CB} = \frac{1}{2\gamma}.$$

For ease of discussion, let F_1 denotes

$$\frac{1}{4} - \gamma q + \frac{\sqrt{qK(1-\delta)}}{\sqrt{\delta q}},$$

and q^{CD} is defined as the zero to F_1.

Now we need to discuss the global optimum, we have following cases.

- If $q^{CB} \leq q_1^C$, then $q^{CA} < q_1^C$ too. Moreover, based on Lemma A.2.2 (ii), we have $q^{CD} \leq q_1^C$. Hence, Π_i^C increases in $q \in [0, q^{CB}]$ and decreases in $q \in (q^{CB}, q_1^C]$, and decreases in $q \in (q_1^C, q_2^C]$ and $q \in (q_2^C, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{CB}$. Moreover, the condition can be rewritten as $\gamma \geq \gamma_2^C$, where

$$\gamma_2^C := \frac{\delta}{32K(1-\delta)}.$$

- If $q^{CB} > q_1^C$ (i.e., $\gamma < \gamma_2^C$), then $q^{CD} > q_1^C$ via Lemma A.2.2 (ii). And we have following cases.
  - If $q^{CA} \leq q_2^C$, then $q^{CD} \leq q_2^C$ via Lemma A.2.2 (iii). Π_i^C increases in $q \in [0, q^{CD}]$, and decreases in $q \in (q^{CD}, q_2^C]$, and decreases in $q \in (q_2^C, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{CD}$.
  - If $q^{CA} > q_2^C$, we have following cases.
    * If $q^{CD} \leq q_2^C$, Π_i^C increases in $q \in [0, q^{CD}]$, and decreases in $q \in (q^{CD}, q_2^C]$, and increases in $q \in (q_2^C, q^{CA}]$, and decreases in $q \in (q^{CA}, \frac{1}{\gamma}]$. Thus, the optimal decision is either $q^* = q^{CD}$ or $q^* = q^{CA}$. Lemma A.2.2 (iv) shows that there exists a threshold γ_1^C such that $\Pi_i^{CA*}\big|_{\gamma=\gamma_1^C} = \Pi_i^{CD*}\big|_{\gamma=\gamma_1^C}$. Therefore, if $\gamma > \gamma_1^C$, the optimal decision is $q^* = q^{CD}$; otherwise, the optimal decision is $q^* = q^{CA}$.
    * If $q^{CD} > q_2^C$, Π_i^C increases in $q \in [0, q^{CA}]$, and decreases in $q \in (q^{CA}, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{CA}$.

Rewrite all conditions, we completes the proof. □

Table A.3: Sensitivity Analysis Results of q^*

Derivative	q^B	q^{FD}	q^{FA}	q^{CD}	q^{CA}
γ	$-\frac{1}{2\gamma^2}$	0	$-\frac{\delta^2-24\delta+32}{4\gamma^2(\delta-4)^2}$	$-\frac{2q^2\sqrt{(\delta-1)\delta(-K)q}}{-\delta K+2\gamma q\sqrt{(\delta-1)\delta(-K)q}+K}$	$-\frac{3\delta-4}{4\gamma^2(\delta-2)}$
δ	0	$-\frac{(\delta-4)(7\delta-4)K}{(\delta-1)^2\delta^2}$	$\frac{4(\delta+2)}{\gamma(\delta-4)^3}$	$\frac{Kq}{\delta\left((\delta-1)K-2\gamma q\sqrt{(\delta-1)\delta(-K)q}\right)}$	$-\frac{1}{2\gamma(\delta-2)^2}$

Proof of Proposition 4 On the basis of Lemma A.2.3, we have following cases.

Case $\gamma > \gamma_2^C$: The quality cost coefficient is so high that the innovator chooses a low quality under both pricing strategies, and the entry is blockaded.

Case $\gamma_2^F < \gamma < \gamma_2^C$: Under committed pricing, the innovator sets a lower quality to deter entry. Under flexible pricing, the innovator monopolizes the market. Obviously, flexible pricing dominates committed pricing.

Case $max(\gamma_1^C, \gamma_1^F) < \gamma < \gamma_2^F$: Lemma A.2.3 (iv) shows that there exists a unique threshold γ_1^{CF} above which $\Pi_i^{FD*} > \Pi_i^{CD*}$.

Case $\gamma < \max(\gamma_1^C, \gamma_1^F)$: Lemma A.2.3 (v) shows that $\Pi_i^{F*} < \Pi_i^{C*}$. □

Proof of Proposition 5 We aim to show that q^{F*} (q^{C*}) decreases in γ, while is non-monotonic in δ. On the basis of Lemma A.2.3, we can derive the equilibrium sequences: (1) Under both pricing strategies, the equilibrium follows a Accommodation → Deterrence → Blockade sequence as γ increases; (2) Under both pricing strategies, the equilibrium follows a Blockade → Deterrence, Blockade → Deterrence → Accommodation → Deterrence or Blockade → Deterrence → Accommodation sequence as δ increases. The sensitivity analysis results for each cases are summarized in Table A.3.

When γ increases, the equilibrium switches from Accommodation → Deterrence → Blockade sequence, note that q^* is continuous under the second switch. We only need to check the first switch, the proof of Lemma 4 has shown that $q^{FA} > q^{FD}$ at $\gamma = \gamma_1^F$. Under committed pricing, $q^{CA} > q_2^C > q^{CD}$ at $\gamma = \gamma_1^C$. Therefore, under both pricing strategies, q^* decreases in γ.

When δ increases, the equilibrium may switch from Deterrence $\rightarrow$ Accommodation. And we have shown that the optimal quality would be higher when the equilibria switches to the duopoly case, i.e., a upward jump.

Lemma A.2.3 (iii) shows that there exists a unique threshold δ^{CF} such that $\gamma_1^C < \gamma_1^F$ if $\delta < \delta^{CF}$ and $\gamma_1^C > \gamma_1^F$ otherwise. Therefore, there are two possible equilibrium sequences when comparing limit quality strategy under different pricing strategies:

(a) If $\delta < \delta^{CF}$, $CA(FA) \xrightarrow{\gamma_1^C} CD(FA) \xrightarrow{\gamma_1^F} CD(FD) \xrightarrow{\gamma_2^F} CD(FB) \xrightarrow{\gamma_2^C} CB(FB)$ as γ increases.

(b) If $\delta > \delta^{CF}$, $CA(FA) \xrightarrow{\gamma_1^F} CA(FD) \xrightarrow{\gamma_1^C} CD(FD) \xrightarrow{\gamma_2^F} CD(FB) \xrightarrow{\gamma_2^C} CB(FB)$ as γ increases.

We next study each subcases:

CA (FA)

$$q^{CA*} - q^{FA*} = \frac{(\delta-1)\delta^2}{2\gamma(\delta-4)^2(\delta-2)} > 0.$$

CD (FA) It is not so obvious when comparing q^{CD*} and q^{FA*}, since q^{CD*} is implicitly decided by $F_1 = 0$. However, since F_1 decreases in q, it is enough to study $F_1|_{q=q^{FA*}}$. And

$$F_1|_{q=q^{FA*}} < F_1|_{q=q^{FA*},\gamma=\gamma_1^F} < 0$$

since $F_1|_{q=q^{FA*}}$ increases in γ. Since q^{CD*} is the zero to F_1, we have $q^{FA*} > q^{CD}$.

CA (FD) Since $q^{CA*} - q^{FD*}$ decreases in γ, we have

$$q^{CA*} - q^{FD*} > q^{CA*} - q^{FD*}|_{\gamma=\gamma_1^C} > q^{CA*} - q^{FD*}|_{\gamma=\gamma_2} > 0.$$

Table A.4: Sensitivity Analysis Results of Profits

	Π_i^{B*}	Π_i^{FA*}	Π_i^{FD*}	Π_i^{CA*}	Π_i^{CD*}
γ	$-\frac{1}{8\gamma^2}$	$-\frac{\left(\delta^2-24\delta+32\right)^2}{32\gamma^2(\delta-4)^4}$	$-\frac{(\delta-4)^4K^2}{2(\delta-1)^2\delta^2}$	$-\frac{(4-3\delta)^2}{32\gamma^2(\delta-2)^2}$	$-\frac{(q^{CD})^2}{2}$
δ	0	$\frac{\delta^3-22\delta^2-16\delta+64}{\gamma(\delta-4)^5}$	$-\frac{(\delta-4)(7\delta-4)K\left((\delta-1)\delta+2\gamma(\delta-4)^2K\right)}{2(\delta-1)^3\delta^3}$	$\frac{4-3\delta}{8\gamma(\delta-2)^3}$	$\frac{8K(2\gamma q^{CD}-1)}{\delta^2(4\gamma q^{CD}-1)}$

CD (FD) Similarly, it is enough to study $F_1|_{q=q^{FD*}}$. Since $F_1|_{q=q^{FD*}}$ decreases in γ, and

$$\begin{cases} F_1|_{q=q^{FD*},\gamma=\gamma_2^F} &= -\frac{3\delta}{4(4-\delta)} < 0 \\ F_1|_{q=q^{FD*},\gamma=\gamma_1^F} &= \frac{3\delta^2+\sqrt{\delta(3\delta^3-16\delta^2-256\delta+512)}-12\delta}{4(\delta-4)^2} > 0 \\ F_1|_{q=q^{FD*},\gamma=\gamma_1^C} &> F_1|_{q=q^{FD*},\gamma=\gamma_2} = \frac{(3\delta-4)(\delta-4)^2}{32(\delta-2)^2\left(\sqrt{-((\delta-2)\delta)}+1\right)} + \frac{\delta-1}{\delta-4} + \frac{1}{4} > 0 \end{cases},$$

there exists a unique threshold $\gamma_2^{CF} = \frac{\delta(1-\delta)(8-5\delta)}{4K(4-\delta)^3}$ such that $q^{FD*} > q^{CD*}$, i.e., $F_1|_{q=q^{FD*}} < 0$, if $\gamma_2^{CF} < \gamma < \gamma_2^F$, and $q^{FD*} < q^{CD*}$ otherwise.

CD (FB) $q^{CD*} < q^{FB*}$.

CB (FB) $q^{CB*} = q^{FB*}$.

Combining the above results, we can see that $q^{F*} \geq q^{C*}$ if and only if $\gamma_1^C < \gamma < \gamma_1^F$ (when δ is not large) or $\gamma_2^{CF} < \gamma < \gamma_2^C$. □

Proof of Corollary 2 Our goal is to show that:

(i) Π_i^{C*}, Π_i^{F*} and Π_i^* increases in q.

(ii) Π_c^{C*}, Π_c^{F*} and Π_c^* increases in q.

(iii) Π_i^{C*}, Π_i^{F*} and Π_i^* decreases in δ.

(iv) Π_c^{C*}, Π_c^{F*} and Π_c^* is non-monotonic in δ.

Table A.5: Sensitivity Analysis Results of CS^{*}

	CS^{B*}	CS^{FA*}	CS^{FD*}	CS^{CA*}
γ	$-\frac{1}{8\gamma^2}$	$-\frac{\left(\delta^2-24\delta+32\right)\left(\delta^2+12\delta+32\right)}{32\gamma^2(\delta-4)^4}$	0	$\frac{(3\delta-8)(3\delta-4)}{32\gamma^2(\delta-2)^3}$
δ	0	$-\frac{\delta^3-148\delta^2-736\delta+640}{8\gamma(\delta-4)^5}$	$-\frac{(\delta-4)(7\delta-4)K}{4(\delta-1)^2\delta^2}$	$\frac{3\left(3\delta^2-12\delta+8\right)}{32\gamma(\delta-2)^4}$

We first derive the derivatives of the innovator's profit to quality cost coefficient γ and imitation efficiency δ and display them in Table A.4. We remark that the innovator's profit is continuous and monotonic in γ and δ due to its first-mover advantage. Suppose Π_i^* is non-monotonic in either γ or δ, that is, Π_i^* jumps upward when equilibrium cases switch. However, since the innovator can choose the optimal one freely, the innovator would adjust quality and pricing strategies to avoid this situation. Therefore, Π_i^* must be continuous and monotonic in γ and δ.

The analysis of Π_c^* would be similar, thus we omit its calculation here. It can be easily verified from the equilibrium sequence that Π_c^* is non-monotonic in δ. When δ is large enough, the innovator may limit quality to a very low extent to deter entry. Under this situation, Π_c^* may decrease in δ. □

Proof of Lemma 6 We first presents derivatives of consumer surplus in Table A.5. Moreover, we have

$$
\begin{aligned}
\frac{dCS^{CD*}}{d\gamma} &= \frac{\partial CS^{CD*}}{\partial q^{CD}}\frac{\partial q^{CD}}{\partial \gamma} \\
&= \frac{\partial q^{CD}}{\partial \gamma}\left(\frac{(\delta-1)K}{\sqrt{(\delta-1)\delta(-K)q^{CD}}}+\frac{5}{8}\right) \\
&= \frac{\partial q^{CD}}{\partial \gamma}\left(\left(\frac{1}{4}-\gamma q^{CD}\right)+\frac{5}{8}\right) \quad (F_1=0) \\
&< (\frac{7}{8}-\frac{1}{2})\frac{\partial q^{CD}}{\partial \gamma} < 0 \quad (q^{CD}\in(\frac{1}{4\gamma},\frac{1}{2\gamma}))
\end{aligned}
$$

Table A.6: Sensitivity Analysis Results of SW^*

	SW^{B*}	SW^{FA*}	SW^{FD*}	SW^{CA*}
γ	$-\frac{3}{8\gamma^2}$	$\frac{\left(\delta^2-24\delta+32\right)\left(5\delta^2+28\delta-96\right)}{32\gamma^2(\delta-4)^4}$	0	$-\frac{(3\delta-4)\left(4\delta^2-21\delta+24\right)}{32\gamma^2(\delta-2)^3}$
δ	0	$\frac{-3\delta^3-580\delta^2+1184\delta+128}{8\gamma(\delta-4)^5}$	$-\frac{3(\delta-4)(7\delta-4)K}{4(\delta-1)^2\delta^2}$	$\frac{7\delta^2+4\delta-24}{32\gamma(\delta-2)^4}$

and

$$\begin{aligned}\frac{dCS^{CD*}}{d\delta} &= \frac{\partial CS^{CD*}}{\partial q^{CD}}\frac{\partial q^{CD}}{\partial \delta}\\ &= \frac{K\left(\delta\left(16\gamma\sqrt{(1-\delta)Kq^5}-5q\sqrt{(\delta-1)(-K)q}\right)+16(\delta-1)K\left(\sqrt{(\delta-1)(-K)q}+2\gamma\sqrt{\delta}q^2-\sqrt{\delta}q\right)\right)}{8\delta^2\sqrt{(\delta-1)(-K)q}\left(-\delta K+2\gamma q\sqrt{(\delta-1)\delta(-K)q}+K\right)}\\ &> \frac{K\left(\frac{1152(\delta-1)^2K^2}{\delta q}-512(\delta-1)K+\frac{91\delta q}{2}\right)}{8\delta^2\sqrt{(\delta-1)(-K)q}\left(-\delta K+2\gamma q\sqrt{(\delta-1)\delta(-K)q}+K\right)} > 0 \quad \text{(quadratic function regarding } \gamma)\end{aligned}$$

The derivatives of social welfare under different cases is listed in Table A.6. Moreover, we have

$$\begin{aligned}\frac{dSW^{CD*}}{d\gamma} &= \frac{\partial SW^{CD*}}{\partial q^{CD}}\frac{\partial q^{CD}}{\partial \gamma}\\ &= \frac{7}{8}\frac{\partial q^{CD}}{\partial \gamma} < 0\end{aligned}$$

and

$$\begin{aligned}\frac{dSW^{CD*}}{d\delta} &= \frac{\partial SW^{CD*}}{\partial q^{CD}}\frac{\partial q^{CD}}{\partial \delta}\\ &= \frac{K\left(16(\delta-1)K+\delta q\left(-32\gamma^2q^2+8\gamma q+7\right)\right)}{4\delta^2\left(2(\delta-1)K+\gamma\delta q^2(1-4\gamma q)\right)}\end{aligned}$$

where the denominator is negative and

$$\begin{aligned}16(\delta-1)K+\delta q\left(-32\gamma^2q^2+8\gamma q+7\right) &= 2\delta q\left(-24\gamma^2q^2+8\gamma q+3\right) \qquad (F_1=0)\\ &> 0 \qquad (q^{CD}\in(\frac{1}{4\gamma},\frac{1}{2\gamma}))\end{aligned}.$$

□

Proof of Proposition 6 We study each cases separately.

Case $CB(FB)$**:** It is easy to see that $CS^{FB*}=CS^{CB*}$ and $SW^{FB*}=SW^{CB*}$.

Case $CA(FA)$**:**

$$\begin{cases} CS^{FA*} - CS^{CA*} & = \frac{\delta\left(\delta^6 - 9\delta^5 - 320\delta^4 + 2280\delta^3 - 5536\delta^2 + 5632\delta - 2048\right)}{32\gamma(\delta-4)^4(\delta-2)^3} > 0 \\ SW^{FA*} - SW^{CA*} & = -\frac{\delta\left(17\delta^6 - 393\delta^5 + 2576\delta^4 - 7544\delta^3 + 10976\delta^2 - 7680\delta + 2048\right)}{32\gamma(\delta-4)^4(\delta-2)^3} > 0 \end{cases}$$

Case $CA(FD)$**:**

$$CS^{CA*} - CS^{FD*} = \frac{\delta\left(-9\delta^3 + 45\delta^2 - 68\delta + 32\right) + 8\gamma(\delta-4)^2(\delta-2)^3 K}{32\gamma(\delta-2)^3(\delta-1)\delta}$$

where

$$\begin{aligned} & \delta\left(-9\delta^3 + 45\delta^2 - 68\delta + 32\right) + 8\gamma(\delta-4)^2(\delta-2)^3 K \\ > & \left(\delta\left(-9\delta^3 + 45\delta^2 - 68\delta + 32\right) + 8\gamma(\delta-4)^2(\delta-2)^3 K\right)\Big|_{\gamma=\gamma_1^C} \qquad \text{Molecular decreases in } \gamma \\ > & \left(\delta\left(-9\delta^3 + 45\delta^2 - 68\delta + 32\right) + 8\gamma(\delta-4)^2(\delta-2)^3 K\right)\Big|_{\gamma=\gamma_2} \qquad \text{Molecular decreases in } \gamma \\ = & \delta\left(-9\delta^3 + 45\delta^2 + \frac{(\delta-4)^2(\delta-2)\left(3\delta^2 - 7\delta + 4\right)}{4\sqrt{-((\delta-2)\delta)} + 4} - 68\delta + 32\right) > 0 \end{aligned}$$

$$SW^{CA*} - SW^{FD*} = \frac{\delta\left(12\delta^4 - 91\delta^3 + 235\delta^2 - 252\delta + 96\right) + 24\gamma(\delta-4)^2(\delta-2)^3 K}{32\gamma(\delta-2)^3(\delta-1)\delta}$$

where

$$\begin{aligned} & \delta\left(12\delta^4 - 91\delta^3 + 235\delta^2 - 252\delta + 96\right) + 24\gamma(\delta-4)^2(\delta-2)^3 K \\ > & \left(\delta\left(12\delta^4 - 91\delta^3 + 235\delta^2 - 252\delta + 96\right) + 24\gamma(\delta-4)^2(\delta-2)^3 K\right)\Big|_{\gamma=\gamma_1^C} \\ > & \left(\delta\left(12\delta^4 - 91\delta^3 + 235\delta^2 - 252\delta + 96\right) + 24\gamma(\delta-4)^2(\delta-2)^3 K\right)\Big|_{\gamma=\gamma_2} \\ = & \frac{3(\delta-2)\delta\left(3\delta^2 - 7\delta + 4\right)(\delta-4)^2}{4\left(\sqrt{-((\delta-2)\delta)} + 1\right)} + \delta\left(12\delta^4 - 91\delta^3 + 235\delta^2 - 252\delta + 96\right) > 0 \end{aligned}$$

Case $CD(FB)$**:**

$$\begin{aligned} CS^{CD*} - CS^{FB} &= CS^{CD*} - CS^{CB*} \\ &> \left(CS^{CD*} - CS^{CB*}\right)\Big|_{\gamma=\gamma_2^C} \qquad \text{(Lemma 6)} \\ &= \left(CS^{CB*} - CS^{CB*}\right)\Big|_{\gamma=\gamma_2^C} = 0 \end{aligned}$$

$$\begin{aligned}SW^{FB*} - SW^{CD*} &= SW^{CB*} - SW^{CD*} \\ &> \left(SW^{CB*} - SW^{CD*}\right)\big|_{\gamma=\gamma_2^C} \qquad \text{(Lemma 6)} \\ &= \left(SW^{CB*} - SW^{CB*}\right)\big|_{\gamma=\gamma_2^C} = 0\end{aligned}$$

Case $CD(FD)$**:**

$$\begin{aligned}CS^{CD*} - CS^{FD*} &> \left(CS^{CD*} - CS^{FD*}\right)\big|_{\gamma=\gamma_2^F} \qquad \text{(Lemma 6)} \\ &= \left(CS^{CD*} - CS^{FB*}\right)\big|_{\gamma=\gamma_2^C} > 0\end{aligned}$$

We aim to show that there exists a unique threshold γ_3^{CF} such that $SW^{CD*} > SW^{FD*}$ if $\gamma < \gamma_3^{CF}$ and $SW^{CD*} < SW^{FD*}$ otherwise. We first notice that $SW^{CD*} - SW^{FD*}$ decreases in γ via Lemma 6. Moreover, we have

$$\begin{cases}\left(SW^{CD*} - SW^{FD*}\right)\big|_{\gamma=\gamma_2^F} &= \left(SW^{CD*} - SW^{FB*}\right)\big|_{\gamma=\gamma_2^F} = \left(SW^{CD*} - SW^{CB*}\right)\big|_{\gamma=\gamma_2^F} < 0 \\ \left(SW^{CD*} - SW^{FD*}\right)\big|_{\gamma=\gamma_1^C} &= \left(\Pi_i^{CD*} - \Pi_i^{FD*}\right)\big|_{\gamma=\gamma_1^C} + \left(CS^{CD*} - CS^{FD*}\right)\big|_{\gamma=\gamma_1^C} > 0 \\ \left(SW^{CD*} - SW^{FD*}\right)\big|_{\gamma=\gamma_1^F} &= \left(\Pi_i^{CD*} - \Pi_i^{FD*}\right)\big|_{\gamma=\gamma_1^F} + \left(CS^{CD*} - CS^{FD*}\right)\big|_{\gamma=\gamma_1^F} > 0\end{cases}$$

and the unique existence of γ_3^{CF} can be ensured via Intermediate value theorem, where

$$\gamma_3^{CF} = \frac{7(1-\delta)\delta\left(11\delta^2 - 2\sqrt{14}(\delta-1)\sqrt{11\delta^2 - 40\delta + 56} - 40\delta + 56\right)}{8\left(11\delta^2 - 40\delta + 56\right)^2 K}.$$

Case $CD(FA)$**:** After tedious algebraic manipulation, we find that $SW^{CD*} - SW^{FA*} < 0$ always holds. In addition, define K_1 be the threshold where $\Pi_i^{CD*}\big|_{K=K_1} = \Pi_i^{CA*}\big|_{K=K_1}$, and K_2 be the threshold where $CS^{CD*}\big|_{K=K_2} = CS^{FA*}\big|_{K=K_2}$, and

$$K_3 = \frac{(1-\delta)\delta\left(2\delta^2 - \sqrt{\delta\left(3\delta^3 - 16\delta^2 - 256\delta + 512\right)} - 16\delta + 32\right)}{4\gamma(\delta-4)^4},$$

$CS^{CD*} - CS^{FA*} > 0$ iff $K_1 < K < K_2$, and $CS^{CD*} - CS^{FA*} < 0$ otherwise.

□

Proof of Lemma 7 and Proposition 7 The analysis of the subgame regarding the copycat remains the same. Using first-order condition, we have $\delta^{F*} = \frac{4}{7}$ and

$\delta^{C*} = \frac{2}{3}$. The lemma and proposition can be easily verified if we substitute δ^* into our previous results. □

Proof of Lemma 8 Lemma A.2.4 presents the optimal uniform pricing under exogenous quality. Note that $q_1^U = q_1^C < q^F$, and

$$q_2^U - q^F = \frac{K\left(5\delta^{3/2} + 8\sqrt{4-3\delta} - 6\delta\sqrt{4-3\delta} - 12\sqrt{\delta}\right)}{\sqrt{\delta}(1-\delta)} > 0.$$

Therefore, we have following cases.

Case $q < q_1^U$: The entry is blockaded under both pricing strategies, and thus the innovator enjoys the same profit.

Case $q_1^U < q < q^F$: Under uniform pricing, the innovator has to set a lower-than-first-best price to deter entry. Under flexible pricing, the innovator achieves the first-best outcome. Obviously, flexible pricing is the dominant one.

Case $q^F < q < q_2^U$: Under uniform pricing, the innovator sets a lower-than-first-best price to deter entry. Under flexible pricing, the innovator accommodates imitation. We have

$$\Pi_i^{UD*} - \Pi_i^{FA*} = \frac{8(\delta-1)K}{\delta} + \frac{4\sqrt{(\delta-1)(-K)q}}{\sqrt{\delta}} - \frac{(\delta^2 - 24\delta + 32)\,q}{4(\delta-4)^2}.$$

Note that the gap is concave in q. Let

$$q^{FU} := \frac{32K(1-\delta)(4-\delta)^2\left(\sqrt{\delta(8+\delta)} - \sqrt{2}\delta + 4\sqrt{2}\right)^2}{\delta(\delta^2 - 24\delta + 32)^2},$$

the gap is is positive if $q^F < q < q^{FU}$ and negative otherwise.

Case $q > q_2^U$: The innovator accommodates imitation under both pricing strategies. We have

$$\Pi_i^{FA*} - \Pi_i^{UA*} = \frac{\delta^2 q(4+5\delta)}{4(4-3\delta)(4-\delta)^2} > 0.$$

□

Proof of Proposition 8 Based on Lemma A.2.7, we have following cases.

Case $\gamma > \gamma_2^U$: The entry is blockaded under both pricing strategies, and thus the innovator enjoys the same profit.

Case $\gamma_2^F < \gamma < \gamma_2^U$: Under uniform pricing, the innovator has to set a lower-than-first-best price to deter entry. Under flexible pricing, the innovator achieves the first-best outcome. Obviously, flexible pricing is the dominant one.

Case $max(\gamma_1^U, \gamma_1^F) < \gamma < \gamma_2^F$: Lemma A.2.7 (iii) shows that $\Pi_i^{F*} > \Pi_i^{U*}$.

Case $\gamma < max(\gamma_1^U, \gamma_1^F)$: Lemma A.2.7 (iv) shows that $\Pi_i^{F*} > \Pi_i^{U*}$. □

A.2 Supporting Results

Lemma A.2.1. *(Exogenous Quality Thresholds) The following statements hold when quality is exogenously given:*

(i) $q_1^C < q^{FD}$ and $q^{FD} < q_2^C$;

(ii) q^{FD} and q_1^C decreases in δ, and q_2^C is convex in δ.

Proof of Lemma A.2.1

Part (i) We have

$$q^{FD} - q_1^C = \frac{3(8-5\delta)}{1-\delta}K > 0,$$

$$q_2^C - q^{FD} = \frac{16\sqrt{2-\delta} - 8\delta\sqrt{2-\delta} - \delta\sqrt{\delta}}{(1-\delta)\sqrt{\delta}}K > \frac{8\sqrt{2-\delta} - \delta\sqrt{\delta}}{(1-\delta)\sqrt{\delta}}K > \frac{8\sqrt{2-\delta} - \sqrt{\delta}}{(1-\delta)\sqrt{\delta}}K > 0.$$

Part (ii) We have

$$\frac{\partial q^{FD}}{\partial \delta} = -\frac{(\delta-4)(7\delta-4)K}{(\delta-1)^2\delta^2} < 0 \quad \text{and} \quad \frac{\partial q_1^C}{\partial \delta} = -\frac{16K}{\delta^2} < 0,$$

Table A.7: Derivatives of q^{CD}

Derivative	δ	γ	K
q^{CD}	$-\frac{qK}{\delta(1-\delta)K+2\delta^{3/2}\gamma q\sqrt{(1-\delta)qK}}(<0)$	$-\frac{2\sqrt{\delta}q^2\sqrt{(\delta-1)(-K)q}}{-\delta K+2\gamma\sqrt{\delta}q\sqrt{(\delta-1)(-K)q}+K}(<0)$	$\frac{(\delta-1)q}{(\delta-1)K-2\gamma\sqrt{\delta}q\sqrt{(\delta-1)(-K)q}}(>0)$

$$\frac{\partial q_2^C}{\partial \delta} = -\frac{8\left(\left(\delta^2-4\delta+2\right)\sqrt{(2-\delta)\delta}+(\delta-2)\delta(2\delta-1)\right)K}{(\delta-1)^2\delta^2\sqrt{(2-\delta)\delta}}.$$

It can be verified that $\left(\delta^2-4\delta+2\right)\sqrt{(2-\delta)\delta}+(\delta-2)\delta(2\delta-1)$ is concave in δ, and thus positive (negative) when δ is small (large). $\square$

Lemma A.2.2. *(Endogenous Quality Thresholds) The following statements hold under committed pricing:*

(i) q^{CD} *decreases in* δ *and* γ*, while increases in* K*.*

(ii) $q^{CB} < q_1^C$ *iff* $q^{CD} < q_1^C$*.*

(iii) If $q^{CB} > q_1^C$*,* $q^{CA} > q^{CD}$*.*

(iv) If $q^{CB} > q_1^C$ *and* $q_1^C < q^{CD} \leq q_2^C$ *and* $q^{CA} > q_2^C$*, there exists a threshold* γ_1^C *such that* $\Pi_i^{CA*}\big|_{\gamma=\gamma_1^C} = \Pi_i^{CD*}\big|_{\gamma=\gamma_1^C}$*.*

<u>**Proof of Lemma A.2.2**</u>

Part (i). See Table A.7.

Part (ii). Firstly, we aim to show that

$$q^{CB} = q_1^C \Leftrightarrow q^{CD} = q_1^C.$$

The left part holds iff $\gamma = \frac{\delta}{32K(1-\delta)}$, which we denote as γ_2^C. And

$$F_1\big|_{\gamma=\gamma_2^C, q=q_1^C} = \left(\frac{1}{4} - \gamma q + \frac{\sqrt{qK(1-\delta)}}{\sqrt{\delta}q}\right)\Bigg|_{\gamma=\gamma_2^C, q=q_1^C} = 0,$$

which shows that $q^{CD} = q_1^C$. Moreover,

$$F_1|_{q=q_1^C} = \left(\frac{1}{4} - \gamma q + \frac{\sqrt{qK(1-\delta)}}{\sqrt{\delta}q}\right)\Bigg|_{q=q_1^C} = \frac{1}{2} - \frac{16(1-\delta)\gamma K}{\delta}$$

is positive (i.e., $q^{CD} > q_1^C$) if $\gamma < \gamma_2^C$ and negative (i.e., $q^{CD} < q_1^C$) if $\gamma > \gamma_2^C$.

Part (iii). To prove $q^{CA} > q^{CD}$ if $q^{CB} > q_1^C$. Notice that

$$\frac{\partial F_1}{\partial q} = -\gamma - \frac{\sqrt{qK(1-\delta)}}{2\sqrt{\delta}q^2} < 0,$$

it is equivalent to show

$$F_1|_{q=q^{CA}} = \left(\frac{1}{4} - \gamma q + \frac{\sqrt{qK(1-\delta)}}{\sqrt{\delta}q}\right)\Bigg|_{q=q^{CA}} < 0$$

for any $\gamma < \gamma_2^C$. Notice that

$$\frac{\partial\, F_1|_{q=q^{CA}}}{\partial \gamma} = \sqrt{\frac{(1-\delta)(2-\delta)K}{\delta\gamma(4-3\delta)}} > 0,$$

it is equivalent to show

$$F_1|_{q=q^{CA},\gamma=\gamma_2^C} = \left(\frac{1}{4} - \gamma q + \frac{\sqrt{qK(1-\delta)}}{\sqrt{\delta}q}\right)\Bigg|_{q=q^{CA},\gamma=\gamma_2^C} = -\frac{1}{4}\left(\frac{2(1-\delta)}{2-\delta} + \sqrt{\frac{2(2-\delta)}{4-3\delta}}\right) < 0.$$

This completes the proof. Moreover, this also means that if $q^{CB} > q_1^C$ and $q^{CD} > q_2^C$, then $q^{CA} > q_2^C$; and if $q^{CB} > q_1^C$ and $q^{CA} < q_2^C$, then $q^{CD} < q_2^C$.

Part (iv). The conditions can be rewritten as

$$\gamma_3\left(:= \frac{(\delta-1)\delta\left(\delta^{3/2} + 3\sqrt{2-\delta}\delta - 4\sqrt{2-\delta} - 2\sqrt{\delta}\right)}{16\left(\sqrt{2-\delta}+\sqrt{\delta}\right)^3(\delta-2)^2K}\right) < \gamma < \gamma_2\left(:= \frac{\delta(1-\delta)(4-3\delta)}{32K(2-\delta)^2(1+\sqrt{\delta(2-\delta)})}\right).$$

Define

$$F_2 := \Pi_i^{CD} - \Pi_i^{CA*} = \frac{1}{4}\left(\frac{16(\delta-1)K}{\delta} + \frac{8\sqrt{(\delta-1)(-K)q}}{\sqrt{\delta}} - 2\gamma q^2 + q\right) - \frac{(4-3\delta)^2}{32\gamma(\delta-2)^2},$$

and thus $\Pi_i^{CD*} - \Pi_i^{CA*} = F_2|_{q=q^{CD}}$. We first show that $F_2|_{q=q^{CD}}$ increases in $\gamma \in [\gamma_3, \gamma_2]$, and then show that $F_2|_{q=q^{CD},\gamma=\gamma_3} < 0$ and $F_2|_{q=q^{CD},\gamma=\gamma_2} > 0$. By envelope theorem, we have

$$\frac{\partial\ F_2|_{q=q^{CD}}}{\partial \gamma} = -\frac{(q^{CD})^2}{2} + \frac{(4-3\delta)^2}{32\gamma^2(2-\delta)^2}.$$

Notice that q^{CD} increases in K, we have

$$\frac{\partial^2\ F_2|_{q=q^{CD}}}{\partial \gamma \partial K} = -q^{CD}\frac{\partial q^{CD}}{\partial K} < 0.$$

If we rewrite the condition $\gamma_3 < \gamma < \gamma_2$ as

$$K_3\left(\frac{(\delta-1)\delta\left(\delta^{3/2}+3\sqrt{2-\delta}\delta-4\sqrt{2-\delta}-2\sqrt{\delta}\right)}{16\gamma\left(\sqrt{2-\delta}+\sqrt{\delta}\right)^3(\delta-2)^2}\right) < K < K_2\left(\frac{\delta\left(3\delta^2-7\delta+4\right)}{32\gamma(\delta-2)^2\left(\sqrt{-((\delta-2)\delta)}+1\right)}\right),$$

it is equivalent to show that $\left.\frac{\partial\ F_2|_{q=q^{CD}}}{\partial \gamma}\right|_{K=K_2} > 0$. When $K = K_2$, $q^{CD}\big|_{K=K_2}$ is the zero to

$$\begin{aligned} F_1|_{K=K_2} &= \left.\left(\frac{\sqrt{(\delta-1)(-K)q}}{\sqrt{\delta}q} + \gamma(-q) + \frac{1}{4}\right)\right|_{K=K_2} \\ &= \frac{1}{8}\left(\frac{2(\delta-1)\sqrt{4-3\delta}}{\left(\sqrt{2-\delta}+\sqrt{\delta}\right)(\delta-2)\sqrt{\gamma q}} - 8\gamma q + 2\right). \end{aligned}$$

It is easy to see that $q = \frac{4-3\delta}{4\gamma(2-\delta)}$ is the unique zero to $\frac{\partial F_2}{\partial \gamma} = 0$ since $\frac{\partial F_2}{\partial \gamma} = 0$ decreases in $q \in (0, \infty)$. Notice that

$$\frac{\partial\ F_1|_{K=K_2}}{\partial q} = -\gamma - \frac{\gamma\sqrt{4-3\delta}(\delta-1)}{8\left(\sqrt{2-\delta}+\sqrt{\delta}\right)(\delta-2)(\gamma q)^{3/2}} < 0$$

and

$$F_1|_{K=K_2, q=\frac{4-3\delta}{4\gamma(2-\delta)}} = -\frac{(\delta-1)\sqrt{\delta}}{2\left(\sqrt{2-\delta}+\sqrt{\delta}\right)(\delta-2)} < 0,$$

which means that for any q such that $F_1|_{K=K_2} = 0$ (i.e., $q = q^{CD}$), we have $q^{CD} < \frac{4-3\delta}{4\gamma(2-\delta)}$. Therefore, $\frac{\partial\ F_2|_{q=q^{CD}}}{\partial \gamma} > 0$ for all possible q^{CD}. This shows that $F_2|_{q=q^{CD}}$ increases in $\gamma \in [\gamma_3, \gamma_2]$.

We next show that $F_2|_{q=q^{CD},\gamma=\gamma_2} > 0$. Since $q = q^{CD}$ is the maximizer, we have

$$F_2|_{\gamma=\gamma_2, q=q^{CD}} > F_2|_{\gamma=\gamma_2, q=q_2^C} = 0.$$

Finally, we show that $F_2|_{q=q^{CD},\gamma=\gamma_3} < 0$. Note that $q^{CD} = q_2^C$ at $\gamma = \gamma_3$, we have

$$F_2|_{\gamma=\gamma_3,q=q^{CD}} = F_2|_{\gamma=\gamma_3,q=q_2^C} = \frac{2\left(\sqrt{2-\delta}+\sqrt{\delta}\right)(1-\delta)K}{\delta^{3/2}+3\sqrt{2-\delta}\delta-4\sqrt{2-\delta}-2\sqrt{\delta}} < 0.$$

□

Lemma A.2.3. *(Quality Cost Thresholds under Flexible and Committed Pricing) The following statements hold:*

(i) $\gamma_2^C > \gamma_2^F$ *and* $\gamma_2^F > \gamma_1^C$.

(ii) γ_2^C *and* γ_2^F *increases in* δ, *and* γ_1^F *and* γ_1^C *is concave in* δ.

(iii) There exists a unique threshold δ^{CF} *such that* $\gamma_1^C < \gamma_1^F$ *if* $\delta < \delta^{CF}$ *and* $\gamma_1^C > \gamma_1^F$ *otherwise.*

(iv) If $max(\gamma_1^C, \gamma_1^F) < \gamma < \gamma_2^F$, *there exists a unique threshold* γ_1^{CF} *such that* $\Pi_i^{FD*}|_{\gamma=\gamma_1^{CF}} = \Pi_i^{CD*}|_{\gamma=\gamma_1^{CF}}$.

(v) If $\gamma < max(\gamma_1^C, \gamma_1^F)$, $\Pi_i^{F*} < \Pi_i^{C*}$.

Proof of Lemma A.2.3

Part (i).

$$\gamma_2^C - \gamma_2^F = \frac{3\delta^2(5\delta-8)}{32(\delta-4)^2(\delta-1)K} > 0$$

The proof of $\gamma_2^F > \gamma_1^C$ is not so obvious. Based on γ_1^C's definition, this is equivalent to show that

$$\Pi_i^{CD*} - \Pi_i^{CA*}|_{\gamma=\gamma_2^F} > 0.$$

From Lemma A.2.2 (iv), we know that $\Pi_i^{CD*} - \Pi_i^{CA*}$ increases in $\gamma \in [\gamma_3, \gamma_2]$ and is positive at $\gamma = \gamma_2$. Thus, it is enough to show that $\gamma_2^F > \gamma_2$. And

$$\gamma_2^F - \gamma_2 = \frac{\delta(1-\delta)}{32(\delta-4)^2(\delta-2)^2\left(\sqrt{-((\delta-2)\delta)}K+K\right)}\left(\delta\left(3\delta^2-12\delta+16\right)+16\sqrt{-((\delta-2)\delta)}(\delta-2)^2\right) > 0.$$

Part (ii). Our goal is to show that γ_1^C is concave in δ. Solving $F_1 = 0$ and $F_2 = 0$, we have

$$\begin{cases} q^{CD} & := 1024(\delta-1)^2(\delta-2)^4K^3 + 272(\delta-1)\delta(\delta-2)^4K^2q^{CD} + \\ & 8\delta^2\left(4\delta^2-13\delta+11\right)(\delta-2)^2K\left(q^{CD}\right)^2 + (\delta-1)\delta^3(3-2\delta)^2\left(q^{CD}\right)^3. \\ \gamma_1^C & = \frac{\sqrt{\left(\frac{1}{\delta}-1\right)K}}{\left(q^{CD}\right)^{3/2}} + \frac{1}{4q^{CD}} \end{cases}$$

Therefore, it is enough to show that $\frac{1}{q^{CD}}$ and $\frac{\sqrt{1-\delta}}{\sqrt{\delta}(q^{CD})^{3/2}}$ are both concave. Solving the implicit function of q^{CD}, we find that q^{CD} can be rewritten as the product of K and a function of δ while independent on K, i.e., $q^{CD}\big|_{\gamma=\gamma_1^C} = KQ(\delta)$. Thus the concavity of $\frac{1}{q^{CD}}$ and $\frac{\sqrt{1-\delta}}{\sqrt{\delta}(q^{CD})^{3/2}}$ can be easily proven.

Part (iii). Note that from the proof of part (ii), we know that

$$\gamma_1^C = \frac{\sqrt{\left(\frac{1}{\delta}-1\right)K}}{\left(q^{CD}\right)^{3/2}} + \frac{1}{4q^{CD}} = \frac{1}{K}\left(\frac{1}{4Q(\delta)} + \frac{\sqrt{1/\delta-1}}{Q(\delta)^{3/2}}\right).$$

Therefore, $\gamma_1^F - \gamma_1^C$ is exactly the product of $1/K$ and a function of δ while independent on K. The lemma thus can be easily proven.

Part (iv). Define

$$F_3 := \frac{(\delta-4)^2K\left((\delta-1)\delta+\gamma(\delta-4)^2K\right)}{2(\delta-1)^2\delta^2} + \frac{1}{4}\left(\frac{16(\delta-1)K}{\delta} + \frac{8\sqrt{(\delta-1)(-K)q}}{\sqrt{\delta}} - 2\gamma q^2 + q\right),$$

and thus $\Pi_i^{CD*} - \Pi_i^{FD*} = F_3|_{q=q^{CD}}$. We first show that $F_3|_{q=q^{CD}}$ decreases in γ, and then show that $F_3|_{q=q^{CD},\gamma=\gamma_2^F} < 0$ and $F_3|_{q=q^{CD},\gamma=\max(\gamma_1^C,\gamma_1^F)} > 0$. By envelope theorem, we have

$$\frac{\partial\, F_3|_{q=q^{CD}}}{\partial\gamma} = \frac{1}{2}\left(\frac{(\delta-4)^4K^2}{(\delta-1)^2\delta^2} - (q^{CD})^2\right).$$

Notice that q^{CD} decreases in γ, we have

$$\frac{\partial^2\, F_3|_{q=q^{CD}}}{\partial\gamma^2} = -q^{CD}\frac{\partial q^{CD}}{\partial\gamma} > 0.$$

Thus, it is enough to show that $\frac{\partial\, F_3|_{q=q^{CD}}}{\partial\gamma}$ is non-negative at $\gamma = \max(\gamma_1^C, \gamma_1^F)$.

When $\gamma = \gamma_1^C$, we have

$$\left.\frac{\partial\, F_3|_{q=q^{CD}}}{\partial\gamma}\right|_{\gamma=\gamma_1^C} \geq \left.\frac{\partial\, F_3|_{q=q^{CD}}}{\partial\gamma}\right|_{\gamma=\gamma_3}.$$

It is easy to see that $q = \frac{K(4-\delta)^2}{\delta(1-\delta)}$ is the unique zero to $\frac{\partial F_3}{\partial \gamma} = 0$ since $\frac{\partial F_3}{\partial \gamma}$ decreases in $q \in (0, \infty)$. Notice that

$$\frac{\partial\, F_1|_{\gamma=\gamma_3}}{\partial q} = -\frac{4\left(6\sqrt{(\delta-1)(-K)q} + \sqrt{\delta}q\right)}{16q^2\sqrt{\delta}} < 0$$

and

$$F_1\Big|_{\gamma=\gamma_3, q=\frac{K(4-\delta)^2}{\delta(1-\delta)}} \geq 0,$$

which means that for any q such that $F_1|_{\gamma=\gamma_3} = 0$ (i.e., $q = q^{CD}$), we have $q^{CD} > \frac{K(4-\delta)^2}{\delta(1-\delta)}$. Therefore, $\frac{\partial\, F_3|_{q=q^{CD}}}{\partial \gamma} < 0$ for all possible q^{CD} when $\gamma > \gamma_1^C$.

When $\gamma = \gamma_1^F$, $q = \frac{K(4-\delta)^2}{\delta(1-\delta)}$ is the unique zero to $\frac{\partial F_3}{\partial \gamma} = 0$ too. Notice that

$$\frac{\partial\, F_1|_{\gamma=\gamma_1^F}}{\partial q} = -\frac{4\sqrt{\delta}\sqrt{(\delta-1)(-K)q} + 2\sqrt{(\delta-1)\delta(-K)q} + \delta q}{4\delta q^2} < 0$$

and

$$F_1\Big|_{\gamma=\gamma_1^F, q=\frac{K(4-\delta)^2}{\delta(1-\delta)}} \geq 0,$$

which means that for any q such that $F_1|_{\gamma=\gamma_1^F} = 0$ (i.e., $q = q^{CD}$), we have $q^{CD} > \frac{K(4-\delta)^2}{\delta(1-\delta)}$. Therefore, $\frac{\partial\, F_3|_{q=q^{CD}}}{\partial \gamma} < 0$ for all possible q^{CD} when $\gamma > \gamma_1^F$.

Taking together, this shows that $F_3|_{q=q^{CD}}$ decreases in γ. We next show that $F_3|_{q=q^{CD}} < 0$ at $\gamma = \gamma_2^F$. Notice that

$$F_3|_{\gamma=\gamma_2^F} = \frac{1}{4}\left(\frac{\left(17\delta^2 - 40\delta + 32\right)K}{(\delta-1)\delta} - \frac{3(\delta-1)\delta q^2}{(\delta-4)^2 K} - q\right)$$

and

$$q = \frac{(\delta-4)\left(\sqrt{205\delta^2 - 488\delta + 400} - \delta + 4\right)K}{6(\delta-1)\delta}$$

is the unique zero to $F_3|_{\gamma=\gamma_2^F}$. Moreover, $F_3|_{\gamma=\gamma_2^F} < 0$ if $q < \frac{(\delta-4)\left(\sqrt{205\delta^2-488\delta+400}-\delta+4\right)K}{6(\delta-1)\delta}$ and positive otherwise. Notice that

$$F_1\Big|_{\gamma=\gamma_2^F, q=\frac{(\delta-4)\left(\sqrt{205\delta^2-488\delta+400}-\delta+4\right)K}{6(\delta-1)\delta}} \leq 0$$

and

$$\frac{\partial\, F_1|_{\gamma=\gamma_2^F}}{\partial q} = -\frac{(\delta-1)^2\left((\delta-4)^2K^2 + \delta^{3/2}q\sqrt{(\delta-1)(-K)q}\right)}{2(\delta-4)^2\sqrt{\delta}((\delta-1)(-K)q)^{3/2}} < 0,$$

which means that for any q such that $F_1|_{\gamma=\gamma_2^F} = 0$ (i.e., $q = q^{CD}$), we have $q^{CD} < \frac{(\delta-4)\left(\sqrt{205\delta^2-488\delta+400}-\delta+4\right)K}{6(\delta-1)\delta}$. Therefore, $F_3|_{\gamma=\gamma_2^F, q=q^{CD}} < 0$ for all possible $q = q^{CD}$. Similarly, we can prove that $F_3|_{\gamma=\gamma_1^C, q=q^{CD}} > 0$ and $F_3|_{\gamma=\gamma_1^F, q=q^{CD}} > 0$.

Part (v). We aim to show that

- If $\gamma < min(\gamma_1^C, \gamma_1^F)$, then $\Pi_i^{FA*} < \Pi_i^{CA*}$;
- If $\gamma_1^C < \gamma < \gamma_1^F$, then $\Pi_i^{FA*} < \Pi_i^{CD*}$;
- If $\gamma_1^F < \gamma < \gamma_1^C$, then $\Pi_i^{FB*} < \Pi_i^{CA*}$.

If $\gamma < min(\gamma_1^C, \gamma_1^F)$, then

$$\Pi_i^{CA*} - \Pi_i^{FA*} = \frac{\delta^2\left(2\delta^4 - 29\delta^3 + 107\delta^2 - 144\delta + 64\right)}{8\gamma(\delta-4)^4(\delta-2)^2} \geq 0.$$

If $\gamma_1^C < \gamma < \gamma_1^F$, we have

$$\Pi_i^{CD*} - \Pi_i^{FA*} \geq \Pi_i^{CD*} - \Pi_i^{FA*} - \left(\Pi_i^{CA*} - \Pi_i^{FA*}\right) = \Pi_i^{CD*} - \Pi_i^{CA*} \geq \Pi_i^{CD*} - \Pi_i^{CA*}\big|_{\gamma=\gamma_1^C} = 0$$

by Lemma A.2.2 (iv).

If $\gamma_1^F < \gamma < \gamma_1^C$, then

$$\Pi_i^{CA*} - \Pi_i^{FB*} = \frac{(4-3\delta)^2}{32\gamma(\delta-2)^2} + \frac{(\delta-4)^2K\left((\delta-1)\delta + \gamma(\delta-4)^2K\right)}{2(\delta-1)^2\delta^2}$$

is convex in γ. And

$$\begin{aligned}\frac{\partial\Pi_i^{CA*} - \Pi_i^{FB*}}{\partial\gamma} &\leq \frac{\partial\Pi_i^{CA*} - \Pi_i^{FB*}}{\partial\gamma}\bigg|_{\gamma=\gamma_1^C} \\ &\leq \frac{\partial\Pi_i^{CA*} - \Pi_i^{FB*}}{\partial\gamma}\bigg|_{\gamma=\gamma_2}. \\ &< 0,\end{aligned}$$

therefore $\Pi_i^{CA*} - \Pi_i^{FB*}$ decreases in γ. And

$$\Pi_i^{CA*} - \Pi_i^{FB*} \geq \left(\Pi_i^{CA*} - \Pi_i^{FB*}\right)\big|_{\gamma=\gamma_1^C} = \left(\Pi_i^{CD*} - \Pi_i^{FB*}\right)\big|_{\gamma=\gamma_1^C} \geq 0$$

by (iv).

□

Lemma A.2.4. *(Optimal Uniform Pricing under Exogenous Quality) Let*

$$q_1^U := \frac{16(1-\delta)}{\delta}K \quad \textit{and} \quad q_2^U := \frac{2(4-3\delta)(2-\delta+\sqrt{\delta(4-3\delta)})}{\delta(1-\delta)}K,$$

the following statements hold.

(i) If $q \leq q_1^U$, the innovator sets $p_i^{U*} = \frac{q}{2}$, and the copycat would not enter the market.

(ii) If $q_1^U < q \leq q_2^U$, the innovator sets $p_i^{U*} = \frac{2\sqrt{Kq(1-\delta)}}{\sqrt{\delta}}$, and the copycat would not enter the market.

(iii) If $q > q_2^U$, the innovator sets $p_i^{U*} = \frac{2q(1-\delta)}{4-3\delta}$, and the copycat enters the market and sets $p_c^{U*} = \frac{\delta}{2}p_i^{U*}$.

Proof of Lemma A.2.4

We solve the game backwards. Consider the case of imitation, after observing p_i^U, the copycat decides p_c^U to maximize its profit, i.e.,

$$\max_{p_c^U} p_c^U \left(\theta_{ic} - \frac{p_c^U}{\delta q}\right).$$

Using the first-order conditions, the optimal decision and profit are

$$p_c^{U*} = \frac{\delta}{2}p_i^U \quad \text{and} \quad \Pi_c^{U*} = \frac{\delta}{4q(1-\delta)}(p_i^U)^2.$$

Under uniform pricing, the innovator can commit a price p_i^{UD} to deter entry, where

$$p_i^{UD} := \frac{2\sqrt{Kq(1-\delta)}}{\sqrt{\delta}}.$$

The innovator's objective function is thus

$$\Pi_i^U = \begin{cases} \Pi_i^{UA} = p_i^U\left(1-\frac{p_i^U}{q}\right) + p_i^U\left(1-\theta_{ic}\right), & \text{if } p_i^U > p_i^{UD}; \\ \Pi_i^{UB} = 2p_i^U\left(1-\frac{p_i^U}{q}\right), & \text{if } p_i^U \leq p_i^{UD}. \end{cases}$$

Let

$$p_i^{UA}\left(:=\frac{2q(1-\delta)}{4-3\delta}\right) \quad < \quad p_i^{UB}\left(:=\frac{q}{2}\right)$$

denote two solutions derived by first-order conditions, respectively. We next study the optimal decision globally.

Case $p_i^{UB} \leq p_i^{UD}$: Since $p_i^{UA} < p_i^{UB}$, we have $p_i^{UA} < p_i^{UD}$ too. Π_i^U increases in $p_i^U \in [0, p_i^{UB}]$, decreases in $p_i^U \in (p_i^{UB}, p_i^{UD}]$, and decreases in $p_i^U \in (p_i^{UD}, \frac{1}{k}]$. Thus, the optimal decision is $p_i^{U*} = p_i^{UB}$.

Case $p_i^{UA} \leq p_i^{UD} < p_i^{UB}$: Π_i^U increases in $p_i^U \in [0, p_i^{UD}]$ and decreases in $p_i^U \in (p_i^{UD}, \frac{1}{k}]$. Thus, the optimal decision is $p_i^{U*} = p_i^{UD}$.

Case $p_i^{UD} < p_i^{UD} < p_i^{UB}$: Π_i^U increases in $p_i^U \in [0, p_i^{UD}]$, increases in $p_i^U \in (p_i^{UD}, p_i^{UA}]$, and decreases in $p_i^U \in (p_i^{UA}, \frac{1}{k}]$. Thus, the optimal decision is $p_i^{U*} = p_i^{UD}$ if $\Pi_i^{UB*} \geq \Pi_i^{UA*}$ and $p_i^{U*} = p_i^{UA}$ otherwise.

Let

$$q_1^U := \frac{16(1-\delta)}{\delta}K \quad \text{and} \quad q_2^U := \frac{2(4-3\delta)(2-\delta+\sqrt{\delta(4-3\delta)})}{\delta(1-\delta)}K,$$

we can derive the lemma. □

Lemma A.2.5. *(Endogenous Quality Thresholds under Uniform Pricing) The following statements hold under committed pricing:*

(i) q^{UD} *decreases in* δ *and* γ*, while increases in* K.

(ii) $q^{UB} < q_1^U$ *iff* $q^{UD} < q_1^U$.

(iii) If $q^{UD} > q_2^U$, $q^{UA} > q_2^U$.

(iv) If $q^{UD} < q_2^U$ *and* $q^{UA} > q_2^U$*, there exists a threshold* γ_1^U *such that* $\Pi_i^{UA*}\big|_{\gamma=\gamma_1^U} = \Pi_i^{UD*}\big|_{\gamma=\gamma_1^U}$.

Proof of Lemma A.2.5

Table A.8: Derivatives of q^{UD}

Derivative	δ	γ	K
q^{UD}	$-\frac{2^{2/3}K}{3\gamma^{2/3}\delta^{4/3}(K-\delta K)^{2/3}}(<0)$	$-\frac{2\ 2^{2/3}\sqrt[3]{K-\delta K}}{3\gamma^{5/3}\sqrt[3]{\delta}}(<0)$	$\frac{2^{2/3}(1-\delta)}{3\gamma^{2/3}\sqrt[3]{\delta}(K-\delta K)^{2/3}}(>0)$

Part (i): See Table A.8.

Part(ii): Firstly, we aim to show that

$$q^{UB} = q_1^U \Leftrightarrow q^{UD} = q_1^U.$$

Note that $q^{UB} = q^{CB}$ and $q_1^U = q_1^C$, thus the left part holds iff $\gamma = \frac{\delta}{32K(1-\delta)}$ via Lemma A.2.2, which we denote as γ_2^U. And $q^{UD} = q_1^U$ iff $\gamma = \gamma_2^U$ too.

Part(iii): Note that $q^{UD} > q_2^U$ iff

$$\gamma < \frac{\delta(1-\delta)^2}{\sqrt{2}\left(\left(\delta - \sqrt{4-3\delta}\sqrt{\delta} - 2\right)(3\delta - 4)\right)^{3/2} K}.$$

And $q^{UA} > q_2^U$ iff

$$\gamma < \frac{(\delta-1)^2\delta}{(4-3\delta)^2\left(-\delta + \sqrt{\delta}\sqrt{4-3\delta} + 2\right) K}.$$

Therefore, it is equivalent to show that

$$\frac{(\delta-1)^2\delta}{(4-3\delta)^2\left(-\delta + \sqrt{\delta}\sqrt{4-3\delta} + 2\right) K} - \frac{\delta(1-\delta)^2}{\sqrt{2}\left(\left(\delta - \sqrt{4-3\delta}\sqrt{\delta} - 2\right)(3\delta - 4)\right)^{3/2} K} > 0.$$

Part(iv): The condition can be rewritten as

$$\frac{\delta(1-\delta)^2}{\sqrt{2}\left(\left(\delta - \sqrt{4-3\delta}\sqrt{\delta} - 2\right)(3\delta - 4)\right)^{3/2} K} < \gamma < \frac{(\delta-1)^2\delta}{(4-3\delta)^2\left(-\delta + \sqrt{\delta}\sqrt{4-3\delta} + 2\right) K}.$$

Consider

$$\Pi_i^{UA*} - \Pi_i^{UD*} = \frac{2\delta^3 - 4\delta^2 + 2\delta - 3\sqrt[3]{2}(4-3\delta)^2\sqrt[3]{\delta}(\gamma(\delta-1)(-K))^{2/3} - 8\gamma(\delta-1)(4-3\delta)^2 K}{\gamma(4-3\delta)^2\delta},$$

define

$$F_4 = 2\delta^3 - 4\delta^2 + 2\delta - 3\sqrt[3]{2}(4-3\delta)^2\sqrt[3]{\delta}(\gamma(\delta-1)(-K))^{2/3} - 8\gamma(\delta-1)(4-3\delta)^2 K.$$

We first show that F_4 decreases in γ. Since

$$\frac{\partial^2 F_4}{\partial \gamma^2} = \frac{2\sqrt[3]{2}(4-3\delta)^2(\delta-1)^2\sqrt[3]{\delta}K^2}{3(\gamma(\delta-1)(-K))^{4/3}} \geq 0,$$

F_4 is convex in γ. Moreover,

$$\frac{\partial F_4}{\partial \gamma} \leq \left.\frac{\partial F_4}{\partial \gamma}\right|_{\gamma=\frac{(\delta-1)^2\delta}{(4-3\delta)^2(-\delta+\sqrt{\delta}\sqrt{4-3\delta}+2)K}} \leq 0.$$

Therefore, F_4 decreases in γ. Moreover,

$$F_4\Big|_{\gamma=\frac{\delta(1-\delta)^2}{\sqrt{2}\left((\delta-\sqrt{4-3\delta}\sqrt{\delta}-2)(3\delta-4)\right)^{3/2}K}} \geq 0,$$

$$F_4\Big|_{\gamma=\frac{(\delta-1)^2\delta}{(4-3\delta)^2(-\delta+\sqrt{\delta}\sqrt{4-3\delta}+2)K}} \leq 0.$$

The unique existence of γ_1^U can be ensured via Intermediate value theorem. □

Lemma A.2.6. *(Optimal Uniform Pricing under Endogenous Quality) Let γ_1^U be the threshold where $\Pi_i^{UD*}\big|_{\gamma=\gamma_1^U} = \Pi_i^{UA*}\big|_{\gamma=\gamma_1^U}$ and*

$$\gamma_2^U := \frac{\delta}{32K(1-\delta)},$$

the following statements hold.

(i) If $\gamma < \gamma_1^U$, the innovator sets $q^{U*} = \frac{2(1-\delta)}{\gamma(4-3\delta)}$ and $p_i^{U*} = \frac{2q^{U*}(1-\delta)}{4-3\delta}$, and the copycat enters the market and sets $p_c^{U*} = \frac{\delta}{2}p_i^{U*}$.

(ii) If $\gamma_1^U \leq \gamma < \gamma_2^U$, the innovator sets $q^{U*} = \frac{2^{2/3}(K-K\delta)^{1/3}}{\gamma^{2/3}\delta^{1/3}}$ and $p_i^{U*} = \frac{2\sqrt{Kq^{U*}(1-\delta)}}{\sqrt{\delta}}$, and the copycat would not enter the market.

(iii) If $\gamma \geq \gamma_2^U$, the innovator sets $q^{U*} = \frac{1}{2\gamma}$ and $p_i^{U*} = \frac{q^{U*}}{2}$, and the copycat would not enter the market.

<u>Proof of Lemma A.2.6</u>

When the quality is endogenized, the innovator chooses q to maximize

$$\Pi_i^U = \begin{cases} \Pi_i^{UA} = \frac{2q(1-\delta)}{(4-3\delta)} - \frac{1}{2}\gamma q^2, & \text{if } q > q_2^U; \\ \Pi_i^{UD} = \frac{8(\delta-1)K}{\delta} + \frac{4\sqrt{(\delta-1)(-K)q}}{\sqrt{\delta}} - \frac{\gamma q^2}{2}, & \text{if } q_1^U < q \leq q_2^U; \\ \Pi_i^{UB} = \frac{q}{2} - \frac{1}{2}\gamma q^2, & \text{if } q \leq q_1^U. \end{cases}$$

Note that Π_i^U is continuous on q. It is easy to see that all objective functions are concave in q. Using the first-order conditions, we derive

$$q^{UA} = \frac{2(1-\gamma)}{\gamma(4-3\delta)} \quad \text{and} \quad q^{UB} = \frac{1}{2\gamma} \quad \text{and} \quad q^{UD} = \frac{2^{2/3}\sqrt[3]{K-\delta K}}{\gamma^{2/3}\sqrt[3]{\delta}}.$$

Now we need to discuss the global optimum, we have following cases.

- If $q^{UB} < q_1^U$, then $q^{UA} < q_1^U$ too. Moreover, based on Lemma A.2.5 (ii), we have $q^{UD} < q_1^U$. Hence, Π_i^U increases in $q \in [0, q^{UB}]$ and decreases in $q \in (q^{UB}, q_1^U]$, and decreases in $q \in (q_1^U, q_2^U]$ and $q \in (q_2^U, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{UB}$. Moreover, the condition can be rewritten as $\gamma \geq \gamma_2^U$, where

$$\gamma_2^U := \frac{\delta}{32K(1-\delta)}.$$

- If $q^{UB} > q_1^U$ (i.e., $\gamma < \gamma_2^U$), then $q^{UD} > q_1^U$ via Lemma A.2.5 (ii). And we have following cases.
  - If $q^{UA} \leq q_2^U$, then $q^{UD} \leq q_2^U$ via Lemma A.2.5 (iii). Π_i^U increases in $q \in [0, q^{UD}]$, and decreases in $q \in (q^{UD}, q_2^U]$, and decreases in $q \in (q_2^U, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{UD}$.
  - If $q^{UA} > q_2^U$, we have following cases.
    * If $q^{UD} \leq q_2^U$, Π_i^U increases in $q \in [0, q^{UD}]$, and decreases in $q \in (q^{UD}, q_2^U]$, and increases in $q \in (q_2^U, q^{UA}]$, and decreases in $q \in (q^{UA}, \frac{1}{\gamma}]$. Thus, the optimal decision is either $q^* = q^{UD}$ or $q^* = q^{UA}$. Lemma A.2.5 (iv) shows that there exists a threshold γ_1^U such that $\Pi_i^{UA*}\big|_{\gamma=\gamma_1^U} = \Pi_i^{UD*}\big|_{\gamma=\gamma_1^U}$. Therefore, if $\gamma > \gamma_1 U$, the optimal decision is $q^* = q^{UD}$; otherwise, the optimal decision is $q^* = q^{UA}$.
    * If $q^{UD} > q_2^U$, Π_i^U increases in $q \in [0, q^{UA}]$, and decreases in $q \in (q^{UA}, \frac{1}{\gamma}]$. Thus, the optimal decision is $q^* = q^{UA}$.

□

Lemma A.2.7. *(Quality Cost Thresholds under Flexible and Uniform Pricing) The following statements hold:*

(i) $\gamma_2^U > \gamma_2^F$ *and* $\gamma_2^F > \gamma_1^U$.

(ii) There exists a unique threshold δ^{UA} *such that* $\gamma_1^U < \gamma_1^F$ *if* $\delta < \delta^{UA}$ *and* $\gamma_1^U > \gamma_1^F$ *otherwise.*

(iii) If $max(\gamma_1^U, \gamma_1^F) < \gamma < \gamma_2^F$, $\Pi_i^{F*} > \Pi_i^{U*}$.

(iv) If $\gamma < max(\gamma_1^U, \gamma_1^F)$, $\Pi_i^{F*} > \Pi_i^{U*}$.

Proof of Lemma A.2.7

Part (i): $\gamma_2^U > \gamma_2^F$ holds since $\gamma_2^U = \gamma_2^C$. Note that

$$\frac{\partial F_4}{\partial \gamma} = 2(4-3\delta)^2(\delta-1)K\left(\frac{\sqrt[3]{2}\sqrt[3]{\delta}}{\sqrt[3]{\gamma(\delta-1)(-K)}} - 4\right)$$

and

$$\frac{\sqrt[3]{2}\sqrt[3]{\delta}}{\sqrt[3]{\gamma(\delta-1)(-K)}} - 4 \geq \left.\frac{\sqrt[3]{2}\sqrt[3]{\delta}}{\sqrt[3]{\gamma(\delta-1)(-K)}} - 4\right|_{\gamma=\frac{(\delta-1)^2\delta}{(4-3\delta)^2\left(-\delta+\sqrt{\delta}\sqrt{4-3\delta}+2\right)K}}$$

$$= \frac{\sqrt[3]{2}\sqrt[3]{(4-3\delta)^2\left(-\delta+\sqrt{\delta}\sqrt{4-3\delta}+2\right)}}{1-\delta} - 4 \geq 0,$$

which means F_4 decreases in $\gamma \in (0, \infty)$. Since γ_1^U is the unique zero to F_4, it is equivalent to show that

$$F_4|_{\gamma=\gamma_2^F} = 2\delta^3 - 4\delta^2 - \frac{3(4-3\delta)^2\left(\frac{(\delta-1)^2\delta}{(\delta-4)^2}\right)^{2/3}\sqrt[3]{\delta}}{\sqrt[3]{2}} + \frac{4(\delta-1)^2(4-3\delta)^2\delta}{(\delta-4)^2} + 2\delta \leq 0.$$

Part (ii): Since $F_4|_{\gamma=\gamma_1^F}$ is a function of δ while independent on K, we can easily verify that there exists a $\delta^{UA} \in (0,1)$ such that $F_4|_{\gamma=\gamma_1^F}$ is negative, i.e., $\gamma_1^U < \gamma_1^F$, if $\delta < \delta^{UA}$, and positive, i.e., $\gamma_1^U > \gamma_1^F$ otherwise.

Part (iii): Note that $\Pi_i^{FD*} - \Pi_i^{UD*} =$

$$-\frac{\gamma K^2\left(-12\sqrt[3]{2}\delta^{4/3}+6\sqrt[3]{2}\delta^{7/3}+6\sqrt[3]{2}\sqrt[3]{\delta}-17\delta^2\sqrt[3]{\gamma(1-\delta)K}+40\delta\sqrt[3]{\gamma(1-\delta)K}-32\sqrt[3]{\gamma(1-\delta)K}\right)}{2\delta(\gamma(\delta-1)(-K))^{4/3}},$$

and thus $\Pi_i^{FD*} - \Pi_i^{UD*} \geq 0$ iff $\gamma \geq -\frac{432(\delta-1)^5\delta}{(17\delta^2-40\delta+32)^3K}$. Therefore, it is enough to show that

$$-\frac{432(\delta-1)^5\delta}{(17\delta^2-40\delta+32)^3K} \leq \min(\gamma_1^F, \gamma_1^U).$$

We first show that the inequality holds for γ_1^F, i.e.,

$$\gamma_1^F - \left(-\frac{432(\delta-1)^5\delta}{(17\delta^2-40\delta+32)^3K}\right) = \frac{(\delta-1)\delta\left(\frac{1728(\delta-1)^4}{(17\delta^2-40\delta+32)^3}+\frac{\sqrt{\delta(3\delta^3-16\delta^2-256\delta+512)}}{(\delta-4)^4}-\frac{2}{(\delta-4)^2}\right)}{4K} \geq 0.$$

Next, since F_4 decreases in γ and γ_1^U is the unique zero to F_4, it is equivalent to show that

$$F_4\big|_{\gamma=-\frac{432(\delta-1)^5\delta}{(17\delta^2-40\delta+32)^3K}} \geq 0.$$

Therefore, $\Pi_i^{F*} > \Pi_i^{U*}$ if $max(\gamma_1^U, \gamma_1^F) < \gamma < \gamma_2^F$.

Part (iv): We consider three cases.

Case $\gamma_1^U < \gamma < \gamma_1^F$: Since $\gamma < \gamma_1^F$, we have $\Pi_i^{FA*} > \Pi_i^{FD*}$. Based on (iii), we have $\Pi_i^{FD*} > \Pi_i^{UD*}$ if $\gamma > \gamma_1^U$. Therefore, $\Pi_i^{FA*} > \Pi_i^{FD*} > \Pi_i^{UD*}$.

Case $\gamma_1^F < \gamma < \gamma_1^U$: Note that

$$\Pi_i^{FD*} - \Pi_i^{UA*} = -\frac{4\delta(\delta-1)^3+\gamma\left(3\delta^2-16\delta+16\right)^2K}{2\gamma(4-3\delta)^2(\delta-1)\delta},$$

i.e., $\Pi_i^{FD*} - \Pi_i^{UA*} > 0$ iff $\gamma > -\frac{4(\delta-1)^3\delta}{(3\delta^2-16\delta+16)^2K}$. It is equivalent to show that

$$\gamma_1^F - \left(-\frac{4(\delta-1)^3\delta}{(3\delta^2-16\delta+16)^2K}\right) = \frac{(\delta-1)\delta\left(-\frac{16(\delta-1)^2}{(3\delta^2-16\delta+16)^2}-\frac{\sqrt{\delta(3\delta^3-16\delta^2-256\delta+512)}}{(\delta-4)^4}+\frac{2}{(\delta-4)^2}\right)}{4K} < 0.$$

Case $\gamma < \min(\gamma_1^U, \gamma_1^F)$: We have

$$\Pi_i^{FA*} - \Pi_i^{UA*} = \frac{-23\delta^6+120\delta^5+2800\delta^4-15104\delta^3+28672\delta^2-24576\delta+8192}{16\gamma(4-3\delta)^2(\delta-4)^4} \geq 0.$$

□

Lemma A.2.8. *(Quality Thresholds with Delay Consumers) The following statements hold:*

(i) $p_{i1}^{CB} > \frac{q}{2}$, $p_{i2}^{CB} < \frac{q}{2}$, *and* $p_{i2}^{CA} < \frac{q}{2}$.

(ii) $p_{i1}^{CB} > p_{i2}^{CB}$, $p_{i1}^{CA} > p_{i2}^{CA}$ *and* $p_{i1}^{CB} > p_{i2}^{CA}$.

(iii) There exists a unique threshold q_2^C *such that* $\Pi_i^{CA*} > \Pi_i^{CD*}$ *if* $q > q_2^C$ *and* $\Pi_i^{CA*} < \Pi_i^{CD*}$ *otherwise.*

Proof of Lemma A.2.8

Part (i): We have

$$\begin{aligned}
p_{i1}^{CB} - \frac{q}{2} &= \frac{3q(1-\rho)}{2(\rho+7)} \geq 0, \\
\frac{q}{2} - p_{i2}^{CB} &= \frac{q(1-\rho)}{2(\rho+7)} \geq 0, \\
\frac{q}{2} - p_{i2}^{CA} &= \frac{q\left(\delta^2(\rho+1) - 3\delta(\rho+1) + 2(\rho-1)\right)}{2(2-\delta)(\delta(\rho+3)-\rho-7)} \geq 0.
\end{aligned}$$

Part (ii): We have

$$\begin{aligned}
p_{i1}^{CB} - p_{i2}^{CB} &= \frac{2q(1-\rho)}{\rho+7} \geq 0, \\
p_{i1}^{CA} - p_{i2}^{CA} &= \frac{(\delta^2 - 5\delta + 8)\, q(\rho-1)}{2(2-\delta)(\delta(\rho+3)-\rho-7)} \geq 0, \\
p_{i1}^{CB} - p_{i2}^{CA} &= \frac{\delta q(\delta(3\rho+5) - 7\rho - 17)}{(2-\delta)(\rho+7)(\delta(\rho+3)-\rho-7)} \geq 0.
\end{aligned}$$

Part (iii): Since

$$\Pi_i^{CA*} - \Pi_i^{CD*} = -\frac{(\delta-1)K(\rho+7)}{\delta} - \frac{(\rho+3)\sqrt{(\delta-1)(-K)q}}{\sqrt{\delta}} + \frac{(\delta-1)q(\delta(\rho+2)-2(\rho+3))^2}{4(\delta-2)^2(\delta(\rho+3)-\rho-7)}$$

is convex in q, and $\Pi_i^{CA*} - \Pi_i^{CD*} < 0$ when $p_{i2}^{CA} = p_{i2}^{CD}$. There exists a unique threshold q_2^C such that $\Pi_i^{CA*} > \Pi_i^{CD*}$ if $q > q_2^C$ and $\Pi_i^{CA*} < \Pi_i^{CD*}$ otherwise. □

Appendix B

Proofs for Chapter 3

B.1 Model Notations

B.2 Proofs of Results in Section 3.4 and Section 3.5

For simplicity of notation, we denote $\beta = \delta\Lambda$. All the subsequent proofs are conducted based on the general regulation conditions: $\alpha \geq 0$, $\beta = \delta\Lambda \leq 1$ and $\theta \in [0, 1/2]$. In the symmetric case, since both firms are identical, we omit the subscripts "A" and "B".

In addition, we further remark that the analysis involves some straightforward yet cumbersome algebraic manipulations. For expositional brevity, we may omit some intermediate calculation steps and some lengthy equations. The complete proof with all the involving equations and formulas will be available from the authors upon request.

Proof of Theorem 1 Given $\beta \leq 1$, it can be verified the existence and uniqueness of the equilibrium by showing that π is concave in p_i (Rosen 1965a). In the case of

Table B.1: Summary of Model Notations

Parameters	
Λ	Total market size
θ_i	Measures the effect of firm j's quality on firm i's demand, $0 \leq \theta_i < \frac{1}{2} \quad i, j \in \{A, B\}, i \neq j$
δ_i	Firm i's learning speed, $i \in \{A, B\}$
q_{it}^k	Firm i's quality level in t period under k pricing model, $q_{it+1}^k = q_{it}^k + \delta D_{it}^k \quad i \in \{A, B\}, k \in \{c, d\}$, where c means committed price, d means dynamic price
α	Substitution parameter, $\alpha \geq 0$, indicates the intensity of price competition
D_{it}^k	Firm i's demand in t period under k pricing model, $D_{it}^k = \frac{1}{2}\left[\Lambda\left(\left(1-\theta_i\right) q_{it}^k + \theta_i q_{jt}^k\right) - p_{it}^k + \alpha\left(p_{jt}^k - p_{it}^k\right)\right] \quad i, j \in \{A, B\}, i \neq j, k \in \{c, d\}$, where c means committed price, d means dynamic price
π_i^k	Firm i's total profit under k pricing model, $i \in \{A, B\}, k \in \{c, d\}$, where c means committed price, d means dynamic price
Decisions	
p_{it}^k	Firm i's pricing decision in t period under k pricing model, $i \in \{A, B\}, k \in \{c, d\}$

committed pricing, we have

$$\frac{\partial^2 \pi^c}{\partial (p_1^c)^2} = \frac{\partial^2 \pi^c}{\partial (p_2^c)^2} = -(1+\alpha) < 0.$$

Solving the system of best response functions, we find the unique symmetric equilibrium prices as follow:

$$\begin{aligned}
p_2^{c*} &= \frac{2\Lambda q(\alpha(\beta+2)+\beta+4)}{4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16} > 0 \\
p_1^{c*} &= \frac{\Lambda q\left(\beta^2(\alpha(2\theta-1)+\theta-1)+2\beta(\alpha(2\theta-1)+\theta-1)+4\alpha+8\right)}{4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16} \\
&\geq \frac{6\alpha\theta+\alpha+3\theta+5}{4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16} > 0
\end{aligned}$$

In the case of dynamic pricing, we have

$$\begin{aligned}
\frac{\partial^2\pi_2^d}{\partial(p_2^d)^2} &= -(1+\alpha)<0\\
\frac{\partial^2\pi^d}{\partial(p_1^d)^2} &= -\frac{-(\alpha+1)\beta^2\left(\alpha^2-(\alpha+2)(2\alpha+1)\theta+4\alpha+2\right)^2+36\alpha^5+228\alpha^4+544\alpha^3+608\alpha^2+320\alpha+64}{4(\alpha+2)^2(3\alpha+2)^2}\\
&\leq -\frac{35\alpha^5+219\alpha^4+516\alpha^3+572\alpha^2+300\alpha+60}{4(\alpha+2)^2(3\alpha+2)^2}<0.
\end{aligned}$$

Solving the system of best response functions, we find the unique symmetric equilibrium prices as follow:

$$\begin{aligned}
p_2^{d*} &= \frac{\left(3\alpha^2+8\alpha+4\right)\Lambda q(\alpha(\beta+2)+\beta+4)}{(\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32}>0\\
p_1^{d*} &= \frac{\Lambda q N_0}{(\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32}\\
&\geq \frac{\alpha^3(6\theta+3)+\alpha^2(21\theta+13)+\alpha(21\theta+22)+6\theta+10}{(\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32}>0
\end{aligned},$$

where

$$\begin{aligned}
N_0 \equiv\ & 2(\alpha+1)\beta\left(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1)\right)+6\alpha^3+28\alpha^2+40\alpha+16+\\
& (\alpha+1)\beta^2\left(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1)\right)
\end{aligned}.$$

□

Proof of Proposition 9 Following the proof of Proposition 1, we have

$$\begin{aligned}
p_2^{c*}-p_1^{c*} &= \frac{\beta\Lambda q(\alpha(-2\beta\theta+\beta-4\theta+4)+\beta(-\theta)+\beta-2\theta+4)}{4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16}\\
&\geq \frac{\beta\Lambda q(4\alpha+\beta+6)}{2(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16)}>0\\
p_2^{d*}-p_1^{d*} &= \frac{(\alpha+1)\beta\Lambda q(\alpha^2\beta+5\alpha^2-(\alpha+2)(2\alpha+1)(\beta+2)\theta+4\alpha\beta+16\alpha+2\beta+8)}{(\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32}\\
&\geq \frac{(\alpha+1)\beta\Lambda q(3\alpha+2)(2\alpha+\beta+6)}{2((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32)}>0
\end{aligned}.$$

□

Proof of Proposition 10 Following the proof of Proposition 1, we have

$$
\begin{aligned}
\frac{\partial p_1^{c*}}{\partial \theta} &= \frac{4\left(2\alpha^2+5\alpha+2\right)\delta\Lambda^2 q(\alpha(\delta\Lambda+2)+\delta\Lambda+4)}{\left(4\alpha^2+\alpha(\delta^2(2\theta-1)\Lambda^2+16)+\delta^2(\theta-1)\Lambda^2+16\right)^2} > 0 \\
\frac{\partial p_2^{c*}}{\partial \theta} &= -\frac{2(2\alpha+1)\delta^2\Lambda^3 q(\alpha(\delta\Lambda+2)+\delta\Lambda+4)}{\left(4\alpha^2+\alpha(\delta^2(2\theta-1)\Lambda^2+16)+\delta^2(\theta-1)\Lambda^2+16\right)^2} < 0 \\
\frac{\partial p_1^{c*}}{\partial \alpha} &= \frac{4\Lambda q\left(\delta^2\Lambda^2(\alpha(-2(\alpha+1)\theta+\alpha+2)+\theta+1)-2(\alpha+2)\delta\Lambda(2(\alpha-1)\theta-\alpha)-4(\alpha+2)^2\right)}{\left(\delta^2\Lambda^2(\alpha(2\theta-1)+\theta-1)+4(\alpha+2)^2\right)^2} \\
&\leq -\frac{4\Lambda q\left(\alpha^2(6\theta+1)+2\alpha(3\theta+5)-9\theta+15\right)}{\left(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16\right)^2} < 0 \\
\frac{\partial p_2^{c*}}{\partial \alpha} &= -\frac{2\Lambda q\left(4\alpha^2(\delta\Lambda+2)+8\alpha(\delta\Lambda+4)+\delta^3\theta\Lambda^3+2\delta^2(3\theta-1)\Lambda^2+32\right)}{\left(4\alpha^2+\alpha(\delta^2(2\theta-1)\Lambda^2+16)+\delta^2(\theta-1)\Lambda^2+16\right)^2} < 0 \\
\frac{\partial p_1^{c*}}{\partial \delta} &= \frac{2\Lambda^2 q(\alpha(2\theta-1)+\theta-1)\left(4\left(\alpha^2+3\alpha+2\right)\beta+4\alpha^2+\beta^2(-2\alpha\theta+\alpha-\theta+1)+16\alpha+16\right)}{\left(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16\right)^2} < 0 \\
\frac{\partial p_2^{c*}}{\partial \delta} &= \frac{2\Lambda^2 q\left(4\alpha^3+20\alpha^2-(\alpha+1)\beta^2(\alpha(2\theta-1)+\theta-1)-4(\alpha+2)\beta(\alpha(2\theta-1)+\theta-1)+32\alpha+16\right)}{\left(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16\right)^2} \\
&\geq \frac{8\Lambda^2 q(\alpha+1)(\alpha+2)^2}{\left(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16\right)^2} > 0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial p_1^{d*}}{\partial \theta} &= \frac{2(\alpha+2)^3\left(6\alpha^3+13\alpha^2+9\alpha+2\right)\beta\Lambda q(\alpha(\beta+2)+\beta+4)}{\left((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32\right)^2} > 0 \\
\frac{\partial p_2^{d*}}{\partial \theta} &= -\frac{(\alpha+2)^2\left(6\alpha^3+13\alpha^2+9\alpha+2\right)\beta^2\Lambda q(\alpha(\beta+2)+\beta+4)}{\left((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32\right)^2} < 0 \\
\frac{\partial p_1^{d*}}{\partial \alpha} &= -\frac{2(\alpha+2)\Lambda q N_1}{\left((\alpha+1)\beta^2(-\alpha^2+(\alpha+2)(2\alpha+1)\theta-4\alpha-2)+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32\right)^2} < 0 \\
\frac{\partial p_2^{d*}}{\partial \alpha} &= -\frac{\Lambda q N_2}{\left((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32\right)^2} < 0 \\
\frac{\partial p_1^{d*}}{\partial \delta} &= -\frac{2(\alpha+1)q\Lambda^2(2(1-\theta)+\alpha^2(1-2\theta)+\alpha(4-5\theta))N_3}{\left((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32\right)^2} < 0 \\
\frac{\partial p_2^{d*}}{\partial \delta} &= \frac{\left(3\alpha^3+11\alpha^2+12\alpha+4\right)q\Lambda^2 N_4}{\left((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32\right)^2} > 0
\end{aligned}
,
$$

where

$$
\begin{aligned}
N_1 &\equiv 18\alpha^5 + 132\alpha^4 + 368\alpha^3 + 480\alpha^2 + (\alpha+1)^2\beta^2(6\alpha(\alpha+1)(\alpha+2)\theta - \\
&\quad 3\alpha(\alpha(\alpha+6)+6)-4) + 288\alpha + 64 + \\
&\quad 2(\alpha+2)\beta((\alpha+2)(\alpha(6\alpha(\alpha+1)-1)-2)\theta - \alpha(\alpha(3\alpha(\alpha+6)+22)+8)) \\
&\geq 9\alpha^5(2\theta+1) + 30\alpha^4(3\theta+2) + \alpha^3(148\theta+195) + 6\alpha^2(13\theta+53) + \alpha(230-12\theta) - 16\theta + 60 \\
&\geq 60 - 16\theta > 0 \\
N_2 &\equiv (\alpha+1)^2\beta^3\left(\alpha^2(\theta+4) + 4\alpha(\theta+1) + 4\theta\right) + 36\alpha^6 + 336\alpha^5 + \\
&\quad 1264\alpha^4 + 2432\alpha^3 + 2496\alpha^2 + 1280\alpha + 256 + \\
&\quad 2(\alpha+2)\beta^2\left(\alpha^3(7\theta+1) + 6\alpha^2(4\theta-1) + 2\alpha(12\theta-5) + 8\theta - 4\right) + \\
&\quad 2(\alpha+2)^3(3\alpha+2)^2\alpha\beta \\
&\geq 2\left(2(\alpha+1)^3\alpha\beta^3 + (\alpha+2)^3(3\alpha+2)^2\alpha\beta + 640\alpha + 128\right) + \\
&\quad 2\left(18\alpha^6 + 168\alpha^5 + 632\alpha^4 + 1216\alpha^3 + 1248\alpha^2\right) + \\
&\quad 2((\alpha+2)\left(\alpha^3 - 6\alpha^2 - 10\alpha - 4\right)\beta^2) \\
&\geq 16(16-\beta^2) > 0 \\
N_3 &\equiv 32 + 96\alpha + 96\alpha^2 + 40\alpha^3 + 6\alpha^4 + 2(\alpha+2)^2\left(3\alpha^2 + 5\alpha + 2\right)\beta - \\
&\quad (\alpha+1)\beta^2\left(\alpha^2(2\theta-1) + \alpha(5\theta-4) + 2(\theta-1)\right) \\
&\geq 2(\alpha+2)^3(3\alpha+2) > 0 \\
N_4 &\equiv 32 + 96\alpha + 96\alpha^2 + 40\alpha^3 + 6\alpha^4 - 4(\alpha+2)\beta\left(\alpha^2(2\theta-1) + \alpha(5\theta-4) + 2(\theta-1)\right) - \\
&\quad (\alpha+1)\beta^2\left(\alpha^2(2\theta-1) + \alpha(5\theta-4) + 2(\theta-1)\right) \\
&\geq 2(\alpha+2)^3(3\alpha+2) > 0
\end{aligned}
.
$$

□

Proof of Proposition 11 Following the proof of Proposition 1, recall that $QI^{k*} = \delta\frac{1}{2}(\Lambda q_1 - p_1^{k*})$, we have

$$
\begin{aligned}
\frac{\partial QI^{k*}}{\partial\theta} &= -\frac{\delta}{2}\frac{\partial p_1^{k*}}{\partial\theta} < 0 \\
\frac{\partial QI^{k*}}{\partial\alpha} &= -\frac{\delta}{2}\frac{\partial p_1^{k*}}{\partial\alpha} > 0 \\
\frac{\partial QI^{k*}}{\partial\delta} &= \frac{1}{2}(\Lambda q_1 - p_1^{k*} - \delta\frac{\partial p_1^{k*}}{\partial\delta}) > 0
\end{aligned}
.
$$

□

Proof of Proposition 12 Following the proof of Proposition 1, we have

$$
\begin{aligned}
\pi_1^{k*} &= \frac{p_1^{k*}}{2}(\Lambda q_1 - p_1^{k*}) \\
\pi_2^{k*} &= \frac{p_2^{k*}}{2}(\Lambda(q_1 + QI^{k*}) - p_2^{k*}) \\
\pi^{k*} &= \pi_1^{k*} + \pi_2^{k*}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial \pi_1^{k*}}{\partial \theta} &= \frac{\partial p_1^{k*}}{\partial \theta}(\frac{\Lambda q_1}{2} - p_1^{k*}) > 0 \\
\frac{\partial \pi_1^{k*}}{\partial \alpha} &= \frac{\partial p_1^{k*}}{\partial \alpha}(\frac{\Lambda q_1}{2} - p_1^{k*}) < 0 \\
\frac{\partial \pi_1^{k*}}{\partial \delta} &= \frac{\partial p_1^{k*}}{\partial \delta}(\frac{\Lambda q_1}{2} - p_1^{k*}) < 0 \\
\frac{\partial \pi_2^{k*}}{\partial \theta} &= \frac{\partial p_2^{k*}}{\partial \theta}(\frac{\Lambda q_1}{2} - p_2^{k*}) + \frac{\Lambda}{2}(\frac{\partial p_2^{k*}}{\partial \theta} QI^{k*} + p_2^{k*}\frac{\partial QI^{k*}}{\partial \theta}) \\
&= \frac{\partial p_2^{k*}}{\partial \theta}(\frac{\Lambda q_1}{2} - p_2^{k*} + \frac{\Lambda}{2} QI^{k*}) - \frac{\beta}{4} p_2^{k*}\frac{\partial p_1^{k*}}{\partial \theta} < 0 \\
\frac{\partial \pi_2^{k*}}{\partial \alpha} &= \frac{\partial p_2^{k*}}{\partial \alpha}(\frac{\Lambda q_1}{2} - p_2^{k*}) + \frac{\Lambda}{2}(\frac{\partial p_2^{k*}}{\partial \alpha} QI^{k*} + p_2^{k*}\frac{\partial QI^{k*}}{\partial \alpha}) \\
&= \frac{\partial p_2^{k*}}{\partial \alpha}(\frac{\Lambda q_1}{2} - p_2^{k*} + \frac{\Lambda}{2} QI^{k*}) - \frac{\beta}{4} p_2^{k*}\frac{\partial p_1^{k*}}{\partial \alpha} \\
&= \frac{2\Lambda^2 q^2(\alpha(\beta+2)+\beta+4)N_5}{(4\alpha^2+\alpha(\beta^2(2\theta-1)+16)+\beta^2(\theta-1)+16)^3} \quad (k=c) \\
&= \frac{\left(3\alpha^2+8\alpha+4\right)q^2\Lambda^2(\alpha(\beta+2)+\beta+4)N_6}{2((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32)^3} \quad (k=d) \\
\frac{\partial \pi_2^{k*}}{\partial \delta} &= \frac{\partial p_2^{k*}}{\partial \delta}(\frac{\Lambda q_1}{2} - p_2^{k*}) + \frac{\Lambda}{2}(\frac{\partial p_2^{k*}}{\partial \delta} QI^{k*} + p_2^{k*}\frac{\partial QI^{k*}}{\partial \delta}) \\
&= \frac{\partial p_2^{k*}}{\partial \theta}(\frac{\Lambda q_1}{2} - p_2^{k*} + \frac{\Lambda}{2} QI^{k*}) - \frac{\Lambda}{4} p_2^{k*}(\Lambda q_1 - p_1^{k*} - \delta\frac{\partial p_1^{k*}}{\partial \delta}) > 0
\end{aligned},
$$

where

$$
\begin{aligned}
N_5 &\equiv -32\alpha - 32\alpha^2 - 8\alpha^3 - 4\left(\alpha^3+\alpha^2-4\alpha-4\right)\beta - 2\beta^2\left(-2\alpha^2\theta+\alpha^2+\alpha\theta+\alpha+4\theta\right) + \\
&\quad (\alpha+1)\beta^3(\alpha(2\theta-1)-\theta-1) \\
\frac{\partial N_5}{\partial \alpha} &\leq 0 \\
N_5|_{(\alpha=0)} &= \beta\left(16-\beta^2(\theta+1)-8\beta\theta\right) > 0 \\
\lim_{\alpha\to+\infty} N_5 &\to -\infty \\
N_6 &\equiv -2(\alpha+2)^3(3\alpha+2)^2\left(\alpha^2-\alpha-2\right)\beta - 36\alpha^7 - \\
&\quad 336\alpha^6 - 1264\alpha^5 - 2432\alpha^4 - 2496\alpha^3 - 1280\alpha^2 - 256\alpha + \\
&\quad 2\left(\alpha^2+3\alpha+2\right)\beta^2\left(\alpha^4(6\theta-3)+\alpha^3(17\theta-22)+6\alpha^2(\theta-5)-12\alpha(\theta+1)-8\theta\right) + \\
&\quad (\alpha+1)^2\beta^3\left(\alpha^4(6\theta-3)+\alpha^3(29\theta-28)+\alpha^2(44\theta-58)+20\alpha(\theta-2)-8\right) \\
\frac{\partial^2 N_6}{\partial \alpha^2} &\leq 0 \\
N_6|_{(\alpha=0)} &= 8\beta\left(16-\beta^2-4\beta\theta\right) > 0 \\
\lim_{\alpha\to+\infty} N_6 &\to -\infty
\end{aligned}.
$$

In addition, following the similar way, we can show that $\frac{\partial \pi^{k*}}{\partial \theta}$ decreases in $\theta \in [0, 1/2]$ and there exists an interior solution for $\frac{\partial \pi^{k*}}{\partial \theta} = 0$; $\frac{\partial \pi^{k*}}{\partial \delta} > 0$; $\frac{\partial \pi^{k*}}{\partial \alpha}$ decreases in α and there exists an interior solution for $\frac{\partial \pi^{k*}}{\partial \alpha} = 0$. The details are omitted for brevity. □

Proof of Proposition 13 Following the proof of Proposition 1, we have

$$\begin{array}{rcl} P^c(\delta,\theta) & = & \frac{\beta(\alpha(\beta+2)+\beta+4)(\alpha(1-2\theta)+1-\theta)}{4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16} > 0 \\ P^d(\delta,\theta) & = & \frac{(\alpha+1)\beta(\alpha(\beta+2)+\beta+4)\left(\alpha^2(1-2\theta)+\alpha(4-5\theta)+2(1-\theta)\right)}{(\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32} > 0 \\ R^c(\delta,\theta) & = & \frac{\beta(\alpha(\beta+2)+\beta+4)(\alpha(1-2\theta)+1-\theta)N_7}{(\alpha+1)(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16)^2} > 0 \\ R^d(\delta,\theta) & = & 1-\frac{2(\alpha+2)^2\left(\alpha^2(-2\beta\theta+\beta+14)+3\alpha^3+\alpha(\beta(4-5\theta)+20)+\beta(2-2\theta)+8\right)N_8}{((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32)^2} > 0 \end{array},$$

where

$$\begin{array}{rcl} N_7 & \equiv & 4\alpha^3+16\alpha^2+\beta^2(-2\alpha\theta+\alpha-\theta+1)-2(\alpha+2)\beta(\alpha(2\theta-1)+\theta-1)+16\alpha > 0 \\ N_8 & \equiv & 16+40\alpha+28\alpha^2+6\alpha^3+2(\alpha+1)\beta\left(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1)\right)+ \\ & & (\alpha+1)\beta^2\left(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1)\right) \end{array}.$$

In addition, conducting sensitivity analysis with respect to θ and δ, we can prove other parts of this proposition. The details are omitted for brevity. □

Proof of Proposition 14 Following the proof of Proposition 1, we have

$$\begin{array}{rl} & p_1^{d*}-p_1^{c*} \\ = & \frac{2\alpha(\alpha+2)\beta q\Lambda(\alpha(\beta+2)+\beta+4)(\alpha(\alpha+1)-(\alpha+2)(2\alpha+1)\theta)}{(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16)((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32)} \\ & p_2^{d*}-p_2^{c*} \\ = & \frac{\alpha\beta^2 q\Lambda(\alpha(\beta+2)+\beta+4)((\alpha+2)(2\alpha+1)\theta-\alpha(\alpha+1))}{(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16)((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32)} \\ & QI^{d*}-QI^{c*} = \frac{\delta}{2}(p_1^{c*}-p_1^{d*}) \\ & \pi^{d*}-\pi^{c*} \\ = & \frac{\alpha\delta q^2(\alpha(\beta+2)+\beta+4)\Lambda^3(\alpha(\alpha+1)-(\alpha+2)(2\alpha+1)\theta)N_9}{2(4\alpha^2+\beta^2(\alpha(2\theta-1)+\theta-1)+16\alpha+16)^2((\alpha+1)\beta^2(\alpha^2(2\theta-1)+\alpha(5\theta-4)+2(\theta-1))+6\alpha^4+40\alpha^3+96\alpha^2+96\alpha+32)^2} \end{array},$$

where N_9 decreases in θ, is positive at $\theta = 0$, and may be negative at $\theta = 1/2$. Given $\beta \leq 1$, all above denominators can be proved to be positive, and it is clear that the sign of $p_1^{d*} - p_1^{c*}$, $p_2^{d*} - p_2^{c*}$ and $QI^{d*} - QI^{c*}$ depend only on $(\alpha(\alpha+1) - (\alpha+2)(2\alpha+1)\theta)$, while the sign of $\pi^{d*} - \pi^{c*}$ depends on both $(\alpha(\alpha+1) - (\alpha+2)(2\alpha+1)\theta)$ and N_9. □

Proof of Proposition 15 Based on the proof of Proposition 14, this proposition can be proved directly by checking the sign of the first derivative. We omit the details for brevity. □

Proof of Proposition 16 Similar to the proof of Proposition 1, given $\beta \leq 1$, it can be verified the existence and uniqueness of the equilibrium by showing that π_i is concave in p_i (Rosen 1965a), which can be done by checking the second derivative. Solving the system of best responses, the equilibrium prices and profits are given respectively:

$$\begin{aligned}
p_{A1}^{c*} &= \frac{q\Lambda N_9}{4(\alpha+1)^2\beta^2(\alpha^2+(\alpha+1)(\alpha+2)\theta^2-(\alpha(\alpha+7)+4)\theta+4\alpha+2)-8(\alpha+2)^2(3\alpha+2)^2} > 0\\
p_{A2}^{c*} &= \frac{\Lambda q\left(\alpha^2(\alpha+1)\beta^2\theta(2\theta-1)+2(\alpha+1)(3\alpha+2)\delta\Lambda((\alpha+2)\theta-2(\alpha+1))-4(\alpha+2)(3\alpha+2)^2\right)}{2(\alpha+1)^2\beta^2(\alpha^2+(\alpha+1)(\alpha+2)\theta^2-(\alpha(\alpha+7)+4)\theta+4\alpha+2)-4(\alpha+2)^2(3\alpha+2)^2} > 0\\
p_{B1}^{c*} &= \frac{q\Lambda N_{10}}{4(\alpha+1)^2\beta^2(\alpha^2+(\alpha+1)(\alpha+2)\theta^2-(\alpha(\alpha+7)+4)\theta+4\alpha+2)-8(\alpha+2)^2(3\alpha+2)^2} > 0\\
p_{B2}^{c*} &= \frac{q\Lambda N_{11}}{2(\alpha+1)^2\beta^2(\alpha^2+(\alpha+1)(\alpha+2)\theta^2-(\alpha(\alpha+7)+4)\theta+4\alpha+2)-4(\alpha+2)^2(3\alpha+2)^2} > 0
\end{aligned}$$

and

$$\begin{aligned}
p_{A1}^{d*} &= \frac{q\Lambda N_{13}}{(3\alpha^2+8\alpha+4)N_{12}} > 0\\
p_{A2}^{d*} &= \frac{q\Lambda N_{14}}{N_{12}} > 0\\
p_{B1}^{d*} &= \frac{q\Lambda N_{15}}{(3\alpha^2+8\alpha+4)N_{12}} > 0\\
p_{B2}^{d*} &= \frac{q\Lambda N_{16}}{N_{12}} > 0
\end{aligned},$$

where

$$
\begin{aligned}
N_9 &\equiv \alpha^2(\alpha+1)\beta^3\theta\left(2\theta^2-3\theta+1\right)+2\left(3\alpha^2+5\alpha+2\right)\beta^2(\theta-1)(\alpha(\theta-2)+2(\theta-1))- \\
&\quad 8(\alpha+2)(3\alpha+2)^2-4(3\alpha+2)\beta\left((\alpha(3\alpha+4)+2)\theta-2(\alpha+1)^2\right) \\
N_{10} &\equiv -8(\alpha+2)(3\alpha+2)^2+\alpha(\alpha+1)^2\beta^3\theta\left(2\theta^2-3\theta+1\right)-4\alpha(\alpha+1)(3\alpha+2)\beta(3\theta-1)+ \\
&\quad 2(\alpha+1)^2\beta^2\left(\alpha((\theta-7)\theta+4)+2(\theta-1)^2\right) \\
N_{11} &\equiv (\alpha+1)(\alpha(\alpha+4)+2)\beta^2(\theta-1)(2\theta-1) \\
&\quad -2(\alpha+1)(3\alpha+2)\beta((\alpha+2)\theta+\alpha)-4(\alpha+2)(3\alpha+2)^2 \\
N_{12} &\equiv -(\alpha+1)^2\beta^2\left(2(\alpha+1)^2(\alpha+2)^2\theta^2-2\alpha(\alpha(\alpha+4)(\alpha+8)+28)\theta\right) \\
&\quad -(\alpha+1)^2\beta^2\left(\alpha(\alpha+2)(\alpha(5\alpha+14)+16)-16\theta+8\right)+ \\
&\quad 54\alpha^6+432\alpha^5+1368\alpha^4+2176\alpha^3+1824\alpha^2+768\alpha+128 \\
N_{13} &\equiv 2(3\alpha+2)^2\left(\alpha^2+3\alpha+2\right)\beta\left(3\alpha^3(\theta-1)+2\alpha^2(5\theta-6)+2\alpha(5\theta-6)+4(\theta-1)\right) \\
&\quad +9264\alpha^4+162\alpha^7+1404\alpha^6+4968\alpha^5 \\
&\quad -\alpha^2(\alpha+1)^3\beta^3(2\theta-1)\left(\alpha^2\left(\theta^2-\theta-2\right)+\alpha\left(4\theta^2-4\theta-2\right)+4(\theta-1)\theta\right) \\
&\quad -(\alpha+2)\left(3\alpha^2+5\alpha+2\right)^2\beta^2(\alpha(\theta-2)+2(\theta-1))^2+9824\alpha^3+5952\alpha^2+1920\alpha+256 \\
N_{14} &\equiv -(\alpha+1)^2\alpha^2\beta^2(2\theta-1)((\alpha+2)\theta+\alpha)+54\alpha^5+324\alpha^4+720\alpha^3+736\alpha^2+352\alpha+64- \\
&\quad (\alpha+1)(\alpha+2)(3\alpha+2)^2\beta((\alpha+2)\theta-2(\alpha+1)) \\
N_{15} &\equiv -(\alpha+1)^2(\alpha+2)(3\alpha+2)\beta^2\left(7\alpha^3+20\alpha^2+(\alpha+1)(\alpha+2)^2\theta^2\right)+256 \\
&\quad -(\alpha+1)^2(\alpha+2)(3\alpha+2)\beta^2\left(-2(\alpha+2)(\alpha(5\alpha+6)+2)\theta+16\alpha+4\right)+5952\alpha^2 \\
&\quad +2\alpha(\alpha+1)^2(\alpha+2)(3\alpha+2)^2\beta(3(\alpha+2)\theta-2)+4968\alpha^5+9264\alpha^4+9824\alpha^3+1920\alpha \\
&\quad -\alpha(\alpha+1)^4\beta^3(2\theta-1)((\alpha+2)\theta+\alpha)((\alpha+2)\theta-2(\alpha+1))+162\alpha^7+1404\alpha^6 \\
N_{16} &\equiv (\alpha+1)(\alpha+2)(3\alpha+2)^2\beta((\alpha+2)\theta+\alpha)+54\alpha^5+324\alpha^4+720\alpha^3 \\
&\quad -(\alpha+1)^2(\alpha(\alpha+4)+2)\beta^2(2\theta-1)((\alpha+2)\theta-2(\alpha+1))+736\alpha^2+352\alpha+64
\end{aligned}
.
$$

Furthermore, for part (ii) and part (iii), we have

$$
\begin{aligned}
p^{c*}_{A2}-p^{c*}_{A1} &= \tfrac{q\beta\Lambda N_{17}}{4(\alpha+1)^2\beta^2(\alpha^2+(\alpha+1)(\alpha+2)\theta^2-(\alpha(\alpha+7)+4)\theta+4\alpha+2)-8(\alpha+2)^2(3\alpha+2)^2}>0 \\
p^{c*}_{B2}-p^{c*}_{B1} &= \tfrac{(\alpha+1)q\beta\Lambda(4+\beta(1-\theta))(\alpha(\alpha+1)\beta\theta(2\theta-1)+2(3\alpha+2)(\alpha(\theta-1)-\theta))}{4(\alpha+1)^2\beta^2(\alpha^2+(\alpha+1)(\alpha+2)\theta^2-(\alpha(\alpha+7)+4)\theta+4\alpha+2)-8(\alpha+2)^2(3\alpha+2)^2}>0 \\
p^{d*}_{A2}-p^{d*}_{A1} &= \tfrac{(\alpha+1)q\beta\Lambda N_{19}}{(\alpha+2)(3\alpha+2)N_{18}}>0 \\
p^{d*}_{B2}-p^{d*}_{B1} &= \tfrac{(\alpha+1)q\beta\Lambda N_{20}}{(\alpha+2)(3\alpha+2)N_{18}}>0
\end{aligned}
,
$$

and

$$\begin{aligned}
p_{A2}^{c*} - p_{B2}^{c*} &= \frac{(\alpha+1)\beta(1-2\theta)\Lambda q\left(\alpha^2(\beta+6)+4\alpha(\beta(-\theta)+\beta+4)+\beta(2-2\theta)+8\right)}{-2(\alpha+1)^2\beta^2(\alpha^2(\theta^2-\theta+1)+\alpha(3\theta^2-7\theta+4)+2(\theta-1)^2)+36\alpha^4+192\alpha^3+352\alpha^2+256\alpha+64} > 0 \\
p_{B1}^{c*} - p_{A1}^{c*} &= \frac{q\beta\Lambda N_{21}}{4(-(\alpha+1)^2\beta^2(\alpha^2(\theta^2-\theta+1)+\alpha(3\theta^2-7\theta+4)+2(\theta-1)^2)+18\alpha^4+96\alpha^3+176\alpha^2+128\alpha+32)} > 0 \\
p_{A2}^{d*} - p_{B2}^{d*} &= \frac{(\alpha+1)\beta\Lambda(1-2\theta)q\left((\alpha+1)\beta\left(\alpha^2(10-4\theta)+3\alpha^3-2\alpha(5\theta-6)-4\theta+4\right)+9\alpha^4+48\alpha^3+88\alpha^2+64\alpha+16\right)}{N_{12}} > 0 \\
p_{B1}^{d*} - p_{A1}^{d*} &= \frac{q(1+\alpha)\beta\Lambda N_{22}}{(3\alpha^2+8\alpha+4)N_{12}} > 0
\end{aligned},$$

where

$$\begin{aligned}
N_{17} &\equiv 4(3\alpha+2)\left((\alpha(4\alpha+7)+4)\theta-4(\alpha+1)^2\right)-\alpha^2(\alpha+1)\beta^2\theta\left(2\theta^2-3\theta+1\right)- \\
&\quad 2(\alpha+1)\beta\left(\alpha^2((\theta-8)\theta+6)+2\alpha(\theta-1)(4\theta-5)+4(\theta-1)^2\right) \\
N_{18} &\equiv (\alpha+1)^2\beta^2\left(2(\alpha+1)^2(\alpha+2)^2\theta^2-2\alpha(\alpha(\alpha+4)(\alpha+8)+28)\theta\right) \\
&\quad (\alpha+1)^2\beta^2\left(\alpha(\alpha+2)(\alpha(5\alpha+14)+16)-16\theta+8\right)-2(\alpha+2)^3(3\alpha+2)^3 \\
N_{19} &\equiv -(\alpha+1)(\alpha+2)(3\alpha+2)\beta\left(\alpha^3((\theta-13)\theta+13)+2\alpha^2(\theta-1)(5\theta-16)+4\alpha(\theta-1)(5\theta-7)\right) \\
&\quad +(\alpha+2)(3\alpha+2)^2((\alpha+2)(\alpha(9\alpha+16)+8)\theta-2(\alpha(\alpha(6\alpha+23)+24)+8)) \\
&\quad -(\alpha+1)(\alpha+2)(3\alpha+2)\beta 8(\theta-1)^2-\alpha^2(\alpha+1)^2\beta^2(2\theta-1)((\alpha+2)\theta+\alpha)((\alpha+2)\theta-2(\alpha+ \\
N_{20} &\equiv (\alpha+1)(\alpha+2)(3\alpha+2)\beta\left((\alpha+2)(\alpha(\alpha+5)+2)\theta^2+(\alpha(\alpha(5\alpha+6)-6)-4)\theta\right) \\
&\quad +(\alpha+2)(3\alpha+2)^2((\alpha+2)(\alpha(3\alpha-2)-4)\theta-\alpha(3\alpha(\alpha+4)+8)) \\
&\quad +(\alpha+1)(\alpha+2)(3\alpha+2)\beta\left(-\alpha(5\alpha(\alpha+2)+4)\right) \\
&\quad -\alpha(\alpha+1)^3\beta^2(2\theta-1)((\alpha+2)\theta+\alpha)((\alpha+2)\theta-2(\alpha+1)) \\
N_{21} &\equiv 16+48\alpha+44\alpha^2+12\alpha^3-16\theta-32\alpha\theta-12\alpha^2\theta-\alpha(\alpha+1)\beta^2(\theta-1)\theta(2\theta-1)+ \\
&\quad 2(\alpha+1)\beta\left(2\alpha^2((\theta-1)\theta+1)+\alpha(\theta(5\theta-7)+4)+2(\theta-1)^2\right) \\
N_{22} &\equiv (\alpha+1)(\alpha+2)(3\alpha+2)\beta\left(5\alpha^3+12\alpha^2+(\alpha+2)^2(2\alpha+1)\theta^2-2(\alpha+2)(\alpha(\alpha+4)+2)\theta+12\alpha\right. \\
&\quad -18\alpha^5\theta-132\alpha^4\theta-(\alpha+1)^2\alpha\beta^2(2\theta-1)((\alpha+2)\theta+\alpha)((\alpha+2)\theta-2(\alpha+1)) \\
&\quad -368\alpha^3\theta-480\alpha^2\theta+54\alpha^6+360\alpha^5+948\alpha^4+1280\alpha^3+960\alpha^2-288\alpha\theta+384\alpha-64\theta+64
\end{aligned}$$

□

Proof of Proposition 17 Note that we have

$$\begin{aligned}
QI_A^{k*} &= \tfrac{\delta}{2}(\Lambda q_1 - p_{A1}^{k*} + \alpha(p_{B1}^{k*} - p_{A1}^{k*})) \\
QI_B^{k*} &= 0 \\
\pi_A^{k*} &= \tfrac{p_{A1}^{k*}}{2}(\Lambda q_1 - p_{A1}^{k*} + \alpha(p_{B1}^{k*} - p_{A1}^{k*})) + \tfrac{p_{A2}^{k*}}{2}(\Lambda(q_1 + QI_A^{k*}) - p_{A2}^{k*} + \alpha(p_{B2}^{k*} - p_{A2}^{k*})) \\
\pi_B^{k*} &= \tfrac{p_{B1}^{k*}}{2}(\Lambda q_1 - p_{B1}^{k*} + \alpha(p_{A1}^{k*} - p_{B1}^{k*})) + \tfrac{p_{B2}^{k*}}{2}(\Lambda q_1 - p_{B2}^{k*} + \alpha(p_{A2}^{k*} - p_{B2}^{k*}))
\end{aligned},$$

and this proposition can be proved directly based on the proof of Proposition 16 via some straightforward yet cumbersome calculations. We omit the details for brevity. □

Proof of Proposition 18 Based on the proofs of Proposition 16 and Proposition 17, this proposition can be proved directly by checking the sign of $p_{A1}^{d*}-p_{A1}^{c*}$, $p_{A2}^{d*}-p_{A2}^{c*}$, $p_{B1}^{d*}-p_{B1}^{c*}$, $p_{B2}^{d*}-p_{B2}^{c*}$, $QI_A^{d*}-QI_A^{c*}$, $\pi_A^{d*}-\pi_A^{c*}$ and $\pi_B^{d*}-\pi_B^{c*}$. We omit the details for brevity. □

Proof of Proposition 19 Similar to the proof of Proposition 1, given $\beta \leq 1$, it can be verified the existence and uniqueness of the equilibrium by showing that π_i is concave in p_i (Rosen 1965a), which can be done by checking the second derivative. Solving the system of best responses, the equilibrium prices and profits are given respectively:

$$\begin{aligned}
p_{A1}^{c*} &= \frac{q\Lambda N_{24}}{N_{23}} > 0 \\
p_{A2}^{c*} &= \frac{2\Lambda q(\alpha(\beta+2)+\beta+4)\left(\left(2\alpha^2+3\alpha+1\right)\beta^2(\theta-1)+36\alpha^2+48\alpha+16\right)}{N_{23}} > 0 \\
p_{B1}^{c*} &= \frac{q\Lambda N_{25}}{N_{23}} > 0 \\
p_{B2}^{c*} &= \frac{2q\Lambda(\alpha(\beta+2)+\beta+4)(36\alpha^2-(2\alpha+1)\beta^2(\theta-1)(\alpha(2\theta-1)+\theta-1)+48\alpha+16)}{N_{23}} > 0
\end{aligned}$$

and

$$\begin{aligned}
p_{A1}^{d*} &= \frac{q\Lambda N_{27}}{N_{26}} > 0 \\
p_{A2}^{d*} &= \frac{q\Lambda\left(3\alpha^2+8\alpha+4\right)(\alpha(\beta+2)+\beta+4)N_{28}}{N_{26}} > 0 \\
p_{B1}^{d*} &= \frac{q\Lambda(N_{27}-2(3\alpha+2)^3\left(2\alpha^3+7\alpha^2+7\alpha+2\right)\beta\theta(\alpha(\beta+2)+\beta+4))}{N_{26}} > 0 \\
p_{B2}^{d*} &= \frac{q\Lambda\left(3\alpha^2+8\alpha+4\right)(\alpha(\beta+2)+\beta+4)(N_{28}-\left(2\alpha^2+3\alpha+1\right)\left(6\alpha^2+7\alpha+2\right)\beta^2(\theta-1)\theta)}{N_{26}} > 0
\end{aligned},$$

where

$$
\begin{aligned}
N_{23} &\equiv \left(2\alpha^2+3\alpha+1\right)\beta^4(\theta-1)(\alpha(2\theta-1)+\theta-1)+16(\alpha+2)^2(3\alpha+2)^2- \\
&\quad 8(\alpha+1)\beta^2\left(\alpha^3+9\alpha^2+(\alpha+2)(2\alpha\theta+\theta)^2-(2\alpha+1)(\alpha(\alpha+6)+4)\theta+12\alpha+4\right) \\
N_{24} &\equiv 16(\alpha+2)(3\alpha+2)^2-4(\alpha+1)\beta^2\left(2(2\alpha\theta+\theta)^2-4\alpha(4\alpha+5)\theta+\alpha(11\alpha+17)-6\theta+6\right)+ \\
&\quad 2(\alpha+1)(2\alpha+1)\beta^3(\theta-1)(\alpha(2\theta-1)+\theta-1)+8(\alpha+1)(3\alpha+2)\beta(4\alpha\theta-3\alpha+2\theta-2)+ \\
&\quad (\alpha+1)(2\alpha+1)\beta^4(\theta-1)(\alpha(2\theta-1)+\theta-1) \\
N_{25} &\equiv 16(\alpha+2)(3\alpha+2)^2-4(\alpha+1)\beta^2\left(11\alpha^2+2(2\alpha\theta+\theta)^2-(2\alpha+1)(5\alpha+4)\theta+17\alpha+6\right)+ \\
&\quad (\alpha+1)(2\alpha+1)\beta^4(\theta-1)(\alpha(2\theta-1)+\theta-1) \\
&\quad +2(\alpha+1)(2\alpha+1)\beta^3(\theta-1)(\alpha(2\theta-1)+\theta-1)+ \\
&\quad 8(3\alpha+2)\beta(\alpha(\alpha(2\theta-3)+\theta-5)-2) \\
N_{26} &\equiv (\alpha+1)^2(2\alpha+1)\beta^4(\theta-1)(\alpha(2\alpha\theta+\alpha+\theta+4)+2)(\alpha(\alpha(4\theta-1)+6\theta-4)+2(\theta-1))+ \\
&\quad 324\alpha^8+3456\alpha^7+15552\alpha^6+38400\alpha^5+56704\alpha^4+51200\alpha^3+27648\alpha^2+8192\alpha+1024 \\
&\quad -2(\alpha+1)(\alpha+2)(3\alpha+2)\beta^2((\alpha+2)(\alpha(5\alpha+8)+4)(2\alpha\theta+\theta)^2 \\
&\quad -(\alpha+2)(2\alpha+1)(\alpha(\alpha+2)(2\alpha+11)+8)\theta+2(\alpha+1)(\alpha(\alpha+4)+2)(\alpha(\alpha+8)+4)) \\
N_{27} &\equiv (\alpha+1)^2(2\alpha+1)\beta^4(\theta-1)(\alpha(2\alpha\theta+\alpha+\theta+4)+2)(\alpha(\alpha(4\theta-1)+6\theta-4)+2(\theta-1))+ \\
&\quad 2(\alpha+1)^2(2\alpha+1)\beta^3(\theta-1)(\alpha(2\alpha\theta+\alpha+\theta+4)+2)(\alpha(\alpha(4\theta-1)+6\theta-4)+2(\theta-1))+ \\
&\quad 4(\alpha+1)(\alpha+2)(3\alpha+2)^3\beta(\alpha(\alpha(2\theta-1)+5\theta-4)+2(\theta-1))-2(\alpha+1)(\alpha+2)(3\alpha+2)\beta^2 \\
&\quad +18528\alpha^4+19648\alpha^3+11904\alpha^2+3840\alpha+512(11\alpha^4+61\alpha^3)+ \\
&\quad 512(96\alpha^2+(\alpha(5\alpha+8)+4)(2\alpha\theta+\theta)^2-(2\alpha+1)(\alpha(\alpha(11\alpha+36)+38)+12)\theta+58\alpha+12) \\
&\quad +324\alpha^7+2808\alpha^6+9936\alpha^5 \\
N_{28} &\equiv \left(2\alpha^2+3\alpha+1\right)\beta^2(\theta-1)\left(\alpha^2(2\theta+1)+\alpha(\theta+4)+2\right)+54\alpha^4+216\alpha^3+288\alpha^2+160\alpha+32
\end{aligned}
$$

Furthermore, for part (ii) and part (iii), we have

$$
\begin{aligned}
p^{c*}_{A2}-p^{c*}_{A1} &= \tfrac{(\alpha+1)\beta\Lambda(\beta+4)qN_{29}}{N_{23}}>0 \\
p^{c*}_{B2}-p^{c*}_{B1} &= \tfrac{q\beta\Lambda N_{30}}{N_{23}}>0 \\
p^{d*}_{A2}-p^{d*}_{A1} &= \tfrac{(\alpha+1)\beta\Lambda qN_{31}}{N_{26}} \\
p^{d*}_{B2}-p^{d*}_{B1} &= \tfrac{(\alpha+1)\beta\Lambda q(N_{31}+(3\alpha+2)^2\left(2\alpha^2+5\alpha+2\right)\theta(\alpha(\beta+2)+\beta+4)(2\alpha(\beta(\theta-1)-3)+\beta(\theta-1)-4))}{N_{26}}>0
\end{aligned}
,
$$

and

$$\begin{array}{rcl} p_{B2}^{c*} - p_{A2}^{c*} &=& \frac{2(1-\theta)\theta\Lambda q(2\alpha\beta+\beta)^2(\alpha(\beta+2)+\beta+4)}{N_{23}} > 0 \\ p_{A1}^{c*} - p_{B1}^{c*} &=& \frac{4\left(6\alpha^2+7\alpha+2\right)\beta\theta\Lambda q(\alpha(\beta+2)+\beta+4)}{N_{23}} > 0 \\ p_{B2}^{d*} - p_{A2}^{d*} &=& \frac{\left(\alpha^2+3\alpha+2\right)\left(6\alpha^2+7\alpha+2\right)^2\beta^2(1-\theta)\theta\Lambda q(\alpha(\beta+2)+\beta+4)}{N_{26}} > 0 \\ p_{A1}^{d*} - p_{B1}^{d*} &=& \frac{2(3\alpha+2)^3\left(2\alpha^3+7\alpha^2+7\alpha+2\right)\beta\theta\Lambda q(\alpha(\beta+2)+\beta+4)}{N_{26}} > 0 \end{array},$$

where

$$\begin{array}{rcl} N_{29} &\equiv& -24\alpha^2\theta + 36\alpha^2 - (2\alpha+1)\beta^2(\theta-1)(\alpha(2\theta-1)+\theta-1) + 2(2\alpha+1)^2\beta(\theta-1)\theta \\ && -28\alpha\theta + 48\alpha - 8\theta + 16 \\ N_{30} &\equiv& (2\alpha+1)\theta\left(\alpha^2(\beta(3\beta(\beta+4)-8)-24) + \alpha(\beta+4)\left(5\beta^2-4\right) + 2\beta^2(\beta+4)\right) + 16(\beta+4) - \\ && (\beta+4)\left((\alpha+1)^2(2\alpha+1)\beta^2 - 4\alpha(3\alpha(3\alpha+7)+16)\right) - \beta(\alpha(\beta(\beta+4)-4) \\ && +\beta(\beta+4))(2\alpha\theta+\theta)^2 \\ N_{31} &\equiv& 5328\alpha^4 + 7136\alpha^3 + 5024\alpha^2 + 1792\alpha + 256 + 2(\alpha+2)(3\alpha+2)^2\beta\left(3\alpha^3+14\alpha^2\right) \\ && -216\alpha^6\theta - 1404\alpha^5\theta - 3528\alpha^4\theta - 4384\alpha^3\theta - 2880\alpha^2\theta - 960\alpha\theta - 128\theta + 270\alpha^6 + 1944\alpha^5 \\ && +2(\alpha+2)(3\alpha+2)^2\beta\left(2(\alpha+1)(2\alpha\theta+\theta)^2 - (2\alpha+1)(\alpha(4\alpha+9)+4)\theta + 14\alpha + 4\right) \\ && -(\alpha+1)(2\alpha+1)\beta^2(\theta-1)\left(-5\alpha^2 + 4(\alpha+1)(2\alpha+1)\theta - 16\alpha - 8\right)(\alpha(2\alpha\theta+\alpha+\theta+4)+2) \\ && -(\alpha+1)(2\alpha+1)\beta^3(\theta-1)(\alpha(2\alpha\theta+\alpha+\theta+4)+2)(\alpha(\alpha(4\theta-1)+6\theta-4)+2(\theta-1)) \end{array}.$$

□

Proof of Proposition 20 Note that we have

$$\begin{array}{rcl} QI_A^{k*} &=& \frac{\delta}{2}(\Lambda q_1 - p_{A1}^{k*} + \alpha(p_{B1}^{k*} - p_{A1}^{k*})) \\ QI_B^{k*} &=& \frac{\delta}{2}(\Lambda q_1 - p_{B1}^{k*} + \alpha(p_{A1}^{k*} - p_{B1}^{k*})) \\ \pi_A^{k*} &=& \frac{p_{A1}^{k*}}{2}(\Lambda q_1 - p_{A1}^{k*} + \alpha(p_{B1}^{k*} - p_{A1}^{k*})) + \frac{p_{A2}^{k*}}{2}(\Lambda(q_1 + QI_A^{k*}) - p_{A2}^{k*} + \alpha(p_{B2}^{k*} - p_{A2}^{k*})) \\ \pi_B^{k*} &=& \frac{p_{B1}^{k*}}{2}(\Lambda q_1 - p_{B1}^{k*} + \alpha(p_{A1}^{k*} - p_{B1}^{k*})) + \frac{p_{B2}^{k*}}{2}(\Lambda(q_1 + QI_B^{k*}) - p_{B2}^{k*} + \alpha(p_{A2}^{k*} - p_{B2}^{k*})) \end{array},$$

and this proposition can be proved directly based on the proof of Proposition 19 via some straightforward yet cumbersome calculations. We omit the details for brevity. □

Proof of Proposition 21 Based on the proofs of Proposition 19 and Proposition 20, this proposition can be proved directly by checking the sign of $p_{A1}^{d*} - p_{A1}^{c*}$, $p_{A2}^{d*} - p_{A2}^{c*}$,

$p^{d*}_{B1} - p^{c*}_{B1}$, $p^{d*}_{B2} - p^{c*}_{B2}$, $QI^{d*}_A - QI^{c*}_A$, $\pi^{d*}_A - \pi^{c*}_A$ and $\pi^{d*}_B - \pi^{c*}_B$. We omit the details for brevity. □

B.3 The General T-Period Model

In this section, we study the dynamic price competition over a T-period model, where firms take learning process into account. We use similar expositions as in the static model, and let the firms' prices ($\mathbf{x}$ and $\mathbf{y}$) and quality ($\mathbf{a}$ and $\mathbf{b}$) to be a T-dimension column vectors. Note that all vectors in this paper are assumed to be column vectors. Subscripts are added to the above notations as time indexes. Parameters for different firms are distinguished by either subscripts or superscripts "a" and "b". We focus on the open-loop equilibrium, where both firms decide their prices over all periods at the beginning of the game. This represents the situations where adjusting the price after the horizon begins is costly for the firms. In the most general setting, we try to establish the existence and uniqueness of a Nash equilibrium point.

We start the analysis by formulating the problems for both firms. Firm A seeks to maximize its own revenue, i.e.

$$\max_{\mathbf{x}\geq 0} \quad \pi_a = \sum_{t=1}^{T} x_t f_t^a, \tag{B.1}$$

subject to the non-negative demand constraint

$$f_t^a = \frac{1}{2}\left(\Lambda((1-\theta_a)a_t + \theta_a b_t) - x_t + \alpha(y_t - x_t)\right) \geq 0, \quad t = 1, 2, \cdots, T;$$

similarly, firm B solves

$$\max_{\mathbf{y}\geq 0} \quad \pi_b = \sum_{t=1}^{T} y_t f_t^b, \tag{B.2}$$

subject to

$$f_t^b = \frac{1}{2}(\Lambda((1-\theta_b)b_t + \theta_b a_t) - y_t + \alpha(x_t - y_t)) \geq 0, \quad t = 1, 2, \cdots, T.$$

For both firms, their maximization problems are also subject to their learning process

$$a_t = a_{t-1} + \delta_a f^a_{t-1}; \quad b_t = b_{t-1} + \delta_b f^b_{t-1}, t = 2, 3, \cdots, T. \tag{B.3}$$

Note that, although the firms' learning speeds δ_a and δ_b may be different, they are assumed to satisfy the basic assumption in this equation.

To facilitate our analysis and presentation, we try to write the firms' objectives in matrix form. To that end, let us first define the following items based on the first order derivatives of firms' demands and quality levels in different periods. For any $t = 1, 2, \cdots, T$ and $k = 0, 1, \cdots, T - t$, let

$$v^x_k = \frac{\partial f^a_{t+k}}{\partial x_t}, \quad \bar{v}^x_k = \frac{\partial a_{t+k}}{\partial x_t}, \quad u^x_k = \frac{\partial f^b_{t+k}}{\partial x_t}, \quad \bar{u}^x_k = \frac{\partial b_{t+k}}{\partial x_t};$$

and

$$v^y_k = \frac{\partial f^a_{t+k}}{\partial y_t}, \quad \bar{v}^y_k = \frac{\partial a_{t+k}}{\partial y_t}, \quad u^y_k = \frac{\partial f^b_{t+k}}{\partial y_t}, \quad \bar{u}^y_k = \frac{\partial b_{t+k}}{\partial y_t}.$$

In addition, we write the above partial derivatives in form of column vectors; let $z^x_k = [v^x_k, u^x_k]'$, $\bar{z}^x_k = [\bar{v}^x_k, \bar{u}^x_k]'$, $z^y_k = [v^y_k, u^y_k]'$ and $\bar{z}^y_k = [\bar{v}^y_k, \bar{u}^y_k]'$. The following lemma reveals the iteration and solves for the above derivatives.

Lemma 9. *Let*

$$A := \begin{bmatrix} \frac{1}{2}\Lambda(1-\theta_a) & \frac{1}{2}\Lambda\theta_a \\ \frac{1}{2}\Lambda\theta_b & \frac{1}{2}\Lambda(1-\theta_b) \end{bmatrix} \text{ and } B := \begin{bmatrix} \delta_a & \\ & \delta_b \end{bmatrix}.$$

For any $t = 1, 2, \cdots, T-1$ and $k = 1, 2, \cdots, T-t$,

$$\begin{cases} z^x_k &= D(I+D)^{k-1} z^x_0, \\ \bar{z}^x_k &= B(I+D)^{k-1} z^x_0 \end{cases}, \quad \begin{cases} z^y_k &= D(I+D)^{k-1} z^y_0, \\ \bar{z}^y_k &= B(I+D)^{k-1} z^y_0 \end{cases},$$

where $D = AB$ and I is the 2×2 identity matrix. Moreover, the initial condition is given by

$$z^x_0 = [-(1+\alpha)/2, \alpha/2]', z^y_0 = [\alpha/2, -(1+\alpha)/2]' \text{ and } \bar{z}^x_0 = \bar{z}^y_0 = 0. \tag{B.4}$$

Based on Lemma 9, we can further define

$$\tilde{H}_a = \begin{bmatrix} v_0^x & 0 & \dots & 0 \\ v_1^x & v_0^x & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ v_{T-1}^x & v_{T-2}^x & \dots & v_0^x \end{bmatrix}, \quad \tilde{H}_b = \begin{bmatrix} u_0^y & 0 & \dots & 0 \\ u_1^y & u_0^y & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ u_{T-1}^y & u_{T-2}^y & \dots & u_0^y \end{bmatrix};$$

$H_a = \tilde{H}_a + \tilde{H}_a'$ and $H_b = \tilde{H}_b + \tilde{H}_b'$;

$$L_a = \begin{bmatrix} v_0^y & 0 & \dots & 0 \\ v_1^y & v_0^y & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ v_{T-1}^y & v_{T-2}^y & \dots & v_0^y \end{bmatrix}, \quad L_b = \begin{bmatrix} u_0^x & 0 & \dots & 0 \\ u_1^x & u_0^x & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ u_{T-1}^x & u_{T-2}^x & \dots & u_0^x \end{bmatrix}.$$

Moreover, let Q be a $2 \times T$ matrix and its column vectors be

$$Q_{\cdot t} = AC^{t-1} \begin{bmatrix} a \\ b \end{bmatrix},$$

where A is defined as in Lemma 9 and

$$C := \begin{bmatrix} 1 + m_a(1-\theta_a) & m_a\theta_a \\ m_b\theta_b & 1 + m_b(1-\theta_b) \end{bmatrix}, m_a = \frac{1}{2}\Lambda\delta_a, m_b = \frac{1}{2}\Lambda\delta_b.$$

Then let S_a and S_b be two column vectors whose transpose are the two row vectors of Q; i.e.

$$Q = \begin{bmatrix} S_a' \\ S_b' \end{bmatrix}.$$

Now, we are ready to write the firms' demand and revenue functions in term of the vectors $\mathbf{x}$ and $\mathbf{y}$.

Lemma 10. *Given the price vectors* $\mathbf{x}$ *and* $\mathbf{y}$*,*

$$[f_1^a, \cdots, f_T^a]' = \tilde{H}_a\mathbf{x} + L_a\mathbf{y} + S_a,$$

and

$$[f_1^b, \cdots, f_T^b]' = \tilde{H}_b\mathbf{y} + L_b\mathbf{x} + S_b.$$

With the help of Lemma 10, we can rewrite both firms' problems based on the objective functions (B.1) and (B.2). Specifically, given firm B's price vector $\mathbf{y}$, firm A solves

$$\begin{aligned}(A) \quad \max_{\mathbf{x}\geq 0} \quad & \pi_a(\mathbf{x};\mathbf{y}) = \mathbf{x}(\tilde{H}_a\mathbf{x} + L_a\mathbf{y} + S_a), \\ \text{s.t.} \quad & \tilde{H}_a\mathbf{x} + L_a\mathbf{y} + S_a \geq 0,\end{aligned}$$

and given firm A's price vector $\mathbf{x}$, firm B solves

$$\begin{aligned}(B) \quad \max_{\mathbf{y}\geq 0} \quad & \pi_b(\mathbf{y};\mathbf{x}) = \mathbf{y}(\tilde{H}_b\mathbf{y} + L_b\mathbf{x} + S_b), \\ \text{s.t.} \quad & \tilde{H}_b\mathbf{y} + L_b\mathbf{x} + S_b \geq 0,\end{aligned}$$

Moreover, we are interested in the first and second order derivatives of the revenue functions. Let $\nabla_{\mathbf{x}}\pi_a$ be the gradient vector with respect to $\mathbf{x}$ and $\nabla_{\mathbf{y}}\pi_b$ be similarly defined. The following lemma, which can be proved directly based on Lemma 10, characterizes the gradient vectors as well as the Hessian matrices of the firms' revenue functions.

Lemma 11. *(i) Given* $0 \leq \mathbf{y} \leq \Lambda$*,*

$$\nabla_{\mathbf{x}}\pi_a = H_a\mathbf{x} + L_a\mathbf{y} + S_a;$$

moreover, the matrix H_a *is the Hessian of* $\pi_a(\mathbf{x})$*.*

(ii) Given $0 \leq \mathbf{x} \leq \Lambda$*,*

$$\nabla_{\mathbf{y}}\pi_b = L_b\mathbf{x} + H_b\mathbf{y} + S_b;$$

moreover, the matrix H_b *is the Hessian of* $\pi_b(\mathbf{y})$*.*

Now, we proceed to show the existence and uniqueness of pure Nash equilibrium for the duopoly game. Based on Theorem 1 in Rosen (1965b), we can prove existence by showing that the firms are engaged in a concave game. From the formulation of the problems (A) and (B), we can see that the set of feasible strategies is

$$\mathcal{F} = \{\mathbf{x} \geq 0, \mathbf{y} \geq 0; \tilde{H}_a\mathbf{x} + L_a\mathbf{y} + S_a \geq 0, \tilde{H}_b\mathbf{y} + L_b\mathbf{x} + S_b \geq 0\},$$

which is convex, closed and bounded. In addition, based on Lemma 11, we can directly apply the result in Rosen (1965b) by checking whether the Hessian matrices H_a and H_b are negative-definite. Although we conjecture that this is true for the general T-period model, proving this result is computationally complicated. Nevertheless, reformulating each firm's revenue maximization problem in matrix form as given by equations (A) and (B) can greatly simplify the analysis, because we only need to check the property of Hessian matrices H_a and H_b for any exogenously given problem instances. In addition, we further identify two sufficient conditions where there exists a Nash equilibrium for the T-period game.

Proposition 22. *The feasible strategy space $\mathcal{F}$ is convex, closed and bounded. Therefore, there exists a Nash equilibrium for the duopoly game if matrices H_a and H_b are negative definite. Particularly, a Nash equilibrium exists if one of the following conditions holds.*

(i) A firm's quality does not affect the other firm's demand; i.e. $\theta_a = \theta_b = 0$.

(ii) One of the firms does not learn; i.e. either $\delta_a = 0$ or $\delta_b = 0$.

To discuss the uniqueness of the equilibrium, we will focus on equilibrium point $(\mathbf{x}^*, \mathbf{y}^*)$ that is an interior point of the feasible region $\mathcal{F}$. Boundary equilibrium, if exists, involves zero prices which are trivial and uninteresting cases. Hence, the equilibrium we are interested in is solved from the first order conditions (FOC). Based on Lemma 11, we write FOC as

$$\begin{cases} H_a\mathbf{x} + L_a\mathbf{y} + S_a &= 0 \\ L_b\mathbf{x} + H_b\mathbf{y} + S_b &= 0 \end{cases}. \tag{B.5}$$

Moreover, define the following matrix:

$$G := \begin{bmatrix} H_a & L_a \\ L_b & H_b \end{bmatrix}.$$

Since the first order conditions (B.5) form a system of linear equations, we can establish the uniqueness of the equilibrium by the following theorem, in which condition (i) guarantees the uniqueness of the solution to (B.5), whereas condition (ii) makes sure that the solution is a feasible pair of pricing schemes.

Theorem 2. *Suppose that H_a and H_b are negative definite. In addition, let* $(\mathbf{x}^*, \mathbf{y}^*) > 0$ *satisfy the system of Equations* (B.5)*. Then* $(\mathbf{x}^*, \mathbf{y}^*)$ *is the unique Nash equilibrium that is an interior point of $\mathcal{F}$ if the following conditions hold:*

(i) G has full rank; i.e. $\det(G) \neq 0$*;*

(ii) $\tilde{H}_a' \mathbf{x}^* < 0$ *and* $\tilde{H}_b' \mathbf{y}^* < 0$.

Now that we have partially established the existence and uniqueness of Nash equilibrium for the general T-period duopoly game, we turn to focus on the main research question: How do competition and cooperation between the two firms affect the game in the presence of experience-based quality improvement? Due to the learning curve, firms' quality levels are, although indirectly, related to the firms' pricing decisions. The next section will study their relationship in a two-period setting.

For example, to facilitate quality improvement, the firms may intentionally set low prices for the early periods; and they may be lower than the prices that firms would set if there were no learning. Moreover, setting low prices also contributes to the demand increase. Hence, it is the potential market expansion along with the quality improvement that drives firms' strategies.

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