STRATEGIC INTERACTIONS IN SUPPLY CHAIN UNDER SUPPLIER CAPITAL CONSTRAINT OVER TWO PERIODS

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PhD

The Hong Kong Polytechnic University
2025

The Hong Kong Polytechnic University
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Strategic Interactions in Supply Chain under Supplier Capital Constraint over Two Periods

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A thesis submitted in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

July 2025

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Abstract

The importance of long-term supply chain interactions can never be overstated. In the industrial world, a large number of retailers establish long-term relationships with suppliers, conducting transactions across multiple periods. Within such relationships, retailers strategically consider future operations when making current decisions, and suppliers, in turn, anticipate retailers' strategic thinking in purchasing when setting prices. This dynamic interaction leads to supply chain performance outcomes that differ significantly from those of single-period transactions. Another noteworthy phenomenon is that many suppliers operate under substantial capital constraint, especially during prevailing global economic downturns. In long-term relationships where suppliers face capital constraint, retailer strategic behavior manifests in two ways. First, strategic inventory, widely studied in the literature, is held to undercut future wholesale prices. Second, strategic purchasing occurs, where strategic retailers purchase more aggressively than myopic ones; the resulting higher early-period wholesale prices facilitate supplier capital accumulation, boosting their capital positions in later periods. This study investigates the role of strategic purchasing and inventory in influencing supply chain performances.

To examine this problem, we develop a model where a retailer purchases from a supplier who faces a capital constraint over a two-period horizon. Their relationship is governed by a contract. Under dynamic contract, in each period, the supplier sets wholesale price, then the retailer purchases, subject to the supplier's capital that caps production. Under commitment contract, the supplier and retailer commit to wholesale prices and purchase quantities for both periods before sales begin. The retailer may hold inventory in period 1 for selling in period 2. The supplier's capital in period 2 is endogenized by early wholesale pricing and retailer purchasing. We solve the research objective under each contract type, and then compare the two contracts.

We first disallow inventory carryover and compare two scenarios under each contract: one with a myopic retailer maximizing imminent profit, and another with a strategic retailer maximizing total profit over two periods. This helps isolate the effects of the retailer's strategic purchasing behavior. We also look into the design of contract to effectively leverage the retailer's strategic behavior for performance enhancement. We show that the retailer employs strategic purchasing when the supplier's production cost is moderate and initial capital position (ICP) is low; furthermore, this phenomenon is more prominent under dynamic contract. The retailer's strategic purchasing benefits the supplier and supply chain, but it may harm the retailer's profit under dynamic contract. From the perspective of increasing sales, dynamic contract is more effective when the ICP is moderate but commitment contract is more effective when it is low. To individual firms, when the production cost is low (high)

and the ICP is moderate (low), the profits to both firms are higher under dynamic (commitment) contract than under the alternative contract. Otherwise, the supplier prefers dynamic contract while the retailer prefers commitment contract. These main results still hold in volatile markets, where strategic purchasing is more frequent, particularly under dynamic contract. In addition to robustness checks, incorporating uncertainty yields interesting insights.

We next allow inventory holding to evaluate the effects of holding inventory by retailers on the operation interactions. Under dynamic contract, the retailer employs both types of strategic behavior; under commitment contract, he engages only in strategic purchasing. Thus, this part focuses primarily on dynamic contract; however, we still briefly compare the two contracts and confirm that key insights regarding their fundamental differences persist irrespective of inventory holding. Our results indicate that in case the supplier's initial capital position (ICP) caps operations, the supplier sets wholesale price to induce the retailer to purchase at full capacity in period 1. The retailer balances sales and inventory by weighing the benefit of enhanced supply and weakened supplier pricing power in period 2 against lowered sales and inventory holding cost in period 1. As a result, the retailer holds inventory either when production cost is not too high and the ICP is high, or when both are moderate; in the latter case, the inventory level can exceed that in the case without capital constraint at the supplier. In this chapter, the combination of the retailer's strategic purchasing and inventory continues to benefit the supplier more than the retailer. Nevertheless, inventory introduces crucial nuances. Inventory generally weakens this profit gain for the supplier. When both the production cost and the ICP are moderate, this effect can be significant that the supplier's profit falls below that when the retailer is myopic. Conversely, in this case, inventory benefits the retailer, creating a new situation where it is profitable for the retailer to behave strategically. Finally, we explore preordering, whereby the retailer preorders from the supplier in period 1 and deploys the preorder quantity for selling in period 2, to mitigate the inefficiency caused by holding inventory at a cost. Preordering generally eases the capital constraint in period 2 more effectively, leading to an improved supply chain profit. However, preordering may lower the profit of either the retailer or the supplier, albeit not simultaneously.

We examine the impact of the supplier's capital constraint, finding robust conclusions regardless of whether inventory holding is allowed. Capital constraint may benefit the supplier, particularly when the production cost is high and the ICP is moderate. Nevertheless, it is detrimental to the retailer and supply chain. Furthermore, a decrease in the ICP, through its effects on firms' operation adaptations, can benefit individual firms and supply chain.

Acknowledgements

First and foremost, I would like to express my sincere gratitude to my Chief Supervisors, Prof. JIANG Li, and Prof. YE Hengqing, for the continuous support of my Ph.D study and research, for their patience, motivation, enthusiasm, and immense knowledge. Prof. JIANG Li, my supervisor for the initial four and a half years, offered profoundly inspiring insights and an exemplary work ethic that will continue to influence my career and life. I am grateful for his constant encouragement, unwavering support, and constructive feedback that significantly improved my research quality. His patient guidance helped me overcome difficulties, and his rigorous, critical thinking deeply enriched my Ph.D study. My sincere appreciation also goes to Prof. YE Hengqing, who provided tremendous support as my supervisor in the concluding stages of my doctorate. His insights during our conversations were consistently enlightening and offered fresh perspectives that greatly benefited my research.

I gratefully acknowledge the professors in Department of Logistics and Maritime Studies for their generous help and support. I extend my thanks to Prof. SONG Miao for her supervision while I served as a tutor for her course. I would also like to thank Prof. YE Hengqing and Prof. GUO Xiaomeng, who were the examiners for my confirmation of registration.

Thanks to Prof. SONG Miao, Prof. JIANG Li, and many others, for their lectures, from which I have learnt the essential knowledge and skills for conducting research.

Then, I would like to acknowledge my classmates and friends: Dr. ZHANG Ping, Dr. YANG Liutao, Dr. LIU Fuzhen, Dr. Qi Jingwen, and many others, for their kind help in my research and daily life. With their accompany, my research and daily life became interesting and colorful.

Last and most importantly, I would like to extend my eternal gratitude to my parents, little brother, and husband for their unconditional and endless love. They always stand by my side to provide encouragement and support.

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Chapter 1

Introduction

1.1 Background

In practice, supplier–retailer transactions typically span multiple periods. Tesla's procurement strategy provides clear examples of such long-term relationships. The company signed a five-year deal with Piedmont Lithium Limited, an American junior mining company, for the supply of spodumene concentrate; this agreement can be extended by mutual consent for a second five-year term [1]. Furthermore, Tesla secured a contract lasting more than seven years with Yahua Industrial Group in Sichuan, China, for raw materials used in electric vehicle batteries (Tang 2023). In the apparel industry, Nike's Futures Ordering Program establishes long-term partnerships with its retailers (McNew 2016; Roy et al. 2022).

In long-term relationships, retailers exhibit strategic behaviors after balancing short- and long-term benefits. Anticipating such behaviors, suppliers make their own decisions accordingly. This dynamic leads to inter-firm interactions and outcomes distinct from those in single-period transactions. Retailers' strategic behaviors can, for instance, include holding strategic inventory. Anand et al. (2008) show that in a two-period model excluding conventional inventory motives, the retailer still maintains inventory to pressure the supplier into lowering the subsequent period's wholesale price. Inventory held solely for such strategic purposes is known as strategic inventory. It provides the retailer with future leverage, increasing his first-period purchases, which the supplier can exploit by raising the wholesale price. Although strategic inventory has received significant attention in the literature, its impact on the supply chain performance when suppliers face capital constraints largely remains unexplored.

Recent empirical evidence indicates that a substantial proportion of suppliers, even those for large firms, are small and medium-sized enterprises (i.e., SMEs). For example, more than two-thirds of Walmart's 18,000 suppliers are SMEs (Waters 2021). These suppliers often operate under capital constraints, a situation exacerbated in global economic downturn. More than 75% of small-business owners express concerns about their ability to secure capital [2]. Capital scarcity leads to production

[1] https://www.businesswire.com/news/home/20200928005208/en/Piedmont-Lithium-Signs-Sales-Agreement-With-Tesla

[2] https://www.goldmansachs.com/insights/articles/more-than-75-percent-of-surveyed-small-businesses-are-worried-about-credit.html

shortfall, deterring suppliers from engaging in active pricing or effectively responding to retailers' strategic moves. Even large suppliers like Yahua can face capital shortage that limits production capacity. At the start of its contract with Tesla, Yahua was unable to meet demand. Through profit accumulation, it has progressively invested in capacity expansion.

Our work is motivated by the above gap between practice and academic research. Building on Anand et al. (2008)'s model, we incorporate the capital constraint at suppliers—an essential extension to capture the real-world challenges in modern supply chains, where financial constraint significantly influences pricing and inventory decisions.

With capital constraints at suppliers, retailers may not only hold strategic inventory but also adopt another tactic, i.e., purchasing more aggressively than myopic ones. This strategic purchasing behavior results in higher early-period wholesale prices, facilitating supplier capital accumulation and improving subsequent capital availability. Toyota exemplifies retailers' such behavior of bolstering future supplier capital in our theoretical framework. Since April 2024, the automaker has increased payments to financially strapped suppliers to help them expand capacity (e.g., hiring workers and improving factories) to meet its surging production demands (Greimel and Okamura 2024).

Given that in long-term interactions where suppliers face capital constraint, retailers exhibit two types of strategic behavior, our objectives are two-fold. One is to unravel the effects of retailers' strategic purchasing and inventory on the interactions between strategic retailers and financially constrained suppliers. The other is to explore the role of capital constraints on suppliers in influencing performances outcomes. Our results generate novel insights into managing interactions between capital-constrained suppliers and retailers engaging in strategic decision-making over the long term.

In line with Anand et al. (2008), we model a two-period supply chain in which a retailer sources from a supplier constrained by limited capital. The retailer may hold inventory from period 1 to period 2 to serve two purposes. One is to weaken the supplier's pricing power in period 2 by intensifying supply competition, and the other is to ease the supplier's capital shortage. We examine a series of related scenarios. In the benchmark scenario B, the retailer strategically purchases over periods and holds inventory, absent capital constraint on the supplier. With capital constraint on the supplier, we examine scenario M, in which the retailer is myopic and maximizes period-by-period profits; scenario ND/NC, in which the retailer is strategic but forsakes inventory; and scenario C, in which the retailer is strategic and holds inventory. The main results and insights are based on a comparative investigation into the performance outcomes across scenarios.

We examine the previously introduced research questions under two contract types. Under dynamic contract (e.g., the contract followed by Tesla and Yahua), the supplier and the retailer take turns to set wholesale price and purchase quantity in each period. Under commitment contract, the supplier fixes wholesale prices for both periods, and the retailer commits to purchase quantities before sales begin. This contract type is exemplified by agreements like those between Nike and its retailers, and Tesla and Piedmont Lithium. Nike commits to fixed prices while retailers place nonrefundable orders five to six months in advance of delivery. Similarly, Tesla's five-year agreement with Piedmont includes explicit commitments regarding both price and purchase quantity, creating a stable framework for the supplier. After analyzing each contract type individually, we compare them to examine how supplier–retailer interactions differ.

1.2 Literature Review

Under the supplier's capital constraint, our work examines the role of holding strategic inventory by the retailer and the retailer's strategic purchasing. We also study how the retailer's strategic behaviors perform differently across contract types. As a result, our study relates to the following streams of literature.

Anand et al. (2008) introduce the notion of strategic inventory, which is later validated by Hartwig et al. (2015) through behavior experiments. Anand et al. (2008) remark that strategic inventory increases price in period 1 but lowers it in period 2, reducing average prices and alleviating double marginalization inherent in supply chains. However, it creates intertemporal price variability, causing supply chain inefficiency. The benefit of reduced double marginalization effect can be dominant, making holding inventory by retailers advantageous for all parties and consumers, when holding cost is not too high.

Subsequent studies explore the impacts of various operational factors on the role of strategic inventory. Arya and Mittendorf (2013) state that the supplier's direct-to-consumer rebate amplifies the upside of strategic inventory (double marginalization alleviation) but lessens downside (intertemporal pricing variability), making it favorable to the supplier, the retailer, and the consumers, regardless of holding cost. Roy et al. (2019) examine the observability of inventory and study its effects on supplier and retailer behavior with periodic supplier pricing. They find that unobservability broadens the range of holding cost, in which inventory is carried. With a low holding cost, inventory is lower when it is unobservable than when it is observable. In contrast, with a high holding cost, the supplier lowers price to induce more inventory, when unobservable. As such, unobservability of inventory can benefit the supplier and the channel, but favors the retailer only when holding cost is high. Dong and Liu (2022) show that the supplier's inventory carryover enhances the retailer's ability to weaken the supplier's monopoly power in period 2 through strategic inventory. This incentivizes the retailer to carry inventory, enabling the supplier to profit from a higher wholesale price in period 1. Guan et al. (2019) show that buyer's inventory can deter the supplier from pursuing an aggressive direct-selling strategy, prompting inventory carryover regardless of the magnitude of holding cost. Mantin and Jiang (2017) examine the effect of quality deterioration on the retailer's decision to hold inventory and show that it restricts the range of holding cost that justifies holding inventory. Arya et al. (2015) find that total inventory in a decentralized purchasing system, wherein divisions purchase separately from the same manufacturer, is lower than in a centralized system. Keskinocak et al. (2008) study the effect of supplier's capacity in the early period on strategic inventory holding. They find that allocating a low capacity to inventory curbs the retailer's sales and limited capacity can even make inventory carryover harmful to individual firms and the supply chain.

The aforementioned literature on strategic inventory focuses on bilateral monopoly. Studies in competitive settings are rather limited. Desai et al. (2010) find that in case two retailers share a single supplier, increased competition lowers retailers' gain from wholesale-price reduction, disincentivizing strategic inventory and potentially leading to a prisoner's dilemma that makes retailers worse off with inventory. When two partially substitutable suppliers sell to a nonexclusive retailer, strategic inventory enables the retailer to leverage supplier competition and make a profit gain. Roy et al. (2022) provide a detailed analysis of the second case, to find that long-term contracts with reciprocal commitments — wholesale price from suppliers and quantity from the retailer — can mitigate supplier competition. Li et al. (2022), like Roy et al. (2022), show that holding inventory intensifies price competition across supply chains, hurting all firms when suppliers' products are highly substitutable.

These studies examine the adaptation of the role of strategic inventory to the addition of new features, along with the implications for inventory holdings, other operations decisions, and ultimately profits. Following a similar vein, we extend Anand et al. (2008) by introducing a capital constraint on the supplier, which restricts the quantity that the supplier can produce to meet the retailer's purchase, with capital availability in the later period depending on the initial capital and net sales revenue in the early period. Keskinocak et al. (2008) examine suppliers' production constraint but focus on physical capacity. In contrast, our model incorporates financial constraint and endogenizes the supplier's capital in period 2, to capture the role of wholesale pricing and retailer purchasing in dynamically shaping the supplier's capacity in the later period, highlighting the lasting impacts of early-period decisions.

Despite the differences in settings, our findings share similar insights with those in Keskinocak et al. (2008). First, allocating a low capacity to inventory could drive out the retailer's sales in period 1. Keskinocak et al. (2008) show that the retailer has no incentive to hold inventory when the capacity falls below a threshold. Likewise, in our work, the retailer balances short-term revenue loss in period 1 due to sales reduction with long-term benefit, when allocating the purchase in period 1 between sales and inventory. However, as the supplier has limited capital in period 2 as well and her capital position depends on operations in period 1, the equilibrium pattern is totally different from that in Keskinocak et al. (2008). Second, suppliers may prefer limited over unlimited capacity. Keskinocak et al. (2008) state that this case arises only when capacity expansion is costly, for otherwise the supplier would opt for abundant capacity. In contrast, we show that the supplier may prefer limited capital even when increasing capital is costless. Third, limited capacity alters the role of holding inventory by retailers in influencing individual firms and the supply chain. Anand et al. (2008) show that strategic inventory benefits the firms and supply chain except when holding cost is extremely high, while Keskinocak et al. (2008) state that inventory carryover can be harmful when the supplier's capacity is low. Our model in Chapter 3 shows that even when the supplier's initial capital is low, inventory carryover may still benefit firms.

By endogenizing the supplier's capital availability in period 2, our study in Chapter 3 yields new insights. First, when the ICP is low, the wholesale price in period 2 is more sensitive to the retailer's inventory than observed by Anand et al. (2008), who assume away the capital constraint on the supplier. Second, inventory weighs on capital accumulation, by either deterring or promoting capital growth, to affect profits. Third, there exist cost-ICP scenarios, where the strategic retailer holds more inventory when the supplier faces a capital constraint than when she faces no such constraint.

The second related stream of literature explores the impact of capacity constraint. Yang et al. (2018) examine the effects of the supplier's capacity on distribution strategy, and show that a moderate capacity leads the retailer to order all and prevent the supplier's market entry through direct selling. It leads the supplier, the retailer, and the consumers to benefit from limited capacity. Erhun et al. (2008) explore the value of an extra trading chance and more demand information in case of limited capacity at the supplier. They note that increased capacity does not always benefit the supplier, even if available at no cost. Our model similarly finds that the supplier can benefit from limited initial capital, while the retailer and supply chain consistently prefer unlimited capital. Dasci and Guler (2019) study a buyer's procurement process design when facing two suppliers with limited capacities. Ghamat et al. (2018) consider a model where a manufacturer outsources to a competitor with limited capacity, while the competitor may prioritize meeting either outsourcing demand or his production need, depending on capacity and competition intensity. We extend this research by analyzing the adaptations made by the supplier and the retailer to fit with the supplier's capital constraint in a two-period horizon, where the operation decisions in the early period shape the supplier's capacity in the later period. Chapter 2 shows that the strategic retailer supports capital accumulation. In Chapter 3, we explore the effects of capital constraint on the presence of strategic inventory, and reveal how the retailer's support behavior interacts with strategic inventory to influence operations and profits.

Under capacity constraint, the existing literature shows that retailers might engage in strategic behavior by purchasing quantities beyond their immediate needs; in some cases, this tactic purchasing quantities beyond their immediate needs (Salop 2004, Eső et al. 2010). In a more recent paper, Jain et al. (2019) show that the dominant retailer, with priority access to supplier capacity, may excessively procure, leaving little or no capacity for the competitor, thereby increasing its market power. Chen et al. (2013) illustrate that the highest-priority retailer may exhibit a similar behavior to exclude other retailers when the supplier allocates limited capacity lexicographically. This behavior can also serve as an effective strategy for competing over scarce resources. Cachon and Lariviere (1999) state that retailers, acting as local monopolies in the end market, order more than needed to secure a more favorable assignment under specific allocation mechanisms. In our work, the strategic retailer is incentivized to purchase beyond what is needed to maximize imminent profit after weighing short-term cost against long-term benefit. As a result, the supplier sets a higher early-period wholesale price, which facilitates capital accumulation and improve future supply chain dynamics.

Another relevant topic in the OM literature is the comparison between dynamic and commitment contract. These pricing contracts have distinct characteristics that influence firms' decisions and ultimately affect profit outcomes. For instance, Gilbert and Cvsa (2003) study the tradeoff between pre-committing to a wholesale price to mitigate opportunism and delaying pricing to buffer against demand uncertainty. Such commitment benefits the retailer and supply chain, but only benefits the supplier when demand uncertainty is low. Kabul and Parlaktürk (2019) study a two-period model similar to Anand et al. (2008), where the retailer may hold strategic inventory; however, consumers are strategic, timing purchases to maximize utility. They show that commitments, intended to deter waiting for price markdowns, aggravate system inefficiency and harm firms under wholesale price contract, but become beneficial under coordinating contract. Liu et al. (2012) examine two forms of price commitment in a multiple-period model: a retail-fixed-markup contract and a price-protection contract. They find that a price-protection contract inhibits supply chain efficiency, while a retail-fixed-markup improve it and can even coordinate the channel. Our work shows that the retailer's strategic behaviors — supporting capital accumulation and holding strategic inventory — perform differently under dynamic and commitment contracts. Chapter 2 finds that strategic purchasing exhibits economies of scale under dynamic contract and diseconomies under commitment contract as ICP rises. As a result, equilibrium patterns differ, and the supplier adjusts pricing in different ways with changes in capital availability, making each contract type more suitable for different conditions. Chapter 3 shows that the retailer does not hold strategic inventory under commitment contract.

Chapter 2

Strategic Purchasing under Supplier's Capital Constraint

2.1 Introduction

In this chapter, we disallow inventory carryover to study the retailer's strategic purchasing behavior—purchasing more aggressively than the myopic retailer to boost the supplier's future capital—under two contract types, which we then compare. Our research questions are: 1) Under each contract, when does such behavior arise, and how does the supplier manage prices in anticipation of this behavior? 2) How does the retailer's strategic purchasing affect the operations and profits, compared to when the retailer acts myopically? 3) How do the underlying distinctions between the two contracts shape different strategic purchasing performance and, ultimately, lead to varied decision-making outcomes? From the perspectives of each firm and the supply chain, we identify the conditions under which each contract is optimal.

As introduced in Chapter 1, we model a two-period supply chain where a strategic retailer sources from a capital-strained supplier and inventory carryover is prohibited. The relationship between the supplier and the retailer is governed by either a dynamic or a commitment contract. Under dynamic contract, the supplier and retailer take turns to decide the wholesale price and purchase quantity in each period. Under commitment contract, the supplier fixes wholesale prices for both periods, and the retailer commits to purchase quantities before sales begin. This chapter examines scenario M, where the retailer is myopic, and scenario ND/NC, where the retailer is strategic but forgoes inventory carryover. We then compare these scenarios to derive our main results. Scenario C, in which the retailer is strategic and carries inventory across periods, is explored in Chapter 3 to assess the role of strategic inventory.

We demonstrate that the retailer engages in strategic purchasing if the supplier's production cost is moderate and initial capital position (abbreviated as ICP) is low. This phenomenon is more prominent under dynamic contract than under commitment contract. On its occurrence, the supplier significantly increases the wholesale price in the early period relative to that for a myopic retailer. With the retailer maintaining purchase full capacity, it enhances the supplier's capital position in the later period, alleviating the capital constraint that restricts operations interactions between firms. The retailer can be harmed by strategic purchasing under dynamic contract, because the significant increase in wholesale price in period 1 outweighs the gain from more sales and higher profit margin in period 2. In contrast, commitment contract shields the retailer from the downsides of strategic purchasing. Regardless, the retailer's strategic purchasing leads to profit gains to the supplier and supply chain, and benefits consumers in the meanwhile.

In the presence of a strategic retailer, the supplier can benefit from the capital constraint, through its impact on the firms' operations adjustments, under dynamic contract. An increase in the supplier's ICP can harm individual firms and supply chain, particularly when commitment contract is enforced.

The contract in use has no consequential effects on the performance outcomes when the retailer is myopic, but it is a crucial factor as the retailer acts strategically. An increase in the ICP has starkly different effects on the retailer's strategic purchasing incentive under the two contracts, thereby influencing the supplier's wholesale prices and the retailer's purchases with subsequent adaptations. With total sales as the measure, dynamic (commitment, resp) contract is more effective when the supplier faces a medium (significant, resp) capital constraint. When the supplier's production cost is low (high, resp) and ICP is medium (low, resp), the advantage of dynamic (commitment, resp) contract in sales generation makes it preferable by the supplier and the retailer over the alternative contract, resulting in a win-win situation. Otherwise, the supplier would prefer dynamic contract, while the retailer would prefer commitment contract. Mechanisms are needed to be developed to reconcile the preferences of the two firms.

The main results largely prevail when market demand is uncertain, albeit with notable exceptions. The prevailing contract is influential to the performance outcomes even when the retailer is myopic in volatile markets, as the supplier and the retailer have the flexibility in tailoring the wholesale price and purchase quantity in the later period to realized market conditions under dynamic contract, but they lack this flexibility under commitment contract. When the retailer is strategic, strategic purchasing is more likely to arise in volatile markets than in deterministic markets, particularly under dynamic contract. As market volatility rises, when the retailer is myopic, he is more likely to prefer commitment contract, whereas the supply chain benefits more from dynamic contract; with a strategic retailer, the divergence in firms' contract preferences aggravates. The phenomenon that the retailer is harmed by strategic purchasing under dynamic contract is most obvious when the market is mediumly volatile.

The remainder of this paper is organized as follows. Section 2.2 introduces model setting. Section 2.3 and Section 2.4 analyze performance outcomes under dynamic contract and commitment contract, respectively. We discuss contract design through a comparative investigation into the two contracts in Section 2.5, and extend to uncertain market in Section 2.6. The paper is concluded in Section 2.7. All the proofs are presented in the Appendix.

2.2 Model Preliminaries

Consider a market comprising a supplier (s) and a retailer (r) in a two-period horizon. We refer to the supplier as "she" and the retailer as "he". The supplier produces a product under capital constraint and sells the product at wholesale price w to the retailer, who incurs a cost, normalized to zero, to sell to consumers at market-clearing price p. We prohibit inventory carryover, which is a reasonable assumption supported by existing studies. Li et al. (2022) state that this can be achieved via a vendor-managed inventory (VMI) system, where suppliers deliver only the required amount for the current period. Furthermore, with drop shipping, a common arrangement between suppliers and retailers in e-commerce, retailers neither physically handle the products nor carry inventory. Arya and Mittendorf (2013) also note that under a just-in-time (JIT) system, retailers only get inputs they sells and cannot carry inventory. This assumption helps to isolate how strategic purchasing shapes outcomes without introducing unnecessary complications. Unsold products have no salvage value and are scrapped at the end of each period.

In period $t \in \{1,2\}$, the market-clearing price follows an inverse demand function as follows:

$$p_t(s_t) = a - bs_t,\ t \in \{1,2\}, \tag{2-1}$$

where a is the maximum price that consumers are willing to pay, s_t is the sales quantity, and $b > 0$ is the sensitivity of price with respect to a marginal increase in quantity. This inverse demand function, which is invariant across periods, is widely used in the literature (e.g., Anand et al. 2008, Keskinocak et al. 2008, Roy et al. 2019, Desai et al. 2010). Following McGuire and Staelin (1983) and Li et al. (2022), we normalize a to 1, i.e., $p_t(s_t) = 1 - bs_t,\ t \in \{1,2\}$. In Section 2.6, we explore the scenario where the inverse demand function has a random intercept, to examine the robustness of the main results and yield additional insights.

2.2.1 Retailer's problem

The strategic retailer manages purchase and sales quantities in period t to optimize the profit below:

$$\underset{(s_t,q_t)}{Max} \sum_{i=t}^{2}(p_i(s_i)s_i - w_i q_i), \text{ s.t., } 0 \le s_t \le q_t \le \frac{K_t}{c},\ t \in \{1,2\}, \tag{2-2}$$

where $\frac{K_t}{c}$ is the maximum production quantity affordable by the supplier in period t and is to be detailed later. In period 2, the retailer purchases from the supplier and sells products to maximize the profit in the period. In period 1, the retailer makes decisions to maximize total profit over the two periods. The retailer always sells all that he has purchased from the supplier in each period, i.e., $s_t = q_t,\ t \in \{1,2\}$. This is proved in the Appendix. So, we rewrite formulation (2-2) as follows:

$$\underset{q_t}{Max} \sum_{i=t}^{2}(p_i(q_i) - w_i)\, q_i, \text{s.t. } 0 \le q_t \le \frac{K_t}{c},\ t \in \{1,2\}, \tag{2-3}$$

We also consider a case where the retailer is myopic, and he decides purchase quantity q_t to maximizes the imminent profit in period t:

$$\underset{q_t}{Max}\, \pi_{r,t} = (p_t(q_t) - w_t)q_t, \text{ s.t. } 0 \le q_t \le \frac{K_t}{c},\ t \in \{1,2\}. \tag{2-4}$$

2.2.2 Supplier's problem

The supplier produces at marginal production cost c, which remains constant over periods to exclude the impact of cost variation. Our results are reinforced when production cost decreases over periods. In period t, the supplier's capital position K_t caps production at $\frac{K_t}{c}$, which is the supplier's capacity. We refer to K_1 as the supplier's initial capital position (abbreviated as ICP). The supplier's capital status in each period is public information, which is a common assumption in the literature. In practice, it is stated in contracts between GM and its suppliers that suppliers should provide official documents, upon request, to verify financial statuses (Jiang and Hao 2014). Given the supplier's wholesale price w_1 and the retailer's purchase q_1 in period 1, the supplier's capital position in period 2 satisfies the following state-transition equation:

$$K_2 = K_1 + (w_1 - c)q_1, \tag{2-5}$$

where $(w_1 - c)q_1$ is the net cash flow generated from transaction in period 1.

We examine two contracts to govern the interactions between the supplier and the retailer: dynamic contract and commitment contract. Anticipating the retailer's responses, the supplier strategically manages wholesale prices to maximize profits. Under dynamic contract, the supplier sets wholesale price w_t in period t to solve the following problem:

$$\underset{w_t}{Max} \sum_{i=t}^{2}(w_i - c)q_i,\ t \in \{1,2\}. \tag{2-6}$$

Under commitment contract, the supplier sets wholesale prices (w_1, w_2) for the two periods prior to the start of the selling horizon to maximize total profit:

$$\underset{(w_1,w_2)}{Max} \sum_{i=1}^{2}(w_i - c)q_i. \tag{2-7}$$

Our work extends Anand et al. (2008) by incorporating the supplier's capital constraint and modeling the retailer's strategic purchasing behavior. In Anand et al. (2008), the two periods are linked through inventory under dynamic contract, but they are independent under commitment contract. In contrast, our model prohibits inventory carryover, linking the periods solely through cash flow from transaction in period 1 under both contracts. The choice of contract determines the firms' flexibility in tailoring decisions to fit with capital status. This chapter focuses on the role of strategic purchasing in supporting supplier capital accumulation to enhance performances. Chapter 3 explores its interaction with inventory carryover.

2.2.3 Decision sequence

Figure 2.1 illustrates decision sequence. In stage 1, the supplier decides wholesale price(s), w_1 for period 1 under dynamic contract but (w_1,w_2) for the two periods under commitment contract. In stage 2, the retailer purchases q_1 under dynamic contract, while he purchases q_1 for period 1 and commits to q_2 for period 2 under commitment contract. Moving into period 2, the supplier decides wholesale price w_2 under dynamic contract but adheres to the wholesale price committed prior to the selling horizon under commitment contract in stage 3. In stage 4, the retailer sets q_2 under dynamic contract, but purchases committed quantity under commitment contract. Finally, profits accrue to the two firms. We adopt a backward procedure to analyze wholesale prices and purchases over periods and derive profits to individual firms and supply chain under the two contract types.

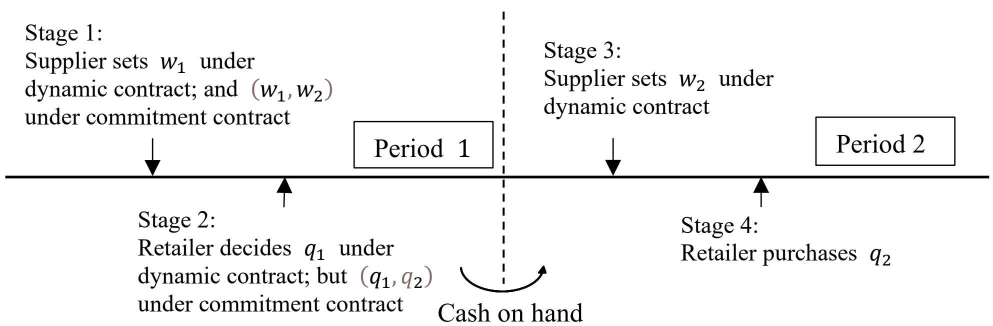


Figure 2.1 Sequence of events

2.3 Dynamic Contract

We examine dynamic contract in this section and discuss commitment contract in the next section.

2.3.1 Myopic retailer (scenario M)

First, we analyze the situation with a myopic retailer. Superscript M on the quantities of interest indicates this scenario. To maximize imminent profit $\pi_{r,t}$ at wholesale price w_t, the retailer purchases $\left(\frac{1-w_t}{2b}\right)^+$ subject to capacity $\frac{K_t}{c}$. Thus, given capital position K_t, the retailer's purchase quantity in period t is:

$$q_t^M(w_t|K_t) = (min\left\{\frac{1-w_t}{2b}, \frac{K_t}{c}\right\})^+. \tag{2-8}$$

In period 2, anticipating the retailer's purchase $q_2^M(w_2|K_2)$, the supplier sets wholesale price w_2 to maximize $\pi_{s,2} = (w_2 - c)q_2^M(w_2|K_2)$ under capital K_2. In equilibrium, their decisions are:

$$\left(w_2^M(K_2), q_2^M(K_2)\right) = \begin{cases} \left(\frac{c-2bK_2}{c}, \frac{K_2}{c}\right) & K_2 \le \frac{(1-c)c}{4b} \\ \left(\frac{1+c}{2}, \frac{1-c}{4b}\right) & K_2 > \frac{(1-c)c}{4b} \end{cases}. \quad (2\text{-}9)$$

Observe that $\frac{(1-c)c}{4b}$ is the threshold capital position, below which the supplier manages a wholesale price to have the retailer purchase at capacity, and above which capital position does not constrain the interactions between firms. In the case when the retailer purchases at capacity, an increase in the capital position enhances the profits to the supplier and the retailer.

In period 1, the supplier manages wholesale price w_1 to maximize total profit over the two periods, anticipating the retailer's purchase $q_1^M(w_1|K_1)$ and the decisions in period 2. Proposition 2.1 presents the performance outcomes. The profits to the supplier, the retailer, and the supply chain are presented in the Appendix.

Proposition 2.1. *Under dynamic contract, when the retailer is myopic, let* $\overline{K}_1^M = \frac{c(1-c)}{4b}$, $\underline{K}_1^M = \frac{(1-\sqrt{1-2c+2c^2})c}{4b}$, *and* $K_2^M = \frac{K_1(c-2bK_1)}{c^2}$, *the supplier's wholesale prices and the retailer's purchases over periods are as follows:*

Condition on ICP	w_1^M	q_1^M	w_2^M	q_2^M
$K_1 \ge \overline{K}_1^M$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$\underline{K}_1^M \le K_1 < \overline{K}_1^M$	$1-\frac{2bK_1}{c}$	$\frac{K_1}{c}$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$K_1 < \underline{K}_1^M$	$1-\frac{2bK_1}{c}$	$\frac{K_1}{c}$	$1-\frac{2bK_2^M}{c}$	$\frac{K_2^M}{c}$

Proposition 2.1 indicates that when the retailer is myopic, the capital availability at the supplier has operational consequences only when the ICP is low (i.e., $K_1 < \overline{K}_1^M$). In this case, the supplier sets the wholesale price in period 1 to induce the retailer to purchase at full capacity. The supplier's capital imposes no restrictions on operations in period 2 when the ICP is moderate (i.e., $\underline{K}_1^M \le K_1 < \overline{K}_1^M$), but caps operations when the ICP is low (i.e., $K_1 < \underline{K}_1^M$). Despite the retailer's myopic purchasing, capital accumulation in period 1 alleviates the supplier's capital burden in period 2.

Corollary 2.1. *Under dynamic contract, when the retailer is myopic, as the supplier's* ICP *increases, the profits to the supplier, the retailer and supply chain,* π_s^M, π_r^M, *and* π_{sc}^M, *increase.*

Premised on the outcomes stated in Proposition 2.1, the firms adapt operations as the supplier's ICP increases. First, the supplier lowers the wholesale prices, leading the retailer to purchase more, in both periods. Then, the supplier lowers the wholesale price in period 1 but stabilizes the wholesale price in period 2. It leads the retailer to order more in period 1, lifting the capital constraint in period 2. Finally, the wholesale prices are stabilized in both periods, when the supplier's ICP is no longer influential. Consequently, the profits to the supplier and the retailer increase, enhancing overall performance.

2.3.2 Strategic retailer without inventory carryover (scenario ND)

Next, we examine the scenario where the retailer is strategic and maximizes total profit over periods but forsakes inventory. In this scenario, given the supplier's capital K_2, interactions in period 2 are the same as in scenario M. Thus, decisions in period 2 are determined by Equation (2-9). In period 1, the retailer bases purchase decision on wholesale price w_1. By the state-transition equation given in Equation (2-5), the supplier's capital in period 2 increases with the retailer's purchase in period 1. Higher capital availability in period 2, in turn, benefits both firms by relaxing the constraint on operations. Anticipating this, the strategic retailer tends to purchase more aggressively than his myopic counterpart, balancing imminent cost increase with the gain in the later period. Lemma 2.1 states the retailer's optimal purchase quantity and identifies when the retailer exhibits such strategic behavior.

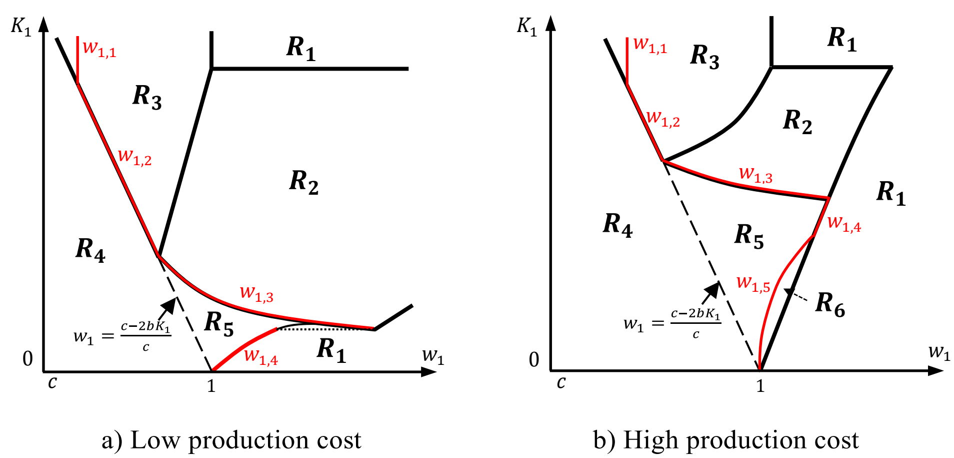
Lemma 2.1. *Under dynamic contract, when the retailer is strategic, given the wholesale price* w_1*:*

1) The purchase quantity $q_1^{ND}(w_1)$ *by the retailer is as follows:*

Region in Figure 2.2	R_1	R_2	R_3	R_4,R_5	R_6
Purchase quantity $q_1^{ND}(w_1)$	0	$\frac{(1-c)c-4bK_1}{4b(w_1-c)}$	$\frac{1-w_1}{2b}$	$\frac{K_1}{c}$	$\frac{c^2(1-w_1)+2bK_1(w_1-c)}{2b(2c-w_1)w_1}$

The regions listed in the table are defined in the Appendix and illustrated in Figure 2.2.

2) $q_1^{ND}(w_1) > q_1^M(w_1|K_1)$ *in regions* R_2,R_5,R_6*, but* $q_1^{ND}(w_1) = q_1^M(w_1|K_1)$ *otherwise.*

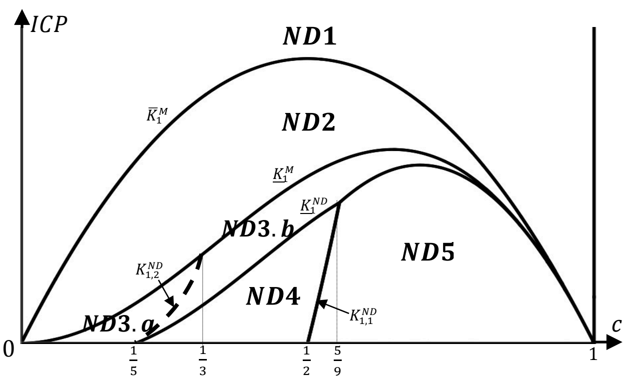


Notes. $w_{1,1}$ to $w_{1,5}$ are the five segments of the supplier's optimal wholesale price w_1^{ND} in period 1.

Figure 2.2 Retailer's best-response purchase quantity $q_1^{ND}(w_1)$

The strategic retailer forgoes purchasing when wholesale price in period 1 is high (region R_1) but purchases at capacity when it is low (region R_4). When the price is medium, the retailer tailors decision to the supplier's ICP. At a high ICP (region R_3), the retailer purchases to maximize the imminent profit as if without capital constraint, resulting in a capital position in period 2 that imposes no constraint. Otherwise (regions R_2, R_5, R_6), the retailer strategically purchases more than a myopic counterpart in period 1. With strategic purchasing, the capital constraint in period 2 is alleviated when the supplier's ICP is mediumly high (region R_2). At a medium ICP (region R_5), the retailer purchases at capacity. At a low ICP (region R_6), the retailer, with a weak purchasing incentive, purchases slightly below capacity to balance the marginal loss and marginal gain in the two periods. In the latter two cases, the capital constraint in period 2 persists to restrict the interactions for firms. A lower production cost enhances the supplier's profit margin and capital accumulation capability. This, in turn, allows the retailer's strategic purchasing to have a more significant impact on the supplier's capital build-up, encouraging the retailer to purchase. As a result, when the production cost is low, region R_6 in Figure 2.2 degenerates. In this situation, the retailer will not purchase below capacity unless a smaller quantity ensures that operations in period 2 are not constrained.

Under dynamic contract, the retailer's strategic purchasing can exhibit an "economy of scale." Specifically, given the retailer's purchase in period 1, as ICP increases, the supplier's capital position in period 2 increases, allowing the retailer to sell more in period 2. As a result, while the retail price in period 2 decreases with higher sales, the wholesale price in period 2 decreases even faster, thereby boosting the retailer's profit margin. This enhances the retailer's gain from increasing sales in period 2, strengthening his incentive to purchase more in period 1 to improve capital availability in period 2.



Notes. $\underline{K}_1^{ND}$, $K_{1,1}^{ND}$ and $K_{1,2}^{ND}$ are defined in the Appendix.

Figure 2.3 Performance outcomes when the retailer is strategic under dynamic contract

Proposition 2.2. *Under dynamic contract, when the retailer is strategic, let* $K_2^{ND} = \frac{w_1^{ND} K_1}{c}$*, referring to Figure 2.3, the wholesale prices and purchases over periods are as follows.*

Conditions	w_1^{ND}	q_1^{ND}	w_2^{ND}	q_2^{ND}
$K_1 \geq \bar{K}_1^M$ (Area $ND1$)	$\frac{1+c}{2}$	$\frac{1-c}{4b}$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$\underline{K}_1^M \leq K_1 < \bar{K}_1^M$ (Area $ND2$)	$1 - \frac{2bK_1}{c}$	$\frac{K_1}{c}$		
$\underline{K}_1^{ND} \leq K_1 < \underline{K}_1^M$ (Area $ND3$)	$\frac{(1-c)c^2}{4bK_1}$			
$K_1 < \underline{K}_1^{ND} \& K_1 \geq K_{1,1}^{ND}$ (Area $ND4$)	$\frac{c^3 - c\sqrt{c^4 - 4bcK_1 + 8b^2K_1^2}}{2bK_1}$		$\frac{c-2bK_2^{ND}}{c}$	$\frac{K_2^{ND}}{c}$
$K_1 < \underline{K}_1^{ND} \& K_1 < K_{1,1}^{ND}$ (Area $ND5$)	$\frac{c^3 + 2bcK_1 - c\sqrt{c^4 - 4b(2-c)cK_1 + 20b^2K_1^2}}{4bK_1}$			

Proposition 2.2 reveals the equilibrium pattern under dynamic contract when the retailer is strategic. Specifically, the supplier's capital has no operational consequences when the ICP is high (i.e., $K_1 \geq \overline{K}_1^M$), while it is inconsequential only in period 2 when the ICP is moderate (i.e., $\underline{K}_1^{ND} \leq K_1 < \overline{K}_1^M$). Relative to when the retailer is myopic, strategic purchasing by the retailer has no effects on performances when $K_1 \geq \underline{K}_1^M$. With $K_1 < \underline{K}_1^M$, myopic purchasing would expose the interactions in both periods to be under the influence of capital constraint, while strategic purchasing can alleviate this influence in period 2 provided the ICP is not too low (i.e., $K_1 \geq \underline{K}_1^{ND}$). However, when the ICP is significantly low (i.e., $K_1 < \underline{K}_1^{ND}$), the supplier's capital restricts operations in both periods, despite the retailer's strategic purchasing behavior.

Recall that the retailer's purchase incentive increases as the supplier's production cost decreases. When production cost is low ($ND4$ in Figure 2.3), a strong incentive drives the retailer to order at full capacity when the wholesale price in period 1 is not extremely high; otherwise, he orders nothing. The supplier sets the price to ensure that ordering is more profitable than abstaining. However, when production cost is high ($ND5$ in Figure 2.3), even when the wholesale price in period 1 is not extremely high, the retailer may order below full capacity, prompting the supplier to adjust the price to encourage full-capacity purchasing.

The profits to the supplier, the retailer, and the supply chain in scenario ND are presented in the Appendix. Corollary 2.2 states the effects of the supplier's ICP on these profits.

Corollary 2.2. *Under dynamic contract, when the retailer is strategic, as the supplier's ICP increases:*

1) *The supplier's profit π_s^{ND}, while increasing in general and sometimes substantially, can decrease when ICP is moderate (i.e., area $ND3$ in Figure 2.3).*
2) *The retailer's profit π_r^{ND} continuously increases.*
3) *The supply chain's profit π_{sc}^{ND} increases, and sometimes substantially.*

Corollary 2.2 reveals that the retailer's and supply chain's profits increase with ICP. However, the supplier's profit decreases when ICP is moderate (i.e., area $ND3$ in Figure 2.3). In this case, strategic purchasing increases the supplier's period-2 capital position just enough to insulate firms' interactions in period 2 from the influence of capital constraint. As ICP increases, less strategic purchasing is needed to achieve this state. This diminishes the retailer's incentive to purchase, making it less likely for the supplier to exploit this behavior. As a result, a higher ICP harms the supplier's profit. Furthermore, as ICP inches above $\underline{K}_1^N$ when $\frac{1}{5} < c < \frac{1}{3}$ (i.e., from $ND4$ to $ND3$), the operational adaptations lead to a significant increase in wholesale price in period 1, abruptly lifting the capital position in period 2, causing jumps in profits to the supplier and the supply chain.

Corollary 2.3. *Under dynamic contract, when the retailer is strategic, the supplier benefits from capital constraint if* $c \geq \sqrt{2} - 1$ *and* $K_{1,3}^{ND} \leq K_1 \leq \frac{(1-c)(3c-1)}{8b}$. $K_{1,3}^{ND}$ *is defined in the Appendix.*

Corollary 2.3 states that the supplier profits from capital constraint when production cost is high and ICP is moderate (i.e., upper part of $ND4 \cup ND5$ and lower part of $ND3$). By Corollary 2.2, as ICP increases, the supplier's profit first increases (in $ND5 \cup ND4$), then decreases (in $ND3$), and eventually increases again (in $ND2 \cup ND1$). As such, were the supplier to make a higher profit under capital constraint, the ICP must be moderate. When production cost is high, the supplier's profit without capital constraint is low since it decreases with production cost. As a result, the supplier's profit is higher with capital constraint at the supplier when production cost is high and ICP is moderate. This finding is consistent with the literature, which states that suppliers can benefit from having limited capacity, weakening their incentive to invest in capacities, even at no additional cost (e.g., Keskinocak et al. 2008, Yang et al. 2018).

2.3.3 Effects of retailer's strategic purchasing behavior

We study the effects of the retailer's strategic purchasing behavior under dynamic contract by comparing performance outcomes when the retailer is strategic relative to when he is myopic.

Proposition 2.3. *Under dynamic contract, when the retailer is strategic, performance outcomes differ from those in the case of a myopic retailer when* $K_1 < \underline{K_1^M}$. *Specifically:*

1) $w_1^{ND} \geq w_1^M$, $q_1^{ND} = q_1^M = \frac{K_1}{c}$, $w_2^{ND} \leq w_2^M$, $q_2^{ND} \geq q_2^M$;
2) $\pi_s^{ND} \geq \pi_s^M$ *and* $\pi_{sc}^{ND} \geq \pi_{sc}^M$; *while* $\pi_r^{ND} \geq \pi_r^M$ *only when production cost* c *is low and* K_1 *is moderate (i.e., area* $ND3.a$ *in Figure 2.3).*

Relative to when the retailer is myopic, strategic purchasing by the retailer has operational consequences when the supplier's ICP is low ($K_1 < \underline{K_1^M}$). With purchase quantity in period 1 fixed at capacity, the supplier raises the wholesale price in period 1, leading to a decrease in wholesale price and a higher purchase in period 2. It improves the profits of the supplier and the supply chain. However, the retailer generally suffers a loss from turning strategic in purchasing, except when the production cost is low and the ICP is moderate (i.e., area $ND3.a$ in Figure 2.3), in which case the gain from enhanced capacity at the supplier in period 2 outweighs the loss from a higher wholesale price in period 1 to benefit the retailer's profit. Regardless, due to increased sales, consumers benefit from the retailer's strategic approach and overall social welfare improves.

Practical examples are ample to show that strategic retailers pay higher wholesale prices early in the season. For instance, since April 2024, Toyota, the Japanese auto manufacturer, has used its record profit to pay high procurement prices to financially strapped parts suppliers, aiming to enhance future capital position. It enables suppliers to recruit more workers and upgrade production facilities, expanding capacities to meet Toyota's "breakneck sales and production pace" (Greimel and Okamura 2024). Toyota's strategic capital transfer is consistent with the behavior captured in our model and offers empirical validation to our results. The myopic behavior modeled in Scenario M is observed in practice as well. For example, Stellantis, the fourth-largest automaker in the world, focuses on short-term profit maximization by significantly improving the company's immediate profit margins. While this approach is detrimental to maintaining supplier relationship, it bolsters Stellantis's short-term financial performance (Wayland 2024). Our research indicates that such short-sightedness may not result in a loss. However, managers acting strategically should be alert to potential win-lose scenarios and be prudent in assessing suppliers' production costs and capital before implementing the strategy. It is imperative to align incentives throughout the supply chain by mechanisms like capping wholesale prices to share initial cost increase and leverage retailers' strategic purchasing. Moreover, managers are advised to perceive procurement not only as a cost-containment tool but as a strategic lever to build supply chain robustness and long-run profitability.

2.4 Commitment Contract

Next, we analyze the performance outcomes of commitment contract and the role of strategic purchasing under it.

2.4.1 Myopic retailer

Similar to the analysis of dynamic contract, we start with a scenario where the retailer is myopic.

Proposition 2.4. *When the retailer is myopic, the performance outcomes are the same under commitment contract as under dynamic contract.*

Proposition 2.4 states that contract type has no consequential effects on performance outcomes over periods when the retailer is myopic. The timing of the supplier's decision on wholesale price for period 2 relative to the myopic retailer's purchase decision in period 1 has no effect on the latter's response. As such, under commitment contract, the supplier achieves the optimal total profit only when profits for the two periods are maximized individually, yielding the same results as under dynamic contract.

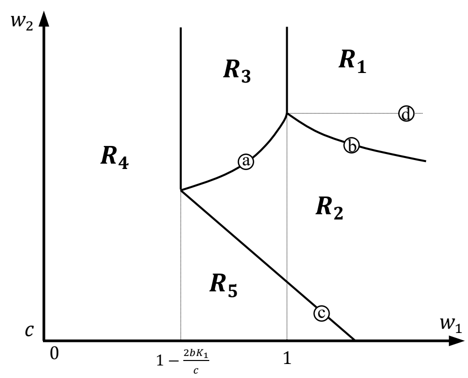
2.4.2 Strategic retailer without inventory carryover (scenario NC)

Given wholesale prices (w_1, w_2), the strategic retailer determines purchase quantities (q_1, q_2) to maximize total profit over the two periods. It is optimal to set $q_2 = max\left\{0, min\left\{\frac{1-w_2}{2b}, \frac{K_2}{c}\right\}\right\}$. As indicated by the state-transition equation (Equation (2-5)), a higher purchase in period 1 increases the supplier's capital in period 2. This can benefit the retailer by enabling larger sales in period 2. Anticipating this, the strategic retailer may purchase more in period 1 than a myopic counterpart, a behavior consistent with his action under dynamic contract. Superscript C indicates commitment contract. Lemma 2.2 presents the retailer's optimal purchase quantity in period 1 in response to (w_1, w_2).

Lemma 2.2. *Under commitment contract, when the retailer is strategic, given wholesale prices for the two periods* (w_1, w_2)*, the retailer's purchase quantity* $q_1^{NC}(w_1, w_2)$ *in period 1 is as follows:*

Region in Figure 2.4	R_1	R_2	R_3	R_4, R_5
Purchase quantity q_1^{NC}	0	$\frac{(1-c)cw_1-(2bK_1+cw_2)(w_1-c)}{2b(c^2+(w_1-c)^2)}$	$\frac{1-w_1}{2b}$	$\frac{K_1}{c}$

The regions listed in the table are defined in the Appendix and illustrated in Figure 2.4.



Notes. The labeled curves are defined as: a) $w_2 = \frac{2(c-bK_1)-(1+c)w_1+w_1^2}{c}$; b) $w_2 = \frac{2bcK_1+((1-c)c-2bK_1)w_1}{c(w_1-c)}$; c) $w_2 = \frac{-2bK_1(c^2+w_1^2-cw_1)+c^2(1-c)w_1}{c^2(w_1-c)}$; d) $w_2 = 1-\frac{2bK_1}{c}$.

Figure 2.4 Retailer's best-response purchase quantity $q_1^{NC}(w_1, w_2)$

The retailer purchases at capacity when the wholesale price in period 1 is low (region R_4), while he refrains from purchasing when the wholesale prices over both periods are high (region R_1). As the wholesale price in period 1 is medium and the wholesale price in period 2 is high (region R_3), the retailer purchases to maximize the imminent profit as if without capital limit, leading to a capital position in period 2 that imposes no constraint. Otherwise, in regions R_2 and R_5, a high wholesale price in period 1 accompanied by a low wholesale price in period 2 is necessary to lead the retailer to strategically purchase more in period 1 than a myopic counterpart. Under commitment contract, the supplier lacks the flexibility in wholesale pricing. Whereas the retailer's sales in period 2 is boosted by strategic purchasing, his profit margin decreases since he has to lower retail price while the wholesale price in period 2 is committed. This tradeoff between more sales and lower profit margin balances at a purchase under capacity in region R_2 but at capacity in region R_5. Despite strategic purchasing, the supplier's capital position in period 2 still restricts firms' interactions.

Under commitment contract, the retailer's strategic purchasing can exhibit a "diseconomy of scale," which is in contrast to the "economy of scale" observed under dynamic contract. Specifically, given the retailer's purchase in period 1, an increase in the supplier's ICP enhances her capital position in period 2, enabling greater retailer sales in period 2. This higher sales lowers the retail price in period 2; consequently, with a committed wholesale price in period 2, the retailer's profit margin shrinks. This decreases the retailer's gain from increasing sales in period 2, weakening his incentive to purchase more in period 1. This phenomenon is evidenced by the shrinkage of regions R_2 and R_5, where the retailer engages in strategic purchasing, as ICP increases (i.e., curves a and b descend).

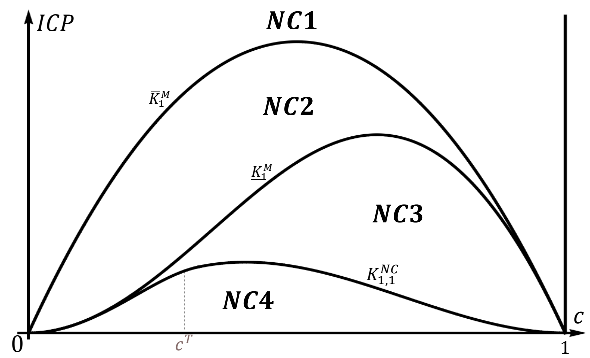


Figure 2.5 Performance outcomes when the retailer is strategic under commitment contract

Proposition 2.5. *Under commitment contract, when the retailer is strategic, let* $K_2^{NC} = \frac{w_1^{NC} K_1}{c}$, *the wholesale prices and purchases over periods are as follows*[3].

<table>
<tr><th>Conditions</th><th>w_1^{NC}</th><th>q_1^{NC}</th><th>w_2^{NC}</th><th>q_2^{NC}</th></tr>
<tr><td>$K_1 \geq \bar{K}_1^M$ (Area $NC1$)</td><td>$\frac{1+c}{2}$</td><td>$\frac{1-c}{4b}$</td><td rowspan="2">$\frac{1+c}{2}$</td><td rowspan="2">$\frac{1-c}{4b}$</td></tr>
<tr><td>$\underline{K}_1^M \leq K_1 < \bar{K}_1^M$ (Area $NC2$)</td><td rowspan="2">$1 - \frac{2bK_1}{c}$</td><td rowspan="3">$\frac{K_1}{c}$</td></tr>
<tr><td>$K_{1,1}^{NC} \leq K_1 < \underline{K}_1^M$ (Area $NC3$)</td><td>$1 - \frac{2bK_2^{NC}}{c}$</td><td rowspan="2">$\frac{K_2^{NC}}{c}$</td></tr>
<tr><td>$0 \leq K_1 < K_{1,1}^{NC}$ (Area $NC4$)</td><td>$\widetilde{w}_1^{NC}$</td><td>$\widetilde{w}_2^{NC}$</td></tr>
</table>

Proposition 2.5 states wholesale prices and purchases under commitment contract when the retailer is strategic. Figure 2.5 illustrates the conditions stated in Proposition 2.5.

The pattern for prices and purchases over periods under commitment contract is similar to that under dynamic contract. When the supplier's ICP is significantly low ($K_1 < K_{1,1}^{NC}$), the retailer engages in strategic purchasing. In this case, the supplier sets equilibrium wholesale prices that lie on Curve c in Figure 2.4; responding to the prices, the retailer purchases at capacity in both periods. The

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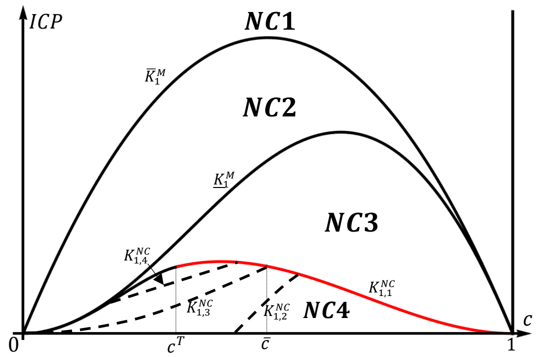
[3] *Notes.* $\widetilde{w}_1^{NC}$ is the largest root to $2bc^3K_1 - 2c^2((1-c)c + 2bK_1)w_1 + c((1-c)c + 8bK_1)w_1^2 - 4bK_1w_1^3 = 0$, and $\widetilde{w}_2^{NC} = \frac{-2bc^2K_1 + c((1-c)c + 2bK_1)w_1 - 2bK_1w_1^2}{c^2(w_1 - c)}$. $K_{1,1}^{C} \in [0, \underline{K}_1^M]$ is the root to $-c^4(1-2c) + 2b(1-c)(2+c)c^2K_1 - 8b^2(2-c)cK_1^2 + 16b^3K_1^3 = 0$ when $0 \leq c \leq c^T \approx 0.316$, and the root to $(1-c)^2c^6 - 4bc^4(1+c^2)K_1 + 4b^2c^2(1+4c-c^2)K_1^2 - 16b^3c(1+c)K_1^3 + 16b^4K_1^4 = 0$ otherwise, the two roots are the same at c^T.

underlying rationale is that the retailer's incentive to enhance the supplier's capital weakens as ICP rises. This reduces the supplier's ability to gain from exploiting such behavior. As a result, the supplier is less inclined to encourage the retailer's strategic purchasing and tends to discontinue this practice, despite potential sales increases in period 2, as ICP exceeds $K_{1,1}^{NC}$. Notably, this ICP threshold for strategic purchasing under commitment contract is lower than its dynamic contract counterpart.

The profits to the supplier, the retailer, and the supply chain in scenario NC are presented in the Appendix. Corollary 2.4 states the effects of the supplier's ICP on these profits.



	π_s^{NC}	π_r^{NC}	π_{sc}^{NC}
$NC1$	$\rightarrow$		
$NC2$	$\uparrow$		
$NC3$	$\uparrow$		
$NC4$	$\uparrow$ if $K_1 \geq K_{1,2}^{NC}$, $\downarrow$ otherwise	$\downarrow$ if $K_1 \geq K_{1,3}^{NC}$, $\uparrow$ otherwise	$\downarrow$ if $K_1 \geq K_{1,4}^{NC}$, $\uparrow$ otherwise

Notes. $K_{1,i}^{NC}$ for $i = 2, \ldots, 4$ are defined in the Appendix. When $c > c^T$, π_r^{NC} and π_{sc}^{NC} jump down as ICP moves above $K_1 = K_{1,1}^{NC}$.

Figure 2.6 Effects of relaxed ICP under commitment contract

Corollary 2.4. *Under commitment contract, when the retailer is strategic, as the supplier's ICP increases:*

1) *The supplier's profit π_s^{NC}, while increasing in general, decreases when production cost c is high and ICP is sufficiently low;*
2) *The retailer's profit π_r^{NC}, while increasing in general, decreases when ICP is moderate, and the profit loss can be substantial;*
3) *The supply chain's profit π_{sc}^{NC}, while increasing in general, decreases when production cost c is high and ICP is moderate, and the profit loss can be substantial.*

An increase in the supplier's ICP exerts mixed effects on the profits to firms and the supply chain in the situation where the retailer overpurchases (i.e., $K_1 \leq K_{1,1}^{NC}$). The supplier's profit decreases when production cost c is high (i.e., to the right of $K_1 = K_{1,2}^{NC}$ in $NC4$). Recall that as K_1 increases, the retailer's overpurchasing incentive weakens, forcing the supplier to decrease w_1, leading to a higher w_2. The reduced w_1 slows supplier's capital accumulation, dampening the effect of overpurchasing. When c is high, capacity $\frac{K_1}{c}$ is scarce, aggravating the loss from a weakened overpurchasing effect, leading it to outweigh the profit gain from a higher w_2, thereby reducing the supplier's profit.

The retailer's profit decreases as K_1 increases in a medium range ($K_{1,3}^{NC} < K_1 \leq K_{1,1}^{NC}$) when $c \leq \bar{c}$ (to the left of $K_1 = K_{1,3}^{NC}$ in $NC4$). Note that as K_1 increases, the retailer benefits from acquiring more products at a lower price in period 1. However, $K_2 = \frac{w_1 K_1}{c}$ decreases since w_1 lowers. When c is low and K_1 is medium, capacity $\frac{K_1}{c}$ is large, exacerbating the impact of a reduced w_1 on clamping down on K_2. Consequently, the retailer's loss in period 2 overrides his gain in period 1, causing his profit to drop. By contrast, when $c > \bar{c}$, the retailer's profit remains increasing with K_1 in region $NC4$, where K_1 is low. Nevertheless, as K_1 inches above threshold $K_{1,1}^{NC}$, the supplier no longer has to commit to wholesale prices at which the retailer overpurchases, leading to a lowered capital position in period 2 that results in a sharp decline in the retailer's profit.

It is noteworthy that an increase in the supplier's ICP leads to improvements in the profits to the supplier and the retailer only when the supplier's production cost c is low and ICP is low as well. Otherwise, it has opposite effects on the profits to the two firms. In case where the profit to the retailer increases but the profit to the supplier decreases (i.e., to the right of $K_1 = K_{1,2}^{NC}$ in $NC4$), the profit gain to the retailer dominates to improve the supply chain profit. Nevertheless, in case where the profit to the retailer decreases but the profit to the supplier increases (i.e., to the left of $K_1 = K_{1,3}^{NC}$ in $NC4$), the supply chain profit increases when the supplier's ICP is mediumly low but decreases otherwise.

2.4.3 Effects of retailer's strategic purchasing behavior

We study the effects of the retailer's strategic purchasing behavior under commitment contract, by comparing performance outcomes when the retailer is strategic relative to when he is myopic.

Proposition 2.6. *Under commitment contract, when the retailer is strategic, performance outcomes differ from those in the case of a myopic retailer when* $K_1 < K_{1,1}^{NC}$. *Specifically:*

1) $w_1^{NC} \geq w_1^M$, $q_1^{NC} = q_1^M = \frac{K_1}{c}$, $w_2^{NC} \leq w_2^M$, $q_2^{NC} \geq q_2^M$;

2) $\pi_s^{NC} \geq \pi_s^M$, $\pi_r^{NC} \geq \pi_r^M$, $\pi_{sc}^{NC} \geq \pi_{sc}^M$.

The retailer's strategic purchasing induces the supplier to raise the wholesale price in period 1, accompanied by a lower wholesale price in period 2 to ensure full capacity purchase in period 1. Purchase quantities in period 1 are the same in two scenarios. As a result, the capital position in period 2 increases, leading the retailer to purchase more. The sales improvement benefits the supplier and the supply chain. All these phenomena echo those under dynamic contract. More important, strategic purchasing by the retailer plays a phenomenal role in increasing the retailer's profit under commitment contract, creating a win-win situation for the firms at stake, than under dynamic contract, when it subsidizes the supplier's profit at the retailer's expense.

2.5 Contract Design

Premised on the analysis of the two contract types when the retailer is strategic, we engage in a comparative investigation into dynamic and commitment contracts from three perspectives: i) the occurrence of strategic purchasing, ii) the impact of the retailer's strategic purchasing on operations adaptations, and iii) overall supply chain performance.

2.5.1 Occurrence of strategic purchasing

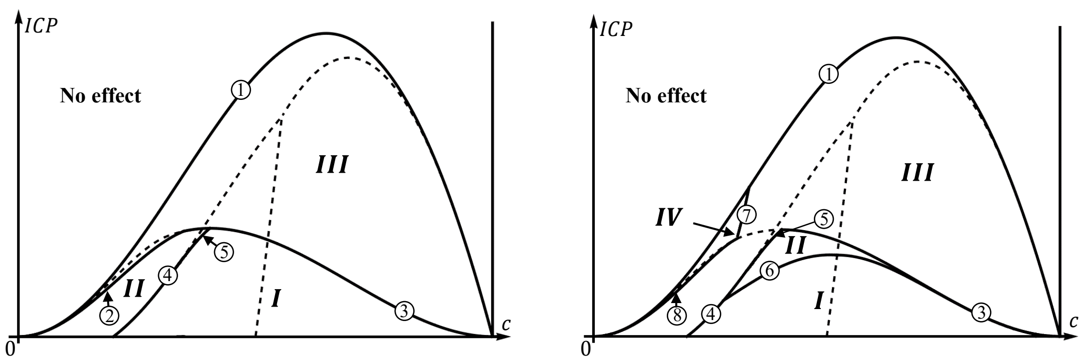
The patterns for the occurrence of strategic purchasing are similar under both contracts: when the supplier's ICP is sufficiently low, the retailer purchases more than $q_1^M(w_1^*|K_1)$—the quantity that maximizes imminent profit at the equilibrium wholesale price w_1^*. While this behavior harms retailer's profit in period 1, it yields a gain in period 2 due to more sales at a lower cost. Recall that the threshold ICP, below which strategic purchasing occurs, is $\underline{K_1^M}$ under dynamic contract, but $K_{1,1}^{NC}$ under commitment contract, with $K_{1,1}^{NC} < \underline{K_1^M}$; strategic purchasing is more likely to occur under dynamic contract than commitment contract. It is traced to the contrast between "economy of scale" under dynamic contract and "diseconomy of scale" under commitment contract in the occurrence of strategic purchasing, i.e., an increase in ICP strengthens the retailer's incentive to strategically purchase in the former case but weakens it in the latter case. When the ICP is high (i.e., $K_1 \geq \underline{K_1^M}$), the retailer has no incentive for strategic purchasing, as the supplier's resulting capital in period 2 is sufficient to alleviate the constraint on operations interactions even if the retailer responds myopically to the wholesale price in period 1. However, a low ICP (i.e., $K_1 < \underline{K_1^M}$) creates an incentive for strategic purchasing by the retailer. Under dynamic contract, when the ICP is extremely low, the capital shortage is so severe that strategic purchasing arises. And as the ICP increases, the retailer's strengthened incentive for this behavior ensures it persists until $\underline{K_1^M}$ is reached. In contrast, under commitment contract, as the ICP increases, the retailer's diminished incentive for strategic purchasing makes it harder for the supplier to encourage such behavior, thereby limiting the supplier's ability to profit from exploiting it. Consequently, once ICP exceeds $K_{1,1}^{NC}$, the supplier no longer sees a benefit in encouraging strategic purchasing and tends to abandon it.

2.5.2 Operations adaptations to the retailer's strategic purchasing

When $K_1 \leq \underline{K_1^M}$, the retailer's strategic purchasing benefits the supplier and supply chain. However, under dynamic contract, it benefits the retailer only when the supplier's production cost is low and ICP is moderate (i.e., area $ND3.a$ in Figure 2.3); otherwise, the supplier's exploitation of this strategic behavior harms the retailer relative to when he is myopic. This downside of the retailer's strategic purchasing degenerates under commitment contract. Specifically, in the area where strategic purchasing occurs (i.e., $K_1 < K_{1,1}^{NC}$), it results in a win-win situation to the supplier and retailer. The reason for this difference is as follows. Under dynamic contract, as the retailer purchases more in period 1, the supplier's wholesale price in period 2 decreases, allowing the retailer to profit from both a higher profit margin and more sales in period 2. In contrast, under commitment contract, the wholesale price in period 2 is committed; while the retailer can still profit from more sales, his profit margin in period 2 shrinks. Thus, purchasing more in period 1 is less attractive to the retailer under commitment contract. This, in turn, makes the supplier less aggressive with her period-1 pricing, avoiding the win-lose situation seen in dynamic contract and creating a more balanced interaction.

2.5.3 Supply chain performances

Next, we examine the effects of contract type on operations and profits when the retailer is strategic.



	I	II	III	
w_1^{ND}	$<$	$\geq$		w_1^{NC}
w_2^{ND}	$\geq$		$<$	w_2^{NC}
q_2^{ND}	$<$	$\geq$		q_2^{NC}

a) Wholesale prices and purchases

	I	II	III	IV	
π_s^{ND}	$<$	$\geq$			π_s^{NC}
π_r^{ND}	$<$			$\geq$	π_r^{NC}
π_{sc}^{ND}	$<$		$\geq$		π_{sc}^{NC}

b) Profits

Notes. The labeled curves are: 1) $K_1 = \underline{K}_1^M$; 2) $K_1 = K_{1,1}^N$; 3) $K_1 = K_{1,1}^{NC}$; 4) $K_1 = \frac{(2-\sqrt{6-12c+10c^2})c}{8b}$; 5) $K_1 = K_{1,2}^N$; 6) $K_1 = K_{1,3}^N$; 7) $K_1 = K_{1,2}^{ND}$; 8) $K_1 = K_{1,4}^N$. $K_{1,i}^N, i \in \{1, \ldots, 4\}$ are defined in the Appendix.

Figure 2.7 Comparisons between dynamic contract and commitment contract

Proposition 2.7. *When the retailer is strategic, the comparison outcomes of wholesale prices* (w_1, w_2) *and purchase quantities* (q_1, q_2) *under dynamic contract and commitment contract are as follows:*

1) $w_1^{ND} < w_1^{NC}$, $w_2^{ND} \geq w_2^{NC}$, *and* $q_2^{ND} < q_2^{NC}$, *when the supplier's production cost is high and the* ICP *is low (i.e., area* I *in Figure 2.7.a);* $w_1^{ND} \geq w_1^{NC}$, $w_2^{ND} < w_2^{NC}$, *and* $q_2^{ND} \geq q_2^{NC}$ *when the supplier's* ICP *is moderate (i.e., area* III *in Figure 2.7.a), while* $w_1^{ND} \geq w_1^{NC}$, $w_2^{ND} \geq w_2^{NC}$ *when both the supplier's production cost and* ICP *are low (i.e., area* II *in Figure 2.7.a*).

2) $q_1^{ND} = q_1^{NC} = q_1^M = min\left\{\frac{K_1}{c}, \frac{1-c}{4b}\right\}$.

When the production cost is high and the ICP is low (i.e., area I in Figure 2.7.a), the retailer engages in strategic purchasing under either contract. Under dynamic contract, the retailer's weak incentive for strategic purchasing compels the supplier to set a lower wholesale price in period 1 than under commitment contract, resulting in a lower capital position and fewer sales in period 2. Note that under commitment contract, the supplier balances wholesale prices across both periods to influence the retailer's purchasing decisions. The supplier can either set a high period-1 and low period-2 price to benefit from enhanced period-2 capital, or a low period-1 and high period-2 price to benefit from a higher period-2 profit margin. In this specific area, the retailer's strong incentive for strategic purchasing leads the supplier to adopt the first approach. This results in a period-1 wholesale price that is higher than under dynamic contract, coupled with a commitment to a lower period-2 price designed to secure a full-capacity order in period 1. When the supplier's ICP is moderate (i.e., area III in Figure 2.7.a), under dynamic contract, the supplier exploits the retailer's strong incentive for strategic purchasing by setting a higher wholesale price in period 1. This boosts the supplier's capital in period 2, which allows for a lower period-2 price. Conversely, under commitment contract, given the retailer's weak incentive for strategic purchasing, it is difficult for the supplier to profit by exploiting this behavior; as a result, the supplier generally abandons inducing strategic purchasing. If the supplier does attempt to induce it, she must balance wholesale prices over periods with a lower wholesale price in period 1 and a higher one in period 2. Nevertheless, a counterintuitive scenario arises when both the supplier's production cost and ICP are low (i.e., area II in Figure 2.7.a): the wholesale prices for both periods are higher under dynamic contract. This is due to the previously mentioned less aggressive pricing under commitment contract when the retailer strategically purchases under either contract. The supplier's weak pricing power prevents him from setting a period-1 wholesale price higher than that under dynamic contract, even with a sufficiently low period-2 wholesale price.

The results of Proposition 2.7 reveal several key insights into the performance of dynamic versus commitment contracts. First, commitment contract is more effective in alleviating capital constraint to facilitate overall sales when the supplier's production cost is high and capital shortage is significant (i.e., area I in Figure 2.7.a), while dynamic contract is better suited for addressing mild capital constraint by easily exploiting the retailer's strategic purchasing behavior (i.e., areas II and III in Figure 2.7.a). Second, balanced pricing under commitment contract acts as a "double-edged sword" for the supplier. On one hand, commitment contract gives the supplier two pricing levers to affect the retailer's strategic purchasing in period 1. By committing to a sufficiently low period-2 price, period-1 price can be very high, boosting period-2 capital. Under dynamic contract, however, the supplier has only one pricing lever and thus can only cater to the retailer's purchase incentive. Consequently, when the production cost is high and the ICP is low (i.e., area I in Figure 2.7.a), although commitment contract is less attractive for the retailer to purchase more in period 1, enhanced capital availability is achieved. On the other hand, the benefit of having an additional pricing lever comes at the cost of less aggressive pricing. This is particularly evident when both the production cost and the ICP are low (i.e., area II in Figure 2.7.a), where wholesale prices across both periods are lower under commitment contract.

Proposition 2.8. *When the retailer is strategic, the comparison outcomes of profits under dynamic contract and commitment contract are as follows:*

1) *The contract type facilitating greater total sales yields more profit to supply chain, i.e.,* $\pi_{sc}^{ND} \geq \pi_{sc}^{NC}$ *if* $q_1^{ND} + q_2^{ND} \geq q_1^{NC} + q_2^{NC}$*, in which case individual firms may suffer profit loss compared to under the alternative contract.*
2) *The supplier profits more under commitment contract, i.e.,* $\pi_s^{NC} \geq \pi_s^{ND}$*, when her production cost is high and the* ICP *is significantly low (i.e., area* I *in Figure 2.7.b). Otherwise, her profit is higher under dynamic contract, in which case total sales may be lower than that under commitment contract.*
3) *The retailer profits more under dynamic contract, i.e.,* $\pi_r^{ND} \geq \pi_r^{NC}$*, when the supplier's production cost is sufficiently low and the* ICP *is moderate (i.e., area* IV *in Figure 2.7.b). Otherwise, his profit is higher under commitment contract, in which case total sales may be lower than under dynamic contract.*

As revealed in Figure 2.7.b, when the production cost is high and the ICP is significantly low (i.e., area I in Figure 2.7.b), the supplier and the retailer are simultaneously better off with commitment contract, which yields more total sales than dynamic contract. However, when the production cost is low and the ICP is moderate (i.e., area IV in Figure 2.7.b), the supplier and the retailer are better off simultaneously with dynamic contract, which yields more total sales than commitment contract. This finding is consistent with the observations on the choice of contracts between capital-constrained suppliers and strategic retailers that maintain long-term relationships. For instance, when Nike faced a severe cash shortage in its early development days, it launched a program committing to future wholesale prices and purchase quantities with its retailers. A parallel exists with Piedmont Lithium, an emerging mining company that significantly lacked the capital for production (Lambert 2023); it secured a contract with Tesla that established long-term commitments for both prices and quantities. Conversely, the agreement between Tesla and Yahua Industrial Group, which is an established chemical company with relatively ample capital and efficient production process, specifies that wholesale prices and purchase quantities should be determined on an annual basis. In these instances, contract design is tailored to specific market conditions and production technology, resulting in a win-win situation for both the supplier and the retailer.

The contract that yields higher overall sales is more effective in alleviating the capital constraint at the supplier. For the retailer, the wholesale price in period 1 is higher under the more effective contract. If it proves too expensive, he would prefer the more cost-efficient alternative, balancing purchasing cost in period 1 with more sales in period 2. For the supplier, the more effective contract typically lowers wholesale price in period 2, demanding her to balance more sales with a lower profit margin. When the production cost is high and the ICP is significantly low or when the production cost is low and the ICP is moderate (i.e., areas I or IV in Figure 2.7.b), the benefit of more sales dominates; thus, both firms prefer the contract that is more effective in alleviating capital constraint to yield more sales. However, when the production cost is high and the ICP is moderately low (i.e., area II in Figure 2.7.b), the supplier is better off with the less effective contract, i.e., dynamic contract, as the benefit of a higher profit margin overweighs the loss of total sales. Moreover, when the production cost is not low and the ICP is moderate (i.e., area III in Figure 2.7.b), the retailer is better off with commitment contract, which is less effective, because the benefit of greater total sales cannot offset high purchasing cost in period 1. Consequently, despite the possibility that the two firms converge in their preferences for the contract that is more effective in sales generation, circumstances exist in which the firms diverge in the contract preferences, with the supplier preferring dynamic contract while the retailer preferring commitment contract.

2.6 Extension: Demand uncertainty

Next, we incorporate demand uncertainty by assuming that the maximum price A in inverse demand function, given in Equation (2-1), is random and follows a uniform distribution $U[1-\delta, 1+\delta]$, where $\delta \in [0,1]$ models the intensity of market uncertainty. The expected value of A is 1, which aligns with the maximum price in the deterministic case. An increase in δ implies a more volatile market. Let a be a realization of A. Once uncertainty is revealed in period 1, the same inverse demand curve applies in period 2. The demand curve is observed after the retailer makes purchase decision before selling in period 1, meaning the period-1 purchase quantity may differ from the selling quantity. We add "$\hat{\cdot}$" on quantities of interest to indicate uncertain demand.

2.6.1 Dynamic contract

We relegate analysis of the myopic and strategic retailer cases under dynamic contract with uncertain demand to the Appendix. Proposition 2.9 presents the main results, confirming the robustness of findings in the deterministic case.

Proposition 2.9. *With demand uncertainty ($\delta > 0$), under dynamic contract, when the retailer is strategic, the retailer engages in strategic purchasing behavior when $K_1 \leq \hat{K}_1^T$, where $\hat{K}_1^T > \underline{K_1}^M$ and $\hat{K}_1^T$ increases with δ.*

With demand uncertainty, the equilibrium pattern is similar to that when the demand is deterministic. When the supplier's ICP is high, the strategic retailer would behave myopically. When the ICP is low, however, he engages in strategic purchasing, purchasing more than a myopic counterpart at a given period-1 wholesale price ($\hat{q}_1^{ND}(\widehat{w}_1^{ND}) > \hat{q}_1^{M}(\widehat{w}_1^{ND})$). Recall that threshold ICP to regulate the rise of the retailer's strategic purchasing in the deterministic case is $\underline{K_1^M}$. We show that $\widehat{K}_1^T > \underline{K_1^M}$, implying that strategic purchasing is more likely to be observed in volatile markets, and it is more obvious as market volatility intensifies.

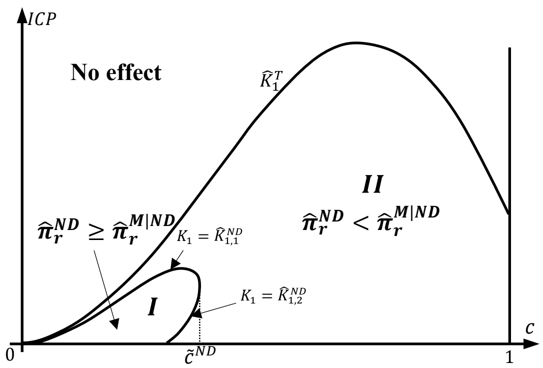


Figure 2.8 Strategic purchasing effect on retailer profit under dynamic contract with uncertain demand

Proposition 2.10. *With uncertain demand ($\delta > 0$), under dynamic contract, when the retailer is strategic, when $K_1 \leq \widehat{K}_1^T$, strategic purchasing benefits the supplier and the supply chain, but may adversely affect the retailer. Specifically, the retailer benefits from this behavior, i.e., $\widehat{\pi}_r^{ND} \geq \widehat{\pi}_r^{M|ND}$, iff $c \leq \tilde{c}^{ND}$ and $\left(\widehat{K}_{1,2}^{ND}\right)^+ \leq K_1 \leq \widehat{K}_{1,1}^{ND}$ (i.e., area I in Figure 2.8), where $\tilde{c}^{ND}$, $\widehat{K}_{1,1}^{ND}$, and $\widehat{K}_{1,2}^{ND}$ are defined in the Appendix.*

Similar to that in the deterministic case, the retailer is better off when the production cost is low and the ICP is moderate (i.e., area I in Figure 2.8). Figure 2.9 illustrates the impact of market uncertainty intensity on the retailer-improvement area (defined as the region where the retailer profits from his strategic purchasing behavior).

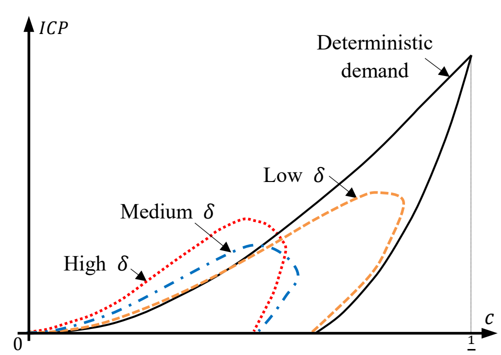


Figure 2.9 Effect of market uncertainty intensity on the retailer-improvement area

Compared to the deterministic case, the retailer-improvement area shrinks under demand uncertainty. Furthermore, as market volatility increases, this area first contracts and then expands. This indicates that demand uncertainty erodes the retailer's profit from strategic purchasing, with the most significant negative impact occurring at moderate volatility. Moreover, as the market becomes more volatile, the production cost threshold below which the retailer benefits from behaving strategically decreases; that is, strategic purchasing becomes profitable only under lower cost conditions.

2.6.2 Commitment contract

We relegate the analysis for the cases with a myopic retailer and a strategic retailer under commitment contract with uncertain demand to the Appendix. Proposition 2.11 state the main results.

Proposition 2.11. *With demand uncertainty ($\delta > 0$), under commitment contract, when the retailer is strategic:*

1) *The retailer engages in strategic purchasing when $K_1 \leq \widehat{K}_{1,1}^{NC}$, where $\widehat{K}_{1,1}^{NC} = K_{1,1}^{NC}$ when $\delta \leq 0.5$, but $\widehat{K}_{1,1}^{NC} > K_{1,1}^{NC}$ when $\delta > 0.5$. Moreover, $\widehat{K}_{1,1}^{NC} < \widehat{K}_1^T$ and $\widehat{K}_{1,1}^{NC}$ increases with δ.*
2) *The supplier, the retailer, and the supply chain benefit from the retailer's strategic purchasing.*

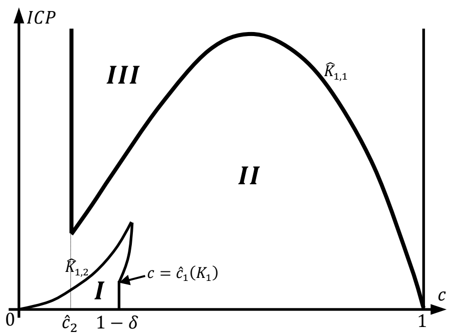
Proposition 2.11 reveals how demand uncertainty affects strategic purchasing under commitment contract. Uncertainty has inconsequential effects when it is low; however, at a high level, it makes strategic purchasing more likely to arise, with its prevalence increasing as uncertainty intensifies. Despite these new dynamics, two findings from the deterministic case remain robust. First, strategic purchasing is less likely to occur under commitment contract than under dynamic contract. Second, when it does occur under commitment contract, the retailer's such strategic behavior benefits all individual firms and the supply chain.

2.6.3 Comparison between dynamic contract and commitment contract

We next compare supplier, retailer, and supply chain profits under dynamic and commitment contracts to examine how contract type influences performance under demand uncertainty. Proposition 2.12 presents the results for a myopic retailer.

Proposition 2.12. *With demand uncertainty ($\delta > 0$), when the retailer is myopic:*

1) *The supplier's profit is higher under dynamic contract, i.e., $\widehat{\pi}_s^{M|ND} \geq \widehat{\pi}_s^{M|NC}$.*
2) *The retailer's profit is higher under dynamic contract, i.e., $\widehat{\pi}_r^{M|ND} > \widehat{\pi}_r^{M|NC}$, when $c > \hat{c}_2$ and $K_1 > \widehat{K}_{1,1}$ (i.e., area III in Figure 2.10), but is higher under commitment contract otherwise.*
3) *The supply chain profit is higher under commitment contract, i.e., $\widehat{\pi}_{sc}^{M|ND} \leq \widehat{\pi}_{sc}^{M|NC}$, when $c \leq \hat{c}_1(K_1)$ and $K_1 \leq \widehat{K}_{1,2}$ (i.e., area I in Figure 2.10), but is higher under dynamic contract otherwise.*



	I	II	III	
$\hat{\pi}_s^{M\|ND}$		$\geq$		$\hat{\pi}_s^{M\|NC}$
$\hat{\pi}_r^{M\|ND}$	$\leq$		$>$	$\hat{\pi}_r^{M\|NC}$
$\hat{\pi}_{sc}^{M\|ND}$	$\leq$	$>$		$\hat{\pi}_{sc}^{M\|NC}$

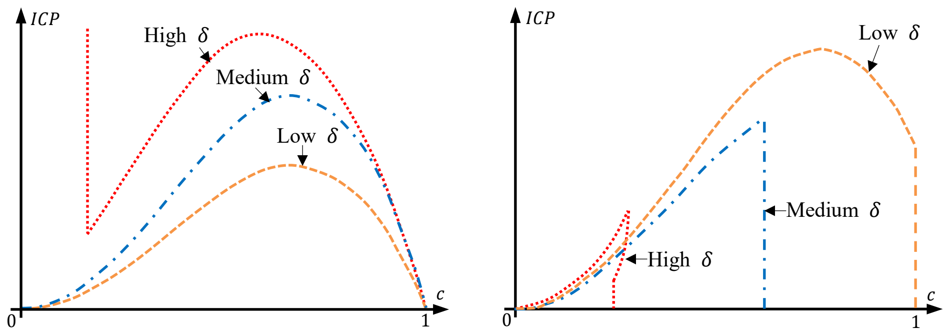
Notes. When $\delta \leq \dot{\delta} \approx 0.624$, the root to $40 - 123\delta + 120\delta^2 - 41\delta^3 = 0$, the threshold $c = \hat{c}_2$ degenerates; then $\hat{\pi}_r^{M|ND} > \hat{\pi}_r^{M|NC}$ iff $K_1 > \hat{K}_{1,1}$, establishing curve $K_1 = \hat{K}_{1,1}$ as the boundary between areas II and III.

Figure 2.10 Comparisons between two contracts with a myopic retailer under demand uncertainty

Figure 2.10 depicts results in Proposition 2.12, with a few additional details. With demand uncertainty, when the retailer is myopic, the performance outcomes are the same in period 1 but differ in period 2 across the two contracts. Under dynamic contract, the supplier tailors the wholesale price in period 2 after learning the realized demand condition, responding to which the retailer purchases. Conversely, both wholesale price and purchase quantity in period 2 are fixed before demand curve is revealed under commitment contract. The supplier utilizes the flexibility in wholesale pricing under dynamic contract to achieve a higher expected profit margin than under commitment contract, thereby consistently earning a higher overall profit.

For the retailer, the ability to tailor period-2 purchase to the realized market condition under dynamic contract avoid the mismatch between the pre-committed quantity and the realized demand that would occur under commitment contract. His contract preference depends on the tradeoff between lower wholesale price under commitment contract and higher purchase flexibility under dynamic contract. Specifically, when the production cost is low, the retailer purchases a significantly high quantity in period 2 under commitment contract to minimize the loss from unmet demand (this quantity exceeds the expected purchase quantity under dynamic contract). However, the wholesale price in period 2 is low enough to offset the cost of excess purchasing. As a result, commitment contract is always preferrable for the retailer regardless of the supplier's ICP. When the production cost is high, the retailer's contract preference depends on the scale of ICP. A higher ICP boosts the supplier's period-2 capital, which allows her to better respond to the retailer's demand. This, in turn, enhances the value of purchase flexibility inherent in dynamic contract. When the ICP is low (i.e., areas I and II in Figure 2.10), the benefit of lower wholesale price dominates, rendering commitment contract more appealing to the retailer. In contrast, when the ICP is high (i.e., area III in Figure 2.10), the value of purchase flexibility becomes high enough that dynamic contract yields more profit.

The supply chain profit is higher under commitment contract when both production cost and ICP are low (i.e., area I in Figure 2.10), because the benefit of making more sales outweighs the cost of excess production.



a) Retailer's profit b) Supply chain's profit

Figure 2.11 Effects of market uncertainty intensity on profit comparisons with a myopic retailer

Figure 2.11 illustrates that as market volatility increases, the area where supply chain profit is higher under dynamic contract than commitment contract (i.e., areas II and III in Figure 2.10) expands, while the area where the retailer's profit is higher under commitment contract than under dynamic contract (i.e., areas I and II in Figure 2.10) expands. Therefore, as the market exhibits stronger volatility, commitment contract is more likely to be a better choice from the retailer's perspective, as wholesale-price commitment for period 2 offers a protection mechanism against market volatility, while dynamic contract is more likely to make the supply chain better off for the enhanced overall sales it results in.

When the retailer is strategic, similar to the deterministic case, commitment contract is more effective in attaining a higher capital position in period 2 when the supplier's production cost is high and ICP is low, but dynamic contract is more effective when the ICP is high. Moreover, both the supplier and the retailer profit more from commitment contract when the supplier's ICP is low, but from dynamic contract when the ICP is high. Divergence in the firms' contract preferences occurs when the supplier's ICP is moderate, and this divergence aggravates as market volatility increases. Generally, the contract that results in a higher period-2 capital position benefits the supply chain more, except in slightly volatile markets.

2.7 Concluding Remarks

This chapter prohibits inventory holding to focus on the retailer's strategic purchasing behavior, i.e., purchasing more aggressively than the myopic retailer. This results in a higher early-period wholesale price, and thus helps to build the supplier's future capital. We consider a bilateral monopoly, consisting of a capital-straddled supplier and a strategic retailer, in a two-period horizon. In each period, the supplier sells a product at a wholesale price to the retailer, who in turn sells the product to consumers. Their relationship is governed either by a dynamic contract or by a commitment contract. Unlike a commitment contract, which fixes wholesale prices and purchase quantities before sales begin, a dynamic contract lets the supplier tailor wholesale prices each period to her capital position, after which the retailer chooses purchase quantities within the supplier's production capacity. The supplier can manage wholesale prices at which the retailer engages in strategic purchasing.

The main tradeoff embedded in strategic purchasing is that the retailer incurs a higher procurement cost in the early period in return for a profit gain from more sales at a lower cost due to the enhanced capital position at the supplier in the later period. Our complete analysis unravels novel insights into the role of the retailer's strategic purchasing in alleviating the supplier's future capital constraint and informs optimal contract design from various stakeholders' perspectives. The main results and insights are summarized as follows.

First, strategic purchasing behavior arises when the supplier's initial capital position (ICP) is low and production cost is moderate; otherwise, the strategic retailer behaves like his myopic counterpart, aiming to maximize the imminent profit in each period. Dynamic contract gives rise to such behavior in a wider range of conditions. Our intuition may suggest that capital constraint has a detrimental effect on the supplier's profit, and an increase in the supplier's ICP should benefit all firms. However, our results indicate that under dynamic contract, the supplier can benefit more from having capital constraint when the retailer engages in strategic purchasing, while a higher capital position at the supplier may harm individual firms and even the supply chain, which is particularly prominent under commitment contract.

Second, compared to when the retailer is myopic, the retailer's strategic purchasing, although benefiting the supplier (who utilizes the first-mover advantage in wholesale pricing to leverage such behavior and efficiently balance the profits over periods) and the supply chain (which benefits from enhanced sales that result from alleviated capital constraint in the later period), can harm the retailer. Specifically, in the case when strategic purchasing arises, the retailer profits less than what his myopic counterpart could make in most circumstances under dynamic contract. In this scenario, the supplier increases the wholesale price from that in the case of myopic retailer in period 1, and flexibly tailors wholesale price in period 2 to fit with enhanced capital position. The profit gain to the retailer in period 2 is insufficient to compensate for the profit loss from a higher wholesale price in period 1. In contrast, under commitment contract, the supplier pairs a high period-1 wholesale price with a low period-2 price: the high initial price leverages the strategic retailer's incentive to support capital accumulation, while the low subsequent price ensures full-capacity purchasing in period 1, yielding a net profit gain for the retailer despite the higher initial cost.

Third, in the presence of a strategic retailer, upon the occurrence of strategic purchasing, the adjustments in wholesale prices, purchase quantities, and profits for the supplier and the retailer depend heavily on the prevailing contract. In period 1, while the retailer purchases at capacity, the wholesale price set by the supplier can be either higher or lower under commitment contract than dynamic contract. Generally, the enhanced (lowered) capital position in period 2 in the former (latter) case leads the supplier to lower (raise) wholesale price in period 2. However, there are circumstances in which the wholesale prices in both periods are higher under dynamic contract. From the supply chain perspective, the contract that yields higher sales is more effective. Dynamic contract is more effective when the supplier's capital shortage is moderate, but commitment contract is more effective when the supplier's capital shortage is severe. From the perspective of individual firms, dynamic (commitment) contract, which is the more effective contract in generating sales, benefits both the supplier and the retailer when the supplier's production cost is low (high) and ICP is medium (low), leading to a win-win situation. Otherwise, conflicts over contract preference exists between the two firms: the supplier favors dynamic contract while the retailer favors commitment contract.

We extend our analysis to a setting with uncertain market demand. In this case, even with a myopic retailer, the performance outcomes differ in the two contracts, because the supplier and the retailer have the flexibility in tailoring period-2 decisions to realized market conditions under dynamic contract, while they lack such flexibility under commitment contract. As demand uncertainty intensifies, when the retailer is myopic, he is more inclined to enforce commitment contract, while the supply chain is more likely to be better off with dynamic contract. When the retailer is strategic, the pattern for the occurrence of strategic purchasing remains robust, while this behavior is more likely to arise in volatile markets, particularly under dynamic contract. Moreover, strategic purchasing is more likely to be detrimental to the retailer's profit under dynamic contract in volatile markets, particularly when market volatility is moderate. Thus, it is unwise for the retailer to be strategic in purchasing when the dynamic contract is in force, particularly in a market that is moderately volatile. With a strategic retailer, the divergence in firms' contract preferences increases.

Chapter 3

Managing Inventory by Strategic Retailers when Suppliers Face Capital Constraint

3.1 Introduction

This chapter introduces inventory holding to examine the role of strategic inventory when the supplier faces capital constraint. Our research questions are: 1) How does strategic inventory influence the subsequent period's operations (compared to Anand et al. 2008), and considering this influence, when does the retailer hold inventory in equilibrium? 2) How does the option to hold inventory influence the retailer's strategic behavior? Answering this helps explain the operational and profit impacts of permitting versus prohibiting inventory carryover. 3) How does the supplier's capital constraint impact equilibrium outcomes and profits?

We investigate the two-period monopoly model from Chapter 1. In this chapter, our primary focus is on the dynamic contract, which serves as our main model (scenario C). We compare scenarios ND and C to assess the effects of allowing strategic inventory, and numerically contrast scenarios M, ND and C to validate and complement analytical results. We also compare our main model to the benchmark scenario (B) without such constraint to analyze the effects of the capital constraint at the supplier. The main findings and insights are summarized below.

Given the strategic option to hold inventory, the retailer indeed takes the option either when the production cost is not excessively high and the ICP is high, or when the production cost and the ICP are moderate. In the former case, the retailer holds inventory to stimulate supply-side competition and entice the supplier to lower wholesale price in period 2, echoing the result in Anand et al. (2008). In the latter case, holding inventory by the retailer mitigates the supplier's capital constraint, besides lowering wholesale price, in period 2.

Inventory holding provides the retailer with an option for strategic behavior. When the supplier faces capital constraint, this capability results in complex effects that depend on the ICP. When the ICP is sufficiently high, the retailer does not engage in strategic purchasing behavior but holds strategic inventory, which increases the wholesale price in period 1 and boosts total sales to the benefit of both the supplier and the retailer. When the ICP is not high enough, inventory holding generally erodes the benefits that the supplier and the supply chain gain from the retailer being strategic in purchasing. This might cause their profits to be lower than when the retailer is myopic. The retailer's profit, however, can increase significantly when the ICP is moderate, potentially surpassing what a myopic retailer would earn. This occurs because the option to hold inventory weakens the retailer's strategic purchasing incentive, which in turn lowers the period-1 wholesale price. Therefore, inventory holding creates an additional region where the retailer benefits from acting strategically, beyond the conditions previously identified when comparing scenarios ND and M.

Our results indicate that the supplier's capital availability has operational consequences only when the ICP is low. It keeps the supplier from sufficiently satisfying the retailer's demand in period 1, enticing the supplier to raise wholesale price and curbing the retailer's purchase to lower sales. The retailer holds less inventory when the supplier faces a capital constraint than when the supplier faces no such constraint, unless the production cost is significantly high or the production cost is moderately high and the ICP is moderate. In the former case, the retailer forsakes inventory regardless of capital constraint on the supplier. In the latter case, the retailer manages to hold more inventory to complement the supplier's capacity, and a higher inventory level leads to a reduced wholesale price but more sales in period 2. Overall, capital constraint on the supplier results in a reduction in total sales.

Despite the option to hold inventory, the retailer suffers a profit loss from the supplier's capital constraint. By contrast, while the supplier is generally worse off as well, she is better off when the production cost is high and the ICP is moderate. Nevertheless, the supplier's profit gain is short of compensating for the retailer's profit loss, resulting in an overall decrease in supply chain profit. It underscores the negative impact of the supplier's capital constraint on market efficiency.

The cost that the retailer incurs to hold inventory harms the overall efficiency. As a remedy, we explore an alternative mechanism, whereby the retailer preorders from the supplier in period 1 and deploys the preorder for selling in period 2. The supplier rations capital to produce for the preorder quantity in period 2 instead of period 1, when the payment for the preorder is made. Preorder plays the same role as carried inventory in enforcing pricing pressure on the supplier and alleviating her capital constraint in period 2. However, the retailer incurs no cost to hold preorder, the quantity of which is not limited by the supplier's capital. Unlike inventory holding, which offers implicit financial support derived from operations decisions, preordering offers an explicit form of buyer-led financial support, with the retailer making advance payment for future delivery. Our results show that strategic purchasing persists even under preordering, revealing its fundamental role in alleviating the supplier's capital constraint. The preorder quantity generally exceeds the amount of inventory that the retailer would otherwise hold, leading to a higher supply chain profit. However, preordering can yield a lower profit for the retailer or the supplier compared to inventory holding, albeit not simultaneously.

The remainder of this chapter is organized as follows. Section 3.2 presents model settings and analyzes the scenario absent capital constraint on the supplier. Section 3.3 examines performance outcomes in various scenarios when the supplier faces a capital constraint and evaluates the effects of inventory holding by the strategic retailer. Section 3.4 investigates the role of capital constraint on the supplier in influencing system performance when the retailer strategically purchases and holds inventory, and studies preordering as an alternative inventory management mechanism. Section 3.5 concludes the paper, providing a summary of the key findings. All the proofs are presented in the Appendix.

3.2 Model Preliminaries

Following Chapter 2, we consider a setting in which a supplier (s) and a retailer (r) interact in a two-period horizon. For ease of exposition, we refer to the supplier as "she" and the retailer as "he". The supplier produces a product subject to capital availability and sells it at wholesale price w to the retailer, who incurs a cost, normalized to zero, to sell the product to consumers at market-clearing price p. In period $t \in \{1,2\}$, the inverse demand function is as follows:

$$p_t(s_t) = a - bs_t,\ t \in \{1,2\}, \tag{3-1}$$

where a is the maximum price that consumers are willing to pay, s_t is sales quantity, and $b > 0$ is the sensitivity of price with respect to marginal increase in sales. We further normalize a to 1, i.e., $p_t(s_t) = 1 - bs_t,\ t \in \{1,2\}$. This inverse demand function, which is invariant across periods, is in consistent with that of Chapter 2.

3.2.1 Retailer's problem

In contrast to Chapter 2, this chapter assumes that the retailer may hold part of or all purchased products in period 1 as inventory for selling in period 2, incurring a marginal holding cost of $h \geq 0$. The products unsold by the end of period 2 have no salvage value.

We analyze the retailer's decisions by backward induction. In period 2, given inventory I carried from period 1 and wholesale price w_2 set by the supplier, the retailer purchases from the supplier and sells products to maximize profit in the period:

$$\underset{(s_2,q_2)}{Max}\ \pi_{r,2} = p_2(s_2)s_2 - w_2 q_2, \text{ s.t., } 0 \leq s_2 \leq q_2 + I,\ 0 \leq q_2 \leq \frac{K_2}{c}, \tag{3-2}$$

where q_2 is retailer's purchase, s_2 is the sales quantity, $p_2(s_2)$ is the market-clearing price as defined in (3-1), K_2 is the supplier's capital at the beginning of period 2 and c is the production cost. The constraint $0 \leq s_2 \leq q_2 + I$ requires that the sales quantity is capped by the number of products that are available from purchase and carried inventory, and the constraint $0 \leq q_2 \leq \frac{K_2}{c}$ caps the purchase quantity below the supplier's production capacity, which is to be discussed in detail later.

In period 1, given wholesale price w_1 imposed by the supplier and her initial capital K_1, the strategic retailer anticipates the decisions in period 2 and manages the sales and inventory to optimize total profit over periods:

$$\underset{(s_1,I)}{Max}\,\pi_r = \sum_{t=1}^{2}(p_t(s_t)s_t - w_t q_t) - hI, \text{ s.t., } q_1 = s_1 + I \le \frac{K_1}{c},\ s_1, I \ge 0, \tag{3-3}$$

where $q_1 = s_1 + I$ is the retailer's purchase in period 1, which is capped by the supplier's production capacity $\frac{K_1}{c}$, and hI is the cost incurred by the retailer to hold inventory.

3.2.2 Supplier's problem

The supplier produces at marginal production cost c, which remains constant over periods to exclude the impact of cost variation. We assert that our results are reinforced when production cost decreases over periods. In period $t \in \{1,2\}$, the supplier's capital K_t caps production capacity at $\frac{K_t}{c}$. We refer to K_1 as the supplier's *initial capital position* (abbreviated as "ICP"). The supplier's capital status in each period is public information, as commonly assumed in the literature. In practice, it is typically stated in contracts between GM and its suppliers that suppliers should provide official documents, upon request, to verify financial statuses (e.g., Jiang and Hao 2014). Given the supplier's wholesale price w_1 and the retailer's purchase q_1 in period 1, the supplier's capital in period 2 satisfies the following state-transition equation:

$$K_2 = K_1 + (w_1 - c)q_1, \tag{3-4}$$

where $(w_1 - c)q_1$ is the net cash flow generated from the transaction in period 1.

Anticipating the retailer's response with the associated impact on capital status, the supplier dynamically manages wholesale prices to maximize total profit. Specifically, she sets wholesale price w_t in period t to solve the following problem:

$$\underset{w_t}{Max}\,\pi_s = \sum_{i=t}^{2}(w_i - c)q_i,\ t \in \{1,2\}. \tag{3-5}$$

Our work extends the model in Anand et al. (2008) to analyze the effects of holding inventory by the retailer as the supplier faces a capital constraint. Anand et al. (2008) show that absent the capital constraint on the supplier, strategic inventory benefits the retailer in period 2 by intensifying supply competition and weakening the supplier's pricing power, but it reduces the retailer's profit in period 1 by inducing the supplier to raise wholesale price and forcing the retailer to bear a holding cost. The retailer weighs this tradeoff in allocating purchase between sales and inventory. The supplier's capital constraint yields additional effects on the retailer's inventory decision. Specifically, holding inventory in period 1 improves the supplier's capital position in period 2, but also reduces the products available for selling in period 1. These additional effects interplay with existing tradeoffs, making the operations and profit impacts of holding inventory more complex as the supplier faces a capital constraint.

3.2.3 Decision sequence

Figure 3.1 depicts the decision framework.

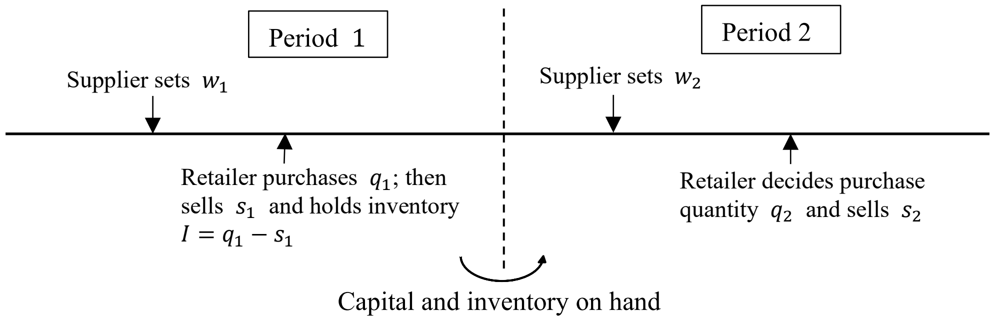


Figure 3.1 Sequence of events

In period 1, the supplier sets wholesale price w_1, and the retailer purchases q_1 and sells $s_1 \leq q_1$. The surplus, $I = q_1 - s_1$, is the inventory carried over into period 2, for which the retailer incurs a holding cost. In period 2, the supplier sets wholesale price w_2, and the retailer purchases q_2 and uses it together with inventory I to sell $s_2 \leq q_2 + I$. Finally, profits are accrued to firms. We analyze the decisions of the supplier and the retailer over the two periods and derive the profits for individual firms and the supply chain.

3.2.4 Benchmark scenario B: No capital constraint at the supplier

In the benchmark scenario B, the supplier has no capital constraint. Proposition 3.1 states the operations decisions. The profits to the supplier, the retailer, and the supply chain are presented in the Appendix.

Proposition 3.1. *Without capital constraint at the supplier, the supplier's wholesale prices and the retailer's sales, inventory, and purchase quantities across periods are as follows:*

Condition	w_1^B	s_1^B	I^B	q_1^B	w_2^B	s_2^B	q_2^B
$c \leq 1 - 4h$	$\frac{9+8c-2h}{17}$	$\frac{4(1-c)+h}{17b}$	$\frac{5(1-c-4h)}{34b}$	$\frac{13(1-c)-18h}{34b}$	$\frac{6+11c+10h}{17}$	$\frac{11(1-c)-10h}{34b}$	$\frac{3(1-c)+5h}{17b}$
$c > 1 - 4h$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$	0	$\frac{1-c}{4b}$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$	$\frac{1-c}{4b}$

The retailer holds inventory when holding and production costs are low (i.e., $h \leq \frac{1}{4}$&$c \leq 1 - 4h$). Conversely, when either cost is high, the supplier sets a high wholesale price to deter the retailer from holding inventory in period 1. These results align with Anand et al. (2008), who assume away production cost. The existence of production cost narrows the range for the holding cost in which the retailer holds inventory, while an increase in production cost further shrinks this range. Additionally, with an increase in the production cost, wholesale prices increase, but inventory, sales, and purchases decrease, harming both firms. We remark that the threshold ICP, below which the supplier's capital limits operation decisions, is $\overline{K}_1^C = max\{cq_1^B, cq_2^B - (w_1^B - c)q_1^B\}$, where cq_1^B is the capital needed to produce the retailer's purchase in period 1, and $cq_2^B - (w_1^B - c)q_1^B$ is the capital needed to produce in period 2, considering capital accumulation in period 1, absent a capital constraint on the supplier.

3.3 Analysis: Capital constraint at supplier

In this section, we derive equilibrium results for the main model, i.e., scenario C, where the strategic retailer holds inventory, and then conduct comparative analysis to generate insights into the role of inventory.

3.3.1 Performance analysis

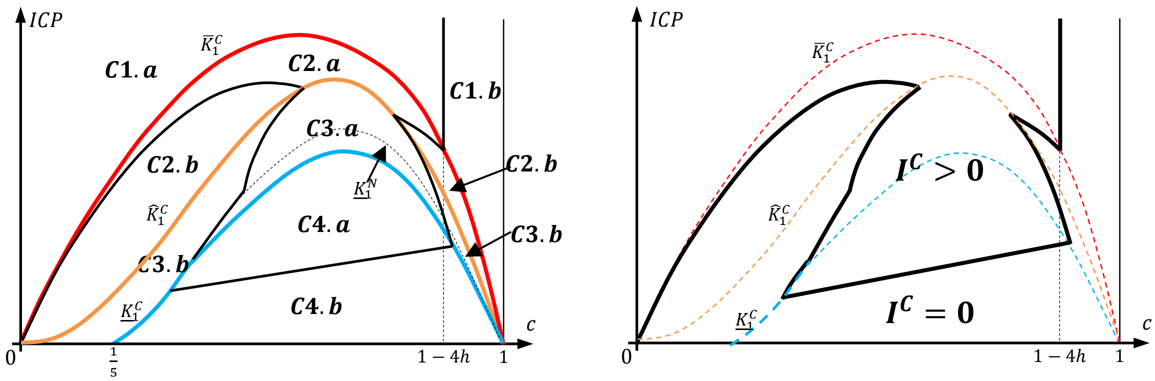
Relative to when forsaking inventory, the strategic retailer now allocates the purchased products in period 1 to yield revenue from selling in period 1 and holding inventory for selling in period 2. The inventory yields long-term benefit at the cost of short-term loss to the retailer. While the retailer may leverage the tradeoff thus arises, it is the supplier who takes advantage of the retailer's strategic thinking to its benefit. We now discuss the tradeoff in the retailer's allocation of purchase between sales and inventory in more detail.

In period 2, given the supplier's capital K_2 and inventory I, the supplier's optimal wholesale price, the retailer's optimal sales and purchase are as follows:

$$\left(w_2^C(K_2, I), s_2^C(K_2, I), q_2^C(K_2, I)\right) = \begin{cases} \left(1 - 2b(I + \frac{K_2}{c}), I + \frac{K_2}{c}, \frac{K_2}{c}\right) & I + \frac{K_2}{c} \leq \frac{1-c+2bI}{4b} \\ \left(\frac{1+c-2bI}{2}, \frac{1-c+2bI}{4b}, \frac{1-c-2bI}{4b}\right) & I + \frac{K_2}{c} > \frac{1-c+2bI}{4b} \end{cases}. \quad (3\text{-}6)$$

Reserving purchased products as strategic inventory in period 1 produces three effects on the retailer. First, it decreases the products available for selling, causing a revenue loss for the retailer in period 1. Second, it leads to a reduction in wholesale price (Anand et al. 2008), lowering the retailer's cost in period 2. Third, inventory can ease the impact of the supplier's capital constraint in period 2, an effect prominent when the supply is low, i.e., a higher inventory increases supply and allows the retailer to sell more in period 2. The retailer benefits more from holding inventory when the total supply is low (i.e., $I + \frac{K_2}{c} \leq \frac{1-c+2bI}{4b}$) than when the total supply is high (i.e., $I + \frac{K_2}{c} > \frac{1-c+2bI}{4b}$). Note that the wholesale price in period 2 decreases with inventory at a higher rate in the former case. An increase in inventory weakens the retailer's incentive to purchase in period 2, prompting the supplier to lower wholesale price more significantly to encourage purchasing at full capacity. In contrast, with a high total supply, the retailer's purchase remains large and the impact on sales would be limited were the retailer to lower purchase. Consequently, the supplier retains a high profit margin by slightly lowering price to avoid a substantial profit loss. Moreover, inventory plays a more important role in alleviating capital constraint when the total supply is low than when it is high.

The retailer's operations response to the supplier's wholesale price w_1 in period 1, the firms' operation decisions and profits over periods are detailed in the Appendix. Proposition 3.2 describes the performance outcomes.



a) Equilibrium partitioning b) Inventory

Notes. Detailed definitions for areas and thresholds are provided in the Appendix. Colored curves are thresholds applicable in scenario C to delimit various equilibrium outcomes. Black curves delimit the areas for the retailer to hold inventory.

Figure 3.2 Performance outcomes when the strategic retailer holds inventory

Proposition 3.2. *With a capital constraint at the supplier, let the decisions in period 1* (w_1^C, s_1^C, I^C) *and capital position in period 2* (K_2^C) *be defined in the Appendix. Referring to Figure 3.2:*

1) *When* $K_1 \geq \overline{K}_1^C$*, the supplier's capital constraint has no impact on firms' decisions.*
2) *When* $K_1 < \overline{K}_1^C$*, the retailer purchases at full capacity in period 1 (i.e.,* $q_1^C = \frac{K_1}{c}$*), and holds inventory (i.e.,* $I^C > 0$*) either when the production cost is not excessively high and* ICP *is high (area* $C2.a$ *in Figure 3.2.a), or when both are moderate (areas* $C3.a$*,* $C4.a$ *in Figure 3.2.a), and,*
   i) K_2^C *exceeds (or precisely matches) the retailer's purchase need in period 2 when* $\widehat{K}_1^C \leq K_1 < \overline{K}_1^C$ *(or* $\underline{K}_1^C \leq K_1 < \widehat{K}_1^C$*), and* $(w_2^C, s_2^C) = (\frac{1+c-2bI^C}{2}, \frac{1-c+2bI^C}{4b})$*;*
   ii) K_2^C *caps the retailer's purchase decision in period 2 when* $K_1 < \underline{K}_1^C$*, and* $(w_2^C, s_2^C) = (1 - 2b(I^C + \frac{K_2^C}{c}), I^C + \frac{K_2^C}{c})$*.*

The supplier's capital availability influences operations only when the ICP is low (i.e., $K_1 < \overline{K}_1^C$). In this case, the supplier manages a wholesale price in period 1 to induce the retailer to purchase at capacity and allocate the purchase to sales and inventory (i.e., $q_1^C = s_1^C + I^C = \frac{K_1}{c}$). When the ICP is moderate (i.e., $\underline{K}_1^C \leq K_1 < \overline{K}_1^C$), the supplier's capital caps the retailer's purchase in period 1 but does not restrict the purchase in period 2. Specifically, the available capital is more than enough to satisfy the retailer's purchase in period 2 when $\widehat{K}_1^C \leq K_1 < \overline{K}_1^C$ (i.e., area $C2$ in Figure 3.2.a). When

$K_1 < \widehat{K}_1^C$, the retailer not only possibly holds inventory but also purchases aggressively for capital accumulation, ordering more for period-1 sales than a myopic retailer would. As a result, the supplier's capital exactly matches the need for period 2 when $\underline{K}_1^C \leq K_1 < \widehat{K}_1^C$ (i.e., area $C3$ in Figure 3.2.a). When the ICP is significantly low (i.e., $K_1 < \underline{K}_1^C$), the supplier's capital caps the retailer's purchase quantities in both periods, despite the retailer's early purchasing efforts.

The thresholds $\overline{K}_1^C$ and $\underline{K}_1^C$ define critical ICPs related to the impact of capital constraint on operations. $\overline{K}_1^C$ is the capital required for the retailer's period-1 purchase to reach the quantity absent the supplier's capital constraint, while $\underline{K}_1^C$ is the minimum ICP at which the supplier's capital does not cap the interactions in period 2. Their concavity shape, as illustrated in Figure 3.2.a, reveals the non-monotone role of production cost in influencing the performance outcomes. Specifically, as production cost increases, it first has no operation consequences, then limits operations in period 1, subsequently caps the retailer's purchases in both periods, then again restricts operations in period 1, and eventually turns negligible. This transition is attributed to the interplay of two effects arising from an increase in production cost. One is that it causes the supplier's capacity to decrease, and the other is that it leads to reductions in the retailer's purchases over periods. These two effects combine to determine whether the supplier's capital can meet the firms' operational needs. The first (second, resp) effect is dominant at a low (high, resp) production cost, making ICP increasingly insufficient (gradually sufficient, resp) to sustain operations.

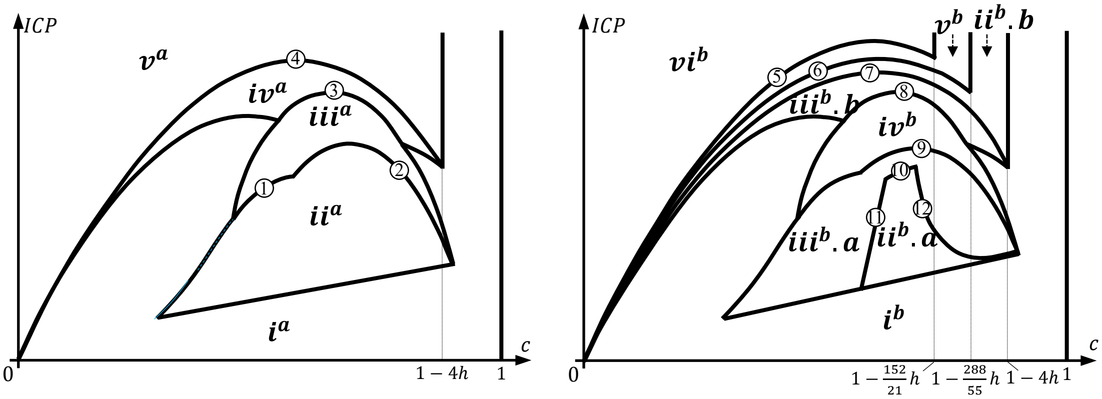
Figure 3.2.b illustrates inventory status. When $K_1 \geq \overline{K}_1^C$, the supplier's capital is ample and the retailer holds inventory if $c \leq 1 - 4h$, which is consistent with the benchmark case. However, when $K_1 < \overline{K}_1^C$, the firms' operations are capital constrained. In period 1, the retailer's purchase is deployed to making sales in period 1 and inventory holding to alleviate the supplier's capital shortage and exert pressure on supplier pricing in period 2. The supplier leverages the retailer's strategic purchase by tailoring wholesale price w_1 to fit the production cost and ICP, ensuring that the retailer consistently purchases at full capacity in period 1. However, the retailer has the liberty to allocate purchase between sales and inventory, and manages inventory to balance the benefit in period 2 from higher sales and weakened supplier pricing power with the loss incurred in period 1 due to sales reduction and holding cost. In areas $C2.a$, $C3.a$, $C4.a$ of Figure 3.2.a, the benefit exceeds the loss, incentivizing the retailer to keep strategic inventory.

3.3.2 Comparative investigation

This section presents a comparative analysis to yield insights, in the presence of capital constraint on the supplier. Recall that in Chapter 2, we compared scenarios ND and M to identify the effects of the retailer's strategic behavior of improving the supplier's future capital availability. Now, we compare scenarios C and ND to evaluate the operations and profit impacts of holding inventory by the strategic retailer. We also engage in a numerical analysis to validate and complement analytical results.

3.3.2.1 Effects of holding inventory by the strategic retailer

We first focus to the role of holding inventory by the retailer in affecting system performances. As shown in Figure 3.2.a, $\underline{K}_1^C \leq \underline{K}_1^{ND}$, which implies that inventory holding more effectively alleviates the supplier's capital constraint. In case the retailer refrains from holding inventory, even when given the option, the performance outcomes are the same as when holding inventory is not an option, i.e., the strategic option of holding inventory is inconsequential.



a) Operations b) Profits

Notes. The labeled curves are defined in the Appendix.

Figure 3.3 Comparisons of performance outcomes with and without inventory carryover

Proposition 3.3. *With a capital constraint at the supplier, compared to when the strategic retailer forsakes inventory, when the retailer indeed holds inventory:*

1) $q_1^C \geq q_1^{ND}$, $s_1^C < s_1^{ND}$, $w_2^C < w_2^{ND}$, $s_2^C > s_2^{ND}$; *referring to Figure 3.3.a, the effects of holding inventory by the retailer on other operations performances are as follows:*

		ii^a	iii^a	iv^a	v^a	
Wholesale price in period 1	w_1^C	>	<	>		w_1^{ND}
Purchase in period 2	q_2^C	>	<			q_2^{ND}
Total sales	$s_1^C + s_2^C$	>	<		>	$s_1^{ND} + s_2^{ND}$

2) *referring to Figure 3.3.b, the effects of holding inventory by the retailer on profits are as follows:*

		ii^b	iii^b	iv^b	v^b	vi^b	
Suppliers' profit	π_s^C	>	<		>		π_s^{ND}
Retailer's profit	π_r^C	<		>	<	>	π_r^{ND}
Supply chain profit	π_{sc}^C	<			>		π_{sc}^{ND}

Operations decisions

Managing strategic inventory by the retailer has various operations implications. It entices the supplier to lower wholesale price in period 1 when the ICP is moderate (area iii^a in Figure 3.3.a) but raise it otherwise. Regardless, it makes the retailer purchase no less in period 1 compared to when inventory holding is not allowed. Specifically, the retailer purchases at capacity, regardless of the existence of inventory carryover as an option, when the ICP is low ($K_1 \leq \overline{K}_1^N$), but purchases more when the ICP is high ($K_1 > \overline{K}_1^N$). Nevertheless, sales in period 1 decrease as the retailer allocates purchased products between sales and inventory.

With a high ICP (i.e., areas iv^a and v^a in Figure 3.3.a), holding inventory strengthens the retailer's purchase incentive in period 1, inducing the supplier to raise wholesale price (Anand et al. 2008). With a low ICP (i.e., areas ii^a and iii^a in Figure 3.3.a), the supplier's capital constraint limits operations in period 2. The retailer strategically purchases to support capital accumulation, weighing a higher imminent cost against gain from enhanced capital in period 2. Holding inventory ushers in intricate effects. On one hand, it grants the retailer flexibility in managing sales over periods to increase profit, termed as the profit-enhancement effect, which incentivizes the retailer to purchase in period 1 and induces the supplier to raise the wholesale price. On the other hand, inventory serves as a substitute for the supplier's capacity in period 2, termed as the substitution effect, which keeps the retailer from purchasing and exerting downward pressure on wholesale price. With a moderately low ICP (i.e., area ii^a in Figure 3.3.a), the profit-enhancing effect dominates, boosting the wholesale price in period 1. In contrast, with a moderate ICP (i.e., area iii^a in Figure 3.3.a), the substitution effect dominates, weakening the retailer's incentive to purchase and enhancing the supplier's capital, leading to a lower wholesale price in period 1.

Inventory exerts pressure on the supplier's wholesale pricing and replenishes sales in period 2, leading to a reduction in wholesale price. Moreover, while the retailer generally purchases less relative to when he forsakes inventory, he consistently sells more. A notable exception occurs when production cost is moderate and the ICP is moderately low (i.e., area ii^a in Figure 3.3.a), in which case a higher period-1 wholesale price improves the supplier's capital position in period 2 and increases the retailer's period-2 purchase. Most literature on strategic inventory states that the increase in period-1 purchase outweighs the decrease in period-2 purchase to boost total sales. Our result indicates that the supplier's capital constraint can restrict the increase in purchase in period 1 and cause an overall reduction in total sales when the ICP is moderately high (i.e., areas iii^a and iv^a in Figure 3.3.a).

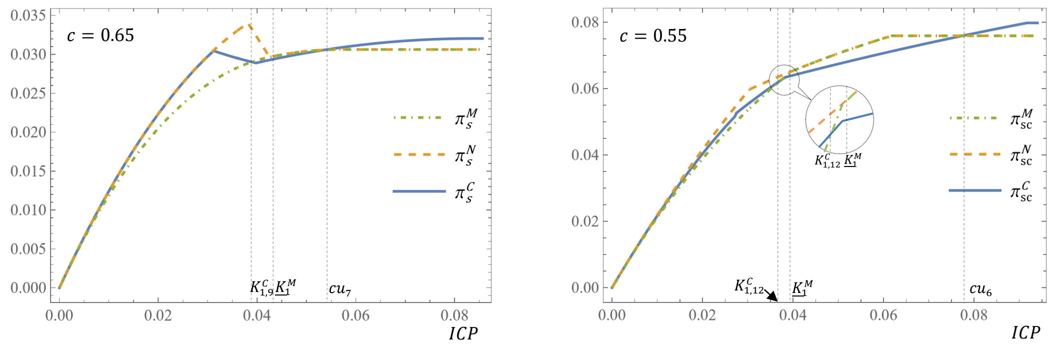
Profits

With a capital constraint on the supplier, the supplier, retailer and supply chain benefit from strategic inventory when the production cost is low (i.e., $c \leq 1 - 4h$, $c \leq 1 - \frac{152}{21}h$, and $c \leq 1 - \frac{288}{55}h$, resp) and ICP is high (i.e., areas $ii^b.b$, v^b, and vi^b for the supplier; area vi^b for the retailer; and areas v^b and vi^b for the supply chain in Figure 3.3.b). The supplier profits from a higher wholesale price in period 1, while the retailer and supply chain benefit from increased total sales. Otherwise, when the production cost is high or the ICP is low (i.e., areas $ii^b - iv^b$ in Figure 3.3.b), strategic inventory harms the supply chain because it fails to generate enough sales to offset holding cost and may even lower total sales. In this case, however, individual firms can still be better off through wholesale-price adjustments. Specifically, when the production cost and the ICP are moderate (area iv^b in Figure 3.3.b), strategic inventory benefits the retailer by clamping down on the wholesale price in period 1 or modestly raising it and enhancing the supplier's period-2 capital to the retailer's advantage. Additionally, when the production cost is moderately high and the ICP is moderately low (i.e., area $ii^b.a$ in Figure 3.3.b), strategic inventory benefits the supplier with a higher wholesale price in period 1 and increased overall sales by capital accumulation, which combine to outweigh the loss from the weakened pricing power in period 2. Keskinocak et al. (2008) assume limited production capacity in period 1 but unlimited capacity in period 2, to find that the firms and supply chain do not benefit from holding inventory when the supplier's capacity is below a threshold. Our results indicate that the opposite can hold.

Anand et al. (2008) identify two key effects of strategic inventory, i.e., double marginalization alleviation and intertemporal price variability. Subsequent literature studies the role of various factors in influencing how these effects weigh on performance. Incorporating the supplier's capital constraint, we find that strategic inventory yields a broader set of conclusions. For instance, with a low ICP (i.e., the area below Curve 3 in Figure 3.3.a), strategic inventory can lead to a higher or a lower wholesale price in period 1, depending on the relative scale of the profit-enhancement and substitution effects that it gives rise to. In case it leads to an increase in the wholesale price in period 1 to facilitate capital accumulation, the purchase in period 2 is boosted. In case the ICP is not sufficiently high (i.e., the area below Curve 7 in Figure 3.3.b), strategic inventory fails to increase total sales, as it does in Anand et al. (2008), and harms supply chain profit, while it can increase the profits of individual firms.

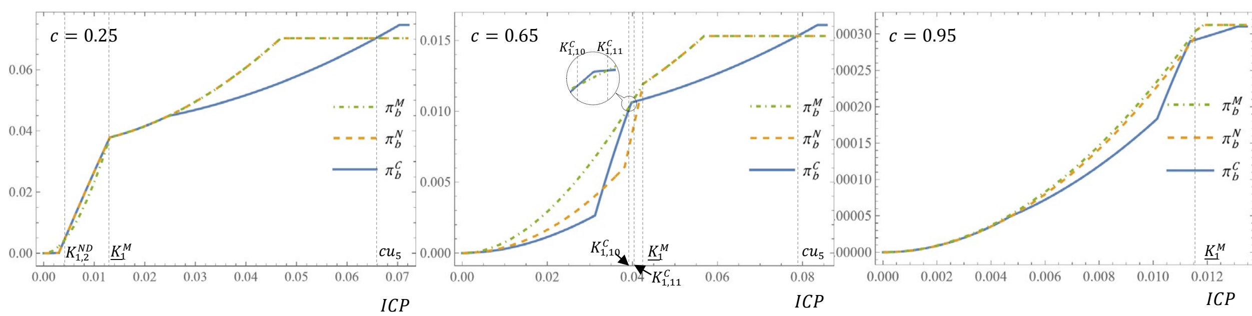
3.3.2.2 Numerical demonstration

We conduct a numerical study to shed more light on the impact of the retailer's strategic purchasing (cf comparing scenarios M and ND) and inventory holding (cf comparing scenarios ND and C), in the presence of a capital constraint on the supplier. Figure 3.4 illustrates the profits of the firms and supply chain. Figure 3.4.a shows that with a low ICP ($K_1 < \underline{K}_1^M$), the retailer acting strategically generally benefits the supplier, but holding inventory does not yield much more value; in fact, it may harm the supplier. Specifically, when the supplier's capital restricts operations, holding inventory by the retailer leads to a lower wholesale price in period 2, while its effects on the wholesale price in period 1 and total sales are mixed (see Proposition 3.3). Even when it leads to a higher wholesale price in period 1 and increased total sales, the gain is insufficient to offset the loss from a lowered wholesale price in period 2, which can be significant (i.e., $K_{1,9} \leq K_1 < \underline{K_1^M}$) and draw the supplier's profit below that when the retailer is myopic. Proposition 3.3 states that the supplier benefits from inventory holding by the retailer when the production cost is moderately high and the ICP is moderately low (i.e., area $ii^b.a$ in Figure 3.3.b), while numerical study reveals that the profit gain is negligible. The performance outcomes when the retailer is strategic in purchasing but forsakes inventory are the same as when the retailer is myopic when the ICP is high (i.e., $K_1 \geq \underline{K_1^M}$). In this case, holding inventory by the retailer slightly reduces the supplier's profit when the ICP is not significantly high (i.e., $\underline{K_1^M} \leq K_1 < cu_7$), but it is ultimately beneficial to the supplier otherwise (i.e., $K_1 \geq cu_7$).



a) Supplier's profit

c) Supply chain's profit



b) Retailer's profit

Notes. The parameters used for numerical analysis are $b = 1$ and $h = 0.01$. The notation cu_i for $i = 5, \ldots, 7$ denotes the threshold corresponding to Curve i in Figure 3.3.

Figure 3.4 Profits with respect to ICP

Figure 3.4.b reveals that when the ICP is low (i.e., $K_1 < \underline{K_1^M}$), the retailer generally suffers a loss from being strategic in purchasing, and the profit loss is further exacerbated by inventory holding, albeit with two exceptions. One occurs when the production cost is low and the ICP is not extremely low, in which case the retailer benefits from strategic purchasing, while inventory holding has no effect since the retailer would forfeit this option. The other occurs when the production cost and the ICP are moderate (i.e., $K_{1,10}^C \leq K_1 < K_{1,11}^C$). In this case, were inventory holding to be allowed, behaving strategically would benefit the retailer, albeit to a small extent. Recall that when $K_{1,9}^C \leq K_1 < \underline{K_1^M}$,

strategic inventory harms the supplier, causing her profit to be lower than when the retailer is myopic. Since $K_{1,9}^{C} < K_{1,10}^{C}$, it follows that whenever holding inventory benefits the retailer, it is detrimental to the supplier's profit. When the ICP is high (i.e., $K_1 > \underline{K}_1^M$), the supplier's capital enforces no limit on operations in period 2, and the retailer no longer purchases aggressively for capital accumulation. In this case, inventory holding benefits the retailer when the production cost is not too high and the ICP is sufficiently high (i.e., $K_1 \geq cu_5$). Otherwise, inventory holding harms the retailer, because the increased wholesale price in period 1 and holding cost outweigh the gains from increased total sales.

Figure 3.4.c reveals that supply chain profit exhibits a trend similar to that of the supplier's profit. With a low ICP (i.e., $K_1 < \underline{K}_1^M$), without inventory holding, strategic purchasing by the retailer yields a profit gain to the supplier more than enough to compensate for the profit loss to the retailer, boosting the supply chain profit. Inventory holding erodes the supplier's profit, resulting in a lower supply chain profit relative to that in the case of myopic retailer when $K_{1,12}^{C} \leq K_1 < \underline{K}_1^M$. With a high ICP (i.e., $K_1 \geq \underline{K}_1^M$), inventory holding by the retailer improves supply chain profit when the ICP is extremely high (i.e., $K_1 \geq cu_6$), but reduces supply chain profit otherwise (i.e., $\underline{K}_1^M \leq K_1 < cu_6$).

Thus, the supplier's production cost and initial capital position (ICP) have intricate effects on the role of inventory holding managed by the retailer in influencing the performance outcomes in the presence of a capital constraint on the supplier. With a low ICP, the retailer's strategic purchasing benefits the supplier, while his inventory holding erodes this gain, potentially resulting in a lower supply chain profit than when the retailer is myopic. The retailer benefits from acting strategically in two conditions. One is when the production cost is low and the ICP is not extremely low, in which case allowing inventory holding has no operational consequences. The other is when the production cost and the ICP are moderate, in which case strategic purchasing and inventory holding yield a profit gain to the retailer, as holding inventory partially substitutes for strategic purchasing and induces a lower wholesale price in period 1. When the ICP is significantly high (i.e., $K_1 \geq cu_5$), strategic inventory managed by the retailer benefits all firms simultaneously (and therefore the supply chain), albeit only when the production cost is not too high.

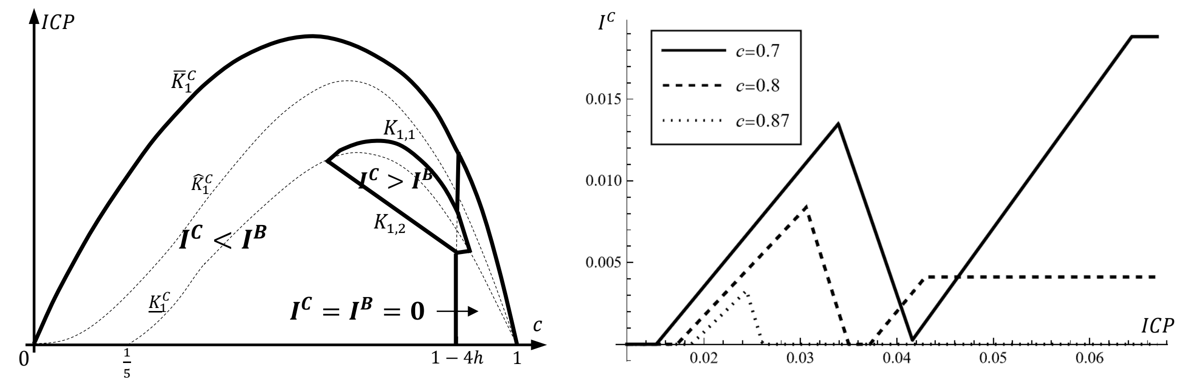
3.4 Discussions

In this section, we examine how the supplier's capital constraint influences operational interactions as the retailer is strategic and holds inventory. Additionally, we investigate the effectiveness of preordering as an alternative mechanism for inventory management.

3.4.1 Effects of supplier's capital constraint

3.4.1.1 Operations decisions

The capital constraint at the supplier weighs on firms' operational decisions when the supplier's ICP is below $\overline{K}_1^C$. Corollary 3.1 details the effects.



a) Change of inventory level due to capital constraint

b) Effects of ICP on inventory level

Notes. $K_{1,1}$ and $K_{1,2}$ are defined in the Appendix. The parameters used to draw Figure 3.5.b are $b = 1$ and $h = 0.043$.

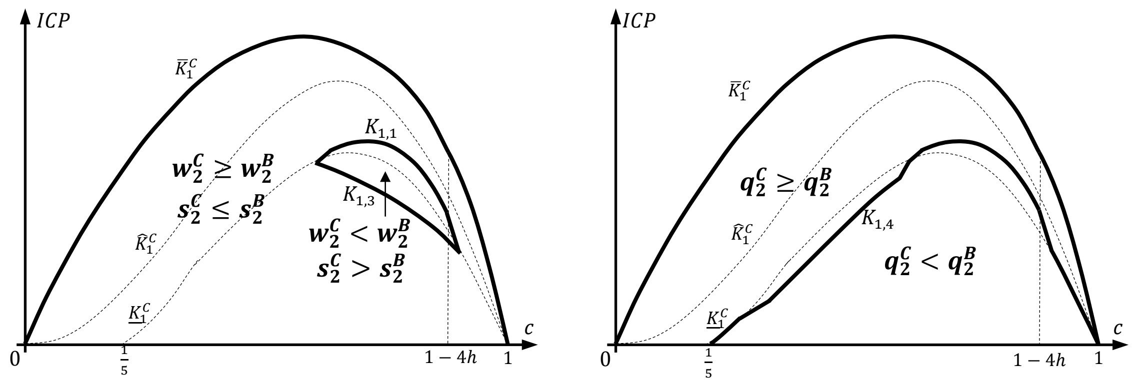
Figure 3.5 Effect of capital constraint on inventory

Corollary 3.1. *When the ICP is low, i.e., $K_1 < \overline{K}_1^C$, the effects of capital constraint on the supplier on the operations decisions are stated as follows:*

w_1^C	s_1^C&q_1^C	I^C	w_2^C	s_2^C	q_2^C	$s_1^C + s_2^C$
↑	↓	*no change if c is high;* *↑ if c is moderately high and K_1 is moderate;* *↓ otherwise*	*↓ if c is moderately high and K_1 is moderate;* *↑ otherwise*	*↑ if c is moderately high and K_1 is moderate;* *↓ otherwise*	*↓ when K_1 is low;* *↑ otherwise*	↓

In period 1, since capital constraint deters the supplier from sufficiently satisfying the retailer's purchase need, the supplier raises wholesale price to curb retailer purchase and lower sales as well. The retailer generally holds less inventory (cf Figure 3.5.a) except when the production cost is high, or when the production cost is moderately high and the ICP is moderate. In the former case, the retailer refrains from holding inventory no matter whether the supplier faces a capital constraint. In the latter case, insufficient capital gives the retailer a strong incentive to maintain inventory, and the associated profit loss in period 1 is limited because the capital shortage is mild. By contrast, in the absence of a capital constraint on the supplier, the retailer holds less inventory since the moderately high production cost discourages inventory holding (i.e., inventory decreases with the production cost). Consequently, inventory level is higher when the supplier has a capital constraint than otherwise.

The supplier's capital constraint generally causes an increase in wholesale price but a decrease in sales in period 2 (see Figure 3.6.a). However, when the production cost is moderately high and the ICP is moderate, the retailer holds more inventory as the supplier faces a capital constraint, lowering the wholesale price but enhancing the sales in period 2. The supplier's capital constraint leads to an increase in the retailer's purchase in period 2 when the ICP is sufficiently high (i.e., $K_1 > K_{1,4}$), but a decrease otherwise (see Figure 3.6.b). This is because with a high ICP, the retailer holds less inventory in period 1 due to the supplier's constraint and must purchase more in period 2 to satisfy demand. In contrast, when the ICP is not sufficiently high, limited capital availability rises as the dominant factor, restricting the retailer's purchase in period 2. Overall, total sales across periods decline as the supplier faces a capital constraint.



a) Wholesale price and sales quantity in period 2 b) Purchase quantity in period 2

Notes. $K_{1,3}$ and $K_{1,4}$ are defined in the Appendix.

Figure 3.6 Effects of supplier's capital constraint ($K_1 < \overline{K}_1^C$) on operations in period 2

Corollary 3.2 states the effect of the supplier's ICP on the retailer's inventory level.

Corollary 3.2. *With a capital constraint at the supplier, as the strategic retailer can hold inventory, when $K_1 < \overline{K}_1^C$, the inventory increases with the supplier's ICP when $K_1 < \underline{K}_1^C$ or $\widehat{K}_1^C \leq K_1 < \overline{K}_1^C$, but decreases when $\underline{K}_1^C \leq K_1 < \widehat{K}_1^C$.*

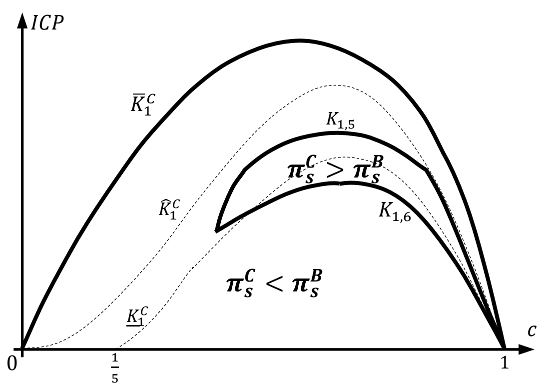
Recall that the strategic retailer holds inventory when production cost and ICP are moderate (i.e., $C3.a$ and $C4.a$ in Figure 3.2.a) or when production cost is not excessively high and ICP is high (i.e., $C2.a$ in Figure 3.2.a). As the ICP increases, carried inventory first increases for $K_1 < \underline{K}_1^C$ (i.e., $C4.a$ in Figure 3.2.a) and then decreases for $\underline{K}_1^C \leq K_1 < \widehat{K}_1^C$ (i.e., $C3.a$ in Figure 3.2.a) in the former case, but continuously increases in the latter case. Figure 3.5.b shows these patterns based on a numerical study. The rationales driving these results are as follows.

As previously discussed, inventory holding has three interrelated effects on the retailer, i.e., the sales in period 1 are reduced (resulting in immediate revenue loss), the wholesale price in period 2 decreases (reducing purchasing cost), and the capital constraint in period 2 is alleviated. An increase in the ICP adjusts the relative magnitude of these effects to influence the retailer's incentive to hold inventory. On one hand, it weakens the role of holding inventory in reducing sales in period 1. On the other hand, it enhances the supplier's capital in period 2, enabling the retailer to purchase more and increasing the total cost saving from inventory-induced wholesale-price reduction. These two effects strengthen the retailer's incentive to hold inventory. Moreover, a higher ICP reduces the necessity to hold inventory. The latter two effects are relevant only when the supplier's capital availability limits the operational interactions between firms.

With a moderately low ICP (i.e., $C4.a$ in Figure 3.2.a), the first two effects dominate the third effect, leading to an increase in inventory as ICP rises. With a high ICP (i.e., $C2.a$ in Figure 3.2.a), inventory growth is driven by the first effect. Notably, inventory increases more slowly when the ICP is moderately low, since the upward incentive is partially offset by the downward pressure arising from the third effect, while no such offset exists when the ICP is high. The third effect dominates when the ICP is moderate (i.e., $C3.a$ in Figure 3.2.a), causing inventory to decrease as the ICP increases, since less inventory is needed to secure adequate supply in period 2.

3.4.1.2 Profits

Corollary 3.3 states the comparison results of the profits of firms and the supply chain when the supplier faces a capital constraint relative to those when the supplier has no capital constraint.



Notes. $K_{1,5}$, and $K_{1,6}$ are defined in the Appendix.

Figure 3.7 Effect of capital constraint ($K_1 < \overline{K}_1^C$) on the supplier's profit

Corollary 3.3. *When the supplier's ICP is low, i.e., $K_1 < \overline{K}_1^C$, capital constraint on the supplier causes the profits for the retailer and supply chain to decrease, while it causes the profit for the supplier to increase when the production cost is high and the ICP is moderate, but decrease otherwise.*

Corollary 3.3 states that the retailer always suffers a loss from the supplier's capital constraint. While the supplier generally experiences a profit loss as well, there are circumstances wherein she is better off. Specifically, the supplier makes a profit gain when the production cost is high and the ICP is moderate (see Figure 3.7). However, the supplier's profit gain is always less than the retailer's profit loss, leading to an overall drop in supply chain profit. It highlights the negative impact of the supplier's capital constraint on overall efficiency.

Note that with capital constraint, the supplier's potential profit gain is driven by the retailer's foresightedness. The retailer engages in strategic purchasing to balance short-term cost with long-term benefit. This behavior leads to a higher wholesale price in period 1, facilitating the supplier's capital accumulation for period 2. However, the capital constraint limits the supplier's capacity and reduces her sales compared to the case without such a constraint, partially offsetting the benefit of a higher wholesale price. When the production cost is high and the ICP is moderate, the supplier's capacity is low and entices the retailer to engage in strategic purchasing, incentivizing the supplier to set a high wholesale price. However, the capital shortage is not too severe to substantially cap sales. As a result, the profit gain from a higher wholesale price in period 1 outweighs the loss from reduced sales, causing the supplier to benefit from her capital constraint.

3.4.2 Preorder (scenario P)

Inventory holding benefits the retailer by weakening the supplier's pricing power and alleviating the supplier's capital constraint on operational interactions, while it has the retailer incur a holding cost, which harms overall supply chain efficiency. Next, we examine preordering, whereby the supplier sets wholesale price w_1, and the retailer sells s_1 and preorders q_p in period 1, with the preorder, like the inventory carried across periods, deployed for selling in period 2, when the supplier decides wholesale price w_2, and the retailer places an at-once order q_2 and sells s_2. The difference between preordering and inventory holding is that the supplier produces to satisfy the preorder in period 2 but satisfies the purchase for inventory by the retailer in period 1, though the retailer pays for preorder or inventory in period 1. Additionally, in contrast to inventory holding, the retailer no longer incurs a holding cost for preorder, and the preorder quantity is not limited by the supplier's production capacity. In contrast to inventory holding that may provide implicit financial support as a byproduct of operations decisions, preordering offers an explicit form of buyer-led financial support, with the retailer making advance payments for future delivery.

3.4.2.1 Performance analysis

When the retailer preorders, the supplier's capital in period 2 is given by: $K_2 = K_1 - cs_1 + w_1(s_1 + q_p)$, where q_p is the retailer's preorder in period 1. Given wholesale price w_2 and preorder q_p, which is to be produced and delivered in period 2, the retailer's problem can be expressed as follows:

$$\underset{(s_2,q_2)}{Max}\ p_2(s_2)s_2 - w_2q_2,\ \text{s.t.},\ 0 \le s_2 \le q_2 + q_p,\ 0 \le q_2 \le \frac{K_2}{c} - q_p, \tag{3-7}$$

where the first constraint caps the sales in period 2 below available products, and the second constraint requires the supplier's production to meet the retailer's preorder in period 1 and purchase in period 2.

Anticipating the retailer's purchase $q_2(w_2|K_2, q_p)$, the supplier decides wholesale price w_2 to maximize period-2 profit given capital K_2 and preorder quantity q_p.

$$\underset{w_2}{Max}(w_2 - c)q_2(w_2|K_2, q_p) - cq_p, \tag{3-8}$$

where note that the supplier incurs the cost to produce and satisfy the preorder.

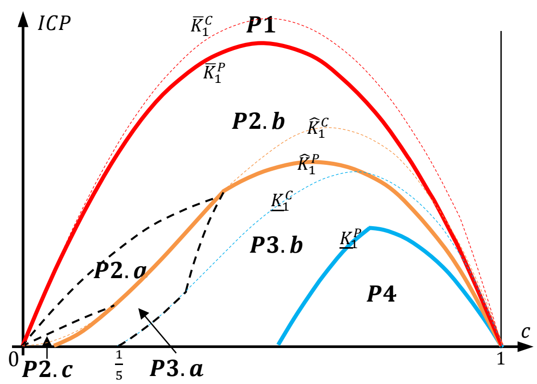
In period 1, given the supplier's wholesale price w_1, anticipating the interactions in period 2, the retailer sells s_1 to market and preorders q_p to maximize total profit over the two periods:

$$\underset{(s_1, q_p)}{Max} \sum_{t=1}^{2} p_t(s_t)s_t - w_1(s_1 + q_p) - w_2q_2, \text{ s.t., } 0 \le s_1 \le \frac{K_1}{c}, \ q_p \ge 0, \tag{3-9}$$

where the supplier's production capacity in period 1 caps the retailer's sales quantity.

Anticipating the retailer's response in period 1 and the subsequent interactions in period 2, the supplier sets wholesale price w_1 to maximize total profit over the two periods:

$$\underset{w_1}{Max}(w_1 - c)(s_1 + q_p) + (w_2 - c)q_2. \tag{3-10}$$



Notes. Colored solid curves are thresholds applicable in scenario P to delimit the areas for various outcomes. Black dashed curves delimit the areas for the retailer to preorder. Colored dotted curves are the thresholds in scenario C.

Figure 3.8 Equilibrium partitioning under preordering

Proposition 3.4. *With a capital constraint at the supplier, as the retailer manages preorders, let equilibrium decisions* (w_1^P, s_1^P, q_p^P) *and the supplier's capital position in period 2* (K_2^P) *be defined in the Appendix, then, referring to Figure 3.8, we have the following results:*

1) *When* $K_1 \ge \bar{K}_1^P$*, the supplier's capital constraint is inconsequential.*

2) *When* $K_1 < \bar{K}_1^P$*, the retailer always purchases at full capacity in period 1 (i.e.,* $s_1^P = \frac{K_1}{c}$*), and preorders in most cases except when the production cost and the* ICP *are low (areas* $P2.a$ *and* $P3.a$ *in Figure 3.8), and,*

   i) K_2^P *exceeds (matches, resp) the required production need in period 2 when* $\hat{K}_1^P \le K_1 < \bar{K}_1^P$ *(*$\underline{K}_1^P \le K_1 < \hat{K}_1^P$*, resp), and* $(w_2^P, s_2^P) = (\frac{1+c-2bq_p^P}{2}, \frac{1-c+2bq_p^P}{4b})$*;*

ii) K_2^P *constrains the decisions in period 2 when* $K_1 < \underline{K}_1^P$, *and* $(w_2^P, s_2^P) = (1 - \frac{2bK_2^P}{c}, \frac{K_2^P}{c})$.

Proposition 4 states the firms' optimal decisions, where superscript P indicates "preorder". The pattern for operations decisions with preorders is similar to that in the case of inventory holding. Specifically, the firms would operate as if without capital constraint on the supplier when the supplier's ICP is significantly high ($K_1 \geq \overline{K}_1^P$). The supplier's capital caps operations in period 1 when the ICP is moderate ($\underline{K}_1^P \leq K_1 < \overline{K}_1^P$), but caps operations in both periods when the ICP is low ($K_1 < \underline{K}_1^P$).

In particular, when $K_1 \leq \widehat{K}_1^P$, the retailer purchases more for sales in period 1 than a myopic retailer would, demonstrating a strategic incentive to enhance the supplier's capital position in period 2 through aggressive early purchasing. As a result of this strategic behavior, the supplier's capital position exactly matches the production requirement for period 2 when $\underline{K}_1^P \leq K_1 < \widehat{K}_1^P$. This result reveals that strategic purchasing remains effective even under an explicit form of financial support.

When $K_1 < \overline{K}_1^P$, the supplier's capital constraint restricts sales s_1, but not preorder q_p. The supplier's wholesale price in period 1 ensures that the retailer always purchases at full capacity. Within a feasible range for wholesale price, the supplier balances between a high-price strategy, aiming for a high profit margin, and a low-price strategy, aiming for a large preorder. Typically, this trade-off leads to a wholesale price that incentivizes the retailer to preorder, especially when either the production cost or the ICP is high. Specifically, when the production cost is high, the supplier's capacity tends to be binding, and the retailer is inclined to preorder to relieve the supplier's capital constraint in period 2. When the ICP is high, capital constraint is not severe. Were the supplier to impose a high wholesale price, the price would still be acceptable for the retailer to preorder. However, the dynamics are more involved when the production cost and ICP are low (i.e., areas $P2.a$, $P2.c$, $P3.a$ in Figure 3.8). In area $P2.c$, the supplier prefers a low-price strategy, and the retailer's sales in period 1 are the same as in the case of a myopic retailer, making preorder the only means to alleviate the supplier's capital constraint in period 2. Besides stimulating preorders in period 1, the supplier adopts a low-price strategy to increase purchase in period 2. In contrast, in areas $P2.a$ and $P3.a$, especially in $P3.a$ where the retailer achieves higher sales in period 1 than a myopic retailer to support capital accumulation, the supplier prefers a high-price strategy, which discourages preorders.

3.4.2.2 Strategic value of preordering versus holding inventory

Premised on the equilibrium outcomes when the strategic retailer holds inventory and preorders, we conduct a comparative investigation into these two mechanisms from three perspectives: equilibrium pattern, operations decisions, and profit performance.

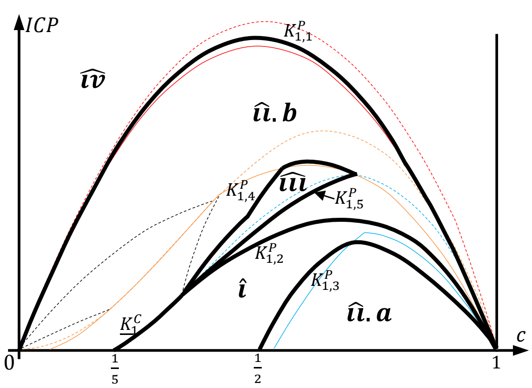
Equilibrium pattern

We can verify that $\overline{K}_1^P < \overline{K}_1^C$, $\widehat{K}_1^P < \widehat{K}_1^C$, and $\underline{K}_1^P < \underline{K}_1^C$ (as shown in Figure 9). Additionally, $\underline{K}_1^C > \widehat{K}_1^P$ when the production cost is high. These results yield notable insights. One is that the supplier's capital constraint has a weaker impact on firms' interactions as the retailer preorders than as the retailer holds inventory ($\overline{K}_1^P < \overline{K}_1^C$). Second, preordering more effectively alleviates the supplier's capital constraint in period 2 ($\underline{K}_1^P < \underline{K}_1^C$), thereby reducing the retailer's need to engage in aggressive purchasing for period-1 sales ($\widehat{K}_1^P < \widehat{K}_1^C$). Notably, when the production cost is high and the ICP is moderate (i.e., $\widehat{K}_1^P < K_1 < \underline{K}_1^C$), the operations in period 2 would be restricted were the retailer to hold inventory but would be exempted from capital constraint were the retailer to preorder. The advantage of preordering over inventory holding in mitigating the impact of capital constraint on the supplier is attributed to the fact that the supplier refrains from rationing capital for production to satisfy preorders but does so for inventory, although the retailer makes payment for inventory or preorder in period 1.

Operations decisions

Corollary 3.4 compares the wholesale prices and sales quantities under preorder and inventory carryover.

Corollary 3.4. *With a capital constraint at the supplier, referring to Figure 3.9, the comparative results of wholesale prices* (w_1, w_2) *and sales* (s_1, s_2) *in the two periods when the strategic retailer holds inventory and preorders are summarized as follows:*



Area	$\hat{i}$	$\widehat{ii}$	$\widehat{iii}$	$\widehat{iv}$	
w_1^C	$<$	$\geq$		$<$	w_1^P
w_2^C	$\geq$		$<$	$\geq$	w_2^P
s_1^C	$\leq$			$>$	s_1^P
s_2^C	$\leq$		$>$	$\leq$	s_2^P
$s_1^C + s_2^C$	$\leq$				$s_1^P + s_2^P$

Notes. $K_{1,i}^P$, $i = 1, \dots, 5$, are defined in the Appendix. The other curves are as defined in Figure 3.8.

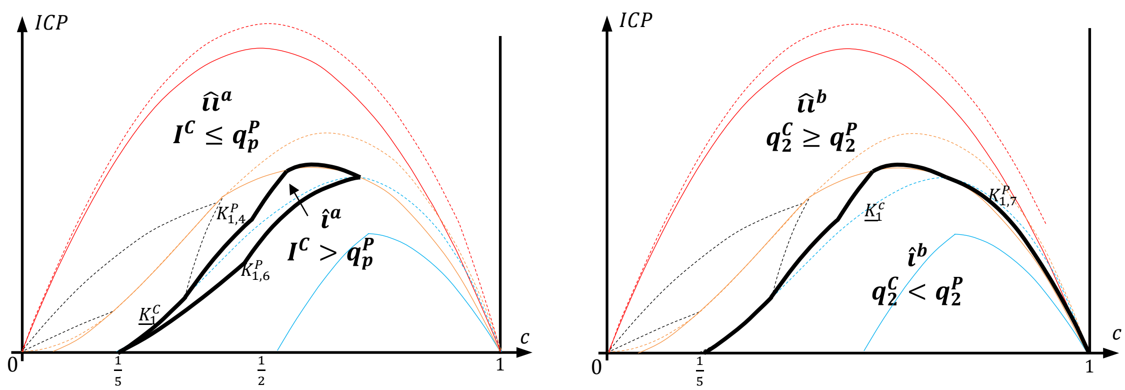
Figure 3.9 Comparisons of wholesale prices and sales between preorder and inventory carryover

As the retailer preorders, the firms operate as if without capital constraint on the supplier when the supplier's ICP is high (i.e., area $\widehat{iv}$ in Figure 3.9). Like carried inventory, preorder stimulates the supply competition in period 2. However, compared to those under inventory carryover, the wholesale price in period 1 is higher but that in period 2 is lower under preorder. The adjustments cause the sales in period 1 to decrease, but those in period 2 to increase. With a low ICP, the supplier's capital burden in period 1 is weak under preorder, which effectively boosts the supplier's capital position in period 2 and leads to a lower wholesale price to increase sales in both periods (i.e., area $\widehat{\iota\iota}$ in Figure 3.9), albeit with two exceptions. When the ICP is moderately low (i.e., area $\hat{\iota}$ in Figure 3.9), the retailer's strategic purchasing has a limited effect on the supplier's capital accumulation, and holding inventory would curb sales in period 1, deterring the retailer from purchasing, while preordering boosts capital in period 2, encouraging purchase and leading to a higher wholesale price in period 1. When the ICP is moderately high (i.e., area $\widehat{\iota\iota\iota}$ in Figure 3.9), the retailer preorders less than the inventory he would hold, resulting in a higher wholesale price and lower sales in period 2.

Corollary 3.5 further examines the sources of sales in period 2 under the two mechanisms.

Corollary 3.5. *The comparison of the sources for sales in period 2 (purchase quantity* q_2 *and inventory* I*, or preorder quantity* q_p*) between inventory carryover and preorder is as follows:*

1) *As shown in Figure 3.10.a,* $I^C \leq q_p^P$*, except when the production cost and* ICP *are medium, where* $I^C > q_p^P$*.*
2) *As shown in Figure 3.10.b,* $q_2^C < q_2^P$ *when the* ICP *is low but* $q_2^C \geq q_2^P$ *otherwise.*



a) Inventory level versus preorder quantity b) Purchase quantity in period 2

Notes. $K_{1,6}^P$ and $K_{1,7}^P$ are defined in the Appendix. The other curves are as defined in Figure 3.8.

Figure 3.10 Comparison of sources of sales in period 2 between preorder and inventory carryover

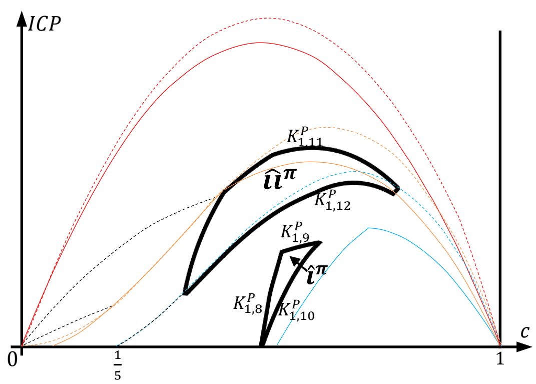
One may intuit that the retailer should preorder more than the inventory he would hold because the preorder quantity is not constrained by ICP, while inventory is. However, an exception occurs when the production cost and the ICP are moderate (i.e., area $\hat{\iota}^a$ in Figure 3.10.a), where the retailer is likely to hold more inventory into period 2 than he would preorder. This occurs because, in this area, were the retailer to hold inventory (see Figure 3.5.b), the retailer would have a strong incentive to hold inventory for sales in period 2. In contrast, were the retailer to preorder, the supplier would set a high wholesale price, lowering the preorder quantity.

The discrepancy between inventory level and preorder quantity also leads to differences in the purchase quantities in period 2. When $K_1 \geq \underline{K}_1^C$, the purchase quantities in period 2 are not constrained by capital constraints in both cases, and the comparison of inventory and preorder quantity exhibits an inverse pattern, i.e., $q_2^C < q_2^P$ when $I^C > q_p^P$, but $q_2^C > q_2^P$ when $I^C < q_p^P$. When $K_1 < \underline{K}_1^C$, preordering has a more pronounced effect in mitigating capital shortage in period 2, leading to $q_2^C < q_2^P$. As such, the purchase in period 2 is generally higher in preordering when the production cost is not too low and the ICP is low (i.e., area $\hat{\iota}^b$ in Figure 3.10.b), but is higher in inventory carryover otherwise (i.e., area $\hat{\iota\iota}^b$ in Figure 3.10.b).

Profit performance

Corollary 3.6 compares supply chain performances between the two mechanisms.

Corollary 3.6. *With a capital constraint at the supplier, compared to when the retailer holds inventory, referring to Figure 3.11, the firms generally perform better and the supply chain is better off when the retailer preorders, except when* (c, K_1) *fall in areas* $\hat{\iota}^\pi$ *and* $\hat{\iota\iota}^\pi$ *of Figure 3.11. Specifically, the retailer performs worse by preordering when* (c, K_1) *fall in area* $\hat{\iota}^\pi$ *of Figure 3.11, while the supplier performs worse when the retailer preorders when* (c, K_1) *fall in area* $\hat{\iota\iota}^\pi$ *of Figure 3.11.*

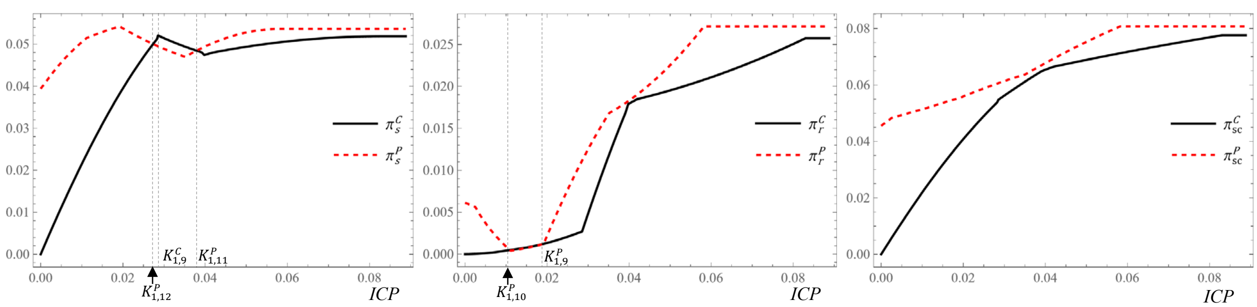


Notes. $K_{1,i}^P$, $i = 8, \ldots, 12$, are defined in the Appendix. The other curves are as defined in Figure 3.8.

Figure 3.11 Comparisons of firms' profits between preorder and inventory carryover

As discussed previously, relative to holding inventory, preordering more effectively alleviates the capital constraint on the supplier, yielding more total sales. Consequently, it benefits both firms in most cases and consistently increases supply chain profit, albeit with two exceptions. One occurs when the production cost is moderate and the ICP is moderately low (i.e., area $\hat{\iota}^\pi$ in Figure 3.11), in which case the retailer profits more by holding inventory, which leads to a lower wholesale price in period 1. The other occurs when the production cost and the ICP are moderate (i.e., area $\hat{\iota\iota}^\pi$ in Figure 3.11), in which case the supplier profits more as the retailer holds inventory than as the retailer preorders. From the supplier's perspective, inventory holding by the retailer offers a price advantage. Compared to that in the case of preordering, where the supplier's ICP only funds sales, the supplier's ICP funds sales and inventory under inventory holding, the capital constraint is more binding, enhancing the supplier's pricing power. In contrast, preordering offers a quantity advantage, enticing the supplier to sell more. In area $\hat{u}^{\pi}$ of Figure 3.11, the preorder quantity would be small, but the price advantage of inventory holding outweighs the quantity advantage of preordering, leading the supplier to profit more when the retailer holds inventory.

Whereas inventory holding worsens supply chain profit relative to preordering, it may improve individual firm profits, although the supplier and the retailer are not simultaneously better off. Area $\hat{\imath}^{\pi}$ in Figure 12 exists when the holding cost h is not over low, and it expands with h, reaching the maximum scale at $h = 0.25$, above which the retailer holds no inventory. Additionally, when holding inventory yields a higher profit for the retailer, the profit gap between the two mechanisms widens with h, i.e., the retailer's profit advantage from holding inventory over preordering increases as the holding cost rises. It is because the retailer relies more on more aggressive purchasing than a myopic retailer would to alleviate the supplier's capital constraint. This shift in purchasing pattern makes inventory holding more attractive to the retailer, reducing the appeal of preordering.



a) Supplier's profit b) Retailer's profit c) Supply chain's profit

Notes. The parameters used to draw this figure are $b = 1$, $h = 0.04$ and $c = 0.55$.

Figure 3.12 Profits when the retailer preorders and holds inventory

Figure 3.12 depicts the firms' profits with respect to the supplier's ICP. The supplier's profit first increases, then decreases, and eventually increases as the ICP increases in either mechanism. Generally, the supplier's profits improve as her capital constraint eases. However, an exception occurs when the ICP is moderate. In this case, as the ICP increases, the retailer's incentive to boost supply in period 2 through strategic purchasing diminishes, weakening the supplier's benefits from exploiting this behavior. It produces a local maximum with respect to ICP. Around this peak in the case of inventory holding ($K_1 = K_{1,9}^C$), the supplier profits more when the retailer holds inventory than when the retailer preorders (cf Figure 3.12.a), which is consistent with the results stated in Corollary 3.6. Moreover, the supplier can profit from operating under a capital constraint under certain conditions in either mechanism.

With an increase in ICP, the retailer's profit increases in the case of inventory holding, while it first decreases and then increases in the case of preordering. Corollary 3.6 states that preordering may harm the retailer relative to inventory holding, while our numerical results (cf Figure 3.12.b) reveal that this negative effect is negligible, i.e., the retailer literally profits more from preordering than holding inventory. Figure 3.12.c further reveals that supply chain profit increases with ICP in either mechanism, but preordering outperforms inventory holding in profit generation.

3.5 Concluding Remarks

We examine a bilateral monopoly with a supplier, who starts with limited capital, and a strategic retailer over a two-period horizon. The supplier sells a product to the retailer, who in turn sells it to consumers. In each period, the supplier sets the wholesale price, and the retailer responds by determining purchase quantity, subject to the supplier's capital availability. The retailer may hold inventory in period 1 for selling in period 2 for two main purposes. One is to intensify supply competition and weaken the supplier's pricing power in period 2, and the other is to eases the supplier's capital shortage in period 2. Anticipating the retailer's strategic responses, the supplier manages wholesale prices to maximize total profit over the two periods. The supplier's period-2 capital is endogenously shaped by early wholesale pricing and retailer purchasing.

Holding strategic inventory yields a benefit for the retailer in period 2, as noted by Anand et al. (2008) and related studies. Nevertheless, under capital constraint, the supplier's capital is insufficient for selling in period 1. Holding inventory would further restrict the retailer's already limited period-1 supply, harming the retailer. The main tradeoff embedded in the retailer's inventory strategy is to balance the benefit in period 2 against the loss in period 1 from reduced sales and holding cost. Our complete analysis unravels that the supplier's capital constraint has significant implications, and the main results and insights are summarized as follows.

In case the supplier's initial capital position (ICP) constrains firms' operations, the supplier strategically sets the wholesale price to induce full-capacity purchases in period 1. Given the option to hold inventory, the retailer indeed allocates the purchased products as inventory when the production cost is not excessively high and the ICP is high or when they are moderate. This decision not only affects supply-side competition to weigh on supplier pricing power but relax the constraint on operations in period 2. Moreover, a decrease in the ICP can raise the inventory level.

Our analysis reveals that inventory holding creates nuanced strategic dynamics when the supplier faces capital constraint. Specifically, at a sufficiently high ICP, strategic purchasing does not emerge but inventory holding does. In this case, the retailer's inventory holding yields a mutually beneficial outcome: the supplier gains from a higher period-1 wholesale price, while the retailer profits from increased sales. Conversely, when the supplier's ICP is insufficient, inventory holding weakens the profitability of the supplier and the supply chain. Therefore, their profits may fall below the levels achieved with a myopic retailer. Most notably, we find that a moderate ICP gives the retailer a unique advantage, weakening his own strategic purchasing incentive to secure a lower wholesale price in period 1. As a result, we identify and characterize a previously unexplored set of conditions where the retailer profits from acting strategically.

The retailer holds less inventory when the supplier faces a capital constraint than when she does not face a constraint, except when production cost is moderately high and the ICP is moderate, in which case the retailer increases inventory holding to adapt to the supplier's capital constraint. It is noteworthy that the supplier can benefit from capital constraint, when the production cost is high and the ICP is moderate in particular. In this case, the retailer engages strategic purchasing to mitigate the adverse effect of capital constraint on the supplier in period 2. The supplier leverages such foresightedness for profit by managing wholesale prices. Nevertheless, the capital constraint on the supplier always makes the retailer worse off. And the supplier's gain never offsets the retailer's loss. Consequently, capital constraint reduce the supply chain's profit, indicating its negative impact on overall system efficiency.

To alleviate the inefficiency arising from holding inventory at a cost, we explore an alternative mechanism, whereby the retailer preorders from the supplier in period 1 and deploys the preorder for selling in period 2. Our results indicate that preorder quantity is generally more than the inventory that the retailer would otherwise hold, because it is not restricted by the supplier's ICP. However, the retailer is more inclined to hold inventory than preorder when production cost and ICP are moderate. Compared to holding inventory, preordering enhances supply chain profit, while it may lead to a profit loss to either the retailer or the supplier, albeit not simultaneously.

Chapter 4

Summary and Future Research

This thesis examines long-term interactions between capital-constrained suppliers and strategic retailers. We develop a two-period bilateral-monopoly model in which, each period, the supplier sells a product at a wholesale price to the retailer, who then sells it to consumers. The retailer's purchases are limited by the supplier's available capital. Chapters 2 and 3 explore settings without and with inventory carryover, respectively. The two firms' relationship can be governed by either a dynamic contract or a commitment contract. This research aims to clarify how strategic purchasing—used to foster supplier capital accumulation—and strategic inventory operate in this setting, as well as how these two motives interact.

Under dynamic contract, we examine a series of scenarios. In the benchmark scenario (B), the supplier has no capital constraint, while the retailer strategically purchases and holds inventory. In the presence of capital constraint at the supplier, we examine scenario M, in which the retailer is myopic and maximizes period-by-period profits; scenario ND, in which the retailer is strategic but forgoes inventory holding; and scenario C, in which the retailer is strategic and holds inventory across periods. In Chapter 2, we first compare scenarios M and ND to isolate the role of the retailer's strategic purchasing. Chapter 3 analyzes scenario C to study how strategic inventory functions under capital constraint and compares it with ND to assess inventory's effects on operations and profits. We then numerically compare scenarios M and C, illustrating the combined impact of strategic purchasing and inventory. Beyond this three-stage analysis, we compare scenario C with the benchmark scenario to highlight how capital constraint alters outcomes. Under commitment contract, the retailer holds no inventory regardless of supplier capital status, and equilibrium outcomes match those when holding inventory is not a strategic option. We analyze scenario NC and compare it to ND in Chapter 2 to examine the underlying distinctions between dynamic and commitment contracts.

Our results highlight long-term supplier–retailer interactions when the supplier faces capital constraint, but the model admits several extensions. First, to avoid unnecessary complexity, we have restricted our attention to a two-period setting to capture the retailer's two significant strategic motives, e.g., supporting supplier capital accumulation and holding inventory to lower the later-period price. Extending to a multi-period framework would be worthwhile; although the core implications should persist, it would likely yield additional insights. Second, our model assumes a linear wholesale pricing contract between the supplier and the retailer. Future work could investigate more sophisticated contracts, for instance the two-part tariff examined by Anand et al. (2008), to assess whether they can achieve the first best profit for the supply chain and their broader impact. Third, we model vertical interactions in a bilateral-monopoly framework. Since real-world supply chains often have diverse structures, future work should extend our analysis of strategic purchasing and inventory to these more complex supply chain configurations. In settings where a monopolist retailer sources from two independent suppliers to serve a common market, or where two supply chains, each comprising a retailer and its exclusive supplier, compete in selling products, Roy et al. (2022) and Li et al. (2022) have shown that, without capital constraint on the supplier, strategic inventory intensifies competition. With supplier capital constraint, however, what role do strategic purchasing and inventory play in influencing competition? Given these effects, we should examine how the interplay between the two motives evolves. When two retailers source from a monopolist supplier, these motives produce a free-rider effect. Building on this, we examine how strategic purchasing and inventory shape competition and, in turn, equilibrium performance.

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Appendix A

Mathematical Proofs and Definitions for Chapter 2

A.1 Mathematical Proofs

Proof of Proposition 2.1. Given K_2, the supplier's and the retailer's decisions are detailed in Equation (2-9). The retailer's and supplier's period-2 profits are:

$$(\pi_{r,2}^M(w_1, q_1), \pi_{s,2}^M(w_1, q_1)) = \begin{cases} \left(b\left(\frac{K_2}{c}\right)^2, \left(\frac{c-2bK_2-c^2}{c}\right)\frac{K_2}{c}\right), & K_2 < \frac{(1-c)c}{4b} \\ \left(\frac{(1-c)^2}{16b}, \frac{(1-c)^2}{8b}\right), & K_2 \geq \frac{(1-c)c}{4b} \end{cases}. \tag{A.1}$$

In period 1, the supplier sets wholesale price w_1^M to maximize total profit π_s over both periods, anticipating the retailer's purchase $q_1^M(w_1|K_1)$ and the decisions in period 2:

$$\max_{w_1} \pi_s = (w_1 - c)q_1^M(w_1|K_1) + \pi_{s,2}^M\left(w_1, q_1^M(w_1|K_1)\right). \tag{A.2}$$

The optimal solution to problem (A.2) is $w_1^M = max\left\{\frac{1+c}{2}, \frac{c-2bK_1}{c}\right\}$. Subsequent decisions can then be determined sequentially. Table A.1 summarizes the equilibrium profits in scenario M.

Table A.1. Equilibrium profits in scenario *M*

Condition	π_r^M	π_s^M	π_{sc}^M
$K_1 \geq \overline{K}_1^M$	$\frac{(1-c)^2}{8b}$	$\frac{(1-c)^2}{4b}$	$\frac{3(1-c)^2}{8b}$
$\underline{K}_1^M \leq K_1 < \overline{K}_1^M$	$\frac{(1-c)^2c^2+16b^2K_1^2}{16bc^2}$	$\frac{(1-c)^2c^2+8b(1-c)cK_1-16b^2K_1^2}{8bc^2}$	$\frac{3(1-c)^2c^2+16b(1-c)cK_1-16b^2K_1^2}{16bc^2}$
$K_1 < \underline{K}_1^M$	$\frac{bK_1^2(c^2+c^4-4bcK_1+4b^2K_1^2)}{c^6}$	$\frac{K_1(c^4(1-c^2)-2bc^2(1+c)K_1+8b^2cK_1^2-8b^3K_1^3)}{c^6}$	$\frac{K_1(c^4(1-c^2)-bc^2(1+2c-c^2)K_1+4b^2cK_1^2-4b^3K_1^3)}{c^6}$

Proof of Corollary 2.1: When $K_1 < \underline{K}_1^M$, $\frac{d\pi_r^M}{dK_1} = \frac{2bK_1(c^2+c^4-6bcK_1+8b^2K_1^2)}{c^6} \geq 0$; $\frac{d\pi_s^M}{dK_1} = \frac{c^4(1-c^2)-4bc^2(1+c)K_1+24b^2cK_1^2-32b^3K_1^3}{c^6} \geq 0$; $\frac{d\pi_{sc}^M}{dK_1} = \frac{c^4(1-c^2)-2bc^2(1+2c-c^2)K_1+12b^2cK_1^2-16b^3K_1^3}{c^6} \geq 0$. When $\underline{K}_1^M \leq K_1 < \overline{K}_1^M$, $\frac{d\pi_r^M}{dK_1} = \frac{2bK_1}{c^2} \geq 0$; $\frac{d\pi_s^M}{dK_1} = \frac{(1-c)c-4bK_1}{c^2} \geq 0$; $\frac{d\pi_{sc}^M}{dK_1} = \frac{(1-c)c-2bK_1}{c^2} \geq 0$. When $K_1 > \frac{(1-c)c}{4b}$, all profits remain constant. Thus, as K_1 increases, π_s^M, π_r^M, and π_{sc}^M increase. □

Proof of Lemma 2.1. Under dynamic contract, interactions in period 2 are the same regardless of whether the retailer is strategic or myopic. Therefore, we have in period 2, $\left(w_2^{ND}(K_2), q_2^{ND}(K_2)\right) = \left(w_2^M(K_2), q_2^M(K_2)\right)$, and the retailer's and supplier's period-2 profits $(\pi_{r,2}^{ND}(w_1, q_1), \pi_{s,2}^{ND}(w_1, q_1)) = (\pi_{r,2}^M(w_1, q_1), \pi_{s,2}^M(w_1, q_1))$.

In period 1, given w_1 and K_1 and anticipating $\pi_{r,2}^{ND}(w_1, q_1)$, the retailer decides sales and purchase quantities, $s_1^{ND}(w_1)$ and $q_1^{ND}(w_1)$, to maximize his total profit π_r:

$$\max_{0 \le s_1 \le q_1 \le \frac{K_1}{c}} \pi_r = (1 - bs_1)s_1 - w_1 q_1 + \pi_{r,2}^{ND}(w_1, q_1). \tag{A.3}$$

The condition $K_2 < \frac{(1-c)c}{4b}$ is equivalent to $q_1 < q_1^T = \frac{(1-c)c - 4bK_1}{4b(w_1 - c)}$.

We first establish the following result regarding the relationship between $s_1^{ND}(w_1)$ and $q_1^{ND}(w_1)$.

Lemma A.1. *In scenario ND, the retailer's purchase and sales quantities in period 1 are equal for a given wholesale price, i.e., $s_1^{ND}(w_1) = q_1^{ND}(w_1)$.*

Proof of Lemma A.1. When $K_1 \ge \frac{(1-c)c}{4b}$, it follows that $K_2 \ge K_1 \ge \frac{(1-c)c}{4b}$, and the retailer's optimization problem can be formulated as $\max_{0 \le s_1 \le q_1 \le \frac{K_1}{c}} \pi_r = (1 - bs_1)s_1 - w_1 q_1 + \frac{(1-c)^2}{16b}$. In this case, it is evident that the best strategy for the retailer is to sell all the purchased products. When $K_1 < \frac{(1-c)c}{4b}$, the retailer's purchase cost is sunk after purchasing $q_1^N(w_1)$, and he maximizes profit by selling $s_1^{ND}(w_1) = min\left\{q_1^{ND}(w_1), \frac{1}{2b}\right\}$. Given that $q_1^{ND}(w_1) \le \frac{K_1}{c} < \frac{1-c}{4b} < \frac{1}{2b}$, we have $s_1^{ND}(w_1) = q_1^{ND}(w_1)$ in this case as well.

Based on Lemma A.1, we simplify problem (A.3) by decomposing it into two constrained optimization subproblems. The original problem can thus be reformulated as:

$$\max \left\{ \begin{aligned} &\max_{\max\{0, q_1^T\} \le q_1 \le \frac{K_1}{c}} (1 - bq_1 - w_1)q_1 + \frac{(1-c)^2}{16b} && \text{(A.3.1)} \\ &\max_{0 \le q_1 \le \min\left\{\frac{K_1}{c}, q_1^T\right\}} (1 - bq_1 - w_1)q_1 + b\left(\frac{K_1 + (w_1 - c)q_1}{c}\right)^2 && \text{(A.3.2)} \end{aligned} \right\}.$$

We first apply the KKT conditions to analyze each subproblem and identify all feasible solutions. Then, by comparing the retailer's profit across these feasible solutions, we derive the optimal purchase quantity $q_1^{ND}(w_1)$, as stated in Lemma 2.1. The regions listed in Lemma 2.1 are defined in Table A.2.

Table A.2. The detailed definitions of the regions in Lemma 2.1

Region	Definition
R_1	$\left(K_1 \ge \frac{(1-c)c}{4b} \wedge w_1 \ge 1\right) \vee \left(K_1 \le \min\left\{\frac{(1-c)c}{4b}, \frac{c^2(w_1-1)}{2b(w_1-c)}\right\} \wedge \left(\frac{(1-c)c^2}{4bw_1} \le K_1 \le \frac{c(4c^2-2c(1+3c)w_1-(1-5c)w_1^2)}{4b(c^2+(w_1-c)^2)} \vee \left(\frac{c^3 - c\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bK_1} \le w_1 \le \min\left\{\frac{c^3+c\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bK_1}, \frac{(1-c)c^2}{4bK_1}\right\}\right)\right)\right)$
R_2	$\max\left\{\frac{(1-c)c^2}{4bw_1}, \frac{2c^2-4c^2w_1-(1-3c)w_1^2}{4bc}, \frac{c(4c^2-2c(1+3c)w_1-(1-5c)w_1^2)}{4b(c^2+(w_1-c)^2)}\right\} \le K_1 \le \min\left\{\frac{(1-c)c}{4b}, \frac{(3-c)c-2(1+c)w_1+2w_1^2}{4b}\right\}$
R_3	$\frac{c-2bK_1}{c} \le w_1 \le \min\left\{1, \frac{1+c+\sqrt{(1-c)(1-3c)+8bK_1}}{2}\right\} \wedge K_1 \ge \frac{(1-c)(3c-1)}{8b}$
R_4	$w_1 \le \frac{c-2bK_1}{c}$

R_5	$\frac{c-2bK_1}{c} \leq w_1 \leq \min\left\{\frac{(1-c)c^2}{4bK_1}, \frac{c^3-c\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bK_1}, \frac{c^3+2bcK_1-c\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{4bK_1}\right\}$
R_6	$\frac{c^3+2bcK_1-c\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{4bK_1} \leq w_1 \leq \frac{c(c-2bK_1)}{c^2-2bK_1} \wedge K_1 \leq \frac{2c^2-4c^2w_1-(1-3c)w_1^2}{4bc}$

Notes. In region R_5, the conditions $w_1 \leq \frac{c^3-c\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bK_1}$ and $w_1 \leq \frac{c^3+2bcK_1-c\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{4bK_1}$ must be satisfied only when $c^4 - 4bcK_1 + 8b^2K_1^2 \geq 0$ and $c^4 - 4b(2-c)cK_1 + 20b^2K_1^2 \geq 0$, respectively.

Proof of Proposition 2.2. In stage 1, anticipating $q_1^{ND}(w_1)$, the supplier sets w_1^{ND} to maximize total profit over both periods π_s:

$$\max_{w_1} \pi_s = (w_1 - c)q_1^{ND}(w_1) + \pi_{s,2}^{ND}(w_1, q_1^{ND}(w_1)). \quad \text{(A.4)}$$

Refer to Figure 2.2 for a detailed visualization of how π_s varies with w_1 under different values of K_1. When $K_1 \geq \frac{(1-c)c}{4b}$, π_s increases with w_1 until $w_1 = \frac{1+c}{2}$, after which it decreases, making the optimal wholesale price $w_1 = \frac{1+c}{2}$. When $K_1 < \frac{(1-c)c}{4b}$, π_s increases in regions $R_4 \cup R_5$, and (weakly) decreases elsewhere. π_s remains continuous, except for a sharp drop when w_1 transitions from region R_5 or R_2 to R_1, and a sharp increase when moving from R_1 to R_5. Given these properties of π_s, the optimal wholesale price w_1^N must lie at the upper boundary of $R_4 \cup R_5$. Accordingly, we identify four feasible solutions: i) $w_1 = 1 - \frac{2bK_1}{c}$; ii) $w_1 = \frac{(1-c)c^2}{4bK_1}$; iii) $w_1 = \frac{c^3-c\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bK_1}$; iv) $w_1 = \frac{c^3+2bcK_1-c\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{4bK_1}$.

By comparing the supplier's profit across these feasible solutions, we determine the optimal wholesale price w_1^{ND}, as stated in Proposition 2.2. Specifically, $\underline{K}_1^{ND} = \begin{cases} \frac{(2-\sqrt{6-12c+10c^2})c}{8b}, & \frac{1}{5} < c \leq \frac{5}{9} \\ \frac{(1+c-\sqrt{5-14c+13c^2})c}{8b}, & \frac{5}{9} < c \leq 1 \end{cases}$, $K_{1,1}^{ND} = \frac{(2c-1)c}{2b}$, $K_{1,2}^{ND} = \frac{(1-\sqrt{1+2c-10c^2})c}{4b}$ in Figure 2.3. Subsequent decisions can then be determined sequentially.

Table A.3 summarizes the equilibrium profits in scenario ND.

Table A.3. Equilibrium Profits in scenario ND

Condition	π_r^{ND}	π_s^{ND}	π_{sc}^{ND}
$ND1$	$\frac{(1-c)^2}{8b}$	$\frac{(1-c)^2}{4b}$	$\frac{3(1-c)^2}{8b}$
$ND2$	$\frac{(1-c)^2c^2+16b^2K_1^2}{16bc^2}$	$\frac{(1-c)^2c^2+8b(1-c)cK_1-16b^2K_1^2}{8bc^2}$	$\frac{3(1-c)^2c^2+16b(1-c)cK_1-16b^2K_1^2}{16bc^2}$
$ND3$	$\frac{c^2-6c^3+5c^4+16bcK_1-16b^2K_1^2}{16bc^2}$	$\frac{1-c^2-8bK_1}{8b}$	$\frac{3(1-c)^2c^2+16b(1-c)cK_1-16b^2K_1^2}{16bc^2}$
$ND4$	$\frac{bK_1^2}{c^2}$	$f_1^{ND}(c, K_1)$	$\frac{bK_1^2}{c^2} + f_1^{ND}(c, K_1)$
$ND5$	$f_2^{ND}(c, K_1)$	$f_3^{ND}(c, K_1)$	$f_4^{ND}(c, K_1)$

Notes. $f_1^{ND}(c,K_1)=\frac{(1-2c)c^3+2b(2-c)cK_1-8b^2K_1^2-(1-2c)c\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bc^2}$,

$f_2^{ND}(c,K_1)=\frac{-c^4+4bcK_1+4b^2K_1^2+(c^2-2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{8bc^2}$,

$f_3^{ND}(c,K_1)=\frac{(1-c)c^3+2bc(3-4c)K_1-12b^2K_1^2+(2bK_1-(1-c)c)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{4bc^2}$,

$f_4^{ND}(c,K_1)=\frac{(2-3c)c^3+16bc(1-c)K_1-20b^2K_1^2+(2bK_1-(2-3c)c)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{8bc^2}$.

Proof of Corollary 2.2.

In $ND5$, $\frac{d\pi_r^{ND}}{dK_1}=\frac{-c^3+2bc(3+c)K_1-20b^2K_1^2+(c+2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{2c^2\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}\geq 0$;

$\frac{d\pi_s^{ND}}{dK_1}=\frac{c^2(2-3c+2c^2)-2bc(11-8c)K_1+40b^2K_1^2+(c(3-4c)-12bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{2c^2\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}\geq 0$;

$\frac{d\pi_{sc}^{ND}}{dK_1}=\frac{2(1-c)^2c^2-2bc(8-9c)K_1+20b^2K_1^2+2(2(1-c)c-5bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{2c^2\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}\geq 0$.

In $ND4$, $\frac{d\pi_r^{ND}}{dK_1}=\frac{2bK_1}{c^2}\geq 0$;

$\frac{d\pi_s^{ND}}{dK_1}=\frac{(c-4bK_1)(1-2c)c+((2-c)c-8bK_1)\sqrt{c^4-4bcK_1+8b^2K_1^2}}{c^2\sqrt{c^4-4bcK_1+8b^2K_1^2}}\geq 0$;

$\frac{d\pi_{sc}^{ND}}{dK_1}=\frac{(1-2c)c(c-4bK_1)+((2-c)c-6bK_1)\sqrt{c^4-4bcK_1+8b^2K_1^2}}{c^2\sqrt{c^4-4bcK_1+8b^2K_1^2}}\geq 0$.

In $ND3$, $\frac{d\pi_r^{ND}}{dK_1}=\frac{c-2bK_1}{c^2}\geq 0$; $\frac{d\pi_s^{ND}}{dK_1}=-1<0$; $\frac{d\pi_{sc}^{ND}}{dK_1}=\frac{(1-c)c-2bK_1}{c^2}\geq 0$.

The patterns in $ND2$ and $ND1$ are identical to those in scenario M. Thus, as K_1 increases, π_r^{ND} and π_{sc}^{ND} increase; π_s^{ND} increases except in region $ND3$.

Additionally, π_r^{ND} is continuous. When $\frac{1}{5}<c<\frac{1}{3}$, π_s^{ND} and π_{sc}^{ND} increase sharply at the boundary between $ND3$ and $ND4$, i.e., $K_1=\frac{(2-\sqrt{2}\sqrt{3-6c+5c^2})c}{8b}$. This is because: $\lim_{K_1\to(\frac{(2-\sqrt{2}\sqrt{3-6c+5c^2})c}{8b})^+}\pi_s^{ND}=\frac{1-2c-c^2+\sqrt{2}c\sqrt{3-6c+5c^2}}{8b}>\lim_{K_1\to(\frac{(2-\sqrt{2}\sqrt{3-6c+5c^2})c}{8b})^-}\pi_s^{ND}=\frac{-1+8c-13c^2+2(2c-1)\sqrt{(3c-1)^2}+\sqrt{2}c\sqrt{3-6c+5c^2}}{8b}$ when $c<\frac{1}{3}$, and the two are equal when $c\geq\frac{1}{3}$. We assume that at the breakpoint $K_1=\frac{(2-\sqrt{2}\sqrt{3-6c+5c^2})c}{8b}$, the equilibrium follows the form in $ND3$. □

Proof of Corollary 2.3. The supplier's profit without capital constraint is identical to that in $ND1$, i.e., $\frac{(1-c)^2}{4b}$. According to the change trend of π_s^{ND}, if its value at $max\left\{\frac{\left((1+c)-\sqrt{5-14c+13c^2}\right)c}{8b}, \frac{(2-\sqrt{2}\sqrt{3-6c+5c^2})c}{8b}, 0\right\}$ exceeds $\frac{(1-c)^2}{4b}$, i.e., when $c \geq \sqrt{2}-1$, then in the range of $K_{1,3}^{ND} \leq K_1 \leq \frac{(1-c)(3c-1)}{8b}$, the supplier's profit with limited capital surpasses that with unlimited capital. Here, $K_{1,3}^{ND}$ is the point where the profit in $ND4 \cup ND5$ equals $\frac{(1-c)^2}{4b}$, and $\frac{(1-c)(3c-1)}{8b}$ marks where the profit in $ND3$ equals $\frac{(1-c)^2}{4b}$. □

Proof of Proposition 2.3. This result follows directly from comparing strategies and profits between scenarios ND and M. Specifically, $\pi_r^{ND} \geq \pi_r^M$ holds when $c \leq \frac{1}{3} \wedge \left(K_{1,2}^{ND}\right)^+ \leq K_1 < \underline{K}_1^M$. □

Proof of Proposition 2.4. Under commitment contract, given (w_1, w_2) set by the supplier, in period t, the myopic retailer responds with purchase quantity, $q_t^M(w_t)$, and sales quantity, $s_t^M(w_t)$, to maximize profit in the period, $(1-bs_t)s_t - w_t q_t$, subject to $0 \leq s_t \leq q_t \leq \frac{K_t}{c}$. The retailer's best strategy is $s_t^M(w_t) = q_t^M(w_t) = max\left\{0, min\left\{\frac{1-w_t}{2b}, \frac{K_t}{c}\right\}\right\}$. In period 1, anticipating the retailer's responses, the supplier sets wholesale prices to maximize profit as follows:

$$\pi_s^M(w_1, w_2) = \begin{cases} (w_1 - c)\frac{K_1}{c} + (w_2 - c)\frac{w_1 K_1}{c^2}, & w_2 \leq \frac{c-2b\left(\frac{w_1 K_1}{c}\right)}{c} \wedge w_1 \leq \frac{c-2bK_1}{c} \\ (w_1 - c)\frac{1-w_1}{2b} + (w_2 - c)\frac{\left(K_1 + (w_1 - c)\frac{1-w_1}{2b}\right)}{c}, & w_2 \leq \frac{c-2b\left(K_1 + (w_1 - c)\frac{1-w_1}{2b}\right)}{c} \wedge \frac{c-2bK_1}{c} < w_1 \leq 1 \\ (w_1 - c)\frac{K_1}{c} + (w_2 - c)\frac{1-w_2}{2b}, & \frac{c-2b\left(\frac{w_1 K_1}{c}\right)}{c} < w_2 \leq 1 \wedge w_1 \leq \frac{c-2bK_1}{c} \\ (w_1 - c)\frac{1-w_1}{2b} + (w_2 - c)\frac{1-w_2}{2b}, & \frac{c-2b\left(K_1 + (w_1 - c)\frac{1-w_1}{2b}\right)}{c} < w_2 \leq 1 \wedge \frac{c-2bK_1}{c} < w_1 \leq 1 \end{cases} \quad \text{(A.5)}$$

We solve this optimization problem using the approach detailed in proving Lemma 2.1. For the four ranges of (w_1, w_2) specified in (A.5), we define four subproblems. Applying KKT conditions to these subproblem helps us identify the local maxima of $\pi_s^M(w_1, w_2)$, which are detailed below.

(1) $\{w_1, w_2\} = \left\{\frac{1+c}{2}, \frac{1+c}{2}\right\}$: This is the stationary point of subproblem 4 and is locally optimal iff $\frac{1+c}{2} > \frac{c-2bK_1}{c} \wedge \frac{1+c}{2} > \frac{c-2b\left(K_1 + \left(\frac{1+c}{2} - c\right)\left(1 - \frac{1+c}{2}\right)\frac{1}{2b}\right)}{c}$.

(2) $\{w_1, w_2\} = \left\{\frac{c-2bK_1}{c}, \frac{1+c}{2}\right\}$: This point, located at boundary $w_1 = \frac{c-2bK_1}{c}$ between subproblems 3 and 4, is the local optimum iff it is optimal for subproblem 3, when $\frac{1+c}{2} > \frac{c-2b\left(\frac{c-2bK_1}{c}\frac{K_1}{c}\right)}{c}$, and for subproblem 4, when $\frac{1+c}{2} > \frac{c-2b\left(K_1 + \left(\frac{c-2bK_1}{c} - c\right)\left(1 - \frac{c-2bK_1}{c}\right)\frac{1}{2b}\right)}{c} \wedge \frac{1+c}{2} \leq \frac{c-2bK_1}{c}$.

(3) $\{w_1, w_2\} = \left\{\frac{c-2bK_1}{c}, \frac{c^3-2bcK_1+4b^2K_1^2}{c^3}\right\}$: This point, located at the intersection of four subproblems, is the local maximum iff it is optimal for all the four subproblems, when $\frac{1+c}{2} \leq \frac{c-2b\left(\frac{w_1K_1}{c}\right)}{c} \wedge \frac{1+c}{2} \leq \frac{c-2bK_1}{c}$. Note that $\frac{c^3-2bcK_1+4b^2K_1^2}{c^3} = \frac{c-2bK_2}{c}$, where $K_2 = \frac{K_1(c-2bK_1)}{c^2}$.

After simplifying the conditions, we identify three mutually exclusive and collectively exhaustive ranges: $K_1 \geq \overline{K}_1^M$, $\underline{K}_1^M \leq K_1 < \overline{K}_1^M$ and $K_1 < \underline{K}_1^M$. In each range, a previously discussed solution emerges as the unique local maximum for the retailer's problem, making it the global optimum. It is evident that the equilibrium outcomes for the scenario with a myopic retailer under commitment contract are identical to those under dynamic contract.

Proof of Lemma 2.2. The retailer's best response in period 2 remains the same as in scenario M. He determines the optimal purchase quantity, $q_1^{NC}(w_1, w_2)$, and sales quantity, $s_1^{NC}(w_1, w_2)$, in period 1 to maximize total profit over periods, $\pi_r^{NC}(s_1, q_1|w_1, w_2)$:

$$\max_{0 \leq s_1 \leq q_1 \leq \frac{K_1}{c}} (1 - bs_1)s_1 - w_1q_1 + \begin{cases} \left(1 - b\frac{K_1+(w_1-c)q_1}{c} - w_2\right)\frac{K_1+(w_1-c)q_1}{c}, & q_1 < \frac{(1-w_2)c-2bK_1}{2b(w_1-c)} \wedge w_2 \leq 1 \\ \frac{(1-w_2)^2}{4b}, & q_1 \geq \frac{(1-w_2)c-2bK_1}{2b(w_1-c)} \wedge w_2 \leq 1 \\ 0, & w_2 > 1 \end{cases}. \quad \text{(A.6)}$$

Lemma A.2. *In scenario* NC*, given wholesale prices, the retailer's purchase and sales quantities in period 1 are equal, i.e.,* $s_1^{NC}(w_1, w_2) = q_1^{NC}(w_1, w_2)$.

Proof of Lemma A.2. When $w_2 > 1$, the statement is obviously true. When $w_2 \leq 1$: i) For $K_1 \geq \frac{(1-w_2)c}{2b}$, the objective function $(1 - bs_1)s_1 - w_1q_1 + \frac{(1-w_2)^2}{4b}$ implies $s_1^{NC}(w_1, w_2) = q_1^{NC}(w_1, w_2)$. ii) For $K_1 < \frac{(1-w_2)c}{2b}$, given $q_1^{NC}(w_1, w_2)$, the retailer sells $s_1^{NC}(w_1, w_2) = min\left\{q_1^{NC}(w_1, w_2), \frac{1}{2b}\right\}$. As $q_1^{NC}(w_1, w_2) \leq \frac{K_1}{c} < \frac{1-w_2}{2b} < \frac{1}{2b}$, it follows that $s_1^{NC}(w_1, w_2) = q_1^{NC}(w_1, w_2)$.

It can be verified that $\pi_r^{NC}(q_1|w_1, w_2)$ is concave, and thus its optimal solution is: i) $q_1 = \frac{1-w_1}{2b}$ iff $max\left\{0, \frac{(1-w_2)c-2bK_1}{2b(w_1-c)}\right\} \leq \frac{1-w_1}{2b} < \frac{K_1}{c}$. ii) $q_1 = \tilde{G}(w_1, w_2) = \frac{(1-c)cw_1-(2bK_1+cw_2)(w_1-c)}{2b(c^2+(w_1-c)^2)}$ iff $0 \leq \tilde{G}(w_1, w_2) < min\left\{\frac{K_1}{c}, \frac{(1-w_2)c-2bK_1}{2b(w_1-c)}\right\} \wedge w_2 \leq 1$. iii) $q_1 = \frac{K_1}{c}$ iff $\frac{(1-w_2)c-2bK_1}{2b(w_1-c)} < \frac{K_1}{c} \leq \frac{1-w_1}{2b} \vee \frac{K_1}{c} \leq min\left\{\frac{(1-w_2)c-2bK_1}{2b(w_1-c)}, \tilde{G}(w_1, w_2)\right\}$. iv) $q_1 = 0$ iff $max\left\{\frac{(1-w_2)c-2bK_1}{2b(w_1-c)}, \frac{1-w_1}{2b}\right\} \leq 0 \vee \tilde{G}(w_1, w_2) \leq 0 \leq \frac{(1-w_2)c-2bK_1}{2b(w_1-c)}$. Through simplification, we define all regions in Table A.4.

Table A.4. The detailed definitions of the regions in Lemma 2.2

Region	Definition
R_1	$w_1 \geq 1 \wedge w_2 \geq \frac{2bcK_1+((1-c)c-2bK_1)w_1}{c(w_1-c)}$
R_2	$\frac{-2bK_1(c^2+w_1^2-cw_1)+c^2(1-c)w_1}{c^2(w_1-c)} \leq w_2 < min\left\{\frac{2(c-bK_1)-(1+c)w_1+w_1^2}{c}, \frac{2bcK_1+((1-c)c-2bK_1)w_1}{c(w_1-c)}\right\}$
R_3	$1-\frac{2bK_1}{c} \leq w_1 < 1 \wedge w_2 \geq \frac{2(c-bK_1)-(1+c)w_1+w_1^2}{c}$
R_4	$w_1 < 1-\frac{2bK_1}{c}$
R_5	$w_1 \geq 1-\frac{2bK_1}{c} \wedge w_2 < \frac{-2bK_1(c^2+w_1^2-cw_1)+c^2(1-c)w_1}{c^2(w_1-c)}$

Proof of Proposition 2.5. In period 1, anticipating the retailer's best responses, $q_1^{NC}(w_1,w_2)$ and $q_2^{NC}(w_1,w_2)$, the supplier sets wholesale prices, w_1^{NC} and w_2^{NC}, to maximize total profit $\pi_s^{NC}(w_1,w_2)$:

$$\max_{w_1,w_2}(w_1-c)q_1^{NC}(w_1,w_2)+(w_2-c)q_2^{NC}(w_1,w_2). \tag{A.7}$$

$q_1^{NC}(w_1,w_2)$ is given in Lemma 2.2 and $q_2^{NC}(w_1,w_2)=max\left\{0, min\left\{\frac{1-w_2}{2b}, \frac{K_1+(w_1-c)q_1^{NC}(w_1,w_2)}{c}\right\}\right\}$. We solve this optimization problem employing the method described before.

If $c > c^T \wedge c^4(2c-1)+2bc^2(1-c)(2+c)K_1-8b^2(2-c)cK_1^2+16b^3K_1^3>0 \wedge (1-c)^3c^3-12b(1-c)^2c^2K_1-60b^2(1-c)cK_1^2+152b^3K_1^3 \geq 0 \wedge K_1 < \underline{K_1^M}$, $\pi_s^{NC}(w_1,w_2)$ is bimodal, offering two local maxima, $\left(\frac{c-2bK_1}{c}, \frac{c^3-2bcK_1+4b^2K_1^2}{c^3}\right)$ and $\left(\widetilde{w}_1^{NC}, \widetilde{w}_2^{NC}\right)$. The best strategy depends on the sign of $\left(c-\widetilde{w}_1^{NC}\right)\left(c^3-2bcK_1+4b^2K_1^2-2bcK_1\widetilde{w}_1^{NC}\right)+c^4\widetilde{w}_1^{NC}$. When it is positive, the first local maximum is globally optimal. In other cases, $\left(\frac{1+c}{2}, \frac{1+c}{2}\right)$, $\left(1-\frac{2bK_1}{c}, \frac{1+c}{2}\right)$, $\left(\frac{c-2bK_1}{c}, \frac{c^3-2bcK_1+4b^2K_1^2}{c^3}\right)$ and $\left(\widetilde{w}_1^{NC}, \widetilde{w}_2^{NC}\right)$ are the unique local optima in regions $K_1 \geq \overline{K}_1^M$, $\underline{K_1^M} \leq K_1 < \overline{K}_1^M$, $((c \leq c^T \wedge c^4(2c-1)+2bc^2(1-c)(2+c)K_1-8b^2(2-c)cK_1^2+16b^3K_1^3>0) \vee (c > c^T \wedge (1-c)^3c^3-12b(1-c)^2c^2K_1-60b^2(1-c)cK_1^2+152b^3K_1^3<0)) \wedge K_1 \leq \underline{K_1^M}$ and $c^4(2c-1)+2bc^2(1-c)(2+c)K_1-8b^2(2-c)cK_1^2+16b^3K_1^3 \leq 0 \wedge K_1 \leq \underline{K_1^M}$, respectively. After simplifying and combining cases, we prove Proposition 2.5. Table A.5 summarizes the equilibrium profits in scenario NC.

Table A.5. Equilibrium Profits in scenario NC

K_1	π_r^{NC}	π_s^{NC}	π_{sc}^{NC}
$NC1$	$\frac{(1-c)^2}{8b}$	$\frac{(1-c)^2}{4b}$	$\frac{3(1-c)^2}{8b}$
$NC2$	$\frac{(1-c)^2c^2+16b^2K_1^2}{16bc^2}$	$\frac{(1-c)^2c^2+8b(1-c)cK_1-16b^2K_1^2}{8bc^2}$	$\frac{3(1-c)^2c^2+16b(1-c)cK_1-16b^2K_1^2}{16bc^2}$
$NC3$	$\frac{bK_1^2(c^2+c^4-4bcK_1+4b^2K_1^2)}{c^6}$	$\frac{K_1(c^4(1-c^2)-2bc^2(1+c)K_1+8b^2cK_1^2-8b^3K_1^3)}{c^6}$	$\frac{K_1(c^4(1-c^2)-bc^2(1+2c-c^2)K_1+4b^2cK_1^2-4b^3K_1^3)}{c^6}$
$NC4$	$\tilde{\pi}_r^{NC}$	$\tilde{\pi}_s^{NC}$	$\frac{K_1(c^2(c-c^2-bK_1)+(1-c)c^2\widetilde{w}_1^{NC}-bK_1\widetilde{w}_1^{NC^2})}{c^4}$

Notes. $\tilde{\pi}_r^{NC} = \frac{K_1(-c^3(c-bK_1)+c^2(c^2+bK_1)\widetilde{w}_1^{NC}-bcK_1\widetilde{w}_1^{NC^2}+bK_1\widetilde{w}_1^{NC^3})}{c^4(\widetilde{w}_1^{NC}-c)}$;

$\tilde{\pi}_s^{NC} = \frac{K_1(c^5 - c^2(c^2+2bK_1)\tilde{w}_1^{NC} + c((1-c)c+2bK_1)\tilde{w}_1^{NC^2} - 2bK_1\tilde{w}_1^{NC^3})}{c^4(\tilde{w}_1^{NC}-c)}$.

Proof of Corollary 2.4. In $NC4$:

(1) If $c < \frac{1}{2}$, $\frac{d\pi_s^{NC}}{dK_1} \geq 0$. If $c \in \left[\frac{1}{2}, \bar{\bar{c}}\right]$, $\frac{d\pi_s^{NC}}{dK_1} \geq 0$ iff $K_1 \geq K_{1,2}^{NC}$. If $c > \bar{\bar{c}}$, $\frac{d\pi_s^{NC}}{dK_1} \leq 0$.

(2) $\frac{d\pi_r^{NC}}{dK_1} \geq 0$ if $K_1 \leq K_{1,3}^{NC}$.

(3) $\frac{d\pi_{sc}^{NC}}{dK_1} \geq 0$ if $K_1 \leq K_{1,4}^{NC}$.

Additionally, when $c \leq c^T$, both π_r^{NC} and π_{sc}^{NC} maintain continuity with K_1. However, when $c > c^T$, they exhibit discontinuity at $K_1 = K_{1,1}^{NC}$. To prove this, we have $\lim_{K_1 \to (K_{1,1}^{NC})^-} \pi_r^{NC} - \lim_{K_1 \to (K_{1,1}^{NC})^+} \pi_r^{NC} = \frac{K_{1,1}^{NC}(-c+2bK_{1,1}^{NC}+c\bar{w}_1)\left(c^5 - bc^2K_{1,1}^{NC} + 2b^2cK_{1,1}^{NC^2} + bK_{1,1}^{NC}\left((1-c)c - 2bK_{1,1}^{NC}\right)\bar{w}_1 + bcK_{1,1}^{NC}\bar{w}_1^2\right)}{c^6(\bar{w}_1 - c)}$, where $\bar{w}_1$ is the value of $\tilde{w}_1^{NC}$ when $K_1 = K_{1,1}^{NC}$. We can show $c^5 - bc^2K_{1,1}^{NC} + 2b^2cK_{1,1}^{NC^2} + bK_{1,1}^{NC}\left((1-c)c - 2bK_{1,1}^{NC}\right)\bar{w}_1 + bcK_{1,1}^{NC}\bar{w}_1^2 > 0$. For $c \leq c^T$, the strategy in $NC4$, $(\tilde{w}_1^{NC}, \tilde{w}_2^{NC})$, converges to that in $NC3$, $\left(\frac{c-2bK_1}{c}, \frac{c^3-2bcK_1+4b^2K_1^2}{c^3}\right)$, at $K_1 = K_{1,1}^{NC}$, ensuring continuity of all equilibrium strategies and profits. For $c > c^T$, $\bar{w}_1 > \frac{c-2bK_{1,1}^{NC}}{c}$, and thus $\lim_{K_1 \to (K_{1,1}^{NC})^-} \pi_r^{NC} > \lim_{K_1 \to (K_{1,1}^{NC})^+} \pi_r^{NC}$, leading to a profit drop at $K_1 = K_{1,1}^{NC}$.

Hence the claim. □

Proof of Proposition 2.6: In $NC4$, $\tilde{w}_1^{NC} \geq \frac{c-2bK_1}{c} = w_1^M$, $q_1^{NC} = q_1^M = \frac{K_1}{c}$, thus $K_2^{NC} \geq K_2^M$, leading to $q_2^{NC} \geq q_2^M$. Moreover, after calculating the differences in w_2 and profits between scenarios NC and M, we have $w_2^{NC} - w_2^M \leq 0$, $\pi_s^{NC} - \pi_s^M \geq 0$, $\pi_r^{NC} - \pi_r^M \geq 0$, and $\pi_{sc}^{NC} - \pi_{sc}^M \geq 0$. Hence the claim. □

Proof of Proposition 2.7. When $K_{1,1}^{NC} \leq K_1 < \underline{K}_1^M$ (in $NC3$), the equilibrium outcomes in scenario NC are the same as those in scenario M. Thus, as stated in Proposition 2.3, $w_1^{ND} > w_1^{NC}$, $w_2^{ND} < w_2^{NC}$, and $q_2^{ND} > q_2^{NC}$. When $K_1 < K_{1,1}^{NC}$, calculations of differences based on Proposition 2.2 and Proposition 2.5 yield the following results.

(1) For the comparison of w_1^{ND} and w_1^{NC}:

In $ND3 \cap NC4$, $w_1^{ND} > \tilde{w}_1^{NC}$.

In $ND4 \cap NC4$, $w_1^{ND} - \widetilde{w}_1^{NC} \geq 0$ iff $K_1 \geq K_{1,2}^N$.

In $ND5 \cap NC4$, $w_1^{ND} < \widetilde{w}_1^{NC}$.

(2) For the comparison of w_2^{ND} and w_2^{NC}:

In $ND3 \cap NC4$, $w_2^{ND} \geq \widetilde{w}_2^{NC}$ iff $K_1 \leq K_{1,1}^N$.

In $ND4 \cap NC4$, $w_2^{ND} > \widetilde{w}_2^{NC}$.

In $ND5 \cap NC4$, $w_2^{ND} > \widetilde{w}_2^{NC}$.

(3) For the comparison of q_2^{ND} and q_2^{NC}:

Under both contract types, when $K_1 < \underline{K}_1^M$, $q_2 = \frac{K_2}{c} = \frac{w_1 K_1}{c^2}$ in equilibrium. Thus, $q_2^{ND} - q_2^{NC} = \frac{K_1(w_1^{ND} - w_1^{NC})}{c^2}$, showing that the comparison result of q_2^{ND} and q_2^{NC} mirrors that of w_1^{ND} and w_1^{NC}.

We produce Figure 2.7.a by consolidating areas with the same characteristics. □

Proof of Proposition 2.8. (1) For the comparison of π_s^{ND} and π_s^{NC}:

When $K_{1,1}^{NC} \leq K_1 < \underline{K}_1^M$, we have $\pi_s^{ND} > \pi_s^{NC}$. When $K_1 < K_{1,1}^{NC}$, we have:

In $ND3 \cap NC4$, $\pi_s^{ND} > \pi_s^{NC}$.

In $ND4 \cap NC4$, $\pi_s^{ND} \geq \pi_s^{NC}$ if $K_1 \geq K_{1,3}^N$.

In $ND5 \cap NC4$, $\pi_s^{ND} \geq \pi_s^{NC}$ if $K_1 \geq K_{1,3}^N$.

(2) For the comparison of π_r^{ND} and π_r^{NC}:

When $K_{1,1}^{NC} \leq K_1 < \underline{K}_1^M$, referring to Proposition 2.3, $\pi_r^{ND} \geq \pi_r^{NC}$ if $c \leq \frac{1}{3} \wedge \left(K_{1,2}^{ND}\right)^+ \leq K_1 < \underline{K}_1^M$ in $ND3 \cap NC3$, while $\pi_r^{ND} < \pi_r^{NC}$ in $ND4 \cap NC3$ and $ND5 \cap NC3$.

When $K_1 < K_{1,1}^{NC}$:

In $ND3 \cap NC4$, $\pi_r^{ND} \geq \pi_r^{NC}$ if $K_1 \geq K_{1,4}^N$.

In $ND4 \cap NC4$, $\pi_r^{ND} < \pi_r^{NC}$.

In $ND5 \cap NC4$, $\pi_r^{ND} < \pi_r^{NC}$.

(3) For the comparison of π_{sc}^{ND} and π_{sc}^{NC}:

In equilibrium, when $K_1 < \underline{K}_1^M$, under either contract, $\pi_{sc} = (1 - b\frac{K_1}{c} - c)\frac{K_1}{c} + (1 - bq_2 - c)q_2$. Thus, $\pi_{sc}^{ND} - \pi_{sc}^{NC} = (q_2^{ND} - q_2^{NC})(1 - c - b(q_2^{ND} + q_2^{NC}))$. We have $q_2^{ND} = min\left\{\frac{K_2^{ND}}{c}, \frac{1-c}{4b}\right\} \leq \frac{1-c}{4b}$. Under commitment contract, $q_2^{NC} = \frac{K_1(c-2bK_1)}{c^2} < \frac{1-c}{4b}$ in $NC3$, and $q_2^{NC} = \frac{\widetilde{w}_1^{NC} K_1}{c^2} < \frac{1-c}{4b}$ in $NC4$. Then, $1 - c - b\left(q_2^{ND} + q_2^{NC}\right) > 0$. Consequently, the comparison result of π_{sc} coincides with that of q_2, and further coincides with w_1.

We produce Figure 2.7.b by merging areas with the same characteristics. □

Decision sequence under dynamic contract when demand is uncertain. In period 1, the supplier and retailer sequentially set the wholesale price w_1 and purchase quantity q_1. After observing a, the retailer sells $s_1 = min\left\{q_1, \frac{a}{2b}\right\}$ in the market. Moving into period 2, given K_2 and the realized a, the supplier sets $w_2 = \begin{cases} max\left\{a - \frac{2bK_2}{c}, \frac{a+c}{2}\right\}, & a > c \\ \infty, & a \leq c \end{cases}$. Then, the retailer purchases and sells $q_2 = \begin{cases} min\left\{\frac{a-c}{4b}, \frac{K_2}{c}\right\}, & a > c \\ 0, & a \leq c \end{cases}$.

Following the outlined sequence, we first present equilibrium outcomes and profits under dynamic contract when demand is uncertain, covering both cases where the retailer is myopic (scenario $M|ND$) and where he is strategic (scenario $\widehat{ND}$).

Equilibrium analysis in scenario $M|ND$. Anticipating the future sales quantity s_1, the myopic retailer purchases q_1 to maximize his expected profit in period 1:

$$\max_{0 \leq q_1 \leq \frac{K_1}{c}} -w_1 q_1 + \begin{cases} q_1 - bq_1^2, & q_1 < \frac{1-\delta}{2b} \\ \frac{8b^3q_1^3 - 12b^2q_1^2(1+\delta) + 6bq_1(1+\delta)^2 - (1-\delta)^3}{24b\delta}, & \frac{1-\delta}{2b} \leq q_1 < \frac{1+\delta}{2b} \\ \frac{3+\delta^2}{12b}, & q_1 \geq \frac{1+\delta}{2b} \end{cases}. \quad \text{(A.8)}$$

We solve all optimization problems in uncertain setting employing the approach used in deterministic setting. Thus, we will only focus on presenting the equilibrium outcomes.

The retailer's purchase quantity in period 1 in response to wholesale price w_1 is: $\hat{q}_1^M(w_1) = (min\left\{\hat{q}_1^o(w_1), \frac{K_1}{c}\right\})^+$, where $\hat{q}_1^o(w_1) = \begin{cases} \frac{1-w_1}{2b}, & w_1 > \delta \\ \frac{1+\delta-2\sqrt{w_1\delta}}{2b}, & w_1 \leq \delta \end{cases}$.

Anticipating future responses, the supplier sets $\widehat{w}_1^{M|ND}$ to maximize her expected total profit over two periods. Since her expected profit in period 2 is positively correlated to profit in period 1, she maximizes $(w_1 - c)\hat{q}_1^M(w_1)$, yielding the following equilibrium outcomes.

Table A.6. Equilibrium Outcomes in scenario $M|ND$

	Conditions on (c, K_1)	$\widehat{w}_1^{M|ND}$	$\hat{q}_1^{M|ND}$
$M1|ND$	$K_1 > \frac{(1-c)c}{4b} \wedge c > 2\delta - 1$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$M2|ND$	$K_1 \leq min\left\{\frac{(1-c)c}{4b}, \frac{(1-\delta)c}{2b}\right\}$	$1 - \frac{2bK_1}{c}$	$\frac{K_1}{c}$
$M3|ND$	$\frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c}{6b} \leq K_1 \wedge c \leq 2\delta - 1$	$\frac{\left((1+\delta)+\sqrt{12c\delta+(1+\delta)^2}\right)^2}{36\delta}$	$\frac{2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}}{6b}$
$M4|ND$	$\frac{(1-\delta)c}{2b} < K_1 < \frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c}{6b}$	$\frac{(c(1+\delta)-2bK_1)^2}{4c^2\delta}$	$\frac{K_1}{c}$

Equilibrium outcomes and profits in scenario $\widehat{\boldsymbol{ND}}$.

(1) The wholesale price set by the supplier and purchase quantity set by the retailer in period 1, $\hat{w}_1^{ND}$ and $\hat{q}_1^{ND}$, are detailed in Table A.7.

Table A.7. Equilibrium Outcomes in scenario $\widehat{\boldsymbol{ND}}$

	Conditions on (c, K_1)	$\hat{w}_1^{ND}$	$\hat{q}_1^{ND}$
$\widehat{ND1}$	$K_1 > \frac{(1-c)c}{4b} \wedge c > 2\delta - 1 \wedge K_1 > \hat{K}_1^T$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$\widehat{ND2}$	$K_1 \le min\left\{\frac{(1-c)c}{4b}, \frac{(1-\delta)c}{2b}\right\} \wedge K_1 > \hat{K}_1^T$	$1 - \frac{2bK_1}{c}$	$\frac{K_1}{c}$
$\widehat{ND3}$	$\frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c_s}{6b} \le K_1 \wedge c \le 2\delta - 1 \wedge K_1 > \hat{K}_1^T$	$\frac{\left((1+\delta)+\sqrt{12c\delta+(1+\delta)^2}\right)^2}{36\delta}$	$\frac{2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}}{6b}$
$\widehat{ND4}$	$\frac{(1-\delta)c}{2b} < K_1 < \frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c}{6b} \wedge K_1 > \hat{K}_1^T$	$\frac{(c(1+\delta)-2bK_1)^2}{4c^2\delta}$	$\frac{K_1}{c}$
$\widehat{ND5}$	$K_1 \le min\left\{\frac{(1-\delta)c}{2b}, \frac{(1-c)c}{4b}, \hat{K}_1^T\right\} \wedge \hat{f}_1^{ND}(\dot{w}_1^{ND}) \ge 0 \wedge \dot{w}_1^{ND} \ge \frac{c^2(1-\delta-c)}{4bK_1}$	$\dot{w}_1^{ND}$	$\frac{K_1}{c}$
$\widehat{ND6}$	$\frac{(1-c)c}{4b} < K_1 \le \hat{K}_1^T \wedge c > 2\delta - 1$	$\ddot{w}_1^{ND}$	$\frac{1-c}{4b}$
$\widehat{ND7}$	$\frac{(2c-1)c}{2b} \le K_1 \wedge \left(3c - (1-\delta)\right)c - 2\sqrt{c^4 - 4bcK_1 + 8b^2K_1^2} \le 0 \wedge \left(!\,\widehat{ND5}\right)$	$\frac{c^3 - c\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bK_1}$	$\frac{K_1}{c}$
$\widehat{ND8}$	$K_1 < min\left\{\frac{(2c-1)c}{2b}, \frac{((1+\delta+c)-\sqrt{13c^2-2c(7-9\delta)+5-6\delta+5\delta^2})c}{8b}\right\}$	$\frac{c^3+2bcK_1-c\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{4bK_1}$	$\frac{K_1}{c}$
$\widehat{ND9}$	$\frac{(1-\delta))c}{2b} \le K_1 \le min\left\{\frac{(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2})c}{6b}, \hat{K}_1^T\right\}$	$\overline{w}_1^{ND}$	$\frac{K_1}{c}$
$\widehat{ND10}$	$c \le 2\delta - 1 \wedge \frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c}{6b} \le K_1 \le \hat{K}_1^T$	$\overline{\overline{w}}_1^{ND}$	$\frac{2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}}{6b}$
$\widehat{ND11}$	$\frac{((1-\delta)-c)c}{4b} \le K_1 \le min\left\{\frac{(1-\delta)c}{2b}, \hat{K}_{1,3}^{ND}\right\}$	$\breve{w}_1^{ND}$	$\frac{K_1}{c}$
$\widehat{ND12}$	$\frac{\left(\sqrt{2}-\sqrt{3+5c^2-6c(1-\delta)-2\delta+\delta^2}\right)c}{4\sqrt{2}b} \le K_1 < min\left\{\frac{((1-\delta)-c)c}{4b}, \hat{K}_{1,4}^{ND}\right\}$	$\tilde{w}_1^{D}$	$\frac{K_1}{c_s}$

Notes. 1) $\dot{w}_1^{ND}$ is the largest root to $-4b^2K_1^2w_1^3 + bcK_1(c(1+\delta-c) + 4bK_1)w_1^2 - c^3(bK_1(1+\delta-c) + c^2\delta)w_1 + c^4(c - 2bK_1)\delta = 0$. $\ddot{w}_1^{ND}$ is the root to $-(1-c)^2w_1^3 + w_1^2\big((1-c)c(4(1-c)+\delta) - 8b(1-c)K_1\big) + w_1\big(-16b^2K_1^2 + 4bcK_1(5(1-c)+\delta) - c^2(5(1-c)^2 + 2(1+c)\delta)\big) + 16b^2cK_1^2 - 4bc^2K_1(3(1-c)+\delta) + c^3(2(1-c)^2 + (3+c)\delta) = 0$. $\overline{w}_1^{ND}$ is the root to $-16b^2K_1^2w_1^3 + 4bcK_1w_1^2(c(1+\delta-c) + 4bK_1) - 4c^3w_1(bK_1(1+\delta-c) + c^2\delta) + c^3(c(1+\delta) - 2bK_1)^2 = 0$. $\overline{\overline{w}}_1^{ND}$ is the root to $-2w_1^3(12c\delta + 5(1+\delta)^2) - w_1^2(48bK_1(1+\delta) - 6c(6(1+\delta)^2 - c(1-11\delta))) + w_1(-72b^2K_1^2 - 6bcK_1(3c - 19(1+\delta)) - 6c^2(7(1+\delta)^2 - c(2-13\delta))) + 72b^2cK_1^2 + 6bc^2K_1(3c - 11(1+\delta)) + c^3(17(1+\delta)^2 - 6c(1-4\delta)) + \sqrt{12c\delta + (1+\delta)^2}(8w_1^3(1+\delta) + w_1^2(24bK_1 + 3c(c - 9(1+\delta))) - w_1(48bcK_1 + 6c^2(c - 5(1+\delta))) + c^2(24bK_1 + c(3c - 10(1+\delta))))) = 0$. $\breve{w}_1^{ND}$ is the root to $\hat{f}_{1,2}^{ND}(w_1) = 0$. $\tilde{w}_1^{ND}$ is the root to $\hat{f}_{1,1}^{ND}(w_1) = 0$. 2) $\hat{K}_{1,3}^{ND}$ is the root to $\hat{f}_2^{ND}(\breve{w}_1^{ND}) = 0$. $\hat{K}_{1,4}^{ND}$ is the root to $\hat{f}_2^{ND}(\tilde{w}_1^{ND}) = 0$. 3) $\hat{f}_{1,1}^{ND}(w_1)$, $\hat{f}_{1,2}^{ND}(w_1)$, $\hat{f}_1^{ND}(w_1)$ and $\hat{f}_2^{ND}(w_1)$ are defined in Appendix A.2.

(2) Profit expressions are detailed below.

The retailer's total expected profit:

$$-w_1 q_1 + \begin{cases} q_1 - bq_1^2, & q_1 < \frac{1-\delta}{2b} \\ \frac{8b^3q_1^3 - 12b^2q_1^2(1+\delta) + 6bq_1(1+\delta)^2 - (1-\delta)^3}{24b\delta}, & \frac{1-\delta}{2b} \le q_1 < \frac{1+\delta}{2b} \\ \frac{3+\delta^2}{12b}, & q_1 \ge \frac{1+\delta}{2b} \end{cases} + E\left[\hat{\pi}_{r,2}^{ND}\right]. \quad \text{(A.9)}$$

We have $E\left[\hat{\pi}_{r,2}^{ND}\right] = \begin{cases} \frac{3(1-c)^2+\delta^2}{48b}, & c \le 1-\delta \wedge K_2 \ge \frac{((1+\delta)-c)c}{4b} \\ \frac{-128b^3K_2^3 + 48b^2cK_2^2(1+\delta-c) - c^3(1-\delta-c)^3}{96bc^3\delta}, & c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le K_2 < \frac{((1+\delta)-c)c}{4b} \\ \frac{bK_2^2}{c^2}, & c \le 1-\delta \wedge K_2 < \frac{((1-\delta)-c)c}{4b} \\ \frac{bK_2^2(3c(1+\delta-c) - 8bK_2)}{6c^3\delta}, & c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le K_2 < \frac{((1+\delta)-c)c}{4b} \\ \frac{(1+\delta-c)^3}{96b\delta}, & c > 1-\delta \wedge K_2 \ge \frac{((1+\delta)-c)c}{4b} \end{cases}.$

The supplier's total expected profit:

$$(w_1 - c)q_1 + E\left[\hat{\pi}_{s,2}^{ND}\right]. \quad \text{(A.10)}$$

$$E\left[\hat{\pi}_{s,2}^{ND}\right] = \begin{cases} \frac{3(1-c)^2+\delta^2}{24b}, & c \le 1-\delta \wedge K_2 \ge \frac{((1+\delta)-c)c}{4b} \\ \frac{64b^3K_2^3 - 48b^2cK_2^2(1+\delta-c) + 12bc^2K_2(1+\delta-c)^2 - c^3(1-\delta-c)^3}{48bc^3\delta}, & c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le K_2 < \frac{((1+\delta)-c)c}{4b} \\ \frac{K_2(c(1-c) - 2bK_2)}{c^2}, & c \le 1-\delta \wedge K_2 < \frac{((1-\delta)-c)c}{4b} \\ \frac{K_2(16b^2K_2^2 - 12bcK_2(1+\delta-c) + 3c^2(1+\delta-c)^2)}{12c^3\delta}, & c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le K_2 < \frac{((1+\delta)-c)c}{4b} \\ \frac{(1+\delta-c)^3}{48b\delta}, & c > 1-\delta \wedge K_2 \ge \frac{((1+\delta)-c)c}{4b} \end{cases}.$$

By substituting the equilibrium wholesale price and purchase quantity in period 1, as detailed in Table A.7, into (A.9) and (A.10), we can derive equilibrium profits.

Proof of Proposition 2.9. The results in Proposition 2.9 follow directly from the equilibrium outcomes in scenario $\widehat{ND}$. Specifically, when $K_1 > \hat{K}_1^T$, the equilibrium outcomes in scenarios $\widehat{ND}$ and $M|ND$ are the same; when $K_1 \le \hat{K}_1^T$, we have $\hat{q}_1^{ND}(\hat{w}_1^{ND}) > \hat{q}_1^{M}(\hat{w}_1^{ND})$. Algebraic calculation confirms that $\hat{K}_1^T > \underline{K_1^M}$ and $\frac{d\hat{K}_1^T}{d\delta} \ge 0$. □

Proof of Proposition 2.10. We derive profits based on equilibrium outcomes and profit expressions in scenarios $\widehat{ND}$ and $M|ND$. Then we have $\hat{\pi}_s^{ND} \ge \hat{\pi}_s^{M|ND}$ and $\hat{\pi}_{sc}^{ND} \ge \hat{\pi}_{sc}^{M|ND}$. $\hat{\pi}_r^{ND} \ge \hat{\pi}_r^{M|ND}$ if $(c, K_1) \in \widehat{ND}5$ and: i) $c \le 1-\delta \wedge \frac{K_1(c-2bK_1)}{c^2} < \frac{((1-\delta)-c)c}{4b} \wedge \hat{f}_3^{ND}(\dot{w}_1^{ND}) \ge 0$; ii) $\frac{((1-\delta)-c)c}{4b} \le \frac{K_1(c-2bK_1)}{c^2} < \frac{((1+\delta)-c)c}{4b} \wedge \hat{f}_4^{ND}(\dot{w}_1^{ND}) \ge 0$. We generate area I in Figure 2.8 by aggregating the two cases together. □

Decision sequence under commitment contract: In period 1, the supplier first sets wholesale prices, (w_1, w_2), and then the retailer determines purchase quantities, (q_1, q_2), for both periods. After observing the realized value of a, the retailer sells $s_t = min\left\{q_t, \frac{a}{2b}\right\}$ in period t.

Following the outlined sequence, we first present equilibrium outcomes and profits under commitment contract when demand is uncertain, covering both cases where the retailer is myopic (scenario $M|NC$) and where he is strategic (scenario $\widehat{NC}$).

Equilibrium analysis in scenario $M|NC$. Given (w_1, w_2), the retailer purchases $\hat{q}_t^M(w_t) = \left(min\left\{\hat{q}_t^o(w_t), \frac{K_t}{c}\right\}\right)^+$, where $\hat{q}_t^o(w_t) = \begin{cases} \frac{1-w_t}{2b}, & w_t > \delta \\ \frac{1+\delta-2\sqrt{w_t\delta}}{2b}, & w_t \leq \delta \end{cases}$, for period t.

Anticipating subsequent responses, the supplier sets (w_1, w_2) to maximize her total profit over two periods: $\max_{w_1, w_2} (w_1 - c)\hat{q}_1^M(w_1) + (w_2 - c)\hat{q}_2^M(w_2)$. The equilibrium outcomes are presented in Table A.8.

Table A.8. Equilibrium Outcomes in scenario $M|NC$

	Conditions on (c, K_1)	$\widehat{w}_1^{M\|NC}$	$\hat{q}_1^{M\|NC}$	$\widehat{w}_2^{M\|NC}$	$\hat{q}_2^{M\|NC}$
$M1\|NC$	$K_1 > \frac{(1-c)c}{4b} \wedge c > 2\delta - 1$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$M2\|NC$	$c > 2\delta - 1 \wedge \underline{K}_1^M \leq K_1 \leq \frac{(1-c)c}{4b}$	$1 - \frac{2bK_1}{c}$	$\frac{K_1}{c}$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$M3\|NC$	$-4b^2K_1^2 + 2bcK_1 - c^3(1-\delta) \leq 0 \wedge K_1 < \underline{K}_1^M$			$1 - \frac{2b\widehat{K}_2^{M\|NC}}{c}$	$\frac{\widehat{K}_2^{M\|NC}}{c}$
$M4\|NC$	$\frac{(1-\sqrt{1-4c(1-\delta)})c}{4b} \leq K_1 \leq min\left\{\frac{(1-\delta)c}{2b}, \hat{f}_1^{NC}(c,\delta)\right\}$			$\frac{(c(1+\delta)-2b\widehat{K}_2^{M\|NC})^2}{4c^2\delta}$	$\frac{\widehat{K}_2^{M\|NC}}{c}$
$M5\|NC$	$\hat{f}_1^{NC}(c,\delta) < K_1 \leq \frac{(1-\delta)c}{2b} \wedge c \leq 2\delta - 1$			$\widetilde{w}$	$\tilde{q}$
$M6\|NC$	$\frac{(1-\delta)c}{2b} < K_1 \leq \hat{f}_2^{NC}(c,\delta)$	$\frac{(c(1+\delta)-2bK_1)^2}{4c^2\delta}$	$\frac{K_1}{c}$	$\frac{(c(1+\delta)-2b\widehat{K}_2^{M\|NC})^2}{4c^2\delta}$	$\frac{\widehat{K}_2^{B\|C}}{c}$
$M7\|NC$	$max\left\{\frac{(1-\delta)c_s}{2b}, \hat{f}_2^{NC}(c,\delta)\right\} < K_1 < \frac{(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2})c}{6b}$			$\widetilde{w}$	$\tilde{q}$
$M8\|NC$	$\frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c}{6b} \leq K_1 \wedge c \leq 2\delta - 1$	$\widetilde{w}$	$\tilde{q}$	$\widetilde{w}$	$\tilde{q}$

Notes. 1) $\widehat{K}_2^{M|NC} = \frac{\widehat{w}_1^{M|NC}K_1}{c}$. 2) $\widetilde{w} = \frac{\left((1+\delta)+\sqrt{12c\delta+(1+\delta)^2}\right)^2}{36\delta}$, $\tilde{q} = \frac{2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}}{6b}$. 3) $\hat{f}_1^{NC}(c,\delta) = \frac{\left(3-\sqrt{12c\sqrt{12c\delta+(1+\delta)^2}+3(3-8c(1+\delta))}\right)c}{12b}$, $\hat{f}_2^{NC}(c,\delta) < \frac{(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2})c}{6b}$ is the root to $-12b^3K_1^3 + 12b^2cK_1^2(1+\delta) - 3bc^2K_1(1+\delta)^2 + 4c^4\delta(1+\delta) - 2c^4\delta\sqrt{12c\delta+(1+\delta)^2} = 0$.

Equilibrium outcomes and profits in scenario $\widehat{NC}$.

(1) When demand is uncertain, the wholesale prices set by the supplier and purchase quantities set by the retailer, $(\widehat{w}_1^{NC}, \widehat{w}_2^{NC})$ and $(\hat{q}_1^{NC}, \hat{q}_2^{NC})$, are detailed in Table A.9.

Table A.9. Equilibrium Outcomes in scenario $\widehat{NC}$

	Conditions on (c, K_1)	$\widehat{w}_1^{NC}$	$\widehat{q}_1^{NC}$	$\widehat{w}_2^{NC}$	$\widehat{q}_2^{NC}$
$\widehat{NC1}$	$K_1 > \frac{(1-c)c}{4b} \wedge c > 2\delta - 1$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$\widehat{NC2}$	$c > 2\delta - 1 \wedge \underline{K}_1^M \le K_1 \le \frac{(1-c)c}{4b}$	$1 - \frac{2bK_1}{c}$	$\frac{K_1}{c}$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$
$\widehat{NC3}$	$-4b^2K_1^2 + 2bcK_1 - c^3(1-\delta) \le 0 \wedge K_1 < \underline{K}_1^M \wedge K_1 \ge \widehat{K}_{1,1}^{NC}$			$1 - \frac{2b\widehat{K}_2^{NC}}{c}$	$\frac{\widehat{K}_2^{NC}}{c}$
$\widehat{NC4}$	$\frac{(1-\sqrt{1-4c(1-\delta)})c}{4b} \le K_1 \le min\left\{\frac{(1-\delta)c}{2b}, \hat{f}_1^{NC}(c,\delta)\right\} \wedge K_1 \ge \widehat{K}_{1,1}^{NC}$			$\frac{(c(1+\delta)-2b\widehat{K}_2^{NC})^2}{4c^2\delta}$	$\frac{\widehat{K}_2^{NC}}{c}$
$\widehat{NC5}$	$\hat{f}_1^{NC}(c,\delta) < K_1 \le \frac{(1-\delta)c}{2b} \wedge c \le 2\delta - 1$			$\widetilde{w}$	$\tilde{q}$
$\widehat{NC6}$	$\frac{(1-\delta)c}{2b} < K_1 \le \hat{f}_2^{NC}(c,\delta) \wedge K_1 \ge \widehat{K}_{1,1}^{NC}$	$\frac{(c(1+\delta)-2bK_1)^2}{4c^2\delta}$	$\frac{K_1}{c}$	$\frac{(c(1+\delta)-2b\widehat{K}_2^{NC})^2}{4c^2\delta}$	$\frac{\widehat{K}_2^{NC}}{c}$
$\widehat{NC7}$	$max\left\{\frac{(1-\delta)c}{2b}, \hat{f}_2^{NC}(c,\delta)\right\} < K_1 < \frac{(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2})c}{6b}$			$\widetilde{w}$	$\tilde{q}$
$\widehat{NC8}$	$\frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c}{6b} \le K_1 \wedge c \le 2\delta - 1$	$\widetilde{w}$	$\tilde{q}$	$\widetilde{w}$	$\tilde{q}$
$\widehat{NC9}$	$\frac{(1-\delta)c}{2b} < K_1 < \widehat{K}_{1,1}^{NC}$	$\dot{w}_1^{NC}$	$\frac{K_1}{c}$	$\dot{w}_2^{NC}$	$\frac{\widehat{K}_2^{NC}}{c}$
$\widehat{NC10}$	$K_1 \le min\left\{\frac{(1-\delta)c}{2b}, \widehat{K}_{1,1}^{NC}\right\} \wedge \ddot{w}_1^{NC} \ge \frac{c^2(1-\delta)}{2bK_1}$	$\ddot{w}_1^{NC}$	$\frac{K_1}{c}$	$\ddot{w}_2^{NC}$	$\frac{\widehat{K}_2^{NC}}{c}$
$\widehat{NC11}$	$K_1 < \widehat{K}_{1,1}^{NC} \wedge \ddot{w}_1^{NC} < \frac{c^2(1-\delta)}{2bK_1}$	$\widetilde{w}_1^{NC}$	$\frac{K_1}{c}$	$\widetilde{w}_2^{NC}$	$\frac{\widehat{K}_2^{NC}}{c}$

Notes. 1) $\widehat{K}_2^{NC} = \frac{\widehat{w}_1^{NC} K_1}{c}$. 2) $\dot{w}_1^{NC}$ is the root to $12b^2K_1^2w_1^4 - 8bcK_1(c(1+\delta) + 3bK_1)w_1^3 + w_1^2(4bK_1(4c(1+\delta) + 3bK_1) + c^2(1+\delta)^2 - 4c^3\delta)c^2 + 2w_1(4c^2\delta - 4bK_1(1+\delta) - c(1+\delta)^2)c^4 - 4bc^4K_1(bK_1 - c(1+\delta)) = 0$, and $\dot{w}_2^{NC} = \frac{4bK_1(bK_1\dot{w}_1^{NC^3} - c\dot{w}_1^{NC^2}(c(1+\delta)+bK_1)+c^3\dot{w}_1^{NC}(1+\delta)-c^3(c(1+\delta)-bK_1))+c^4\dot{w}_1^{NC}(1+2(1-2c)\delta+\delta^2)}{4c^4(\dot{w}_1^{NC}-c)\delta}$. $\ddot{w}_1^{NC}$ is the root to $12b^2K_1^2w_1^4 - 8bcK_1(c(1+\delta) + 3bK_1)w_1^3 + w_1^2\left(4bK_1(4c(1+\delta) + 3bK_1) + c^2((1+\delta)^2 - 4c\delta)\right)c^2 + 2w_1\left(4c^2\delta - c(1+\delta)^2 - 4bK_1(1+\delta)\right)c^4 + (c(1-\delta)^2 + 8bK_1\delta)c^5 = 0$, and $\ddot{w}_2^{NC} = \frac{4b^2K_1^2\ddot{w}_1^{NC^3} - 4bcK_1(c(1+\delta)+bK_1)\ddot{w}_1^{NC^2} + \ddot{w}_1^{NC}(4bK_1(1+\delta)-4c^2\delta+c(1+\delta)^2)c^3 - c^4(c(1-\delta)^2+8bK_1\delta)}{4c^4(\ddot{w}_1^{NC}-c)\delta}$.

(2) Profit expressions are detailed below.

The retailer's total expected profit:

$$\sum_{t=1}^{2} -w_t q_t + \begin{cases} q_t - bq_t^2, & q_t < \frac{1-\delta}{2b} \\ \frac{8b^3q_t^3 - 12b^2q_t^2(1+\delta) + 6bq_t(1+\delta)^2 - (1-\delta)^3}{24b\delta}, & \frac{1-\delta}{2b} \le q_t < \frac{1+\delta}{2b}. \\ \frac{3+\delta^2}{12b}, & q_t \ge \frac{1+\delta}{2b} \end{cases} \quad \text{(A.11)}$$

The supplier's total profit:

$$(w_1 - c)q_1 + (w_2 - c)q_2. \quad \text{(A.12)}$$

By substituting the equilibrium outcomes, as detailed in Table A.9, into (A.11) and (A.12), we can derive equilibrium profits.

Proof of Proposition 2.11: (1) The results in Proposition 2.11 follow directly from the equilibrium outcomes in scenario $\widehat{NC}$. Specifically, when $K_1 > \widehat{K}_{1,1}^{NC}$, the equilibrium outcomes in scenarios $\widehat{NC}$

and $M|NC$ are the same; when $K_1 \leq \widehat{K}_{1,1}^{NC}$, we have $\hat{q}_1^{NC}(\widehat{w}_1^{NC}, \widehat{w}_2^{NC}) = \frac{K_1}{c} > \hat{q}_1^{M}(\widehat{w}_1^{NC})$. Algebraic operations confirm that $\widehat{K}_{1,1}^{NC} < \widehat{K}_1^{T}$ and $\frac{d\widehat{K}_{1,1}^{NC}}{d\delta} \geq 0$.

(2) We divide the area where $K_1 \leq \widehat{K}_{1,1}^{NC}$ into four subareas: $M3|NC \cap \widehat{NC10}$, $M3|NC \cap \widehat{NC11}$, $M4|NC \cap \widehat{NC10}$, and $M6|NC \cap \widehat{NC9}$, to compare profits. Profits vary across these subareas. The comparison results show that $\widehat{\pi}_s^{NC} \geq \widehat{\pi}_s^{M|NC}$, $\widehat{\pi}_r^{NC} \geq \widehat{\pi}_r^{M|NC}$ and $\widehat{\pi}_{sc}^{NC} \geq \widehat{\pi}_{sc}^{M|NC}$.

Hence the claim. □

Proof of Proposition 2.12: Referring to Tables A.6 and A.8, we establish key regional correspondences between scenarios $M|ND$ and $M|NC$. Specifically:

- $M1|NC$ aligns with $M1|ND$.
- $(M2|NC) \cup (M3|NC) \cup (M4|NC) \cup (M5|NC)$ aligns with $M2|ND$.
- $M8|NC$ aligns with $M3|ND$.
- $(M6|NC) \cup (M7|NC)$ aligns with $M4|ND$.

Based on these alignments, we then compare profits under scenario $M|ND$ versus scenario $M|NC$ within each corresponding $Mi|NC$ region (for $i = 1, \dots, 8$) to identify the following patterns.

(1) $\widehat{\pi}_s^{M|ND} \geq \widehat{\pi}_s^{M|NC}$ in all cases.

(2) For the comparison of $\widehat{\pi}_r^{M|ND}$ and $\widehat{\pi}_r^{M|NC}$:

In $M1|NC$, $\widehat{\pi}_r^{M|ND} \leq \widehat{\pi}_r^{M|NC}$ iff $\left(c \leq 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \hat{f}_1(K_1) \leq 0\right) \vee \left(c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \hat{f}_2(K_1) \leq 0\right)$, where $\widehat{K}_2^M = \widehat{K}_2^{M|ND} = \widehat{K}_2^{M|NC} = \frac{(1-c)^2+8bK_1}{8b}$.

In $M2|NC$, $\widehat{\pi}_r^{M|ND} \leq \widehat{\pi}_r^{M|NC}$ iff $\left(c \leq 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \hat{f}_3(K_1) \leq 0\right) \vee \left(c > 1-\delta \wedge \hat{f}_4(K_1) \leq 0\right)$, where $\widehat{K}_2^M = \frac{K_1(c-2bK_1)}{c^2}$.

In $M3|NC$, $\widehat{\pi}_r^{M|ND} \leq \widehat{\pi}_r^{M|NC}$.

In $M4|NC$, $\widehat{\pi}_r^{M|ND} < \widehat{\pi}_r^{M|NC}$.

In $M5|NC$, $\widehat{\pi}_r^{M|ND} \leq \widehat{\pi}_r^{M|NC}$ iff $\left(\widehat{K}_2^M \geq \frac{((1+\delta)-c)c}{4b} \wedge c \leq min\{1-\delta, \bar{c}\}\right) \vee \left(c \leq 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \hat{f}_5(K_1) \leq 0\right) \vee \left(c > 1-\delta \wedge \hat{f}_6(K_1) \leq 0\right)$, where $\widehat{K}_2^M = \frac{K_1(c-2bK_1)}{c^2}$.

In $M6|NC$, $\widehat{\pi}_r^{M|ND} < \widehat{\pi}_r^{M|NC}$.

In $M7|NC$, $\widehat{\pi}_r^{M|ND} \leq \widehat{\pi}_r^{M|NC}$ iff $c \leq min\{1-\delta, \bar{c}\} \vee \left(c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \hat{f}_7(K_1) \leq 0\right) \vee \left(c > 1-\delta \wedge \widehat{K}_2^M \geq \frac{((1+\delta)-c)c}{4b} \wedge c \leq \bar{\bar{c}}\right)$, where $\widehat{K}_2^M = \frac{K_1(c(1+\delta)-2bK_1)^2}{4c^3\delta}$.

In $M8|NC$, $\widehat{\pi}_r^{M|ND} \leq \widehat{\pi}_r^{M|NC}$ iff $c \leq min\{1-\delta, \bar{c}\} \vee \left(c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \hat{f}_8(K_1) \leq 0\right) \vee \left(c > 1-\delta \wedge \widehat{K}_2^M \geq \frac{((1+\delta)-c)c}{4b} \wedge c \leq \bar{\bar{c}}\right)$,

where $\widehat{K}_2^M = \frac{216bK_1\delta+2(1+\delta)((1+\delta)^2-36c\delta)+2(12c\delta+(1+\delta)^2)^{3/2}}{216b\delta}$.

(3) For the comparison of $\widehat{\pi}_{sc}^{M|ND}$ and $\widehat{\pi}_{sc}^{M|NC}$:

Note that $\widehat{K}_2^M$ for each case has been detailed when comparing $\widehat{\pi}_r^{M|ND}$ and $\widehat{\pi}_r^{M|NC}$.

In $M1|NC$, $\widehat{\pi}_{sc}^{M|ND} \leq \widehat{\pi}_{sc}^{M|NC}$ iff $c \leq 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \check{f}_1(K_1) \leq 0$.

In $M2|NC$, $\widehat{\pi}_{sc}^{M|ND} \leq \widehat{\pi}_{sc}^{M|NC}$ iff $c \leq 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \check{f}_2(K_1) \leq 0$.

In $M3|NC$, when $c \leq 1-\delta$, $\widehat{\pi}_{sc}^{M|ND} \leq \widehat{\pi}_{sc}^{M|NC}$. Specifically, $\widehat{\pi}_{sc}^{M|ND} = \widehat{\pi}_{sc}^{M|NC}$ if $\widehat{K}_2^M = \frac{K_1(c-2bK_1)}{c^2} < \frac{((1-\delta)-c)c}{4b}$, and $\widehat{\pi}_{sc}^{M|ND} \leq \widehat{\pi}_{sc}^{M|NC}$ if $\frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}$. When $c > 1-\delta$, $\widehat{\pi}_{sc}^{M|ND} > \widehat{\pi}_{sc}^{M|NC}$.

In $M4|NC$, $\widehat{\pi}_{sc}^{M|ND} > \widehat{\pi}_{sc}^{M|NC}$ iff $c > 1-\delta \wedge \check{f}_3(K_1) > 0$.

In $M5|NC$, $\widehat{\pi}_{sc}^{M|ND} \leq \widehat{\pi}_{sc}^{M|NC}$ iff $\left(c \leq 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \check{f}_4(K_1) \leq 0\right) \vee \left(c > 1-\delta \wedge \check{f}_5(K_1) \leq 0\right)$.

In $M6|NC$, $\widehat{\pi}_{sc}^{M|ND} \leq \widehat{\pi}_{sc}^{M|NC}$ iff $\check{f}_6(K_1) \leq 0$.

In $M7|NC$, $\widehat{\pi}_{sc}^{M|ND} \leq \widehat{\pi}_{sc}^{M|NC}$ iff $c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \leq \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b} \wedge \check{f}_7(K_1) \leq 0$.

In $M7|NC$, $\widehat{\pi}_{sc}^{M|ND} > \widehat{\pi}_{sc}^{M|NC}$.

We construct Figure 2.10 by aggregating regions that share similar characteristics. □

A.2 Definitions

A.2.1 Definitions in Chapter 2

Corollary 2.3. $K_{1,3}^{ND} \in [0, \underline{K_1^M}]$ is the root to $-c^2(1-4c+5c^2)+4b(2-c)cK_1-16b^2K_1^2-2c(1-2c)\sqrt{c^4-4bcK_1+8b^2K_1^2}=0$ if $\sqrt{2}-1 \leq c \leq \frac{22-\sqrt{3}}{37}$, it is the root to $(1-c)c^2(2c-1)-2bc(4c-3)K_1-12b^2K_1^2+(2bK_1-(1-c)c)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}=0$ if $\frac{22-\sqrt{3}}{37} < c \leq 1$.

Figure 2.6. $K_{1,i}^{NC}$ for $i = 2, \ldots, 4$ are defined below.

(1) $K_{1,2}^{NC} \in [0, K_{1,1}^{NC}]$ is the root to $(1-c)^2c^3(2c-1)(2c-\widetilde{w}_1^{NC})\widetilde{w}_1^{NC} - 16b^3K_1^3\left(c^2+2c\widetilde{w}_1^{NC}+\right.$

$4\widetilde{w}_1^{NC^2}\Big) + 8b^2cK_1^2\Big(c^3 + c^2(1 - 4\widetilde{w}_1^{NC}) + c(5 - 7\widetilde{w}_1^{NC})\widetilde{w}_1^{NC} + 5\widetilde{w}_1^{NC^2}\Big) - 2b(1-c)c^2K_1\Big(6c^3 + 6c(1 - \widetilde{w}_1^{NC})\widetilde{w}_1^{NC} + 2\widetilde{w}_1^{NC^2} - c^2(1 + 4\widetilde{w}_1^{NC})\Big) = 0.$

(2) $K_{1,3}^{NC} \in [0, \underline{K_1^M}]$ is the smallest root to $-(1-c)^2c^3(2c - \widetilde{w}_1^{NC})\widetilde{w}_1^{NC} + 8b^3K_1^3(3c + 10\widetilde{w}_1^{NC}) + 2b(1-c)cK_1\Big(c^3 + c^2(4 - 8\widetilde{w}_1^{NC}) - 5\widetilde{w}_1^{NC^2} + c\widetilde{w}_1^{NC}(10 + \widetilde{w}_1^{NC})\Big) + 4b^2K_1^2\Big(3c^3 + 5\widetilde{w}_1^{NC^2} - c\widetilde{w}_1^{NC}(20 + \widetilde{w}_1^{NC}) + c^2(18\widetilde{w}_1^{NC} - 7)\Big) = 0.$

(3) $K_{1,4}^{NC} \in [0, \underline{K_1^M}]$ is the smallest root to $c^2((1-c)^2c^2 + 3b(1-c)cK_1 - 8b^2K_1^2) - 2c((1-c)^2c^2 + 3b(1-c)cK_1 - 9b^2K_1^2)\widetilde{w}_1^{NC} + 10bK_1((1-c)c - 2bK_1)\widetilde{w}_1^{NC^2} = 0.$

Figure 2.7. $K_{1,i}^N$ for $i \in \{1, \ldots, 4\}$ are defined below.

(1) $K_{1,1}^N \in [0, \underline{K_1^M}]$ is the root to $-c^2(c(1+c) - 4bK_1) + c(c(3c-1) - 4bK_1)\widetilde{w}_1^{NC} + 4bK_1\widetilde{w}_1^{NC^2} = 0.$

(2) $K_{1,2}^N \in [0, \underline{K_1^M}]$ is the root to $c^3 - c\sqrt{c^4 - 4bcK_1 + 8b^2K_1^2} - 2bK_1\widetilde{w}_1^{NC} = 0.$

(3) $K_{1,3}^N \in [0, K_{1,1}^{NC}]$; in $ND4 \cap NC4$, $K_{1,3}^N$ is the root to $c^3(c^3(2c-1) - 4bcK_1 + 8b^2K_1^2) - c^2(c^3(2c-1) - 4bcK_1 + 4b^2K_1^2)\widetilde{w}_1^{NC} - 2bcK_1(c(1-c) + 2bK_1)\widetilde{w}_1^{NC^2} + 4b^2K_1^2\widetilde{w}_1^{NC^3} + \sqrt{c^4 - 4bcK_1 + 8b^2K_1^2}(\widetilde{w}_1^{NC} - c)(2c-1)c^3 = 0$, and in $ND5 \cap NC4$, $K_{1,3}^N$ is the root to $c^3(-(1-c)c^3 + 2bc(2c-3)K_1 + 12b^2K_1^2) + c^2\Big((1-c)c^3 - 2bc(2c-3)K_1 - 4b^2K_1^2\Big)\widetilde{w}_1^{NC} - 4bcK_1((1-c)c + 2bK_1)\widetilde{w}_1^{NC^2} + 8b^2K_1^2\widetilde{w}_1^{NC^3} + \sqrt{c^4 - 4b(2-c)cK_1 + 20b^2K_1^2}\Big((1-c)c^4 - 2bc^3K_1 - c^2((1-c)c - 2bK_1)\widetilde{w}_1^{NC}\Big) = 0.$

(4) $K_{1,4}^N \in [0, \underline{K_1^M}]$ is the root to $-(1-c)(1-5c)c^5 + c^2((1-c)(1-5c)c^2 + 16bK_1(c(1-c) - 2bK_1))\widetilde{w}_1^{NC} - 16b^2K_1^2(\widetilde{w}_1^{NC} - c)\widetilde{w}_1^{NC^2} = 0.$

Proposition 2.9.
$$\hat{K}_1^T = \begin{cases} \frac{(1-c)(3c-1)+2c\delta}{8b}, & K_1 > \frac{(1-c)c}{4b} \wedge c > 2\delta - 1 \\ \frac{\left(1-\sqrt{1+2c^2-2c(1+\delta)}\right)c}{4b}, & K_1 \le min\left\{\frac{(1-c)c}{4b}, \frac{(1-\delta)c}{2b}\right\} \\ \hat{f}_1^{ND}(c,\delta), & \frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c}{6b} \le K_1 \wedge c \le 2\delta - 1 \\ \hat{f}_2^{ND}(c,\delta), & \frac{(1-\delta)c}{2b} < K_1 < \frac{\left(2(1+\delta)-\sqrt{12c\delta+(1+\delta)^2}\right)c}{6b} \end{cases},$$

where $\hat{f}_1^{ND}(c,\delta) = \frac{-1-3(1-21c+9c^2)\delta-3(1-21c)\delta^2-\delta^3-(12c\delta+(1+\delta)^2)^{3/2}}{108b\delta}$ and $\hat{f}_2^{ND}(c,\delta)$ is the root to $4b^3K_1^3 - 4b^2cK_1^2(1+\delta) + bc^2K_1(1+\delta)^2 - c^4(1+\delta-c)\delta = 0.$

Figure 2.8. The boundary of area I is define as $\hat{f}_3^{ND}(\dot{w}_1^{ND})=0$ if $K_1<\frac{(1-\sqrt{1+2c^2-2c(1-\delta)})c}{4b}$, and as $\hat{f}_4^{ND}(\dot{w}_1^{ND})=0$ if $K_1\geq\frac{(1-\sqrt{1+2c^2-2c(1-\delta)})c}{4b}$.

■ $\hat{f}_3^{ND}(w_1)$ and $\hat{f}_4^{ND}(w_1)$ are defined below.

$\hat{f}_3^{ND}(w_1)=-128b^3K_1^3w_1^3+48b^2c^2K_1^2(1+\delta-c)w_1^2-96bc^5K_1\delta w_1+96bK_1(c-2bK_1)(c^4-bK_1(c-2bK_1))\delta-c^6(1-\delta-c)^3.$

$\hat{f}_4^{ND}(w_1)=-8b^2c^2K_1^2w_1^2+bcK_1(3c^3(1+\delta-c)-8bK_1(c-2bK_1))w_1+32b^3K_1^3(c-bK_1)-2b^2c^2K_1^2(4-3c^2+3c(1+\delta))+3bc^4K_1(1+\delta-c)-6c^7\delta.$

Proposition 2.11. $\widehat{K}_{1,1}^{NC}$ is defined below.

When $\delta\leq 0.5$, $\widehat{K}_{1,1}^{NC}=K_{1,1}^{NC}$. When $\delta>0.5$, $\widehat{K}_{1,1}^{NC}>K_{1,1}^{NC}$, and:

(1) If $-4b^2K_1^2+2bcK_1-c^3(1-\delta)\leq 0\wedge K_1<\underline{K_1^M}$, $\widehat{K}_{1,1}^{NC}$ is the root to $4b^2K_1^2\ddot{w}_1^{NC^4}-4bcK_1(c+bK_1+c\delta)\ddot{w}_1^{NC^3}+c^3\big(4bK_1(1+\delta)+c((1+\delta)^2-4c\delta)\big)\ddot{w}_1^{NC^2}+\ddot{w}_1^{NC}\Big(32b^3K_1^3\delta-32b^2cK_1^2\delta+8bc^2(1+c-c^2)K_1\delta-c^4(c(1+\delta^2)+2(2-c)\delta)\Big)+4c(c-2bK_1)(c^3-2bcK_1+4b^2K_1^2)\delta=0$ when $\ddot{w}_1^{NC}\geq\frac{c^2(1-\delta)}{2bK_1}$, and the root to $2bcK_1\widetilde{w}_1^{NC^2}-\big(c^3(1-c)-2bK_1(c(1-c)-2bK_1)\big)\widetilde{w}_1^{NC}+c(c^3-2bcK_1+4b^2K_1^2)=0$ otherwise.

(2) If $\frac{(1-\sqrt{1-4c(1-\delta)})c}{4b}\leq K_1\leq min\left\{\frac{(1-\delta)c}{2b},\hat{f}_1^{NC}(c,\delta)\right\}$, $\widehat{K}_{1,1}^{NC}$ is the root to $4b^2c^2K_1^2\ddot{w}_1^{NC^3}+4bcK_1\Big(b(1-c)cK_1-2b^2K_1^2-c^3(1+\delta)\Big)\ddot{w}_1^{NC^2}+\ddot{w}_1^{NC}\Big(16b^4K_1^4-8b^3(2-c)cK_1^3+4b^2c^2K_1^2(1+c+2c\delta)-4b(1-c)c^4K_1(1+\delta)-c^6(4c\delta-(1+\delta)^2)\Big)-c\big(c^3(1+\delta)-2bK_1(c-2bK_1)\big)^2=0.$

(3) If $\frac{(1-\delta)c}{2b}<K_1\leq\hat{f}_2^{NC}(c,\delta)$, $\widehat{K}_{1,1}^{NC}$ is the root to $-16b^2c^4K_1^2\dot{w}_1^{NC^3}\delta^2-4bc^2K_1\delta\Big(4b^3K_1^3-4b^2cK_1^2(1+\delta)-bc^2K_1(-1-2\delta+4c\delta-\delta^2)-4c^4\delta(1+\delta)\Big)\dot{w}_1^{NC^2}+\dot{w}_1^{NC}\Big(-16b^6K_1^6+32b^5cK_1^5(1+\delta)+8b^3c^3K_1^3(1+\delta)^3+8b^4c^2K_1^4(-3-6\delta+2c\delta-3\delta^2)+4c^8\delta^2(-1-2\delta+4c\delta-\delta^2)-4bc^6K_1\delta(1+\delta)(-1-2\delta+4c\delta-\delta^2)-b^2c^4K_1^2(1+\delta)^2(1+2\delta+12c\delta+\delta^2)\Big)+c\Big(4b^3K_1^3-4b^2cK_1^2(1+\delta)+bc^2K_1(1+\delta)^2-2c^4\delta(1+\delta)\Big)^2=0.$

Figure 2.10. $\hat{c}_1(K_1)$, $\hat{c}_2$, $\widehat{K}_{1,1}$, and $\widehat{K}_{1,2}$ are defined below.

(1) If $\delta\leq\frac{2}{3}$, $\hat{c}_1(K_1)=1-\delta$.

If $\delta > \frac{2}{3}$, $\hat{c}_1(K_1) = 1-\delta$ when $K_1 \le \frac{(1-\sqrt{1-4(1-\delta)^2})(1-\delta)}{4b}$, and $\hat{c}_1(K_1)$ is the inverse function of $\widehat{K}_{1,3}$ when $K_1 > \frac{(1-\sqrt{1-4(1-\delta)^2})(1-\delta)}{4b}$, where $\widehat{K}_{1,3}$ is the root to $0 = \begin{cases} \check{f}_3(K_1), & B_4|C \wedge c_s > 1-\delta \\ \check{f}_6(K_1), & B_6|C \end{cases}$.

(2) $\hat{c}_2 = \begin{cases} 0, & \delta \le \dot{\delta} \\ \bar{c}, & \dot{\delta} < \delta \le \bar{\delta} \\ \bar{\bar{c}}, & \delta > \bar{\delta} \end{cases}$. $\dot{\delta} \approx 0.624$ is the root to $40 - 123\delta + 120\delta^2 - 41\delta^3 = 0$; $\bar{\delta} \approx 0.965$ is the root to $5 - 48\delta + 15\delta^2 - 13\delta^3 + \sqrt{1+14\delta-11\delta^2}(5 - 2\delta + 17\delta^2) = 0$; $\bar{c}$ is the root to $20 - 183\delta + 81c^2\delta + 60\delta^2 - 61\delta^3 - 18c\delta(5-4\delta) + \sqrt{12c\delta + (1+\delta)^2}(20(1+\delta)^2 - 48c\delta) = 0$; $\bar{\bar{c}}$ is the root to $67 - 447\delta + 201\delta^2 - 149\delta^3 - 9c\big((1+\delta)(9-7\delta) - 9c(1+\delta) + 3c^2\big) + 8(5(1+\delta)^2 - 12c\delta)\sqrt{12c\delta + (1+\delta)^2} = 0$.

(3) $\widehat{K}_{1,1}$ is the root to $\begin{cases} \hat{f}_1(K_1), & M1|NC \wedge \left(c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \\ \hat{f}_2(K_1), & M1|NC \wedge \left(c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \\ \hat{f}_3(K_1), & M2|NC \wedge \left(c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \\ \hat{f}_4(K_1), & M2|NC \wedge (c > 1-\delta) \\ \hat{f}_5(K_1), & M5|NC \wedge \left(c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \\ \hat{f}_6(K_1), & M5|NC \wedge (c > 1-\delta) \\ \hat{f}_7(K_1), & M7|NC \wedge \left(c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \\ \hat{f}_8(K_1), & M8|NC \wedge \left(c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \end{cases} = 0$.

(4) $\widehat{K}_{1,2}$ is the root to $\begin{cases} \check{f}_1(K_1), & M1|NC \wedge \left(c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \\ \check{f}_2(K_1), & M2|NC \wedge \left(c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \\ \check{f}_4(K_1), & M5|NC \wedge \left(c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \\ \check{f}_5(K_1), & M5|NC \wedge (c > 1-\delta) \\ \check{f}_7(K_1), & M7|NC \wedge \left(c > 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} \le \widehat{K}_2^M < \frac{((1+\delta)-c)c}{4b}\right) \end{cases} = 0$.

$\widehat{K}_2^B$ for each area is detailed in proof of Proposition 2.12.

■ $\hat{f}_i(K_1)$ for $i = 1, \ldots, 8$ are defined below.

$\hat{f}_1(K_1) = -512b^3K_1^3 - 192b^2(1 - 3c + 2c^2 - c\delta)K_1^2 - 24b(1-c)^2(1 - 4c + 3c^2 - 2c\delta)K_1 + 9c^5(1-\delta) + 3c^4(4\delta(1+\delta) - 11) + 2c^3(23 + 3\delta - 6\delta^2 + 2\delta^3) - 6c^2(5 + 2\delta) + 3c(3+\delta) - 1$.

$\hat{f}_2(K_1) = -512b^3K_1^3 - 192b^2K_1^2(1 - 3c + 2c^2 - c\delta) - 24b(1-c)^2K_1(1 - 4c + 3c^2 - 2c\delta) - (1-c)^2((1-c)^3(1-4c) + 3c(-1 + 2c + 7c^2)\delta)$.

$\hat{f}_3(K_1) = 1024b^6K_1^6 - 1536b^5cK_1^5 + 192b^4c^2K_1^4(4 + c(1 + \delta - c)) - 64b^3c^3K_1^3(2 + 3c(1 + \delta - c)) + 48b^2c^5K_1^2(1 + \delta - c) - c^9((1-c)^3 + 3(1-c)^2\delta + 3(1-c)\delta^2 - \delta^3)$.

$\hat{f}_4(K_1) = 512b^6K_1^6 - 768b^5cK_1^5 + 96b^4c^2K_1^4(4 + c(1 + \delta - c)) - 32b^3c^3K_1^3(2 + 3c(1 + \delta - c)) + 24b^2c^5K_1^2(1 + \delta - c) - 3(1 - c)^2c^9\delta.$

$\hat{f}_5(K_1) = 27648b^6K_1^6 - 41472b^5cK_1^5 + 5184b^4c^2K_1^4(4 + c(1 + \delta - c)) - 1728b^3c^3K_1^3(2 + 3c(1 + \delta - c)) + 1296b^2c^5K_1^2(1 + \delta - c) + c^9(13 - 447\delta + 39\delta^2 - 149\delta^3 + 9c(9 - 2\delta + 25\delta^2) - 81c^2(1 - \delta) + 27c^3) - 8c^9(12c\delta - 5(1 + \delta)^2)\sqrt{12c\delta + (1 + \delta)^2}.$

$\hat{f}_6(K_1) = 3456b^6K_1^6 - 5184b^5cK_1^5 + 648b^4c^2K_1^4(4 + c(1 + \delta - c)) - 216b^3c^3K_1^3(2 + 3c(1 + \delta - c)) + 162b^2c^5K_1^2(1 + \delta - c) + c^9(5 - 66\delta + 18c\delta + 15\delta^2 + 18c\delta^2 - 22\delta^3) - c^9(12c\delta - 5(1 + \delta)^2)\sqrt{12c\delta + (1 + \delta)^2}.$

$\hat{f}_7(K_1) = -3456b^9K_1^9 + 10368b^8cK_1^8(1 + \delta) - 12960b^7c^2K_1^7(1 + \delta)^2 - 432b^6c^3K_1^6(3c^2\delta - 3c\delta(1 + \delta) - 20(1 + \delta)^3) + 648b^5c^4K_1^5(1 + \delta)(4c^2\delta - 4c\delta(1 + \delta) - 5(1 + \delta)^3) - 648b^4c^5K_1^4(1 + \delta)^2(3c^2\delta - 3c\delta(1 + \delta) - (1 + \delta)^3) + 54b^3c^6K_1^3(1 + \delta)^3(12c^2\delta - 12c\delta(1 + \delta) - (1 + \delta)^3) + 81b^2c^8K_1^2(1 + \delta - c)\delta(1 + \delta)^4 + 8c^{12}\delta^3(5 - 66\delta + 15\delta^2 - 22\delta^3 + 18c\delta(1 + \delta)) - 8c^{12}\delta^3(12c\delta - 5(1 + \delta)^2)\sqrt{12c\delta + (1 + \delta)^2}.$

$\hat{f}_8(K_1) = -10077696b^3K_1^3\delta^3 - 139968b^2K_1^2\delta^2(27c^2\delta - 99c\delta(1 + \delta) + 2(1 + \delta)^3) - 2592bK_1\delta(-63c\delta(1 + \delta)^4 + 2(1 + \delta)^6 - 108c^3\delta^2(9 - 7\delta) + 27c^2\delta(1 + \delta)^2(1 + 101\delta)) - 8(69984c^5\delta^4 - 189c\delta(1 + \delta)^7 + 4(1 + \delta)^9 + 81c^2\delta(1 + \delta)^5(1 + 35\delta) + 5832c^4\delta^3(1 + \delta)(12 - 41\delta) - 162c^3\delta^2(9 + 1102\delta + 1794\delta^2 + 3234\delta^3 + 589\delta^4)) + \sqrt{12c\delta + (1 + \delta)^2}(-279936b^2K_1^2\delta^2(12c\delta + (1 + \delta)^2) - 2592bK_1\delta(12c\delta + (1 + \delta)^2)(27c^2\delta - 99c\delta(1 + \delta) + 2(1 + \delta)^3) - 8(-213c\delta(1 + \delta)^6 + 4(1 + \delta)^8 + 1296c^4\delta^3(-27 + 16\delta) + 27c^2\delta(1 + \delta)^4(3 + 155\delta) + 108c^3\delta^2(1 + \delta)^2(-18 + 667\delta))).$

- $\check{f}_i(K_1)$ for $i = 1, \ldots, 7$ are defined below.

$\check{f}_1(K_1) = -64b^2K_1^2(1 + \delta - c) - 16bK_1(1 + \delta - c)(1 + 3c^2 - 2c(2 + \delta)) - (1 - c)^3(1 - 3c)^2 - (1 - c)^2(1 - 3c)(1 - 7c)\delta + 4(1 - c)c(1 - 4c)\delta^2 + 4c^2\delta^3.$

$\check{f}_2(K_1) = -64b^4K_1^4(1 + \delta - c) + 64b^3cK_1^3(1 + \delta - c) - 16b^2c^2K_1^2(1 + \delta - c)(1 - c^2 + c(1 + \delta)) + 8bc^4K_1(1 + \delta - c)^2 - c^6((1 - c)^3 + 3(1 - c)^2\delta + 3(1 - c)\delta^2 - \delta^3).$

$\check{f}_3(K_1) = 64b^6K_1^6 - 96b^5cK_1^5 + 48b^4c^2(1 + c^2)K_1^4 - 8b^3c^3(1 + 6c^2)K_1^3 + 12b^2c^6K_1^2(1 - c^2 + 2c(1 - \delta)) + 6bc^8K_1(c - 2(1 - \delta)) + c^9(1 - \delta)^3.$

$\check{f}_4(K_1) = -5184b^4K_1^4(1 + \delta - c) + 5184b^3cK_1^3(1 + \delta - c) - 1296b^2c^2K_1^2(1 + \delta - c)(1 - c^2 + c(1 + \delta)) + 648bc^4K_1(1 + \delta - c)^2 + c^6(81c^3 - 243c^2(1 - \delta) + 9c(27 + 58\delta + 139\delta^2) - 65 - 357\delta - 195\delta^2 - 119\delta^3) - 16c^6(24c\delta - (1 + \delta)^2)\sqrt{12c\delta + (1 + \delta)^2}.$

$\check{f}_5(K_1) = -648b^4K_1^4(1 + \delta - c) + 648b^3cK_1^3(1 + \delta - c) - 162b^2c^2K_1^2(1 + \delta - c)(1 - c^2 + c(1 + \delta)) + 81bc^4K_1(1 + \delta - c)^2 + c^6(2 - 3(25 - 42c)\delta + 6(1 + 21c)\delta^2 - 25\delta^3) - 2c^6(24c\delta - (1 +$

$\delta)^2)\sqrt{12c\delta + (1+\delta)^2}$.

$\check{f}_6(K_1) = -64b^9K_1^9 + 192b^8cK_1^8(1+\delta) - 240b^7c^2K_1^7(1+\delta)^2 + 32b^6c^3K_1^6(3c^2\delta + 5(1+\delta)^3) - 12b^5c^4K_1^5(1+\delta)(16c^2\delta + 5(1+\delta)^3) + 12b^4c^5K_1^4(1+\delta)^2(12c^2\delta + (1+\delta)^3) + b^3c^6K_1^3(48c^4\delta^2 - 96c^3(1-\delta)\delta^2 - 48c^2\delta(1+\delta)^3 - (1+\delta)^6) - 6b^2c^9K_1^2\delta(1+\delta)(8c^2\delta - 16c(1-\delta)\delta - (1+\delta)^3) + 12bc^{11}K_1\delta^2(1+\delta)^2(c - 2(1-\delta)) + 8c^{12}(1-\delta)^3\delta^3$.

$\check{f}_7(K_1) = -1296b^6K_1^6(1+\delta-c) + 2592b^5cK_1^5(1+\delta-c)(1+\delta) - 1944b^4c^2K_1^4(1+\delta-c)(1+\delta)^2 - 648b^3c^3K_1^3(1+\delta-c)(c^2\delta - c\delta(1+\delta) - (1+\delta)^3) + 81b^2c^4K_1^2(1+\delta-c)(1+\delta)(8c^2\delta - 8c\delta(1+\delta) - (1+\delta)^3) + 162bc^6K_1(1+\delta-c)^2\delta(1+\delta)^2 + 8c^8\delta^2(2 - 75\delta + 6\delta^2 - 25\delta^3 + 126c\delta(1+\delta)) - 16c^8\delta^2(24c\delta - (1+\delta)^2)\sqrt{12c\delta + (1+\delta)^2}$.

A.2.2 Definitions in Appendix A.1

Table A.7. $\hat{f}_{1,1}^{ND}(w_1)$, $\hat{f}_{1,2}^{ND}(w_1)$, $\hat{f}_1^{ND}(w_1)$ and $\hat{f}_2^{ND}(w_1)$ are defined below.

$\hat{f}_{1,1}^{ND}(w_1) = -128b^3K_1^3w_1^3 + 48b^2c^2K_1^2w_1^2(1+\delta-c) - 96bc^5K_1w_1\delta + c^4(96bcK_1\delta - 192b^2K_1^2\delta - c^2(1-\delta-c)^3)$.

$\hat{f}_{1,2}^{ND}(w_1) = -8b^2K_1^2w_1^3 + 3bc^2K_1w_1^2(1+\delta-c) + 8b^2c^3K_1^2 - 3bc^4K_1(1-c+3\delta) + 6c^5(1-w_1)\delta$.

$$\hat{f}_1^{ND}(w_1) = \begin{cases} \hat{f}_{1,1}^{ND}(w_1), & c \le 1-\delta \wedge K_1 \le \frac{((1-\delta)-c)c}{4b} \\ \hat{f}_{1,2}^{ND}(w_1), & (c \le 1-\delta \wedge \frac{((1-\delta)-c)c}{4b} < K_1 \le \frac{((1+\delta)-c)c}{4b}) \vee c > 1-\delta \end{cases}.$$

$\hat{f}_2^{ND}(w_1) = -4b^2K_1^2w_1^3 + bcK_1(4bK_1 + c(1+\delta-c))w_1^2 - c^3(bK_1(1+\delta-c) + c^2\delta)w_1 + c^4(c - 2bK_1)\delta$.

Appendix B

Mathematical Proofs and Definitions for Chapter 3

B.1 Mathematical Proofs

Proof of Proposition 3.1. We solve this sequential game backward. In period 2, unsold products have no salvage value and are scrapped. Thus, $q_2 = (s_2 - I)^+$. Given inventory I and wholesale price w_2, the retailer decides sales quantity $s_2^B(w_2, I)$ to maximize period-2 profit $\pi_{r,2}$. Solving problem (3-2) in the main text excluding the constraint $q_2 \leq \frac{K_2}{c}$, we have:

$$s_2^B(w_2, I) = \begin{cases} \frac{1-w_2}{2b}, & w_2 \leq 1 - 2bI \\ min\left\{I, \frac{1}{2b}\right\}, & w_2 > 1 - 2bI \end{cases}. \tag{B.1}$$

Anticipating $q_2^B(w_2, I) = (s_2^B(w_2, I) - I)^+$, the supplier sets $w_2^B(I)$ to maximize period-2 profit $\pi_{s,2}$ by solving problem (3-5) in the main text. The supplier's optimal wholesale price, the retailer's optimal sales, and purchases in period 2 are:

$$(w_2^B(I), s_2^B(I), q_2^B(I)) = \begin{cases} \left(\frac{1+c-2bI}{2}, \frac{1-c+2bI}{4b}, \frac{1-c-2bI}{4b}\right), & I \leq \frac{1-c}{2b} \\ \left(\infty, min\left\{I, \frac{1}{2b}\right\}, 0\right), & I > \frac{1-c}{2b} \end{cases}. \tag{B.2}$$

The retailer's and supplier's period-2 profits are:

$$(\pi_{r,2}^B(I), \pi_{s,2}^B(I)) = \begin{cases} \left(\frac{(1-c)^2+4b(3+c)I-12b^2I^2}{16b}, \frac{(1-c-2bI)^2}{8b}\right), & I \leq \frac{1-c}{2b} \\ \left(\left(1 - bmin\left\{I, \frac{1}{2b}\right\}\right) min\left\{I, \frac{1}{2b}\right\}, 0\right), & I > \frac{1-c}{2b} \end{cases}. \tag{B.3}$$

In period 1, anticipating decisions in period 2, the retailer sets sales quantity $s_1^B(w_1)$ and inventory level $I^B(w_1)$ to maximizes total profit π_r, given the supplier's wholesale price w_1. Note that when the retailer sets $I > \frac{1-c}{2b}$, $w_2^B(I)$ is then will be ∞. In this case, the retailer's total profit across both periods decreases with I. Thus, in stage 2, any $I > \frac{1-c}{2b}$ is suboptimal, limiting our following analysis to $I \leq \frac{1-c}{2b}$. The retailer's problem in stage 2 is:

$$\max_{s_1 \geq 0, I \geq 0} \pi_r = (1 - bs_1)s_1 - w_1(s_1 + I) - hI + \frac{(1-c)^2+4b(3+c)I2-12b^2I^2}{16b}. \tag{B.4}$$

By solving the problem with the Lagrangian and complementary slackness conditions, we have:

$$(s_1^B(w_1), I^B(w_1)) = \left(\left(\frac{1-w_1}{2b}\right)^+, \left(\frac{3+c-4h-4w_1}{6b}\right)^+\right). \tag{B.5}$$

The supplier sets wholesale price w_1^B to maximize total profit over periods π_s, anticipating the retailer's response $(s_1^B(w_1), I^B(w_1))$ in period 1 and the subsequent decisions in period 2:

$$\max_{w_1} \pi_s = (w_1 - c)(s_1^B(w_1) + I^B(w_1)) + \pi_{s,2}^B(I^B(w_1)). \tag{B.6}$$

The optimal solution to problem (B.6) is $w_1 = \begin{cases} \frac{9+8c-2h}{17}, c \le 1-4h \\ \frac{1+c}{2}, \quad c > 1-4h \end{cases}$. Subsequent decisions can then be determined sequentially. Table B.1 summarizes the equilibrium profits in scenario B.

Table B.1. Equilibrium profits in scenario B

Condition	π_r^B	π_s^B	π_{sc}^B
$c \le 1-4h$	$\frac{155(1-c)^2-118(1-c)h+304h^2}{1156b}$	$\frac{9(1-c)^2-4(1-c)h+8h^2}{34b}$	$\frac{461(1-c)^2-254(1-c)h+576h^2}{1156b}$
$c > 1-4h$	$\frac{(1-c)^2}{8b}$	$\frac{(1-c)^2}{4b}$	$\frac{3(1-c)^2}{8b}$

Proof of Proposition 3.2. The results in Proposition 3.2 follow from the equilibrium outcomes in scenario C, which we derive below.

In period 2, given the capital position K_2, inventory I, and wholesale price w_2, the retailer solves problem (3-2) with $q_2 = (s_2 - I)^+$. The resulting sales quantity in period 2 $s_2^C(w_2, K_2, I)$ is:

$$s_2^C(w_2, K_2, I) = \begin{cases} \frac{K_2}{c} + I, & w_2 \le 1 - 2b(I + \frac{K_2}{c}) \\ \frac{1-w_2}{2b}, & 1 - 2b\left(I + \frac{K_2}{c}\right) < w_2 \le 1 - 2bI. \\ min\left\{I, \frac{1}{2b}\right\}, & w_2 > 1 - 2bI \end{cases} \tag{B.7}$$

Anticipating $q_2^C(w_2, K_2, I)$, the supplier sets $w_2^C(K_2, I)$ to maximize period-2 profit $\pi_{s,2}$. Solving problem (3-5), we obtain the supplier's optimal wholesale price and the retailer's optimal sales and purchase quantities in period 2:

$$(w_2^C(K_2,I), s_2^C(K_2,I), q_2^C(K_2,I)) = \begin{cases} \left(1 - 2b(I + \frac{K_2}{c}), I + \frac{K_2}{c}, \frac{K_2}{c}\right), I \le \frac{1-c}{2b} \wedge I + \frac{K_2}{c} \le \frac{1-c+2bI}{4b} \\ \left(\frac{1+c-2bI}{2}, \frac{1-c+2bI}{4b}, \frac{1-c-2bI}{4b}\right), I \le \frac{1-c}{2b} \wedge I + \frac{K_2}{c} > \frac{1-c+2bI}{4b} \\ \left(\infty, min\left\{I, \frac{1}{2b}\right\}, 0\right), \quad I > \frac{1-c}{2b} \end{cases}. \tag{B.8}$$

We restrict the subsequent analysis to the case $I \le \frac{1-c}{2b}$, consistent with the rationale provided in the proof of Proposition 3.1. Thus, the retailer's and supplier's period-2 profits are:

$$(\pi_{r,2}^C(w_1, s_1, I), \pi_{s,2}^C(w_1, s_1, I)) = \begin{cases} \left(\frac{c^2(1-bI)I + bK_2^2}{c^2}, (1 - 2b(I + \frac{K_2}{c}) - c)\frac{K_2}{c}\right), I + \frac{K_2}{c} \le \frac{1-c+2bI}{4b} \\ \left(\frac{(1-c)^2 + 4b(3+c)I - 12b^2I^2}{16b}, \frac{(1-c-2bI)^2}{8b}\right), \quad I + \frac{K_2}{c} > \frac{1-c+2bI}{4b} \end{cases},$$

The condition $I + \frac{K_2}{c} \le \frac{1-c+2bI}{4b}$ is equivalent to $b\left(\frac{2w_1-c}{c}\right)I + \frac{2b(w_1-c)}{c}s_1 - \frac{1-c}{2} + \frac{2bK_1}{c} \le 0$.

Retailer's best response in period 1

In period 1, given w_1 and K_1, the retailer sets $s_1^C(w_1)$ and $I^C(w_1)$ to maximize total profit π_r:

$$\max_{s_1, I\geq 0, s_1+I\leq \frac{K_1}{c}} \pi_r = (1-bs_1)s_1 - w_1(s_1+I) - hI + \pi_{r,2}^C(w_1, s_1, I). \quad \text{(B.9)}$$

By solving problem (B.9), we obtain the retailer's optimal sales quantity $s_1^C(w_1)$ and inventory level $I^C(w_1)$ in response to w_1, as stated in Lemma B.1. The corresponding purchase quantity $q_1^C(w_1) = s_1^C(w_1) + I^C(w_1)$.

Lemma B.1. *In scenario C, given the supplier's wholesale price w_1:*

1) The sales quantity $s_1^C(w_1)$ and inventory level $I^C(w_1)$ by the retailer are as follows:

Region	Sales quantity $s_1^C(w_1)$	Inventory level $I^C(w_1)$
R_1	0	0
R_2	$\frac{1-w_1}{2b}$	0
R_3	$\frac{1-w_1}{2b}$	$\frac{3+c-4h-4w_1}{6b}$
R_4	$\frac{(1-c+4h)c+6bK_1}{14bc}$	$\frac{-(1-c+4h)c+8bK_1}{14bc}$
R_5	$\frac{K_1}{c}$	0
R_6	$\frac{-(1-c)c^2+2bK_1(c+2w_1)}{2bc^2}$	$\frac{(1-c)c^2-4bK_1w_1}{2bc^2}$
R_7	$\frac{ch+2bK_1}{4bc}$	$\frac{-ch+2bK_1}{4bc}$
R_8	$\frac{K_1}{c}$	0
R_9	$\frac{(1-c)c-4bK_1}{4b(w_1-c)}$	0
R_{10}	$\bar{g}(w_1)$	$\tilde{g}(w_1)$
R_{11}	$\frac{-c^2(1-w_1-h)+2c(bK_1-hw_1)-w_1(2bK_1-hw_1)}{2b(c^2-4cw_1+2w_1^2)}$	$\frac{-c^2(1-w_1)+2c(bK_1+hw_1)-w_1(2bK_1+hw_1)}{2b(c^2-4cw_1+2w_1^2)}$
R_{12}	$\frac{c^2(1-w_1)+2bK_1(w_1-c)}{2b(2c-w_1)w_1}$	0

Where $\bar{g}(w_1) = \frac{-c^2(2-c-2h)+6bcK_1+(c(2+c-6h)-6bK_1)w_1+(1-3c+4h)w_1^2}{2b(4c^2-10cw_1+7w_1^2)}$, $\tilde{g}(w_1) = \frac{2c^2(c-2h)+4bcK_1+(2c(1-3c+4h)-8bK_1)w_1-(1-c+4h)w_1^2}{2b(4c^2-10cw_1+7w_1^2)}$, *and the regions in the table are defined in Table B.2 and illustrated in Figure B.1.*

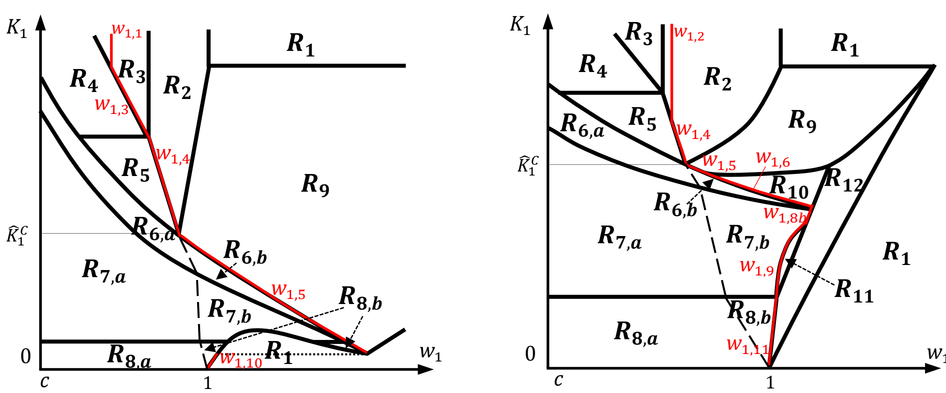
2) $s_1^C(w_1) > q_1^M(w_1|K_1)$ in regions $R_{6,b}$, $R_{7,b}$, $R_{8,b}$ and R_i, $i = 9, \ldots, 12$; but $s_1^C(w_1) \leq q_1^M(w_1|K_1)$ otherwise.

Table B.2. The detailed definitions of the regions in Lemma B.1

Region	Definition
R_1	$\left(w_1 > 1 \wedge K_1 > \frac{(1-c)c}{4b}\right) \vee \left(K_1 \leq min\left\{\frac{(1-c)c}{4b}, \frac{c^2}{2b}\right\} \wedge w_1 \geq \frac{(c-2bK_1)c}{c^2-2bK_1} \wedge (\neg R_6) \wedge (\neg R_7) \wedge (\neg R_8) \wedge (\neg R_9) \wedge (\neg R_{10})\right)$
R_2	$max\left\{1-\frac{2bK_1}{c}, \frac{3+c-4h}{4}\right\} \leq w_1 \leq min\left\{1, \frac{1+c+\sqrt{(1-c)(1-3c)+8bK_1}}{2}\right\}$
R_3	$\frac{6c+c^2-4ch-6bK_1}{7c} \leq w_1 < min\left\{\frac{3+c-4h}{4}, \frac{3(1+c)-2h+\sqrt{(3-4c)^2-2(6-c)h+4h^2+42bK_1}}{7}\right\}$
R_4	$\frac{c((2(1-c)+h)c-2bK_1)}{7bK_1} \leq w_1 < \frac{6c+c^2-4ch-6bK_1}{7c} \wedge K_1 > \frac{c(1-c+4h)}{8b}$
R_5	$\frac{(1-c)c^2}{4bK_1} \leq w_1 < 1-\frac{2bK_1}{c} \wedge K_1 \leq \frac{c(1-c+4h)}{8b}$

R_6	$\frac{c((2(1-c)+h)c-2bK_1)}{8bK_1} \le w_1 < min\left\{\frac{(1-c)c^2}{4bK_1}, \frac{c((2(1-c)+h)c-2bK_1)}{7bK_1}, \frac{(c(4-5c+2h)-4bK_1+\sqrt{c^2(2-12c+11c^2+2(1-3c)h+4h^2)+4bc(6+3c-4h)K_1-40b^2K_1^2})c}{14bK_1}\right\} \wedge g_5^C(w_1) \le 0$
R_7	$w_1 < \frac{c((2(1-c)+h)c-2bK_1)}{8bK_1} \wedge K_1 > \frac{ch}{2b} \wedge g_{12}^C(w_1) > 0 \wedge g_{13}^C(w_1) > 0 \wedge \begin{cases} g_9^C(w_1) > 0, & \hat{R}_{12} \\ N/A, & Others \end{cases}$
R_8	$w_1 < \frac{(1-c)c^2}{4bK_1} \wedge K_1 \le \frac{ch}{2b} \wedge g_{10}^C(w_1) > 0 \wedge g_{11}^C(w_1) > 0$
R_9	$K_1 \le \frac{(1-c)c}{4b} \wedge w_1 \ge \frac{(1-c)c^2}{4bK_1} \wedge g_1^C(w_1) > 0 \wedge g_2^C(w_1) > 0 \wedge g_3^C(w_1) > 0 \wedge g_4^C(w_1) > 0$
R_{10}	$g_1^C(w_1) \le 0 \wedge g_5^C(w_1) > 0 \wedge g_6^C(w_1) > 0 \wedge g_7^C(w_1) > 0 \wedge g_8^C(w_1) > 0 \wedge \begin{cases} g_{14}^C(w_1) > 0, & \hat{R}_{12} \\ N/A, & Others \end{cases}$
R_{11}	$w_1 \le 2c \wedge c^2 - 4cw_1 + 2w_1^2 \le 0 \wedge w_1 \le \frac{(c+2h)c-2bK_1-\sqrt{c^2(c^2-4(1-c)h+4h^2)-4bc^2K_1+4b^2K_1^2}}{2h} \wedge w_1 \ge \frac{\left(c^2+2bK_1-\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}\right)c}{4bK_1} \wedge g_7^C(w_1) \le 0$
R_{12}	$\hat{R}_{12} \wedge (\neg R_7) \wedge (\neg R_{10})$

Notes. $\hat{R}_{12}$ is $max\left\{\frac{\left(c^2+2bK_1-\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}\right)c}{4bK_1}, \frac{(c+2h)c-2bK_1-\sqrt{c^2(c^2-4(1-c)h+4h^2)-4bc^2K_1+4b^2K_1^2}}{2h}\right\} \le w_1 \le min\left\{2c, \frac{c(c-2bK_1)}{c^2-2bK_1}\right\} \wedge g_3^C(w_1) \le 0$. $g_i^C(w_1)$ for $i = 1, \ldots, 14$ are defined in Appendix B.2.



a) Low production cost

b) High production cost

Notes. The red marked curves, $w_{1,1}$ to $w_{1,11}$, denote segments of the supplier's optimal wholesale price w_1^C in period 1.

Figure B.1. Retailer's best-response sales quantity $s_1^C(w_1)$ and inventory level $I^C(w_1)$

Supplier's optimal wholesale price in period 1

Anticipating $s_1^C(w_1)$ and $I^C(w_1)$, the supplier determines w_1^C to maximize total profit over both periods π_s:

$$\max_{w_1} \pi_s = (w_1 - c)(s_1^C(w_1) + I^C(w_1)) + \pi_{r,2}^C(w_1, s_1^C(w_1), I^C(w_1)). \quad \text{(B.10)}$$

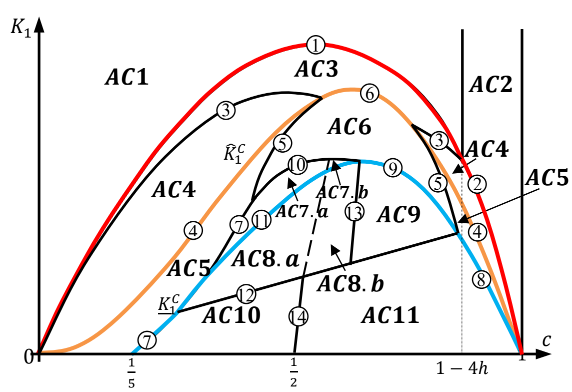
When $K_1 \ge \bar{K}_1^C$, the equilibrium outcomes coincide with the benchmark case. When $K_1 < \bar{K}_1^C$, π_s increases in regions where $s_1^C(w_1) + I^C(w_1) = \frac{K_1}{c}$, but (weakly) decreases elsewhere. π_s remains continuous, except for sharp drops when w_1 transitions from regions R_6, R_7, or R_8 to R_1, or from regions R_6 or R_7 to R_{12}, and sharp increases when moving from R_1 to R_7 or R_8. As a result, the optimal wholesale price must lie at the upper boundary of the regions where $s_1^C(w_1) +$

$I^C(w_1) = \frac{K_1}{c}$. Based on this, we identify all feasible solutions of w_1, and determine the optimal wholesale price w_1^C by comparing the supplier's profit across these candidates. Subsequent decisions are then derived sequentially. Proposition B.1 summarizes the optimal decisions in scenario C.

Proposition B.1. *In scenario C, referring to Figure B.2, the equilibrium wholesale prices (w_1^C, w_2^C), sales quantities (s_1^C, s_2^C), and inventory level I^C are given as follows.*

Area	w_1^C	s_1^C	I^C	w_2^C	s_2^C
$AC1$	$\frac{9+8c-2h}{17}$	$\frac{4(1-c)+h}{17b}$	$\frac{5(1-c-4h)}{34b}$	$\frac{1+c-2bI^C}{2}$	$\frac{1-c+2bI^C}{4b}$
$AC2$	$\frac{1+c}{2}$	$\frac{1-c}{4b}$	0		
$AC3$	$\frac{c(6+c-4h)-6bK_1}{7c}$	$\frac{(1-c+4h)c+6bK_1}{14bc}$	$\frac{-(1-c+4h)c+8bK_1}{14bc}$		
$AC4$	$1-\frac{2bK_1}{c}$	$\frac{K_1}{c}$	0		
$AC5$	$\frac{(1-c)c^2}{4bK_1}$	$\frac{K_1}{c}$	0		
$AC6$	$\frac{c(c(4-5c+2h)+6bK_1+\sqrt{\Delta_1})}{28bK_1}$	$\frac{-c(3-2c-2h)+20bK_1+\sqrt{\Delta_1}}{14bc}$	$\frac{c(3-2c-2h)-6bK_1-\sqrt{\Delta_1}}{14bc}$		
$AC7.a$	$\frac{(c(4-5c+2h)-4bK_1+\sqrt{\Delta_2})c}{14bK_1}$	$\frac{c(1-3c+4h)+6bK_1+2\sqrt{\Delta_2}}{14bc}$	$\frac{-c(1-3c+4h)+8bK_1-2\sqrt{\Delta_2}}{14bc}$		
$AC7.b$	$w_{1,7b}$	$\frac{-(1-c)c^2+2bK_1(c+2w_{1,7b})}{2bc^2}$	$\frac{(1-c)c^2-4bK_1w_{1,7b}}{2bc^2}$		
$AC8.a$	$\frac{(2c^2-\sqrt{2}\sqrt{\Delta_3})c}{4bK_1}$	$\frac{ch+2bK_1}{4bc}$	$\frac{-ch+2bK_1}{4bc}$	$1-2b(I^C+\frac{K_2^C}{c})$	$I^C+\frac{K_2^C}{c}$
$AC8.b$	$w_{1,8b}$				
$AC9$	$\frac{(c^2+2bK_1-\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2})c}{4bK_1}$				
$AC10$	$\frac{c^3-c\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bK_1}$	$\frac{K_1}{c}$	0		
$AC11$	$\frac{(c^2+2bK_1-\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2})c}{4bK_1}$				

Notes. 1) $K_2^C = \frac{w_1^C K_1}{c}$. 2) $w_{1,7b} = Root[g_{1,7b}(w_1),3]$ and $w_{1,8b} = Root[g_{1,8b}(w_1),2]$, where $Root[f(x),k]$ represents the k th real root of $f(x)=0$, $g_{1,7b}(w_1) = -c^4(c-2bK_1)^2 - 2c^3\big(c^2\big((2-c)(1-2c)+2(1-c)h\big) - 6b(1-c)cK_1 + 4b^2K_1^2\big)w_1 + c^2(c^2(2-4c+c^2+2(1-c)h) + 8bc(3-4c+2h)K_1 - 28b^2K_1^2)w_1^2 - 4bcK_1(c(4-5c+2h)+10bK_1)w_1^3 + 28b^2K_1^2w_1^4$, $g_{1,8b}(w_1) = -2c^4(c-2bK_1)^2 + 2c^3(c^2(2c+h^2) + 4bc(1-c-h)K_1 - 4b^2K_1^2)w_1 - c^2(c^2(2c^2+h^2) + 4bc(2(1+c)-h)K_1 - 4b^2K_1^2)w_1^2 + 8bcK_1(c^2+2bK_1)w_1^3 - 8b^2K_1^2w_1^4$. 3) $\Delta_1 = c^2(4-5c+2h)^2 - 4bc(16-27c+22h)K_1 + 148b^2K_1^2$; $\Delta_2 = c^2(11c^2 - 6c(2+h) + 2(1+h+2h^2)) + 4bc(6+3c-4h)K_1 - 40b^2K_1^2$; $\Delta_3 = (2c^2-h^2)c^2 - 4bc(2-h)K_1 + 12b^2K_1^2$.



Notes. 1) The labeled curves are defined as: (1) $K_1 = \frac{c(13(1-c)-18h)}{34b}$; (2) $K_1 = \frac{(1-c)c}{4b}$; (3) $K_1 = \frac{c(1-c+4h)}{8b}$; (4) $K_1 = \frac{(1-\sqrt{1-2c+2c^2})c}{4b}$; (5) $K_1 = \frac{((1-3c+4h)+\sqrt{(3-c)(3c-1)-8(1+c)h+16h^2})c}{8b}$; (6) $K_1 = \frac{c(3(2+c)-4h-\sqrt{57c^2-12c(1+4h)+4(3-2h)^2})}{12b}$; (7) $K_1 = \frac{(2-\sqrt{2}\sqrt{3-6c+5c^2})c}{8b}$; (8) $K_1 = \frac{((1+c)-\sqrt{5-14c+13c^2})c}{8b}$; (9) $K_1 = \frac{((2+8c-7h)-2\sqrt{4(1-c)+25c^2-2(2+17c)h+13h^2})c}{6b}$; (10) $K_1 = \widetilde{K}_{1,1}$; (11) $K_1 = \widetilde{K}_{1,2}$; (12) $K_1 = \frac{ch}{2b}$; (13) $K_1 = \frac{c(2(1+\sqrt{2})c-\sqrt{2}(2-h))}{4b}$; (14) $K_1 = \frac{(2c-1)c}{2b}$. 2) The dashed curve is $f_1(c, K_1) = 0$. 3) $\widetilde{K}_{1,1}$, $\widetilde{K}_{1,2}$, and $f_1(c, K_1)$ are defined in Appendix B.2.

Figure B.2. Partitioning of space to sustain various equilibrium scenarios in Proposition B.1

Table B.3 summarizes the equilibrium profits in scenario C.

Table B.3. Equilibrium Profits in scenario C

	π_r^C	π_s^C	π_{sc}^C
$AC1$	$\frac{155(1-c)^2-118(1-c)h+304h^2}{1156b}$	$\frac{9(1-c)^2-4(1-c)h+8h^2}{34b}$	$\frac{461(1-c)^2-254(1-c)h+576h^2}{1156b}$
$AC2$	$\frac{(1-c)^2}{8b}$	$\frac{(1-c)^2}{4b}$	$\frac{3(1-c)^2}{8b}$
$AC3$	$\frac{c^2((1-c)^2+h(1-c+2h))+6b^2K_1^2}{14bc^2}$	$\frac{2(c^2(2(1-c)+h)^2+bc(13(1-c)-18h)K_1-17b^2K_1^2)}{49bc^2}$	$f_1^C(c, K_1)$
$AC4$	$\frac{(1-c)^2c^2+16b^2K_1^2}{16bc^2}$	$\frac{(1-c)^2c^2+8b(1-c)cK_1-16b^2K_1^2}{8bc^2}$	$\frac{3(1-c)^2c^2+16b(1-c)cK_1-16b^2K_1^2}{16bc^2}$
$AC5$	$\frac{(1-c)c^2(1-5c)+16bcK_1-16b^2K_1^2}{16bc^2}$	$\frac{1-c^2-8bK_1}{8b}$	$\frac{3(1-c)^2c^2+16b(1-c)cK_1-16b^2K_1^2}{16bc^2}$
$AC6$	$f_2^C(c, K_1)$	$f_3^C(c, K_1)$	$f_4^C(c, K_1)$
$AC7.a$	$\frac{bK_1^2}{c^2}$	$f_5^C(c, K_1)$	$\frac{bK_1^2}{c^2} + f_5^C(c, K_1)$
$AC7.b$	$g_{15}^C(w_{1,7b})$	$\frac{K_1(c^3(w_{1,7b}-c)+2bK_1w_{1,7b}^2)}{c^4}$	$g_{16}^C(w_{1,7b})$
$AC8.a$	$\frac{bK_1^2}{c^2}$	$f_6^C(c, K_1)$	$\frac{bK_1^2}{c^2} + f_6^C(c, K_1)$
$AC8.b$	$g_{17}^C(w_{1,8b})$	$\frac{K_1(-2c^4+(c^2(2+h)-2bcK_1)w_{1,8b}-4bK_1w_{1,8b}^2)}{2c^4}$	$g_{18}^C(w_{1,8b})$
$AC9$	$f_7^C(c, K_1)$	$f_8^C(c, K_1)$	$f_9^C(c, K_1)$
$AC10$	$\frac{bK_1^2}{c^2}$	$f_{10}^C(c, K_1)$	$\frac{bK_1^2}{c^2} + f_{10}^C(c, K_1)$
$AC11$	$f_{11}^C(c, K_1)$	$\frac{(c(1-c)-2bK_1)(c^2+6bK_1-\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2})}{4bc^2}$	$f_{12}^C(c, K_1)$

Notes. $f_i^C(c, K_1)$, $i = 1, \ldots, 12$, and $g_i^C(w_1)$, $i = 15, \ldots, 18$, are defined in Appendix B.2.

Proof of Proposition 3.3. We compare the operations and profits between scenarios C and ND and identify the following patterns:

(1) $q_1^C > q_1^{ND}$, $s_1^C < s_1^{ND}$, $w_2^C < w_2^{ND}$, and $s_2^C > s_2^{ND}$ always hold.

(2) $w_1^C < w_1^{ND}$ may occur in the following regions: 1) Always holds in $AC6 \cap ND3$; 2) Holds if $K_1 > K_{1,1}^C$ in $AC6 \cap ND4$; 3) Holds if $K_1 > K_{1,2}^C$ in $AC6 \cap ND5$; 4) Holds if $K_1 < \frac{((6+c-4h)-\sqrt{43c^2+4(3-2h)^2-2c(15+4h)})c}{12b}$ in $AC3 \cap ND3$; 5) Holds if $K_1 < K_{1,3}^C$ in $AC3 \cap ND5$.

Combining all regions where this inequality holds yields area iii^a in Figure 3.3.a.

(3) When $K_1 \leq \widehat{K}_1^C$, we have $q_2 = \frac{K_2}{c}$ in both scenarios. In area ii^a of Figure 3.3.a, since $w_1^C > w_1^{ND}$ and $q_1^C = q_1^{ND} = \frac{K_1}{c}$, capital accumulation is greater in scenario C, leading to $q_2^C > q_2^{ND}$.

In area iii^a, the opposite holds: $q_2^C < q_2^{ND}$. When $K_1 > \widehat{K}_1^C$, we also have $q_2^C < q_2^{ND}$. Hence, $q_2^C > q_2^{ND}$ occurs only in area ii^a of Figure 3.3.a.

(4) When $K_1 \leq \widehat{K}_1^C$, we have $s_1 + s_2 = \frac{K_1}{c} + \frac{K_2}{c}$. In area ii^a (iii^a, resp.) in Figure 3.3.a, a larger (smaller, resp.) K_2^C leads to more (less) total sales in scenario C. When $K_1 > \widehat{K}_1^C$, $s_1^C + s_2^C < s_1^{ND} + s_2^{ND}$ may occur in the following regions: 1) Always hold in $AC3 \cap ND2$, $AC3 \cap ND3$, and $AC3 \cap ND5$; 2) Holds if $K_1 < \frac{c(3(1-c)-2h)}{10b}$ in $AC3 \cap ND1$. Combining all regions where this inequality holds yields areas $iii^a \cup iv^a$ in Figure 3.3.a.

(5) $\pi_s^C > \pi_s^{ND}$ may occur in the following regions: 1) Holds if $c \leq 1 - 4h$ and $K_1 > \frac{c(15(1-c)+4h)}{64b}$ in $AC1 \cap N1$, $AC3 \cap N1$, and $AC3 \cap N2$; 2) Holds if $K_1 < K_{1,7}^C$ in $AC8.b \cap N5$; 3) Holds if $K_1 < K_{1,8}^C$ in $AC9 \cap N5$. Combining all regions where this inequality holds yields areas $ii^b \cup v^b \cup vi^b$ in Figure 3.3.b.

(6) $\pi_r^C > \pi_r^{ND}$ may occur in the following regions: 1) Holds if $c \leq 1 - \frac{152h}{21}$ and $K_1 > \frac{c\sqrt{3-4h-8h^2+3c^2-2c(3-2h)}}{2\sqrt{6}b}$ in $AC1 \cap ND1$ and $AC3 \cap ND1$; 2) Holds if $K_1 < K_{1,4}^C$ in $AC3 \cap ND3$ and $AC3 \cap ND5$; 3) Always holds in $AC6 \cap ND3$ and $AC6 \cap ND4$; 4) Holds if $K_1 > K_{1,5}^C$ in $AC6 \cap ND5$. Combining all regions where this inequality holds yields areas $iv^b \cup vi^b$ in Figure 3.3.b.

(7) $\pi_{sc}^C > \pi_{sc}^{ND}$ occurs only in $AC1 \cap ND1$ and $AC3 \cap ND1$, holds if $c < 1 - \frac{288}{55}h$ and $K_1 > \frac{(4(13(1-c)-18h)-7\sqrt{2}\sqrt{13-52h+72h^2+13c^2-26c(1-2h)})c}{52b}$. Hence, $\pi_{sc}^C > \pi_{sc}^{ND}$ occurs in areas $v^b \cup vi^b$ of Figure 3.3.b.

Hence the claim of Proposition 3.3. □

Proof of Corollary 3.1. We compare the operations between scenarios C and B and identify the following patterns:

(1) $w_1^C > w_1^B$, $s_1^C < s_1^B$, $q_1^C > q_1^B$, and $s_1^C + s_2^C > s_1^B + s_2^B$ always hold.

(2) $I^C > I^B$ may occur in regions $AC6$, $AC7.b$, $AC8.b$, and $AC9$. When $c \leq 1 - 4h$: $I^C > I^B$ holds in $AC6$ if $K_1 < \frac{((4(4+h)-33c)+\sqrt{57c^2+8c(75+74h)+16(91h^2-3h-23)})c}{68b}$; $I^C > I^B$ holds in $AC7.b$ if $K_1 < \widetilde{K}_{1,3}$; $I^C > I^B$ holds in $AC8.b \cup AC9$ if $K_1 > \frac{c(10(1-c)-23h)}{34b}$. When $c > 1 - 4h$, $I^C > I^B$ holds in $AC6 \cup AC9$, where $I^C > 0 = I^B$.

(3) $w_2^C < w_2^B$ may occur in regions $AC6$, $AC7.b$, $AC8.b$, and $AC9$. When $K_1 \geq \underline{K}_1^C$, $w_2 = \frac{1+c-2bI}{2}$ in $AC6 \cup AC7.b$, consistent with the benchmark. Since w_2 decreases as I increases, it follows that $w_2^C < w_2^B$ if $I^C > I^B$. In contrast, in $AC8.b$ and $AC9$, $w_2^C < w_2^B$ holds if $K_1 > K_{1,3}$.

(4) Since $s_2 = \frac{1-w_2}{2b}$ in both scenarios, it follows that $s_2^C > s_2^B$ if $w_2^C < w_2^B$, and vice versa.

(5) $q_2^C < q_2^B$ may occur in regions $AC6$, $AC7.b$, $AC8.a$, $AC8.b$, $AC9$, $AC10$, and $AC11$. When $K_1 \geq \underline{K}_1^C$, $q_2 = \frac{1-c-2bI}{4b}$ in $AC6 \cup AC7.b$, consistent with the benchmark. Therefore, $q_2^C < q_2^B$ if $I^C > I^B$. In $AC8.a$, $q_2^C < q_2^B$ if $K_1 < \frac{(17(2-h)-2\sqrt{343-414c+360c^2-(109+690c)h+439h^2})c}{102b}$. In $AC8.b$, $q_2^C < q_2^B$ if $K_1 < \widetilde{K}_{1,4}$. In $AC9$, $q_2^C < q_2^B$ if $K_1 < \frac{(22+12c-37h-\sqrt{4(193-216c+312c^2)-4(167+802c)h+2169h^2})c}{68b}$. In $AC10$, when $c \leq 1-4h$, $q_2^C < q_2^B$ if $K_1 < \frac{(17-\sqrt{361-552c+480c^2+40(6-23c)h+200h^2})c}{68b}$; when $c > 1-4h$, $q_2^C < q_2^B$ always holds. In $AC11$, $q_2^C < q_2^B$ always holds.

Hence the claim. □

Proof of Corollary 3.2. When $K_1 < \underline{K}_1^C$, the retailer carries inventory in equilibrium in regions $AC8.a$, $AC8.b$, and $AC9$ of Figure A.4, where $\frac{dI^C}{dK_1} = \frac{1}{2c} > 0$. When $\underline{K}_1^C \leq K_1 < \widehat{K}_1^C$, the retailer carries inventory in regions $AC6$, $AC7.a$, and $AC7.b$, where $\frac{dI^C}{dK_1} < 0$. When $\widehat{K}_1^C \leq K_1 < \overline{K}_1^C$, the retailer carries inventory in region $AC3$, where $\frac{dI^C}{dK_1} = \frac{4}{7c} > 0$. Notably, the rate of increase in this latter range is higher than in the earlier case where $K_1 < \underline{K}_1^C$. Hence the claim. □

Proof of Corollary 3.3. We compare the profits between scenarios C and B and identify the following patterns:

(1) $\pi_r^C < \pi_r^B$ and $\pi_r^C + \pi_s^C < \pi_r^B + \pi_s^B$ always hold.

(2) $\pi_s^C > \pi_s^B$ may occur in regions $AC5$, $AC6$, $AC7.a$, $AC7.b$, $AC8.a$, $AC8.b$, $AC9$, $AC10$ and $AC11$. Specifically, in $AC5$ and $AC6$, $\pi_s^C > \pi_s^B$ if $K_1 < K_{1,5}$; in other regions, $\pi_s^C > \pi_s^B$ if $K_1 > K_{1,6}$.

Hence the claim. □

Proof of Proposition 3.4. The results in Proposition 3.4 follow from the equilibrium outcomes in scenario P, which we derive below.

Operations in period 2

In period 2, given the capital position K_2, q_p, and wholesale price w_2, the retailer solves problem (3-8) with $q_2^P(w_2, K_2, q_p) = \left(s_2^P(w_2, K_2, q_p) - q_p\right)^+$. The resulting sales quantity in period 2 $s_2^P(w_2, K_2, q_p)$ is:

$$s_2^P(w_2|K_2, q_p) = \begin{cases} \frac{K_2}{c}, & w_2 \leq 1 - \frac{2bK_2}{c} \\ \frac{1-w_2}{2b}, & 1 - \frac{2bK_2}{c} < w_2 \leq 1 - 2bq_p. \\ min\left\{q_p, \frac{1}{2b}\right\}, & w_2 > 1 - 2bq_p \end{cases} \quad \text{(B.11)}$$

Anticipating $q_2^P(w_2|K_2, q_p)$, the supplier sets $w_2^P(K_2, q_p)$ to maximize period-2 profit $\pi_{s,2}$. Solving problem (3-9), we obtain the supplier's optimal wholesale price and the retailer's optimal sales and purchase quantities in period 2:

$$\left(w_2^P(K_2, q_p), s_2^P(K_2, q_p), q_2^P(K_2, q_p)\right) = \begin{cases} \left(1 - \frac{2bK_2}{c}, \frac{K_2}{c}, \frac{K_2}{c} - q_p\right), & q_p < \frac{1-c}{2b} \wedge K_2 \leq \frac{c(1-c+2bq_p)}{4b} \\ \left(\frac{1+c-2bq_p}{2}, \frac{1-c+2bq_p}{4b}, \frac{1-c-2bq_p}{4b}\right), & q_p < \frac{1-c}{2b} \wedge K_2 > \frac{c(1-c+2bq_p)}{4b} \\ \left(\infty, min\left\{q_p, \frac{1}{2b}\right\}, 0\right), & q_p \geq \frac{1-c}{2b} \end{cases} \quad \text{(B.12)}$$

Retailer's best response in period 1

In period 1, given w_1 and K_1, the retailer sets $s_1^P(w_1)$ and $q_p^P(w_1)$ to maximize total profit π_r. Solving problem (3-10), we have Lemma B.2.

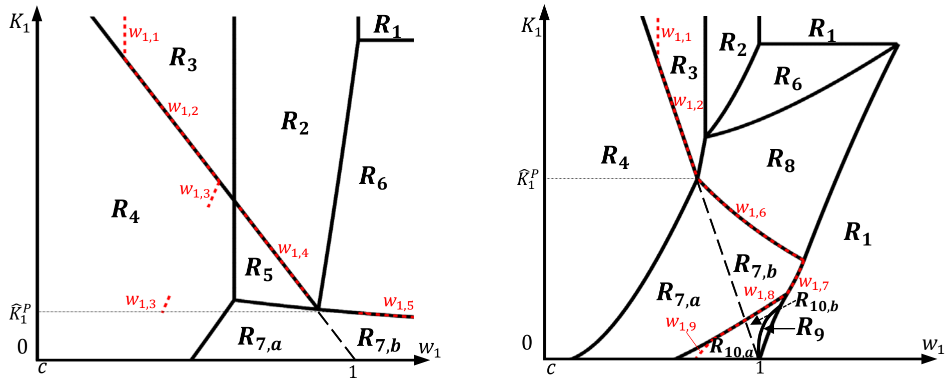
Lemma B.2. *In scenario P, given the supplier's wholesale price w_1:*

1) The sales quantity $s_1^P(w_1)$ and preorder quantity $q_p^P(w_1)$ by the retailer are as follows:

Region	Sales quantity $s_1^P(w_1)$	Pre-ordered quantity $q_p^P(w_1)$
R_1	0	0
R_2	$\frac{1-w_1}{2b}$	0
R_3	$\frac{1-w_1}{2b}$	$\frac{3+c-4w_1}{6b}$
R_4	$\frac{K_1}{c}$	$\frac{3+c-4w_1}{6b}$
R_5	$\frac{K_1}{c}$	0
R_6	$\frac{(1-c)c-4bK_1}{4b(w_1-c)}$	0
R_7	$\frac{K_1}{c}$	$\frac{(1-c)c^2-4bK_1w_1}{2bc(2w_1-c)}$
R_8	$\frac{-(2-c)c^2+6bcK_1+(c(2+c)-6bK_1)w_1+(1-3c)w_1^2}{2b(4c^2-10cw_1+7w_1^2)}$	$\frac{2c^3+4bcK_1+(2c(1-3c)-8bK_1)w_1-(1-3c)w_1^2}{2b(4c^2-10cw_1+7w_1^2)}$
R_9	$\frac{(c^2-2bK_1)w_1-c(c-2bK_1)}{2b(c^2-4cw_1+2w_1^2)}$	$\frac{(c^2-2bK_1)w_1-c(c-2bK_1)}{2b(c^2-4cw_1+2w_1^2)}$
R_{10}	$\frac{K_1}{c}$	$\frac{c^3-c(c^2+2bK_1)w_1+2bK_1w_1^2}{2bc(2c-w_1)w_1}$

Where the regions in the table are defined in Table B.4 and illustrated in Figure B.3.

2) $s_1^P(w_1) > q_1^M(w_1|K_1)$ *in regions* R_6, $R_{7,b}$, R_8, R_9 *and* $R_{10,b}$; *but* $s_1^P(w_1) \leq q_1^M(w_1|K_1)$ *otherwise.*



a) Low production cost b) High production cost

Notes. The red marked curves, $w_{1,1}$ to $w_{1,9}$, denote segments of the supplier's optimal wholesale price w_1^P in period 1.

Figure B.3. Retailer's best-response sales quantity $s_1^P(w_1)$ and pre-ordered quantity $q_p^P(w_1)$

Table B.4. The detailed definitions of the regions in Lemma A.4

Region	Definition
R_1	$\left(K_1 > \frac{(1-c)c}{4b} \wedge w_1 \geq 1\right) \vee \left(K_1 \leq min\left\{\frac{(1-c)c}{4b}, \frac{c^2}{2b}\right\} \wedge w_1 \geq \frac{c(c-2bK_1)}{c^2-2bK_1} \wedge (\neg R_6) \wedge (\neg R_7) \wedge (\neg R_8) \wedge (\neg R_{10})\right)$
R_2	$max\left\{1 - \frac{2bK_1}{c}, \frac{3+c}{4}\right\} \leq w_1 < min\left\{1, \frac{1+c+\sqrt{(1-c)(1-3c)+8bK_1}}{2}\right\}$
R_3	$1 - \frac{2bK_1}{c} \leq w_1 < min\left\{\frac{3+c}{4}, \frac{3(1+c)+\sqrt{(3-4c)^2+42bK_1}}{7}\right\}$
R_4	$w_1 < min\left\{\frac{3+c}{4}, 1 - \frac{2bK_1}{c}, \frac{3c(1+c)+6bK_1+\sqrt{c^2(3-5c)^2+36bc(1+c)K_1+36b^2K_1^2}}{8c}\right\}$
R_5	$max\left\{\frac{(1-c)c^2}{4bK_1}, \frac{3+c}{4}\right\} \leq w_1 < 1 - \frac{2bK_1}{c}$
R_6	$K_1 \leq \frac{(1-c)c}{4b} \wedge w_1 \geq \frac{(1-c)c^2}{4bK_1} \wedge g_1^P(w_1) \geq 0 \wedge g_2^P(w_1) \geq 0 \wedge g_3^P(w_1) \geq 0 \wedge g_4^P(w_1) \geq 0$
R_7	$w_1 < \frac{(1-c)c^2}{4bK_1} \wedge w_1 \geq \frac{(1-c)c^2}{(1-c)c+2bK_1} \wedge g_5^P(w_1) \geq 0 \wedge g_6^P(w_1) \geq 0 \wedge g_7^P(w_1) \geq 0 \wedge g_8^P(w_1) \geq 0$
R_8	$4c^2 - 10cw_1 + 7w_1^2 \geq 0 \wedge g_1^P(w_1) < 0 \wedge g_5^P(w_1) < 0 \wedge g_9^P(w_1) \leq 0 \wedge g_{10}^P(w_1) \geq 0 \wedge g_{11}^P(w_1) \geq 0$
R_9	$c^2 - 4cw_1 + 2w_1^2 \leq 0 \wedge \frac{\left(c^2+6bK_1-\sqrt{c^4-4bc(4-3c)K_1+36b^2K_1^2}\right)c}{8bK_1} \leq w_1 \leq min\left\{2c, \frac{c(c-2bK_1)}{c^2-2bK_1}\right\} \wedge g_{10}^P(w_1) < 0$
R_{10}	$w_1 \leq 2c \wedge g_7^P(w_1) < 0 \wedge g_{12}^P(w_1) < 0 \wedge g_{13}^P(w_1) \geq 0$

Notes. $g_i^P(w_1)$ for $i = 1, \ldots, 13$ are defined in Appendix B.2.

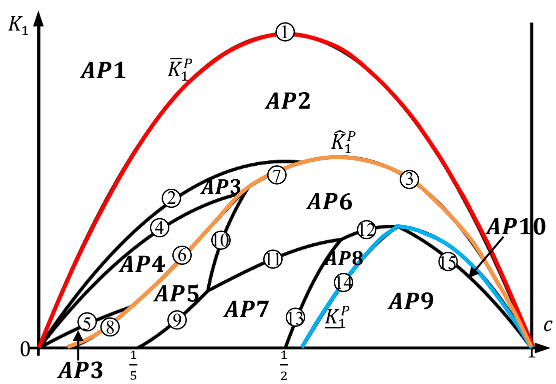
Supplier's optimal wholesale price in period 1

Anticipating the retailer's response outlined in Lemma B.2, the supplier sets wholesale price w_1^P to maximize total profit π_s by solving problem (3-11). Subsequent decisions are then derived sequentially. Proposition B.2 summarizes the optimal decisions in scenario P.

Proposition B.2. *In scenario* P*, referring to Figure B.4, the equilibrium wholesale prices* (w_1^P, s_1^P)*, sales quantities* (s_1^P, s_2^P)*, and pre-ordered quantity* q_p^P *are as follows.*

Area	w_1^P	s_1^P	q_p^P	w_2^P	s_2^P
$AP1$	$\frac{9+8c}{17}$	$\frac{4(1-c)}{17b}$	$\frac{5(1-c)}{34b}$		
$AP2$	$1-\frac{2bK_1}{c}$	$\frac{K_1}{c}$	$\frac{-c(1-c)+8bK_1}{6bc}$		
$AP3$	$\frac{(9+7c)c+18bK_1}{16c}$	$\frac{K_1}{c}$	$\frac{(1-c)c-6bK_1}{8bc}$		
$AP4$	$1-\frac{2bK_1}{c}$	$\frac{K_1}{c}$	0		
$AP5$	$\frac{(1-c)c^2}{4bK_1}$	$\frac{K_1}{c}$	0	$\frac{1+c-2bq_p^P}{2}$	$\frac{1-c+2bq_p^P}{4b}$
$AP6$	$\frac{c((2+c)c+14bK_1+\sqrt{\Delta_1})}{2(-c(1-3c)+14bK_1)}$	$\frac{K_1}{c}$	$\frac{-(1-c)c^2(1-3c)+2bc(5-8c)K_1-28b^2K_1^2-2bK_1\sqrt{\Delta_1}}{2bc((3-2c)c+\sqrt{\Delta_1})}$		
$AP7$	$\widehat{w}_1$	$\frac{K_1}{c}$	$\frac{(1-c)c^2(5c-1)-2bc(5+c)K_1+32b^2K_1^2-4bK_1\sqrt{\Delta_2}}{2bc((3-c)c+6bK_1+2\sqrt{\Delta 2})}$		
$AP8$	$\frac{((3c^2+2bK_1)+\sqrt{\Delta_3})c}{2((3c-1)c-2bK_1)}$	$\frac{K_1}{c}$	$\frac{(1-c)c^3(3c-1)-2bc^2(1+2c)K_1-4b^2cK_1^2-2bcK_1\sqrt{\Delta_3}}{2bc^2(c+4bK_1+\sqrt{\Delta_3})}$		
$AP9$	$\widetilde{w}_1$	$\frac{K_1}{c}$	$\frac{c^3-c(c^2+2bK_1)\widetilde{w}_1+2bK_1\widetilde{w}_1^2}{2bc(2c-\widetilde{w}_1)\widetilde{w}_1}$	$1-\frac{2bK_2^P}{c}$	$\frac{K_2^P}{c}$
$AP10$	$\frac{((c^2+6bK_1)-\sqrt{c^4-4bc(4-3c)K_1+36b^2K_1^2})c}{8bK_1}$	$\frac{K_1}{c}$	$\frac{K_1}{c}$		

Notes. 1) $K_2^P = w_1^P(\frac{K_1}{c} + q_p^P)$. 2) $\widehat{w}_1 = \frac{c(-(1-c)c^2(1+2c)+2bc(1+2c)K_1-16b^2K_1^2+(2bK_1-c(1-c))\sqrt{\Delta_2})}{(1-c)c^2(1-5c)+4bc(1+c)K_1-44b^2K_1^2}$, and $\widetilde{w}_1 = Root[\tilde{g}^P(w_1), 1]$, where $\tilde{g}^P(w_1) = 2c^5 - 3c^4w_1 + c(2c^2(1-c+c^2) - 8b^2K_1^2)w_1^2 - (c^2(1-3c+3c^2) + 4bc(1-2c)K_1 + 4b^2K_1^2)w_1^3$. 3) $\Delta_1 = c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2$; $\Delta_2 = c^2(3-6c+4c^2) + 12bcK_1 - 24b^2K_1^2$; $\Delta_3 = c^2(2-3c)^2 + 4bc(2+3c)K_1 + 4b^2K_1^2$.



Notes. The labeled curves are defined as: (1) $K_1 = \frac{4(1-c)c}{17b}$; (2) $K_1 = \frac{7(1-c)c}{50b}$; (3) $K_1 = \frac{(8-3c-\sqrt{36-48c+37c^2})c}{28b}$; (4) $K_1 = \frac{(7+2\sqrt{2})c(1-c)}{82b}$; (5) $K_1 = \frac{(7-2\sqrt{2})c(1-c)}{82b}$; (6) $K_1 = \frac{(1-\sqrt{1-2c+2c^2})c}{4b}$; (7) $K_1 = \widetilde{K}_{1,1}^P$; (8) $K_1 = \frac{(2\sqrt{2}\sqrt{9+36c-13c^2}-7c-9)c}{18b}$; (9) $K_1 = \frac{(2-\sqrt{2}\sqrt{3-6c+5c^2})c}{8b}$; (10) $K_1 = \frac{(1-3c+\sqrt{-3+10c-3c^2})c}{8b}$; (11) $K_1 = \widetilde{K}_{1,2}^P$; (12) $K_1 = \widetilde{K}_{1,3}^P$; (13) $K_1 = \widetilde{K}_{1,4}^P$; (14) $K_1 = \widetilde{K}_{1,5}^P$; (15) $K_1 = \widetilde{K}_{1,6}^P$. Where $\widetilde{K}_{1,i}^P$ for $i = 1, \ldots, 6$ are defined in Appendix B.2.

Figure B.4. Partitioning of space to sustain various equilibrium scenarios in Proposition B.2

Table B.5 summarizes the equilibrium profits in scenario P.

Table B.5. Equilibrium Profits in scenario P

	π_r^P	π_s^P	π_{sc}^P
$AP1$	$\frac{155(1-c)^2}{1156b}$	$\frac{9(1-c)^2}{34b}$	$\frac{461(1-c)^2}{1156b}$
$AP2$	$\frac{(1-c)^2c^2-4b(1-c)cK_1+28b^2K_1^2}{12bc^2}$	$\frac{(1-c)^2c^2+32b(1-c)cK_1-68b^2K_1^2}{18bc^2}$	$\frac{5(1-c)^2c^2+52b(1-c)cK_1-52b^2K_1^2}{36bc^2}$
$AP3$	$\frac{19(1-c)^2c^2+76b(1-c)cK_1-436b^2K_1^2}{256bc^2}$	$\frac{9((1-c)c+2bK_1)^2}{64bc^2}$	$\frac{55(1-c)^2c^2+220b(1-c)cK_1-292b^2K_1^2}{256bc^2}$
$AP4$	$\frac{(1-c)^2c^2+16b^2K_1^2}{16bc^2}$	$\frac{(1-c)^2c^2+8b(1-c)cK_1-16b^2K_1^2}{8bc^2}$	$\frac{3(1-c)^2c^2+16b(1-c)cK_1-16b^2K_1^2}{16bc^2}$
$AP5$	$\frac{(1-c)c^2(1-5c)+16bcK_1-16b^2K_1^2}{16bc^2}$	$\frac{1-c^2-8bK_1}{8b}$	$\frac{3(1-c)^2c^2+16b(1-c)cK_1-16b^2K_1^2}{16bc^2}$
$AP6$	$\frac{\hat{f}_1^P(c,K_1)}{8bc^2(\hat{f}_2^P(c,K_1))^2}$	$\frac{\hat{f}_3^P(c,K_1)}{2bc^2(\hat{f}_2^P(c,K_1))^2}$	$\frac{\hat{f}_1^P(c,K_1)+4\hat{f}_3^P(c,K_1)}{8bc^2(\hat{f}_2^P(c,K_1))^2}$
$AP7$	$\frac{bK_1^2}{c^2}$	$\frac{\hat{f}_4^P(c,K_1)}{2bc^2(\hat{f}_5^P(c,K_1))^2}$	$\frac{bK_1^2}{c^2}+\frac{\hat{f}_4^P(c,K_1)}{2bc^2(\hat{f}_5^P(c,K_1))^2}$
$AP8$	$\frac{\hat{f}_6^P(c,K_1)}{8bc^2(\hat{f}_7^P(c,K_1))^2}$	$\frac{\hat{f}_8^P(c,K_1)}{2bc^2(\hat{f}_7^P(c,K_1))^2}$	$\frac{\hat{f}_6^P(c,K_1)+4\hat{f}_8^P(c,K_1)}{8bc^2(\hat{f}_7^P(c,K_1))^2}$
$AP9$	$\hat{g}_1^P(\widetilde{w}_1)$	$\hat{g}_2^P(\widetilde{w}_1)$	$\hat{g}_1^P(\widetilde{w}_1)+\hat{g}_2^P(\widetilde{w}_1)$
$AP10$	$\frac{\hat{f}_9^P(c,K_1)}{4bc^2\hat{f}_{10}^P(c,K_1)}$	$\frac{\hat{f}_{11}^P(c,K_1)}{bc^2(\hat{f}_{10}^P(c,K_1))^2}$	$\frac{\hat{f}_9^P(c,K_1)\hat{f}_{10}^P(c,K_1)+4\hat{f}_{11}^P(c,K_1)}{4bc^2(\hat{f}_{10}^P(c,K_1))^2}$

Notes. $\hat{f}_i^P(c,K_1)$, $i=1,\dots,11$, and $\hat{g}_i^P(w_1)$, $i=1,2$, are defined in Appendix B.2.

Proof of Corollary 3.4. (1) $w_1^C < w_1^P$ may occur in regions $AC1 \cap AP1$, $AC2 \cap AP1$, $AC3 \cap AP1$, $AC4 \cap AP1$, $AC8.a \cap AP6$, $AC8.a \cap AP7$, $AC8.b \cap AP6$, $AC8.b \cap AP7$, $AC8.b \cap AP8$, $AC9 \cap AP6$, $AC9 \cap AP7$, $AC9 \cap AP8$, $AC9 \cap AP9$, $AC9 \cap AP10$, $AC10 \cap AP6$, $AC10 \cap AP7$, $AC11 \cap AP6$, $AC11 \cap AP7$, $AC11 \cap AP8$, $AC11 \cap AP9$, and $AC11 \cap AP10$. Similarly, $w_2^C < w_2^P$ may occur in regions $AC6 \cap AP6$, $AC7.a \cap AP6$, $AC8.a \cap AP6$, $AC8.b \cap AP6$, $AC9 \cap AP6$, and $AC6 \cap AP2$. Each case is examined individually, and we group the relevant regions where $w_1^C < w_1^P$ ($w_2^C < w_2^P$, resp.) to obtain areas $\hat{\imath}$ and $\widehat{\imath v}$ (area $\widehat{\imath\imath\imath}$, resp.), as shown in Figure 3.9.

(2) Regarding quantities, $s_1^C > s_1^P$ if $K_1 > K_{1,1}^P$, while $s_1^C + s_2^C \leq s_1^P + s_2^P$ always holds. Since $s_2 = \frac{1-w_2}{2b}$, it follows that $s_2^C > s_2^P$ if $w_2^C < w_2^P$, and vice versa. □

Proof of Corollary 3.5. (1) $I^C > q_p^P$ may occur in regions $AC\ 8.a \cap AP6$, $AC8.a \cap AP7$, $AC7.a \cap AP6$, $AC7.b \cap AP6$, $AC6 \cap AP6$, and $AC9 \cap AP6$. Each of these cases is examined individually to construct Figure 3.10.a.

(2) When $K_1 \geq \underline{K}_1^C$, $q_2^C = \frac{1-c-2bI^C}{4b}$ and $q_2^P = \frac{1-c-2bq_p^P}{4b}$, implying that $q_2^C < q_2^P$ if $I^C > q_p^P$. In contrast, when $K_1 < \underline{K}_1^C$, $q_2^C < q_2^P$ may occur in regions $AC8.a \cap AP6$, $AC8.b \cap AP6$, $AC9 \cap AP6$, $AC10 \cap AP6$, $AC11 \cap AP6$, $AC8.a \cap AP7$, $AC8.b \cap AP7$, $AC9 \cap AP7$, $AC10 \cap AP7$, $AC11 \cap AP7$, $AC8.a \cap AP8$, $AC8.b \cap AP8$, $AC9 \cap AP8$, $AC11 \cap AP8$, $AC8.b \cap AP9$, $AC9 \cap AP9$, $AC11 \cap AP9$, $AC9 \cap AP10$, $AC11 \cap AP10$, $AC6 \cap AP2$, $AC9 \cap AP2$, and $AC11 \cap AP2$. Each of these regions is analyzed case by case to generate Figure 3.10.b. □

Proof of Corollary 3.6. $\pi_r^C > \pi_r^P$ may occur in regions $AC8.b \cap AP6$, $AC9 \cap AP6$, $AC11 \cap AP6$, $AC8.b \cap AP7$, $AC9 \cap AP7$, $AC11 \cap AP7$, $AC8.b \cap AP8$, $AC9 \cap AP8$, and $AC11 \cap AP8$. $\pi_s^C > \pi_s^P$ may occur in regions $AC5 \cap AP6$, $AC6 \cap AP6$, $AC7.a \cap AP6$, $AC8.a \cap AP6$, $AC8.b \cap AP6$, $AC9 \cap AP6$, $AC10 \cap AP6$, $AC11 \cap AP6$, $AC5 \cap AP2$, $AC6 \cap AP2$, $AC5 \cap AP3$, $AC6 \cap AP3$, $AC9 \cap AP2$, and $AC11 \cap AP2$. Figure 3.11 is constructed by systematically examining each case and aggregating regions that share similar characteristics. □

B.2 Definitions

B.2.1 Definitions in Chapter 3

Figure 3.3. In Figure 3.3 in Chapter 3, the labeled curves are defined as: (1) $K_1 = K_{1,1}^C$; (2) $K_1 = K_{1,2}^C$; (3) $K_1 = K_{1,3}^C$; (4) $K_1 = \frac{c(3(1-c)-2h)}{10b}$; (5) $K_1 = \frac{c\sqrt{3-4h-8h^2+3c^2-2c(3-2h)}}{2\sqrt{6}b}$; (6) $K_1 = \frac{(4(13(1-c)-18h)-7\sqrt{2}\sqrt{13-52h+72h^2+13c^2-26c(1-2h)})c}{52b}$; (7) $K_1 = \frac{c(15(1-c)+4h)}{64b}$; (8) $K_1 = K_{1,4}^C$; (9) $K_1 = K_{1,5}^C$; (10) $K_1 = K_{1,6}^C$; (11) $K_1 = K_{1,7}^C$; (12) $K_1 = K_{1,8}^C$.

■ $K_{1,i}^C, i \in \{1, \dots, 8\}$ are defined below.

(1) $K_{1,1}^C \in AC6 \cap ND4$ and $K_{1,1}^C = Root[f_{13}^C(c, K_1), 1]$.

(2) $K_{1,2}^C \in AC6 \cap ND4$ and $K_{1,2}^C = Root[f_{14}^C(c, K_1), 1]$.

(3) $K_{1,3}^C = \begin{cases} \frac{((6+c-4h)-\sqrt{43c^2+4(3-2h)^2-2c(15+4h)})c}{12b}, & AC3 \cap ND3 \\ Root[f_{15}^C(c, K_1), 1], & AC3 \cap ND5 \end{cases}$.

(4) $K_{1,4}^C = \begin{cases} \frac{(14-\sqrt{2}\sqrt{93-40h-80h^2+135c^2-10c(13-4h)})c}{40b}, & AC3 \cap ND3 \\ Root[f_{16}^C(c, K_1), 1], & AC3 \cap ND5 \end{cases}$

(5) $K_{1,5}^C = \begin{cases} \widetilde{K}_{1,1}, & AC6 \cap ND4 \\ Root[f_{17}^C(c, K_1), 1], & AC6 \cap ND5 \end{cases}$.

(6) $K_{1,6}^C = \begin{cases} \widetilde{K}_{1,2}, & AC8.b \cap ND5 \\ \frac{((2+8c-7h)-2\sqrt{4(1-c)+25c^2-2(2+17c)h+13h^2})c}{6b}, & AC9 \cap ND5 \end{cases}$.

(7) $K_{1,7}^C \in AC8.b \cap ND5$ and $K_{1,7}^C = Root[f_{18}^C(c, K_1), 1]$.

(8) $K_{1,8}^C \in AC9 \cap ND5$ and $K_{1,8}^C = Root[f_{19}^C(c, K_1), 1]$.

■ $f_i^C(c, K_1)$ for $i = 13, \dots, 19$ are defined below.

$f_{13}^C(c, K_1) = -(1-c)c^3(4-3c+2h)(4-19c+2h) + bc^2(201c^2 - 4c(29+19h) - 4(40 - 14h - h^2))K_1 + 4b^2c(171 - 36c - 14h)K_1^2 - 604b^3K_1^3$;

$f_{14}^{C}(c,K_1) = 2c(2-6c+h) - 8bK_1 + 7\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2} +$
$\sqrt{c^2(4-5c+2h)^2-4bc(16-27c+22h)K_1+148b^2K_1^2}$;
$f_{15}^{C}(c,K_1) = -7c^4 + 2bc(12-5c-8h)K_1 - 24b^2K_1^2 + 7c^2\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}$;
$f_{16}^{C}(c,K_1) = 11c^4 - 4c^3(2+h) + 4c^2(1+h+2h^2) - 28bcK_1 - 4b^2K_1^2 - 7(c^2 -$
$2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}$;
$f_{17}^{C}(c,K_1) = 4c^2(h^2-3(1+h)+c^2+2c(2+h)) + 4bc(27-25c+10h)K_1 - 224b^2K_1^2 -$
$7(c^2-2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2} + (c(4-5c+2h) -$
$14bK_1)\sqrt{c^2(4-5c+2h)^2-4bc(16-27c+22h)K_1+148b^2K_1^2}$;
$f_{18}^{C}(c,K_1) = -(1-c)c^5 - 2bc^3(3-2c)K_1 + 12b^2c^2K_1^2 + c^2((1-c)c -$
$2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2} + 2bcK_1(c(2+h)-2bK_1)w_{1,8b} - 8b^2K_1^2w_{1,8b}^2$;
$f_{19}^{C}(c,K_1) = c^3h - 2bc(c+h)K_1 + 4b^2K_1^2 - (c(2(1-c)+h) -$
$6bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2} + 2((1-c)c-2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}$.

Figure 3.5. $K_{1,1}$ and $K_{1,2}$ are defined below.

(1) $K_{1,1} = \begin{cases} \frac{\left((4(4+h)-33c)+\sqrt{57c^2+8c(75+74h)+16(91h^2-3h-23)}\right)c}{68b}, & AC6 \wedge c \leq 1-4h \\ \frac{((1-3c+4h)+\sqrt{(3-c)(3c-1)-8(1+c)h+16h^2})c}{8b}, & AC6 \wedge c > 1-4h \\ \widetilde{K}_{1,3}, & AC7.b \wedge c \leq 1-4h \end{cases}$, where $\widetilde{K}_{1,3} =$ $Root[f_5(c,K_1),1]$, and $f_5(c,K_1) = c^4(3(1-c)+5h)^2(479c^2-2c(88+345h)-2(7-175h+10h^2)) - 34bc^3(3(1-c)+5h)(58c^2+c(377-590h)-2(73-125h-90h^2))K_1 - 289b^2c^2(360c^2-180c(4+h)+71+180h-700h^2)K_1^2 - 19652b^3c(11+6c-10h)K_1^3 + 334084b^4K_1^4$.

(2) $K_{1,2} = \begin{cases} \frac{c(10(1-c)-23h)}{34b}, & (AC8.b \vee AC9) \wedge c \leq 1-4h \\ \frac{ch}{2b}, & AC9 \wedge c > 1-4h \end{cases}$.

Figure 3.6. $K_{1,3}$ and $K_{1,4}$ are defined below.

(1) $K_{1,3} = \begin{cases} Root[1+\frac{h}{2}-\frac{bK_1}{c}-\frac{2bK_1w_{1,8b}}{c^2}-\frac{6+11c+10h}{17},1], & AC8.b \\ \frac{(10-61c+11h+\sqrt{-384+496c+2489c^2+8(44-197c)h+112h^2})c}{34b}, & AC9 \wedge c \leq 1-4h. \\ \frac{(3(h-c)+\sqrt{-1+4c+6c^2-2(1+7c)h+8h^2})c}{2b}, & AC9 \wedge c > 1-4h \end{cases}$

(2) $K_{1,4} = \begin{cases} K_{1,1}, & K_1 \geq \underline{K_1^C} \\ \frac{\left(17(2-h)-2\sqrt{343-414c+360c^2-(109+690c)h+439h^2}\right)c}{102b}, & AC8.a \\ \widetilde{K}_{1,4}, & AC8.b \\ \frac{\left(22+12c-37h-\sqrt{4(193-216c+312c^2)-4(167+802c)h+2169h^2}\right)c}{68b}, & AC9 \\ \frac{\left(17-\sqrt{361-552c+480c^2+40(6-23c)h+200h^2}\right)c}{68b}, & AC10 \end{cases}$, where $\widetilde{K}_{1,4} = Root[\frac{K_1 w_{1,8b}}{c^2} - \frac{3(1-c)+5h}{17b}, 1]$.

Figure 3.7. $K_{1,5}$, and $K_{1,6}$ are defined below.

(1) $K_{1,5} = \begin{cases} \frac{(1-c)(53c-19)+16(1-c)h-32h^2}{136b}, & AC5 \wedge c \leq 1-4h \\ \frac{(1-c)(3c-1)}{8b}, & AC5 \wedge c > 1-4h \\ Root[f_6(c,K_1),1], & AC6 \wedge c \leq 1-4h \\ Root[f_7(c,K_1),1], & AC6 \wedge c > 1-4h \end{cases}$.

(2) $K_{1,6} = \begin{cases} Root[f_8(c,K_1),1], & AC7.a \\ Root\left[\frac{K_1(c^3(w_{1,7b}-c)+2bK_1w_{1,7b}^2)}{c^4} - \frac{9(1-c)^2-4(1-c)h+8h^2}{34b}, 1\right], & AC7.b \\ Root[f_9(c,K_1),1], & AC8.a \\ Root[\frac{K_1(-2c^4+(c^2(2+h)-2bcK_1)w_{1,8b}-4bK_1w_{1,8b}^2)}{2c^4} - \frac{9(1-c)^2-4(1-c)h+8h^2}{34b}, 1], & AC8.b \\ Root[f_{10}(c,K_1),1], & AC9 \wedge c \leq 1-4h \\ Root[f_{11}(c,K_1),1], & AC9 \wedge c > 1-4h \\ Root[f_{12}(c,K_1),1], & AC10 \wedge c \leq 1-4h \\ Root[f_{13}(c,K_1),1], & AC10 \wedge c > 1-4h \\ Root[f_{14}(c,K_1),1], & AC11 \wedge c \leq 1-4h \\ Root[f_{15}(c,K_1),1], & AC11 \wedge c > 1-4h \end{cases}$.

■ $f_i(c,K_1)$ for $i = 6,\dots,15$ are defined below.

$f_6(c,K_1) = -c^2(305-780c+526c^2-(332-247c)h+358h^2) - 17bc(4+65c+16h)K_1 + 782b^2K_1^2 + (17c(2+c+h)+51bK_1)\sqrt{\Delta_1}$;

$f_7(c,K_1) = -c^2(33-86c+59c^2-2(8-3c)h-4h^2) - 2bc(4+65c+16h)K_1 + 92b^2K_1^2 + (2c(2+c+h)+6bK_1)\sqrt{\Delta_1}$;

$f_8(c,K_1) = -c^2(135-474c+424c^2-2(251-200c)h+256h^2) - 34bc(4+37c+16h)K_1 - 408b^2K_1^2 + (17c(8-3c+4h)-136bK_1)\sqrt{\Delta_2}$;

$f_9(c,K_1) = 2c^2(2(9-35c+43c^2)-(8+9c)h-h^2) - 68bc(4-3c-2h)K_1 + 408b^2K_1^2 + \sqrt{2}(17c(2-4c+h)-34bK_1)\sqrt{\Delta_3}$;

$f_{10}(c,K_1) = -c^2(2(1-c)(18-35c)-(16+c)h+32h^2) + 34bc(6-9c-h)K_1 - 340b^2K_1^2 - (17c(2(1-c)+h)-102bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}$;

$f_{11}(c,K_1)=-c^2(2(1-c)(1-2c)-ch)+2bc(6-9c-h)K_1-20b^2K_1^2-(c(2(1-c)+h)-6bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}$;

$f_{12}(c,K_1)=c^2(9-35c+43c^2-4(1-c)h+8h^2)-34b(2-c)cK_1+136b^2K_1^2+17c(1-2c)\sqrt{c^4-4bcK_1+8b^2K_1^2}$;

$f_{13}(c,K_1)=c^2(1-4c+5c^2)-4b(2-c)cK_1+16b^2K_1^2+2c(1-2c)\sqrt{c^4-4bcK_1+8b^2K_1^2}$;

$f_{14}(c,K_1)=c^2\big((1-c)(18-35c)-8(1-c)h+16h^2\big)-34bc(3-4c)K_1+204b^2K_1^2+(17(1-c)c-34bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}$;

$f_{15}(c,K_1)=(1-c)c^2(1-2c)-2bc(3-4c)K_1+12b^2K_1^2+\big((1-c)c-2bK_1\big)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}$.

Figure 3.9. $K_{1,i}^P$ for $i=1,\dots,5$ are defined below.

(1) $K_{1,1}^P=\begin{cases}\frac{c(39(1-c)-68h)}{102b}, & c\le\frac{15-68h}{15}\\ \bar{K}_1^P, & c>\frac{15-68h}{15}\end{cases}.$

(2) $K_{1,2}^P=\begin{cases}Root[f_1^P(c,K_1),1], & AC8.a\cap AP6\\ Root[f_2^P(c,K_1),1], & AC8.b\cap AP6\\ Root[f_3^P(c,K_1),1], & AC9\cap AP6\\ Root[f_4^P(c,K_1),1], & AC10\cap AP6\\ Root[f_5^P(c,K_1),1], & AC11\cap AP6\end{cases}.$

(3) $K_{1,3}^P=\begin{cases}Root[f_6^P(c,K_1),1], & AC11\cap AP8\\ Root[f_7^P(c,K_1),1], & AC8.b\cap AP8\\ Root[f_8^P(c,K_1),1], & AC9\cap AP8\\ Root\left[\widetilde{w}_1-\frac{\left(c^2+2bK_1-\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}\right)c}{4bK_1},1\right], & AC9\cap AP9\\ Root\left[\widetilde{w}_1-\frac{\left(c^2+2bK_1-\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}\right)c}{4bK_1},1\right], & AC11\cap AP9\end{cases}.$

(4) $K_{1,4}^P=\begin{cases}Root[f_9^P(c,K_1),1], & AC7.a\cap AP6\\ Root[f_{10}^P(c,K_1),1], & AC6\cap AP6\\ Root[f_{11}^P(c,K_1),1], & AC6\cap AP2\end{cases}.$

(5) $K_{1,5}^P=\begin{cases}Root[f_{12}^P(c,K_1),1], & AC8.a\cap AP6\\ Root[f_{13}^P(c,K_1),1], & AC8.b\cap AP6.\\ Root[f_{14}^P(c,K_1),1], & AC9\cap AP6\end{cases}$

■ $f_i^P(c,K_1)$ for $i=1,\dots,14$ are defined below.

$f_1^P(c,K_1) = -2c^3(3c-1) - 2bc(13c-2)K_1 + 28b^2K_1^2 + \sqrt{2}(c(3c-1) +$

$14bK_1)\sqrt{c^2(2c^2-h^2) - 4bc(2-h)K_1 + 12b^2K_1^2} +$

$2bK_1\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2};$

$f_2^P(c,K_1) = w_{1,8b} - \frac{c((2+c)c+14bK_1+\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2})}{2(14bK_1-c(1-3c))};$

$f_3^P(c,K_1) = c(3c-1)(c^2+6bK_1) - (c(3c-1)+14bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2} -$

$2bK_1\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2};$

$f_4^P(c,K_1) = -c^3(3c-1) - bc(13c-2)K_1 + 14b^2K_1^2 + (c(3c-1) +$

$14bK_1)\sqrt{c^4-4bcK_1+8b^2K_1^2} + bK_1\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2};$

$f_5^P(c,K_1) = c(3c-1)(c^2+6bK_1) - (c(3c-1)+14bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2} -$

$2bK_1\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2};$

$f_6^P(c,K_1) = c^3(3c-1) - 2bc(1+c)K_1 - 8b^2K_1^2 + (c(1-3c) +$

$2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2} - 2bK_1\sqrt{c^2(2-3c)^2+4bc(2+3c)K_1+4b^2K_1^2};$

$f_7^P(c,K_1) = w_{1,8b} - \frac{((3c^2+2bK_1)+\sqrt{c^2(2-3c)^2+4bc(2+3c)K_1+4b^2K_1^2})c}{2((3c-1)c-2bK_1)};$

$f_8^P(c,K_1) = c^3(3c-1) - 2bc(1+c)K_1 - 8b^2K_1^2 + (c(1-3c) +$

$2bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2} - 2bK_1\sqrt{c^2(2-3c)^2+4bc(2+3c)K_1+4b^2K_1^2};$

$f_9^P(c,K_1) = c^2((3c-1)(5c-4)+4(2c-3)h) + 2bc(48c-23)K_1 + 196b^2K_1^2 + (c(3c-1-$

$4h) + 22bK_1)\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2} +$

$\sqrt{c^2(2-12c+11c^2-2(3c-1)h+4h^2)+4bc(6+3c-4h)K_1-40b^2K_1^2}(2c(-3+2c) -$

$2\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2});$

$f_{10}^P(c,K_1) = c^2((5c-4)^2+2(2c-3)h) + 4bc(31c-22)K_1 + 196b^2K_1^2 + (c(3-2c-2h) +$

$8bK_1)\sqrt{c^2(12-24c+13c^2) + 12bc(5c-4)K_1 + 84b^2K_1^2} +$

$\sqrt{c^2(4-5c+2h)^2+4bc(-16+27c-22h)K_1+148b^2K_1^2}(c(2c-3) -$

$\sqrt{c^2(12-24c+13c^2)+12bc(5c-4)K_1+84b^2K_1^2});$

$f_{11}^P(c,K_1) = c(-16+13c+6h) + 74bK_1 +$

$3\sqrt{c^2(4-5c+2h)^2+4bc(-16+27c-22h)K_1+148b^2K_1^2};$

$f_{12}^P(c,K_1) = c^2((2-c)(1-3c)+(3-2c)h) + 4bc(1-3c)K_1 - 28b^2K_1^2 + \sqrt{2}c(3-$

$2c)\sqrt{c^2(2c^2-h^2) - 4bc(2-h)K_1 + 12b^2K_1^2} +$

$\sqrt{c^2(12-24c+13c^2)+12bc(5c-4)K_1+84b^2K_1^2}(c(1-3c+h)-4bK_1+$

$\sqrt{2}\sqrt{c^2(2c^2-h^2)-4bc(2-h)K_1+12b^2K_1^2});$

$f_{13}^P(c,K_1)=w_{1,8b}-$

$$\frac{c\left(c^2\big((1-c)(2+c)+(3-2c)h\big)+4bc(1-3c)K_1-28b^2K_1^2+(c(1-c+h)-4bK_1)\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}\right)}{4bK_1\left(c(3-2c)+\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}\right)};$$

$f_{14}^P(c,K_1)=c^2(2-4c+c^2+(3-2c)h)-2bc(1+4c)K_1-28b^2K_1^2+c(3-$

$2c)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}+$

$\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(-c(-1+2c-h)-6bK_1+$

$\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}).$

Figure 3.10. $K_{1,6}^P$ and $K_{1,7}^P$ are defined below.

(1) $K_{1,6}^P=\begin{cases}Root[f_{15}^P(c,K_1),1], & AC8.a\cap AP7\\ Root[f_{16}^P(c,K_1),1], & (AC8.a\cap AP6)\cup(AC9\cap AP6)\end{cases}.$

(2) $K_{1,7}^P=\begin{cases}Root[f_{17}^P(c,K_1),1], & K_1<\underline{K_1^C}\cap(AC8.a\cap AP6)\\ Root[f_{18}^P(c,K_1),1], & K_1<\underline{K_1^C}\cap(AC8.b\cap AP6)\\ Root[f_{19}^P(c,K_1),1], & K_1<\underline{K_1^C}\cap(AC9\cap AP6)\\ Root[f_{20}^P(c,K_1),1], & K_1<\underline{K_1^C}\cap(AC10\cap AP6)\\ Root[f_{21}^P(c,K_1),1], & K_1<\underline{K_1^C}\cap(AC9\cap AP2)\\ Root[f_{22}^P(c,K_1),1], & K_1<\underline{K_1^C}\cap(AC11\cap AP2)\\ K_{1,4}^P, & K_1\geq\underline{K_1^C}\end{cases}.$

■ $f_i^P(c,K_1)$ for $i=15,\dots,22$ are defined below.

$f_{15}^P(c,K_1)=-c^2(2(1-c)(1-5c)-(3-c)h)-2bc(13+c-3h)K_1+52b^2K_1^2+2(ch-$

$6bK_1)\sqrt{c^2(3-6c+4c^2)+12bcK_1-24b^2K_1^2};$

$f_{16}^P(c,K_1)=c^2(2(1-c)(1-3c)-(3-2c)h)-14bc(1-2c)K_1+56b^2K_1^2-(ch-$

$6bK_1)\sqrt{c^2(12-24c+13c^2)-12b(4-5c)cK_1+84b^2K_1^2};$

$f_{17}^P(c,K_1)=c^2(4-3c)(1-3c)-2bc(5-8c)K_1+28b^2K_1^2+\sqrt{2}c(3-$

$2c)\sqrt{c^2(2c^2-h^2)-4bc(2-h)K_1+12b^2K_1^2}+$

$\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c(1-3c)+2bK_1+$

$\sqrt{2}\sqrt{c^2(2c^2-h^2)-4bc(2-h)K_1+12b^2K_1^2});$

$$f_{18}^P(c,K_1)=\frac{K_1w_{1,8b}}{c^2}-\frac{(1-c)c^2(4-5c)-2bc(5-8c)K_1+28b^2K_1^2+((1-c)c+2bK_1)\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}}{4bc(c(3-2c)+\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2})};$$

$f_{19}^{P}(c,K_1) = c^2(4-12c+7c^2) - 4bc(4-5c)K_1 + 28b^2K_1^2 + c(3-2c)\sqrt{c^2(c^2+4bK_1) - 4bc(2-h)K_1 + 12b^2K_1^2} + \sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2}(c(1-2c) + \sqrt{c^2(c^2+4bK_1) - 4bc(2-h)K_1 + 12b^2K_1^2});$

$f_{20}^{P}(c,K_1) = c^2(4-3c)(1-3c) - 2bc(5-8c)K_1 + 28b^2K_1^2 + 2c(3-2c)\sqrt{c^4 - 4bcK_1 + 8b^2K_1^2} + \sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2}(c(1-3c) + 2bK_1 + 2\sqrt{c^4 - 4bcK_1 + 8b^2K_1^2});$

$f_{21}^{P}(c,K_1) = -c(4-7c) + 14bK_1 - 3\sqrt{c^2(c^2+4bK_1) - 4bc(2-h)K_1 + 12b^2K_1^2};$

$f_{22}^{P}(c,K_1) = -c(4-7c) + 14bK_1 - 3\sqrt{c^4 - 4b(2-c)cK_1 + 20b^2K_1^2}.$

Figure 3.11. $K_{1,i}^{P}$ for $i = 8,\ldots,12$ are defined below.

(1) $K_{1,8}^{P} = \begin{cases} Root[f_1(c,K_1),1], & AC8.b \cap AP7 \\ \frac{(2c-1)c}{2b}, & AC11 \cap AP7 \end{cases}.$

(2) $K_{1,9}^{P} = \begin{cases} Root[f_{23}^{P}(c,K_1),1], & AC8.b \cap AP6 \\ Root[f_{24}^{P}(c,K_1),1], & AC9 \cap AP6 \\ Root[f_{25}^{P}(c,K_1),1], & AC11 \cap AP6 \end{cases}.$

(3) $K_{1,10}^{P} = \begin{cases} Root[f_{26}^{P}(c,K_1),1], & AC11 \cap AP8 \\ Root[f_{27}^{P}(c,K_1),1], & AC8.b \cap AP8. \\ Root[f_{28}^{P}(c,K_1),1], & AC9 \cap AP8 \end{cases}$

(4) $K_{1,11}^{P} = \begin{cases} \frac{(1-3c+\sqrt{-3+10c-3c^2})c}{8b}, & AC5 \cap AP6 \\ \frac{\left(2\sqrt{2}\sqrt{(3-c)(3+13c)}-9-7c\right)c}{18b}, & AC5 \cap AP3 \\ \frac{(16-7c-3\sqrt{19-40c+30c^2})c}{68b}, & AC5 \cap AP2 \\ Root[f_{29}^{P}(c,K_1),1], & AC6 \cap AP3 \\ Root[f_{30}^{P}(c,K_1),1], & AC6 \cap AP2 \end{cases}.$

(5) $K_{1,12}^{P} = \begin{cases} Root[f_{31}^{P}(c,K_1),1], & AC10 \cap AP6 \\ Root[f_{32}^{P}(c,K_1),1], & AC8.a \cap AP6 \\ Root[f_{33}^{P}(c,K_1),1], & AC8.b \cap AP6 \\ Root[f_{34}^{P}(c,K_1),1], & AC9 \cap AP6 \\ Root[f_{35}^{P}(c,K_1),1], & AC11 \cap AP6 \\ \frac{c(10c-1)}{178b}, & AC11 \cap AP2 \\ Root[f_{36}^{P}(c,K_1),1], & AC9 \cap AP2 \\ Root[f_{37}^{P}(c,K_1),1], & AC7.a \cap AP6 \\ Root[f_{38}^{P}(c,K_1),1], & AC6 \cap AP6 \end{cases}.$

■ $f_i^P(c,K_1)$ for $i=23,\ldots,38$ are defined below.

$f_{23}^P(c,K_1)=c^6((1-c)^2c(24-23c)+(21-36c+17c^2)h^2)-4bc^5(6-15c+5c^2+(21-36c+17c^2)h+3(4-5c)h^2)K_1+4b^2c^4(78-162c+127c^2+12(4-5c)h+21h^2)K_1^2-48b^3c^3(17-37c+7h)K_1^3+2016b^4c^2K_1^4+(-8bc^5(21-36c+17c^2)K_1+96b^2c^4(4-5c)K_1^2-672b^3c^3K_1^3)w_{1,8b}+(8b^2c^2(21-36c+17c^2)K_1^2-96b^3c(4-5c)K_1^3+672b^4K_1^4)w_{1,8b}^2+\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c^5(7(1-c)^2c+2(3-2c)h^2)-2bc^4((3+c)(1-3c)+4(3-2c)h)K_1+4b^2c^3(12+13c)K_1^2+168b^3c^2K_1^3-16bc^4(3-2c)K_1w_{1,8b}+16b^2c(3-2c)K_1^2w_{1,8b}^2)$;

$f_{24}^P(c,K_1)=c^4(2c(12-46c+53c^2-20c^3)+(21-36c+17c^2)h^2)-2bc^3(2(27-51c+10c^2+15c^3)+(21-36c+17c^2)h+6(4-5c)h^2)K_1+4b^2c^2(14(12-21c+10c^2)+6(4-5c)h+21h^2)K_1^2-24b^3c(64-94c+7h)K_1^3+2688b^4K_1^4+\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}(c^4(21-36c+17c^2)-2bc^2(21-12c-13c^2)K_1+12b^2c(8-3c)K_1^2-168b^3K_1^3)+\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c^3(c(7-20c+11c^2)+2(3-2c)h^2)+2bc^2(-15+16c+3c^2-2(3-2c)h)K_1+4b^2c(24+5c)K_1^2+168b^3K_1^3+2c(3-2c)(c^2-2bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2})$;

$f_{25}^P(c,K_1)=2c^5(12-46c+53c^2-20c^3)-4bc^3(27-51c+10c^2+15c^3)K_1+56b^2c^2(12-21c+10c^2)K_1^2-48b^3c(32-47c)K_1^3+2688b^4K_1^4+\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}(c^4(21-36c+17c^2)-2bc^2(21-12c-13c^2)K_1+12b^2c(8-3c)K_1^2-168b^3K_1^3)+\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c^4(7-20c+11c^2)-2bc^2(15-16c-3c^2)K_1+4b^2c(24+5c)K_1^2+168b^3K_1^3+2c(3-2c)(c^2-2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2})$;

$f_{26}^P(c,K_1)=2c^4(1-2c-6c^2+15c^3-9c^4)+4bc^3(5-11c+10c^2-9c^3)K_1+8b^2c^2(14-17c+14c^2)K_1^2+16b^3c(16+15c)K_1^3+288b^4K_1^4+\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}(c^4(5-12c+9c^2)-2bc^2(5-20c+3c^2)K_1-4b^2c(8+c)K_1^2-40b^3K_1^3)+\sqrt{c^2(2-3c)^2+4bc(2+3c)K_1+4b^2K_1^2}(c^3(2-7c+6c^2-3c^3)+2b(1-c)c^2(3+7c)K_1+4b^2c(10+3c)K_1^2+120b^3K_1^3+2(c^2-2bK_1)(c+4bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2})$;

$f_{27}^P(c,K_1)=c^6((1-c)^2(2-9c^2)+(5-12c+9c^2)h^2)+4bc^5(10-23c+23c^2-6c^3-(5-12c+9c^2)h+(4+3c)h^2)K_1+4b^2c^4(34+2c+15c^2-4(4+3c)h+5h^2)K_1^2+16b^3c^3(13+9c-5h)K_1^3+128b^4c^2K_1^4-8bK_1(c^2(5-12c+9c^2)+4bc(4+3c)K_1+20b^2K_1^2)w_{1,8b}(c^3-bK_1w_{1,8b})+\sqrt{c^2(2-3c)^2+4bc(2+3c)K_1+4b^2K_1^2}(c^5((1-c)^2(2-3c)+2h^2)+2bc^4((1+$

$c)(7-3c)-4h+4h^2)K_1+4b^2c^3(14+3c-8h)K_1^2+56b^3c^2K_1^3-16bK_1(c+$
$4bK_1)w_{1,8b}(c^3-bK_1w_{1,8b}));$

$f_{28}^P(c,K_1)=c^4(2(1-2c-6c^2+15c^3-9c^4)+(5-12c+9c^2)h^2)+2bc^3(2(5-11c+$
$10c^2-9c^3)-(5-12c+9c^2)h+2(4+3c)h^2)K_1+4b^2c^2(2(14-17c+14c^2)-2(4+$
$3c)h+5h^2)K_1^2+8b^3c(32+30c-5h)K_1^3+288b^4K_1^4+(c^2-$
$2bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}(c^2(5-12c+9c^2)+4bc(4+3c)K_1+20b^2K_1^2)+$
$\sqrt{c^2(2-3c)^2+4bc(2+3c)K_1+4b^2K_1^2}(c^3(2-7c+6c^2-3c^3+2h^2)+2bc^2(3+4c-7c^2-$
$2h+4h^2)K_1+4b^2c(10+3c-4h)K_1^2+120b^3K_1^3+2(c^2-2bK_1)(c+$
$4bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2});$

$f_{29}^P(c,K_1)=-c^2(185-690c+601c^2-32(8-3c)h-64h^2)-4bc(473+79c+128h)K_1-$
$292b^2K_1^2+(32c(2+c+h)+$
$96bK_1)\sqrt{c^2(4-5c+2h)^2-4bc(16-27c+22h)K_1+148b^2K_1^2};$

$f_{30}^P(c,K_1)=c^2(23+44c-94c^2+9(8-3c)h+18h^2)-bc(1604-983c+144h)K_1+$
$3746b^2K_1^2+(9c(2+c+h)+$
$27bK_1)\sqrt{c^2(4-5c+2h)^2-4bc(16-27c+22h)K_1+148b^2K_1^2};$

$f_{31}^P(c,K_1)=c^4(7-39c+91c^2-89c^3+32c^4)-bc^3(104-245c+262c^2-129c^3)K_1+$
$2b^2c^2(213-370c+201c^2)K_1^2-4b^3c(206-191c)K_1^3+952b^4K_1^4+c(1-$
$2c)\sqrt{c^4-4bcK_1+8b^2K_1^2}(c^2(21-36c+17c^2)-12bc(4-5c)K_1+84b^2K_1^2)+$
$\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c^3(2-9c+16c^2-7c^3)-bc^2(25-$
$18c-3c^2)K_1+4b^2c(13-4c)K_1^2+28b^3K_1^3+2c^2(3-2c)(1-2c)\sqrt{c^4-4bcK_1+8b^2K_1^2});$

$f_{32}^P(c,K_1)=\sqrt{2}(c(2-4c+h)-2bK_1)\sqrt{c^2(2c^2-h^2)-4bc(2-h)K_1+12b^2K_1^2}(c^2(21-$
$36c+17c^2)-12bc(4-5c)K_1+84b^2K_1^2)+2(c^4(2(7-39c+91c^2-89c^3+32c^4)-c(21-$
$36c+17c^2)h-(21-36c+17c^2)h^2)+4bc^3(-52+133c-149c^2+73c^3+(21-24c+$
$2c^2)h+3(4-5c)h^2)K_1+4b^2c^2(2(96-179c+107c^2)-3(16-13c)h-21h^2)K_1^2-$
$112b^3c(13-13c-3h)K_1^3+1568b^4K_1^4)+$
$\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(2\sqrt{2}c(3-2c)(c(2-4c+h)-$
$2bK_1)\sqrt{c^2(2c^2-h^2)-4bc(2-h)K_1+12b^2K_1^2}-4(c^3(-2+9c-16c^2+7c^3+c(3-2c)h+$
$(3-2c)h^2)+bc^2(25-24c+c^2-4(3-2c)h)K_1-8b^2(5-c)cK_1^2-28b^3K_1^3));$

$f_{33}^P(c,K_1)=c^2((1-c)^2c^4(7-4c-2c^2)-bc^3(20-53c+38c^2-9c^3)K_1+2b^2c^2(33-$
$64c+49c^2)K_1^2-4b^3c(26-71c)K_1^3+280b^4K_1^4)-bcK_1(c(2+h)-2bK_1)(c^2(21-36c+$
$17c^2)-12bc(4-5c)K_1+84b^2K_1^2)w_{1,8b}+4b^2K_1^2(c^2(21-36c+17c^2)-12bc(4-5c)K_1+$

$84b^2K_1^2)w_{1,8b}^2 + \sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c^2((1-c)^2c^3(2+c) -$
$bc^2(1+c)(1-3c)K_1 + 4b^2c(1+4c)K_1^2 + 28b^3K_1^3) - 2bc^2(3-2c)K_1(c(2+h) -$
$2bK_1)w_{1,8b} + 8b^2c(3-2c)K_1^2w_{1,8b}^2);$

$f_{34}^P(c,K_1) = c^4(2(1-c)(14-43c+40c^2-13c^3) - c(21-36c+17c^2)h) - 2bc^3(166 -$
$475c + 466c^2 - 163c^3 - (21-12c-13c^2)h)K_1 + 4b^2c^2(5(63-130c+75c^2) - 3(8 -$
$3c)h)K_1^2 - 8b^3c(298-397c-21h)K_1^3 + 2800b^4K_1^4 + (2(1-c)c + ch -$
$6bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}(c^2(21-36c+17c^2) - 12bc(4-5c)K_1 +$
$84b^2K_1^2) + 2\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c(3-2c)(2(1-c)c + ch -$
$6bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2} - (c^3(c(3-2c)h - 2(1-c)(2-4c+c^2)) +$
$2bc^2(19-29c+7c^2-(3-2c)h)K_1 - 4b^2c(17-2c)K_1^2 - 56b^3K_1^3));$

$f_{35}^P(c,K_1) = (1-c)c^4(14-43c+40c^2-13c^3) - 2bc^3(83-227c+215c^2-73c^3)K_1 +$
$4b^2c^2(168-331c+181c^2)K_1^2 - 56b^3c(23-29c)K_1^3 + 1568b^4K_1^4 + ((1-c)c -$
$2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}(c^2(21-36c+17c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2) +$
$2\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c(3-2c)((1-c)c -$
$2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2} + ((1-c)c^3(2-4c+c^2) - bc^2(19-26c+5c^2)K_1 +$
$8b^2(5-c)cK_1^2 + 28b^3K_1^3));$

$f_{36}^P(c,K_1) = -c^2(2(1-c)(2-11c) - 9ch) - 2bc(10+17c+9h)K_1 + 92b^2K_1^2 + (54bK_1 -$
$9c(2(1-c)+h))\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2};$

$f_{37}^P(c,K_1) = -c^4(35-270c+554c^2-444c^3+115c^4+6(3-2c)(21-36c+17c^2)h +$
$8(21-36c+17c^2)h^2) + bc^3(52-427c+626c^2-27c^3+8(192-351c+158c^2)h +$
$96(4-5c)h^2)K_1 + 2b^2c^2(921-1744c+1843c^2-12(127-122c)h-336h^2)K_1^2 -$
$4b^3c(1394-3335c-672h)K_1^3 + 15736b^4K_1^4 +$
$\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2}(c^3(-10+69c-102c^2+53c^3-12(3-$
$2c)^2h - 16(3-2c)h^2) + bc^2(64(3-2c)h - (1-3c)(1+81c))K_1 + 4b^2c(85+172c)K_1^2 +$
$1372b^3K_1^3) +$
$\sqrt{c^2(2-12c+11c^2+2(1-3c)h+4h^2)+4bc(6+3c-4h)K_1-40b^2K_1^2}(2c(3-$
$2c)(8bK_1 - c(8-3c+4h))\sqrt{c^2(12-24c+13c^2)-12bc(4-5c)K_1+84b^2K_1^2} + (8bK_1 -$
$c(8-3c+4h))(c^2(21-36c+17c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2));$

$f_{38}^P(c,K_1) = c^4(175-468c+390c^2-78c^3-13c^4-(8-3c)(21-36c+17c^2)h - 2(21-$
$36c+17c^2)h^2) - 2bc^3(256-496c+243c^2-90c^3-2(180-300c+113c^2)h - 12(4-$
$5c)h^2)K_1 + 4b^2c^2(351-572c+615c^2-3(120-101c)h-42h^2)K_1^2 - 8b^3c(319-1048c -$

$168h)K_1^3 + 9856b^4K_1^4 + \sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2}(c^3((2+c)(25 - 52c + 29c^2) - 2(3-2c)(8-3c)h - 4(3-2c)h^2) - bc^2(25 + 116c - 279c^2 + 32(-3 + 2c)h)K_1 - 8b^2c(10-121c)K_1^2 + 1372b^3K_1^3) - \sqrt{c^2(4-5c+2h)^2 - 4bc(16-27c+22h)K_1 + 148b^2K_1^2}(2c(3-2c)(c(2+c+h) + 3bK_1)\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2} + (c(2+c+h) + 3bK_1)(c^2(21 - 36c + 17c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2)).$

B.2.2 Definitions in Appendix B.1

Table B.2. $g_i^C(w_1)$ for $i = 1, \dots, 14$ are defined below.

$g_1^C(w_1) = -2c(c(c-2h) + 2bK_1) + 2(4bK_1 - c(1-3c+4h))w_1 + (1-3c+4h)w_1^2;$

$g_2^C(w_1) = (3-c)c - 4bK_1 - 2(1+c)w_1 + 2w_1^2;$

$g_3^C(w_1) = -2c(c-2bK_1) + 4c^2w_1 + (1-3c)w_1^2;$

$g_4^C(w_1) = -4c^2(c-2bK_1) + 2c(c(1+3c) - 4bK_1)w_1 + (c(1-5c) + 4bK_1)w_1^2;$

$g_5^C(w_1) = c^2(c(2-3c+2h) - 2bK_1) - c(c(4-5c+2h) + 6bK_1)w_1 + 14bK_1w_1^2;$

$g_6^C(w_1) = c(6-c-2h) - 6bK_1 - 2(3(1+c) - 2h)w_1 + 7w_1^2;$

$g_7^C(w_1) = -c^2(2+c-2h) + 2bcK_1 + (c(7c-4h) + 2bK_1)w_1 + (2(1-3c) + h)w_1^2;$

$g_8^C(w_1) = c^4(c^2 - 4(1-c+h)(1-h)) + 4bc^3(6-c-2h)K_1 - 28b^2c^2K_1^2 + (-2c^3(5c^2 - 2(1+h)(1-2h) - 2c(1+3h)) - 8bc^2(3(1+c) - 2h)K_1 + 40b^2cK_1^2)w_1 + (c^2(11c^2 - 6c(2 + h) + 2(1+h+2h^2)) + 28bc^2K_1 - 28b^2K_1^2)w_1^2;$

$g_9^C(w_1) = -2c^4(c-2bK_1)^2 + 2c^3(c^2(2c+h^2) + 4bc(1-\mathrm{cs}-h)K_1 - 4b^2K_1^2)w_1 - c^2(c^2(2c^2+h^2) + 4bc(2(1+c) - h)K_1 - 4b^2K_1^2)w_1^2 + 8bcK_1(c^2 + 2bK_1)w_1^3 - 8b^2K_1^2w_1^4;$

$g_{10}^C(w_1) = c^2(c-2bK_1) - c(c^2 + 2bK_1)w_1 + 2bK_1w_1^2;$

$g_{11}^C(w_1) = c^2(c-2bK_1) - c^3w_1 + bK_1w_1^2;$

$g_{12}^C(w_1) = c^2(c(2-h) - 2bK_1) - 2c(c^2 + 2bK_1)w_1 + 4bK_1w_1^2;$

$g_{13}^C(w_1) = c^2(c^2h^2 + 4bc(2-h)K_1 - 12b^2K_1^2) - 8bc^3K_1w_1 + 8b^2K_1^2w_1^2;$

$g_{14}^C(w_1) = -4c^2(c-2bK_1)^2 + 2c(c^2(1+c^2+4c(2-h) + 4h^2) - 4bc(1+3c+2h)K_1 + 8b^2K_1^2)w_1 + (c^2(5-4c(4-7h) - 25c^2 - 8h - 20h^2) - 4bc(1-3c-10h)K_1 - 16b^2K_1^2)w_1^2 + 2(c(21c^2 - c(7+12h) + 4h(1+2h)) - 2b(1-3c+4h)K_1)w_1^3 - 2(1-3c(2+h) + 9c^2 + h + 2h^2)w_1^4.$

Figure B.2. $\widetilde{K}_{1,1}$, $\widetilde{K}_{1,2}$, and $f_1(c, K_1)$ are defined below.

(1) $f_1(c,K_1)=\begin{cases} f_{1,1}(c,K_1), c\le \hat{c} \\ f_{1,2}(c,K_1), c>\hat{c}\end{cases}$, where $\hat{c}=Root[f(c),2]$ with $f(c)=46c^4-4c^3(19+119h)+2c^2(-7+432h+189h^2)-2c(-36+252h+275h^2+57h^3)-(7-36h+7h^2)(4+4h+9h^2)$. $\hat{c}$ denotes the horizontal coordinate where the curve $f_1(c,K_1)=0$ intersects with curve 11 in Figure B.2.

(2) $\widetilde{K}_{1,1}\in[0,\bar{K}_1^C]$ and $\widetilde{K}_{1,1}=\begin{cases} Root[f_2(c,K_1),1], f_1(c,K_1)\le 0 \\ Root[f_3(c,K_1),1], f_1(c,K_1)>0\end{cases}$.

(3) $\widetilde{K}_{1,2}\in[0,\bar{K}_1^C]$ and $\widetilde{K}_{1,2}=\begin{cases} \frac{((14+6c-9h)-4\sqrt{31c^2-24c(1+h)+2(9-5h+9h^2)})c}{46b}, & f_1(c,K_1)\le 0 \\ Root[f_4(c,K_1),1], & f_1(c,K_1)>0\end{cases}$.

$f_1(c,K_1)\le 0$ indicates the left side of the dashed curve in Figure B.2.

■ $f_{1,1}(c,K_1)$, $f_{1,2}(c,K_1)$, and $f_i(c,K_1)$ for $i=2,\dots,4$ are defined below.

$f_{1,1}(c,K_1)=-2(c^4-2bc(1+c)K_1+4b^2K_1^2)+\sqrt{2}(c^2-2bK_1)\sqrt{c^2(2c^2-h^2)-4bc(2-h)K_1+12b^2K_1^2}$;

$f_{1,2}(c,K_1)=(1-c)c^6(1-c+h)-2bc^4(c^2-c(7+2h)+2(3+2h))K_1-2b^2c^2(10c^2-2c(3+h)-6h-17)K_1^2-8b^3c(15-2c+2h)K_1^3+104b^4K_1^4$;

$f_2(c,K_1)=-c^2(11c^2-6c(2+h)+2(1+h+2h^2))-4bc(3+5c-2h)K_1+28b^2K_1^2-(c(4-5c+2h)-14bK_1)\sqrt{c^2(11c^2-6c(2+h)+2(1+h+2h^2))+4bc(6+3c-4h)K_1-40b^2K_1^2}$;

$f_3(c,K_1)=c^4(4-5c+2h)^2(9c^2-3c(2+h)+1+h+2h^2)+2bc^3(393c^3-c^2(281+564h)+2c(-29+202h+180h^2)-4(-9+18h+51h^2+26h^3))K_1+4b^2c^2(362c^2-c(442+669h)+412+447h+222h^2)K_1^2+8b^3c(-1409+1091c-736h)K_1^3+20608b^4K_1^4+(c^3(4-5c+2h)(9c^2-3c(2+h)+1+h+2h^2)-2bc^2(30c^2+c(29-45h)-13+15h+30h^2)K_1+4b^2c(-101+107c-12h)K_1^2+1344b^3K_1^3)\sqrt{c^2(4-5c+2h)^2-4bc(16-27c+22h)K_1+148b^2K_1^2}$;

$f_4(c,K_1)=c^4(2(1-c)+h)^2(36c^2-12c(2+h)+4+4h+9h^2)-8bc^3(2(1-c)+h)(48c^2-2c(2+17h)-(2+3h)(2-9h))K_1+8b^2c^2(12c^2+8c(26-23h)-20+108h+127h^2)K_1^2-32b^3c(2(31+24c)+23h)K_1^3+3088b^4K_1^4$.

Table B.3. $f_i^C(c,K_1)$ for $i=1,\dots,12$ and $g_i^C(w_1)$ for $i=15,\dots,18$ are defined below.

$f_1^C(c,K_1)=\frac{c^2(23(1-c)(1-c+h)+18h^2)+4bc(13(1-c)-18h)K_1-26b^2K_1^2}{98bc^2}$;

$f_2^C(c,K_1)=\frac{-c^2(12-16c+3c^2+4(3-2c)h-4h^2)+4bc(34-25c+10h)K_1-196b^2K_1^2+(c(4-5c+2h)-14bK_1)\sqrt{\Delta_1}}{56bc^2}$;

$f_3^C(c,K_1) = \frac{c^2(4-5c+2h)(2+c+h)-bc(4+65c+16h)K_1+46b^2K_1^2+(c(2+c+h)+3bK_1)\sqrt{\Delta_1}}{98bc^2};$

$f_4^C(c,K_1) =$

$\frac{-c^2(52-88c+41c^2+4(13-11c)h-36h^2)+24bc(39-40c+9h)K_1-1188b^2K_1^2+(c(18(2+h)-31c)-86bK_1)\sqrt{\Delta_1}}{392bc^2};$

$f_5^C(c,K_1) = \frac{c^2(18-24c+c^2+6(3-2c)h+8h^2)-2bc(4+37c+16h)K_1-24b^2K_1^2+(c(8-3c+4h)-8bK_1)\sqrt{\Delta_2}}{98bc^2};$

$f_6^C(c,K_1) = \frac{2c^2(2c(1-2c)+ch+h^2)+4bc(4-3c-2h)K_1-24b^2K_1^2+(2bK_1-c(2(1-2c)+h))\sqrt{2}\sqrt{\Delta_3}}{8bcs^2};$

$f_7^C(c,K_1) = \frac{-c^2(c-h)(c+h)+2bc(2-h)K_1+4b^2K_1^2+(c^2-2bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}}{8bcs^2};$

$f_8^C(c,K_1) = \frac{c^3(2(1-c)+h)+2bc(6-9c-h)K_1-20b^2K_1^2-(c(2(1-c)+h)-6bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}}{8bc^2};$

$f_9^C(c,K_1) = \frac{c^2(c(2-3c)+ch+h^2)+2bc(8-9c-2h)K_1-16b^2K_1^2-(c(2-3c+h)-4bK_1)\sqrt{c^4-4bc(2-c-h)K_1+12b^2K_1^2}}{8bc^2};$

$f_{10}^C(c,K_1) = \frac{c^3(1-2c)+2b(2-c)cK_1-8b^2K_1^2-c(1-2c)\sqrt{c^4-4bcK_1+8b^2K_1^2}}{2bc^2};$

$f_{11}^C(c,K_1) = \frac{-c^4+4bcK_1+4b^2K_1^2+(c^2-2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{8bcs^2};$

$f_{12}^C(c,K_1) = \frac{c^3(2-3c)+16b(1-c)cK_1-20b^2K_1^2-(c(2-3c)-2bK_1)\sqrt{c^4-4b(2-c)cK_1+20b^2K_1^2}}{8bcs^2}.$

And:

$g_{15}^C(w_1) = \frac{-(1-c)c^4(1-c+h)+2bc^2K_1((2-c)c+(4-5c+2h)w_1)-2b^2K_1^2(c^2+4cw_1+7w_1^2)}{2bc^4};$

$g_{16}^C(w_1) = \frac{-(1-c)c^4(1-c+h)+4b(1-c)c^3K_1-2b^2c^2K_1^2+(4bc^2(2(1-c)+h)K_1-8b^2cK_1^2)w_1-10b^2K_1^2w_1^2}{2bcs^4};$

$g_{17}^C(w_1) = \frac{c^4h^2+4bc^3(2-h)K_1-4b^2c^2K_1^2-8bc^3K_1w_1+8b^2K_1^2w_1^2}{8bc^4};$

$g_{18}^C(w_1) = \frac{c^4h^2+4bc^3(2(1-c)-h)K_1-4b^2c^2K_1^2+(4bc^2(2(1-c)+h)K_1-8b^2cK_1^2)w_1-8b^2K_1^2w_1^2}{8bc^4}.$

Table B.4. $g_i^P(w_1)$ for $i = 1, \ldots, 13$ are defined below.

$g_1^P(w_1) = -2c(c^2 + 2bK_1) - 2(c(1-3c) - 4bK_1)w_1 + (1-3c)w_1^2;$

$g_2^P(w_1) = (3-c)c - 4bK_1 - 2(1+c)w_1 + 2w_1^2;$

$g_3^P(w_1) = -2c(c - 2bK_1) + 4c^2w_1 + (1-3c)w_1^2;$

$g_4^P(w_1) = -4c^2(c - 2bK_1) + 2c((1+3c)c - 4bK_1)w_1 + ((1-5c)c + 4bK_1)w_1^2;$

$g_5^P(w_1) = -c^2((2-c)c + 2bK_1) + c(c(2+c) + 14bK_1)w_1 + (c(1-3c) - 14bK_1)w_1^2;$

$g_6^P(w_1) = (3-c)c^2 - 3(c(1+c) + 2bK_1)w_1 + 4cw_1^2;$

$g_7^P(w_1) = -c^3 + c(3c^2 + 2bK_1)w_1 + (c(1-3c) + 2bK_1)w_1^2;$

$g_8^P(w_1) = -2c^2((1-c)c^2 - 2bcK_1 + 4b^2K_1^2) + 2c((1-c)c^2(1+2c) - 2bc(1+2c)K_1 + 16b^2K_1^2)w_1 + ((1-c)c^2(1-5c) + 4bc(1+c)K_1 - 44b^2K_1^2)w_1^2;$

$g_9^P(w_1) = -(6-c)c + 6bK_1 + 6(1+c)w_1 - 7w_1^2;$

$g_{10}^P(w_1) = -c^2(2+c) + 2bcK_1 + (7c^2 + 2bK_1)w_1 + 2(1-3c)w_1^2;$

$g_{11}^P(w_1) = c^2(c^2(c^2 - 4(1-c)) + 4b(6-c)cK_1 - 28b^2K_1^2) + 2c(c^2(2(1+c) - 5c^2) - 12bc(1+c)K_1 + 20b^2K_1^2)w_1 + (c^2(2(1-6c) + 11c^2) + 28bK_1(c^2 - bK_1))w_1^2;$

$g_{12}^P(w_1) = -c^3 + c(c^2 + 6bK_1)w_1 - 4bK_1w_1^2;$

$g_{13}^P(w_1) = c^4 - 2c(c^3 - 2bcK_1 + 8b^2K_1^2)w_1 + (c^4 - 4bc^2K_1 + 12b^2K_1^2)w_1^2.$

Figure B.4. $\widetilde{K}_{1,i}^P$ for $i = 1, \ldots, 6$ are defined below.

(1) $\widetilde{K}_{1,1}^P$ is the root to $7(1-c)^2c^4(5 + 28c - 31c^2) - 4b(1-c)c^3(241 - 9c - 182c^2)K_1 + 8b^2c^2(291 - 263c + 11c^2)K_1^2 - 16b^3c(289 - 286c)K_1^3 + 5936b^4K_1^4 + \sqrt{c^2(12 - 24c + 13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2}(2(1-c)^2c^3(5 + 34c) - 8b(1-c)c^2(31 + 26c)K_1 - 8b^2c(11 - 82c)K_1^2 + 896b^3K_1^3) = 0.$

(2) $\widetilde{K}_{1,2}^P$ is the root to $-(1-c)^2c^5(24 - 23c) + 12b(1-c)c^3(16 - 13c)K_1 - 4b^2c^2(237 - 390c + 178c^2)K_1^2 + 48b^3c(43 - 52c)K_1^3 - 3024b^4K_1^4 - (7(1-c)^2c^4 - 6b(1-c)c^2(9 + c)K_1 + 4b^2c(30 + c)K_1^2 + 168b^3K_1^3)\sqrt{c^2(12 - 24c + 13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2} = 0.$

(3) $\widetilde{K}_{1,3}^P$ is the root to $(c^2(1-3c)(4-5c) + 2bc(13 + 10c)K_1 + 28b^2K_1^2) + \big((1-3c)c + 14bK_1\big)\sqrt{c^2(12 - 24c + 13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2} = 0.$

(4) $\widetilde{K}_{1,4}^P$ is the root to $(1-c)^2c^4(3 - 4c - 6c^2) + 4b(1-c)c^4(4 + 3c)K_1 + 4b^2c^2(9 - 56c + 48c^2)K_1^2 + 16b^3c(33 + 5c)K_1^3 - 864b^4K_1^4 + \sqrt{c^2(3 - 6c + 4c^2) + 12bcK_1 - 24b^2K_1^2}((2 - 3c)(1-c)^2c^3 - 2b(1-3c)(1-c)c^2K_1 + 4b^2c(4 + 3c)K_1^2 + 152b^3K_1^3).$

(5) $\widetilde{K}_{1,5}^P$ is the root to $c^3(8 - 36c + 81c^2 - 81c^3) + 36bc^2K_1 + 12b^2c(1 + 33c)K_1^2 - 16b^3K_1^3 - \sqrt{(2-3c)^2c^2 + 4bc(2 + 3c)K_1 + 4b^2K_1^2}(-c^2(4 - 39c + 63c^2) - 2b(7 - 57c)cK_1 + 8b^2K_1^2).$

(6) $\widetilde{K}_{1,6}^P$ is the root to $(1-c)c^5 - 4bc^2(2 - c + c^2)K_1 + 4b^2c(41 - 25c)K_1^2 - 416b^3K_1^3 + (c^2 - 10bK_1)((1-c)c - 8bK_1)\sqrt{c^4 - 4bc(4-3c)K_1 + 36b^2K_1^2}.$

Table B.5. $\hat{f}_i^P(c, K_1)$ for $i = 1, \ldots, 11$ and $\hat{g}_i^P(w_1)$ for $i = 1,2$ are defined below.

$\hat{f}_1^P(c,K_1) = -(1-c)^2c^5(24-23c) + 12b(1-c)c^3(16-13c)K_1 - 12b^2c^2(65-106c+48c^2)K_1^2 + 336b^3c(5-6c)K_1^3 - 2352b^4K_1^4 - (7(1-c)^2c^4 - 6b(1-c)c^2(9+c)K_1 + 36b^2c(2+c)K_1^2 + 168b^3K_1^3)\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2}$;

$\hat{f}_2^P(c,K_1) = (3-2c)c + \sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2}$;

$\hat{f}_3^P(c,K_1) = (1-c)^2c^4(7-4c-2c^2) - b(1-c)c^3(20+9c-25c^2)K_1 + 2b^2c^2(33-16c-11c^2)K_1^2 - 4b^3c(26-29c)K_1^3 + 280b^4K_1^4 + ((1-c)^2c^3(2+c) - b(1-c)c^2(1+11c)K_1 + 4b^2c(1+4c)K_1^2 + 28b^3K_1^3)\sqrt{c^2(12-24c+13c^2) - 12bc(4-5c)K_1 + 84b^2K_1^2}$;

$\hat{f}_4^P(c,K_1) = (1-c)^2c^4(7-6c+2c^2) + 2b(1-c)c^3(28-31c+13c^2)K_1 + 4b^2c^2(9-16c-6c^2)K_1^2 - 8b^3c(38-11c)K_1^3 + 160b^4K_1^4 + \sqrt{c^2(3-6c+4c^2) + 12bcK_1 - 24b^2K_1^2}((1-c)^2c^3(4+c) + 12b(2-c)(1-c)c^2K_1 - 44b^2c^2K_1^2 - 64b^3K_1^3)$;

$\hat{f}_5^P(c,K_1) = (3-c)c + 6bK_1 + 2\sqrt{c^2(3-6c+4c^2) + 12bcK_1 - 24b^2K_1^2}$;

$\hat{f}_6^P(c,K_1) = (1-c)^2c^4(9c^2-2) - 4b(1-c)c^4(1+6c)K_1 - 4b^2c^2(1-4c)(7-6c)K_1^2 - 16b^3c(7+12c)K_1^3 - 208b^4K_1^4 + (-(1-c)^2c^3(2-3c) + 2b(1-c)c^2(1-3c)K_1 - 12b^2c^2K_1^2 - 88b^3K_1^3)\sqrt{c^2(2-3c)^2 + 4bc(2+3c)K_1 + 4b^2K_1^2}$;

$\hat{f}_7^P(c,K_1) = c + 4bK_1 + \sqrt{c^2(2-3c)^2 + 4bc(2+3c)K_1 + 4b^2K_1^2}$;

$\hat{f}_8^P(c,K_1) = (1-c)^2c^4(2-3c) + 3b(1-c)c^3(4-5c+5c^2)K_1 + 2b^2c^2(13-14c-5c^2)K_1^2 + 4b^3c(6-5c)K_1^3 + 8b^4K_1^4 + \sqrt{c^2(2-3c)^2 + 4bc(2+3c)K_1 + 4b^2K_1^2}((1-c)^2c^3 + b(5-c)(1-c)c^2K_1 + 4b^2c(2-3c)K_1^2 + 4b^3K_1^3)$;

$\hat{f}_9^P(c,K_1) = -c^8 + 8b(2-c)c^5K_1 - 4b^2c^2(8+c^2)K_1^2 + 16b^3c(2+3c)K_1^3 - 64b^4K_1^4 + c(c^2-2bK_1)(c^3 - 4b(2-c)K_1)\sqrt{c^4 - 4bc(4-3c)K_1 + 36b^2K_1^2}$;

$\hat{f}_{10}^P(c,K_1) = c^4 - 4b(2-c)cK_1 + 4b^2K_1^2 - (c^2 - 2bK_1)\sqrt{c^4 - 4bc(4-3c)K_1 + 36b^2K_1^2}$;

$\hat{f}_{11}^P(c,K_1) = (1-c)c^{11} - 4bc^8(5-9c+5c^2)K_1 + 8b^2c^5(10-27c+36c^2-16c^3)K_1^2 + 16b^3c^3(2-25c+31c^2-16c^3)K_1^3 + 16b^4c^2(10-5c+11c^2)K_1^4 - 64b^5c(6-7c)K_1^5 - 384b^6K_1^6 + \sqrt{c^4 - 4bc(4-3c)K_1 + 36b^2K_1^2}(-(1-c)c^9 + 2bc^6(6-11c+7c^2)K_1 - 4b^2c^3(4-12c+23c^2-11c^3)K_1^2 + 8b^3c^2(2+3c-c^2)K_1^3 + 64b^4c(1-2c)K_1^4 + 64b^5K_1^5)$;

And:

$\hat{g}_1^P(w_1) = \frac{c^4 - 2c(c^3 - 2bcK_1 + 4b^2K_1^2)w_1 + (c^4 - 4bc^2K_1 + 8b^2K_1^2)w_1^2}{4bc^2(2c - w_1)w_1}$;

$\hat{g}_2^P(w_1) = \frac{-c^4 + 2c^2(c + 2b(1-2c)K_1)w_1 - (c^2(1+c-c^2) + 4bc(1-2c)K_1 + 4b^2K_1^2)w_1^2 + (1-c)c^2w_1^3}{2bcw_1(2c-w_1)^2}$.