SHIP REPOSITIONING OPTIMIZATION IN TRAMP SHIPPING UNDER UNCERTAINTY

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Ship Repositioning Optimization in Tramp Shipping Under Uncertainty

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CERTIFICATE OF ORIGINALITY

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Abstract

In tramp shipping, the spot market is characterized by intense competition and volatility. Securing profitable spot cargoes for a tramp shipping company is challenging. To handle this issue, this thesis addresses a ship repositioning problem in the dry bulk spot market, aiming to provide short-term operational planning for a tramp shipping company. The goal is to maximize profits by optimizing destination decisions, speed, and cargo selection upon arrival. This thesis considers two types of uncertainties: (1) the availability of future cargoes and (2) the behavior of competitors in the spot market. To handle high competition and market volatility, the cargo selection mechanism considers competitors' movements, a priority principle, and cargo-worthiness, along with these uncertainties. We propose a two-stage stochastic programming model to solve this problem. The first stage determines the destination and arrival time for each empty ship, while the second stage involves cargo acquisition based on first-stage decisions, revealed uncertainties, and the cargo selection mechanism. We use the Sample Average Approximation (SAA) method to solve the stochastic model. Extensive experiments validate the SAA method's performance, and the Value of Stochastic Solution (VSS) is calculated by comparing it with a deterministic model. Finally, a sensitivity analysis based on a case study informs the development of operational policies for tramp

shipping companies.

Keywords: ship repositioning; tramp shipping; spot market; (mixed) integer programming; sample average approximation

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Chapter 1

Introduction

1.1 Introduction

Maritime transport is the backbone of international trade and globalization. It impressively handled 11 billion tons in 2022 and accounts for a staggering 80% of all goods transported by volume, with dry bulk shipping responsible for nearly half of this volume (UNCTAD, 2023). Tramp shipping, the most prevalent method in dry bulk shipping, operates based on customer demand. Shipping companies serve as carriers, moving cargo according to client instructions, earning revenue from freight charges. The spot market generates spot cargoes for short-term transactions, characterized by intense competition, high volatility, and significant uncertainty, presenting both challenges and opportunities for tramp shipping companies.

Ship repositioning is crucial for securing profitable spot cargoes in the tramp spot market by providing short-term operational planning to navigate market volatility. It involves two core decisions: determining the next destination for relocating ships and then selecting spot cargoes at the arrived destinations. Ship speed plays a significant role in ship repositioning because it may have opposite impacts on fuel costs and cargo profitability. Firstly, fuel costs, which are largely dependent on the ship's speed, constitute 50% of the total operational costs in a shipping company (De et al., 2021), making speed a critical factor in cost management. In addition, high speed generates high fuel consumption costs. Secondly, speed impacts the arrival date at the targeted destination. Due to the volatility of the spot market, different arrival dates can significantly affect the availability and types of spot cargoes generated. A high speed may increase opportunities to obtain profitable cargoes by allowing earlier arrivals at the destination, but results in higher fuel consumption costs. Therefore, speed optimization is difficult yet important for shipping companies when deciding the next destination for relocating ships.

To secure available cargoes in the spot market, the cargo selection mechanism must be carefully considered when a tramp ship arrives at its destination. The company must decide whether to accept the current cargo or wait for a potentially more valuable one. Firstly, competitor movements need to be considered, as the number of competing ships may exceed cargo demand. Modern ship tracking systems like the Automatic Identification System (AIS) (Wright et al., 2019; Yang et al., 2021) help monitor and forecast these movements. Secondly, a priority principle should be incorporated to ensure effective functioning in real-world scenarios. Shippers often choose the first available service, making a first-come, first-served approach primary. Additionally, ships arriving on the same day as competitors are given lower priority. Thirdly, cargo suitability is considered, as ships vary in capacity and cargoes differ in weight requirements, requiring different capacity levels for carriers.

In tramp shipping, two types of uncertainties significantly impact cargo profitability in ship repositioning optimization within the spot market. These uncertainties are inherent to tramp shipping (Pantuso et al., 2014), and slim profit margins are particularly sensitive to them (Kalouptsidi, 2014). The first type involves the daily generation of cargoes at different destinations, influenced by factors such as weather. The second type, more critical to the tramp spot market, concerns the uncertain movements of competitors, which can lead to changes in destination, arrival dates, or target cargo types. Both uncertainties should be considered together. To address these challenges, methodologies such as stochastic programming, robust optimization, and dynamic routing are essential. These strategies enhance decision-making under uncertainty, ensure effective solutions across various scenarios, and adapt to changing conditions (Besbes & Savin, 2009; Y. Wu et al., 2023), thereby enhancing the robustness and adaptability of ship repositioning optimization models.

While maritime transport decision-making is typically structured into strategic, tactical, and operational levels (Ksciuk et al., 2023), most existing studies focus on long-term planning, especially Contract of Affreightment (COA) cargoes, which generate the majority of revenue for large tramp shipping companies. In such contexts, spot cargoes are often treated as given parameters, with limited attention paid to their acquisition process. However, this overlooks the inherent complexity of the spot market, including high volatility, intense competition, and the strategic behavior of competitors. For tramp shipping companies that rely solely on spot cargoes, decisions regarding spot market engagement are not merely operational—they are central to revenue generation and survival. Despite this, research that explicitly models decision-making under spot market uncertainty remains scarce. There is a clear gap in understanding how to obtain profitable spot cargoes in such a highly challenging environment. Addressing this gap is essential for improving short-term planning and enhancing the responsiveness of tramp shipping operations.

This study aims to address these gaps by optimizing ship repositioning with a cargo selection mechanism that considers competitors' movements, a priority principle, and cargo-worthiness in tramp shipping, while accounting for uncertainties in cargo demand and competitors' movements. We employ a two-stage stochastic programming approach to tackle this complex and realistic problem. Following this, we will outline our key contributions and provide a roadmap for the subsequent chapters of this thesis.

1.2 Summary of Our Scientific Contributions

Our main scientific contributions can be summarized as follows:

1. To address the deterministic version of the ship repositioning optimization problem in tramp shipping, we developed a Mixed Integer Linear Programming (MILP) model. Additionally, we introduced the M-tightening technique to strategically enhance the model's efficiency.

2. Considering the uncertainties in both cargo demands and competitors' movements, we tackle this problem using a two-stage stochastic programming methodology. This approach allows us to derive optimal average solutions under scenarios that account for the joint distribution of cargo demands and competing ships. To facilitate problem resolution, we employ the Sample Average Approximation (SAA) method.

3. We conduct comprehensive numerical experiments to evaluate the effectiveness of the M-tightening techniques and the performance of the SAA method. Finally, we perform several data sensitivity analyses to provide valuable insights and inform policy decisions.

1.3 Outline

The rest of the thesis is structured as follows: Chapter 2 reviews relevant literature. Chapter 3 describes the problem, introduces the deterministic model and a two-stage stochastic programming model for uncertainties. Chapter 4 details the Sample Average Approximation (SAA) method. Chapter 5 presents experiments and results. Finally, Chapter 6 summarizes findings and suggests future research areas.

Chapter 2

Literature Review

The ship repositioning problem in tramp spot market encompasses four key areas within maritime transportation. First, we address ship repositioning in the dry bulk tramp shipping sector. Second, we aim to help tramp shipping companies secure profitable spot cargoes. Third, the first stage of our problem focuses on determining the target destination and arrival date, taking into account fuel consumption based on different ship speeds. Finally, the second stage involves selecting the available profitable cargoes. To comprehensively cover these aspects, we review literature on repositioning decisions, spot cargoes, ship speed optimization, and cargo selection in tramp shipping, detailed in Sections 2.1 to 2.4 respectively.

2.1 Review of Studies on Repositioning Decisions in Maritime Transportation

Repositioning decisions in maritime transportation predominantly focus on liner shipping, with limited research on tramp shipping. Repositioning is discussed in three contexts: container repositioning, fleet repositioning, and repositioning voyages within ship routing problems. Container repositioning (Xu et al., 2024; Zhang et al., 2017) is crucial due to the economic and environmental impacts of managing empty containers, as highlighted by Song and Carter (2009) and Khakbaz and Bhattacharjya (2014), who emphasize strategic management, and by Braekers et al. (2013) and Cai et al. (2022), who focus on cost minimization. Fleet repositioning in liner shipping has been extensively studied, with Müller and Tierney (2016) and Tierney et al. (2015) underscoring the importance of decision support systems and optimization models. However, these concepts are under-explored in tramp shipping, indicating potential for future research. Repositioning voyages within ship routing problems are another critical area, with Vilhelmsen et al. (2017) demonstrating that optimizing routing and scheduling can significantly impact operational efficiency. Despite the well-defined nature of "repositioning" in maritime literature, its application to tramp shipping is less examined, presenting an opportunity for future research to develop targeted strategies for tramp shipping.

There is only one exception in the literature, which is the work by Omholt-Jensen et al. (2025). This study considers fleet repositioning in tramp shipping and addresses the uncertainty of future market prospects by repositioning the vessels at the end of the planning period.

2.2 Review of Studies on Spot Cargoes in Tramp Shipping

In tramp shipping, companies use two main types of cargo contracts: contracts of affreightment (COA) and spot cargoes. COAs provide stability and predictability, while spot cargoes are based on immediate market conditions (Fagerholt et al., 2011; Korsvik et al., 2010; Li et al., 2022). Handling spot cargoes complements COAs by filling schedule gaps and maximizing operational efficiency (Fagerholt et al., 2011; Li et al., 2022). This integration adapts to market demands and maintains service levels (Huang et al., 2024; Zhao & Yang, 2018). Spot cargoes can significantly impact profitability due to higher rates (Pollaris, 2018).

Although tramp shipping literature is extensive, specific studies on the spot market are limited. Existing research often treats spot cargoes as parameters within broader frameworks (Fan et al., 2019; Pollaris, 2018). L. Wu et al. (2021) considered fleet adjustment, cargo selection, and ship routing in tramp shipping but treated spot cargoes as special COA cargoes. The unique dynamics of securing spot cargoes warrant further investigation (El Noshokaty, 2019). Nurminarsih et al. (2024) studies a decision model for ocean freight carriers to manage slot allocation, focusing on determining a protection level for spot sales and incorporating overbooking.

The impact of competitor behavior on the availability of spot cargo has been largely overlooked in the existing literature. To the best of our knowledge, only Zhao and Yang (2018) explicitly addressed this issue by modeling competitors through shippers' choice inertia.

In conclusion, while COAs provide stable revenue, strategically handling spot cargoes is essential for maximizing fleet utilization and profitability. Focused research on the spot market is needed to provide valuable strategies for tramp shipping companies. In addition, the consideration of competitor decisions remains limited in existing studies, with only a few works addressing this important factor.

2.3 Review of Studies on Ship Speed Optimization in Tramp Shipping

Ship speed optimization in tramp shipping is crucial for operational efficiency, reducing fuel consumption, minimizing costs, and mitigating environmental impacts. Speed influences ship repositioning due to lengthy voyages and fuel costs, which constitute 50% of total operational costs.

Baştürk and Erol (2023) examined optimizing ship speed based on cargo and wind-sea conditions for sustainable growth and climate change mitigation. Li et al. (2022) addressed decarbonization by examining stochastic tramp ship routing with speed optimization under uncertain future cargo scenarios. Moon et al. (2015) integrated speed optimization into tramp ship routing and scheduling, highlighting its importance in fleet deployment and network design.

In summary, optimizing ship speed is essential for sustainable and cost-effective maritime operations.

2.4 Review of Studies on Cargo Selection in Tramp Shipping

Cargo selection in tramp shipping focuses on spot cargoes, competitors' influence, and priority principles like First-Come First-Served (FCFS). Spot cargoes are a significant revenue source, complementing long-term Contracts of Affreightment (COAs). Effective cargo selection strategies are necessary to adapt to fluctuating market conditions (Hemmati et al., 2015; L. Wu et al., 2021). Competitors' influence is significant, requiring strategic decision-making to differentiate from rivals and maintain market share (Durgut & Şakar, 2023). The FCFS principle is relevant, with timely decision-making impacting operational success (El Noshokaty, 2019). Combining FCFS and competitor behavior in cargo selection is unique, especially for spot cargoes. Understanding these factors provides a competitive edge, optimizing operations and revenue streams.

In summary, cargo selection in tramp shipping, particularly for spot cargoes, is influenced by competitive dynamics and principles like FCFS. Effective decision-making in this context is essential for success in a competitive market.

2.5 Research Gaps and Contributions

This study optimizes ship repositioning in the dry bulk tramp shipping spot market, aiming to secure profitable spot cargoes amid competition and market volatility. This problem incorporates a cargo selection mechanism considering competitors' movements, a priority principle, and uncertainties in cargo demand and competitors' behavior.

This study builds on L. Wu et al. (2021), who considered fleet adjustment, cargo selection, and ship routing but treated spot cargoes as special COA cargoes. By focusing exclusively on the spot market, we offer a more applicable framework for competitive environments. Unlike Zhao and Yang (2018), who modeled competitors through shippers' choice inertia, we incorporate competitors' movements and assume rational shippers prioritizing the earliest service. Additionally, we integrate uncertainties for a comprehensive model. Our work diverges from Li et al. (2022), who focused on decarbonization and speed optimization under uncertain cargo availability, by emphasizing a cargo selection mechanism tailored to the spot market's competitive dynamics and shorter planning horizon. Omholt-Jensen et al. (2025) tackled a tramp ship routing and scheduling problem with bunker optimization problem using a two-stage stochastic programming approach over an 80-day planning horizon. The first stage involves cargo selection, ship routing, scheduling, and bunkering, while the second stage considers repositioning costs. In contrast, our work differs by incorporating repositioning in the first stage and considering a much shorter planning horizon.

Based on the above literature review, the following research gaps are identified:

1. Focus on liner shipping: Most existing studies on ship repositioning concentrate on liner shipping (container shipping, fleet shipping), while research on ship repositioning in tramp shipping remains limited.

2. Underexplored spot market dynamics: Studies focusing on the spot market are scarce. The spot market is often treated as a parameter within broader long-term planning models, and its unique dynamics—such as volatility,

competition, and strategic behavior of competitors—are not well captured.

3. Cargo selection mechanisms: The integration of first-come-first-served (FCFS) principles with competitor analyses in cargo selection is rarely considered in the literature.

By integrating these elements, our study provides a comprehensive and practical approach to ship repositioning optimization in the dry bulk tramp shipping spot market, addressing both the competitive dynamics and uncertainties inherent in this environment. Our contributions to the literature are threefold:

1. We focus on ship repositioning optimization in tramp shipping, addressing a significant gap in the literature.
2. We provide a dedicated study on the spot market, offering actionable insights and strategies for maximizing profitability in this volatile and competitive environment.
3. We introduce a cargo selection mechanism that considers competitors' movements and a priority principle, enhancing decision-making under uncertainty.

Chapter 3

Problem Formulation

This chapter developed the mathematical models for both deterministic problem and the problem under uncertainties for dry bulk tramp shipping.

3.1 Problem Description

3.1.1 Problem Background

The notation used for deterministic ship repositioning problem is provided in Table 3.1.

Table 3.1: Notation for deterministic ship repositioning optimization problem.

Data	
Notation	Meaning
$\mathcal{I}$	a set of our ships
$\mathcal{T}$	a set of periods, representing a finite and discrete-time planning horizon.
$\mathcal{A}$	a collection of potential destinations for repositioning
$\mathcal{K}_a$	a set of cargoes generated by area $a \in \mathcal{A}$
$\mathcal{K}_{a,i}$	a set of cargoes generated by area $a \in \mathcal{A}$ which are cargo-worthy for ship $i \in \mathcal{I}$
$\mathcal{I}_{a,k}$	a set of our ships which can select type-k cargoes at destination a where $k \in \mathcal{K}_a$ and $a \in \mathcal{A}$
$n_{a,t,k}$	the number of requests of type-k, where $k \in \mathcal{K}_a$, generated in period $t \in \mathcal{T}$ in area $a \in \mathcal{A}$
$R_{i,k}$	the reward for ship i to serve a type-k request
$\mathcal{T}_{i,a}$	the set of feasible arrival times of ship $i \in \mathcal{I}$ at area $a \in \mathcal{A}$, $\mathcal{T}_{i,a} \subseteq \mathcal{T}$
$C_{i,a,t}$	the cost of repositioning ship $i \in \mathcal{I}$ to area $a \in \mathcal{A}$ with arrival time $t \in \mathcal{T}_{i,a}$
D	the unit idling cost for a ship waiting for requests in an area in each period
$m_{a,t,k}$	the number of competing ships that arrive at area $a \in \mathcal{A}$ in period $t \in \mathcal{T}$
$M_{i,a,t,k}$	a very large number for $i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i}$
Variables	
Notation	Meaning
$x_{i,a,t}$	a binary variable, it equals 1 only when ship $i \in \mathcal{I}$ is repositioned to $a \in \mathcal{A}$ and the ship arrives in period $t \in \mathcal{T}$
$y_{i,a,t,k}$	a binary variable, it equals 1 only when ship $i \in \mathcal{I}$ accepts a type-k request from area $a \in \mathcal{A}$ in period $t \in \mathcal{T}$, for $k \in \mathcal{K}_{a,i}$
$\alpha_{i,a,t}$	a binary variable, it is equal to 1 only when ship $i \in \mathcal{I}$ has arrived at area $a \in \mathcal{A}$ by the period $t \in \mathcal{T}$
$\beta_{i,a,t}$	a binary variable, it equals 1 only when ship $i \in \mathcal{I}$ has accepted a request at area $a \in \mathcal{A}$ by the period $t \in \mathcal{T}$

We tackle the ship repositioning problem for a dry bulk tramp shipping company that owns a fleet of ships, denoted by $\mathcal{I}$. These ships need to be repositioned within a finite, discrete-time planning horizon of $\mathcal{T}$ periods (measured in days) among a set of potential repositioning destinations, denoted by $\mathcal{A}$, which are areas with transportation requests.

Ships operate within given speed range. For ship $i \in \mathcal{I}$, the set of feasible arrival times at area a is denoted by $\mathcal{T}_{i,a}$, which is determined by the given feasible speed range and the distance from the initial location to destination a. Given arrival time $t \in \mathcal{T}_{i,a}$, the corresponding cost of repositioning ship i to area a is represented by $C_{i,a,t}$.

Destination $a \in \mathcal{A}$ generates several types of cargo, denoted as the set $\mathcal{K}_a$. For cargo type $k \in \mathcal{K}_a$, the amount is $n_{a,t,k}$ in period $t \in \mathcal{T} \cup \{0\}$, where 0 represents the beginning of the planning horizon. All types of requests across all areas are different, denoted as $\mathcal{K}$, where $\mathcal{K} = \bigcup_{a \in \mathcal{A}} \mathcal{K}_a$. The cargoes are all in full-shipload contracts, meaning the entire cargo is allocated to a single ship, and the cargo must fit within the ship's capacity. If the weight of type-k cargo generated in destination $a \in \mathcal{A}$ exceeds the capacity of ship $i \in \mathcal{I}$, then type-k cargo is not feasible for ship i. The types of cargoes suitable for ship i in area a are included in the set $\mathcal{K}_{a,i}$. By serving a type-k request, ship i can obtain the freight revenue, denoted by $R_{i,k}$, if the cargo is available and suitable for the ship. In addition to the previously introduced cost of repositioning the ship, there is also a unit idling cost for a ship waiting for requests in an area during each period, represented by a constant D.

When canvassing requests from areas in $\mathcal{A}$, ships in $\mathcal{I}$ must face competition from ships in $\mathcal{J}$ that are also seeking transportation requests from areas in $\mathcal{A}$. The number of competing ships that arrive at area $a \in \mathcal{A}$ in period $t \in \mathcal{T}$ and compete for type-k requests, where $k \in \mathcal{K}_a$, is denoted by $m_{a,t,k}$. Not all ships in $\mathcal{I} \cup \mathcal{J}$ need to be homogeneous in ship size.

For clarity, "competing ships" ($j \in \mathcal{J}$) refer to ships from other companies, while "our ships" ($i \in \mathcal{I}$) denote the ships of the shipping company we assist in decision-making. This distinction differentiates external competition from our operations.

3.1.2 Key Assumptions

Based on the practical challenges observed in ship repositioning, we have established the following fundamental assumptions to define the scope of our study:

Asm.1 Each ship can target at most one type of cargo in at most one area.

Asm.2 Requests originating from a single destination are assigned to the ships that have already arrived.

Asm.3 The shippers' main objective is to ensure the rapid transportation of their cargo. Consequently, they inquire with arriving ships in the order of their arrival to determine their willingness to transport the goods.

Asm.4 Competing ships that reach the same destination no later than our ships will be accorded priority in cargo pickup over our ships.

Asm.5 All ships are restricted to selecting cargo that aligns with their individual carrying capacity. A ship is deemed cargo-worthy for a specific cargo only when the ratio of the cargo's weight to the ship's carrying capacity lies within the interval $(0, 1]$.

Justification for the assumptions is as follows.

1. Asm.2 is practical for several reasons. Firstly, it promotes efficiency by allowing ships already at the destination to load cargo quickly, minimizing waiting times. Secondly, it is cost-effective as it reduces idle time and the associated costs. Thirdly, this approach simplifies logistics, making it easier

to plan loading and unloading operations. Lastly, it ensures fairness in the distribution of cargo among the ships.

2. Asm.4 aims to ensure the robustness of the proposed plan. Given the uncertainty in the arrival order of our ship and competing ships on the same day, assuming our ship is prioritized might lead to choosing a later arrival time to minimize time spent at the destination. However, if the actual situation is less favorable, the plan could fail. Therefore, we make decisions based on conservative assumptions.

3. Asm.5 is both obvious and realistic, as a ship's load capacity is limited. If a cargo's weight exceeds this capacity, the ship cannot transport it.

3.2 The Deterministic Model

3.2.1 The Mixed Integer Linear Programming Model

Given the problem background as outlined in Section 3.1, we are required to make two decisions for each ship to optimize total profit. The first decision, symbolized by the variable $\boldsymbol{x}$, pertains to determining the arrival time and location for each ship. So we define $x_{i,a,t}$ as a binary variable that equals 1 if and only if ship $i \in \mathcal{I}$ is repositioned to $a \in \mathcal{A}$ and arrives in period $t \in \mathcal{T}$. The second decision, represented by $\boldsymbol{y}$, involves selecting a cargo from the demand pool at each ship's destination, based on the outcomes of the first decision. Therefore, we let $y_{i,a,t,k}$ be a binary variable that equals 1 if and only if ship $i \in \mathcal{I}$ accepts a type-k request from area $a \in \mathcal{A}$ in period $t \in \mathcal{T}$. To facilitate modeling, we introduce two auxiliary variables, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. The former denotes whether ship $i \in \mathcal{I}$ arrives at area $a \in \mathcal{A}$ before or in period $t \in \mathcal{T}$, while the latter denotes whether ship $i \in \mathcal{I}$ has accepted a request at area $a \in \mathcal{A}$ before or in period $t \in \mathcal{T}$.

The notation used for this deterministic problem is provided in Table 3.1. Based on the decision variables, we can formulate the deterministic ship repositioning optimization problem as [**DP**].

$$[\ \mathbf{DP}\ ] \quad \max \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}_{a,i}} R_{i,k} y_{i,a,t,k} - \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}_{i,a}} C_{i,a,t} x_{i,a,t} - \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}} D(\alpha_{i,a,t} - \beta_{i,a,t}) \quad (3.1)$$

$$s.t. \sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}_{i,a}} x_{i,a,t} \leq 1, \quad \forall i \in \mathcal{I}, \quad (3.2)$$

$$\sum_{a\in\mathcal{A}} \sum_{t\in(\mathcal{T}\setminus\mathcal{T}_{i,a})} x_{i,a,t} = 0, \quad \forall i \in \mathcal{I}, \quad (3.3)$$

$$\alpha_{i,a,t} = \sum_{t'=1}^{t} x_{i,a,t'}, \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, \quad (3.4)$$

$$\beta_{i,a,t} = \sum_{k\in\mathcal{K}_{a,i}} \sum_{t'=1}^{t} y_{i,a,t',k}, \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, \quad (3.5)$$

$$\sum_{t'=0}^{t} n_{a,t',k} - \sum_{t'=1}^{t} m_{a,t',k}(1 - \alpha_{i,a,t'} + x_{i,a,t'}) - \sum_{t'=1}^{t} \sum_{i'\in\mathcal{I}_{a,k}} y_{i',a,t',k} \geq M_{i,a,t,k}(y_{i,a,t,k} - 1), \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i}, \quad (3.6)$$

$$\beta_{i,a,t} \leq \alpha_{i,a,t}, \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, \quad (3.7)$$

$$x_{i,a,t} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, \quad (3.8)$$

$$y_{i,a,t,k} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i}, \quad (3.9)$$

$$\alpha_{i,a,t} \in \{0, 1\}, \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, \quad (3.10)$$

$$\beta_{i,a,t} \in \{0,1\}, \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}. \tag{3.11}$$

Equation (3.1) defines our objective function, which aims to maximize total profit. This function comprises three components: total revenue, repositioning cost, and the cost incurred by waiting for cargoes.

Please note that the second term of the objective function, $\sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}_{i,a}} C_{i,a,t}x_{i,a,t}$, reflects the speed optimization component of the model. Each $t \in \mathcal{T}_{i,a}$ corresponds to a feasible arrival day, which is mapped from a specific sailing speed within a speed range. For each arrival time $t \in \mathcal{T}_{i,a}$, the associated speed v determines the fuel consumption cost $C_{i,a,t}$, which is calculated using a cubic function of speed. This cost is treated as the repositioning cost in the objective function. The construction of the feasible arrival time set $\mathcal{T}_{i,a}$ and the corresponding fuel cost $C_{i,a,t}$ is detailed in Section 5.1.2.

As for the second term of the objective function, we introduce the expression $(\alpha_{i,a,t} - \beta_{i,a,t})$ to indicate indicates if the ship $i \in \mathcal{I}$ at destination $a \in \mathcal{A}$ is waiting during period $t \in \mathcal{T}$.

Constraints (3.2) stipulate that the arrival date of ship $i \in \mathcal{I}$ to area $a \in \mathcal{A}$ must be at most one of its feasible arrival dates, denoted by $\mathcal{T}_{i,a}$. Constraints (3.3) set the value of $x_{i,a,t}$ as 0 when ship i cannot reach destination a at time t. Constraints (3.4) and (3.5) establish the relationships between variables $x_{i,a,t}$ and $\alpha_{i,a,t}$, and between variables $y_{i,a,t,k}$ and $\beta_{i,a,t}$, respectively.

Constraints (3.6) illustrate the cargo selection mechanism, which is designed in accordance with Assumptions Asm.2 and Asm.4. Specifically, if a ship $i \in \mathcal{I}$ selects a cargo $k \in \mathcal{K}_{a,i}$ from destination $a \in \mathcal{A}$ on day $t \in \mathcal{T}$, there must be sufficient remaining cargoes available on that day. The remaining cargoes by day

t are calculated as the total accumulated cargoes minus those obtained by high-priority competing ships and our own ships. In the second term on the left-hand side of the inequality, the expression $1 - \alpha_{i,a,t'} + x_{i,a,t'}$ indicates whether time $t \in \mathcal{T}$ is not later than the arrival day of ship i at destination a, which is consistent with the cargo selection mechanism.

Inequalities (3.7) ensure that each ship begins loading cargo only after it arrives at its destination. Among these, the domain for $\boldsymbol{y}$ in (3.9) imposes constraints that ship $i \in \mathcal{I}$ can only select cargo that is deemed cargo-worthy, which is based on Assumption Asm.5. Finally, the remaining constraints (from (3.8) to (3.11)) define the domains of variables $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\beta}$.

Constraints (3.6) are the most important in this problem. There are two things we need to explain. One is to show why the formulation of constraints (3.6) is correct; the other is to determine the value for $M_{i,a,t,k}$. We will deal with them in the next two subsections.

3.2.2 Validation of Key Constraints

Proposition 1. *Ship $i \in \mathcal{I}$ will pick up at most one cargo, which is assured in the model **DP**.*

Proof. Based on Constraints (3.7), given data $T = \max_{t \in \mathcal{T}}\{t\}$, we can have the following correct formulation:

$$\beta_{i,a,T} \leq \alpha_{i,a,T}, \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, \tag{3.12}$$

Based on (3.12), it is easy to derive the following correct formulation:

$$\sum_{a \in \mathcal{A}} \beta_{i,a,T} \leq \sum_{a \in \mathcal{A}} \alpha_{i,a,T}, \quad \forall i \in \mathcal{I} \tag{3.13}$$

Based on Constraints (3.4) and (3.5), we can derive the following correct formulation:

$$\sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}_{i,a}} y_{i,a,t,k} \leq \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}_{i,a}} x_{i,a,t}, \quad \forall i \in \mathcal{I} \tag{3.14}$$

These constraints represent the meaning of Proposition 1. □

Proposition 2. *The model* ***DP*** *ensures that the optimal solution aligns with the cargo selection mechanism, the principles that are rooted in Assumptions Asm.1 to Asm.5.*

Proof. There are two cases that we need to prove.

Case 1) When our ship $i \in \mathcal{I}$ obtains a type-k cargo at destination $a \in \mathcal{A}$, the competing ships which also target type-k cargo and which arrived before ship i had already obtained their cargoes.

Case 2) When our ship $i \in \mathcal{I}$ obtains a type-k cargo at destination $a \in \mathcal{A}$, the competing ships which also target type-k cargo and which arrived at the same time as ship i had already obtained their cargoes.

We need to prove that if our ship $i \in \mathcal{I}$ obtains type-k cargo at time $t \in \mathcal{T}$, then at time $t \in \mathcal{T}$, the total demand minus the number of competing ships that arrive before our ship i and the total number of competing ships that arrive at the same time as our ship is greater than 0. This can be expressed as formula (3.15).

$$\sum_{t'=0}^{t} n_{a,t',k} - \sum_{t'=1}^{t} m_{a,t',k}(1 - \alpha_{i,a,t'} + x_{i,a,t'}) > 0,$$
$$\forall i \in \mathcal{I}, \text{ given } a, t \text{ and } k \text{ which make } y_{i,a,t,k} = 1 \tag{3.15}$$

We prove the correctness of formula (3.15) based on Constraints (3.6). At this time, we have $y_{i,a,t,k} = 1$ and $\sum_{t'=1}^{t} \sum_{j\in\mathcal{I}} y_{j,a,t',k} \geq 1$. Then Constraints (3.6) is transformed into the following equation (3.16).

$$\sum_{t'=0}^{t} n_{a,t',k} - \sum_{t'=1}^{t} m_{a,t',k}(1 - \alpha_{i,a,t'} + x_{i,a,t'}) \geq 1,$$
$$\forall i \in \mathcal{I}, \text{ given } a, t \text{ and } k \text{ which make } y_{i,a,t,k} = 1 \tag{3.16}$$

Obviously, formula (3.15) can be proven using equation (3.16). □

3.2.3 M-tightening Techniques for the Deterministic Problem

In mathematical optimization, the parameter $M_{i,a,t,k}$ in (3.6) is often referred to as a “big-M” parameter. Choosing an appropriate value for the big-M parameter is crucial. It should be large enough to ensure the constraint is valid under all feasible conditions. However, it should also be as small as possible to avoid numerical instability and to maintain the linearity of the constraint.

When $y_{i,a,t,k} = 0$, the value of $M_{i,a,t,k}$ must be taken into account. That is, the smallest value of $M_{i,a,t,k}$ that is large enough must be determined for the formula (3.17) to hold.

$$-\sum_{t'=0}^{t} n_{a,t',k} + \sum_{t'=1}^{t} m_{a,t',k}(1 - \alpha_{i,a,t'} + x_{i,a,t'}) + \sum_{t'=1}^{t} \sum_{i' \in \mathcal{I}_{a,k}} y_{i',a,t',k} \leq M_{i,a,t,k},$$
$$\forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i} \quad (3.17)$$

Based on (3.17), we can determine the proper value for $M_{i,a,t,k}$, which can be obtained using (3.18).

$$M_{i,a,t,k} = \text{Maximize} \sum_{t'=1}^{t} m_{a,t',k}(1 - \alpha_{i,a,t'} + x_{i,a,t'}) + \sum_{t'=1}^{t} \sum_{i' \in \mathcal{I}_{a,k}} y_{i',a,t',k} - \sum_{t'=0}^{t} n_{a,t',k},$$
$$\forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i} \quad (3.18)$$

The maximal value of the right-hand side can be attained when we allocate 1 to $(1 - \alpha_{i,a,t'} + x_{i,a,t'})$ and assign $|\mathcal{I}_{a,k}|$ to $\sum_{t'=1}^{t} \sum_{i' \in \mathcal{I}_{a,k}} y_{i',a,t',k}$. As a consequence, the value for $M_{i,a,t,k}$ can be determined as defined in (3.19).

$$M_{i,a,t,k} = \sum_{t'=1}^{t} m_{a,t',k} - \sum_{t'=0}^{t} n_{a,t',k} + |\mathcal{I}_{a,k}|, \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i} \quad (3.19)$$

Subsequently, the constraints delineated by (3.6) will transition to the form presented in (3.20).

$$\sum_{t'=0}^{t} n_{a,t',k} y_{i,a,t,k} - \sum_{t'=1}^{t} m_{a,t',k}(y_{i,a,t,k} - \alpha_{i,a,t'} + x_{i,a,t'})$$
$$- \sum_{t'=1}^{t} \sum_{i' \in \mathcal{I}_{a,k}} y_{i',a,t',k} - |\mathcal{I}_{a,k}| y_{i,a,t,k} \geq -|\mathcal{I}_{a,k}|,$$
$$\forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i} \tag{3.20}$$

3.3 Two-stage Stochastic Programming Models

The notation for the stochastic ship repositioning problem is displayed in Table 3.2 to facilitate a quick understanding.

Table 3.2: Notation for the stochastic ship repositioning problem.

Data	
Notation	Meaning
Θ	A set of scenarios for the joint distribution of cargo demand and the number of competing ships.
θ	A scenario from the joint distribution of cargo demand and the number of competing ships, where $\theta = (\theta^n, \theta^m) \in \Theta$.
$P(\theta)$	The joint probability of scenario $\theta \in \Theta$, where $\sum_{\theta \in \Theta} P(\theta) = 1$,
$\boldsymbol{n}(\theta)$	The vector of the number of requests under the scenario θ.
$n_{a,t,k}(\theta)$	The number of requests for type-k cargoes at destination $a \in \mathcal{A}$ in period $t \in \mathcal{T}$ under the scenario θ.
$\boldsymbol{m}(\theta)$	The vector of the number of arrived competing ships under the scenario θ.
$m_{a,t,k}(\theta)$	The number of competing ships for type-k cargoes that arrived at destination $a \in \mathcal{A}$ in period $t \in \mathcal{T}$ under the scenario θ.
$M_{i,a,t,k}(\theta)$	a very large number for $i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i}, \theta \in \Theta$
Variables	
Notation	Meaning
$y_{i,a,t,k}(\theta)$	Variable $y_{i,a,t,k}$ under the scenario $\theta \in \Theta$.
$\beta_{i,a,t}(\theta)$	Variable $\beta_{i,a,t}$ under the scenario $\theta \in \Theta$.

3.3.1 Uncertainties

Maritime shipping operations are inherently uncertain due to a multitude of factors such as market fluctuations, seasonal variations, changes in consumer preferences, economic conditions, and unpredictable competitor actions. These uncertainties will have an important impact on the following two factors:

1. The quantity of requests for each cargo type at each destination during every period.

2. The quantity of competing ships targeting each cargo type at each destination during every period.

For the listed two aspects, we will introduce two uncertainty factors, $\boldsymbol{n}(\theta)$ and $\boldsymbol{m}(\theta)$ for the listed two aspect respectively, into the deterministic model, to reflect the real-world operations of a maritime shipping company. These factors form a joint distribution, Θ, representing scenarios for both cargo demand and the number of competing ships. Each scenario $\theta \in \Theta$ has a probability of occurrence $P(\theta)$, with $\sum_{\theta\in\Theta} P(\theta) = 1$.

Specifically, for the first factor, $\boldsymbol{n}(\theta)$, represents the discrete distribution of cargo requests, with $n_{a,t,k}(\theta)$ denoting the number of requests for type-k cargoes at destination a in period t under scenario $\theta \in \Theta$. While for the second factor, $\boldsymbol{m}(\theta)$, accounts for the number of competing ships, with $m_{a,t,k}(\theta)$ denoting the number of competing ships for type-k cargoes arriving at destination a in period t under scenario $\theta \in \Theta$.

In practice, tramp shipping companies manage cargo demand uncertainty through COA contracts and flexible fleet deployment, allowing them to secure predictable volumes while maintaining adaptability. To address competition uncertainty, they rely on market intelligence, shipbrokers, and dynamic routing decisions based on real-time freight rates and vessel positions. These strategies reflect a need for responsive decision-making under uncertainty. Our model formalizes this process by jointly capturing both types of uncertainty through scenario-based stochastic programming, enabling robust planning and aligning with industry practices.

Given the uncertainties in cargo demand and the number of competing ships, our goal is to optimize the average objective function value across all scenarios. We will propose a two-stage stochastic programming model based on scenario $\theta \in \Theta$ in the following sections.

3.3.2 Stochastic Models

In this section, we construct a two-stage stochastic programming formulation, denoted as [**SP**], for the ship repositioning problem to handle uncertainties in the number of cargoes and the movements of competing ships. The notation is displayed in Table 3.2.

$$[\,\mathbf{SP}\,] \quad \max_{(\boldsymbol{x},\boldsymbol{\alpha})} \quad \mathbb{E}_{\theta}[\mathcal{Q}(\boldsymbol{x},\boldsymbol{\alpha})] - \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}_{i,a}} C_{i,a,t}x_{i,a,t} - \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}} D\alpha_{i,a,t} \tag{3.21}$$

$s.t.$ Constraints $(3.2), (3.3), (3.4), (3.8)$ and (3.10).

The objective function (3.21) maximizes the expected profit, where the profit for each scenario θ is defined as the revenue minus the fuel consumption cost and idling cost. The constraints (3.2), (3.3), (3.4), (3.8) and (3.10) determine the destination and arrival time for each ship $i \in \mathcal{I}$, represented by the variables $\boldsymbol{x}$ and $\boldsymbol{\alpha}$, without any information about the number of cargoes or the competing ships. The second-stage function $\mathbb{E}_\theta[\mathcal{Q}(\boldsymbol{x}, \boldsymbol{\alpha})]$ evaluates the expected objective value of the second-stage problem over all scenarios in Θ, which can be calculated by equation (3.22).

$$\mathbb{E}_\theta[\mathcal{Q}(\boldsymbol{x}, \boldsymbol{\alpha})] = \sum_{\theta \in \Theta} \boldsymbol{P}(\theta) Q(\boldsymbol{x}, \boldsymbol{\alpha}, \boldsymbol{n}(\theta), \boldsymbol{m}(\theta)) \tag{3.22}$$

In equation (3.22), $\boldsymbol{P}(\theta)$ can be regarded as given data which can be obtained by random experiments.

As a result, we need to focus on the value of $Q(\boldsymbol{x}, \boldsymbol{\alpha}, \boldsymbol{n}(\theta), \boldsymbol{m}(\theta))$ so that $\mathbb{E}_\theta[\mathcal{Q}(\boldsymbol{x}, \boldsymbol{\alpha})]$ can be finally obtained. $Q(\boldsymbol{x}, \boldsymbol{\alpha}, \boldsymbol{n}(\theta), \boldsymbol{m}(\theta))$ is the objective function value of the maximization problem defined in (3.23) to (3.28).

$$\begin{aligned} Q(\boldsymbol{x}, \boldsymbol{\alpha}, &\boldsymbol{n}(\theta), \boldsymbol{m}(\theta)) = \\ &\max_{(\boldsymbol{y}(\theta), \boldsymbol{\beta}(\theta))} \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{K}_{a,i}} R_{i,k} y_{i,a,t,k}(\theta) \\ &\qquad + \sum_{i \in \mathcal{I}} \sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}} D \beta_{i,a,t}(\theta) && (3.23) \\ s.t. \quad & \beta_{i,a,t}(\theta) = \sum_{k \in \mathcal{K}_{a,i}} \sum_{t'=1}^{t} y_{i,a,t',k}(\theta), \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, && (3.24) \\ & \sum_{t'=0}^{t} n_{a,t',k}(\theta) - \sum_{t'=1}^{t} m_{a,t',k}(\theta)(1 - \alpha_{i,a,t'} + x_{i,a,t'}) \end{aligned}$$

$$-\sum_{t'=1}^{t}\sum_{i'\in\mathcal{I}_{a,k}} y_{i',a,t',k}(\theta) \geq M_{i,a,t,k}(\theta)(y_{i,a,t,k}(\theta)-1),$$

$$\forall i\in\mathcal{I}, a\in\mathcal{A}, t\in\mathcal{T}, k\in\mathcal{K}_{a,i}, \quad (3.25)$$

$$\beta_{i,a,t}(\theta) \leq \alpha_{i,a,t}, \ \ \forall i\in\mathcal{I}, a\in\mathcal{A}, t\in\mathcal{T}, \quad (3.26)$$

$$y_{i,a,t,k}(\theta) \in \{0,1\}, \ \ \forall i\in\mathcal{I}, a\in\mathcal{A}, t\in\mathcal{T}, k\in\mathcal{K}_{a,i}, \quad (3.27)$$

$$\beta_{i,a,t}(\theta) \in \{0,1\}, \ \ \forall i\in\mathcal{I}, a\in\mathcal{A}, t\in\mathcal{T}. \quad (3.28)$$

The constraints above are analogous to constraints (3.5), (3.20), (3.7), (3.9) and (3.11) in the deterministic model DP (defined in Section 3.2), but with decision variables marking with the symbol θ to signify their association with a particular scenario θ.

3.3.3 M(θ)-tightening Techniques for the Stochastic Problem

The $M_{i,a,t,k}(\theta)$ parameter in (3.25) must be large enough to ensure that the constraint (3.25) is not violated. However, if $M_{i,a,t,k}(\theta)$ is too large, it will cause the problem to be difficult to solve. As a result, the value of $M_{i,a,t,k}(\theta)$ should be determined in a careful manner. We employ tightening techniques to determine its value.

When $y_{i,a,t,k}(\theta) = 0$, the value of $M_{i,a,t,k}(\theta)$ needs to be considered. That is, the smallest value of $M_{i,a,t,k}(\theta)$ that is large enough needs to be determined for formula (3.29) to hold.

$$-\sum_{t'=0}^{t} n_{a,t',k}(\theta) + \sum_{t'=1}^{t} m_{a,t',k}(\theta)(1-\alpha_{i,a,t'}+x_{i,a,t'}) + \sum_{t'=1}^{t}\sum_{i'\in\mathcal{I}_{a,k}} y_{i',a,t',k}(\theta)$$

$$\leq M_{i,a,t,k}(\theta), \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i} \tag{3.29}$$

Based on (3.29), we can determine the proper value for $M_{i,a,t,k}$, which can be obtained using (3.30).

$$M_{i,a,t,k}(\theta) = \text{Maximize } \sum_{t'=1}^{t} m_{a,t',k}(\theta)(1 - \alpha_{i,a,t'} + x_{i,a,t'}) + \sum_{t'=1}^{t} \sum_{i' \in \mathcal{I}_{a,k}} y_{i',a,t',k}(\theta) - \sum_{t'=0}^{t} n_{a,t',k}(\theta), \quad \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i} \tag{3.30}$$

The value of the right-hand side can be maximized when we assign 1 to $(1 - \alpha_{i,a,t'} + x_{i,a,t'})$ and $|\mathcal{I}_{a,k}|$ to $\sum_{t'=1}^{t} \sum_{i' \in \mathcal{I}_{a,k}} y_{i',a,t',k}(\theta)$. As a result, the value for $M_{i,a,t,k}(\theta)$ can be determined as in (3.31).

$$M_{i,a,t,k}(\theta) = \sum_{t'=1}^{t} m_{a,t',k}(\theta) - \sum_{t'=0}^{t} n_{a,t',k}(\theta) + |\mathcal{I}_{a,k}|, \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i} \tag{3.31}$$

At last, Constraints (3.25) will transition to the form presented in (3.32).

$$\sum_{t'=0}^{t} n_{a,t',k}(\theta) y_{i,a,t,k}(\theta) - \sum_{t'=1}^{t} m_{a,t',k}(\theta)(y_{i,a,t,k}(\theta) - \alpha_{i,a,t'} + x_{i,a,t'}) - \sum_{t'=1}^{t} \sum_{i' \in \mathcal{I}_{a,k}} y_{i',a,t',k}(\theta) - |\mathcal{I}_{a,k}| y_{i,a,t,k}(\theta) \geq -|\mathcal{I}_{a,k}|,$$

$$\forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, k \in \mathcal{K}_{a,i} \quad (3.32)$$

Chapter 4

Sample Average Approximation Method (SAA)

The Sample Average Approximation (SAA) method is a powerful tool for tackling stochastic discrete optimization problems. It approximates the expected value of a stochastic optimization problem using a finite number of scenarios, useful for complex problems with unknown or difficult-to-handle distributions. Its efficacy in shipping-related challenges involving uncertainties is demonstrated by Meng et al., 2012 and Lai et al., 2022.

SAA method is employed to address our two-stage stochastic ship repositioning problem, managing uncertainties in cargo demand (n) and the number of competing ships (m). This approach optimizes operations while accounting for unpredictable factors, particularly useful in tramp shipping.

4.1 The Procedure of SAA

The procedure for the SAA method for our ship repositioning problem is outlined in the following three steps:

SAA method

1. Generate samples of $\boldsymbol{n}(\theta_1), \ldots, \boldsymbol{n}(\theta_S)$ and $\boldsymbol{m}(\theta_1), \ldots, \boldsymbol{m}(\theta_S)$ of S scenarios as Θ_S from the joint distribution of the random cargo request $\boldsymbol{n}$ and the number of competing ships $\boldsymbol{m}$
2. Solve the following **SAA problem** expressed as (4.1).

SAA problem :

$$\max_{(\boldsymbol{x},\boldsymbol{\alpha},\boldsymbol{y}(\theta_1),\boldsymbol{\beta}(\theta_1),\ldots,\boldsymbol{y}(\theta_S),\boldsymbol{\beta}(\theta_S))} \frac{1}{S}\sum_{\theta=\theta_1}^{\theta_S}\left(\sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}_{a,i}} R_{i,k}y_{i,a,t,k}(\theta) + \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}} D\beta_{i,a,t}(\theta)\right)$$
$$-\sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}_{i,a}} C_{i,a,t}x_{i,a,t} - \sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}} D\alpha_{i,a,t}$$

s.t. $$\beta_{i,a,t}(\theta) = \sum_{k\in\mathcal{K}_{a,i}}\sum_{t'=1}^{t} y_{i,a,t',k}(\theta), \quad \forall i\in\mathcal{I}, a\in\mathcal{A}, t\in\mathcal{T}, \theta\in\Theta_S,$$

$$\sum_{t'=0}^{t} n_{a,t',k}(\theta)y_{i,a,t,k}(\theta) - \sum_{t'=1}^{t} m_{a,t',k}(\theta)(y_{i,a,t,k}(\theta) - \alpha_{i,a,t'} + x_{i,a,t'})$$
$$-\sum_{t'=1}^{t}\sum_{i'\in\mathcal{I}_{a,k}} y_{i',a,t',k}(\theta) - |\mathcal{I}_{a,k}|y_{i,a,t,k}(\theta) \geq -|\mathcal{I}_{a,k}|,$$
$$\forall i\in\mathcal{I}, a\in\mathcal{A}, t\in\mathcal{T}, k\in\mathcal{K}_{a,i}, \theta\in\Theta_S,$$

$$\beta_{i,a,t}(\theta) \leq \alpha_{i,a,t}, \quad \forall i\in\mathcal{I}, a\in\mathcal{A}, t\in\mathcal{T}, \theta\in\Theta_S,$$

$$y_{i,a,t,k}(\theta) \in \{0,1\}, \quad \forall i\in\mathcal{I}, a\in\mathcal{A}, t\in\mathcal{T}, k\in\mathcal{K}_{a,i}, \theta\in\Theta_S,$$

$$
\begin{aligned}
&\beta_{i,a,t}(\theta) \in \{0,1\}, \ \ \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, \theta \in \Theta_S, \\
&\sum_{a \in \mathcal{A}} \sum_{t \in \mathcal{T}_{i,a}} x_{i,a,t} \leq 1, \ \ \forall i \in \mathcal{I}, \\
&\sum_{a \in \mathcal{A}} \sum_{t \in (\mathcal{T} - \mathcal{T}_{i,a})} x_{i,a,t} = 0, \ \ \forall i \in \mathcal{I}, \\
&\alpha_{i,a,t} = \sum_{t'=1}^{t} x_{i,a,t'}, \ \ \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, \\
&x_{i,a,t} \in \{0,1\}, \ \ \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}, \\
&\alpha_{i,a,t} \in \{0,1\}, \ \ \forall i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}
\end{aligned} \tag{4.1}
$$

3. Drive an optimal solution ($\boldsymbol{x}^*$, $\boldsymbol{\alpha}^*$, $\boldsymbol{y}^*(\theta_1)$, ..., $\boldsymbol{y}^*(\theta_S)$, $\boldsymbol{\beta}^*(\theta_1)$, ..., $\boldsymbol{\beta}^*(\theta_S)$).

4.2 Explanation for SAA

The SAA method operates by approximating the expected value of a stochastic objective function through the average of a finite number of sampled scenarios. This technique is grounded in the principles of Monte Carlo simulation, which enables the modeling of uncertainty by generating random samples from the underlying probability distributions of the uncertain parameters.

The first step in the Sample Average Approximation (SAA) method for our ship repositioning problem involves preparing scenarios of uncertain future cargo requests, denoted as $\boldsymbol{n}$, and the movements of competing ships, denoted as $\boldsymbol{m}$.

To ensure the generated samples closely reflect real-world conditions, it is imperative to use appropriate data sources. Samples can be generated using historical data, which provides a reliable reflection of past trends and patterns. Alternatively, predictive models, such as time series analyses or regression models, can be employed to forecast future values. In scenarios where neither historical data nor predictive models are available, random generation of samples from known probability distributions, such as through Monte Carlo methods, can be utilized. The choice of method largely depends on the nature of the problem and the availability of data. Utilizing historical data is often the most reliable approach, while predictive models serve as a valuable alternative when historical data is insufficient. Random generation, though less ideal, remains a viable option in the absence of other data sources.

By using these samples, the SAA method transforms the original stochastic problem into a deterministic one, where the objective function is replaced with its sample average, thus facilitating the optimization process. As a result, the original two-stage stochastic problem is transformed into a deterministic problem called the "SAA problem", which is obtained by combining the first-stage and second-stage model together, using sample Θ_S of S scenarios of uncertain future cargo requests and the movements of competing ships to approximate all scenarios in Θ.

Since the problem has transformed into a deterministic integer programming problem, we can solve it using a MILP solver, such as CPLEX and Gurobi. By step 3 of the SAA method, we can obtain the optimal solution for both the first-stage $(\boldsymbol{x}^*, \boldsymbol{\alpha}^*)$ and second-stage solution $(\boldsymbol{y}^*(\theta_1), \ldots, \boldsymbol{y}^*(\theta_S), \boldsymbol{\beta}^*(\theta_1), \ldots, \boldsymbol{\beta}^*(\theta_S))$.

4.3 SAA Practical Implementation

To implement the optimal solution $(\boldsymbol{x}^*, \boldsymbol{\alpha}^*, \boldsymbol{y}^*(\theta_1), \ldots, \boldsymbol{y}^*(\theta_S), \boldsymbol{\beta}^*(\theta_1), \ldots, \boldsymbol{\beta}^*(\theta_S))$ obtained by SAA in the real environment, we can use the first-stage solution $(\boldsymbol{x}^*, \boldsymbol{\alpha}^*)$ to determine the ship's repositioning plan immediately. After the ships arrive at their destination and the real cargo requests and competitors' movements are revealed, we need to identify which scenario of Θ_S occurred. For example, if scenario $\theta_r \in \Theta_S$ occurs, we use the corresponding pair decisions $(\boldsymbol{y}^*(\theta_r), \boldsymbol{\beta}^*(\theta_r))$ to obtain cargoes.

There might be special occasions when $\theta_r \notin \Theta_S$, that is, a new scenario happened in the real environment. In such cases, we can directly solve the second-stage recourse problem and obtain $Q(\boldsymbol{x}^*, \boldsymbol{\alpha}^*, \boldsymbol{n}(\theta_r), \boldsymbol{m}(\theta_r))$.

Chapter 5

Numerical Experiments

The implementation of all experiments was carried out in the Julia programming language. The computational experiments were performed on a workstation equipped with an Intel(R) Core(TM) i7-8700K CPU operating at 3.70 GHz with 64.0 GB of RAM.

5.1 Instance Generation

5.1.1 Details for Cargo and Competitors

Six potential destinations includes Port Hedland, Esperance, Samarinda, Belawan, Abbot Point Port and Newcastle. The first two for iron ore while the last four for coal. Freight revenue of transporting a cargo as reward of a cargo $R_{i,k}$ is obtained by Eq. (5.1) and the related data is based on Shanghai Shipping Exchange (2024). The source data and the calculated data for Reward of a cargo is shown in Table

A.2.

$$\textbf{Reward} \text{ (per cargo)} = \text{Cargo Weight} \times \text{Freight Rate} \tag{5.1}$$

Competitive intensity is a key indicator of market conditions, defined as the ratio between the total number of cargoes and the total number of competing ships, that is, $r = \frac{\sum \boldsymbol{n}}{\sum \boldsymbol{m}}$, where r ranges from r_{min} to r_{max}. Here, $\sum \boldsymbol{n}$ and $\sum \boldsymbol{m}$ represent the total number of cargoes and competing ships, respectively. We use r to distinguish between favorable and unfavorable market conditions. Favorable market conditions are characterized by $r \in [1, 1.5]$ and $\sum \boldsymbol{m} \in [882, 1000]$, while unfavorable market conditions correspond to $r \in [0.8, 1]$ and $\sum \boldsymbol{m} \in [800, 882]$. In subsequent sections, the values of $\sum \boldsymbol{n}$ are generated based on the selected r values.

The number of cargoes, $n_{a,t,k}$, is calculated as $\frac{r_{\text{min}} \sum \boldsymbol{m}}{T} \times r1$, where $r1$ ranges from 0.75 to 1.25, and T (decision period) is 21 days. $\bar{n}_k$ is the average shipment number of cargo-k (per day), which is calculated by Eq. (A.1). The source data and the calculated data for the Average Shipment Number (per day) are shown in Table A.2.

5.1.2 Ship Details

Firstly, we introduce ship types. We consider a shipping company that operates a fleet of Handymax, Capesize, and Panamax ships, with the fleet size ranging from 1 to 10. The company makes assignment decisions on a three-week basis. The size of the ships can be generated randomly according to the set of bulk ship types labeled as $\mathcal{B} = \{1, 2, 3, 4, 5, 6, 7, 8\}$. This set represents different capacities of ships.

Secondly, we introduce cargo-related data $\mathcal{K}_{a,i}$ and $\mathcal{I}_{a,k}$. These can be obtained from $\mathcal{K}_a$, $\mathcal{K}_b$, and $\mathcal{B}_i$, where $\mathcal{B}_i$ is the bulk ship type of each ship uniformly generated from $\mathcal{B}$. $\mathcal{K}_b$ is the set of cargo types that can be carried by bulk ship type $b \in \mathcal{B}$.

Thirdly, we introduce $\mathcal{T}_{i,a}$, the set of possible days for ship $i \in \mathcal{I}$ arriving at area $a \in \mathcal{A}$. This set is obtained by $\mathcal{T}_{i,a} = \{t_{i,a,\lambda}\}_{\lambda \in \Lambda}$, where $t_{i,a,\lambda} = f_{(i,a)}(v = \mathcal{V}_\lambda)$. Here we define the mapping from ship speed v to t as $f(v)$, shown in formula (5.4) as follows:

$$\mathcal{T}_{i,a} = \{t_{i,a,\lambda}\}_{\lambda \in \Lambda}, \ \ i \in \mathcal{I}, a \in \mathcal{A} \tag{5.2}$$

$$t_{i,a,\lambda} = f_{(i,a)}(v = \mathcal{V}_\lambda), \ \ \lambda \in \Lambda, i \in \mathcal{I}, a \in \mathcal{A} \tag{5.3}$$

$$f_{(i,a)}(v) := \lceil \tau_i^p + \Delta_{i,a}/v \rceil, \ \ v \in \mathcal{V}, i \in \mathcal{I}, a \in \mathcal{A} \tag{5.4}$$

$$\tau_i^p = (\Delta_i^p / V_i^p + H_i^p) \times P_i^p, \ \ i \in \mathcal{I} \tag{5.5}$$

The speed v belongs to the set $\mathcal{V}$, which is defined as as the integers from 7 to 16 based on Gunes (2023). The set Λ represents the indices of $\mathcal{V}$ for the current voyage.

As for the detailes of the preceding voyage of ship i, τ_i^p is the time cost, calculated by Equation (5.5). V_i^p is the speed, with distances Δ_i^p. Speeds are 13 knots unloaded and 12 knots loaded. H_i^p is the handling time at unloading ports, uniformly [1, 3] days. P_i^p is the rest process, uniformly [0, 1] days.

In terms of distances Δ_i^p and $\Delta_{i,a}$ for ship $i \in \mathcal{I}$ and destination $a \in \mathcal{A}$, O_i^p and D_i^p are the origin and destination ports of the preceding voyage. Δ_i^p is the distance from O_i^p to D_i^p, and $\Delta_{i,a}$ is from D_i^p to $a \in \mathcal{A}$, both from Table A.3. The pair (O_i^p, D_i^p) is uniformly generated from iron ore or coal voyages.

Lastly, $C_{i,a,t}$ (mt) is the fuel consumption cost for ship $i \in \mathcal{I}$ to area $a \in \mathcal{A}$ arriving on day $t \in \mathcal{T}_{i,a}$, obtained by 5.6. Here, $f^{-1}_{(i,a)}(t)$ is described in (5.7) and $g(\delta, v)$ in (5.8), based on Luan Thanh Le and Woo (2020). The fuel price rate is PR(\$/mt) $\in [430, 500]$.

$$C_{i,a,t} = g\left(\delta = \Delta_{i,a}, v = f^{-1}_{(i,a)}(t)\right), \; i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}_{i,a} \tag{5.6}$$

$$f^{-1}_{(i,a)}(t) := \left\lceil \frac{\Delta_{i,a}}{t - \tau_i^p} \right\rceil, \; i \in \mathcal{I}, a \in \mathcal{A}, t \in \mathcal{T}_{i,a} \tag{5.7}$$

$$g(\delta, v) := (0.0141 \times v^3 + 32.1584) \times PR \times \delta / v \tag{5.8}$$

5.1.3 Uncertainty Setting

Market volatility is defined as a perturbation parameter that controls the percentage deviation of daily cargoes and competing ships from their nominal values.

The uncertainty includes changes in the quantity of cargoes $n_{a,t,k}$ and the number of competing ships $m_{a,t,k}$ arriving daily. We consider daily fluctuations of ± 0.3 times their original values for both cargoes and competing ships. This means we set the market volatility as 0.3.

5.2 Experiments for M-Tightening Techniques for Deterministic Model and SAA Algorithm

To verify computational efficiency of M-tightening method, we conduct experiments in this section.

5.2.1 Experiment Setting

We evaluated deterministic and stochastic models using small (5-10 ships) and medium (12-15 ships) scales, with fixed parameters for destinations (6), decision period length (21 days), and cargo categories (14). We created 40 instances for each model. For the stochastic model, we added scenarios (30, 50, 80, 100, 200). We compared objective values with and without the M-tightening method, using CPLEX for the deterministic model and the SAA algorithm for the stochastic model, recording run times and objective values.

5.2.2 Experiment Results

The results of the performance of the M-tightening method for deterministic and stochastic models are shown in the first 40 rows and last 40 rows in Table A.4, respectively.

Results for deterministic models: The first 20 rows of Table A.4 compare the outcomes of solving small-scale deterministic problems with and without the M-tightening technique. The results consistently show identical objective values, confirming the technique's effectiveness. Additionally, the optimal solution is achieved more quickly when the M-tightening technique is applied, demonstrating its efficiency. Regardless of its use, the computational time for small-scale problems remains impressively short, not exceeding 10 seconds. The M-tightening technique further optimizes this process, reducing the maximum solution time to just 2 seconds. For deterministic problems involving a fleet of 5 ships, solutions can be obtained in less than 1 second. Rows 21 to 40 of Table A.4 present similar findings for medium-scale deterministic models (10 to 15 ships). These results validate the correctness, effectiveness, and efficiency of the M-tightening technique for such models.

Results for stochastic models: The data presented in rows 41 to 80 of Table A.4 demonstrate that the implementation of M-tightening techniques results in identical objective function values. This outcome confirms the accuracy and efficacy of M-tightening when applied to two-stage stochastic programming for ship repositioning using the sample average approximation (SAA) method. Notably, in comparison to deterministic problems, the application of M-tightening techniques substantially reduces computational time.

5.3 Experimental Evaluation of the Value of the Stochastic Solution (VSS)

5.3.1 Experiment Design for VSS Experiment

The Value of the Stochastic Solution (VSS) measures the benefit of using stochastic models. We record VSS_{ins} for instance ins following steps 1 to 4. Note that in ship repositioning, future scenarios S_f may differ from historical data S_h.

Step 1: Generate two samples S_h and S_f with the same distribution.

Step 2: Obtain first-stage decisions $\boldsymbol{x}^*_{SAA}$ and $\boldsymbol{\alpha}^*_{SAA}$ using SAA on S_h, and $\boldsymbol{x}^*_d$ and $\boldsymbol{\alpha}^*_d$ using the mean of S_h with the deterministic model.

Step 3: For each scenario θ in S_f, calculate RP_θ using $\boldsymbol{x}^*_{SAA}$ and EV_θ using $\boldsymbol{x}^*_d$. RP_θ and EV_θ are calculated by (5.9) and (5.10) with $\boldsymbol{Cx}$ and $D\boldsymbol{\alpha}$ for $\sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}_{i,a}} C_{i,a,t}x_{i,a,t}$ and $\sum_{i\in\mathcal{I}}\sum_{a\in\mathcal{A}}\sum_{t\in\mathcal{T}} D\alpha_{i,a,t}$. $Q(\boldsymbol{x},\boldsymbol{\alpha},\boldsymbol{n}(\theta),\boldsymbol{m}(\theta))$ is the second-stage objective value described in (3.23) to (3.28).

$$\text{RP}_\theta = Q(\boldsymbol{x}^*_{SAA}, \boldsymbol{\alpha}^*_{SAA}, \boldsymbol{n}(\theta), \boldsymbol{m}(\theta)) - \boldsymbol{C}\boldsymbol{x}^*_{SAA} - D\boldsymbol{\alpha}^*_{SAA} \tag{5.9}$$

$$\text{EV}_\theta = Q(\boldsymbol{x}^*_{d}, \boldsymbol{\alpha}^*_{d}, \boldsymbol{n}(\theta), \boldsymbol{m}(\theta)) - \boldsymbol{C}\boldsymbol{x}^*_{d} - D\boldsymbol{\alpha}^*_{d} \tag{5.10}$$

$$\text{VSS}_{ins} = \frac{\sum_{\theta \in S_f} \text{RP}_\theta}{|S_f|} - \frac{\sum_{\theta \in S_f} \text{EV}_\theta}{|S_f|} \tag{5.11}$$

Step 4: Calculate VSS for instance ins as (5.11).

We generated 48 instances to verify VSS, considering $|\mathcal{I}| \in \{10, 15\}$ and $S \in \{50, 100, 200\}$ with $(|\mathcal{A}|, |\mathcal{T}|, |\mathcal{K}|) = (6, 21, 14)$. For each pair of $|\mathcal{I}|$ and S, we generated 8 instances.

5.3.2 Result and Discussion for VSS Experiment

All the raw results are shown in Tables A.5.

The VSS values were analyzed across 50, 100, and 200 SAA scenarios, each with 16 VSS values.

Please note that in some instances, the deterministic model may make decisions that result in no cargoes being obtained. This leads to zero profit but incurs high costs, which can cause the VSS to become large.

- For 50 scenarios: mean VSS is 30,587.61, median is 10,087.37, standard deviation is 43,500.97, and 18.75% of values are zero.

- For 100 scenarios: mean VSS is 64,867.11, median is 28,102.40, standard deviation is 101,712.89, with no zero values.

- For 200 scenarios: mean VSS is 110,304.45, median is 27,438.42, standard deviation is 187,950.34, with no zero values.

Based on the VSS values and their implications for the SAA algorithm, the set with 200 SAA scenarios stands out as the most suitable candidate, followed by the set with 100 SAA scenarios. The set with 50 SAA scenarios, with its lower VSS values and presence of zero values, might not be as effective for applying the SAA algorithm. These analyses helps in identifying the sets of scenarios where the SAA algorithm can be most beneficial, ensuring optimal application and results.

5.4 Sensitivity Analyses for Objective Value in Ship Repositioning

This section analyzes how different parameters affect optimal profits using the SAA method in tramp shipping. The goal is to identify the most impactful parameters and provide insights for decision-makers in varying spot market conditions.

5.4.1 Experiment Design and Results

We conducted several one-way sensitivity analyses to evaluate how varying parameters affect optimal profits in tramp shipping using the SAA method. By adjusting one parameter at a time while keeping others constant, we isolated each parameter's impact on the optimal solution. This helps identify key parameters and provides insights for decision-makers in different spot market conditions.

The parameters analyzed were reward ($R_{i,a}$), fuel consumption (PR), charter rate as unit idle cost (D), competitive intensity, and market volatility, chosen for their significant impact on operational and financial outcomes. Here, competitive intensity is defined as the ratio of the total number of cargoes to the total number of competing ships. Market volatility is defined as a perturbation parameter that controls the percentage deviation of daily cargoes and competing ships from their nominal values.

We set ten levels for each parameter, ranging from favorable to unfavorable conditions, with baseline values at the median level. For reward, fuel consumption, and charter rate, the change rates range from -0.045 to 0.045. Competitive intensity ranges from 0.775 to 1.225, and market volatility from 0.12 to 0.48. All levels are linearly spaced between the lower and upper bounds.

The experimental raw results are recorded in Table A.6. Their detailed analyses are represented in the next following subsections.

5.4.2 Impact of Reward Values, Fuel Consumption, and Chartering Fees on Objective Value

The related columns in Table A.6 illustrates how variations in reward values (related to cargo revenue), fuel consumption (related to repositioning cost), and chartering fees (related to idle cost), each across 10 levels, affect the Objective Value in the tramp spot market.

- **Reward Values**: There is a positive correlation between reward changes and the Objective Value. As reward values decrease from level 1 to level 10, the Objective Value drops from 693,831.42 to 292,575.42. Higher reward values lead to a higher Objective Value.

- **Fuel Consumption**: Fuel consumption negatively impacts the Objective Value. As the fuel consumption change rate decreases from 0.045 at level 1 to -0.045 at level 10, the Objective Value falls from 669,387.26 to 317,019.57. Higher fuel consumption results in a lower Objective Value.

- **Chartering Fees**: Chartering fees also show a negative correlation with the Objective Value. As chartering fees decrease from level 1 to level 10, the Objective Value reduces from 495,453.42 to 490,953.42. Higher chartering fees lead to a lower Objective Value.

These trends highlight the significant impact of reward values, fuel consumption, and chartering fees on the Objective Value, emphasizing the importance of considering these factors in daily operations and decision-making.

5.4.3 Competitive Intensity's Impact on Objective Value

The Competitive Intensity column in Table A.6 shows how competition affects the Objective Value. As competition decreases from level 1 to level 10, the Objective Value also drops. Lower competitive intensity means stronger competition, defined by the ratio of cargo to competing ships. When competitive intensity decreases from 1.225 to 0.775, the Objective Value falls from 591,083.33 to 395,323.50. The change is stepwise: within ± 0.125 around the median, the Objective Value stays stable. Outside this range, it shifts. In summary, in the tramp spot market, less competition (fewer cargoes relative to ships) lowers the Objective Value.

5.4.4 Market Volatility's Impact on Objective Value

The Market Volatility column in Table A.6 shows how market volatility at 10 levels affects the Objective Value. As volatility rises from level 1 to level 10, the Objective Value drops. Higher volatility means greater daily fluctuations in cargo availability and competitor numbers. There is a negative correlation between volatility and the Objective Value. As volatility decreases from 1.225 to 0.775, the Objective Value declines from 597,455.95 to 156,224.34. Initially, the Objective Value remains stable at 597,455.95 for levels 1 to 3, then decreases moderately to 493,203.42 for levels 4 to 6, and drops sharply to 156,224.34 from level 6 onwards. In summary, in the tramp spot market, higher market volatility leads to a decrease in the Objective Value, with larger fluctuations having a more pronounced effect.

5.4.5 Conclusion for Objective Value Sensitivity Analyses

Sensitivity analyses show that reward, fuel consumption, charter rate, competitive intensity, and market volatility are key factors affecting profit. Competitive intensity and market volatility have the most significant impact. When market volatility is low, profit is high, followed by periods of intense competition. Therefore, acquiring market information, especially on competitive intensity and market volatility, is crucial for maximizing profit in tramp shipping.

5.5 Sensitivity Analyses for the Value of Stochastic Solution (VSS)

We also perform several one-way sensitivity analyses on VSS, following a methodology similar to Section 5.4. We analyze the same five parameters (reward $R_{i,a}$, fuel consumption PR, charter rate D, competitive intensity, and market volatility) as in the sensitivity analyses of SAA's objective value in Section 5.4.1, using the same values and levels as Section 5.4.1. The raw results are presented in Table A.6, with detailed explanations of these parameters provided in the subsequent subsections.

5.5.1 Impact of Reward Values, Fuel Consumption, and Chartering Fees on VSS

The related columns in Table A.6 illustrate how variations in reward values, fuel consumption, and chartering fees, each across 10 levels from -0.045 to 0.045, affect the VSS in the tramp spot market.

- **Reward Values**: The VSS is highly sensitive to reward levels, with all values being positive, indicating the consistent benefit of the SAA approach for two-stage ship repositioning optimization. VSS values increase from 355,242.31 to 480,561.66 as rewards progress from Level 1 to Level 10, showing a negative relationship between reward and VSS.
- **Fuel Consumption**: As the rate of change in fuel consumption increases, the VSS rises from 331,353.17 to 984,675.42, indicating a positive relationship between fuel consumption and VSS.
- **Chartering Fees**: As the rate of change in chartering fees rises, the VSS climbs from 332,194.26 to 465,550.31, indicating a positive relationship between chartering fees and VSS.

In summary, reward values, fuel consumption, and chartering fees significantly influence VSS, with the SAA approach offering greater benefits in markets with smaller rewards, higher fuel consumption, and higher chartering fees. Besides, all VSS values are positive, showing the consistent advantage of the SAA approach for two-stage ship repositioning optimization.

5.5.2 Competitive Intensity's Impact on VSS

The analyses show that all VSS values remain positive, highlighting the consistent utility of the SAA approach across varying competitive intensity levels. Competitive intensity significantly influences VSS values, with a notable decrease from 1,079,475.10 to 148,865.67 as intensity escalates from Level 1 to Level 10, indicating a negative correlation. Competitive intensity is defined as Competitive Intensity $= \sum n / \sum m$, where a lower ratio implies more intense competition. VSS values remain stable within certain intensity ranges (Levels 1-2 and 3-5).

In conclusion, while VSS values are stable within certain levels, the overall trend shows that higher competitive intensity leads to higher VSS values, indicating greater value from the stochastic solution in more competitive markets.

5.5.3 Market Volatility's Impact on VSS

There is an upward trend in VSS values as market volatility increases. As volatility rises from 0.12 at Level 1 to 0.48 at Level 10, VSS values range from 287,859.37 to 1,460,421.77, indicating a positive relationship. This relationship strengthens beyond the baseline level, with a substantial rise in VSS from Level 6 to Level 10. All VSS values are positive, showing the consistent effectiveness of the SAA approach.

In summary, higher market volatility leads to increased VSS values, highlighting the SAA approach's effectiveness in volatile markets.

5.5.4 Policy Recommendations for Bulk Shipping Companies

Based on the sensitivity analyses, the following policy recommendations are suggested for bulk shipping companies regarding the use of the SAA (Sample Average Approximation) approach:

1. **Adopt SAA in Adverse Market Conditions**: The analyses indicate that VSS values are higher in "bad" market conditions (Levels 6 to 10). Companies should prioritize using the SAA approach during periods of high market volatility, low competitive intensity, and unfavorable rewards and chartering fees. This maximizes optimization benefits under uncertainty, ensuring robust decision-making in challenging market environments.

2. **Monitor Fuel Consumption**: The sensitivity analyses show that higher fuel consumption correlates with larger VSS values. Companies should use the SAA approach when fuel prices are high or expected to rise. This strategy aids in making cost-effective decisions by optimizing operations to mitigate rising fuel costs.

3. **Evaluate Competitive Intensity**: The analyses show that higher competitive intensity leads to higher VSS values. Companies should use the SAA approach in highly competitive markets to achieve better VSS outcomes. This enhances their competitive edge, enabling more informed and strategic decisions in the face of heightened competition.

4. **Adjust for Market Volatility**: Given that market volatility significantly impacts VSS, companies should be vigilant during high volatility periods. The

SAA approach is particularly beneficial then, as it helps manage risks and uncertainties. By adopting the SAA approach during volatile conditions, companies can improve resilience and adaptability, ensuring more stable and predictable outcomes.

5. **Regular Sensitivity Analyses**: To maintain optimal decision-making, companies should regularly perform sensitivity analyses to understand how changes in key parameters affect optimization outcomes. These ongoing analyses provide valuable insights into market dynamics, enabling informed decisions about when to apply the SAA approach. Regular sensitivity analyses help companies stay ahead of market trends and adjust strategies proactively.

These recommendations help bulk shipping companies leverage the SAA approach to optimize operations under varying spot market conditions. By adopting these strategies, companies can enhance efficiency, reduce costs, and improve market performance. The SAA approach provides a robust framework for navigating the bulk shipping market, ensuring long-term sustainability and profitability.

Chapter 6

Summary and Future Research

6.1 Conclusions

In the tramp spot market, a shipping company usually faces significant challenges in securing profitable spot cargoes due to intense competition and volatility. The stochastic shipping repositioning problem arises in this context. This study presents a two-stage stochastic programming model to optimize ship repositioning in tramp shipping for profit maximization. The first stage involves deciding arrival times and destinations for empty ships, while the second stage focuses on selecting cargo types based on cargo-worthiness. The model accounts for uncertainties in cargo requests and competing ships, using the Sample Average Approximation (SAA) method. The model's effectiveness is validated through experiments, and the Value of Stochastic Solution (VSS) is compared with a deterministic model. Sensitivity analyses from a case study help develop operational policies, contributing to future research and practical applications. We provide specific and general management advice for tramp shipping companies. Specifically, for tramp shipping companies aiming to secure profitable spot cargoes, it is essential to employ the Sample Average Approximation (SAA) method to enhance profitability, particularly under adverse market conditions. More broadly, we recommend that decision-making processes be grounded in key information. The more critical information that is gathered and utilized, the higher the likelihood of achieving favorable outcomes. This approach is especially crucial for navigating and surviving in challenging market environments.

6.2 Future Research

Future research should integrate advanced methodologies, address larger-scale problems, and incorporate energy-efficient and green shipping initiatives to enhance ship repositioning precision and effectiveness. This includes conducting more extensive experimental analyses, considering larger-scale scenarios for SAA, and developing effective solution methods like decomposition techniques.

In addition, alternative ways to address uncertainties could be explored. For instance, AIS (Automatic Identification System) data can be used to track the movements of competitor vessels, providing real-time insights into market competition. Similarly, demand forecasting can be improved through dynamic market analyses, leveraging historical trends, regional economic indicators, and commodity flow data. These data-driven approaches can complement the proposed stochastic framework and offer practical enhancements for real-time decision-making in tramp shipping operations.

Moreover, the possibility of using different methodological frameworks such as robust optimisation deserves further investigation. Unlike stochastic programming, which relies on scenario probabilities, robust optimisation focuses on worst-case performance under bounded uncertainty sets. This approach may be particularly useful when probability distributions are difficult to estimate or when decision-makers prefer conservative strategies. Exploring hybrid models that combine stochastic and robust elements could lead to more resilient and practical solutions for ship repositioning under deep uncertainty.

Appendix A

Materials for Chapter 2

A.1 The List of Acronyms

Table A.1: List of acronyms in this thesis

Acronym	Description
MILP	Mixed Integer Linear Programming
SAA	Sample Average Approximation Method
VSS	Value of Stochastic Solution
RP	Recourse Problem
EV	Expected Value Solution
EEV	Expected Result of Using EV Solution
SP	Stochastic Problem
DP	Deterministic Problem
FCFS	First-Come-First-Served
COA	Contracts of Affreightment
UNCTAD	United Nations Conference on Trade and Development
AIS	Automatic Identification System

A.2 The Tables and Equations for Instance Generation

The tables and calculation equations for instance generation in Section 5.1 are given here.

Eq. (A.1) is used for the calculation Average Shipment Number in Section 5.1.1.

$$
\begin{aligned}
\textbf{Average Shipment Number}\text{ (per day)} =\ & \text{World Annual Shipments} \\
& \times \text{World Market Shares} \\
& \times \text{Proportion of Tramp Shipments} \\
& \times \text{Proportion of Cargo} \\
& /\ \text{Cargo Weight} \\
& /\ \text{Number of Days in a Year}
\end{aligned} \tag{A.1}
$$

Cargo types and their shipping Details for Section 5.1.1 are show in the following Table A.2.

Table A.2: Cargo types and shipping details for Section 5.1.1.

Cargo-type ID	1	2	3	4	5	6	7
Major Bulk Type	Iron ore	Iron ore	Coal	Coal	Coal	Coal	Coal
Destination	Western Australia	Western Australia	Indonesia	Indonesia	Indonesia	Eastern Australia	Eastern Australia
World Annual Shipments (10^6 tons)	1517	1517	1232	1232	1232	1232	1232
World Market Shares	0.58	0.58	0.35	0.35	0.35	0.29	0.29
Proportion of Tramp Shipments	0.7	0.7	0.7	0.7	0.7	0.7	0.7
Proportion of Cargo	0.5	0.5	0.3	0.4	0.3	0.5	0.5
Cargo Weight (10^3 tons)	60	170	50	60	70	80	130
Average Shipment Number ($\bar{n}_k$, per day)	14	5	5	6	4	4	3
Freight Rate (USD per ton)	20.88	9.17	14	11	9	14	13
Reward ($R_{i,k}$, 10^3 USD per cargo)	1252.8	1558.9	700	660	630	1120	1690

Source from AIS system and Shanghai Shipping Exchange (2024).

Sailing distances (nautical mile) between different areas are shown in Table A.3.

Table A.3: Sailing Distances (nautical mile) between Importers and Exporters.

	Western Australia (iron ore)		Indonesia (coal)		Eastern Australia (coal)	
	Port Hedland	Esperance	Samarinda	Belawan	Abbot Point Port	Newcastle
China (iron ore)						
Qingdao Port	3583	4825	2453	2838	4140	4806
Tianjin Port	3868	5110	2738	3139	4425	5039
Japan (iron ore)						
Yokohama Port	3613	5003	2641	3267	3928	4272
Kobe Port	3496	4875	2513	3066	3997	4351
China (coal)						
Guangzhou Port	2857	4099	1730	1918	3664	4522
India (coal)						
Kandla	4071	4803	3883	2470	6063	6438
Mumbai	3702	4437	3517	2104	5697	6072
Cochin	3125	3855	2935	1522	5115	5490

Source: Sea Distances (n.d.)

Computational results of impacts of the M-tightening technique for both the deterministic model and the SAA Algorithm for Section 5.2 is shown in the following Table A.4.

Table A.4: Computational results of impacts of the M-tightening technique for both the deterministic model and the SAA Algorithm

Instance	$\|\mathcal{I}\|$	$\|\mathcal{A}\|$	$\|\mathcal{T}\|$	$\|\mathcal{K}\|$	S	M-tightening technique		$M = 999$	
						Objective Value	Time (s)	Objective Value	Time (s)
1	5	6	21	14	1	292118.14	0.00	292118.14	9.00
2	5	6	21	14	1	97789.67	0.00	97789.67	1.00
3	5	6	21	14	1	246626.25	0.00	246626.25	1.00
4	5	6	21	14	1	0.00	0.00	0.00	1.00
5	5	6	21	14	1	181788.48	0.00	181788.48	1.00
6	5	6	21	14	1	84301.75	0.00	84301.75	1.00
7	5	6	21	14	1	89952.25	0.00	89952.25	4.00
8	5	6	21	14	1	422658.68	0.00	422658.68	1.00

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Instance	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{K}\rvert$	S	M-tightening technique		$M = 999$	
						Objective Value	Time (s)	Objective Value	Time (s)
9	5	6	21	14	1	305826.69	0.00	305826.69	1.00
10	5	6	21	14	1	179366.35	0.00	179366.35	1.00
11	10	6	21	14	1	493700.92	1.00	493700.92	2.00
12	10	6	21	14	1	0.00	1.00	0.00	2.00
13	10	6	21	14	1	1282294.34	1.00	1282294.34	2.00
14	10	6	21	14	1	57934.30	1.00	57934.30	2.00
15	10	6	21	14	1	313972.35	1.00	313972.35	2.00
16	10	6	21	14	1	0.00	1.00	0.00	2.00
17	10	6	21	14	1	0.00	1.00	0.00	3.00
18	10	6	21	14	1	0.00	1.00	0.00	4.00
19	10	6	21	14	1	558100.51	1.00	558100.51	3.00
20	10	6	21	14	1	151322.94	2.00	151322.94	4.00
21	12	6	21	14	1	241242.03	2.00	241242.03	3.00
22	12	6	21	14	1	233172.52	1.00	233172.52	2.00
23	12	6	21	14	1	534410.36	1.00	534410.36	3.00
24	12	6	21	14	1	391818.34	1.00	391818.34	2.00
25	12	6	21	14	1	633490.25	2.00	633490.25	3.00
26	12	6	21	14	1	818289.38	2.00	818289.38	3.00
27	12	6	21	14	1	94574.28	1.00	94574.28	2.00
28	12	6	21	14	1	0.00	1.00	0.00	2.00
29	12	6	21	14	1	787579.46	1.00	787579.46	1.00
30	12	6	21	14	1	208800.45	1.00	208800.45	2.00

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Instance	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{K}\rvert$	S	M-tightening technique		$M = 999$	
						Objective Value	Time (s)	Objective Value	Time (s)
31	15	6	21	14	1	754562.74	1.00	754562.74	2.00
32	15	6	21	14	1	469936.22	1.00	469936.22	2.00
33	15	6	21	14	1	0.00	2.00	0.00	3.00
34	15	6	21	14	1	931514.94	3.00	931514.94	4.00
35	15	6	21	14	1	1190621.34	1.00	1190621.34	2.00
36	15	6	21	14	1	887504.37	2.00	887504.37	3.00
37	15	6	21	14	1	691953.66	3.00	691953.66	4.00
38	15	6	21	14	1	756883.08	1.00	756883.08	2.00
39	15	6	21	14	1	228739.49	3.00	228739.49	4.00
40	15	6	21	14	1	405126.08	3.00	405126.08	4.00
1	5	6	21	14	30	292118.14	4.00	292118.14	4.00
2	5	6	21	14	30	97789.67	2.00	97789.67	3.00
3	5	6	21	14	50	51044.03	12.00	51044.03	15.00
4	5	6	21	14	50	0.00	17.00	0.00	27.00
5	5	6	21	14	80	181788.48	17.00	181788.48	21.00
6	5	6	21	14	80	83019.89	7.00	83019.89	15.00
7	5	6	21	14	100	89952.25	10.00	89952.25	19.00
8	5	6	21	14	100	512658.68	16.00	512658.68	14.00
9	5	6	21	14	200	84874.82	77.00	84874.82	146.00
10	5	6	21	14	200	179366.35	129.00	179366.35	142.00
11	10	6	21	14	30	298622.74	37.00	298622.74	44.00
12	10	6	21	14	30	82987.26	22.00	82987.26	24.00

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Table A.4 – continued from previous page

Instance	$\|\mathcal{I}\|$	$\|\mathcal{A}\|$	$\|\mathcal{T}\|$	$\|\mathcal{K}\|$	S	M-tightening technique		$M = 999$	
						Objective Value	Time (s)	Objective Value	Time (s)
13	10	6	21	14	50	1282294.34	68.00	1282294.34	76.00
14	10	6	21	14	50	7934.30	45.00	7934.30	48.00
15	10	6	21	14	80	313972.35	78.00	313972.35	125.00
16	10	6	21	14	80	0.00	171.00	0.00	176.00
17	10	6	21	14	100	345675.96	120.00	345675.96	131.00
18	10	6	21	14	100	18135.15	166.00	18135.15	188.00
19	10	6	21	14	200	129192.50	442.00	129192.50	449.00
20	10	6	21	14	200	302708.85	486.00	302708.85	513.00
21	12	6	21	14	30	291242.03	35.00	291242.03	41.00
22	12	6	21	14	30	195682.15	34.00	195682.15	38.00
23	12	6	21	14	50	356991.58	94.00	356991.58	111.00
24	12	6	21	14	50	260853.75	45.00	260853.75	73.00
25	12	6	21	14	80	117353.59	125.00	117353.59	179.00
26	12	6	21	14	80	867190.37	131.00	867190.37	186.00
27	12	6	21	14	100	161922.89	319.00	161922.89	335.00
28	12	6	21	14	100	431135.75	224.00	431135.75	230.00
29	12	6	21	14	200	315471.85	426.00	315471.85	530.00
30	12	6	21	14	200	145501.64	147.00	145501.64	1305.00
31	15	6	21	14	30	545571.17	65.00	545571.17	85.00
32	15	6	21	14	30	352709.09	98.00	352709.09	107.00
33	15	6	21	14	50	354021.31	172.00	354021.31	182.00
34	15	6	21	14	50	691514.94	180.00	691514.94	182.00

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Table A.4 – continued from previous page

Instance	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{K}\rvert$	S	M-tightening technique		$M = 999$	
						Objective Value	Time (s)	Objective Value	Time (s)
35	15	6	21	14	80	1190621.34	212.00	1190621.34	277.00
36	15	6	21	14	80	792918.78	189.00	792918.78	206.00
37	15	6	21	14	100	524831.68	198.00	524831.68	255.00
38	15	6	21	14	100	401967.01	162.00	401967.01	284.00
39	15	6	21	14	200	178739.49	621.00	178739.49	763.00
40	15	6	21	14	200	514814.78	760.00	514814.78	1085.00

The results for VSS analysis for the extensive computational experiments for Section 5.3 is shown in the following Table A.5.

Table A.5: VSS analysis for the extensive computational experiments

Instance	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{K}\rvert$	S	Deterministic obj	Stochastic obj	VSS
1	10	6	21	14	50	492116.45	508931.15	16814.70
2	10	6	21	14	50	959426.20	959426.20	0.00
3	10	6	21	14	50	366398.70	390676.45	24277.75
4	10	6	21	14	50	391788.25	391788.25	0.00
5	10	6	21	14	50	267986.17	356295.58	88309.41
6	10	6	21	14	50	149312.45	153420.89	4108.44
7	10	6	21	14	50	141763.42	287742.47	145979.05
8	10	6	21	14	50	548805.06	557969.86	9164.80
9	15	6	21	14	50	386280.44	387092.76	812.32

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Table A.5 – continued from previous page

Instance	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{K}\rvert$	S	Deterministic obj	Stochastic obj	VSS
10	15	6	21	14	50	112377.73	123387.67	11009.94
11	15	6	21	14	50	515282.05	521914.51	6632.46
12	15	6	21	14	50	434881.35	462008.81	27127.46
13	15	6	21	14	50	551955.34	554902.05	2946.71
14	15	6	21	14	50	372263.89	372263.89	0.00
15	15	6	21	14	50	625586.54	665159.56	39573.02
16	15	6	21	14	50	73522.53	186168.22	112645.69
17	10	6	21	14	100	402861.13	431015.36	28154.23
18	10	6	21	14	100	153441.88	219408.67	65966.79
19	10	6	21	14	100	292072.68	300626.26	8553.57
20	10	6	21	14	100	85670.48	89950.96	4280.49
21	10	6	21	14	100	138540.73	147665.13	9124.40
22	10	6	21	14	100	246702.25	249561.55	2859.30
23	10	6	21	14	100	20044.29	50737.46	30693.17
24	10	6	21	14	100	31656.22	59706.79	28050.57
25	15	6	21	14	100	95412.56	512132.37	416719.81
26	15	6	21	14	100	846291.18	858269.63	11978.45
27	15	6	21	14	100	883552.06	911586.23	28034.16
28	15	6	21	14	100	93717.22	274345.42	180628.20
29	15	6	21	14	100	363011.25	468949.84	105938.59
30	15	6	21	14	100	351937.55	355083.28	3145.73
31	15	6	21	14	100	322077.35	357943.13	35865.78
32	15	6	21	14	100	422759.16	500639.70	77880.54

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Table A.5 – continued from previous page

Instance	$\lvert\mathcal{I}\rvert$	$\lvert\mathcal{A}\rvert$	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{K}\rvert$	S	Deterministic obj	Stochastic obj	VSS
33	10	6	21	14	200	489728.42	583829.16	94100.74
34	10	6	21	14	200	310789.98	312240.83	1450.85
35	10	6	21	14	200	437973.73	449589.74	11616.01
36	10	6	21	14	200	70286.14	87718.91	17432.77
37	10	6	21	14	200	120313.99	154382.48	34068.49
38	10	6	21	14	200	478250.16	503962.87	25712.71
39	10	6	21	14	200	252510.83	299193.87	46683.04
40	10	6	21	14	200	118114.02	392236.73	274122.71
41	15	6	21	14	200	914787.09	943951.22	29164.12
42	15	6	21	14	200	434097.84	435475.18	1377.34
43	15	6	21	14	200	201509.08	475821.19	274312.11
44	15	6	21	14	200	374304.81	397099.28	22794.46
45	15	6	21	14	200	678997.53	857831.10	178833.57
46	15	6	21	14	200	113255.60	863477.61	750222.01
47	15	6	21	14	200	447814.15	449452.81	1638.66
48	15	6	21	14	200	620961.52	622303.20	1341.68

Shown below are the results for sensitivity analysis of the objective value by sample average approximation (SAA) and the value of stochastic solution (VSS) for Sections 5.4 and 5.5, respectively.

Table A.6: Sensitivity analysis results

Level	Objective Value by Sample Average Approximation (SAA)					Value of Stochastic Solution (VSS)				
	Reward	Fuel consumption	Chartering fee	Competitive intensity	Market volatility	Reward	Fuel consumption	Chartering fee	Competitive intensity	Market volatility
1	693831.42	669387.26	495453.42	591083.33	597455.95	355242.31	331353.17	332194.26	148865.67	287858.37
2	649247.42	630235.30	494953.42	591083.33	597455.95	394617.74	337478.47	332610.93	148865.67	296191.71
3	604663.42	591083.33	494453.42	493203.42	597455.95	402917.31	400519.05	333027.60	422683.64	391635.30
4	560079.42	551931.37	493953.42	493203.42	493203.42	411216.88	407907.25	420750.31	422683.64	407212.22
5	515495.42	512779.40	493453.42	493203.42	493203.42	419516.44	415295.45	421716.98	422683.64	415182.73
6	470911.42	473627.43	492953.42	493203.42	493203.42	425375.39	457468.54	423650.31	748431.77	1136381.77
7	426327.42	434475.47	492453.42	493203.42	407455.95	440932.56	466627.39	424616.98	770098.44	1228381.77
8	381743.42	395323.50	491953.42	493203.42	307455.95	456489.73	475786.24	425100.31	1018251.77	1259531.77
9	337159.42	356171.54	491453.42	395323.50	296224.34	469779.14	972522.28	425583.64	1058751.77	1311181.77
10	292575.42	317019.57	490953.42	395323.50	156224.34	480561.66	984675.42	426550.31	1079475.10	1460421.77

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