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**SYNERGISTIC INTEGRATION OF
PIEZOELECTRIC–ELECTROMAGNETIC–TRIBOELECTRIC
GENERATORS: SYSTEM DESIGN, PERFORMANCE
EVALUATION, AND OPTIMIZATION ANALYSIS**

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The Hong Kong Polytechnic University
Department of Civil and Environmental Engineering

**Synergistic Integration of
Piezoelectric–Electromagnetic–Triboelectric Generators:
System Design, Performance Evaluation, and Optimization Analysis**

JIAMEI WANG

**A thesis submitted in partial fulfillment of the requirements for
the degree of Doctor of Philosophy**

August 2025

CERTIFICATE OF ORIGINALITY

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WANG JIAMEI (Name of student)

Dedicated to my family

ABSTRACT

The swift advancement of the Internet of Things (IoT) and Industry 5.0, along with the surging demand for self-powered real-time environmental monitoring and AI-driven automation systems, has increased interest in vibration-based energy harvesting technologies. Conventional power sources like batteries suffer from limited lifespans, high maintenance requirements, and environmental issues, highlighting the need for sustainable and maintenance-free energy solutions. Among various energy harvesting methods, piezoelectric generators (PEG), electromagnetic generators (EMG), and triboelectric nanogenerators (TENG) have become prominent due to their capacity to transform ambient mechanical energy into electrical power. However, each of these methods has its own limitations, such as restricted frequency adaptability, low power density, and durability concerns. To tackle these technical challenges, a triple hybridization of energy harvesting systems that integrate multiple transduction mechanisms can be created, enabling a broader operational bandwidth, improved efficiency, and superior energy conversion performance.

This research explores nonlinear multi-stable tri-hybrid energy harvesting systems that integrate PEG, EMG, and TENG units, with an emphasis on their system design, performance evaluation, and optimization strategies. In contrast to single-mode and dual-mode energy harvesters, tri-hybrid energy harvesting systems leverage the complementary advantages of each mechanism, resulting in higher power output, greater frequency adaptability, and improved durability. Specifically, PEG units effectively convert high-frequency mechanical vibrations into electrical energy, EMG units boost power generation in response to large-amplitude and low-frequency vibrations, and TENG units facilitate energy harvesting from time-varying, low-amplitude, and low-frequency mechanical excitations. By interacting these three mechanisms, the system achieves a multi-stability-based frequency up-conversion approach, enabling continuous and stable power generation across diverse environmental conditions. Despite the advantages of multi-stable hybridization, several significant challenges persist. Key concerns include optimizing the interaction among the three energy harvesting units, reducing negative interference, and ensuring that changes to one unit do not adversely affect the performance of the others. Furthermore, the complexity of the design escalates considerably when integrating multiple transduction mechanisms, thereby requiring careful

attention to geometric configurations, material properties, and coupling effects. In addition, the spatial limitations of compact electronic devices require efficient structural designs to maximize power output while minimizing the overall footprint.

To address these issues, this study employs a combined experimental and simulation methodology to systematically evaluate the performance of tri-hybrid vibration-driven energy harvesters. First, the role of triboelectric components (friction) within the overall system is analyzed, focusing on their effects on power generation, durability, and frequency response. Next, the auxetic piezoelectric structural unit is examined, highlighting its improved mechanical flexibility, strain distribution, and output efficiency. We also explore the effect of magneto-multi-stable oscillators into tri-hybrid design. Finally, strategies to enhance power density and optimize the integration of the hybrid nonlinear system are explored. The results offer valuable insights into the interactions between different energy harvesting mechanisms and offer guidelines for designing next-generation hybrid energy harvesters with improved efficiency and robustness. Experimental validations involve the fabrication and testing of tri-hybrid energy harvester prototypes, followed by controlled vibration and electromechanical characterization. The performance of tri-hybrid energy harvesters is evaluated under periodic and chaotic behaviors by analyzing voltage and current output at different vibration frequencies, the efficiency of multi-stable energy conversion mechanisms, the effects of material properties on charge generation, and the stability and durability of power output.

Going beyond structural design, optimization techniques are essential for enhancing the performance of tri-hybrid vibration-driven energy harvesters. While conventional optimization techniques (e.g., finite element method) are commonly used, they tend to be computationally intensive and require experimental validation. On the other hand, advanced machine learning methods (e.g., deep reinforcement learning), have been investigated for energy harvesting applications, but they mainly emphasize adaptive control instead of direct optimization. In contrast, Physics-informed neural networks (PINNs) offer an efficient data-driven optimization framework by directly integrating governing physical laws into a neural network architecture. Unlike conventional machine learning models that depend solely on labeled datasets, PINNs allow for accurate predictions with minimal or scarce experimental data. This work demonstrates that the integration of an advanced time-marching PINN-based (AT-PINN)

method with a genetic algorithm, forming “AT-PINN-GA approach”, can enhance and optimize critical design parameters. The arrangement of magnets in EMG is optimized by modifying the distance between them, which affects the restoring force and multi-stability. This adjustment enables transitions into tri-stable or quad-stable nonlinear system states, resulting in increased displacement and improved energy conversion efficiency. For the piezoelectric plate configuration, the method is used to predict optimal structural designs that maximize strain-induced charge generation while minimizing material degradation and mechanical losses. Furthermore, the performance of TENG units can also be enhanced. Compared to traditional numerical simulations and experimental tuning, the PINN-based approach significantly lowers computational costs, allowing for real-time optimization through dynamic adjustments of system parameters. This ensures that the tri-hybrid energy harvester operates at peak efficiency under varying environmental conditions, making it a highly effective and adaptable solution for energy harvesting applications. In addition, traditional methods typically require determining a single parameter first and then roughly setting the ranges for four additional parameters. After an initial calculation, each parameter must be adjusted sequentially until an optimal configuration is achieved for effective energy harvesting at a specific frequency. This process involves multiple iterations, making it time-consuming and inefficient. The integration of PINN-based optimization greatly enhances calculation efficiency, resulting in a substantial reduction in computational time compared to conventional tuning methods.

However, the increased complexity of tri-hybrid vibration-based energy harvesting structures leads to manufacturing and scalability issues, resulting in advanced microfabrication and materials engineering solutions. Power management and circuit integration present difficulties, as each transduction mechanism produces electrical output with varying voltage, current, and impedance characteristics. This requires sophisticated rectifiers, impedance matching circuits, and DC-DC converters for effective energy storage and utilization. Furthermore, nonlinear behaviors caused by electric hysteresis, charge leakage, and unpredicted distortion must be carefully managed to minimize energy loss and enhance long-term stability. Addressing these problems is essential for further optimizing the tri-hybrid energy harvester for practical applications. Future research should concentrate on developing advanced materials and structural configurations to improve durability, flexibility, and efficiency, ensuring the long-term stability and adaptability of tri-hybrid energy harvesters.

Besides, incorporating self-adaptive power management circuits will facilitate real-time adjustments of energy harvesting parameters based on environmental conditions, optimizing performance across varying operational scenarios. Finally, applying tri-hybrid vibration-based energy harvesters in wearable IoT-connected electronics and remote sensor networks will enable them to utilize their self-sustaining capabilities, making them suitable for a wide array of real-world engineering applications.

In conclusion, the present work offers a thorough analysis of tri-hybrid energy harvesting systems, showcasing their superior performance compared to conventional single and dual-mode harvesters. By integrating experimental, simulation and optimization approaches, this research contributes to the advancement of self-powered energy harvesting technologies, paving an efficient way for highly efficient, adaptive, and sustainable energy harvesting solutions.

LIST OF PUBLICATIONS

List of Journal Papers:

- J.M. Wang, Y.T. Zhang, Z.L. Chen, C. Wang, and S.K. Lai, Exploring the impact of auxetic structures in a hybrid design approach to low-frequency vibration energy harvesting, *Smart Materials and Structures*, 34, 065005, 2025.
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- C. Wang, S.K. Lai, J.M. Wang, J.J. Feng and Y.Q. Ni, An ultra-low-frequency, broadband and multi-stable tri-hybrid energy harvester for enabling the next-generation sustainable power, *Applied Energy*, 291, 116825, 2021.
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- Z.L. Chen, S.K. Lai, and J.M. Wang, On the role of hard-constraint strategies in PINNs for vibration analysis, *Proceedings of the 7th Cross-Strait Symposium on Dynamics, Vibration and Control*, Hong Kong and Macau, China, 16–22 December 2024.
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- S.K. Lai, Y.T. Zhang, J.M. Wang, S.Y. Leu and J.Q. Sun, Investigation of a smart noise barrier for dual active and passive control of construction noise, *Proceedings of Inter-Noise 2020 E-Congress*, Seoul, Korea, 23-26 Aug 2020.

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LIST OF SYMBOLS

Chapter 3

The main symbols used in this chapter are listed below:

A_b	Cross-sectional area of cantilever
A_p	Cross sectional area of piezoelectric layers
$b(t)$	Base displacement
$\ddot{b}$	Base acceleration
B_{ext-B}	External magnetic flux densities of magnets B
C	Arbitrary amplitude that is derived from the corresponding orthonormality conditions
d	Horizontal distance from the center point of magnet A to magnets B and C
d_{31}	Piezoelectric constant
E_b	Elasticity modulus of beam
E_p	Elasticity modulus of MFC layers
f	Separation of magnets B and C
F_f	Non-conservative force (friction force)
F_m	Magnetic force
g	Acceleration of gravity
H_{y1}	Magnitudes of the magnetic field intensity in y -direction at the center s of the front surface of magnet A
H_{y2}	Magnitudes of the magnetic field intensity in y -direction at the centers of the back surface of magnet A
I_b	Area moment of inertia of beam
I_p	Area moment of inertia of MFC layers
K_m	Surface current density of magnet A
l_A	Length of magnet A
L_b	Length of cantilever beam
L_p	Length of piezoelectric layers
m	Mass of magnet A
m_A	Magnetization of magnet A

n	A unit vector in the direction outward from the surface
$q(t)$	Associated time-varying generalized coordinates
S	Surface area of magnet A
$sgn(\cdot)$	Signum function
T	Kinetic energy
t_1	Initial time
t_2	Final time
T_m	Kinetic energy of magnet A
U	Potential energy
U_{b+p}	Potential energy of the cantilever beam with piezoelectric layers
U_m	Elastic potential energy
W	Kinetic energy resulting from force
$w(x, t)$	Deflection of beam relative to its supporting base
w_A	Width of magnet A
θ	Angle between AO_3 and horizontal direction
μ	Friction coefficient
μ_0	Permeability
ζ	Damping ratio
ρ_b	Density of cantilever
ρ_p	Density of piezoelectric layers
$\delta(x - L_b)$	Dirac's delta function
$\phi(x)$	Mode shape function
ω	Undamped natural frequency of the first mode of the beam without the tip mass

Chapter 4

The main symbols used in this chapter are listed below:

c_e	Electrical damping coefficient due to the relative displacement between magnetic mass and movable coil group
c_i	Additional damping coefficient due to impact effect
c_v	Linear viscous damping coefficient
d	Separation distance
$d\varphi/dx$	Rate of change of magnetic flux
F_d	Dry friction force
F_m	Magnetic force on the center magnetic slider
F_l	Lateral pressure exerted by contact surface on a slider
g	Acceleration of gravity
k	Spring stiffness
k_e	Effective stiffness
m	Mass of magnet
R_C	Coil resistance
R_L	External load
μ	Friction coefficient
φ	Total magnetic flux

Chapter 5

The main symbols used in this chapter are listed below:

$C(x_r)$	Capacitance between two electrodes with respect to displacement
C_e	Electrical damping coefficient due to the relative displacement between magnetic mass and movable coil group
C_i	Additional damping coefficient due to the impact effect
C_v	Linear viscous damping coefficient
d	Separation distance
D	Bending modulus per unit width of beam
d_1, d_2	Width of magnet A and B with Type I
d_{31}	Piezoelectric strain constant
d_g	Initial deformation
d_s	Separation gap between slider and folded piezoelectric spring
$d\phi/dx_r$	Rate of change of magnetic flux
$E(z, x)$	Induced electric field
E_p	Young's modulus of PZT-5A patches
E_s	Young's modulus of beryllium bronze substrate
F_l	Lateral pressure acting on the slider
F_m	Magnetic force
F_{quad}	Restoring force (quad-stable mode)
F_{tri}	Restoring force (tri-stable mode)
g	Acceleration of gravity
h	Width between magnets
k	Spring stiffness
k_e	Effective stiffness
l_1	Effective length of PZT-5A patches
l_2	Effective length of beryllium bronze substrate
m	Mass of magnet
M	Magnetic slider
$M(z)$	Bending moment at z
N	Effective number of turns
Q_{sc}	Short-circuit transferred charge

$R(z)$	Radius of curvature
R_C	Coil resistance
R_L	External load
t_n	Position of the neutral axis
t_p	Thickness of PZT-5A patches
t_s	Thickness of beryllium bronze substrate
V_{oc-EMG}	Open-circuit voltage of EMG
V_{oc-F}	Open-circuit voltage of TENG-F
V_{oc-PEG}	Open-circuit voltage of PEG
w	Width of beryllium bronze substrate
$x_b(t)$	Input excitation
$x_r(t)$	Relative displacement
ΔS	Change of contact area
ε_0	Permittivity of free space
ε_r	Dielectric constant of PZT-5A
σ	Surface charge density on the slider
$\sigma(z, x)$	Lateral stress on the PZT-5A patches at the position (z, x)
σ_{CS}	Surface charge density on the folded piezoelectric spring electrodes
μ	Friction coefficient
φ	Total magnetic flux

Chapter 6

The main symbols used in this chapter are listed below:

$B[w(x, t)]$	Differential operator on the boundary
c	Damping coefficient
c_1 and c_2	Constants of the strong Wolfe conditions
d_k	Searching direction
$D[w(x, 0)]$	Differential operator at the initial time
F_d	Dry friction force
F_m	Magnetic force
g	Acceleration of gravity
H_k	Array
k	Spring stiffness
k_i	Stiffness coefficients
$L(\theta)$	Loss function
$L_1^n(\theta)$	Initial displacement loss
$L_2^n(\theta)$	Initial velocity loss
$L_r(\theta), L_b(\theta)$ and $L_i(\theta)$	PDE loss
$L_r^n(\theta)$	Governing equation residual
m	Mass of magnet
$N[w(x, t)]$	Differential operator within the interior of the domain
N_b	Number of sampling points corresponding to the boundary condition loss
N_i	Number of sampling points corresponding to the initial condition loss
N_r	Number of sampling points corresponding to the PDE loss
w	Displacement of the magnet
$w(x, t)$	Solution function of the partial differential equation
$\tilde{w}(x, t)$	Continuously differentiable function
$\ddot{x}_b$	External excitation
Δx	Time interval of the n^{th} time period
α_k	Step size

β	Amplification factor
γ	Magnification factor
ε	Convergence coefficient
θ	Trainable parameters
θ^*	Optimal trainable parameter
λ_1	Weight of initial displacement loss
λ_2	Weight of initial velocity loss
λ_b	Weight of boundary condition loss
λ_i	Weight of initial condition loss
μ	Friction coefficient
Ω	Bounded domain in space
$\partial\Omega$	Boundary

LIST OF ABBREVIATIONS

AC	Alternating Current
AT-PINN	Advanced Time-Marching Physics-Informed Neural Network
Cu	Copper
DC	Direct Current
EMG	Electromagnetic Generator
FEM	Finite Element Modeling
GA	Genetic Algorithm
GNN	Graph Neural Network
MEMS	Micro-Electro-Mechanical Systems
PEG	Piezoelectric Generator
PDE	Partial Differential Equation
PINN	Physics-Informed Neural Network
PTFE	Polytetrafluoroethylene
PVDF	Polyvinylidene Fluoride
PZT	Lead Zirconate Titanate Piezoelectric Ceramics
RMS	Root Mean Square
STD PINN	Standard Physics-Informed Neural Network
TENG	Triboelectric Nanogenerator
WEC	Wave Energy Converter

CHAPTER 1

- INTRODUCTION

1.1 Background Information

The swift pace of urbanization and population increase has led to a need for more smart and sustainable cities. The Internet of Things (IoT) has surfaced as a groundbreaking technology, presenting remarkable opportunities to improve urban operations and services. The numerous possibilities offered by IoT, which creates a framework for the electronic oversight and control of physical objects, has sparked a wave of innovation. The integration of IoT and artificial intelligence (AI) in smart cities has driven significant advancements in wireless sensor networks, thereby highlighting the crucial importance of mobile or distributed power sources. The implementation of IoT connects extensive networks of devices, sensors, and actuators that gather, analyze, and share real-time data to boost efficiency, safety, and overall quality of life (Chataut et al., 2023). Its uses extend across various fields, such as smart infrastructure, environmental monitoring, healthcare systems, industrial automation, and transportation networks.

As the core component of IoT-connected systems, low-powered sensors serve as essential components for capturing environmental, mechanical, or biological signals, facilitating informed decision-making and accurate control. However, the exponential growth of IoT-enabled devices, frequently installed in remote or difficult-to-access locations, presents considerable challenges regarding power supply. Conventional batteries, although commonly utilized, are constrained by limited lifespans, high maintenance needs, and environmental concerns related to chemical waste.

In this context, vibration-based energy harvesting techniques are essential for overcoming the power challenges faced by IoT-connected systems. These devices convert ambient mechanical energy into electrical energy, such as vibrations from buildings, bridges, vehicles, and industrial machinery, providing a self-sustaining power source for sensors and wireless

communication modules. Energy harvesting techniques not only prolong the operational lifespan of IoT-connected devices but also reduce dependence on traditional batteries, thereby supporting environmental sustainability and economic feasibility. As IoT continues to grow, incorporating vibration-based energy harvesting technologies into smart electronic systems is crucial for developing scalable, efficient, and sustainable solutions for the future.

Energy harvesting techniques provide an alternative and sustainable way for powering devices by harnessing renewable energy sources from the ambient environment. These technologies can transform different types of energy, including solar energy, wave energy, thermal energy, radio-frequency (RF) energy, and vibrational energy, into electrical energy. Hence, a continuous and environmentally friendly power supply for contemporary applications can be provided. Each energy source offers distinct benefits that make it ideal for particular situations. For example:

- Solar energy: Solar harvesting is particularly effective in outdoor settings with ample sunlight, making it well-suited for applications such as outdoor sensors and solar-powered charging stations (Luo et al., 2018).
- Wave energy: A prevalent method entails utilizing energy derived from ocean surface waves, a concept that has garnered significant attention owing to its abundance and capacity to operate independently of temporal, meteorological, or seasonal fluctuations (Tang et al., 2025).
- Thermal energy: Waste heat produced during industrial processes or human activities can be harnessed to power devices in thermally active environments, such as industrial facilities or wearable technologies (Zhu, 2016).
- Radio-frequency energy: RF harvesting utilizes available wireless signals, allowing low-power devices to function in environments where radio signals are prevalent, such as urban areas with extensive wireless infrastructure (Ibrahim et al., 2022).

Of these options, vibrational energy stands out as a highly versatile and sustainable resource, particularly in urban and industrial contexts (Daud et al., 2020). It can be harvested from a range of ambient mechanical sources, including building motions, bridge vibrations, vehicle movements, industrial machinery, and even human-motion activities. This energy source is readily accessible in various sources, providing a reliable and consistent energy

supply. Moreover, vibrational energy has a high-power density relative to other energy forms, enabling efficient energy generation even under low-frequency, small-amplitude, and time-varying excitation conditions (Dong et al., 2019).

The advantages of vibration-based energy harvesting techniques are particularly evident in its broad application ranges, for example:

- Smart infrastructure monitoring: Vibrational energy harvesters can power wireless sensors embedded in bridges, high-rise buildings, or tunnels, enabling real-time assessment of structural health monitoring under various environmental loads (e.g., wind, traffic, or seismic activity).
- Industrial automation: Machinery vibrations in factories can be harvested to power IoT sensors for predictive maintenance, operational efficiency, and safety monitoring.
- Transportation systems: Vibrations induced by trains, buses, and automobiles can be harnessed to generate energy for monitoring systems within smart transportation networks.
- Healthcare devices: Human motions resulting from different body positions can be used as a vibrational energy source for wearable and portable devices to allow real-time healthcare monitoring. Furthermore, micro-sized self-powered implantable medical devices can lessen the necessity for battery replacements.
- Consumer electronics: Portable and wearable devices can take advantage of vibration energy harvesting, enabling them to operate continuously without the need for frequent recharging.

The versatility of vibrational energy harvesting techniques makes it a crucial element of energy-efficient systems in both urban and remote environmental settings. Its capacity to capture mechanical energy from a variety of common sources guarantees a sustainable power supply for IoT-driven devices, furthering the objectives of smart cities and sustainable development. Vibration energy harvesting techniques harness ambient energy to generate electrical power, and several transduction mechanisms have been developed for this purpose, including piezoelectric, electromagnetic, and triboelectric mechanisms. Each of these mechanisms presents unique advantages and challenges, shaped by their fundamental physics principles, material properties and practical uses. Therefore, a thorough understanding of these

mechanisms is essential for optimizing their design and integrating them into hybrid systems for wider and more efficient applications.

Piezoelectric energy harvesting relies on the principle that certain materials (like lead zirconate titanate (PZT)) to generate an electric charge when they undergo mechanical deformation. This phenomenon, known as the piezoelectric effect, has been extensively utilized due to the highpower density and efficient energy conversion capabilities of piezoelectric materials under dynamic loads (He et al., 2024). Furthermore, piezoelectric systems are particularly well-suited for micro- and nanoscale applications because they can be miniaturized without significant loss of performance. However, the brittle nature of piezoelectric materials presents technical challenges for their durability, particularly in environments with substantial deformations or repeated impacts. In addition, their working performance may decline under extreme environmental conditions, e.g., high humidity or temperature fluctuations. These can limit their applicability in certain situations.

Electromagnetic energy harvesting is based on Faraday's law of electromagnetic induction. This principle indicates that when there is relative motion between a magnetic field and a coil, an electric current is produced (Muscat et al., 2022). This method is highly durable and reliable since it does not depend on mechanical contact, making it well-suited for long-term and high-frequency applications. Electromagnetic systems are frequently used in large-scale energy harvesting applications, e.g., industrial machinery or transportation systems, where significant amplitudes and high-frequency vibrations occur. However, the relatively large components, including magnets and coils, render this approach less appropriate for small-scale or lightweight devices. Its efficiency decreases in low-frequency or low-amplitude environments.

Triboelectric energy harvesting, which takes advantage of contact electrification and electrostatic induction to produce power, was first proposed by Fan et al. (2012). When two materials possessing distinct triboelectric characteristics make contact and subsequently separate, an electrical charge is exchanged, resulting in a potential difference that can be captured as electrical energy. This approach has garnered interest due to its versatility, as it can employ a wide variety of materials, including flexible and lightweight options, and is adaptable to irregular, low-frequency vibrations. Triboelectric systems hold particular promise for

wearable technologies and environmental monitoring, where their flexibility and material diversity provide significant benefits (Wang et al., 2015). However, the physical contact between materials can lead to wear and degradation over time, which diminishes their long-term reliability. Moreover, the power output of triboelectric-based systems is often lower than that of piezoelectric or electromagnetic methods, limiting their applicability in scenarios that require high energy density.

The limitations of individual transduction mechanisms have led to the creation of hybrid energy harvesters that combine two or more methods. For instance, merging piezoelectric and electromagnetic mechanisms allows the system to capture the high power density from piezoelectric materials at smaller scales while benefiting from the stability of electromagnetic systems for larger applications. Similarly, combining piezoelectric and triboelectric methods takes advantage of the piezoelectric efficiency at medium frequencies alongside the triboelectric adaptability to irregular or low-frequency vibrations (He et al., 2018). These hybridization designs effectively broaden the operational bandwidth and improve energy conversion efficiency. However, integrating multiple mechanisms together increases system design complexity and often leads to larger size and higher costs, presenting new engineering challenges.

Integrating all three mechanisms, including piezoelectric, triboelectric, and electromagnetic, into a single energy harvesting system significantly explores its application potentials (Zhang et al., 2020b). These tri-hybrid systems offer exceptional versatility and resilience, enabling them to capture energy from a wide range of vibrations. For example, piezoelectric elements perform well in medium-frequency environments, triboelectric components are effective with irregular and low-frequency sources, and electromagnetic systems optimize energy output from high-frequency or large-amplitude vibrations. This combination ensures continuous and efficient energy harvesting across various conditions, making tri-hybrid systems particularly well-suited for complex urban and industrial settings. Additionally, the integration of these three mechanisms can achieve higher power density and broader applicability while ensuring durability and adaptability, further promoting sustainable development and the growth of self-powered systems.

1.2 Problem Statement

The widespread implementation of IoT technologies and the rapid advancement of smart infrastructures have created a pressing demand for efficient, self-powered vibration energy harvesting systems. Enhancing the spatial efficiency of energy harvesters has become a vital design consideration. Achieving higher energy density and output efficiency in limited spaces while ensuring system reliability and stability poses a significant challenge in current research.

Nonlinear dynamics is essential in determining the spatial efficiency of energy harvesters. In multimodal hybrid systems, the dynamic responses of piezoelectric, electromagnetic, and triboelectric mechanisms are closely linked to their geometric designs. For instance, piezoelectric elements require adequate bending or tensile deformation for effective energy conversion; however, reducing their size may result in insufficient deformation and lower efficiency. Similarly, the arrangement of magnets and coils in electromagnetic harvesters must take into account the spatial limitations of magnetic field strength and the trajectory of relative motions. In contrast, triboelectric harvesters heavily rely on contact area and mode to achieve optimal output performance. Optimizing nonlinear coupling designs is crucial for enhancing energy harvesting efficiency within confined spatial conditions.

Numerical simulations and experimental validations pose additional challenges in optimizing the performance of multimodal hybrid systems. The highly nonlinear coupling behavior in restricted spaces increases complexity. During simulations, unknown parameters (e.g., damping coefficients and material properties) often lead to discrepancies between simulated and experimental results. These inconsistencies necessitate considerable time for parameter adjustments and calibrations. Developing effective optimization methods to streamline this tuning process and improve the alignment between simulations and experiments remains a key area of research focus.

Enhancing the power density of energy harvesters requires innovative design strategies. Integrated structural designs can combine multiple energy harvesting mechanisms into a single physical framework. For example, layering piezoelectric materials with flexible triboelectric films or embedding electromagnetic components within piezoelectric substrates can save space while improving coupling effects. Multimodal structural optimization can further enhance

material utilization and energy conversion efficiency by adjusting the geometric arrangement of components, such as using folded, stacked, or honeycomb structures. In addition, broadband response designs can expand the operational frequency range of energy harvesters, utilizing nonlinear coupling for frequency tuning or gradient stiffness structures to capture multiple vibrational modes.

Emerging technologies also offer potential improvements in spatial efficiency. For instance, nanomaterials and micro-nano fabrication techniques enable the creation of high-efficiency energy harvesting components within minimal volumes, significantly reducing system size while increasing power density. When combined with intelligent optimization algorithms, such as genetic algorithms or machine learning approaches, these strategies can further enhance the spatial utilization and dynamic performance of energy harvesters.

In short, improving the spatial utilization of energy harvesters necessitates a comprehensive approach that addresses nonlinear dynamics, multimodal coupling designs, material innovations, and simulation validation. This study will focus on tri-hybrid (piezoelectric–electromagnetic–triboelectric) multi-stable nonlinear systems, exploring strategies for efficient energy harvesting in confined spaces. Through a combination of theoretical analysis and experimental investigation, this work aims to propose design solutions that optimize spatial utilization. The findings will provide essential support for the development of compact self-powered systems, advancing their applications in IoT technologies and smart infrastructures.

Physics-Informed Neural Networks (PINNs) represent a category of deep learning architectures that integrate underlying physical principles into the training of neural networks (Raissi et al., 2019). The rationale behind a PINN is that it is trained not only on available data but also on the residuals of physics equations (e.g., partial differential equations or conservation laws) that can describe such systems. By embedding these equations into the loss function, the network is “informed” by physics knowledge and is guided to produce solutions that adhere to established and known scientific principles. This approach effectively integrates (potentially noisy) observational data with mathematical models (Karniadakis et al., 2021), ensuring that

the learned solutions respect underlying physical constraints. PINNs were first brought to prominence by Raissi and his co-workers (2019) as a unified framework for solving forward problems (predicting system behavior) and inverse problems (parameter optimization or state estimations) involving nonlinear PDEs. Since then, the method has gained considerable interests, with researchers demonstrating PINNs on applications ranging from fluid dynamics and structural mechanics to quantum physics. The appeal of PINNs lies in their ability to leverage well-established physical theories within deep learning models, this can merge data-driven learning with domain knowledge in a fully trainable end-to-end framework.

A key motivation for using PINNs in optimization is their ability to overcome limitations of traditional black-box or purely data-driven approaches. In conventional optimization of engineering systems, one often relies on iterative simulations (e.g., finite element analyses) or empirical models, combined with numerical optimization algorithms (gradient-based optimizers, genetic algorithms, etc.). These approaches can be computationally expensive and typically require extensive data or simulation runs. Moreover, standard machine learning models that ignore physics usually struggle in regimes with sparse data and provide no guarantee that their predictions obey physical laws. In contrast, PINNs shine in data-scarce environments: by incorporating prior knowledge of physics, they constrain the solution space to only physically plausible solutions. This makes PINNs highly data-efficient and robust.

As reported by Raissi et al. (2019), when faced with limited data, most modern ML techniques lack robustness and “*fail to provide any guarantees of convergence*”, whereas leveraging physical laws as a form of regularization can dramatically improve reliability. In practice, this implies that PINNs require far fewer training samples to achieve accurate results, since the physics-informed loss guides the model even in areas with scarce data. Another merit of PINNs is their ability to generalize and extrapolate beyond the training domain. By building physics knowledge directly into a neural network model, PINNs can often make accurate predictions for scenarios outside the original data range, as the network’s behavior is anchored by the governing equations rather than just the data points. This leads to improved interpretability and trustworthiness of the model. Indeed, physics-guided machine learning approaches tend to offer better generalizability and even extrapolation capability compared to purely data-driven models (Xu et al., 2023).

From an optimization perspective, PINNs offer a seamless way to handle complex constraints and multi-physics couplings. Traditional optimization methods might require simplifying assumptions or decoupling of physics to make a problem tractable. In contrast, a PINN can naturally incorporate multiple coupled field equations into its loss function, handling them simultaneously. Because the PINN essentially serves as a mesh-free simulator, it can evaluate objective functions and constraints continuously in space and time without needing an explicit mesh or iterative solver at every optimization step. This has practical computational benefits: once a PINN is trained to represent a system's behavior, it can act as a fast surrogate model for optimization queries. Studies have shown that PINN surrogates can achieve comparable accuracy to high-fidelity simulations at a fraction of the computation cost. For example, a physics-informed neural network trained to emulate a finite element model can predict system responses much faster than running a new finite element analysis for each parameter set, thereby accelerating design optimization loops.

Furthermore, PINNs benefit from automatic differentiation, meaning sensitivities (gradients) with respect to design parameters can be obtained analytically from the network. This eliminates the need for finite-difference or adjoint sensitivity analyses typically required in conventional optimization. In summary, by blending physics into the learning process, PINNs reduce the reliance on large data sets, ensure adherence to physical laws, and often streamline the optimization of complex systems. These advantages position PINNs as a powerful tool for engineering optimization tasks that are beyond the reach of traditional methods or would otherwise require prohibitive computational effort.

Vibration-based energy harvesting systems, particularly those that integrate multiple transduction mechanisms like piezoelectric, electromagnetic, and triboelectric conversions, provide a promising opportunity for PINN-based optimization. These systems are inherently multi-physics problems, as they involve mechanical vibrations, electrical dynamics, material nonlinearities, and cross-coupling effects. Optimizing an energy harvester often means balancing these interacting physics to maximize power output under various conditions. PINNs are well-suited to this challenge because they can capture the full physics of the device in a single trainable model. Recent literature shows a growing interest in using PINNs and physics-informed AI for improving energy harvesting devices. For instance, researchers have

developed PINN frameworks to perform parameter identification in piezoelectric energy harvesters. In one study, a PINN was trained on limited experimental data from a piezoelectric harvester and was able to accurately infer key device parameters by ensuring the neural network's predictions satisfy the piezoelectric governing equations (Bai et al., 2024). This approach offers a way to calibrate and optimize harvesters using real-world data without resorting to exhaustive trial-and-error experiments.

Another promising direction is the inverse design and topology optimization of energy harvesters using physics-informed neural networks. A striking example is in triboelectric nanogenerators (TENGs), where the design space (choice of materials, surface microstructures, etc.) is complex and not amenable to straightforward optimization. Jiao et al. (2023) introduced a physics-informed AI inverse design framework for TENGs that leverages PINN concepts. By integrating the fundamental triboelectric effect equations into a neural network, their method can intelligently search for device configurations that yield improved electrical outputs. This approach, termed “Complete PINN-based Topology Optimization (CPINNTO)” (Jeong et al., 2023), demonstrates how physics-informed networks can discover non-intuitive designs for nanogenerators that traditional methods might miss. The use of PINNs in this context addresses the challenge that conventional TENG models face: uncertainties and complex nonlinear behavior that make purely analytical optimization difficult. With a PINN-based model, one can explore a wide design space (e.g., new geometries or material distributions) with the guidance of physical laws, thereby finding optimized designs that achieve higher energy harvesting efficiency.

Beyond triboelectric devices, the same principles have been applied to other harvesters. Researchers have reported PINN applications for vibration-based harvesters and hybrid systems, where the network learns the coupling between mechanical motion and electrical output. In thermo-electric energy harvesters (another complex multiphysics system involving heat transfer and electric generation), PINNs have been used as surrogates to reduce simulation time.

In summary, PINNs can make a substantial progress in our approach to modeling and optimizing complex engineering systems. By weaving the fabric of physical laws into the

structure of neural networks, PINNs mitigate the data requirements and physical inconsistencies that often plague conventional optimization methods. They bring notable advantages – data efficiency, enforcement of physical constraints, ability to handle coupled multi-physics problems, and integration of simulation and optimization, which have been demonstrated in a range of fields from fluid mechanics to structural design. The concept of a triple hybridization design (piezoelectric–electromagnetic–triboelectric generator) exemplifies a modern multi-domain system that can benefit greatly from PINN-based optimization.

1.3 Research Motivation and Objectives

In terms of research motivation, there are four primary reasons for integrating PEG–EMG–TENG transduction mechanisms:

- First, piezoelectric materials typically exhibit a high natural frequency of vibration, while flexible materials tend to have a low electromechanical coupling coefficient. To effectively harvest low-frequency energy, a relatively large proof mass is necessary in the generator to match the frequency range and capture vibrational energy. A magnet can serve as this proof mass, enabling the formation of an EMG.
- Second, this energy harvester will utilize a multi-stable nonlinearity-enhanced mechanical frequency up-conversion mechanism, which can improve power density under low-amplitude excitation for various applications. Implementing this mechanism will depend on magnetic forces, making magnets essential.
- Third, while both PEG and TENG generate high voltage outputs, EMG is more suitable for applications requiring large currents. Integrating all three into a single hub can lead to enhanced electrical performance compared to using any one of them alone.
- Finally, the integration of these three mechanisms can reinvigorate limitless and ingenious design strategies to achieve enhanced efficiency. For example, as a dual function, an electromagnetic approach can also be used for designing an efficient system for wireless charging from a standard wireless charger through resonant inductive coupling in an air medium.

In this connection, this research will focus on two main research objectives, each targeting essential aspects of tri-hybrid vibration-based energy harvesters to improve their performance and expand their applicability in low-frequency, low-amplitude, and time-varying

environments. The first objective is to explore the structural optimization of a hybrid energy harvester under nonlinear dynamic vibrations and to evaluate the individual contributions of its components through experimental and simulation methods. Hybrid energy harvesters integrate piezoelectric, electromagnetic, and triboelectric mechanisms, each offering distinct advantages and limitations, to capture energy across a broad spectrum of vibrational frequencies and amplitudes. Understanding the role of each component in the overall output of the system is crucial for optimizing performance. This study constructs the output response of the hybrid energy harvester under external excitations and systematically assesses the contributions of the triboelectric, and piezoelectric components. By analyzing their interactions and individual performances, the research identifies key optimization strategies to enhance energy conversion efficiency and bandwidth. Experimental validations are conducted to confirm the simulated results, ensuring the practical viability of the optimized design.

The second objective tackles a more complex challenge: designing a hybrid energy harvester that can achieve optimal output in the low-frequency range when key parameters such as material dimensions, spacing, and elastic coefficients are unknown. Traditional design methods often depend on predefined parameters, which limits adaptability and efficiency in environments with uncertain or variable conditions. To address this limitation, this study utilizes PINNs, which incorporate physical laws into the neural network training process. By leveraging PINNs, the research develops a data-driven yet physics-informed model to predict and optimize the performance of the hybrid energy harvester under varying parameter conditions. This approach significantly reduces the computational costs associated with conventional optimization methods and provides a robust framework for managing uncertainties in system design. Through iterative learning and training, the PINN-based model identifies optimal configurations, ensuring high output efficiency even in challenging low-frequency vibrational environments.

Together, these objectives contribute to a comprehensive understanding of tri-hybrid energy harvesters by integrating experimental validation, theoretical modeling, and advanced optimization techniques. The completion of this work will facilitate the development of tri-hybrid energy harvesting techniques, and enable its use in self-powered systems for IoT-connected devices in various low-frequency operational contexts.

1.4 Thesis Outline

The dissertation is structured into seven chapters. A concise outline of the dissertation is provided as follows:

- Chapter 1 provides an overview of the current study and outlines the research goals.
- Chapter 2 reviews the relevant literature on hybrid energy harvester system, nonlinear vibration-based energy harvesting techniques, and PINN-based optimization methods.
- Chapter 3 examines the effect of friction on a tri-hybrid energy harvesting system. The chapter will discuss theoretical modeling and experimental characterization to evaluate the influence of frictional force in a tri-hybrid multi-stable energy harvesting system.
- Chapter 4 explores the impact of auxetic structures in enhancing hybrid energy harvesters. The chapter will evaluate how auxetic structures influence the piezoelectric energy harvester part and cooperate with other components under the low-frequency excitation of the hybrid energy harvester system.
- Chapter 5 investigates the role of multi-stable oscillators in vibration-based energy harvesting, with a focus on magnetic interactions and nonlinear resonance. This chapter will discuss optimization strategies for improving energy efficiency and working stability.
- Chapter 6 introduces PINN-based optimization approaches for tri-hybrid vibration-based energy harvesters. The section will discuss the advantages of using advanced PINNs over conventional optimization techniques, and their application in the design of energy harvesting system parameters.
- Chapter 7 provides a detailed summary of the key findings from the research, critically evaluates its strengths and limitations, and proposes recommendations for potential directions in future studies.

CHAPTER 2

- LITERATURE REVIEW

2.1 Overview of this Research

This chapter offers an in-depth examination of essential subjects in vibration-based energy harvesting methods. It starts with a general introduction to the principles of nonlinear energy harvesting. The chapter then delves into the basic workings and practical uses of piezoelectric, electromagnetic, and triboelectric energy conversion technologies. Additionally, it examines the operation and design of dual hybrid energy harvesting systems. An in-depth discussion will cover the emerging concept of triple hybridization systems that integrate piezoelectric, electromagnetic, and triboelectric mechanisms to enhance energy conversion efficiency. Finally, the chapter reviews the role of PINNs in optimizing tri-hybrid energy harvesting systems by highlighting its advantages in performance prediction and design optimization.

2.1.1 Fundamentals of Nonlinear Energy Harvesting Techniques

Energy harvesting involves capturing the surrounding energy in the environment and transforming it into electrical power for electronic devices. Traditional linear-type energy harvesters operate efficiently near their resonant frequencies but exhibit significant performance degradation when the excitation frequency deviates from resonance, limiting their applicability in environments with variable or broadband frequency spectra (Harne et al., 2013). In addition, research studies also offered insights into the challenges associated with linear energy harvesters and discussed various approaches to achieve broadband energy harvesting frequency ranges, including the use of nonlinear dynamic mechanisms and multi-modal designs (Tang et al., 2010). To address these limitations, nonlinear energy harvesting mechanisms introduce intentional nonlinearities into the dynamics of systems. Common nonlinear approaches include bi-stable, tri-stable, and multi-stable configurations, which enable broader frequency operation and enhanced energy conversion efficiency (Daqaq et al., 2014), as shown in Fig. 2.1.

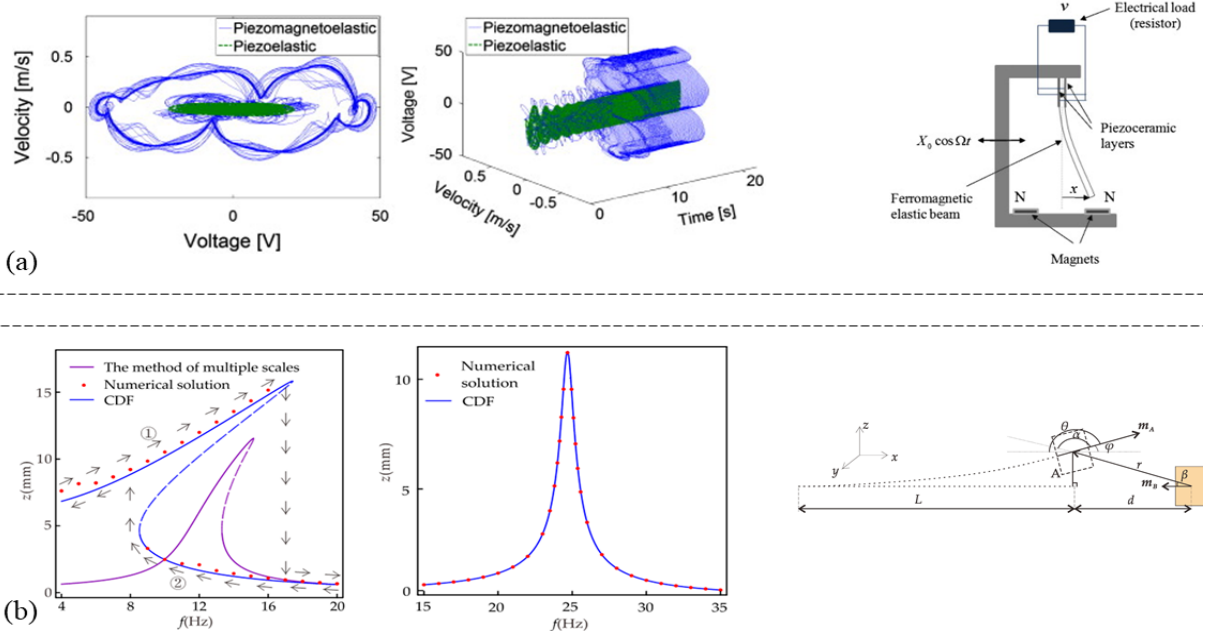


Fig. 2.1: (a) Experimental voltage versus velocity trajectories of nonlinear and linear systems (Erturk et al., 2011); and (b) displacement versus frequency of nonlinear and linear systems (Zhang et al., 2021b).

Unlike linear dynamic systems, nonlinear energy harvesters can exhibit complex behaviors, including jump phenomenon, hysteresis effect, and chaotic motions. When properly controlled such dynamic behavior, the performance of energy harvesters in power generation can be greatly enhanced (Erturk et al., 2009). For instance, a bi-stable magneto-piezo-elastic harvester was developed that demonstrated reversible hysteresis, allowing for broadband vibration energy harvesting by harnessing inter-well oscillations (Stanton et al., 2009). Their study highlighted those bi-stable dynamic structures can significantly enhance power output compared to traditional linear harvesters. Similarly, the occurrence of stochastic resonance was investigated in bi-stable energy harvesters and demonstrated that nonlinear coupling can increase power conversion efficiency by up to 250% in noisy conditions (Cottone et al., 2009).

Figure 2.1 illustrates the experimental trajectories of voltage versus velocity, as well as displacement versus frequency, for both nonlinear and linear systems, demonstrating that nonlinear systems can effectively broaden the bandwidth. Figure 2.2 illustrates that the nonlinear harvester can exhibit mono-stable, bi-stable, or tri-stable behavior by adjusting the distance parameters between the magnets. In contrast to a triple-welled harvester, whose

performance is highly sensitive to its initial conditions, a harvester with a single-well potential can attain a relatively large amplitude near the resonant frequency. This research (Lai et al., 2019) also proved that this harvesters with a triple-well potential offer several advantages for energy conversion, including increased efficiency, broader operational bandwidth, and effectiveness at lower frequencies.

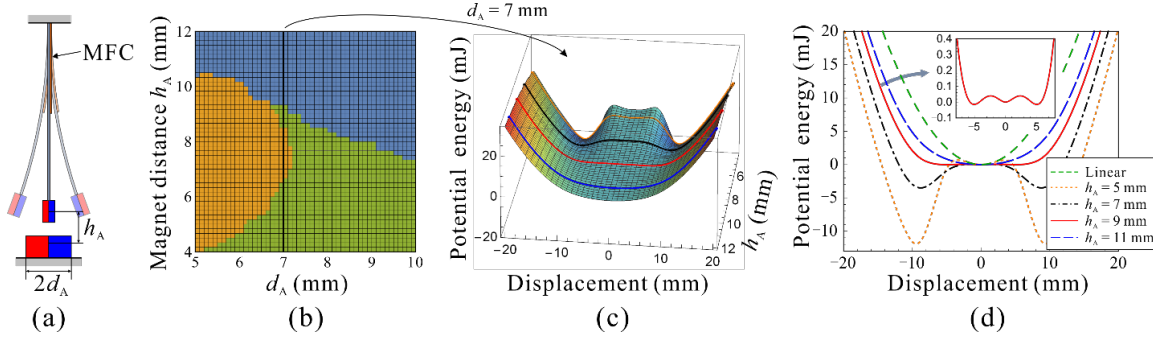


Fig. 2.2. (a) Schematic of the nonlinear harvester; (b) parameter regions: mono-stable (blue region), bi-stable (yellow region), tri-stable (green region); (c) potential energy varies with separation distance; and (d) the potential energy functions highlighted in (c) (Lai et al., 2019).

2.1.2 Key Advantages Compared to Conventional Energy Harvesting Systems

One of the primary advantages of nonlinear vibration energy harvesters is their ability to operate effectively over a wide range of frequencies. This broadband response is particularly beneficial in environments with unpredictable or time-varying excitation frequencies. In addition, nonlinear vibration energy harvesters can achieve higher energy conversion efficiency under specific conditions compared to their linear counterparts (Andò et al., 2023; Yu, 2020).

Upon reviewing the literature, Liu et al. (2012) designed a frequency-up-conversion energy harvester that utilized a cantilever stopper mechanism to achieve nonlinear frequency shifts. Their findings indicated that nonlinear effects allowed the device to generate power across a frequency range more than three times wider than that of conventional designs. Masana et al. (2012) explored the potential of parametric resonance in vibration-based harvesters, concluding that nonlinear parametric excitation can enhance energy conversion efficiency by over 40% compared to conventional piezoelectric designs. Another significant advantage of nonlinear systems is their ability to generate larger amplitude oscillations, leading to increased

power output. For example, Fu et al. (2023) developed a multi-stable energy harvester that leveraged magnetically coupled oscillators to achieve high-amplitude, low-frequency responses. Their experimental results confirmed that multi-stable harvesters exhibit superior performance under random excitations compared to linear harvesters. Moreover, nonlinear harvesters tend to be less sensitive to damping effects, which commonly limit the efficiency of traditional linear systems. Furthermore, Ferrari et al. (2010) investigated chaotic vibration energy harvesters and found that chaotic motion decreases sensitivity to damping losses, which can enhance long-term stability and power generation.

2.1.3 Applications in Self-Powered IoT Systems

The growing demand for self-powered electronics in applications such as wearable devices, IoT sensors, and biomedical implants has driven interest in nonlinear energy harvesting solutions. Unlike battery-dependent systems, nonlinear energy harvesters can provide continuous, maintenance-free power by utilizing ambient mechanical energy sources (Wang et al., 2023a). Figure 2.3 shows the wireless sensors for different real-time monitoring systems across multi-disciplinary applications.

Reviewing existing works, Iannacci et al. (2014) demonstrated a nonlinear MEMS-based energy harvester capable of powering wireless IoT nodes under ambient vibrations. Their findings showed that nonlinear designs can increase power density by over 35% compared to linear MEMS harvesters, making them ideal for autonomous sensor networks. Likewise, Qian et al. (2020) developed a bio-inspired nonlinear harvester for electronic wearables, proving that soft nonlinear structures can efficiently harness the energy of biomechanical motions to power low-energy devices. In structural health monitoring, low-powered sensors equipped with nonlinear energy harvesters can continuously monitor the variation of infrastructure integrity without requiring the supply of external power sources. Similarly, in healthcare applications, researchers have explored using nonlinear piezoelectric harvesters to extract energy from human motions, reducing or eliminating the need for frequent battery replacements (Ding et al., 2023). Figure 2.4 shows the trends in the number of IoT-based device connections, indicating that a significant number of IoT systems are currently being deployed and will continue to be in the future. It also highlights the growing demand for wireless sensors in self-powered systems.



Fig. 2.3: Schematic illustration of wireless sensors used for environmental monitoring, human monitoring, industrial manufacturing, and daily application (Li et al., 2023b).

Total number of device connections (incl. Non-IoT)

20.0Bn in 2019– expected to grow 13% to 41.2Bn in 2025

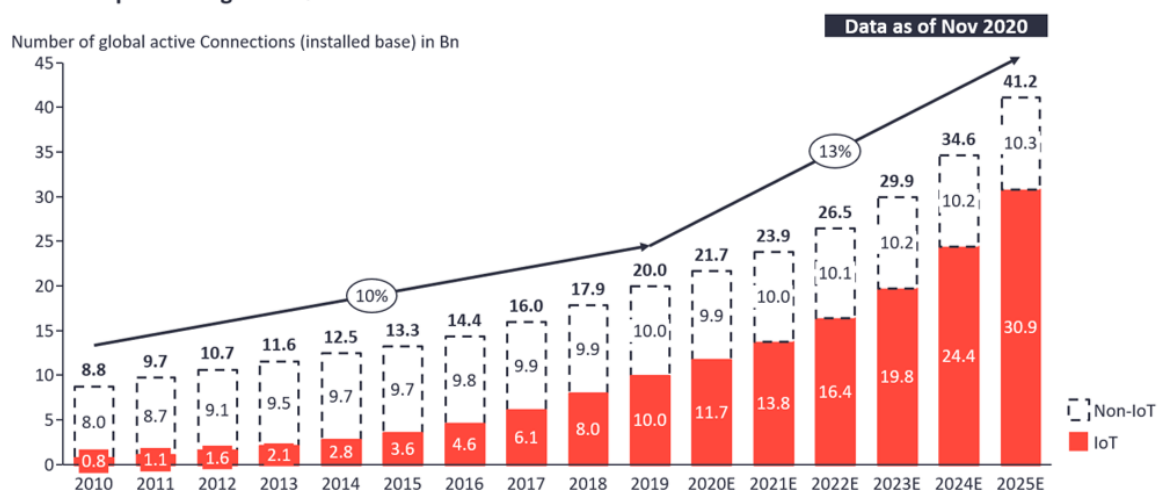


Fig. 2.4 Trends in the number of IoT-based device connections (Kang et al., 2022).

2.2 Various Energy Harvesting Working Mechanisms

The demand for self-sustaining electronic devices has driven advancements in energy harvesting techniques. These methods enable electronic devices to operate independently without reliance on traditional batteries, making them ideal for applications in structural health monitoring, biomedical implants, and autonomous sensors (Beeby et al., 2006). Piezoelectric, electromagnetic, and triboelectric energy harvesters represent three major transduction mechanisms for converting mechanical energy into electrical energy (Xiao et al., 2022). Each method has their distinct characteristics in terms of energy conversion efficiency, scalability, and environmental adaptability. This section provides a comparative review of these three harvesting techniques, highlighting their advantages and limitations to guide future research and application development.

Piezoelectric energy generators (PEGs) generate electricity through mechanical stress-induced polarization in piezoelectric materials. The conversion mechanism is based on the direct piezoelectric effect, where applied mechanical deformation induces an electric charge (Cook-Chennault et al., 2008). PEGs have numerous benefits. First, they offer high energy conversion efficiency, making them suitable for applications requiring low-power, continuous energy harvesting (Anton et al., 2007). Second, they can be miniaturized and integrated into microelectromechanical systems (MEMS), enabling compact energy harvesting solutions (Challa et al., 2008). Third, piezoelectric materials such as lead zirconate titanate (PZT) and polyvinylidene fluoride (PVDF) are widely available and can be engineered for specific applications (Chen et al., 2022). Despite these benefits, PEGs also pose several limitations. They face several limitations, notably the brittleness of ceramic-based materials like PZT, which affects their durability in dynamic environments. For instance, Ogbonna et al. (2022) noted that while PZT materials exhibit high energy-generating performance, they are prone to mechanical failure and compromised performance in harsh environments.

In addition, Wang et al. (2021b) observed that under impact loads, PZT-5H ceramics developed micro-cracks, leading to increased dielectric loss and potential material degradation. Also, their performance is highly dependent on excitation frequency, requiring resonance tuning to achieve optimal efficiency (Tang et al., 2010). Furthermore, PEGs often produce a high output voltage, although the current levels are low. This trait stems from the fundamental

characteristics of piezoelectric materials, which effectively transform mechanical strain into electrical charge, leading to high voltage outputs but restricted current capacity. To utilize this energy efficiently, specialized power conditioning circuits are necessary. These circuits tackle issues like impedance matching, voltage regulation, and energy storage, ensuring that the harvested energy is converted into a usable form for powering electronic devices. The design and optimization of such circuits are critical to maximize the efficiency and reliability of piezoelectric energy harvesting systems (Skow et al., 2016). Figure 2.5 presents various types of PEGs.

Electromagnetic energy generators (EMGs) function according to the widely recognized principle of Faraday's law of electromagnetic induction. This law states that when there is relative movement between a conductor and a magnetic field, an electric current is generated as a result (Duffy et al., 2004). This method is commonly used in vibration energy harvesting and rotational energy conversion systems. One prime advantage of EMGs is their ability to generate high current outputs, making them ideal for powering electronic devices with moderate power requirements (Roundy et al., 2003). In addition, they exhibit strong mechanical robustness due to the use of durable materials such as copper coils and permanent magnets (Phan et al., 2020). Moreover, EMGs can function effectively over a broad frequency range, allowing them to be employed in variable environmental conditions (Feng et al., 2021). However, EMGs also have notable disadvantages. However, the major limitation is their relatively large size due to the requirement of a magnet-coil assembly, which hinders their integration into small-scale or wearable devices ((Digregorio et al., 2024). Another technical challenge is electromagnetic interference, which can affect nearby sensitive electronic systems (Ma et al., 2024). Hence, the fabrication of high-efficiency EMGs can be complex and expensive due to precision requirements in coil winding and magnetic component alignment (Muscat et al., 2022). Typical structural configurations for EMGs are shown in Fig. 2.6.

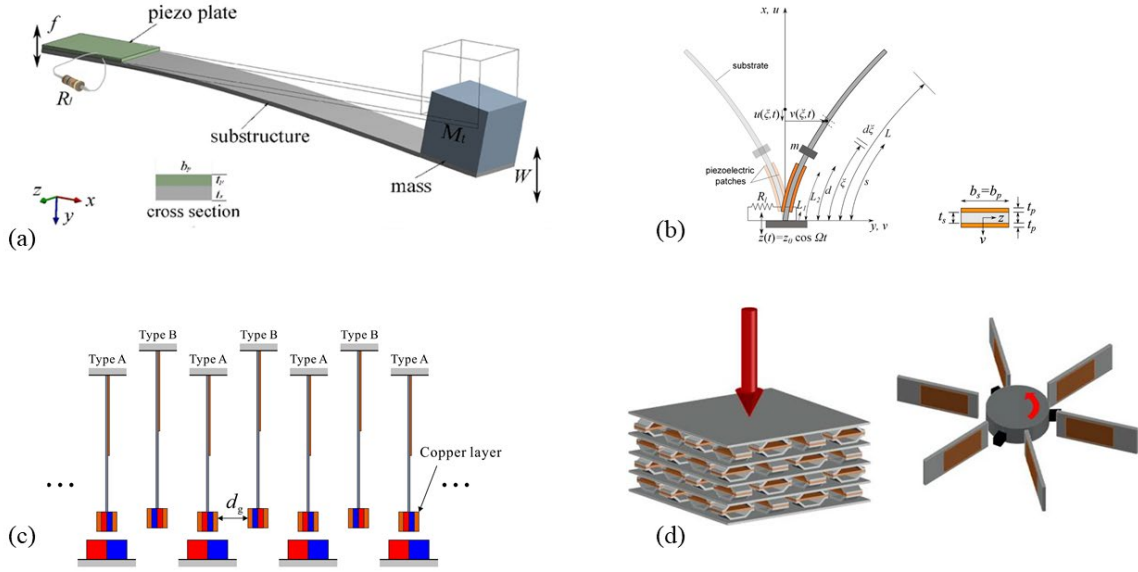


Fig. 2.5: Schematic diagram of PEGs: (a) single beam PEG by Yang (2016); (b) combination parametric and internal resonance PEG by Garg et al. (2020); (c) multi-stable beam array PEGs by Lai et al. (2019); (d) press and rotary PEGs by Bernat-Maso et al. (2023).

TENGs utilize the triboelectric effect and electrostatic induction for transforming mechanical energy into electrical energy. This process involves charge transfer between materials of differing electron affinities during contact and separation. TENGs offer several advantages. First, they allow for material flexibility, enabling their integration into stretchable, foldable, and wearable devices (Fan et al., 2012). Second, TENGs can be fabricated using low-cost polymers and natural materials, making them economically feasible for large-scale production (Shi et al., 2024). Third, they generate high open-circuit voltages, which are useful for sensing applications and energy storage solutions (Chen et al., 2023). Despite these advantages, TENGs have limitations that affect their practicality. One drawback is their low current output, which restricts their ability to power devices requiring sustained high energy consumption. In addition, the durability of TENGs is a concern, as prolonged contact and friction can lead to material degradation over time (Uddin et al., 2025).

Environmental factors such as humidity and temperature variations can also influence their charge transfer efficiency, making them less reliable in certain operating conditions (Zhao et al., 2023). TENGs work via four primary working modes: contact-separation, sliding, single-

electrode, and freestanding. In contact-separation mode, two materials come into contact and then separate, generating charge via the triboelectric effect. Sliding mode involves continuous relative motion between the materials, generating charge through friction. Single-electrode mode uses one electrode to harvest charge generated when the material interacts with an external electrode. In freestanding mode, one part of the system is suspended, and relative motion with the triboelectric material generates electricity. These modes offer versatile energy harvesting solutions for various applications, such as wearable electronics and self-powered sensors (Zhou et al., 2020). The performance comparison of various working mechanisms is presented in Table 2.1. Figure 2.7 shows the typical structural configurations for TENGs, and the four operating modes of TENGs are presented in Fig. 2.8.

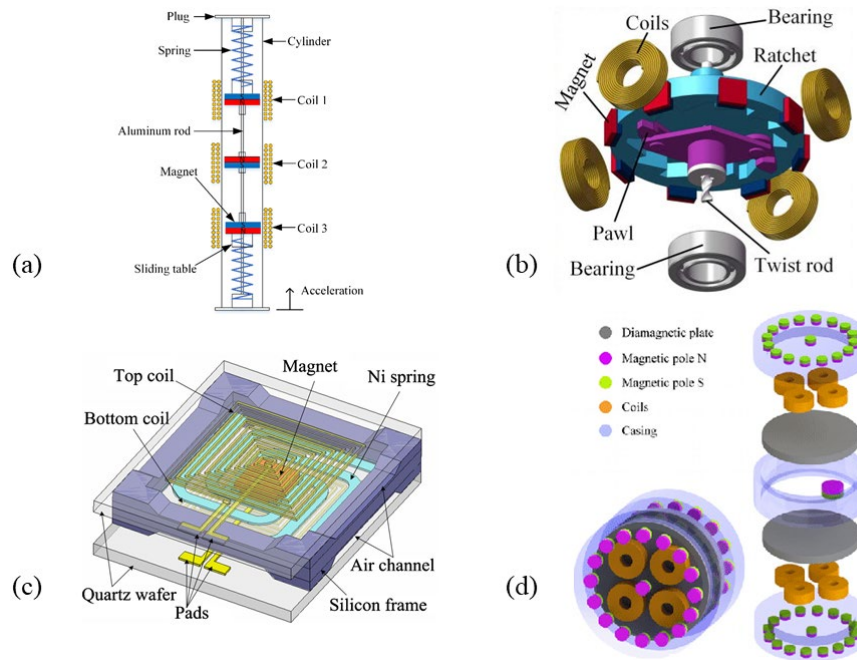


Fig. 2.6: Typical structural configurations for EMGs: (a) Magnetically Coupled EMGs (Li et al., 2021b); (b) rotational EMG (Zhang et al., 2020a); (c) sandwiched structure and air channel EMG (Wang et al., 2012b); (d) magnetically modulated orbit EMG (Zhao et al., 2019).

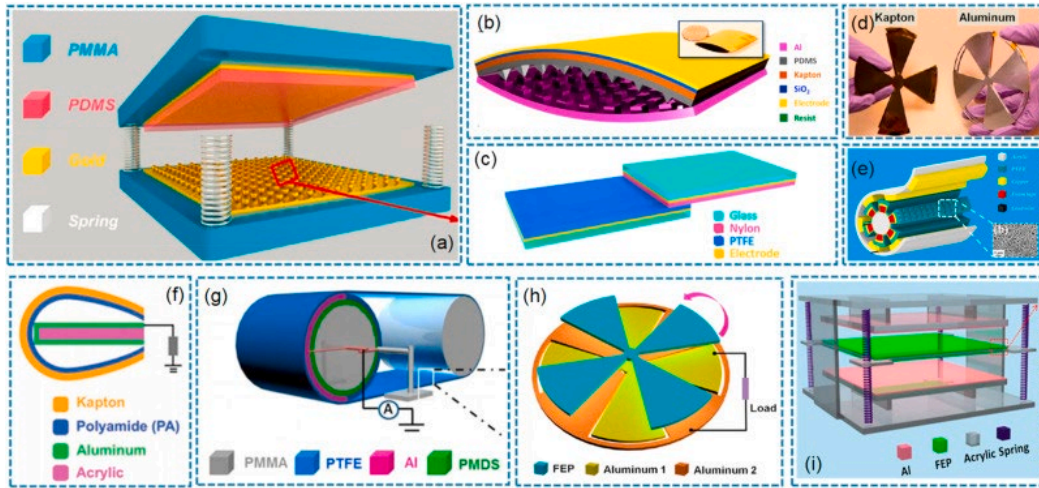


Fig. 2.7: Typical structural configurations for TENGs including (a)–(b) contact-separation (Wang et al., 2012a; Zhu et al., 2013), (c)–(e) lateral-sliding (Bai et al., 2013; Lin et al., 2013; Wang et al., 2013), (f)–(g) single-electrode (Li et al., 2015; Yang et al., 2013b), and (h)–(i) freestanding triboelectric-layer modes (Lin et al., 2014; Wang et al., 2014a).

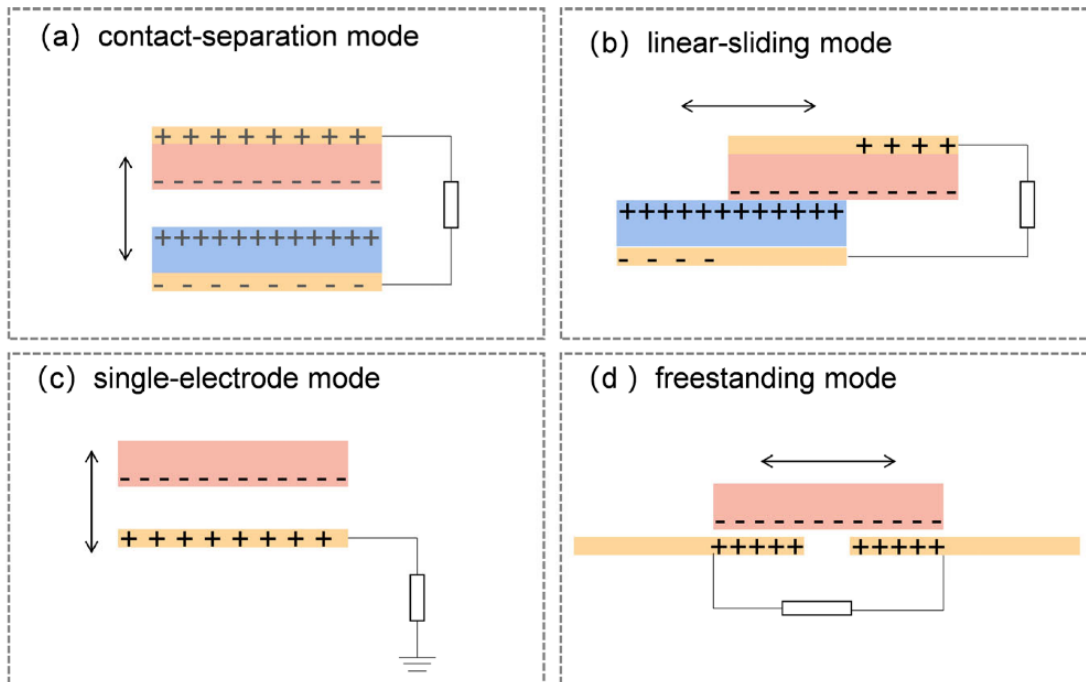


Fig. 2.8: Schematic illustrations of four basic TENG modes.

Table 2.1: Comparison of various energy harvesting mechanisms

Technique	Electromagnetic Generator (EMG)	Triboelectric Nanogenerator (TENG)	Piezoelectric Generator (PEG)
Working Principle	Electromagnetic induction	Contact electrification and electrostatic induction	Piezoelectric effect
Advantages	• High energy conversion efficiency • Low output impedance • Easy to scale up, works with common metals and magnets • High-power and current output	• High output voltage. • Simple fabrication process. • Low weight	• High energy conversion efficiency • Consistent output, making it easy for application • High-frequency response
Disadvantages	• Generally large and bulky in size • Efficiency decreases at lower frequencies	• Limited current output • Low durability • High device impedance	• Narrow operational frequency range, only works at specific resonant frequencies • Requires specific materials for optimal performance • High device impedance
Applications	• Rotational system • Windmill	• Wearable technology • Biomedical implants	• Flexible applications • Suitable for implantable

2.3 Double Hybrid Energy Harvesting Systems

Conventional single-mode energy harvesters, whether piezoelectric, electromagnetic, or triboelectric, frequently encounter limitations imposed by environmental conditions and the specific characteristics of energy sources. In response to these challenges, dual hybrid energy harvesting systems have arisen as a potential solution, combining two distinct energy harvesting methods into one device. This section investigates two significant dual hybrid energy harvesting systems: the piezoelectric-electromagnetic hybrid system and the

piezoelectric-triboelectric hybrid system. It analyzes their underlying principles, recent developments, and inherent constraints.

Piezoelectric materials transfer mechanical stress into electrical energy, while electromagnetic harvesters utilize Faraday's law of electromagnetic induction to transform magnetic flux changes into electrical power. Combining these two mechanisms allows for effective harvesting of ambient vibrational energy across a broader frequency spectrum. Recent studies have focused on developing efficient piezoelectric-electromagnetic hybrid energy harvesters (P-EHEHs). For example, Han et al. (2023) highlighted the high output power, simple structure, and ease of miniaturization associated with P-EHEHs, making them suitable for different applications, covering devices on human body and monitoring sensors. Shaukat et al. (2023) explored the possibilities of hybrid energy harvesting systems for sustainable uses, highlighting their significance in self-sustaining wireless sensor networks. In practical implementations, a type of magnetically coupled piezoelectric-electromagnetic energy harvester designed for low-frequency applications has been created. These devices exhibit the ability to collect energy from various directions and produce greater power output when subjected to low-frequency environmental vibrations. For example, a study showcased a ring-shaped structure with cantilever beams equipped with permanent magnets, enabling simultaneous energy harvesting through both piezoelectric and electromagnetic mechanisms. Further advancements include the integration of nonlinear magnetic forces to enhance energy harvesting efficiency (Xie et al., 2014). A review by Zhu et al. (2022) discussed the role of magnetic coupling in broadening the operational bandwidth and improving the energy conversion efficiency of piezoelectric harvesters. In addition, hybrid energy harvesters combining piezoelectric and electromagnetic mechanisms have been explored for their potential to harness low-frequency vibrations effectively.

Despite their advantages, piezoelectric-electromagnetic hybrid systems face several challenges:

- integrating both piezoelectric and electromagnetic components can lead to increased device complexity and size, this potentially limits their applications in compact or flexible electronic devices;

- the hybrid approach aims to broaden the operational bandwidth, both piezoelectric and electromagnetic harvesters still exhibit peak efficiencies near their respective resonant frequencies, and deviations from these frequencies can result in significant performance drops; and
- the varying material properties and structural requirements of piezoelectric and electromagnetic components demand careful design to achieve optimal performance and mechanical stability.

The combination of piezoelectric-electromagnetic hybrid energy harvester enhances power generation efficiency and allows energy capture from a wider range of mechanical excitations (Zhu et al., 2022). The synergy between different effects has been extensively studied, with advancements in microstructure design, material modifications, and integration strategies aimed at improving electrical output (Ali et al., 2024a). However, the combination of two types harvesting mechanisms also face several limitations. One primary issue is environmental sensitivity; triboelectric charge accumulation is highly dependent on factors such as humidity and temperature, which can cause fluctuations in performance. Moreover, triboelectric components tend to degrade over time due to repeated contact and separation cycles, reducing long-term durability (Niu et al., 2013). Fabrication complexity also remains a concern, as optimizing both piezoelectric and triboelectric layers requires precise material engineering and surface modifications, which can increase manufacturing (Sriphan et al., 2022). Despite these challenges, research into hybrid energy harvesting systems continues to advance, with ongoing efforts to address these limitations through material innovation, device miniaturization, and enhanced energy management strategies (Kang et al., 2024). Figure 2.9 shows the schematic of two types hybrid energy harvesting devices. Figure 2.10 illustrates a spring-augmented multi-stable hybrid-integrated vibration energy harvester designed for ultra-low-frequency excitations.

The incorporation of various energy harvesting mechanisms into a single device offers several advantages, including increased power output, extended operational bandwidth, and improved adaptability to varying environmental conditions (Shaukat et al., 2023). These hybrid systems are particularly beneficial in applications where energy sources are unpredictable or variable, such as in wearable electronics, implantable medical devices, and remote sensor

networks. Within the domain of wearable electronics, hybrid energy harvesters can harness biomechanical energy from human motion, providing a continuous power source for devices like health monitors and fitness trackers (Xu et al., 2024). The study demonstrated the use of a piezoelectric-electromagnetic hybrid energy harvester to power wireless sensors for monitoring structural vibrations, highlighting its effectiveness in real-world applications.

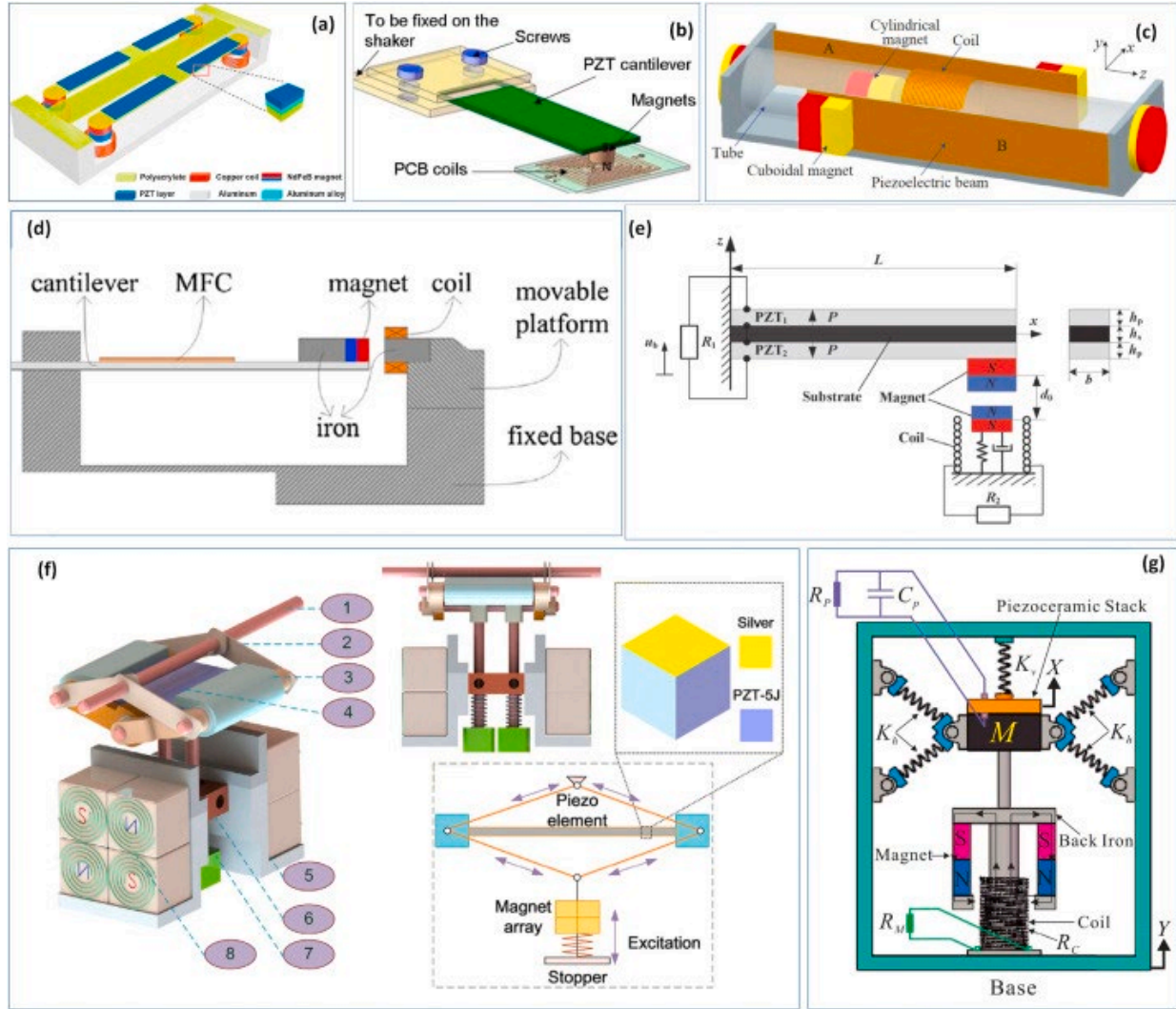


Fig. 2.9: Various types of hybrid energy harvesting devices (Liu et al., 2021).

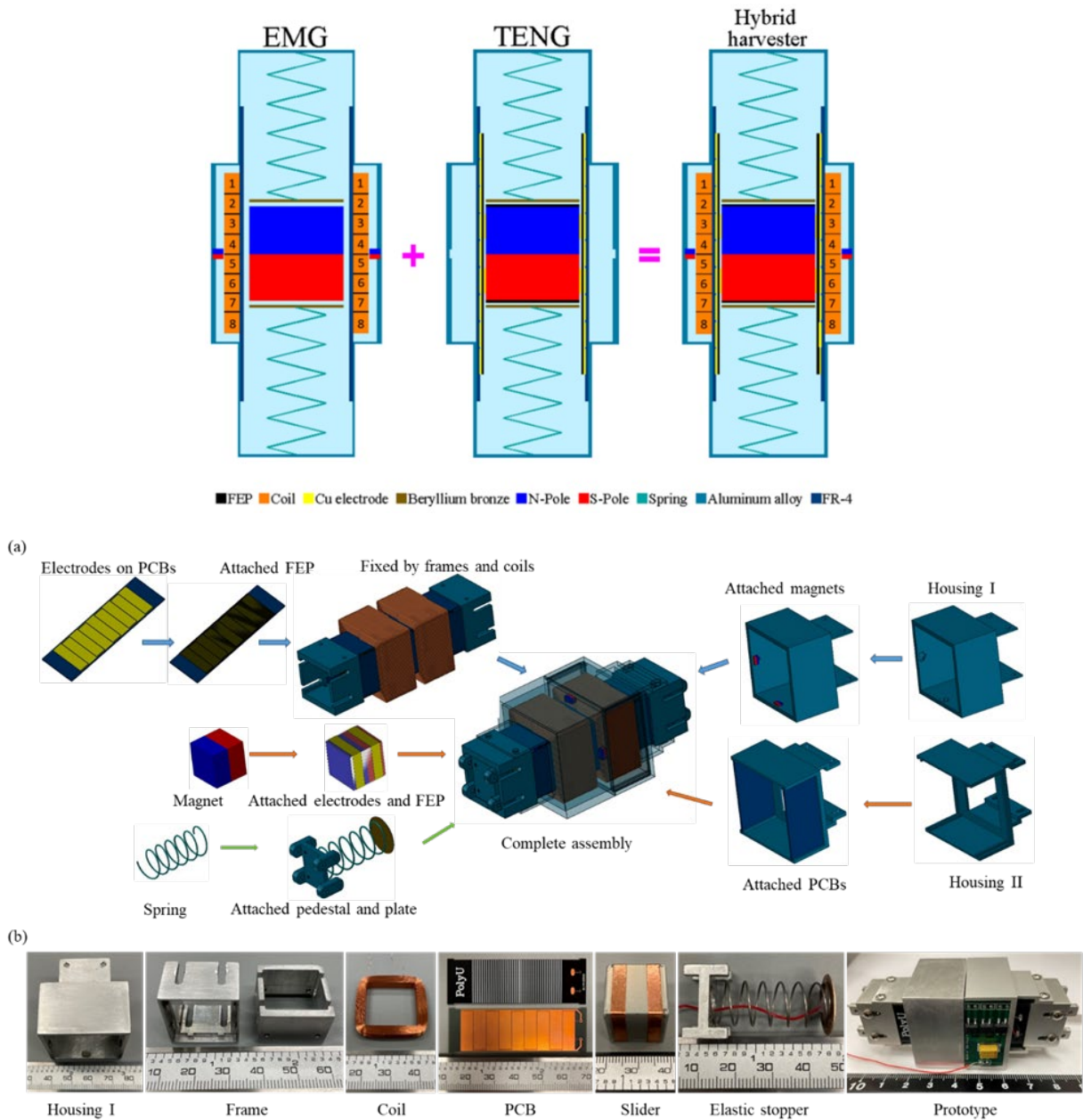


Fig. 2.10: Integrated design of a hybrid energy harvester (Yang et al., 2022).

2.4 Triple Hybrid Energy Harvesting Systems

Triple hybrid (piezoelectric–electromagnetic–triboelectric) energy harvesting systems represent an innovative approach to enhancing power generation by integrating multiple energy conversion mechanisms. These systems combine piezoelectric harvesting, which generates electricity from mechanical strain through materials like lead zirconate titanate (PZT) and polyvinylidene fluoride (PVDF), electromagnetic harvesting, which exploits Faraday’s law of

induction using magnets and coils, and triboelectric harvesting, which transfers mechanical motion into electrical energy by contact electrification and electrostatic induction. This combination enhances power generation efficiency and allows energy capture from a wider range of mechanical excitations (Zhu et al., 2014). Recent studies have demonstrated that hybridizing these three mechanisms can achieve significant improvements in energy harvesting efficiency compared to standalone systems. Yang et al. (2022) discussed a tri-hybrid energy harvester that combines piezoelectric, electromagnetic, and triboelectric mechanisms. The results indicated a significant broadband response, with the maximum RMS power output recorded at 40.2 mW under specific conditions. While Yan et al. (2023b) introduced a rocking boat-style triboelectric nanogenerator design for harvesting low-frequency wave energy. This design effectively captures low-frequency mechanical energy, demonstrating the potential of triboelectric nanogenerators in harnessing low-frequency environmental energy. One of the primary advantages of triple hybrid systems is their broadband frequency response, a crucial factor in environments with unpredictable or variable excitation sources.

Conventional vibration-driven energy harvesters, particularly piezoelectric one, suffer from narrow operational bandwidths and limited energy capture outside their resonance frequencies. However, the integration of electromagnetic induction, which excels in low-frequency operation, and triboelectric effects, which generate high voltages at mid-frequencies, enables these systems to function efficiently over a broad range of mechanical stimulation (Liu et al., 2021). Zhou et al. (2025) investigated the stochastic response of an impact-based hybrid energy harvesting system, focusing on its ability to widen the operating frequency bandwidth and reduce the impact-induced damage under random excitations. Basaran (2022) presented a hybrid energy harvesting system utilizing electromagnetic actuators to induce vibrations, aiming to enhance energy conversion efficiency under non-rigid ground connections. In addition to broadband adaptability, another significant advantage of triple hybrid systems is their improved structural durability and reliability. A hybrid energy harvester combining piezoelectric, electromagnetic, and triboelectric generators (P-WISE system) significantly enhances power output through tri-stable nonlinear frequency up-conversion (Wang et al., 2024a). Another study (Liu et al., 2025) introduces a hybrid low-frequency wave energy converter (WEC) that integrates a tri-stable electromagnetic generator with a triboelectric

generator. This setup yield a maximum power output of 311 mW at 2 Hz and effectively powering small electronic devices. These schematic figures are presented in Fig. 2.11.

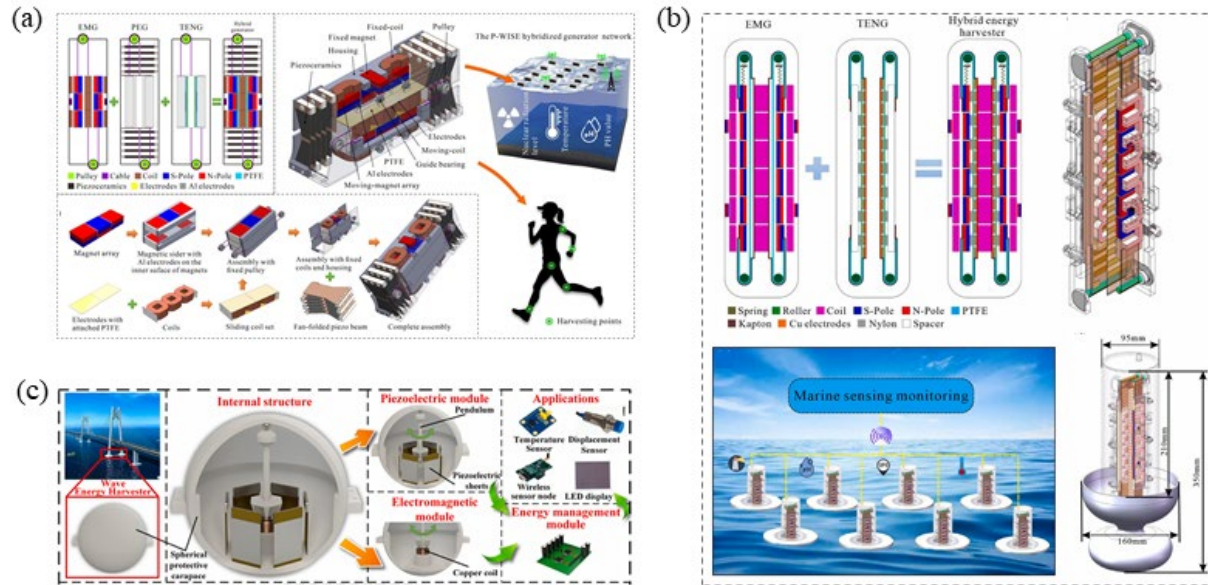


Fig. 2.11: Hybrid wave energy harvesting systems: (a) P-WISE system (Wang et al., 2024a); (b) hybrid low-frequency WEC system (Liu et al., 2025); (c) omnidirectional WEC system (Lu et al., 2023b).

Piezoelectric materials, while efficient, degrade over time due to fatigue and depolarization, and triboelectric harvesters experience charge dissipation and wear from repeated contact-separation cycles. However, electromagnetic mechanisms are highly stable and do not suffer from these degradation effects, thus enhancing the overall lifespan of the system. explores the potential of rotational electromagnetic harvesters as a complement to triboelectric and piezoelectric systems, aiming to enhance energy harvesting efficiency and stability. These studies (Ali et al., 2024b; Rolo et al., 2024) highlight the benefits of integrating multiple energy harvesting mechanisms to achieve more stable and efficient energy conversion, which is crucial for long-term applications. Despite these advantages, triple hybrid energy harvesting systems face several challenges, primarily related to structural complexity and fabrication difficulties. Designing an efficient harvester requires precise alignment of the three transduction mechanisms to optimize energy transfer, which increases manufacturing costs and scalability concerns (Shaukat et al., 2023). Moreover, power management and circuit integration pose additional challenges, as each transduction mechanism generates electricity with different

voltage, current, and impedance characteristics, requiring sophisticated rectifiers, impedance matching circuits, and DC-DC converters to effectively combine the outputs into a stable power supply (Huang et al. 2024).

Nonlinear behavior and energy losses further complicate the performance of these systems. Hysteresis effects, charge leakage, and parasitic capacitance can reduce efficiency, particularly in triboelectric and piezoelectric components. Zhang et al. (2021a) highlighted that addressing these issues requires advanced materials and optimization strategies, such as machine learning-based tuning of system parameters, to dynamically adapt to environmental variations and maximize energy harvesting efficiency. Future research in this field should focus on several key areas, including the development of advanced materials and structural configurations to enhance durability and efficiency. Song et al. (2024) proposed using high-durability piezoelectric films, nanostructured triboelectric layers, and high-efficiency electromagnetic cores to improve overall performance. In addition, optimizing power management circuits through intelligent energy storage and conversion strategies is essential. Zou et al. (2021) presented a low-power, high-efficiency adaptive energy harvesting circuit specifically designed for broadband piezoelectric vibration energy harvesters. The proposed circuit employs an enhanced double synchronized switch harvesting (EDSSH) technique, which adaptively adjusts to varying vibration frequencies, resulting in improved energy harvesting performance. Another emerging research direction is the application of triple hybrid energy harvesters in wearable and biomedical devices, where flexible, biocompatible materials are required. A study Turar et al. (2025) conducted by discusses the integration of piezoelectric and triboelectric nanogenerators, highlighting the use of advanced materials and structural designs to improve performance. Figure 2.12 shows the hybridized energy harvesting structures and basic working modes. Figure 2.13 presents a low-frequency, broadband, and tri-hybrid energy harvester that incorporates a septuple-stable nonlinearity-enhanced mechanical frequency up-conversion mechanism.

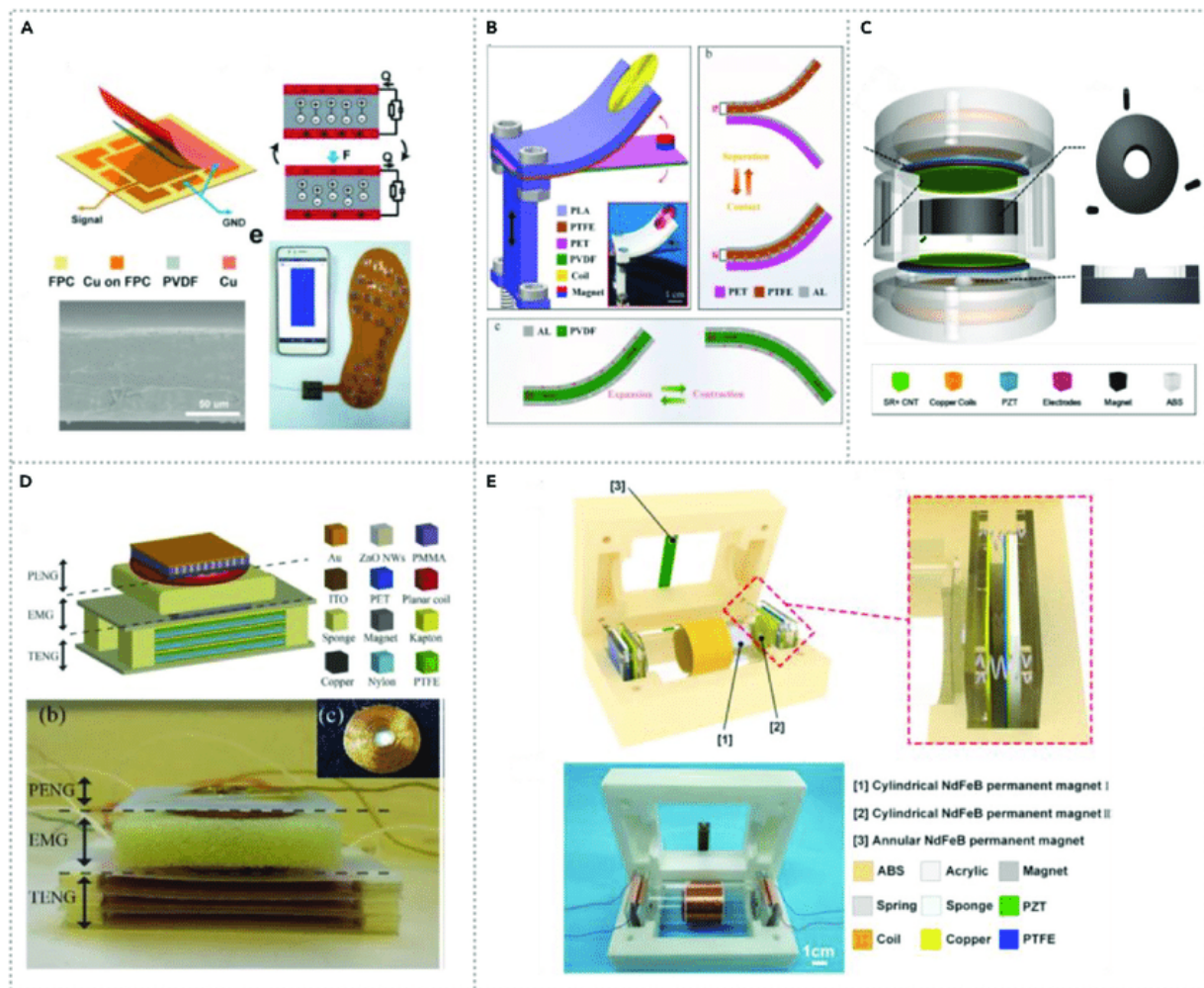


Fig. 2.12: Hybridized energy harvesting structures and basic working modes (Zhang et al., 2020b).

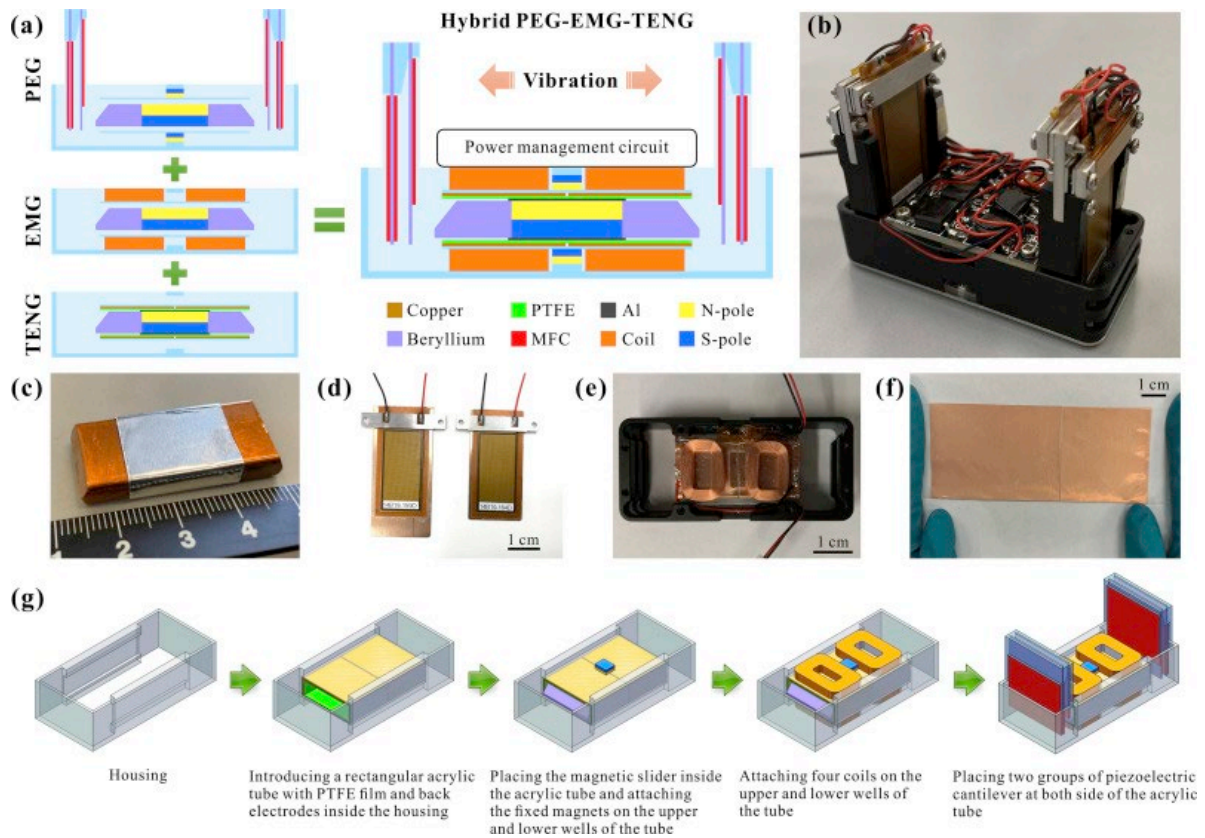


Fig. 2.13: Schematic of the tri-hybrid energy harvester (Wang et al., 2019).

In short, the triple hybridization design energy harvesting system (i.e., piezoelectric–electromagnetic–triboelectric) represents a significant advancement in energy harvesting technology, leveraging the strengths of multiple transduction mechanisms to achieve higher power output, broader frequency response, and greater durability compared to single-mode or dual-mode harvesters. However, challenges related to fabrication complexity, power management, and nonlinear losses must be addressed through innovative materials, intelligent optimization techniques, and advanced circuit design. Continued research in this field will drive the development of highly efficient, self-powered electronic devices for IoT and structural monitoring applications.

2.5 Physics-Informed Neural Networks (PINNs) for Parametric Optimization

2.5.1 Fundamental Principle and Variants

As computational resources become more accessible and artificial intelligence continues to evolve, deep learning has become increasingly prominent in numerous fields, including natural language processing, autonomous driving, and computer vision. In recent years, a growing body of research has focused on applying deep learning techniques to solve partial differential equations (PDEs) (Cuomo et al., 2022). Among the many deep learning approaches, physics-informed neural networks (PINNs) have emerged as a promising and widely recognized method for solving these equations. Figure 2.14 shows the published years (from 2019 to 2025) of PINN papers and the top 16 fields related to these papers according to the Web of Science. PINNs were initially introduced by Raissi et al. (2019) to address various classical problems, such as those in quantum mechanics and water wave propagation. The fundamental concept of PINNs is that they integrate physical information, such as governing equations, initial conditions, and boundary conditions, directly into the neural network's loss function. By this, the model is capable of learning how to map the spatiotemporal domain to the solutions of partial differential equations. The framework of a standard PINN (STD-PINN) approach is presented in Fig. 2.15.

In contrast to traditional deep learning methods that rely heavily on large datasets, PINNs are particularly beneficial as they do not require a vast amount of labeled data. They offer distinct advantages, such as physical interpretability and enhanced generalization, making them highly effective in solving complex problems where understanding the physical context is crucial (Moseley et al., 2020b; Raissi et al., 2019). Figure 2.16 presents a comparative depiction of the connection between the data-driven approaches and directly physics-based solutions for the solid mechanical PDEs. A variety of methods rooted PINN have been effectively applied to numerous types of PDEs, encompassing both steady-state and transient equations, as well as more specialized forms such as fractional PDEs (Pang et al., 2019), stochastic PDEs (Yang et al., 2020), and even integro-differential equations (Yuan et al., 2022). These approaches have demonstrated their utility in diverse areas including fluid mechanics, structural dynamics, biomedical sciences, and wave mechanics (Hu et al., 2024).

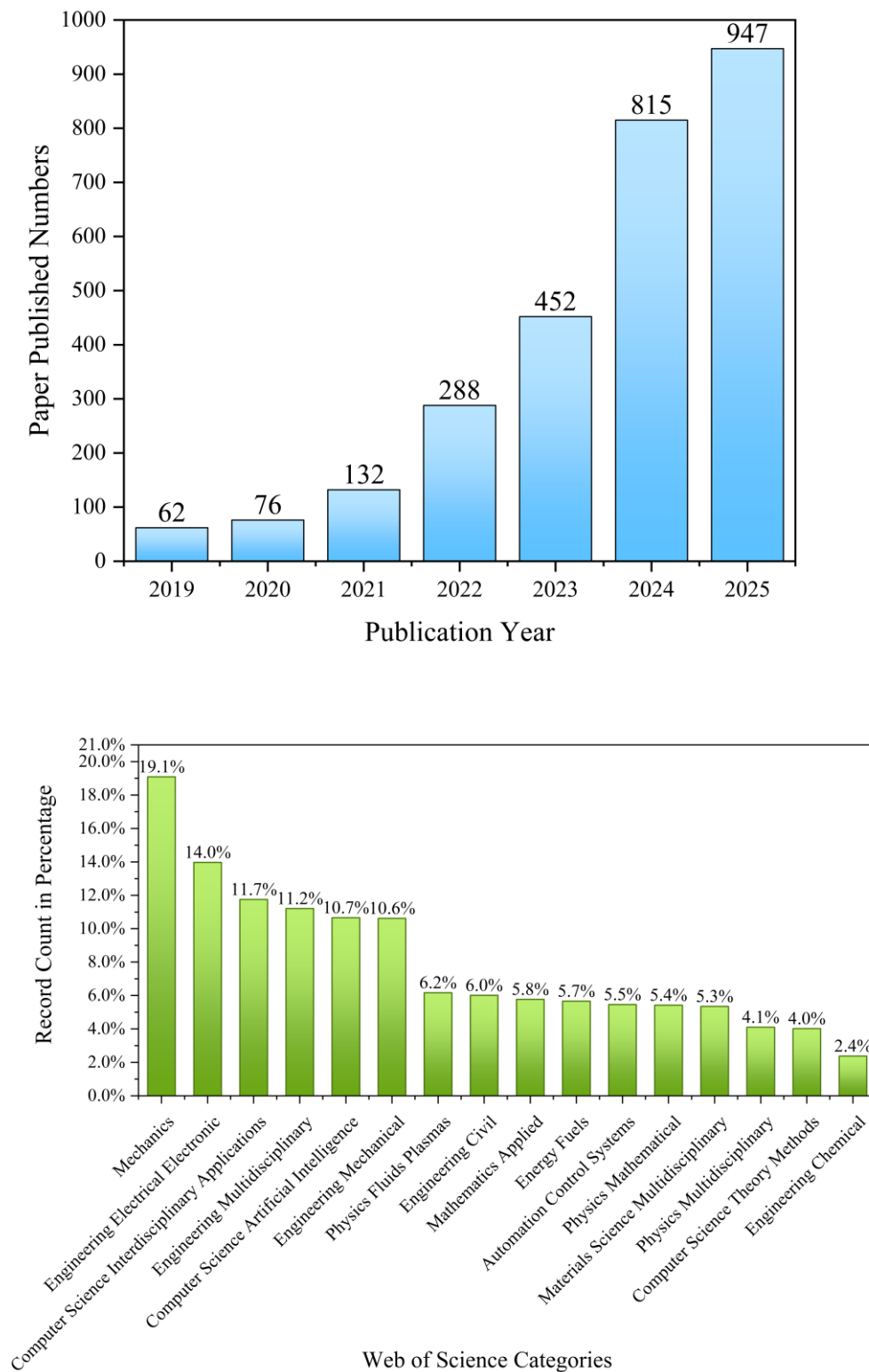


Fig. 2.14: Overview and categorization of PINNs research articles from 2019 to 2025 (data from Web of Science as of 18 Nov 2025).

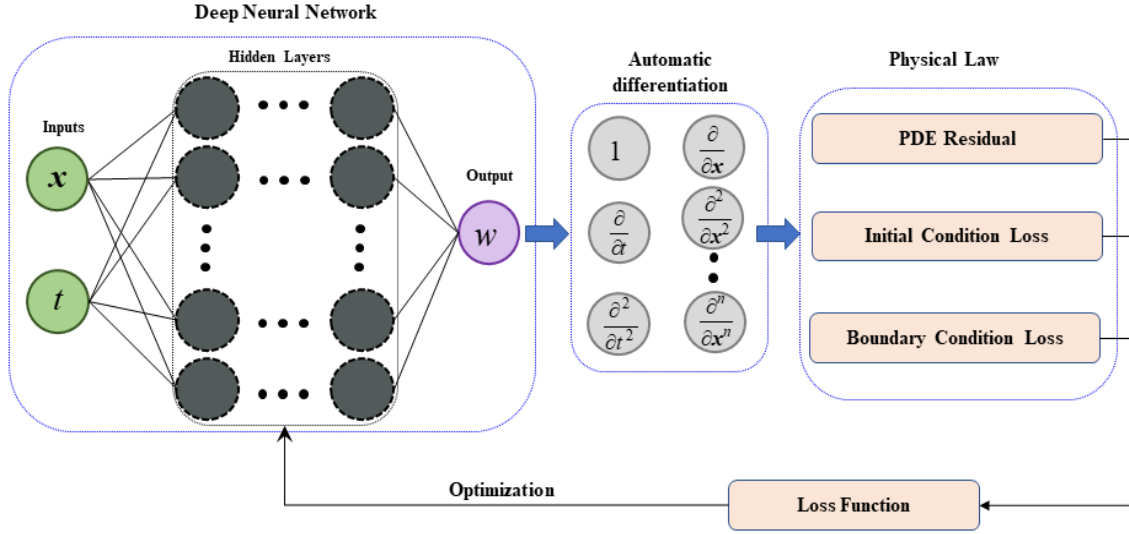


Fig. 2.15: Framework of a standard PINN method.

To enhance the performance of PINNs, various extensions and refinements have been proposed in recent years. These advancements primarily focus on improving accuracy, efficiency, and robustness through innovative techniques. **Modified Network Architectures:** researchers have explored alternative neural architectures such as extended Physics-Informed Neural Networks (XPINNs) (Ameya et al., 2020) and Physics-Informed Point Net (PINN) (Kashefi et al., 2022) to enhance PINN accuracy in handling complex physics problems. To reduce numerical errors in large-scale simulations, multi-domain PINNs partition the problem domain into smaller sub-domains that are trained independently before being integrated (Dolean et al., 2024). **Adaptive Sampling Techniques:** efficient sampling strategies such as residual-based adaptive refinement (RAAR) (Lu et al., 2021) and importance sampling (Li et al., 2025) have been developed to strategically place training points in regions with high error gradients, improving convergence. **Enhanced Loss Functions:** alternative loss functions, including weighted residual loss (Wang et al., 2022b) and physics-constrained Sobolev norms (Difonzo et al., 2024), have been designed to ensure better numerical stability and convergence in PINN training. **Hybrid Physics-Data Approaches:** combining PINNs with experimental or synthetic data improves accuracy, particularly in complex multi-physics problems (Karniadakis et al., 2021).

The STD-PINN approach proposed by Raissi et al. (2019) is limited in solution accuracy, training efficiency and application range. To overcome the limitations of STD-PINN, numerous enhancements and various PINN methods have been proposed. These modifications include the adoption of novel network architectures, the segmentation of the spatiotemporal domain into sub-domains, along with sophisticated sampling approaches for input data, precise enforcement of boundary and initial conditions (Wang et al., 2021c), adapting the loss function weights dynamically and introducing new activation functions. A comparison of various PINN methods is shown in Table 2.2, illustrating that each variant method corresponds to distinct problems. With the continuous contributions to the development of PINN, numerous open-source libraries have been created to implement customized PINN frameworks, as summarized in a comprehensive review paper (Hu et al., 2024), see Table 2.3.

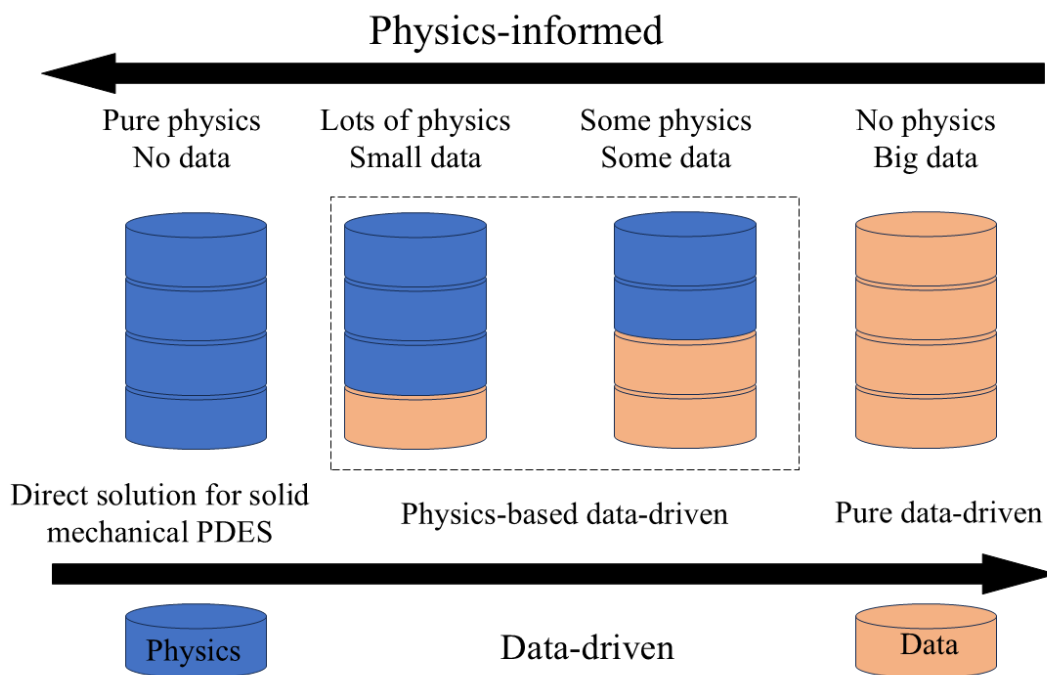


Fig. 2.16: Illustration of the connection between data-driven approach and direct physical solutions for solid mechanics problems (Hu et al., 2024).

Table 2.2: A list of various PINN methods

Method	Designated Problems	Reference
STD PINN	Solved PDE equations in general forms (e.g., Burgers equation).	(Raissi et al., 2019)
FBPINN	Solved both small and larger, multi-scale problems.	(Moseley et al., 2023)
EPINN	Reduced training time for solid mechanics problems.	(Wang et al., 2023c)
PINN (integrated framework)	Solved finite-strain elasto-plasticity problems.	(Niu et al., 2023)
PINN (quantum control)	Solved state-to-state transfer problems with high probabilities, short-time evolution.	(Norambuena et al., 2024)
BC-cPINN	Solved neutron diffusion problems with unsmooth solutions.	(Wang et al., 2022a)
sPINN	Solved large deformation problems of hypoelastic materials.	(Luo et al., 2023)
AT-PINN	Solved structural vibration problems in long-duration simulation.	(Chen et al., 2024a)
AT-PINN-HC	Incorporated hard-constraint strategies to AT-PINN for predicting structural behavior under dynamic loads	(Chen et al., 2025)
SA-PINNs	Provided more accurate solutions for wave PDE.	(Mcclenny et al., 2023)
XPINNs	Solved inverse problems in supersonic flows.	(Jagtap et al., 2022)
PINN (ensemble)	Solved two-dimensional elasticity equations.	(Mouratidou et al., 2024)
PINN (mitigating gradient)	Solved dynamic weights of the loss function problems.	(Wang et al., 2021c)
CAN-PINN	Solved fluid dynamics problems.	(Chiu et al., 2022)
cPINN	Applied to forward and inverse problems.	(Jagtap et al., 2020c)
B-PINN	Solved forward and inverse PDE problems with noisy data.	(Yang et al., 2021b)
PINN (optimal control)	Solved optimal control problems with PDE constraints.	(Mowlavi et al., 2023)
TL-PINN	Predicted nuclear reactor transient results.	(Prantikos et al., 2023)
SK-PINN	Solved problems in the computational mechanics of complex domains.	(Pan et al., 2025)
Δ -PINN	Solved complex geometries problems.	(Sahli Costabal et al., 2024)
Bayesian PINN	Solved seasonal endemic infectious disease problems.	(Linka et al., 2022)
ST-PINN	Addressed low accuracy and convergence problems.	(Yan et al., 2023a)

gPINNs	Achieved higher accuracy and fewer iterations.	(Yu et al., 2022)
PSO-PINN	Afforded the possibility of robust predictions with quantified uncertainty.	(Davi et al., 2022)
NSGA-PINN	Solved ordinary and partial differential equation problems.	(Lu et al., 2023a)
PINNs-MPF	Handled multi-phase-field simulations of microstructure evolution.	(Elfetni et al., 2025)

Table 2.3: Open-source libraries of various PINNs and solvers (Hu et al., 2024)

Name	Open-source Code	Program	Reference
NeuralPDE	https://github.com/SciML/NeuralPDE.jl.git	Julia	(Zubov et al., 2021)
Neural Solvers	https://github.com/Photon-AI-Research/NeuralSolvers.git	Python	(Stiller et al., 2020)
IDRLnet	https://github.com/idrl-lab/idrlnet.git	Python	(Peng et al., 2021)
PINA	https://github.com/mathLab/PINA.git	Python	(Coscia et al., 2024)
TensorDiffEq	https://github.com/SciML/TensorFlowDiffEq.jl.git	Julia	(Mcclenny et al., 2021)
DeepINN	https://github.com/praksharma/DeepINN.git	Jupyter Notebook	(Zhang et al., 2021c)
NeuralUQ	https://github.com/Crunch-UQ4ML/neuraluq.git	Python	(Zou et al., 2024)
NeurodiffEq	https://github.com/NeuroDiffGym/neurodiffEq.git	Python	(Chen et al., 2020a)
DeepXDE	https://github.com/lululxvi/deepxde	Python	(Lu et al., 2021)
SciANN	https://github.com/sciann/sciann.git	Python	(Haghighat et al., 2021)
SimNet	https://developer.nvidia.com/simnet	Python	(Hennigh et al., 2021)
ADCME	https://github.com/kailaix/ADCME.jl.git	Julia	(Xu et al., 2020)
PDEBench	https://github.com/pdebench/PDEBench.git	Jupyter Notebook	(Takamoto et al., 2022)
PINNs-Torch	https://github.com/rezaakb/pinns-torch.git	Python	(Bafghi et al., 2023)
Neural Eikonal solver	https://github.com/sgrubas/NES.git	Jupyter Notebook	(Grubas et al., 2023)
PinnDE	https://github.com/JB55Matthews/PinnDE	Python	(Matthews et al., 2024)
MLniCS	https://github.com/MLniCS/MLniCS	Jupyter Notebook	(Chen et al., 2021b)
PINNLib	https://github.com/m-raj/PINNLib	Python	(Rajaei Harandi et al., 2023)
Δ -PINN	https://github.com/fsahli/Delta-PINNs	Python	(Sahli Costabal et al., 2024)

2.5.2 AT-PINN (Advanced Time-marching Physics-informed Neural Network)

PINNs have achieved successfully in a wide range of fields, such as fluid mechanics (Cai et al., 2022), structural mechanics (Ghalambaz et al., 2024), biomedical modeling (Katsikis et al., 2022), and wave propagation problems (Rasht-Behesht et al., 2022). However, despite these advancements, a key challenge remains in dealing with time-dependent PDEs, where the error tends to accumulate as the simulation progresses (Ameya et al., 2020). This issue poses a significant challenge when applying PINNs to long-duration simulations, making it a critical area for further research and development.

The conventional PINN approach solves PDEs over the entire spatiotemporal domain, which makes it inefficient for handling evolving physical systems such as structural vibrations and transient wave equations (Ameya et al., 2020). To overcome this, researchers have introduced sequential learning strategies, such as Time-Marching PINNs (TM-PINNs), which train neural networks sequentially over smaller time segments. Lawal et al. (2022) pioneered this approach by dividing the entire time domain into successive intervals, training the PINN separately on each segment, and using the final state from one segment as the initial condition for the next. This approach significantly improved the accuracy and stability of PINNs for solving time-dependent PDEs compared to standard PINNs. While TM-PINNs have demonstrated better accuracy and efficiency, they still suffer from accumulated numerical errors, particularly when solving hyperbolic PDEs in structural vibration problems. Errors in the approximate initial conditions at each time segment progressively increase, leading to divergence over extended time horizons.

Considering the constraints of current PINNs in solving vibration problems over long durations, an improved PINN framework is required. To tackle the limitations of existing PINNs in long-duration vibration simulations, Chen et al., 2024a proposed the advanced time-marching PINN (AT-PINN), which improves accuracy and efficiency in both forced and free vibration analyses. As shown in Fig. 2.17, AT-PINN incorporates four key techniques:

- (1) Spatiotemporal normalization: It avoids domain offsets in each time segment and can enhance solution accuracy and training efficiency;
- (2) Reactivating optimization: It reactivates the optimization process after convergence to reduce loss and accumulated error;

- (3) Transfer learning: It leverages the enhanced similarity between time segments to accelerate training; and
- (4) Sine activation function: It outperforms conventional activations in solving structural vibration problems.

The AT-PINN method addresses the limitations of the conventional PINN methods when analyzing vibration issues in long-duration simulation. Additionally, it is anticipated to serve as a proficient approach for resolving other time-dependent PDEs.

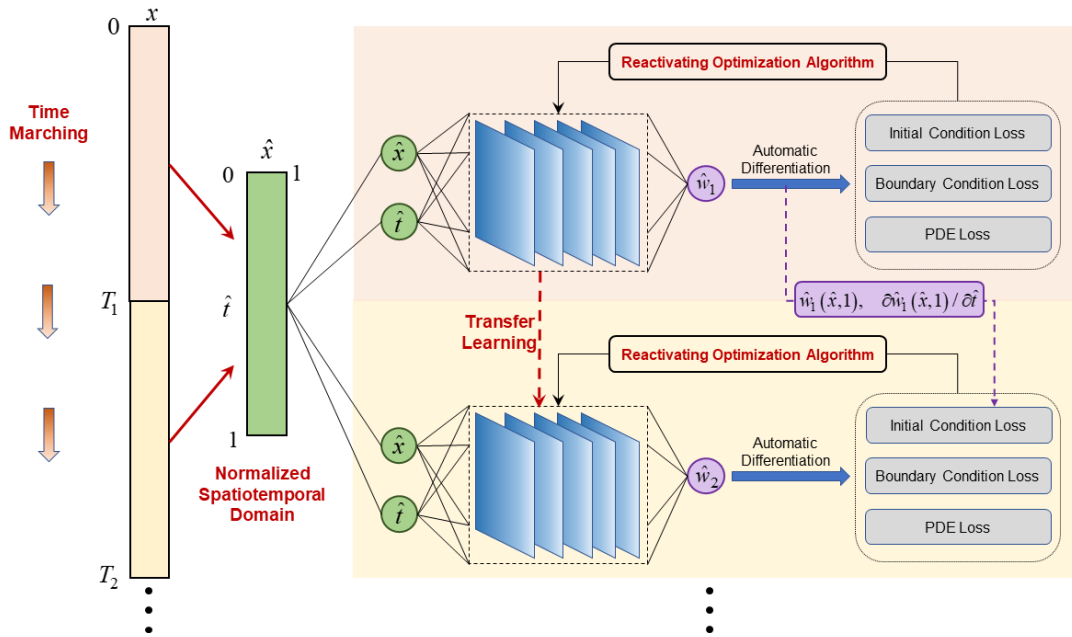


Fig. 2.17: Basic framework of the AT-PINN method (Chen et al., 2024a).

2.5.3 AT-PINN-HC (Advanced Time-marching Physics-informed Neural Network incorporating Hard-constraint Strategies)

In the STD-PINN method, boundary conditions and initial conditions are typically embedded within the loss function. Through training the neural network to minimize the loss function, the boundary and initial conditions can be approximately met. This process is referred to as soft constraints. Since it is generally impossible to reduce the loss function to exactly zero, soft constraints cannot strictly enforce the boundary and initial conditions. In some complex problems, neural network training becomes challenging, and reducing the loss function proves difficult, potentially leading to significant errors in the initial and boundary conditions. To

address the issue, research has been conducted on imposing hard constraints for boundary and initial conditions by modifying the neural network output using auxiliary (Wang et al., 2023c). The modified output under hard constraints can strictly satisfy the boundary and initial conditions, thereby enhancing the accuracy of PINN when solving partial differential equations. Furthermore, the loss function in PINN consists of various components associated with partial differential equations, boundary conditions, and initial conditions. The allocation of weights to each part of the loss function plays a critical role in neural network training and significantly influence solution precision and training performance.

While adaptive methods for adjusting these weights have been explored (Mcclenny et al., 2023),, manually selecting the weights through repeated trials often yields more accurate solutions. Hard constraints guarantee that boundary and initial conditions are inherently fulfilled, thus removing the necessity for specific parts of the loss function associated with these conditions. Therefore, incorporating hard constraints can effectively reduce the complexity and computational workload associated with adjusting the weights of the loss function. However, most existing PINNs that utilize hard constraints have primarily focused on time-independent problems or short-time time-dependent problems, making them unsuitable for long-time simulations, such as the vibration problems under investigation. Chen et al. (2025) has further extended their previous work (Chen et al., 2024) by introducing three novel advanced time-sequential PINNs embedded with hard constraints, collectively referred to as AT-PINN-HC, as shown in Fig. 2.18. This method is also capable of solving time-dependent problems over extended durations while providing enhanced solution accuracy and training efficiency compared to AT-PINN.

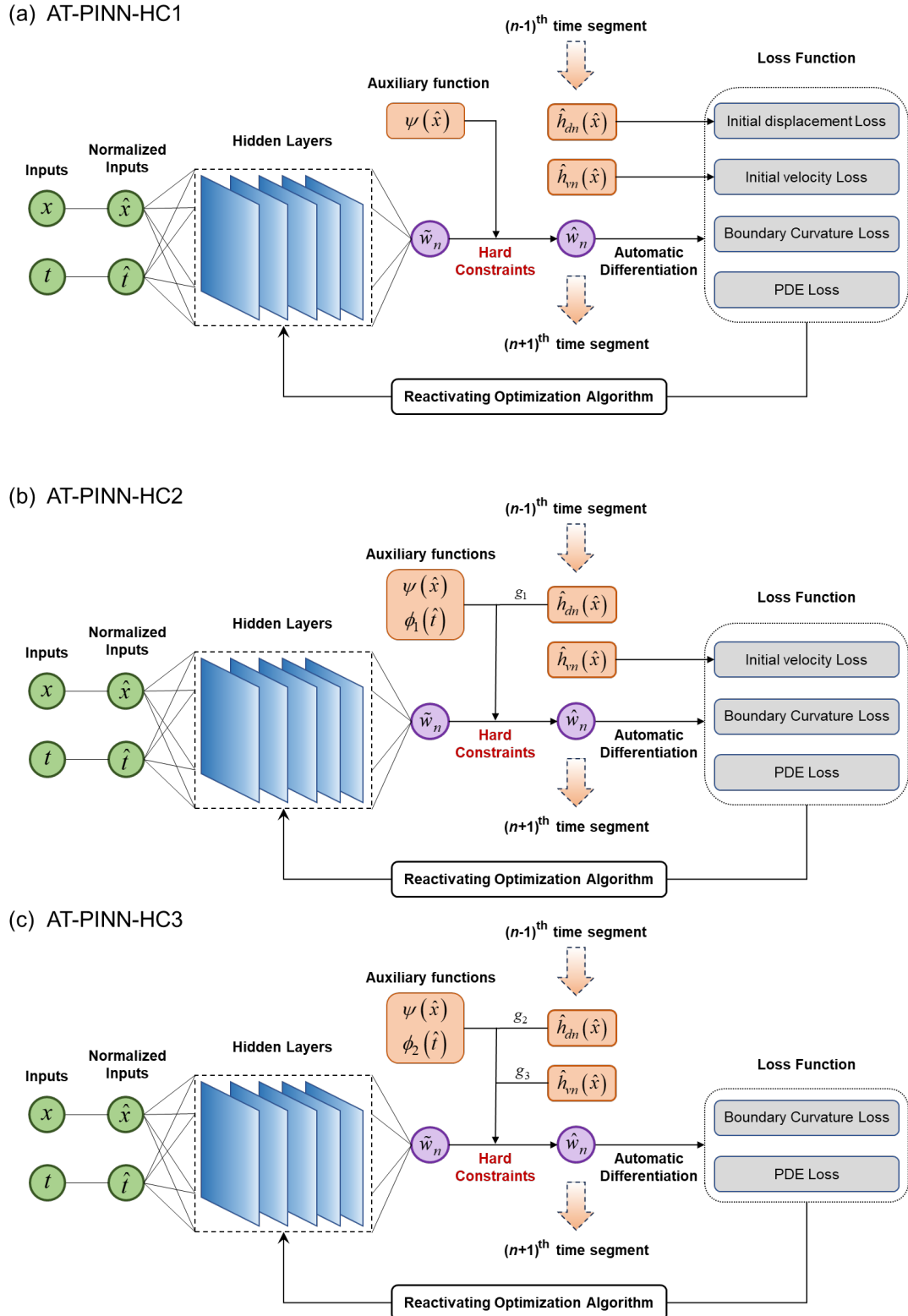


Fig. 2.18: Schematic representations of the AT-PINN-HC methods in the n^{th} time segment (Chen et al., 2025): (a) AT-PINN-HC1 featuring hard constraints on boundary displacement; (b) AT-PINN-HC2 incorporating hard constraints on both boundary and initial displacement; and (c) AT-PINN-HC3 applying hard constraints on boundary displacement, initial displacement, and velocity.

2.5.4 PINN for Optimization Design of Energy Harvesters

Hybrid energy harvesters integrating EMG, PEG, and TENG mechanisms offer significant advantages in capturing broadband mechanical energy. However, optimizing such complex multi-physics systems remains a challenge due to their high-dimensional, nonlinear parameter space. Machine learning and artificial intelligence offer promising solutions for real-time optimization of hybrid energy harvesters. Abouzarkhanifard et al. (2024) employs a machine learning approach using a graph neural network (GNN) model trained with finite element analysis data to optimize the performance of MEMS energy harvesters. The GNN model facilitates efficient design adjustments to achieve optimal energy harvesting under varying conditions. Grossmann et al. (2024) highlighted the limitations of FEM-based optimization for complex energy systems and how machine learning-based PINN methods can overcome these computational inefficiencies. PINNs offer a promising approach to accelerate hybrid harvester optimization by integrating physical laws (e.g., electromechanical coupling, magnetic force interactions, triboelectric charge transfer) directly into deep learning frameworks. Unlike conventional neural networks that rely purely on labeled data, PINNs incorporate governing equations (e.g., Maxwell's equations, nonlinear oscillator equations, and electrostatic boundary conditions), making them ideal for hybrid energy systems where data collection is costly or impractical (Nohra et al., 2024).

In a tri-hybrid vibration-based energy harvester, multiple factors contribute to the overall energy output. PINN-based optimization enables simultaneous tuning of multiple design parameters, significantly reducing computational cost compared to traditional iterative methods. The key parameters influencing hybrid energy harvesting efficiency include:

- Magnet spacing in EMG subsystems: Adjusting the distance between magnets directly affects the restoring force and bi-stability, which in turn influences the harvester's ability to transition into a tri-stable state, achieving higher displacement and greater energy conversion (Yang et al., 2022).
- Piezoelectric plate design: Using PINN can predict optimal the maximum displacement within the strength limitation (Wang et al., 2025a).
- Triboelectric surface roughness: By integrating materials and friction mass conditions into PINN loss functions, the framework can enhance charge transfer efficiency (Jiao et al., 2023).

Hybridizing energy harvesters can achieve multi-stability (bistable, tri-stable or quad-stable behavior) through magnetic interactions, enabling wider bandwidth operation. Compared to conventional data-driven deep learning and numerical optimization, PINNs provide the following key advantages in the optimization of hybrid energy harvesting systems in Table 2.4.

Table 2.4: Different optimization methods in solving energy harvesting systems.

Optimization Method	Computational Cost	Physics Awareness	Generalization Ability
Finite Element Analysis	High	Explicit	Limited to predefined models
Genetic Algorithms	Moderate	No physics constraints	Dependent on training data
Deep Learning (CNN/RNN)	Moderate-High	No explicit physics constraints	Requires large datasets
PINN-Based Optimization	Low	Fully physics-informed	Strong generalization, low data dependency

As shown, among various optimization methods, finite element method (FEM) offers explicit physics integration but is computationally expensive and limited to predefined models. Genetic algorithms (GA) require moderate computational resources but lack inherent physics constraints and rely on training data for generalization. Deep Learning (CNN/RNN) provides flexible modeling but demands large datasets and lacks explicit physics awareness, making it computationally intensive. In contrast, PINNs achieve low computational cost while fully embedding physical laws, enabling strong generalization with minimal data dependence. This makes PINNs particularly effective for physics-driven optimization problems where efficiency and accuracy are critical. Among various PINN approaches, AT-PINN-HC is proved to be an effective method for long-duration vibration analysis, therefore, it is promising to optimize the parameters of the hybrid energy harvesting system based on AT-PINN-HC.

2.6 Concluding Remarks

The literature review examined in this chapter underscores significant progress in nonlinear energy harvesting, hybrid energy conversion mechanisms (including single, double, and triple systems), and the use of PINNs to optimize tri-hybrid vibration-based energy

harvesters. While traditional linear energy harvesters are effective within limited frequency ranges, they encounter substantial limitations in environments characterized by variable or broadband frequency spectra. To overcome these challenges, researchers have investigated nonlinear mechanisms such as bistable, tri-stable, and multi-stable configurations, which facilitate a wider frequency response and improved energy conversion efficiency. The advent of nonlinear energy harvesters has shown enhanced power generation by utilizing complex system dynamics, including hysteresis effects, chaos, and stochastic resonance, all of which contribute to greater adaptability in practical applications.

Among the leading energy harvesting mechanisms, piezoelectric, electromagnetic, and triboelectric systems have emerged as essential technologies, each offering distinct advantages and facing inherent limitations. PEG units provide high conversion efficiency but are limited by their dependence on resonance tuning and the brittleness of their materials. EMGs deliver high power output and robust mechanical strength, but they are hindered by relatively large size requirements and susceptibility to electromagnetic interference. TENGs benefit from material flexibility and cost-effectiveness, yet they face challenges in maintaining consistent charge transfer efficiency and durability. Double hybrid energy harvesting systems, which integrate PEG–EMG or PEG–TENG mechanisms, have shown improved operational bandwidth and higher power output compared to single-mode harvesters. The development of triple hybrid energy harvesting systems, which combine piezoelectric, electromagnetic, and triboelectric conversion mechanisms, marks a significant advancement in energy harvesting technology.

On the other hand, a significant advancement in the optimization of hybrid energy harvesting has been the implementation of PINNs. Unlike traditional numerical methods (e.g., FEM, GA, and DRL approaches), PINNs integrate governing physical equations directly into the training of neural networks, allowing for real-time optimization of complex energy harvesting systems. Frameworks based on PINNs have shown enhanced computational efficiency and accuracy in adjusting critical system parameters, including magnet spacing in EMGs, stress analysis of piezoelectric plates, and surface roughness in triboelectric systems. Furthermore, the AT-PINN has emerged as a promising approach to address numerical instabilities encountered in long-duration simulations of time-dependent PDEs. By utilizing

higher-order time integration, adaptive subdomain partitioning, and hybrid physics-driven data prediction mechanism, the AT-PINN approach offers a more reliable and efficient method for optimizing multi-stable tri-hybrid energy harvesters.

CHAPTER 3

- EFFECT OF FRICTION ON TRI-HYBRID VIBRATION-BASED ENERGY HARVESTING DESIGN

3.1 Introduction

Friction is a critical and underestimated factor in vibration-based energy harvesters that incorporate piezoelectric, electromagnetic, and triboelectric mechanisms. In these dynamical systems, friction occurs inherently at interfaces, e.g., the effect of sliding contact in triboelectric nanogenerators (TENG), which can significantly affect device performance. On one hand, friction is integral to triboelectric energy conversion, enabling charge transfer between contacting dielectrics during relative motions. On the other hand, it induces dissipative forces and nonlinear stick–slip dynamics that can alter the vibration dynamics and energy output of energy harvesters. TENG initially reported in 2012 (Fan et al., 2012) can convert mechanical energy into electrical power through a combination of triboelectrification and electrostatic induction. TENG-based energy harvesters offer the benefits of being lightweight, easy to manufacture, and cost-effective, rendering them suitable for a wide range of potential uses. Surface charge density is one of the most critical factors determining the performance of TENG. It can be increased by choosing suitable charging materials and enlarging contact surfaces (Jin et al., 2016). Research on various TENG applications has primarily focused on advances in material surface processing techniques (Busolo et al., 2019; He et al., 2015; Hu et al., 2023).

Electrical energy transferred from a single mechanical excitation can be enhanced by combining multiple energy conversion processes, allowing the benefits of different conversion mechanisms to be utilized in a single device (Bolat et al., 2019; Chen et al., 2024b; Chen et al., 2021a; Liu et al., 2021; Wang et al., 2023b; Wang et al., 2021a; Yang et al., 2022; Zhao et al., 2022). This type of integration boosts a device's energy density, broadening the spectrum of its potential uses. Considering the low-frequency vibration excitation, various conversion methods have been developed to create structures for resonating. The first piezoelectric–

electromagnetic–triboelectric (“*tri-hybrid*”) energy harvester can be traced back to 2017 (He et al., 2017), providing inspiration for other research. Various approaches, including employing impact-driven piezoelectric generators (Wang et al., 2021a), frequency up-conversion mechanism (Wang et al., 2019), and adding external non-conservative forces (Yang et al., 2022), have been proposed to enhance output performance under low-excitation conditions.

Unlike piezoelectric and electromagnetic energy harvesters, TENG-driven devices are constructed from readily available materials and exhibit high-voltage and low-current output traits (Cheng et al., 2019; Song et al., 2022). Generally, there are four different working modes, namely a vertical contact-separation mode (Wardhana et al., 2022), an in-plane sliding mode (Li et al., 2023a; Lin et al., 2019), a single-electrode mode (Yang et al., 2013a), and a freestanding triboelectric-layer mode (Wang et al., 2014b). In the four fundamental working modes of TENG, sliding-mode TENG is an important one (Karapetyan, 2010), and this phenomenon is initiated by the relative sliding between triboelectric layers.

Mu et al. (2022) fabricated an arrayed contact-separation structure that increased the output performance of a hybrid enhanced TENG by 104%. The latest literature on advances in energy harvesting using micro- and nano-scale generators that exploit piezoelectric and frictional electric effects has been extensively reviewed (Chandrasekaran et al., 2019). Tan et al. (2021) conducted a numerical dynamic analysis to examine the sliding-mode friction acting on a bird-shaped cantilever. Fu et al. (2020) have devised a sliding-mode triboelectric energy harvester to analyze the effect of friction in terms of structural dynamics and static electricity. Their study is a rare example of an investigation on the dynamics of energy harvesters in relation to frictional forces, making it a valuable contribution to recent literature. The researchers further studied the slip–stick behavior to determine the effect of friction on energy-harvesting efficiency. Although the presentation of analytical models in previous studies, the investigation into the friction effect of multi-stable systems and its mechanism is still scarce. In addition, experimental conditions, such as humidity, temperature, spillage of material, and the process used to fabricate a prototype affect the friction coefficient and energy-harvesting performance of a device (Biswas et al., 1992; Pan et al., 2019). Despite numerous numerical studies that have been conducted, there remains a need for practical experimental investigations on sliding-mode triboelectric energy harvesters. In many hybrid

harvesting structures, the inclusion of friction is essential (Zobova, 2016) and poses a significant challenge when attempting to isolate its effects. Hence, it is crucial to design a comprehensive model that concurrently investigates the influence of friction on the dynamics aspect by analyzing both simulation and experimental data.

Many theoretical models simplify or neglect friction for tractability, but experimental studies have shown that even moderate friction can change the operating regime of multi-stable harvesters. For instance, the presence of dry sliding friction (with coefficients in the range ~ 0.2 to 0.5) was observed to modify the distribution of tri-stable equilibrium positions in a hybrid harvester, and its impact on output voltage became pronounced at lower excitation levels. At higher vibration amplitudes (e.g., $1\text{--}1.5\text{ g}$), the impact of friction becomes less significant, yet it still plays an important role in establishing the threshold for inter-well oscillations and the overall efficiency.

Accurately modeling frictional effects is therefore essential for reliable design and performance prediction of hybrid energy harvesters. This chapter represents both analytical and empirical approaches to represent friction in these systems. A common idealization is the Coulomb friction model, which assumes a constant friction force opposing motion; this captures the basic dry sliding behavior and is often a first approximation in dynamic simulations. For instance, a tri-stable piezoelectric–electromagnetic–triboelectric harvester requires a higher excitation amplitude to achieve inter-well oscillations than predicted by a frictionless model, this can be explained by frictional resistance at the interfaces. By tuning the dry friction coefficient in the model to match the experimental response, we can get a deeper understanding of how friction is influencing multi-well potential transitions and energy dissipation.

The presence of friction significantly affects both vibration dynamics and working efficiency. Friction introduces an additional dissipation mechanism for mechanical energy, frequently converting a portion of vibrational energy into heat rather than electricity, which in turn reduces the overall energy harvesting efficiency. In addition, this force effect can induce stick-slip oscillations and constrain small-amplitude motions. For example, a lightly excited

system may remain “stuck” in one potential well until sufficient force overcomes static friction, enabling a transition to another state. This behavior can restrict bandwidth or necessitate higher excitation thresholds for nonlinear benefits. Conversely, in triboelectric subsystems, controlled friction is beneficial and indeed necessary for charge generation. Excessive friction damps out vibrations, whereas too little friction would undermine triboelectric output. Ultimately, accounting for friction leads to more accurate predictions of system behavior and informs optimization strategies. By incorporating frictional models and validating them experimentally, this chapter demonstrates a model in the form of a cantilever structure with a tip mass to evaluate the effect of frictional force on tri-hybrid energy harvesting. The main contributions of this study include a structural dynamics approach is adopted to model and investigate a triboelectric energy harvester with sliding-mode operation. Different types of stability resulting from a magnetic field are identified by using the magnetizing current method for force calculations. Besides, the effect of the friction coefficient on the dynamics of the entire structure is investigated through numerical simulations. Finally, the multi-stability dynamics of the model is examined through an experimental approach, with a major emphasis on the impact of friction at low frequencies (2 to 8 Hz).

3.2 Structural design and working mechanism

3.2.1 Structural configuration

The configuration of the tri-hybrid vibration-based energy harvesting model is presented in Fig. 1(a). All of the generator units are integrated onto a base that is embedded on a vibration shaker, which is controlled by a power amplifier. The piezoelectric generator (PEG) unit has a copper cantilever beam that is clamped onto the base. Two macro fiber composite (MFC) patches (MFC-2807-P2) are mounted on the beam. These two MFC patches and the surfaces on both sides of the adhered copper beam constitute PEG1 and PEG2 units, respectively. Magnet A is attached to the free end of the beam, while external magnets B and C are positioned symmetrically on the base to create a restoring force. The dimensions of the beryllium bronze beam are $89 \times 12 \times 0.25 \text{ mm}^3$, the two external magnets (i.e., magnets B and C) have dimensions of $7.5 \times 9.5 \times 1.5 \text{ mm}^3$, and the fixed magnet A has dimensions of $9.8 \times 9.8 \times 4 \text{ mm}^3$.

The electromagnetic generator (EMG) unit consists of magnet A and four copper coils. The four coils are placed between the external magnets and magnet A, and are labeled 1, 2, 3 and 4 as shown in Fig. 3.1, i.e., coil 1 is symmetric with coil 4, and coil 2 is symmetric with coil 3. Coils 1 and 4 are connected in series (i.e., “EMG1”), while coils 2 and 3 are in series (i.e., “EMG2”). The sliding contact area of the TENG unit is a printed circuit board with an 80- μm polytetrafluoroethylene film coating, interdigitated parallel copper electrodes, and a glass epoxy base. A rectangular resin plate with parallel arrayed copper electrodes is fixed onto the bottom of magnet A. A schematic diagram of the preset positions of the magnets is shown in Fig. 3.1(b). The centers of mass of the magnets are denoted A, B, and C. The separation of magnets B and C is denoted f , and the horizontal magnet interval between point A and the B–C plane is denoted d . In Table 3.1, the governing parameters are presented.

When magnet A oscillates due to base acceleration, its motion is affected by the restoring force generated by external magnets B and C. Triboelectrification occurs in the contact area due to motion of the bottom magnet A (i.e., copper electrodes) relative to the negatively charged triboelectric material (i.e., polytetrafluoroethylene film). That is, during relative sliding motion, the copper electrodes (at the bottom of magnet A) are positively charged, whereas the polytetrafluoroethylene film is negatively charged. The resulting electrostatic equilibrium means that there is no charge transfer between the copper electrodes and polytetrafluoroethylene film during the movement. As magnet A moves, electrons flow from the top electrode to the bottom electrode, resulting in current flowing in the opposite direction. The process repeats as the slider oscillates and represents a half cycle of the TENG unit. Since the mechanisms of many harvesters have rarely been studied because of dry friction between the tip magnet and base. It is found that this hybrid tri-stable energy harvesting model exhibits better performance than traditional bi-stable or tri-stable cantilever beam harvesting models.

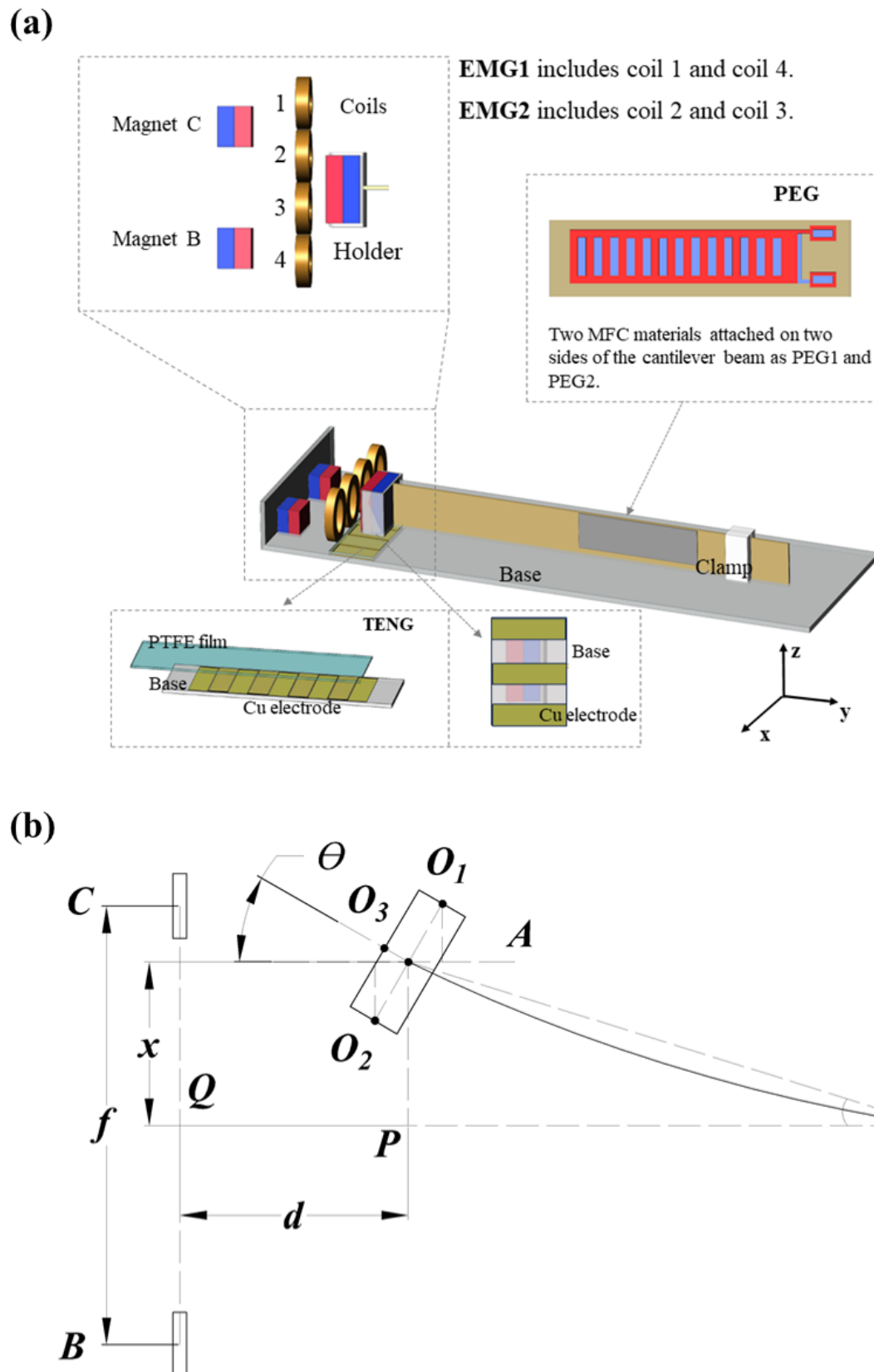


Fig. 3.1: (a) Schematic of the tri-hybrid energy harvesting model; and (b) schematic for the preset positions of magnets A, B, and C.

Table 3.1: Material parameters of the tri-hybrid energy harvesting model.

Item	Parameters	Quantities
Cantilever beam	Length (L_b) $\times$ Width (W_b) $\times$ Thickness (T_b)	$89 \times 12 \times 0.25 \text{ mm}^3$
	Elasticity modulus (E_b)	100 GPa
	Density (ρ_b)	7765 kg/m^3
MFC patch	Length (l_b) $\times$ Width (w_b) $\times$ Thickness (t_b)	$37 \times 10 \times 0.27 \text{ mm}^3$
	Elasticity modulus (E_p)	100 GPa
	Density (ρ_p)	5.44 g/cm^3
	Piezoelectric constant (d_{31})	-190 pC/N
Magnets	Dimensions of Magnet A	$9.8 \times 9.8 \times 4 \text{ mm}^3$
	Dimensions of Magnet B and Magnet C	$7.5 \times 9.5 \times 1.5 \text{ mm}^3$
	Density (ρ_A, ρ_B, ρ_C)	7500 kg/m^3
	Magnetization (M_A, M_B, M_C)	$7.4 \times 10^{-5} \text{ A/m}$
Four copper coils	Length $\times$ Width $\times$ Thickness	$12 \times 16.5 \times 3 \text{ mm}^3$
	Number of turns	650
TENG unit	PTFE: Length $\times$ Width $\times$ Thickness	$60 \times 12.5 \times 0.35 \text{ mm}^3$
	Copper film thickness	0.02 mm

3.2.2 Theoretical model and working mechanism

The governing equations for modeling the mechanical system depicted in Fig. 3.1 can be derived using the Lagrange–d’Alembert principle and Hamilton principle (Feeny, 2006) in the presence of external forces (i.e., friction) (Fang et al., 2017), we have

$$\int_{t_1}^{t_2} [\delta(T - U) + \delta W] dt = 0 \quad (3.1)$$

where T and U are the kinetic and potential energy of the system, respectively; W is the kinetic energy resulting from the nonconservative (frictional) force; and t_1 and t_2 are the initial time and final time, respectively.

$$W = \int_0^{L_b} F_f \delta(x - L_b) w(x, t) dx \quad (3.2)$$

where L_b is the length of the cantilever beam, F_f is the nonconservative force (friction force), $\delta(x - L_b)$ is Dirac's delta function, $w(x, t)$ is the deflection of the beam relative to the base. The kinetic energy of the cantilever beam and piezoelectric layers is given by

$$T_{b+p} = \frac{1}{2} \rho_b A_b \int_0^{L_b} \left[\frac{\partial w(x, t)}{\partial t} + \frac{\partial b(t)}{\partial t} \right]^2 dx + \frac{1}{2} \rho_p A_p \int_0^{L_p} \left[\frac{\partial w(x, t)}{\partial t} + \frac{\partial b(t)}{\partial t} \right]^2 dx \quad (3.3)$$

where ρ_b and A_b are the density and cross-sectional area of the cantilever, respectively; ρ_p , A_p , and L_p are the density, cross sectional area, and length of the piezoelectric layers, respectively; and $b(t)$ is the base displacement. The kinetic energy of magnet A (tip magnet) is

$$T_m = \frac{1}{2} m \left[\frac{\partial w(x, t)}{\partial t} + \frac{\partial b(t)}{\partial t} \right]_{x=L_b}^2 \quad (3.4)$$

where m is the mass of magnet A. In general, the moment of inertia should be considered. However, in this design, the moment of inertia is deemed insignificant due to the low mass of magnet A's tip. The total kinetic energy T of the system can be obtained from Eqs. (3.3) and (3.4).

The potential energy of the cantilever beam with piezoelectric layers is given by

$$U_{b+p} = \frac{1}{2} E_b I_b \int_0^{L_b} \left[\frac{\partial^2 w(x, t)}{\partial x^2} \right]^2 dx + E_p I_p \int_0^{L_p} \left[\frac{\partial^2 w(x, t)}{\partial x^2} \right]^2 dx \quad (3.5)$$

where E_b and E_p are the Young's modulus of the beam and the MFC layers, respectively; and I_b and I_p are the area moments of inertia of the beam and the MFC layers, respectively. The elastic potential energy provided by the magnetic field is given by

$$U_m = \int (F_{B-x} + F_{C-x}) dw \quad (3.6)$$

A three-dimensional reference system is established for the structure. Magnet A is fixed on the top of the copper beam, which is placed on the x - y plane and moves along the x axis. A magnetizing current method (Leng et al., 2017) is adopted in the current magnet model. Magnetic forces generated by the two external fixed magnets B and C (F_{B-x} and F_{C-x}) act on

magnet A to move it in the x direction. In addition, magnets B and C generate magnetic forces in the x -direction. F_{B-x} and F_{C-x} are written as follows:

$$\begin{aligned}\vec{F}_{B-x} &= \iint_S \vec{K}_m \times \vec{B}_{ext-B} dS \\ \vec{F}_{C-x} &= \iint_S \vec{K}_m \times \vec{B}_{ext-C} dS\end{aligned}\tag{3.7}$$

where K_m is the surface current density of magnet A; B_{ext-B} and B_{ext-C} are the external magnetic flux densities of magnets B and C, respectively; and S is the surface area of magnet A. K_m is given by

$$\vec{K}_m = \vec{m}_A \times \vec{n}\tag{3.8}$$

where m_A is the magnetization of magnet A and n is a unit vector in the direction outward from the surface. The magnetizing current densities on the top and bottom surfaces of magnet A can be ignored, owing to the magnetization K_m of magnet A being in the x -direction. The magnet flux density of an isotropic medium is expressed as follows:

$$\vec{B} = \mu_0 \vec{H}\tag{3.9}$$

where μ_0 is the permeability, and H is the magnetic field strength. In this work, we only consider the x -direction in adopting the magnetizing current method with calculation of the magnetic forces F_{B-x} and F_{C-x} , i.e.,

$$F_x = \mu_0 M_A S (H_{y1} - H_{y2})\tag{3.10}$$

where M_A is the magnitude of magnet A's magnetization; and H_{y1} and H_{y2} are the magnitudes of the magnetic field intensity in the y -direction at the centers of the front and back surfaces of magnet A, respectively.

The magnetic field strength at a point (x, y, z) in the y -direction is written in the form (Agashe and Arnold, 2008)

$$\begin{aligned}
 H_y(x, y, z) = \frac{M_C}{4\pi} & \left[\tan^{-1} \left(\frac{x_p z_p}{y \sqrt{x_p^2 + z_p^2 + y^2}} \right) + \tan^{-1} \left(\frac{x_n z_n}{y \sqrt{x_n^2 + z_n^2 + y^2}} \right) \right. \\
 & \left. - \tan^{-1} \left(\frac{x_n z_p}{y \sqrt{x_n^2 + z_p^2 + y^2}} \right) - \tan^{-1} \left(\frac{x_p z_n}{y \sqrt{x_p^2 + z_n^2 + y^2}} \right) \right]
 \end{aligned} \quad (3.11)$$

where $x_p = x + l_c/2$, $x_n = x - l_c/2$, $z_p = z + w_c/2$, and $z_n = z - w_c/2$, in which l_c and w_c are the length at the x direction and the width at the z direction of magnet C, respectively. M_C is the magnitude of magnet C's magnetization. The magnetic force generated by magnets B and C and acting on magnet A in the x -direction is

$$F_{B-x} + F_{C-x} = \mu_0 M_A S (H_{B-y1} - H_{B-y2} + H_{C-y1} - H_{C-y2}) \quad (3.12)$$

where H_{B-y1} and H_{B-y2} are the magnitudes of the magnetic field intensity in the y -direction generated by magnet B at the centers of the front and back surfaces of magnet A, respectively. H_{C-y1} and H_{C-y2} are the magnitudes of the magnetic field in the y -direction generated by magnet C at the centers of the front and back surfaces of magnet A, respectively.

By setting the coordinate origin at the center of magnet C and taking into account the arc-shaped trajectory of the tip magnet's movement, we found that the spatial coordinates of points O_1 and O_2 are expressed as follows

$$(f/2 - x - l_A/2 \times \cos \theta, d + l_A \times (1 - \cos \theta) + w_A/2 \times \sin \theta, 0) \quad (3.13a)$$

and

$$(f/2 - x + l_A/2 \times \cos \theta, d + l_A \times (1 - \cos \theta) - w_A/2 \times \sin \theta, 0) \quad (3.13b)$$

where θ is the angle between AO_3 and the horizontal direction (see Fig. 3.1(b)). In above equations, d is the horizontal distance from the center point of magnet A to magnets B and C. l_A and w_A are the length and width of magnet A, respectively. Similarly, setting the coordinate origin at the center of magnet B, we found that the spatial coordinates of points O_1 and O_2 are, respectively, written as

$$(-f/2 - x - w_A/2 \times \cos \theta, d + l_A \times (1 - \cos \theta) + w_A/2 \times \sin \theta, 0) \quad (3.14a)$$

and

$$(-f/2 - x + w_A/2 \times \cos \theta, d + l_A \times (1 - \cos \theta) - w_A/2 \times \sin \theta, 0) \quad (3.14b)$$

The cantilever beam is assumed to be a uniform Euler–Bernoulli beam and the transverse displacement of the beam is approximated as follows:

$$w(x, t) = \phi(x)q(t) \quad (3.15)$$

where $q(t)$ denotes the associated time-varying generalized coordinates and $\phi(x)$ is the mode shape function. In this work, we only consider the first-order mode as this is a dominant vibration mode. The latter is obtained by solving the eigenvalue problem for the first-order mode of an undamped and linearized cantilever beam with a tip magnet, that is

$$\phi(x) = C \left\{ \left[\cos\left(\frac{\lambda}{L}x\right) - \cosh\left(\frac{\lambda}{L}x\right) \right] + \sigma \left[\sin\left(\frac{\lambda}{L}x\right) - \sinh\left(\frac{\lambda}{L}x\right) \right] \right\} \quad (3.16)$$

where C is an arbitrary amplitude that is derived from the corresponding orthonormality conditions (with the corresponding formulas not given here), and σ and λ are determined from the associated eigensystem. We substitute Eq. (3.15) into the previously mentioned potential (Eqs. (3.3) and (3.4)) and kinetic (Eqs. (3.5) and (3.6)) energy formulas, and then apply the resulting equations to Eq. (3.17). A Lagrange function can be derived for the entire system using modal coordinates and a mode shape function.

Following the d'Alembert principle, which is equivalent to the Euler–Lagrange equations with external forces, the governing equation of the mechanical system is

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = F_f \phi(L_b) \quad (3.17)$$

where $L = T - U$ with T and U are the kinetic energy and potential energy of the system, respectively. F_f is the kinetic frictional force acting on the tip magnet as given by

$$F_f = -\mu mg \operatorname{sgn}(\dot{q} \phi(L_b)) \quad (3.18)$$

where μ is the friction coefficient, m is the mass of magnet A, g is the acceleration of gravity, and $\operatorname{sgn}(\cdot)$ is the signum function. Through applying the orthonormality conditions and including damping, we have

$$\ddot{q} + 2\zeta\omega\dot{q} + \omega^2q - \theta v = \alpha\ddot{b} + F_f\phi(L_b) + F_x\phi(L_b) \quad (3.19)$$

where ζ is the damping ratio; ω is the undamped natural frequency of the first mode of the beam without the tip mass; $\ddot{b}$ is the base acceleration; and F_x is the magnetic force exerted by the magnetic field at the tip.

3.2.3 Experimental setup

Figure 3.2(a) shows the experimental setup. Harmonic excitations were generated by a vibration exciter (*SPEKTRA APS 113*). The model was fixed on the shaking table using a screw link. A signal generator (*Keysight 33500B*) and power amplifier (*APS 125*) were used to generate excitation signals. Acceleration and frequency data were recorded using an accelerometer (*Dytran 3023A3H*), which was connected to a data acquisition system (*Dewesoft SIRIUSi-8xACC*), and observed on a computer equipped with bundled software. The output voltage data were recorded using an oscilloscope (*Keysight DSOX3014T*).

Based on the schematic model, a preliminary test and numerical simulation needed to be conducted before analysis of the output experimental results. First, the restoring force was measured to obtain the magnet spacings (i.e., f and d as indicated in Fig. 3.1(b)), which greatly affected the steady state and output of the structure. A digital dynamometer and micrometer were used to measure the forces acting on the magnetic slider under different displacements, as shown in Fig. 3.2(b). Figure 3.2(c) shows the three stable positions of the beam with the tip magnet under an external magnetic force, which corresponds to two symmetric static equilibria and one central static equilibrium. Magnet spacings (d and f) needed to be adjusted until the tri-stable state was achieved.

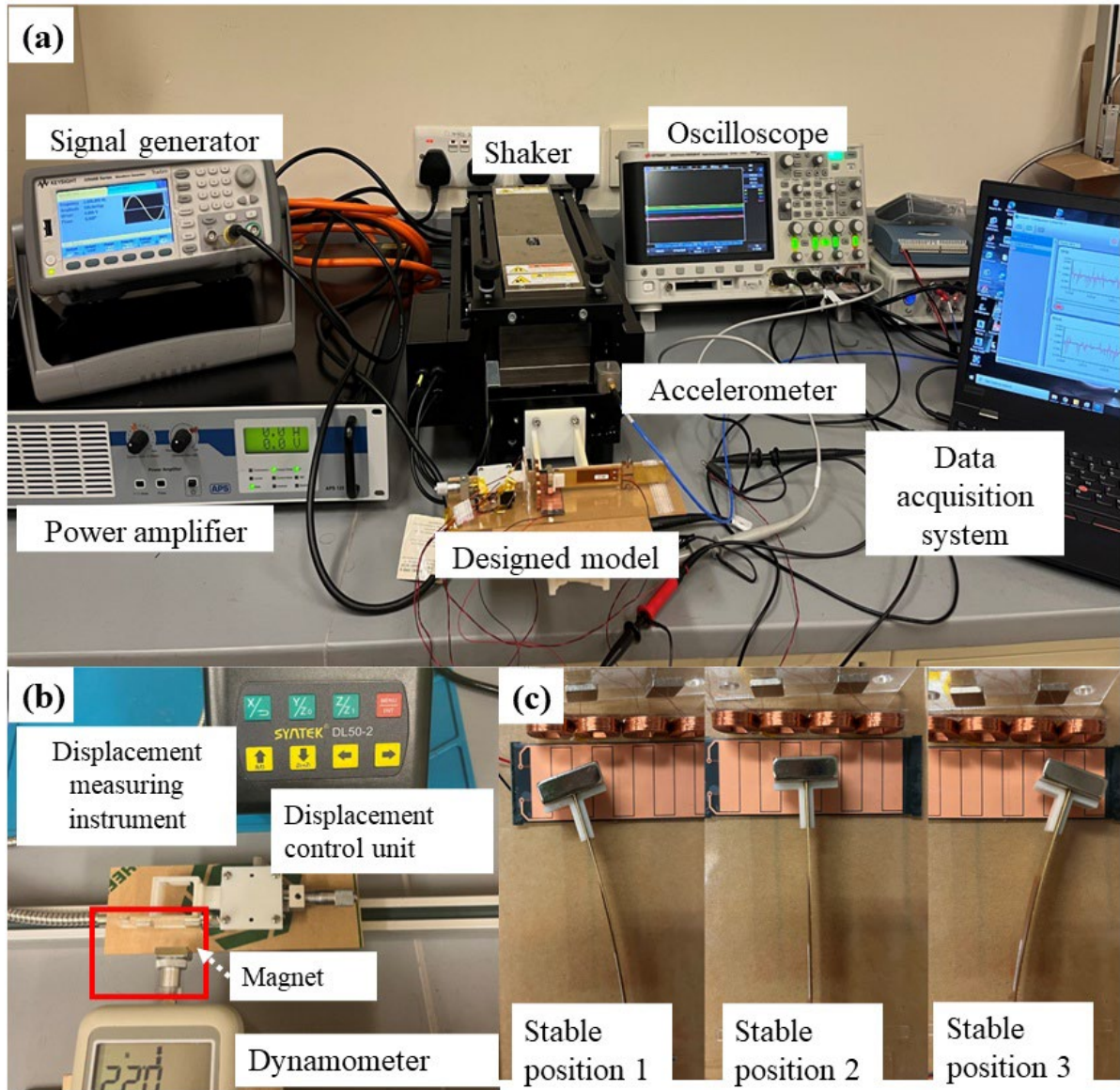


Fig. 3.2: (a) Experiment setup; (b) magnet force measurement; and (c) three equilibrium points.

Figure 3.3(a) shows the magnet force F_m calculation and results of tests of energy harvesters with two different tri-stable modes (i.e., the first one is: $d = 12.5$ mm, $f = 17$ mm; and the second one is: $d = 14.5$ mm, $f = 17$ mm). The results obtained using the above method for calculating the magnet force are almost identical to the experimental test results. The calculated two restoring force results which consist of magnet force and elastic force of the beam are shown in Fig. 3.3(b). After confirming the feasibility of the calculation method, a potential energy analysis was carried out as shown in Fig. 3.3(c) with the data of the model with $d=12.5$ mm. It was found that the nonlinear system may require a low excitation frequency to pass through the potential well and obtain a greater response than that of high excitation, due

to the shallow depth of the potential well. Consequently, the output performance of the PEG unit was improved under low-frequency and low-intensity vibrations. The frequency responses of the PEG, EMG, and TENG units under various base accelerations (0.5 g, 1 g, and 1.5 g) were investigated in the next section. Peak-to-peak (P-P) open-circuit voltage levels were experimentally measured in the frequency range of 2–8 Hz.

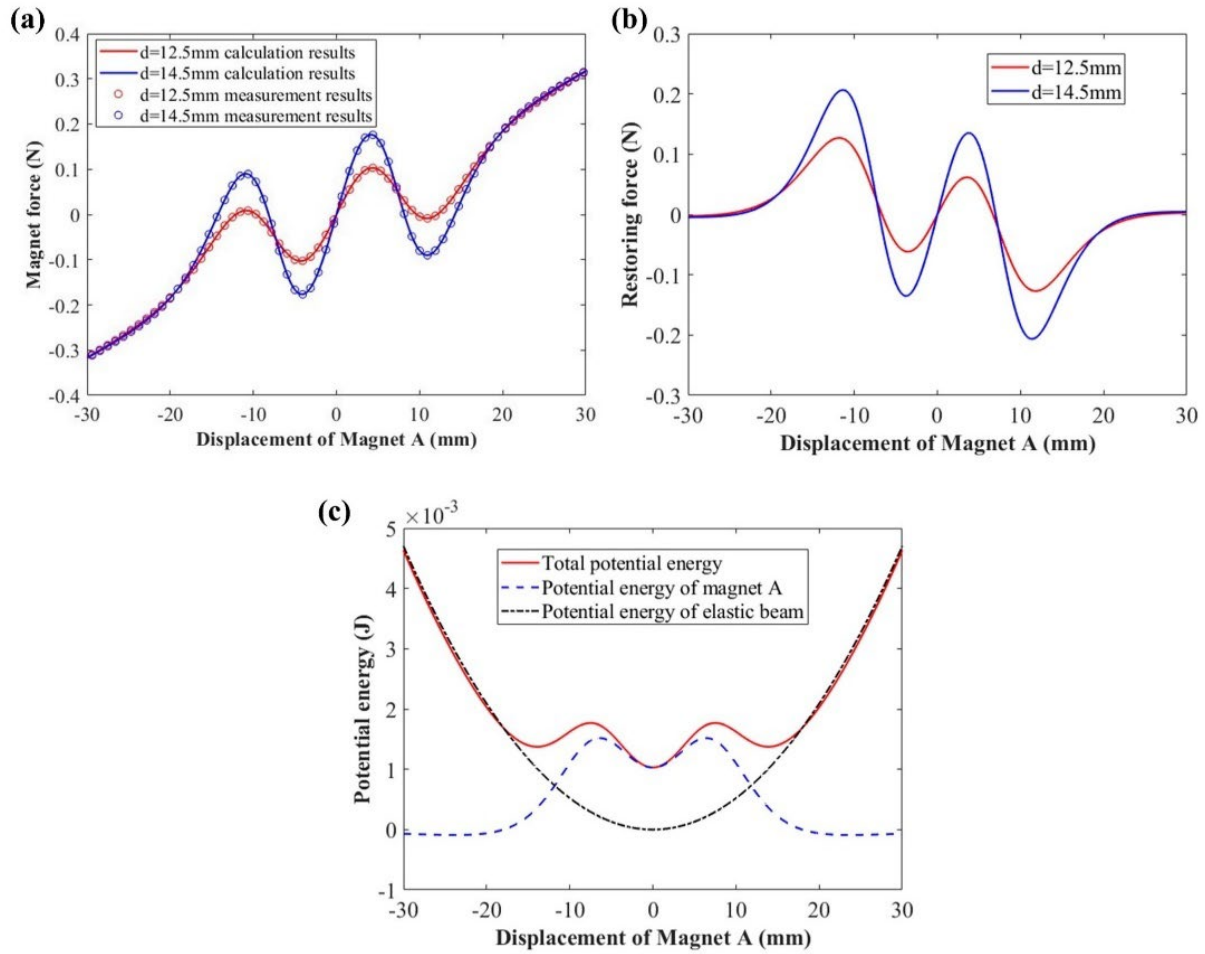


Fig. 3.3: (a) Calculation and measurement results of magnet force with $d=12.5\text{mm}$ and $d=14.5\text{mm}$; (b) the restoring force of magnet A under $d=12.5\text{mm}$ and $d=14.5\text{mm}$; and (c) the total potential energy of magnet A and elastic beam.

3.3 Results and discussion

3.3.1 Simulation results

Numerical simulations are indispensable for fully validating the dynamic effect of friction on multi-stable nonlinear structures. In energy harvesting systems, the intricate relationship

between friction and output is heightened by the various configurations of frictional materials. From the perspective of material characteristics, the coefficient of friction is highly correlated to various factors such as external load (normal load), sliding speed, contact temperature, friction time, and curing temperature. It implies that the coefficient of friction is hard to be identified as a precise value because it may change with time and the surrounding environment.

In this study, we can calculate the external load of magnet A: $F_{nA} = \rho_A \times V_A \times g = 0.02824$ N, and the pressure provided by magnet A is $P_A = F_{nA}/S = 720$ Pa, where S is the lower surface (contact surface) area. Based on the analysis of the previous works (Biswas and Vijayan, 1992; Fu, Ouyang, and Benjamin Davis, 2020; Pan et al., 2019; Unal et al., 2004), it can be inferred that the range of coefficients falls within approximately 0.3 based on F_{nA} and P_A at 1-g to 1.5-g acceleration. To observe the effect of frictional forces, we thus set $\mu = 0.2$ – 0.5 in numerical simulations. Figure 3.4 illustrates the tip vibration amplitudes during a frequency sweeping process of up-conversion (1 to 15 Hz) and down-conversion (15 to 1 Hz) for $\mu = 0.2$ – 0.5 , at two different initial phases (i.e., $y_{initial} = 4$ mm and 20 mm). The initial state of the structure exerts a significant influence on dynamic analysis. An increase in the friction coefficient (μ) results in decreased vibration amplitudes, leading to reduced inter-well motion, fewer tri-stable states, yet more monostable and bistable states. In the absence of frictional effects, the inter-trap motion becomes more stable. Under the given excitation, the presence of tri-stable states becomes more pronounced, facilitating inter-well transitions. This finding provides further evidence for the detrimental effect of friction on the output.

At an excitation acceleration of 1 g and frequency of 2 Hz, owing to the effect of the frictional force, the dynamic properties of the tip magnet transitioned from multi-stable states to a monostable state as shown in Fig. 3.5. It is found that as the excitation level decreases, there is a reduction in the frequency bandwidth of inter-well oscillations. In addition, the amplitude decreased. The outputs of the piezoelectric material (MFC plates) positively correlated with the displacement of the tip magnet. Figure 3.6 shows that at an excitation acceleration of 1 g and frequency of 5 Hz, owing to the effect of friction, there was a slight decrease in amplitude without a substantial change in stability. Figure 3.7 illustrates that when subjected to an excitation of 1.5 g acceleration and 5 Hz, the displacement of the tip magnet is slightly constrained by friction, leading to a marginal reduction in the inter-trap motion

bandwidth. Nonetheless, the system still exhibits a tri-stable state. When the excitation is high (Figs. 3.7(a) and (b)), friction has a relatively minor impact compared to when the excitation is low (Figs. 5(a) and (b)). The overall trends of Figs. 3.6(a) and (b) are similar to that of Figs. 3.7(a) and (b). However, due to a higher acceleration excitation, a relatively larger displacement is observed in Fig. 3.7.

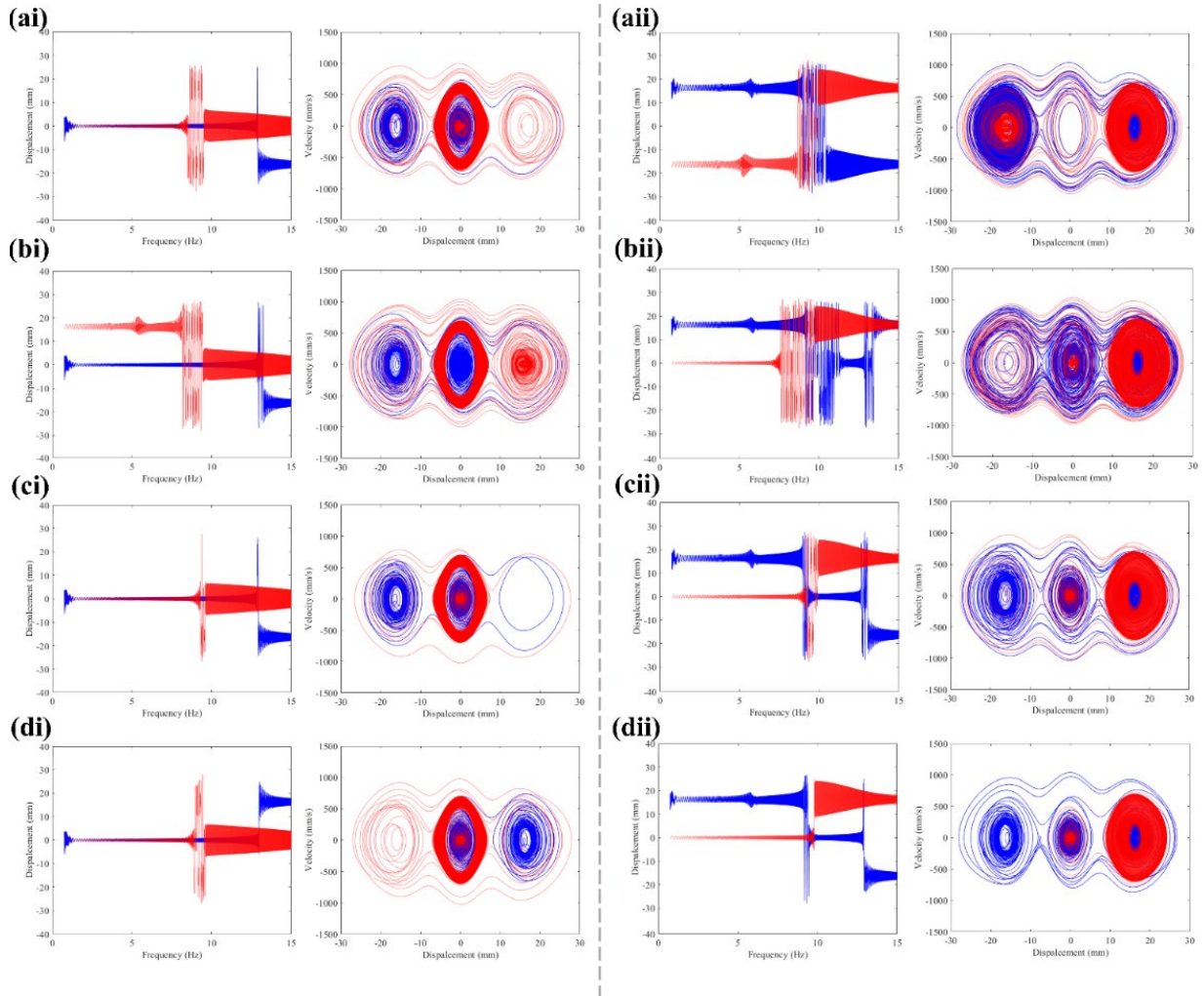


Fig. 3.4: Time-domain analysis of tip magnet vibration amplitudes and phase portraits at an excitation acceleration of 1 g: (ai) $\mu = 0.2$, $y_{\text{initial}}=4$ mm; (aai) $\mu = 0.2$, $y_{\text{initial}}=20$ mm; (bi) $\mu = 0.3$, $y_{\text{initial}}=4$ mm; (bii) $\mu = 0.3$, $y_{\text{initial}}=20$ mm; (ci) $\mu = 0.4$, $y_{\text{initial}}=4$ mm; (cii) $\mu = 0.4$, $y_{\text{initial}}=20$ mm; (di) $\mu = 0.5$, $y_{\text{initial}}=4$ mm; and (dii) $\mu = 0.5$, $y_{\text{initial}}=20$ mm. (Note: Blue line: up-conversion frequency sweeping results; and red line: down-conversion frequency sweeping results).

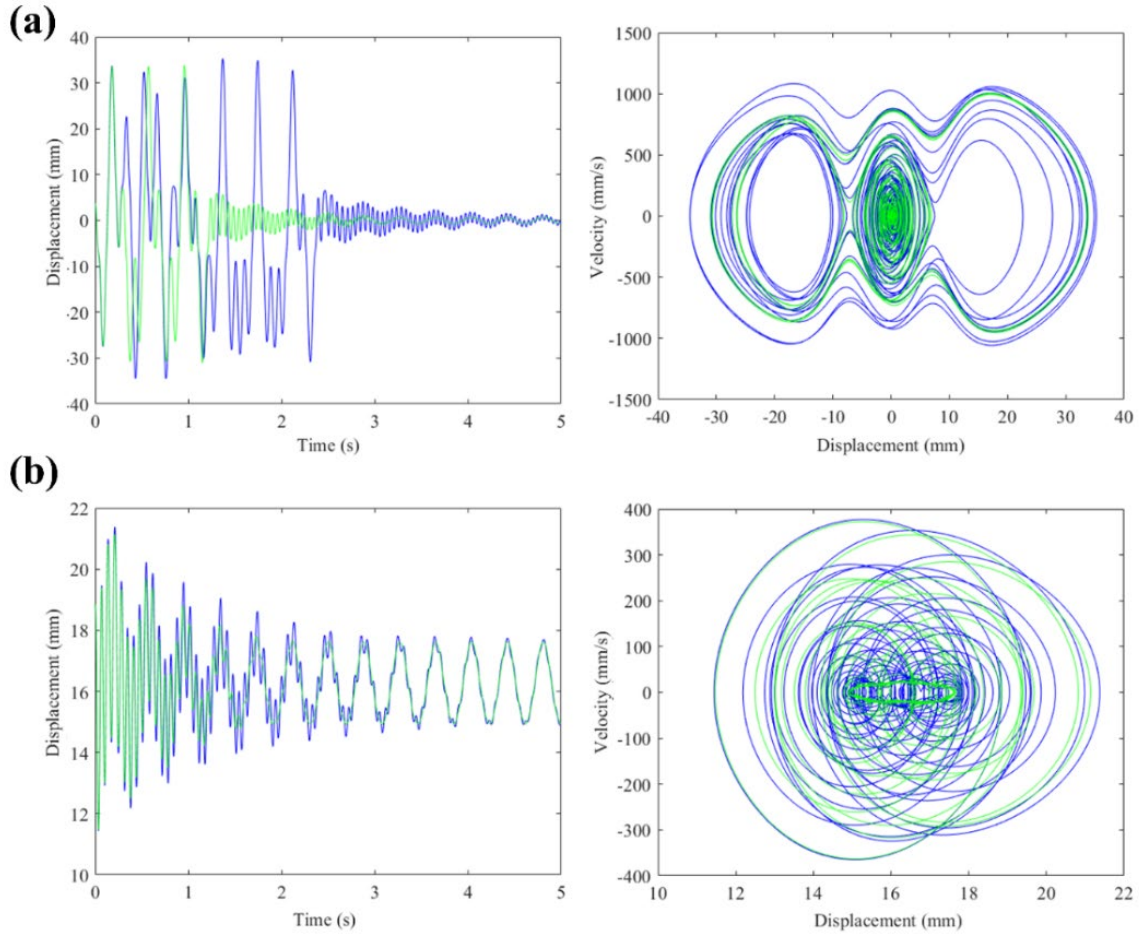


Fig. 3.5: Phase diagrams for the dynamic motion of magnet A: (a) $y_{initial} = 4$ mm and (b) $y_{initial} = 20$ mm without and with friction under an excitation acceleration of 1 g and 2 Hz. (Note: Blue line: without friction; and green line: with friction ($\mu = 0.3$).)

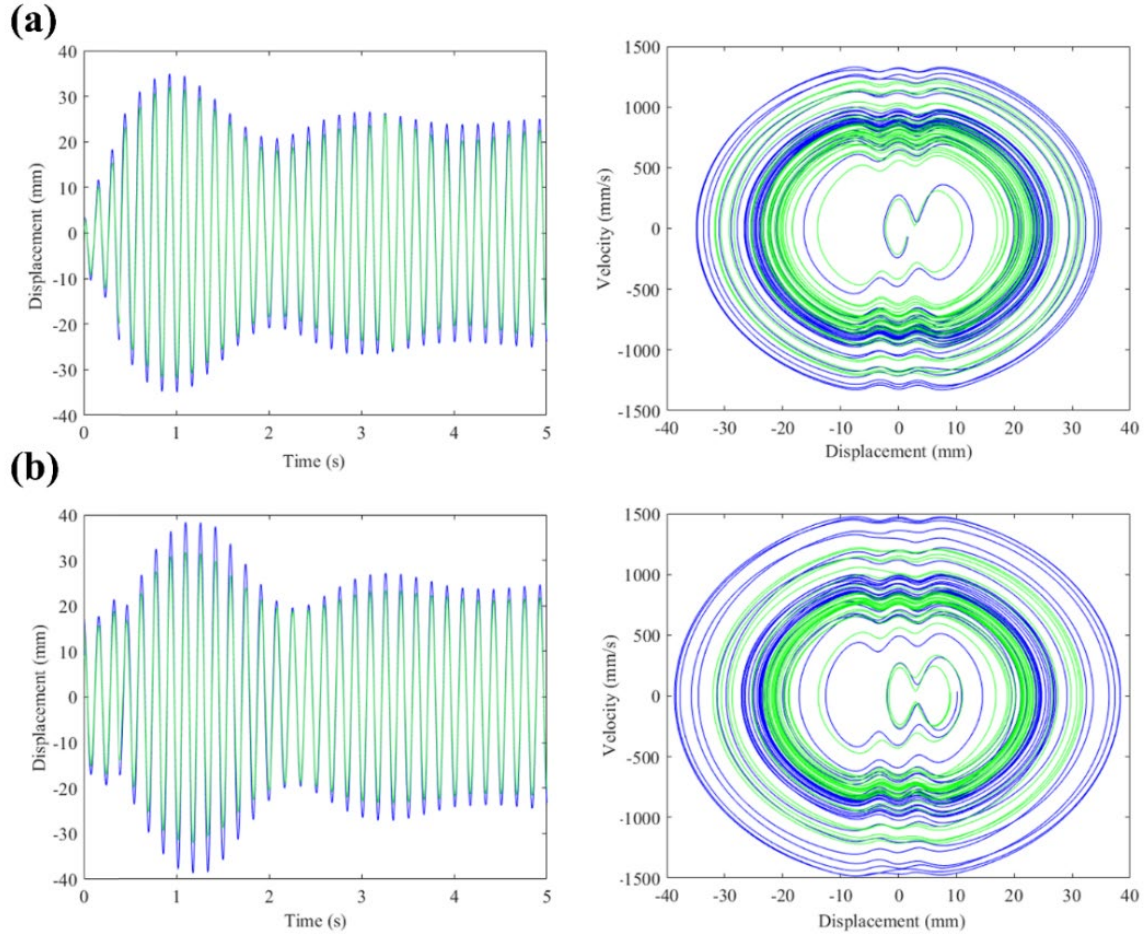


Fig. 3.6: Phase diagrams for the dynamic motion of magnet A: (a) $y_{initial} = 4$ mm and (b) $y_{initial} = 20$ mm without and with friction under excitation acceleration of 1 g and 5 Hz. (Note: Blue line: without friction; and green line: with friction ($\mu = 0.3$).)

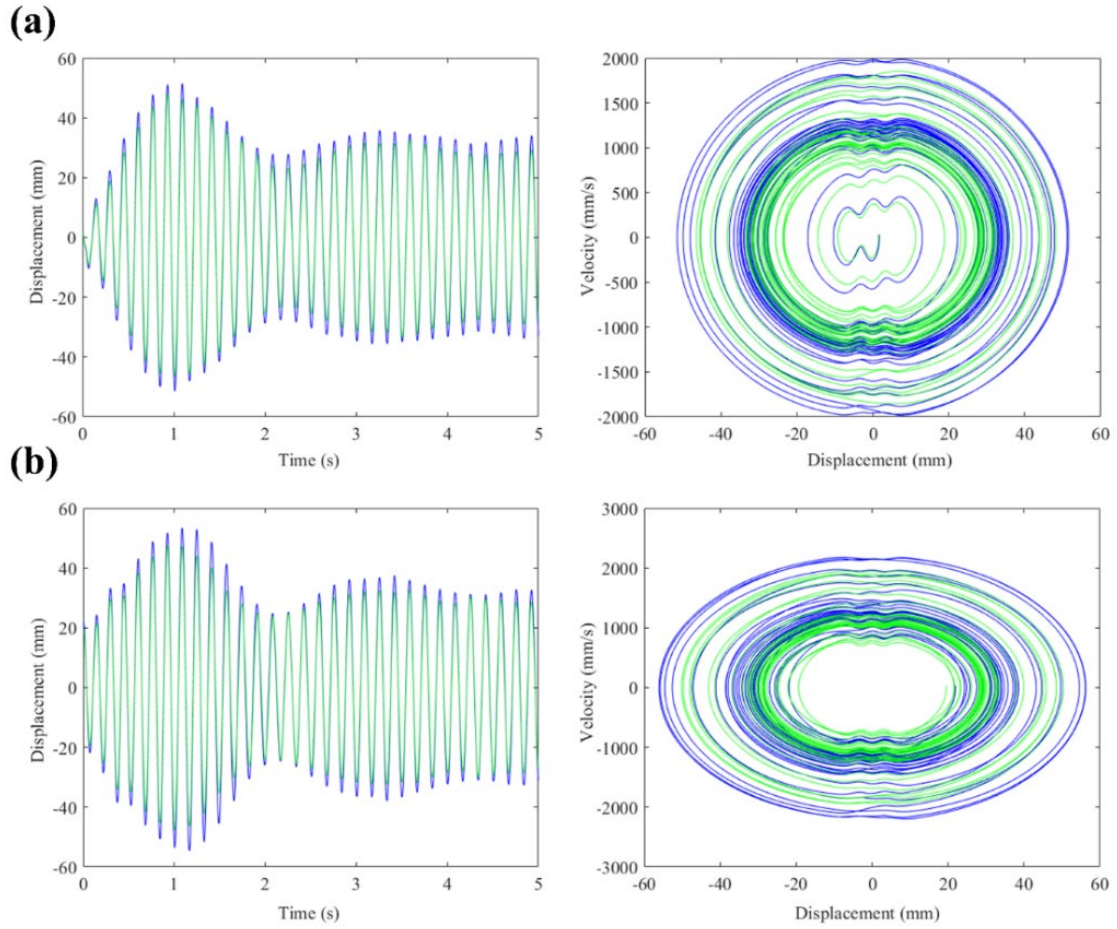


Fig. 3.7: Phase diagrams for the dynamic motion of magnet A: (a) $y_{\text{initial}} = 4 \text{ mm}$ and (b) $y_{\text{initial}} = 20 \text{ mm}$ without and with friction under an excitation acceleration of 1.5 g and 5 Hz . (Note: Blue line: without friction; and green line: with friction ($\mu = 0.3$).)

3.3.2 Experimental results

Owing to friction, the tip magnet A struggled to produce an efficient output at an excitation acceleration of 0.5 g , and thus no measurements were made on the PEG and EMG units under the effect of the TENG at an excitation acceleration of less than 0.5 g . In addition, at an excitation acceleration of 0.5 g , it was difficult for the tip magnet to move between wells, and the measurement range was thus from 2 to 8 Hz . As all of the harvester units in the prototype were driven by identical mechanical motions, their open-circuit voltages had similar broadband responses based on the selected multi-stable nonlinearity mode. In addition, owing to symmetry, the opposite side of the beam theoretically should have produced an equivalent output. However, to ensure experimental accuracy, comparative tests were conducted on the symmetric

elements. In the following, we investigated the presence and absence of the sliding-mode friction effect, i.e., with and without the TENG unit.

The experimental open-circuit voltage P-P values of two PEG units (with two sides) at 2–8 Hz are presented in Fig. 3.8(a). The PEG unit exhibited a higher output under conditions of high acceleration compared to low acceleration. As the acceleration increased, the maximum response frequency gradually increased and then rapidly declined. As the input frequency increased, the slider exhibited large-amplitude inter-well oscillations, leading to high-voltage responses. The maximum single-side PEG unit's output was about 90 V at an excitation acceleration of 1.5 g and frequency of 6–7 Hz. It is noted that the tip magnet struggled to achieve inter-well motion when subjected to 2 Hz excitation and 1.5 g acceleration. Hence, experimental data were recorded from 3 to 8 Hz in the case of 1.5 g. Moreover, PEG1 and PEG2 data were essentially consistent, indicating that the experimental structure was symmetrical, and thus only PEG2 data were used for comparison in subsequent studies.

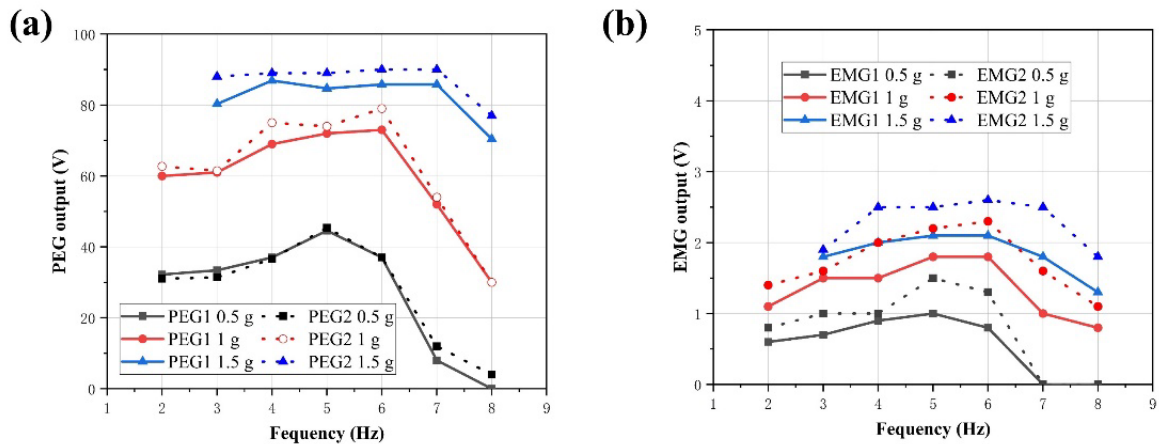


Fig. 3.8: Open-circuit voltage outputs of PEG and EMG at 0.5 g, 1 g, and 1.5 g: (a) PEG1 and PEG2 outputs; and (b) EMG1 and EMG2 outputs.

The voltage outputs of EMG1 and EMG2 under different excitations (0.5 g, 1 g, and 1.5 g) are presented in Fig. 3.8(b). EMG1 comprised two coils located on the outer side, whereas EMG2 comprised two coils located in the center. Regardless of the excitation applied, the output of EMG2 surpassed that of EMG1, owing to a more rapid change in the magnetic field intensity at the center than on the outer side. The output trends of EMG2 and EMG1 exhibited similarity, demonstrating an ascending pattern at lower frequencies (2–4 Hz), a smoother

trajectory at 5–6 Hz, and a decline trend after 6 Hz or 7 Hz. The maximum open-circuit voltage output of EMG2 was about 2.6 V at an excitation acceleration of 1.5 g and frequency of 6 Hz.

The TENG unit reduced the PEG2 P-P output. This phenomenon may be attributable to the frictional force, which increased the damping factor and restricted beam movement to some extent, resulting in a reduced voltage output. The experimental open-circuit voltage P-P values of PEG2 in the frequency range of 2–8 Hz in the presence and absence of the TENG are presented in Fig. 3.9(a). Owing to the effect of friction at an excitation acceleration of 0.5 g, the open-circuit P-P voltage output of PEG2 approached to zero. Therefore, only results for excitation accelerations of 1 g and 1.5 g are presented. The figure reveals that friction did not always impede the movement of magnet A at the beam's tip. At an excitation acceleration of 1 g and frequency of 4 Hz, the P-P voltage in the presence of friction exceeded that in the absence of friction, whereas at an excitation acceleration of 1 g and frequency of 5 Hz, friction appeared to have no effect. At other excitation frequencies, friction had a detrimental effect. At an excitation acceleration of 1.5 g and frequency of 2 Hz, outputs with or without friction were negligible and are thus not shown in the figure. The effect of friction was negligible at an excitation acceleration of 1.5 g and frequencies of 3–6 Hz. However, the open circuit P-P voltage was reduced by friction at an excitation acceleration of 1.5 g and frequency of 7 Hz. Overall, the effect of friction was smaller at an excitation acceleration of 1.5 g than at an excitation acceleration of 1 g. Due to the effect of friction, the excitation points with the maximum response in the figure shifted to the left, indicating a decrease in the excitation frequency at the point of maximum response.

The generators produced large P-P open-circuit voltages at low frequencies, owing to the resonant inter-well oscillations around various local oscillation frequencies at 5–6 Hz. As the structure and trajectory of the harvester were symmetric, the measurements for EMG1 and EMG2 were similar. As coil 2 was on the edge of the motion curve, the output P-P value exhibited asymmetry. Coil 1 was closer to the center stable point than coil 2, and at a higher rate of magnetic induction change, coil 1 performed better in terms of the output P-P value than coil 2. When the excitation frequency was set to 2 and 8 Hz, respectively, the output was reduced by the inter-well motions of the tip magnet.

The open-circuit P-P voltage outputs of the TENG unit at excitation accelerations of 1 g and 1.5 g are presented in Fig. 3.9(b). As the frequency increased, the output increased until it reached the point of maximum response. The maximum responses were obtained at an excitation acceleration of 1 g and frequency of 5 Hz, and at an excitation acceleration of 1.5 g and frequency of 6 Hz. The outputs corresponding to the maximum-response points were 62 V and 80 V, respectively. Figures 3.9(c) and 3.9(d) show the friction effect on the open-circuit P-P voltages of the two EMG units with different input signals. The output results of EMG1 indicate that the presence of and maximum response frequency. The general trend for EMG2 was similar to that for EMG1, and the observed frictional negative effect of EMG2 was significantly higher than that of EMG1. The output of EMG2 experienced a notable decrease at 7 Hz.

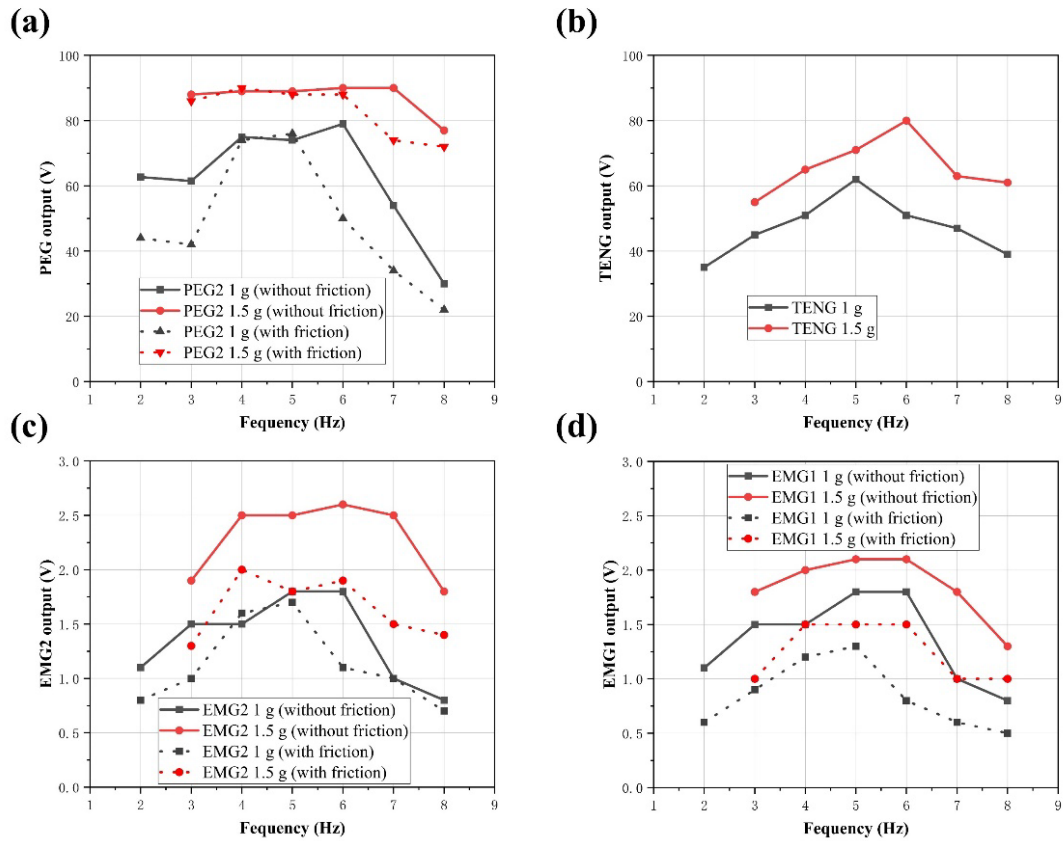


Fig. 3.9: Open-circuit outputs of different units at excitation accelerations of 0.5 g, 1 g, and 1.5 g: (a) PEG2 output (with and without friction effect); (b) TENG output; (c) EMG2 output (with and without friction effect); and (d) EMG1 output (with and without friction effect).

Figure 3.10 shows the percentage of open circuit voltage output between PEG2, EMG1, and EMG2 units (with and without friction effect). The results clearly demonstrated that friction can generally reduce the voltage output. EMG1 exhibited the most significant disparity influenced by friction, and all output units under both excitations were more susceptible to its effect. In the analysis, the effect of friction was pronounced at 6 Hz and 7 Hz under 1 g and 1.5 g, respectively.

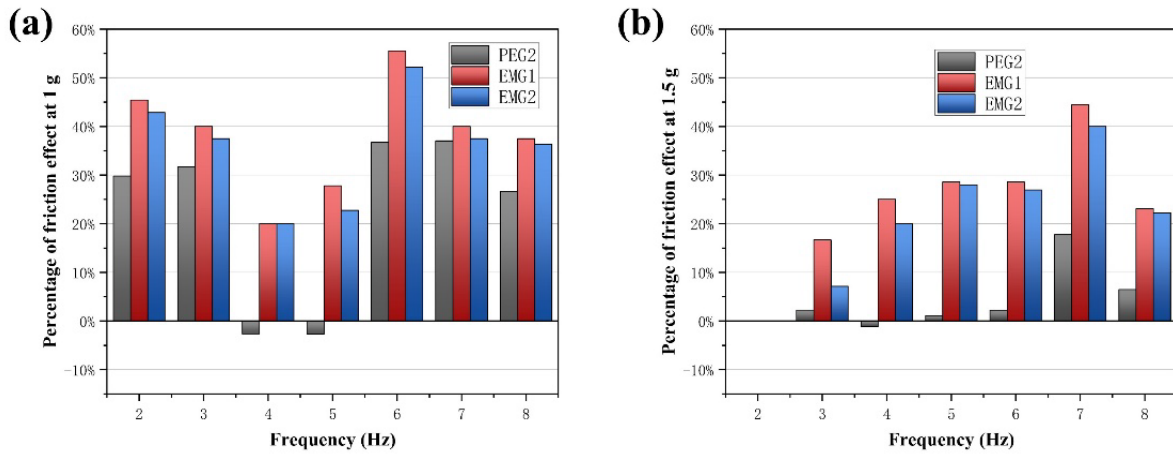


Fig. 3.10: Percentage of open circuit voltage outputs of PEG2, EMG1, and EMG2 units (with and without friction effect). (Note: Percentage = (Voltage output (without friction) – Voltage output (with friction))/ Voltage output (without friction)×100%).

To investigate the output performance of the proposed model, the maximum output powers of the TENG, EMG, and PEG units were measured at 5 Hz under 1 g acceleration as shown in Fig. 3.11. Under the influence of friction, the maximum power for the PEG unit was 48.4 mW across an optimal impedance of ~80 kΩ. The maximum power output of the PEG unit was 52 mW (without friction), representing an increase of 7.4% compared to PEG output with friction. For the EMG unit, we rectified and connected EMG1 and EMG2 units in series to obtain the overall output of EMG. At an impedance of 220 Ω, the maximum EMG output power was 5.87 mW. For the TENG unit, due to its higher internal resistance, the measured optimal matching impedance was 21 MΩ with a maximum power of 51.23 μW. Regardless of the presence of friction, the EMG output remained constant at 5.85 mW at 270 Ω, implying that friction has a negligible impact on EMG results. The peak power output of the hybrid model (with friction) was 54.26 mW, while the hybrid model (without friction) was 58.75 mW.

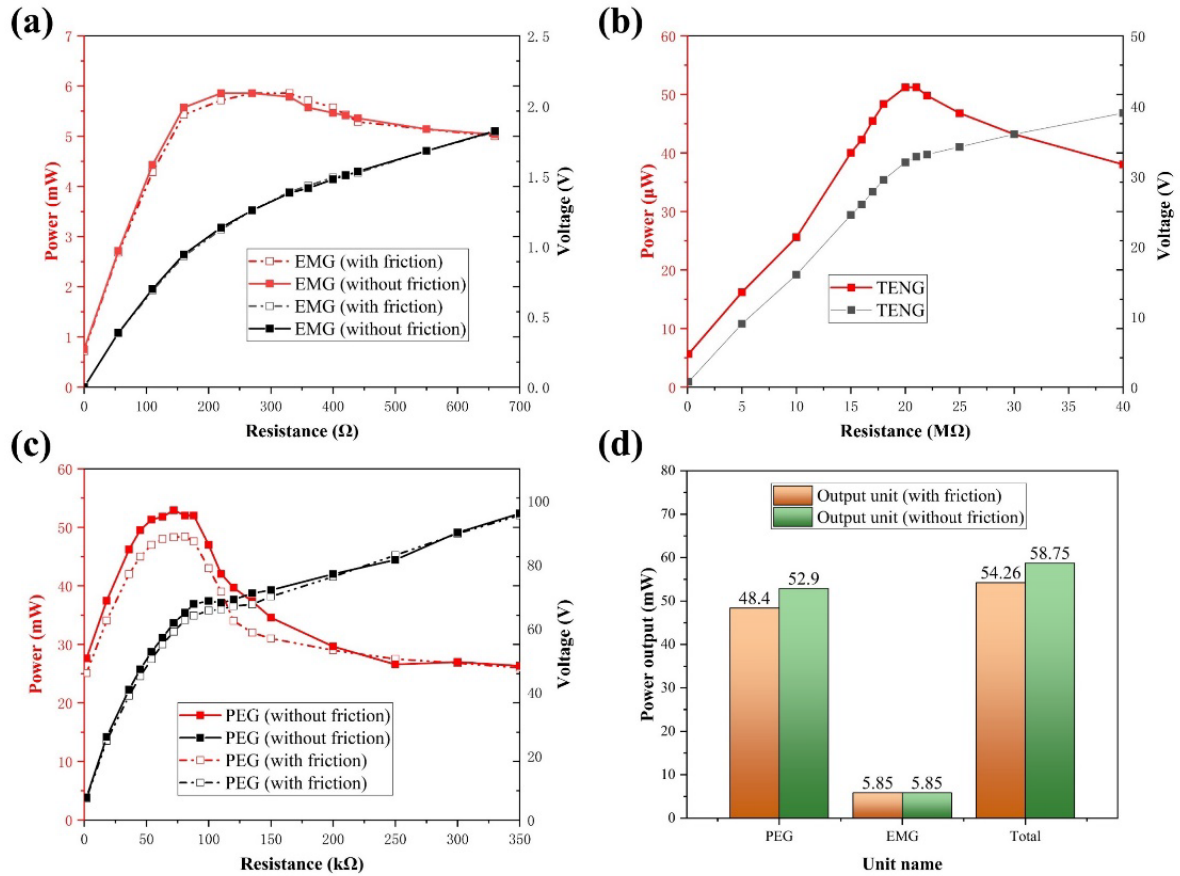


Fig. 3.11: Performance comparison for (a) EMG unit; (b) TENG unit; (c) PEG unit; and (d) EMG, PEG and hybrid all units' output differences. (All results were measured at 1 g and 5 Hz).

According to the simulation results, it can be observed that the movement of the tip magnet is hindered by frictional force under various excitation signals and coefficients of friction. Experimental findings indicate that frictional force significantly affects the open-circuit output. The output of the designed TENG unit demonstrates notable variations in different experimental environments. To enhance the TENG output, control measures such as temperature regulation, pre-charging techniques, and nano-scale surface treatment of materials can be implemented. This experiment aims to investigate the impact of friction on dynamic motion states; hence, complex friction structures were not employed in this study.

3.4 Concluding Remarks

A tri-hybrid vibration-based energy harvesting model featuring a triboelectric unit operating in sliding mode has been investigated in this chapter (Wang et al., 2024b). We developed a model for the mechanical vibrations of a cantilever beam with a tip mass, subjected to magnetic and frictional forces under base excitation. In addition, we analyzed the open-circuit peak-to-peak voltages of the sliding-mode tri-stable energy harvester. Moreover, the analysis of dynamic simulations was conducted for different values of μ (0.2–0.5) and initial phases $y_initial$ (4 mm and 20 mm). The following findings can be identified from the investigation of friction on the multi-stable dynamical system.

- In the absence of frictional influence, the EMG unit exhibited a maximum output power of 5.85 mW under 1 g and 5 Hz excitation, while the PEG unit demonstrated a maximum output power of 52.9 mW. In the presence of frictional effect, the measured output powers of EMG, PEG, and TENG were 5.85 mW, 48.4 mW, and 51.23 μ W, respectively.
- Both simulation and experimental results indicated that, under most excitations of the cantilever beam structure, friction adversely affected the outputs of the PEG unit. At low excitation levels (e.g., 0.5 g acceleration), the frictional force within the structure completely hindered output.
- However, friction is not always detrimental. At higher excitation accelerations, (e.g., 1.5 g compared to 1 g), the impact of friction was relatively minor. The TENG unit achieved a maximum open circuit voltage output of 80 V and a load power output of 51.23 μ W. Friction also caused a shift in the distribution of multi-stability, resulting in decreased inter-well motion and increased intra-well motion.
- Furthermore, friction reduced both the amplitude of the tip magnet and the frequency at which the structure reached its maximum response. The dynamic characteristics of the system can be significantly affected by the magnitude of the friction coefficient.

CHAPTER 4

- IMPACT OF AUXETIC STRUCTURES ON TRI-HYBRID VIBRATION-BASED ENERGY HARVESTING DESIGN

4.1 Introduction

The integration of auxetic structures into vibration-based energy harvesting systems, particularly in piezoelectric components, has attracted considerable attention due to their unique mechanical properties and potential to enhance device performance. Auxetic materials, characterized by a negative Poisson's ratio, expand laterally when stretched and contract laterally when compressed, unlike conventional materials. This distinctive behavior can result in improved strain distribution and enhanced energy absorption capabilities within energy harvesting devices. In the context of tri-hybrid energy harvesters that combine piezoelectric, electromagnetic, and triboelectric mechanisms, the incorporation of auxetic structures offers several advantages. For example, auxetic substrates can improve the mechanical flexibility of piezoelectric layers, promoting more uniform strain distribution and potentially increasing energy conversion efficiency. Furthermore, the unique deformation characteristics of auxetic materials can broaden operational bandwidths, enabling the harvester to efficiently capture energy across a wider range of vibrational frequencies.

Moreover, integrating auxetic structures facilitates the miniaturization of energy harvesting devices without sacrificing performance. By optimizing the design of auxetic components, compact configurations can be achieved while maintaining or even enhancing energy output, which is critical for applications in constrained environments such as wearable electronics and small-scale sensors. However, successfully implementing auxetic structures in tri-hybrid energy harvesters necessitates meticulous consideration of design parameters and material properties. Factors including the geometry of the auxetic pattern, the type of piezoelectric material employed, and the overall structural configuration must be carefully optimized to ensure effective integration and performance enhancement. In addition, the manufacturing processes for creating auxetic structures, particularly at small scales, can be

intricate and may pose significant challenges regarding cost-effectiveness and scalability. In short, integrating auxetic structures into tri-hybrid energy harvesters offers a promising approach to enhance performance, broaden operational bandwidth, and facilitate miniaturization.

Unlike conventional materials, which generally contract laterally under tension and expand under compression, auxetic materials display the opposite behavior, expanding laterally when stretched and contracting when compressed (Cho et al., 2019; Ferguson et al., 2018). Previous research has investigated the incorporation of auxetic materials with a negative Poisson's ratio, which can expand laterally when stretched, into energy harvesting devices (Chen et al., 2021a; Gao et al., 2023; Mazloomi et al., 2018; Zhang et al., 2024). Auxetic materials enhance the mechanical compliance and energy absorption capacity of harvesters, resulting in improved efficiency and robustness. Gao et al. (2022) proposed an analytical framework to predict the in-plane mechanical properties of auxetic structures, utilizing a modified analytical model that simplifies boundary conditions and yields accurate results compared to numerical simulations. When paired with piezoelectric materials, auxetic structures can further boost energy output by enhancing mechanical deformation under vibrational loads. Experimental findings have validated the advantages of auxetic designs in energy harvesters, demonstrating significant improvements in both power output and frequency bandwidth compared to traditional designs (Tabak et al., 2024). Ren et al. (2018) further reviewed the applications of auxetic metamaterials in energy harvesting, emphasizing their potential to enhance the performance of piezoelectric harvesters. Their study showed that auxetic structures could improve the mechanical properties of harvesters, leading to higher energy conversion efficiency and increased durability. Another study corroborated these findings through finite element method-based simulations, indicating that integrating auxetic structures into piezoelectric energy harvesters resulted in substantial enhancements in power output and operational bandwidth (Tikariha et al., 2022).

This work is novel in its approach to improving the integration of auxetic lattices with hybrid energy harvesting systems by confronting two aspects in low-frequency vibration energy harvesting, i.e., deformation amplification and frequency adaptability. While prior studies have emphasized the benefits of auxetic structures with single-plate piezoelectric

materials, this research broadens the scope by systematically analyzing their behavior under low-frequency dynamic loads and optimizing their interaction with piezoelectric components. The proposed approach exploits on the unique deformation characteristics of auxetics to amplify strain transfer, leading to improved energy output and operational efficiency. Furthermore, this research offers new insights into the design of hybrid systems, investigating how auxetic structures can be fine-tuned to achieve enhanced energy conversion and a wider frequency bandwidth. Previous research has highlighted the advantages of auxetic lattices over non-auxetic configurations, particularly in their coupling with energy harvesting mechanisms. In this study, the integration of auxetic structures with piezoelectric materials, hybrid harvesting mechanisms, and multi-stable designs provides a comprehensive solution to the challenges associated with low-frequency vibration energy harvesting. This approach enhances energy density by substituting restoring force springs with foldable piezoelectric beams and incorporating an auxetic structure on each beam to increase deformation. This study compares the performance of both conventional newly designed folded beam auxetic structures with piezoelectric materials. The experimental results for open-circuit voltage output and power output are presented. For small-scale energy harvesting prototypes, enhancing the output efficiency of the device poses a significant challenge due to space constraints. This research introduces a novel approach that utilizes auxetic structures to further improve the output efficiency of piezoelectric units.

4.2 Simulation analysis of auxetic structures

To illustrate the benefits of auxetic structures compared to conventional piezoelectric beams, we conduct a thorough study that combined experimental data with simulation analysis. Using traditional materials, a positive Poisson's ratio results in lateral contraction when subjected to longitudinal tensile stress and lateral expansion under compressive stress. In contrast, auxetic materials, which have a negative Poisson's ratio, display the opposite behavior; they expand laterally under longitudinal tensile stress. When incorporated into a piezoelectric energy harvester, this characteristic enables the piezoelectric material to experience greater displacement for the same applied force, significantly enhancing the strain distribution within the piezoelectric material and increasing energy output. The unique deformation behavior of the auxetic structure leads to a more uniform strain distribution, which in turn improves the overall energy conversion efficiency by maximizing the strain that the piezoelectric material experiences under mechanical excitation.

Numerous studies have been conducted on the properties of auxetic structures, yielding valuable insights for design optimization. Before developing a prototype in this research, it is crucial to identify the most appropriate auxetic structure. Referring to previous studies (Chen et al., 2022a, 2022b; Li et al., 2022), three distinct auxetic designs and their corresponding displacements under identical stress conditions are investigated. The structures exhibiting the most distinct geometries are selected for the assessment of auxetic properties. By applying a uniform load of 25,000 Pa to the right end of three auxetic beams with identical dimensions, while constraining the left end, the deformation displacement results are obtained. COMSOL simulations are used to evaluate three auxetic designs: (a) a cube structure; (b) an arrow structure; and (c) a re-entrant structure, as presented in Fig. 4.1. In these structures, the auxetic design (c) exhibits a higher peak displacement (maximum displacement: 2.58 mm) when comparing to the designs (a) (maximum displacement: 2.16 mm) and (b) (maximum displacement: 2.24 mm). Furthermore, considering the dimensions of the prototype and the precision of material processing, structure (c) is ultimately chosen. The dimensions of the re-entrant structure (c) are illustrated in Fig. 4.2.

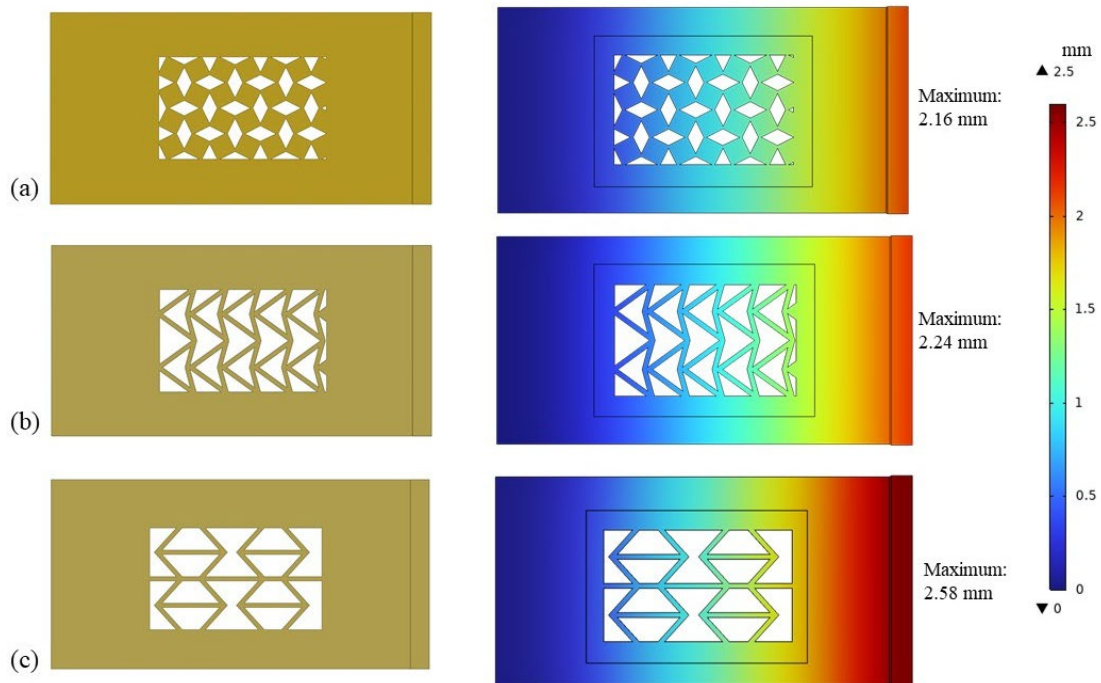


Fig. 4.1: Displacement analysis of three types of auxetic structures under a uniform load: (a) a cube structure; (b) an arrow structure; and (c) a re-entrant structure.

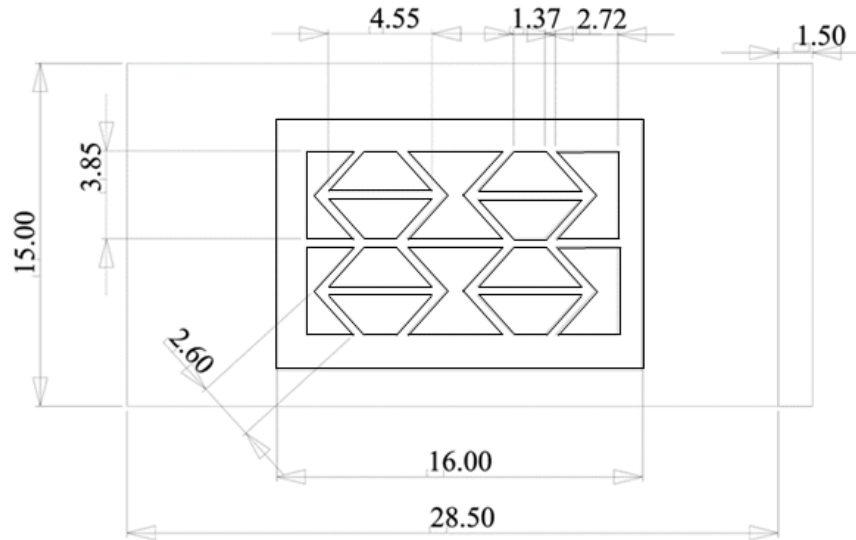


Fig. 4.2: Dimensions of the re-entrant auxetic structure.

Figure 4.3(a) illustrates a schematic of the auxetic beam and its integration with a PZT plate. Figures 4.3(b) and (c) present the COMSOL simulation results for stress and displacement in both auxetic and conventional piezoelectric beams of same dimensions. The left end is fixed, while a vertical downward load is applied to the upper surface of the right end, which is equipped with a copper block. All other regions are free, with an initial displacement set to zero. The analysis employs a finely refined mesh and incorporates coupling between the piezoelectric and mechanical fields. In these simulations, red indicates areas of higher displacement or stress, while blue denotes regions with lower values. The samples consist of 0.2 mm thick copper sheets with PZT-5A material attached to the upper end. When a force of 25,000 Pa is applied at the edge area, the simulation results in Fig. 4.3(b) reveal the stress distribution across various positions. While the overall stress distribution is rather similar for both beams, there are significant differences in the stress experienced by the piezoelectric plates on the surface. In addition, the displacement results indicate that the auxetic structure achieves a maximum displacement of 1.67 mm at the tip, compared to 1.05 mm for the conventional structure. The maximum stress and displacement values of auxetic and conventional beams are presented in Table 4.1. In the case without a PZT plate, the auxetic beam exhibits higher stress and displacement. However, when a PZT plate is present, the maximum stress in both the auxetic and conventional beams is comparable, while the maximum displacement in the auxetic case increases 27.4%. These findings highlight the

superior deformation capabilities of the auxetic structure, which enhances its performance in energy harvesting applications.

In this study, we examine different sample sizes to find that piezoelectric materials are less prone to damage when a covering extension of approximately 1.5–2 mm is maintained around their edges on the beam. Consider the working efficiency and mechanical strength performance, the total area of two PZT plates occupies approximately 80 % of the beam area (on one side). This arrangement facilitates repeated testing without significant damage. The maximum strain in a cantilever beam occurs near the fixed end. In a folded beam unit, the ends of the single-layer beam are constrained and can be considered simply supported. This study seeks to provide a clearer comparison of strain behavior between auxetic and conventional structures under identical loading conditions, concentrating exclusively on simulating cantilever beams. Ideally, to enhance the efficiency of the piezoelectric material, the beam would be completely covered. However, the fragility of piezoelectric ceramics makes it challenging to achieve full coverage for experimental testing. Consequently, the material is applied only to the center of the beam.

Table 4.1: Maximum stress and displacement of the auxetic and conventional beams.

Item	Maximum stress (Pa)	Maximum displacement (m)
Auxetic Cu beam	1.80×10^8	2.09×10^{-3}
Conventional Cu beam	1.08×10^8	1.74×10^{-3}
Auxetic Cu beam (with PZT plate)	2.25×10^7	1.44×10^{-3}
Conventional Cu beam (with PZT plate)	2.14×10^7	1.13×10^{-3}

The simulation and experimental results of the auxetic and conventional beams with a PZT plate are presented in Fig. 4.4. When a 40 kΩ resistance load is connected, the auxetic one generates a maximum power of 28.7 mW, while the conventional beam produces 18.5 mW under the same load. The output of the auxetic beam is 1.55 times greater than that of the conventional beam. This combined approach of experimental validation and simulation analysis provides strong evidence supporting the improved performance of auxetic structures in piezoelectric energy harvesting applications.

4.3 Prototype design

4.3.1 Theoretical design of the hybrid energy harvester

The present design features a hybrid energy harvester that includes a folded auxetic beam piezoelectric generator (PEG), an electromagnetic generator (EMG), a sliding mode triboelectric nanogenerator (TENG_S), and a contact-separate mode triboelectric nanogenerator (TENG_CS), as shown in Figs. 4.5 and 4.6. The EMG unit includes two fixed small magnets (B and C) and a moving central magnet (A) (see Fig. 4.6(b)), with six copper coils positioned on either side of the central moving magnet. Two folded beam springs, each with a stiffness of k , are located at both ends of magnet A along the x -axis (see Fig. 4.5(e)). The distance from the center of each fixed magnet to the coordinate origin is represented as d . The geometrical dimensions of the proposed design are outlined in Table 4.2.

Figures 4.6(a)–(d) depict the detailed composition of the three energy harvesting units. Piezoelectric materials are integrated with an auxetic copper beam structure, with piezoelectric materials attached to both sides of the beam. A rectangular aluminum block is secured at one end of the beam, and screw holes are drilled into the copper block to allow for the connection of each copper beam with screws, forming the PEG unit. Subsequently, copper coils and external magnets are arranged on either side of the central magnet to assemble the EMG unit. Two sets of copper electrodes are alternately positioned beneath the PTFE film. In addition, two parallel copper (Cu) films, acting as triboelectric materials, are affixed to the top and bottom of the central magnet to facilitate triboelectric motion, creating the TENG_S unit. For the TENG_CS unit, the PTFE film is attached to the left and right ends of the central magnet as the triboelectric material, while another electrode, a circular beryllium copper plate, is connected to both ends of the spring. Figure 4.6(e) shows the electronic circuit diagram for the whole device.

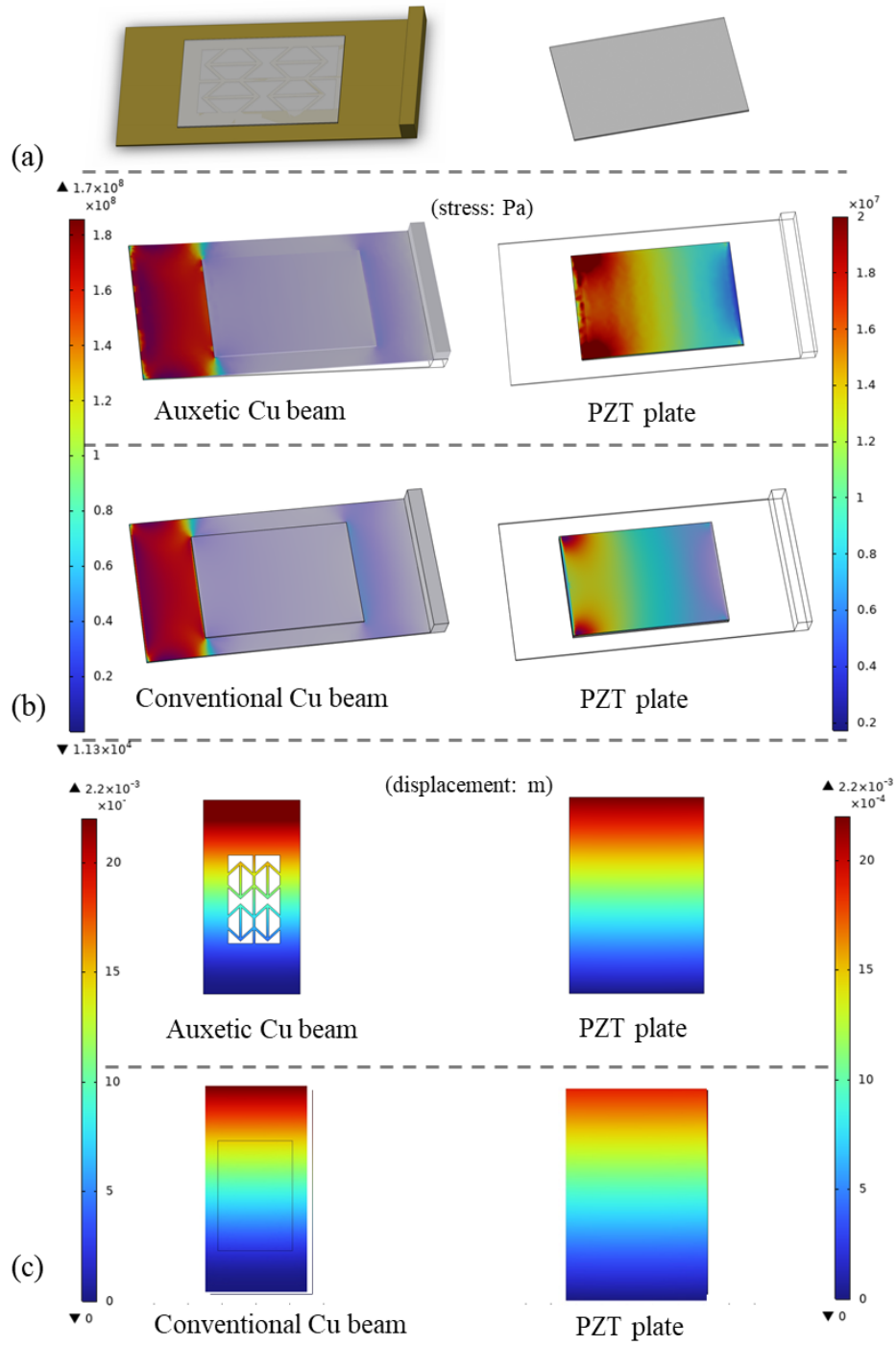


Fig. 4.3: Analysis of stress and displacement in the auxetic structure: (a) schematic diagrams of the auxetic beam with a PZT plate; (b) the stress performance of the auxetic beam and a conventional beam; and (c) the displacement of the auxetic beam and a conventional beam.

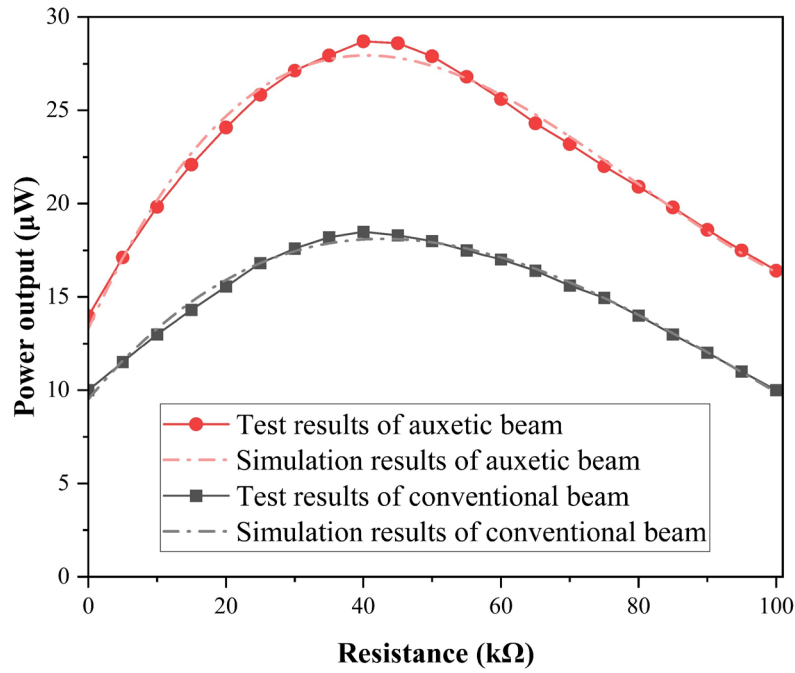


Fig. 4.4: Simulation and test results of auxetic and conventional beams with PZT.

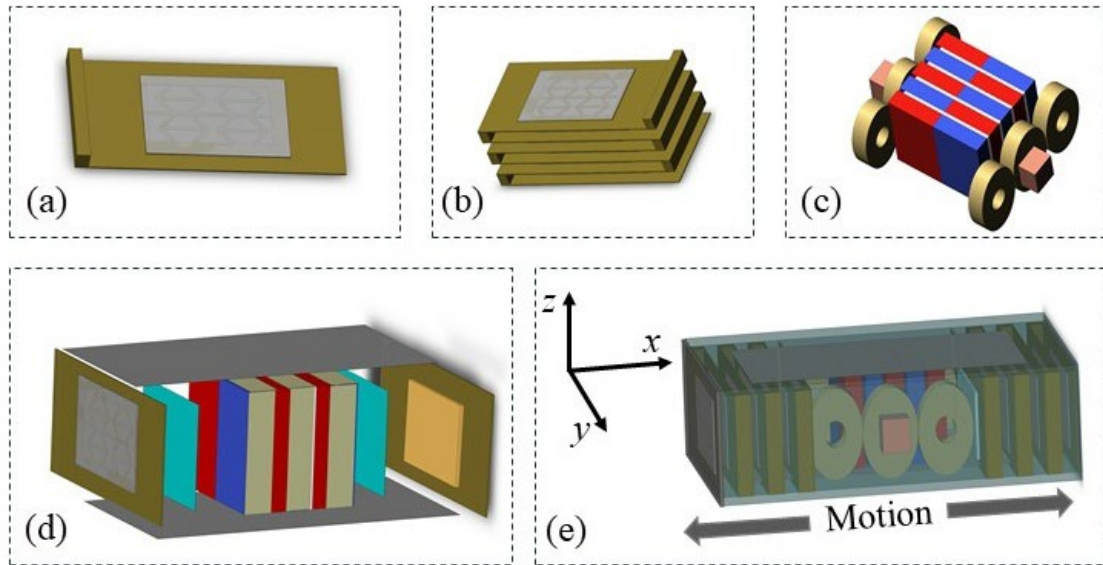


Fig. 4.5: Schematic representations of the proposed design include: (a) an auxetic Cu beam with a piezoelectric plate (PZT-5A); (b) a folded beam structure with PZT; (c) magnets and coils; (d) triboelectric materials; and (e) a fully assembled device.

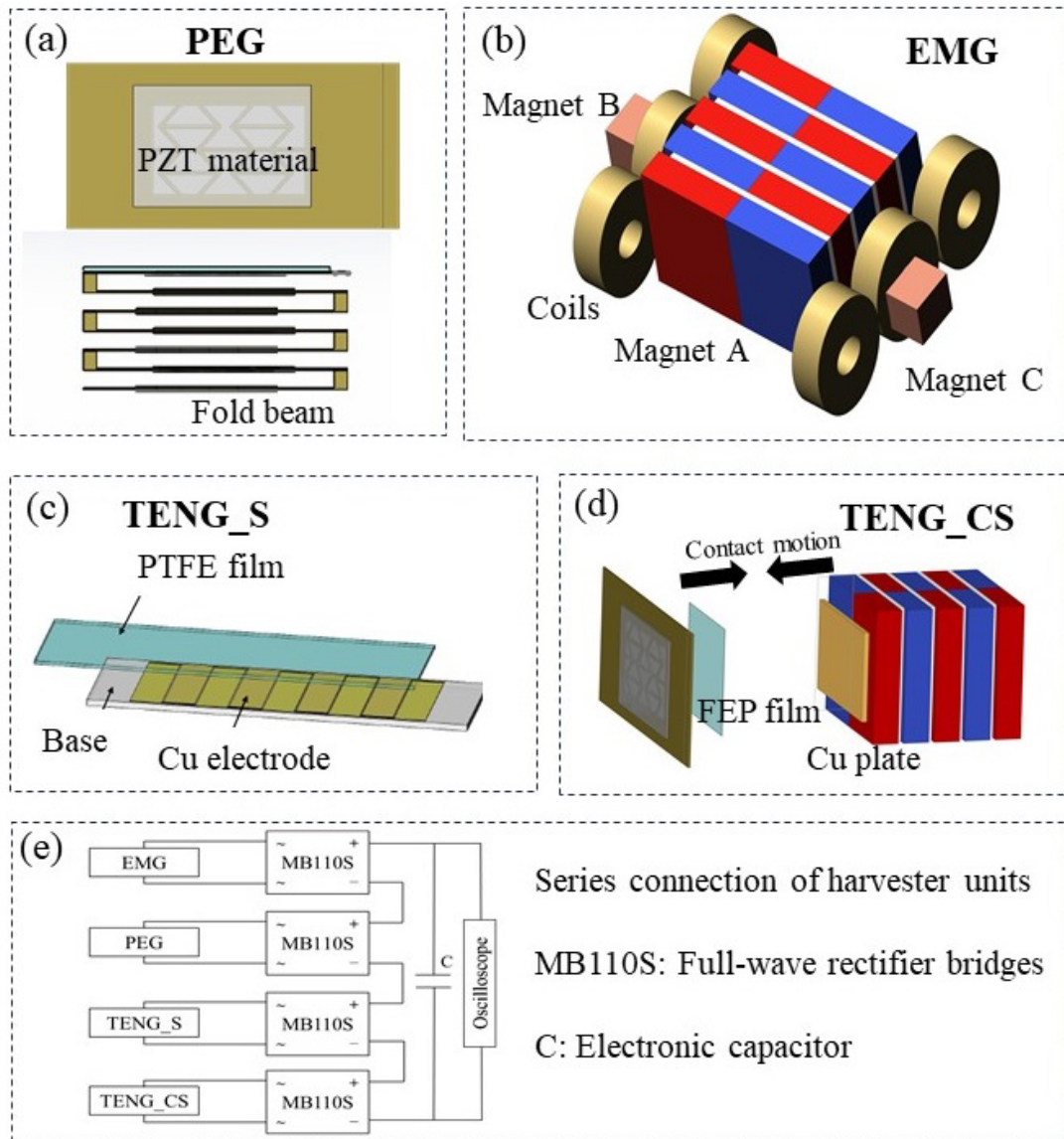


Fig. 4.6: Various energy harvesting units: (a) PEG unit; (b) EMG unit; (c) TENG_S unit; (d) TENG_CS unit; and (e) the electronic circuit design of the whole system.

Table 4.2: Geometric information of the present model.

Item	Parameters	Quantities
Cantilever beam	Length $\times$ Width $\times$ Thickness	$15 \times 30 \times 0.25 \text{ mm}^3$
PZT-5A plate	Length $\times$ Width $\times$ Thickness	$11 \times 16 \times 0.27 \text{ mm}^3$
Magnets	Magnet array mass	30.5 g
	Dimensions of Magnet A (magnet array)	$22 \times 21 \times 10 \text{ mm}^3$
	Dimensions of Magnet B and Magnet C	$4 \times 3 \times 1.5 \text{ mm}^3$
	Length $\times$ Width $\times$ Thickness	$12 \times 16.5 \times 3 \text{ mm}^3$
Copper coils	Number of turns	800
TENG unit	PTFE: Length $\times$ Width $\times$ Thickness	$20 \times 12 \times 0.35 \text{ mm}^3$
	Copper film thickness	0.02 mm

An equivalent mechanical system is introduced for dynamic analysis to demonstrate the working principle of the present model. When the system experiences base excitation $\ddot{x}_b$, the central magnet with mass m shifts in relation to the magnetic coil, compressing the folded beam spring. The stiffness k of this custom-designed folded beam is measured using a force gauge. The equations of motion for this mechanical system in both horizontal and vertical configurations are presented as follows (Wang et al., 2023b; Yang et al., 2022) :

$$m\ddot{x} + (c_v + c_e + c_i)\dot{x} + k_e(|x| - d_s) + F_m + F_d = -m\ddot{x}_b \quad (4.1)$$

$$m\ddot{x} + (c_v + c_i + c_e)\dot{x} + F_m + F_d + k_e x = -m\ddot{x}_b \quad (4.2)$$

It is noted that the feedback effect of the piezoelectric transducer on the dynamic system is negligible and can thus be disregarded in the equations. In the equations provided above, F_m represents the magnetic force on the center magnetic slider, and F_d denotes the dry friction force on the slider, which can be described by the Coulomb friction force F_d as follows:

$$F_d = \frac{\dot{x}}{|\dot{x}|} \mu F_l \quad (4.3)$$

where μ is the dry friction coefficient, F_l is the lateral pressure exerted by the contact surface on the slider, c_v is the linear viscous damping coefficient, c_i is the additional damping coefficient due to the impact effect, and c_e is the electrical damping coefficient due to the relative displacement between the magnetic mass and the movable coil group (Toyabur et al., 2020):

$$c_e = \left(\frac{d\varphi}{dx} \right)^2 \left(\frac{1}{R_C + R_L} \right) \quad (4.4)$$

In Eq. (4), the total magnetic flux through the coils is φ . The rate of change of magnetic flux with respect to the relative displacement is thus $d\varphi/dx$. The resistances of the coils and the external load are R_C and R_L , respectively. The effective stiffness (k_e) during the impact and non-impact modes can be analyzed in a piecewise manner (Toyabur et al., 2020):

$$k_e = \begin{cases} -k, & x < -d \\ 0, & -d \leq x \leq d \\ k, & x > d \end{cases} \quad (4.5)$$

where k and d are the spring stiffness and the separation distance, respectively.

The magnetic force F_m exerted on the magnetic slider and the elastic force of the fan-folded piezoelectric spring were individually measured at different displacements using a dynamometer, resulting in a folded beam stiffness $k = 1334$ N/m. Figure 4.7(a) illustrates how the restoring force varies with the displacement of the magnetic slider, depicting the restoring force curve of the energy harvester. By integrating the derived expression, the potential energy curve can be obtained, as shown in Fig. 4.7(b). This indicates that the energy of the harvester remains below 0.15 mJ in the horizontal direction (quad-stable mode) and below 0.4 mJ in the vertical direction (tri-stable mode), within a displacement range of -6 to 6 mm. A comparison shows that as gravitational force increases, the system shifts from a quad-stable state to a tri-stable state, leading to a deeper potential well. This suggests that more energy is required for interwell motion in the vertical direction. The nonlinear behavior observed in the hybrid system with auxetic structures underscores the deformation amplification mechanism under low-frequency excitations. This behavior enhances the coupling between mechanical deformation and the conversion of piezoelectric, electromagnetic, and triboelectric energy, thereby improving the overall performance of the hybrid system.

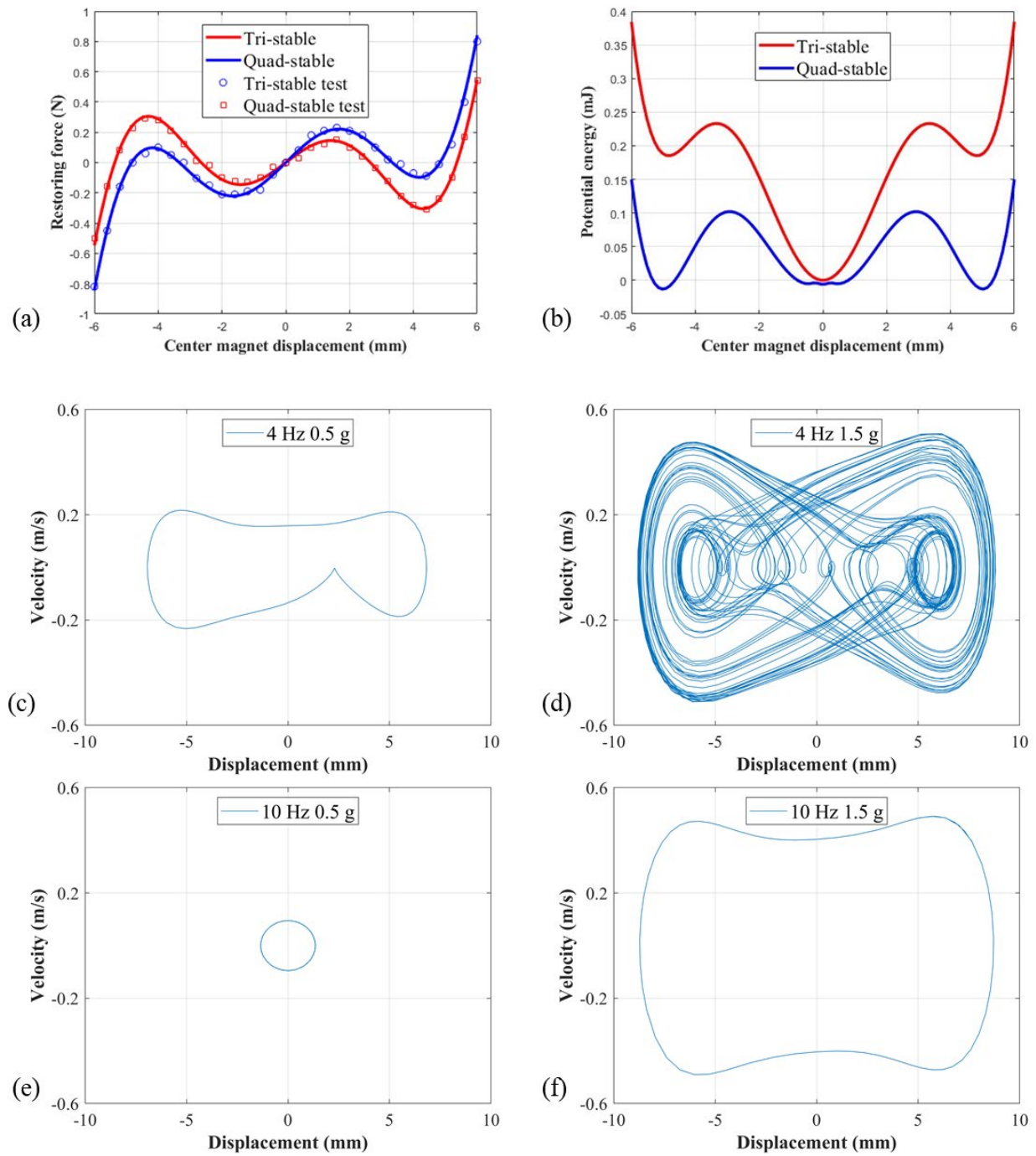


Fig. 4.7: (a) Results of force measurements; (b) potential energy of the harvester in two directions with two state conditions; (c) and (d) phase portraits of the harvester under excitations of 4 Hz at 0.5g and 1.5g, respectively; (e) and (f) phase portraits of the hybrid energy harvester under excitations of 10 Hz at 0.5 g and 1.5 g, respectively.

Furthermore, it is important to note that the central magnet is more conducive to oscillation at lower excitation levels in the horizontal direction, facilitating the generation of electrical signals. To examine its dynamic characteristics, simulations were performed under excitations at low frequencies of 4 Hz and slightly higher frequencies of 10 Hz, using two acceleration amplitudes, 0.5 g and 1.5 g, as references. Figures 4.7(c)–(f) show the amplitude variations under different excitation levels. Both the initial displacement and velocity were set to zero. Under 4 Hz and 0.5 g excitation, the magnet matrix exhibits high-speed inter-well motions. In comparison, the 4 Hz 1.5 g excitation results in a more vigorous response, with more pronounced inter-well motion due to the larger acceleration and response amplitude. When comparing outputs at the same acceleration, the 10 Hz excitation demonstrates stronger nonlinearity than the 4 Hz output.

4.3.2 Prototype fabrication and experimental setup

This section presents experimental verification to validate the simulation and numerical results. A prototype was prepared and fabricated, as illustrated in Fig. 4.8. The prototype consisted of four components: (i) two PEG units affixed to either side of the device; (ii) an EMG featuring six coils, one moving magnet, and two fixed magnets; (iii) a TENG_CS unit positioned on either side of the moving magnet; and (iv) a TENG_S unit located on the upper and lower surfaces of the moving magnet for sliding friction. For a single PEG unit, five bimorph auxetic beams were connected end-to-end using aluminum alloy blocks. Each piezoelectric beam measured $15 \times 30 \times 0.25 \text{ mm}^3$ and was constructed from a phosphor bronze substrate with two piezoelectric plates (PZT-5A) sized at $11 \times 16 \times 0.27 \text{ mm}^3$.

The auxetic structures were prepared by metal fabrication. The coils in the EMG unit were secured to the base using 3D-printed acrylic slots and are insulated with tape to prevent short circuits. The central movable slider was made of beryllium bronze blocks and an array of magnets, with a 0.5 mm gap between each magnet. This configuration of the magnet array can produce a greater magnetic flux (Choi et al., 2008) compared to magnets of the same size, effectively enhancing the output of the EMG unit. In the TENG_S unit, copper tape (0.05 mm thick) was applied to both the upper and lower surfaces of the central moving magnet array, functioning as one electrode. An interdigitated electrode was then coated with a 50- μm thick PTFE film on the copper surface. In the TENG_CS unit, Fluorinated Ethylene Propylene (FEP)

films were used at the ends of the slider as the triboelectric material, while a beryllium bronze plate was affixed to the ends of the spring, serving as the other contact material.

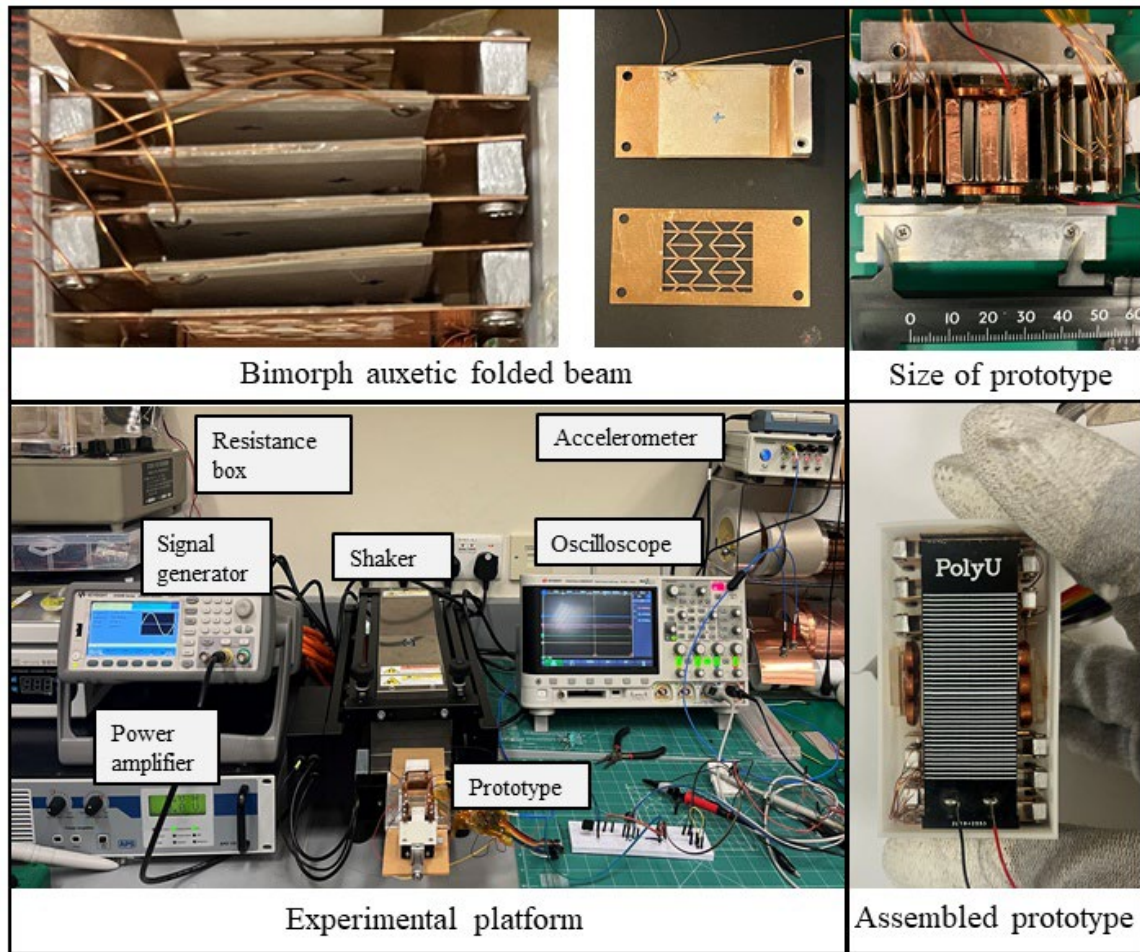


Fig. 4.8: Pictures of the experimental prototype and testing platform.

4.4 Results and discussion

4.4.1 Mechanical vibration and human motion tests

The dynamic behavior of the device was evaluated at amplitudes of 0.5 g, 1 g, and 1.5 g. The open-circuit voltage of each generator (measured in both horizontal and vertical directions) was analyzed via experimental studies across a frequency range of 3–10 Hz, as presented in Figs. 4.9 and 4.10. Figure 4.9 indicates that all generator units in the horizontal direction (quadrable) produced high output voltages within the 3–6 Hz frequency range under 0.5 g. However, when the excitation frequency increased to 7 Hz, nearly all harvesting units exhibited a declining output trend. Both PEG and EMG units showed strong output between 3–7 Hz, while

the TENG's output decreased at 7 Hz. In the prototype, as all harvesting units were powered by the identical mechanical motions, their open-circuit voltage levels displayed comparable broadband responses influenced by the chosen multi-stable nonlinearity mode. The output of each unit was maximized under 1.5 g excitation, with the peak output occurring at 6 or 7 Hz. Under this excitation, the open-circuit voltages for PEG, EMG, TENG_CS, and TENG_S were 64 V, 5.4 V, 68 V, and 45 V, respectively. Resonant frequency behavior was observed, with peak output occurring at approximately 5–6 Hz in the horizontal direction and 6–7 Hz in the vertical direction.

When the prototype functions in tri-stable mode (i.e., vertical orientation), the magnet array required more energy to achieve intra-well motion when comparing to the horizontal orientation. The PEG exhibited minimal performance variation across different orientations (Figs. 4.10(a)). In the vertical position, the effect of gravity impeded some of the motion, leading to reduced displacement and lower output. In addition, the TENG_CS unit struggled to make effective contact with the dielectric surface, resulting in decreased output. Under 1 g and 1.5 g excitations, there is a noticeable drop in overall output after 6 Hz (Figs. 4.10(a), (c), and (d)). This further illustrates that the quad-stable mode can offer a wider operating frequency range, facilitating more efficient inter-well motion. Since PEG units can be evaluated based on the peak voltage density per acceleration (V/g/cm^3), Table 4.3 presents the performance of the PEG's open circuit output in the vertical direction.

To evaluate the electrical performance of various individual units, the electrical characteristics (including open-circuit voltage, load current, and power output) under vibration excitation at 6 Hz with 1 g acceleration were measured, using the methods outlined in previous studies (Wang et al., 2021a; Wang et al., 2019). A single PEG unit achieved a peak open-circuit voltage of 50 V, as shown in Fig. 4.9(a). As load resistance increased, the PEG unit delivered a maximum power output of 26.50 mW at an optimal load resistance of 56 k Ω (Fig. 4.11(a)), resulting in a maximum current of 0.70 mA. The EMG unit exhibited a peak open-circuit voltage of 5.1 V due to the instantaneous change in velocity during impact, as illustrated in Fig. 4.9(b). The maximum power obtained at an optimal load resistance of 650 Ω was 33.97 mW (Fig. 4.11(b)), with a maximum current of 7.20 mA. In Figs. 4.9(c) and (d), the peak open-circuit voltage levels for the TENG_CS and TENG_S units were around 42 V and 33.5 V under

1 g at 6 Hz, respectively. Under an optimal load resistance of 30 M Ω , the TENG_S unit produced a maximum power of 14.10 μ W with a maximum load current of 0.69 μ A. The TENG_CS unit delivered a maximum power of 38.72 μ W with a maximum load current of 1.76 μ A under an optimal load resistance of 12.5 M Ω (Fig.4.11(c)). Of various units, the EMG unit produced a higher output current but had a comparatively lower load voltage because of its low internal resistance. The PEGs and TENGs delivered higher voltage outputs but lower currents across the load due to their higher internal resistance. The charge performance generated by each harvester and the entire device is shown in Fig. 4.11(d).

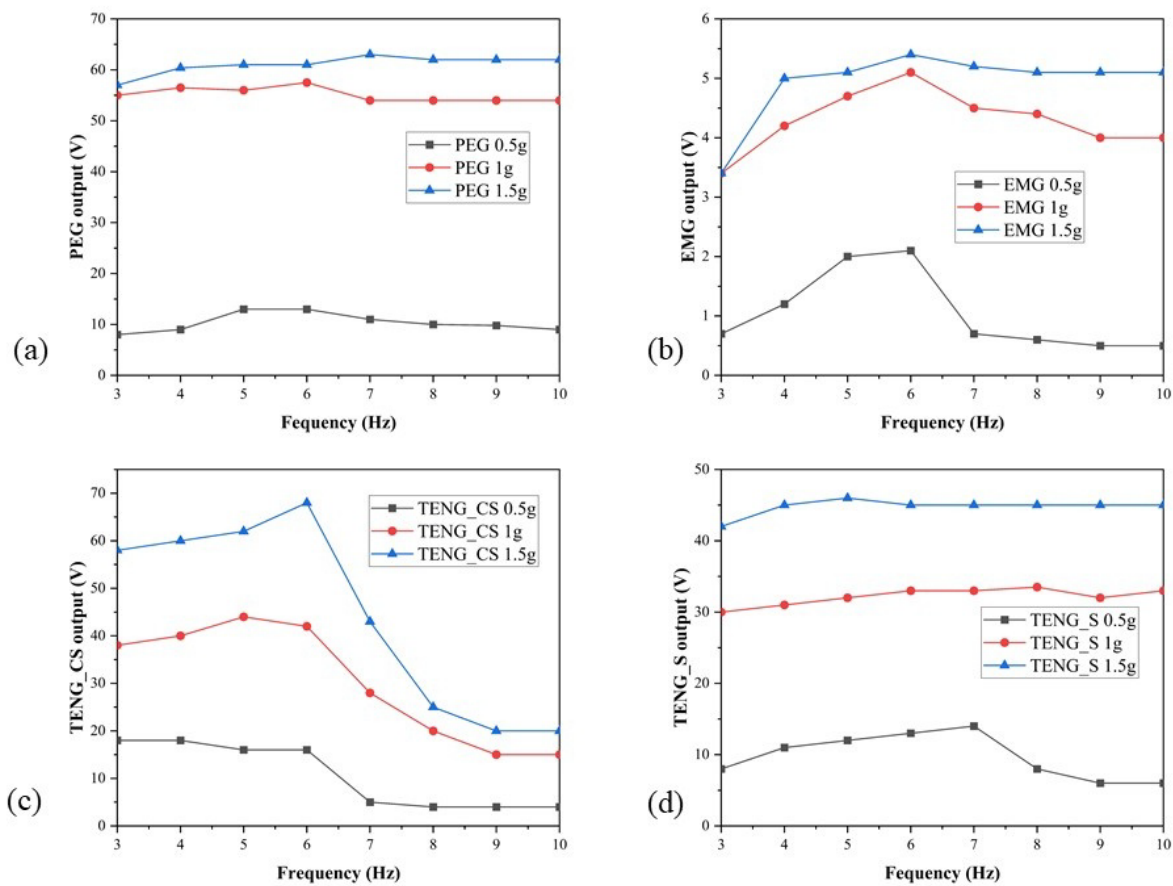


Fig. 4.9: Experimental results of open-circuit voltages for different units in the horizontal direction (quad-stable mode) at 0.5–1.5 g.

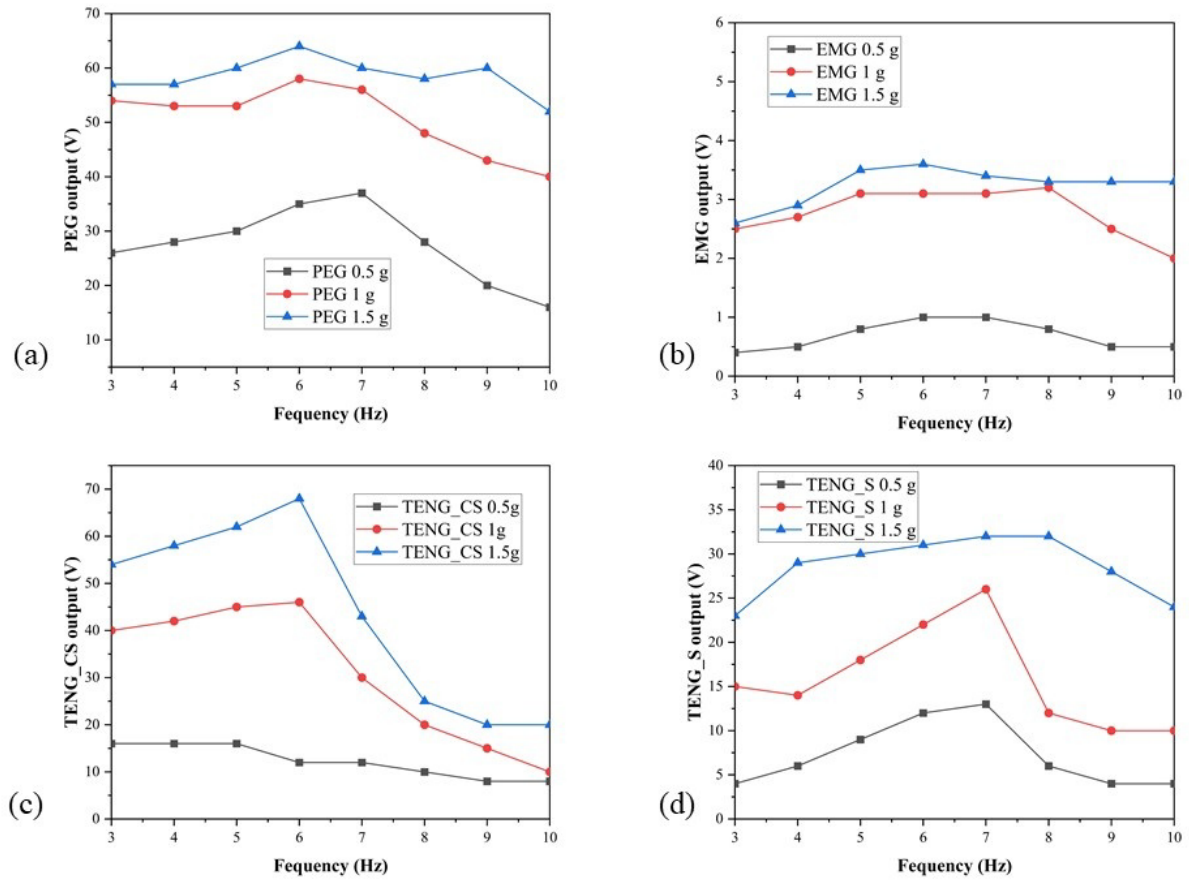


Fig. 4.10. Experimental results of open-circuit voltages for different units in the vertical direction (tri-stable mode) at 0.5–1.5 g.

Table 4.3. Peak voltage density per acceleration ($V/g/cm^3$) of PEG unit

Frequency	Acceleration		
	0.5 g	1 g	1.5 g
3 Hz	7.22	7.50	5.28
4 Hz	7.78	7.36	5.28
5 Hz	8.33	7.36	5.56
6 Hz	9.72	8.06	5.93
7 Hz	10.28	7.78	5.56
8 Hz	7.78	6.67	5.37
9 Hz	5.56	5.97	5.56
10 Hz	4.44	5.56	4.81

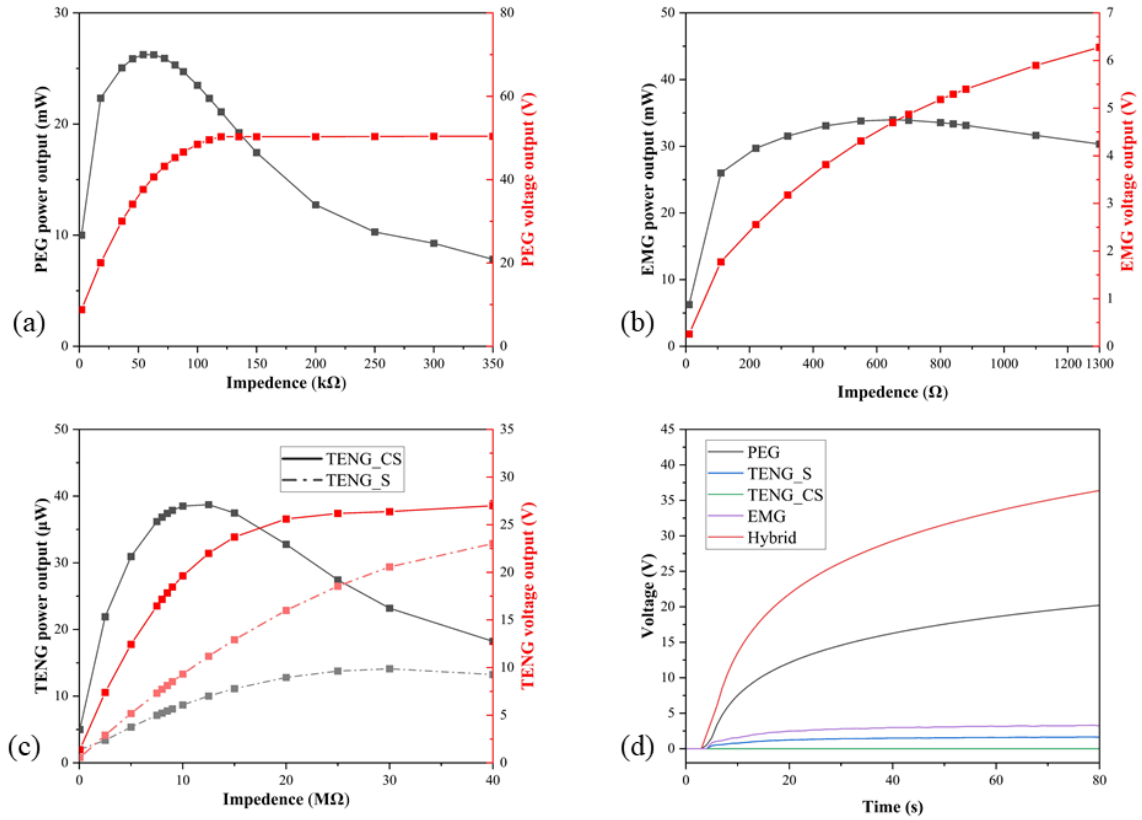


Fig. 4.11: (a)-(c): Electrical performance of various units against external load resistances; and (d) charging performance.

The EMG unit yielded a greater power output compared to the individual PEGs and TENGs, although it occupied more volume and weight in this prototype. Figure 4.11(d) presents the charging performance of each harvester unit. We observe that the hybrid device surpasses the performance of the individual units. Furthermore, the PEG unit demonstrated improved electrical performance compared to the others. The whole prototype can deliver a peak output power of 87.02 mW, resulting in a normalized power density of $3.56 \text{ mW cm}^{-3} \text{ g}^{-2}$ under 1 g at 5 Hz. Figure 4.12 illustrates the volume distribution of individual units. The normalized power density (in mW/cm^3) was also used to evaluate the performance of each unit. The PEG unit exhibited the highest performance density at 7.36 mW/cm^3 , followed by the EMG unit at 3.42 mW/cm^3 . The PEG unit achieved its high density due to the improvement of folded auxetic beam structure. The TENG_S and TENG_CS units comprised only 0.57% of the total volume, primarily due to the thin materials used in the TENG units. The normalized power density of the TENG_S unit was the lowest among all, primarily due to the low surface charge density of the unmodified PTFE. Recent studies have tackled this issue using various

approaches (e.g., Refs. (Yang et al., 2021a) and (Zhang et al., 2023)), including contact enhancement, surface modification, and material selection, which can enhance charge density and boost output power.

As illustrated in Fig. 4.12, the power-to-volume data indicates that PEG contributed 68.26% of the total output, whereas EMG made up 31.72%. In Fig. 4.11, under an excitation of 1 g and 5 Hz, PEG accounted for 60.9% of the overall power output, while EMG contributed 37.04%. It is evident that PEG contributed the largest portion of the total output for the prototype. Optimizing the component with the highest output share and improving the energy harvesting efficiency of PEG will have the most substantial effect on the overall system. Although the TENG units generated significantly lower power output, it is important to highlight that they can produce a high voltage level (see Fig. 4.11(c)), and their low friction coefficient along with high wear resistance can effectively safeguard the coils (Wang et al., 2023d).

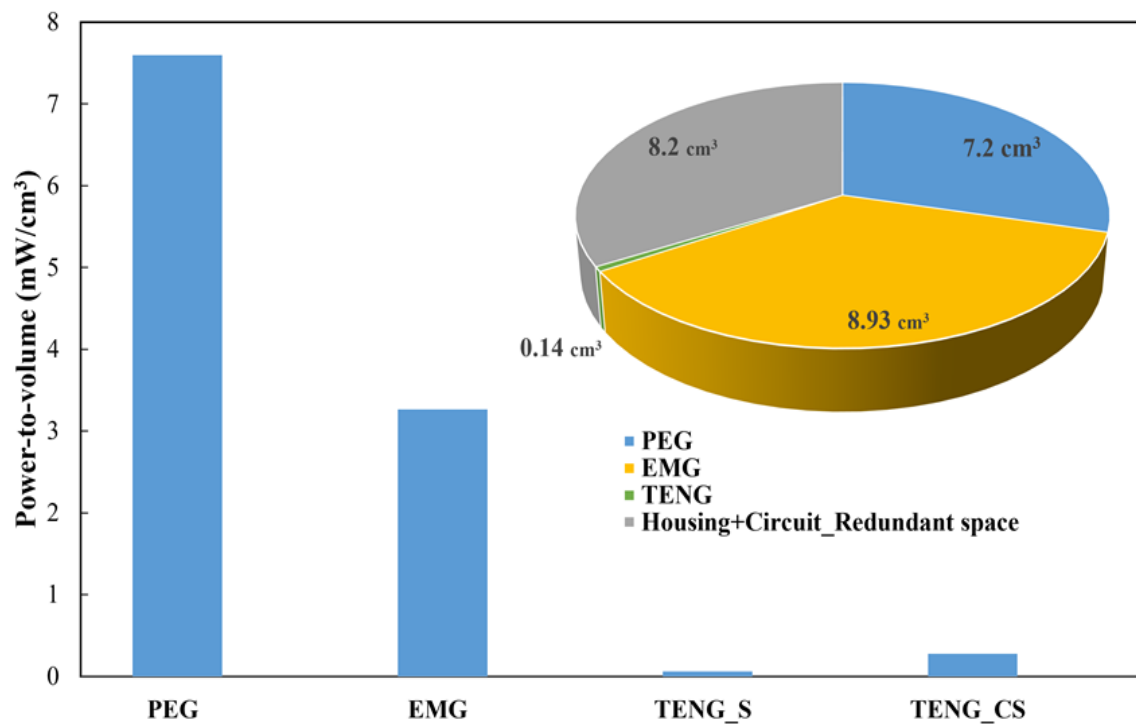


Fig. 4.12: Power-to-volume ratio of different harvester units.

4.4.2 Practical application to electronic devices

To assess the feasibility of self-powering in this prototype, the ability to generate direct current (DC) for powering LED bulbs was tested. Vibrational signals produced by handshaking motions were converted from alternating current (AC) to DC using a rectifier bridge. The rectified power was subsequently stored in a capacitor and utilized to power the LED bulbs. The prototype successfully powered 110 costumed LED bulbs simultaneously, as shown in Fig. 4.13. In addition, this energy harvesting prototype can power a wireless Bluetooth sensor capable of measuring displacement, angle, and temperature. By removing the built-in battery of the sensor and connecting its circuit to the energy harvesting prototype under 6 Hz and 1.5 g, real-time data from the sensor can be displayed on a smartphone, as shown in Fig. 4.14.

Table 4.4 presents a performance comparison between the present design and other hybrid energy harvesting devices (He et al., 2018; Rahman et al., 2020b; Wang et al., 2019; Yang et al., 2022). The current device exhibits good performance in terms of operational bandwidth and normalized power density at low frequencies. However, it also has notable limitations that require further investigation. Specifically, the multi-stable nonlinear mechanism shows potential for biomechanical energy harvesting, providing an extended and efficient operational bandwidth that aligns well with the frequencies of human motion vibrations. It is important to note that the energy harvesting prototype discussed in this work can also be further optimized. One issue is that the piezoelectric materials do not completely cover the copper beam, it may result in suboptimal efficiency. Given the inherent fragility of piezoelectric ceramics, a more precise design approach during the machining process could help enhance working efficiency.

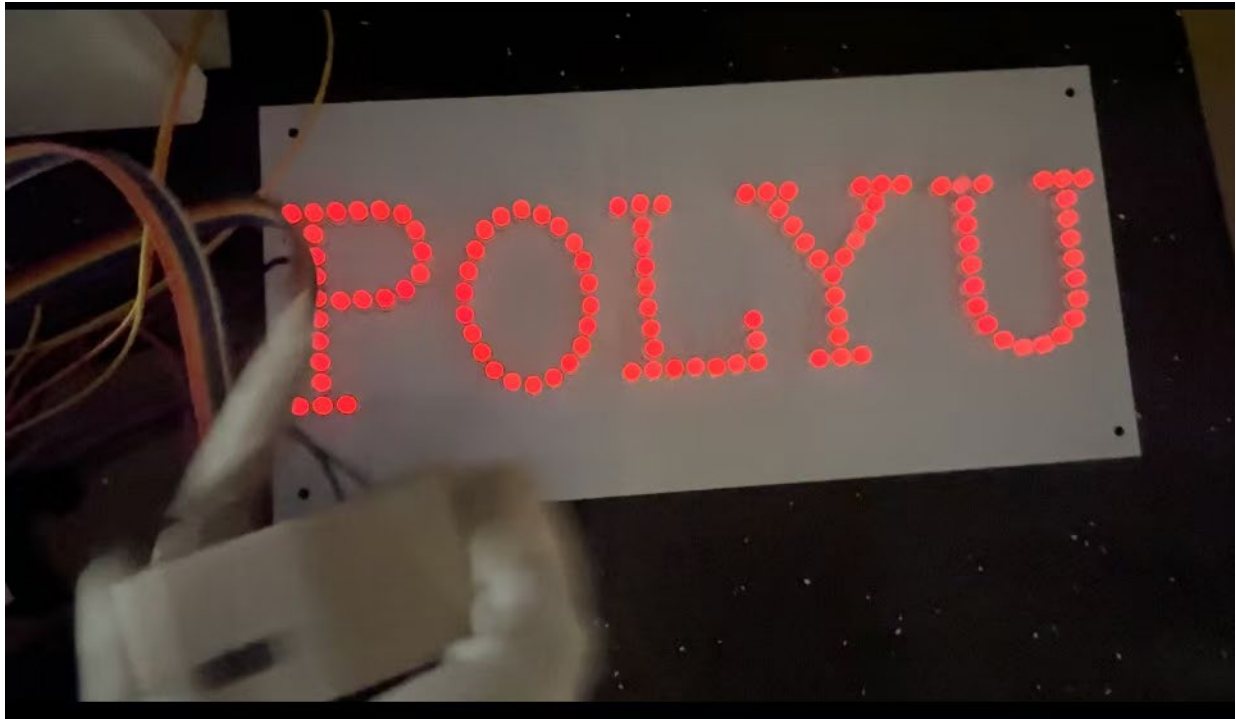


Fig. 4.13: 110 LED bubbles powered by the energy harvester through handshaking motions.

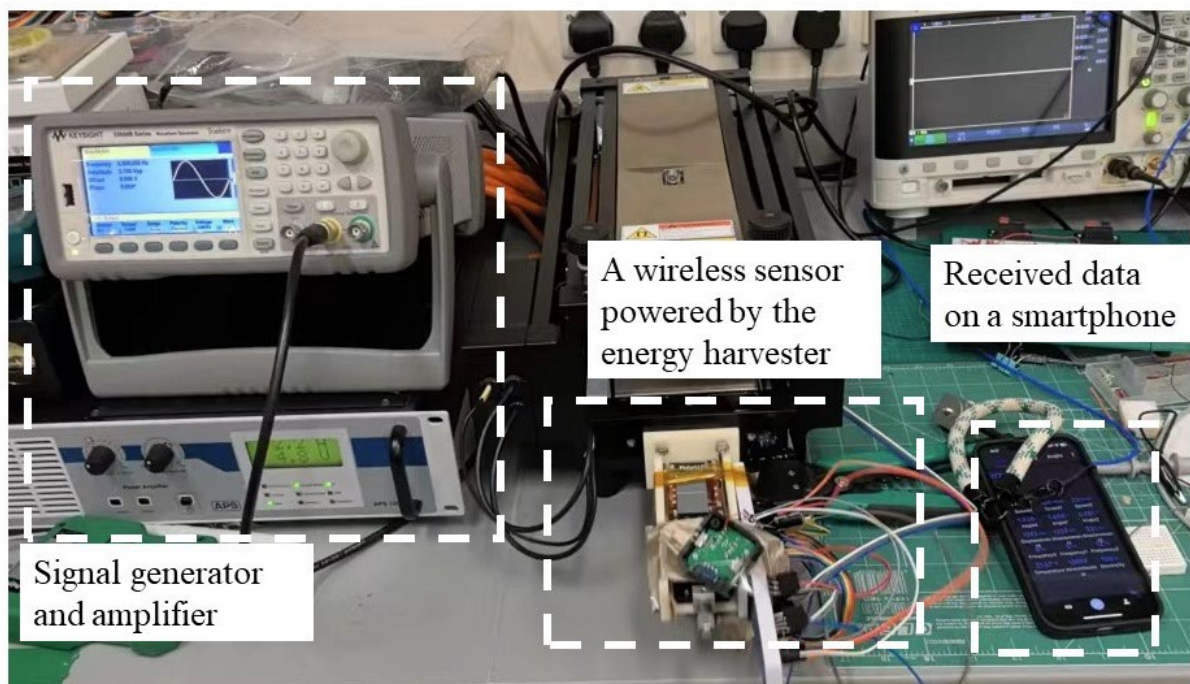


Fig. 4.14: A wireless sensor powered by the energy harvester for real-time data transmission on a smartphone through Bluetooth connectivity under 6 Hz and 1.5 g.

Table 4.4: Performance comparison of various hybrid-type energy harvesters.

Reference (Working Modes)	Operational Conditions	Performance Density
(Tan et al., 2019) (PEG + EMG + TENG)	1–5 Hz	2 mW cm ⁻³ g ⁻¹
(Wang et al., 2019) (PEG + EMG + TENG)	2–12.5 Hz, 1 g	1.97 mW cm ⁻³ g ⁻² , 5 Hz
(Rahman et al., 2020b) (EMG + TENG)	1–15 Hz, 1 g	3.13 mW cm ⁻³ g ⁻² , 6 Hz
(Yang et al., 2022) (EMG + TENG)	1–15 Hz, 1 g	2.26 mW cm ⁻³ g ⁻² , 5 Hz
Present Work (PEG + EMG + TENG)	3–10 Hz, 1 g	3.56 mW cm ⁻³ g ⁻² , 5 Hz

4.5 Concluding Remarks

In this chapter, we presented a folded auxetic beam hybrid energy harvester that optimizes the strengths of various components (Wang et al., 2025b). By combining the PEG, EMG, and TENG units with a folded auxetic beam as a substitute for a traditional spring, a compact multi-stable nonlinear system can be realized. Although the use of auxetic structures in hybrid energy harvesters has been somewhat restricted, their potential to enhance output efficiency warrants further investigation. This study has uncovered the impact of auxetic structures on a nonlinear multi-stable hybrid energy harvester, as detailed below:

- Three distinct auxetic structure patterns were compared, considering factors such as maximum displacement and ease of fabrication. The most efficient auxetic structure was chosen for this energy harvesting system.
- The integration of an auxetic structure with piezoelectric material resulted in a 1.55-fold increase in output efficiency compared to a traditional piezoelectric beam, as shown by simulation results. This marks a significant improvement in enhancing the efficiency of hybrid energy harvesters through the design of auxetic structures.
- Mechanical testing of the prototype revealed that the power outputs of the PEG, EMG, TENG_S and TENG_CS units reached 53.0 mW, 33.97 mW, 14.10 μ W, and 38.72 μ W,

respectively at 1 g and 5 Hz. The entire prototype achieved a peak output of about 87.02 mW, resulting in a normalized power density of $3.56 \text{ mW cm}^{-3} \text{ g}^{-2}$.

- By applying an external magnetic force in conjunction with a non-conservative force (friction), an energy harvesting system capable of operating under low excitation conditions was developed. This approach effectively broadens the frequency range, specifically testing from 3 Hz to 10 Hz at various acceleration excitations (0.5–1.5 g).
- During handshaking motions, the current device effectively powered 110 custom LED bulbs. Furthermore, a wireless vibration sensor operates under an excitation of 6 Hz at 1.5 g, delivering real-time data to a smartphone via Bluetooth connectivity.

In this work, the experimental results of the fully assembled hybrid energy harvester, both with and without the auxetic structure, were not compared. The reason is that the auxetic structure can modify the stiffness of each copper beam plate, affecting the overall stiffness (k) of the PEG. Under identical conditions for other parameters, this change in stiffness k leads to different steady-state characteristics between two devices. Consequently, output performance cannot definitively indicate whether differences arise from the auxetic structure or variations in steady-state behavior. For example, if one structure exhibits a tri-stable state while the other shows a bi-stable state under the same conditions, this discrepancy significantly impacts output performance. Therefore, this work first establishes that the individual auxetic beam performs better under the same force conditions, providing a basis for concluding that the overall hybrid energy harvesting system is enhanced.

CHAPTER 5

-ANALYSIS OF MAGNETO-MULTI-STABLE OSCILLATORS IN TRI-HYBRID VIBRATION-BASED ENERGY HARVESTING DESIGN

5.1 Introduction

The integration of magneto-multi-stable oscillators into tri-hybrid energy harvesting systems significantly enhances their performance by broadening operational bandwidth and improving energy conversion efficiency. These oscillators utilize multiple stable states to capture energy from a wider range of vibrational frequencies, which is particularly beneficial for low-frequency and broadband applications. Incorporating such oscillators into tri-hybrid systems, which combine piezoelectric, electromagnetic, and triboelectric mechanisms, allows for the simultaneous harvesting of energy across different frequency ranges. This multi-modal approach ensures that the system can effectively capture and convert ambient vibrations from various sources, enhancing overall energy output. The compact design enabled by the integration of magneto-multi-stable oscillators is crucial for applications in wearable and portable devices, where space constraints are a significant consideration. By optimizing the spatial arrangement of the energy harvesting components, these systems can achieve high power density without compromising on size, making them suitable for integration into small-scale electronic devices.

The presence of magneto-multi-stable oscillators in tri-hybrid energy harvesting systems can improve working bandwidth, increase energy conversion efficiency, and facilitate compact designs, thereby resulting in effective and versatile solutions for a range of potential applications. The application of multi-stable nonlinearity-enhanced mechanisms in energy harvesting technology also plays a crucial role in improving the working performance of vibration-based energy harvesters under low excitation levels. Consequently, extensive

research efforts have targeted the expansion of the operational bandwidth nonlinear energy harvesters, possessing mono-stable (Kumar et al., 2018), bi-stable (Gu et al., 2020), tri-stable (Zhou et al., 2014), quad-stable (Wang et al., 2018) and even quin-stable (Wang et al., 2017) characteristics. Under certain conditions, achieving multiple robust equilibrium states, such as those provided by a septuple-stable mechanical frequency up-conversion scheme enhanced by nonlinearity, facilitates higher oscillation amplitudes while lowering the excitation threshold (Wang et al., 2019). Recent studies have introduced a tri-stable energy harvester array featuring multiple degrees of freedom, which enables the combination of advantages from multi-degree-of-freedom architectures and nonlinear operational principles (Lai et al., 2019). While tri-stable responses and higher-order multi-stability have demonstrated notable advantages for vibration energy harvesting in both low-frequency and broadband regimes (Lallart et al., 2020), the integration of such features into portable energy harvesters has rarely been explored. In view of the inherently time-varying behavior of human-induced excitations, employing an advanced energy conversion scheme within a high-order multi-stable mechanical framework is a promising route to improve the performance of portable devices.

The problem of scavenging sufficient energy from low-frequency human movements stems from the fact that the power produced by a resonant generator is directly proportional to vibration frequency, making it insufficient for practical use under low-frequency excitation (Gu et al., 2011). For an energy harvester to resonate with low-frequency excitations, a lower stiffness and/or greater inertial mass is often necessary, inevitably resulting in larger displacement amplitudes and greater space requirements. This challenge has motivated the development of numerous solutions, including vibration-to-rotation conversion mechanisms (Fan et al., 2020) and frequency up-conversion (FUC) schemes, to link low-frequency inputs with high-frequency responses of the transducer. Typically, FUC is implemented using a variety of mechanical configurations, i.e., mechanical impact (Huang et al., 2019), mechanical plucking (Kathpalia et al., 2017), pulsed magnetic force (Tang et al., 2012), pulsed acceleration (Han et al., 2015), and internal resonance (Wu et al., 2018). These configurations enable high-frequency excitation of the transducer units by channeling mechanical energy from oscillators operating in non-resonant or low-frequency regimes (Liu et al., 2012). However, a relatively large displacement stroke is still essential to ensure that the use of low-frequency oscillators can harness sufficient energy from an ambient low-frequency vibration source. This reduces the compactness and power density of these energy harvesters inevitably. To fix this problem,

a (FUC) scheme incorporating quin-stability enhancement has been introduced, in which the displacement stroke of the low-frequency vibration stage is designed to partially overlap with that of the mechanical energy transfer stage through inter-well motion (Wang et al., 2017; Wang et al., 2018). This partial-overlap strategy prolongs and strengthens the mechanical energy transfer phase while shortening the overall displacement stroke, thereby enabling broadband energy harvesting from weak, low-frequency vibration sources. Using this mechanism, the power density can be increased by nearly 32-fold relative to a conventional FUC design (Wang et al., 2019; Wang et al., 2017). Nevertheless, the performance of this FUC configuration deteriorates in the presence of gravitational effects, leading to reduced harvesting efficiency under low-intensity vertical excitations (Wang et al., 2019). Therefore, developing a more compact FUC-based architecture is necessary for wearable and portable energy harvesting application.

In this work, we propose a highly miniaturized, ultra-low-frequency, broadband, multi-stable, FUC, and tri-hybrid portable energy harvester designed to sustainably power wearable and portable electronics (Wang et al., 2021a). This harvester is developed using a novel multi-stability-based FUC approach, achieved through the compact integration of two innovative configurations of magneto-multi-stable oscillators. When operated vertically, the mechanical system of the harvester degrades into a tri-stable system, maintaining a wide bandwidth and high efficiency comparable to its horizontal state. Moreover, by integrating two impact-driven piezoelectric generators, an array-based electromagnetic generator, a sliding-type triboelectric nanogenerator, and a contact-separation triboelectric nanogenerator in a highly compact configuration, significantly enhanced power generation can be achieved from a single mechanical motion, thereby further improving the power density. Through comprehensive theoretical analysis and experimental investigations, the mechanical characteristics and electrical performance of the portable energy harvester are systematically evaluated within a low-frequency range that encompasses typical human-induced motions. In addition, the practical applicability of the proposed harvester is demonstrated experimentally through applications, e.g., charging storage units, illuminating LEDs, and powering thermohydrometers.

5.2 System design and working mechanism

5.2.1 Configuration of the multi-stable oscillators

More critically, although human-induced motions often exhibit greater intensity in the vertical direction, many of these tri-stable systems have overlooked gravitational effects during the design stage. When operating along the gravitational axis, their potential functions can become asymmetrically distorted and, in some cases, collapse into mono-stable profiles with a single deep asymmetric potential well, restricting the frequency response to narrow bands and producing only small-amplitude oscillations.

To address the above issues, we design a new tri-hybrid energy harvester, as shown in Fig. 5.1. Two new multi-stable configurations, namely Type I and Type II, are first proposed in this work. In Fig. 5.1(a), the configuration Type I consists of a fixed magnet A with a width of d_1 and a parallel moving-magnet B poled reversely with a width of d_2 ($d_2 > d_1$). The initial distance between the center of two magnets is h . Several spacers A and B are attached on the moving-magnet as a proof mass, where the spacer A has the same dimensions with that of magnet B. Under the force of gravity, the proof mass will compress the lower linear spring with an initial deformation of d_g , which is equal to the gap distance between the free end of the upper linear spring and the upper spacer A. For comparison, the configuration Type II has two magnets B poled in the same direction as magnet A and it is separated by two spacers B and one spacer A, as shown in Fig. 5.1(e). By varying the parameters d_2 and h , both configurations can conduct the dynamic behavior of mono-stable, bi-stable or tri-stable states, as shown in Fig. 5.1(b) and Fig. 5.1(f), respectively. In the case of $d_2 = 6.35$ mm, with the increase of h , the potential function of the configuration Type I can pass through a tri-stable state to a bi-stable state, and further to a mono-stable state, as shown in Fig. 5.1(c). For the configuration Type II, the transition path to a mono-stable state directly comes from a tri-stable state, as shown in Fig. 5.1(g). In both transition stages mentioned above, the increase of h can reduce the potential barrier(s) so that the inter-well oscillations can occur more easily, but it also decreases the distance between two outermost stable equilibrium positions significantly, resulting in the reduction of inter-well motion amplitudes. Therefore, the value of h should be properly chosen by taking the characteristics of ambient vibration sources into consideration.

Given that human-induced motions typically occur at low frequencies (Cai et al., 2020; Chen et al., 2017), a high-magnification FUC scheme incorporating a magnet array arrangement has been developed to match the operating characteristics of energy transducers with such motions. The similarity in the distribution of multi-stable parameter regions between Types I and II enables the direct assembly of a hybrid configuration with optimized dynamic behavior. As illustrated in Fig. 5.1(i), the vertical layout comprises two Type I and two Type II units sharing common fixing magnets. In this arrangement, the alternating-polarity magnet arrays generate a phase shift in magnetic flux density during the slider's upward and downward motions, thereby enhancing the efficiency of the electromagnetic transducer.

The stiffness of the linear spring increases to $4k$ to further strengthen the magnification of FUC. In the case of $d_2 = 6.35$ mm, the potential energy function of the combined configuration can transform from a tri-stable state to a bi-stable state, and then return back to a tri-stable state, and finally reach to a mono-stable state, as shown in Fig. 5.1(k). Particularly, for $h = 4.8$ mm, the Type I configuration possesses a double-well potential with the depth of $\Delta U_1 = 0.204$ mJ (Fig. 5.1(d)), while the Type II configuration possesses a single-well potential (Fig. 1(h)). The combination of these two configurations forms a triple-well potential with a shallower depth, i.e., $\Delta U_2 = 0.151$ mJ, which can reduce the threshold of excitation levels for interwell motions.

The current design, characterized by a triple-well potential, offers benefits for energy harvesting, including improved efficiency and an expanded low-frequency operating bandwidth (Zhou et al., 2014; Zhou et al., 2018). When the combined configuration operates in a horizontal direction, a separation distance, d_s , can be set up between the sliding magnets and linear springs. In Fig. 5.1(n), with the increase of d_s , a quadruple-well potential arises from 0 to 6.4×10^{-2} mm. The negative value of d_s denotes that both springs can be compressed by the sliding magnets at the initial stage. In the case of $d_s = 0.045$ mm, the well depth of the quadruple-well potential is only 0.042 mJ (see Fig. 5.1(o)), which is significantly smaller than that of the triple-well potential in the same horizontal arrangement. By fine-tuning the separation distance, the potential energy distribution in the combined configuration can be made more uniform, thereby mitigating energy imbalances arising from assembly tolerances.

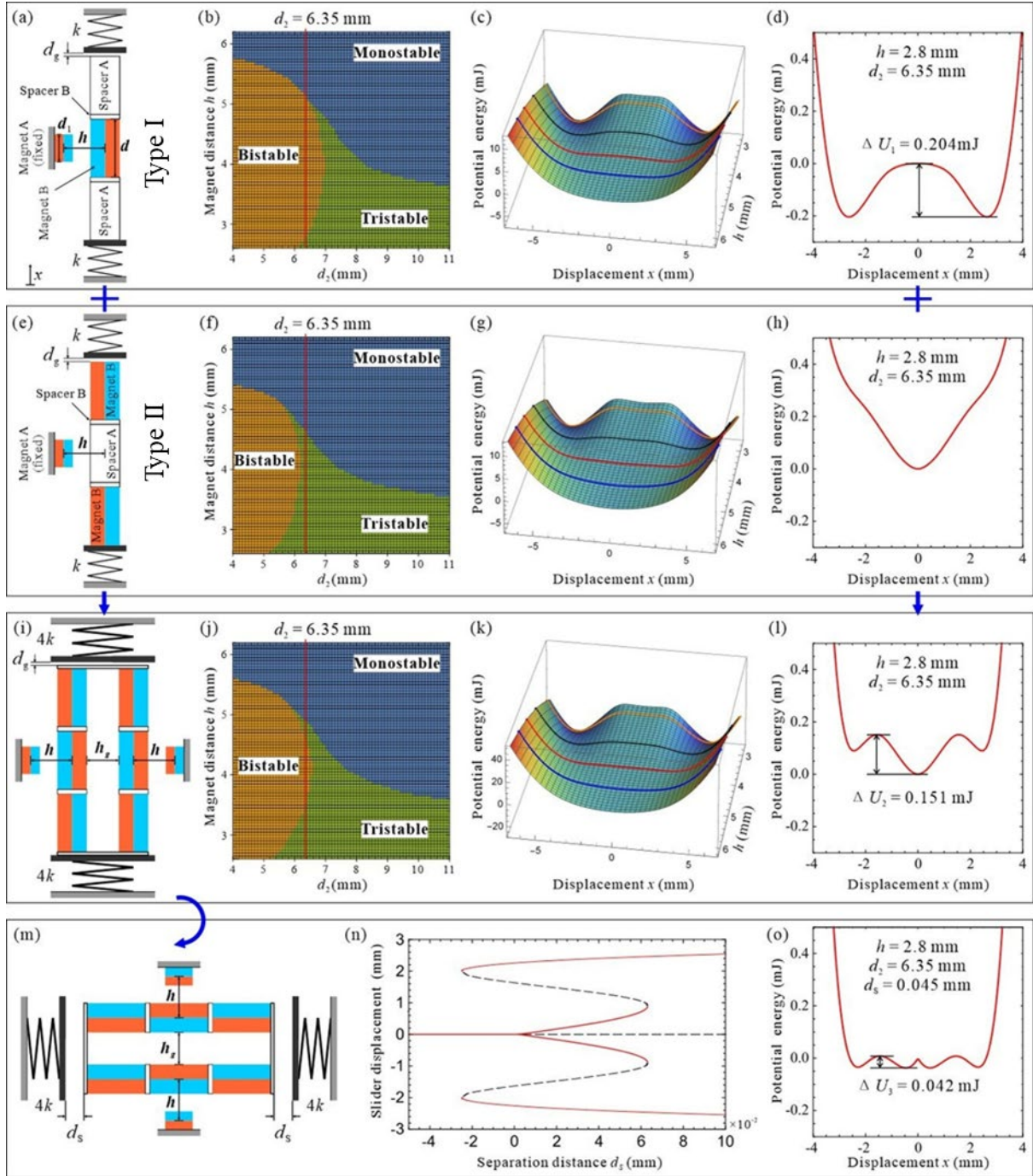


Fig. 5.1: Schematic of the multi-stable oscillators: structural diagram of (a) configuration Type I and (e) configuration Type II; (b, f, j) numerically categorized different types of stability on the parametric plane; (c, g, k) potential energy varies with separation distance h ; (d, h, l, o) the potential energy functions under the specified parameters; and (n) the bifurcation diagram of equilibrium solutions, the thin lines and dotted lines denote the stable and unstable equilibria, respectively.

5.2.2 Working principle of the present energy harvester

In the current harvester design, a multi-stable mechanical framework integrates four functional modules: an array-based EMG, a frequency-up-converted PEG, a contact–separation mode TENG-CS, and a freestanding-mode TENG (TENG-F), as illustrated in Fig. 5.2(a). The EMG module incorporates a rectangular magnet array mounted on an outer frame that serves simultaneously as a slider and inertial mass. Within the air gap between the magnet arrays and the frame, two small magnets and copper (Cu) coils are fixed to the harvester housing, functioning to maintain separation between the arrays and the frame. Each array contains three block magnets separated by spacers, enhancing the magnetic flux density across the plane facing the magnets. For the frequency-up-converted PEG module, the slider is supported by two folded springs. Each folded spring is composed of one thick beryllium bronze beam and five bimorph piezoelectric beams stacked in a folded arrangement. The outermost piezoelectric beam is anchored to the harvester housing, while the adjacent thick metal beam is positioned with an 80 μm clearance from the slider.

For the TENG module, the left and right sections are constructed from printed circuit boards (PCBs). A 1.2 mm-thick glass epoxy layer serves as the substrate, onto which four copper (Cu) electrodes are patterned in parallel arrays on its surface. Two rectangular-shaped PCBs with a power management circuit on the outer surface and eleven parallel arrayed Cu electrodes on the inner surface are embedded into the harvester housing as left and right sidewalls. Fluorinated Ethylene Propylene (FEP) films with a thickness of 80 μm are attached to the Cu electrodes on sidewalls and both ends of the slider, it can serve as triboelectric layers incorporate materials carrying negative triboelectric charges. The completed prototype is presented in the photograph of Fig. 5.2(b), and its primary fabrication stages are illustrated in Fig. 5.2(c). Additional details on the fabrication process can be found in the experimental section.

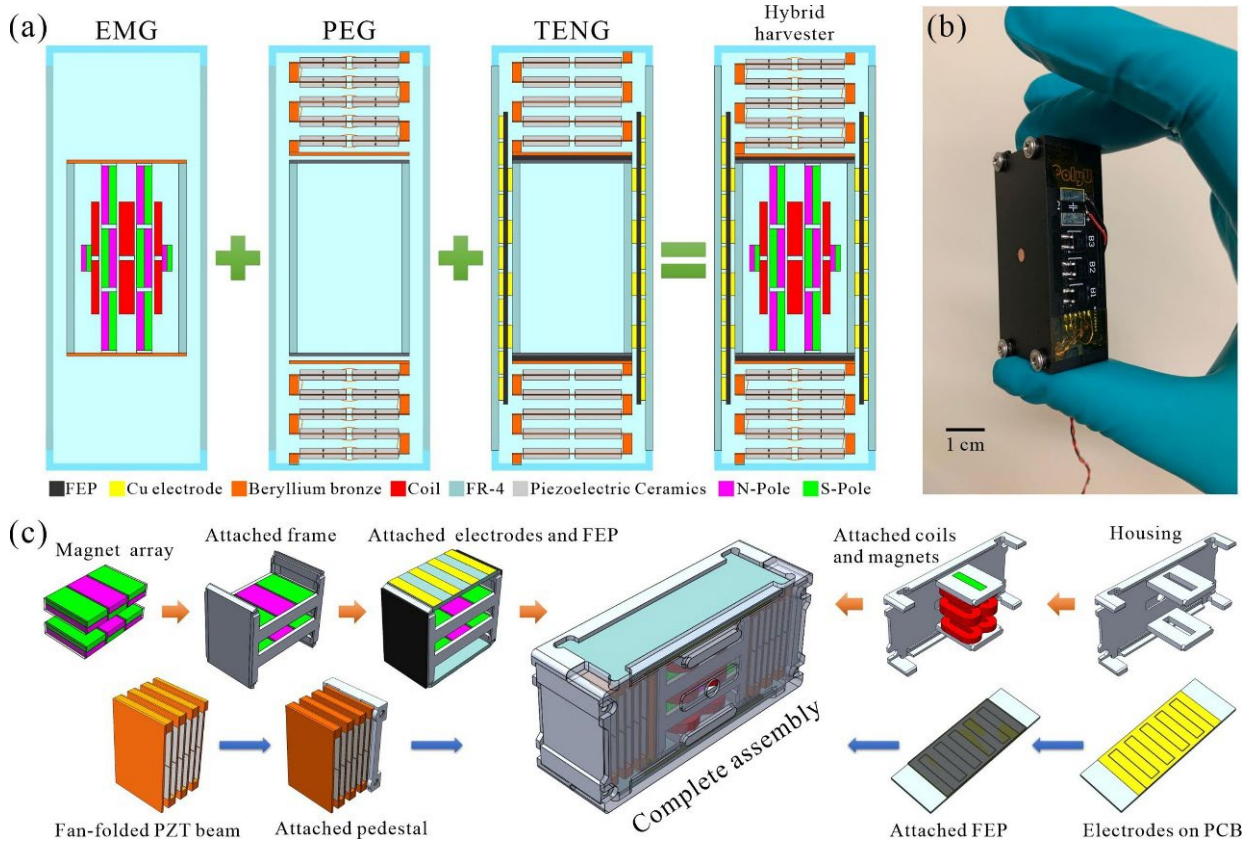


Fig. 5.2: Proposed tri-hybrid PEG–EMG–TENG energy harvester: (a) schematic diagram; (b) fully packaged prototype; and (c) major fabrication steps.

Figure 5.3 illustrates the operating principle of the tri-hybrid energy harvester, which can be divided into three parts: the electromagnetic part, the piezoelectric part and the triboelectric part (including a sliding mode and a contact-separation mode). When an external excitation is applied on the working direction of the harvester, the slider (proof mass) starts to oscillate between the sidewalls (TENG-F) and contact, compress and separate from the folded springs alternately (TENG-CS and PEG). For the triboelectric part, the contact areas between the slider, both folded piezoelectric springs and sidewalls of the harvester housing are significant. This results in an electrostatic induction when surface contact occurs. According to the triboelectric series, Cu has a lower electron affinity than FEP, leading to positive triboelectric charges on the Cu surface and negative triboelectric charges on the FEP surface.

In initial stage (stage I), when the slider is centered, the beryllium bronze electrodes on both piezoelectric springs share the same potential, thereby preventing charge transfer. At the same time, the Cu electrodes on the slider align with one set of sidewall electrodes, creating

electrical equilibrium. As the slider moves right (stage II), for the TENG-F unit, the Cu electrodes on the slider leave the overlapping state, electrons flow from the leaving electrodes to the forward electrodes due to the potential drop, and consequently generating triboelectric current through the external connection between the electrode groups. Meanwhile, for the TENG-CS unit, the triboelectric layer on the left side starts to separate from the Cu electrode, while the triboelectric charges are retained on the surfaces. This separation creates a potential difference between the FEP film and the beryllium bronze electrode, driving electrons from the right to the left electrode and producing a triboelectric current to the right.

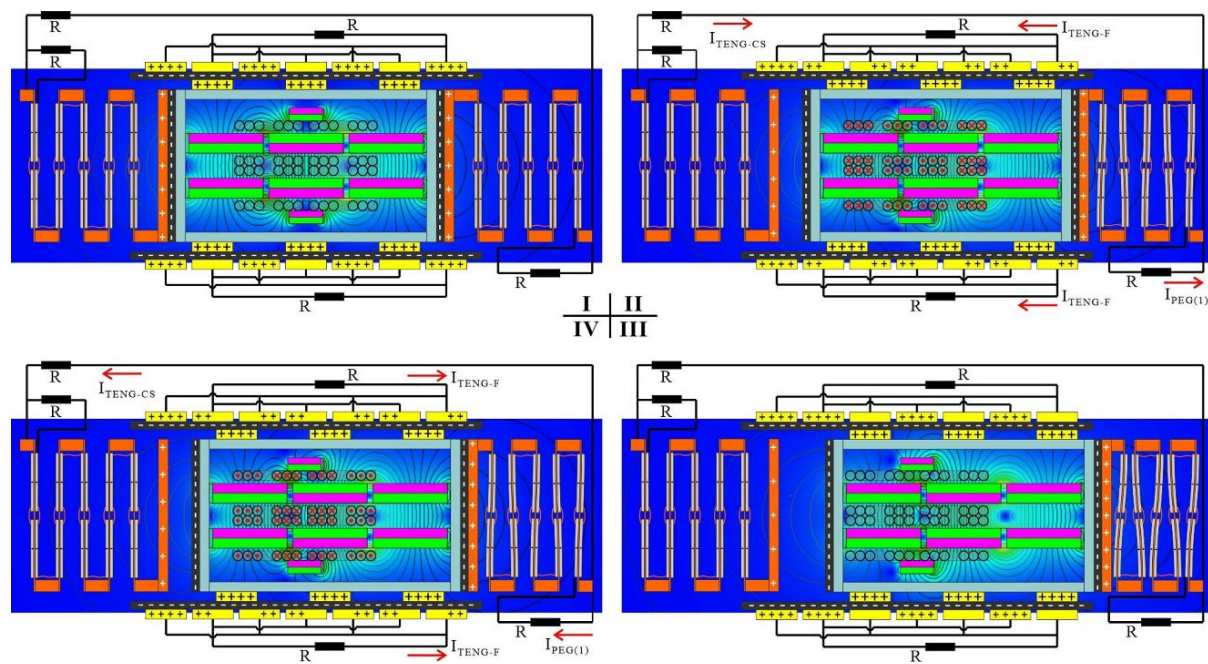


Fig. 5.3: Working principle of the tri-hybrid energy harvester in a half cycle.

In the PEG unit, slider motion compresses the right folded spring, straining the PZT sheet and generating opposing piezoelectric potentials that drive current through the external circuit. Simultaneously, the motion of the magnetic slider alters the magnetic flux linkage across the coils, thereby inducing a current in the series-connected coils of the EMG unit. The triboelectric, piezoelectric, and electromagnetic outputs persist until the slider reaches its maximum displacement (stage III). Subsequently, as the slider moves reversely (stage IV), for the TENG-F unit, electrons flow from the leaving electrodes to the forward electrodes, generating an opposite current from stage II. For the TENG-CS unit, the triboelectric potential on the left beryllium bronze electrode increases with the slider gets closer. Positive charges flow back

from the right beryllium bronze electrode to the left one. Meanwhile, the release of the internal strain in the PZT sheets and the change of the magnetic flux across the coils can generate an opposite current from stage II. Then, the slider reverts to stage I and repeats the above processes on the other side. Then, electric current can be constantly generated by all these units.

5.2.3 Theoretical modeling

Two simplified mechanical system models of the hybrid energy harvester in the horizontal and vertical directions are presented in Fig. 5.4. When an input excitation ($x_b(t)$) is applied to the harvester, the magnetic slider (M) will oscillate with a relative displacement of $x_r(t)$ with respect to the harvester. At the initial stage, there is a separation gap (d_s) between the slider and folded piezoelectric spring(s). Turning to the vertical orientation, $d_s = Mg/k$ where g is the gravitational acceleration and k is the stiffness of the folded piezoelectric spring. In the case of impact with the folded piezoelectric springs, different types of forces are exerted on the slider, including gravitational force, magnetic force, coulomb's friction force, viscous friction force, and impact force.

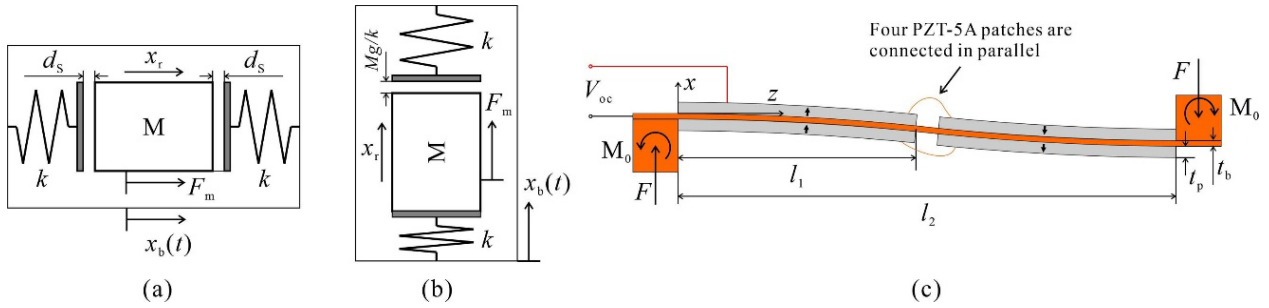


Fig. 5.4: Mechanical models of the proposed energy harvester in (a) horizontal and (b) vertical directions, respectively. (c) An equivalent mechanical model of one bimorph piezoelectric beam in the folded piezoelectric spring.

According to the Newton second law of motion, the governing equations of the mechanical systems in the horizontal and vertical directions are, respectively, given by ((Haroun et al., 2015; Rahman et al., 2020a).

$$M\ddot{x}_r + (C_i + C_v + C_e)\dot{x}_r + k_r(|x_r| - d_s) + F_m + \frac{\dot{x}_r}{|\dot{x}_r|}\mu F_l = -M\ddot{x}_b \quad (5.1)$$

$$M\ddot{x}_r + (C_i + C_v + C_e)\dot{x}_r + k\alpha_r + F_m + \frac{\dot{x}_r}{|\dot{x}_r|}\mu F_l = -M\ddot{x}_0 \quad (5.2)$$

where μ is the coefficient of the dry friction. F_m and F_l are the magnetic force and lateral pressure acting on the slider, respectively. C_i is the additional damping coefficient when the slider is in contact with the folded piezoelectric spring, C_v is the linear viscous damping coefficient, and C_e is the electrical damping coefficient that depends on the following electromagnetic parameters

$$C_e = \frac{(d\varphi/dx_r)^2}{R_L + R_C} \quad (5.3)$$

where φ is the total magnetic flux across the coils, $d\varphi/dx_r$ denotes the change rate of the magnetic flux with the relative displacement. R_L and R_C are the resistance of the external load and the coils, respectively. Moreover, the effective stiffness of the folded piezoelectric springs (k_e) takes the value of k during impact and zero when they are not during impact, then

$$k_e = \begin{cases} -k, & x_r < -d_s \\ 0, & -d_s \leq x \leq d_s \\ k, & x > d_s \end{cases} \quad (5.4)$$

It is noted that the feedback effect of the piezoelectric transducer on the dynamic system is negligible and can thus be disregarded in the governing equations.

For the PEG unit, consider the symmetry of the folded piezoelectric spring, an equivalent mechanical model of one bimorph piezoelectric beam shown in Fig. 5.4(b) can be used to simplify complicated calculations. The bimorph piezoelectric beam consists of a beryllium bronze substrate with Young's modulus E_s , effective length l_2 , width w , and thickness t_s . Four PZT-5A patches, with Young's modulus E_p , length l_1 , width w , and thickness t_p , are attached on the upper and lower surfaces of the beryllium bronze layer and connected in parallel. Let the z -axis be along the length direction, with $z = 0$ located at the left end. While the x -axis is along the thickness direction, with $x = 0$ at the upper surface of the beryllium bronze substrate. According to the vibration mode and symmetry of the structure, one obtains

$$\frac{Fl_2}{2} - M_0 = 0 \quad (5.5)$$

Then, the radius of curvature $R(z)$ of the beam at the location z can be expressed as

$$\left| \frac{1}{R(z)} \right| = \frac{M(z)}{wD} = \frac{F(l_2 - 2z)}{2wD} \quad (5.6)$$

where $M(z)$ is the bending moment at z , and D is the bending modulus per unit width of the beam that can be obtained as

$$D = \begin{cases} \frac{E_s t_s^3}{12} + \frac{E_p t_s}{6} (4t_p^2 + 6t_s t_p + 3t_p^2), & z < l_1 \text{ or } z > l_2 - l_1 \\ \frac{E_s t_s^3}{12}, & l_1 < z < l_2 - l_1 \end{cases} \quad (5.7)$$

Due to the structural symmetry, the position of the neutral axis is given by

$$t_n = -\frac{t_s}{2} \quad (5.8)$$

Then, the lateral stress on the PZT-5A patches at the position (z, x) can be written as

$$\sigma(z, x) = \left| \frac{1}{R(z)} \right| (x - t_n) E_p = \frac{F E_p}{2wD} (l_2 - 2z)(x - t_n) \quad (5.9)$$

The induced electric field $E(z, x)$ in the thickness direction at (z, x) on the PZT-5A patches is

$$E(z, x) = \left(\frac{-d_{31}}{\varepsilon_r \varepsilon_0} \right) \times \sigma(z, x) = -\frac{d_{31} F E_p}{2\varepsilon_r \varepsilon_0 wD} (l_2 - 2z)(x - t_n) \quad (5.10)$$

where d_{31} , ε_0 , and ε_r are the piezoelectric strain constant, the permittivity of free space, and the dielectric constant of PZT-5A, respectively. Hence, the generated open-circuit voltage of the PEG unit can be given by

$$V_{oc-PEG} = \frac{1}{l_1} \int_0^{l_1} \int_0^{t_p} E(z, x) dx dz = -\frac{d_{31} F E_p t_p l_1}{4\varepsilon_r \varepsilon_0 wD} (t_s + t_p)(l_2 - l_1) \quad (5.11)$$

For the EMG unit, as described by Faraday's law of electromagnetic induction, the relationship of the open-circuit voltage V_{oc-EMG} is presented as

$$V_{oc-EMG} = -N \frac{d\phi}{dt} \quad (5.12)$$

where N is the effective number of turns of the coil group.

For the TENG-F unit, V_{oc-F} can be expressed as follows

$$V_{oc-F} = \frac{Q_{sc}}{C(x_r)} = \frac{\Delta S \cdot \sigma}{C(x_r)} \quad (5.13)$$

where Q_{sc} , $C(x_r)$, and σ are the short-circuit transferred charge, the capacitance between two electrodes with respect to the displacement, and the surface charge density on the slider, respectively. Since the transferred charge is induced by the triboelectric charge on the slider, the potential difference between the two groups of electrodes is proportional to the change of contact area (ΔS). While for the TENG-CS unit, the generated output voltage can be obtained by the V - Q - x relationship for the metal-freestanding-layers, which is expressed by

$$V_{oc-CS} = \frac{\sigma_{CS} x_r}{\epsilon_0} \quad (5.14)$$

where σ_{CS} is the surface charge density on the folded piezoelectric spring electrodes.

5.2.4 Performance characterization

Considering the present design, a numerical analysis was performed to characterize the dynamic behavior of the current design before experimental verification. Static tests determined the nonlinear restoring force coefficients, which were then used to establish the potential energy functions for both tri-stable and quad-stable modes of the energy harvester. As shown in Fig. 5.5(a), a force acting on the magnetic slider was measured at different displacements by using a digital force gauge. Figure 5(b) shows the variation of the restoring force with the displacement of the magnetic slider. Then, the fitted polynomial expressions for the restoring forces of both tri-stable and quad-stable modes can be, respectively, written as:

$$F_{tri} = 415.172x_r - 2.405 \times 10^8 x_r^3 + 3.630 \times 10^{13} x_r^5 - 1.249 \times 10^{18} x_r^7 \quad (5.15)$$

and

$$F_{quad} = \begin{cases} -424.827x_r - 2.405 \times 10^8 x_r^3 + 3.630 \times 10^{13} x_r^5 \\ -1.249 \times 10^{18} x_r^7, & (for |x_r| \leq 1.1 \times 10^{-4}) \\ -9.460 \times 10^{-2} \text{sgn}(x_r) + 435.172x_r - 2.405 \times 10^8 x_r^3 \\ + 3.630 \times 10^{13} x_r^5 - 1.249 \times 10^{18} x_r^7, & (for |x_r| > 1.1 \times 10^{-4}) \end{cases} \quad (5.16)$$

From the potential energy diagram, it is obvious that the quad-stable system has lower potential energy than that of the tri-stable one, as shown in Fig. 5.5(c). This implies the quad-

stable system may require a lower excitation level to pass through potential wells than the tri-stable one. However, the outmost potential wells in the quad-stable mode are deeper than the one in the tri-stable mode, resulting in a higher local oscillation frequency in the quad-stable mode (Panyam and Daqaq, 2017; Wang et al., 2018). In addition, the folded piezoelectric spring has an equivalent stiffness of $\sim 3.12 \times 10^3 \text{ N/m}$ and a natural frequency of 173 Hz. By coupling a strong magnetic force, the maximum restoring force of the multi-stable system is lower than 0.21 N in the displacement range from -2.5 to 2.5 mm. Consequently, the output performance of the PEG unit can be improved under low-frequency and low-intensity vibrations.

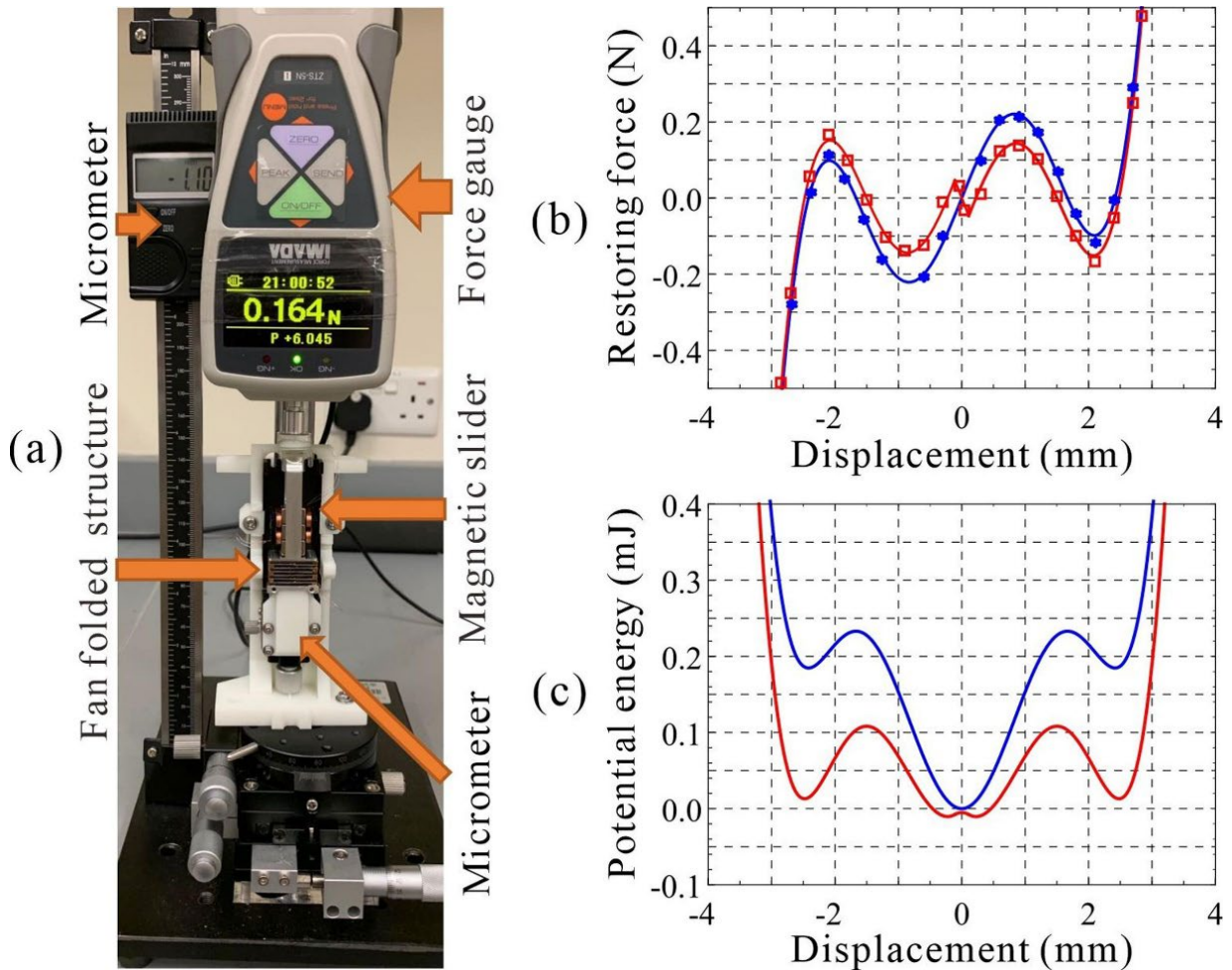


Fig. 5.5: Experimental setup: (a) measurement of restoring force; (b) variation of restoring force with displacement; and (c) potential energy functions of the tri-stable and quad-stable systems.

Figure 5.6 presents the amplitude–frequency responses and the corresponding phase portraits of the tri-stable and quad-stable systems under different constant-frequency excitations (ranging from 1 Hz to 20 Hz) with an acceleration amplitude of 1 g. The discrete points represent displacement amplitudes at each excitation frequency within one cycle, while multiple points at a frequency indicate period-doubling or chaotic motion. As shown in Fig. 5.6(a), the tri-stable system oscillates with a large-amplitude of inter-well motions from the starting frequency of 1 Hz to the cut-off frequency of 11.6 Hz. This shows the performance of period-doubling behaviour at low frequencies. After the cut-off frequency, the tri-stable system exhibits a chaotic motion with a relatively small amplitude in the frequency range of 11.6–15.8 Hz. In contrast, the quad-stable system can perform large-amplitude inter-well oscillations in a wider frequency range of 1–12.3 Hz, and the corresponding chaotic motion (in the range of 11.6–16.1 Hz) exhibits large-amplitude oscillations, leading to a better energy harvesting efficiency than that in the tri-stable system.

The starting-up processes of the tri-stable and quad-stable systems are investigated under a sinusoidal excitation at 3 Hz as shown in Fig. 5.7. The acceleration amplitude is changed from 0.1 g to 1.5 g. In the tri-stable system (Fig. 5.7(a)), the slider is trapped in the middle potential well at stage I (0.1–0.48 g), and can only perform small-amplitude intra-well oscillations, as shown in the time-varying phase diagram accordingly. With the acceleration amplitude increases over 0.48 g, the slider is activated into inter-well oscillations after an amplitude jumping phenomenon. The displacement amplitude continues to enlarge as the excitation level increases. In the quad-stable system (Fig. 5.7(b)), at the very beginning of stage I (0.1–0.52 g), the slider can only perform small-amplitude intra-well oscillations in one of the middle potential wells. As the acceleration amplitude increases to 0.19 g, the slider begins to oscillate between the two middle potential wells after a small amplitude jumping. When the acceleration amplitude further increases to 0.52–0.62 g, the slider can jump out of the two middle potential wells but be trapped in the one of the outer wells as shown in the phase diagram, due to the deeper outer well depth and higher local oscillation frequencies. After the acceleration amplitude increases over 0.62 g, the slider passes through a chaotic region and finally enters into large-amplitude inter-well oscillations. The above starting-up processes and phase diagrams clearly illustrate that the large-amplitude motions of the tri-stable and quad-stable systems are highly dependent on the level of input excitations. Particularly, even though the potential wells of the quad-stable system are globally shallower than that of the tri-stable

one, its threshold of inter-well oscillation motions is higher than that of the tri-stable one under the low-frequency sinusoidal excitation of 3 Hz, due to the higher local oscillation frequencies of the outer wells.

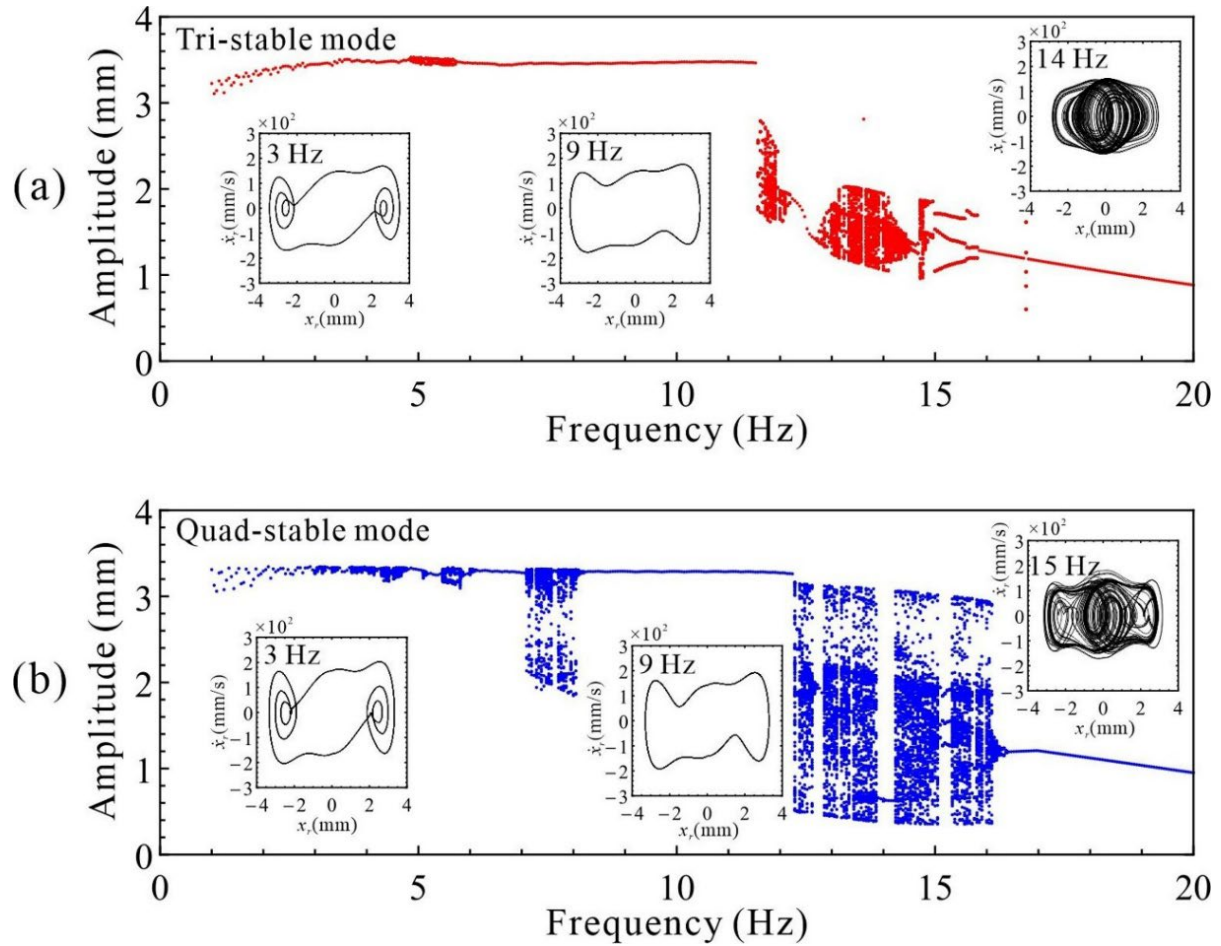


Fig. 5.6: Simulated amplitude-frequency responses and corresponding phase portraits under various frequency excitations with 1 g acceleration: (a) tri-stable mode; and (b) quad-stable mode.

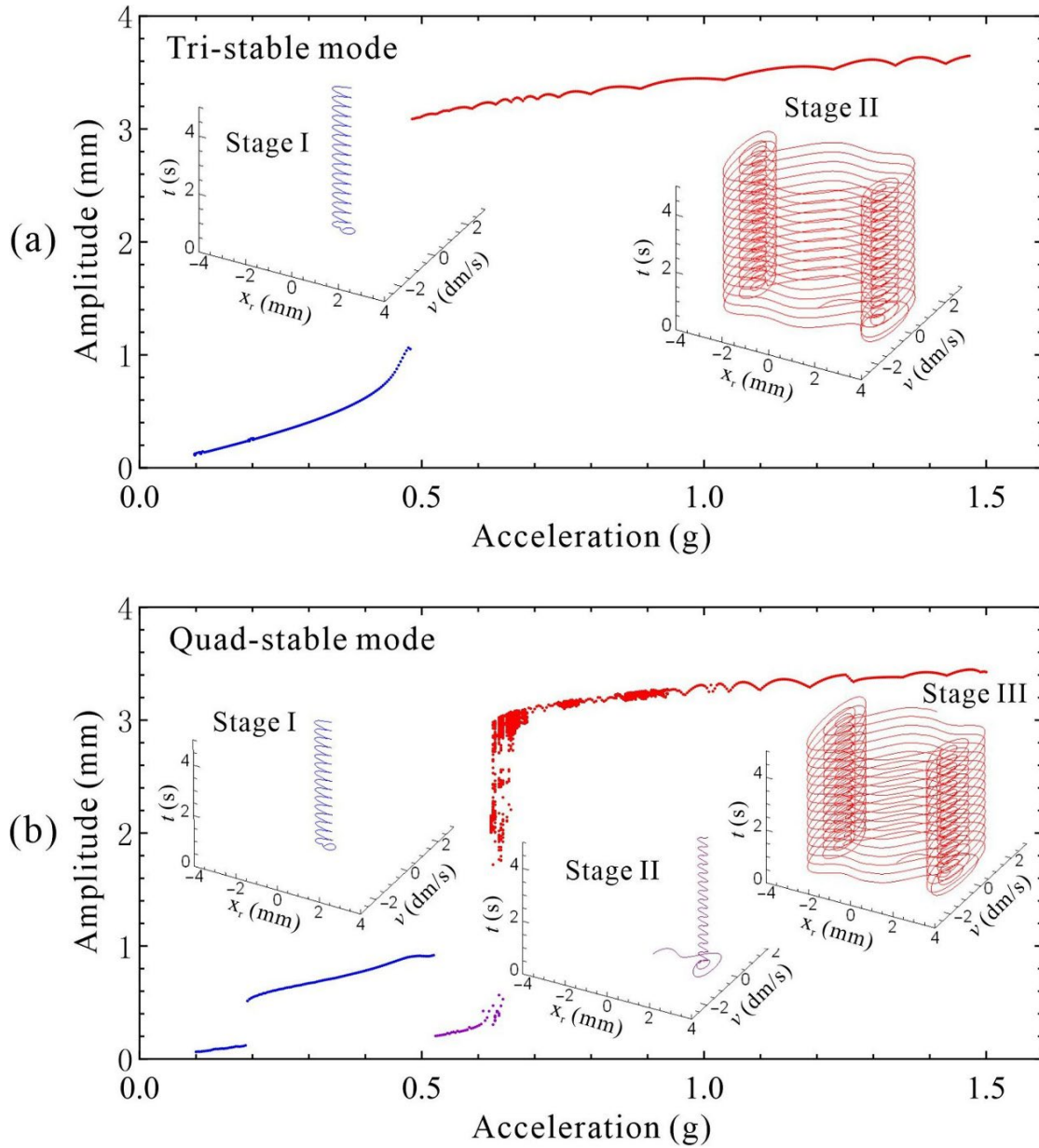


Fig. 5.7. Starting-up processes under a sinusoidal excitation of 3 Hz: (a) tri-stable mode; and (b) quad-stable mode.

5.2.5 Prototype fabrication and experimental setup

To verify the performance of the proposed energy harvester, a miniaturized prototype was fabricated and tested under different excitation levels. Photographs of the fabricated prototype before assembling components are presented in Fig. 5.8(a). The fabricated prototype consisted of two aluminum alloy housing structures with dimensions of $1.8 \times 2 \times 6 \text{ cm}^3$. A dual magnet-array was fixed by a beryllium bronze frame ($2.86 \times 2.19 \times 1.43 \text{ cm}^3$). Each magnet-array ($2 \times 1.27 \times 0.3 \text{ cm}^3$) was composed of three NdFeB (N35 grade) block magnets with two

acrylic spacers ($1.27 \times 0.3 \times 0.05 \text{ cm}^3$) in between. Each copper coil was wound into an ellipse shape ($1.3 \times 0.6 \times 0.2 \text{ cm}^3$), in a position facing the magnet-array. An air gap of 0.3 mm between the magnet-array and coil surface was maintained. Each six-beam-fan-folded structure ($2.06 \times 1.33 \times 1.21 \text{ cm}^3$) was mounted on an aluminum alloy pedestal, which can be fixed to the housing structure by bolted connections. Each beam ($2 \times 1.33 \text{ cm}^2$) consisted of a 0.3 mm thick beryllium bronze substrate and four 0.13 mm thick piezoelectric patches (PSI-5A4E PZT) supplied by PIEZO SYSTEMS, INC. The links ($1.33 \times 0.2 \times 0.15 \text{ cm}^3$) made of phosphor bronze can be used to connect the beams.

In addition, two PCBs ($44.8 \times 11.2 \text{ mm}^2$) with one side etched into the parallel arrayed electrodes and the other side as a rectifying circuit were inserted into the housing structure. FEP films were attached on the parallel arrayed electrodes and the top surface of the fan-folded structure to act as a triboelectric material. An initial gap ($\sim 0.05 \text{ mm}$) was maintained between the slider and the FEP film to prevent excess friction. An electronic circuit with five rectifier ICs on the PCBs was used to connect all harvester units in parallel. Such an electronic design configuration can avoid the opposite phase problem due to different outputs of the units. The rectified DC voltages from all harvester units were stored in a $560\text{-}\mu\text{F}$ ceramic capacitor on one PCB. After the final assembly, the total volume and weight of the prototype are $5.28 \times 2.54 \times 1.73 \text{ cm}^3$ and 85.7 g, respectively.

Figure 5.8(b) shows the stable equilibrium positions of the fabricated prototype in the vertical (tri-stable) and horizontal (quad-stable) directions, respectively. The experimental setup for testing the fabricated prototype is illustrated in Fig. 5.8(c). In this setup, the prototype was mounted on an electrodynamic shaker (SPEKTRA APS 113) that can generate a harmonic base excitation in both horizontal and vertical directions with various frequencies and accelerations (Fig. 5.8(d)). The shaker was powered by an amplifier (SPEKTRA APS 125) in conjunction with a signal generator (Keysight 33500B) to provide different operating frequencies to the shaker, while the base acceleration can be monitored by using a reference accelerometer (Dytran 3093 M10) that was connected to an IEPE acceleration sensor amplifier (MonoDAQ MONODAQ-E-ACC-1). The output terminals of all harvester units were connected to the probe of a digital oscilloscope (Keysight DSOX3014T), which can

record and display the output voltage of each harvester unit across a load resistor or in an open circuit. The geometrical dimensions of the proposed design are outlined in Table 5.1.

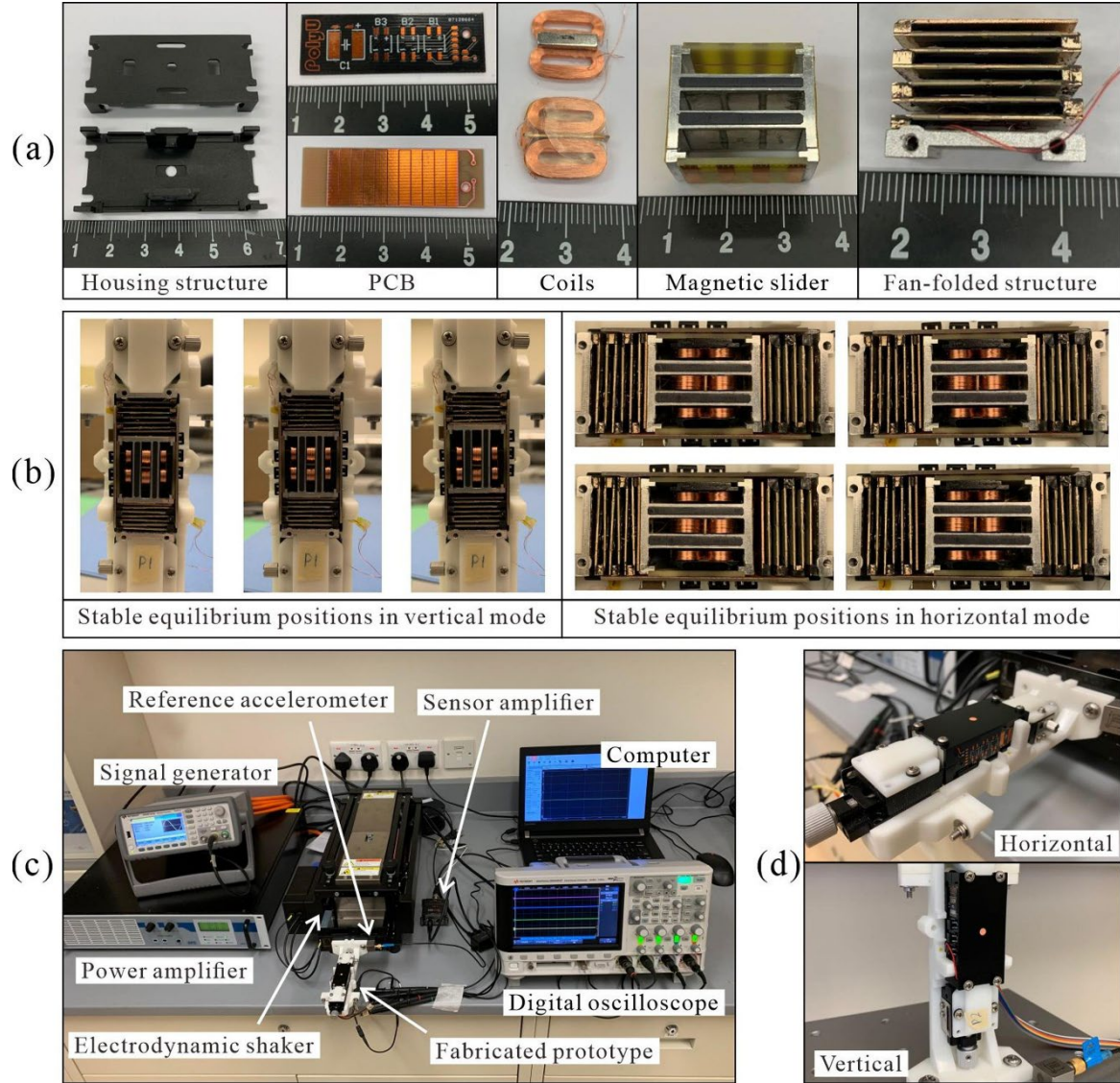


Fig. 5.8: Experimental investigation: (a) the fabricated prototype before assembling components; (b) stable equilibrium positions for the fabricated prototype in both horizontal and vertical modes, respectively; and (c) experimental setup for testing the fabricated prototype; and (d) the prototype was mounted on an electrodynamic shaker in both horizontal and vertical directions.

Table 5.1: Geometric information of the model.

Item	Parameters	Quantities
Alloy housing structures	Length $\times$ Width $\times$ Thickness	$1.8 \times 2 \times 6 \text{ cm}^3$
Beryllium bronze frame	Length $\times$ Width $\times$ Thickness	$2.86 \times 2.19 \times 1.43 \text{ cm}^3$
Beryllium bronze substrate	Thickness	0.3 mm
Copper coils	Length $\times$ Width $\times$ Thickness	$1.3 \times 0.6 \times 0.2 \text{ cm}^3$
Six-beam-fan-folded structure	Length $\times$ Width $\times$ Thickness	$2.06 \times 1.33 \times 1.21 \text{ cm}^3$
Cantilever beam	Length $\times$ Width	$2 \times 1.33 \text{ cm}^2$
Piezoelectric patches (PSI-5A4E PZT)	Thickness	0.13 mm
The links (phosphor bronze)	Length $\times$ Width $\times$ Thickness	$1.33 \times 0.2 \times 0.15 \text{ cm}^3$
PCBs	Length $\times$ Width	$44.8 \times 11.2 \text{ mm}^2$
Total prototype	Weight	85.7 g
	Length $\times$ Width $\times$ Thickness	$5.28 \times 2.54 \times 1.73 \text{ cm}^3$

5.3 Results and discussion

5.3.1 Mechanical vibration test

Under a sinusoidal acceleration with various acceleration amplitudes (0.5 g, 1 g, and 1.5 g), the frequency responses of various harvester units in the tri-stable (vertical) and quad-stable (horizontal) systems are investigated, respectively. The simulated and experimental open-circuit voltage values of various harvester units in the frequency range of 1–20 Hz are presented in Fig. 5.9. Because all harvester units in the prototype can be driven by the same mechanical motions, their open-circuit voltage values show similar broadband responses according to the selected multi-stable nonlinearity mode. In the tri-stable state, as shown in Fig. 5.9(a, b, d, f, h), all harvester units can produce a high voltage output in the bandwidth of ~ 1 to ~ 7 Hz under 0.5 g, except for a narrow frequency band around 4 Hz. Under a 4 Hz excitation, the slider can be eventually caught in the outer well of one side, which will disable the TENG-CS and PEG units on the other side (the upper PEG unit in this experiment).

When the excitation level increased to 1 g, the slider can perform large-amplitude inter-well oscillations in the bandwidth of ~ 1 to ~ 11 Hz, resulting in high voltage responses in all

harvester units. However, the experimental results of TENGs (Fig. 5.9(f, h)) are significantly lower than the predicted ones, because the measuring circuit in experiment cannot reach to an ideal open-circuit state. When the excitation level was set at 1.5 g, the cut-off frequency of all harvester units further increased to ~ 13 Hz. To compare the relatively stable voltage responses of PEG and TENG-CS, whose open-circuit voltage values linearly depend on the displacement. Both EMG and TENG-F units encountered a sudden drop in voltage during the transition process from a period-doubling/chaotic motion to a periodic motion (i.e., 6–9 Hz). This transition process may cause a change in the form of motion, resulting in the change of velocity distributions and the reduction of voltage responses.

When the prototype was operated at a quad-stable mode, the slider cannot perform large-amplitude inter-well oscillations throughout the excitation level of 0.5 g, as shown in Fig. 5.9(c, e, g, i). Only relatively low energy output can be generated by the harvester units when the slider oscillated between the two middle wells at the frequencies between 2 and 6 Hz and higher than 13 Hz. Note that, for the TENG-CS unit (Fig. 5.9(g)), the intra-well oscillation of the slider in the outer well (i.e., 1–2 Hz and 7–12 Hz) may result in a wrong prediction. In the theoretical model of the TENG-CS unit, V_{oc} that is simple linearly depends on the displacement (x_r). Nevertheless, the TENG-CS unit has lost its output properties, because the dielectric surface on the slider cannot make contact with the two electrodes alternately.

When the acceleration increased to 1 g, the excitation can activate the large-amplitude inter-well oscillation of the slider in the frequency range of ~ 1 to ~ 12 Hz, which is wider than that of the tri-stable mode. Meanwhile, at the low frequencies, the voltage responses of the individual harvester units in the quad-stable state are higher than that in the tri-stable state. When the excitation level further increased to 1.5 g, all harvester units generated large-amplitude voltage responses over a wider range of frequencies (1–16 Hz). In addition, the measured open-circuit voltage levels of the individual harvester units are lower than the calculated ones. This discrepancy is mainly due to the fact that the impact between the slider and the folded piezoelectric springs is elastic and the mass effect of the folded piezoelectric springs is neglected. Indeed, the impact effect cannot be strictly inelastic and the mass effect of the piezoelectric springs can cause a velocity mutation based on the momentum theorem.

Both numerical and experimental results, a tri-stable mode is more easily to be triggered into large-amplitude inter-well motions for generating a higher energy output under a sinusoidal excitation. When the external excitation level is high enough to activate a quad-stable mode for inter-well motions, it shows superior wideband and low-frequency performance over the tri-stable mode. Furthermore, both nonlinear dynamical modes can offer a high-efficiency operating bandwidth that covers the vibration frequencies of general human motions (1–10 Hz) under 1 g acceleration.

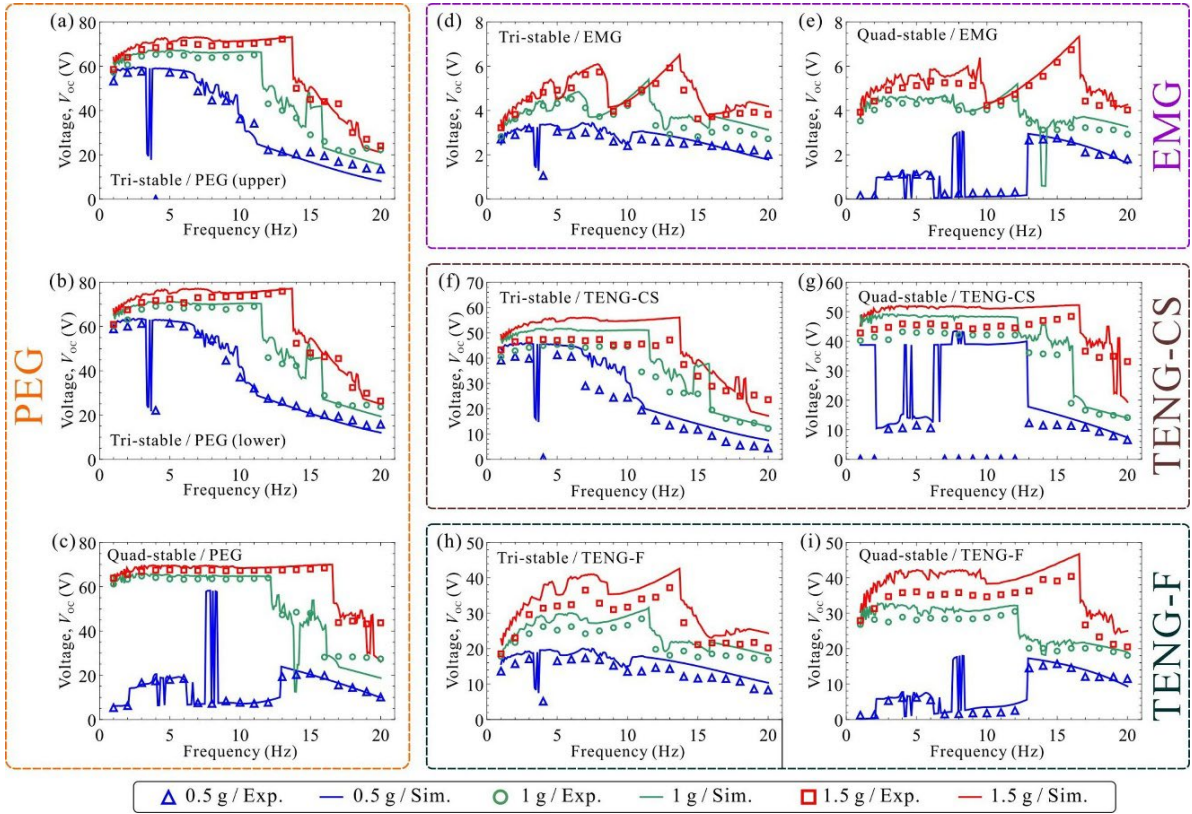


Fig. 5.9: Open-circuit voltage of various harvester units against different frequencies in the tri-stable and quad-stable modes under 0.5 g, 1 g and 1.5 g: (a, b, c) PEG; (d, e) EMG; (f, g) TENG-CS; and (h, i) TENG-F.

To characterize the electrical performance of the individual PEG, EMG, TENG-F and TENG-CS units, the open-circuit voltage, load current, and electrical output versus external load were all measured and calculated under the vibration excitation at 3 Hz with 1 g acceleration. For the PEG unit, a high peak open-circuit voltage of 64.1 V (root-mean-square of peaks over multiple cycles, hereinafter inclusive) can be generated during the compressed

stage, as shown in Fig. 5.10(a). At the released stage, only a relatively small reverse voltage can be generated and would slowly return to zero due to the hysteresis characteristics of piezoceramics. With the increase of load resistance, the maximum power of 25.37 mW was delivered from a PEG unit across an optimum load resistance of 35 k Ω (Fig. 5.10(b)), corresponding to the maximum current of 0.85 mA (Fig. 5.10(c)). For the EMG unit, due to the instantaneous change in velocity at the time of impact, a smaller peak occurred at the peak open-circuit voltage of 4.26 V as shown in Fig. 5.10(d). The obtained maximum power across an optimum load resistance of 210 Ω was 35.08 mW (Fig. 5.10(e)), while the corresponding maximum current was 12.92 mA (Fig. 5.10(f)). In Fig. 5.10(g), the peak open-circuit voltage levels of the TENG-F and TENG-CS units were 27.4 V and 40.3 V, respectively. Under an optimum load resistance of 19 M Ω , the maximum power of the TENG-F unit was 27.12 μ W with a maximum load current of 1.19 μ A, while the TENG-CS unit can reach to the maximum power of 60.99 μ W with a 3.59 μ A maximum load current at an optimum load resistance of 17 M Ω (Fig. 5.10(h) and 5.10(i)). Among the transducer units, the EMG unit produces higher output current but lower load voltage due to its low internal resistance, while the PEG and TENG generate higher load voltages but lower currents because of their high internal resistance. Although the EMG delivers greater power output than either the PEG unit or the TENG unit, it does so with increased volume and weight, as shown in Fig. 5.10(j) and 10(k).

In Fig. 5.10, the results show that the PEG units have the highest normalized power density, whether it is calculated based on the volume ratio (7.69 mW cm⁻³) or weight ratio (2.44 mW g⁻¹). The EMG unit ranks second with the normalized power density of 4.62 mW cm⁻³ and 1.63 mW g⁻¹. For TENGs, even though they account for only 0.65% and 0.51% of the total volume and weight, respectively, their normalized power density is quite low due to the relative-low charge density on the surface of plain FEP. Many methods have been reported to increase the surface charge density of a dielectric material, such as material selection (Wei et al., 2014), surface modification (Wang et al., 2016) and contact improvement (Tang et al., 2015), which could increase the charge density to hundreds of μ C m⁻² and boost the output power tens of times. Furthermore, coupling EMG with PEG and TENG takes good advantages under low frequencies in energy storage. Under an ultra-low frequency excitation, the EMG unit can only provide a low output voltage which might not be sufficient to efficiently drive an electronic load, but the PEG (with FUC approach) and TENG units can achieve a satisfactory output voltage level under such frequencies. This can solve the problem of voltage

insufficiency in ultra-low-frequency applications. Storing the harvested energy in a storage unit such as capacitors or rechargeable batteries, the low output voltage of EMG limits the maximum voltage stored in capacitors that can be optimized by hybridizing with the PEG (with the FUC approach) and TENG mechanisms.

It is worth noting that the normalized harvested power density of electromagnetic harvesters grows proportionally to the square of dimensions, but piezoelectric harvesters do not vary with dimensions (Dal Bo et al., 2020). Therefore, the proposed energy harvester can be further miniaturized to compare with those existing ones that mainly rely on EMG units. Owing to the full space sharing between PEG and EMG units, there is no need to provide additional space for the mechanical motion of inertia components. Under 1 g sinusoidal acceleration at 3 Hz, the overall normalized power density of $3.70 \text{ mW cm}^{-3} \text{ g}^{-2}$ can be resulted. Note that a considerable amount of volume and weight occupied in the fabricated prototype can be used to adjust the experimental parameters. Excluding the redundant volume and weight will further increase the power density over 10%.

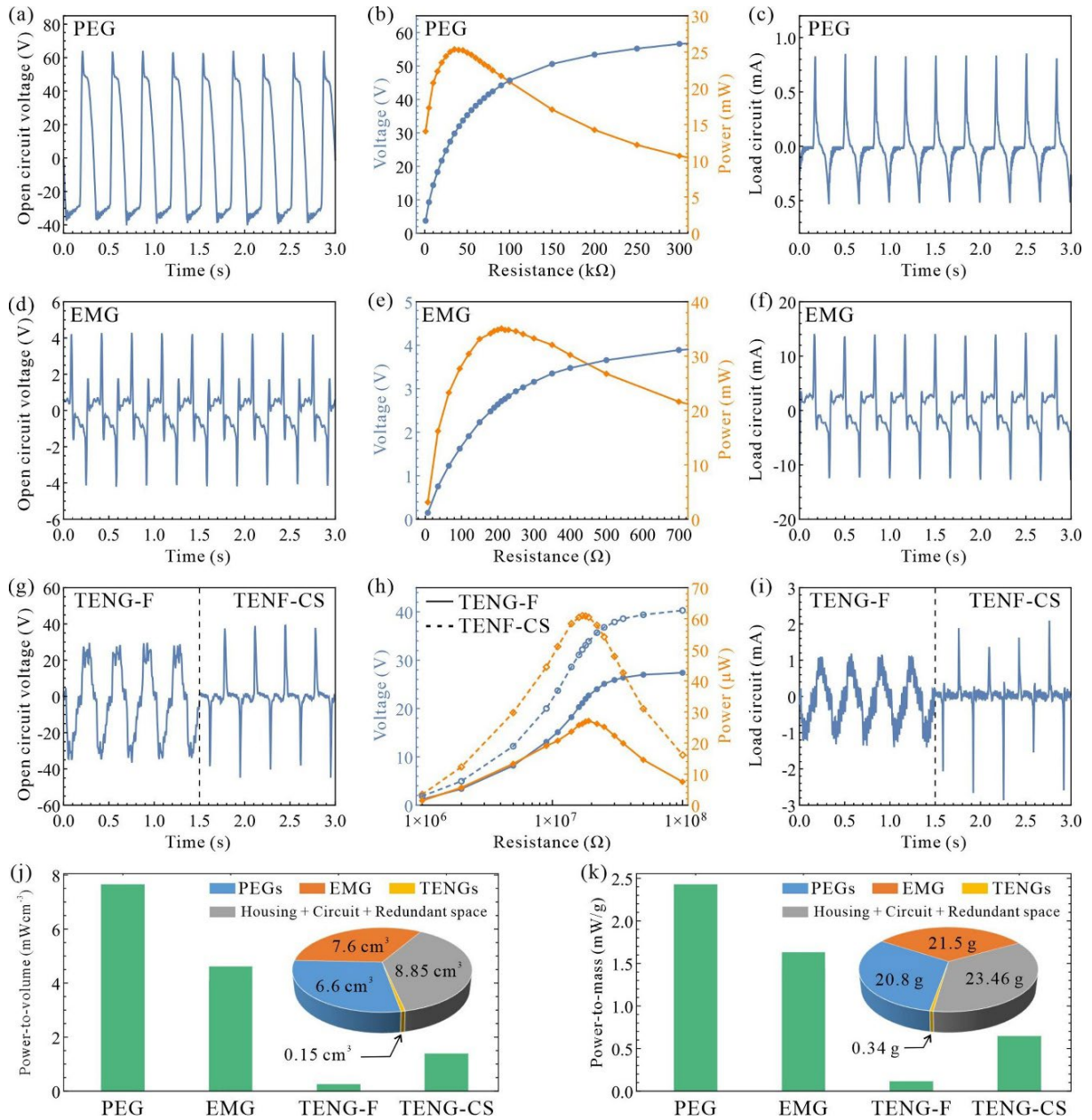


Fig. 5.10: Electrical performance of the tri-hybrid energy harvester: measured (a, d, g) open-circuit voltage and (c, f, i) load current for the individual units; (b, e, h) Dependence of the output voltage and maximum power of the individual units on variable external load resistance values; (j) Power-to-volume ratios of the PEG, EMG and TENG units across various optimum loads; and (k) Power-to-mass ratios of the PEG, EMG and TENG units. All the results were conducted under 1 g sinusoidal acceleration at 3 Hz in horizontal direction provided by an electrodynamic shaker.

5.3.2 Human motion test

To evaluate the energy generation performance of the present energy harvester, a series of experiments were conducted to investigate the capacitor charging behaviour under typical human motions, e.g., walking (5 km/h) and slow running (8 km/h). The fabricated prototype placed horizontally/vertically in a smartphone armband can be attached to arm, wrist, knee and ankle, as shown in Fig. 5.11(a, e, i, and m). When the harvester units of the prototype are working simultaneously, a rectifier bridge circuit is required for output hybridization to avoid a large internal power consumption (He et al., 2018). In this work, five bridge rectifier ICs (MB110S) were utilized to convert the AC outputs of each harvester unit into DC signals, and subsequently the rectified outputs were connected in parallel to a 560 μF storage capacitor (Fig. 5.12(a)).

The capacitor charging curves under various human motions were recorded by a digital oscilloscope, and the acceleration levels generated by the human-body-induced vibration were also measured by a 3-axis accelerometer (Dytran 3093 M10) simultaneously. The measured 3-axis acceleration waveforms, corresponding to different wearable positions under the walking and slow running motions, are shown in Fig. 5.11(b, f, j, and n) and Fig. 5.11(c, g, k, and o), respectively. In these figures, we observe that the slow running motion can generate a higher acceleration level along all the three axes to compare with the walking motion. Moreover, the lower wearable positions (i.e., ankle and knee) generated a higher acceleration level than the upper wearable positions (i.e., wrist and upper-arm). In this case, the fabricated prototype in the horizontal position (with a quad-stable mode) mainly utilized kinetic energy along the x - and y -axes. While in the vertical position, the fabricated prototype (with a tri-stable mode) mainly depended on the z -axis excitation.

Under the walking mode, the acceleration levels along all the three axes at the upper-arm position are lower than 0.5 g with a dominant frequency around 1 Hz. The fabricated prototype can only perform small-amplitude intra-well motions with a low voltage output. After a 64 s walking motion, the 560 μF capacitor can only be charged up to 2.9 and 8.3 V for the horizontal and vertical positions, respectively. Switching to the slow running mode, the peak acceleration levels along the x - and z -axes at the upper-arm position are higher than 1 g with the dominant frequencies of 1.5 and 2.4 Hz, respectively. The fabricated prototype can

successfully perform large-amplitude inter-well motions and the 560 μF capacitor can be charged up to 16 and 23.7 V for the horizontal and vertical positions, respectively.

At the wrist position, the acceleration levels along x - and z -axes under walking motions are similar. Both of them are over 0.8 g with the dominant frequencies of 1 and 1.5 Hz, respectively. Therefore, the 560 μF capacitor can be charged up to a similar level of 14.1 and 13.8 V for the horizontal and vertical positions, respectively. The vertical case shows a quicker charging rate at the early stage due to the higher working frequency, while the horizontal case performs a better long-term charging rate due to the higher voltage output at low frequencies. Under the slow running motion, the peak-to-peak acceleration levels at the wrist position increased to about 5 g and 3.5 g along the x - and z -axes with a dominant frequency of 2.4 and 1.5 Hz, respectively. These results show 22.1 and 19.2 V on the 560 μF capacitor for horizontal and vertical cases, respectively. For the lower wearable positions, the fabricated prototype is capable of performing large-amplitude inter-well motions under both walking and slow running motions. After a charging time of 64 s, all the cases can charge the 560 μF capacitor up to over 18 V (see Fig. 5.11((l) and (p)).

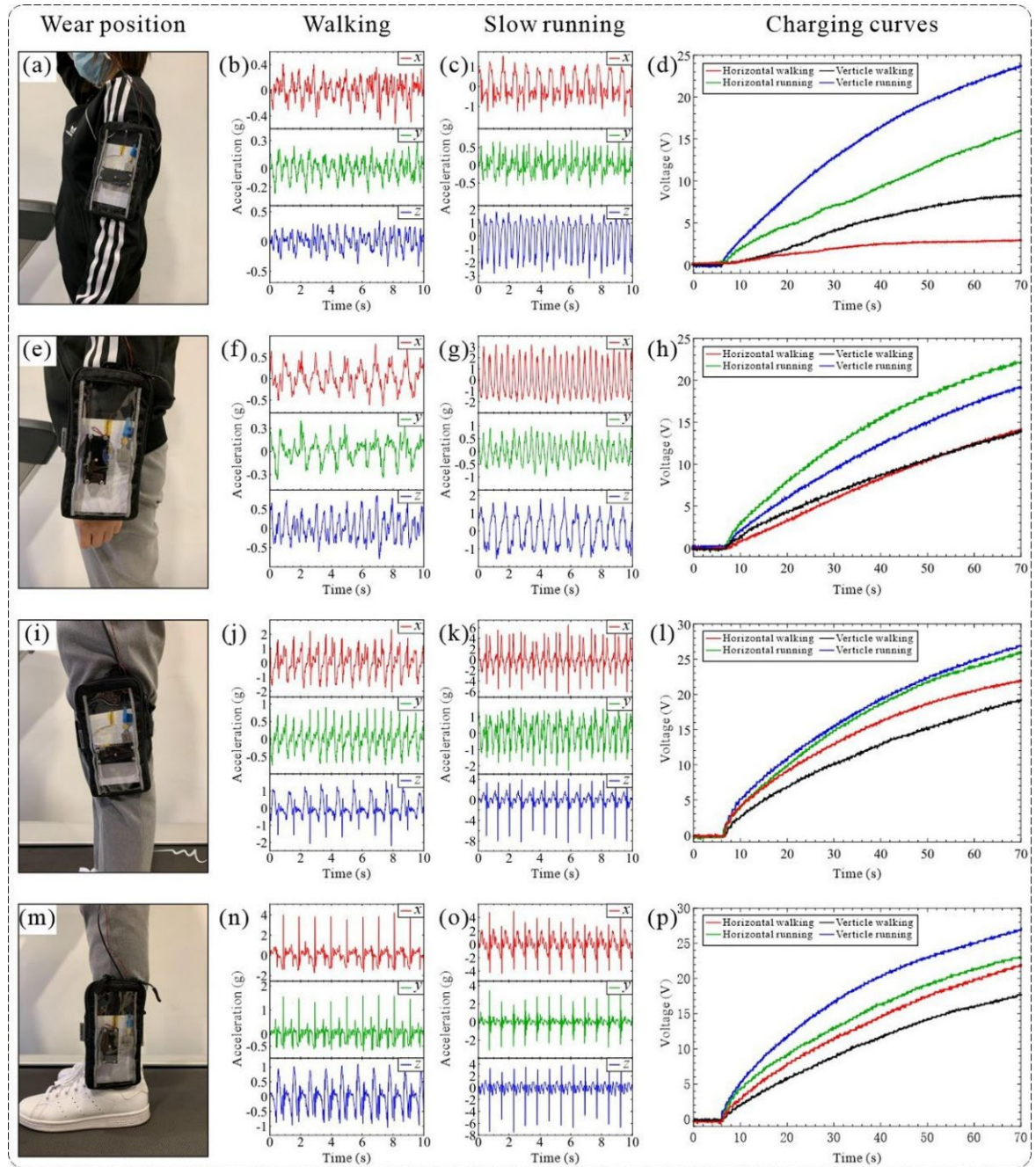


Fig. 5.11: Practical performance of the tri-hybrid energy harvester: (a, e, i, and m) photographs of the human motions on a treadmill with the prototype placed at different wearable positions; (b, f, j, and n) measured acceleration waveforms in walking motions with the prototype placed at different wearable positions; and (c, g, k, and o) measured acceleration waveforms in slow running motions with the prototype placed at different wearable positions.

5.3.3 Practical application to electronic devices

To evaluate the feasibility of the tri-hybrid energy harvester as a practical power source, the fabricated prototype was employed to drive several commercial electronic devices by harvesting human-induced biomechanical energy. In these demonstrations, five full-bridge rectifiers (MB110S) were used to convert the AC outputs of the individual transducers into DC signals. The rectified electricity was then stored in a capacitor (C), which was connected in parallel with the commercial electronics, as illustrated in Fig. 5.12(a). Figure 5.12(b) presents the charging profiles of a 560 μF capacitor powered by the individual harvesting units and by the hybrid harvester at 3 Hz under a 1 g acceleration. The present hybrid harvester offers significantly better charging capability than any individual unit. Its charging process consists of two stages:

- A turbulent charging stage where the EMG unit rapidly increases voltage due to high current output; and
- A trickle charging stage where the PEG and TENG units contribute with high voltage output despite low current.

Obviously, the TENG charges more slowly but maintains a consistently stable charging rate throughout.

The demonstration of powering three electronic thermohygrometers simultaneously using the fabricated prototype worn on the wrist during walking motion is shown in Fig. 5.12(b). In the demonstration, the charging characteristics of a 470 μF storage capacitor connected in parallel with the electronic thermohygrometers are shown in Fig. 5.12(c) and 12(d). It can be observed that these thermohygrometers can be turned on when the storage capacitor is up to 1.3 V after 8 s. However, due to the unstable trigger of inter-well motions under this excitation level, it can induce the fluctuation of the supply voltage. Sometimes, an excessive supply voltage level will cause an abnormal display on the electronic thermohygrometers. Therefore, a Zener diode was connected in parallel with the electronic thermohygrometers to maintain the supply voltage below 2 V. In addition, by handshaking, the present harvester can successfully brighten 296 LEDs (Fig. 5.12(e)) and power up 20 electronic thermohygrometers simultaneously (Fig. 5.12(f)).

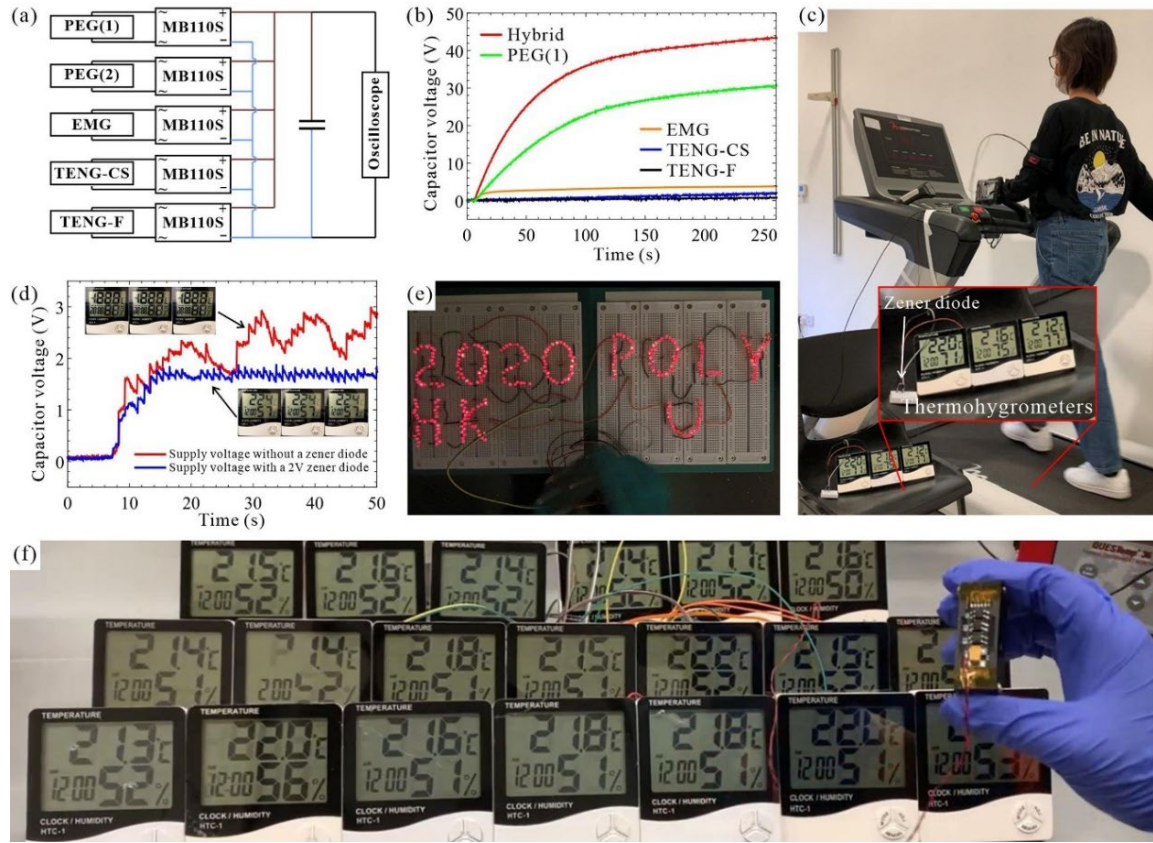


Fig. 5.12: Practical applications of the tri-hybrid energy harvester: (a) electrical circuit management design; (b) capacitor charging characteristics of the PEGs, EMG, TENGs and hybrid energy harvester at 3 Hz under 1 g acceleration; (c) three electronic thermohygrometers powered by the fabricated prototype worn on the wrist under walking (5 km/h) on a treadmill; (d) charging curves of a 470 μF capacitor while driving three electronic thermohygrometers connected in parallel with and without a Zener diode; (e) 296 LEDs driven simultaneously using the prototype under fast handshaking motions; and (f) 20 thermohygrometers powered by the prototype under slow handshaking motions.

A performance comparison between the proposed energy harvester and those recently reported hybrid energy harvesters (He et al., 2018; Rahman et al., 2020a; Rahman et al., 2020b; Tan et al., 2019; Wang et al., 2019) is presented in Table 5.2. The present device shows outstanding performance compared with other devices in terms of the operation bandwidth and normalized power density at low frequencies. The multi-stable nonlinear mechanism not only performs well in biomechanical energy harvesting applications, and also offers a high-efficiency operating bandwidth that can cover the vibration frequencies of general human

motions. In terms of durability, no significant damage of the fan-folded beams or magnetic slider is observed under a long period of experimental time.

Table 5.2: Comparison of the proposed tri-hybrid energy harvester with similar works.

Reference (Working Modes)	Operating Conditions	Performance Density
(Wang et al., 2019) (PEG + EMG + TENG)	2–12.5 Hz, 1 g	1.97 mW cm ⁻³ g ⁻² , 5 Hz
(Rahman et al., 2020b) (EMG + TENG)	3.5–7 Hz, 1 g	3.67 mW cm ⁻³ g ⁻² , 5 Hz
(Tan et al., 2019) (PEG + EMG + TENG)	1–5 Hz	2 mW cm ⁻³ g ⁻¹
(Tan et al., 2019) (PEG + EMG + TENG)	1–5 Hz, 1–4 g	2 mW g ⁻¹ , 5 Hz
(Rahman et al., 2020a) (EMG + TENG)	4–9 Hz, 1 g	3.13 mW cm ⁻³ g ⁻² , 6 Hz
Present Work (PEG + EMG + TENG)	1–11 Hz, 1 g	3.70 mW cm ⁻³ g ⁻² , 3 Hz

5.4 Concluding Remarks

In this chapter, we present a novel miniaturized tri-hybrid vibration-based energy harvester (Wang et al., 2021a). A multi-stability-based frequency up-conversion approach, utilizing a compact combination of two new magneto-multi-stable oscillator configurations, was proposed to enhance the dynamic performance of the energy harvester under ultra-low-frequency and broadband vibrations. By integrating various units into a single multi-stable mechanical system, significant improvements in power generation capability can be achieved with the same mechanical motions. Theoretical models have been developed to analyze the dynamic and electrical characteristics of the harvester in tri-stable and quad-stable modes. The fabricated prototype was tested under electrodynamic shaker and human-body-induced vibrations in both tri-stable (vertical) and quad-stable (horizontal) states. The key findings of this study are summarized as follows:

- In the electrodynamic shaker test, both dynamical modes provide a high-efficiency operating bandwidth covering 1–11 Hz at 1 g acceleration. The tri-stable mode is more easily triggered into large-amplitude inter-well motions under low-intensity excitation, resulting in higher energy output. Conversely, when the external excitation is sufficiently high to activate the quad-stable mode, it outperforms the tri-stable state.

- Under a 3 Hz excitation with an acceleration of 1 g, the PEG, EMG, TENG-F and TENG-CS units can generate the maximum output power of 25.37 mW, 35.08 mW, 27.12 μ W and 60.99 μ W across an optimum load resistance of 35 k Ω , 210 Ω , 19 M Ω and 17 M Ω , respectively. The whole prototype can reach the maximum output power of 85.9 mW, corresponding to the normalized power density of 3.70 mW cm⁻³ g⁻² at such excitation conditions.
- Among all units, the PEG unit has the highest normalized power density, whether it is calculated either on the volume ratio (7.69 mW cm⁻³) or the weight ratio (2.44 mW g⁻¹). While the EMG ranks second with the normalized power density of 4.62 mW cm⁻³ (volume ratio) and 1.63 mW g⁻¹ (weight ratio). Owing to the highly efficient PEGs, the proposed energy harvester is flexible for miniaturization to compare with the mainstream vibration-based energy harvesting techniques that mainly rely on the function of EMG units.
- During tests of human motions, the energy harvester was positioned both horizontally and vertically for different body parts (arm, wrist, knee, and ankle). It effectively charged a 560- μ F storage capacitor to specific voltage levels due to the acceleration from walking and slow running. Additionally, the harvester was able to continuously power 20 thermohygrometers and 296 LEDs under various random handshaking motions.
- The tri-stable mode is more readily triggered into large-amplitude inter-well motions, resulting in higher energy output under sinusoidal excitation. When the external excitation is sufficient to activate the quad-stable mode, it demonstrates superior wideband and low-frequency performance compared to the tri-stable mode. Moreover, the quad-stable mode provides a more uniform distribution of potential energy, which can mitigate imbalances resulting from assembly errors. Consequently, the choice between these two modes should depend on the operating environment and the precision of manufacturing.

CHAPTER 6

-PINN-BASED OPTIMIZATION FOR TRI-HYBRID ENERGY HARVESTING DESIGN

6.1 Introduction

Driven by the rapid advancements in artificial intelligence and high-performance computing hardware, deep learning has made remarkable strides in fields, for example language translation and processing (Hameed et al., 2025), graphical vision and identification (Liu et al., 2023), and autonomous driving (Zhao et al., 2025). In recent years, its application has swiftly extended into scientific computing, where neural network-based methods for solving partial differential equations (PDEs) have emerged as a powerful tool. The foundational theory of Physics-informed Neural Networks (PINNs) was introduced by Raissi and his research associates (Raissi et al., 2019). This work introduced two forms of the PINN method, i.e., the discrete form and the continuous form. These two forms were then applied to solve a series of classical partial differential equations in quantum mechanics and fluid mechanics. At their core, PINNs embed physical constraints, including PDEs, initial conditions, and boundary conditions, into the loss function of a neural network. This integration constrains the training process of the neural network and enables it to learn the mapping relationship from the spatial-temporal domain to the solution of PDEs. Notably, this approach requires only a minimal amount of labeled data or can even operate without any labeled data. In contrast to purely data-driven deep learning methods for solving PDEs (Moseley et al., 2020b; Prachaseree and Lejeune, 2022), PINNs can mitigate the reliance on labeled data while maintaining strong physical interpretability and generalization capabilities. To date, after several years of development, PINNs have been utilized to solve a wide range of PDEs, including fractional (Pang et al., 2019), stochastic (Karumuri et al., 2020; Yang et al., 2020), and integro-differential problems (Yuan et al., 2022). Furthermore, it has found extensive applications in various engineering fields, including fluid mechanics (Mao et al., 2020), structural mechanics (Gu et al., 2022; Viana et al., 2021), biomedicine (Liu et al., 2020; Sahli Costabal

et al., 2020), wave dynamics (Moseley et al., 2020a; Ovadia et al., 2021), metamaterials (Chen et al., 2020; Fang et al., 2019), and solid mechanics (Wei et al., 2023).

To improve the training efficiency, method robustness, and solution accuracy of PINNs, researchers have introduced a variety of extensions and enhancements built upon the original PINN framework (Raissi et al., 2019). These refinements mainly involve adopting novel network architectures (Wang et al., 2021c; Wang et al., 2021d; Yang et al., 2021b), decomposing the equation domain into multiple subdomains (Karniadakis, 2020; Moseley et al., 2023), modifying the sampling strategy for input data (Nabian et al., 2021; Zhang et al., 2022), adaptively adjusting the weights of the loss function (Linka et al., 2022; Maddu et al., 2022; McClenny et al., 2023), constructing diverse forms of loss functions (Li et al., 2021a; Samaniego et al., 2020; Zhuang et al., 2021), improving the activation functions of neural networks (Huang et al., 2021; Jagtap et al., 2020b; Liang et al., 2024), and enforcing initial and boundary conditions as hard constraints (Moseley et al., 2020b; Pang et al., 2019; Rao et al., 2021; Sheng et al., 2021; Zhang et al., 2022). These strategies have contributed to varying degrees of performance improvement in the PINN method, leading to advanced variants such as A-PINN (Yuan et al., 2022), B-PINN (Yang et al., 2021b), FBPINN (Moseley et al., 2023), GW-PINN (Zhang et al., 2022), PPINN (Meng et al., 2020), VPINN (Kharazmi et al., 2021), and XPINN (Karniadakis, 2020). Consequently, these developments enable the PINN method to address more complex and challenging problems.

In the study of applying neural networks to solve PDEs, methods that utilize a purely data-driven approach have initially gained considerable attention. As shown in Fig. 6.1, this type of method requires prior knowledge of certain solutions to PDEs. By training a neural network to approximate these known solutions, the resulting model can serve as a surrogate for physical systems, thereby enabling the solution of the equations. However, purely data-driven methods often suffer from two major issues:

- First, the methods heavily rely on data samples. Developing an accurate surrogate model for a complex physical system generally requires an extensive dataset. However, obtaining a large number of data samples in reality often comes at a very high cost or may even be infeasible.

- Second, such algorithms sometimes lack robustness, cannot guarantee the convergence of the algorithm, and the generalization ability of the model is poor. In addition, the results of purely data-driven methods often exhibit limited physical interpretability after training.

To mitigate this reliance on extensive data and enhance model interpretability, physics-driven approaches including PINNs, Physics-Guided Neural Networks (PGNN) (Gupta et al., 2024), and Physics-Encoded Neural Networks (PENNN) (Faroughi et al., 2022) have gained significant attention. During the training process of PINNs, by minimizing the loss function, the neural network ensures adherence to the governing physical laws. Consequently, PINNs can solve PDEs and provide solutions at any location within the domain using very few data samples, or in some cases, without any data samples, as illustrated in Fig. 6.2. Compared with purely data-driven neural network methods, PINNs reduce dependence on large datasets while preserving robust physical interpretability. Therefore, this approach represents a highly promising direction in the field of artificial intelligence.

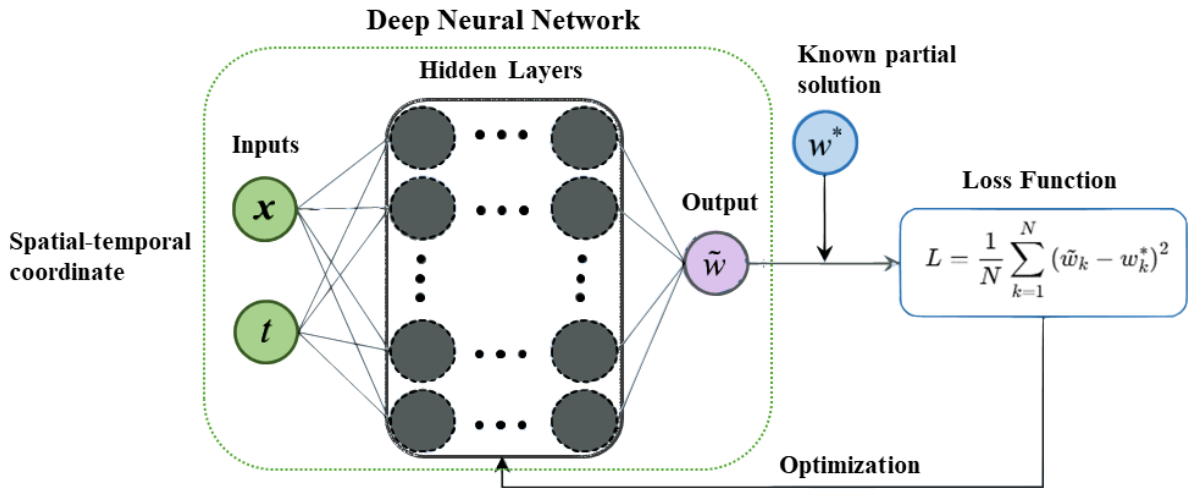


Fig. 6.1: Schematic diagram of a purely data-driven neural network method.

Despite their advantages, the standard PINN (STD-PINN) approach still encounters technical problems in the analysis of long-duration dynamic systems, where cumulative numerical errors may accumulate over time, compromising prediction accuracy. This limitation is especially significant for real-world engineering challenges that demand dependable long-term predictions, such as the control and optimization of complex dynamical systems. A representative case is vibration-based energy harvesting, which involves the interaction of multiple physical domains, including structural vibrations, nonlinear transduction mechanisms, and material-dependent electrical behavior. Traditional optimization techniques for such systems typically resort to time-consuming iterative simulations or empirical surrogate modeling, which often lack scalability and physical interpretability.

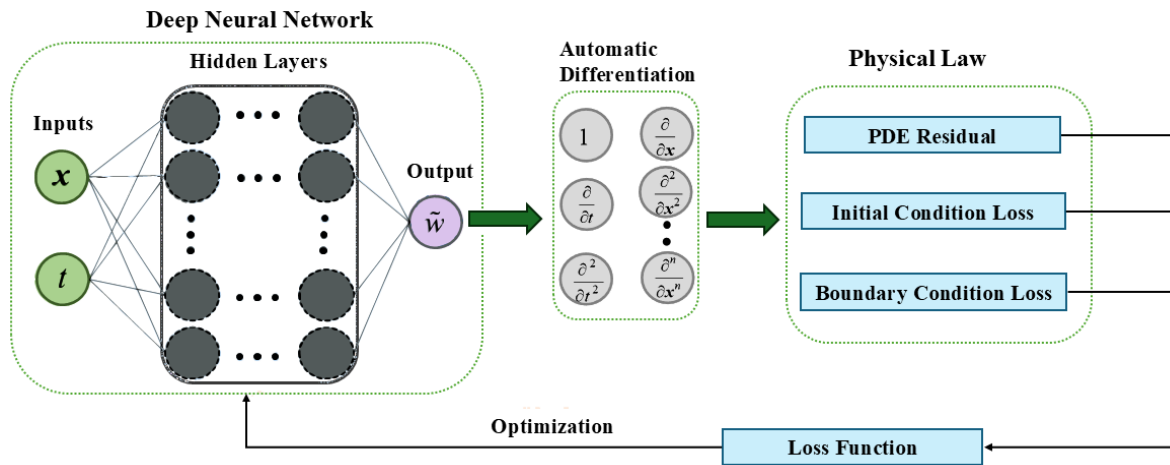


Fig. 6.2: Schematic diagram of a PINN framework.

PINNs offer a mesh-free and fully differentiable surrogate framework capable of capturing coupled physics information while simultaneously enabling efficient optimization. This makes them well-suited for modeling hybrid energy harvesters, where nonlinearities and multi-physical couplings are dominating factors. Recent studies have employed PINNs for both inverse design and parameter identification in these systems, demonstrating that physically constrained learning can reveal design configurations that are non-intuitive yet highly efficient. By leveraging automatic differentiation, PINNs can also provide precise sensitivity information with respect to design parameters, eliminating the need for traditional finite-difference or adjoint methods in gradient-based optimization workflows. For example, in the modeling of tri-hybrid energy harvesters, PINNs can be used to capture the interactions across multiple

energy domains and to identify parameter sets that maximize output power under varying excitation conditions. These physics-informed models not only reduce the dependency on computationally intensive solvers but also improve generalization and robustness in previously unexplored design regions. Therefore, the integration of PINNs into the modeling and optimization of energy harvesting systems offers a powerful strategy to overcome the limitations of conventional approaches. Their capacity to unify data, information, and optimization in a single end-to-end framework highlights their transformative potential for the development of next-generation smart energy devices and the broader adoption of physics-guided machine learning in multi-coupling engineering applications.

In what follows, Section 6.2 introduces the standard PINN method and improved AT-PINN method. Section 6.3 discusses the combination of the AT-PINN and genetic algorithm. Section 6.4 provides the prediction results and analysis of the design parameters for the energy harvesting system using the AT-PINN method. Finally, Section 6.5 summarizes the key research findings and outlines future prospects of this work.

6.2 Theoretical frameworks of PINN approaches

6.2.1 Standard PINN (STD-PINN)

In this section, the standard PINN approach is briefly reviewed. Consider the following PDE associated with its boundary and initial conditions:

$$\begin{aligned} N[w(\mathbf{x}, t)] &= f(\mathbf{x}, t), & (\mathbf{x}, t) &\in \Omega \times (0, T] \\ \mathcal{B}[w(\mathbf{x}, t)] &= g(\mathbf{x}, t), & (\mathbf{x}, t) &\in \partial\Omega \times (0, T] \\ \mathcal{D}[w(\mathbf{x}, 0)] &= h(\mathbf{x}), & \mathbf{x} &\in \Omega \end{aligned} \tag{6.1}$$

In the equation, Ω denotes a bounded domain in space, and $\partial\Omega$ represents its boundary; $\mathbf{x}$ is the spatial coordinate vector; t denotes the time coordinate; $\Omega \times (0, T]$ defines the space-time domain, which serves as the domain of definition for the partial differential equation; $N[w(\mathbf{x}, t)]$ represents the differential operator within the interior of the domain, $\mathcal{B}[w(\mathbf{x}, t)]$ denotes the differential operator on the boundary, and $\mathcal{D}[w(\mathbf{x}, 0)]$ corresponds to the differential operator at the initial time; $w(\mathbf{x}, t)$ is the solution function of the partial differential equation.

The neural network defines the mapping relationship from input to output. In the PINN method, it defines the mapping relationship from the spatiotemporal coordinates $(\mathbf{x}, t)$ to the approximate solution $\tilde{w}$ of the partial differential equation. $\tilde{w}(\mathbf{x}, t)$ is a continuously differentiable function represented by a neural network. The automatic differentiation technique of the neural network (Baydin et al., 2018) can be used to calculate the derivatives of all orders of the network output $\tilde{w}(\mathbf{x}, t)$ with respect to each input variable, and then substitute them into the known partial differential equation and its initial and boundary conditions in Eq. (6.1) to construct the loss function. Generally speaking, the loss function of the PINN method consists of three parts: the loss of the partial differential equation, the loss of the boundary conditions, and the loss of the initial conditions. The loss function is often defined by the mean squared error as follows

$$L(\theta) = L_r(\theta) + \lambda_b L_b(\theta) + \lambda_i L_i(\theta) \quad (6.2)$$

where

$$L_r(\theta) = \frac{1}{N_r} \sum_{k=1}^{N_r} \{\mathcal{N}[\tilde{w}(\mathbf{x}_k^r, t_k^r)] - f(\mathbf{x}_k^r, t_k^r)\}^2, (\mathbf{x}_k^r, t_k^r) \in \Omega \times (0, T] \quad (6.3)$$

$$L_b(\theta) = \frac{1}{N_b} \sum_{k=1}^{N_b} \{\mathcal{B}[\tilde{w}(\mathbf{x}_k^b, t_k^b)] - g(\mathbf{x}_k^b, t_k^b)\}^2, (\mathbf{x}_k^b, t_k^b) \in \partial\Omega \times [0, T] \quad (6.4)$$

$$L_i(\theta) = \frac{1}{N_i} \sum_{k=1}^{N_i} \{\mathcal{D}[\tilde{w}_n(\mathbf{x}_k^i, 0)] - h(\mathbf{x}_k^i)\}^2, \mathbf{x}_k^i \in \Omega \quad (6.5)$$

where $L_r(\theta)$, $L_b(\theta)$ and $L_i(\theta)$ are the PDE loss, the boundary condition loss and the initial condition loss, respectively. The hyperparameters λ_b and λ_i are the weights of boundary and initial condition losses, which play a key role in balancing each loss function component. N_r , N_b and N_i are the numbers of sampling points corresponding to the PDE loss, the boundary condition loss and the initial condition loss respectively. $(\mathbf{x}_k^r, t_k^r)$, $(\mathbf{x}_k^b, t_k^b)$ and $(\mathbf{x}_k^i, 0)$ are the coordinates of the sampling points for calculating the PDE loss, the boundary condition loss and the initial condition loss, respectively. With the goal of minimizing the loss function $L(\theta)$, the trainable parameters θ of the neural network are continuously adjusted through an optimization algorithm to find the optimal parameter θ^* , that is

$$\theta^* = \arg \min L(\theta) \quad (6.6)$$

When $L(\theta)$ is very small, it can be considered that the mapping relationship represented by this neural network can approximately satisfy the partial differential equation and the initial and boundary value conditions given in Eq. (6.1). Using this neural network, the value of $\tilde{w}(\mathbf{x}, t)$ corresponding to any spatiotemporal coordinates $(\mathbf{x}, t)$ within the domain of definition can be obtained, and it is taken as the approximate solution of Eq. (6.2).

6.2.2 Advanced Time-Marching PINN (AT-PINN)

Existing PINN methods suffer from the problem of addressing PDEs in long-time simulations. As a time-dependent issue, vibration problems fall under hyperbolic PDEs and typically demand precise solutions over extended temporal scales. Currently, there are only a limited number of studies focused on solving vibration equations using PINNs. While some researchers have begun to explore the application of PINNs in vibration analysis, their findings predominantly remain restricted to determining natural vibration characteristics (Jagtap et al., 2020a; Li et al., 2021a). To address the limitations of existing PINNs for vibration analysis in long-term simulations, the advanced time-marching PINN (AT-PINN) approach (Chen et al., 2024; Chen et al., 2025) has been recently proposed. This method exhibits remarkable accuracy and efficiency in both forced and free vibration analyses during extended simulations. The following four techniques are integrated into this method:

- **Spatiotemporal Domain Normalization:** This approach can stabilize the spatiotemporal domain within each time interval by applying spatiotemporal domain normalization, which greatly improves both accuracy and training efficiency.
- **Reactivating Optimization Algorithm:** To prevent the issue of getting stuck in locally optimal solutions, an effective optimization algorithm is implemented to restart the optimization process after convergence to further decrease the loss value and minimize the cumulative error.
- **Transfer Learning Technique:** To reduce training time, using spatiotemporal domain normalization enhances the consistency of vibration analysis across various time intervals, this facilitates smooth integration with transfer learning.
- **Appropriate Activation Function:** Selection of an appropriate activation function can affect the solution accuracy and computational efficiency of predicted results in dynamic analysis. This combination of techniques guarantees the robustness and effectiveness of the AT-PINN method in solving complex dynamic equations over long simulation durations.

This section will provide a brief review of the fundamental principles underlying the AT-PINN method, as well as the above four techniques. A graphical illustration of the AT-PINN method is depicted in Fig. 6.3.

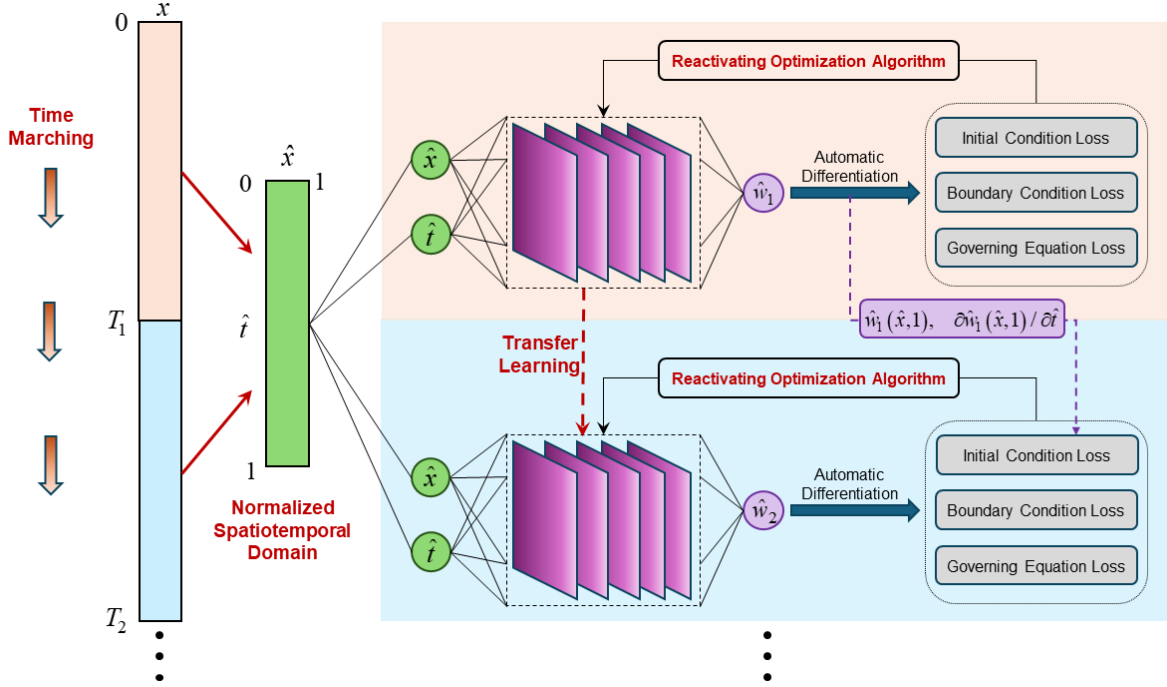


Fig. 6.3: Schematic diagram of the AT-PINN method

When solving PDEs involving time variables, as the analysis duration T increases, the spatial-temporal domain progressively expands. This expansion leads to a significant increase in the complexity of the mapping relationship between inputs and outputs, necessitating a higher sampling density to maintain accuracy. Consequently, this not only intensifies the challenges in neural network training but also dramatically escalates the demand for computational resources. As a result, the solution accuracy of the STD-PINN method typically diminishes with the prolongation of the analysis duration, thereby restricting its applicability for long-term simulations. To mitigate this limitation, the optimized method has been introduced (Krishnapriyan et al., 2021; Matthey et al., 2022; Zhao, 2021). Considering the AT-PINN method, the entire spatiotemporal domain is divided into M consecutive time intervals, as illustrated in Fig. 6.4.

$$[T_0 = 0, T_1], [T_1, T_2], \dots, [T_{n-1}, T_n], \dots, [T_{M-1}, T_M = T] \quad (6.7)$$

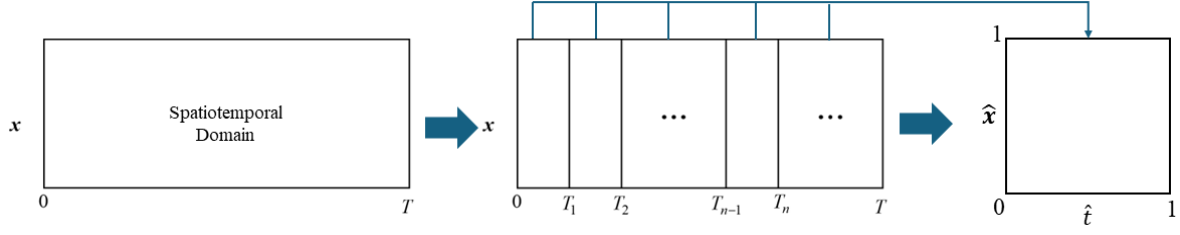


Fig. 6.4: Normalized spatiotemporal domains of the AT-PINN method.

This method does not train the neural network over the entire spatiotemporal domain at once. Instead, it sequentially trains the neural network for each individual time period. The core idea is to train the network and solve the partial differential equation within a single time period at each step, using the solution obtained at the end of the previous time period as the initial condition for the subsequent one. However, the cumulative error in solving equations tends to increase progressively over time. This phenomenon has been documented in several studies (Krishnapriyan et al., 2021; Matthey et al., 2022; Wang et al., 2024a; Zhao, 2021). Matthey and Ghosh (Matthey et al., 2022) noted that the error at the end of the n th time interval is propagated to the $(n+1)^{\text{th}}$ time interval, thereby affecting the accuracy of the initial conditions for that interval. As a result, the overall error accumulates gradually with each successive time interval. Normalization is a widely utilized preprocessing technique in data-driven deep learning methods, aimed at handling raw data with varying scales. To mitigate spatiotemporal domain offsets, the *min-max* normalization approach, transforming the spatiotemporal domain of each time period into a standardized unit spatiotemporal domain is presented in Fig. 6.4, which represents the spatial and temporal coordinates of the normalized domain, respectively. For the n^{th} time period, these coordinates are defined as follows:

$$\begin{cases} \hat{x} = \frac{x - x_{\min}}{\Delta x} \\ \hat{t} = \frac{t - T_{n-1}}{\Delta T_n} \end{cases} \quad (6.8)$$

In Eq. (6.8), $\Delta x = x_{\max} - x_{\min}$, $\Delta T_n = T_n - T_{n-1}$ is the time interval of the n^{th} time period. After adopting this normalization technique, each time period has the same normalized spatiotemporal domain, $\hat{x} \in [0, 1]$, $\hat{t} \in [0, 1]$. After obtaining the solution $\hat{w}_n$ of the n^{th} time period by the AT-PINN method, the solution of the partial differential equation in the original space-time domain can be obtained as

$$\begin{cases} w(x, t) = \hat{w}_n(\hat{x}, \hat{t}) \\ x = \hat{x}\Delta x + x_{min} \\ t = \hat{t}\Delta T_n + T_{n-1} \end{cases} \quad (6.9)$$

Basically, solving PDEs using PINNs is an optimization problem focused on minimizing the loss function. For complex problems, PINNs may converge to a local optimum instead of the global optimum. In some cases, this local optimum can closely approximate the exact solution; however, in other instances, it may significantly deviate from the global optimum, resulting in considerable errors. To address this challenge, the AT-PINN method incorporates a reactivation optimization algorithm. When the optimization process meets the convergence criterion but becomes trapped in a local optimum, the algorithm reactivates the optimization by selectively relaxing the line search conditions. This allows the method to escape the local optimum and continue searching for an improved solution. Figure 6.5 illustrates the details of the reactivation optimization algorithm.

Definition: $L_{current}$ is the current value of the loss function. $L_{previous}$ is the last saved loss function value and initial value is 0. ε is the convergence coefficient of this algorithm.

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while the current iteration number is less than the maximum iteration number do
|   execute the L-BFGS optimization algorithm and the strong Wolfe line search condition to obtain  $L_{current}$ 
|   if meet the convergence conditions of the L-BFGS optimization algorithm then
| |   if  $|L_{previous} - L_{current}| > \varepsilon L_{previous}$  then
| | |    $L_{previous} = L_{current}$ 
| | |    $L_{previous} = (1+\gamma) L_{current}$ 
| |   else
| | |   break
| |   end
|   end
end

```

Fig. 6.5: Reactivating optimization algorithm.

Taking the L-BFGS optimizer with the strong Wolfe line search embedded as an example (Tankaria et al., 2022), the strong Wolfe line search conditions at the k^{th} iteration are

$$L(\theta_k + \alpha_k d_k) \leq L(\theta_k) + c_1 \alpha_k d_k^T \nabla L(\theta_k) \quad (6.10)$$

$$|d_k^T \nabla L(\theta_k + \alpha_k d_k)| \leq c_2 |d_k^T \nabla L(\theta_k)| \quad (6.11)$$

where $d_k = -H_k^{-1} \nabla L(\theta_k)$ is the searching direction, H_k is an array, c_1 and c_2 ($0 < c_1 < c_2 < 1$) are the constants of the strong Wolfe conditions. Based on this, we can search for the step size α_k , which can ensure the objective function value $L(\theta_k + \alpha_k d_k)$ at the next step less than the objective function value $L(\theta_k)$ at this step. If the optimization process has fallen into a local optimal solution at the start of the k th iteration, then the step size α_k that satisfies Eq. (6.10) and Eq. (6.11) cannot be found, and the optimization process will stop. To restart the optimization process, we introduce a magnification factor γ (slightly greater than zero), which modifies Eq. (6.10) to (Chen et al, 2024)

$$L(\theta_k + \alpha_k d_k) \leq (1 + \gamma) L(\theta_k) + c_1 \alpha_k d_k^T \nabla L(\theta_k) \quad (6.12)$$

In Eq. (6.12), it is a more relaxed condition than Eq. (6.10). This allows $L(\theta_k + \alpha_k d_k)$ to be slightly greater than $L(\theta_k)$ so that it is easier to find the step size α_k that satisfies both Eq. (6.11) and Eq. (6.12). It should be noted that Eq. (6.12) and Eq. (6.11) are only used when reactivating the optimization process. After the optimization process is activated, Eq. (6.10) and Eq. (6.11) are still used to find the step size α_k to continue the subsequent optimization process. As shown in Fig. 6.6, the schematic diagrams of the reactivation optimization algorithm are graphically illustrated.

The reactivation optimization algorithm can be applied iteratively within a single optimization problem. As illustrated in Fig. 6.6, the introduction of the reactivation mechanism alters the termination criterion of the original optimization process. Specifically, the convergence condition of the L-BFGS algorithm no longer serves as the stopping criterion for the entire optimization procedure; instead, it functions as the trigger for initiating the reactivation optimization algorithm. To determine when to terminate the overall optimization process, we introduce a small convergence coefficient ε in Fig. 6.5, which is used to evaluate the effectiveness of each reactivation step. If the difference between the loss function values at two consecutive reactivations falls below $\varepsilon L_{\text{previous}}$, it suggests that further reactivation of the L-BFGS optimizer

will not yield significant improvements, and thus the optimization process is terminated. Compared with the standard L-BFGS optimizer equipped with the strong Wolfe line search, the proposed reactivation optimization algorithm can more effectively avoid local optima, achieve a lower value of the loss function, and consequently provide a more accurate solution to the partial differential equation. Moreover, although the current implementation of the reactivation strategy is based on the L-BFGS optimizer with strong Wolfe line search, similar approaches can be adapted to other line search-based optimization algorithms by relaxing the line search conditions accordingly.

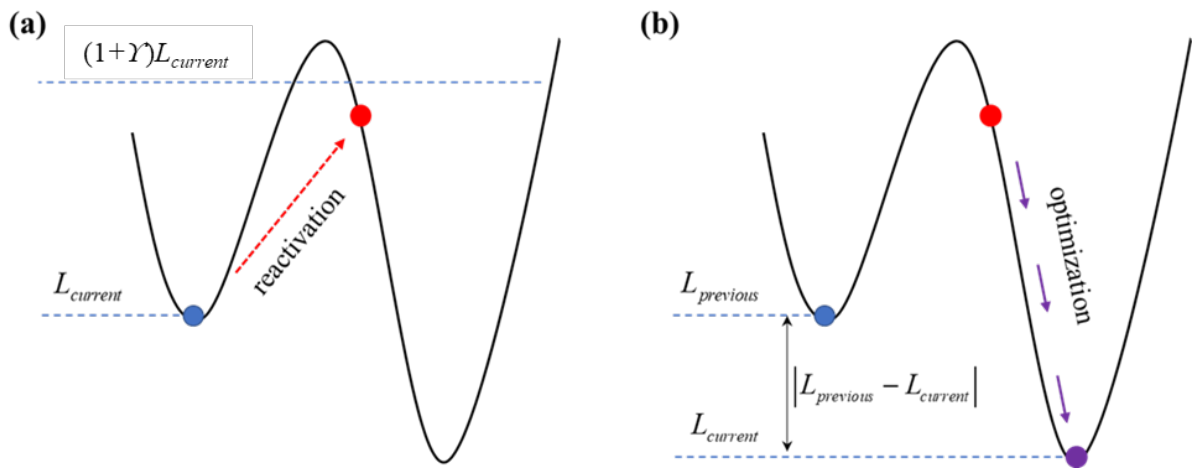


Fig. 6.6: Schematic diagrams of optimization algorithm: (a) reactivating optimization; (b) continue optimization.

When the AT-PINN method divides the entire spatial-temporal domain using a fixed time interval ΔT , the only differences between consecutive time intervals are the external forces and initial conditions. The partial differential equations, boundary conditions, and coordinates of the sampling points remain unchanged across each time interval. Consequently, there exists a certain degree of similarity in the input-output mapping relationships between adjacent time intervals. Leveraging this similarity, it proposes the application of transfer learning techniques (Hosna et al., 2022) within the AT-PINN framework to enhance training efficiency and reduce computational cost. As shown in Fig. 6.7, after completing the training of the neural network in the $(n-1)^{\text{th}}$ time period, the trainable parameter θ of the neural network is extracted and transferred for application in the n^{th} time period as the initial value of the neural network. With this transfer learning strategy, only in the first time period is it necessary to start training from a

standard initialized neural network, and in other time periods, training can start from a similar neural network, thus significantly reducing the training time. It should be noted that in the STD-PINN method, the spatiotemporal domain of each time period is different, so the input of the neural network is different. Therefore, the mapping relationship between the input and output among different time periods will vary greatly. As a result, when transfer learning is applied to the STD-PINN method, the effect is not obvious. Prior to applying this transfer learning strategy, the entire spatiotemporal domain must be segmented using consistent time intervals, and each time segment must be normalized. This also highlights the merit of AT-PINN to show its strong compatibility with the transfer learning technique.

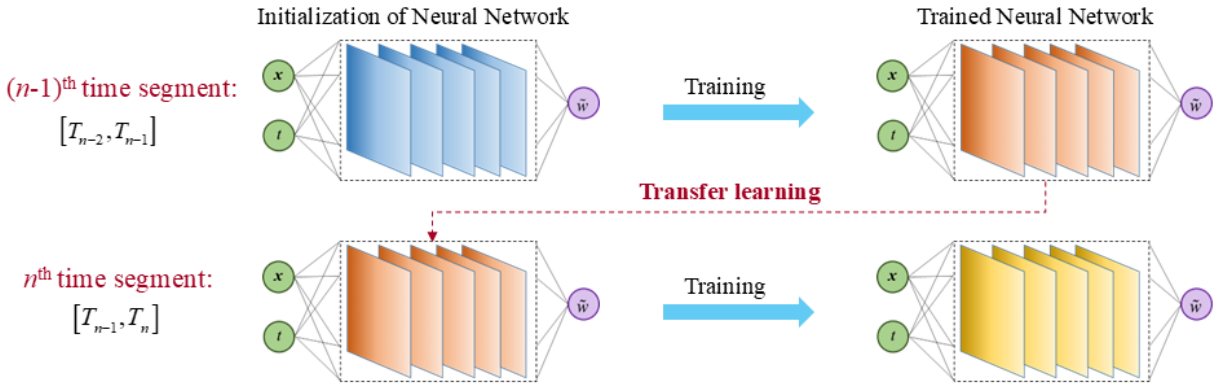


Fig. 6.7: Transfer learning strategy used in AT-PINN.

When applying PINNs to solve partial differential equations, the choice of the activation function σ significantly influences the accuracy of model performance. The hyperbolic tangent (*tanh*) function is most commonly used in PINNs due to its continuous differentiability. Other frequently used activation functions are *sine*, *tanhshrink*, *sigmoid*, and *softplus*. Notably, the *sine* activation function and all of its higher-order derivatives exhibit periodicity, which offers distinct advantages when solving partial differential equations involving time variables using PINNs. Therefore, the AT-PINN method employs the “*sine*” activation function. Given that a comprehensive comparison of different activation functions used for structural dynamic analysis has been presented (Chen et al, 2024), the analysis is not repeated in this section.

6.3 AT-PINN-GA parametric optimization technique

In this study on optimizing tri-hybrid energy harvesting systems using PINNs, the Genetic Algorithm (GA) is incorporated for optimizing system parameters. Tri-hybrid energy harvesters that combine multiple transduction mechanisms involve complex multi-physics coupling interactions. To maximize energy conversion efficiency, it is necessary to optimize factors like material properties, geometric configurations, and the arrangement of mechanical and electrical components. Traditional optimization methods often depend on iterative simulations that are computationally intensive and time-consuming, and they may struggle to accurately model the nonlinear and coupled behaviors characteristic of these systems. By emulating natural selection, GA efficiently explores large, nonlinear, and high-dimensional design spaces, increasing the likelihood of identifying globally optimal or near-optimal solutions. Additionally, in energy harvesting applications, GA can optimize parameters such as the dimensions of harvesting components, the placement of magnets and coils, and the contact area in triboelectric harvesters. Unlike gradient-based methods, which are more susceptible to becoming trapped in local minima, GAs conduct a broad search across the solution space, ensuring the discovery of the best-performing configurations.

Using GA is a class of optimization techniques inspired by the process of natural evolution. This method operates through a population of candidate solutions (called chromosomes) that evolve over successive generations (Katoch et al., 2021). The successive steps of GA include the following procedures:

- **Selection:** Individual samples from the population are selected according to their fitness, which is assessed using an objective function. The higher the fitness, the more likely the sample will be selected to reproduce. This mimics the process of natural selection, where the fittest samples have a greater likelihood of surviving and transmitting their genes to the next generation.
- **Crossover:** The selected individuals undergo crossover, where parts of their genetic material are exchanged to produce offspring. This step combines the traits of parent solutions, allowing the algorithm to explore new regions of the search space and discover potentially better solutions.

- **Mutation:** Occasionally, minor random alterations are applied to the offspring's genetic material. This promotes diversity within the population and helps prevent the algorithm from prematurely converging on suboptimal solutions.
- **Fitness Evaluation:** Each candidate solution is evaluated based on its effectiveness as determined by the objective function. In the optimization of energy harvesting systems, the fitness function typically evaluates the energy conversion efficiency or the output power of a tri-hybrid energy harvester.

The iterative process of selection, crossover, mutation, and fitness evaluation continues for several generations, with each generation producing solutions that are more refined and optimized than the previous one. The AT-PINN method provides the surrogate model of the tri-hybrid energy harvesting system for evaluating the objective function value of the GA. Figure 6.8 shows the schematic of the *AT-PINN-GA parametric optimization technique*, while Fig. 6.9 presents the flow chart of it.

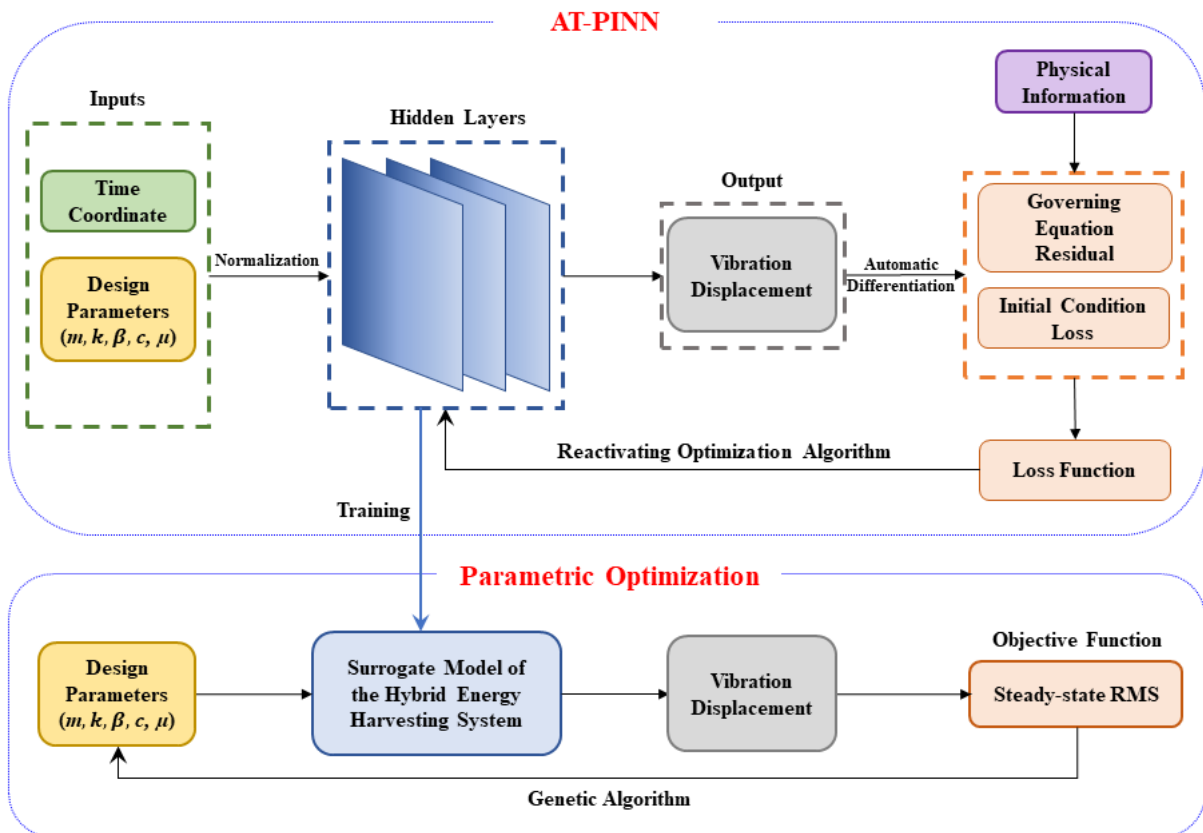


Fig. 6.8: Schematic of the AT-PINN-GA parametric optimization technique.

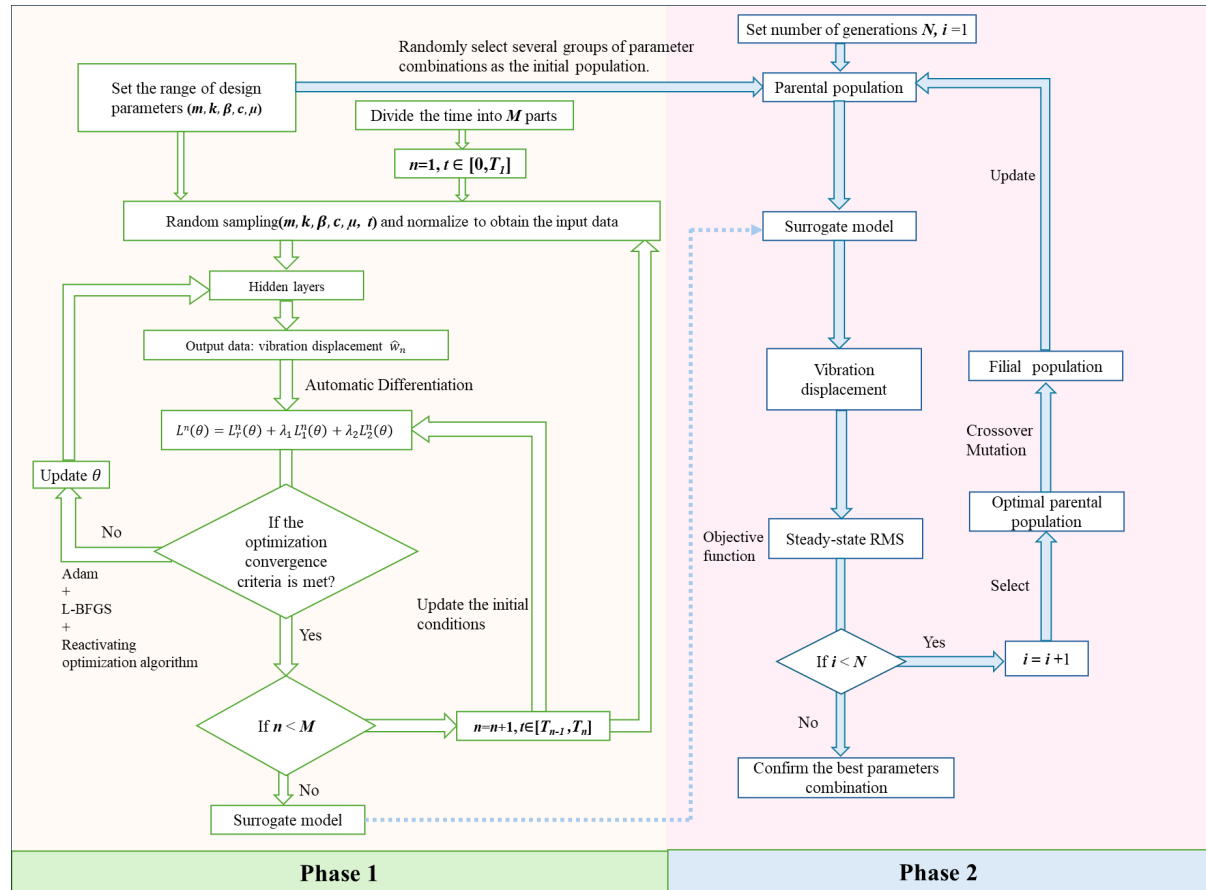


Fig. 6.9: Flow chart of the AT-PINN-GA parametric optimization technique.

6.4 Illustrative example using AT-PINN-GA

Nonlinear dynamics is essential in determining the spatial efficiency of vibration energy harvesters. In multimodal hybrid systems, the dynamic behaviors of piezoelectric, electromagnetic, and triboelectric mechanisms are closely influenced by their geometric configurations. For example, piezoelectric elements require sufficient bending or tensile deformation to convert energy effectively. However, reducing their size can lead to inadequate deformation and decreased efficiency. Similarly, the placement of magnets and coils in electromagnetic harvesters must consider spatial constraints related to magnetic field strength and the paths of relative motion. In contrast, triboelectric harvesters depend heavily on the contact area and mode to achieve optimal output performance. Therefore, optimizing the nonlinear coupling designs is essential for maximizing energy harvesting efficiency within limited spatial conditions.

Numerical simulations and experimental validations present additional challenges in optimizing the performance of multimodal hybrid systems. The highly nonlinear coupling behavior within confined spaces adds to the complexity. During simulations, unknown parameters (e.g., damping coefficients and material properties) often cause discrepancies between simulated and experimental results. Addressing these inconsistencies requires significant time for parameter adjustment and calibration. Consequently, developing efficient optimization methods to streamline this tuning process and enhance the agreement between simulations and experiments remains a critical area of research.

Taking the tri-hybrid vibration-driven energy harvesting system in **Chapter 4** for consideration as shown in Fig. 6.10, The piezoelectric material is integrated with an auxetic copper beam structure, with piezoelectric elements mounted on both sides of each copper beam. A rectangular aluminum block is attached to one end of the copper beam. Additionally, screw holes are machined into the copper beams to allow secure connection via screws, thereby forming a PEG unit. On either side of the central magnet, a copper coil and an external magnet are arranged to constitute an EMG unit. Furthermore, two sets of copper electrodes are alternately positioned beneath the PTFE film. In the TENG part, two parallel copper (Cu) films serve as friction materials and are mounted on the top and bottom surfaces of the central magnet to enable triboelectrification through relative motion, forming a TENG_S (sliding) unit. In the TENG_CS (contact and separate) unit, the PTFE film acts as a friction component and is fixed at the left and right ends of the central magnet. Meanwhile, circular beryllium copper plates function as electrodes and are connected to both ends of the spring. Considering Eq. (4.1), we can simplify the governing equation of the energy harvesting system (in horizontal mode) to

$$m\ddot{w} + c\dot{w} + kw + F_m + F_d = -m\ddot{x}_b \quad (6.13)$$

where m is the magnet mass, k is the stiffness, c is the damping effect, F_m is the magnetic force, F_d is the dry friction force, w is the displacement of the magnet, and $\ddot{x}_b$ is the external excitation.

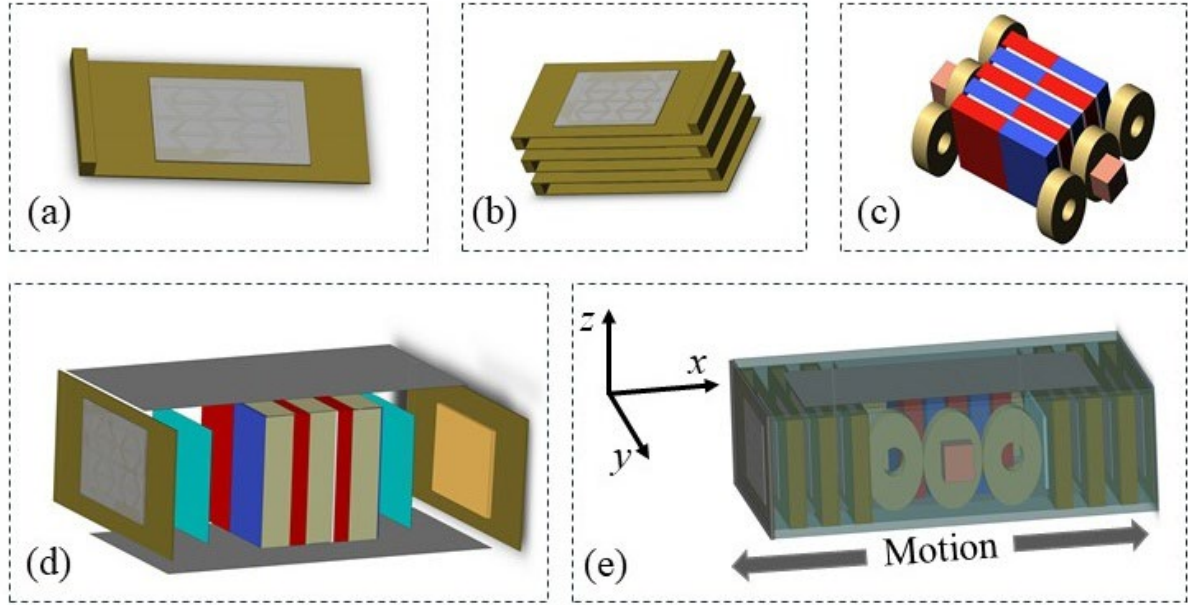


Fig. 6.10: Schematic diagrams of the hybrid energy harvester design include: (a) a Cu beam with a piezoelectric plate (PZT-5A); (b) a folded beam structure with PZT; (c) magnets and coils; (d) triboelectric materials; and (e) a fully assembled device.

During the design process, the primary objective is to enhance the output power density across a broader frequency range, particularly under low-frequency excitation. As illustrated in Fig. 6.11, the schematic diagram of the prototype demonstrates that the design requirements can be effectively satisfied. First, the size of the magnet, denoted as m , must be approximately determined. The selection of m directly influences the performance of the TENG_S unit. Second, m also significantly affects the magnetic force F_m . By applying the magnetization current method to expand F_m , the interaction between the central moving magnet and the outer fixed magnet can be modeled, leading to an expression for the magnetic force

$$F_m = \beta(k_1 w + k_3 w^3 + k_5 w^5 + k_7 w^7) \quad (6.14)$$

where β is an amplification factor and k_i ($i = 1, 3, 5, 7$) are stiffness coefficients.

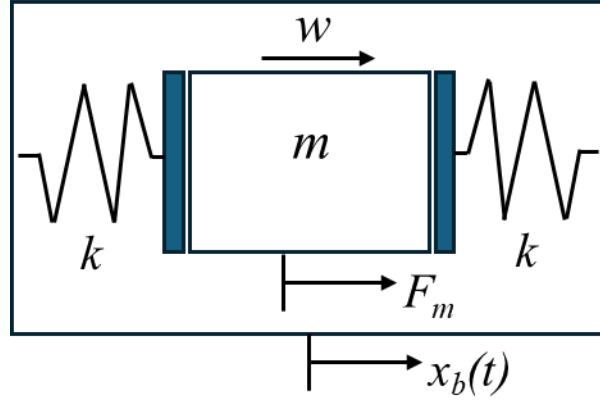


Fig. 6.11: Equivalent mechanical model of the tri-hybrid vibration-driven energy harvesting system in Fig. 6.10.

An analogous mechanical system is proposed for dynamic analysis to elucidate the operational mechanism of the current model. Under a base excitation $\ddot{x}_b$, the central magnet, with a mass of m , moves relative to the magnetic coil, resulting in compression of the folded beam spring. The stiffness k of this specially designed folded beam is determined experimentally using a force gauge. The damping c represents the total damping coefficient including the linear viscous damping coefficient, the additional damping coefficient caused by the impact effect, and the electrical damping coefficient resulting from the relative displacement between the magnetic mass and the movable coil assembly.

The dry friction force can be characterized using the Coulomb friction model, expressed as $\frac{\dot{w}}{|\dot{w}|} \mu mg$, where μ is the dry friction coefficient. Because piecewise functions cannot be used in the PINN programming code, the original expression of the dry friction F_d is thus approximated by the following continuous function:

$$F_d = \frac{\dot{w}}{|\dot{w}|} \mu mg \approx \mu mg \tanh(1000\dot{w}) \quad (6.15)$$

A comparison between the piecewise and continuous representations in Eq. (6.15) is depicted in Fig. 6.12. Hence, the motion governing the dynamic behavior of this mechanical system becomes

$$m\ddot{w} + c\dot{w} + kw + \beta(k_1w + k_3w^3 + k_5w^5 + k_7w^7) + \mu mg \tanh(1000\dot{w}) = -m\ddot{x}_b \quad (6.16)$$

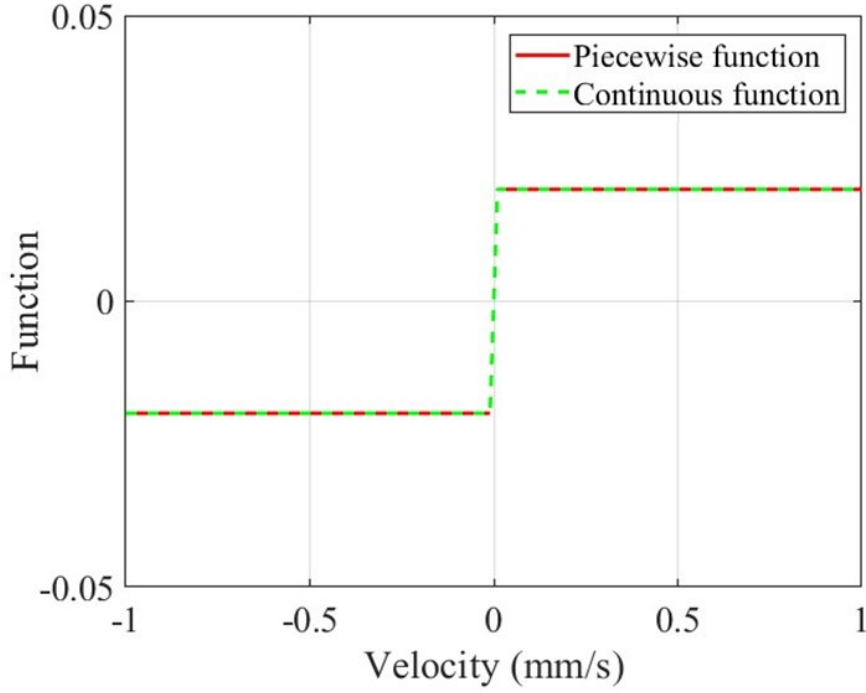


Fig. 6.12: Comparison of piecewise and continuous representations of the dry friction force.

According to Eqs. (6.2)–(6.5), the loss function of AT-PINN for the n^{th} time segment $[T_{n-1}, T_n]$ is

$$L(\theta) = L_r^n(\theta) + \lambda_1 L_1^n(\theta) + \lambda_2 L_2^n(\theta) \quad (6.17)$$

where

$$L_r^n(\theta) = \frac{1}{N_r^n} \sum_{k=1}^{N_r^n} \left\{ \frac{\partial^2 \hat{w}_n(\hat{t}_k^r)}{\Delta T_n^2 \partial \hat{t}^2} + \frac{c}{m} \frac{\partial \hat{w}_n(\hat{t}_k^r)}{\Delta T_n \partial \hat{t}} + \frac{k[\hat{w}_n(\hat{t}_k^r)]}{m} \right. \\ \left. + \frac{\beta[k_1[\hat{w}_n(\hat{t}_k^r)] + k_3[\hat{w}_n(\hat{t}_k^r)]^3 + k_5[\hat{w}_n(\hat{t}_k^r)]^5 + k_7[\hat{w}_n(\hat{t}_k^r)]^7]}{m} \right\} \quad (6.18)$$

$$+ \mu g \tanh \left[1000 \frac{\partial \hat{w}_n(\hat{t}_k^r)}{\Delta T_n \partial \hat{t}} \right] + \frac{\partial^2 x_b(\hat{t}_k^r \Delta T_n + T_{n-1})}{\Delta T_n^2 \partial \hat{t}^2} \Bigg\}^2, t_k^r \in [0, 1]$$

$$L_1^n(\theta) = [\hat{w}_n(0) - \hat{w}_{n-1}(1)]^2 \quad (6.19)$$

$$L_2^n(\theta) = \left[\frac{\partial \hat{w}_n(0)}{\partial \hat{t}} - \frac{\partial \hat{w}_{n-1}(1)}{\partial \hat{t}} \right]^2 \quad (6.20)$$

where $L_r^n(\theta)$, $L_1^n(\theta)$ and $L_2^n(\theta)$ are the governing equation residual, initial displacement loss and initial velocity loss, respectively. The hyperparameters λ_1 and λ_2 are respectively the weights of the initial displacement loss and the initial velocity loss, which balance the interplay

of different components of the loss function. N_r^n , N_1^n and N_2^n are the sample points for the governing equation residual, initial displacement loss and initial velocity loss, respectively. $\hat{w}_n$ is the output of the neural network in the n^{th} time segment, and $\hat{w}_{n-1}$ is the output of the neural network in the $(n-1)^{\text{th}}$ time segment. In the first time segment, where $T_{n-1} = T_0 = 0$, $\hat{w}_{n-1}(1)$ and $\partial \hat{w}_n(1)/\partial t$ are determined by the given initial conditions. The neural network of AT-PINN consists of 6 input neurons (i.e., 5 design parameters of the energy harvesting system (m , k , β , c , μ) and time t) and 1 output neuron (vibration displacement of the magnet slider). The neural network of the AT-PINN method is pre-trained by the ADAM optimizer and then trained by the L-BFGS optimizer with the reactivating optimization algorithm. Selected parameters of the AT-PINN approach are listed in Table 6.1. The parameters presented in the table were fine-tuned iteratively through extensive experimentation until the model achieved the desired level of accuracy as specified in this study.

Table 6.1: Parameters of the AT-PINN approach

Parameter	Value
No. of neurons in the input layer	6
No. of neurons in the output layer	1
No. of hidden layers	6
No. of neurons in each hidden layer	64
No. of sample points for governing equation residual	100,000
No. of sample points for initial condition loss	1,000
Learning rate of the ADM optimizer	0.001
Initial learning rate of the L-BFGS optimizer	1.0

Moreover, the mass m impacts the magnetic induction intensity, which is closely related to the output of the EMG unit. Another critical parameter is k , which provides a linear restoring force. Combined with the magnetic force F_m , the resultant force acting on the system can be calculated. The total elastic restoring force curve and the integrated potential energy curve provide clear visualizations of the equilibrium point of the system (where the resultant force is equal to zero) and the characteristics of the potential energy well. These curves determine whether the moving magnet can escape from the potential energy well under low excitation

levels and subsequently generate an electrical signal. At this stage, the width of the potential energy well is associated with the maximum displacement of the central magnet, which in turn affects the output of the PEG unit. Based on the properties of piezoelectric materials, the spring replacement structure composed of multi-layer bimorph piezoelectric cantilever beams exhibits a positive correlation with the displacement of the magnet. Specifically, a greater displacement results in a higher output from the PEG. The interrelationships among the various parameters are highly significant. In earlier experiments, determining all optimal design parameters simultaneously proved difficult, requiring iterative adjustments and ongoing debugging. To address this issue, the AT-PINN method can be utilized to greatly improve computational efficiency.

For the energy harvester prototype discussed in this research, a detailed analysis of several key variables is conducted. The most critical parameter is the mass of the central magnet, denoted by m , which also serves as a representative size parameter. Another important variable is the damping coefficient c , which varies across different environments, systems, and conditions. This coefficient needs to be determined based on empirical knowledge and reference literature, and it remains a parameter that requires continuous adjustment throughout the optimization process. Regarding the magnetic force, the typically long and complex formulas are approximated by a seventh-degree polynomial function, with a scaling factor β introduced for targeted adjustments. This simplification allows for a more manageable calculation while maintaining accuracy in modeling the magnetic characteristics of the system. In addition, the elastic coefficient k is another essential parameter in the design of energy harvesters. Traditionally, k is directly derived from the properties of the spring used in the system. However, in this design, k must be measured experimentally. This is due to the complexity of the system's construction, including the connection method between cantilever beams and the multi-layered structure. These factors make it difficult to “customize” the value k through traditional means. To modify k , the only viable approach is to adjust the configuration of the cantilever beams, i.e., either by adding or reducing the number of beams in the system. Moreover, different materials and the size of the contact area significantly influence the output of the triboelectric nanogenerator (TENG). In this study, the coefficient μ is used to represent this parameter. Thus, five critical parameters must be considered simultaneously: the central magnet mass m , the damping coefficient c , the elastic coefficient k , the magnetic force

coefficient (adjustment through β), and the material/contact area coefficient μ . As a result, the design involves solving a five-dimensional parameter space.

Basically, solving the governing equations and using an exhaustive search method to find the maximum displacement (which corresponds to higher power output) would be time-consuming. However, the use of AT-PINN-GA simplifies this process significantly. With the calculation time fixed at 10 seconds, the use of AT-PINN-GA method allows for more accurate and faster results compared to traditional methods. Based on the preceding research presented in **Chapters 3–5**, the ranges for the five variable parameters of the energy harvesting prototype in this study are as follows:

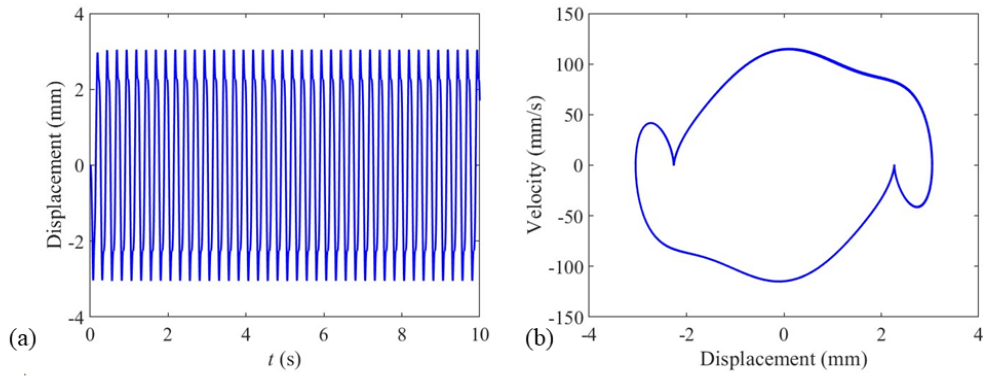
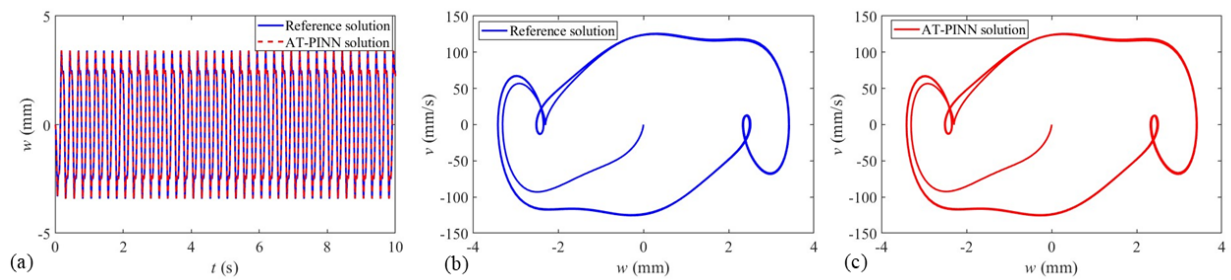
- mass m (0.05–0.08 kg)
- spring stiffness k (50–350 N/m)
- amplification factor β (0.8–1.2)
- total damping coefficient c (2–5 N·s/m)
- friction coefficient μ (0.04–0.07)

We will take various combination sets for parametric optimization analysis.

These parameters are chosen based on their direct influence on the performance of the energy harvester and the need for practical tunability under varying system conditions. The selected ranges reflect typical variations encountered in experimental setups and real-world applications. Four sets of parameters are listed in Table 6.2. Set I is regarded as initial parameters of the energy harvesting system before parameter optimization. Figure 6.13 displays the vibration displacement and phase diagram of the central magnet for set I, and the results are calculated by using the traditional Runge–Kutta method. Sets II–IV are used to compare the AT-PINN results and the reference solutions obtained by using the traditional Runge–Kutta method as shown in Figs. 6.14, 6.15, and 6.16, highlighting the performance of the AT-PINN method in this optimization task. These three sets demonstrate that, within the chosen parameter ranges, the AT-PINN method can accurately and efficiently generate results consistent with the reference solutions. As seen in Figs. 6.14(a), 6.15(a), and 6.16(a), the outcomes from both methods closely align, validating the accuracy of AT-PINN and the surrogate model of the energy harvesting system. This conclusion is further reinforced by the phase diagram results.

Table 6.2: Four sets of parameter combinations for optimization analysis

Parameter	Value
Simulation time	10 s
Base excitation	1 g and 4 Hz
Parameter combination set I	$m = 0.08 \text{ kg}$, $k = 200 \text{ N/m}$, $\beta = 1.2$, $c = 2.5 \text{ N}\cdot\text{s/m}$, $\mu = 0.05$
Parameter combination set II	$m = 0.05 \text{ kg}$, $k = 60 \text{ N/m}$, $\beta = 0.8$, $c = 2 \text{ N}\cdot\text{s/m}$, $\mu = 0.04$
Parameter combination set III	$m = 0.07 \text{ kg}$, $k = 128 \text{ N/m}$, $\beta = 1.1$, $c = 3.8 \text{ N}\cdot\text{s/m}$, $\mu = 0.05$
Parameter combination set IV	$m = 0.07 \text{ kg}$, $k = 290 \text{ N/m}$, $\beta = 1.2$, $c = 4.4 \text{ N}\cdot\text{s/m}$, $\mu = 0.06$


Fig. 6.13: Parameter combination set I: displacement responses and phase portraits of the magnetic slider.

Fig. 6.14: Parameter combination set II: comparison of the reference and AT-PINN solutions of the magnetic slider.

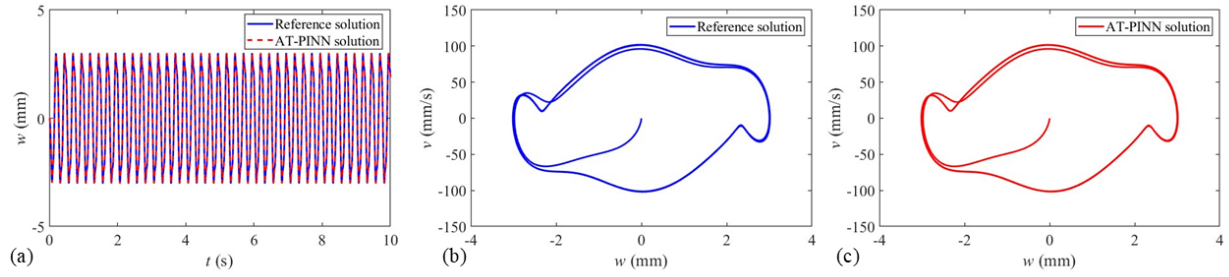


Fig. 6.15: Parameter combination set III: comparison of the reference and AT-PINN solutions of the magnetic slider.

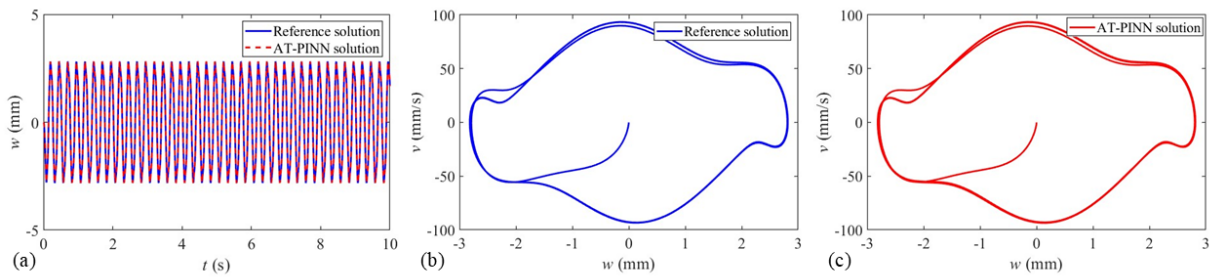


Fig. 6.16: Parameter combination set IV: comparison of the reference and AT-PINN solutions of the magnetic slider.

This research primarily focuses on the dynamic behavior of the harvester under low-frequency excitations, as these more accurately reflect natural environmental vibrations. To simulate typical operating conditions, the excitation input is set to a gravitational acceleration of 1 g at a frequency of 4 Hz, with analysis conducted over a 10-second sampling period. The objective is to maximize energy conversion efficiency in response to low-frequency vibrations, where the system may encounter challenges related to nonlinear interactions arising from the interplay of multiple transduction mechanisms and their geometric configurations. To assess system performance, both the AT-PINN method and reference solutions are used to determine the maximum and minimum system responses across various parameter configurations. The system demonstrates different stability behaviors depending on the parameter combination sets, exhibiting either tri-stable or bi-stable states. These results highlight the critical importance of precise parameter tuning in hybrid energy harvesting systems, where the response is highly sensitive to subtle design variations.

Comparative analysis reveals that the AT-PINN method offers superior accuracy and efficiency in addressing complex nonlinear optimization problems. Unlike traditional approaches, which often rely on time-consuming trial-and-error experiments or extensive simulations, AT-PINN directly integrates physical laws into the optimization process. This capability makes it an especially effective tool for optimizing energy harvesting systems, enabling the identification of design configurations that maximize energy output under real-world conditions. Once the neural network has been trained using the AT-PINN method, the next challenge is to efficiently explore a large dataset to identify the most promising parameter combination set that can yield optimal solutions. This task, which seeks to find the precise solution in the shortest possible time, is effectively addressed by employing GA. As illustrated in Fig. 6.17, the vertical axis represents the objective function value, while the horizontal axis indicates the number of genetic iterations. In this example, the GA process completes 500 iterations in approximately 9.4 seconds, with the best solution (RMS value: 2.70 mm) found after about 150 iterations. The parameter combination set corresponding to this optimal solution is: $m=0.079$ kg, $k = 50.01$ N/m, $\beta = 0.803$, $c = 3.343$ N·s/m, and $\mu = 0.042$. Compared to traditional search methods, this approach significantly accelerates the identification of optimal solutions.

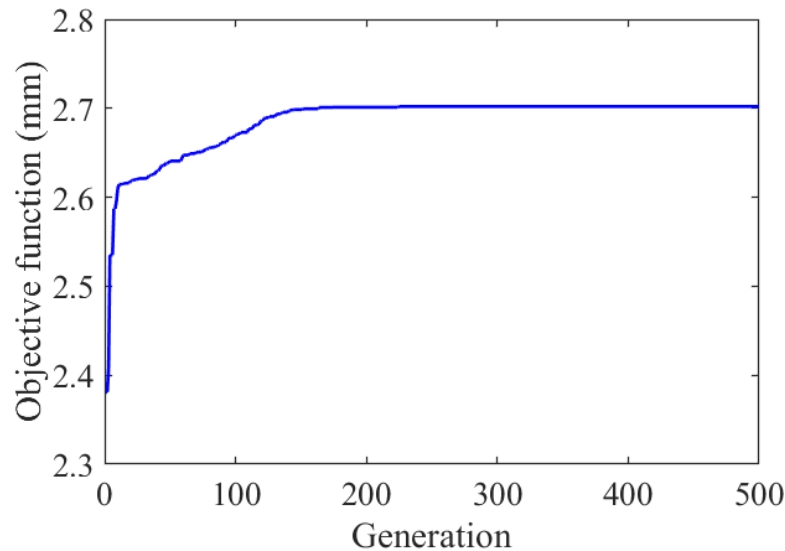


Fig. 6.17: Objective function and generation in GA analysis.

Figure 6.18 shows that, through optimization using the AT-PINN-GA approach, the maximum magnet displacement achieved is 1.39 times greater than that obtained by solving with the initial parameter set I using the Runge–Kutta method in MATLAB (4.07 mm versus 3.09 mm). The integration of GA with the AT-PINN method facilitates a seamless transition from solving the governing equations to selecting the most suitable solutions. This hybrid approach not only reduces computational time but also enhances the overall optimization process, thereby improving operational efficiency.

By integrating the accuracy and physics-consistency of AT-PINN with the efficient parameter space exploration capabilities of GA, the optimization process is significantly accelerated without compromising solution quality. This approach is particularly beneficial for applications such as energy harvesting systems, where identifying optimal configurations within a vast design space can be computationally demanding. The ability to rapidly pinpoint high-performance solutions ensures that the system is efficient both in computational resource usage and time, which is essential for practical engineering applications.

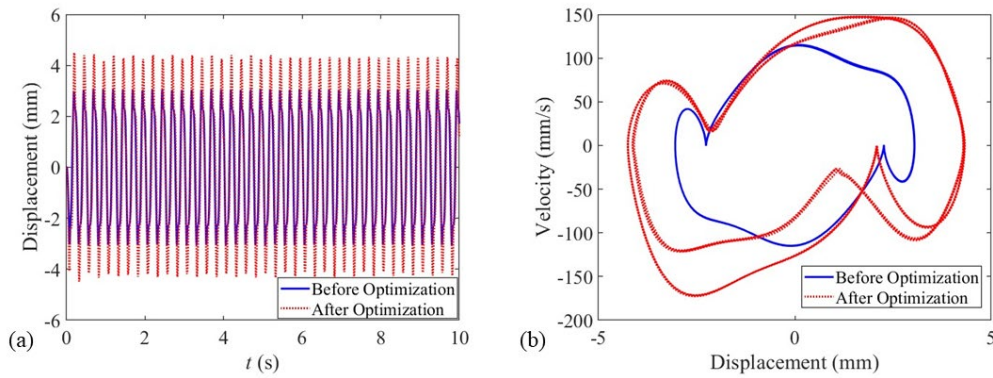


Fig. 6.18: Displacement response and phase portrait: comparison between AT-PINN-GA optimization results and reference solutions using parameter combination set I.

6.5 Concluding Remarks

This work explored the use of AT-PINN to optimize tri-hybrid energy harvesting systems integrating piezoelectric, electromagnetic, and triboelectric mechanisms. The AT-PINN method is effective for the analysis of nonlinear coupling and multi-physics interactions involving complex interdependencies between different energy conversion mechanisms. By leveraging

advanced techniques such as spatiotemporal domain normalization, reactivation optimization algorithms, and transfer learning, AT-PINN enhances the accuracy and efficiency of solving dynamic problems over extended periods, overcoming the limitations of conventional PINNs. The major contribution of this research further combined AT-PINN with GA to form the AT-PINN-GA parametric optimization technique, which can significantly accelerate the search for optimal solutions, as the capability of GA can complement the strength of AT-PINN in parameter optimization. Together, this hybrid AT-PINN-GA computational framework offers a powerful and efficient approach for optimizing tri-hybrid energy harvesters, enabling high-performance and non-intuitive designs that cannot be achieved by traditional methods.

CHAPTER 7

- CONCLUSIONS, LIMITATIONS, AND PERSPECTIVES

7.1 Conclusions

This research is focused on advancing vibration-based energy harvesting systems by developing a tri-hybrid system model incorporating PEG, EMG, and TENG mechanisms. The study explores the integration of nonlinear multi-stable dynamics to enhance power output and broaden operational frequency ranges. The following key findings summarize the main achievements of this work:

- ***Analysis of friction impact on multi-stable energy harvesting system:*** The study examined how friction affects the performance of a tri-stable hybrid energy harvester. While friction suppressed PEG output at low excitation levels (0.5 g), it had a minimal impact at higher excitations (1.5 g), shifting the nonlinear behavior of the system from inter-well to intra-well oscillations. The friction coefficient influenced the amplitude and frequency response of the system, suggesting that friction can be beneficial or detrimental depending on excitation levels. In particular, the TENG unit benefited from frictional force under moderate to high excitations.
- ***Integration of auxetic structure for performance enhancement:*** The use of auxetic structures was explored to improve the efficiency of a tri-hybrid energy harvesting system. The selected auxetic design significantly enhanced power output, delivering a 55% increase in output efficiency compared to conventional designs. The experimental results showed that the maximum power output of the prototype can be up to 87 mW with a normalized power density of $3.56 \text{ mW cm}^{-3} \text{ g}^{-2}$. The ability of the auxetic structure to enhance displacement and maintain system stability is a key factor in optimizing power generation, especially at low-frequency excitations.
- ***Development of multi-stability-based frequency up-converted approach:*** A novel miniaturized tri-hybrid harvester was developed with frequency up-conversion using multi-stable dynamics. The system, tested in both tri-stable and quad-stable modes,

demonstrated superior performance in the range of 1–11 Hz at 1 g excitation. The prototype achieved the maximum power output of ~86 mW, with $3.70 \text{ mW cm}^{-3} \text{ g}^{-2}$ normalized power density. The tri-stable mode was optimal for low-intensity excitations, while the quad-stable mode provided better performance under high excitation conditions. The energy harvester successfully powered LEDs and thermohygrometers, showcasing its real-world applicability for wearable and portable devices.

- ***Implementation of physics-informed parametric optimization analysis:*** An efficient AT-PINN method was combined with GA, forming the AT-PINN-GA parametric optimization approach, to refine the design of a tri-hybrid energy harvesting system. This combined deep learning strategy facilitated the swift identification of optimal design configurations through efficient analysis of multiple parameters. In principle, the AT-PINN method incorporated physics-based learning and reduced dependence on large datasets, while GA accelerated parameter space exploration that can greatly improve computational efficiency.

This research advances the design of tri-hybrid energy harvesters by introducing multi-stable dynamics, auxetic structures, and the integration of AT-PINN and GA for parametric optimization. The design of tri-hybrid energy harvesters can be advanced to achieve higher output and broader bandwidth compared to traditional systems, particularly in low-frequency, low-amplitude, and time-varying scenarios. This work also introduced new optimization strategies for energy harvesting system design, which can significantly improve working efficiency and reducing computational effort. The hybrid approach introduced in this work could transform the design of self-powered devices to allow for more efficient and autonomous energy harvesting from ambient vibrations.

7.2 Current limitations and future perspectives

The progress toward tri-hybrid vibration-based energy harvesting is still fraught with challenges. A previous study (Xiahou et al., 2023) systematically reviewed the low-frequency energy harvesting performance enhanced through the modulation of material compositions, structures, and devices, and provided corresponding recommendations. Building upon the modeling, optimization, and experimental findings of this research, several key directions emerge for further improving the design of tri-hybrid energy harvesting systems. The following

future works are proposed to enhance the practicality, robustness, and performance of system design:

- ***Advanced energy management circuits:*** The current prototypes employed basic rectification and simple capacitor-based storage units. Although these components are functional, they cause significant energy losses. The next critical step is to design dedicated power management circuitry tailored to multi-source harvester units. This includes high-efficiency AC/DC rectifiers, impedance matching networks, and DC-DC converters or charge pumps that can adapt to varying inputs from piezoelectric, triboelectric, and electromagnetic sources. For instance, integrating a reconfigurable rectifier and smart switching interface could maximize energy transfer from the triboelectric nanogenerator under irregular vibrations. By reducing voltage drops and actively managing the energy flow, these circuits would significantly improve storage efficiency and provide stable output for electronics. Overall, effective power management circuits are vital for transforming the raw output of hybrid harvesters into usable power with minimal losses. Future research should focus on developing such integrated circuits (potentially CMOS-based) that support multi-modal inputs and include features like adaptive rectification, maximum power point tracking, and low-dropout regulation to enable efficient energy storage and delivery in practical deployments.
- ***Built-in low-power wireless communication:*** While the present system successfully powered an off-the-shelf Bluetooth sensor, a fully self-powered wireless node requires a more seamless integration of the communication module. Future work should develop or incorporate an ultra-low-power Bluetooth Low Energy (BLE) transmitter tailored to the harvester output. This means selecting or designing a BLE system-on-chip that can operate on intermittent, milliwatt-level power and leveraging strategies like duty-cycling and energy-aware scheduling to transmit sensor data. BLE is a promising choice for battery-less devices due to its low energy consumption and built-in features for low-power operation. Through the co-design of the energy harvester and communication electronics, the device can efficiently manage its energy budget by storing charge and activating the system only for brief data transmissions when enough energy has been accumulated. A custom integrated module would eliminate the need for a separate sensor pack, enabling a compact, self-contained wireless sensor node. Research in this direction includes developing custom firmware/hardware for BLE that can cold-start quickly, handle intermittent power (e.g., using a supercapacitor as buffer), and reliably send data once

powered. This will transform the prototype into a truly autonomous wireless sensor system capable of continuous, untethered operation.

- ***Enhanced material durability:*** The longevity of tri-hybrid energy harvesters is currently limited by material fragility and degradation. Piezoelectric elements (often ceramic-based like PZT) are efficient but brittle and expensive, risking crack or fracture under prolonged stress. Similarly, common triboelectric materials can wear out or lose effectiveness due to repeated contact, humidity, and contamination. Future studies should explore alternative materials that offer flexibility, toughness, and cost-effectiveness without sacrificing performance. One strategy involves using flexible piezoelectric materials or composites (e.g., polyvinylidene fluoride (PVDF) polymers or polymer-ceramic composites), which can withstand bending and stretching while maintaining performance. These flexible piezoelectric retain functionality under deformation and could vastly improve resilience to mechanical fatigue. For triboelectric nanogenerators, research should investigate durable polymer pairs and surface coatings that resist wear and environmental degradation. Emerging self-healing and stretchable polymers, along with advanced encapsulation techniques, can help preserve triboelectric performance over millions of cycles. In addition, substituting scarce or expensive materials with more abundant alternatives (e.g., lead-free piezoelectric ceramics or low-cost dielectrics for TENG layers) would enhance the economic feasibility of tri-hybrid energy harvesters. It is important to prioritize systematic durability testing of new materials and designs to ensure that any proposed alternatives can endure real-world conditions, including vibration, temperature fluctuations, and humidity, over extended periods.
- ***Improved triboelectric performance:*** In the current design, the triboelectric component contributes a relatively modest portion of the total energy output. To fully leverage the triboelectric mechanism, future work can focus on novel materials and surface engineering to boost the efficiency of triboelectric nanogenerators. This could involve identifying new material combinations with a larger difference in triboelectric series affinity, such that a higher charge density per contact cycle can be generated. Material modification strategies, including surface micro- and nano-texturing, chemical functionalization to enhance charge trapping, or the incorporation of high-dielectric nanoparticles, can substantially boost the charge transfer and output current of TENG. Research studies should also explore the direction of material treatments that can improve surface charge retention (e.g., fluorination or electron beam charging of triboelectric layers), as well as advanced

structural designs (e.g., multilayer or interdigital configurations) to increase the effective contact area. Furthermore, integrating a small bias charge or using ferroelectric materials in triboelectric layers can enhance the polarization effect and output. Any improvements in TENG output should be balanced with the earlier focus on durability. For example, introducing lubricating coatings or optimized contact modes might increase output without adding excessive wear. By systematically experimenting with these material and design innovations, the working performance of triboelectric subsystem can be elevated, thereby improving the overall power density and reliability.

In summary, pursuing these future directions will advance the design of tri-hybrid energy harvesters toward practical use in various engineering fields. Improved power electronics will enable efficient energy capture and storage, while custom low-power wireless modules will support autonomous sensor operation for IoT applications. Advanced material designs focused on flexibility, durability, and power output will enhance lifespan and performance. Building on the simulation, experimental, and machine-learning insights from the present work, these efforts collectively pave the way for a robust, efficient, and fully integrated self-powered energy harvesting system for next-generation smart IoT-connected sensors and devices.

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