ON THE VALUE OF CUSTOMER INCENTIVE PROGRAMS

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PhD

The Hong Kong Polytechnic University

2025

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On the Value of Customer Incentive Programs

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A thesis submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy

August 2025

CERTIFICATE OF ORIGINALITY

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Abstract

This thesis investigates the optimal design and implementation of customer incentive programs, including coalition loyalty programs (CLPs) and take-back programs (TBPs), as tools to enhance customer engagement and build firm competitiveness. Both program types are intended to shape customer behavior beyond a single transaction, thereby generating long-term value for both firms and their customers.

The first study explores CLPs, where firms partner with other brands to offer joint loyalty initiatives. Despite their increasing popularity, CLPs remain understudied in comparison to proprietary loyalty programs (PLPs) managed by individual firms. We investigate CLPs and compare them to PLPs using an analytical framework. In an infinite time horizon setting, n firms within a CLP offer nondurable products to a continuum of heterogeneous customers. We analyze the design of CLPs and show that CLPs can significantly expand the range of market conditions under which offering reward programs is desirable. That is, firms have incentives to join CLPs even when PLPs are ineffective since the price discrimination can be executed more efficiently. The study also reveals the pivotal role of market composition, customer discounting, and coalition size in the effectiveness of CLPs. In a market with high heterogeneity in customers' valuations, larger CLPs are not always preferred, offering one explanation for the struggles of some CLPs in their expansion. Conversely, in a market where customer valuations are more homogeneous, the per-firm profit may increase or decrease monotonically with coalition size, or exhibit a non-monotonic

relationship, indicating the existence of an optimal intermediate coalition size.

The second study examines in-store TBPs, in which firms incentivize customers to return used products or packaging at retail locations. Although TBPs are increasingly adopted by environmentally conscious firms, their implementation remains optional and presents operational and financial challenges. In this study, we investigate the strategic implementation and economic implications of TBPs in competitive markets by comparing three scenarios: no TBP implementation, partial TBP implementation, and full TBP implementation. Customers derive psychological satisfaction from recycling, but also incur hassle costs from the return process. Our analysis reveals that partial TBP implementation can emerge as an equilibrium. Notably, even asymmetric adoption of TBPs can result in win-win outcomes, allowing firms to differentiate themselves by appealing to either environmentally conscious or price-sensitive customers. This, in turn, enhances overall profitability while advancing sustainability goals.

Collectively, these studies provide strategic insights into the design and implementation of customer incentive programs. By uncovering key trade-offs and market dynamics associated with CLPs and TBPs, this thesis offers actionable guidance for firms seeking to leverage such programs to remain competitive and responsive to evolving societal expectations.

Keywords: customer incentive, program design, customer heterogeneity, price discrimination, profit foci.

Publications Arising from the Thesis

1. Y. Liu, J. Wang, Y. Wang and D. Zhang. On the Value of Coalition Loyalty Programs. Under a major revision at *Marketing Science*, 2025.

Acknowledgments

I extend my heartfelt gratitude to PolyU and the Department of Logistics and Maritime Studies for providing such a supportive and stimulating environment for my academic development. Over the past three years, my experiences on campus have broadened my horizons and fostered a spirit of open-mindedness, profoundly enriching my university life.

I am immensely fortunate to have been guided by my esteemed supervisors, Prof. Yan Liu and Prof. Yulan Wang. Their mentorship has not only shaped my approach to research but also my outlook on life, grounding me in thoughtful reflection and practical wisdom. Whenever I encountered challenges, their professional insights and effective advice enabled me to overcome obstacles and gain new perspectives. Prof. Liu excels at uncovering the underlying essence of complex phenomena, and I am consistently impressed by his meticulous and rigorous calculations. Prof. Wang is both professional and approachable. Thanks to her efforts and our department's invitations, I had the opportunity to attend presentations by numerous distinguished scholars, including Prof. Baojun Jiang, Prof. Bin Hu, Prof. Christoph H. Loch, Prof. Christopher Tang, Prof. Guoming Lai, Prof. Jianqing Chen, Prof. Ming Hu, Prof. Yi Zhu, and so on. Under the guidance of these professors, I have learned to think critically, be academically rigorous, and take a holistic view of academia.

My sincere appreciation also goes to everyone who has contributed to my PhD journey. I am deeply grateful to Prof. Baozhuang Niu, Prof. Dan Zhang, Prof. Chiu

Yu Ko, and Dr C.L. Johnny Wan for their invaluable assistance in expanding my knowledge across diverse fields such as game theory, revenue management, microeconomic theory, and business analytics. I also wish to thank Prof. Zhuoyu Long, Prof. Yimin Yu, and Prof. Miao Song for their time, insightful comments, and valuable feedback. My thanks extend to Priscilla Choy, Becky Kam, Irene Lam, and other administrative staff for their kind assistance and efficiency throughout my studies.

To my dear friends, your unwavering support and companionship have been instrumental in shaping who I am today. To Yu Xiao, your presence has brightened both my present and future, and for that, I am truly grateful. Finally, I owe my deepest appreciation to my beloved family for their unconditional love and selfless support. The road ahead is long, but I firmly believe that inner peace is the new success. I aspire to become an independent and impactful researcher. By embracing OM thinking, I strive to discover the true meaning of life.

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Chapter 1

Introduction

1.1 Research Background

In today's increasingly competitive and sustainability-conscious marketplace, firms are turning to innovative customer incentive programs such as coalition loyalty programs (CLPs) and take-back programs (TBPs) to encourage repeat behavior, foster customer loyalty, and achieve strategic differentiation. While these programs differ in form and objectives, they share a core purpose: influencing customer behavior beyond individual transactions to generate long-term value for both firms and customers.

CLPs involve collaboration among multiple firms, providing customers with the opportunity to collect and redeem rewards throughout a network of partner brands. This multi-brand structure accelerates reward redemption and enhances customer convenience, contributing to the growing popularity of CLPs in industries such as retail, travel, and dining. However, despite their increasing prevalence, CLPs remain relatively understudied in contrast to proprietary loyalty programs (PLPs), which are offered by individual firms. In particular, little is known about how coalition size, customer heterogeneity, and customer discounting interact with the effectiveness of CLPs. Key questions about optimal CLP design and the conditions under which

CLPs outperform PLPs remain largely unanswered.

TBPs, on the other hand, are sustainability-driven initiatives that encourage customers to return used products or packaging. These programs support circular economy practices and have been adopted by prominent retailers such as H&M, Marks & Spencer, and L'Occitane. By offering TBPs, firms not only encourage green consumption but also boost in-store traffic and reinforce their environmentally responsible images. However, the implementation of TBPs poses several challenges, including recycling-related operational costs and the heterogeneity in customer participation stemming from return hassle sensitivity. Given their relatively limited adoption and potential strategic implications, a deeper understanding of TBPs in competitive market settings is both timely and necessary.

While distinct in operational design, CLPs focus on multi-brand loyalty and TBPs on incentivized recycling. Both programs engage in strategic reward design to influence customer decision-making over time. Both face trade-offs between customer inclusion and exclusion, and require careful incentive design to balance profitability and effectiveness. Both CLPs and TBPs influence market dynamics by altering customer behavior and competitive interactions. CLPs affect how customers perceive the value of rewards across multiple brands, while TBPs shape environmentally driven purchasing behavior. Moreover, both programs exhibit structural similarities in how customer heterogeneity impacts program profitability and effectiveness. Notably, both studies highlight that customer incentive programs may yield win-win outcomes for firms and customers, but can also lead to lose-lose outcomes if ineffectively executed.

This thesis aims to deepen the understanding of customer incentive programs through rigorous analytical modeling. The first study models a CLP with n firms, each selling a nondurable product to heterogeneous customers in an infinite-horizon setting. We analyze the design of CLPs and show that CLPs can significantly expand the range of market conditions under which offering reward programs is desirable.

That is, firms have incentives to join CLPs even when PLPs are ineffective since the price discrimination can be executed more efficiently. The study also reveals the pivotal role of market composition, customer discounting, and coalition size in the effectiveness of CLPs. In a market with high heterogeneity in customers' valuations, larger CLPs are not always preferred, offering one explanation for the struggles of some CLPs in their expansion. In contrast, in a market where customer valuations are more homogeneous, the per-firm profit may either increase or decrease monotonically in the coalition size n. Additionally, it is possible for the per-firm profit to first increase and then decrease in n, implying the existence of an optimal intermediate coalition size n.

The second study investigates the strategic implementation and economic implications of TBPs in competitive markets. We model in-store TBPs using a standard circular spatial competition framework, featuring two firms located on a unit circle. Customers who choose to recycle a used product must physically return the item to the nearest store, gaining psychological satisfaction from recycling, but also incurring hassle costs associated with the return process. Customers make recycling and purchasing decisions by maximizing their individual utilities. We compare three scenarios: no TBP implementation (NN case), partial TBP implementation (YN case, which is equivalent to NY case due to symmetry), and full TBP implementation (YY case). Our findings indicate that all three scenarios can arise as equilibrium outcomes. Notably, even the asymmetric implementation of TBPs can create a win-win outcome for competing firms. By differentiating themselves to appeal to either environmentally conscious or price-sensitive customers, firms can enhance overall profitability while simultaneously advancing sustainability goals.

Through this thesis, we offer practical insights for firms considering the design or adoption of customer incentive programs. By examining the nuanced trade-offs and market dynamics associated with CLPs and TBPs, we contribute to a richer understanding of how such programs can be leveraged effectively, ensuring they remain competitive and responsive to both market and societal changes.

1.2 The Layout of the Thesis

The remainder of the thesis is organized as follows. Chapter 2 develops an analytical model of CLPs and provides a foundational analysis on comparing the performance and effectiveness of CLPs with PLPs. Chapter 3 focuses on in-store TBPs. By comparing firm profits across scenarios of no TBP implementation, partial TBP implementation, and full TBP implementation, we identify the equilibrium outcomes and the value of TBPs in the competitive markets. Chapter 4 concludes this thesis and highlights potential avenues for future research. All proofs are provided in the appendix.

Chapter 2

On the Value of Coalition Loyalty Programs

2.1 Introduction

Coalition loyalty programs (CLPs) bring multiple firms into a partnership, whereby customers earn rewards from purchases made at any participating firm and can redeem these rewards for future purchases across all coalition partners. The concept of CLPs is not novel; its roots can be traced to the launch of AAdvantage, the first frequent flyer program by American Airlines in 1981 (Walsman and Dixon 2020). In response to rapidly intensifying competition among proprietary loyalty programs (PLPs), AAdvantage expanded by forming partnerships with hotels and rental car companies, marking the early stages of CLP development (InsideFlyer 2006). Since then, CLPs have gained significant momentum across various industries, including hospitality, dining, and retail. The recent surge in digital marketing has further fueled their growth. As an example, the Air Miles program in Canada allows customers to earn or redeem rewards while booking flights or hotels, shopping for groceries, fueling their cars, or making online purchases. Similarly, the Payback program includes partners that enable customers to earn or redeem rewards on travel bookings, movie tickets, and financial services. In these examples, customers benefit from the convenience of using a single loyalty card or app that works across multiple brands.

CLPs stand in contrast to PLPs, where customers earn and redeem rewards solely with a single firm. Compared to PLPs, CLPs offer customers a faster path to earn and redeem rewards, potentially enhancing customer engagement and boosting sales for participating firms. It is perhaps not surprising that a customer survey conducted by Salesforce reveals that 55 percent of customers prefer CLPs over PLPs (Antavo 2024). However, CLPs remain significantly understudied compared to PLPs, which are the focus of most existing research on customer reward programs.

There is a small strand of research on CLPs, much of which is empirical studies. However, as pointed out by Dorotic et al. (2021), "the empirical work cannot make causal claims about the impact of joining the partnership relative to not being part of it (i.e., whether firms should join CLPs and cease sole PLPs)." Taylor and Dong (2023) also acknowledge that "estimating the effectiveness of a CLP is challenging when the network of participating retailers is larger and changes over time". Additionally, Ngwe et al. (2022) highlight a key limitation, namely, the lack of data available to firms before joining the program. Even with access to rich data sets, observations are typically limited to transactions within a specific CLP and to a certain period. These constraints leave significant gaps in understanding the relative effectiveness of CLPs compared to PLPs and the trade-offs involved in CLP design. Addressing these gaps is crucial for business practitioners who are interested in launching or joining a CLP. Yet, to the best of our knowledge, no analytical studies have thoroughly investigated these issues.

Although firms typically introduce loyalty programs to increase purchase frequency, expand basket size, and strengthen customer loyalty, their mixed performance in recent years suggests that many programs have struggled to deliver sustained behavioral loyalty. For example, the discontinuation of the Plenti loyalty program by American

Express in 2018 illustrates this broader challenge, after most participating firms, such as Macy's, Chili's, and AT&T, reverted to PLPs (Forbes 2018). Similar difficulties faced by Aeroplan in Canada (Global News 2017) and Nectar in the UK (Evolve Politics 2017) further reinforce the concern that loyalty programs often fail to maintain long-term customer engagement. Consequently, marketers continue to debate the relative performance of CLPs and PLPs in terms of engaging customers and boosting profit, which affects firms' strategic choices between these two types of programs (The Wise Marketer 2021).

Empirical studies also suggest that many loyalty programs are not particularly effective in cultivating customer loyalty. For instance, Sharp and Sharp (1997) find no evidence that reward programs increase customers' average purchase frequency, while Liu (2007) indicates that such programs have no significant impact on heavy buyers regarding purchase frequencies and transaction sizes. These findings indicate that the primary benefit of loyalty programs may lie in their ability to help firms discriminate among customers with different valuations and purchasing behaviors, rather than in enhancing loyalty itself. Supporting this view, Biyalogorsky et al. (2001) show that loyalty programs can improve profitability even without increasing repeat purchases by enabling differential pricing, and Caminal and Claici (2007) demonstrate their effectiveness in segmenting customers for price discrimination. By optimally designing the reward programs, firms can extract greater surplus from high-valuation segments without necessarily increasing purchase frequency. Consequently, the profitability of loyalty programs may derive more from facilitating price discrimination than from fostering customer loyalty. Motivated by this perspective, this study abstracts from loyalty-enhancing effects and focuses exclusively on the price discrimination role of loyalty programs.

Building on this foundation, we develop an analytical model of CLPs and compare their performance with PLPs under exogenous customer arrival rates. Specifically, we investigate the following research questions:

1. How do CLPs facilitate price discrimination through the optimal design of price, reward amount, and coalition size?

2. What is the value of CLPs in enabling price discrimination relative to PLPs and no-reward programs? Under what market conditions do CLPs enable more effective price discrimination than PLPs?

3. How do factors such as market composition, customer discounting, and coalition size affect the ability of CLPs to implement price discrimination?

To address this, we consider a CLP comprising n independent firms, each offering a nondurable product to heterogeneous customers in an infinite-horizon setting. Customers differ in their shopping intensity and valuation toward the product, aiming to maximize their total discounted surplus. The arrival of customers at each firm is modeled as an independent Poisson process, so the total shopping intensity for the coalition is the sum of the intensities at each firm. When a customer purchases from any coalition member without redeeming a reward, she earns a reward that can be redeemed at any participating firm prior to expiration; otherwise, expired rewards are forfeited. We assume that the CLP's objective is to maximize the long-run average profit rate per firm (hereafter "per-firm profit").

We begin by analyzing customer purchasing behavior under a given CLP and demonstrate that a customer will make a purchase from the coalition whenever the sum of her product valuation and the expected value of the reward exceeds the price. Building on the customers' purchase decision problem, we derive the optimal price and reward decisions to maximize the per-firm profit. Furthermore, we investigate the performance of CLPs relative to no-reward programs or only offering PLPs.

Our analysis reveals several insights regarding the optimal design and operation of CLPs. First, an optimally designed CLP excludes low-valuation-infrequent customers; otherwise, joining the CLP does not improve the profit of participating firms. Second, a CLP facilitates price discrimination based on customers' shopping intensity rather than their valuation. Third, the coalition size n (i.e., the number of firms in the coalition) significantly impacts the effectiveness of price discrimination in CLPs. An improperly sized CLP can perform worse than no-reward programs at all.

Market composition plays a critical role in the effectiveness of CLPs. On the one hand, as the coalition size n increases, each customer visits the coalition more often and thus is more likely to make a purchase. This helps reduce the reward breakage rate and alleviate the effect of customer discounting. On the other hand, this might also reduce the effectiveness of discriminating against customers based on their shopping intensity, because both frequent and high-valuation-infrequent customers purchase intensively when the coalition is large. These countervailing forces drive the following results as the coalition size increases. In a market with high heterogeneity in customers' valuations, expanding a CLP does not necessarily lead to a higher per-firm profit. Conversely, in a market where customer valuations are more homogeneous, the per-firm profit may either increase or decrease monotonically in the coalition size n. Moreover, the per-firm profit may first increase and then decrease in n, implying the existence of an optimal intermediate coalition size n. These results highlight the crucial importance of market composition.

To examine the value of CLPs, we first compare them with no-reward programs (either CLPs or PLPs). We demonstrate that participation in a CLP does not enhance a firm's profit when customers' product valuations are positively correlated with their shopping intensities. Only when the correlation is negative can a properly designed CLP benefit the firm. Moreover, under certain conditions, joining a CLP not only helps a firm boost its profit but also improves the aggregate customer surplus, leading to a win-win outcome for both the firm and customers. A lose-lose outcome, however, can also occur when a CLP is inappropriately sized, highlighting the importance of proper CLP sizing.

We further investigate the performance of CLPs relative to PLPs. Our analysis shows that CLPs can significantly expand the range of market conditions under which offering reward programs (either CLPs or PLPs) is more desirable than not offering them. A firm's capability to benefit from reward programs, however, is rather limited if only PLPs are considered. More importantly, firms have incentives to join CLPs even when their PLPs are ineffective, providing one explanation for the observed popularity of CLPs in practice. However, we also show that CLPs can perform worse than PLPs under certain conditions.

Note that our base model assumes customers' arrival rates are exogenous and do not depend on the attractiveness of CLPs. This assumption may be appropriate for coalitions centered around non-discretionary products and services, where consumption is necessity-driven. However, for discretionary products and services, customer arrival rates may be influenced by CLP participation. For such a situation, we provide a comprehensive discussion regarding the impact of endogenous arrival rates on our main results[1]. We find that the profit structure remains the same as the base model because CLPs still attract frequent and high-valuation-infrequent customers while excluding low-valuation-infrequent customers. However, the comparison between CLPs and no-reward programs hinges on the extent of changes in the arrival rates in CLPs. Furthermore, CLPs can still be used to price discriminate between frequent and infrequent customers, although the discrimination power may be weakened under certain conditions.

It is also worth noting that our base model does not consider point accumulation. This setup aligns with the practice of The Guestbook, which offers customers up to 15% cash rewards redeemable for a future stay at any participating hotel within 60 days. However, we acknowledge that point accumulation is also common in practice, and thus, we also discuss how incorporating point accumulation into our model may affect the main results. We find that point accumulation makes CLPs more attrac-

[1]Note that the model does not capture potential changes in customers' price sensitivity or purchase frequency, possible competition among coalition members, or nonlinear reward structures (e.g., tiered memberships) commonly used in practice.

tive and reinforces their advantages over PLPs, because a CLP allows customers to accumulate points more quickly by shopping across multiple participating brands, whereas in a PLP, points must be earned through repeated purchases at a single brand. Although CLPs become more appealing in this context, the negative correlation between valuation and shopping intensity remains a necessary condition for CLPs to outperform no-reward programs.

Finally, we consider several model extensions to check the robustness of our results. First, we consider a scenario in which a new reward is allowed to be earned upon redemption, which affects the reward's effectiveness at differentiating frequent and infrequent customers. Second, we consider an extension where customers do not discount their future surplus and show that there exists a flat region in which the per-firm profit remains constant. This result implies that whether or not customers discount their future surplus critically affects CLPs' design and effectiveness. Third, we extend our analytical framework to a situation in which customers' arrival rates are asymmetric across firms. We show that several key results still hold.

Before proceeding, we note that our model does not consider certain spillover effects, such as cross-purchase among partner firms in CLPs, which received considerable attention in the empirical literature (Lemon and Wangenheim 2009, Schumann et al. 2014, Stourm and Bradlow 2023). While the literature offers mixed results on the impact of such spillover effects, we note that many of those effects considered in the literature strengthen the advantage of CLPs compared with PLPs. One of our key results is that CLPs tend to perform better than PLPs, explicitly modeling these effects would further strengthen the positive result. Of course, our model also ignores elements that may negatively impact CLPs. One such element is the cost of operating CLPs, which conceivably would be more expensive than operating PLPs, since operating CLPs requires coordination across firm boundaries. From this perspective, our results are best viewed as a baseline subject to adjustments based on practical considerations.

The rest of the study is organized as follows. Section 2.2 reviews the related literature on customer reward programs. Section 2.3 introduces the model setup and analyzes the customers' purchase decision problem under a given CLP. Building on this, Section 2.4 derives the coalition's decisions regarding price, reward amount, and coalition size to maximize the per-firm profit. Moreover, we study how model parameters influence the effectiveness of CLPs. Section 2.5 investigates the value of CLPs by comparing them to no-reward programs or only offering PLPs. Several model extensions are presented in Section 2.6. Section 2.7 provides concluding remarks.

2.2 Literature Review

Our work is closely related to the research on customer reward programs, much of which focuses on PLPs offered by individual firms. There is also a growing but mostly empirical research stream on CLPs.

The design and implementation of reward programs have attracted considerable interest in marketing and operations management, with particular focus on reward types (e.g., Kim et al. 2001, Chun and Ovchinnikov 2019, Walsman and Dixon 2020, Shirai 2023), referral rewards (e.g., Biyalogorsky et al. 2001, Kornish and Li 2010, Wolters et al. 2020), redemption hurdles (e.g., Breugelmans and Liu-Thompkins 2017, Sun and Zhang 2019, Liu et al. 2021), and point currencies (e.g., Chun et al. 2020, Chung et al. 2022, Lim et al. 2024). For instance, Kim et al. (2001) examine the optimal choice between cash rewards and free products, while Chun and Ovchinnikov (2019) explore the design of quantity-based, spending-based, and combined rewards. Kornish and Li (2010) investigate the role of recommendations and determine the optimal reward amount for referral reward programs. In the context of point reward programs, Chun et al. (2020) examine the optimal policy for adjusting the exchange rate between cash prices and points, and Chung et al. (2022) further investigate the redemption availability and point requirements. Different from the current paper, the aforementioned studies focus on PLPs.

Naturally, much of the literature on PLPs investigates the effects of reward programs on firms and customers, where the comparison baseline is not offering PLPs (see, e.g., Kim et al. 2004, Singh et al. 2008, Liu and Yang 2009, Caillaud and De Nijs 2014, Qiu and Rao 2020, Kuksov and Zia 2021, Rossi and Chintagunta 2023, Lei et al. 2024). Kim et al. (2004) demonstrate that reward programs can enhance profitability by providing the flexibility to adjust capacities in response to fluctuating market demand. Singh et al. (2008) show that a firm offering a reward program can remain profitable even when a competitor adopts a low-pricing strategy, owing to differentiated market positions. Rossi and Chintagunta (2023) reveal how firms in the gasoline industry strategically increase prices in later periods of a reward program, following an initial period of lower pricing aimed at capturing market share. Overall, the literature provides many mechanisms through which PLPs can benefit firms. Our work further shows that CLPs often dominate PLPs by allowing customers to earn and redeem the reward more frequently, thereby reducing reward breakage rate and alleviating customer discounting.

The literature presents mixed results on the impact of PLPs on customers (see, e.g., Kivetz 2003, Liu 2007, Kopalle et al. 2012, Wang et al. 2016, Rossi 2018, Liu and Ansari 2020, Son et al. 2020, Gopalakrishnan et al. 2021, Orhun et al. 2022, Kadiyala et al. 2024). For instance, Liu (2007) finds that light buyers change their behavior more than heavy buyers in reward programs regarding purchase frequencies and transaction sizes. Kopalle et al. (2012) uncover that price-sensitive customers value frequency rewards, whereas service-sensitive customers prioritize higher reward tiers. Rossi (2018) shows that most customers' purchase behavior is unaffected by reward points, but a small group of frequent customers is more sensitive to rewards than price changes. In line with these works, our model explicitly considers customer heterogeneity in shopping intensity and product valuation and shows that market composition critically affects the effectiveness of CLPs. Moreover, following the previous literature (e.g., Kopalle et al. 2012, Rossi 2018, Gopalakrishnan et al. 2021, Orhun et al. 2022), we incorporate customer discounting to capture the customer tradeoff between the future surplus and present value.

Our study contributes to the small but growing stream of research on CLPs, much of which is empirical studies. Few studies analytically examine cross-market discounts. For instance, Gans and King (2006) consider two markets with two competing firms in each market and demonstrate that coalitions are not more profitable. Building on this, Goić et al. (2011) examine a monopolist selling both groceries and gasoline, showing that cross-market discounts can increase profits by extracting greater customer surplus from the two markets. Similarly, in the context of bundled discounts of groceries and gasoline, Brito and Vasconcelos (2015) analyze a scenario where the differentiation with respect to groceries is more vertical than horizontal. They find that only a coalition of high-quality firms is more profitable in a competitive context. Expanding the same setting as Gans and King (2006), Gardete and Lattin (2018) introduce customer segments loyal to specific firms and allow for endogenous pricing and discount decisions. They show that cross-market coalitions unambiguously lead to increased profits. In contrast to these studies, our work considers a CLP with n firms and an infinite horizon model, in which customers make repeated purchases across coalition partners, and a reward can be redeemed in a future purchase across all participating firms. This unique setup enables us to investigate how coalition size impacts the performance of CLPs. The studies on CLPs face unique challenges due to the participation of multiple firms, including potential cross-vendor effects and ongoing debates regarding their effectiveness compared to PLPs. Lemon and Wangenheim (2009) identify a positive cross-buy effect between a firm and a complementary coalition partner. Schumann et al. (2014) investigate the negative impact of a service failure by one coalition partner on customer responses to other firms in the CLP. Dorotic et al. (2021) study both cross-purchase effects and the cannibalization of sales among 33 coalition partners from 16 industry sectors. Additionally, Stourm and Bradlow (2023) examine how rewards offered by one firm influence customers' purchases at other coalition partners, factoring in product category overlap and geographic distance. Although we do not consider the cross-over effect by assuming n independent firms, our results imply that the cross-over effect can probably further enhance the profitability of CLPs. On the other hand, Dorotic et al. (2011) find no evidence that the joint promotions in CLPs outperform those offered by individual firms, while Shirai (2023) demonstrates through online experiments that customers prefer CLPs over PLPs, particularly when a utilitarian-dominant firm offers the program. Despite these insights, as noted by Dorotic et al. (2021), empirical studies are often limited in their ability to make causal claims about whether firms should switch from PLPs to CLPs. This is because even with extensive data sets, observations are typically confined to transactions within a specific CLP and limited to a given time frame. Our study addresses this gap by developing an analytical model that compares the performance of CLPs with PLPs. We show that firms have incentives to join CLPs even when PLPs are ineffective, corroborating the increasing popularity of CLPs in recent years.

A critical element of our analysis is how the entry of a new partner affects customer spending at existing firms within a CLP. This question is examined empirically in the recent literature. Using difference-in-differences and Bayesian structural time series approaches, Ngwe et al. (2022) find that the addition of a new partner benefits pre-existing firms by increasing transactions, basket sizes, and aggregate sales. Conversely, Taylor and Dong (2023) observe that the individual marginal effects on customer spending are largely negligible when most coalition partners join or leave. Our analysis shows that the impact of adding more firms to a CLP depends critically on the market composition. In a market with high heterogeneity in customers' valuations, expanding a CLP does not necessarily lead to a higher per-firm profit. However, in a market where customer valuations are more homogeneous, the per-firm profit may either increase or decrease monotonically in the coalition size n. Furthermore, the per-firm profit may first increase and then decrease in n, implying the existence of an optimal intermediate coalition size n. Therefore, it is possible to reconcile our results with the results in the aforementioned papers.

2.3 Model Setup

We consider a CLP with n independent firms, each selling a nondurable product to a continuum of infinitesimal customers in an infinite-horizon setting. The market size is normalized to 1, and the product's marginal cost is set to zero for simplicity. Time is continuous. Customer arrivals at each firm follow an independent Poisson process, and we refer to the arrival rate as customers' *shopping intensity*. Thus, the total shopping intensity for a customer visiting firms in the coalition is the sum of the shopping intensity across all individual firms. Customers are heterogeneous in two dimensions: product valuation and shopping intensity. Specifically, a fraction α of customers are high-valuation types with valuation v_H, while the remaining fraction $1-\alpha$ are low-valuation types with valuation $v_L < v_H$. Regarding shopping intensity, a fraction β are frequent shoppers with intensity λ_F, and the remaining fraction $1-\beta$ are infrequent shoppers with intensity $\lambda_I < \lambda_F$. To account for the correlation between valuation and shopping intensity, we assume that a fraction γ of customers are high-valuation-frequent (HF) customers. Accordingly, $\alpha - \gamma$ are high-valuation-infrequent (HI) customers, $\beta - \gamma$ are low-valuation-frequent (LF) customers, and the remaining $1-\alpha-\beta+\gamma$ are low-valuation-infrequent (LI) customers. Table 2.1 summarizes these customer segments. The value of γ relative to $\alpha\beta$ indicates the nature of the correlation: $\gamma < \alpha\beta$ implies a negative correlation, $\gamma > \alpha\beta$ indicates a positive correlation, and $\gamma = \alpha\beta$ reflects no correlation between product valuation and shopping intensity.

A CLP is defined by a quartet (p, r, μ, n), where p is the price, r is the reward

Table 2.1: The Four Customer Segments

	$\lambda_{\mathbf{F}}(\beta)$	$\lambda_{\mathbf{I}}(\mathbf{1}-\beta)$
$\mathbf{v}_{\mathbf{H}}(\alpha)$	$\mathbf{HF}(\gamma)$	$\mathbf{HI}(\alpha-\gamma)$
$\mathbf{v}_{\mathbf{L}}(\mathbf{1}-\alpha)$	$\mathbf{LF}(\beta-\gamma)$	$\mathbf{LI}(1-\alpha-\beta+\gamma)$

amount, μ is the expiration rate of the rewards, and n is the coalition size.[2] Price p is assumed to be constant over time, and size n is an integer. For analytical tractability, we assume the reward's expiration time is exponentially distributed with rate μ.[3] Each purchase from any coalition member, provided no existing rewards are redeemed, grants the customer a reward of value r, which can be redeemed at any participating firm before it expires. Our model setup is consistent with the practice of many CLPs. For example, customers booking hotel rooms on The Guestbook (https://theguestbook.com/) can earn up to 15% cash rewards redeemable for a future stay at any participating hotel within 60 days. We naturally assume $r \leq p$, meaning the reward cannot exceed the price paid for the product. Additionally, our model does not incorporate reward accumulation or redemption thresholds, aligning with the operational practices of programs such as The Guestbook. An investigation of reward accumulation or the redemption threshold for CLPs is an important topic that deserves a separate study.

[2]Here, we do not explicitly model the price competition among firms in the coalition. However, it can be shown that firms in the coalition have no incentive to deviate from an optimally set price p.

[3]In practice, reward expiration terms are usually deterministic. Our treatment here uses an exponentially distributed quantity to approximate a deterministic quantity within a continuous-time model. Such an approximation is widely adopted in the queuing literature. For example, an M/M/1 queue is often used to approximate an M/D/1 queue, where a deterministic service time is approximated by an exponentially distributed service time. An exponentially distributed expiration date can also reflect customers' forgetfulness, as there is a certain chance that a customer may forget to use the reward on any purchase occasion.

2.3.1 Customers' Purchase Decision

Customers are heterogeneous in their shopping intensity and product valuation. When analyzing customers' purchase decision problem, we consider a generic customer with product valuation v and shopping intensity λ. For brevity, we refer to such a customer as a (v, λ)-customer. The customer visits the coalition with rate $n\lambda$ over time. Our model here implicitly assumes the n firms are symmetric in terms of customers' product valuation and shopping intensity. We consider an extension with asymmetric firms in Section 2.6.2. Upon arrival, the customer decides whether or not to make a purchase. The customer discounts the future surplus with a continuous-time discount rate $\delta > 0$. We also consider the case in which customers do not discount the future surplus (i.e., $\delta = 0$) in Section 2.6.3 to investigate the impact of customer discounting on our results.

The customer's decision problem can be modeled as a continuous-time infinite-horizon discounted reward dynamic program. Let $i \in \{0, 1\}$ denote the state, where $i = 0$ (1) denotes that the customer does not (does) hold a valid reward when she visits a firm in the coalition.[4]

Let $u(\cdot)$ denote the value function representing the customer's maximum total discounted surplus. The optimality equations are then given by

$$u(1) = \frac{n\lambda}{\delta + n\lambda + \mu} \max\Big\{v - p + r + u(0), u(1)\Big\} + \frac{\mu}{\delta + n\lambda + \mu} u(0), \tag{2.1}$$

$$u(0) = \frac{n\lambda}{\delta + n\lambda + \mu} \max\Big\{v - p + u(1), u(0)\Big\} + \frac{\mu}{\delta + n\lambda + \mu} u(0). \tag{2.2}$$

In state 1, if the customer arrives before the reward expires (with probability $\frac{n\lambda}{\delta+n\lambda+\mu}$) and chooses to make a purchase using the reward, she pays a net price of $p-r$.[5] Upon

[4]Due to the memoryless property of the exponential distribution, we do not need to track the time until reward expiration in the continuous-time model; therefore, it is sufficient to use $\{0, 1\}$ to denote the customer's state.

[5]We also consider a scenario in which a new reward is earned upon redemption in Section 2.6.1.

redeeming the reward, she transitions to state 0, as she cannot earn a new reward in the same transaction, resulting in a surplus of $v - p + r + u(0)$. If no purchase is made, she remains in state 1. If the reward expires before her arrival (with probability $\frac{\mu}{\delta+n\lambda+\mu}$), then she moves to state 0. Similarly, in state 0, if the customer makes a purchase upon her arrival, she pays the full price p, earns a new reward, and moves to state 1, yielding a surplus of $v - p + u(1)$; otherwise, she remains in state 0. The second term on the right-hand side of equation (2.2) is a fictitious transition that returns to state 0.[6]

By solving the optimality equations, we demonstrate that a customer will always make a purchase whenever she visits a firm in the coalition if the sum of her product valuation v and the expected value of her reward $\frac{n\lambda}{\delta+2n\lambda+\mu}r$ exceeds price p.[7] Otherwise, the customer will not remain in the market over the long run and therefore does not contribute to the per-firm profit. In summary, holding a valid reward in a CLP increases a customer's willingness to pay, thereby raising her probability of making a purchase.

2.4 The Optimal Design of CLPs

Building on the customers' purchase decision problem, we derive the coalition's decisions regarding price, reward amount, and coalition size to maximize the per-firm profit. Because the firms are symmetric, the coalition's total profit is distributed equally among all members. In practice, the reward expiration term is often exogenous due to competitive pressure or industry norms. It is also undesirable for a CLP to adjust the expiration term frequently, which will lead to customer confusion and

[6] Here, we add the fictitious transition to make the total transition rates in the two states the same.

[7] We provide a detailed analysis of the customers' decision-making process, including explicit solutions to the optimality equations (2.1)–(2.2) on page 73 in Online Appendix.

discourage their engagement. Moreover, rewards that expire very quickly may also be viewed unfavorably by customers, making the adoption of an extremely short expiration term impractical. Overall, the idea of adjusting the reward expiration term as a CLP grows in size may not be desirable in practice. Therefore, we focus on analyzing the situation in which the expiration term is exogenous with rate μ.

A CLP will not be sustainable if each firm's profit in the coalition is lower than that with no-reward programs. When no-reward programs are adopted, each firm can implement one of the following two strategies: a full market coverage strategy with price v_L or a partial market coverage strategy with price v_H.

- Full market coverage with price v_L: The price is v_L, at which all arriving customers make a purchase. The per-firm profit is $\pi_1 = (\beta\lambda_F + (1-\beta)\lambda_I)v_L$.

- Partial market coverage with price v_H: The price is v_H, at which only high-valuation customers make a purchase. The per-firm profit is $\pi_2 = (\gamma\lambda_F + (\alpha - \gamma)\lambda_I)v_H$.

In the following, we first characterize the optimal price, reward amount, and per-firm profit of the CLP by assuming that the coalition size n is given. We then determine the optimal size for the coalition. Along the way, we derive a necessary condition for a firm to participate in the CLP by comparing the per-firm profit under the CLP with that of no-reward programs.[8]

Lemma 2.1. *A CLP with size n improves the per-firm profit relative to no-reward programs only if*

$$(\alpha-\gamma)\lambda_I\Big(\lambda_F\mu - \lambda_I(\delta+\mu)\Big)(2n\lambda_F+\mu) > \beta\lambda_F^2\delta(2n\lambda_I+\mu). \tag{2.3}$$

[8]We note that it is also possible for each firm to offer a PLP. We compare joining a CLP with offering a PLP in Section 2.5.2.

When condition (2.3) holds, the optimal per-firm profit in the CLP are

$$\pi^* = \underbrace{\beta\lambda_F\Big(v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu}r^* - \frac{n\lambda_F}{2n\lambda_F + \mu}r^*\Big)}_{\text{profit from HF and LF customers}} + \underbrace{(\alpha-\gamma)\lambda_I\Big(v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu}r^* - \frac{n\lambda_I}{2n\lambda_I + \mu}r^*\Big)}_{\text{profit from HI customers}}. \tag{2.4}$$

We highlight three key observations regarding the optimal design of CLPs. First, an optimally structured CLP excludes LI customers; otherwise, the CLP is less profitable than no-reward programs with a full market coverage strategy and price v_L. Since the highest effective price (i.e., the price net of the expected value of the reward) that LI customers can afford does not exceed v_L, the profit contribution from each customer segment cannot surpass v_L if LI customers are included. Consequently, under the optimal CLP, all frequent customers (HF and LF) and HI customers make purchases, while LI customers do not participate. The profit expression $\pi^*(n)$ in (2.4) reflects this, with the first term representing profit from frequent customers and the second from HI customers.

Second, the CLP discriminates against customers based on their shopping intensity rather than their valuation. Customers who shop more frequently have a greater likelihood of earning and redeeming rewards, resulting in a lower effective price over time. Both HF and LF customers pay the same effective price in the long run, which is less than that paid by HI customers.

Third, the CLP should be adopted only when condition (2.3) holds, which can be rewritten as

$$\beta\lambda_F\Big(\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_F}{2n\lambda_F + \mu}\Big) + (\alpha-\gamma)\lambda_I\Big(\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_I}{2n\lambda_I + \mu}\Big) > 0. \tag{2.5}$$

The terms on the left-hand side correspond to the coefficients of reward r in the per-firm profit π^* in (2.4). Only when this coefficient is positive will the firm have the incentive to offer reward r; otherwise, $r^* = 0$ and the CLP should not be adopted, as π^* is less than the profit π_1 earned from the full market coverage with price v_L. Since $\frac{n\lambda_F}{\delta+2n\lambda_F+\mu} < \frac{n\lambda_F}{2n\lambda_F+\mu}$, the first term in (2.4) indicates that the effective price for

HF and LF customers is below v_L, while the effective price for HI customers can exceed v_L. Therefore, for a CLP to outperform no-reward programs, the proportion of frequent customers β must not be excessively large relative to the proportion of HI customers $\alpha - \gamma$. This balance is necessary because the CLP must offset the cost of rewards redeemed by frequent customers with the additional profit generated from HI customers.

Lemma 2.1 builds the foundation for us to analyze the optimal sizing for the coalition. Before moving forward, we define the following critical threshold on size:

$$\tilde{n} := \frac{\left(1 - \frac{\lambda_I}{\lambda_F}\sqrt{\frac{(\alpha-\gamma)[\lambda_F\mu - \lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]}}\right)\mu}{2\lambda_I\left(\sqrt{\frac{(\alpha-\gamma)[\lambda_F\mu - \lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]}} - 1\right)}.$$

Proposition 2.1 (The Optimal Sizing)**.**

(a) When $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} < \frac{v_H}{v_L}$*, the per-firm profit* $\pi^*(n)$ *does not increase monotonically in the coalition size* n*. In particular,* $\pi^*(n)$ *decreases in* n *when* n *is sufficiently large.*

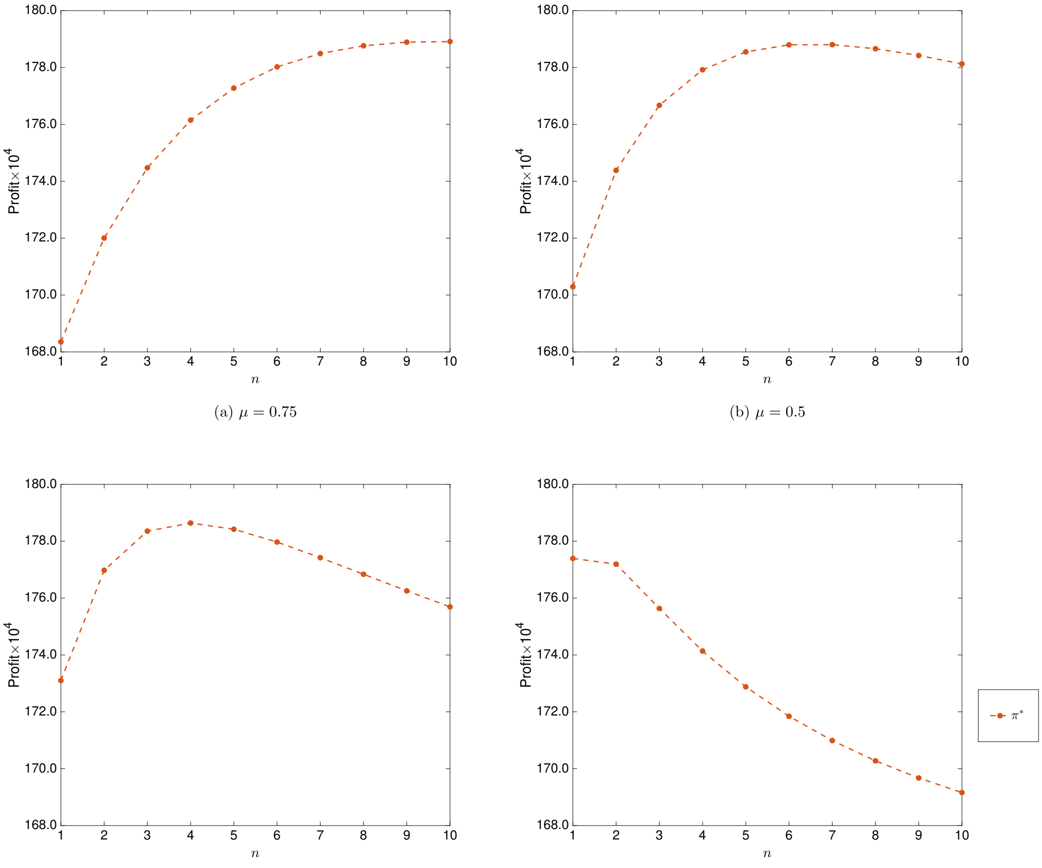
(b) When $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} \geq \frac{v_H}{v_L}$*, under different conditions, the per-firm profit* $\pi^*(n)$ *can increase monotonically, decrease monotonically, or first increase and then decrease in the coalition size* n*, respectively. Specifically, the profit* $\pi^*(n)$ *increases monotonically in the coalition size* n *if* $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \leq 1$*; consequently, larger CLPs lead to a higher per-firm profit. If* $1 < \frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} < \left(\frac{\lambda_F}{\lambda_I}\right)^2$*, the profit* $\pi^*(n)$ *first increases in* n *for* $n \leq \tilde{n}$ *and then decreases in* n*; the optimal size* $n^* = \lfloor \tilde{n} \rfloor$ *is* $\lceil \tilde{n} \rceil$*, where* $\lfloor \cdot \rfloor$ *and* $\lceil \cdot \rceil$ *are the floor and ceiling functions, respectively. Otherwise, the profit* $\pi^*(n)$ *decreases monotonically in the coalition size* n*; consequently, larger CLPs lead to a lower per-firm profit.*

Intuitively, as more firms join the coalition, the likelihood that customers will redeem their rewards before expiration increases, thereby enhancing customer surplus. This allows the coalition to extract greater profit from customers by adjusting the price p and the reward r. Additionally, a larger coalition size n raises the probability that customers will visit and make purchases within the coalition, meaning that repeat purchases occur sooner and rewards are less affected by discounting. Therefore, one might expect that the per-firm profit increases in the coalition size n.

However, Proposition 2.1 shows that this intuition is not completely correct. Specifically, Proposition 2.1(b) shows that the approach of "the more the merrier" works well only if $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \leq 1$. Moreover, if $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \geq \left(\frac{\lambda_F}{\lambda_I}\right)^2$, expanding a CLP actually reduces per-firm profit. The key underlying reason lies in the price discrimination role of the CLP.

Note that the optimal price takes the minimum value between the highest prices acceptable to HI and LF customers. The optimal reward r is chosen to bring the two prices as close as possible. Ideally, r is set so that both HI and LF customers pay their respective highest acceptable prices, achieving perfect price discrimination. However, in a market with high heterogeneity in customers' valuations (i.e., v_L is much smaller than v_H), or in a market with high heterogeneity in shopping intensity (i.e., $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} < \left(\frac{\lambda_F}{\lambda_I}\right)^2$), the highest acceptable price for LF customers is below that for HI customers for any reward r less than price p. This leaves an uncollected surplus from HI customers, implying that price discrimination is carried out imperfectly. As the coalition size n increases, the highest acceptable price for LF customers increases faster than that for HI customers, because LF customers visit the coalition and redeem the reward more frequently, and thus their utility increases faster than that of HI customers. That is, the gap between the two acceptable prices shrinks, and the CLP can extract more surplus from HI customers and achieve better price discrimination. This explains why the profit increases in the coalition size n in these scenarios.

However, when the coalition becomes sufficiently large, the optimal reward r can be set so that the effective prices for LF and HI customers are equal, meaning both customer segments pay their highest acceptable prices and perfect price discrimination is realized. As a result, the difference in effective prices between these two customer segments diminishes, weakening the price discrimination power of the CLP. Consequently, the per-firm profit decreases as the coalition size n increases.



(a) $\mu = 0.75$ (b) $\mu = 0.5$

(c) $\mu = 0.3$ (d) $\mu = 0.1$

Figure 2.1: Coalition's Optimal Per-firm Profit Across Size n (Parameters $(v_H, v_L, \lambda_F, \lambda_I, \alpha, \beta, \gamma, \delta) = (1, 0.45, 0.1, 0.03, 0.6, 0.2, 0.06, 0.005)$)

Figure 2.1 illustrates how the per-firm profit π^* changes with the coalition size n under different reward expiration rates μ. The values of all other parameters are fixed. Note that when $n = 1$, a CLP reduces to a PLP. Recall that a larger μ corresponds to a shorter expiration term. Accordingly, the larger the value of μ, the stronger the positive effect of reducing the reward breakage rate as the coalition size n increases. When $\mu = 0.75$, Figure 2.1(a) shows that the per-firm profit keeps increasing in the coalition size n for $n \leq 10$. Figure 2.1(b) shows that the negative effect of a larger coalition size n (i.e., eroding the firm's discrimination power) emerges once $n \geq 7$ when $\mu = 0.5$. However, the negative effect is mild, and the profit decreases slowly as the coalition size n keeps increasing. Figure 2.1(c) shows that the negative effect dominates for $n \geq 4$ when $\mu = 0.3$. Figure 2.1(d) shows that the maximum per-firm profit is reached at $n = 1$ when $\mu = 0.1$, meaning that forming a CLP is undesirable compared to offering PLPs. When $\mu = 0.1$, the expiration term is long enough that the positive effect of reducing the reward breakage rate cannot fully compensate for the negative effect of eroding the firms' price discrimination power, even for a small CLP.

2.5 The Value of CLPs

In this section, we examine the value of CLPs by comparing them with no-reward programs (Section 2.5.1) and PLPs (Section 2.5.2), in terms of both the firm profit and customer surplus.

2.5.1 Comparison with No-Reward Programs

Recall that when a firm does not offer a reward program, the full market coverage strategy with price v_L yields a profit of $\pi_1 = \big(\beta\lambda_F + (1-\beta)\lambda_I\big)v_L$, while the partial market coverage strategy with price v_H yields a profit of $\pi_2 = \big(\gamma\lambda_F + (\alpha-\gamma)\lambda_I\big)v_H$.

Proposition 2.2. *By comparing a CLP and no-reward programs, we have the following results:*

(a) When customers' product valuation and shopping intensity are positively correlated, a CLP, regardless of its size, always yields a lower profit for the firm than no-reward programs.

(b) *The aggregate customer surplus increases when firms switch from a partial market coverage strategy with price v_H to forming a CLP, while the aggregate customer surplus decreases when firms switch from a full market coverage strategy with price v_L to forming a CLP.*

Table 2.2: Effective Prices Paid by the Four Customer Segments

	CLP	No-Reward Programs	
		Full Market Coverage	Partial Market Coverage
HF	$\leq v_L$	v_L	v_H
LF	$\leq v_L$	v_L	-
HI	$\leq v_H$	v_L	v_H
LI	-	v_L	-

Proposition 2.2(a) states that a negative correlation between customers' product valuation and shopping intensity (i.e., $\gamma > \alpha\beta$) is necessary for firms to have the incentive to join a CLP compared to opting for no-reward programs. [9] To understand the rationale, we need to recall the effective prices paid by each customer segment in Table 2.2. Relative to a full market coverage strategy at price v_L, the coalition can charge HI customers a higher effective price, though LI customers are excluded from making purchases. Compared to a partial market coverage strategy with price v_H,

[9] The correlation between customer valuation and shopping intensity has been explored in both industry surveys and academic research. For instance, a 2017 survey of 1,455 primary household grocery shoppers in the U.S. found that households with annual incomes above \$150,000 (a proxy for higher customer valuation) tend to shop less frequently, with 60% of this group buying groceries once a week or less (Source: Statista). Academic studies also support this negative correlation: Kim and Rossi (1994) and Anslie and Rossi (1997) report that frequent grocery shoppers are generally more price-sensitive, while Bell and Lattin (1998) find that high-valuation customers prefer infrequent but well-researched purchases.

the coalition is able to retain LF customers, but HF customers pay a lower effective price. As a result, for the CLP to outperform scenarios with no-reward programs, the HI and LF customer segments must be sufficiently large, which suggests a negative correlation between product valuation and shopping intensity.

Proposition 2.2(b) discusses the implications of CLPs on customer surplus. Under the partial market coverage strategy with price v_H, only high-valuation customers purchase, and the aggregate customer surplus is 0, which is less than the aggregate customer surplus under a CLP. Under a CLP, the effective price paid by HF and LF customers does not exceed v_L, [10] which means these customers enjoy a greater surplus compared to the full market coverage strategy with price v_L. Conversely, HI customers in a CLP always pay an effective price above v_L, [11] resulting in a lower surplus for HI customers. LI customers do not make purchases under the CLP, so their surplus remains zero, the same as in the full market coverage strategy with price v_L. Proposition 2.2(b) reveals that the CLP leads to a lower aggregate customer surplus than that in the full market coverage strategy with price v_L. This is because the adoption of a CLP requires a small fraction of frequent customers (who redeem the rewards frequently and thus hurt the firm) and a large fraction of HI customers (who pay a higher effective price and contribute to the firm's profit), as indicated by condition (2.5). In this situation, the surplus increment of frequent customers in the CLP cannot compensate for the surplus loss of HI customers, resulting in a lower aggregate customer surplus.

Proposition 2.2 indicates that transitioning from no-reward programs to a CLP can hurt both the firm and customers if there is a positive correlation between product

[10] In fact, the effective price for LF customers cannot exceed v_L; otherwise, they would opt not to purchase, resulting in lower per-firm profit for the CLP compared to that of no-reward programs with the partial market coverage strategy.

[11] The effective price for HI customers must be above v_L; otherwise, all three segments contribute no more than v_L, making the CLP's per-firm profit no greater than that of no-reward programs with the full market coverage strategy.

valuation and shopping intensity (or negative, but the coalition size is inappropriate), resulting in a lose-lose outcome. A win-win outcome is possible only when there is a negative correlation and the coalition size is optimally set. This underscores the critical role of customer composition in determining the success of CLPs, highlighting its significance not just for firms, but also for social welfare (i.e., the sum of the firms' profits and the aggregate customer surplus).

2.5.2 Comparison with PLPs

This section examines the value of joining a CLP compared with offering a PLP. To facilitate the discussion, let π_p denote the optimal profit of a PLP, and $\pi^*(n^*)$ denote the CLP's optimal per-firm profit under the optimally chosen coalition size n^*. Additionally, for a meaningful comparison, we require that the CLP consist of at least two firms, i.e., $n \geq 2$. If not, $\pi^*(n^*)$ will always be (weakly) greater than π_p, since a PLP is simply a CLP with $n = 1$.

Similar to the condition (2.3) required for the adoption of a CLP, if

$$(\alpha - \gamma)\lambda_I\Big(\lambda_F\mu - \lambda_I(\delta + \mu)\Big)(2\lambda_F + \mu) > \beta\lambda_F^2\delta(2\lambda_I + \mu). \tag{2.6}$$

does not hold, then adopting a PLP is no more profitable than offering no-reward programs. It can be verified that condition (2.3) is less stringent than condition (2.6). Therefore, when offering a PLP cannot lead to a higher profit than that of no-reward programs (i.e., condition (2.6) is not satisfied), firms still have incentives to join a CLP to boost their profits. Hereafter, for comparison purposes, we assume that the conditions stated in (2.3) and (2.6) both hold.

The relative performance of a PLP versus a CLP depends on how the CLP's per-firm profit changes with coalition size n. If the per-firm profit increases as the size n expands from 1 to 2, then a CLP with the optimal size n^* will perform better than a PLP. In other words, only when the per-firm profit decreases in the coalition size n

in the very beginning can a PLP outperform a CLP.

By utilizing the results stated in Proposition 2.1, we derive the following results in Proposition 2.3.

Proposition 2.3. *By comparing a CLP and a PLP, we have the following results:*

(a) When $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} < \frac{v_H}{v_L}$*, an optimally sized CLP does not necessarily lead to a higher per-firm profit than a PLP.*

(b) When $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} \geq \frac{v_H}{v_L}$*, a PLP yields a higher per-firm profit than an optimally sized CLP when either of the following conditions holds:*

(i) $1 < \frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} < (\frac{\lambda_F}{\lambda_I})^2$ *and* $\tilde{n} \leq 1$*;*

(ii) $1 < \frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} < (\frac{\lambda_F}{\lambda_I})^2$*,* $1 < \tilde{n} < 2$*, and* $\pi^*(1) > \pi^*(2)$*;*

(iii) $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \geq (\frac{\lambda_F}{\lambda_I})^2$*.*

Proposition 2.3(a) shows that a larger coalition does not necessarily benefit the participating firms. Recall that there may exist two countervailing effects as the coalition size n increases. On one hand, higher shopping intensity in a CLP leads to a lower reward breakage rate and less discounting of customer surplus, allowing the firm to extract more surplus from customers. On the other hand, as the difference in shopping intensity between frequent and HI customers diminishes, the CLP's ability to price discriminate is weakened. If the reward expiration period is long enough (i.e., μ is very small) and the discount rate δ is also small, the negative impact may outweigh the positive, making the PLP potentially more profitable than a CLP of optimal size. Proposition 2.3(b) characterizes the conditions under which such a situation arises. Even when the conditions are not satisfied, a CLP may still perform worse than the PLP if it is not properly sized. Recall that the per-firm profit may first increase and then decrease in the coalition size n. Therefore, there may be a threshold n' such that when $n > n'$, the per-firm profit under the CLP, $\pi^*(n)$, falls below that of the PLP, π_p. This again highlights the critical role of coalition size in determining the effectiveness of CLPs.

As expected, PLPs and CLPs generate the same social welfare, since all customer segments except LI customers make purchases under both programs. Consequently, higher profits in each program are offset by lower customer surplus, and vice versa.

These findings provide useful guidelines for managers to understand and design reward programs. First, CLPs can significantly expand the range of market conditions under which offering reward programs is more desirable than not offering them. Firms' capability to benefit from reward programs is rather limited if only PLPs are considered. There exists a broad range of parameter space in which PLPs cannot improve a firm's profit, but CLPs can. This is because binding different participating firms together through the CLP helps reduce the reward breakage rate and alleviate customer discounting. The effectiveness of CLPs is evident in the case of Sainsbury's (Sky News 2023). Struggling with its PLP, Sainsbury's Rewards (which faces difficulty competing with Tesco's Clubcard and other more versatile programs in the UK market), Sainsbury's made a strategic shift to join the Nectar program, which includes a wide range of partners such as British Airways, eBay, and Esso. This move not only broadens the appeal of their loyalty offerings but also leverages the collective strength of multiple brands, creating a more compelling value proposition and enhancing profitability. Second, the market composition and the sizing decision for CLPs are crucial for their success. In particular, larger CLPs are not necessarily preferred. Third, reward programs are most effective when the customer heterogeneity in product valuation is moderate. Otherwise, pricing alone is sufficient to extract the most value from the market. For example, when the variation in customers' product valuation is sufficiently small (i.e., v_L is very close to v_H), pricing at v_L can bring the firm a higher profit than adopting reward programs. Likewise, when the variation in customers' product valuation is sufficiently large (i.e., v_L is very small), pricing at v_H is more effective. Last but not least, reward programs benefit from heterogeneity in customers' shopping intensity. As expected, reward programs help firms discriminate against customers based on their shopping intensity. Consider an extreme situation where $\lambda_I = \lambda_F$, under which there is no customer heterogeneity in shopping intensity. Then, the reward programs lose their price discrimination capability and are not profitable.

2.6 Model Extensions

In this section, we consider three extensions of the base model to check the robustness of our results. Section 2.6.1 considers a scenario in which a new reward is allowed to be earned upon redemption. Section 2.6.2 extends the base model to asymmetric firms. Section 2.6.3 considers the situation in which customers do not discount the future surplus.

2.6.1 A New Reward is Earned upon Redemption

In the base model, customers cannot earn a reward upon redemption. In this section, we assume that customers are allowed to earn a new reward with each purchase, even when they redeem an existing reward. The optimality equations are given by

$$u(1) = \frac{n\lambda}{\delta + n\lambda + \mu} \max\Big\{v - p + r + u(1), u(1)\Big\} + \frac{\mu}{\delta + n\lambda + \mu} u(0), \tag{2.7}$$

$$u(0) = \frac{n\lambda}{\delta + n\lambda + \mu} \max\Big\{v - p + u(1), u(0)\Big\} + \frac{\mu}{\delta + n\lambda + \mu} u(0). \tag{2.8}$$

Following a similar analysis as in the base model, we first derive the customer's optimal purchasing decision and then analyze the coalition's pricing and reward decisions. The detailed analysis is relegated to the appendix. We find that the general structure of the optimal price, reward, and per-firm profit closely resembles that of the base model. More importantly, the comparison between CLPs and no-reward programs yields the same result as in the base model, demonstrating the robustness of our result in Proposition 2.2. Before moving forward, we note that the two ratios $\frac{\lambda_F}{\lambda_I}$ and $\frac{v_H}{v_L}$ measure the degree of customer heterogeneity in shopping intensity and product valuation, respectively. When $\frac{v_H}{v_L} \geq \frac{\lambda_F}{\lambda_I}$, the valuation heterogeneity is more pronounced, and accordingly, we call the market a *valuation-driven (VD) market*; otherwise, we call the market an *intensity-driven (ID) market*.

Proposition 2.4. *Suppose that customers are allowed to earn a new reward during the redemption of an existing one.*

(a) In a VD Market, the profit π^* increases in the coalition size n. That is, larger CLPs lead to a higher per-firm profit.

(b) In an ID Market, the profit π^* increases monotonically in the coalition size n if $T \leq 1$; consequently, larger CLPs lead to a higher per-firm profit. Otherwise, the profit π^* first increases in n for $n \leq \hat{n}$ and then decreases in n; the optimal size $n^* = \lfloor \hat{n} \rfloor$ or $\lceil \hat{n} \rceil$, where $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ are the floor and ceiling functions, respectively.[12]

Unlike Proposition 2.3(a), which suggests that a larger coalition does not necessarily benefit the participating firms, Proposition 2.4(a) reveals that, when a new reward is allowed to be earned upon redemption, a larger CLP is always preferred in a VD market. The left of Figure 2.2(a)-(b) shows that profit decreases when n is sufficiently large in the base model, while the right panel illustrates that profit increases with size n in the VD market in the new setting. In particular, the left panel of Figure 2.2(b) indicates that the PLP can outperform the CLP, which is not possible in the VD market in the new setting. Proposition 2.4(b) aligns with Proposition 2.3(b): an optimally sized CLP can lead to a higher per-firm profit than a PLP

[12] Please refer to the detailed expressions of T and $\hat{n}$ in the Proof of Proposition 2.4 in the appendix.

in many cases, but there do exist conditions under which a PLP yields a higher profit than an optimally sized CLP. One difference is that the optimal coalition size is larger in the new setting. For instance, the optimal coalition size in this context is 8 (see the right panel of Figure 2.2c), compared to 4 in the base model (see the left panel of Figure 2.2c). This suggests that the optimal coalition size tends to be larger in the ID market compared to the base model.

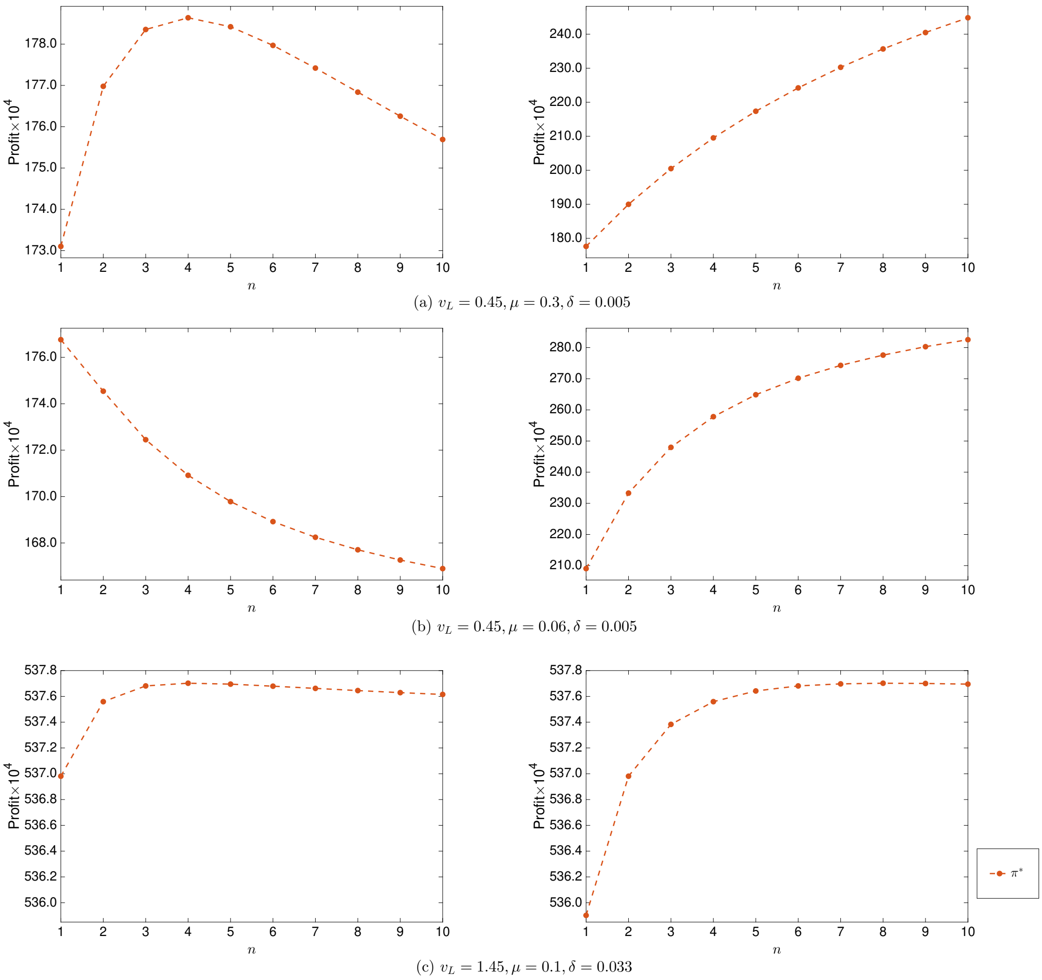


Figure 2.2: Coalition's Optimal Per-firm Profit Across Size n. The left panel illustrates the scenario in the base model, while the right panel depicts the situation where a new reward is earned upon redemption (Parameters $(v_H, \lambda_F, \lambda_I, \alpha, \beta, \gamma) = (1.6, 0.1, 0.03, 0.6, 0.2, 0.06)$)

In the new setting, the reward is more effective at differentiating frequent and infrequent customers since rewards can be applied to every purchase. This allows the CLP greater capacity to differentiate the two customer groups, meaning that only a sufficiently large n can diminish the CLP's differentiation power. In contrast, the reward in the base model is less effective at differentiating customers, as the fraction of purchases using a reward cannot exceed $\frac{1}{2}$. Consequently, the new setting provides a relatively greater capacity for the CLP to differentiate the two customer groups, under which the negative effect of a larger coalition size n (i.e., eroding the coalition's discrimination power) emerges more slowly. This explains why profit cannot decrease in Proposition 2.4(a) and why the optimal coalition size is larger in Proposition 2.4(b).

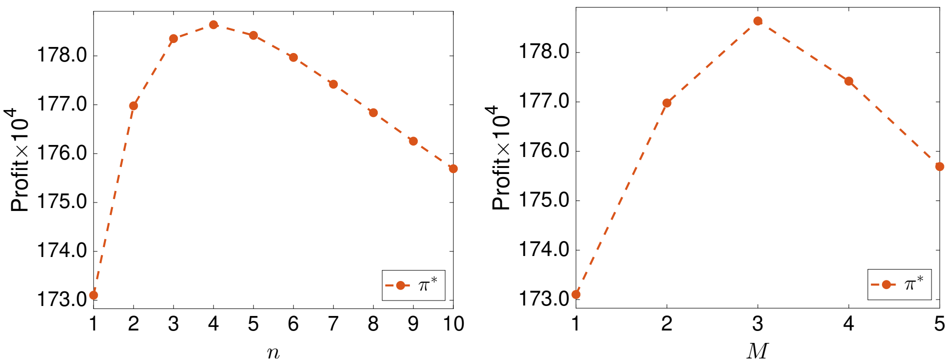
2.6.2 Asymmetric Firms

The firms in the base model are assumed to be symmetric in terms of customers' product valuation and shopping intensity in the interest of analytical tractability. We now relax this assumption to check the robustness of our main results. Regardless of whether the firms are symmetric, as more firms join the coalition, each customer visits the coalition more frequently. This leads to the positive effect of reducing the reward breakage rate and alleviating customer discounting. In the meantime, both frequent and infrequent customers have more opportunities to redeem earned rewards in a larger CLP, weakening the power of shopping-intensity-based price discrimination. These effects persist even if firms are asymmetric. Therefore, we expect our main results to hold qualitatively even when firms are asymmetric.

To illustrate this point in a concrete setting, consider a CLP consisting of two independent firms (firms 1 and 2) with asymmetric shopping intensities. In particular, we assume that frequent and infrequent customers visit firm i with shopping intensity $n_i\lambda_F$ and $n_i\lambda_I$, respectively, where $i = 1, 2$ and $n_1 \neq n_2$ are two positive integers. We find that for such a CLP, the total profit is equal to that of a CLP with $n_1 + n_2$

symmetric firms, where frequent and infrequent customers' shopping intensities for each symmetric firm are λ_F and λ_I, respectively, as in the base model. The following proposition summarizes this result.

Proposition 2.5. *The total profit of a CLP consisting of the above two asymmetric firms 1 and 2 is equal to that of a CLP with $n_1 + n_2$ symmetric firms in the base model.*



(a) Symmetric case (b) Asymmetric case

Figure 2.3: Coalition's Optimal Average Profit Across Size M (Parameters $(v_H, v_L, \lambda_F, \lambda_I, \alpha, \beta, \gamma, \mu, \delta, n_1, n_2, n_3, n_4, n_5) = (1, 0.45, 0.1, 0.03, 0.6, 0.2, 0.06, 0.3, 0.005, 1, 1, 2, 3, 3)$)

Proposition 2.5 can be easily extended to a setting with M ($M \geq 2$) asymmetric firms. Then, the results of the base model can be applied to examine the value of CLPs in this asymmetric setting. Here, we define the coalition's average profit as the coalition's total profit divided by $\sum_{i=1}^{M} n_i$.[13] Consider an example with five asymmetric firms where $(n_1, n_2, n_3, n_4, n_5) = (1, 1, 2, 3, 3)$, and the other parameters have the same value as those in Figure 2.1(c). The relationship between the average profit and size can be derived from Figure 2.1(c). For ease of comparison, we replicate

[13]We acknowledge that the profit allocation among asymmetric firms is an interesting but non-trivial topic, which apparently deserves a separate study.

Figure 2.1(c) in Figure 2.3(a). Figure 2.3(b) depicts how the average profit changes as the five asymmetric firms join the CLP sequentially from firm 1 to firm 5. Unsurprisingly, a closer look at Figures 2.3(a) and (b) reveals that the average profit of the asymmetric CLP with $M = 2$ $(3, 4, 5)$ equals the per-firm profit of the symmetric CLP with $n = \sum_{i=1}^{M} n_i = 2$ $(4, 7, 10)$. In Figure 2.3(b), the CLP comprising firms 1 and 2 achieves a higher average profit than both PLPs and no-reward programs. However, when firm 3 joins the CLP, the average profit drops below that of no-reward programs, although it remains higher than that of PLPs. When firm 4 joins, the CLP is outperformed by both no-reward programs and PLPs. This figure illustrates that while an optimally designed CLP can surpass the performance of PLPs and no-reward programs, a non-optimally designed CLP may end up performing worse than both. It confirms that the main driving forces in our base model continue to carry over; thus, the main results continue to be valid in an asymmetric setting.

2.6.3 No Customer Discounting

In the base model, customers discount the future surplus with rate $\delta > 0$. To examine the impact of customer discounting on the value of CLPs, this section analyzes the scenario where customers do not discount future surplus ($\delta = 0$), aiming to maximize the long-run average customer surplus. The decision problem for a generic (v, λ)-customer can be formulated as an infinite-horizon average-reward dynamic program. Let g^* denote the optimal average customer surplus and $h(\cdot)$ represent the bias function. The optimality equations are given by

$$g^* + h(1) = \frac{n\lambda}{n\lambda + \mu} \max\left\{v - p + r + h(0), h(1)\right\} + \frac{\mu}{n\lambda + \mu} h(0), \tag{2.9}$$

$$g^* + h(0) = \frac{n\lambda}{n\lambda + \mu} \max\left\{v - p + h(1), h(0)\right\} + \frac{\mu}{n\lambda + \mu} h(0). \tag{2.10}$$

Following a similar approach to that for the base model, we first solve the customer's purchase decision problem and then analyze the coalition's pricing and reward decisions.[14] The following proposition mirrors the result in Proposition 2.1.

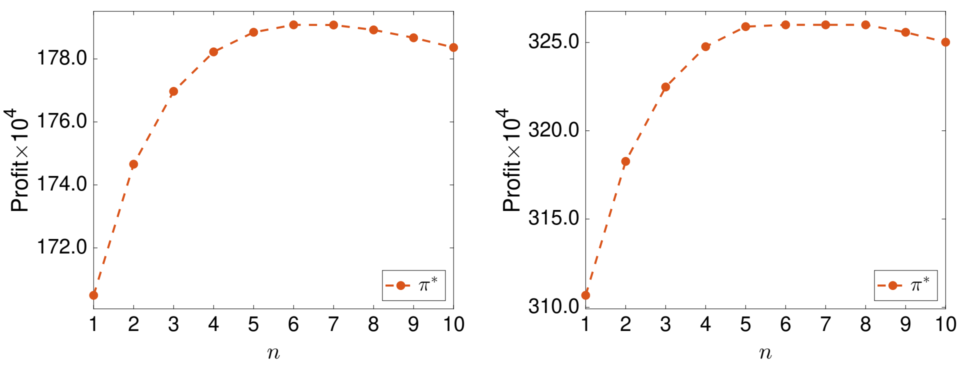
Proposition 2.6. *Suppose that customers do not discount the future surplus. For any fixed coalition size n, the optimal price, reward amount, and per-firm profit in the CLP are, respectively,*

$$p^* = v_L + \frac{n\lambda_F}{2n\lambda_F + \mu} r^*,$$

$$r^* = \min\left\{\frac{(v_H - v_L)(2n\lambda_F + \mu)(2n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)}, \frac{\mu + 2n\lambda_F}{\mu + n\lambda_F} v_L\right\},$$

$$\pi^* = \beta\lambda_F v_L + (\alpha - \gamma)\lambda_I \min\left\{v_H, \frac{(2n\lambda_F + \mu)(n\lambda_I + \mu)}{(2n\lambda_I + \mu)(n\lambda_F + \mu)} v_L\right\}. \tag{2.11}$$

Moreover, if $\Delta > 0$ and $n_1 > 0$, the per-firm profit π^* first increases in the coalition size n for $n < n_1$ and remains a constant for $n_1 < n < n_2$ and then decreases in the coalition size n for $n \geq n_2$[15]; if $\Delta > 0, n_2 < 0$ or $\Delta \leq 0$, the per-firm profit π^* first increases in the coalition size n when $n < \frac{\mu}{\sqrt{2\lambda_F \lambda_I}}$ and then decreases in n.



(a) $v_L = 0.45$ (b) $v_L = 0.82$

Figure 2.4: Coalition's Optimal Per-firm Profit Across Size n without Customer Discounting (Parameters $(v_H, \lambda_F, \lambda_I, \alpha, \beta, \gamma, \mu) = (1, 0.1, 0.03, 0.6, 0.2, 0.06, 0.5)$)

[14]Please refer to the details on page 96 in the appendix.

[15]For detailed expressions of the thresholds Δ, n_1, and n_2, please see the proof of Proposition 2.6 in the appendix.

Proposition 2.6 shows that when customers do not discount the future surplus, there exists a flat region (i.e., the per-firm profit remains constant) when $5 < n < 9$, as illustrated by Figure 2.4(b). This is in sharp contrast to that when customers discount the further surplus, in which the per-firm profit never flattens out (see Proposition 2.1 and Figure 2.1).

To understand why the per-firm profit without customer discounting exists in a flat region, we consider two terms: $\frac{n\lambda}{2n\lambda+\mu}$ (without discounting) and $\frac{n\lambda}{\delta+2n\lambda+\mu}$ (with discounting). According to a customer's profit contribution stated in (A2), the first term represents the probability of a customer redeeming the reward before expiration. Accordingly, the expected firm profit is reduced by $\frac{n\lambda}{2n\lambda+\mu}r$ due to the rewards offered to customers. However, the expected value of the reward to a generic customer is $\frac{n\lambda}{\delta+2n\lambda+\mu}r$ rather than $\frac{n\lambda}{2n\lambda+\mu}r$ due to customer discounting. Hence, the highest acceptable price to a generic customer is $v + \frac{n\lambda}{\delta+2n\lambda+\mu}r$, and the effective price paid by her is $v + \frac{n\lambda}{\delta+2n\lambda+\mu}r - \frac{n\lambda}{2n\lambda+\mu}r$. One can verify that the difference between $\frac{n\lambda_F}{\delta+2n\lambda_F+\mu} - \frac{n\lambda_F}{2n\lambda_F+\mu}$ and $\frac{n\lambda_I}{\delta+2n\lambda_I+\mu} - \frac{n\lambda_I}{2n\lambda_I+\mu}$ decreases in the coalition size n when n becomes sufficiently large. It implies that the CLP's price discrimination power decreases when the coalition size n exceeds a certain threshold. This explains why the firm's profit may only decrease in size n when n exceeds a threshold, when customers discount the future surplus. In contrast, without customer discounting, there is no discrepancy between the expected firm profit deduction and the expected value of the reward to the customer. Thus, the effective price paid by a generic customer is simply v. A closer look at the optimal per-firm profit π^* in (2.11) shows that, when customers do not discount, the CLP enables the firm to collect the maximum profit from frequent customers, whose highest acceptable effective price is v_L. The firm charges an effective price $\min\left\{v_H, \frac{(2n\lambda_F+\mu)(n\lambda_I+\mu)}{(2n\lambda_I+\mu)(n\lambda_F+\mu)}v_L\right\}$ to HI customers, which increases until it reaches the maximum v_H (the highest effective price acceptable to HI customers) and thus creates a flat region.

This result reveals the critical impact of customer discounting on a CLP's sizing decision and its effectiveness. Without customer discounting, the sizing decision could be taken without worrying about the immediate erosion of price discrimination power across a broader range. In contrast, in the presence of customer discounting, a larger-sized CLP may have a weaker price discrimination power, leading to an earlier decline in per-firm profit.

2.7 Discussion

CLPs have witnessed considerable growth in recent years, yet there remains a significant gap in research that thoroughly explains the drivers behind their widespread adoption. Much of the research on customer reward programs focuses on PLPs, with a growing but primarily empirical body of work on CLPs. Our study advances the field by offering an analytical framework. Rather than merely supplementing existing empirical findings, we provide a rigorous theoretical analysis that deepens our understanding of CLPs. Moreover, there is a lack of clear understanding among marketers regarding the relative merit of CLPs and PLPs in terms of boosting profit and engaging customers, which hinders their ability to choose between the two types of programs. In this work, we examine the value of CLPs and compare them to PLPs and no-reward programs (either CLPs or PLPs). We show that firms often have incentives to join CLPs even when PLPs are ineffective, corroborating the increasing popularity of CLPs in recent years. Under certain conditions, joining a CLP not only helps a firm boost its profit but also improves the aggregate customer surplus, leading to a win-win outcome for both the firm and its customers. A lose-lose outcome, however, can also occur when a CLP is inappropriately designed. To our knowledge, we are the first to build an analytical model to examine the popularity and value of CLPs compared to PLPs. Our work sheds light on the rationale for adopting CLPs and provides useful guidelines for their optimal design and implementation in practice.

Chapter 3

On the Value of In-Store Take-Back Programs

3.1 Introduction

In response to the growing environmental awareness among customers, many firms are incorporating sustainability into their business strategies by incentivizing customers to return used products or packaging. One widely adopted initiative is the take-back program (TBP), in which firms offer rewards to customers for returning end-of-life items. These programs aim to support circular economy practices by facilitating responsible product disposal and encouraging sustainable customer behavior.

Leading retailers such as H&M, Marks & Spencer, and L'Occitane have instituted TBPs that encourage customers to return unwanted clothes, cosmetic containers, and other packaging materials directly to the nearest store. These initiatives not only foster recycling participation but also reinforce the brand image as environmentally responsible. For instance, H&M's Garment Collecting program encourages customers to return unwanted clothes, which are then sorted for reuse, recycling into new garments, or industrial downcycling (H&M 2025). Similarly, Marks & Spencer's

Shwopping initiative invites customers to drop off used clothing regardless of brand at the Shwop box in M&S clothing stores. The returned items are sorted for resale, reuse, or recycling. Besides, L'Occitane offers an in-store TBP that allows customers to return cleaned and dried beauty empties, including those from other brands, to collection bins located in its retail stores. The returned packaging is then processed and recycled into new materials, reinforcing the brand's commitment to sustainability and responsible consumption. By making recycling easy and rewarding, L'Occitane appeals to environmentally conscious customers and promotes sustainable consumption patterns.

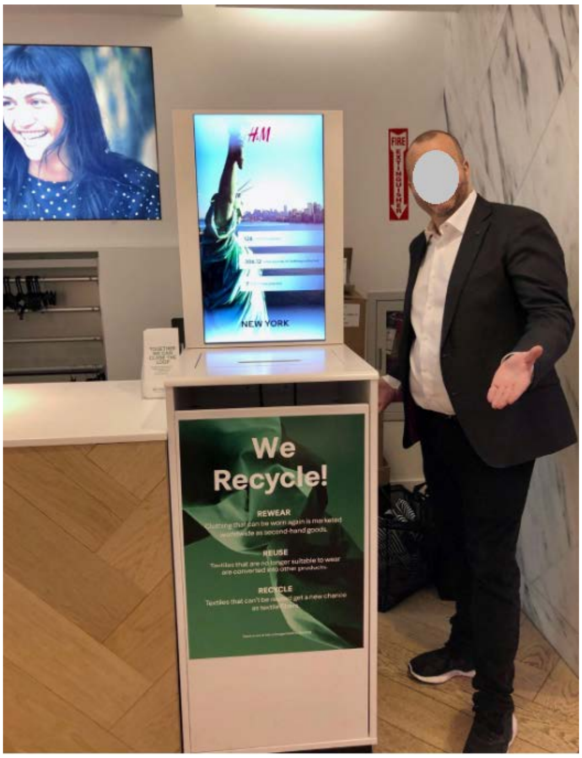


Figure 3.1: H&M's In-Store Take-Back Program

Despite the prevalence of such programs, the adoption and implementation of TBPs as a standard business practice remains optional for many firms worldwide. In countries like the UK, many firms still do not participate in recycling initiatives (EU Business School 2021). This inconsistency is partly due to the operational challenges and financial burdens associated with recycling, including collection, transportation, storage, and processing costs. However, as recycling volumes increase, firms can benefit from economies of scale, reducing the total recycling costs. Moreover, these costs can be offset by the potential revenue from the recycled units. For example, recycled units can be resold, transformed into raw materials for new products, or used in industrial applications. Hence, while recycling entails upfront costs, revenue generation and potential economies of scale contribute to the long-term financial sustainability of recycling efforts.

This study focuses on in-store TBPs, where returns must be made physically at firm-operated locations. This TBP encourages customers to visit stores and thus does not include a mailing component. We examine the strategic implementation and economic implications of TBPs in competitive markets, addressing the following research questions:

1. How do customers make recycling and purchasing decisions?
2. What are the optimal pricing strategies for competing firms?
3. What is the optimal TBP implementation strategy?

To answer these questions, we model in-store TBPs using a standard circular spatial competition framework, with two firms located on a unit circle. Customers who choose to recycle a used product must physically return the item to the store, gaining psychological satisfaction from recycling but also incurring hassle costs associated with the return process. Customers make both recycling and purchasing decisions by maximizing their individual utilities.

We compare three scenarios: no TBP implementation (NN case, where neither firm adopts TBPs), partial TBP implementation (YN case, which is equivalent to the NY case due to symmetry, where only one firm implements a universal TBP), and full TBP implementation (YY case, where both firms adopt TBPs). Our findings indicate that all three scenarios can emerge as equilibrium outcomes. Notably, even asymmetric implementation of TBPs can lead to win-win outcomes for competing firms. By differentiating themselves to appeal to either environmentally conscious or price-sensitive customers, firms can enhance overall profitability while simultaneously advancing sustainability goals.

We begin by deriving the equilibrium results for each scenario using backward induction. First, we analyze customer recycling and purchasing decisions by characterizing their utility-maximizing behavior and determining the resulting market segmentation. Next, we determine the optimal pricing decisions and firm profits. Finally, we identify the conditions under which both firms have incentives to choose no TBP implementation, partial TBP implementation, or full TBP implementation.

Our key findings are as follows. First, all three TBP implementation strategies can arise as equilibrium outcomes. Second, customer segmentation is driven by the interplay between the recycling inclination effect (measured by customer sensitivity to the return hassle cost) and the recycling economic effect (measured by the net unit cost or benefit of processing returned items). When hassle sensitivity is low, all customers choose to recycle. As sensitivity increases, however, some customers perceive recycling as burdensome. Customers begin to split between the recycling decisions depending on their location and the incentives offered. Third, even with asymmetric TBP implementation, both firms can achieve a win-win outcome by facilitating strategic market segmentation. Firms can differentiate themselves by targeting environmentally conscious or price-sensitive customers, thereby enhancing overall profitability and advancing sustainability goals.

The remainder of this study proceeds as follows. Section 3.2 provides an overview of the relevant literature on TBPs and environmentally conscious customers. Section 3.3 introduces the sequence of events, focusing on the partial TBP implementation scenario, with all relevant modeling, customer decision-making process, and firm pricing strategy analysis. In Section 3.4, we introduce the no TBP implementation scenario as a benchmark to identify the possible win-win region. We also analyze the full TBP implementation scenario, enabling a comparative analysis across all three scenarios to derive the equilibrium implications of TBPs and highlight the value of TBPs. In Section 3.5, we summarize this study and explore potential avenues for future research in this area.

3.2 Literature Review

Our study is related to the research on TBPs and environmentally conscious customers. Section 3.2.1 provides an overview of TBPs and the role of recycling regulations in shaping recycling initiatives. Section 3.2.2 explores research on environmentally conscious customer behavior and their decision-making process.

3.2.1 Literature on Take-Back Programs

Recent operations management studies with a similar emphasis on TBPs have explored several important topics, including: (i) The impact of recycling cost structures and competition on the efficiency of TBPs; (ii) The cost allocation mechanisms among stakeholders involved in TBPs; and (iii) The effects of TBPs on sustainable product design and innovation.

Prior research has explored various aspects of TBP operations and their impact on stakeholders, including manufacturers, recyclers, and customers. For instance, Toyasaki et al. (2011) compare the competitive and monopolistic take-back schemes, examining the role of recycling fees set by contracted recyclers or non-profit collection organizations. Their findings reveal a win-win situation for all stakeholders and emphasize the value of recycler consolidation due to the scale economies. In their seminal work, Atasu and VanWassenhove (2012) provide a comprehensive framework of TBPs, highlighting the critical interplay between policy choices, producer operational choices, and economic impacts on different stakeholders. They offer insights drawn from diverse industries and identify practical research needs to optimize TBP efficiency. Atasu et al. (2013) further explore the distinctions between manufacturer-operated TBP (where manufacturers are responsible for collection, recycling, and related costs) and state-operated TBP (where the state collects funds from manufacturers to oversee collection and recycling activities). They show the misaligned incentives between the welfare-maximizing tax model and the benefits of manufacturers, customers, and the environment. Tian et al. (2019) address product heterogeneity and (dis)economies of scale in multiproduct markets, analyzing stable recycling structures under various models, including all-inclusive, market-based, firm-based, and product-based structures, and their suitability for different market contexts. Our work contributes to this branch of literature by examining how recycling cost structures and market competition affect the efficiency of TBPs.

The allocation of recycling costs has become a focal point of research, particularly in light of Extended Producer Responsibility (EPR) legislation. Scholars have investigated strategies to enhance the efficiency and distribution of recycling costs within cooperative networks, aiming to improve both environmental sustainability and operational performance. Gui et al. (2016) refine recycling cost allocation mechanisms within a cooperative collection and recycling network. Their work aims to enhance the efficacy of collective EPR legislation by streamlining the distribution of recycling costs among stakeholders. This approach seeks to improve the overall performance of TBPs while balancing the economic interests of producers, recyclers, and customers. Building on this foundation, Gui et al. (2018) integrate non-cooperative design decision-making with a cooperative recycling network framework. They provide further insights into recycling cost allocation mechanisms, placing particular emphasis on product design incentives and group incentive compatibility. While much of the existing research has focused on cooperative networks, there remains a gap in the exploration of retailer-operated TBPs, where the primary interaction occurs directly between the retailer and the customer. In such cases, the retailer typically takes on responsibility for the collection and management of returned products, which in turn influences its strategic decisions and those of its competitors.

In recent years, researchers have increasingly emphasized the complex relationship between product design, including attributes such as durability and recyclability, and the implementation of TBPs. Huang et al. (2019) examine the balance between product durability and recyclability in the design process, demonstrating that strict EPR legislative requirements can lead to reductions in either durability or recyclability when these attributes are in conflict. Similarly, Alev et al. (2020) analyze the effects of stringent legislative targets on durable goods, particularly within the context of producers' secondary markets. They reveal that these targets tend to focus on durable products rather than nondurable ones, given the potential for product lifespan extension through secondary markets. While these studies shed light on the influence of TBPs on sustainable product design and innovation, our research adopts a more targeted approach by examining the firms' optimal TBP implementation strategies and pricing decisions to maximize the value of a TBP in a competitive business environment.

3.2.2 Literature on Green Customers

Customers' environmental consciousness significantly influences their decision-making process, making it a primary focus for customer-oriented businesses (Yang and Thøgersen 2022). A growing body of operations management research examines how the presence of green (or environmentally conscious) customers affects product pricing and marketing strategies. For instance, Chen (2001) reveals that green customers are willing to pay a premium for products with positive environmental attributes. Agrawal et al. (2012) compare customers' willingness to pay for new products with their willingness to pay for old (off-lease or used) products, revealing key differences in perceived value. Similarly, Sheu and Li (2014) explore how customers' green attitudes towards green-service value affect their willingness to pay for environmentally conscious transportation services offered by airlines.

Other studies emphasize customer heterogeneity in environmental preferences. Atasu et al. (2008) identify the marketing potential of green customers for remanufacturers, acknowledging the presence of a secondary market segment of customers who do not discount remanufactured products, while Shi et al. (2020) find that customers generally have a lower willingness to pay for remanufactured products compared to new ones. Guo et al. (2016) investigate a heterogeneous customer population, including individuals who are willing to pay for social responsibility initiatives and those who are not. The diversity of customers' sustainability concerns has also been empirically validated, as supported by Bhattacharya and Sen (2004) and Galbreth and Ghosh (2013). Yang and Chen (2018) examine how different retail contracts impact manufacturers' carbon reduction efforts and profitability under carbon taxation and varying degrees of customer environmental awareness. Similarly, Guo et al. (2023) categorize customers into ordinary and green segments when studying customer behavior in express packaging recycling.

Building on these works, we integrate both the psychological benefits and the perceived hassle cost of returning used products into customers' utility functions. While most prior studies focus on one-dimensional customer heterogeneity and assume symmetric competition, we introduce an asymmetric setting in which only one firm adopts a TBP. This allows us to capture the competitive dynamics that emerge when firms strategically respond to heterogeneous customer inclinations for sustainability and the operational costs associated with recycling initiatives.

3.3 Model Setup

We begin our analysis of in-store TBPs using a standard circular spatial competition framework (Salop 1979), featuring two firms located on a unit circle. Specifically, Firm A is located at $x_A = 0$ and Firm B is located at $x_B = \frac{1}{2}$. This framework effectively captures horizontal differentiation in retail markets. We abstract from vertical differentiation and focus on a setting where both firms sell non-durable products that are symmetric in terms of product valuation.[16] For simplicity, we set the marginal production cost for both firms to zero.

3.3.1 Sequence of Events

Figure 3.2 outlines the sequence of events. In Stage 1, the two firms choose among three TBP implementation options: No TBP Implementation, Partial TBP Implementation, and Full TBP Implementation. In Stage 2, we determine the equilibrium prices, demands, and profits corresponding to each TBP scenario. Drawing upon prior literature (e.g., Huang et al. (2001), Agrawal et al. (2012), and Alev et al. (2020)), we assume both firms commit to stationary strategies and thus focus on the steady-state version of the problem. In each period, both firms simultaneously set their product prices $p_j > 0$, where $j \in \{A, B\}$, aiming to maximize their average per-period profits. Given the possible recycled option and the prices set by both firms, customers then decide whether to return their used product and from which firm to purchase a new one.

[16]The assumption of symmetric product valuation is widely adopted in the literature, e.g., Galbreth and Ghosh (2013), Chen and Guo (2014), Li (2021), Sharma and Mehra (2021), Kim et al. (2025), and facilitates analytical tractability.

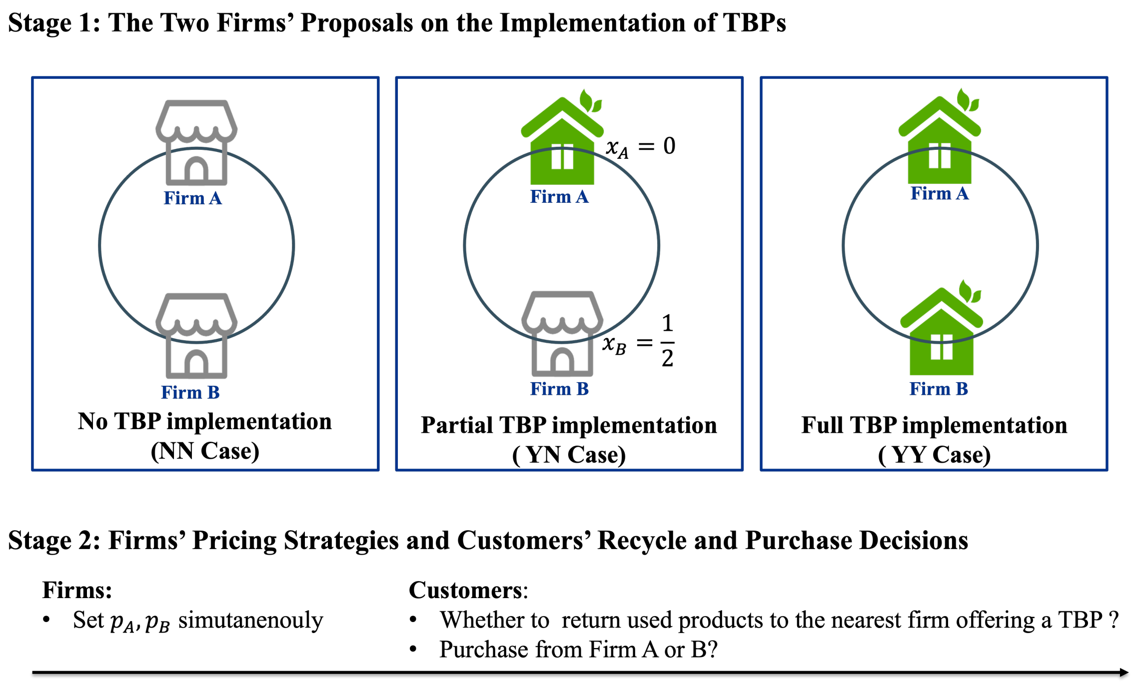


Figure 3.2: Sequence of Events

3.3.2 Analysis of Partial TBP Implementation

We first investigate the YN Case in which Firm A implements a universal TBP, while Firm B does not. This asymmetry mirrors common industry practice, where only a subset of firms adopt recycling initiatives. By examining this setting, we aim to capture not only the direct impact of the TBP on the implementing firm but also its competitive spillover effects on the rival. Firm A's TBP is universal in the sense that it accepts any used products made of the same recyclable material, including those originally sold by Firm B. As a result, customers' recycling decisions are independent of their previous purchase choices. This assumption is consistent with observed practice and ensures that recycling behavior is primarily driven by incentive structures rather than brand loyalty.

3.3.2.1 Customers' Decision Problem

Customers are uniformly distributed along a circle with a circumference of 1. Each customer is indexed by her location $x \in [0, 1]$. Owing to the symmetry of the spatial model, we focus without loss of generality on customers located in the segment $x \in [0, \frac{1}{2}]$ for the subsequent analysis. Customers derive the value v from consuming either firm's product, with $v_A = v_B = v$, where $v \gg 0$ ensures positive net utility and market participation.[17]

Customers who choose to recycle a used product at Firm A must physically return the item to the store. On one hand, these customers receive intrinsic psychological satisfaction from engaging in environmentally responsible behavior, which we model as a fixed utility gain of $g = 1$. On the other hand, they incur a return hassle cost that is proportional to the distance traveled during the recycling process. For example, carrying used or bulky items, such as bags of worn clothing, electronics, or packaging, can be physically inconvenient, particularly for customers relying on walking, biking, or public transportation. The total return hassle cost is denoted by $\beta|x - x_A|$, where $\beta > 1$ is a cost sensitivity parameter reflecting the customer's aversion to the distance of the recycling point, and $|x - x_A|$ represents the (shortest) distance from the customer's location to the collection point in the store.[18]

Importantly, the return hassle cost reflects not only spatial distance but also the broader marginal effort per unit distance associated with recycling. For instance, recycling may require a dedicated trip to the store, which imposes an opportunity cost, especially for customers who are not planning to shop. The parameter β can also serve as a behavioral proxy for a customer's sensitivity to recycling-related ef-

[17] The assumption $v \gg 0$ is standard in spatial competition models to guarantee an interior solution for customer choices. A similar assumption can be found in Gao and Su (2017), Mehra et al. (2018), and Gao et al. (2022).

[18] If multiple collection points exist (e.g., n evenly spaced drop-off locations), the return hassle cost is given by $min\{\beta|x|, \beta|x - \frac{1}{n}|\}$.

forts. Customers with lower values of β are typically more environmentally conscious and perceive the marginal effort per unit distance as less burdensome, and are consequently more inclined to participate in the TBP. In contrast, customers with higher values of β are more effort-averse and may perceive even small logistical barriers as significant deterrents.

Customers make their decisions regarding recycling and purchasing by maximizing their individual utilities. Following Singh et al. (2008), Subramaniam and Gal-Or (2009), Shin and Sudhir (2010), Adner et al. (2020), and Tang et al. (2023), we make the full market coverage assumption. That is, every customer purchases at most one unit of the product from either Firm A or Firm B. Given this, customers engage in one of the four possible behaviors: (1) Recycling and purchasing from Firm A; (2) Recycling and purchasing from Firm B; (3) Not recycling and purchasing from Firm A; and (4) Not recycling and purchasing from Firm B. The corresponding customer utility function, denoted by U_j^i, depends on whether the customer recycles ($i \in \{R, N\}$, where R represents recycling and N represents not recycling) and from which firm they make a purchase ($j \in \{A, B\}$, where A represents Firm A and B represents Firm B), is given in Table 3.1.

Table 3.1: Customer Utilities Based on Recycling and Purchasing Behavior in the YN Case

	Firm A	Firm B
Return to A	$U_A^R(x) = 1-\beta\lvert x-x_A\rvert+v-p_A$	$U_B^R(x) = 1-\beta\lvert x-x_A\rvert+v-p_B-\lvert x-x_B\rvert$
Not Return to A	$U_A^N(x) = v - p_A - \lvert x - x_A\rvert$	$U_B^N(x) = v - p_B - \lvert x - x_B\rvert$

The utility for a customer who returns a used product to Firm A and purchases from the same firm, U_A^R, comprises five components: the psychological satisfaction from recycling ($g = 1$), the hassle cost incurred when returning the used product to Firm A ($\beta|x - x_A|$), the value derived from Firm A's product (v), and the price paid for the product (p_A). The utility for a customer who returns the used products to Firm A but purchases from Firm B, U_B^R, includes the psychological satisfaction from recycling ($g = 1$) and the hassle cost incurred when returning the used product to Firm A ($\beta|x - x_A|$). However, the customer derives product value from Firm B (v), pays Firm B's price (p_B), and incurs an additional transportation cost $|x - x_B|$ to travel from her location to Firm B's store. This captures the disutility from making two separate trips.[19] For customers who opt not to recycle, the utility of purchasing from Firm j, U_j^N, is simply the value derived from Firm j's product, minus the price paid (p_j), and the transportation cost ($|x - x_j|$) to reach the store.

Given these utility specifications, customers choose the behavior–recycling or not, and from which firm to purchase–that yields the highest utility. Let d_{ij} denote the demand associated with behavior $(i, j) \in \{R, N\} \times \{A, B\}$, defined as the measure of customers $x \in [0, \frac{1}{2}]$ who satisfy $max\{U_A^R(x), U_B^R(x), U_A^N(x), U_B^N(x)\} = U_j^i(x)$. To characterize the boundaries between customer segments, we identify the indifference points at which the utility from two behaviors is equal. Since each pair of utilities

[19] In-store recycling entices customers to make an immediate purchase from Firm A after recycling, leveraging the convenience of a single trip. In contrast, customers who recycle at Firm A but subsequently purchase from Firm B incur both the hassle cost of recycling and the additional cost of traveling to a different location for the purchase. As interpurchase timing is assumed irrelevant, we omit the cost of returning home after the recycling trip.

can be equated, there are a total of six distinct indifference points to consider:

$$U_A^R(x) = U_B^R(x) \iff x_{RA,RB} = \frac{1}{2} - p_A + p_B, \tag{3.1}$$

$$U_A^N(x) = U_B^N(x) \iff x_{NA,NB} = \frac{1}{4} - \frac{p_A - p_B}{2}, \tag{3.2}$$

$$U_A^R(x) = U_A^N(x) \iff x_{RA,NA} = \frac{1}{\beta - 1}, \tag{3.3}$$

$$U_A^R(x) = U_B^N(x) \iff x_{RA,NB} = \frac{3 - 2(p_A - p_B)}{2(\beta + 1)}, \tag{3.4}$$

$$U_B^R(x) = U_A^N(x) \iff x_{RB,NA} = \frac{2(p_A - p_B) + 1}{2(\beta - 2)}, \tag{3.5}$$

$$U_B^R(x) = U_B^N(x) \iff x_{RB,NB} = \frac{1}{\beta}. \tag{3.6}$$

These indifference points segment customer behavior across the circular city and form the basis for determining equilibrium market shares and firm profits. We analyze customers' recycling and purchasing decisions according to their position on the circle. Let $x_{RA,RB}$ denote the location of the recycling indifferent customer. That is, recycling customers located to the left of $x_{RA,RB}$ strictly prefer to purchase from Firm A, while those located to the right prefer Firm B. Similarly, let $x_{NA,NB}$ denote the location of the non-recycling indifferent customer. That is, non-recycling customers to the left of $x_{NA,NB}$ prefer to purchase from Firm A, while those to the right prefer Firm B. Based on these indifference points, we identify eight distinct cases, as summarized in Table 3.2.[20]

[20] We verify that these cases are mutually exclusive and collectively exhaustive with respect to both firms' prices and customers' sensitivity to hassle costs, as shown in Table B1 in the appendix.

Table 3.2: Market Composition under Eight Cases in the YN Case

Market Composition	Constraints Based on Indifference Point Orderings	Demand Functions
Case 1: Only RA	(1.1) $x_{\text{RA,RB}} \geq \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{RA,NB}} \geq \frac{1}{2}$ (1.2) $x_{\text{RA,RB}} \geq \frac{1}{2}, x_{\text{NA,NB}} \geq \frac{1}{2}, x_{\text{RA,NA}} \geq \frac{1}{2}$	$d_{RA} = 1$
Case 2: Only RB	(2.1) $x_{\text{RA,RB}} \leq 0, x_{\text{NA,NB}} \leq 0, x_{\text{RB,NB}} \geq \frac{1}{2}$	$d_{RB} = 1$
Case 3: RA+NA	(3.1) $x_{\text{RA,RB}} \geq \frac{1}{2}, x_{\text{NA,NB}} \geq \frac{1}{2}, x_{\text{RA,NA}} < \frac{1}{2}$	$d_{RA} = 2x_{RA,NA}, d_{NA} = 1 - 2x_{RA,NA}$
Case 4: RB+NB	(4.1) $x_{\text{RA,RB}} \leq 0, x_{\text{NA,NB}} \leq 0, x_{\text{RB,NB}} < \frac{1}{2}$	$d_{RB} = 2x_{RB,NB}, d_{NB} = 1 - 2x_{RB,NB}$
Case 5-1: RA+NB	(5.1.1) $0 < x_{\text{RA,RB}} < \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{RA,NB}} \leq x_{\text{NA,NB}}$ (5.1.2) $x_{\text{RA,RB}} \geq \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{RA,NB}} \leq x_{\text{NA,NB}}$	$d_{RA} = 2x_{NA,NB}, d_{NB} = 1 - 2x_{NA,NB}$
Case 5-2: RA+NB	(5.2.1) $0 < x_{\text{RA,RB}} < \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{NA,NB}} < x_{\text{RA,NB}} < x_{\text{RA,RB}}$ (5.2.2) $x_{\text{RA,RB}} \geq \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{NA,NB}} < x_{\text{RA,NB}} < \frac{1}{2}$	$d_{RA} = 2x_{RA,NB}, d_{NB} = 1 - 2x_{RA,NB}$
Case 5-3: RA+NB	(5.3) $0 < x_{\text{RA,RB}} < \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{RA,NB}} \geq x_{\text{RA,RB}}, x_{\text{RB,NB}} \leq x_{\text{RA,RB}}$	$d_{RA} = 2x_{RA,RB}, d_{NB} = 1 - 2x_{RA,RB}$
Case 6: RA+RB	(6.1) $0 < x_{\text{RA,RB}} < \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{RA,NB}} \geq x_{\text{RA,RB}}, x_{\text{RB,NB}} \geq \frac{1}{2}$	$d_{RA} = 2x_{RA,RB}, d_{RB} = 1 - 2x_{RA,RB}$
Case 7: RA+NA+NB	(7.1) $0 < x_{\text{RA,RB}} < \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} < x_{\text{NA,NB}}$ (7.2) $x_{\text{RA,RB}} \geq \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} < x_{\text{NA,NB}}$	$d_{RA} = 2x_{RA,NA}, d_{NA} = 2(x_{NA,NB} - x_{RA,NA}), d_{NB} = 1 - 2x_{NA,NB}$
Case 8: RA+RB+NB	(8.1) $0 < x_{\text{RA,RB}} < \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{RA,NB}} \geq x_{\text{RA,RB}}, x_{\text{RA,RB}} < x_{\text{RB,NB}} < \frac{1}{2}$	$d_{RA} = 2x_{RA,RB}, d_{RB} = 2(x_{RB,NB} - x_{RA,RB}), d_{NB} = 1 - 2x_{RB,NB}$

3.3.2.2 Recycling Structure

We first introduce the recycling structure of Firm A and then formulate the profit functions for both firms. Firm A encourages customers to return used products, regardless of brand, to its store. For each unit of returned product or packaging, Firm A incurs a unit processing cost denoted by $\hat{c} > 0$. However, as the total volume of collected items increases, Firm A can benefit from economies of scale in recycling operations. To capture this feature, we adopt a quadratic cost adjustment function, following Gui et al. (2016) and Tian et al. (2019). Specifically, the total cost adjustment is modeled as $(d_{RA} + d_{RB})^2$, where d_{RA} and d_{RB} denote the quantities of recycled customers who purchase from Firms A and B, respectively. We normalize the coefficient of the quadratic term to unity for simplicity. Accordingly, the total recycling cost borne by Firm A is $\hat{c}(d_{RA} + d_{RB}) - (d_{RA} + d_{RB})^2$.[21] Meanwhile, Firm A retrieves a unit recovery value $s \in (0, p)$ when recycled items are reused in manufacturing. Therefore, the net unit economic impact of recycling, denoted by $c = \hat{c} - s \in \mathbb{R}$, captures the trade-off between recycling management costs and potential revenue streams. If $\hat{c} > s$, then $c > 0$, indicating that Firm A incurs a net cost per unit of recycling. Conversely, if $\hat{c} < s$, then $c < 0$, suggesting a net gain from recycling activities.

The profit functions of the two firms are as follows.

$$\begin{aligned}
\pi_A &= \underbrace{p_A(d_{RA} + d_{NA})}_{\text{product revenues}} - [\underbrace{\hat{c}(d_{RA} + d_{RB})}_{\text{processing cost}} - \underbrace{(d_{RA} + d_{RB})^2}_{\text{economies of scale}}] + \underbrace{s(d_{RA} + d_{RB})}_{\text{recovery value}} \\
&= p_A(d_{RA} + d_{NA}) - c(d_{RA} + d_{RB}) + (d_{RA} + d_{RB})^2, \qquad (3.7) \\
\pi_B &= p_B(d_{RB} + d_{NB}). \qquad (3.8)
\end{aligned}$$

We substitute the relevant demand functions into the profit equations (3.7)–(3.8) and derive the corresponding profit functions. Under each case, the two firms choose their optimal prices simultaneously that satisfy the inequality constraints to maximize their respective profits.

3.3.2.3 Illustrative Example

Case 5 provides an illustrative example. If the indifference points satisfy the conditions $0 < x_{\text{RA,RB}} < \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{NA,NB}} < x_{\text{RA,NB}} < x_{\text{RA,RB}}$, the market is segmented into two customer types: (i) RA customers who recycle and

[21] Note that $\hat{c} > 1$ to ensure that the overall recycling costs $c(d_{RA} + d_{RB}) - (d_{RA} + d_{RB})^2$ are nonpositive when $0 \leq d_{RA} + d_{RB} \leq 1$.

purchase from Firm A, and (ii) NB customers who do not recycle and purchase from Firm B. The demand functions are

$$d_{RA} = 2x_{RA,NB} = \frac{3 - 2(p_A - p_B)}{\beta + 1},$$

$$d_{NB} = 1 - 2x_{RA,NB} = \frac{\beta - 2 + 2(p_A - p_B)}{\beta + 1}.$$

Substituting the demands into the profit functions (3.7)–(3.8) yields the simultaneous optimal problems for two firms.

$$\max \ \pi_{A5}(p_A, p_B) = \frac{(2p_A - 2p_B - 3)\left(-(\beta - 1)p_A - 2p_B + \beta c + c - 3\right)}{(\beta + 1)^2},$$

$$\max \ \pi_{B5}(p_B, p_A) = \frac{p_B\left(2p_A + \beta - 2p_B - 2\right)}{\beta + 1}.$$

They are subject to the shared constraints:

$$\begin{cases} 0 < p_A - p_B < \frac{1}{2} - \frac{1}{\beta}, \\ p_A - p_B > \frac{\beta - 5}{2(\beta - 1)}, \\ p_A > 0, p_B > 0, \beta > 1, \end{cases}$$

The following proposition summarizes the optimal prices that satisfy all KKT conditions and the corresponding customer demands and firm profits.

Proposition 3.1. *If the market consists of RA and NB customer segments, the equilibrium results are as follows.*

(a) When $\beta > 5$ and $c < \frac{\beta^2 - 3\beta - 2}{2\beta - 2}$, the optimal prices, demands, and firm profits are

$$p_A^* = \frac{-\beta^2 + \beta + 8}{2 - 2\beta}, p_B^* = \frac{-\beta^2 + 2\beta + 3}{2 - 2\beta},$$

$$d_{RA}^* = \frac{2}{\beta - 1}, d_{NB}^* = \frac{\beta - 3}{\beta - 1},$$

$$\pi_{A5}^* = \frac{\beta^2 - \beta - 2(\beta - 1)c - 4}{(\beta - 1)^2}, \pi_{B5}^* = \frac{(\beta - 3)^2(\beta + 1)}{2(\beta - 1)^2}.$$

(b) When $2 < \beta \leq 5$ *and* $c < \frac{\beta^2-4\beta+7}{2\beta+2}$*, the optimal prices, demands, and firm profits are*

$$p_A^* = \frac{\beta-2}{2}, p_B^* = \frac{\beta-2}{2},$$

$$d_{RA}^* = \frac{3}{\beta+1}, d_{NB}^* = \frac{\beta-2}{\beta+1},$$

$$\pi_{A5}^* = \frac{3\left(\beta^2-\beta-2(\beta+1)c+4\right)}{2(\beta+1)^2}, \pi_{B5}^* = \frac{(\beta-2)^2}{2(\beta+1)}.$$

(c) When (i) $2 < \beta \leq 5$ *and* $\frac{\beta^2-4\beta+7}{2\beta+2} < c < \frac{\beta^2-2\beta+2}{2\beta}$*; or (ii)* $\beta > 5$ *and* $\frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{\beta^2-2\beta+2}{2\beta}$*, the optimal prices, demands, and firm profits are*

$$p_A^* = \frac{\beta^2+\beta+4(\beta+1)c-12}{6\beta-2}, p_B^* = \frac{(\beta+1)(2\beta+2c-5)}{6\beta-2},$$

$$d_{RA}^* = \frac{\beta-2c+4}{3\beta-1}, d_{NB}^* = \frac{2\beta+2c-5}{3\beta-1},$$

$$\pi_{A5}^* = \frac{(\beta-1)(\beta-2c+4)^2}{2(1-3\beta)^2}, \pi_{B5}^* = \frac{(\beta+1)(2\beta+2c-5)^2}{2(1-3\beta)^2}.$$

(d) When $\beta > 2$ *and* $c > \frac{\beta^2-2\beta+2}{2\beta}$*, the optimal prices, demands, and firm profits are*

$$p_A^* = -\frac{3}{\beta}+c+1, p_B^* = -\frac{2}{\beta}+c+\frac{1}{2},$$

$$d_{RA}^* = \frac{2}{\beta}, d_{NB}^* = \frac{\beta-2}{\beta},$$

$$\pi_{A5}^* = \frac{2(\beta-1)}{\beta^2}, \pi_{B5}^* = \frac{(\beta-2)(\beta+2\beta c-4)}{2\beta^2}.$$

Several key observations emerge from these equilibrium results. First, the market exhibits clear customer segmentation. The introduction of TBP amplifies the asymmetry between the two firms' profit foci. Specifically, all customers who choose to recycle purchase from Firm A, while all non-recycling customers purchase from Firm B. Second, the two firms adopt distinct pricing strategies that depend on both the net recycling cost c and the hassle sensitivity parameter β. As illustrated in Figure 3.4(a), the parameter c captures the direct economic impact of recycling cost on

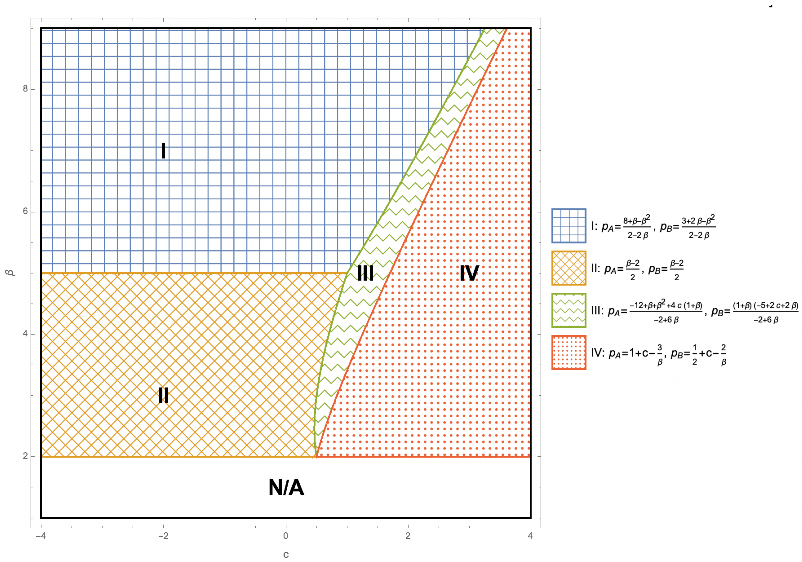
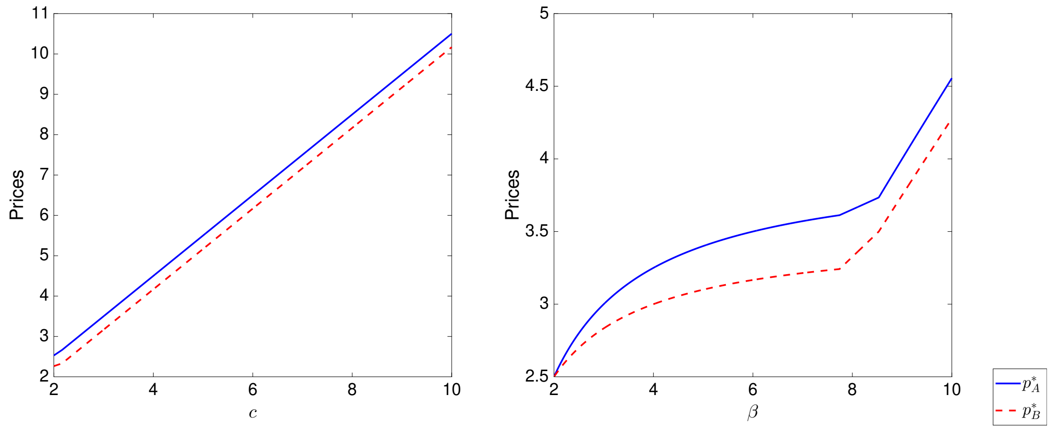


Figure 3.3: Pricing Strategies When the Market Consists of RA and NB Segments

the firms' pricing decisions. As c increases, Firm A's marginal cost rises above that of Firm B. To maintain profitability, Firm A must raise its price p_A^*, passing a portion of the increased cost onto its customers. This creates a strategic tension for Firm A, which must balance the need to cover higher costs with the necessity of keeping its TBP attractive to customers. Firm B, by contrast, does not offer recycling but competes for non-recycled customers. In contrast, Firm B, which does not offer recycling, competes solely for non-recycling customers. This dynamic can lead to a competitive spillover, whereby Firm B benefits when Firm A's TBP becomes less effective, such as when recycling costs are high or recycling benefits are insufficient to offset the associated hassle. Consequently, Firm B must strategically set its price p_B to attract customers who are not fully "locked in" by Firm A's TBP. Since the incremental cost of recycling is largely passed on to both segments in a comparable way, the price gap between p_A and p_B remains relatively stable as c increases. Figure 3.4(b) highlights the role of the hassle sensitivity parameter β, which captures how customers' perceived inconvenience of recycling affects their behavior. As β increases, fewer customers find it worthwhile to engage in recycling, as the inconvenience outweighs the



(a) $\beta = 6$

(b) $c = 3$

Figure 3.4: Pricing Strategies of Both Firms Across the Parameters c and β

psychological benefit. This leads to a contraction of the RA segment and an expansion of the NB segment. However, the remaining RA customers are those who place a high value on recycling despite the inconvenience, making them less price-sensitive and more willing to pay a premium for TBP. Firm A can exploit this by charging a premium. Meanwhile, Firm B, facing reduced competition for the NB segment, can also raise its price. Notably, the price gap between p_A and p_B first widens as Firm A can charge a premium to a shrinking but motivated RA segment, but then narrows as the segment becomes too small and costly to serve, and Firm B's price also increases due to reduced competition.

3.3.2.4 Firms' Profit Maximization

Following the same logic as in the illustrative example, we determine the optimal prices and corresponding profits across eight cases.[22] By comparing the optimal profits of two firms in each case, we identify the optimal customer segmentation in the following proposition.

Proposition 3.2. *Among the eight possible customer segmentations, i.e., RA, RB,*

[22] We summarize the equilibrium results of eight cases in Table B3 in the appendix.

RA+RB, RA+NA, RB+NB, RA+NB, RA+RB+NB, and RA+NA+NB, the optimal segmentation that yields the highest profits for both firms necessarily involves the coexistence of both firms in the market. Specifically, only the segmentations RA+RB, RA+NB, RA+RB+NB, and RA+NA+NB can be optimal.

Proposition 3.2 highlights that market dominance by a single firm (e.g., RA, RB, RA+NA, or RB+NB segmentation) cannot maximize joint profits. Instead, both firms can sustain profitable operations by focusing on their respective customer segments. This coexistence arises because each firm is able to specialize and cater to the distinct preferences of different customer groups, making it unprofitable for either firm to serve all segments alone. Thus, the market structure remains robust, and competitive dynamics ensure that both firms maintain a presence across the optimal segmentations identified.

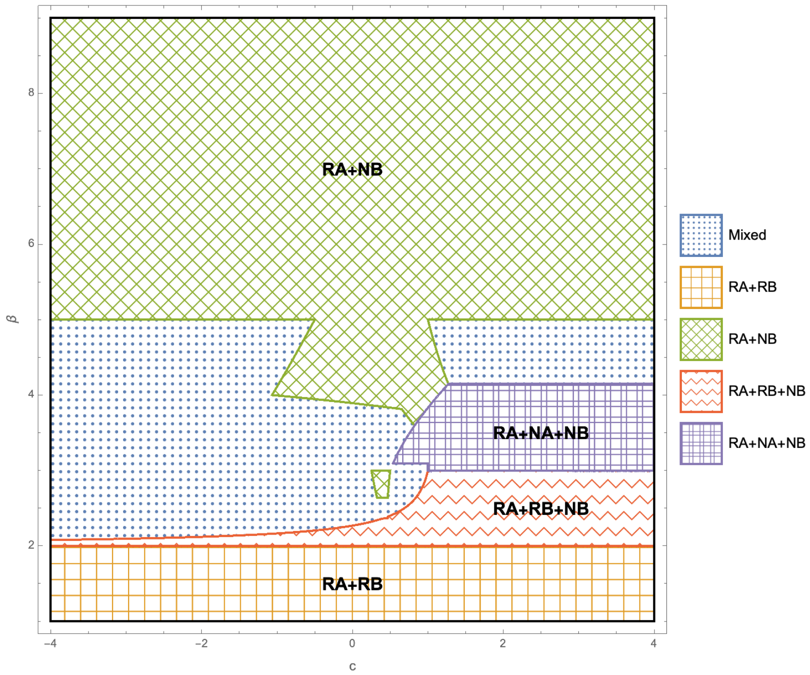


Figure 3.5: The Optimal Customer Segmentations in the YN Case

Figure 3.5 demonstrates how the interaction between the net unit impact of recycling c and the recycling hassle sensitivity β shapes the optimal customer segmentations. When β is low, recycling is perceived as convenient and all customers are incentivized to recycle, leading to a straightforward RA+RB segmentation. As we move to intermediate values of β, the market supports more nuanced segmentation. Both firms can attract a broader mix of customer types, resulting in "Mixed" segmentations, where firms might probabilistically choose between all feasible segmentations. Three distinct customer segments can coexist, such as RA+RB+NB or RA+NA+NB. These segmentations arise only within narrow parameter ranges, where neither the recycling cost nor the hassle is extreme. As β increases further, recycling becomes increasingly burdensome for customers. In this regime, the RA+NB segmentation prevails: only the most motivated customers (RA) choose to recycle, while the majority (NB) opt not to recycle. Firm A captures all recycling customers, potentially charging a premium for their commitment, while Firm B attracts all non-recycling customers by offering a lower price, specifically those who are not fully "locked in" by Firm A's TBP. This leads to a clear and distinct market segmentation, with each firm specializing in serving a particular customer segment.

3.4 Comparative Analysis and the Value of TBPs

In this section, we begin by analyzing the NN case as a benchmark to assess the strategic implications of TBP implementation. By comparing the equilibrium results in the YN case with this benchmark, we identify the conditions under which both firms achieve mutual benefit (win-win) or where one firm gains at the expense of the other (win-lose). Then, we incorporate the YY case and compare the equilibrium profits across the NN, YN/NY, and YY cases to determine the optimal TBP implementation strategies for both firms.

3.4.1 Analysis of No TBP Implementation

In the absence of either firm adopting TBPs, the profit functions of the two profit-maximizing firms follow a straightforward structure.

$$\pi_A = p_A d_A,$$

$$\pi_B = p_B d_B.$$

By solving the first-order conditions for both firms, we obtain the optimal prices, demands, and profits, which are summarized in the following lemma.

Lemma 3.1. *When neither firm adopts a TBP, the optimal prices for both firms are $p_A^* = p_B^* = \frac{1}{2}$; the optimal demands are $d_A^* = d_B^* = \frac{1}{2}$; and the optimal profits are $\pi_A^* = \pi_B^* = \frac{1}{4}$.*

In the NN case, the equilibrium results reflect a state of perfect competition. This equilibrium is characterized by homogeneous prices, with both Firm A and Firm B setting their prices at $p_A^* = p_B^* = \frac{1}{2}$. Consequently, customers are indifferent between purchasing from either firm, leading to equal demands for both firms. Thus, the equilibrium demands are $d_A^* = d_B^* = \frac{1}{2}$, signifying a symmetric distribution of market share. As a result of this equilibrium configuration, both firms achieve identical profits, with $\pi_A^* = \pi_B^* = \frac{1}{4}$.

3.4.2 Comparison with the Benchmark

In the NN case, each firm earns a profit of $\frac{1}{4}$. By comparing the optimal profits of two firms in the YN case with this benchmark, we establish the following proposition.

Proposition 3.3. *Compared with the benchmark, the asymmetric adoption of TBP leads to two possible outcomes.*[23]

[23] The thresholds β_1 through β_{10} are defined in the appendix.

(a) ***Win-Win Region:*** *Both firms benefit from the asymmetric adoption of TBP when the following conditions on beta and c are satisfied: (1)* $\beta_1 < \beta \le \frac{11}{3}, \frac{-9\beta^3+43\beta^2+5\beta+25}{4\beta^3+4\beta^2-4\beta-4} \le c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$*; (2)* $\frac{11}{3} < \beta \le \beta_2, \frac{-9\beta^3+43\beta^2+5\beta+25}{4\beta^3+4\beta^2-4\beta-4} \le c < \frac{-\beta^2+2\beta+31}{8\beta-8}$*; (3)* $\beta_2 < \beta \le \beta_3, \frac{3\beta^4-9\beta^3+5\beta^2-59\beta-4}{2\beta^3-10\beta^2-2\beta+10} \le c < \frac{-\beta^2+2\beta+31}{8\beta-8}$*; (4)* $\beta_3 < \beta \le 4, \frac{3\beta^4-9\beta^3+5\beta^2-59\beta-4}{2\beta^3-10\beta^2-2\beta+10} \le c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$*; (5)* $4 < \beta \le \beta_4, \frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$*; (6)* $\beta_4 < \beta \le 5, \frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c < \frac{4}{\beta-1}$*; (7)* $5 < \beta < 2\sqrt{2}+4$*; (8)* $2\sqrt{2}+4 \le \beta < 9, c < \frac{\beta+4}{2} - \frac{\sqrt{\frac{9\beta^2-6\beta+1}{\beta-1}}}{2\sqrt{2}}$*; (9)* $9 \le \beta \le \beta_5, c < \frac{3\beta^2-2\beta-17}{8\beta-8}$*; (10)* $\beta_5 < \beta < \beta_6$*,* $c \le 1$ *or* $\frac{\beta^3-10\beta^2-3\beta+4}{2\beta^2-4\beta+2} \le c < \frac{3\beta^2-2\beta-17}{8\beta-8}$*; (11)* $\beta \ge \beta_6, c \le 1$.

(b) ***Win-Lose Region:*** *Firm A benefits from its TBP, but Firm B is worse off, when: (1)* $1 < \beta \le 2, c < \frac{35}{36}$*; (2)* $2 < \beta \le \beta_7, \frac{7\beta^2-36}{9\beta^2-18\beta} \le c < \frac{144-\beta^2}{72\beta}$*; (3)* $\beta_7 < \beta \le \beta_8, \frac{23\beta^4-35\beta^3+32\beta^2-144\beta-72}{18\beta^3-18\beta^2-36\beta} \le c < \frac{144-\beta^2}{72\beta}$*; (4)* $\beta_8 < \beta \le \beta_9, \frac{\beta^2-4\beta+7}{2\beta+2} \le c < \frac{144-\beta^2}{72\beta}$*; (5)* $\beta_9 < \beta < 3\sqrt{97}-27, \frac{7\beta^2-36}{9\beta^2-18\beta} \le c < \frac{144-\beta^2}{72\beta}$*; (6)* $\frac{\sqrt{109}+37}{18} \le \beta \le 3, \frac{5-\beta}{2\beta+2} \le c < \frac{\beta^2-4\beta+7}{2\beta+2}$*; (7)* $3 < \beta < \beta_{10}, 1 < c < \frac{-\beta^2+2\beta+31}{8\beta-8}$*; (8)* $\beta_{10} \le \beta < \frac{11}{3}, \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c < \frac{-\beta^2+2\beta+31}{8\beta-8}$.

Figure 3.6 characterizes how the asymmetric adoption of a TBP influences the profits of both firms, compared to the benchmark where neither firm implements TBPs. The outcomes are driven by two key factors: (1) the recycling economic effect, measured by the net unit cost or benefit of recycling c, and (2) the recycling inclination effect, captured by the hassle sensitivity parameter β. The interaction of these two forces determines whether TBP implementation results in mutual gains or unilateral benefits.

First, the win–win region primarily corresponds to the RA+NB segmentation, where market segmentation is highly efficient: Firm A serves recycling customers, while Firm B targets price-sensitive, non-recycling customers. The introduction of a TBP by Firm A enables both firms to better segment the market, allowing Firm A to capture the recycling premium and Firm B to focus on non-recyclers. As a result, both firms achieve higher profits compared to the benchmark. Second, the win-lose

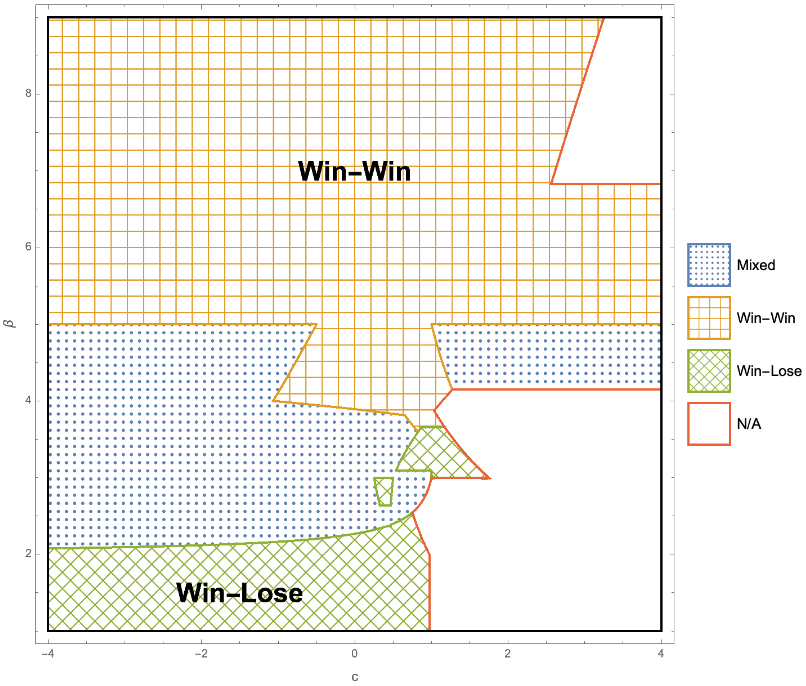


Figure 3.6: Comparison with the Benchmark

region mainly aligns with the RA+RB segmentation, which occurs when both β and c are low, making recycling attractive and convenient for all customers. In this case, Firm A benefits by capturing the segment of motivated recyclers, while Firm B is left with the less profitable non-recycling segment, resulting in a profit gain for Firm A but a loss for Firm B relative to the benchmark. Third, the mixed region corresponds to the mixed segmentation, where the market structure is more complex and may involve unstable customer segmentation. In these intermediate regions, small changes in β and c can shift the market structure, leading to unpredictable or sensitive profit outcomes.

3.4.3 Analysis of Full TBP Implementation

In the competitive retail landscape, both firms can adopt in-store TBPs. In this case, the maximum distance from any customer to her nearest collection point is $\frac{1}{4}$. The customer utility function U_j^i, where $i \in \{R, N\}$ represents recycling or not recycling, $j \in \{A, B\}$ indicates the firm from which the purchase is made, is given as follows.

$$U_j^R = 1 - \beta \min\{|x - x_j|, |x - \frac{1}{4}|\} + v - p_j,$$
$$U_j^N = v - p_j - |x - x_j|.$$

The profit function for each profit-maximizing firm j is:

$$\pi_j = p_j(d_{Rj} + d_{Nj}) - c_j d_{Rj} + d_{Rj}^2.$$

To simplify the analysis, we assume $c_A = c_B = c$. The demand functions d_{ij} are derived based on customers selecting the option that provides them with the highest utility from the set $\{U_A^R, U_B^R, U_A^N, U_B^N\}$. By solving the first-order conditions for each firm's profit maximization problem, we obtain the equilibrium prices, demands, and profits, as detailed in the following lemma.

Lemma 3.2. *Suppose both firms adopt TBPs with the same net unit economic impact of recycling c. Then,*

- *If $1 < \beta \leq 5$, the equilibrium prices are $p_A^* = p_B^* = \frac{\beta+2c-2}{2}$; the equilibrium demands are $d_{RA}^* = d_{RB}^* = \frac{1}{2}$; and the equilibrium profits are $\pi_A^* = \pi_B^* = \frac{\beta-1}{4}$.*
- *If $\beta > 5$, the equilibrium prices are $p_A^* = p_B^* = \frac{1}{2}$; the equilibrium demands are $d_{RA}^* = d_{RB}^* = \frac{2}{\beta-1}$ and $d_{NA}^* = d_{NB}^* = \frac{1}{2} - \frac{2}{\beta-1}$; and the equilibrium profits are $\pi_A^* = \pi_B^* = \frac{4-2(\beta-1)c}{(\beta-1)^2} + \frac{1}{4}$.*

In the YY case, the profit structures of the two firms are identical due to symmetry, which leads to coinciding optimal pricing decisions. When $1 < \beta \leq 5$, the equilibrium prices for both firms are equal and are determined by the parameters c and β. In this regime, all customers are incentivized to recycle, and each firm captures exactly half of the market demand. The equilibrium profit increases with the return hassle sensitivity parameter β, as firms can charge a recycling premium. However, when $\beta > 5$, some customers become reluctant to recycle. The equilibrium prices drop to $\frac{1}{2}$, and the demand of recycle customers per firm decreases to $\frac{2}{\beta-1}$. Overall, as the return hassle sensitivity β increases, the market dynamics shift from a symmetric split of recycled customers at higher prices to more aggressive price competition and a reallocation of demand between recycled and non-recycled customers.

3.4.4 Equilibrium Outcomes

Now, we compare the equilibrium profits in the NN, YN/ NY, and YY cases in Stage 2 to determine the equilibrium outcomes in Stage 1.

Proposition 3.4. *The equilibrium outcomes are as follows.*

- *No TBP Implementation is an equilibrium when* $\pi_A^{NN} \geq \pi_A^{YN}, \pi_B^{NN} \geq \pi_B^{NY}, \pi_A^{NN} \geq \pi_A^{YY}, \pi_B^{NN} \geq \pi_B^{YY}$.
- *Partial TBP Implementation is an equilibrium when* $\pi_A^{YN} \geq \pi_A^{NN}, \pi_B^{YN} \geq \pi_B^{YY}$ *or* $\pi_A^{NY} \geq \pi_A^{YY}, \pi_B^{NY} \geq \pi_B^{NN}$.
- *Full TBP Implementation is an equilibrium when* $\pi_A^{YY} \geq \pi_A^{NY}, \pi_B^{YY} \geq \pi_B^{YN}, \pi_A^{YY} \geq \pi_A^{NN}, \pi_B^{YY} \geq \pi_B^{NN}$.

The conditions for the YN equilibrium ensure that neither firm has an incentive to deviate to either the NN or YY equilibrium. Analogous reasoning applies to the conditions governing other equilibrium outcomes. Figure 3.7 illustrates the parameter space in which the NN, YN/NY, and YY equilibria described in Proposition 3.4 can arise. As shown in Figure 3.7, partial TBP implementation can constitute an equilibrium when β and c are both small, or when β is sufficiently large. This observation further confirms the existence of the win-win region depicted in Figure 3.6. The underlying intuition is that, under asymmetric adoption, both firms benefit from improved customer segmentation: Firm A is able to capture the recycling premium, while Firm B can target non-recyclers more effectively.

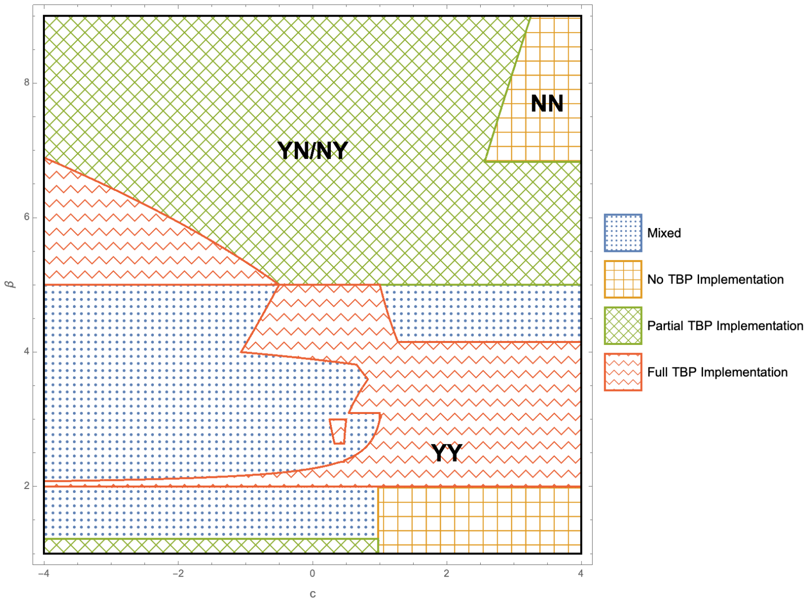


Figure 3.7: Equilibrium Outcomes

3.5 Discussion

This study examines the strategic implementation and economic impact of TBPs in a competitive retail environment. Customers who choose to recycle their used products are required to physically return them to the store, which provides psychological satisfaction but also imposes hassle costs associated with the return process. We model customers' recycling and purchasing decisions as utility-maximizing choices, and analyze three distinct TBP implementation strategies: no TBP implementation, partial TBP implementation, and full TBP implementation. For each case, we derive the optimal pricing decisions and firm profits.

Our analysis demonstrates that any of these TBP implementation strategies can serve as an equilibrium, contingent on customers' sensitivity to the return hassle cost and the net unit cost or benefit of processing returned items. This result helps explain the diversity of TBP implementation observed in practice, where some firms fully embrace recycling initiatives, others implement them selectively, and some choose not to adopt them at all. We further identify the parameter regions in which each equilibrium is sustainable, offering practical guidance for managers evaluating TBP implementation. Importantly, our results show that even partial TBP implementation can lead to win-win outcomes for competing firms. Essentially, TBPs function as a strategic tool for strategic market segmentation. They enable firms to cater to environmentally conscious or price-sensitive customers without engaging in price wars. By leveraging TBPs, firms can effectively differentiate themselves and increase profitability while promoting circular economy goals.

Chapter 4

Conclusions

This thesis offers a comprehensive exploration of how customer incentive programs, especially CLPs and TBPs, can serve as strategic tools for enhancing customer engagement, driving competitive advantage, and informing optimal program design across varying market conditions.

The first study makes three key contributions. First, to our knowledge, we are among the first to analytically study CLPs and examine their price discrimination role based on customers' shopping intensity. This perspective provides a novel rationale for offering CLPs and sheds light on the optimal design of such programs in practice. Second, our work compares the performance of CLPs and PLPs, identifying the market conditions under which CLPs enable more effective price discrimination. Third, we demonstrate how factors such as customer composition, customer discounting, and the number of firms in the coalition affect the ability of CLPs to differentiate among customers and extract surplus.

This study represents a first step in understanding CLPs for many industries. While CLPs may be theoretically optimal under certain conditions, real-world challenges can make PLPs a more viable choice in some scenarios. First, forming a coalition requires significant coordination among partner firms, particularly in designing the rules of earning and redeeming rewards. Disagreements or misaligned priorities can weaken the coalition's effectiveness or deter firms from joining. Second, firms often consider customer data a strategic asset and may be reluctant to share it with coalition partners, making CLPs less appealing for firms that prioritize data protection. Third, concerns about brand dilution may discourage firms from participating in a CLP, as they may prefer to maintain a distinct brand identity within a PLP. Fourth, customers often earn points from everyday purchases (e.g., groceries, gas) but redeem them for discretionary or aspirational products (e.g., travel, entertainment). This redemption asymmetry can create tension among coalition partners, making CLPs less attractive to some firms.

This study opens several promising avenues for future research. First, our work focuses on reward programs where the reward can be earned on every purchase, and customers do not need to accumulate reward points over time. An important future direction is to consider reward accumulation and redemption thresholds in point reward programs. On one hand, empirical studies have documented the point pressure phenomenon (also known as point acceleration behavior), where a customer's likelihood of making a purchase rises as their reward points near the redemption threshold or expiration (Inman and McAlister 1994, Lewis 2004, Kivetz et al. 2006, Hartmann and Viard 2008, Kopalle et al. 2012). On the other hand, determining the optimal redemption threshold requires the coalition manager to balance the effective price customers pay with their probability of making a purchase. In addition, customers who are able to successfully earn and redeem rewards may also alter their purchase behavior, warranting further investigation. Second, our work assumes that customer visits are non-strategic. An important direction for future research is to examine scenarios in which customers strategically decide when to visit and make purchases. For example, introducing randomness into the arrival process or assuming that each customer faces a stochastic outside option in each period could result in customer behavior that is influenced by the reward expiration and the realization of the outside option. Third, our model assumes that the product's price remains constant regardless of the customer's state. Exploring customized pricing strategies would be an interesting extension to our work. Fourth, the increasing digitalization of loyalty programs presents a fertile area for study. Future research could examine how technologies such as mobile apps, digital wallets, and blockchain platforms affect customer engagement, reward tracking, redemption behavior, and coalition management. Finally, CLPs are often managed by specialized program administrators who might be rewarded by sales-based commissions and hence may always have an incentive to enlist more firms. This creates an incentive misalignment issue between program administrators and participating firms. Such misalignment of incentives is sometimes offered as a reason for the struggles some CLPs face in their expansion. In our work, we do not explicitly model program administrators. Designing appropriate incentives for program administrators is another important topic for future research.

The second study contributes by analyzing the strategic implementation and economic implications of in-store TBPs in competitive markets. We are among the first to develop an analytical model that integrates customer recycling behavior, hassle sensitivity, and competitive firm strategy. By explicitly capturing both the psychological satisfaction and the hassle costs associated with recycling, our framework provides firms with actionable guidance on optimal pricing and TBP implementation. Furthermore, we highlight the dual role of TBPs as both a sustainability initiative and a strategic differentiation tool, enabling firms to optimize customer targeting and maximize returns from circular practices.

This study also opens new directions for future research. One promising direction is to examine the role of regulatory frameworks and policy interventions in shaping TBP adoption. Governments and industry bodies can play a pivotal role in promoting environmental stewardship, and understanding their influence is critical for designing effective policies. Incorporating product differentiation represents another important direction, as it can provide deeper insights into how firms balance profitability, environmental objectives, and competitive positioning in markets with heterogeneous customer preferences. For example, a high-quality firm may prefer an in-store TBP that reinforces its brand image, enhances perceived value, and appeals to environmentally conscious consumers, whereas a lower-tier firm may pursue alternative strategies to attract price-sensitive customers. Finally, future research could explore alternative recycling incentive mechanisms. Firms might offer loyalty points or tiered rewards to customers who participate in recycling initiatives, thereby encouraging environmentally responsible behavior while simultaneously enhancing customer engagement and long-term loyalty.

In sum, this thesis highlights the strategic potential of customer incentive programs that extend beyond transactional rewards. Despite their growing real-world adoption, CLPs and TBPs remain understudied in the academic literature. By developing rigorous analytical models and offering practical implications, the thesis lays the foundation for deeper inquiry into these increasingly relevant mechanisms. We hope this thesis inspires continued research into the dynamic interplay between customer behavior, firm strategy, and broader societal goals in the evolving landscape of incentive-based marketing.

Appendix A

Appendix for Chapter 2

The online appendices provide supplemental materials for Chapter 2 and include four sections:

- Section A.1 provides proofs of propositions and lemmas in the base model.
- Section A.2 provides a detailed analysis of the model extension in which a new reward is earned upon redemption.
- Section A.3 provides the proof of Proposition 2.5 in the case with asymmetric firms.
- Section A.4 provides a detailed analysis of the model extension without customer discounting.

A.1 Proofs of Propositions and Lemmas in the Base Model

Lemma A1 provides a condition under which it is optimal for the customer to always make a purchase in state 0 when visiting a firm in the coalition. If this condition is not met, the customer will eventually exit the market and will not contribute to the per-firm profit in the long run. The optimality equations yield an explicit solution, as stated in Lemma A1.

Lemma A1. *If*

$$v - p + \frac{n\lambda}{\delta + 2n\lambda + \mu} r \geq 0, \tag{A1}$$

then it is optimal for a (v, λ)*-customer to make a purchase at every visit to a firm in the coalition, and the solution to the optimality equations* (2.1)–(2.2) *is:*

$$u(0) = \frac{n\lambda}{\delta}\left(v - p + \frac{n\lambda}{\delta + 2n\lambda + \mu} r\right), \quad u(1) = \frac{n\lambda}{\delta}\left(v - p + \frac{\delta + n\lambda}{\delta + 2n\lambda + \mu} r\right).$$

Note that the term $\frac{n\lambda}{\delta+2n\lambda+\mu} r$ can be interpreted as the perceived value of the reward, accounting for both reward expiration and discounting effects. Condition (A1) thus requires that the sum of the customer's product valuation v and the perceived value of the reward exceeds the price p. When this condition holds, the customer will always make a purchase upon each visit.

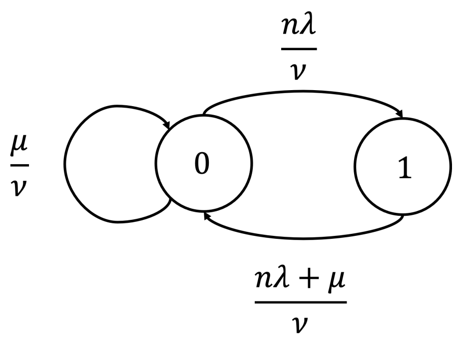


Figure A.1: Diagram of State Transitions

The state of a customer who makes a purchase whenever she visits a firm in the coalition follows a Markov chain, as illustrated in Figure A.1. Let q_0 and q_1 denote the stationary probabilities of states 0 and 1, respectively. By solving the balance equations, we find that $q_0 = \frac{n\lambda+\mu}{2n\lambda+\mu}$ and $q_1 = \frac{n\lambda}{2n\lambda+\mu}$. Recall that when a customer makes a purchase, she pays price $p-r$ in state 1 and price p in state 0. Accordingly, when condition (A1) is satisfied, the customer contributes a profit to the coalition at rate

$$n\lambda\Big(q_0 p + q_1(p-r)\Big) = n\lambda\Big(p - \frac{n\lambda}{2n\lambda+\mu}r\Big). \tag{A2}$$

Proof of Lemma A1

We first show that if $v-p+u(1) \geq u(0)$, then it must also be true that $v-p+r+u(0) \geq u(1)$. Suppose, for the sake of contradiction, that $v-p+r+u(0) < u(1)$. Then,

$$u(1) - u(0) = \frac{n\lambda}{\delta+n\lambda+\mu}\Big\{u(1) - [v-p+u(1)]\Big\} = \frac{n\lambda}{\delta+n\lambda+\mu}(-v+p) > 0,$$

which implies $v < p$. Now, consider,

$$v-p+u(1)-u(0) = v-p+\frac{n\lambda}{\delta+n\lambda+\mu}(-v+p) = \frac{\delta+\mu}{\delta+n\lambda+\mu}(v-p) < 0,$$

which contradicts our initial assumption that $v-p+u(1) \geq u(0)$. Therefore, our assumption must be false, and it follows that $v-p+r+u(0) \geq u(1)$. Next, under the condition $v-p+u(1) \geq u(0)$, equations (2.1) and (2.2) reduce to

$$u(1) = \frac{n\lambda}{\delta+n\lambda+\mu}[v-p+r+u(0)] + \frac{\mu}{\delta+n\lambda+\mu}u(0),$$
$$u(0) = \frac{n\lambda}{\delta+n\lambda+\mu}[v-p+u(1)] + \frac{\mu}{\delta+n\lambda+\mu}u(0).$$

Solving these equations yields

$$u(1) = \frac{n\lambda}{\delta}\left(v-p+\frac{\delta+n\lambda}{\delta+2n\lambda+\mu}r\right), \text{ and } u(0) = \frac{n\lambda}{\delta}\left(v-p+\frac{n\lambda}{\delta+2n\lambda+\mu}r\right).$$

Substituting these expressions into the condition $v-p+u(1) \geq u(0)$ gives

$$v-p+\frac{n\lambda}{\delta+2n\lambda+\mu}r \geq 0,$$

which guarantees that the customer will choose to purchase each time she visits a firm within the coalition. This completes the proof.

Proof of Lemma 2.1

To calculate the per-firm profit in the CLP, aggregated across the four customer segments, we examine how the price p compares to the values $v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r$, $v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r$, $v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r$, and $v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r$. This comparison allows us to characterize customer purchasing decisions within each segment. It can be verified that $v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r < \min\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} \leq \max\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} < v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r$. Based on the relative value of the price p, we distinguish six possible cases:

<u>Case 1:</u> $p \leq v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r < \min\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} \leq \max\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} < v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r$.

All customers will make a purchase. The optimization problem is thus reduced to

$$\max_{p\geq r\geq 0} \ \beta\lambda_F\Big(p - \frac{n\lambda_F}{2n\lambda_F+\mu}r\Big) + (1-\beta)\lambda_I\Big(p - \frac{n\lambda_I}{2n\lambda_I+\mu}r\Big)$$
$$s.t.\ p \leq v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r.$$

Clearly,

$$(p^*, r^*) = (v_L, 0),$$

$$\pi^* = \beta\lambda_F v_L + (1-\beta)\lambda_I v_L,$$

Hence, in this case, adopting the CLP is never optimal.

<u>Case 2:</u> $v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r < p \leq \min\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} \leq \max\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} < v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r$.

All customers except the LI segment will make a purchase. The optimization problem is thus reduced to:

$$\max_{p\geq r\geq 0} \ \beta\lambda_F\Big(p - \frac{n\lambda_F}{2n\lambda_F+\mu}r\Big) + (\alpha-\gamma)\lambda_I\Big(p - \frac{n\lambda_I}{2n\lambda_I+\mu}r\Big)$$
$$s.t.\ v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r < p \leq \min\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\}.$$

Clearly, $p^* = \min\left\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\right\}$. We then have the following two subcases.

Subcase 1: Suppose

$$v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu}r \le v_H + \frac{n\lambda_I}{\delta + 2n\lambda_I + \mu}r.$$

It follows that $p^* = v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r$. Reorganizing the supposition yields $r \le \frac{(v_H-v_L)(\delta+2n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)}{n(\lambda_F-\lambda_I)(\delta+\mu)}$. Moreover, $p \ge r$ gives $r \le \frac{\delta+2n\lambda_F+\mu}{\delta+n\lambda_F+\mu}v_L$. So, the optimization problem can be rewritten as

$$\max_{r\ge 0}\ \beta\lambda_F\Big[v_L + (\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_F}{2n\lambda_F + \mu})r\Big] + (\alpha-\gamma)\lambda_I\Big[v_L + (\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_I}{2n\lambda_I + \mu})r\Big]$$
$$s.t.\ r \le \min\Big\{\frac{(v_H - v_L)(\delta + 2n\lambda_F + \mu)(\delta + 2n\lambda_I + \mu)}{n(\lambda_F - \lambda_I)(\delta + \mu)}, \frac{\delta + 2n\lambda_F + \mu}{\delta + n\lambda_F + \mu}v_L\Big\}.$$

Note that if and only if

$$\beta\lambda_F(\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_F}{2n\lambda_F + \mu}) + (\alpha-\gamma)\lambda_I(\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_I}{2n\lambda_I + \mu}) > 0, \qquad \text{(A3)}$$

the coefficient for r above is non-negative. Therefore, at optimality, we have

$$r^* = \min\Big\{\frac{(v_H - v_L)(\delta + 2n\lambda_F + \mu)(\delta + 2n\lambda_I + \mu)}{n(\lambda_F - \lambda_I)(\delta + \mu)}, \frac{\delta + 2n\lambda_F + \mu}{\delta + n\lambda_F + \mu}v_L\Big\}.$$

Reorganizing inequality (A3) gives condition (2.3) in Lemma 2.1.

Subcase 2: Suppose

$$v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu}r \ge v_H + \frac{n\lambda_I}{\delta + 2n\lambda_I + \mu}r.$$

It follows that $p^* = v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r$. Reorganizing the supposition yields $r \ge \frac{(v_H-v_L)(\delta+2n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)}{n(\lambda_F-\lambda_I)(\delta+\mu)}$. Moreover, $p \ge r$ gives $r \le \frac{\delta+n\lambda_I+\mu}{\delta+\mu}v_H$. So, the optimization problem can be rewritten as

$$\max_{r\ge 0}\ \beta\lambda_F\Big[v_H + (\frac{n\lambda_I}{\delta + 2n\lambda_I + \mu} - \frac{n\lambda_F}{2n\lambda_F + \mu})r\Big] + (\alpha-\gamma)\lambda_I\Big[v_H + (\frac{n\lambda_I}{\delta + 2n\lambda_I + \mu} - \frac{n\lambda_I}{2n\lambda_I + \mu})r\Big]$$
$$s.t.\ \frac{(v_H - v_L)(\delta + 2n\lambda_F + \mu)(\delta + 2n\lambda_I + \mu)}{n(\lambda_F - \lambda_I)(\delta + \mu)} \le r \le \frac{\delta + n\lambda_I + \mu}{\delta + \mu}v_H.$$

Note that $\frac{n\lambda_I}{\delta+2n\lambda_I+\mu} < \frac{n\lambda_I}{2n\lambda_I+\mu} < \frac{n\lambda_F}{2n\lambda_F+\mu}$, so the profit decreases in r, and thus

$$r^* = \frac{(v_H - v_L)(\delta + 2n\lambda_F + \mu)(\delta + 2n\lambda_I + \mu)}{n(\lambda_F - \lambda_I)(\delta + \mu)},$$

under which

$$v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} r^* = v_H + \frac{n\lambda_I}{\delta + 2n\lambda_I + \mu} r^*.$$

Hence, the profit in Subcase 2 is strictly less than that in Subcase 1.

Case 3: $v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu} r < v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu} r < p \le v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu} r < v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu} r.$

Only high-valuation customers will make a purchase. The optimization problem is thus reduced to

$$\max_{p \ge r \ge 0} \ \gamma\lambda_F\Big(p - \frac{n\lambda_F}{2n\lambda_F + \mu} r\Big) + (\alpha - \gamma)\lambda_I\Big(p - \frac{n\lambda_I}{2n\lambda_I + \mu} r\Big)$$

$$s.t.\ v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} r < p \le v_H + \frac{n\lambda_I}{\delta + 2n\lambda_I + \mu} r.$$

Clearly,

$$(p^*, r^*) = (v_H, 0),$$

$$\pi^* = \gamma\lambda_F v_H + (\alpha - \gamma)\lambda_I v_H,$$

Hence, in this case, adopting the CLP is never optimal.

Case 4: $v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu} r < v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu} r < p \le v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu} r < v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu} r.$

Only frequent customers will make a purchase. The optimization problem is thus reduced to

$$\max_{p \ge r \ge 0} \ \beta\lambda_F\Big(p - \frac{n\lambda_F}{2n\lambda_F + \mu} r\Big)$$

$$s.t.\ v_H + \frac{n\lambda_I}{\delta + 2n\lambda_I + \mu} r < p \le v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} r.$$

Clearly,

$$p^* = v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} r^*,$$

$$\pi^* = \beta\lambda_F\Big[v_L + \big(\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_F}{2n\lambda_F + \mu}\big) r^*\Big] < \beta\lambda_F v_L < \pi_1.$$

Hence, in this case, adopting the CLP is never optimal.

Case 5: $v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r < \min\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} \leq \max\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} < p \leq v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r$.

Only high-valuation frequent customers will make a purchase. The optimization problem is thus reduced to

$$\max_{p\geq r\geq 0} \ \gamma\lambda_F\Big(p - \frac{n\lambda_F}{2n\lambda_F+\mu}r\Big)$$
$$s.t. \ p \leq v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r.$$

Clearly,

$$p^* = v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r^*,$$
$$\pi^* = \gamma\lambda_F\Big[v_H + (\frac{n\lambda_F}{\delta+2n\lambda_F+\mu} - \frac{n\lambda_F}{2n\lambda_F+\mu})r^*\Big] < \gamma\lambda_F v_H < \pi_2.$$

Hence, in this case, adopting the CLP is never optimal.

Case 6: $v_L + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r < \min\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} \leq \max\Big\{v_L + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r, v_H + \frac{n\lambda_I}{\delta+2n\lambda_I+\mu}r\Big\} < v_H + \frac{n\lambda_F}{\delta+2n\lambda_F+\mu}r < p$.

No customers will make a purchase. Clearly, in this case, adopting the CLP is never optimal.

Taking the above six cases into consideration, we can see that only Subcase 1 in Case 2 is possibly optimal. This completes the proof.

Proof of Proposition 2.1

Part (a): One can show that when $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} < \frac{v_H}{v_L}$ holds,

$$\frac{\delta+2n\lambda_F+\mu}{\delta+n\lambda_F+\mu}v_L < \frac{(v_H-v_L)(\delta+2n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)}{n(\lambda_F-\lambda_I)(\delta+\mu)}.$$

Hence, according to Lemma 2.1,

$$r_a^* = \frac{\delta + 2n\lambda_F + \mu}{\delta + n\lambda_F + \mu} v_L,$$

$$p_a^* = v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu}\frac{\delta + 2n\lambda_F + \mu}{\delta + n\lambda_F + \mu} v_L = \frac{\delta + 2n\lambda_F + \mu}{\delta + n\lambda_F + \mu} v_L = r_a^*,$$

$$\begin{aligned}\pi_a^* &= \beta\lambda_F\Big[v_L + \big(\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_F}{2n\lambda_F + \mu}\big)\frac{\delta + 2n\lambda_F + \mu}{\delta + n\lambda_F + \mu} v_L\Big] \\ &\quad + (\alpha - \gamma)\lambda_I\Big[v_L + \big(\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_I}{2n\lambda_I + \mu}\big)\frac{\delta + 2n\lambda_F + \mu}{\delta + n\lambda_F + \mu} v_L\Big] \\ &= \beta\lambda_F v_L + (\alpha - \gamma)\lambda_I v_L - \beta\lambda_F \frac{n\lambda_F\delta}{2n\lambda_F + \mu}\frac{v_L}{\delta + n\lambda_F + \mu} \\ &\quad + (\alpha - \gamma)\lambda_I \frac{n[\lambda_F\mu - \lambda_I(\delta + \mu)]}{2n\lambda_I + \mu}\frac{v_L}{\delta + n\lambda_F + \mu}.\end{aligned}$$

Taking the derivative of π_a^* with respect to n yields

$$\begin{aligned}\frac{d\pi_a^*}{dn} &= -\beta\lambda_F^2\delta v_L\Big(\frac{n}{(2n\lambda_F + \mu)(\delta + n\lambda_F + \mu)}\Big)' + (\alpha - \gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta + \mu)]v_L\Big(\frac{n}{(2n\lambda_I + \mu)(\delta + n\lambda_F + \mu)}\Big)' \\ &= -\beta\lambda_F^2\delta v_L \frac{(2n\lambda_F + \mu)(\delta + n\lambda_F + \mu) - n[2\lambda_F(\delta + n\lambda_F + \mu) + \lambda_F(2n\lambda_F + \mu)]}{(2n\lambda_F + \mu)^2(\delta + n\lambda_F + \mu)^2} \\ &\quad + (\alpha - \gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta + \mu)]v_L \frac{(2n\lambda_I + \mu)(\delta + n\lambda_F + \mu) - n[2\lambda_I(\delta + n\lambda_F + \mu) + \lambda_F(2n\lambda_I + \mu)]}{(2n\lambda_I + \mu)^2(\delta + n\lambda_F + \mu)^2} \\ &= -\beta\lambda_F^2\delta v_L \frac{\mu(\delta + n\lambda_F + \mu) - n\lambda_F(2n\lambda_F + \mu)}{(2n\lambda_F + \mu)^2(\delta + n\lambda_F + \mu)^2} + (\alpha - \gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta + \mu)]v_L \frac{\mu(\delta + n\lambda_F + \mu) - n\lambda_F(2n\lambda_I + \mu)}{(2n\lambda_I + \mu)^2(\delta + n\lambda_F + \mu)^2} \\ &= \frac{v_L}{(\delta + n\lambda_F + \mu)^2}\Big(\frac{(\alpha - \gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta + \mu)][\mu(\delta + n\lambda_F + \mu) - n\lambda_F(2n\lambda_I + \mu)](2n\lambda_F + \mu)^2}{(2n\lambda_F + \mu)^2(2n\lambda_I + \mu)^2} \\ &\quad - \frac{\beta\lambda_F^2\delta[\mu(\delta + n\lambda_F + \mu) - n\lambda_F(2n\lambda_F + \mu)](2n\lambda_I + \mu)^2}{(2n\lambda_F + \mu)^2(2n\lambda_I + \mu)^2}\Big) \qquad \text{(A4)} \\ &> \frac{v_L}{(\delta + n\lambda_F + \mu)^2}\Big(\frac{(\alpha - \gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta + \mu)][\mu(\delta + n\lambda_F + \mu) - n\lambda_F(2n\lambda_F + \mu)](2n\lambda_F + \mu)}{(2n\lambda_F + \mu)(2n\lambda_I + \mu)^2} \\ &\quad - \frac{\beta\lambda_F^2\delta[\mu(\delta + n\lambda_F + \mu) - n\lambda_F(2n\lambda_F + \mu)](2n\lambda_I + \mu)\frac{2n\lambda_I + \mu}{2n\lambda_F + \mu}}{(2n\lambda_F + \mu)(2n\lambda_I + \mu)^2}\Big) \\ &> \frac{v_L}{(\delta + n\lambda_F + \mu)^2}[\mu(\delta + n\lambda_F + \mu) - n\lambda_F(2n\lambda_F + \mu)]\frac{(\alpha - \gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta + \mu)](2n\lambda_F + \mu) - \beta\lambda_F^2\delta(2n\lambda_I + \mu)}{(2n\lambda_F + \mu)(2n\lambda_I + \mu)^2} \\ &> 0,\end{aligned}$$

where the first inequality holds since $2n\lambda_I + \mu < 2n\lambda_F + \mu$, the second inequality holds since $\frac{2n\lambda_I+\mu}{2n\lambda_F+\mu} < 1$ and $\mu(\delta + n\lambda_F + \mu) - n\lambda_F(2n\lambda_F + \mu) > 0$, and the last inequality holds due to condition (2.3) and $\mu(\delta+n\lambda_F+\mu)-n\lambda_F(2n\lambda_F+\mu) > 0$ when $n < \frac{\sqrt{2\mu(\delta+\mu)}}{2\lambda_F}$. This establishes that when n is smaller than a threshold, the profit π_a^* is increasing in size n.

On the other hand, by (A4), we have

$$\begin{aligned}
\frac{d\pi_a^*}{dn} &= \frac{v_L}{(\delta+n\lambda_F+\mu)^2} \times \Big(\frac{(\alpha-\gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta+\mu)][\mu(\delta+n\lambda_F+\mu) - n\lambda_F(2n\lambda_I+\mu)](2n\lambda_F+\mu)^2}{(2n\lambda_F+\mu)^2(2n\lambda_I+\mu)^2} \\
&\qquad - \frac{\beta\lambda_F^2\delta[\mu(\delta+n\lambda_F+\mu) - n\lambda_F(2n\lambda_F+\mu)](2n\lambda_I+\mu)^2}{(2n\lambda_F+\mu)^2(2n\lambda_I+\mu)^2}\Big) \\
&= \frac{v_L}{(\delta+n\lambda_F+\mu)^2} \times \Big(\frac{(\alpha-\gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta+\mu)][\mu(\delta+n\lambda_F+\mu) - n\lambda_F(2n\lambda_I+\mu)]}{(2n\lambda_I+\mu)^2} \\
&\qquad - \frac{\beta\lambda_F^2\delta[\mu(\delta+n\lambda_F+\mu) - n\lambda_F(2n\lambda_F+\mu)]}{(2n\lambda_F+\mu)^2}\Big) \\
&\to \frac{v_L}{(\delta+n\lambda_F+\mu)^2} \times \Big((\alpha-\gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta+\mu)](-\frac{\lambda_F}{2\lambda_I}) - \beta\lambda_F^2\delta(-\frac{1}{2})\Big) \qquad \text{[as } n \to \infty\text{]} \\
&= \frac{v_L}{2(\delta+n\lambda_F+\mu)^2} \times \Big(-(\alpha-\gamma)\lambda_F[\lambda_F\mu - \lambda_I(\delta+\mu)] + \beta\lambda_F^2\delta\Big) \\
&< 0,
\end{aligned}$$

where the last inequality holds since as $n \to \infty$, condition (2.3) becomes

$$\begin{aligned}
&(\alpha-\gamma)\lambda_I\Big(\lambda_F\mu - \lambda_I(\delta+\mu)\Big)(2n\lambda_F+\mu) > \beta\lambda_F^2\delta(2n\lambda_I+\mu) \\
&(\alpha-\gamma)\lambda_I\Big(\lambda_F\mu - \lambda_I(\delta+\mu)\Big)\frac{2n\lambda_F+\mu}{2n\lambda_I+\mu} > \beta\lambda_F^2\delta \\
&(\alpha-\gamma)\lambda_I\Big(\lambda_F\mu - \lambda_I(\delta+\mu)\Big)\frac{\lambda_F}{\lambda_I} > \beta\lambda_F^2\delta \\
&(\alpha-\gamma)\lambda_F\Big(\lambda_F\mu - \lambda_I(\delta+\mu)\Big) > \beta\lambda_F^2\delta
\end{aligned}$$

This implies that when n is sufficiently large, the profit π_a^* decreases in n. Therefore, the profit does not increase monotonically in the coalition size n.

<u>Part (b):</u> One can show that when $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} \geq \frac{v_H}{v_L}$ hold,

$$\frac{\delta+2n\lambda_F+\mu}{\delta+n\lambda_F+\mu}v_L \geq \frac{(v_H-v_L)(\delta+2n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)}{n(\lambda_F-\lambda_I)(\delta+\mu)}.$$

Hence, according to Lemma 2.1,

$$r_b^* = \frac{(v_H - v_L)(\delta + 2n\lambda_F + \mu)(\delta + 2n\lambda_I + \mu)}{n(\lambda_F - \lambda_I)(\delta + \mu)},$$

$$p_b^* = v_L + \frac{n\lambda_F}{\delta + 2n\lambda_F + \mu}\frac{(v_H - v_L)(\delta + 2n\lambda_F + \mu)(\delta + 2n\lambda_I + \mu)}{n(\lambda_F - \lambda_I)(\delta + \mu)}$$
$$= \frac{\lambda_F(\delta + 2n\lambda_I + \mu)v_H - \lambda_I(\delta + 2n\lambda_F + \mu)v_L}{(\lambda_F - \lambda_I)(\delta + \mu)},$$

$$\pi_b^* = \beta\lambda_F\Big[v_L + (\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_F}{2n\lambda_F + \mu})\frac{(v_H - v_L)(\delta + 2n\lambda_F + \mu)(\delta + 2n\lambda_I + \mu)}{n(\lambda_F - \lambda_I)(\delta + \mu)}\Big]$$
$$+ (\alpha - \gamma)\lambda_I\Big[v_L + (\frac{n\lambda_F}{\delta + 2n\lambda_F + \mu} - \frac{n\lambda_I}{2n\lambda_I + \mu})\frac{(v_H - v_L)(\delta + 2n\lambda_F + \mu)(\delta + 2n\lambda_I + \mu)}{n(\lambda_F - \lambda_I)(\delta + \mu)}\Big]$$
$$= \beta\lambda_F v_L + (\alpha - \gamma)\lambda_I v_L - \beta\lambda_F\frac{\delta + 2n\lambda_I + \mu}{2n\lambda_F + \mu}\frac{\lambda_F\delta(v_H - v_L)}{(\delta + \mu)(\lambda_F - \lambda_I)}$$
$$+ (\alpha - \gamma)\lambda_I\frac{[\lambda_F\mu - \lambda_I(\delta + \mu)](v_H - v_L)}{(\delta + \mu)(\lambda_F - \lambda_I)}\frac{\delta + 2n\lambda_I + \mu}{2n\lambda_I + \mu}.$$

Taking the derivative of π_b^* with respect to n yields

$$\frac{d\pi_b^*}{dn} = -\frac{v_H - v_L}{(\lambda_F - \lambda_I)(\delta + \mu)}\Big\{\beta\lambda_F^2\delta[\frac{\delta + 2n\lambda_I + \mu}{2n\lambda_F + \mu}]' - (\alpha - \gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta + \mu)][\frac{\delta + 2n\lambda_I + \mu}{2n\lambda_I + \mu}]'\Big\}$$
$$= -\frac{v_H - v_L}{(\lambda_F - \lambda_I)(\delta + \mu)}\Big\{2\beta\lambda_F^2\delta\frac{\lambda_I\mu - \lambda_F(\delta + \mu)}{(2n\lambda_F + \mu)^2} - 2(\alpha - \gamma)\lambda_I[\lambda_F\mu - \lambda_I(\delta + \mu)]\frac{-\lambda_I\delta}{(2n\lambda_I + \mu)^2}\Big\}$$
$$= \frac{2(v_H - v_L)\delta}{(\lambda_F - \lambda_I)(\delta + \mu)}\Big\{\beta\lambda_F^2\frac{\lambda_F(\delta + \mu) - \lambda_I\mu}{(2n\lambda_F + \mu)^2} - (\alpha - \gamma)\lambda_I^2\frac{\lambda_F\mu - \lambda_I(\delta + \mu)}{(2n\lambda_I + \mu)^2}\Big\}.$$

Let

$$\tilde{n} = \frac{\Big(1 - \frac{\lambda_I}{\lambda_F}\sqrt{\frac{(\alpha-\gamma)[\lambda_F\mu - \lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu) - \lambda_I\mu]}}\Big)\mu}{2\lambda_I\Big(\sqrt{\frac{(\alpha-\gamma)[\lambda_F\mu - \lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu) - \lambda_I\mu]}} - 1\Big)}.$$

(i) Consider the case where $\frac{(\alpha-\gamma)[\lambda_F\mu - \lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu) - \lambda_I\mu]} \leq 1$. This implies that

$$(\alpha - \gamma)[\lambda_F\mu - \lambda_I(\delta + \mu)] \leq \beta[\lambda_F(\delta + \mu) - \lambda_I\mu].$$

Observe that

$$\frac{\lambda_I^2}{(2n\lambda_I + \mu)^2} - \frac{\lambda_F^2}{(2n\lambda_F + \mu)^2} = \frac{[4n\lambda_F\lambda_I + \mu(\lambda_I + \lambda_F][\mu(\lambda_I - \lambda_F]]}{(2n\lambda_I + \mu)^2(2n\lambda_F + \mu)^2} < 0,$$

which leads to

$$(\alpha - \gamma)\lambda_I^2\frac{\lambda_F\mu - \lambda_I(\delta + \mu)}{(2n\lambda_I + \mu)^2} < \beta\lambda_F^2\frac{\lambda_F(\delta + \mu) - \lambda_I\mu}{(2n\lambda_F + \mu)^2}.$$

Therefore, $\frac{d\pi_b^*}{dn} > 0$ for any n.

(ii) Now suppose $1 < \frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} < \left(\frac{\lambda_F}{\lambda_I}\right)^2$. In this case, the definition of $\tilde{n}$ ensures that $\tilde{n}$ is positive. It can be verified that $\frac{d\pi_b^*}{dn} = 0$ when $n = \tilde{n}$; $\frac{d\pi_b^*}{dn} > 0$ for $n < \tilde{n}$; and $\frac{d\pi_b^*}{dn} < 0$ for $n > \tilde{n}$. Thus, the profit π_b^* increases with coalition size n up to $n = \tilde{n}$ and decreases thereafter.

(iii) Finally, if $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \geq \left(\frac{\lambda_F}{\lambda_I}\right)^2$, then

$$(\alpha-\gamma)\lambda_I^2[\lambda_F\mu-\lambda_I(\delta+\mu)] > \beta\lambda_F^2[\lambda_F(\delta+\mu)-\lambda_I\mu].$$

Since $\frac{1}{(2n\lambda_I+\mu)^2} > \frac{1}{(2n\lambda_F+\mu)^2}$, it follows that

$$(\alpha-\gamma)\lambda_I^2\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{(2n\lambda_I+\mu)^2} > \beta\lambda_F^2\frac{\lambda_F(\delta+\mu)-\lambda_I\mu}{(2n\lambda_F+\mu)^2}.$$

Therefore, $\frac{d\pi^*}{dn} < 0$ for all n, meaning that the profit π_b^* decreases as the coalition size n increases.

In summary, if $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \leq 1$, then π_b^* increases monotonically in n; if $1 < \frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} < \left(\frac{\lambda_F}{\lambda_I}\right)^2$, π_b^* increases with n for $n \leq \tilde{n}$ and decreases for $n > \tilde{n}$, so the optimal size $n^* = \lfloor\tilde{n}\rfloor$ or $\lceil\tilde{n}\rceil$; otherwise, π_b^* decreases monotonically as n increases. This completes the proof.

Proof of Proposition 2.2

The profit $\pi^*(n)$ in (2.4) consists of the profit collected from frequent customers and HI customers. Observe that $\frac{n\lambda_F}{\delta+2n\lambda_F+\mu} < \frac{n\lambda_F}{2n\lambda_F+\mu}$, which means that the effective price paid by HF and LF customers is less than v_L. On the other hand, the effective price for HI customers can exceed v_L but remains below v_H. This leads to the upper bound on the per-firm profit that $\pi^* < \beta\lambda_F v_L + (\alpha-\gamma)\lambda_I v_H$.

Part (a): Now, we show $\pi^* < \max\{\pi_1, \pi_2\}$ when $\gamma > \alpha\beta$. We can show that

$$\pi^* - \pi_1 < \beta\lambda_F v_L + (\alpha-\gamma)\lambda_I v_H - [\beta\lambda_F + (1-\beta)\lambda_I]v_L = [(\alpha-\gamma)v_H - (1-\beta)v_L]\lambda_I$$

$$< [\alpha(1-\beta)v_H - (1-\beta)v_L]\lambda_I = (1-\beta)(\alpha v_H - v_L)\lambda_I.$$

We can also show that

$$\pi^* - \pi_2 < \beta\lambda_F v_L + (\alpha-\gamma)\lambda_I v_H - [\gamma\lambda_F + (\alpha-\gamma)\lambda_I]v_H = [\beta v_L - \gamma v_H]\lambda_F < \beta(v_L - \alpha v_H)\lambda_F.$$

Observe that if $\alpha v_H \geq v_L$, then $\pi^* < \pi_2$; otherwise, $\pi^* < \pi_1$. Hence, $\pi^* < \max\{\pi_1, \pi_2\}$.

Part (b): Table A1 presents customer welfare in each segment under CLPs, and compares them with that under the full market coverage strategy with price v_L.

	CLP ($\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} < \frac{v_H}{v_L}$)	vs.	Full Market Coverage	vs.	CLP ($\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} \geq \frac{v_H}{v_L}$)
HF	$v_H - \frac{(n\lambda_F+\mu)(\delta+2n\lambda_F+\mu)}{(2n\lambda_F+\mu)(\delta+n\lambda_F+\mu)} v_L$	$>$	$v_H - v_L$	$<$	$(v_H - v_L)\left[1 + \frac{\delta+2n\lambda_I+\mu}{2n\lambda_F+\mu} \frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F-\lambda_I)}\right]$
LF	$v_L - \frac{(n\lambda_F+\mu)(\delta+2n\lambda_F+\mu)}{(2n\lambda_F+\mu)(\delta+n\lambda_F+\mu)} v_L$	$>$	0	$<$	$(v_H - v_L)\frac{\delta+2n\lambda_I+\mu}{2n\lambda_F+\mu} \frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F-\lambda_I)}$
HI	$v_H - \frac{(n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(2n\lambda_I+\mu)(\delta+n\lambda_F+\mu)} v_L$	$<$	$v_H - v_L$	$>$	$(v_H - v_L)\left[1 - \frac{\delta+2n\lambda_I+\mu}{2n\lambda_I+\mu} \frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{(\delta+\mu)(\lambda_F-\lambda_I)}\right]$
LI	0	$=$	0	$=$	0

Table A1: Customer Welfare Comparison in Each Customer Segment Between CLP and Full Market Coverage with No Reward

When the firm implements the partial market coverage strategy with price v_H, the aggregate customer surplus is 0. It is important to emphasize that, under CLPs, the customer surplus for each purchasing segment is nonnegative; otherwise, those customers would not make a purchase. Indeed, Table A1 shows that the aggregate customer surplus under CLPs is strictly positive. Consequently, CLPs yield a higher aggregate customer surplus than the partial market coverage strategy with price v_H. Next, we compare the total customer surplus under CLPs with that under the full market coverage strategy with price v_L. We consider the following two cases.

Case 1: Suppose $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} < \frac{v_H}{v_L}$.

By Table A1, the aggregate customer surplus associated with CLPs in this case minus that associated with the full market coverage strategy with price v_L equals

$$\begin{aligned}
&\gamma\lambda_F\Big\{v_H - \frac{(n\lambda_F+\mu)(\delta+2n\lambda_F+\mu)}{(2n\lambda_F+\mu)(\delta+n\lambda_F+\mu)}v_L - (v_H-v_L)\Big\}\\
&\quad+(\beta-\gamma)\lambda_F\Big\{v_L - \frac{(n\lambda_F+\mu)(\delta+2n\lambda_F+\mu)}{(2n\lambda_F+\mu)(\delta+n\lambda_F+\mu)}v_L\Big\}\\
&\quad+(\alpha-\gamma)\lambda_I\Big\{v_H - \frac{(n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(2n\lambda_I+\mu)(\delta+n\lambda_F+\mu)}v_L - (v_H-v_L)\Big\}\\
=&\beta\lambda_F\Big\{v_L - \frac{(n\lambda_F+\mu)(\delta+2n\lambda_F+\mu)}{(2n\lambda_F+\mu)(\delta+n\lambda_F+\mu)}v_L\Big\} + (\alpha-\gamma)\lambda_I\Big\{v_L - \frac{(n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(2n\lambda_I+\mu)(\delta+n\lambda_F+\mu)}v_L\Big\}\\
=&\frac{nv_L}{\delta+n\lambda_F+\mu}\Big\{\beta\lambda_F\frac{\lambda_F\delta}{2n\lambda_F+\mu} - (\alpha-\gamma)\lambda_I\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{2n\lambda_I+\mu}\Big\}\\
<&0,
\end{aligned}$$

where the last inequality holds due to condition (2.3).

Case 2: Suppose $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} \geq \frac{v_H}{v_L}$.

By Table A1, the aggregate customer surplus associated with CLPs in this case minus that associated with the full market coverage strategy with price v_L equals

$$\begin{aligned}
&\gamma\lambda_F\Big\{(v_H-v_L)\Big[1+\frac{\delta+2n\lambda_I+\mu}{2n\lambda_F+\mu}\frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F-\lambda_I)}\Big] - (v_H-v_L)\Big\}\\
&\quad+(\beta-\gamma)\lambda_F\Big\{(v_H-v_L)\frac{\delta+2n\lambda_I+\mu}{2n\lambda_F+\mu}\frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F-\lambda_I)}\Big\}\\
&\quad+(\alpha-\gamma)\lambda_I\Big\{(v_H-v_L)\Big[1-\frac{\delta+2n\lambda_I+\mu}{2n\lambda_I+\mu}\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{(\delta+\mu)(\lambda_F-\lambda_I)}\Big] - (v_H-v_L)\Big\}\\
=&\beta\lambda_F\Big\{(v_H-v_L)\frac{\delta+2n\lambda_I+\mu}{2n\lambda_F+\mu}\frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F-\lambda_I)}\Big\} - (\alpha-\gamma)\lambda_I\Big\{(v_H-v_L)\frac{\delta+2n\lambda_I+\mu}{2n\lambda_I+\mu}\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{(\delta+\mu)(\lambda_F-\lambda_I)}\Big\}\\
=&(v_H-v_L)\frac{\delta+2n\lambda_I+\mu}{(\delta+\mu)(\lambda_F-\lambda_I)}\Big\{\beta\lambda_F\frac{\lambda_F\delta}{2n\lambda_F+\mu} - (\alpha-\gamma)\lambda_I\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{2n\lambda_I+\mu}\Big\}\\
<&0,
\end{aligned}$$

where the last inequality holds due to condition (2.3). This completes the proof.

Proof of Proposition 2.3

Part (a): Suppose $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} < \frac{v_H}{v_L}$. By Proposition 2.1 (a), we have

$$\pi^c(n) = \beta\lambda_F v_L + (\alpha-\gamma)\lambda_I v_L - \beta\lambda_F \frac{n\lambda_F\delta}{n\lambda_F+\mu}\frac{v_L}{\delta+\mu} + (\alpha-\gamma)\lambda_I \frac{n[\lambda_F\mu - \lambda_I(\delta+\mu)]}{n\lambda_I+\mu}\frac{v_L}{\delta+\mu},$$

$$\pi_p = \beta\lambda_F v_L + (\alpha-\gamma)\lambda_I v_L - \beta\lambda_F \frac{\lambda_F\delta}{\lambda_F+\mu}\frac{v_L}{\delta+\mu} + (\alpha-\gamma)\lambda_I \frac{\lambda_F\mu - \lambda_I(\delta+\mu)}{\lambda_I+\mu}\frac{v_L}{\delta+\mu}.$$

Note that $\pi_p = \pi^c(1)$. Since $\pi^c(n)$ increases in n when $n < \frac{\sqrt{2\mu(\delta+\mu)}}{2\lambda_F}$ and decreases in n when n is sufficiently large, it follows that an optimally sized CLP does not necessarily lead to a higher per-firm profit than a PLP.

Part (b): Suppose $\frac{(\delta+n\lambda_I+\mu)(\delta+2n\lambda_F+\mu)}{(\delta+n\lambda_F+\mu)(\delta+2n\lambda_I+\mu)} \geq \frac{v_H}{v_L}$. By Proposition 2.1 (b),

(i) If $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \leq 1$, the profit $\pi^c(n)$ increases monotonically in the coalition size n. It follows immediately that a PLP always leads to a lower per-firm profit than a CLP.

(ii) If $1 < \frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} < \left(\frac{\lambda_F}{\lambda_I}\right)^2$, the profit $\pi^c(n)$ first increases in n for $n \leq \tilde{n}$ and then decreases in n. It follows immediately that $\pi_p > \pi^c(n^*)$ for any exogenous μ if either $\tilde{n} \leq 1$ or $1 < \tilde{n} < 2$ and $\pi^c(1) > \pi^c(2)$.

(iii) If $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \geq \left(\frac{\lambda_F}{\lambda_I}\right)^2$, the profit $\pi^c(n)$ decreases monotonically in the coalition size n. It follows immediately that a PLP always leads to a higher per-firm profit than a CLP.

This completes the proof.

A.2 A New Reward is Earned upon Redemption

Following a similar process as in the base model, we first derive the customer's optimal purchasing decision and then analyze the coalition's pricing and reward decisions, by assuming that the coalition size n is given.

Lemma A2. *If*

$$v - p + \frac{n\lambda}{\delta + n\lambda + \mu} r \geq 0, \tag{A5}$$

then it is optimal for a (v, λ)-customer to make a purchase whenever she visits a firm in the coalition, and the solution to the optimality equations (2.7)–(2.8) is given by

$$u(0) = \frac{n\lambda}{\delta + n\lambda + \mu}[v - p + u(1)] + \frac{\mu}{\delta + n\lambda + \mu} u(0),$$
$$u(1) = \frac{n\lambda}{\delta + n\lambda + \mu}[v - p + r + u(1)] + \frac{\mu}{\delta + n\lambda + \mu} u(0).$$

Proof of Lemma A2

We first show that if $v - p + u(1) \geq u(0)$, then it must also be true that $v - p + r + u(1) \geq u(1)$. Suppose, for contradiction, that $v - p + r + u(1) < u(1)$. Then,

$$u(1) - u(0) = \frac{n\lambda}{\delta + n\lambda + \mu}\Big\{u(1) - [v - p + u(1)]\Big\} = \frac{n\lambda}{\delta + n\lambda + \mu}(-v + p) > 0,$$

which implies $v < p$. Now, consider

$$v - p + u(1) - u(0) = v - p + \frac{n\lambda}{\delta + n\lambda + \mu}(-v + p) = \frac{\delta + \mu}{\delta + n\lambda + \mu}(v - p) < 0,$$

which contradicts our initial assumption that $v - p + u(1) \geq u(0)$. Therefore, our assumption must be false, and it follows that $v - p + r + u(1) \geq u(1)$. Next, under the condition $v - p + u(1) \geq u(0)$, equations (2.7) and (2.8) reduce to

$$u(1) = \frac{n\lambda}{\delta + n\lambda + \mu}[v - p + r + u(1)] + \frac{\mu}{\delta + n\lambda + \mu} u(0),$$
$$u(0) = \frac{n\lambda}{\delta + n\lambda + \mu}[v - p + u(1)] + \frac{\mu}{\delta + n\lambda + \mu} u(0).$$

Solving these equations yields:

$$u(1) = \frac{n\lambda}{\delta}(v - p + \frac{\delta + n\lambda}{\delta + n\lambda + \mu} r), \text{ and } u(0) = \frac{n\lambda}{\delta}(v - p + \frac{n\lambda}{\delta + n\lambda + \mu} r).$$

Substituting these expressions into the condition $v - p + u(1) \geq u(0)$ gives:

$$v - p + \frac{n\lambda}{\delta + n\lambda + \mu} r \geq 0,$$

which guarantees that the customer will choose to purchase each time she visits a firm within the coalition. This completes the proof.

By solving the balance equations $q_0 \frac{n\lambda}{\delta+n\lambda+\mu} = q_1 \frac{\mu}{\delta+n\lambda+\mu}$ and $q_0 + q_1 = 1$, we find that $q_0 = \frac{\mu}{n\lambda+\mu}$ and $q_1 = \frac{n\lambda}{n\lambda+\mu}$. Accordingly, when condition (A5) is satisfied, the customer contributes a profit to the coalition at rate

$$n\lambda\Big(q_0 p + q_1(p - r)\Big) = n\lambda\Big(p - \frac{n\lambda}{n\lambda + \mu} r\Big). \tag{A6}$$

To determine the per-firm profit in the CLP across the four customer segments, we examine how the price p compares to $v_H + \frac{n\lambda_F}{\delta+n\lambda_F+\mu} r$, $v_H + \frac{n\lambda_I}{\delta+n\lambda_I+\mu} r$, $v_L + \frac{n\lambda_F}{\delta+n\lambda_F+\mu_F} r$, and $v_L + \frac{n\lambda_I}{\delta+n\lambda_I+\mu} r$, which allows us to characterize customer purchasing behavior in each segment. Lemma A3 provides a summary of the optimal price, reward amount, and per-firm profit.

Lemma A3. *A CLP with size n improves the per-firm profit relative to no-reward programs only if*

$$(\alpha - \gamma)\lambda_I\Big(\lambda_F\mu - \lambda_I(\delta + \mu)\Big)(n\lambda_F + \mu) > \beta\lambda_F^2\delta(n\lambda_I + \mu). \tag{A7}$$

When condition (A7) is satisfied, the optimal price, reward amount, and per-firm profit in the CLP are as follows:

(a) When either $\frac{v_H}{v_L} \geq \frac{\lambda_F}{\lambda_I}$ or $\frac{v_H}{v_L} < \frac{\lambda_F}{\lambda_I}$ but $n < \frac{(v_H - v_L)(\delta+\mu)}{v_L\lambda_F - v_H\lambda_I}$ holds, the optimal price, reward amount, and per-firm profit in the CLP are

$$p^* = r^* = \frac{\delta + \mu + n\lambda_F}{\delta + \mu} v_L;$$

$$\pi^* = \beta\lambda_F v_L + (\alpha - \gamma)\lambda_I v_L - \beta\lambda_F \frac{n\lambda_F\delta}{n\lambda_F + \mu}\frac{v_L}{\delta + \mu} + (\alpha - \gamma)\lambda_I \frac{n[\lambda_F\mu - \lambda_I(\delta + \mu)]}{n\lambda_I + \mu}\frac{v_L}{\delta + \mu}.$$

(b) When $\frac{v_H}{v_L} < \frac{\lambda_F}{\lambda_I}$ *and* $n \geq \frac{(v_H - v_L)(\delta+\mu)}{v_L\lambda_F - v_H\lambda_I}$ *hold, the optimal price, reward amount, and per-firm profit in the CLP are*

$$p^* = \frac{\lambda_F(\delta+\mu+n\lambda_I)v_H - \lambda_I(\delta+\mu+n\lambda_F)v_L}{(\lambda_F-\lambda_I)(\delta+\mu)},$$

$$r^* = \frac{(v_H - v_L)(\delta+n\lambda_F+\mu)(\delta+n\lambda_I+\mu)}{n(\lambda_F-\lambda_I)(\delta+\mu)},$$

$$\pi^* = \beta\lambda_F v_L + (\alpha-\gamma)\lambda_I v_L - \beta\lambda_F \frac{\delta+n\lambda_I+\mu}{n\lambda_F+\mu}\frac{\lambda_F\delta(v_H-v_L)}{(\delta+\mu)(\lambda_F-\lambda_I)}$$
$$+ (\alpha-\gamma)\lambda_I \frac{[\lambda_F\mu - \lambda_I(\delta+\mu)](v_H-v_L)}{(\delta+\mu)(\lambda_F-\lambda_I)}\frac{\delta+n\lambda_I+\mu}{n\lambda_I+\mu}.$$

Proof of Lemma A3

The proof of Lemma A3 is very similar to that of Lemma 2.1. To avoid repetition, we omit the analysis of the non-optimal cases (one can verify that the profits in the non-optimal cases are dominated by those with no-reward programs). Below, we just present the details of the optimal case. Suppose

$$v_L + \frac{n\lambda_F}{\delta+n\lambda_F+\mu}r \leq v_H + \frac{n\lambda_I}{\delta+n\lambda_I+\mu}r.$$

It follows that $p^* = v_L + \frac{n\lambda_F}{\delta+n\lambda_F+\mu}r$.
Reorganizing condition (A5) gives $r \leq \frac{(v_H-v_L)(\delta+n\lambda_F+\mu)(\delta+n\lambda_I+\mu)}{n(\lambda_F-\lambda_I)(\delta+\mu)}$. Moreover, $p \geq r$ yields $r \leq \frac{\delta+n\lambda_F+\mu}{\delta+\mu}v_L$. So,

$$r \leq \min\Big\{\frac{(v_H-v_L)(\delta+n\lambda_F+\mu)(\delta+n\lambda_I+\mu)}{n(\lambda_F-\lambda_I)(\delta+\mu)}, \frac{\delta+n\lambda_F+\mu}{\delta+\mu}v_L\Big\}. \tag{A8}$$

The profit in this case becomes

$$\beta\lambda_F\Big[v_L + (\frac{n\lambda_F}{\delta+n\lambda_F+\mu} - \frac{n\lambda_F}{n\lambda_F+\mu})r\Big] + (\alpha-\gamma)\lambda_I\Big[v_L + (\frac{n\lambda_F}{\delta+n\lambda_F+\mu} - \frac{n\lambda_I}{n\lambda_I+\mu})r\Big]. \tag{A9}$$

Note that if and only if

$$\beta\lambda_F(\frac{n\lambda_F}{\delta+n\lambda_F+\mu} - \frac{n\lambda_F}{n\lambda_F+\mu}) + (\alpha-\gamma)\lambda_I(\frac{n\lambda_F}{\delta+n\lambda_F+\mu} - \frac{n\lambda_I}{n\lambda_I+\mu}) > 0, \tag{A10}$$

$r^* \neq 0$. Therefore, if condition (A10) holds, by inequality (A8),

$$r^* = \min\Big\{\frac{(v_H-v_L)(\delta+n\lambda_F+\mu)(\delta+n\lambda_I+\mu)}{n(\lambda_F-\lambda_I)(\delta+\mu)}, \frac{\delta+n\lambda_F+\mu}{\delta+\mu}v_L\Big\}.$$

Reorganizing inequality (A10) yields condition (A7). Comparing the two terms in r^* gives the two cases in Lemma A3.

We next compare CLPs with no-reward programs in terms of per-firm profit and customer surplus. Lemma A4 shows that the comparison result between CLPs and no-reward programs in the base model still holds when a new reward is earned upon redemption.

Lemma A4. *By comparing a CLP and no-reward programs, we have the following:*

(a) When customers' product valuation and shopping intensity are positively correlated, i.e., $\gamma > \alpha\beta$, a CLP, regardless of its size, always yields a lower profit for the firm than no-reward programs; that is, $\pi^*(n) < \max\{\pi_1, \pi_2\}$ for any coalition size n.

(b) The aggregate customer surplus increases when firms switch from a partial market coverage strategy with price v_H to forming a CLP, while the aggregate customer surplus decreases when firms switch from a full market coverage strategy with price v_L to forming a CLP.

Proof of Lemma A4

Part (a): The profit $\pi^*(n)$ in (A9) consists of the profit collected from frequent customers and HI customers. Note that the effective price paid by HF and LF customers is smaller than v_L, while the effective price paid by HI customers cannot exceed v_H. This leads to an upper bound on the per-firm profit, $\pi^*(n) < \beta\lambda_F v_L + (\alpha - \gamma)\lambda_I v_H$, which is the same as that in the base model. Similar to the proof of Proposition 2.2(a), one can show that $\pi^*(n) < \max\{\pi_1, \pi_2\}$ when $\gamma > \alpha\beta$.

Part (b): Table A2 presents customer welfare in each segment under CLPs, and compares them with that under the full market coverage strategy with price v_L.

	CLP ($\frac{v_H}{v_L} \geq \frac{\lambda_F}{\lambda_I}$)	vs.	Full Market Coverage	vs.	CLP ($\frac{v_H}{v_L} < \frac{\lambda_F}{\lambda_I}$)
HF	$v_H - \frac{\mu(\delta+n\lambda_F+\mu)}{(n\lambda_F+\mu)(\delta+\mu)}v_L$	$>$	$v_H - v_L$	$<$	$(v_H - v_L)\Big[1 + \frac{\delta+n\lambda_I+\mu}{n\lambda_F+\mu}\frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F-\lambda_I)}\Big]$
LF	$v_L - \frac{\mu(\delta+n\lambda_F+\mu)}{(n\lambda_F+\mu)(\delta+\mu)}v_L$	$>$	0	$<$	$(v_H - v_L)\frac{\delta+n\lambda_I+\mu}{n\lambda_F+\mu}\frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F-\lambda_I)}$
HI	$v_H - \frac{\mu(\delta+n\lambda_F+\mu)}{(n\lambda_I+\mu)(\delta+\mu)}v_L$	$<$	$v_H - v_L$	$>$	$(v_H - v_L)\Big[1 - \frac{\delta+n\lambda_I+\mu}{n\lambda_I+\mu}\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{(\delta+\mu)(\lambda_F-\lambda_I)}\Big]$
LI	0	$=$	0	$=$	0

Table A2: Customer Welfare Comparison in Each Customer Segment Between CLP and Full Market Coverage when A New Reward is Earned upon Redemption

When the firm implements the partial market coverage strategy with price v_H, the aggregate customer surplus is 0. In contrast, under CLPs, the surplus for each purchasing segment is nonnegative, since customers would otherwise choose not to buy. As shown in Table A2, the aggregate customer surplus in CLPs is strictly positive. This means that CLPs generate a greater aggregate customer surplus compared to the partial market coverage strategy with price v_H. Next, we compare the aggregate customer surplus under CLPs with that under the full market coverage strategy with price v_L, considering the following two cases.

<u>Case 1:</u> Suppose $\frac{v_H}{v_L} \geq \frac{\lambda_F}{\lambda_I}$.

By Table A2, the aggregate customer surplus associated with CLPs in a VD market minus that associated with the full market coverage strategy with price v_L equals

$$\begin{aligned}
&\gamma\lambda_F\Big\{v_H - \frac{\mu(\delta+n\lambda_F+\mu)}{(n\lambda_F+\mu)(\delta+\mu)}v_L - (v_H - v_L)\Big\} + (\beta-\gamma)\lambda_F\Big\{v_L - \frac{\mu(\delta+n\lambda_F+\mu)}{(n\lambda_F+\mu)(\delta+\mu)}v_L\Big\} \\
&\quad + (\alpha-\gamma)\lambda_I\Big\{v_H - \frac{\mu(\delta+n\lambda_F+\mu)}{(n\lambda_I+\mu)(\delta+\mu)}v_L - (v_H - v_L)\Big\} \\
=&\beta\lambda_F\Big\{v_L - \frac{\mu(\delta+n\lambda_F+\mu)}{(n\lambda_F+\mu)(\delta+\mu)}v_L\Big\} + (\alpha-\gamma)\lambda_I\Big\{v_L - \frac{\mu(\delta+n\lambda_F+\mu)}{(n\lambda_I+\mu)(\delta+\mu)}v_L\Big\} \\
=&\frac{nv_L}{\delta+\mu}\Big\{\beta\lambda_F\frac{\lambda_F\delta}{n\lambda_F+\mu} - (\alpha-\gamma)\lambda_I\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{n\lambda_I+\mu}\Big\} \\
<&0,
\end{aligned}$$

where the last inequality holds due to condition (A7).

Case 2: Suppose $\frac{v_H}{v_L} < \frac{\lambda_F}{\lambda_I}$.

By Table A2, the aggregate customer surplus associated with CLPs in an ID market minus that associated with the full market coverage strategy with price v_L equals

$$\begin{aligned}
&\gamma\lambda_F\Big\{(v_H - v_L)\Big[1 + \frac{\delta + n\lambda_I + \mu}{n\lambda_F + \mu}\frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F - \lambda_I)}\Big] - (v_H - v_L)\Big\} \\
&\quad + (\beta - \gamma)\lambda_F\Big\{(v_H - v_L)\frac{\delta + n\lambda_I + \mu}{n\lambda_F + \mu}\frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F - \lambda_I)}\Big\} \\
&\quad + (\alpha - \gamma)\lambda_I\Big\{(v_H - v_L)\Big[1 - \frac{\delta + n\lambda_I + \mu}{n\lambda_I + \mu}\frac{\lambda_F\mu - \lambda_I(\delta+\mu)}{(\delta+\mu)(\lambda_F - \lambda_I)}\Big] - (v_H - v_L)\Big\} \\
=&\beta\lambda_F\Big\{(v_H - v_L)\frac{\delta + n\lambda_I + \mu}{n\lambda_F + \mu}\frac{\lambda_F\delta}{(\delta+\mu)(\lambda_F - \lambda_I)}\Big\} - (\alpha-\gamma)\lambda_I\Big\{(v_H - v_L)\frac{\delta + n\lambda_I + \mu}{n\lambda_I + \mu}\frac{\lambda_F\mu - \lambda_I(\delta+\mu)}{(\delta+\mu)(\lambda_F - \lambda_I)}\Big\} \\
=&(v_H - v_L)\frac{\delta + n\lambda_I + \mu}{(\delta+\mu)(\lambda_F - \lambda_I)}\Big\{\beta\lambda_F\frac{\lambda_F\delta}{n\lambda_F + \mu} - (\alpha-\gamma)\lambda_I\frac{\lambda_F\mu - \lambda_I(\delta+\mu)}{n\lambda_I + \mu}\Big\} \\
<&0,
\end{aligned}$$

where the last inequality holds due to condition (A7). This completes the proof.

Proof of Proposition 2.4

Similar to the condition (A7) required for the adoption of a CLP, if

$$(\alpha - \gamma)\lambda_I\big(\lambda_F\mu - \lambda_I(\delta+\mu)\big)(\lambda_F + \mu) > \beta\lambda_F^2\delta(\lambda_I + \mu) \tag{A11}$$

does not hold, then adopting a PLP is no more profitable than offering no-reward programs.

Part (a): In a VD market where $\frac{v_H}{v_L} \geq \frac{\lambda_F}{\lambda_I}$. Recall from Part (a) in Lemma A3 that

$$\pi^* = \beta\lambda_F v_L + (\alpha-\gamma)\lambda_I v_L - \beta\lambda_F\frac{n\lambda_F\delta}{n\lambda_F + \mu}\frac{v_L}{\delta+\mu} + (\alpha-\gamma)\lambda_I\frac{n[\lambda_F\mu - \lambda_I(\delta+\mu)]}{n\lambda_I + \mu}\frac{v_L}{\delta+\mu}.$$

Taking the derivative of π^* with respect to n yields

$$\begin{aligned}
\frac{d\pi^*}{dn} &= -\frac{v_L}{\delta+\mu}\Big\{\beta\lambda_F \frac{\lambda_F\delta\mu}{(n\lambda_F+\mu)^2} - (\alpha-\gamma)\lambda_I \frac{\mu[\lambda_F\mu - \lambda_I(\delta+\mu)]}{(n\lambda_I+\mu)^2}\Big\} \\
&= -\frac{v_L\mu}{\delta+\mu}\frac{\beta\lambda_F^2\delta(n\lambda_I+\mu)^2 - (\alpha-\gamma)\lambda_I[\lambda_F\mu-\lambda_I(\delta+\mu)](n\lambda_F+\mu)^2}{(n\lambda_F+\mu)^2(n\lambda_I+\mu)^2} \\
&= \frac{v_L\mu}{\delta+\mu}\frac{(\alpha-\gamma)\lambda_I[\lambda_F\mu-\lambda_I(\delta+\mu)](n\lambda_F+\mu)^2 - \beta\lambda_F^2\delta(n\lambda_I+\mu)^2}{(n\lambda_F+\mu)^2(n\lambda_I+\mu)^2} \\
&= \frac{v_L\mu}{\delta+\mu}\frac{(\alpha-\gamma)\lambda_I[\lambda_F\mu-\lambda_I(\delta+\mu)](n\lambda_F+\mu) - \beta\lambda_F^2\delta(n\lambda_I+\mu)\frac{n\lambda_I+\mu}{n\lambda_F+\mu}}{(n\lambda_F+\mu)(n\lambda_I+\mu)^2} \\
&> \frac{v_L\mu}{\delta+\mu}\frac{(\alpha-\gamma)\lambda_I[\lambda_F\mu-\lambda_I(\delta+\mu)](n\lambda_F+\mu) - \beta\lambda_F^2\delta(n\lambda_I+\mu)}{(n\lambda_F+\mu)(n\lambda_I+\mu)^2} \\
&> 0,
\end{aligned}$$

where the last inequality holds due to condition (A7). Hence, the profit π^* increases in the coalition size n in this case.

Part (b): In an ID market where $\frac{v_H}{v_L} < \frac{\lambda_F}{\lambda_I}$. When $n < \frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F - v_H\lambda_I}$, the profit is the same as that in Part (a), otherwise, recall from Part (b) in Lemma A3 that

$$\pi^* = \beta\lambda_F v_L + (\alpha-\gamma)\lambda_I v_L - \beta\lambda_F\frac{\delta+n\lambda_I+\mu}{n\lambda_F+\mu}\frac{\lambda_F\delta(v_H-v_L)}{(\delta+\mu)(\lambda_F-\lambda_I)}$$
$$+ (\alpha-\gamma)\lambda_I\frac{[\lambda_F\mu-\lambda_I(\delta+\mu)](v_H-v_L)}{(\delta+\mu)(\lambda_F-\lambda_I)}\frac{\delta+n\lambda_I+\mu}{n\lambda_I+\mu}.$$

Taking the derivative of π^* with respect to n yields

$$\begin{aligned}
\frac{d\pi^*}{dn} &= -\frac{v_H-v_L}{(\lambda_F-\lambda_I)(\delta+\mu)}\Big\{\beta\lambda_F^2\delta\frac{\lambda_I\mu-\lambda_F(\delta+\mu)}{(n\lambda_F+\mu)^2} - (\alpha-\gamma)\lambda_I[\lambda_F\mu-\lambda_I(\delta+\mu)]\frac{-\lambda_I\delta}{(n\lambda_I+\mu)^2}\Big\} \\
&= \frac{(v_H-v_L)\delta}{(\lambda_F-\lambda_I)(\delta+\mu)}\Big\{\beta\lambda_F^2\frac{\lambda_F(\delta+\mu)-\lambda_I\mu}{(n\lambda_F+\mu)^2} - (\alpha-\gamma)\lambda_I^2\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{(n\lambda_I+\mu)^2}\Big\}.
\end{aligned}$$

Let

$$\bar{n} = \frac{\Big(1-\frac{\lambda_I}{\lambda_F}\sqrt{\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]}}\Big)\mu}{\lambda_I\Big(\sqrt{\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]}}-1\Big)} = 2\tilde{n}.$$

We define the following critical threshold on size:

$$\hat{n} := \max\left\{\frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F-v_H\lambda_I}, \frac{\Big(1-\frac{\lambda_I}{\lambda_F}\sqrt{\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]}}\Big)\mu}{\lambda_I\Big(\sqrt{\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]}}-1\Big)}\right\}.$$

(i) Suppose $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \leq 1$. Then,

$$(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)] \leq \beta[\lambda_F(\delta+\mu)-\lambda_I\mu].$$

Note that

$$\frac{\lambda_I^2}{(n\lambda_I+\mu)^2} - \frac{\lambda_F^2}{(n\lambda_F+\mu)^2} = \frac{2n\lambda_F\lambda_I\mu(\lambda_I-\lambda_F)+(\lambda_I^2-\lambda_F^2)\mu^2}{(n\lambda_I+\mu)^2(n\lambda_F+\mu)^2} < 0,$$

so,

$$(\alpha-\gamma)\lambda_I^2\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{(n\lambda_I+\mu)^2} < \beta\lambda_F^2\frac{\lambda_F(\delta+\mu)-\lambda_I\mu}{(n\lambda_F+\mu)^2}.$$

Hence, $\frac{d\pi^*}{dn} > 0$ for any n. Recall from Part (a) that π^* increases in n when $\frac{v_H}{v_L} < \frac{\lambda_F}{\lambda_I}$ and $n < \frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F-v_H\lambda_I}$. Therefore, in an ID market, π^* always increases in n in this case.

(ii) Suppose $1 < \frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} < \left(\frac{\lambda_F}{\lambda_I}\right)^2$, then, by the definition of $\bar{n}$, it follows that $\bar{n}$ is positive. One can check that $\frac{d\pi^*}{dn} = 0$ when $n = \bar{n}$; $\frac{d\pi^*}{dn} > 0$ when $n < \bar{n}$; and $\frac{d\pi^*}{dn} < 0$ when $n > \bar{n}$. Therefore, the profit π^* first increases in the coalition size n when $n \leq \bar{n}$ and then decreases in n in an ID market when $n \geq \frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F-v_H\lambda_I}$. Recall from Part (a) that π^* increases in n when $n < \frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F-v_H\lambda_I}$. To summarize, in an ID market, π^* first increases when $n \leq \max\{\bar{n}, \frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F-v_H\lambda_I}\} = \hat{n}$ and then decreases in this case.

(iii) Suppose $\frac{(\alpha-\gamma)[\lambda_F\mu-\lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu)-\lambda_I\mu]} \geq \left(\frac{\lambda_F}{\lambda_I}\right)^2$. Then,

$$(\alpha-\gamma)\lambda_I^2[\lambda_F\mu-\lambda_I(\delta+\mu)] > \beta\lambda_F^2[\lambda_F(\delta+\mu)-\lambda_I\mu].$$

Note that $\frac{1}{(n\lambda_I+\mu)^2} > \frac{1}{(n\lambda_F+\mu)^2}$, so

$$(\alpha-\gamma)\lambda_I^2\frac{\lambda_F\mu-\lambda_I(\delta+\mu)}{(n\lambda_I+\mu)^2} > \beta\lambda_F^2\frac{\lambda_F(\delta+\mu)-\lambda_I\mu}{(n\lambda_F+\mu)^2}.$$

Hence, $\frac{d\pi^*}{dn} < 0$ for any n. That is, the profit π^* decreases in the coalition size n when $n \geq \frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F-v_H\lambda_I}$. Recall from Part (a) that π^* increases in n when $n < \frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F-v_H\lambda_I}$. To summarize, in an ID market, π^* first increases when $n \leq \frac{(v_H-v_L)(\delta+\mu)}{v_L\lambda_F-v_H\lambda_I}$ and then decreases in this case. Note that in this case, $\frac{(v_H - v_L)(\delta+\mu)}{v_L\lambda_F - v_H\lambda_I} > \bar{n}$, so it can be rewritten as π^* first increases when $n \le \max\{\bar{n}, \frac{(v_H - v_L)(\delta+\mu)}{v_L\lambda_F - v_H\lambda_I}\} = \hat{n}$ and then decreases.

Let $T = \frac{(\alpha-\gamma)[\lambda_F\mu - \lambda_I(\delta+\mu)]}{\beta[\lambda_F(\delta+\mu) - \lambda_I\mu]}$. In short, if $T \le 1$, the profit π^* increases monotonically in the coalition size n. Then, the optimal size n^* is as large as possible; otherwise, π^* first increases in n for $n \le \hat{n}$ and then decreases in n. Therefore, the optimal size $n^* = \lfloor \hat{n} \rfloor$ or $\lceil \hat{n} \rceil$, where $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ are the floor and ceiling functions, respectively. This completes the proof.

A.3 Asymmetric Firms

A generic customer visits firms 1 and 2 with shopping intensities $n_1\lambda$ and $n_2\lambda$, where $n_1 \ne n_2$ are positive integers, respectively. Since firms 1 and 2 are independent, the total shopping intensity for the two firms in the coalition is thus $n_1\lambda + n_2\lambda$. Hence, the optimality equations can be written as

$$u(1) = \frac{n_1\lambda + n_2\lambda}{\delta + n_1\lambda + n_2\lambda + \mu} \max\Big\{v - p + r + u(0), u(1)\Big\} + \frac{\mu}{\delta + n_1\lambda + n_2\lambda + \mu} u(0), \quad \text{(A12)}$$

$$u(0) = \frac{n_1\lambda + n_2\lambda}{\delta + n_1\lambda + n_2\lambda + \mu} \max\Big\{v - p + u(1), u(0)\Big\} + \frac{\mu}{\delta + n_1\lambda + n_2\lambda + \mu} u(0). \quad \text{(A13)}$$

By following the same approach as in the base model, we can derive explicit solutions to the optimality equations above. Specifically, if

$$v - p + \frac{(n_1+n_2)\lambda}{\delta + 2(n_1+n_2)\lambda + \mu} r \ge 0, \quad \text{(A14)}$$

then it is optimal for a customer to make a purchase whenever she visits a firm in the coalition. In this case, the optimality equations (A12)–(A13) have the following solutions:

$$u(1) = \frac{(n_1+n_2)\lambda}{\delta}\Big\{v - p + \frac{\delta + (n_1+n_2)\lambda}{\delta + 2(n_1+n_2)\lambda + \mu} r\Big\},$$

$$u(0) = \frac{(n_1+n_2)\lambda}{\delta}\Big\{v - p + \frac{(n_1+n_2)\lambda}{\delta + 2(n_1+n_2)\lambda + \mu} r\Big\}.$$

Given the customer's purchasing behavior, we can now determine the optimal price, the optimal reward amount, and the coalition's total profit as follows. Let π_1^A and π_2^A denote the respective optimal profits of firm 1 and firm 2. Given any exogenous expiration rate μ, offering a CLP makes the two asymmetric firms better off only if

$$(\alpha-\gamma)\lambda_I\Big(\mu\lambda_F-\lambda_I(\delta+\mu)\Big)(2(n_1+n_2)\lambda_F+\mu)>\beta\lambda_F^2\delta(2(n_1+n_2)\lambda_I+\mu),$$

under which the optimal price, reward amount, and the coalition's profit are

$$p^*=v_L+\frac{(n_1+n_2)\lambda_F}{\delta+2(n_1+n_2)\lambda_F+\mu}r^*, \tag{A15}$$

$$r^*=\min\Big\{\frac{(v_H-v_L)(\delta+2(n_1+n_2)\lambda_F+\mu)(\delta+2(n_1+n_2)\lambda_I+\mu)}{(n_1+n_2)(\lambda_F-\lambda_I)(\delta+\mu)},\frac{(\delta+\mu+2(n_1+n_2)\lambda_F)}{\delta+\mu+(n_1+n_2)\lambda_F}v_L\Big\}, \tag{A16}$$

$$\begin{aligned}\pi_1^A+\pi_2^A=&\beta(n_1+n_2)\lambda_F\Big(v_L+\frac{(n_1+n_2)\lambda_F}{\delta+2(n_1+n_2)\lambda_F+\mu}r^*-\frac{(n_1+n_2)\lambda_F}{2(n_1+n_2)\lambda_F+\mu}r^*\Big)\\&+(\alpha-\gamma)(n_1+n_2)\lambda_I\Big(v_L+\frac{(n_1+n_2)\lambda_F}{\delta+2(n_1+n_2)\lambda_F+\mu}r^*-\frac{(n_1+n_2)\lambda_I}{2(n_1+n_2)\lambda_I+\mu}r^*\Big).\end{aligned}$$

Otherwise, a CLP should not be offered.

According to Lemma 2.1, for a CLP consisting of n_1+n_2 symmetric firms, the price and reward are the same as those stated in (A15) and (A16), and the per-firm profit is

$$\begin{aligned}\pi^*(n_1+n_2)=&\beta\lambda_F\Big(v_L+\frac{(n_1+n_2)\lambda_F}{\delta+2(n_1+n_2)\lambda_F+\mu}r^*-\frac{(n_1+n_2)\lambda_F}{2(n_1+n_2)\lambda_F+\mu}r^*\Big)\\&+(\alpha-\gamma)\lambda_I\Big(v_L+\frac{(n_1+n_2)\lambda_F}{\delta+2(n_1+n_2)\lambda_F+\mu}r^*-\frac{(n_1+n_2)\lambda_I}{2(n_1+n_2)\lambda_I+\mu}r^*\Big).\end{aligned}$$

Clearly, $\pi^*(n_1+n_2)=\frac{\pi_1^A+\pi_2^A}{n_1+n_2}$. This completes the proof.

A.4 No Customer Discounting

It is important to note that the solution for $h(\cdot)$ is not unique. By normalizing with $h(0)=0$, we can solve the customer's dynamic programming equations (2.9)–(2.10) to obtain explicit expressions for the optimal average customer surplus g^* and the bias function $h(1)$. The results are summarized as follows. If

$$v - p + \frac{n\lambda}{2n\lambda + \mu} r \geq 0,$$

then it is optimal for a (v, λ)-customer to make a purchase whenever she visits a firm in the coalition, and a solution to the optimality equations (2.9)–(2.10) is given by

$$g^* = \frac{n\lambda}{n\lambda + \mu}(v - p + \frac{n\lambda}{2n\lambda + \mu} r), \quad h(0) = 0, \quad h(1) = \frac{n\lambda}{2n\lambda + \mu} r.$$

Proof of Proposition 2.6

To determine the per-firm profit in the CLP across the four customer segments, we examine how the price p compares to $v_H + \frac{n\lambda_F}{2n\lambda_F+\mu} r$, $v_H + \frac{n\lambda_I}{2n\lambda_I+\mu} r$, $v_L + \frac{n\lambda_F}{2n\lambda_F+\mu} r$, and $v_L + \frac{n\lambda_I}{2n\lambda_I+\mu} r$, which allows us to characterize customer purchasing behavior in each segment. This analysis closely parallels the proof of Lemma 2.1. Therefore, to avoid repetition, we omit the analysis of the non-optimal cases (one can verify that the profits in the non-optimal cases are dominated by those with no-reward programs). Below, we just present the details of the optimal case.

Suppose

$$v_L + \frac{n\lambda_F}{2n\lambda_F + \mu} r \leq v_H + \frac{n\lambda_I}{2n\lambda_I + \mu} r.$$

It follows that $p^* = v_L + \frac{n\lambda_F}{2n\lambda_F+\mu} r$.
Reorganizing the supposition gives $r \leq \frac{(v_H - v_L)(n\lambda_F + \mu)(n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)}$. Moreover, $p \geq r$ yields $r \leq \frac{n\lambda_F+\mu}{\mu} v_L$. Hence,

$$r \leq \min\left\{\frac{(v_H - v_L)(2n\lambda_F + \mu)(2n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)}, \frac{2n\lambda_F + \mu}{n\lambda_F + \mu} v_L\right\}. \tag{A17}$$

The profit in this case becomes

$$\beta\lambda_F v_L + (\alpha - \gamma)\lambda_I\Big[v_L + (\frac{n\lambda_F}{2n\lambda_F + \mu} - \frac{n\lambda_I}{2n\lambda_I + \mu})r\Big].$$

Note that

$$\frac{n\lambda_F}{2n\lambda_F+\mu}-\frac{n\lambda_I}{2n\lambda_I+\mu}>0,$$

so, by inequality (A17), we have

$$r^*=\min\Big\{\frac{(v_H-v_L)(2n\lambda_F+\mu)(2n\lambda_I+\mu)}{n\mu(\lambda_F-\lambda_I)},\frac{2n\lambda_F+\mu}{n\lambda_F+\mu}v_L\Big\}.$$

Plugging r^* into the profit function yields

$$\pi^*=\beta\lambda_F v_L+(\alpha-\gamma)\lambda_I\min\Big\{v_H,\frac{(2n\lambda_F+\mu)(n\lambda_I+\mu)}{(2n\lambda_I+\mu)(n\lambda_F+\mu)}v_L\Big\}.$$

(1) When $\frac{(n\lambda_I+\mu)(2n\lambda_F+\mu)}{(n\lambda_F+\mu)(2n\lambda_I+\mu)}<\frac{v_H}{v_L}$ holds,

$$\frac{2n\lambda_F+\mu}{n\lambda_F+\mu}v_L<\frac{(v_H-v_L)(2n\lambda_F+\mu)(2n\lambda_I+\mu)}{n\mu(\lambda_F-\lambda_I)}.$$

Hence,

$$\begin{aligned}
r_a^*&=\frac{2n\lambda_F+\mu}{n\lambda_F+\mu}v_L,\\
p_a^*&=v_L+\frac{n\lambda_F}{2n\lambda_F+\mu}\frac{2n\lambda_F+\mu}{n\lambda_F+\mu}v_L=\frac{2n\lambda_F+\mu}{n\lambda_F+\mu}v_L=r_a^*,\\
\pi_a^*&=\beta\lambda_F\Big[v_L+\big(\frac{n\lambda_F}{2n\lambda_F+\mu}-\frac{n\lambda_F}{2n\lambda_F+\mu}\big)\frac{2n\lambda_F+\mu}{n\lambda_F+\mu}v_L\Big]\\
&\quad+(\alpha-\gamma)\lambda_I\Big[v_L+\big(\frac{n\lambda_F}{2n\lambda_F+\mu}-\frac{n\lambda_I}{2n\lambda_I+\mu}\big)\frac{2n\lambda_F+\mu}{n\lambda_F+\mu}v_L\Big]\\
&=\beta\lambda_F v_L+(\alpha-\gamma)\lambda_I v_L\frac{(2n\lambda_F+\mu)(n\lambda_I+\mu)}{(2n\lambda_I+\mu)(n\lambda_F+\mu)}.
\end{aligned}$$

Taking the derivative of π_a^* with respect to n yields

$$\begin{aligned}
\frac{d\pi_a^*}{dn}&=(\alpha-\gamma)\lambda_I v_L\frac{[(2n\lambda_F+\mu)(n\lambda_I+\mu)]\prime(2n\lambda_I+\mu)(n\lambda_F+\mu)-[(2n\lambda_I+\mu)(n\lambda_F+\mu)]\prime(2n\lambda_F+\mu)(n\lambda_I+\mu)}{(2n\lambda_I+\mu)^2(n\lambda_F+\mu)^2}\\
&=(\alpha-\gamma)\lambda_I v_L\frac{[(4n\lambda_F\lambda_I+\mu(2\lambda_F+\lambda_I)](2n\lambda_I+\mu)(n\lambda_F+\mu)-[(4n\lambda_F\lambda_I+\mu(\lambda_F+2\lambda_I)](2n\lambda_F+\mu)(n\lambda_I+\mu)}{(2n\lambda_I+\mu)^2(n\lambda_F+\mu)^2}\\
&=(\alpha-\gamma)\lambda_I v_L\frac{\mu(\lambda_F-\lambda_I)(\mu^2-2\lambda_F\lambda_I n^2)}{(2n\lambda_I+\mu)^2(n\lambda_F+\mu)^2}.
\end{aligned}$$

Then, $\frac{d\pi_a^*}{dn} > 0$ if $n < \frac{\mu}{\sqrt{2\lambda_F \lambda_I}}$; and $\frac{d\pi_a^*}{dn} < 0$ if $n > \frac{\mu}{\sqrt{2\lambda_F \lambda_I}}$. Therefore, the profit π_a^* first increases in the coalition size n when $n < \frac{\mu}{\sqrt{2\lambda_F \lambda_I}}$ and then decreases in n.

(2) When $\frac{(n\lambda_I+\mu)(2n\lambda_F+\mu)}{(n\lambda_F+\mu)(2n\lambda_I+\mu)} \geq \frac{v_H}{v_L}$ hold,

$$\frac{2n\lambda_F + \mu}{n\lambda_F + \mu} v_L \geq \frac{(v_H - v_L)(2n\lambda_F + \mu)(2n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)}.$$

Hence,

$$r_b^* = \frac{(v_H - v_L)(2n\lambda_F + \mu)(2n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)},$$

$$\begin{aligned} p_b^* &= v_L + \frac{n\lambda_F}{2n\lambda_F + \mu} \frac{(v_H - v_L)(2n\lambda_F + \mu)(2n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)} \\ &= \frac{\lambda_F(2n\lambda_I + \mu)v_H - \lambda_I(2n\lambda_F + \mu)v_L}{(\lambda_F - \lambda_I)\mu}, \end{aligned}$$

$$\begin{aligned} \pi_b^* &= \beta\lambda_F \Big[v_L + \big(\frac{n\lambda_F}{2n\lambda_F + \mu} - \frac{n\lambda_F}{2n\lambda_F + \mu}\big) \frac{(v_H - v_L)(2n\lambda_F + \mu)(2n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)}\Big] \\ &\quad + (\alpha - \gamma)\lambda_I \Big[v_L + \big(\frac{n\lambda_F}{2n\lambda_F + \mu} - \frac{n\lambda_I}{2n\lambda_I + \mu}\big) \frac{(v_H - v_L)(2n\lambda_F + \mu)(2n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)}\Big] \\ &= \beta\lambda_F v_L + (\alpha - \gamma)\lambda_I v_H. \end{aligned}$$

Clearly, the profit π_b^* remains a constant for any n.

Then, note that $\pi^* = \min\Big\{\pi_a^*, \pi_b^*\Big\}$. Let

$$\begin{aligned} \Delta_r = r_b^* - r_a^* &= \frac{(v_H - v_L)(2n\lambda_F + \mu)(2n\lambda_I + \mu)}{n\mu(\lambda_F - \lambda_I)} - \frac{2n\lambda_F + \mu}{n\lambda_F + \mu} v_L \\ &= \frac{2n\lambda_F + \mu}{n\mu(\lambda_F - \lambda_I)(n\lambda_F + \mu)} \Big[(n\lambda_F + \mu)(2n\lambda_I + \mu)v_H - (n\lambda_I + \mu)(2n\lambda_F + \mu)v_L\Big] \\ &= \frac{(2n\lambda_F + \mu)v_L}{n\mu(\lambda_F - \lambda_I)(n\lambda_F + \mu)} \Big[2\lambda_F\lambda_I(\frac{v_H}{v_L} - 1)n^2 + \mu\Big((2\lambda_I + \lambda_F)\frac{v_H}{v_L} - (2\lambda_F + \lambda_I)\Big)n \\ &\qquad + \mu^2(\frac{v_H}{v_L} - 1)\Big]. \end{aligned}$$

For the quadratic equation with respect to n

$$2\lambda_F\lambda_I(\frac{v_H}{v_L} - 1)n^2 + \mu\Big((2\lambda_I + \lambda_F)\frac{v_H}{v_L} - (2\lambda_F + \lambda_I)\Big)n + \mu^2(\frac{v_H}{v_L} - 1) = 0,$$

$A = 2\lambda_F \lambda_I (\frac{v_H}{v_L} - 1) > 0$,

$B = \mu\Big((2\lambda_I + \lambda_F)\frac{v_H}{v_L} - (2\lambda_F + \lambda_I)\Big) > 0$ if $\frac{v_H}{v_L} > \frac{2\lambda_F+\lambda_I}{2\lambda_I+\lambda_F}$; and $B < 0$ if $\frac{v_H}{v_L} < \frac{2\lambda_F+\lambda_I}{2\lambda_I+\lambda_F}$,

$C = \mu^2(\frac{v_H}{v_L} - 1) > 0$.

Then, $\Delta = B^2 - 4AC = \mu^2\Big((2\lambda_I + \lambda_F)\frac{v_H}{v_L} - (2\lambda_F + \lambda_I)\Big)^2 - 8\lambda_F\lambda_I(\frac{v_H}{v_L} - 1)^2\mu^2$.

(i) If $\Delta > 0$. Let $n_1 = \frac{-B-\sqrt{\Delta}}{2A}$ and $n_2 = \frac{-B+\sqrt{\Delta}}{2A}$. One can verify that $n_1 < 0, n_2 > 0$ cannot happen. When $n_1 > 0$, $\pi^* = \pi_a^*$ if $n < n_1$ or $n > n_2$ and $\pi^* = \pi_b^*$ if $n_1 < n < n_2$. When $n_2 < 0$, $\pi^* = \pi_b^*$ if $n < n_2$ and $\pi^* = \pi_a^*$ if $n > n_2$.

(ii) If $\Delta \leq 0$, $\Delta_r > 0$ for any n, then $\pi^* = \pi_a^*$.

In summary, if $\Delta > 0$ and $n_1 > 0$, One can verify that $n_1 < \frac{\mu}{\sqrt{2\lambda_F\lambda_I}} < n_2$. Thus, the profit first increases in the coalition size n for $n < n_1$ and remains a constant for $n_1 < n < n_2$ and then decreases in the coalition size n for $n \geq n_2$; if $\Delta > 0$ and $n_2 < 0$ or $\Delta \leq 0$, the profit first increases in the coalition size n when $n < \frac{\mu}{\sqrt{2\lambda_F\lambda_I}}$ and then decreases in n. This completes the proof.

Appendix B

Appendix for Chapter 3

B.1 Proofs of Propositions and Lemmas in the Base Model

To analyze customer decisions, we first examine the relative positions of the key points: $0, x_{RA,RB}, x_{NA,NB}$, and $\frac{1}{2}$. Note that $x_{RA,RB} = 2x_{NA,NB}$, as derived from equations(3.1) and (3.2). Given this structural relationship, the possible orderings of these points yield four mutually exclusive base scenarios. Building on these base scenarios, we incorporate additional indifferent points $x_{RA,NA}, x_{RB,NB}, x_{RA,NB}$ and $x_{RB,NA}$ to further refine the characterization of customer choices.

Consider, for example, Scenario 1, in which both $x_{RA,RB} \leq 0$ and $x_{NA,NB} \leq 0$. Under this condition, we have $U_A^R \leq U_B^R$ and $U_A^N \leq U_B^N$ for all customers in the interval $x \in [0, \frac{1}{2}]$. Thus, all customers prefer Firm B, regardless of whether they choose to recycle. To determine actual behavior, we further compare U_B^R and U_B^N, and identify the relevant indifference point $x_{RB,NB} = \frac{1}{\beta}$. This leads to two sub-scenarios:

- Scenario 1.1: If $x_{RB,NB} < \frac{1}{2}$, customers to the left of $x_{RB,NB}$ prefer recycling and purchasing from Firm B, while those to the right prefer not recycling and purchasing from Firm B.

- Scenario 1.2: If $x_{RB,NB} \geq \frac{1}{2}$, then all customers in the interval $x \in [0, \frac{1}{2}]$ prefer to recycle and purchase from Firm B.

By systematically applying this logic across all scenarios and comparing all relevant indifferent points, we distinguish fourteen sub-scenarios, as illustrated in Figure B.1. The full set of relevant constraints on the orderings of these indifference points arises from the utilities of the four customer segments, which depend on both firms' prices and the customers' sensitivity to hassle costs. In each scenario, these constraints determine the demand for each customer segment (RA, RB, NA, NB).

	Sub-scenario 1	Sub-scenario 2	Sub-scenario 3	Sub-scenario 4	Sub-scenario 5	Sub-scenario 6
Scenario 1 $x_{RA,RB} \leq 0$, $x_{NA,NB} \leq 0$,	$x_{RB,NB} < \frac{1}{2}$.	$x_{RB,NB} \geq \frac{1}{2}$.				
	$d_{RB} = 2x_{RB,NB}$, $d_{NB} = 2(\frac{1}{2} - x_{RB,NB})$.	$d_{RB} = 1$.				
Scenario 2 $0 < x_{RA,RB} < \frac{1}{2}$, $0 < x_{NA,NB} < \frac{1}{2}$,	$x_{RA,NA} < x_{NA,NB}$.	$x_{RA,NA} \geq x_{NA,NB}$, $x_{RA,NB} \leq x_{NA,NB}$.	$x_{RA,NA} \geq x_{NA,NB}$, $x_{NA,NB} < x_{RA,NB} < x_{RA,RB}$.	$x_{RA,NA} \geq x_{NA,NB}$, $x_{RA,NB} \geq x_{RA,RB}$, $x_{RB,NB} \leq x_{RA,RB}$.	$x_{RA,NA} \geq x_{NA,NB}$, $x_{RA,NB} \geq x_{RA,RB}$, $x_{RA,RB} < x_{RB,NB} < \frac{1}{2}$.	$x_{RA,NA} \geq x_{NA,NB}$, $x_{RA,NB} \geq x_{RA,RB}$, $x_{RB,NB} \geq \frac{1}{2}$.
	$d_{RA} = 2x_{RA,NA}$, $d_{NA} = 2(x_{NA,NB} - x_{RA,NA})$, $d_{NB} = 2(\frac{1}{2} - x_{NA,NB})$.	$d_{RA} = 2x_{NA,NB}$, $d_{NB} = 2(\frac{1}{2} - x_{NA,NB})$.	$d_{RA} = 2x_{RA,NB}$, $d_{NB} = 2(\frac{1}{2} - x_{RA,NB})$.	$d_{RA} = 2x_{RA,RB}$, $d_{NB} = 2(\frac{1}{2} - x_{RA,RB})$.	$d_{RA} = 2x_{RA,RB}$, $d_{RB} = 2(x_{RB,NB} - x_{RA,RB})$, $d_{NB} = 2(\frac{1}{2} - x_{RB,NB})$.	$d_{RA} = 2x_{RA,RB}$, $d_{RB} = 2(\frac{1}{2} - x_{RA,RB})$.
Scenario 3 $x_{RA,RB} \geq \frac{1}{2}$, $0 < x_{NA,NB} < \frac{1}{2}$,	$x_{RA,NA} < x_{NA,NB}$.	$x_{RA,NA} \geq x_{NA,NB}$, $x_{RA,NB} \leq x_{NA,NB}$.	$x_{RA,NA} \geq x_{NA,NB}$, $x_{NA,NB} < x_{RA,NB} < \frac{1}{2}$.	$x_{RA,NA} \geq x_{NA,NB}$, $x_{RA,NB} \geq \frac{1}{2}$.		
	$d_{RA} = 2x_{RA,NA}$, $d_{NA} = 2(x_{NA,NB} - x_{RA,NA})$, $d_{NB} = 2(\frac{1}{2} - x_{NA,NB})$.	$d_{RA} = 2x_{NA,NB}$, $d_{NB} = 2(\frac{1}{2} - x_{NA,NB})$.	$d_{RA} = 2x_{RA,NB}$, $d_{NB} = 2(\frac{1}{2} - x_{RA,NB})$.	$d_{RA} = 1$.		
Scenario 4 $x_{RA,RB} \geq \frac{1}{2}$, $x_{NA,NB} \geq \frac{1}{2}$,	$x_{RA,NA} < \frac{1}{2}$.	$x_{RA,NA} \geq \frac{1}{2}$.				
	$d_{RA} = 2x_{RA,NA}$, $d_{NA} = 2(\frac{1}{2} - x_{RA,NA})$.	$d_{RA} = 1$.				

Figure B.1: Fourteen Scenarios Based on Indifference Point Orderings

We group the same market composition and identify eight cases in Table 3.2. We verify that these cases are mutually exclusive and collectively exhaustive if they are segmented based on the price gap between p_A and p_B and the hassle sensitivity parameter β, as summarized in Table B1.

Table B1: Eight Cases Divided by the Price Gap and the Hassle Cost

(1) Region 1: $1 < \beta \leq 2$	(2) Region 2: $2 < \beta \leq 3$	(3) Region 3: $\beta > 3$
$p_A - p_B \leq 0$ (Case 1)	$p_A - p_B \leq \frac{2-\beta}{2}$ (Case 1)	$p_A - p_B \leq -\frac{1}{2}$ (Case 3)
$0 < p_A - p_B < \frac{1}{2}$ (Case 6)	$\frac{2-\beta}{2} < p_A - p_B < \frac{\beta-2}{2\beta}$ (Case 5-2)	$-\frac{1}{2} < p_A - p_B < \frac{\beta-5}{2(\beta-1)}$ (Case 7)
$p_A - p_B \geq \frac{1}{2}$ (Case 2)	$p_A - p_B = \frac{\beta-2}{2\beta}$ (Case 5-3)	$p_A - p_B = \frac{\beta-5}{2(\beta-1)}$ (Case 5-1)
-	$\frac{\beta-2}{2\beta} < p_A - p_B < \frac{1}{2}$ (Case 8)	$\frac{\beta-5}{2(\beta-1)} < p_A - p_B < \frac{\beta-2}{2\beta}$ (Case 5-2)
-	$p_A - p_B \geq \frac{1}{2}$ (Case 4)	$p_A - p_B = \frac{\beta-2}{2\beta}$ (Case 5-3)
-	-	$\frac{\beta-2}{2\beta} < p_A - p_B < \frac{1}{2}$ (Case 8)
-	-	$p_A - p_B \geq \frac{1}{2}$ (Case 4)

Proof of Proposition 3.1

Case 5: $d_{RA} = 2x_{RA,NB}, d_{NB} = 1 - 2x_{RA,NB}$

The profit maximization problems for two firms are as follows.

$$\max\ \pi_{A5}(p_A, p_B) = \frac{(2p_A - 2p_B - 3)\left(-(\beta-1)p_A - 2p_B + \beta c + c - 3\right)}{(\beta+1)^2},$$

$$\max\ \pi_{B5}(p_B, p_A) = \frac{p_B\left(2p_A + \beta - 2p_B - 2\right)}{\beta + 1}.$$

They are subject to the shared constraints:

$$0 < x_{\text{RA,RB}} < \frac{1}{2}, 0 < x_{\text{NA,NB}} < \frac{1}{2}, x_{\text{RA,NA}} \geq x_{\text{NA,NB}}, x_{\text{NA,NB}} < x_{\text{RA,NB}} < x_{\text{RA,RB}},$$

which can be simplified as:

$$p_A - p_B > 0, p_A - p_B < \frac{1}{2} - \frac{1}{\beta}, p_A - p_B > \frac{\beta - 5}{2(\beta-1)}, \beta > 2.$$

Denote the constraint functions as

$$g_1(p_A, p_B) = -(p_A - p_B) \leq 0,$$

$$g_2(p_A, p_B) = p_A - p_B - (\frac{1}{2} - \frac{1}{\beta}) \leq 0,$$

$$g_3(p_A, p_B) = \frac{\beta - 5}{2(\beta - 1)} - (p_A - p_B) \leq 0,$$

$$g_4(p_A, p_B) = -p_A \leq 0,$$

$$g_5(p_A, p_B) = -p_B \leq 0.$$

Note that $\frac{\partial^2 - \pi_{A5}}{\partial p_A^2} = \frac{4(\beta-1)}{(\beta+1)^2} > 0$ and $\frac{\partial^2 - \pi_{B6}}{\partial p_B^2} = \frac{4}{\beta+1} > 0$. $-\pi_{A5}(p_A, p_B)$ is strictly convex in p_A and $-\pi_{B6}(p_B, p_A)$ is strictly convex in p_B.

Firm A's Lagrangian is $L_A(p_A, \lambda) \triangleq -\pi_{A5}(p_A, p_B) + \sum_{i=1}^{5} \lambda_i g_i$, where $\lambda_i \geq 0$ and $i \in \{1, 5\}$.

Firm B's Lagrangian is $L_B(p_B, \eta) \triangleq -\pi_{B5}(p_B) + \sum_{i=1}^{5} \eta_i g_i$, where $\eta_i \geq 0$ and $i \in \{1, 5\}$.

We solve by writing down the Karush–Kuhn–Tucker (KKT) conditions for both firms simultaneously, and find the equilibrium point satisfying each firm's optimality under shared constraints.

- Stationarity Conditions: We take derivatives of each Lagrangian with respect to each firm's decision variables and set them to zero.

$$\begin{cases} \frac{\partial L_A}{\partial p_A} = -\frac{\partial \pi_{A5}}{\partial p_A} + \sum_{i=1}^{5} \lambda_i \frac{\partial g_i}{\partial p_A} = 0, \\ \frac{\partial L_B}{\partial p_B} = -\frac{\partial \pi_{B5}}{\partial p_B} + \sum_{i=1}^{5} \eta_i \frac{\partial g_i}{\partial p_B} = 0. \end{cases}$$

- Primal feasibility: $g_i(p_A, p_B) \leq 0, i = 1, \cdots, 5$.

- Dual feasibility: $\lambda_i \geq 0, \eta_i \geq 0, i = 1, \cdots, 5$.

- Complementary slackness: For each i: $\lambda_i g_i(p_A, p_B) = 0, \eta_i g_i(p_A, p_B) = 0$.

By inspecting all possible solutions, we find the following solutions that satisfy all KKT conditions.

(i)The interior candidate (i.e., $\lambda_i = \eta_i = 0$).

Solve $\frac{\partial \pi_{A5}}{\partial p_A} = 0, \frac{\partial \pi_{B5}}{\partial p_B} = 0$. We derive the optimal solutions:

$$p_A = \frac{\beta^2 + \beta + 4(\beta+1)c - 12}{6\beta - 2}, p_B = \frac{(\beta+1)(2\beta+2c-5)}{6\beta-2}.$$

One can check all $g_i < 0, i \in \{1, 5\}$ when $2 < \beta \le 5$ and $\frac{\beta^2-4\beta+7}{2\beta+2} < c < \frac{\beta^2-2\beta+2}{2\beta}$; or $\beta > 5$ and $\frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{\beta^2-2\beta+2}{2\beta}$.

(ii) The boundary candidates with the active constraint $g_1 = 0$.

For each possible subset $S \subset \{1, \cdots, 5\}$ of active constraints, set

$$g_i = 0, i \in S; \lambda_i > 0 \quad \text{and /or} \quad \eta_i > 0, i \in S; \lambda_i = \eta_i = 0, i \notin S,$$

and resolve the reduced system of stationarity plus those equalities.

Solve $g_1 = 0$ with stationarity conditions. We derive the optimal solutions:

$$p_A = \frac{\beta-2}{2}, p_B = \frac{\beta-2}{2}.$$

One can check that the solution satisfies all KKT conditions when $2 < \beta < 5$ and $c < \frac{\beta^2-4\beta+7}{2\beta+2}$. In particular, for primal feasibility, $g_1 = 0$ and all other constraints are satisfied with slack (i.e., $g_i < 0, i = 2, 4, 5$) at the interior of the admissible region. For dual feasibility, $\lambda_2 = \frac{2(\beta+1)p_A - 3\beta - 2(\beta+1)c + 9}{(\beta+1)^2} > 0$ and all other $\lambda_i = 0, \eta_i = 0$. For complementary slackness, all $\lambda_i g_i = 0$ and $\eta_i g_i = 0$.

(iii) The boundary candidates with the active constraint $g_2 = 0$.

Solve $g_2 = 0$ with stationarity conditions. We derive the optimal solutions:

$$p_A = -\frac{3}{\beta} + c + 1, p_B = -\frac{2}{\beta} + c + \frac{1}{2}.$$

One can check that the solution satisfies all KKT conditions when $\beta > 2$ and $c > \frac{\beta^2-2\beta+2}{2\beta}$.

(iv) The boundary candidates with the active constraint $g_3 = 0$.

Solve $g_3 = 0$ with stationarity conditions. We derive the optimal solutions:

$$p_A = \frac{-\beta^2+\beta+8}{2-2\beta}, p_B = \frac{-\beta^2+2\beta+3}{2-2\beta}.$$

One can check that the solution satisfies all KKT conditions when $\beta > 5$ and $c < \frac{\beta^2-3\beta-2}{2\beta-2}$.

Therefore, the optimal equilibrium results in Case 5 are

(1) when (i) $2 < \beta \leq 5$ and $\frac{\beta^2-4\beta+7}{2\beta+2} < c < \frac{\beta^2-2\beta+2}{2\beta}$ or (ii) $\beta > 5$ and $\frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{\beta^2-2\beta+2}{2\beta}$, the optimal prices and firm profits are

$$p_A^* = \frac{\beta^2+\beta+4(\beta+1)c-12}{6\beta-2}, p_B^* = \frac{(\beta+1)(2\beta+2c-5)}{6\beta-2},$$

$$\pi_{A5}^* = \frac{(\beta-1)(\beta-2c+4)^2}{2(1-3\beta)^2}, \pi_{B5}^* = \frac{(\beta+1)(2\beta+2c-5)^2}{2(1-3\beta)^2};$$

(2) when $2 < \beta < 5$ and $c < \frac{\beta^2-4\beta+7}{2\beta+2}$, the optimal prices and firm profits are

$$p_A^* = \frac{\beta-2}{2}, p_B^* = \frac{\beta-2}{2},$$

$$\pi_{A5}^* = \frac{3\left(\beta^2-\beta-2(\beta+1)c+4\right)}{2(\beta+1)^2}, \pi_{B5}^* = \frac{(\beta-2)^2}{2(\beta+1)};$$

(3) when $\beta > 2$ and $c > \frac{\beta^2-2\beta+2}{2\beta}$, the optimal prices and firm profits are

$$p_A^* = -\frac{3}{\beta}+c+1, p_B^* = -\frac{2}{\beta}+c+\frac{1}{2},$$

$$\pi_{A5}^* = \frac{2(\beta-1)}{\beta^2}, \pi_{B5}^* = \frac{(\beta-2)(\beta+2\beta c-4)}{2\beta^2}.$$

(4) when $\beta > 5$ and $c < \frac{\beta^2-3\beta-2}{2\beta-2}$, the optimal prices and firm profits are

$$p_A^* = \frac{-\beta^2+\beta+8}{2-2\beta}, p_B^* = \frac{-\beta^2+2\beta+3}{2-2\beta},$$

$$\pi_{A5}^* = \frac{\beta^2-\beta-2(\beta-1)c-4}{(\beta-1)^2}, \pi_{B5}^* = \frac{(\beta-3)^2(\beta+1)}{2(\beta-1)^2}.$$

This completes the proof.

Following the same logic as in the above-mentioned illustrative example, we determine the optimal prices and corresponding profits across eight cases as follows.

<u>Case 1</u>: $d_{RA} = 1$

The profit maximization problems for two firms are given by

$$\max\ \pi_{A1}(p_A, p_B) = p_A - c + 1,$$

$$\max\ \pi_{B1}(p_B, p_A) = 0,$$

subject to the following shared constraints:

$$\begin{cases} p_A - p_B \leq 0, & \text{if } 1 < \beta \leq 2, \\ p_A - p_B \leq \dfrac{2-\beta}{2}, & \text{if } 2 < \beta \leq 3. \end{cases}$$

(1.1) When $1 < \beta \leq 2$, Firm A's best response is to choose $p_A = p_B$. Firm B has an incentive to limit the competitor's profits and choose $p_B = 0$;

(1.2) When $2 < \beta \leq 3$, Firm A's best response is to choose $p_A = p_B + \frac{2-\beta}{2}$. Firm B has an incentive to limit the competitor's profits and choose $p_B = \frac{\beta-2}{2}$.

Therefore, the optimal equilibrium results in Case 1 are:

(1.1) when $1 < \beta \leq 2$, the optimal prices and firm profits are

$$p_A^* = 0, p_B^* = 0,$$

$$\pi_{A1}^* = 1 - c, \pi_{B1}^* = 0.$$

(1.2) when $2 < \beta \leq 3$, the optimal prices and firm profits are

$$p_A^* = 0, p_B^* = \frac{\beta - 2}{2},$$

$$\pi_{A1}^* = 1 - c, \pi_{B1}^* = 0.$$

Case 2: $d_{RB} = 1$

The profit maximization problems for two firms are given by

$$\max \ \pi_{A2}(p_A, p_B) = 1 - c,$$

$$\max \ \pi_{B2}(p_B, p_A) = p_B,$$

subject to the following shared constraints:

$$p_A - p_B \geq \frac{1}{2}, \text{if } 1 < \beta \leq 2.$$

Firm B's best response is to choose $p_B = p_A - \frac{1}{2}$. Firm A has an incentive to limit the competitor's profits and choose $p_A = \frac{1}{2}$. Therefore, the optimal equilibrium results in Case 2 are:

(2.1) when $1 < \beta \leq 2$, the optimal prices and firm profits are

$$p_A^* = \frac{1}{2}, p_B^* = 0,$$

$$\pi_{A2}^* = 1 - c, \pi_{B2}^* = 0.$$

Case 3: $d_{RA} = 2x_{RA,NA}, d_{NA} = 1 - 2x_{RA,NA}$

The profit maximization problems for two firms are given by

$$\max \ \pi_{A3}(p_A, p_B) = p_A + \frac{4 - 2(\beta - 1)c}{(\beta - 1)^2},$$

$$\max \ \pi_{B3}(p_B, p_A) = 0,$$

subject to the following shared constraints:

$$p_A - p_B \leq -\frac{1}{2}, \text{if } \beta > 3.$$

Firm A's best response is to choose $p_A = p_B - \frac{1}{2}$. Firm B has an incentive to limit the competitor's profits and choose $p_B = \frac{1}{2}$. Therefore, the optimal equilibrium results in Case 3 are

(3.1) when $\beta > 3$, the optimal prices and firm profits are

$$p_A^* = 0, p_B^* = \frac{1}{2},$$

$$\pi_{A3}^* = \frac{4 - 2(\beta - 1)c}{(\beta - 1)^2}, \pi_{B3}^* = 0.$$

<u>Case 4</u>: $d_{RB} = 2x_{RB,NB}, d_{NB} = 1 - 2x_{RB,NB}$

The profit maximization problems for two firms are as follows.

$$\max \ \pi_{A4}(p_A, p_B) = \frac{4 - 2\beta c}{\beta^2},$$

$$\max \ \pi_{B4}(p_B, p_A) = p_B,$$

subject to the following shared constraints:

$$p_A - p_B \geq \frac{1}{2}, \text{if } \beta > 2.$$

Firm B's best response is to choose $p_B = p_A - \frac{1}{2}$. Firm A has an incentive to limit the competitor's profits and choose $p_A = \frac{1}{2}$. Therefore, the optimal equilibrium results in Case 4 are

(4.1) when $\beta > 2$, the optimal prices and firm profits are

$$p_A^* = \frac{1}{2}, p_B^* = 0,$$

$$\pi_{A4}^* = \frac{4 - 2\beta c}{\beta^2}, \pi_{B4}^* = 0.$$

Case 5-1: $d_{RA} = 2x_{NA,NB}, d_{NB} = 1 - 2x_{NA,NB}$

The profit maximization problems for two firms are as follows.

$$\max\ \pi_{A5-1}(p_A, p_B) = \frac{1}{4}\left(2p_A - 2p_B - 1\right)\left(-2p_B + 2c - 1\right),$$
$$\max\ \pi_{B5-1}(p_B, p_A) = \frac{1}{2}p_B\left(2p_A - 2p_B + 1\right),$$

subject to the following shared constraints:

$$p_A - p_B = \frac{\beta - 5}{2\beta - 2}, \text{if } \beta > 3.$$

Then, for each firm, we now solve the one-dimensional constrained problem with the binding constraint enforced by substitution $p_A - p_B = \frac{\beta-5}{2\beta-2}$.

For Firm A, solving $\frac{\partial \pi_{A10}}{\partial p_A} = 0$ and $p_B = p_A - \frac{\beta-5}{2\beta-2}$ derives $p_A = c - \frac{2}{\beta-1}, p_B = c - \frac{1}{2}$. Thus, $\pi_{\text{A5-1}} = 0, \pi_{\text{B5-1}} = \frac{(\beta-3)(2c-1)}{2(\beta-1)}$.

For Firm B, solving $\frac{\partial \pi_{B10}}{\partial p_B} = 0$ and $p_A = p_B + \frac{\beta-5}{2\beta-2}$ derives $p_A = \frac{11-3\beta}{2-2\beta}, p_B = \frac{\beta-3}{\beta-1}$. Thus, $\pi_{\text{A5-1}} = \frac{3\beta-2(\beta-1)c-7}{(\beta-1)^2}, \pi_{\text{B5-1}} = \frac{(\beta-3)^2}{(\beta-1)^2}$.

Therefore, the optimal equilibrium results in Case 5-1 are

(5.1.1) when $\beta > 3$ and $c < \frac{3\beta-7}{2\beta-2}$, the optimal prices and firm profits are

$$p_A^* = \frac{11 - 3\beta}{2 - 2\beta}, p_B^* = \frac{\beta - 3}{\beta - 1},$$
$$\pi_{A5-1}^* = \frac{3\beta - 2(\beta - 1)c - 7}{(\beta - 1)^2}, \pi_{B5-1}^* = \frac{(\beta - 3)^2}{(\beta - 1)^2}.$$

(5.1.2) when $\beta > 3$ and $c > \frac{3\beta-7}{2\beta-2}$, the optimal prices and firm profits are

$$p_A^* = c - \frac{2}{\beta - 1}, p_B^* = c - \frac{1}{2},$$
$$\pi_{A5-1}^* = 0, \pi_{B5-1}^* = \frac{(\beta - 3)(2c - 1)}{2(\beta - 1)}.$$

Case 5-2: $d_{RA} = 2x_{RA,NB}, d_{NB} = 1 - 2x_{RA,NB}$

The profit maximization problems for two firms are given by

$$\max\ \pi_{A5-2}(p_A, p_B) = \frac{(2p_A - 2p_B - 3)\left(-(\beta-1)p_A - 2p_B + \beta c + c - 3\right)}{(\beta+1)^2},$$

$$\max\ \pi_{B5-2}(p_B, p_A) = \frac{p_B\left(2p_A + \beta - 2p_B - 2\right)}{\beta+1},$$

subject to the following shared constraints:

$$\begin{cases} \frac{2-\beta}{2} < p_A - p_B < \frac{\beta-2}{2\beta}, & \text{if } 2 < \beta \leq 3, \\ \frac{\beta-5}{2\beta-2} < p_A - p_B < \frac{\beta-2}{2\beta}, & \text{if } \beta > 3. \end{cases}$$

(5.2.1) When $2 < \beta \leq 3$:

Denote the constraint functions as

$$g_1(p_A, p_B) = \frac{2-\beta}{2} - (p_A - p_B) \leq 0,$$

$$g_2(p_A, p_B) = p_A - p_B - \frac{\beta-2}{2\beta} \leq 0,$$

$$g_3(p_A, p_B) = -p_A \leq 0,$$

$$g_4(p_A, p_B) = -p_B \leq 0.$$

Note that $\frac{\partial^2 -\pi_{A5-2}}{\partial p_A^2} = \frac{4(\beta-1)}{(\beta+1)^2} > 0$ and $\frac{\partial^2 -\pi_{B5-2}}{\partial p_B^2} = \frac{4}{\beta+1} > 0$. $-\pi_{A5-2}(p_A, p_B)$ is strictly convex in p_A and $-\pi_{B5-2}(p_B, p_A)$ is strictly convex in p_B.

Firm A's Lagrangian is $L_A(p_A, \lambda) \triangleq -\pi_{A5-2}(p_A, p_B) + \sum_{i=1}^{4} \lambda_i g_i$, where $\lambda_i \geq 0$ and $i \in \{1, 4\}$.

Firm B's Lagrangian is $L_B(p_B, \eta) \triangleq -\pi_{B5-2}(p_B) + \sum_{i=1}^{4} \eta_i g_i$, where $\eta_i \geq 0$ and $i \in \{1, 4\}$. We solve by writing down the Karush–Kuhn–Tucker (KKT) conditions for both firms simultaneously, and find the equilibrium point satisfying each firm's optimality under shared constraints.

- Stationarity Conditions: We take derivatives of each Lagrangian with respect to each firm's decision variables and set them to zero.

$$\begin{cases} \frac{\partial L_A}{\partial p_A} = -\frac{\partial \pi_{A5-2}}{\partial p_A} + \sum_{i=1}^{4} \lambda_i \frac{\partial g_i}{\partial p_A} = 0, \\ \frac{\partial L_B}{\partial p_B} = -\frac{\partial \pi_{B5-2}}{\partial p_B} + \sum_{i=1}^{4} \eta_i \frac{\partial g_i}{\partial p_B} = 0. \end{cases}$$

- Primal feasibility: $g_i(p_A, p_B) \leq 0, i = 1, \cdots, 4$.

- Dual feasibility: $\lambda_i \geq 0, \eta_i \geq 0, i = 1, \cdots, 4$.

- Complementary slackness: For each i: $\lambda_i g_i(p_A, p_B) = 0, \eta_i g_i(p_A, p_B) = 0$.

By inspecting all possible solutions, we find the following solutions that satisfy all KKT conditions.

(i) The interior candidate (i.e., $\lambda_i = \eta_i = 0$).

Solve $\frac{\partial \pi_{A5-2}}{\partial p_A} = 0, \frac{\partial \pi_{B5-2}}{\partial p_B} = 0$. We derive the optimal solutions:

$$p_A = \frac{\beta^2 + \beta + 4(\beta+1)c - 12}{6\beta - 2}, p_B = \frac{(\beta+1)(2\beta + 2c - 5)}{6\beta - 2}.$$

One can check all $g_i < 0, i \in \{1, 4\}$ when $2 < \beta \leq 3$ and $\frac{-\beta^2 - \beta + 12}{4\beta + 4} < c < \frac{\beta^2 - 2\beta + 2}{2\beta}$.

(ii) The boundary candidates with the active constraint $g_2 = 0$.

Solve $g_2 = 0$ with stationarity conditions. We derive the optimal solutions: $p_A = -\frac{3}{\beta} + c + 1, p_B = -\frac{2}{\beta} + c + \frac{1}{2}$ when $2 < \beta \leq 3$ and $c > \frac{\beta^2 - 2\beta + 2}{2\beta}$.

(5.2.2) When $\beta > 3$:

Denote the constraint functions as

$$g_1(p_A, p_B) = \frac{\beta - 5}{2\beta - 2} - (p_A - p_B) \leq 0,$$

$$g_2(p_A, p_B) = p_A - p_B - \frac{\beta - 2}{2\beta},$$

$$g_3(p_A, p_B) = -p_A \leq 0,$$

$$g_4(p_A, p_B) = -p_B \leq 0.$$

By inspecting all possible solutions, we find the following solutions that satisfy all KKT conditions.

(i) The interior candidate (i.e., $\lambda_i = \eta_i = 0$).

Solve $\frac{\partial \pi_{A5-2}}{\partial p_A} = 0$, $\frac{\partial \pi_{B5-2}}{\partial p_B} = 0$. We derive the optimal solutions:

$$p_A = \frac{\beta^2 + \beta + 4(\beta + 1)c - 12}{6\beta - 2}, p_B = \frac{(\beta + 1)(2\beta + 2c - 5)}{6\beta - 2}.$$

One can check all $g_i < 0, i \in \{1, 4\}$ when $3 < \beta \leq \frac{1}{2}\left(\sqrt{33} + 1\right)$ and $\frac{-\beta^2 - \beta + 12}{4\beta + 4} < c < \frac{\beta^2 - 2\beta + 2}{2\beta}$; or $\beta > \frac{1}{2}\left(\sqrt{33} + 1\right)$ and $\frac{\beta^2 - 3\beta - 2}{2\beta - 2} < c < \frac{\beta^2 - 2\beta + 2}{2\beta}$.

(ii) The boundary candidates with the active constraint $g_1 = 0$.

We derive the optimal solutions: $p_A = \frac{-\beta^2 + \beta + 8}{2 - 2\beta}, p_B = \frac{-\beta^2 + 2\beta + 3}{2 - 2\beta}$ when $\beta > \frac{1}{2}\left(\sqrt{33} + 1\right)$ and $c < \frac{\beta^2 - 3\beta - 2}{2\beta - 2}$.

(iii) The boundary candidates with the active constraint $g_2 = 0$.

We derive the optimal solutions: $p_A = -\frac{3}{\beta} + c + 1, p_B = -\frac{2}{\beta} + c + \frac{1}{2}$ when $\beta > 3$ and $c > \frac{\beta^2 - 2\beta + 2}{2\beta}$.

Therefore, the optimal equilibrium results in Case 5-2 are

(5.2.1) when (i) $2 < \beta \leq \frac{1}{2}\left(\sqrt{33} + 1\right)$ and $\frac{-\beta^2 - \beta + 12}{4\beta + 4} < c < \frac{\beta^2 - 2\beta + 2}{2\beta}$; or (ii) $\beta >$

$\frac{1}{2}\left(\sqrt{33}+1\right)$ and $\frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{\beta^2-2\beta+2}{2\beta}$, the optimal prices and firm profits are

$$p_A^* = \frac{\beta^2+\beta+4(\beta+1)c-12}{6\beta-2}, p_B^* = \frac{(\beta+1)(2\beta+2c-5)}{6\beta-2},$$

$$\pi_{A5-2}^* = \frac{(\beta-1)(\beta-2c+4)^2}{2(1-3\beta)^2}, \pi_{B5-2}^* = \frac{(\beta+1)(2\beta+2c-5)^2}{2(1-3\beta)^2};$$

(5.2.2) when $\beta > 2$ and $c > \frac{\beta^2-2\beta+2}{2\beta}$, the optimal prices and firm profits are

$$p_A^* = -\frac{3}{\beta}+c+1, p_B^* = -\frac{2}{\beta}+c+\frac{1}{2},$$

$$\pi_{A5-2}^* = \frac{2(\beta-1)}{\beta^2}, \pi_{B5-2}^* = \frac{(\beta-2)(\beta+2\beta c-4)}{2\beta^2};$$

(5.2.3) when $\beta > \frac{1}{2}\left(\sqrt{33}+1\right)$ and $c < \frac{\beta^2-3\beta-2}{2\beta-2}$, the optimal prices and firm profits are

$$p_A^* = \frac{-\beta^2+\beta+8}{2-2\beta}, p_B^* = \frac{-\beta^2+2\beta+3}{2-2\beta},$$

$$\pi_{A5-2}^* = \frac{\beta^2-\beta-2(\beta-1)c-4}{(\beta-1)^2}, \pi_{B5-2}^* = \frac{(\beta-3)^2(\beta+1)}{2(\beta-1)^2}.$$

Case 5-3: $d_{RA} = 2x_{RA,RB}, d_{NB} = 1-2x_{RA,RB}$

The profit maximization problems for two firms are as follows.

$$\max\ \pi_{A5-3}(p_A,p_B) = (2p_A-2p_B-1)(p_A-2p_B+c-1),$$

$$\max\ \pi_{B5-3}(p_B,p_A) = -2p_B(p_B-p_A),$$

subject to the following shared constraints:

$$p_A - p_B = \frac{\beta-2}{2\beta}, \text{if } \beta > 2.$$

Then, for each firm, we now solve the one-dimensional constrained problem with the binding constraint enforced by substitution $p_A - p_B = \frac{\beta-2}{2\beta}$.

For Firm A, solving $\frac{\partial \pi_{A5-3}}{\partial p_A} = 0$ and $p_B = p_A - \frac{\beta-2}{2\beta}$ derives $p_A = c - \frac{3}{\beta}, p_B = -\frac{2}{\beta}+c-\frac{1}{2}$. Thus, $\pi_{\text{A5-3}} = -\frac{2}{\beta^2}, \pi_{\text{B5-3}} = \frac{(\beta-2)(\beta(2c-1)-4)}{2\beta^2}$.

For Firm B, solving $\frac{\partial \pi_{B5-3}}{\partial p_B} = 0$ and $p_A = p_B + \frac{\beta-2}{2\beta}$ derives $p_A = \frac{\beta-2}{\beta}, p_B = \frac{1}{2} - \frac{1}{\beta}$. Thus, $\pi_{\text{A5-3}} = \frac{2-2c}{\beta}, \pi_{\text{B5-3}} = \frac{(\beta-2)^2}{2\beta^2}$.

By comparison, $\frac{2-2c}{\beta} > -\frac{2}{\beta^2}, \frac{(\beta-2)^2}{2\beta^2} > \frac{(\beta-2)(\beta(2c-1)-4)}{2\beta^2}$ when $\beta > 2$ and $c < \frac{\beta+1}{\beta}$. Therefore, the optimal equilibrium results in Case 5-3 are

(5.3.1) when $\beta > 2$ and $c < \frac{\beta+1}{\beta}$, the optimal prices and firm profits are

$$p_A^* = \frac{\beta-2}{\beta}, p_B^* = \frac{1}{2} - \frac{1}{\beta},$$
$$\pi_{A5-3}^* = \frac{2-2c}{\beta}, \pi_{B5-3}^* = \frac{(\beta-2)^2}{2\beta^2}.$$

(5.3.2) when $\beta > 2$ and $c > \frac{\beta+1}{\beta}$, the optimal prices and firm profits are

$$p_A^* = c - \frac{3}{\beta}, p_B^* = -\frac{2}{\beta} + c - \frac{1}{2},$$
$$\pi_{A5-3}^* = -\frac{2}{\beta^2}, \pi_{B5-3}^* = \frac{(\beta-2)(\beta(2c-1)-4)}{2\beta^2}.$$

In summary, the market consists of RA and NB customer segments in Cases 5-1, 5-2, and 5-3. For each subcase, the conditions are disjoint. Next, we need to find, for each (β, c) in the overlapping regions, which subcase gives the highest profit for both firms simultaneously. We show the detailed steps in Table B2. Note that the key boundaries in c are: $c_1 = \frac{-\beta^2-\beta+12}{4\beta+4}, c_2 = \frac{\beta^2-3\beta-2}{2\beta-2}, c_3 = \frac{3\beta-7}{2\beta-2}, c_4 = \frac{\beta^2-2\beta+2}{2\beta}, c_5 = \frac{\beta+1}{\beta}$. β_{12} is the third root of the polynomial equation $x^3 - 5x^2 - 2x + 2 = 0$.

Table B2: The Feasible Sub-Cases in Each Sub-Region of Case 5

Region	Sub-region	Condition	Feasible Sub-cases
$2 < \beta \leq 3$	1.1	$c < c_1$	5.3.1
	1.2	$c_1 < c < c_4$	5.2.1, 5.3.1
	1.3	$c_4 < c < c_5$	5.2.2, 5.3.1
	1.4	$c > c_5$	5.2.2, 5.3.2
$3 < \beta \leq \frac{1}{2}\left(\sqrt{33}+1\right)$	2.1	$c < c_1$	5.1.1, 5.3.1
	2.2	$c_1 < c < c_3$	5.1.1, 5.2.1, 5.3.1
	2.3	$c_3 < c < c_4$	5.1.2, 5.2.1, 5.3.1
	2.4	$c_4 < c < c_5$	5.1.2, 5.2.2, 5.3.1
	2.5	$c > c_5$	5.1.2, 5.2.2, 5.3.2
$\frac{1}{2}\left(\sqrt{33}+1\right) < \beta \leq 4$	3.1	$c < c_2$	5.1.1, 5.2.3, 5.3.1
	3.2	$c_2 \leq c < c_3$	5.1.1, 5.2.1, 5.3.1
	3.3	$c_3 < c < c_4$	5.1.2, 5.2.1, 5.3.1
	3.4	$c_4 < c < c_5$	5.1.2, 5.2.2, 5.3.1
	3.5	$c > c_5$	5.1.2, 5.2.2, 5.3.2
$4 < \beta \leq 5$	4.1	$c < c_2$	5.1.1, 5.2.3, 5.3.1
	4.2	$c_2 \leq c < c_3$	5.1.1, 5.2.1, 5.3.1
	4.3	$c_3 < c < c_5$	5.1.2, 5.2.1, 5.3.1
	4.4	$c_5 < c < c_4$	5.1.2, 5.2.1, 5.3.2
	4.5	$c > c_4$	5.1.2, 5.2.2, 5.3.2
$5 < \beta \leq \beta_{12}$	5.1	$c < c_3$	5.1.1, 5.2.3, 5.3.1
	5.2	$c_3 \leq c < c_2$	5.1.2, 5.2.3, 5.3.1
	5.3	$c_2 \leq c \leq c_5$	5.1.2, 5.2.1, 5.3.1
	5.4	$c_5 < c < c_4$	5.1.2, 5.2.1, 5.3.2
	5.5	$c > c_4$	5.1.2, 5.2.2, 5.3.2
$\beta_{12} < \beta \leq \frac{1}{2}\left(\sqrt{41}+7\right)$	6.1	$c < c_3$	5.1.1, 5.2.3, 5.3.1
	6.2	$c_3 \leq c < c_5$	5.1.2, 5.2.3, 5.3.1
	6.3	$c_5 \leq c \leq c_2$	5.1.2, 5.2.3, 5.3.2
	6.4	$c_2 < c < c_4$	5.1.2, 5.2.1, 5.3.2
	6.5	$c > c_4$	5.1.2, 5.2.2, 5.3.2
$\beta > \frac{1}{2}\left(\sqrt{41}+7\right)$	7.1	$c < c_5$	5.1.1, 5.2.3, 5.3.1
	7.2	$c_5 \leq c < c_3$	5.1.1, 5.2.3, 5.3.2
	7.3	$c_3 \leq c \leq c_2$	5.1.2, 5.2.3, 5.3.2
	7.4	$c_2 < c < c_4$	5.1.2, 5.2.1, 5.3.2
	7.5	$c > c_4$	5.1.2, 5.2.2, 5.3.2

Lemma B1. *The optimal equilibrium results in Case 5 are as follows.*[24]

[24]Note that β_{13} is the fourth root of the polynomial equation $x^4 - 6x^3 + 5x^2 + 12x - 4 = 0$. β_{14}

(a) *Subcase 5.1.1 is optimal for both firms when (i)* $\beta_{13} < \beta \leq 5$ *and* $c < \frac{\beta^2-3\beta-2}{2\beta-2}$; *(ii)* $\beta > 5$ *and* $c < \frac{3\beta-7}{2\beta-2}$.

(b) *Subcase 5.1.2 is optimal for both firms when (i)* $3 < \beta \leq 5$ *and* $c \geq \frac{\beta^2-2\beta+2}{2\beta}$; *(ii)* $5 < \beta \leq \frac{1}{2}\left(\sqrt{41}+7\right)$ *and* $\frac{3\beta-7}{2\beta-2} \leq c < \frac{\beta^2-3\beta-2}{2\beta-2}$; *(iii)* $5 < \beta \leq \frac{1}{2}\left(\sqrt{41}+7\right)$ *and* $c \geq \frac{\beta^2-2\beta+2}{2\beta}$; *(iv)* $\frac{1}{2}\left(\sqrt{41}+7\right) < \beta < \beta_{14}$ *and* $c \geq \frac{\beta^2-2\beta+2}{2\beta}$; *(v)* $\beta \geq \beta_{14}$ *and* $\frac{\beta^2-2\beta+2}{2\beta} \leq c < \frac{5\beta^3-18\beta^2+23\beta-2}{4\beta(\beta+1)} - \frac{1}{4}\sqrt{\frac{9\beta^6-168\beta^5+514\beta^4-684\beta^3+633\beta^2-260\beta+36}{\beta^2(\beta+1)^2}}$; *(vi)* $\beta \geq \beta_{14}$ *and* $c > \frac{5\beta^3-18\beta^2+23\beta-2}{4\beta(\beta+1)} + \frac{1}{4}\sqrt{\frac{9\beta^6-168\beta^5+514\beta^4-684\beta^3+633\beta^2-260\beta+36}{\beta^2(\beta+1)^2}}$.

(c) *Subcase 5.2.1 is optimal for both firms when (i)* $2 < \beta \leq 3$ *and* $\frac{1}{2}\sqrt{\frac{9\beta^4-42\beta^3+61\beta^2-28\beta+4}{\beta^2(\beta+1)}} + \frac{1}{2}(5-2\beta) < c < \frac{\beta^2-2\beta+2}{2\beta}$; *(ii)* $3 < \beta \leq \beta_{15}$ *and* $\frac{1}{2}\sqrt{\frac{9\beta^4-42\beta^3+61\beta^2-28\beta+4}{\beta^2(\beta+1)}} + \frac{1}{2}(5-2\beta) < c < \frac{3\beta-7}{2\beta-2}$; *(iii)* $\beta_{15} < \beta < 5$ *and* $\frac{\beta^2-3\beta-2}{2\beta-2} \leq c < \frac{3\beta-7}{2\beta-2}$.

(d) *Subcase 5.3.1 is optimal for both firms when (i)* $2 < \beta \leq 3$ *and* $c < \frac{-\beta^2-\beta+12}{4\beta+4}$; *(ii)* $3 < \beta < \frac{1}{2}\left(\sqrt{33}+1\right)$ *and* $\frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{-\beta^2-\beta+12}{4\beta+4}$.

(e) *Subcase 5.3.2 is optimal for both firms when* $2 < \beta \leq 3$ *and* $c > \frac{\beta+1}{\beta}$.

(f) *A mixed strategy emerges as a solution where firms might probabilistically choose between other feasible subcases in the remaining region.*

Case 6: $d_{RA} = 2x_{RA,RB}, d_{RB} = 1 - 2x_{RA,RB}$

The profit maximization problems for two firms are given by

$$\max\ \pi_{A6}(p_A, p_B) = p_A\left(2p_B + 1\right) - 2p_A^2 - c + 1,$$

$$\max\ \pi_{B6}(p_B, p_A) = 2p_B\left(p_A - p_B\right),$$

subject to the following shared constraints:

$$0 < p_A - p_B < \frac{1}{2}, \text{if } 1 < \beta \leq 2.$$

is the first root of the polynomial equation $x^3 - 16x^2 + 13x - 18 = 0$. β_{15} is the third root of the polynomial equation $x^5 - 6x^4 + 9x^3 - 4x^2 + 12x - 4 = 0$.

Denote the constraint functions as

$$g_1(p_A, p_B) = -(p_A - p_B) \leq 0,$$

$$g_2(p_A, p_B) = p_A - p_B - \frac{1}{2} \leq 0,$$

$$g_3(p_A, p_B) = -p_A \leq 0,$$

$$g_4(p_A, p_B) = -p_B \leq 0.$$

Note that $-\pi_{A6}(p_A, p_B)$ is strictly convex in p_A and $-\pi_{B6}(p_B, p_A)$ is strictly convex in p_B.

Firm A's Lagrangian is $L_A(p_A, \lambda) \triangleq -\pi_{A6}(p_A, p_B) + \sum_{i=1}^{4} \lambda_i g_i$, where $\lambda_i \geq 0$ and $i \in \{1, 4\}$.

Firm B's Lagrangian is $L_B(p_B, \eta) \triangleq -\pi_{B6}(p_B) + \sum_{i=1}^{4} \eta_i g_i$, where $\eta_i \geq 0$ and $i \in \{1, 4\}$.

By inspecting all possible solutions, we find the following solutions that satisfy all KKT conditions.

(i) The interior candidate (i.e., $\lambda_i = \eta_i = 0$).

Solve $\frac{\partial \pi_{A6}}{\partial p_A} = 0, \frac{\partial \pi_{B6}}{\partial p_B} = 0$. We derive the optimal solutions:

$$p_A = \frac{1}{3}, p_B = \frac{1}{6}.$$

One can check all $g_i < 0, i \in \{1, 4\}$ when $1 < \beta \leq 2$.

Therefore, the optimal equilibrium results in Case 6 are:

(6.1) when $1 < \beta \leq 2$, the optimal prices and firm profits are

$$p_A^* = \frac{1}{3}, p_B^* = \frac{1}{6},$$

$$\pi_{A6}^* = \frac{11}{9} - c, \pi_{B6}^* = \frac{1}{18}.$$

Case 7: $d_{RA} = 2x_{RA,NA}, d_{NA} = 2(x_{NA,NB} - x_{RA,NA}), d_{NB} = 1 - 2x_{NA,NB}$

The profit maximization problems for two firms are as follows.

$$\max \ \pi_{A7}(p_A, p_B) = p_A\left(p_B + \frac{1}{2}\right) - p_A^2 + \frac{4-2(\beta-1)c}{(\beta-1)^2},$$

$$\max \ \pi_{B7}(p_B, p_A) = \frac{1}{2}p_B\left(2p_A - 2p_B + 1\right),$$

subject to the following shared constraints:

$$-\frac{1}{2} < p_A - p_B < \frac{\beta-5}{2\beta-2}, \text{if } \beta > 3.$$

Denote the constraint functions as

$$g_1(p_A, p_B) = -\frac{1}{2} - (p_A - p_B) \leq 0,$$

$$g_2(p_A, p_B) = p_A - p_B - \frac{\beta-5}{2\beta-2} \leq 0,$$

$$g_3(p_A, p_B) = -p_A \leq 0,$$

$$g_4(p_A, p_B) = -p_B \leq 0.$$

Note that $-\pi_{A7}(p_A, p_B)$ is strictly convex in p_A and $-\pi_{B7}(p_B, p_A)$ is strictly convex in p_B.

Firm A's Lagrangian is $L_A(p_A, \lambda) \triangleq -\pi_{A7}(p_A, p_B) + \sum_{i=1}^{4} \lambda_i g_i$, where $\lambda_i \geq 0$ and $i \in \{1, 4\}$.

Firm B's Lagrangian is $L_B(p_B, \eta) \triangleq -\pi_{B7}(p_B) + \sum_{i=1}^{4} \eta_i g_i$, where $\eta_i \geq 0$ and $i \in \{1, 4\}$.

By inspecting all possible solutions, we find the following solutions that satisfy all KKT conditions.

(i) The boundary candidates with the active constraint $g_2 = 0$.

Solve $g_2 = 0$ with stationarity conditions. We derive the optimal solutions:

$$p_A = \frac{2}{\beta - 1}, p_B = -\frac{\beta - 9}{2(\beta - 1)}.$$

One can check that the solution satisfies all KKT conditions when $3 < \beta < 5$.

Therefore, the optimal equilibrium results in Case 7 are

(7.1) when $3 < \beta < 5$, the optimal prices and firm profits are

$$p_A^* = \frac{2}{\beta - 1}, p_B^* = -\frac{\beta - 9}{2(\beta - 1)},$$

$$\pi_{A7}^* = \frac{8 - 2(\beta - 1)c}{(\beta - 1)^2}, \pi_{B7}^* = \frac{(9 - \beta)(\beta - 3)}{2(\beta - 1)^2}.$$

(7.2) when $\beta > 5$, the optimal prices and firm profits are

$$p_A^* = \frac{1}{2}, p_B^* = \frac{1}{2},$$

$$\pi_{A7}^* = \frac{4 - 2(\beta - 1)c}{(\beta - 1)^2} + \frac{1}{4}, \pi_{B7}^* = \frac{1}{4}.$$

<u>Case 8</u>: $d_{RA} = 2x_{RA,RB}, d_{RB} = 2(x_{RB,NB} - x_{RA,RB}), d_{NB} = 1 - 2x_{RB,NB}$

The profit maximization problems for two firms are as follows.

$$\max \ \pi_{A8}(p_A, p_B) = p_A (2p_B + 1) - 2p_A^2 + \frac{4 - 2\beta c}{\beta^2},$$

$$\max \ \pi_{B8}(p_B, p_A) = 2p_B (p_A - p_B),$$

subject to the following shared constraints:

$$\frac{\beta - 2}{2\beta} < p_A - p_B < \frac{1}{2}, \text{if } \beta > 2.$$

Denote the constraint functions as

$$g_1(p_A, p_B) = \frac{\beta - 2}{2\beta} - (p_A - p_B) \leq 0,$$

$$g_2(p_A, p_B) = p_A - p_B - \frac{1}{2} \leq 0,$$

$$g_3(p_A, p_B) = -p_A \leq 0,$$

$$g_4(p_A, p_B) = -p_B \leq 0.$$

Note that $-\pi_{A8}(p_A, p_B)$ is strictly convex in p_A and $-\pi_{B8}(p_B, p_A)$ is strictly convex in p_B.

Firm A's Lagrangian is $L_A(p_A, \lambda) \triangleq -\pi_{A8}(p_A, p_B) + \sum_{i=1}^{4} \lambda_i g_i$, where $\lambda_i \geq 0$ and $i \in \{1, 4\}$.

Firm B's Lagrangian is $L_B(p_B, \eta) \triangleq -\pi_{B8}(p_B) + \sum_{i=1}^{4} \eta_i g_i$, where $\eta_i \geq 0$ and $i \in \{1, 4\}$.

By inspecting all possible solutions, we find the following solutions that satisfy all KKT conditions.

(i) The interior candidate (i.e., $\lambda_i = \eta_i = 0$).

Solve $\frac{\partial \pi_{A8}}{\partial p_A} = 0$, $\frac{\partial \pi_{B8}}{\partial p_B} = 0$. We derive the optimal solutions:

$$p_A = \frac{1}{3}, p_B = \frac{1}{6}.$$

One can check all $g_i < 0, i \in \{1, 4\}$ when $2 < \beta < 3$.

(ii) The boundary candidates with the active constraint $g_1 = 0$.

Solve $g_1 = 0$ with stationarity conditions. We derive the optimal solutions:

$$p_A = \frac{\beta - 2}{\beta}, p_B = \frac{1}{2} - \frac{1}{\beta}.$$

One can check that the solution satisfies all KKT conditions when $\beta > 3$.

Therefore, the optimal equilibrium results in Case 8 are

(8.1) when $2 < \beta < 3$, the optimal prices and firm profits are

$$p_A^* = \frac{1}{3}, p_B^* = \frac{1}{6},$$
$$\pi_{A8}^* = \frac{4 - 2\beta c}{\beta^2} + \frac{2}{9}, \pi_{B8}^* = \frac{1}{18};$$

(8.2) when $\beta > 3$, the optimal prices and firm profits are

$$p_A^* = \frac{\beta - 2}{\beta}, p_B^* = \frac{1}{2} - \frac{1}{\beta},$$
$$\pi_{A8}^* = \frac{2 - 2c}{\beta}, \pi_{B8}^* = \frac{(\beta - 2)^2}{2\beta^2}.$$

The optimal profits in the eight cases mentioned above are summarized in Table B3.

Table B3: The Optimal Profits in the Eight Cases

Case 1 (Only RA):	(1.1) $1 < \beta \leq 2$: $p_A^* = 0, p_B^* = 0$	$\pi_{A1-1}^* = 1 - c, \pi_{B1-1}^* = 0$;
	(1.2) $2 < \beta \leq 3$: $p_A^* = 0, p_B^* = \frac{\beta-2}{2}$	$\pi_{A1-2}^* = 1 - c, \pi_{B1-2}^* = 0$.
Case 2 (Only RB):	(2.1) $1 < \beta \leq 2$: $p_A^* = \frac{1}{2}, p_B^* = 0$	$\pi_{A2}^* = 1 - c, \pi_{B2}^* = 0$.
Case 3 (RA+NA):	(3.1) $\beta > 3$: $p_A^* = 0, p_B^* = \frac{1}{2}$	$\pi_{A3}^* = \frac{4-2(\beta-1)c}{(\beta-1)^2}, \pi_{B3}^* = 0$.
Case 4 (RB+NB):	(4.1) $\beta > 2$: $p_A^* = \frac{1}{2}, p_B^* = 0$	$\pi_{A4}^* = \frac{4-2\beta c}{\beta^2}, \pi_{B4}^* = 0$.
Case 5-1 (RA+NB):	(5.1.1) $\beta > 3$ and $c < \frac{3\beta-7}{2\beta-2}$: $p_A^* = \frac{11-3\beta}{2-2\beta}, p_B^* = \frac{\beta-3}{\beta-1}$	$\pi_{A5-1-1}^* = \frac{3\beta-2(\beta-1)c-7}{(\beta-1)^2}, \pi_{B5-1-1}^* = \frac{(\beta-3)^2}{(\beta-1)^2}$;
	(5.1.2) $\beta > 3$ and $c > \frac{3\beta-7}{2\beta-2}$: $p_A^* = c - \frac{2}{\beta-1}, p_B^* = c - \frac{1}{2}$	$\pi_{A5-1-2}^* = 0, \pi_{B5-1-2}^* = \frac{(\beta-3)(2c-1)}{2(\beta-1)}$.
Case 5-2 (RA+NB):	(5.2.1) (i) $2 < \beta \leq \frac{1}{2}\left(\sqrt{33}+1\right)$ and $\frac{-\beta^2-\beta+12}{4\beta+4} < c < \frac{\beta^2-2\beta+2}{2\beta}$ or (ii) $\beta > \frac{1}{2}\left(\sqrt{33}+1\right)$ and $\frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{\beta^2-2\beta+2}{2\beta}$: $p_A^* = \frac{\beta^2+\beta+4(\beta+1)c-12}{6\beta-2}, p_B^* = \frac{(\beta+1)(2\beta+2c-5)}{6\beta-2}$	$\pi_{A5-2-1}^* = \frac{(\beta-1)(\beta-2c+4)^2}{2(1-3\beta)^2}, \pi_{B5-2-1}^* = \frac{(\beta+1)(2\beta+2c-5)^2}{2(1-3\beta)^2}$;
	(5.2.2) $\beta > 2$ and $c > \frac{\beta^2-2\beta+2}{2\beta}$: $p_A^* = -\frac{3}{\beta} + c + 1, p_B^* = -\frac{2}{\beta} + c + \frac{1}{2}$	$\pi_{A5-2-2}^* = \frac{2(\beta-1)}{\beta^2}, \pi_{B5-2-2}^* = \frac{(\beta-2)(\beta+2\beta c-4)}{2\beta^2}$;
	(5.2.3) $\beta > \frac{1}{2}\left(\sqrt{33}+1\right)$ and $c < \frac{\beta^2-3\beta-2}{2\beta-2}$: $p_A^* = \frac{-\beta^2+\beta+8}{2-2\beta}, p_B^* = \frac{-\beta^2+2\beta+3}{2-2\beta}$	$\pi_{A5-2-3}^* = \frac{\beta^2-\beta-2(\beta-1)c-4}{(\beta-1)^2}, \pi_{B5-2-3}^* = \frac{(\beta-3)^2(\beta+1)}{2(\beta-1)^2}$.
Case 5-3 (RA+NB):	(5.3.1) $\beta > 2$ and $c < \frac{\beta+1}{\beta}$: $p_A^* = \frac{\beta-2}{\beta}, p_B^* = \frac{1}{2} - \frac{1}{\beta}$	$\pi_{A5-3-1}^* = \frac{2-2c}{\beta}, \pi_{B5-3-1}^* = \frac{(\beta-2)^2}{2\beta^2}$.
	(5.3.2) $\beta > 2$ and $c > \frac{\beta+1}{\beta}$: $p_A^* = c - \frac{3}{\beta}, p_B^* = -\frac{2}{\beta} + c - \frac{1}{2}$	$\pi_{A5-3-2}^* = -\frac{2}{\beta^2}, \pi_{B5-3-2}^* = \frac{(\beta-2)(\beta(2c-1)-4)}{2\beta^2}$.
Case 6 (RA+RB):	(6.1) $1 < \beta \leq 2$: $p_A^* = \frac{1}{3}, p_B^* = \frac{1}{6}$	$\pi_{A6}^* = \frac{11}{9} - c, \pi_{B6}^* = \frac{1}{18}$.
Case 7 (RA+NA+NB):	(7.1) $3 < \beta < 5$: $p_A^* = \frac{2}{\beta-1}, p_B^* = -\frac{\beta-9}{2(\beta-1)}$	$\pi_{A7-1}^* = \frac{8-2(\beta-1)c}{(\beta-1)^2}, \pi_{B7-1}^* = \frac{(9-\beta)(\beta-3)}{2(\beta-1)^2}$;
	(7.2) $\beta > 5$: $p_A^* = \frac{1}{2}, p_B^* = \frac{1}{2}$	$\pi_{A7-2}^* = \frac{4-2(\beta-1)c}{(\beta-1)^2} + \frac{1}{4}, \pi_{B7-2}^* = \frac{1}{4}$.
Case 8 (RA+RB+NB):	(8.1) $2 < \beta < 3$: $p_A^* = \frac{1}{3}, p_B^* = \frac{1}{6}$	$\pi_{A8-1}^* = \frac{4-2\beta c}{\beta^2} + \frac{2}{9}, \pi_{B8-1}^* = \frac{1}{18}$;
	(8.2) $\beta > 3$: $p_A^* = \frac{\beta-2}{\beta}, p_B^* = \frac{1}{2} - \frac{1}{\beta}$	$\pi_{A8-2}^* = \frac{2-2c}{\beta}, \pi_{B8-2}^* = \frac{(\beta-2)^2}{2\beta^2}$.

Proof of Proposition 3.2

Next, we compare all feasible scenarios in each sub-region. We need to find, for each (β, c) in the overlapping regions, which case gives the highest profit for both firms simultaneously. We show the details in Table B4. [25]

[25] Note that β_{16} is the second root of the polynomial equation $4x^4 - 19x^3 + 22x^2 - 7x - 4 = 0$. β_{17} is the first root of the polynomial equation $x^3 - 4x^2 + 3x - 4 = 0$. β_{18} is the second root of the polynomial equation $4x^4 - 23x^3 + 41x^2 - 33x - 5 = 0$. β_{19} is the first root of the polynomial

Table B4: The Optimal Customer Segmentation in Each Sub-Region

Region	Sub-region	Condition	Feasible Customer Segment
$1 < \beta \leq 2$	1.1	$c \leq 1$	1, 2, 6
	1.2	$1 < c \leq \frac{11}{9}$	6
$2 < \beta \leq 3$	2.1	$c < \frac{\beta^2-4\beta+7}{2\beta+2}$	1, 4, 5-1-1, 8-1
	2.2	$\frac{\beta^2-4\beta+7}{2\beta+2} \leq c \leq \frac{2}{\beta}$	1, 4, 8-1
	2.4	$1 < c \leq \frac{2}{\beta} + \frac{\beta}{9}$	8-1
$3 < \beta \leq \beta_{16}$	3.1	$\frac{3\beta^4-15\beta^3+23\beta^2-5\beta+26}{2\beta^3-10\beta^2-2\beta+10} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	3, 4, 5-1-1, 7, 8-2
	3.2	$\frac{\beta^2-4\beta+7}{2\beta+2} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$	3, 4, 5-3-1, 7, 8-2
	3.3	$\frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c \leq \frac{2}{\beta}$	3, 4, 7, 8-2
	3.4	$\frac{2}{\beta} < c \leq \frac{2}{\beta-1}$	3, 7, 8-2
	3.5	$\frac{2}{\beta-1} < c \leq 1$	7, 8-2
	3.6	$1 < c \leq \frac{4}{\beta-1}$	7
$\beta_{16} < \beta \leq \frac{1}{2}\left(\sqrt{33}+1\right)$	4.1	$\frac{3\beta^4-15\beta^3+23\beta^2-5\beta+26}{2\beta^3-10\beta^2-2\beta+10} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	3, 4, 5-1-1, 7, 8-2
	4.2	$\frac{\beta^2-4\beta+7}{2\beta+2} < c \leq \frac{2}{\beta}$	3, 4, 5-3-1, 7, 8-2
	4.3	$\frac{2}{\beta} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$	3, 5-3-1, 7, 8-2
	4.4	$\frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c \leq \frac{2}{\beta-1}$	3, 7, 8-2
	4.5	$\frac{2}{\beta-1} < c \leq 1$	7, 8-2
	4.6	$1 < c \leq \frac{4}{\beta-1}$	7
$\frac{1}{2}\left(\sqrt{33}+1\right) < \beta \leq \beta_{17}$	5.1	$\frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	3, 4, 5-1-1, 7, 8-2
	5.2	$\frac{\beta^2-4\beta+7}{2\beta+2} < c \leq \frac{2}{\beta}$	3, 4, 5-3-1, 7, 8-2
	5.3	$\frac{2}{\beta} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$	3, 5-3-1, 7, 8-2
	5.4	$\frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c \leq \frac{2}{\beta-1}$	3, 7, 8-2
	5.5	$\frac{2}{\beta-1} < c \leq 1$	7, 8-2
	5.6	$1 < c \leq \frac{4}{\beta-1}$	7
$\beta_{17} < \beta \leq \beta_{18}$	6.1	$\frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c \leq \frac{2}{\beta}$	3, 4, 5-1-1, 7, 8-2
	6.2	$\frac{2}{\beta} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	3, 4, 5-1-1, 7, 8-2
	6.3	$\frac{\beta^2-4\beta+7}{2\beta+2} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$	3, 5-3-1, 7, 8-2
	6.4	$\frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c \leq \frac{2}{\beta-1}$	3, 7, 8-2
	6.5	$\frac{2}{\beta-1} < c \leq 1$	7, 8-2
	6.6	$1 < c \leq \frac{4}{\beta-1}$	7
$\beta_{18} < \beta \leq \beta_{11}$	7.1	$\frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c \leq \frac{2}{\beta}$	3, 4, 5-1-1, 7, 8-2
	7.2	$\frac{2}{\beta} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	3, 5-1-1, 7, 8-2
	7.3	$\frac{\beta^2-4\beta+7}{2\beta+2} < c \leq \frac{2}{\beta-1}$	3, 5-3-1, 7, 8-2
	7.4	$\frac{2}{\beta-1} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$	5-3-1, 7, 8-2

equation $x^3 - 5x^2 + 7x - 11 = 0$.

Region	Sub-region	Condition	Feasible Customer Segment
	7.5	$\frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c \leq 1$	7, 8-2
	7.6	$1 < c \leq \frac{4}{\beta-1}$	7
$\beta_{11} < \beta \leq \beta_{19}$	8.1	$\frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c \leq \frac{2}{\beta}$	3, 4, 5-1-1, 7, 8-2
	8.2	$\frac{2}{\beta} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	3, 5-1-1, 7, 8-2
	8.3	$\frac{\beta^2-4\beta+7}{2\beta+2} < c \leq \frac{2}{\beta-1}$	3, 5-3-1, 7, 8-2
	8.4	$\frac{2}{\beta-1} < c \leq 1$	5-3-1, 7, 8-2
	8.5	$1 < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$	5-3-1, 7
	8.6	$\frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c \leq \frac{4}{\beta-1}$	7
$\beta_{19} < \beta \leq \beta_4$	9.1	$\frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c \leq \frac{2}{\beta}$	3, 4, 5-1-1, 7, 8-2
	9.2	$\frac{2}{\beta} < c \leq \frac{2}{\beta-1}$	3, 5-1-1, 7, 8-2
	9.3	$\frac{2}{\beta-1} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	5-1-1, 7, 8-2
	9.4	$\frac{\beta^2-4\beta+7}{2\beta+2} < c \leq 1$	5-3-1, 7, 8-2
	9.5	$1 < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$	5-3-1, 7
	9.6	$\frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c \leq \frac{4}{\beta-1}$	7
$\beta_4 < \beta \leq \sqrt{5}+2$	10.1	$\frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c \leq \frac{2}{\beta}$	3, 4, 5-1-1, 7, 8-2
	10.2	$\frac{2}{\beta} < c \leq \frac{2}{\beta-1}$	3, 5-1-1, 7, 8-2
	10.3	$\frac{2}{\beta-1} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	5-1-1, 7, 8-2
	10.4	$\frac{\beta^2-4\beta+7}{2\beta+2} < c \leq 1$	5-3-1, 7, 8-2
	10.5	$1 < c \leq \frac{4}{\beta-1}$	5-3-1, 7
	10.6	$\frac{4}{\beta-1} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$	5-3-1
$\sqrt{5}+2 < \beta \leq 5$	11.1	$\frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c \leq \frac{2}{\beta}$	3, 4, 5-1-1, 7, 8-2
	11.2	$\frac{2}{\beta} < c \leq \frac{2}{\beta-1}$	3, 5-1-1, 7, 8-2
	11.3	$\frac{2}{\beta-1} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$	5-1-1, 7, 8-2
	11.4	$\frac{\beta^2-4\beta+7}{2\beta+2} < c \leq 1$	5-3-1, 7, 8-2
	11.5	$1 < c \leq \frac{4}{\beta-1}$	5-3-1, 7
	11.6	$\frac{4}{\beta-1} < c < \frac{2\beta^3-7\beta^2+7\beta+4}{2\beta^2+2\beta}$	5-3-1
$\beta > 5$	12.1	$c \leq \frac{2}{\beta}$	3, 4, 5-2-3, 8-2
	12.2	$\frac{2}{\beta} < c \leq \frac{2}{\beta-1}$	3, 5-2-3, 8-2
	12.3	$\frac{2}{\beta-1} < c \leq 1$	5-2-3, 8-2
	12.4	$1 < c < \frac{\beta^2-3\beta-2}{2\beta-2}$	5-2-1, 5-2-3
	12.5	$\frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{\beta^2-2\beta+2}{2\beta}$	5-2-1
	12.6	$c > \frac{\beta^2-2\beta+2}{2\beta}$	5-2-2

By comparing the optimal profits of two firms in eight cases with the respective feasible regions, we identify the optimal customer segmentation as follows.[26]

[26]Note that β_{20} is the first root of the polynomial equation $3x^5 - 9x^4 + x^3 - 39x^2 - 20 = 0$. β_{21} is the second root of the polynomial equation $2x^4 + x^3 - 31x^2 + 3x - 39 = 0$. β_{22} is the first root of the polynomial equation $x^3 - 5x^2 + 7x - 11 = 0$. β_{23} is the third root of the polynomial equation

(a) RA+RB is the optimal customer segmentation when $1 < \beta \le 2$.

(b) RA+NB is the optimal customer segmentation when any of the following conditions are satisfied: (1) $\beta > 5, c > \frac{\beta^2-3\beta-2}{2\beta-2}$; (2)$\frac{1}{18}(\sqrt{109}+37) \le \beta \le 3, \frac{5-\beta}{2\beta+2} \le c < \frac{\beta^2-4\beta+7}{2\beta+2}$; (3) $\beta_{20} \le \beta \le 4, \frac{3\beta^4-9\beta^3+5\beta^2-59\beta-4}{2\beta^3-10\beta^2-2\beta+10} \le c \le \frac{2}{\beta}$; (4) $4 < \beta \le 5, \frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c \le \frac{2}{\beta}$; (5) $\beta_{21} < \beta \le \beta_{20}, \frac{3\beta^4-9\beta^3+5\beta^2-59\beta-4}{2\beta^3-10\beta^2-2\beta+10} \le c < \frac{\beta^2-4\beta+7}{2\beta+2}$; (6) $\beta_{20} < \beta \le 5, \frac{2}{\beta} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$; (7) $5 < \beta \le \beta_{23}, c < \frac{\beta^2-3\beta-2}{2\beta-2}$; (8) $\beta > \beta_{23}, \frac{\beta^3-10\beta^2-3\beta+4}{2\beta^2-4\beta+2} \le c < \frac{\beta^2-3\beta-2}{2\beta-2}$; (9) $\beta > \beta_{23}, c \le 1$; (10) $\beta_{24} < \beta \le \beta_{25}, \frac{-9\beta^3+43\beta^2+5\beta+25}{4\beta^3+4\beta^2-4\beta-4} \le c \le \frac{2}{\beta-1}$; (11) $\beta_{25} < \beta < \beta_{22}, \frac{\beta^2-4\beta+7}{2\beta+2} < c \le \frac{2}{\beta-1}$; (12) $\beta_{26} < \beta \le \beta_{24}, \frac{-9\beta^3+43\beta^2+5\beta+25}{4\beta^3+4\beta^2-4\beta-4} \le c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (13) $\beta_{24} < \beta \le \beta_{27}, \frac{2}{\beta-1} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (14) $\beta_{27} < \beta \le \beta_{22}, \frac{2}{\beta-1} < c \le 1$; (15) $\beta_{22} < \beta < 5, \frac{\beta^2-4\beta+7}{2\beta+2} < c \le 1$; (16) $\beta_{27} < \beta \le \beta_{28}, 1 < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (17) $\beta_{28} < \beta < 5, 1 < c \le \frac{4}{\beta-1}$.

(c) RA+RB+NB is the optimal customer segmentation when any of the following conditions are satisfied: (1) $2 < \beta \le \beta_{29}, c \ge \frac{7\beta^2-36}{9\beta^2-18\beta}$; (2) $\beta_{29} < \beta \le \beta_{30}, c \ge \frac{23\beta^4-35\beta^3+32\beta^2-144\beta-72}{18\beta^3-18\beta^2-36\beta}$; (3) $\beta_{30} < \beta \le \beta_{31}, c \ge \frac{\beta^2-4\beta+7}{2\beta+2}$; (4) $\beta_{32} < \beta \le 3, c \ge \frac{7\beta^2-36}{9\beta^2-18\beta}$.

(d) RA+NA+NB is the optimal customer segmentation when any of the following conditions are satisfied: (1) $3 < \beta < \beta_{33}, c > 1$; (2) $\beta_{33} \le \beta < \beta_4$.

(e) A mixed strategy emerges as a solution where firms might probabilistically choose among feasible customer segmentation options in the remaining region.

This completes the proof.

$x^3 - 12x^2 + x + 2 = 0$. β_{24} is the first root of the polynomial equation $9x^3 - 35x^2 + 11x - 17 = 0$. β_{25} is the second root of the polynomial equation $2x^4 + x^3 - 31x^2 + 3x - 3 = 0$. β_{26} is the second root of the polynomial equation $8x^4 - 37x^3 + 47x^2 - 55x - 27 = 0$. β_{27} is the third root of the polynomial equation $4x^3 - 21x^2 + 22x - 1 = 0$. β_{28} is the second root of the polynomial equation $4x^4 - 23x^3 + 37x^2 - 41x - 9 = 0$. β_{29} is the first root of the polynomial equation $23x^3 - 49x^2 + 18x - 72 = 0$. β_{30} is the second root of the polynomial equation $14x^4 + 19x^3 - 103x^2 - 18x - 72 = 0$. β_{31} is the first root of the polynomial equation $9x^4 - 68x^3 + 121x^2 - 54x + 72 = 0$. β_{32} is the first root of the polynomial equation $9x^4 - 68x^3 + 121x^2 - 54x + 72 = 0$. β_{33} is the first root of the polynomial equation $x^4 - 9x^3 + 20x^2 - 6x + 2 = 0$.

Proof of Lemma 3.1

In the benchmark scenario, neither firms adopt TBPs. The customer utility function, denoted by U_j, where $j \in A, B$ represents which firm they make a purchase from. The utility functions are given by:

$$U_A(x) = v - p_A - |x - x_A|,$$

$$U_B(x) = v - p_B - |x - x_B|.$$

$U_A(x) = U_B(x)$ yields $x_{AB} = \frac{1}{4} - \frac{p_A - p_B}{4}$. That is, the demand functions are $d_A = 2x_{AB}$ and $d_B = 1 - 2x_{AB}$. We substitute them into the profit equations and derive the corresponding profit functions as follows.

$$\pi_A = p_A \left(\frac{1}{2} - \frac{p_A - p_B}{2} \right),$$

$$\pi_B = p_B \left(\frac{1}{2} + \frac{p_A - p_B}{2} \right).$$

To obtain the optimal decisions, we solve the first-order conditions: $\frac{d\pi_A}{dp_A} = 0, \frac{d\pi_B}{dp_B} = 0$. Solving these equations yields $p_A = \frac{1}{2}, p_B = \frac{1}{2}$. The resulting demands are $d_A = \frac{1}{2}, d_B = \frac{1}{2}$. Therefore, the optimal profits are $\pi_A = \frac{1}{4}, \pi_B = \frac{1}{4}$.

This completes the proof.

Proof of Proposition 3.3

Comparing the optimal firm profits under the YN case with the NN case, we derive the following outcomes. [27]

[27]Note that β_1 is the second root of the polynomial equation $8x^4 - 37x^3 + 47x^2 - 55x - 27 = 0$; The roots are ordered by increasing real part; if two roots have the same real part, then by increasing imaginary part. β_2 is the second root of the polynomial equation $2x^4 + x^3 - 31x^2 + 3x - 39 = 0$. β_3 is the second root of the polynomial equation $17x^4 - 92x^3 + 146x^2 - 164x - 35 = 0$. β_4 is the second root of the polynomial equation $4x^4 - 23x^3 + 37x^2 - 41x - 9 = 0$. β_5 is the third root of

(a) The win-win region occurs (i.e., both firms benefit from TBP) when any of the following conditions are satisfied: (1) $\beta_1 < \beta \leq \frac{11}{3}, \frac{-9\beta^3+43\beta^2+5\beta+25}{4\beta^3+4\beta^2-4\beta-4} \leq c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (2) $\frac{11}{3} < \beta \leq \beta_2, \frac{-9\beta^3+43\beta^2+5\beta+25}{4\beta^3+4\beta^2-4\beta-4} \leq c < \frac{-\beta^2+2\beta+31}{8\beta-8}$; (3) $\beta_2 < \beta \leq \beta_3, \frac{3\beta^4-9\beta^3+5\beta^2-59\beta-4}{2\beta^3-10\beta^2-2\beta+10} \leq c < \frac{-\beta^2+2\beta+31}{8\beta-8}$; (4) $\beta_3 < \beta \leq 4, \frac{3\beta^4-9\beta^3+5\beta^2-59\beta-4}{2\beta^3-10\beta^2-2\beta+10} \leq c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (5) $4 < \beta \leq \beta_4, \frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (6) $\beta_4 < \beta \leq 5, \frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c < \frac{4}{\beta-1}$; (7) $5 < \beta < 2\sqrt{2}+4$; (8) $2\sqrt{2}+4 \leq \beta < 9, c < \frac{\beta+4}{2} - \frac{\sqrt{\frac{9\beta^2-6\beta+1}{\beta-1}}}{2\sqrt{2}}$; (9) $9 \leq \beta \leq \beta_5, c < \frac{3\beta^2-2\beta-17}{8\beta-8}$; (10) $\beta_5 < \beta < \beta_6$, $c \leq 1$ or $\frac{\beta^3-10\beta^2-3\beta+4}{2\beta^2-4\beta+2} \leq c < \frac{3\beta^2-2\beta-17}{8\beta-8}$; (11) $\beta \geq \beta_6, c \leq 1$.

(b) The win-lose region occurs (i.e., Firm A benefits from its TBP, but Firm B hurts) when any of the following conditions are satisfied: (1) $1 < \beta \leq 2, c < \frac{35}{36}$; (2) $2 < \beta \leq \beta_7, \frac{7\beta^2-36}{9\beta^2-18\beta} \leq c < \frac{144-\beta^2}{72\beta}$; (3) $\beta_7 < \beta \leq \beta_8, \frac{23\beta^4-35\beta^3+32\beta^2-144\beta-72}{18\beta^3-18\beta^2-36\beta} \leq c < \frac{144-\beta^2}{72\beta}$; (4) $\beta_8 < \beta \leq \beta_9, \frac{\beta^2-4\beta+7}{2\beta+2} \leq c < \frac{144-\beta^2}{72\beta}$; (5) $\beta_9 < \beta < 3\sqrt{97}-27, \frac{7\beta^2-36}{9\beta^2-18\beta} \leq c < \frac{144-\beta^2}{72\beta}$; (6) $\frac{\sqrt{109}+37}{18} \leq \beta \leq 3, \frac{5-\beta}{2\beta+2} \leq c < \frac{\beta^2-4\beta+7}{2\beta+2}$; (7) $3 < \beta < \beta_{10}, 1 < c < \frac{-\beta^2+2\beta+31}{8\beta-8}$; (8) $\beta_{10} \leq \beta < \frac{11}{3}, \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2} < c < \frac{-\beta^2+2\beta+31}{8\beta-8}$.

(c) The YN case is infeasible (i.e., Firm A hurts) when any of the following conditions are satisfied: (1)$1 < \beta \leq 2, c > \frac{35}{36}$; (2) $2\left(\sqrt{2}+2\right) < \beta \leq 9, \frac{\beta+4}{2} - \frac{\sqrt{\frac{9\beta^2-6\beta+1}{\beta-1}}}{2\sqrt{2}} < c < \frac{\beta^2-2\beta+2}{2\beta}$; (3) $\beta > 9, \frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{\beta^2-2\beta+2}{2\beta}$;(4)$\beta > 2\left(\sqrt{2}+2\right), c > \frac{\beta^2-2\beta+2}{2\beta}$; (5) $9 < \beta \leq \beta_5, \frac{3\beta^2-2\beta-17}{8\beta-8} < c < \frac{\beta^2-3\beta-2}{2\beta-2}$; (6) $\beta_5 < \beta \leq \beta_6, \frac{3\beta^2-2\beta-17}{8\beta-8} < c < \frac{\beta^2-3\beta-2}{2\beta-2}$; (7) $\beta > \beta_6, \frac{\beta^3-10\beta^2-3\beta+4}{2\beta^2-4\beta+2} \leq c < \frac{\beta^2-3\beta-2}{2\beta-2}$; (8) $2 < \beta \leq \beta_7, c > \frac{144-\beta^2}{72\beta}$; (9)$\beta_8 < \beta \leq \beta_9, c > \frac{144-\beta^2}{72\beta}$; (10) $\beta_9 < \beta \leq 3\sqrt{97}-27, c > \frac{144-\beta^2}{72\beta}$; (11) $3\sqrt{97}-27 < \beta \leq 3, c \geq \frac{7\beta^2-36}{9\beta^2-18\beta}$; (12) $\beta_7 < \beta \leq \beta_8, c > \frac{144-\beta^2}{72\beta}$; (13) $3 < \beta < \beta_{10}, c > \frac{-\beta^2+2\beta+31}{8\beta-8}$; (14)$\beta_{10} \leq \beta \leq \beta_{11}, c > \frac{-\beta^2+2\beta+31}{8\beta-8}$; (15)$\beta_{11} < \beta \leq \beta_3, c > \frac{-\beta^2+2\beta+31}{8\beta-8}$; (16)$\beta_3 < \beta < \beta_4, c > \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$.

(d) A mixed strategy emerges as a solution where firms might probabilistically choose among feasible customer segmentation options in the remaining region of the YN case, making direct comparisons with the NN case challenging.

This completes the proof.

the polynomial equation $x^3 - 12x^2 + x + 2 = 0$. β_6 is the first root of the polynomial equation $x^3 - 35x^2 + 3x - 1 = 0$. β_7 is the first root of the polynomial equation $23x^3 - 49x^2 + 18x - 72 = 0$. β_8 is the second root of the polynomial equation $14x^4 + 19x^3 - 103x^2 - 18x - 72 = 0$. β_9 is the first root of the polynomial equation $9x^4 - 68x^3 + 121x^2 - 54x + 72 = 0$. β_{10} is the first root of the polynomial equation $x^4 - 9x^3 + 20x^2 - 6x + 2 = 0$. β_{11} is the third root of the polynomial equation $4x^3 - 21x^2 + 22x - 1 = 0$.

Proof of Lemma 3.2

In the YY case, both firms can adopt in-store TBPs. In this case, the maximum distance from any customer to her nearest collection point is $\frac{1}{4}$. The customer utility function U_j^i, where $i \in \{R, N\}$ represents recycling or not recycling, $j \in \{A, B\}$ indicates the firm from which the purchase is made, is given as follows.

$$U_j^R = 1 - \beta \min\{|x - x_j|, |x - \frac{1}{4}|\} + v - p_j,$$

$$U_j^N = v - p_j - |x - x_j|.$$

The profit function for each profit-maximizing firm j is:

$$\pi_j = p_j(d_{Rj} + d_{Nj}) - c_j d_{Rj} + d_{Rj}^2.$$

To simplify the analysis, we assume $c_A = c_B = c$. Then, the profit structures of the two firms are identical due to symmetry, which leads to coinciding optimal pricing decisions.

(i) If $x_{Rj,Nj} \geq \frac{1}{4}$, that is, $1 < \beta \leq 5$, all customers are incentivized to recycle.

$U_A^R(x) = U_B^R(x)$ yields $x_{RA,RB} = \frac{-2p_A + \beta + 2p_B}{4\beta}$. The demand functions are $d_{RA} = 2x_{AB}$ and $d_{RB} = 1 - 2x_{AB}$. We substitute them into the profit equations and derive the corresponding profit functions as follows.

$$\pi_A = -\frac{(-2p_A + \beta + 2p_B)\left(-2(\beta - 1)p_A - 2p_B + \beta(2c - 1)\right)}{4\beta^2},$$

$$\pi_B = \frac{(2p_A + \beta - 2p_B)\left(2p_A + \beta + 2(\beta - 1)p_B - 2\beta c\right)}{4\beta^2}.$$

To obtain the optimal decisions, we solve the first-order conditions: $\frac{d\pi_A}{dp_A} = 0$, $\frac{d\pi_B}{dp_B} = 0$. Solving these equations yields $p_A = p_B = \frac{1}{2}(\beta + 2c - 2)$. The resulting demands are $d_{RA} = d_{RB} = \frac{1}{2}$. Therefore, the optimal profits are $\pi_A = \pi_B = \frac{\beta - 1}{4}$.

(ii) If $x_{Rj,Nj} < \frac{1}{4}$, that is, $\beta > 5$, the market consists of recycled and non-recycled customers.

The demand functions are $d_{RA} = 2x_{RA,NA}, d_{NA} = 2(x_{NA,NB} - x_{RA,NA}), d_{NB} = 2(x_{RB,NB} - x_{NA,NB}), d_{RB} = 1 - 2x_{RB,NB}$. We substitute them into the profit equations and derive the corresponding profit functions as follows.

$$\pi_A = p_A\left(p_B + \frac{1}{2}\right) - p_A^2 + \frac{4 - 2(\beta - 1)c}{(\beta - 1)^2},$$

$$\pi_B = \left(p_A + \frac{1}{2}\right)p_B - p_B^2 + \frac{4 - 2(\beta - 1)c}{(\beta - 1)^2}.$$

To obtain the optimal decisions, we solve the first-order conditions: $\frac{d\pi_A}{dp_A} = 0$, $\frac{d\pi_B}{dp_B} = 0$. Solving these equations yields $p_A = p_B = \frac{1}{2}$. The resulting demands are $d_{RA} = \frac{2}{\beta-1}, d_{NA} = \frac{1}{2} - \frac{2}{\beta-1}, d_{NB} = \frac{1}{2} - \frac{2}{\beta-1}, d_{RB} = \frac{2}{\beta-1}$. Therefore, the optimal profits are $\pi_A = \pi_B = \frac{4-2(\beta-1)c}{(\beta-1)^2} + \frac{1}{4}$.

This completes the proof.

Proof of Proposition 3.4

Comparing the optimal firm profits under the NN, YN, and YY cases, we derive the following equilibrium outcomes.[28]

(i) NN is an equilibrium when $\pi_A^{NN} \geq \pi_A^{YN}, \pi_B^{NN} \geq \pi_B^{NY}, \pi_A^{NN} \geq \pi_A^{YY}, \pi_B^{NN} \geq \pi_B^{YY}$. That is, (1) $1 < \beta \leq 2, c \geq \frac{35}{36}$; (2) $2\left(\sqrt{2}+2\right) < \beta < 9, \frac{\beta+4}{2} - \frac{\sqrt{\frac{9\beta^2-6\beta+1}{\beta-1}}}{2\sqrt{2}} \leq c < \frac{\beta^2-2\beta+2}{2\beta}$; (3) $\beta \geq 9, \frac{\beta^2-3\beta-2}{2\beta-2} < c < \frac{\beta^2-2\beta+2}{2\beta}$; (4) $\beta \geq 2\left(\sqrt{2}+2\right), c > \frac{\beta^2-2\beta+2}{2\beta}$; (5) $9 < \beta \leq \beta_{34}, \frac{3\beta^2-2\beta-17}{8\beta-8} \leq c < \frac{\beta^2-3\beta-2}{2\beta-2}$; (6) $\beta_{34} < \beta \leq \beta_{35}, \frac{3\beta^2-2\beta-17}{8\beta-8} \leq c < \frac{\beta^2-3\beta-2}{2\beta-2}$; (7) $\beta > \beta_{35}, \frac{\beta^3-10\beta^2-3\beta+4}{2\beta^2-4\beta+2} \leq c < \frac{\beta^2-3\beta-2}{2\beta-2}$.

(ii) YN is an equilibrium when $\pi_A^{YN} \geq \pi_A^{NN}, \pi_B^{YN} \geq \pi_B^{YY}$ or $\pi_A^{NY} \geq \pi_A^{YY}, \pi_B^{NY} \geq \pi_B^{NN}$. That is, (1) $1 < \beta \leq \frac{11}{9}, c \leq \frac{35}{36}$; (2) $5 < \beta \leq 2\sqrt{2}+4, \frac{1}{8}\left(-2\beta^2+9\beta+1\right) \leq c < \frac{\beta^2-3\beta-2}{2\beta-2}$; (3) $5 < \beta \leq$

[28] Note that β_{34} is the third root of the polynomial equation $x^3 - 12x^2 + x + 2 = 0$. β_{35} is the first root of the polynomial equation $x^3 - 35x^2 + 3x - 1 = 0$. β_{36} is the second root of the polynomial equation $2x^4 + x^3 - 31x^2 + 3x - 39 = 0$. β_{37} is the first root of the polynomial equation $23x^3 - 49x^2 + 18x - 72 = 0$. β_{38} is the first root of the polynomial equation $9x^4 - 68x^3 + 121x^2 - 54x + 72 = 0$. β_{39} is the second root of the polynomial equation $14x^4 + 19x^3 - 103x^2 - 18x - 72 = 0$. β_{40} is the first root of the polynomial equation $x^4 - 9x^3 + 20x^2 - 6x + 2 = 0$.

$2\sqrt{2}+4, c > \frac{\beta^2-3\beta-2}{2\beta-2}$; (4) $2\sqrt{2}+4 < \beta < 9, \frac{1}{8}\left(-2\beta^2+9\beta+1\right) \leq c \leq \frac{\beta+4}{2} - \frac{\sqrt{\frac{9\beta^2-6\beta+1}{\beta-1}}}{2\sqrt{2}}$; (5) $9 \leq \beta \leq \beta_{34}, \frac{1}{8}\left(-2\beta^2+9\beta+1\right) \leq c \leq \frac{3\beta^2-2\beta-17}{8\beta-8}$; (6) $\beta_{34} < \beta \leq \beta_{35}, \frac{1}{8}\left(-2\beta^2+9\beta+1\right) \leq c \leq 1$; (7) $\beta_{34} < \beta \leq \beta_{35}, \frac{\beta^3-10\beta^2-3\beta+4}{2\beta^2-4\beta+2} \leq c \leq \frac{3\beta^2-2\beta-17}{8\beta-8}$; (8) $\beta > \beta_{35}, \frac{1}{8}\left(-2\beta^2+9\beta+1\right) \leq c \leq 1$.

(iii) YY is an equilibrium when $\pi_A^{YY} \geq \pi_A^{NY}, \pi_B^{YY} \geq \pi_B^{YN}, \pi_A^{YY} \geq \pi_A^{NN}, \pi_B^{YY} \geq \pi_B^{NN}$. That is, (1) $\frac{1}{18}(\sqrt{109}+37) \leq \beta \leq 3, \frac{5-\beta}{2\beta+2} \leq c < \frac{\beta^2-4\beta+7}{2\beta+2}$; (2) $\beta_{20} \leq \beta \leq \beta_{22}, \frac{2}{\beta} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$; (3) $\beta_{25} \leq \beta < \beta_{20}, \frac{3\beta^4-9\beta^3+5\beta^2-59\beta-4}{2\beta^3-10\beta^2-2\beta+10} \leq c < \frac{\beta^2-4\beta+7}{2\beta+2}$; (4) $\beta_{22} < \beta \leq 5, \frac{2}{\beta-1} < c < \frac{\beta^2-4\beta+7}{2\beta+2}$; (5) $\beta_{20} < \beta < 4, \frac{3\beta^4-9\beta^3+5\beta^2-59\beta-4}{2\beta^3-10\beta^2-2\beta+10} \leq c \leq \frac{2}{\beta}$; (6) $4 \leq \beta \leq 5, \frac{\beta^3-6\beta^2+\beta-4}{2\beta^2-2} < c \leq \frac{2}{\beta}$; (7) $\beta_{22} < \beta \leq 5, \frac{2}{\beta} < c < \frac{2}{\beta-1}$; (8) $5 < \beta \leq \beta_{23}, c \leq \frac{1}{8}(-2\beta^2+9\beta+1)$; (9) $\beta > \beta_{23}, c \leq \frac{1}{8}(-2\beta^2+9\beta+1)$; (10) $\beta_{24} \leq \beta < \beta_{25}, \frac{-9\beta^3+43\beta^2+5\beta+25}{4\beta^3+4\beta^2-4\beta-4} \leq c \leq \frac{2}{\beta-1}$; (11) $\beta_{24} < \beta \leq \beta_{27}, \frac{2}{\beta-1} < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (12) $\beta_{26} < \beta < \beta_{24}, \frac{-9\beta^3+43\beta^2+5\beta+25}{4\beta^3+4\beta^2-4\beta-4} \leq c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (13) $\beta_{36} < \beta < \beta_{22}, \frac{\beta^2-4\beta+7}{2\beta+2} < c \leq \frac{2}{\beta-1}$; (14) $\beta_{22} < \beta < 5, \frac{\beta^2-4\beta+7}{2\beta+2} < c \leq 1$; (15) $\beta_{27} < \beta \leq \beta_{28}, 1 < c < \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$; (16) $\beta_{27} < \beta \leq \beta_{22}, \frac{2}{\beta-1} < c \leq 1$; (17) $\beta_{28} < \beta < 5, 1 < c \leq \frac{4}{\beta-1}$; (18) $2 < \beta \leq \beta_{37}, c \geq \frac{7\beta^2-36}{9\beta^2-18\beta}$; (19) $\beta_{38} < \beta \leq 3, c \geq \frac{7\beta^2-36}{9\beta^2-18\beta}$; (20) $\beta_{37} < \beta \leq \beta_{39}, c \geq \frac{23\beta^4-35\beta^3+32\beta^2-144\beta-72}{18\beta^3-18\beta^2-36\beta}$; (21) $\beta_{39} < \beta \leq \beta_{38}, c \geq \frac{\beta^2-4\beta+7}{2\beta+2}$; (22) $3 < \beta < \beta_{40}, c > 1$; (23) $\beta_{40} \leq \beta < \beta_{28}, c > \frac{4\beta^3-19\beta^2+26\beta+1}{2\beta^2+4\beta+2}$.

This completes the proof.

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