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INSTABILITIES AND CONTROL STRATEGIES IN HYPERSONIC BOUNDARY-LAYER FLOWS

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Instabilities and Control Strategies in Hypersonic Boundary-layer Flows

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CERTIFICATE OF ORIGINALITY

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Name of student: CHEN YIFENG

*Measure what is measurable,
and make measurable what is not so.*

— GALILEO GALILEI

*To the hands that lifted me,
the minds that challenged me,
and the hearts that believed.*

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Abstract

Boundary-layer instability and transition remain significant and unresolved challenges for hypersonic vehicles. Transition prediction and control require a comprehensive understanding of the character, energy source, and interaction of instabilities. These instabilities include both modal and nonmodal disturbances in linear and nonlinear stages that lead to transition. However, the aforementioned fundamental aspects remain incompletely understood and warrant further systematic investigation.

The primary objective of this investigation is to elucidate the fundamental energy mechanisms underlying various boundary-layer instabilities and to evaluate potential transition control approaches, especially for a passive control method—acoustic metasurface.

By employing three established energy-based methodologies—the momentum potential theory, inviscid Lagrangian energy analysis, and relative phase analysis—a comprehensive unified framework for the source terms governing the exponential evolution of the second mode is developed. For the Mach 6 flow condition under investigation, all three methodologies converge on identical local energy amplification pathways driven by two dominant source mechanisms: the dilatation term in the near-wall region and either the Reynolds thermal stress term or heat exchange term in the outer layer region, contingent upon the chosen energy norm. By extending relative phase analysis to multiple instability modes and influencing factors, the analysis reveals that obliqueness results in a strengthened first mode while a weakened second mode via Reynolds shear stress. The wall cooling stabilizes the first mode via thermoacoustic phase opposition but destabilizes the second mode through a newly generated in-phase region of wall-normal internal energy transfer. Acoustic metasurfaces mimic wall heating to suppress the second mode while amplifying oblique first modes through in-phase shear production. Furthermore, the wall-normal transport of internal energy at the critical layer demonstrates increased phase deviation for the substantially weaker supersonic mode, explaining its reduced contribution to energy growth.

Resolvent analysis of hypersonic flow over blunt wedges identifies two distinct growth regimes in the entropy layer (pattern A) and boundary layer (pattern B) of travelling waves, distinguished by their energy gain characteristics between optimal and sub-optimal disturbances. Both regimes

exhibit energy growth dominated by nonmodal processes, which may subsequently facilitate first-mode instability development under adiabatic wall conditions. Wall cooling suppresses Pattern-B's modal/nonmodal growth but minimally affects Pattern A. Through the combined analysis of slow acoustic wave receptivity and resolvent response, a plausible mechanistic explanation was proposed for the experimentally observed "transition reversal" phenomenon. Furthermore, the analysis identifies concurrent Orr and lift-up mechanisms governing nonmodal growth.

Finally, transition triggered by multiple dominant instabilities is examined, integrating linear and nonlinear, modal and nonmodal growth stages. The acoustic metasurface effect is modelled using a rigorously validated time-domain impedance boundary condition (TDIBC). The resolvent analysis identifies two distinct optimal disturbance peaks corresponding to the oblique first mode and planar Mack second mode. The critical finding reveals that acoustic metasurface placement profoundly influences transition delay and skin friction reduction. Positioning within the second mode's linearly unstable region delays the transition by suppressing streaks in the oblique breakdown scenario. However, during late transition stages, the metasurface increases skin friction overshoot attributable to intensified shear-induced dissipation from reinforced de-tuned modes associated with combination resonance. This adverse effect can be mitigated by restricting metasurface placement upstream of the overshoot region.

This study provides fundamental insights into instability mechanisms and transition processes in hypersonic boundary-layer flows while advancing effective control strategies. The findings offer significant potential to accelerate the development of next-generation hypersonic vehicles through improved transition prediction and control capability.

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List of Nomenclatures

a	local speed of sound
A	metasurface admittance
c	phase speed
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
C_f	skin friction coefficient
e	total energy per unit mass
f	frequency
M	Mach number
Re	Reynolds number
ρ	density
T	temperature
p	pressure
Pr	Prandtl number
$\mathbf{m}'$	momentum density
$\mathbf{m}'_A$	the acoustic component of momentum density
$\mathbf{m}'_B$	the vortical component of momentum density
$\mathbf{m}'_T$	the thermal component of momentum density
R_n	nose radius
s	entropy
$\overline{\mathbf{S}}$	viscous stress tensor
t	time
u	streamwise velocity
v	normal velocity
w	spanwise velocity
x	streamwise coordinate

ξ	body-fitted streamwise coordinate
y	normal coordinate
η	body-fitted normal coordinate
z	spanwise coordinate
Z	metasurface impedance
$\mathbf{u}$	velocity vector
α	acceleration vector
α	streamwise wavenumber
β	spanwise wavenumber
ω	circular frequency
$\boldsymbol{\omega}$	vorticity vector
σ	growth rate
σ^2	optimal energy gain
γ	specific heat ratio

Superscripts

'	fluctuation quantity
—	mean quantity
*	dimensional quantity
^	eigen function
T	transpose
†	complex conjugate

Subscripts

ad	adiabatic wall condition
e	boundary-layer edge quantity
w	wall quantity
∞	freestream quantity

List of Abbreviations

BMD	Bispectral mode decomposition
DAP	Divergence of acoustic power
DNS	Direct numerical simulation
FFT	Fast Fourier transform
GIP	Generalised inflection point
HE	Heat exchange
LST	Linear stability theory
MFD	Mean flow distortion
MPT	Momentum potential theory
NPSE	Nonlinear-parabolized stability equations
N–S	Navier–Stokes
PAE	Parcel acoustic energy
PSE	Parabolic stability equation
RMS	Root mean square
RPA	Relative phase analysis
RSS	Reynolds shear stress
RTS	Reynolds thermal stress
TFE	Total fluctuating enthalpy
T–S	Tollmien–Schlichting
TDIBC	Time–domain impedance boundary condition

Chapter 1

Introduction

This chapter provides a concise review of the literature pertinent to the research problems addressed in this thesis and is organized into the following sections:

- Introduction
- Boundary-layer instability and transition
- Energy budget of unstable modes
- Boundary-layer control strategies
- Aims and outline

Section [1.1](#) provides a concise overview of the significance of boundary-layer transition in both academic research and engineering applications. Section [1.2](#) elaborates on the primary and secondary instabilities responsible for boundary-layer transition. Section [1.3](#) reviews three prominent energy budget analysis methods utilized in this study. Section [1.4](#) introduces various boundary-layer transition control strategies, with a particular focus on a passive control method: the acoustic metasurface. Finally, Section [1.5](#) summarizes the research gap, objectives, and aims of this study. Relevant literature corresponding to specific chapters or sections of the thesis has been appropriately cited throughout.

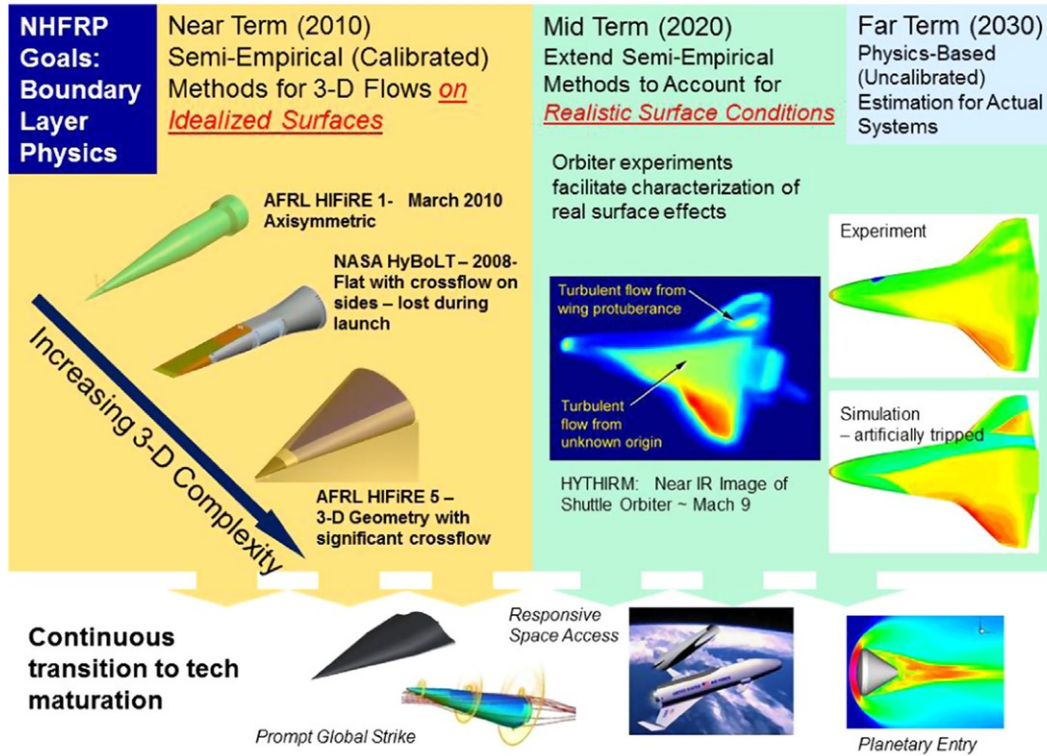


FIGURE 1.1: Visualization of collaborative government research frameworks pursuing laminar-turbulent transition objectives under American National Hypersonic Foundational Research Plan (NHFRP) boundary layer initiatives [1].

1.1 Background

The boundary layer transition problem has been studied for over a century, with significant progress achieved in low-speed flows. However, as aerospace technology advances, low-speed aircraft can no longer meet the demands of space exploration, national defence, and emerging commercial markets, such as those targeted by SpaceX. In recent decades, increasing attention has been directed toward transition phenomena in supersonic and hypersonic boundary layers. Despite advances in experimental, numerical, and theoretical studies, fundamental gaps persist in understanding the physical mechanisms governing boundary layer transition [15, 16]. A primary challenge in hypersonic vehicle design is predicting and controlling laminar-turbulent transition, as turbulence leads to substantially higher aerothermal loads and skin friction drag compared to laminar flows [1]. Consequently, understanding boundary-layer transition mechanisms and developing effective laminar flow control technologies are crucial for enhancing hypersonic vehicle performance and operational viability, particularly in optimizing thermal protection systems. As illustrated in figure 1.1, the laminar-turbulent transition remains a key unresolved challenge in developing next-generation hypersonic vehicles. Accurately predicting

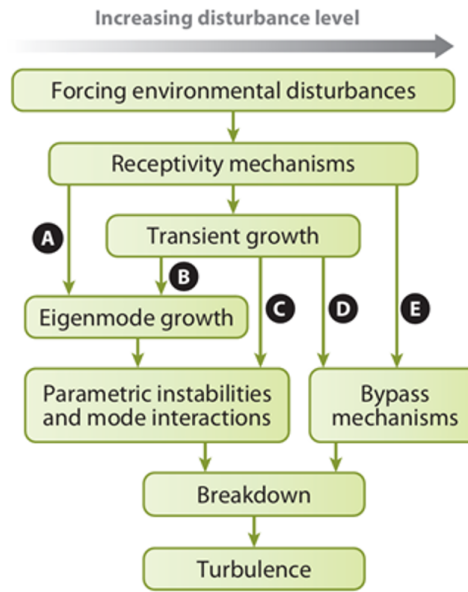


FIGURE 1.2: Paths to boundary-layer transition with respect to the disturbance amplitude [2].

and delaying boundary-layer transition enables precise thermal protection system sizing, which would eliminate excessive safety margins in heat shield design, thereby reducing material waste and enhancing design efficiency.

Accurate transition prediction requires a comprehensive understanding of both modal and non-modal instabilities that may initiate the transition process. Figure 1.2 illustrates typical transition pathways categorized by disturbance amplitude. In flight environments, where freestream turbulence intensity is relatively low, transition typically results from the exponential amplification of unstable modes in hypersonic boundary layers over symmetric models at zero angle of attack, corresponding to routes A and B in figure 1.2. For blunt-tipped configurations, modal instabilities within the boundary layer are suppressed (shifting downstream with increasing bluntness) due to the presence of an overlying entropy layer. In such cases, recent studies have identified nonmodal disturbances (transient growth) as dominant [17, 18], typically leading to transition via route C in figure 1.2 when nose bluntness exceeds a critical threshold, triggering transition before modal instabilities emerge.

While progress has been made in understanding the receptivity mechanisms and linear growth laws of unstable modes, several fundamental questions remain unresolved. These include: (1) the energy source sustaining modal disturbance growth, (2) the combined effects of wall temperature and Mach number, and (3) the control efficacy of traditional porous materials

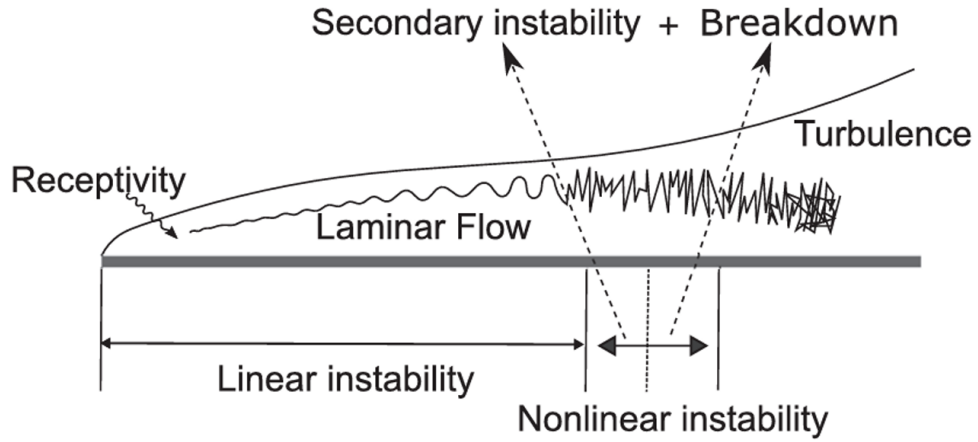


FIGURE 1.3: Diagram for flow instability during the laminar turbulent transition in a boundary layer [3].

under broadband disturbances. Furthermore, the nonmodal characteristics of blunt-body flows—particularly the influence of wall temperature and bluntness on boundary-layer and entropy-layer optimal response—require clarification. The role of transient growth mechanisms (including Orr and lift-up processes) in transition initiation also demands further investigation.

1.2 Boundary-layer instability and transition

Under realistic cruise conditions, free-stream disturbances typically exhibit low amplitudes. In such cases, transition over boundary layers with sharp leading edge or small bluntness primarily results from the exponential growth of unstable modes within the boundary layer. These modes correspond to Tollmien-Schlichting waves in low-speed flows, but shift to first-mode and second-mode instabilities in supersonic and hypersonic boundary layers [19–21].

As illustrated in figure 1.3, disturbance-induced transition generally progresses through four distinct stages: (1) the receptivity process, which determines the initial frequency and amplitude of unstable boundary-layer modes; (2) linear instability, characterized by exponential wave growth; (3) nonlinear instability, involving interactions between saturated waves; and (4) breakdown leading to complete transition and subsequent turbulence development.

1.2.1 Primary instability

For incompressible boundary layers, Tollmien [22] and Schlichting [23] successively employed asymptotic matching methods to solve the linear stability equations for parallel flows under small-perturbation assumptions, specifically the Orr-Sommerfeld (O-S) equation. Their solutions revealed unstable waves generated by viscous effects, which were subsequently designated as Tollmien-Schlichting (T-S) waves. Later, Lees & Lin [24] extended linear stability theory to compressible boundary layers. They established a necessary and sufficient condition for the existence of neutral waves, requiring that the phase speed at the generalized inflection point (GIP) must match that of the base flow. The wall-normal location of the GIP is defined by:

$$\frac{d}{dy} \left(\bar{\rho} \frac{d\bar{u}}{dy} \right) = 0. \quad (1.1)$$

The presence of a GIP serves as a sufficient condition for unstable wave existence. As Mach number increases, the most unstable boundary layer mode shifts from T-S waves to first-mode instabilities, which behave as vorticity mode (similar to T-S mode) near the boundary-layer edge and demonstrate greater energy at oblique angles [25]. In hypersonic flows over asymmetric geometries, Mack's second-mode induced transition has been extensively confirmed in wind tunnel experiments [26, 27] and remains a subject of significant research interest. As reported by Mack [25], the second mode belongs to the inviscid mode family and manifests as the most energetic planar wave.

The Mack's second mode [19], characterized by acoustic waves trapped between the wall and the relative sonic line, represents the dominant linear instability in hypersonic boundary layers when the Mach number exceeds a threshold value (typically around 4). The modal synchronization mechanism, first proposed by Fedorov and colleagues [28–30] and subsequently validated through direct numerical simulations by Ma & Zhong [20, 31, 32], is widely accepted as the receptivity mechanism for second-mode excitation.

As illustrated in figure 1.4, near the leading edge, the phase speeds (c/u_e) of fast (mode F) and slow modes (mode S) asymptotically approach those of freestream fast ($c/u_e = 1 + 1/M_e$) and slow ($c/u_e = 1 - 1/M_e$) acoustic waves, respectively. Note that mode S refers to the first mode when it is unstable. The approaching of phase speed, known as the synchronization process, generates a new discrete spectrum branch that excites the second mode through intermodal conversion between mode S and mode F. Higher-order modes, including the Mack third mode,

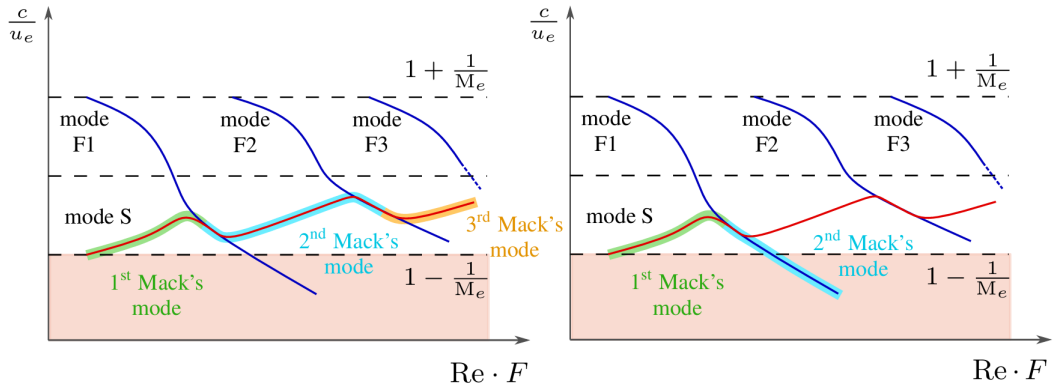


FIGURE 1.4: Sketch of phase-speed diagrams (c/u_e) showing modes synchronizations associated to higher Mack's mode instabilities, for two different spectrum-branching topologies. The colored area refers to the region where a mode is supersonic [4]. Here, M_e is the Mach number at the boundary-layer edge, F is disturbance reduced frequency, Reynolds number Re represents spatial location.

may emerge from synchronization between the second mode and fast mode II (denoted as mode F2 in figure 1.4). These higher-order modes exhibit lower energy content compared to the second mode and have consequently received limited research attention due to their diminished role in transition dynamics. Additionally, supersonic modes with phase speeds below $1 - 1/M_e$ can develop under specific conditions, such as cooled walls [33, 34] and high-enthalpy flows [35, 36]. These supersonic modes exhibit an energy-redistribution phenomenon arising from interactions between discrete eigenmodes and the slow-acoustic continuous spectrum, leading to pressure wave emission from the boundary layer into the freestream.

For blunt-nosed configurations, entropy-layer disturbances induce alternative second-mode generation with distinct synchronization characteristics. Wan *et al.* first reported this phenomenon in a receptivity simulation over a blunt cone at Mach 6 [37]. Later, Chen *et al.* identified these disturbances as nonmodal through direct numerical simulation (DNS) of a blunt wedge [38]. Notably, these disturbances differ fundamentally from the entropy-layer modes predicted by linear stability theory (LST).

Although the receptivity mechanisms of T-S waves, oblique first modes, second modes, and supersonic modes are well established, their growth mechanisms - particularly the energy-sustaining processes enabling local exponential growth - remain inadequately explained and require further theoretical justification.

The leading surfaces of hypersonic vehicles are intentionally blunted to reduce heat transfer rates, which results in the formation of a detached bow shock and an entropy layer characterized by a

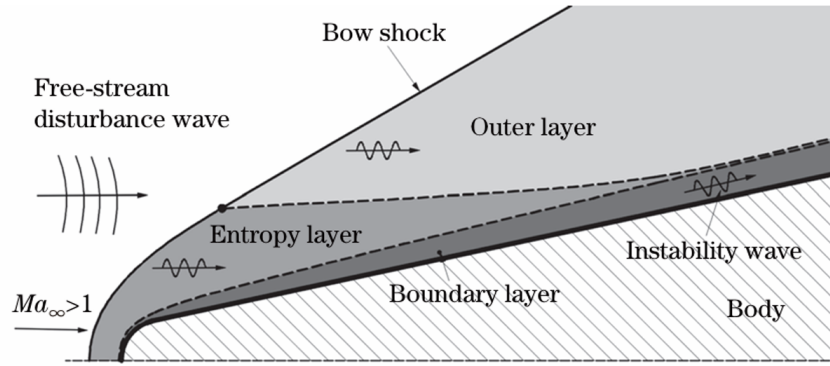


FIGURE 1.5: Conceptual illustration showing the receptivity mechanism in a blunt cone boundary layer [5].

strong entropy gradient, as shown in figure 1.5. This entropy layer, located above the boundary layer, decreases the local Reynolds number at the boundary layer's edge, thereby stabilizing the boundary layer. The presence of the entropy layer may influence the transition mechanism within the boundary layer. As reported by wind tunnel experiments [39, 40], the transition location initially shifts downstream with an increase in nose radius, but subsequently moves upstream once the nose radius exceeds a certain threshold. This downstream movement of the transition location aligns with the results of stability analyses [41, 42], which indicate that both the first and second modes become highly stabilized as the nose radius increases. Furthermore, the newly generated instability associated with the entropy layer mode, observed alongside the bow shock, has been reported to be insignificant in triggering the transition due to its low growth rate and limited growth region near the leading edge [5, 38].

Despite the fact that the "transition reversal" phenomenon [39, 40] was first reported 40 years ago, its underlying mechanisms remain unclear. Recent studies have examined the characteristics of nonmodal disturbances, which are regarded as a potential cause for the transition, particularly when modal disturbances are insufficiently strong to trigger the transition [17, 43]. According to the resolvent analysis and numerical simulations conducted by Paradise *et al.* [17, 18], nonmodal disturbances within the entropy layer can penetrate further into the boundary layer, ultimately causing the transition. Harman *et al.* [44] investigated the nonlinear breakdown process on a blunt cone using DNS, indicating that when linear unstable oblique disturbance waves initiated in the entropy layer possess sufficiently large initial amplitudes, a rapid transition may occur prior to the instability of the second mode. Additionally, similar Schlieren images were observed in wind tunnel experiments conducted by Kennedy *et al.* [6, 45], validating the critical role of nonmodal disturbances within the entropy layer during the transition process. As shown in

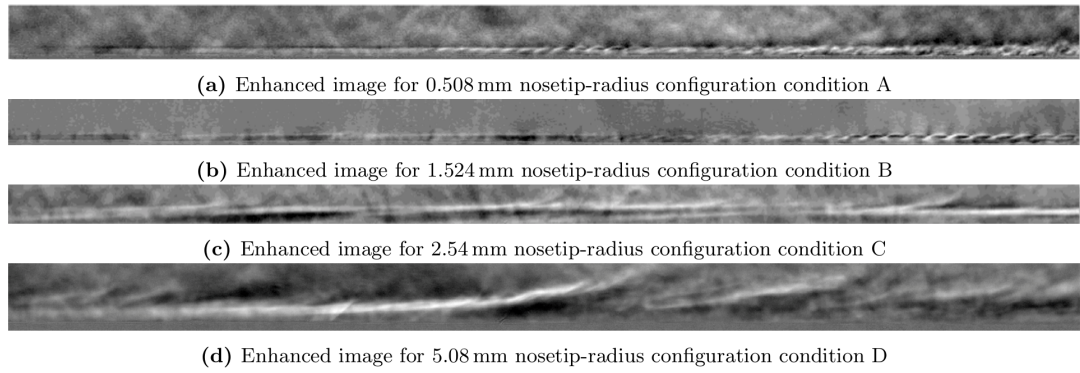


FIGURE 1.6: Enhanced schlieren visualizations generated by subtracting a mean flow image from the image of interest [6].

figure 1.6, under condition A, a rope-like structure resembling the second mode appears prior to the eventual breakdown. With an increase in bluntness, the second mode is suppressed and appears further downstream in condition B. Subsequently, an elongated structure emerges near the boundary edge in condition C, with a nose radius of 2.54 mm, potentially associated with the first mode. Upon further increasing the nose radius to 5.08 mm, a dominant Schlieren feature appears as a wisp structure above the boundary-layer edge (located within the entropy layer), suggesting a transition scenario dominated by nonmodal disturbances.

Although progress has been made in understanding the role of optimal perturbations—those disturbances that gain the most energy within a given time frame—and nonmodal disturbances in the transition process of blunt-tip hypersonic boundary layers, the characteristics of these nonmodal disturbances and the underlying transient growth mechanisms remain unclear and require further clarification, particularly in configurations with moderate bluntness where the second mode is significantly suppressed. Furthermore, while existing resolvent analyses can provide a mathematical characterization of nonmodal disturbances, their physical dependence on wall temperature and bluntness warrants further investigation.

1.2.2 Secondary instability

Generally, after the amplitude of the primary wave saturates, secondary instability arises, leading to breakdown. Conventional breakdown scenarios—extensively studied in the literature—include fundamental, subharmonic, and oblique types. These scenarios are characterized by the emergence of additional spectral peaks at multiples or fractions of the primary frequency when nonlinear effects dominate.

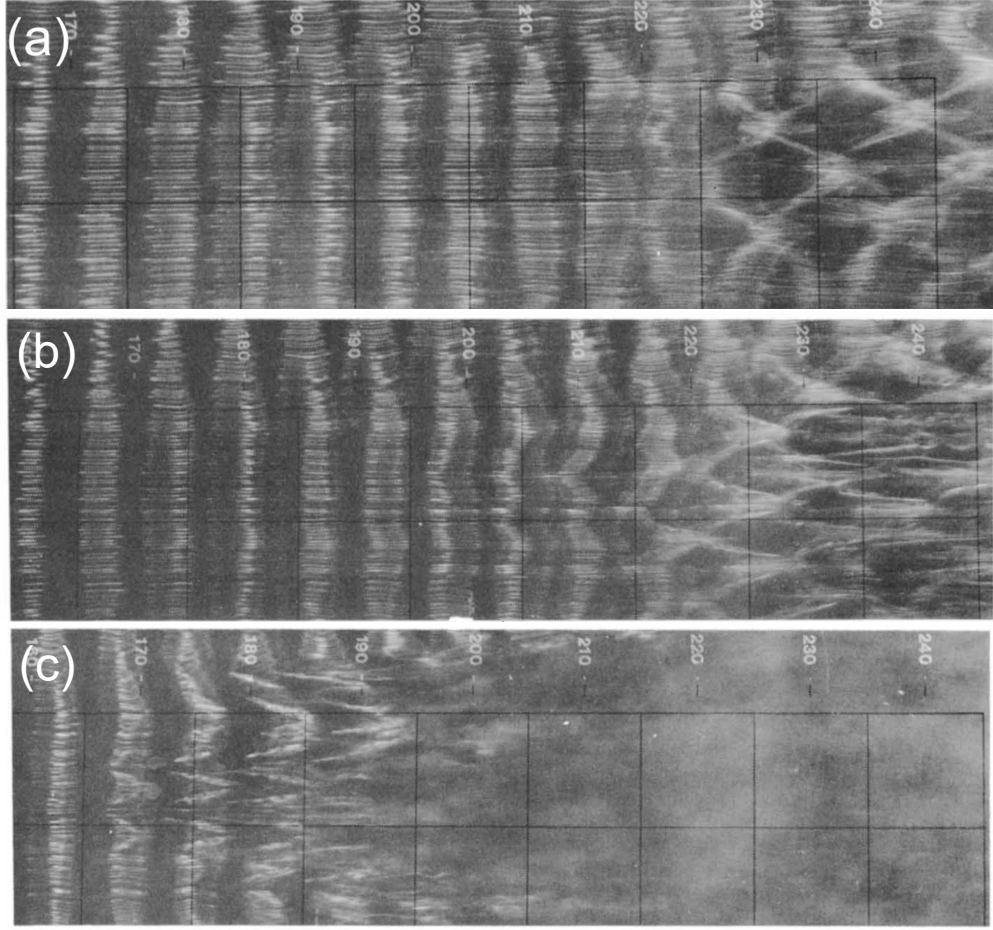


FIGURE 1.7: Experimental visualization of (a) C-type resonance, (b) H-type breakdown, and (c) K-type breakdown [7].

Early efforts to investigate the physical mechanisms governing nonlinear transition phases primarily focused on experimental studies of controlled transitions in incompressible flat-plate flows. According to Herbert [46], secondary instability is associated with the three-dimensional distortion of the transitional boundary layer, induced by the parametric resonance of oblique disturbances via finite-amplitude Tollmien-Schlichting (T-S) waves. Multiple mechanisms were identified based on the excited harmonics and the resulting perturbed flow-field patterns. These patterns are marked by the spatial distribution of regions with amplified (peaks) and attenuated (valleys) disturbance amplitudes of the modulated T-S wave, often manifesting as vortex-like structures.

The primary mechanisms initially identified within the context of the secondary instability of incompressible T-S waves are known as subharmonic and fundamental breakdowns. Additionally, a further mechanism, termed oblique breakdown, was subsequently discovered [47]. The key characteristics of these phenomena are outlined briefly below.

When the fundamental T–S wave reaches sufficient amplitude, the transitional flow field develops a staggered vortex arrangement, as shown in figure 1.7(b). This configuration features alternating peaks and valleys, causing hot-wire sensors to detect signals at twice the T–S period (or equivalently, at half the T–S frequency). In such cases, oblique subharmonic modes ($f_o/2, \pm\beta$) become predominantly excited and, together with the two-dimensional T–S wave, serve as initial conditions in controlled transition experiments conducted by Ermolaev *et al.* [48]. Subsequent DNS simulations performed by Mayer & Fasel [49] confirmed the existence and feasibility of subharmonic resonances. A notable manifestation of subharmonic breakdown is the Craik triad resonance (denoted as C-type breakdown and illustrated in figure 1.7(a)), which requires both primary and oblique waves to maintain identical, infinitesimal amplitudes. While experimental configurations can successfully reproduce this scenario, C-type breakdown rarely occurs in natural flow conditions.

In the low-speed boundary layer, as the amplitude of the fundamental T–S wave increases, experiments reveal an aligned vortex pattern (as shown in Figure 1.7(c)). These conditions closely resemble those described by Klebanoff *et al.* [50], leading to this mechanism being termed K-type breakdown, also known as fundamental resonance. In this scenario, the most strongly excited oblique disturbances ($f_o, \pm\beta$) share the same frequency as the planar wave ($f_p, 0$). The vorticity components of freestream disturbances, such as turbulence, play a critical role in this breakdown mechanism. For hypersonic flow, fundamental resonance was first reported on a flared cone during the early transition stage in the Boeing/AFOSR Mach 6 Quiet Tunnel (BAM6QT) at Purdue University [51]. Subsequent research confirmed its significance in natural transition scenarios [52]. The characteristic "hot streak" pattern observed experimentally was later reproduced in DNS through forced fundamental breakdown simulations [53]. Additionally, second-mode dominant breakdown was documented in quiet tunnel experiments using a flared cone model at Mach 6 at Peking University [27].

The oblique breakdown is triggered by a pair of oblique waves ($f_o, \pm\beta$) sharing the same frequency but having opposite wave angles relative to the streamwise direction, which generates rapidly growing steady longitudinal vortices. This mechanism was first identified through DNS conducted by Thumm *et al.* [54] and Fasel *et al.* [47]. For incompressible flows, Schmid & Henningson [55] demonstrated that both nonlinear and transient linear mechanisms contribute to the growth of longitudinal vortices, whose amplitude may surpass that of the primary oblique modes. Nonlinear mechanisms become particularly dominant in supersonic boundary layers governed by a first Mack-mode instability, where the most unstable waves exhibit inherent

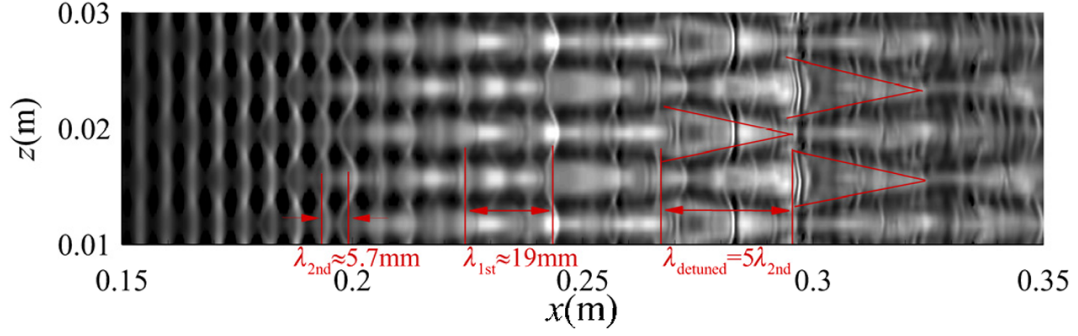


FIGURE 1.8: Instantaneous skin friction coefficient contour for a Mach 6 flat-plate boundary transition induced jointly by optimal first and Mack second modes. Here, λ_{1st} , λ_{2nd} and $\lambda_{detuned}$ denote the streamwise wavelengths of the first, second and detuned modes, respectively [8].

three-dimensionality [56]. For a blunt cone at Mach 6, DNS conducted by Hartman *et al.* [44] demonstrate that introducing a pair of oblique waves in the entropy layer can trigger rapid oblique breakdown in the downstream boundary layer.

Supersonic/hypersonic boundary-layer transition over axisymmetric models at zero angle of attack is typically initiated by the growth of unstable modes—namely, the first mode (associated with vorticity disturbances) and the Mack second mode (of acoustic nature). Under realistic flight or wind tunnel conditions, these two instabilities often coexist and compete. Their nonlinear interaction can lead to a novel breakdown scenario. This mode-mode interaction is considered universal because multiple instabilities inherently exist in real-world boundary layers. The nonlinear mechanism governing interactions between low-frequency waves (the first mode or Görtler mode) and Mack-mode instabilities was systematically investigated by Chen *et al.* using nonlinear-parabolized stability equations (NPSE) [57]. Recent DNS by Guo *et al.* [8] have revealed, for the first time, the critical role and universal nature of non-conventional combination resonance induced by modal detuning in high-speed boundary-layer transition. Their simulations identified that the detuned mode, characterized by distinct Λ -shaped structures (see figure 1.8), becomes the dominant feature governing near-wall dynamics during the late transitional stage.

1.3 Energy budget of unstable modes

In hypersonic flows, both the second mode and supersonic mode have garnered significant research interest in the context of high-Mach-number thermochemical non-equilibrium flows. As depicted in figure 1.9, the characteristic structures of these instability modes are presented schematically. The wall-normal region can be categorized into three distinct zones based on the

relative Mach number $\bar{M}_r(y) = \frac{\bar{u}(y)-c}{\bar{a}(y)}$ (denoted as $\bar{M}$ in figure 1.9), where $\bar{u}$ represents the base flow streamwise velocity, c denotes the disturbance phase speed, and $\bar{a}$ is the local sound speed: (1) a near-wall supersonic region, (2) an intermediate subsonic region, and (3) a potential off-wall supersonic region. These zones are intrinsically defined by the phase speed characteristics of the disturbances. Experimental observations in wind-tunnel studies [27, 45] have revealed the presence of rope-like structures in second-mode-dominant flows. Nevertheless, the energy source that sustains the growth of the second mode, as well as the physical mechanism underlying supersonic mode radiation remains to be elucidated from an energy budget standpoint.

The energy source and growth mechanism of the second mode have been extensively investigated through various stability analyses and energy equation formulations. Kuehl [58] employed an inviscid Lagrangian approach to analyze the acoustic impedance of the second mode in the wall-normal direction, revealing that the impedance peaks near the sonic line. The interaction generates an acoustic impedance well bounded by the sonic line and wall, providing the necessary conditions for second-mode wave resonance to persist. The energy maintaining these resonant standing waves is primarily supplied by thermoacoustic Reynolds stress and thermodynamic effort, with the significance of thermodynamic effort being further corroborated by experimental studies [59].

Analogous to acoustic wave generation in Rijke tubes, where the phase of the resonance process is crucial, the second mode exhibits trapped wave behavior between the relative sonic line and the wall. This similarity suggests that the phase distribution of source terms may be critical for sustaining the second mode's growth. Tian & Wen [60] conducted a comprehensive phase analysis by examining the relationship between perturbation rates and source terms, elucidating the growth mechanisms of second-mode instability. Their findings demonstrate that fluctuating internal energy is maintained through two key processes: advection of perturbed thermal energy near the critical layer, and dilatation fluctuations in the wall vicinity.

To comprehensively elucidate the energy production and transfer mechanisms of the second mode, physical-based feature extraction methods offer valuable analytical insights. For uniform base flow fields, Kovasznay [61] pioneered the decomposition of fluctuations into three fundamental components: vortical, acoustic, and thermal (entropic) modes. However, in non-uniform base flows, velocity and temperature gradients induce coupling between these modes, significantly complicating the application of Kovasznay's decomposition to hypersonic boundary layers characterized by strong mean flow gradients.

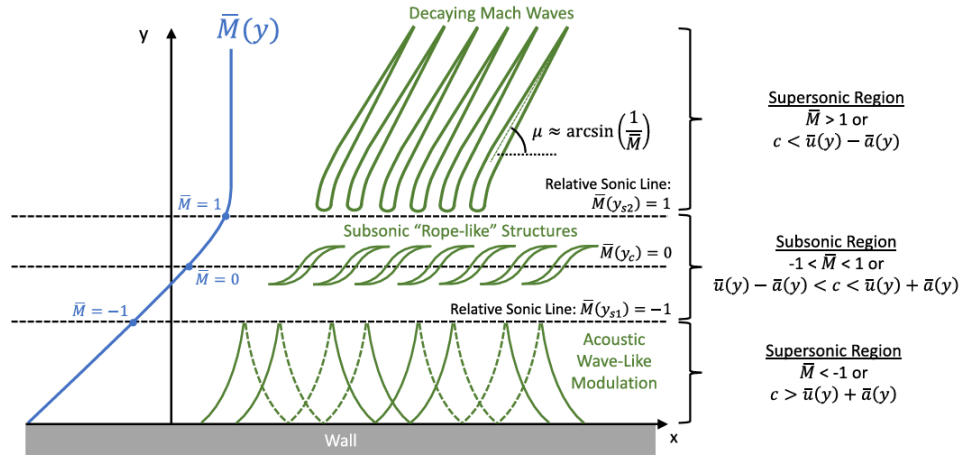


FIGURE 1.9: Visualization of neutral supersonic mode in the large wavenumber limit. The first sonic line is denoted by $\bar{M}(y_s) = 1$, the critical layer by $\bar{M}(y_c) = 0$, and the second sonic line by $\bar{M}(y_{s2}) = 1$. The reflection at the first sonic line transforms waves from compression to expansion and vice versa [9].

This limitation is addressed by the momentum potential theory (MPT) developed by Doak [62], which extends analytical capability to general time-stationary flows. The efficacy of MPT has been demonstrated in studies of cold Mach 1.3 jet flows [63], where its vortical component successfully captured shear layer growth and breakdown processes. Recent work by Unnikrishnan & Gaitonde [12] has applied MPT to decompose the second mode into its constituent vortical, acoustic, and entropic components, revealing clear interaction patterns between these components and the base flow. Notably, their analysis identified the entropic component as the dominant energy source for acoustic perturbation growth, providing critical theoretical foundations for understanding second-mode dynamics in hypersonic flows.

Subsequent work by Long [64] established a more explicit formulation of Doak's energy budget equation, incorporating the interaction terms among the three MPT components. This formulation distinctly reveals two key aspects: (1) the influence of various source terms on each energy flux component, and (2) the specific energy transfer mechanisms between different MPT components. Notably, the analysis identified a strong positive acoustic source term originating from the vortical component, with maximum intensity observed near the critical layer. Furthermore, the outward transport of the acoustic component was found to correlate directly with the supersonic mode's sound radiation characteristics.

While MPT has provided valuable new perspectives on boundary layer instability mechanisms, certain limitations persist due to the approach's inherent complexity. A particular discrepancy arises with the pressure dilatation term - known to be physically significant through relative

phase analysis (RPA), Lagrangian methods, and experimental validation - which does not appear explicitly in the MPT governing equations. This omission creates a theoretical gap between MPT and other established analytical frameworks, suggesting the need for further refinement and cross-methodological analysis to achieve complete consistency.

The current theoretical landscape reveals several unresolved discrepancies and fundamental questions requiring further investigation. First, the mechanisms underlying modal obliqueness remain unclear, particularly why it manifests exclusively in the most unstable first mode. Second, opposing stabilization trends emerge under different boundary conditions: while wall cooling stabilizes the oblique first mode while destabilizing the second mode [21], porous coatings exhibit precisely the inverse behavior [65]. Third, the governing physics controlling supersonic-mode instability remains to be fully elucidated. These unresolved issues highlight the pressing need for a more comprehensive and self-consistent theoretical framework to explain the energy sources driving these instability modes.

1.4 Boundary-layer control strategies

Boundary-layer transition control strategies are an essential technology to deliver the occurrence of the transition, which can be classified as positive methods, such as wall cooling and heating [66], and passive methods, for example, porous coating surface [67–69]. The passive method requires no extra energy input and is flexible to implement. Therefore, it has attracted more attention from the academics and researchers. Furthermore, there are also some studies showing the effectiveness of a combination of different control strategies.

1.4.1 The acoustic metasurface

Acoustic metasurfaces, encompassing absorptive, impedance-near-zero, and reflection-controlled configurations, have emerged as promising solutions for controlling the acoustically-dominated second mode instability [10]. These passive control techniques provide implementation flexibility without requiring external energy input, making them particularly attractive for both fundamental research and industrial applications. Among these approaches, porous coatings represent a pioneering passive control method, originally conceptualized by Federov [65] through analogy with sound wave propagation in elongated tubes.

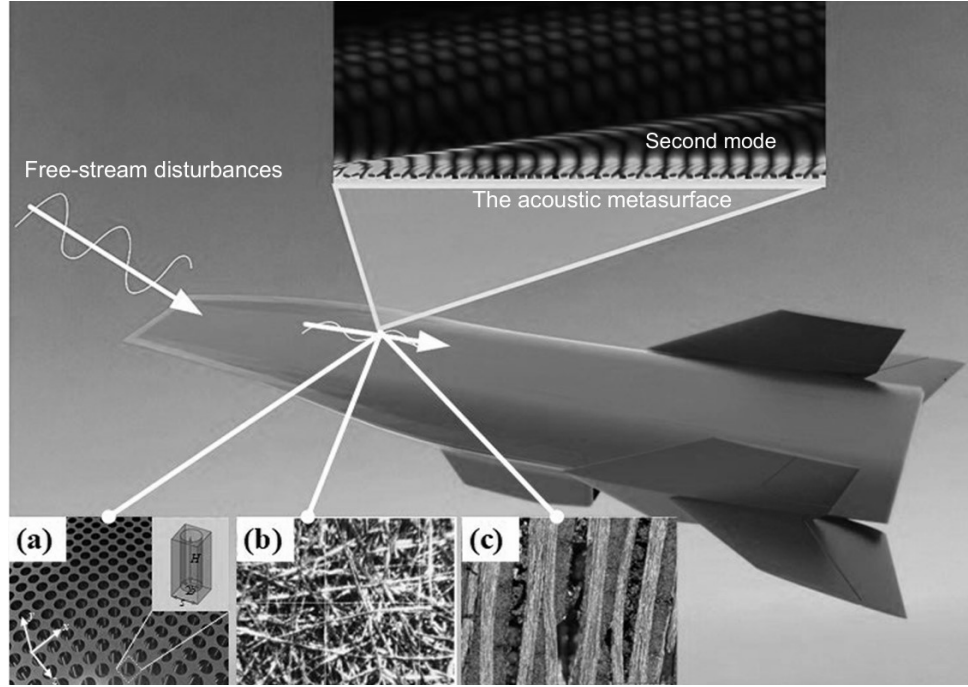


FIGURE 1.10: Schematic drawing of suppression of the second mode by the acoustic metasurface of (a) regular metasurface, (b) random felt metasurface, and (c) C/C composite surface, adapted from Ref. [10].

The efficacy of porous coatings has been rigorously validated through three complementary research approaches: theoretical analyses [60, 64, 65], numerical simulations [69, 70], and experimental investigations [71, 72], all demonstrating significant transition delay through second-mode suppression. Figure 1.10 schematically illustrates this suppression mechanism under freestream disturbance conditions. Physically, the second mode appears as a compression-expansion wave train bounded between the relative sonic line and the wall [25], characterized by a streamwise wavelength approximately double the local boundary-layer thickness. This distinctive wave structure makes it particularly susceptible to manipulation through acoustic metasurface interventions.

In the linear stability stage, the effect of porous coating can be theoretically described and satisfactorily predicted using linear stability theory (LST) with the deduced acoustic impedance boundary condition [73]. The physical mechanism by which the porous coating influences the linear stage has been widely studied [60, 74, 75]). It is known that viscous effects within the pores of the porous coating can dissipate the second-mode instability. Acoustic wave reflection is also linked to regimes with phase cancellation or reinforcement [76].

While previous studies have primarily examined instability and transition induced by single-wave disturbances, realistic freestream conditions involve broadband disturbance spectra [77].

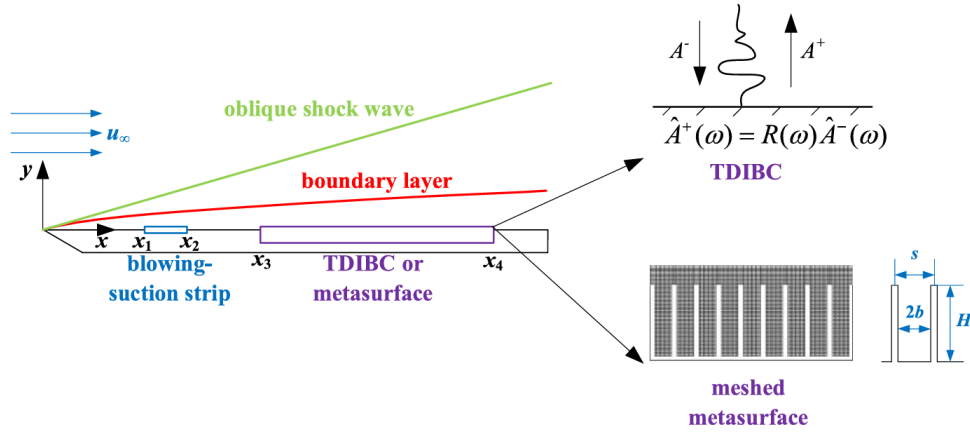


FIGURE 1.11: Schematic drawing of the physical problem of boundary-layer transition control using the acoustic metasurface [11].

Although direct numerical simulation (DNS) incorporating broadband disturbances with actual metasurface microstructures is feasible, it remains computationally prohibitive. To efficiently simulate multi-mode interactions between acoustic metasurfaces and disturbance fields, impedance boundary conditions offer a practical alternative.

Previous acoustic impedance models for metasurfaces, developed in the frequency domain for single-frequency disturbances [69], require extension for hypersonic boundary layer transition studies. To enhance the multi-pole broadband impedance model, Li *et al.* [78] developed piecewise linear recursive convolution technique for integration into the DNS solver. It should be noted that Li *et al.*'s model was derived from linearized Euler equations assuming parallel mean flow, whereas viscous effects become essential for metasurfaces with microstructures. Recent investigations have demonstrated the application of time-domain impedance boundary condition (TDIBC) in hypersonic boundary layer studies [11, 79], as illustrated in Figure 1.11. These studies have established TDIBC's effectiveness in suppressing broadband disturbances, particularly for high-frequency components. However, existing analyses remain confined to linear stability regimes, leaving the nonlinear evolution and breakdown stages unexplored. Consequently, comprehensive three-dimensional direct numerical simulations are required to fully reveal the metasurface's role in transition processes, with particular emphasis on elucidating the nonlinear interactions among multiple primary disturbances.

1.4.2 Other control strategies

Wall cooling or heating represents another viable active control strategy. Masad & Abid [21] demonstrated that wall cooling stabilizes the first mode while destabilizing the second. A similar result was confirmed in DNS with local wall cooling and heating strips [66]. Their energy budget analysis revealed that this stabilization mechanism depends critically on mean shear and Reynolds thermal stress. For nonmodal perturbations in the entropy layer, temperature fluctuations dominate. Thus, wall cooling/heating likely plays a pivotal role in controlling such perturbations, though its efficacy requires further investigation.

Suction, another widely adopted control method, has been shown to attenuate optimal disturbance energy [80]. Nonmodal perturbations in blunt-tip boundary layers exhibit similarities to those in flat-plate boundary layers. Consequently, blow-suction could suppress perturbation growth, thereby delaying the transition. Hader & Fasal [81] recently demonstrated that blowing/suction strips effectively delay hot streaks triggered by fundamental resonance. Notably, simulations with multiple control strips revealed a cumulative suppression effect. The hot streaks and thus transition were eliminated within the computational domain with three strips.

In supersonic flow, the leading-edge geometry can significantly alter the resulting bow shock and the intensity of the off-shock entropy layer, thereby affecting flow instability and the transition process. According to the research of Ba *et al.* [82], the receptivity coefficient for the first and second instability modes vary depending on the leading-edge geometry. As demonstrated by Borovoy *et al.* [83], the transition location varies with different leading-edge geometries, including cylindrical, elliptical, flat face, and "smoothed" cylindrical. Moreover, Kennedy *et al.* [6] reported that instability characteristics and transition scenarios change with increasing nose radius.

1.5 Aims and outline

Boundary-layer instability and transition control have emerged as critical topics in the development of next-generation hypersonic vehicles. Instabilities and potential control strategies over boundary-layer flow are of interest in this study. While progress has been made in understanding the receptivity mechanisms and linear growth laws of unstable modes, several fundamental questions remain unresolved. These include: (1) the energy source sustaining modal disturbance

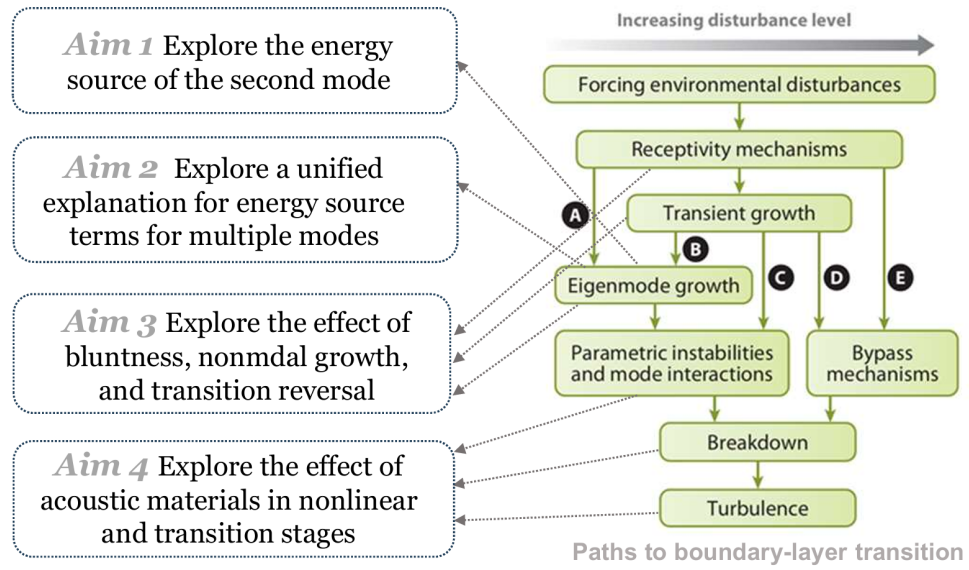


FIGURE 1.12: A flowchart depicting the connection between the research aims and paths to boundary-layer transition. The right part is based on the figure in Ref. [2].

growth of the second mode, (2) the effects of wall temperature, obliqueness, Mach number, and acoustic metasurface on energy sources, (3) the effects of nonmodal growth and bluntness, and the reason for "transition reversal", and (4) the effect of acoustic materials in nonlinear and transition stages initialised by multiple primary disturbances. In this study, four research aims are proposed to explore instabilities and potential control strategies over boundary-layer flows. Their connection with paths to boundary-layer transition is depicted in figure 1.12.

- *Aim 1:* Explore the underlying physics of the growth of the second modes in hypersonic boundary layers by comparing the formulation and physical nature of dominant sources terms in relative phase analysis, momentum potential theory, and Lagrangian equations.
- *Aim 2:* Investigate the dominant energy sources terms for T–S mode, oblique first mode, second mode, and supersonic mode. To propose a unified energy explanation by evaluating the effect of Mach number, wall temperature, obliqueness, and the acoustic metasurface on the dominant energy source terms.
- *Aim 3:* Explore the optimal responses and growth patterns in hypersonic blunt-wedge flow. Explore the nonmodal growth mechanisms, i.e., Orr/Lift up mechanism in blunt-tip models. Explore the reason for "transition reversal" by considering the receptivity process.
- *Aim 4:* Explore the impact of the acoustic metasurface on the nonlinear mode–mode interaction and the resulting breakdown process, with the main objective of revealing the mechanism of

transition delay by acoustic metasurface subject to multiple primary instabilities.

For Mack's second mode, the momentum potential theory and Lagrangian equations are employed and examined in conjunction with direct numerical simulations. A consistent explanation of energy sources will be provided, considering three influential energy analysis methods. For other modal disturbances, the phase of each term in the linear stability equation is examined to identify the significant physical sources for a wide range of Mach numbers and wall temperature ratios, including the classical unstable modes such as T-S mode, the first mode, the second mode, and the supersonic mode. In the meantime, the effect of obliqueness and porous coating will be examined by phase analysis. As for nonmodal disturbances, resolvent analysis and parabolized stability equation analysis are performed to investigate the response pattern, the parametric dependence, and the possible control strategy of the nonmodal instability over blunt tip wedges for hypersonic flow. Regarding the nonlinear mode-mode interaction and breakdown process, the transition induced by jointly low-frequency first mode and high-frequency second mode is focused, the effect of the acoustic metasurface in the whole process is evaluated in detail, and the transition delay mechanism is uncovered.

The structure of this thesis is organized as follows and illustrated in figure 1.13. To achieve research aims 1-4, formulation deduction, stability analysis, and direct numerical simulation are conducted. For the problem under consideration, the base flow is initially investigated. Following this, stability analyses are conducted to elucidate the unstable characteristics of the system. Meanwhile, unsteady flow can be initiated through the introduction of external forcings, such as the blowing-suction actuator. The analysis of energy sources is grounded in the unsteady characteristics of the normal modes. A specific boundary condition, namely a slow acoustic wave, is introduced in the receptivity simulation to mimic the noisy wind tunnel condition. Additionally, the transition control effect of the acoustic metasurface is examined using a modelled impedance boundary condition.

Chapter 2 describes the approach and analysis tools applied in the present study, which include DNS, LST, parabolized stability equation (PSE), and resolvent analysis. Besides, the energy budget analysis methods, including RPA, MPT, and Lagrangian equations, are included. Furthermore, the time-domain impedance boundary condition (TDIBC), which models the effect of the acoustic metasurface, is introduced.

Chapter 3 is based on an article published in the *Physics of Fluids*, titled "Consistent energy-based framework of amplification mechanisms for the second mode in hypersonic boundary layers"

Thesis outline

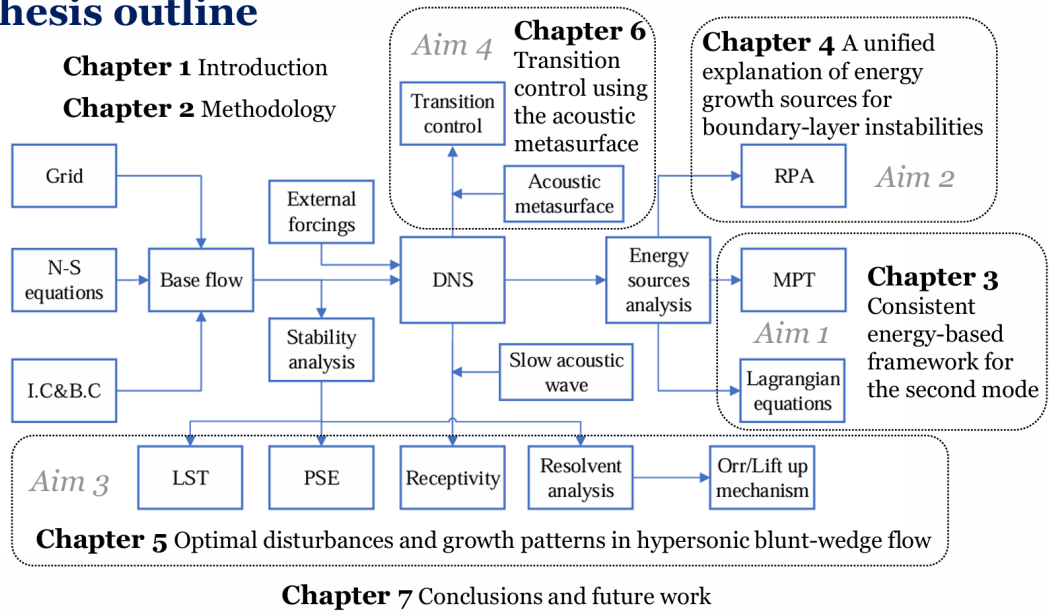


FIGURE 1.13: A flowchart depicting the structure of the present thesis and connections among different chapters regarding the method. Here, I.C. is initial condition, B.C. is boundary condition, DNS is direct numerical simulation, RPA is relative phase analysis, MPT is momentum potential theory, LST is linear stability theory, and PSE is parabolized stability equation.

[84]. This study addresses *Aim 1* by conducting a numerical simulation of hypersonic unsteady flow over a flat plate at a freestream Mach number of 6. Three influential fundamental approaches are employed to investigate the energy source terms that drive the amplification of the second mode: the Lagrangian equations based on Chu's energy density, the MPT based on the transport of total fluctuating enthalpy, and the RPA based on internal energy analysis. Meanwhile, a unified indicator is defined to evaluate the contribution of the significant source terms quantitatively. Depending on the selected energy norm, all three methods consistently indicate the same local energy amplification pathway in the Mach 6 flow state considered. It shows that the second mode is driven by two prominent source terms: the Reynolds thermal stress term, or heat exchange term, in the outer layer region, and the dilatation term in the near-wall region. The mathematical forms of the associated sources are either explicitly described or implicitly included. Results from both simulations and theoretical analyses provide a cohesive understanding of the local energy amplification processes associated with the second mode.

Chapter 4 adapts material from our publication in the *Journal of Fluid Mechanics*, titled "A unified explanation of energy growth sources for unstable modes in flat-plate boundary layers" [75]. This research addresses *Aim 2* through comprehensive LST and DNS analyses of flat-plate boundary layers. To identify the significant physical sources across a wide range of Mach numbers and wall temperature ratios, this study examines the relative phase of the source term in

the linear stability equation. The investigation covers multiple instability modes, including T-S waves, oblique first modes, Mack's second modes, and supersonic modes. A unified framework was revealed through RPA to understand their energy amplification mechanisms. Notably, this RPA framework successfully explains the effects of key parameters including Mach number variation, wall cooling, obliqueness, and acoustic metasurface on instability development.

Chapter 5 is based on an article published in the *Chinese Journal of Aeronautics*, titled "Optimal disturbances and growth patterns in hypersonic blunt-wedge flow" [85]. This study addresses *Aim 3* by conducting resolvent analysis and receptivity simulations on hypersonic blunt wedges with various bluntness. This study investigates the response patterns, parametric dependencies, and impacts of bluntness and wall temperatures on the instabilities of blunt-tip wedges at Mach 5.9. The resolvent analysis and parabolized stability equation are employed, which provide potential control options. Notably, two competing travelling wave growth patterns are identified, which emerge within the boundary layer and the entropy layer, respectively. These patterns are characterized by the intersection of the energy gains of optimal and sub-optimal disturbances. Subsequently, this chapter provides a comprehensive explanation of the nature of optimal disturbances and control methods for the amplification of modal and nonmodal instabilities. Furthermore, a potential explanation for "transition reversal" is proposed by integrating receptivity to slow acoustic waves with resolvent analysis.

Chapter 6 is based on an article submitted to the *Journal of Fluid Mechanics*, titled "Role of acoustic metasurface in the nonlinear mode–mode interaction and breakdown of hypersonic boundary layer" [86]. This study addresses *Aim 4* by conducting stability analysis and numerical simulations on a hypersonic flat plate at Mach 6. In this work, DNS is employed to investigate the transition to turbulence in a hypersonic boundary layer, focusing on the impact of acoustic metasurfaces modelled by a time-domain impedance boundary condition (TDIBC). Both high-frequency second-mode and low-frequency first-mode instabilities evident in the resolvent analysis are introduced in the simulation. The response of mode-mode interactions and the subsequent breakdown mechanisms to the acoustic metasurface is elucidated in DNS for the first time. Furthermore, energy budget and bi-Fourier analyses are employed to evaluate the increased skin friction observed during the late transition period, which has also been reported in wind tunnel studies. Meanwhile, the dominant mode–mode interaction is measured by bispectral mode decomposition. This study elucidates the underlying transition control mechanisms with the acoustic metasurface and the significant nonlinear mode-mode interaction processes.

The thesis concludes in chapter 7 by summarising the key findings from chapters 3-6, along with a brief discussion on the possible future studies that may evolve from the observations reported in this thesis.

Chapter 2

Research methodology

This study contains three kinds of methods to fully understand the instability character and transition process of the boundary layer, namely, direct numerical simulation, stability analysis, and energy source analysis methods. The result of numerical simulation would be utilized to study the evolution of both modal and nonmodal disturbances, and stability tools would be employed to identify the instability character. Moreover, three classical energy sources analysis methods, including relative phase analysis, momentum potential theory, and Lagrangian equations are employed to identify the main energy sources.

2.1 Governing equations

The flow dynamics are governed by the following conservation-form compressible Navier-Stokes equations:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} + \frac{\partial \mathbf{H}}{\partial z} = \mathbf{B} \mathbf{f}', \quad (2.1)$$

where $\mathbf{U} = (\rho, \rho u, \rho v, \rho w, \rho e)^T$ denotes the vector of conservation variables, and $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ represent vectors of inviscid and viscous fluxes. Detailed expressions can be found in Anderson [87]. In addition, ρ represent density, and u , v , and w denote velocity in the x , y , and z directions, respectively. Total energy per unit mass is denoted by e . The matrix $\mathbf{B}$ constrains the forcing vector ($\mathbf{f}'$) to be added at a certain region.

For thermodynamic modeling, the simulation employed an ideal gas approximation with $\gamma = 1.4$. Transport properties were evaluated through Sutherland's viscosity law, with thermal conductivity

being determined by maintaining the Prandtl number at 0.72 throughout the calculations. The calculation is constituted in two steps. In the first step, the base flow is calculated, and the right-hand-side term of equation (2.1) is set to zero. Subsequently, the stability analysis or DNS is performed on the converged base flow. It is assumed that the vector of conservation variable can be decomposed into its base-flow and perturbation parts:

$$\mathbf{U}(x, y, z, t) = \bar{\mathbf{U}}(x, y) + \mathbf{U}'(x, y, z, t). \quad (2.2)$$

Hereafter, the overbar and prime represent mean variables and perturbation variables, respectively. Hereafter, the overbar and prime represent base variables and perturbation variables, respectively. Substituting equation (2.2) into equation (2.1) and then subtracting the base-flow equation and dropping the higher-order terms yields the following linearized governing equation for small disturbances:

$$\frac{\partial \mathbf{U}'}{\partial t} + \frac{\partial \mathbf{F}'}{\partial x} + \frac{\partial \mathbf{G}'}{\partial y} + \frac{\partial \mathbf{H}'}{\partial z} = \mathbf{B}\mathbf{f}', \quad (2.3)$$

2.2 Computational method and boundary conditions

As for two-dimensional simulation (Chapter 3-5), the base flow is calculated using an in-house multi-block parallel finite-volume solver called PHAROS, which has been successfully employed in many configurations, such as compression corner, double wedge, and double cone models [88–90]. The inviscid flux is calculated by a modified Steger-Warming scheme, and the second-order central difference scheme is utilized for viscous flux calculation. Pseudo-time advancing was accomplished by the implicit line relaxation method. More detailed information for the PHAROS can be found in the Ref. [90]. Here, the left boundary of the computational domain is symmetric for blunt-tip cases, while the freestream condition is applied for flat plate cases. In addition, the wall was set to be non-slip and isothermal or adiabatic, depending on the case.

As for three-dimensional simulation (Chapter 6), the DNS is performed using a multi-block parallel finite difference solver called OpenCFD, which has been successfully employed in high-speed transitional and turbulent simulations [91, 92]. The inviscid flux splitting is implemented using the Steger-Warming scheme, and a seven-order weighted essential non-oscillation (WENO) scheme is employed for the reconstruction of the split flux. The sixth-order central difference scheme is utilized for viscous flux discretization, and a third-order Runge-Kutta method is employed for temporal integration. With regard to the boundary conditions, the left boundary of

the computational domain is set to freestream, and the wall is configured to be isothermal, non-slip, and no-penetration, or to use TDIBC in a certain region. The upper and outer boundaries are addressed by extrapolation, while the spanwise boundary is set to be periodic.

2.3 Stability tools

In this Section, the stability analysis methods, including linear stability theory, parabolic stability equation, and resolvent analysis are introduced. Specifically, the linear stability theory (LST) has a parallel base flow assumption and only consider small amplitude of disturbances, and the parabolic stability equation (PSE) can cover small nonparallel flow and nonlinear effects. As for resolvent analysis which aim at finding the response of the system that maximizes the energy gain, can capture both modal and nonmodal disturbances (transient growth).

2.3.1 Linear stability theory

For instabilities in hypersonic boundary-layer flow, the first and second modes cloud be the most energetic unstable modes that lead to the transition with low background noisy and axisymmetric configuration, depending on the Mach number and wall temperature conditions. The profiles of these modes can be obtained by solving the local eigen-value system, starting from the governing equation of small-amplitude perturbations with linear and parallel based flow assumption, i.e., equation 2.3. In that formulation, the physical quantity is decomposed into a time-averaged base-flow term and a small disturbance. The base flow is assumed to be a self-similar boundary layer. The disturbance is assumed to possess the form of a travelling wave as

$$\phi = \bar{\phi} + \phi', \phi' = \hat{\phi}(y)e^{i(\alpha x + \beta z - \omega t)} + \text{c.c.} \quad (2.4)$$

Where $\phi = (u, v, w, T, p)^T$ and $\hat{\phi}$ represents the corresponding complex eigenfunction. In addition, T and p denote temperature and pressure, respectively. The solution form depends on time t , angular frequency ω , and spatial wavenumbers (α : streamwise, β : spanwise), and c.c. indicates the complex conjugate component. In the spatial analysis adopted in this study, α is complex, and its image part refers to the spatial growth rate (the mode solution with negative α_i is an exponential growth unstable wave). In this research, β and ω are real numbers. Substituting equation 2.4 into equation 2.3, neglecting the nonparallel base flow terms and terms representing

the nonlocal effect of shape function and streamwise wavenumber, we may obtain the governing equation of LST. The boundary condition of the eigen-value system are:

$$\begin{aligned}\hat{u} = \hat{w} = \hat{T} = 0, \hat{v} = A\hat{p}, y = 0 \\ \hat{u} = \hat{v} = \hat{w} = \hat{T} = 0, y \rightarrow \infty\end{aligned}\tag{2.5}$$

where A denotes the admittance of porous coatings (metasurface). For the smooth solid wall, A equals to zero.

2.3.2 Parabolic stability equation

To confirm the appearance of modal disturbance obtained by resolvent analysis, LST and PSE analysis were employed. Specifically, the LST provides the initial inlet profiles (eigen functions solved by linear perturbation equations with small amplitude assumption) for PSE, and PSE can obtain the evolution of modal disturbance without parallel basic flow assumption. In PSE, the disturbance is expressed by:

$$\boldsymbol{\psi}'(x, y, z, t) = \hat{\boldsymbol{\psi}}(x, y) \exp\left(i \int_{x_0}^x \alpha d\tilde{x} + i\beta z - i\omega t\right),\tag{2.6}$$

where the $\boldsymbol{\psi} = (\rho, u, v, w, T)^T$, $\hat{\boldsymbol{\psi}}$ and α are the shape function and the complex streamwise wavenumber, respectively, the superscript "T" denotes the transpose, and x_0 is the inflow location of the PSE. Substituting equation (2.6) into (2.3), we may obtain the governing equation of linear PSE (LPSE).

$$(\mathbf{\Gamma}_0 + \mathbf{\Gamma}_1)\hat{\boldsymbol{\psi}} + \mathbf{\Gamma}_2 \frac{\partial \hat{\boldsymbol{\psi}}}{\partial x} + \frac{\partial \alpha}{\partial x} \mathbf{\Gamma}_3 \hat{\boldsymbol{\psi}} = \mathbf{0}.\tag{2.7}$$

The effects of locally parallel flow, non-parallel base flow, non-local shape function, and non-local streamwise wavenumber, respectively, give birth to the base-flow-related operators $\mathbf{\Gamma}_0$, $\mathbf{\Gamma}_1$, $\mathbf{\Gamma}_2$, and $\mathbf{\Gamma}_3$. An eigenvalue issue related to LST will be produced by keeping only the local operator $\mathbf{\Gamma}_0$. The detailed formulation was given by Paredes [93]. The calculation was performed by in-house code CHASES, which integrates the LST, LPSE, NPSE (nonlinear PSE), and sensitivity analyses. The code has been validated by a series of cases for LST and LPSE compared with both theoretical [94–96] and DNS [8, 97] results. The detailed computational scheme can be found in the above-mentioned references.

2.3.3 Resolvent analysis

Equation (2.3) can be reformulated as:

$$\frac{\partial \mathbf{U}'}{\partial t} = \mathbf{A}\mathbf{U}' + \mathbf{B}\mathbf{f}'. \quad (2.8)$$

Where matrix $\mathbf{A}$ is the Jacobian matrix constituted by the base-flow variables. The harmonic assumption was utilized for small amplitude perturbation vector

$$\mathbf{U}'(x, y, z, t) = \hat{\mathbf{U}}(x, y) \exp(i\beta z - i\omega t) + \text{c.c.} \quad (2.9)$$

Where $\hat{\mathbf{U}}$ is the complex eigenfunction, β is the spanwise wave number, ω is the angular frequency, and c.c. denotes complex conjugate. Similar, the harmonic forcing can be written as

$$\mathbf{f}'(x, y, z, t) = \hat{\mathbf{f}}(x, y) \exp(i\beta z - i\omega t) + \text{c.c.} \quad (2.10)$$

Substituting equations (2.9) and (2.10) into (2.8), it gives

$$\hat{\mathbf{U}} = \mathbf{R}\mathbf{B}\hat{\mathbf{f}}, \quad \mathbf{R} = (-i\omega\mathbf{I} - \mathbf{A})^{-1}. \quad (2.11)$$

where $\mathbf{R}$ represent response matrix, indicating the relationship between external forcing and linear response of the system. Here, the identity operator is represented by $\mathbf{I}$. The resolvent framework identifies maximum achievable energy amplification, i.e. the optimal gain, which can be evaluated through parametric space constituted by β and ω of the input (forcings) and output (response) of the systems

$$\sigma^2(\beta, \omega) = \max_{\hat{\mathbf{f}}} \frac{\|\hat{\mathbf{U}}\|_E}{\|\mathbf{B}\hat{\mathbf{f}}\|_E}. \quad (2.12)$$

Where E is the integral over the whole domain. Here, Chu's energy [98] is utilized for the calculation of energy norm

$$\|\hat{\mathbf{U}}\|_E = \iint_{\Omega} \mathbf{U}^\perp \mathbf{M} \hat{\mathbf{U}} dx dy, \quad (2.13)$$

where Ω represents the computational domain for resolvent analysis, the superscript $\perp$ refers to the conjugate transpose, and $\mathbf{M}$ is the weight operator given by Bugeat *et al.* [99]. The dimensionless expression of Chu's energy at a local station is

$$E_{Chu}(x) = \frac{1}{2} \int_0^\infty \left[\bar{\rho}(u'^2 + v'^2 + w'^2) + \frac{\bar{T}}{\gamma M a_\infty^2 \bar{\rho}} \rho'^2 + \frac{\bar{\rho}}{\gamma(\gamma - 1) M a_\infty^2 \bar{T}} T'^2 \right] dy. \quad (2.14)$$

The optimisation problem in equation (2.12) can be transformed into an eigenvalue problem, as demonstrated by Sipp & Marquet [100]. The discretized eigenvalue problem is numerically solved via ARPACK's Arnoldi iteration algorithm for prescribed spanwise wavenumber β and angular frequency ω corresponding to regular instability modes [101]. The optimal forcing is utilised to initiate the unsteady flow in direct numerical simulation. The in-house code has been validated in our previous research [102]. The numerical implementation of our resolvent analysis framework, including solver validation and verification procedures, follows the approach developed by Hao *et al.* [88].

2.4 Energy analysis method

In this Section, three influential energy sources analysis methods, including relative phase analysis, momentum potential theory, and Lagrangian equation are introduced.

2.4.1 Relative phase analysis

Relative phase analysis (RPA) employed here is initially utilized to interpret the generation of acoustic waves in the Rijke tube (Rayleigh). Similar to the generation of acoustic waves in the Rijke tube, the second mode behaves as a trapped wave between the relative sonic line and the wall. Consequently, the resonance mechanism plays a vital role in sustaining its exponential energy growth. From the resonance standpoint, it is contended that the reliance on phase surpasses that of amplitude in terms of its significance. Specifically, only those source terms that are in phase with the left-hand side term are deemed to contribute to the amplification of the corresponding momentum or internal perturbation energy. Tian & Wen [60] analysed the local phase difference between the rate of change of fluctuations and right-hand-side source terms. The purpose is to identify the pronounced sources and explain the growth mechanisms of the second mode.

To obtain the phase, linear stability analysis is performed based on the LST introduced in Section 2.3.1. The Reynolds number $Re = \rho_{\infty}^* u_{\infty}^* l^* / \mu_{\infty}^*$ is defined based on the length scale $l^* = \sqrt{\mu_{\infty}^* x^* / (\rho_{\infty}^* u_{\infty}^*)}$, where μ^* is the dynamic viscosity. This length scale is also used for nondimensionalisation in this paper. In the present study, the source terms for the momentum and internal energy fluctuations based on the linear stability theory are concentrated on. The

base spanwise velocity is set to $\bar{w} = 0$. The governing equations of LST, mainly the momentum equations in x and y directions and the internal energy equation can be written as:

$$\underbrace{i(\alpha\bar{u} - \omega)\frac{\hat{u}}{\bar{T}}}_{\bar{\rho}Du'/Dt} = \underbrace{-\frac{d\bar{u}}{dy}\frac{\hat{v}}{\bar{T}}}_{T_{u,1}} \underbrace{-i\alpha\hat{p}}_{T_{u,2}} + T_{u,3}, \quad (2.15)$$

$$\underbrace{i(\alpha\bar{u} - \omega)\frac{\hat{v}}{\bar{T}}}_{\bar{\rho}Dv'/Dt} = \underbrace{-\frac{d\hat{p}}{dy}}_{T_{v,1}} + T_{v,2}, \quad (2.16)$$

$$\underbrace{i(\alpha\bar{u} - \omega)\frac{\hat{T}}{\bar{T}}}_{\bar{\rho}DT'/Dt} = \underbrace{-\frac{d\bar{T}}{dy}\frac{\hat{v}}{\bar{T}}}_{T_{T,1}} \underbrace{-(\gamma - 1)(i\alpha\bar{u} + \frac{d\hat{v}}{dy} + i\beta\hat{w})}_{T_{T,2}} + T_{T,3}, \quad (2.17)$$

where $\bar{\rho}Du'/Dt$, $\bar{\rho}Dv'/Dt$, and $\bar{\rho}DT'/Dt$ represent the rates of change in fluctuations of streamwise velocity, wall-normal velocity, and internal energy, respectively. The right-hand-side sources $T_{u,1}$ and $T_{T,1}$ represent the production by mean shear and mean temperature gradient, which are associated with Reynolds shear stress (RSS) and Reynolds thermal stress (RTS), respectively. $T_{u,2}$, $T_{v,1}$ and $T_{T,2}$ represent the streamwise gradient of pressure fluctuations, the normal gradient of pressure fluctuations and the dilatation fluctuations, respectively. $T_{u,3}$ and $T_{v,2}$ refer to viscous stresses, and $T_{T,3}$ corresponds to the viscous dissipation and thermal conduction. The specific expression of $T_{u,3}$, $T_{v,2}$, and $T_{T,3}$ refer to the Appendix 2.5.1. The linear stability analysis is performed by the in-house code, which has been fully validated by comparison with theoretical, computational, and experimental results.

In the framework of the relative phase analysis, the left-hand-side Lagrangian rate of change in the momentum or internal energy fluctuation can be regarded as the ‘output’ of the linear system. The right-hand-side terms are the source ‘inputs’ induced by the interaction between the base flow and the existing fluctuation itself, such as the mean shear and the wall-normal velocity fluctuation in $T_{u,1}$. With appropriate phase angles of local source inputs (such as dilatation in near-wall regions), sustainable energy growth can be excited. These eligible inputs that force the outputs to be in a coherent phase are regarded as ‘significant’. The phase analysis is applied to identify which term or terms are significant at different wall-normal heights of the laminar boundary layer. The corresponding results may provide possible insights into what source or sources should be controlled for different types of instability modes. Meanwhile, the crucial region of the laminar boundary layer may be identified for control.

2.4.2 Momentum potential theory

For uniform base flow field, Kovasznay [61] decomposed the fluctuation into three components, i.e., vortical, acoustic, and thermal (or entropic) characteristics. When it comes to non-uniform baseflow, the gradient of velocity and temperature resulting from viscosity will attribute to the coupling between all three Kovasznay's modes, making it hard to extend the application of Kovasznay's decomposition to the problem in hypersonic boundary layers with strong mean flow gradients. The MPT framework developed by Doak [62] successfully circumvents the constraints present in Kovasznay's approach when applied to stationary general flow conditions. In the framework of MPT, the flow field is assumed to be time-stationary. For such cases, any physical variable may be represented as a superposition of its mean value and temporal fluctuations. The fluctuating momentum density, $\mathbf{m}' = (\rho \mathbf{u})'$, is split into its vortical, acoustic, and thermal components, where $\mathbf{u}$ is a velocity vector. Accordingly, the fluctuating momentum density $\mathbf{m}'$ consists of three physically distinct modes: vortical component $\mathbf{m}'_B$, the acoustic component $\mathbf{m}'_A$, and the thermal component $\mathbf{m}'_T$:

$$\mathbf{m}' = \mathbf{m}'_B + \mathbf{m}'_A + \mathbf{m}'_T. \quad (2.18)$$

Vortical component $\mathbf{m}'_B$ is solenoidal, isobaric, and isentropic.

The acoustic component $\mathbf{m}'_A$ corresponds to the isentropic, irrotational portion of momentum fluctuations, directly coupled with pressure variations p' . Meanwhile, the thermal component $\mathbf{m}'_T$ embodies the isobaric, irrotational part associated with entropy fluctuations s' in the flow field. The explicit expressions of the three components can be found in the reference [64]. Consider the conservation equation of the momentum fluctuation:

$$\frac{\partial \mathbf{u}'}{\partial t} + \nabla H' = -\boldsymbol{\alpha}', \quad (2.19)$$

where H' denotes the total fluctuating enthalpy (TFE) per unit mass and $\boldsymbol{\alpha}'$ is the "acceleration" vector:

$$H' = \left(\frac{\mathbf{u}^2}{2} + c_p T \right)', \quad \boldsymbol{\alpha}' = (-\boldsymbol{\Omega} \times \mathbf{u} + T \nabla s + \frac{1}{\rho} \nabla \cdot \bar{\bar{\mathbf{S}}})', \quad (2.20)$$

where c_p represents specific heat at constant pressure, $\mathbf{\Omega}$ is vorticity, and $\bar{\mathbf{S}}$ is the viscous stress tensor. The mean transport equation of the TFE due to $\mathbf{m}'$ can be obtained as:

$$\nabla \cdot (\overline{\mathbf{m}'H'}) = -(\overline{\mathbf{m}'\alpha'}) + \frac{\overline{p'} \partial s'}{R \partial t}. \quad (2.21)$$

where s denotes entropy and R represents gas constant.

2.4.3 Lagrangian equations

Since the second mode belongs to the family of inviscid Mack modes, the inviscid Lagrangian framework is applicable. The inviscid linearized N-S equation under the locally parallel-flow assumption can be expressed by

$$\frac{\partial \rho'}{\partial x} + v' \frac{\partial \bar{\rho}}{\partial y} + \bar{\rho} \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial x} \right) = 0, \quad (2.22)$$

$$-\frac{\partial u'}{\partial t} + \bar{u} \frac{\partial u'}{\partial x} + v' \frac{\partial \bar{u}}{\partial y} = -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial x}, \quad (2.23)$$

$$-\frac{\partial v'}{\partial t} + \bar{u} \frac{\partial v'}{\partial x} = -\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial y}, \quad (2.24)$$

$$\frac{\partial s'}{\partial t} + \bar{u} \frac{\partial s'}{\partial x} + v' \frac{\partial \bar{s}}{\partial y} = 0. \quad (2.25)$$

The relationship between the entropy fluctuation and the pressure and density fluctuations is given by

$$s' = c_v \frac{p'}{\bar{p}} - c_p \frac{\rho'}{\bar{\rho}}. \quad (2.26)$$

Substituting equation (2.26) into equation (2.25) yields

$$\frac{c_p}{\bar{\rho}} \frac{\partial \rho'}{\partial t} + \bar{u} c_p \frac{\partial}{\partial x} \left(\frac{\rho'}{\bar{\rho}} \right) + \frac{v' c_p}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial y} = \frac{c_v}{\bar{p}} \frac{\partial p'}{\partial t} + \bar{u} c_v \frac{\partial}{\partial x} \left(\frac{p'}{\bar{p}} \right) + \frac{v' c_v}{\bar{p}} \frac{\partial \bar{p}}{\partial y}, \quad (2.27)$$

where c_v represents specific heat at constant volume. With the parallel base flow assumption, equation (2.27) can be rewritten as

$$\frac{\partial \rho'}{\partial t} + \bar{u} \frac{\partial \rho'}{\partial x} + v' \frac{\partial \bar{\rho}}{\partial y} = \frac{1}{\bar{a}^2} \left(\frac{\partial p'}{\partial t} + \bar{u} \frac{\partial p'}{\partial x} \right). \quad (2.28)$$

Here, $\bar{a}$ denotes the local speed of sound. Combining equation (2.22) and equation (2.28) gives rise to

$$\frac{1}{\bar{\rho}\bar{a}^2}\left(\frac{\partial p'}{\partial t} + \bar{u}\frac{\partial p'}{\partial x}\right) + \left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}\right) = 0. \quad (2.29)$$

Then, taking $u' \cdot$ equation (2.23) + $v' \cdot$ equation (2.24) gives

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\frac{\bar{\rho}(u'u' + v'v')}{2} + u'v'\bar{\rho}\frac{\partial \bar{u}}{\partial y} = u'\frac{\partial p'}{\partial x} + v'\frac{\partial p'}{\partial y} \quad (2.30)$$

Finally, combining equation (2.29) and equation (2.30) and taking time average, we obtain

$$\bar{u}\frac{\partial}{\partial x}\underbrace{\left(\bar{\rho}\frac{u'u' + v'v'}{2} + \frac{p'p'}{2\bar{\rho}\bar{a}^2}\right)}_{\text{PAE}} = \underbrace{-\frac{\partial(p'u')}{\partial x} - \frac{\partial(p'v')}{\partial y}}_{\text{DAP}} - \underbrace{u'v'\bar{\rho}\frac{\partial \bar{u}}{\partial y}}_{\text{RSS}}. \quad (2.31)$$

The left-hand side of equation (2.31) represents the rate of change in particle acoustic energy (PAE). The first two terms on the right-hand side represent the divergence of acoustic power (DAP), and the third term on the right-hand side is the Reynolds shear stress (RSS). The above equation is equivalent to the formulation based on the acoustic energy norm by Kuehl [58]. The difference is that RSS is now considered in the present paper.

2.5 Appendix

2.5.1 Detailed formulae in stability equations

The expressions of $T_{u,3}$, $T_{v,2}$ and $T_{T,3}$ are shown as follows, where Stokes' hypothesis for the bulk viscosity is applied:

$$\begin{aligned} T_{u,3} = & \frac{\bar{\mu}}{Re} \left[-\frac{4}{3}\alpha^2\hat{u} + \frac{1}{3} \left(i\alpha\frac{d\hat{u}}{dy} - \alpha\beta\hat{w} \right) + \frac{d^2\hat{u}}{dy^2} - \beta^2\hat{u} \right] \\ & + \frac{1}{Re} \left[\frac{d\bar{\mu}}{dy} \left(\frac{d\hat{u}}{dy} + i\alpha\hat{v} \right) + \frac{d\bar{\mu}}{dT} \left(\frac{d^2\bar{u}}{dy^2}\hat{T} + \frac{d\bar{u}}{dy}\frac{d\hat{T}}{dy} \right) \right. \\ & \left. + \frac{d^2\bar{\mu}}{dT^2} \frac{d\bar{T}}{dy} \frac{d\bar{u}}{dy} \hat{T} \right], \end{aligned} \quad (2.32)$$

$$\begin{aligned} T_{v,2} = & \frac{\bar{\mu}}{Re} \left[-\alpha^2\hat{v} + \frac{1}{3} \left(i\alpha\frac{d\hat{u}}{dy} + i\beta\frac{d\hat{w}}{dy} \right) + \frac{4}{3}\frac{d^2\hat{v}}{dy^2} - \beta^2\hat{v} \right] \\ & + \frac{1}{Re} \left[\frac{d\bar{\mu}}{dT} \left(i\alpha\frac{d\bar{u}}{dy} + i\beta\frac{d\bar{w}}{dy} \right) \hat{T} - \frac{2}{3}\frac{d\bar{\mu}}{dy} \left(i\alpha\hat{u} + i\beta\hat{w} - 2\frac{d\hat{v}}{dy} \right) \right], \end{aligned} \quad (2.33)$$

$$\begin{aligned}
T_{T,3} = & \frac{\bar{\mu}\gamma}{RePr} \left[-\alpha^2 \hat{T} + \frac{d^2 \hat{T}}{dy^2} - \beta^2 \hat{T} + \frac{2}{\bar{\mu}} \frac{d\bar{\mu}}{dy} \frac{d\hat{T}}{dy} \right] \\
& + \frac{\gamma}{RePr} \frac{d^2 \bar{\mu}}{dy^2} \hat{T} + (\gamma - 1) M^2 \frac{1}{Re} \frac{d\bar{\mu}}{d\bar{T}} \left[\left(\frac{d\bar{u}}{dy} \right)^2 + \left(\frac{d\bar{w}}{dy} \right)^2 \right] \hat{T} \\
& + 2(\gamma - 1) M^2 \frac{\bar{\mu}}{Re} \left[\frac{d\bar{u}}{dy} \left(\frac{d\hat{u}}{dy} + i\alpha\hat{v} \right)^2 + \frac{d\bar{w}}{dy} \left(i\beta\hat{v} + \frac{d\hat{w}}{dy} \right) \right].
\end{aligned} \tag{2.34}$$

Chapter 3

Consistent energy-based framework of amplification mechanisms for the second mode

[Published paper I¹]

3.1 Preface to the chapter

This chapter is reorganized based on the conference published in the *Physics of Fluids*, titled "Consistent energy-based framework of amplification mechanisms for the second mode in hypersonic boundary layers" [84]. The hypersonic boundary layer instability has been a hot topic in fluid mechanics in recent years, especially the second-mode instability. However, a long-term debate exists on the detailed energy sources that sustain the modal exponential growth. In this chapter, three fundamental approaches are employed and examined with direct numerical simulations. These include the relative phase analysis based on internal energy analysis, the momentum potential theory based on the transport of total fluctuating enthalpy, and the Lagrangian equations based on Chu's energy density. The present research focuses on the simulation and analysis of a hypersonic (Mach 6) boundary layer developing over a flat plate. The main finding of this chapter is that the three popular fundamental approaches agree on the energy amplification mechanism

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of the second mode. The explicit expression of each energy source is derived, and despite the differences in mathematical form, the physical implications are consistent. It is believed that this chapter has provided a comprehensive and in-depth understanding of the growth mechanism of the second mode.

3.2 Abstract

The second mode is of significant interest in hypersonic boundary layer flows due to its crucial role in the transition to turbulence. However, a longstanding debate exists regarding the detailed energy sources that sustain modal exponential growth. Currently, three influential energy-based approaches reveal different significant energy sources due to their dissimilar mathematical formulations: momentum potential theory, inviscid Lagrangian energy analysis, and relative phase analysis. In this study, these three fundamental approaches are employed and examined in conjunction with direct numerical simulations. The aim is to seek a possible unified explanation for the source terms that dominate the exponential evolution of the second mode. In the considered Mach 6 flow state, all three approaches consistently indicate the same local energy amplification route driven by two pronounced source terms: the dilatation term in the near-wall region and the Reynolds thermal stress term or heat exchange term across the outer layer region, depending on the selection of the specific energy norm. The mathematical forms of the corresponding sources are derived or discussed explicitly. Both theoretical and simulation results provide a unified understanding of the local energy amplification mechanisms of the second mode.

3.3 Introduction

To fully understand the energy production and transfer associated with the second mode, the physics-based feature extraction method [103] provides valuable insights [104, 105]. For a uniform base flow field, Kovasznay [61] decomposed the fluctuation into three components: vortical, acoustic, and thermal (or entropic). In a non-uniform base flow, the gradients of velocity and temperature contribute to the coupling among all three of Kovasznay's modes. This coupling complicates the extension of Kovasznay's decomposition to problems in hypersonic boundary layers with strong mean flow gradients. The momentum potential theory (MPT) proposed by Doak [62, 106] overcomes the limitations of Kovasznay's approach for general time-stationary flows. MPT has been employed to explain the generation process of sound waves in a cold Mach 1.3 jet flow, where its vortical component adequately captures the shear layer growth and breakdown process [63]. A recent investigation by Unnikrishnan and Gaitonde [12] leveraged MPT to decompose the second mode into vortical, acoustic, and entropic elements. These components were observed to interact with the base flow in a well-defined manner, with the entropic component recognized as the most likely source of energy for acoustic perturbation

growth. The findings of Unnikrishnan and Gaitonde are particularly instructive. Subsequently, Long *et al.* [64, 107] derived a more explicit formulation of Doak's energy budget equation, which briefly included the interaction terms among the three MPT components. The effects of various source terms on each energy flux were explicitly shown, and the energy transfer among MPT components was detailed. Vortical-to-acoustic energy conversion processes were particularly enhanced adjacent to the critical layer, creating concentrated regions of positive acoustic sources. The observed outward transport of acoustic perturbations was directly attributed to sound wave radiation caused by supersonic mode oscillations. Although MPT has provided new insights into the boundary layer instability problem, confusion persists due to the complex formulation of this approach. For instance, the pressure dilatation term, which has been shown to be significant in RPA and Lagrangian approaches as well as in experimental results, is not readily apparent in the governing equations of MPT. This disconnects MPT from other well-established methods. Further analysis may yield improved explanations.

Remarkable progress has been made through the analysis of classical acoustic energy [58], relative phase analysis from the perspective of resonance [60, 75], and the feature extraction approach of momentum potential theory [12]. However, existing theories have yet to reach a consensus due to differences in the specific forms of governing equations. Therefore, a more comprehensive and self-consistent explanation of the energy sources responsible for the growth of the second mode is both required and necessary. In the present study, the effects of source terms related to the local destabilization of the second mode are theoretically analyzed alongside direct numerical simulations. Comparative studies on the source terms from three influential theoretical approaches are conducted, and the connections between these approaches are discussed in detail. Ultimately, a unified explanation of significant energy sources for the local amplification of the second mode is deduced. This paper incorporates the MPT tool into the energy-based theoretical framework. Furthermore, a detailed derivation of the Lagrangian equations to include the heat exchange effect is provided for the first time, facilitating a complete consideration from energy perspectives.

The remainder of the paper is organized as follows. Section 3.4 describes the three energy analysis methods. Numerical simulation strategies is introduced in Section 3.5. Section 3.6 presents the base flow and the evolution of the second-mode instability. Specifically, the dominant energy source terms of the second mode are derived or identified for three energy analysis methods. Section 3.7 describes a consistent energy-based framework of amplification mechanisms for the second mode. Section 3.8 gives concluding remarks.

3.4 Revisit of three energy analysis methods

3.4.1 Relative phase analysis

RPA employed here is originally used to interpret how acoustic waves are generated in the Rijke tube through a resonance process [108]. Similarly, the second mode demonstrates reflective acoustic behavior, acting as compression and expansion waves between the relative sonic line and the wall. To identify the pronounced sources and explain the growth mechanisms of the second mode, Tian & Wen [60] employed RPA to study the local phase discrepancy between the rate of change of fluctuations and the right-hand-side source terms. Since the two-dimensional second mode is more energetic than the three-dimensional one, the former is considered in our study. Note that in the spatial analysis, α is a complex number whose imaginary part corresponds to the spatial growth rate, while ω is a real number. The solution of the disturbance can be either extracted from downstream DNS data or provided by LST, where an eigenvalue problem is solved using our well-validated in-house LST code [94–96, 109]. In this chapter, the DNS data is mainly used for RPA, which serves to confirm the LST results by Tian & Wen [60].

Generally, for the second mode under adiabatic wall conditions, the magnitude of the eigenfunction of temperature fluctuation is observed to be significantly larger than that of velocity fluctuation, as reported by Erlebacher & Hussaini [110]. Therefore, in the relative phase analysis, the change of internal energy perturbation is of interest. The reduced dimensionless linearized internal energy equation obtained by Tian & Wen [60] is employed. For reading convenience, it is rewritten in dimensional form as:

$$\underbrace{i(\alpha\bar{u} - \omega)\frac{\hat{T}}{\bar{T}}}_G = \underbrace{i\alpha\bar{u}\frac{\hat{T}}{\bar{T}}}_{G_1} - \underbrace{i\omega\frac{\hat{T}}{\bar{T}}}_{G_2} = \underbrace{-\frac{d\bar{T}}{dy}\frac{\hat{v}}{\bar{T}}}_{P_1} - \underbrace{(\gamma - 1)(i\alpha\hat{u} + \frac{d\hat{v}}{dy})}_{P_2} + \underbrace{\frac{\kappa(\gamma - 1)}{\bar{p}}\frac{d^2\hat{T}}{dy^2}}_{P_3} + \varepsilon, \quad (3.1)$$

where G_1 and G_2 represent the advection and the time rate of change of internal energy perturbation, respectively. Three source terms are denoted as P_1 , P_2 , and P_3 , corresponding to the wall-normal transport of internal energy (related to Reynolds thermal stress), dilatation fluctuation, and thermal conduction, respectively. Additionally, κ denotes the coefficient of heat conduction, and ε represents the residual term, which is negligible. In RPA, the source terms that demonstrate an approaching phase with the left-hand-side term of the governing equation positively contribute to the growth of perturbation energy.

3.4.2 Momentum potential theory

To connect MPT with growth rate of the instabilities of interest, the transportation of TFE is focused. The equation (2.21) can also be written as

$$\frac{\overline{\partial(\rho u)'H'}}{\partial x} + \frac{\overline{\partial(\rho v)'H'}}{\partial y} = -(\overline{m'_B \alpha'} + \overline{m'_A \alpha'} + \overline{m'_T \alpha'}) + \frac{\overline{p'}}{R} \frac{\partial s'}{\partial t}. \quad (3.2)$$

The left-hand side of equation (3.2) represents the transport of TFE by momentum density fluctuations. The first three terms of the right-hand side term represent the source terms due to interactions between α' and different MPT components. The last term on the right-hand side represents the source term due to the thermal diffusion process. Equation (3.2) undergoes y -integration to quantitatively link MPT parameters with second-mode instability growth rates. According to Ref. [107], we have $\overline{\partial(\rho u)'H'/\partial x} = -2\alpha_i \overline{(\rho u)'H'}$, and denote $\int_0^{+\infty} \overline{(\rho u)'H'} dy$ as Q_x . Since the integral of $\overline{\partial(\rho v)'H'/\partial y}$ in the y direction is zero, eventually we may obtain the growth rate of the second mode contributed by different source terms:

$$-\alpha_i = \sigma_B + \sigma_A + \sigma_T + \sigma_s. \quad (3.3)$$

Here,

$$\begin{aligned} \sigma_B &= \frac{\int_0^{+\infty} -\overline{m'_B \alpha'} dy}{2Q_x}, \sigma_A = \frac{\int_0^{+\infty} -\overline{m'_A \alpha'} dy}{2Q_x}, \\ \sigma_T &= \frac{\int_0^{+\infty} -\overline{m'_T \alpha'} dy}{2Q_x}, \sigma_s = \frac{\int_0^{+\infty} \overline{\frac{p'}{R} \frac{\partial s'}{\partial t}} dy}{2Q_x}. \end{aligned} \quad (3.4)$$

3.4.3 Lagrangian equations

Despite the fact that the second mode is of acoustic nature, the effect of the heat exchange term [61] should be important in hypersonic boundary layers. However, this term is neglected in their paper. Considering this heat exchange effect with a complete Chu's energy norm, equation (2.31) is rewritten as:

$$\bar{u} \frac{\partial}{\partial x} \underbrace{\left[\bar{\rho} \frac{u'u' + v'v'}{2} + \frac{p'p'}{2\bar{\rho}\bar{a}^2} + \frac{\gamma-1}{2\gamma} \bar{p} \left(\frac{s'}{R} \right)^2 \right]}_{\text{Chu's energy density}} = \underbrace{-\frac{\overline{\partial(p'u')}}{\partial x} - \frac{\overline{\partial(p'v')}}{\partial y}}_{\text{DAP}} - \underbrace{u'v'\bar{\rho} \frac{\partial \bar{u}}{\partial y}}_{\text{RSS}} - \underbrace{\frac{\bar{p}}{\bar{T}} \frac{\partial \bar{T}}{\partial y} v's'}_{\text{HE}}. \quad (3.5)$$

The third term on the right-hand side represents the effect of heat exchange, noted as HE in the following analysis.

3.5 Numerical setup

This paper investigates the hypersonic boundary layer over a flat plate with a sharp leading edge at a zero attack angle. Two-dimensional patterns of the base flow and disturbances are considered since the most unstable second mode is two-dimensional. The basic state of the hypersonic boundary layer is calculated at a Mach number of 6 and a Reynolds number of 2×10^6 . The wall temperature condition is nearly adiabatic, with $T_w/T_\infty = 7.04$, which is the same as that by Unnikrishnan & Gaitonde [12]. In the dimensionless Navier-Stokes equations, the primitive quantities are normalized by the corresponding freestream quantities, except that the pressure is normalized by $\rho_\infty u_\infty^2$. Additionally, the perfect gas model is adopted with a specific heat ratio of $\gamma = 1.4$ and a Prandtl number of $Pr = 0.72$. The temperature dependence of the dynamic viscosity is modeled by $\mu/\mu_\infty = (T/T_\infty)^{0.7}$.

After the base flow is converged, a wall-blowing-suction actuator [66] is utilized to initiate the second mode. As indicated by Wang & Zhong [111], the actuator upstream of the corresponding synchronization point ($x \sim 0.14$ m) would excite the unstable evolution of the second mode. The fluctuation of the wall-normal mass flow rate q_w is given by

$$q_w(x, t) = \delta \rho_\infty u_\infty \sin(2\pi \frac{x - x_1}{x_2 - x_1}) \sin(\omega t), \quad x_1 \leq x \leq x_2, \quad (3.6)$$

where $x_1 = 0.00716$ m and $x_2 = 0.0099$ m. The dimensionless angular frequency $\omega L/u_\infty$ is chosen to be 260, following Unnikrishnan & Gaitonde [12]. The equivalent dimensional frequency equals 163.5 kHz. The forcing amplitude $\delta = 0.0006$ is adopted to ensure the linear evolution of the disturbances.

Regarding the simulation program, our earlier study [66] has fully validated the finite difference method of the DNS code. Under the same flow conditions, excellent convergence of mesh resolution and time step selection was demonstrated. A fifth-order upwind compact scheme is applied to discretize the inviscid flux derivatives, while a sixth-order central difference method is utilized to discretize the viscous terms. The numerical solution advances in time through a third-order accurate Runge-Kutta approach.

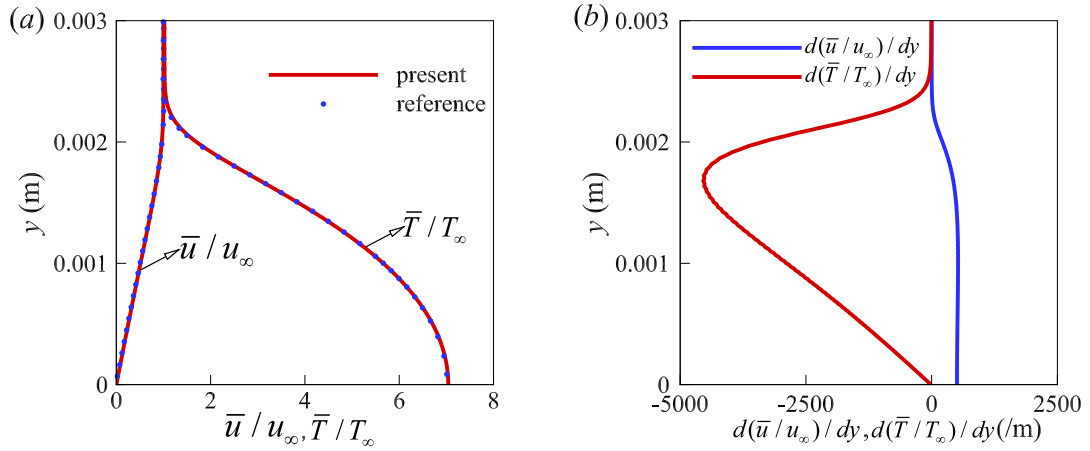


FIGURE 3.1: Mean flow solution of (a) streamwise velocity and (b) temperature at $x = 0.18$ m compared with Ref. [12].

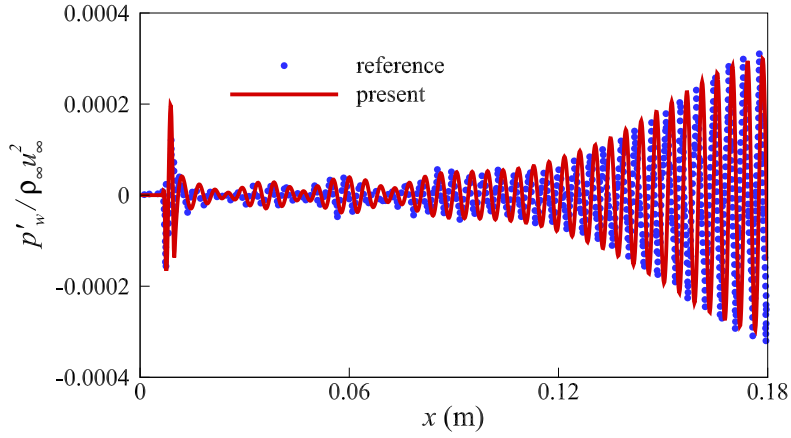


FIGURE 3.2: Instantaneous pressure fluctuations along the wall compared with Ref. [12].

3.6 Results and analysis

3.6.1 Results of base flow and unsteady field

As shown in figure 3.1(a), the base flow solution obtained by the DNS code is in good agreement with that of Unnikrishnan & Gaitonde [12]. Figure 3.1(b) provides the wall-normal gradients of mean streamwise velocity and temperature, which contribute to the formation of Reynolds shear stress and Reynolds thermal stress, respectively. It is indicated that, compared to the gradient of velocity, the temperature gradient is more prominent, with a peak near the boundary layer. The wall pressure fluctuation is also compared with the reference data in figure 3.2. It is shown that the disturbance undergoes exponential growth starting from about $x = 0.1$ m. The consistent results presented, compared with references, verify the eligibility of the computational

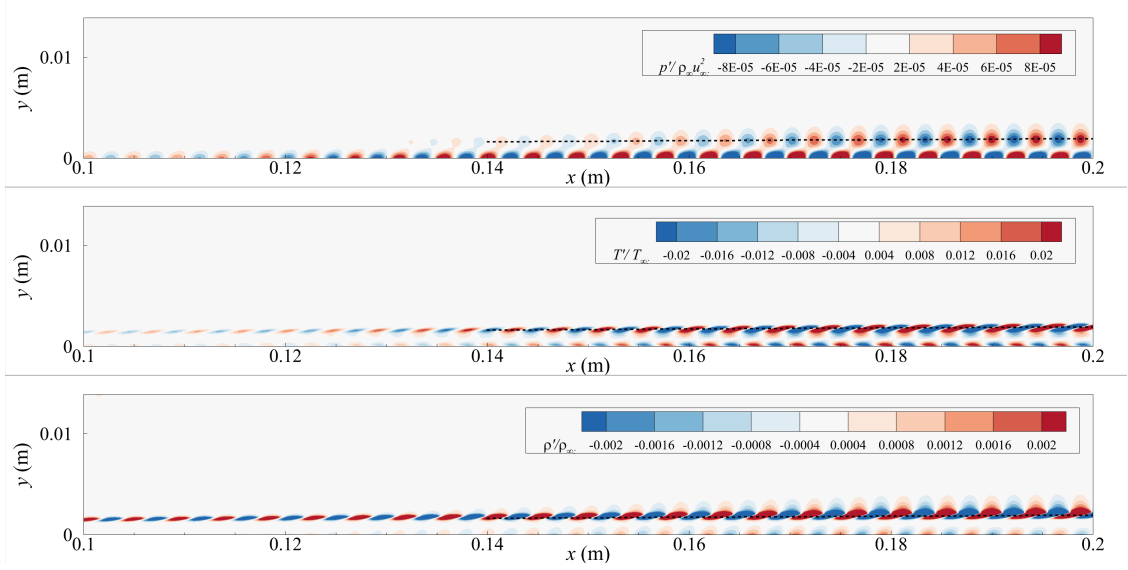


FIGURE 3.3: Contours of pressure (top), temperature (middle), and density (bottom) fluctuations. Dashed line represents the location of the critical layer ($y = y_c$).

grid adopted. Figure 3.3 displays the contours of p' , T' , and ρ' from DNS. Evident amplification is observed mainly in the near-wall region and near the critical layer. Specifically, pressure fluctuations are primarily distributed near the wall, while density and temperature fluctuations tend to be concentrated near the critical layer ($y = y_c$). Here, y_c represents the location of the critical layer where the nondimensional phase speed of the second mode equals the local base streamwise velocity. Therefore, crucial energy sources are likely distributed in these two regions, which are responsible for the growth of perturbation energy of the second modes.

3.6.2 Results of relative phase analysis

Instantaneous DNS data are first transformed into the frequency domain via Fourier transform. For the disturbed result with a period, a temporal fast Fourier transform (FFT) analysis is performed to split the disturbances into harmonic components:

$$\phi'(x, y, t) = \hat{\phi}(x, y)e^{-i\omega t} + c.c. \quad (3.7)$$

Here, ten periods of the harmonic signal are used for FFT. Thus, the second-mode (fixed) frequency is 10 times the resolved lowest frequency. Meanwhile, the second-mode (single) frequency is 1/8 of the Nyquist frequency, which enables the extraction of the desired second-mode component. This setup is sufficient for this simple single-frequency disturbance case. Subsequently, the amplitude and phase profiles of the terms in equation (3.1) are obtained for the

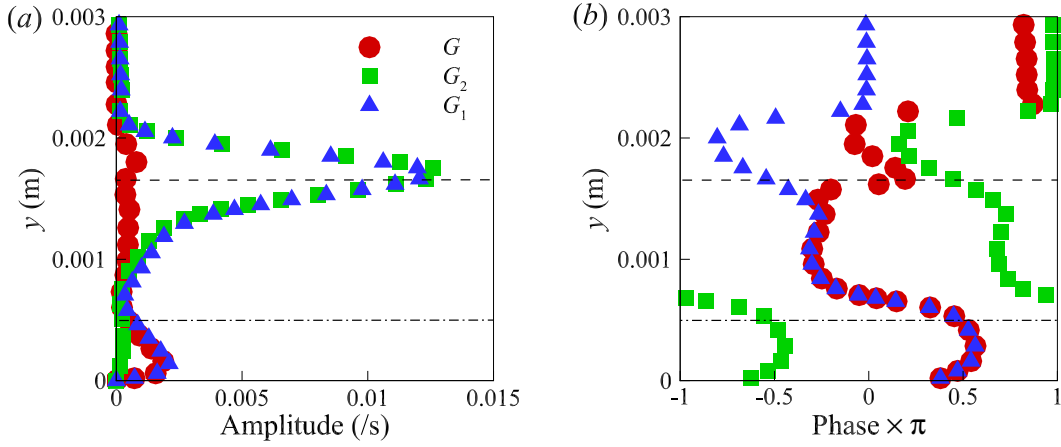


FIGURE 3.4: (a) Amplitude and (b) phase profiles of G , G_1 , and G_2 in equation (3.1) at $x = 0.16$ m. The dashed line indicates y_c , while the dash-dotted line corresponds to $y = 0.3y_c$.

actuator frequency and are depicted in figures 3.4 and 3.5. Figure 3.5 indicates that the phase discrepancy between the time rate of change (G_1) and the advection (G_2) of the internal energy fluctuation remains nearly π , and their amplitudes peak near the critical layer. The antiphase relationship between G_1 and G_2 cancels each other, resulting in a small-amplitude total energy growth. In terms of the relative phase, figure 3.5 demonstrates that the Reynolds-thermal-stress-related term P_1 and the dilatational term P_2 are in phase with the left-hand side G in the near-wall region (between the wall and $y = 0.3y_c$) and in the outer layer (between $y = 0.3y_c$ and $y = y_c$), respectively. Consequently, the main energy sources that sustain the second-mode growth are the dilatation effect (P_2) in the near-wall region and the Reynolds thermal stress effect (P_1) in the outer layer. The distribution of amplitude and phase is consistent with that of Tian & Wen [60] obtained by LST. Therefore, the nonparallel effect is negligible in the phase analysis regarding the second mode.

3.6.3 Results of momentum potential theory

Based on the DNS data, the MPT is employed to analyze the distribution of energy sources and their contribution to the growth rate via the decomposition in equation (3.3). The computational domain has a size of $0.2 \text{ m} \times 0.05 \text{ m}$. Near the upper boundary, the magnitudes of the three components of MPT are found to decay to less than 10^{-6} times the maxima in the local streamwise profile. A sponge layer is also used next to the outflow boundary to minimize the reflection of disturbances [109]. Precursor numerical simulations with an extended computational domain show no visible difference in terms of the MPT decomposition results. Figure 3.6 provides

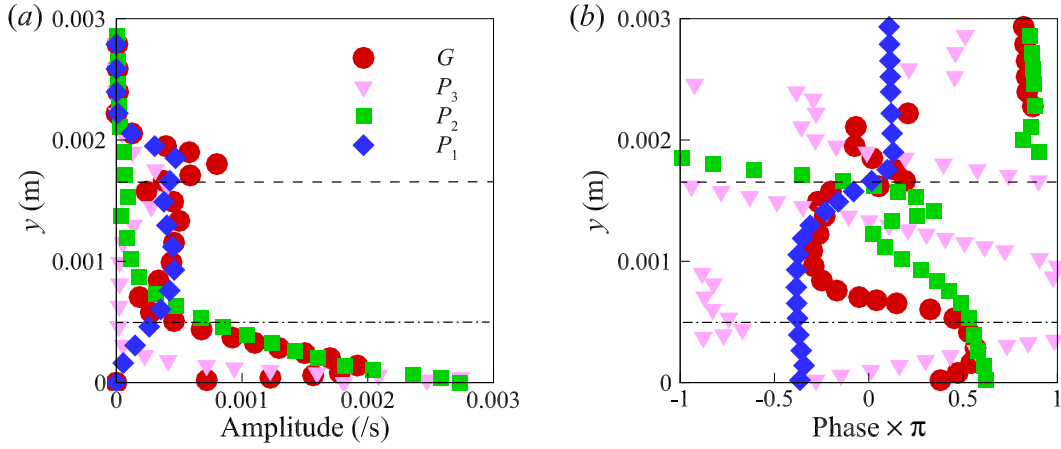


FIGURE 3.5: (a) Amplitude and (b) phase profiles of G , P_1 , P_2 , and P_3 in equation (3.1) at $x = 0.16$ m. The dashed line indicates y_c , while the dash-dotted line corresponds to $y = 0.3y_c$.

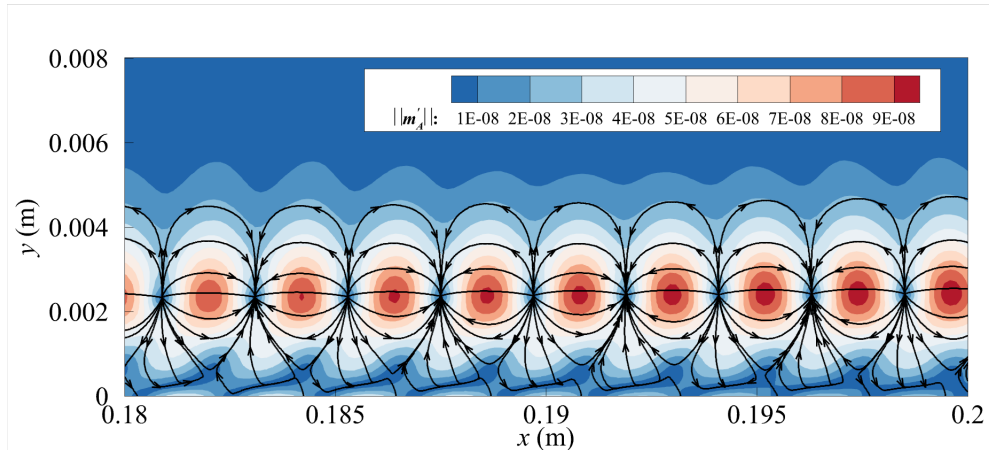


FIGURE 3.6: Flux line and the amplitude magnitude (contour) of the acoustic component ($\text{kg/m}^2/\text{s}$).

the amplitude distribution of the acoustic component and its flux lines, suggesting a waveguide behavior regarding energy transport. This waveguide behavior is most clearly manifested in the acoustic component, displaying alternating monopoles along an essentially horizontal line at approximately the height of the critical layer. As a result, acoustic fluctuations are essentially "locked" between the wall and the critical layer. These findings are consistent with the overall features of the Mack mode [30].

Based on equation (3.3) in Section 3.4.2, the growth rate of the second mode arises from different source terms, as shown in figure 3.7. It is found that the peak position of the diffusion-related growth rate σ_s coincides with the synchronization point, in the vicinity of which the second mode is generated. Thus, the diffusion term is quite significant to the second-mode energy growth. However, the diffusion term has never been explicitly linked with the terms in either

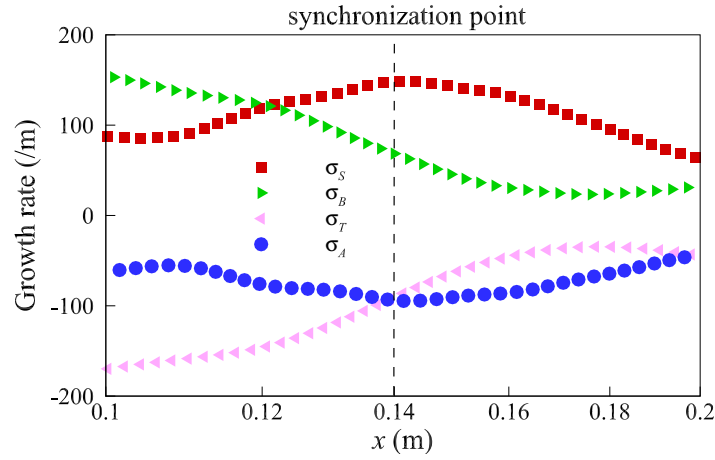


FIGURE 3.7: Different MPT source terms contribute to the second mode's growth rate, with the dashed line denoting the synchronization point..

the conventional energy budget or the linear stability equation, which possess clearer physical meanings. In other words, the role of the diffusion term is not clarified from the usual viewpoint. In this paper, the relation between the diffusion term and the second mode growth mechanism is elucidated. As shown in Appendix 3.9.1, the expression of the thermal diffusion term can be decomposed into

$$\overline{(p'/R)(\partial s'/\partial t)} = \underbrace{\frac{\gamma}{\gamma-1} \overline{p' \nabla \cdot \mathbf{u}'}}_{T_1} + \underbrace{\frac{\gamma}{\gamma-1} \frac{\nabla \cdot \bar{\mathbf{u}}}{\bar{\rho}} \overline{p' \rho'}}_{T_2} + \underbrace{\frac{\gamma}{\gamma-1} \frac{\overline{p'}}{\bar{\rho}} \nabla \bar{\rho} \cdot \mathbf{u}'}_{T_3} + \underbrace{\frac{\gamma}{\gamma-1} \frac{\overline{p'}}{\bar{\rho}} \nabla \rho' \cdot \bar{\mathbf{u}}}_{T_4}. \quad (3.8)$$

In equation (3.8), the term $\frac{\gamma}{\gamma-1} \overline{p' \nabla \cdot \mathbf{u}'}$, noted as T_1 , refers to the dilatation effect, while $\frac{\gamma}{\gamma-1} \frac{\nabla \cdot \bar{\mathbf{u}}}{\bar{\rho}} \overline{p' \rho'}$ is noted as T_2 , represents the work done on the mean divergence by the thermodynamic fluctuations. With the parallel base flow assumption, T_3 can be rewritten as

$$\frac{\gamma}{\gamma-1} \frac{\overline{p'}}{\bar{\rho}} \nabla \bar{\rho} \cdot \mathbf{u}' = \frac{\gamma}{\gamma-1} \frac{\overline{p'}}{\bar{\rho}} \left(\frac{d\bar{\rho}}{dy} \right) v' = - \frac{\gamma}{\gamma-1} \frac{\overline{p'}}{\bar{T}} \left(\frac{d\bar{T}}{dy} \right) v', \quad (3.9)$$

which contributes to the energy transported by the wall-normal velocity fluctuation. This term is referred to as the Reynolds thermal stress term in MPT, and the effect of Reynolds thermal stress is represented by P_1 in RPA. For the fourth term, T_4 can be split into

$$\underbrace{\frac{\gamma}{\gamma-1} \frac{\overline{p'}}{\bar{\rho}} \nabla \rho' \cdot \bar{\mathbf{u}}}_{T_4} = \underbrace{\frac{\gamma}{\gamma-1} \frac{\bar{\mathbf{u}}}{\bar{\rho}} \overline{p' \frac{\partial p'}{\partial x}}}_{T_{41}} - \underbrace{\frac{\gamma}{\gamma-1} \frac{\bar{\mathbf{u}}}{\bar{T}} \overline{p' \frac{\partial T'}{\partial x}}}_{T_{42}}. \quad (3.10)$$

The profiles of the terms in equation (3.10) are depicted in figure 3.8(a), where $x = 0.16$ m falls

in the unstable second-mode region. It is indicated that T_4 equals T_{42} approximately. Note that the advection of perturbation of internal energy (G_1) in RPA is contained in T_{42} . Specially, G_1 corresponds to $(\bar{u}/\bar{T}) \partial T'/\partial x$ in real number space, whereas the production of T_{42} results from time average of $(\bar{u}/\bar{T}) \partial T'/\partial x$ and pressure fluctuation. However, the advection of perturbation of internal energy should be on the left-hand side of the energy equation and treated as a generated term. Its value is mainly distributed near the critical layer, perfectly canceled by the time rate of change of the internal energy term (G_2), as shown in figure 3.4. Therefore, to compare the result of MPT with that of RPA, this T_4 term is not treated as the source term. Further analysis will be performed on the other source terms, i.e., T_1 , T_2 , and T_3 .

To explore which term plays a dominant role in contributing to the growth of the second mode, the distribution of different source terms obtained from the diffusion term of MPT is important, as depicted in figure 3.8(b). The pronounced dilatation effect near the wall is successfully reproduced by the subterm T_1 of the thermal diffusion in MPT, which also shares some similarities with the significance of near-wall dilatation observed in the wind tunnel experiment [59]. The energy transported by the wall-normal velocity fluctuation is evident in the outer region between $y = 0.3y_c$ and $y = y_c$, resembling the distribution in figure 3.5(a). The peak near the critical layer is not evident in figure 3.8(b). In conclusion, the significance of the dilatation effect is ultimately detected in the MPT framework, while that of the thermal stress effect is partly found. The remaining thermal stress effect may originate from the terms σ_B , σ_A , and σ_T in equation (3.3), which are not easily shown explicitly. However, these observations do not contradict existing knowledge because the acoustic energy amplification of the second mode due to the near-wall mechanism has ultimately manifested in the thermal diffusion term.

3.6.4 Results of Lagrangian analysis

Based on the data from DNS, the time-averaged quantities of source terms in equation (3.5) are depicted in figure 3.9(a). It is indicated that in the near-wall region, the dominant source term that destabilizes the second mode is the DAP. The RSS term maintains a nearly constant positive value inside the boundary layer, indicating a moderate destabilization effect on the second mode. However, the positive contribution of the RSS term is nearly canceled by the DAP in the outer region. Near the critical layer, the heat exchange (HE) term shows a pronounced peak, indicating a substantial positive contribution to the growth of Chu's energy. The moderate distribution of RSS and the distinguished role of HE may result from the diverse profile of wall-normal

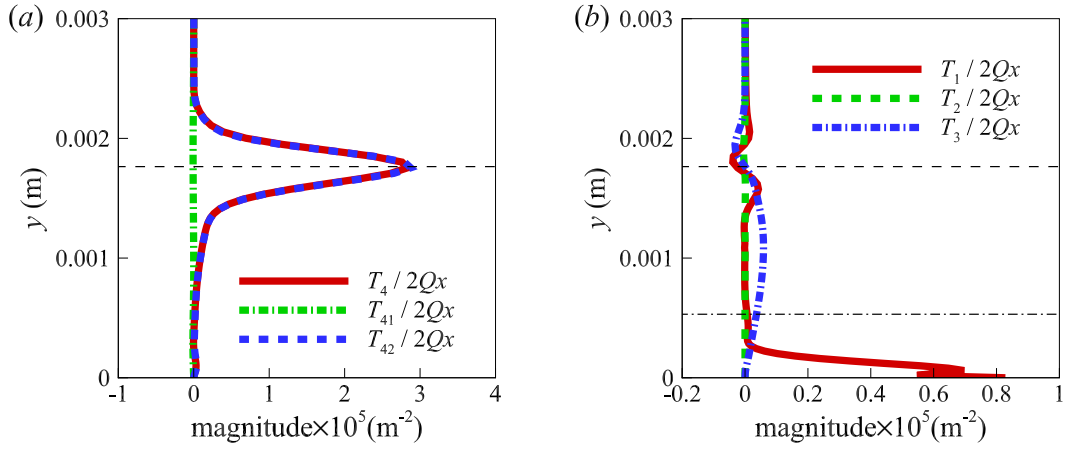


FIGURE 3.8: Distribution of (a) T_4 , T_{41} , and T_{42} and (b) T_1 , T_2 , and T_3 decomposed from the thermal diffusion term of MPT at $x = 0.16$ m. The dashed line indicates y_c , while the dash-dotted line corresponds to $y = 0.3y_c$.

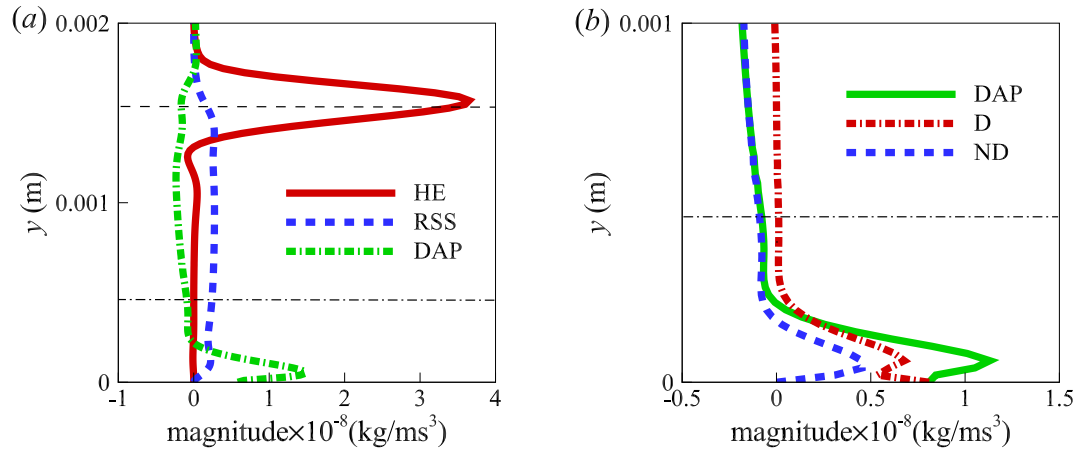


FIGURE 3.9: Distribution of (a) DAP and RRS in equation (3.5) and (b) D and ND in equation (3.11) at $x = 0.16$ m. The dashed line indicates y_c , while the dash-dotted line corresponds to $y = 0.3y_c$.

gradients of streamwise velocity and temperature, as shown in figure 3.1(b). The wall-normal gradient of mean temperature, which is crucial for the Reynolds thermal stress in RPA and the thermal diffusion term in MPT, is closely related to the heat exchange term in the Lagrangian equation.

To further identify the dominant source term in the near wall region, the divergence acoustic energy term is further decomposed into two terms:

$$\underbrace{\overline{\nabla \cdot (p' \mathbf{u}')}}_{\text{DAP}} = \underbrace{\overline{p' \nabla \cdot \mathbf{u}'}}_{\text{D}} + \underbrace{\overline{\nabla p' \cdot \mathbf{u}'}}_{\text{ND}}. \quad (3.11)$$

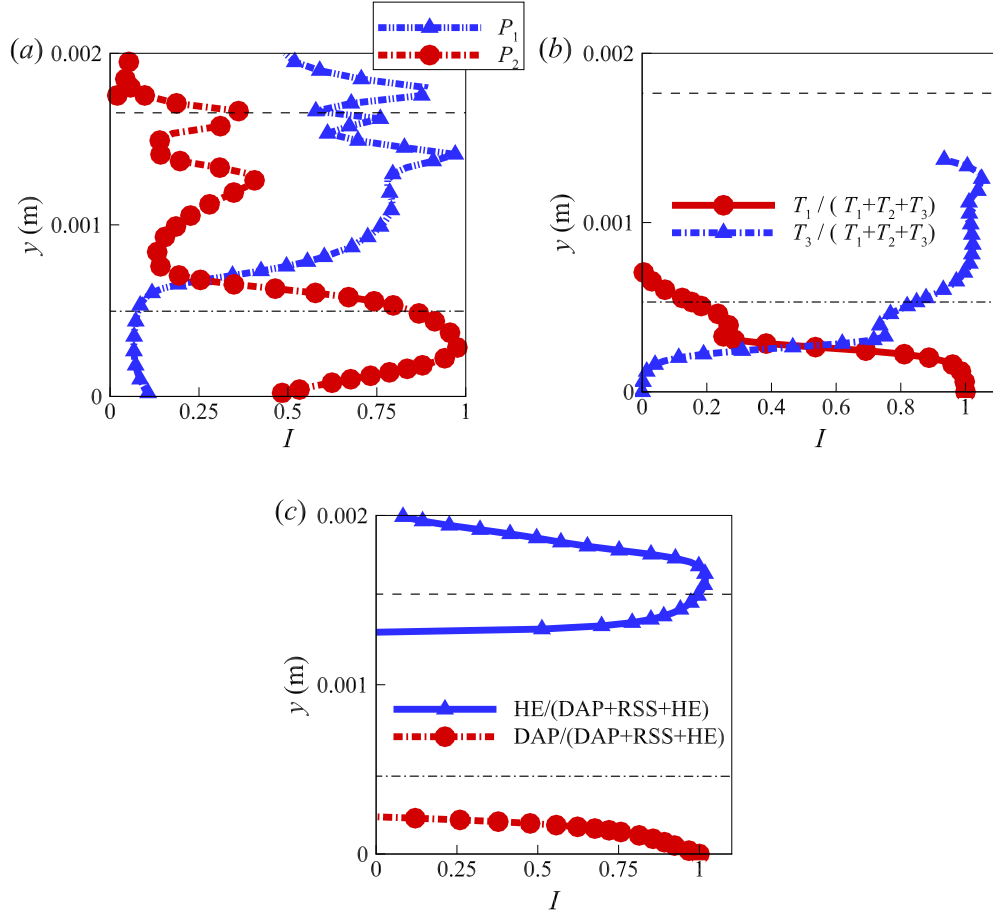


FIGURE 3.10: I indicators distribution in (a) RPA, (b) MPT, and (c) Lagrangian equation at $x = 0.16$ m, respectively. The dashed line indicates y_c , while the dash-dotted line corresponds to $y = 0.3y_c$.

Here, $\overline{p' \nabla \cdot \mathbf{u}'}$ is the pressure dilatation term (D), which also represents the transport of the velocity fluctuation by the pressure fluctuation. This term vanishes in the incompressible limit with $\nabla \cdot \mathbf{u}' = 0$ and becomes important due to compressibility. By contrast, $\overline{\nabla p' \cdot \mathbf{u}'}$ is a non-dilatation term (ND), which exists in the incompressible flow. Figure 3.9(b) depicts the distribution of different source terms decomposed from DAP at a location after the synchronization point. The result shows that the value of the non-dilatation term equals zero on the wall and is always less than the dilatation term. Therefore, the pressure dilatation effect is dominant near the wall, while the non-dilatation one is lesser. This dilatation term eventually results in the amplification of acoustic energy of the second mode, which is consistent with the above RPA and MPT analyses.

3.7 Consistent energy-based framework of amplification mechanisms for the second mode

A unified indicator I is defined to evaluate the contribution of the significant source terms quantitatively for the three energy source analysis methods mentioned above. In RPA, the indicator is defined as

$$I = \exp[-20 |(\Theta_R - \Theta_L) / \Theta_L|]. \quad (3.12)$$

where Θ is the phase, and the subscripts L and R denote the left-hand-side and right-hand-side terms. When the two phases are close, the indicator value approaches unity; otherwise, the value approaches zero. The distributions of the indicator I for significant source terms in RPA, i.e., the Reynolds thermal stress (P_1) and dilatation term (P_2), are shown in figure 3.10(a). The dominance of dilatation in the near-wall region and Reynolds thermal stress near the critical layer is prominent as the I indicators approach 1.0 in the corresponding regions, respectively.

In the MPT and Lagrangian approaches, the indicator I is defined as the ratio of each source term to the sum of the source terms. As shown in figure 3.10(b), the I indicators of the dilatation term and the RTS term in MPT approach 1.0 in the near-wall region and outer layer, respectively. Additionally, figure 3.10(c) indicates that the I indicators of the DAP (dilatation-related term) and HE term in the Lagrangian equation approach 1.0 in the near-wall region and near the critical layer, respectively. Due to the sum of T_1 , T_2 , and T_3 approaching 0 around the critical layer, the I indicators are not valid in this region and, therefore, are not shown.

Through the above analyses, a unified explanation of the energy source for the amplified second mode over hypersonic boundary layers is proposed. Figure 3.11 explicitly shows the dominant energy source terms that drive the amplification of the second mode based on the three fundamental approaches, including RPA based on the internal energy fluctuation, MPT based on the transport of TFE, and the Lagrangian equation based on Chu's energy density. Essentially, RPA examines the phase coherence between the rate-of-change term and the source term, while MPT and the Lagrangian equation start from the analysis of time-averaged source terms. Through a detailed decomposition and comparison of the source terms, it is found that all three approaches point to source terms with essentially the same physical meanings. In the near-wall region below the relative sonic line, the responsible energy source is the dilatation effect, manifested as $-(\gamma - 1)(i\alpha\bar{u} + d\hat{v}/dy)$ in RPA and $-\overline{p'(\partial u'/\partial x + \partial v'/\partial y)}$ in MPT and Lagrangian equations. The critical role of the dilatation term in the energy transport near the wall was also reported in

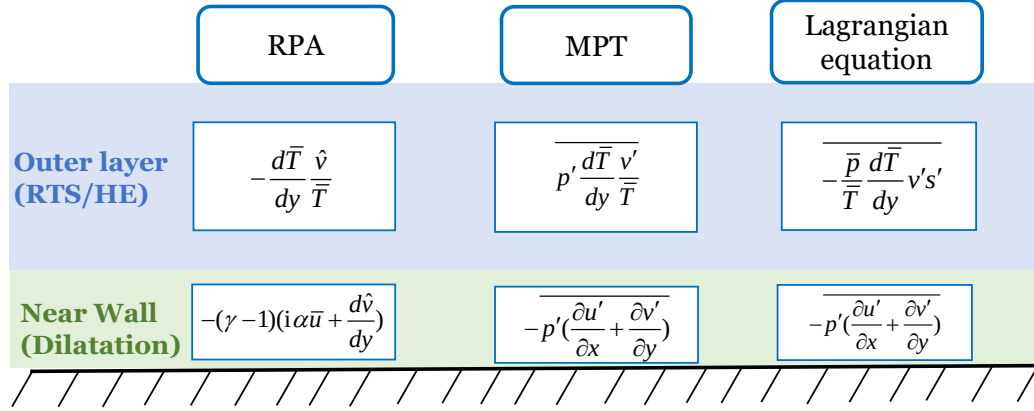


FIGURE 3.11: A schematic of the dominant energy source terms that drive the second mode's amplification derived from three methods.

the flow–thermodynamics interactions analysis by Sharama & Girimaji using Helmholtz decomposition [112]. In the outer layer, both PRA and MPT report significance of the Reynolds thermal stress, which manifests as $-(d\bar{T}/dy)\hat{v}/\bar{T}$ in RPA and $\overline{(d\bar{T}/dy)p'v'/\bar{T}}$ in MPT. The dominant source term near the critical layer in the Lagrangian equation is found to be the heat exchange term, which is not considered in Kuehl's thermoacoustic interpretation. The expression of the HE term $-\overline{(d\bar{T}/dy)v's'\bar{p}/\bar{T}}$ also indicates the importance of the mean temperature gradient.

Furthermore, the Reynolds shear stress term, which appears as $-\overline{u'v'\bar{\rho}(\partial\bar{u}/\partial y)}$ in Lagrangian equations, appears to be nearly constant and moderate in the outer layer. According to the relative phase analysis of Tian & Wen [60] based on linear stability theory, the RSS term is essential in the streamwise transport of kinetic energy fluctuation of the second mode. Regarding MPT, the RSS term is probably implicitly included in the source term due to the interaction between momentum density fluctuation ($\mathbf{m}'$) and fluctuation acceleration term (α') instead of the analyzed thermal diffusion term, since the diffusion term does not contain the mean shear $\partial\bar{u}/\partial y$.

3.8 Conclusions

By performing theoretical analysis and numerical simulation, the growth mechanism of the second mode across a hypersonic flat-plate boundary layer is examined. The energy source terms that drive the amplification of the second mode are investigated using three influential fundamental approaches: the relative phase analysis based on internal energy analysis, the momentum

potential theory based on the transport of total fluctuating enthalpy, and the Lagrangian equations based on Chu's energy density. Comparative studies on the source terms from these three influential theoretical approaches are conducted, and the connections among them are discussed and clarified in detail. The detailed derivation of the Lagrangian equations to include the heat exchange effect is provided for the first time, facilitating more complete considerations from energy perspectives. Meanwhile, a unified indicator is defined to evaluate the contribution of the significant source terms quantitatively.

The main finding of this study is that the three popular fundamental approaches reach an agreement on the energy amplification mechanism of the Mack second mode. The explicit expression of each energy source is derived, and despite differences in mathematical forms, the physical meanings behind them are consistent. Generally, the dilatation effect due to compressibility is responsible for acoustic energy production near the wall, while the base temperature gradient is significant in the wall-normal transport of internal energy in the outer region of the boundary layer. Moreover, this study focuses on the second-mode instability under the adiabatic wall condition. Whether momentum potential theory can be extended to other forms of instabilities in identifying significant source terms merits further investigation.

3.9 Appendix

3.9.1 Derivation of equation (3.8)

The decomposition of the instantaneous quantity gives

$$\rho = \bar{\rho} + \rho', \mathbf{u} = \bar{\mathbf{u}} + \mathbf{u}'. \quad (3.13)$$

By substituting the above equation into the continuity equation, subtracting the base flow equation and dropping the nonlinear terms, it yields

$$\frac{\partial \rho'}{\partial t} + \bar{\rho} \nabla \cdot \mathbf{u}' + \nabla \bar{\rho} \cdot \mathbf{u}' + \rho' \nabla \cdot \bar{\mathbf{u}} + \nabla \rho' \cdot \bar{\mathbf{u}} = 0. \quad (3.14)$$

Therefore,

$$\frac{p'}{R} \frac{\partial s'}{\partial t} = \frac{p'}{R} \frac{\partial}{\partial t} \left(c_v \frac{p'}{\bar{p}} - c_p \frac{\rho'}{\bar{\rho}} \right) = -\frac{\gamma}{\gamma - 1} \frac{p'}{\bar{p}} \frac{\partial \rho'}{\partial t}. \quad (3.15)$$

Substituting equation (3.14) into equation (3.15) yields

$$\overline{\frac{p'}{R} \frac{\partial s'}{\partial t}} = \frac{\gamma}{\gamma - 1} \overline{\frac{p'}{\bar{\rho}} (\bar{\rho} \nabla \cdot \mathbf{u}' + \nabla \bar{\rho} \cdot \mathbf{u}' + \rho' \nabla \cdot \bar{\mathbf{u}} + \nabla \rho' \cdot \bar{\mathbf{u}})}, \quad (3.16)$$

i.e., equation (3.8).

Chapter 4

A unified explanation of energy growth sources for unstable modes in flat-plate boundary layers

[Published paper II¹]

4.1 Preface to the chapter

This chapter includes a part of an article published in the *Journal of Fluid Mechanics*, titled "A unified explanation of energy growth sources for unstable modes in flat-plate boundary layers" [75]. In this Chapter, Relative phase theory (RPA) is extended to the flat-plate boundary layers with a wide range of Mach numbers and wall temperature ratios. Since the phase result using LST agrees with that of DNS in Chapter 3, LST is employed mainly for discussion in this Chapter to save computational cost. Different instability modes (including T-S mode, oblique first mode, second mode, and supersonic mode) are included and discussed. Interestingly, a unified explanation has been found for the energy growth sources by RPA in these considered cases. The effects of Mach number, wall cooling, obliqueness, and porous coatings can all be understood and explained in the current framework of RPA.

¹Y. Chen, P. Guo and C. Y. Wen (2023). A unified explanation of energy growth sources for unstable modes in flat-plate boundary layers. *Journal of Fluid Mechanics*, 972, A5.

4.2 Abstract

The detailed energy sources that sustain the eigenmodal exponential growth in boundary layers remain unclear. In the present study, the phases of each term in the linear stability equation are examined to identify the significant physical sources across a wide range of Mach numbers and wall temperature ratios. The instability characteristics of boundary layer flows demonstrate notable similarities across different speed regimes: Tollmien-Schlichting waves in incompressible flow, oblique first modes in supersonic flow, and Mack second modes/supersonic modes in hypersonic flow. Notably, the maximum instability of the first mode correlates with its oblique nature and amplified Reynolds shear stress. Conversely, reduced Reynolds thermal stress suppresses the oblique second mode from becoming the dominant instability. The stabilization mechanisms differ substantially between instability modes under wall cooling conditions. For the oblique first mode, stabilization occurs through phase opposition between Reynolds thermal stress/dilatation fluctuations and internal energy variations. Conversely, second mode destabilization arises from intensified wall-normal internal energy transport below the second generalized inflection point. Interestingly, porous coatings exhibit inverse effects - destabilizing the first mode via augmented mean shear production while mimicking wall heating's stabilizing effect on the second mode. The supersonic mode demonstrates relatively weak instability, primarily due to phase cancellation in wall-normal transport near the critical layer, leading to diminished internal energy growth. These findings establish important connections with prior inviscid thermoacoustic theory (Kuehl 2018), revealing consistent physical interpretations for both second and supersonic modes. Notably, the concentrated energy sources observed along the critical layer and near-wall regions offer a unified framework for understanding local amplification mechanisms of inviscid modes in hypersonic boundary layers.

4.3 Introduction

Eigenmodal boundary-layer instability has been extensively investigated due to its fundamental and industrial significance. In the low-speed limit, the T-S mode may first undergo linear amplification to induce boundary layer transition [113]. With the necessary presence of the viscous wall boundary condition, the T-S mode can be unstable in a zero-pressure-gradient boundary layer. As the Mach number increases, the dominant linear instability becomes the oblique first mode, which can be regarded as an extension of the T-S mode in low-supersonic

flow [114]. The difference is that, with the presence of the generalized inflection point, the inviscid first-mode instability becomes possible, eliminating dependence on the no-slip boundary condition. When the Mach number exceeds a threshold value, a family of inviscid Mack modes evolves [115], among which the second mode is dominant. The most unstable Mack second mode is two-dimensional and behaves like acoustic waves trapped between the wall and the relative sonic line [19, 30]. Regarding the origin of the second mode, the synchronization of the phase speeds of the fast and slow modes plays a significant role. Near the synchronization point, the second mode is excited through the intermodal energy exchange mechanism, proposed by Fedorov and Khokhlov [116] and verified by Ma and Zhong [20, 31, 32] using DNS. It is generally accepted that wall cooling stabilizes the first mode while destabilizing the second mode. If the level of wall cooling is high, a new unstable branch called the supersonic mode can be generated by the synchronization between the second mode and the slow acoustic wave of the continuous spectrum [33, 117]. This supersonic mode may radiate slow acoustic waves outside the boundary layer and also enlarge the unstable frequency band. Due to different physical characteristics, the instability mechanisms of various eigenmodes in subsonic, supersonic, and hypersonic boundary layers are generally discussed separately. However, the growth mechanism—specifically, how to sustain the local exponential growth of energy—still requires convincing explanations. In the high-speed transition community, Kuehl [58] performed an energy analysis on the second mode using an inviscid Lagrangian formulation. The energy required to maintain the resonant second-mode waves is believed to be supplied by the thermoacoustic Reynolds stress. Tian and Wen [60] performed relative phase analyses on the second mode, concluding that the change in fluctuating internal energy is sustained by the advection of perturbed thermal energy in the vicinity of the critical layer and by dilatation fluctuations near the wall.

Clearly, existing theories have not yet reached a consensus. Meanwhile, several interesting questions merit consideration: Why does obliqueness only appear for the most unstable first mode? Why does wall cooling stabilize the oblique first mode while destabilizing the second mode [21], whereas porous coating behaves reversely [65]? What are the determining factors for supersonic-mode instability? A more comprehensive and self-consistent explanation of the energy sources responsible for the unstable modes is currently lacking. In this study, phase analysis is performed on the T–S mode, the oblique first mode, the second mode, and the supersonic mode based on LST. We aim to identify the significant physical sources for different types of boundary layer instabilities. Additionally, for the second and supersonic modes in hypersonic states, connections and consistency with the current Lagrangian inviscid theory are

demonstrated.

4.4 Numerical setup

Since this chapter focuses solely on the linear stage, the primary results are derived from LST, conducted at a fixed Reynolds number ($Re = 1183.2$) across varying Mach numbers and wall temperatures. The dominant energy source terms are identified using RPA introduced in 2.4.1. For validation and deeper analysis, two-dimensional DNS data are employed to examine the source terms in the Lagrangian energy equations and verify the phase results obtained via LST. Two DNS cases are simulated here to investigate the Mach 6 flat-plate boundary layers under the adiabatic wall (case 1) and cold wall (case 2) conditions with the same unit Reynolds number 1×10^7 . Perfect gas is assumed with a specific heat ratio $\gamma = 1.4$ and Prandtl number $Pr = 0.72$. In case 1, we follow the flow conditions of Egorov *et. al* [118], the ratio of the wall temperature to the adiabatic wall ('ad') value is $T_w/T_{ad} = 1$. In case 2, we follow the settings of Chuvakhov & Fedorov [33] with $T_w/T_{ad} = 0.07$, which could generate the supersonic mode. Other details can be found in the references. A wall blowing-suction actuator [118] is utilised to initiate unstable modes, which perturbs the wall-normal mass flow rate $\dot{m}_w$ in the following form:

$$\dot{m}_w(x, t) = \epsilon \sin(2\pi \frac{x - x_1}{x_2 - x_1}) \sin(2\pi f t). \quad (4.1)$$

In case 1, we set x_1 and x_2 corresponding to $Re = 267.6$ and 313.7 , respectively. The forcing frequency is fixed at $f^* = 160.52$ kHz. The forcing amplitude $\epsilon = 6 \times 10^{-4}$ is adopted. In case 2, the corresponding Re of x_1 and x_2 are 1481.1 and 1596.2 , respectively. Additionally, $f^* = 435.11$ kHz and $\epsilon = 1 \times 10^{-4}$ are employed. The utilised DNS code based on the finite difference method has been fully validated by previous work [66]. An excellent convergence of mesh resolution and time step selection was confirmed under the considered conditions. The inviscid flux derivatives are discretized using a fifth-order upwind compact scheme, while the viscous components are discretized using a sixth-order central difference scheme. The temporal discretization is performed using a third-order Runge-Kutta algorithm.

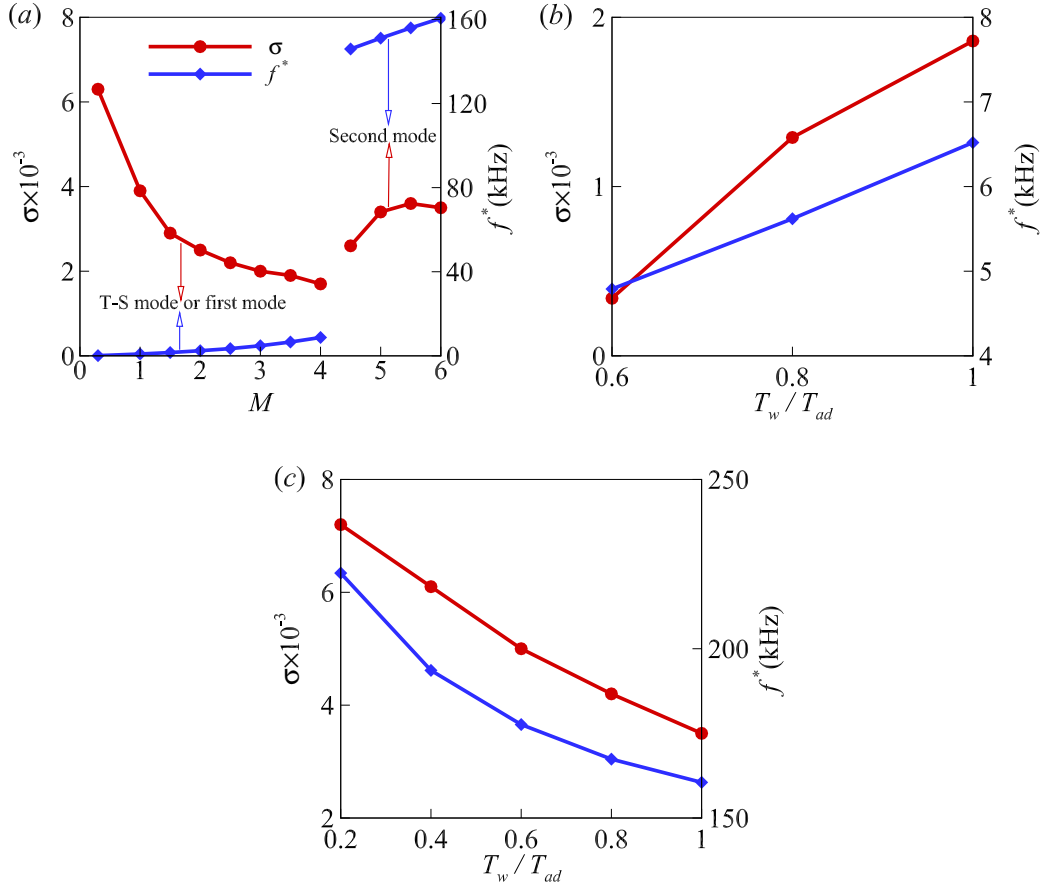


FIGURE 4.1: Dimensionless growth rate and dimensional frequency of the most unstable mode versus (a) M , (b) wall temperature ratio at $M = 3.5$ (first mode), and (c) wall temperature ratio at $M = 6$ (second mode).

4.5 Results of relative phase analysis

In this section, LST is used to reveal the instability features of the most unstable mode. The maximum growth rate $\sigma = -\alpha_i$ is sought and determined in the parameter space of ω and β . We concentrate on a fixed Reynolds number $Re = 1183.2$. This value is designated corresponding to the station $x^* = 0.14$ m in the DNS of case 1 to facilitate convenient comparisons. The effects of Mach number and the wall temperature ratio on the growth rate and the dimensional frequency of the most unstable mode are shown in figure 4.1. In figure 4.1(a), the most unstable modes at Mach 0.3, 1.0–4.0, and 4.5–6.0 correspond to the T–S mode, the oblique first mode, and the second mode, respectively. The considered interval for Mach number is $\delta M = 0.5$ for $M \in [1, 6]$. Figures 4.1(b,c) show that wall cooling would stabilise the oblique first mode while destabilise the second mode, which is consistent with existing knowledge. For figure 4.1(b), the cases $T_w/T_{ad} = 0.2$ and $T_w/T_{ad} = 0.4$ do not report unstable first modes, and thus they are not

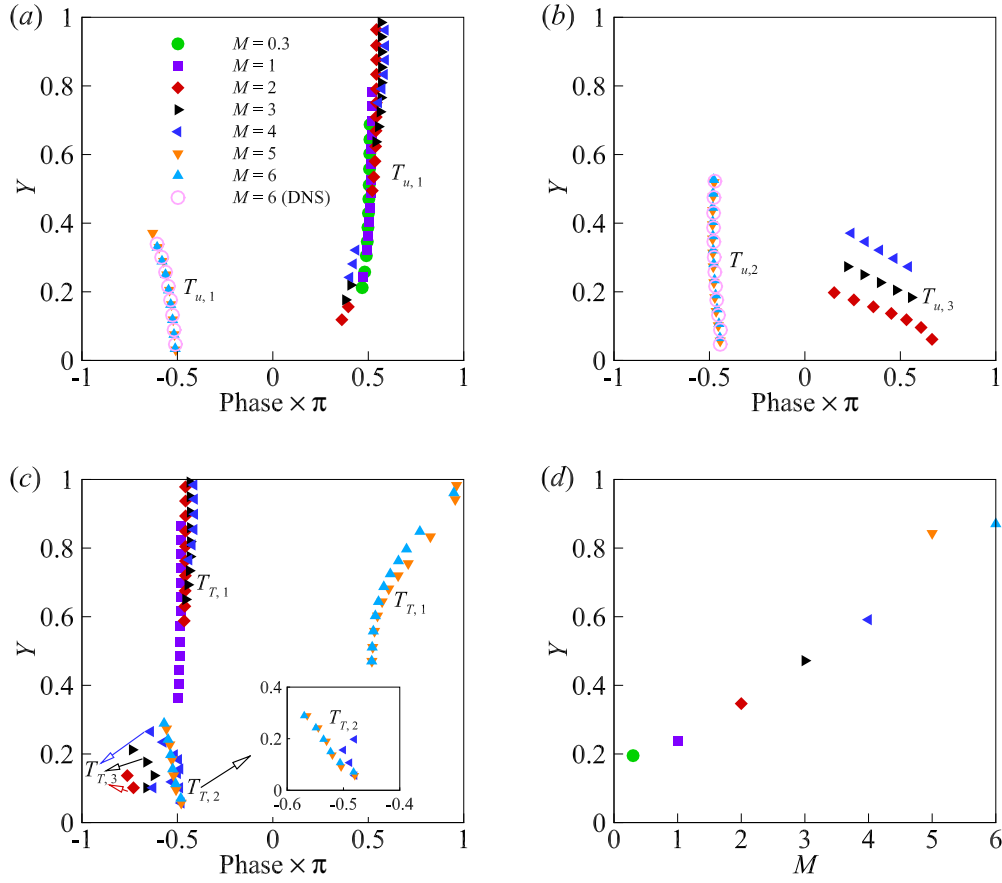


FIGURE 4.2: Phase profiles of significant source terms (a) $T_{u,1}$ and (b) $T_{u,2}$ and $T_{u,3}$ in equation (2.15), (c) the terms in equation (2.17), and (d) location of the critical layer with various Mach numbers.

shown.

As interpreted in § 2.4.1, the phase profiles will be analyzed first. It should be noted that the significant source terms identified by the relative phase analysis require a sufficient magnitude of amplitude input. Therefore, the amplitude profiles of the source terms are also examined in Appendix 4.8.1.

4.5.1 Effects of Mach number and obliqueness

In this section, the effects of Mach number and obliqueness on the most unstable modes are investigated according to the phase distribution of the terms in equations (2.15-2.17). Throughout this paper, the results of the eigenfunctions are normalized by the pressure eigenfunction at the wall to depict the phase. Figure 4.2 shows the wall-normal distribution of the significant source terms and the location of the critical layer at various Mach numbers under the adiabatic wall

condition. These terms, marked by symbols, are determined by retaining those whose phases differ minimally from that of the left-hand-side rates of change in equations (2.15) and (2.17). Thus, these symbols highlight the locally significant physical sources that are in phase with the rate of change of fluctuations. Here, this minor tolerance of phase difference is set to 0.1π , which has shown qualitative convergence of significant source terms. The vertical axis is given by $Y = \frac{y}{\delta}$, where δ is the local boundary-layer thickness determined by $\bar{u}_{y=\delta} = 0.99$. For all cases below, the change in wall-normal velocity fluctuation is consistently dominated by the normal gradient of the pressure fluctuation. Thus, only results related to fluctuations of streamwise velocity and internal energy are displayed. DNS data are also transformed into the frequency domain and used to obtain the phase in figures 4.2(a,b) for Mach 6. The favorable agreement between LST and DNS rules out the possible influence of high-order effects in the relative phase analysis.

Figure 4.2 indicates that different types of modal instabilities share some similarities. The productions induced by mean shear ($T_{u,1}$) and mean temperature gradient ($T_{T,1}$) are always present, except that the internal energy fluctuation is not considered for the T-S mode. Mean shear production is pronounced across the boundary layer for low-frequency disturbances (T-S and oblique first modes) and beneath the critical layer for high-frequency disturbances (second mode). Mean temperature gradient production is active in the outer regions for all eligible modes. In the near-wall region, the significant sources switch from viscous stress ($T_{u,3}$) and viscous dissipation and thermal conduction ($T_{T,3}$) for the first mode at $M \in [2, 4]$ to the streamwise gradient of pressure fluctuation ($T_{u,2}$) and dilatation fluctuation ($T_{T,2}$) for the second mode at $M \in [5, 6]$. This observation highlights the acoustic nature of the second mode and the increasing significance of compressibility in hypersonic states. The importance of near-wall dilatation for the second mode has also been observed in experimental studies [59]. It should also be noted that the dilatation fluctuation ($T_{T,2}$) becomes notable near the wall for the oblique first mode at $M = 4$. This phase feature indicates that the dilatational contribution is augmented not only by the trapped thermoacoustic resonance of the second mode but also by compressibility. In general, the significant source terms are largely concentrated near the wall for the second mode, suggesting that wall control techniques, such as ultrasonic absorptive coatings, are promising. However, for the first mode, both the active mean shear ($T_{u,1}$) and mean temperature gradient ($T_{T,1}$) productions are distributed across the boundary layer. Wall manipulation might be less efficient in controlling the first mode than the second mode. For the first mode, a global adjustment of the mean flow (e.g., the pressure gradient) to reduce the velocity gradient $d\bar{u}/dy$ may be an

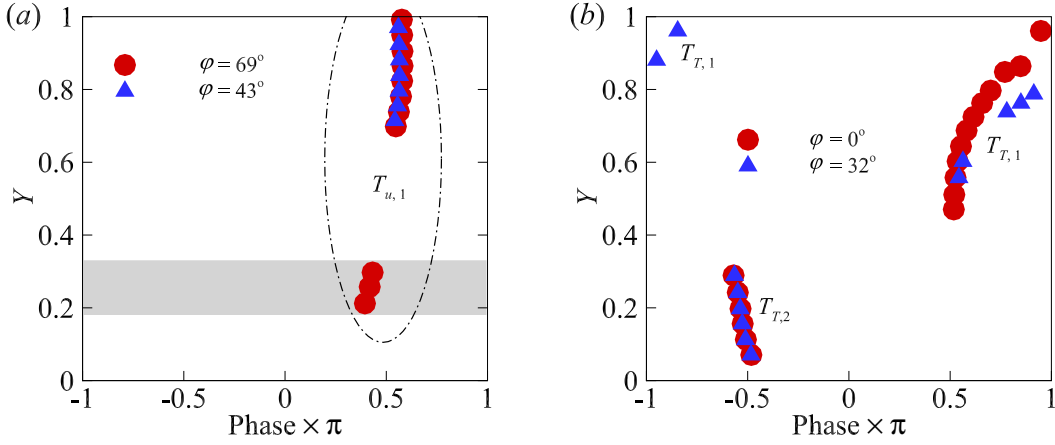


FIGURE 4.3: Effect of obliqueness on the phase profiles of the (a) oblique first mode ($M = 3.5$) and (b) second mode ($M = 6$). The grey region in (a) marks the enlarged in-phase region of the most unstable state $\varphi = 69^\circ$ compared to the marginally unstable state $\varphi = 43^\circ$.

alternative control strategy to consider for regions away from the wall. Wall temperature control to alter the temperature profile $\bar{T}(y)$ and reduce $d\bar{T}/dy$ also merits consideration [119].

Interestingly, for the low-speed cases at $M = 0.3$ and 1 in figure 4.2, the viscous stress ($T_{u,3}$) competes strongly with the Reynolds shear stress ($T_{u,1}$) and streamwise gradient of pressure fluctuations ($T_{u,2}$) according to their absolute amplitude, resulting in the disappearance of dominators in the near-wall region. This does not contradict the knowledge that the existence of viscosity (a no-slip wall) is a prerequisite for T-S instability in a zero-pressure-gradient boundary layer. Once the necessary viscosity is present, u' and v' are nonorthogonal, and a nonzero Reynolds shear stress is generated. Then the Reynolds shear stress becomes the driver for the energy production of the T-S mode. The observed two regions for energy amplification, i.e., the outer region in the vicinity of the critical layer and the near-wall region, will be further checked by the inviscid Lagrangian equations.

The effect of obliqueness on the oblique first mode and second mode can also be interpreted by figure 4.3. The obliqueness is evaluated by the wave angle $\varphi = \arctan(\beta/\alpha_r)$. The most unstable first mode has a wave angle of $\varphi = 69^\circ$, while the most unstable second mode is two-dimensional ($\varphi = 0^\circ$). For comparison, another two marginally unstable states are displayed, where $\varphi = 43^\circ$ for the first mode and $\varphi = 32^\circ$ for the second mode. Note that the terms which do not show evident differences are not displayed, such as $T_{u,2}$, $T_{T,1}$ and $T_{T,2}$ in figure 4.3(a). As shown in figure 4.3(a), the optimal first mode has an enlarged region (marked by the grey box) of the mean shear term ($T_{u,1}$) which is in phase with the rate of change of the streamwise velocity fluctuation.

As shown in figure 4.3(a), the most unstable first mode with $\varphi = 69^\circ$ at $M = 3.5$ possesses two significant regions of the mean shear term ($T_{u,1}$), which is in phase with $\bar{\rho}Du'/Dt$. The two regions are located above and below the critical layer $Y = 0.53$, respectively. This feature is also found for the most unstable first mode at other Mach numbers. The two in-phase regions contributed by mean shear also correspond to the peak amplitude of $\bar{\rho}Du'/Dt$ on the two sides of the critical layer in figure 4.9(a). In comparison, the marginally unstable state with $\varphi = 43^\circ$ reports the absence of the lower significant region (marked by the grey box) in figure 4.3(a). This indicates that the strong growth of the most unstable oblique wave via mean shear is decreased when gradually reducing to the two-dimensional pattern. The growth rate is increased from 0.5×10^{-3} at $\varphi = 43^\circ$ to 1.9×10^{-3} at $\varphi = 69^\circ$. The peak modulus of the eigenfunction $|\hat{u}|$, $|\hat{v}|$ and $|\hat{w}|$ is increased by a factor of 5.5, 2.4 and 3.3 from $\varphi = 43^\circ$ to $\varphi = 69^\circ$, respectively, which is obviously inhomogeneous. The largest amplification in the streamwise velocity fluctuation agrees with the augmented mean-shear effect in the streamwise velocity equation. As the wave angle φ is changed, the phase modulation induced by fluctuation energy redistribution among u' , v' and w' should be the contributor, since the base-flow quantity such as $d\bar{u}/dy$ is unchanged in the mean shear term. At an appropriate oblique wave angle, the phase coherence between the u' -related $\bar{\rho}Du'/Dt$ and the v' -related $-\bar{\rho}v'd\bar{u}/dy$ (i.e., $T_{u,1}$) reaches the maximum. To suppress the first-mode growth, spanwise fluctuation may be introduced to redistribute u' , v' , and w' and disrupt the phase coherence at a certain wave angle, such as spanwise flow oscillation via electromagnetic body force [120]. By contrast, as shown in figure 4.3(b), Reynolds thermal stress ($T_{T,1}$) becomes out of phase with the internal energy fluctuation for the oblique second mode compared to the planar counterpart, indicating a weaker contribution to the growth of internal energy fluctuation. This provides an explanation of why obliqueness would strengthen the first-mode instability while weakening the second-mode instability.

4.5.2 Effects of wall cooling and porous coating

Figures 4.4(a,b) show the streamwise velocity and temperature of the base flow in conjunction with their wall-normal gradients. The location for the generalised inflection point (GIP) is shown in figure 4.4(c). Clearly, wall cooling results in a significant change in the magnitude and sign of the base temperature gradient. This profile distortion would affect the vertical transport of internal energy mostly by $T_{T,1}$ in equation (2.17). Figure 4.5 depicts the phase profiles of significant source terms, which reflect the impact of wall cooling. The decrease of wall temperature has no evident effect on the streamwise velocity fluctuations, i.e., the significant

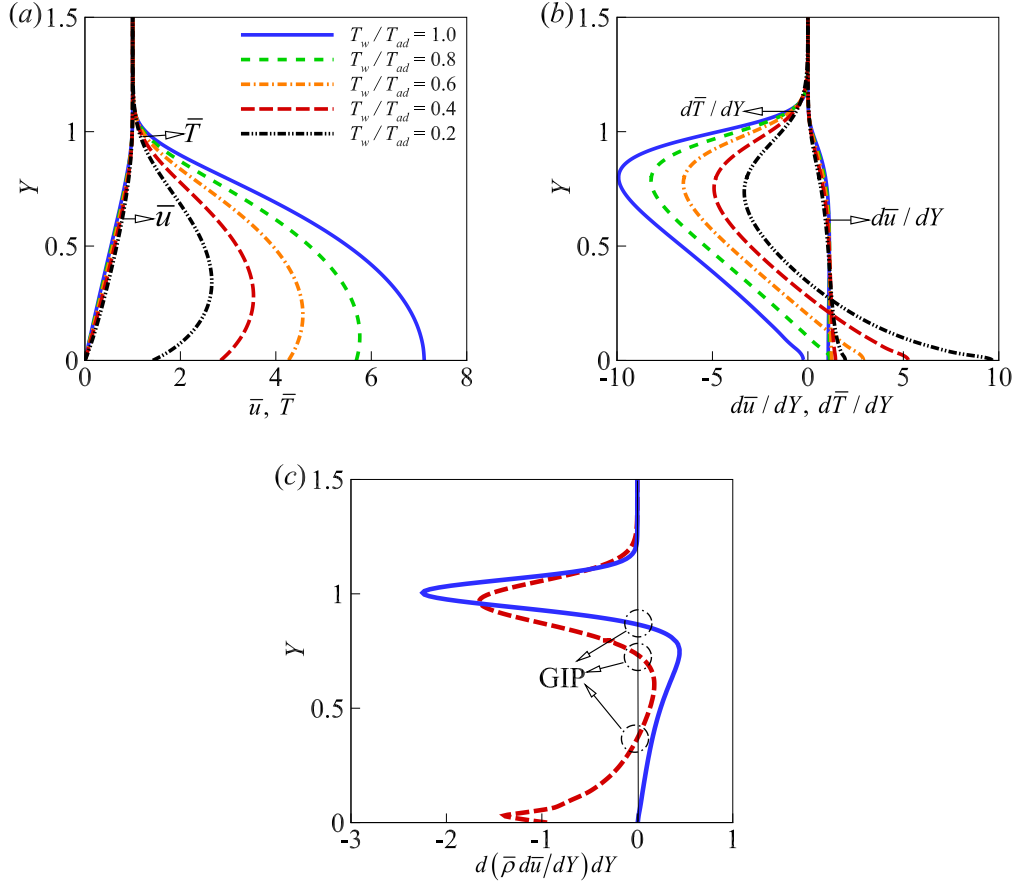


FIGURE 4.4: (a) Mean streamwise velocity and temperature profiles and (b) their normal gradient, and (c) the distribution of GIP for $M = 6$ boundary layers with various wall temperature ratios.

regions for $T_{u,1}$ and $T_{u,2}$ are nearly invariant, as shown in figure 4.5(a) for $M = 6$ and also for $M = 3.5$ (not shown here). For internal energy fluctuations under the adiabatic wall condition, in figure 4.5(b), the vertical transport of internal energy ($T_{T,1}$) is in phase with $\bar{\rho}DT'/Dt$ only when it is away from the wall ($Y > 0.45$). Note that the relative sonic line is located in the range between $Y = 0.48$ and $Y = 0.5$ for all the cases at $M = 6$. Meanwhile, the acoustic energy of the second mode is mostly trapped beneath the relative sonic line. Thus, under the adiabatic wall condition, figure 4.5(b) demonstrates that the RTS ($T_{T,1}$) is almost not responsible for the acoustic energy production essentially below the relative sonic line ($Y < 0.48$). By contrast, the dilatation work ($T_{T,2}$), which is active between the wall and the relative sonic line, is significant in the acoustic energy growth of the second mode.

What is new with wall cooling is that, for cold walls with $T_w/T_{ad} \leq 0.8$, a new branch of symbols related to the mean temperature gradient effect ($T_{T,1}$) appears in the near-wall region of $Y < 0.9Y_{\text{GIP,lower}}$. Here, $Y_{\text{GIP,lower}}$ denotes the lower generalized inflection point (GIP) appearing under

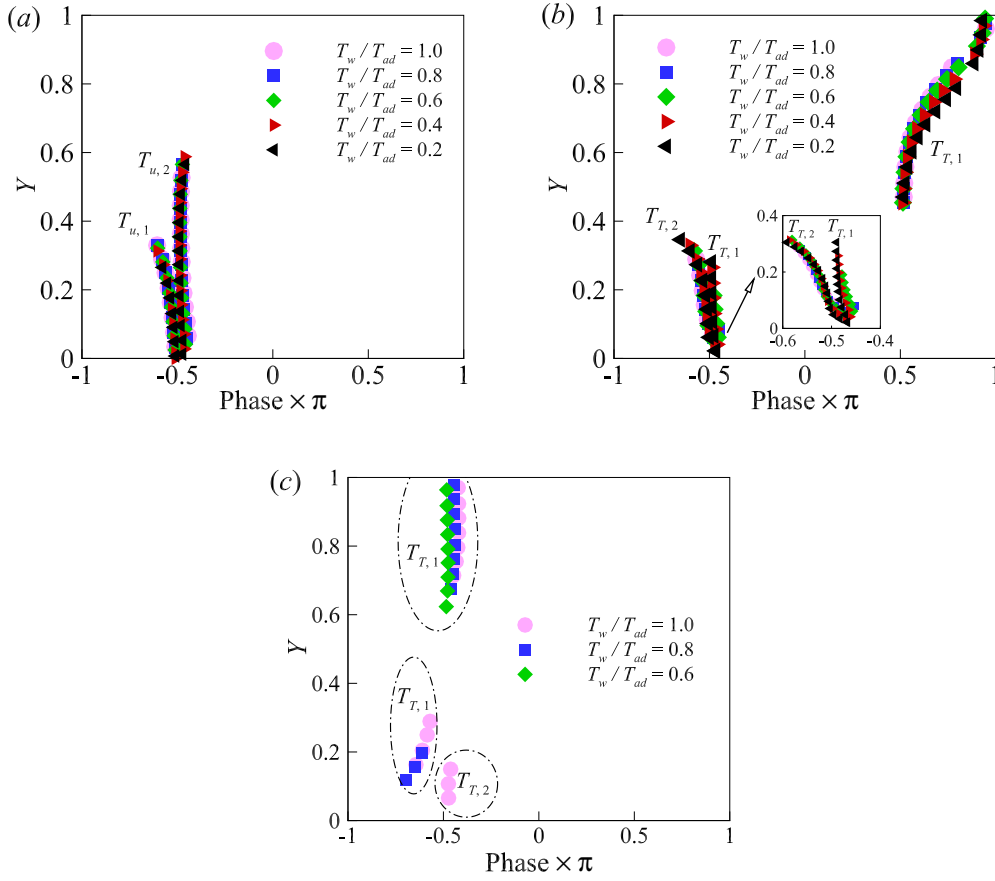


FIGURE 4.5: Phase profiles of significant source terms of (a) equation (2.15) and (b) equation (2.17) for $M = 6$, and (c) equation (2.17) for $M = 3.5$ with various wall temperature ratios.

wall cooling (see figure 4.4(c)). Figure 4.4(b) illustrates that wall cooling increases the absolute magnitude of the base temperature gradient $\frac{d\bar{T}}{dy}$ in the near-wall region. Consequently, wall cooling results in an enlarged region for the vertical transport of internal energy $-\bar{\rho}v'T'd\bar{T}/dy$ through the augmented base temperature gradient. This internal energy behavior manifests as the newly detected phase coupling between $\bar{\rho}DT'/Dt$ and $-\bar{\rho}v'd\bar{T}/dy$ (i.e., $(T_{T,1})$) beneath the second GIP $Y_{\text{GIP,lower}}$ in figure 4.5(b). Clearly, this region is further extended as wall temperature is decreased, which is relevant to a more unstable second mode. This extended source in phase leads to an enhancement in the internal energy growth, which will be reconfirmed by the following Lagrangian energy analysis. In comparison, for the oblique first mode, figure 4.5(c) shows that wall cooling suppresses the effects of the dilatation fluctuation and Reynolds thermal stress (RTS) near the wall. This provides an explanation of why wall cooling stabilizes the first mode while destabilizing the second mode.

For hypersonic flows, porous coatings have proven to be a promising passive control strategy

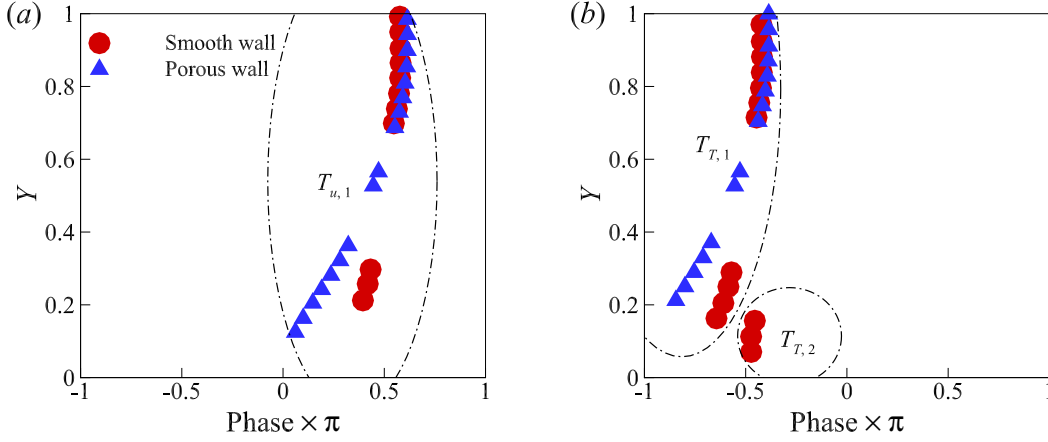


FIGURE 4.6: Phase profiles of significant source terms of equation (2.15) in (a), and equation (2.17) in (b) at $M = 3.5$ with smooth and porous walls.

for the second mode. Tian & Wen [60] demonstrated that the effect of the porous coating on the second mode is to distort the phase of the mean temperature gradient effect $T_{T,1}$ at Mach 6, thereby reducing the internal energy growth induced by wall-normal transport. This mechanism resembles that of wall heating, as inferred from figure 4.5(b). They also investigated the two-dimensional first mode at Mach 6, but the results warrant further examination due to the lack of obliqueness. Figure 4.6 illustrates the effect of the porous coating on the oblique first mode at Mach 3.5. The same boundary condition $v' = Ap'$ as imposed by Tian & Wen [60] is utilized, where A represents the wall acoustic admittance. When the wall transitions from smooth ($A = 0$) to porous ($A = -4$), the growth rate of the most unstable oblique first mode increases from 1.86×10^{-3} to 3.35×10^{-3} . Compared to the phase distribution of the smooth-wall case, a significantly enlarged region of the Reynolds shear stress (RSS), related to $T_{u,1}$ in phase, enhances the production of the streamwise velocity fluctuation. This analysis of the streamwise momentum equation was not considered in Tian & Wen [60]. Regarding internal energy, the Reynolds thermal stress (RTS), related to $T_{T,1}$ in phase, is slightly strengthened, while the dilatation is somewhat suppressed for the porous wall case, which aligns with the findings of Tian & Wen [60] at Mach 6. In summary, phase variations in both RSS and RTS account for the destabilizing effect of the porous wall on the first mode in the presence of obliqueness. As previously mentioned, the oblique first mode is characterized by pronounced mean shear production across the boundary layer. Although the dilatational contribution is suppressed by the porous wall, the reinforced mean shear effect is more pronounced at moderate Mach numbers, except in special cases, such as when the phase of wall admittance A approaches $\pi/2$ [121].

For the cold-wall condition, the supersonic mode is also found to be important. The phase distribution of equations (2.15–2.17) is investigated under the cold wall condition for both the second and supersonic modes, as shown in figures 4.7(a,b). Two critical Reynolds numbers are examined: $Re = 2000$ marks the second mode region, whereas $Re = 2200$ identifies the supersonic mode domain. The second mode synchronises with the slow acoustic wave at $Re = 2120$, which evolves into the supersonic mode downstream [33]. As shown in figure 4.7(c), the supersonic mode radiates acoustic waves towards the far field, whose wave structure is essentially different from the second mode above the dashed line in figure 4.7(c). This region roughly corresponds to $Y > 0.3$ in figure 4.7(b). Phase analysis reveals that the supersonic mode's wall-normal internal energy transport shows weaker phase correlation with $(T_{T,1})$. This decreased contribution of $T_{T,1}$ may be linked with the weaker instability of the supersonic mode compared to the second mode, which will be further checked by the Lagrangian approach.

4.6 Connection with inviscid Lagrangian interpretation

Based on the DNS data, the time-averaged quantities of the source terms for case 1 and case 2 in equation (3.5) are depicted in figure 4.8. For case 1, as shown in figure 4.8(a), the dominant source term destabilizing the second mode near the wall appears to be the DAP. The expression for DAP, representing the divergence of acoustic power, contains components of fluctuations in dilatation and pressure gradient from equations (2.15–2.17). Therefore, the fact that DAP contributes to the growth of disturbance energy of the second mode near the wall is consistent with the significant role of fluctuations in dilatation and pressure gradient in RPA, as shown in figures 4.2(a–c). The Reynolds shear stress (RSS) term maintains a nearly constant positive value inside the boundary layer, indicating a destabilizing effect on the second mode. The most pronounced contribution comes from the heat exchange term near the critical layer. This term is closely related to the internal energy fluctuation due to vertical transport, i.e., $T_{T,1}$ (RTS) in equation (2.17). By replacing Kuehl's [58] acoustic energy norm with Chu's energy norm, we obtain a more comprehensive understanding that internal energy is primarily amplified along the critical layer, while acoustic energy is augmented in near-wall regions. Consistency with the RPA is also observed.

For case 2, as shown in figures 4.8(b,c), the distribution of Reynolds shear stress (RSS) for cold walls shows limited differences compared to the second mode in case 1. This finding is consistent with RPA, which indicates that wall cooling has no significant effect on RSS, as shown

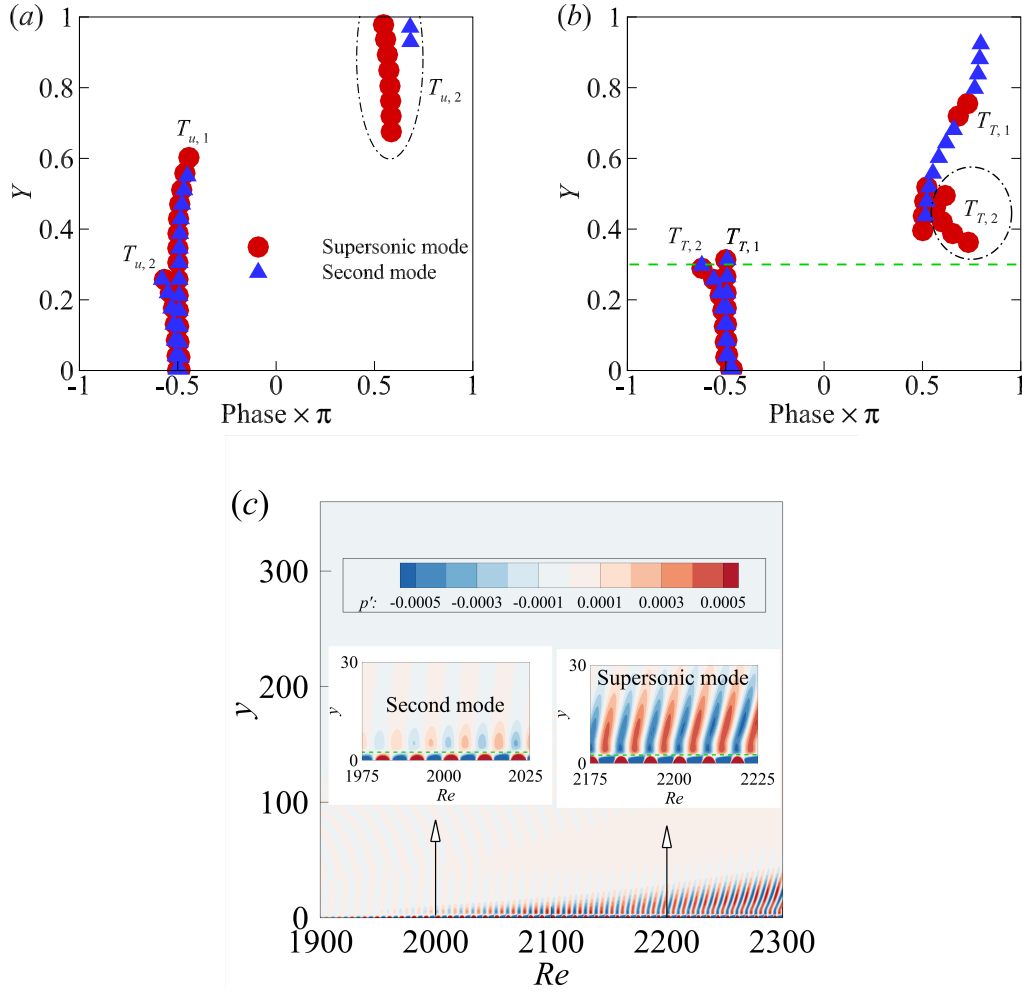


FIGURE 4.7: Phase profiles of significant source terms in the regions of the second mode ($Re = 2000$) and the supersonic mode ($Re = 2200$) of (a) equation (2.15) and (b) equation (2.17). (c) Contour of pressure fluctuation of case 2. Dashed green line in (b,c) represents the location above which the structures of the supersonic and second modes differ evidently.

in figure 4.5(a). Regarding the DAP, it is observed that the region of DAP becomes relatively narrow under the cold wall condition, which may be related to the emergence of the second generalized inflection point in the very near-wall region (see figure 4.4(c)). The contribution of the heat exchange (HE) effect is not distinguished under the cold wall condition for both the second and supersonic modes near the critical layer. This may be relevant to the decreased magnitude of the base temperature gradient in the outer region, as shown in figure 4.4(b). Furthermore, the HE effect is augmented by wall cooling near the wall, which is consistent with the near-wall resonance between $\bar{\rho}DT'/Dt$ and $T_{T,1}$ in figure 4.5(b). The considerable reduction in the HE of the supersonic mode near the critical layer is also consistent with the explanation provided by RPA in figure 4.7(b). The transition from a strong second-mode instability to a weak supersonic-mode instability may be associated with the fact that the wall-normal heat

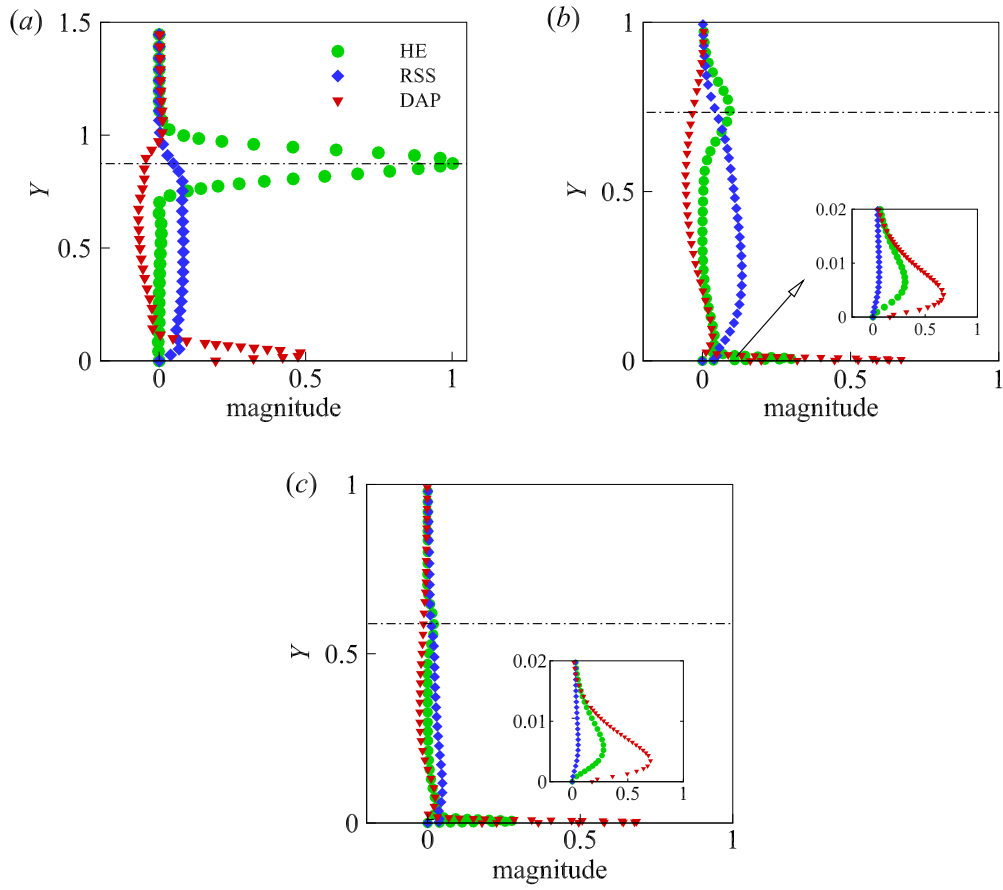


FIGURE 4.8: Distribution of source terms in equation (3.5) normalised by each maximum left-hand side term of equation (3.5) at (a) $Re = 1183$ for case 1, and (b) $Re = 2000$ and (c) $Re = 2200$ for case 2. The critical layer location is denoted by the dash-dotted line.

exchange (also $T_{T,1}$) gradually becomes out of phase with the internal energy fluctuation. These observations indicate that RPA and Lagrangian energy analysis point to the same mechanisms for the inviscid high-frequency modes in hypersonic boundary layers.

4.7 Conclusion

The energy source terms of unstable modes over flat-plate boundary layers are investigated using RPA across various Mach numbers and wall temperature ratios. Through phase analysis of source terms and changes in momentum and internal energy perturbations, the leading source terms of the T-S mode, the oblique first mode, the second mode, and the supersonic mode are clarified, considering porous coatings and obliqueness. It is found that the contribution of the Reynolds shear stress is strengthened along with the distinctive oblique characteristics of the most unstable first mode, whereas that of the Reynolds thermal stress is weakened for the second

mode when considering obliqueness. Wall cooling causes fluctuations in Reynolds thermal stress and dilatation to become out of phase with the rate of change of internal energy fluctuations, thereby stabilizing the oblique first mode. However, a newly generated prominent region of wall-normal internal energy transfer, located beneath the second generalized inflection point, destabilizes the second mode under wall cooling. In terms of the effect of porous coating, it stabilizes the second mode in a manner comparable to wall heating while destabilizing the oblique first mode by enhancing mean shear production in phase. Moreover, the wall-normal transport of internal energy at the critical layer becomes more out of phase for the significantly weaker supersonic mode, which accounts for the decreased contribution to energy growth. Connections and consistencies are highlighted with the inviscid thermoacoustic interpretation for the second and supersonic modes. The distinct sources in the critical layer and near-wall regions provide an in-depth understanding of the regional energy amplification processes of the inviscid modes in hypersonic boundary layers. Possible control strategies aimed at different modal properties are discussed preliminarily.

4.8 Appendix

4.8.1 Amplitude profiles

The amplitude distribution of the terms in equations (2.15–2.17) is also examined for further reference. The variation of the amplitude profiles across the considered wide parameter space is calculated. In general, the amplitude distribution supports the existence of the significant source terms reported by the relative phase analysis in each parametric state. This finding is not surprising, as consistency between RPA and the Lagrangian interpretation has been established. The criterion of the Lagrangian interpretation is based on the magnitude of the amplitude.

In this section, the amplitude profiles of two representative states are presented: the oblique first mode under adiabatic wall conditions in figure 4.9 and the second mode under cold wall conditions in figure 4.10. Figure 4.9 illustrates that the amplitudes of mean shear ($T_{u,1}$) and Reynolds thermal stress ($T_{T,1}$) are pronounced across the boundary layer for the oblique first mode. The amplitudes of viscous stress ($T_{u,3}$) and viscous dissipation and thermal conduction ($T_{T,3}$) become comparable to the rate of change in fluctuations below the critical layer. It is concluded that these distributions of amplitude magnitudes support the significant source terms shown with RPA in figure 4.2 in the corresponding wall-normal region. For the behavior of

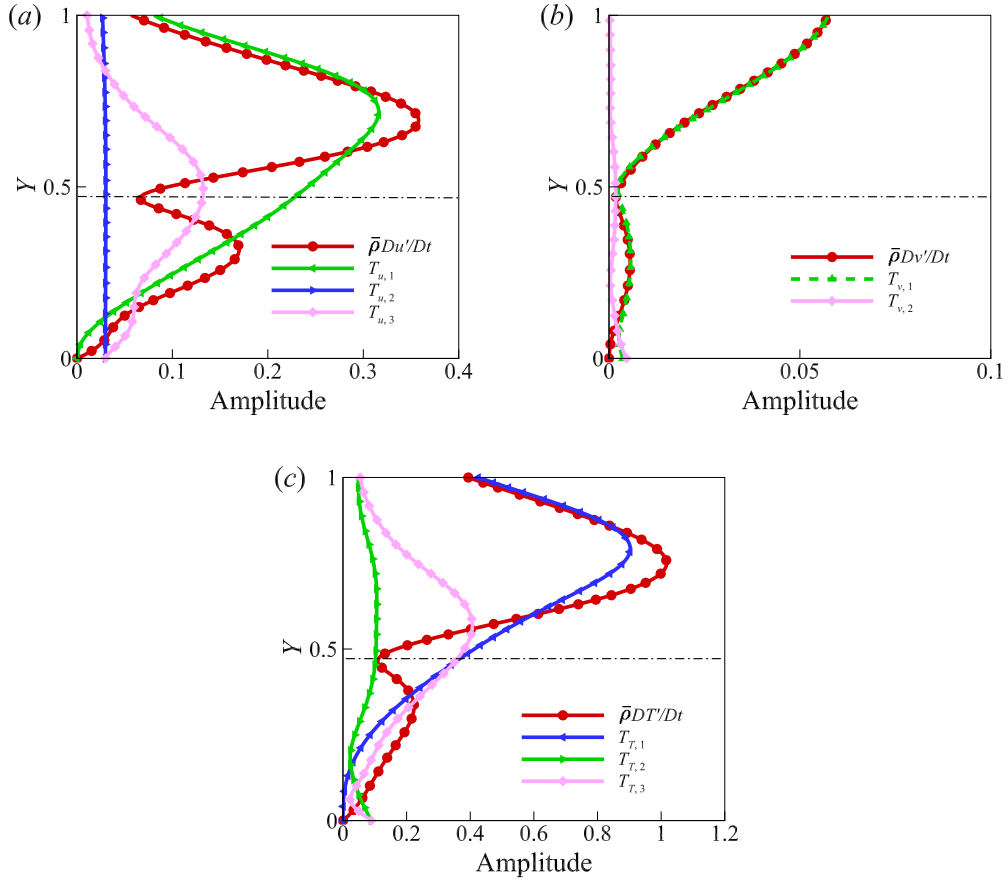


FIGURE 4.9: Amplitude distribution of the terms in (a) equation (2.15), (b) equation (2.16), and (c) equation (2.17) for the oblique first mode at $M = 3.5$ and $T_w / T_{ad} = 1.0$. Dash-dotted line marks the location of the critical layer.

the second mode in figure 4.10, mean shear ($T_{u,1}$) and the streamwise gradient of pressure fluctuations ($T_{u,2}$) exhibit relatively large amplitudes below the critical layer compared to the amplitude of $\bar{\rho}Du'/Dt$. Regarding the sources of internal energy fluctuations, the amplitude of Reynolds thermal stress ($T_{T,1}$) near the critical layer and dilatation ($T_{T,2}$) near the wall are comparable to that of $\bar{\rho}DT'/Dt$. Additionally, the newly generated source term ($T_{T,1}$) near the wall due to wall cooling (see figure 4.5(b)) also shows a moderate amplitude. The amplitudes of the viscous-associated terms become negligible for the second mode. All these amplitude observations show no inconsistency with the conclusions drawn from RPA.

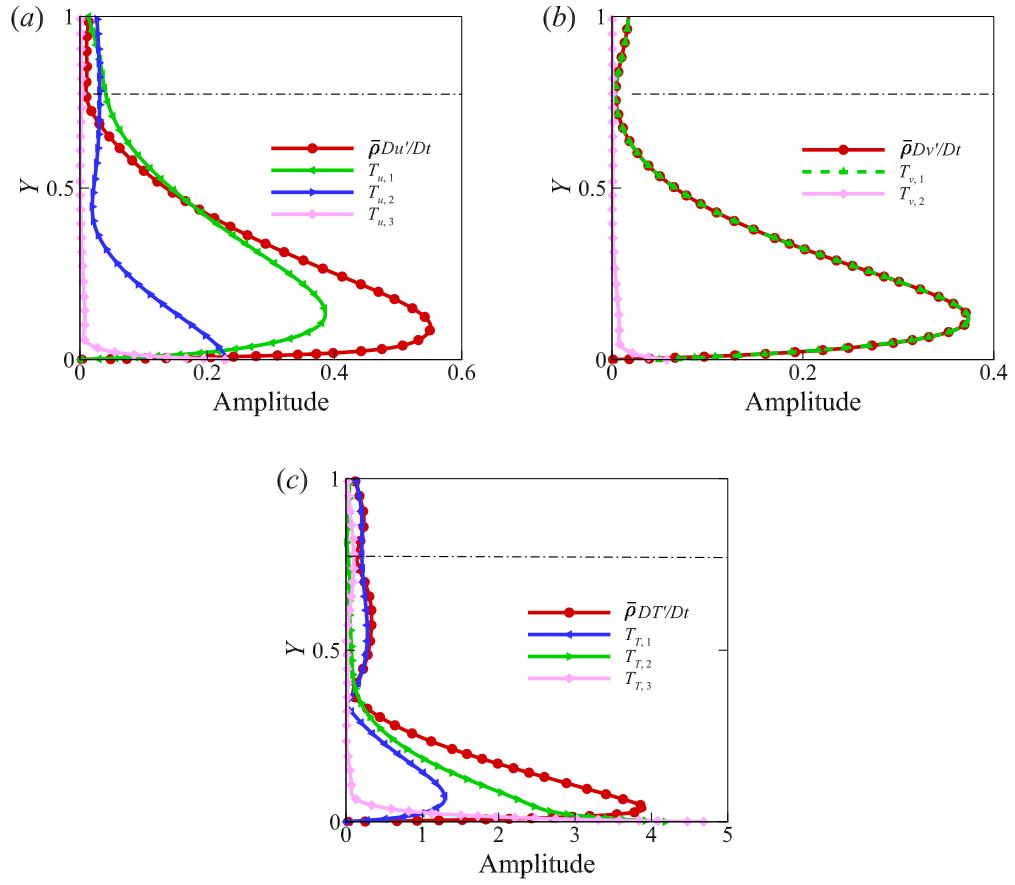


FIGURE 4.10: Amplitude distribution of the terms in (a) equation (2.15), (b) equation (2.16), and (c) equation (2.17) for the second mode at $M = 6$ and $T_w / T_{ad} = 0.2$. Dash-dotted line marks the location of the critical layer.

Chapter 5

Optimal disturbances and growth patterns in hypersonic blunt-wedge flow

[Published paper III¹]

5.1 Preface to the chapter

This chapter is reorganized based on the conference published in the *Chinese Journal of Aeronautics*, titled "Optimal disturbances and growth patterns in hypersonic blunt-wedge flow" [85]. Following the modal instabilities analysis in Chapters 3-4, this Chapter considers the non-modal disturbance with the appearance of bluntness using resolvent analysis. In the context of resolvent analysis, the optimal disturbance corresponds to the instability which undergoes the largest energy amplification for the system. For hypersonic boundary layers, the optimal disturbance is notably contributed by normal-mode instabilities, such as Mack second mode. However, recent experimental and numerical efforts have demonstrated the dominance of non-modal growth in hypersonic flows with the presence of moderate nose bluntness. In this Chapter, resolvent analysis and parabolized stability equation analysis are performed to investigate the response pattern, the parametric dependence, and the effects of bluntness and wall temperature on the instabilities

¹**Y. Chen**, T. Ma, P. Guo, J. Hao and C. Y. Wen (2025). Optimal disturbances and growth patterns in hypersonic blunt-wedge flow. *Chinese Journal of Aeronautics*, 38(7),103461.(**Highlighted Article**)

over a blunt-tip wedge which suggests potential control strategies. Interestingly, two growth patterns of travelling waves which appear respectively inside the boundary layer and the entropy layer are identified to compete, whose nature is the intersection of the energy gain of optimal and sub-optimal disturbances. Moreover, a detailed discussion about the character of optimal disturbances and control strategies on the amplification of modal and nonmodal instabilities in hypersonic boundary layers over blunt-tip models has been provided in this chapter. Furthermore, a potential explanation for "transition reversal" is proposed by integrating the receptivity to the slow acoustic wave and resolvent analysis.

5.2 Abstract

In hypersonic boundary layers, the optimal disturbance is notably contributed by normal-mode instabilities, such as Mack second mode. However, recent experimental and numerical efforts have demonstrated the dominance of nonmodal growth in hypersonic flows with the presence of moderate nose bluntness. In this study, resolvent analysis and parabolized stability equation analysis are performed to investigate the instabilities over a blunt-tip wedge. Main parameters include Mach number 5.9, unit Reynolds number $91.5 \times 10^6/\text{m}$, half wedge angle 5 degree, and nose radii ranging from 2.54 mm to 15.24 mm. Two novel growth patterns of travelling waves are identified to compete, whose nature is the intersection of the energy gain of optimal and sub-optimal disturbances. Pattern A with large spanwise wavelengths has the signature of slow energy amplification over a long distance which concentrates in the entropy layer. By contrast, pattern B with relatively small spanwise wavelengths presents rapid transient growth inside the boundary layer. A systematic study is performed on the growth/attenuation mechanism of disturbance patterns and the effects of wall temperature and nose radius. Wall cooling is found to be an alternative control strategy aiming at nonmodal instabilities. The receptivity to slow acoustic waves is considered when studying the effect of bluntness. An estimated amplitude response favorably reproduces the reversal-like phenomenon. Finally, the lift-up/Orr mechanism analysis provides an explanation of energy growth for nonmodal responses.

5.3 Introduction

Recent research of instabilities over Ogive-Cylinder models [122–124] reported three disturbance patterns: rope-like, elongated, and wisp-like structure disturbances corresponding to the second mode, the first mode, and the disturbance inside the entropy layer, respectively, accompanying with the increase of nose bluntness. However, with the perturbation imposed downstream of the bow shock, direct numerical simulation presented a monotonous delay of the transition onset as bluntness was increased. In contrast, considering the freestream receptivity process by introducing forcings upstream of the bow shock, the importance of non-Mack-mode disturbances was also reported in hypersonic flow over blunt flat plates beyond the critical reversal nose radius [125]. The unstable first or second mode is generated by a resonance process due to the synchronization of the phase speed of discrete modes [19]. In general, the first and second modes correspond to signals with physical frequencies of tens and hundreds of kilohertz, respectively.

In hypersonic flows over increasingly blunted models, the most energetic disturbance presents a shift from high-frequency bands toward low-frequency ones [42, 126], which might imply the significance of the first-mode or nonmodal instabilities [42]. In the noisy wind-tunnel conditions, the disturbance in the incoming freestream is dominated by the slow acoustic wave [127], which has been identified in wind-tunnel experiments [128–130] and widely recognized in numerical simulations [28, 115, 131] and applied in improved transition prediction methods [69, 132]. Acoustic waves are predominantly generated by turbulent boundary layer radiation along the nozzle wall, necessitating the inclusion of slow-wave receptivity in bluntness effect studies.

Despite the above reviews regarding the significant role of nonmodal disturbances, the distribution patterns of different optimal disturbances (streaks, oblique waves, and planar waves) and their dependence on external factors, such as wall temperature and bluntness, remain scarcely discussed and poorly understood. The appearance of the most amplified disturbances in either the entropy layer or the boundary layer eventually results in different physical mechanisms of transition to turbulence. The information about the wall-normal locations of optimal disturbances would benefit the proposal of possible control techniques as well. Therefore, it is fundamentally crucial to determine in which region the most energetic disturbance undergoes energy amplification. In this paper, a resolvent analysis of a blunt wedge flow is performed to reveal the strongest response to external forcings. In the linear instability stage, the resolvent analysis has been proven by the authors to have an equal accuracy as the direct numerical simulation [133]. Meanwhile, a modal parabolized stability equation (PSE) analysis is employed to reveal the region of normal-mode instabilities. The classical Orr and lift-up nonmodal growth mechanisms will be examined to interpret how the nonmodal instabilities are amplified in the entropy/boundary layer.

Another noteworthy issue is the control of nonmodal disturbances over such blunted models. Wall manipulations, such as acoustic metasurfaces implemented by porous coatings, have been confirmed by the authors to effectively suppress the second-mode instability [11, 124, 126]. As for the wall cooling effect, its suppression on the first mode and destabilization on the second mode have been illustrated both in early stability analysis [21, 132] and recent energy source analysis [133]. It is found that wall cooling stabilizes the first mode by weakening the Reynolds thermal stress and dilatation fluctuations. Meanwhile, wall cooling destabilizes the second mode by producing a new, active zone of wall-normal internal energy transport beneath the second generalized inflection point. The effects of local heating and cooling on the nonlinear and breakdown stages are systematically reported by Zhou *et al.* [134]. For control strategies for crossflow

mode induced by generalized inflection point of spanwise profiles, Nie *et al.* [132] indicated that a flat pressure distribution and a short leading-edge flow acceleration zone are beneficial for controlling cross-flow instabilities. However, to the best knowledge of the authors, rare attention has been paid to controlling the nonmodal instability over moderately or largely blunted models. Obviously, it is of engineering significance to control the dominant nonmodal instabilities over blunt bodies. A relevant and inspiring study by Tempelmann *et al.* [135] on swept plate models suggested that wall cooling could stabilize crossflow modes while destabilizing disturbances of nonmodal nature. Regarding transient growth in compressible boundary-layer flows, the wall temperature effect was studied on the optimal disturbances over a flat plate [133] and a sphere [136]. The literatures reported that wall cooling could destabilize the low-Mach-number boundary layer. Nevertheless, a detailed energy analysis from the transient growth mechanism is lacking and necessary. The eligibility of the wall cooling strategy merits further considerations for nonmodal instabilities over the blunt wedge, and the control mechanism needs to be comprehended. Currently, it is well-recognized that wall cooling would stabilize the first mode. The energy analysis of our previous work [133] indicated that the stabilization mechanism is highly related to the mean shear reduction. In terms of the concerned nonmodal disturbances, the dominant component appears to be the temperature perturbation [30]. Therefore, it will be of further interest in this paper to evaluate the effect of wall cooling or heating as a potential controlling technique for nonmodal instabilities over blunt bodies.

This chapter is intended to deepen the understanding of the characteristics of modal and nonmodal instabilities in hypersonic flows over blunt bodies. It is expected to throw light on the effects of wall temperature, nose radius, and the underpinning transient growth mechanism. The resolvent analysis, parabolized stability equation, and direct numerical simulation are employed. The optimal responses inside the entropy layer or boundary layer are systematically examined. As for the effect of bluntness, only receptivity and linear growth stages are investigated and discussed, while nonlinear interaction and transitional stages will be studied in the future.

This chapter is organized as follows. Section 5.4 provides the investigated model, the parametric setup, and the numerical methods for base flow calculation and stability analysis. Section 5.5 shows the results and discussions, which are divided into 5 parts. The base flow over the blunt wedge is presented in Section 5.5.1. The distribution patterns of the optimal responses with different frequencies and spanwise wavenumbers are depicted in Section 5.5.2. Subsequently, the effects of wall cooling and bluntness are examined in Section 5.5.3 and 5.5.4, respectively. The Orr and lift-up mechanisms are explored from the perspective of energy and vorticity

evolutions in Section 5.5.5. Finally, the main findings are summarized in Section 5.6. In particular, the appendix Section provides an example for understanding the competition among different types of disturbances, which are manifested as the optimal response and a series of sub-optimal disturbances.

5.4 Numerical setup

5.4.1 Geometric configuration and flow conditions

The freestream flow condition investigated here follows that by Paredes *et al.* [17] at Mach 5.9 and a unit Reynolds number of $91.5 \times 10^6/\text{m}$. The freestream static and total temperatures are 76.74 K and 611 K, respectively. The baseline wall temperature is 300 K with $T_w/T_{ad} = 0.57$, where the subscripts w and ad refer to quantities at the wall and under the adiabatic wall condition, respectively. Here, T_{ad} is the laminar recovery temperature. The studied model is a blunt wedge with a 5-degree half-wedge angle. Three nose radii (R_n^*) are studied, including 2.54 mm, 5.08 mm, and 15.24 mm. The asterisk represents dimensional quantities. Based on the experimental data [17, 122], the reversal Reynolds number based on the nose radius is around 1×10^6 for the cone. Thus, the radii 2.54 mm and 15.24 mm can be regarded as moderate and large blunted cases, respectively. The grid independence is verified and shown in Appendix 5.7.1, which demonstrates the results of both the base flow and resolvent analysis are converged under the considered grid resolution.

5.4.2 Base flow and slow acoustic wave model

The base flow is calculated using an in-house multi-block parallel finite-volume solver called PHAROS, which has been successfully employed in various configurations, such as the compression corner, the double wedge, and the double cone models [88–90]. The inviscid flux is calculated by MUSCL scheme and a modified Steger-Warming scheme, and the second-order central difference scheme is utilized for viscous flux discretization. Pseudo-time advancing is achieved by the implicit line relaxation method. More detailed information for the PHAROS can be found in the Ref. [88]. In terms of the boundary condition, the $y = 0$ boundary of the computational domain is set to be symmetric, the upper boundary is farfield, the right boundary is outflow via extrapolation, and the wall boundary is no-slip and isothermal or adiabatic,

depending on the case. To explore the effect of nose bluntness (Section 5.5.4), the receptivity process to free stream slow acoustic waves is considered. We imposed a slow acoustic wave disturbance at the computational domain's freestream boundary. The dimensionless fluctuation of the plane slow acoustic wave [5] is

$$\begin{pmatrix} \rho' \\ u' \\ v' \\ p' \end{pmatrix} = \begin{pmatrix} A_\infty Ma_\infty^2 \\ -A_\infty Ma_\infty^2 \\ 0 \\ A_\infty \end{pmatrix} e^{i(\alpha x - \omega t)} + c.c., \quad (5.1)$$

where A_∞ is the small amplitude of the freestream disturbance, ω is the angular frequency, α is the stream-wise wavenumber, and c.c. denotes complex conjugate. In this study, the incident angle of the slow acoustic wave is set to zero and the dispersion relation is $\alpha = \omega/(1 - 1/Ma_\infty)$. In addition, u , v , and w are the velocities in the x , y , and z directions, which denote streamwise, wall-normal, and spanwise directions, respectively. Meanwhile, a body-fitted orthogonal coordinate system is constructed, which is represented by (ξ, η, z) . In the simulation of the receptivity process, the three-stage third-order Runge-Kutta method is employed for time advancement.

5.5 Results

In this Section, the results of the numerical simulation and stability analysis will be presented and discussed. The base flows for the considered three nose radii are shown in Section 5.5.1. The characteristics of different optimal disturbances will be focused on in Section 5.5.2 for a moderate bluntness $R_n^* = 2.54$ mm with $T_w/T_{ad} = 0.57$. Following that, the effects of wall temperature and bluntness will be examined in Sections 5.5.3 and 5.5.4, respectively. Finally, the transient growth mechanism will be analyzed in Section 5.5.5.

5.5.1 Base flow

The contours of the Mach number for cases with different nose radii are shown in figure 5.1 with $T_w/T_{ad} = 0.57$. The criterion to determine the boundary layer edge is set to the location where $h_t/h_{t,\infty} = 0.995$, where h_t is the total enthalpy. The entropy-layer edge is defined as the wall-normal location where the entropy increment equals 0.25 times the entropy increment at

the local wall. The entropy increment is defined as

$$\Delta S^* = c_p^* \ln(\bar{T}^*/\bar{T}_\infty^*) - R^* \ln(\bar{p}^*/\bar{p}_\infty^*), \quad (5.2)$$

where c_p^* is the specific heat at constant pressure, and R^* is the specific gas constant. The entropy-layer effect over a wedge [38] is stronger than that over a cone [5] under the same freestream conditions, as the streamline keeps nearly parallel to the wall downstream [137]. As shown in figure 5.1, as the nose radius is increased, the entropy-layer region is enlarged. According to the stability analysis [42], the first/second mode usually appears downstream, and the entropy-layer mode emerges upstream. The latter has a relatively low growth rate, which is not likely to dominate the transition process [38]. On the other hand, the enhanced entropy layer may have an effect on the nonmodal disturbance generated herein. The effect of bluntness on the optimal disturbance will be discussed in Section 5.5.4.

5.5.2 Features of optimal disturbances

After the converged base flow is obtained, the resolvent analysis is performed. The characteristics of the optimal responses, including streaks, plane, and oblique traveling disturbance, will be discussed in this Section.

Figure 5.2 shows the plots of the optimal gain for different frequencies and spanwise wavenumbers and the N -factor distributions along the x direction for typical optimal oblique disturbances and plane waves. The N -factor is defined by $N = 0.5 \ln(E_{Chu}/E_{Chu,0})$, where $E_{Chu,0}$ is measured at $x = x_0$ and N is set to be 0 at x_0 . For brevity, (0.3, 0) represents the optimal response for a Fourier mode with $\omega = 0.3$ and $\beta = 0$, and similarly for other optimal disturbances. Generally, in figure 5.2(a), the maximum energy gain exhibits a monotonous de-creasing trend as the frequency is increased. An interesting finding is that as β increases, the optimal response switches from pattern A (open symbols) to pattern B (solid symbols). The former pattern is mainly distributed inside the entropy layer between the entropy-layer edge and boundary-layer edge, while the latter shows its signature inside the boundary layer. As shown in figure 5.2(b), the N -factor of the pattern-A response (including plane waves and oblique waves when β is relatively small) grows at a moderate rate till the end of the computational domain. In contrast, pattern-B responses with large β undergo a rapid transient growth immediately downstream of the forcing location, followed by a decay process. Note that the pattern-B waves were not identified in some similar studies on the optimal disturbances such as by Paredes *et al.* [17].

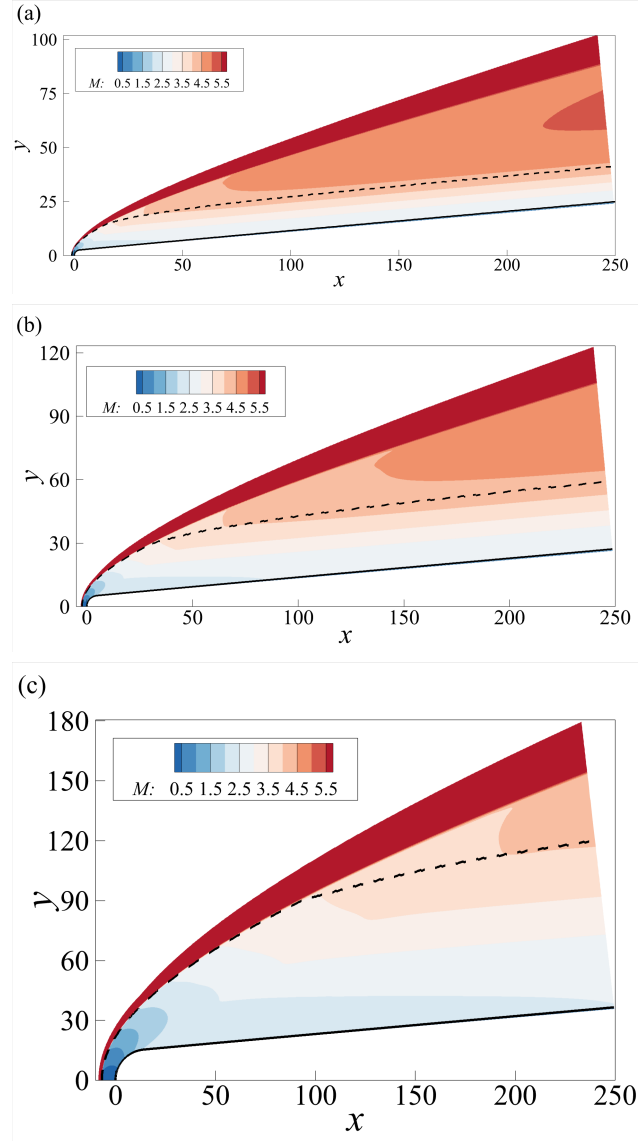


FIGURE 5.1: Contours of Mach number for blunt wedges for (a) $R_n^* = 2.54$ mm, (b) $R_n^* = 5.08$ mm, and (c) $R_n^* = 15.24$ mm. The boundary layer edge is represented by the solid black line, whereas the dashed black line signifies the entropy layer edge.

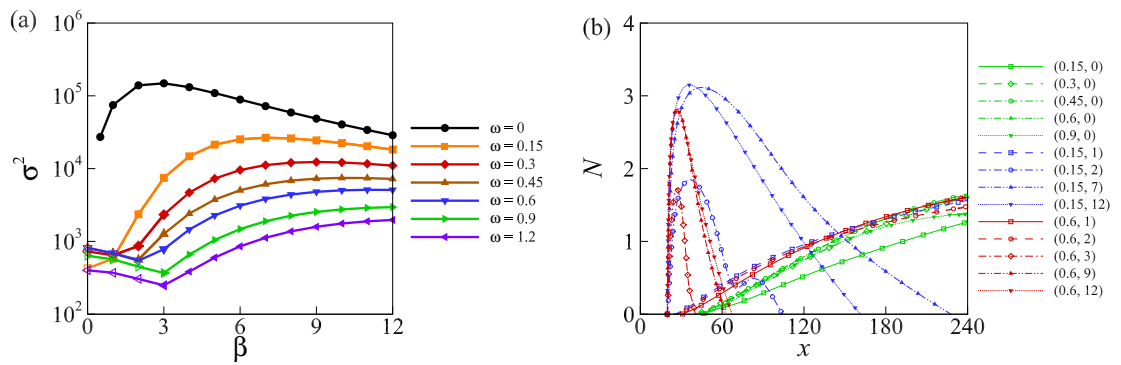


FIGURE 5.2: Optimal gains for different frequencies and spanwise wavenumbers (open symbols represent pattern A and solid symbols represent pattern B), and (b) N -factor distributions for typical optimal disturbances along the x direction.

This is probably due to the different definitions of the energy gain. Paredes *et al.* [17] used the ratio of energy at two different streamwise locations as the optimization object function, whereas the present work considers the energy integrated over the whole domain (see Eqs. (2.9, 2.10)). The former definition may miss the possible energy overshoot, which was also realized by Paredes *et al.* [126]. The energy overshoot may occur over configurations such as hypersonic blunt forebodies [138], which are manifested as pattern-B waves in this paper. In this study, the optimal response pattern A and pattern B, are the products obtained from a selection process. In detail, the maximum energy gain is sought for different β at a fixed frequency. Two different patterns exactly appear in the respective entropy and boundary layers, corresponding to small- and large- wavenumber responses, respectively. The natural external disturbance is a combination of broadband frequencies and various wavenumbers in wind tunnels [77]. In realistic conditions, these patterns usually coexist subject to broadband incoming disturbances. They can be distributed in different regions (boundary layer and entropy layer).

The turning of the gain curve in figure 5.2(a) as the state transitions from pattern A to B may indicate a competition of different growth patterns. To confirm this deduction, the Arnoldi iteration method is further employed in the resolvent analysis to find a series of sub-optimal disturbances. An illustration of the competitive physical image between different growth patterns is shown in the Appendix 5.7.2. The turning point of the optimal gain curve in figure 5.2(a) is actually the intersection of the gain between optimal and the first sub-optimal responses. For brevity, only the optimal response is discussed in the main body. When the nose bluntness exceeds a threshold, modal instabilities tend to be too weak to trigger the transition, and transient growth of low-frequency disturbances is of particular significance [126]. Therefore, for blunt wedges with moderate or large nose radius, high-frequency disturbances are less considered. In fact, no pronounced second-mode instabilities are observed in the case considered here. In the following analysis, three kinds of representative forcings and associated optimal responses of the system, i.e., streaks, planar waves, and oblique waves, will be presented and discussed. The contributions of different parts of energy are quantified based on Chu's energy. For example, the contribution of the streamwise velocity fluctuation is evaluated by the indicator I_u

$$I_u(x) = \int_0^\infty \left(\overline{\rho u'^2} \right) dy \Big/ 2E_{Chu}. \quad (5.3)$$

Similar indicators can be obtained for the fluctuations of the wall-normal velocity, the spanwise velocity and thermodynamic quantities.

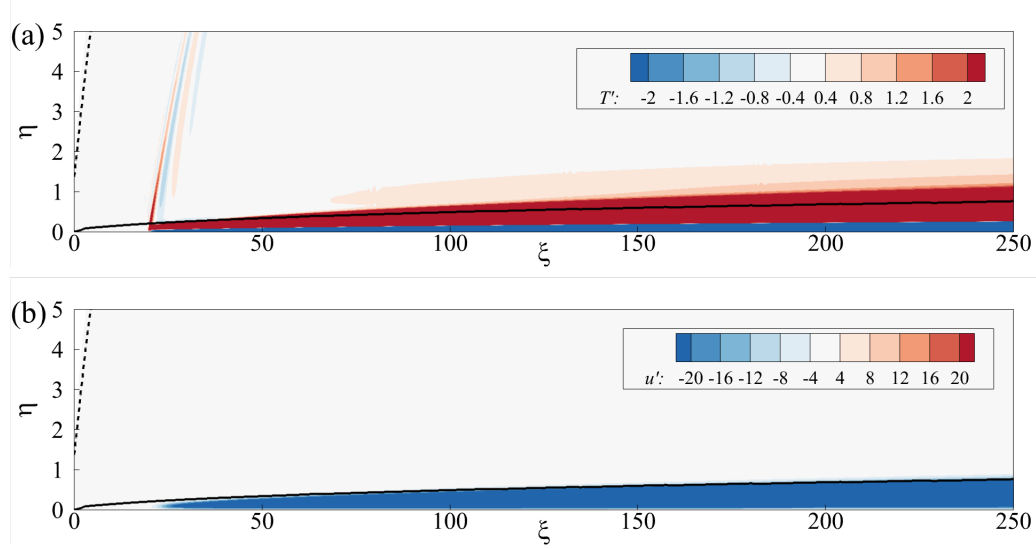


FIGURE 5.3: Contours of normalized (a) temperature and (b) streamwise velocity perturbations of the optimal response for streaks (0, 2). The solid and dashed black lines mark the locations of the boundary layer edge and the entropy layer edge, respectively.

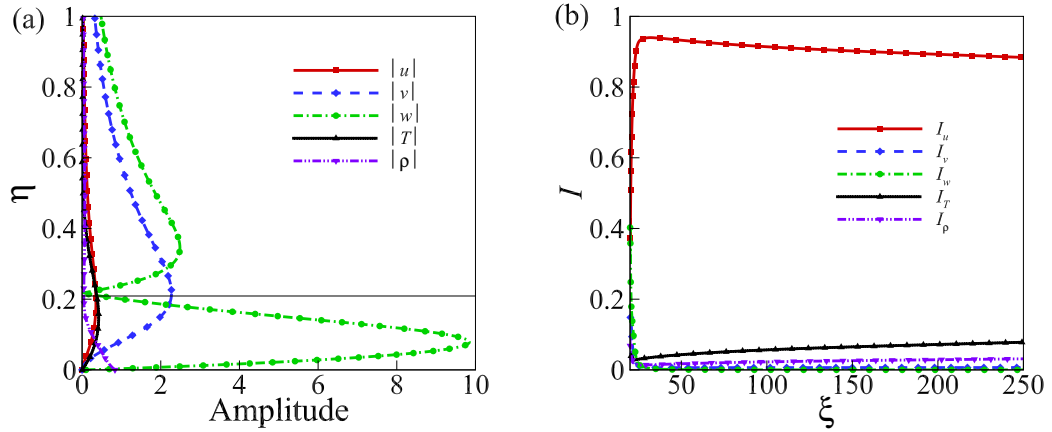


FIGURE 5.4: (a) Input forcing profiles and (b) I indicators of Chu energy of optimal response for streaks (0, 2).

Figure 5.3 provides contours of streamwise temperature and velocity fluctuations of streaks (0, 2), which are mainly distributed inside the boundary layer. The input eigenfunction profiles and I indicators of Chu energy for streak (0, 2) are shown in figure 5.4. It is found that the main component of the forcing is constituted by the spanwise forcing. In terms of the downstream response, the streamwise velocity fluctuation (I_u) grows rapidly as it evolves downstream. A marginal contribution comes from the temperature and density fluctuations. The transfer of the dominant component indicates a classical lift-up mechanism, which will be discussed in detail in Section 5.5.5.

As shown in figure 5.5, the evolution of the plane wave (0.45, 0) shows a different signature

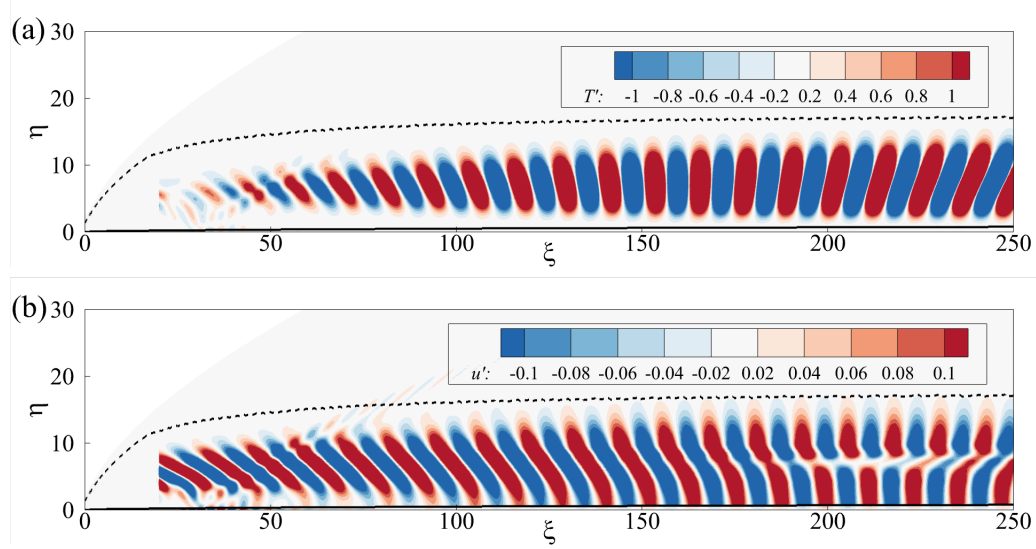


FIGURE 5.5: Contours of normalized (a) temperature and (b) streamwise velocity perturbations of optimal response for the plane wave (0.45, 0). The solid and dashed black lines mark the locations of the boundary layer edge and the entropy layer edge, respectively.

from that of the streaks, which is a representative pattern-A response and is mainly distributed inside the entropy layer. Similar growth patterns can be obtained for the optimal response when β is small, such as (0.6, 1) and (0.6, 2), as indicated by the N -factor in figure 5.2(b). The prominent role of disturbance inside the entropy layer for moderately blunt models was also reported in the experiments for ogive-cylinder [139] and blunt cones [6]. However, as mentioned above, the blunt wedge configuration encounters a stronger entropy layer, and the entropy swallowing phenomenon present in the cone model moves downstream in the wedge model [137]. This difference may delay the disturbance evolution inside the entropy layer to penetrate the boundary and, in turn, affect the transition process. Figure 5.6 plots the input eigenfunction profiles and I indicators of Chu energy for the plane wave (0.45, 0). The forcing component in the streamwise velocity equation constitutes the main component of the input forcings. The streamwise velocity response experiences a gradual decay downstream, while the internal energy components (temperature and density fluctuation) continue to grow and become dominant. Meanwhile, the tilting structure may indicate the presence of the Orr mechanism, which will be discussed in detail in Section 5.5.5.

As for the other pattern-B response (oblique waves when β is large enough), the streamwise velocity and temperature fluctuations are mainly distributed inside the boundary layer, as shown in figure 5.7. The input forcing profiles and I indicators of Chu energy for Fourier mode (0.3, 7) are shown in figure 8. It appears that the distribution resembles that of the streak (0, 2), indicating a possible unsteady-streak signature for (0.3, 7). This kind of optimal response

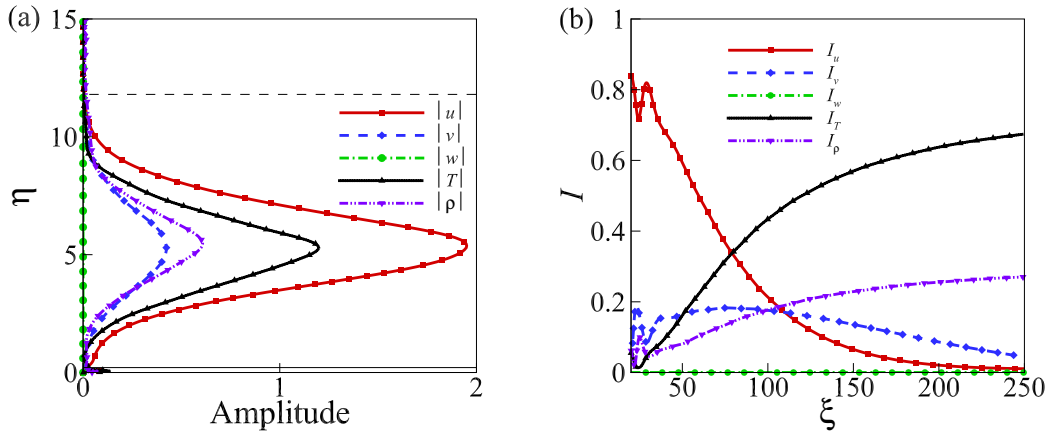


FIGURE 5.6: (a) Input forcing profiles and (b) I indicators of Chu energy of optimal response for the plane wave (0.45, 0).

undergoes rapid growth and attenuation near the input forcing location, which has rarely been reported in previous optimal disturbance analyses on the blunt cone models [17, 43]. One possibly similar observation is the disturbance reported by Hartman *et al.* [44] using direct numerical simulation, which shows growth near the leading edge before the appearance of the second mode. The elongated structure (shown in figure 5.7) inside the boundary layer may indicate the occurrence of a modal instability. To clarify this point, the integrated N -factor is obtained by PSE that is initialized by the upstream stable mode S (normal-mode solution). The modal PSE result manifests a purely modal instability evolution. Given that the linear PSE only offers a relative growth in spatial analysis without amplitude information, the N -factor curve of PSE is shifted to match that of resolvent analysis. This operation enables a comparison of the modal evolution with respect to the curve slope, i.e., the energy growth rate. As shown in figure 5.9, a good agreement is reached for both N -factor and fluctuation profiles between resolvent analysis and PSE after an initial transient growth region when β is not large. Thus, the energy attenuation of pattern-B waves is associated with the stable normal mode.

In summary, the evolution patterns of optimal streaks, plane waves, and oblique waves are studied using resolvent analysis for a moderate bluntness $R_n^* = 2.54$ mm with $T_w/T_{ad} = 0.57$. The energetic region of the resolvent response and the associated N -factor and optimal gain indicate the existence of different growth patterns. These patterns are competitive to be the most energetic one under a certain combination of ω and β , which may contribute to a competitive routine in triggering the transition. However, which routes (patterns) will finally stand out and play a dominant role in triggering the transition further depends on the receptivity process [140, 141] of the disturbance in the Fourier space, as well as the nonlinear instability stage.

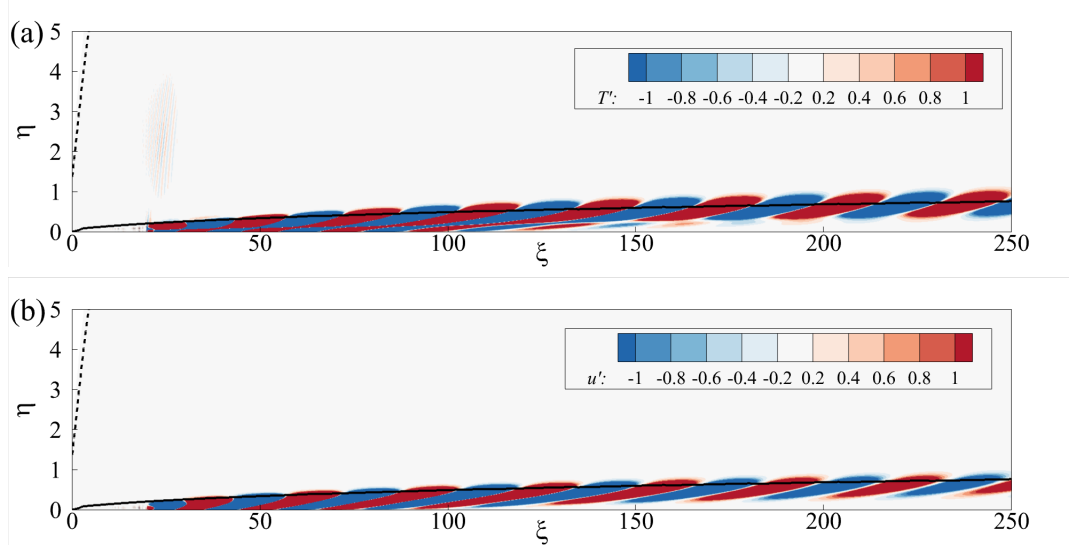


FIGURE 5.7: Contours of normalized (a) temperature and (b) streamwise velocity perturbations of optimal response for the oblique wave (0.3, 7) (pattern B). The solid and dashed black lines mark the locations of the boundary layer edge and the entropy layer edge, respectively.

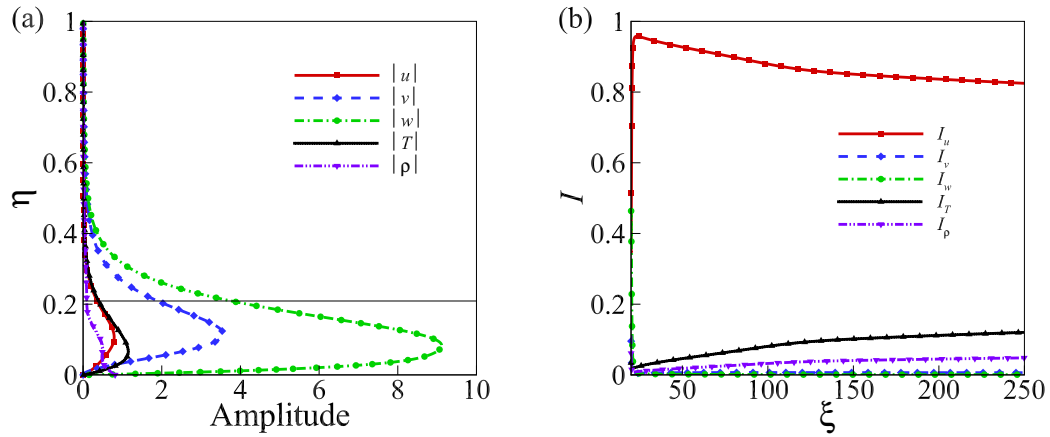


FIGURE 5.8: (a) Input forcing profiles and (b) I indicators of Chu energy of optimal response for the oblique wave (0.3, 7).

5.5.3 Effect of wall temperature

In this subsection, the adiabatic-wall case is further considered. The stabilization and destabilization effects of wall cooling on the first and second modes, respectively, have been recently explained from the perspective of energy analysis [75]. However, the effect of wall cooling on hypersonic flows where nonmodal instability is pronounced remains less understood fundamentally. As for a disturbance of nonmodal nature, wall cooling was reported to destabilize the disturbance in compressible boundary layers over swept flat plates [135], spheres [136], and flat plates [133]. For the present hypersonic flow over a blunt wedge, figure 5.10 shows the optimal

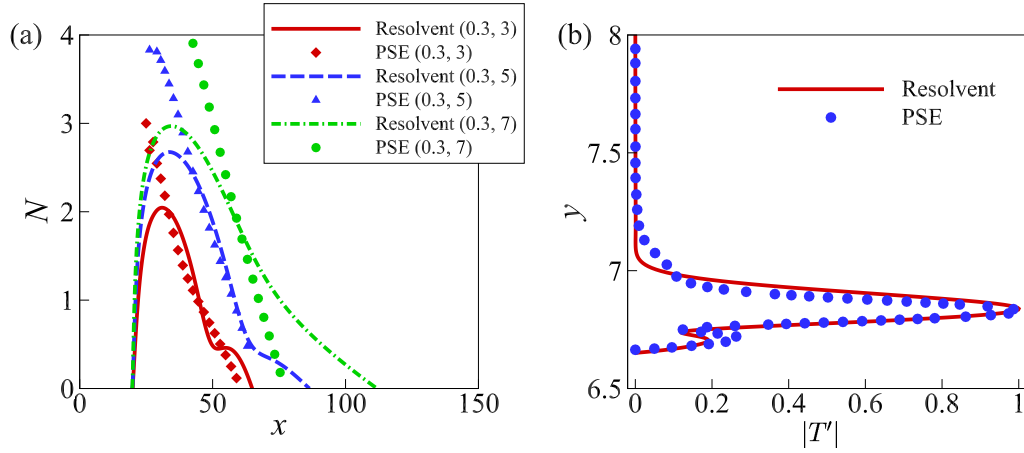


FIGURE 5.9: Comparison of (a) integrated N factor and (b) fluctuation temperature profile for (0.3, 5) at $x = 50$ (normalized by its maximal value inside the boundary layer) obtained by PSE and resolvent analysis.

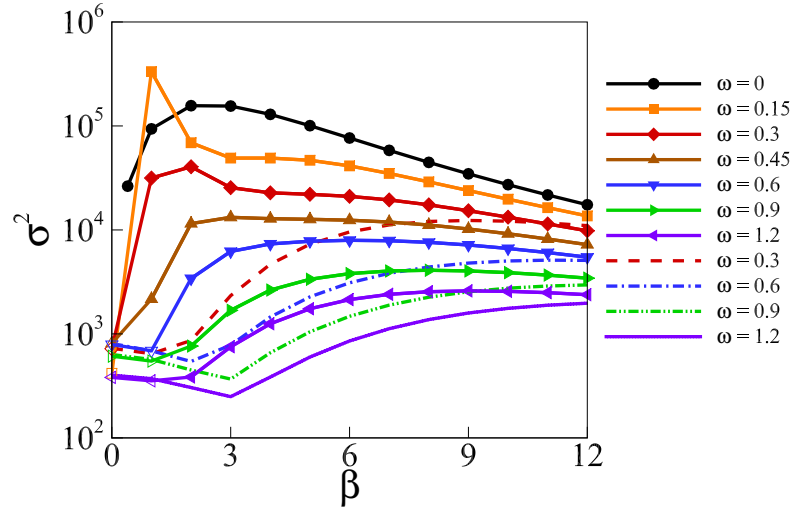


FIGURE 5.10: Optimal gains for different frequencies and spanwise wavenumbers for Ma 5.9 flow over the blunt wedge with $R_n^* = 2.54$ mm (solid symbols represent pattern A and hollow ones represent pattern B). Solid lines represent cases for adiabatic wall and dashed lines represent cases for $T_w/T_{ad} = 0.57$.

gain for different frequencies and spanwise wavenumbers for the adiabatic wall (solid line) and baseline cold wall (dashed line) cases. Compared with the cold-wall cases, the optimal gain is generally larger under the adiabatic condition. In particular, a peak (0.15, 1) arises with a low frequency and low spanwise wavenumber, whose optimal gain exceeds those of the stationary streak, plane wave and other pattern-B oblique waves with the same frequency. The comparison for gain contour between cold-wall and adiabatic wall conditions can be seen in Appendix 5.7.3. Figure 5.11 shows the contours of the normalized temperature and streamwise velocity perturbations for the optimal response (0.15, 1). The fluctuation is mainly distributed near the

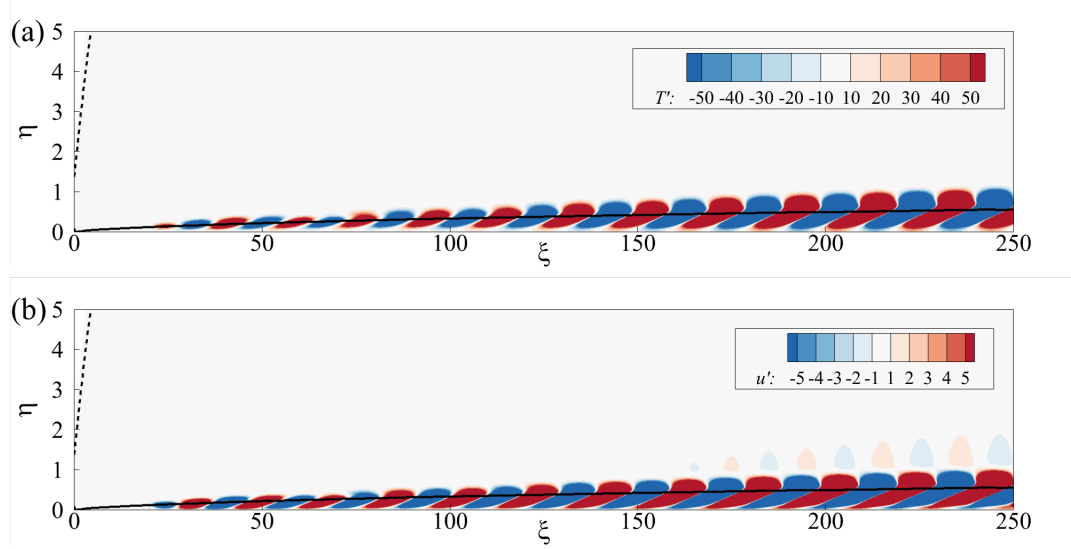


FIGURE 5.11: Contours of normalized (a) temperature and (b) streamwise velocity perturbation components of optimal response for Fourier mode (0.15, 1) under the adiabatic wall. The solid and dashed black lines mark the locations of the boundary layer edge and the entropy layer edge, respectively.

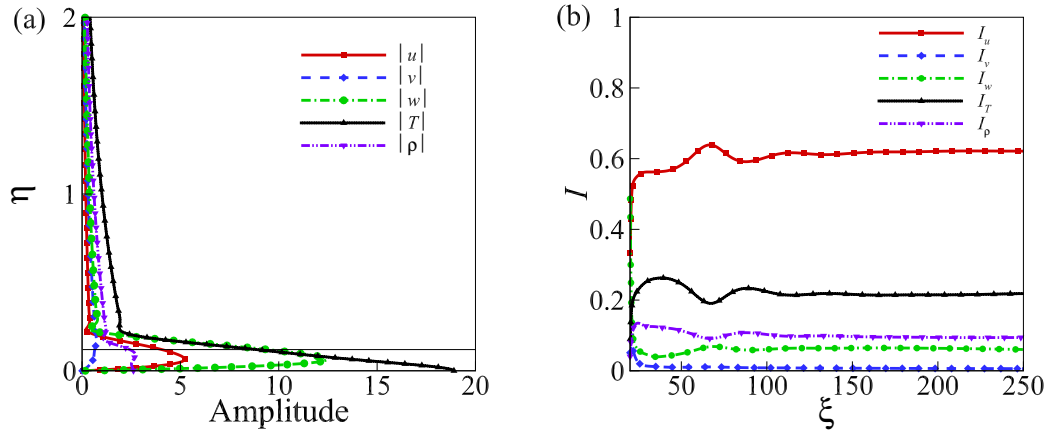


FIGURE 5.12: (a) Input forcing profiles and (b) I indicators of Chu energy of optimal response for Fourier mode (0.15, 1) under the adiabatic wall.

outer boundary layer, which resembles the contours of the first mode. The input forcing of the Fourier mode (0.15, 1) is shown in figure 5.12(a). The temperature fluctuation is dominant inside the boundary layer, which is different from the dominant w' for the Fourier mode (0.3, 7) shown in figure 5.8. The evolution of the I indicator of (0.15, 1) indicates the importance of streamwise velocity and temperature perturbations, which reach around 0.6 and 0.2 downstream, respectively.

To evaluate the role of the modal instability around the peak gain in figure 5.10, the PSE analysis is employed. As shown in figure 5.13, the initial transient growth stage is not captured by PSE

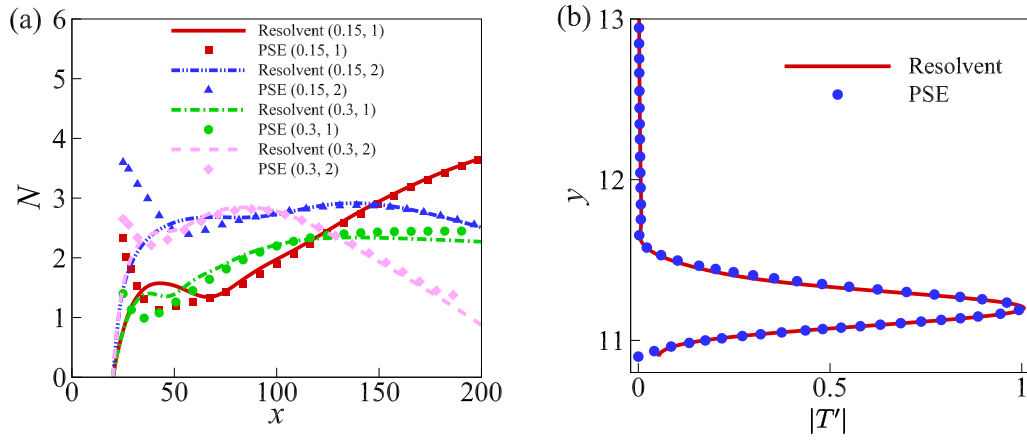


FIGURE 5.13: Comparison of (a) integrated N factors, and (b) fluctuation temperature profiles for (0.15, 1) at $x = 100$ (nor-malized by its maximal value inside the boundary layer) obtained by PSE and resolvent analysis.

initialized by mode S. However, in the second-stage growth, a good agreement is reached for both N factors and fluctuation profiles between resolvent analysis and PSE, which indicates the presence of the unstable oblique first mode evolving from mode S. This comparison confirms the appearance of the first mode with a low frequency and low spanwise wavenumber under the adiabatic wall condition.

Figure 5.14 provides the N -factor of the streak, the plane and the oblique optimal disturbances considering three different wall temperature conditions. Here, cases with a fixed frequency $\omega = 0.3$ (corresponding to 50 kHz) are selected, which correspond to a representative low frequency. It is indicated that wall cooling has nearly no effect on the plane wave while suppressing the transient growth of streaks and oblique waves, especially for lower β . When β increases and exceeds a critical value, the effect of wall temperature becomes minor for both optimal streaks and oblique waves.

The different effects of the wall temperature on the optimal disturbance can be attributed to their different growth patterns, as shown in Section 5.5.2. Specifically, the optimal plane wave is mainly distributed in the entropy layer, while the wall cooling effect only changes the mean variables inside the boundary layer, as indicated in figure 5.15. Therefore, wall cooling only affects the optimal oblique waves, which mainly lie inside the boundary layer and contain a pronounced temperature fluctuation. One special case is that the pattern-A response (0.3, 1) is located inside the entropy layer for the baseline cold wall, whereas the response switches to the growth pattern B (oblique wave inside the boundary layer) as the wall temperature increases, as shown in figure 5.14 (b). This indicates that the optimal response pattern not only depends on β

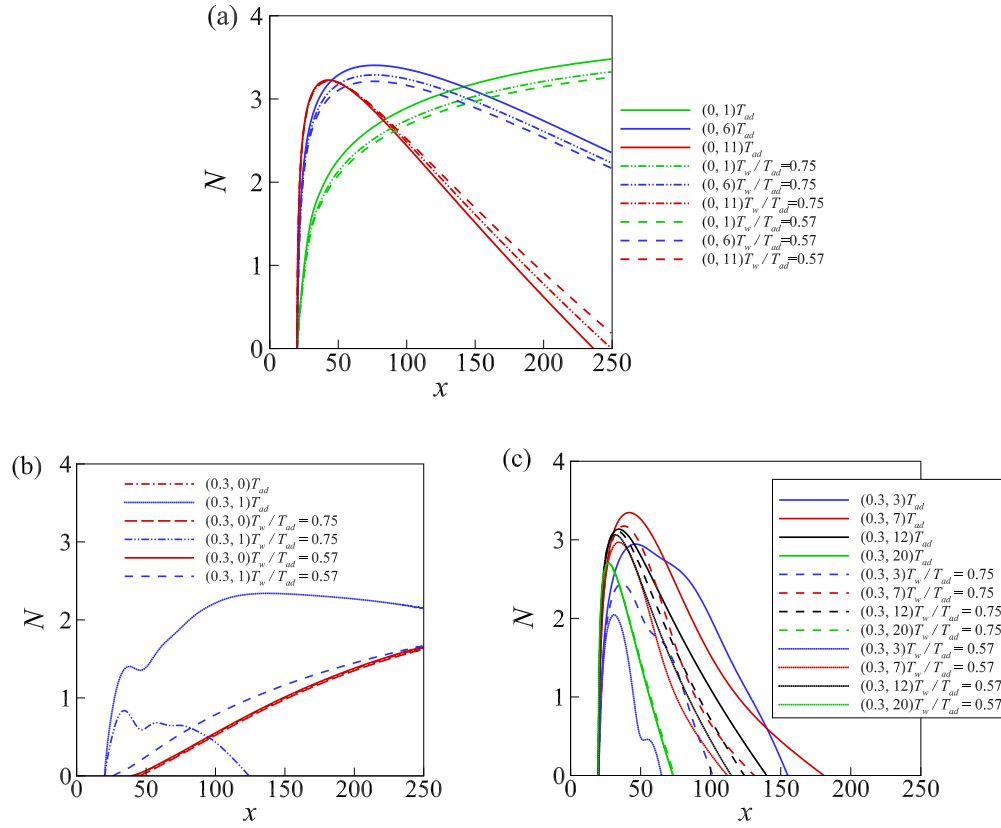


FIGURE 5.14: N -factor evolution of (a) streaks, (b) planar and oblique optimal waves with small β , and (c) oblique optimal waves with larger β along x .

for a fixed frequency but also may change with other parameters, such as the wall temperature. Consequently, wall cooling may serve as a potential control strategy for pronounced nonmodal disturbances over blunt bodies, which has rarely been mentioned before.

5.5.4 Effect of bluntness

The "transition reversal" for high-speed flows over blunt bodies was first reported in 1983 [39]. However, the physical mechanism behind it remains a debate currently. When the nose radius is approaching the reversal value, the modal instability has been confirmed to be of insufficient significance by linear stability analysis to trigger the transition, including the newly generated inflection instability arising from the entropy layer (entropy layer mode) [5, 38, 142, 143]. However, the nonmodal disturbance inside the entropy layer has been recognized to be possible to penetrate the boundary layer and excite a strong instability [37, 38, 82]. Recently, the nonmodal disturbance in the entropy layer has shown an alternative oblique transition route when there is no considerable second modal growth [18, 44]. The nonmodal instability, when the nose radius

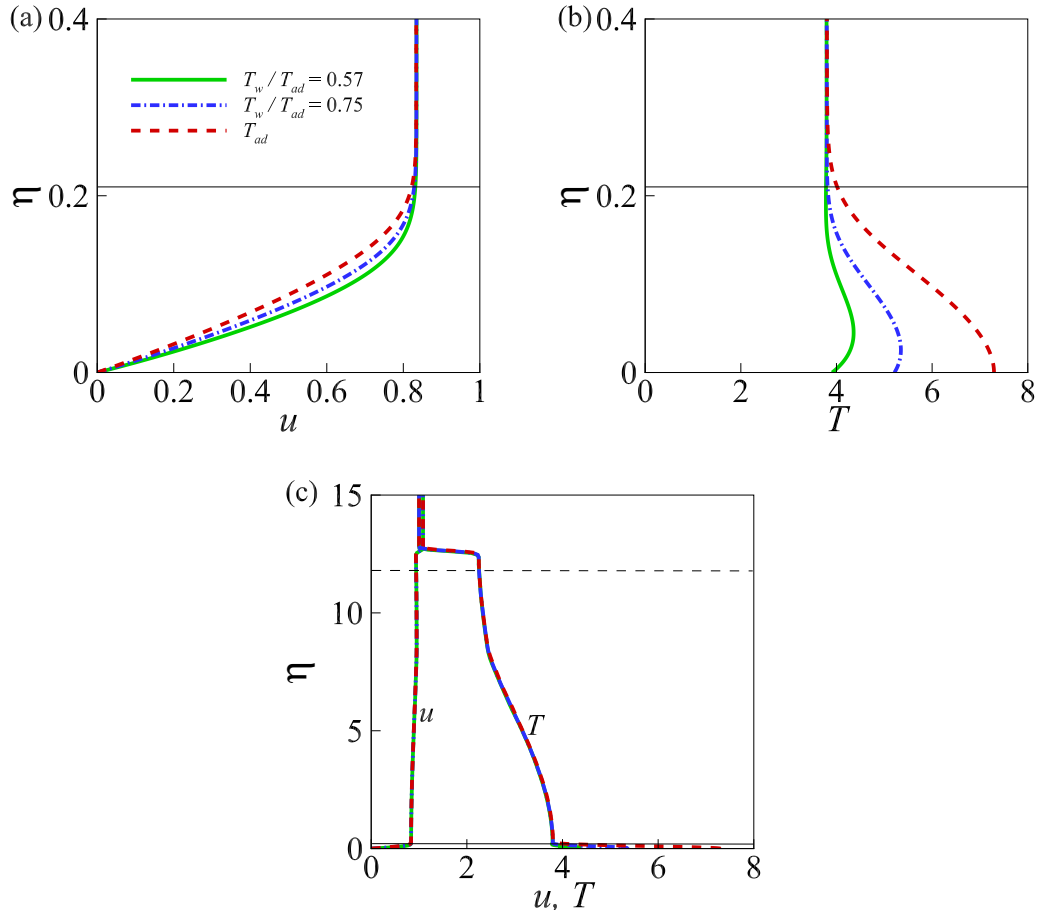


FIGURE 5.15: Mean (a) stream-wise velocity and (b) temperature profile distributions near the wall, and (c) mean profiles outside the boundary layer at $\xi = 20$. The solid and dashed black lines mark the locations of the boundary layer edge and the entropy layer edge, respectively.

exceeds a certain value, may provide a potential explanation for the "transition reversal" from the perspective of receptivity and linear instability.

Figure 5.16 provides a comparison of the N -factor of low-frequency (e.g., $\omega = 0.15$) optimal oblique waves and stationary streaks with the same ω and β yet different nose radii. The forcing location is fixed at $x = 40$ for all cases. Here, only pattern-B responses are displayed, because pattern-A responses lose their dominance as the increment of bluntness. These pattern-A responses correspond to no positive N -factor in figure 5.16 for $R_n^* = 5.08$ mm and $R_n^* = 15.24$ mm. The unstable first mode (0.15, 1) gets suppressed as the increment of bluntness for both modal instability and transient growth, as shown in figure 5.16(b). For other oblique waves, the suppression effect of bluntness becomes weaker as β increases. Finally, the N factor of oblique waves can be increased with growing bluntness when β exceeds a critical value. However, in the meantime, the N factor is also smaller for a large β . To conclude, increasing bluntness

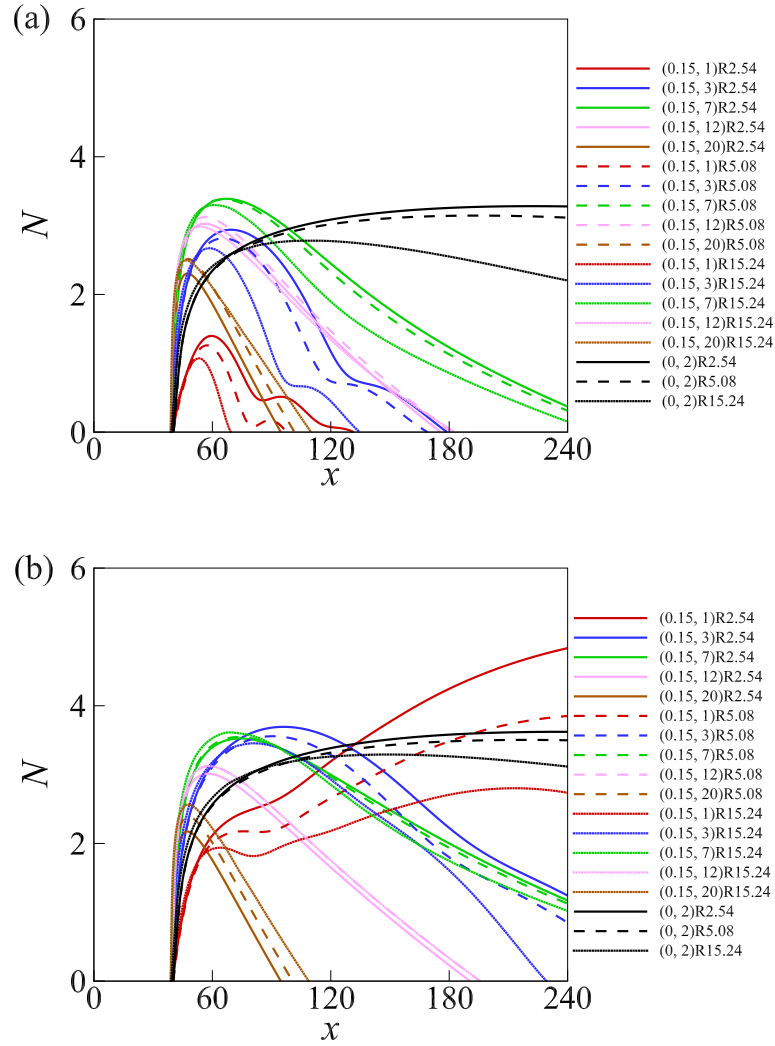


FIGURE 5.16: N -factor distributions of optimal disturbance with different bluntness for (a) $T_w/T_{ad} = 0.57$ and (b) $T_w = T_{ad}$. R2.54 refers to the case with $R_n^* = 2.54$ mm, and the same applies to other cases.

can suppress or strengthen the optimal response, depending on the spanwise wavenumber. Moreover, optimal disturbances obtained by resolvent analysis here exhibit no visible reversal-related phenomenon for a certain combination of ω and β with the increment of bluntness. An expected reversal-related result is that the N -factor increases as the nose radius varies from 2.54 mm to 5.08 mm then decreases as it further rises to 15.24 mm. This observation agrees with the observation by Nair & Unnikrishnan [144], which reported no "transition reversal" phenomenon when the white noise is initialized downstream of the shock. Furthermore, transient growth near the nose region is examined in Appendix 5.7.4, which also shows a decrease trend with increment of nose radius. Consequently, the interaction between the bow shock and freestream disturbances - the receptivity process - merits further investigation.

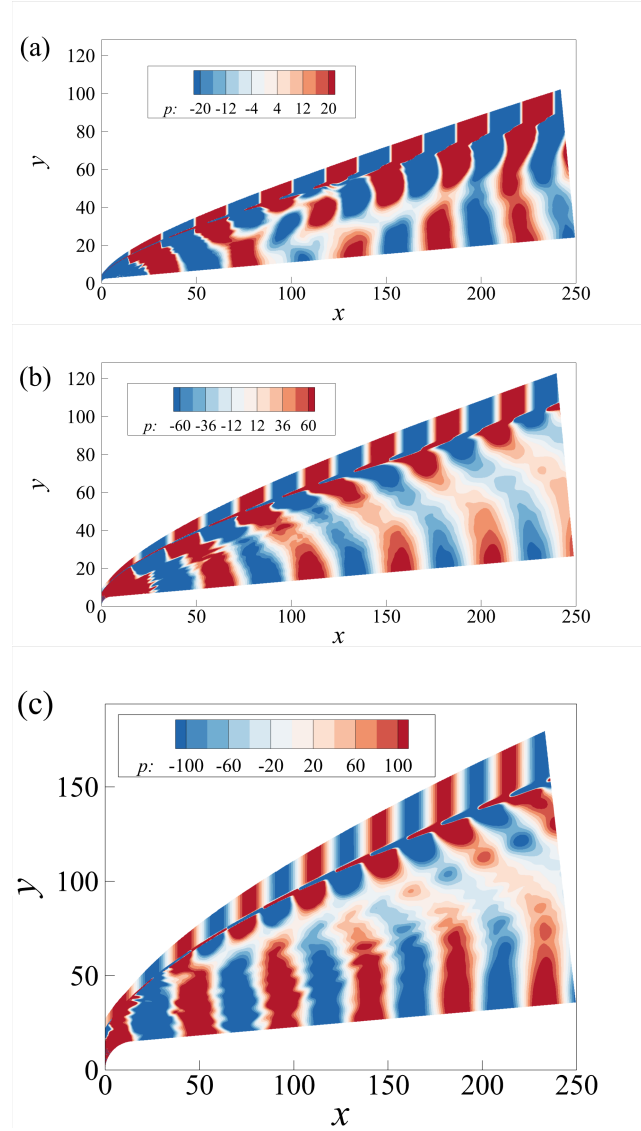


FIGURE 5.17: Contours of instantaneous dimensional pressure fluctuations (unit: Pa) for (a) $R_n^* = 2.54$ mm, (b) $R_n^* = 5.08$ mm, and (c) $R_n^* = 15.24$ mm.

The receptivity process provides the initial frequency and amplitude of the optimal responses at the forcing location. Considering this physical process, the effect of nose radii is revisited. The slow acoustic wave is added to the free stream to produce the response of the boundary layer. Note that only a representative single low-frequency ($\omega = 0.15$) forcing is adopted here with the same amplitude for blunt-wedge flows with different nose radii. The corresponding physical frequency is 25 kHz, which has shown its significance in the above resolvent analysis. This frequency magnitude also falls within the dominant frequency range of wind-tunnel experiments for blunt cones¹⁵ and Ogive-Cylinders [139]. For brevity, only adiabatic wall condition cases are displayed.

As indicated by figure 5.17, the free-stream slow acoustic wave first interacts with the bow shock,

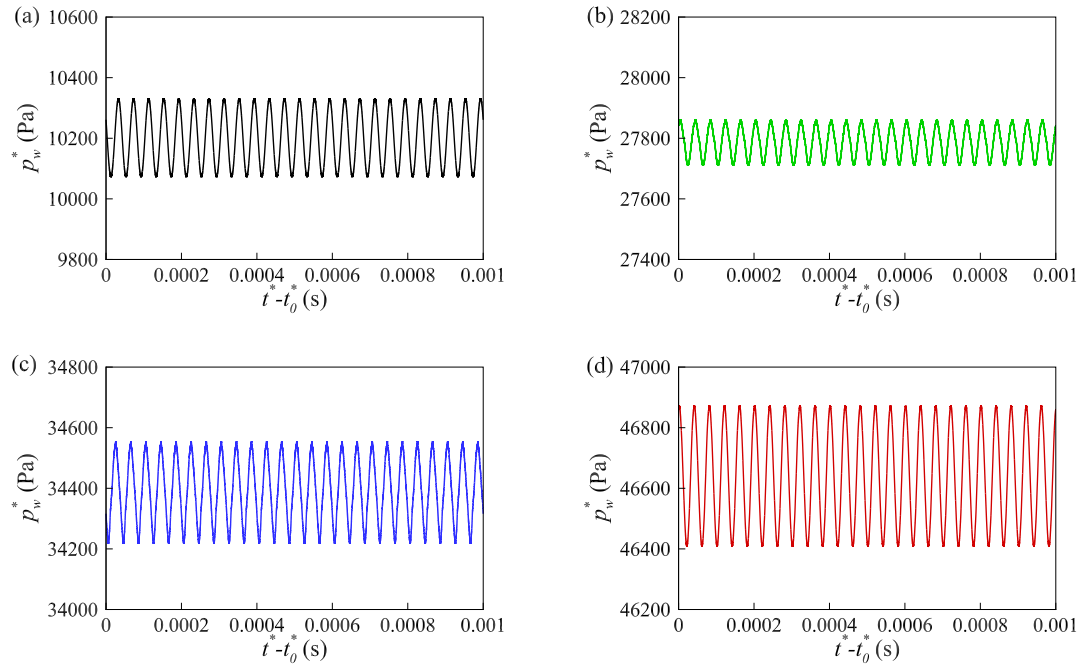


FIGURE 5.18: The instantaneous pressure at (a) free stream and $x = 40$ along the wall for (b) $R_n^* = 2.54$ mm, (c) $R_n^* = 5.08$ mm, and (d) $R_n^* = 15.24$ mm.

and then the resulting disturbance penetrates the entropy layer and excites responses inside the boundary layer. The pressure history in the free stream and on the wall for three blunt wedges is displayed in figure 5.18. Here, t_0^* is the time when the statistical stationary process begins. In terms of the response at the forcing location, the amplitude of the pressure fluctuation at $x = 40$ becomes larger as the increment of nose bluntness. The root mean square (RMS) of pressure at $x = 40$, normalized by its free-stream value, is 0.554, 1.159, and 1.811 for $R_n^* = 2.54$ mm, $R_n^* = 5.08$ mm, and $R_n^* = 15.24$ mm, respectively. This indicates a stronger receptivity process with an increase of nose radius, which may result from a stronger bow shock and thereby intensified shock-disturbance interaction.

With the amplitude of disturbance obtained by the receptivity process, the initial forcing amplitude of optimal disturbances at $x = 40$ is replaced. Figure 5.19 (a) illustrates the normalized RMS of pressure fluctuations along the wall for different optimal disturbances. Here, the amplitude at $x = 40$ is obtained by the receptivity simulation, and the further amplitude evolution downstream is obtained from resolvent analysis (see figure 5.17 (b)). Subsequently, the two amplitude ratio is multiplied to figure out the final normalized RMS with respect to the freestream value. Physically, figure 5.19 (a) suggests the response magnitude under the same level of wind-tunnel noise in real-life situations. Compared with figure 5.17 (b), as the increment of bluntness, the

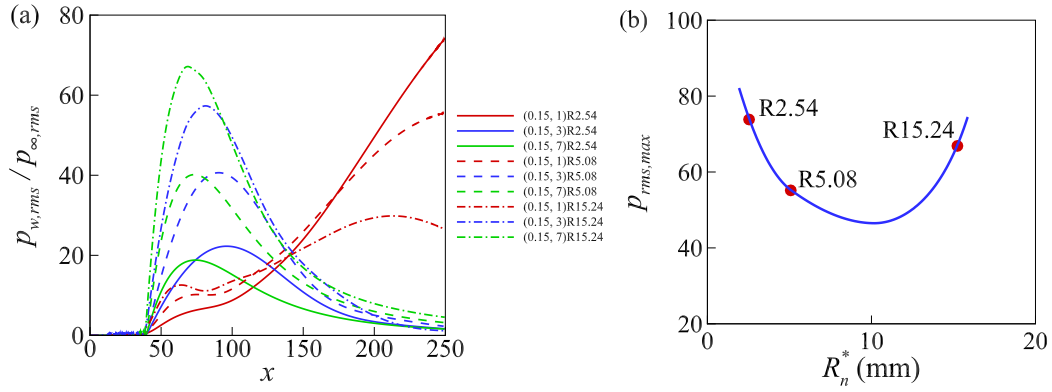


FIGURE 5.19: (a) The normalized root mean square of pressure along the wall, and (b) the maximize root mean square of pressure fluctuations in the computational domain among all the calculated Fourier modes for different blunt wedges. Line in (b) represents the data trend from fitting. The experimental transition reversal radius is around 10 mm.

strength of the oblique wave (0.15, 7) upstream becomes comparable to the first mode (0.15, 1) downstream.

Figure 5.19 (b) depicts the maximum RMS of pressure fluctuations in the computational domain among all the calculated Fourier modes for different nose radii, which can be seen in figure 5.19 (a). The blue line is obtained from a nonlinear fitting procedure to show the data trend. It indicates that as increment of nose radius, the maximized root mean square of pressure decreases and then increases after bluntness exceeds its critical value. The experimental reversal Reynolds number based on the nose radius is around 1×10^6 [17]. The resulting critical nose radius is about 10 mm. As a result, a qualitative agreement is reached in the data trend. The above observation may demonstrate the significance of nonmodal disturbances upstream when the modal components are suppressed as the nose radius increases. Specifically, the oblique waves with relatively large spanwise wave-number show their potential in triggering the transition rather than the suppressed first mode downstream for large bluntness cases. The strengthening of receptivity and transient growth of oblique waves upstream might provide some insights into the "transition reversal" phenomenon observed in wind-tunnel experiments, which merits further research considering nonlinear instability and breakdown stages.

This Section only provides the evolution of energy magnitude of optimal disturbances over the blunt wedge from a small to large nose radius. The parametric study may benefit a further investigation to fully understand the reason for the "transition reversal" phenomenon using a three-dimensional simulation of the complete stage.

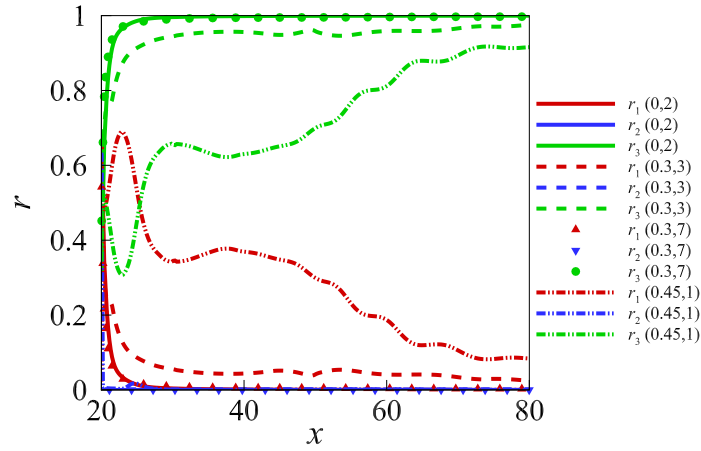


FIGURE 5.20: The evolution of enstrophy ratios for Fourier modes (0, 2), (0.3, 3), and (0.3, 7).

5.5.5 Orr/lift up mechanism analysis

In this Section, two classical mechanisms of transient growth, i.e., Orr and lift-up mechanisms, are analyzed. Disturbances are amplified by Orr mechanism when they have streamlines tilted against the mean shear and become stronger when the base flow reorients them [145]. The lift-up effect arises when streamwise vortices form streaky structures by displacing fluid particles in the wall-normal direction, which maintains their horizontal momentum [146]. To quantify the intensity of the vorticity transfer, the streamwise evolution of the integrated enstrophy ratios [147, 148] is analyzed, which is characterized by

$$r_l(x) = \Omega_l \left/ \sum_i \Omega_i \right., \Omega_i(x) = \int_0^\infty \varpi_i^2 dy \quad i = x, y, z; l = 1, 2, 3 \quad (5.4)$$

where ϖ_x , ϖ_y , and ϖ_z represent the streamwise, wall-normal, and spanwise vorticity, respectively. Figure 5.20 gives the integrated enstrophy ratio for some representative combinations of ω and β . Overall, these combinations show the same trend: the decrease of streamwise vorticity components and the increase of spanwise vorticity. Note that the transfer of vorticity from the streamwise direction to the spanwise direction is associated with the lift-up mechanism. Among them, pattern-B disturbances (0.3, 7) and (0, 2) nearly collapse onto the same curve, which probably indicates that (0.3, 7) shares the characteristics of unsteady streaks. Interestingly, the vorticity transfer of the pattern-A wave (0.45, 1) is weaker than the other optimal disturbances, which is characterized by a tardy vorticity transfer near the forcing location. This indicates that

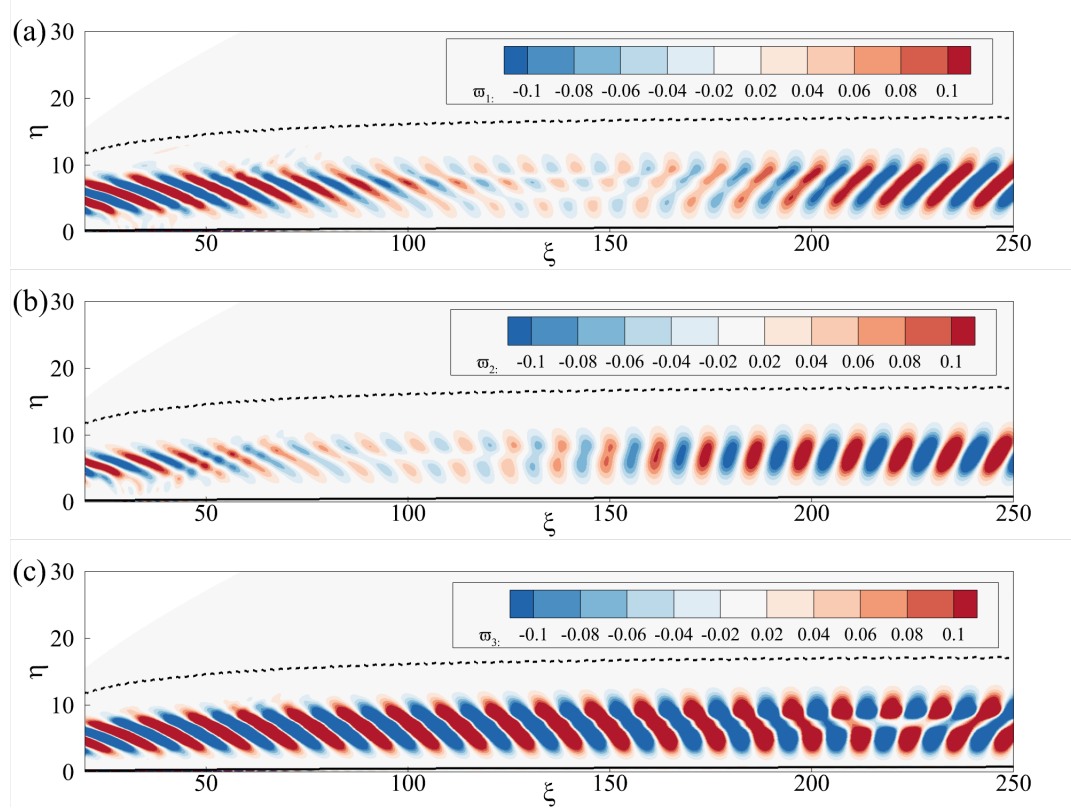


FIGURE 5.21: Contours of (a) streamwise, (b) wall-normal, and (c) spanwise vorticity of the pattern-A response (0.45, 1). The solids and dashed black lines mark the locations of the boundary layer edge and the entropy layer edge, re-spectively.

pattern-B responses possess a more efficient lift-up mechanism than pattern-A responses, which can also be seen in Appendix 5.7.2.

Figure 5.21 depicts the streamwise, wall-normal, and spanwise vorticity components of the optimal pattern-A wave (0.45, 1). The gradual change of the vorticity orientation indicates the existence of the Orr mechanism, where tilted vortices extract energy from the mean flow while being erected [135, 149]. For incompressible flows, nonmodal growth due to the Orr mechanism is ascribed to the extraction of energy from the mean shear by conveying momentum along the mean momentum gradient via the Reynolds shear stress [150, 151]. However, the energy amplification observed in this hypersonic scenario is primarily due to the temperature gradient within the entropy layer. As illustrated in figure 5.15(c), the characteristic of the hypersonic blunt wedge flow is that the mean temperature gradient is substantially more prominent than the streamwise velocity gradient. Thus, the interaction between the disturbance and the mean flow via the Reynolds thermal stress in the entropy layer appears to be significant for the Orr mechanism.

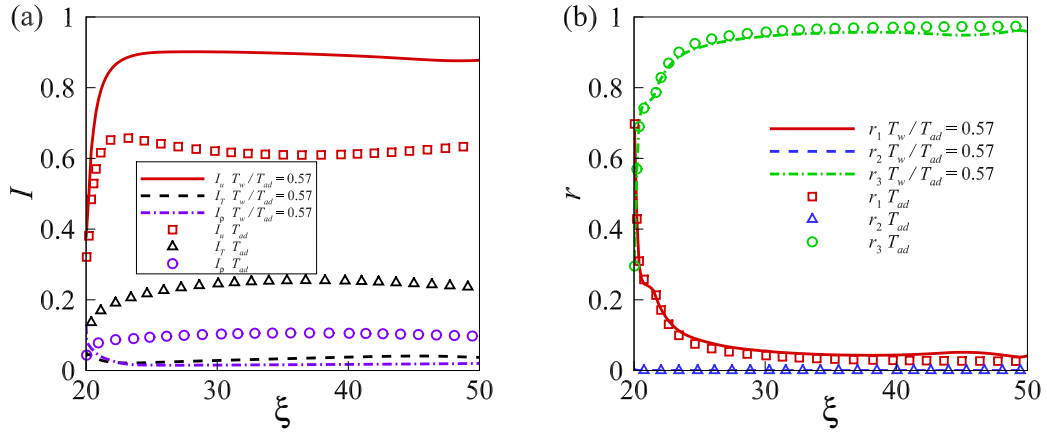


FIGURE 5.22: (a) I indicators of Chu energy and (b) enstrophy ratios of the pattern-B response (0.3, 3).

Individual energy contribution analysis is then performed to characterize the effect of wall temperature on the optimal disturbance. As shown in figure 5.22, reducing wall temperature would lead to a more substantial streamwise velocity disturbance component and weaker temperature and density components. This suggests that when wall cooling is applied, the thermodynamic part of energy becomes less significant compared to the kinetic part. These may be associated with the modified mean flow gradient inside the boundary layer. The adiabatic wall condition results in a steeper temperature gradient, which in turn strengthens thermodynamic fluctuations components and eventually leads to a stronger Pattern-B wave. As for the enstrophy ratios, wall cooling almost does not affect the evolution of their values. This indicates that the vorticity transfer process is weakly affected by the wall temperature. The same curve and conclusion can be obtained for streaks, not shown here for brevity.

5.6 Conclusions

In this work, resolvent analysis is utilized to study the optimal disturbance of the hypersonic flow over a blunt wedge at Mach 5.9 and a unit Reynolds number of $91.5 \times 10^6 \text{m}^{-1}$. With the absence of the second-mode disturbance, the low-frequency (within 50 kHz) associated modal and nonmodal disturbances are identified and examined with the parabolic stability equation analysis. The growth pattern of optimal disturbances is investigated with consideration of the effects of wall temperature ($T_w/T_{ad} = 0.57$, $T_w/T_{ad} = 0.75$, and adiabatic wall) and nose radius ($R_n^* = 2.54 \text{ mm}$, 5.08 mm , and 15.24 mm). The lift-up/Orr transient growth mechanisms

sustaining the nonmodal amplification of the optimal response are also studied. The main conclusion is summarized as follows:

1) For a fixed frequency, with the increment of the spanwise wavenumber, competitive patterns of optimal disturbances are identified. This competition is due to the appearance of nearly equally significant optimal and sub-optimal solutions. Specifically, the optimal response of the system is firstly shown as disturbances propagating inside the entropy layer (pattern A), and then switches to the pattern inside the boundary layer (pattern B). The former pattern A experiences a long-distance energy growth, while the latter pattern B under-goes a rapid transient growth followed by a significant decay. The significant decay can be interpreted as the steady oblique mode S when the spanwise wave number is not larger than 5 in the studied cases.

2) Compared to the baseline cold wall condition ($T_w/T_{ad} = 0.57$), the adiabatic wall condition supports the appearance of the unstable oblique first mode. The optimal oblique wave first experiences a rapid transient growth and then a modal growth, resulting in a higher N -factor than other kinds of disturbances, including streaks. Moreover, the cold wall condition tends to weaken the transient growth of oblique waves inside the boundary layers (pattern B), especially for low-spanwise-wavenumber cases. For optimal patterns travelling inside the entropy layer (pattern A), the effect exerts no influence since the base flow inside the entropy layer is hardly affected by the wall temperature.

3) Regarding the influence of the nose radius, the resolvent analysis shows no discernible improvement in an understanding of either modal or nonmodal growth. Nonetheless, considering the receptivity to the slow acoustic wave, the amplitude of nonmodal oblique waves within the boundary layer increases with the nose bluntness. The enhanced nonmodal oblique wave in the upstream region approaches that of the first mode downstream. The strengthened receptivity of oblique waves with relatively large spanwise wavenumber (say, $\beta = 7$ for $R_n^* = 15.24$ mm) may potentially cause the transition to move upstream. An estimated amplitude response favorably reproduces the reversal-like phenomenon. A three-dimensional simulation of the complete stage (from receptivity to transition) is expected in the future.

4) Both Orr and lift-up mechanisms play a significant role in the transient growth of optimal disturbances. This deduction particularly holds true for pattern-A optimal responses, which are manifested as tilted waves inside the entropy layer. The Orr mechanism is more likely to be accomplished by energy transfer via the mean temperature gradient rather than the mean shear. This is because the base velocity gradient in the entropy layer is ignorable compared

to the temperature gradient. Furthermore, pattern-B responses possess a more efficient lift-up mechanism than pattern-A responses according to an enstrophy ratio criterion.

The base flow is limited to two-dimensional cases in this study, and the linear assumption is applied to disturbance equations. Furthermore, the resolvent analysis only yields the optimal disturbances that experience the largest energy gain in the studied computational domains, which serve as the upper boundary or the most dangerous scenario subject to external forcings. It may not be realisable by wind-tunnel experiments under specific environments. In the future, the role of these disturbances of various patterns should be adequately considered in the Fourier space for a transitional hypersonic flow over blunt bodies. The corresponding transition to turbulence under the realistic wind tunnel condition with broadband noise may be a combinational consequence of various kinds of disturbance which are selected to be considerably amplified.

5.7 Appendix

5.7.1 Verification of grid independence

Computational grids are constructed with two different grid resolutions, including 750×300 (coarse) and 1000×400 (fine) in the streamwise and wall-normal directions, respectively. The grid near the wall and the bow shock was locally refined to improve resolution. Here, only is the case for $R_n^* = 2.54$ mm and $T_w/T_{ad} = 0.57$ shown for brevity. The same conclusion can be obtained for other cases. Figure 5.23 compares the mean temperature contour between the coarse and fine grids, and figure 5.24 compares the N -factor distribution of representative optimal responses for two sets of grids. It is indicated that the coarse grid is sufficient for both the base-flow simulation and the resolvent analysis for the frequency of interest in this study.

5.7.2 The competitive patterns of optimal and sub-optimal disturbances

The optimal disturbance can be viewed as an upper bound of the response of the system, while realistic external disruptions typically lead to a suboptimal transient growth. As shown in figure 5.25, the optimal response switches from pattern A to pattern B as β increases, the competition between these two patterns can be revealed by extracting both the optimal and sub-optimal responses. The sub-optimal disturbance can be captured using an Arnoldi iteration to search

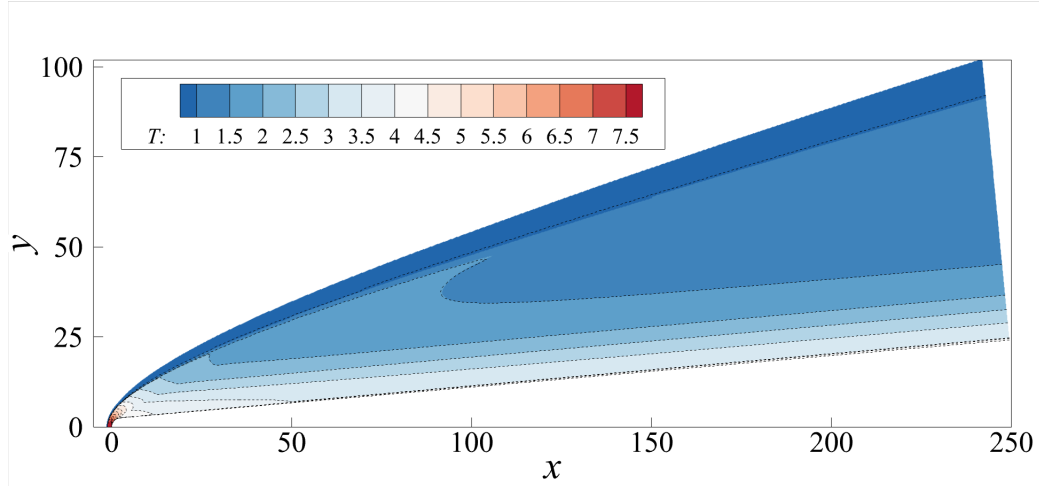


FIGURE 5.23: Contour of mean temperature of fine grid (contour) and coarse grid (dashed line) for $R_n^* = 2.54$ mm and $T_w/T_{ad} = 0.57$.

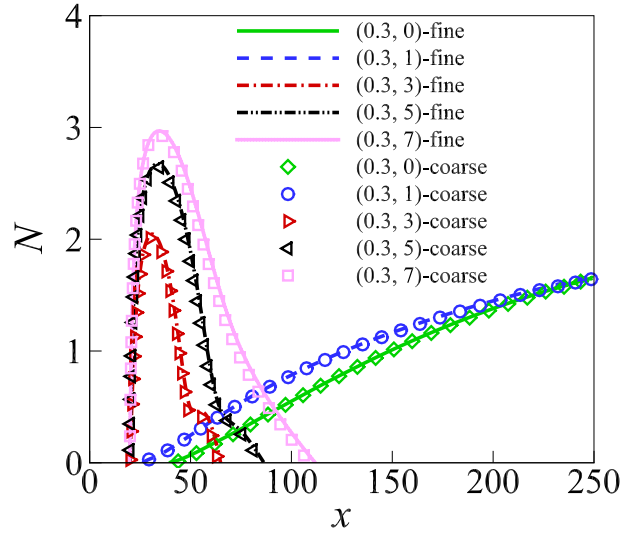


FIGURE 5.24: Evolution of (a) N -factor of representative optimal responses for $R_n^* = 2.54$ mm and $T_w/T_{ad} = 0.57$.

for more eigenvalues. Here, we display the optimal (pattern-B family) and sub-optimal (pattern-A family) disturbances for the Fourier mode (0.3, 2). This mode is typical to facilitate an understanding of the competition between different patterns.

As shown in figure 5.26 (a), the optimal response demonstrates a rapid growth near the forcing location, while the other non-optimal disturbances experience a gradual growth downstream. Note that the optimal gain decreases monotonically from the optimal response to the first, second, third, and fourth sub-optimal responses. According to figure 5.26 (b), the vorticity transfer from the streamwise direction to the spanwise direction is more efficient for optimal

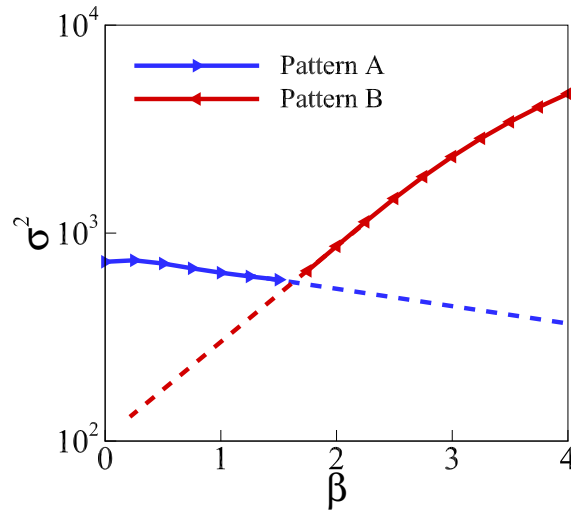


FIGURE 5.25: Optimal gain (solid line) along spanwise wavenumber for $\omega = 0.3$. Dashed lines indicate the extension of solid lines.

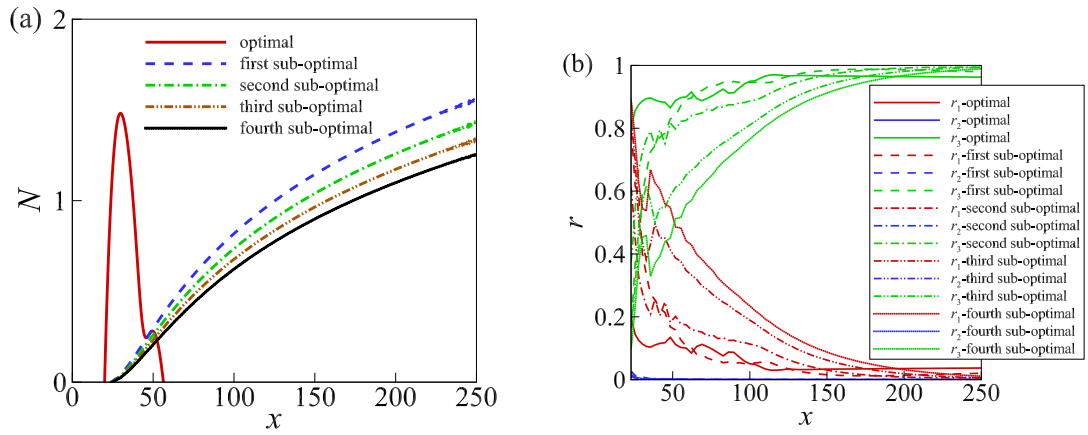


FIGURE 5.26: Evolution of (a) N -factor and (b) entropy ratios of optimal and the first, second, third and fourth sub-optimal responses for the Fourier mode $(0.3, 2)$.

disturbances, followed by the first, second, third, and fourth sub-optimal responses. This is because the optimal response achieves a rapid r_1 -dominated state downstream of the forcing location. Therefore, the lift-up mechanism may play a significant role in the competition between optimal and relevant sub-optimal disturbances.

The temperature fluctuation contours for optimal and four sub-optimal disturbances for $(0.3, 2)$ are shown in figure 5.27. The optimal disturbance is distributed inside the boundary layer, while the sub-optimal disturbances show their signature inside the entropy layer. The non-optimal disturbances present a tilting structure that seems to be associated with the Orr mechanism. When neither the first nor second instability modes prevail, the wisp-like structures visualized in wind

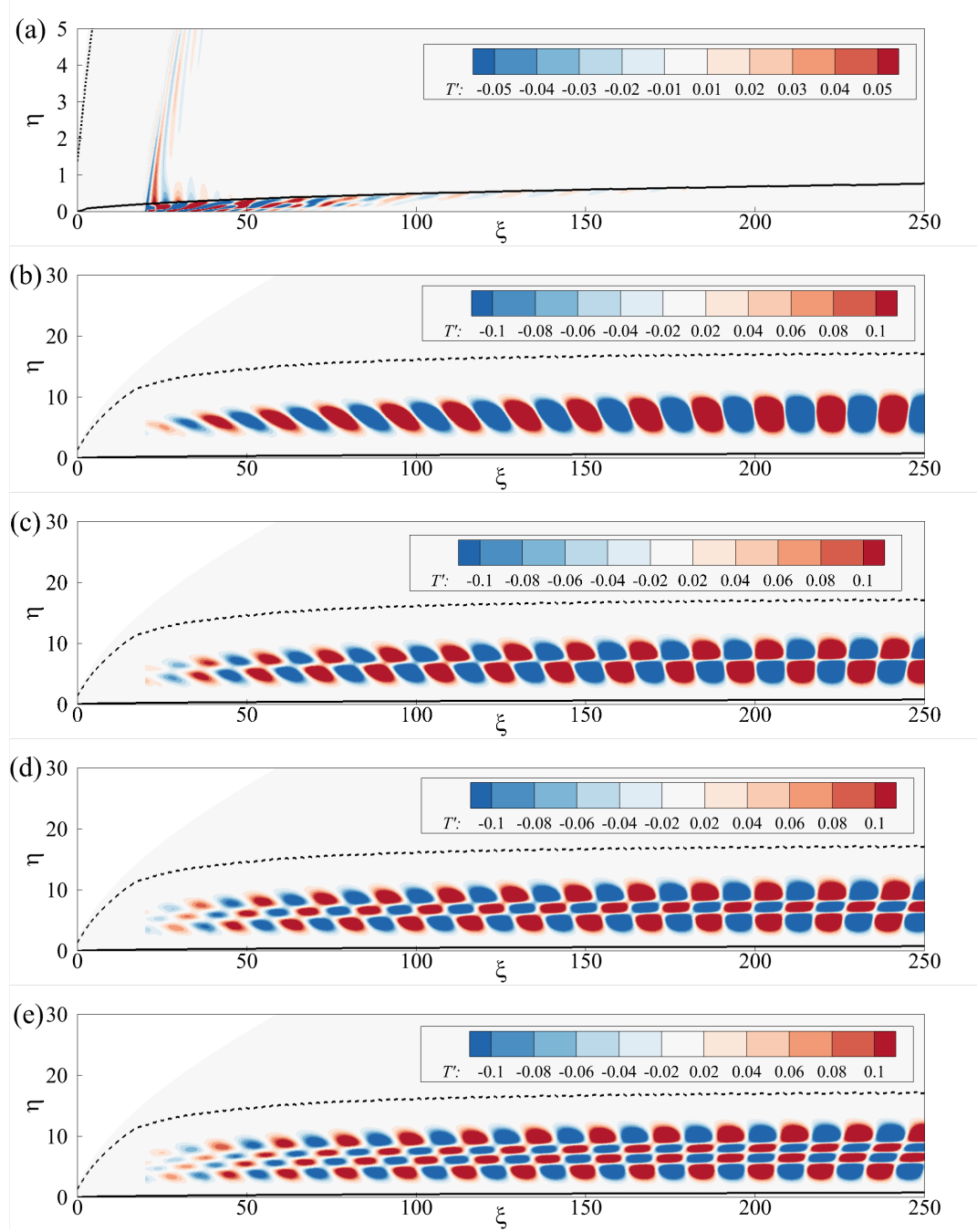


FIGURE 5.27: Contours of normalized temperature fluctuations for (a) Optimal response and the (b) first, (c) second, (d) third and (e) fourth sub-optimal responses for (0.3, 2). The boundary layer edge is represented by the solid black line, whereas the dashed black line signifies the entropy layer edge.

tunnel experiments may originate from disturbances developing in the entropy layer [45, 139]. The nonlinear evolution and potential transition process induced by these disturbances merits further investigation.

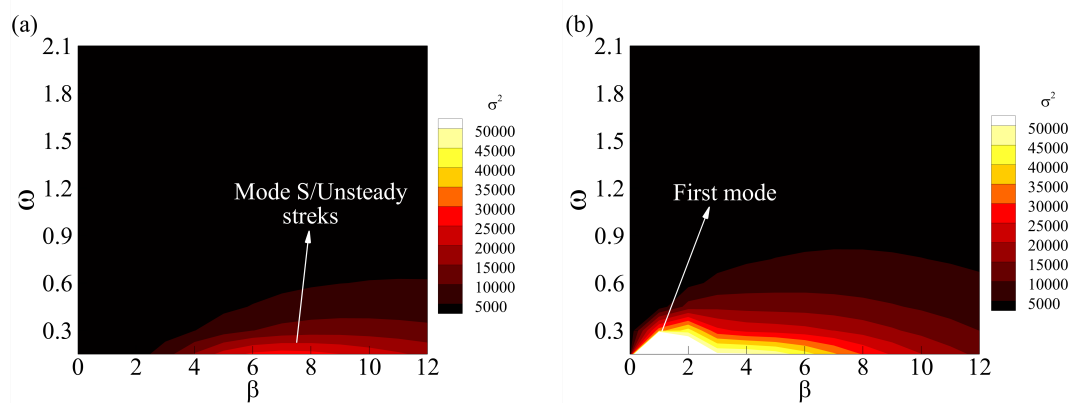


FIGURE 5.28: Gain contour for $R_n^* = 2.54$ mm with (a) $T_w/T_{ad} = 0.57$, and (b) $T_w = T_{ad}$. The forcing location $x_0 = 20$.

5.7.3 The gain contour for optimal disturbance

The gain contour comparison between the cold-wall condition and the adiabatic wall is illustrated in figure 5.28. Under adiabatic wall conditions, the first mode is prominently observed in the gain contour. However, when the wall temperature is reduced, the first mode is significantly suppressed, and the dominant optimal response shifts to mode S or unsteady streaks, as discussed in Section 5.5.2.

The gain contour comparison among different nose radii under adiabatic wall conditions is depicted in figure 5.29. Under adiabatic wall conditions, the first mode is prominently observed in the gain contour. However, as the nose radius increases, the first mode is significantly suppressed, and the optimal energy gain decreases, as discussed in Section 5.5.4.

5.7.4 Transient growth near the nose

As discussed in Section 5.5.4, when considering only the transient growth downstream of the nose region, no reversal phenomenon is observed. Therefore, transient growth near the nose region may be significant. Figure 5.30 illustrates the N -factor distribution of significant low-frequency disturbances under adiabatic wall conditions for various nose radii. Note that the forcing is introduced at around $x = 0$ (near the stagnation point). It is indicated that as the nose radius increases, the transient growth downstream of the shoulder region diminishes. This result excludes the possibility that the transient growth near the nose region is responsible for the "transition reversal". Consequently, the interaction between the bow shock and freestream disturbances - specifically, the receptivity process - may warrant further investigation.

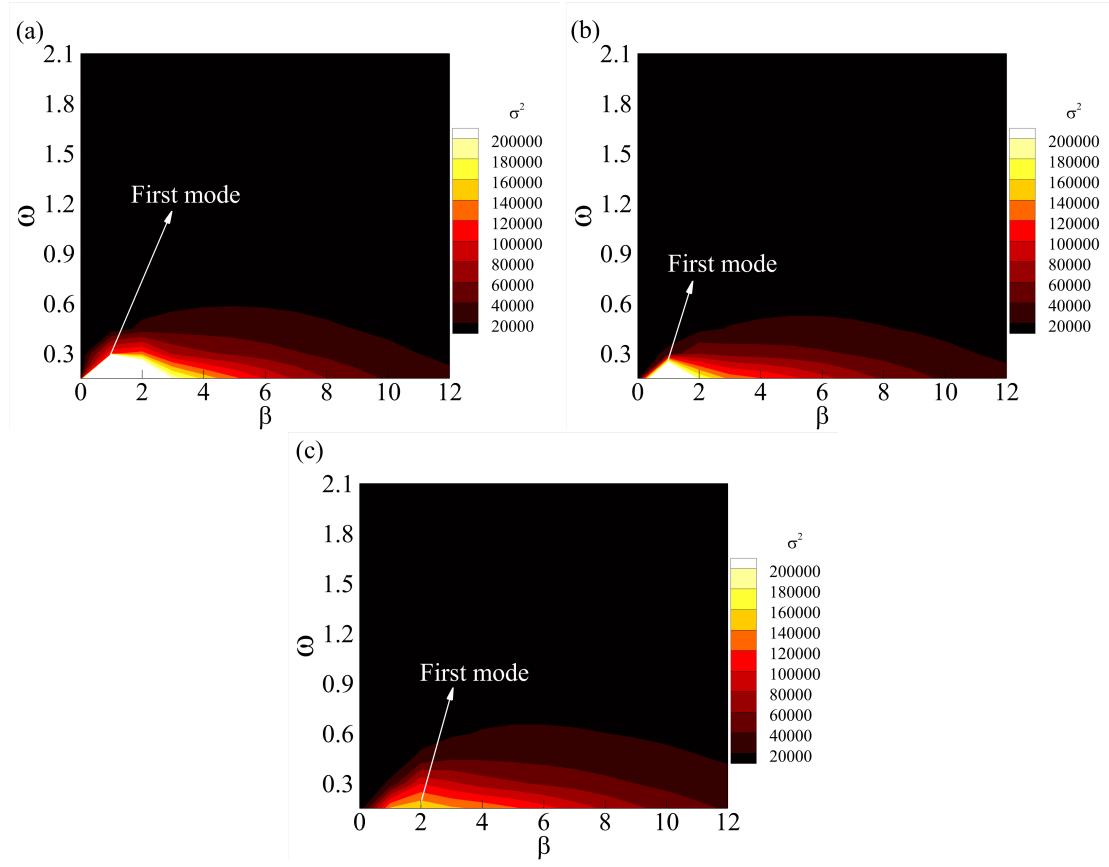


FIGURE 5.29: Gain contour at adiabatic wall condition for (a) $R_n^* = 2.54$ mm, (b) $R_n^* = 5.08$ mm, and (c) $R_n^* = 15.24$ mm. The forcing location $x_0 = 40$.

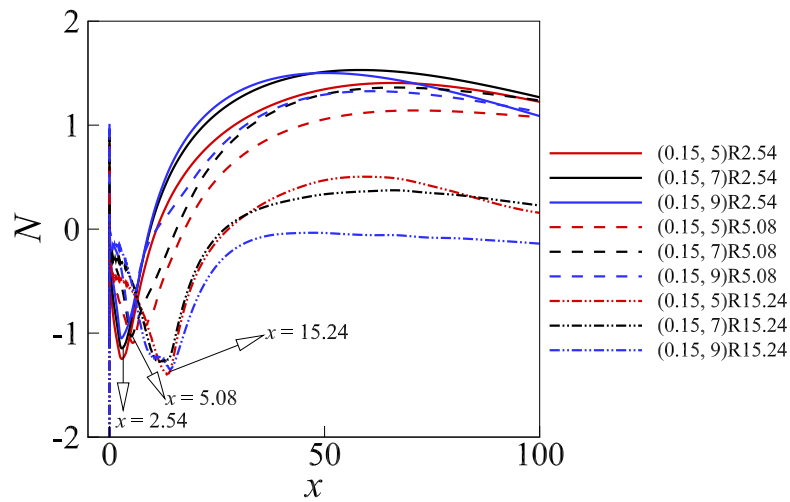


FIGURE 5.30: N -factor distributions of optimal disturbance with different bluntness for $T_w = T_{ad}$. R2.54 refers to the case with $R_n^* = 2.54$ mm, and the same applies to other cases. The starting point of the shoulder is pointed out by arrows.

Chapter 6

Hypersonic boundary-layer transition control using the acoustic metasurface

[Unpublished paper (under review) IV¹]

6.1 Preface to the chapter

This chapter is reorganized based on the paper submitted to *Journal of Fluid Mechanics*, titled "Role of acoustic metasurface in the nonlinear mode–mode interaction and breakdown of hypersonic boundary layer" [86]. Following a systematic study on the instabilities character and their influencing factors on the linear stage (see Chapters 3-5), the nonlinear and breakdown stages are of interest. In this Chapter, the transition to turbulence in a hypersonic boundary layer is studied using DNS, with a particular focus on the influence of acoustic metasurfaces modelled by a time-domain impedance boundary condition (TDIBC). Both low-frequency first-mode and high-frequency second-mode instabilities pronounced in resolvent analysis are involved. For the first time, the mode–mode interactions and the resulting breakdown mechanisms response to the acoustic metasurface are elucidated in DNS. Furthermore, the increased skin friction observed during the late transition stage, which has also been corroborated by wind tunnel experiments, is examined and analysed through energy budget and bi-Fourier analysis and bispectral mode decomposition. This chapter uncovers the dominant nonlinear mode-mode interaction processes and underlying transition control mechanisms using the acoustic metasurface. It is believed that

¹Y. Chen, P. Guo and C. Y. Wen (2025). Role of acoustic metasurface in the nonlinear mode–mode interaction and breakdown of hypersonic boundary layer. *Journal of Fluid Mechanics* (Accepted).

this chapter provides novel insights into the control mechanisms of acoustic metasurfaces on transition scenarios induced by multiple primary disturbances, thereby bridging the gap between theoretical design and engineering applications.

6.2 Abstract

Boundary-layer instability and transition control have emerged as critical topics in the development of next-generation hypersonic vehicles. The acoustic metasurface has become a promising passive control method due to its straightforward implementation and lack of requirement for external energy input. In this study, the TDIBC is utilized to model the acoustic metasurface during the transitional stage of a flat-plate boundary layer at Mach 6. Resolvent analysis is conducted to identify the optimal disturbances, which reveal two peaks: the oblique first mode and the planar Mack second mode. Results from single-frequency simulations indicate that, compared to the solid wall case, the acoustic metasurface effectively suppresses the high-frequency second mode while not significantly affecting the low-frequency first mode. Subsequently, both low-frequency and high-frequency optimal forcings are introduced upstream, leading to a dominant detuned mode prior to breakdown. The interaction between modes and the resulting transition process with the acoustic metasurface is revealed for the first time. The study indicates that the location of the acoustic metasurface is critical in delaying transition, with placement within the unstable region of the second mode being more efficient for transition control. The bispectral mode decomposition (BMD) results reveal that the dominant energy transfer routine along the streamwise direction: from primary modes to low-frequency detuned modes. By employing the acoustic metasurface, the nonlinear triadic interaction between high- and low-frequency primary modes is effectively suppressed, ultimately delaying transition onset; whereas the late interaction related to lower-frequency detuned modes is reinforced, promoting the late skin friction. Meanwhile, in the later stages of transition, the acoustic metasurface induces an undesirable increase in skin friction overshoot due to the strengthening of shear-induced dissipation work, primarily arising from detuned modes related to combination resonance. By restricting the allocation of the metasurface upstream of the overshoot region, this undesirable increase in skin friction can be mitigated. Consequently, the reasonable placement of the metasurface is crucial for damping early instability while minimizing negative impacts in the later transitional stage.

6.3 Introduction

Several breakdown or resonance mechanisms have been confirmed by numerous DNS and experimental studies. These include subharmonic resonance [152], first-mode dominated oblique breakdown [56], second-mode oblique breakdown [44], second-mode dominated fundamental

resonance [45, 53], and second-mode subharmonic resonance [153, 154]. These conventional transition scenarios are characterized by tuned modes with frequencies that are integer multiples of $f_{1/2}$. Here, $f_{1/2}$ refers to half of the frequency of the primary instability wave. The dominating transition scenario depends on the specific numerical forcing with given flow and geometric conditions. In a realistic disturbance environment, several primary instabilities can be excited simultaneously and coexist. In this case, complex mode–mode interactions emerge, and detuned modes may become pronounced [155]. As reported by Chen *et al.* [57] for the flared-cone model at Peking University, multiple primary instabilities give rise to complex nonlinear interactions and significant energy transfer between low- and high-frequency components. A possible combination resonance is also indicated, which differs from that induced by a single primary instability [53, 154]. Nonetheless, direct evidence in the transitional stage was not provided. Recently, Guo *et al.* [8] have demonstrated the significance and generality of the non-conventional combination resonance associated with modal detuning in high-speed boundary layer transition. The nonlinear interaction between low-frequency first mode and high-frequency second mode can produce a new breakdown scenario. This multiple-primary-instabilities scenario is believed to be noteworthy, as multiple instabilities are seeded in realistic boundary layers.

Boundary-layer transition control strategies are essential technologies for delaying the occurrence of transition. These strategies can be classified into active methods, such as wall cooling and heating [156], and passive methods, such as porous coatings or acoustic metasurfaces [10, 73]. Acoustic metasurfaces, including absorptive acoustic metasurfaces, impedance-near-zero acoustic metasurfaces, and reflection-controlled acoustic metasurfaces, are primarily proposed for controlling the second mode of acoustic nature [10]. The passive approach is flexible for implementation and requires no additional energy, which has attracted increased interest from both academia and industry. Experimental observations have shown that porous coatings can successfully postpone the initiation of transition [74, 157–159]. However, porous coatings lead to an undesirable increase in wall friction and flux during the late stage of transitional flow. This phenomenon has been observed experimentally [160] but remains not fully understood.

The porous coating effect in the nonlinear regime has also been the subject of early research [161, 162]. It was discovered that the second-mode fundamental breakdown caused a delayed transition commencement. However, these investigations are restricted to single-frequency perturbations and typical breakdown situations, which may not accurately represent transitional flow with broadband disturbances in wind tunnels or flight testing. Recently, Sousa *et al.* [163] conducted a numerical simulation of hypersonic transition delay over broadband wall

impedance using dynamic large-eddy simulation. Good agreement was observed with reference experimental data. The corresponding wind tunnel experiment features a high-enthalpy flow with highly cooled wall conditions, where the first mode is significantly suppressed. Consequently, the nonlinear interaction between the low-frequency first mode and the high-frequency second mode was not considered in their simulation. The resulting breakdown is more likely to be a fundamental breakdown induced by the second mode, which represents the only dominant instability under such cooling wall conditions.

Triadic interactions serve as the fundamental mechanism governing energy transfer in nonlinear flow fields, playing a well-documented role in laminar-turbulent transition. The seminal work by Craik [164] established that resonant Tollmien–Schlichting wave triads can significantly accelerate transition in wall-bounded shear flows. Building on this, Rigas *et al.* [165] employed resolvent analysis incorporating nonlinear triadic interactions, demonstrating that transition dynamics can be accurately captured using only a limited number of harmonics and their triadic couplings. To quantify such interactions, bispectral mode decomposition (BMD) proposed by Schmidt [166] has emerged as a powerful diagnostic tool, enabling identification of dominant triads and their associated energy transfer pathways in three-dimensional nonlinear flows. For instance, Schmidt & Oberleithner [167] applied bispectral analysis to two-phase swirling flows, revealing that triadic resonance generates a broad spectrum of secondary modes. Recent advances by Craig *et al.* [168] further extended BMD to hypersonic boundary layers, where bispectral analysis of experimental data identified multiple quadratic phase-coupled sum and difference interactions. Their findings suggest that under quiet wind tunnel conditions, the nonlinear transition mechanism is predominantly governed by high-frequency second-mode-induced fundamental breakdown. Nevertheless, this framework may not fully explain transition scenarios involving mutual interactions between low-frequency first modes and high-frequency second modes, revealing the need for further investigations on multimodal triadic coupling effects.

Even though the linear-stage stabilisation or destabilisation mechanisms are well-established, how metasurfaces modulate nonlinear interactions between coexisting first and second modes—a scenario routinely encountered in practice—remains unknown. Moreover, the undesired increase of late-stage skin friction observed experimentally has yet to be explained through first-principles simulations. The current work examines the impact of porous coatings on the nonlinear mode–mode interaction and the resulting breakdown process, with the main objective of revealing the mechanism of transition delay by acoustic metasurfaces subject to multiple primary instabilities. The effect of the metasurface is evaluated using a modeled impedance boundary condition (IBC)

to avoid meshing the cavity, which is flexible and can save computational costs. The TDIBC [79, 102, 169, 170], which can be efficiently incorporated into the Navier-Stokes solver and can handle broadband disturbances, has been proven for its accuracy and efficiency in simulating late transitional and turbulent boundary layer flow [163, 171, 172]. It is anticipated that the mode–mode interaction and the resulting breakdown scenario with the acoustic metasurface will be revealed in detail for the first time. Moreover, regarding the enhanced skin friction in the late transition stage, which has also been observed in experiments by Wagner *et al.* [158, 159], energy budget analysis and bi-Fourier analysis are performed to explore the underlying physics.

The framework of this chapter is as follows. Section 6.4 provides an overview of the investigated model and flow parameters, the formulation of TDIBC, stability tools, and numerical methods for DNS. Section 6.5 presents the fitting results of TDIBC using a theoretical model, the results of the stability analysis, validation of the modeled TDIBC code, and the initialization of disturbances obtained through stability analysis in DNS. The results of the transition simulation induced by multiple disturbances with a solid wall and the acoustic metasurface are shown in Section 6.6, including the analysis of the instantaneous and time-averaged flow field, Bi-Fourier analysis, results of BMD, and energy budget analysis. Finally, Section 6.7 provides concluding remarks and discussion.

6.4 Numerical setup

The freestream flow condition investigated here is based on a wind tunnel condition in Bountin *et al.* [173] for a Mach 6 flow, with a unit Reynolds number of $1.05 \times 10^7 \text{ m}^{-1}$. The freestream temperature T_∞ is 43.18 K, and the isothermal wall temperature $T_w = 293$ K is employed. In this study, subscripts ∞ and w respectively refer to values at freestream and wall. The freestream variables are utilized for non-dimensionalization, except that $\rho_\infty u_\infty u_\infty$ is used for pressure. Unless otherwise stated, the reference length L_{ref} is taken as 1 mm. Without unit markings, the following physical quantities are presented in their dimensionless form.

6.4.1 Time domain impedance boundary condition

DNS with realistic metasurface microstructures can be conducted to investigate the transition control mechanism. However, meshing numerous micro-cavities is computationally expensive. To efficiently and simultaneously simulate the interactions between acoustic metasurfaces and

multi-modal waves, the application of an impedance boundary condition is recommended. The early acoustic impedance models of metasurfaces were all constructed in the frequency domain and applied with a single disturbance frequency in hypersonic boundary-layer transition studies [73, 174], which are not suitable for transitions involving broadband disturbances. In this study, a multi-pole broadband impedance model with a piecewise linear recursive convolution technique proposed by Fung & Ju [169] will be revised and incorporated into the current DNS code. This embedded boundary condition enables efficient simulations of broadband disturbance propagation by transforming the acoustic impedance boundary condition from the frequency domain into the time domain.

The reflective coefficient $\tilde{R}$ in the frequency domain is defined as

$$\tilde{R}(\omega) = A^{out}(\omega) / A^{in}(\omega), \quad (6.1)$$

where A^{in} and A^{out} are the ingoing and outgoing waves, respectively. The ingoing and outgoing waves are related to the physical variables of the flow field (pressure and velocity) at the wall [169]. Their amplitudes are defined as

$$A^{in}(t) = v'(t) - p'(t)Ma_\infty\sqrt{T_w}, A^{out}(t) = v'(t) + p'(t)Ma_\infty\sqrt{T_w}. \quad (6.2)$$

The softness is defined as

$$\tilde{S}(\omega) = 1 + \tilde{R}(\omega). \quad (6.3)$$

Physically $\tilde{S} = 2$ and $\tilde{S} = 1$ when the wall is totally reflective (rigid wall) and absorptive (soft wall), respectively. Combining equations (6.1) and (6.3) gives rise to

$$A^{out}(\omega) = -A^{in}(\omega) + \tilde{S}(\omega)A^{in}(\omega). \quad (6.4)$$

The softness can be approached by a multi-oscillator model proposed by Fung & Ju [169].

$$\tilde{S}(s) = \sum_{k=1}^{n_o} \left[\frac{\mu_k}{s - p_k} + \frac{\mu_k^\dagger}{s - p_k^\dagger} \right] + C_0, \quad s = i\omega, \quad (6.5)$$

where p_k and $p_k^\dagger$ are poles of the pole base function, $\mu_k = i \cdot \text{Residue}[\tilde{S}(s), p_k]$ and similarly for $\mu_k^\dagger$. The superscript $\dagger$ refers to the complex conjugate, and n_o is the total number of fitting pairs. In addition, C_0 is a real number. The utilization of this constant number to improve the accuracy of the model was also reported by Fiévet *et al.* [79]. Due to the requirement of the

reality of the signal [175], the softness in the time domain must be real and $\tilde{S}^\dagger(s) = \tilde{S}(-s)$. The values of poles and residues are obtained from a nonlinear fitting process [170] to approximate the softness calculated from a well-validated impedance model, which considers high-order diffraction [13]. The authors have fully validated the impedance model by comparing it with the meshed metasurface [102, 176]. Then, the softness in the time domain can be obtained using the inverse Fourier transform:

$$\tilde{S}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{S}(\omega) e^{i\omega t} d\omega. \quad (6.6)$$

Using the Residue theorem, one may obtain

$$\tilde{S}(t) = \left(\sum_{k=1}^{n_o} \mu_k e^{p_k t} + \sum_{k=1}^{n_o} \mu_k^\dagger e^{p_k^\dagger t} \right) H(t) + C_0 \delta(t). \quad (6.7)$$

Heaviside function $H(t)$ indicates that the causality concentration is satisfied, and δ is the Dirac delta function. Taking the inverse Fourier transform of equation (6.4), one may obtain

$$A^{out}(t) = (C_0 - 1)A^{in}(t) + \int_0^\infty \tilde{S}(\tau) A^{in}(t - \tau) d\tau. \quad (6.8)$$

Substituting equation (6.7) into equation (6.8), it yields

$$A^{out}(t) = (C_0 - 1)A^{in}(t) + \int_0^\infty \sum_{k=1}^{n_o} (\mu_k e^{p_k \tau} + \mu_k^\dagger e^{p_k^\dagger \tau}) A^{in}(t - \tau) d\tau. \quad (6.9)$$

For the next time step $t + \Delta t$, equation (6.9) can be written as

$$A^{out}(t + \Delta t) = (C_0 - 1)A^{in}(t + \Delta t) + \int_0^\infty \sum_{k=1}^{n_o} (\mu_k e^{p_k \tau} + \mu_k^\dagger e^{p_k^\dagger \tau}) A^{in}(t + \Delta t - \tau) d\tau. \quad (6.10)$$

Here, the convolution in equation (6.10) can be written as G_k^{in} , that is

$$G_k^{in}(t + \Delta t) = \int_0^\infty \mu_k e^{p_k \tau} A^{in}(t + \Delta t - \tau) d\tau. \quad (6.11)$$

According to Fung & Ju [169], equation (6.11) can be calculated by a recursive scheme

$$G_k^{in}(t + \Delta t) = z_k G_k^{in}(t) + \mu_k \Delta t [w_{k0} A^{in}(t + \Delta t) + w_{k1} A^{in}(t)]. \quad (6.12)$$

Here $z_k = e^{p_k \Delta t}$, and

$$\begin{cases} w_{k0} = \frac{z_k - 1}{p_k^2 \Delta t^2} - \frac{1}{p_k \Delta t}, \\ w_{k1} = -\frac{z_k - 1}{p_k^2 \Delta t^2} + \frac{z_k}{p_k \Delta t}. \end{cases} \quad (6.13)$$

With a first-order assumption, the ingoing wave at $t + \Delta t$ can be written as

$$A^{in}(t + \Delta t, y) \simeq A^{in}(t, y) + \sqrt{T_w}/Ma_\infty \Delta t \frac{\partial}{\partial y} \left(-v'(t) + Ma_\infty \sqrt{T_w} p'(t) \right). \quad (6.14)$$

Then, combining equation (6.10) and equation (6.14), we may obtain the fluctuation wall-normal velocity at $t + \Delta t$:

$$v'(t + \Delta t) = \left(A^{in}(t + \Delta t) + A^{out}(t + \Delta t) \right) / 2 = \text{Real} \left(\sum_{k=1}^{n_o} G_k^{in}(t + \Delta t) \right) + C_0 A^{in}(t + \Delta t). \quad (6.15)$$

Finally, the fluctuation pressure at $t + \Delta t$ is

$$p'(t + \Delta t) = \left[v'(t + \Delta t) - A^{in}(t + \Delta t) \right] / \left(Ma_\infty \sqrt{T_w} \right). \quad (6.16)$$

In the simulation where the metasurface exists locally, the pressure and normal velocity on the wall are updated based on equations (6.15) and (6.16) at each time step, and the incoming wave amplitude $A^{in}(t)$ is initialised using equation (6.2). The density is calculated by the updated pressure from the equation of state for perfect gases. Although the TDIBC was derived from linear approximations, it remains valid for nonlinear disturbance dynamics in our system due to the strongly viscous-dominated regime inside the porous media [163], [177].

6.4.2 Case initialisation for direct numerical simulation

To explore the effect of the acoustic metasurface in hypersonic transitional flow, three cases subject to the same forcing but with different wall boundary conditions are considered here. In the present Mach 6 state, Guo *et al.* [102] reported two dominant optimal disturbances: a low-frequency oblique wave and a high-frequency planar wave. These two kinds of forcing are considered in the transition simulation. The mathematical form of the forcing in DNS is given by

$$f'(x_0, y, z, t) = f'_p(x_0, y, z, t) + f'_o(x_0, y, z, t), \quad (6.17)$$

where $x_0 = 0.04$ m is the forcing location, and subscripts p and o represent the optimal plane wave and oblique wave obtained by resolvent analysis, respectively. According to our previous study [102], the additional background noise term produces a negligible effect on the flow field and is thus not considered here. The planar wave forcing is expressed as:

$$f'_p(x_0, y, z, t) = \varepsilon_p \hat{f}_p(x_0, y) \exp(i\beta_p z - i\omega_p t) + \text{c.c.}, \quad (6.18)$$

where ε is the amplitude coefficient. A pair of oblique waves with an opposite spanwise wavenumber is employed, which is expressed by

$$f'_o(x_0, y, z, t) = \varepsilon_o \left[\hat{f}_o(x_0, y) \exp(i\beta_o z - i\omega_o t) + \hat{f}_{-o}(x_0, y) \exp(-i\beta_o z - i\omega_o t) \right] + \text{c.c.} \quad (6.19)$$

In all transition simulation cases, the initial amplitude $(\rho u)'_{max} = 0.002$ remains the same for the planar wave (ω_p, β_p) and a pair of oblique waves $(\omega_o, \pm\beta_o)$ at $x = 0.045$ m. This configuration excites the first and second modes with equal importance downstream, ensuring an evident linear, nonlinear, and eventually breakdown region within the computational domain. As reported by Guo *et al.* [102], the resulting nonlinear mechanism (combination resonance) is independent of both the initial amplitude ratio and the absolute amplitude of the oblique and planar waves. Therefore, this study focuses on one of these cases without loss of generality.

The wall boundary conditions for the DNS cases are presented in table 6.1. The boundary condition is of primary interest in this study. In Case 1, the solid wall boundary condition is employed. For Case 2, with the acoustic metasurface, TDIBC is employed starting from $x = 0.115$ m to the end of the computational domain, i.e., 0.6 m. It is important to note that $x = 0.115$ m corresponds to the neutral point of the optimal plane wave (10, 0), downstream from which the modal growth begins. In Case 3, during the late stage of transitional flow ($x > 0.34$ m), the TDIBC is replaced by solid walls to mitigate undesirable higher wall friction and flux in the late transitional stage. The selection of $x = 0.34$ m in Case 3 also considers the modal growth region of the second mode. As illustrated in figure 6.2 (b), the modal growth of the optimal plane wave (10, 0) terminates at approximately $x = 0.34$ m. A detailed discussion and explanation will be provided in the subsequent sections.

In the direct numerical simulation, the computation starts from $x = 0$ to $x = 0.6$ m, with grid points of $3060 \times 260 \times 60$, denoting the grid points in the streamwise, wall-normal, and spanwise directions, respectively. The corresponding dimensionless grid spacings are $\Delta x^+ \approx 3.13$, $\Delta z^+ \approx$

Cases	Solid wall	TDIBC
1	0-0.6 m	-
2	0-0.115 m	0.115-0.6 m
3	0-0.115 & 0.34-0.6 m	0.115-0.34 m

TABLE 6.1: Wall boundary conditions for DNS cases

5.9, and $\Delta y^+ \approx 0.30$. Grid convergence was confirmed for optimal disturbance amplitudes during instability stages [102]. A good agreement is reached between the direct numerical simulation and the reference values or stability analysis results.

6.4.3 Bispectral mode decomposition

Based on the DNS data, bi-Fourier analysis and bispectral analysis are performed. Bi-Fourier analysis demonstrates the significant amplitude evolution for multiple disturbances, while bispectral decomposition reveals the energy transfer mechanism. The BMD method developed by Schmidt [166] is employed to elucidate the triadic interactions arising from quadratic nonlinearities in the Navier-Stokes equations, which govern energy transfer between scales in the flow. As shown in Schmidt [166], BMD examines triadic interactions originating from the quadratic nonlinearity inherent in the Navier-Stokes equations. The perturbation dynamics about a base flow are governed by

$$\frac{\partial \mathbf{q}'}{\partial t} = \mathbf{L}\mathbf{q}' + \mathbf{Q}(\mathbf{q}', \mathbf{q}'), \quad (6.20)$$

where $\mathbf{q}'(\mathbf{x}, t)$ represents the flow perturbation (typically velocity components or pressure fluctuations), $\mathbf{L}$ is the linear operator, $\mathbf{Q}(\cdot, \cdot)$ denotes the quadratic nonlinearity arising from terms like $(\mathbf{u}' \cdot \nabla)\mathbf{u}'$ in the convective term.

The expectation operator $E[\cdot]$, central to BMD's statistical formulation, performs ensemble averaging over independent flow realisations. Specifically, BMD identifies phase-coupled frequency triads $\{f_k, f_l, f_m\}$ satisfying $f_m = f_k \pm f_l$ through the bispectrum. The bispectrum is defined as the double Fourier transform of the third-order moment $E[q(t)q(t - \tau_k)q(t - \tau_l)]$:

$$S_{qqq}(f_k, f_l) = \lim_{T \rightarrow \infty} \frac{1}{T} E [\hat{q}^\dagger(f_k) \hat{q}^\dagger(f_l) \hat{q}(f_k + f_l)], \quad (6.21)$$

where $\hat{q}(\mathbf{x}, f)$ is the Fourier transform of $\mathbf{q}(\mathbf{x}, f)$ and $\dagger$ indicates complex conjugate. The classical bispectrum S_{qqq} serves as a conceptual foundation for detecting quadratic phase coupling. Its spatial generalisation, the mode bispectrum ϕ , is achieved by replacing the temporal expectation

$E[\cdot]$ with a spatially weighted ensemble average over Fourier realisations, thereby extending pointwise statistics to coherent structures. For multidimensional datasets, BMD constructs the bispectral density tensor.

$$\mathbf{D} = \frac{1}{N_{\text{blk}}} \hat{\mathbf{Q}}_{k \circ l}^H \mathbf{W} \hat{\mathbf{Q}}_{\mathbf{k}+\mathbf{l}}, \quad (6.22)$$

with $\mathbf{W}$ containing spatial quadrature weights, N_{blk} the number of data blocks, and $\hat{\mathbf{Q}}_{k \circ l} = \hat{\mathbf{Q}}_k \circ \hat{\mathbf{Q}}_l$ being the Hadamard product of Fourier realisations. Here, superscript H denotes the Hermitian transpose (conjugate transpose). The matrix $\mathbf{Q}$ organises the Fourier coefficients of $\mathbf{q}'(\mathbf{x}, t)$ in a block form, where each column contains a realisation of $\hat{\mathbf{q}}(\mathbf{x}, f)$ for spectral estimation. The dominant triadic interactions are identified by maximising the numerical radius of $\mathbf{D}$, which involves solving the eigenvalue problem for the Hermitian matrix $\mathbf{H}(\theta) = \frac{1}{2}(\mathbf{e}^{i\theta} \mathbf{D} + \mathbf{e}^{-i\theta} \mathbf{D}^H)$. The complex mode bispectrum $\phi(f_k, f_l)$, which is defined as the maximum numerical radius of $\mathbf{D}$, quantifies the triad interaction strength through its magnitude $|\phi|$. The magnitude of the mode bispectrum scales with the product of the absolute amplitudes of the triad components. This preserves the energy-transfer scaling inherent to quadratic nonlinearities.

The BMD method and open-source code proposed by Schmidt (2020) were employed in this study, and a detailed introduction for the formulation and definition can be found in Schmidt (2020).

6.4.4 A summary of analytical techniques

The present analysis framework systematically combines existing techniques to elucidate the effects of the metasurface across different transition stages. The investigation progresses from linear mechanisms, in which the resolvent analysis identifies optimal disturbances and LST/PSE quantifies modal growth rates, to the nonlinear transition dynamics captured via DNS visualisations of vortex formation and bi-Fourier quantifications of mode evolutions with or without the metasurface. The complete energy pathway is resolved through BMD of triadic interactions and energy budget diagnosis of dissipation mechanisms. Crucially, DNS visualisations provide physical grounding for spectral analyses, with instantaneous flow structures associated with BMD-detected triadic interactions and streak spacing matching bi-Fourier analysis. This represents the first coupled application of BMD and hypersonic DNS to resolve metasurface-modified triads while maintaining a direct correlation between flow structures and spectral-space dynamics.

6.5 Linear instability stage

In this section, the linear instability of dominant primary disturbances and the effect of the acoustic metasurface are of interest. In particular, the fitting result of softness using the theoretical model is presented. The correctness of the embedded TDIBC code is examined in comparison with the results obtained by directly meshing cavities. The resolvent analysis is employed to capture the optimal response of the boundary layer. The effect of the metasurface on single-frequency disturbances of low- and high-frequency nature is demonstrated, respectively. The preferred starting location for TDIBC is discussed.

6.5.1 Fitting result of softness and validation of TDIBC

Regarding the metasurface employed, the dimensionless half-width $b/L_{ref} = 0.196$, the unit-cell periods are $s/L_{ref} = 0.52$, and the depth $H/L_{ref} = 1.642$ are all consistent with those in Zhao *et al.* [176]. The geometric setting corresponds to a porosity of $\varphi = 2b/s = 0.75$. These parameters were designed to control the second mode and have demonstrated their effectiveness in linear stability theory and two-dimensional numerical simulations [176]. Figure 6.1 compares the softness obtained by a fitting procedure and the theoretical model considering high-order diffraction [13]. The fitting result was based on the method proposed by Douasbin *et al.* [170], using equation (2.13), with $C_0 = -0.05$ and poles and residues shown in Appendix 6.8.1. It is indicated that the fitting result agrees well with the theoretical model within a broadband frequency domain, i.e., 0–500 kHz, which enables accurate simulation subject to both high- and low-frequency disturbances using the acoustic metasurface. The comparison between the results using the developed TDIBC and the meshed cavity subject to broadband disturbances, as well as a constant impedance model ($v' = Ap'$), can be seen in Appendix 6.8.2. A good agreement is reached, validating the correctness of the TDIBC code in this study.

6.5.2 Result of stability analysis

In resolvent analysis, the interval 0–0.5 m is utilised, and the forcing is introduced at $x_0 = 0.04$ m. The N -factor is defined by $N = 0.5 \log(E_{Chu}/E_{Chu,0})$, where $E_{Chu,0}$ is measured at $x = x_0$ and N is 0 at x_0 .

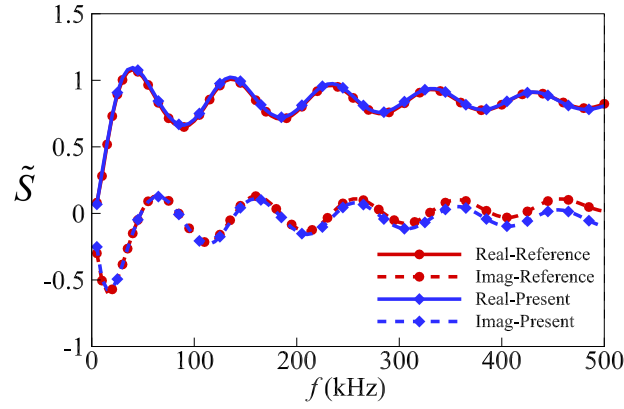


FIGURE 6.1: Comparison of wall softness between the impedance model [13] and the multi-pole fitting results (present).

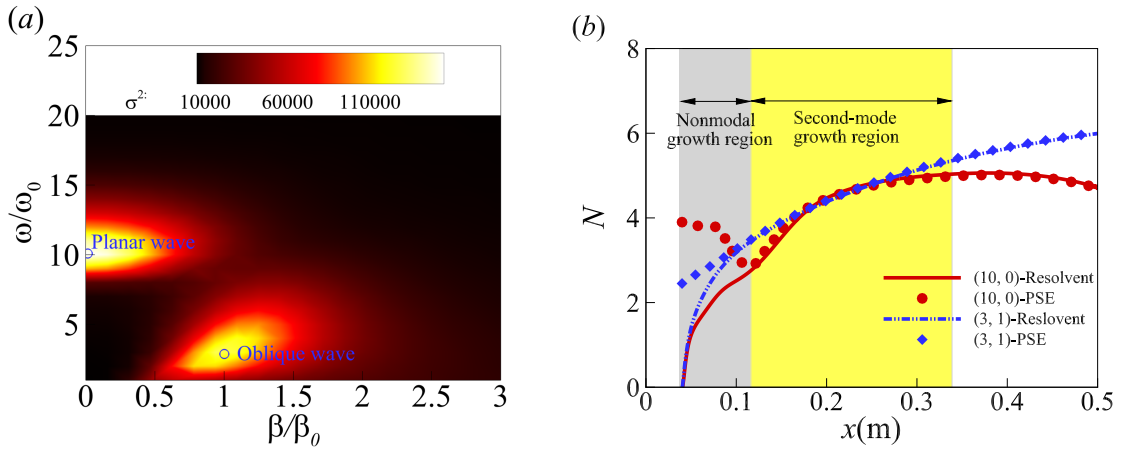


FIGURE 6.2: Optimal energy amplification contours mapped across the spanwise wavenumber-frequency domain, where $\omega_0 L_{ref}/u_\infty = 0.1$ and $\beta_0 L_{ref} = 0.8$. (b) comparison of N factors between PSE and resolvent analysis. N factor of PSE is shifted to be compared with resolvent analysis.

The gain contour for a range of spanwise wavenumbers and angular frequencies is shown in figure 6.2(a). It demonstrates that the low-frequency oblique first mode $(\omega/\omega_0, \beta/\beta_0) = (3, 1)$ and the high-frequency plane second mode $(\omega/\omega_0, \beta/\beta_0) = (10, 0)$ are prominent in the gain contour. In this study, $\omega/\omega_0 = 10$ corresponds to a dimensional frequency of 125.8 kHz, which has also been shown to be significant under the considered experimental conditions [173]. For brevity, the optimal response for a Fourier mode with $\omega/\omega_0 = 10$ and $\beta/\beta_0 = 0$ is designated as mode (10, 0), and the same applies to other optimal disturbances. In the next DNS, the three-dimensional transition to turbulence will be initiated using the optimal forcings determined by resolvent analysis. As shown in figure 6.2(b), the optimal response captured by resolvent analysis undergoes a transient growth downstream of the forcing location, which cannot be captured in PSE when a purely modal profile is introduced upstream. The transient growth

region is determined by the interval between the forcing location and the neutral point of the normal mode, where dN/dx from the resolvent analysis converges with that from the PSE. For instance, the interval $x \sim [0.04 \text{ m}, 0.115 \text{ m}]$ corresponds to the transient growth region of Fourier mode (10, 0). The PSE results indicate that modal growth begins at approximately $x = 0.115$ and concludes at about $x = 0.34 \text{ m}$ for (10, 0), as depicted in figure 6.2(b). As for the optimal oblique wave (3, 1), its integral N factor exceeds that of the plane wave (10, 0) downstream, owing to a longer growth region, despite a lower maximum growth rate upstream.

In the numerical simulation, both modal and nonmodal growth are included. Given the broadband nature of freestream disturbances in wind tunnels and realistic flying conditions, both the first and second modes should be considered in the investigation of hypersonic boundary-layer instability and transition. In this study, optimal disturbances $(\omega_p, \beta_p) = (10, 0)$ and $(\omega_o, \beta_o) = (3, 1)$ were utilized to initiate unsteady flow for the cases listed in table 6.1. These disturbances enable complicated nonlinear mode–mode interactions and rapid spectral broadening [102].

6.5.3 TDIBC for single-frequency disturbance

In this section, the single-frequency disturbance is introduced in a precursor simulation to study its linear evolution individually. The validation of initialisation for optimal forcing obtained by resolvent analysis in OpenCFD can be seen in Appendix 6.8.3.

Figure 6.3 (a) compares the evolution of the Fourier modes (10, 0) with solid wall and TDIBC starting from $x = 0.04 \text{ m}$ and $x = 0.115 \text{ m}$. The acoustic metasurface may suppress the modal growth of the second mode while enhancing the transient growth. Note that $x = 0.115 \text{ m}$ is the starting point of modal growth for Fourier mode (10, 0) according to the PSE result in figure 6.2 (b). When TDIBC is employed in the range of 0.04–0.2 m, the total Chu’s energy is augmented (see figure 6.3 (a) around $x = 0.1 \text{ m}$), while the wall pressure fluctuation shows a suppression effect in figure 6.3 (b). This indicates that the wall pressure fluctuation may not reflect the overall impact of the metasurface individually. The result agrees with the numerical simulation of Wang & Zhong [178] and the wind tunnel experiment of Lukashevich *et al.* [179], which reported that the acoustic metasurface is efficient only within the unstable region of the second mode. The detailed effect of the acoustic metasurface on each variable downstream and upstream of the synchronization point of the second mode is compared in figure 6.4.

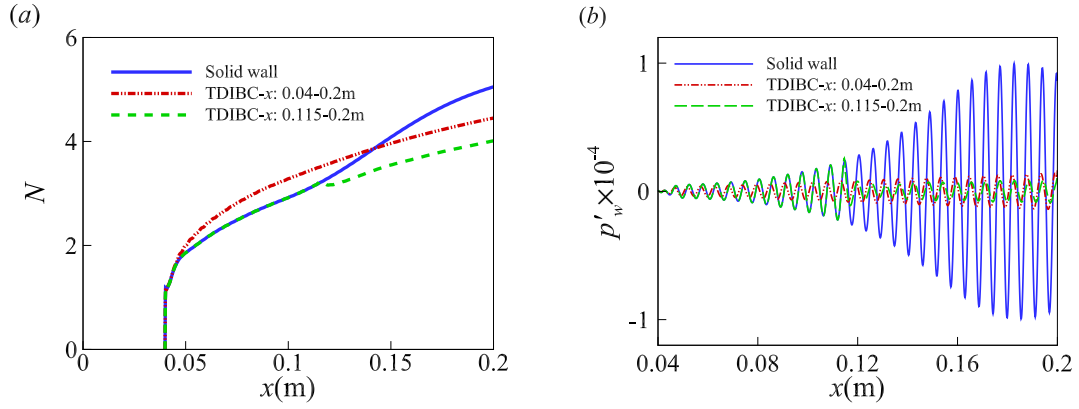


FIGURE 6.3: (a) N -factor evolution and (b) dimensionless fluctuation wall pressure for different wall boundary conditions subject to optimal forcing of mode (10, 0).

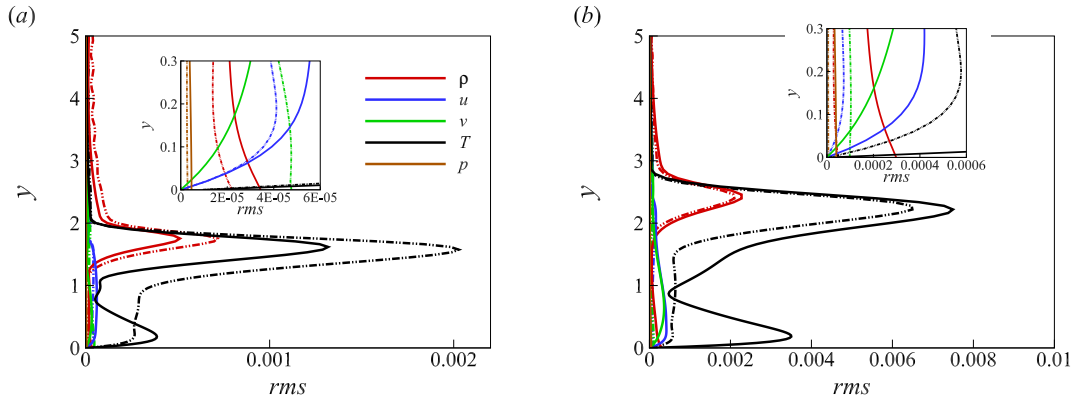


FIGURE 6.4: Comparison of dimensionless rms magnitude between cases using solid wall (solid line) and TDIBC (dash dot-dot line) at (a) $x = 0.08$ m and (b) $x = 0.16$ m initialised by optimal mode (10, 0). The TDIBC is applied within the range of 0.04–0.2 m.

Figure 6.4 compares the root-mean-square (rms) values of primary variables between the results obtained with a solid wall and the acoustic metasurface (realized by TDIBC). It indicates that the rms of wall pressure is smaller with TDIBC at $x = 0.08$ m, which agrees with the fluctuation result shown in figure 6.3 (b). However, the density and temperature are significantly enhanced near the boundary layer edge (around $y = 2$). This leads to a larger total energy and destabilization effect on the plane wave (10, 0) in this region, namely a larger N , as shown in figure 6.3 (a). Downstream of the neutral point of the second mode, the disturbances are completely suppressed, as shown in figure 6.4 (b). On the contrary, the acoustic metasurface slightly destabilizes the oblique optimal disturbance (3, 1), as shown in figure 6.5.

Based on that, the starting location of TDIBC for Case 2 and Case 3 was selected as $x = 0.115$ m to avoid unnecessary amplification of unstable waves. Additionally, TDIBC ends at $x = 0.34$ m in Case 3 because the second mode is stable downstream of $x = 0.34$ m. It should be noted

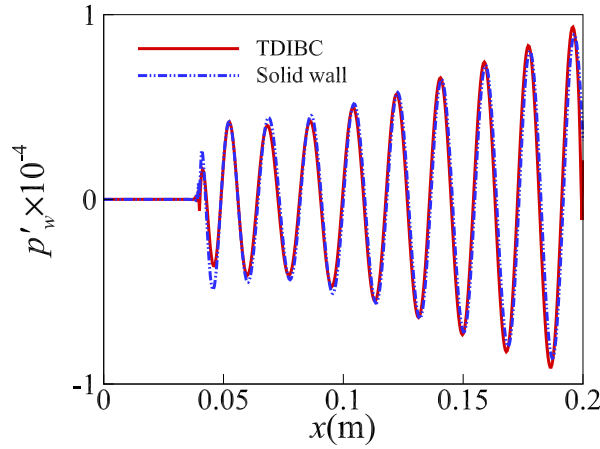


FIGURE 6.5: Comparison of dimensionless fluctuation pressure at the wall for the optimal wave (3, 1) between solid wall condition and TDIBC.

that no prior reports have examined the impact of the metasurface on the transient growth stage. More research is necessary to determine how the metasurface affects the Orr/Lift-up process that underlies the transient growth. More importantly, it is not yet understood how the transition initiated by the first and second modes responds to the acoustic metasurface. In the next investigation, the effect of the acoustic metasurface in combination resonance [8] will be focused on, which requires the involvement of both first and second modes.

6.6 Nonlinear mode–mode interaction and breakdown

6.6.1 Transition onset and end

Following the verification of the initialization of optimal responses and the embedded TDIBC code, the nonlinear interaction between the low-frequency and high-frequency components and the resulting transition process with the acoustic metasurface are of concern. In this section, optimal disturbances with two prominent frequencies and the same amplitude are applied to initiate the transition. Three cases with different wall boundary conditions listed in table 6.1 are simulated and compared. The effect of the acoustic metasurface on the combination resonance will be evaluated in detail for statistical averages and instantaneous flow fields, especially for mode–mode interaction mechanisms using bi-Fourier analysis and energy budget analysis.

The comparison of vortex structures between Case 1 and Case 2 is plotted in figure 6.6. The spanwise-aligned structure observed in Case 1, representing the planar-wave mode (10, 0),

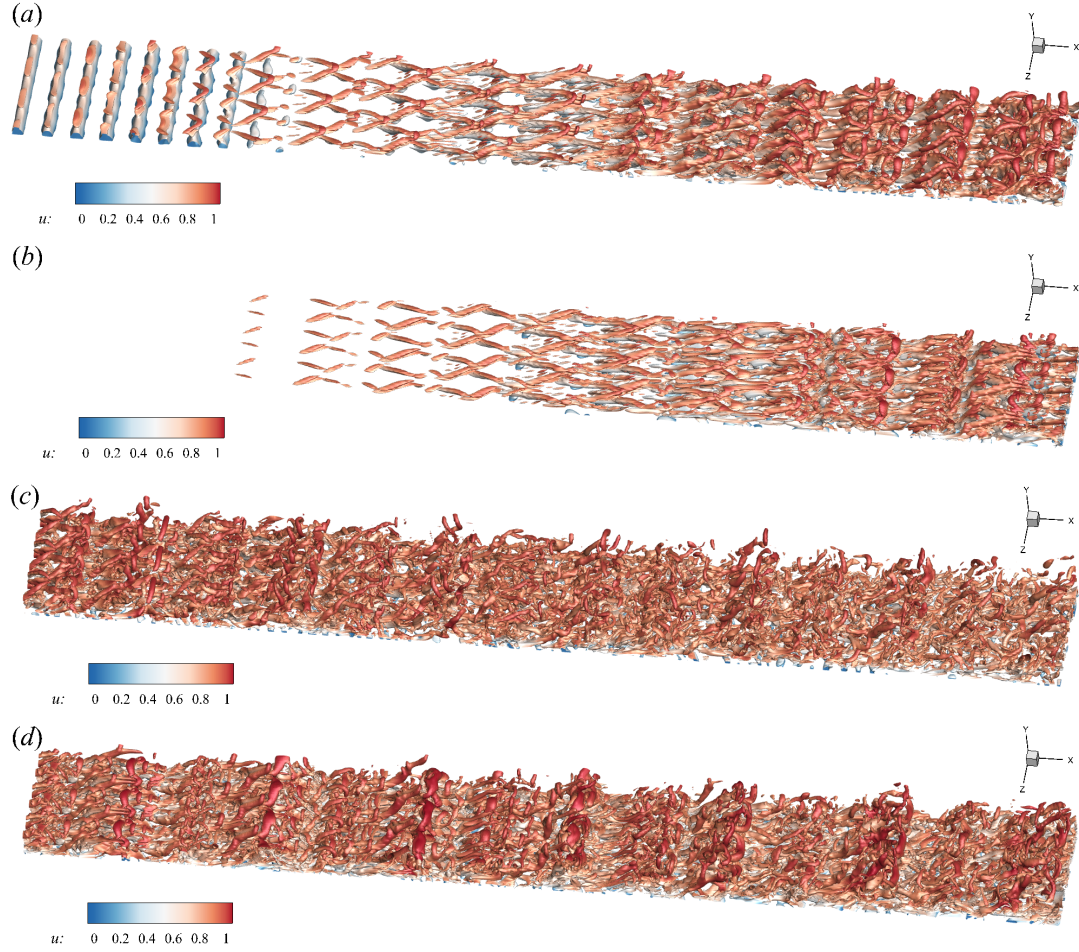


FIGURE 6.6: Vortical flow features identified by $(L_{ref}/u_{\infty})^2 Q = 0.005$ iso-surfaces (velocity-colored) are compared in the range of $0.2 \text{ m} < x < 0.4 \text{ m}$ for (a) Case 1 and (b) Case 2, and of $0.4 \text{ m} < x < 0.6 \text{ m}$ for (c) Case 1 and (d) Case 2.

vanishes in Case 2 due to the presence of the acoustic metasurface. This indicates that mode (10, 0) is highly suppressed by the acoustic metasurface. Further downstream, the staggered structure appears later in Case 2 than in Case 1, which is caused by the oblique wave $(3, \pm 1)$. Additionally, the hairpin vortex, indicating the late stage of transition, is also delayed in its appearance in Case 2 compared to Case 1. Until the end of the computational domain, the vortex structure in Case 1 has broken down into small-scale structures, indicating fully developed turbulence. This is consistent with the result of the skin friction coefficient illustrated in figure 6.7, in which the curve collapses into the van Driest correlation near the end of the computational domain.

Figure 6.7 illustrates the spanwise- and time-averaged skin friction coefficient for three cases, accompanied by the van Driest II correlation. It is shown that the increment of the skin friction coefficient is delayed for Case 2 and Case 3 compared with Case 1. By estimating the transition onset location based on the minimum skin friction coefficient, the transition onset occurs at

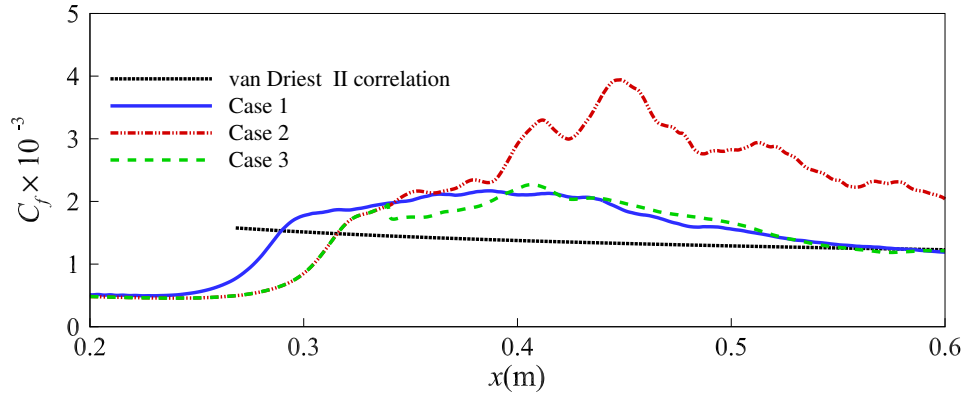


FIGURE 6.7: Comparison of spanwise- and time-averaged skin friction coefficient with the result of van Driest II formula. The van Driest II formula is applied following the procedure of Guo *et al.* [14].

approximately $x_{onset} = 0.22$ m in Case 1, while it shifts downstream to around $x_{onset} = 0.24$ m in Case 2 and Case 3. The transition ends at around $x = 0.57$ m for Case 1 and Case 3, as the skin friction coefficient collapses to the van Driest II correlation. The transition delay efficiency is evaluated by η , which is defined by

$$\eta = \frac{x_{onset, Case 2 or 3} - x_{onset, Case 1}}{x_{onset, Case 1}}. \quad (6.23)$$

The transition delay efficiency is about 9% based on the onset location for Cases 2 and 3 with the acoustic metasurface. This value is not prominent compared with the wind tunnel experiment of Wagner *et al.* [158], where the second mode is dominant under a cooling wall condition at Mach 7.5, with the first mode highly suppressed. Nevertheless, this finding confirms the effectiveness of the acoustic metasurface in delaying the transition induced jointly by the low-frequency first mode and the high-frequency second mode. However, in Case 2, the skin friction coefficient is higher than its counterpart in Case 1 in the late transitional region, which is downstream around $x = 0.34$ m. This phenomenon was also reported in the wind tunnel experiment at DLR conducted by Wagner *et al.* [158–160]. The reason for the higher skin friction in the late transitional region is of general interest and will be explored using bi-Fourier analysis and the energy budget equation later. In Case 3, the undesirable increase in wall friction is eliminated when the wall boundary condition reverts to a solid wall downstream of $x = 0.34$ m. The comparative study demonstrates that positioning the acoustic metasurface within the unstable region of the second mode is more effective for controlling flow transition.

Figure 6.8 provides the contour of the time-averaged skin friction coefficient. The streak spacing

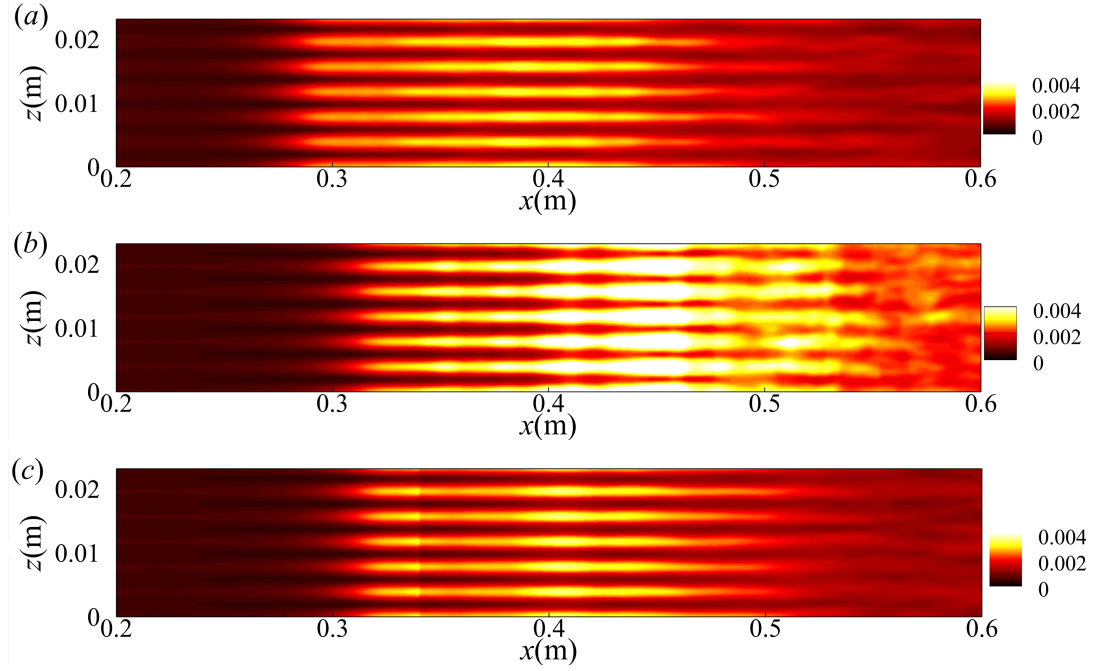


FIGURE 6.8: Contour of time-averaged skin friction coefficient for (a) Case 1, (b) Case 2, and (c) Case 3.

corresponds to the spanwise wavenumber of the steady streamwise mode $(0, 2)$, which can be nonlinearly produced by the interaction of the forced oblique waves $(3, \pm 1)$. It is evident that, compared to the solid-wall case, the appearance of streaks is delayed in the case with TDIBC. Meanwhile, the maximum skin friction induced by streaks is more pronounced for Case 2, consistent with the spanwise-averaged result shown in figure 6.7. Compared to the results of Case 1, Case 3 exhibits a purely delayed effect in the contour of time-averaged skin friction. Additionally, these two cases converge to fully developed turbulence nearly at the same location, as shown in figure 6.7. Interestingly, the skin friction drops slightly where the acoustic metasurface is replaced by solid wall conditions at $x = 0.34$ m, and it gradually develops into a consistent result with Case 1. This indicates that the acoustic metasurface only produces a local effect. The undesirable increment of wall friction in Case 2 downstream of $x = 0.34$ m is caused by the acoustic metasurface. The underlying physics may relate to the strengthening of mean shear near the wall, as reported in the study of permeable walls over the supersonic boundary layer by Chen & Scalo [172, 177].

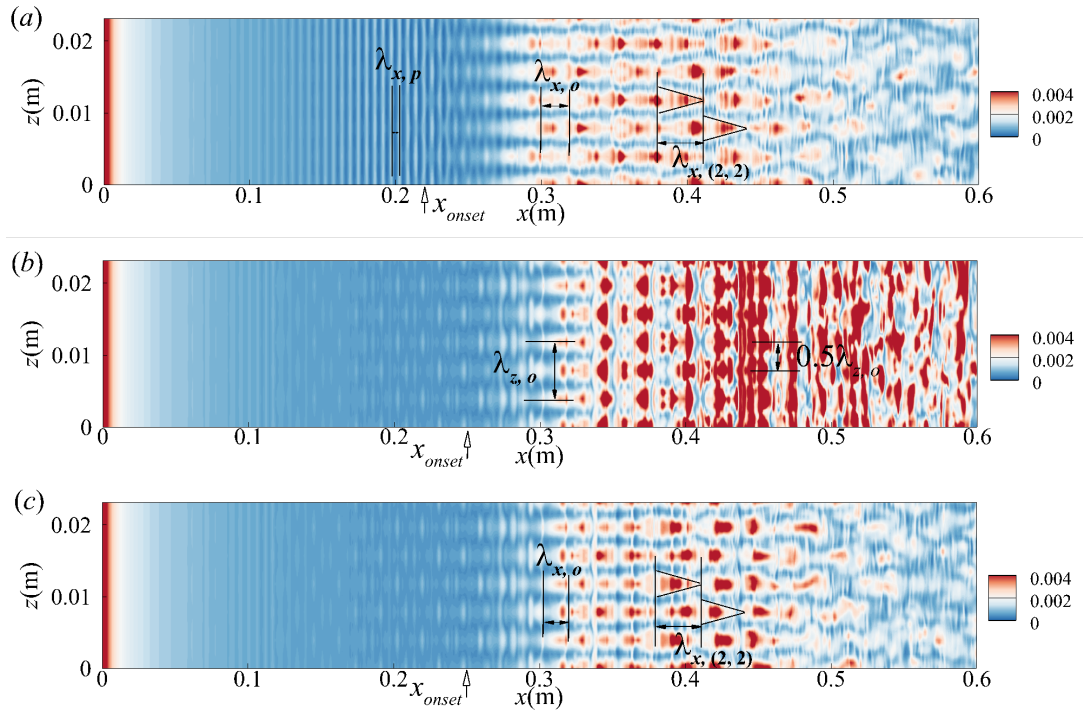


FIGURE 6.9: Instantaneous skin friction coefficient C_f for (a) Case 1, (b) Case 2, and (c) Case 3, where x_{onset} refers to the evaluated starting location of the transition.

6.6.2 Evolution and contribution of Fourier modes

To facilitate the understanding of the dominant nonlinear interaction mechanism and to explore the reasons why transition onset is delayed by the acoustic metasurface, as well as why wall friction is higher in the late transitional stage with the acoustic metasurface (see figure 6.7), the instantaneous near-wall flow structure is examined. The instantaneous skin friction coefficient C_f is plotted, as shown in figure 6.9. The streamwise scale of the plane wave $\lambda_{x,p}$, oblique wave $\lambda_{x,o}$, and large-scale Λ -vortices related to the detuned mode $\lambda_{x,(2,2)}$ can be clearly identified in Case 1, as shown in figure 6.9 (a). The detuned mode (2, 2) is generated through

$$\begin{aligned}
 (3, 1) + (3, -1) &\rightarrow (6, 0), \\
 (10, 0) - (3, -1) &\rightarrow (7, 1), \\
 (7, 1) - (6, 0) &\rightarrow (1, 1), \\
 (1, 1) + (1, 1) &\rightarrow (2, 2).
 \end{aligned} \tag{6.24}$$

The generation of detuned mode (2, 2) is confirmed by wave vectors (α_r, β) of the considered triads [155], and details can be seen in Appendix 6.8.4.

The detuned mode, shown as large-scale Λ -vortices [8], can also be observed in Case 3, which indicates that the flow field can be recovered as the wall boundary shifts to the solid wall condition and that the acoustic metasurface only produces a local effect. In contrast, the spanwise-aligned structure of the second mode (at around $x = 0.2$ m) loses its signature in Case 2 and Case 3 due to the suppression effect of the acoustic metasurface. In Case 2, the distinct spanwise-aligned structure near $x = 0.45$ m indicates the importance of the planar wave $(2, 0)$, as shown in figure 6.9 (b). Furthermore, the spanwise scale approaches $0.5\lambda_{z,o}$, which is identical to the detuned mode $(2, 2)$. The enhanced pattern related to detuned modes may be responsible for the increment of skin friction in the late transitional stage with the acoustic metasurface (see figure 6.7). A further decomposition of skin friction is needed to identify the dominant contributor. In the subsequent investigation, temporal and spanwise bi-Fourier analysis and energy budget methods will be applied to investigate the contribution of wall friction in the late transitional zone.

The Chu's energy integral in the wall-normal direction includes kinetic and internal energy. Its streamwise evolution of multiple Fourier modes can comprehensively reflect the dominant modes in the transitional stage, which already includes the effect of nonlinear interaction. As depicted in figure 6.10, the growth of Chu's energy of the second-mode-related mode $(10, 0)$ is suppressed by the acoustic metasurface downstream of $x = 0.115$ m. Meanwhile, the first mode is slightly promoted near $x = 0.2$ m, which is consistent with previous theoretical and simulation research [74, 178]. In Case 1, the planar wave $(10, 0)$ saturates at around $x = 0.18$ m, and its amplitude begins to decrease due to energy transfer via nonlinear interaction with modes $(5, 1)$, $(10, 2)$, and $(2, 2)$ or $(2, 0)$ through subharmonic resonance, fundamental resonance, and combination resonance, respectively. The oblique breakdown associated with mode $(3, 1)$, the fundamental resonance associated with mode $(10, 0)$, and the subharmonic resonance associated with mode $(10, 0)$ can be described as

$$\begin{aligned} (3, 1) - (3, -1) &\rightarrow (0, 2), \\ (10, 0) + (0, 2) &\rightarrow (10, 2), \\ (10, 0) - (5, 1) &\rightarrow (5, -1), \end{aligned} \tag{6.25}$$

respectively. The aforementioned modes, other than modes $(3, \pm 1)$ and $(10, 0)$, known as secondary instabilities, undergo a rapid growth process that leads the flow to develop into the late stage of transition and breakdown. The specific interaction and nonlinear mode-mode mechanism were systematically studied and reported by Guo *et al.* [8]. In this study, the effect of the acoustic metasurface is emphasized. In figure 6.10 (a), one can see that the second-mode-related mode $(10, 0)$ in Case 2 grows gradually to nearly the same amplitude as in Case 1 but

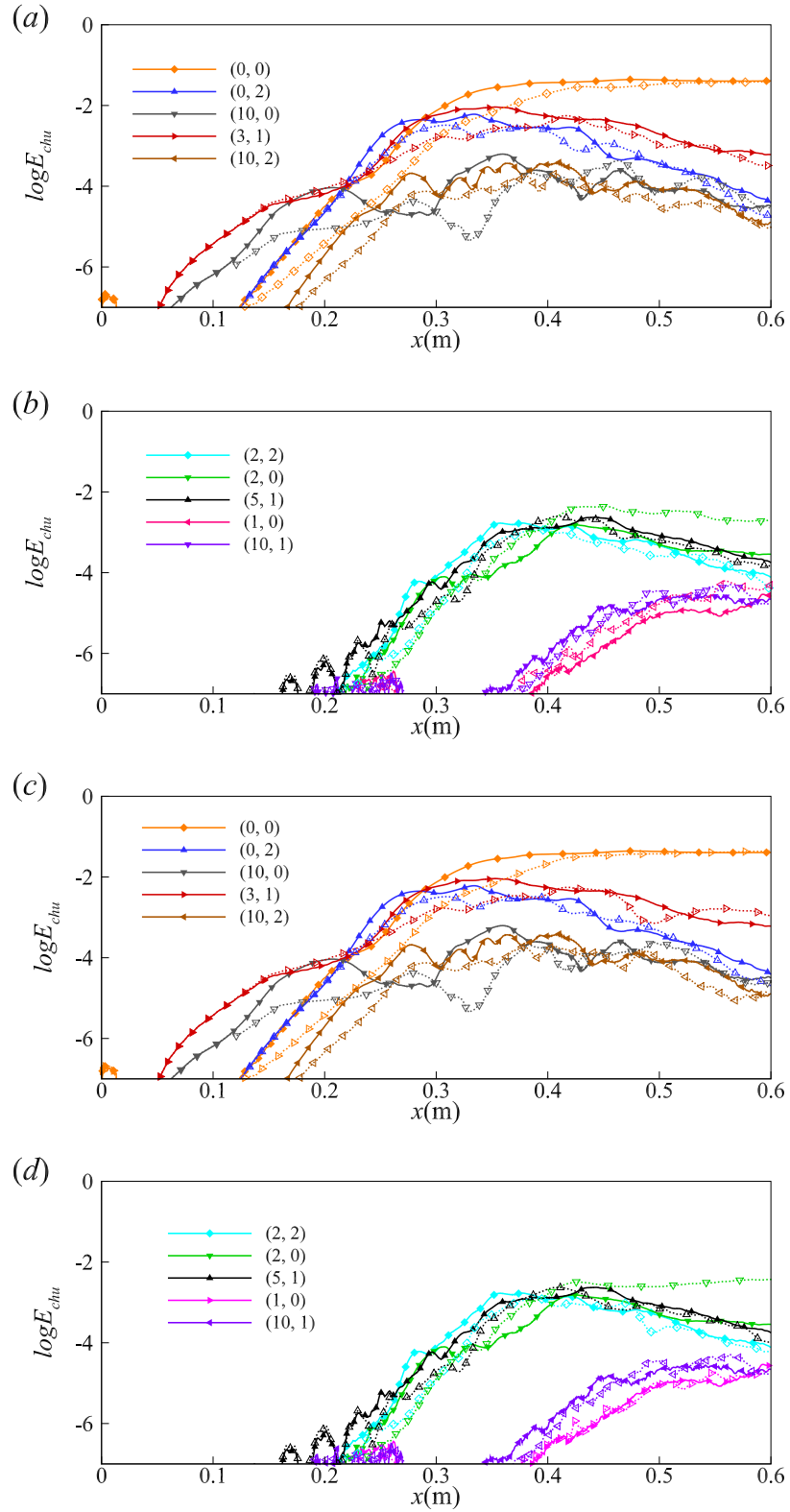


FIGURE 6.10: Streamwise growth of disturbances evaluated using Chu's energy for (a) and (b) for Case 1 and Case 2, and (c) and (d) for Case 1 and Case 3. The modes in Case 1 are represented by solid lines with filled symbols, and modes in Case 2 and Case 3 are represented by dotted lines with open symbols.

saturates downstream. Consequently, the secondary growth of the primary oblique wave (3, 1) shows suppression or a lower growth rate in Case 2 at around $x = 0.28$ m compared with that in Case 1. This result is somewhat consistent with the study of Chen *et al.* [57], where the amplitude of the second mode saturates and the "phase lock" sets in, supporting a secondary growth stage of the primary low-frequency mode. Meanwhile, the second mode decays due to feedback from the rapidly growing secondary modes. Most importantly, the dominant detuned modes (2, 0) are apparently more energetic with the acoustic metasurface in Case 2 and Case 3 compared with the solid wall case. Additionally, the low-frequency modes generated in the later transitional stage, such as (1, 0), are also more evident than their counterparts in the solid-wall case.

To identify the dominant Fourier mode contributing to the increment of higher skin friction in Case 2 and reveal the underlying nonlinear mode–mode interaction mechanism. The maximum absolute contribution from mode (m, n) to the instantaneous skin friction is quantified as

$$\Delta C_{f,(m,n)}(x) = \max_{(z,t)} |C_{f,(m,n),unsteady} - C_{f,laminar}|, \quad (6.26)$$

where $C_{f,(m,n),unsteady}$ represents the skin friction generated by the combined effect of the laminar flow and mode $(m, \pm n)$, while $C_{f,laminar}$ corresponds to the skin friction produced solely by the laminar flow. This indicator is introduced to identify the Fourier modes that significantly influence the skin friction distribution, as illustrated in figure 6.9. Figure 6.11 compares the maximum absolute modal contribution to the instantaneous skin friction coefficient C_f for cases with the acoustic metasurface (Case 2 and Case 3) and the solid wall (Case 1). Apparently, the second mode (10, 0) is suppressed in both Case 2 and Case 3 with the acoustic metasurface, which agrees with the disappearance of the aligned structure in vortex analysis (shown in figure 6.6) and instantaneous wall friction (shown in figure 6.9). As shown in figure 6.11 (a), the detuned modes (2, 2) and (2, 0) demonstrate a considerably higher contribution, indicating a significantly strengthened combination resonance. In addition, the subharmonic-related mode (5, 1) and oblique mode (3, 1) in Case 2 also show a more pronounced contribution than in Case 1, indicating the promotion of these two nonlinear mechanisms in the late transitional stage using the acoustic metasurface. To quantitatively evaluate the difference in the maximum absolute contribution of mode (m, n) between Case 1 and Case 2, and to identify the main contributor for the strengthened skin friction in the late transitional region in Case 2, the integral effect is

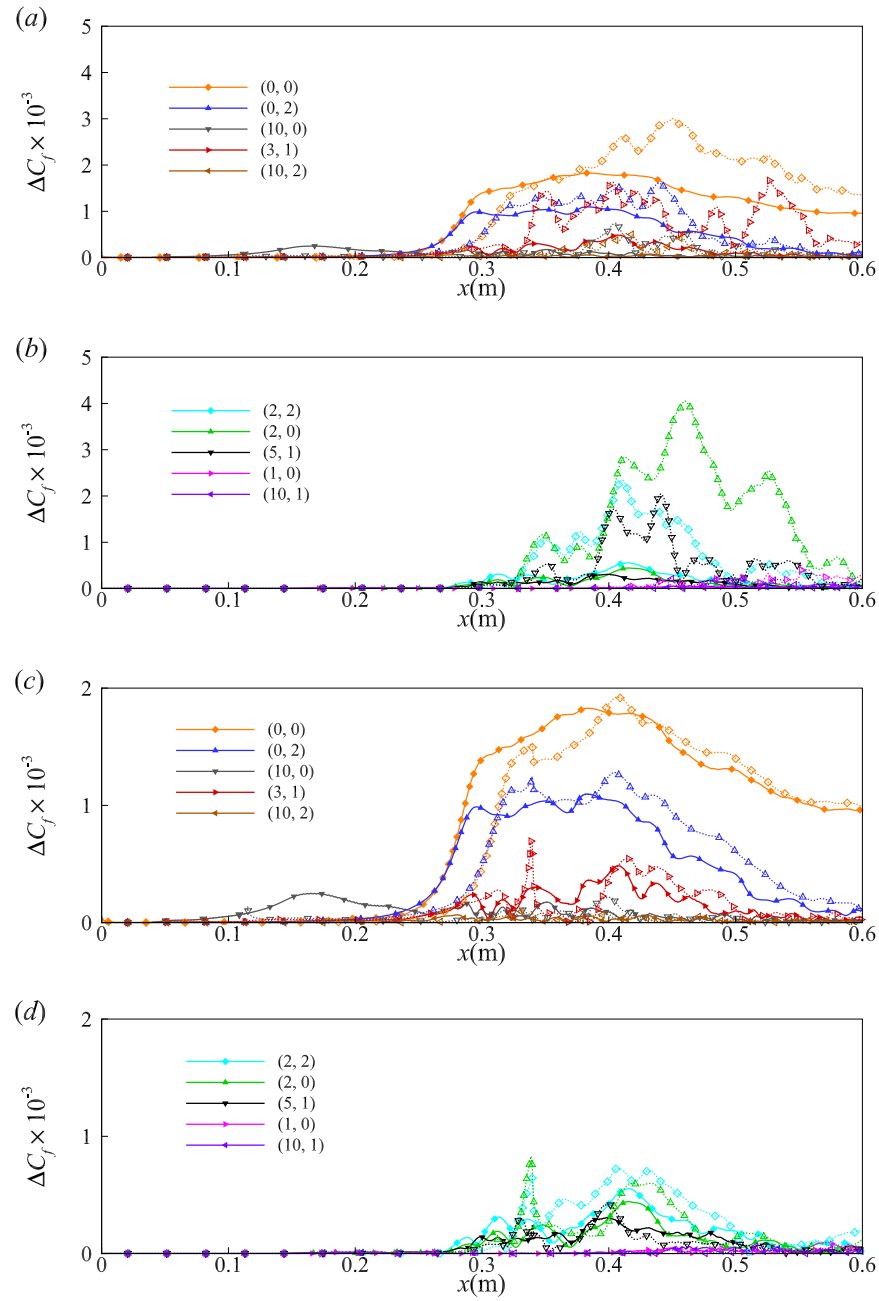


FIGURE 6.11: Assessment of dominant modal impacts on the instantaneous skin friction coefficient C_f for (a) and (b) for Case 1 and Case 2, and (c) and (d) for Case 1 and Case 3. The modes in Case 1 are represented by solid lines with filled symbols, and modes in Case 2 and Case 3 are represented by dotted lines with open symbols.

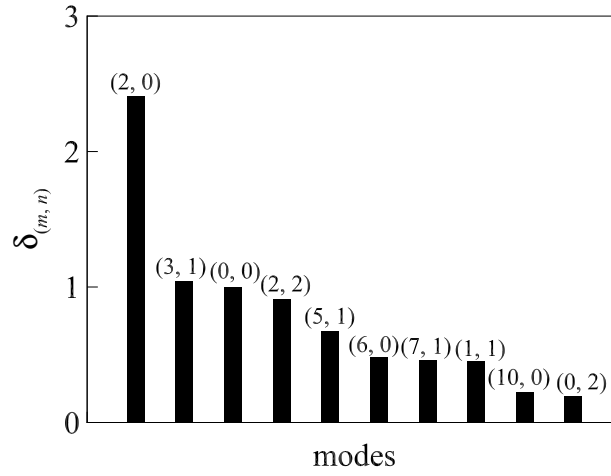


FIGURE 6.12: Comparison of $\delta_{(m,n)}$ among different Fourier modes. The value $\delta_{(0,0)}$ is utilised for normalisation.

calculated using the following equation:

$$\delta_{(m,n)} = \int_{x_1}^{x_2} (\Delta C_{f,(m,n), \text{Case 2}} - \Delta C_{f,(m,n), \text{Case 1}}) dx, \quad (6.27)$$

where $x_1 = 0.34$ m and $x_2 = 0.6$ m. The comparison of $\delta_{(m,n)}$, normalised by $\delta_{(0,0)}$, among different Fourier modes is presented in figure 6.12. The results indicate that the dominant contributor comes from the detuned mode (2, 0), which aligns with the planar structure observed at approximately $x = 0.45$ m in figure 6.9(b).

As for Case 2 and Case 3, the modal contribution to the instantaneous skin friction coefficient shows a delayed increment compared with Case 1. This results from the saturation of the amplitude of the plane wave (10, 0), and the resulting nonlinear interaction is delayed due to the suppression of the acoustic metasurface. Interestingly, the amplification of the skin friction coefficient for secondary modes in the late transitional region in Case 2 (see figure 6.11) is evidently higher, while the corresponding Chu's energy shows little difference (except for mode (2, 0) shown in figure 6.10), which means the acoustic metasurface results in a significant impact near the wall while exerting less impact on the whole energy.

Regarding the question of why transition onset is delayed, the increasing contribution of the mean flow distortion (0, 0) should be discussed. In the range $x \in [0.25, 0.32]$ m, where the transition begins in figure 6.7, the presence of the metasurface leads to the postponed growing contribution of (0, 0) in figure 6.11(a). In this range, only the streak mode (0, 2) is notable in terms of magnitude. As a result, the delayed transition onset can be attributed to the postponed

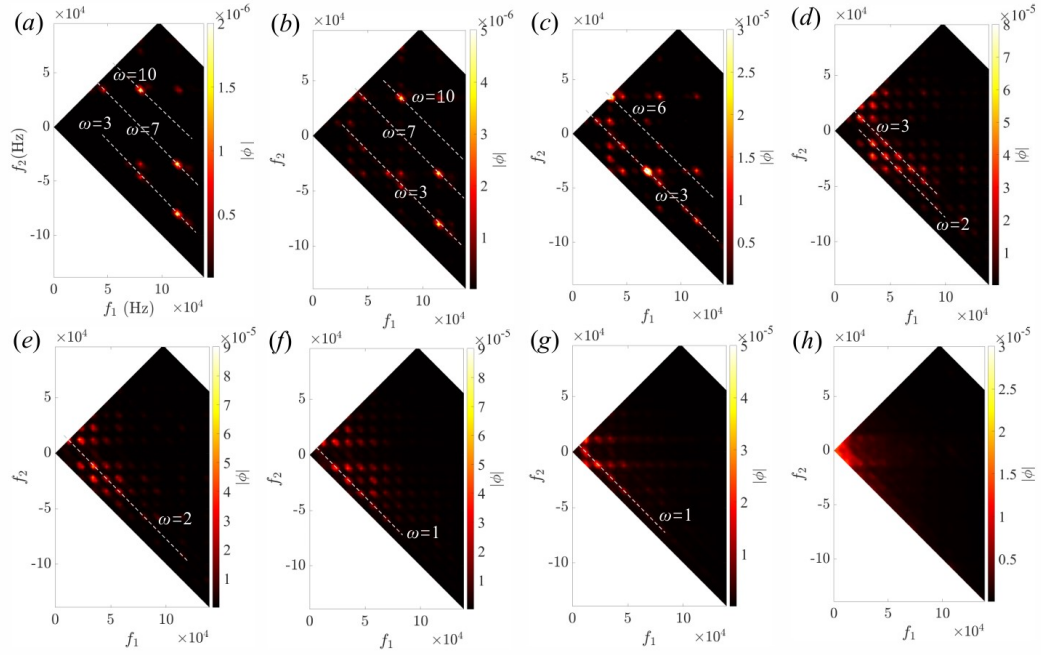


FIGURE 6.13: (Magnitude) mode bispectrum of Case 1 for various streamwise regions (a) 0.2 – 0.225 m, (b) 0.25 – 0.275 m, (c) 0.3 – 0.325 m, (d) 0.35 – 0.375 m, (e) 0.4 – 0.425 m, (f) 0.45 – 0.475 m, (g) 0.5 – 0.525 m, and (h) 0.55 – 0.575 m at $z = z_0/2$, where z_0 is the spanwise width of the computational domain.

growth of mode (0, 2). This finding is interesting because the manifested control effect by the metasurface does not lie in the second mode directly. Given the nonlinearity, the Fourier mode (0, 2) is the direct contributor to the delayed transition onset instead. Upstream of $x = 0.3$ m, the contributions of other secondary modes are not evident in figure 6.11(a). Consequently, the delayed (0, 2) is mainly caused by the suppressed oblique-wave resonance (see the first interaction equation in equation (6.25)) via the relatively large-amplitude primary instability wave (3, 1). As interpreted in the discussion of figure 6.10, mode (3, 1) is stabilized by the metasurface in the nonlinear stage, which is likely achieved by the less energy received from the suppressed Mack second mode. Therefore, the second mode plays an indirect role in the transition delay mechanism of the acoustic metasurface.

6.6.3 Mode–mode interaction

Through complementary spectral analyses, the bi-Fourier decomposition quantifies disturbance amplitude development, while the bispectral analysis elucidates the underlying energy transfer

pathways governing these evolutionary patterns. To clarify the energy transfer mechanism, BMD was performed on the mid- xy plane of the computational domain, where the dominant triadic interactions are clearly discernible through the bispectrum value distribution. As illustrated in figure 6.13, the bispectrum maps at various streamwise sections reveal distinct nonlinear interaction patterns, with the magnitude of bispectrum values ϕ directly correlating with the strength of triadic coupling. The upper half and lower half planes of each map correspond to sum and difference triadic interactions respectively. The coordinates (k, l) and $(m, -l)$ represent the sum interaction ($\omega_k + \omega_l \rightarrow \omega_m$) and the difference interaction ($\omega_m - \omega_l \rightarrow \omega_k$). Here, the subscripts k, l, m denote nondimensional angular frequencies (e.g., ω_{10} represents Fourier modes of $\omega = 10$, equivalent to the physical frequency $f = 125.8$ kHz) and satisfy the triad resonant condition $k + l = m$. Diagonal dashed lines demarcate triadic interactions generating constant frequencies, as annotated by their respective angular frequencies ω . These dashed lines in the figures cover all the triad interactions that generate the mode with the corresponding angular frequency ω . The initial stage exhibits prominent interactions between the high-frequency component (ω_{10}) and low-frequency mode (ω_3), generating ω_7 and ω_{13} modes through difference ($\omega_{10} - \omega_3 \rightarrow \omega_7$) and sum ($\omega_{10} + \omega_3 \rightarrow \omega_{13}$) triadic interactions respectively, as displayed in figure 6.13 ($a - b$). Concurrently, sum-self-interaction of ω_3 produces its harmonic ω_6 , while subsequent difference interaction between ω_7 and ω_3 yields ω_4 . Downstream evolution demonstrates progressive attenuation of high-frequency components accompanied by amplification of low-frequency modes, particularly the detuned ω_2 and ω_1 modes generated through $\omega_3 - \omega_1 \rightarrow \omega_2$ and $\omega_3 - \omega_2 \rightarrow \omega_1$ interactions (see figure 6.13(f)). The late transitional region ($x \in [0.55, 0.575]$) exhibits emergent near-zero-frequency components with the absence of dominant spectral peaks, indicative of fully developed turbulent flows. This energy transfer pathway, where triadic interactions mediate progressive energy transfer from high- to low-frequency modes culminating in near-zero-frequency components, contrasts with previous hypersonic boundary layer studies employing bicoherence analysis. While Craig *et al.* [168] and Zhang & Shi [180] reported dominant self-sum interactions of the second mode (ω_{10}) and its harmonic in quiet wind tunnel conditions, the present results highlight stronger detuned mechanisms involving first-mode interactions. This discrepancy likely stems from the relative suppression of second-mode self-sum interactions under the current configuration and flow condition, combined with enhanced first-mode activity that is typically attenuated in the quiet wind tunnel experimental environments.

To confirm the spanwise wavenumber for the dominant triadic interactions, the spatial structure

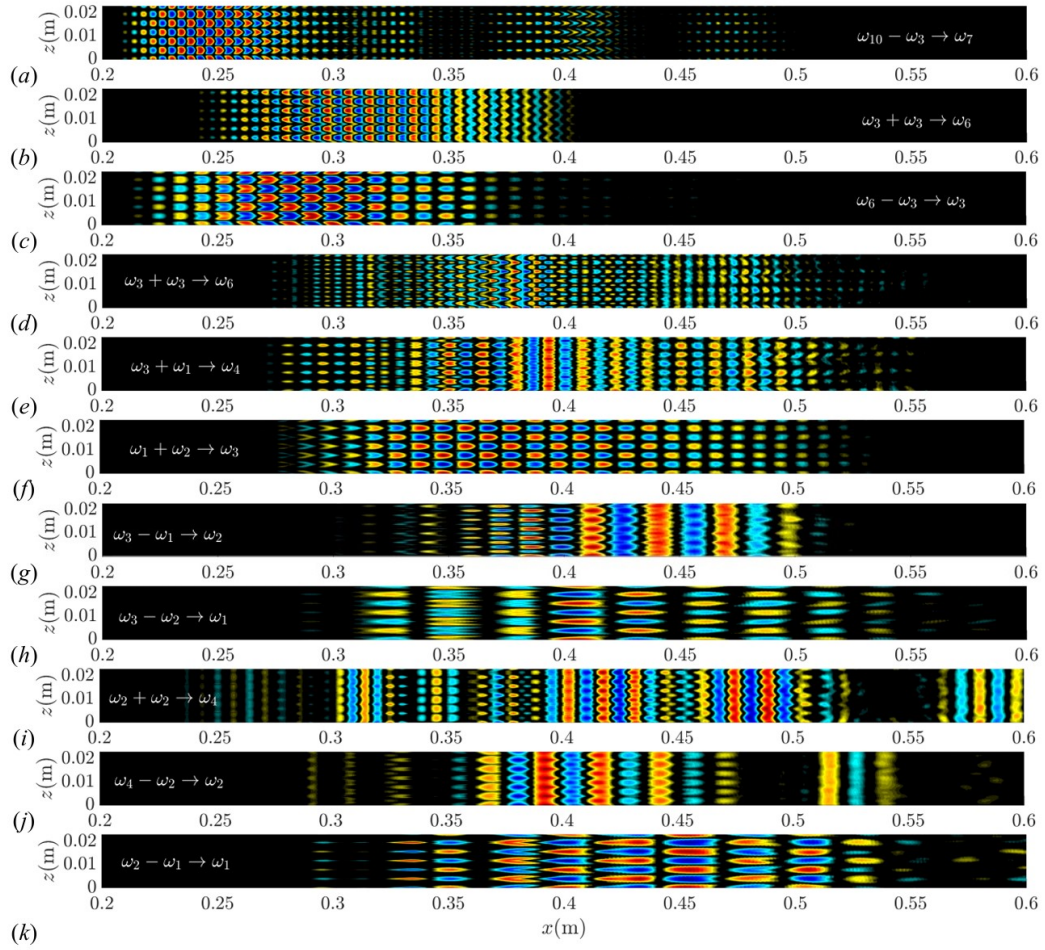


FIGURE 6.14: Bispectral mode (real part of density component) marked with the associated sum and difference triadic interactions at $y = 3.2$ mm (a – c), $y = 1.6$ mm (d – h), and $y = 0$ mm (i – k).

of bispectral modes was revealed using BMD. Figure 6.14 presents the xz -plane bispectral modes characterised by density at three characteristic wall-normal positions: the wall surface ($y = 0$ mm), an intermediate layer ($y = 1.6$ mm), and the boundary-layer edge vicinity ($y = 3.2$ mm). The results reveal that the triadic interaction associated with detuned modes (corresponding to ω_1 and ω_2) exhibits high energy density near the wall, whereas the first-mode (ω_3) and its harmonic (ω_6) are more energetic in the off-wall region (based on the bispectrum map at three characteristic wall-normal positions, not shown here). This spatial distribution is consistent with both the vortical structure characteristics of the first instability mode near the boundary-layer edge and the dominant influence of detuned modes on skin friction generation during late transitional stages, as documented in Guo *et al.* [8]. Notably, the spanwise bispectral mode distribution serves as an effective indicator of the dominant spanwise wavenumber for specific frequencies. For instance: modes (1, 1), (3, 1), (6, 0), and (7, 1) are identified as primary contributors to ω_1 ,

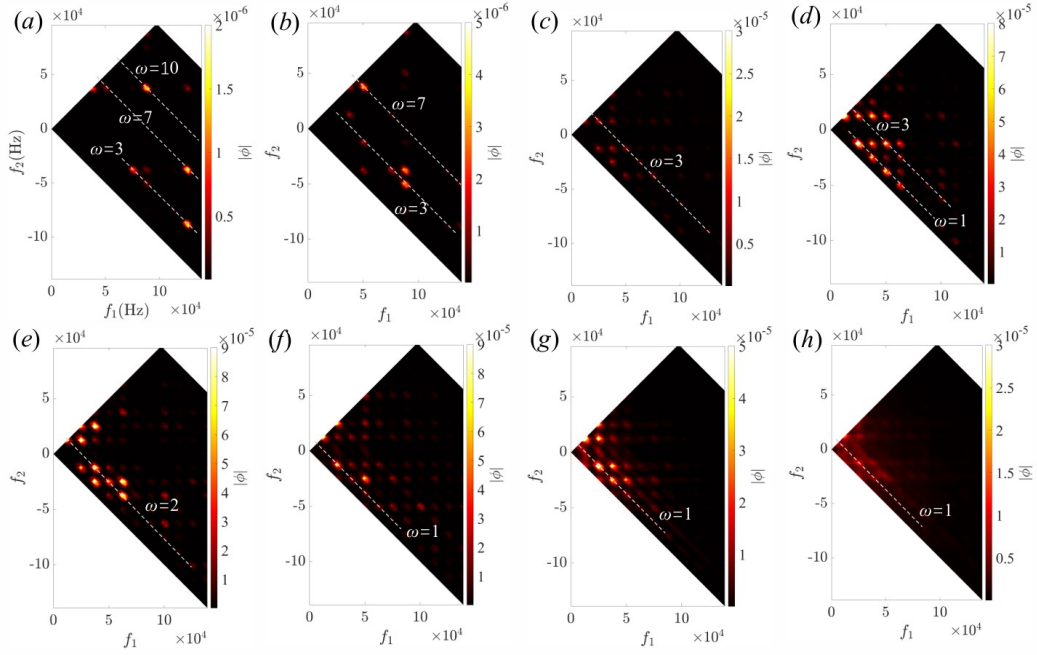


FIGURE 6.15: (Magnitude) mode bispectrum of Case 2 for various streamwise regions (a) 0.2 – 0.225 m, (b) 0.25 – 0.275 m, (c) 0.3 – 0.325 m, (d) 0.35 – 0.375 m, (e) 0.4 – 0.425 m, (f) 0.45 – 0.475 m, (g) 0.5 – 0.525 m, and (h) 0.55 – 0.575 m at $z = z_0/2$, where z_0 is the spanwise width of the computational domain.

ω_3 , ω_6 , and ω_7 , respectively. Modes (2, 2) and (2, 0) jointly govern ω_2 formation, with (2, 0) exhibiting greater dominance due to the weak spanwise periodicity (see figure 6.14(j)). These detuned modes are further responsible for the large-scale Lambda-structures observed near $x = 0.4$ m in figure 6.14. In summary, the BMD successfully resolves the dominant spanwise features of transitional flow dynamics, involving both primary (first and second) instability modes as well as secondary modes. The triadic interaction mechanisms are quantitatively elucidated during the transition process.

Figure 6.15 presents the bispectrum distribution at different streamwise locations for Case 2. To highlight the influence of the acoustic metasurface on nonlinear mode–mode interactions, the colour bar levels are maintained consistent with the solid-wall case in figure 6.13 for a direct comparison. The introduction of the acoustic metasurface (Case 2) leads to a pronounced suppression of second-mode-dominated triadic interactions in the region $x \in [0.25 \text{ m}, 0.275 \text{ m}]$ (see figure 6.15(b), compared with figure 6.13(b)). This suppression subsequently weakens the triadic interaction between ω_6 and ω_3 in $x \in [0.3 \text{ m}, 0.325 \text{ m}]$ (see figure 6.15(c), compared with figure 6.13(c)), owing to reduced energy transfer from ω_{10} . This observation also echoes with the

deduction in Section 6.6.2. As the generation triad for ω_3 is suppressed, the resulting stationary streak mode is less energetic, and the MFD growth and transition onset in figure 6.11 are thus delayed by the metasurface. Further downstream, the amplified presence of ω_1 and ω_2 (see figure 6.15(e–g), compared with figure 6.13(e–g)) indicates stronger nonlinear interactions involving detuned modes with the presence of the metasurface. In other words, the metasurface effect on the low-frequency dynamics is evidently different, where the low-frequency mode interactions are enhanced in the controlled case. This finding is consistent with the elevated skin friction observed for Case 2 beyond $x = 0.34$ m (figure 6.7). The enhanced triadic interactions generating the detuned modes result in an increased contribution to the skin friction coefficient, as shown in figure 6.11. This effect may originate from the shear-induced dissipation observed in figure 6.16 from the perspective of mean-flow statistics. Notably, the underlying triadic interaction mechanism remains unaltered, as low-frequency detuned modes (ω_2 and ω_1) continue to emerge in later transitional stages. Interestingly, residual light points persist in the triadic maps for Case 2 (figure 6.15(h)), suggesting that the flow in the controlled case may not yet be fully developed by the end of the computational domain. Further analysis of the flow state will be provided in subsequent sections.

6.6.4 Energy budget analysis

To better understand the effect of acoustic metasurface in delaying transition, the transport of internal energy is of interest [27]. The influence of the acoustic metasurface on the energy budget terms is analysed. The viscous dissipation (which is decomposed into contribution induced by shear Φ_ϖ and dilatation Φ_θ) and pressure dilatation work (T_p) near the wall are analysed in the following study. The expressions of Φ_ϖ and Φ_θ are

$$\Phi_\varpi = \langle \overline{\mu \varpi_k \varpi_k} \rangle, \quad \Phi_\theta = \left\langle \frac{4}{3} \mu \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 \right\rangle. \quad (6.28)$$

In this section, $\overline{(\cdot)}$ and $\langle \cdot \rangle$ denote time and spanwise averaging, respectively. For a Cartesian coordinate system, the vorticity component ϖ_k is expressed as

$$\varpi_1 = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \quad \varpi_2 = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \quad \varpi_3 = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \quad (6.29)$$

Subsequently, the shear-induced dissipation is decomposed into four parts: $\Phi_{\varpi} = \Phi_{\varpi 0} + \Phi_{\varpi 1} + \Phi_{\varpi 2} + \Phi_{\varpi 3}$, representing the effects of the time- and spanwise-averaged field, the second-order moment of the cross correlation between fluctuation in dynamic viscosity and vorticity, the second-order moment of the vorticity self-correlation, and the third-order moment, respectively. The specific expressions of these four terms are

$$\begin{aligned}\Phi_{\varpi 0} &= \langle \bar{\mu} \rangle \langle \overline{\varpi_k} \rangle \langle \overline{\varpi_k} \rangle, \quad \Phi_{\varpi 1} = 2 \langle \overline{\varpi_k} \rangle \langle \overline{\mu' \varpi'_k} \rangle, \\ \Phi_{\varpi 2} &= \langle \bar{\mu} \rangle \langle \overline{\varpi'_k \varpi'_k} \rangle, \quad \Phi_{\varpi 3} = \langle \overline{\mu' \varpi'_k \varpi'_k} \rangle.\end{aligned}\tag{6.30}$$

The dimensionless time- and spanwise-averaged pressure dilatation term (T_P), shown as

$$T_P = -\langle \bar{p} \rangle \left\langle \frac{\partial u_i}{\partial x_i} \right\rangle - \left\langle p' \frac{\partial u'_i}{\partial x_i} \right\rangle.\tag{6.31}$$

The instantaneous quantity is expressed as $\phi = \langle \bar{\phi} \rangle + \phi'$, where $\langle \bar{\phi} \rangle$ represents the base flow superimposed by mean flow distortion (MFD), and ϕ' denotes the disturbance excluding MFD. By incorporating MFD into the base flow, the formulation reflects the overall effect of the time- and spanwise-averaged field.

Figure 6.16 provides the comparison of streamwise development of the aforementioned terms between Cases 1, 2, and 3. The magnitude of shear-induced dissipation Φ_{ϖ} in Case 2 is evidently higher than in the other cases downstream of $x = 0.34$ m. The most prominent contribution comes from $\Phi_{\varpi 2}$ (shown in figure 6.16(b)), which refers to the second-order moment of vorticity self-correlation $\langle \bar{\mu} \rangle \langle \overline{\varpi'_k \varpi'_k} \rangle$. Also, the reinforcement of time- and spanwise-averaged dissipation $\Phi_{\varpi 0} = \langle \bar{\mu} \rangle \langle \overline{\varpi_k} \rangle \langle \overline{\varpi_k} \rangle$ is significant downstream of $x = 0.4$ m, as shown in figure 6.16(a). The other two terms are not displayed due to their negligible contributions. Furthermore, the dissipation induced by dilatation (Φ_{ϑ}) is suppressed by the acoustic metasurface upstream at $x = 0.34$ m, as illustrated in figure 6.16(c). However, the magnitude of Φ_{ϑ} is significantly lower compared to that of shear-induced dissipation (Φ_{ϖ}). These results suggest that shear-induced dissipation plays a dominant role in internal energy transport, implying that the first-mode oblique breakdown may be critical in the late-stage transitional flow. The oblique breakdown is strengthened by the acoustic metasurface in Case 2. This finding aligns with the transition simulation by Tullio *et al.* [181], who directly resolved the flow within the pores of the acoustic metasurface and demonstrated that the first mode was promoted. As for Case 3, the magnitude

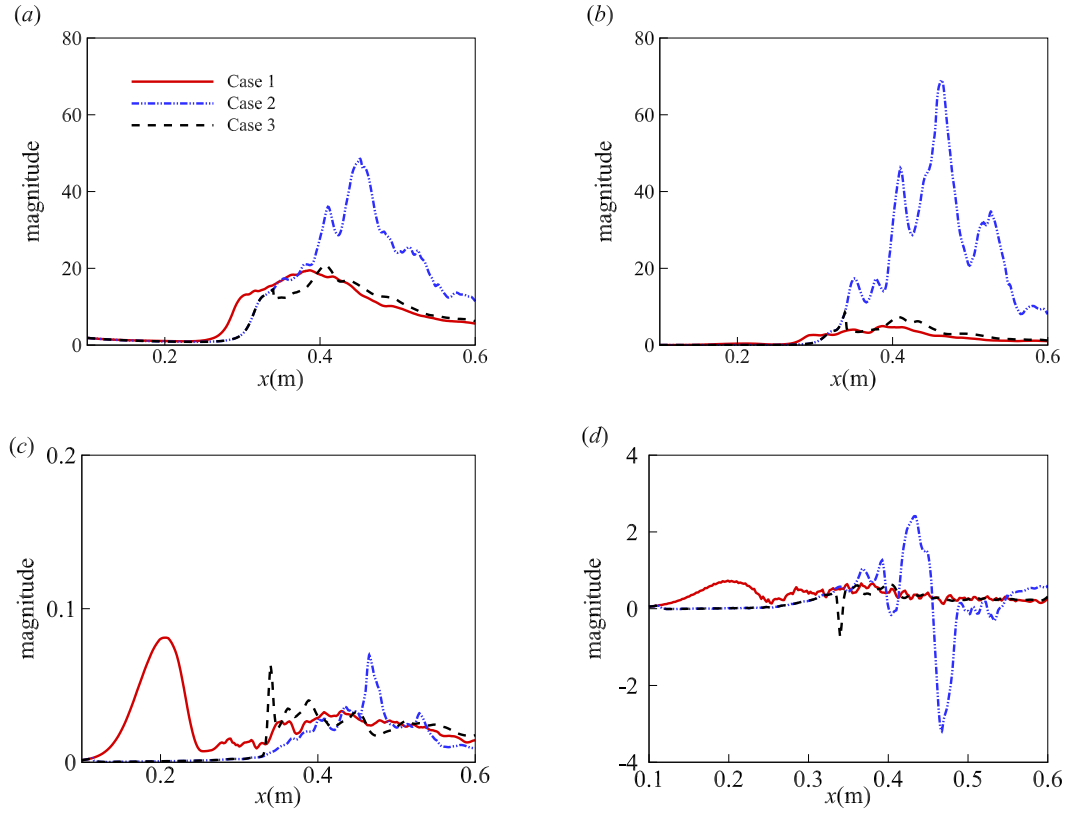


FIGURE 6.16: Comparison of energy budget terms of (a) $\Phi_{\varpi 0}$, (b) $\Phi_{\varpi 2}$, (c) Φ_{ϑ} , and (d) T_p in the internal energy transport equation on the wall between Case 1, Case 2, and Case 3. Here, $\Phi_{\varpi 1}$ and $\Phi_{\varpi 3}$ are not shown due to their negligible amplitude in comparison with that of $\Phi_{\varpi 0}$ and $\Phi_{\varpi 2}$.

of energy budget terms gradually agrees with that in Case 1 downstream of $x = 0.34$ m, as the boundary shifts from the acoustic metasurface to a solid wall condition.

The terms associated with dilatational work, including dilatation-induced dissipation in figure 6.16(c) and pressure dilatation work shown in figure 6.16(d), are suppressed significantly by the acoustic metasurface at around $x = 0.2$ m. The underlying physics probably relates to the second mode of acoustic nature [84, 182], whose main energy source near the wall is dilatational work and can be suppressed by the acoustic metasurface.

In summary, dilatation-related budget terms are suppressed while the shear-related terms are strengthened in cases with the acoustic metasurface. The results of the statistical energy budget analysis agree with the energy source analysis using the relative phase analysis [75]. In that work, it was concluded that the acoustic metasurface stabilizes the second mode by suppressing the dilatation near the wall while destabilizing the first mode by strengthening the mean shear.

While the energy budget here focuses on spatially averaged terms to highlight global trends, future work could decompose these budgets by wavenumber pairs to explicitly resolve triad interactions. Nevertheless, the current analysis suffices to demonstrate that the metasurface's trade-off—between suppressing dilatational work (stabilising the second mode) and amplifying shear dissipation (probably related to destabilising detuned modes, see following analysis)—is the key driver of higher skin friction in the late transitional stage.

6.6.5 Paths to turbulent flow

In this section, the spanwise- and time-averaged flow field is focused and analysed. To evaluate the flow state and its progression, the van-Driest-transformed velocity [183] U_{VD}^+ profiles for three cases at various streamwise stations are plotted in figure 6.17. The influence of mean density variations due to the effect of compressibility, which can be removed by Van Driest transformation for the mean velocity

$$U_{VD}^+ = \int_0^{\bar{u}^+} \sqrt{\frac{\bar{\rho}}{\bar{\rho}_w}} d\bar{u}^+(y), \quad (6.32)$$

where friction velocity $\bar{u}_\tau$ is utilised for normalisation

$$\bar{u}^+ = \frac{\bar{u}}{\bar{u}_\tau}, \quad \bar{u}_\tau = \sqrt{\frac{\bar{\tau}_w}{\bar{\rho}_w}}. \quad (6.33)$$

The viscous sublayer law ($u^+ = y^+$) and the incompressible log-layer law ($u^+ = 5.8 + 1/\kappa \ln y^+$, $\kappa = 0.4$) are given as references, represented by a black dashed line. At $x = 0.2$ m, the results of the three cases agree well with the reference, showing a consistent and well-resolved viscous sublayer. Downstream at $x = 0.3$ m, Case 1 shows its initial establishment of the log layer, while in Case 2 and Case 3, velocity profiles just begin to deviate from the sublayer reference at around $y^+ = 10$. Further downstream, the velocity profile gradually approaches the reference log layer, and the transformed velocity profile is nearly the same for Case 1 and Case 3 at $x = 0.6$ m, indicating that fully developed turbulence is established, even if the slope shows a slight difference due to the high Mach number employed. A similar difference was also reported in supersonic/hypersonic simulations by Zhou *et al.* [184] and Guo *et al.* [14]. As for Case 2, the slope of the transformed velocity is lower downstream of $x = 0.3$ m (see figure 6.17 (c–f)). This is qualitatively consistent with the permeable wall over the supersonic boundary layer studied by Chen & Scalo [172, 177].

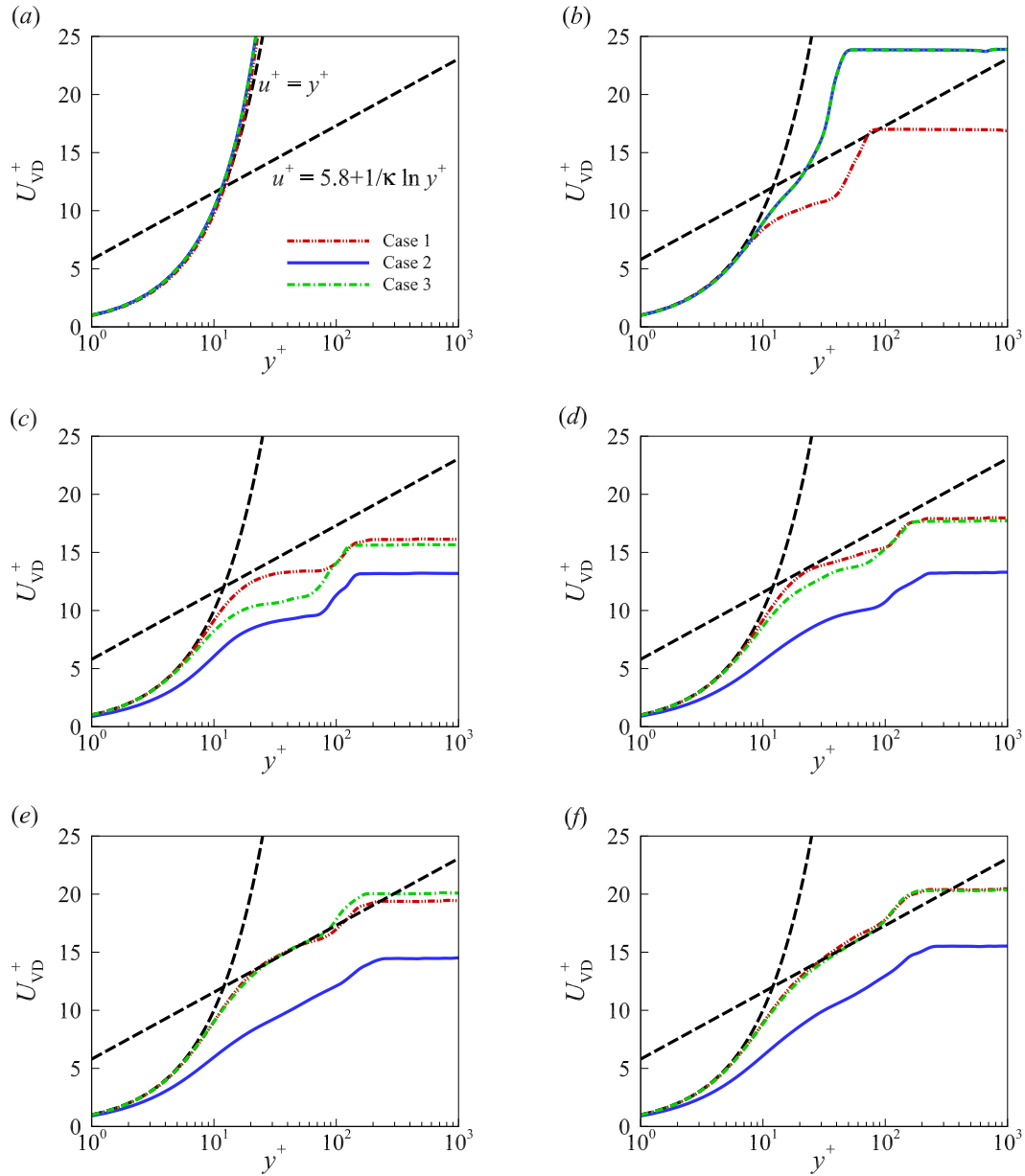


FIGURE 6.17: Van Driest velocity profiles for Case 1, Case 2, and Case 3 at (a) $x = 0.2$ m, (b) $x = 0.3$ m, (c) $x = 0.4$ m, (d) $x = 0.5$ m, (e) $x = 0.55$ m, (f) $x = 0.6$ m.

Owing to the augmented near-wall shear-induced dissipation in the late transitional region for Case 2, the van-Driest-transformed velocity profiles deviate from the viscous law near the wall. To evaluate the flow state, the PSD of wall pressure fluctuations at the downstream location is analyzed. As illustrated in figure 6.18, a good agreement is reached with the reference scale law for Case 1, indicating the establishment of a fully developed turbulent state at $x = 0.6$ m. In contrast, the comparison with the $\omega^{-5/3}$ law in the inertial subrange for Case 2 shows less consistency compared to Case 1. Moreover, the peak frequency ($f = 25.2$ kHz) corresponding to detuned modes (2, 0) and (2, 2) is observed at $x = 0.6$ m for Case 2. This indicates that

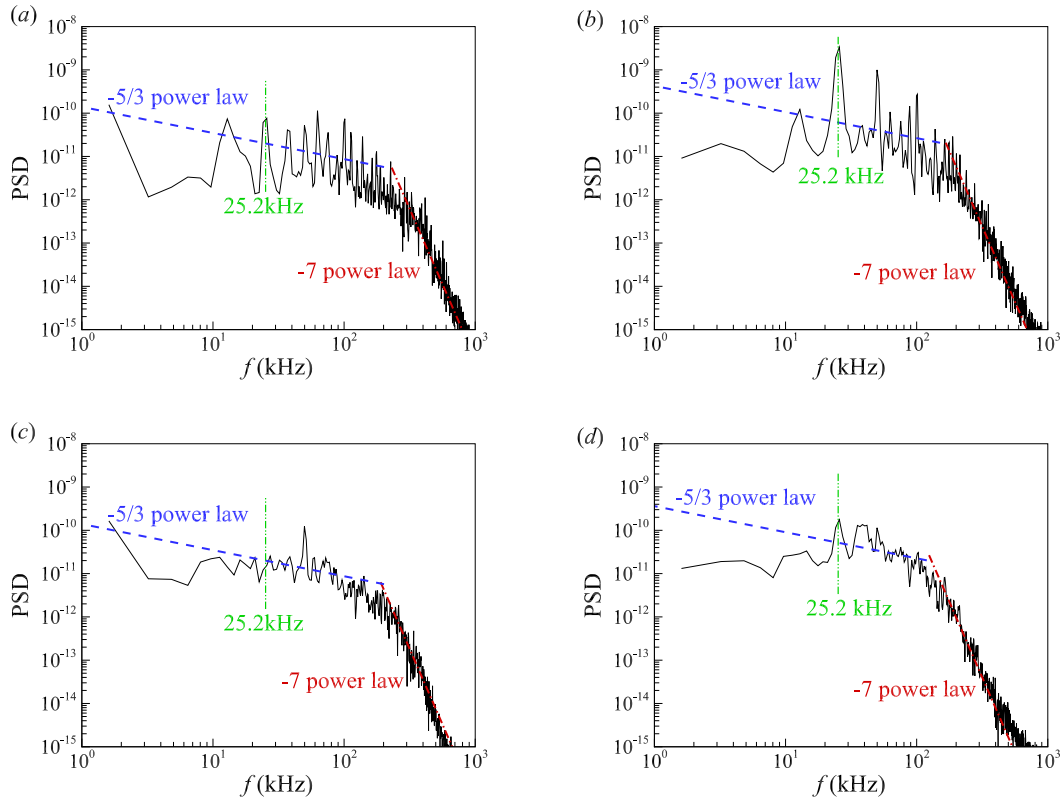


FIGURE 6.18: Power spectral density (PSD) of wall pressure fluctuation at $x = 0.45$ m for (a) Case 1 and (b) Case 2, and at $x = 0.6$ m for (c) Case 1 and (d) Case 2, in comparison with the law of $\omega^{-5/3}$ in the inertial subrange and the law of ω^{-7} in the dissipation scale. Frequencies corresponding to the (2, 0) and (2, 2) detuned modes are identified by the green dash-dot-dot line.

spectral broadening is incomplete at this location, and fully developed turbulence has not yet been established.

In summary, the transition onset was delayed by the implementation of the acoustic metasurface, and fully developed turbulence, as observed in the solid wall case, does not appear to have been established within the computational domain using the acoustic metasurface.

6.7 Conclusion remarks and discussion

Hypersonic boundary-layer transition control using a promising passive technique, i.e., the acoustic metasurface, is investigated. The purpose of this work is to elucidate the mechanism of transition delay by the acoustic metasurface subject to multiple primary instabilities and to explain the enhancement of skin friction during the late transitional stage. The TDIBC, which models the effect of the acoustic metasurface in the time domain, is successfully incorporated

Tool	Primary Objective	Key Outcome
Resolvent analysis	To identify optimal disturbances	Revealing linear competing mode growth: first and second modes
DNS visualisations	To track spatiotemporal transition physics	Documenting vortex dynamics and main flow structures
BMD	To quantify triadic interactions	Measuring possible energy transfer direction: from primary waves to detuned modes
Bi-Fourier analysis	To resolve modal evolution	Correlating modal growth with MFD: An indirect oblique breakdown delay mechanism
Energy budgets	To diagnose mean-flow dissipation mechanisms	Linking shear-induced dissipation to the oblique breakdown

TABLE 6.2: A combined framework where each tool targets specific physical questions.

into the Navier-Stokes solver OpenCFD. The resolvent analysis is performed to obtain the optimal disturbances of the boundary layer, and both the low-frequency first mode and the high-frequency second mode are involved in the primary instabilities. Several analytical tools are employed in this study, including resolvent analysis, DNS visualisation, BMD, bi-Fourier analysis, and energy budget analysis. The diverse physical questions addressed by these methods are synthesised in table 6.2, demonstrating how their combined use advances our understanding of the transition mechanism affected by the metasurface.

For the transition induced jointly by the first and second modes, the effectiveness of the metasurface in delaying the onset of transition has been demonstrated through our direct numerical simulations. The vortex visualisation indicates that the spanwise-aligned structure associated with the second mode disappears when the acoustic metasurface is employed. The staggered and hairpin structures, as well as the streamwise streak, are postponed with the presence of the metasurface. The fully developed turbulence observed in the solid-wall case seems not yet established when the acoustic metasurface is utilised. Notably, the acoustic metasurface induces higher wall friction during the late transitional stage, which can be mitigated by replacing the acoustic impedance boundary with a solid wall condition in this region. A bi-Fourier analysis of wall friction is conducted to elucidate the role of the acoustic metasurface in delaying the transition onset and increasing the late-stage skin friction. It turns out that the saturation of the second mode is delayed in cases involving the acoustic metasurface, which consequently attenuates the secondary growth of the first mode and the growth of the resulting streamwise streak. Eventually, the transition onset is delayed due to the postponed contribution of mean flow distortion. Figure 6.19 briefly summarises the effect of the acoustic metasurface (marked

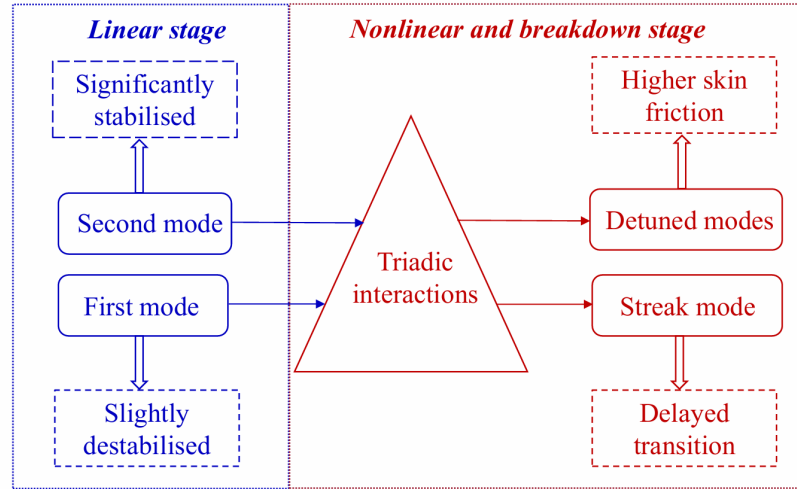


FIGURE 6.19: A schematic diagram revealing the effect of the acoustic metasurface (marked in dashed rectangular box) in the linear and nonlinear mode–mode interaction and breakdown of hypersonic boundary layers.

in dashed rectangular box) in the linear and nonlinear mode–mode interaction and breakdown processes.

Next, we briefly summarise the main contributions of this study:

(i) Regarding the reinforced skin friction in the late stage, different secondary modes participate in the physical process. Among them, the detuned modes primarily contribute to the increased wall friction with the metasurface. The mean internal energy transport suggests that the dilatational work is weaker prior to the transition, whereas the dissipation induced by shear is stronger in the cases with the acoustic metasurface. The latter is responsible for the higher wall friction from the perspective of the energy budget. The comparative study implies that the strategic placement of the acoustic metasurface is significant in minimising the skin friction and probably also the heat flux.

(ii) Bispectral mode decomposition (BMD) quantifies the energy transfer process: from primary disturbances to low-frequency detuned modes during the transition to turbulence, through intermediate modes generated directly by sum and difference triadic interactions of primary modes. The comparative analysis indicates that the acoustic metasurface significantly suppresses second-mode-related triadic interactions, while enhancing triadic interactions related to the detuned mode generation process. The affected second mode is in the early stage, which is related to the transition onset delay. The affected detuned mode leads to elevated mean skin

friction in late transitional stages. This dual mechanism explains the experimentally observed trade-off between transition delay and late-stage friction penalties.

(iii) Unlike previous works that typically focused on single-mode scenarios, we demonstrate how metasurfaces modify the complete transition process through nonlinear triadic interactions between these competing instabilities (evidenced via bispectral analysis), revealing a previously unreported mechanism where second-mode suppression indirectly weakens first-mode growth. Notably, the DNS result reveals that the metasurface's stabilisation effect in multi-mode transition is indirect: by attenuating the second mode, it reduces energy transfer to the first mode, thereby delaying streak formation—a fundamentally different mechanism from single-mode control reported by Tullio *et al.* [181] and Hader *et al.* [161]; [162]. Moreover, the coupled DNS-TDIBC framework provides the first first-principles explanation for experimentally observed higher skin friction [158]; [159] on the metasurface by linking it to enhanced combination resonance or shear-induced dissipation, while the derived design guidelines offer solutions for achieving transition delay without late-stage penalties.

While natural transition typically involves broadband wave packets with random spatiotemporal distributions, our study employs controlled monochromatic wave excitation to isolate and elucidate the combination resonance control mechanism. As the identified resonance mechanism—being independent of absolute initial amplitude or their ratios—suggests robustness against disturbance variations [8], the conclusions derived in this study should have broader implications for natural transition control using the acoustic metasurface. The present nonlinear interaction scenario may constitute a building block in a natural transition with a wide and continuous frequency range, i.e. with significantly increased Fourier components. However, the presence of intermittent turbulent spots or wave packets may further complicate or add to the interaction mechanisms.

6.8 Appendix

6.8.1 Poles and residues for pole base functions

Table 6.3 lists 20 pairs of dimensionless poles and residues employed to calculate softness using equation (6.5), with $C_0 = -0.05$. The reference length is 1 m. The resulting softness, dependent

Pair number (k)	Poles (p_k)	Residues (μ_k)
1	(-1504294.553, 4810.197)	(677409.898, -158275.251)
2	(-466.785, 52.763)	(1161.854, 4177.756)
3	(-419.365, 1007.668)	(596.328, 365.519)
4	(-517.687, 1882.378)	(1153.962, -317.971)
5	(-513.669, 2834.064)	(-119.973, -572.596)
6	(-1032.603, 2120.59)	(-2407.724, 3249.423)
7	(-445.944, 3554.34)	(64.41, -212.655)
8	(-75, 14.263)	(-489.092, -992.871)
9	(-1443.028, 5837.405)	(-78.771, 5.575)
10	(-75.01, 34.641)	(387.44, 191.596)
11	(-173.145, 4272.261)	(-19.664, 22.645)
12	(-107.785, 4217.54)	(9.436, 28.795)
13	(-102.821, 4091.752)	(-90.437, -39.358)
14	(-75, 4102.434)	(-69.528, -19.905)
15	(-78.478, 4165.123)	(19.763, 22.55)
16	(-138.227, 4111.577)	(3.371, -12.674)
17	(-75, 4122.214)	(26.314, 2.057)
18	(-75.867, 4118.516)	(36.017, 4.191)
19	(-85.535, 4099.064)	(49.002, 2.039)
20	(-81.259, 4095.32)	(61.568, 1.498)

TABLE 6.3: Conjugate pairs of dimensionless poles and residues.

on the frequency, is illustrated in figure 6.1.

6.8.2 Verification of TDIBC code

The contours of pressure and wall-normal velocity fluctuation are compared between the results of the meshed metasurface simulation and the TDIBC for broadband disturbance in figure 6.20. The parameters for broadband disturbance are the same as those in Guo *et al.* [102]. It is indicated that the TDIBC embedded solver can reproduce nearly the same flow structure and amplitude (see figure 6.21(a)), thereby serving as an efficient tool in studying the effect of the metasurface for broadband disturbances, avoiding directly meshing the cavity and thus reducing the computational cost. For three-dimensional simulations with oblique waves, the result between a constant admittance model ($v' = Ap'$) and TDIBC for oblique wave (3.8, 1.25) is plotted in figure 6.21(b), where the admittance $A = (-17.041809, -2.57 \times 10^{-4})$. Mode (3.8, 1.25) is representative as it approaches the peak of the oblique wave in the gain contour of figure 6.2(a). Note that the constant admittance model has been verified in previous research and utilized for simulation with single-frequency disturbance [176, 185]. The good agreement indicates the

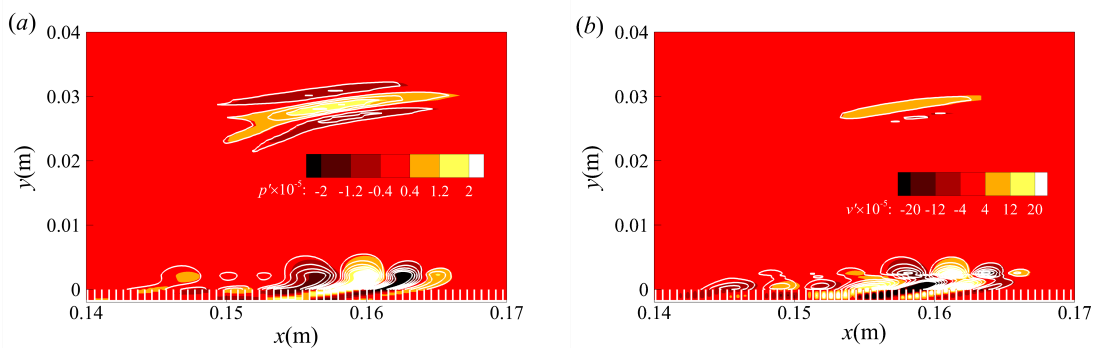


FIGURE 6.20: Comparison of (a) dimensionless fluctuation pressure and (b) fluctuation wall-normal velocity field between results using meshed cavity (contour) and modelled TDIBC (solid line) subject to broadband disturbances.

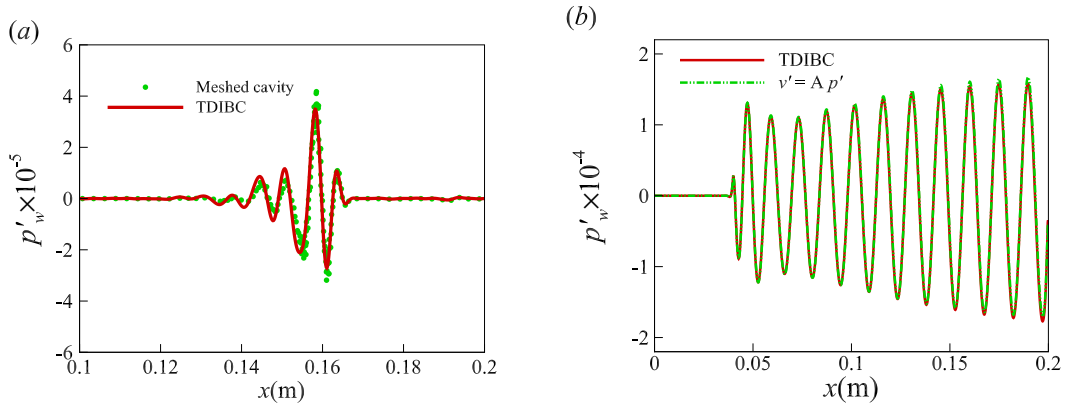


FIGURE 6.21: Comparison of dimensionless wall fluctuation pressure between results (a) using meshed cavity and modelled TDIBC subject to broadband disturbances, and (b) between results of using $v' = A p'$ and TDIBC for oblique wave (3.8, 1.25).

accuracy of TDIBC in modeling the effect of the acoustic metasurface on oblique waves in three-dimensional simulations.

6.8.3 Verification of initialisation of OpenCFD

The evolutions of optimal responses between the result of resolvent analysis and N-S solver agree well with each other in the fluctuation density and N -factor integral of Chu energy density, as shown in figure 6.22.

6.8.4 The resonate state

Resonance occurs when the wavevector triad $(\alpha_r, \beta, \omega)$ satisfies the closure condition [164]:

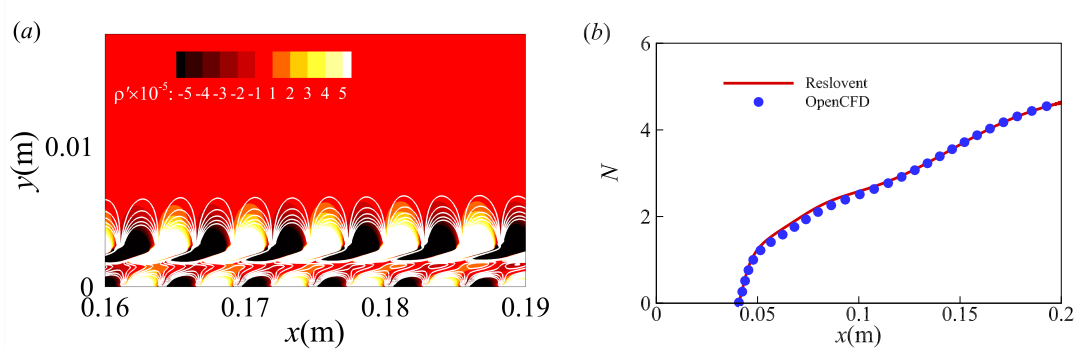


FIGURE 6.22: Comparison of (a) dimensionless fluctuation density and (b) N -factor between the result of resolvent analysis (contour) and OpenCFD (solid line) for optimal mode (10, 0). No acoustic metasurface is applied.

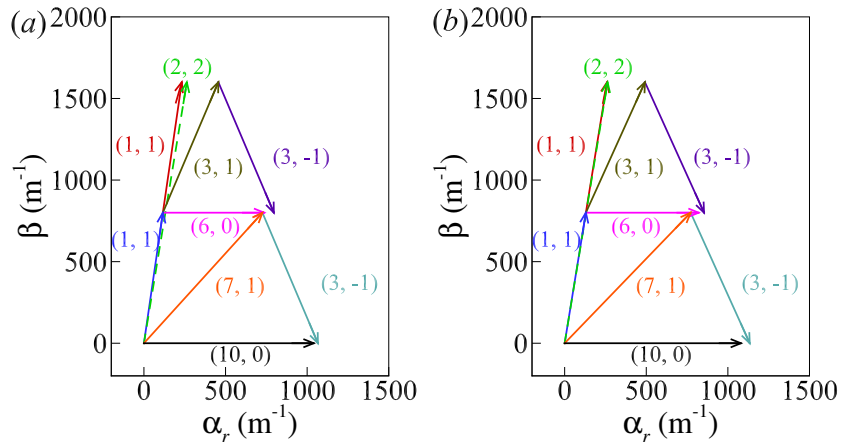


FIGURE 6.23: The mode vectors (α_r, β) in equation (6.24) at $x = 0.27$ m for (a) Case 1 and (b) Case 2.

$$\chi_1 + \chi_2 = \chi_3, \chi = \alpha_r, \beta, \omega. \quad (6.34)$$

The streamwise wavenumber is calculated by $\alpha_{r,(m,n)} = \partial \theta_{(m,n)} / \partial x$, and $\theta_{(m,n)}$ denotes the phase angle of the wall pressure for mode (m, n) using DNS data. Figure 6.23 demonstrates wave vectors of the four interactions in equation (6.24) at $x = 0.27$ m for Case 1 and Case 2. The closed triangle is nearly satisfied and the resonance condition for the generation of detuned mode (2, 2) is generally fulfilled at $x = 0.27$ m for both Case 1 and Case 2.

Chapter 7

Conclusions

7.1 Summary and conclusions

This thesis investigates the energy sources and potential control strategies for instabilities within boundary layers, encompassing the T–S mode in incompressible flow, the oblique first mode in supersonic flow, and the planar second and supersonic modes in hypersonic flow. The energy analysis methods employed include relative phase analysis derived from internal energy analysis, momentum potential theory based on the transport of total fluctuating enthalpy, and Lagrangian equations rooted in Chu’s energy density. Comparative studies of the source terms derived from three influential theoretical approaches are conducted, and the relationships between these approaches are thoroughly discussed and clarified. Instability analysis tools, including linear stability theory, parabolized stability equations, and resolvent analysis, are employed alongside direct numerical simulations and examined in conjunction. Specific control strategies are examined, particularly the effects of wall cooling, Mach number, bluntness, obliqueness, and the acoustic metasurface. Both nonmodal (Chapters 5-6) and modal (Chapters 3-6) disturbances are discussed. Regarding nonlinear processes, the transition induced by the combined effects of the low-frequency first mode and the high-frequency second mode is investigated in the presence of the acoustic metasurface (Chapter 6).

As outlined in Chapter 1, this study focuses on boundary-layer instabilities and potential control strategies. Building on this objective, four sub-studies are formulated. Chapters 3-6 provide detailed discussions on the findings and conclusions derived from each sub-study conducted within the scope of this thesis. This chapter consolidates and reiterates these key findings in the

context of the original aims of this thesis. The aims of this thesis, as originally stated in Chapter 1, along with the corresponding key findings, are summarized as follows:

Aim 1: Investigate the underlying physics governing the growth of second modes in hypersonic boundary layers by comparing the formulations and physical interpretations of dominant source terms in relative phase analysis, momentum potential theory, and Lagrangian equations.

- A unified indicator is established to quantitatively evaluate the contributions of significant source terms. The primary finding is that the three widely recognized fundamental approaches converge in their explanation of the energy amplification mechanism associated with the Mack second mode. The explicit expressions for each energy source are derived, and despite differences in their mathematical forms, the underlying physical interpretations remain consistent. In general, the dilatation effect resulting from compressibility drives acoustic energy production near the wall, whereas the base temperature gradient plays a critical role in the wall-normal transport of internal energy within the outer region of the boundary layer.

Aim 2: Investigate the dominant energy source terms for the T-S mode, oblique first mode, second mode, and supersonic mode. Evaluate the effects of Mach number, wall temperature, obliqueness, and the acoustic metasurface on the dominant energy source terms.

- The energy source terms associated with unstable modes in flat-plate boundary layers are investigated using relative phase analysis across a range of Mach numbers and wall temperature ratios. It is found that the increase in Reynolds shear stress coincides with the distinct emergence of obliqueness in the most unstable first mode. In contrast, the reduction in Reynolds thermal stress inhibits the oblique second mode from reaching the most unstable state. Wall cooling stabilizes the oblique first mode by causing Reynolds thermal stress and dilatation fluctuations to become out of phase with internal energy fluctuations. It destabilizes the second mode through the formation of a newly pronounced region of wall-normal internal energy transport below the second generalized inflection point. Conversely, the porous coating stabilizes the second mode similarly to wall heating, while destabilizing the oblique first mode by significantly enhancing mean shear production. Moreover, the relatively weak supersonic mode is characterized by phase cancellation in wall-normal transport near the critical layer, resulting in a minimal contribution to internal energy growth.

Aim 3: Investigate the optimal responses and growth patterns in hypersonic blunt-wedge flows. Examine the underlying mechanisms responsible for "transition reversal".

- Two competing growth patterns of travelling waves are identified, appearing within the boundary layer and the entropy layer, respectively. Their behaviour is characterized by the intersection of energy gains from optimal and sub-optimal disturbances. Furthermore, we provide a detailed discussion of the characteristics of optimal disturbances and control strategies for mitigating the amplification of modal and nonmodal instabilities in hypersonic boundary layers over blunt-tip models. Additionally, a potential explanation for "transition reversal" is proposed by integrating receptivity to slow acoustic waves with resolvent analysis. The amplitude of nonmodal oblique waves within the boundary layer increases with larger nose bluntness. The amplified nonmodal oblique waves in the upstream region exhibit equivalent amplitude with that of the first mode further downstream. The enhanced receptivity of oblique waves with increasing nose radius may contribute to an upstream movement in the transition location.

Aim 4: Investigate the influence of the acoustic metasurface on nonlinear mode–mode interactions and the resulting breakdown process, with the primary objective of elucidating the mechanisms underlying transition delay by the acoustic metasurface subject to multiple primary instabilities.

- For the first time, the mode–mode interactions and the resulting transition process involving the acoustic metasurface have been elucidated. Our direct numerical simulations demonstrate that the transition, jointly induced by the first and second modes, is effectively delayed by the metasurface. Vortex visualization reveals that the spanwise-aligned structure associated with the second mode vanishes upon employing the acoustic metasurface. Furthermore, the staggered and hairpin structures, as well as the streamwise streaks, are postponed in the presence of the metasurface. In contrast to the solid-wall case, where fully developed turbulence is observed, the turbulence regime remains unestablished when the acoustic metasurface is utilized. The results demonstrate that the placement of the acoustic metasurface is critical for delaying transition, with positioning within the unstable region of the second mode proving particularly effective for transition control. However, during the late transitional stage, the metasurface induces an undesirable increase in skin friction overshoot, attributed to enhanced shear-induced dissipation caused by detuned modes from combination resonance. By restricting the metasurface placement upstream of the overshoot region, this adverse effect can be mitigated. Thus, optimal positioning of the metasurface is essential to suppress early instabilities while minimizing detrimental impacts in the late transitional stage.

7.2 Possible directions for future work

The findings presented in this thesis suggest several promising directions for future research:

I. Resolvent analysis incorporating receptivity mechanisms:

- The current resolvent analysis method fails to account for the generation of disturbances, as it does not incorporate any initial bias toward specific spanwise wavenumbers or frequencies. This limitation renders the approach inconsistent with realistic free-stream conditions. For example, in wind tunnel experiments, the intensity of free-stream disturbances demonstrates a decreasing trend as frequency increases. Resolvent analysis that incorporates the spectral characteristics of free-stream disturbances under realistic conditions can significantly improve the authenticity and practical applicability of interpreting wind tunnel scenarios.

II. Nonlinear evolution and interaction of optimal disturbances in hypersonic blunt-wedge boundary layers:

- As discussed in Chapter 5, resolvent analysis effectively captures the linear evolution of optimal disturbances. However, focusing solely on the receptivity and evolution of single-frequency disturbances is insufficient to fully understand the phenomenon of "transition reversal". The nonlinear evolution and potential interactions between different growth patterns may emerge downstream, ultimately triggering the transition. Therefore, incorporating broadband disturbances and their nonlinear interactions is essential to deepen the understanding of "transition reversal".

III. Possible control strategies for nonmodal disturbances:

- Chapter 5 of this thesis identifies two growth patterns of nonmodal nature. The pattern distributed within the entropy layer may remain unaffected by wall-boundary manipulations, such as wall cooling or acoustic metasurfaces. A potential control strategy involves modifying the geometry of the leading edge. The off-body bow shock can be altered by adjusting the leading-edge geometry. Consequently, the intensity of disturbances within the entropy layer, induced by the bow shock, can also be modified.

IV. Transition control using the acoustic metasurface subject to broadband/stochastic freestream disturbances:

- In Chapter 6, the transition is triggered by the optimal planar wave and a pair of optimal oblique waves, derived from resolvent analysis. The objective is to elucidate the role of the acoustic metasurface in the nonlinear mode–mode interaction process. However, in realistic wind tunnel experiments, free-stream disturbances exhibit a broadband nature. Therefore, future work should incorporate a realistic forcing environment to enable meaningful comparisons between numerical simulations and wind tunnel experiments, thereby bridging the gap between theoretical design and engineering applications.

Publications

The following items show the conference papers and journal papers published or accepted during my Ph.D. program.

Journal paper:

Y. Chen, P. Guo and C. Y. Wen (2023). A unified explanation of energy growth sources for unstable modes in flat-plate boundary layers. *Journal of Fluid Mechanics*, 972, A5.

Y. Chen, P. Guo and C. Y. Wen (2023). Consistent energy-based framework of amplification mechanisms for the second mode in hypersonic boundary layers. *Physics of Fluids*, 35(12), 124107.

Y. Chen, T. Ma, P. Guo, J. Hao and C. Y. Wen (2025). Optimal disturbances and growth patterns in hypersonic blunt-wedge flow. *Chinese Journal of Aeronautics*, 36(1), 016115. (**Highlighted Article**)

Y. Chen, P. Guo and C. Y. Wen. Role of acoustic metasurface in the nonlinear mode–mode interaction and breakdown of hypersonic boundary layer. *Journal of Fluid Mechanics* (Accepted).

Conference paper:

Y. Chen, P. Guo and C. Y. Wen (2023). A unified explanation of the growth mechanism of the second mode over hypersonic boundary layer. *Conference on PRSC2023*, Hong Kong, 2043.

Y. Chen, P. Guo, J. Hao and C. Y. Wen (2024). Characteristics of optimal disturbances for hypersonic flows over blunt wedges. *12th International Conference on Computational Fluid Dynamics (ICCFD12)*, Kobe, C000204.

Y. Chen, P. Guo and C. Y. Wen (2025). Hypersonic boundary-layer transition control using acoustic metasurface. *The 35th International Symposium on Shock Waves (ISSW35)*, Brisbane, 053.

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