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**BEHAVIOUR AND MODELLING OF
EARLY-AGE 3D-PRINTED
CONCRETE**

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PhD

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The Hong Kong Polytechnic University

Department of Civil and Environmental Engineering

**BEHAVIOUR AND MODELLING OF
EARLY-AGE 3D-PRINTED
CONCRETE**

RUI LONG LI

**A Thesis Submitted in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy**

April 2025

CERTIFICATE OF ORIGINALITY

I hereby declare that this thesis is my own work and that, to the best of my knowledge and belief, it reproduces no material previously published or written, nor material that has been accepted for the award of any other degree or diploma, except where due acknowledgement has been made in the text.

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Rui-Long LI (Name of student)

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I declare that Generative AI tools have been used to assist in the preparation of this thesis. The tools were used exclusively for revising and providing feedback on my drafts. The specific tools and their respective uses are as follows:

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- English grammar verification
- Programming code syntax verification

Anthropic Claude

- English grammar verification
- British English conventions and usage checking
- Assessment of sentence structure and logical flow
- Recommendations for alternative vocabulary
- Programming code syntax verification

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To My Family

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ABSTRACT

The advent of extrusion-based 3D concrete printing technology marks a paradigm shift in construction methodologies, opening avenues for automated construction, on-demand customisation, and the ambitious goal of extraterrestrial constructions. At the heart of this technology is the printing stage, during which soft, fresh concrete is deposited layer-by-layer to form a temporarily weak structure that gradually hardens as concrete cures over several days. Existing modelling approaches for fresh concrete structures have yet to produce reliable predictive models, as significant discrepancies persist between experimental results and predictions. These inconsistencies stem from a lack of understanding of the mechanical behaviour of fresh 3D-printed concrete (3DPC).

To address these knowledge gaps, this thesis aims to develop a comprehensive understanding of the mechanical behaviour of early-age 3DPC and its implications for fresh concrete structures during construction. The research programme integrates three key components: experimental studies, theoretical developments, and numerical simulations to achieve this objective.

Through a series of pioneering experimental studies, the mechanical behaviour of fresh 3DPC was systematically characterised. The material behaviour under biaxial compression, arguably the most pertinent stress condition for printed material during construction, was investigated. This comprehensive experimental study spanned concrete ages ranging from a few minutes to six hours and included biaxial stress ratios of 0 (uniaxial compression), 0.33, 0.5, 0.7, and 1 (equibiaxial compression). It addressed a significant gap in existing literature, which had primarily focused on uniaxial compression and shear testing. Moreover, innovative uniaxial compression tests were designed to investigate the material's elasticity, viscous behaviour (e.g., creep, stress relaxation), and healing properties. In particular, the cyclic compression tests revealed that the actual elastic modulus of fresh 3DPC is 5 to 20 times higher than previously reported values. Multi-step creep-recovery experiments established the predominantly nonlinear and non-recoverable nature of creep deformation, whilst material healing tests provided groundbreaking evidence that equivalent plastic strain

can decrease due to ongoing hydration - a phenomenon that fundamentally challenges conventional plasticity theory.

Analysis of the biaxial compression test data demonstrated that the widely used von Mises and Mohr-Coulomb criteria inadequately represent the strength behaviour of fresh 3DPC. In response, two complementary strength models were developed: a simplified model for rapid assessment of biaxial-to-uniaxial strength ratios at different ages, and a comprehensive multi-axial strength model. To capture the complete stress-strain response, an elastoplastic model was developed, capable of accurately simulating the uniaxial and biaxial compressive behaviour of fresh 3DPC across different ages. The framework was further enhanced through viscoplasticity theory to incorporate material creep effects, enabling the accurate simulation of nonlinear creep behaviour that conventional linear viscoelastic approaches failed to capture.

Finally, a series of finite element (FE) simulations connected the enhanced material-level knowledge to the structural-level response of printed structures during construction. FE models incorporating the proposed elastoplastic model outperformed existing numerical models, revealing two distinct instability failure mechanisms: elastic buckling and plastic buckling - both previously misidentified due to the adoption of inappropriate material models. Moreover, the research highlighted the critical role of complex interactions between material ageing, creep, and printing speed on the mechanics of printed structures. This work not only provides deeper insights into the structural responses during concrete printing but also establishes a reliable framework for toolpath and printing parameter optimisation, advancing both the theoretical understanding and practical applications of 3D concrete printing technology.

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Be curious, be determined, overcome the odds.

- Stephen Hawking

From childhood, I have been endlessly curious about how things operate, always seeking to understand the underlying mechanisms of the world around me. This unwavering curiosity ultimately led me to embark on my Ph.D. journey. When I began this endeavour five years ago, I could not have anticipated the profound challenges that lay ahead, yet the experience has been deeply rewarding.

Throughout my research journey, I have been fortunate to receive tremendous support, inspiration and encouragement from a remarkable group of individuals. As I complete this thesis - a pivotal milestone in my academic journey - I wish to convey my deepest gratitude to them.

Pursuing a Ph.D. is not solely about completing a research project; more importantly, it is about developing the mindset and discipline required for conducting research; that is, the *philosophy of research*. Therefore, I would like to begin by expressing my heartfelt gratitude to my advisors, who played pivotal roles in shaping my research philosophy.

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LIST OF ACRONYMS

2D	Two-dimensional
3D	Three-dimensional
3DPC	Three-dimensional printed concrete
a/b	Aggregate-to-binder ratio
BC	Biaxial compression or biaxial compressive
C-S-H	Calcium silicate hydrate
FE	Finite element
GHL	Generalised Hooke's law
NLF	Number of layers at failure of a 3D-printed concrete structure during construction
UC	Uniaxial compression or biaxial compressive
w/b	Water-to-binder ratio

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NOTATION

$\mathbf{C}$	Cauchy elasticity tensor (a fourth order tensor with its component given as C_{ijkl})
$\mathbf{C}^t$	Tangent elasticity tensor (a fourth order tensor with its component given as C_{ijkl}^t)
E	Young's modulus
E_{ap}	Apparent Young's modulus derived from uniaxial tests
E_{ci}	Young's modulus of fresh 3D-printed concrete at age zero minutes
E_0	A parameter representing the initial gradient of the normalised yield stress-normalised equivalent plastic strain curve
F	Strength function
f	Yield function
f_{co}	Uniaxial compressive strength
f_{ci}	Uniaxial compressive strength of fresh 3D-printed concrete at age zero minutes
f_{bc}	Biaxial compressive strength
f_{ebc}	Equibiaxial compressive strength
g	Flow potential function
h	Hardening component of the strain hardening function

h_0	Proportional limit
J	Creep compliance
q	Strain hardening function
s	Softening component of the strain hardening function
α	Volumetric dilation parameter
α_k	Volumetric dilation parameter at ultimate yield stress (strength)
α_k^{uc}	Volumetric dilation parameter at ultimate yield stress under uniaxial compression
α_k^*	A parameter accounting for the influence of biaxial confining stress on the volumetric dilation parameter at ultimate yield stress
α_0	Volumetric dilation parameter at initial yield
β_s	Biaxial stress ratio
ϵ	Strain tensor (a second order tensor, with its component given as ϵ_{ij})
ϵ^e	Elastic strain tensor
ϵ^p	Plastic strain tensor
θ	The lode angle, one of the three Haigh-Westergaard coordinates
κ	Equivalent plastic strain
κ_k	Equivalent plastic strain at ultimate yield stress (strength)

κ_k^{uc}	Equivalent plastic strain at ultimate yield stress under uniaxial compression
κ_k^*	A parameter accounting for the influence of biaxial confining stress on the equivalent plastic strain at ultimate yield stress
ξ	The hydrostatic length, one of the three Haigh-Westergaard coordinates
ρ	The deviatoric length, one of the three Haigh-Westergaard coordinates
σ	Stress tensor (a second order tensor, with its component given as σ_{ij})
σ_l	Lateral (hydrostatic) confinement stress (in case of triaxial compression)
τ	Shear stress
τ_s	Shear strength
Y modulus	A conversion factor between the apparent and the actual Young's modulus
ϕ	Mohr-Coulomb friction angle

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

The advent of extrusion-based 3D concrete printing (Khoshnevis, 2004) represents a significant advancement in both construction methodology and design capabilities. Compared to traditional concrete casting, which is highly labour intensive and time-consuming, this new approach has the potential to substantially reduce labour costs while increasing productivity (Mohan *et al.*, 2021a; Wangler *et al.*, 2019). Furthermore, 3D concrete printing eliminates the need for conventional formworks, offering unprecedented geometric flexibility in construction. This geometric freedom enables the fabrication of topologically optimised structures (Li *et al.*, 2024), allowing designers to maximise structural efficiency while minimising material usage. The practical viability and potential for wider adoption of this technology have been demonstrated through several successful small-scale construction projects, as illustrated in Figure 1.1.

Despite its considerable advantages, this novel construction approach presents distinctive challenges in structural engineering. During the construction stage, a 3D concrete-printed structure is fabricated by sequential deposition of layers of fresh concrete filaments along predetermined toolpaths without formwork support. In the absence of formwork, the fresh 3D-printed concrete (3DPC) must support its own weight. However, the initial strength and stiffness of 3DPC are quite low, making the structure highly prone to collapse during printing, as evidenced by prior research (Wolfs *et al.*, 2019; Jayathilakage *et al.*, 2020; Suiker *et al.*, 2020) and depicted in Figure 1.3. Consequently, a 3D concrete-printed structure requires structural analyses for both the construction stage when the concrete is in its fresh state and the post-construction stage after the concrete is cured. This design philosophy is markedly different from that of conventional construction practice, where structural assessment typically focuses solely on the hardened state.

Whereas significant research progress has been made in areas including 3DPC mixture designs (Arunothayan *et al.*, 2020; Zhang *et al.*, 2021), pumping and extrusion of 3DPC (Jayathilakage *et al.*, 2021; Mohan *et al.*, 2021b) and structural analysis of hardened elements (Van Den Heever *et al.*, 2021; Yan *et al.*, 2024; Li *et al.*, 2024), research into the structural analysis of fresh concrete during construction, despite numerous efforts, remains comparatively limited and has yet to yield reliable predictive models (Chang *et al.*, 2023a; Wolfs, 2024). As a result, most 3D concrete printing projects still rely on trial-and-error approaches (Wolfs, 2024), leading to material waste and impeding the widespread adoption of this technology in construction.

1.2 STRUCTURAL ANALYSIS OF 3D-PRINTED STRUCTURES DURING CONSTRUCTION

The structural analysis of fresh 3D-printed concrete structures presents significant technical challenges (Kamhawi and Aghaei Meibodi, 2024; Wolfs, 2024), stemming from multiple interrelated factors. Firstly, both the structural geometry and loading conditions evolve continuously throughout the construction process. Secondly, the mechanical properties of 3DPC are highly age-dependent due to the ongoing hydration. These characteristics fundamentally distinguish the analysis from that of hardened concrete structures, where both geometry and material properties remain constant. The complexity is further compounded by the influence of printing parameters, such as printing speed and toolpath design. Consequently, comprehensive structural analysis of fresh concrete structures demands careful consideration of these interdependent factors.

Existing approaches for analysing the mechanical behaviour of fresh 3D-printed concrete structures can be broadly categorised into analytical models and numerical simulations (Y. Chen *et al.*, 2024). Analytical models (Suiker, 2018, 2022) idealise structures as fundamental elements such as columns, plates, or shells, making them particularly suitable for parametric studies and structures with simple geometries. Conversely, numerical simulations (Wolfs *et al.*, 2018; Chang *et al.*, 2021; Wijaya *et al.*, 2024) offer greater versatility in handling complex geometries and can effectively account for the intricate interplay between evolving geometry, time-dependent material properties, and process parameters.

Through analytical and numerical approaches, researchers have successfully identified and verified two primary failure mechanisms: structural instability and material failure (Wolfs *et al.*, 2019; Suiker *et al.*, 2020). For certain structural configurations, these models have demonstrated good accuracy in predicting failure modes and the associated conditions (Wolfs *et al.*, 2019; Suiker *et al.*, 2020; An *et al.*, 2024).

However, significant limitations in their predictive capabilities have been reported in several studies (Wolfs *et al.*, 2019; Ooms *et al.*, 2021; Liu *et al.*, 2022; B. Chen *et al.*, 2024). For instance, the error in predicting the number of layers at failure can exceed 40% in some cases. Furthermore, existing models have shown limited success in accurately simulating structural deformations during the printing process (Wolfs *et al.*, 2018; Liu *et al.*, 2022).

The accuracy of both analytical and numerical approaches critically depends on the fidelity of the constitutive models of fresh 3DPC. The limitations of existing structural analysis approaches suggest fundamental gaps in existing understanding of the mechanical response of 3DPC during printing.

1.3 MECHANICAL BEHAVIOUR OF EARLY-AGE 3D-PRINTED CONCRETE

During 3D concrete printing, fresh 3DPC transitions from a fluid-state during mixing, pumping, and extrusion to a static-state after deposition when the material is at rest (Wolfs *et al.*, 2018; Kamhawi and Aghaei Meibodi, 2024). Therefore, the mechanical behaviour of static-state 3DPC is pertinent to the response of fresh concrete structures during construction.

Research on the mechanical behaviour of fresh 3DPC can be categorised into rheology-based and solid mechanics-based studies (Mechtcherine *et al.*, 2022). Rheology-based studies examine the flow behaviour of fluid-state concrete, while solid mechanics-based studies investigate the stress-strain behaviour of static-state concrete. Historically, extensive rheological-based studies have investigated the behaviour of fluid-state concrete (Kaplan *et al.*, 2005; Roussel, 2011; Mohan *et al.*, 2021b), driven by the

requirements of concrete mixing and pumping in conventional concrete casting. In contrast, the static-state behaviour of fresh concrete, analysed within the solid mechanics framework (Wolfs *et al.*, 2018; Pott *et al.*, 2022; Chang *et al.*, 2023b), has gained prominence with the emergence of 3D concrete printing.

The mechanical response of static-state 3DPC has been investigated under various loading conditions, including compression, shear and tension. These pioneering studies have revealed that the material behaviour is characterised by three key dependencies: age (Wolfs *et al.*, 2018; Pott *et al.*, 2023), stress state (Mettler *et al.*, 2015; Wolfs *et al.*, 2019; Jacquet *et al.*, 2021), and strain rate (Casagrande *et al.*, 2020; Kolawole *et al.*, 2021; Xia *et al.*, 2024).

Building upon these findings, many constitutive models have been developed. The majority of these models adopt a strain rate-independent elastoplastic framework (Wolfs *et al.*, 2018; Jayathilakage *et al.*, 2020; Ooms *et al.*, 2021; Liu *et al.*, 2022; Nguyen-Van *et al.*, 2022; Wijaya *et al.*, 2022; An *et al.*, 2024), while a few incorporate strain-rate-dependent theory (Esposito *et al.*, 2021; Reinold *et al.*, 2022; Chang *et al.*, 2023b; Xia *et al.*, 2024). These models have demonstrated some success in capturing specific aspects of 3DPC material behaviour.

Despite the contributions of these pioneering works, several fundamental limitations exist in the current understanding of fresh 3DPC behaviour. These limitations are attributable to the underdevelopment of both experimental programmes and constitutive models.

Firstly, it is questionable whether the stress states examined in existing material tests are representative of those encountered during 3D concrete printing. During construction, as shown in Figure 1.4, fresh 3DPC experiences vertical compressive stress from gravity. While the material can freely expand in the filament width direction, causing visible bulging, its expansion along the printing direction is restrained by adjacent material, resulting in significant confining stress. This indicates that fresh 3DPC is primarily under biaxial compression during construction (Kruger *et al.*, 2020). However, biaxial compression tests for fresh concrete within the typical printing

timeframe (minutes to hours after deposition) are notably absent from the literature. Consequently, existing constitutive models, though developed from uniaxial and triaxial compression, shear, and tension data, remain unverified under the most relevant loading condition - biaxial compression.

Secondly, experimental investigations into the strain-rate-dependent behaviour of 3DPC have yielded contradictory results, as will be detailed in Chapter 2. Furthermore, existing strain-rate-dependent constitutive models for fresh 3DPC remains incomprehensive, particularly in their ability to capture complex viscous phenomena, such as nonlinear creep, and plasticity behaviour within a consistent framework.

Thirdly, the current understanding of the elasticity behaviour of fresh concrete is incomplete. As will be demonstrated in Chapter 2, significant discrepancies exist between experimental measurements and model predictions of elastic deformation. This limitation primarily stems from insufficient systematic investigation into the relative contributions of elastic and plastic deformations in fresh concrete.

Finally, during construction, fresh 3DPC simultaneously experiences two competing effects: a damaging effect due to gravitational loading and a material healing effect due to material ageing. The interaction between these two effects remains unexplored, despite their potentially critical role in the structural response during construction.

1.4 AIMS, SCOPE AND LAYOUT OF THIS THESIS

Based on the identified research gaps, this Ph.D. research programme aims to develop a comprehensive understanding of the mechanical behaviour of 3DPC and establish a reliable numerical simulation framework for analysing structural responses during the construction phase. This research programme encompasses experimental investigations, constitutive modelling, and numerical simulations to address the following objectives:

- (1) To characterise the mechanical behaviour of fresh 3DPC under biaxial compression, representing the primary stress condition during construction;

- (2) To understand the elastic behaviour of fresh 3DPC and determine the relative contributions of elastic and plastic deformations;
- (3) To investigate the strain-rate-dependent phenomena, including creep, stress relaxation, and rate-dependent stress-strain relationships;
- (4) To examine the influence of material healing on the mechanical properties of fresh 3DPC;
- (5) To develop constitutive models that incorporate the aforementioned material characteristics;
- (6) To implement numerical models that integrate these material characteristics for analysing fresh concrete structures during construction.

This thesis comprises eight chapters, structured as follows:

Chapter 1 introduces the research background and establishes the motivations underpinning this Ph.D. research project.

Chapter 2 presents a comprehensive literature review examining the current understanding of fresh concrete's mechanical behaviour. The chapter begins by establishing fundamental concepts in material constitutive modelling, thereby providing the theoretical foundation for subsequent discussions. Previous experimental works into the mechanical behaviour of fresh concrete are critically analysed. Furthermore, the chapter systematically evaluates existing material constitutive models, encompassing both strain-rate-dependent and independent frameworks, whilst identifying their theoretical constraints and practical limitations. Through this analysis, specific research needs pertaining to the mechanical characterisation and modelling of fresh 3DPC are identified.

Chapters 3 to 5 are concerned with the mechanical behaviour of 3DPC under biaxial compression. All analyses are performed within the framework of strain-rate-independent elastoplasticity.

Chapter 3 details the development of an extensive experimental study designed to characterise the mechanical response of 3DPC under uniaxial and biaxial compression.

An innovative biaxial compression testing apparatus is proposed and verified through finite element simulations. The concrete ages span from tens of minutes to six hours, corresponding to typical printing durations in 3D concrete construction. This pioneering study examines the BC behaviour under both proportional and non-proportional loading conditions to elucidate the stress path-dependency of mechanical responses. The experimental results and their analyses are presented in subsequent Chapters 4 and 5.

Chapter 4 examines the biaxial compressive strength characteristics of fresh 3DPC, building upon the experimental study outlined in Chapter 3. The investigation begins with a detailed analysis of failure modes and stress-strain relationships. Comprehensive strength analyses are conducted to establish BC strength envelopes during the hydration process. Existing strength models from the literature are evaluated against the experimental data, leading to the development and validation of refined strength models for predicting BC strength.

Chapter 5 extends the investigation from strength analysis to a detailed examination of stress-strain responses under biaxial compression. The chapter presents and analyses experimental data describing the relationships between stress and various strain components (axial, lateral, and volumetric). Detailed analyses focus on three key aspects: the evolution of material stiffness, plastic dilation behaviour, and hardening-softening characteristics. Based on these findings, a multiaxial elastoplastic model incorporating a non-associated flow rule is proposed and validated against the experimental stress-strain data from the BC tests.

Chapter 6 presents a series of innovative uniaxial compression experiments designed to characterise the strain-rate-dependent behaviour, elasticity, and material healing phenomena of fresh 3D-printed concrete (3DPC). The experimental study encompasses several distinct investigations: variable displacement rate tests to examine strain-rate-dependent stress-strain relationships; stress relaxation tests at multiple strain levels to quantify time-dependent stress decay; and multi-step creep-recovery tests to elucidate both nonlinear creep behaviour and the relative contributions of recoverable and non-recoverable creep deformations. The elasticity behaviour of fresh 3DPC is studied by the loading-unloading-reloading tests. The material healing phenomena are examined

through comparative analyses of virgin and reloaded specimens. These comprehensive investigations yield fundamental insights into the complex mechanical responses of fresh 3DPC.

Chapter 7 integrates the advanced understanding of fresh 3DPC material behaviour into structural-level analyses through FE simulations of benchmark structures. The numerical investigation comprises two distinct simulation groups, each implementing a different constitutive model. The first group employs the elastoplastic model developed in Chapter 5 to validate its efficacy in structural-level analyses. The simulation results demonstrate that the proposed material model yields superior performance compared to previous FE models based on rate-independent frameworks. The second group investigates the influence of material creep on structural response through an advanced elastoviscoplastic model, which extends the elastoplastic framework established in Chapter 5. This enhanced model represents a significant advancement over existing strain-rate-dependent constitutive models by incorporating both viscous and plastic behaviours within a unified theoretical framework. The incorporation of creep phenomena enables accurate simulation of both deformation patterns and failure mechanisms in benchmark structures. Furthermore, the combined results from both simulation groups provide critical insights into fresh concrete structural behaviour that were not previously identified in the literature. These findings enhance the understanding of structural response during the critical early-age period of 3D concrete printing.

Chapter 8 concludes the thesis by presenting a comprehensive synthesis of key findings from the preceding chapters and establishing directions for future research.

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Figure 1.1 Examples of construction projects utilising the extrusion-based 3D concrete printing technology: (a) 3D-printed bicycle bridge at the Eindhoven University of Technology [extracted from Salet *et al.* (2018)]; (b) 3D-printed IT server hotel by COBOD [extracted from Symonenko (2024)]; (c) 3D-printed Chinese courtyard by WinSun Limited [extracted from WinSun (2015)].



Figure 1.2 A topologically optimised reinforced 3D-printed concrete bridge, where the concrete was designed to be in pure compression [extracted from Li *et al.* (2024)].



Figure 1.3 Failure of 3D-printed structures during construction: (a) A free-standing wall [extracted from Wolfs *et al.* (2019)]; (b) A cylindrical wall [extracted from Jayathilakage *et al.* (2020)]; (c) A rectangular wall [extracted from Suiker *et al.* (2020)].

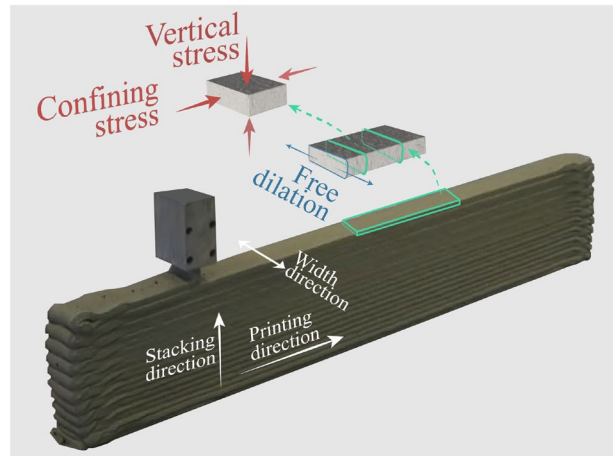


Figure 1.4 Schematic of the biaxial compression of 3DPC during construction
[modified from Wolfs *et al.* (2019)].

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CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter reviews the state-of-the-art of the experimental and theoretical research on the mechanical behaviour of fresh concrete, with a primary focus on its compressive behaviour. Before proceeding with the detailed review, it is crucial to establish that no fundamental distinction exists between 3DPC and conventional concrete in terms of their mechanical behaviour at early ages. Previous studies (Wangler *et al.*, 2019; Mechtcherine *et al.*, 2020) demonstrate that the compressive strength of 3DPC immediately after extrusion varies significantly: from less than 200 Pa in the flowable filament approach, where the filament undergoes substantial deformation after extrusion, to hundreds or thousands of Pa in the infinite brick approach, where the filament remains nearly rigid. Within these broad criteria, many conventional concretes, such as those examined by Voigt *et al.* (2006) and Alexandridis and Gardner (1981), could be classified as 3DPC. Furthermore, whilst most 3DPC mixtures are classified as mortar without gravel (Rehman and Kim, 2021), some incorporate coarse aggregates (Kolawole *et al.*, 2020; Rahul and Santhanam, 2020), and others consist of paste without sand or gravel (Chen *et al.*, 2020; Jacquet *et al.*, 2021b). Therefore, throughout this chapter, the term ‘fresh concrete’ encompasses both conventional concrete and 3DPC, with ‘fresh 3DPC’ being used only when discussing studies specifically focused on 3D concrete printing applications.

The chapter begins by introducing fundamental concepts in continuum mechanics, alongside strain rate-independent and strain-rate-dependent constitutive models, which form the theoretical foundation for subsequent analyses. Following this, an overview of a database containing 45 key experimental studies on the mechanical behaviour of fresh concrete is presented, examining the various stress conditions investigated in each study. Drawing from this database, comprehensive analyses of fresh concrete’s mechanical behaviour are conducted and discussed throughout the remainder of this

chapter. The material behaviour is examined from both strain rate-independent (Sections 2.4 and 2.5) and strain-rate-dependent (Section 2.6) perspectives.

Section 2.4 presents detailed strength analyses, followed by a critical review of existing strength models. Whilst these analyses and models primarily focus on stress behaviour, they do not address deformation characteristics. Section 2.5 bridges this gap by investigating the stress-strain behaviour of fresh concrete, including a discussion of existing strain-rate-dependent constitutive models. Section 2.6 explores various viscous behaviours, including creep, stress relaxation, and rate-dependent stress-strain responses, demonstrating the significant implications of viscous responses, particularly creep behaviour, for structural analysis of fresh concrete structures during the printing process. Existing strain-rate-dependent models are critically reviewed based on these experimental observations.

The chapter concludes with a summary of the key findings and the identification of research gaps within the existing body of literature, providing the rationale for the research undertaken in this Ph.D. research project.

2.2 A BRIEF INTRODUCTION TO CONSTITUTIVE MODELLING

Unless otherwise specified, the material considered throughout this thesis is assumed to be isotropic and homogeneous, and all analyses are conducted within the framework of continuum mechanics. This section provides a brief introduction to key concepts in constitutive modelling, which form the foundation for the subsequent discussions. Initially, fundamental concepts related to stress and strain tensors, which quantify internal forces and deformations within materials, will be addressed. The focus will then shift to constitutive models, which mathematically describe material behaviour by relating stress to strain. In this thesis, *elasticity* and *plasticity* are defined as classes of constitutive models, with elasticity describing material responses characterised by fully recoverable deformations under mechanical loading, and plasticity accounting for responses characterised by non-recoverable deformations, specifically under mechanical loading. These models will first be examined in the context of strain rate-

independent behaviour, followed by an exploration of their extension to rate-dependent (viscous) responses.

2.2.1 General Concepts

Stress is a measure of the intensity of internal forces in response to external loads. In continuum mechanics, stress is characterised by a stress tensor, which consists of nine components, as illustrated in Eq. (2.1). However, because the stress tensor is symmetric (i.e. $\sigma_{ij} = \sigma_{ji}$), it can be fully described by six independent components: σ_{11} , σ_{22} , σ_{33} , σ_{12} , σ_{13} , σ_{23} . The components σ_{11} , σ_{22} , σ_{33} are referred to as normal stresses, and σ_{12} , σ_{13} , σ_{23} are known as shear stresses.

$$\boldsymbol{\sigma} = \sigma_{ij} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \quad (2.1)$$

Geometrically, the state of stress in a material can be represented as a point in a six-dimensional stress space. Alternatively, the stress tensor can be transformed into a coordinate system where the shear stress components vanish, known as the principal stress space. In this system, the state of stress is described by only three principal stresses, denoted as σ_1 , σ_2 , σ_3 , represented in a Cartesian coordinate system, as shown in Figure 2.1a. For stress analysis, it is also common practice to express stress in the Haigh-Westergaard stress space, which is an inclined cylindrical coordinate system, also depicted in Figure 2.1a. Readers unfamiliar with the definitions of principal stress and Haigh-Westergaard stress spaces are encouraged to refer to Appendix 2.1.

Strain is a measure of deformability of material. Similarly, strain is characterised by a strain tensor, as shown in Eq. (2.2), which has nine components with six of which are independent. The components ϵ_{11} , ϵ_{22} , ϵ_{33} are referred to as normal strains, and ϵ_{12} , ϵ_{13} , ϵ_{23} are known as shear strains.

$$\boldsymbol{\varepsilon} = \varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \quad (2.2)$$

Similar to the stress tensor, the state of strain in a material can be represented in the three-dimensional principal strain space or the Haigh-Westergaard strain space. It is important to note that, throughout this thesis, compressive stress and strain are designated as positive.

Having introduced the concepts of stress and strain, a constitutive model can be defined as a mathematical representation of the relationship between these two quantities (Belytschko *et al.*, 2013). Given the variation in mechanical responses across different materials, various classes of material constitutive models have been formulated to simulate these behaviours. The following sections will introduce elasticity and plasticity models, considering both strain rate-independent and strain-rate-dependent behaviours. These models are considered the most relevant to understanding the mechanical behaviour of fresh concrete.

2.2.2 Rate-Independent Elasticity

Existing elastic constitutive models can be categorised as *Cauchy elastic*, *hypoelastic*, and *hyperelastic models*. These formulations are fundamental not only for understanding rate-independent elasticity but also for exploring viscoelasticity.

In Cauchy elastic models, the current stress state is determined exclusively by the present strain, leading to a constitutive law expressed in terms of total stress and strain. For isotropic materials, the generalised Hooke's law is commonly applied, as indicated by Eq. (2.3):

$$\boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon} \quad (2.3)$$

Here, $\mathbf{C} = C_{ijkl}$ denotes the fourth-order elastic stiffness tensor, which is a function of the Young's modulus (E) and Poisson's ratio (ν). The Poisson's ratio ($\nu = -\epsilon_l/\epsilon_a$)

typically ranges from 0 to 0.5 for most materials in the elastic regime. A Poisson's ratio of zero implies that no lateral deformation occurs when the material is loaded axially, whereas a material is considered incompressible when $\nu = 0.5$, as it exhibits no volumetric change. The generalised Hooke's law has been broadly utilised to model the elastic stress–strain behaviour across a variety of materials.

Hypoelastic models are typically expressed in incremental or rate form, as shown in Eq. (2.4).

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}^t : \dot{\boldsymbol{\varepsilon}} \quad (2.4)$$

Unlike Cauchy elastic models, the hypoelastic model is commonly employed to describe materials whose elastic behaviour depends not only on the current stress or strain state but also on the stress or strain history or other influencing parameters (Chen, 1994). Consequently, the elastic tangent stiffness tensor C_{ijkl}^t can be a function of various parameters. For instance, C_{ijkl}^t may be expressed as a function of the current hydrostatic pressure and void ratio to simulate the pressure-dependent elastic behaviour of soils (Lashkari, 2010). Moreover, these hypoelastic models can be easily integrated with plasticity within the framework of incremental plasticity.

Despite the widespread application of the Cauchy elastic and the hypoelastic models, these approaches exhibit limitations in generating or dissipating energy during certain loading cycles. This limitation contravenes the thermodynamic definition of elastic behaviour, which states that elasticity is a reversible process characterised by zero energy dissipation in any loading-unloading cycle (Buehler, 2008). The hyperelastic model, grounded in thermodynamics, restricts the elastic behaviour of a material through a strain energy function (or free energy function in thermodynamic terms). The constitutive relations are then derived from the Clausius–Duhem inequality.

Examples of strain rate-independent hyperelastic models include the neo-Hookean model, which generalises the generalised Hooke's law to finite strain regimes and is extensively employed for polymers (Hossain *et al.*, 2009). Other models include the

pressure-dependent hyperelastic model for soils (Einav and Puzrin, 2004; Houlsby *et al.*, 2005) and the ageing hyperelastic model for shotcrete (Meschke, 1996).

2.2.3 Rate-Independent Plasticity

In rate-independent plasticity, the material's elastic response is governed by the elasticity module, while its inelastic behaviour is described by the plasticity module, which consists of three key components: a yield criterion, a hardening function, and a flow rule (Chen and Han, 2007).

2.2.3.1 Strength and yield criteria

Before introducing the yield criterion, it is logical to discuss the strength criterion. The *strength function*, denoted as F , is a scalar function of the stress tensor ($\boldsymbol{\sigma}$) that characterises the ultimate load-carrying capacity of a material. Traditionally, the strength criteria have been formulated as a function of the stress invariants, or equivalently, the Haigh-Westergaard coordinates (ξ , ρ and θ) for mathematical convenience, with its general form expressed as:

$$F(\boldsymbol{\sigma}) = F(\xi, \rho, \theta) \leq 0 \quad (2.5)$$

A failure criterion, as specified in Eq. (2.5), can be graphically represented as a surface in the three-dimensional principal stress space (Figure 2.1a), known as a *failure surface*. A stress point may lie either on this surface, indicating failure, or within the surface, signifying that the material remains intact. Under plane stress conditions, the stress state of the material is represented by the two in-plane principal stresses, and its strength is characterised by an envelope in the plane stress space, which effectively serves as a projection of the three-dimensional failure surface into the plane stress space.

The mathematical formulation of the failure criterion depends on the specific mechanical behaviour of the material. For instance, the strength of metals is independent of hydrostatic pressure (Batdorf and Budiansky, 1949; Hill, 1998). This independence is attributed to the fact that the strength of metals is primarily governed

by the motion of dislocations (Rice and Thomson, 1974), which is influenced by the deviatoric part of the stress tensor (Buehler, 2008). Consequently, strength models for metals, such as those proposed by Tresca (1869) and von Mises (1913), are designed to be independent of the hydrostatic component ξ . In contrast, granular materials, such as cured concrete (Mills and Zimmerman, 1970), soils (Lade *et al.*, 1973), exhibit pressure-dependent strength behaviour due to the aggregate interlocking mechanism. Accordingly, strength models for these materials, such as the Mohr-Coulomb (Mohr, 1900) and Drucker-Prager (Drucker and Prager, 1952) models, incorporate a dependency on ξ .

While the strength criterion defines the ultimate load-carrying capacity, the *yield function* (f) is a mathematical model that defines the stress threshold, known as yield stress, beyond which plastic deformation occurs. Analogous to the failure criterion, the yield criterion for multiaxial stress states is also a function of the stress tensor $\boldsymbol{\sigma}$ and, therefore, is dependent on the Haigh-Westergaard coordinates. Notably, the yield stress of a material is not constant but varies during loading due to changes in the internal structures - a phenomenon known as hardening and softening (Jirasek and Bazant, 2001). Specifically, hardening refers to an increase in yield stress, while softening denotes a decrease. For the sake of convenience, the term 'hardening behaviour' will be used in this thesis in a broader sense to encompass both hardening and softening, unless otherwise specified.

The isotropic and kinematic hardening hypotheses are two fundamental approaches to accounting for the hardening behaviour of materials. Among these, isotropic hardening is extensively applied in cases of predominantly monotonic loading, whereas kinematic hardening is utilised for more complex loading conditions (Chen and Han, 2007). Given that 3D concrete printing primarily involves a monotonic loading process, this thesis focuses exclusively on the isotropic hardening hypothesis. In this context, the evolution of yield criterion is governed by a scalar hardening function (q) driven by a history variable (κ), both of which will be discussed in subsequent subsections. Thus, the yield criterion can be expressed as

$$f(\boldsymbol{\sigma}, \kappa) = f(\xi, \rho, \theta, \kappa) \leq 0 \quad (2.6)$$

At the maximum yield stress, the yield criterion converges with the strength criterion. Moreover, in some engineering practices, the hardening behaviour is neglected for simplicity, and the material is said to exhibit *perfectly plastic* behaviour. In such cases, the strength criterion serves as the yield criterion. Therefore, it can be concluded that the yield criterion is a more general concept that encompasses the strength criterion.

Graphically, Eq. (2.6) represents a collection of surfaces in the three-dimensional principal stress space, referred to as yield surfaces. In rate-independent plasticity, a stress point can either lie on a yield surface ($f = 0$) indicating that the material has yielded, or inside the yield surface ($f < 0$) indicating that the material remains within the elastic regime.

Unlike material strength, which can be straightforwardly identified through experiments, determining yield stress is non-trivial. As a result, a yield criterion is often derived from the strength criterion of the material, typically adopting a similar mathematical form (Borja and Lee, 1990).

2.2.3.2 Hardening function

As established in the previous section, the yield criterion (f) governs the onset of plastic deformation and is a more general concept that converges with the strength criterion at maximum yield stress. The evolution of the yield criterion during loading is controlled by a scalar *hardening function* (q), which is driven by a scalar history variable known as the *equivalent plastic strain* (κ).

Physically, κ represents the accumulated change in the internal structure of a material. Historically, κ has been derived from the plastic strain tensor (ϵ^p), though different studies have employed various approaches (Odqvist, 1933; Taylor and Quinney, 1931). Despite these differing definitions, $\kappa \geq 0$ reflects the irreversible nature of the damage of internal structures. However, this property may not hold for fresh concrete, as will be explored later.

The hardening function q ranges between zero and one, encapsulating the transition from initial yielding to ultimate strength. When $\kappa = 0$, q equals to the proportional limit h_0 , defining the initial yield surface. At $\kappa = \kappa_k$, where κ_k is the equivalent plastic strain at ultimate yield stress, $q = 1$, the corresponding yield surface aligns with the strength surface.

In plasticity models, the material tangential stiffness, which represents the slope of the stress-strain curve in the inelastic region, is governed by the hardening function. Given that the stability of a structure is heavily influenced by its tangential stiffness (Timoshenko and Gere, 1961), accurately predicting this stiffness through the hardening function is crucial for structural stability analysis.

For completeness, it is important to note that the hardening function q can depend on additional parameters in addition to κ . For example some researchers have introduced dependencies on confining pressures to account for their impact on the deformation of materials such as cured concrete (Jiang and Teng, 2007; Yu *et al.*, 2010a, 2010b).

2.2.3.3 Flow rule

Following the discussion on the yield criterion and the hardening function, the flow rule determines the rate and direction of plastic deformation in materials. Mathematically, the flow rule is expressed as:

$$\dot{\boldsymbol{\epsilon}}^{pl} = \dot{\lambda} \frac{\partial g}{\partial \boldsymbol{\sigma}} \quad (2.7)$$

Here, $\dot{\lambda}$ is the plastic multiplier, which governs the magnitude of the plastic strain rate, while the term $\partial g / \partial \boldsymbol{\sigma}$ defines the flow direction, specifying the direction of plastic strain rate.

The flow direction is defined by the gradient of a function g , known as the *flow potential function*. Typically, the flow potential function depends on the current stress state ($\boldsymbol{\sigma}$), as well as the hardening variable (κ), as given by:

$$g(\boldsymbol{\sigma}, \kappa) = g(\xi, \rho, \theta, \kappa) \quad (2.8)$$

In metal plasticity (Chen and Han, 2007), the yield criterion is often used as the flow potential function, meaning $g = f$. This scenario is known as an *associated flow rule*, where the plastic strain rate direction is aligned with the gradient of the yield function. However, it is widely recognised that the associated flow rule is not suitable for granular materials such as soils (Zienkiewicz *et al.*, 1975) and cured concrete (Jirasek and Bazant, 2001). For these materials, the flow potential function g must differ from the yield function f , leading to what is known as a *non-associated flow rule*.

2.2.4 Viscoelasticity

Viscoelasticity and *viscoplasticity* are extensions of rate-independent elasticity and plasticity respectively to account for strain-rate-dependent material behaviours. These models were developed primarily for simulating time-dependent viscous behaviour (Banks *et al.*, 2011) including creep (increasing strain under constant stress) and stress relaxation (decreasing stress under constant strain), as well as the mechanical behaviour of materials under impact loading (Winnicki *et al.*, 2001). This subsection focuses on introducing key concepts of viscoelasticity, whilst viscoplasticity will be discussed in the subsequent subsection.

To date, the majority of viscoelastic models have been developed within the framework of linear viscoelasticity. The cornerstone of linear viscoelasticity is the principle of superposition in time, originally proposed by Boltzmann (1878). This principle posits that the response of a material to a complex loading history can be treated as the sum of its responses to individual loading events. Mathematically, this implies that the total strain at any given time is the cumulative effect of all past stress increments, each considered independently, or vice versa. This principle enables the representation of material behaviour through linear operators, facilitating the development of integral formulations for stress-strain relationships.

To this end, the strain of a material can be expressed as

$$\varepsilon(t) = \int_0^t J^e(t, t') \frac{d\sigma(t')}{dt'} dt' \quad (2.9)$$

where $J^e(t, t')$ is a linear operator called the elastic creep compliance function. The variable t represents the current time, and t' represents the time corresponding to the time when application of stress $\sigma(t')$.

Alternatively, the stress of a material can be expressed as a function of strain history:

$$\sigma(t) = \int_0^t R^e(t, t') \frac{d\varepsilon(t')}{dt'} dt' \quad (2.10)$$

where $R^e(t, t')$ is known as the elastic relaxation modulus.

Notably, the same linear viscoelastic model can be either be expressed in the form of Eq. (2.9) or Eq. (2.10), and the functions $J^e(t, t')$ and $R^e(t, t')$ are interconvertible (Park and Schapery, 1999). The choice of formulation is typically dependent on the available test data. Generally, Eq. (2.9) is most convenient when creep test data is available, whilst Eq. (2.10) is more suitable when stress relaxation data is available.

In many studies, the viscoelastic behaviour of a material is represented by the rheologic models (Banks *et al.*, 2011), which can be visualised as a combination of linear springs and dashpots (Figure 2.2a). Each spring i is characterised by a stiffness parameter (E_i), whereas each dashpot i is characterised by a viscosity parameter (η_i). Notably, the viscous behaviour is controlled by the properties of the dashpot. Figure 2.2b illustrates two elementary units: the Kelvin unit, consisting of a spring and a dashpot arranged in parallel, and the Maxwell unit, comprising a spring and dashpot connected in series. Many rheologic viscoelastic models comprise combinations of these units, with the Kelvin chain and Maxwell chain being the most general representations, as depicted in Figure 2.2c (Cedolin and Bažant, 2010). It is worth noting that while the Kelvin unit exhibits purely viscoelastic behaviour, a single Maxwell unit can generate permanent deformation and is thus not strictly viscoelastic.

Whilst the aforementioned linear viscoelastic models adequately describe many viscous material behaviours, they assume age-independent material properties. However, certain materials, such as concrete (Pirtz, 1968), exhibit evolving material properties over time due to ongoing chemical reactions, a phenomenon known as material ageing. This ageing leads to different viscoelastic responses at different ages, necessitating an extension of linear viscoelastic theory. This extension has led to a class of models known as ageing viscoelasticity. The common practice in ageing viscoelasticity is to make the elastic creep compliance function (Bažant and Prasannan, 1989) or the elastic relaxation function (Bažant and Wu, 1974) dependent on the material age. Notably, in ageing viscoelasticity, the strain of the material cannot be fully recoverable upon unloading. However, such models are still termed as viscoelastic in this thesis, as the ageing-induced inelastic strain is due to chemical reactions rather than mechanical loading.

Equations (2.9) and (2.10) have primarily addressed linear viscoelastic behaviour under uniaxial stress states, which are applicable only to simple structures such as trusses. However, in most structures, materials are subjected to multi-axial stress conditions, necessitating the generalisation of uniaxial viscoelastic models to three-dimensional stress states. Using Eq. (2.9) as an example, this generalisation can be achieved through two approaches: (1) employing two independent elastic creep compliance functions for the volumetric and deviatoric parts of the stress tensor (Schapery, 1969), or (2) utilising a single elastic creep compliance function with the assumption that Poisson's ratio is strain rate-invariant (Bažant and Jirásek, 2018). In the existing literature (Bazant and Yu, 2012; Lai and Bakker, 1996), the latter approach is more prevalent, primarily due to the scarcity of comprehensive three-dimensional viscous behaviour test data required for the former approach (Bažant and Jirásek, 2018).

It has been well established that the theory of linear viscoelasticity is applicable to simulating the viscous response of many materials, such as cured concrete (Bažant and Prasannan, 1989) and polymers (Lai and Bakker, 1996), in the low stress regime. However, this theory has limitations in simulating nonlinear viscous behaviour, such as nonlinear creep. Depending on the sources of the nonlinearity, nonlinear viscous behaviour can be simulated by nonlinear viscoelasticity (Green and Rivlin, 1957;

Leaderman, 1943; Schapery, 1969), viscoplasticity (Adachi and Oka, 1982), or a combination of both theories (You *et al.*, 2012).

For completeness, a viscoelastic model can also be formulated from the thermodynamic point of view, resulting in a strain-rate-dependent hyperelastic formulation (Belytschko *et al.*, 2013), similar to the hyperelastic model in rate-independent elasticity. Consequently, it is important to note that rate-independent elasticity and viscoelasticity are two inter-related theories which should not be treated in isolation.

2.2.5 Viscoplasticity

Viscoplasticity, an extension of rate-independent plasticity theory, emerged as a framework to model materials exhibiting strain-rate-dependent inelastic behaviour. The development of viscoplastic models has resulted in two primary classifications: rheologic viscoplastic models and three-dimensional viscoplastic models. Rheologic models, often based on spring-dashpot analogies, provide intuitive representations of simple material behaviour under uniaxial stress conditions. In contrast, three-dimensional models offer more comprehensive descriptions that are applicable to complex, multi-axial loading scenarios. This subsection will examine the fundamental principles and mathematical frameworks of both types, highlighting how they build upon the foundations of rate-independent plasticity while addressing the additional complexities introduced by strain-rate effects.

2.2.5.1 Rheologic viscoplastic models

Rheologic viscoplastic models can be categorised based on their coupling with other material behaviours. These categories include models based solely on viscoplasticity (purely viscoplastic), models coupled with elasticity (elastoviscoplastic), and models coupled with viscoelasticity (viscoelastic-viscoplastic).

Purely viscoplastic models describe materials exhibiting completely non-recoverable deformation. A classic example is the Bingham model (Bingham, 1917), which consists of a linear dashpot and a plastic slider arranged in parallel (Figure 2.3b). This model is

characterised by a yield stress (σ_0), below which the material behaves as a rigid body. Beyond this limit, the stress (σ) becomes linearly dependent on the strain rate ($\dot{\epsilon}$):

$$\sigma(\dot{\epsilon}) = \sigma_0 + \eta\dot{\epsilon} \quad (2.11)$$

where η represents the plastic viscosity of the dashpot. Notably, the relationship between stress and strain rate is nonlinear in general (Chaboche, 2008). This observation led to the development of the Herschel-Bulkley model (Herschel and Bulkley, 1926), a purely viscoplastic model that accounts for the nonlinear dependence of stress on strain rate. Both the Bingham and Herschel-Bulkley models have been extensively applied in rheology, particularly for fluid-state materials (Roussel, 2011a).

Most solid-state materials exhibit some degree of elastic or viscoelastic behaviour, leading to the coupling of elasticity or viscoelasticity with viscoplasticity. In elastoviscoplastic models, the viscous response is solely attributed to inelastic deformation. The modified Bingham model (Liingaard *et al.*, 2004), illustrated in Figure 2.3c, is an example of such a rheologic model. Conversely, in viscoelastic-viscoplastic models, both recoverable and non-recoverable deformation contribute to the viscous response. The Burgers model (Figure 2.3c) is a typical rheologic model of this type.

It is important to acknowledge the inherent limitations of rheologic viscoplastic models. As discussed in Section 2.2.3, material plasticity behaviours are dependent on triaxial stress states (tensorial dependence) and deformation history (e.g., hardening behaviour). Therefore, viscoplastic behaviour is expected to exhibit these dependencies (Bažant and Jirásek, 2018; Jirasek and Bazant, 2001; Mazzotti and Savoia, 2003). However, rheologic models are unable to capture these complexities, prompting the development of more comprehensive three-dimensional viscoplastic models, discussed in the next section.

2.2.5.2 Three-dimensional viscoplastic models

Three-dimensional viscoplastic models provide a robust constitutive framework capable of capturing the complex interplay between plasticity and viscous behaviour in materials. These models retain the three core components of rate-independent plasticity - *yield criterion*, *hardening function* and *flow rule* - while introducing modifications to account for strain-rate effects. Viscoplasticity theory, based on these modifications, can be broadly classified into two main classes: overstress theory and Non-Stationary Flow Surface (NSFS) theory.

The *overstress theory*, developed by Perzyna (1963) and Duvaut and Lions (1972), provides the foundational framework for most 3D viscoplastic models. Unlike rate-independent plasticity, where the stress point is confined to lie on or within the yield surface, as shown in Eq. (2.6), the overstress theory allows for the stress point to lie outside the yield surface. To distinguish it from rate-independent theory, the yield criterion in overstress theory is typically referred to as the *static yield criterion* (Adachi and Oka, 1982), denoted as f_s , which specifies the yield stress corresponding to a static condition with zero strain rate. At maximum yield stress, the static yield surface converges with the *static strength surface*.

The flow rule, as described in Eq. (2.7) for rate-independent plasticity, remains applicable in overstress theory. However, in overstress theory, the plastic multiplier $\dot{\lambda}$ is not determined through the consistency condition. Instead, it is explicitly defined by an overstress function Φ . This overstress function typically depends on f_s , and it quantifies the extent to which the current stress state exceeds the static yield surface. Calibration of Φ is necessary and is typically achieved through utilising data from viscous-based experiments, such as creep tests. The hardening function from rate-independent plasticity remains valid in overstress theory, maintaining its role in describing the evolution of static yield surfaces.

The *NSFS theory*, initially introduced by Naghdi and Murch (1963) and subsequently refined by Olszak and Perzyna (1970), introduces a different approach. Compared to the rate-independent theory, Eq. (2.6), the NSFS theory modifies the yield criterion by

incorporating an additional parameter (ω). This is achieved by making the hardening function q dependent on ω , allowing the yield stress of the material to vary with ω even when σ and κ remain constant, as expressed in Eq. (2.12):

$$f_{NSFS} = f_{NSFS}(\sigma, \kappa, \omega) \quad (2.12)$$

Furthermore, the flow rule from rate-independent theory is directly adopted in NSFS theory, with the plastic multiplier $\dot{\lambda}$ determined through the consistency condition, similar to the rate-independent approach.

Researchers have adopted various approaches to define the parameter ω . For instance, Matsui and Abe (1985) and Naghdi and Murch (1963) defined ω as the loading time, while others, such as Lu *et al.* (2020) and Shahbodagh *et al.* (2020), expressed it as a function of the viscoplastic strain rate. Wang *et al.* (1997) referred to the latter approach as ‘consistency models’. Both methodologies effectively capture time-dependent viscous responses, including creep and stress relaxation (Liingaard *et al.*, 2004).

In 3D viscoplasticity, as in rate-independent plasticity, the elastic response of the material must be modelled by elasticity. As discussed in Section 2.2.5.1, the elastic component can be modelled with either a rate-independent elastic model (resulting in an elastoviscoplastic model) or a viscoelastic model (resulting in a viscoelastic-viscoplastic model). The selection of the appropriate elasticity model typically depends on the material’s characteristics. For instance, elastoviscoplastic models are more commonly applied to soils (Liingaard *et al.*, 2004) and mild steel at elevated temperatures (Rusinek *et al.*, 2007), where the viscous response is predominantly plastic. In contrast, viscoelastic-viscoplastic models are more suitable for materials like asphalt concrete (Abu Al-Rub *et al.*, 2009; Huang *et al.*, 2011), which exhibit significant contributions from both recoverable and non-recoverable deformations in their overall viscous response.

2.3 OVERVIEW OF EXISTING EXPERIMENTAL STUDIES

This section provides an overview of key experimental works on the mechanical behaviour of fresh concrete, forming the foundation for subsequent analyses. These studies are categorised into two primary approaches: *rheology-based* and *solid mechanics-based*, each offering distinct perspectives on the mechanical behaviour of fresh concrete.

Rheology-based studies primarily focus on the flow behaviour of *fluid-state* fresh concrete under continuous shearing stress. Additionally, specific studies, such as oscillatory tests (Moeini *et al.*, 2020; Roussel *et al.*, 2012), offer valuable insights into the material's response at extremely small strains, as well as the hydration mechanisms of fresh concrete. In contrast, *solid mechanics-based studies* examine the strength and deformation of *static-state* fresh concrete under conditions of compression, tension, and shear.

Table 2.1 summarises a total of 45 experimental works in chronological order, detailing the range of concrete ages and the specific stress conditions evaluated in each study. It is noteworthy that the definition of concrete age varies significantly across studies. Some studies define concrete age as the time from the initial contact between water and binder materials (Ivanova and Mechtcherine, 2020), others from the time immediately after mixing (Tripathi *et al.*, 2022) or casting (Wolfs *et al.*, 2019), and some from the earliest possible testing time in the apparatus (Wolfs *et al.*, 2018). To ensure consistency in comparisons across studies, this chapter standardises the definition of concrete age as the time counted from the completion of casting for solid mechanics-based tests, or from the completion of the pre-shear procedure for rheology-based tests.

Research on the mechanical behaviour of fresh concrete dates back to the 1960s. While the concept of extrusion-based 3D concrete printing was introduced in the early 21st century (Khoshnevis, 2004), the majority of studies before 2018 focused on traditional casting methods. These early investigations primarily aimed to understand the pressure exerted by fresh concrete on formwork during casting and its flow behaviour during pumping. Consequently, the materials in these studies were primarily subjected to compression and shear stresses.

Since 2018, there has been a marked increase in research on extrusion-based 3D concrete printing technology, leading to a significant focus on the mechanical behaviour of fresh concrete at early ages - specifically within the first few minutes to a few hours, which corresponds to the typical timeframe of a 3D printing session. Studies in this area have sought to understand the behaviour of 3DPC during pumping, extrusion, and construction phases. During pumping and extrusion, where the material is in a fluid state and subjected to shear stress, rheometers have been widely employed. Conversely, for static-state fresh concrete during construction, mechanical tests have been conducted under various stress states, including compression, tension, and shear. Notably, among the compression tests - most relevant to the stress conditions during construction stage - the majority have focused on uniaxial compression, with a few exploring triaxial compression. Only one study by Dong *et al.* (2016) has examined equibiaxial compressive strength at later ages (greater than six hours).

Of the 45 studies reviewed, 28 have focused on the mechanical behaviour of the same fresh concrete mixture under a single stress state. In contrast, 17 studies have examined multiple stress states at the same time. Among these, only four have specifically investigated both uniaxial and multiaxial compression simultaneously. The remaining studies primarily addressed combinations of uniaxial compression, tension, and shear modes. This distribution indicates a predominant focus on singular stress conditions within the literature, with limited exploration of combined stress states, particularly in the context of compression. This poses challenges for comparing the mechanical behaviour of fresh concrete under different stress conditions, especially within compression modes.

Building upon the database described in this section, comprehensive analyses have been conducted to evaluate the strength, deformation, and viscous behaviours of fresh concrete. The findings from these analyses will be detailed in subsequent sections. While results from both solid mechanics-based and rheology-based tests will be discussed, particular emphasis will be placed on solid mechanics-based tests, especially those focusing on compression that is the most relevant to the stress state of 3DPC during the construction phase.

2.4 STRENGTH BEHAVIOUR AND STRENGTH MODELS

Utilising the experimental database outlined in the preceding section, a comprehensive analysis of the strength behaviour of fresh concrete has been undertaken. This section begins by presenting the findings of the strength analysis, followed by a critical evaluation of existing strength models for fresh concrete.

2.4.1 Experimental Observations

2.4.1.1 Strength growth due to material ageing

Figures 2.4 to 2.6 illustrate the evolution of uniaxial compressive (f_{co}), uniaxial tensile (f_{to}) and shear (τ_s) strengths with concrete ages ranging from 0 minutes to 7 hours for fresh concrete in different studies. Evidently, these strengths increase monotonically with concrete age.

Strength monitoring within the first five minutes is achievable only through rheological shear tests, as depicted in Figure 2.6a. It is evident that shear strength increases significantly during approximately the first three minutes before transitioning to a nearly linear growth rate over the twenty-minute period. Kruger *et al.* (2019a) assert that rapid strength growth is initially attributable to a re-flocculation process, which is reversible. Conversely, subsequent strength enhancement is ascribed to the structuration process, resulting from irreversible chemical reactions. For concrete with less than 20 minutes of curing, the rate of strength growth in the re-flocculation phase is two to ten times that observed in the structuration phase, varying with the concrete mixture. Although no attempts have been made to understand the re-flocculation process in fresh concrete under other stress conditions, it is reasonable to hypothesise that this process also contributes to strength development under diverse stress conditions during the initial minutes. Thus, the re-flocculation process is pivotal in developing initial strength in freshly extruded 3DPC. Meanwhile, the structuration process is the most critical to the 3D printing process, ensuring the material possesses sufficient strength to counteract gravitational forces during printing. At the conclusion of the re-flocculation period, freshly extruded 3DPC typically achieves an initial compressive strength (f_{co}) ranging from 0.5 to 20 kPa (Figure 2.4b), which is essential

for optimal buildability immediately after extrusion (Mechtcherine *et al.*, 2020; Wangler *et al.*, 2019).

After the re-flocculation period and before approximately 1 to 2 hours, depending on concrete mixtures, the material demonstrates an approximate linear growth in strength (refer to Figure 2.4b for uniaxial compressive strength, Figure 2.5 for uniaxial tensile strength, and Figure 2.6b for shear strength). According to the current database, the strength growth rate in this regime ranges from 0.05 to 0.6 kPa/min for specimens under uniaxial compression, and 0.02 to 0.15 kPa/min for fresh 3DPC under either uniaxial tension or shear. In other words, the strengths associated with different stress conditions increase at varying rates. The implications of this will be discussed in Section 2.4.1.2. It is worth noting that while some believe that the strength growth in this period is due to the C-S-H formation (Roussel *et al.*, 2012), the results of x-ray diffraction tests (Pott *et al.*, 2023) show that the strength development in this period is primarily due to the Ettringite formation.

Beginning approximately two hours of ageing, the strengths corresponding to all three stress conditions exhibit an increasing growth rate with the ageing of the concrete, displaying an exponential trend (refer to Figure 2.4a for uniaxial compressive strength, Figure 2.5 for uniaxial tensile strength, and Figure 2.6b for shear strength). This increase can be attributed to the onset of the accelerating period of hydration, characterised by the silicate reaction (Pott *et al.*, 2023). This reaction results in substantial production of C-S-H, which possesses greater strength than Ettringite, substantially filling the internal pores and markedly densifying the material.

Depending on concrete mixtures, the strength starts growing at a decreasing rate after approximately 10 hours and approaching its cured strength (Pott *et al.*, 2023; Voigt *et al.*, 2006).

2.4.1.2 Comparison of strengths under different stress states

In the concluding subsection, the relationship between strength and its age was explored, establishing that the rate of strength development under varied stress states differs. This

segment aims to juxtapose the strength under distinct stress states across different ages. Such comparisons offer critical insights into the evolution of the strength envelope at varying ages. Notably, the strength evolution for fresh concrete with varying mixture proportions exhibits significant differences, as illustrated from Figure 2.4 to 2.6 . Consequently, comparisons of strength should be confined to concretes with identical mixture formulations; however, research in this area remains relatively scant, as outlined in Table 2.1.

Figure 2.7a illustrates the ratio of uniaxial tensile strength to uniaxial compressive strength ($f_{to}:f_{co}$) for two different fresh concrete mixtures aged less than 6 hours. Notably, the ratio $f_{to}:f_{co}$ for fresh concrete exceeds the typical value for cured concrete, which is 0.1 (Kupfer *et al.*, 1969). Specifically, the ratio derived from strength test results reported by Mettler *et al.* (2016) decreases from approximately 1.0 at 120 minutes to around 0.15 at 300 minutes, slightly higher than the ratio for cured concrete. This suggests that the material may exhibit similar compressive and tensile strength at early ages, with the symmetry between these strengths gradually diminishing due to microstructural changes from hydration. Conversely, the ratios calculated from strength data presented by Jacquet *et al.* (2021a) range from 0.2 to 0.6 , without showing a consistent trend with concrete age. Their findings suggest that the asymmetry between compressive and tensile strengths occurs even in fresh concrete at earlier ages. The disparity in results between these two studies can be attributed to two factors: (a) the Dumbbell-shaped device used in Jacquet *et al.* (2021a) may be more prone to stress concentration than the strip-shaped device used by Mettler *et al.* (2016), potentially leading to an underestimation of tensile strength and therefore a smaller $f_{to}:f_{co}$; (b) the ratio $f_{to}:f_{co}$ may vary depending on the concrete mixture.

Figure 2.7b depicts the ratio of shear strength to uniaxial compressive strength ($\tau_s:f_{co}$) at various stages of curing. Generally, this ratio decreases as the concrete ages over the first 5 hours. Specifically, findings by Wolfs *et al.* (2018) suggest that the $\tau_s:f_{co}$ ratio remains roughly constant at 0.5 during the initial 100 minutes. Results from Panda *et al.* (2019) follow a similar trend, except for the initial measurement at 5 minutes, where the ratio is higher at 0.75. Further comparisons of τ_s and f_{co} across the age range of 100 to 300 minutes can be based on data from Mettler *et al.* (2016). It was observed

that the ratio stays constant at about 0.7 for ages up to 160 minutes, but gradually decreases thereafter. At 300 minutes, the $\tau_s:f_{co}$ ratio reaches approximately 0.17, marginally higher than the value for cured concrete, marked as 0.09 by Kupfer *et al.* (1969) and indicated by a dashed line in the graph. While the $\tau_s:f_{co}$ ratio typically ranges from 0.4 to 0.7 according to the three aforementioned studies, results from Bos *et al.* (2021) indicate that the ratio lies between 0.2 and 0.4 for concrete aged less than 65 minutes. This variance could be attributed to differences in mixture proportions.

Figure 2.8 displays the relationship between the ratio of equibiaxial compressive strength to uniaxial compressive strength ($f_{ebc}:f_{co}$) and concrete age, from 6 hours to 28 days, as presented by Dong *et al.* (2016). As illustrated, the $f_{ebc}:f_{co}$ ratio decreases from approximately 2.5 to 4.5 at 6 hours to around 1.2 - a typical value for cured concrete (Kupfer *et al.*, 1969) - by the age of 3 days, and subsequently remains roughly constant. These results suggest that the compressive strength of fresh concrete may be significantly sensitive to lateral confining stress. As established in Chapter 1, fresh 3DPC is primarily subjected to biaxial compression during printing, implying that this confinement sensitivity could significantly influence the behaviour of a printed structure. Unfortunately, existing literature does not provide data on the biaxial compressive strength with varying confining stress ratios at younger ages. This absence underscores the need for future research into the biaxial compression behaviour of fresh 3DPC.

To study the confining pressure-dependent strength behaviour of fresh concrete, the Mohr-Coulomb friction angle (ϕ) from various studies is presented in Figure 2.9¹. The selection of ϕ , rather than the ratio of triaxial-to-uniaxial compressive strength ($f_{cc}:f_{co}$), as the indicator follows from two considerations: (1) f_{co} was not measured in some studies, rendering the calculation of $f_{cc}:f_{co}$ impossible; (2) only the value of ϕ was provided in certain studies. As shown, the values of ϕ in these studies span a wide range. Specifically, the values of ϕ for the fresh concrete in Olsen (1968) and Wolfs *et al.*

¹ The friction angles from Alexandridis and Gardner (1981) were recalculated using total stress analysis, rather than their original effective stress analysis to facilitate direct comparison with other studies.

(2019) range from 1 to 7 degrees, significantly lower than the typical value for cured concrete (34 degree, showing as a dash line in the figure), suggesting minimal sensitivity to confining pressure. Conversely, the results provided by Alexandridis and Gardner (1981) indicates that ϕ is greater than 25 degrees. This indicates a clear dependence of fresh concrete strength on confining pressure challenging the belief that fresh concrete is insensitive to confining pressure (Mettler *et al.*, 2016; Wolfs *et al.*, 2019). Moreover, the ϕ of fresh concrete in Olsen (1968) and Wolfs *et al.* (2019) is evidently affected by ageing. Initially, the strength is less sensitive to pressure due to the highly fluid cement paste acting as a lubricant, reducing aggregate interlocking. As hydration progresses, the cement paste hardens, increasing contact points between aggregates and enhancing interlocking, leading to greater pressure sensitivity. However, Alexandridis and Gardner (1981) found that ageing had minimal impact on the growth of ϕ for their fresh concrete. As will be discussed in Section 2.4.1.3, this discrepancy may be attributed to the variations in mixture proportions across different studies.

Furthermore, existing triaxial compression tests on fresh concrete adhere to the soil mechanics convention, setting confining pressure arbitrarily without reference to uniaxial compressive strength. Table 2.2 clarifies the ratio of confining pressure to uniaxial compressive strength ($\sigma_l: f_{co}$) in these studies, revealing significantly high confining levels. Given that the material is not actively confined during 3D concrete printing, the feasibility of such high confining levels is questionable. Future triaxial compression tests should focus on lower confinement levels, more relevant to 3D printing applications.

2.4.1.3 Influence of mixture proportions on strength

As shown in Figures 2.4 to 2.6, the strength at the same age can significantly vary for fresh concrete across different studies. This can be primarily attributed to the differences in mixture proportions of these fresh concretes. Given that fresh concrete is a complex composite, each constituent can impact its mechanical properties. However, a detailed analysis of mixture proportions on strength is beyond the scope of this study. This section discusses the most significant factors influencing strength; that is the water-to-binder ratio (w/b) as well as the aggregate-to-binder ratio (a/b).

In fresh concrete, aggregates are chemically inert and non-cohesive, whereas the cement paste, a cohesive mixture of water and binder, undergoes hydration and primarily contributes to strength development. Lower w/b ratios have been associated with higher compressive, tensile, and shear strengths in fresh concrete (Assaad *et al.*, 2014; Hu, 2005; Jacquet *et al.*, 2021a; Lu *et al.*, 2008). Notably, the strengths corresponding to different stress states increase at a different rate with decreasing w/b. Figures 2.1 a and b respectively demonstrates the ratios $f_{to}:f_{co}$ and $\tau_s:f_{co}$ for fresh concrete with w/b ranging from 0.4 to 0.55, calculated from the results of Jacquet *et al.* (2021a). A bifurcation in strength ratios occurs at a critical w/b ratio (0.48 in their study). Below this critical ratio, these ratios remain nearly constant and close to the corresponding values for cured concrete, as indicated by dashed lines in the figures. Above the critical ratio, there is a significant increase in the ratio $f_{to}:f_{co}$ and a slight increase in $\tau_s:f_{co}$. This implies that symmetric compression-tension strength behaviour can only be achieved for early-age fresh concrete with w/b greater than the critical ratio. Relatively few research has been focused on the impact of w/b on the pressure-dependent strength behaviour. Based on the triaxial compression test results of (Ritchie, 1962), it was found that the w/b has marginal influence on the pressure-dependent strength behaviour of fresh concrete.

In addition to the w/b, aggregate content significantly influences the strength characteristics of fresh concrete. This influence can be quantitatively described using the aggregate-to-binder ratio (a/b). Research has shown that the shear strength increases with a higher a/b (Hu, 2005; Ivanova and Mechtcherine, 2020; Jacquet *et al.*, 2021b; Mohan *et al.*, 2021). Moreover, as shown in Figure 2.11, a higher a/b tends to lead to a higher friction angle, implying that the strength becomes more sensitive to confining pressure. With the w/b ratio held constant, an increase in the a/b ratio reduces the average interparticle distance (Lu and Wang, 2011), which facilitates more contact points between aggregates, thereby enhancing granular interlocking. However, the friction angle cannot increase indefinitely with increasing a/b. Instead, the friction angle of fresh concrete should approach the that of the aggregate with increasing a/b, which is evident in Figure 2.11 where the value of ϕ approaching 35 degrees as a/b exceeds 6. Notably, the friction angle of sand generally fall within the range of 35 to 45 degrees (Schnaid *et al.*, 2001). This mechanism has been clarified by Lu and Wang (2011) with

a theoretical model. Conversely, ϕ decreases as a/b decreases. When $a/b=0$, the friction angle of the material should correspond to that of the cement paste. Regarding the impact of a/b on the uniaxial tensile strength, Jacquet *et al.* (2021b) observed that there exists a critical a/b ratio below which an increase in sand content results in a higher tensile strength. Conversely, surpassing this critical a/b ratio leads to a reduction in tensile strength. It is noteworthy that, to the best of the author's knowledge, there is currently no investigation into how sand content influences the strength ratios ($f_{to}:f_{co}$ and $\tau_s:f_{co}$), which are important information for strength analysis. This gap in the research highlights a significant opportunity for future studies.

In summary, mixture proportions can significantly influence both the size and shape of the failure envelope in fresh concrete. This contrasts with cured concrete, where the size of the strength envelope is dependent on the mixture, but its shape remains relatively consistent across different concrete mixtures.

2.4.2 Existing Strength Models for Fresh Concrete

The strength behaviour of fresh concrete exhibits three distinct characteristics based on experimental findings: (a) it depends on material age; (b) it is influenced by the stress state; and (c) it is dependent on the concrete mixture composition. To accommodate these factors, the size of the strength surface should be adjustable (referred to as flexibility in size) to account for strength growth, and its shape also needs to be adjustable (referred to as flexibility in shape) to account for the different stress-state dependent strength at different stages of hydration and mixture dependent strength behaviour. In this section, existing strength models for fresh concrete will be comprehensively reviewed and evaluated.

These models can be categorised into uniaxial stress state and multi-stress state models. Uniaxial stress state models focus on tracking strength evolution under specific stress conditions over time, whereas multi-axial stress state models elucidate strength variations under diverse stress conditions during different stages of hydration.

2.4.2.1 Uniaxial stress state models

Uniaxial stress state models have been predominantly developed to simulate the uniaxial compressive and shear strengths of fresh concrete at different ages. The simplest yet most extensively used model for fresh 3DPC is the linear model (Jayathilakage *et al.*, 2020; Roussel, 2006; Roussel *et al.*, 2012; Suiker, 2018; Suiker *et al.*, 2020; Wolfs *et al.*, 2019, 2018). In this model, strength, denoted by σ_u (representing either f_{co} and τ_s), increases linearly with concrete age as defined in Eq. (2.13):

$$\sigma_u(t) = \sigma_{ui} + \zeta_{\sigma_u} t \quad (2.13)$$

Here, σ_{ui} , represents a hypothetical strength at age zero, and ζ_{σ_u} is a curing factor controlling the strength growth rate, both of which are calibrated from experimental data. Physically, σ_{ui} represent the strength after the re-flocculation period (Figure 2.6a). The rationale behind the linear model is the observed linear evolution of strengths at different stress states within the concrete age range of approximately 5 to 100 minutes, (Figures 2.4 to 2.6), which aligns with the typical construction period. This explains the popularity of this model in the field of 3D concrete printing.

Over a longer age range, such as from 5 minutes to 6 hours, the strength growth rate increases with time leading to a nonlinear strength evolution (see Figures 2.4 to 2.6). Additionally, when curing is accelerated by rising temperature (Wolfs, 2019) or the use of accelerators (Suiker, 2022), the strength evolution may be nonlinear within the 5 to 100-minute range. Consequently, some researchers propose nonlinear models to account for this accelerated, nonlinear strength increase. For instance, strength growth has been modelled using exponential laws (Jayathilakage *et al.*, 2019a; Mettler *et al.*, 2016; Panda *et al.*, 2019; Perrot *et al.*, 2016), power laws (Reiter *et al.*, 2020; Suiker, 2022; Voigt *et al.*, 2006) and quadratic forms (Wolfs and Suiker, 2019).

Within the initial age range of 0 to approximately 3 minutes, the strength of fresh concrete increases non-linearly due to the process of reflocculation (Figure 2.6a). Although this period is crucial for early strength development, it is less significant over the extended duration of the typical construction processes. Consequently, strength

models focusing on the reflocculation period have primarily garnered interest in rheology-based investigations. Kruger *et al.* (2019a) proposed a bilinear model that has been widely adopted in subsequent rheological studies (Kruger *et al.*, 2019a, 2019b, 2020a, 2020b; Zhu *et al.*, 2021). Similarly, the effects of reflocculation have been captured using nonlinear models, as demonstrated by Suiker (2018) and Ma *et al.* (2018).

2.4.2.2 Multi-axial stress state models

During 3D concrete printing, the material is subjected to multi-axial stress conditions; therefore, its strength must be assessed using a multi-axial stress state model.

The von Mises model (Mises, 1913) model, as expressed in Eq. (2.14), has been predominantly adopted in rheology-based studies (Roussel and Coussot, 2005; Wangler *et al.*, 2016; Roussel, 2018; Kruger *et al.*, 2019b; Jacquet *et al.*, 2021a; Reinold *et al.*, 2022).

$$F(\rho) = \rho - \sqrt{2}\tau_s = 0 \quad (2.14)$$

In these studies, the shear strength (τ_s) of fresh concrete was measured at various ages using rheometers. From these data, a uniaxial stress state model was calibrated to establish a continuous relationship between shear strength and concrete age, represented as $\tau_s = \tau_s(t)$. Based on this model, Eq. (2.14) can be calibrated to predict the strength under other stress conditions.

However, while the von Mises strength surface can be adjusted in size, its shape remains fixed. This is illustrated by plotting the von Mises model in the 3D principal stress space (Figures 2.12a-c) as well as the plane stress space (Figure 2.12d), for fresh concrete at three different ages. As shown, the strength envelope at each age resemble a cylinder in the 3D principal stress space (Figure 2.12a), and an ellipse in the plane stress space (Figure 2.12d). As hydration progresses, the size of the envelope increases, but its shape remains unchanged. Consequently, the strength ratios, including $f_{to}:f_{co}$, $\tau_s:f_{co}$ and $f_{ebc}:f_{co}$, remain constant with concrete age, as depicted in Figure 2.12d, which contradicts the experimental observations (Figures 2.7 to 2.90). In particular, the von

Mises model predicts equal f_{co} and f_{to} and a constant $\tau_s:f_{co}$ of 0.58 (Figure 2.12d). This suggests that this model can only give accurate predictions on the $f_{to}:f_{co}$ and $\tau_s:f_{co}$ for certain concrete mixtures at a very early age (Figure 2.8). For more general cases, this model tends to give overestimation on these two strength ratios.

Additionally, the von Mises criterion predicts equal equibiaxial compressive and uniaxial compressive strength, that is $f_{ebc}:f_{co} = 1$ (Figure 2.12d). This prediction has not been verified for fresh concrete aged less than 6 hours due to a lack of experimental data. However, it underestimates the biaxial compressive strength ratios for concrete aged more than 6 hours (Figure 2.8) including cured concrete (Kupfer *et al.*, 1969). Furthermore, the von Mises criterion, being independent of the first stress invariant, predicts pressure-independent strength behaviour (Figure 2.12b), which is not true for fresh concrete, whose strength is clearly pressure sensitive (Figure 2.9).

To account for frictional nature of fresh concrete, Wolfs *et al.* (2018) proposed using the Mohr-Coulomb criterion (Mohr, 1900), given as Eq. (2.15), for fresh 3DPC.

$$F(\xi, \rho, \theta) = \sqrt{3}\rho \sin\left(\theta + \frac{\pi}{3}\right) + \rho \cos\left(\theta + \frac{\pi}{3}\right) \sin \phi \quad (2.15)$$

$$-\sqrt{2}\xi \sin \phi - \sqrt{6}c \cos \phi = 0$$

This criterion has become the most widely used model in numerical simulations of fresh concrete structures during the printing process (Wolfs *et al.*, 2019; Jayathilakage *et al.*, 2020; Vantighem and Ooms, 2020; Ooms *et al.*, 2021; Nguyen-Van *et al.*, 2022; An *et al.*, 2024). The Mohr-Coulomb model offers flexibility in both the size and, to some extent, the shape of the strength surface. Specifically, the size of the strength surface is primarily determined by the cohesion parameter (c), whereas its shape is solely controlled by the Mohr-Coulomb friction angle (ϕ). For fresh 3DPC, the effect of material ageing on strength growth is incorporated by relating c to concrete age, while ϕ can be adjusted to account for changes in pressure-dependent strength behaviour due to hydration. However, it is noteworthy that in all existing studies (Wolfs *et al.*, 2019; Jayathilakage *et al.*, 2020; Vantighem and Ooms, 2020; Ooms *et al.*, 2021; Nguyen-Van *et al.*, 2022; An *et al.*, 2024) a constant ϕ has been used, neglecting the age-

dependent variation in pressure-dependent strength behaviour, as illustrated in Figure 2.9. Figures 2.13a-d present schematics of the Mohr-Coulomb failure envelope at three distinct stages of hydration, with the values of the friction angle used for plotting the envelope indicated in the figure. Unlike the von Mises model, the meridians (Figure 2.13b) of the Mohr-Coulomb model are not parallel to the ξ axis, a crucial characteristic to represent the pressure-dependent strength behaviour. Moreover, the ratio $f_{to}:f_{co}$ and $\tau_s:f_{co}$ can be adjusted in this model via modifying ϕ , as shown in Figure 2.13d. However, altering ϕ affects the strength prediction under triaxial stress states (Figure 2.13b), which is inappropriate. In the context of 3D concrete printing, the major drawback of the Mohr-Coulomb model is that it predicts equal values for f_{bc} and f_{co} (Figure 2.13d), which contradicts experimental results for fresh concrete aged more than 6 hours.

Some researchers (Di Carlo *et al.*, 2013; Wijaya *et al.*, 2022, 2024) suggests using the Drucker-Prager model (Drucker and Prager, 1952), given by Eq. (2.16), for fresh concrete. The shape and size of this model is governed by the friction (χ) and cohesion (c^{DP}) parameters, respectively.

$$F(\xi, \rho) = \rho - \chi\sqrt{2}\xi - c^{DP} = 0 \quad (2.16)$$

Similar to the Mohr-Coulomb model, χ and c^{DP} can be made function of concrete age to account for the impact of material ageing on strength growth and pressure-dependent strength, respectively. In existing studies (Di Carlo *et al.*, 2013; Wijaya *et al.*, 2024, 2022), however, only the age-dependence of c_{DP} was considered with χ assumed constant during hydration. Figures 2.14a-d exhibit the schematics of the ageing Drucker-Prager model in 3D and 2D stress spaces at three different stages of hydration. To facilitate a comparison between the Drucker-Prager and the Mohr-Coulomb model, the parameter α_{DP} is converted from the friction angle of the corresponding age in Figure 2.13b. Similar to the Mohr-Coulomb model, the meridians of the Drucker-Prager model can be inclined, reflecting the impact of confinement on strength. Moreover, the strength behaviour in the tension and shear regime, Figure 2.14d, can be adjusted by the friction parameter α_{DP} , but this adjustment also impact the pressure-dependent behaviour (Figure 2.13b), as in the Mohr-Coulomb model. Unlike the Mohr-Coulomb

model, this model offer flexibility in adjusting the biaxial compressive strength behaviour (Figure 2.13d). When $\chi=0$, the model reduces to the von Mises model, predicting the same equibiaxial compressive strength to uniaxial compressive strength. By increasing the value of χ , the ratios of biaxial compressive strength to uniaxial compressive strength $f_{bc}:f_{co}$ corresponding to different biaxial stress ratios ($\beta_s = \sigma_2:\sigma_1$) increase.

Recently, Liu *et al.* (2022) suggests using the extended Drucker-Prager model in Abaqus (Dassault Systemes, 2023), Eq. (2.17), for fresh 3DPC.

$$F(\xi, \rho, \theta) = \frac{1}{2} \left[1 + \frac{1}{K} - \left(1 - \frac{1}{K} \right) \cos(3\theta) \right] \rho - \chi \sqrt{2\xi} - c^{DP} \quad (2.17)$$

Compared to the original Drucker-Prager model, Eq. (2.16), the extended model includes an additional parameter K . The parameter K , varying between 0.78 and 1, controls the deviatoric shape of the strength surface, producing a circular section when $K = 1$, as in the original model, and a non-circular section when $K < 1$ (Figure 2.16a). In Liu *et al.* (2022), the parameter K was set 0.80. Figures 2.15a-d exhibit schematics of the extended model with $K=0.80$ at three distinct stages of hydration with the friction parameters maintained consistent with those in Figure 2.14b. Unlike the original Drucker-Prager model (Figure 2.14c) the extended model displays a non-circular deviatoric section (Figure 2.15c). Under biaxial compression (Figure 2.15d) the envelope is not following an elliptical shape. This is further explained in Figure 2.16b, which indicates that the strength envelope in the biaxial compression quadrant gradually deviate the elliptical shape with decreasing K . In particular, the ratio $f_{ebc}:f_{co}$ is less than 1 for small friction parameters. Despite the lack of biaxial compressive strength data for fresh concrete, studies on cured concrete suggests that the strength envelope in the biaxial compression regime should resemble an elliptical shape (Kupfer *et al.*, 1969). Additionally, for fresh concrete aged greater than 6 hours, $f_{ebc}:f_{co} > 1$. Hence, the extended Drucker-Prager model is deemed inappropriate for representing the biaxial compressive strength behaviour of 3DPC. Furthermore, both the parameters

K and χ affects the ratios of $f_{to}:f_{co}$ and $\tau:f_{co}$, implying that its strength behaviour in shear, tension, and compression modes are coupled in this model.

Furthermore, it has been observed that the ageing Mohr-Coulomb, Drucker-Prager, and extended Drucker-Prager models share a common limitation in that the predicted strength within the triaxial tension regime may not increase monotonically, as illustrated in Figures 2.13b, 2.14b and 2.15b. This is attributable to the increasing slope of the meridians as concrete ages, resulting in a decreasing intersection with the ξ axis. However, since the fresh concrete is predominantly under compression during printing, this issue may not be significant. However, given that fresh concrete predominantly experiences compressive forces during the printing process, this issue may be of limited significance. To fully address this limitation, the adoption of a more advanced strength surface model or a multi-surface strength model is recommended (Chen and Han, 2007).

2.5 STRESS-STRAIN BEHAVIOUR AND CONSTITUTIVE MODELS

Following the discussion on the strength behaviour of fresh concrete in the previous section, this section delves into the stress-strain behaviour. Subsection 2.5.1 analyses relevant experimental data, while subsection 2.5.2 evaluates existing constitutive models for fresh concrete. It is important to note that this section focuses primarily on strain-rate-independent mechanical behaviour. Viscous behaviour, such as creep, will be addressed in the next section.

2.5.1 Experimental Observations

2.5.1.1 Stress-strain response

For fresh concrete under uniaxial compression, the axial stress-axial strain relationship has been extensively studied (Di Carlo *et al.*, 2013; Mettler *et al.*, 2016; Wolfs *et al.*, 2018; Panda *et al.*, 2019; Ding *et al.*, 2020; Bos *et al.*, 2021; Pott *et al.*, 2022; Xia *et al.*, 2024). Figures 2.17a and 2.18a demonstrate typical axial stress-axial strain curves for fresh concrete at different stages of hydration under uniaxial compression. It is evident that these stress-strain curves generally comprise three regimes: (a) a linear regime

wherein stress escalates approximately linearly with strain; (b) a hardening regime where stress increases at a diminishing rate as strain augments until reaching peak stress; succeeded by (c) a post-peak regime where the material exhibits either stress plateauing (where stress remains nearly constant) or strain softening (where stress diminishes with increasing strain). This suggests that the yield stress changes during the loading process; thus, the material exhibits *strain hardening* behaviour. Moreover, due to the on-going hydration, the yield stress changes with concrete age, a phenomenon referred to as *age hardening*.

The response in each regime can be explained by the crack patterns of the specimens. Figures 2.17 b1-b2 and c1-c2 illustrate the evolution of cracks during loading for specimen aged 30 and 120 minutes, respectively. The crack patterns at test completion for specimens at three different ages are shown in Figures 2.17 b3, c3 and 2.25b. At early stages of loading, the change of internal microstructure is minimal due to the low stress level. This is evident from Figures 2.17b1 and c1 that the material shows no visible cracks. Thus, a linear correlation between stress and strain is observed. With increasing stress level, changes in the material microstructure become more significant, including crack opening and propagation resulted from the gliding of granular particles, which deteriorates the integrity of the material. This is evident in Figures 2.17b2 and c2, where the specimen aged 30 minutes show minor cracks, and the specimen aged 120 minutes demonstrates larger diagonal cracks. Consequently, the material tangential stiffness keeps decreasing with increasing strain in the hardening regime. At peak stress, the material reaches its ultimate load bearing capacity (i.e. the strength). In the post-peak regime, the material shows plateauing in stress if the cracks remain distributed (Figure 2.17b3) or descending if the cracks get localised (Figures 2.17c3 and 2.18b).

Furthermore, the stress-strain response of fresh concrete transitions from ductile to brittle as hydration progresses (Wolfs *et al.*, 2018; Wangler *et al.*, 2022). Specifically, the pre-peak material stiffness increases while the axial strain corresponding to the peak stress (known as the peak strain) decreases. For fresh concrete under uniaxial compression, the peak axial strain reduces from 15-35% at 5 minutes to less than 5% at 6 hours (Figure 2.19), compared to 0.1% for cured concrete (CEB-FIP, 2013). This suggests that fresh concrete is significantly more deformable than cured concrete. In

the post-peak regime, early-age specimens show a stress plateau indicating excellent ductility. This ductility diminishes with age, shown by a steeper softening branch and crack patterns transitioning from distributed minor shear cracks (Figure 2.17b3) to localised shear (Figure 2.17c3) and tensile cracks (Figure 2.18b).

Notably, this age-induced ductile-to-brittle transition in stress-strain response has also been observed for fresh concrete under various stress states, including triaxial compression (Wolfs *et al.*, 2019), uniaxial tension (Hannant *et al.*, 1999; Mettler *et al.*, 2016) and shear (Wolfs *et al.*, 2018). However, the deformation response exhibits clear stress-state dependence. Figure 2.20 shows that at 5 minutes of age, the peak tensile strain is approximately an order of magnitude smaller than the peak compressive strain. By age of 6 hours, the peak tensile strain is about two orders of magnitude smaller than the peak compressive strain, indicating that the material behaves more brittle under tension compared to compression. Additionally, triaxial compression tests reveal that confinement impacts the stress-strain behaviour of fresh concrete (Alexandridis, 1979; Alexandridis and Gardner, 1981; Ritchie, 1962; Wolfs *et al.*, 2019). Specifically, Wolfs *et al.* (2019) observed that, for concrete aged 15 minutes, both the pre- and post-peak stiffness are unaffected by confining pressure, although the peak compressive strain increases (Figure 2.21a). Conversely, for specimens aged 60 minutes, increasing confining stress results in increased pre-peak material stiffness, decreased peak axial strain, and an unchanged slope of the post-peak branch (Figure 2.21b). The findings by Alexandridis (1979) and Ritchie (1962) also confirm the trend of increasing pre-peak stiffness and decreasing peak axial strain with rising pressure. Unlike the results in Wolfs *et al.* (2019), they found that the slope of the post-peak regime tends to steepen as pressure increases. This ductile-to-brittle transition in stress-strain behaviour with increasing pressure is markedly different from that of cured concrete, where ductility improves with confining pressure (Imran and Pantazopoulou, 1996; Lim and Ozbakkaloglu, 2015).

It has been revealed that the stress-strain response of fresh concrete is dependent on the mixture composition (Jacquet *et al.*, 2021a). With a constant aggregate-to-binder ratio, the pre-peak stiffness increases, the axial peak strain decreases, and the slope of the post-peak branch becomes steeper as the water-to-binder ratio decreases.

Furthermore, as discussed in Section 2.2.3.2, the history variable κ increases monotonically in conventional plasticity, indicating that the damage to the internal structures of materials is irreversible. However, due to the ongoing hydration of fresh concrete, this internal damage has the potential to be healed (Liu *et al.*, 2002), implying that $\dot{\kappa} < 0$ may be permissible. Unfortunately, there is no systematic experimental investigation available on the impact of material healing on the stress-strain behaviour of fresh concrete.

2.5.1.2 Material stiffness in the linear stress-strain regime

It has been established that the stress-strain relationship of fresh concrete remains approximately linear at low stress levels. Typically, this linear regime extends to a few percent of axial strain for fresh concrete under uniaxial compression. The slope of the stress-strain curve in this linear regime reflects the initial stiffness of the material, which is crucial for the stability analysis of fresh concrete structures.

In most existing research (Wolfs *et al.*, 2018; Casagrande *et al.*, 2020; Ding *et al.*, 2020; Xia *et al.*, 2024), material behaviour in the linear regime is generally treated as linear elastic, with the slope of the stress-strain curve in this regime considered the Young's modulus. However, this definition of Young's modulus can be misleading. Figure 2.22 illustrates the ratio of elastic strain to total strain for the fresh 3DPC mixture in Wolfs *et al.* (2018), where the elastic strain was calculated from the slope of the linear regime. Based on this approach, more than 20% of the deformation is recoverable. Although all existing uniaxial compression tests have utilised a monotonic loading scheme, preventing an accurate analysis of the recoverable strain, the creep-recovery test results from Chang *et al.* (2023c) indicate that only around 3% of the total strain is recoverable upon stress removal. This suggests that the actual elastic modulus should be much larger than the slope of the linear regime of the stress-strain curve. Moreover, the uniaxial compression test results by Tripathi *et al.* (2022) reveal that the stress-strain response in the linear regime is actually bi-linear, with the slope in the first segment, which extends to around 0.25% strain, being 4 to 18 times the slope in the second segment, which extends to a few percent strain. This observation aligns with the results from small amplitude shear tests in rheology (Kolawole *et al.*, 2020; Moeini *et al.*, 2020;

Ranjbar *et al.*, 2021), where the stress-strain relationship is linear only when shear strain is below a few tenths of a percent. These findings suggest that the modulus corresponding to strains of a few percent should not be considered the actual Young's modulus (E), but rather as a parameter defining material stiffness corresponding to few percent strain. Thus, this modulus will be referred to as the apparent Young's modulus (Pott *et al.*, 2022), denoted as E^{ap} in this thesis. To clarify the relationship between E and E_{ap} , future research should focus on cyclic compression tests for fresh concrete.

Figure 2.23a and b illustrate the evolution of E^{ap} for fresh concrete with concrete age in the range of 0 to 7 hours and 0 to 100 minutes, respectively. Similar to the development of uniaxial compressive strength (Figure 2.4), the evolution of E_{ap} from 5 minutes to the first 1 to 2 hours is approximately linear, followed by a drastically increase thereafter. Typically, E^{ap} ranges from 30 to 250 kPa at 5 minutes of age, with a growth rate between 0.3 to 15 kPa/min during the initial 1 to 2 hours.

Existing triaxial compression tests on fresh concrete have revealed that its apparent Young's modulus depends on confining pressure (Alexandridis and Gardner, 1981; Ritchie, 1962; Wolfs *et al.*, 2019), a behaviour that has also been observed in soils (Dano and Hicher, 2003; Hicher, 1996; Verdugo and Ishihara, 1996). Figure 2.24 compares the apparent Young's modulus calculated from triaxial compression test results at different confining pressure levels to that calculated from uniaxial compression test results. Except for the specimens aged 15 minutes in Wolfs *et al.* (2019), which show no dependency on confining pressure, E_{ap} increases with increasing confining pressure. Specifically, the apparent Young's modulus for specimens under a confining pressure level ($\sigma_l: f_{co}$) of 1 is 1.5 ~ 5 times greater than that with no confinement. However, in the cured stage, concrete does not exhibit this pressure-dependent stiffness (Lim and Ozbakkaloglu, 2015).

2.5.1.3 Volumetric dilation behaviour

In this thesis, the term *volumetric dilation behaviour* will be used in a boarder sense to represent the change of volume of a material. During loading, the volume of a material can increase, decrease, or remain constant, and these are termed volumetric dilation,

volumetric contraction, and volumetric conservation, respectively. This subsection aims to discuss the factors impacting the volumetric dilation behaviour of fresh concrete.

Figure 2.25c exhibits volumetric strain–axial strain curves of fresh concrete at three distinct ages. The corresponding axial stress–axial strain curves, displayed on a semi-logarithmic scale, and lateral strain–axial strain curves are depicted in Figures 2.25a and b, respectively, with the location of peak stress on each curve indicated by a marker. Figure 2.25b demonstrates that the lateral strain monotonically increases with increasing axial strain across all three ages, thereby illustrating Poisson’s effect. Notably, this increase in lateral strain results from the rearrangement of granular particles and the opening and propagation of cracks, as indicated by the crack patterns during loading (Figures 2.17b1-b3, c1-c3).

Conversely, the volumetric strain exhibits a distinct contraction-to-dilation pattern during loading, as depicted in Figure 2.25c. The initial volumetric contraction at lower stress levels indicates a decrease in inter-particle distance, likely due to pore collapse within the internal structure (Chang *et al.*, 2023c). Prior research has established that cured cement paste typically exhibits exclusively volumetric contraction under compression until failure (Shah and Chandra, 1968; Kohees *et al.*, 2020), with volumetric dilation occurring only in mixtures containing aggregates (Shah and Chandra, 1968; Imran and Pantazopoulou, 1996). This dilation is attributed to the gliding of aggregates, creating internal voids and cracks, as observed in direct shear tests of fresh concrete (Jayathilakage *et al.*, 2019b). Consequently, the volumetric dilation behaviour of fresh concrete is significantly influenced by the deformation history, which dictates the extent and nature of internal structural changes during loading.

From Figure 2.25c, it is observed that earlier-aged specimens exhibit a higher volumetric contraction than those at later ages, suggesting that the younger specimens may have a higher pore volume ratio. Moreover, the volumetric contraction rate upon initial loading, as indicated by the slope of the $\epsilon_v - \epsilon_a$ curves at zero axial strain, is similar for specimens at these three ages. Since the initial contraction rate is governed by the elastic Poisson’s ratio (ν), this finding suggests that ν remains almost constant

in the first 7 hours of ageing for this fresh concrete. Notably, for fresh 3DPC, ν was found to be 0.3 for ages less than 180 minutes (Wolfs *et al.*, 2018, 2019; Pott *et al.*, 2022). In contrast, for cured concrete without and with coarse aggregates, ν is 0.19 and 0.16, respectively (Anson and Newman, 1966). Thus, ν should reduce to the value corresponding to its cured stage with increasing concrete age, although existing studies suggest that this decrease may not be monotonic (Byfors *et al.*, 1980; De Schutter and Taerwe, 1996). Following the volumetric contraction, fresh concrete exhibits volumetric dilation. The volumetric dilation rate is found to increase as hydration progresses (Figure 2.25c). This suggests that specimens at later ages exhibit a lower resistance to cracking or a more brittle behaviour, as indicated by their stress-strain curves.

For cured concrete (Lim and Ozbakkaloglu, 2015) and soils (Schnaid *et al.*, 2001), it has been established that the magnitude of volumetric contraction increases with increasing confining pressure. Additionally, a higher confining pressure leads to a lower volumetric dilation rate. The increase in volumetric compaction can be explained by the decrease in void ratio as confining pressure increases, while the decrease in volumetric dilation rate can be attributed to the slower crack propagation rate under elevated confining pressure. Currently, there is no experimental data on the volumetric dilation behaviour of fresh concrete at the same age under different confining pressures, preventing a definitive conclusion for fresh concrete. However, it is anticipated that fresh concrete would exhibit a similar trend to that observed in cured concrete and soil.

Furthermore, based on the strength and deformation data from triaxial compression tests, Wolfs *et al.* (2019) demonstrated that the flow rule of fresh concrete is non-associated, similar to the behaviour observed in soils and cured concrete.

2.5.2 Existing Constitutive Models

2.5.2.1 Elasticity model

Due to the hydration process, the elastic stress-strain response is age-dependent (Section 2.5.1.2), indicating the necessity of considering an ageing elasticity model to accurately describe its mechanical behaviour in the elastic regime.

Most existing studies (Suiker, 2018; Wolfs *et al.*, 2019; Vantighem *et al.*, 2021; Liu *et al.*, 2022; An *et al.*, 2024) modified the generalised Hooke's law (GHL), Eq. (2.3), to account for the material ageing effect in the elastic regime, given as:

$$\boldsymbol{\sigma} = \mathbf{C}(t) : \boldsymbol{\varepsilon} \quad (2.18)$$

Here, $\mathbf{C}(t)$ denotes an age-dependent elastic stiffness tensor, derived from the Young's modulus and Poisson's ratio at different ages. This approach is referred to as the total form ageing GHL, given that the total stress is dependent on the total strain. Notably, the majority of existing simulation works on fresh concrete structures (Wolfs *et al.*, 2019; Vantighem *et al.*, 2021; Liu *et al.*, 2022; An *et al.*, 2024) employed the FEA software Abaqus, where the ageing elastic response is captured by modifying the built-in linear elastic model (Dassault Systemes, 2023). The current author found that this approach is equivalent to the total form GHL, Eq. (2.18), whether coupled with a plasticity model or not.

According to the total form GHL, an ageing material exhibits increasing stress when subjected to a constant strain (Figure 2.26a¹), while showing decreasing strain under a constant stress (Figure 2.26b) as the loading time increases. However, these behaviours have been shown to be physically implausible (Bažant, 1977, 1979; Bazant and L'Hermite, 1988; Benboudjema and Torrenti, 2008; Nedjar, 2021a). Thus, despite the widespread use of the total form ageing GHL, it remains conceptually flawed.

To address the limitations inherent in the total form ageing GHL, some research (Jayathilakage *et al.*, 2020; Nedjar, 2021b) has employed an incremental form of the generalised ageing Hooke's law, given as:

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}^t(t) : \dot{\boldsymbol{\varepsilon}} \quad (2.19)$$

¹ In this numerical example, the relationship between Young's modulus and concrete age was assumed to be $E(t) = 39.5 + 1.7t$, consistent with the relationship derived from uniaxial compression tests of fresh 3DPC in Wolfs *et al.* (2019).

For an ageing material, this incremental form of the ageing GHL correctly predicts a constant stress under constant strain (Figure 2.26a), and a constant strain under constant stress (Figure 2.26b). Notably, when plasticity is not accounted for, the total deformation predicted by the total form ageing GHL is composed solely of elastic strain. In contrast, the strain predicted by the incremental form ageing GHL comprises both elastic strain and an ageing-induced non-recoverable strain.

Given that the incremental form ageing GHL is fundamentally a hypoelastic model, as described in Eq. (2.4), it may violate thermodynamic principles. However, as discussed in Section 2.5.1.2, fresh concrete exhibits minimal elastic strain, rendering the associated energy error negligible (Belytschko *et al.*, 2013). This rationale also accounts for the widespread use of hypoelastic formulations in the modelling of soils (Borja and Kavazanjian, 1985; Nova, 2012), which, similar to fresh concrete, experience large total strains but relatively small elastic strain.

Some researchers (Reinold and Meschke, 2022; Wijaya *et al.*, 2022) have proposed the use of hyperelastic models for fresh concrete to ensure conformity with thermodynamic principles. In these models, the elastic energy potential was made dependent on the concrete age to account for the impact of ageing on elasticity.

Noteworthy, the formulations of ageing hyperelastic models are generally more complex than that of incremental form ageing GHL model, and their integration with plasticity is less straightforward. Consequently, for engineering purposes, it may be sufficient to employ the incremental form ageing GHL, given that the elastic deformation of fresh concrete is negligibly small compared to its plastic strain.

2.5.2.2 Yield criterion

As demonstrated in Section 2.5.1.1, fresh concrete exhibits hardening behaviour, necessitating a well-defined yield criterion to capture changes in the elastic domain due to strain hardening from mechanical loading and age hardening from hydration.

In existing plasticity models for fresh concrete, the yield criterion has been constructed based on the failure criteria discussed in Section 2.4.2.2. Specifically, in the von Mises model, the parameter τ_s , or the cohesion parameter in the Mohr-Coulomb (c) and Drucker-Prager (c^{DP}) models, is scaled by a strain hardening function q .

Figure 2.27 provides an example of the evolution of the Mohr-Coulomb yield surfaces during strain hardening and softening with a fixed concrete age. While the deviatoric shape remains identical to that of the failure surface, the size varies to simulate changes in yield stress throughout loading. The evolution of yield surfaces for the von Mises, Drucker-Prager, and extended Drucker-Prager models during strain hardening and softening follows a similar pattern and is therefore not depicted here.

In this approach, the age-hardening effect is accounted for through the ageing strength envelope, whereas the strain hardening phenomenon is governed by the strain-hardening law. As these yield criteria are derived from strength criteria, they share the same strengths and limitations.

2.5.2.3 Strain hardening function

Based on the experimental stress-strain response of fresh concrete (Section 2.5.1.1), the strain hardening law should possess the following three characteristics. First, it should be a function of the equivalent plastic strain (κ) to reflect the dependency on deformation history. Secondly, it should be age-dependent to account for the ductile-to-brittle transition due to hydration. Finally, it should be a function of the stress tensor to capture the impact of the stress state.

Table 2.3 summarises the characteristics of existing strain hardening laws, which have been developed exclusively based on uniaxial compressive stress-strain data. As indicated, the majority of these models treat fresh concrete as a perfectly plastic material, thereby neglecting strain hardening behaviour. In other words, the corresponding strain hardening law of these models is independent of κ . This treatment clearly contradicts the experimental observations for fresh concrete (Section 2.5.1.1). A critical limitation of these models is that they fail to predict the reduction of material

tangential stiffness due to hardening. From the perspective of structural analysis, these perfectly plastic models can lead to unconservative predictions for fresh concrete structures that fail by buckling. Due to this reason, a few studies have considered strain hardening (κ dependency) in their plasticity models. Of these models, Di Carlo *et al.* (2013) and Wijaya *et al.* (2022) & (2024) only consider hardening response, while Liu *et al.* (2022) consider both hardening and softening responses.

Noteworthy, only the strain hardening function proposed by Liu *et al.* (2022) is age-dependent considering the age-induced ductile-to-brittle transition in stress-strain response observed in experiments. Furthermore, none of the existing hardening functions is stress state dependent; thus, none of them are able to consider the impact of confinement on the hardening behaviour contradicting the experimental observation discussed in Section 2.4.1.

2.5.2.4 Flow potential function

Experimental results in Section 2.5.1.3 have demonstrated that the volumetric dilation behaviour of fresh concrete depends on the deformation history, confining stress, and concrete age. Moreover, these results suggest the necessity of using a non-associated flow rule.

In plasticity, the volumetric dilation behaviour of a material can be characterised by a volumetric dilation parameter α which specifies the inclination of $\dot{\epsilon}^p$ in the Haigh-Westergaard coordinate system, mathematically expressed as:

$$\tan \alpha = \frac{\partial g / \partial \xi}{\partial g / \partial \rho} \quad (2.20)$$

To accurately reflect the dependence of volumetric dilation on deformation history, confining stress, and concrete age, α must be dependent on κ , the stress state and concrete age.

Table 2.3 summarises the flow rules of plasticity models for fresh concrete in various studies, revealing that none of the models satisfy all three criteria. In Tanigawa and

Mori (1989) and Reinold *et al.* (2019, 2022) the flow rule is associated with the von Mises yield criterion, which predicts no plastic volumetric deformation, a response that contradicts experimental observations. Other researchers (Di Carlo *et al.*, 2013; Wijaya *et al.*, 2022, 2024) adopted a flow rule associated with the Drucker-Prager yield criterion. Furthermore, although non-associated flow rules have been implemented in a few studies, these models employ a constant volumetric dilation parameter, neglecting the impact of deformation history, stress state and concrete age.

Therefore, the flow potential function in existing plasticity models for fresh concrete remains incomplete, necessitating further investigation.

2.6 VISCOUS BEHAVIOUR AND STRAIN-RATE-DEPENDENT CONSTITUTIVE MODELS

In Sections 2.4 and 2.5, the mechanical behaviour of fresh concrete is assumed to be strain rate-independent, meaning the material response at a given age remains unaffected by loading duration. However, this assumption is an idealisation because the mechanical response of a material arises from the rearrangement of its microstructure, a process that takes time (Jirasek and Bazant, 2001). Most materials, including cured concrete (Xiao *et al.*, 2008), soils (Leroueil *et al.*, 1985), and polymers (Lai and Bakker, 1996), exhibit strain-rate-dependent (or viscous) behaviour. Therefore, rate-independent constitutive models apply only in limited conditions, while viscous-based models better capture actual mechanical responses.

This section explores three viscous behaviours of fresh concrete: creep, stress relaxation, and rate-dependent stress-strain response. In the context of 3D concrete printing, where each layer exerts continuous loading on the one beneath it, understanding creep behaviour, particularly under compression, is crucial. Therefore, greater emphasis will be placed on creep behaviour, especially under compression.

2.6.1 Experimental Observations

2.6.1.1 Creep behaviour

To date, experimental investigations into the creep behaviour of fresh concrete remain limited. Table 2.4 summarises these studies, detailing the concrete age ranges and test setups employed. As illustrated, the concrete ages examined in these investigations typically span from 5 minutes to 3 hours. Notably, the majority of existing studies have focused on creep behaviour under shear stress using rheometers, with only three studies (Esposito *et al.*, 2021; Chang *et al.*, 2023c; Xia *et al.*, 2024) examining uniaxial compressive creep behaviour, which is more relevant to the stress condition of fresh concrete during printing. Furthermore, while some researchers have investigated creep behaviour under a single stress level, others have explored multiple stress levels, enabling the assessment of nonlinear creep behaviour. Additionally, several studies provide strain recovery data for fresh concrete. However, among the studies focusing on compressive creep behaviour, only Chang *et al.* (2023c) provides limited insights into the strain recovery behaviour of fresh concrete under compression.

When a viscous material is subjected to a sustained stress exceeding a critical stress limit, the creep strain becomes unbounded, a phenomenon known as *creep failure* - a distinctive type of material failure. This critical stress limit typically ranges from 70% to 90% of the short-term strength determined by quasi-static material tests, as evidenced by creep test results for soils (Arulanandan *et al.*, 1971) and cured concrete (Mazzotti and Savoia, 2003; Rusch, 1960). Following the discussion of viscoplasticity in Section 2.2.5.2, this stress limit is referred to as the *static strength*, which denotes the material strength at zero strain rate. Whilst creep failure has been observed in shear creep tests for fresh concrete (Qian and Kawashima, 2016), the relative magnitude of the static strength compared to the short-term strength remains undetermined.

To avoid the 3DPC from creep failure during printing, it is crucial to ensure that the stress within a structure remains below the static strength. Consequently, the creep behaviour of fresh concrete at stress levels lower than the static strength is of particular interest. Figure 2.28a illustrates a typical creep curve under these conditions. As depicted, the strain of the material increases at a decreasing rate as the loading time

progresses. The total creep strain appears to comprise two components: an ‘instantaneous’ strain, followed by a delayed strain that evolves with loading time. A critical aspect often overlooked in many studies (Kolawole *et al.*, 2020; Esposito *et al.*, 2021; Chang *et al.*, 2023c) is that the ‘instantaneous’ strain does not occur truly instantaneously; rather, it represents strain increase over a very short period (Xia *et al.*, 2024). Furthermore, it has been shown that the aggregates are primarily responsible for the ‘instantaneous’ strain whereas the delayed strain is mainly due to cement paste (Neville, 1964; Bernachy-Barbe and Bary, 2019; Kolawole *et al.*, 2020).

In most existing studies, creep test results are presented as *creep compliance* curves, which are derived by normalising the creep strain by the applied stress. These curves provide an objective measure of the change in material stiffness over time. As creep progresses, the compliance increases, indicating a corresponding decrease in material stiffness as the load duration extends (see Figure 2.28b for an example). For virgin fresh concrete specimens that have not been subjected to pre-stress, the creep compliance typically ranges from a few to tens of MPa^{-1} (Chen *et al.*, 2020; Esposito *et al.*, 2021; Xia *et al.*, 2024). In contrast, for cured concrete at 28 days of age, the compliance is significantly lower, in the order of 10^{-5} to 10^{-4} MPa^{-1} (Bažant and Jirásek, 2018). This stark difference indicates that fresh concrete exhibits a much more pronounced creep behaviour than cured concrete. Notably, fresh concrete shows substantial creep within just a few minutes of loading, a period that aligns with the typical duration of a 3D concrete printing session - ranging from minutes to hours (Wangler *et al.*, 2022). Given that creep leads to a reduction in material stiffness over time, the rapid creep nature of fresh concrete suggests a risk of creep-induced buckling during printing. Therefore, it is crucial to account for creep in the structural analysis of fresh concrete structures, a philosophy that contrasts sharply with the structural analysis of cured concrete, where creep is primarily a concern for serviceability design (Jirasek and Bazant, 2001).

Creep behaviour is classified as linear when the creep compliance curves corresponding to different stress levels are identical. Conversely, nonlinear creep is observed when these curves vary across stress levels. In materials such as cured concrete (Bažant and Prasannan, 1989) and polymers (Schapery, 1969), a critical stress limit defines this transition: below this threshold, creep remains linear, while above it, the behaviour

turns nonlinear. For cured concrete, this critical stress is about 40% of the short-term strength, close to the service load limit. In contrast, soils lack a clear stress limit and can exhibit nonlinear creep even below 40% of their short-term strength (Arulanandan *et al.*, 1971; Adachi and Oka, 1982). In the nonlinear domain of cured concrete, soils, and polymers, higher stress leads to greater creep compliance, indicating a reduction in material stiffness with increasing stress. This observation is consistent with the stress-strain curves of these materials where the material tangential stiffness decreases as stress increases.

Research has shown that fresh concrete exhibits nonlinear creep behaviour, as indicated by varying creep compliance under different stress levels (Qian and Kawashima, 2016; Chen *et al.*, 2020; Chang *et al.*, 2023c; Xia *et al.*, 2024). Similar to soils, the studies by Chen *et al.* (2020) and Xia *et al.* (2024) suggest that nonlinear creep can occur even at stress levels below 30% of the short-term strength, with no clear limit below which creep remains linear. Experimental results reveal two distinct trends in nonlinear creep: rheologic shear creep tests (Qian and Kawashima, 2016; Chen *et al.*, 2020; Xia *et al.*, 2024) show that creep compliance increases with applied stress, consistent with other materials (see Figure 2.29a). In contrast, compressive creep tests by Chang *et al.* (2023c) demonstrate a decrease in creep compliance with increasing stress (Figure 2.29b). Chang *et al.* (2023c) attribute this difference to pore collapse under high compressive stress, leading to a denser microstructure less prone to creep, while shear creep remains purely deviatoric without altering pore structures. Moreover, the specimens of the compressive creep tests in Chang *et al.* (2023c) were pre-stressed before testing, which may also contribute to the unusual compressive creep behaviour observed. Further research is necessary to clarify the nonlinear creep behaviour under shear and compression.

As demonstrated in Section 2.5.1, the material stiffness of fresh concrete increases as hydration progresses. Consequently, creep compliance decreases as the concrete ages, a trend observed in multiple studies (Esposito *et al.*, 2021; Kolawole *et al.*, 2021; Chang *et al.*, 2023c; Xia *et al.*, 2024), and is similar to the behaviour of cured concrete (Bažant and Prasannan, 1989).

Building upon the preceding analysis, it is evident that creep compliance is governed by three primary factors: load duration, concrete age, and stress level. At a constant stress level, the creep compliance curves corresponding to varying load durations and concrete ages combine to form what is termed the *creep compliance surface* (Chang *et al.*, 2023c), as illustrated in Figure 2.30. Furthermore, due to the nonlinear characteristics of creep in fresh concrete, the configuration of this surface changes across different stress levels.

Upon stress removal, the creep strain of fresh concrete shows an ‘instantaneous’ reduction followed by a gradual decrease over time, a process known as *strain recovery*. Notably, only viscoplastic strain contributes to strain recovery, as viscoplastic deformation is non-recoverable. analysing the proportion of recoverable to non-recoverable strain during the strain recovery phase provides valuable insights into the mechanisms underlying creep deformation during the creep period.

Research indicates significant differences in strain recovery behaviour under shear and compression. Compressive creep-recovery tests conducted by Chang *et al.* (2023c) reveal that, for a virgin fresh concrete specimen, only about 3% of the creep deformation is recoverable (Figure 2.31b). This suggests that compressive creep strain is primarily due to viscoplasticity, which is inherently inelastic. In contrast, shear creep-recovery tests (Qian and Kawashima, 2016; Kolawole *et al.*, 2020, 2021) show that between 20% to 100% of the creep strain is recoverable (see Figure 2.31a as an example), suggesting that most of the creep strain is of viscoelastic nature.

The disparity in strain recovery between compression and shear remains unclear, underscoring the need for further research to clarify this phenomenon. It is hypothesised that the unique testing protocols for shear creep tests in rheometers may introduce residual stress prior to testing, potentially affecting the strain recovery response. Given that fresh concrete is predominantly under compression during the construction phase of printing, this thesis will focus primarily on compressive creep behaviour. Accordingly, it can be concluded that creep behaviour is predominantly attributable to viscoplasticity.

2.6.1.2 Stress relaxation behaviour

Under a constant strain, the stress in a viscous material decreases over time, a phenomenon known as stress relaxation. Although the material is merely under a condition of constant strain during printing, the results from these tests can still provide some valuable insight into the viscous behaviour of a material.

To the best of the author's knowledge, there are only two experimental investigations (Kolawole *et al.*, 2020, 2021) into the relaxation behaviour of fresh 3DPC, both conducted using a shear rheometer. Results from these studies indicate that, under constant strain, there is an initial drastic drop in stress followed by a gradual decrease, eventually approaching a constant value, referred to as *equilibrium stress* in the current study. Furthermore, these studies showed that the ratio of the equilibrium stress after 10 minutes loading to the initial stress at the beginning of relaxation is in the range of 0.65 to 0.85, indicating a 15% to 35% stress reduction during the tests. Notably, this property was found to be independent of concrete age, from 3 to 127 minutes.

Similar to creep compliance, the stress relaxation modulus is defined as the ratio of the measured stress to the applied strain. Figure 2.32 shows a typical example of the relationship between the relaxation modulus and load duration for fresh concrete. Evidently, the relaxation modulus decreases over time, indicating a reduction in material stiffness over time, an observation consistent with the results of creep tests.

Notably, both viscoelasticity (Banks *et al.*, 2011) and viscoplasticity (Liingaard *et al.*, 2004) can contribute to stress relaxation. In materials treated as elastoviscoplastic, such as soils (Liingaard *et al.*, 2004; Qiao *et al.*, 2016), the equilibrium stress corresponds to a stress point on the static yield surface (see Section 2.2.5.2 for its definition). On the other hand, if the material is purely viscoelastic, the equilibrium stress reflects the long-term stiffness. Therefore, stress relaxation tests can potentially provide insight into the magnitude of the static strength, as well as long-term material stiffness.

2.6.1.3 Strain-rate-dependent stress-strain behaviour

Both creep and stress relaxation are manifestations of viscous behaviour, rooted in the strain-rate-dependent mechanical responses of materials (Leroueil *et al.*, 1985). Consequently, the stress-strain behaviour of such strain-rate-dependent materials - including soils (Adachi and Oka, 1982), cured concrete (Suaris and Shah, 1985; Xiao *et al.*, 2008), mild steel (Rusinek *et al.*, 2007) and polymers (Thiruppukuzhi and Sun, 2001) - exhibits dependence on the applied strain rate.

Several experimental investigations have been conducted to study the strain-rate-dependent stress-strain behaviour of fresh concrete. Casagrande *et al.* (2020) examined the uniaxial compressive behaviour of fresh 3DPC under two distinct axial strain rates differing by an order of magnitude. They found that the strength and apparent stiffness of specimens tested at the higher strain rate were, on average, 40% and 6% higher, respectively, than those tested at the lower strain rate. Similarly, Jacquet *et al.* (2021b) demonstrated that a higher strain rate leads to increased tensile strength, and they further showed that the axial strain at peak stress decreases as the applied strain rate increases. In the field of rheology, numerous studies have shown that the shear strength of fresh concrete increases with the shear strain rate (Chen *et al.*, 2020; Mohan *et al.*, 2021; Zhang *et al.*, 2018), which supports the use of the Bingham model to describe this behaviour. The observed increase in strength and stiffness with higher strain rates is consistent with findings for other materials (Adachi and Oka, 1982; Suaris and Shah, 1985). Notably, this increase in strength at higher strain rates is a typical characteristic of viscoplastic response (Perzyna, 1963).

On the other hand, some studies have reported different trends. Jayathilakage *et al.* (2019b) found that the shear strength of fresh 3DPC is only minimally affected by the loading rate. More recently, Pott *et al.* (2022) observed that the apparent Young's modulus decreases as the strain rate increases, which is the opposite of the trend reported by Casagrande *et al.* (2020). Specifically, Pott *et al.* (2022) found that the apparent Young's modulus for specimens under a strain rate of 64.8%/min was, on average, 33% lower than that for specimens tested at a strain rate of 10.8%/min.

The contradictory observations regarding the strain-rate-dependent stress-strain behaviour of fresh 3DPC remain unclear. One possible explanation is the variability in material properties, as different batches of fresh concrete were used for tests at different strain rates. This variability can be significant, with relative standard deviations ranging from 10% to 45% across batches (Casagrande *et al.*, 2020; Pott *et al.*, 2022; Wolfs *et al.*, 2018). Additionally, a fundamental assumption in this research is that concrete ageing does not affect material properties during testing. However, when a lower strain rate is applied, the longer test duration may allow concrete ageing to influence the results. Future investigations are needed to clarify the impact of strain rate on the stress-strain behaviour of fresh concrete.

2.6.2 Existing Strain-Rate-Dependent Constitutive Models

From Section 2.6.1, it is evident that fresh concrete exhibit both viscoelastic and viscoplastic behaviours, necessitating a strain-rate-dependent constitutive framework for modelling its mechanical response. Notably, such viscous-based constitutive models are still limited and under active development (Mechtcherine *et al.*, 2022). This section is aimed at providing a critical review of these strain-rate-dependent constitutive models.

2.6.2.1 Viscoelastic models

To the best of the author's knowledge, all existing viscoelastic models for fresh 3DPC have been developed within the framework of linear viscoelasticity.

Nedjar (2021a) introduced a simple rheologic viscoelastic model for fresh 3DPC, comprising a single spring and Maxwell unit in parallel - essentially a simplified Maxwell chain model (Figure 2.2c). To account for material ageing, both the spring stiffness and the plastic viscosity of dashpot were made age-dependent. The model was further extended to multi-axial stress states by assuming that the Poisson's ratio is strain-rate-independent. However, as demonstrated in Figure 2.33, the model failed to provide an accurate fit for creep test data, largely due to the limitation of employing only one Maxwell unit.

Chang *et al.* (2023c) recently applied the well-known double power law which was originally proposed by Bažant and Osman (1976) for cured concrete, to fresh concrete, given by Eq. (2.21):

$$J^e(t, t') = \frac{1}{\tilde{E}} + \frac{\varphi}{\tilde{E}} t'^{-m} (t - t')^n \quad (2.21)$$

where $\tilde{E}$, m , n and φ are material parameters.

This compact equation was shown to effectively fit their creep test data for fresh concrete at various ages. However, the model is currently limited to uniaxial stress states and has not been generalised to three-dimensional stress conditions in this study. In subsequent studies (Chang *et al.*, 2023a, 2023b), attempts were made to extend the uniaxial model to multi-axial stress states using a lattice model analogy.

Due to the inherent limitations of the linear viscoelasticity framework, the two aforementioned models are unable to accurately simulate the nonlinear creep behaviour observed in experiments. Additionally, as viscoelastic models, they fundamentally assume that the viscous response of fresh concrete is elastic, meaning the deformation is recoverable. However, as discussed in Section 2.6.1.1.1, the deformation of fresh 3DPC should be predominantly viscoplastic in nature, highlighting the inappropriateness of purely viscoelastic models. More importantly, the tensorial and deformation history-dependent plasticity behaviours of fresh concrete, as discussed in Sections 2.4 and 2.5, cannot be addressed within the framework of viscoelasticity.

2.6.2.2 Viscoplastic models

To date, most viscoplastic models for fresh concrete are rheologic viscoplastic models (as detailed in Section 2.2.5.1). Historically, the Bingham (Figure 2.3b) and Herschel-Bulkley models, both of which are purely viscoplastic, have been extensively employed to represent the strain-rate-dependent behaviour of fluid-state fresh concrete under steady-state flow conditions in rheology (Kaplan *et al.*, 2005; Roussel, 2011b; Perrot *et al.*, 2021).

However, both models are inadequate for creep modelling. Considering the Bingham model as an example, the creep strain is derived by integrating Eq. (2.11), yielding the following equation:

$$\varepsilon(t) = \frac{\sigma - \sigma_0}{\eta} t \quad ; \forall \sigma > \sigma_0 \quad (2.22)$$

As evidenced by this equation, the Bingham model predicts a linear relationship between creep strain and loading time. Similarly, it can be demonstrated that the Herschel-Bulkley model also predicts a linear evolution of creep strain. However, the predictions made by both models are clearly contradicted by the results of creep tests conducted on fresh 3DPC (Figure 2.28a). Therefore, despite their prevalence in concrete rheology, both models do not accurately capture the creep behaviour of fresh concrete.

Andersen *et al.* (2020) proposed a rheologic elastoviscoplastic model incorporating a single Maxwell unit for fresh 3DPC (Figure 2.2b). However, in the context of creep modelling, this model is constrained to predicting a linear relationship between creep strain and loading time due to its dependence on a single Maxwell unit (Cedolin and Bažant, 2010), which is similarly inadequate.

Subsequently, Esposito *et al.* (2021) proposed the use of the Burgers model (Figure 2.3c), a rheological viscoelastic-viscoplastic model, to simulate the creep behaviour of fresh 3DPC. The material parameters of the Burgers model were adjusted to be age-dependent, thereby accounting for the influence of ageing on creep. The proposed model was shown to provide a reasonably accurate fit for compressive creep data obtained by the same authors across various concrete ages. However, subsequent research by Xia *et al.* (2024) argued that the original version of the Burgers model could generate unbounded viscoplastic strain due to the dashpot in series - a condition that is not physically feasible for fresh concrete, given the ongoing hydration process. This limitation motivated them to propose a modified Burgers model.

As highlighted in Section 2.2.5.1, rheologic viscoplastic models are restricted to uniaxial stress state and fail to account for the tensorial and deformation history dependent nature of material plasticity. In an attempt to address these limitations, Wang *et al.* (2023) integrated the Burgers model with a three-dimensional rate-independent plasticity model. However, while this approach partially mitigates some of the issues, it does not establish a consistent framework that simultaneously incorporates both the tensorial and deformation history dependent aspects of plasticity with the viscous inelastic behaviour.

The three-dimensional viscoplastic models offer a consistent framework for addressing these limitations. To the best of the author's knowledge, all existing 3D viscoplastic models for fresh concrete (Tanigawa and Mori, 1989; Reinold *et al.*, 2019, 2022) were based on overstress theory and employ the von Mises yield criterion with an associated flow rule. Additionally, fresh concrete has consistently been modelled as a perfectly plastic material, neglecting the strain hardening behaviour. These models have also failed to account for the effects of material ageing. Drawing on the discussion from Section 2.5.2, it is evident that the yield criterion, flow rule, and hardening function in these models are incomplete. Therefore, the attention of the subsequent discussions will be given to the characteristics of overstress functions (Φ) of these models which governs their creep prediction.

In both Tanigawa and Mori (1989) and Reinold *et al.* (2019), the overstress function adopted results in a linear relationship between the stress tensor and viscoplastic strain rate. Consequently, this model can be seen as a generalisation of the uniaxial Bingham model, Eq. (2.11), to multiaxial stress conditions. However, for fresh concrete, the relationship between stress and strain rate has been proven to be nonlinear (Roussel, 2011b). Therefore, Reinold *et al.* (2022) proposed an overstress function that produces a nonlinear relationship between stress and viscoplastic strain rate.

Despite these advancements, all these 3D viscoplastic models predict a linear relationship between creep strain and time in the context of creep modelling. Through a series of numerical tests (as detailed in APPENDIX 2.2), it was found that this limitation is attributed to absence of a hardening function in these models.

2.7 CONCLUDING REMARKS

Understanding the mechanical behaviour of fresh concrete, particularly under compression, is critically important for structural analysis of 3D concrete-printed structures during the construction phase. Accordingly, this chapter has been dedicated to reviewing the state-of-the-art knowledge concerning the mechanical properties of fresh concrete, with a particular focus on compressive behaviour.

The chapter began by introducing key concepts in constitutive modelling, which are fundamental for the discussions in the rest of the thesis. Following this, an overview was provided of a database comprising 45 key experimental studies that investigated the mechanical response of fresh concrete. Utilising this database, comprehensive analyses of the strength, stress-strain, and viscous behaviours of fresh concrete were subsequently presented. The findings from these analyses can be summarised as follows:

- The strength behaviour of fresh concrete is influenced by both the concrete age and the applied stress conditions. As hydration progresses, the strength of fresh concrete under various stress conditions increases, leading to an expansion of the strength envelope. Meanwhile, the rate of strength development differs along different stress paths, resulting in changes to the shape of the strength envelope. The strength behaviour was also found to be dependent on the mixture composition, with two key parameters, the water-to-binder ratio and the aggregate-to-binder ratio, being identified as the most significant influences.
- Fresh concrete exhibits hardening behaviour, which leads to variations in material yield stress. Two hardening mechanisms were identified: age hardening due to hydration and strain hardening resulting from mechanical loading. The strain hardening phenomenon is significantly affected by the age of the concrete, as evidenced by the ductile-to-brittle transition in stress-strain curves, as well as the stress state of the material.
- The modulus associated with a few percent of axial strain in a uniaxial compressive test (apparent Young's modulus) has often been regarded as the

Young's modulus in existing studies. But this may be misleading. It was shown that the actual Young's modulus of fresh concrete may be much greater than the apparent Young's modulus.

- The volumetric dilation behaviour of fresh concrete depends on the concrete age, the deformation history, and the confining stress.
- Fresh concrete exhibits pronounced creep within a period of several minutes to tens of minutes, a timeframe closely associated with the construction phase of 3D-printed concrete structures. This creep behaviour can result in a gradual reduction of material stiffness over time, underscoring the importance of incorporating creep effects into the structural analysis of fresh concrete. Moreover, the creep response of fresh concrete has been shown to depend on both concrete age and applied stress levels. Notably, it was shown that the creep deformation under compressive loading is predominantly inelastic, challenging the commonly accepted notion that this creep is primarily viscoelastic.
- Results of stress relaxation tests confirmed that the material stiffness of fresh concrete decreases over time under a constant strain due to its viscous nature. Furthermore, it has been found that the ratio of the equilibrium stress to the initial stress in relaxation tests is in the range of 0.65 to 0.85, a ratio remains age-independent.
- The stress-strain behaviour of fresh concrete is affected by the applied strain rate.

Upon reviewing these experimental studies, the following research gaps have been identified:

- The most critical gap is the lack of experimental studies on the biaxial compressive behaviour of fresh concrete, a stress condition considered as the most relevant during the construction phase, as explained in Chapter 1.

- Cyclic compression tests are required to determine the actual Young's modulus and to clarify its relationship with the apparent Young's modulus; however, such studies are currently unavailable.
- Due to ongoing hydration, the internal damage in fresh concrete caused by external loading may be healed. No systematic studies have yet focused on the influence of this material healing phenomenon on the mechanical behaviour of fresh concrete.
- The observations regarding the nonlinear creep and strain recovery behaviours of fresh concrete under shear and compressive loading have been contradictory. Further research is needed to clarify these behaviours, particularly under compressive loading, which is more relevant to 3D concrete printing.
- The existing stress relaxation tests have been focused on shear under a single strain level, leaving the compressive stress relaxation response unexplored. Moreover, the roles of viscoelasticity and viscoplasticity in stress relaxation as well as the implications of the equilibrium stress in these tests remain unclear.
- The results on the strain-rate-dependent stress-strain behaviour of fresh concrete from different studies appear to be contradictory. This discrepancy may be attributed to the fact that the mechanical tests conducted at different strain rates were performed on specimens cast from different batches of concrete, where significant inter-batch variability in mechanical properties can occur. Therefore, further research is required to clarify the specific impact of strain rate on the stress-strain response of fresh concrete.

Based on the updated understandings from the analyses of the test results, this chapter also presented a critical review of existing strain-rate-independent and dependent constitutive models for fresh concrete. The findings from the critical evaluation are summarised as follows:

- For material within the elastic regime, the widely used total form ageing generalised Hooke's law is not appropriate. Instead, the elasticity model can be formulated as hypoelastic or hyperelastic, with the hypoelastic ageing model has the benefit of simpler formulation and easier coupling with plasticity.
- Existing yield criteria for fresh concrete have been modified from the strength criteria. To accurately simulate the strength behaviour of fresh concrete, both the size (to account for strength growth) and shape (to account for stress state- and mixture-dependence) need to be adjustable. The strengths and limitations of existing strength models have been demonstrated. Of the existing models, the Drucker-Prager model has the potential to represent the biaxial compressive strength behaviours of different fresh concretes across different ages.
- Most existing constitutive models, whether rate-dependent or independent, consider concrete as a perfectly plastic material neglecting the strain hardening behaviour, which have been shown to be inappropriate. It was further pointed out that a comprehensive strain hardening function for fresh concrete needs to be a function of concrete age, stress state, and the equivalent plastic strain. None of the existing models consider these three factors simultaneously.
- The flow rule for fresh concrete should be non-associated and the volumetric dilation parameter of the flow potential function needs to be a function of concrete age, stress state, and the equivalent plastic strain. While most studies adopted a non-associated flow rule, the volumetric dilation parameters were assumed to be constant in these studies, which is inappropriate.
- Strain-rate-dependent constitutive models are essential for simulating viscous behaviours, such as creep and strain relaxation. Most existing rate-dependent models are rheological and limited to uniaxial stress conditions, failing to capture the tensorial nature and deformation history-dependent plasticity of fresh concrete. These key material characteristics require modelling through three-dimensional viscoplasticity frameworks. Although several three-dimensional viscoplasticity models have been proposed, they remain

incomplete, highlighting the need for the development of a more comprehensive strain-rate-dependent constitutive model for fresh concrete.

This thesis addresses critical research gaps in both experimental investigation and theoretical development of fresh concrete behaviour. To overcome the current limitations in biaxial compression test data, Chapter 3 presents a comprehensive experimental study investigating the biaxial compression response of fresh concrete. Using these test data, both strength and stress-strain analyses have been conducted, which will be presented in Chapters 4 and 5, respectively. Based on these analyses, novel strength models are developed in Chapter 4, whilst a complete rate-independent constitutive model is introduced in Chapter 5. These models are optimised for biaxial compressive stress conditions but can also be generalised to multi-axial stress states. Chapter 6 examines the elasticity behaviour, strain-rate-dependent response and material healing characteristics through specialised uniaxial compression tests.

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Table 2.1 Summary of concrete age and stress conditions in 45 key studies on fresh concrete.

Literature	Concrete Age ¹	Solid Mechanics-based					Rheology
		Compression			Tension	Shear	
		Uniaxial	Biaxial	Triaxial	Uniaxial	Direct Shear	Rheologic
Ritchie (1962)	Not specified	×	×	✓	×	×	×
Olsen (1968)	20 ~ 180 mins	×	×	✓	×	×	×
Alexandridis and Gardner (1981); Alexandridis (1979) ²	40 ~ 160 mins	✓	×	✓	×	×	×
Hannant <i>et al.</i> (1999)	60 ~ 300 mins	×	×	×	✓	×	×
Voigt <i>et al.</i> (2006)	20 mins ~ 36 hrs	✓	×	×	×	×	×
Assaad <i>et al.</i> (2009)	5, 125 mins	×	×	✓	×	×	×
Lu and Wang (2010)	Not specified	×	×	×	×	✓	✓
Lu and Wang (2011)	Not specified	×	×	×	×	✓	✓
Assaad and Harb (2012)	Not specified	✓	×	✓	×	×	×
Roussel <i>et al.</i> (2012)	5 ~ 25 mins	×	×	×	×	×	✓
Di Carlo <i>et al.</i> (2013)	6 ~ 283 mins	✓	×	×	×	×	×
Assaad <i>et al.</i> (2014)	< 10 mins	×	×	×	×	✓	✓
Mettler <i>et al.</i> (2016)	93 ~ 322 mins	✓	×	×	✓	✓	×
Dong <i>et al.</i> (2016)	6 hrs ~ 28 days	✓	✓	×	×	×	×
Qian and Kawashima (2016)	Not specified	×	×	×	×	×	✓
Nguyen <i>et al.</i> (2017)	3 ~ 9 hrs	×	×	×	✓	×	×
Yuan <i>et al.</i> (2017)	15 ~ 135 mins	×	×	×	×	×	✓
Panda and Tan (2018)	< 30 mins	✓	×	×	×	×	✓

Table 2.1 (continued)

Literature	Concrete Age ¹	Solid Mechanics-based					Rheology
		Compression			Tension	Shear	Rheologic
		Uniaxial	Biaxial	Triaxial	Uniaxial	Direct Shear	
Wolfs <i>et al.</i> (2018); Wolfs (2019) ³	5 ~ 95 mins	✓	×	×	×	✓	×
Zhang <i>et al.</i> (2018)	5 ~ 65 mins	✓	×	×	×	✓	✓
Jayathilakage <i>et al.</i> (2019b)	4 mins	×	×	×	×	✓	×
Kruger <i>et al.</i> (2019a)	1 ~ 60 mins	×	×	×	×	×	✓
Wolfs <i>et al.</i> (2019); Wolfs (2019) ³	5 ~ 90 mins	✓	×	✓	×	×	×
Panda <i>et al.</i> (2019)	5 ~ 150 mins	✓	×	×	×	×	✓
Casagrande <i>et al.</i> (2020)	3 ~ 63 mins	✓	×	×	×	×	×
Ding <i>et al.</i> (2020)	30 ~ 150 mins	✓	×	×	×	×	×
Ivanova and Mechtcherine (2020)	10 ~ 70 mins	×	×	×	×	×	✓
Jayathilakage <i>et al.</i> (2020)	5 ~ 65 mins	✓	×	×	×	✓	✓
Kolawole <i>et al.</i> (2020)	3 ~ 127 mins	×	×	×	×	×	✓
Moeini <i>et al.</i> (2020)	< 25 mins	×	×	×	×	✓	✓
Proske <i>et al.</i> (2020)	10 ~ 90 mins	×	×	×	×	×	✓
Rahul and Santhanam (2020)	5 ~ 425 mins	✓	×	×	×	×	✓
Chen <i>et al.</i> (2020)	5 mins	×	×	×	×	×	✓
Bos <i>et al.</i> (2021)	5 ~ 95 mins	✓	×	×	×	✓	✓
Jacquet <i>et al.</i> (2021a)	6 ~ 324 mins	✓	×	×	✓	×	✓
Jacquet <i>et al.</i> (2021b)	Not specified	×	×	×	✓	×	✓

Table 2.1 (continued)

Literature	Concrete Age ¹	Solid Mechanics-based					Rheology
		Compression			Tension	Shear	
		Uniaxial	Biaxial	Triaxial	Uniaxial	Direct Shear	Rheologic
Kolawole <i>et al.</i> (2021)	3 ~ 127 mins	×	×	×	×	×	✓
Zhu <i>et al.</i> (2021)	10 ~ 60 mins	✓	×	×	×	×	✓
Esposito <i>et al.</i> (2021)	3 ~ 63 mins	✓	×	×	×	×	×
Marcucci <i>et al.</i> (2022), Kompella <i>et al.</i> (2024) ⁴	25 ~ 85 mins	×	×	×	✓	✓	✓
Tripathi <i>et al.</i> (2022)	35 ~ 125 mins	✓	×	×	×	×	×
Pott <i>et al.</i> (2022)	30 ~ 120	✓	×	×	×	×	×
Pott <i>et al.</i> (2023)	30 ~ 120	✓	×	×	×	×	×
Chang <i>et al.</i> (2023c)	< 90 mins	✓	×	×	×	×	×
Xia <i>et al.</i> (2024)	10 ~ 180 mins	✓	×	×	×	×	✓

Note:

¹ For calculating the concrete age, the duration for casting and/or the time for mixing is required, depending on the definition of concrete age in these studies. For studies based on solid mechanics, when the casting time is not specified, it is assumed to be 5 minutes for calculating the concrete age. Additionally, if the time required for mixing is not specified, a duration of 10 minutes is assumed.

² Alexandridis and Gardner (1981) is the published version while Alexandridis (1979) is the corresponding Ph.D. thesis.

³ Wolfs *et al.* (2018) and Wolfs *et al.* (2019) are the published version while Wolfs (2019) is the corresponding Ph.D. thesis.

⁴ Marcucci *et al.* (2022) is a conference paper and Kompella *et al.* (2024) is the corresponding journal paper.

Table 2.2 A summary of triaxial compression testing results of fresh and cured mixtures.

Literature	Approx. Confining Level ¹ ($\sigma_l: f_{co}$)
Ritchie (1962)	0.3 ~ 15
Olsen (1968)	1.1 ~ 14.3
Alexandridis and Gardner (1981)	3.09 ~ 43.48
Assaad <i>et al.</i> (2009)	3.9 ~ 10.3
Assaad and Harb (2012)	2.7 ~ 18.5
Wolfs <i>et al.</i> (2019)	0.7 ~ 3.0

Notes:

¹ For studies that do not present the uniaxial compressive strength, f_{co} was estimated from the Mohr-Coulomb cohesive (c) as $f_{co} = 2c$.

Table 2.3 Characteristics of the strain-hardening function and flow rule of plasticity models for fresh concrete in various studies.

Literature ¹	Strain-Hardening Function			Flow Rule			
	κ Dependent	Age- Dependent	Stress-State Dependent	Non- Associated	κ Dependent	Age- Dependent	Stress-State Dependent
Di Carlo <i>et al.</i> (2013)	Yes ²	No	No	No	N/A		
Suiker (2018); Suiker <i>et al.</i> (2020)	No	No	No	N/A ³			
Wolfs <i>et al.</i> (2018); Wolfs and Suiker (2019)	No	No	No	Yes	No	No	No
Jayathilakage <i>et al.</i> (2020)	No	No	No	Yes	No	No	No
Ooms <i>et al.</i> (2021); Vantighem <i>et al.</i> (2021)	No	No	No	Yes	No	No	No
Liu <i>et al.</i> (2022)	Yes	Yes	No	Yes	No	No	No
Wijaya <i>et al.</i> (2022, 2024)	Yes ²	No	No	No	N/A		
An <i>et al.</i> (2024)	No	No	No	Yes	No	No	No
Tanigawa and Mori (1989) ⁴	No	No	No	No	No	No	No
Reinold <i>et al.</i> (2019, 2022) ⁴	No	No	No	No	No	No	No

Note:

¹ Research from the same research group is grouped into the same row.

² Only strain-hardening was considered.

³ Analytical model. Dilation behaviour was not considered.

⁴ Viscoplasticity models.

Table 2.4 A summary of existing creep tests for fresh concretes.

Literature	Concrete Age Range (min)	Stress States		Test Setups		
		UC	Shear	Single Stress Level	Multiple Stress Levels	Strain Recovery
Qian and Kawashima (2016)	Not specified		✓		✓	✓
Chen <i>et al.</i> (2020)	5		✓		✓	×
Esposito <i>et al.</i> (2021)	3 ~ 63	✓			✓	×
Kolawole <i>et al.</i> (2021)	3 ~ 127		✓	✓		✓
Chang <i>et al.</i> (2023c)	10 ~ 70	✓			✓	✓
Xia <i>et al.</i> (2024)	20 ~ 180		✓	✓		×
	10 ~ 60	✓			✓	×

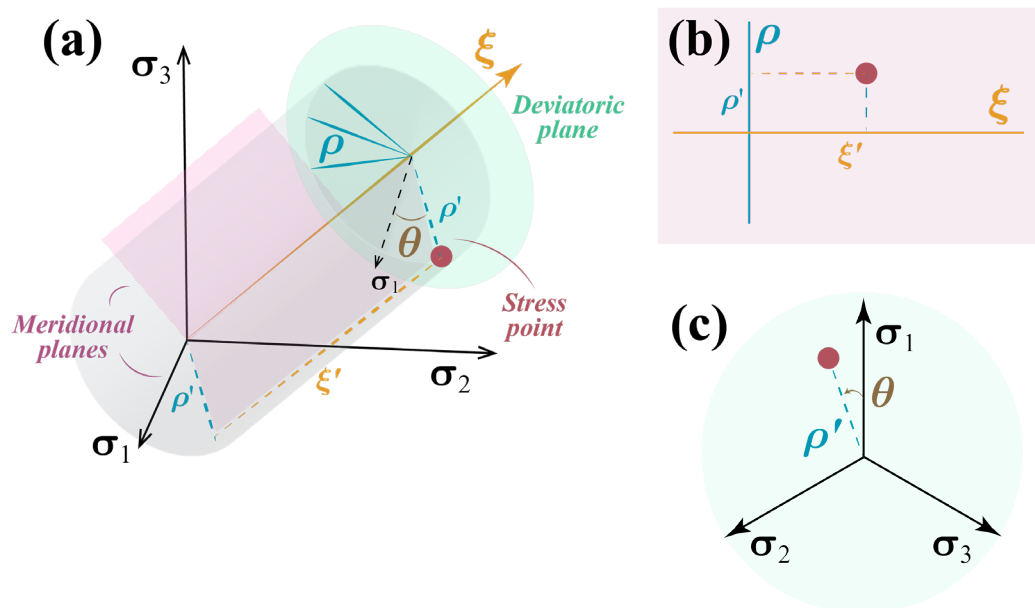


Figure 2.1 Schematic representation of a stress point in the principal stress space and the Haigh-Westergaard stress space: (a) 3D view; (b) Projection of the stress point onto the meridional plane. (c) Projection of the stress point onto the deviatoric plane.

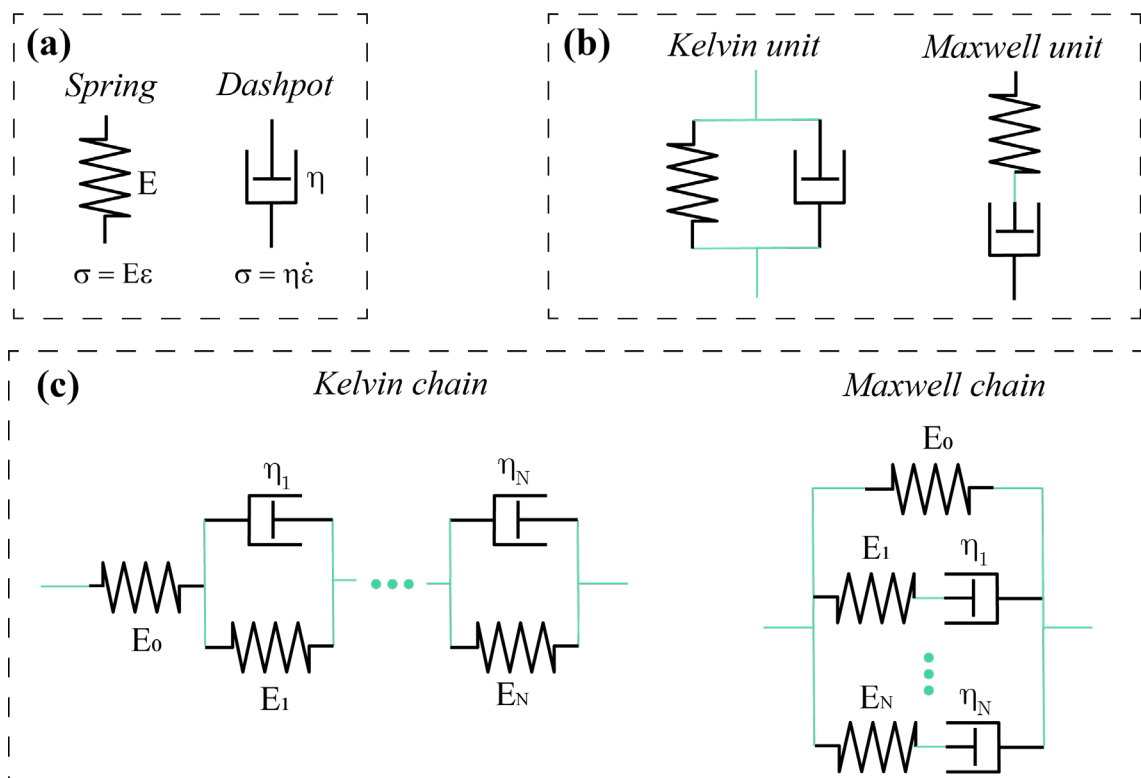


Figure 2.2 Rheologic viscoelastic models: (a) Basic rheologic units; (b) Kelvin unit and Maxwell unit; (c) Rheologic chains.

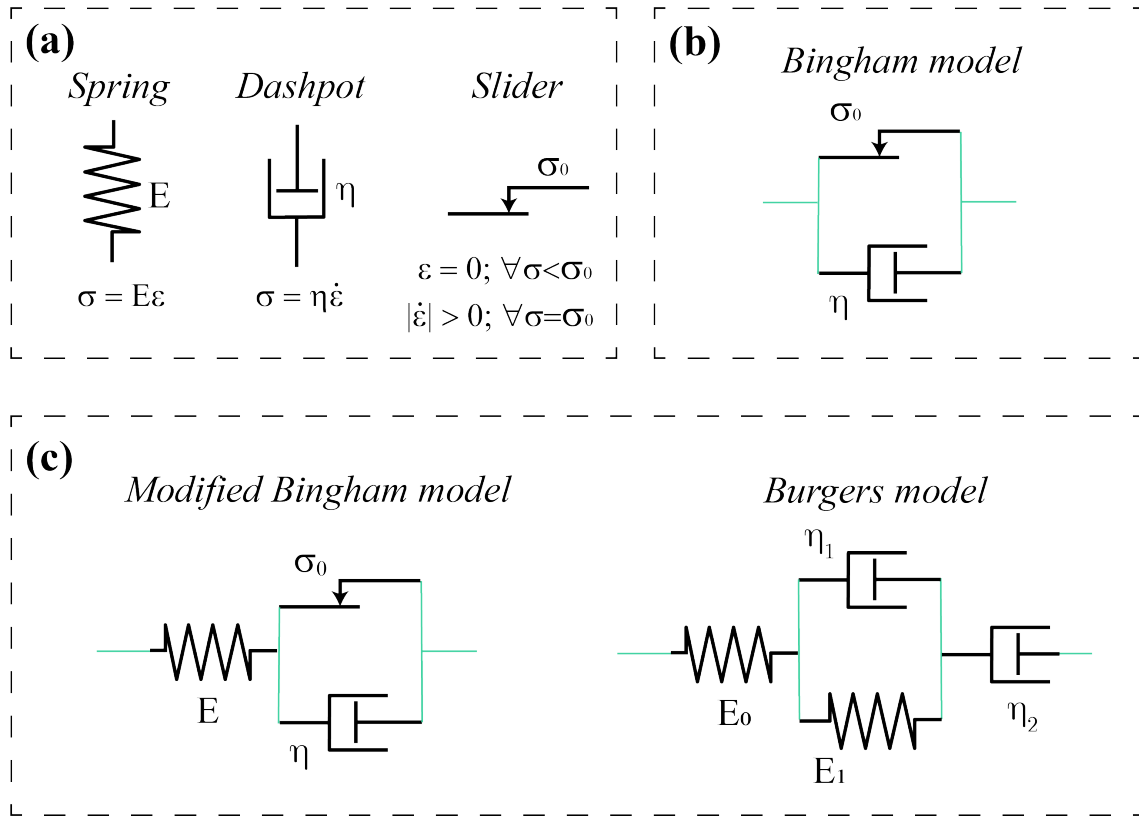


Figure 2.3 Rheologic viscoplastic models: (a) Basic rheologic units; (b) Purely viscoplastic model: Bingham model; (c) Elastoviscoplastic model: Modified Bingham model, viscoelastic-viscoplastic model: Burgers model (Kelvin-type).

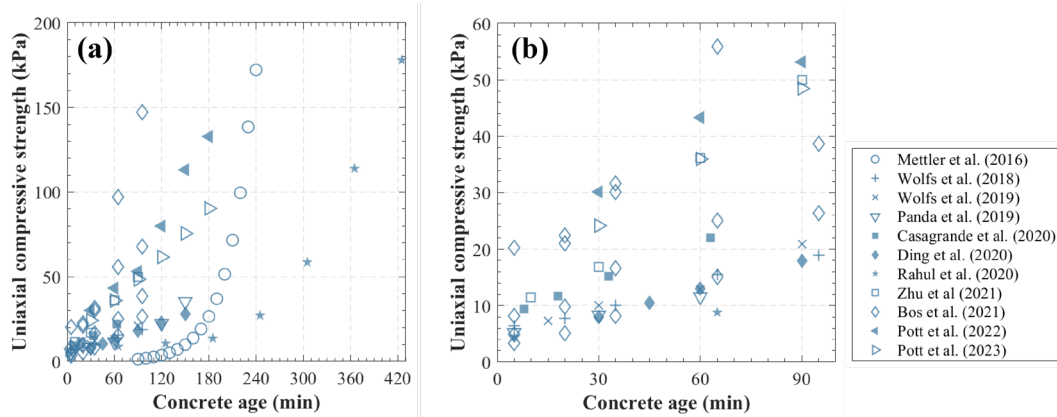


Figure 2.4 Evolution of uniaxial compressive strength with concrete age for different fresh concretes in literature: (a) Over the age range of 0 to 7 hours; (b) Over the age range of 0 to 100 minutes.

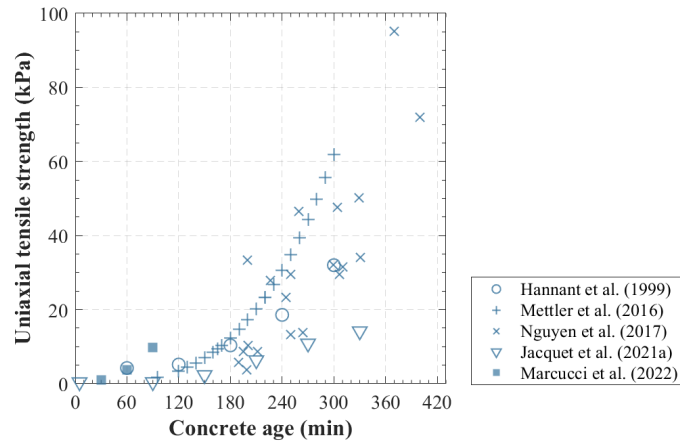


Figure 2.5 Evolution of uniaxial tensile strength with concrete age for different fresh concretes in literature.

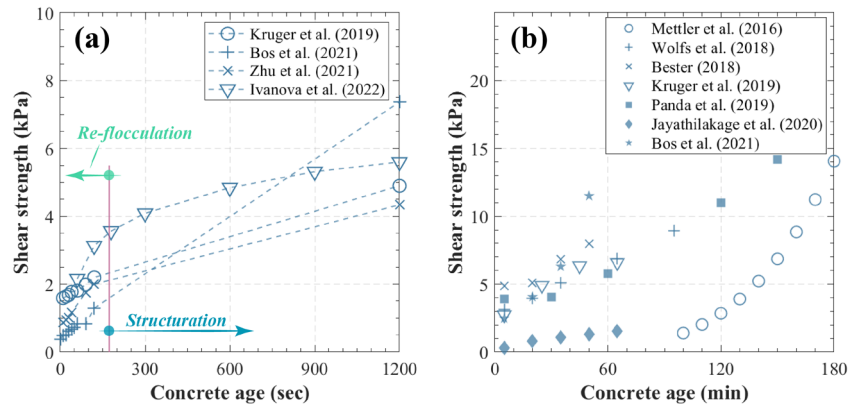


Figure 2.6 Evolution of shear strength with concrete age for different fresh concretes in literature: (a) Over short resting time; (b) Over long resting time.

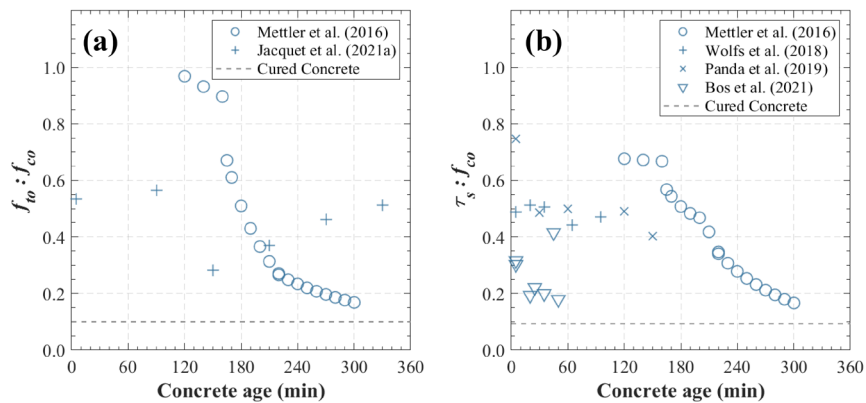


Figure 2.7 Variation of strength ratios with concrete age: (a) Ratios of uniaxial tensile strength to uniaxial compressive strength; (b) Ratios of shear strength to uniaxial compressive strength.

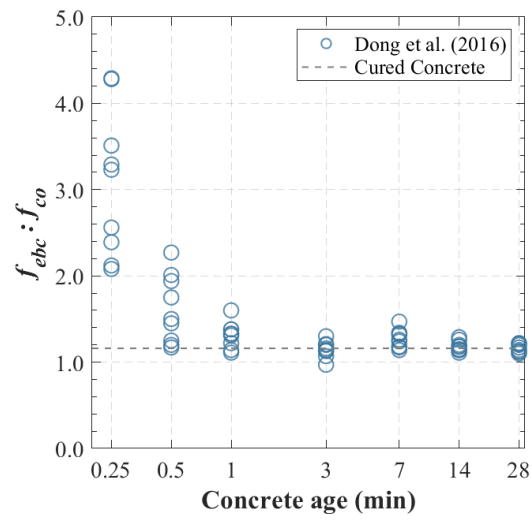


Figure 2.8 Ratios of equibiaxial compressive to uniaxial compressive strength of fresh concrete.

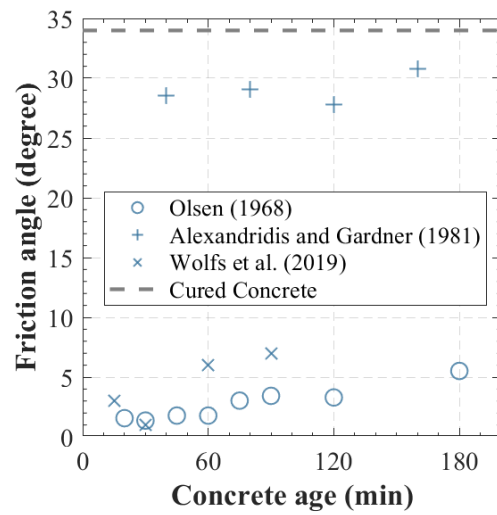


Figure 2.9 Age-dependent evolution of the Mohr-Coulomb friction angle in fresh concrete, comparing results across multiple studies.

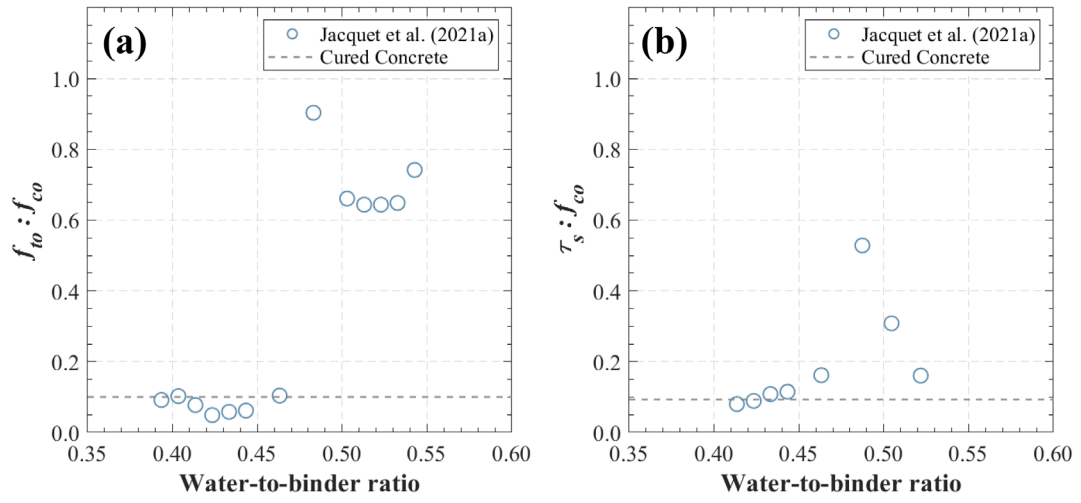


Figure 2.10 Influence of water-to-binder ratio on the strength ratios of fresh concrete: (a) Ratios of uniaxial tensile to uniaxial compressive strength; (b) Ratios of shear strength to uniaxial compressive strength.

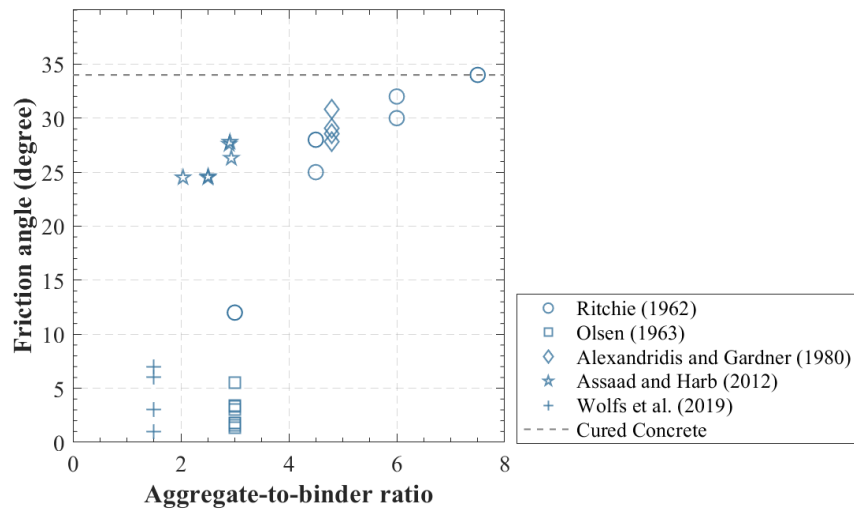


Figure 2.11 Influence of aggregate-to-binder ratio on the Mohr-Coulomb friction angle of fresh concrete, as observed across multiple experimental studies.

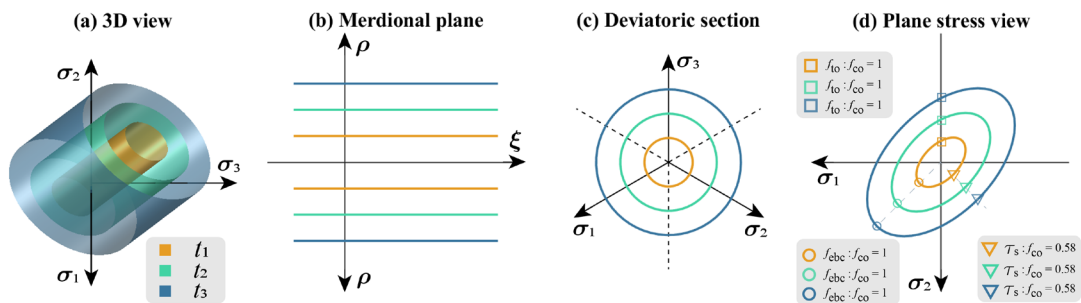


Figure 2.12 Schematics of the ageing von Mises criterion (for $t_3 > t_2 > t_1$).

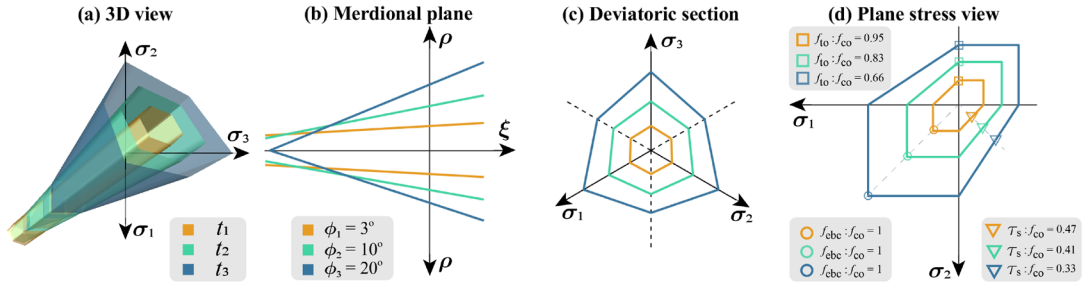


Figure 2.13 Schematics of the ageing Mohr-Coulomb criterion (for $t_3 > t_2 > t_1$).

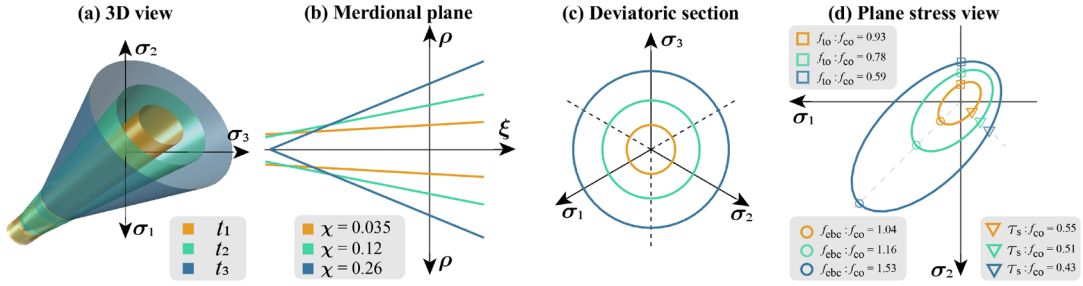


Figure 2.14 Schematics of the ageing Drucker-Prager criterion (for $t_3 > t_2 > t_1$).

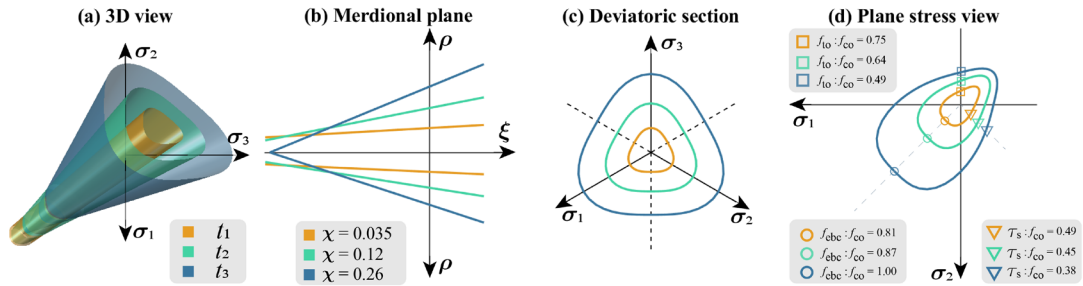


Figure 2.15 Schematics of the ageing extended Drucker-Prager criterion (for $t_3 > t_2 > t_1$).

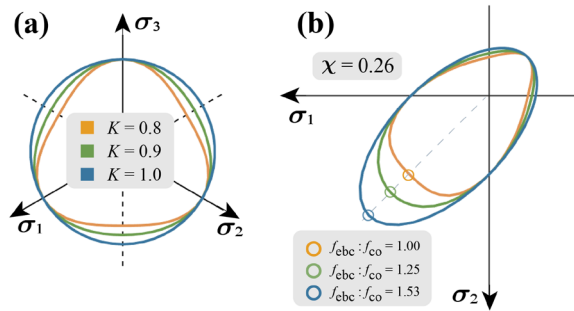


Figure 2.16 Impact of the parameter K on the shape of the Abaqus extended Drucker-Prager surface: (a) Deviatoric plane; (b) Plane stress space.

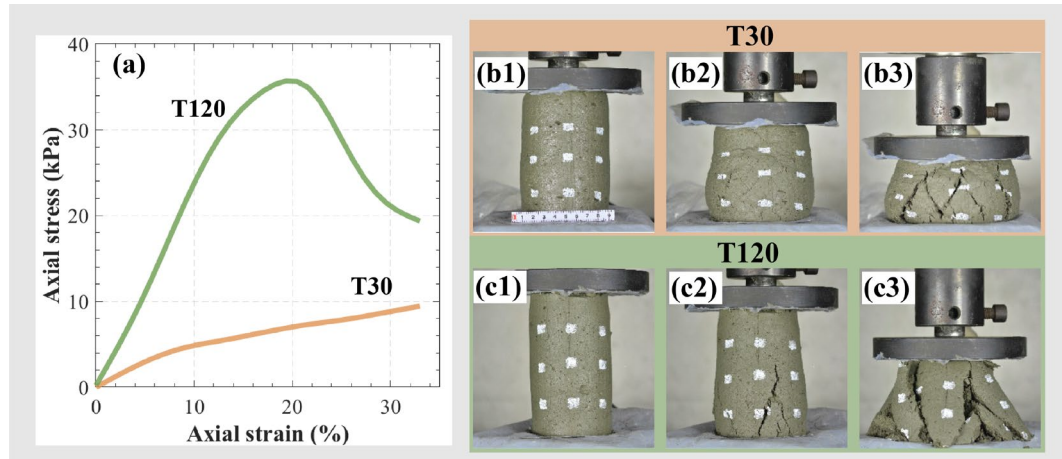


Figure 2.17 Compressive behaviour and progressive crack development in fresh concrete specimens at relatively early hydration stages: (a) True stress-engineering strain relationships¹; (b) Crack evolution in 30-minute-aged specimen [modified from Ding *et al.* (2020)]: (b1) initial, (b2) intermediate, and (b3) final loading stages; (c) Crack evolution in 120-minute-aged specimen [modified from Ding *et al.* (2020)]: (c1) initial, (c2) intermediate, and (c3) final loading stages.

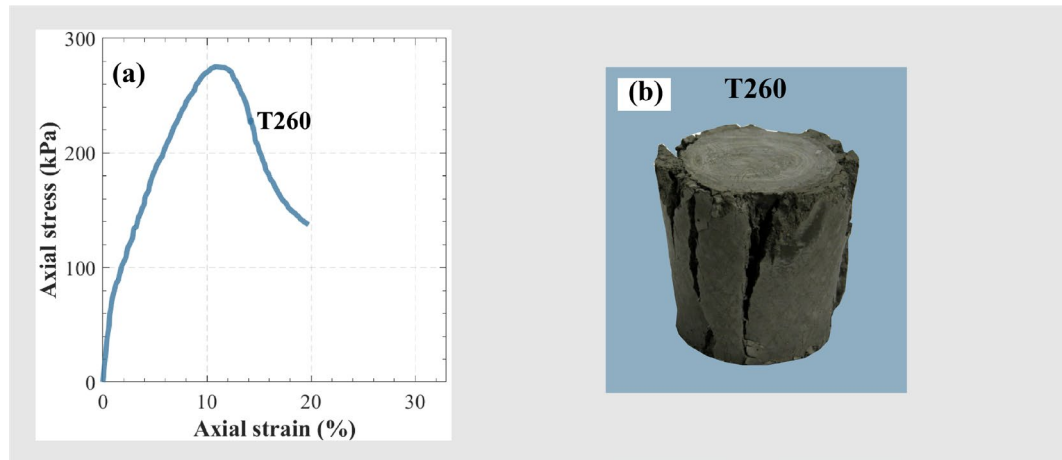


Figure 2.18 Compressive behaviour and progressive crack development in fresh concrete specimens at relatively early hydration stages: (a) Engineering stress-strain relationship²; (b) Final crack distribution pattern observed upon test completion [modified from Mettler *et al.* (2016)].

¹ Stress-strain data in Figure 2.17a were read from Figure 9 in Ding *et al.* (2020).

² Stress-strain data in Figure 2.18a were converted from the force-displacement data read from Figure 2 in Mettler *et al.* (2016)

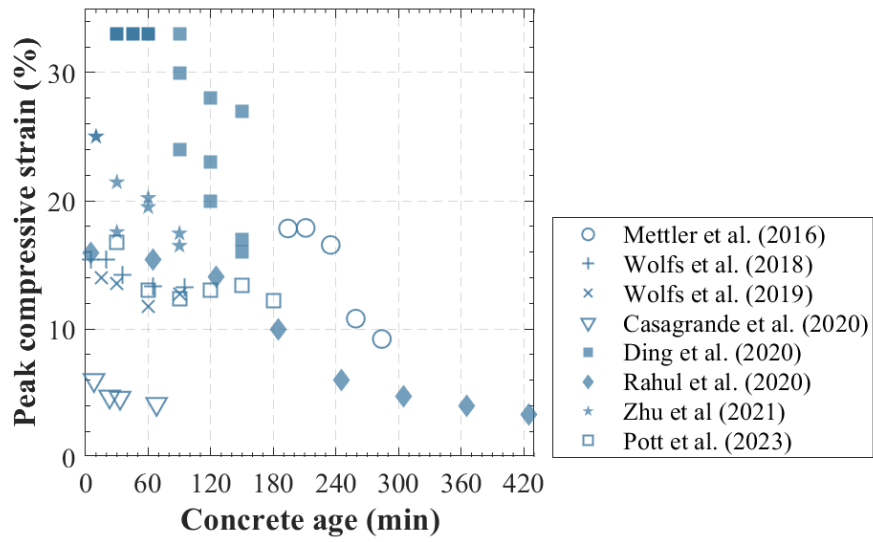


Figure 2.19 Influence of material ageing on the peak compressive strain.

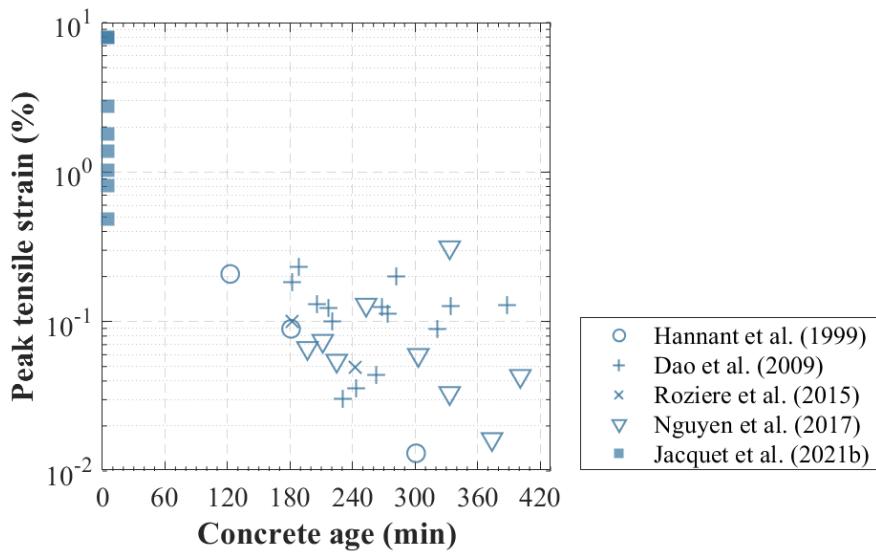


Figure 2.20 Influence of material ageing on the peak tensile strain.

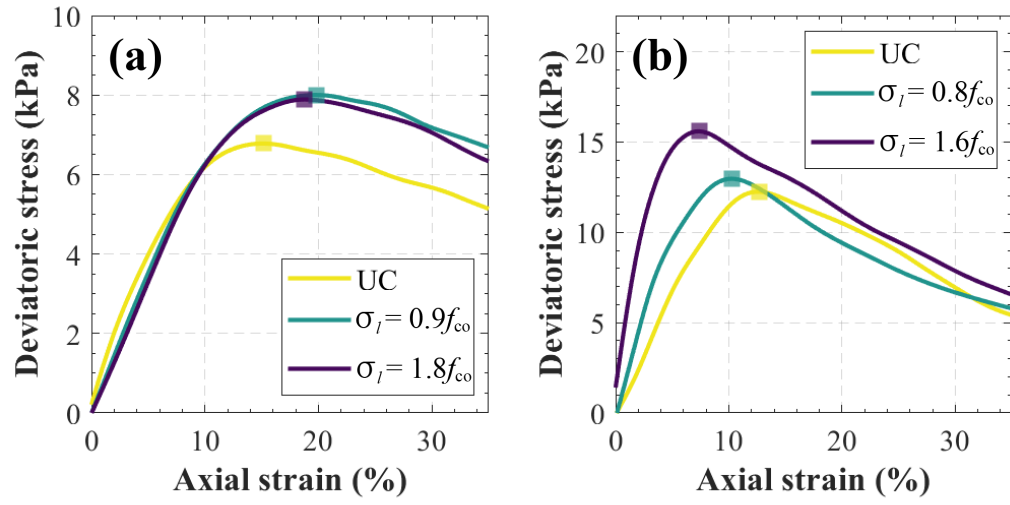


Figure 2.21 Deviatoric stress-axial strain curves of fresh concrete under triaxial compression: (a) age = 15 minutes; (b) age = 60 minutes¹.

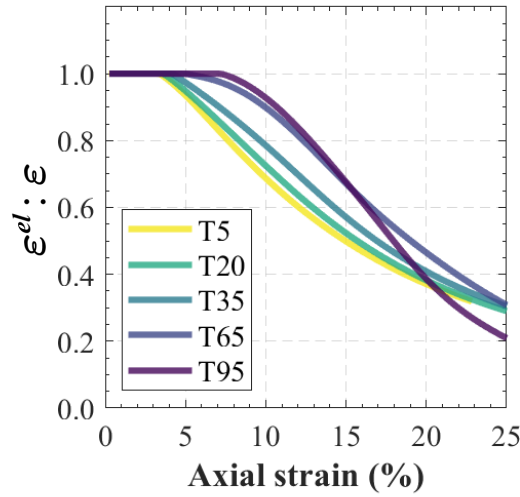


Figure 2.22 Ratio of elastic strain to total strain².

¹ Data were read from Figure 10 in Wolfs *et al.* (2019).

² The ratios of elastic-to-total strain were derived from the stress-strain data read from Figure 8, utilising the age-dependent apparent Young's modulus formulated Eq. (5) in Wolfs *et al* (2018).

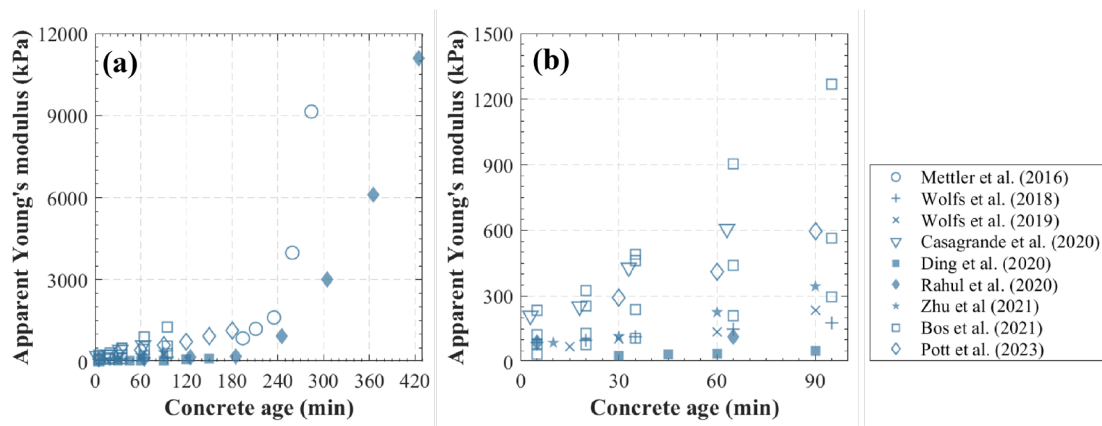


Figure 2.23 Evolution of apparent Young's modulus with concrete age for fresh concrete in different studies: (a) Over the age range of 0 to 7 hours; (b) Over the age range of 0 to 100 minutes.

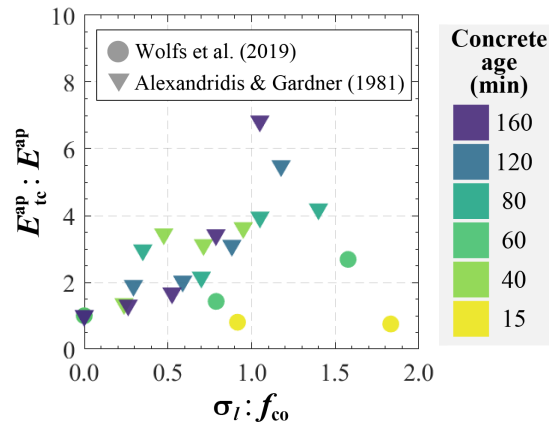


Figure 2.24 Ratios of apparent Young's modulus derived from triaxial compression tests at different confining stresses (E_{tc}^{ap}) to that derived from uniaxial compression tests.

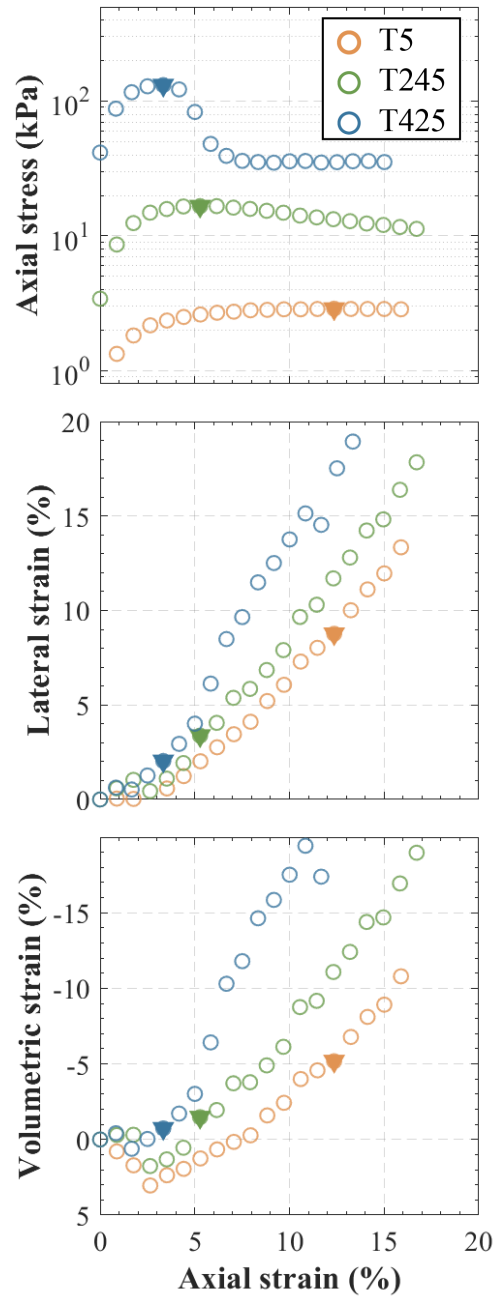


Figure 2.25 Evolution of mechanical response in fresh concrete under uniaxial compression at progressive hydration stages¹: (a) True axial stress versus engineering axial strain relationship; (b) Engineering lateral-to-axial strain relationship; (c) Engineering volumetric-to-axial strain relationship.

¹ The data in (a) were converted from the stress-displacement data read from Figure 7 in Rahul and Santhanam (2020). The data in (b) and (c) were derived from the stress-displacement data read from Figure 7 and force-displacement curves read from Figure 8 from the same study.

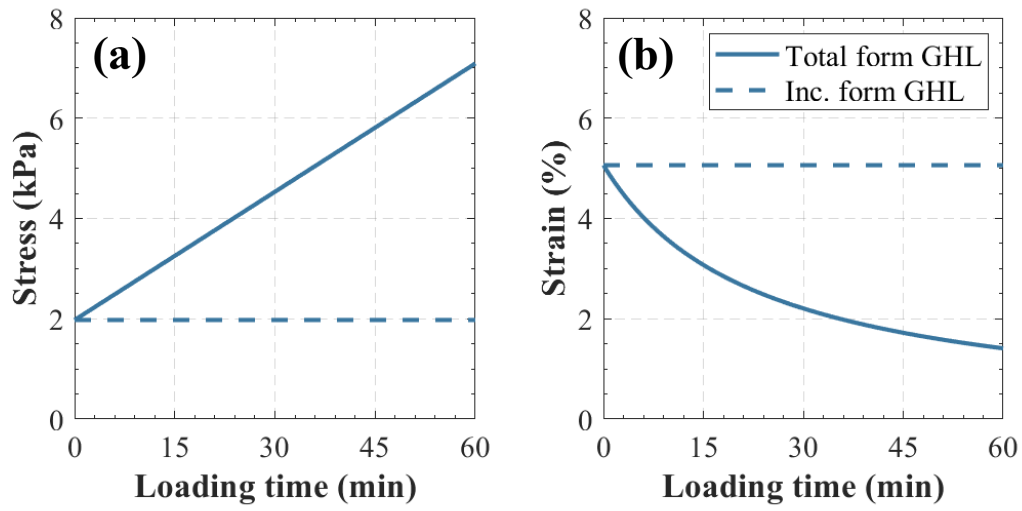


Figure 2.26 Material responses calculated from total form GHL and incremental form GHL models: (a) When subjected to a constant strain of 1%; (b) When subjected to a constant stress of 2 kPa.

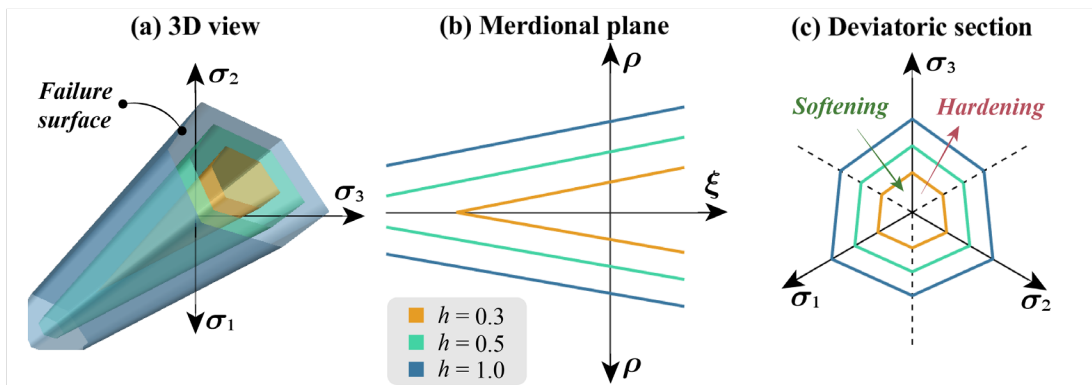


Figure 2.27 Evolution of the Mohr-Coulomb yield surfaces during strain hardening and softening (at a constant concrete age).

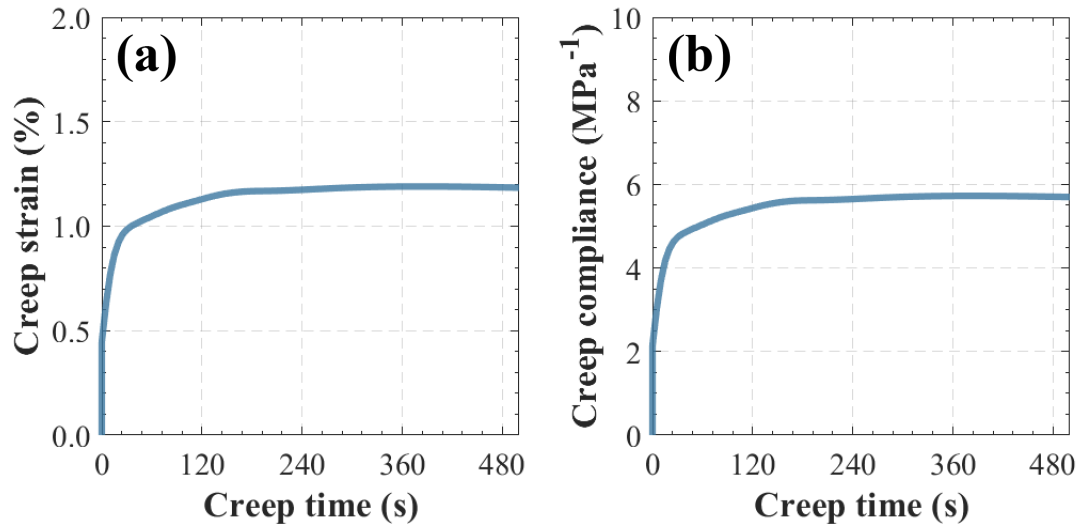


Figure 2.28 Creep behaviour of fresh 3DPC under uniaxial compression¹: (a) A typical creep curve; (b) The corresponding creep compliance curve.

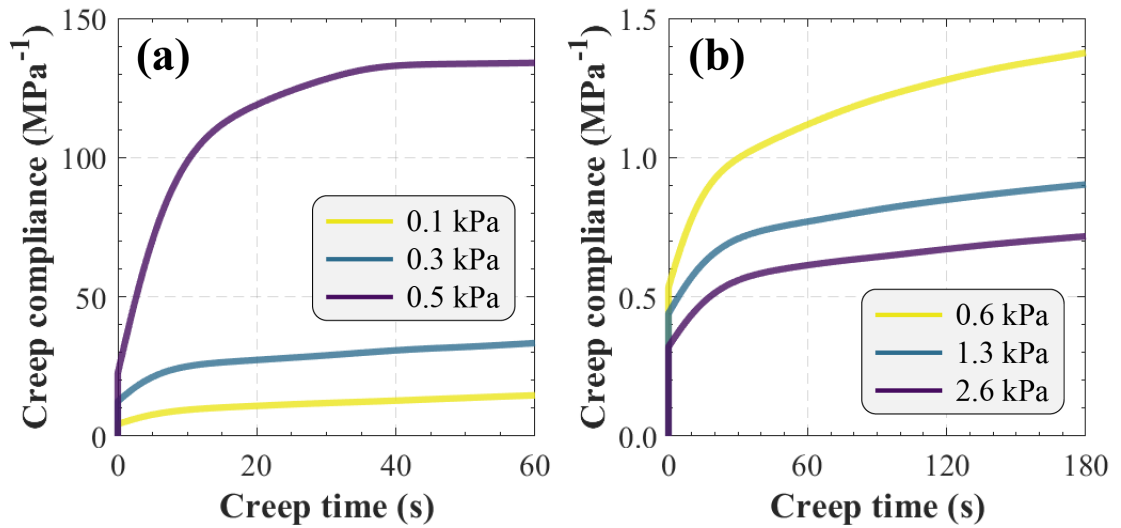


Figure 2.29 Creep compliance curves at different force levels²: (a) Rheologic shear creep tests; (b) Uniaxial compressive creep tests.

¹ Data in Figure 2.28a was read from Figure 7 (specimen aged 30 minutes) in Esposito *et al.* (2021). Data in Figure 2.28b was calculated from the extracted creep data.

² Data in Figure 2.29a were read from Figure 11 in Chen *et al.* (2020), and the data in Figure 2.29b were read from Figure 18 in Chang *et al.* (2023c).

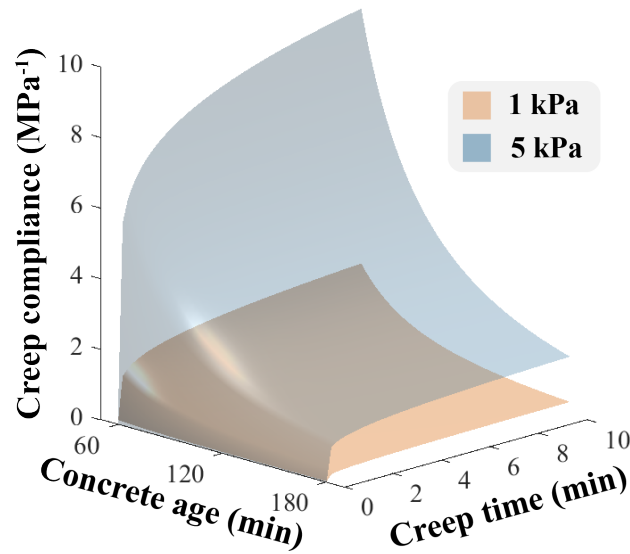


Figure 2.30 Creep compliance surfaces at two different stress levels¹.

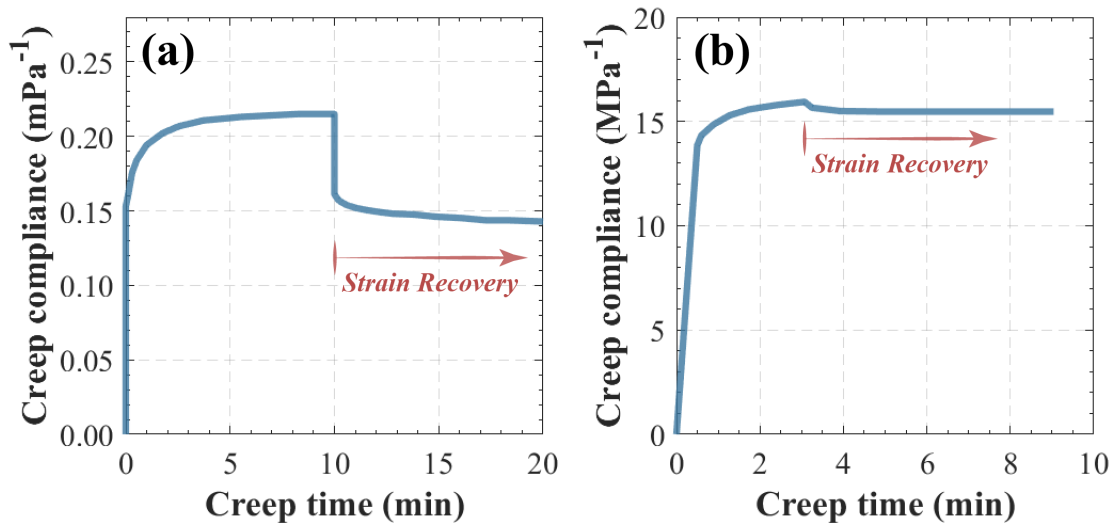


Figure 2.31 Strain recovery behaviour of fresh concrete²: (a) Shear creep test; (b) Uniaxial compressive creep test.

¹ Each surface was derived from the creep test data read from Figure 11 in Xia *et al.* (2024) by fitting a surface to the creep data corresponding to a single stress level at three concrete ages.

² Data in Figure 2.31a were read from Figure 17 (specimen CVS) in Kolawole *et al.* (2021), and data in Figure 2.31b were converted from the creep displacement-time data read from Figure 6b in Chang *et al.* (2023c).

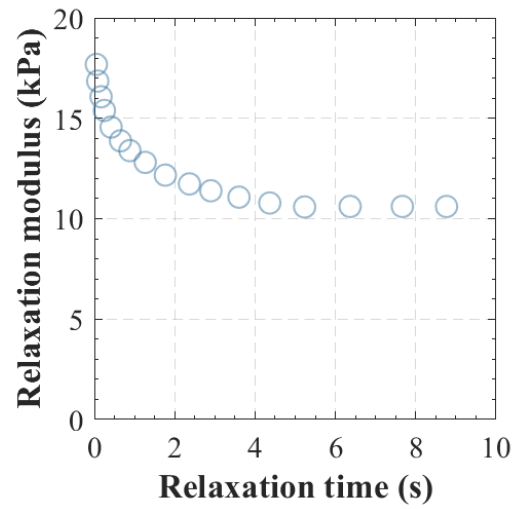


Figure 2.32 Characteristic relationship between relaxation modulus and relaxation time for fresh concrete¹.

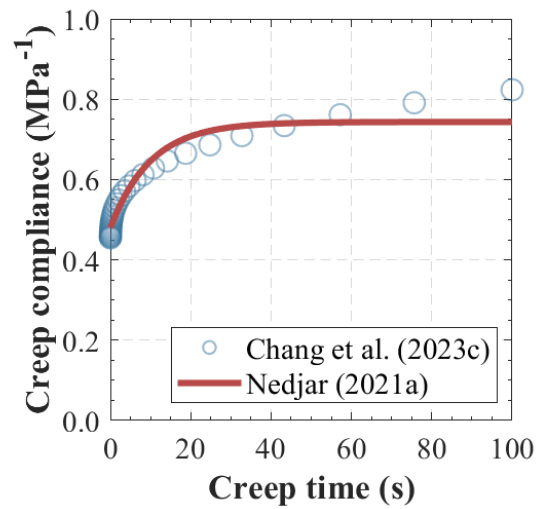


Figure 2.33 Assessment of predictive capability of Nedjar's (2021a) rheologic model in fitting creep test data.

¹ Data were read from Figure 12 (specimen C) in Kolawole et al. (2020).

APPENDIX 2.1 EXPLANATION OF PRINCIPAL STRESS AND HAIGH–WESTERGAARD SPACES

The representation of the state of stress can be visualized through two distinct coordinate systems: the principal stress space (a Cartesian coordinate system) and the Haigh-Westergaard stress space (an inclined cylindrical coordinate system), as depicted in Figure 2.1a. In the principal stress space, the stress point is characterised by three coordinates, which are the maximum, intermediate and minimum principal stresses ($\sigma_1, \sigma_2, \sigma_3$) of the stress tensor ($\boldsymbol{\sigma}$).

In the Haigh-Westergaard stress space, a stress point is defined by three Haigh-Westergaard coordinates:

$$\begin{cases} \xi(\boldsymbol{\sigma}) = I_1 / \sqrt{3} \\ \rho(\boldsymbol{\sigma}) = \sqrt{2J_2} \\ \theta(\boldsymbol{\sigma}) = \frac{1}{3} \cos^{-1} \left(\frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{1.5}} \right) \end{cases} \quad (2.23)$$

Here, I_1 is the first invariant of the stress tensor, while J_2 and J_3 are respectively the second and third invariant of the deviatoric stress tensor. These quantities remain invariant with respect to the chosen coordinate system and can be expressed as functions of the three principal stresses:

$$\begin{cases} I_1(\boldsymbol{\sigma}) = \sigma_1 + \sigma_2 + \sigma_3 \\ J_2(\boldsymbol{\sigma}) = \frac{1}{6} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \\ J_3(\boldsymbol{\sigma}) = \left(\sigma_1 - \frac{I_1}{3} \right) \left(\sigma_2 - \frac{I_1}{3} \right) \left(\sigma_3 - \frac{I_1}{3} \right) \end{cases} \quad (2.24)$$

Figure 2.1 illustrates the relationship between the principal stress space (σ_1 - σ_2 - σ_3) and the Haigh-Westergaard stress space (ξ - ρ - θ). Notably, the symbols ξ' , ρ' and θ' denote the specific coordinate values of the stress point. As shown, the *hydrostatic length* (ξ) represents the distance from the origin to the projection of the stress point onto the hydrostatic axis, where $\sigma_1=\sigma_2=\sigma_3$. The *deviatoric length* (ρ) measures the distance from the stress point to its projection on the hydrostatic axis. A *meridional plane* is any plane that contains the hydrostatic axis, as shown in Figure 2.1a. Specifically, for illustration, the meridional plane passing through the stress point is depicted in Figure 2.1b. The *deviatoric plane* is defined as the plane perpendicular to the hydrostatic axis, as illustrated in Figure 2.1a. The *lode angle* (θ) describes the orientation of the stress point within the deviatoric plane, measured from the projection of the positive direction of the σ_1 -axis, as illustrated in Figure 2.1c.

APPENDIX 2.2 IMPACT OF HARDENING FUNCTION IN VISCOPLASTIC MODEL ON CREEP PREDICTION.

To illustrate the impact of the hardening function in a viscoplastic model on its prediction of creep, two numerical tests were conducted. In both tests, a material point was subjected to a constant uniaxial compressive stress for 60 seconds. The first test employed a viscoplastic model without strain hardening, while the second utilised a model with strain hardening. The hardening functions q for the viscoplastic models used in both tests are summarised in Table A.2.1 and are plotted in Figure A.2.1a. Notably, in the case of uniaxial compression, the axial viscoplastic strain (ϵ_1^{vp}) can be directly used as the history variable, that is $\kappa = \epsilon_1^{vp}$.

The static yield criterion under uniaxial compression can be simply expressed as:

$$f_s = \sigma - f_{co.s} q(\epsilon_1^{vp}) \quad (2.25)$$

Here, $f_{co.s}$ represent the static uniaxial compressive strength. By adopting the overstress theory, the axial viscoplastic strain rate ($\dot{\epsilon}_1^{vp}$) can be expressed using a power law, which has been widely used in existing studies (Lazari *et al.*, 2019), as follows:

$$\dot{\epsilon}_1^{vp} = \gamma \left(\frac{f_s}{\sigma_{y0}} \right)^{N_0} \quad (2.26)$$

Here, the parameters γ and N_0 govern the strain-rate-dependent behaviour, and their values are summarised in Table A.2.1. The parameter σ_0 in the denominator represents the initial yield stress, with $\sigma_{y0} = h_0 f_{co.s}$, where h_0 is the proportional limit, assumed to be 0.1.

Substituting Eq. (2.25) into Eq. (2.26) yields the following equation:

$$\dot{\epsilon}_1^{vp} = \gamma \left[\frac{\Gamma_{cr} - q(\epsilon_1^{vp})}{h_0} \right]^{N_0} \quad (2.27)$$

Here, the parameter $\Gamma_{cr} = \sigma: f_{co.s}$ represents the creep stress level. In both numerical tests, Γ_{cr} was taken as 0.3.

Figure A.2.1b presents the creep curves predicted by both models. Evidently, the viscoplastic model without strain hardening predicts a linear relationship between creep strain and time, whereas the model with hardening accurately predicts a nonlinear evolution of creep strain, consistent with experimental observations.

Table A.2.1 Material parameters of models used in both numerical tests.

Parameter/function	w/o Hardening	w Hardening
q	$q(\epsilon_1^{vp}) = 0.27$	$q(\epsilon_1^{vp}) = -90\epsilon_1^{vp^2} + 18\epsilon_1^{vp} + 0.1$
γ	1×10^{-3}	
N_0	1.5	
h_0	0.1*	0.1

Note: * h_0 was assumed to be 0.1 here to be consistent with the model with hardening.

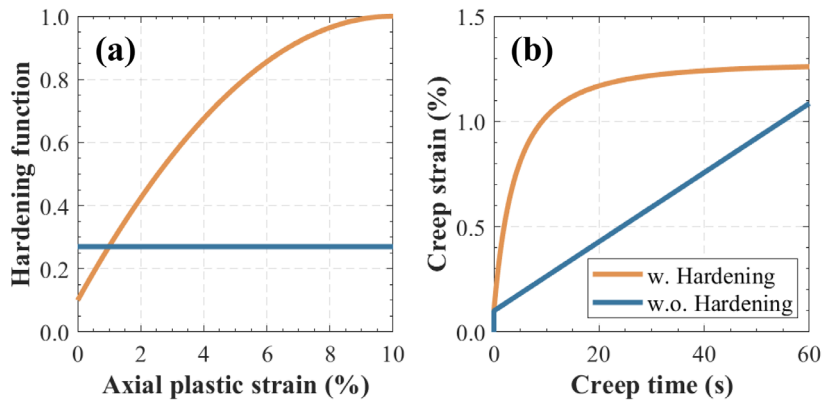


Figure A.2.1 (a) Hardening function adopted in the numerical example; (b) Prediction of creep by viscoplastic models with and without strain hardening. Note: an elastic strain of 0.1% was assumed in both tests.

CHAPTER 3

A BIAXIAL COMPRESSION TESTING PROGRAMME FOR FRESH 3D-PRINTED CONCRETE

3.1 INTRODUCTION

The comprehensive literature review presented in Chapter 2 highlighted a significant research gap: the absence of experimental data characterising fresh concrete behaviour under biaxial compression. To address this limitation, this chapter presents an extensive experimental study designed to investigate the mechanical behaviour of fresh 3D-printed concrete (3DPC) under biaxial loading conditions.

The experimental investigation spans concrete ages from minutes to hours, corresponding to the critical timeframe during 3D concrete printing operations. Central to this investigation is the development of an innovative testing apparatus specifically engineered for conducting biaxial compression tests on fresh 3DPC specimens. The reliability and accuracy of this apparatus is validated through comprehensive finite element analyses. Additionally, this chapter details the mixture proportions employed and establishes rigorous protocols for specimen preparation and testing procedures.

The experimental data obtained from this investigation provide fundamental insights into the biaxial compressive behaviour of fresh concrete, establishing a robust foundation for the development of advanced constitutive models. Detailed presentations and analyses of the experimental results are elaborated in subsequent chapters.

3.2 MATERIAL AND SPECIMENS

3.2.1 Mixture Proportions

The 3DPC utilised in the current study comprises binder materials, fillers, tap water, a superplasticiser, and a viscosity-modifying agent. The specific mixture proportions are detailed in Table 3.1, which has been adapted from the concrete mixture in Teng *et al.* (2019). The binder consists of Portland cement grade P.II 52.5R conforming to the Chinese national standard GB175-2007 (SAC, 2007), along with silica fume. The filler materials comprise quartz powder with a mean particle diameter of 7.47 μm and river sand with a maximum particle diameter of 1.18 mm ($d_{\text{max}} = 1.18 \text{ mm}$). Notably, this 3DPC mixture excludes coarse aggregates, aligning with widespread practices in real-life 3D concrete printed construction projects (Rehman and Kim, 2021). The mixture maintains specific proportions: a filler-to-binder ratio (f/b) of 1.1 and a water-to-binder ratio (w/b) of 0.26. These proportions fall within the typical ranges established for 3DPC mixtures without coarse aggregates, which, according to a comprehensive review by Rehman and Kim (2021), are characterised by w/b ratios of 0.16-0.49, f/b ratios of 0.38-2.5, and maximum particle diameters (d_{max}) below 2.5 mm. To enhance the workability of 3DPC during mixing, a powder form polycarboxylate superplasticiser was added to the mixture. A Hydroxypropyl methylcellulose viscosity modifying agent (VMA), which has been widely used in previous studies (Marchon *et al.*, 2018; Zhu *et al.*, 2021), was employed to achieve sufficient initial compressive strength.

Three supplies of cement and river sand were used in the present study due to the large quantity of specimens. Despite being sourced from the same supplier, variations in the material properties of the cement and river sand were observed. To minimise the influence on material properties made of different supplies of cement and river sand, the amount of VMA used in the mixtures was adjusted while the proportions of the other raw materials were kept constant. This adjustment was performed to achieve a consistent uniaxial compressive strength at age of approximately 20 minutes for materials made from different supplies.

3.2.2 Specimen Preparation

Given the absence of a specific testing standard for fresh 3DPC, the shape of the specimen has been designed according to the standard for cohesive soils, ASTM D2166 (ASTM International, 2016), in existing studies (Casagrande *et al.*, 2020; Ivanova *et al.*, 2022; Wolfs *et al.*, 2019). According to this standard, the minimum dimension of the specimen should be at least six times greater than the maximum aggregate size (d_{max}). In the current investigation, cubic specimens with a 75 mm side length were adopted for uniaxial (UC) and biaxial compression (BC) tests. This results in a side length to d_{max} ratio of 64, which satisfies the requirement of ASTM D2166 that the sample size should be 6 times that of the maximum coarse aggregate size. This requirement ensures that the sample encompasses a uniform distribution of particles and is therefore representative of the mechanical performance of the material.

The mixing process included the following steps: (1) Dry binder materials, quartz powder, and superplasticiser were first homogenised in a mixer operating at 148 rpm for 5 minutes; (2) Water was subsequently added to the homogenised dry materials in three incremental stages and mixed at 244 rpm for 7 minutes, followed by 1 minute at 480 rpm. Upon completion of this step, a consistent and flowable cement paste was achieved. (3) River sand was introduced into the cement paste and mixed at 244 rpm for 1 minute, and then at 480 rpm for an additional minute. This yielded a consistent and flowable mortar. (4) VMA was incorporated and mixed at speeds of 148 rpm for 1 minute, 244 rpm for 1 minute, and 480 rpm for 1 minute. The total mixing duration for the 3DPC in all experiments was around 13 minutes. The fresh 3DPC was subsequently cast into concrete moulds in three layers, each compacted 15 times using a rod. The concrete age was counted immediately after casting, and it was counted to the time point when concrete strength was reached during testing. This definition is considered to be more accurate than counting age to the beginning of testing, as discussed in existing studies (Pott *et al.*, 2022; Wolfs *et al.*, 2019), especially considering that reaching the strength may take up to 5 minutes from the beginning of the test. Demoulding was typically performed approximately 8 minutes after casting.

3.3 TEST SETUP

As shown in Figure 3.1b, under biaxial compression, a fresh concrete specimen is subjected to vertical compression along axis-1 and horizontal compression along axis-2, while remaining stress-free along axis-3. With compression defined as positive, the vertical and horizontal stresses represent the maximum (σ_1) and minimum (σ_2) in-plane principal stresses in the two-dimensional principal stress space. The biaxial stress ratio is denoted as $\beta_s = \sigma_2 : \sigma_1$. A value of $\beta_s = 0$ indicates uniaxial compression, while $\beta_s > 0$ signifies biaxial compression. To investigate the biaxial compressive behaviour of fresh concrete under different biaxial stress conditions, biaxial compression tests were conducted with proportional loading at constant stress ratios of $\beta_s = 0.33, 0.50, 0.70$, and 1.0.

However, during the 3D concrete printing process, fresh concrete experiences passive confinement, resulting in a varying lateral confining stress that depends on the dilation of fresh concrete. This variation leads to a changing stress ratio, indicative of non-proportional loading. It has been well established that the mechanical behaviour of cured concrete is independent of the stress path. For instance, Lan and Guo (1997) demonstrated that concrete specimens subjected to different stress paths reach similar strength. Consequently, it is crucial to ascertain whether the mechanical behaviour of fresh concrete exhibits similar path independence. If so, test data obtained from proportional loading can be directly utilised to analyse the response of 3DPC during printing. However, if the behaviour of fresh concrete is path-dependent, it will be necessary to assess its mechanical properties under stress paths that closely resemble those encountered during the printing process to ensure accurate analysis. Moreover, while it is qualitatively understood that fresh concrete experiences passively confined biaxial compression during printing (see Figure 1.4), a quantitative analysis of the variation in the biaxial stress ratio would provide additional valuable insights.

To this end, a series of passively confined biaxial compression tests were carried out in which the specimen was compressed vertically while lateral dilation in the horizontal direction was restricted. Consequently, the horizontal platens applied a varying horizontal stress to the specimen and causing β_s to vary. The passively confined tests

were conducted to identify the level of horizontal stress in fresh 3DPC when subjected to very high lateral confining stiffness, which is a good estimate of the upper bound for the biaxial stress ratio of fresh concrete during the 3DCP process. Moreover, comparing the test data of passively confined tests to the proportional loading BC tests reveals whether the mechanical behaviour of 3DPC is stress path-dependent.

3.3.1 Loading System

All tests were conducted using a GDS True Triaxial testing machine (GDS Instruments, 2023), equipped with four electro-mechanical actuators: two aligned with the vertical axis 1 and two with the horizontal axis 2, as shown in Figure 3.1a. This enables independent control of load and displacement along each axis. For the concrete specimens within the studied age range, the peak load in axis 1 during the tests varied from approximately 0.1 kN to over 10 kN. Therefore, each actuator was equipped with a 20 kN load cell with a nominal accuracy of 25 N. Preliminary tests confirmed that the actual accuracy of the load cells was sufficient for specimens with a capacity below 1 kN, with further details available in Appendix 3.1. Each actuator has a stroke of 10 mm. Given that the side length of the specimen was 75 mm, the stroke allowed for a maximum engineering strain of 26.7%, which was sufficiently large for specimens with ages of tens of minutes to reach their strength as demonstrated in existing UC tests (Wolfs *et al.*, 2018). The actuator displacement was measured with internal Linear Variable Differential Transformers (LVDT) with a range of 50 mm and a nominal accuracy of 0.05 mm.

3.3.2 Design of Loading Platens

Extensive studies have been conducted to study the biaxial behaviour of cured concrete (Kupfer *et al.*, 1969; Wang *et al.*, 1987). In these tests, the specimens were loaded using a platen system similar to the one depicted in Figure 3.1b. Notably, the platens in these studies were designed to not fully cover the loaded surfaces of the cubes, i.e., W_1 and W_2 were slightly smaller than L_1 and L_2 , to avoid collision between platens during the biaxial loading process. Unlike cured concrete, 3DPC is much more deformable as previously shown in Chapter 2, such that up to 25% of axial and horizontal strain is

expected at ages within an hour. To accommodate this deformability, W_1 and W_2 must be considerably smaller than L_1 and L_2 to prevent collisions. However, if the platens are too small, the uniformity of stress and strain distribution within the cubic specimen may be compromised, raising concerns about the reliability of the calculated stress and strain values.

In the present study, three sets of platens (as summarised in Table 3.2) were designed based on the results of preliminary tests in which cube specimens were loaded under different biaxial stress ratios, reaching up to 25% axial strain without collision. For UC tests, only the two vertical platens (Set 1 in Table 3.2) were used. These platens had a rectangular loading surface of 90×120 mm, compared to the 75×75 mm loading surface of the cube specimen. The larger platen face ensured that the specimen loading surfaces were completely covered throughout the loading process. For BC tests with $\beta_s < 1$ and the passively confined biaxial compression tests, both vertical and horizontal platens (Set 2 in Table 3.2) were used, and the out-of-plane width (W_3) was kept to 120 mm. However, to avoid collision, W_1 and W_2 were reduced to 70 and 55 mm, respectively. Finally, for the equibiaxial compression tests (Set 3 in Table 3.2), the in-plane widths of the platens (W_1 and W_2) were set to 55 mm, while the out-of-plane width (W_3) was kept 120 mm. To minimise friction during testing, PTFE films were utilised at the interfaces between the specimens and the loading platens.

3.3.3 Stress and Strain Measurement

The stress and strain were calculated from the force (F) and displacement (u) data recorded by the testing machine. In this chapter, true stress and engineering strain are utilised.

Figure 3.2 shows the stress and strain conditions of specimens in uniaxial compression and biaxial compression tests. Under UC, as shown in Figure 3.2b, the specimen is subjected to positive axial stress ($\sigma_1 > 0$) and strain ($\epsilon_1 > 0$) and negative lateral strains ($\epsilon_2 = \epsilon_3$). Under BC, as shown in Figure 3.2c, the specimen is subjected to positive stresses on side 1 and side 2 with $\sigma_1 \geq \sigma_2 > 0$. In this case, ϵ_1 is always

positive, while ϵ_2 could be negative when σ_2 is significantly smaller than σ_1 . Meanwhile, the specimen exhibits a negative ϵ_3 .

In UC tests, axial strain (ϵ_1) along axis 1 was determined from the relative displacement of actuators along axis 1 (u_1). In BC tests, both the axial strain (ϵ_1) along axis 1 and the horizontal strain (ϵ_2) along axis 2 were calculated from the relative displacement of the corresponding actuators (u_1, u_2). However, accurately measuring the free-dilating lateral deformation (ϵ_l), specifically $\epsilon_l = \epsilon_2 \approx \epsilon_3$ in UC tests and $\epsilon_l = \epsilon_3$ in BC tests, proves to be challenging during the loading process. The use of contact-based displacement measurement devices (e.g., LVDT) inevitably introduces an extra confining stress along axis 3. Although non-contact digital image correlation methods have been employed in existing literature to measure the lateral deformation of 3DPC under triaxial compression (Wolfs *et al.*, 2019) and uniaxial compression (Zhu *et al.*, 2021; Pott *et al.*, 2022), this approach is not applicable to the current test setup. This is because the image-based method requires the analysis of photos taken at an angle to the measuring direction. In the current configuration, however, the view at an angle to axis 3 will be partial obstructed, as the dimension of the loading platens (L_3) exceeds that of the specimen (W_3) along axis 3. While more advanced displacement measuring system (e.g., laser displacement sensor) could potentially resolve this issue, their implementation is time-consuming and could delay the earliest age at which compression testing can commence, which is a primary concern of this investigation.

Therefore, the free-dilating lateral strains (ϵ_l) during the loading process was calculated via an interpolation approach. To be specific, dimensions along the free dilating direction(s) of the deformed specimens at the conclusion of the loading process were measured by using a calliper with an accuracy of ± 0.03 mm. For specimens tested under UC, the maximum dimensions on sides 2 and 3 (Figure 3.2a) were recorded. On the other hand, both the maximum and minimum dimensions on side 3 were measured for specimens under BC. This disparity arises from the relatively uniform free-dilating deformation in UC tests, attributed to the absence of frictional stress from side platens, whereas the side platens introduce non-uniformities in free-dilating deformation for specimens tested under BC.

Using the measured deformed dimensions, the free-dilating lateral strain at the end of the loading process ($\epsilon_{l.u}$) was calculated. In UC tests, ϵ_2 and ϵ_3 are slightly different; thus, $\epsilon_{l.u}$ was taken as the average. For BC tests, $\epsilon_{l.u}$ was simply taken as ϵ_3 calculated from the averaged dimensions of side3. The intermediate strains throughout the loading process were reconstructed via interpolation by assuming a relationship between the free-dilating lateral strain and the axial strain as follows in Eq. (3.1):

$$\epsilon_l = A\epsilon_1^2 - \nu^* \epsilon_1 \quad (3.1)$$

where the coefficient A was determined by using $\epsilon_{l.u}$ and the corresponding ϵ_1 . The parameter ν^* is the apparent Poisson's ratio which depends on the stress state of the specimen, as detailed in Appendix 3.2. The validity of the proposed model is demonstrated in Figure 3.3, showing that the predicted axial-lateral strain relationship closely aligns with experimental data from both uniaxial (Chang *et al.*, 2023) and triaxial compression tests (Wolfs *et al.*, 2019) for fresh 3DPC.

3.3.4 Strain Rate

In all tests, the axial load along axis 1 was applied via displacement control at a constant rate. For equibiaxial compression tests with $\beta_s = 1.0$, the load along axis 2 was also applied using displacement control at an identical rate to that of axis 1. For the other biaxial compression tests with $\beta_s < 1.0$, the load along axis 2 was applied via load control, maintaining a constant ratio β_s throughout the testing process. In the passively confined biaxial compression tests, the platens along axis 2 were kept stationary, allowing the specimens to be passively confined along axis 2.

For the loading scheme described, selecting an appropriate axial strain rate (displacement rate) is crucial. As previously demonstrated in Chapter 2, the mechanical behaviour of fresh concrete is influenced by both its age and the applied strain rate. A higher strain rate leads to a shorter test duration, which is desirable to minimise the influence of material ageing during testing. However, higher strain rates may lead to misleading measurements, as explained below. Due to the viscous nature of 3DPC, its mechanical response is affected by the applied strain rate. The strain rate effect can be

quantified by an effective strain rate $\dot{\epsilon}_{eff}$, defined as the Euclidean norm of the applied strain rate tensor $\dot{\epsilon}_{ap}$. For instance, $\dot{\epsilon}_{eff}$ is equal to $\dot{\epsilon}_1$ under uniaxial compression and $\dot{\epsilon}_{eff} = \sqrt{\dot{\epsilon}_1^2 + \dot{\epsilon}_2^2}$ under biaxial compression. This indicates that, despite identical axial strain rates, the effective strain rates, which govern the rate-dependent behaviour of the material, differ between specimens under uniaxial and biaxial compression. In the case of equibiaxial compression, which is the most critical case, $\dot{\epsilon}_{ap}$ is 1.4 times that of uniaxial compression with the same axial strain rate. This discrepancy in effective strain rate can result in non-objective comparisons between test results from UC and BC tests.

Therefore, the strain rate should be judiciously determined to minimise the effects of ageing and rate-dependence. A series of preliminary UC tests were conducted under different axial engineering strain rates, $\dot{\epsilon} = 5, 12.5, 25, 50$ %/min, at various ages. Figure 3.4 shows the influence of strain rate on the strengths normalised to that at 5%/min. When the strain rate ranges from 5 %/min to 25 %/min, the strength increases by less than 5%. But it increases over 10% from 25 %/min to 50 %/min. Therefore, for conservative purposes, an axial strain rate of 5 %/min was selected for the current experimental program. It is anticipated that the influence of the difference in effective strain rate between UC and biaxial compression (BC) tests on the results is less than 1% (or at most 5%).

3.4 VERIFICATION OF THE PROPOSED TEST SETUP

Although the platen design successfully avoided collision, the absence of direct contact between the cube specimen corners and the platens potentially generated non-uniform stress distributions within the specimens, which could affect the measured data. To verify the efficacy of the proposed test setup, a series of finite element (FE) analyses was conducted using Abaqus (Dassault Systemes, 2023) to simulate the biaxial compression of cube specimens.

The numerical simulations replicated both uniaxial compression and biaxial compression conditions with four distinct biaxial stress ratios ($\beta_s=0.33, 0.50, 0.70$ and 1.0), corresponding to the experimental investigation. The strengths of the simulated specimens were determined from the extracted reaction force-displacement curves,

employing identical data processing methodologies to those utilised in the physical experiments.

The investigation employed two distinct platen configurations: Configuration 1 implemented the proposed platen system with dimensions specified in Table 3.2, whilst Configuration 2 utilised extended platens that fully encompassed the specimens (a condition achievable in FE analyses without physical interference). The validity of the proposed platen design was assessed through comparative analysis of the predicted strengths from both configurations against the strength parameters of the input constitutive model.

3.4.1 Geometry, Boundary Conditions and Mesh Configuration

Due to symmetry, only one-eighth of the specimen and one-quarter of the testing device were modelled. Figures 3.5 a and b depict the schematics for the FE models with platen configuration 1 and configuration 2, respectively. The 3DPC was assumed to be deformable, whereas the platens were assumed to be rigid. This assumption is justified, as the elastic modulus and strength of 3DPC are at least two orders of magnitude lower than those of steel. Consequently, the 3DPC cube was simulated using 3D linear solid elements (C3D8) with a full integration scheme, while the platens were treated as hollow shells (for models with platen configuration-1) or flat shell (for models with platen configuration-2) and modelled using 3D rigid elements (R3D4). The optimal mesh size was determined through a mesh convergence study; in this investigation, a global mesh size of $1\text{ mm} \times 1\text{ mm} \times 1\text{ mm}$ was employed. A reference point was assigned to each platen for the purpose of force/displacement assignment or extraction.

3.4.2 Contact Behaviour

The interfaces between 3DPC and platens are defined as contact pairs. The contact in the normal direction is defined as ‘hard contact’, while the tangential contact was defined using ‘penalty friction’. It was assumed that there was no friction between the specimen and loading platens; therefore, the friction coefficient was set to zero.

3.4.3 Load Application

The servo-controlled algorithm of the GDS testing machine, which ensures that the true stresses along the two loading axes remain proportional, was integrated into Abaqus through a user-defined amplitude subroutine called VUAMP. In this subroutine, the reaction force (F_1) and displacement (u_1) of the platen along axis 1 as well as the displacement (u_2) of platen along axis 2 in the last increment were used to calculate the updated load-bearing area (A_i) along each axis. The applied force of platen along axis 2 was then calculated by using the following equation:

$$F_2 = \frac{F_1 A_1}{\beta_s A_2} \quad (3.2)$$

In this way, the loading process in FE simulation is analogous to that in the experiments. Similar to the experiments, the mechanical behaviour of 3DPC was studied under uniaxial compression and biaxial compression with $\beta_s = 0.33, 0.50, 0.70, 1.00$.

3.4.4 Material Constitutive Model

In the finite element analyses, the loading platens were modelled as rigid bodies, requiring only the definition of the material constitutive behaviour for 3DPC. Noteworthy, the precise accuracy of the material model for 3DPC employed in these simulations is not critical. This is because the purpose of this simulation aims to evaluate whether the proposed platen design can effectively capture the intrinsic material behaviour, regardless of the specific material properties assigned. The validation approach compares three sets of material properties: those derived from simulation results using the proposed platen design (Configuration 1), those obtained from the idealised full-contact platen design (Configuration 2), and the properties defined in the input constitutive model. Agreement amongst these properties would validate that the proposed platen design can reliably measure the actual material behaviour of 3DPC, irrespective of its specific characteristics.

For the numerical implementation, the mechanical behaviour of 3DPC was simulated using a combination of linear elastic properties and the Concrete Damage Plasticity (CDP) model available in Abaqus (Dassault Systemes, 2023). The CDP model incorporates a yield criterion developed by Lubliner *et al.* (1989), implementing a non-associated flow rule and allowing user-defined hardening functions for both uniaxial compression and tension responses.

The elastic component of the constitutive model was defined by the Young's modulus (E) and the Poisson's ratio (ν). The plastic component, however, requires defining relevant parameters for the yield function, flow rule and hardening function. The shape of the yield criterion in the plane stress space is controlled by the ratio of equal-biaxial compressive strength to uniaxial compressive strength ($f_{ebc}:f_{bc}$), and the ratio of uniaxial tensile strength to uniaxial compressive strength ($f_{to}:f_{co}$). The flow rule is primarily governed by the dilation angle (φ) which is essentially equivalent to the dilation parameter discussed in Chapter 2. In this study, only the hardening behaviour of 3DPC under compression was considered, with the material assumed to be elastic-perfectly plastic in tension. It should be noted that the behaviour of 3DPC in tension is not expected to significantly affect the results, as the 3DPC is subjected to compression-dominant loads.

Under uniaxial compression, the evolution of the yield stress is expressed as $\sigma_c(\epsilon_1^{pl}) = f_{co} \cdot q(\epsilon_1^{pl})$, where ϵ_1^{pl} is the axial plastic strain and $q(\epsilon_1^{pl})$ is given by Eq. (3.3). The ascending branch of the model was adopted from Etse and Willam (1994)'s model, while the descending branch was derived from Papanikolaou and Kappos (2007)'s model.

$$q(\epsilon_1^{pl}) = \begin{cases} h_0 + (1 - h_0) \sqrt{1 - \left(1 - \frac{\epsilon_1^{pl}}{\epsilon_{1,k}^{pl}}\right)^2} & \forall \epsilon_1^{pl} \leq \epsilon_{1,k}^{pl} \\ \left[1 + \left(\frac{\epsilon_1^{pl} - \epsilon_{1,k}^{pl}}{\kappa_s}\right)^{1.5}\right]^{-1.5} & \forall \epsilon_1^{pl} > \epsilon_{1,k}^{pl} \end{cases} \quad (3.3)$$

Here, h_0 is the proportional limit, $\epsilon_{1,k}^{pl}$ is the equivalent plastic strain at peak stress, and κ_s is a parameter controlling the slope of the descending branch.

The model parameters used in the simulations were summarised in Table 3.3. For parameters not explicitly mentioned in this table, default values were employed. Note that the model parameters were tailored to simulate specimen who shows both hardening and softening behaviour under UC. The FE results for material show plateauing in stress under UC were similar, and therefore are not shown here.

3.4.5 Results of Simulation

Figure 3.6 presents a comparative analysis of normalised strength envelopes obtained from three sources: the input constitutive model, FE simulations with the idealised full-contact platens (Configuration 2), and FE simulations with the proposed platen design (Configuration 1). With platen configuration 2, the simulated strength envelope aligns with the input envelope. It is evident that the results with configuration 2 can serve as the benchmark for comparison. The strength envelope obtained using the proposed platen design (Configuration 1) shows strong correlation with Configuration 2 across most biaxial stress ratios ($\beta_s=0.33, 0.50$ and 0.70), with only a minor deviation observed under equibiaxial loading conditions.

A detailed quantitative comparison of the axial compressive strengths (along axis 1) between the two platen configurations is presented in Table 3.4. The results demonstrate remarkable consistency, with relative differences of less than 1% for most loading conditions ($\beta_s=0.33, 0.50$ and 0.70). Even under the most demanding case of equibiaxial compression ($\beta_s = 1.00$), the proposed platen system underestimates the strength by only 4%. These findings provide strong evidence that the proposed platen system, despite its geometric modifications to prevent collision, can accurately measure the mechanical behaviour of 3DPC under biaxial compression conditions.

3.5 DESIGN OF THE TEST MATRIX

Characterising the biaxial compressive behaviour of 3DPC at a specific age requires both UC test data and BC test data with different stress ratios, all conducted on the same concrete material at the same age. This is particularly challenging since only one specimen can be tested at a time. Consequently, specimens cast from the same batch had to be tested at different ages, and specimens tested at the same age had to be cast from different batches. Specimens from different batches can exhibit considerable variability in material properties. Specifically, previous research (Wolfs *et al.*, 2018; Pott *et al.*, 2022) has demonstrated that inter-batch variability significantly affects compressive strength but has a relatively minimal impact on deformation responses such as peak compressive strain.

To minimise the influence of inter-batch variability on strength analysis, the test matrix was designed to consist of groups of specimens cast from the same batch. In each group, UC and BC strengths were measured on specimens at different ages. To establish their relationship at the same ages, multiple UC strength measurements at different ages were utilised to construct a continuous UC strength–age curve through interpolation and extrapolation, as illustrated in Figure 3.7. Consequently, the measured BC strengths were normalised by the UC strengths at corresponding ages to establish the BC-to-UC strength ratios, which were then used to develop the biaxial strength envelope.

However, due to constraints in specimen fabrication and testing equipment, each test group cast from the same batch was limited to a maximum of eight specimens, typically comprising both UC and BC tests. In total, 42 groups consisting of 302 specimens were tested. Figure 3.8 illustrates the distribution of specimens based on stress ratios and ages. Among them, 155 specimens were tested under UC and 147 under BC with various stress ratios. The BC and UC test data within the same age ranges will be utilised to analyse the mechanical behaviour of the 3DPC at that age range. Furthermore, the specimens with a stress ratio $\beta_s = 0.70$ and the passively confined specimens were tested up to age of 120 minutes.

Each specimen was assigned a two-part designation to facilitate identification. The first part specifies the age (e.g., ‘T15’ for specimen aged 15 minutes), while the second part indicates the stress ratio (e.g., ‘B0’ for uniaxial compression and ‘B1.0’ for equal biaxial

compression). In the remain of this thesis, the notation ‘Txx’ refers to the concrete age in xx minutes, and BC-xx denotes biaxial compression tests with stress ratio $\beta_s = xx$.

3.6 CONCLUDING REMARKS

This chapter has presented a comprehensive experimental investigation into the biaxial compressive behaviour of fresh three-dimensional printed concrete (3DPC), addressing a significant gap in existing research. The chapter began by outlining the mixture proportions of the fresh 3DPC, which have been shown to be representative of typical 3D concrete printing applications, and the procedures for preparing the cube specimens. Subsequently, the test setup of the experimental investigation was explained. A key contribution of this study is the development of an innovative loading platen system, specifically engineered to accommodate the highly deformable nature of fresh concrete under biaxial compression whilst preventing platen collision. Measuring techniques for stresses and strains were described, including an indirect approach for capturing the lateral deformation during uniaxial and biaxial compression tests. To minimise the impact of material ageing and strain rate on the test results, a series of preliminary uniaxial compression tests loaded with different displacement rates was conducted to determine the optimal loading rate. The validity of the proposed test setup was subsequently verified through finite element analyses, which demonstrated that the system can accurately capture the material’s mechanical response under biaxial loading conditions.

The experimental study comprised 302 concrete cube specimens subjected to three distinct loading conditions: uniaxial compression, proportionally loaded biaxial compression with stress ratios ranging from 0.33 to 1.0, and passively confined non-proportionally loaded biaxial compression. These specimens were tested at ages ranging from a few tens of minutes to six hours, encompassing the critical period relevant to practical 3D concrete printing applications.

The extensive dataset generated from this experimental study provides fundamental insights into the mechanical response of fresh 3DPC under biaxial compression, establishing a crucial foundation for developing a comprehensive constitutive model.

The analysis of these experimental results is presented in two subsequent chapters: Chapter 3 examines the strength characteristics, whilst Chapter 4 focuses on the stress-strain behaviour.

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Table 3.1 Mixture proportions (in g/L).

Supply	Binders		Fillers		Water	PCE	VMA
	Cement	SF	QP	RS			
A	830	207	207	913	270	6	9.6
B							9.0
C							3.0

Note: SF: silica fume; QP: quartz powder; RS: river sand; PCE: Polycarboxylate superplasticiser;
VMA: Hydroxypropyl methylcellulose viscosity modifying agent.

Table 3.2 Dimensions of the loading platens for specimens loaded at different stress ratios.

Set	$\beta_s = \sigma_2\!:\!\sigma_1$	Width of the platen [mm]			Thickness
		Vertical platens W ₁	Side platens W ₂	Out of plane W ₃	t [mm]
1	0	90	N/A	120	15
2	0.33	70	55		
	0.50				
	0.70				
	Passively confined				
3	1.0	55	55		

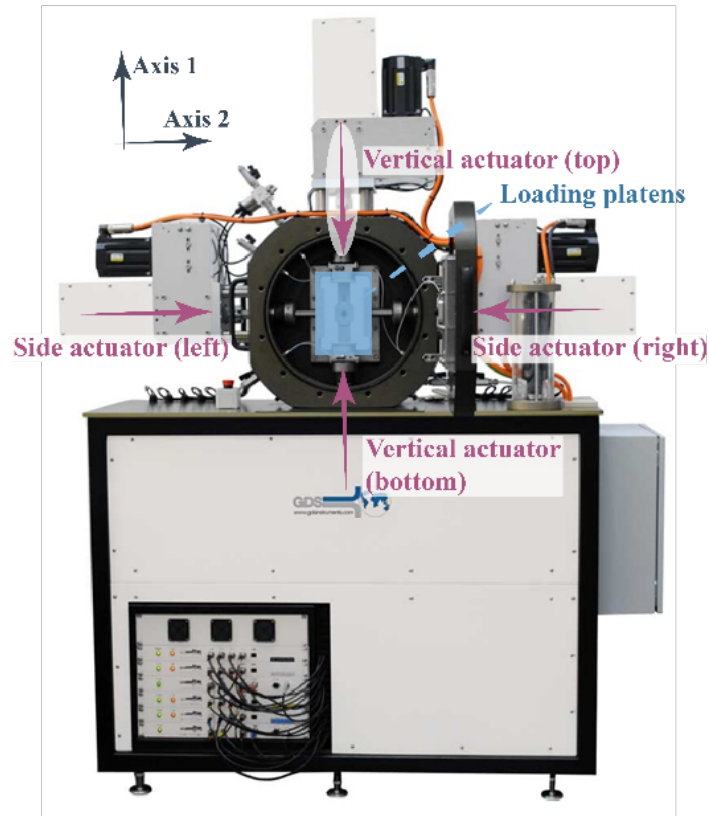
Table 3.3 Parameters of the constitutive model for 3DPC.

Parameters	Values
E (kPa)	639
ν	0.3
$f_{ebc}:f_{co}$	1.4
φ	25°
f_{co} (kPa)	50.5
f_{to} (kPa)	25.2
h_0	0.4
$\epsilon_{1,k}^{pl}$	0.043
κ_s	0.34

Table 3.4 FE simulation results of models with Config-1 and Config-2 platen designs.

$\beta_s = \sigma_2:\sigma_1$	Axial compressive strength (kPa)		Difference (%)
	<i>Config-1</i>	<i>Config-2</i>	
0.33	65.28	65.72	-0.67
0.50	71.77	71.66	0.16
0.70	76.68	76.72	-0.06
1.00	67.90	71.01	-4.38

(a) Loading system



(b) Loading platens

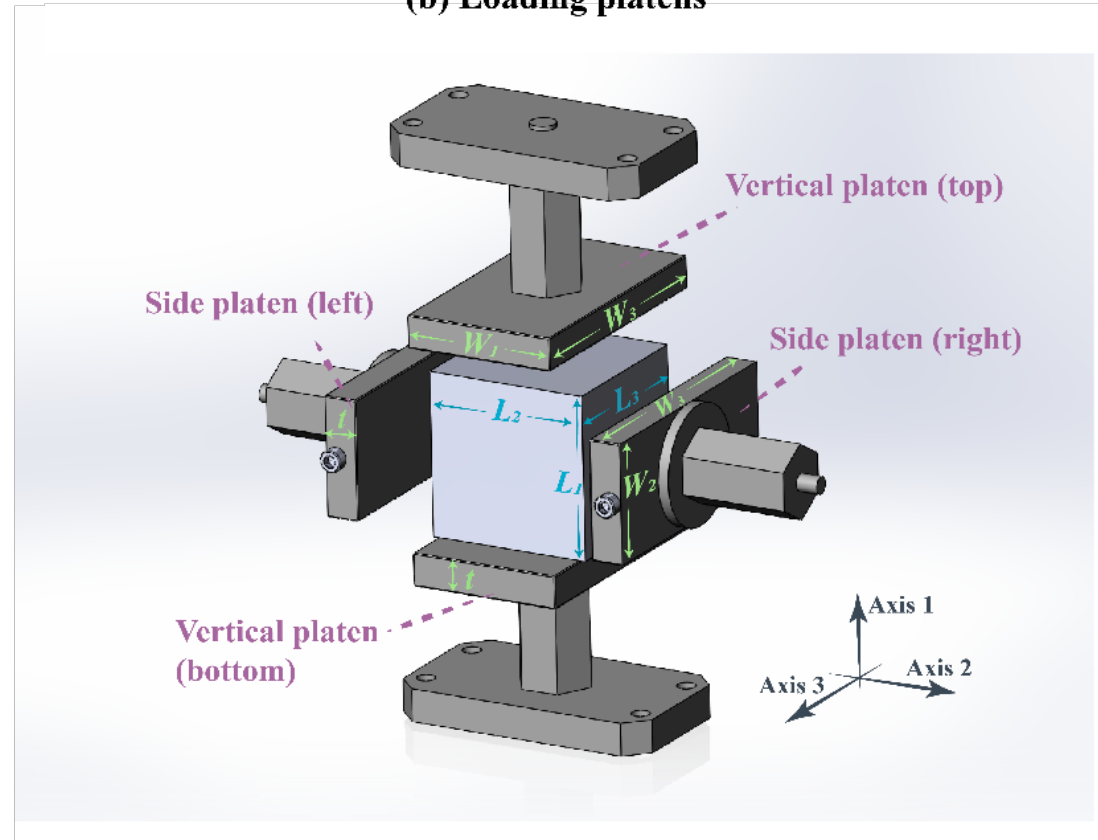


Figure 3.1 Schematic of the biaxial compression test setup: (a) Loading system [modified from GDS Instruments (2023)]; (b) Loading platens.

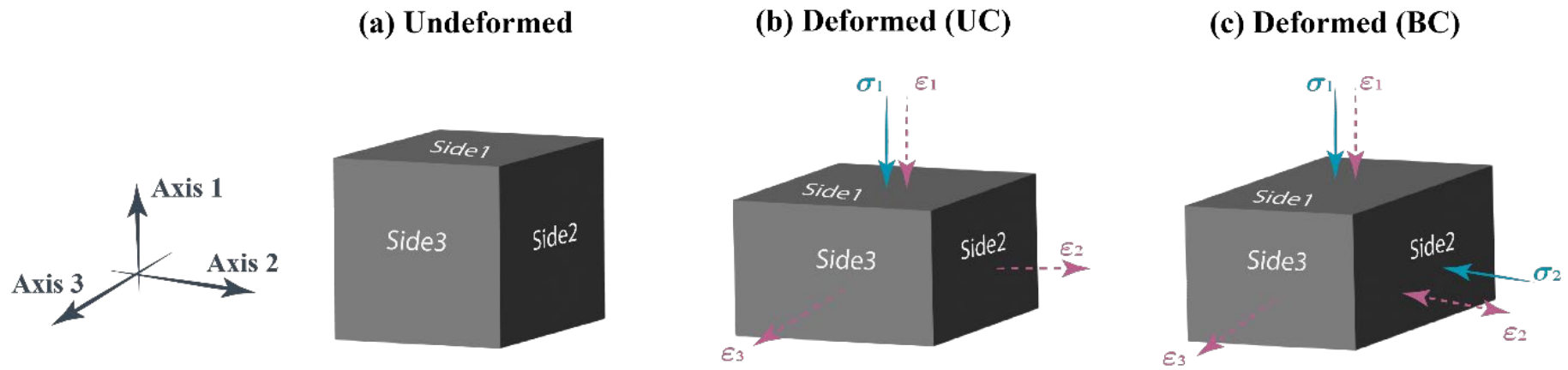


Figure 3.2 Stress and strain conditions of the specimens: (a) Undeformed; (b) Uniaxial compression; (c) Biaxial compression.

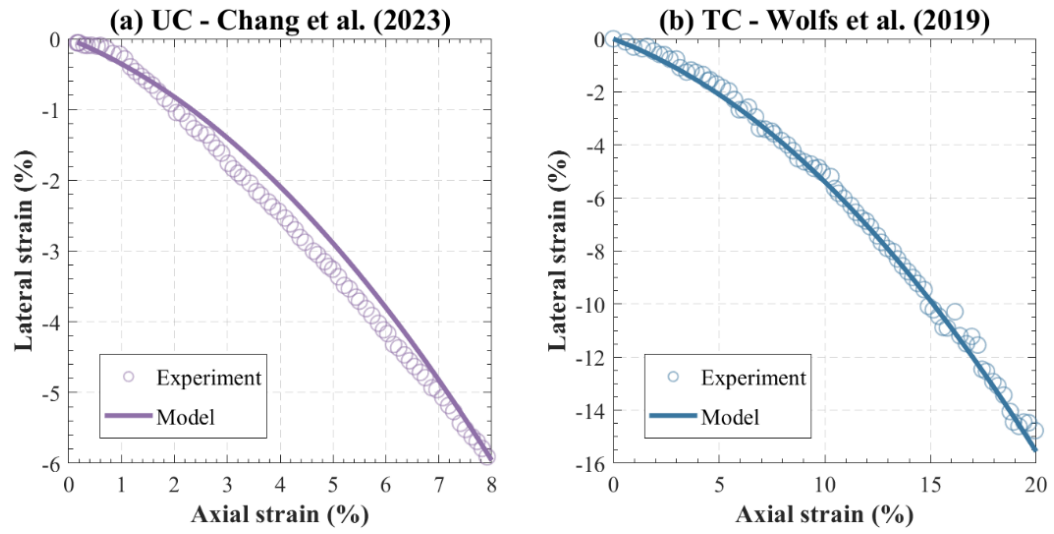


Figure 3.3 Comparison between the proposed axial-lateral strain relationship with test data: (a) Uniaxial compression; (b) Triaxial compression.

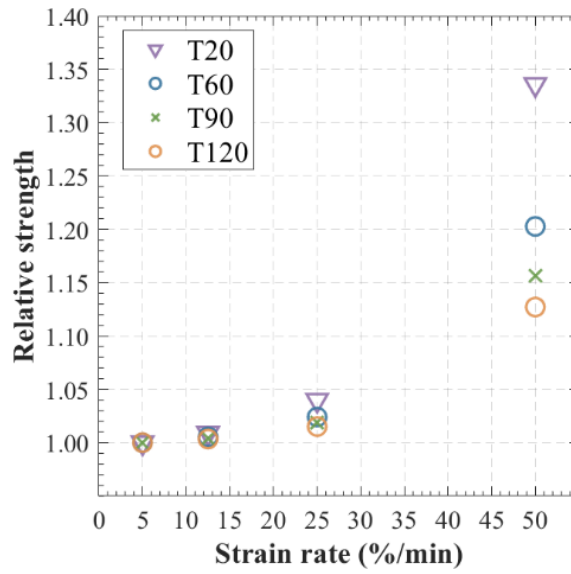


Figure 3.4 Influence of strain rate on strength.

(a) Configuration 1

(b) Configuration 2

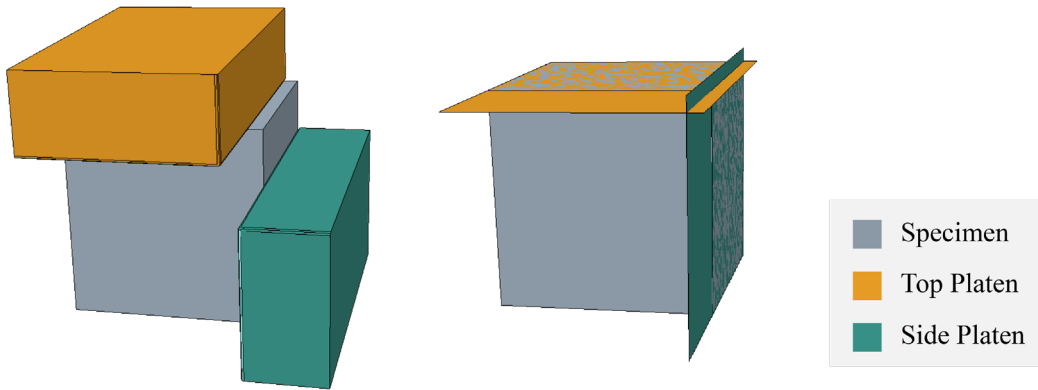


Figure 3.5 Schematics of the finite element models: (a) Model with platen configuration 1; (b) Model with platen configuration 2.

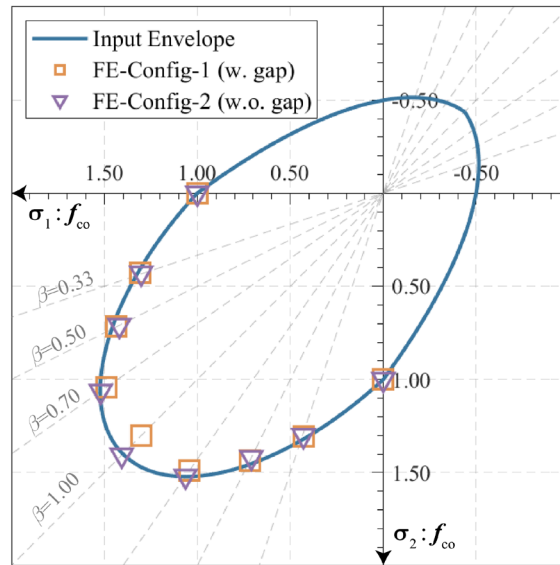


Figure 3.6 Comparison of the strength envelopes predicted by numerical simulations to the input envelope.

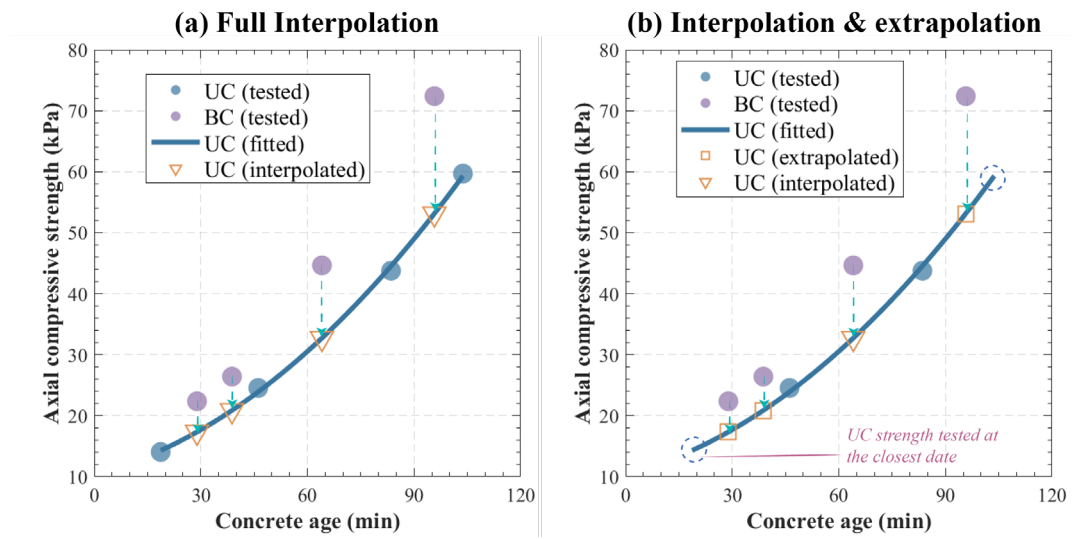


Figure 3.7 Estimation of uniaxial compressive strength.

Number of tests										Total
Biaxial stress ratio	PC	2	2	2	2	0	0	0	0	→ 8
	1.00	4	4	6	5	3	2	5	5	→ 37
	0.70	2	3	3	3	0	0	0	0	→ 11
	0.50	3	5	6	6	6	3	8	5	→ 46
	0.33	4	5	5	5	7	6	7	3	→ 45
	0	13	22	22	27	11	17	21	10	→ 155
Concrete age (min)										

APPENDIX 3.1 ACTUAL ACCURACY OF LOAD CELL

Since all four 20 kN load cells attached to the four actuators of the GDS testing machine are the same, only the accuracy of the bottom load cell (see Figure 3.1a) was verified using scale weights of varying mass. Initially, the mass of each scale weight was measured using a digital scale with a 5 kg capacity and an accuracy of 0.01g. These mass measurements were then converted to forces (F_a) by assuming a gravitational acceleration of 9.81 m/s^2 . Subsequently, the scale weights were positioned on the loading platen, which was connected to the 20 kN load cell, and the corresponding force readings were recorded (F_b). The error is calculated as $err = \frac{F_b - F_a}{F_a}$, and the results are summarised in Figure A.3.1. It is evident that the error of the load cell decreases as load increases and is below 1% when the load exceeds 30 N.

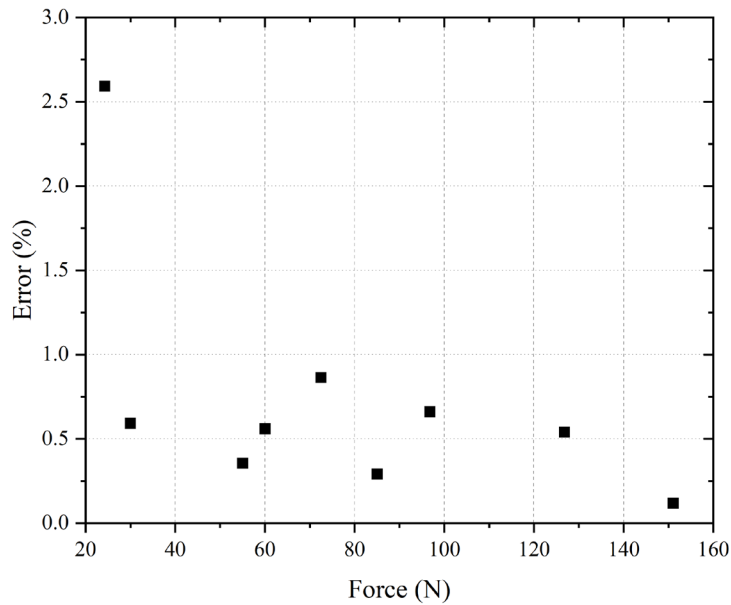


Figure A.3.1 Actual accuracy of the load cell.

Additionally, it is noteworthy that the load cell capacity in this study falls within the range commonly utilised in previous research (Table A.3.1).

Table A.3.1 Load cell capacity in existing studies

Publication	Load Capacity (kN)
Panda and Tan (2018)	50
Wolfs <i>et al.</i> (2018)	5
Wolfs <i>et al.</i> (2019)	0.2
Panda <i>et al.</i> (2019)	50
Casagrande <i>et al.</i> (2020)	10
Rahul and Santhanam (2020)	5
Zhu <i>et al.</i> (2021)	300
Ivanova <i>et al.</i> (2022)	50

APPENDIX 3.2 CALCULATION OF THE APPARENT POISSON'S RATIO

In the case of uniaxial and triaxial compression, the *apparent* Poisson's ratio (ν^*) reflects the initial slopes the $\epsilon_2 - \epsilon_1$ curve, or equivalently, the $\epsilon_3 - \epsilon_1$ curve. For biaxial compression, ν^* represents the initial slope of the $\epsilon_3 - \epsilon_1$ curve. This parameter was derived from the theory of elasticity, as explained below.

In the absence of shear stress, the strain components can be related to the normal stresses using Eq. (3.4):

$$\begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} \quad (3.4)$$

where the subscripts of the strain and stress indicate the directions in the coordinate system as illustrated in Figure 3.2b. The parameters ν and E represent the elastic Poisson's ratio and the Young's modulus, respectively. In this study, the elastic Poisson's ratio was assumed to be 0.3, based on the findings of Wolfs *et al.* (2018). Since E is cancelled out during the derivation of ν^* , defining its specific value is unnecessary in this context.

A3.2.1. Uniaxial compression

Under uniaxial compression, where $\sigma_2 = \sigma_3 = 0$, the lateral strains are defined by the relation $\epsilon_2 = \epsilon_3 = -\nu\epsilon_1$. This leads to the conclusion that ν^* is equal to ν under uniaxial compression.

A3.2.2. Biaxial compression

In biaxial compression, where $\sigma_3 = 0$, the stress ratio between the two active axes is defined as $\beta_s = \sigma_2/\sigma_1$. Using this ratio, the axial strain (ϵ_1) and lateral strain (ϵ_3) can be formulated as follows:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{E}(\sigma_1 - \nu\sigma_2) \\ \varepsilon_3 &= \frac{-\nu}{E}(\sigma_1 + \sigma_2)\end{aligned}\tag{3.5}$$

From this, the parameter ν^* is calculated by the expression:

$$\nu^* = -\frac{d\varepsilon_3}{d\varepsilon_1} = -\frac{d\varepsilon_3}{d\sigma_1} \frac{d\sigma_1}{d\varepsilon_1} = \frac{\nu(1 + \beta_s)}{1 - \nu\beta_s}\tag{3.6}$$

This expression follows from:

$$\begin{aligned}\frac{d\varepsilon_1}{d\sigma_1} &= \frac{1}{E}(1 - \nu\beta_s) \\ \frac{d\varepsilon_3}{d\sigma_1} &= \frac{-\nu}{E}(1 + \beta_s)\end{aligned}\tag{3.7}$$

A3.2.3. Triaxial compression with uniform confinement

In triaxial compression with uniform confinement, where $\sigma_2 = \sigma_3 = \sigma_c$, the specimen is first subjected to hydrostatic pressure, that is $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_c$. After reaching the desired confining pressure, $\sigma_2 = \sigma_3$ are held constant, and σ_1 is increased until the specimen fails, as described by Wolfs *et al.* (2019). Under these conditions, the axial and lateral strains are expressed as:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{E}(\sigma_1 - 2\nu\sigma_c) \\ \varepsilon_2 = \varepsilon_3 &= \frac{1}{E}[-\nu\sigma_1 + (1 - \nu)\sigma_c]\end{aligned}\tag{3.8}$$

The apparent Poisson's ratio ν^* can be computed using the chain rule:

$$\nu^* = -\frac{d\varepsilon_3}{d\varepsilon_1} = -\frac{d\varepsilon_3}{d\sigma_1} \frac{d\sigma_1}{d\varepsilon_1} = \nu\tag{3.9}$$

which is derived from:

$$\begin{aligned}\frac{d\varepsilon_1}{d\sigma_1} &= \frac{-\nu}{E} \\ \frac{d\varepsilon_3}{d\sigma_1} &= \frac{1}{E}\end{aligned}\tag{3.10}$$

CHAPTER 4

BIAXIAL COMPRESSIVE STRENGTH BEHAVIOUR AND STRENGTH MODELS OF FRESH 3D-PRINTED CONCRETE

4.1 INTRODUCTION

Chapter 2 revealed a significant knowledge gap in the understanding of the biaxial compressive strength behaviour of fresh concrete. Although several strength models have been developed for 3D-printed concrete (3DPC), the absence of experimental data under biaxial compression has prevented their validation under multiaxial stress states. To address this limitation, Chapter 3 presented an extensive experimental investigation comprising over 300 fresh concrete specimens tested under both uniaxial and biaxial compression, spanning ages from tens of minutes to six hours. Building upon this experimental foundation, the present chapter focuses on characterising and modelling the biaxial compressive strength behaviour of fresh 3DPC throughout its early-age curing process.

The investigation begins with a detailed discussion of experimental observations, examining both failure modes and stress-strain relationships. This is followed by comprehensive strength analyses that provide crucial insights into the biaxial strength characteristics of fresh 3DPC. These experimental findings serve as a basis for evaluating existing strength models and highlight their limitations in predicting biaxial compression behaviour. In response to these limitations, two new strength models are proposed: a simplified model correlating biaxial strengths with uniaxial strength, and an advanced age-dependent multiaxial strength formulation suitable for integration into plasticity-based constitutive models. These developments represent significant advances in modelling the mechanical behaviour of fresh 3DPC under complex loading conditions.

4.2 EXPERIMENTAL OBSERVATIONS

Throughout this chapter, true stress was used to accurately capture the stress condition of fresh concrete as it undergoes large deformation. This choice of this stress measure is essential for accurate strength analyses. On the other hand, engineering strain is used to describe deformation for ease of discussion, as it is the most intuitive measure of deformation. As strength analyses, which are the primary focus of the current chapter, are essentially stress analyses, the choice of strain measure will not affect the results of these analyses.

4.2.1 Failure Modes

Figures 4.1 to 4.4 depict the 3DPC cube specimens at the conclusion of the uniaxial and biaxial compression tests. Figure 4.1 illustrates the failure modes of 3DPC cubes under uniaxial compression at various ages: 15, 44, 83, 174, 231, and 311 minutes. The cubes at 15 minutes (T15) and 44 minutes (T44), as shown in Figure 4.1a, underwent 25% axial compressive strain in axis 1, displaying significant dilation along axis 2 and noticeable bulging, yet without any noticeable cracking. This suggests that the fresh concrete was highly deformable at these early stages, a condition referred to as the rheological phase (R phase). At 83 minutes (T83) and 174 minutes (T174), as depicted in Figure 4.1b, the cubes experienced 25% and 8% axial compressive strain, respectively, with inclined cracking observed. These inclined cracks, occurring under the combined influence of compressive normal stress and shear stress, suggest shear failure in the fresh concrete. Notably, the dilation in the T83 cube was substantially greater than in the T174 cube, though both exhibited similar cracking patterns, indicating a transition from the rheological state to a less deformable solid state, referred to as the R-S phase. Finally, the cubes at 231 minutes (T231) and 311 minutes (T311) underwent 6% and 4% axial strain, respectively, displaying near-vertical cracks and spalling, as illustrated in Figure 4.1c. These splitting cracks indicate tensile failure in the fresh concrete. The dilation observed in these cubes was comparable to each other but significantly less than in the earlier rheological phase. The failure mode of these later-stage cubes closely resembled that of fully cured concrete, as depicted in Figure

4.1d by Gao and Hu (2021). Consequently, they are referred to as the solid phase (S phase).

Figures 4.2 and 4.3 present the failure modes of specimens subjected to biaxial compression at stress ratios of 0.33 and 0.50, respectively, across various ages. In both scenarios, cracks predominantly manifest on side 2, while sides 1 and 3 exhibit either no cracking or only minor fractures. The age-dependent cracking patterns are similar for specimens under these two stress ratios. R phase specimens (T38-B0.33, T34-B0.50) display only minor shear cracks. R-S phase specimens (T96/T181-B0.33, T93/T181-B0.50) demonstrate localised, pronounced shear cracking. In the S phase specimens (T301-B0.33, T301-B0.50), tensile splitting cracks are evident, and the cracking pattern closely resembles that of cured concrete subjected to biaxial compression, as illustrated in Figure 4.2d by Gao and Hu (2021). The progression of the crack pattern aligns with those observed in UC tests.

The failure modes for specimens under equibiaxial compression (stress ratio of 1.0) are presented in Figure 4.4. In these specimens, cracks predominantly manifest on side 2 and propagate along axis 3, akin to those observed in BC-0.33 and BC-0.50. Despite experiencing significant out-of-plane dilation exceeding 70%, the R phase specimen (T33-B1.00) exhibits only minor cracking on side 2. In contrast to R-S phase specimens under BC-0.33 and BC-0.50, the R-S specimen under BC-1.0 (T92/T182-B1.00) displays distributed cracking rather than pronounced localised fractures. This is probably due to the higher level of confinement in axis 3 resulting from friction between the specimen and loading platens on sides 1 and 2. For the S phase specimen (T301-B1.00), tensile splitting cracks can be observed as those in cured concrete by Gao and Hu (2021) (Figure 4.2d).

In summary, the progression of failure modes demonstrates that fresh concrete initially exists in the R phase, characterised by high deformability and an absence of cracking under excessive compression. This is followed by a transition through an intermediate R-S phase, which is marked by inclined shear cracking upon compression failure, ultimately reaching the S phase, characterised by vertical cracks and spalling. In the present study, the R phase for the developed concrete mixtures persisted for

approximately 45 minutes, while the transition to the R-S phase occurred between 45 and 230 minutes, at which point the S phase commenced. It is hypothesised that different 3D-printable concretes undergo this three-phase process; however, the durations of each phase may differ depending on the specific mixture.

4.2.2 True Stress–Engineering Strain Curves

Figure 4.5a demonstrates the axial stress against the axial and lateral strains for all 155 cubes tested under uniaxial compression, ranging from T10 to T380. The axial stress (σ_1) is plotted in logarithmic scale, as the maximum axial stress increases by two orders of magnitude from 10 to 380 minutes of age. Each curve was color-coded according to the concrete age as indicated by the colour bar. For ease of discussion, exemplary stress-strain curves at seven distinct ages from the R phase to the S phase are illustrated in Figure 4.5b.

In general, the shape of the stress-strain curves undergoes significant changes as concrete ages. For R phase specimens (e.g., T17, T35 in Figure 4.5b), the stress-strain curves show plateauing without a distinct peak stress point. The ductile behaviour is embodied in the high crack resistance as shown in Figure 4.1a. In the R-S phase (e.g., T83, T172 in Figure 4.5b), each stress-strain curve reaches a peak stress point followed by a gradual post-peak softening branch. These specimens show reduced ductility, embodied in the localised shear cracks in Figure 4.1b. S phase specimens (e.g., T191, T231, T311 in Figure 4.5b) exhibit a steep post-peak softening branch. The brittle material response is represented by the localised tensile cracks (see Figure 4.1c). The axial strain (ϵ_1) at maximum axial stress decrease from 25% at T15 to less than 4% at T360, undergoing a ductile-to-brittle transition consistent with the failure mode progression. This observed age-induced ductile-to-brittle transition aligns with the observations in existing uniaxial compression tests as discussed in Chapter 2.

Figures 4.6 and 4.7 show the stress-strain curves for specimens tested under BC-0.33 and BC-0.50, respectively, with ages ranging from T15 to T330. In both tests, the maximum axial and horizontal stresses grow two orders of magnitude from T15 to T330, and the corresponding axial strains decrease as age increases. A ductile-to-brittle

transition with age is evident akin to specimens under UC. The R phase specimens (e.g., T35-B0.33, T40-B0.50) show a plateau in stress without subsequent softening, indicating a ductile response. This ductile response is accompanied by excellent resistance to cracking as shown in the R phase specimens in Figures 4.2a and 4.3a. The R-S phase specimens (e.g., T65-B0.33 to T182-B0.33 and T73-B0.50 to T182-B0.50) display reduced ductility, featuring gradual post-peak softening behaviour. The slope of the softening branch becomes steeper as the concrete age increases, consistent with the patterns observed under UC. The reduction in ductility is manifested in the localised shear cracks on side 2 (see T96/T181-B0.33 in Figure 4.2b and T93/T181-B0.50 in Figure 4.3b as examples). For S phase specimens (e.g., T302-B0.33, T301-B0.50) a more brittle behaviour is observed, characterised by a significant decrease in stress in the post-peak regime and the development of tensile cracking patterns, see Figure 4.2c (T301-B0.33) and Figure 4.3c (T301-B0.50) for illustrative examples.

Under equibiaxial compression, the stress-strain response is different from those under UC and BC with lower stress ratios. As presented in Figure 4.8, although the stress-strain curves show a ductile-to-brittle transition as age increases, only S phase specimens with ages beyond T185 exhibit post-peak softening, corresponding to the localised tensile cracks at the end of testing as illustrated in Figure 4.4c (T301-B1.00). In contrast, for R phase specimens (e.g., T33-B1.00) and R-S phase specimens (e.g., T63-B1.00 to T182-B1.00), the stress-strain curve shows an ascending second branch. However, this secondary hardening behaviour is considered not due to the inherent material behaviour but a result of the testing method; hence, it will be referred to as pseudo-hardening. Specifically, the two sides of the concrete cube subjected to compressive stress inevitably experience friction, which practically serves as confinement to the material. Therefore, the material is practically subjected to a triaxial compressive stress state, although friction reduction measures have been taken place using the PTEE films. In other words, while the actual stress ratio of the specimen is $\sigma_1 : \sigma_2 : \sigma_3$ where σ_3 is the confining stress due to friction, only the stress ratio of the maximum and intermediate principal stress values ($\sigma_1 : \sigma_2$) is presented in this paper. It was assumed that σ_3 due to friction is negligibly small compared to σ_1 and σ_2 ; hence it will be neglected in strength analyses. To validate that the pseudo-hardening is due to friction, a series of FE simulations were performed as detailed in Appendix 4.1. The

results show that in the absence of friction, the stress-strain curve displays a clear peak stress. However, with an increased friction coefficient (e.g., 0.1), the pseudo-hardening phenomenon emerges. Consequently, the secondary ascending branch is not representative of the inherent behaviour and is thus excluded from strength evaluation. This unusual behaviour is embodied in the distributed cracks in the cubes shown in Figure 4.4, for instance, at T33, T92, and T182.

Integrating the findings from both uniaxial and biaxial compression tests, it is clear that the maximum stress that the concrete can withstand increases by two orders of magnitude within the first six hours. Alongside this increase in strength, the material exhibits a marked transition from ductile to brittle behaviour. This shift is characterised by a decrease in axial strain at maximum stress and the emergence of a steeper softening branch in the stress-strain curves as the material ages. The evolution of the mechanical properties is related to the changing patterns of cracks. At earlier ages, there are almost no cracks visible, but as the concrete ages, localised cracks become more pronounced. A notable observation is that specimens subjected to equibiaxial compression display significant differences compared to those under lower stress ratios. In these specimens, the stress-strain curves show, as previously discussed, pseudo-hardening due to testing conditions rather than an inherent material property. This phenomenon, along with its implications on the strength analyses, will be further explored in subsequent sections. These findings highlight the complex nature of the mechanical behaviour of fresh concrete.

4.3 STRENGTH ANALYSES

4.3.1 Definition of Strength

In uniaxial compression tests, the uniaxial compressive strength, f_{co} , was determined from the axial stress-axial strain curve as the maximum stress achieved during the test. For biaxial compression tests with proportional loading, the maximum stresses along both axes were reached simultaneously. Here, the biaxial strength f_{bc} was defined as the higher value between the axial and horizontal strengths. However, it is noted that stress-strain curves of the R phase specimens in both UC and BC tests did not exhibit a

clear peak stress. Moreover, in the R and transition R-S phase specimens under BC-1.00, pseudo-hardening behaviour was identified.

The axial stress-axial strain curves of the specimens can be categorised into three types: 1) curves with plateauing or with slight hardening (Type 1) – characteristic of R phase specimens with low confining stress ratio ($\beta_s = 0, 0.33, 0.50$), where the plateau value is taken as the strength (Figure 4.9a); 2) curves with a clear peak stress (Type 2) – typical of R-S and S phase specimens with low confining stress ratio ($\beta_s = 0, 0.33, 0.50$), and S phase specimens with high confining stress ratio ($\beta_s = 1.0$), with the peak stress defining the strength (Figure 4.9b); 3) curves with pseudo-hardening (Type 3) – observed in R and R-S phase specimens with high confining stress ratio ($\beta_s = 1.0$), featuring a secondary ascending branch (Figure 4.9c). Strength for Type 3 stress-strain behaviour is defined as the axial stress level at the inflection point on the ascending branch, when the second derivative equals zero (Figure 4.9c).

4.3.2 Uniaxial Compressive Strength Analysis

Figure 4.10 illustrates the evolution of the uniaxial compressive strength of 3DPC produced from three distinct supplies of raw materials, displayed on a semi-logarithmic scale. There is evident scattering in the uniaxial compressive strengths among specimens of the same age, which can be quantified as the ratio between the maximum and minimum values. For specimens cast from the same supply, the inter-batch variability was found to be less than 2. This level of variability aligns with findings in existing studies (Mettler *et al.*, 2016; Wolfs *et al.*, 2018; Pott *et al.*, 2022), suggesting that such scattering is inevitable. In contrast, when comparing specimens produced from different supplies of raw materials, the inter-supply variability is observed to be less than 2.5 for concrete ages under 180 minutes (T180) and less than 4 for ages exceeding T180. Factors such as material variations and ambient conditions (temperature and humidity) during experimentation, which are challenging to control completely, contribute to this inter-supply variability.

During the first 120 minutes, strength evolution follows an exponential decay pattern on the semi-logarithmic scale, indicating that the relationship between uniaxial

compressive strength and concrete age at this stage adheres to a linear trend. Following this period, from approximately 120 minutes, the evolution of strength manifests as a linear pattern on the semi-logarithmic scale, reflecting an exponential correlation between uniaxial compressive strength and concrete age. This suggests that strength evolution accelerates during this stage. These observations align with those found in existing studies, which have been systematically reviewed in Chapter 2. As discussed in Chapter 2, the accelerating evolution of strength post-120 minutes is attributed to the accelerating period of hydration (Pott *et al.*, 2023).

For ease of analysis and representation, the evolution of uniaxial compressive strength was effectively represented by a single exponential function Eq. (4.1), corresponding to a linear trend in a semi-logarithmic plot, as indicated by the dashed line in Figure 4.10. This simplification encapsulates the essential correlation between the uniaxial compressive strength of the concrete and its age, providing a mathematically convenient model for strength development over time:

$$f_{co} = f_{ci} \cdot e^{\xi_s t} \quad (4.1)$$

where f_{ci} represents the fictional initial strength at age 0 minutes, ξ_s is a coefficient representing the uniaxial strength growth rate. For the material developed in the present study, f_{ci} is around 11 kPa, and η is approximately 1.015 min^{-1} .

Furthermore, the uniaxial compressive strengths of fresh 3DPC at various ages, as measured in previous studies (Mettler *et al.*, 2016; Wolfs *et al.*, 2018; Pott *et al.*, 2022) are plotted in Figure 4.10 as a reference. It indicates that the developed 3DPC exhibits a strength evolution comparable to that reported in the literature.

4.3.3 Biaxial Compressive Strength Analysis

Figure 4.11 presents the measured UC and BC strengths in the plane stress space. This space is symmetric, with UC strengths plotted along the σ_1 and σ_2 axes, and the BC strengths under biaxial stress ratios $\beta_s = 0.33, 0.5, 0.7$ and 1.0 within the quadrant between these two axes. Strength data for concrete ages T15-T120, T120-T190, and

T190-T330 are illustrated in Figures 4.11a, b, and c, respectively, with each data point colour-coded to represent its corresponding age. Similar to the UC strengths, the BC strengths exhibit a substantial increase of two orders of magnitude as the concrete ages from T10 to T330. Notably, the BC strengths consistently exceed the UC strengths at the same ages, irrespective of the confining stress ratio.

To further elucidate the relationship between biaxial and uniaxial strengths, each biaxial strength was normalised against its corresponding uniaxial strength at the same age. Figure 4.12a displays the normalised strength data across all ages in the plane stress space, indicating that the shape of the normalised biaxial strength envelope exhibits a marked transformation as fresh 3DPC matures. As discussed in Chapter 2, although strength increases monotonically under various stress conditions for fresh concrete, the growth rates differ across these conditions, resulting in a change in the shape of the strength envelope. Further analysis is presented in Figures 4.12b to 19e, where the biaxial-to-uniaxial strength ratio, denoted as $i_b = f_{bc}/f_{co}$, is plotted against concrete age t for four different stress ratios. For stress ratios of 0.33, 0.50, and 1.0, a clear trend emerges, showing that i_b increases with concrete age up to 180 minutes (denoted as the *transition age*), followed by a continuous decrease. A similar trend in i_b is observed under the stress ratio of 0.70, though this occurs within the 120-minute testing window.

At each stress ratio, a curve was fitted to the relationship between i_b and t , as indicated in the figure. The evolution of i_b throughout the R, R-S, and S phases reveals a tendency for i_b to increase during the R phase and decrease during the S phase as the concrete matures. In the R-S phase, these opposing dynamics coexist, resulting in a peak i_b value. A comparative analysis of the evolution of failure modes, stress-strain behaviour, and biaxial-to-uniaxial strength in the R, R-S, and S phases indicates that two distinct chemical-physical mechanisms dominate the R and S phases while interacting within the R-S phase. Notably, the transition age within the R-S phase aligns with the conclusion of the dormant period and the onset of the accelerating period of fresh concrete (Ghourchian *et al.*, 2021; Pott *et al.*, 2023). Specifically, it has been suggested by Pott *et al.* (2023) that the strength prior to the transition age is predominantly influenced by Ettringite, whereas the strength following this age is primarily attributed to the C-S-H formed through silicate reactions. Therefore, it is reasonable to

hypothesise that the change in shape of the biaxial compressive strength envelope signifies a transition from an Ettringite-dominant to a C-S-H-dominant mechanical response. However, the intricacies of these underlying mechanisms and their interplay warrant further experimental investigation before more definitive conclusions can be drawn.

By combining the fitted i_b - t relationship and the uniaxial strength growth represented by Eq. (4.1), the biaxial strength envelopes of the fresh 3DPC tested in this study spanning from 30 minutes to 300 minutes are plotted in Figure 4.13, where the subplots (a) and (b) present the strength envelopes at ages ranging from age T30 to T150 and from T180 to T300, respectively. It is evident that the strength envelopes not only increase in size but also undergo changes in shape as the concrete matures.

4.3.4 Passively Confined Biaxial Compression Tests

Figure 4.14 presents representative axial stress (σ_1), lateral stress (σ_2), and the corresponding stress ratio, $\beta_s = \sigma_2/\sigma_1$, plotted against axial strain for specimens tested under passively confined biaxial compression at ages T37 (subplot a), T64 (subplot b), and T94 (subplot c). The axial stress is actively applied to the cube specimen, while the lateral stress is a confining stress passively applied to the platens due to the confined dilation of the cube specimens. Since a 5 N seating load was applied to both axial and lateral directions to ensure contact between specimens and platens, the value of β_s for the initial portion of the loading process was disregarded. During the subsequent phases of loading, the stress ratio β_s was initially around 0.44, 0.38, and 0.31 for concrete at ages of T37, T64, and T94. The stress ratios continue to increase to approximately 0.5 at the defined strengths (as discussed in Section 4.3.1). This provides a good estimate of the stress ratio range of 0.3~0.5 for fresh 3DPC during the construction process, while in a real 3D-printed concrete structure, the confinement of concrete from the neighbouring material should be lower than that in the passively confined tests where the lateral platens were kept still. It is important to note that the continuous increase of the biaxial stress ratio in the pseudo-hardening branch is disregarded.

Figure 4.15 compares the biaxial strengths of the 3DPC obtained from proportionally loaded BC tests with those from the non-proportional loaded passively confined BC tests. The dashed lines represent the stress paths ($\sigma_1 - \sigma_2$) that the specimens experienced during the passively confined tests, with each endpoint indicating the measured biaxial strength. Specimens with an age difference of less than 3 minutes were grouped in the same subplot. Using the average age of the two specimens in each group, the strength envelopes obtained from proportional loading tests were calculated (using the fitted $i_b - t$ relationship in Figures 4.12b to e) and are plotted as the triangular scatters. It appears that the strengths acquired from both proportional and non-proportional loading tests are very similar. This finding suggests that the biaxial compressive strength of fresh concrete should be stress path independent. However, it is noted that the non-proportional stress paths of the passively confined tests in the current study do not undergo significant changes, remaining within the range of $\beta_s = 0.33$ to 0.50 . Therefore, more experimental data are needed to further validate this behaviour of 3DPC. For the scope and purpose of the current study, the strength of fresh concrete is considered to be stress path independent.

4.4 EVALUATION OF EXISTING STRENGTH MODELS

As discussed in Chapter 2, three strength models have been widely used for fresh concrete in existing research: the von Mises, the Mohr-Coulomb (M-C), and the Drucker-Prager (D-P) models. Of these, the von Mises model is particularly prevalent in rheology-based research (Roussel and Coussot, 2005; Roussel, 2018; Jacquet *et al.*, 2021), whereas both the M-C (Wolfs *et al.*, 2019; Jayathilakage *et al.*, 2020; Ooms *et al.*, 2021; An *et al.*, 2024) and D-P (Di Carlo *et al.*, 2013; Wijaya *et al.*, 2024) have been employed for modelling static-state fresh 3DPC during the construction process. The Mohr-Coulomb model is the most widely used in this context. However, the accuracy of these models in representing the biaxial compressive strength of 3DPC has not been validated due to a lack of test data. Building upon the strength analyses presented in the previous section, the current section aims to evaluate these three strength models under biaxial compression.

Figure 4.16 compares the normalised strength envelopes at eight different ages, spanning from 30 to 300, with the three strength models. Despite the widespread use of M-C model for modelling static-state 3DPC in existing literature, this model predicts identical uniaxial and biaxial strengths, resulting in substantial underestimation for BC strengths with all stress ratios across all ages. The von Mises criterion has been considered suitable for very early-age concrete (Mettler *et al.*, 2016). However, it only provides an acceptable prediction for BC strength when the biaxial stress ratio is below 0.33. It considerably underestimates BC strength with a stress ratio above 0.33, especially for equibiaxial compression. Assuming a Mohr-Coulomb friction angle of 17 degrees based on the triaxial compression tests conducted by Wolfs *et al.* (2019), the D-P model can provide reasonably good predictions on the BC strengths when the age is less than T240. As highlighted in Chapter 2, the D-P model is the only one amongst these whose shape is adjustable in the biaxial compression quadrant by tuning the value of its friction parameter. Therefore, by calibrating the D-P model with the test data at different ages, an effective representation of the normalised biaxial strength envelopes at all ages can be achieved.

4.5 BIAXIAL COMPRESSIVE STRENGTH MODELS

Based on the biaxial strengths measured across a range of stress ratios (0, 0.33, 0.5, 0.7, and 1.0) and concrete ages (ranging from 10 minutes to 330 minutes), two biaxial compressive strength models were proposed. A simple biaxial-to-uniaxial strength ratio model was proposed for fresh 3DPC with a biaxial stress ratio of up to 0.5 and a concrete age of up to 300 minutes. This model provides a straightforward way to estimate the biaxial strength of early-age 3DPC. In addition, a multi-axial strength envelope was developed. The implementation of this strength model to an elastoplastic constitutive model for 3DPC is elaborated in the next chapter.

4.5.1 A Simple Biaxial Compressive Strength Model

Figures 4.17 a and b illustrate the relationship between biaxial-to-axial strength ratio, $i_b = f_{bc}/f_{co}$, and the biaxial stress ratio, $\beta_s = \sigma_2/\sigma_1$, specifically for β_s ranging from 0 to 0.5. This range is particularly relevant to 3D concrete printing, as demonstrated in

the passively confined biaxial compression tests, which exhibit an upper limit of biaxial stress ratio (β_s) of 0.55. It is clear that i_b is linearly proportional to β_s in this range, while the slope changes with concrete age. Therefore, i_b can be represented by the following equation:

$$i_b = \psi(t) \beta_s + 1 \quad (4.2)$$

where ψ is the slope dependent on concrete age, t . To determine the relationship between ψ and t , the values of ψ at 30, 60, 90, 120, 150, 180, 240, and 300 minutes were calculated and shown in Figure 4.17c. As shown, a parabolic curve can well represent the relationship between the slope ψ and concrete age t , given as:

$$\psi(t) = \varsigma_\psi (t - t_{c,\psi})^2 + \psi_c \quad (4.3)$$

where $t_{c,\psi}$ denotes the critical transition age at which ψ reaches its maximum value, ψ_c , and ς_ψ is a curing coefficient. Putting together Eqs. (4.2) and (4.3), the biaxial compressive strength ratio can be mathematically presented as follows:

$$i_b = \left[\varsigma_\psi (t - t_{c,\psi})^2 + \psi_c \right] \beta_s + 1 \quad (4.4)$$

It is hypothesised that for other 3D-printed concrete materials, the biaxial compressive strength ratio can be well represented with Eq. (4.4), albeit with potentially different values for ς_ψ , $t_{c,\psi}$ and ψ_c , which would require calibration. For the 3DPC developed in the present study, using the values $t_{c,\psi}=175$ minutes, $\psi_c=0.95$, and $\varsigma_\psi=-2 \times 10^{-5} \text{ min}^{-2}$ effectively describes the biaxial compressive strength response.

4.5.2 Age-Dependent Multi-Axial Strength Model

As previously discussed in Section 4.4, the shape of the Ducker-Prager model (Drucker and Prager, 1952) in the biaxial compression quadrant is adjustable. This feature is essential for modelling the biaxial compressive strength of 3DPC at different ages, as evidenced by the observation that the strength envelope changes shape as hydration

progresses (see Figure 4.12). Based on the Drucker-Prager model, an age-dependent multi-axial strength model was developed, which is given as:

$$f(\boldsymbol{\sigma}, t) = \bar{\rho} - \chi(t) \sqrt{2\bar{\xi}} - \frac{\sqrt{6}}{3} [1 - \chi(t)] \quad (4.5)$$

Here, $\bar{\rho} = \rho / f_{co}(t)$ and $\bar{\xi} = \xi / f_{co}(t)$ are the normalised Haigh-Westergaard coordinates. It is noteworthy that the friction parameter χ controls the shape of the envelope in the biaxial compression quadrant. The parameter χ was related to t so that the shape of the envelope in this quadrant changes as concrete ages.

The relationship between χ and t needs to be calibrated from BC strength data at different ages. For concrete ages less than T90, $\chi(t)$ was calibrated from data with BC-0.33 and -0.50, while data obtained from BC-0.33, -0.50, -1.00 were used otherwise. This is because the proposed strength envelope cannot fit the data points with low (BC-0.33, -0.50) and high (BC-0.70, -1.00) confining stress ratios simultaneously when the age is less than T90. Since it was established that the BC strength with $\beta_s \leq 0.50$ is the most relevant to the stress state of 3DPC during the construction process, it was deemed acceptable to sacrifice some accuracy in the case of high confining stress ratio for ages less than T90. When $t \geq 90$ minutes, however, the proposed model demonstrates reasonably accurate predictions for BC strength under both low and high confining stress ratios; thus, all available data were used for calibration.

To obtain the relation between χ and t , the values of χ at eight different ages (30, 60, 90, 120, 150, 180, 240 and 300 minutes) were first determined and are shown in Figure 4.18c. Evidently, the relationship can be predicted using a parabolic equation, Eq. (4.6), with relatively good accuracy.

$$\chi(t) = \varsigma_{\chi} (t - t_c)^2 + \chi_c \quad (4.6)$$

Similar to Eq. (4.3) of the closed form model, the parameter t_c represents a critical age at which the parameter χ reaches its peak (χ_c), and ς_{χ} is a curing factor. For the 3DPC used in the current study, $\varsigma_{\chi} = -5 \times 10^{-6} \text{ min}^{-2}$, $t_c = 170$ minutes and $\chi_c = 0.2$. It is assumed

that Eq. (4.6) is applicable to other 3DPC mixtures, while the model parameters ζ_χ , t_c and χ_c needs to be calibrated accordingly.

Figure 4.18a and b compare the normalised strength data with the proposed strength envelope at eight ages. Although there is an underestimation of BC strength with high confinement stress ratios ($\beta_s=0.70, 1.00$) at ages less than T90, the proposed model provides reasonably accurate predictions in all other cases.

4.6 CONCLUDING REMARKS

This chapter has focused on the biaxial compressive strength behaviour of fresh 3D-printed concrete (3DPC) with ages ranging from tens of minutes to six hours, based on the results from the biaxial compressive testing programme detailed in the preceding chapter.

The test results revealed a complex three-phase evolution in the early life of 3DPC, with each phase exhibiting distinct mechanical behaviours. The initial ‘rheological phase’ is characterised by high deformability and low strength. This is succeeded by the ‘transition phase’, a period defined by rapidly diminishing ductility and a notable increase in strength. The concluding ‘solid phase’ is where the mechanical behaviour of fresh concrete begins to mirror that of its cured counterpart, predominantly exhibiting brittle failure. Across these phases, the failure modes, stress-strain curves, and biaxial-to-uniaxial compressive strength collectively suggest the presence of two evolving chemical-physical mechanisms governing the mechanical behaviour of fresh concrete: one dominant in the rheological phase, likely due to Ettringite formation, and the other in the solid phase, probably attributed to C-S-H formation. The transition phase intriguingly represents a complex intermingling of these mechanisms, underscoring the intricate evolution of fresh concrete’s mechanical behaviour.

Passively confined tests provided particularly informative findings. These tests showed that fresh concrete, even when subjected solely to vertical loading, experiences substantial horizontal stress if laterally confined. This is a crucial observation for understanding the conditions of 3DPC during the construction process, where fresh

concrete often experiences biaxial compression due to confinement from neighbouring material. The test results indicated an upper limit of the biaxial stress ratio of around 0.55, suggesting that in practical 3D printing scenarios, horizontal stress due to confinement should bellow around one-half of the vertical gravitational stress. Furthermore, the similar strength observed between non-proportionally loaded and proportionally loaded biaxial compression tests suggested that the strength behaviour of fresh concrete is likely independent of the stress path. This finding is significant as it validates the use of proportionally loaded tests to study the mechanical behaviour of fresh concrete in actual 3D printing processes, where non-proportional loading is prevalent.

From the amassed test data, biaxial strength envelopes for the developed 3DPC were constructed up to six hours of age, employing a series of mathematical treatments. A critical assessment of commonly used strength models, including the von Mises and Mohr-Coulomb, revealed their inadequacy in accurately describing the biaxial strength of fresh 3DPC. To address this gap, a novel, straightforward biaxial compressive strength model was proposed. This model, with its adjustable parameters, is designed to calculate the ratio between biaxial and uniaxial compressive strengths for ages up to six hours and stress ratios up to 0.5, particularly relevant for 3D concrete printing scenarios. To further aid the development of a constitutive model for fresh 3DPC that can be used in numerical simulations, an age-dependent multiaxial strength model based on the Ducker-Prager model was introduced. The findings from this study are instrumental in advancing the understanding of the mechanical behaviour of fresh concrete and lay a robust foundation for future research.

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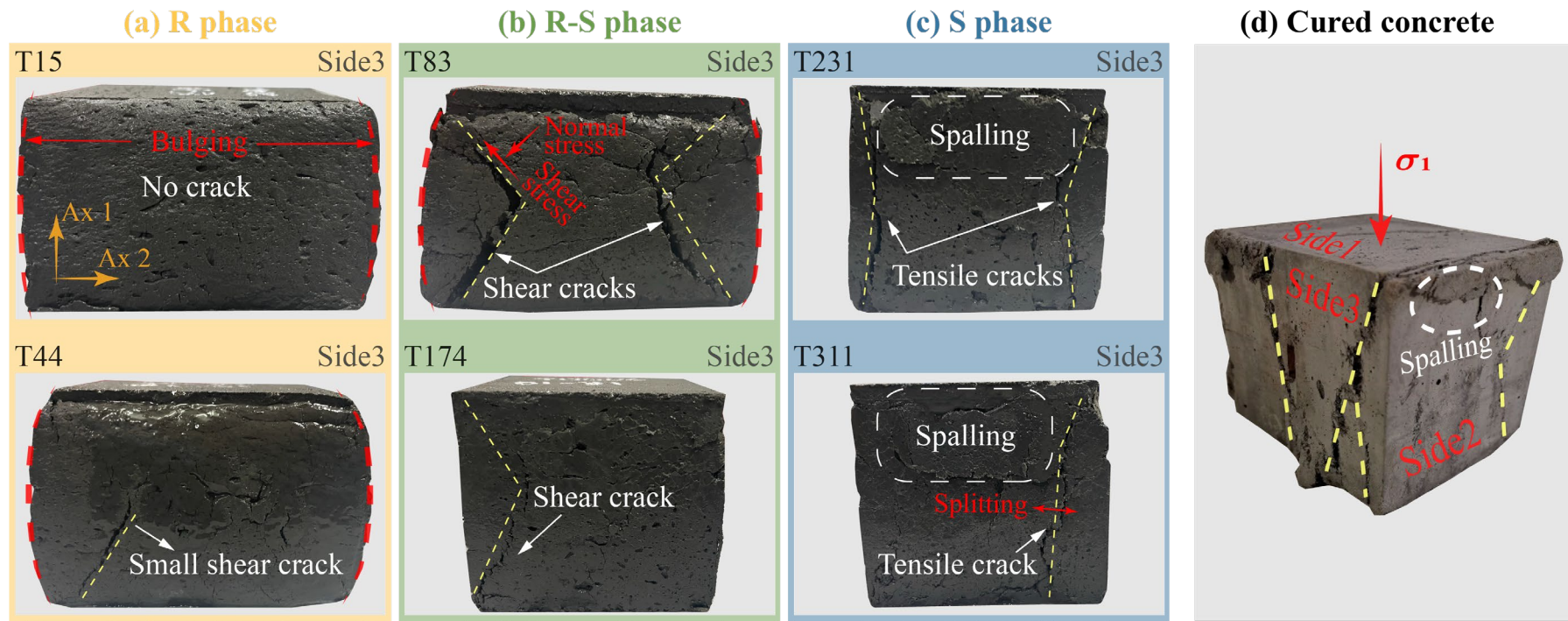


Figure 4.1 Crack patterns of specimens at the end of uniaxial compression tests: (a)-(c) Fresh 3DPC specimens; (d) Cured concrete specimen [modified from Gao and Hu (2021)].

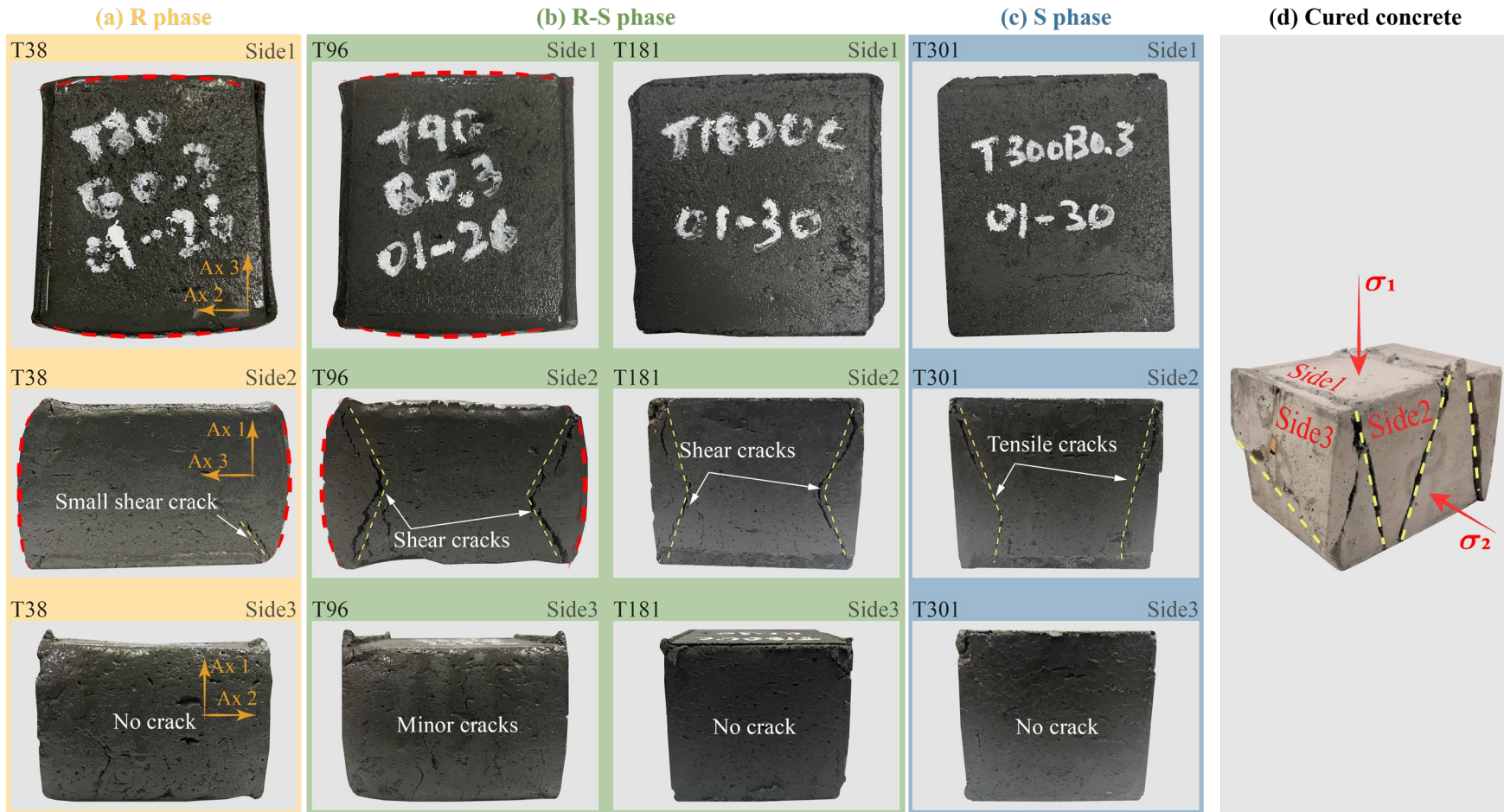


Figure 4.2 Crack patterns of specimens at the end of biaxial compression tests: (a)-(c) Fresh 3DPC specimens with BC-0.33; (d) Cured concrete specimen with BC-0.80 [modified from Gao and Hu (2021)].

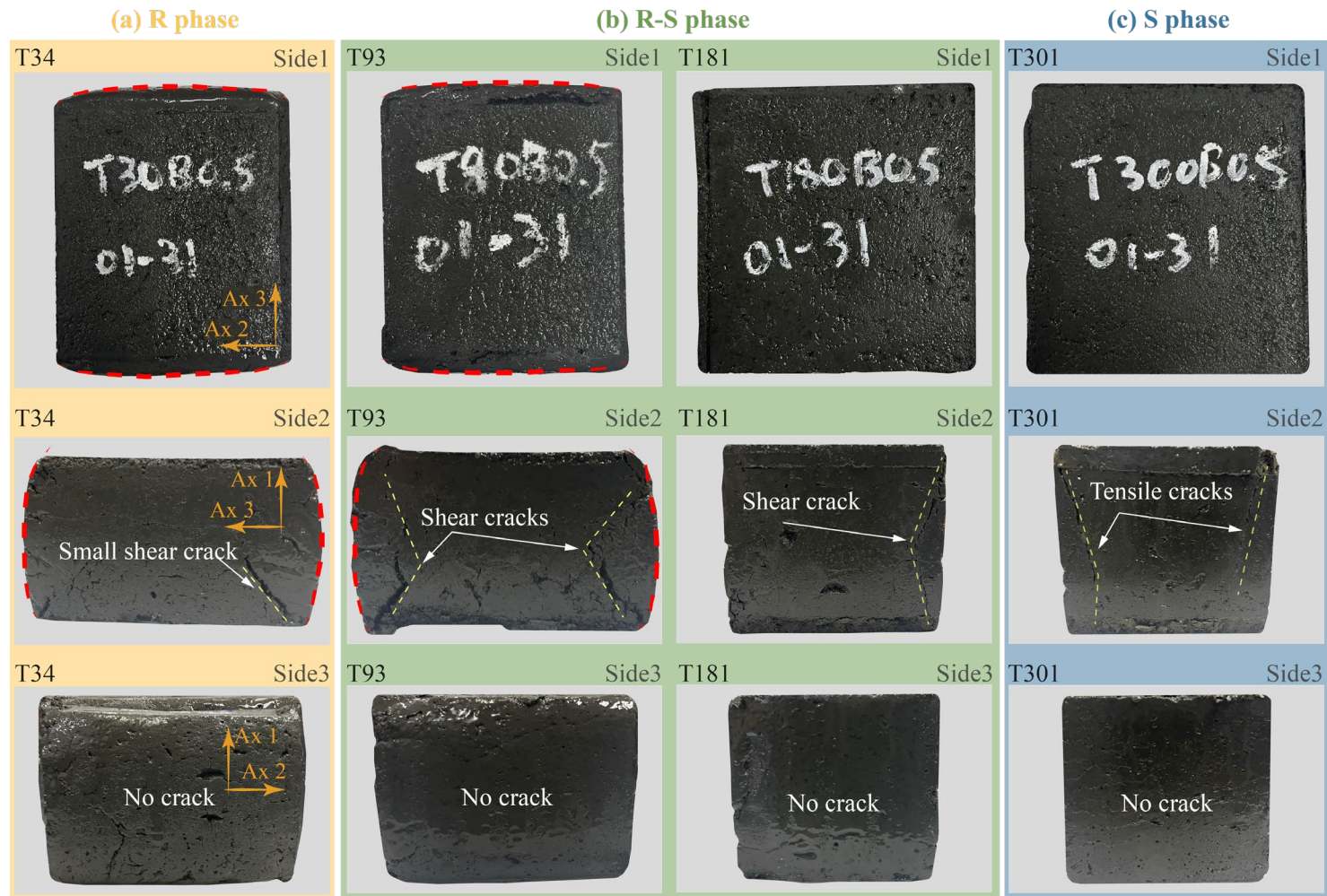


Figure 4.3 Crack patterns of specimens at the end of biaxial compression tests: BC-0.50.

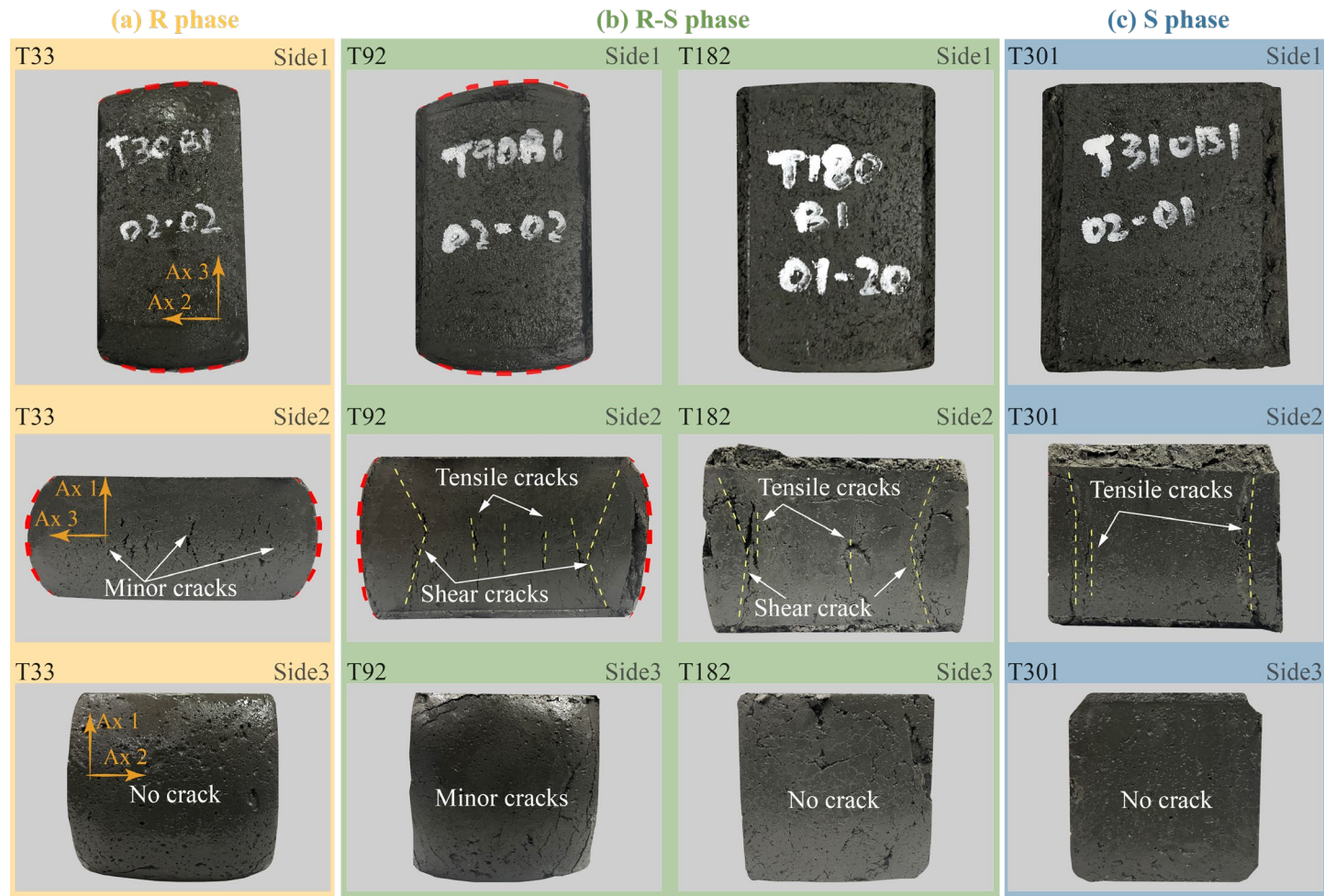


Figure 4.4 Crack patterns of specimens at the end of biaxial compression tests: BC-1.00.

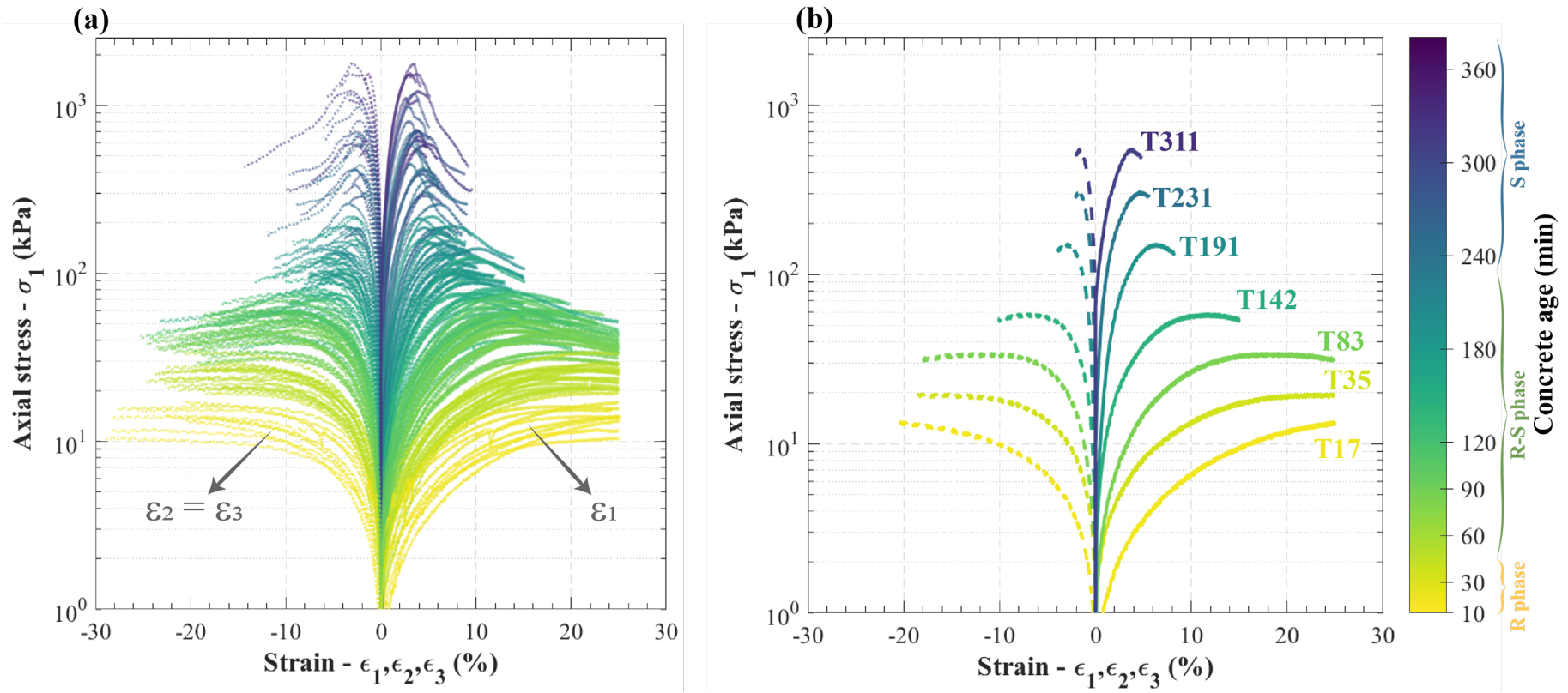


Figure 4.5 True stress-engineering strain curves of UC specimens: (a) All results, (b) Exemplary results.

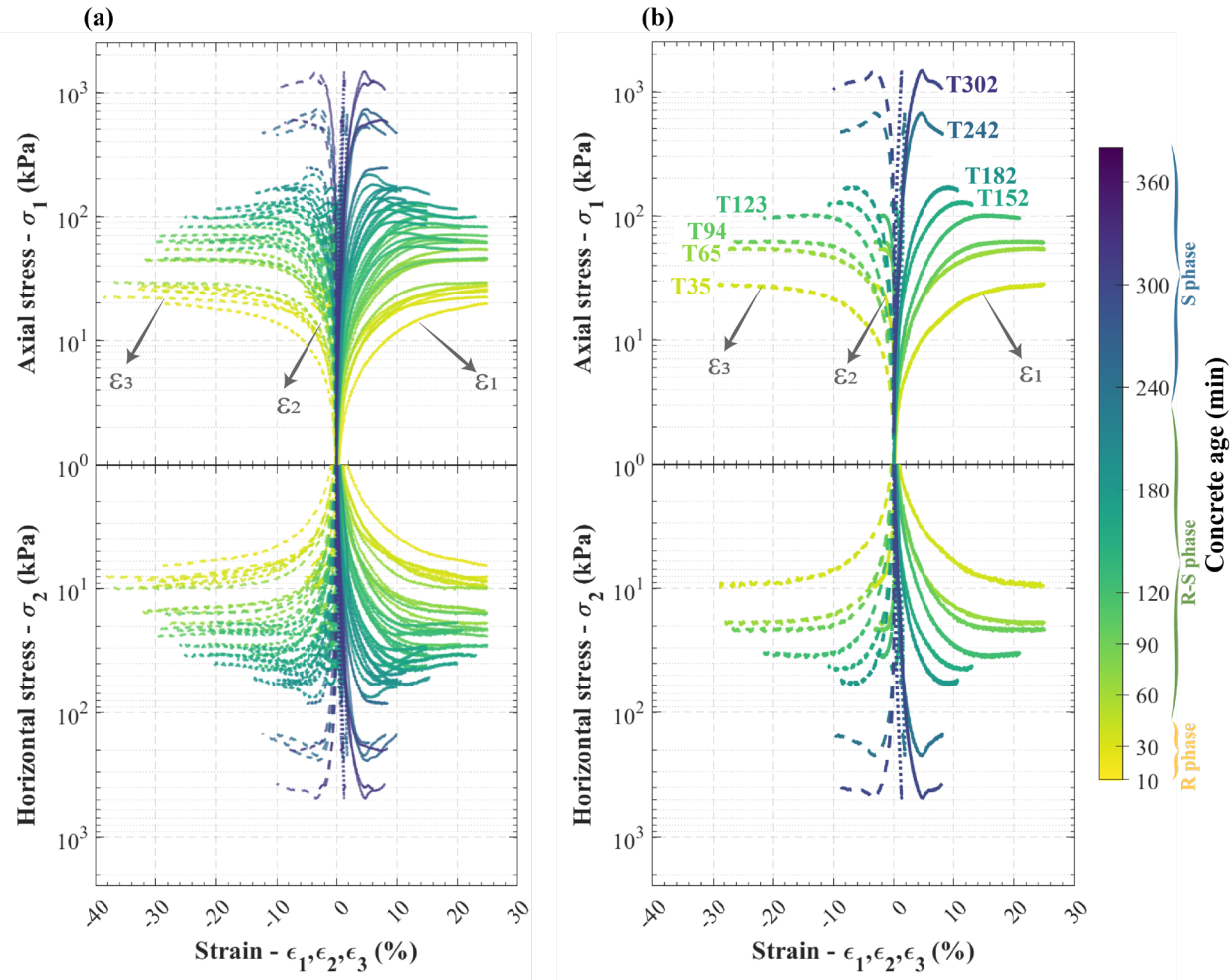


Figure 4.6 True stress-engineering strain curves of BC-0.33 specimens: (a) All results, (b) Exemplary results.

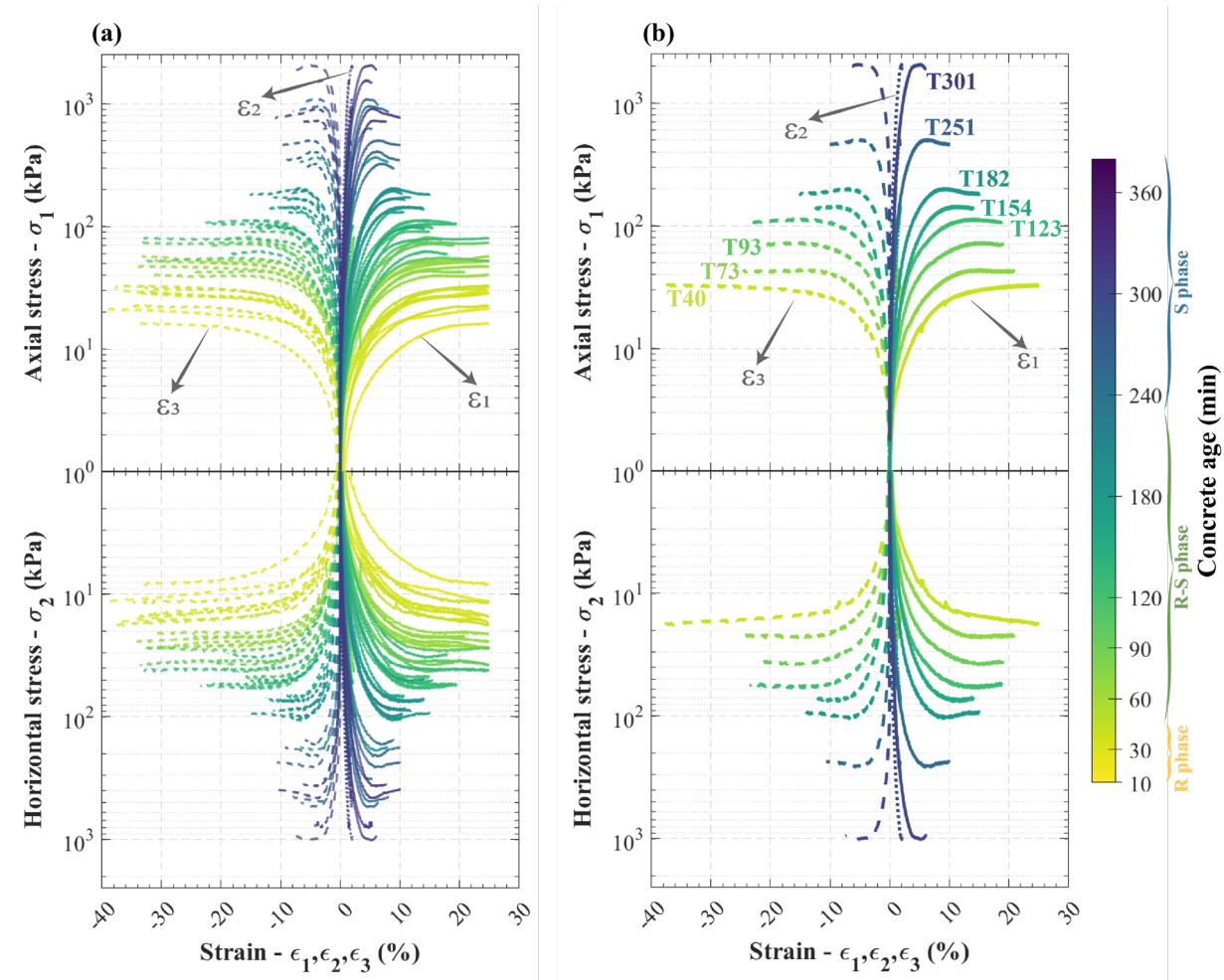


Figure 4.7 True stress-engineering strain curves of BC-0.50 specimens: (a) All results, (b) Exemplary results.

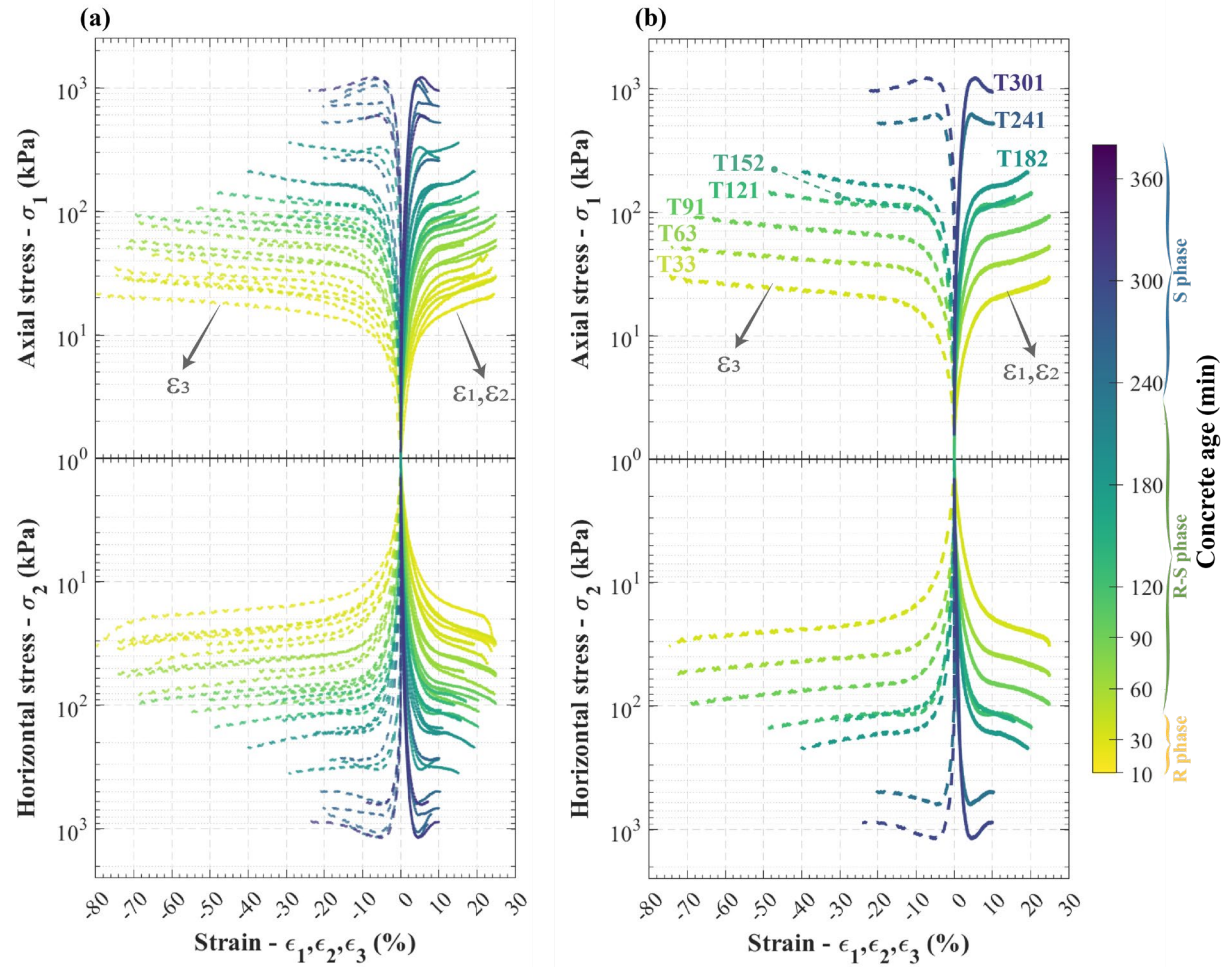


Figure 4.8 True stress-engineering strain curves of equibiaxial compression specimens: (a) All results, (b) Exemplary results.

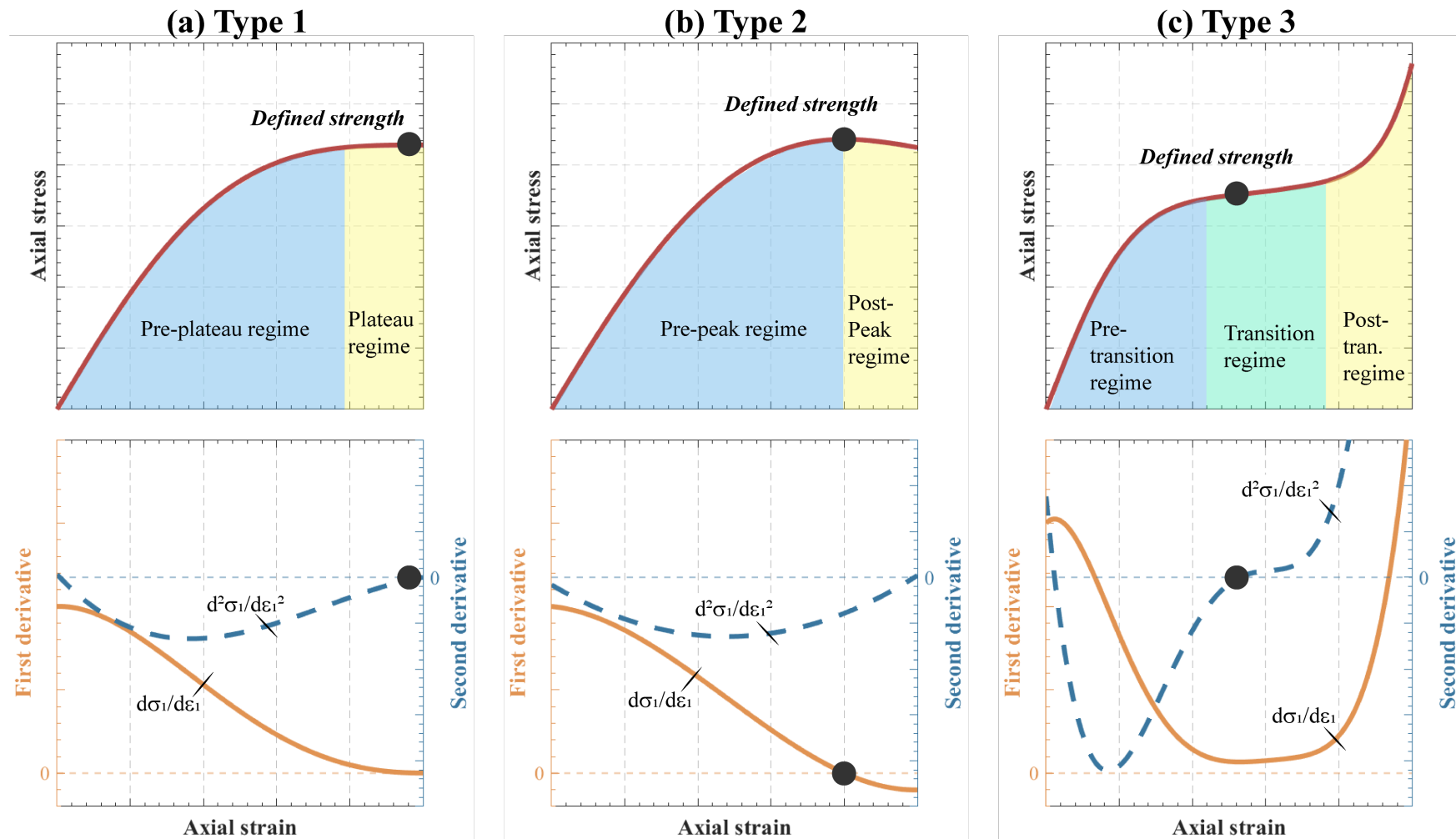


Figure 4.9 Three typical types of axial stress-axial strain curves.

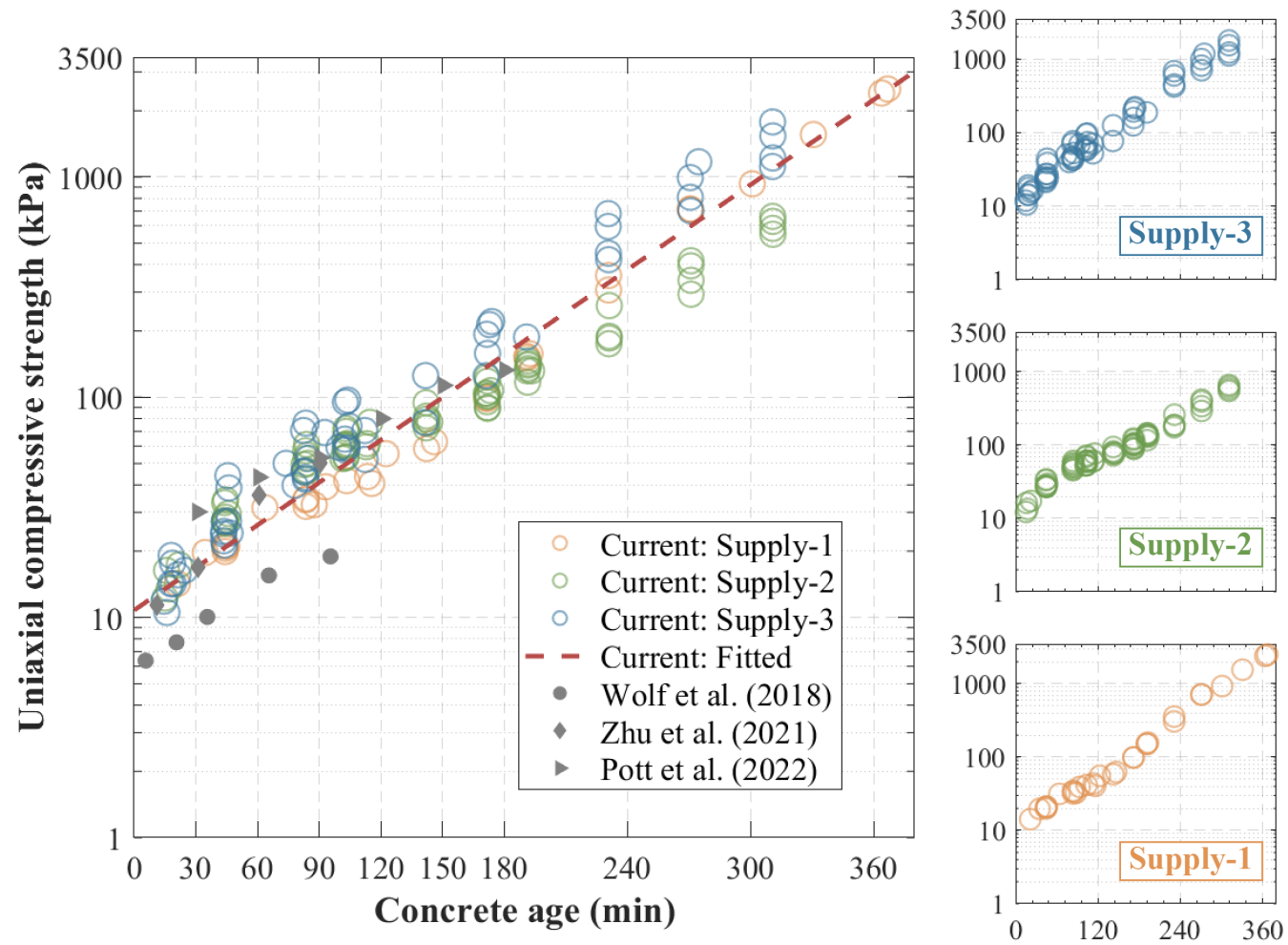
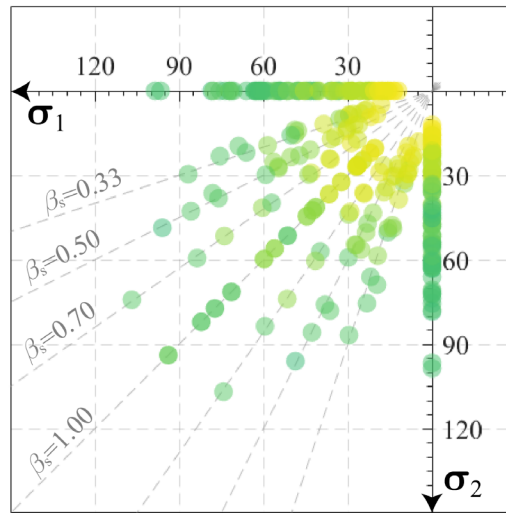
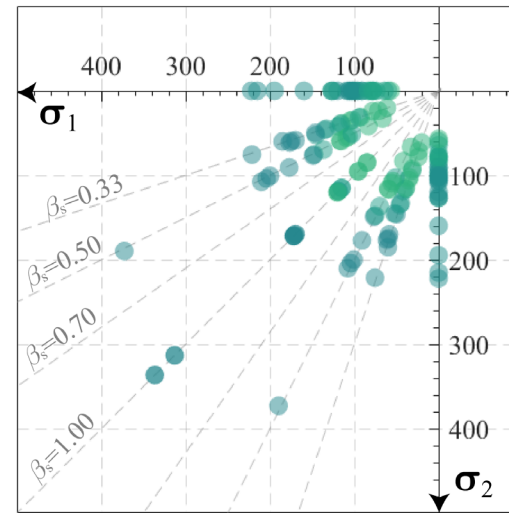


Figure 4.10 Evolution of uniaxial compressive strength with concrete age.

(a) T15 - T120



(b) T120 - T190



(c) T190 - T330

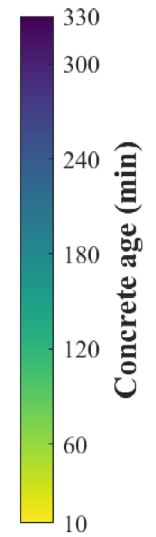
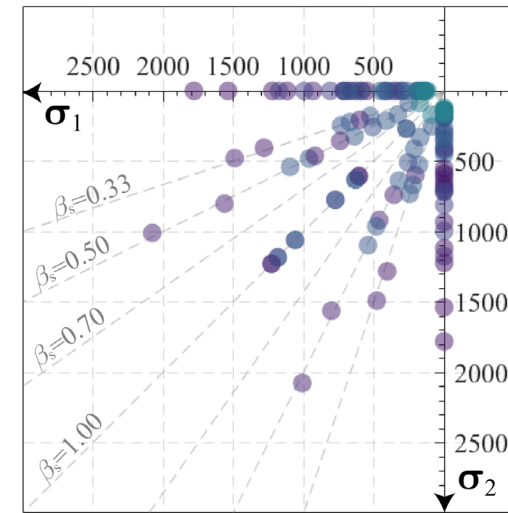


Figure 4.11 Strength data in the plane stress space: results of experiments (unit in kPa).

(a) Normalised biaxial strength data

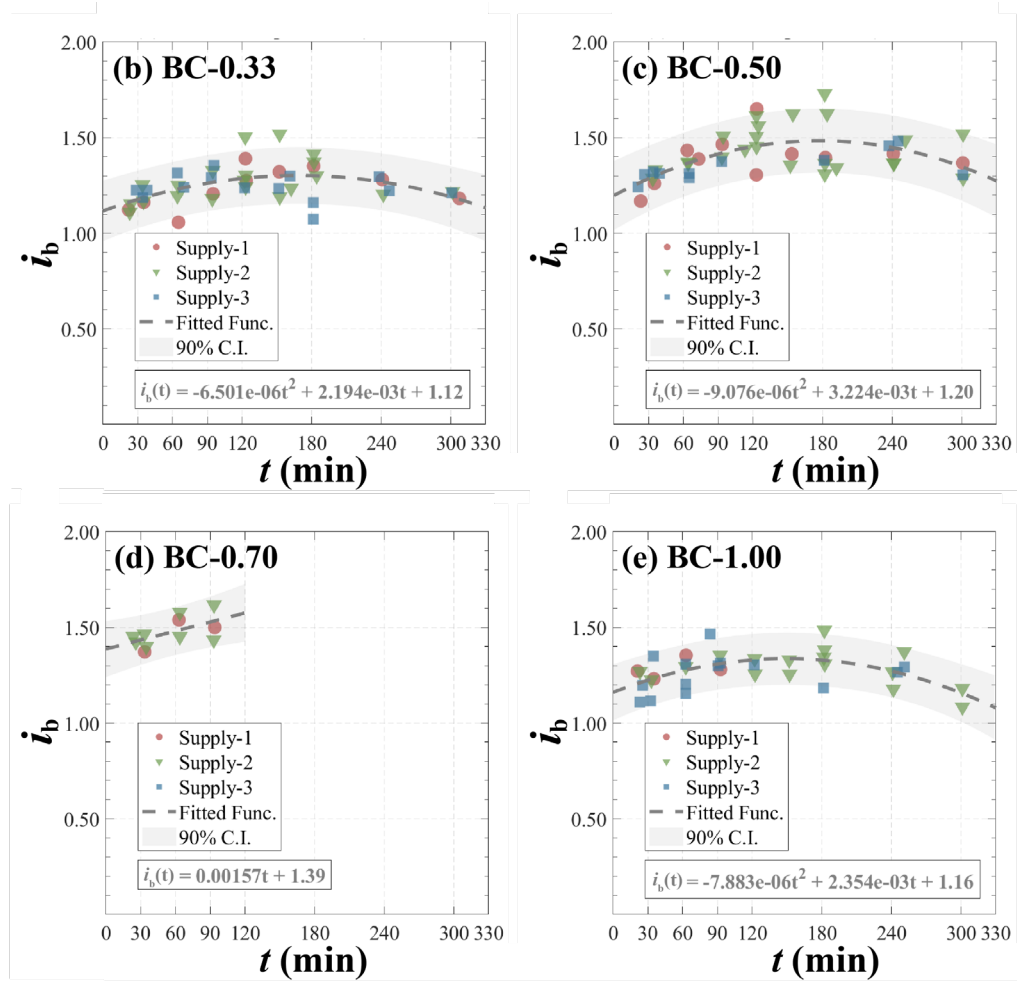
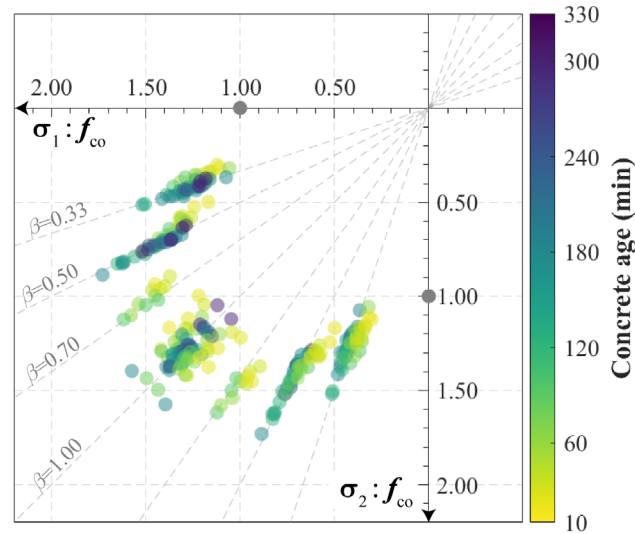


Figure 4.12 Evolution of normalised strength with concrete age: results of experiments.

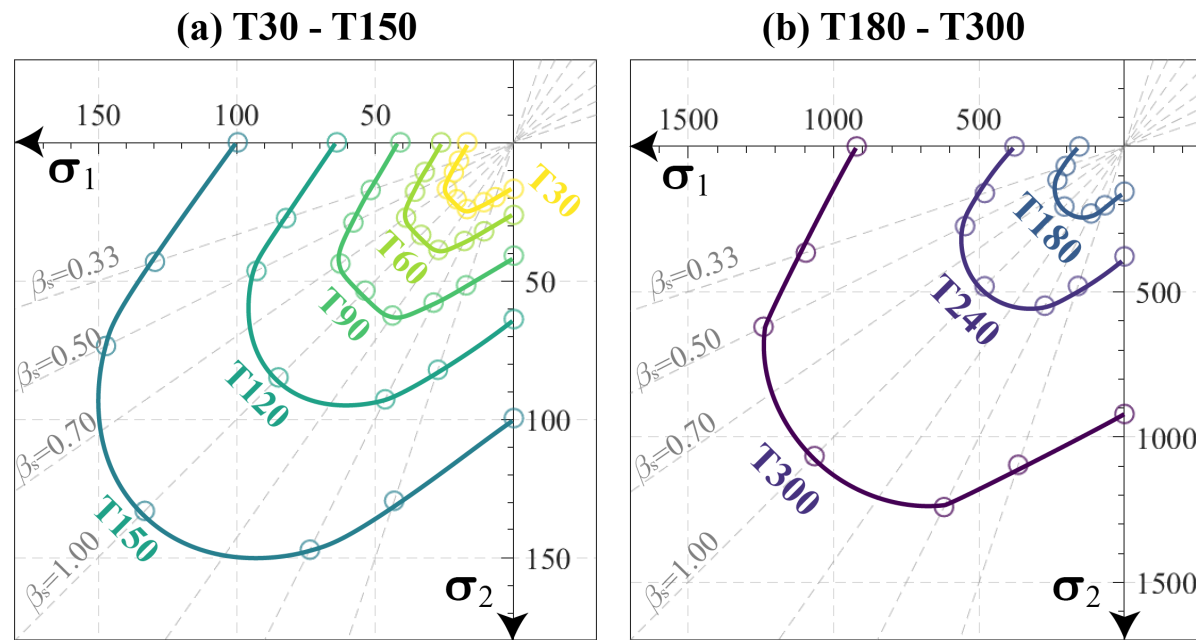


Figure 4.13 Biaxial compressive strength: average of experimental results (unit in kPa).

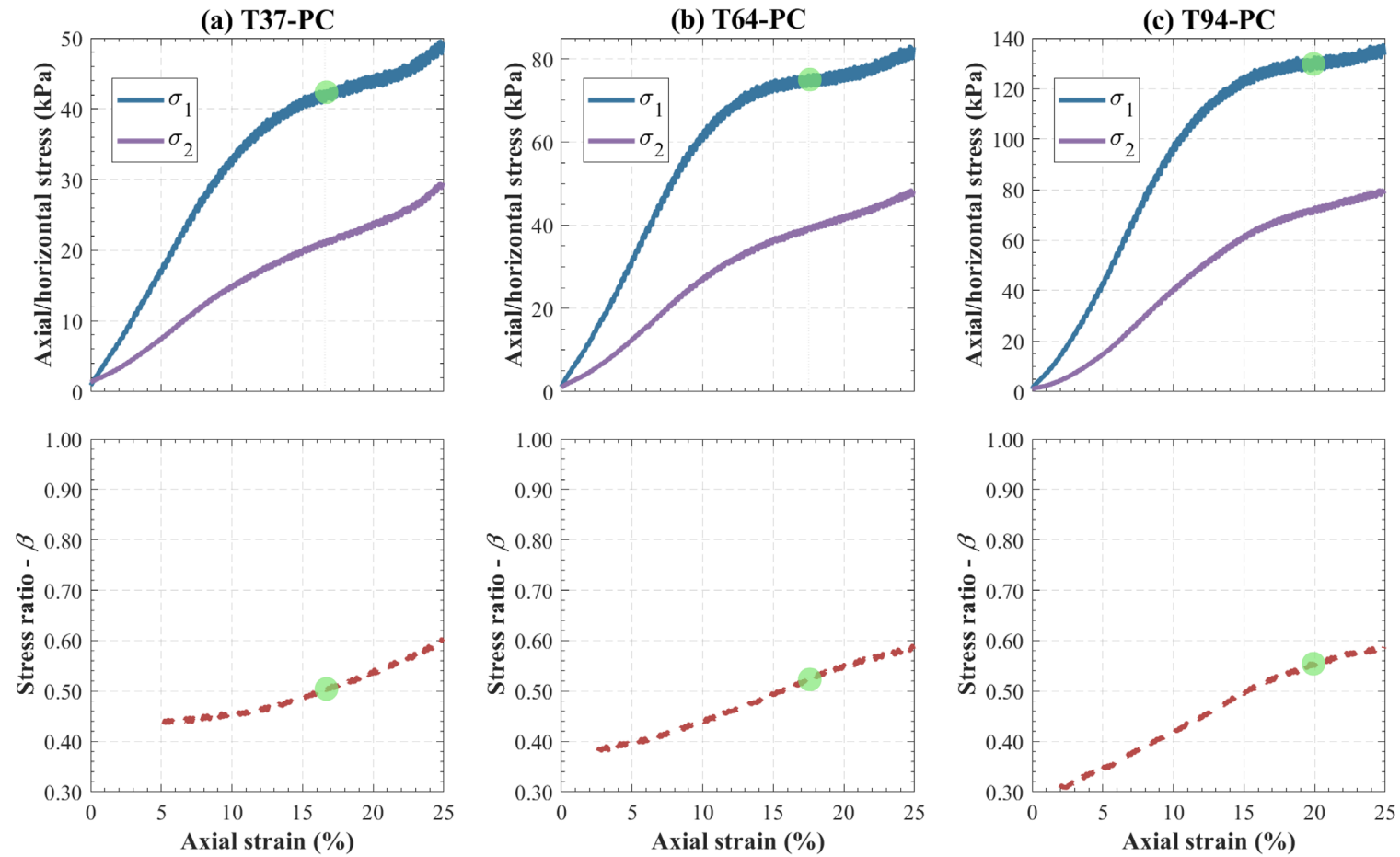


Figure 4.14 Representative stress-strain curves and evolution of stress ratio in passively confined biaxial compression (PC) tests.

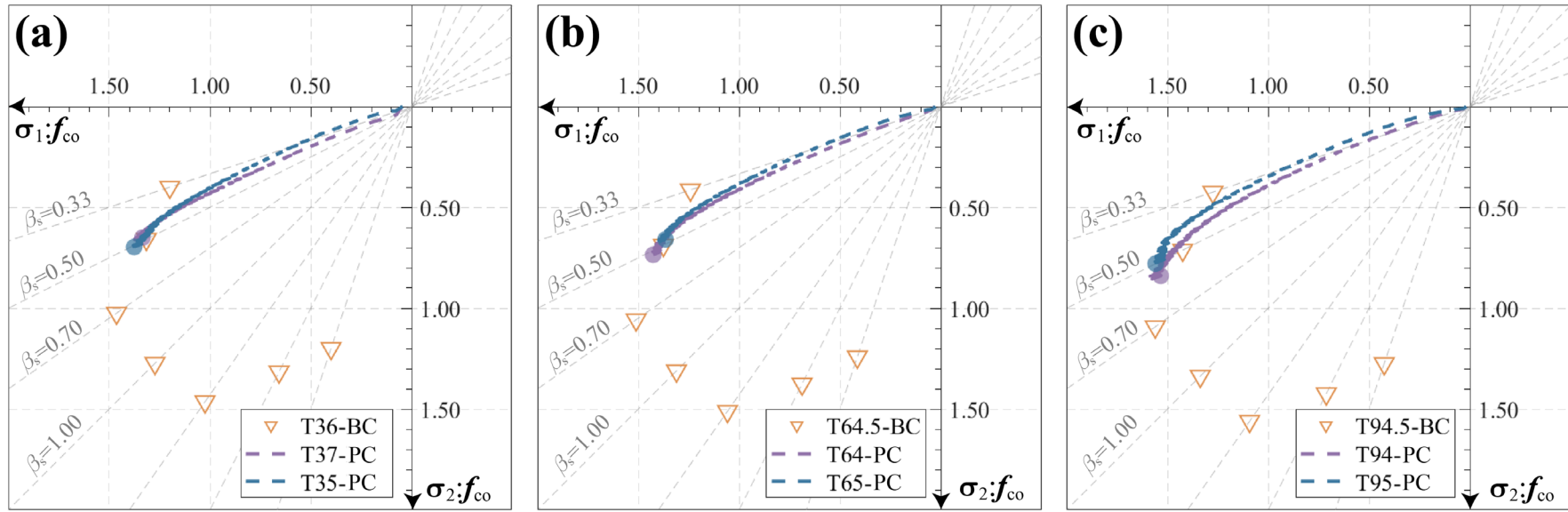


Figure 4.15 Comparison of strength data obtained from passively confined biaxial compression (PC) tests and from proportionally loaded BC tests.

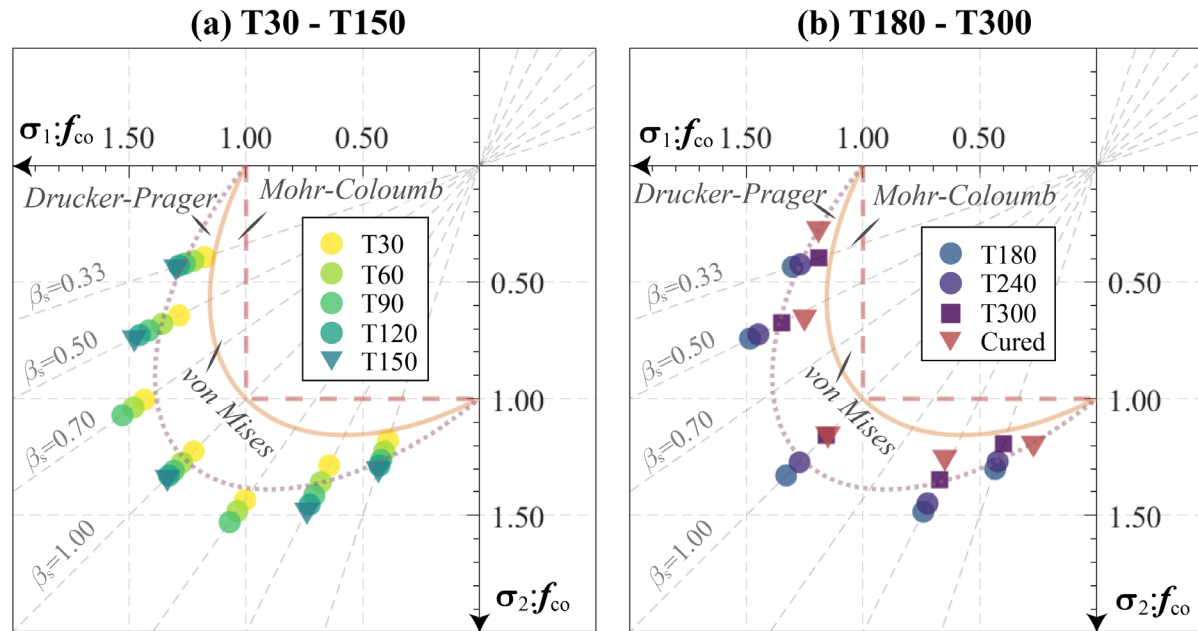


Figure 4.16 Comparison of the tested normalised strength envelope with existing strength models.

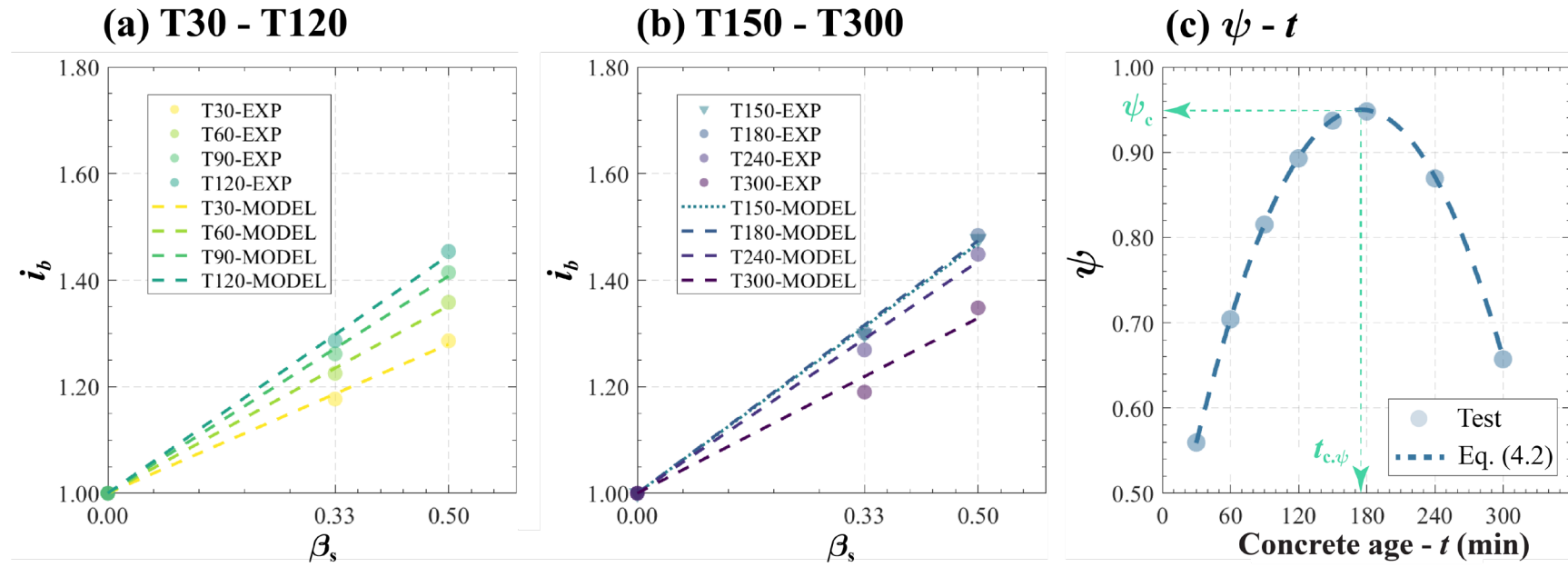
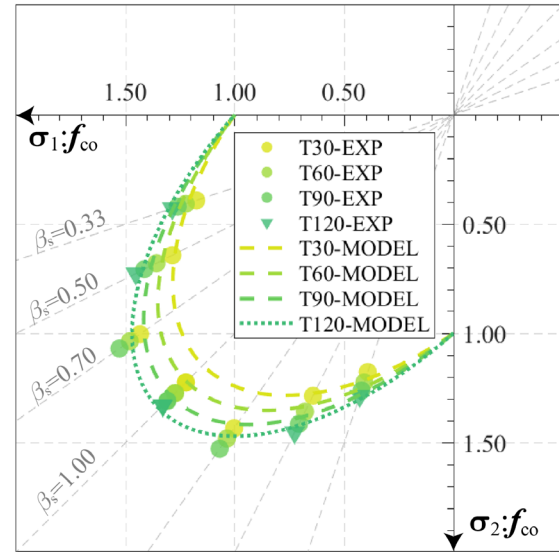
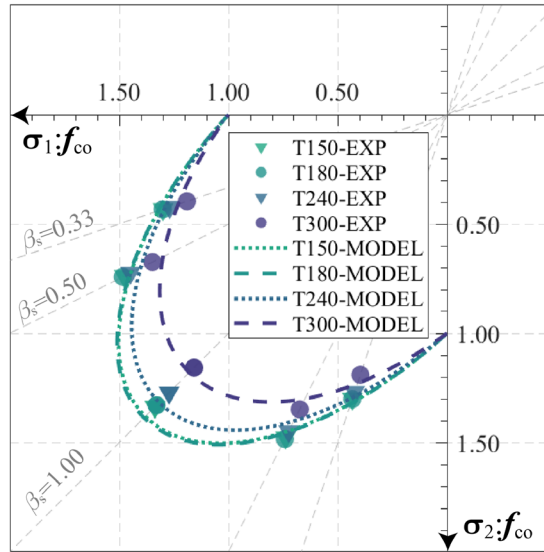


Figure 4.17 Biaxial-to-axial strength ratio: (a) Up to 120 mins; (b) Up to 300 mins; (c) Evolution of the slope ψ .

(a) T30 - T120



(b) T150 - T300



(c) $\chi - t$

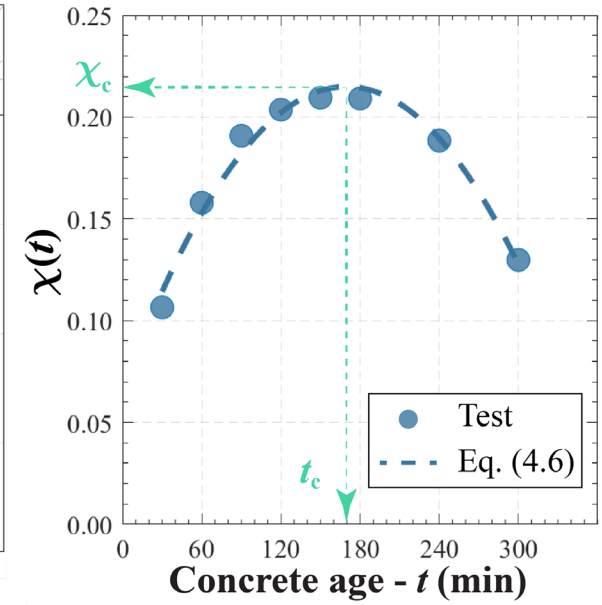


Figure 4.18 Performance of the age-dependent strength envelope: (a) Up to 120 mins; (b) Up to 300 mins; (c) Evolution of the parameter χ with concrete age.

APPENDIX 4.1 FINITE ELEMENT SIMULATION: IMPACT OF FRICTION ON STRENGTH MEASUREMENT

As presented in Section 4.2.2, the stress-strain curves of R and R-S phase specimens under biaxial compression with a high confining stress ratio ($\beta_s = 1.0$) are characterised by a secondary ascending branch. This phenomenon was believed to be due to friction and was thus referred to as pseudo-hardening behaviour. To validate this hypothesis, a series of finite element (FE) simulations were conducted on specimens under equibiaxial compression, with different friction coefficients assigned to the interfaces between the specimens and the loading platens. Specifically, the FE models were developed based on model with platen configuration 1 in Section 3.4, whereas three different friction coefficients: 0 (frictionless), 0.05 (low friction), and 0.1 (high friction) for the interfaces were investigated.

Figure A.4.1a presents the axial stress-axial strain curves for models with the three different friction coefficients under BC-1.00. The circles indicate the defined strengths as discussed in 4.3.1. Figure A.4.1b depicts the ratio of the averaged stress along axis 3 ($\sigma_{3.av}$) to that along axis 1 ($\sigma_{1.av}$). The averaged stress was computed by taking the mean of a specific stress component across all elements of the specimen. As indicated, the model without friction demonstrates a type 2 stress-strain curve (see Figure 4.9b), whereas the descending branch becomes less steep in the case of $\mu = 0.05$. For the model with $\mu = 0.1$, the stress-strain curve becomes type 3 (see Figure 4.9c), clearly revealing *pseudo-hardening behaviour*.

Examining the ratio $\sigma_{3.av} : \sigma_{1.av}$, it is evident that this metric increases dramatically after the defined strength is reached. For the model with low friction ($\mu = 0.05$), the ratio increases significantly after the peak point, implying an increasing confinement in the axis 3 direction, which results in less pronounced softening compared to the frictionless case. For the specimen with high friction ($\mu = 0.1$), the dramatic increase occurs before pseudo-hardening happens, suggesting that the degree of confinement along axis 3 is sufficiently high to induce this phenomenon. It is evident that the defined strength is the threshold stress value exceed which the influence of friction becomes significant.

Figure A.4.2 shows the deformed shape of the specimen in the FE simulation at the defined strength for model with $\mu = 0.1$. It can be observed that localised cracks have been formed on the specimens with cracking strain exceeding 40%. These localised cracks indicate that the specimen should not be considered as a continuum when the defined strength is reached; therefore, the specimen at this stage should be considered to have failed.

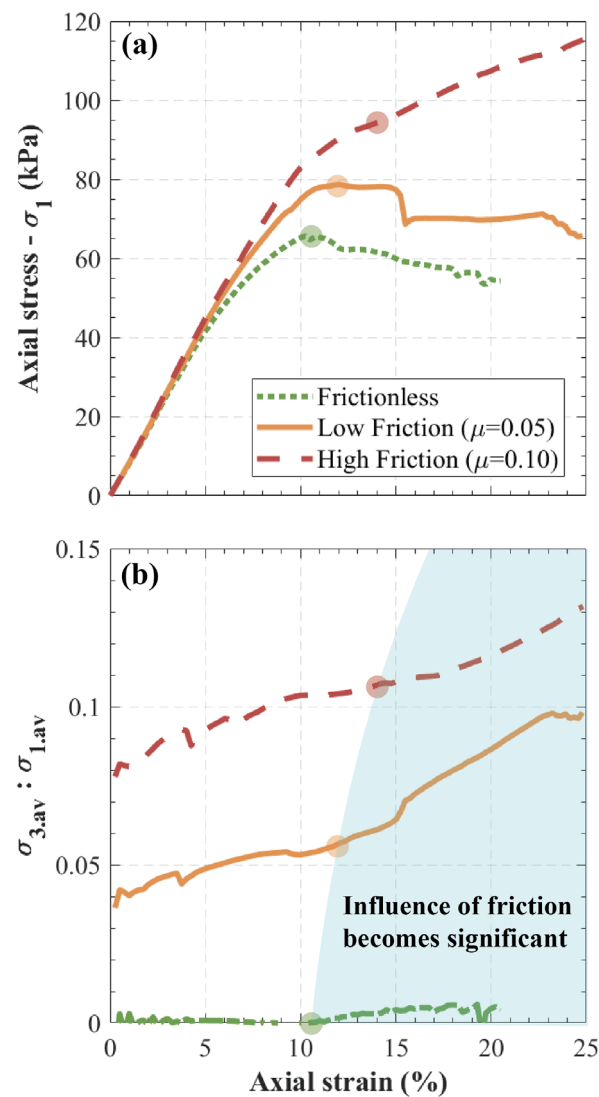


Figure A.4.1 Influence of friction on the stress strain behaviour of specimen under BC-1.00: (a) Axial stress-axial strain curve; (b) Ratio of average stress along axis 3 to that along axis 1.

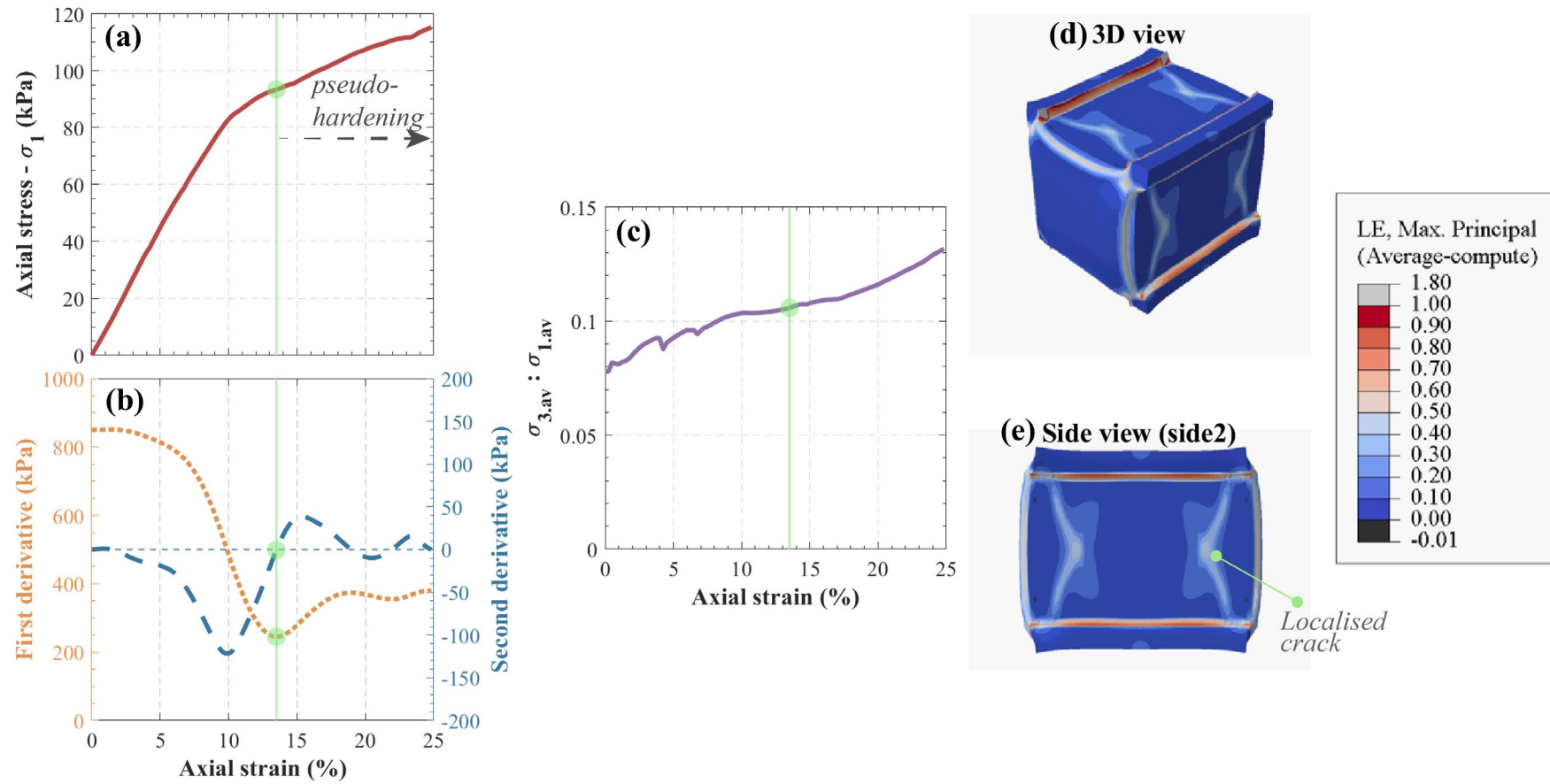


Figure A.4.2 FE simulation: deformation of specimen at the defined strength (BC-1.00, $\mu = 0.1$)

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CHAPTER 5

STRESS-STRAIN BEHAVIOUR AND CONSTITUTIVE MODEL OF FRESH 3D- PRINTED CONCRETE

5.1 INTRODUCTION

While Chapter 4 established fundamental insights into the strength behaviour of fresh 3D-printed concrete (3DPC) under uniaxial and biaxial compression, a comprehensive understanding of its mechanical properties requires detailed characterisation of its complete stress-strain relationships. Therefore, this chapter advances the investigation beyond strength characteristics to examine the full stress-strain response under both uniaxial and biaxial loading conditions.

The investigation begins with a discussion of stress-strain interactions, encompassing axial, lateral and volumetric strain responses, based on data from the biaxial testing programme presented in Chapter 3. Then, comprehensive stress-strain analyses, focusing on three key aspects: material stiffness evolution, plastic dilation characteristics, and hardening behaviour, are performed within the framework of strain-rate-independent elastoplasticity. These analyses provide crucial insights into the fundamental mechanisms governing the mechanical response of fresh 3DPC, establishing a robust foundation for constitutive modelling.

Building upon these analytical results, a comprehensive constitutive model for fresh 3DPC is proposed. This model provides a mathematical framework for characterising the stress-strain response under both uniaxial and multiaxial loading conditions throughout the critical early-age period. Validation against the experimental data demonstrates the model's high predictive accuracy, supporting its potential application in analysing the response of fresh concrete structures during construction.

5.2 TEST RESULTS

Whilst engineering strain served as the measure of deformation in the preceding chapter owing to its simplicity and intuitive nature, this chapter employs logarithmic strain (true strain) to establish the true stress-true strain constitutive relationship. This transition is particularly crucial as the finite element software Abaqus (Dassault Systèmes, 2023), which will be utilised in Chapter 7 to investigate the structural behaviour of fresh concrete during printing, requires constitutive relationships to be defined in terms of true stress and true strain. Hence, throughout this chapter, unless otherwise specified, all references to ‘strain’ (and therefore the symbol ϵ_i) denote logarithmic strain. The relationship between true strain (ϵ_{tr}^i) and engineering strain (ϵ_{eng}^i) along a particular axis i is given by Eq. (5.1), where compressive strain designated as positive.

$$\epsilon_{tr}^i = -\ln(1 - \epsilon_{eng}^i) \quad \forall i = 1 \dots 3 \quad (5.1)$$

Furthermore, the term ‘peak stress’ refers to the material strength as defined in the previous chapter.

5.2.1 True Axial Stress–True Axial Strain Curves

The true axial stress–true axial strain curves for specimens subjected to uniaxial and biaxial compression are respectively depicted in Figures 5.1a and 5.2 a1-a3. Axial stress (σ_1) is presented on a logarithmic scale, while axial strain (ϵ_1) is shown on a linear scale. Each curve is colour-coded according to the colour bar displayed, which corresponds to the age of the specimen. Furthermore, the strength of each specimen - determined as per the methodologies described in the previous chapter - is illustrated through a scatter plot, alongside the associated axial strain. As observed, the shape of these stress-strain curves is similar to those presented in the previous chapter, where engineering strains were employed.

It is evident that the stress-strain response is influenced by both the concrete age (t) and stress ratio (β_s). Specifically, under a given stress ratio β_s , the true stress–true strain curves exhibit a ductile-to-brittle transformation as the concrete ages. Rheological

phase specimens (R-phase, $t \leq 45$ mins) display plateauing or slight hardening, transitional phase specimens (R-S phase, $45 \text{ mins} < t \leq 230 \text{ mins}$) show a gradual post-peak descent, and solid phase specimens (S-phase, $t > 230 \text{ mins}$) demonstrate a significant post-peak descent. At a specific age, the slope of the descending branch becomes more gradual as β_s increases. Additionally, since true strain is employed in this chapter, the magnitude of compressive strain is larger, while that of tensile strain is smaller compared to those in the previous chapter, where engineering strain was used.

5.2.2 Lateral Strains–Axial Strain Curves

Under uniaxial compression, the free-dilation lateral strain (the average of ϵ_2 and ϵ_3) is plotted against the axial strain (ϵ_1) in Figure 5.1b, with triangles indicating the corresponding values at peak stress. These curves are depicted as dash lines since the lateral deformation was only measured at test completion, whereas the intermediate strains are interpolated according to Eq. (3.1). As shown, the lateral strains increase monotonically with increasing axial strain due to the Poisson's effect as well as cracking. The maximum lateral strain decreases from around 25% at T30 to less than 15% for age larger than T300. Despite highly scattered data, the slope of the curves, indicating the rate of dilation, seems to decrease gradually in the first 180 minutes and then increase afterward. This transition in rate of dilation is attributed to the change in microstructure due to material ageing which results in the change in deformation mechanism, as evident in the photos of failure patterns (Figures 4.1 to 4.4).

Under biaxial compression, ϵ_2 was calculated from the relative displacement of the horizontal platens. Figures 5.2b1-b3 show the ϵ_2 - ϵ_1 relationship corresponding to BC-0.33, BC-0.50 and BC-1.00, respectively. Under BC-0.33, as depicted in Figure 5.2b1, specimens aged less than approximately 90 minutes predominantly exhibit dilation along axis 2 throughout the loading process, with negligible contraction, despite a compressive stress equal to one-third of the axial stress being applied along axis 2. The accumulated dilation in axis 2 for these specimens ranges from approximately 2% to 5% when the axial strain reaches 28% at the end of the loading process. With increasing age, the specimens demonstrate greater contraction before dilation along axis 2. Consequently, the cumulative dilation in axis 2 up to peak stress gradually decreases

with increasing age. At around T150, the cumulative dilation along axis 2 up to peak stress is approximately 0%, and by T300, it transitions to approximately 1% contraction. The maximum contraction along axis 2 increases from 0% at approximately T30 to around 1.8% at T300. The ϵ_2 - ϵ_1 relationship for BC-0.50, as depicted in Figure 5.2b2, exhibits a markedly different pattern. Specimens across all ages demonstrate a contraction-to-dilation pattern. The axial strain associated to the turning point decreases from approximately 15 % at around T30 to approximately 5% at around T300. The maximum contractive strain is primarily in the range of 1 to 2.5%, largely independent of age. Unlike the case with BC-0.33, the accumulative strain along axis 2 up to peak stress is positive (contractive), with the exception of one R-phase specimen with age T23, which can be considered an outlier. The higher tendency in contraction along axis 2 can be due to the higher confining stress ratio, which suppresses the tendency of crack opening along this direction. Under BC-1.00, the specimens are equally compressed along axis 1 and 2; thus, the ϵ_2 - ϵ_1 curves manifest as 45-degree straight lines in Figure 5.2b3.

Figures 5.2c1-c3 delineate the ϵ_3 - ϵ_1 relationships corresponding to BC-0.33, BC-0.50 and BC-1.00, respectively. In all cases, the material consistently exhibits dilation, as indicated by the interpolated dashed lines. For BC-0.33, the ϵ_3 at test completion decreases from approximately 33% at around T30 to less than 10% for ages greater than T300. For BC-0.50, the magnitude of ϵ_3 is marginally greater than that observed with BC-0.33, due to the larger confinement ratio along axis 2, making crack opening along axis 3 more likely than along axis 2. In the case of equal biaxial compression, BC-1.00, cracks can only manifest along axis 3. The magnitude of ϵ_3 at test completion, shown in Figure 5.2c3, decreases from around 60% at around T30 to less than 25% for ages greater than T300, which is approximately twice of that of specimens with BC-0.33 and BC-0.50. Despite the differences of confining stress ratio, S-phase specimens show higher rate of dilation as evident from the steeper slope of the ϵ_3 - ϵ_1 curves. However, the difference between the S-phase and that of R- and R-S phase specimens under BC is less pronounced than that in UC tests. This is likely due to the frictional stress from the horizontal platens inhibiting crack opening along axis 3. As discussed in the previous chapter, this frictional stress is believed to be the cause of the pseudo-hardening phenomenon observed in BC-1.00.

5.2.3 Volumetric Strain–Axial Strain Curves

Based on the three principal strains, the volumetric strain can be calculated ($\epsilon_v = \sum \epsilon_i$) representing the change of volume during the loading process. For specimens under UC, the volumetric strain (ϵ_v)-axial strain (ϵ_1) curves are depicted in Figure 5.1c, whereas those for specimens under BC are depicted in Figures 5.2d1-d3. The volumetric strain at peak stress (ϵ_v^k) of each specimen is identified by a scatter.

Although the lateral strains-axial strain relationship exhibits distinct patterns across specimens subjected to different stress ratios, similar pattern in the ϵ_v - ϵ_1 curves can be found. Specifically, ϵ_v initially demonstrates a positive trend, indicating volumetric contraction during the early loading stages, before transitioning to a negative trend (volumetric dilation) with increasing axial strain. This behavioural pattern aligns with previous findings from uniaxial and triaxial tests conducted on both fresh 3DPC (Wolfs *et al.*, 2019) and cured concrete (Imran and Pantazopoulou, 1996). The volumetric compaction can be attributed to the compression-induced reduction in void ratio within the material's microstructure, whilst the subsequent volumetric dilation may be due to the relative displacement of granular particles, leading to an increase in inter-particle void spaces.

The rate of volumetric dilation, as indicated by the slope of the portion of the ϵ_v - ϵ_1 curve during the dilation phase, is influenced by both concrete age and applied stress ratio. At a fixed stress ratio, the dilation rate exhibits a non-monotonic temporal evolution: it decreases during the initial 180 minutes of concrete age before subsequently increasing. Notably, S-phase specimens aged beyond 240 minutes demonstrate substantially higher volumetric dilation rates compared to their R- and R-S phase counterparts. Furthermore, at any given concrete age, specimens subjected to BC exhibit lower dilation rates than those under UC, which can be attributed to the crack-inhibiting effect of the additional horizontal stress. A comprehensive analysis of these dilation characteristics will be presented in the subsequent section.

Figure 5.3 compares the volumetric strain at peak stress (ϵ_v^k) for specimens under different stress ratios. Under uniaxial compression (UC), specimens predominantly

exhibit volumetric expansion ($\epsilon_v^k < 0$) at peak stress. The magnitude of ϵ_v^k demonstrates a clear age-dependent behaviour, decreasing from approximately +20% at 15 minutes to approximately 0% at 240 minutes - a value that aligns with published data for hardened concrete under UC (Imran and Pantazopoulou, 1996) - before stabilising at this level. Specimens under BC show less dilation at peak stress compared to their UC counterparts. Furthermore, for specimens aged less than 240 minutes (R and R-S phase), the ϵ_v^k decreases with increasing β_s . Conversely, S-phase specimens maintain comparable ϵ_v^k values across different stress ratios under UC conditions.

5.3 STRESS-STRAIN ANALYSES

In this section, a comprehensive stress-strain analysis based on the test results will be conducted within the framework of rate-independent elastoplasticity.

5.3.1 Evaluation of Stiffness Parameters

As illustrated in Figure 5.1, the stress-strain relationship under uniaxial compression exhibits an approximately linear response at lower stress levels. As discussed in Chapter 2, this linear segment of the stress-strain curve has been widely interpreted as elastic behaviour in existing literature (Wolfs *et al.*, 2018; Casagrande *et al.*, 2020), leading researchers to derive Young's modulus from the slope of this linear portion. However, subsequent studies have clarified that this derived parameter should be interpreted solely as a stiffness parameter (termed apparent modulus or E_{ap}), which is different from the actual Young's modulus of the material. Indeed, as will be demonstrated in the subsequent chapter, the actual elastic modulus (denoted as E) for R-phase specimens can exceed E_{ap} by an order of magnitude.

A comprehensive deformation analysis requires the isolation of elastic deformation from plastic deformation, which, in turn, requires the acknowledgment of the actual elastic modulus. However, the actual elastic modulus cannot be directly obtained from the stress-strain curves with monotonic loading. Therefore, in this study, the actual elastic modulus (E) is estimated from the apparent modulus (E_{ap}) by using Eq. (5.2).

$$E = \Upsilon E_{ap} \quad (5.2)$$

where Υ is a function representing the ratio of the actual modulus to apparent modulus of the fresh 3DPC, which is given by the following equation.

$$\Upsilon(t) = \Upsilon_0 + \varsigma_r \left(\frac{t}{t_f} \right)^\vartheta \quad (5.3)$$

Here, Υ_0 denotes the ratio $E:E_{ap}$ at age zero, ς_r and ϑ are curing parameters that govern the shape of the function, and t_f represents an assumed age factor. Based on the loading-unloading-reloading test results conducted between 20 and 120 minutes (presented in the subsequent chapter) and assuming $E:E_{ap} = 1.5$ at age 360 minutes, the following parameter set was found to provide optimal fit: $\Upsilon_0 = 132$, $\varsigma_r = -130$ and $\vartheta = 0.0312$. Further research is warranted to investigate the relationship between E and E_{ap} over a broader age range and for different fresh 3DPC.

Figure 5.4b demonstrates the evolution of the E_{ap} with concrete age for fresh 3DPC made of three supplies of materials. Similar to the trend observed in f_{co} (Figure 5.4a), E_{ap} exhibits a significant increase within the first six hours, indicating a substantial rise in material stiffness. The degree of data scatter for E_{ap} and f_{co} shows notable similarity. While the variability within batches remains relatively minor, variability between different supplies is more pronounced but remains within a reasonable range, as discussed in the preceding chapter.

The estimated elastic modulus at different ages is illustrated in Figure 5.4c. Similar to E_{ap} , E increases drastically as concrete age, attributable to the progressive stiffening of the material's microstructures. For R-phase specimens, E exceeds E_{ap} by an order of magnitude, though this disparity diminishes progressively with material maturation. A comprehensive analysis of this phenomenon will be presented in the subsequent chapter.

In the previous chapter, it was established that a constant Poisson's ratio ($\nu = 0.3$) for fresh 3DPC is a valid assumption within the studied age range. Consequently, the elastic

deformation behaviour under both uniaxial and biaxial compression can be fully characterised using the generalized Hooke's law, incorporating the age-dependent modulus of elasticity (E) and the assumed Poisson's ratio (ν).

5.3.2 Plastic Dilation Behaviour

As discussed in Chapter 2, the dilation behaviour of fresh concrete is influenced by three factors: (1) stress state, (2) deformation history, and (3) concrete age. This section focuses on analysing the plastic dilation behaviour of fresh concrete using data from uniaxial compression (UC) and biaxial compression (BC) tests.

To analyse plastic deformation, the plastic strains were computed by subtracting the elastic strains from the total strains, that is $\epsilon_i^p = \epsilon_i - \epsilon_i^e, \forall i = 1, 2, 3$. Subsequently, the plastic strain rate throughout the loading process is calculated using the central difference method. From this, the volumetric and deviatoric dilation behaviour can be investigated.

5.3.2.1 Volumetric dilation behaviour

Denote α as the volumetric dilation parameter, which is defined as

$$\alpha = -\frac{d\xi(\boldsymbol{\epsilon}^p)}{d\rho(\boldsymbol{\epsilon}^p)} \quad (5.4)$$

Mechanically, α represents the plastic volumetric dilation rate, where a larger α signifies a greater rate of volumetric expansion. From a mathematical perspective, this parameter specifies the orientation of the plastic strain increment vector - $d\boldsymbol{\epsilon}^p$ - within the meridional plane. In line with the established sign convention, a positive α corresponds to plastic volumetric expansion rate, while a negative α indicates plastic volumetric contraction rate.

Figure 5.5 illustrates the dilation parameters at peak stress (α_k) for specimens subjected to UC and BC at various ages. The results for 3DPC made of different supplies of raw

materials are differentiated with the used of different scatters. It is observed that data variation is notably greater for specimens aged beyond 180 minutes compared to those tested within the initial 180-minute period. This increased variability likely stems from the increased sensitivity of later aged specimens to environmental parameters, particularly temperature fluctuations. Specimens produced from the same supply of materials demonstrate significantly lower variability compared to those fabricated using materials from different supplies, a trend that consistent with the observed variations in strength measurements.

Despite the data variability, it is evident that α_k clearly exhibits dependence on both t and β_s . Across all stress ratios, α_k follows a distinctive pattern, initially decreasing and then increasing with concrete age. The critical time (t_c) at which α_k reaches its minimum occurs at approximately 180 minutes, which aligns with the critical time for the maximum biaxial-to-uniaxial strength ratio discussed in the previous chapter. This temporal alignment suggests an intricate relationship between the strength and deformation behaviour of fresh 3D-printed concrete, both primarily influenced by microstructural evolution during material ageing. At any given age, specimens under BC exhibit lower α_k values compared to those under UC, a phenomenon also observed in cured concrete (Kupfer *et al.*, 1969). This behaviour can be attributed to the horizontal confining stress present in BC, which inhibits crack propagation in the horizontal direction, thereby reducing the rate of volumetric dilation.

To examine the volumetric dilation behaviour throughout the loading process, this study analysed the relationship between the volumetric dilation rate and the equivalent plastic strain (κ) was examined. As discussed in Chapter 2, κ serves as an internal state variable characterising the progressive material damage during loading. While various definitions of κ are possible, this study defines it as the cumulative plastic strain, as expressed in Eq. (5.5). Theoretically, the increment of equivalent plastic strain, $d\kappa$, needs to be integrated to compute κ . However, experimental measurements yield only discrete values of $d\kappa$ at specific time intervals, rather than a continuous function. In light of this, the integral is approximated using Riemann summation, as shown in the second part of Eq. (5.5), where the subscript n stands for the n^{th} data point.

$$\kappa_n = \int d\kappa \approx \sum_{i=1}^n d\kappa_n \quad (5.5)$$

Here, the n^{th} equivalent plastic strain increment is evaluated as the Euclidean norm of the plastic strain increment tensor.

$$d\kappa_n = \|d\boldsymbol{\epsilon}_n^p\| \quad (5.6)$$

Figures 5.7a-d exhibit the variation of the dilation rate (α) throughout the loading process for specimens under UC and BC. The normalised equivalent plastic strain ($\bar{\kappa}$) shown in the x axes of these figure is computed by normalising the equivalent plastic strain with the value at peak stress, that is $\bar{\kappa} = \kappa : \kappa_k$. Across all concrete ages and stress ratios, α initially present a negative value and then increases drastically as $\bar{\kappa}$ increases until the peak stress is reached ($\bar{\kappa} = 1$) after which α grows at a relatively small constant rate. These results align with the experimental observations that the material undergoes volumetric contraction followed by volumetric dilation throughout the loading process, as shown in Figures 5.1d and 5.2 d1-d3. Moreover, for the same β_s , the dilation parameter responding to $\bar{\kappa} = 0$, denoted as the initial volumetric dilation rate (α_0), is approximately the same for specimens at different ages, whereas the dilation parameter at peak stress (α_k) shows obvious age-dependence.

5.3.2.2 Deviatoric dilation behaviour

To examine the deviatoric dilation behaviour, the orientation of the plastic strain increment vector ($d\boldsymbol{\epsilon}^p$) at peak stress within the deviatoric plane is illustrated in Figures 5.8a-d across six age ranges of fresh concrete. Each $d\boldsymbol{\epsilon}^p$ is represented by an arrow of constant length. To facilitate comparison of the direction of $d\boldsymbol{\epsilon}^p$ corresponding to different β_s , the starting point of each arrow is translated to a unit circle, with the starting point position determined by the lode angle of the stress tensor, $\theta_\sigma = \theta(\boldsymbol{\sigma})$. The arrow orientation, and consequently its endpoint, is governed by the lode angle of $d\boldsymbol{\epsilon}^p$, denoted as $\theta_\epsilon = \theta(d\boldsymbol{\epsilon}^p)$. The geometrical interpretation of these two lode angles is illustrated in Figure 5.8a. For uniaxial compression, $\theta_\epsilon = \theta_\sigma = 0^\circ$, whereas for

equibiaxial compression, $\theta_\epsilon = \theta_\sigma = 60^\circ$; thus, only the data for specimens under BC-0.33 ($\theta_\sigma = 19^\circ$) and BC-0.50 ($\theta_\sigma = 30^\circ$) is presented.

For R-phase specimens, as depicted in Figure 5.8a, $d\epsilon^p$ is approximately normal to the circle and $\theta_\epsilon \approx \theta_\sigma$. For S-phase specimens (Figures 5.8e and f), particularly for those aged beyond 300 minutes, $d\epsilon^p$ is not perpendicular to the circular with $\theta_\epsilon > \theta_\sigma$, as shown in Figure 5.8f. For R-S phase specimens, as depicted in Figures 5.8b-d, the disparity between θ_ϵ and θ_σ progressively increases with concrete age. These observations indicate that whilst the deviatoric shape of the flow potential surface maintains approximate circularity for R-phase specimens, it develops increasing non-circularity as the concrete matures. Notably, Zheng and Teng (2022) have demonstrated that the deviatoric section of flow potential surface of cured concrete exhibits non-circularity.

5.3.2.3 Limitation of the current approach

The deformation analysis provides robust insights into the plastic dilation behaviour of fresh 3DPC; however, the author acknowledges a limitation in the current approach. In the current approach, lateral strains ($\epsilon_l = \epsilon_2 = \epsilon_3$ in the case of UC, $\epsilon_l = \epsilon_3$ in the case of BC) were derived indirectly by assuming a parabolic relationship between ϵ_1 and ϵ_l . Whilst this method effectively captures overall deformation patterns, it may not fully represent instantaneous slope changes during brittle failure, particularly in S-phase specimens. The accuracy of true stress analyses remains robust, as these calculations depend primarily on ϵ_l . However, the derived lateral strain relationships warrant consideration in analyses of dilation rates, which are sensitive to the gradients of ϵ_l - ϵ_1 curves. Future research would benefit from implementing high-precision, non-contact deformation monitoring systems to capture the comprehensive deformation field data during loading.

5.3.3 Hardening-Softening Behaviour

The hardening-softening behaviour of fresh 3DPC can be characterised through the relationship between yield stress and equivalent plastic strain. Under proportional

loading conditions, which is the case of the BC testing programme in Chapter 3, the axial and horizontal yield stresses maintain a constant ratio; hence, the axial yield stress serves as an indicator of the material yield stress. Figures 5.9 and 5.10 present the axial yield stress-equivalent plastic strain (σ_{1y} - κ) curves for specimens under UC and BC, respectively. Each curve is colour-coded to indicate its concrete age, and the peak axial stress is highlighted by a scatter. These relationships demonstrate that the hardening-softening behaviour is influenced by both concrete age (t) and confining stress ratio (β_s). Moreover, two hardening mechanisms are observed: strain hardening and age hardening.

At any given age, a specimen exhibits *strain hardening* behaviour in the pre-peak regime with increasing yield stress at a decreasing rate as κ increases. The post-peak behaviour varies: R phase specimens with $\beta_s = 0, 0.33, 0.50$ exhibit plateauing or slight hardening; R-S, S- phase specimens with $\beta_s = 0, 0.33, 0.50$ and S phase specimens with $\beta_s = 1.00$ show strain softening, whereas R and R-S phase specimens with $\beta_s = 1.00$ demonstrate pseudo-hardening due to friction. The softening branch gradient steepens with increasing concrete age, indicating progressive development of brittle behaviour. Conversely, this gradient becomes more gradual with increasing stress ratio. At peak axial yield stress, the σ_{1y} - κ curves approximately zero slope.

The equivalent plastic strain at peak stress (κ_k) demonstrates a clear age-dependent behaviour, as illustrated in Figure 5.11a. Specifically, κ_k decreases from above 40% to below 15% across all stress ratios during the initial five hours of ageing, indicating a pronounced ductile-to-brittle transition. At any given age, specimens with higher biaxial confinement - β_s - tends to exhibit greater κ_k , a phenomenon also observed in cured concrete (Kupfer *et al.*, 1969).

Figure 5.12 presents the relationship between normalised axial stress (computed as the ratio of axial stress to axial strength) and normalised equivalent plastic strain (κ/κ_k). Whilst the σ_{1y} - κ relationships vary significantly with age, the normalised curves demonstrate remarkable similarity in pattern. These curves will serve as the basis for developing the strain hardening-softening function, which will be discussed in Section 5.4.4.

Furthermore, under constant κ and β_s , the axial yield stress increases with concrete age, demonstrating *age-hardening* behaviour. As discussed in Chapter 2, this phenomenon is attributed to the progressive microstructural development during material ageing.

5.4 CONSTITUTIVE MODEL

Drawing upon the stress-strain analyses presented in this chapter and the strength envelope established in the preceding chapter, a constitutive model was formulated within the formwork of rate-independent incremental plasticity. The model encompasses four critical components, a hypoelastic stress-strain relation, a yield criterion, a hardening-softening function, and a flow rule, each of which will be systematically examined in this section.

5.4.1 Strain Decomposition

The strain rate tensor was decomposed additively, where the total strain rate tensor ($\dot{\boldsymbol{\epsilon}}$) comprises the sum of the elastic strain rate tensor ($\dot{\boldsymbol{\epsilon}}^e$) and plastic strain rate tensor ($\dot{\boldsymbol{\epsilon}}^p$).

$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^e + \dot{\boldsymbol{\epsilon}}^p \quad (5.7)$$

5.4.2 Hypoelastic Stress-Strain Relationship

As in conventional plasticity, stress is entirely attributed to the elastic strain. Chapter 2 established that the Cauchy elastic model, which relates current total stress to current total strain, is unsuitable for ageing materials such as fresh concrete. In this study, the hypoelastic formulation is adopted, wherein the stress rate tensor ($\dot{\boldsymbol{\sigma}}$) is related to the elastic strain rate tensor ($\dot{\boldsymbol{\epsilon}}^e$), as given by

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}^t : \dot{\boldsymbol{\epsilon}}^e = \mathbf{C}^t : (\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}^p) \quad (5.8)$$

Here, $\mathbf{C}^t$ is a fourth order tangent elasticity tensor, which is constructed based on the Young's modulus and Poisson's ratio at a particular concrete age.

$$\mathbf{C}'(t) = \mathbf{C}(E(t), \nu(t)) \quad (5.9)$$

5.4.3 Yield Criterion

The strength envelope established in Chapter 4 serves as the foundation for the yield criterion. Adopting the isotropic hardening hypothesis, the yield surface maintains the same shape as the strength envelope whilst its size varies during hardening and softening processes. To account for both strain hardening and age hardening, the yield function is given as follows

$$f(\boldsymbol{\sigma}, \kappa, t) = \rho - \chi(t)\sqrt{2}\xi - k(\kappa, \boldsymbol{\sigma}, t) \quad (5.10)$$

Here, $\chi(t)$ represents the friction parameter that characterises the influence of confining stress, as discussed in the last chapter. The hardening parameter $k = k(\kappa, \boldsymbol{\sigma}, t)$ is defined by:

$$k(\kappa, \boldsymbol{\sigma}, t) = \frac{\sqrt{6}}{3} [1 - \chi(t)] \cdot f_{co}(t) \cdot q(\kappa, \boldsymbol{\sigma}, t) \quad (5.11)$$

where $f_{co}(t)$ denotes the uniaxial compressive strength and $q(\kappa, \boldsymbol{\sigma}, t)$ is the strain hardening function.

5.4.4 Strain Hardening Function

The strain hardening behaviour of fresh 3DPC exhibits both age and stress-state dependence. For constitutive models of cured concrete, researchers have addressed the influence of stress state on strain hardening-softening behaviour through two primary approaches: implementing a stress-state-dependent hardening function (Yu *et al.*, 2010) or adjusting the evolution of the equivalent plastic strain rate (Grassl *et al.*, 2013; Zheng and Teng, 2022). This study employs the former approach, wherein the strain hardening function incorporates dependencies on both concrete age and stress tensor.

To be specific, the hardening function (q) is decomposed into two components as presented in Eq. (5.12), a hardening component (h) and a softening component (s), which account for strain hardening and softening behaviours, respectively:

$$q(\kappa, \sigma, t) = \begin{cases} h(\kappa, \sigma, t) & \forall \kappa \leq \kappa_k \\ s(\kappa, \sigma, t) & \forall \kappa > \kappa_k \end{cases} \quad (5.12)$$

The adoption of proportional loading in the BC testing programme enables the function q to be constructed based on the normalised axial yield stress relationships presented in Figure 5.12.

In the strain hardening regime ($\kappa/\kappa_k < 1$), the hardening component is expressed as a cubic function:

$$h(\kappa, \sigma, t) = h_a \left(\frac{\kappa}{\kappa_k} \right)^3 + h_b \left(\frac{\kappa}{\kappa_k} \right)^2 + h_c \left(\frac{\kappa}{\kappa_k} \right) + h_d \quad (5.13)$$

Here, h_a to h_d are given by Eq. (5.14) which ensures several essential mathematical constraints $h(0) = h_0$, $h'(0) = E_0$, $h(1) = 1$, $h'(1) = 0$, where h_0 denotes proportional limit. The parameter E_0 denotes the slope of h at $\kappa = 0$, which needs to be calibrated from the normalised relation shown in Figure 5.12. Additionally, and gradient of the h at ultimate yield stress is assumed to be zero.

$$\begin{cases} h_a = E_0 + 2(h_0 - 1) \\ h_b = -2E_0 - 3(h_0 - 1) \\ h_c = E_0 \\ h_d = h_0 \end{cases} \quad (5.14)$$

Notably, the shape of the hardening component h , and therefore the pre-peak material stiffness, is primarily controlled by two parameters: E_0 and κ_k . Figure 5.13a illustrates the impact of E_0 on the shape of the function h . As depicted, a higher E_0 leads to a stiffer strain hardening response in the pre-peak regime, with other parameters held

constant. The influence of the parameter κ_k on h is demonstrated in Figure 5.13b. Focusing on the pre-peak regime, κ_k not only determines the equivalent plastic strain at peak stress but also significantly influence on the material stiffness in this regime.

The strain softening function s for $\kappa/\kappa_k > 1$ was modified from the one in Zheng and Teng (2022), and is given by:

$$s(\kappa, \sigma, t) = \left[1 + \left(\frac{\kappa/\kappa_k - 1}{\Lambda} \right)^{1.5} \right]^{-1.5} \quad (5.15)$$

In this expression, Λ is a softening parameter, governing the slope of q in the post-peak regime. Figure 5.13c illustrates that with κ_k held constant, increasing Λ results in a corresponding increase in the gradient of the descending branch of q . A notable special case occurs when $\Lambda = \infty$, wherein the model reduces to a perfect plastic formulation with $s = 1$. Moreover, as shown in Figure 5.13b, the slope of the softening branch is also affected by κ_k . When Λ remains constant, a reduction in κ_k produces a steeper descend in the post-peak regime.

The equivalent plastic strain at peak stress (κ_k) exhibits dependencies on both concrete age and stress states, as shown in Figure 5.11. This study proposes a decoupled approach to account for these influences through the following equation:

$$\kappa_k(\sigma, t) = \kappa_k^{uc}(t) + \kappa_k^*(\sigma) \quad (5.16)$$

Here, κ_k^{uc} specifically represents the relationship between equivalent plastic strain at peak stress and concrete age determined from UC tests results. This baseline relationship is modified by a factor (κ_k^*) to account for the influence of biaxial confining stress.

5.4.5 Evolution of Equivalent Plastic Strain

Following the strain hardening hypothesis, the equivalent plastic strain rate is defined as follows:

$$\dot{\kappa} = \|\dot{\boldsymbol{\epsilon}}^p\| = \dot{\lambda} n(\boldsymbol{\sigma}) \geq 0 \quad (5.17)$$

where $n(\boldsymbol{\sigma}) = \|\partial g / \partial \boldsymbol{\sigma}\|$ represents the norm of the flow potential function gradient.

An important assumption in this formulation is that material ageing does not influence the evolution of equivalent plastic strain. Consequently, the phenomenon of material healing, which will be examined in detail in Chapter 6, is not incorporated in the present analysis. This assumption ensures that the rate of κ is non-negative.

5.4.6 Flow Potential Function

To accurately characterise the plastic dilation behaviour of fresh concrete, a non-associated flow rule was implemented:

$$\dot{\boldsymbol{\epsilon}}^p = \dot{\lambda} \frac{\partial g}{\partial \boldsymbol{\sigma}} \quad (5.18)$$

Based on experimental findings from this study and previous research discussed in Chapter 2, the flow potential function (g) exhibits dependencies on the stress tensor, equivalent plastic strain, and concrete age, expressed as $g = g(\boldsymbol{\sigma}, \kappa, t)$. The proposed flow rule represents an extension of the classical Drucker-Prager model:

$$g(\boldsymbol{\sigma}, \kappa, t) = \rho \cdot r(\theta, e) - \bar{\alpha} \xi \quad (5.19)$$

where $r(\theta, e)$ is given by:

$$r(\theta, e) = \frac{4(1-e^2)\cos^2 \theta + (2e-1)^2}{2(1-e^2)\cos \theta + (2e-1)\sqrt{4(1-e^2)\cos^2 \theta + 5e^2 - 4e}} \quad (5.20)$$

The function $r(\theta, e)$, adopted from Menetrey and Willam (1995), controls the deviatoric section shape of g and consequently determines the deviatoric dilation behaviour. For simplicity, the deviatoric section shape is assumed constant across different hydrostatic pressures. The parameter e ranges from 0.5 to 1 with its influence on the flow potential surface in the deviatoric plane illustrated in Figure 5.14. At $e = 0.5$, the section exhibits a triangular shape, while $e = 1$ produces a circular section. Adjusting the value of e enables modelling of varying deviatoric dilation behaviour throughout the hydration process.

The volumetric dilation behaviour is controlled by the parameter $\bar{\alpha}^l$, given by:

$$\bar{\alpha}(\kappa, \sigma, t) = \alpha(\kappa, \sigma, t) \cdot r(\theta, e) \quad (5.21)$$

where α represents volumetric dilation parameter showing deformation history, stress-state and age dependences. The evolution of the dilation parameter α with κ , referred to Figure 5.7 for the test results, is represented by a simple hyper-tangent model:

$$\alpha(\sigma, \kappa) = \alpha_0(\sigma) + \frac{\alpha_k - \alpha_0(\sigma)}{\tanh(\gamma_\alpha)} \tanh\left(\gamma_\alpha \frac{\kappa}{\kappa_k}\right) \quad (5.22)$$

Here, γ_α is a curvature controlling factor, which is taken as 1.8 in the current investigation. This equation assumes that α increases from an initial value (α_0) at initial yield ($\kappa = 0$) to the peak value (α_k) at peak stress ($\kappa = \kappa_k$). Detailed calculation for the initial volumetric dilation parameter (α_0) can be found in Appendix 5.1. On the other hand, the volumetric dilation rate at peak stress (α_k) was found to be stress state and concrete age dependent (Figure 5.5). Similar to Eq. (5.16), these influences are decoupled:

¹ To ensure the parameter $\bar{\alpha}$ only contribute to the volumetric dilation behaviour, it is treated as a constant when calculating the flow direction $\mathbf{m} = \partial g / \partial \sigma$.

$$\alpha_k(\sigma, t) = \alpha_k^{uc}(t) + \alpha_k^*(\sigma) \quad (5.23)$$

The first term represents the volumetric dilation rate without horizontal confinement at different ages, whilst the second term accounts for confining stress effects.

5.4.7 Calibration

The material parameters in Sections 5.4.1 to 5.4.6 can be classified as: *elastic stiffness parameters*, *parameters related to the yield criterion*, *parameters related to the strain-hardening function* and *parameters related to the flow rule*. The calibration procedures for these parameters are elaborated in this section.

5.4.7.1 Elastic stiffness parameters

The relationship between elastic modulus and concrete age demonstrates a linear trend when plotted on a semi-logarithmic scale, as illustrated in Figure 5.4c. This observation suggests that the $E - t$ relationship can be represented by an exponential function:

$$E(t) = E_{ci} \cdot e^{\zeta_E t} \quad (5.24)$$

Here, E_{ci} is a fictional Young's modulus at age zero minutes, whilst ζ_E denotes a curing coefficient characterises the curing rate of E . For the fresh 3DPC examined in the current investigation, the calibrated parameters are $E_{ci}=839.37$ kPa and $\zeta_E=1.437 \times 10^{-2} \text{ min}^{-1}$. The model's performance is demonstrated in Figure 5.4c, where the predicted Young's modulus shows strong agreement with experimental data.

As will be elaborated in Chapter 6, elastic deformation proves negligible compared to plastic deformation for fresh 3DPC, indicating that dilation behaviour is primarily governed by plastic mechanisms. Consequently, the precise value of the elastic Poisson's ratio (ν) becomes less critical to the overall material response. Therefore, for simplicity, ν is assumed constant at 0.3 across all concrete ages.

5.4.7.2 Parameters related to the yield criterion

The relationship between uniaxial compressive strength and concrete age follows the exponential function established in the last chapter.

$$f_{co}(t) = f_{ci} \cdot e^{\zeta_s t} \quad (5.25)$$

Recall that f_{ci} denotes the fictional initial strength at age zero minutes and ζ_s represents the curing rate of uniaxial compressive strength. For the material developed in the present study, the parameters were calibrated to $f_{ci} = 12.7$ kPa, and $\zeta_s = 1.380 \times 10^{-2} \text{ min}^{-1}$, with the model performance validated against experimental data as shown in Figure 5.4a.

The friction parameter $\chi(t)$ in Eq. (5.10) is only a function of concrete age. Under the isotropic hardening hypothesis, the friction parameter maintains consistency between the yield criterion and failure criterion, thereby enabling the adoption of the same $\chi - t$ relation as the failure criterion established for the failure criterion:

$$\chi(t) = \chi_c + \zeta_\chi (t - t_c)^2 \quad (5.26)$$

Here, parameter t_c represents a critical age at which the parameter χ attains its peak value (χ_c), and ζ_χ is a time sensitivity factor. Based on the strength analyses in Chapter 4, the parameters for the investigated 3DPC were determined to be $\zeta_\chi = -5 \times 10^{-6} \text{ min}^{-2}$, $t_c = 170$ minutes and $\chi_c = 0.2$.

5.4.7.3 Parameters related to the strain-hardening function

The parameter E_0 in Eq. (5.13) corresponding to a specific age t was determined as the tangential modulus corresponding to $h = 0.2$ and 0.4 of the $h - \kappa$ curve. Figure 5.15 demonstrates the relationship between E_0 and t for specimens under different stress ratios. The analysis reveals that E_0 tends to be stress state independent while demonstrating a roughly linear relationship with t . To quantify the impact of material ageing on E_0 , the following relationship was established:

$$E_0(t) = E_{0i} + \varsigma_{E_0} t \quad (5.27)$$

where E_{0i} represents the value of E_0 at zero minutes age, while ς_{E_0} is a curing parameter. Figure 5.15 illustrates that the model calibrated with $E_{0i} = 2.04$ and $\varsigma_{E_0} = -2.49 \times 10^{-3} \text{ min}^{-1}$ (represented by a solid line) aligns well with the trend of the test data, (depicted as scattered points).

The equivalent strain at peak stress (κ_k) follows an exponential decay trend as concrete age, as evident in Figure 5.11a. In the case of uniaxial compression, the relationship between the equivalent plastic strain at peak stress (κ_k^{uc}) and t can be modelled by the following equation.

$$\kappa_k^{uc}(t) = \kappa_{ki}^{uc} \cdot e^{\varsigma_{\kappa} t} \quad (5.28)$$

For the concrete examined in the present study, the κ_k^{uc} corresponding to age zero minutes (κ_{ki}^{uc}) is 0.45, while the curing parameter ς_{κ} is $-7.76 \times 10^{-3} \text{ min}^{-1}$.

Under biaxial stress conditions, the shape of the κ_k - t curve is assumed to be the same as that under uniaxial compression, while the baseline curve derived from UC test data shifts upward to account for the influence of confining stress, as represented by Eq. (5.16). Figure 5.11b plots the values of the shifting parameter κ_k^* as scatters under different stress ratios¹. Theoretically, the increase of β_s should lead to an increase in κ_c at a given age, therefore, the following monotonically increase function is proposed for κ_k^* .

$$\kappa_k^*(\sigma) = \varsigma_1^{\kappa} \left(e^{\varsigma_2^{\kappa} \cdot \beta_s(\sigma)} - 1 \right) \quad (5.29)$$

Here, ς_1^{κ} and ς_2^{κ} are two parameters that control the shape of the function. It was found that $\varsigma_1^{\kappa} = 9.03 \times 10^{-3}$ and $\varsigma_2^{\kappa} = 1.57$ can give reasonably good predictions on the κ_k^* - β_s

¹ The four data points corresponding to BC-0.50 with age less than 35 minutes highlighted in Figure 5.11a were considered as outliers and were not used in determining the values of κ_k^* .

relationship, as depicted in Figure 5.11b. Figure 5.11a confirms that the calibrated model effectively predicts the κ_k for specimens under different β_s and t with relatively good accuracy.

Regarding the softening parameter Λ , a constant value of 3 was found to provide satisfactory results for 3DPC across different stress ratios and ages. This simplification is justified by the relatively minor importance of post-peak behaviour in collapse analysis of 3D-printed concrete structures. However, if greater precision is required, Λ can be formulated to incorporate dependencies on both confining stress and concrete age.

5.4.7.4 Parameters related to the flow rule

Under uniaxial compression, the dilation parameter at peak stress (α_k^{uc}) in Eq. (5.23) can be approximated using the following parabolic equation.

$$\alpha_k^{uc}(t) = \alpha_{kc}^{uc} + \varsigma_\alpha (t - t_c')^2 \quad (5.30)$$

Here, t_c' denotes the transition age when α_k^{uc} reaches its critical minimum value (α_{kc}^{uc}), and ς_α serving as a curing factor. For the 3DPC investigated, the transition age for the dilation parameter at peak stress roughly coincides with that of the friction parameter, yielding $t_c' = t_c = 170$ minutes. Moreover, α_{kc}^{uc} and ς_α were calibrated to 0.20 and $2.91 \times 10^{-6} \text{ min}^{-2}$, respectively. Following the approach established in Eq. (5.29), the influence of confining stress is incorporated through the shifting factor:

$$\alpha_k^*(\sigma) = \varsigma_1^\alpha \left(e^{\varsigma_2^\alpha \cdot \beta_s(\sigma)} - 1 \right) \quad (5.31)$$

In this equation, the parameters ς_1^α and ς_2^α were determined to be 0.20 and -3.22, respectively. Figure 5.6 demonstrates the agreement between predicted and experimental shifting factors, while Figure 5.5 validates the performance of Eq. (5.23) for specimens under both under UC and BC.

The parameter e in Eq. (5.20) is modelled to decrease linearly from an initial value e_0 at age t_0 to a final value (e_h) at age t_h . Furthermore, it is assumed that the value of e is bounded by e_0 and e_h .

$$e(t) = e_0 + \frac{e_0 - e_h}{t_h - t_0}(t - t_h); \quad e_0 \leq e \leq e_h \quad (5.32)$$

For this investigation, $e_0 = 1.0$ and $t_0 = 15$ minutes. As demonstrated in the previous chapter, the biaxial compressive strength envelope at 300 minutes closely resembles that of hardened concrete. This suggests that fresh 3DPC at 300 minutes exhibits behaviour similar to cured concrete. Therefore, e_h was set to 0.85, based on the value recommended for mature concrete by Zheng and Teng (2022), and $t_h = 300$ minutes.

Table 5.1 presents a comprehensive summary of model parameters for the fresh concrete examined in this study. It should be noted that these parameters may vary for 3DPC mixtures with different proportions, necessitating further experimental investigation to characterise the deformation behaviour of various fresh 3DPC formulations.

5.5 VERIFICATION

The proposed constitutive model was integrated into Abaqus via a custom user-defined material subroutine (UMAT) to enable structural analysis of fresh concrete elements. The numerical integration of the constitutive equations employs the closest-point projection algorithm (Simo and Taylor, 1985), a well-established method in computational plasticity (Grassl *et al.*, 2002; Zheng and Teng, 2022). A detailed description of the algorithmic implementation is presented in Appendix 5.2.

This section presents the material-level validation of the proposed model under uniaxial and biaxial compression conditions. The validation process employed single-element tests in Abaqus using eight-node solid elements with reduced integration (C3D8R). These elements were configured using the UMAT subroutine with material parameters

specified in Table 5.1. Appropriate boundary conditions were implemented to achieve uniform stress states under both uniaxial and biaxial compression loading scenarios.

In experiments, the concrete age of each specimen is different. To facilitate meaningful validation analyses, the experimental results were systematically grouped according to specimen age. Each category comprises specimens tested within a ten-minute interval, based on the established assumption that fresh concrete exhibits consistent mechanical properties within this short temporal window. For each group, the mean specimen age was utilised as the reference time point for material model predictions. Notably, these groups were deliberately selected to encompass specimens in the R, R-S, and S phases.

5.5.1 Uniaxial Compression

The experimental and numerical results for specimens under UC at various concrete ages are presented in Figures 5.16 to 5.21, with experimental data shown as thin lines and numerical predictions as thick lines.

Figures 5.16a to 5.21a display the stress-strain behaviour. To facilitate meaningful comparisons of deformation behaviour between experimental and numerical results, the axial stress for each specimen was normalised by its corresponding uniaxial compressive strength. The axial strain (ϵ_1) and lateral strains (ϵ_2 and ϵ_3) are differentiated using distinct colours. The proposed model demonstrates excellent capability in capturing the age-dependent mechanical behaviour, accurately reflecting the experimental observations: specimens at later ages exhibit decreased peak strain values, enhanced pre-peak material stiffness, and more pronounced post-peak stress degradation as hydration progresses.

Subplots (b) and (c) illustrate the axial-horizontal strain and axial-volumetric strain relationships, respectively. The model yields satisfactory predictions across all concrete ages, accurately capturing both the continuous dilation along lateral axes (2 and 3) and the transition from contraction to dilation in volumetric behaviour with increasing axial strain. However, a slight underestimation in volumetric strain is observed in R-phase specimens (as seen in Figures 5.16c, 5.17c), primarily stems from the α_k - t function

underestimating the dilation parameter at peak stress for younger specimens under UC (as shown in Figure 5.5a). Whilst refinement of the α_k-t function could potentially improve predictions, the significant data scatter observed in this study suggests that such modifications may not substantially enhance the actual accuracy. Therefore, further optimisation of the model parameters should be a subject of future research.

5.5.2 Biaxial Compression

Figures 5.22 to 5.27 compare the biaxial compression test results with those predicted by the proposed model, where the average concrete ages range from 34 minutes to 301 minutes. The subplots in the first-, second- and third- row of each figure represent the results corresponding to stress ratios (β_s) of 0.33, 0.50 and 1.00, respectively.

Subplots (a) in each figure represent the stress-strains behaviour. Here, the axial stress has been normalised by the corresponding specimen's uniaxial compressive strength. The proposed model demonstrates relatively good accuracy in predicting both the biaxial-to-axial compressive strength ratio and the corresponding strains, particularly for lower confining stress ratios (BC-0.33, BC-0.50), which are most relevant to the 3D printing process. For BC-1.00 specimens aged less than 230 minutes (R- and R-S phases), the model predicts post-peak softening behaviour, whilst experimental results show pseudo-hardening attributed to friction effects. Furthermore, the model gives accurate predictions on ϵ_2 and ϵ_3 suggesting that the model correctly reproduces the deviatoric dilation behaviour.

Subplots (b) further illustrate the axial strain-horizontal strain behaviour. The model accurately captures the impact of stress ratio and concrete age on the ϵ_2 - ϵ_1 relationship. With respect to concrete age, the model successfully identifies that R-phase specimens predominantly exhibit dilating ϵ_2 under BC-0.33 (Figure 5.22b1), whereas the ϵ_2 of R-S phase specimens (Figures 5.24b1 and 5.25b1) and S phase specimens (Figures 5.26b1 and 5.27b1) show a contraction-to-dilation transition as ϵ_1 increases. Concerning the impact of stress ratio, the model correctly predicts that the material exhibits increasing contractive ϵ_2 as the stress ratio increases.

Figures 5.22c to 5.27c demonstrate the axial strain-volumetric strain behaviour. The model accurately reproduces the experimental observations of initial volumetric contraction followed by dilation as ϵ_1 increases. With increasing confining stress ratio, the model correctly predicted a increased tendency for volumetric contraction. Moreover, the model effectively predicts the ϵ_v - ϵ_1 curves for specimens at different ages, demonstrating its capability to represent the effects of material ageing on volumetric deformation characteristics.

5.6 CONCLUDING REMARKS

Based on the uniaxial and biaxial compression test results on fresh 3D-printed concrete, this chapter presents extensive stress-strain analyses for the material aged from tens of minutes to six hours. The analyses focus on three aspects of the material: change in material stiffness, volumetric and deviatoric dilation behaviour, and hardening-softening behaviour, the results of which are summarised as follows:

- (1) Similar to the development of strength, the material stiffness, including the apparent and actual Young's modulus, increases by approximately two orders of magnitude in the first six hours.
- (2) The volumetric dilation behaviour exhibits complex dependencies on deformation history, stress states, and concrete age. Under both uniaxial and biaxial compression, specimens demonstrate a characteristic transition from volumetric contraction to dilation during loading at any given age. Increased biaxial confinement promotes greater volumetric contraction whilst reducing the volumetric dilation rate. As hydration progresses, the volumetric dilation rate at peak stress follows a distinctive pattern across all biaxial stress ratios: decreasing within the initial 180-minute window, followed by a subsequent increase. This critical transition age of 180 minutes aligns with the observed changes in biaxial compressive strength behaviour discussed in Chapter 4, reinforcing the conclusion that fresh 3D-printed concrete's mechanical response is governed by two evolving chemical-physical mechanisms.

- (3) The deviatoric dilation response, which characterises the relative strain ratios along different axes, demonstrates age dependency. Throughout the hydration process, the flow potential surface undergoes a fundamental transformation in its deviatoric plane, evolving from an approximately circular shape to a distinctly bulged triangular configuration.
- (4) The hardening behaviour of fresh 3DPC is governed by two distinct mechanisms: strain hardening and age hardening. Strain hardening manifests as variations in material yield stress during loading at any given concrete age, with its characteristics being dependent on both the concrete age and stress conditions. As concrete age increases, the material undergoes a progressive transition from ductile to brittle behaviour, evidenced by a monotonic decrease in equivalent plastic strain at peak stress and an increasing gradient of the post-peak softening branch. With a higher biaxial stress ratio, material exhibits enhanced ductility, with increased equivalent plastic strain at peak stress. Age hardening, conversely, describes the evolution of material yield stress driven by ongoing hydration processes.

Grounded on the results of stress-strain analyses, a constitutive model for fresh 3DPC within the framework of rate-independent elastoplasticity has been proposed, the characteristics of which are summarised as follows:

- (1) The age-dependent yield criterion was developed based on the strength envelope established in the Chapter 4.
- (2) Regarding the volumetric and deviatoric dilation behaviour, a non-associated flow rule was proposed. The proposed flow potential surface represents an extension of the Drucker-Prager model. Its deviatoric section is adjustable, allowing it to be either circular or non-circular, to account for changes in deviatoric dilation behaviour with ageing. The volumetric dilation parameter is linked to concrete age, equivalent plastic strain, and stress states to account for their impacts.

- (3) The strain hardening-softening function depends on concrete age to account for the age-induced ductile-to-brittle transition. Additionally, it is linked to the confining stress to account for the impact of confining stress on material hardening.

The proposed constitutive model has been calibrated using test data from uniaxial and biaxial compression tests and subsequently implemented in Abaqus using a closest-point projection numerical scheme. Validation against experimental results under both uniaxial and biaxial compression demonstrates its capability to accurately predict material strength and deformation behaviour across various ages and stress states.

Whilst the current model focuses on compressive loading conditions, its framework can be extended to incorporate tensile behaviour through either multi-surface plasticity theory (Cervenka and Papanikolaou, 2008) or non-local modelling approaches (Feng and Wu, 2018). Additionally, the present strain rate-independent formulation can be enhanced to account for viscous effects, an aspect that will be elaborated in Chapter 7.

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Table 5.1 Summary of model parameters for the fresh 3DPC in the current study.

Equation Number	Parameters	Values
<i>Elastic stiffness parameters</i>		
Eq. (5.24)	E_t	839.37 kPa
	ζ_E	$1.437 \times 10^{-2} \text{ min}^{-1}$
Eq. (5.9)	ν	0.3
<i>Yield criterion</i>		
Eq. (4.1)	f_{ci}	12.7 kPa
	ζ_s	$1.380 \times 10^{-2} \text{ min}^{-1}$
Eq. (4.6)	χ_c	0.2
	ζ_χ	$-5 \times 10^{-6} \text{ min}^{-2}$
	t_c	170 mins
<i>Hardening/softening function</i>		
Eq. (5.27)	E_{0i}	2.04
	ζ_{E_0}	$-2.49 \times 10^{-3} \text{ min}^{-1}$
Eq. (5.28)	κ_{ki}^{uc}	0.45
	ζ_κ	$-7.76 \times 10^{-3} \text{ min}^{-1}$
Eq. (5.29)	ζ_1^κ	9.03×10^{-3}
	ζ_2^κ	1.57
	Λ	3
	h_0	5×10^{-2}
<i>Flow rule</i>		
Eq. (5.30)	α_{kc}^{uc}	0.20
	ζ_α	$2.91 \times 10^{-6} \text{ min}^{-2}$
	t'_c	170 mins
Eq. (5.31)	ζ_1^α	0.20
	ζ_2^α	-3.22
Eq. (5.32)	e_0	1.00
	t_0	15 mins
	e_h	0.85
	t_h	300 mins

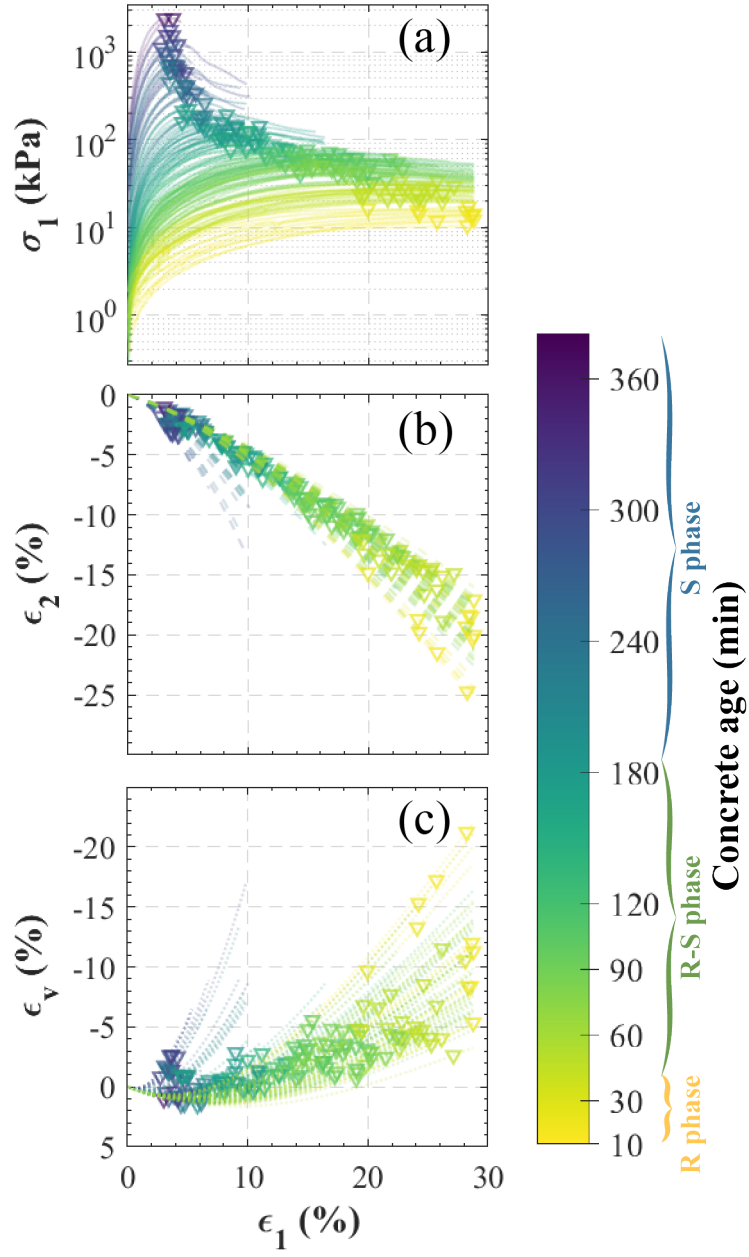


Figure 5.1 Deformation data of specimens under UC: (a) σ_1 - ϵ_1 curves; (b) ϵ_2 - ϵ_1 curves; (c) ϵ_3 - ϵ_1 curves; (d) ϵ_v - ϵ_1 curves.

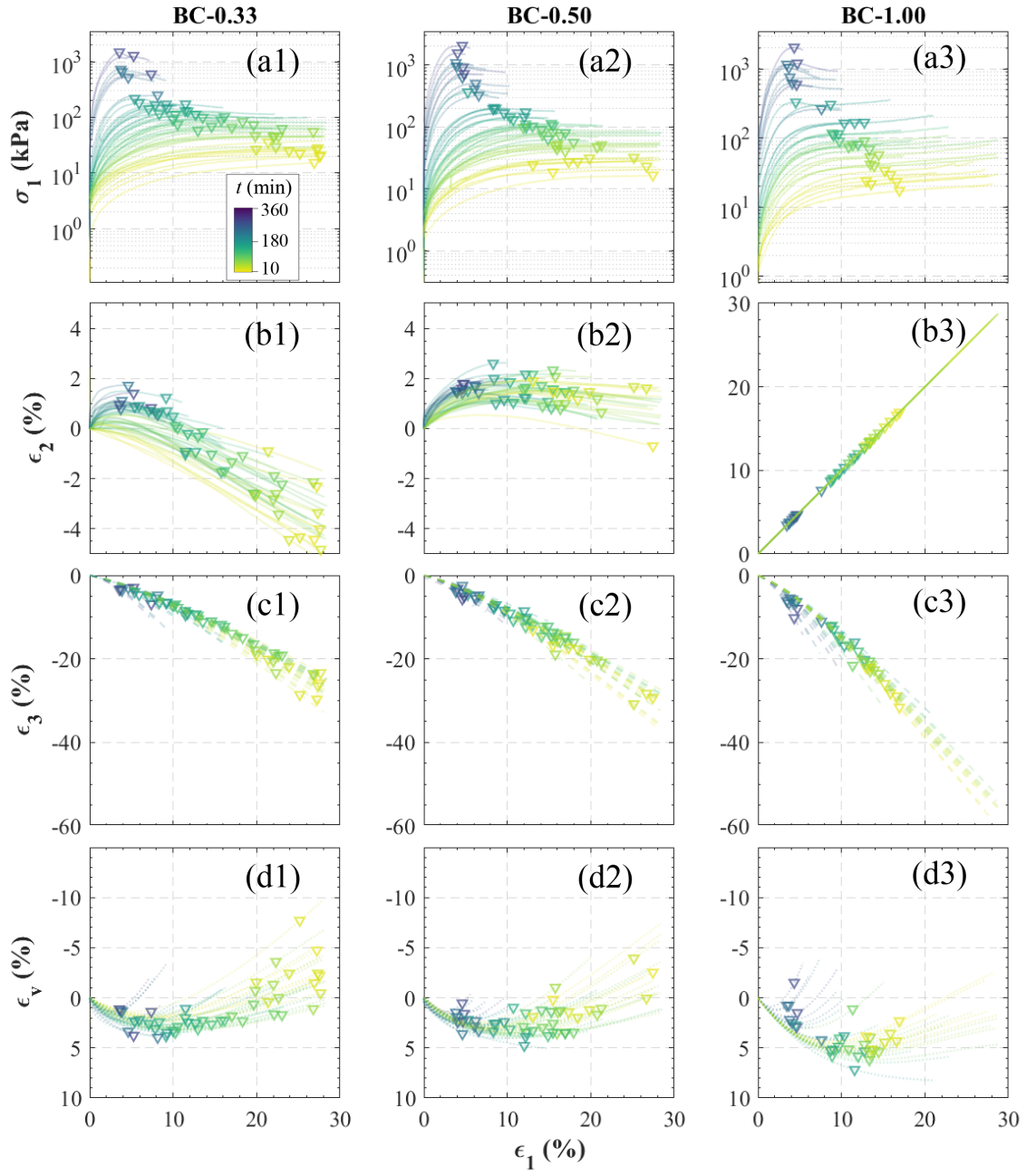


Figure 5.2 Deformation data for specimens under BC: (a1)-(a3) σ_1 - ϵ_1 curves; (b1)-(b3) ϵ_2 - ϵ_1 curves; (c1)-(c3) ϵ_3 - ϵ_1 curves; (d1)-(d3) ϵ_v - ϵ_1 curves.

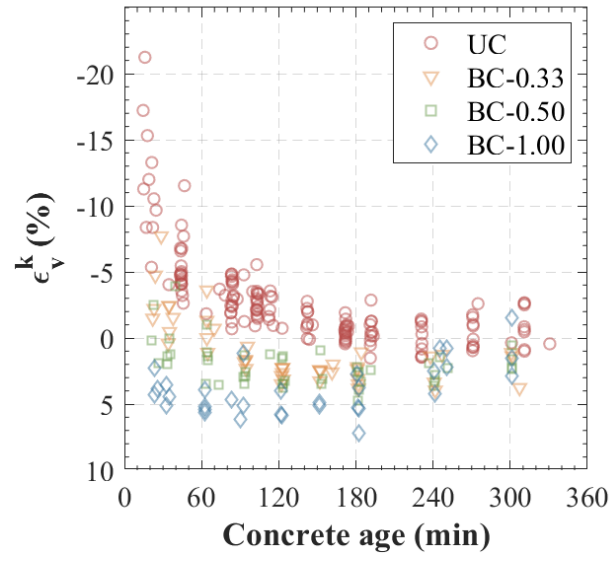


Figure 5.3 Volumetric strain at peak stress for specimens under UC and BC.

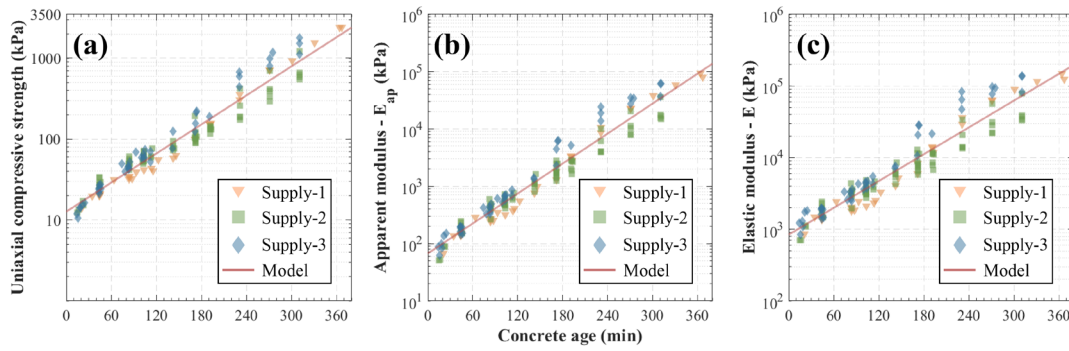


Figure 5.4 Evolution of material parameters obtained from UC tests: (a) Uniaxial compressive strength; (b) Apparent elastic modulus; (c) Estimated elastic modulus.

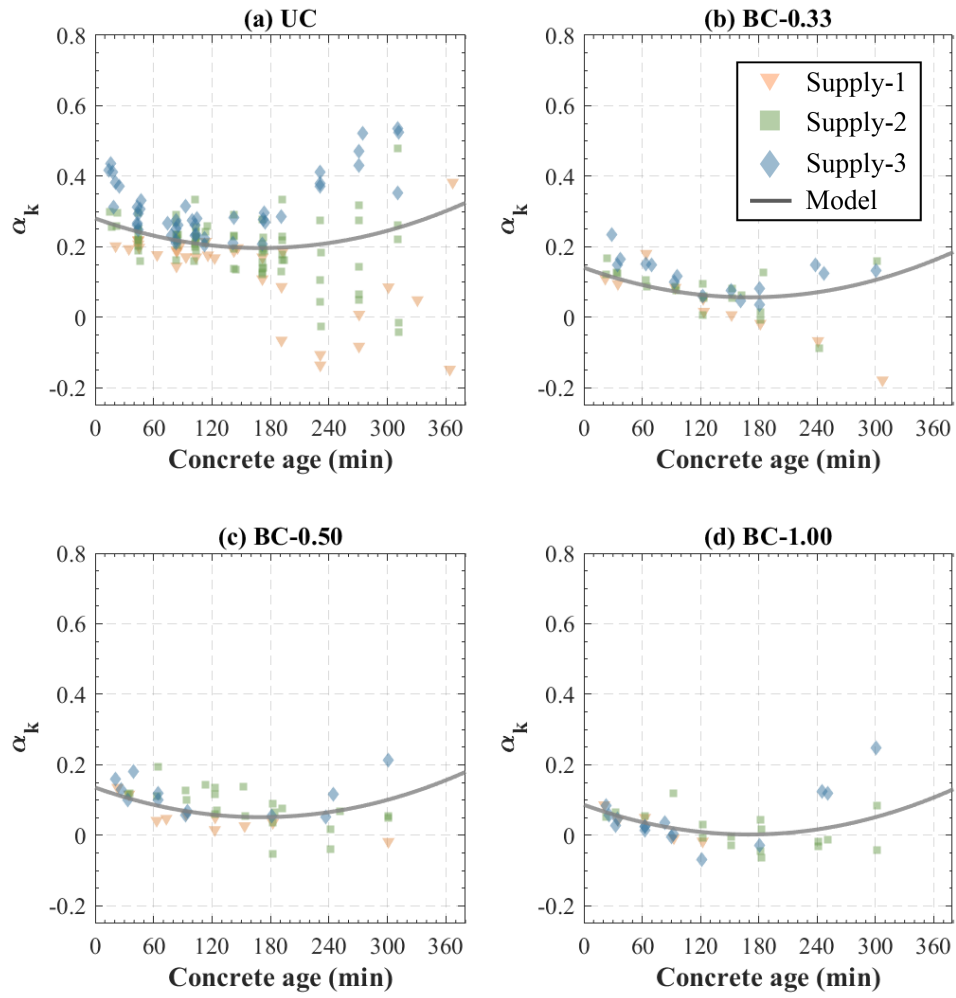


Figure 5.5 Change in volumetric dilation parameter at peak stress during hydration:
(a) Uniaxial compression tests; (b)-(d) Biaxial compression tests.

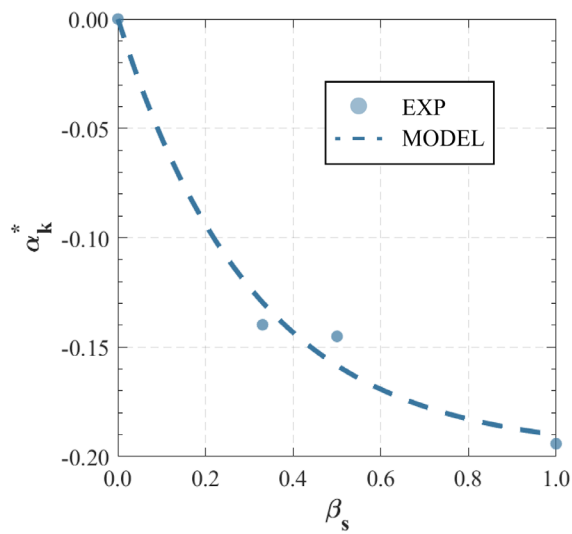


Figure 5.6 Shifting parameters accounting for the influence of stress ratios on dilation parameters.

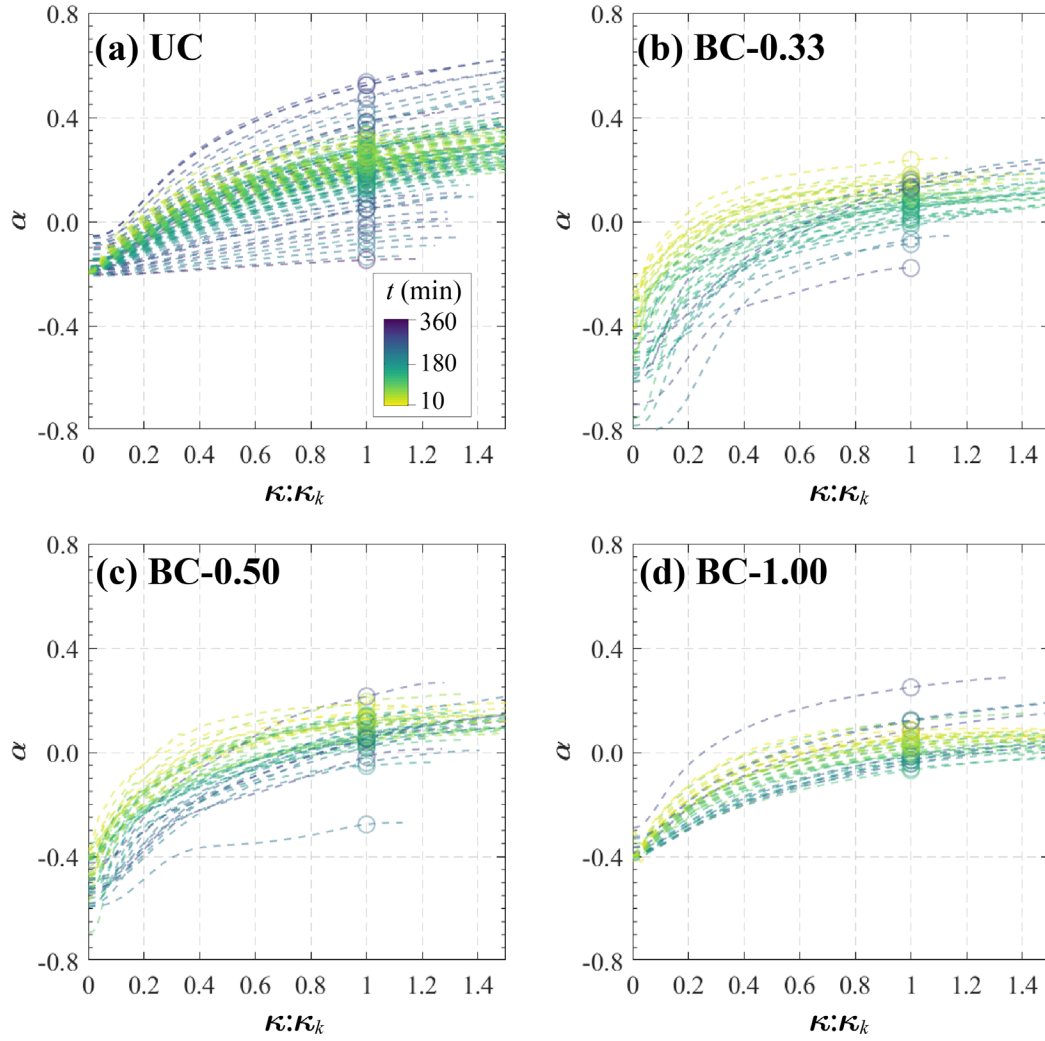


Figure 5.7 Variation in the volumetric dilation parameter during loading for specimens under UC and BC.

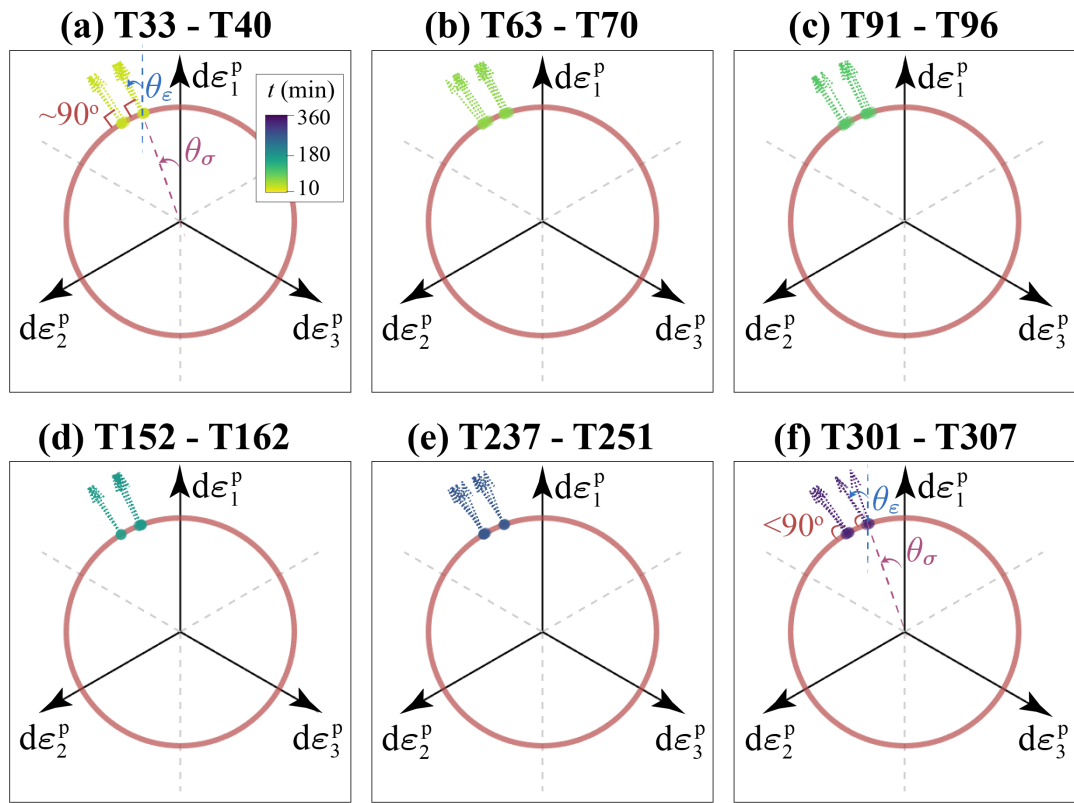


Figure 5.8 Orientation of plastic strain increment vector at peak stress in the deviatoric plane.

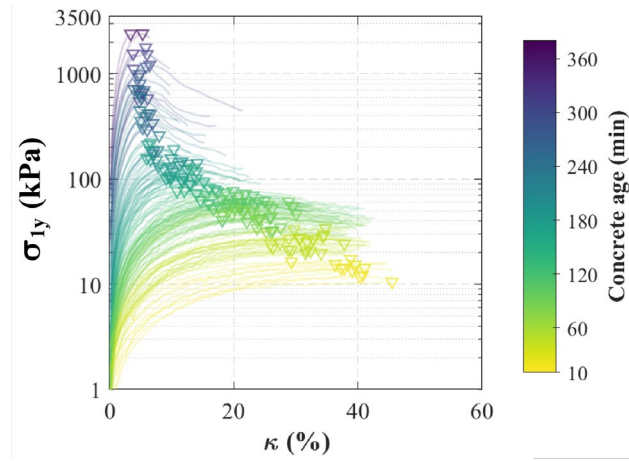


Figure 5.9 Axial yield stress-equivalent plastic strain relationships for specimens under UC.

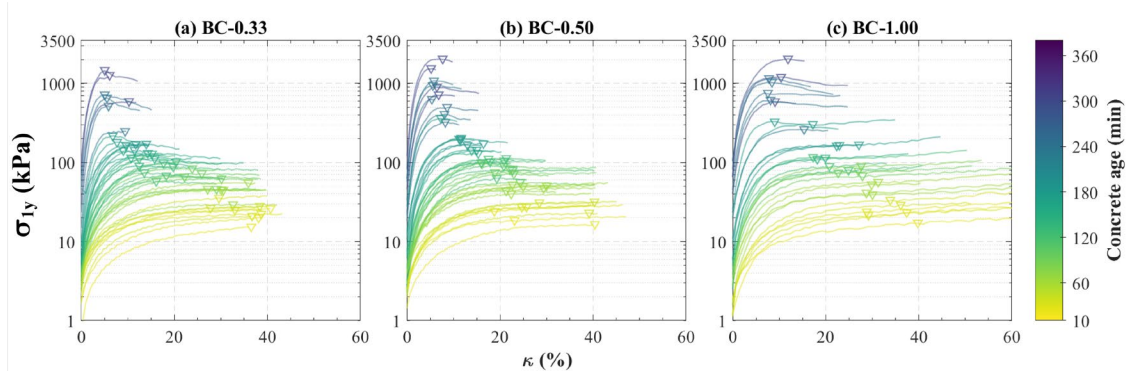


Figure 5.10 Axial yield stress-equivalent plastic strain relationships for specimens under BC: (a) BC-0.33; (b) BC-0.50; (c) BC-1.00.

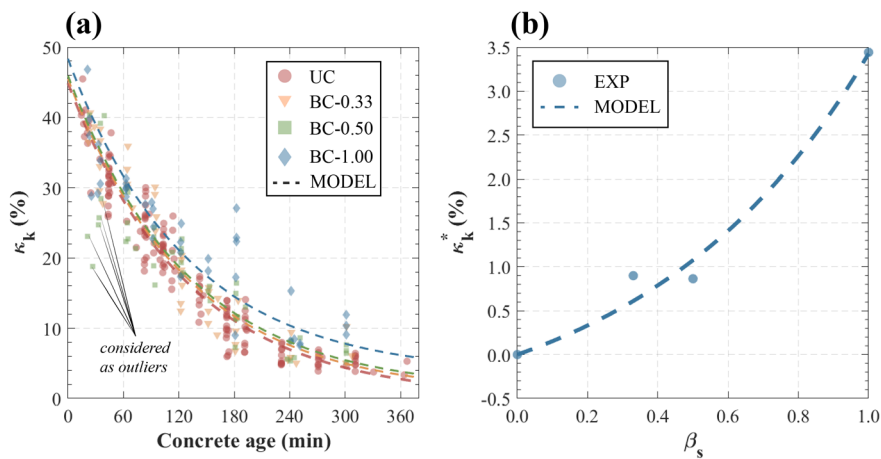


Figure 5.11 Evolution of equivalent plastic strain at peak stress: (a) Results for specimens under UC and BC; (b) Values of the shifting parameter for equivalent plastic strain at peak stress for specimens under different stress ratios.

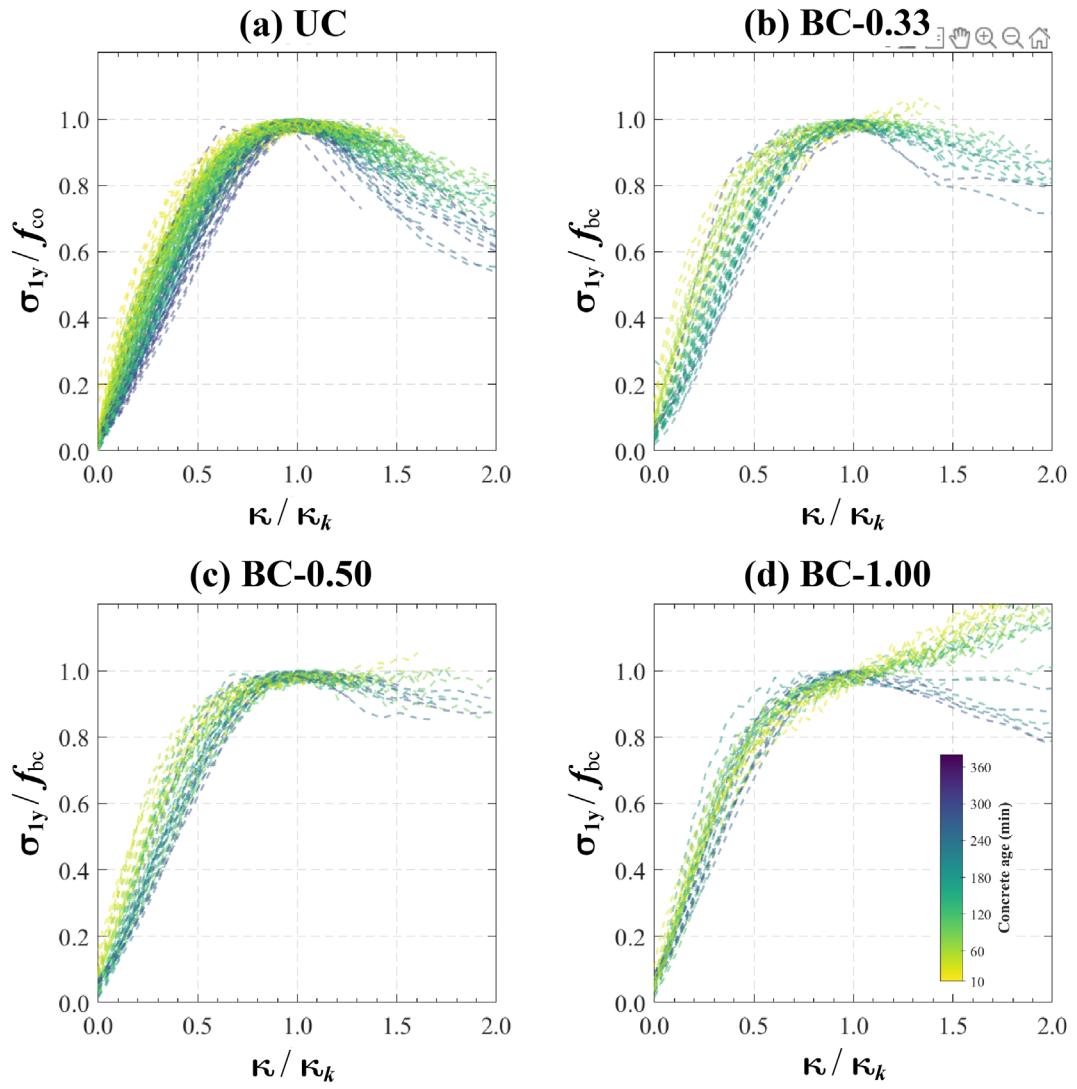


Figure 5.12 Normalised axial yield stress vs normalised equivalent strain relationships for specimens under UC and BC.

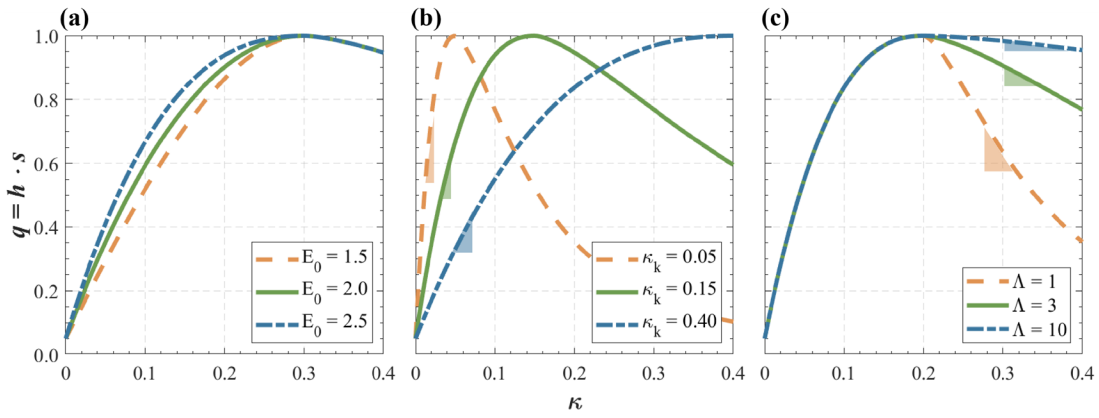


Figure 5.13 Influence of model parameters on the shape of the strain hardening function: (a) Influence of E_0 , $\kappa_k = 0.3$, $\Lambda = 3$; (b) Influence of κ_k , $E_0 = 2$, $\Lambda = 3$; (c) Influence of Λ , $E_0 = 2$, $\kappa_k = 0.2$. $h_0 = 0.05$ in all three examples.

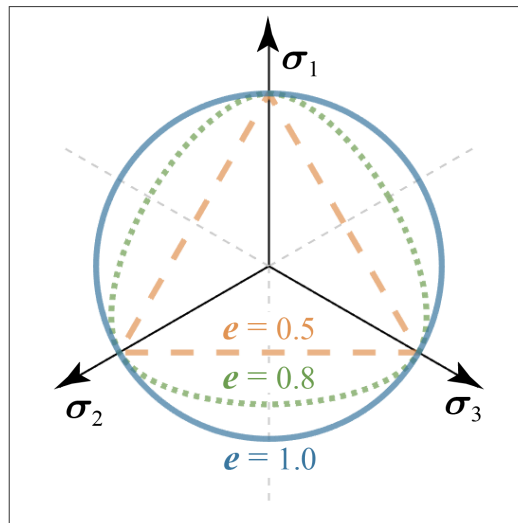


Figure 5.14 Influence of the parameter e on the shape of the proposed flow potential surface in the deviatoric plane.

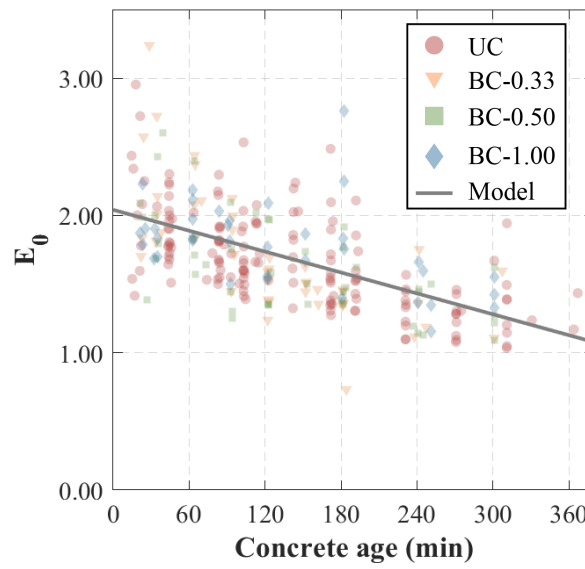


Figure 5.15 Relationship between the parameter E_0 with concrete age.

Average concrete age = 17 min

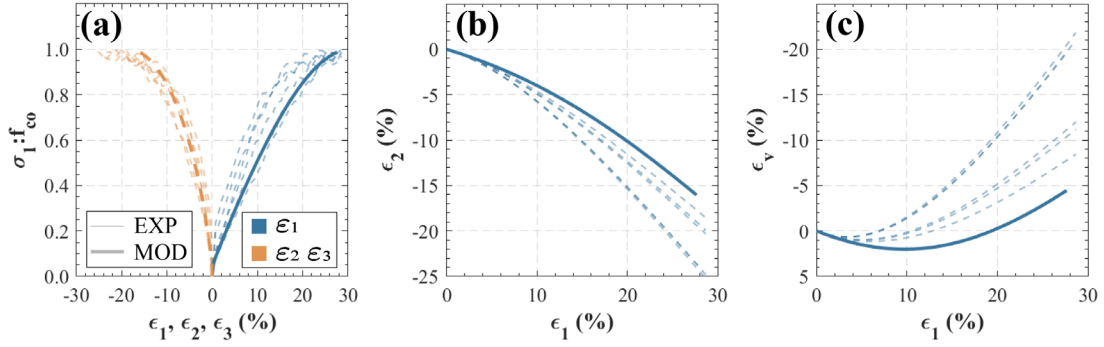


Figure 5.16 Comparison of the proposed model with UC test results (average concrete age = 17 min): (a) Normalised stress-strains curves; (b) Axial strain-horizonal strain curves; (c) Axial strain-volumetric strain curves.

Average concrete age = 45 min

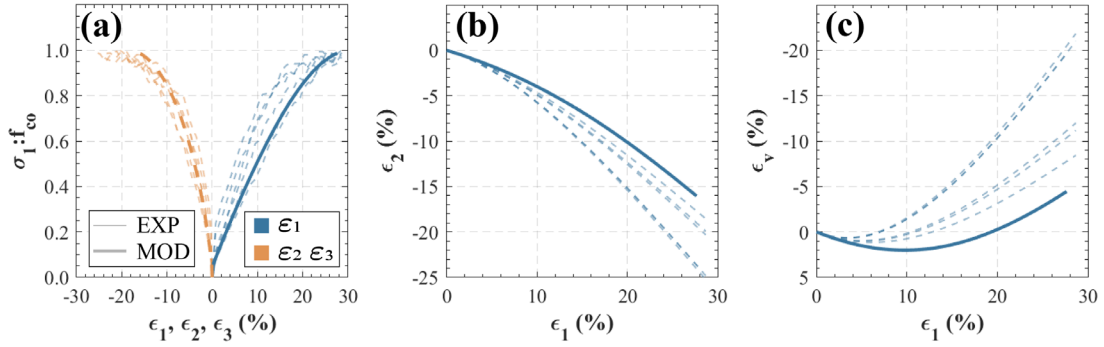


Figure 5.17 Comparison of the proposed model with UC test results (average concrete age = 45 min): (a) Normalised stress-strains curves; (b) Axial strain-horizonal strain curves; (c) Axial strain-volumetric strain curves.

Average concrete age = 103 min

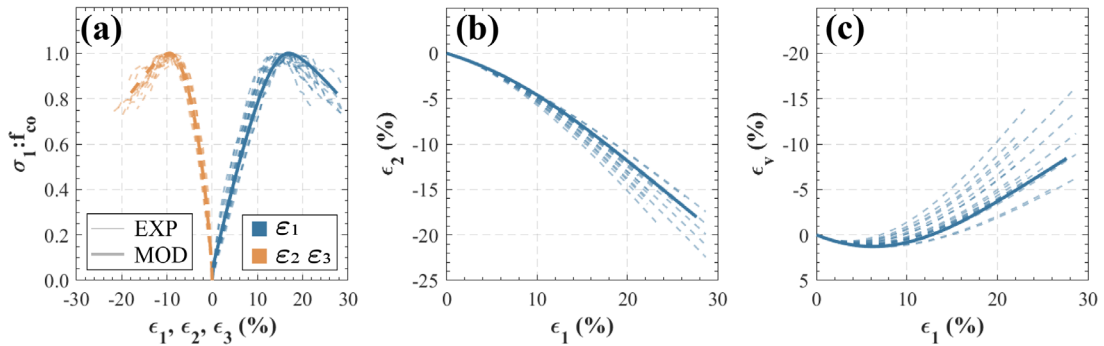


Figure 5.18 Comparison of the proposed model with UC test results (average concrete age = 103 min): (a) Normalised stress-strains curves; (b) Axial strain-horizonal strain curves; (c) Axial strain-volumetric strain curves.

Average concrete age = 172 min

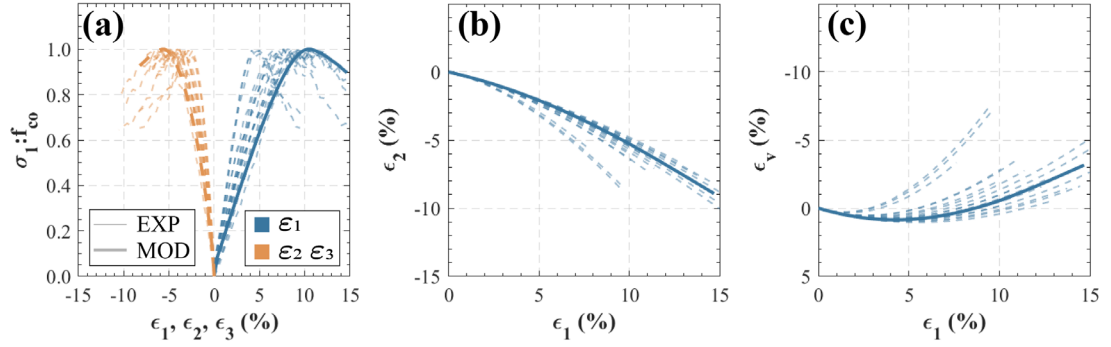


Figure 5.19 Comparison of the proposed model with UC test results (average concrete age = 172 min): (a) Normalised stress-strains curves; (b) Axial strain-horizontal strain curves; (c) Axial strain-volumetric strain curves.

Average concrete age = 231 min

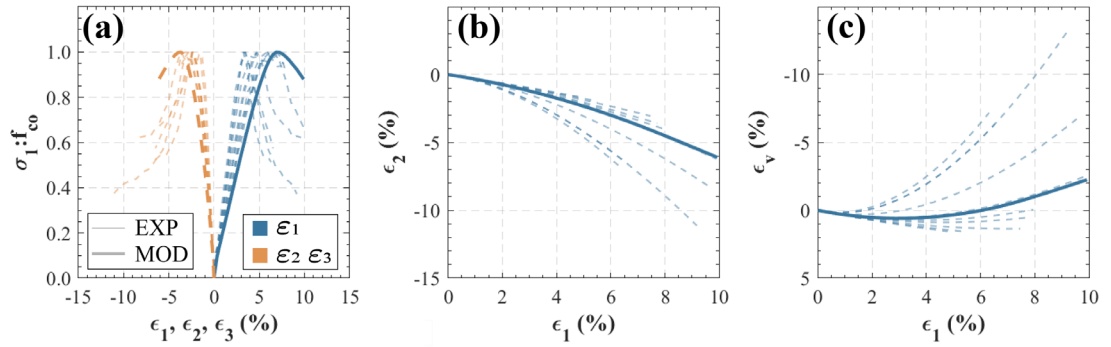


Figure 5.20 Comparison of the proposed model with UC test results (average concrete age = 231 min): (a) Normalised stress-strains curves; (b) Axial strain-horizontal strain curves; (c) Axial strain-volumetric strain curves.

Average concrete age = 310 min

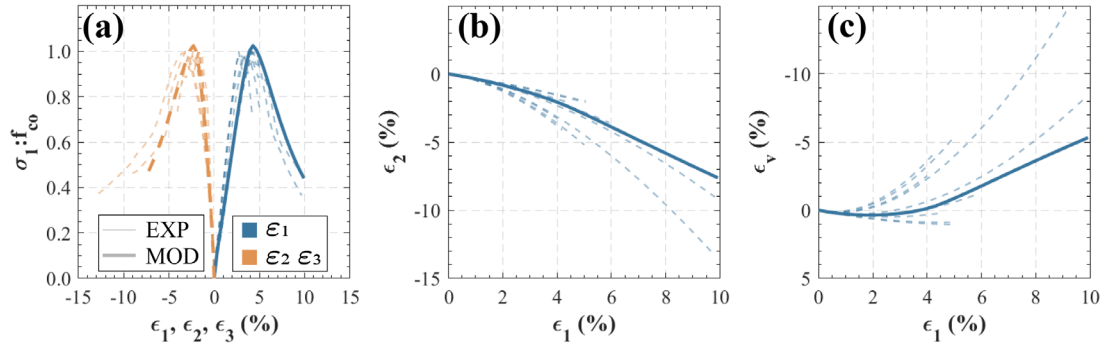


Figure 5.21 Comparison of the proposed model with UC test results (average concrete age = 310 min): (a) Normalised stress-strains curves; (b) Axial strain-horizontal strain curves; (c) Axial strain-volumetric strain curves.

Average concrete age = 34 min

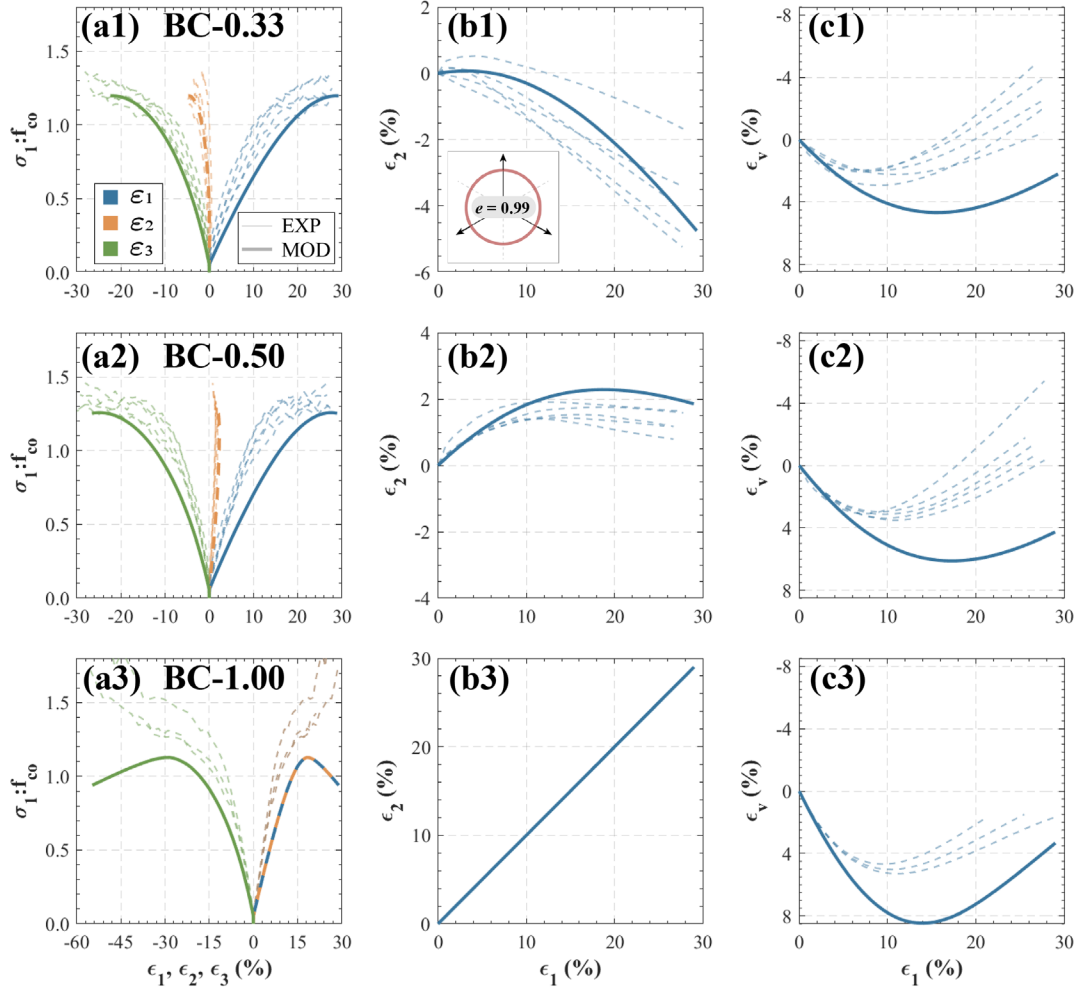


Figure 5.22 Comparison of the proposed model with BC test results (average concrete age = 34 min): (a1)-(a3) Normalised stress-strains curves; (b1)-(b3) Axial strain-horizontal strain curves; (c1)-(c3) Axial strain-volumetric strain curves.

Average concrete age = 63 min

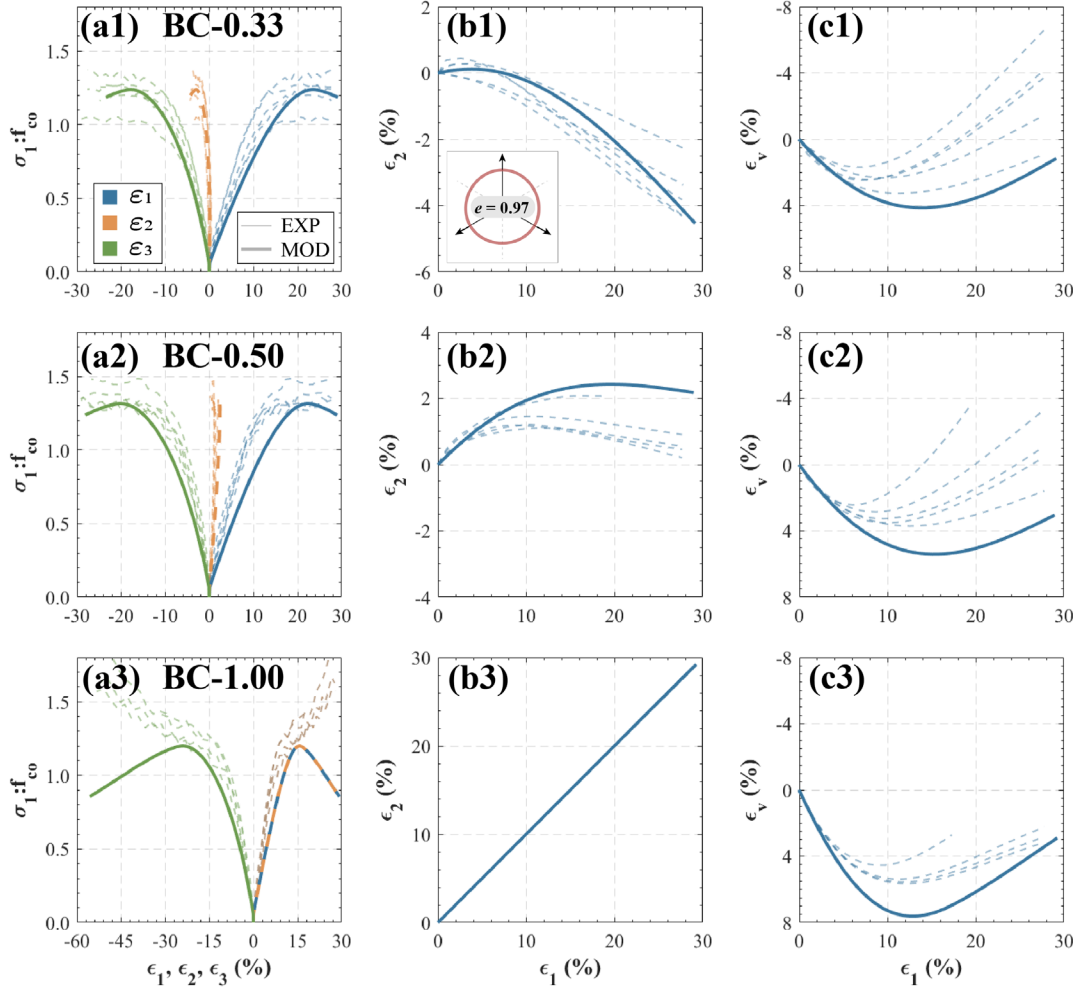


Figure 5.23 Comparison of the proposed model with BC test results (average concrete age = 63 min): (a1)-(a3) Normalised stress-strains curves; (b1)-(b3) Axial strain-horizontal strain curves; (c1)-(c3) Axial strain-volumetric strain curves.

Average concrete age = 92 min

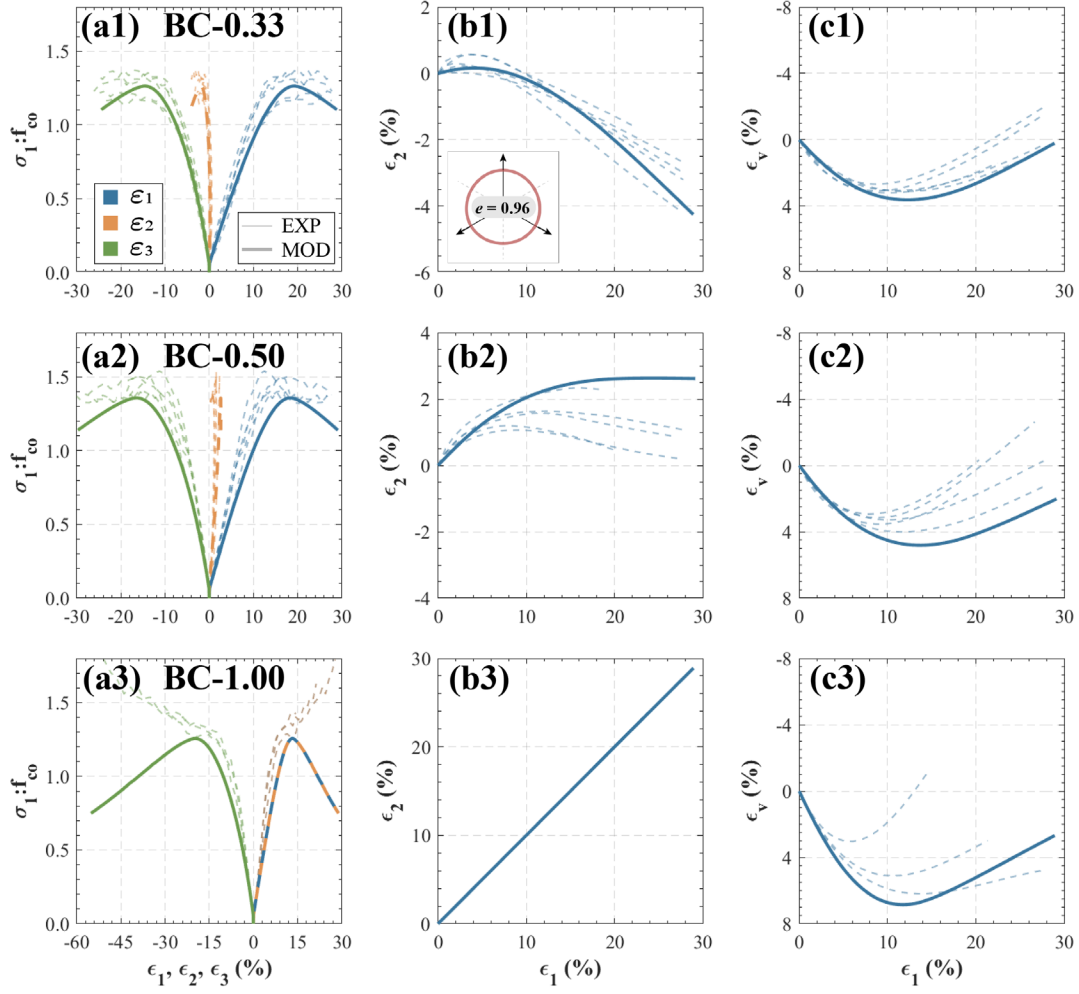


Figure 5.24 Comparison of the proposed model with BC test results (average concrete age = 92 min): (a1)-(a3) Normalised stress-strains curves; (b1)-(b3) Axial strain-horizontal strain curves; (c1)-(c3) Axial strain-volumetric strain curves.

Average concrete age = 152 min

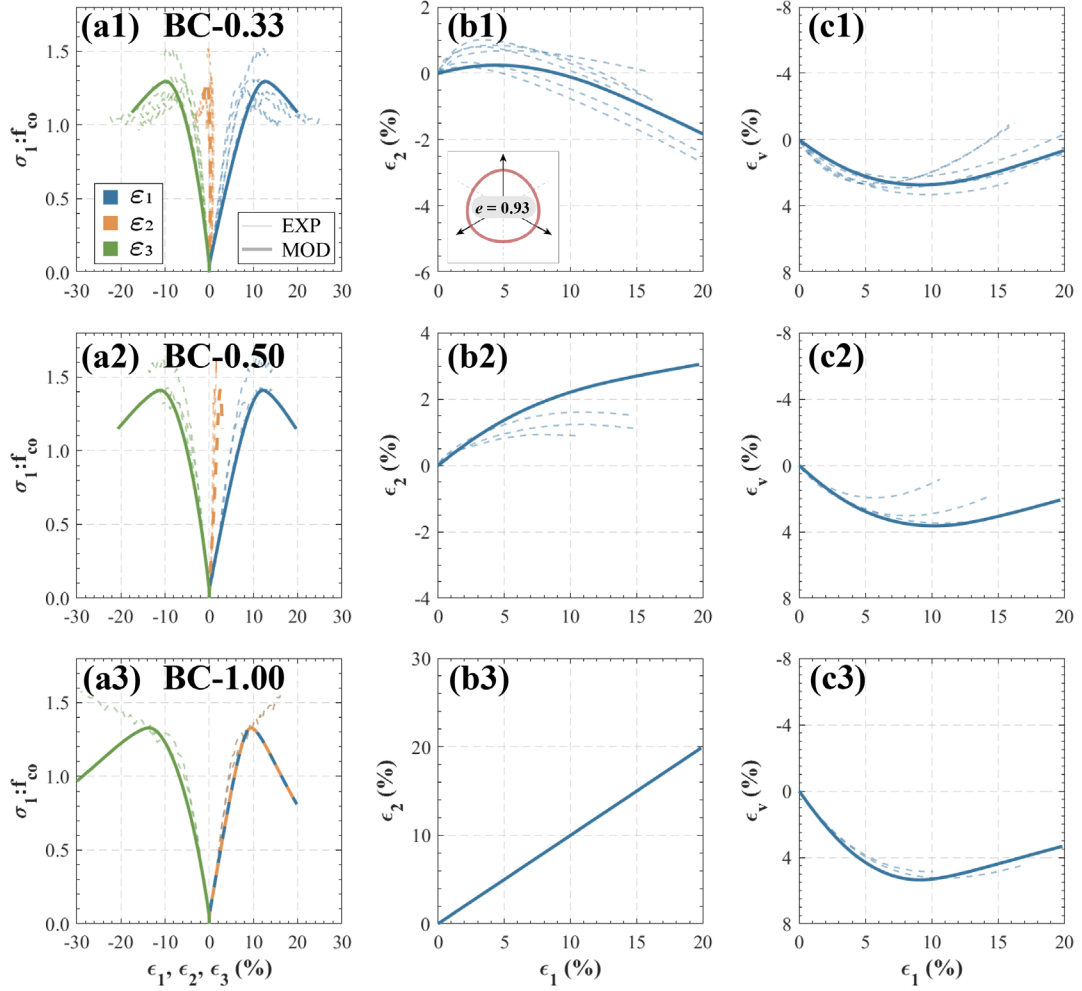


Figure 5.25 Comparison of the proposed model with BC test results (average concrete age = 152 min): (a1)-(a3) Normalised stress-strains curves; (b1)-(b3) Axial strain-horizontal strain curves; (c1)-(c3) Axial strain-volumetric strain curves.

Average concrete age = 246 min

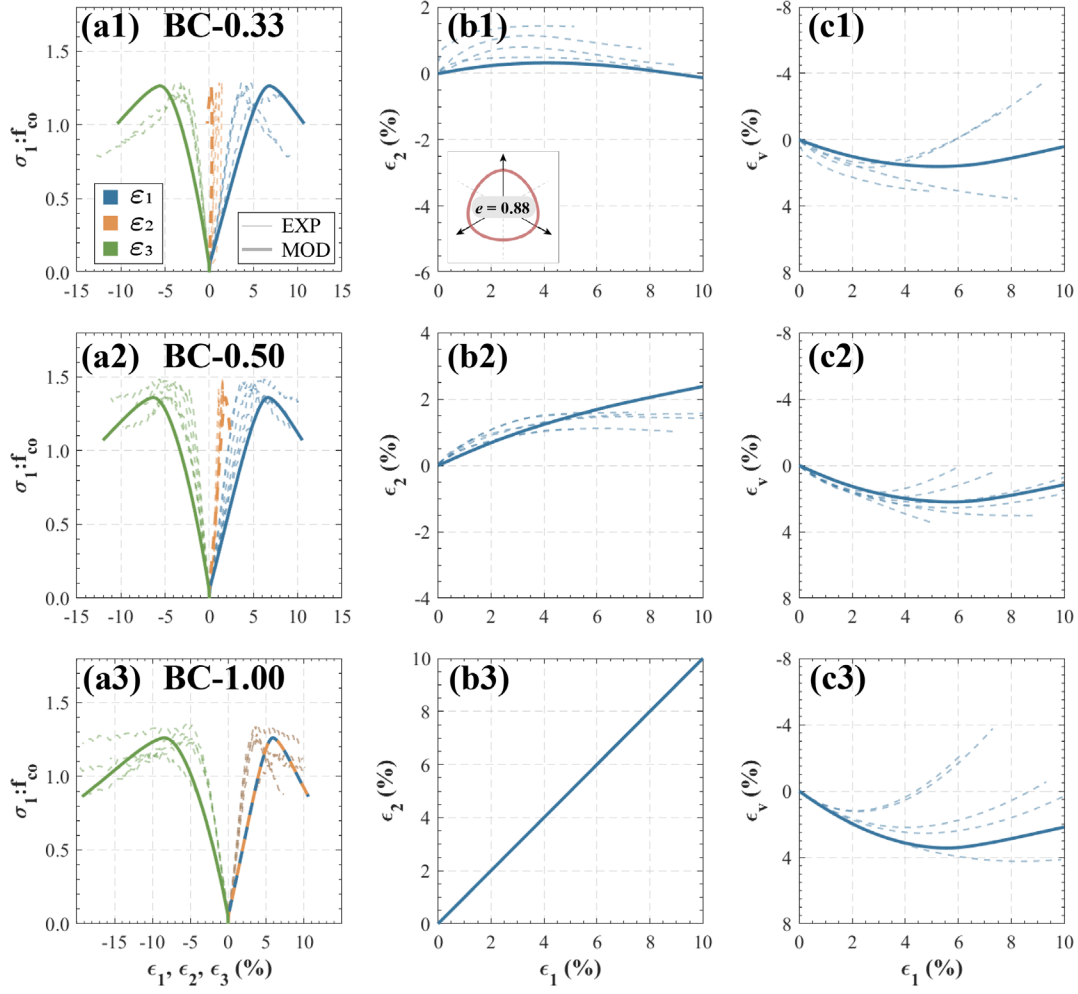


Figure 5.26 Comparison of the proposed model with BC test results (average concrete age = 246 min): (a1)-(a3) Normalised stress-strains curves; (b1)-(b3) Axial strain-horizontal strain curves; (c1)-(c3) Axial strain-volumetric strain curves.

Average concrete age = 301 min

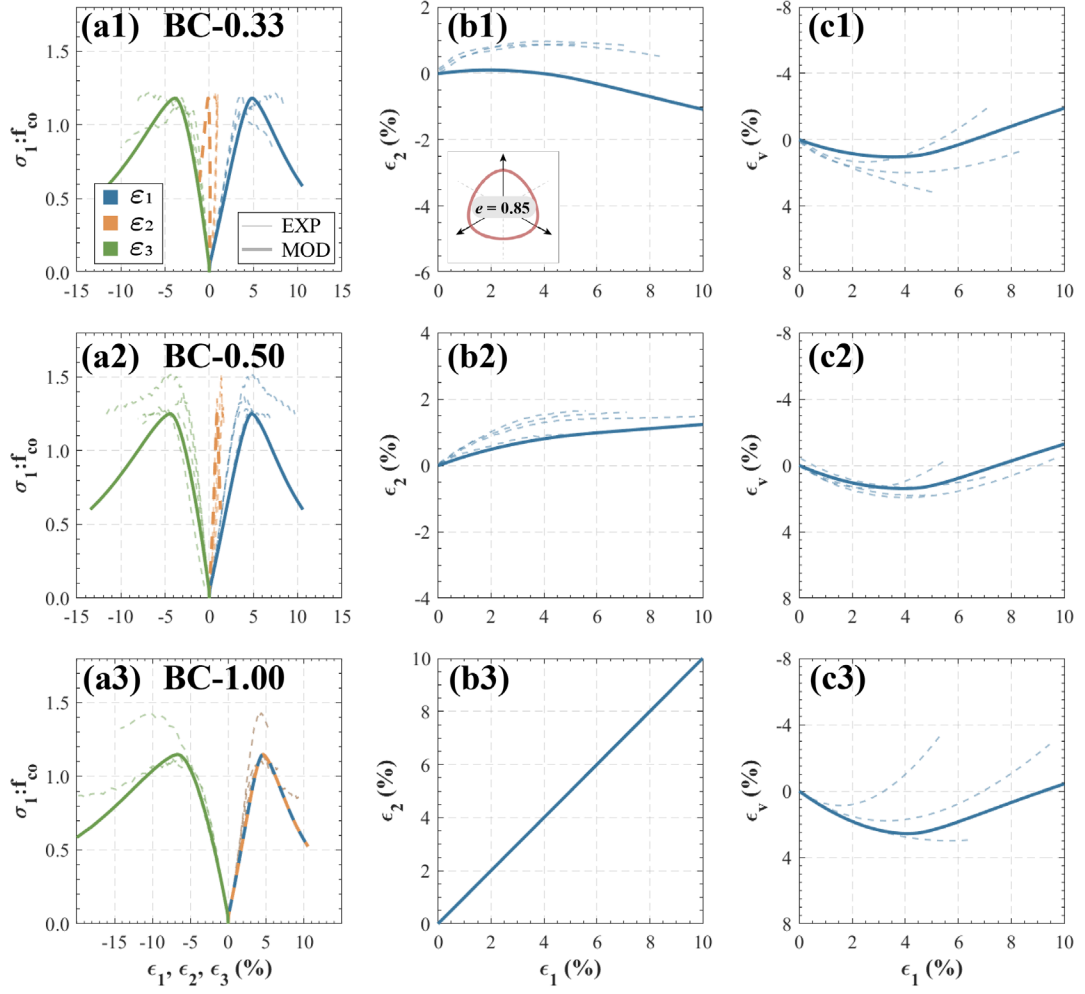


Figure 5.27 Comparison of the proposed model with BC test results (average concrete age = 301 min): (a1)-(a3) Normalised stress-strains curves; (b1)-(b3) Axial strain-horizontal strain curves; (c1)-(c3) Axial strain-volumetric strain curves.

APPENDIX 5.1 INITIAL DILATION PARAMETER

The initial dilation parameter represents the volumetric dilation rate at the on-set of plastic deformation ($\kappa_c = 0$). Physically, the internal microstructures of the material should remain largely intact at this stage. Consequently, it is reasonable to assume that the volumetric dilation rate at the on-set of initial yield is the same as those in the elastic stage.

The elastic strains can be evaluated from the generalised Hooke's law as

$$\boldsymbol{\varepsilon}^e = \mathbf{C}^{-1} : \boldsymbol{\sigma} \quad (5.33)$$

In the case of plane stress $\sigma_2 = \beta_s \sigma_1$; $\sigma_3 = 0$, the principal elastic strains are as follows.

$$\begin{cases} \varepsilon_1^e = \frac{\sigma_1}{E}(1 - \nu\beta_s) \\ \varepsilon_2^e = \frac{\sigma_1}{E}(-\nu + \beta_s) \\ \varepsilon_3^e = \frac{\sigma_1}{E}(-\nu - \nu\beta_s) \end{cases} \quad (5.34)$$

From this, the first strain invariant and second deviatoric strain

$$\begin{cases} I_1(\boldsymbol{\varepsilon}^e) = \frac{\sigma_1}{E}(1 - 2\nu)(1 + \beta_s) \\ J_2(\boldsymbol{\varepsilon}^e) = \frac{1}{6} \left(\frac{\sigma_1}{E} \right)^2 (1 + \nu)^2 \left[(1 - \beta_s)^2 + \beta_s^2 + 1 \right] \end{cases} \quad (5.35)$$

$$\begin{cases} \xi(\boldsymbol{\varepsilon}^e) = \frac{1}{\sqrt{3}} \frac{\sigma_1}{E} (1 - 2\nu)(1 + \beta_s) \\ \rho(\boldsymbol{\varepsilon}^e) = \frac{1}{\sqrt{3}} \frac{\sigma_1}{E} (1 + \nu) \sqrt{(1 - \beta_s)^2 + \beta_s^2 + 1} \end{cases} \quad (5.36)$$

The friction parameter is then evaluated as

$$\alpha_0 = -\frac{d\xi(\boldsymbol{\varepsilon}^e)}{d\rho(\boldsymbol{\varepsilon}^e)} = -\frac{(1-2\nu)(1+\beta_s)}{(1+\nu)\sqrt{(1-\beta_s)^2 + \beta_s^2 + 1}} \quad (5.37)$$

Based on Eq. (5.37), the relationship between the initial dilation parameter and stress ratio is plotted in Figure A. 5.1. Apparently, a larger β_s leads to a lower value of initial dilation parameter indicating that the material demonstrates higher tendency to volumetric contraction as the confining stress increases.

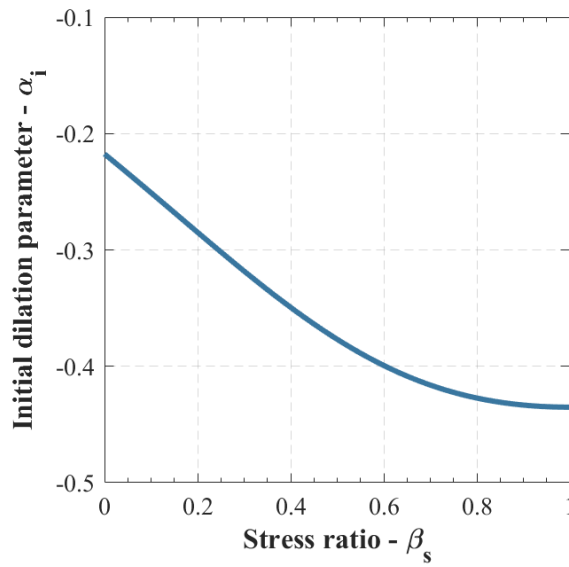


Figure A. 5.1 Relationship between initial dilation parameters with stress ratio.

APPENDIX 5.2 NUMERICAL IMPLEMENTATION OF THE PROPOSED CONSTITUTIVE MODEL

At increment j of a typical FE analysis, the total strain increment ($\Delta\epsilon_j$) and concrete age (t) is given, whereas the stress (σ_{j-1}) and equivalent plastic strain (κ_{j-1}) at the previous equilibrated step ($j - 1$) is known. The algorithm is summarised in the following steps:

- (i) Calculate the trial stress (σ^{tr})

$$\sigma^{tr} = \sigma_j + \mathbf{C}^t : \Delta\epsilon \quad (5.38)$$

- (ii) Check for material yielding

If $f(\sigma, \kappa, t) < 0$, the material is within the elastic regime. The variables at step j are updated as follows: $\sigma_j = \sigma^{tr}$, $\kappa_j = 0$; go to step (iv). Otherwise, the material is in the plastic regime; go to step (iii).

- (iii) Perform stress return

The vector of unknowns, Eq. (5.39), contains all the quantities that need to be solved for by Newton iteration. Note that the subscript j indicate the increment number, whereas the superscript (i) represent the iteration number.

$$\mathbf{v}_j^{(i)} = \begin{Bmatrix} \sigma_j^{(i)} \\ \kappa_j^{(i)} \\ \Delta\lambda_j^{(i)} \end{Bmatrix} \quad (5.39)$$

In order to satisfy the constitutive equations, the vector of residual $\mathbf{r}$, given in Eq. (5.40), needs to be minimised.

$$\mathbf{r}_j^{(i)} = \begin{bmatrix} r_\sigma^{(i)} \\ r_\kappa^{(i)} \\ r_f^{(i)} \end{bmatrix} = \begin{bmatrix} \mathbf{C}^{-1} : (\boldsymbol{\sigma}_j^r - \boldsymbol{\sigma}_j^{(i)}) - \Delta\lambda_j^{(i)} \mathbf{m} \\ \kappa_j^{(i)} - \kappa_{j-1} - \Delta\lambda_j^{(i)} n(\boldsymbol{\sigma}_j^{(i)}) \\ f(\boldsymbol{\sigma}_j^{(i)}, \kappa_j^{(i)}, t) \end{bmatrix} \quad (5.40)$$

The vector of unknown corresponding to a new iteration is calculated following the following equation based on Newton iteration.

$$\mathbf{v}^{(i+1)} = \mathbf{v}^{(i)} - \left\{ \frac{\partial \mathbf{r}^{(i)}}{\partial \mathbf{v}^{(i)}} \right\}^{-1} \mathbf{r}^{(i)} \quad (5.41)$$

Three tolerances were designed as convergence criteria, which are given as follows.

$$\begin{aligned} \text{tol}_\sigma &= \max \left(\left| \frac{\mathbf{v}_j^{(i+1)} - \mathbf{v}_j^{(i)}}{\mathbf{v}_j^{(i)}} \right|; \forall j = 1 \dots 3 \right) \\ \text{tol}_\kappa &= \left| \frac{\mathbf{v}_4^{(i+1)} - \mathbf{v}_4^{(i)}}{\mathbf{v}_4^{(i)}} \right| \\ \text{tol}_f &= \left| \mathbf{v}_5^{(i+1)} \right| \end{aligned} \quad (5.42)$$

In the current investigation, the convergence is said to be reached when all three tolerances are less than 10^{-3} . When this is the case, the stress and equivalent plastic strain are updated.

(iv) Compute the consistent tangential operator

The consistent tangential operator ($\partial \Delta \boldsymbol{\sigma} / \partial \Delta \boldsymbol{\epsilon}$) is crucial for obtaining quadratic convergence rate in implicit FE analysis (Dassault Systemes, 2023). When the material is not yield, the elastic stiffness material ($\mathbf{C}^t$) is taken as $\partial \Delta \boldsymbol{\sigma} / \partial \Delta \boldsymbol{\epsilon}$. On the other hand, $\partial \Delta \boldsymbol{\sigma} / \partial \Delta \boldsymbol{\epsilon}$ was evaluated numerically using the central difference method.

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CHAPTER 6

EXPERIMENTAL INVESTIGATIONS INTO THE VISCOUS, ELASTICITY AND MATERIAL HEALING BEHAVIOURS OF FRESH 3D- PRINTED CONCRETE

6.1 INTRODUCTION

As established in Chapter 2, 3D-printed concrete (3DPC) exhibits distinct strain-rate-dependent behaviours, including creep, stress relaxation and strain-rate-dependent stress-strain relationships. However, existing experimental investigations into these viscous properties remain limited, with contradictory findings across different studies. To address this knowledge gap, three comprehensive experimental programmes have been developed: variable displacement rate tests, stress relaxation tests, and multi-step creep-recovery tests.

Furthermore, Chapter 2 highlighted that a comprehensive understanding of fresh concrete's elasticity behaviour has yet to be established. To address this limitation, this chapter presents a systematic investigation through loading-unloading-reloading tests. These experiments specifically aim to characterise the elasticity behaviour of fresh 3DPC and quantify the relative contributions of elastic and plastic deformations.

During 3D concrete printing operations, the material simultaneously experiences two competing mechanisms: a 'destructive force' from gravitational loading and a 'healing force' from ongoing hydration reactions. An experimental study has been designed to investigate this material healing phenomenon through comparative analyses of virgin and reloaded specimens, providing crucial insights into the material healing behaviour.

6.2 MATERIAL AND SPECIMEN PREPARATION

Four mixture proportions for 3DPC, as detailed in Table 6.1, were prepared in the current experimental program. The raw materials are identical to those used in the biaxial compression tests discussed in Chapter 3; however, the proportions of the raw materials differ slightly. Moreover, all specimens are cube-shaped with a side length of 75 mm. The procedures for concrete mixing and specimen preparation were consistent with those described in Chapter 3.

6.3 TEST PROCEDURE

All experiments were conducted under uniaxial compression conditions using the GDS True Triaxial Apparatus (GDS Instruments, 2023) previously detailed in Chapter 3, which offers both displacement-controlled and force-controlled modes. Notably, the apparatus does not permit direct specification of loading rates; instead, it requires the input of target displacement (or force) values and test duration, whereupon the system implements a linear progression to achieve these terminal values.

The axial stress and axial strain were derived from the actuator force and displacement measurements. Lateral deformation, perpendicular to the loading axis, was measured post-test using callipers, whilst intermediate values were derived through the interpolation method detailed in Chapter 3. True stress and true strain were adopted to account for the significant specimen deformation. The concrete age was defined as the duration between the completion of casting and the point of strength attainment.

Table 6.2 provides a summary of the five proposed experimental programs, detailing the concrete ages and corresponding mixture proportions employed. The subsequent section will be dedicated to elucidating the test procedures for each program.

6.3.1 Variable Displacement Rate Tests

Previous experimental investigations into the uniaxial compression behaviour of fresh 3DPC have predominantly employed constant displacement rate loading protocols.

This conventional methodology requires multiple test groups at distinct displacement rates to characterise strain-rate dependence at a specific concrete age (Casagrande *et al.*, 2020; Pott *et al.*, 2022). A significant limitation of this approach is the necessity to fabricate specimens from different concrete batches for each test group, introducing inherent inter-batch variability in mechanical properties, as highlighted in Chapter 3 and 4. In response to this challenge, this study presents a novel experimental procedure that enables the investigation of rate-dependent behaviour using a single batch of material.

The experimental setup comprised five stages (Table 6.3), with stages 1, 3, and 5 maintaining the same displacement rate of 0.31 mm/s, serving as the reference rate. This rate was chosen to achieve an engineering strain rate of 25%/min, akin to those used in existing uniaxial compression tests (Wolfs *et al.*, 2018; Esposito *et al.*, 2021). Stages 2 and 4 had displacement rates respectively higher and lower by an order of magnitude compared to the reference rate. By using the stress-strain data in stages 1, 3, and 5, a reference stress strain curve can be established. The stress response in stage 2 and 4 can be compared to this reference stress-strain curve to study the impact of displacement rate (and therefore strain rate) on the mechanical behaviour of the material. The test duration for each specimen thus amounted to 2.6 minutes and a total of four specimens were tested.

6.3.2 Stress Relaxation Tests

Two sets of stress relaxation tests were conducted. In Group-1, as delineated in Table 6.4, specimens were loaded to 18.75 mm at a constant displacement rate of 0.31 mm/s, followed by a stress relaxation stage lasting 5 minutes. Consequently, the test duration for each specimen in Group-1 amounted to 6 minutes. Conversely, Table 6.5 summarises the testing stages for specimens in Group-2, encompassing 5 loading stages and 5 stress relaxation stages. A constant displacement rate of 0.31 mm/s was employed during each loading stage, with each followed by a relaxation stage lasting for 50 seconds. Moreover, stress relaxation occurred at five pre-defined displacements (3.75 mm, 7.5 mm, 11.25 mm, 15 mm, 18.75 mm), resulting in a total test duration of 5.2

minutes for specimens in this group. Eight relaxation tests were conducted in total, evenly split between Group-1 and Group-2.

6.3.3 Multi-Step Creep-Recovery Test

The multi-step creep-recovery test comprises four creep-recovery cycles, as outlined in Table 6.6. Both the creep and recovery phases were executed under load control, with each phase lasting for 66 seconds. During each creep phase, the machine automatically adjusted the load to achieve the predefined force within the first 6 seconds, maintaining it constant for the subsequent 60 seconds. Likewise, in each recovery phase, the specimen was unloaded to a force of 5N within 6 seconds, followed by a 60-second period where the force was held constant. The multi-step creep-recovery test was performed on one specimen at 30 minutes after casting. To determine the strength development, two additional specimens from the same concrete batch were tested under uniaxial compression at 15 and 42 minutes after casting. The strength of the creep specimen was then estimated by interpolating between these two compression test results.

6.3.4 Loading-Unloading-Reloading Tests

Two groups of loading-unloading-reloading tests were conducted, with specimens subjected to cyclic loading at five predetermined displacement levels (3.75 mm, 7.5 mm, 11.25 mm, 15 mm, 18.75 mm) across 15 phases. Group-1 specimens (Table 6.7) were tested with a constant displacement rate of 0.0625 mm/s during both loading and reloading phases, whilst Group-2 specimens (Table 6.8) underwent loading at 0.104 mm/s and reloading at 0.0625 mm/s. For both groups, the unloading phases were executed under force control over 13 seconds, reducing the axial force to 10 N. The total test duration was 7.7 minutes for Group-1 and 3.7 minutes for Group-2, with the shorter duration of Group-2 specifically designed to minimise the effects of material ageing. The experimental study comprised five specimens in total, with four allocated to Group-1 and one to Group-2.

6.3.5 Material Healing Tests

The experimental investigation comprised two sets of material healing tests, each containing test and control series. The control series established baseline relationships between uniaxial compressive strength and concrete age using specimens without pre-loading (virgin specimens). The test series investigated material healing through a three-step protocol: initial loading of virgin specimens to failure, a prescribed rest period for material healing, and subsequent reloading to failure (reloaded specimens). Here, the specimens were intentionally loaded to failure during the pre-stress phase to amplify material damage, thereby facilitating a clearer observation of damage recovery during the reloading phase. All tests employed displacement control, with Group-1 and Group-2 utilising displacement rates of 0.104 mm/s and 0.0625 mm/s, respectively. Group-1 included two specimens in the test series and six in the control series, whilst Group-2 comprised one test specimen and four control specimens.

For clarity in specimen identification, virgin specimens are designated as 'Txx', where 'xx' denotes the concrete age in minutes at testing. Reloaded specimens follow the notation 'Txx@yy', where 'xx' indicates the age at initial loading and 'yy' represents the age at reloading.

6.4 RESULTS AND DISCUSSION

6.4.1 Variable Displacement Rate Tests

The tested stress-strain curves for specimens aged 23 to 122 minutes are depicted as blue solid lines in Figure 6.1a to Figure 6.4a, with corresponding displacement rate variations, determined through numerical differentiation, shown in subplot b of each figure. Each test stage is designated as 'Sx', where 'x' represents the sequential stage number. Notably, during testing, the displacement rate in stage 4 were lower than intended (10 times the reference displacement rate), reaching only 3.7 to 4.7 times the reference rate. Therefore, to maintain the designated test duration, the displacement rate in stage 5 was automatically adjusted to 1.2 to 1.4 times the reference rate. Moreover, for specimens aged 122 minutes, $\dot{u}_1 = \dot{u}_2 = 0.0625 \text{ mm/s}$ due to machine errors. Despite

these experimental constraints, the obtained data provides significant insights into the strain-rate-dependent behaviour of the material.

A systematic post-processing methodology was implemented to evaluate the influence of displacement rate (and consequently strain rate) on the stress response. Reference stress-strain curves were constructed for comparative analysis: for stage 2 (lower displacement rate), the reference curve was established from stages 1 and 3, whilst for stage 4 (higher displacement rate), data from stages 3 and 5 were utilised¹. These reference curves, depicted as green dashed lines in Figures 6.1a-6.4a, provide baseline responses for quantifying rate-dependent behaviour in stages 2 and 4. Additionally, for specimens tested at ages 23, 62 and 92 minutes, the stage 2 data were fitted to establish a continuous stress-strain relationship, shown as red dashed lines in Figures 6.1a-6.4a, for ease of comparison

The experimental results demonstrate that fresh 3DPC exhibits distinct strain-rate-dependent behaviour, with stress responses in stages 2 and 4 consistently lower and higher than the reference curves, respectively. Within the investigated displacement rate range, the material shows greater stress sensitivity to rates exceeding the reference rate compared to those below it.

Table 6.9 shows the ratios of the tested stress to the reference stress in stages 2 and 4 for all four specimens. In stage 2, where the displacement rate is one tenth of the reference rate, the decrease in stress is less than 12%. Conversely, the stress increases by more than 70% in stage 4 where the displacement rate is only 3.7 to 4.7 times the reference rate.

Furthermore, the stress-strain curves in stages 2 and 4 maintain approximately parallel trajectories to their corresponding reference curves. This finding aligns with previous

¹ The reference line spanning stages 3 and 5 was justified by the assumption that the response in stage 5, with a displacement rate of only 1.2 to 1.4 times the reference rate, approximates the material behaviour at the reference rate

experimental observations on other rate-dependent materials, including soils (Yin and Graham, 1999) and metal (Rusinek *et al.*, 2007).

6.4.2 Stress Relaxation Tests

Figure 6.5a-d presents the stress-strain responses for Group-1 specimens, with the specimen aged 91 minutes loaded only to 15 mm displacement due to equipment limitations. Each stress-strain curve exhibits a characteristic stress reduction during constant-strain periods, occurring gradually rather than instantaneously. The stress-time relationship of this relaxation behaviour is illustrated in Figure 6.6a-d, where the relaxation time is measured from the initiation of each constant-strain phase. All specimens demonstrate a consistent pattern: the stress decreases at a diminishing rate before asymptotically approaching a steady-state value.

To quantify the magnitude of stress relaxation, each curve was normalised against its initial stress value at the onset of the relaxation phase. The normalised stress-relaxation time relationships are presented in Figure 6.7a-d, with key values at 50 and 300 seconds summarised in Table 6.10. Analysis reveals that the specimens underwent average stress reductions of 44% and 53% after 50 and 300 seconds of relaxation, respectively.

Figure 6.8a-d presents the stress-strain responses for Group-2 specimens as solid blue lines. Similar to Group-1, stress reduction is observed during all five constant-strain relaxation cycles. The stress values at the conclusion of these relaxation cycles are connected by a smooth spine line, depicted as an orange dashed line in each subplot. Notably, these spine lines mirror the shape of the corresponding stress-strain curves without relaxation, albeit at lower stress magnitudes.

The stress-relaxation time relationships are illustrated in Figure 6.9a-d, with colour-coding to distinguish individual relaxation cycles. Each cycle exhibits a pattern consistent with Group-1: stress decreases at a diminishing rate before asymptotically approaching a steady-state value. Notably, the stress-relaxation curve corresponding to each relaxation cycle appears to parallel to each other. The characteristic can be better visualised by the normalised stress-relaxation relationships shown in Figure 6.10a-d,

which demonstrate remarkably similar profiles across all relaxation cycles. The normalised stress corresponding to a relaxation time of 50 seconds for all four specimens are summarised in Table 6.10. At any given age, the normalised stress at the end of each relaxation cycle is almost the same. This uniform stress reduction explains the observed phenomenon in Figure 6.8, where the smooth spine line connecting the post-relaxation stresses represents a proportionally scaled version of the non-relaxed stress-strain curve.

The combined analysis of Group-1 and Group-2 results, summarised in Table 6.11, reveals age-dependent variations in stress relaxation behaviour. The stress reduction after 50 seconds of relaxation shows a modest decrease from approximately 45% at 20 minutes to 41% at 120 minutes of concrete age, indicating a diminishing influence of viscous behaviour with increasing age.

6.4.3 Multi-Step Creep-Recovery Test

Figures 6.11a and b depict the temporal evolution of axial strain and axial stress, respectively, during the creep-recovery test. As illustrated in Figure 6.11b, the stress is not strictly constant during the creep and recovery stages (indicated as *i*, *ii*), which can be attributed to two primary factors. Firstly, the servo-controlled system applying the load may not achieve the target force within the specified timeframe (i.e., 6 seconds), a limitation previously documented in similar testing systems (Chang *et al.*, 2023; Esposito *et al.*, 2021). Secondly, under a constant axial load, the cross-sectional area of the specimen increases, leading to a decrease in true stress during the creep period. Moreover, the final strain recovery cycle, indicated as *iii*, was compromised due to equipment malfunction.

During each creep stage, as shown in Figure 6.11a, the strain exhibited a sharp initial increase, followed by a gradual rise at a continuously decreasing rate. Upon unloading, the material displayed initial significant decrease in strain, succeeded by a gradual decline at a diminishing rate. This strain recovery behaviour evidences the material's viscoelastic nature. Moreover, consistent to the observation in Chang *et al.* (2023), the recoverable portion of the total axial strain remains notably small, suggesting that while

both viscoelastic and viscoplastic mechanisms contribute to the creep response, the behaviour is predominantly viscoplastic, leading to significant permanent deformation.

Figures 6.12a and b illustrate the incremental creep compliance curves¹ during the four creep and recovery periods, with the creep stress assumed constant at the average value for each period (Table 6.12). These creep stresses range from 32% to 93% of the material's uniaxial compressive strength. Figure 6.12a reveals that fresh 3DPC exhibits nonlinear creep, as evidenced by the stress-dependent variation in creep compliance curves. From the second creep cycle, higher stress levels consistently produced higher creep compliance values. However, during the first two cycles, an exceptional behaviour was observed where higher compliance occurred at lower stress levels. This phenomenon may be attributed to pore collapse under initial compression loading (Chang *et al.*, 2023). The pore collapse mechanism may result in a softer material response compared to the particle rearrangement that dominates later loading stages, thus explaining the unusually high creep compliance observed in the first creep cycle. In subsequent creep periods, the increased compliance observed at higher stress levels likely results from progressive material damage, consistent with findings from shear creep tests on fresh concrete (Chen *et al.*, 2020; Xia *et al.*, 2024). The initial nonlinear creep response during the first two cycles, characterised by higher compliance at lower stress, is uncommon. However, the stress-dependent compliance increase observed after the second cycle follows a more typical pattern, similar to that documented in both soil (Arulanandan *et al.*, 1971) and cured concrete (Roll, 1964).

The creep compliance curves derived from the recovery data are roughly one order of magnitude lower than those from the creep period, indicating substantial non-recoverable creep strain. Notably, specimens subjected to higher prior creep stresses exhibited lower recovered compliance values, suggesting increased material stiffness following exposure to higher stress levels.

¹ The incremental creep compliance curve for cycle i was calculated as $\Delta J_i(t) = \frac{\varepsilon_i(t) - \varepsilon_{i-1}(t_{end})}{\sigma_i - \sigma_{i-1}}$, where t is the time measured from the beginning of each cycle; $\varepsilon_{i-1}(t_{end})$ represents the strain at the end of the last cycle; σ_i and σ_{i-1} are the applied creep stresses at the current and previous cycles, respectively.

6.4.4 Loading-Unloading-Reloading Tests

Figures 6.13 a-e present the stress-strain curves for specimens at five distinct ages in Group-1 (a-d) and Group-2 (e) loading-unloading-reloading tests. For ease of discussion, the modulus corresponding to the initial linear regime is referred to as apparent Young's modulus (E_{ap}), while E_i denotes the re-loading modulus at stage i . The difference between these two moduli is illustrated in Figure 6.13a.

Across all ages, the fresh concrete exhibits negligible recoverable strain during all unloading stages, consistent with observations of the creep-recovery test (Section 6.4.3). Notably, significant plastic deformation occurs during the initial unloading-reloading cycle at stress levels below 10% of the uniaxial compressive strength. This notably low proportional limit (<10%) contrasts markedly with conventional cured concrete, which typically maintains elastic behaviour up to 30-50% of its compressive strength (Chen 1994). Furthermore, specimens tested at both high (Group-2) and low (Group-1) displacement rates display similar stress-strain patterns, suggesting that the material stiffness in unloading-reloading cycles reflects an intrinsic material property rather than age-related effects during loading.

Table 6.13 summarises E_{ap} and E_i for all specimens. For specimens aged 22 and 48 minutes, E_1 could not be accurately determined because the re-loading branch is too small, whilst for the specimen at age 114 minutes, E_6 is unavailable as testing concluded after the fifth cycle. The re-loading modulus exhibits slight degradation with increasing axial strain for specimens T22 and T48. On the other hand, specimens aged 63 to 114 minutes display an initial increase followed by a decrease in modulus with rising axial strain. The initial increase in E_i during loading can be attributed to pore collapse within the microstructure caused by the compression load, which increases the effective load bearing area and therefore a high material stiffness, while the stiffness degradation at later stage of loading is attributed to material damage. The stiffness degradation exhibits pronounced age dependency: the specimen tested at 22 minutes shows a 17% reduction from its maximum reloading modulus, whilst the specimen tested at 114 minutes displays a more significant 42% decrease. These reductions,

however, remain substantially lower than those observed in hardened concrete, where stiffness degradation can exceed 80% (Sinha *et al.*, 1964).

Notably, the reloading modulus (E_i) represents the material's actual elastic modulus during the loading process. For the purpose of comparative analysis, the maximum reloading modulus across the six reloading stages is defined as the actual Young's modulus (E) in this study. Figure 6.14 shows how the ratio of actual to apparent Young's modulus varies with concrete age. The results demonstrate that E is approximately 13 times E_{ap} at 22 minutes of age, decreasing to 6 times E_{ap} at age 114 minutes. These findings align with the monotonic uniaxial compression study by Tripathi et al (2022), whose results are included in Figure 6.14 for comparison. This significant difference between apparent and actual moduli indicates that using the apparent Young's modulus as the elastic modulus, as done in several existing studies (Wolfs *et al.*, 2018; Casagrande *et al.*, 2020; Xia *et al.*, 2024) is inappropriate.

6.4.5 Material Healing Tests

Figures 6.15 and 6.16 present the stress-strain curves for specimens in Group-1 and Group-2, respectively, with virgin specimens shown as solid lines and reloaded specimens as dashed lines. For comparison, stress-strain curves of virgin specimens tested at ages similar to the reloaded specimens are also included.

The reloaded specimens (T72@T373, T251@T377 in Group-1 and T45@T199 in Group-2) demonstrate higher strength and pre-peak stiffness compared to their initial testing states (T72, T251 in Group-1 and T45 in Group-2). This enhancement in mechanical properties during the healing period can be attributed to continued hydration, which likely filled existing cracks with hydration products. However, these reloaded specimens exhibit lower strength and pre-peak stiffness compared to virgin specimens of similar age, indicating that the initial loading induced permanent damage that could not be fully recovered within the healing period.

Table 6.14 compares the strength, apparent Young's modulus, and peak strain of reloaded specimens with estimated values for virgin specimens at equivalent ages.

These virgin specimen properties were derived through interpolation or extrapolation of data from specimens tested at different ages (see Appendix 6.1).

Noteworthy, specimens subjected to pre-loading at earlier ages (T72@T373, T45@T199) showed less recovery in strength and stiffness compared to the specimen pre-loaded at a later age (T251@T377), despite having a longer healing period. Specifically, specimen T72@T373, which experienced a 6-hour healing period, exhibited strength and stiffness values 58.2% and 84.7% lower than the corresponding virgin specimen. In contrast, specimen T251@T377, with only 2 hours of healing, showed smaller reductions of 35.8% and 46.9% in strength and stiffness, respectively. This difference may be attributed to the larger deformation experienced by specimens pre-loaded at earlier ages, resulting in greater inter-particle distances that require more hydration products to fill internal defects. The peak compressive strain behaviour also showed age-dependent patterns. Specimens pre-loaded at earlier ages (T72@T373, T45@T199) exhibited higher peak compressive strains than their virgin counterparts, whilst the specimen pre-loaded at a later age showed reduced peak compressive strain compared to its virgin equivalent.

These findings have significant implications for 3D concrete printing, where fresh concrete filaments often experience high stress levels at early ages. Consequently, the cured final product may exhibit lower strength and stiffness compared to values obtained from virgin specimen tests. Further research is needed to investigate the relationship between pre-stress levels and material healing behaviour of fresh concrete, particularly at early ages.

Despite being previously loaded into the post-peak regime, the reloaded specimens (T72@T373, T251@T377 in Group-1, and T45@T199 in Group-2) exhibited both strain hardening and strain softening behaviours. In the context of plasticity, this response suggests that the equivalent plastic strain (κ), which controls the evolution of yield stress, decreases during the healing process (refers to Section 6.5.2 for a more detailed discussion).

6.5 JUXTAPOSING RESULTS FROM DIFFERENT TESTS

6.5.1 Contribution of Viscoelasticity and Viscoplasticity to the Mechanical Behaviour of Fresh Concrete

6.5.1.1 *Role of viscoelasticity and viscoplasticity in deformation response*

The multi-step creep-recovery test results (Figure 6.11a) reveal that fresh concrete exhibits both viscoelastic and viscoplastic responses, where the strain recovery is attributed to viscoelastic mechanisms, whilst the creep response stems from combined viscoelastic and viscoplastic effects. Notably, both the creep test (Figure 6.11a) and loading-unloading-reloading tests (Figure 6.13) demonstrate minimal recoverable strain upon unloading, even at low stress levels, indicating that viscoplastic mechanisms predominantly govern the deformation behaviour of 3DPC during loading phases (such as creep).

The nonlinear characteristics of both creep and strain recovery, as evidenced in Figure 6.12, can be attributed to two distinct mechanisms. The first mechanism manifests as nonlinear viscoelastic behaviour, evidenced by both the stress history-dependent reloading modulus in the loading-unloading-reloading tests and the nonlinear creep compliance observed during strain recovery. Secondly, the predominantly non-recoverable nature of the creep strain suggests that viscoplastic mechanisms are the primary contributors to the nonlinear creep response during loading phases.

6.5.1.2 *Role of viscoelasticity and viscoplasticity in stress relaxation*

Soil and fresh concrete share similar mechanical characteristics, exhibiting both creep and stress relaxation behaviour. In soil mechanics, the elastic-viscoplastic framework has been widely adopted to model these behaviours, wherein elastic response is considered rate-independent (Adachi and Oka, 1982; Yin and Graham, 1999; Shahbodagh *et al.*, 2020). Within this framework, the converged stress in stress relaxation tests is interpreted as the elastic limit or yield stress (Liingaard *et al.*, 2004; Qiao *et al.*, 2016).

However, interpreting the converged stress as the elastic limit appears unsuitable for fresh concrete. To explain this incompatibility, it is essential to distinguish between static strength (material strength at zero strain rate) and dynamic strength (strength under non-zero strain rate). The stress relaxation test results (Table 6.11) suggest that, according to the soil mechanics approach, the static strength would be 40-45% of the dynamic strength (at 0.31 mm/s displacement rate). Utilising this static strength in an elastic-viscoplastic model would predict creep rupture (material failure) when the applied stress exceeds this static strength threshold.

Experimental evidence contradicts this prediction. Figure 6.12a demonstrates that creep rupture does not occur even at 79% of the dynamic strength at 0.31 mm/s displacement rate, indicating that the material's static strength must exceed 45% of the dynamic strength. This discrepancy suggests that the observed stress reduction is predominantly attributed to viscoelastic rather than viscoplastic mechanisms.

To validate this hypothesis, normalised stress-relaxation time curves were derived from two sources: creep compliance data during strain recovery stages (Figure 6.12b) and Group-2 relaxation tests. The conversion methodology from creep compliance to normalised stress is detailed in Appendix 6.2. Figure 6.17 demonstrates that these independently derived curves align reasonably well. Since strain recovery is purely viscoelastic, this correlation confirms that viscoelasticity predominantly governs the stress relaxation behaviour of fresh concrete.

6.5.2 Age Hardening, Material Healing and Material Damage

The preceding chapters have established that fresh 3DPC exhibits two distinct hardening mechanisms: *strain hardening*, where the yield stress varies due to mechanical loading, and *age-hardening*, where the yield stress evolves due to material ageing. Notably, the strain hardening-softening process, resulting from changes in microstructures, is irreversible for most materials. Thus, in conventional plasticity, the evolution of equivalent plastic strain (κ) is non-reversible, i.e., $\dot{\kappa} \geq 0$ (Chen and Han, 2007). However, material healing tests have shown that κ can decrease (that is $\dot{\kappa} < 0$ is permissible) for fresh concrete, contrary to conventional plasticity theory. To maintain

clarity in subsequent discussions, this work distinguishes between material healing and age-hardening: material healing specifically refers to the recovery of κ , whilst age-hardening pertains to age-dependent yield development.

A series of single-element numerical tests were conducted to illustrate the interaction between age-hardening and material healing. Each element, with a length of 100 mm, was subjected to uniaxial compression, assuming an elastic-plastic material response. The complete numerical modelling details and material parameters are documented in Appendix 6.3.

Figure 6.18a illustrates the yield stress (σ_y)-equivalent plastic strain (κ) relationship for virgin specimens at two distinct ages (t_0 and t_1 where age $t_1 > t_0$). Three numerical tests were performed, each comprising three distinct steps:

- (1) Loading a specimen aged t_0 to a displacement of 15 mm, reaching the yield stress at point *ii* in Figure 6.18a
- (2) Complete unloading of the specimen
- (3) Reloading the specimen at age t_1 to an accumulative displacement of 25 mm

These tests were designed to examine three healing scenarios: no healing, partial healing (where an 80% recovery of κ was assumed at the beginning of step 3), and full healing.

Figure 6.18b compares the stress-strain responses of these three scenarios against virgin specimens aged t_0 and t_1 . In the absence of healing, the specimen exhibits linear elastic behaviour until reaching the yield stress at point *iv* in Figure 6.18a, followed by strain softening. This behaviour arises because whilst age hardening increases the yield stress (from point *ii* to *iv* in Figure 6.18a), the equivalent plastic strain (κ) can only increase monotonically in the no-healing scenario. This response is clearly different from the experimental observations shown in Figures 6.15 and 6.16, that the reloaded specimens clearly exhibit strain hardening before softening.

The fully healed specimen demonstrates behaviour identical to the virgin specimen at t_1 , as κ resets to zero after step 2. The partially healed specimen demonstrates initial strain hardening upon reloading, followed by strain softening, with a peak strain lower than that of the virgin specimen at t_1 . These responses align with experimental observations (e.g., T72@T373 and T251@377 in Figure 6.15), suggesting that experimental specimens underwent only partial healing. However, a notable discrepancy exists between simulation and experimental results: both fully and partially healed specimens in simulations achieve strengths identical to virgin specimens at t_1 , whereas experimental specimens consistently show reduced strength upon reloading. This discrepancy suggests that pre-stressing induces irreversible damage to the material's internal structure, which persists despite the healing period. Collectively, these experimental and numerical investigations demonstrate that fresh concrete exhibits concurrent age-hardening, material damage, and material healing mechanisms, whose complex interplay governs the material's overall mechanical behaviour.

6.6 CONCLUDING REMARKS

This chapter presents and analyses the results of five novel uniaxial compression testing protocols developed for fresh 3D-printed concrete (3DPC). These protocols comprise variable displacement rate tests, stress relaxation tests, and multi-step creep-recovery tests to characterise the material's viscous behaviour, whilst loading-unloading-reloading tests investigate its elastic response. Furthermore, material healing tests were conducted to examine the effects of pre-loading on the mechanical properties of fresh concrete. Through comparative analysis of these experimental results, supplemented by numerical simulations, comprehensive insights into the mechanical behaviour of fresh 3DPC were obtained. The following sections present key findings derived from this integrated experimental and numerical investigation:

- (1) **Viscoelastic-viscoplastic nature:** Fresh 3D-printed concrete is a viscoelastic-viscoplastic material. This finding is revolutionary, as fresh concrete has traditionally been regarded as either viscoelastic or viscoplastic in the existing literature.

- (2) **Actual versus Apparent Young's Modulus and Proportional Limit:** The actual Young's modulus of fresh concrete within the initial 120 minutes of ageing exceeds the apparent Young's modulus (secant modulus at a few percent strain) by a factor of 5 to 20. Notably, during this period, the material's proportional limit remains below 10% of its compressive strength. These findings demonstrate that the widespread practice among existing studies of adopting the apparent Young's modulus as the actual elastic modulus significantly overestimates the elastic strain component, leading to potentially inaccurate material characterisation.
- (3) **Creep Strain and Viscoplasticity:** The experimental results demonstrate that viscoplastic deformation constitutes the primary component of creep strain in fresh 3DPC, with the observed nonlinear creep behaviour predominantly arising from viscoplastic mechanisms. This finding fundamentally challenges the conceptual validity of viscoelasticity-based creep models widely adopted for fresh 3DPC.
- (4) **Stress Relaxation and Viscoelasticity:** While the creep behaviour is mainly due to viscoplasticity, the stress relaxation behaviour is primarily attributed to viscoelasticity.
- (5) **Material Healing and Damage:** Fresh concrete specimens loaded to failure at an early age demonstrate partial recovery of strength and stiffness following a rest period. This recovery involves two distinct mechanisms: age-hardening, where continuous hydration leads to increasing yield stress, and material healing, whereby the equivalent plastic strain (controlling strain hardening behaviour) partially recovers during the rest period. However, the initial loading induces permanent material damage, manifesting as irreversible alterations to the internal structure and resulting in degraded mechanical properties. Whilst existing constitutive models for fresh concrete typically incorporate age-hardening effects, they notably fail to account for both material healing and damage mechanisms.

Whilst this work establishes fundamental material behaviour through uniaxial compression testing, fresh concrete in practical 3D printing applications experiences complex multi-axial stress states. Future research should therefore extend these investigations in two key directions. Firstly, the characterisation of viscous behaviour, material damage, and healing mechanisms should be evaluated under diverse stress conditions to better reflect printing conditions. Secondly, a comprehensive experimental study should be performed to systematically investigate the material's mechanical response under varying pre-loading intensities and healing durations, thereby establishing a more complete understanding of these mechanisms.

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Table 6.1 Mixture proportions (in g/L).

Mixture	Binder		Filler		TW	PCE	VMA
	Cement	SF	QP	RS			
A	830	207	207	913	300	6.5	9.6
B					270	6.0	3.0
C					300	6.5	4.5
D					270	6.0	9.0

Note: SF: silica fume; QP: quartz powder; RS: river sand; TW: tap water; PCE: Polycarboxylate superplasticizer; VMA: Hydroxypropyl methylcellulose viscosity modifying agent.

Table 6.2 A summary of specimens in each testing program.

Testing program		Concrete age (min)	Mixture	Number of specimens
Variable Displacement Rate		23, 62, 92, 122	A	4
Stress relaxation	Group-1	21, 61, 91, 120	A	4
	Group-2	24, 63, 92, 122	A	4
Multi-step creep-recovery		38*	B	1
Loading-unloading-reloading	Group-1	22, 48, 94, 114	B	4
	Group-2	63	B	1
Material healing	Group-1	<i>Test</i> 72 (virgin) – 373 (reload) 251 (virgin) – 377 (reload) <i>Control</i> 30, 100, 130, 160, 190, 370	C	2 (test) 6 (control)
	Group-2	<i>Test</i> 45 (virgin) – 199 (reload) <i>Control</i> 170, 230, 270, 310	D	1 (test) 4 (control)

Note: Since the strength was not reached during the creep test, the time duration from after casting to the end of the last creep cycle was used to estimate the age.

Table 6.3 Testing procedures for the variable displacement rate tests.

Stage	1	2	3	4	5
$\dot{u}$ (mm/s)	0.31	0.031	0.31	3.1	0.31
Duration (s)	12	120	12	1.2	12
Cumulative Displacement (mm)	3.75	7.5	11.25	15	18.75

Table 6.4 Testing procedures for the stress relaxation tests (Group-1).

Stage	1	2
Type	Load	Relax
$\dot{u}$ (mm/s)	0.31	0
Duration (s)	60	300
Cumulative Displacement (mm)	18.75	18.75

Table 6.5 Testing procedures for the stress relaxation tests (Group-2).

Stage	1	2	3	4	5	6	7	8	9	10
Type	Load	Relax	Load	Relax	Load	Relax	Load	Relax	Load	Relax
$\dot{u}$ (mm/s)	0.31	0	0.31	3.1	0.44	0	0.31	0	0.31	0
Duration (s)	12	50	12	50	12	50	12	50	12	50
Cumulative Displacement (mm)	3.75	3.75	7.5	7.5	11.25	11.25	15	15	18.75	18.75

Table 6.6 Testing procedures for the multi-step creep-recovery test.

Stage	1	2	3	4	5	6	7	8
Type	Creep	Recovery	Creep	Recovery	Creep	Recovery	Creep	Recovery
Duration (s)	66	66	66	66	66	66	66	66
Force (N)	90	5	175	5	260	5	345	5

Table 6.7 Testing procedures for the loading-unloading-reloading test (Group-1).

Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Type	L	UL	ReL	L	UL	ReL	L	UL	ReL	L	UL	ReL	L	UL	ReL
$\dot{u}$ (mm/s)	0.0625	L.C.	0.0625	0.0625	L.C.	0.0625	0.0625	L.C.	0.0625	0.0625	L.C.	0.0625	0.0625	L.C.	0.0625
Duration (s)	60	12	N/A	60	12	N/A	60	12	N/A	60	12	N/A	60	12	N/A
Cumulative Displacement (mm)	3.75	N/A	3.75	7.5	N/A	7.5	11.25	N/A	11.25	15	N/A	15	18.75	N/A	18.75

Table 6.8 Testing procedures for the loading-unloading-reloading test (Group-2).

Stage	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Type	L	UL	ReL	L	UL	ReL	L	UL	ReL	L	UL	ReL	L	UL	ReL
$\dot{u}$ (mm/s)	0.104	L.C.	0.0625	0.104	L.C.	0.0625	0.104	L.C.	0.0625	0.104	L.C.	0.0625	0.104	L.C.	0.0625
Duration (s)	36	12	N/A	36	12	N/A	36	12	N/A	36	12	N/A	36	12	N/A
Cumulative Displacement (mm)	3.75	N/A	3.75	7.5	N/A	7.5	11.25	N/A	11.25	15	N/A	15	18.75	N/A	18.75

Notations:

L: Loading stage.

UL: Unloading stage.

ReL: Reloading stage.

L.C.: Load controlled.

Table 6.9 Comparison of the tested axial stress with the reference stress in stage 2 and stage 4.

Concrete age (min)	$\sigma_{s2}:\sigma_{ref}$ (at 8% strain)	$\sigma_{s4}:\sigma_{ref}$ (at 18% strain)
23	0.89	1.82
62	0.96	1.82
92	0.96	1.77
122	N/A	1.79

Table 6.10 A summary of normalised stress in Group-1 and Group-2 relaxation tests.

Concrete age (mm)	Strain level (%)	Normalised stress	
		At 50 s	At 300 s
21	28.7	0.58	0.48
61	28.7	0.55	0.46
91	22.3	0.55	0.47
120	28.7	0.57	0.48
24	5.1	0.49	
	10.5	0.56	
	16.3	0.54	
	22.3	0.55	
	28.7	0.58	
63	5.1	0.55	
	10.5	0.58	
	16.3	0.58	
	22.3	0.55	
	28.7	0.58	
92	5.1	0.63	
	10.5	0.62	
	16.3	0.61	
	22.3	0.57	
	28.7	0.60	
122	5.1	0.64	
	10.5	0.60	
	16.3	0.59	
	22.3	0.58	
	28.7	0.58	

Table 6.11 Average stress reduction after 50 seconds of stress relaxation.

Specimens	Average stress reduction
T21(Group-1), T24(Group-2)	45%
T61(Group-1), T63(Group-2)	44%
T92(Group-1), T92(Group-1)	40%
T120(Group-1), T122(Group-2)	41%

Table 6.12 Average stress during the creep and recovery period.

Cycle	Creep period		Recovery period
	Average stress (kPa)	Creep stress levels $\sigma: f_{co}$	Average stress (kPa)
1	14.90	0.32	1.02
2	26.21	0.57	0.97
3	36.11	0.79	0.90
4	42.64	0.93	N.A.*

Note: the last recovery period was not successful due to a machine error.

Table 6.13 Comparison of apparent Young's modulus and reloading modulus at different reloading stages.

Specimens	Material stiffness (kPa)						
	E_{ap}	E_1	E_2	E_3	E_4	E_5	E_6
T22	190.05	N/A	2418.09	2340.66	2327.14	2220.40	2011.83
T48	363.46	N/A	3542.93	3514.36	3213.88	3291.40	2919.02
T63	497.75	1209.47	2769.39	3699.93	3812.65	3487.56	3152.04
T94	884.75	2620.62	5115.11	6885.01	6027.37	5116.50	4247.94
T114	1542.21	5186.86	8012.23	9605.36	7850.90	5554.22	N.A.

Table 6.14 Comparison of uniaxial compressive strength, apparent Young's modulus and axial peak strain for virgin and reloaded specimens.

Specimens	Uniaxial compressive strength (kPa)			Apparent Young's modulus (MPa)			Axial peak strain (%)		
	Virgin*	Reload	Difference (%)	Virgin*	Reload	Difference (%)	Virgin*	Reload	Difference (%)
T72@T373	1378.4	576.1	-58.2	75.4	11.6	-84.7	3.00	4.86	62.3
T251@T377	1453.8	933.0	-35.8	81.6	42.4	-42.6	2.96	2.61	-11.6
T45@T199	137.60	69.40	-49.6	2.26	0.511	-77.4	8.83	16.7	89.4

Note: these values were estimated based on interpolation or extrapolation of results of virgin specimens at different ages.

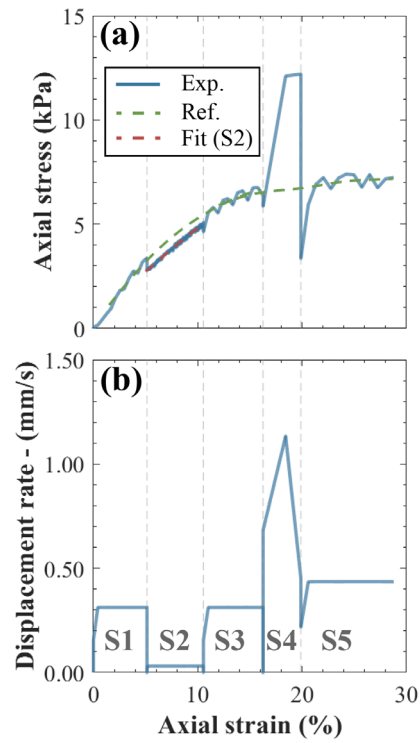


Figure 6.1 Results of variable displacement test of specimen aged 23 minutes: (a) Stress-strain curves; (b) Variation in displacement rate.

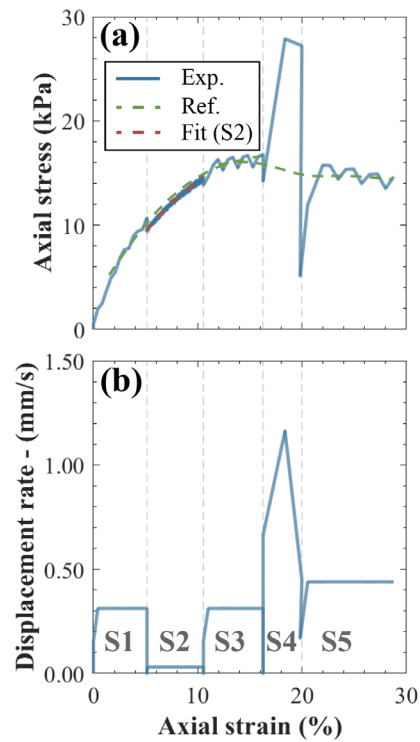


Figure 6.2 Results of variable displacement test of specimen aged 62 minutes: (a) Stress-strain curves; (b) Variation in displacement rate.

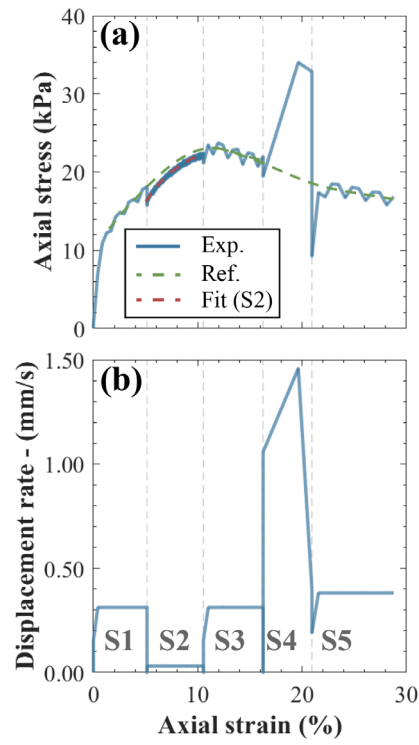


Figure 6.3 Results of variable displacement test of specimen aged 92 minutes: (a) Stress-strain curves; (b) Variation in displacement rate.

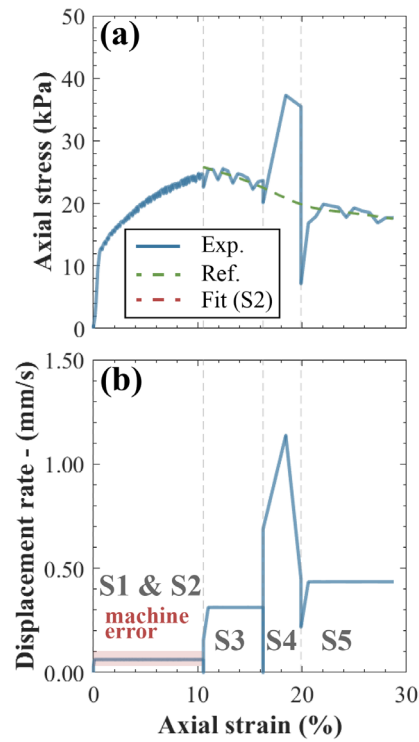


Figure 6.4 Results of variable displacement test of specimen aged 122 minutes: (a) Stress-strain curves; (b) Variation in displacement rate.

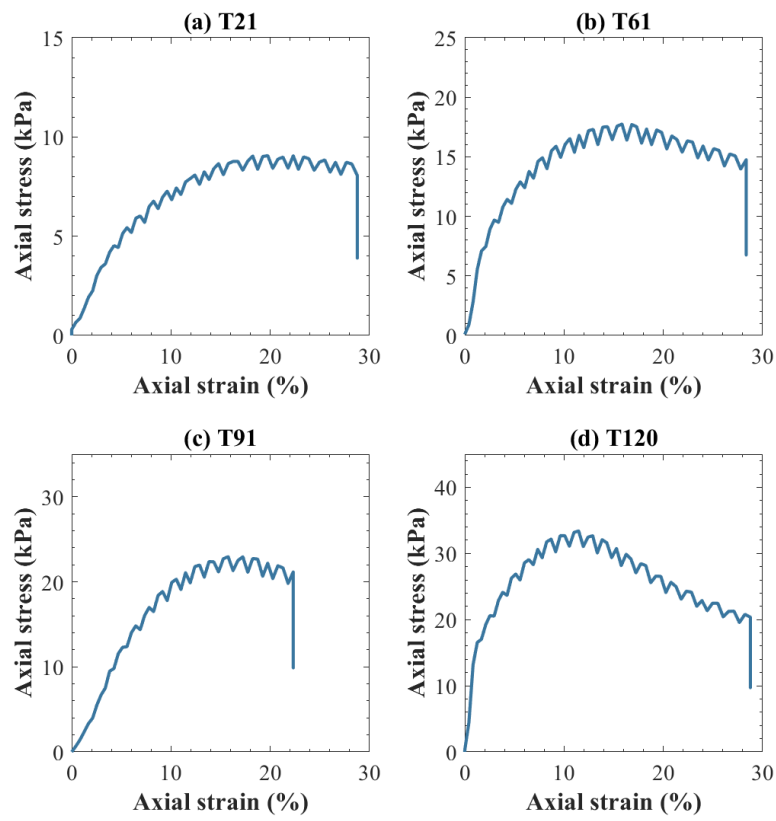


Figure 6.5 Stress-strain curves - Group-1 relaxation tests.

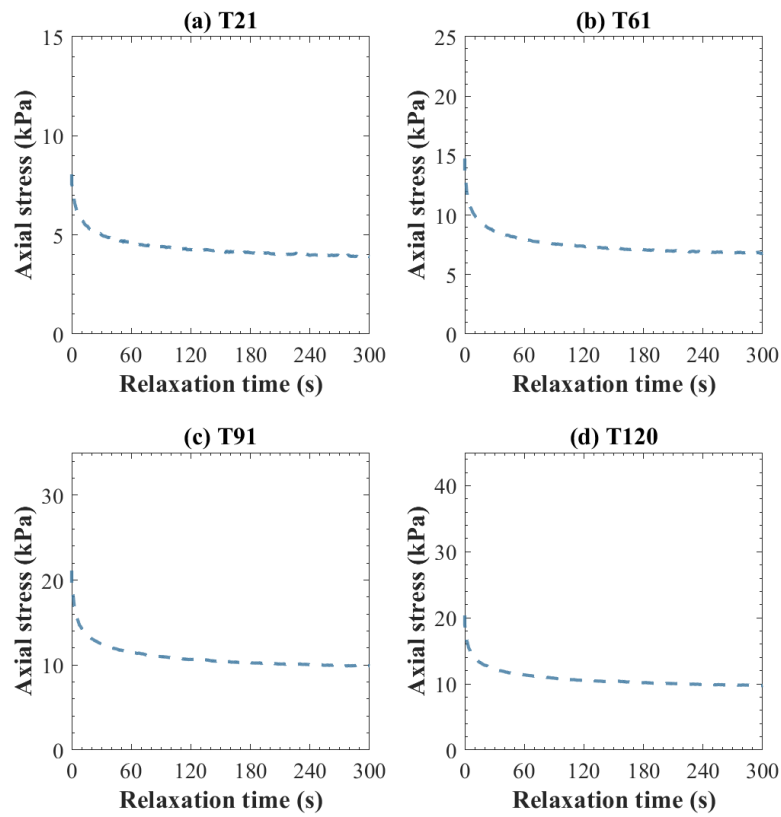


Figure 6.6 Stress-time curves - Group-1 relaxation tests.

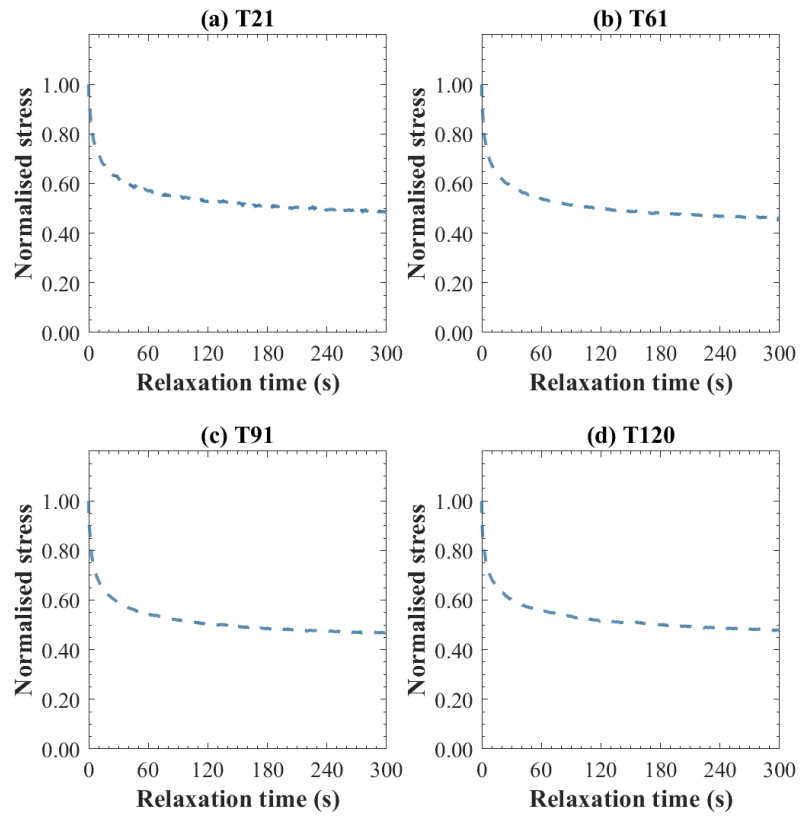


Figure 6.7 Normalised stress-time curves - Group-1 relaxation tests.

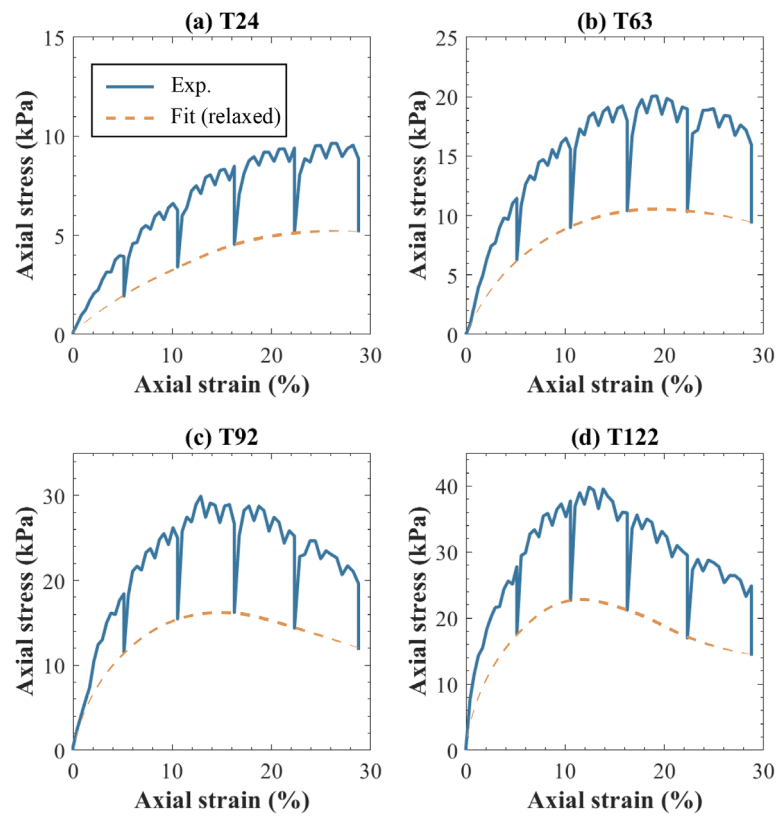


Figure 6.8 Stress-strain curves - Group-2 relaxation tests.

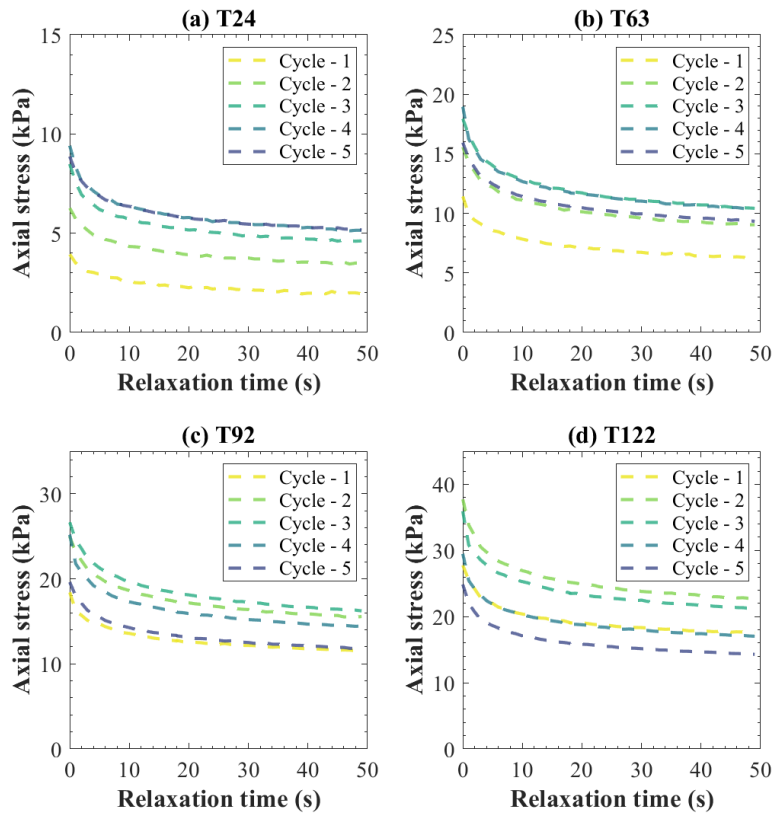


Figure 6.9 Stress-time curves - Group-2 relaxation tests.

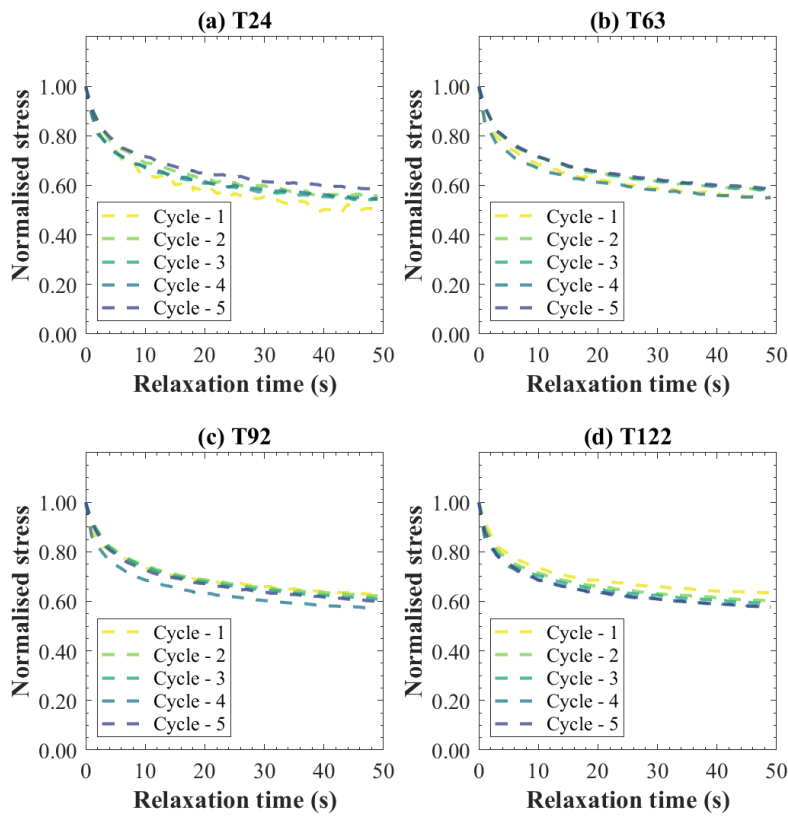


Figure 6.10 Normalised stress-time curves - Group-2 relaxation tests

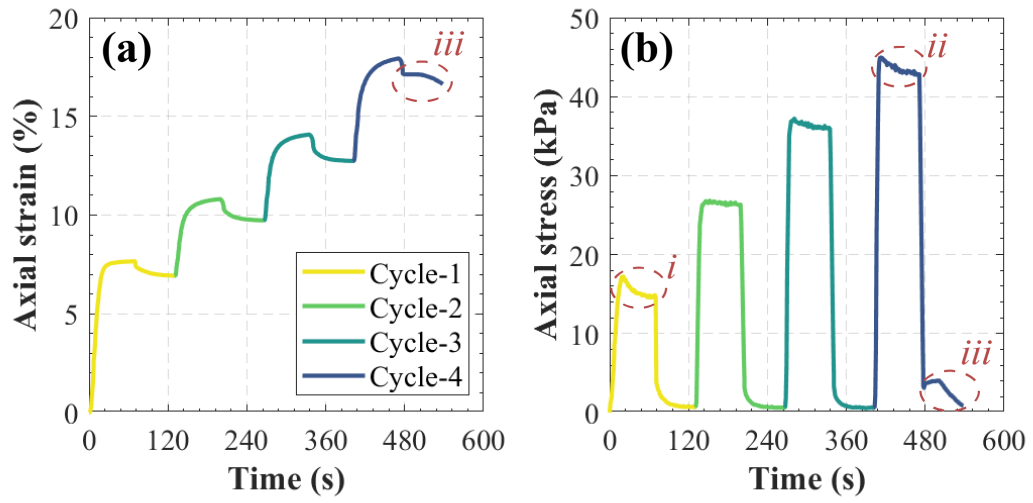


Figure 6.11 Results of multi-step creep-recovery test: (a) Creep strain curve; (b) Change in creep stress.

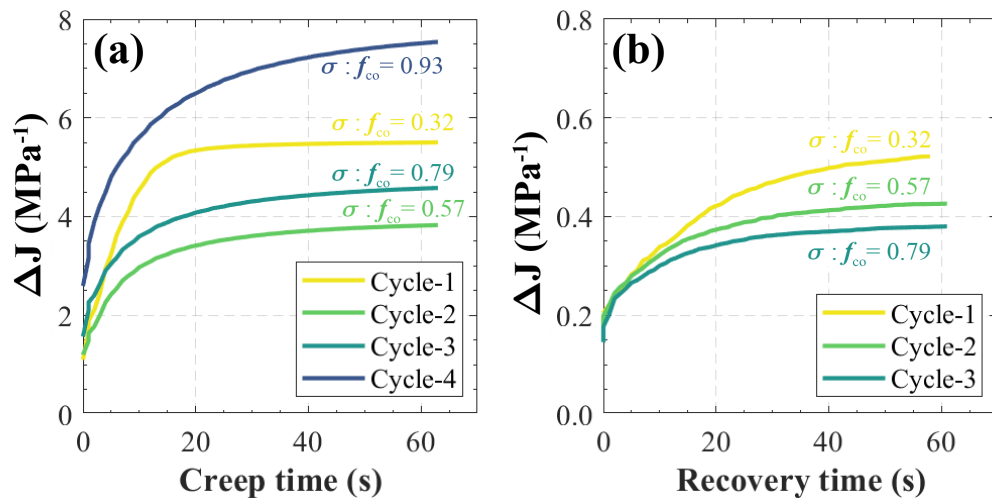


Figure 6.12 Incremental creep compliance curves at different stress levels: (a) Calculated based on data in the creep period; (b) Calculated based on data in the recovery period.

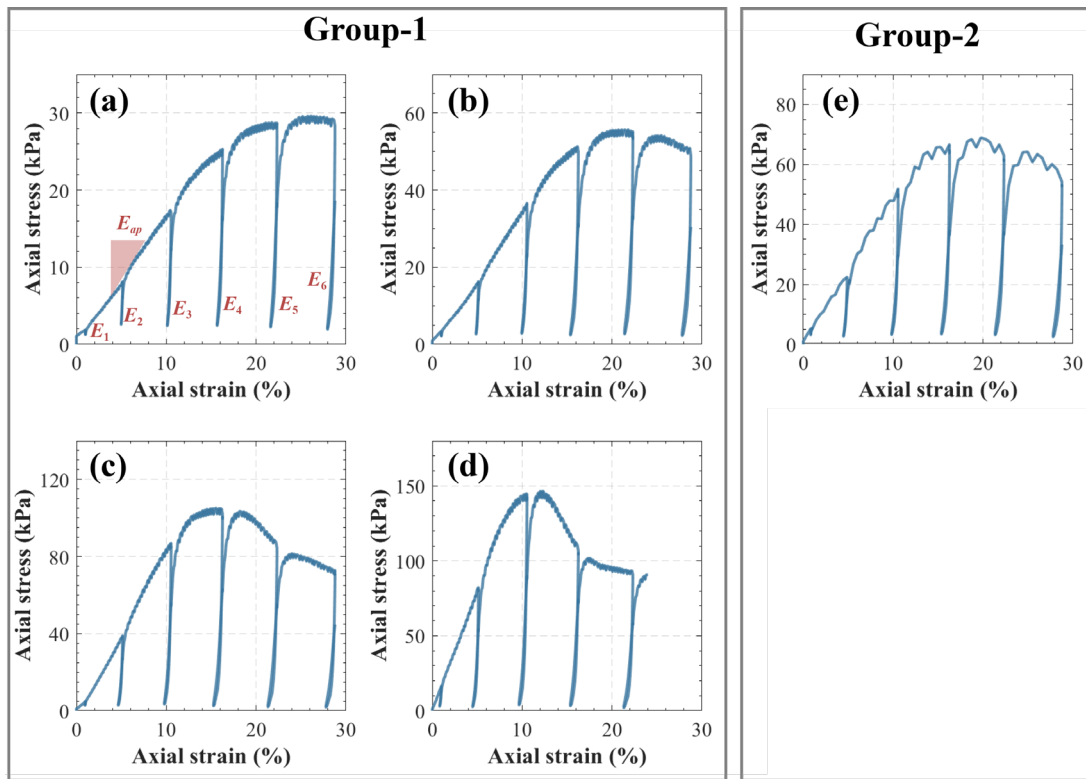


Figure 6.13 Stress strain curves in loading-unloading-reloading tests: (a) T22, (b) T48; (c) T94; (d) T114; (e) T63.

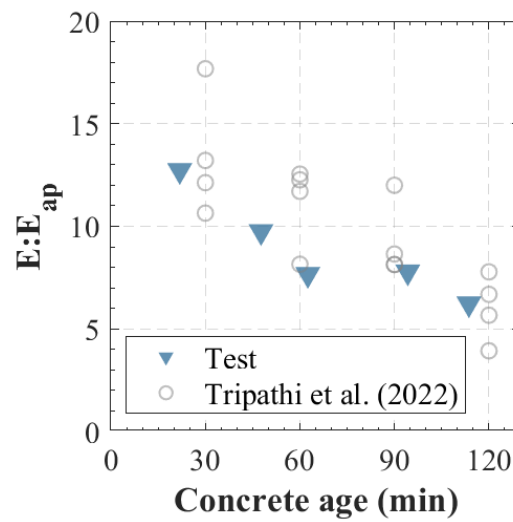


Figure 6.14 Ratios of actual Young's modulus (E) to apparent Young's modulus (E_{ap}) at different ages.

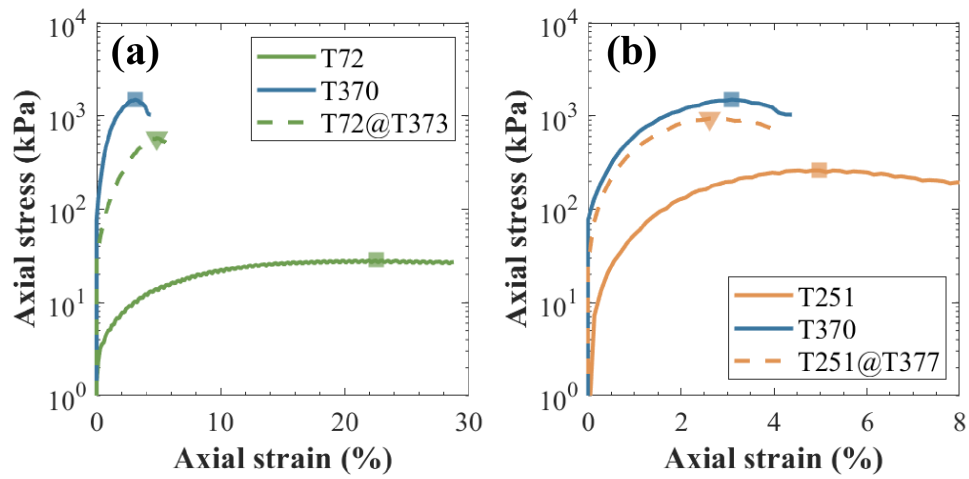


Figure 6.15 Stress-strain curves of virgin and reloaded specimens (Group-1).

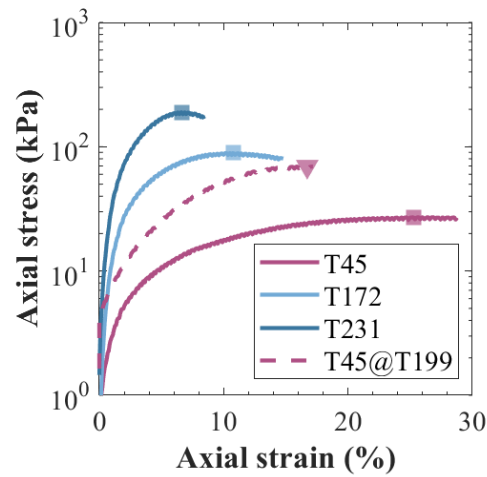


Figure 6.16 Stress-strain curves of virgin and reloaded specimens (Group-2).

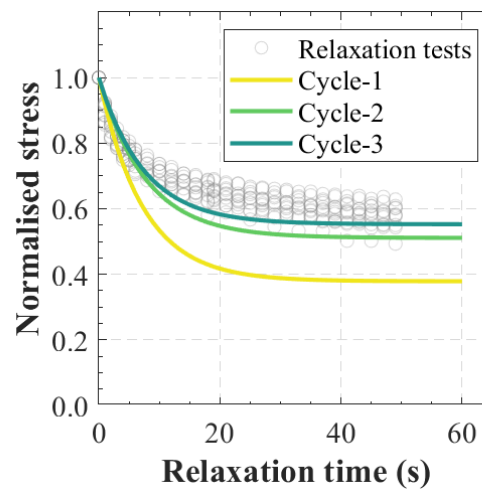


Figure 6.17 The normalised stress-relaxation time relationship derived from the creep compliance data in the strain recovery stages of the creep-recovery test.

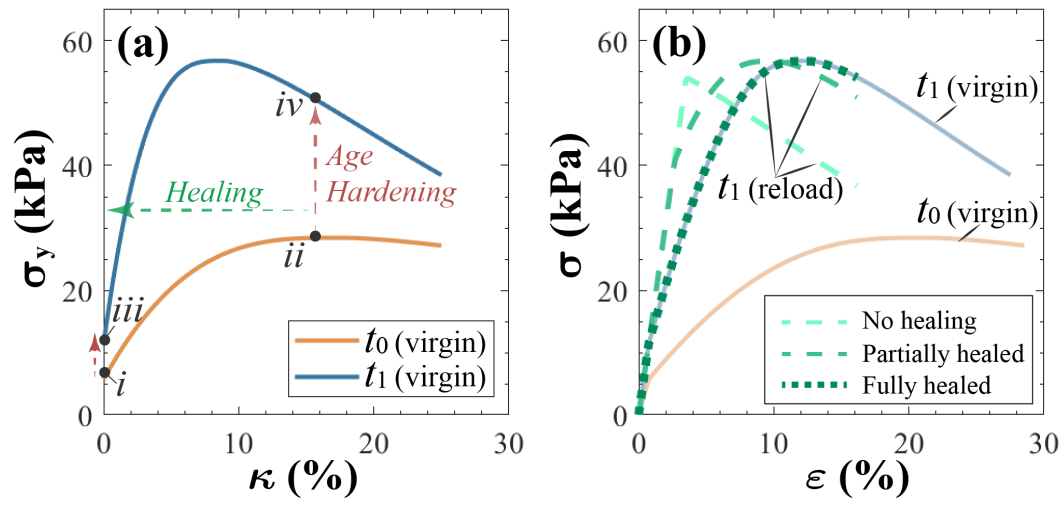


Figure 6.18 Interaction between age hardening and material healing¹: (a) Yield stress vs equivalent plastic strain curves; (b) Stress-strain curves.

¹ Concrete age $t_1 > t_0$.

APPENDIX 6.1 DETAILED RESULTS OF THE MATERIAL HEALING TESTS

The uniaxial compressive strength (f_{co}), apparent Young's modulus (E_{ap}), and peak compressive strain (ϵ_{co}) for virgin and reloaded specimens in the Group-1 and Group-2 material healing tests are summarised in Figure A.6.1 and Figure A.6.2, respectively. To determine these three material parameters for the virgin specimens at the same ages as the reloaded specimens, a relationship between each material parameter and concrete age was established by curve fitting the data from the virgin specimens at various ages. These relationships are depicted as grey solid lines in each subplot.

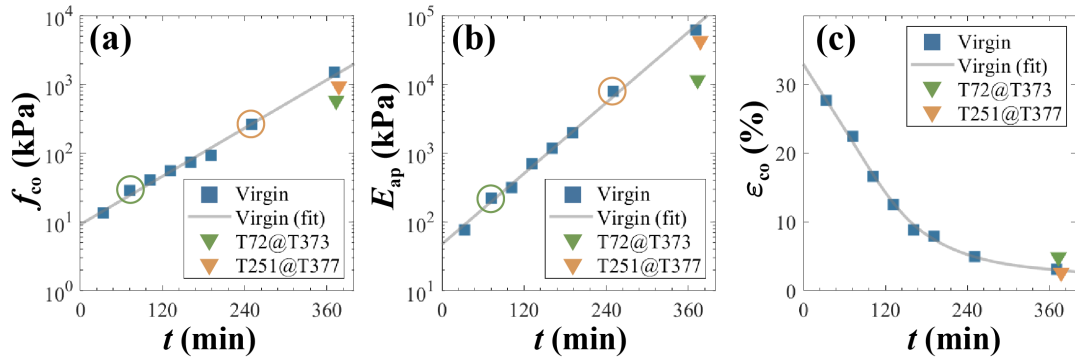


Figure A.6.1 Results for specimens in Group-1 material healing test: (a) Evolution of uniaxial compressive strength; (b) Evolution of apparent Young's modulus; (c) Evolution of peak compressive strain.

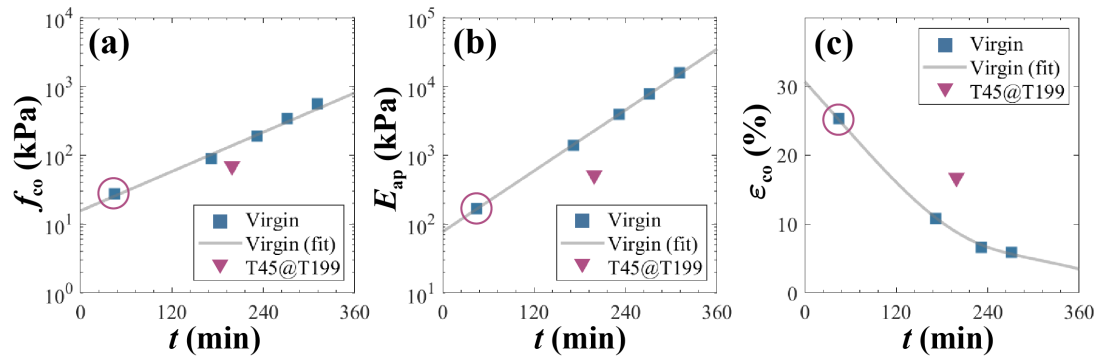


Figure A.6.2 Results for specimens in Group-2 material healing test: (a) Evolution of uniaxial compressive strength; (b) Evolution of apparent Young's modulus; (c) Evolution of peak compressive strain.

APPENDIX 6.2 METHODOLOGY FOR TRANSFORMING CREEP COMPLIANCE INTO RELAXATION MODULUS

Based on the theory of viscoelasticity, the following relationships hold between the creep compliance function and the relaxation function (Park and Schapery, 1999).

$$\int_0^t E(t-t') \frac{dJ(t')}{dt'} dt' = 1 \quad \forall t > 0 \quad (6.1)$$

Remind that the creep compliance function, $J(t)$, represents the change in deformation with time under a unit constant stress, whereas the relaxation function, $E(t)$, represents the change in stress with time under a unit constant strain.

To evaluate the integral in Eq. (6.1) analytically, $J(t)$ needs to be a continuous and differentiable function. However, experimental measurements are discrete. In light of this, it is assumed that the creep compliance function follows a simple exponential law.

$$J(t) = \alpha_J e^{b_J t} + c_J \quad (6.2)$$

where α_J , b_J and c_J are material parameters. These parameters were determined by curve fitting of the creep compliance data. Table A.6.1 summarises the fitted parameters corresponding to the first three strain recovery stages, and Figure A.6.3 reveals that the Eq. (6.2) can provide an excellent fit to the data.

From the results of the relaxation tests, as illustrated in Figures 6.6 and 6.9, the stress-time relationship approximately follow an exponential trend. To this end, it is assumed that the relaxation modulus follows the exponential function below.

$$E(t) = a_E e^{b_E t} + c_E \quad (6.3)$$

To obtain the parameters (a_E, b_E, c_E) in this equation, Eq. (6.1) must be used. In this study, Eq. (6.1) was enforced at nine sample time points ($t_{smp}=0.1, 1, 1.5, 10, 20, 30, 40, 50, 60$ seconds) covering the entire range of the time range of the creep compliance

data, and the parameters (a_E , b_E , c_E) were determined using the optimisation algorithm ‘fmincon’ in MATLAB. The parameters for the relaxation function determined from the creep compliance data at the three recovery stages are summarised in Table A.6.1. After the relaxation function was determined, the normalised stress can be calculated by normalising the function values at different times with its value at time zero.

Table A.6.1 A summary of parameters for the assumed creep compliance and relaxation function.

Parameters	Step-1	Step-2	Step-3
Creep compliance function (unit: MPa ⁻¹)			
a_J	-0.335	-0.210	-0.170
b_J	-0.0526	-0.0663	-0.0744
c_J	0.539	0.429	0.380
Relaxation function (unit: MPa)			
a_E	3.05	2.23	2.13
b_E	-0.139	-0.130	-0.135
c_E	1.86	2.33	2.63

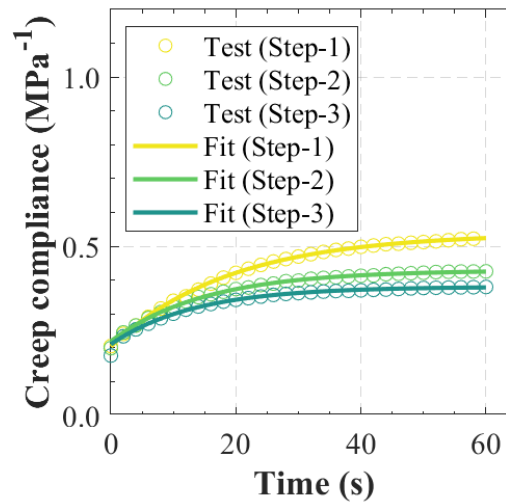


Figure A.6.3 Comparison between the creep compliance function obtained from the strain recovery data and the fitted creep compliance curves.

APPENDIX 6.3 NUMERICAL SIMULATION: UNIAXIAL COMPRESSION OF FRESH CONCRETE WITH CONSIDERATION OF MATERIAL HEALING

A total of three single-element numerical simulations were performed to understand the interaction between age-hardening and material healing. As discussed, all tests were performed under uniaxial compression in three steps. The material was assumed to be at age t_0 in steps 1 and 2, and at age t_1 in step 3. A one-dimensional incremental elastic-plastic model with additive strain rate decomposition ($\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^p$) was adopted.

To this end, the stress-strain relation is described by an age-dependent hypoelastic model as, $\dot{\sigma} = E(t)\dot{\epsilon}$. The values of Young's modulus at t_0 and t_1 is summarised in Table A.6.2. Given that the model is for uniaxial condition, the yield criterion is simply $f = |\sigma| - \sigma_y = 0$, where σ_y is the yield stress. Furthermore, the rate of the equivalent plastic strain is simply given as $\dot{\kappa} = \dot{\epsilon}^p$. To demonstrate the effect of material healing, the evolution of κ is augmented by a healing parameter (H), and the evolution law for κ is specified as follows:

$$\kappa = (1 - H) \cdot \int \dot{\kappa} \quad (6.4)$$

H is set to 0 for no healing, 0.8 for partial healing and 1 for full healing.

The evolution of yield stress is governed by the following function.

$$\sigma_y(\kappa, t) = \begin{cases} f_{co}(t) \cdot h(\kappa, t) & \forall \kappa \leq \kappa_k \\ f_{co}(t) \cdot s(\kappa, t) & \forall \kappa > \kappa_k \end{cases} \quad (6.5)$$

Noteworthy, the age-hardening effect is accounted for by the age-dependent strength parameter (f_{co}), whereas the strain hardening-softening effect is considered through the hardening (h) and softening (s) functions whose mathematical forms were provided in the previous chapter. The material parameters related to the hardening function are summarised in Table A.6.2.

The complete stress-strain curves for specimens in the numerical tests are presented in Figure A.6.4. Each curve is composed of three segments: a preloaded segment, an unloaded segment, and a reloaded segment, corresponding to the three steps in each numerical test. The stress-strain responses of the reloaded specimens, as shown in Figures 6.18, were calculated by shifting the third segment of each curve to the origin.

Table A.6.2 A summary of material parameters for fresh concrete at age t_0 and t_1 .

Material parameters	Concrete age	
	t_0 (virgin)	t_1 (virgin)
E (kPa)	768.2	1536.4
f_{co} (kPa)	28.4	56.8
h_0	0.2	0.2
E_0	2.5	2.5
κ_k	0.18	0.09
Λ	4	4

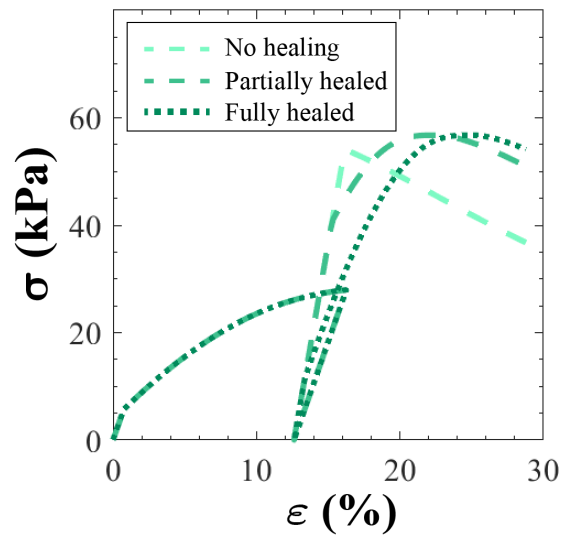


Figure A.6.4 Complete stress-strain curves of the numerical tests.

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CHAPTER 7

NUMERICAL SIMULATIONS OF FRESH CONCRETE STRUCTURES DURING THE CONSTRUCTION STAGE

7.1 INTRODUCTION

The previous chapters have established a comprehensive understanding of the mechanical behaviour of fresh 3D-printed concrete (3DPC) at the material level. This chapter explores the numerical simulations of fresh concrete structures, leveraging the updated knowledge on the constitutive behaviour of 3DPC established.

Research has identified two failure mechanisms in 3D-printed structures: stability-based and strength-based failures (Suiker *et al.*, 2020). Given the predominance of thin-walled elements in 3D-printed structures, stability failure has emerged as the dominant mode in experimental investigations (Suiker, 2018; Wolfs *et al.*, 2018, 2019; Zhu *et al.*, 2021). Consequently, this chapter focuses on structures that exhibit instability failure.

Whilst existing literature demonstrates various approaches to simulate fresh concrete structures, significant challenges persist in achieving reliable predictions. Most studies have employed elastoplastic models whilst neglecting creep effects (Wolfs *et al.*, 2019; Jayathilakage *et al.*, 2020; Vantighem and Ooms, 2020; Ooms *et al.*, 2021; Nguyen-Van *et al.*, 2022; An *et al.*, 2024), with only a few incorporating creep behaviour (Nedjar, 2021; Chang *et al.*, 2023; Wang *et al.*, 2023). For both types of models, notable discrepancies between numerical predictions and experimental results remain. For instance, whilst the finite element (FE) model proposed by Wolfs *et al.* (2019) accurately predicted the failure of their 1-metre free-standing wall, it underestimated the number of layers at failure (NLF) by more than 40% for a larger rectangular wall using identical 3DPC mixture. Furthermore, multiple studies (Wolfs *et al.*, 2018; Ooms *et al.*, 2021; Liu *et al.*, 2022) overestimated the NLF of a benchmark cylindrical wall

by more than 40%. As critically reviewed in Chapter 2, these discrepancies likely stem from the adoption of incomprehensive constitutive models in their simulations.

In this chapter, two groups of simulations were designed, each with a distinct constitutive model. The first group utilises the elastoplastic model proposed in Chapter 5. The primary objective of these simulations is to verify the performance of this constitutive model in structural analysis. Results demonstrate that these FE models significantly outperform previous simulation works that used other strain rate-independent material models. The second group investigates the role of material creep through an advanced elastoviscoplastic model, which extends the proposed elastoplastic model. This elastoviscoplastic is capable of capturing nonlinear creep and strain-rate-dependent plasticity behaviours - phenomena not addressed by current linear viscoelastic models (Nedjar, 2021; Chang *et al.*, 2023; Wang *et al.*, 2023). The results reveal significant impacts of material creep on structural stability and elucidate the complex interaction between creep and material ageing. This investigation makes a significant contribution by bridging the gap between material-level behaviour and structural performance, whilst highlighting critical mechanisms that influence structural response during 3D concrete printing.

7.2 DESCRIPTION OF THE FINITE ELEMENT MODEL

This study utilises seven 3D-printed concrete structures as benchmarks for finite element (FE) simulations. Table 7.1 delineates the geometrical characteristics of these structures, comprising three free-standing walls, two rectangular walls, and two cylindrical walls. Each structure is assigned a unique identifier that encodes key information: an acronym representing the researcher [e.g., Sk18 for Suiker (2018)], followed by dimension(s) in metres. For free-standing walls, the number indicates length; for rectangular walls, 'LS×SS' represents long and short side dimensions; for cylindrical walls, 'Cyn' precedes the centreline diameter. The FE simulations were performed using the commercial FE software Abaqus (Dassault Systemes, 2023).

In experiment, the printed structures experience non-uniform gravitational loads. Figure 7.1a illustrates this phenomenon: as concrete filaments are extruded along a pre-

defined path, layers stack progressively, resulting in height disparities ($H_1 > H_2$). Consequently, Segment 1 of the bottom filament bears a greater gravitational load than Segment 2, exemplifying the non-uniform load distribution inherent in the printing process.

Following the approach in Wolfs *et al.* (2018), this research simplifies the construction process by assuming direct layer stacking without explicit modelling of each layer's extrusion in FE simulation¹ (see Figure 7.1b). This approach results in a uniform gravitational load distribution. Results of previous numerical study by Ooms *et al.* (2021) justify this simplification, demonstrating that models with and without non-uniform gravitational loads yield similar numbers of layers at failure, provided geometric imperfections are considered.

7.2.1 Geometry and Boundary Conditions

To optimise computational efficiency, symmetry was leveraged in the simulations. For free-standing walls, only half of the structure was modelled, whilst for rectangular and cylindrical walls, a quarter of the structure sufficed. Symmetric boundary conditions were applied to the corresponding planes of symmetry.

While the fresh concrete is deformable, the print bed for each structure was modelled as a rigid shell with all degrees of freedom fixed. The interaction between the print bed and the first printed layer was simulated using a 'hard contact' model in the normal direction and a friction penalty model in the tangential direction, with a friction coefficient of 0.6.

To trigger the potential instability of structures, a geometric imperfection was introduced to each fresh concrete structure. The imperfection profile was based on the

¹ The impact of gravitational load non-uniformity during printing may be negligible in many cases. For instance, in a free-standing wall structure printed to 21 layers (Figure 7.1a), the stress difference between Segment 1 and Segment 2 is approximately 5%. This difference diminishes further with increased wall height. Given that the benchmark structures typically fail beyond 20 layers (Table 7.2), neglecting this non-uniformity in simulations is deemed reasonable.

first buckling mode obtained from a linear buckling analysis. Following the approach of Wolfs *et al.* (2019), the maximum amplitude of the imperfection was set to 1/1000 of the filament's width.

7.2.2 Layer-by-Layer Printing Process

To model the layer-by-layer 3D concrete printing process, an element birth and death approach, called 'model change', was implemented. This method involves grouping elements into sets corresponding to individual concrete layers. Initially, all element sets are deactivated. During simulation, each set is progressively activated to replicate the construction process, effectively mimicking the layer-by-layer nature of 3D printing. Furthermore, the concrete age of each layer is tracked from the moment of activation. Thus, although the material properties of each layer are isotropic, the properties of the overall wall are anisotropic due to differential ageing.

Early simulation studies (Wolfs *et al.*, 2018, 2019) assumed perfect bonding between filaments using 'tie constraint'. However, this approach has significant limitations. The primary issue stems from the activation of new layers based on undeformed coordinates, which creates gaps with the already deformed structure. When perfect bonding is assumed, this leads to unrealistic element distortion as shared nodes are pulled towards the deformed structure (Ooms *et al.*, 2021; Liu *et al.*, 2022; Wang *et al.*, 2023). Noteworthy, this fictitious deformation can also alter the plastic strain of the 3DPC, whose mechanical behaviour is deformation-history dependent, thus compromising the accuracy of the simulation.

To address this issue, surface-to-surface interaction has been recommended to model the interfacial behaviour between layers (Ooms *et al.*, 2021), an approach adopted in the current investigation. Specifically, the 'hard contact' model and the 'friction penalty' model with a coefficient of 0.6 are used to define the contact behaviour in the normal and tangential directions, respectively. By employing this approach, a layer is activated according to the undeformed coordinates and falls onto the previously printed layer due to gravitation without causing element distortion (see Figure 7.1b as an illustrative example).

7.2.3 Element Selection and Mesh Convergence

The stability of thin-walled structures is primarily governed by their out-of-plane bending rigidity (Timoshenko and Gere, 1961), underscoring the importance of accurately representing this stiffness in FE simulations. The choice of element formulation significantly impacts the simulated bending stiffness of a structure. First-order solid elements with full integration (C3D8) have been widely employed in previous simulation works (Ooms *et al.*, 2021; Wang *et al.*, 2023; An *et al.*, 2024). However, these elements typically overestimate bending stiffness due to the shear locking phenomenon that occurs in bending modes (Cook, 2002). Other research adopted reduced integrated first-order elements (C3D8R) for computational efficiency (Nguyen-Van *et al.*, 2022; Chen *et al.*, 2024). Compared to C3D8, C3D8R mitigate the limitation of shear locking; however, it can still produce unreliable results due to hourglass modes in bending, even when regularisation approaches are applied (Dassault Systemes, 2017). The limitations of both types of elements can be alleviated by increasing the element density in the wall thickness direction. However, maintaining element aspect ratios necessitates a corresponding increase in element density along other directions, significantly elevating computational costs. This effect is particularly pronounced for structures with high length-to-thickness (free-standing and rectangular walls) or diameter-to-thickness (cylindrical walls) ratios.

To balance accuracy in modelling bending rigidity with computational efficiency, this research employs 20-node quadratic solid elements with reduced integration (C3D20R) (Dassault Systemes, 2023). Mesh convergence studies revealed an optimal configuration: one element in the filament thickness direction, two elements in the wall thickness direction (filament width direction), and an element side length ratio of 1:1 on the plane along the printing direction.

7.2.4 Nonlinear Analysis and Solution Strategy

To accurately capture the mechanical response of the printed structures, both geometric and material nonlinearities were incorporated into the analysis. Geometric nonlinearity was considered by activating the ‘Nlgeom’ option in Abaqus, whilst material

nonlinearity, representing the nonlinear stress-strain behaviour, was addressed using two user-defined material subroutines (UMAT). These subroutines were developed based on the constitutive models described in Section 7.3.

To solve the resulting highly nonlinear system of equations, the ‘Dynamic Implicit’ solver in Abaqus was employed. Previous research (Chen *et al.*, 2015) has demonstrated that this solver offers higher efficiency and can overcome potential convergence issues when dealing with highly nonlinear systems, compared to the ‘static’ solver.

7.3 MATERIAL CONSTITUTIVE MODELS

The current studies pursue two primary objectives: firstly, to demonstrate the efficacy of the proposed elastoplastic model (in Chapter 5) in simulating fresh concrete structures during construction, and secondly, to examine the influence of creep on structural instability. To achieve these aims, two constitutive models are employed: (1) an elastoplastic model, as proposed in Chapter 5, and (2) an elastoviscoplastic model, wherein the elastic component remains rate-independent whilst the plastic component exhibits rate-dependency. This elastoviscoplastic model represents an extension of the elastoplastic model, which is detailed in Section 7.3.2. Additionally, for ease of calibration, a minor modification to the elastoplastic model is delineated in Section 7.3.1.

7.3.1 Elastoplastic Model

In the original formulation of the constitutive model, the equivalent plastic strain rate ($\dot{\kappa}$) was defined as the Euclidean norm of the plastic strain rate tensor ($\dot{\epsilon}^p$), as expressed in Eq. (5.17). Taking a specimen under uniaxial compression as an example, the calculation of $\dot{\kappa}$ necessitates accounting for both the axial plastic strain rate ($\dot{\epsilon}_1^p$) and the lateral plastic strain rate ($\dot{\epsilon}_2^p$ or $\dot{\epsilon}_3^p$). This approach was feasible in the experimental investigations in this thesis, where both axial and lateral strains were recorded. However, the published results of uniaxial compression tests on the 3DPC used for printing the benchmark structures give no information on lateral deformation, rendering the original

method for calculating $\dot{\kappa}$ inapplicable. This limitation poses challenges in calibrating the constitutive model for these 3DPCs using their available test results.

To address this issue, a scaling factor C was introduced:

$$\dot{\kappa} = C \|\dot{\epsilon}^p\|; \quad C = \sqrt{\frac{(\sqrt{2} - \alpha)^2}{3(\alpha^2 + 1)}} \quad (7.1)$$

Remind that α , given by Eq. (5.22), is the volumetric dilation parameter. This modification ensures that $\dot{\kappa}$ is consistent with the axial plastic strain - $\dot{\epsilon}_1^p$ - in the case of uniaxial compression. Consequently, this adaptation allows for direct calibration of the strain hardening function using the experimentally obtained axial stress-axial strain curves from uniaxial compression tests, without the need for lateral strain data. A detailed derivation of the scaling factor is presented in Appendix 7.1.

7.3.2 Elastoviscoplastic Model

To account for the viscous effect, the plastic component of the elastoplastic model is augmented by the overstress theory (Perzyna, 1963), resulting in an elastoviscoplastic model. This approach is predicated on the fundamental assumption that creep deformation is non-recoverable. The validity of this assumption is substantiated by the experimental observations presented in Section 6.4.4, which demonstrate that the deformation of 3DPC is predominantly inelastic in nature.

In this model, the total strain rate tensor is decomposed into two components: an elastic strain rate tensor and a viscoplastic strain rate tensor, expressed as $\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^{vp}$. Consistent with the elastoplastic model discussed earlier, the elastic stress-strain response is modelled using the hypoelastic formulation, as previously demonstrated in Eq. (5.8).

The yield criterion of the elastoviscoplastic model maintains the same form as that of the elastoplastic model. However, to distinguish it from the rate-independent model,

this yield criterion is referred to as the static yield criterion (f_s), with its corresponding yield surface termed the static yield surface. This distinction is crucial in understanding the fundamental differences between rate-independent and rate-dependent plasticity theories.

In the elastoplastic model, as demonstrated in Figure 7.2a, a stress point can only lie within (point P_1) or on (point P_2) the yield surface; such that $f \leq 0$. The elastoviscoplastic model, however, extends this concept by allowing a stress point P to exist in three distinct regions relative to the static yield surface (as shown in Figure 7.2b):

- (1) Inside the static yield surface ($f_s < 0$): The material remains elastic, consistent with rate-independent theory.
- (2) On the static yield surface ($f_s = 0$): The material yields without viscous effects, termed ‘static yield’.
- (3) Outside the static yield surface ($f_s > 0$): The material undergoes ‘dynamic yield’, where viscous effects occur.

This extension, that is the allowance for stress points to lie outside the static yield surface, enables the model to account for rate-dependent behaviour, including creep, which is crucial in modelling the mechanics of 3D-printed structures. Appendix 7.2.1 presents a detailed discussion on how this extension in the yield criterion accounts for creep behaviour.

While the formulation of the static yield criterion - given by Eq. (7.2) - mirrors that of the yield criterion in the rate-independent model, it embodies significant conceptual distinctions:

$$f_s(\boldsymbol{\sigma}, \kappa, t) = \rho - \chi(t)\sqrt{2}\xi - k_s(\kappa, \boldsymbol{\sigma}, t) \quad (7.2)$$

Here, k_s represents the static hardening parameter in viscoplasticity, a concept different from the hardening parameter (k) in rate-independent plasticity. Specifically, k_s is defined as:

$$k_s(\kappa, \sigma, t) = \frac{\sqrt{6}}{3} [1 - \chi(t)] \cdot f_{co.s}(t) \cdot q(\kappa, \sigma, t) \quad (7.3)$$

where $f_{co.s}$ is the static uniaxial compressive strength, representing the material strength at zero strain rate. The strain hardening function q retains the same form as in the elastoplastic model, accounting for the impact of deformation history on yield stress. The evolution law for κ remains consistent with that described in Eq. (7.1).

It is important to note that k_s is a theoretical value, which can only be evaluated through material test at zero strain rate that is not possible in reality. Despite varying stress-strain responses at different strain rates, k_s remains invariant. The model accounts for rate-dependent stress by allowing stress points to exist outside the static yield surface defined by k_s . This approach contrasts with the classical yield criterion in elastoplasticity, where the hardening parameter k is derived directly from the stress-strain curve at a prescribed strain rate.

The flow rule of the elastoviscoplastic model is expressed as:

$$\dot{\epsilon}^{vp} = \dot{\lambda} \frac{\partial g}{\partial \sigma} ; \forall \dot{\lambda} = \Phi \quad (7.4)$$

In this formulation, the flow potential function (g) remains consistent with that of the elastoplastic model. However, a crucial distinction lies in the determination of the plastic multiplier ($\dot{\lambda}$). Unlike in rate-independent theory, where $\dot{\lambda}$ is derived from the consistency condition, in the elastoviscoplastic model, it is explicitly defined by the overstress function (Φ).

The overstress function is a critical component that governs the strain rate-dependence of the material. It is defined by the following piecewise function:

$$\Phi(f_s) = \begin{cases} 0 & \forall f_s < 0 \\ \gamma \left(\frac{f_s}{\sigma_{y.\max}} \right)^{N_0} & \forall f_s \geq 0 \end{cases} \quad (7.5)$$

This formulation encapsulates two key behaviours:

- (1) when $f_s < 0$ the material is within the elastic regime, resulting in no plastic flow ($\Phi = 0$).
- (2) When $f_s \geq 0$, the material enters the plastic regime, and the overstress function adopts a power law form, widely employed in existing studies (Chaboche and Rousselier, 1983; Heeres *et al.*, 2002).

A notable modification in this study pertains to the denominator of the power law term. Conventionally, the initial yield stress was used. However, given the very small yield stress of fresh concrete, this approach can lead to numerical instability. To address this issue, the present study employs the ultimate yield stress $\sigma_{y.max}$ in the denominator, defined as:

$$\sigma_{y.max} = \frac{\sqrt{6}}{3}(1 - \chi)f_{co.s} \quad (7.6)$$

Compared to the elastoplastic model described in Section 7.3.1, the elastoviscoplastic model introduces two additional parameters: γ and N_0 . These parameters are crucial in accounting for the material's viscous behaviour, particularly in creep modelling, where they control the creep strain rate and therefore the curvature of the creep curve (a detailed discussion is presented in Appendix 7.2.2). For accurate representation of a material's creep response, these parameters should be calibrated using creep test results.

It is important to note that while the overstress function is assumed to be independent of concrete age, the creep behaviour predicted by the proposed model is age-dependent, as detailed in Appendix 7.2.3. Further investigation in future research is recommended to verify the validity of this assumption.

The implementation of the proposed elastoviscoplastic model into Abaqus was achieved through another User-defined Material subroutine (UMAT). This implementation employs a stress return algorithm similar to that used in the elastoplastic model (as detailed in Appendix 5.2) for solving the constitutive equations.

7.4 CALIBRATION AND VERIFICATION

7.4.1 Elastoplastic Model

7.4.1.1 Calibration of material parameters

In Chapter 5, the material parameters of the elastoplastic model were classified into four groups: *elastic stiffness parameters*, *yield criterion's parameters*, *strain hardening function's parameters*, and *flow rule's parameters*. These parameters have been calibrated for the 3DPC developed in this thesis. However, the 3DPCs used for the benchmark structures possess a different material composition from that utilised in the BC tests in this thesis. This discrepancy necessitates the recalibration of the material parameters using results from tests performed on the actual 3DPC, thereby ensuring that the proposed constitutive model accurately represents its mechanical behaviour.

As mentioned previously, the material tests on these 3DPCs have solely focused on the axial stress-axial strain behaviour under uniaxial compression with a monotonic loading procedure. This poses two challenges to the recalibration process. Firstly, due to the monotonic loading test setup, these material tests can only provide the apparent Young's modulus (E_{ap}) for the fresh concrete, leaving the actual Young's modulus (E), which is required by the proposed elastoplastic model, unknown. Secondly, some material parameters require both uniaxial and biaxial compression test data, the latter of which is unavailable for these 3DPCs. To address these challenges, this research proposes a calibration method, detailed in Appendix 7.3, to estimate the material parameters that are not obtainable from the monotonic uniaxial compression tests.

Furthermore, as shown in Table 7.2, the total printing time for the benchmark structures ranges from 3.4 to 76.5 minutes, suggesting that the behaviour of 3DPC within this age range is most pertinent for structural analysis of these structures. Therefore, the material model was specifically calibrated using the material test data within this age range.

Elastic stiffness parameters

The actual Young's modulus (E) was estimated from the apparent Young's modulus by using the following relationship: $E = YE_{ap}$, where Y is a conversion factor detailed in Appendix 7.3.1. It then remains to establish the relationship between E_{ap} and concrete age. Within the studied age range, the evolution of E_{ap} can be represented by the following linear function:

$$E_{ap} = E_{ai} + \zeta_{E_a} t \quad (7.7)$$

Where the parameter E_{ai} represents the apparent Young's modulus at age zero, and ζ_{E_a} is a curing parameter, both of which were calibrated from the uniaxial compression tests conducted on the 3DPC used for printing.

Yield criterion's parameters

Similarly, the evolution of UC strength within the investigated age range is modelled by a linear function:

$$f_{co} = f_{ci} + \zeta_s t \quad (7.8)$$

where f_{ci} and ζ_s are, respectively, the initial uniaxial compressive strength and the curing rate. Both parameters were determined from the results of uniaxial compression tests in these studies.

Due to the absence of biaxial compressive strength data for the 3DPCs, the friction parameter χ was estimated using the reference age approach in Appendix 7.3.2.

Strain-hardening function's parameters

As shown in Figure 5.11, the equivalent plastic strain at ultimate yield stress (κ_k) in the first two hours of age is minimally affected by the biaxial compressive stress ratio (β_s). Given that the current study is concerned solely with fresh concrete aged less than 80

minutes, it was deemed reasonable to neglect the impact of biaxial compressive stress on the strain hardening behaviour for simplicity. For the 3DPC used in printing the benchmark structures, the evolution of κ_k in the first 80 minutes of aging can be effectively represented by the following equation:

$$\kappa_k(t) = \kappa_{k1} \cdot e^{\zeta_k t} + \kappa_{k2} \quad (7.9)$$

Here, κ_{k1} , κ_{k2} and ζ_k are material coefficients that require calibration from the UC test results.

Furthermore, within the studied age range, it was found that the parameters E_0 and Λ of the hardening function can be set as constants without compromising the accuracy of the model within this specific age range. This simplification aids in reducing the complexity of the calibration process while maintaining the reliability of the model.

Flow potential function's parameters

Due to the lack of lateral deformation data under uniaxial and biaxial compression, both the peak volumetric dilation parameters under UC (α_k^{uc}) as well as the deviatoric dilation parameter e were determined using the reference age approach detailed in Appendix 7.3.2. Furthermore, the function α_k^* , which reflects the impact of biaxial confining stress on volumetric dilation behaviour, was demonstrated to be independent of concrete age (refer to Chapter 5 for details). It is assumed that the calibrated function based on the 3DPC in the BC tests in this thesis is applicable to the fresh concrete used for printing.

Tables 7.3 to 7.5 summarise the calibrated material parameters for three 3DPC mixtures utilised in constructing the benchmark structures. Notably, the same commercial 3DPC (designated as Weber 3D 115-1) was employed for printing in both Suiker (2018) and Wolfs *et al.* (2018); consequently, an identical set of material parameters was applied for these two studies. Figures 7.3a-c demonstrate the axial stress-axial strain curves of

these three 3DPCs, obtained from UC tests¹, juxtaposed with predictions from the calibrated elastoplastic model. The results demonstrate that the calibrated model accurately captures the stress-strain response of these 3DPCs across various concrete ages, thus indicating the efficacy of the proposed model.

7.4.1.2 An approach to account for variability in material

Previous research (Wolfs *et al.*, 2018; Pott *et al.*, 2022), corroborated by the biaxial compression tests in this thesis, has demonstrated significant variability in the mechanical properties of fresh concrete. These studies indicate that a relative standard deviation of approximately 20% can be expected. Figure 7.4, which presents stress-strain curves for fresh concrete at 35 minutes of age from Wolfs *et al.* (2018), clearly illustrates this variability, showing considerable scatter in both strength and material stiffness. This variability can potentially affect the results of structural analysis, necessitating a robust approach to account for it.

In the current investigation, the variability in two key parameters - uniaxial compressive strength (f_{co}) and Young's modulus (E) - was considered. Each fresh concrete structure was subjected to three separate analyses, each employing a distinct set of material parameters: (1) Mean parameters²: The averaged f_{co} and E values obtained from UC tests were utilised. (2) Lower bound parameters: Values of f_{co} and E were set at 20% below the averaged values. (3) Upper bound parameters: Values of f_{co} and E were set at 20% above the averaged values.

Figure 7.4 compares the stress-strain curves produced by these three sets of material parameters to those obtained experimentally. It is evident that the stress-strain curves

¹ The axial stress of each experimental stress-strain curve was scaled by (f_{co}/f_{co}^*) , where f_{co} is the UC strength predicted by Eq. (7.8) and f_{co}^* is the UC strength from the unscaled curve. This scaling reconciles individual stress-strain measurements with the averaged strength response in Eq. (7.8), which was derived from multiple tests at each age. It aligns experimental curves with the overall strength evolution model for accurate verification of the calibrated constitutive model.

² In Zhu *et al.* (2021), only a group of specimens was tested, with one specimen evaluated at each concrete age. Consequently, these results are treated as representative of mean parameters, despite the absence of multiple samples per age to establish statistical variance.

predicted by these three parameter sets encompass the variability of the material's stress-strain response, indicating the effectiveness of this method. By conducting analyses at the mean, lower bound, and upper bound of the material properties, the potential impact of property variations on the structural performance of the printed structures could be systematically evaluated.

7.4.2 Elastoviscoplastic Model

To elucidate the impact of creep on the structural response of 3D-printed structures during the construction phase, a series of numerical simulations were conducted on the cylindrical wall Wf18-Cyn0.5 utilising the elastoviscoplastic (E-VP) model. This necessitated the calibration of material parameters against the fresh concrete employed for printing in the study by Wolfs *et al.* (2018).

7.4.2.1 Calibration of the overstress function

A crucial component of the E-VP model is the overstress function, Eq. (7.5). Calibration of this function typically requires creep test data specific to the 3DPC under investigation. However, as such data were unavailable, the results from our uniaxial compressive creep test, detailed in Chapter 6, were utilised for calibration. This approach rests on the premise that the overstress functions of the two fresh 3DPC materials are comparable.

Within the framework of the E-VP model, creep deformation is attributed solely to viscoplastic strain. Consequently, in the context of a uniaxial compressive creep test, the axial creep strain rate ($\dot{\epsilon}_1$) is equivalent to the axial viscoplastic strain rate ($\dot{\epsilon}_1^{vp}$). In addition, under uniaxial compression, the overstress function Eq. (7.5) can be simplified to Eq. (7.10):

$$\dot{\epsilon}_1^{vp} = \gamma \left(\frac{\sigma_1 - \sigma_{1y.s}(\epsilon_1^{vp})}{f_{co.s}} \right)^{N_0} \quad (7.10)$$

where ϵ_1^{vp} and $\dot{\epsilon}_1^{vp}$ denote the axial viscoplastic strain and its rate, respectively. Furthermore, $\sigma_{1y,s}$ represents the axial static yield stress, and $f_{co,s}$ signifies the static uniaxial compressive strength. Based on this simplified overstress function, the parameters γ and N_0 can be calibrated using the creep test results presented in Section 6.4.3. A detailed explanation of the calibration process is presented in Appendix 7.4.

Figure 7.5 juxtaposes tested creep strain data with that predicted by the calibrated E-VP model¹. The results demonstrate that the proposed model not only accurately predicts strain during creep phases, but also provides reasonably accurate predictions of deformation during strain recovery phases. This suggests that the proposed model can effectively represent both the elastic and plastic deformations of fresh 3DPC simultaneously.

7.4.2.2 Calibrations of other parameters

With the overstress function calibrated, it remains necessary to calibrate the yield criterion, the hardening function, and the flow potential function. In this study, it is assumed that the parameters of the hardening function (q) as well as the flow potential function (g) remains unaffected by strain rate. Consequently, the material parameters related to these two functions are maintained as those employed in the rate-independent model (Table 7.3).

The calibration of the static yield function (f_s) requires knowledge of the static uniaxial compressive strength ($f_{co,s}$), as evident in Eqs. (7.2) and (7.3). However, $f_{co,s}$ is a fictional strength corresponding to the uniaxial compressive strength with zero strain rate, which cannot be directly obtained from uniaxial compressive tests where the strain rate must be greater than zero. Therefore, $f_{co,s}$ was assumed to be a fraction of the uniaxial compressive strength (f_{co}) obtained through UC tests in Wolfs *et al.* (2018), expressed as $f_{co,s} = \varrho f_{co}$. The parameter ϱ was determined through an iterative trial-

¹ This verification was conducted on a single solid element under stress-controlled loading conditions. To ensure accurate representation of variations in creep stress, the axial creep stress recorded in the experimental study was replicated in the FE simulation.

and-error process, adjusting it until the predicted stress-strain curves matched the experimental ones loaded at the same displacement rate. In the current investigation, q was determined to be 0.75.

Table 7.6 summarises the calibrated material parameters of the E-VP model. Figure 7.6 demonstrates that the calibrated E-VP model can accurately predict the stress-strain behaviour of the 3DPC studied by Wolfs *et al.* (2018) when subjected to identical displacement rates.

7.5 RESULTS OF SIMULATIONS

7.5.1 Results of the Elastoplastic Model

7.5.1.1 Performance of the elastoplastic model

The number of layers at failure (NLF) served as a quantifiable metric to evaluate the accuracy of the proposed FE models in predicting the structural behaviour of 3D-printed concrete structures. Table 7.7 presents a comparative analysis of the NLF values observed in experiments against those from FE analysis by the elastoplastic model, utilising lower bound, mean, and upper bound material parameters. For experimental investigations involving multiple walls with identical dimensions and test setups (denoted by the same wall ID), the NLF is presented as a range; otherwise, a singular value is reported for each wall ID. Moreover, a wall is considered to be not failed in simulation if it remains intact with a number of printed layers exceeding 1.5 times the experimental NLF.

Finite element models employing mean material parameters generally yield accurate predictions for the NLF of the benchmark structures. The experimental NLF for the majority of tested walls fall within the range of predictions generated by lower bound and upper bound analyses, indicating the robust performance of the model across various scenarios. However, two notable exceptions exist: the walls Zhu21-Cyn0.25 and Wf18-Cyn0.5.

The wall Zhu21-Cyn0.25 deviates from the prediction using mean parameters, whilst still falling within the range of lower and upper bound analyses. This discrepancy can be attributed to the limited material testing conducted in the study by Zhu *et al.* (2021). Specifically, they performed only a single group of material tests on their 3DPC at various ages; consequently, the derived material parameters may not accurately represent the actual mean material properties. Nevertheless, the inherent variability in material properties is accounted for through the lower and upper bound analyses, explaining why the NLF of Zhu21-Cyn0.25 falls within this broader predictive range.

Conversely, the wall Wf18-Cyn0.5 presents a more significant deviation, falling outside the range defined by both lower and upper bound analyses. As will be discussed in detail in Section 7.5.1.3, this discrepancy may be attributed to the material's creep which will be accounted for in FE model using an elastoviscoplastic model in Section 7.5.2.

7.5.1.2 Further discussions on the walls in Wolfs *et al.* (2019)

The three printed walls presented in Wolfs *et al.* (2019), all constructed using the same 3DPC mixture, merit further examination. Whilst the FE model proposed by Wolfs *et al.* (2019) - a simulation framework subsequently widely adopted in the field - accurately predicted the failure of wall Wf19-1.0, it significantly underpredicted the NLFs of walls Wf19-5.0 and Wf19-5.0×0.2 by 15% and 43%, respectively. In contrast, the current FE model, with a single calibration, yields relatively accurate predictions for all three walls. To elucidate the primary deficiencies of Wolfs *et al.*'s (2019) model and demonstrate the superiority of our approach, it is essential to analyse the failure mechanisms of these structures in detail.

Although all three structures were found to be failed by instability in experiments, the current simulation results reveal markedly different failure mechanisms for each. Figures 7.7a to 7.9a depict the deformed shapes of walls Wf19-1.0, Wf19-5.0, and Wf19-5.0×0.2, respectively, captured two layers prior to failure when buckling has initiated. To elucidate the mechanical response of the material during the construction process, the evolution of equivalent plastic strain (κ) in two representative elements

from the second layer of each structure, which are highlighted in Figures 7.7a to 7.9a, was analysed. Figures 7.7b to 7.9b illustrate the progression of κ throughout the construction process for each of the three walls.

For Wf19-1.0, as illustrated in Figure 7.7b, the κ of both elements initially increases linearly with the addition of layers. However, upon reaching layer 17, further deposition of filament leads to an accelerating increase in κ . Throughout the construction process, the vertical stress increases linearly due to the constant printing speed. Consequently, the accelerating increase in κ can be attributed to the decrease in material stiffness due to strain hardening. In this scenario, the plasticity behaviour of the material governs the structural response, and the failure mechanism is classified as plastic buckling. Notably, although both elements are subjected to identical vertical stress, Element 2 exhibits a higher κ than Element 1. This discrepancy is attributed to the location of Element 1 near the plane of symmetry, where it experiences higher confining stress due to the presence of neighbouring material, in contrast to Element 2, which is situated farther from the plane of symmetry.

The evolution of κ for wall Wf19-5.0 $\times$ 0.2, as depicted in Figure 7.9b, exhibits a markedly different trend. Throughout the construction process, κ increases at a progressively decreasing rate. Moreover, the evolution of κ essentially ceases after layer 20 for Element 1 and layer 30 for Element 2. This phenomenon can be primarily attributed to the age-hardening effect, which increases the yield stress of the printed material over time. The significance of this age-hardening effect depends on both the structural dimensions and printing speed. For smaller structures with high printing speeds, such as Wf19-1.0, the effect may be negligible as the growth rate of stress in the material exceeds that of the yield stress. However, for larger structures requiring extended printing times, like Wf19-5.0 $\times$ 0.2, the effect becomes pronounced as the growth rate of stress in the material is slower than that of the yield stress. Consequently, the material becomes in an elastic state, as evidenced by the plateau of κ in Figure 7.9b, when the structural failure is governed by elasticity rather than plasticity. Given these observations, it can be inferred that the structure in this case fails due to elastic buckling.

The behaviour of wall Wf19-5.0, as illustrated in Figure 7.8b, can be classified as plastic buckling, given that the κ continues to increase (indicating strain hardening) throughout the construction process. However, a closer examination reveals a complex interplay between age-hardening and strain-hardening effects. Notably, the growth rate of κ diminishes after layer 12, despite a constant printing speed. This deceleration in κ can be attributed to the increased rate of yield stress growth, which is a primary manifestation of the age-hardening effect.

The preceding analyses indicate that the accuracy of the FE model in simulating these three structures is contingent upon the robustness of the constitutive model in capturing two key aspects: the elastic response and the evolution of yield stress in the material. The accuracy of the elasticity model governs the former, while the latter is predominantly determined by the precision of the yield criterion, strain hardening law, and material ageing model. As the current elastoplastic model addresses these critical issues, it results in satisfactory simulation outcomes across all three structural configurations.

A comprehensive analysis of the constitutive model presented by Wolfs *et al.* (2019) reveals several critical deficiencies:

- (1) The use of apparent Young's modulus (E_{ap}) in place of the actual Young's modulus (E) resulting in a significant underestimation of the material's elastic modulus. This error directly accounts for the substantial underestimation of the number of layers at failure (NLF) for wall Wf19-5.0×0.2, which failed due to elastic buckling.
- (2) The application of the Mohr-Coulomb yield criterion, which, as demonstrated in Chapter 4, proves inadequate for accurately modelling the biaxial compressive (BC) strength of fresh concrete.
- (3) The adoption of an elastic-perfectly plastic framework, which precludes the incorporation of strain hardening effects. This omission explains the misclassification of failure mechanisms of these three walls as elastic buckling in their study.

7.5.1.3 Further discussions on the wall Wf18-Cyn0.5

The FE models employing an elastoplastic constitutive model failed to accurately predict the failure of wall Wf18-Cyn0.5. Whilst the average number of layers at failure (NLF) in experiments was 29.5, the current FE model with mean material parameters predicted failure at 41 layers, 39% higher than the experimental result. Even with lower bound material parameters, the model predicted failure at layer 33, 12% higher than the average test results. This discrepancy necessitates further investigation into the root cause of this inaccuracy.

Notably, the NLF of this wall has been consistently overestimated in previous numerical studies utilising elastoplastic constitutive models. Wolfs *et al.* (2018), Ooms *et al.* (2021), and Liu *et al.* (2022) predicted that wall failure would occur at 46, 53, and 42 layers, respectively, significantly overestimating the NLF compared to experimental results. Although Liu and Sun (2021) claimed accurate failure prediction for this cylindrical wall, they achieved this by reducing the filament width, which may be methodologically questionable. Comparatively, the current model outperforms these previous models, which can be attributed to two primary factors:

- (1) The incorporation of strain hardening, which was neglected in Wolfs *et al.* (2018) and Ooms *et al.* (2021).
- (2) The formulation of the current constitutive model using true stress-true strain relationships, as required by Abaqus, rather than the true stress-engineering strain relationships adopted in previous studies. Since engineering strain is smaller than true strain under compression, the use of true stress-engineering strain relationships in previous simulation works potentially overestimated the material stiffness.

To elucidate the possible cause of overestimation, Figures 7.10a-c compare the experimentally measured radial displacement profiles of the wall with those predicted by the model using mean parameters at layers 23, 26, and 29, respectively. Similarly, Figures 7.11a-c depict the results using lower bound parameters. Both models significantly underestimated the radial deformation, suggesting an overestimation of

the wall's bending rigidity. This overestimation is hypothesised to stem from the neglect of material creep, which can lead to decreasing material stiffness under constant stress over time. The validity of this hypothesis will be examined in the subsequent section.

7.5.2 Results of the Elastoviscoplastic Model

7.5.2.1 Predictions of wall Wf18-Cyn0.5 considering creep

The implementation of the proposed elastoviscoplastic model in simulations allowed for the consideration of material creep. With this refined approach, the wall Wf18-Cyn0.5 was predicted to fail at 30, 36, and 42 layers for FE models using lower bound, mean, and upper bound material parameters, respectively. These predictions demonstrate improved agreement with experimental results, particularly for the model using lower bound parameters, which predicted failure at 30 layers, falling within the experimental range of 25 to 31 layers.

Figures 7.10 and 7.11 depict the radial displacement predicted by FE models incorporating creep, using mean and lower bound material parameters, respectively. Compared to the FE model without creep, these models predict greater radial displacement in the middle height of the wall, resulting in radial deformation profiles that more closely align with experimental observations. Notably, the model employing lower bound parameters, which accurately predicted the Number of Layers at Failure (NLF), also provides a reasonably good estimation of the wall's radial displacement.

An intriguing observation is that while the FE models considering creep offer improved estimations of radial displacement, their predictions of total wall height remain similar to those of the model neglecting creep. This finding suggests that creep in fresh concrete does not simply magnify vertical settlement but plays a critical role in amplifying bending deflection due to the reduction in structural bending stiffness over time. Consequently, the creep response of fresh concrete influences not only the settlement of the structures but also significantly impacts their stability.

7.5.2.2 Impact of printing speed

The creep strain of a material is intrinsically linked to load duration. In 3D concrete printing, the load duration of printed filaments is primarily governed by printing speed. Consequently, printing speed can significantly influence material creep and, and therefore structural stability. Furthermore, as 3DCP materials exhibit ageing properties, printing speed also affects material age, thereby altering structural response. This complex interplay between material creep and ageing necessitates careful examination.

To elucidate these intricate interactions, a series of numerical case studies were conducted on the Wf18-Cyn0.5 wall, implementing the elastoviscoplastic model with lower bound parameters. Three printing speeds were investigated:

- (1) Normal speed: 83.3 mm/s (equivalent to experimental conditions)
- (2) Slow speed: 41.7 mm/s (half of normal speed)
- (3) Fast speed: 166.6 mm/s (twice the normal speed)

Two simulation groups were established to isolate different effects. Group-1 maintained a constant material age of 9.4 minutes (the time required to print 30 layers at normal speed) across all printing speeds, isolating the impact of printing speed on structural stability due to creep whilst excluding material ageing effects. Group-2 incorporated material ageing, allowing for an examination of the interplay between creep and material ageing on structural stability.

It is important to note that this study considers only printing-phase stability - that is, the maximum number of layers that can be deposited before structural collapse occurs during the printing process. However, freshly printed concrete structures may also fail after printing due to continued creep under self-weight loading, which falls outside the scope of the present study.

Table 7.8 summarises the results of both groups. In Group-1, where material ageing was excluded, higher printing speeds led to greater numbers of layers at failure (NLF). This phenomenon can be attributed to the reduced load duration associated with faster

printing, which allows less time for creep strains to develop. Consequently, the material exhibits stiffer behaviour during the construction process, rendering the structure less prone to instability. The introduction of material ageing in Group-2 revealed more intricate behaviour. For the wall printed at a slower speed, the NLF surpassed that at normal speed, indicating that material ageing exerted a greater influence on structural stability than creep in this scenario. Conversely, the wall printed at high speed also achieved higher capacity than normal speed, suggesting that the reduction in creep accumulation time outweighed the reduction in material ageing time. This complex relationship underscores the delicate balance between ageing and creep in determining the overall structural stability of 3D-printed concrete elements.

While this study focused on a single wall structure examined at three distinct printing speeds, the results strongly suggest that this intricate interaction between ageing and creep is likely to be present in various fresh concrete structures. This observation highlights the critical need for further comprehensive investigations to fully understand and optimise 3D concrete printing processes across a wider range of geometries and printing parameters.

7.6 CONCLUDING REMARKS

Numerical simulation has emerged as a promising tool for understanding the mechanical response of fresh 3D-printed concrete structures during the construction phase. Previous studies have developed various simulation models for fresh concrete structures. However, the accuracy of these models is limited to specific structural types, and significant discrepancies between experimental and simulation results have been observed for other structures. These deficiencies can be attributed to the inadequate representation of the material constitutive behaviour.

This chapter has presented a series of numerical simulations on a group of benchmark fresh concrete structures, which were observed to fail due to instability in experiments, using two advanced constitutive models: an elastoplastic model and an elastoviscoplastic model. These models have been developed based on the updated

knowledge of the mechanical behaviour of fresh concrete as discussed in the previous chapters.

The simulations using the elastoplastic model were conducted to evaluate the performance of the proposed constitutive formulation in structural analysis. The results indicate that finite element (FE) models incorporating this elastoplastic model provide accurate predictions of the behaviour of fresh concrete structures, outperforming earlier simulation studies that relied on less comprehensive elastoplastic formulations. The primary limitations of previous elastoplastic models, which led to inaccurate simulation results, were also identified. Additionally, the proposed elastoplastic model revealed two distinct instability mechanisms: elastic buckling and plastic buckling, both resulting from the interaction between strain-hardening and age-hardening. These mechanisms had been overlooked in prior research, where both were incorrectly classified as elastic buckling due to the use of inadequate constitutive models.

The influence of creep on structural instability was investigated through simulations using an elastoviscoplastic model, which is an extension of the proposed elastoplastic model. This elastoviscoplastic model demonstrated superiority over previous creep models based on linear viscoelasticity, which are unable to capture nonlinear creep behaviour or the interaction between viscous and plastic responses. The simulation results suggested that material creep has a significant impact on the structural response, affecting both the number of layers printed before failure and the deformation profile. Additionally, the results revealed a complex interaction between material creep and ageing, which can critically influence structural stability.

This research has bridged the gap between the material-level behaviour of fresh concrete and the structural-level response of fresh concrete structures, highlighting the critical role of accurate constitutive models in structural analysis. However, some limitations of the current work should be acknowledged. Firstly, while the focus has been on structural instability, which has been identified as the most common failure mode in existing printing tests, the capacity of the models to simulate strength-based failure needs further exploration when more experimental data becomes available. Secondly, as discussed in Chapter 6, fresh concrete exhibits both material healing and

damage behaviours. The effects of these phenomena on fresh 3D-printed concrete structures have not been examined in this study, yet they may play a significant role in future studies. Additionally, further research is required to better understand the interactions between different material responses, such as creep, ageing, healing, and damage, in fresh concrete. Investigating these behaviours across a wider range of geometries and printing configurations (e.g., varying printing speeds, interruptions during printing) will offer deeper insights into the mechanical behaviour of fresh concrete during printing. Such future research will contribute to the development of more robust and efficient designs, both in terms of the geometry of the printed structures and the optimisation of the printing workflow.

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Table 7.1 Geometrical characteristics of benchmark fresh 3D-printed concrete structures from literature.

Publications	Geometry of structure			Geometry of filament		Wall ID
	Free Standing - <i>Length</i> (m)	Rectangular - <i>Long side</i> × <i>short side</i> (m)	Cylindrical - <i>Diameter</i> (m)	Width (mm)	Thickness (mm)	
Suiker (2018)	0.8			44	9.2	Sk18-0.8
		0.625 × 0.250				Sk18-0.625×0.25
Wolfs <i>et al.</i> (2018)			0.50	40	10	Wf18-Cyn0.5
Wolfs <i>et al.</i> (2019)	1.0			60	9.5	Wf19-1.0
	5.0					Wf19-5.0
		5.0 × 0.20				Wf19-5.0×0.2
Zhu <i>et al.</i> (2021)			0.25	25	12	Zhu21-Cyn0.25

Table 7.2 Printing parameters for the benchmark structures.

Publications	Wall ID	Number of layers at failure	Printing speed (mm/s)	Printing time of each layer (s)	Printing time of each structure (min)
Suiker (2018)	Sk18-0.8	22.0	83.3	9.6	3.5
	Sk18-0.625×0.25	33.0		21.0	11.6
Wolfs <i>et al.</i> (2018)	Wf18-Cyn0.5	29.0	83.3	18.85	9.1
Wolfs <i>et al.</i> (2019)	Wf19-1.0	21.5	104.2	9.6	3.4
	Wf19-5.0	27.0		48.0	21.6
	Wf19-5.0×0.2	46.0		99.9	76.5
Zhu <i>et al.</i> (2021)	Zhu21-Cyn0.25	49.0	34.1	23.0	18.8

Table 7.3 Material parameters of the inviscid elastoplastic model for the fresh concrete used in Suiker (2018) and Wolfs *et al.* (2018).

Equation Number	Parameters	Values
<i>Stiffness parameters</i>		
Eq. (7.7)	E_{ai}	71.78 kPa
	ζ_{Ea}	1.227 kPa/min
<i>Yield criterion</i>		
Eq. (7.8)	f_{ci}	5.25 kPa
	ζ_s	0.147 kPa/min
<i>Hardening/softening function</i>		
Eq. (7.9)	κ_{k1}	4.57×10^{-2}
	κ_{k2}	1.4×10^{-1}
	ζ_κ	-4.27×10^{-2}
	E_0	2.5
	Λ	3.5

Table 7.4 Material parameters of the inviscid elastoplastic model for the fresh concrete used in Wolfs *et al.* (2019)

Equation Number	Parameters	Values
<i>Stiffness parameters</i>		
Eq. (7.7)	E_{ai}	10.32 kPa
	ζ_{Ea}	2.94 kPa/min
<i>Yield criterion</i>		
Eq. (7.8)	f_{ci}	2.32 kPa
	ζ_s	0.285 kPa/min
<i>Hardening/softening function</i>		
Eq. (7.9)	κ_{k1}	2.1×10^{-1}
	κ_{k2}	1.6×10^{-1}
	ζ_κ	-4.49×10^{-2}
	E_0	2.5
	Λ	1.5

Table 7.5 Material parameters of the inviscid elastoplastic model for the fresh concrete used in Zhu *et al.* (2021).

Equation Number	Parameters	Values
<i>Stiffness parameters</i>		
Eq. (7.7)	E_{ai}	94.9 kPa
	ζ_{Ea}	0.93 kPa/min
<i>Yield criterion</i>		
Eq. (7.8)	f_{ci}	8.67 kPa
	ζ_{fc}	0.274 kPa/min
<i>Hardening/softening function</i>		
Eq. (7.9)	κ_{k1}	2.1×10^{-1}
	κ_{k2}	8.48×10^{-2}
	ζ_{κ}	-1.04×10^{-2}
	E_0	2.8
	Λ	3.5

Table 7.6 Material parameters of the elastoviscoplastic model for the fresh concrete used in Wolfs *et al.* (2018).

Equation Number	Parameters	Values
<i>Stiffness parameters</i>		
Eq. (7.7)	E_{ai}	71.78 kPa
	ζ_{Ea}	1.227 kPa/min
<i>Static Yield criterion</i>		
Eq. (7.8)	f_{ci}	5.25 kPa
	ζ_s	0.147 kPa/min
	$q=f_{co,s}:f_{co}$	0.75
<i>Hardening/softening function</i>		
Eq. (7.9)	κ_{k1}	4.57×10^{-2}
	κ_{k2}	1.4×10^{-1}
	ζ_{κ}	-4.27×10^{-2}
	E_0	2.5
	Λ	3.5
<i>Overstress function</i>		
Eq. (7.5)	γ	1.62
	N_0	3.82×10^{-2}

Table 7.7 Comparison of the number of layers at failure for 3D-printed concrete structures: experimental results vs. simulation predictions using the elastoplastic model.

Wall ID	Number of Layers at Failure			
	Experiment	Simulation		
		Lower Bound	Mean	Upper Bound
Sk18-0.8	22	22	23	25
Sk18-0.625×0.25	33	28	35	Not failed ²
Wf18-Cyn0.5	25 ~ 31 ¹	33	41	Not failed ²
Wf19-1.0	21 ~ 22 ¹	18	19	21
Wf19-5.0	27	27	30	37
Wf19-5.0×0.2	46	43	45	48
Zhu21-Cyn0.25	49	47 ³	65	Not failed ²

Note:

- ¹. Multiple walls were tested, the range was presented.
- ². The walls were not failed during simulation even with the number of layers greater than 1.5 times the number in experiments.
- ³. In Zhu *et al.* (2021), only a group of specimens was tested.

Table 7.8 Impact of printing speed on the printing-phase failure of wall Wf18-Cyn0.5 considering material creep: results of model with and without considering material ageing.

Simulation Group ¹	Printing speed (mm/s)		
	41.7 (Slow)	83.3 (Normal)	166.6 (Fast)
Group-1	30	32	55
Group-2	33	30	51

Note:

- ¹. Group-1 represents results without considering ageing, while Group-2 represents results considering ageing.

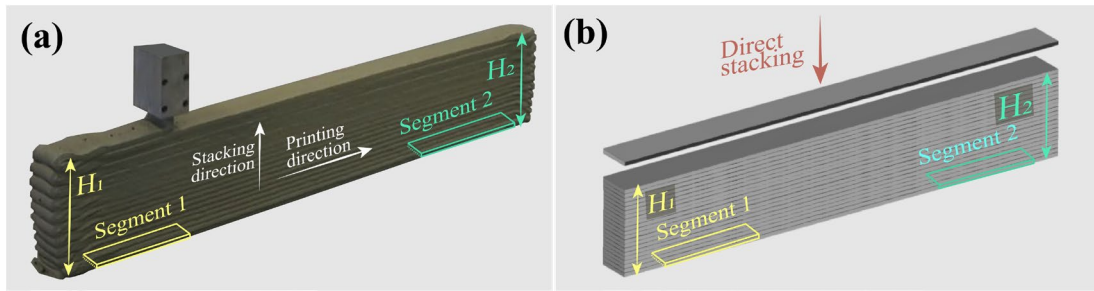


Figure 7.1 Schematic representation of the construction process of a printed wall: (a) Experimental setup with non-uniform gravitational load distribution [modified from Wolfs *et al.* (2019)]; (b) Simplified FE model with uniform gravitational load.

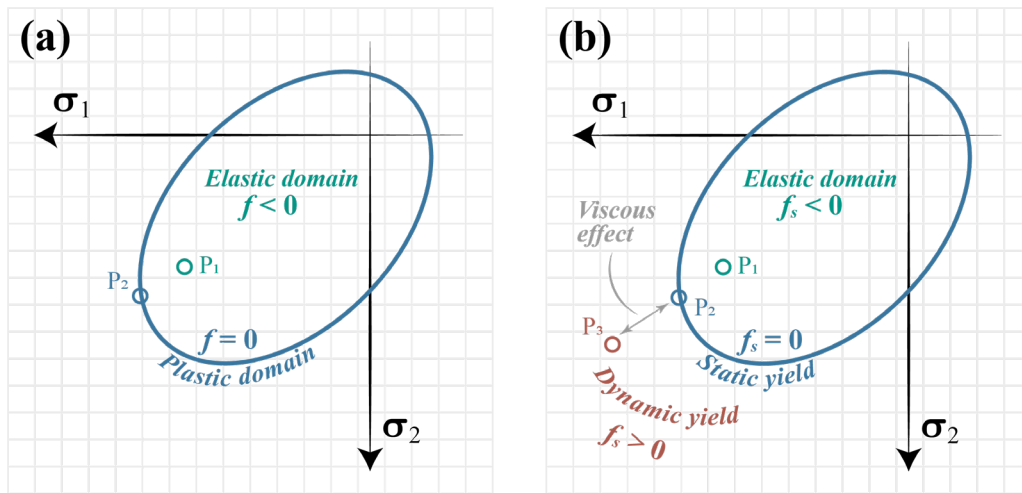


Figure 7.2 Comparison of yield surfaces in elastoplastic and elastoviscoplastic models: (a) Elastoplastic model (b) Elastoviscoplastic model.

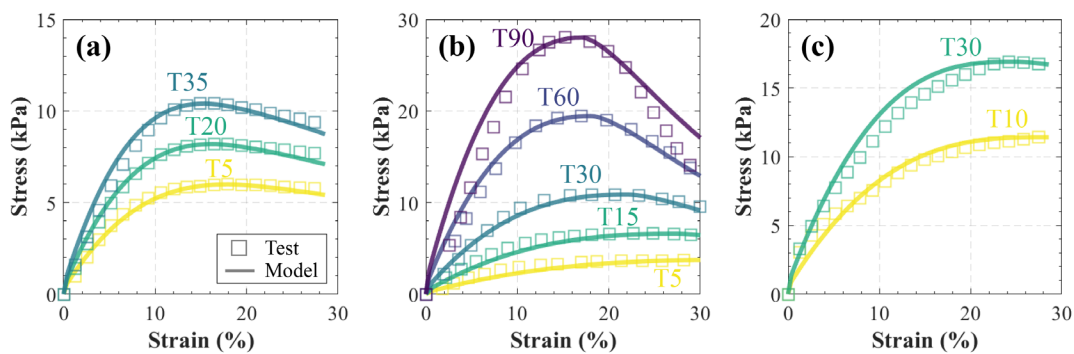


Figure 7.3 Verification of the calibrated elastoplastic model against uniaxial compression test data for different fresh concretes: (a) Suiker (2018) and Wolfs *et al.* (2018); (b) Wolfs *et al.* (2019); (c) Zhu *et al.* (2021).

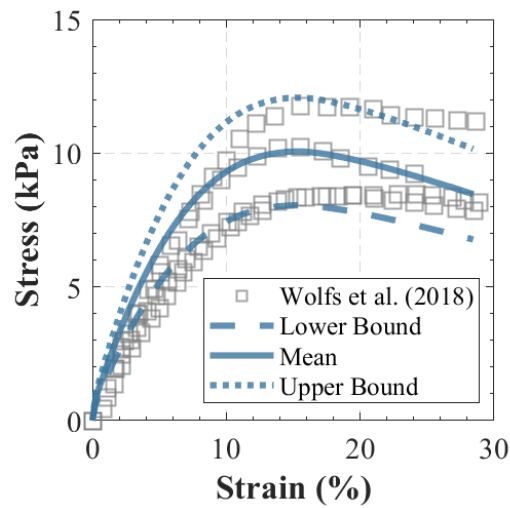


Figure 7.4 Comparison of stress-strain curves in experiments¹ to predictions by elastoplastic model with lower bound, mean and upper bound material parameters.

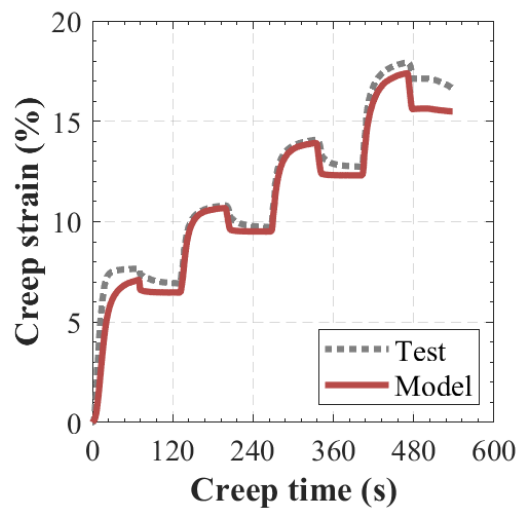


Figure 7.5 Comparison of creep curves: experiment² versus prediction by the elastoviscoplastic model.

¹ Test data were read from Figure 7 in Wolfs *et al.* (2018).

² Test data were based on the multi-step creep-recovery test result presented in Chapter 6.

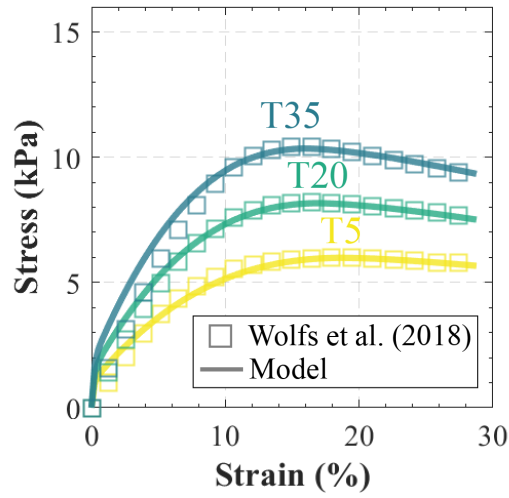


Figure 7.6 Comparison of stress-strain curves: experiments versus predictions by the elastoviscoplastic model.

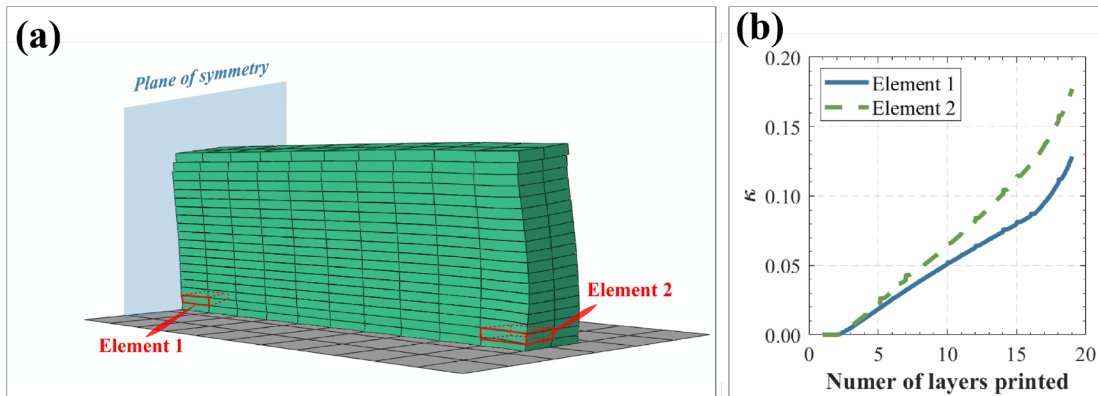


Figure 7.7 Wall Wf19-1.0: (a) Deformed shape; (b) Evolution of equivalent plastic strain of two elements of the second filament during construction.

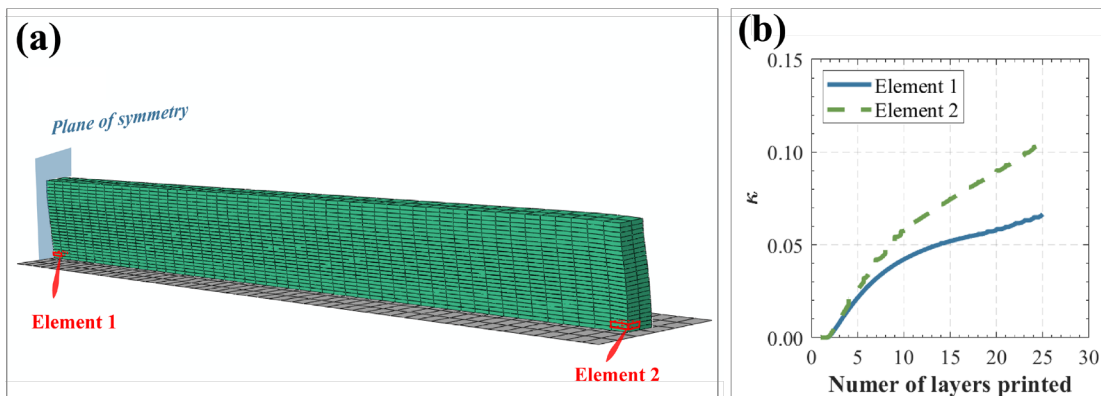


Figure 7.8 Wall Wf19-5.0: (a) Deformed shape; (b) Evolution of equivalent plastic strain of two elements of the second filament during construction.

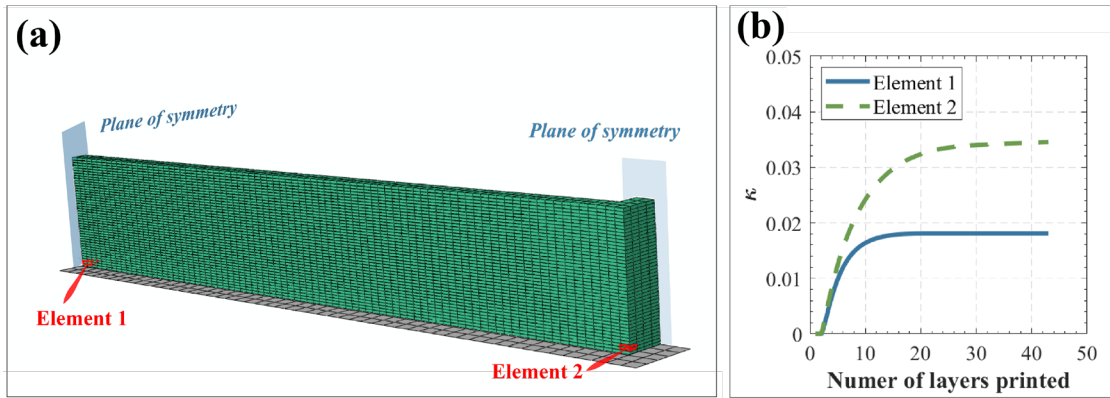


Figure 7.9 Wall Wf19-5.0 $\times$ 0.2: (a) Deformed shape; (b) Evolution of equivalent plastic strain of two elements of the second filament during construction.

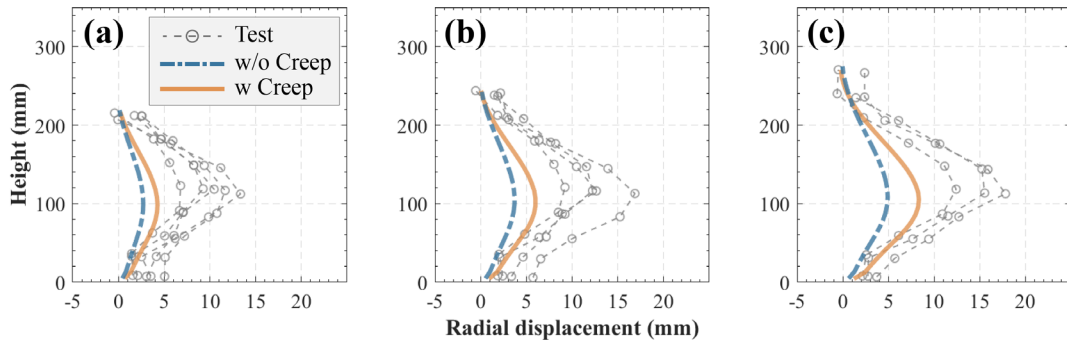


Figure 7.10 Radial displacement profiles of wall Wf18-Cyn0.5: experiments versus simulations with mean material parameters: (a) Printed to layer 23; (b) Printed to layer 26; (c) Printed to layer 29.

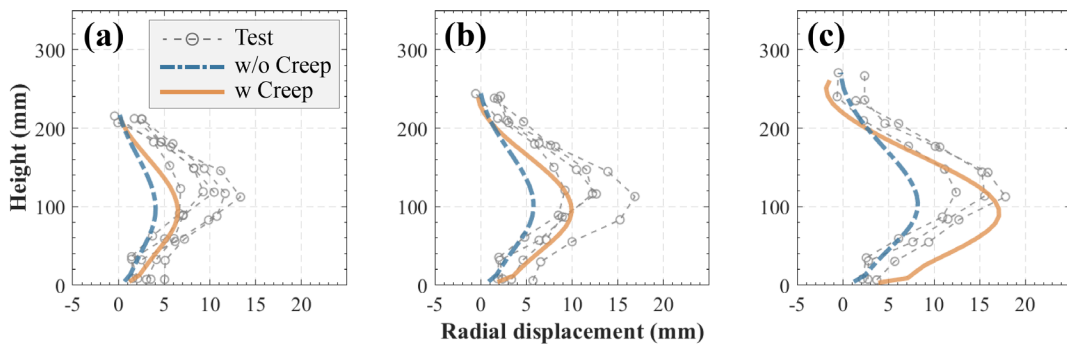


Figure 7.11 Radial displacement profiles of wall Wf18-Cyn0.5: experiments versus simulations with lower bound material parameters: (a) Printed to layer 23; (b) Printed to layer 26; (c) Printed to layer 29.

APPENDIX 7.1 DERIVATION OF THE SCALING FACTOR FOR THE EQUIVALENT PLASTIC STRAIN

In the case of uniaxial compression, the flow potential function, as given by Eq. (5.19), reduces to

$$g = \rho - \alpha \xi \quad (7.11)$$

From this simplified form, we can calculate the plastic strain rate along the three principal stress directions using the flow rule flow rule $\dot{\epsilon}^p = \dot{\lambda} \partial g / \partial \sigma$:

$$\begin{cases} \dot{\epsilon}_1^p = \dot{\lambda} \left[\frac{\sqrt{3}}{3} \frac{\partial g}{\partial \xi} + \frac{1}{3\sqrt{2J_2}} (2\sigma_1 - \sigma_2 - \sigma_3) \frac{\partial g}{\partial \rho} \right] \\ \dot{\epsilon}_2^p = \dot{\lambda} \left[\frac{\sqrt{3}}{3} \frac{\partial g}{\partial \xi} + \frac{1}{3\sqrt{2J_2}} (2\sigma_2 - \sigma_1 - \sigma_3) \frac{\partial g}{\partial \rho} \right] \\ \dot{\epsilon}_3^p = \dot{\lambda} \left[\frac{\sqrt{3}}{3} \frac{\partial g}{\partial \xi} + \frac{1}{3\sqrt{2J_2}} (2\sigma_3 - \sigma_1 - \sigma_2) \frac{\partial g}{\partial \rho} \right] \end{cases} \quad (7.12)$$

Under uniaxial compression, there is no confining stress, such that $\sigma_2 = \sigma_3 = 0$. Additionally, from Eq. (7.11), it can be deduced that $\partial g / \partial \rho = 1$, $\partial g / \partial \xi = -\alpha$. Substituting these into Eq. (7.12) yields the following simplified equations:

$$\begin{cases} \dot{\epsilon}_1^p = \dot{\lambda} \left[-\frac{\sqrt{3}}{3} \alpha + \frac{2}{3\rho} \sigma_1 \right] \\ \dot{\epsilon}_2^p = \dot{\epsilon}_3^p = \dot{\lambda} \left[-\frac{\sqrt{3}}{3} \alpha - \frac{1}{3\rho} \sigma_1 \right] \end{cases} \quad (7.13)$$

From these expressions, we can calculate the rate of the equivalent plastic strain, given by:

$$\begin{aligned}
\dot{\kappa} &= C \|\varepsilon_{ij}\| \\
&= C \dot{\lambda} \sqrt{\alpha^2 + \frac{2}{3\rho^2} \sigma_1^2}
\end{aligned} \tag{7.14}$$

Imposing the condition that $\dot{\kappa} = \dot{\epsilon}_1^p$ under uniaxial compression, we can determine the scaling factor C :

$$C = \sqrt{\frac{(\sqrt{2} - \alpha)^2}{3(\alpha^2 + 1)}} \tag{7.15}$$

APPENDIX 7.2 FURTHER EXPLANATIONS OF THE ELASTOVISCOPLASTIC MODEL IN PREDICTING CREEP

A7.2.1 Evolution of static yield surfaces during creep

For ease of illustration, a material under biaxial stress conditions is considered, whereby static yield surfaces can be effectively represented graphically by their projections on the plane stress space. The impact of material ageing is neglected in the following discussion for simplicity.

The evolution of static yield surfaces during creep is demonstrated in Figure A.7.1. The process can be described as follows:

- Initial state: Prior to load application, the initial static yield surface is denoted as YS0, with a corresponding equivalent plastic strain of κ_0 .
- Creep initiation: Creep is initiated when the applied stress (σ_{cr}) is sufficient to cause dynamic yield, i.e., $f_s(\sigma_{cr}, \kappa_0) > 0$. This is graphically represented by stress point P lying outside YS0. The conjugated static yield stress on YS0 is represented by point P' .
- Viscoplastic strain development: Given that $f_s(\sigma_{cr}, \kappa_0) > 0$, a positive viscoplastic strain rate is produced according to Eqs. (7.4) and (7.5). This results in increasing strain over time, which is manifested as creep.
- Strain hardening: The increase in plastic strain drives the growth of equivalent plastic strain (κ) from κ_0 to κ_1 . This strain hardening effect leads to an evolved static yield surface, denoted as YS1.
- Creep rate decrease: As the static yield surface expands, the distance between point P and the yield surface is decreased. Mathematically, this is expressed as $f_s(\sigma_{cr}, \kappa_1) < f_s(\sigma_{cr}, \kappa_0)$. Consequently, the creep strain rate is decreased over time, which is consistent with observations in creep tests.

- Creep termination: With further strain hardening, the static yield surface (YS2) eventually meets the stress point P , resulting in $f_s(\sigma_{cr}, \kappa_2) = 0$. At this point, the creep strain rate becomes zero, and no further creep deformation is generated.

A7.2.2 Impact of the overstress function on creep behaviour

During creep, when $f_s \geq 0$, only the power function component of the overstress function, as given in Eq. (7.16), is effective.

$$\Phi(f_s) = \gamma \left(\frac{f_s}{\sigma_{y.\max}} \right)^{N_0} \quad (7.16)$$

The term inside the parenthesis represents a scalar measure of the distance between the stress point representing the creep stress and the current static yield surface. For ease of discussion, this term is denoted as OS .

The influence of parameters N_0 and γ on the overstress function and resultant creep behaviour is demonstrated in Figure A.7.2a and Figure A.7.3a. It is observed that N_0 controls the curvature of the overstress function, with a higher N_0 leading to increased curvature. Conversely, γ governs the global slope, where a higher γ results in a steeper global slope.

To elucidate the effects of these parameters on the predicted creep curve, a series of numerical uniaxial compressive creep tests was conducted. For simplicity, elastic behaviour was neglected, and the material was assumed to be age-independent. The following material parameters were adopted for the uniaxial viscoplastic model: $h_0=0.01$, $E_0=2$, $\kappa_k=0.2$, $\Lambda=3.5$, $f_{co.s}=4.5$ kPa (under UC, the other parameters related to yield criterion and flow rule that are not mentioned here are not effective). The axial static yield stress-axial plastic strain ($\sigma_{1y.s} - \epsilon_{1p}$) curve of the material is presented in Figure A.7.4. The numerical creep tests were performed under a creep stress of 3 kPa, corresponding to an axial plastic strain of 0.083 according to the $\sigma_{1y.s} - \epsilon_{1p}$ curve.

This strain value represents the asymptotic creep strain (denoted as ϵ_{aym}) under the given creep stress of the material.

The creep curves corresponding to different values of N_0 and γ are shown in Figure A.7.2b and Figure A.7.3b, respectively. It is evident that the creep strain of all curves approaches ϵ_{aym} with increasing creep time, but at different rates. At a constant value of γ , an increasing N_0 leads to a lower creep strain rate. With N_0 held constant, an increase in γ results in a higher creep strain rate. In other words, the parameters N_0 and γ effectively control the rate at which the static yield surface expands during creep, thus influencing the creep strain rate and the time required to reach the asymptotic creep strain.

A7.2.3 Age-dependent creep behaviour predicted by the proposed model

A series of numerical uniaxial compressive creep tests was conducted with an axial creep stress of 3 kPa on a fresh concrete specimen. The material parameters used in these tests are summarised in Table 7.6. As illustrated in Figure A.7.5, the creep curves for specimens at later ages are lower than those at earlier ages, an observation that is consistent with experimental findings.

As demonstrated in Section A7.2.2, the overstress function only influences the creep strain rate, while the asymptotic creep strain (ϵ_{aym}) under a specific creep stress is governed by the relationship between yield stress and equivalent plastic strain. Although the overstress function of the proposed elastoviscoplastic model is age-independent, the model is still capable of correctly predicting lower creep strain in older specimens. This capability is attributed to the incorporation of the ageing phenomenon in the yield stress-equivalent plastic strain relationship through the consideration of age-dependent strength and age-dependent strain-hardening law.

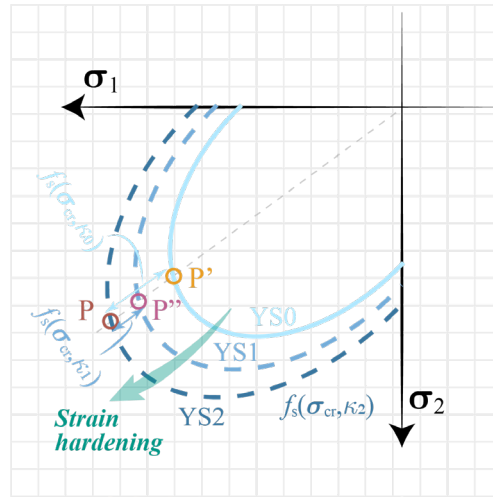


Figure A.7.1 Schematic representation of the evolution of static yield surfaces during creep in the plane stress space.

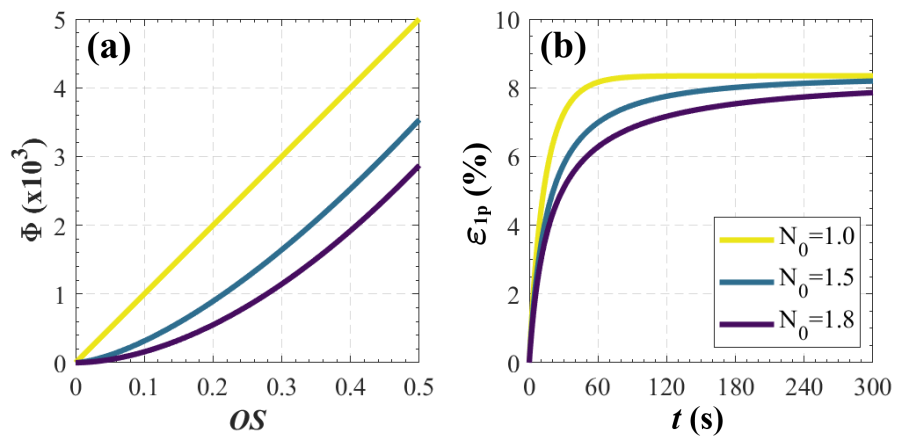


Figure A.7.2 Impact of the parameter N_0 (with $\gamma=1\times10^{-3}$) on: (a) The overstress function; (b) The creep curve

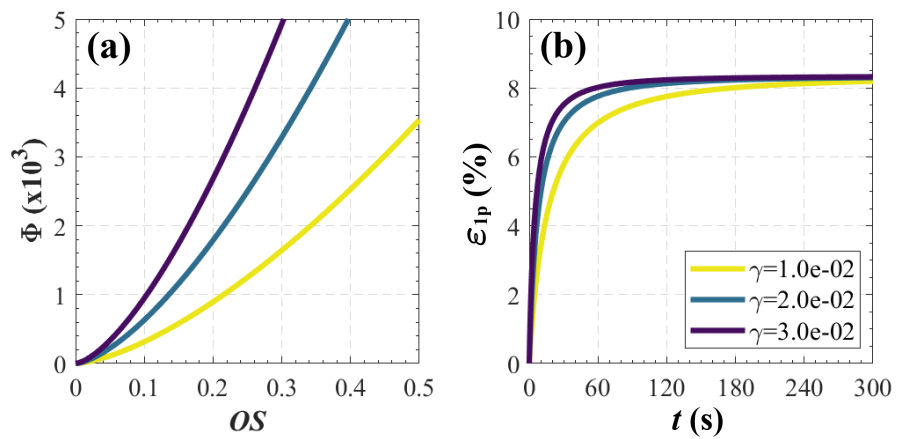


Figure A.7.3 Impact of the parameter γ (with $N_0=1.5$) on: (a) The overstress function; (b) The creep curve.

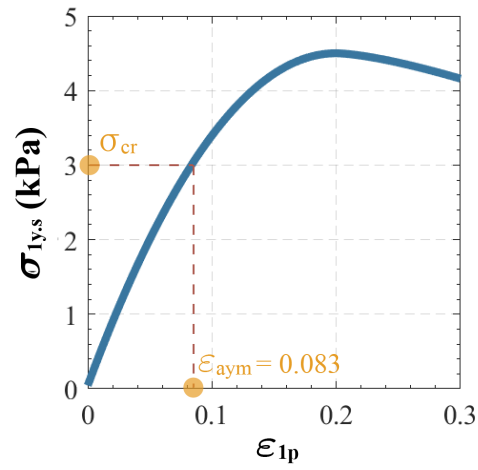


Figure A.7.4 Axial static yield stress-axial plastic strain curve for the numerical creep tests in A7.2.2.

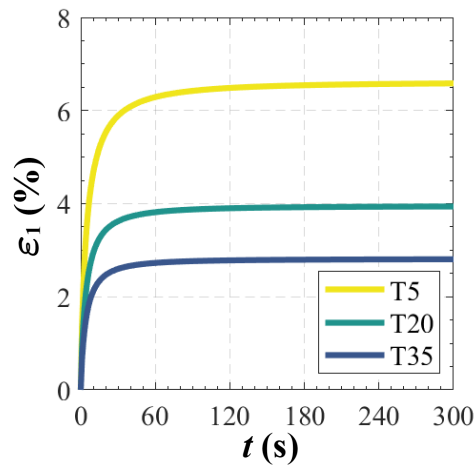


Figure A.7.5 Creep curves at different concrete ages predicted by the proposed elastoviscoplastic model.

APPENDIX 7.3 APPROACHES FOR ESTIMATING THE MATERIAL PARAMETERS NOT OBTAINABLE FROM MONOTONIC UNIAXIAL COMPRESSION TEST

This appendix presents two approaches for estimating material parameters that cannot be obtained directly from monotonic uniaxial compression tests of fresh concrete. Both approaches are predicated on the assumption that fresh concretes with identical uniaxial compressive strengths exhibit similar mechanical responses. The validity of this assumption will be elucidated in Section A7.3.3.

A7.3.1 Conversion factor between the apparent and actual Young's modulus

As illustrated in Figure 6.14, the parameter Υ , representing the ratio of actual to apparent Young's modulus is a function of concrete age. In the present study, Υ was reformulated as a function of the uniaxial compressive strength (f_{co}), expressed as:

$$\Upsilon(f_{co}) = \Upsilon_1 + \Upsilon_2 (f_{co})^{\Upsilon_3} \quad (7.17)$$

Based on the results of the loading-unloading-reloading tests described in Section 6.4.4, the parameters Υ_1 , Υ_2 and Υ_3 were determined to be 14.9, -2.85×10^{-2} and 6.33, respectively. Figure A.7.6 demonstrates that this equation accurately represents the relationship between Υ and f_{co} .

A7.3.2 Reference age approach

To determine parameters related to the yield criterion and flow rule that are not ascertainable from uniaxial compression tests, a reference age approach was proposed, analogous to that employed by Mettler *et al.* (2016). This approach establishes a relationship between the uniaxial compressive strength (f_{co}^{rf}) and concrete age of a reference concrete mixture, referred to as the master curve.

In the current study, the 3D-printed concrete developed in Chapter 3 was used as the reference mixture. The master curve is mathematically expressed as: $f_{co}^{rf} = f_{ci}^{rf} \cdot e^{\zeta_s^{rf} t}$, where $f_{ci}^{rf} = 12.7$ kPa and $\zeta_s^{rf} = 1.380 \times 10^{-2} \text{ min}^{-1}$ following the results of strength analysis in Section 4.3.3. Based on this master curve, the material properties of a 3DPC with a uniaxial compressive strength of f_{co} is characterised by an age parameter denoted as reference age (t^{rf}), determined using the following relationship:

$$t^{rf} = \begin{cases} 0 & ; \forall f_{co} \leq f_{ci}^{rf} \\ \ln\left(\frac{f_{co}}{f_{ci}^{rf}}\right) / \zeta_s^{rf} & ; \forall f_{co} > f_{ci}^{rf} \end{cases} \quad (7.18)$$

Notably, if the uniaxial compressive strength of 3DPC in other studies is less than the fictional compressive strength at zero age (f_{ci}^{rf}) of the reference mixture, the reference age is assumed to be zero. This assumption prevents excessive extrapolation of results.

By substituting the reference age into the equations provided in Section 5.4.7, the pertinent parameters for the yield criterion and flow potential function can be estimated. For example, the friction parameter of yield criterion can be estimated as $\chi = \chi(t^{rf})$.

A7.3.3 Validation of the assumption of ‘same f_{co} same properties’

Table A.7.1 compares the tested ¹ and predicted values of volumetric dilation parameters at ultimate yield stress (denoted as α_k^{uc}) a fresh concrete mixture reported in Wolfs *et al.* (2018). The predicted values were calculated using the reference age approach based on the f_{co} of these specimens. As demonstrated, the average of the predicted values aligns reasonably well with that of the tested ones, lending credence

¹ Notably, the tested α_k^{uc} were not directly presented in Wolfs *et al.* (2018); instead, they were calculated from the lateral strain-axial strain relationships, as shown in Figure A.7.7, which were derived from the force-displacement and true stress-true strain relationships of these specimens presented in the original study.

to the assumption that identical uniaxial compressive strengths correspond to similar material properties.

Table A.7.1 Validation of the reference age approach for the peak volumetric dilation parameter.

Specimen Number	f_{co} (kPa)	Peak volumetric dilation parameter	
		Tested	Predicted
1	8.45	-0.09	0.27
2	8.40	0.15	0.27
3	11.79	0.28	0.26
4	10.19	0.37	0.27
5	9.75	0.35	0.27
6	11.48	0.08	0.26
Mean ¹		0.28	0.27

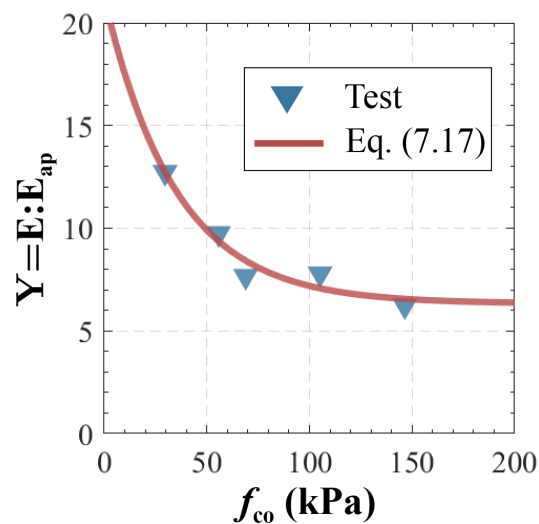


Figure A.7.6 Conversion function between the actual Young's modulus and the apparent Young's modulus².

¹ The result of specimens 1 and 6 were excluded in calculating the mean due to the large deviation from other results

² The test data were derived from the loading-unloading-reloading test results presented in Chapter 6.

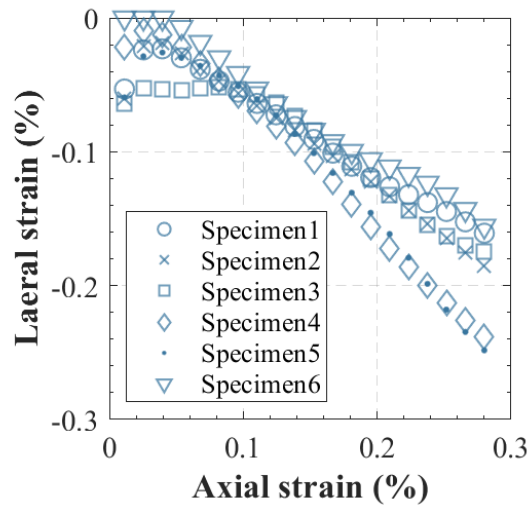


Figure A.7.7 Lateral strain-axial strain relationships for fresh concrete under uniaxial compression¹.

¹ The data were calculated from the force-displacement curves read from Figure 6 and the stress-strain curves read from Figure 7 in Wolfs *et al.* (2018).

APPENDIX 7.4 CALIBRATION OF THE OVERSTRESS FUNCTION

For ease of discussion, let the term inside the parentheses of Eq. (7.10) be denoted as OS . To determine the parameters γ and N_0 , the relationship between ϵ_1^{vp} and OS needs to be established.

As previously discussed, the creep strain of fresh concrete is assumed to be entirely inelastic in this study. Consequently, the axial viscoplastic strain ϵ_1^{vp} during the creep process can be calculated from the uniaxial creep curve using the central difference method.

The calculation of OS requires acknowledging the relationship between the axial static yield stress ($\sigma_{1y.s}$) and the axial plastic strain (ϵ_1^{vp}), as well as the value the static uniaxial compressive strength ($f_{co.s}$). Theoretically, the relationship $\sigma_{1y.s}-\epsilon_1^{vp}$ needs to be obtained through a uniaxial compression test with zero strain rate, which is not feasible in reality. For calibration purposes, the stress and plastic strain values at the end of each creep period were treated as points on the $\sigma_{1y.s}-\epsilon_1^{vp}$ curve (summarised in Table A.7.2). The relationship $\sigma_{1y.s}-\epsilon_1^{vp}$ can be obtained through interpolation of these data points. This approach is predicated on the assumption that the static yield surface meets the stress point representing the creep stress at the end of each creep cycle. As the creep strain rate is negligibly small at the end of each creep cycle, this assumption is deemed appropriate (refer to Appendix 7.2.1 for a detailed discussion on the relationship between stress state and static yield surface). Furthermore, $f_{co.s}$ was estimated to be 45.9 kPa, based on the results of uniaxial compressive tests on specimens with age close to the creep specimen.

Consequently, the relationship between ϵ_1^{vp} and OS corresponding to the four creep cycles can be calculated, which is shown in Figure A.7.8 as scatters. The parameters γ and N_0 of Eq. (7.10) can be determined by curve fitting these data. Notably, except for the first creep cycle, the data correlates well with each other. The higher creep strain rate in the first cycle can be explained by the high porosity of the material in this cycle

which does not reflect the general viscous behaviour of the material; therefore, the data in this cycle will be neglected for calibrating the overstress function.

Figure A.7.8 illustrates that the overstress function, plotted as a solid red line with γ and N_0 equal to 3.82×10^{-2} and 1.62, respectively, can accurately predict the tested relationship between $\dot{\epsilon}_1^{vp}$ and OS .

Table A.7.2. Axial static yield stress and the corresponding axial plastic strain calculated from the creep test results presented in Section 6.4.3.

Creep cycle	ϵ_1^{vp}	$\sigma_{1y.s}$
0 ¹	0.000	0.45
1	0.070	14.60
2	0.096	26.34
3	0.125	36.03
4	0.160	42.84

Note: ¹ The initial yield stress was estimated as 1% of the static uniaxial compressive strength of the fresh concrete.

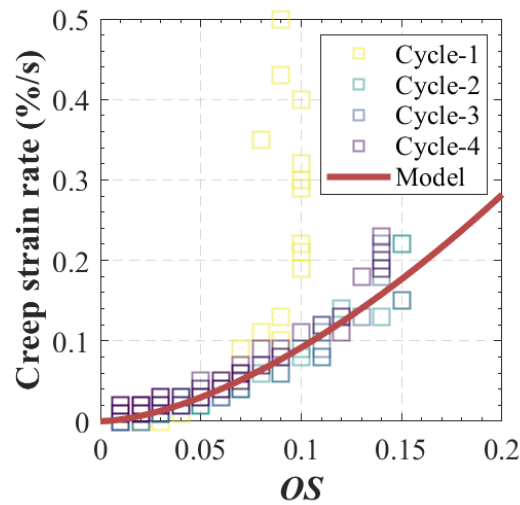


Figure A.7.8 Comparison of relationship between the axial creep strain rate and the parameter OS : experiment vs calibrated model.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

8.1 INTRODUCTION

Extrusion-based three-dimensional (3D) concrete printing represents a transformative advancement in construction technology, offering unprecedented advantages in automation, design flexibility, and resource efficiency. However, this innovative construction method presents a distinctive challenge to structural engineering: the necessity to analyse fresh concrete structures during construction, when they are vulnerable to collapse. Despite significant research efforts, existing approaches to modelling fresh concrete structures have not yet yielded reliable predictive models, with substantial discrepancies reported between experimental results and predictions. These limitations suggest fundamental gaps in existing understanding of the mechanical behaviour of early-age 3D-printed concrete (3DPC) during the construction process.

This thesis has addressed these knowledge gaps through a comprehensive series of experimental investigations, theoretical developments, and numerical simulations. The research programme aimed to develop a thorough understanding of both the material-level mechanical behaviour of early-age 3DPC and its implications for structural-level response during construction.

A systematic literature review in Chapter 2 identified four critical limitations in the current understanding of fresh concrete behaviour:

- (1) The mechanical response of fresh concrete under biaxial compression (BC), identified as the predominant stress condition during construction, remained largely unexplored in both experimental characterisation and material modelling.

- (2) Existing studies on the strain-rate-dependent (viscous) behaviour of fresh concrete have reported contradictory findings. Moreover, existing strain-rate-dependent constitutive models proposed in the literature lacked comprehensive treatment of key phenomena.
- (3) The elastic behaviour of fresh concrete under compression lacked systematic investigation.
- (4) The phenomenon of material healing, whereby existing damage can be restored through ongoing hydration, remained poorly understood in the context of fresh concrete.

To address these limitations, this thesis has presented a structured research programme comprising several interconnected studies. Chapters 3 through 5 focused on the first limitation, presenting an extensive BC testing programme and analysing both strength characteristics and general stress-strain behaviour under BC loading. Chapter 6 addressed the remaining limitations through innovative uniaxial compression testing protocols. While these chapters concentrated on material-level behaviour, Chapter 7 extended the investigation to structural-level response during construction through finite element (FE) simulations that integrated the enhanced understanding of material behaviour. The key findings from these studies are summarised in the following sections.

8.2 MECHANICAL BEHAVIOUR OF EARLY-AGE 3D-PRINTED CONCRETE UNDER BIAXIAL COMPRESSION

A comprehensive BC testing programme was developed to investigate the mechanical behaviour of fresh 3DPC. This study involved the design and validation of a specialised loading platen system capable of testing highly deformable fresh concrete while preventing platen collision. A total of 302 specimens were tested under two loading configurations: proportional loaded BC with biaxial stress ratios (β_s) ranging from 0 (uniaxial compression) to 1.0 (equibiaxial compression), and passively confined non-proportional loaded BC. The testing ages spanned from minutes to six hours after casting, encompassing the critical period during 3D concrete printing construction. This

experimental investigation revealed several fundamental aspects of fresh 3DPC behaviour.

The investigation demonstrated that material age is a dominant factor affecting mechanical behaviour. As concrete ages, the BC strength envelope evolves in both size and shape, suggesting that the strengths corresponding to different β_s grow at varying rates. The stress-strain responses and failure modes revealed an age-induced ductile-to-brittle transition. These age-dependent mechanical responses were attributed to the evolution of the material's internal structure during hydration, likely governed by two distinct chemical-physical mechanisms. The first mechanism, possibly linked to Ettringite formation, dominates the early 'rheological phase' characterised by low strength and high deformability. The second mechanism, presumably associated with C-S-H formation, controls the later 'solid phase' where the material exhibits behaviour resembling hardened concrete. Between these two distinct phases lies a 'transition phase', representing the complex interaction of these competing mechanisms.

Biaxial confinement emerged as another crucial factor influencing mechanical behaviour. The axial strength under BC consistently exceeded that under uniaxial compression at all ages, regardless of the confining stress ratio. Increasing biaxial stress ratios led to enhanced material ductility and reduced volumetric dilation, effects attributed to improved crack resistance and aggregate interlocking under confinement.

The study revealed significant deformation history dependence in the mechanical response of fresh 3DPC. Under both uniaxial and biaxial compression, specimens exhibited a contraction-to-dilation pattern in volumetric strain throughout the loading process. This behaviour was further substantiated by loading-unloading-reloading tests in Chapter 6, which demonstrated evolving yield stress during the loading process. The observed deformation history dependence was attributed to progressive damage of the material's internal structure under external loading.

The passively-confined BC tests yielded two particularly significant findings. First, they demonstrated that the mechanical behaviour of fresh concrete could be independent of stress path, validating the use of proportional loading tests to

characterise material behaviour during actual 3D printing processes where non-proportional loading predominates. Second, these tests provided an empirical estimate of the maximum biaxial stress ratio (approximately 0.55) that fresh 3DPC experiences during the printing process.

A critical evaluation of strength data revealed limitations in commonly used models, with both von Mises and Mohr-Coulomb criteria proving inadequate for representing the BC strength of fresh concrete. In response, an age-dependent Drucker-Prager model was developed to capture the BC strength behaviour. While the model slightly underestimated BC strength at high confinement stress ratios ($\beta_s=0.70, 1.00$) for ages below 90 minutes, it provided accurate predictions across all other conditions. Since the passively confined tests have indicated fresh 3DPC rarely experiences such high confinement during printing, the model was deemed suitable for practical applications. Additionally, a closed-form BC strength model was developed to enable rapid assessment of biaxial-to-uniaxial compressive strength ratios at different ages.

Building upon the strength model, a comprehensive constitutive model was developed to fully characterise the stress-strain behaviour of fresh concrete under biaxial compression. The model addresses a significant gap in existing literature by incorporating three key factors: concrete age, biaxial confining stress, and deformation history. The formulation extends the proposed age-dependent Drucker-Prager failure criterion as yield criterion and incorporates a non-associated flow rule to capture volumetric and deviatoric dilation behaviour. A novel strain hardening law was developed to account for both the age-induced ductile-to-brittle transition and the effects of biaxial confinement. The model was implemented in Abaqus through a User-defined Material subroutine (UMAT) to facilitate FE analysis of fresh concrete structures. Validation against experimental data demonstrated the model's capability to accurately predict stress-strain behaviour across different ages under both uniaxial and biaxial compression conditions.

8.3 ELASTICITY BEHAVIOUR

A systematic investigation of fresh concrete's elasticity behaviour was conducted through uniaxial compression tests incorporating six unloading-reloading cycles on specimens aged less than two hours.

The experimental results fundamentally challenged the conventional understanding of elastic properties of fresh 3DPC. Previous studies defined Young's modulus as the modulus measured at several percent strain, but this investigation demonstrated that such measurements yield only an apparent modulus that significantly underestimates the actual elastic stiffness. For the investigated 3DPC, this apparent modulus was found to be 5 to 20 times smaller than the actual Young's modulus during the first two hours of ageing.

This study also revealed that fresh concrete exhibits a remarkably low proportional limit, often below 10% of its compressive strength. This characteristic indicates that plastic deformation begins at very low stress levels. Consequently, the combination of low proportional limit and high actual Young's modulus results in deformation behaviour dominated by non-recoverable plastic components, with recoverable elastic deformation comprising only a minor fraction of the total deformation.

8.4 STRAIN-RATE-DEPENDENT BEHAVIOUR

Chapter 6 presented a comprehensive investigation into three distinct strain-rate-dependent mechanical behaviours of fresh concrete (age < 2 hours): rate-dependent stress-strain, stress relaxation, and creep behaviours.

The rate-dependent stress-strain behaviour was examined through innovative variable displacement rate uniaxial compression tests. Unlike previous studies that tested different concrete batches at varying displacement rates, this study employed a novel approach of subjecting individual specimens to three different displacement rates. This methodology eliminated inter-batch variability, thereby providing more reliable data.

The results conclusively demonstrated that stress increases with increasing displacement rate and therefore strain rate.

This investigation pioneered the exploration of stress relaxation behaviour in fresh 3DPC through uniaxial compression tests incorporating intermediate stress relaxation phases. The experiments revealed significant stress reduction during relaxation periods, with subsequent analysis indicating that this behaviour primarily stems from the material's viscoelastic response.

During 3D concrete printing, each deposited layer experiences progressive compressive loading as subsequent layers are added, causing the material to undergo a continuous creep process. Whilst previous research has examined the compressive creep behaviour of fresh 3DPC, these studies were limited in number and failed to adequately characterise nonlinear creep characteristics or elucidate the underlying deformation mechanisms. To address these limitations, an innovative multi-step creep-recovery test protocol was developed, incorporating five distinct creep stress levels, each followed by an unloading phase. This comprehensive testing revealed two significant findings: the creep compliance differs at different stress levels, indicating nonlinear creep behaviour; secondly, the predominance of non-recoverable deformation, confirmed through complementary loading-unloading-reloading tests, indicated the significant role of viscoplastic mechanisms. These findings demonstrate that conventional linear viscoelastic models, widely adopted in existing literature, are fundamentally inadequate for characterising the creep behaviour of fresh 3DPC.

Building upon these experimental insights, Chapter 7 introduced an advanced elastoviscoplastic model that extends the rate-dependent constitutive framework developed in Chapter 5. While primarily developed to facilitate FE simulations to study the material creep effects on fresh concrete structures, this model represents the first unified theoretical framework that coherently integrates both creep and material plasticity. The model's efficacy in simulating nonlinear creep behaviour was successfully validated against the experimental data presented in Chapter 6.

8.5 MATERIAL HEALING BEHAVIOUR

During 3D concrete printing, the printed material simultaneously experiences two competing mechanisms: a destructive effect due to external loading and a material healing effect due to continuous hydration. To investigate the complex interaction between these opposing processes, Chapter 6 presented an innovative experimental investigation examining the material healing phenomenon in fresh 3DPC. The methodology involved a sequence of loading phases: the initial loading of virgin specimens to failure, complete unloading, a controlled resting period, and subsequent reloading of the specimens.

These pioneering experiments yielded significant insights into the unique mechanical behaviour of fresh 3DPC. Notably, the results provided definitive evidence that the equivalent plastic strain (κ) can decrease during rest periods due to material healing, a phenomenon attributable to continued concrete hydration. This finding fundamentally challenges a core assumption of conventional plasticity theory - that equivalent plastic strain cannot decrease ($\dot{\kappa} \geq 0$) - and necessitates the development of modified theoretical frameworks for fresh cementitious materials. However, the experimental data also revealed that material healing is partial rather than complete; certain mechanical properties, particularly material strength and stiffness, exhibit permanent degradation that persists despite ongoing hydration processes.

8.6 NUMERICAL SIMULATIONS OF 3D PRINTED CONCRETE STRUCTURES DURING CONSTRUCTION

Chapter 7 presented two comprehensive sets of numerical simulations examining fresh concrete structures during construction, incorporating advanced understanding of material behaviour. The first set employed the elastoplastic model developed in Chapter 5 to validate its efficacy in structural-level analysis. The second set investigated the influence of material creep on structural response using an elastoviscoplastic model. Both simulation groups focused primarily on structural instability failure modes, which previous research has identified as the predominant failure mechanism in 3D concrete printing.

Validation against seven benchmark structures demonstrated that finite element models incorporating the proposed elastoplastic framework significantly outperformed existing models based on strain rate-independent constitutive theories. Notably, the proposed model successfully captured the failure of two benchmark wall structures which previous numerical models have failed to accurately predict. These simulations revealed two distinct instability mechanisms - elastic and plastic buckling - which prior research had incorrectly classified uniformly as elastic buckling due to inadequate constitutive modelling.

However, the elastoplastic model showed limitations in accurately simulating a cylindrical wall structure, a challenge also documented in previous studies using rate-dependent material models. This limitation was subsequently overcome through the implementation of the elastoviscoplastic model incorporating material creep effects. The enhanced model demonstrated superior capability in accurately predicting both deformation patterns and failure mechanisms in the cylindrical wall structure, highlighting the critical role of material creep in structural response. Further parametric studies examining the influence of printing speed revealed complex interactions between material creep and material ageing phenomena, identifying these as crucial factors governing structural stability during construction.

8.7 FUTURE RESEARCH

This thesis has advanced the fundamental understanding of fresh 3DPC behaviour and elucidated the critical role of material properties in structural response during construction. While these findings represent significant progress toward the practical application of this novel technology, several important research areas warrant further investigation. The following sections outline key research needs in three interconnected domains: material characterisation, constitutive modelling, and structural analysis.

8.7.1 Material Characterisation

The experimental investigations presented in this thesis, whilst comprehensive within their scope, highlight several critical areas requiring further research for complete material characterisation of fresh 3DPC.

Firstly, this Ph.D. research project has not examined the influence of the aggregate-to-binder (a/b) and water-to-binder (w/b) ratios in mixture design on the mechanical behaviour of fresh 3DPC. However, as outlined in Chapter 2, these factors can significantly affect the material's behaviour. Consequently, future research should systematically explore the impact of these mixture design parameters on the various mechanical properties of fresh concrete. This will allow the proposed material models to be calibrated over a wider range of mixtures, thereby enhancing their practical applicability.

Secondly, although this research prioritised biaxial compression as the primary stress condition during printing, materials near bottom filaments may experience non-uniform triaxial compression due to friction from the print bed (Kruger *et al.*, 2020). Future true-triaxial compression tests could characterise this behaviour (Lade *et al.*, 1973), enabling refinement of the proposed constitutive models.

Thirdly, while Chapter 6 investigated creep behaviour under uniaxial compression, fresh concrete experiences multi-axial compression during actual construction. Future research should therefore examine multi-axial compressive creep behaviour, particularly under biaxial stress conditions, to better represent realistic loading scenarios. Such experimental data would enable validation of the elastoviscoplastic framework proposed in Chapter 7 for modelling multi-axial creep behaviour. Additionally, whereas the multi-step creep-recovery tests in this thesis were limited to a single concrete age, future studies should examine creep behaviour across various ages to validate the age-independent assumption of the overstress function in the proposed elastoviscoplastic model.

Moreover, material healing behaviour warrants extensive future investigation. In this research, specimens were first loaded to failure and then cured without external stresses. However, in actual printing, pre-stress levels are typically lower, and healing takes place under sustained loading. Previous research on mature concrete (age > 9 days) has demonstrated that sustained compressive stresses applied during curing can enhance material healing (Liu *et al.*, 2002). Therefore, future studies should investigate healing behaviour under various uni- and multi-axial compression states, encompassing different pre- and healing stress levels, concrete ages, and healing periods to comprehensively characterise this phenomenon in fresh 3DPC.

Furthermore, understanding the relationship between microscopic mechanisms and macroscopic behaviour represents a significant knowledge gap. While this thesis has characterised the mechanical behaviour of fresh 3DPC at the macroscopic level, these observed phenomena - including material ageing and plasticity - fundamentally arise from microstructural evolution. Future research should bridge the gap between microscopic mechanisms and macroscopic behaviour, not only to deepen understanding of fresh 3DPC mechanics but also to provide theoretical underpinning for the current findings. Advanced techniques such as in-situ micro-computed tomography (micro-CT) offer promising capabilities for monitoring microstructural evolution during mechanical testing (Zhao *et al.*, 2017). Complementing experimental approaches, computational methods employing discrete element modelling (Wu *et al.*, 2022) could enable systematic investigation of how granular particle and binder properties influence the material's macroscopic response.

Finally, the development of non-destructive testing methodologies requires attention. The destructive tests employed in this Ph.D. research project, whilst valuable for investigating material responses under complex loading conditions, are time-consuming and challenging to integrate into production lines for in-line quality monitoring. Non-destructive measuring techniques, including ultrasound measurements (Voigt *et al.*, 2006; Wolfs *et al.*, 2018) and small amplitude oscillating tests (Roussel *et al.*, 2012), present more suitable alternatives for in-line measurements (Wolfs, 2024). However, the parameters obtained through non-destructive testing cannot directly characterise the mechanical behaviour of fresh 3DPC during printing.

Future research should therefore focus on establishing comprehensive correlations between destructive and non-destructive experimental methodologies.

8.7.2 Constitutive Modelling

This thesis has developed two distinct material modelling frameworks for fresh 3DPC: an elastoplastic model neglecting creep and its extension into an elastoviscoplastic model incorporating creep behaviour. Whilst the elastoplastic model effectively characterises material responses at a specific range of strain rates, the elastoviscoplastic framework provides a more comprehensive approach, critical for accurate analysis of fresh concrete structures where creep behaviour significantly influences structural performance. Several key areas warrant further investigation to enhance these modelling approaches.

Firstly, these two models have been designed specifically for the biaxial compression behaviour of fresh concrete. Future investigations should examine and validate these frameworks under broader stress conditions, including triaxial compression and tension states. Such extensions would enable potential refinements to enhance their comprehensiveness.

The material healing phenomenon, whilst experimentally characterised in Chapter 6, remains to be incorporated into both constitutive frameworks. This significant material characteristic requires integration into future model developments, contingent upon the availability of more comprehensive experimental data.

Whilst this thesis characterises the evolution of mechanical properties using concrete age as a parameter, the underlying ageing phenomenon fundamentally originates from the cement hydration process. Rather than being solely governed by concrete age, material development is primarily controlled by the degree of hydration, which progresses through temperature-dependent chemical reactions (De Schutter and Taerwe, 1996). Consequently, future research should focus on developing coupled chemo-thermo-mechanical models (Wang *et al.*, 2020, 2023) that incorporate these

fundamental physicochemical processes, thereby enabling more accurate predictions of material evolution during the printing process.

A further focus lies in ensuring thermodynamic rigour. Although the current phenomenon-driven constitutive models are effective, they deviate from the Generalised Standard Material (GSM) framework (Halphen and Nguyen, 1975; Flaschel et al., 2025). Therefore, additional dissipation analyses should be performed to assess their thermodynamic consistency (Jirásek and Bažant, 2001).

8.7.3 Structural Analysis

Accurate modelling of fresh concrete behaviour during the printing process represents a critical component for quality control and process optimisation in three-dimensional concrete printing. FE simulation has emerged as a reliable tool for structural-level modelling, as demonstrated in Chapter 7, enabling prediction of structural responses during the construction process.

Whilst the simulations presented in Chapter 7 effectively characterised structures with relatively simple geometries, future research should expand the scope of the developed FE model through comprehensive parametric investigations. Such investigations should encompass a broader range of printing process parameters, material characteristics, and structural geometries to elucidate the complex interactions between process variables, material behaviour, and structural performance. This expanded understanding would facilitate the development of more robust printing strategies for complex architectural designs.

Toolpath planning, which determines the location and sequence of material deposition, presents another crucial area for investigation. Whilst toolpath design primarily enables the production of topology-optimised structures at the hardened stage, it also plays a fundamental role in optimising the printing process itself (Zhuang *et al.*, 2024). Specifically, strategic infill patterns implemented through toolpath planning (see Figure 8.1 as an example) can potentially minimise deformation and prevent structural instability during construction (Kamhawi and Aghaei Meibodi, 2024). The

development of reliable toolpath design methodologies necessarily depends upon comprehensive understanding of structural responses to various deposition sequences. Consequently, future research should focus on developing integrated toolpath planning frameworks that utilise FE simulations to predict and optimise fresh concrete structures for the construction stage.

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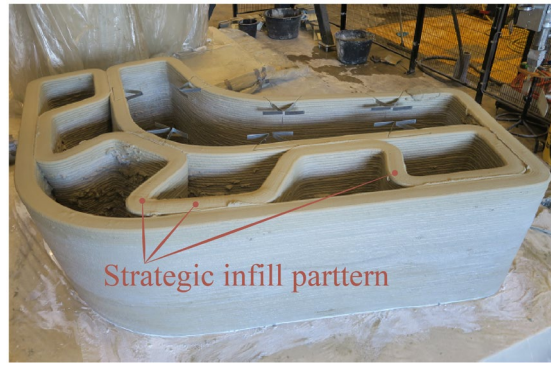


Figure 8.1 A 3D-printed concrete wall with a strategic infill pattern [modified from Wolfs (2019)].