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**SYSTEM IDENTIFICATION OF BRIDGES
INTEGRATING PHYSICS-BASED AND
DATA-DRIVEN MODELS**

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Department of Civil and Environmental Engineering

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Department of Bridge Engineering

System Identification of Bridges
Integrating Physics-based and Data-driven Models

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A thesis submitted in partial fulfilment of the requirements for the degree
of **Doctor of Philosophy**

October 2025

CERTIFICATE OF ORIGINALITY

I hereby declare that this thesis is my own work and that, to the best of my knowledge and belief, it produces no material previously published or written, nor material that has been accepted for the award of any other degree or diploma, except where due acknowledgement has been made in the text.

_____(Signed)

LUO Lanxin (Name of student)

To My Family

ABSTRACT

Bridge system identification aims to establish a mechanical model and identify characteristic parameters of the structure based on measurement data. Existing methods can be categorized as either physics-based or data-driven. Recent advancements in artificial intelligence have promoted the development of physics–data co-driven approaches, which exploit the complementary strengths of both paradigms. Meanwhile, structural health monitoring systems collect abundant data for bridge system identification, though these multi-source data introduce significant uncertainties. In this context, this thesis attempts to develop advanced techniques for bridge system identification by integrating physics-based and data-driven models.

First, a physics-based finite element model updating (FEMU) method using static and dynamic measurement data is developed for structural parameter identification. The theoretical and measured static responses are efficiently obtained by fusing traffic videos, vehicle weight data, and structural responses through computer vision-based load identification and signal processing techniques. A parallel particle swarm optimization algorithm is employed to extract the stiffness and mass parameters in an optimal manner. The method is applied to a three-span bridge model.

Next, the hierarchical Bayesian theorem is introduced into FEMU for uncertainty quantification. A hierarchical Bayesian FEMU method using static and dynamic data is then developed. The hyperparameters for both updating parameters and error functions are introduced to provide a comprehensive probabilistic representation. A two-stage sampling strategy, combining maximum a posteriori estimation with Markov Chain Monte Carlo sampling, is proposed to compute the posterior distribution. The method is applied to a three-span bridge model. Later, the influences of temperature and traffic loads on structural characteristics are incorporated into the hierarchical Bayesian FEMU

framework. A linear relationship between temperature and stiffness parameters is established to explain the observed correlation between temperature and natural frequencies. The equivalent vehicle loads estimated from the weigh-in-motion data are introduced into the FE model to capture the traffic-induced variations in dynamic characteristics. The method is validated on a long-span arch bridge. The comparative analysis of the three proposed FEMU methods highlights the importance of considering operational loads and their associated uncertainties for FEMU and long-term condition assessment.

Subsequently, a physics-based sparse Bayesian learning method for joint load–parameter identification is developed to address unknown loading conditions. The spatial sparsity of both loads and damage is embedded as prior information within the Bayesian framework. The expectation–maximization algorithm and a modified Metropolis–Hastings sampling are used to iteratively estimate damage indices, loads, and precision variables. The method is applied to both numerical and experimental bridge models.

Finally, two physics–data co-driven approaches are proposed to enhance model generalizability and flexibility. A physics-encoded neural network is proposed for joint load–parameter identification. The FEMU module for parameter identification and the autoencoder for load identification are seamlessly integrated within the framework of the Newmark-beta method. The gradient information for optimization is automatically computed through backpropagation on the deep learning platform. This approach requires only the forward formulation to be explicitly defined, thereby eliminating the repeated and complex inverse derivations required by conventional methods. The applications of this method to numerical and experimental examples demonstrate that it outperforms a filter-based method in both accuracy and efficiency. Subsequently, a hybrid finite element (FE)–neural network approach is developed for nonlinear boundary condition (NBC) identification without prior knowledge of the nonlinear types. Multilayer perceptrons are used to represent the mechanical behaviors of unknown NBCs and are integrated into the FE model. The stabilized central difference algorithm is adopted as the network architecture. Numerical studies demonstrate that the proposed approach can accurately identify both elastic and hysteretic NBCs, and remains robust under varying noise levels,

sensor layouts, and nonlinear characteristics.

In summary, this thesis addresses three key issues in bridge system identification: multi-source data fusion, uncertainty quantification, and the integration of physics-based and data-driven models. The proposed techniques effectively enhance the accuracy, robustness, and generalizability of system identification under scenarios with varying prior knowledge and data availability. These developments provide a sound basis for long-term structural health monitoring, condition assessment, and decision-making in bridge maintenance and management.

PUBLICATIONS

Journal Papers

1. **Luo LX**, Xia Y, Wang A, Lei XM, Jian XD, and Sun LM. (2022). Finite element model updating method for continuous girder bridges using monitoring responses and traffic videos. *Structural Control and Health Monitoring*, 29(11), e3062.
2. **Luo LX**, Song MM, Zhong HQ, He TT, and Sun LM. (2024). Hierarchical Bayesian model updating of a long-span arch bridge considering temperature and traffic loads. *Mechanical Systems and Signal Processing*, 210, 111152.
3. **Luo LX**, Song MM, Li YX, and Sun LM. (2024). A hierarchical Bayesian model updating method for bridge structures by fusing multi-source information. *Structural Health Monitoring*, 24(2), 1292-1310.
4. **Luo LX**, Sun LM, Li YX, and Xia Y. (2025). Structural nonlinear boundary condition identification using a hybrid physics data-driven approach. *Nonlinear Dynamics*, 113(9), 9605-9623.
5. **Luo LX**, Sun LM, Song MM, Liu JX, Li YX, and Xia Y. (2025). Joint load–parameter–response identification using a physics-encoded neural network. *Mechanical Systems and Signal Processing*, 230, 112597.
6. **Luo LX**, Sun LM, Li YX, and Xia Y. (2026). Sparse Bayesian Learning for Joint Load–Parameter–Response Identification. *Reliability Engineering & System Safety*, 268, 111831.
7. Li YX, **Luo LX**, Wang XY, Zhou XQ, Sun LM, and Xia Y. (2025). State-of-art review on load identification and response reconstruction to realize digital twin of infrastructures. *Advances in Structural Engineering*, 28(10), 1731-1765.

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LIST OF SYMBOLS

Symbols

x, y, z	Cartesian coordinates
$\boldsymbol{\theta}$	Vector of the updating parameter
$\mathbf{F}$	Load matrix
$\tilde{\mathbf{D}}, \mathbf{D}$	Measured and theoretical static deflection
$\tilde{\mathbf{f}}, \mathbf{f}$	Measured and theoretical natural frequency
$\tilde{\boldsymbol{\Phi}}, \boldsymbol{\Phi}$	Measured and theoretical mode shape
J	Objective function
$\mathcal{N}$	Normal distribution
$\boldsymbol{\mu}$	Mean vector
$\boldsymbol{\Sigma}$	Covariance matrix
σ^2	Variance
$\mathbf{e}$	Error function
Θ	Set of updating parameters
$\hat{\boldsymbol{\theta}}, \hat{\mathbf{F}}, \hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\Sigma}}, \sigma^2$	Maximum a posteriori estimates of updating parameter vector, load matrix, mean vector, covariance matrix, and variance
$\tilde{\boldsymbol{\theta}}, \tilde{\mathbf{e}}, \tilde{\mathbf{f}}, \tilde{\mathbf{D}}$	Predicted vectors of the updating parameter, error function, natural frequency, and static deflection
$\tilde{\mathbf{M}}_t$	Vehicle stream weight on each node in the t -th dataset

C	Vehicle weight scaling variable
$\mathbf{T}_t$	Temperature vector in the t -th test
$\mathbf{a}, \mathbf{b}$	Intercept and slope vectors of the linear relationship
$\tilde{\mathbf{d}}, \mathbf{d}$	Measured and theoretical displacement of the expansion joint
$\hat{\mathbf{u}}, \mathbf{u}, \bar{\mathbf{u}}$	Response surface model-predicted responses, FE model-predicted responses, and the mean value of the FE model-predicted response
R^2	Coefficient of determination
$\mathbf{M}, \mathbf{C}, \mathbf{K}$	Structural mass, damping, and stiffness matrices
$\mathbf{a}(t), \mathbf{v}(t), \mathbf{d}(t), \mathbf{f}(t)$	Acceleration, velocity, displacement, and force vectors at time t
$\mathbf{S}_p$	Input selection matrix
$\bar{\mathbf{K}}_i$	Standard stiffness matrix of the i -th element
$\mathbf{x}(t)$	State vector at time t
$\mathbf{x}_k, \mathbf{f}_k$	State and force vectors at the k -th time step
$\mathbf{y}_k, \tilde{\mathbf{y}}_k, \hat{\mathbf{y}}_k$	Theoretical, measured, and true observation vectors at the k -th time step
Δt	Time step
$\mathbf{S}_\varepsilon, \mathbf{S}_d, \mathbf{S}_v, \mathbf{S}_a$	Response selection matrices for strain, displacement, velocity, and acceleration
Υ	Shape function matrix that converts the displacement to strain
$\mathbf{G}, \mathbf{L}$	State-to-observation and load-to-observation matrices
$\tilde{\mathbf{Y}}, \mathbf{Y}$	Stacked measured and theoretical responses at all time steps
$\mathbf{H}$	Transmissibility matrix

α, β, γ	Precision factors for responses, damage indices, and loads
ξ	Collection of hyperparameters
$\mathbf{Z}$	Latent variables
$\mathbb{E}$	Expectation
$\mathbf{X}_0, \mathbf{X}_F$	Candidate samples for damage indices and loads
$\hat{\mathbf{K}}$	Equivalent stiffness matrix
L	Loss function
$\mathbf{F}^r$	Structural restoring force
$\hat{\mathbf{M}}$	Revised mass matrix
$\mathbf{K}^s, \mathbf{F}^s$	Stiffness matrix and the restoring force vector of all nonlinear boundary conditions
$\mathbf{K}^T$	Tangent stiffness matrix
k^s, f^s, d^s	Stiffness, restoring force, and displacement of a single nonlinear boundary condition

Abbreviations

1D	One-Dimensional
2D	Two-Dimensional
ARD	Automatic Relevance Determination
CC	Correlation Coefficient
CCD	Central Composite Design
CI	Confidence Interval
CNN	Convolutional Neural Network
DDS	Difference in Deflection Shape

DED	Difference in Extreme Deflection
DEI	Dynamic Error Indicator
DEJ	Displacement of Expansion Joint
DF	Difference in Frequency
DI	Damage Index
DKF	Dual Kalman Filter
DL	Deep Learning
DMS	Difference in Mode Shape
DOE	Design of Experiments
DOF	Degree of Freedom
DRAM	Delayed Rejection Adaptive Metropolis
EKF	Extended Kalman Filter
EM	Expectation–Maximization
E-step	Expectation Step
FDD	Frequency Domain Decomposition
FE	Finite Element
FEMU	Finite Element Model Updating
GA	Genetic Algorithm
GNN	Graph Neural Network
KF	Kalman Filter
LHS	Latin Hypercube Sampling
LOWESS	Locally Weighted Regression
LSTM	Long Short-Term Memory
MAC	Modal Assurance Criterion
MAP	Maximum A Posteriori
MCMC	Markov Chain Monte Carlo

MH	Metropolis–Hastings
ML	Machine Learning
MPV	Most Probable Value
MRE	Mean Relative Error
MSE	Mean Squared Error
M-step	Maximization Step
NBC	Nonlinear Boundary Condition
NN	Neural Network
NR	Noise Ratio
ODE	Ordinary Differential Equation
PCC	Partial Correlation Coefficient
PDE	Partial Differential Equation
PDF	Probability Density Function
PENN	Physics-encoded Neural Network
PINN	Physics-informed Neural Network
PSO	Particle Swarm Optimization
RAE	Residual Autoencoder
RNN	Recurrent Neural Network
RRMSE	Relative Root Mean Squared Error
RSM	Response Surface Model
SBL	Sparse Bayesian Learning
SCD	Stabilized Central Difference
SCI	Structural Condition Indicator
SEI	Static Error Indicator
SHM	Structural Health Monitoring
SSI	Stochastic Subspace Identification

UKF	Unscented Kalman Filter
VAE	Variational Autoencoder
WIM	Weigh-in-motion

CHAPTER 1

INTRODUCTION

1.1 Research Background

Bridges, as critical components of transportation networks, play an indispensable role in economic development and regional integration. Since the Industrial Revolution, a vast number of bridges have been constructed worldwide. Many of these structures have remained in service for several decades, or even centuries, resulting in varying degrees of deterioration. For instance, the United States owns approximately 623,218 bridges, with an average age of about 47 years. Among them, only 44.1% are rated in “good” condition, while 49.1% are classified as “fair”, a proportion that continues to grow each year (Asce, 2025). Structural degradation can lead to safety incidents, affect public wellbeing, and cause economic losses. To ensure structural safety and reliability, accurate structural condition assessment and timely maintenance are essential. In this process, developing an accurate structural mechanical model is a fundamental requirement.

Structural system identification (Ljung, 1998) is such a process of establishing accurate structural models based on measured loads and responses, and it is inherently an inverse problem. The resulting models characterize both the static and dynamic behavior of structures, thereby supporting structural assessment, maintenance, and management. Key steps in system identification include data collection, model selection, parameter estimation, and model validation. Current research in this field encompasses parameter identification (Wu et al., 2007), equivalent model establishment (Fang et al., 2012), and modal identification (Au, 2012b), etc. This study primarily focuses on parameter identification and equivalent model establishment.

In recent years, structural health monitoring (SHM) systems (Li et al., 2016; Sun et al., 2020c; Xu et al., 2011) have been widely implemented on bridges to improve the accuracy, coverage, and efficiency of structural condition assessment. A typical SHM system consists of several functional modules, including data acquisition, transmission, storage, analysis, and warning. Its core function is to monitor and record key physical quantities, such as environmental conditions, external loads, and structural responses. The large volume of multi-source monitoring data collected by SHM systems provides a comprehensive and reliable foundation for the system identification of operational bridges.

Traditional system identification methods are broadly classified into physics-based and data-driven approaches, based on their underlying modeling paradigms. In physics-based approaches, finite element (FE) models serve as the foundation for identifying unknown parameters and mechanical behaviors. Finite element model updating (FEMU) has emerged as a presentive technique, which adjusts model parameters to minimize discrepancies between simulated and measured responses (Mottershead et al., 2011). These methods can be extended to cases with unknown input conditions using transmissibility matrices (Zhu et al., 2014) or dynamic response sensitivities (Lu et al., 2007a). Furthermore, system identification inherently involves uncertainties arising from measurement noise, operational loads, and modeling errors. Probabilistic identification techniques, such as Bayesian FEMU methods (Beck et al., 1998; Katafygiotis et al., 1998) and filter-based approaches (Bishop et al., 2001), have therefore been developed to thoroughly quantify these uncertainties. While physics-based approaches provide element-level parameter identification with high interpretability, their effectiveness relies heavily on the accuracy of the FE model and prior knowledge of loading conditions.

In contrast, data-driven approaches directly learn the input–output relationships from measured data without explicit physical modeling. Traditional regression-based techniques, such as autoregressive models, are computationally efficient and grounded in control theory. However, their ability to capture complex nonlinear dynamics remains

limited (Xia et al., 1997). To address these limitations, machine learning (ML) techniques have been introduced to extract features and recognize patterns using flexible function approximators, including support vector machines, kernel methods, and decision trees (Zhou, 2021). More recently, deep learning (DL) (Lecun et al., 2015) has emerged as a powerful tool for system identification in high-dimensional and nonlinear scenarios, benefiting from abundant trainable parameters and well-designed architectures. Various neural network (NN) architectures, such as convolutional neural networks (CNNs), recurrent neural networks (RNNs), and transformers, have been applied to capture spatial and temporal features from observed data. Although data-driven models demonstrate remarkable performance in modeling complex systems, they suffer from limited physical interpretability and typically require large volumes of high-quality training data.

To address the limitations of both physics-based and data-driven paradigms, physics–data co-driven approaches have been proposed. These approaches integrate physical laws with data-driven models through data feature extraction, physics-based constraints, and network architecture design. A widely adopted technique is the physics-informed neural network (PINN) (Raissi et al., 2019), which incorporates residuals of governing equations into the loss function as soft constraints. Through simultaneous minimization of the data loss and physical loss, PINNs are trained to align with both empirical data and physical laws. However, the trained PINNs may not strictly satisfy the underlying physical laws because the physics constraints are imposed softly. To address this limitation, physics-encoded neural networks (PENNs) have been developed by inseparably embedding the physical laws into the network architecture or computational graphs as hard constraints (Faroughi et al., 2024). Physics–data co-driven methods improve the model interpretability and generalizability, and reduce dependency on large-scale datasets. Therefore, physics–data co-driven methods are a promising direction for structural system identification and merit further investigation.

In summary, system identification plays a pivotal role in developing accurate mechanical models and supporting the condition assessment of bridges. The recent widespread deployment of SHM systems provides abundant, diverse, and timely data for system

identification. To effectively estimate structural properties across scenarios with varying prior knowledge and data availability, system identification methods—including physics-based, data-driven, and physics–data co-driven approaches—have been developed. However, current studies on the topic still have the following challenges that merit further development:

- (1) Effective fusion of multi-source monitoring data in the FEMU process is needed. The multi-source data collected by SHM systems dynamically capture the structural state from multiple perspectives—including static and dynamic characteristics, time- and frequency-domain features, and global and local behaviors—thus providing valuable information for system identification. Existing methods typically utilize only a limited range of data types in FEMU, resulting in insufficient target information for optimization. Effectively integrating these diverse datasets can enhance the accuracy and robustness of FEMU.
- (2) System uncertainty needs to be thoroughly quantified in the FEMU process. System identification intrinsically involves uncertainties arising from various sources, including measurement noise, varying operational loads, and modeling errors. These uncertainties may become more pronounced as data diversity increases, operational periods extend, and structural complexity grows. The hierarchical Bayesian theorem can be introduced into FEMU for comprehensive uncertainty quantification.
- (3) Prior knowledge of the structural system can be efficiently incorporated into FEMU. System identification is a typical inverse problem and often exhibits ill-posedness due to the imbalance between the dimensionality of the measured data and the number of parameters to be identified. In practical scenarios, early-stage structural damage typically occurs at limited and localized regions, and loads are often applied at limited locations. This spatial sparsity of damage and loading can be incorporated in FEMU through sparse Bayesian learning, thereby mitigating the ill-posedness of the system identification process.
- (4) In-depth integration of physics-based and data-driven models requires further exploration. PINNs do not strictly enforce physical laws, and existing PENNs are not specifically developed for structural system identification. In particular, they are unable to identify element-level behaviors. A flexible strategy to establish PENNs for

system identification is to embed NNs within classical physics-based frameworks—such as the Newmark-beta method, Kalman filter, or Runge–Kutta method—to approximate unknown mapping relationships and component behaviors. Through this modular approach, physical knowledge and data-driven learning can be integrated in a flexible and structured manner.

1.2 Research Objectives

This study aims to develop advanced techniques for bridge system identification integrating physics-based and data-driven models. The research objectives are:

- 1) To develop an FEMU method that integrates static and dynamic measurements by effectively utilizing video, time-series, and tabular monitoring data collected during the operational phase;
- 2) To develop hierarchical Bayesian FEMU methods for comprehensive system uncertainty quantification:
 - Establish a framework that uses both static and dynamic responses by assigning separate updating parameters to static and dynamic tests and applying the Markov Chain Monte Carlo (MCMC) method;
 - Account for the influences of the operational loads by establishing the temperature–parameter relationship and considering the vehicle-induced equivalent weights;
- 3) To develop a sparse Bayesian learning method for joint load–parameter identification under various loading conditions by combining the expectation–maximization (EM) algorithm with the MCMC method;
- 4) To develop modular PENNs capable of identifying structural behavior at the element level:
 - Establish a PENN for joint load–parameter identification by integrating an FEMU module and a load identification network within the framework of the

Newmark-beta method;

- Develop a hybrid modeling technique for nonlinear boundary condition (NBC) identification that integrates deep learning models with nonlinear dynamic solvers.

1.3 Thesis Organization

The thesis is organized into nine chapters, as outlined in Figure 1.1. Chapter 1 introduces the research background and objectives of the thesis in general. Chapter 2 reviews the related literature on system identification from three perspectives: physics-based, data-driven, and physics–data co-driven approaches. Chapters 3 to 6 focus on the newly developed physics-based approaches. Chapters 7 and 8 propose innovative physics–data co-driven approaches.

In detail, Chapter 3 proposes an FEMU method that utilizes both static and dynamic measurements for parameter identification. Multi-source monitoring data, including video, time series, and tabular monitoring data, are effectively fused through computer vision-based vehicle load identification and signal processing techniques. A parallel particle swarm optimization (PSO) algorithm is employed to minimize the objective function.

A hierarchical Bayesian FEMU method is developed in Chapter 4 to improve the uncertainty quantification ability. Hyperparameters are introduced for both updating parameters and error functions to comprehensively consider multiple sources of uncertainty. A two-stage MCMC approach is developed to estimate the posterior distribution.

Chapter 5 further extends the hierarchical Bayesian FEMU framework to account for the

effects of temperature and traffic loads on structural properties. A linear relationship between temperature and stiffness parameters is embedded in the prior function, and the equivalent vehicle weights are incorporated into the FE model.

Chapter 6 proposes a sparse Bayesian learning (SBL) method for joint load–parameter identification under unknown loading conditions. The spatial sparsity of both damage and loading is incorporated as prior information into the Bayesian framework. The EM algorithm and a modified Metropolis–Hastings (MH) sampler are used to iteratively estimate damage indices, loads, and hyperparameters.

Chapter 7 proposes a PENN for joint load–parameter identification. The FEMU module, an autoencoder, and the Newmark-beta method are seamlessly integrated into the network architecture for parameter estimation, load identification, and response prediction, respectively. Both identification efficiency and model interpretability are enhanced through this integrated framework.

In Chapter 8, a hybrid physics–data approach is developed for identifying NBCs without prior knowledge of their nonlinear types. MLPs are used to simulate the unknown NBCs within the FE model. A stabilized central difference (SCD) algorithm is adopted as the network architecture.

Chapter 9 concludes the thesis and discusses the future work.

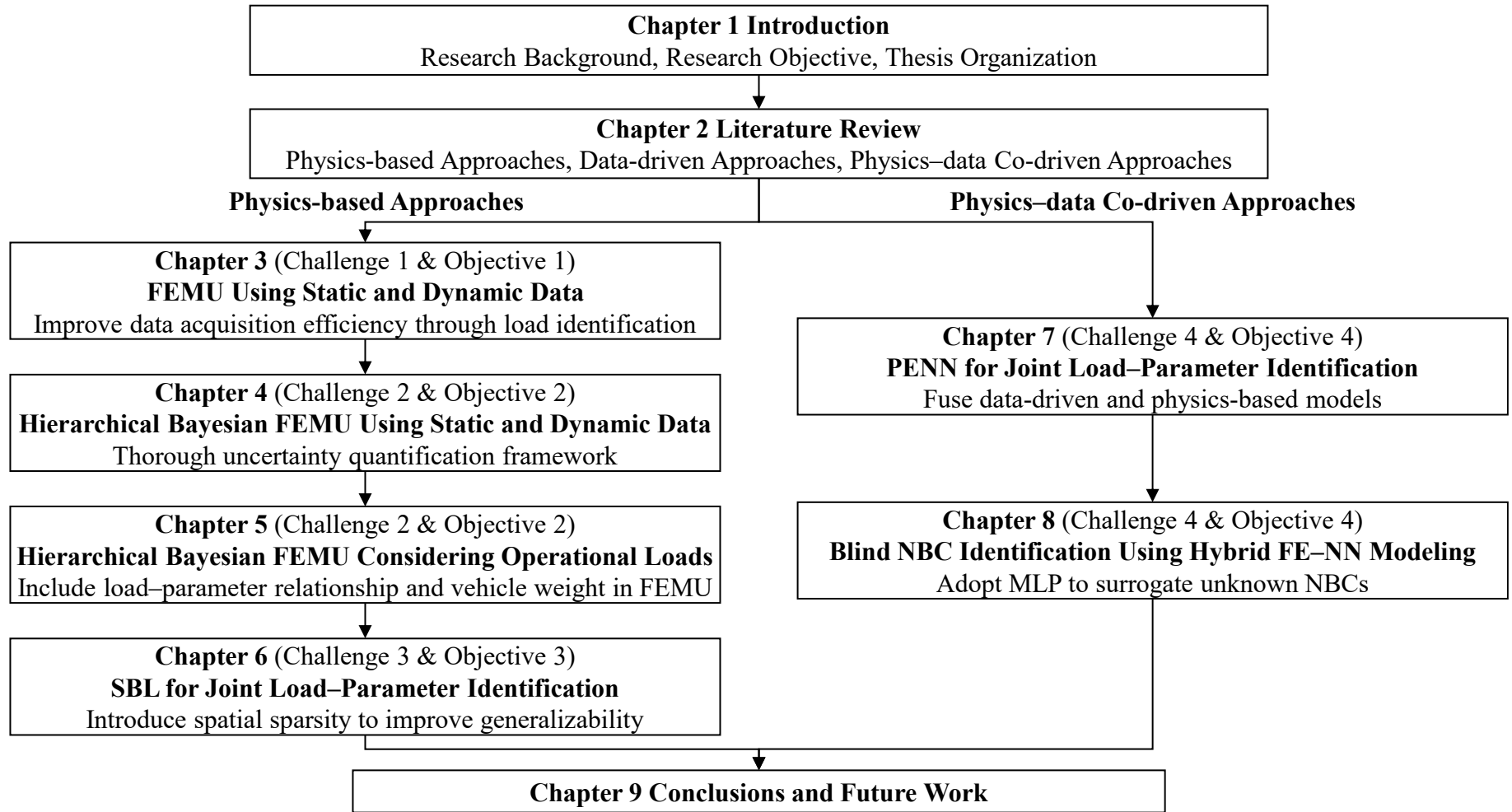


Figure 1.1 Organization of Thesis

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

System identification refers to the process of constructing or updating mathematical models of systems based on the measured input–output data (Ljung, 1998). Originally developed in control theory, system identification has evolved into a fundamental tool for establishing baseline models across a wide range of engineering disciplines. In the field of structural engineering, the main research areas of system identification include parameter identification (Wu et al., 2007), equivalent model development (Fang et al., 2012), damage detection (Hou et al., 2021), and modal identification (Au, 2012b). This review primarily focuses on the recent advances in physical parameter identification and the equivalent model establishment.

Existing system identification studies are typically categorized based on the type of model employed, resulting in three major classes: physics-based, data-driven, and physics–data co-driven approaches. Physics-based approaches rely on fundamental physical principles and commonly adopt FE models as a baseline to estimate unknown parameters and identify mechanical behaviors. In contrast, data-driven approaches employ statistical learning models and ML techniques to directly extract the input–output relationships from measured data, without explicitly modeling the underlying physics. Meanwhile, physics–data co-driven approaches leverage the complementary strengths of physics-based and data-driven methods to improve model interpretability, enhance generalizability, and reduce reliance on large-scale training datasets. These diverse methods are applied to a wide range of system identification scenarios with varying data availability and prior

knowledge, each offering distinct advantages and limitations. This review focuses on system identification methods most relevant to the present research, without attempting exhaustive coverage.

2.2 Physics-based Approaches

Physics-based approaches rely solely on physical models to identify unknown parameters or system behaviors, offering high physical interpretability. FE models are commonly used as the baseline models of bridges due to their large scale, complex configurations, and high degrees of freedom (DOFs). Physics-based methods can be broadly categorized into deterministic and stochastic approaches, depending on whether system uncertainties are considered in the identification process. Deterministic approaches mainly consist of deterministic finite element model updating (FEMU) techniques. In contrast, stochastic approaches encompass perturbation methods (Hua et al., 2008; Xia et al., 2003; Xia et al., 2002), Bayesian FEMU, and filter-based methods. Among these, Bayesian FEMU and filter-based approaches are reviewed in detail in this subsection due to their theoretical robustness and broad applicability.

2.2.1 Deterministic FEMU

The initial FE models of bridges are typically established based on design documents. However, these initial models often exhibit significant discrepancies from the actual structures in both the static and dynamic characteristics due to discrepancies in design parameters, limitations in modeling theory, and degradation of structural components (Soize, 2005). To this end, FEMU techniques have been developed to improve the accuracy of theoretical models by iteratively adjusting the model parameters (Mottershead et al., 2011). A typical FEMU process involves defining an objective function based on the discrepancies between the predicted and measured responses, and then minimizing this function using optimization algorithms. Early matrix-based methods

directly modify the entries of structural stiffness, mass, and damping matrices. Although these methods minimize the objective function, they often fail to preserve the connectivity, symmetry, and sparsity of the system matrices, thereby rendering the updated models physically unrealistic (Hou et al., 2021). Consequently, recent FEMU approaches primarily focus on updating element-level parameters, such as the material elastic modulus and density. A comprehensive review of FEMU was recently provided by Ereiz et al. (2022). In this subsection, three key aspects of FEMU relevant to this study are reviewed: (1) the types of data used for updating, (2) optimization algorithms, and (3) surrogate modeling techniques.

2.2.1.1 Data Type

Both dynamic and static responses, either individually or in combination, can be employed in the FEMU process. The objective function is typically formulated using these responses or features extracted from them.

(1) Dynamic data

The dynamic data can be categorized into frequency- and time-domain types. The most widely used frequency-domain data are modal parameters, including natural frequencies and mode shapes. These parameters are identified from vibration measurements and represent intrinsic structural characteristics. Their variations can reflect changes in stiffness and mass properties, making them effective features for model updating.

Discrepancies between measured and simulated natural frequencies are commonly quantified using the relative error indicator (Xiao et al., 2015). Mode shape differences are typically evaluated using the modal assurance criterion (MAC), a correlation-based indicator ranging from 0 to 1. A value of 1 indicates complete consistency between the measured and simulated shapes (Allemang, 2003). Error indicators can alternatively be

established using residuals between normalized predicted and measured mode shapes (Behmanesh et al., 2015; Wang et al., 2020b). This approach enables pointwise estimation of mode shape discrepancies and facilitates the prediction of mode shapes considering modeling errors. To further exploit the structural information, indicators that combine modal parameters and stiffness matrices have been proposed, including the modal flexibility residual (Cui et al., 2012; Jaishi et al., 2006) and modal strain energy (Jaishi et al., 2007; Yang et al., 2020). These unified indicators effectively integrate multiple types of modal data, eliminating the need for weight assignment to individual indicators.

However, objective functions based on modal parameters are inadequate for nonlinear system identification, because these modal parameters are extracted under linear assumptions. Moreover, the modal identification process involves signal filtering, noise reduction, and mode truncation, which reduce the information related to high-frequency components and nonlinear behaviors. These components, however, are critical for identifying localized damage and capturing nonlinear dynamic characteristics.

In contrast to modal parameters, the time-domain responses preserve the full information of the structural dynamic behavior and are better suited for representing nonlinear systems. Therefore, the raw dynamic responses are also directly used to establish the objective function (Ebrahimian et al., 2017; Lam et al., 2020; Lin et al., 2020b; Tian et al., 2021; Zheng et al., 2020). This direct formulation eliminates errors arising from modal identification and extends FEMU applicability to both linear and nonlinear systems. For example, Lam et al. (2020) updated a linear rail–sleeper–ballast system using the hammer-induced acceleration data. The location and severity of ballast damage were successfully identified. Zheng et al. (2020) developed a fiber element-based FE model of a pedestrian cable-stayed bridge. The model accounts for both material and geometric nonlinearities and was updated using time-domain acceleration and strain responses. Lin et al. (2020b) utilized acceleration and displacement responses to update the nonlinear FE model of a long-span cable-stayed bridge. A cluster-computing-aided PSO algorithm was implemented to enhance the computational efficiency. Tian et al. (2022) applied substructuring techniques to condense a numerical planar frame with nonlinear base

isolators; the reduced model was then updated using acceleration time histories.

The dynamic times history response can also be applied to the joint load–parameter identification because they inherently contain complete information about both the structural parameters and external excitations. The core issue in these studies is estimating the unknown loads from the measured responses. Early studies can be broadly divided into two categories: sensitivity-based approaches and transmissibility-based approaches. The first category utilizes the sensitivity matrix of dynamic responses with respect to both structural parameters and unknown loads to calculate the gradient information. The loads and parameters are then simultaneously identified using the gradient-based optimization algorithms (Lu et al., 2007a; Lu et al., 2007b; Zhang et al., 2011). The second category formulates the mapping relationship between unknown loads and measured responses through transmissibility matrices that depend on the updating parameters. This matrix is then iteratively optimized to minimize the discrepancies between the predicted and measured dynamic responses, thereby simultaneously estimating the input loads and the updating parameters (Ni et al., 2014; Pan et al., 2019; Zhu et al., 2014; Zhu et al., 2015). However, most existing joint identification methods assume that the loading locations and types are known, which limits their applicability in real-world scenarios.

To mitigate the ill-posedness of the load identification problem, various parameterization techniques, such as Chebyshev polynomial expansion (Zhang et al., 2011) and sine basis functions (Lu et al., 2007b), have been introduced to parametrically represent input loads in the time domain using only a few unknowns. Meanwhile, considering that structural damage and external loads are often spatially sparse, various regularization techniques, including ℓ_1 (Hou et al., 2019a), ℓ_2 (Li et al., 2022b), and ℓ_p (Pan et al., 2019) norms, have been incorporated into the objective function to improve the stability and mitigate the ill-posedness in inverse problems. However, the selection of regularization coefficients remains largely empirical and lacks solid theoretical guidelines.

(2) Static data

It's reported that the mass and stiffness parameters may compensate for each other when the objective function is established solely based on modal parameters (Kim et al., 2019). This is because modal parameters are primarily correlated to the relative values between mass and stiffness parameters, rather than their absolute values. In contrast, static data are only sensitive to structural stiffness parameters and can be measured with high accuracy. As a result, FEMU approaches based on static data can estimate the stiffness parameters with higher accuracy. Consequently, various types of static measurements, such as deflections (Liu et al., 2017; Sanayei et al., 1997), rotations (Kim et al., 2019), and influence lines (Liao et al., 2012), have attracted increasing attention in recent FEMU studies.

Ren et al. (2011) updated the stiffness parameters of a numerical beam and an experimental continuous bridge using static deflection measurements. Wang et al. (2022b) proposed an FEMU method that integrates the Kriging surrogate model to identify stiffness parameters of a deteriorated long-span continuous bridge using static deflection data. Guan et al. (2024) developed an FEMU approach using the long-term statistical characteristics of traffic flows and static responses. The probabilistic distributions of structural stiffness parameters are updated by minimizing the discrepancies between the measured and predicted probabilistic distributions of structural responses. The proposed method was applied to a numerical example and a real-world suspension bridge.

However, FEMU methods using static data still face several limitations. First, the static data are often obtained through vehicle load tests, which require traffic control and preparation, making them impractical for real-time monitoring. Secondly, the quantity of static data collected in a single test is limited.

(3) Combination of static and dynamic data

Static and dynamic data can be integrated into the FEMU process to enhance the identification accuracy and reliability (Nagesh et al., 2020). In this case, the optimization problem can be formulated either as a multi-objective optimization problem, where different data types are used to define independent objective functions, or as a single-objective problem, where multiple objectives are combined into an overall objective function through weighted summation.

Wang et al. (2013) updated a multi-scale FE model of the Runyang suspension bridge using the multi-objective optimization framework. Natural frequencies, mode shapes, static displacements, and stresses obtained from field tests were used to establish the independent objective functions. Sanayei et al. (2015) employed static strains, displacements, slopes, dynamic natural frequencies, and mode shapes from nondestructive tests to estimate the stiffness and mass parameters of a laboratory bridge model. The updating accuracy among different combinations of responses is also compared. Xiao et al. (2015) integrated modal frequencies and multi-scale static influence lines to update the FE model of the Stonecutters Bridge. The influence line was derived from time-domain displacements induced by moving vehicle loads. Wu et al. (2017) identified the local bending stiffness, density, and boundary condition parameters of a highway bridge using static long-gauge strain data and the first-order natural frequency. Kim et al. (2019) proposed a sequential FEMU approach, in which the boundary conditions were first updated using static rotation measurements, followed by the identification of main structural parameters using static displacements and modal data. The results of the above studies demonstrate that the updated models are capable of accurately predicting both static and dynamic responses.

However, several challenges remain in the fusion of static and dynamic data for FEMU. In the single-objective formulations, the weights for different objective functions are chosen based on engineering judgment or trial-and-error, which introduces subjectivity

and lacks physical rigor. Improper weighting can misguide the optimization process and result in suboptimal updating results. In the multi-objective formulations, a set of Pareto-optimal solutions is obtained, each representing a trade-off among conflicting objectives. The final solution is then selected from the Pareto front based on engineering preferences, prior experience, or specific requirements. As a result, the decision-making process also remains partially subjective.

2.2.1.2 Optimization Algorithms

Once the objective functions are formulated and the updating parameters are selected, optimization techniques are employed to minimize the objective functions by adjusting the updating parameters. Existing approaches can be classified into sensitivity-based methods and nature-inspired metaheuristic algorithms.

In sensitivity-based FEMU techniques, the nonlinear relationship between the objective functions and the updating parameters is linearized using the first-order Taylor expansion. The optimization problem is thus transformed into the solution of linear equations. The unknowns are the increment of the updating parameters, the coefficient matrix corresponds to the sensitivity matrix, i.e., the Jacobian of the objective function with respect to the updating parameters, and the constant term is the current objective function value. This linear system is iteratively solved until the predefined convergence criterion is satisfied (Mottershead et al., 2011). Rezaiee-Pajand et al. (2020) identified the mass and stiffness parameters using the sensitivities of modal kinetic energy and modal strain energy with respect to the model parameters. While sensitivity-based methods offer clear physical interpretability and are computationally efficient for small-scale problems, the sensitivity matrix needs to be re-derived for each new FEMU problem (Ren et al., 2010). Moreover, the optimization results are highly dependent on the initial values of the updating parameters; poor initialization may lead to divergence or suboptimal solutions.

In contrast, evolutionary algorithms draw inspiration from various natural phenomena,

such as swarm intelligence, biological evolution, and physical or chemical processes (Yang, 2020). Once the objective function is defined, these algorithms search for optimal parameters through heuristic mechanisms rather than deterministic gradient-based calculations. Commonly used nature-inspired optimization algorithms in FEMU include genetic algorithms (GA) (Hao et al., 2002), PSO (Kang et al., 2012), simulated annealing (Kourehli, 2017), artificial bee colony (Sun et al., 2013), harmony search, and others.

GA is inspired by biological evolution principles, employing selection, crossover, and mutation operations to simulate natural evolution and guide optimal solution searches. GA can address multi-objective optimization problems when fitness evaluation is designed to handle conflicting objectives through Pareto-based ranking and fitness sharing. Cui et al. (2018) updated a multi-scale FE model of a large shell structure using multi-objective functions and the non-dominated sorting genetic algorithm-II. Sun et al. (2020a) applied GA to obtain a set of Pareto-optimal solutions for updating parameters and employed the fuzzy outranking technique to select the most satisfactory solution.

The PSO algorithm is inspired by the social behavior of bird flocking during food search. It identifies optimal solutions through learning from personal experience and population knowledge. PSO is known for its computational efficiency and global search capability. Gökdağ et al. (2012) successfully detected the damage of a Timoshenko beam using modal parameters and the PSO algorithm. Seyedpoor (2012) applied PSO to update the FE models of a cantilever beam and a planar truss based on modal strain energy. Since each particle in PSO is generated and evaluated independently, parallel computing can be employed to evaluate the fitness functions of all particles simultaneously, significantly improving computational efficiency (Lin et al., 2020a).

2.2.1.3 Surrogate Model

FEMU is computationally expensive because it requires a large number of FE simulations

to search for the optimal solution. This issue becomes more pronounced when dealing with large-scale structures, where individual simulations demand substantial computational resources. To this end, surrogate modeling techniques have been introduced to accelerate the updating process. These techniques aim to construct computationally efficient models that approximate the relationship between input and output data. The implementation of surrogate models consists of two stages: design of experiments (DOE) and model optimization.

The DOE stage is to strategically select a limited number of sample points in the input parameter space to efficiently represent the system's behavior. Commonly employed DOE techniques include central composite design (CCD) (Montgomery, 2017), Latin hypercube sampling (LHS) (Mckay et al., 2000), orthogonal design (Cui et al., 2018), and uniform design (Ren et al., 2011). The simulation results generated at these sampled points are used to train the surrogate models.

The widely used surrogate models are the response surface model (RSM) and the multilayer perceptron (MLP). RSMs typically employ the polynomial functions (Kudela et al., 2022), radial basis functions (Zhou et al., 2016), or Kriging models (Wang et al., 2017) as the basis functions. Among these models, the Kriging model can additionally quantify the model uncertainty, making it well-suited for stochastic FEMU (Mao et al., 2020). These surrogate models are trained on designed samples through optimization algorithms.

Ren et al. (2010) compared an RSM-based FEMU method with a sensitivity-based FEMU method on a simply supported beam and a continuous girder bridge. They found that the RSM-based method converged faster under the same convergence criterion. To further enhance surrogate accuracy, Ren et al. (2011) proposed an adaptive sample reduction strategy that dynamically adjusts the sampling range of data points during the surrogate model training process.

The MLP, a type of NN, has also been applied to surrogate FE models due to its ability to capture complex and nonlinear relationships (Tan et al., 2017). Sabamehr et al. (2018) employed an MLP to establish the relationship between structural frequencies and parameters of bridge sections. The GA was used to optimize the MLP. Similarly, Park et al. (2017) adopted an MLP to represent the relationship between the structural responses and rotational spring stiffness in a bridge. The trained MLP was subsequently used to identify the boundary condition using measured response data.

2.2.2 Bayesian FEMU

Structural system identification is subject to uncertainties arising from measurement noise, modeling errors, and environmental variability. Although the uncertainty introduced by any single factor may be relatively minor, the combined effects can lead to significant deviations in structural responses, ultimately reducing the accuracy of system identification. To address this issue, probabilistic methods such as Bayesian FEMU have been proposed through comprehensive uncertainty modeling.

The Bayesian approach, rooted in the foundational work of Thomas Bayes and Pierre-Simon Laplace (Bayes, 1958), integrates prior knowledge (i.e., subjective beliefs derived from experience or theory) with observational data using Bayes' theorem. The Bayesian framework provides a rigorous mathematical foundation for uncertainty analysis in complex engineering systems by balancing subjective assumptions with objective evidence (Huang et al., 2018). This subsection reviews Bayesian FEMU methods from two perspectives: the formulation framework and the computational techniques.

2.2.2.1 Bayesian FEMU Frameworks

The frameworks of the Bayesian FEMU methods are classified based on whether the hyperparameters are introduced for the random variables, leading to the classic Bayesian

and hierarchical Bayesian frameworks.

(1) Classic Bayesian framework

Beck and Katafygiotis (Beck et al., 1998; Katafygiotis et al., 1998) were the first to introduce Bayesian theory into system identification. In this framework, the posterior distribution of parameters is calculated by multiplying the likelihood function and prior distribution, then normalizing by the evidence. The discrepancies between the measured and model-predicted data are treated as the error function. The likelihood function is defined as the probability of observing the measured data given a specific set of parameters. Its probability density function (PDF) is derived based on the assumed PDF for the error function using PDF transformation rules. The prior PDF of the updating parameters is defined based on engineering experience or selected from the conjugate priors, which simplifies the posterior derivation. The evidence function represents the marginal distribution of measured data, obtained by integrating the likelihood function over the parameter space. Over the past two decades, the classic Bayesian framework has been extensively developed and applied in various domains, including FEMU (Yuen et al., 2001) and modal identification (Au, 2012a, 2012b).

However, classical Bayesian methods typically assume that the PDFs of both the error functions and updating parameters remain constant across different tests. This assumption may not hold for operational structures where structural dynamic characteristics are significantly affected by time-varying operational conditions, such as temperature, traffic loads, and wind. As a result, fixed PDF assumptions may lead to the underestimation of uncertainties associated with model parameters and modeling errors (Sedehi et al., 2019).

(2) Hierarchical Bayesian framework

To overcome the limitations of the classic Bayesian framework, Behmanesh et al. (2015)

introduced the hierarchical Bayesian theory into FEMU. The primary distinction between the hierarchical and classical Bayesian methods lies in the treatment of the updating parameters and their priors. In the classical Bayesian framework, the measured data in different tests are assumed to originate from the same structural state, and the updating parameters across all tests are drawn from a common and fixed distribution. In contrast, the hierarchical Bayesian method considers that the data are generated under independent structural states, and the updating parameters from different tests are modeled as independent random variables. Moreover, hyperparameters are introduced for the distributions of the error functions and updating parameters to yield a flexible and comprehensive probabilistic representation of system uncertainty. Consequently, the hierarchical Bayesian approach removes the reliance on fixed-distribution assumptions and significantly improves the capability for uncertainty quantification. For example, Sedehi et al. (2019) proposed a time-domain hierarchical Bayesian FEMU framework to improve the reliability of Bayesian approaches for uncertainty quantification and propagation in structural dynamics.

The hierarchical Bayesian framework focuses on estimating the posterior distributions of hyperparameters rather than solely the model parameters. Therefore, the hierarchical Bayesian framework is capable of quantifying the variability of structural properties under varying operational conditions by introducing hyperparameters to capture the relationships between the updating parameters and the influencing factors. For example, Behmanesh et al. (2016) modeled the structural stiffness parameters as a function of ambient temperature and vibration amplitude in the prior function. The uncertainties arising from the damage, environmental variability, and external loads are therefore effectively decoupled and quantified. This approach was applied to model calibration, response prediction, and damage identification of the Dowling Hall footbridge at Tufts University. Song et al. (2019) incorporated a linear relationship between the stiffness parameters and the excitation amplitude into the hierarchical Bayesian framework to update the FE model of a reinforced concrete building with masonry infills. The updated model effectively captures frequency variations caused by loads and accurately predicts dynamic responses.

Given the spatial sparsity of structural damage and external loads, sparse priors have been incorporated into the hierarchical Bayesian framework, leading to a specialized variant known as the sparse Bayesian method (Huang et al., 2018). In this approach, the automatic relevance determination (ARD) priors (Tipping, 2001) are assigned to the updating parameters. These priors act as adaptive constraints that promote the sparsity in the solution. The method has demonstrated promising performance in tasks such as damage identification (Hou et al., 2019b; Wang et al., 2020b, 2020c) and load identification (Li et al., 2023c).

2.2.2.2 Computational Techniques

Although Bayesian frameworks provide a theoretically rigorous foundation for uncertainty quantification, obtaining the analytical solutions of the posterior PDFs remains intractable. This difficulty arises from the high-dimensional and nonlinear nature of the integrals involved in computing the evidence. To address this issue, a range of approximation and numerical techniques have been developed for Bayesian inference.

(1) Laplace approximation

Laplace approximation expands the target distribution around its maximum a posteriori (MAP) estimate using a second-order Taylor series. This results in a Gaussian approximation of the posterior PDF, where the covariance matrix is given by the inverse of the Hessian matrix evaluated at the MAP point.

The Laplace approximation was first applied to Bayesian FEMU by Katafygiotis et al. (1998) and Beck et al. (1998). Yuen et al. (2006a) further extended its application to FEMU using incomplete modal data without mode matching. Wang et al. (2020a) incorporated the Laplace approximation into the sparse Bayesian framework to identify the updating parameters and precision factors. More recently, Chen et al. (2023) utilized

it to estimate structural parameters, moving forces, and precision factors within a sparse Bayesian joint identification framework based on dynamic response sensitivity. However, the Laplace approximation assumes that the distribution is unimodal and approximately symmetric, and it performs well only when the identification problem is globally identifiable. When the true posterior is multi-modal or highly skewed, the Gaussian approximation may fail to capture its characteristics accurately.

(2) EM algorithm

The EM algorithm is widely used in Bayesian inference problems involving latent variables. It consists of two iterative steps: the expectation step (E-step) and the maximization step (M-step). In the E-step, the expectation of the complete-data log-likelihood with respect to the latent variables is calculated as a function of the hyperparameters. In the M-step, the hyperparameters are optimized by maximizing this expectation. These two steps are iteratively called until definite criteria are satisfied.

Ching and Beck (2004) applied the EM algorithm within a Bayesian FEMU framework to estimate the most probable value (MPV) of stiffness parameters using incomplete measurement data. The EM algorithm reformulates the original non-convex and nonlinear optimization problem into two coupled concave subproblems and outperforms the conventional nonlinear optimization methods. Hou et al. (2019b) proposed a sparse Bayesian learning framework for structural damage detection, where the EM algorithm was employed to iteratively estimate latent damage indices. Wang et al. (2021) proposed the Sparse Bayesian factor analysis approach for structural damage detection under unknown environmental conditions. The EM algorithm was employed to infer the influences of different environmental factors on the natural frequencies. Teymouri et al. integrated augmented (Teymouri et al., 2022) and extended Kalman filters (Teymouri et al., 2023) into the Bayesian framework for joint load-parameter identification. The external loads, structural parameters, as well as the process and measurement noises, are identified using the EM algorithm. Despite its conceptual simplicity, the EM algorithm is

sensitive to initial values and may converge slowly in high-dimensional or highly nonlinear problems.

(3) Variational inference

Variational inference reformulates the Bayesian inference task as an optimization problem by introducing a tractable PDF to approximate the intractable target posterior. The tractable PDF is optimized by minimizing its Kullback–Leibler divergence from the true posterior. The method can be further combined with the mean-field theory for efficient and scalable inference, where the posterior is approximated by a factorized family of distributions. Wang et al. (2020c) applied the variational inference together with the delayed rejection adaptive metropolis (DRAM) algorithm within the sparse Bayesian framework for structural damage identification. The approach is capable of capturing both standard and non-standard posterior distributions, making it suitable for complex identification problems involving non-Gaussian uncertainties.

(4) MCMC

MCMC approaches generate samples from the posterior distribution by constructing Markov chains that converge to the target distribution. This approach can unbiasedly estimate complex posteriors, including those with multiple modes or asymmetric confidence intervals (Beck et al., 2002; Yang et al., 2022). MCMC methods avoid the need to calculate the intractable evidence function, as they rely solely on the proportional posterior PDF. Commonly used MCMC techniques include the MH algorithm, Gibbs sampling, the DRAM algorithm, and the transitional MCMC method.

The MH algorithm generates candidate samples from a proposal distribution and accepts them based on the calculated acceptance probability. While flexible, its performance is sensitive to the design of the proposal distribution and the choice of stopping criteria

(Roberts et al., 2001). The DRAM algorithm improves upon the classical MH method by incorporating the delayed rejection step and the adaptive covariance updating step, which significantly enhance sampling efficiency, particularly in high-dimensional problems (Haario et al., 2006). Gibbs sampling iteratively samples from the conditional distributions of each parameter while keeping the others fixed. Although the acceptance rate in Gibbs sampling is theoretically 100%, it requires that the conditional distributions be explicitly expressed (Robert et al., 1999). The transitional MCMC method approximates the target posterior through a sequence of intermediate distributions that gradually transition from prior to posterior using temperature parameters. While effective for challenging sampling tasks, the transitional MCMC method is often computationally expensive due to its multi-stage sampling procedure (Ching et al., 2007).

2.2.3 Filter-based Approaches

In addition to Bayesian FEMU approaches, filter-based methods represent another important class of stochastic system identification approaches. Among them, the Kalman filter (KF) is a representative technique. The structural parameters are incorporated into the state vector and are recursively updated using the noisy measurements (Bishop et al., 2001). The standard KF assumes linear system dynamics and Gaussian-distributed noise. It operates through a two-step process: a prediction step, which calculates the state estimates using the governing equations, and an update step, which refines the state estimates and associated uncertainties based on the latest measurements.

Several variants of KF have been developed to address system nonlinearity and improve identification accuracy. The extended Kalman filter (EKF) deals with nonlinear problems by linearizing the process and measurement equations around the current state through a first-order Taylor series expansion (Hoshiya et al., 1984). Although computationally efficient, the EKF performs poorly on highly nonlinear systems because it neglects higher-order terms during the linearization process. Unlike EKF, the unscented Kalman filter (UKF) introduces a set of sigma points to approximate the prior distribution of the

state vector. These sigma points are propagated through the nonlinear process model, and the transformed points are used to compute a weighted mean and covariance. This method provides an accurate approximation of the posterior distribution without the need to compute Jacobians (Wang et al., 2022a; Xie et al., 2012). As a result, the UKF typically performs better in highly nonlinear identification tasks than the EKF. Another variant, the dual Kalman filter (DKF), employs two interdependent KFs to recursively estimate system states and unknown parameters (Sen et al., 2017). The latest state estimates are input into one filter to update the parameters, which are then fed to another filter for state estimation.

Besides parameter identification, KF and its variants have also been applied to the joint identification of system states, structural parameters (e.g., stiffness and damping), and external loads (e.g., wind or seismic excitations). In such approaches, all these quantities are augmented into the state vector and updated recursively through the predict–update cycle. A range of KF variants have been employed for this purpose, including the EKF (Huang et al., 2019; Naets et al., 2015), UKF (Wang et al., 2022a), and DKF (Astroza et al., 2019; Yuan et al., 2024). For example, Ebrahimian et al. (2018) developed an EKF-based method for the joint identification of seismic inputs and unknown parameters of a nonlinear reinforced concrete structure. Song et al. applied both window-based EKF (Song et al., 2023b) and UKF (Song et al., 2022) to simultaneously estimate wind loads and stiffness parameters of a wind turbine. Regularization techniques have also been integrated into the KF framework to promote the sparsity in the identification results (Huang et al., 2019).

Despite their flexibility, KF-based methods are highly sensitive to the settings of the process and measurement noise covariance matrices. These matrices are often unknown in practice and require careful tuning through trial-and-error procedures or adaptive algorithms, which can be both computationally expensive and time-consuming (Aucejo et al., 2019; Wang et al., 2024).

To address some of these limitations, the Gillijns De Moor filter (Gillijns et al., 2007) has been proposed. The structural parameters, external excitations, and system states are augmented into an extended state vector, which is then updated recursively without assuming Gaussianity or stationarity. This characteristic makes it particularly suitable for applications involving nonstationary or impulsive excitations. Moreover, the filter does not require probabilistic noise models and offers enhanced robustness under uncertain environments. However, its computational complexity increases exponentially with the number of parameters to be identified, which limits its applicability to high-dimensional systems and large-scale structures.

Physics-based system identification methods provide clear physical interpretability during the identification process. However, their performance may decline in the presence of modeling errors, such as unknown boundary conditions or multi-physics interactions. In such cases, data-driven methods can be used as alternatives, since they capture system behaviors without requiring accurate physical models.

2.3 Data-driven Approaches

Data-driven methods directly extract the input–output mapping relationships from the multi-source measured data. Traditional data-driven techniques include regression analysis, time series modeling, statistical pattern recognition, and machine learning (Sun et al., 2020c). Recently, DL techniques, typically realized through deep NNs, have shown strong capabilities in learning complex patterns from large datasets (Lecun et al., 2015). Given their effectiveness and the increasing research interest, this subsection focuses on DL-based approaches for system identification. The discussion is organized by network architecture, covering the applications of MLPs, CNNs, RNNs, and autoencoders in tasks such as parameter identification, damage detection, and response prediction.

2.3.1 MLP

MLP consists of multiple layers of neurons, where each node is connected to those in adjacent layers. Each connection is associated with a weight, a bias term, and an activation function. These parameters enable the MLP to learn fundamental structural patterns from training data, while the activation functions further allow it to capture nonlinear behaviors (Avci et al., 2021). MLPs have been widely used for parameter identification and damage detection. These studies typically follow a similar workflow. First, FE simulations are employed to generate the response/feature–parameter training datasets. Then, the MLP is trained using supervised learning to approximate the FE model. Finally, the trained network is applied to measured data to identify structural parameters or detect damage.

Modal parameters are commonly used as inputs to the MLP. González et al. (2008) used an MLP to establish the relationship between the first flexural mode parameters (frequencies and mode shapes) and the structural mass and stiffness properties. The method was validated using a numerical five-story building model. Mehrjoo et al. (2008) trained an MLP to estimate the damage severity of joints in a truss bridge. The simulated natural frequencies and mode shapes were used as inputs, and the damage degrees of joints were used as outputs. A substructuring technique was applied in the FE simulation to reduce the number of unknown parameters. Hakim et al. (2015) proposed an ensemble MLP architecture for damage identification of a laboratory beam. An individual MLP was assigned to each vibration mode, taking the corresponding modal parameters as input and the damage severity vector as output. The outputs of these MLPs were then aggregated by a final MLP to generate an improved damage prediction. Bakhary et al. (2010) developed a multi-stage MLP-based damage identification method incorporating substructuring to improve computational efficiency. In the first stage, damaged substructures were detected by an MLP using global modal parameters as inputs and substructure frequencies as outputs. In the second stage, the damage location and severity within the damaged substructure were further identified by an MLP using the substructure's modal parameters as input and the damage degrees of each element as output.

The optimization of MLP architecture has also attracted attention because a well-designed model should balance prediction accuracy, computational efficiency, and model complexity to mitigate the risks of underfitting and overfitting. Yuen et al. (2006b) applied Bayesian theory to select the optimal model class of the MLP, including the number of hidden layers and neurons per layer. Lam et al. (2008) extended this approach to select both the number of hidden neurons and the activation functions within the hidden layers.

In addition to modal parameters, time-series data were also used directly or in combination with feature extraction techniques as inputs to the MLP. Barai et al. (1997) used dynamic displacement histories of a numerical railway bridge as inputs to the MLP to estimate stiffness parameters. De Lautour et al. (2010) fit the structural acceleration time histories in both undamaged and damaged scenarios using autoregressive models. The autoregressive coefficients were used as input features to an MLP, which was trained to classify damage cases and estimate stiffness parameters. Alhamaydeh et al. (2013) developed an MLP to represent the hysteretic behaviors of the buckling-restrained braces by incorporating historical forces and deformations into the input. Dang et al. (2021) compared the performance of MLP, RNN, one-dimensional (1D) CNN, and two-dimensional (2D) CNN for damage detection using time-series data. The results demonstrated that the 2D CNN achieved the highest accuracy, while the MLP, due to its simple architecture, was suitable as a baseline model.

While MLP is a powerful and straightforward tool for system identification, it has several limitations. First, its fully connected architecture limits its capacity to extract spatial and temporal features. As a result, most MLP-based studies primarily focus on low-degree-of-freedom (DOF) structures and are validated through numerical cases. In addition, most existing approaches rely on large amounts of labeled data from both damaged and undamaged scenarios, which are difficult or even impossible to obtain in practice. Although FE simulations are widely used to generate training datasets, the resulting data may fail to adequately capture the characteristics of actual structures due to modeling errors. This limitation can reduce the generalizability and reliability of the trained models in real-world applications.

2.3.2 CNN

CNNs are a class of NNs characterized by the use of convolutional operations, which can effectively learn and extract both temporal and spatial features from input data (Krizhevsky et al., 2017). These capabilities make CNNs suitable for a variety of system identification tasks, including parameter estimation and equivalent model establishment.

CNNs are typically trained in a supervised learning framework for the purpose of structural damage detection and parameter identification. For instance, Abdeljaber et al. (2017) developed a 1D CNN to detect bolt loosening in a steel grandstand, using accelerations measured at the joints as input. Lin et al. (2017) applied a 1D CNN to detect damage of a numerical beam model, where acceleration time histories were used as the input to predict the damage-level vector. Their study revealed that the convolutional layers acted as distinct band-pass filters. Yu et al. (2018) proposed a 2D CNN that uses frequency spectra from multiple accelerometers as input and a damage vector as output. This approach was validated using a five-story numerical benchmark building subjected to earthquake excitation. Avci et al. (2018) assigned separate 1D CNNs to each sensor in a wireless sensor network. Each CNN received a time-history signal as input and output a local damage classification vector. These outputs were then integrated to compute a global structural health indicator. Cofre-Martel et al. (2019) used images of the transmissibility function as input to a 2D CNN for feature extraction. The resulting features were then passed to a multilayer perceptron (MLP) to predict elemental damage severities. Duan et al. (2019) proposed a CNN-based approach to detect damage in the hangers of a tied-arch bridge. The Fourier amplitude spectra of measured acceleration signals were used as input, and the damage indices of each hanger were set as output.

CNNs have been employed to model the relationship between external loads and structural responses, owing to their strong regression capabilities in dealing with both spatial and temporal data. Xue et al. (2021) applied a CNN to predict the time-history response of a transmission tower subjected to complex wind loading. In their study, both

temporal and spatial wind speed data were transformed into a single-channel image, and the tower's top displacement was used as the output. Sun et al. (2023) proposed a hierarchical CNN model to predict bearing displacements by incorporating multiple types of loads, including temperature, wind, and vehicle loads, as input variables. The proposed method was validated using one month of monitoring data from a cable-stayed bridge.

CNNs have also been used for response reconstruction, where they are trained to establish direct mappings between structural responses at different DOFs. Wu et al. (2019) employed a CNN to predict dynamic responses in both linear and nonlinear numerical systems, as well as in a laboratory three-story steel frame. Fan et al. (2021) introduced a densely connected CNN architecture for acceleration response reconstruction. Skip connections and dense blocks were incorporated in the network to enhance information flow, improve training efficiency, and increase feature extraction accuracy. Later, Fan et al. (2023) developed a generative adversarial network with an embedded self-attention mechanism to capture the intrinsic correlations between acceleration responses. Huang et al. (2025) further integrated a CNN with a bidirectional gated recurrent unit to reconstruct acceleration responses under varying environmental temperatures. The proposed method was validated using both simulated data from a numerical model and field measurements from the Guangzhou Tower.

Although CNNs are efficient in extracting features and processing time-series data, their ability to capture long-term temporal dependencies is limited. This is because the receptive field of CNNs is constrained by the kernel size, making them less effective than RNNs and other sequential architectures that are specifically designed for temporal dependency modeling.

2.3.3 RNN and LSTM

RNNs and their enhanced variant, LSTM models, are specifically designed to handle

sequential data such as time series. These networks are constructed with recurrent architectures that retain information from previous states. They process current inputs based on historical information and effectively represent temporal dynamics and dependencies in system identification tasks.

RNNs adopt a cyclic connection structure to retain information from the previous time step. Graf et al. (2010) employed an RNN to predict the displacement of a numerical reinforced concrete plate, using structural stress as input. External loads and environmental factors, including temperature and humidity, were incorporated into the simulation. A fuzzy logic mechanism was integrated into the training process to address measurement uncertainty. Perez-Ramirez et al. (2019) proposed a nonlinear autoregressive exogenous model-based RNN to reconstruct acceleration responses of a large-scale structure. A mutual information index was introduced to determine the optimal number of neurons in the hidden layer. However, standard RNNs exhibit limited capacity for modeling long-term dependencies and are susceptible to issues such as exploding and vanishing gradients.

LSTM networks address these limitations by introducing gated memory cells that regulate the flow of information via input, output, and forget gates. The model selectively retains or discards information over time through these gates, thereby preserving relevant signals across long sequences and mitigating gradient-related problems. Zhang et al. (2019) proposed two LSTM architectures to model nonlinear structural responses under seismic excitation. The networks were trained using ground motion as input and structural responses as output. A K-means-based unsupervised clustering method was applied to classify the training and validation datasets, thereby improving prediction accuracy. Peng et al. (2021a) integrated convolutional operations into the LSTM architecture to enhance its capability in spatial feature extraction. Li et al. (2021) developed an LSTM model to simulate bridge buffeting responses. The model was validated using monitoring data from the Xihoumen Bridge. The measured wind parameters, including horizontal wind speed, wind direction, and wind attack angle, were used as the inputs, and vertical structural displacement was used as the output. Peng et al. (2021b) conducted a comparative study

on the performance of MLP, piecewise linear least squares method, and LSTM networks in simulating nonlinear dynamic responses under periodic, impact, and seismic loads. The results showed that both MLP and LSTM models demonstrated strong robustness to noise, and the prediction accuracy of the LSTM model was the highest.

2.3.4 Deep Autoencoder

The deep autoencoder is an NN architecture designed to efficiently learn compressed representations of input data. It consists of an encoder and a decoder, each typically implemented using MLPs, CNNs, or RNNs. The encoder compresses the input into a low-dimensional latent space, while the decoder reconstructs the original data from this compressed representation. Deep autoencoders have been widely applied in structural damage detection and response reconstruction tasks due to their strong feature extraction capabilities.

In damage detection, Shang et al. (2020) proposed a method based on a deep convolutional denoising autoencoder. The model takes multi-dimensional cross-correlation functions of vibration data as input. Then, exponentially weighted moving average control charts were established using the features from the latent space to detect minor structural damage. Coraça et al. (2023) introduced an unsupervised framework that integrates a variational autoencoder (VAE) with a hidden Markov model to learn the degradation behavior of bearings. Ma et al. (2020) employed a VAE to extract latent features from structural dynamic responses, which were then used to identify bridge damage under moving vehicle loads. Pathirage et al. (2018a) applied an autoencoder for structural damage identification, where natural frequencies and mode shapes were used as inputs, and structural stiffness reduction factors were predicted as outputs. The proposed method was validated using both numerical and laboratory-scale frame models. In a follow-up study, Pathirage et al. (2018b) introduced a sparse autoencoder to promote sparsity in the damage localization results.

In response reconstruction, Li et al. (2022a) employed an autoencoder with skip connections to learn the mapping relationships across different types of structural responses. The model was trained using responses generated by an FE model. Lei et al. (2021) developed a deep convolutional generative adversarial network, in which the generator was implemented as an autoencoder, to reconstruct missing strain data of a medium-span bridge. Lei et al. (2022) proposed a residual autoencoder to estimate bridge displacement responses under multiple loads. The model was validated using monitoring data from a long-span cable-stayed bridge. The input features were environmental and operational parameters, including temperature, wind, and vehicle loads, while the output targets were the displacement responses at the mid-span and the tops of pylons.

2.3.5 Other Architectures

In addition to the aforementioned NN architectures, recent and effective models such as transformers and graph neural networks (GNNs) have also been employed in system identification tasks.

Transformers are designed to process sequential data by leveraging self-attention mechanisms, which assign different weights to elements of the input sequence. This allows the model to effectively capture and interpret temporal dependencies. Bao et al. (2023) applied a transformer to identify structural damage in a jacket offshore platform, using strain data as input and damage labels as output. Triviño et al. (2023) employed a transformer-based approach to localize damage in the supports of offshore wind turbines. Dao et al. (2025) introduced a multi-scale transformer for response reconstruction. A dilation operation was incorporated to reduce redundant computation, and the proposed method was validated using ambient vibration data from the Canton Tower. Xu et al. (2026) proposed a hybrid LSTM-transformer network to predict the expansion displacement at the girder end. This model captures multi-scale spatiotemporal dependencies in temperature and displacement data by combining local recurrence with global attention mechanisms. He et al. (2025) developed an automated signal recovery

framework based on a symmetric transformer–generative adversarial network. The generator employs a symmetric encoder-decoder transformer with positional encoding and residual connections for long-sequence modeling, while the discriminator evaluates generated signals in both time and frequency domains. Li et al. (2025) introduced a nonstationary transformer that reconstructs nonstationary data using a de-stationary attention mechanism, which dynamically adapts to changing temporal patterns. Wang et al. (2025) proposed a two-stage deep learning model: a bidirectional gated recurrent unit predicts deformation data, and a transformer models the prediction errors. The preliminary predictions and error corrections are integrated to generate high-precision deformation forecasts.

GNNs are designed to aggregate and propagate information through graph-structured data, where nodes and edges represent entities and their relationships. GNNs have been increasingly adopted in civil engineering for structural response simulation and intelligent design, because the topology of FE models can be naturally represented as graphs. Kuo et al. (2024) proposed a GNN–LSTM model for structural surrogate modeling. In this approach, the structural topology is represented as a graph and encoded into a low-dimensional feature map using a GNN. The resulting feature maps are then combined with ground motion data and input into an LSTM network to predict full-field structural responses. Shan et al. (2025) proposed a semi-supervised model integrating a GNN and an LSTM model to reconstruct full-field temperature distributions from sparse measurements. The mesh, data, and essence of the FE model were embedded into the GNN. The LSTM component was employed to capture the temporal evolution of temperature fields. Eslamlou et al. (2025) proposed a GNN model with conformal prediction for damage detection and uncertainty quantification. The model is built based on the structural topologies, and trained on simulated modal data, with damage location and severity as outputs. Jian et al. (2025b) proposed a GNN-based DL approach to identify modal parameters, including natural frequencies, damping ratios, and mode shapes, for a population of structures using the power spectral density of vibration measurements. Furthermore, Jian et al. (2025a) introduced the modal superposition theory into a DL framework that integrates GNNs and transformers for population-based

modal identification. The transformer is trained to decompose multi-DOF measurements into the modal space, enabling the identification of natural frequencies and damping ratios, while the GNN encodes structural topology to infer corresponding mode shapes. The loss function is constructed based on modal superposition theory and mode independence.

DL-based data-driven models have demonstrated strong capabilities in feature extraction and prediction accuracy, providing significant advantages in system identification tasks. However, their application still faces several limitations. First, purely data-driven models act as black boxes and provide limited physical interpretability. Second, these models are typically trained in a supervised manner, which demands large amounts of labeled data from multiple structural conditions. This requirement significantly limits their generalization to unseen or varying structural states. Third, these models cannot reconstruct structural responses at unmeasured locations, restricting their ability to simulate full-field responses. To address these issues, researchers have proposed integrating the interpretability of physics-based approaches with the strong representational capacity of data-driven approaches during the modeling and training processes, resulting in physics–data co-driven approaches.

2.4 Physics–Data Co-driven Approaches

Physics–data co-driven approaches have been developed to leverage the complementary strengths of purely physics-based and data-driven methods. In these approaches, domain knowledge or physical laws, such as partial differential equations (PDEs), model topologies, and prior information, are embedded into deep NNs to improve physical interpretability, enhance model generalizability, and reduce the amount of required training data. Rueden et al. (2023) provided a comprehensive summary of the sources, representations, and integration strategies for incorporating prior knowledge into learning systems, as illustrated in Figure 2.1. Haywood-Alexander et al. (2024) conducted an in-depth review of recent advances in hybrid physics-data models for structural dynamics. They also defined a methodological spectrum to classify existing approaches. For

simplicity, this subsection categorizes current physics–data co-driven methods based on whether physical constraints are strictly enforced, leading to two main groups: PINNs and PENNs. These two categories will be reviewed separately in the following sections.

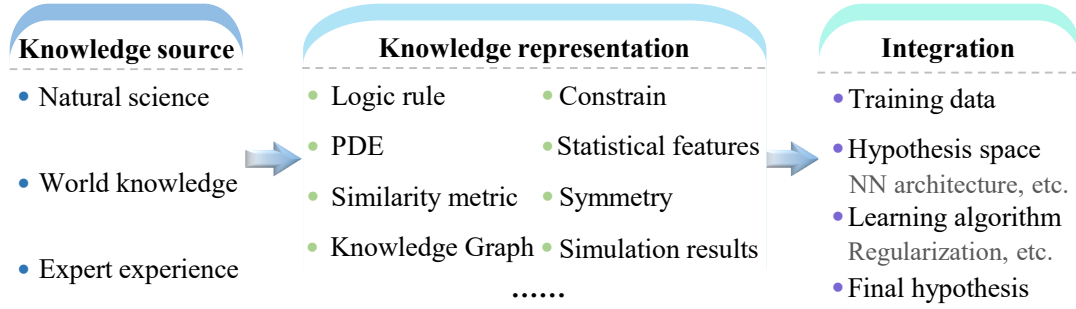


Figure 2.1 Knowledge source, representation, and integration methods of physics–data co-driven approaches

2.4.1 PINN

PINN was termed by Raissi et al. (2019). It refers to a framework that incorporates underlying physical laws into the training process of NNs, thereby ensuring that the outputs remain consistent with established physical principles. The most widely adopted implementation strategy is to augment the loss function with a weighted physics-based penalty term, which applies weak penalties to the network when its predictions violate the physical constraints (Faroughi et al., 2024).

PINNs were initially proposed in computational science, particularly in fluid and solid mechanics, to infer or discover underlying PDEs (Ren et al., 2023; Ren et al., 2022). Raissi et al. (2019) implemented both a continuous-time model and a discrete-time model to solve and discover classical PDEs in their seminal work. Automatic differentiation was employed to compute the derivatives of the network outputs with respect to their input coordinates (i.e., space and time), which were used to construct the physics loss. The proposed methods were validated on both forward and inverse problems involving various boundary conditions, strong nonlinearities, and complex-valued solutions. Chen

et al. (2021) introduced a PINN framework combined with sparse regression to discover governing PDEs of nonlinear spatiotemporal systems from sparse and noisy data. This approach integrates the expressive power of NNs for rich representation learning, the embedding of physical constraints to approximate system dynamics, and sparse regression to identify key derivative terms and parameters in the governing equations. Yang et al. (2021) proposed a Bayesian PINN to address forward and inverse problems involving nonlinear PDEs and noisy data. In this method, a Bayesian NN serves as the data-driven component. The prior is defined by the distribution of the Bayesian network. The likelihood function is modeled using observed data. The physical laws are embedded in the loss function. Hamiltonian Monte Carlo and variational inference are employed to estimate the posterior distribution. Yu et al. (2022) improved the accuracy of standard PINNs by incorporating both the residual and gradient information of the PDEs into the loss function. Chen et al. (2024) developed a PINN for full-field reconstruction of rotational flow beneath nonlinear periodic water waves, and the Euler equation was formulated as part of the physics-informed loss.

In structural engineering, PINNs have been applied to identify unknown structural parameters. Guo et al. (2023) proposed a PINN to map the relationship between modal parameters and structural stiffness parameters. A regression function between natural frequencies and stiffness was used to construct the physics-based loss. However, since the physics loss was derived from a data-driven regression model, it remains uncertain whether the embedded physical constraints can provide meaningful guidance during training. Liu et al. (2023) developed a PINN for identifying parameters in nonlinear structural systems. The physics loss was constructed using structural dynamic equations and a heat transfer-based damping model, where the damping behavior was modeled as a thermalization process. Teloli et al. (2025) proposed a PINN to identify damping and stiffness parameters and estimate beam displacement. The residual of the fourth-order PDE of the Euler–Bernoulli beam was used to build the physics loss. Four independent MLPs were used to predict structural parameters and displacements, using structural coordinates as inputs.

PINNs have also been applied for surrogate modeling and structural response reconstruction. Zhang et al. (2020b) developed a PINN to simulate structural responses subjected to seismic excitation. The physics loss was constructed using the residual of the dynamic equilibrium equation, which was computed based on the network-predicted responses and known structural mass, damping, and stiffness matrices. Zhang et al. (2020c) further presented a PINN-based metamodel for nonlinear structural systems, consisting of multiple LSTM models. Physics-informed constraints, derived from the dynamic equilibrium equation, state-dependent relationships, and hysteretic constitutive behavior, were embedded in the loss function. Ni et al. (2022b) developed a PINN to estimate structural responses subjected to seismic excitation. A multi-channel autoencoder was employed to predict full-field structural responses, while the physics-informed loss was formulated based on the dynamic equilibrium equation. In another study, Ni et al. (2022a) applied a PINN to reconstruct vehicle-induced displacements from measured strain and acceleration data. The differential relationship between displacement and acceleration was formulated as the physics loss. Xu et al. (2024) applied this method to reconstruct responses of plate structures. Qu et al. (2024) employed a temporal convolutional network for dynamic response reconstruction. Discrepancies between the predicted and measured data in both the time and frequency domains were incorporated into the loss function. The proposed method was validated using deflection data from a mid-span bridge.

While PINNs demonstrate strong potential for modeling and identifying dynamic systems, several limitations remain. First, physics knowledge is incorporated only during the training process, with the resulting trained models still functioning as black boxes. Consequently, the physics interpretability of both the prediction process and its results remains limited. Furthermore, physics constraints typically function as soft regularizations rather than strict rules. This limitation means that predictions may not fully comply with physical laws, particularly when dealing with noisy or incomplete data. Last but not least, there are currently no standardized guidelines for selecting the optimal weights assigned to the data loss and physics loss terms. These weights are typically selected manually or based on empirical experience. Improper weight settings may result

in competition between different loss terms during training, thereby degrading both training stability and model accuracy.

2.4.2 PENN

Unlike PINNs, which incorporate physics knowledge as soft constraints through loss functions, PENNs embed physical laws directly into the network architecture or computational graph. The physical laws are inseparably integrated into the PENNs' forward propagation process so that the model strictly complies with governing equations. Classical computational methods, such as the Euler forward method, Runge–Kutta method, and Newmark-beta method, are often adopted to build the architectures of PENNs.

Early PENNs originated from computational science. Rao et al. (2022) proposed a PENN to discover spatiotemporal PDEs from sparse and noisy data. A recurrent CNN was used to mimic the forward Euler time-stepping process. A “ π -block” was introduced to approximate the PDE operator via polynomial combinations. Sparse regression was then applied to the reconstructed data to identify the explicit form of the governing equations. Building on this work, Rao et al. (2023) further validated the PENN framework in applications such as data-driven modeling and the discovery of the reaction–diffusion processes and other PDE-governed systems. Chen et al. (2018) introduced neural ordinary differential equations (ODEs) to model continuous-time systems and identify physical parameters. In this approach, the PDEs are represented using NNs and solved using ODE solvers. The ODE solving process is hard-coded into the computational graph, whereby the trained model inherently satisfies the underlying physical laws. Dulny et al. (2022) extended the neural ODE framework to learn the dynamics systems governed by PDEs. Greydanus et al. (2019) proposed the Hamiltonian NN, which learns the Hamiltonian function (i.e., total energy) of physical systems from measured data. The loss function is constructed as the residual of Hamilton's equations using automatic differentiation. These pioneering works have inspired the development of PENNs in structural engineering.

PENNs have been applied to establish surrogate models in structural engineering. As physical laws are embedded into the network architecture, the required volume of training data is significantly reduced. In some cases, the PENNs can be trained even in an unsupervised manner. Yu et al. (2020) employed a deep residual RNN to simulate the system equation in state-space representation, with an MLP embedded to model the observation equation. Sadeghi Eshkevari et al. (2021) developed a recursive NN based on the Newmark-beta method to simulate full-field dynamic responses of nonlinear structural systems. The Newton–Raphson iteration was approximated using recurrent residual blocks. Zhai et al. (2023) mimicked the procedure of the fourth-order Runge–Kutta method to design a PENN for modeling nonlinear dynamic systems. The physics loss is computed by evaluating the governing PDEs with simulated responses and trainable system parameters. The model and system parameters were simultaneously optimized by minimizing the sum of the data loss and physics loss. Song et al. (2023a) proposed an unsupervised GNN to surrogate the elastic static analysis. Structural topology, constitutive relationships, and member configurations were encoded as graph data. The internal force equilibrium equation was used to build the loss function. Liao et al. (2025) introduced a mechanics-informed GNN to simulate the dynamic responses of linear elastic structures. The mode superposition method was embedded into the transformer’s multi-head attention mechanism. The adjacency matrix of the GNN was replaced by the structural stiffness matrix to enforce deformation compatibility.

Various types of PENNs have been proposed to reconstruct structural dynamic responses, and some approaches can reconstruct full-field responses. Lai et al. (2021) introduced a neural ODE to solve the structural dynamic problem. Unlike the original formulation, the derivative in their model was computed as a combination of a physics-based term and an NN-predicted term, which facilitates the incorporation of prior knowledge into the modeling process. Lai et al. (2022) further combined the modal decomposition method with the neural ODE framework to model the dynamics of high-dimensional linear and mildly nonlinear systems. Liu et al. (2022) developed a hybrid modeling framework that combines a physics-based state-space model with a data-driven Deep Markov Model to capture nonlinear system behavior. Liu et al. (2024) further applied NNs within the EKF

framework to model both process dynamics and sensor observations. Li et al. (2023a) proposed a PENN for structural parameter identification by augmenting the governing PDE with NNs. The stiffness and damping parameters were predicted using two independent NNs with structural coordinates as input. These predicted structural parameters were then used to compute structural responses by solving the PDEs. The loss function was defined as the discrepancy between measured and simulated responses, and was minimized to indirectly train the two networks. Following a similar route, Li et al. (2024) employed a transmissibility matrix-based dynamic response simulation framework to design a PENN for response reconstruction. Unknown excitation forces were predicted using a trainable CNN, with measured responses as input. These predicted forces were then combined with known structural matrices to generate full-field responses. Liu et al. (2025) further incorporated the sparsity of the moving force into the framework by separately identifying the load time history and vehicle velocity. The ill-posedness of the inverse problem was thus reduced significantly. Wang et al. (2024) developed a KalmanNet based on the KF framework for noise estimation. An RNN was used to compute Kalman gain coefficients under non-Gaussian noise conditions.

In addition to modeling entire systems, NNs have also been used to simulate component-level behavior, such as material constitutive models. Ghaboussi et al. (1991) were the first to use NNs to learn nonlinear constitutive models from paired strain–stress datasets obtained in static experiments. Ghaboussi et al. (1998) further developed autoprogressive models (Shin et al., 2000; Yun et al., 2008) to expand the static load–deflection experimental data into virtual strain–stress datasets for training the material NN. Huang et al. (2020) integrated the material NN into the nonlinear Poisson equation and utilized the residual of the governing function as the loss function to indirectly train the NN model, eliminating the need for paired strain–stress data. However, this method requires full-field displacement measurements. To facilitate learning with limited measurements, Liu et al. (2021) and Xu et al. (2021) further employed the discrepancy between incomplete measurements and predictions as the training loss. These studies effectively integrated the nonlinear modeling capabilities of NNs with the physical knowledge of FE models to identify material constitutive models from measured responses. However, these studies

are limited to static problems.

PENNs effectively combine the expressive power of DL models with the interpretability of physics-based models, making them a promising framework for efficiently modeling complex systems. However, existing research has predominantly focused on modeling structural systems at the global level, with relatively limited attention given to the identification of structural parameters and component behaviors.

2.5 Summary and Challenges

Physics-based, data-driven, and physics–data co-driven approaches for structural system identification have been comprehensively reviewed. In summary, physics-based approaches construct white-box models solely based on physical laws, such as FE formulations, dynamic equilibrium equations, and state-space representations. These models offer high interpretability but perform poorly when applied to complex structures with significant modeling errors. Data-driven approaches directly learn black-box mapping relationships between inputs and outputs based on training data. In this review, particular attention is given to methods based on DL techniques, due to their high modeling capacity and recent popularity. While they can model complex systems accurately, they typically require large volumes of labeled training data. Physics–data co-driven approaches have been proposed to build grey-box models by integrating the complementary strengths of purely physics-based and data-driven methods. Among them, PINNs and PENNs are representative techniques. PINNs incorporate physical laws as soft constraints in the loss function, whereas PENNs embed physics knowledge into the network architecture or computational graph.

Despite the significant progress achieved by these approaches, the following challenges remain to be addressed:

- (1) Static data for most existing FEMU methods are typically obtained through live-load

tests. The implementation of such field tests involves meticulous preparation, including the design of loading scenarios, sensor installation, and traffic lane closures. These requirements make the testing process labor-intensive, time-consuming, and costly. As a result, the current FEMU method using static data is not suitable for timely structural condition assessment. Although SHM systems have been increasingly deployed to support continuous data acquisition, multi-source monitoring data from these systems are often not effectively incorporated into the FEMU process. For example, time-history responses collected by SHM systems, such as displacement, inclination, and strain, are rarely used in FEMU because the corresponding theoretical responses cannot be computed without accurate load measurements.

- (2) The FEMU process is subject to uncertainties arising from measurement noise, modeling errors, parameter variability, and environmental influences. However, the system uncertainty within the FEMU framework using both static and dynamic data has not been thoroughly quantified. In addition, SHM systems operate under varying environmental and operational conditions, which amplify uncertainty in the acquired data. Besides, current Bayesian FEMU methods generally assume that the PDFs of both modeling errors and updating parameters remain fixed over time. This assumption may not hold in real-world scenarios, where structural dynamic characteristics are significantly influenced by time-varying factors such as temperature, traffic loads, and wind loads. As a result, the fixed-PDF assumption can lead to inaccurate uncertainty quantification, which in turn may mislead structural condition assessment.
- (3) The ill-posedness of current joint load–parameter identification methods is often alleviated through predefining loading patterns or locations. However, such prior knowledge is typically unavailable in real-world applications. Regularization techniques are frequently employed for both loads and structural parameters to promote solution sparsity. The identification accuracy and convergence performance are highly sensitive to the choice of regularization weights. However, these weights are typically determined through empirical adjustment or trial and error in existing studies, which reduces the robustness and generalizability of the identification algorithms.
- (4) The integration of physics-based and data-driven models in current physics–data co-driven identification approaches remains relatively shallow. For instance, PINNs

incorporate governing equations into the loss function only as weak physical constraints during training. As a result, the trained models retain black-box characteristics, require large volumes of training data, and exhibit limited generalization capability. Besides, most existing PENN-based approaches focus on modeling the system at the global level, without the capability to capture component-level behaviors or element-level structural parameters. These limitations reduce their effectiveness in localized damage detection and detailed structural condition assessment.

CHAPTER 3

FINITE ELEMENT MODEL UPDATING USING STATIC AND DYNAMIC MEASUREMENTS THROUGH MULTI-SOURCE DATA FUSION

3.1 Introduction

Integrating both static and dynamic data into the FEMU process can improve identification accuracy (Xiao et al., 2015). However, static data are typically obtained through loading tests, which require extensive preparation and traffic management. As a result, using static and dynamic data in FEMU is impractical for regular condition assessment. To address this issue, this chapter proposes an FEMU method that incorporates both static and dynamic measurements through multi-source data fusion. The framework of the proposed method is shown in Figure 3.1. In this method, the traditional step of “designing the experimental loads” is replaced by “identifying the operational vehicle loads”. The proposed method is validated on a three-span laboratory continuous bridge. The main contributions and innovations of this study are as follows: (1) a computer vision-based load identification technique is introduced to generate load distributions for the static FE simulation, while a smoothing-based signal processing method is employed to extract the measured static responses from the raw monitoring data; (2) an objective function is formulated by integrating vehicle-induced static responses and modal data; and (3) a parallel PSO algorithm is employed to minimize the objective function.

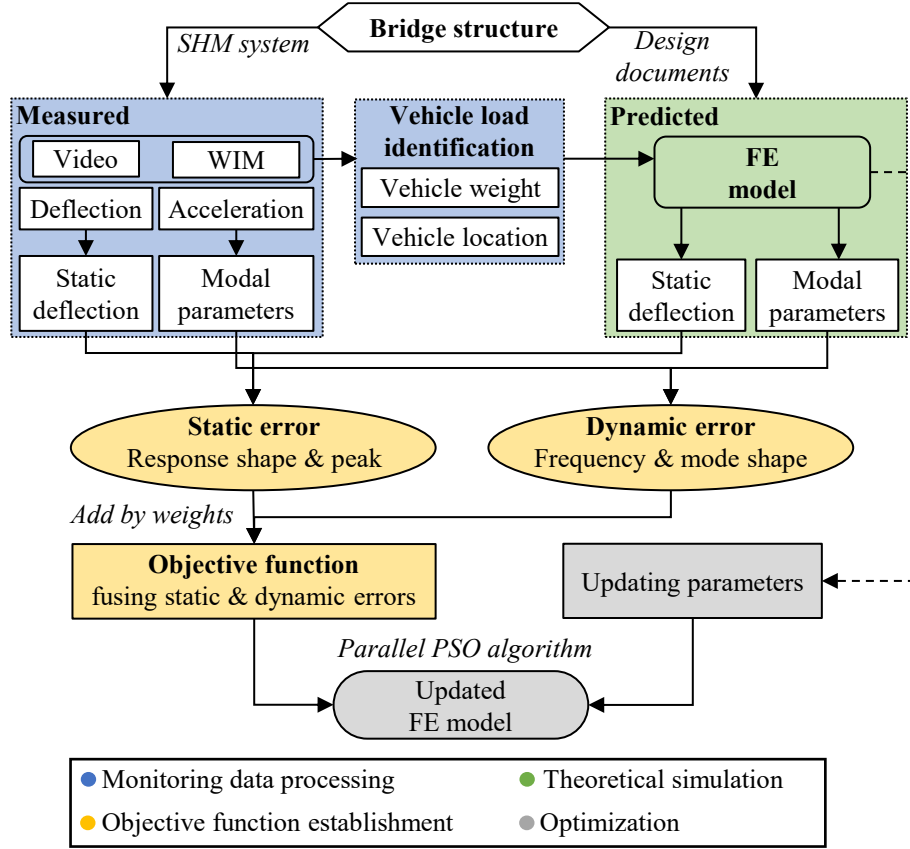


Figure 3.1 Framework of the FEMU method using static and dynamic measurements through multi-source data fusion

3.2 Vehicle-induced Static Response Analysis

3.2.1 Theoretical Response Simulation

The flowchart of simulating theoretical vehicle-induced responses is presented in Figure 3.2. First, the vehicle load identification technique is applied to determine the loading locations and corresponding weights of the vehicles. The vehicle locations are captured using a computer vision-based target tracking algorithm, while the vehicle weights are measured through the weigh-in-motion (WIM) system. The identified loads are subsequently input to the FE model to simulate the corresponding static responses. A detailed description of this process is provided below.

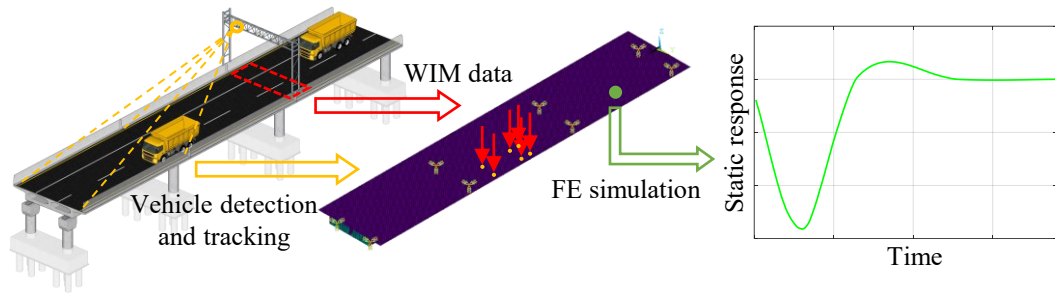
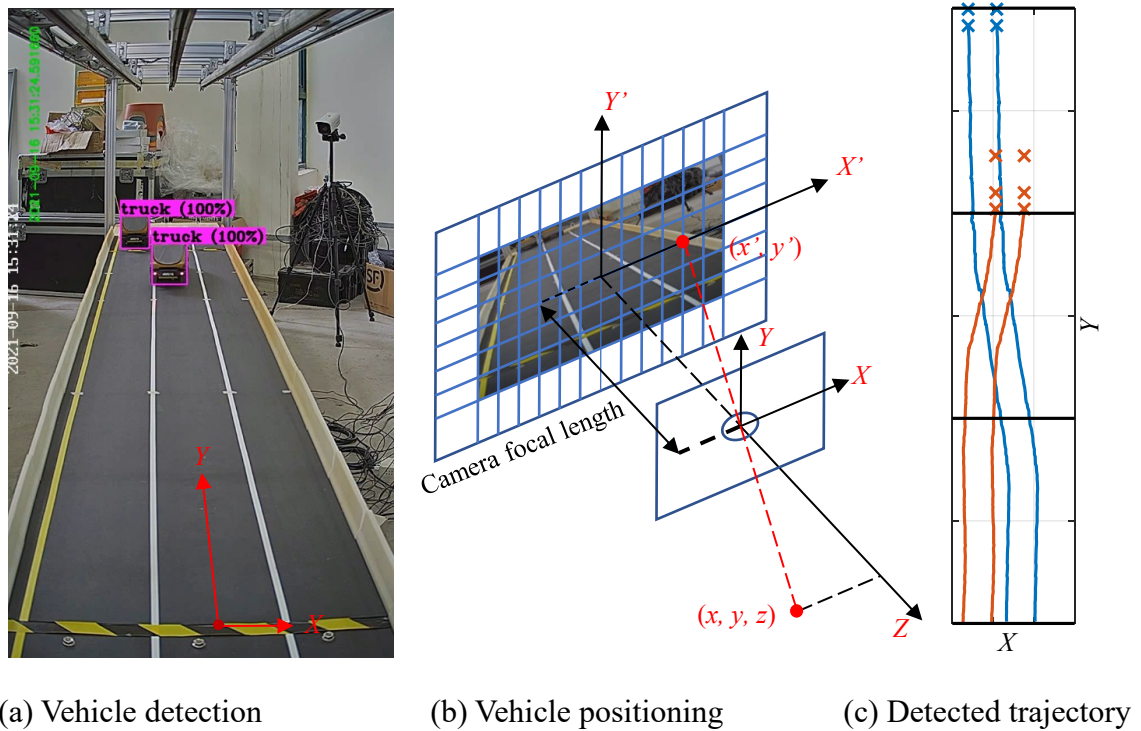


Figure 3.2 Flowchart of vehicle-induced static response simulation

A mature DL-based vehicle trajectory detection method is adopted in this study (Jian et al., 2019; Xia et al., 2019). Its accuracy and efficiency have been validated through practical applications. The overall procedure includes vehicle detection and localization, as illustrated in Figure 3.3.



(a) Vehicle detection (b) Vehicle positioning (c) Detected trajectory

Figure 3.3 Computer vision-based vehicle detection and tracking

First, the YOLOv4 algorithm is employed to detect vehicles from recorded video frames, as shown in Figure 3.3(a). YOLO is a CNN-based single-stage object detection

framework designed for real-time applications. It enables fast and accurate detection and classification of multiple objects within an image. The YOLOv4 model follows a supervised learning paradigm. In the training images, vehicles are annotated with bounding boxes that include information such as pixel coordinates, geometric dimensions, and category labels. Once trained, the YOLOv4 model is used to perform real-time inference on video streams or static images. It outputs vehicle locations (in pixel coordinates), classification confidence scores, object dimensions, and visualization results. Further details on the architecture and deployment of the YOLOv4 algorithm can be found in the original work (Bochkovskiy et al., 2020).

A coordinate transformation based on the principle of geometric similarity is employed to convert pixel coordinates into physical coordinates in real-world space, as illustrated in Figure 3.3(b). Detailed methodologies are available in related references (Jian et al., 2019; Xia et al., 2019; Xu et al., 1996). The complete trajectory of a vehicle crossing the bridge is obtained by performing vehicle detection and localization on a frame-by-frame basis, as shown in Figure 3.3(c).

In practical applications, cameras are typically installed at the entrance to the bridge to capture both top and rear views of approaching vehicles. When a vehicle enters the bridge, a pre-trained YOLOv4 model is employed to identify its type and geometric dimensions. As the vehicle moves across the bridge, the midpoint of the lower edge of its detection bounding box is continuously tracked. The corresponding loading locations are then inferred from the bounding box positions and the known geometric dimensions of the vehicle. The WIM system is typically installed near the bridge entrance to record the axle weight and entry time of each vehicle. By synchronizing the timestamps recorded by the vision system and the WIM system, the vehicle's weight data and its spatial position on the bridge can be accurately fused. As a result, the load distribution on the bridge deck can be effectively characterized, providing essential input for the static FE simulation.

In the FE model, the identified wheel loads are applied as concentrated forces, and the

static analysis is conducted step by step. For linear elastic structures, the static response at each time step is independent of the others. Therefore, the time step can be adjusted based on the desired accuracy and the computational capacity of the hardware. The resulting theoretical static responses at the measurement points are subsequently used in the FEMU process.

3.2.2 Measured Response Extraction

The process of extracting the measured static displacement from raw monitoring data is described in this subsection. The raw displacement response can be decomposed into three main components: vehicle-induced static displacement, vehicle-induced dynamic displacement, and other displacement components primarily caused by environmental influences and noise. Both time-domain and frequency-domain analysis techniques are commonly used for signal extraction and noise reduction in bridge SHM. Representative methods include band-pass filtering and wavelet transform. The band-pass filtering method utilizes the Fourier transform to analyze the frequency spectrum of the displacement signal. A specific frequency band is selected to filter out low-frequency disturbances caused by environmental factors and high-frequency structural vibrations. The static displacement component is then obtained by applying an inverse Fourier transform to the filtered frequency band (Christiano et al., 2003). However, improper selection of frequency thresholds may result in amplitude attenuation or phase distortion in the reconstructed signal. In contrast, the wavelet transform method decomposes the signal into multiple scales in the wavelet domain. The static displacement is then reconstructed using approximation coefficients that represent the quasi-static components of the signal. This process involves several key steps, including the selection of appropriate wavelet basis functions, identification of relevant coefficients, and application of adaptive noise thresholding. As a result, the method is computationally demanding (Kijewski et al., 2003).

Therefore, a conceptually intuitive and procedurally straightforward signal smoothing

method, locally weighted regression (LOWESS) (Cleveland, 1981; Cleveland et al., 1988), is employed in this study for static displacement extraction. This method directly filters out other displacement components and vehicle-induced dynamic responses in the time domain, thereby isolating the vehicle-induced static displacement. The main steps of the LOWESS algorithm are summarized as follows:

- (1) Perform locally weighted linear regression within a moving window of length L , centered at a given instant t , to generate a fitted line within the window.
- (2) Move the window center to each data instant, and repeat step (1) to obtain a regression line at each instant.
- (3) Connect the midpoints of all fitted lines to form the LOWESS curve.

The smoothing effect of the algorithm is primarily governed by the choice of window length L . Figure 3.4 illustrates the extraction process using the measured displacement data from a bridge model as an example. For displacement signals sampled at 200 Hz, a window length of $L = 10 \times 200 = 2,000$ is first applied to eliminate low-frequency displacement components caused by environmental factors, whose variation period typically exceeds 10 s. Subsequently, a shorter window length of $L = 1 \times 200 = 200$ is used to further separate vehicle-induced static displacement from its dynamic counterpart, as the period of static displacement variation typically exceeds 1 s for medium-span bridges. The resulting vehicle-induced static displacement is then compared with the theoretically calculated static response to compute the static error function.

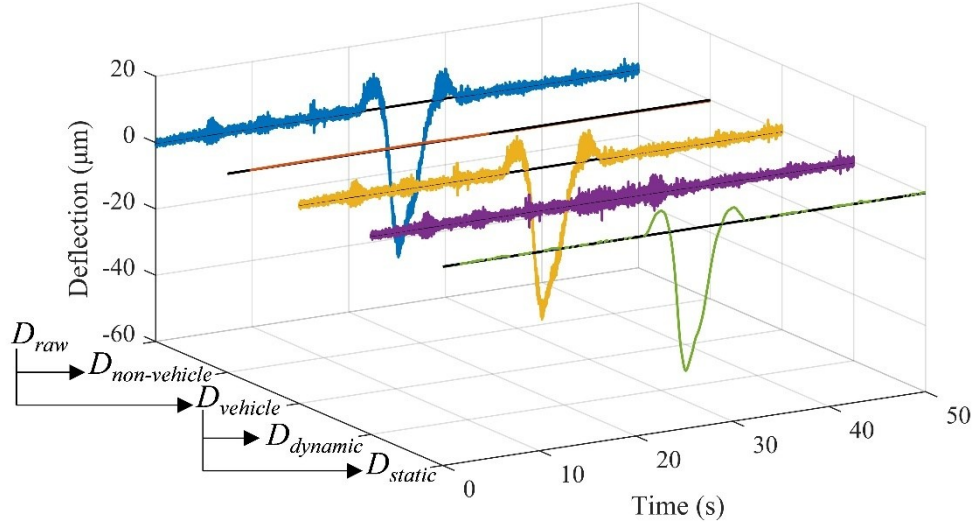


Figure 3.4 Extraction of vehicle-induced static displacement from raw displacement signals

3.3 FEMU Framework Fusing Static and Dynamic Data

The proposed objective function for FEMU and the adopted optimization algorithm are introduced in this section. The overall objective function consists of two components: the static error indicator (SEI) and the dynamic error indicator (DEI). These components are integrated through a weighted summation to form the overall objective. A parallel PSO algorithm is employed to minimize the overall objective function.

3.3.1 SEI

SEI quantifies the discrepancy between the measured and theoretical static deflections by considering two aspects: the difference in deflection shape (DDS) and the difference in extreme deflection (DED). The SEI is defined as

$$SEI(\boldsymbol{\theta}) = \frac{\eta_1}{n} \sum_{i=1}^n DDS_i(\boldsymbol{\theta}) + \sum_{i=1}^n \sum_{j=1}^{m_i} \frac{\eta_2}{nm_i} DED_{i,j}(\boldsymbol{\theta}), \quad (3.1)$$

$$\text{DDS}_i(\boldsymbol{\theta}) = 1 - \frac{\left(\left\{ \tilde{\mathbf{D}}_i \right\}^T \left\{ \mathbf{D}_i(\boldsymbol{\theta}, \mathbf{F}) \right\} \right)^2}{\left(\left\{ \tilde{\mathbf{D}}_i \right\}^T \left\{ \tilde{\mathbf{D}}_i \right\} \right) \cdot \left(\left\{ \mathbf{D}_i(\boldsymbol{\theta}, \mathbf{F}) \right\}^T \left\{ \mathbf{D}_i(\boldsymbol{\theta}, \mathbf{F}) \right\} \right)}, \quad (3.2)$$

$$\text{DED}_{i,j} = \left| 1 - \frac{\mathbf{D}_{i,j}}{\tilde{\mathbf{D}}_{i,j}(\boldsymbol{\theta}, \mathbf{F})} \right|, \quad (3.3)$$

where $\boldsymbol{\theta}$ denotes the vector of updating parameters; i is the index of displacement measurements (n in total); j represents the index of extreme points in the i -th curve (m_i in total); η_1 and η_2 are the weighting factors of DDS and DED, respectively; $\tilde{\mathbf{D}}$ and $\mathbf{D}$ stand for the measured and theoretical static deflection, respectively; and $\mathbf{F}$ represents the load distribution.

3.3.2 DEI

DEI combines the difference in frequency (DF) and the difference in mode shape (DMS), and it is expressed as

$$\text{DEI}(\boldsymbol{\theta}) = \frac{\eta_3}{k} \sum_{l=1}^k \text{DF}_l(\boldsymbol{\theta}) + \frac{\eta_4}{k} \sum_{l=1}^k \text{DMS}_l(\boldsymbol{\theta}), \quad (3.4)$$

$$\text{DF}_l(\boldsymbol{\theta}) = \left| 1 - \frac{\mathbf{f}_l(\boldsymbol{\theta})}{\tilde{\mathbf{f}}_l} \right|, \quad (3.5)$$

$$\text{DMS}_l(\boldsymbol{\theta}) = 1 - \frac{\left(\left\{ \tilde{\boldsymbol{\Phi}}_l \right\}^T \left\{ \boldsymbol{\Phi}_l(\boldsymbol{\theta}) \right\} \right)^2}{\left(\left\{ \tilde{\boldsymbol{\Phi}}_l \right\}^T \left\{ \tilde{\boldsymbol{\Phi}}_l \right\} \right) \left(\left\{ \boldsymbol{\Phi}_l(\boldsymbol{\theta}) \right\}^T \left\{ \boldsymbol{\Phi}_l(\boldsymbol{\theta}) \right\} \right)}, \quad (3.6)$$

where l stands for the mode order (k in total); η_3 and η_4 are the weighting factors of DF and DMS, respectively; $\tilde{\mathbf{f}}$ and $\mathbf{f}$ denote the measured and theoretical natural frequencies, respectively; and $\tilde{\boldsymbol{\Phi}}$ and $\boldsymbol{\Phi}$ represent the measured and theoretical mode shapes, respectively. The minimal value of SEI, as well as those of DDS, DED, DEI, DF, and DMS, is zero. Smaller values indicate a closer agreement between the FE model and the actual structure in terms of static and dynamic behaviors.

3.3.3 Overall Objective Function

The overall objective function $J(\boldsymbol{\theta})$ is composed of SEI and DEI, and is expressed as

$$J(\boldsymbol{\theta}) = \eta_5 \text{SEI}(\boldsymbol{\theta}) + \eta_6 \text{DEI}(\boldsymbol{\theta}), \quad (3.7)$$

where η_5 and η_6 are the weighting factors of SEI and DEI, respectively. By substituting Eqs. (3.1)–(3.6) into Eq.(3.7), the final form of the overall objective function is expressed as

$$J(\boldsymbol{\theta}) = \frac{\eta_5}{n} \sum_{i=1}^n \left[\eta_1 \left(1 - \frac{\left(\{\tilde{\mathbf{Y}}_i\}^T \{\mathbf{Y}_i(\boldsymbol{\theta})\} \right)^2}{\left(\{\tilde{\mathbf{Y}}_i\}^T \{\tilde{\mathbf{Y}}_i\} \right) \cdot \left(\{\mathbf{Y}_i(\boldsymbol{\theta})\}^T \{\mathbf{Y}_i(\boldsymbol{\theta})\} \right)} \right) + \frac{\eta_2}{m_i} \sum_{j=1}^{m_i} \left| 1 - \frac{\mathbf{Y}_{i,j}}{\tilde{\mathbf{Y}}_{i,j}(\boldsymbol{\theta})} \right| \right] + \frac{\eta_6}{k} \sum_{l=1}^k \left[\eta_3 \left| 1 - \frac{\mathbf{f}_l(\boldsymbol{\theta})}{\tilde{\mathbf{f}}_l} \right| + \eta_4 \left(1 - \frac{\left(\{\tilde{\boldsymbol{\Phi}}_l\}^T \{\boldsymbol{\Phi}_l(\boldsymbol{\theta})\} \right)^2}{\left(\{\tilde{\boldsymbol{\Phi}}_l\}^T \{\tilde{\boldsymbol{\Phi}}_l\} \right) \cdot \left(\{\boldsymbol{\Phi}_l(\boldsymbol{\theta})\}^T \{\boldsymbol{\Phi}_l(\boldsymbol{\theta})\} \right)} \right) \right]. \quad (3.8)$$

In this study, all weighting coefficients $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$, and η_6 in the objective function are set to 1 because the error indicators fall within comparable ranges. This setting assigns equal importance to each indicator. In this way, the FEMU problem using both static and dynamic data is formulated as a single-objective optimization problem. The PSO algorithm is then used to minimize the total objective function $J(\boldsymbol{\theta})$ by adjusting the updating parameters $\boldsymbol{\theta}$.

3.3.4 Parallel PSO Algorithm

The PSO algorithm is a global optimization method inspired by the social behaviors observed in bird flocking. It simulates the search process of a particle swarm, where each particle represents a candidate solution. The velocity and position of each particle are dynamically updated based on its own best historical position (p_{best}) and the global best position (g_{best}) found by the entire swarm. PSO is easy to implement, fast in convergence,

and requires only a few hyperparameters. As a result, it has been widely used in function optimization, neural network training, and engineering design problems (Lin et al., 2020b). The procedure of the PSO algorithm is summarized as follows:

- (1) Initialize the algorithm parameters, including the population size M , maximum number of iterations $T_{\max}$, maximum and minimum inertia weights ($w_{\max}$, $w_{\min}$), and acceleration coefficients ζ_1 and ζ_2 .
- (2) Randomly generate the positions and velocities of all particles.
- (3) Calculate the objective function values of each particle.
- (4) For the s -th particle, select the position with the minimum objective function value before the t -th iteration as $p_{t,s}^{\text{best}}$. Select the position with the minimum objective function value among all particles before the t -th iteration as g_t^{best} .
- (5) Update the velocities and positions of each particle as follows:

$$\mathbf{v}_{t+1,s} = w_t \mathbf{v}_{t,s} + \zeta_1 r_1 (\mathbf{p}_{t,s}^{\text{best}} - \mathbf{x}_{t,s}) + \zeta_2 r_2 (\mathbf{g}_t^{\text{best}} - \mathbf{x}_{t,s}), \quad (3.9)$$

$$w_t = w_{\max} - \frac{w_{\max} - w_{\min}}{T_{\max}} \times t, \quad (3.10)$$

$$\mathbf{x}_{t+1,s} = \mathbf{x}_{t,s} + \mathbf{v}_{t+1,s}, \quad (3.11)$$

where $\mathbf{v}_{t,s}$ and $\mathbf{x}_{t,s}$ are the velocity and position vectors of the s -th particle in the t -th iteration, respectively; w_t is the inertial weight in the t -th iteration; and r_1, r_2 are random numbers uniformly distributed in the range $[0, 1]$.

- (6) Repeat steps (3)–(5) until the maximum number of iterations is reached.

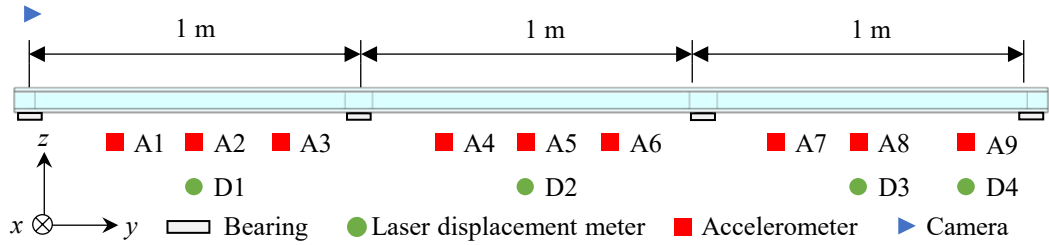
Parallel computing has significantly improved the computational efficiency of many optimization algorithms with the widespread adoption of multi-core processors (Lin et al., 2020a). This technique divides the computational workload into multiple independent subtasks, which are executed concurrently on different processor cores. In the context of PSO, the evaluation of the objective function for each particle within an iteration is independent of others. Therefore, parallel computing is applied in step (3) to evaluate all particles' objective function values simultaneously. As a result, the time cost of FEMU is

substantially reduced.

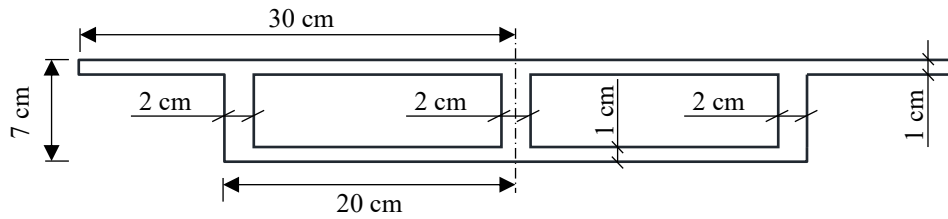
3.4 Experimental Study

3.4.1 Experiment Setup

A three-span continuous bridge model, illustrated in Figure 3.5(a), is used to validate the proposed FEMU method. Its span arrangement is $1\text{ m} \times 3 = 3\text{ m}$, and it adopts a single-box and double-cell cross-section, as shown in Figure 3.5(b). The material of the main girder is polymethyl methacrylate, and all structural plates are bonded using chemical adhesives. Lead blocks are uniformly placed inside the box cells to serve as counterweights, so that the fundamental structural frequency falls within a measurable range.



(a) Configuration and sensor layout



(b) Cross-section dimensions

Figure 3.5 Schematic diagram and cross-sectional view of the three-span continuous bridge model

Three types of sensors, including cameras, accelerometers, and laser displacement meters, were installed on the bridge, as shown in Figure 3.5(a). The camera was mounted at the entrance to continuously record the vehicles as they crossed the bridge. Nine accelerometers (A1–A9) and four laser displacement meters (D1–D4) were positioned beneath the central web. The camera operated at a frame rate of 25 frames per second, while the accelerometers and displacement meters were sampled at 200 Hz. The experimental setup is presented in Figure 3.6.

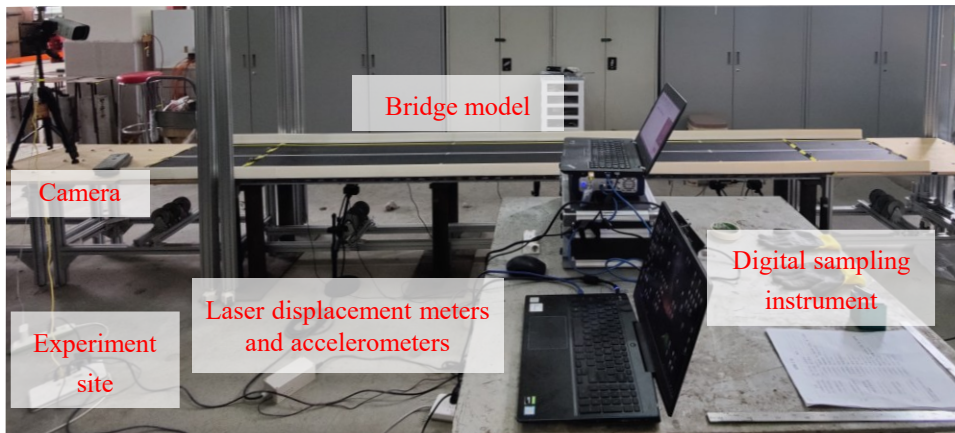


Figure 3.6 Experimental setup of the three-span continuous bridge model

3.4.2 Experimental Scenario

Two three-axle remote-controlled truck models, as shown in Figure 3.7, were used as the test vehicles in the experiment. The distances between the axles were 18 cm and 8 cm, respectively. Clump weights were added to the vehicle compartments to simulate different gross vehicle weights. The total weights and individual axle weights of the two trucks were measured using digital balances. The first vehicle weighed 6,452.3 g, with axle weights of 1,823.5 g, 2,112.5 g, and 2,516.3 g, respectively. The second vehicle weighed 7,181.7 g, with axle weights of 1,972.7 g, 2,404.3 g, and 2,804.7 g, respectively. These measured axle weights, combined with the identified axle positions, were used to construct the vehicle load distribution, which was applied as input to the static FE simulation.

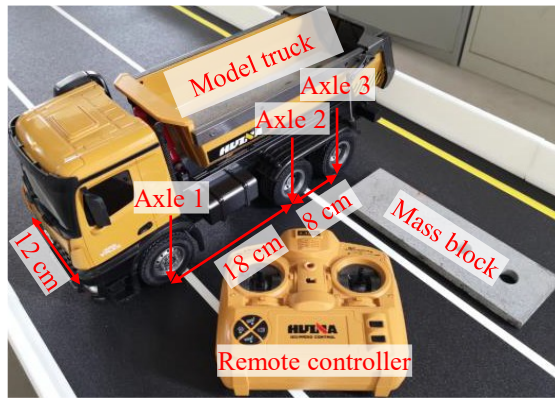


Figure 3.7 Configuration of the truck model

Four vehicle driving scenarios were designed to simulate real-world traffic conditions, with the corresponding experimental details summarized in Table 3.1. Among them, Scenario I, which involves a single vehicle driving straight, represents the most typical and straightforward case. Therefore, the data from this scenario were selected for FEMU. The remaining scenarios were used to validate the accuracy and generalizability of the updated FE model. In practice, the SHM system records sufficient video and response data, allowing operators to select appropriate datasets for FEMU based on actual traffic conditions.

Table 3.1 Scenario description

Scenario No.	Vehicle number	Description	Data usage
I	1	Go straight	Update FE model
II	1	Change lane	Validate the updated model
III	2	Go straight	
IV	2	Change lane	

In addition, a random impact test was conducted on the bridge deck to simulate the white noise excitation. The resulting structural vibration data were used for modal identification.

3.4.3 Initial FE Model

The initial FE model is built based on the design documents. The main girder is modelled using shell elements, with the material density set to $1.39 \times 10^3 \text{ kg/m}^3$, the elastic modulus to $3.3 \times 10^9 \text{ Pa}$, and Poisson's ratio to 0.37. The nodes on all bearings are constrained in the longitudinal, lateral, and vertical DOFs. The rotational stiffness of the bearings around the transverse axis of the bridge is simulated using elastic rotational springs, each with an initial stiffness of $10^4 \text{ N}\cdot\text{m/rad}$. The masses of the lead blocks are modelled using the lumped mass elements. The FE model contains 35,910 elements and 35,715 nodes in total.

The modal parameters and static responses predicted by the initial model are then compared with the measured results. Stochastic subspace identification (SSI) (Peeters et al., 1999) and frequency domain decomposition (FDD) methods were employed to identify the natural frequencies and mode shapes from the acceleration data obtained during the random impact test. The identification results are exhibited in Table 3.2. Only the first-order bending mode was identified due to the relatively high stiffness and damping of the bridge. The first-order frequencies identified by FDD and SSI methods were 38.84 Hz and 38.77 Hz, respectively, with a relative error of 0.19%, indicating the mode is accurately identified. The result from the SSI method was adopted as the reference value. The first-order frequency predicted by the initial FE model is 45.50 Hz, resulting in a relative error of 17.34%.

Table 3.2 Measured and FE model-predicted frequencies

Modal result type		Frequency (Hz)	Relative error (%)	MAC
Measured	FDD	38.84	0.19	1.00
	SSI (Reference)	38.77	-	-
Theoretical	Initial FE Model	45.50	17.34	0.98
	Model updated using static and dynamic data	38.77	0.01	1.00
	Model updated using only dynamic data	38.77	0.00	1.00

The identified and predicted mode shapes are illustrated in Figure 3.8. The MAC value between the mode shapes identified by the two methods is 1.00, while the MAC value between the theoretical and measured results is 0.98, as shown in Table 3.2. These results indicate that the stiffness and mass parameters of the initial FE model require updating.

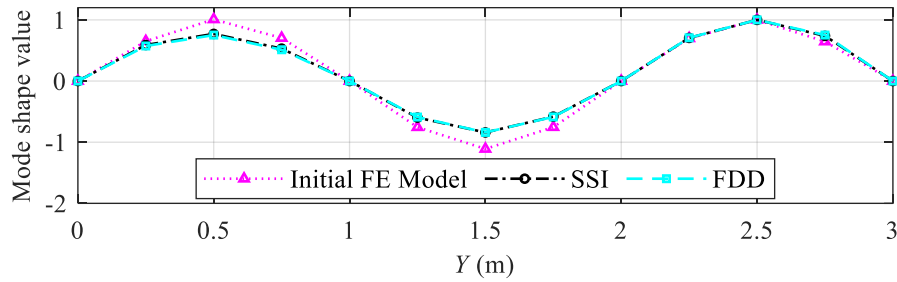


Figure 3.8 Identified and initial FE model-predicted mode shapes

The static deflections predicted by the initial model and corresponding measured responses under Scenario I are compared in Figures 3.9(a)–(d). The trajectories of the wheels corresponding to axle 3 are shown in Figure 3.9(e). Although the overall shapes of the predicted and measured deflection curves are similar, the predicted amplitudes are significantly smaller, indicating that the stiffness parameters in the initial FE model are overestimated.

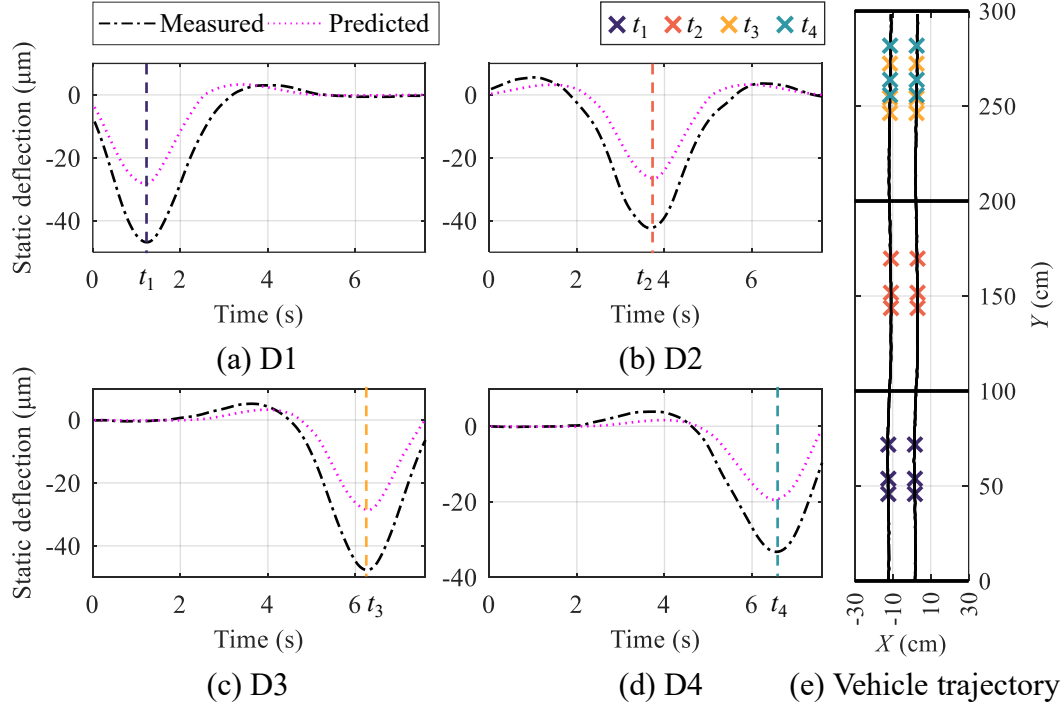


Figure 3.9 Measured and initial FE model-predicted static deflections under Scenario I

3.4.4 Updating Results

The density of the main girder, the transverse rotational stiffness of the four bearings, and the elastic modulus of the girders in the three spans are selected as the updating parameters, resulting in a total of eight updating parameters. All parameters are normalized with respect to their standard values. The objective function is established based on the natural frequency and mode shape of the first mode, as well as the static displacements at measurement points D1, D2, and D3 in Scenario I. The static response at D4 is used to validate the updated FE model. Both the identified load distributions and the measured static responses are downsampled to 2.5 Hz to ensure a consistent time step between the measured and the predicted static responses and to improve computational efficiency. The weighting coefficients η_1 , η_2 , η_3 , η_4 , η_5 , and η_6 in the objective function are all set to 1. In the parallel PSO algorithm, the swarm size M is set to 48, and the maximum number of iterations $T_{\max}$ is set to 300. The evolution of the objective function values for all particles during the optimization process is illustrated in Figure 3.10. The

objective function values continuously decrease as the iterations progress. After approximately 100 iterations, the value of the global best particle stabilizes, indicating convergence of the optimization.

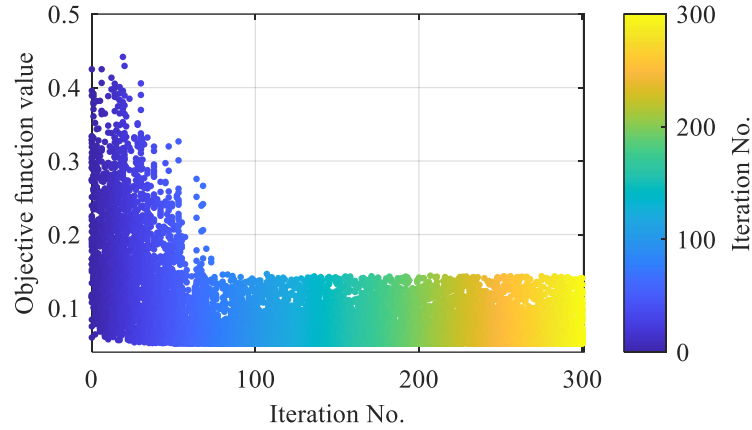


Figure 3.10 Iteration process of the parallel PSO algorithm

The updated parameters are $[0.77, 1.89, 9.57, 7.56, 3.73, 0.61, 0.60, 0.60]$. The normalized density of the main girder is adjusted to 0.77, indicating that the initial material density or the lead block counterweights were overestimated. The rotational stiffness values of the bearings on the middle piers are larger than those on the side piers. The updated elastic modulus of the main girders in all three spans is approximately 0.60, which is consistent with the fact that the girders across all spans share identical cross-sections and are fabricated from the same material. This updated modulus is notably lower than the design value because the polymethyl methacrylate plates are bonded with chemical adhesive, which significantly reduces the overall structural rigidity.

The updated FE model is subsequently utilized to simulate both the static and dynamic structural responses. The first-order natural frequency of the updated model is 38.77 Hz, with a relative error of only 0.01% compared to the measured value, as shown in Table 3.2. The first-order mode shape of the updated FE model is shown in Figure 3.11, and the MAC value between the measured and predicted mode shapes is 1.00. These results

demonstrate that the updated model provides an improved representation of the system's dynamic behavior compared to the initial FE model.

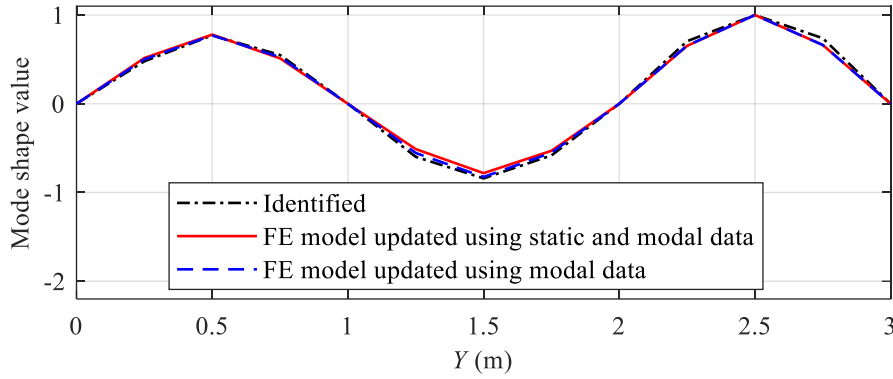


Figure 3.11 First-order mode shape of the updated FE model

To obtain a smooth static displacement curve, the sampling frequency of the theoretical static response during the prediction process is set to 25 Hz. The static deflections predicted by the updated model under Scenario I are shown in Figures 3.12(a)–3.12(d), and they align well with the measured ones. The average relative error at the peak points is 4.88%, which meets the engineering accuracy requirements. Notably, the static response at D4 is accurately predicted, even though the deflection at D4 was not included in the FEMU process. This indicates that the updated model provides an accurate representation of the static behavior of the actual structure. The prediction errors in the static deflections are primarily attributed to two factors: (1) the wheel loads are simplified as concentrated forces in the FE analysis, whereas the vehicle loads are spatially distributed across the contact area in reality; and (2) inaccuracies in vehicle positioning may cause errors in the applied loading locations.

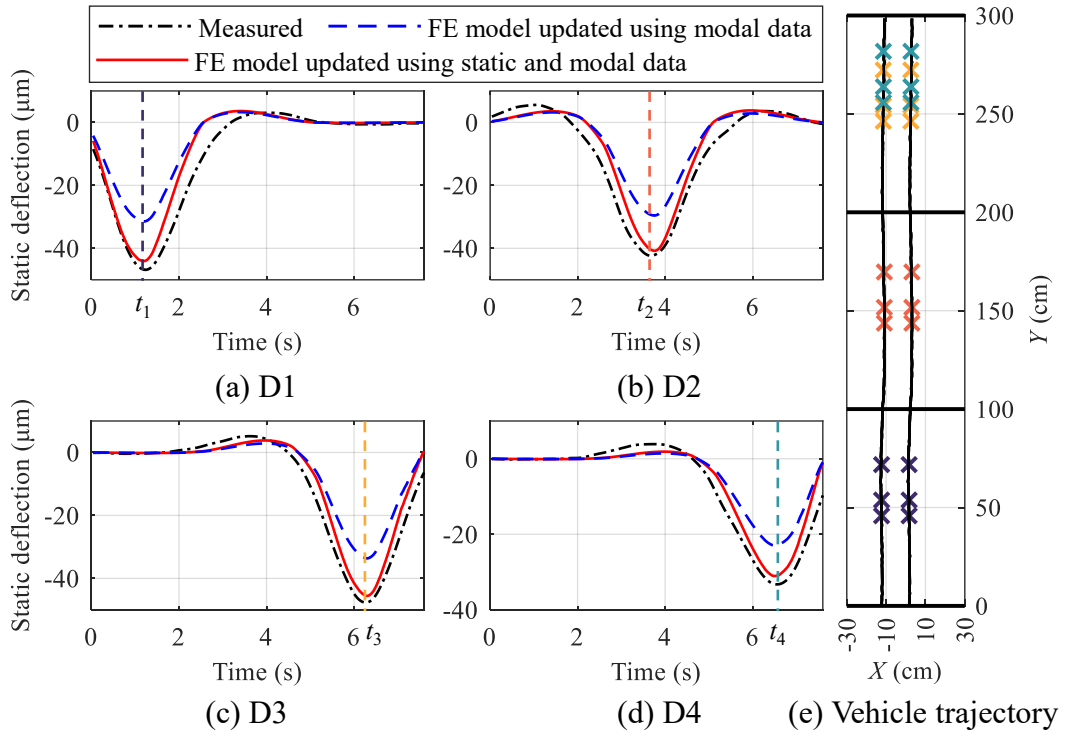
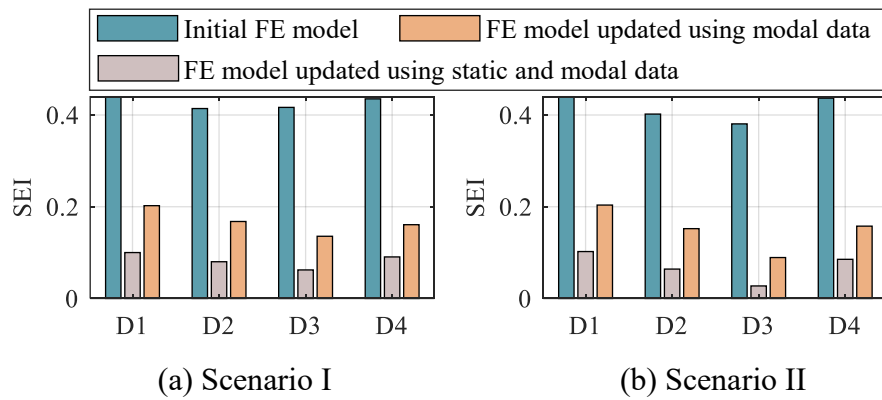


Figure 3.12 Measured and updated model-predicted static deflections under Scenario I

The SEIs for each deflection measurement under Scenario I are calculated according to Eq. (3.1) and are presented in Figure 3.13(a). Compared with the initial FE model, the SEIs of the updated model are significantly reduced, with all values below 0.1.



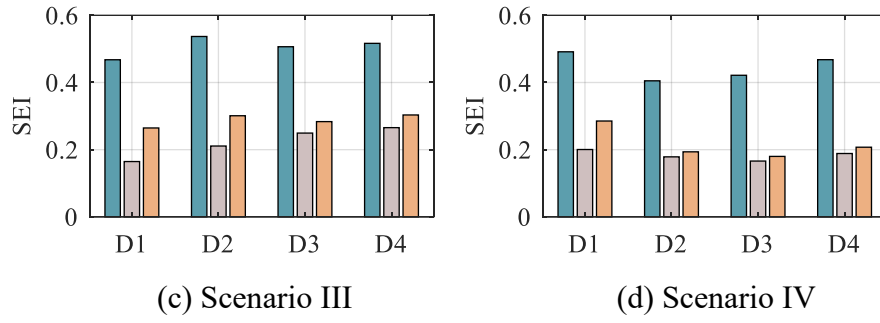


Figure 3.13 SEIs for all scenarios

The predicted static deflections and the corresponding vehicle trajectories under the remaining scenarios are shown in Figures 3.14–3.15. The predicted static responses exhibit a shape similar to that of the measured responses, and their peak values are also in close agreement. Figures 3.13(b)–3.13(d) present the SEIs at each measurement point across all scenarios. Most SEIs are below 0.2, which is significantly lower than those obtained using the initial FE model. These results indicate that the updated model can accurately predict static responses under varying traffic conditions. The SEIs for Scenarios III and IV are higher than those for Scenarios I and II. This is because the accuracy of the vehicle position identification decreases due to vehicle occlusion when multiple vehicles are present on the bridge.

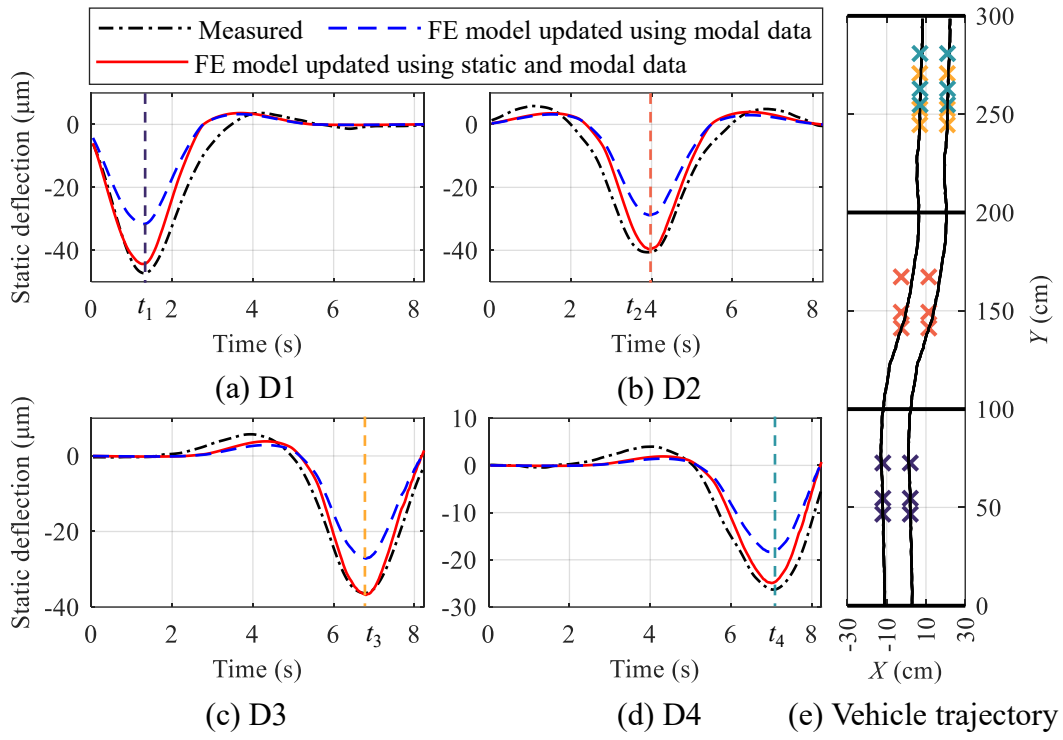


Figure 3.14 Measured and updated model-predicted static deflections under Scenario II

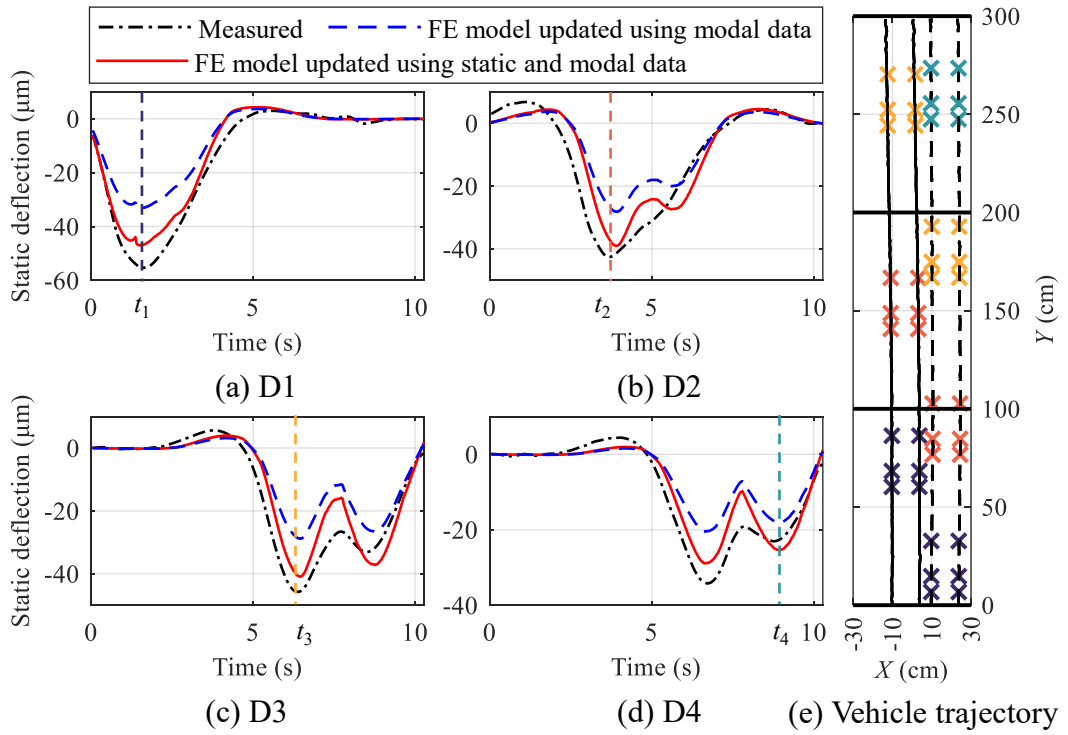


Figure 3.15 Measured and updated model-predicted static deflections under Scenario III

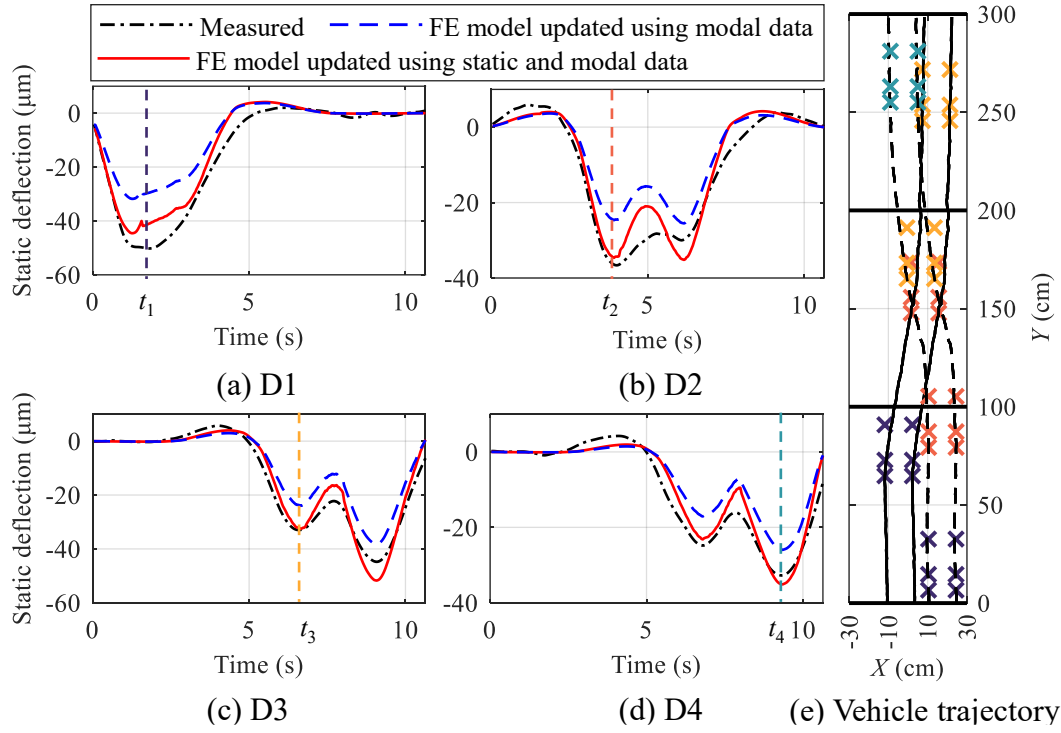


Figure 3.16 Measured and updated model-predicted static deflections under Scenario IV

The overall results indicate that the FE model updated using the proposed method is more accurate than the initial FE model in both static and dynamic characteristics.

3.4.5 Comparison with the Results Using Modal Data Only

The proposed method is compared with a conventional FEMU approach that utilizes only modal parameters, including natural frequencies and mode shapes. The updating parameters and other settings remain consistent with those described in subsection 3.4.4. The updated parameters obtained by the modal-based method are [1.04, 5.45, 3.42, 7.69, 6.10, 0.70, 0.68, 0.65]. The updated material density parameter is 1.04, which is significantly higher than the value of 0.77 obtained using the proposed method. The normalized elastic modulus of the main girders is 0.70, 0.68, and 0.65, respectively. These values are not only higher than the value of 0.60 identified by the proposed method, but also exhibit variation among different spans.

The first-order natural frequency predicted by the model updated using only modal data is 38.77 Hz, as shown in Table 3.2, which exactly matches the measured value. The corresponding predicted mode shape is shown in Figure 3.11, and the MAC value between the predicted and identified mode shapes is 1.00, indicating that the updated model closely matches the actual structure in terms of dynamic characteristics. The static responses predicted by the model updated using only modal parameters are presented in Figure 3.12, and Figures 3.14–3.16. These responses are significantly lower than the measured values across all scenarios because the stiffness parameters are overestimated. The corresponding SEIs, as shown in Figure 3.13, are significantly higher than those obtained from the model updated using both static and dynamic data.

The inaccuracy of the FE model updated using only modal data arises because, for beam-type structures, natural frequencies and mode shapes are determined by the relative ratio of stiffness and mass parameters. When only modal data are used in FEMU, these two parameters can compensate for each other. In contrast, when both static and modal data are incorporated into FEMU, as in the proposed method, the stiffness parameters can be more reliably identified by minimizing the static error indicators, while the mass parameters are simultaneously determined by minimizing the dynamic error indicators.

3.5 Summary

This chapter proposes an FEMU method that integrates both static and dynamic measurements by fusing multi-source monitoring data. A computer vision-based load identification technique and the LOWESS method are employed to facilitate the acquisition of theoretical and measured static responses, respectively. The objective function is established by quantifying the discrepancies between predicted and measured static responses and modal parameters, with a parallel PSO algorithm employed for model updating.

The proposed method was applied to a laboratory three-span bridge model. A random impact test was conducted to excite white noise responses, while four vehicle driving scenarios were conducted to generate vehicle-induced responses. The updated model can accurately predict natural frequencies, mode shapes, and vehicle-induced static deflections at both instrumented and non-instrumented locations under various traffic conditions. Compared with a conventional FEMU method using only modal parameters, the parameters identified by the proposed method are more reasonable, and the predicted static responses are more accurate. However, discrepancies in the predicted static responses persist, indicating the presence of system uncertainties, which will be quantified in the next chapter.

CHAPTER 4

HIERARCHICAL BAYESIAN MODEL UPDATING USING STATIC AND DYNAMIC MEASUREMENTS

4.1 Introduction

Chapter 3 proposed a deterministic FEMU method that integrates static and dynamic measurements through multi-source data fusion. However, system uncertainties were not considered in this approach. In addition, the weighting coefficients in the objective function were manually assigned, which potentially limits the method's applicability. To address these limitations, this chapter proposes a hierarchical Bayesian FEMU method for uncertainty quantification, which extends the framework introduced in Chapter 3. The overall methodology is illustrated in Figure 4.1. Hyperparameters are introduced for both updating parameters and error functions to enhance the uncertainty quantification ability. Each time step of vehicle loading is treated as an independent static test. Error functions are constructed based on the discrepancies between the measured and predicted static responses, as well as modal parameters. The weights of the static and dynamic error terms in the objective function are determined through their respective covariance matrices, which are automatically updated via Bayesian inference. The proposed method is validated on a three-span outdoor continuous bridge model. The main innovations and contributions of this study are as follows: (1) a hierarchical Bayesian FEMU framework using static and dynamic measurements is developed; (2) distinct updating parameters are assigned to the static and dynamic tests to account for the varying sensitivities between parameters and response types; and (3) a two-stage MAP–MCMC sampling approach is proposed to improve the sampling efficiency.

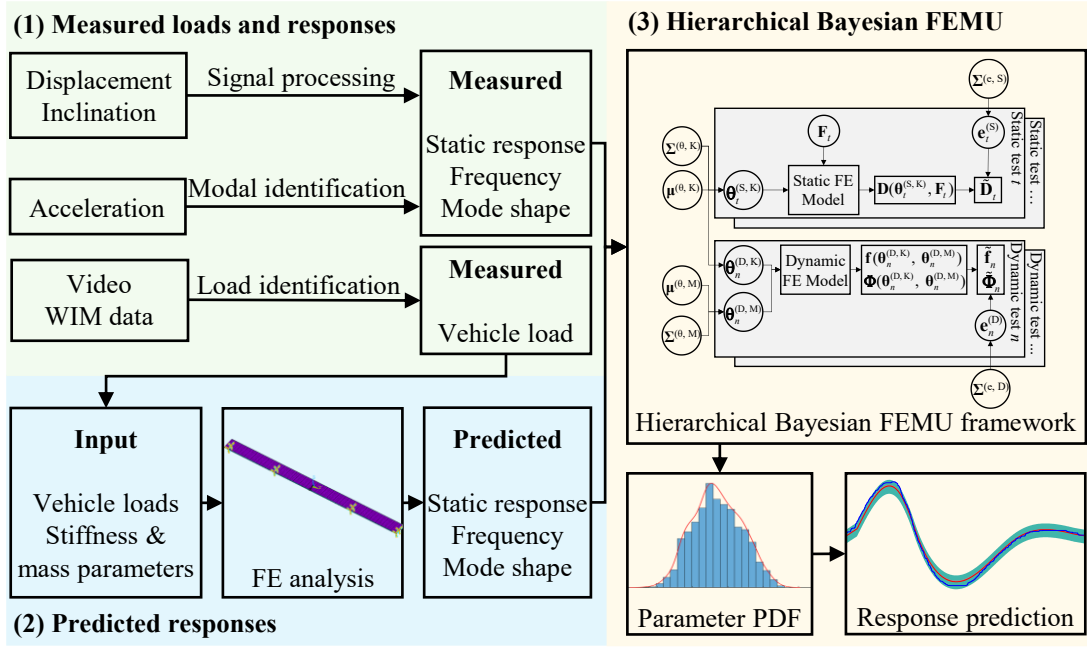


Figure 4.1 Framework of the proposed hierarchical Bayesian FEMU method

4.2 Hierarchical Bayesian FEMU Method Using Static and Dynamic Data

As described in subsection 2.2.2, the hyperparameters are introduced for the error functions and prior distributions in the hierarchical Bayesian framework. Compared with the classical Bayesian method, this framework does not rely on fixed assumptions about the distribution of the measured data and provides enhanced capabilities for uncertainty quantification. This section introduces the proposed hierarchical Bayesian FEMU framework from four perspectives: prior distribution, likelihood function, posterior distribution, and Bayesian inference.

4.2.1 Prior Distribution for Updating Parameters

The static responses are solely related to the stiffness parameters given the load distributions, whereas the modal parameters are influenced by both stiffness and mass parameters. Accordingly, the updating parameters in static tests comprise only stiffness

parameters $\boldsymbol{\theta}_t^{(S, K)}$, where the superscript ‘S’ denotes ‘static’, ‘K’ represents ‘stiffness’, and the subscript t indicates the index of the static experiment (N_a in total). In contrast, the updating parameters in the dynamic tests include both stiffness parameters $\boldsymbol{\theta}_n^{(D, K)}$ and mass parameters $\boldsymbol{\theta}_n^{(D, M)}$, where the superscript ‘D’ denotes ‘dynamic’, ‘M’ stands for ‘mass’, and the subscript n refers to the index of the dynamic experiment (N_b in total). Both $\boldsymbol{\theta}_t^{(S, K)}$ and $\boldsymbol{\theta}_n^{(D, K)}$ represent structural stiffness parameters and should therefore follow the same distribution. They are separated into two groups to explicitly demonstrate the individual contributions of the static and dynamic data to parameter identification and uncertainty quantification. Finally, based on the principle of maximum information entropy, both the stiffness and mass updating parameters are assumed to follow the multivariate normal distributions expressed as

$$\boldsymbol{\theta}_t^{(S, K)} \sim \mathcal{N}(\boldsymbol{\mu}^{(\theta, K)}, \boldsymbol{\Sigma}^{(\theta, K)}), \boldsymbol{\theta}_n^{(D, K)} \sim \mathcal{N}(\boldsymbol{\mu}^{(\theta, K)}, \boldsymbol{\Sigma}^{(\theta, K)}), \quad (4.1)$$

$$\boldsymbol{\theta}_n^{(D, M)} \sim \mathcal{N}(\boldsymbol{\mu}^{(\theta, M)}, \boldsymbol{\Sigma}^{(\theta, M)}), \quad (4.2)$$

where $\boldsymbol{\mu}^{(\theta, K)}$ and $\boldsymbol{\mu}^{(\theta, M)}$ denote the mean vectors of stiffness and mass parameters, respectively; and $\boldsymbol{\Sigma}^{(\theta, K)}$ and $\boldsymbol{\Sigma}^{(\theta, M)}$ represent their corresponding covariance matrices. All these parameters are treated as random variables within the hierarchical Bayesian framework.

The updating parameters are assumed to be independent. As a result, $\boldsymbol{\Sigma}^{(\theta, K)}$ and $\boldsymbol{\Sigma}^{(\theta, M)}$ are taken as diagonal matrices, which are expressed as

$$\boldsymbol{\Sigma}^{(\theta, K)} = \begin{bmatrix} \ddots & & 0 \\ & (\sigma_h^{(\theta, K)})^2 & \\ 0 & & \ddots \end{bmatrix}_{Ng \times Ng}, \boldsymbol{\Sigma}^{(\theta, M)} = \begin{bmatrix} \ddots & & 0 \\ & (\sigma_i^{(\theta, M)})^2 & \\ 0 & & \ddots \end{bmatrix}_{Nk \times Nk}, \quad (4.3)$$

where $(\sigma_h^{(\theta, K)})^2$ denotes the variance of the h -th stiffness parameter (N_g in total), and $(\sigma_i^{(\theta, M)})^2$ denotes the variance of the i -th mass parameter (N_k in total).

4.2.2 Likelihood Function Fusing Static and Dynamic Data

The likelihood functions for the static and dynamic errors are introduced separately in this subsection. Each instant of vehicle loading is regarded as an independent static test. The static error $\mathbf{e}^{(s)}$ is defined as the discrepancy between the measured and theoretical static responses, and it is expressed as

$$\mathbf{e}_{t,j}^{(s)} = \tilde{\mathbf{D}}_{t,j} - \mathbf{D}_j(\boldsymbol{\theta}_t^{(s,K)}, \mathbf{F}_t), \quad (4.4)$$

where $\mathbf{e}_{t,j}^{(s)}$ represents the static error at the j -th measurement (N_c in total) in the t -th static test (N_a in total); $\tilde{\mathbf{D}}$ and $\mathbf{D}$ are the measured and predicted static responses, respectively, which are obtained through the response simulation and signal processing techniques introduced in Chapter 3; and $\mathbf{F}_t$ denotes the identified vehicle load in the t -th static test. The static errors primarily arise from the modelling errors, measurement noise, and response extraction inaccuracies. Modeling error serves as a “trash bin” for uncertainties that cannot be fully explained by parameter uncertainty alone. Bayesian inference provides a systematic framework to allocate these two types of uncertainty based on the available data and the chosen model. When observational data are limited, uncertainty quantification becomes sensitive to outliers or extreme values, making it difficult to reliably distinguish between parameter uncertainty and modeling error. However, as the amount of data increases, the distributions of both parameters and modeling errors tend to stabilize, allowing for a clearer distinction between them.

According to the central limit theorem, the static error $\mathbf{e}_t^{(s)}$ is assumed to follow a zero-mean multivariate normal distribution, which is expressed as

$$\mathbf{e}_t^{(s)} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}^{(e,s)}), \quad (4.5)$$

where $\boldsymbol{\Sigma}^{(e,s)}$ denotes the covariance matrix of the static error function. The static errors at different measurement points are assumed to be independent; therefore, $\boldsymbol{\Sigma}^{(e,s)}$ is taken as a diagonal matrix, which is expressed as

$$\mathbf{\Sigma}^{(e,S)} = \begin{bmatrix} \ddots & & 0 \\ & (\sigma_j^{(e,S)})^2 & \\ 0 & & \ddots \end{bmatrix}_{N_c \times N_c}, \quad (4.6)$$

where $(\sigma_j^{(e,S)})^2$ represents the variance of the static error at the j -th measurement point (N_c in total).

Based on the principle of PDF transformation, the likelihood function for the t -th static test is expressed as

$$\begin{aligned} p(\tilde{\mathbf{D}}_t | \boldsymbol{\theta}_t^{(S,K)}, \mathbf{\Sigma}^{(e,S)}, \mathbf{F}_t) &= \frac{\partial \mathbf{e}_t^{(S)}}{\partial \tilde{\mathbf{D}}_t} p(\mathbf{e}_t^{(S)} | \boldsymbol{\theta}_t^{(S,K)}, \mathbf{\Sigma}^{(e,S)}, \mathbf{F}_t) = p(\mathbf{e}_t^{(S)} | \boldsymbol{\theta}_t^{(S,K)}, \mathbf{\Sigma}^{(e,S)}, \mathbf{F}_t) \\ &= \frac{1}{(2\pi)^{N_c/2} |\mathbf{\Sigma}^{(e,S)}|^{1/2}} \exp\left[-\frac{1}{2} (\mathbf{e}_t^{(S)})^T (\mathbf{\Sigma}^{(e,S)})^{-1} \mathbf{e}_t^{(S)}\right] \end{aligned} \quad (4.7)$$

The dynamic error function $\mathbf{e}_n^{(D)}$ in the n -th dynamic test (N_b in total) consists of two components: the frequency error $\mathbf{e}_n^{(f)}$ and the mode shape error $\mathbf{e}_n^{(\Phi)}$, which are defined as

$$\mathbf{e}_{n,m}^{(f)} = \tilde{\mathbf{f}}_{n,m} - \mathbf{f}_m(\boldsymbol{\theta}_n^{(D,K)}, \boldsymbol{\theta}_n^{(D,M)}), \quad (4.8)$$

$$\mathbf{e}_{n,m}^{(\Phi)} = \frac{\tilde{\Phi}_{n,m}}{\|\tilde{\Phi}_{n,m}\|} - a_{n,m} \frac{\Phi_m(\boldsymbol{\theta}_n^{(D,K)}, \boldsymbol{\theta}_n^{(D,M)})}{\|\Phi_m(\boldsymbol{\theta}_n^{(D,K)}, \boldsymbol{\theta}_n^{(D,M)})\|}, \quad (4.9)$$

$$a_{n,m} = \frac{\tilde{\Phi}_{n,m}^T}{\|\tilde{\Phi}_{n,m}\|} \cdot \frac{\Phi_m(\boldsymbol{\theta}_n^{(D,K)}, \boldsymbol{\theta}_n^{(D,M)})}{\|\Phi_m(\boldsymbol{\theta}_n^{(D,K)}, \boldsymbol{\theta}_n^{(D,M)})\|}, \quad (4.10)$$

where $\mathbf{f}$ and Φ represent the predicted frequencies and mode shapes, respectively; $\tilde{\mathbf{f}}$ and $\tilde{\Phi}$ are the measured counterparts; $\|\cdot\|$ denotes the Euclidean norm of a vector; m is the modal order; and $a_{n,m}$ is the mode shape scaling factor.

Similar to the static error function, the dynamic error function is assumed to follow a zero-mean multivariate normal distribution, and it is expressed as

$$\mathbf{e}_n^{(D)} = \begin{bmatrix} \mathbf{e}_n^{(f)} \\ \mathbf{e}_n^{(\Phi)} \end{bmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}^{(e, D)}), \quad (4.11)$$

where $\mathbf{\Sigma}^{(e, D)}$ denotes the covariance matrix of the dynamic error function. The dynamic errors are assumed to be independent; therefore, $\mathbf{\Sigma}^{(e, D)}$ is modeled as a diagonal matrix:

$$\mathbf{\Sigma}^{(e, D)} = \begin{bmatrix} \ddots & & 0 \\ & (\sigma_r^{(e, D)})^2 & \\ 0 & & \ddots \end{bmatrix}_{Nd \times Nd}, \quad (4.12)$$

where $(\sigma_r^{(e, D)})^2$ is the variance associated with the r -th dynamic error (N_d in total).

Consequently, the likelihood function for the n -th dynamic test is expressed as

$$p(\tilde{\mathbf{f}}_n, \tilde{\Phi}_n | \boldsymbol{\theta}_n^{(D, K)}, \boldsymbol{\theta}_n^{(D, M)}, \mathbf{\Sigma}^{(e, D)}) = \frac{1}{(2\pi)^{Nd/2} |\mathbf{\Sigma}^{(e, D)}|^{1/2}} \exp\left[-\frac{1}{2} (\mathbf{e}_n^{(D)})^T (\mathbf{\Sigma}^{(e, D)})^{-1} \mathbf{e}_n^{(D)}\right]. \quad (4.13)$$

4.2.3 Posterior PDF

The relationships among the parameters in the proposed hierarchical Bayesian framework are illustrated in Figure 4.2.

According to the hierarchical Bayesian theorem, the joint posterior PDF of the updating parameters and hyperparameters is proportional to the product of the likelihood functions, prior distributions, and hyperprior distributions. Therefore, the final posterior PDF is expressed as

$$\begin{aligned}
& p(\Theta^{(S,K)}, \Theta^{(D,K)}, \Theta^{(D,M)}, \mu^{(\theta,K)}, \Sigma^{(\theta,K)}, \mu^{(\theta,M)}, \Sigma^{(\theta,M)}, \Sigma^{(e,S)}, \Sigma^{(e,D)} | \mathbf{F}, \tilde{\mathbf{D}}, \tilde{\mathbf{f}}, \tilde{\Phi}) \\
& \propto p(\tilde{\mathbf{D}} | \Theta^{(S,K)}, \Sigma^{(e,S)}, \mathbf{F}) p(\Theta^{(S,K)} | \mu^{(\theta,K)}, \Sigma^{(\theta,K)}) p(\tilde{\mathbf{f}}, \tilde{\Phi} | \Theta^{(D,K)}, \Theta^{(D,M)}, \Sigma^{(e,D)}) \\
& \quad p(\Theta^{(D,K)} | \mu^{(\theta,K)}, \Sigma^{(\theta,K)}) p(\Theta^{(D,M)} | \mu^{(\theta,M)}, \Sigma^{(\theta,M)}) \\
& \quad p(\mu^{(\theta,K)}) p(\Sigma^{(\theta,K)}) p(\mu^{(\theta,M)}) p(\Sigma^{(\theta,M)}) p(\Sigma^{(e,S)}) p(\Sigma^{(e,D)}) \\
& \propto \prod_{t=1}^{N_a} \left\{ p(\tilde{\mathbf{D}}_t | \theta_t^{(S,K)}, \Sigma^{(e,S)}, \mathbf{F}_t) p(\theta_t^{(S,K)} | \mu^{(\theta,K)}, \Sigma^{(\theta,K)}) \right\} \\
& \quad \prod_{n=1}^{N_b} \left\{ p(\tilde{\mathbf{f}}_n, \tilde{\Phi}_n | \theta_n^{(D,K)}, \theta_n^{(D,M)}, \Sigma^{(e,D)}) p(\theta_n^{(D,K)} | \mu^{(\theta,K)}, \Sigma^{(\theta,K)}) p(\theta_n^{(D,M)} | \mu^{(\theta,M)}, \Sigma^{(\theta,M)}) \right\} \\
& \quad p(\mu^{(\theta,K)}) p(\Sigma^{(\theta,K)}) p(\mu^{(\theta,M)}) p(\Sigma^{(\theta,M)}) p(\Sigma^{(e,S)}) p(\Sigma^{(e,D)})
\end{aligned} \tag{4.14}$$

where $\Theta^{(S,K)} = [\theta_1^{(S,K)}, \dots, \theta_t^{(S,K)}, \dots, \theta_{N_a}^{(S,K)}]$ is the set of all the stiffness updating parameters in the static tests; and $p(\Sigma^{(\theta,K)}) = \prod_h^{Ng} p[(\sigma_h^{(\theta,K)})^2]$, $p(\Sigma^{(\theta,M)}) = \prod_i^{Nk} p[(\sigma_i^{(\theta,M)})^2]$, $p(\Sigma^{(e,S)}) = \prod_j^{Nc} p[(\sigma_j^{(e,S)})^2]$, and $p(\Sigma^{(e,D)}) = \prod_r^{Nd} p[(\sigma_r^{(e,D)})^2]$ because these covariance matrices are assumed to be diagonal.

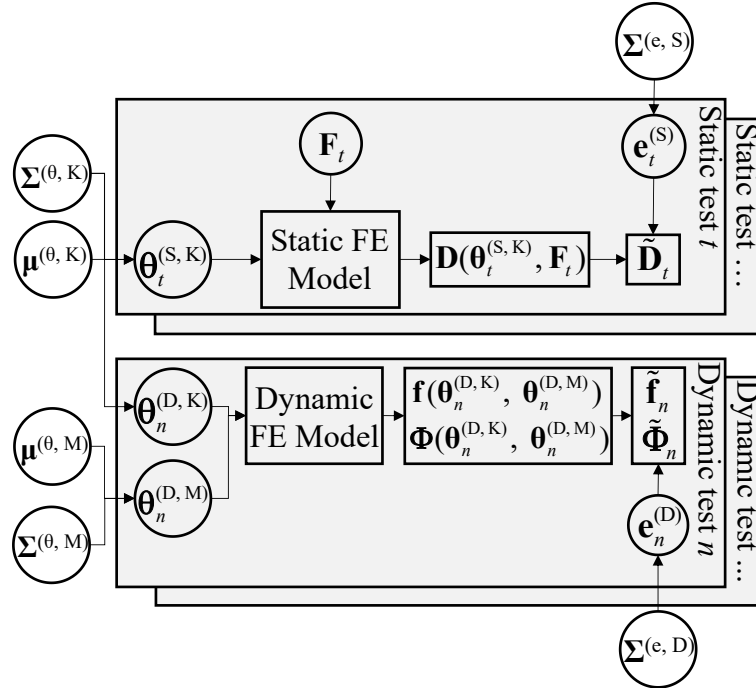


Figure 4.2 Parameter relationships in the proposed hierarchical Bayesian framework

The conjugate priors are adopted for the hyperparameters to facilitate analytical derivation and simplify the sampling process. Uniform distributions are adopted for $p[\boldsymbol{\mu}^{(\theta, K)}]$ and $p[\boldsymbol{\mu}^{(\theta, M)}]$ due to the lack of prior information about the mean vectors. The scaled inverse- χ^2 distribution, which is commonly adopted as the prior distribution for the variance of the normal distribution, is assigned to $p[(\sigma_h^{(\theta, K)})^2]$, $p[(\sigma_i^{(\theta, M)})^2]$, $p[(\sigma_j^{(e, S)})^2]$, and $p[(\sigma_r^{(e, D)})^2]$. The PDF of the scaled inverse- χ^2 distribution is given by:

$$p(u | \lambda, \tau_0) = \frac{\left(\frac{\lambda \tau_0}{2}\right)^{\frac{\lambda}{2}} \exp\left(-\frac{\lambda \tau_0}{2u}\right)}{\Gamma\left(\frac{\lambda}{2}\right) x^{\frac{\lambda}{2}+1}}, \quad (4.15)$$

where u denotes the random variable (i.e., the hyperparameters of the variances in this study); Γ denotes the gamma function; λ refers to the number of degrees of freedom; and τ_0 is the scaling parameter. The scaled inverse- χ^2 distribution is parameterized by λ and τ_0 . The parameter λ is an integral and reflects the confidence incorporated into the prior. A larger λ results in a more concentrated prior, while a smaller λ yields a more diffuse prior. The parameter τ_0 represents the guess for the variance. If historical data or domain knowledge is available, τ_0 can be set to the sample variance; otherwise, in the absence of prior information, τ_0 can be assigned a relatively large value to ensure a weakly informative prior.

After substituting Eqs. (4.1), (4.2), (4.7), and (4.13) into Eq. (4.14), the final posterior PDF is expressed as

$$p(\boldsymbol{\Theta}^{(S, K)}, \boldsymbol{\Theta}^{(D, K)}, \boldsymbol{\Theta}^{(D, M)}, \boldsymbol{\mu}^{(\theta, K)}, \boldsymbol{\Sigma}^{(\theta, K)}, \boldsymbol{\mu}^{(\theta, M)}, \boldsymbol{\Sigma}^{(\theta, M)}, \boldsymbol{\Sigma}^{(e, S)}, \boldsymbol{\Sigma}^{(e, D)} | \mathbf{F}, \tilde{\mathbf{D}}, \tilde{\mathbf{f}}, \tilde{\Phi})$$

$$\propto |\boldsymbol{\Sigma}^{(\theta, K)}|^{-\frac{N_a + N_b + v + 2}{2}} |\boldsymbol{\Sigma}^{(\theta, M)}|^{-\frac{N_b + v + 2}{2}} |\boldsymbol{\Sigma}^{(e, S)}|^{-\frac{N_a + v + 2}{2}} |\boldsymbol{\Sigma}^{(e, D)}|^{-\frac{N_b + v + 2}{2}}, \quad (4.16)$$

$$\exp \left\{ -\sum_{t=1}^{N_a} (J_t^{(e, S)} + J_t^{(\theta, S, K)}) - \sum_{n=1}^{N_b} (J_n^{(e, D)} + J_n^{(\theta, D, K)} + J_n^{(\theta, D, M)}) \right.$$

$$\left. - \sum_{h=1}^{N_g} \frac{\lambda \tau_{0h}^{(\theta, K)}}{2(\sigma_h^{(\theta, K)})^2} - \sum_{i=1}^{N_k} \frac{\lambda \tau_{0i}^{(\theta, M)}}{2(\sigma_i^{(\theta, M)})^2} - \sum_{j=1}^{N_c} \frac{\lambda \tau_{0j}^{(e, S)}}{2(\sigma_j^{(e, S)})^2} - \sum_{r=1}^{N_d} \frac{\tau_{0r}^{(e, D)}}{2(\sigma_r^{(e, D)})^2} \right\}$$

$$J_t^{(e, S)} = \frac{(\mathbf{e}_t^{(S)})^T (\boldsymbol{\Sigma}^{(e, S)})^{-1} (\mathbf{e}_t^{(S)})}{2}, \quad (4.17)$$

$$J_t^{(\theta, S, K)} = \frac{(\boldsymbol{\theta}_t^{(S, K)} - \boldsymbol{\mu}^{(\theta, K)})^T (\boldsymbol{\Sigma}^{(\theta, K)})^{-1} (\boldsymbol{\theta}_t^{(S, K)} - \boldsymbol{\mu}^{(\theta, K)})}{2}, \quad (4.18)$$

$$J_n^{(e, D)} = \frac{(\mathbf{e}_n^{(D)})^T (\boldsymbol{\Sigma}^{(e, D)})^{-1} (\mathbf{e}_n^{(D)})}{2}, \quad (4.19)$$

$$J_n^{(\theta, D, K)} = \frac{(\boldsymbol{\theta}_n^{(D, K)} - \boldsymbol{\mu}^{(\theta, K)})^T (\boldsymbol{\Sigma}^{(\theta, K)})^{-1} (\boldsymbol{\theta}_n^{(D, K)} - \boldsymbol{\mu}^{(\theta, K)})}{2}, \quad (4.20)$$

$$J_n^{(\theta, D, M)} = \frac{(\boldsymbol{\theta}_n^{(D, M)} - \boldsymbol{\mu}^{(\theta, M)})^T (\boldsymbol{\Sigma}^{(\theta, M)})^{-1} (\boldsymbol{\theta}_n^{(D, M)} - \boldsymbol{\mu}^{(\theta, M)})}{2}. \quad (4.21)$$

4.2.4 Two-stage MCMC Sampling Approach

The MCMC method is used to generate samples from the posterior PDF. Among various sampling techniques, Gibbs sampling is preferred in the hierarchical Bayesian framework because it guarantees a 100% acceptance rate, which remains effective even when the number of parameters to be identified is relatively large. However, Gibbs sampling requires that the conditional probability distributions of the parameters follow standard forms. In practice, not all parameters satisfy this requirement. Therefore, a joint Gibbs–MH sampling strategy is adopted. Gibbs sampling is used for parameters whose conditional PDFs can be explicitly expressed in standard forms, and MH sampling is applied for parameters with proportional PDFs.

Meanwhile, to avoid the inefficiency of the burn-in period, the MAP estimates of these parameters are first computed and then used as the initial values of MCMC sampling. According to the posterior PDFs given in Eq. (4.16)–(4.21), the conditional posterior PDFs for each parameter are expressed as

$$p(\boldsymbol{\theta}_t^{(S, K)} | \cdot) \propto \exp(-J_t^{(e, S)} - J_t^{(\theta, S, K)}), \quad (4.22)$$

$$p([\boldsymbol{\theta}_n^{(D, K)}, \boldsymbol{\theta}_n^{(D, M)}] | \cdot) \propto \exp(-J_n^{(e, D)} - J_n^{(\theta, D, K)} - J_n^{(\theta, D, M)}), \quad (4.23)$$

$$p(\boldsymbol{\mu}^{(\theta, K)} | \cdot) = \mathcal{N}\left(\frac{1}{N_a + N_b} \left(\sum_{t=1}^{N_a} \boldsymbol{\theta}_t^{(S, K)} + \sum_{n=1}^{N_b} \boldsymbol{\theta}_n^{(D, K)} \right), \frac{\boldsymbol{\Sigma}^{(\theta, K)}}{N_a + N_b}\right), \quad (4.24)$$

$$p[(\sigma_h^{(\theta, K)})^2 | \cdot] = \text{Inverse} - \chi^2(N_a + N_b + \lambda, \frac{\lambda \tau_{0h}^{(\theta, K)} + \sum_{t=1}^{N_a} (\boldsymbol{\theta}_{t,h}^{(S, K)} - \boldsymbol{\mu}_h^{(\theta, K)})^2 + \sum_{n=1}^{N_b} (\boldsymbol{\theta}_{n,h}^{(D, K)} - \boldsymbol{\mu}_h^{(\theta, K)})^2}{N_a + N_b + \lambda}), \quad (4.25)$$

$$p(\boldsymbol{\mu}^{(\theta, M)} | \cdot) = \mathcal{N}(\frac{1}{N_b} \sum_{n=1}^{N_b} \boldsymbol{\theta}_n^{(D, M)}, \frac{\boldsymbol{\Sigma}^{(\theta, M)}}{N_b}), \quad (4.26)$$

$$p[(\sigma_i^{(\theta, M)})^2 | \cdot] = \text{Inverse} - \chi^2(N_b + \lambda, \frac{\lambda \tau_{0i}^{(\theta, M)} + \sum_{n=1}^{N_b} (\boldsymbol{\theta}_{n,i}^{(D, M)} - \boldsymbol{\mu}_i^{(\theta, M)})^2}{N_b + \lambda}), \quad (4.27)$$

$$p[(\sigma_j^{(e, S)})^2 | \cdot] = \text{Inverse} - \chi^2(N_a + \lambda, \frac{\lambda \tau_{0j}^{(e, S)} + \sum_{t=1}^{N_a} (\mathbf{e}_{t,j}^{(S)})^2}{N_a + \lambda}), \quad (4.28)$$

$$p[(\sigma_r^{(e, D)})^2 | \cdot] = \text{Inverse} - \chi^2(N_b + \lambda, \frac{\lambda \tau_{0r}^{(e, D)} + \sum_{n=1}^{N_b} (\mathbf{e}_{n,r}^{(D)})^2}{N_b + \lambda}). \quad (4.29)$$

At the initial stage, the MAP estimates of each parameter are calculated through the following iterative procedure:

- (1) Initialize the hyperparameters ${}^0 \hat{\boldsymbol{\mu}}^{(\theta, K)}$, ${}^0 \hat{\boldsymbol{\mu}}^{(\theta, M)}$, ${}^0 \hat{\boldsymbol{\Sigma}}^{(\theta, K)}$, ${}^0 \hat{\boldsymbol{\Sigma}}^{(\theta, M)}$, ${}^0 \hat{\boldsymbol{\Sigma}}^{(e, S)}$, and ${}^0 \hat{\boldsymbol{\Sigma}}^{(e, D)}$.
- (2) Update the MAP estimate of $\boldsymbol{\theta}_t^{(S, K)}$ in the l -th iteration using ${}^{l-1} \hat{\boldsymbol{\mu}}^{(K)}$, ${}^{l-1} \hat{\boldsymbol{\Sigma}}^{(\theta, K)}$, and ${}^{l-1} \hat{\boldsymbol{\Sigma}}^{(e, S)}$: ${}^l \hat{\boldsymbol{\theta}}_t^{(S, K)} = \arg \min_{\boldsymbol{\theta}_t^{(S, K)}} (J_t^{(e, S)} + J_t^{(\theta, S, K)})$.

- (3) Update the MAP estimates of $\boldsymbol{\theta}_n^{(D, K)}$ and $\boldsymbol{\theta}_n^{(D, M)}$ in the l -th iteration using ${}^{l-1} \hat{\boldsymbol{\mu}}^{(\theta, K)}$, ${}^{l-1} \hat{\boldsymbol{\Sigma}}^{(\theta, K)}$, ${}^{l-1} \hat{\boldsymbol{\mu}}^{(\theta, M)}$, ${}^{l-1} \hat{\boldsymbol{\Sigma}}^{(\theta, M)}$, and ${}^{l-1} \hat{\boldsymbol{\Sigma}}^{(e, D)}$:

$$[{}^l \hat{\boldsymbol{\theta}}_n^{(D, K)}, {}^l \hat{\boldsymbol{\theta}}_n^{(D, M)}] = \arg \min_{\boldsymbol{\theta}_n^{(S, K)}, \boldsymbol{\theta}_n^{(D, K)}} (J_n^{(e, D)} + J_n^{(\theta, D, K)} + J_n^{(\theta, D, M)}).$$

- (4) Update the MAP estimates of the mean vectors:

$${}^l \hat{\boldsymbol{\mu}}^{(\theta, K)} = \frac{1}{N_a + N_b} (\sum_{t=1}^{N_a} {}^l \hat{\boldsymbol{\theta}}_t^{(S, K)} + \sum_{n=1}^{N_b} {}^l \hat{\boldsymbol{\theta}}_n^{(D, K)}), \quad {}^l \hat{\boldsymbol{\mu}}^{(\theta, M)} = \frac{1}{N_b} (\sum_{n=1}^{N_b} {}^l \hat{\boldsymbol{\theta}}_n^{(D, M)}).$$

- (5) Update the MAP estimates of the variance hyperparameters:

$$\begin{aligned}
({\sigma}_h^{(\theta, K)})^2 &= \frac{\lambda \tau_{0h}^{(\theta, K)} + \sum_{t=1}^{N_a} ({}^l \hat{\boldsymbol{\theta}}_{t,h}^{(S, K)} - {}^l \hat{\boldsymbol{\mu}}_h^{(\theta, K)})^2 + \sum_{n=1}^{N_b} ({}^l \hat{\boldsymbol{\theta}}_{n,h}^{(D, K)} - {}^l \hat{\boldsymbol{\mu}}_h^{(\theta, K)})^2}{N_a + N_b + \lambda + 2}, \\
({\sigma}_i^{\theta, M})^2 &= \frac{\lambda \tau_{0i}^{(\theta, M)} + \sum_{n=1}^{N_b} ({}^l \hat{\boldsymbol{\theta}}_{n,i}^{(D, M)} - {}^l \hat{\boldsymbol{\mu}}_i^{(\theta, M)})^2}{N_b + \lambda + 2}, \\
({\sigma}_j^{e, S})^2 &= \frac{\lambda \tau_{0j}^{(e, S)} + \sum_{t=1}^{N_a} ({}^l \mathbf{e}_{t,j}^{(S)})^2}{N_a + \lambda + 2}, \\
({\sigma}_r^{e, D})^2 &= \frac{\lambda \tau_{0r}^{(e, D)} + \sum_{n=1}^{N_b} ({}^l \mathbf{e}_{n,r}^{(D)})^2}{N_b + \lambda + 2}.
\end{aligned}$$

(6) Repeat steps (2) to (5) until the mean ℓ_1 norm of the difference between the updating parameters in two consecutive iterations is below 10^{-3} .

After the MAP estimates of each parameter are obtained, they are used as the initial values for the subsequent MCMC sampling. The samples are generated using the joint Gibbs–MH sampling based on Eqs. (4.22)–(4.29). Gibbs sampling is applied to parameters with standard conditional PDFs, including $\boldsymbol{\mu}^{(\theta, K)}$, $\boldsymbol{\mu}^{(\theta, M)}$, $\boldsymbol{\Sigma}^{(\theta, K)}$, $\boldsymbol{\Sigma}^{(\theta, M)}$, $\boldsymbol{\Sigma}^{(e, S)}$, and $\boldsymbol{\Sigma}^{(e, D)}$. MH sampling is used for parameters whose conditional distributions cannot be expressed explicitly, including $\boldsymbol{\theta}_t^{(S, K)}$, $\boldsymbol{\theta}_n^{(D, K)}$, and $\boldsymbol{\theta}_n^{(D, M)}$. The sampling process is repeated until the required number of samples is obtained.

4.2.5 Probabilistic Response Prediction

The structural natural frequencies and static responses are predicted based on the generated posterior samples according to the definitions of the error functions:

$$\tilde{\mathbf{f}}_{n,m} = \mathbf{f}_m(\tilde{\boldsymbol{\theta}}_n^{(D, K)}, \tilde{\boldsymbol{\theta}}_n^{(D, M)}) + \tilde{\mathbf{e}}_{n,m}^{(f)}, \quad (4.30)$$

$$\tilde{\mathbf{D}}_{t,j} = \mathbf{D}_j(\tilde{\boldsymbol{\theta}}_t^{(S,K)}, \mathbf{F}_t) + \tilde{\mathbf{e}}_{t,j}^{(S)}. \quad (4.31)$$

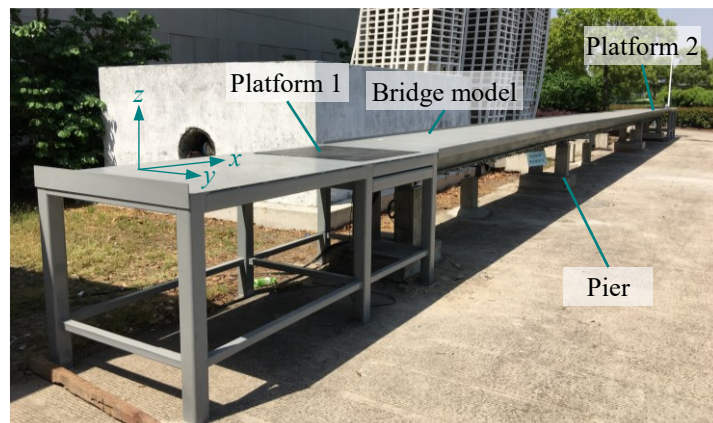
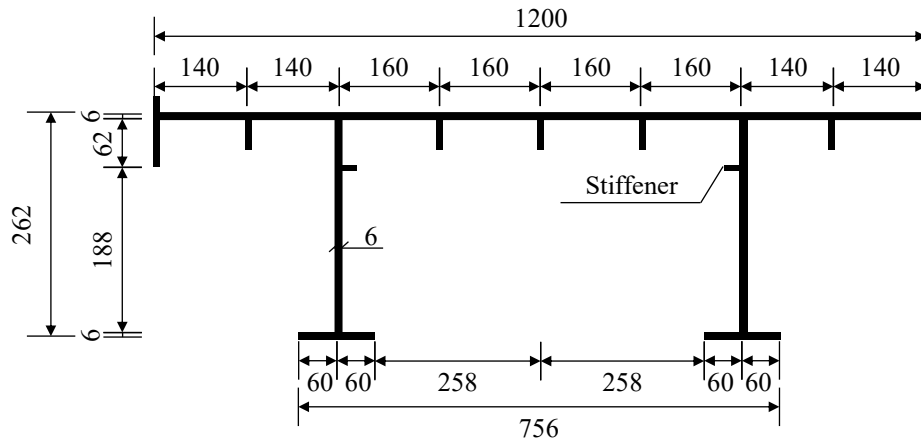
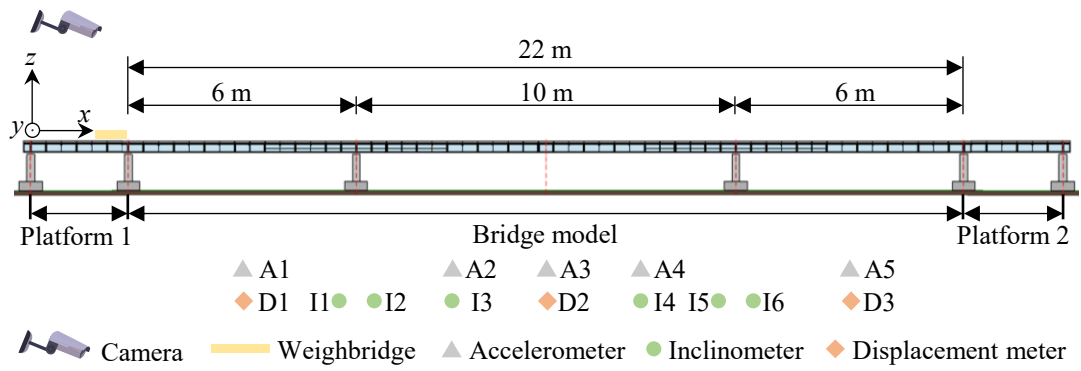
where $\tilde{\mathbf{f}}_{n,m}$ and $\tilde{\mathbf{D}}_{t,j}$ represent the predicted natural frequencies and static responses, respectively; $\tilde{\boldsymbol{\theta}}$ represents samples of the updating parameters generated from $\mathcal{N}(\boldsymbol{\mu}^{(\theta,K)}, \boldsymbol{\Sigma}^{(\theta,K)})$ and $\mathcal{N}(\boldsymbol{\mu}^{(\theta,M)}, \boldsymbol{\Sigma}^{(\theta,M)})$; and $\tilde{\mathbf{e}}$ denotes the samples of the error functions generated from $\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}^{(e,S)})$ and $\mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}^{(e,D)})$. The system uncertainties arising from both parameter and modelling errors are considered in Eqs. (4.30) and (4.31). If the error function term $\tilde{\mathbf{e}}$ is neglected in the prediction process, only the uncertainty associated with the updating parameters is considered in the predicted responses.

The 95% confidence intervals (CIs) for the predicted static responses are estimated through the following procedure: (1) generate 100 posterior samples of both the updating parameters and the modelling errors from their conditional posterior PDFs, (2) compute the predicted static responses at each time step using Eq. (4.31) for all 100 samples, (3) sort the predicted responses in descending order at each time step, and (4) select the 3rd and 97th values from the sorted list at each time step as the upper and lower bounds of the predicted CIs, respectively.

4.3 Experimental Study

4.3.1 Experiment Setup

A three-span bridge model equipped with a complete SHM system is used to validate the method, as illustrated in Figure 4.3. The span arrangement is $6 + 10 + 6 = 22$ m, and the main girder adopts a π -shaped cross-section, as exhibited in Figure 4.4. Diaphragms are installed at 0.4-m intervals along the x -axis to minimize torsional deformation. Platforms are constructed on both sides of the bridge to provide sufficient distance for the test vehicle to start and brake. These platforms are structurally independent of the main girder. The site situation of the bridge model is exhibited in Figure 4.5.

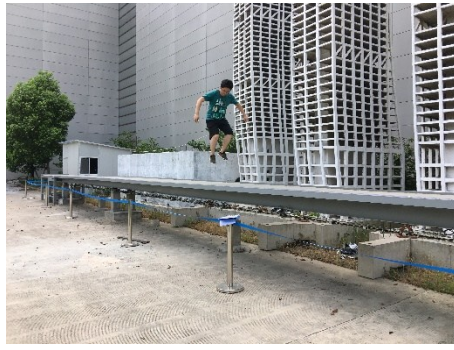


The equipped sensors include a camera, a weighbridge, three laser displacement meters, five accelerometers, and six inclinometers. The camera and weighbridge are installed at the entrance of the bridge to measure the vehicle location and axle loads, respectively. The remaining sensors are placed at positions where structural responses are significant. The sampling frequencies of the displacement meters, accelerometers, and inclinometers are all set to 200 Hz, while the frame rate of the camera is 25 frames per second.

4.3.2 Experimental Scenario

Manual excitation and vehicle driving experiments were conducted on the bridge model, as illustrated in Figure 4.6. In the manual excitation tests, the experimenter randomly walked or jumped on the bridge deck to induce dynamic responses. A total of twelve tests were conducted, each lasting for 60 s. In the vehicle driving tests, the experimenter drove a model car across the bridge. The dimensions of the model car are shown in Figure 4.7, and its weight can be adjusted by attaching additional mass blocks. Four vehicle driving scenarios were designed, and the corresponding information is summarized in Table 4.1.

The acceleration data collected during the manual excitation tests were used for modal identification. The vehicle weights and the video data were used to estimate the vehicle load distributions, which were then input into the FE model to predict static responses. The measured static responses were extracted from the raw displacement and inclination data obtained during the vehicle driving tests.



(a) Manual excitation test

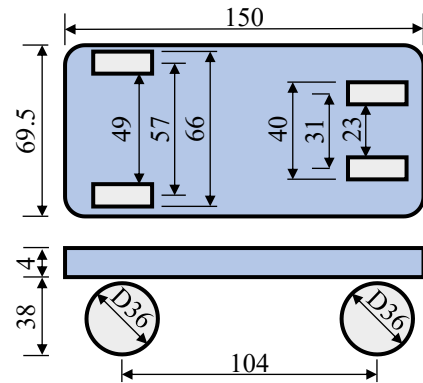


(b) Vehicle driving test

Figure 4.6 Experimental scenarios



(a) Model car appearance



(b) Dimensions of the model car (unit: cm)

Figure 4.7 Information about the model car

Table 4.1 Vehicle driving scenario settings

Scenario	Front axle weight (kg)	Rear axle weight (kg)	Gross weight (kg)	Passing time (s)
I	156	230	386	50
II	156	230	386	43
III	187	395	582	51
IV	217	375	592	31

4.3.3 Initial FE Model

In the initial FE model, the main girder is built using beam elements with an element length of 0.1 m. The initial values of the elastic modulus and material density of the main girder are set to 2.1×10^{11} Pa and 7.8×10^3 kg/m³, respectively. The mass properties of the diaphragms are modeled using lumped mass elements.

The modal parameters and static responses predicted by the initial model are then compared with the measured results. The SSI algorithm is used to identify the modal parameters from the twelve sets of acceleration data. Two modes are identified: the first-order symmetric vertical bending mode (SV1) and the first-order antisymmetric vertical bending mode (ASV1), with their natural frequencies listed in Table 4.2. The relative errors between the identified and the predicted natural frequencies exceed 10% for both modes. The identified mode shapes (averaged across all data sets) and predicted mode shapes are compared in Figure 4.8. The mean MAC values between the two sets of mode shapes are all close to 1, indicating that there is no local damage along the *x*-axis. These results indicate that the mass and stiffness parameters of the initial FE model need to be updated to ensure that the predicted modal parameters, particularly the natural frequencies, are consistent with the measured ones. Therefore, the elastic modulus and the density of the main girder are selected as the updating parameters in the dynamic tests.

Table 4.2 Identified and initial model-predicted natural frequencies

Mode	Method	Natural frequency (Hz)	Relative error (%)	MAC
SV1	SSI	9.47	11.61	1.00
	Initial model	10.67		
ASV1	SSI	19.90	13.25	0.99
	Initial model	22.64		

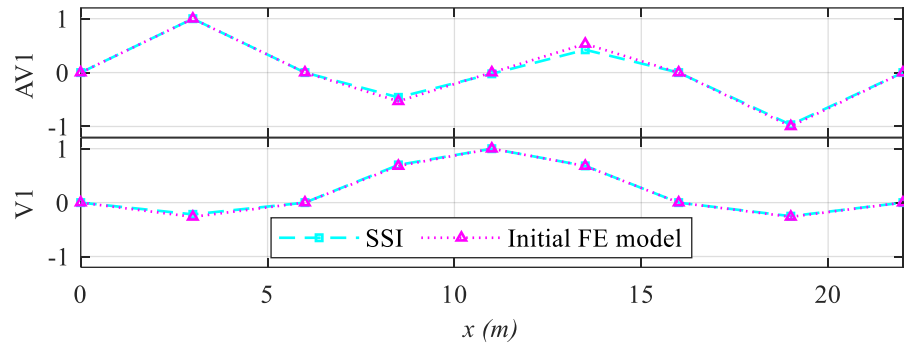


Figure 4.8 Identified and initial model-predicted mode shapes

The measured and predicted static responses under Scenario IV are shown in Figure 4.9. Although both curves are similar in shape, the predicted values are significantly lower than the measured ones, with a maximum relative error of 15%, indicating the stiffness parameters of the initial FE model are overestimated. Consequently, the overall stiffness parameter of the main girder is selected as the updating parameter in the static tests.

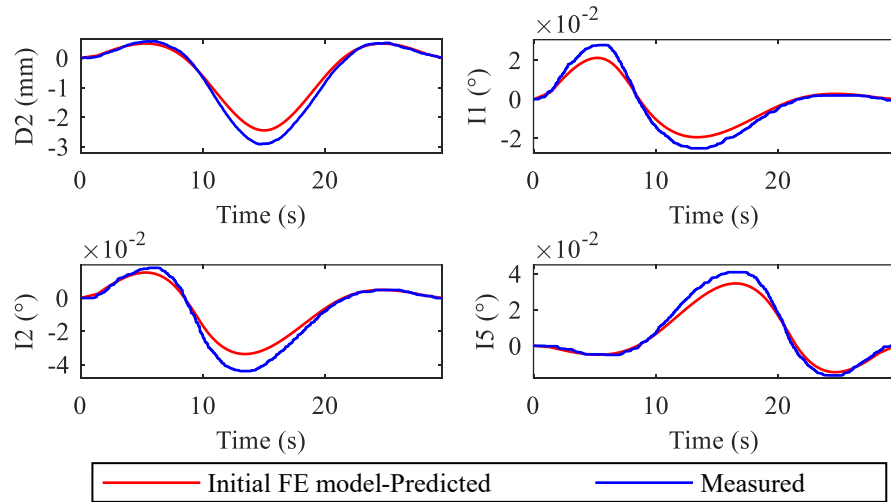


Figure 4.9 Measured and initial model-predicted static responses under Scenario IV

4.4 Updating Results

The identified frequencies and mode shapes are used as the dynamic data, and the static

responses measured at D2, I1, I2, and I5 under Scenario IV are used as static data in the model updating process. The static responses from the remaining sensors are used to validate the updated model. The measured and predicted static responses are both downsampled to 2.5 Hz to ensure a consistent time step and improve the updating efficiency. In total, 75 sets of static data and 12 sets of dynamic data are incorporated into the hierarchical FEMU process. The elastic modulus and density of the main girder, normalized by their respective initial values and denoted as θ_E and θ_ρ , are selected as the updating parameters. In this study, due to the lack of prior information about the variances, λ is set to 1 for all prior PDFs and τ_0 is assigned a relatively large value. Since the absolute values of the updating parameters in the error functions differ, $\tau_{0h}^{(\theta, K)}$ and $\tau_{0i}^{(\theta, M)}$ are set to 10^{-3} , and $\tau_{0j}^{(e, S)}$ and $\tau_{0r}^{(e, D)}$ are set to 10^{-7} . When the number of monitoring datasets used in FEMU is large, adjustments to these parameters do not significantly influence the updating results.

4.4.1 MAP Estimations

The MAP estimates of the hyperparameters associated with the updating parameters are presented in Table 4.3 Case I. The mean value of the elastic modulus parameter is 0.83, which is lower than the initial value of 1, primarily due to the stiffness reduction caused by spot welding between the steel plates. Additionally, the mean value of the density parameter is slightly greater than 1 because the mass of sensors, tracks, and strings was not considered in the initial FE model.

Table 4.3 MAP estimates of hyperparameters associated with updating parameters

Updating parameter	Case I: both static and dynamic data		Case II: only static data		Case III: only dynamic data	
	Mean	Variance	Mean	Variance	Mean	Variance
θ_E	0.83	1.10×10^{-5}	0.82	1.31×10^{-5}	0.85	1.23×10^{-4}
θ_ρ	1.08	2.25×10^{-4}	-	-	1.11	6.99×10^{-5}

Two additional cases are examined to assess the effect of integrating static and dynamic data in the FEMU process: Case II, which uses only static data, and Case III, which uses only dynamic data. The MAP estimates of each parameter are summarized in Table 4.3. The mean values of the stiffness parameter in Cases I and II are relatively close. However, in Case III, the mean values of both the stiffness and mass parameters are slightly higher compared to those in Cases I and II. This is because the stiffness and mass parameters may exhibit compensatory effects when only modal data are used in FEMU. In contrast, in Cases I and II, static data are introduced to provide independent constraints on the stiffness parameters, thereby preventing these compensating effects. This phenomenon has also been discussed in Chapter 3.

The variance of the stiffness parameter is smaller in Cases I and II than in Case III, because the measured static responses are more accurate than the identified modal parameters, which are subject to greater uncertainty due to identification errors resulting from the non-ideal white noise excitation. Additionally, the variance of the mass parameter in Case I is slightly higher than in Case III. This is because the reduced uncertainty of the stiffness parameter in Case I is compensated by an increase in the variance of the mass parameter, thereby maintaining the overall uncertainty within the dynamic dataset.

4.4.2 Sampling Results

A total of 1,000 samples of the updating parameters and hyperparameters are generated using the two-stage MCMC approach. In the subsequent analysis, only Case I, where both static and dynamic data are used, is examined in detail. The histograms of the samples associated with the updating parameters are shown in Figure 4.10. The black solid lines represent the kernel density estimates obtained using the normal kernel function. The samples of the mean parameters approximately follow normal distributions, while the samples of the variance parameters exhibit right-skewed distributions, consistent with the characteristics of the scaled inverse- χ^2 distributions. These posterior samples are in

agreement with the conjugate prior assumptions adopted in the hierarchical Bayesian framework.

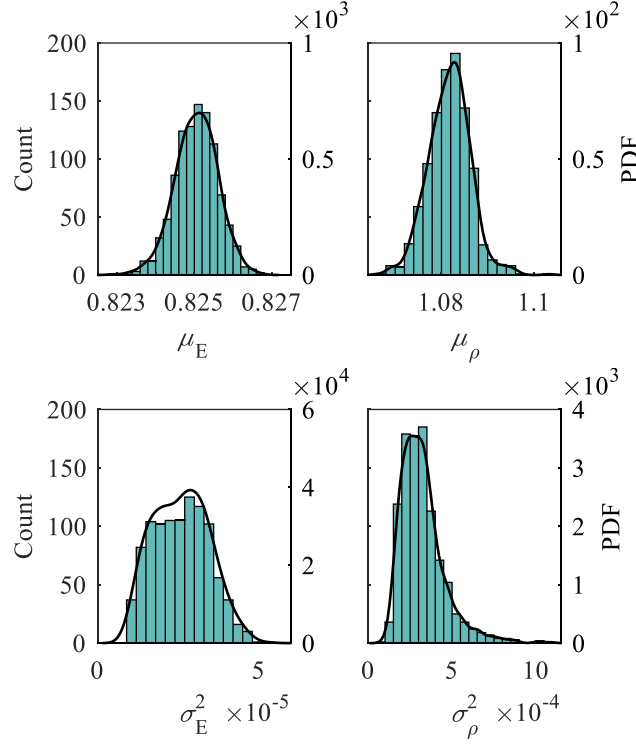


Figure 4.10 Histograms of samples associated with the structural parameters

4.4.3 Probabilistic Response Prediction

4.4.3.1 Modal Parameters

One hundred parameter samples are selected for probabilistic frequency prediction based on Eq. (4.30). The identified and predicted natural frequencies are summarized in Table 4.4. The natural frequencies of the updated model are calculated as the mean values of the predicted frequencies, which consider both parameter and modelling error uncertainties. The relative frequency errors of the updated model are significantly smaller than those of the initial model, indicating that the updated model can accurately simulate the structural dynamic properties. Meanwhile, the variances of the model-predicted

natural frequencies are consistent with the identified values when the error function is considered. In contrast, when the error function is excluded, the frequency variance of the SV1 mode is substantially reduced, while the variance of the ASV1 mode remains nearly unchanged. These results indicate that the updated FE model can accurately predict the natural frequency of the ASV1 mode, but its predictive accuracy for the SV1 mode is relatively low.

Table 4.4 Predicted frequencies by the updated model

Mode	Method	Mean of frequency (Hz)	Relative error (%)	Variance	
				Overall uncertainties	Parameter uncertainty
SV1	SSI	9.47	-	1.37×10^{-2}	-
	Updated model	9.34	1.37	2.29×10^{-2}	5.43×10^{-3}
ASV1	SSI	19.90	-	2.02×10^{-2}	-
	Updated model	19.89	0.03	2.45×10^{-2}	2.43×10^{-2}

Note: Overall uncertainties denote that both parameter and modelling errors are considered.

4.4.3.2 Static Responses

The static responses and their corresponding 95% CIs are simulated based on Eq. (4.31) using the same set of parameter samples. The predicted and measured responses at the observed DOFs (i.e., D2, I1, I2, and I5) under Scenario IV are shown in Figure 4.11. The medians of the predicted responses align closely with the measured responses in terms of both curve shapes and extreme values. The measured responses generally fall within the predicted CIs when the uncertainties from both parameters and error functions are considered. In contrast, the measured responses often fall outside the predicted CIs when the error function is excluded. These results suggest that the uncertainties in the predicted static responses are dominated by the error function, which is primarily attributed to factors such as the simplified load type, inaccurate vehicle positioning, and measurement noise.

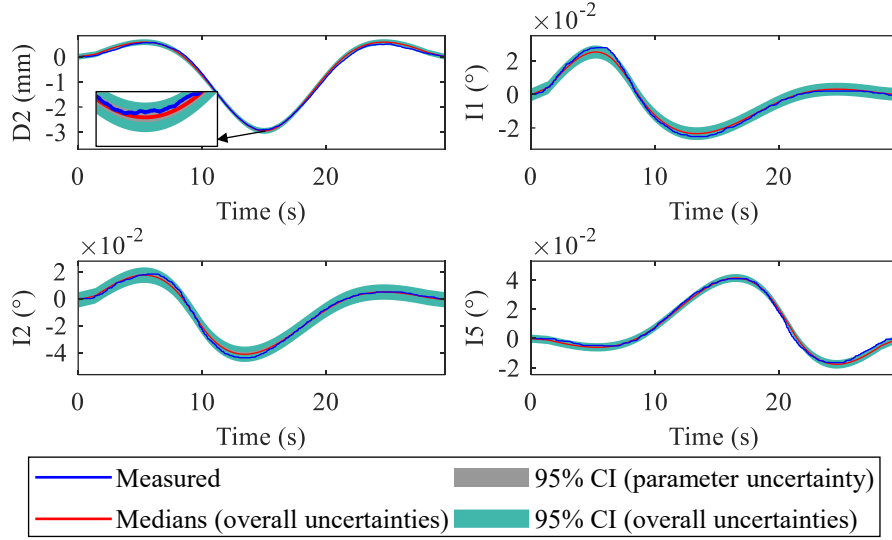


Figure 4.11 Predicted static responses at observed DOFs under Scenario IV

The predicted static responses and corresponding 95% CIs at unobserved DOFs (i.e., D1, D3, I4, and I6) are shown in Figure 4.12. The error function for these unobserved responses was not directly estimated using the proposed method because these responses were not included in the FEMU process. To this end, the mean error variance computed from the observed DOFs with the same response type is assigned to the unobserved DOFs for prediction purposes. When the error function is considered in the prediction, the measured responses generally fall within the predicted CIs. These results demonstrate the effectiveness of the updated model in accurately predicting full-field responses and quantifying associated uncertainties.

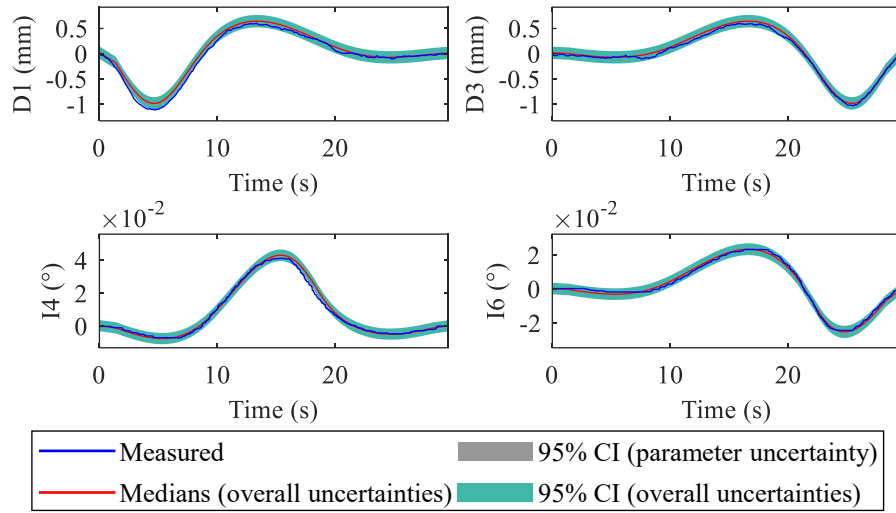


Figure 4.12 Predicted static responses at unobserved DOFs under Scenario IV

The model updated using data from Scenario IV is used to predict static responses under the other three driving scenarios. The predicted responses and CIs are presented in Figures 4.13–4.15. The predicted responses show good agreement with the measured data, and the predicted CIs consistently encompass the measured responses in all scenarios. The results indicate that the updated model is capable of reconstructing full-field static responses and quantifying associated uncertainties under various driving conditions.

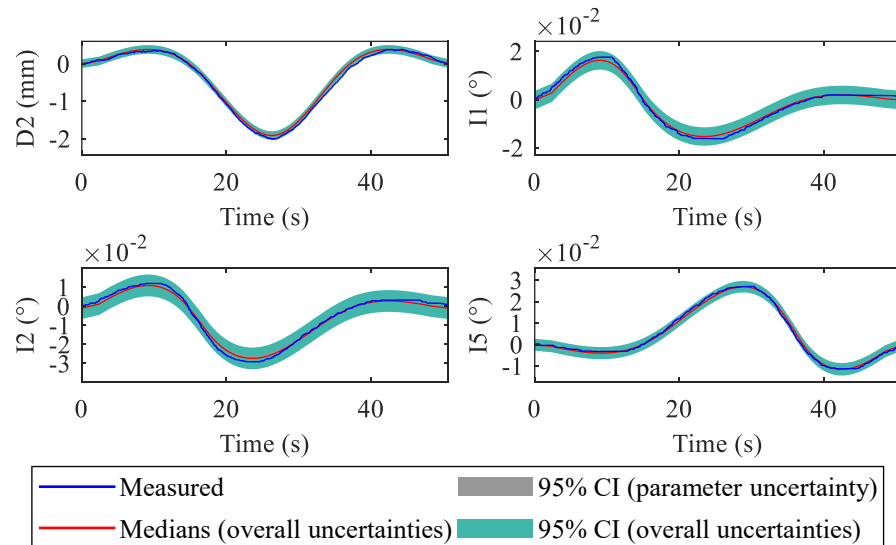


Figure 4.13 Predicted static responses under Scenario I

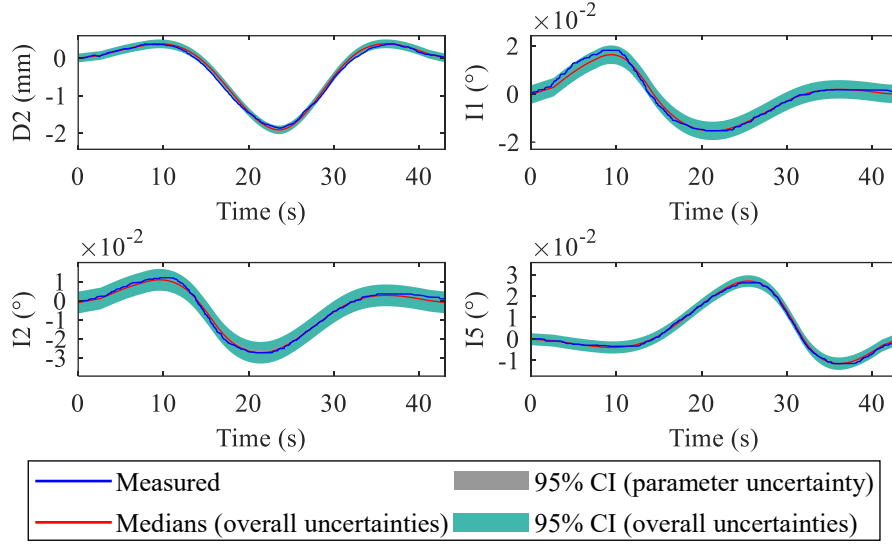


Figure 4.14 Predicted static responses under Scenario II

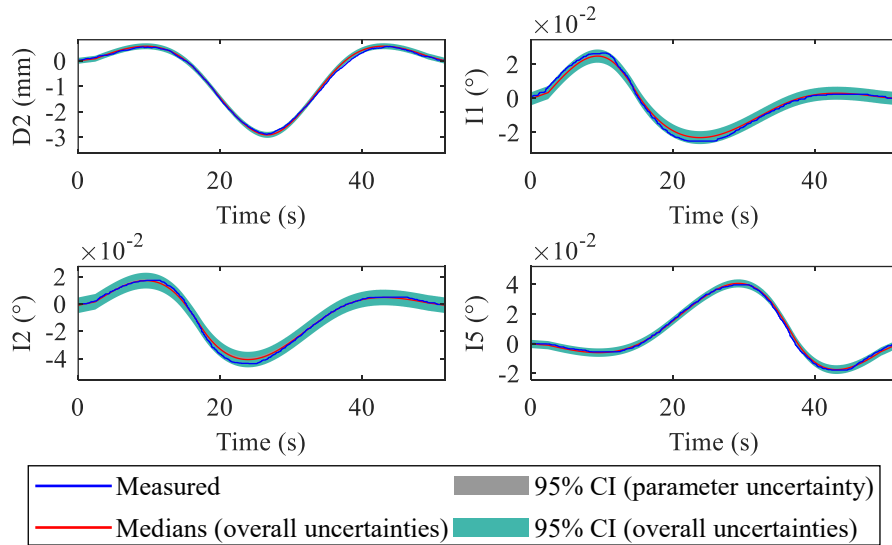


Figure 4.15 Predicted static responses under Scenario III

4.5 Summary

This chapter presents a hierarchical Bayesian FEMU framework that integrates static and dynamic measurements, extending the methodology introduced in Chapter 3. Hyperparameters are assigned to both updating parameters and error functions for comprehensive uncertainty modelling. A two-stage MCMC sampling approach is

developed to efficiently generate posterior samples for probabilistic response prediction.

The proposed method is applied to a three-span bridge model. The updated FE model demonstrates relative errors of less than 2% in natural frequencies, and the variances of predicted natural frequencies are consistent with the measured data. Meanwhile, both observed and unobserved static responses under various driving scenarios generally fall within the predicted 95% CIs when uncertainties from both structural parameters and error functions are considered. Comparative studies using different dataset combinations reveal that incorporating both static and dynamic data in the FEMU process significantly improves parameter identification accuracy and uncertainty quantification reliability.

CHAPTER 5

HIERARCHICAL BAYESIAN MODEL UPDATING AND STRUCTURAL CONDITION ASSESSMENT CONSIDERING OPERATIONAL LOADS

5.1 Introduction

The FEMU methods presented in Chapters 3 and 4 neglect the effects of operational loads. However, field monitoring studies have demonstrated that the structural dynamic characteristics are significantly affected by operational loads such as temperature, traffic loads, and wind (Cross et al., 2013; Mao et al., 2018; Teng et al., 2021; Xia et al., 2012). Furthermore, distinguishing damage-induced changes in structural properties from those caused by operational loads is critical for structural condition assessment. To this end, this chapter develops a hierarchical Bayesian FEMU and condition assessment method that explicitly incorporates the effects of environmental temperature and operational loads. The overall framework is shown in Figure 5.1. A long-span arch bridge is used as the testbed. The main innovations and contributions of this study are as follows: (1) a linear temperature–stiffness parameter relationship and an equivalent vehicle weight estimation method are integrated into the hierarchical Bayesian framework to model the influence of operational loads on structural characteristics; (2) an RSM and a two-stage MCMC sampling strategy are employed to accelerate the FEMU process; and (3) a structural condition indicator is proposed to quantitatively assess the structural condition under varying operational conditions.

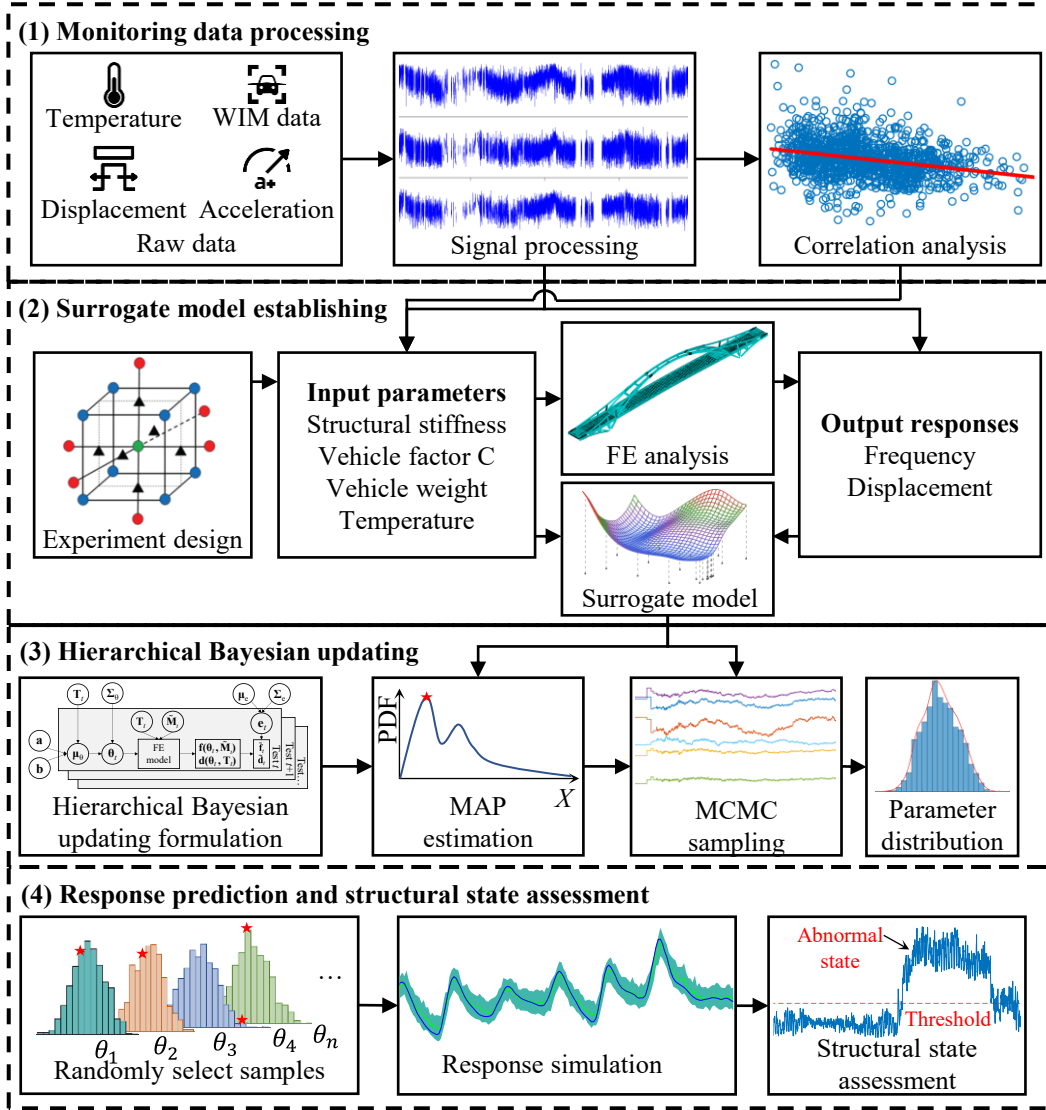


Figure 5.1 Framework of the hierarchical Bayesian FEMU method considering operational loads

5.2 Mingzhou Bridge and Its SHM System

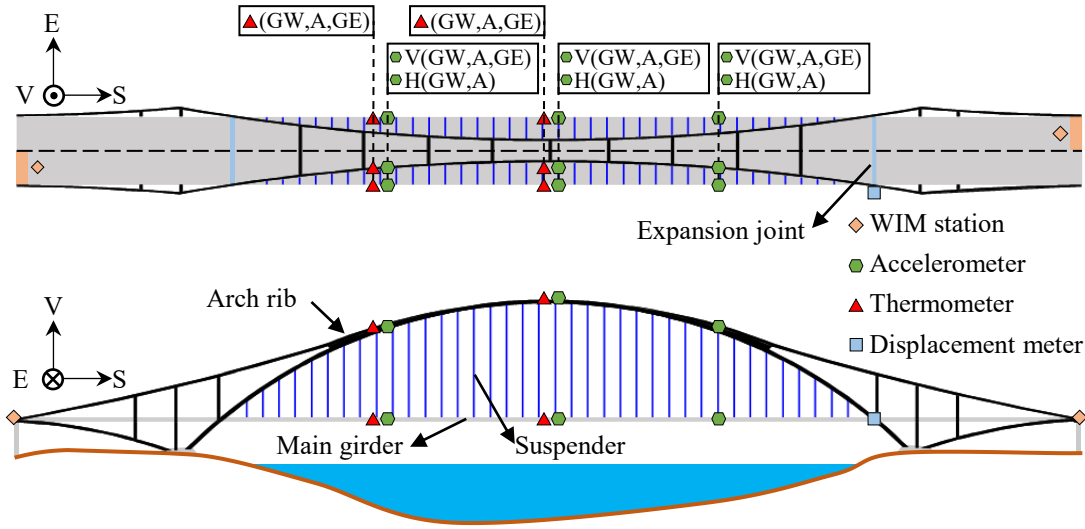
The Mingzhou Bridge, located in Ningbo, Zhejiang Province, China, is used as the testbed in this chapter. It is a half-through arch bridge with a span configuration of 100 m + 450 m + 100 m = 650 m. A southwest view of the arch bridge is shown in Figure 5.2. The main arch is composed of two parallel ribs connected by transverse braces. Both ends of the main girder in the middle span are simply supported on the cross beams at the arch-

girder joints. The main girders in the side spans are pinned at the arch-girder joints and simply supported at the side bearings.



Figure 5.2 Southwest view of the Mingzhou Bridge

A comprehensive SHM system is deployed on the Mingzhou Bridge. The collected acceleration, displacement of the expansion joint (DEJ), structural temperature, and WIM data are used for FEMU. The layout of the adopted sensors is shown in Figure 5.3. Two WIM stations are deployed at both entrances of the bridge and record the axle weights, gross weights, and speeds of all passing vehicles. The thermometers and accelerometers are arranged at three cross-sections along the bridge. At each cross-section, two accelerometers are installed on the main girder, and one is installed on the west arch rib. The accelerometers on the west side of the girder and arch are biaxial, measuring both vertical and transverse vibrations. The accelerometer on the east side of the girder is uniaxial and measures only the vertical vibration. The thermometers are placed adjacent to six of the accelerometers. The displacement meter is installed at the southern expansion joint to monitor its displacement variation.



Note: 'G', 'A', 'V', 'H', 'W', and 'E' denote 'Girder', 'Arch', 'Vertical', 'Horizontal', 'West', and 'East', respectively.

Figure 5.3 Sensor layout of the Mingzhou Bridge

5.3 Monitoring Data Analysis

The monitoring data of the Mingzhou Bridge collected between 2017 and 2018 are first presented in this section. Subsequently, partial correlation analysis and regression analysis are performed to quantitatively evaluate the influences of temperature and vehicle loads on the structural responses. These analyses provide guidance for the selection of loads to be considered in the FEMU process. The data are processed using a one-hour interval, and each dataset includes four features: equivalent vehicle weight, mean structural temperature, mean DEJ, and natural frequencies. Any datasets containing missing features are excluded from the FEMU dataset.

5.3.1 Modal Identification

The natural excitation technique-based eigensystem realization algorithm is employed to identify the modal parameters from the measured acceleration data (Juang et al., 1985). Three modes are identified: the first-order antisymmetric vertical bending mode (V1), the

second-order symmetric vertical bending mode (V2), and the third-order antisymmetric vertical bending mode (V3). The MAC values between the identified mode shapes and those predicted by the initial FE model are shown in Figure 5.4. The initial FE model will be presented in the following section.

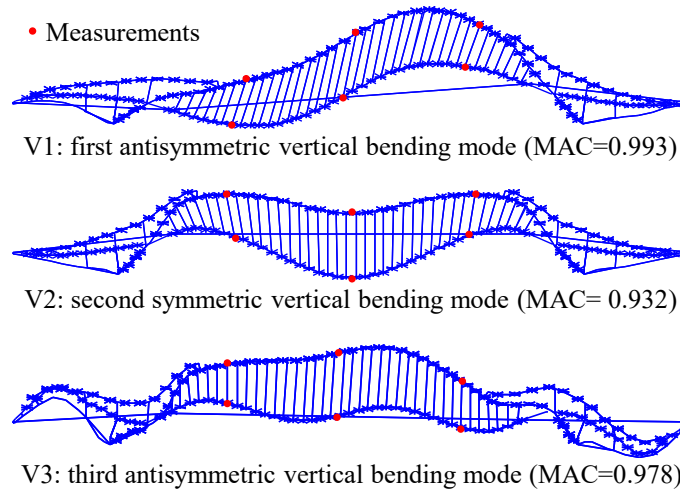


Figure 5.4 Identified and predicted mode shapes

For a comprehensive illustration, the identified natural frequencies from 2016 to 2019 are shown in Figure 5.5, where the gaps indicate missing or corrupted data. The identified frequencies exhibit a clear cyclic pattern, with lower values observed in the middle of each year and higher values at the beginning and end of each year. This cyclic behavior is primarily caused by temperature fluctuations because the elastic modulus of both concrete and steel decreases with rising temperature (Zhou et al., 2019).

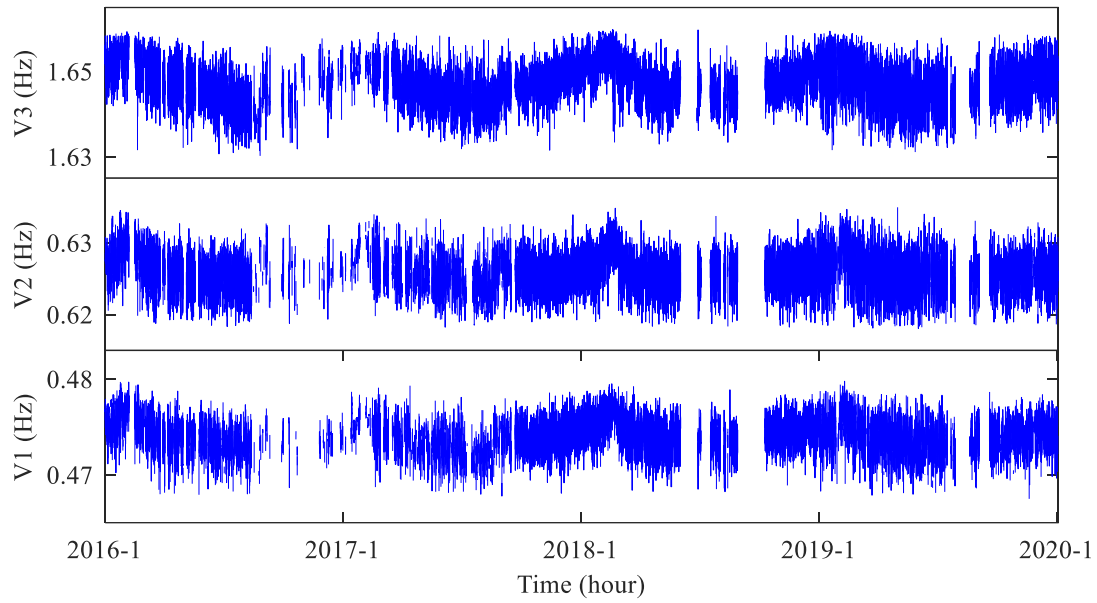


Figure 5.5 Identified natural frequencies from 2016 to 2019

5.3.2 Vehicle Weight Estimation

The hourly summed vehicle weights from 2017 to 2018 based on the WIM data are presented in Figure 5.6. A noticeable reduction in traffic loads is observed during the Chinese Spring Festival period.

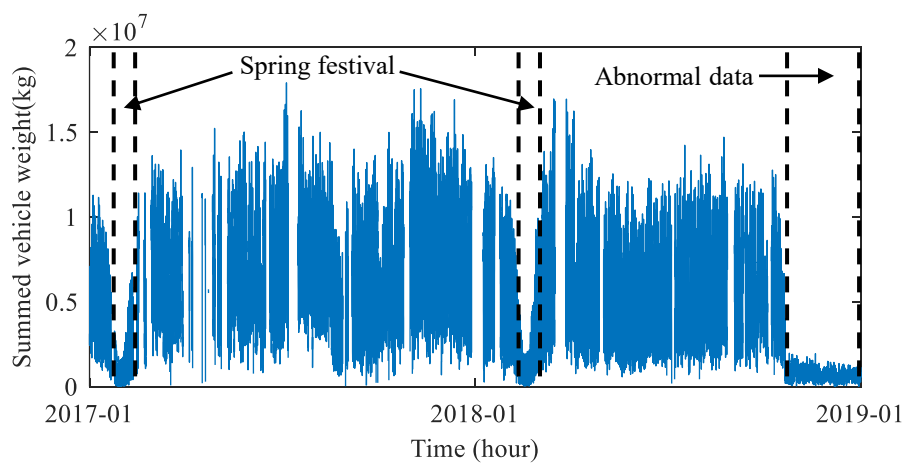


Figure 5.6 Hourly summed vehicle weights from 2017 to 2018

However, the hourly summed vehicle weights are not suitable for natural frequency simulation because they do not align with the natural frequencies in either the spatial or temporal domains. If the hourly summed vehicle weights are directly used in the FE simulation, the effects of traffic loads on structural dynamic characteristics will be overestimated. To address this issue, an equivalent vehicle weight estimation method is proposed for modal analysis. Three assumptions are introduced for the calculation of the equivalent load: (1) the vehicles travel at constant speeds and do not change lanes while crossing the bridge; (2) the equivalent vehicle weight is proportional to both the weight and its crossing time on the bridge; and (3) the equivalent weight is uniformly distributed along the main girder. The equivalent vehicle weight for the t -th dataset is calculated as follows:

- (1) Calculate the equivalent weight of a single vehicle $\bar{m}_{t,s}$:

$$\bar{m}_{t,s} = \frac{L}{v_{t,s}T} \times m_{t,s}, \quad (5.1)$$

where s denotes the vehicle index, L is the bridge length (650 m), T denotes the analysis interval (one hour), and $m_{t,s}$ and $v_{t,s}$ denote the gross weight and velocity of the vehicle as recorded at the WIM station, respectively.

- (2) Calculate the equivalent weight of the vehicle stream at each node $\tilde{\mathbf{M}}_t$:

$$\tilde{\mathbf{M}}_t = C \sum_{s=1}^{V_t} \bar{m}_{t,s} / N_n, \quad (5.2)$$

where V_t is the total number of vehicles in the t -th dataset, N_n is the number of nodes along the main girder, and C is a scaling variable to be updated in the FEMU process. The uncertainty in parameter C arises from the assumptions made in the equivalent weight calculation. In each simulation, the lumped mass elements with value $\tilde{\mathbf{M}}_t$ are applied at all nodes along the main girder to simulate the influence of traffic loading on the structural dynamic properties.

5.3.3 Structural Mean Temperature and DEJ

The mean temperature of the main girder from 2017 to 2018 is shown in Figure 5.7, exhibiting a cyclical variation between 0 °C and 50 °C. The DEJ value measured at the temperature of bridge completion (24.7 °C) is used as the reference point in both data processing and FE simulation so that the measured and simulated DEJs are comparable. The processed DEJ measurements are also plotted in Figure 5.7. The DEJ and the average temperature exhibit a similar variation pattern, indicating a strong correlation between them.

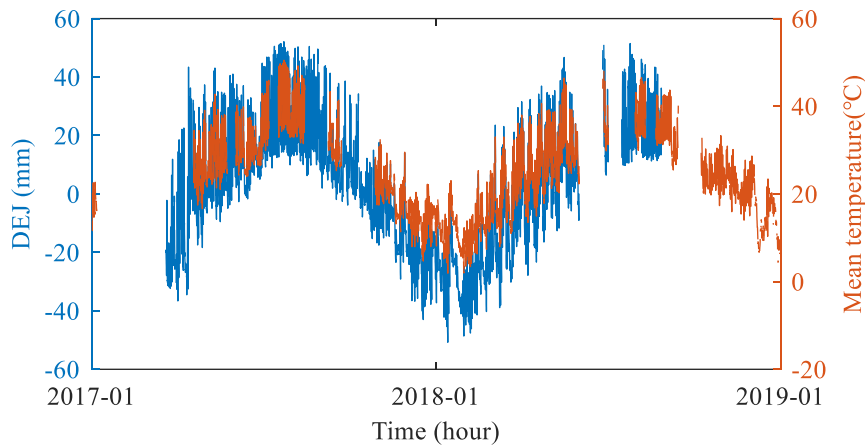


Figure 5.7 Hourly mean temperature and DEJ from 2017 to 2018

5.3.4 Quantitative Analysis between Loads and Responses

Partial correlation analysis is employed to quantify the relationships between the operational loads and structural responses. This method evaluates the correlation between two variables with the influence of other variables eliminated (Bao et al., 2013). The correlation coefficient (CC) and the first-order partial correlation coefficient (PCC) are calculated as follows:

$$CC_{U,V} = \frac{\mathbb{E}[(U - \mu_U)(V - \mu_V)]}{\sigma_U \sigma_V}, \quad (5.3)$$

$$PCC_{U, V|W} = \frac{CC_{U, V} - CC_{U, W}CC_{V, W}}{\sqrt{(1 - CC_{U, W}^2)(1 - CC_{V, W}^2)}}, \quad (5.4)$$

where $CC_{U, V}$ denotes the standard CC between the variables U and V ; μ and σ represent the mean and standard deviations of the random variable, respectively; and $PCC_{U, V|W}$ refers to the PCC between U and V with the influences of W removed. The ranges of the CCs and PCCs are both $[-1, 1]$. A positive value indicates a positive correlation between the variables, whereas a negative value indicates a negative correlation. The correlation strength is classified based on the absolute value of the coefficients (Evans, 1996): very strong (0.8–1), strong (0.6–0.8), moderate (0.4–0.6), weak (0.2–0.4), and negligible (0–0.2).

The PCCs between the considered operational loads and responses are summarized in Table 5.1. All PCCs associated with the natural frequencies are negative, indicating that natural frequencies tend to decrease as the operational loads increase. Meanwhile, the PCC between DEJ and temperature is close to one, indicating that DEJ is primarily influenced by temperature.

Table 5.1 PCCs between operational loads and responses

Data features	Mean temperature	Equivalent vehicle weight
V1	−0.54	−0.41
V2	−0.27	−0.26
V3	−0.79	−0.36
DEJ	0.99	0.14

Regression analysis is subsequently performed to further examine the relationship type between the responses and the considered operational loads. The scatter plots and linear regression results for natural frequencies versus operational loads are shown in Figure 5.8. $C = 1$ is temporarily used in calculating the equivalent vehicle weight, and it will be estimated later in FEMU. The identified natural frequencies generally exhibit a linear

decrease with increasing temperature. The decreasing slope of V3 is the largest among all frequencies. The percentage change rates of the three natural frequencies with respect to temperature are 2.02×10^{-4} , 1.16×10^{-4} , and 2.2×10^{-4} , respectively. These values are approximately half of the temperature-induced variation rate in the elastic modulus of steel, which is 1.8×10^{-4} , and they are consistent with the findings reported by Xia et al. (2006). In addition, the natural frequencies also decrease with increasing traffic loads due to the additional mass introduced by the vehicles.

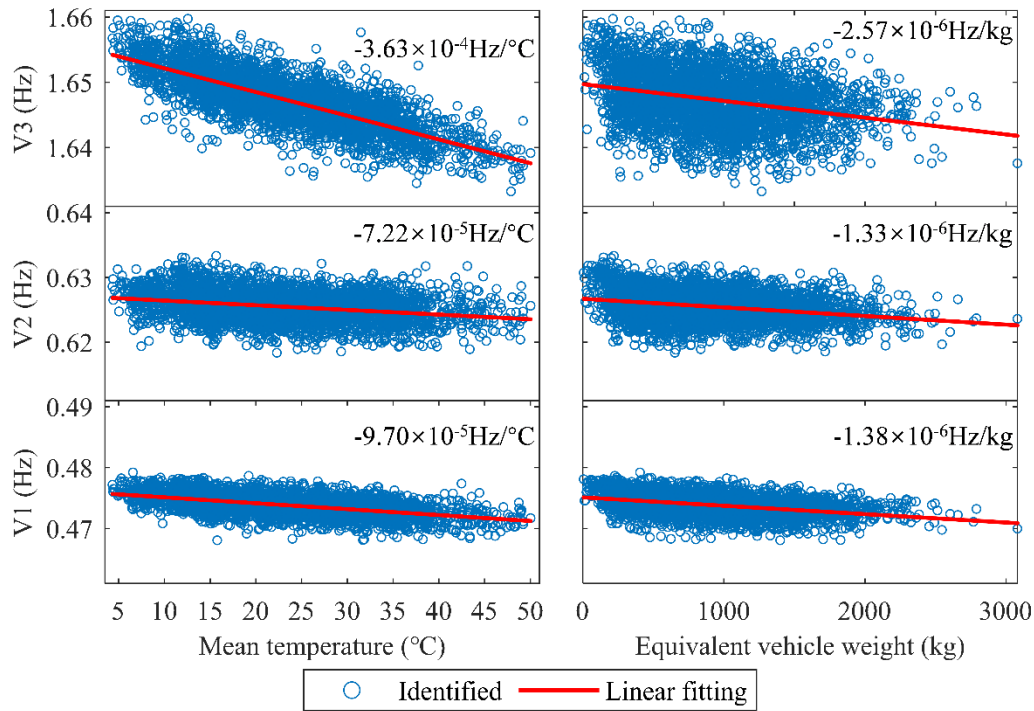


Figure 5.8 Linear regression between operational loads and natural frequencies

The scatter plots and regression results for DEJ versus operational loads are shown in Figure 5.9. A clear linear relationship is observed between the temperature and DEJ, which is attributed to the thermal expansion of the main girder. In contrast, no significant relationship is found between DEJ and the equivalent vehicle weight.

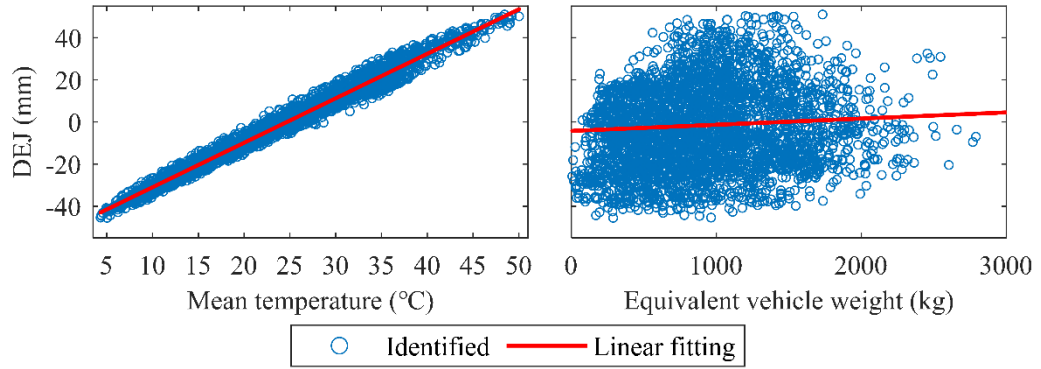


Figure 5.9 Linear regression between operational loads and DEJ

Based on the data analysis results, both the temperature and vehicle loads should be considered in the simulation of natural frequencies. The identified frequencies show a clear linear trend with temperature. Therefore, a linear temperature–stiffness relationship is assumed to consider temperature effects because structural frequencies change linearly with stiffness parameters in the linear range (Xia, 2002). However, this assumption may not hold when the frequency–temperature relationship is nonlinear. For instance, when the temperature drops below freezing, frequencies can increase sharply due to moisture freezing within the concrete (Behmanesh et al., 2016) and changes in material properties (Peeters et al., 2001). In such cases, more appropriate material models, such as multi-linear or exponential models, should be considered. The influence of vehicle loads is represented by adding additional mass elements at the nodes along the main girder. In contrast, only the temperature load is considered in the simulation of DEJ because no apparent relationship between the DEJ and the vehicle loads is observed.

5.4 Hierarchical Bayesian FEMU Method Considering Operational Loads

5.4.1 Prior Distribution for Updating Parameters

The updating parameters adopted in this study are defined as $\boldsymbol{\theta} = [E_g, E_a, K_e, C]^T$, where E_g and E_a represent the elastic modulus of the main girder and main arch, respectively;

K_e denotes the longitudinal stiffness of the expansion joint; and C is the equivalent vehicle weight factor. These parameters are normalized with respect to their initial values to avoid the numerical ill-conditioning problem. The updating parameters in the t -th experiment $\boldsymbol{\theta}_t$ are assumed to follow a multivariate normal distribution with a mean vector $\boldsymbol{\mu}_\theta$ and a covariance matrix $\boldsymbol{\Sigma}_\theta$:

$$\boldsymbol{\theta}_t \sim \mathcal{N}(\boldsymbol{\mu}_\theta, \boldsymbol{\Sigma}_\theta). \quad (5.5)$$

Based on the analysis of the monitoring data, the mean parameter vector $\boldsymbol{\mu}_\theta$ is assumed to follow a linear relationship with the temperature vector $\mathbf{T}_t$, which is expressed as

$$\boldsymbol{\mu}_\theta(\mathbf{T}_t) = \mathbf{a} + \mathbf{b}\mathbf{T}_t, \quad (5.6)$$

where the first two items of $\mathbf{T}_t$ represent the mean temperature of the main girder and the main arch, respectively, while the third and fourth items are set to zero because K_e and C are not significantly influenced by temperature; and $\mathbf{a}$ and $\mathbf{b}$ denote the intercept and slope vectors, respectively. The reference value for the temperature-induced variation rate in the elastic modulus of steel is recommended to be within the range of $[-5.5 \times 10^{-4}, -2.3 \times 10^{-4}]/^\circ\text{C}$ (En-1993-1-2, 2005), and a value of $-3.6 \times 10^{-4}/^\circ\text{C}$ is adopted in this study. The slope components of $\mathbf{b}$ associated with K_e and C are set to zero.

The covariance matrix $\boldsymbol{\Sigma}_\theta$ is assumed to be a diagonal matrix, which is expressed as

$$\boldsymbol{\Sigma}_\theta = \begin{bmatrix} \cdot & \cdot & & 0 \\ & \sigma_{\theta,i}^2 & & \\ 0 & & \cdot & \cdot \\ & & & \cdot \end{bmatrix}_{N_q \times N_q}, \quad (5.7)$$

where $\sigma_{\theta,i}^2$ denotes the variance of the i -th updating parameter (N_q in total). The diagonal assumption for $\boldsymbol{\Sigma}_\theta$ is based on two considerations: (1) the correlations between the updating parameters are negligible because the operational conditions have already been explicitly considered in the hierarchical Bayesian framework, and (2) the diagonal matrix assumption reduces the total number of hyperparameters and improves computational efficiency.

Finally, the prior PDF of the updating parameters in the t -th dataset is expressed as

$$p(\boldsymbol{\theta}_t | \mathbf{a}, \mathbf{b}, \boldsymbol{\Sigma}_\theta, \mathbf{T}_t) = \frac{1}{(2\pi)^{Nq/2} |\boldsymbol{\Sigma}_\theta|^{1/2}} \exp\left[-\frac{1}{2}(\boldsymbol{\theta}_t - \mathbf{a} - \mathbf{b}\mathbf{T}_t)^T \boldsymbol{\Sigma}_\theta^{-1} (\boldsymbol{\theta}_t - \mathbf{a} - \mathbf{b}\mathbf{T}_t)\right]. \quad (5.8)$$

5.4.2 Likelihood Function Using Natural Frequencies and DEJ

The error function in the t -th dataset $\mathbf{e}_t$ consists of two components: the natural frequency error $\mathbf{e}_t^f$ and the DEJ error $\mathbf{e}_t^d$. These errors primarily arise from measurement noise, modeling error, and modal identification error. According to the central limit theorem, the error function is assumed to follow a multivariate normal distribution with a mean vector $\boldsymbol{\mu}_e$ and a covariance matrix $\boldsymbol{\Sigma}_e$:

$$\mathbf{e}_t = \begin{bmatrix} \mathbf{e}_t^f \\ \mathbf{e}_t^d \end{bmatrix} \sim \mathcal{N}(\boldsymbol{\mu}_e, \boldsymbol{\Sigma}_e). \quad (5.9)$$

The covariance matrix of the error function $\boldsymbol{\Sigma}_e$ is assumed to be diagonal and is expressed as

$$\boldsymbol{\Sigma}_e = \begin{bmatrix} \ddots & & 0 \\ & \sigma_{e,j}^2 & \\ 0 & & \ddots \end{bmatrix}_{Nr \times Nr}, \quad (5.10)$$

where $\sigma_{e,j}^2$ is the variance of the j -th error item (N_r in total).

The frequency error $\mathbf{e}_t^f$ is calculated as

$$\mathbf{e}_{t,m}^f = \frac{\tilde{\mathbf{f}}_{t,m} - \mathbf{f}_m(\boldsymbol{\theta}_t, \tilde{\mathbf{M}}_t)}{\tilde{\mathbf{f}}_{t,m}}, \quad (5.11)$$

where $\tilde{\mathbf{f}}_{t,m}$ denotes the measured natural frequency of the m -th mode in the t -th experiment, and $\mathbf{f}_m(\boldsymbol{\theta}_t, \tilde{\mathbf{M}}_t)$ is the corresponding frequency predicted by the FE model

using the updating parameter vector $\boldsymbol{\theta}_t$ and the equivalent weight of the vehicle stream $\tilde{\mathbf{M}}_t$. The temperature vector is not explicitly included in the calculation of the theoretical natural frequencies because its effects have already been considered in the updating parameters $\boldsymbol{\theta}_t$.

The error function of the DEJ in the t -th test is defined as

$$\mathbf{e}_t^d = \frac{\tilde{\mathbf{d}}_t - \mathbf{d}(\boldsymbol{\theta}_t, \mathbf{T}_t)}{\tilde{\mathbf{d}}_{\max}}, \quad (5.12)$$

where $\tilde{\mathbf{d}}_t$ and $\mathbf{d}(\boldsymbol{\theta}_t, \mathbf{T}_t)$ denote the measured and predicted DEJs, respectively; and $\tilde{\mathbf{d}}_{\max}$ represents the maximum value of the measured DEJ. The normalization step is applied in the definitions of both $\mathbf{e}_t^f$ and $\mathbf{e}_t^d$ to mitigate potential ill-conditioning issues in the FEMU process.

Finally, the likelihood function for the t -th dataset is expressed as

$$\begin{aligned} & p(\tilde{\mathbf{f}}_t, \tilde{\mathbf{d}}_t \mid \boldsymbol{\mu}_e, \boldsymbol{\Sigma}_e, \mathbf{T}_t, \tilde{\mathbf{M}}_t) \\ &= \frac{1}{(2\pi)^{Nr/2} |\boldsymbol{\Sigma}_e|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{e}_t - \boldsymbol{\mu}_e)^\top \boldsymbol{\Sigma}_e^{-1}(\mathbf{e}_t - \boldsymbol{\mu}_e)\right], \end{aligned} \quad (5.13)$$

5.4.3 Posterior PDF

The parameter relationships in the proposed hierarchical Bayesian FEMU method are summarized in Figure 5.10.

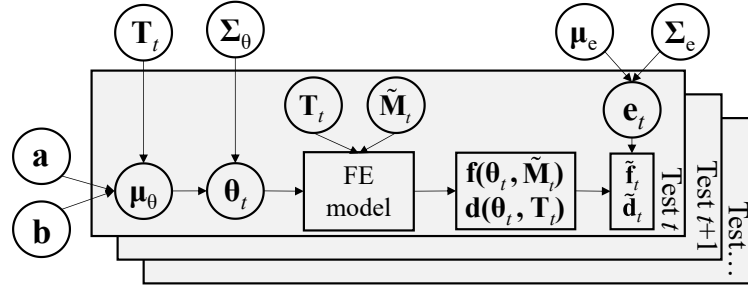


Figure 5.10 Parameter relationships in the proposed hierarchical Bayesian method

According to the hierarchical Bayesian theorem, the joint posterior PDF of the updating parameters and hyperparameters is expressed as

$$\begin{aligned}
 & p(\boldsymbol{\Theta}, \mathbf{a}, \mathbf{b}, \boldsymbol{\Sigma}_\theta, \boldsymbol{\mu}_e, \boldsymbol{\Sigma}_e | \tilde{\mathbf{f}}, \tilde{\mathbf{d}}, \tilde{\mathbf{M}}, \mathbf{T}) \\
 & \propto p(\tilde{\mathbf{f}}, \tilde{\mathbf{d}} | \boldsymbol{\Theta}, \boldsymbol{\mu}_e, \boldsymbol{\Sigma}_e, \tilde{\mathbf{M}}, \mathbf{T}) p(\boldsymbol{\Theta} | \mathbf{a}, \mathbf{b}, \boldsymbol{\Sigma}_\theta, \mathbf{T}) \\
 & \quad p(\mathbf{a}) p(\mathbf{b}) p(\boldsymbol{\Sigma}_\theta) p(\boldsymbol{\mu}_e) p(\boldsymbol{\Sigma}_e) \quad , \quad (5.14) \\
 & \propto \prod_{t=1}^{Nd} \left[p(\tilde{\mathbf{f}}_t, \tilde{\mathbf{d}}_t | \boldsymbol{\theta}_t, \boldsymbol{\mu}_e, \boldsymbol{\Sigma}_e, \tilde{\mathbf{M}}_t, \mathbf{T}_t) p(\boldsymbol{\theta}_t | \mathbf{a}, \mathbf{b}, \boldsymbol{\Sigma}_\theta, \mathbf{T}_t) \right] \\
 & \quad p(\mathbf{a}) p(\mathbf{b}) p(\boldsymbol{\Sigma}_\theta) p(\boldsymbol{\mu}_e) p(\boldsymbol{\Sigma}_e)
 \end{aligned}$$

where N_d denotes the total number of tests; $\boldsymbol{\Theta} = [\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_t, \dots, \boldsymbol{\theta}_{Nd}]$ represents the set of updating parameters in all tests; and $p(\boldsymbol{\Sigma}_\theta) = \prod_i^{N_q} p(\sigma_{\theta,i}^2)$, $p(\boldsymbol{\Sigma}_e) = \prod_j^{N_r} p(\sigma_{e,j}^2)$ because these covariance matrices are assumed to be diagonal.

Conjugate priors are employed to simplify the derivation of the posterior PDF and facilitate MCMC sampling. Specifically, uniform distributions are assigned to $p(\mathbf{a})$, $p(\mathbf{b})$, and $p(\boldsymbol{\mu}_e)$, while scaled inverse- χ^2 distributions are assigned to $p(\sigma_{\theta,i}^2)$ and $p(\sigma_{e,j}^2)$. The scaled inverse- χ^2 distribution is commonly used as the prior function for the variance in normal distributions, and its definition is provided in Eq. (4.15).

After substituting Eqs. (5.5), (5.8), and (5.13) into Eq. (5.14), the final posterior PDF is expressed as

$$p(\boldsymbol{\Theta}, \mathbf{a}, \mathbf{b}, \boldsymbol{\Sigma}_\theta, \boldsymbol{\mu}_e, \boldsymbol{\Sigma}_e | \tilde{\mathbf{f}}, \tilde{\mathbf{d}}, \tilde{\mathbf{M}}, \mathbf{T}) \propto |\boldsymbol{\Sigma}_\theta|^{-\frac{N_d + \lambda + 2}{2}} |\boldsymbol{\Sigma}_e|^{-\frac{N_d + \lambda + 2}{2}} \exp[-\sum_{t=1}^{N_d} (J_{\theta,t} + J_{e,t}) - \sum_{i=1}^{N_q} \frac{\lambda \tau_{0,\theta,i}}{2\sigma_{\theta,i}^2} - \sum_{j=1}^{N_r} \frac{\lambda \tau_{0,e,j}}{2\sigma_{e,j}^2}], \quad (5.15)$$

$$J_{\theta,t} = \frac{(\boldsymbol{\theta}_t - \mathbf{a} - \mathbf{b}\mathbf{T}_t)^T \boldsymbol{\Sigma}_\theta^{-1} (\boldsymbol{\theta}_t - \mathbf{a} - \mathbf{b}\mathbf{T}_t)}{2}, \quad (5.16)$$

$$J_{e,t} = \frac{(\mathbf{e}_t - \boldsymbol{\mu}_e)^T \boldsymbol{\Sigma}_e^{-1} (\mathbf{e}_t - \boldsymbol{\mu}_e)}{2}. \quad (5.17)$$

5.4.4 Two-stage MCMC Sampling Approach

The two-stage MCMC sampling approach proposed in Chapter 4 is used to generate posterior samples of both the updating parameters and the hyperparameters. The conditional posterior PDFs of each parameter are expressed as

$$p(\boldsymbol{\theta}_t | \cdot) \propto \exp(-J_{\theta,t} - J_{e,t}), \quad (5.18)$$

$$(\mathbf{a} | \cdot) \sim \mathcal{N}\left(\frac{1}{N_d} \sum_{t=1}^{N_d} (\boldsymbol{\theta}_t - \mathbf{b}\mathbf{T}_t), \frac{\boldsymbol{\Sigma}_\theta}{N_d}\right), \quad (5.19)$$

$$p(\mathbf{b} | \cdot) \propto \exp(-\sum_{t=1}^{N_d} J_{\theta,t}), \quad (5.20)$$

$$(\sigma_{\theta,i}^2 | \cdot) \sim \text{Inverse-}\chi^2(N_d + \lambda, \frac{\lambda \tau_{0,\theta,i} + \sum_{t=1}^{N_d} (\boldsymbol{\theta}_{t,i} - \boldsymbol{\mu}_{\theta,i})^2}{N_d + \lambda}), \quad (5.21)$$

$$(\boldsymbol{\mu}_e | \cdot) \sim \mathcal{N}\left(\frac{1}{N_d} \sum_{t=1}^{N_d} \mathbf{e}_t, \frac{\boldsymbol{\Sigma}_e}{N_d}\right), \quad (5.22)$$

$$(\sigma_{e,j}^2 | \cdot) \sim \text{Inverse-}\chi^2(N_d + \lambda, \frac{\lambda \tau_{0,e,j} + \sum_{t=1}^{N_d} (\mathbf{e}_{t,j} - \boldsymbol{\mu}_{e,j})^2}{N_d + \lambda}). \quad (5.23)$$

The MAP estimates for each parameter are calculated using the following procedure:

- (1) Initialize parameters ${}^0\hat{\mathbf{a}}$, ${}^0\hat{\mathbf{b}}$, ${}^0\hat{\boldsymbol{\mu}}_e$, ${}^0\hat{\boldsymbol{\Sigma}}_\theta$ and ${}^0\hat{\boldsymbol{\Sigma}}_e$.

(2) Update the MAP estimate of $\boldsymbol{\theta}_t$ in the l -th iteration using $^{l-1}\mathbf{a}$, $^{l-1}\mathbf{b}$, $^{l-1}\hat{\boldsymbol{\mu}}_e$, $^{l-1}\hat{\boldsymbol{\Sigma}}_\theta$, and

$$^{l-1}\hat{\boldsymbol{\Sigma}}_e : ^l\hat{\boldsymbol{\theta}}_t = \arg \min_{\boldsymbol{\theta}} (J_{\theta,t} + J_{e,t}).$$

(3) Update the MAP estimates of $\mathbf{a}$ and $\mathbf{b}$ using $^l\hat{\boldsymbol{\theta}}_t$ and $^{l-1}\hat{\boldsymbol{\Sigma}}_\theta : ^l\hat{\mathbf{a}}, ^l\hat{\mathbf{b}} = \arg \min_{\mathbf{a}, \mathbf{b}} \left(\sum_{t=1}^{N_d} J_{\theta,t} \right).$

(4) Update the MAP estimate of $\boldsymbol{\mu}_e$ using $^{l-1}\hat{\boldsymbol{\theta}}_t$ and $^{l-1}\hat{\boldsymbol{\Sigma}}_e : ^l\hat{\boldsymbol{\mu}}_e = \arg \min_{\boldsymbol{\mu}_e} \left(\sum_{t=1}^{N_d} J_{e,t} \right).$

(5) Update the MAP estimates of $\boldsymbol{\Sigma}_\theta$ and $\boldsymbol{\Sigma}_e$:

$$^l\hat{\sigma}_{\theta,i}^2 = \frac{\lambda\tau_{0,\theta,i} + \sum_{t=1}^{N_d} (^l\hat{\boldsymbol{\theta}}_{t,i} - ^l\hat{\mathbf{a}}_i - ^l\hat{\mathbf{b}}_i \mathbf{T}_t)^2}{N_d + \lambda + 2}, \quad ^l\hat{\sigma}_{e,j}^2 = \frac{\lambda\tau_{0,e,j} + \sum_{t=1}^{N_d} (^l\mathbf{e}_{t,j} - ^l\hat{\boldsymbol{\mu}}_{e,j})^2}{N_d + \lambda + 2}.$$

(6) Repeat steps (2) to (5) until the mean ℓ_1 norm of the difference between the updating parameters in two consecutive iterations is below 10^{-3} .

The resulting MAP estimates are then used as the initial values for the MCMC sampling. The Gibbs–MH sampling approach is employed to generate samples based on Eq. (5.18)–(5.23). The samples of $\boldsymbol{\theta}_t$ and $\mathbf{b}$ are generated using the MH algorithm because their conditional posterior distributions are not of standard form. Samples of the remaining parameters, including $\mathbf{a}$, $\boldsymbol{\mu}_e$, $\sigma_{\theta,i}^2$, and $\sigma_{e,j}^2$, are generated using Gibbs sampling because their conditional posterior PDFs correspond to standard forms. The sampling procedure is repeated until the required number of samples is obtained.

5.5 FE Model and Surrogate Model

5.5.1 FE Model

The FE model of the Mingzhou Bridge is illustrated in Figure 5.11. The main girder, arch,

and transverse braces are built using beam elements, while the suspenders are modeled using link elements. These components are made of steel, with the initial elastic modulus set to 2.1×10^{11} Pa, and the thermal expansion coefficient set to 1.2×10^{-5} . The expansion joints connecting the middle spans and the side spans are modeled with linear spring elements with an initial longitudinal stiffness of 2.25×10^{11} N/m. The mass properties of the pavement, transverse girders, and the equivalent vehicle load are simulated using lumped mass elements. The FE model consists of 903 nodes and 971 elements in total.

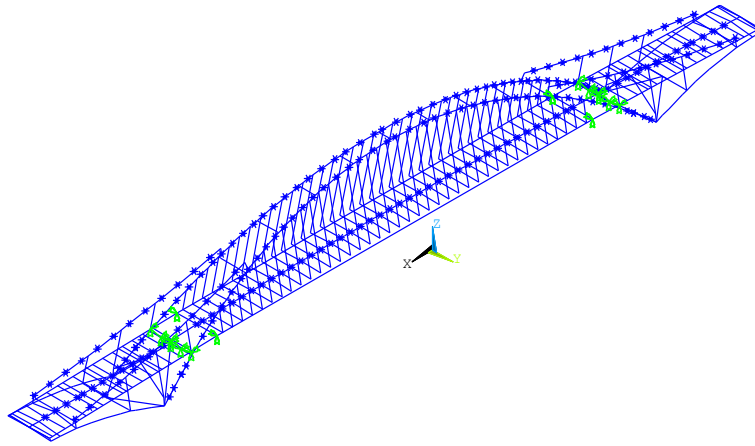


Figure 5.11 FE model of the Mingzhou Bridge

The predicted and identified natural frequencies under a structural temperature of 24.7 °C and zero equivalent vehicle weight are compared in Table 5.2. The relative errors between predicted and identified frequencies are below 4%, indicating that the FE model can accurately simulate the dynamic characteristics of the Mingzhou Bridge.

Table 5.2 Model-predicted and identified natural frequencies and DEJ

Response	Identified	Predicted	Relative error (%)
V1 (Hz)	0.48	0.49	2.98
V2 (Hz)	0.63	0.65	3.03
V3 (Hz)	1.65	1.69	2.42
DEJ (mm)	42.86	42.03	1.92

Note: DEJ is calculated with an actual temperature of 45 °C and a reference temperature of 24.7 °C.

The simulation process of DEJ in the t -th experiment is illustrated in Figure 5.12. The main steps are as follows:

- (1) Set the updating parameters in the FE model to θ_t , apply a global temperature load of $T_0 = 24.7\text{ }^\circ\text{C}$, and perform a static analysis, as shown in Figure 5.12(a).
- (2) Extract the longitudinal coordinates of the two nodes of the expansion joint, denoted as x_{N1} and x_{N2} . Calculate the reference DEJ as $\hat{d}(\theta_t, T_0) = x_{N2}(\theta_t, T_0) - x_{N1}(\theta_t, T_0)$.
- (3) Apply the operational temperature load T_t , and calculate the absolute DEJ as $\hat{d}(\theta_t, T_t) = x_{N2}(\theta_t, T_t) - x_{N1}(\theta_t, T_t)$, as shown in Figure 5.12(b).
- (4) Calculate the relative DEJ with the updating vector θ_t and the temperature load T_t as $d(\theta_t, T_t) = \hat{d}(\theta_t, T_0) - \hat{d}(\theta_t, T_t)$.

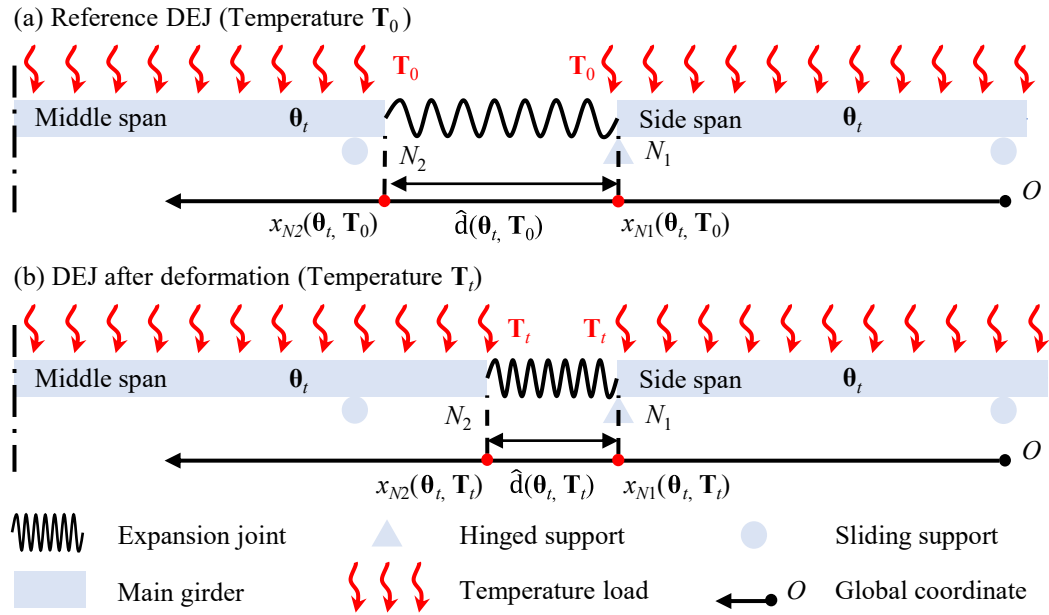


Figure 5.12 Simulation process of DEJ

The measured and predicted DEJs under a temperature load of $45\text{ }^\circ\text{C}$ and a reference temperature of $24.7\text{ }^\circ\text{C}$ are presented in Table 5.2. The relative error is less than 2%, indicating the proposed method can accurately simulate the thermally-induced DEJ.

5.5.2 Response Surface Surrogate Model

To improve computational efficiency, an RSM (Myers et al., 2016) based on the Gaussian radial basis function (Sun et al., 2020b) is adopted to build the surrogate model of the FE model. The mathematical formulation of the RSM is expressed as

$$\hat{\mathbf{u}} = \sum_{h=1}^{N_w} \varsigma_h e^{-\frac{\|\mathbf{r}-\mathbf{S}_h\|^2}{F^2}} + \sum_{k=1}^{N_z} p_k g_k(\mathbf{r}), \quad (5.24)$$

where $\hat{\mathbf{u}}$ denotes the RSM-predicted responses, $\mathbf{r}$ stands for the input vector, $\mathbf{S}_h$ is the h -th sample point (N_w in total), ς_h is the coefficient of the h -th Gaussian radial basis function, F is the shape factor, and p_k is the coefficient of the k -th polynomial basis function $g_k(\mathbf{r})$ (N_z in total). The parameters ς_h , F , and p_k are unknown and need to be optimized using the training samples. The training procedure of the RSM includes the following steps: (1) generating training samples using an appropriate DOE method; (2) defining an objective function based on the discrepancies between the RSM-predicted and the FE model-predicted responses; and (3) optimizing the parameters ς_h , F , and p_k to minimize the objective function.

Two separate RSMs are developed to simulate the natural frequencies and DEJs because these responses are influenced by different types of loads. The input variables for the frequency RSM are $[E_g, E_a, K_e, C, \mathbf{M}_t]$, and the outputs are the first three natural frequencies. The temperature load is excluded from the input because its influence on the stiffness parameters has already been accounted for in the stiffness parameters E_g and E_a . In the DEJ RSM, the input variables are $[E_g, E_a, K_e, \mathbf{T}_t]$, and the output is the DEJ. A total of 200 training samples are generated using the LHS method (Mckay et al., 2000). An additional 25 samples are generated using the CCD method to validate the trained RSMs (Montgomery, 2017). The performance of the trained RSMs is evaluated using the coefficient of determination (R^2), which is defined as

$$R^2 = 1 - \frac{\sum_{h=1}^{N_w} (\hat{\mathbf{u}}_h - \mathbf{u}_h)^2}{\sum_{h=1}^{N_w} (\hat{\mathbf{u}}_h - \bar{\mathbf{u}})^2}, \quad (5.25)$$

where $\hat{\mathbf{u}}$ and $\mathbf{u}$ are the RSM-predicted and FE model-predicted responses, respectively; and $\bar{\mathbf{u}}$ is the mean value of the FE model-predicted responses.

The objective function used for training the RSMs is defined as the sum of the relative errors between RSM-predicted and FE model-predicted responses. The GA is used to optimize the parameters of the two RSMs. The RSM-predicted and the FE model-predicted responses are compared in Table 5.3. The relative errors between the two models are all below 0.2%, and the R^2 values for the validation samples are close to 1, indicating the RSMs provide high approximation accuracy. Additionally, the computational time for a single test (including calculating both frequencies and DEJ) is reduced from 30.56 seconds using the FE model to just 0.11 seconds using the RSM. Therefore, incorporating the RSM into the hierarchical Bayesian FEMU framework can significantly improve computational efficiency without sacrificing accuracy.

Table 5.3 FE model-predicted and RSM-predicted responses

Response	FE model-predicted	RSM-predicted	Relative error (%)	R^2
V1 (Hz)	0.49	0.49	0.14	0.99
V2 (Hz)	0.65	0.65	-0.0051	0.99
V3 (Hz)	1.69	1.69	0.026	0.99
DEJ (mm)	42.03	42.03	0.00028	0.99

Note: DEJ is calculated with an actual temperature of 45 °C and a reference temperature of 24.7 °C.

5.6 Application to the Mingzhou Bridge

The proposed hierarchical Bayesian FEMU method is then applied to the Mingzhou Bridge. A total of 3,000 datasets collected from January 2017 to June 2018 are used as

the training set, while another 409 datasets collected from July 2018 to January 2019 are used for model validation. The hyperparameters of the prior distributions are set as $\lambda = 1$, $\tau_{0,\theta,i} = 10^{-2}$, and $\tau_{0,\epsilon,j} = 10^{-6}$. A relatively large value is chosen for $\tau_{0,\theta,i}$ to reflect the high initial uncertainty in the structural parameters and to minimize the influences of the priors on the posterior distribution. A small value is assigned to $\tau_{0,\epsilon,j}$ to impose a strong constraint on the posterior PDF of the mean vector of the error function, thereby facilitating the convergence of the stochastic sampling process.

5.6.1 Updating Results

The MAP estimates of the model parameters are first obtained through the iterative process, and the results are summarized in Table 5.4. a_{Eg} and a_{Ea} denote the intercepts of the linear temperature–elastic modulus relationship for the main girder and arch. Their values are slightly less than 1, indicating that the elastic modulus of the steel material is slightly lower than the initial value of 210 GPa under zero temperature and traffic loads. a_{K_e} , the mean value of the normalized longitudinal stiffness of the expansion joint, is less than 1, indicating that the initial value of K_e is overestimated. a_C represents the mean of the vehicle weight scaling variable and is slightly higher than one.

Table 5.4 MAP estimates of the updating parameters

Updating parameter	a	b ($10^{-4}/^{\circ}\text{C}$)	$\log(\sigma_{\theta i}^2)$	Response	μ_{ϵ} (%)	$\log(\sigma_{\epsilon j}^2)$
E_g	0.96	−13.30	−5.47	V1	0.21	−9.48
E_a	0.94	−3.88	−4.49	V2	−0.29	−5.03
K_e	0.72	-	−2.87	V3	−0.06	−5.54
C	1.12	-	−4.48	DEJ	−2.24	−2.61

The MAP estimate for the slope term in the temperature–elastic modulus relationship of the main girder, $b_{Eg} = -13.3 \times 10^{-4}/^{\circ}\text{C}$, is significantly larger in magnitude than the standard value of $-3.6 \times 10^{-4}/^{\circ}\text{C}$. This is because the main girder is composed of steel,

asphalt, and concrete, and the recommended elastic modulus reduction factors of concrete and asphalt are within the ranges of $[-18 \times 10^{-4}, -47 \times 10^{-4}]/^{\circ}\text{C}$ (En-1992-1-2, 2004) and $[-160 \times 10^{-4}, -190 \times 10^{-4}]/^{\circ}\text{C}$ (Espersson, 2014), respectively, which are significantly larger in magnitude than that of steel. The MAP estimate for the slope term of the main arch b_{Ea} is relatively close to the standard value of $-3.6 \times 10^{-4}/^{\circ}\text{C}$, because the arch is entirely made of steel.

The mean and variance of the frequency errors are minor, indicating the updated FE model can accurately capture the dynamic characteristics of the Mingzhou Bridge. However, the mean error value of the DEJ reaches 2.24%, and the associated variance is relatively large, suggesting the accuracy of the FE model in simulating DEJ is relatively low. This is because the selected reference temperature in the DEJ simulation may be inaccurate, resulting in a constant offset between the simulated and measured DEJs.

Subsequently, 1,000 samples for each parameter are generated using the joint Gibbs–MH sampling method. The histograms of the posterior samples are shown in Figures 5.13–5.14. The samples of $\mathbf{a}$, $\boldsymbol{\mu}_{\text{e}}$, $\sigma_{\theta, i}^2$ and $\sigma_{\text{e}, j}^2$ roughly follow normal distributions, and the samples of $\mathbf{b}$ exhibit skewed tails.

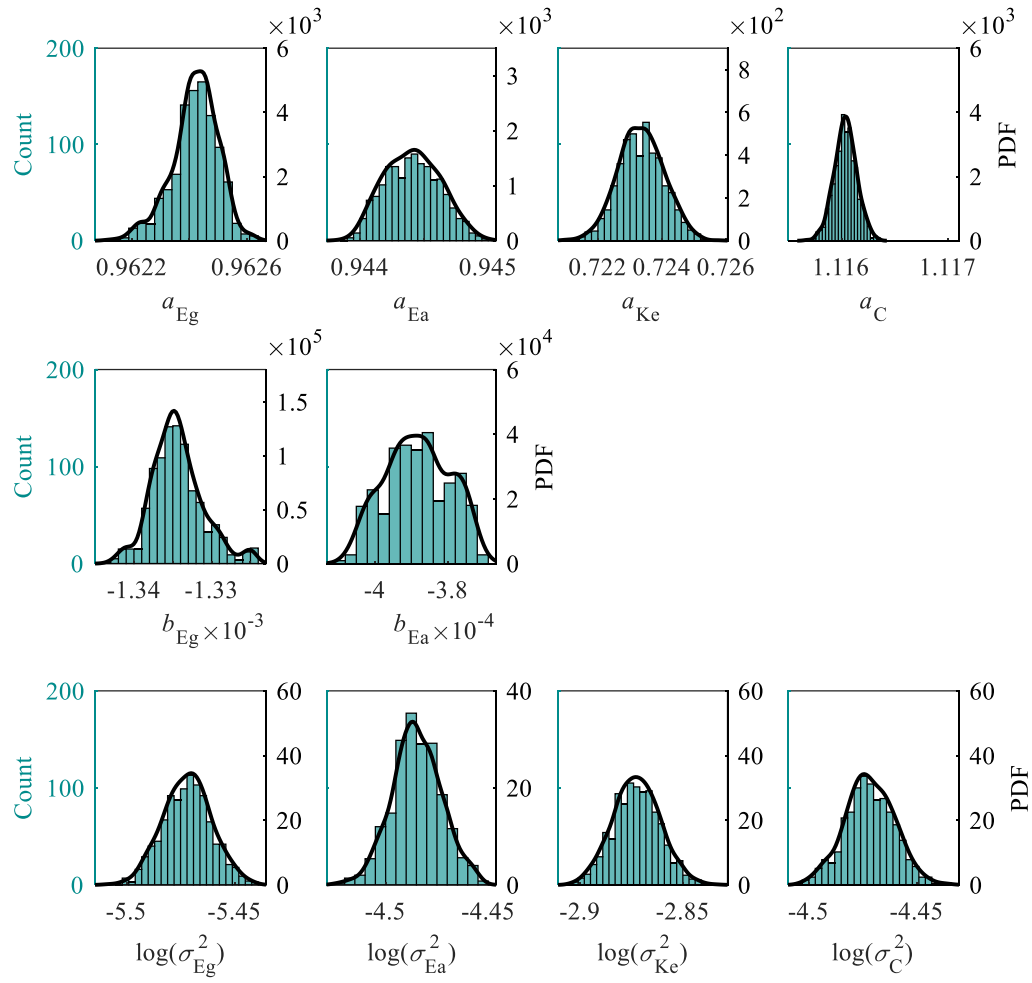


Figure 5.13 Histograms of samples associated with updating parameters

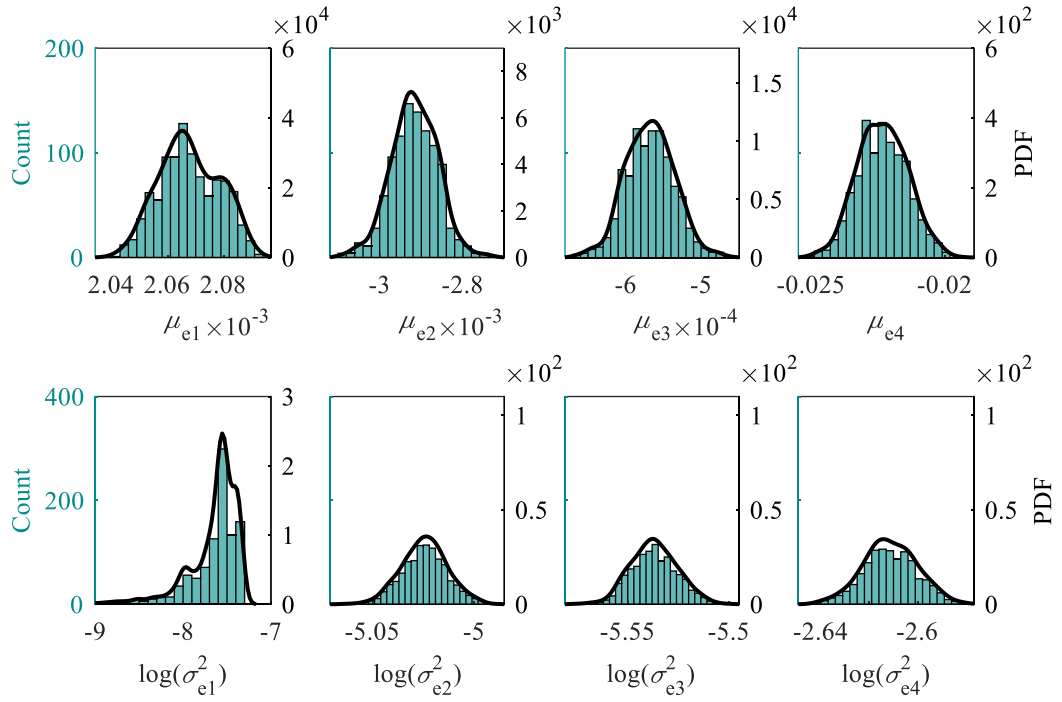


Figure 5.14 Histograms of samples associated with error functions

5.6.2 Probabilistic Response Prediction

A subset of 100 samples is selected from the generated samples to calculate probabilistic responses based on the procedure described in subsection 4.2.5. The predicted natural frequencies of modes V1, V2, and V3 are presented in Figures 5.15–5.17, respectively. The predicted frequencies exhibit the same variation patterns as the identified ones, demonstrating that the updated model accurately simulates the influence of operational loads on the structure's dynamic behavior. In the upper subfigures, both the uncertainties arising from the updating parameters and error functions are considered in the predictions. Most measured values fall within the predicted 95% CIs. In the lower subfigures, where only the parameter uncertainty is considered in the predictions, some of the measured frequencies fall outside the predicted CIs.

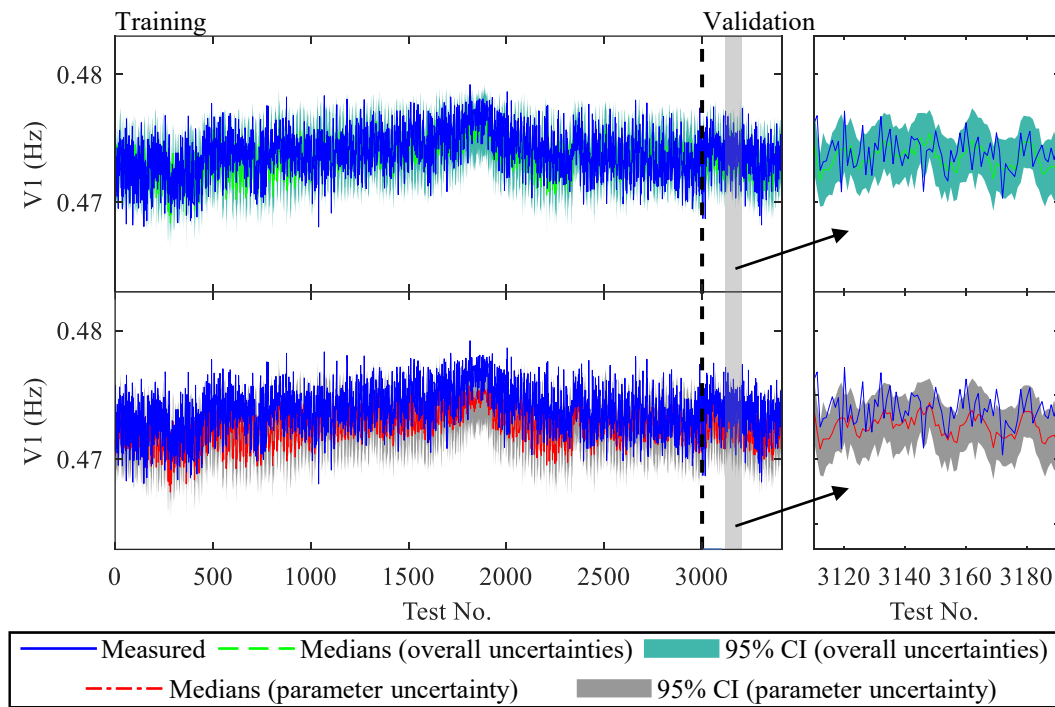


Figure 5.15 Predicted natural frequencies of mode V1

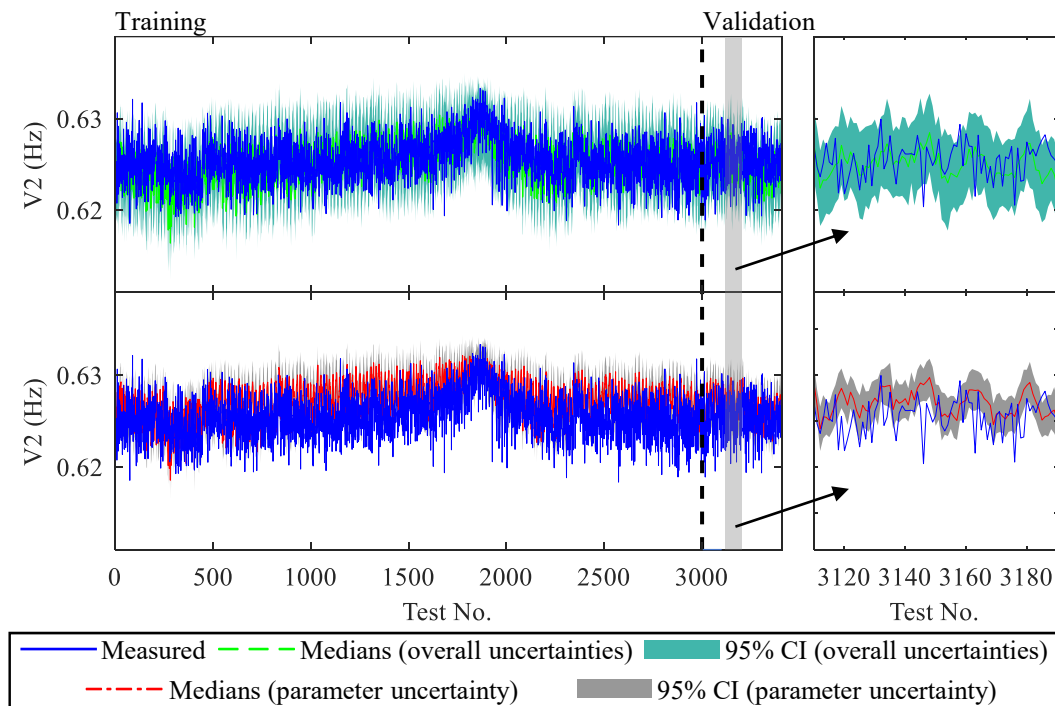


Figure 5.16 Predicted natural frequencies of mode V2

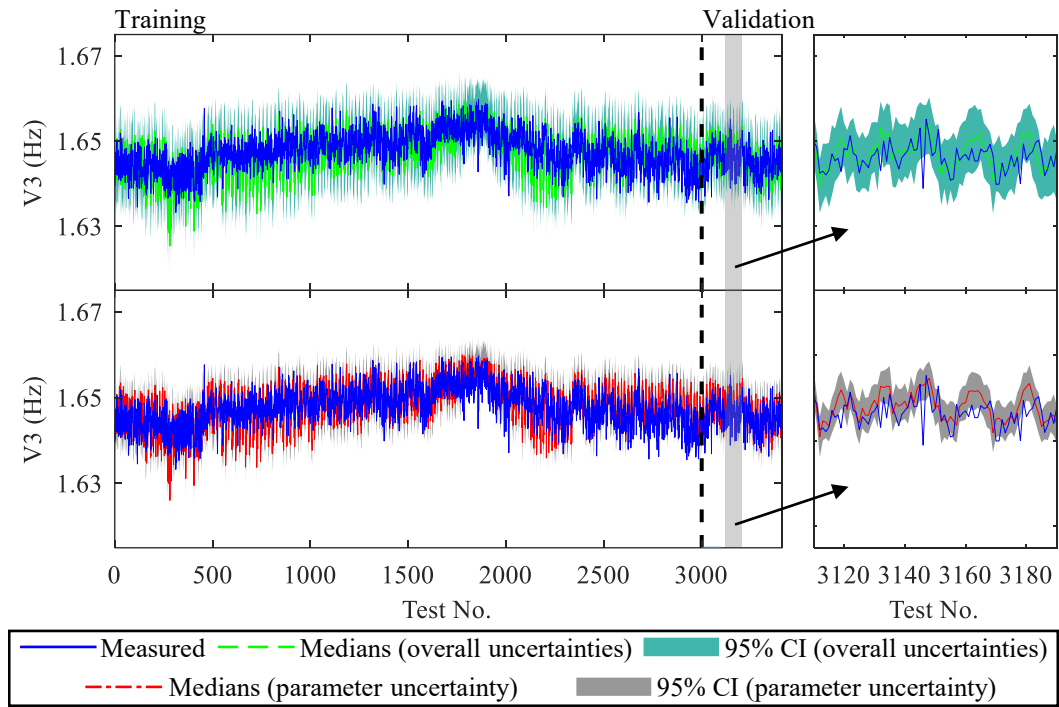


Figure 5.17 Predicted natural frequencies of mode V3

The comparison between the predicted and measured DEJs is shown in Figure 5.18. In the lower subfigure, where the error function is excluded, the predicted 95% CIs are very narrow and fail to encompass the majority of the measured DEJ values. In the upper subfigure, where the uncertainties arising from both the updating parameters and error functions are incorporated, the predicted CIs become wider and successfully encompass most of the measured DEJs. The result indicates that the dominant source of uncertainty in DEJ predictions arises from the modeling error.

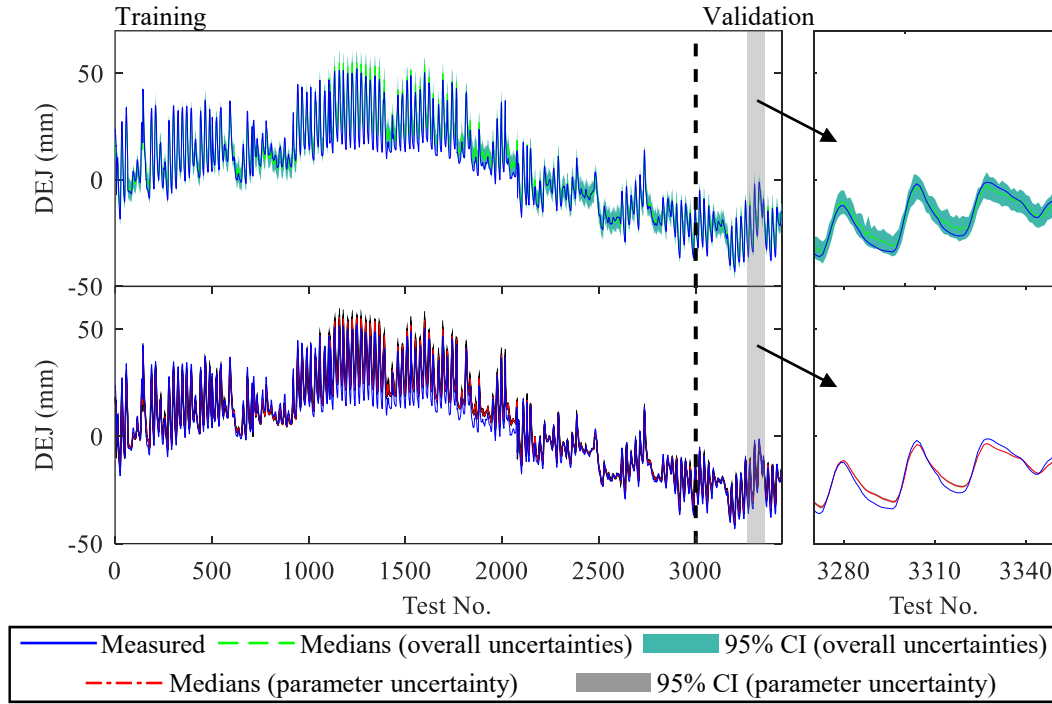


Figure 5.18 Predicted DEJs

In the hierarchical Bayesian framework, the uncertainties in the predicted responses arise from two primary sources: parameter uncertainty and modeling error uncertainty. The relative contribution of each source to the total predictive uncertainty depends on the accuracy of the FE model and the level of measurement noise. If the FE model is highly accurate, most of the uncertainty is attributed to the parameter uncertainty (Σ_{θ}), while only a small portion arises from the uncertainty of the error function (Σ_e), as in the case of the natural frequency prediction in this study. Conversely, when the modeling errors are significant, the dominant source of uncertainty shifts to the error function, as demonstrated in the case of the DEJ simulation. Therefore, it is crucial to incorporate the error function into probabilistic response predictions, particularly when the estimated variance of the error function is relatively large.

5.6.3 Structural Abnormal State Identification

The predicted response CIs have already considered the uncertainties arising from the

operational loads, structural parameters, and modeling errors. Therefore, if the measured responses consistently fall outside the predicted CIs over an extended period, the structure can be qualitatively considered to be in an abnormal state. The Mingzhou Bridge underwent a pavement replacement from January 2021 to March 2021. During this period, the mass properties of the main girder changed significantly. This event provides a valuable dataset to validate the effectiveness of the proposed method in structural condition assessment.

The measured and predicted natural frequencies during this period are shown in Figure 5.19. Due to changes in mass properties caused by pavement movement and reinstallation, the measured structural frequencies initially increase and then decrease. The measured frequencies fall outside the predicted 95% CIs for nearly 800 hours during the pavement replacement period, indicating that the structure was likely in an abnormal state.

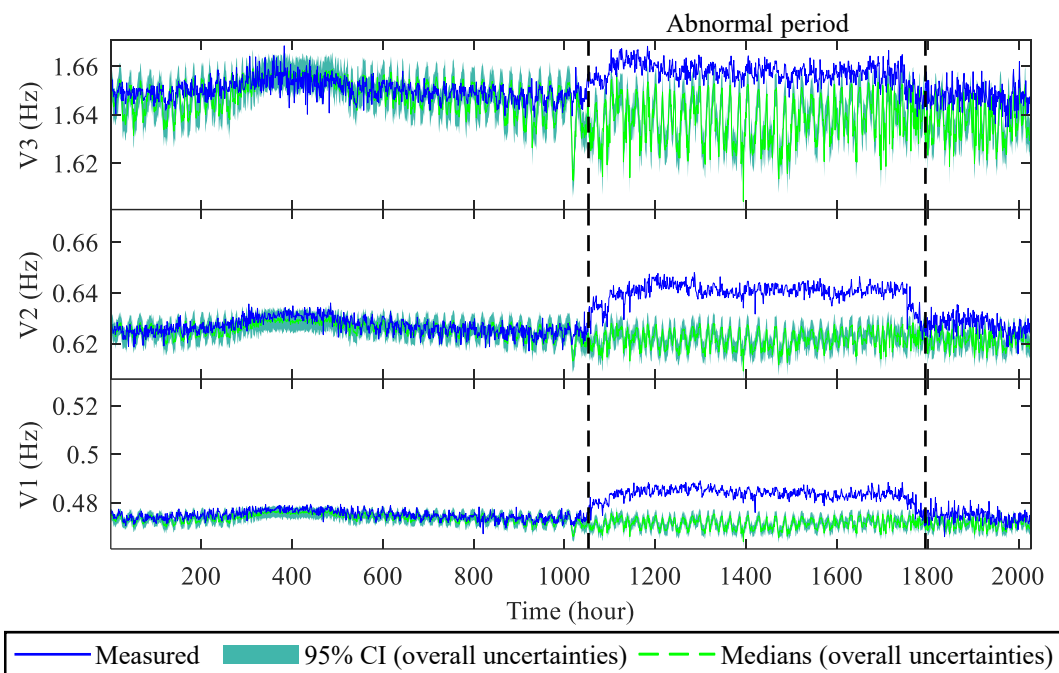


Figure 5.19 Natural frequency predictions during the pavement replacement period

A structural condition indicator (SCI), established using the identified and predicted

natural frequencies, is further proposed to quantitatively assess the abnormality. The structure is considered to be in an abnormal state when the measured frequency lies outside the 3σ range of the predicted response distribution (i.e., the 99.7% confidence interval). The process for calculating SCI is exhibited in Figure 5.20, and the key steps are summarized as follows:

- (1) Calculate SCI for the m -th mode in the t -th experiment as $SCI_{t,m} = (\tilde{f}_{t,m} - \mu_{t,m}^f) / (3\sigma_{t,m}^f)$, where $\tilde{f}_{t,m}$ represents the measured natural frequency, and $\mu_{t,m}^f$ and $\sigma_{t,m}^f$ denote the mean and standard deviation of the predicted responses.
- (2) Calculate the weighted SCI using a triangular time window of length W , which smooths out the short-term noise in the measurements.
- (3) Average the weighted SCIs across all modes to obtain the final condition indicator SCI_t .

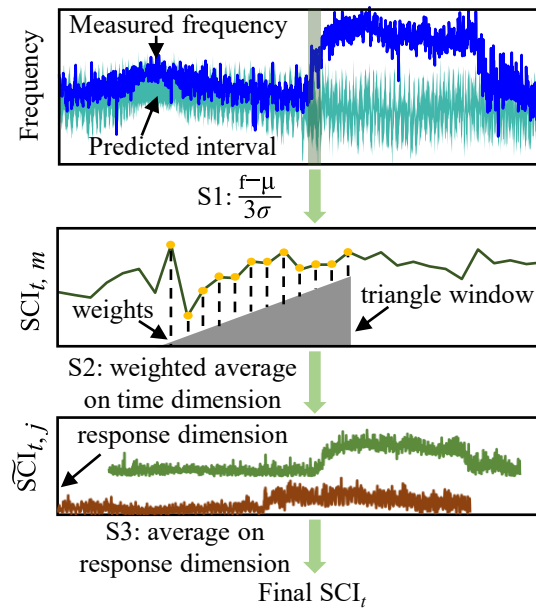


Figure 5.20 Steps for calculating SCI

When SCI is larger than 1, the structure is considered to be in an abnormal state. The SCIs obtained using different window lengths are compared in Figure 5.21. The SCI curves

become smoother as the window length increases. The moment when SCI exceeds 1 is close to the onset of the maintenance period, indicating the proposed indicator can accurately detect the structural abnormal state. Overall, the proposed SCI serves as a primitive yet effective indicator for quantitative structural condition assessment. The hyperparameters, including the threshold value, window type, and window length, can be flexibly adjusted to improve detection performance based on practical monitoring scenarios and operational requirements. For risk-based decision-making, a multi-level warning system can be implemented, with each warning level triggering corresponding inspection protocols and intervention measures. Thresholds should be calibrated using historical data and engineering experience to accurately reflect the actual risks and critical points of the structure.

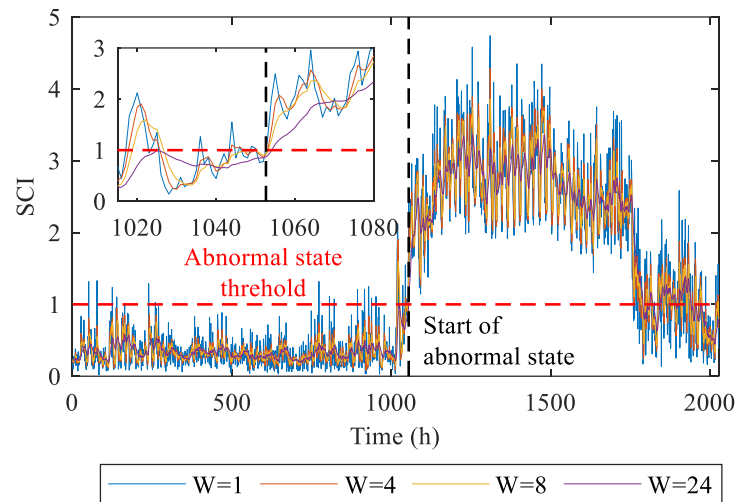


Figure 5.21 Comparison of SCIs obtained using different window lengths

5.6.4 Comparison between Different FEMU Methods

The methods proposed in Chapters 3–5, referred to as Methods I, II, and III, are comparatively evaluated in this subsection. The Mingzhou Bridge is used as the testbed. The monitoring data collected from 2017 to 2018 are employed to update the FE model. To ensure consistency across the methods, only the first three natural frequencies are used

for FEMU in this comparison. The measured frequencies during the pavement replacement period are used for model validation. In Methods I and II, the updating parameters include the elastic modulus of the main girder and the main arch, denoted as $[E_g, E_a]^T$. Method III additionally incorporates the vehicle weight scaling factor C , resulting in an updating parameter vector of $[E_g, E_a, C]^T$.

The MAP estimates of the key parameters are summarized in Table 5.5. The mean values of the updating parameters from the three methods are generally consistent. Method I is a deterministic method and cannot quantify the system uncertainty. In contrast, Methods II and III are probabilistic and can estimate the variances of both the updating parameters and modeling errors. The variances of updating parameters estimated by Method II are larger than those from Method III. This is because Method II does not consider the operational loads. As a result, the response variability caused by the load changes is incorrectly attributed to the parameter uncertainty. The variances of the error functions estimated by Methods II and III are both small and similar, indicating the FE model can accurately simulate the natural frequencies.

Table 5.5 MAP estimates of the key parameters by the three methods

Methods	E_g	$\log(\sigma_{E_g}^2)$	E_a	$\log(\sigma_{E_a}^2)$	$\log(\sigma_{e,1}^2)$	$\log(\sigma_{e,2}^2)$	$\log(\sigma_{e,3}^2)$
I	0.92	—	0.91	-	-	-	-
II	0.93	-2.70	0.91	-3.79	-9.47	-4.94	-8.09
III	0.93	-6.48	0.94	-4.14	-9.48	-4.85	-5.18

Note: Stiffness parameters for Method III are calculated using $\mathbf{a} + \mathbf{b}\mathbf{T}_t$ with $\mathbf{T}_t = 24.7^\circ\text{C}$.

Subsequently, the predicted natural frequencies during the pavement replacement process are compared in Figure 5.22. For Method I, the first three predicted natural frequencies are fixed at 0.47, 0.63, and 1.65 Hz, which are nearly identical to the mean values of the measured frequencies in the training dataset. Since Method I only provides deterministic predictions, the SCI cannot be calculated. In contrast, both the uncertainties arising from the parameter errors and modeling errors are considered in the predicted CIs of Methods

II and III. For Method II, the predicted CIs remain constant across different datasets because the influence of operational loads is not considered. During the normal state, the measured values fall within the predicted CIs. However, these CIs are relatively wide and contain redundant uncertainty. In addition, the breakthrough points, where the measured frequencies exceed the predicted CIs, occur significantly later than those predicted by Method III. This delay is caused by the misinterpretation of uncertainty: the response variability induced by operational loads, which is independent across tests, is incorrectly attributed to parameter uncertainty, which affects all tests consistently. As a result, the response uncertainty arising from the updating parameters is overestimated, and the sensitivity and accuracy of the early warnings are thus reduced. Therefore, incorporating the operational loads into the FEMU process is essential for long-term operational structures.

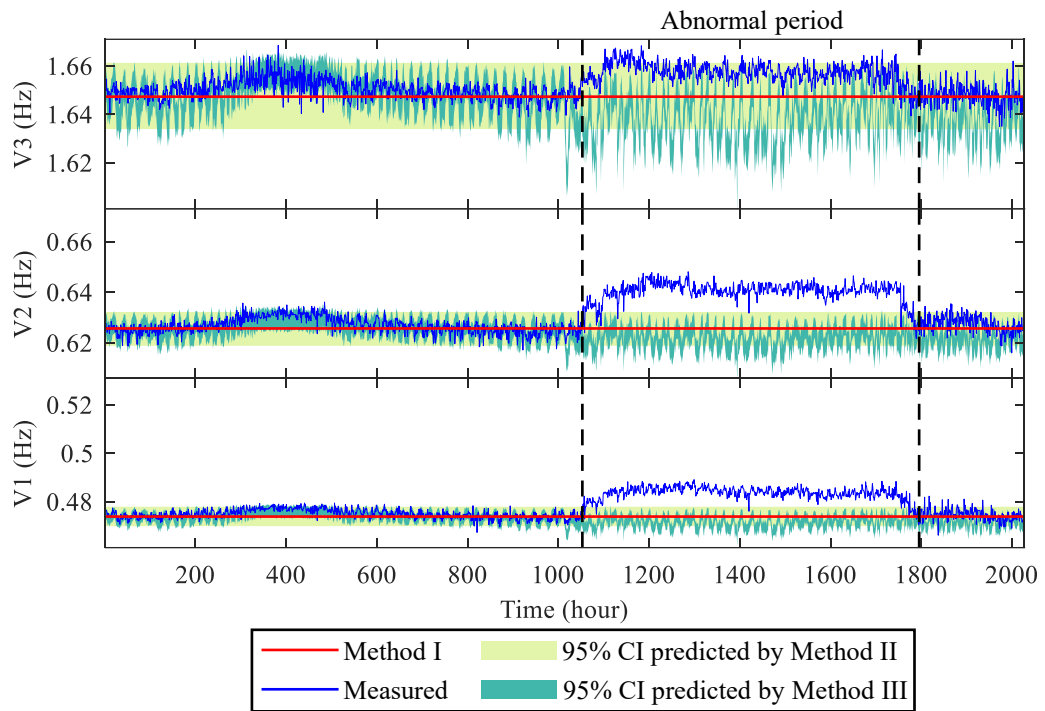


Figure 5.22 Predicted frequencies by the three methods

Finally, the SCIs for Methods II and III, using a time window length of 8, are calculated and shown in Figure 5.23. The SCI values from Method II are significantly lower than

those from Method III because the CIs predicted by Method II are relatively wide. In addition, the early warning time predicted by Method II lags behind that of Method III by approximately 60 hours. These results further highlight the importance of considering the influence of operational loads in the FEMU process and the condition assessment of long-term operational structures.

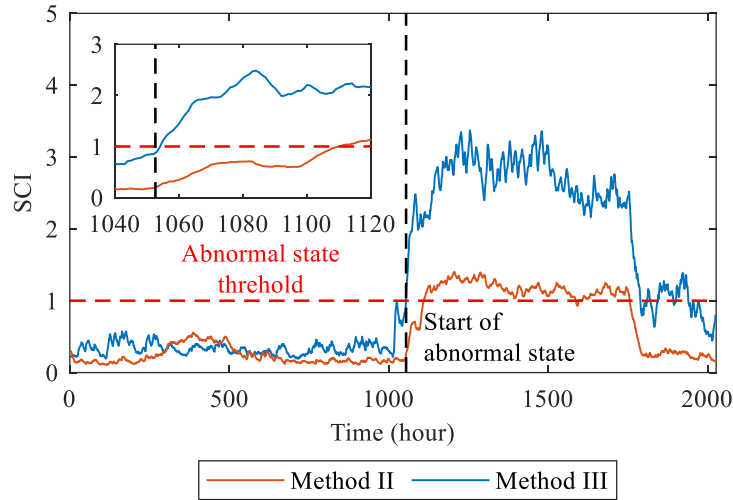


Figure 5.23 SCIs calculated by Methods II and III

5.7 Summary

This chapter proposes a hierarchical Bayesian FEMU method considering the effects of temperature and traffic loads on structural characteristics. A linear relationship between structural temperature and elastic modulus is established to explain the observed decreasing trend between temperature and measured natural frequencies. A vehicle weight estimation approach is developed to consider the effects of traffic loads on structural properties. A two-stage MCMC method is employed to obtain the posterior samples of both updating parameters and hyperparameters. Finally, an SCI is defined to quantitatively assess the structural condition based on the predicted response CIs.

The Mingzhou Bridge is used as the testbed to validate the proposed method. The updated model accurately simulates the response variations induced by temperature and vehicle loads. The predicted 95% CIs encompass most measured natural frequencies and DEJs when the uncertainties of both updating parameters and modeling errors are considered. The proposed SCI successfully identifies the pavement replacement period as an abnormal state and provides a reliable indicator for quantitative structural condition assessment. Comparative analyses with the methods presented in Chapters 3–5 further emphasize the importance of incorporating operational loads into the FEMU process and condition assessment of long-term operational structures.

CHAPTER 6

SPARSE BAYESIAN LEARNING FOR JOINT LOAD– PARAMETER IDENTIFICATION

6.1 Introduction

Chapters 3 to 5 address parameter identification and uncertainty quantification when the loads are known. This chapter further investigates system identification problems under unknown loading conditions. Existing joint load–parameter methods often rely on *ad hoc* hyperparameter settings and assume that the loading locations are known, which limits their practical applicability. Given that both structural damage and external loads are typically sparse in the spatial domain, their sparsity can be effectively incorporated as prior information within the Bayesian framework. To this end, this chapter proposes a sparse Bayesian joint load–parameter identification method based on the transmissibility function. The contributions of this study can be summarized as follows: (1) the transmissibility matrix is employed to simulate dynamic responses, and the method is thus applicable to various types of unknown loads; (2) the EM algorithm is used to iteratively estimate damage indices, loads, and precision factors; and (3) a revised MH algorithm is developed to jointly sample structural parameters and loads.

6.2 Sparse Bayesian Modeling for Joint Identification

6.2.1 Governing Equations

The governing equation of a linear dynamic system with N DOFs is given by:

$$\mathbf{M}\mathbf{a}(t) + \mathbf{C}\mathbf{v}(t) + \mathbf{K}\mathbf{d}(t) = \mathbf{S}_p \mathbf{f}(t), \quad (6.1)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K} \in \mathbb{R}^{N \times N}$ are the structural mass, damping, and stiffness matrices, respectively; $\mathbf{a}(t)$, $\mathbf{v}(t)$, $\mathbf{d}(t) \in \mathbb{R}^{N \times 1}$, and $\mathbf{f}(t) \in \mathbb{R}^{N_f \times 1}$ denote the acceleration, velocity, displacement, and force vectors at time t , respectively; N_f is the number of forces; and $\mathbf{S}_p \in \mathbb{R}^{N \times N_f}$ is the input selection matrix. The structural mass matrix $\mathbf{M}$ is assumed to be known, while the stiffness matrix $\mathbf{K}$ is unknown and parameterized by assembling the elemental stiffness matrices:

$$\mathbf{K}(\boldsymbol{\theta}) = \sum_{i=1}^{N_e} (1 - \theta_i) \bar{\mathbf{K}}_i, \quad (6.2)$$

where N_e is the number of elements; $\bar{\mathbf{K}}_i$ is the standard stiffness matrix of the i -th element ($i=1, 2, \dots, N_e$); $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_i, \dots, \theta_{N_e}]^T$ is the damage index vector; and θ_i is the damage index (DI) associated with the i -th element. Each θ_i ranges from zero to one, where zero indicates no damage to the element and one denotes complete damage.

Then, Eq. (6.1) is reformulated as the system equation in the state space:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{S}_p \end{bmatrix} \mathbf{f}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \mathbf{f}(t), \quad (6.3)$$

where $\mathbf{x}(t) = \begin{bmatrix} \mathbf{d}(t) \\ \mathbf{v}(t) \end{bmatrix}$ is the state vector. The time-discretized recursive form of Eq. (6.3) is further expressed as

$$\mathbf{x}_{k+1} = e^{\mathbf{A}_c \Delta t} \mathbf{x}_k + \mathbf{A}_c^{-1} (e^{\mathbf{A}_c \Delta t} - \mathbf{I}) \mathbf{B}_c \mathbf{f}_k = \mathbf{A} \mathbf{x}_k + \mathbf{B} \mathbf{f}_k, \quad (6.4)$$

where $\mathbf{x}_k$ is the state vector at the instant $t_k = k \cdot \Delta t$ ($k = 0, 1, 2, \dots, N_t - 1$); Δt is the time step; N_t is the total number of time steps; and $\mathbf{f}_k = [f_{k,1}, f_{k,2}, \dots, f_{k,N_f}]^T$ is the time-discretized force vector with the zero-order hold assumption.

The theoretical observation vector at the k -th time step, $\mathbf{y}_k \in \mathbb{R}^{Nm \times 1}$, is defined as

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{S}_\varepsilon \boldsymbol{\gamma} & \mathbf{0} \\ \mathbf{S}_d & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_v \\ -\mathbf{S}_a \mathbf{M}^{-1} \mathbf{K} & -\mathbf{S}_a \mathbf{M}^{-1} \mathbf{D} \end{bmatrix} \mathbf{x}_k + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{S}_a \mathbf{M}^{-1} \mathbf{S}_p \end{bmatrix} \mathbf{f}_k = \mathbf{G} \mathbf{x}_k + \mathbf{L} \mathbf{f}_k, \quad (6.5)$$

where N_m is the total number of sensors; $\mathbf{S}_\varepsilon \in \mathbb{R}^{N_\varepsilon \times N}$, $\mathbf{S}_d \in \mathbb{R}^{N_d \times N}$, $\mathbf{S}_v \in \mathbb{R}^{N_v \times N}$, and $\mathbf{S}_a \in \mathbb{R}^{N_a \times N}$ are the response selection matrices for strain, displacement, velocity, and acceleration, respectively; the sum of N_ε , N_d , N_v , and N_a is N_m ; $\boldsymbol{\gamma} \in \mathbb{R}^{N \times N}$ is the shape function matrix that converts the displacement to strain; $\mathbf{G} \in \mathbb{R}^{N_m \times 2N}$ is the state-to-observation (indirect feedthrough) matrix; and $\mathbf{L} \in \mathbb{R}^{N_m \times N_f}$ is the load-to-observation (direct feedthrough) matrix.

Without loss of generality, the initial system state is assumed to be zero ($\mathbf{x}_0 = \mathbf{0}$). Subsequently, Eqs. (6.4)–(6.5) are represented as follows with the data at all sampling instants being collected:

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \vdots \\ \mathbf{y}_{N_t-1} \end{bmatrix} = \begin{bmatrix} \mathbf{L} & & & & \\ \mathbf{GB} & \mathbf{L} & & & \\ \mathbf{GAB} & \mathbf{GB} & \mathbf{L} & & \\ \mathbf{GA}^2\mathbf{B} & \mathbf{GAB} & \mathbf{GB} & \mathbf{L} & \\ \vdots & \vdots & & & \ddots \\ \mathbf{GA}^{N_t-2}\mathbf{B} & \mathbf{GA}^{N_t-3}\mathbf{B} & \dots & \mathbf{GAB} & \mathbf{GB} & \mathbf{L} \end{bmatrix} \begin{bmatrix} \mathbf{f}_0 \\ \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \\ \vdots \\ \mathbf{f}_{N_t-1} \end{bmatrix} = \mathbf{H}(\boldsymbol{\theta}) \mathbf{F} \quad (6.6)$$

where $\mathbf{Y} \in \mathbb{R}^{(N_m \times N_t) \times 1}$ and $\mathbf{F} \in \mathbb{R}^{(N_f \times N_t) \times 1}$ are stacked theoretical response and load vectors of time steps, respectively; and $\mathbf{H}(\boldsymbol{\theta}) \in \mathbb{R}^{(N_m \times N_t) \times (N_f \times N_t)}$ represents the transmissibility matrix as a function of the damage index vector $\boldsymbol{\theta}$. Transmissibility-based response simulation can be time-consuming when processing large-scale, long-duration datasets. In this case, model reduction techniques such as modal superposition and substructuring can be employed to reduce computational cost. Additionally, damage can be first identified using small data segments, as damage indices generally remain stable over short periods. Loads can then be estimated segment by segment using short-duration responses and the identified damage indices, avoiding the need to construct the

transmissibility matrix for the entire dataset.

6.2.2 Sparse Bayesian Framework

The structural dynamic responses are assumed to be measured, from which the structural damage and input can be jointly estimated. The Bayesian formulation for the joint load–parameter identification is expressed as

$$p(\boldsymbol{\theta}, \mathbf{F} | \tilde{\mathbf{Y}}) = p(\tilde{\mathbf{Y}} | \boldsymbol{\theta}, \mathbf{F})p(\boldsymbol{\theta})p(\mathbf{F}) / p(\tilde{\mathbf{Y}}), \quad (6.7)$$

where $p(\boldsymbol{\theta}, \mathbf{F} | \tilde{\mathbf{Y}})$ is the posterior PDF of the structural damage index vector $\boldsymbol{\theta}$ and input vector $\mathbf{F}$ given the measured responses $\tilde{\mathbf{Y}}$; $p(\tilde{\mathbf{Y}} | \boldsymbol{\theta}, \mathbf{F})$ is the likelihood function representing the PDF of $\tilde{\mathbf{Y}}$ given $\boldsymbol{\theta}$ and $\mathbf{F}$; $p(\boldsymbol{\theta})$ and $p(\mathbf{F})$ are prior PDFs for the damage and input vectors, respectively; and $p(\tilde{\mathbf{Y}})$ is the evidence function and is usually simplified as a constant factor. The likelihood, prior, and posterior PDFs represent data-derived information, initial assumptions, and updated knowledge, respectively. In such a Bayesian formulation, empirical observations and prior beliefs are integrated via probabilistic reasoning.

6.2.3 Likelihood Function of Observation

The measurement error $\mathbf{e}$ is defined as the discrepancy between the measured response $\tilde{\mathbf{Y}}$ and theoretical response $\mathbf{Y}$:

$$\mathbf{e} = \tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F}. \quad (6.8)$$

In this study, only the acceleration data are used for joint identification. For simplicity, $\mathbf{e}$ is assumed to follow the homogeneous and independent Gaussian distribution:

$$\mathbf{e} \sim \mathcal{N}(\mathbf{0}, \mathbf{I}/\alpha), \quad (6.9)$$

where α is the precision factor for the measurement error. If additional types of measurement are included, or the noise levels of various sensors are different, different precision factors can be assigned for the covariance matrix of $\mathbf{e}$.

Finally, the likelihood function can be expressed as

$$p(\tilde{\mathbf{Y}} | \boldsymbol{\theta}, \mathbf{F}, \alpha) = \left(\frac{\alpha}{2\pi}\right)^{N_t \times N_m / 2} \exp\left(-\frac{\alpha}{2}(\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F})^T (\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F})\right). \quad (6.10)$$

6.2.4 Prior Distributions for Damage and Load

For practical infrastructures, the damage is usually sparsely distributed, and the external loads are applied to limited positions. Therefore, the ARD priors following the Gaussian distribution are used for $p(\boldsymbol{\theta})$ and $p(\mathbf{F})$. The ARD priors serve as adaptive regularization terms to deal with the ill-posed problem in the joint identification.

The DI of the i -th element θ_i is assumed to follow a zero-mean Gaussian distribution with the precision of β_i :

$$\theta_i \sim \mathcal{N}(0, 1/\beta_i), (i=1, 2, \dots, N_e). \quad (6.11)$$

The precision coefficients β_i of different elements are independent. Therefore, the prior distribution of the damage index vector $\boldsymbol{\theta}$ can be represented as

$$p(\boldsymbol{\theta} | \boldsymbol{\beta}) = \prod_{i=1}^{N_e} p(\theta_i) = \frac{1}{(2\pi)^{N_e/2} |\boldsymbol{\Sigma}_{\boldsymbol{\theta}}|^{1/2}} \exp\left(-\frac{1}{2} \boldsymbol{\theta}^T \boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1} \boldsymbol{\theta}\right), \quad (6.12)$$

where $\boldsymbol{\beta} = [\beta_1, \beta_2, \dots, \beta_{N_e}]^T$ is the collection of precision coefficients; and $\boldsymbol{\Sigma}_{\boldsymbol{\theta}}^{-1}$ is the inverse of the covariance matrix, calculated by:

$$\mathbf{\Sigma}_0^{-1} = \begin{bmatrix} \beta_1 & & & 0 \\ & \beta_2 & & \\ & & \ddots & \\ 0 & & & \beta_{N_e} \end{bmatrix}_{N_e \times N_e} . \quad (6.13)$$

The determinant of $\mathbf{\Sigma}_0$ can be directly calculated as

$$|\mathbf{\Sigma}_0| = \prod_{i=1}^{N_e} \frac{1}{\beta_i} . \quad (6.14)$$

Similarly, the input force at the p -th candidate loading location and the k -th time step is assumed to follow a zero-mean Gaussian distribution governed by the precision parameter γ_p :

$$f_{k,p} \sim \mathcal{N}(0, 1/\gamma_p), (p=1, 2, \dots, N_f). \quad (6.15)$$

The nodal forces at each DOF are assumed to be independent. Thus, $p(\mathbf{F})$ can be expressed as the product of $N_t \times N_f$ independent Gaussian distributions as follows:

$$p(\mathbf{F}|\boldsymbol{\gamma}) = \prod_{k=0}^{N_t-1} \prod_{p=1}^{N_f} p(f_{k,p}) = \frac{1}{(2\pi)^{N_t \times N_f/2}} \frac{1}{|\mathbf{\Sigma}_F|^{1/2}} \exp\left(-\frac{1}{2} \mathbf{F}^T \mathbf{\Sigma}_F^{-1} \mathbf{F}\right), \quad (6.16)$$

where $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \dots, \gamma_{N_f}]^T$ is the precision vector of the candidate forces; and $\mathbf{\Sigma}_F$ is the covariance matrix of the load. The inverse of $\mathbf{\Sigma}_F$, which is the precision matrix, can be expressed as

$$\mathbf{\Sigma}_F^{-1} = \begin{bmatrix} \text{diag}(\boldsymbol{\gamma}) & & & \mathbf{0} \\ & \text{diag}(\boldsymbol{\gamma}) & & \\ & & \ddots & \\ \mathbf{0} & & & \text{diag}(\boldsymbol{\gamma}) \end{bmatrix}_{N_t \times N_t} . \quad (6.17)$$

The determinant of Σ_F is calculated as

$$|\Sigma_F| = \left(\prod_{p=1}^{N_f} \frac{1}{\gamma_p} \right)^{N_t}. \quad (6.18)$$

6.2.5 Posterior PDF

The posterior PDF $p(\theta, F | \tilde{Y}, \xi)$ is obtained by substituting Eqs. (6.10), (6.12), and (6.16) into Eq. (6.7):

$$\begin{aligned} p(\theta, F | \tilde{Y}, \xi) &= c^{-1} p(\tilde{Y} | \theta, F, \alpha) p(\theta | \beta) p(F | \gamma) \\ &= c^{-1} \frac{\alpha^{N_t \times N_m / 2}}{(2\pi)^{(N_t \times N_m + N_t \times N_f + N_e) / 2}} \frac{1}{|\Sigma_\theta|^{1/2}} \frac{1}{|\Sigma_F|^{1/2}} \exp\left(-\frac{J(\theta, F)}{2}\right), \end{aligned} \quad (6.19)$$

where $J(\theta, F) = \alpha(\tilde{Y} - H(\theta)F)^T(\tilde{Y} - H(\theta)F) + F^T \Sigma_F^{-1} F + \theta^T \Sigma_\theta^{-1} \theta$, and $\xi = \{\alpha, \beta, \gamma\}$ is the collection of hyperparameters. The posterior PDF will be used to estimate the MPVs of the damage indices, loads, and hyperparameters in the next section.

6.3 Bayesian Inference Based on EM Algorithm and MH Sampling

After the probabilistic model of the dynamic system is defined, the damage indices, loads, and hyperparameters are identified in this section. The set $\{\theta, F, \tilde{Y}\}$ is named as the complete data, and $Z = \{\theta, F\}$ are the latent variables. The MPVs of the damage index vector θ and the input vector F can be simultaneously determined through the MAP estimates of the posterior PDF $p(\theta, F | \tilde{Y}, \xi)$. After omitting the irrelevant variables with θ and F , the optimization problem is formulated as

$$\hat{\theta}, \hat{F} = \arg \min_{\theta, F} J(\theta, F), \quad (6.20)$$

where $\hat{\theta}$ and $\hat{F}$ are the MAP estimates of θ and F , respectively.

The partial derivatives of $J(\boldsymbol{\theta}, \mathbf{F})$ with respect to $\boldsymbol{\theta}$ and $\mathbf{F}$ are individually derived as

$$\begin{aligned}\frac{\partial J(\boldsymbol{\theta}, \mathbf{F})}{\partial \boldsymbol{\theta}} &= 2\alpha \frac{\partial \mathbf{H}^T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} (\mathbf{H}(\boldsymbol{\theta})\mathbf{F} - \tilde{\mathbf{Y}}) + 2\boldsymbol{\Sigma}_0^{-1}\boldsymbol{\theta}, \\ \frac{\partial J(\boldsymbol{\theta}, \mathbf{F})}{\partial \mathbf{F}} &= 2\alpha \mathbf{H}^T(\boldsymbol{\theta})(\mathbf{H}(\boldsymbol{\theta})\mathbf{F} - \tilde{\mathbf{Y}}) + 2\boldsymbol{\Sigma}_F^{-1}\mathbf{F}.\end{aligned}\quad (6.21)$$

The derivatives in Eq. (6.21) should equal zero to minimize $J(\boldsymbol{\theta}, \mathbf{F})$. Notably, a closed-form solution for $\hat{\boldsymbol{\theta}}$ does not exist because the relationship between $\mathbf{H}(\boldsymbol{\theta})$ and $\boldsymbol{\theta}$ is nonlinear. The closed-form solution for $\mathbf{F}$ can be derived as a function of $\boldsymbol{\theta}$ as follows:

$$\mathbf{F}(\boldsymbol{\theta}) = (\mathbf{H}^T(\boldsymbol{\theta})\mathbf{H}(\boldsymbol{\theta}) + \boldsymbol{\Sigma}_F^{-1} / \alpha)^{-1} \mathbf{H}^T(\boldsymbol{\theta})\tilde{\mathbf{Y}}. \quad (6.22)$$

To reduce the optimization dimension, Eq. (6.22) is substituted into Eq. (6.20), and then, the MAP estimates of $\boldsymbol{\theta}$ and $\mathbf{F}$ can be represented as

$$\begin{aligned}\hat{\boldsymbol{\theta}} &= \arg \min_{\boldsymbol{\theta}} \bar{J}(\boldsymbol{\theta}), \\ \hat{\mathbf{F}} &= (\mathbf{H}^T(\hat{\boldsymbol{\theta}})\mathbf{H}(\hat{\boldsymbol{\theta}}) + \boldsymbol{\Sigma}_F^{-1} / \alpha)^{-1} \mathbf{H}^T(\hat{\boldsymbol{\theta}})\tilde{\mathbf{Y}},\end{aligned}\quad (6.23)$$

where $\bar{J}(\boldsymbol{\theta}) = \alpha[\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F}(\boldsymbol{\theta})]^T[\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F}(\boldsymbol{\theta})] + \mathbf{F}(\boldsymbol{\theta})^T \boldsymbol{\Sigma}_F^{-1} \mathbf{F}(\boldsymbol{\theta}) + \boldsymbol{\theta}^T \boldsymbol{\Sigma}_0^{-1} \boldsymbol{\theta}$. $\hat{\boldsymbol{\theta}}$ and $\hat{\mathbf{F}}$ are solved using the parallel particle swarm optimization method in the MATLAB optimization toolbox (Kennedy et al., 1995).

The hyperparameter set ξ in Eq. (6.23) also needs to be determined. The MPVs of these hyperparameters can be estimated by maximizing $p(\xi | \tilde{\mathbf{Y}})$, which can be represented as

$$p(\xi | \tilde{\mathbf{Y}}) = \frac{p(\tilde{\mathbf{Y}} | \xi) p(\xi)}{p(\tilde{\mathbf{Y}})} \propto p(\tilde{\mathbf{Y}} | \xi), \quad (6.24)$$

where the prior PDFs of hyperparameters $p(\xi)$ are assumed to be uniformly distributed due to the lack of hyper-prior knowledge. Consequently, the hyperparameters can be estimated by maximizing $p(\tilde{\mathbf{Y}} | \xi)$, which is calculated by integrating $p(\boldsymbol{\theta}, \mathbf{F}, \tilde{\mathbf{Y}} | \xi)$ with respect to $\boldsymbol{\theta}$ and $\mathbf{F}$ as follows:

$$p(\tilde{\mathbf{Y}}|\xi) = \iint p(\boldsymbol{\theta}, \mathbf{F}, \tilde{\mathbf{Y}}|\xi) d\boldsymbol{\theta} d\mathbf{F} = \iint p(\tilde{\mathbf{Y}}|\boldsymbol{\theta}, \mathbf{F}, \xi) p(\boldsymbol{\theta}|\boldsymbol{\beta}) p(\mathbf{F}|\boldsymbol{\gamma}) d\boldsymbol{\theta} d\mathbf{F}. \quad (6.25)$$

The high-dimensional integration in Eq. (6.25) is intractable, and the relationship between the measurements $\tilde{\mathbf{Y}}$ and the latent variables $\mathbf{Z}$ is nonlinear. As a result, the explicit expression of $p(\tilde{\mathbf{Y}}|\xi)$ is intractable, and asymptotic or numerical methods are needed to estimate the MPVs of all hyperparameters ξ . Therefore, the EM algorithm is adopted in this study to maximize the natural log evidence $\ln p(\tilde{\mathbf{Y}}|\xi)$ iteratively, which will be detailed in the following subsection.

6.3.1 EM Algorithm

EM is widely used in computational Bayesian inference when the latent variables are involved. It consists of two steps: an E-step to infer the expectation with respect to the latent variables, and an M-step to update the hyperparameters. The E- and M-steps are iteratively operated to estimate the MPVs of all parameters (Hou et al., 2020; Hou et al., 2019b). The dual-phase mechanism in EM provides a stable convergence toward the maximum of the posterior PDF with computational tractability and statistical rigor.

In order to estimate the MPVs of the hyperparameters using EM, the natural log-likelihood function of the complete data is needed and expressed as

$$\begin{aligned} \ln p(\boldsymbol{\theta}, \mathbf{F}, \tilde{\mathbf{Y}}|\xi) &= \ln p(\tilde{\mathbf{Y}}|\boldsymbol{\theta}, \mathbf{F}, \xi) + \ln p(\boldsymbol{\theta}|\xi) + \ln p(\mathbf{F}|\xi) \\ &= -\frac{N_t \times N_m + N_t \times N_f + N_e}{2} \ln 2\pi + \frac{N_t \times N_m}{2} \ln \alpha + \frac{1}{2} \ln \left(\prod_{i=1}^{N_e} \beta_i \right) + \frac{N_t}{2} \ln \left(\prod_{p=1}^{N_f} \gamma_p \right). \quad (6.26) \\ &\quad -\frac{\alpha}{2} (\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F})^T (\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F}) - \frac{1}{2} \sum_{i=1}^{N_e} (\beta_i \theta_i^2) - \frac{1}{2} \sum_{p=1}^{N_f} \left[\gamma_p \sum_{k=0}^{N_t-1} f_{k,p}^2 \right] \end{aligned}$$

As the direct maximization of $\ln p(\boldsymbol{\theta}, \mathbf{F}, \tilde{\mathbf{Y}}|\xi)$ with respect to ξ is difficult, the EM algorithm proposes to maximize the expectation of $\ln p(\boldsymbol{\theta}, \mathbf{F}, \tilde{\mathbf{Y}}|\xi)$ with respect to the

latent variables $\mathbf{Z}$, which is

$$\begin{aligned} & \mathbb{E}_{\mathbf{Z}} \left\{ \ln p(\mathbf{\theta}, \mathbf{F}, \tilde{\mathbf{Y}} | \xi) \right\} \\ &= -\frac{N_t \times N_m + N_t \times N_f + N_e}{2} \ln 2\pi + \frac{N_t \times N_m}{2} \ln \alpha + \frac{1}{2} \ln \left(\prod_{i=1}^{N_e} \beta_i \right) + \frac{N_t}{2} \ln \left(\prod_{p=1}^{N_f} \gamma_p \right) \quad (6.27) \\ & - \frac{\alpha}{2} \mathbb{E}_{\mathbf{Z}} \left[(\tilde{\mathbf{Y}} - \mathbf{H}(\mathbf{\theta})\mathbf{F})^\top (\tilde{\mathbf{Y}} - \mathbf{H}(\mathbf{\theta})\mathbf{F}) \right] - \frac{1}{2} \sum_{i=1}^{N_e} \left[\beta_i \mathbb{E}_{\mathbf{Z}} (\theta_i^2) \right] - \frac{1}{2} \sum_{p=1}^{N_f} \left[\gamma_p \mathbb{E}_{\mathbf{Z}} \left(\sum_{k=1}^{N_t-1} f_{k,p}^2 \right) \right] \end{aligned}$$

In practice, the real distributions of latent variables $\mathbf{\theta}$ and $\mathbf{F}$ are unavailable, so they should be estimated by the posterior PDF $p(\mathbf{\theta}, \mathbf{F} | \tilde{\mathbf{Y}}, \xi)$. In the E-step, given the current values of hyperparameters ξ^{old} , the posterior PDF of the latent variables $p(\mathbf{\theta}, \mathbf{F} | \tilde{\mathbf{Y}}, \xi^{old})$ is used to calculate the expectation of the complete data $\mathbb{E}_{\mathbf{Z}} \left\{ \ln p(\mathbf{\theta}, \mathbf{F}, \tilde{\mathbf{Y}} | \xi) \right\}$. In the consecutive M-step, the hyperparameters are updated as ξ^{new} by maximizing $\mathbb{E}_{\mathbf{Z}} \left\{ \ln p(\mathbf{\theta}, \mathbf{F}, \tilde{\mathbf{Y}} | \xi) \right\}$ with respect to ξ . By setting the partial derivatives of $\mathbb{E}_{\mathbf{Z}} \left\{ \ln p(\mathbf{\theta}, \mathbf{F}, \tilde{\mathbf{Y}} | \xi) \right\}$ with respect to the hyperparameters in set ξ to zero, i.e., $\partial \mathbb{E}_{\mathbf{Z}} \left\{ \ln p(\mathbf{\theta}, \mathbf{F}, \tilde{\mathbf{Y}} | \xi) \right\} / \partial \xi = 0$, the hyperparameters can be calculated as follows:

$$\alpha = \frac{N_t \times N_m}{\mathbb{E}_{\mathbf{Z}} \left[(\tilde{\mathbf{Y}} - \mathbf{H}(\mathbf{\theta})\mathbf{F})^\top (\tilde{\mathbf{Y}} - \mathbf{H}(\mathbf{\theta})\mathbf{F}) \right]}, \quad (6.28)$$

$$\beta_i = \frac{1}{\mathbb{E}_{\mathbf{Z}} (\theta_i^2)}, \quad (6.29)$$

$$\gamma_p = \frac{N_t}{\mathbb{E}_{\mathbf{Z}} \left(\sum_{k=0}^{N_t-1} f_{k,p}^2 \right)}. \quad (6.30)$$

The expectations in Eqs. (6.28)–(6.30) are approximated using the samples of $\mathbf{\theta}$ and $\mathbf{F}$ generated from $p(\mathbf{\theta}, \mathbf{F} | \tilde{\mathbf{Y}}, \xi)$ (Hou et al., 2019b). The probabilistic information in $p(\mathbf{\theta}, \mathbf{F} | \tilde{\mathbf{Y}}, \xi)$ is thus represented by the posterior samples $\mathbf{\theta}^{(l)}$ and $\mathbf{F}^{(l)}$ ($l = 1, 2, \dots, N_s$), where N_s is the number of samples. Consequently, the expectations are approximated as

$$\begin{aligned}
& \mathbb{E}_{\mathbf{Z}} \left[(\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F})^T (\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F}) \right] \\
&= \iint (\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F})^T (\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta})\mathbf{F}) p(\boldsymbol{\theta}, \mathbf{F} \mid \tilde{\mathbf{Y}}, \boldsymbol{\xi}) d\boldsymbol{\theta} d\mathbf{F}, \\
&\approx \frac{1}{N_s} \sum_{l=1}^{N_s} \left[(\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta}^{(l)})\mathbf{F}^{(l)})^T (\tilde{\mathbf{Y}} - \mathbf{H}(\boldsymbol{\theta}^{(l)})\mathbf{F}^{(l)}) \right]
\end{aligned} \tag{6.31}$$

$$\mathbb{E}_{\mathbf{Z}}(\theta_i^2) = \iint \theta_i^2 p(\boldsymbol{\theta}, \mathbf{F} \mid \tilde{\mathbf{Y}}, \boldsymbol{\xi}) d\boldsymbol{\theta} d\mathbf{F} \approx \frac{1}{N_s} \sum_{l=1}^{N_s} (\theta_i^{(l)})^2, \tag{6.32}$$

$$\mathbb{E}_{\mathbf{Z}} \left(\sum_{k=0}^{N_t-1} f_{k,p}^2 \right) = \iint \sum_{k=0}^{N_t-1} f_{k,p}^2 p(\boldsymbol{\theta}, \mathbf{F} \mid \tilde{\mathbf{Y}}, \boldsymbol{\xi}) d\boldsymbol{\theta} d\mathbf{F} \approx \frac{1}{N_s} \sum_{l=1}^{N_s} \sum_{k=0}^{N_t-1} (f_{k,p}^{(l)})^2. \tag{6.33}$$

6.3.2 Modified MH Algorithm for Joint Load–Parameter Sampling

The MCMC sampling approach is employed to generate the posterior samples of the latent variables because their posterior distribution does not follow a Gaussian distribution perfectly. Commonly adopted MCMC approaches in Bayesian inference include the MH algorithm and Gibbs sampling. The MH algorithm employs a proposal–acceptance mechanism to flexibly explore target distributions, and it is widely applied in various scenarios (Roberts et al., 2001). Gibbs sampling generates acceptable samples from explicit conditional PDFs (Robert et al., 1999). Given the flexibility of the MH algorithm, a modified MH algorithm is proposed in this study for the joint load–parameter sampling.

The modified MH algorithm is listed in Table 6.1, which improves the joint sampling efficiency. Initially, the MAP estimate of $\boldsymbol{\theta}$ is adopted as the initial mean vector of the proposal Gaussian distribution to eliminate the burn-in period, as detailed in step 1. In step 2, $\boldsymbol{\theta}$ and $\mathbf{F}$ are jointly sampled. Given the difficulty of directly generating acceptable samples of $\mathbf{F}$ from the high-dimensional proposal distribution, the samples of $\mathbf{F}$ are directly computed through Eq. (6.22) using the generated sample of $\boldsymbol{\theta}$. It's assumed that the samples of $\mathbf{F}$ are generated from a symmetric proposal distribution because the sampling process is stochastic. In step 3, the PDFs related to the proposal distributions

are omitted in the acceptance ratio calculation to reduce the computational cost, which is reasonable because the proposal distributions of $\boldsymbol{\theta}$ and $\mathbf{F}$ are both symmetric, and forward and backward transition probabilities are equal. Steps 4 and 5 follow the standard MH algorithm. Finally, the proposal covariance matrix for $\boldsymbol{\theta}$ is periodically adjusted in step 6 through an adaptive module to improve the sampling efficiency.

Table 6.1 Modified MH algorithm for joint input–parameter sampling

-
1. Define the initial proposal distribution for $\boldsymbol{\theta}$, i.e., $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\mu} = \hat{\boldsymbol{\theta}}$, and $\boldsymbol{\Sigma} = \boldsymbol{\Sigma}^{(0)}$.
Let $l = 1$.
 2. Generate candidate samples of the DIs, $\mathbf{X}_\theta \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$; Generate candidate samples of the load $\mathbf{X}_F = (\mathbf{H}^T(\mathbf{X}_\theta)\mathbf{H}(\mathbf{X}_\theta) + \boldsymbol{\Sigma}_F^{-1} / \alpha)^{-1} \mathbf{H}^T(\mathbf{X}_\theta) \tilde{\mathbf{Y}}$.
 3. Calculate the acceptance ratio of the candidate samples using the following equation
$$\zeta = \min \left\{ 1, \frac{\exp(-J(\mathbf{X}_\theta, \mathbf{X}_F) / 2)}{\exp(-J(\boldsymbol{\theta}^{(l)}, \mathbf{F}^{(l)}) / 2)} \right\}.$$
 4. Randomly generate δ from the uniform distribution $\mathcal{U}(0, 1)$.
 5. Determine the accepted samples: $\{\boldsymbol{\theta}^{(l+1)}, \mathbf{F}^{(l+1)}\} = \begin{cases} \{\boldsymbol{\theta}^{(l)}, \mathbf{F}^{(l)}\}, & \zeta \leq \delta \\ \{\mathbf{X}_\theta, \mathbf{X}_F\}, & \zeta > \delta \end{cases}$.
 6. Adjust the proposal covariance matrix for $\boldsymbol{\theta}$:
$$\boldsymbol{\Sigma} = \begin{cases} \boldsymbol{\Sigma}, & l \leq N_c \\ S_d \text{cov}(\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, \dots, \boldsymbol{\theta}^{(l+1)}) + S_d \tau \mathbf{I}, & l > N_c \end{cases},$$

where N_c is the bound number of samples before the adjustment of the covariance matrix; cov is the covariance calculator; S_d is a scaling parameter associated with the dimension of the covariance matrix; S_d is generally set to $2.4^2/N_e$ (Haario et al., 2006); and τ is a close-zero constant.
 7. Let $\boldsymbol{\mu} = \boldsymbol{\theta}^{(l+1)}$.
 8. Let $l = l + 1$, repeat steps 2–7 until $l = N_s$.
 9. The samples following the posterior PDF $p(\boldsymbol{\theta}, \mathbf{F} | \tilde{\mathbf{Y}}, \xi)$ is obtained as $(\boldsymbol{\theta}^{(1)} \sim \boldsymbol{\theta}^{(N_s)})$ and $(\mathbf{F}^{(1)} \sim \mathbf{F}^{(N_s)})$.
-

6.3.3 Summary of the Proposed Method

The flowchart of the proposed method is summarized in Figure 6.1. After hyperparameters are initialized, the MAP estimates of the latent variables are estimated by solving Eq. (6.23). Next, the modified MH algorithm is employed to generate samples for the latent variables, which are then used to calculate the expectations in Eqs. (6.31)–(6.33). Finally, the hyperparameters are updated according to Eqs. (6.28)–(6.30) based on the EM algorithm. The iteration is repeated until the ℓ_2 norm of the difference between the structural parameters in two consecutive iterations is smaller than 10^{-6} .

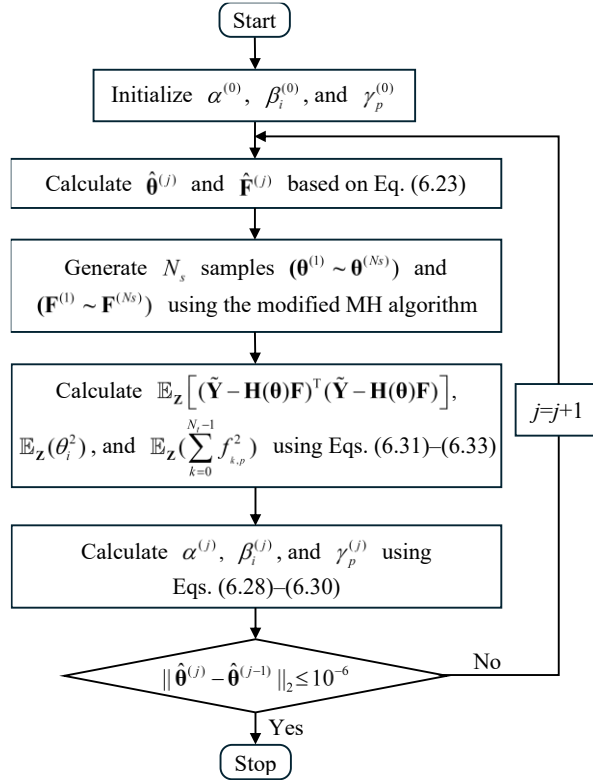


Figure 6.1 Flowchart of sparse Bayesian learning for joint load–parameter identification

6.4 A Numerical Example

A numerical simply-supported beam, as illustrated in Figure 6.2, is used to verify the proposed method. The beam is 10 m long, 0.1 m high, and 0.25 m wide. The material density is $7,850 \text{ kg/m}^3$, and the elastic modulus is 200 GPa. The beam is equally divided into 20 beam elements, each with a length of 0.5 m, resulting in $N_e = 20$. Only the in-plane deformation is considered, resulting in a total of 40 DOFs. The bending stiffness of elements 5, 10, and 14 is reduced by 35%, 25%, and 40%, respectively, to simulate damage, while the mass parameters remain unchanged. The first two natural frequencies of the undamaged beam are 2.29 Hz and 9.16 Hz. Rayleigh damping is adopted, with 1% damping ratios for the first two modes.

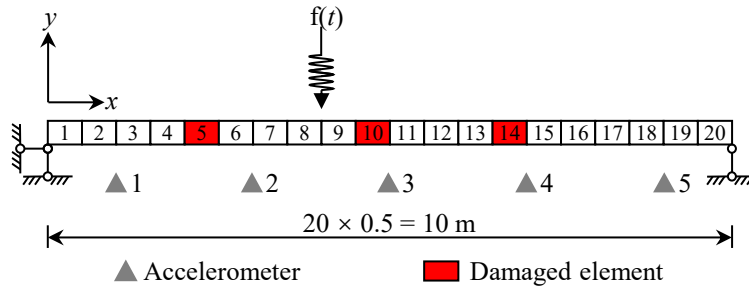


Figure 6.2 Configuration of the numerical simply supported beam

A concentrated white noise load is applied at $x = 4 \text{ m}$ to excite the beam vibration, and it is within the range of $\pm 1 \text{ kN}$. Five evenly distributed accelerometers are adopted to measure the vertical acceleration responses, as depicted in Figure 6.2. The sampling frequency of all sensors is 100 Hz.

Measurement noise is inevitable in practice, and it can lead to the overfitting effect. Therefore, the calculated responses are contaminated by zero-mean Gaussian noise. The noisy measurement is simulated as follows:

$$\tilde{\mathbf{Y}} = \hat{\mathbf{Y}} + \mathbf{NR} \cdot \mathcal{N}(\mathbf{0}, \frac{1}{N_m} \sum_{q=1}^{N_m} \text{var}(\hat{\mathbf{y}}_q) \mathbf{I}), \quad (6.34)$$

where $\tilde{\mathbf{Y}}$ represents the noisy measurements, $\hat{\mathbf{Y}}$ is the ground truth responses without noise, NR is the noise ratio, q denotes the sensor index, and N_m represents the total number of measurements of the same type.

The relative root mean squared error (RRMSE) is employed to quantify the difference between the measured and reconstructed response vectors, and it is defined as

$$\text{RRMSE} = \sqrt{\sum_{k=0}^{N_t-1} \|\mathbf{y}_k - \hat{\mathbf{y}}_k\|^2} / \sqrt{\sum_{k=0}^{N_t-1} \|\hat{\mathbf{y}}_k\|^2} \times 100\%, \quad (6.35)$$

where $\mathbf{y}$ and $\hat{\mathbf{y}}$ represent the reconstructed and true response time history vectors, respectively.

6.4.1 Baseline Scenario

In the baseline scenario, the response time history lasting for 2 s is used for the joint identification, and it is contaminated by 5% noise. The stiffness reduction factors for all twenty elements are selected as the DIs, and the loading location is assumed to be known, resulting in the dimensions of $\boldsymbol{\theta}$, $\mathbf{F}$, $\tilde{\mathbf{Y}}$, and $\mathbf{H}$ being 20, $200 \times 1 = 200$, $200 \times 5 = 1,000$, and $1,000 \times 200$, respectively. The hyperparameters are initialized as $\alpha^{(0)} = 100$, $\beta_i^{(0)} = 1 \times 10^{-2}$, and $\gamma_p^{(0)} = 0$. Small initial values are assigned to all DIs' precision factors to ensure that all elements are damaged element candidates. In the modified MH sampling algorithm, the number of samples N_s is set to 1,000, and other parameters are set as $N_c = 100$, and $\tau = 1 \times 10^{-6}$. The initial proposal covariance matrix $\boldsymbol{\Sigma}^{(0)}$ is assumed to be a diagonal matrix $1 \times 10^{-4} \mathbf{I}$. The proposal covariance matrix $\boldsymbol{\Sigma}$ is subsequently adjusted according to the adaptive module in the proposed MH algorithm. Finally, the proposed Bayesian method is used to iteratively estimate $\boldsymbol{\theta}$, $\mathbf{F}$, α , $\boldsymbol{\beta}$, and $\boldsymbol{\gamma}$.

The identification process converges within ten iterations. The iteration process of the hyperparameters is depicted in Figure 6.3. In Figure 6.3(a), α converges to 31.51, and it

is close to the reciprocal of the true variance of measurement noise, 26.00. In Figure 6.3(b), γ stabilizes at 2.74×10^{-6} , and it closely matches the reciprocal of the input variance, 2.79×10^{-6} . Precision factors for DIs β_i are exhibited in Figure 6.3(c), and only five elements, including the three damaged elements, are shown here for clear observation. Although all β_i are initialized identically, β_5, β_{10} , and β_{14} eventually converge to relatively small values, whereas others approach infinity, resulting in their associated DIs being zero. This suggests that the ARD priors effectively impose sparse constraints on the DIs in the identification process.

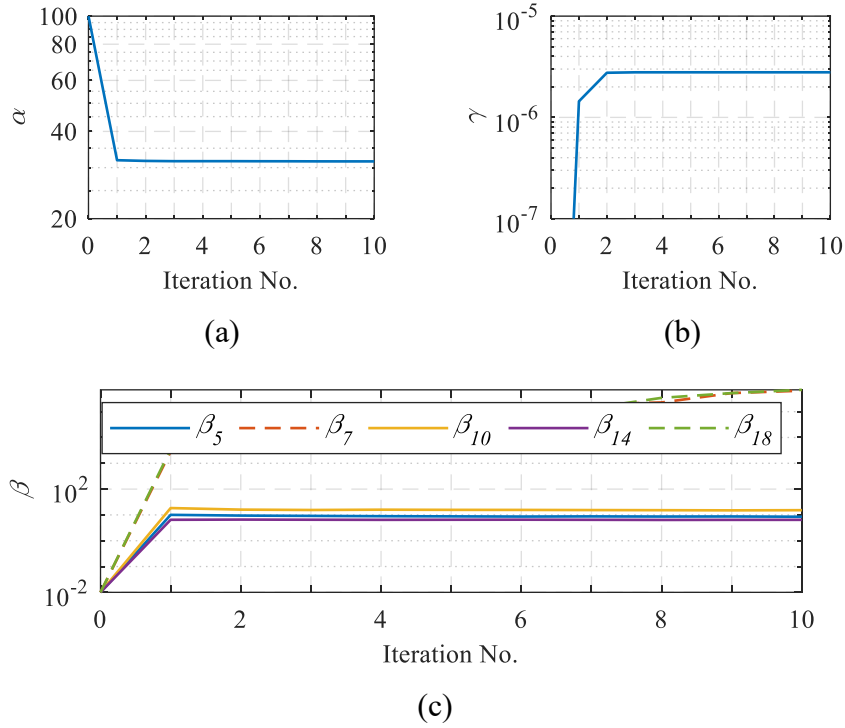


Figure 6.3 Iteration process of the hyperparameters in the baseline scenario

Moreover, the identified DIs of 20 elements are exhibited in Figure 6.4. The θ_i of the undamaged elements are zero, indicating that the sparse regularization is successfully applied. θ_5, θ_{10} , and θ_{14} are precisely estimated as 0.34, 0.26, and 0.39, respectively, with the ground truth of 0.35, 0.25, and 0.4, respectively. This result illustrates that both damage locations and degrees are accurately identified, demonstrating the high performance of the proposed approach.

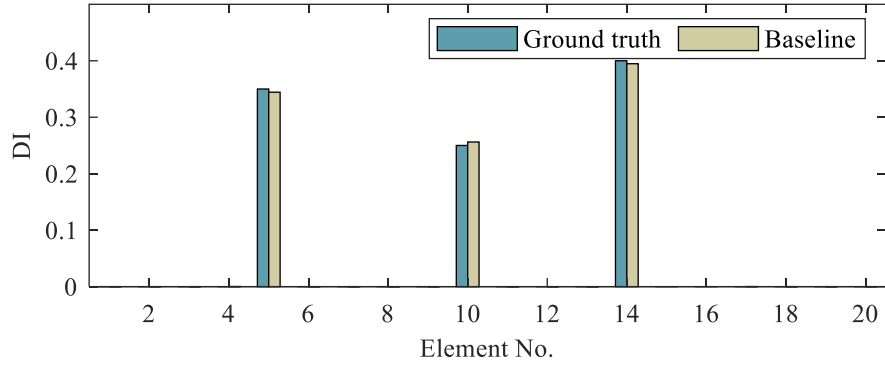


Figure 6.4 Identified DIs in the baseline scenario

The identified load time history is displayed in Figure 6.5. It aligns well with the ground truth and deviates only at some peak values. The tiny deviation is mainly caused by the measurement noise, which will be detailed in the next subsection. The RRMSE between the real and identified loads is 9.16%, indicating that the proposed method can accurately identify the load from noisy measurements.

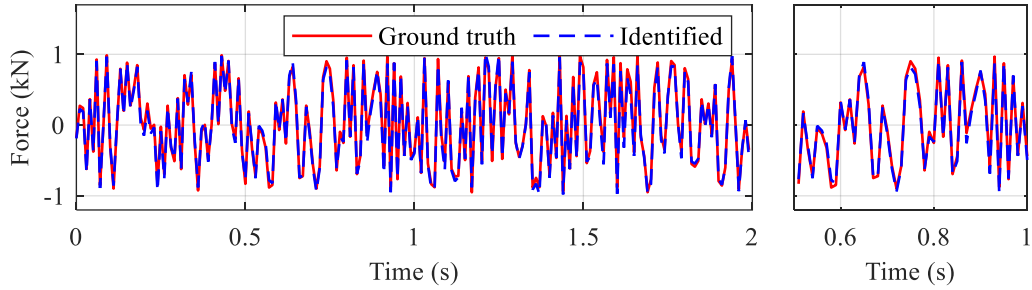


Figure 6.5 Identified load time history in the baseline scenario

The identified DIs and load time history are then used to reconstruct the full-field responses of the beam. The real and reconstructed acceleration, velocity, and deflection at $x = 6$ m are shown in Figure 6.6. These reconstructed responses match the ground truth in both shape and peak values because the load and DIs have already been accurately identified. The average RRMSEs of acceleration, velocity, and deflection at all DOFs are 4.42%, 6.86%, and 13.20%, respectively, demonstrating satisfactory reconstruction accuracy. The RRMSE of deflection is higher than that of velocity and acceleration. This

is because only acceleration data are used in the identification process, and low-frequency drift can appear in the reconstructed displacement time history. The reconstruction accuracy of deflection can be further improved by including responses with high-quality low-frequency components, such as strain and displacement data, in the identification process.

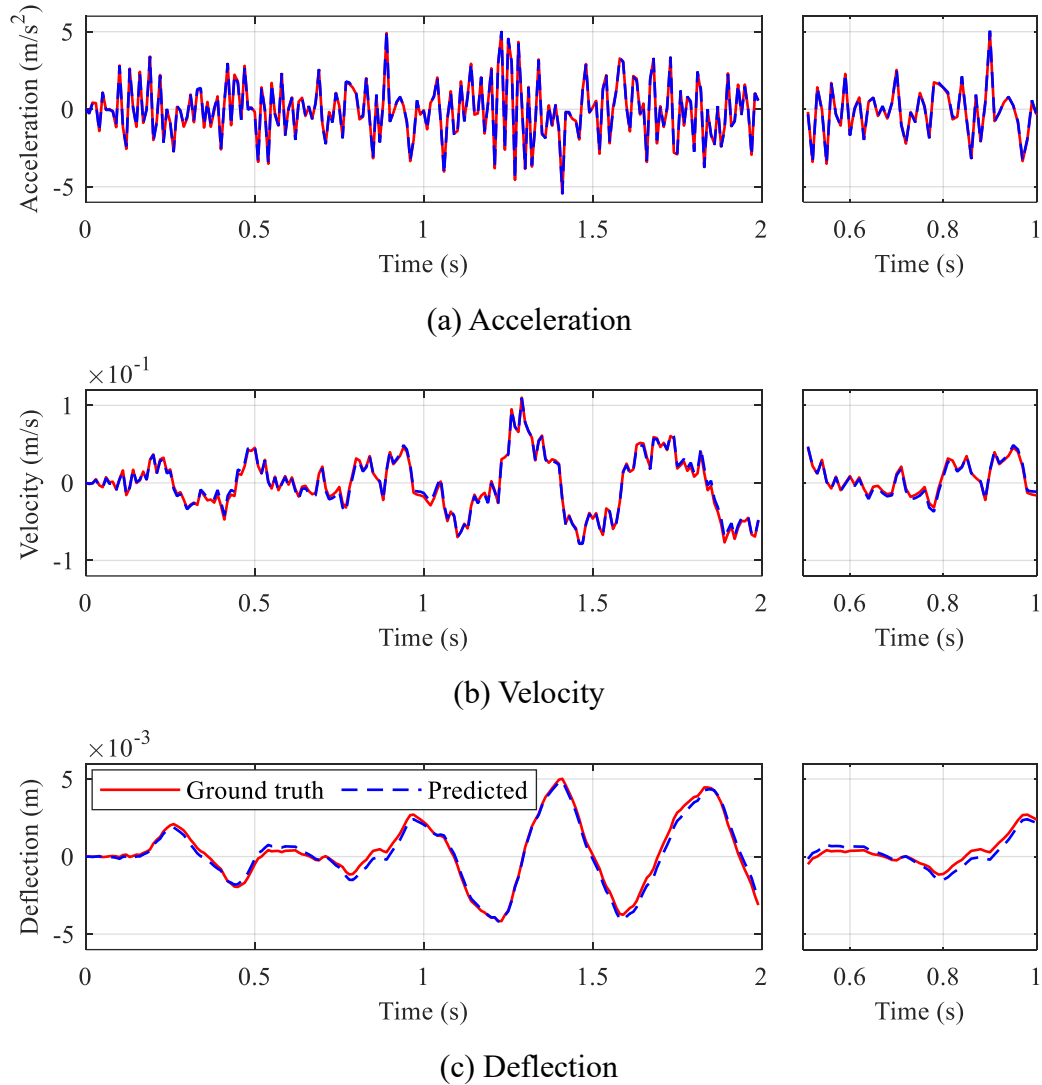


Figure 6.6 Reconstructed responses at $x = 6$ m in the baseline scenario

The RRMSEs of the reconstructed responses along the beam are calculated and displayed in Figure 6.7. The RRMSEs of acceleration between $x = 3.5$ m and $x = 4.5$ m are higher than at other locations because the load is applied around this region. The RRMSEs of

the deflection are consistent and are larger than those of the acceleration and velocity. This is again because an accurate reconstruction of deflection requires measurement data containing both sufficient low and high frequency components.

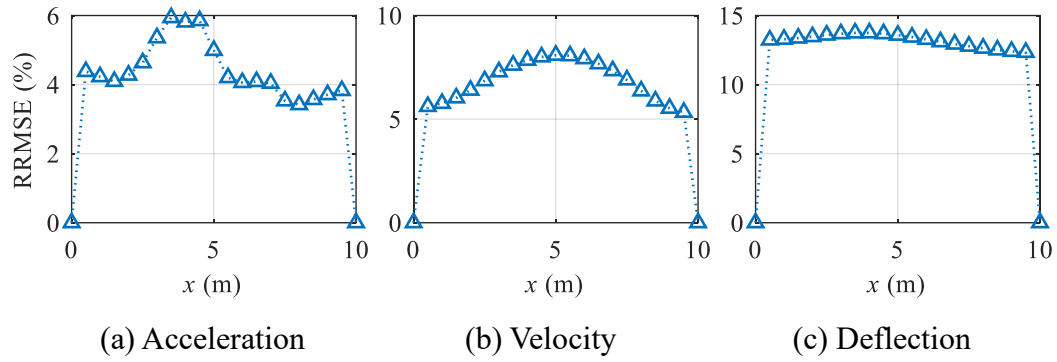


Figure 6.7 RRMSEs of the reconstructed responses at all locations

The applicability of the proposed method is discussed in this subsection. First, the influence of the noise ratios (NRs) on the identification performance is compared. Then, the impulse and moving load cases are studied to discuss the influence of the load characteristics on the joint identification accuracy. Finally, the proposed method is tested with unknown loading locations, which is the most challenging case. The NRs, load types, and loading locations in all scenarios are summarized in Table 6.2.

Table 6.2 Scenario settings

Scenario	NR (%)	Load type	Loading location
Baseline	5		
N1	1	White noise	Known
N2	10		
L1	5	Impulse force	Known
L2	5	Harmonic moving force	Speed known
U1	5	White noise	Unknown

6.4.2 Influence of Measurement Noise

In this subsection, the calculated response data are contaminated by 1%, 5%, and 10% white noise to discuss the influence of noise level, while other settings remain consistent with those in the baseline scenario. The identified DIs with different NRs are shown in Figure 6.8. The identified DIs are sparsely distributed, where the DIs of the undamaged elements are close to zero. The average DI errors of the damaged elements under 1%, 5%, and 10% noise levels are 0.11%, 0.57%, and 0.87%, respectively. As the NR increases, the average DI error slightly increases because the identified DIs and loads are both related to the measurement noise. Nevertheless, the DIs can still be accurately identified when the NR reaches 10%, verifying the robustness of the proposed method to measurement noise.

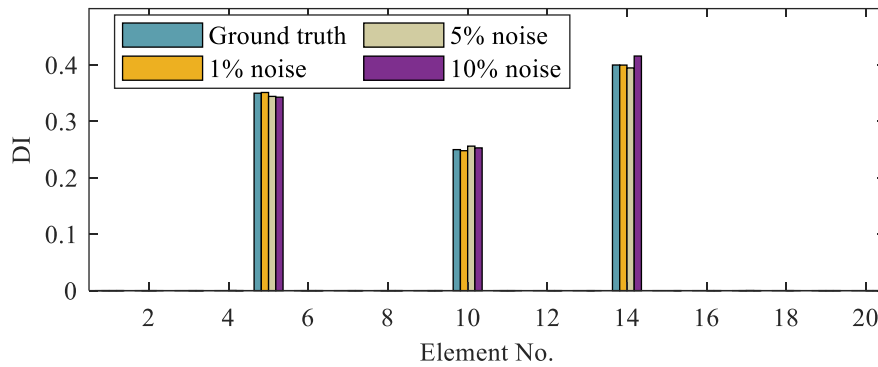


Figure 6.8 Identified DIs from measurements with different NRs

The RRMSEs of the identified loads and reconstructed responses are listed in Table 6.3. The RRMSEs increase with higher NRs, indicating that the noise degrades the joint identification accuracy. The RRMSEs of deflection are larger than those of the velocity and acceleration, as discussed in the baseline scenario. When the NR reaches 10%, all average RRMSEs remain below 20%, demonstrating that the full-field responses are accurately reconstructed.

Table 6.3 RRMSE (%) of the identified forces and reconstructed responses

Scenario	NR (%)	Acceleration	Velocity	Deflection	Load
Baseline	5	4.42	6.86	13.20	9.16
N1	1	0.87	1.99	7.50	5.11
N2	10	8.52	10.62	19.55	17.28
L1	5	3.79	1.75	2.12	6.27
L2	5	4.91	3.14	12.55	11.09
U1	5	4.61	6.76	14.27	10.86

6.4.3 Influence of Load Type

The performance of the proposed method under the impulse force and the harmonic moving force is further investigated. An impulse load with a magnitude of 1 kN is applied at $x = 4$ m. The moving force, whose time history is $f(t) = 1 \times \sin(10\pi t) + 8 \times \sin(20\pi t)$ kN, moves from the left to the right at a constant speed of 10 m/s. The other settings remain unchanged, as detailed in Table 6.2 L1–L2.

The identified DIs under all three load conditions are presented in Figure 6.9. The damages are accurately located in all scenarios. The average DI errors of the damaged elements in the baseline, L1, and L2 load cases are 0.57%, 2.01%, and 2.18%, respectively. The identification error is lowest under the white noise load because its power spectrum is broadband to excite multiple modes of responses.

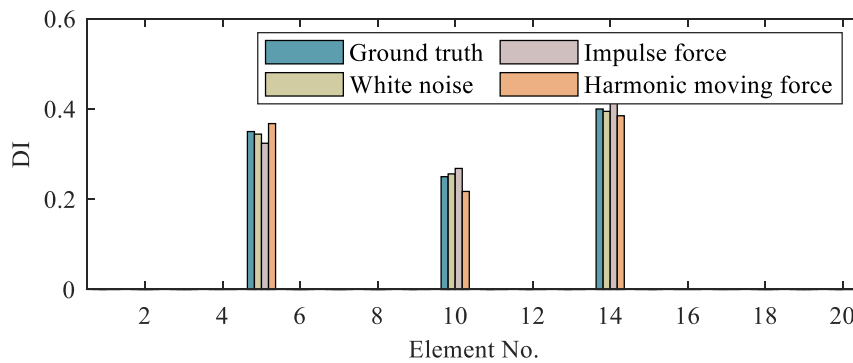


Figure 6.9 Identified DIs under different load types

The identified impulse and harmonic forces are exhibited in Figure 6.10. They align well with the ground truth, with the RRMSEs of 6.27% and 11.09%. In Figure 6.10(b), there is a noticeable deviation between the true and identified forces at the last few moments, which is primarily caused by two reasons: (1) The measured acceleration data contain more information about the loads at the beginning moments than those at the later moments; (2) When the moving force reaches the bearing, the excited structural responses are minor and thus can be easily obscured by the measurement noise. The RRMSEs of the reconstructed responses are listed in Table 6.3. The accuracy of the reconstructed full-field responses in L1 is the highest among the L1, L2, and baseline scenarios.

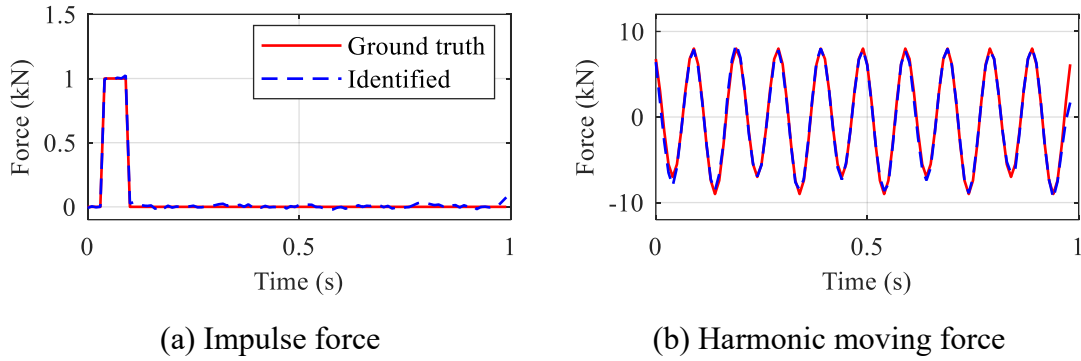


Figure 6.10 Identified forces in L1 and L2

6.4.4 Loading Location

In this subsection, the load location is assumed to be unknown in advance, and four candidate loading locations are assigned at $x = 2$ m, 4 m, 6 m, and 8 m. $\gamma_p^{(0)}$ is set to 10^{-2} due to multiple candidate loading locations, and other settings remain consistent with those in subsection 6.4.1. The ARD priors are imposed on the load and DIs to provide their spatial sparsity. The iteration process of γ is exhibited in Figure 6.11. The initial input covariance matrix is a 4×4 identity matrix without any prior information about the loading location. After 15 iterations, γ_2 converges to 2.78×10^{-6} , while γ_1 , γ_3 , and γ_4 become significantly large, indicating that the forces at the second candidate location are non-zero while the others are zero. Consequently, the location and time history of the

applied force are precisely identified with the RRMSE of 10.86%.

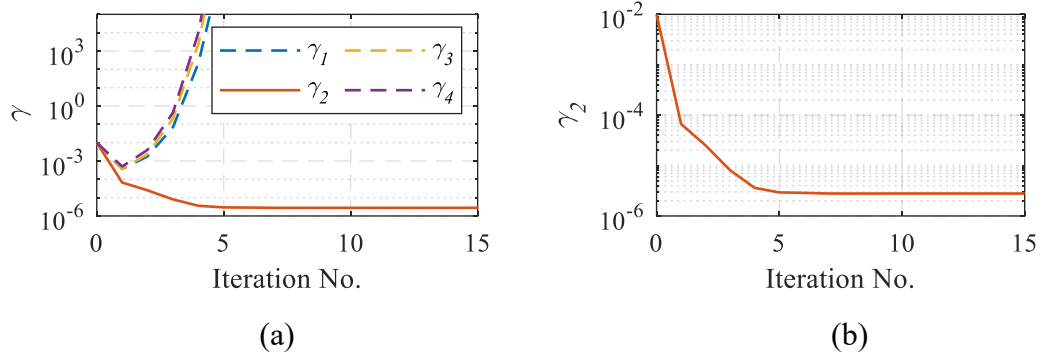


Figure 6.11 Convergence process of γ

The identified DIs are exhibited in Figure 6.12. The mean identification errors of the three damaged elements in the baseline scenario and U1 are 0.57% and 1.04%, respectively. Moreover, the RRMSEs of the identified force and responses do not increase significantly in U1, as reported in Table 6.3. These results indicate that the structural damage indices, loads, and full-field responses can be jointly identified with high accuracy even when the loading location is unknown.

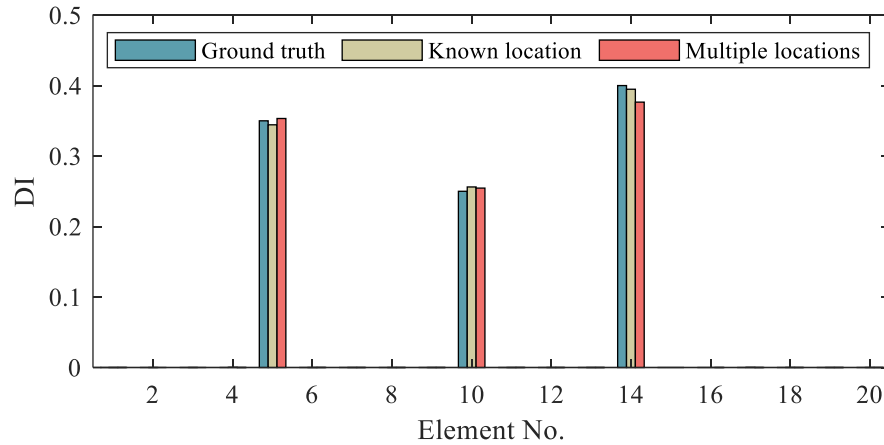


Figure 6.12 Identified DIs when the loading location is known and unknown

6.5 Experimental Studies

6.5.1 A Three-story Frame Model

6.5.1.1 Experimental Setup

The proposed approach is then applied to a laboratory-tested three-story frame model for verification, as shown in Figure 6.13. The model is made of aluminum alloy with an elastic modulus of 68.9 GPa and a density of $2.80 \times 10^3 \text{ kg/m}^3$. It was mounted on a strong floor through a thick plate to simulate the fixed boundary. The bending stiffness of the columns in the y -direction is significantly larger than that in the x -direction; thus, the vibration along the y -axis is neglectable. Consequently, the shear frame is simplified to a three-DOF system. An impact force was applied to the first floor in the x -direction to excite the structural responses. Three accelerometers (A1–A3) were installed on each floor to measure the structural acceleration at a sampling frequency of 100 Hz. The first three natural frequencies are 3.32 Hz, 9.54 Hz, and 13.95 Hz, respectively, and the associated damping ratios are 0.25%, 0.22%, and 0.20%, which were identified from the measured responses using the eigensystem realization algorithm (Juang et al., 1985).

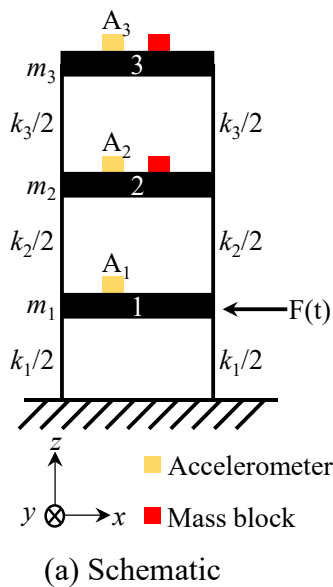


Figure 6.13 Laboratory-tested three-story frame

The initial FE model of the frame has been updated by using the natural frequencies and mode shapes (Wang et al., 2024), and it is directly adopted in this study. The mass of the columns was evenly assigned to their adjacent slabs; thus, the mass of the whole structure was concentrated on the floor slabs. The mass of the accelerometers was also counted on each floor. Consequently, the mass parameters for the three floors are calculated as $m_1 = m_2 = 2.58$ kg, and $m_3 = 2.45$ kg. The updated stiffness parameters are $k_1 = 5.28$ kN/m, $k_2 = 6.22$ kN/m, and $k_3 = 6.08$ kN/m.

Two damage cases were designed to verify the proposed method. The damage was simulated by placing additional mass blocks on different floors while the stiffness parameters remained unchanged, as depicted in Figure 6.13(a) and detailed in Table 6.4. In D1, a mass block weighing 0.55 kg was added to the third floor. In D2, a mass block weighing 0.60 kg was consecutively added to the second floor. The identified natural frequencies in the undamaged (D0) and two damaged cases (D1 and D2) are reported in Table 6.4.

Table 6.4 Mass parameters and structural frequencies in all cases

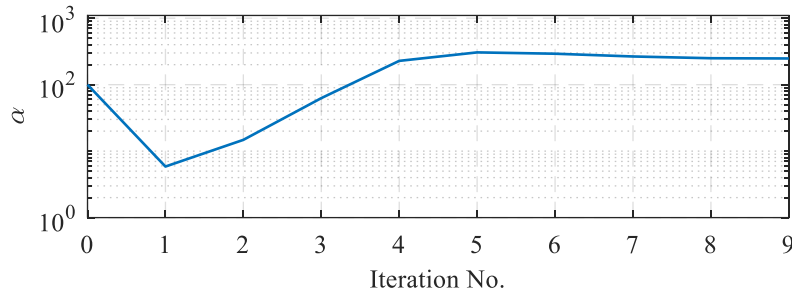
Items		D0	D1	D2
Mass (kg)	m_1	2.58	2.58	2.58
	m_2	2.58	2.58	3.18
	m_3	2.45	3.00	3.00
Natural frequencies (Hz)	1st	3.32	3.15	3.04
	2nd	9.54	9.22	9.11
	3rd	13.95	13.78	13.03

6.5.1.2 Joint Identification

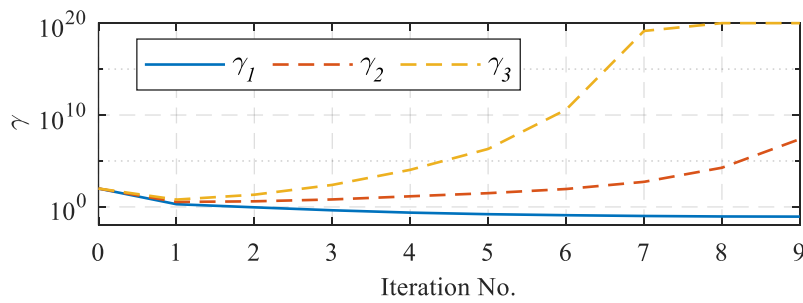
Given that the measurement noise and modeling error are considered in the proposed method, the raw acceleration data with a length of two seconds are directly used to jointly

identify the structural parameters and input force without data cleaning. The mass increment factors of each floor are selected as the DIs. The load position is assumed to be unknown; thus, all three floors are selected as the possible loading locations.

In D1, the acceleration data of A2 and A3 are used for joint identification, and in D2, the data of A1–A3 are all used. The hyperparameters are initialized as $\alpha^{(0)} = 100$, $\beta_i^{(0)} = 1 \times 10^{-2}$, and $\gamma_p^{(0)} = 100$. The iteration process of the hyperparameters in D1 is illustrated in Figure 6.14. α converges to 247.80 after seven iterations. The damage precision factors for the first and second floors β_1 and β_2 , as well as the force precision factors for the second and third floors γ_2 and γ_3 , quickly approach infinity, indicating that the sparse constraints are applied to the undamaged elements and the unloaded DOFs.



(a)



(b)

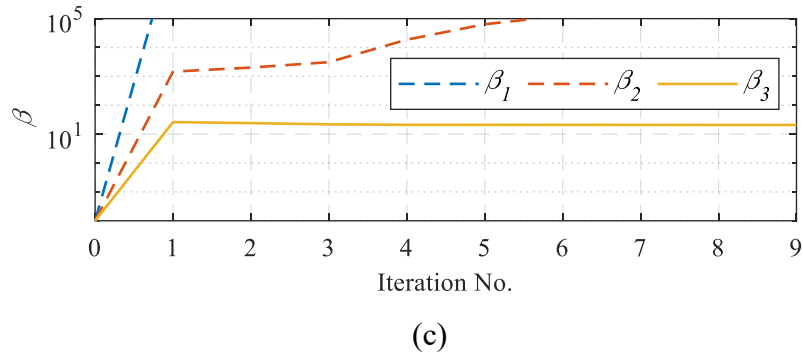


Figure 6.14 Iteration process of the hyperparameters in D1

The identified DIs in D1 and D2 are illustrated in Figure 6.15. The damage locations are accurately identified in noisy environments, where the DIs of non-damaged elements are correctly identified as zero. This indicates that the sparse constraints are successfully applied to the damage identification result. The DI errors of damaged elements in both cases are within 1.5%, demonstrating the high accuracy of the damage identification results.

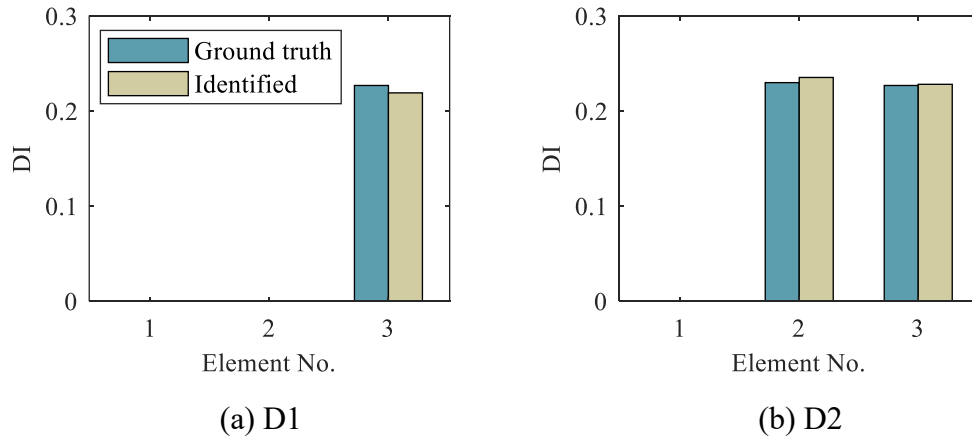


Figure 6.15 Identified DIs

The identified forces at all three floors are displayed in Figure 6.16. The identified forces at the second and third floors are all nearly zero, indicating that the loading locations are successfully identified. The identified forces at the first floor have a peak within the initial 0.5 s, and the remaining values are around zero, which accords well with an impulse force.

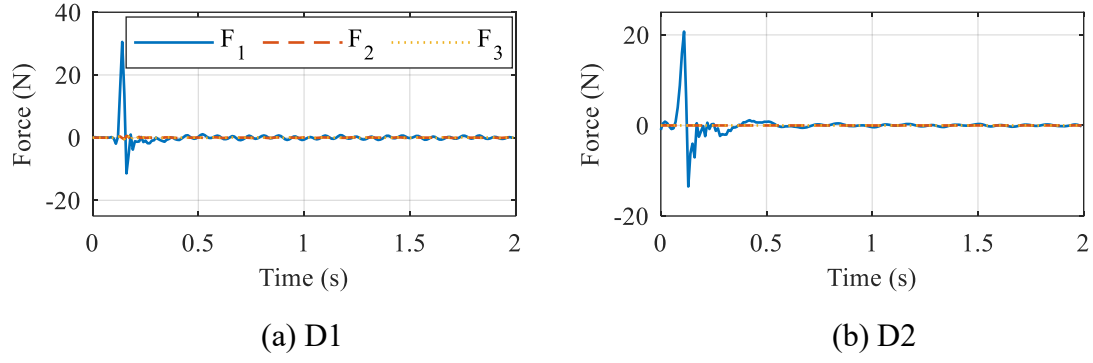


Figure 6.16 Reconstructed forces at all floors

The identified DIs and forces are then used to reconstruct the full-field responses. The measured and reconstructed accelerations at the second floor in D1 and D2 are displayed in Figures 6.17(a) and 6.17(b). The reconstructed responses align closely with the measurements, with only minor deviations at the peaks. The average RRMSEs of the acceleration responses in D1 and D2 are 6.37% and 6.50%, respectively, which suggests that the jointly identified DIs and forces are sufficient to calculate accurate structural responses.

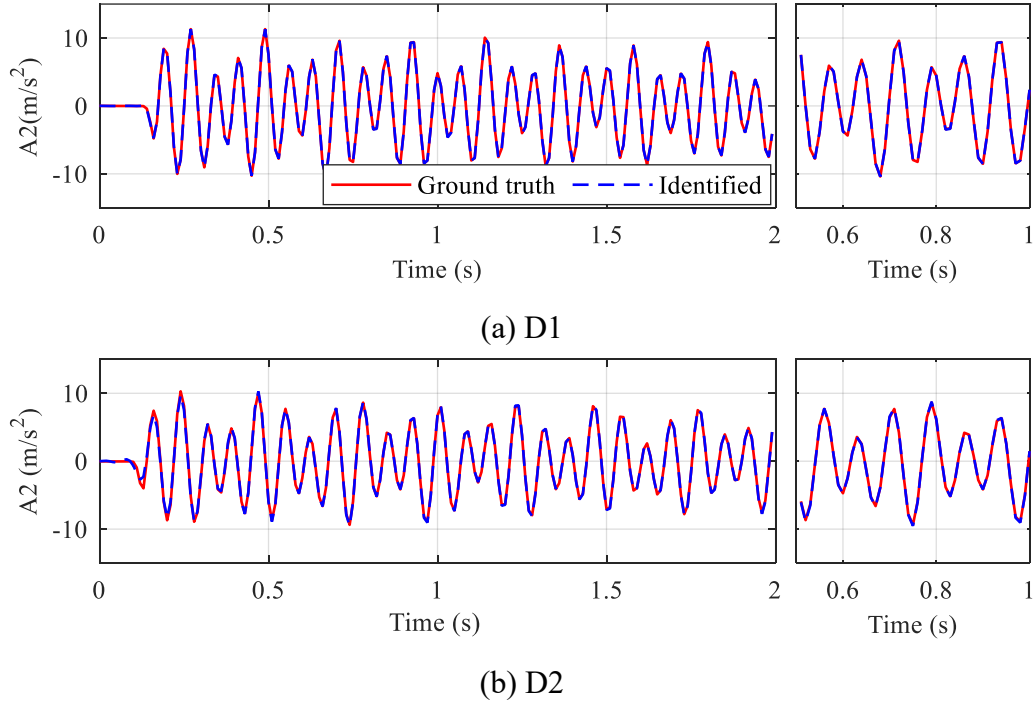


Figure 6.17 Reconstructed acceleration responses at the second floor

6.5.2 An Outdoor Bridge

The outdoor bridge model used in Chapter 4, as depicted in Figure 6.18(a), is further tested to verify the proposed method. The model layout and sensor placement are shown in Figure 6.18(c). A vehicle bump test was conducted on the bridge model: the front wheels of a vehicle were first positioned on a wooden block, and they were subsequently driven off the block to impose a vertical impact force at $x = 11$ m, as shown in Figures 6.18(b) and 6.18(c). Five accelerometers A1–A5 were installed on the bottom flanges to measure the vertical vibrations. The sampling frequency was set at 200 Hz. In the experiment, the bottom flanges were removed to simulate the structural damage. The damage center was at $x = 8.9$ m, and the damage length was 0.10 m.

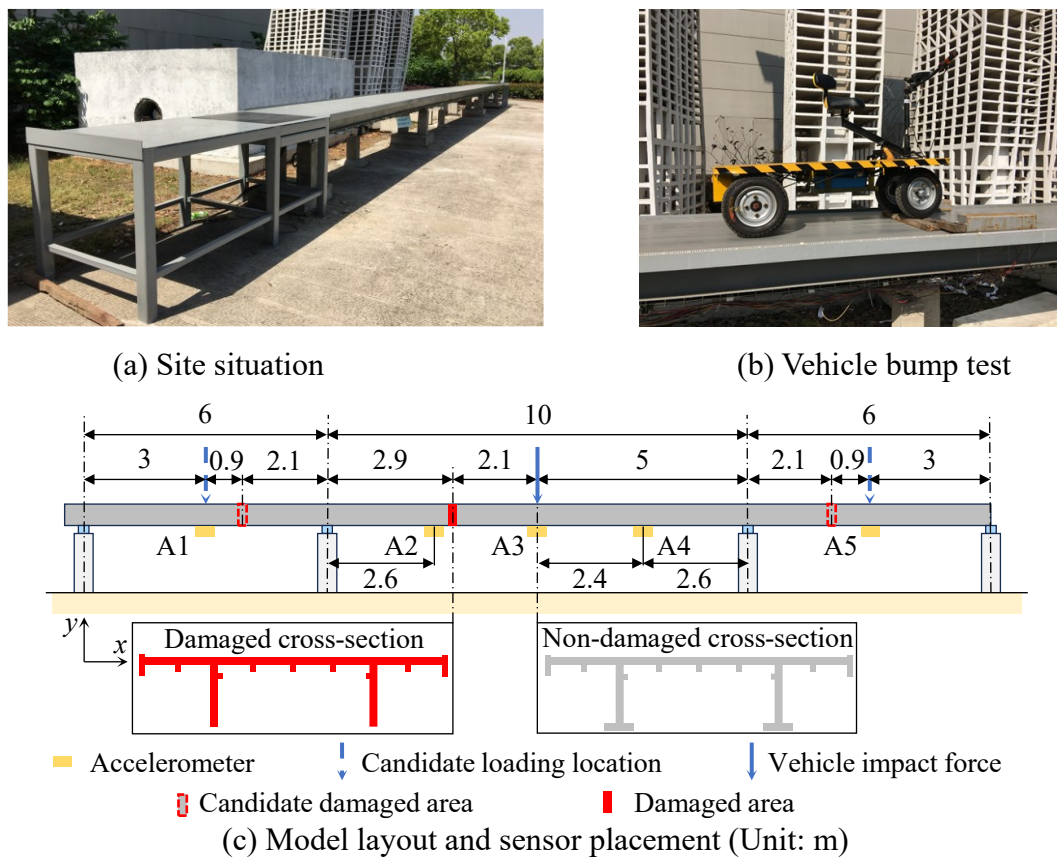


Figure 6.18 Experiment configuration

The baseline beam FE model is built with an element length of 0.1 m, resulting in a total of 220 elements. The elastic modulus is 173.25 GPa, the material density is $8.43 \times 10^3 \text{ kg/m}^3$, and the first two damping ratios are set to 1.73% and 2.40%, as calibrated and identified in previous studies (Li et al., 2023b). Three regions, $x \in [3.8, 4.0]$, $[8.8, 9.0]$, and $[18.0, 18.2]$ m, are considered as the candidate damage areas. The region of $x \in [8.8, 9.0]$ m corresponds to the real damaged area. The element stiffness of each candidate damage region is assumed to be spatially uniform. The stiffness reduction factors for these three candidate damage regions are selected as the DIs, resulting in three DIs to be identified. Nodes at $x = 3, 11$, and 19 m are selected as the candidate loading locations. Consequently, six unknowns should be identified by using five sensors in this experiment.

In the impulse test, two-second acceleration data of A1–A5 are used for the joint identification. The identified DIs are $[0, 0.392, 0]$, which is close to the real stiffness reduction factors of $[0, 0.376, 0]$, indicating that both the damage location and degree are accurately identified. The identified load time histories at the three candidate loading locations are shown in Figure 6.19(a). The force at the second candidate loading location is nonzero while the others are zero, indicating that the loading location is accurately identified. The static component of the vehicle load cannot be identified because only acceleration data are used, while the dynamic load component is well estimated. The identified excitation force aligns with the four-stage bump excitation model, which includes separation, free-fall, contact, and impulse, and coupled vibration phases (Zhang et al., 2020a). Before t_1 , the nonzero forces are mainly caused by the movement of the vehicle and operator during the startup. Next, the separation phase is observed between t_1 and t_2 , during which the contact force gradually decreases. The free-fall stage is not observed because the vehicle moves slowly and does not separate from the bridge deck. Subsequently, the contact and impulse phase occurs between t_2 and t_3 , with the contact force gradually increasing to the peak at t_3 . Finally, the vehicle-bridge-coupled vibration starts, and the identified force fluctuates with a reducing magnitude due to the damping effect.

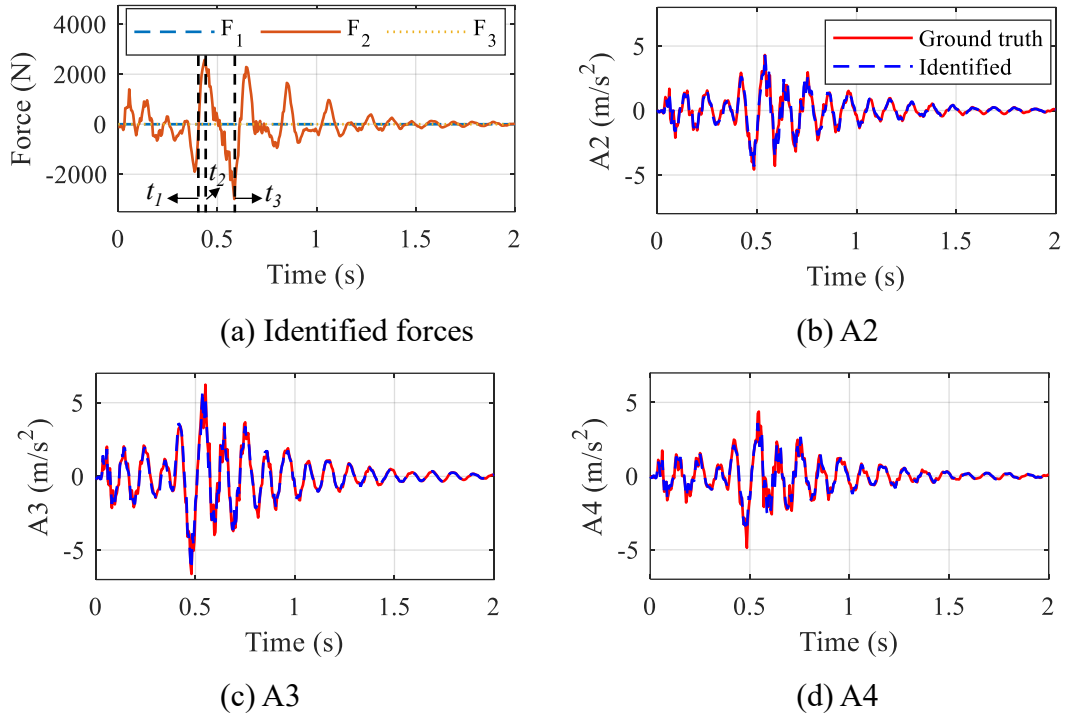


Figure 6.19 Identified load and reconstructed responses in the bump test

Finally, the identified DIs and loads are used to reconstruct the acceleration responses. The acceleration time histories at A2 to A4 are shown in Figures 6.19(b)–6.19(d), where the reconstructed responses align well with the measurements. The MSEs are calculated as 0.057, 0.056, and 0.064, demonstrating that the identified loads and DIs are sufficiently accurate for response reconstruction.

6.6 Summary

This chapter develops a sparse Bayesian learning method for joint load–parameter identification using noisy acceleration measurements. The spatial sparsity of structural damage indices and loads is incorporated as prior information within the Bayesian framework. The damage indices, forces, and hyperparameters are iteratively estimated using the EM algorithm, while the posterior samples of the latent variables are generated through a modified MH sampling approach.

The proposed method is validated through both numerical and experimental studies. Independent Gaussian priors are assigned to the loads and damage indices at different locations, enforcing spatial sparsity in the identified results. As a result, the damage indices of undamaged elements and the loads at unloaded nodes are identified as zero. The method's applicability is further evaluated under varying noise ratios, loading conditions, and damage levels. Results demonstrate that the damage indices, loads, and hyperparameters can be accurately identified without prior knowledge of damage locations or loading positions.

CHAPTER 7

JOINT LOAD–PARAMETER IDENTIFICATION USING A PHYSICS–ENCODED NEURAL NETWORK

7.1 Introduction

This chapter further investigates the joint load–parameter identification problem from a physics–data co-driven perspective. Existing physics-based methods for joint identification rely on complex analytical derivations to establish the relationship among structural parameters, loads, and responses. In contrast, purely data-driven approaches suffer from limited physical interpretability. To this end, this chapter proposes a PENN for joint load–parameter identification and full-field response reconstruction. FEMU, CNN, and Newmark-beta modules are integrated in PENN for parameter identification, load estimation, and response simulation, respectively. The main contributions of this study are summarized as follows: (1) FE models are formulated in tensor form on the DL platform, and the backpropagation algorithm is employed to compute gradients for joint identification; and (2) structural loads and parameters are simultaneously identified using sparse measurements within a self-supervised learning framework.

7.2 Methodology

7.2.1 Newmark-beta Method for Linear Structure Analysis

For an elastic linear structure with n DOFs, the dynamic equilibrium equation is expressed as

$$\mathbf{M}\mathbf{a}(t)+\mathbf{C}\mathbf{v}(t)+\mathbf{K}\mathbf{d}(t)=\mathbf{f}(t), \quad (7.1)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K} \in \mathbb{R}^{n \times n}$ represent the mass, damping, and stiffness matrices, respectively; and $\mathbf{a}(t)$, $\mathbf{v}(t)$, $\mathbf{d}(t)$, and $\mathbf{f}(t) \in \mathbb{R}^n$ denote acceleration, velocity, displacement, and input vectors at time t , respectively.

Eq. (7.1) can be solved by explicit or implicit methods. The Newmark-beta method is adopted in this study to calculate dynamic responses at discrete time steps due to its unconditional stability. The detailed procedures are listed in Table 7.1. The initial structural displacement and velocity are assumed to be zero.

Table 7.1 Newmark-beta method

1. Calculation for the initial time step

1.1. Define the time step Δt .

1.2. Define the initial condition.

$$\mathbf{d}_0 = 0, \mathbf{v}_0 = 0, \mathbf{a}_0 = \mathbf{M}^{-1} (\mathbf{f}_0 - \mathbf{C}\mathbf{v}_0 - \mathbf{K}\mathbf{d}_0).$$

1.3. Calculate constants.

$$a_0 = 1 / (\beta \Delta t^2), \quad a_1 = \delta / (\beta \Delta t), \quad a_2 = 1 / (\beta \Delta t), \quad a_3 = 1 / 2\beta - 1, \quad a_4 = \delta / \beta - 1, \\ a_5 = \Delta t / [2(\delta / \beta - 2)], \quad a_6 = \Delta t(1 - \delta), \quad a_7 = \delta \Delta t.$$

1.4. Calculate the equivalent stiffness matrix $\hat{\mathbf{K}} = a_0 \mathbf{M} + a_1 \mathbf{C} + \mathbf{K}$.

2. Calculation for remaining time steps

2.1. for $t = 0$ to $N_t - 1$, do:

2.2. $\hat{\mathbf{f}}_{t+1} = \mathbf{f}_{t+1} + \mathbf{M}(a_0 \mathbf{u}_t + a_2 \mathbf{v}_t + a_3 \mathbf{a}_t) + \mathbf{C}(a_1 \mathbf{u}_t + a_4 \mathbf{v}_t + a_5 \mathbf{a}_t).$

2.3. $\mathbf{u}_{t+1} = \hat{\mathbf{K}}^{-1} \hat{\mathbf{f}}_{t+1}.$

2.4. $\mathbf{a}_{t+1} = a_0 (\mathbf{u}_{t+1} - \mathbf{u}_t) - a_2 \mathbf{v}_t - a_3 \mathbf{a}_t.$

2.5. $\mathbf{v}_{t+1} = \mathbf{v}_t + a_6 \mathbf{a}_t + a_7 \mathbf{a}_{t+1}.$

2.6. end for.

3. Return: $\mathbf{d}$, $\mathbf{v}$, $\mathbf{a}$.

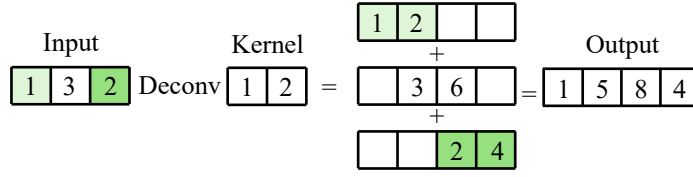
In the Newmark-beta method, Δt is the time step; N_t denotes the number of total steps; and δ and β are hyperparameters of the Newmark-beta method. In this study, δ is set to 0.5 and β is set to 0.25 in accordance with the linear acceleration assumption.

7.2.2 Deep Learning Modules

Two adopted deep learning modules, namely CNN and residual autoencoder (RAE), are introduced in this subsection. They are used to map the measured responses and identified loads. CNN is a feedforward neural network with strong feature extraction capability and high computational efficiency. It is widely applied to time series processing, including response reconstruction (Li et al., 2022a), data anomaly detection (Jian et al., 2021), and damage identification (Duran et al., 2025). The convolution and deconvolution operators are used for feature extraction and upsampling in CNNs, as presented in Figure 7.1. Figure 7.1(a) shows the 1D convolution. It involves element-wise multiplication between the kernel and input sub-blocks, followed by a summation operation to obtain the output element. The kernel moves horizontally across the input time series, and the mentioned procedure repeats. Figure 7.1(b) shows the deconvolution, an inverse operation of convolution. It can increase the resolution and create larger outputs from the input features. A broadcast multiplication between each input element and the kernel is first conducted, and then, the kernel moves over the input, repeating such multiplication. The products are finally summed up to obtain the output. The kernels in the convolution and deconvolution operators act as filters to compress and expand the features, respectively. In the training process, the filter weights and biases are adjusted to learn the underlying features of the dataset.

$$\begin{array}{c} \text{Input} \\ \boxed{1} \boxed{3} \boxed{0} \boxed{2} \end{array} \text{ Conv } \begin{array}{c} \text{Kernel} \\ \boxed{1} \boxed{2} \end{array} = \begin{array}{c} \boxed{1} \boxed{3} \boxed{0} \\ \boxed{6} \boxed{0} \boxed{4} \end{array} = \begin{array}{c} \text{Output} \\ \boxed{7} \boxed{3} \boxed{4} \end{array}$$

(a) Convolution



(b) Deconvolution

Figure 7.1 Convolution and deconvolution diagrams of 1D CNN

The RAE (Ni et al., 2022a) introduces residual architecture into an autoencoder to avoid the feature loss and gradient vanishing problems in deep networks, as exhibited in Figure 7.2. An autoencoder consists of an encoder and a decoder. An encoder compresses the input into a latent-space representation, and a decoder reconstructs the input from this latent representation. The residual learning mechanism introduces skip connections to transmit features over multiple layers. Therefore, it can enhance gradient backpropagation and mitigate the gradient vanishing problem. Skip connections are classified as additive or concatenative skips. The former sums up the output from two layers, preserving the original output size. Concatenative skips stack the outputs up, expanding the size of the feature map. In this study, the additive skip connections are used, given their dimension-conserving properties. The convolutional RAE is adopted in this study.

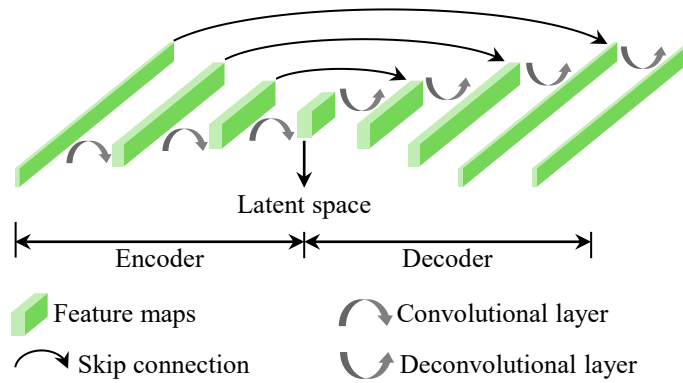


Figure 7.2 Schematic of an RAE

7.2.3 Joint Identification Based on PENN

The schematic of the proposed physics–data co-driven method is illustrated in Figure 7.3, consisting of the physics-based and data-driven components. The physics-based parts include an FEMU module and a Newmark-beta dynamic solver. The data-driven part is a convolutional RAE for equivalent load identification. The equivalent load refers to the load vector to which the reconstructed structural responses at the sensor locations are identical to the measured ones. The physics-based and data-driven parts are integrated to jointly estimate the unknown loads, system parameters, and responses. The trainable parameters in the proposed PENN include the structural stiffness parameters in the FEMU module and the weights and biases in the RAE.

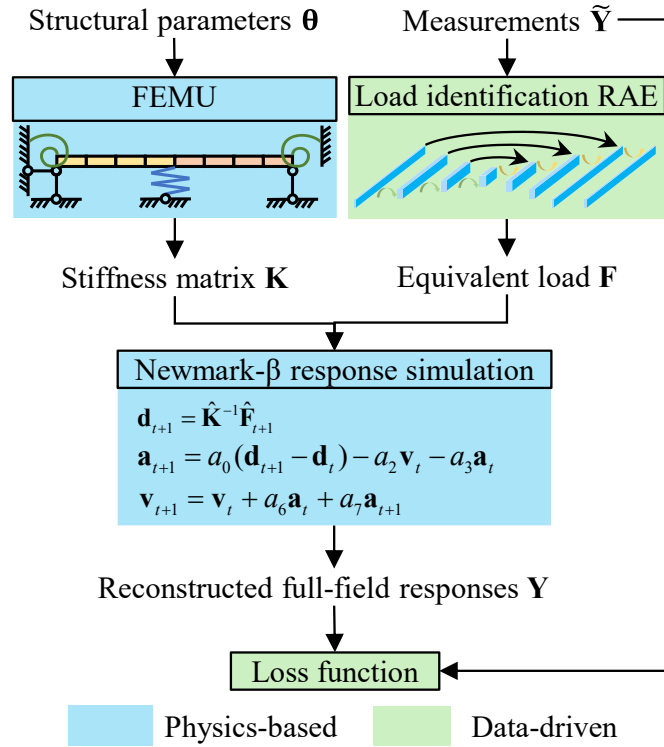


Figure 7.3 Schematic of PENN for joint load–parameter identification

In the FEMU module, an FE model is established based on the pre-selected structural parameters. The mass matrix and damping ratios are assumed to be known beforehand,

and they remain constant in the learning process. The global stiffness matrix $\mathbf{K}$ is assembled from the element stiffness matrices:

$$\mathbf{K} = \sum_{i=1}^{N_i} \theta_i \bar{\mathbf{K}}_i, \quad (7.2)$$

where θ_i is the i -th element stiffness parameter, N_i is the number of elements, and $\bar{\mathbf{K}}_i$ is the standard stiffness matrix of the i -th element. The FE model is directly built on the deep learning platform, where the stiffness, damping, and mass matrices, and force vector are represented as tensors. These tensors encapsulate both data values and gradients within the computation graph. The associated gradients enable inverse identification by using the gradient backpropagation algorithm on the deep learning platform. Therefore, the structural parameters $\boldsymbol{\theta}$ can be automatically updated when training the data-driven model, as will be introduced later.

The load identification RAE is detailed in Figure 7.4. It can identify the equivalent load from the measured structural response time history. The RAE comprises four convolutional layers and four deconvolutional layers with identical hyperparameters: four kernels, two strides, and one padding. Besides, the LeakyReLU function is adopted as the activation function. Additive skip connections are employed to transmit the extracted features from shallow layers to deep layers, preventing gradient vanishing when the network deepens. The network input is the measured response data with a size of $b \times N_s \times 512$, corresponding to the batch size, number of sensors, and segment length, respectively. The network output is the identified equivalent load with a size of $b \times N_d \times 512$, where N_d denotes the dimension of the reconstructed load and is associated with the load candidate positions.

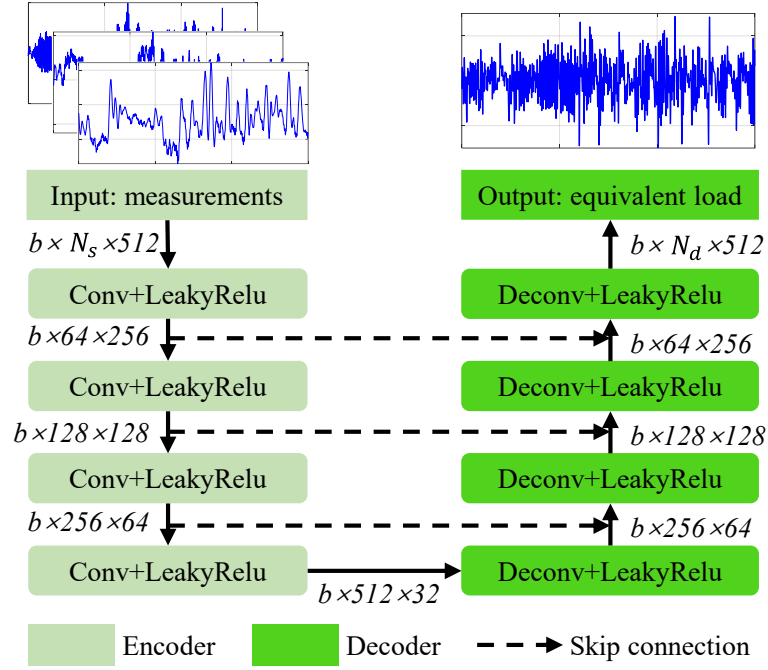


Figure 7.4 Detailed architecture of the load identification RAE

In the encoder part, the input response data are compressed in the time dimension through four convolutional operators, resulting in a latent space with a size of $b \times 512 \times 32$. Meanwhile, the dimension of features increases from N_m to 256 to mine the high-level features. In the decoder part, the mined features are expanded by four deconvolutional operators to estimate the equivalent force matrix. The features extracted from the shallow layers are also transmitted to the deep layers via skip connections to keep the completeness of the information. When the load location is known, the identified load closely approximates the real load. If the load location is unknown and varying, the RAE aims to generate an equivalent load, which ensures the calculated responses match the measured ones at the sensor locations. The NN architecture is not limited to CNN; any model with sufficient data fitting and regression capability, such as LSTM or RNN, can be used in the proposed framework.

7.2.4 Model Training

The FE model-extracted matrices $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and RAE-estimated equivalent load are used to calculate the dynamic responses via the Newmark-beta solver. The calculated responses at observed DOFs should be consistent with the measurements, and their difference is used to formulate the loss function. In this study, the loss function is defined as the MSE between the reconstructed and measured responses at observed DOFs. Moreover, ℓ_1 norms on the structural parameters and load vector are added to the loss function as the regularization terms to impose a spatial sparsity property on these variables. The final loss function L is expressed as

$$L = \frac{1}{N_s N_t} \sum_{s=1}^{N_s} \sum_{t=1}^{N_t} \|\mathbf{Y}_{s,t} - \tilde{\mathbf{Y}}_{s,t}\|^2 + \kappa_1 \frac{1}{N_i} \sum_{i=1}^{N_i} |1 - \theta_i| + \kappa_2 \frac{1}{N_d N_t} \sum_{d=1}^{N_d} \sum_{t=1}^{N_t} |\mathbf{F}_{d,t}|, \quad (7.3)$$

where s denotes the sensor index, t is the time step index, d represents the load index, κ_1 and κ_2 are the regularization coefficients, and $\mathbf{Y}$ and $\tilde{\mathbf{Y}} \in \mathbb{R}^{N_s \times N_t}$ represent the measured and calculated responses at the sensor locations, respectively.

The developed deep learning framework is implemented on the PyTorch platform. The parameters to be identified include the stiffness parameters and the weights and biases of the RAE. These parameters are built as tensors and indirectly linked to the measured responses through a gradient-embedded computational graph. In each epoch, the loss is calculated via forward propagation from the measured responses to the predicted ones, and then, the gradients of loss with respect to stiffness parameters and RAE parameters are calculated through the backpropagation algorithm. The response-based loss function is minimized using gradient descent methods to update the stiffness parameters and train the load identification RAE simultaneously. In such a way, the whole framework is trained in a self-supervised manner, where only sparse output-only data is needed to estimate loads and structural parameters. Compared to the supervised methods that require collecting and labeling load or damage datasets, the proposed approach is much more efficient and physically interpretable.

7.3 A Numerical Study

A numerical two-span continuous beam is utilized to verify the proposed method, as illustrated in Figure 7.5. The beam is $8 + 8 = 16$ m long, the cross-section is 0.15 m high and 0.25 m wide. The material density is $7,800 \text{ kg/m}^3$, and the elastic modulus is 210 GPa. The FE model is built using 32 beam elements and 33 nodes. Only the in-plane deformation is considered, resulting in a total of 64 DOFs. Two rotational springs with the stiffness of $1 \times 10^4 \text{ kN/rad}$ are installed at the end supports; a vertical spring with the stiffness of $5 \times 10^4 \text{ kN/m}$ is set at the middle support. Then, structural damage is simulated by changing stiffnesses at five locations: the left-side rotational spring, left-span beam, middle vertical spring, right-span beam, and right-side rotational spring. The normalized stiffness parameters in the changed state are $\theta = [0.80, 0.75, 0.9, 0.85, 1.2]$. The resulting first two natural frequencies of the beam are 4.738 Hz and 7.120 Hz. Rayleigh damping is adopted, with the first two damping ratios set as 3%.

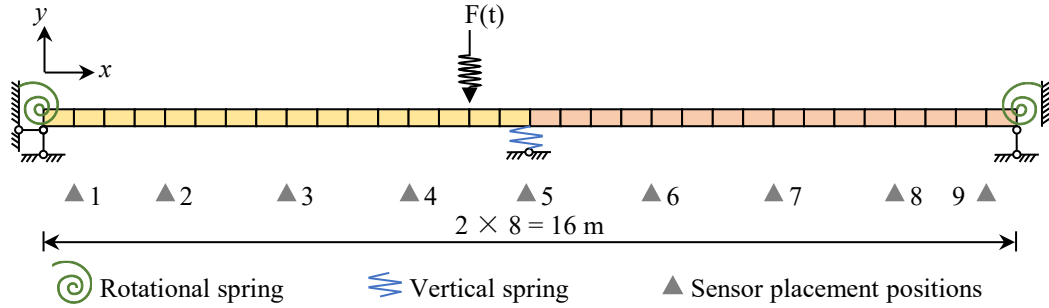


Figure 7.5 Configuration of the numerical two-span beam

A concentrated force comprising static, dynamic, and random components is applied at the vertical direction of the 15th node ($x = 7$ m), and it is expressed as

$$F(t) = w + w \cdot \sin(2\pi f \cdot t) + w \cdot \varepsilon, \quad (7.4)$$

$$w \in [980, 4900], \quad f \in [1, 5], \quad \varepsilon \in [0, 1],$$

where w represents the static load, f denotes the frequency of the harmonic component, and ε is a stochastic variable accounting for the load uncertainty. It is assumed that $w, f,$

and ε all follow uniform distributions. w is set in the range of [980 N, 4,900 N] to provide that the amplitudes of beam responses keep consistent with those in the real world. f is set between 1 and 5 Hz because the load frequency of civil structures is usually low. $\varepsilon \in [0, 1]$ will introduce a large white noise component into the load to evaluate the system identification performance of the developed PENN.

The sensors are assumed to be installed at nine candidate locations to measure the vertical vibration responses, as depicted in Figure 7.5. Three types of sensors are used for comparison, including strain gauges, accelerometers, and displacement transducers. The sampling frequency of all sensors is 100 Hz. The RRMSE indicator defined in Eq. (6.35) is employed to quantify the difference between the measured and reconstructed responses.

7.3.1 Baseline Scenario

In the baseline scenario N0 ($\theta = [0.80, 0.75, 0.90, 0.85, 1.2]$), the noise-free strain data at nine locations (1–9) are used to identify the structural parameters and load time history. The load position is constant and assumed to be known; therefore, the output channel of the load RAE N_d is one. A total of 600 numerical samples with a segment length of 512 are calculated. The training–validation–test ratio is 4:1:1. The hybrid model is implemented on the PyTorch platform and trained by the Adam optimizer with a batch size of 128. The initial stiffness parameters are all set to 1. Moreover, varied learning rates are adopted for different modules. Specifically, the learning rates for the stiffness parameters and the load RAE are 0.01 and 0.001, respectively. The regularization coefficients κ_1 and κ_2 are set to zero because the model class is precise in the numerical study. The model is trained on a computer with an Intel Core i7-13700 CPU, 64 GB of RAM, and an NVIDIA 4070 12 GB graphics card. In the N0 scenario, the training time is 2.27 s per epoch, and 10,000 epochs are included in the whole training process, resulting in a 6.31-hour training time.

The training and validation losses are presented in Figure 7.6(a). Both losses decrease

rapidly at the beginning of training epochs and start oscillating after around 200 epochs. After 4,000 epochs, the losses decrease at a slower rate and closely align, indicating that neither overfitting nor underfitting occurs in the training process. The updating process of the stiffness parameters is shown in Figure 7.6(b). The parameters converge to their ground truth in the first 300 epochs. After 300 epochs, they slightly oscillate around the ground truth. A comparison of Figures 7.6(a) and 7.6(b) indicates that the loss reduction before 300 epochs is associated with the optimization of both the stiffness parameters and the load RAE. After 300 epochs, the decrease in loss is primarily attributed to the further optimization of the load RAE.

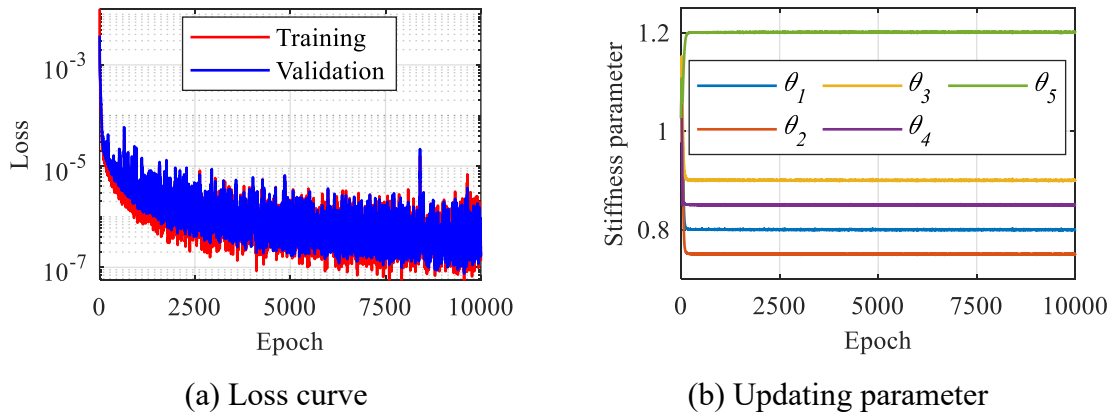


Figure 7.6 Loss curves and parameter estimation in the training process of N0

The trained model with the lowest validation loss is employed to estimate structural parameters and load time history. The identified stiffness parameters are [0.80, 0.75, 0.90, 0.85, 1.20] with an average error of 0.04%, demonstrating a high identification accuracy. A new test sample is used to verify the performance of the trained PENN in the load–response reconstruction. The identified load time history is displayed in Figure 7.7. It aligns well with the ground truth and deviates only at the peak values. The average RRMSE between the reconstructed and real load across all test samples is 3.39%. This indicates that the well-trained hybrid model can accurately identify the load with a high generalization ability for loads it has never seen.

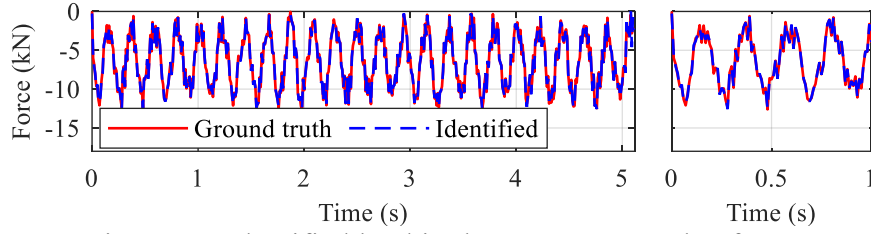
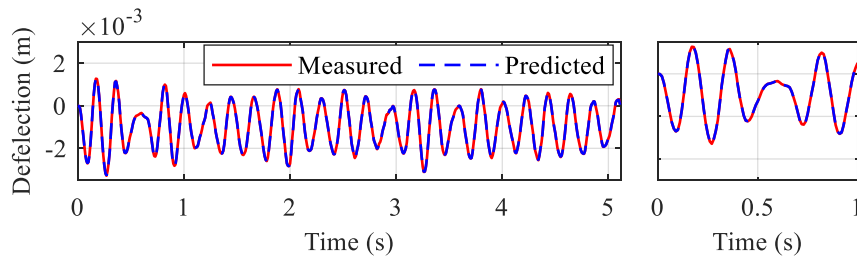


Figure 7.7 Identified load in the new test sample of N0

The trained hybrid model is then used to reconstruct the full-field responses using the updated parameters and the identified load time history. The beam responses to the real load, which are calculated by using the Newmark-beta method, are used to compute the response reconstruction error. The predicted and real responses at the center of the left span ($x = 4$ m) are illustrated in Figure 7.8. They are identical in shape and peak values because the load and stiffness parameters are already identified accurately. The average RRMSEs for displacement, strain, velocity, and acceleration among all test samples are 0.35%, 0.45%, 0.89%, and 5.44%, respectively, indicating satisfactory reconstruction accuracy. The RRMSE of acceleration is higher than the other responses because acceleration, as a high-order response with respect to time, is very sensitive to the prediction errors in the identified load. The average RRMSE distributions of the vertical vibrations along the beam in the test sets are displayed in Figure 7.9. Most of the RRMSEs are at a low level. Notably, the errors at $x = 7$ m are higher than at other locations because the load is applied to this location.



(a) Deflection

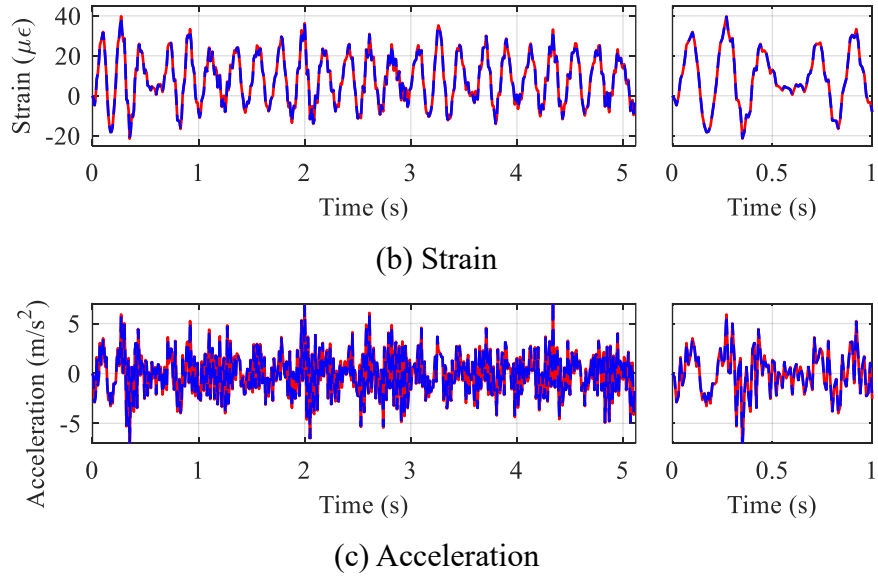


Figure 7.8 Reconstructed responses at $x = 4$ m in the new test sample of N0

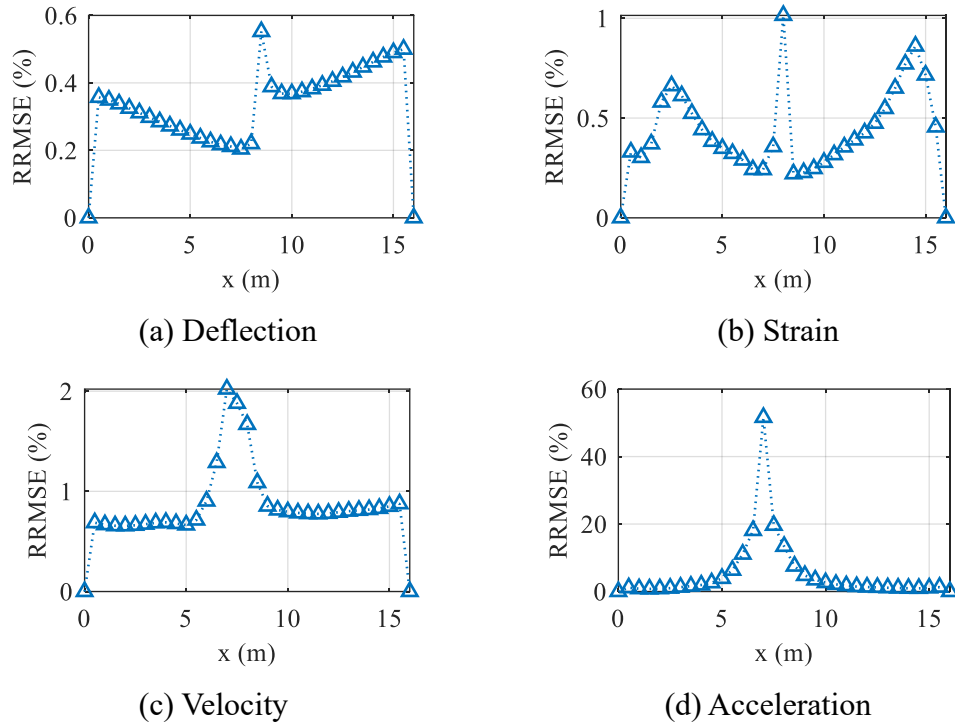


Figure 7.9 Distribution of the average RRMSE along the beam in N0

The applicability of the proposed method is examined in this subsection. First, different levels of artificial noise are added to responses to verify the robustness of the method.

Then, the sensor location is changed to study its influence on the identification accuracy. Finally, the whole framework is implemented without knowing the load position. The NR and sensor configurations in all scenarios are summarized in Table 7.2.

Table 7.2 Scenario settings

Scenario	Measurement	NR (%)	Loading location
Baseline		0	
N1	S1, S2, S3, S4, S5, S6, S7, S8, S9	5	Known
N2		10	
N3		20	
P1	D1, D2, D3, D4, D5, D6, D7, D8, D9		
P2	A1, A2, A3, A4, A5, A6, A7, A8, A9	5	Known
P3	S1, S2, A3, S4, S5, S6, A7, S8, S9		
U1	S1, S2, S3, S4, S5, S6, S7, S8, S9	5	Unknown

Note: Letters ‘S’, ‘D’, and ‘A’ represent strain, displacement, and acceleration measurements, respectively. The number following each letter corresponds to the sensor index.

7.3.2 Influence of Measurement Noise

In this subsection, the calculated response data are contaminated by 5%, 10%, and 20% white noise to discuss the influence of noise level, while other settings remain consistent with those in the baseline scenario, as shown in Table 7.2 N1–N3. The measurement contaminated by white Gaussian noise is defined in Eq. (6.34).

The identified structural parameters are reported in Table 7.3. The estimation errors do not increase significantly with the increase in noise level. The maximum estimation errors in all scenarios are below 0.1%. This phenomenon indicates that the proposed method is insensitive to the observation noise in identifying the structural parameters. The averaged testing RRMSEs of the load and response from all scenarios are listed in Table 7.4. The RRMSEs increase with rising noise levels, suggesting that the noise affects the force

identification performance. Moreover, the RRMSEs of velocity and acceleration are highly related to the noise level because they are high-order responses with respect to time, and they are thus sensitive to errors in the identified loads. When the noise level is less than 20%, the average RRMSEs are all below 40%, indicating full-field responses are accurately reconstructed. When the noise level is larger than 20%, the average RRMSEs for acceleration are larger than 40%. Considering that the noise level of 20% is quite high in actual applications, the overall reconstruction accuracy is satisfactory.

Table 7.3 Updated parameters in different scenarios

Scenario	θ_1	θ_2	θ_3	θ_4	θ_5	Average relative error (%)
Ground truth	0.80	0.75	0.90	0.85	1.20	-
Baseline	0.80	0.75	0.90	0.85	1.20	0.04
N1	0.80	0.75	0.90	0.85	1.20	0.03
N2	0.80	0.75	0.90	0.85	1.20	0.03
N3	0.80	0.75	0.90	0.85	1.20	0.05
P1	0.80	0.75	0.90	0.85	1.20	0.08
P2	0.80	0.75	0.90	0.85	1.20	0.14
P3	0.80	0.75	0.90	0.85	1.20	0.02
U1	0.80	0.74	0.92	0.84	1.20	0.93

Table 7.4 Average RRMSE (%) of the reconstructed responses in the test set

Scenario	Displacement	Strain	Velocity	Acceleration	Load
N0	0.35	0.45	0.89	5.44	3.39
N1	1.95	2.93	5.70	20.29	6.16
N2	2.63	4.65	9.17	34.91	9.93
N3	3.88	7.87	15.94	72.76	23.44
P1	9.97	32.21	66.59	188.40	33.92
P2	244.36	213.67	31.29	19.07	288.13
P3	10.23	14.19	23.55	35.63	11.98
U1	6.60	8.77	12.62	27.55	9.62

7.3.3 Influence of Sensor Type

In this subsection, the influence of sensor types is investigated. The displacement, strain, and acceleration responses are used with the noise ratio being 5%. The sensors are installed at nine locations, as detailed in Table 7.2 P1–P3.

The identified structural parameters are presented in Table 7.3, where the estimation error is less than 0.1% across all sensor placement schemes. This indicates that the stiffness parameters are identifiable given the adopted sensor placements.

The load–response identification results are reported in Table 7.4. The RRMSE between the reconstructed and real loads in P1 is far more than that in N1. This is because only the displacement data are used in P1. Displacement contains more quasi-static and low-frequency dynamic components compared to acceleration and strain because the derivatives with respect to time and space would amplify high-frequency components. Consequently, the PENN model prioritizes reconstructing the low-frequency components of the load, while the high-frequency features are not well learned under P1. This inference is also demonstrated by the fact that the RRMSEs of the velocity and acceleration are far greater than those of the displacement and strain. In P2, only the acceleration data is used. The load is incorrectly identified because the RAE cannot learn the low-frequency features from the acceleration data, leading to the low-frequency drifting problem in the identified load and responses. Therefore, the acceleration and velocity have lower errors than the displacement and strain. The identification accuracy in P2 could be further improved by adding ℓ_1 norm on the load to the loss function or adding displacement and strain data to the training dataset. In P3, the strain and acceleration data are both used, and the reconstructed load is accurate. Nevertheless, the overall accuracy does not improve compared to N1. This fact suggests that more strain gauges should be used to improve the identification accuracy. Moreover, in the investigated sensor placement schemes, the full-field responses can only be correctly reconstructed in N1 and P3. Therefore, it is not recommended to use displacement or

acceleration data alone.

7.3.4 Influence of Unknown Loading Position

The performance of the proposed method is further studied when the input location is unknown. All 16 vertical DOFs of the left span are assumed to be the candidate loading locations, resulting in an output size of $b \times 16 \times 512$ for the load identification RAE. Nine strain gauges (S1 to S9) are used for the identification task in this scenario. The NR is 5%, as detailed in Table 7.2.

The identified structural parameters are reported in Table 7.3. The average error of all parameters is 0.929%. It is larger than those in the other cases because the load location is unknown in advance, resulting in the joint load–parameter–response identification being an ill-conditioned problem. The real load is applied to the left span whose property is closely associated with θ_2 and θ_3 . Consequently, θ_2 and θ_3 could compensate for the errors of the identified load. Therefore, the identification accuracies of θ_2 and θ_3 are lower than those of the other parameters. Nevertheless, the small error is acceptable in practice.

The real and identified loads in a new test sample are displayed in Figure 7.10. The identified load is concentrated at $x = 7$ m, with little noise appearing at other locations. Moreover, the reconstructed load time history at $x = 7$ m aligns well with the ground truth with an RRMSE of 9.62%, as reported in Table 7.4. This result demonstrates that the load can be accurately identified even with unknown load positions.

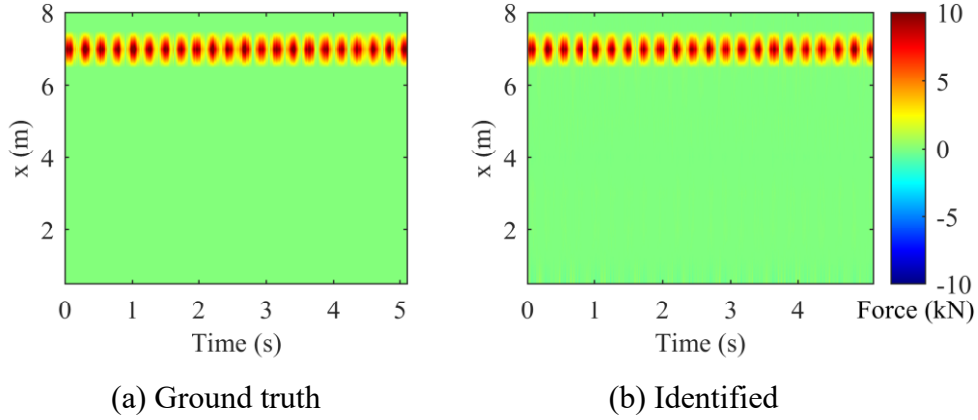


Figure 7.10 Time–space distributions of the real and identified loads

The full-field responses are calculated using the identified loads and structural parameters. Their average RRMSEs are reported in Table 7.4 with high accuracy. The full-field deflection time history is illustrated in Figure 7.11(a). It consists of both static and dynamic components. The response magnitude at the left span is greater than that at the right span because the force is applied on the left span. The error map is displayed in Figure 7.11(b), where all errors are below 5%. The maximum error appears at the ending instant due to the boundary effects of CNNs, because the zero padding operation in CNNs introduces invalid information, reducing the prediction accuracy at the beginning and ending moments of a time series.

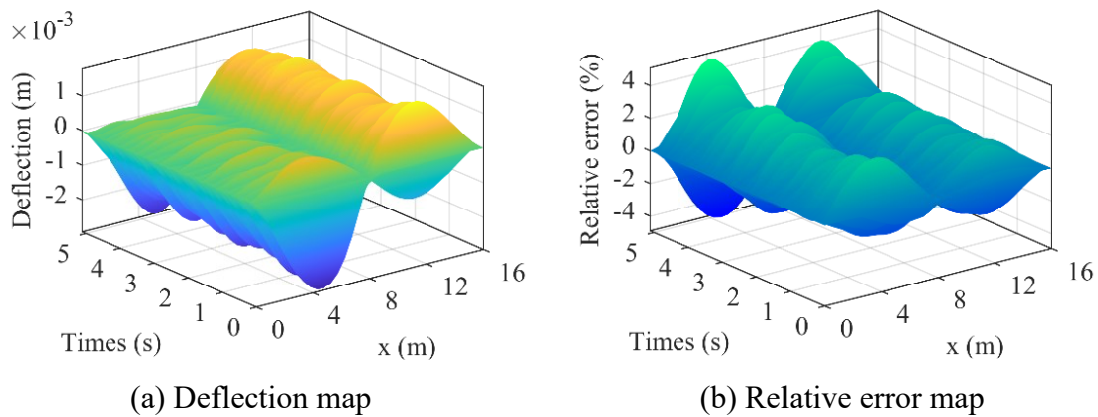


Figure 7.11 Time–space distributions of the reconstructed deflection and the associated error map

7.4 An Experimental Study

7.4.1 Experimental Setup

The proposed approach is applied to a laboratory-tested iron cantilever beam model. The results will be compared with those using the physics-based EKF method (Song et al., 2023b). The model configuration is illustrated in Figure 7.12. The beam was mounted on a strong floor through a thick plate to simulate the fixed boundary. The beam is 1,200 mm long and has a cross-section size of 50 mm $\times$ 3 mm. It is divided into eight beam elements in the initial 2D FE model, resulting in 16 DOFs because the longitudinal response is neglected. The material density is 7,550 kg/m³, and the initial elastic modulus is set to 150 GPa. Eight strain gauges (S1–S8) and eight accelerometers (A2–A9) were used to measure the structural dynamic responses. The mass of the accelerometers is considered in the FE model. A laser displacement transducer (D9) was installed to capture the horizontal displacement of the top end. An instrumented hammer was used to strike the beam's top to excite the beam's vibration. Figure 7.13 demonstrates the experimental setup and sensors used in the test.

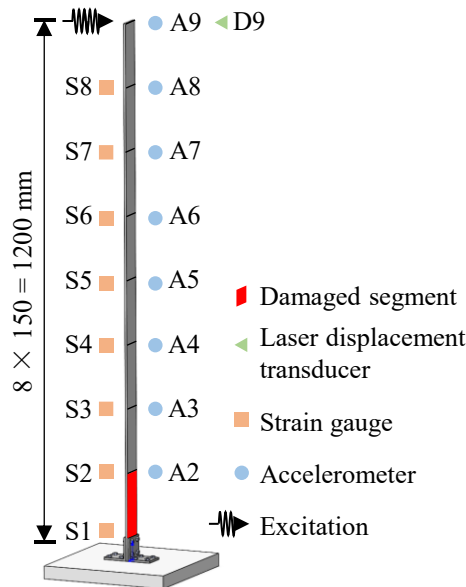


Figure 7.12 Laboratory cantilever beam and sensor layout

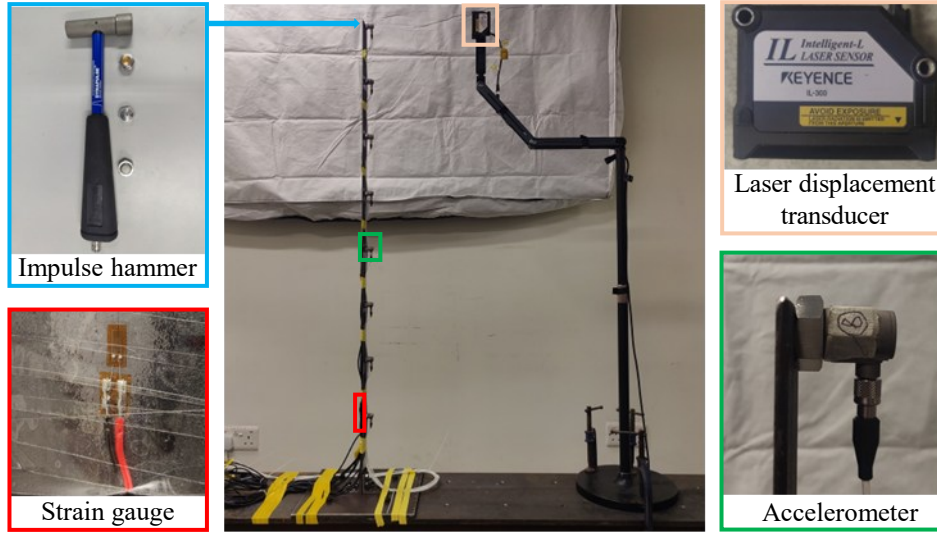


Figure 7.13 Experimental setup

Four types of loads were applied to the model, including (1) enforced displacement, (2) single-impulse, (3) multiple-impulse, and (4) continuous excitation. In case 1, the beam top was imposed by an arbitrary displacement and released to generate the free-vibration response. In case 2, a single strike was applied to the beam top by the hammer. In case 3, the hammer repeatedly struck the beam's top to generate a steady enforced vibration response. In the last case, a dynamic force was applied by pressing the beam top with a fluctuating force. The first two cases repeated 50 times, each lasting for 50 s; and the last two cases repeated 10 times, each for 3 minutes. The sampling frequency was 200 Hz.

The first two natural frequencies of the intact beam are identified as 1.304 Hz and 8.526 Hz, and the associated damping ratios are 1.027% and 0.509%, from the measured impulse response data by the rational fraction polynomial method (Richardson et al., 1982). Rayleigh damping is adopted to calculate the damping matrix.

7.4.2 Joint Load–Parameter–Response Identification

The measured strain data at S1, S3, S5, and S7 are used as the input, and the full-field responses are calculated at the output layer of the PENN. The MSE between the measured and reconstructed responses at S1, S3, S5, and S7 is formulated as the loss function. The

data of the remaining sensors are not involved in training but are used to verify the accuracy of the reconstructed full-field responses. The measured data is divided into segments by using a moving window with a length of 800 and a stride of 400, yielding 2,117 training and 530 validation samples. The global elastic rigidity is the parameter to be updated, and its initial value is normalized in a range of $[0.5, 1.1]$. The loading position is assumed unknown, and Nodes 2–9 are set as the candidate locations. The learning rates of the stiffness parameters and the load RAE are 0.01 and 0.001, respectively. The batch size is 128, and the PENN is trained for 2,000 epochs. The regularization coefficients κ_1 and κ_2 are set to 10^{-4} to impose a sparsity property on identified structural parameters and load because of the modeling errors and measurement noise.

The training and validation losses in the training process are shown in Figure 7.14(a). At the beginning epochs, the training and validation losses decrease steadily and start to fluctuate after 50 epochs, implying the complexity of the identification task. The learning process of the stiffness parameter is exhibited in Figure 7.14(b). The stiffness parameter stabilizes at 0.895 after 250 epochs. Its ground truth is 0.894 as estimated by the existing FEMU method using the modal data, verifying the high accuracy of the proposed approach. The PENN model with the smallest validation loss is used to generate the elastic modulus, recover the input load, and reconstruct full-field responses.

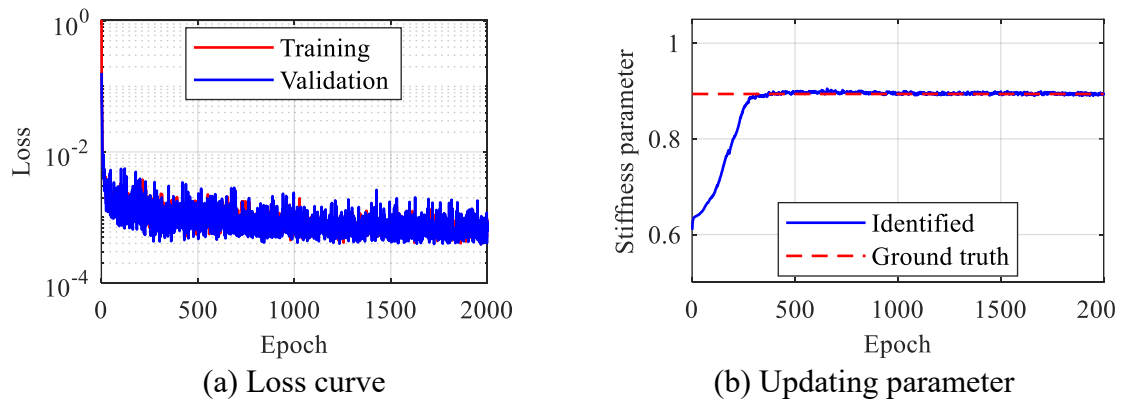


Figure 7.14 Loss curves and parameter updating in the training process

The well-trained PENN model is used to identify the load and reconstruct the full-field responses using the new test samples, which are regenerated under all four loading conditions. Under load case No. 1, the identified force map is shown in Figure 7.15(a), and the identified force at Node 9 is detailed in Figure 7.16(a). The identified load is close to zero, consistent with the real load. The displacement, strain, and acceleration response measurements, which are not used for training, are reconstructed and shown in Figures 7.15(b)–7.15(d). The reconstructed responses are highly consistent with the measured ones. Their RRMSEs are listed in Table 7.5. The RRMSE of the acceleration response is slightly higher than that of other types of responses because acceleration is more sensitive to the noise in the identified loads. Overall, the identified structural parameters and loads are accurate to reconstruct the full-field responses in the enforced displacement loading case.

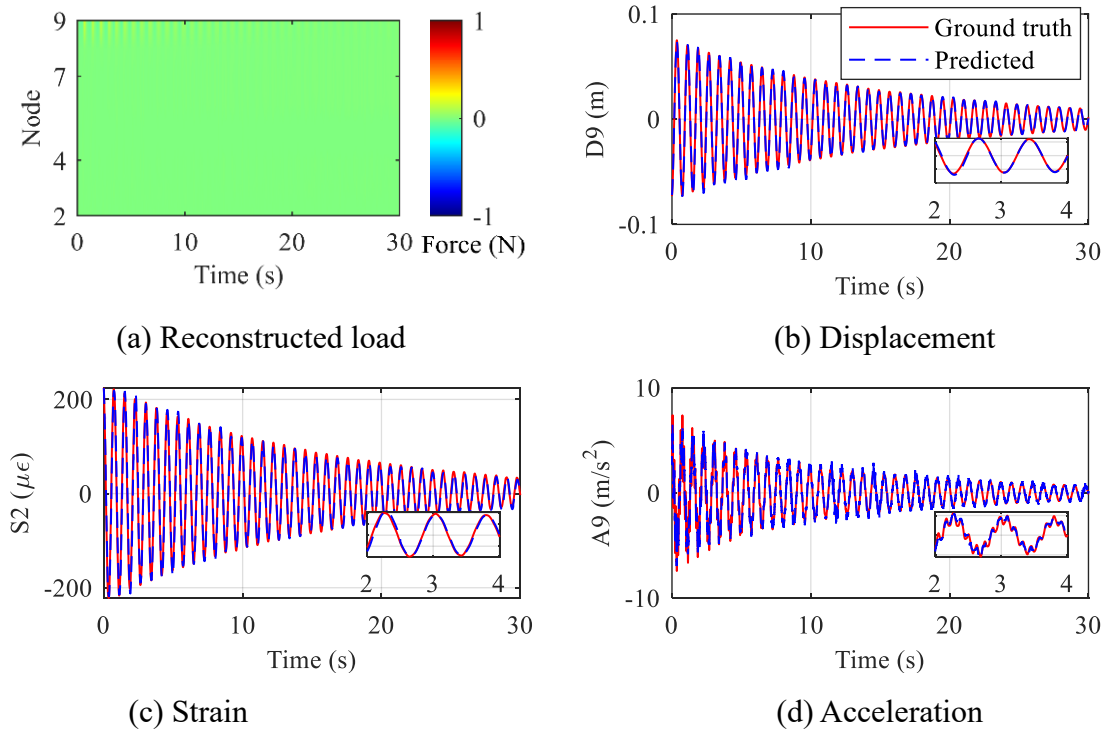


Figure 7.15 Reconstructed load and responses in enforced displacement case

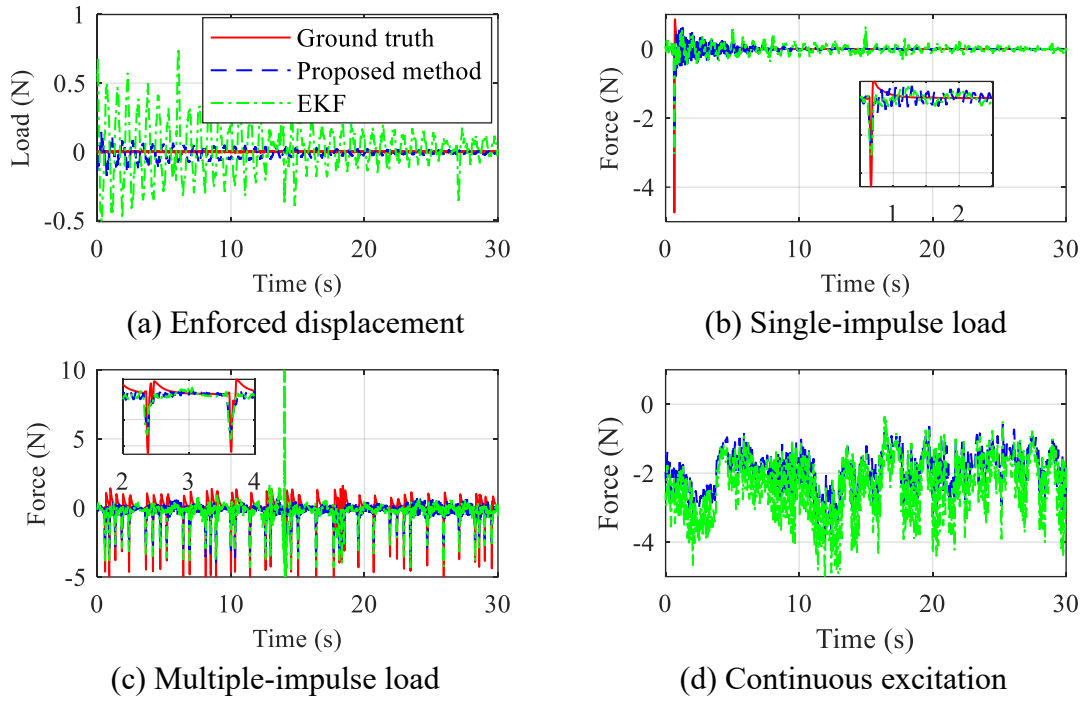


Figure 7.16 Reconstructed force at Node 9

Scenario	Loading case	Method	Displacement	Strain	Acceleration
Da0	Enforced displacement	Proposed	7.62	16.98	33.79
		EKF	19.43	21.51	40.86
	Single-impulse	Proposed	8.35	11.82	83.66
		EKF	26.01	23.67	86.39
	Multiple-impulse	Proposed	12.23	10.11	96.37
		EKF	36.83	35.15	101.43
	Continuous excitation	Proposed	3.94	3.85	91.28
		EKF	3.54	4.51	93.12
Da1	Enforced displacement	Proposed	9.49	17.34	32.41

The reconstructed loads in the single-impulse and multiple-impulse loading cases are shown in Figure 7.17(a) and Figure 7.18(a), and they are correctly concentrated at Node 9. The measured and identified force time histories at Node 9 are compared in Figures

7.16(b) and 7.16(c), where the force peaks appear nearly at the same instant. However, the magnitudes of the real loads are higher than those of the identified forces because RAEs tend to produce smooth load distributions. The displacement and strain at unmeasured DOFs are accurately reconstructed, as exhibited in Figures 7.17(b)–7.17(c) and Figures 7.18(b)–7.18(c). Again, the reconstructed acceleration contains more error since they are more sensitive to the noise in the identified load. The average RRMSEs of all types of responses are summarized in Table 7.5, demonstrating the high accuracy of the proposed method in reconstructing the full-field strain and displacement responses.

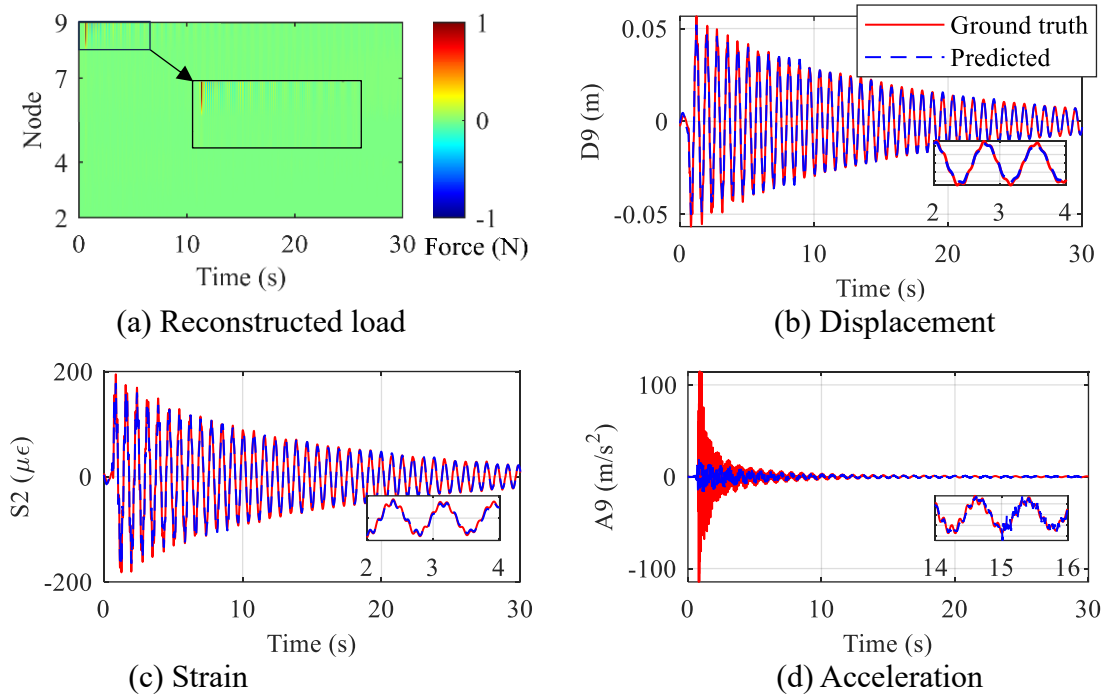
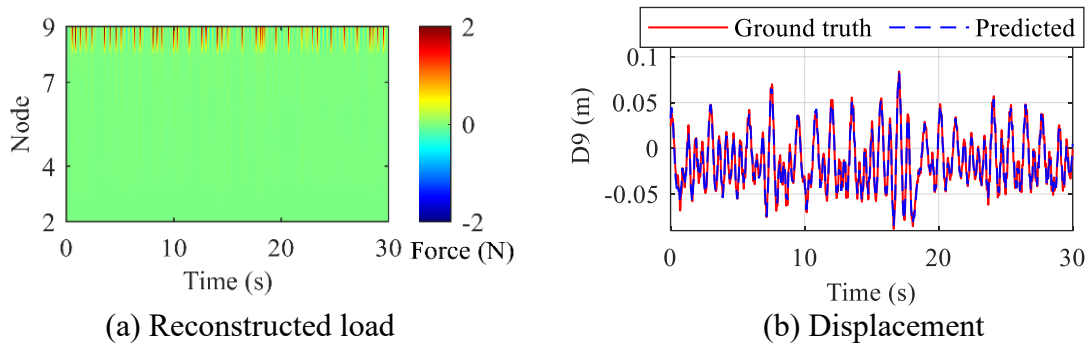


Figure 7.17 Reconstructed load and responses in single-impulse loading case



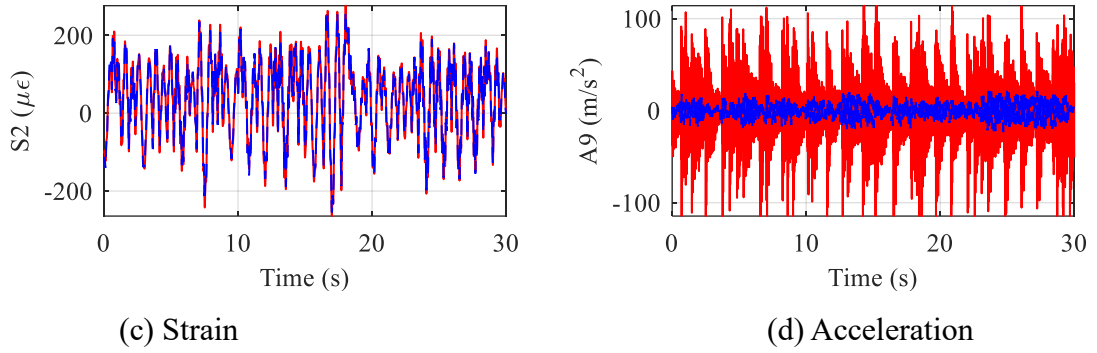


Figure 7.18 Reconstructed load and responses in multiple-impulse loading case

The reconstruction results in load case No. 4 are shown in Figure 7.19, where the loading position is correctly identified near Node 9. The identified force time history at Node 9 is illustrated in Figure 7.16(d), and it aligns well with the continuous excitation pattern. The reconstructed responses are exhibited in Figures 7.19(b)–7.19(d), and the associated RRMSEs are listed in Table 7.5. The small reconstruction errors demonstrate that full-field strain and displacement responses are accurately reconstructed under continuous excitation load.

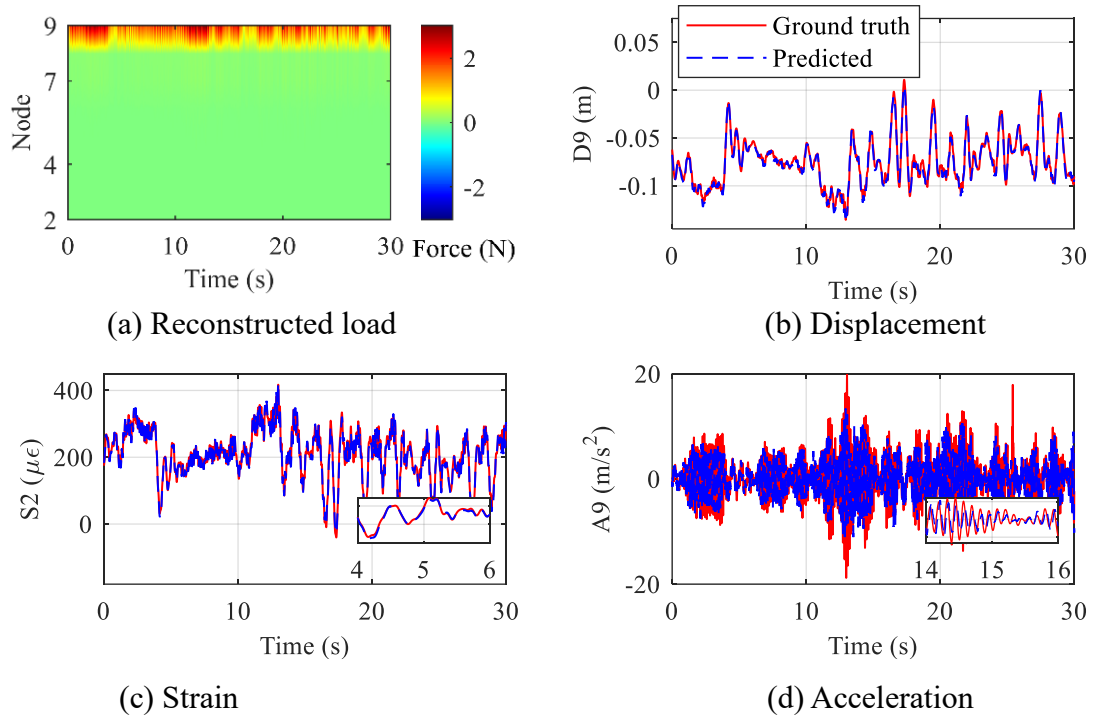


Figure 7.19 Reconstructed load and responses in continuous loading case

In summary, the proposed method can accurately identify the global structural parameter, load distributions, and reconstruct full-field strain and displacement responses across all loading cases. However, the full-field acceleration is accurately reconstructed only in the enforced displacement loading case due to its high sensitivity to force prediction errors.

7.4.3 Comparison with EKF Method

A window-based EKF (Song et al., 2023b) is also applied to the example for comparison. The loading location is assumed to be known to avoid divergence in the EKF approach. The identified stiffness parameters in all four loading cases are 1.079, 1.024, 1.027, and 1.091. These values are significantly larger than the actual value of 0.894, resulting in an average error of 18.06%. The identified force time histories in four loading cases are shown in Figures 7.16(a)–7.16(d), exhibiting the same variation pattern as those by the proposed PENN but with larger amplitudes and fluctuations. The results indicate that EKF-identified parameters and forces could compensate for each other.

The RRMSEs of the reconstructed responses are compared in Table 7.5. The RRMSEs of PENN are significantly smaller than those of EKF in most cases. This is attributed to several reasons: (1) The proposed PENN is trained using the response data from multiple samples across four loading cases, enabling comprehensive feature extraction. In contrast, the EKF uses only a single sample; and (2) EKF requires more hyperparameters than the proposed PENN, such as the covariance matrices for the observation and process noises. An inappropriate hyperparameter could lead to errors in estimation and difficulty in convergence. In addition, the EKF requires a total of 17 minutes and 8 seconds to reconstruct all the needed responses, while the well-trained PENN requires only 4.11 seconds. Although training a PENN requires additional time, the PENN is still efficient over a long period.

7.5 Damage Identification

The proposed approach is further used for damage identification in the experiment. The associated condition is denoted as Da1. The damage was introduced by cutting a notch at the middle of the first segment, as shown in Figure 7.12. The notch is 70 mm long and 23.6 mm wide. The damage extent is 47.24%, calculated by (cut width/ beam width) $\times$ 100%. The elastic rigidities in the central 70-mm regions of all eight segments are set as the structural parameters (θ_i) to be identified. The elastic rigidities of the remaining parts in the FE model are fixed at their ground truth and kept untrainable. The range of the normalized structural parameters is [0.45, 1]. The strain data at S1 and S3, and acceleration data at A2, A3, A4, A6, and A8 are used to train the PENN. The enforced displacement loading scenario is adopted for damage identification.

The damage degree is defined as $(1-\theta_i) \times 100\%$. The damage identification results are shown in Figure 7.20, where the damage location and degree are accurately identified. The trained PENN is then used to reconstruct the full-field responses of a new test sample. The RRMSEs of the reconstructed responses are summarized in Table 7.5. The full-field strain, displacement, and acceleration responses are precisely reconstructed given the damaged beam model. The overall accuracy in Da1 is lower than that in Da0 because the identification task is more ill-conditioned in Da1 with more stiffness parameters to be identified.

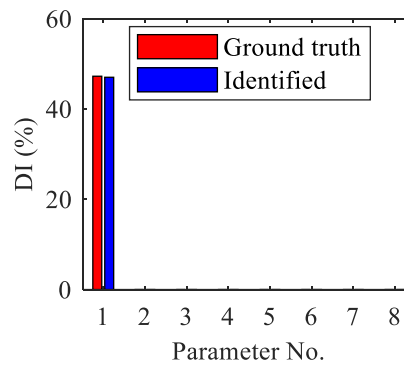


Figure 7.20 Damage identification results in Da1

7.6 Summary

This chapter proposes a PENN for joint load–parameter identification. An FEMU module for parameter identification and an RAE for load identification are integrated within the Newmark-beta method framework. The gradient information for optimization is automatically computed through backpropagation on the deep learning platform. The entire method is trained in a self-supervised manner using output-only response data. The discrepancy between the measured and predicted responses is used to build the loss function, which is then minimized to indirectly train both the structural parameters and the load RAE.

The proposed method is applied to a numerical two-span beam and a laboratory cantilever beam. It accurately identifies both the structural parameters and loads, and reconstructs the full-field responses. The method exhibits robustness under varying noise levels, sensor types, and loading conditions. Compared with purely physics-based or data-driven approaches, the PENN framework offers improved accuracy, computational efficiency, and physical interpretability.

CHAPTER 8

BLIND NONLINEAR BOUNDARY CONDITION IDENTIFICATION USING FE–NN HYBRID MODELING

8.1 Introduction

The preceding chapters primarily focus on parameter identification problems. However, critical bridge components, such as bearings, often exhibit significant changes in their mechanical behavior during the operational phase, due to material degradation (Gu et al., 2010), object blockage (Teng et al., 2021), and extreme loads. These changes often introduce unknown nonlinearities into components, which makes traditional linear parameter identification algorithms unsuitable. Additionally, collecting restoring force–response datasets of these components during the operational phase is difficult, making the direct identification of their behaviors impractical. To address these issues, this study proposes an FE–NN hybrid approach for NBC identification. The main contributions of this chapter are: (1) NNs representing the unknown NBCs are embedded into the FE model, resulting in a hybrid FE–NN model, as illustrated in Figure 8.1; (2) the SCD method is employed as the network architecture; and (3) the hybrid model is trained in a self-supervised manner using structural input loads and measured responses.

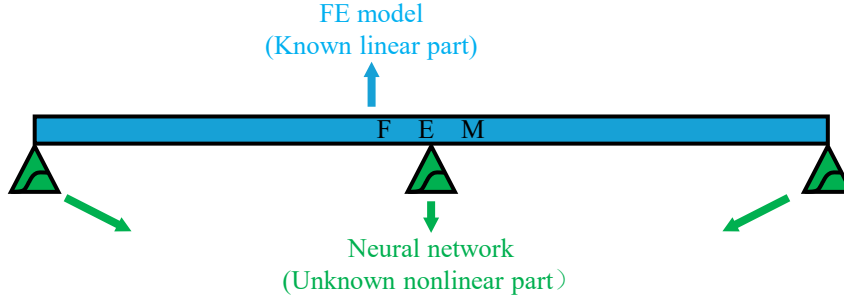


Figure 8.1 Schematic diagram of FE–NN hybrid modeling

8.2 Methodology

8.2.1 Stabilized Central Difference Method

The governing equation for a nonlinear dynamic system describes the relationships between the structural property, states, and external forces, and it is expressed as

$$\mathbf{M}\mathbf{a}(t) + \mathbf{C}\mathbf{v}(t) + \mathbf{F}^r(\mathbf{v}(t), \mathbf{d}(t)) = \mathbf{f}(t), \quad (8.1)$$

where $\mathbf{d}$, $\mathbf{v}$, and $\mathbf{a}$ denote vectors of nodal displacement, velocity, and acceleration, respectively; $\mathbf{f}$ represents the external load matrix; and $\mathbf{F}^r$ denotes the structural restoring force, which is associated with the structural state variables. In elastic linear systems, $\mathbf{F}^r(\mathbf{v}(t), \mathbf{d}(t)) = \mathbf{K}\mathbf{d}(t)$, where $\mathbf{K}$ is the stiffness matrix.

Time-domain analysis methods are commonly used in nonlinear analysis to solve Eq. (8.1), and they are generally classified into explicit and implicit methods. Implicit methods, such as Newmark-beta and Runge–Kutta methods, typically require iterative procedures (often employing the Newton–Raphson algorithm) at each time step to estimate the structural state at the next step. These iterations can be computationally expensive and may suffer from convergence issues. In contrast, explicit methods, such as the central difference method, compute the state variables at the next time step directly from those at the current step, thereby avoiding iterative calculations. However, most explicit methods are conditionally stable because the time step is highly sensitive to the

longest period of the analyzed structure.

To reduce computational cost and ensure numerical stability, an explicit method, the SCD method (Soares et al., 2018), as detailed in Table 8.1, is adopted for the dynamic response simulation of systems with NBCs. The SCD method incorporates a selective mass scaling technique, which introduces artificial mass to the system to suppress inaccurate high-frequency components while preserving the accuracy of low-frequency responses (Olovsson et al., 2005).

Table 8.1 SCD method for dynamic systems with nonlinear boundaries

1. Calculation for the initial time step
1.1. $\mathbf{F}_0^r = \mathbf{K}\mathbf{u}_0 + \mathbf{F}^s(\mathbf{x}_0^s)$.
1.2. $\mathbf{K}_0^T = \mathbf{K} + \mathbf{K}^s(\mathbf{x}_0^s)$.
1.3. $\hat{\mathbf{M}}_0 = (\mathbf{M} + \frac{1}{2}\Delta t\mathbf{C} + \alpha_0\mathbf{K}_0^T\Delta t^2)$, $\alpha_0 = \frac{1}{4}\tanh(\frac{1}{4}\omega_0^m\Delta t)$.
1.4. $\mathbf{a}_0 = (\frac{1}{2}\Delta t\mathbf{C} + \hat{\mathbf{M}}_0)^{-1}(\mathbf{f}_0 - \mathbf{C}\mathbf{v}_0 - \mathbf{F}_0^r)$, $\mathbf{d}_{-1} = \mathbf{d}_0 - \mathbf{v}_0\Delta t + \frac{(\Delta t)^2}{2}\mathbf{a}_0$.
2. Calculation for each time step
2.1. For $i = 0$ to $N_i - 1$, do:
2.2. $\mathbf{F}_i^r = \mathbf{K}\mathbf{u}_i + \mathbf{F}^s(\mathbf{x}_i^s)$, $\mathbf{K}_i^T = \mathbf{K} + \mathbf{K}^s(\mathbf{x}_i^s)$.
2.3. $\hat{\mathbf{M}}_i = (\mathbf{M} + \frac{1}{2}\Delta t\mathbf{C} + \alpha_i\mathbf{K}_i^T\Delta t^2)$, $\alpha_i = \frac{1}{4}\tanh(\frac{1}{4}\omega_i^m\Delta t)$.
2.4. $\mathbf{d}_{i+1} = 2\mathbf{d}_i - \mathbf{d}_{i-1} + (\frac{1}{2}\Delta t\mathbf{C} + \hat{\mathbf{M}}_i)^{-1}[\Delta t^2(\mathbf{f}_i - \mathbf{F}_i^r) - \Delta t\mathbf{C}(\mathbf{d}_i - \mathbf{d}_{i-1})]$,
$\mathbf{v}_i = \frac{\mathbf{d}_{i+1} - \mathbf{d}_{i-1}}{2\Delta t}$, $\mathbf{a}_i = \frac{\mathbf{d}_{i+1} - 2\mathbf{d}_i + \mathbf{d}_{i-1}}{(\Delta t)^2}$.
2.5 End for.
3. Return $\mathbf{d}$, $\mathbf{v}$, and $\mathbf{a}$.

In the SCD algorithm, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ denote the mass, damping, and stiffness matrices of

the known linear part of the structure, respectively; $\mathbf{K}^T$ represents the structural tangent stiffness matrix; $\hat{\mathbf{M}}$ is the revised mass matrix; Δt denotes the time step; $\mathbf{x}^s$ is the state vector (including displacement and velocity vectors) of the NBC; and ω_i^m is the maximum natural frequency at the i -th step, which is obtained through the modal analysis. $\mathbf{K}^s$ and $\mathbf{F}^s$ are the stiffness and restoring force matrices of NBCs, respectively. They are defined as $\mathbf{K}^s(r, r) = k_r^s$ and $\mathbf{F}^s(r) = f_r^s$, respectively, with all other positions being zero. r represents the DOF index of the NBC, and k_r^s and f_r^s are the stiffness parameter and restoring force of the r -th NBC, respectively. A partial derivative relationship exists between k_r^s and f_r^s , expressed as $k_{i,r}^s = \partial f_{i,r}^s / \partial d_{i,r}^s$, where d^s denotes the displacement of the NBC. Meanwhile, it is assumed that the structure is initially in its equilibrium position, indicating that the initial displacement and velocity of the system are both set to zero.

When the mechanical model of NBCs is unknown, the structural dynamic responses cannot be directly solved. Therefore, NNs are employed in this chapter to approximate the mechanical behaviors of the NBCs (i.e., the displacement–restoring force relationships), thereby enabling the forward dynamic simulations. However, the initially established NNs cannot accurately replicate the behavior of the NBCs and need to be optimized using training data. The modeling and training process will be detailed in the following content.

8.2.2 Multilayer Perceptron

The MLP is adopted in this study to approximate the NBC. The MLP is a feedforward NN that comprises multiple layers, as illustrated in Figure 8.2 (a). Each layer consists of multiple neurons connected to adjacent layers through weights, bias, and activation functions, as shown in Figure 8.2(b). The inclusion of nonlinear activation functions enables the MLP to effectively learn and model nonlinear behaviors, while the high-dimensional weights and bias allow the MLP to accurately fit complex models and

datasets.

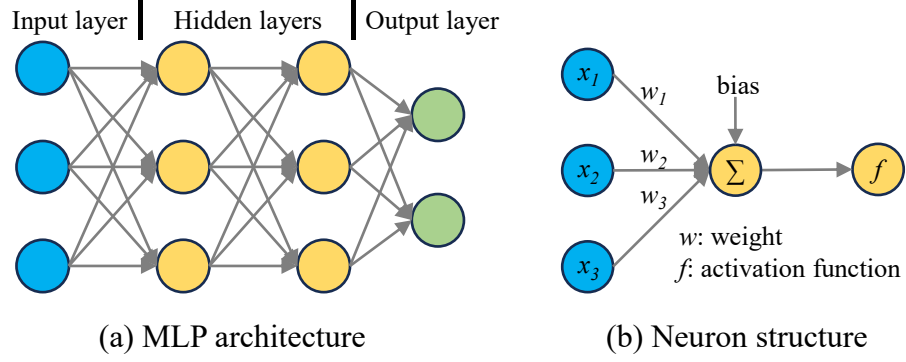


Figure 8.2 Schematic diagram of MLP

MLPs for simulating elastic and hysteretic NBCs are presented in Figures 8.3(a) and 8.3(b), respectively. For elastic components, the restoring force depends solely on the current displacement d_i^s . Therefore, the MLP designed for elastic NBCs takes only the current displacement d_i^s as input, as shown in Figure 8.3(a). In contrast, hysteretic behaviors are path-dependent. Consequently, the boundary displacements from the last q steps and the boundary restoring force from the last step are taken as the input of the MLP for hysteretic models, as shown in Figure 8.3(b). The output of both MLP architectures is the restoring force of the NBC at the current time step, denoted as f_i^s . Although velocity and acceleration responses of the NBCs are not included in the MLP input, they can be learned from the MLP from displacement data because they are first-order and second-order derivatives of displacement with respect to time, respectively.

Due to the limited understanding of the time-dependent characteristics of hysteretic models, the q value is unknown in advance. To balance computational efficiency and identification accuracy, it is recommended to begin model training with a small q value (e.g., $q = 2$). If the loss function does not reach the expected value, the q value can be gradually increased until the training results meet expectations.

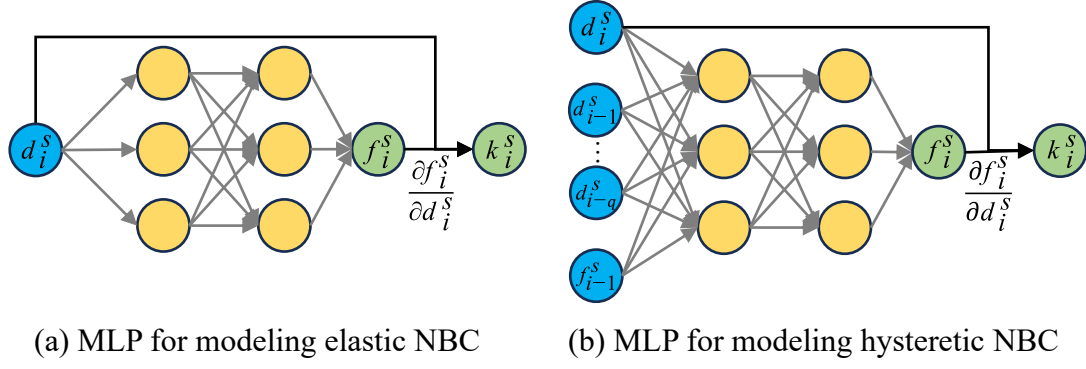


Figure 8.3 MLPs representing the NBCs

Subsequently, the stiffness parameters of the NBCs are obtained by calculating the partial derivatives of the output restoring force with respect to the input boundary displacement through the automatic differentiation technique. However, the initially established MLP cannot accurately represent NBCs and needs to be trained to approximate the true mechanical model.

8.2.3 NBC Identification Based on FE–NN Hybrid Modeling

The general framework of the proposed NBC identification method is illustrated in Figure 8.4. In this framework, the MLPs are employed to simulate the mechanical behavior of the unknown NBCs, while the FE method is used to model the linear structural components. The mechanical matrices from both parts are then integrated to form the complete hybrid model. The SCD stepper module shown in Figure 8.4 corresponds to steps 2.1–2.5 in the SCD algorithm. The discrepancies between the predicted responses $\mathbf{Y}$ and the measured responses $\tilde{\mathbf{Y}}$ are then used to build the loss function, which is minimized to indirectly train the MLPs to approximate the true mechanical behavior of the NBCs. The proposed method does not require direct measurements of the displacement and restoring force of the NBCs during training. This makes it particularly suitable for operational scenarios in which the restoring force of NBCs is difficult or impossible to measure directly.

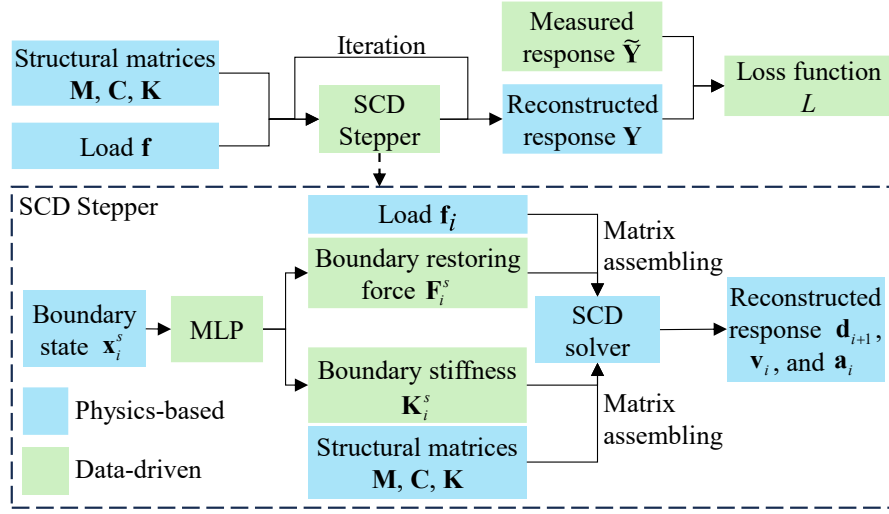


Figure 8.4 Framework of the NBC identification method based on the FE–NN hybrid model

The normalized mean square error (MSE) between the predicted and measured responses is used to build the loss function, which is expressed as

$$L = \frac{1}{N_m} \sum_{m=1}^{N_m} \frac{\sum_{i=1}^{N_i} (\mathbf{Y}_{m,i} - \tilde{\mathbf{Y}}_{m,i})^2}{\text{var}(\tilde{\mathbf{Y}}_m)}, \quad (8.2)$$

where N_m represents the total number of measurements, m is the index of the response channel, N_i represents the total number of time steps, i denotes the time step, var is the variance calculator, and $\mathbf{Y}$ and $\tilde{\mathbf{Y}} \in \mathbb{R}^{Nm \times Ni}$ are predicted and measured responses, respectively.

8.3 Case I: A Three-DOF Spring-mass System with a Nonlinear Spring

In this section, a nonlinear spring-mass system with three DOFs, as depicted in Figure 8.5, is used to verify the proposed method. The three masses are $m_1 = m_2 = m_3 = 1 \times 10^6$ kg, and they are connected by two linear springs. m_1 is connected to the ground by a linear spring and an elastic nonlinear spring. The stiffness parameters of the three linear springs are $k_1 = k_2 = k_3 = 4 \times 10^5$ kN/m. Three types of elastic nonlinear springs, namely, softening,

hardening, and anisotropic, are used in this case. Their corresponding restoring force–response curves are shown in Figures 8.6(a)–8.6(c), respectively. The primary case utilizes the softening spring for illustration, while the hardening and anisotropic springs will be employed for comparative analysis. The elastic softening boundary condition shown in Figure 8.6(a) is frequently observed in vibration isolators. The overall softening characteristic with quasi-zero stiffness behavior far from the static equilibrium point can enhance the low-frequency vibration isolation performance (Anastasio et al., 2024).

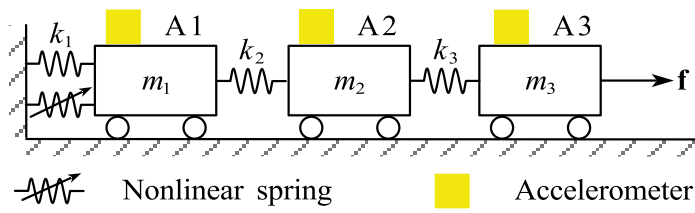


Figure 8.5 Three-DOF spring-mass system with a nonlinear spring

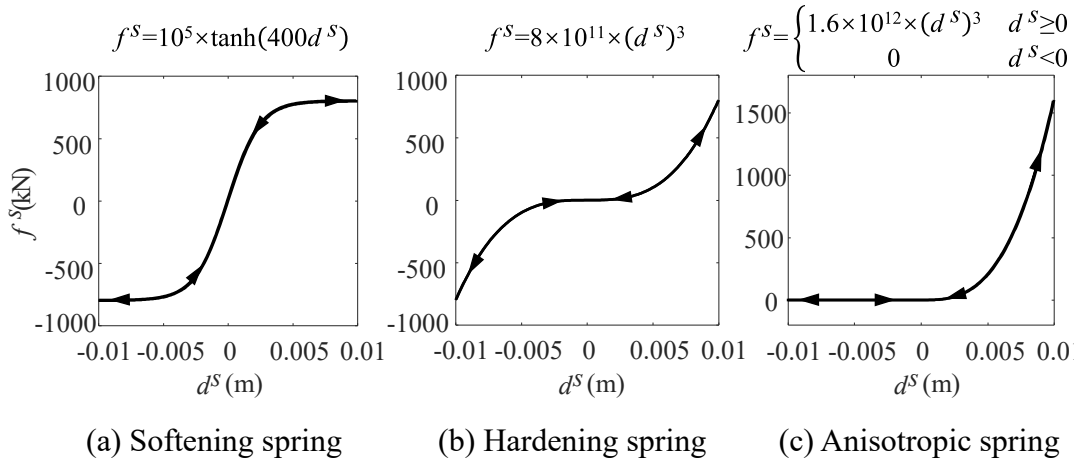


Figure 8.6 Elastic nonlinear springs used in Case I

In the baseline scenario, the natural frequencies of the spring-mass system are 1.417, 3.969, and 5.736 Hz. Rayleigh damping is assumed for the whole structure, and the first two damping ratios are both 2%. A white noise load is applied to m_3 to excite the dynamic responses. In this case, three accelerometers are installed at a sampling frequency of 100 Hz, corresponding to a 0.01-second time step in the FE analysis. The acceleration and

input force measurements are used to identify the nonlinear model.

The CC and the mean relative error (MRE) are used to measure the shape correlation and deviation of predictions and measurements:

$$CC = \frac{\text{cov}(\mathbf{y}, \hat{\mathbf{y}})}{\sqrt{\text{var}(\mathbf{y})\text{var}(\hat{\mathbf{y}})}}, \quad (8.3)$$

$$\text{MRE} = \frac{\text{avg}(|\mathbf{y} - \hat{\mathbf{y}}|)}{\max(\hat{\mathbf{y}})}, \quad (8.4)$$

where $\mathbf{y}$ and $\hat{\mathbf{y}}$ are prediction and ground truth vectors, respectively, which can represent either the structural responses or boundary conditions; $\text{cov}(\cdot)$ represents the covariance of two vectors; $\text{avg}(\cdot)$ represents the mean value; and $\max(\cdot)$ represents the maximum. In practice, the two vectors are regarded as highly correlated when $CC > 0.8$, and the prediction accuracy is considered acceptable when $\text{MRE} < 5\%$.

8.3.1 Baseline Scenario

Measurement noise is not considered in the baseline scenario. An MLP with four hidden layers is used to simulate the nonlinear spring, as shown in Table 8.2. The spring is elastic, and the input number n is therefore set as 1. The hybrid model and algorithm are implemented on the PyTorch platform. A total of 1,024 numerical samples are collected in the training dataset. Each sample segment is 512 long, namely, its duration is 5.12 s (512×0.01 s). The training–validation ratio is 8:2. The hybrid model is trained using the Adam optimizer with a batch size of 128 and a learning rate of 1×10^{-3} . The computer configuration includes an Intel Core i7-13700 CPU, 64 GB RAM, and an NVIDIA 4070 12 GB graphics card. The training time is 7.4 s per epoch in this case.

Table 8.2 Architecture of the MLP

Layer number	Layer type	Size	Activation function
1	Input	n (input number)	-
2	Linear	500	Tanh
3	Linear	500	Tanh
4	Linear	500	Tanh
5	Linear	500	Tanh
6	Output	1	-

The entire training process consists of 1,000 epochs and is illustrated in Figure 8.7. Initially, the training and validation losses decrease sharply. After around 400 epochs, they exhibit significant oscillations. The training and validation loss curves closely align, suggesting no overfitting or underfitting during the training process.

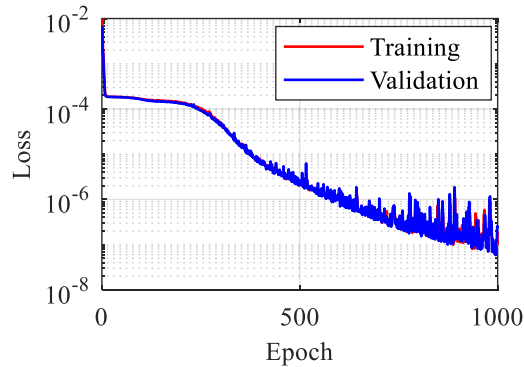


Figure 8.7 Training and validation losses

The trained MLP with the smallest validation loss is selected as the final surrogate model. The learned and real boundary conditions are presented in Figure 8.8. When the displacement is within training range, the predicted nonlinearity is close to the ground truth. CC within the training data range is close to 1, and the MRE is 1.449%, indicating a high accuracy of the identified nonlinear boundary condition. However, when the measured displacement exceeds the training range, the MLP-predicted restoring force becomes less accurate. This phenomenon can be attributed to the inherent limitations of

data-driven models, which are designed to learn only the behaviors represented within the training datasets. In such cases, new nonlinear data can be used to further fine-tune the pre-trained model. The trained model can thus adapt to previously unseen nonlinear behaviors, thereby improving its predictive capability for extreme events.

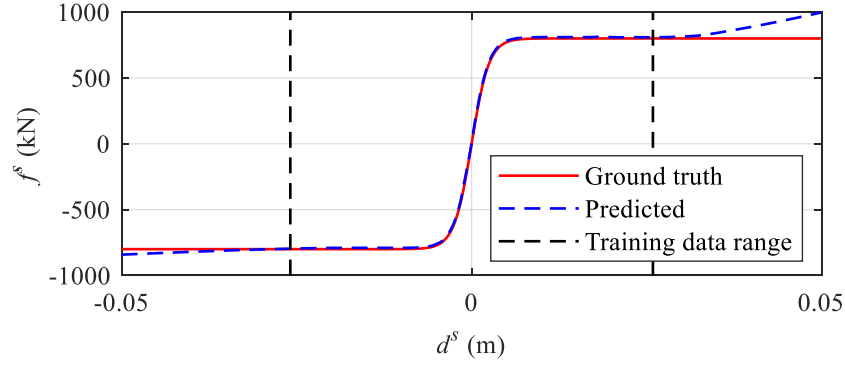


Figure 8.8 Learned NBC using measurements without measurement noise

To verify the performance of the trained MLP in predicting structural dynamic responses, the histograms of MREs between predicted and measured structural responses in the validation sets, including the displacement, velocity, and acceleration of all masses and the restoring force of the nonlinear spring, are shown in Figure 8.9. All the MREs are smaller than 5%. Moreover, the average CC and MRE listed in Table 8.3 are close to 1 and smaller than 5%, respectively. These results indicate that the reconstructed responses are very accurate because the nonlinear boundary condition is well learned by the MLP.

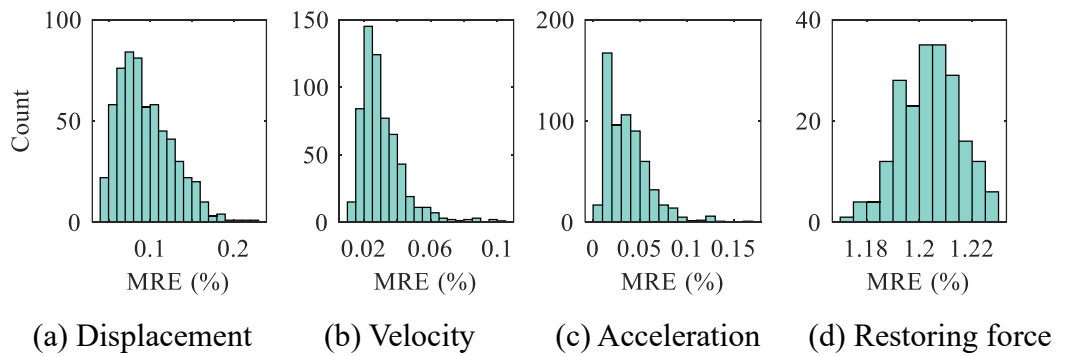
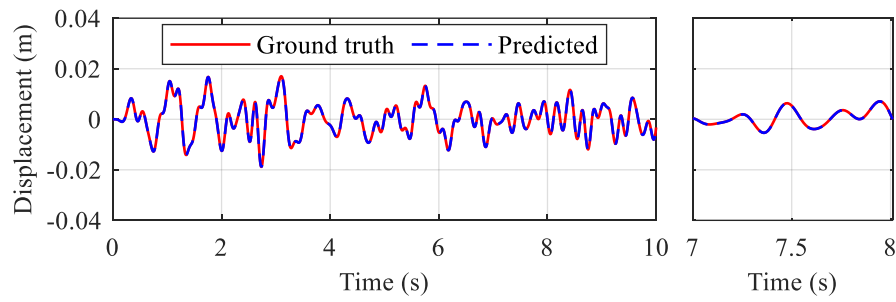


Figure 8.9 MRE histogram in the baseline scenario

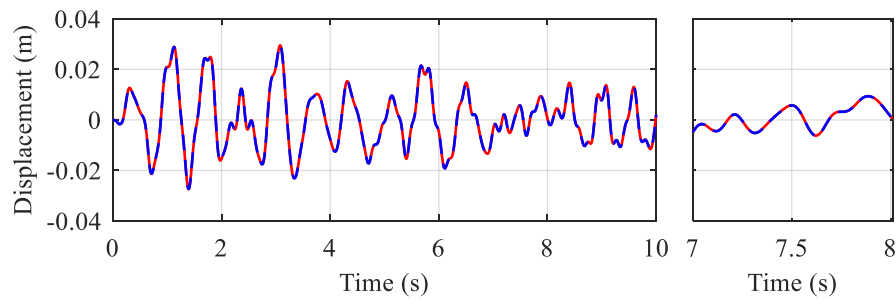
Table 8.3 Average response evaluation indicators in the baseline scenario

Response	CC	MRE (%)
Displacement	1.00	0.10
Velocity	1.00	0.03
Acceleration	1.00	0.04
Restoring force	1.00	1.20

The trained MLP model is also used to generate the full-field structural responses of a new test sample that was not included in the training set. The test sample lasts for 10 s. The predicted and ground truth displacements of all three masses are illustrated in Figure 8.10, where they closely align with each other in overall shape and peak values. The results indicate a high generalization ability of the trained MLP.



(a) m_1



(b) m_2

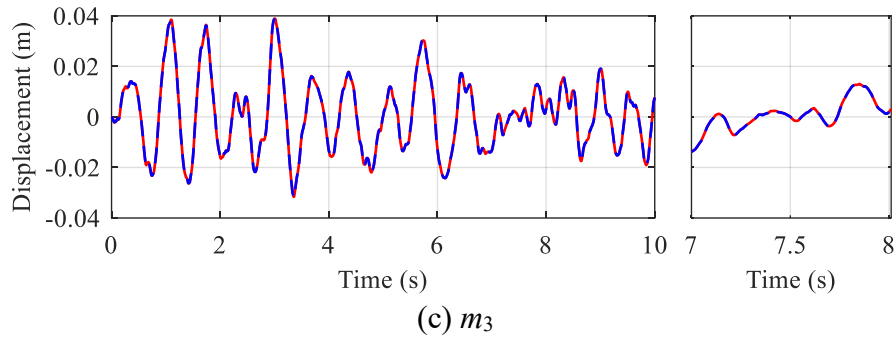


Figure 8.10 Predicted displacements of a test sample in the baseline scenario

8.3.2 Influence of Measurement Noise

The robustness of the proposed method against measurement noise is evaluated in this subsection. Four noise ratios, 5%, 10%, 20%, and 30%, are studied. The noisy measurements are generated using Eq. (6.34). The training settings are consistent with those in the baseline scenario.

The calculated CC and MRE at different noise levels are presented in Table 8.4. All CCs are close to 1, and MREs are less than 5%, indicating that the predicted boundary conditions are accurate in shape and time history. With the increase of noise ratio, the MRE also increases but remains within 5%, and the highest MRE is 2.085%, suggesting that the influence of Gaussian noise is acceptable. These results all indicate that the proposed method exhibits strong robustness to noise.

Table 8.4 Average response evaluation indicators under different NRs

Scenario	NR (%)	CC	MRE (%)
Baseline	0	1.00	1.45
N1	5	1.00	1.52
N2	10	1.00	1.85
N3	20	1.00	1.96
N4	30	1.00	2.09

The nonlinear boundary condition identified from the 30% noise-contaminated dataset is used to predict responses in a new test set. Figure 8.11 shows the predicted, measured, and real displacements. The predicted response time history aligns with the ground truth, although the noise of the measurements is significant. This suggests that the nonlinear boundary condition identified with noisy measurements is sufficiently accurate to predict the real responses.

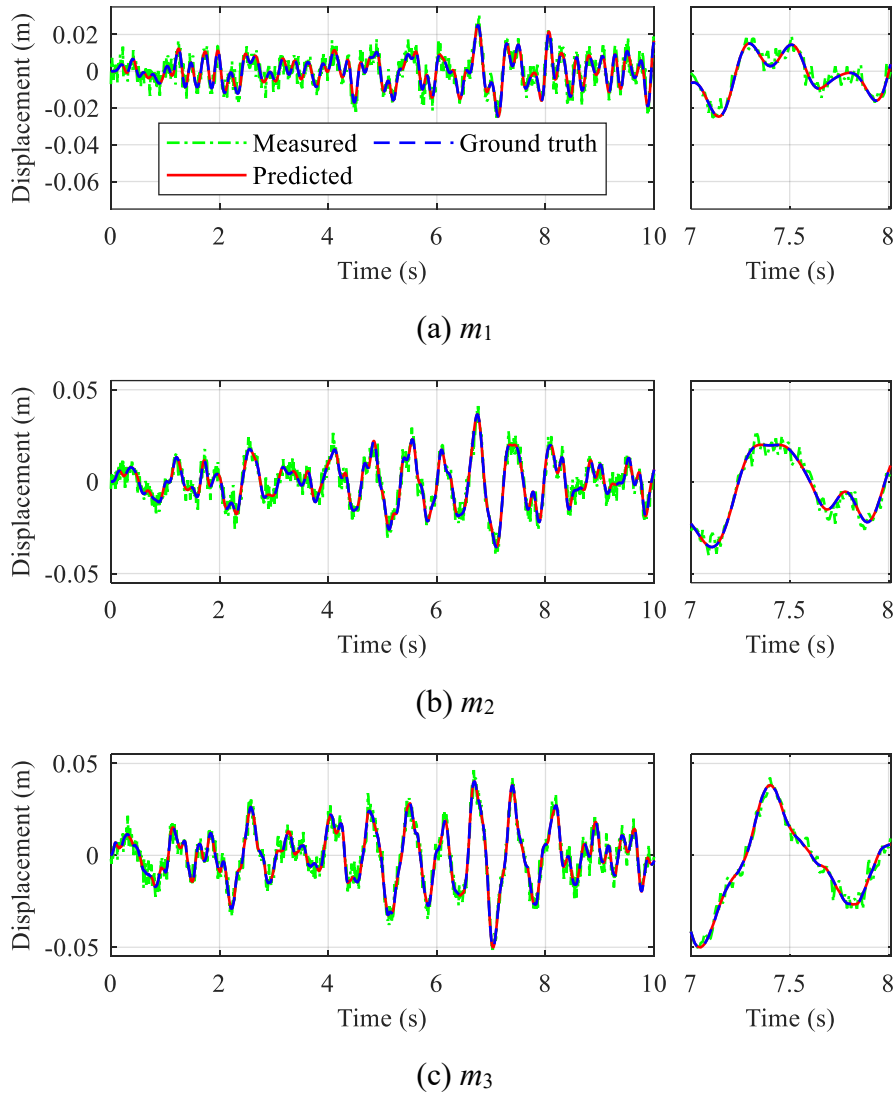


Figure 8.11 Predicted displacements of a test sample in N4

8.3.3 Influence of Sensor Placement

In this subsection, different accelerometer arrangements are used to investigate the influence of sensor placement. All possible sensor placement schemes for the three-DOF spring-mass system are considered, as shown in Table 8.5. In this subsection, the noise ratio NR is set as 5%.

Table 8.5 Average response evaluation indicators in different sensor placement schemes

Scenario	Sensor placement	CC	MRE (%)
P1	A1	1.00	2.42
P2	A2	1.00	2.80
P3	A3	1.00	4.85
P4	A1, A2	1.00	1.67
P5	A1, A3	1.00	2.60
P6	A2, A3	1.00	1.81
Baseline	A1, A2, A3	1.00	1.52

The comparison of indicators CC and MRE is presented in Table 8.5. For all sensor placement schemes, CCs are close to 1, and the MRE is consistently less than 5%, indicating a high accuracy of the learned nonlinear models. The calculated MRE decreases with the increase in sensor number. This suggests that the MLP can better learn the boundary condition with more optimization targets (sensor-measured data) being satisfied. Additionally, given only one accelerometer, the estimation MRE is the lowest when the data of A1 is used. This is because the nonlinear spring is installed between the ground and m_1 , and the response measured from A1 can provide more information about the nonlinearity model. The sensors should therefore be placed near the nonlinear component of the structure when arranging the sensor configuration. Notably, the nonlinear boundary condition can be accurately identified even using A1 or A2.

8.3.4 Influence of Nonlinear Types

Two additional types of nonlinear springs are investigated in this section: the hardening and anisotropic springs. Their restoring force–response curves are shown in Figures 8.6(b) and 8.6(c). The elastic hardening boundary is simulated by using a Duffing spring that can describe the force–displacement relationships of disc springs (Miyamoto et al., 2023) and the elastic suspension spring pairs (Ibrahim, 2008) in vibration isolation systems. The elastic anisotropic boundary condition is used to model tension-only boundary conditions, such as cable connections in cable-restrained high-damping rubber bearings (Fang et al., 2022), as well as compression-only boundary conditions, such as elastic foundations (Pellicano et al., 1997).

The identified boundary conditions are presented in Figures 8.12(a) and 8.12(b). The identified models are consistent with their ground truth. The accuracy indicators of the two identified models are: CCs close to 1, and MREs of 0.66% and 0.37%, respectively. The MRE of the anisotropic model is the smallest because the left part of the boundary condition is zero, similar to the initial state of the MLP where the weight and bias are all set as zero, which will be seen again in Case III. Consequently, the training difficulty would decrease sharply in the anisotropic spring case, leading to a more accurate identified result.

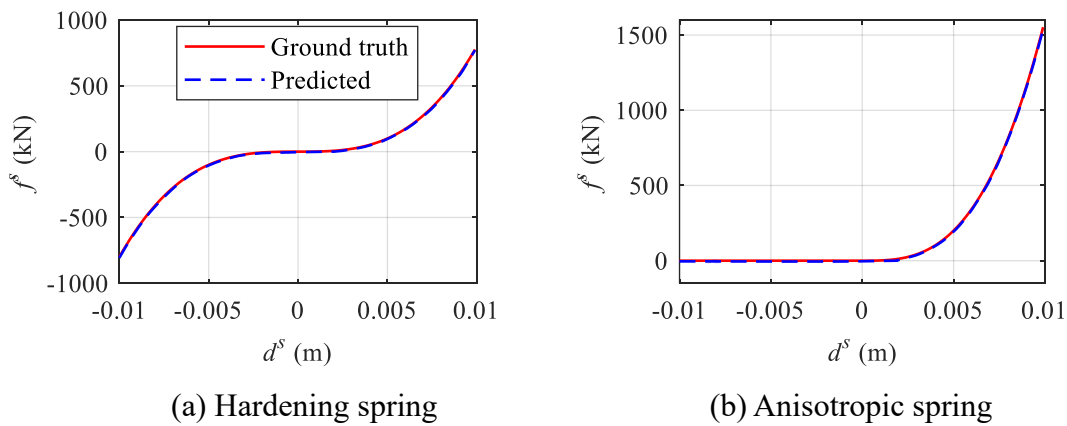


Figure 8.12 Predicted boundary conditions of the hardening and anisotropic springs

8.4 Case II: A Three-DOF Spring-mass System with a Hysteretic Spring

The hysteretic boundary condition is further investigated in this section. The same three-DOF mass-spring system in Case I is used again. Here, the elastic spring connecting m_1 and the ground is replaced by a bilinear hysteretic spring. The hysteretic curve is depicted in Figure 8.13. The bilinear hysteretic model (Katsaras et al., 2008) is defined by three parameters: initial stiffness k_0 , yielding displacement d_y^s , and stiffness reduction factor ς . In this case, k_0 is 4×10^5 kN/m, d_y^s is 5 mm, and ς is 0.1. The initial stiffness is adopted when unloading. The bilinear boundary condition is commonly used to describe the hysteretic behaviors of lead rubber bearings and laminated rubber bearings in earthquake engineering (Zordan et al., 2014).

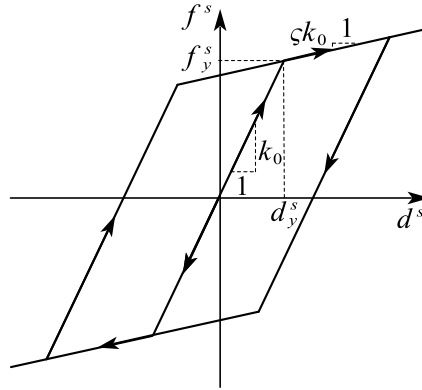


Figure 8.13 Hysteretic curve of the bilinear model

The MLP architecture exhibited in Figure 8.3(b) is utilized to identify the hysteretic boundary condition. The MLP input should include the historical restoring force and displacement because the hysteretic model is path-dependent. The restoring force of a bilinear model theoretically relates merely to the restoring force, velocity, and displacement at the previous time step, and the hyperparameter q should therefore be 1. However, such prior knowledge is unknown when learning the nonlinear model, and q should therefore be set as a relatively large number. In Case II, q is taken as 2 and n is

taken as 4, which includes the restoring force from the previous step and the displacements from the last three steps, to yield satisfactory results. The measurement noise ratio adopts 5%, and the remaining hyperparameters for training are identical to those in the baseline scenario.

The predicted and real hysteretic curves are displayed in Figure 8.14 for a new test set. The predicted hysteretic curve exhibits the same variational pattern, including loading, yielding, and unloading behaviors, as the ground truth. This indicates that the MLP has well learned the relationship between the loading history and the restoring force through its powerful nonlinear modeling capability. The observed discrepancy at sharp corners in the hysteretic curve arises because the MLP simulates the derivative through the backpropagation algorithm, as introduced in subsection 6.2.2, which yields only finite derivatives. However, the derivative of the restoring force with respect to displacement in the bilinear model at sharp corners is theoretically infinite. Consequently, it is challenging for the MLP to accurately fit the sharp turning points. The applicability of the MLP to the non-smooth nonlinearities can be improved by adding the training dataset, reducing the training noise level, increasing the network depth, and adjusting the activation functions, which may cause a higher computational burden.

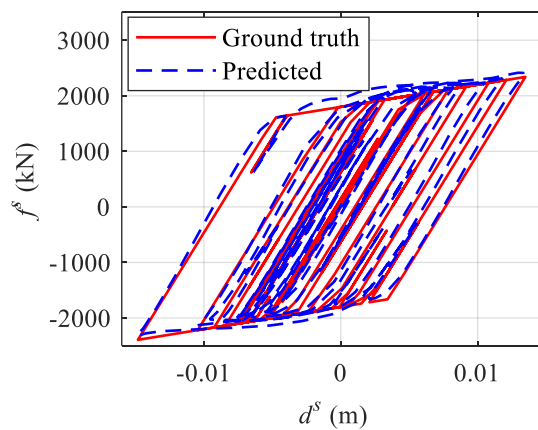


Figure 8.14 Predicted hysteretic curve of a test sample

The MREs of different types of response time histories in the validation set are displayed

in Figure 8.15. Nearly all MREs are smaller than 5%, indicating that the errors of reconstructed responses and restoring forces are acceptable. The mean values of CC and MRE in the validation set are also reported in Table 8.6, suggesting that the predicted responses are accurate.

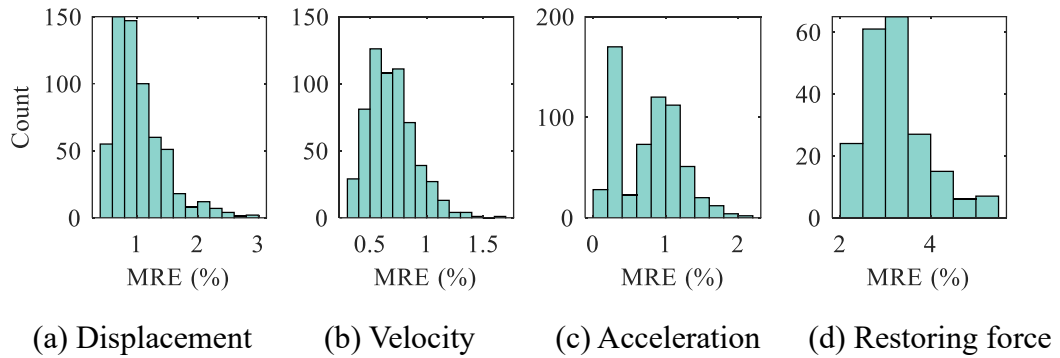


Figure 8.15 MRE histograms of responses in Case II

Table 8.6 Average response evaluation indicators in Case II

Response	CC	MRE (%)
Displacement	1.00	1.03
Velocity	1.00	0.69
Acceleration	1.00	0.77
Restoring force	1.00	3.23

The trained MLP, representing the hysteretic boundary condition, is then used to predict structural responses. The predicted displacement time histories of the test set are displayed in Figure 8.16, which fit well with the measured ones. This indicates that the hybrid model, combining the FE model and the well-trained MLP, can accurately simulate the responses of the considered hysteretic system.

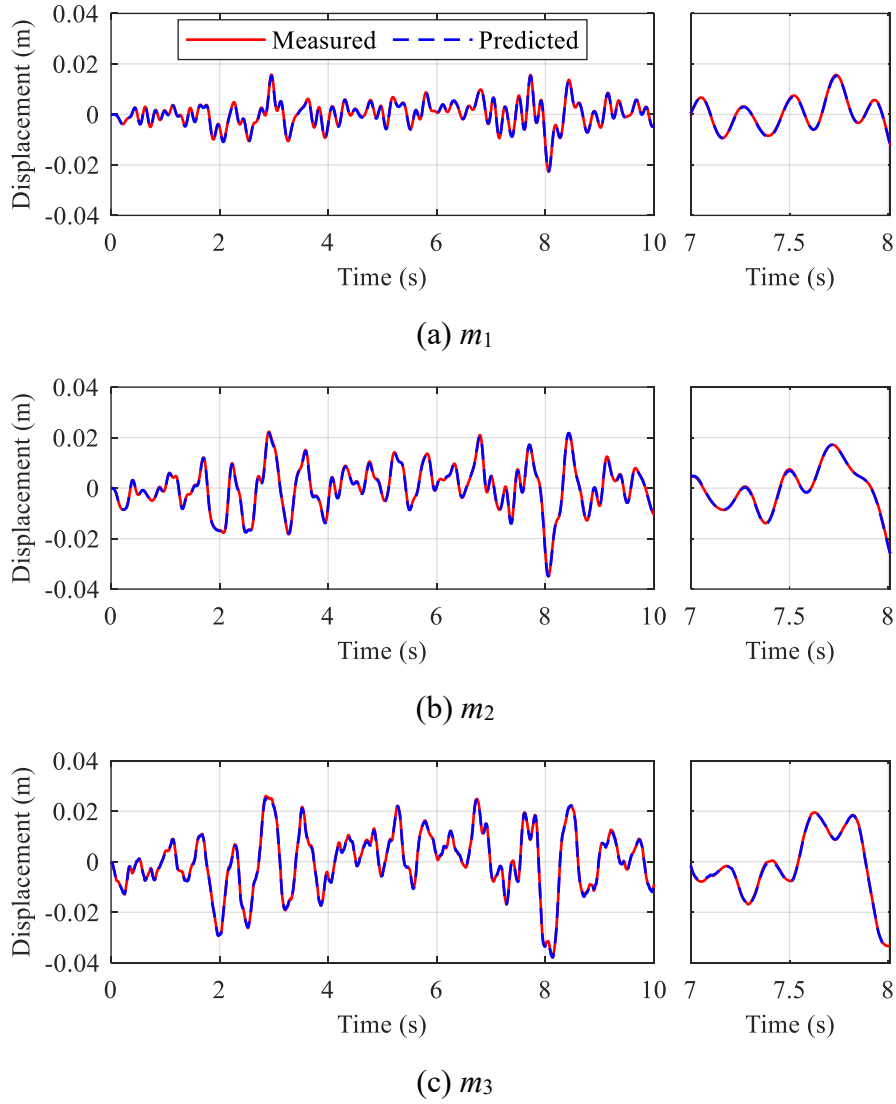


Figure 8.16 Predicted displacement of a test sample

8.5 Case III: A Beam Model with Multiple Elastic Nonlinear Springs

A beam model is used to verify the proposed method with nonlinearity at multiple positions, as shown in Figure 8.17. The beam is divided into eight elements with only the in-plane deformation being considered, resulting in 16 DOFs. The bending stiffness is $1.477 \times 10^4 \text{ kN} \cdot \text{m}^2$, and the mass per unit length is $1.178 \times 10^3 \text{ kg/m}$. The first two natural frequencies of the beam model are 2.748 and 10.997 Hz, and the associated damping ratios are both 2%. Subsequently, the linear damping matrix can be established. An elastic rotational softening nonlinear spring is connected to the left beam end, and a vertical

hardening nonlinear spring is placed at the beam center. The functions between the restoring force and the displacement of the rotational and vertical springs are $f^s = 4 \times 10^4 \times \tanh(130 d^s)$ and $f^s = 8 \times 10^6 \times (d^s)^3$, respectively. The maximum frequency ω_i^m with zero displacement is 5,621.42 rad/s, and it is reduced by 24.25% when the selective mass scaling technique is applied. The white noise loads are applied to the nodes on the two spans. The accelerometers, inclinometers, and displacement transducers are used to measure structural responses at a sampling frequency of 1,000 Hz.

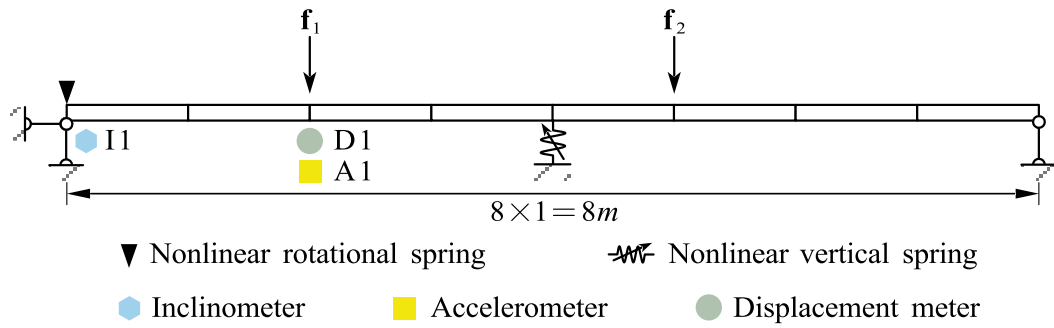


Figure 8.17 Schematic of the beam model in Case III

The FE–NN hybrid model is built using the mass, damping, and linear stiffness matrices and two MLPs associated with the two nonlinear springs. The model was trained for 5,000 epochs, and the other settings are consistent with those in Case I. The loss curve and the corresponding evolution process of the MLP representing the rotational nonlinear spring are displayed in Figure 8.18. Initially, the learned nonlinear model is a straight line with a constant value of zero, because the initial weights and biases of the MLP are set to 0. Then, the training and validation losses sharply drop, and after about 500 epochs, they oscillate below 3×10^{-4} . At around the 1,000th epoch, the learned boundary condition curve exhibits a similar shape to the ground truth but still has deviations. After 1,000 training epochs, the reduction in the training and validation losses is insignificant, whereas the predicted boundary condition curve continues to converge towards its ground truth. At this stage, the loss function, which represents the disparity between the reconstructed and measured responses, is primarily related to measurement noise and thus

cannot be further reduced. However, the MLP is still making efforts to identify the true boundary condition with the noisy data. Finally, after 5,000 epochs, the learned boundary conditions approach the true boundary conditions, as shown in the right-bottom subfigure in Figure 8.18.

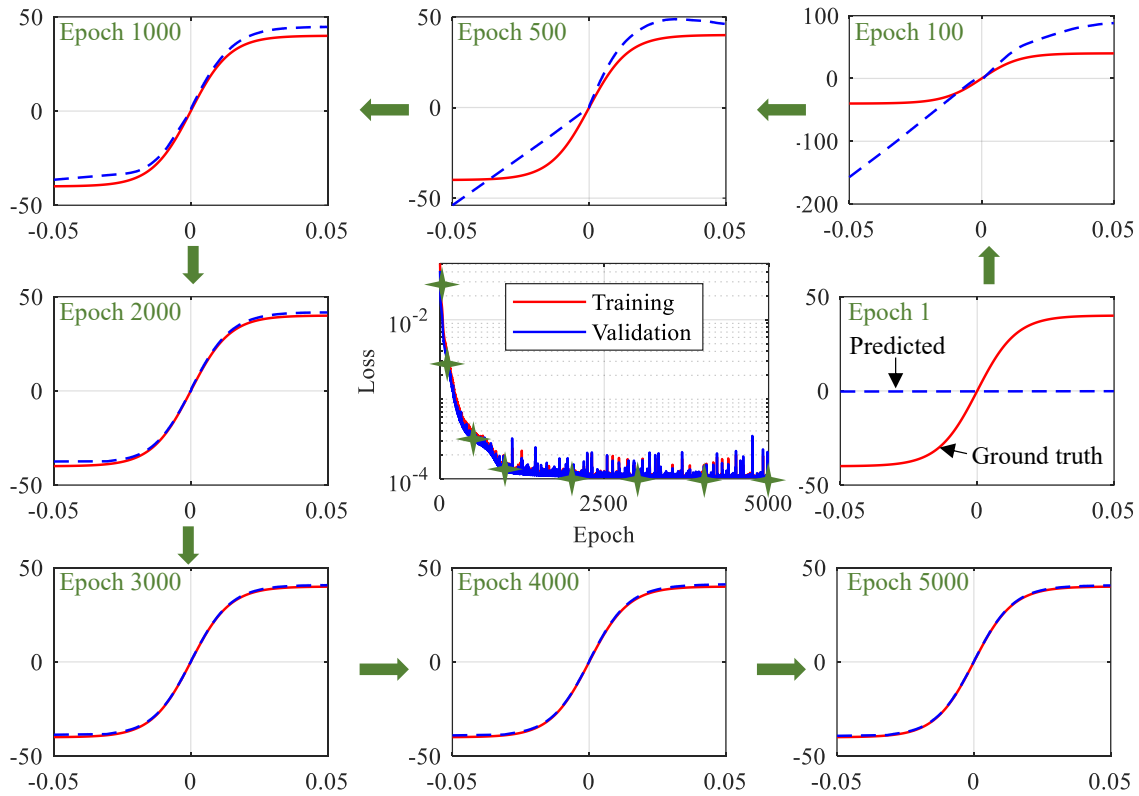


Figure 8.18 Evolution process of the MLP representing the rotational nonlinear spring

In the training process, the MLP learns the middle section (close to the origin of the horizontal coordinate) of the boundary condition more rapidly than other sections. This is because the structural responses to white noise excitation are normally distributed. As a result, the middle section in a boundary condition curve, which is close to zero, is more easily trained, benefiting from the concentration of training data. Correspondingly, the remaining sections of the boundary condition model are learned more slowly. Compared with Cases I and II, Case III requires more training iterations, indicating that with more DOFs and nonlinear models involved, the nonlinearity identification process would become challenging and time-consuming.

The learned boundary condition models are displayed in Figure 8.19, and the evaluation indicators of the two models are: CC close to 1, and MREs of 0.15% and 1.07%, respectively. The results demonstrate that the accuracy of the identified nonlinear springs is satisfactory, and the proposed approach can identify multiple nonlinear models simultaneously.

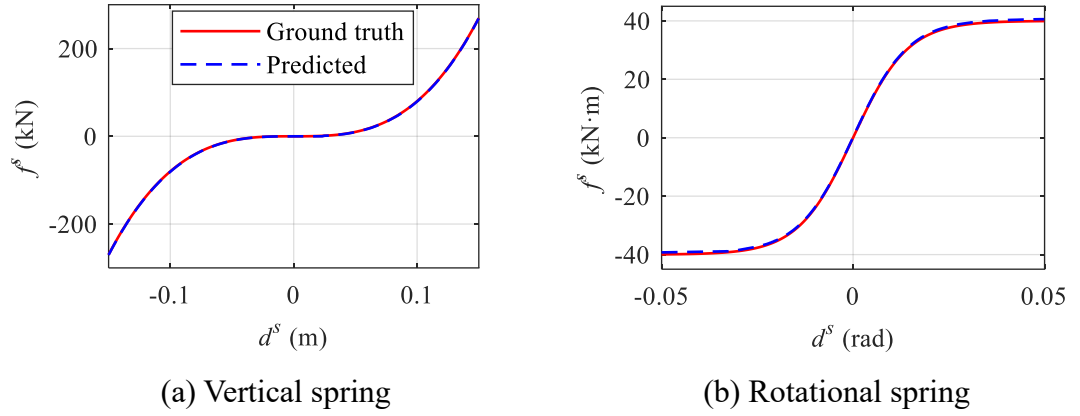


Figure 8.19 Predicted boundary conditions of two nonlinear springs in Case III

The MREs of the estimated response time histories in the validation dataset are exhibited in Figure 8.20, and they are smaller than 1% in most cases. The mean values of the CC and MRE are also presented in Table 8.7. These results collectively affirm the effectiveness of the hybrid model, incorporating the two well-trained MLPs, in accurately identifying the nonlinear structure and predicting its full-field responses.

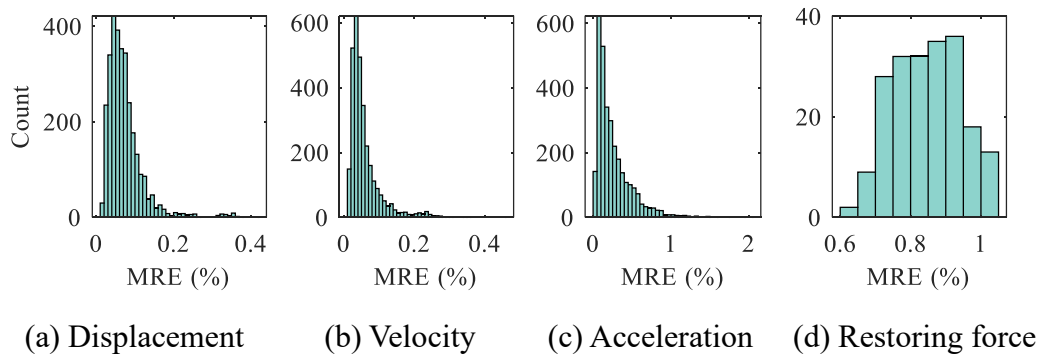


Figure 8.20 MRE histogram of responses in Case III

Table 8.7 Average response evaluation indicators in Case III

Response	CC	MRE (%)
Displacement	1.00	0.073
Velocity	1.00	0.059
Acceleration	1.00	0.26
Restoring force	1.00	0.85

8.6 Summary

This chapter proposes an FE–NN hybrid method for identifying NBCs. MLPs are employed to represent NBCs with unknown mechanical behavior, while the FE method is utilized to model the known linear components. These two components are integrated to construct the hybrid model. The SCD solver is then used to simulate the full-field dynamic responses. The difference between the predicted and measured responses is used to establish the loss function, which is then minimized to indirectly train MLPs to approximate the NBCs.

The proposed method is validated through a three-DOF spring-mass system and a two-span continuous beam. The method successfully identifies both elastic and bilinear hysteretic NBCs. The trained FE–NN hybrid model can accurately predict the full-field structural responses. The applicability of the proposed method is further evaluated under various noise ratios, sensor configurations, and nonlinear types.

CHAPTER 9

CONCLUSIONS AND FUTURE WORK

9.1 Conclusions

Bridge system identification methods integrating physics-based and data-driven models are developed in this study. First, a physics-based FEMU method fusing multi-source data is proposed for accurate parameter identification. Then, the hierarchical Bayesian theorem is introduced into FEMU to quantify parameter uncertainties. The influences of operational loads on structural parameters are further considered in the hierarchical Bayesian FEMU framework. Next, a sparse Bayesian learning technique is developed for joint load-parameter identification. Finally, two physics-data co-driven techniques are developed to enhance model generalizability and flexibility. The Newmark-beta and SCD methods are adopted as network architectures for joint load-parameter identification and blind NBC identification, respectively. The results and findings are summarized as follows.

The FEMU method using both static and dynamic measurements has been proposed, which can accurately estimate structural stiffness and mass parameters. The theoretical and measured static responses are efficiently obtained by fusing traffic video, vehicle weight data, and structural responses via load identification and signal processing techniques. This approach eliminates the time-consuming and costly traffic interruptions required by conventional methods.

The hierarchical Bayesian FEMU methods have been developed to quantify system

uncertainties.

1. The hierarchical Bayesian FEMU method using static and dynamic data accurately identifies structural parameters and their associated uncertainties. The predicted responses exhibit good agreement with the measured data. The predicted static responses generally fall within the 95% CIs when both parameter and modeling error uncertainties are incorporated.
2. The hierarchical Bayesian FEMU method incorporating operational loads effectively captures temperature-induced variations in stiffness parameters and vehicle load effects on girder mass distributions. The structural condition indicator calculated from the predicted CIs and measured responses accurately detects abnormal structural states. Comparative analysis of the three proposed FEMU methods demonstrates that considering operational loads and their associated uncertainties is critical for FEMU and condition assessment of long-term operational structures.

The sparse Bayesian method for joint load–parameter identification has been proposed, which successfully imposes independent Gaussian priors on loads and damage indices to enforce spatial sparsity. This method effectively identifies both loads and damage from sparse noisy acceleration measurements without requiring prior knowledge of damage or loading locations.

The physics–data co-driven methods for element-level system identification have been proposed. Both methods are implemented on the DL platform, and the identification process and results strictly adhere to physical laws.

1. The PENN for joint load–parameter identification effectively integrates FEMU and NN modules within the framework of the Newmark-beta method. The method accurately identifies both structural parameters and loads under various noise levels, sensor types, and loading conditions. The proposed method outperforms a filter-based method in both accuracy and efficiency.
2. The hybrid FE–NN method successfully identifies both elastic and hysteretic NBCs without prior knowledge of the nonlinear type. The structural full-field responses are

subsequently accurately reconstructed. The method demonstrates robustness against varying noise levels, sensor configurations, and nonlinear types.

This thesis presents a systematic framework and comprehensive methodology for bridge system identification. In the short term, the proposed approaches provide regular and accurate assessment of structural conditions, offering a solid basis for early warning in bridge management. Meanwhile, the precise structural models developed through these methods support visualization and the creation of digital twins, promoting the integration of physical and digital infrastructure management. In the long term, the methodology supports the development of structural degradation models, which are essential for effective maintenance planning and decision-making. Overall, these advancements provide a strong foundation for safer, more economical, and intelligent infrastructure management systems.

9.2 Future Work

Despite the improvements made, the methods developed in this thesis can be improved and extended in the following aspects:

1. The proposed system identification methods are primarily designed for offline settings and exhibit limitations in handling streaming data for real-time monitoring scenarios. In addition, their computational efficiency is insufficient for rapid identification, and this limitation becomes increasingly pronounced as the structural DOF increases. Lightweight algorithms, online identification architectures, and edge computing strategies can be employed to facilitate practical deployment.
2. The current physics–data co-driven approaches operate within a deterministic paradigm, leaving the associated system uncertainties unquantified. Probabilistic modeling techniques, including Bayesian NNs and VAEs, can be applied to account

for uncertainties arising from various sources. Variational inference methods can be employed to model the propagation of uncertainty.

3. The ill-posed nature of system identification stems from the mismatch between the low dimensionality of measured data and the high dimensionality of the parameters to be identified. A potential solution for mitigating the ill-posedness of the inverse problem is to increase the spatial resolution of observable data. Full-field monitoring technologies, such as those based on LiDAR and computer vision techniques, can be used to acquire high-spatial-resolution structural responses. Concurrently, system identification algorithms specifically tailored to full-field responses require further exploration.

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