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LATERAL AND TORSIONAL VIBRATION CONTROL OF LONG SPAN BRIDGE DECK USING NOVEL TUNED LIQUID COLUMN DAMPERS

By

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B.Eng.

A thesis submitted for the Degree of Doctor of Philosophy

Department of Civil and Structural Engineering
The Hong Kong Polytechnic University

2004
Dedicated to my parents

In appreciation for their love and support for their children
DECLARATION

I hereby declare that this thesis entitled "Lateral and Torsional Vibration Control of Long Span Bridge Deck Using Novel Tuned Liquid Column Dampers" is my own work and that, to the best of my knowledge and belief, it reproduces no material previously published or written, nor material that has been accepted for the award of any other degree or diploma, except where due acknowledgement has been made in the text.

SIGNED

SHUM, Kei Man
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Abstract of Thesis Entitled

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Submitted by
SHUM, Kei Man
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at The Hong Kong Polytechnic University in September 2004

ABSTRACT

Long span cable-supported bridges are rapidly increasing nowadays not only in number but also in the length of central-span because of their inherent economical and technical advantages. However, these long span bridges are very flexible and lightly damped and vulnerable to wind-induced vibration. In particular, during the erection, long span bridge deck lacks continuity from pylon to pylon and its rigidity is much lower than that of a completed bridge. Serious buffeting vibration has been observed from a few long span cable-supported bridges during construction. Reviews of the existing literature, however, show that few studies have examined the suppression of lateral and torsional vibration of long span bridges. This thesis thus focuses on the development and application of novel tuned liquid column dampers for suppressing lateral and torsional vibration of long span cable-supported bridges during construction and at completion stage.

The first part of the thesis, consisting of Chapters 3 and 4, presents a combined experimental and theoretical investigation on the performance of multiple tuned liquid column damper (MTLCD) for reducing torsional vibration of structures in comparison with single-tuned liquid column damper (STLCD), with particular focus on the sensitivity of its performance to frequency tuning. A large structure model simulating the torsional vibration of a bridge deck and several STLCDs and MTLCDs of different configurations are designed and constructed. A series of harmonically forced vibration tests are conducted to evaluate the effectiveness of MTLCD in reducing torsional
vibration of the structure. An averaging method is also developed to identify the head loss coefficients of STLCD and MTLCD in conjunction with free vibration test technique. An analytical model for the torsional vibration of the structure with an MTLCD under either harmonic excitation or white noise excitation is then developed and verified using the obtained experimental results together with the identified head-loss coefficient. The performance of MTLCD in reducing torsional displacement is further investigated through extensive parametric studies using the verified theoretical model. It is found that the sensitivity of an optimized MTLCD to the frequency tuning ratio is less than that of an optimized STLCD and it can be further improved by increasing the bandwidth but at the cost of smaller torsional vibration reduction.

The frequency of STLCD or MTLCD depends solely on the length of liquid column, which imposes certain restrictions on its application to long span bridges. The short period of torsional vibration of long span bridges may require short liquid column length to have a proper frequency tuning. Consequently, a large number of such small TLCD containers are required, which leads to a higher cost of installation and maintenance. Multiple pressurized tuned liquid column dampers (MPTLCD) are thus studied in Chapter 5 to facilitate torsional vibration reduction of a structure and to improve the sensitivity under mistuning. The MPTLCD container is sealed with an air chamber at its two ends. The frequency tuning can be adjusted by manipulating static pressure inside the air chamber while the length of liquid column is fixed. An analytical model is developed for torsional vibration of a structure with a MPTLCD under either harmonic or white noise excitation. The nonlinear damping due to orifice and the nonlinear restoring force due to air pressure in the MPTLCD are linearised in the frequency domain. After such linearization is proved to be satisfactory through a comparison with a nonlinear analysis in the time domain, extensive parametric studies are finally carried out in the frequency domain to find the beneficial parameters by which the maximum torsional vibration reduction can be achieved. The investigations demonstrate that MPTLCD can provide a greater flexibility for application in practice and achieve a high degree of vibration reduction.

To control vibration of a structure with high natural frequency, the MPTLCD with a longer liquid column length can be used to replace the STLCD or MTLCD with a
shorter liquid column length. However, for a long span cable-supported bridge during construction, its very low natural frequency may require a STLCD or a MTLCD with very long liquid column length, which may not be possible in practice. Moreover, natural frequencies of a long span bridge vary during its construction stage. Hence, tuned liquid column dampers with adaptive frequency tuning capacity are developed in Chapter 6 for suppressing lateral or torsional vibration of a structure using a semi-active control technology. The natural frequency of the semi-active tuned liquid column dampers (SATLCD) studied herein can be adjusted by active control of air pressures at the two chambers of a PTLCD. Analytical models are developed for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD, respectively, under either harmonic or white noise excitation. The nonlinear damping property of SATLCD is linearized using an equivalent linearization technique. Extensive parametric studies are carried out in the frequency domain to find the beneficial parameters by which the maximum vibration reduction can be achieved. The investigations demonstrate that the SATLCD can provide a greater flexibility for its application in practice and achieve a high degree of vibration reduction. The sensitivity of SATLCD to the frequency offset between the damper and the structure can be improved by adapting its frequency precisely to the measured structural frequency.

Wind-induced vibration of a long span bridge involves many modes of vibration. Large vibration may result from coupling of different modes of vibration. Most of previous studies pertaining to the suppression of wind-induced vibration of a bridge focused on the coupling of vertical and torsional vibrations. With an increase in the span length and complexity of a bridge deck, significant mechanical and aerodynamic coupling may exist between the first lateral and torsional vibration under turbulent winds. However, little information is available on this topic. The use of MTLCD for reducing the coupled lateral and torsional vibration of a bridge deck is therefore explored using mode-by-mode spectral approach in Chapter 7. The equations of motion for coupled lateral and torsional vibration of the bridge deck are formulated. The efficiency of MTLCD in reducing the coupled lateral and torsional vibration is investigated through extensive parametric studies. The results show that the MTLCD can reduce both the lateral and torsional vibrations of the bridge deck effectively if the parameters are properly selected. The aeroelastic effects due to the interaction between turbulent wind and
bridge motion is a crucial factor which affects the performance of MTLCD in reducing buffeting responses of the bridge deck.

The flexibility of MPTLCD in frequency tuning offers wider choice of container configurations, which makes it easier to be installed in a real long span bridge in its completion stage. The use of MPTLCD for reducing the coupled lateral and torsional vibration might be an alternative solution for some long span cable-supported bridges with relatively high lateral or torsional frequency. The performance of MPTLCD for the suppression of lateral and torsional vibration of a long span bridge deck at the completion stage is investigated using finite element based approach in Chapter 8. The prediction of buffeting response of long span bridge is usually done by the finite element method in addition with the aerodynamic characteristics of the bridge obtained from wind tunnel tests. A finite element model of MPTLCD is thus developed and incorporated into the finite element model of a long span bridge for predicting the buffeting response of the coupled MPTLCD-bridge system in the time domain. The investigations show that the MPTLCD not only provides great flexibility for selecting liquid column length but also significantly reduces the lateral and torsional displacement response of a long span bridge under wind excitation.

The configuration of a long span bridge varies from different construction stages and so do its natural frequencies. It is thus difficult to apply TLCD with a fixed configuration to the bridge during construction or it is not economical to design a series of TLCD with different liquid column length to suit for various construction stages. The use of SATLCD with a fixed container configuration is thus studied in Chapter 9 for the suppression of the lateral and torsional vibration of a long span cable-supported bridge during construction. The finite element model of SATLCD is also developed for the prediction of buffeting response of the coupled SATLCD-bridge system and the assessment of the control performance of SATLCD in the time domain. Five different construction stages of the bridge are selected for the study of the SATLCD performance and adaptability. It is found that with a fixed container configuration, SATLCD can effectively reduce the lateral and torsional vibration of the bridge deck under all the five construction stages.
LIST OF PUBLICATIONS


Y.L. Xu and K.M. Shum, Multiple tuned liquid column dampers for torsional vibration control of structures: theoretical investigation, Earthquake engineering and structural dynamics, 32(2), 2003, 309-328.

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LIST OF ABBREVIATIONS

b = deck width

d = thickness of liquid column damper

d_k = thickness of the kth liquid column damper

e = eccentricity of bridge

f_0 = central frequency

f_i = natural frequency of water in the i\textsuperscript{th} TLCD

f_s = natural frequency of structure

f_l = the lowest natural frequency among all TLCD units in MTLCD

f_n = the highest natural frequency among all TLCD units in MTLCD

g = acceleration due to gravity

h = air chamber height of PTLCD

h_k = vertical displacement of the bridge deck at a given position

h_k = height of the air chambers of the kth PTLCD

i = imaginary unit

k = reduced frequency

m_d = total mass of liquid column dampers

m_k = mass of the kth liquid column damper

m_s = mass of structure

m_{ij} = modal mass of bridge

m_{ij} = mass of liquid column dampers tuned to the jth lateral vibration

mode of bridge

m_{ij} = mass of liquid column dampers tuned to the jth torsional vibration

mode of bridge

\bar{m}(y) = mass of the bridge deck per unit length

n = total number of points where wind speeds are simulated

p = lateral displacement of the bridge deck at a given position

r = structural radius of gyration

u(t) = control force of SATLCD

v_y = wind speed components in the horizontal direction

v_b = nodal dynamic displacement of bridge
\[ \ddot{v}_b = \text{nodal dynamic displacement of bridge} \]
\[ \ddot{v}_b = \text{nodal dynamic displacement of bridge} \]
\[ w(t) = \text{wind speed components in the vertical direction} \]
\[ x = \text{lateral displacement of structure} \]
\[ \dot{x} = \text{lateral velocity of structure} \]
\[ \ddot{x} = \text{lateral acceleration of structure} \]
\[ z = \text{vertical offset distance between elastic centre and mass centre of bridge} \]
\[ A = \text{cross-sectional area of liquid column damper} \]
\[ A_k = \text{cross-sectional area of the kth liquid column damper} \]
\[ A_i^* (i=1~6) = \text{non-dimensional flutter derivatives} \]
\[ B = \text{horizontal width of liquid column damper} \]
\[ B_c = \text{horizontal width of liquid column in the central unit of MTLCD} \]
\[ B_k = \text{horizontal width of the kth liquid column damper} \]
\[ C = \text{coherence function between points j and m} \]
\[ C_{ae} = \text{aeroelastic damping matrix} \]
\[ C_t = \text{torsional damping coefficient of structure} \]
\[ C_\theta = \text{torsional damping coefficient of structure} \]
\[ = \text{modal torsional damping coefficient of bridge} \]
\[ C_x = \text{lateral damping coefficient of structure} \]
\[ = \text{modal lateral damping coefficient of bridge} \]
\[ C_D = \text{drag coefficient of bridge deck section model} \]
\[ C_L = \text{lift coefficient of bridge deck section model} \]
\[ C_M = \text{moment coefficient of bridge deck section model} \]
\[ C_D' = \text{slope of } C_D \text{ at angle } \alpha \]
\[ C_L' = \text{slope of } C_L \text{ at angle } \alpha \]
\[ C_M' = \text{slope of } C_M \text{ at angle } \alpha \]
\[ D_0(t) = \text{buffeting drag on bridge deck of unit span length at a given position} \]
\[ F_s = \text{external force excitation} \]
\[ = \text{generalised force acting on elastic centre of bridge} \]
\[ G = \text{liquid mass moment in liquid column damper} \]
\( G_k \) = liquid mass moment in the \( k \)th liquid column damper

\( H \) = distance from the centre line of the bottom tube of liquid column damper to the rotational axis of structure

\( H_k \) = distance from the center line of the bottom tube of the \( k \)th liquid column damper to the rotational axis of structure

\( \bar{H} \) = distance from the mass center of liquid inside liquid column damper to the rotational axis of structure

\( \bar{H}_k \) = distance from the mass center of liquid inside the \( k \)th liquid column damper to the rotational axis of structure

\( H_i^*(i=1-6) \) = non-dimensional flutter derivatives

\( I_d \) = total mass moment of inertia of liquid inside liquid column dampers about the rotational axis of the structure/bridge

\( I_j \) = mass moment inertia of liquid column damper per unit mass tuned to the \( j \)th torsional mode of vibration of bridge

\( I_s \) = mass moment of inertia of structure including mass moment of inertia of containers about the rotational axis of structure

\( I_v \) = modal mass moment of inertia of bridge about elastic centre

\( I_u \) = turbulence intensity in alongwind direction

\( I_w \) = turbulence intensity in vertical direction

\( \bar{I}(y) \) = mass moment inertia of the bridge deck per unit length

\( L \) = total length of liquid column damper

\( L_k \) = total length of the \( k \)th liquid column damper

\( L_b(t) \) = buffeting lift on bridge deck of unit span length at a given position

\( L_c \) = total length of liquid column in the central unit of MTLCD

\( L_p \) = liquid column length of passive TLCD

\( K_{ae} \) = aeroelastic stiffness matrix

\( K_x \) = lateral stiffness of structure

\( K_s \) = modal lateral stiffness of bridge

\( K_t \) = torsional stiffness of structure

\( K_{t0} \) = modal torsional stiffness of bridge

\( M_0 \) = harmonic excitation amplitude
\( M_b(t) \) = buffeting moment on bridge deck of unit span length at a given position

\( M_s \) = external moment excitation

\( M_g \) = generalised moment

\( N \) = total number of frequency interval

\( P_0 \) = static pressure of compressed air inside air chamber of PTLCD/SATLCD

\( P_{0,k} \) = static pressure of compressed air inside air chamber of the kth PTLCD/SATLCD

\( P \) = net pressure acting on liquid

\( P_{\text{buff}} \) = buffeting force vector of bridge

\( P_{\text{max}} \) = maximum pressure among all PTLCD units in MPTLCD

\( P_{\text{se}} \) = self-excited force vector of bridge

\( P_L \) = pressure in the left air chamber

\( P_R \) = pressure in the right air chamber

\( P'_i (i=1-6) \) = non-dimensional flutter derivatives

\( R_1 \) = displacement response index

\( R_1 \) = response ratio of lateral displacement

\( R_t \) = response ratio of torsional displacement

\( S \) = constant displacement feedback gain of SATLCD

\( S_o \) = spectral intensity level of zero-mean stationary white noise moment excitation

\( S_{11}(\omega) \) = spectral density function of the drag force

\( S_{44}(\omega) \) = spectral density function of the moment

\( S_{14}(\omega) \) = cross-spectral density function of the drag force and moment

\( S_0(\omega) \) or \( S_{uu}(\omega) \) = spectral density function of the horizontal turbulent wind

\( S_w(\omega) \) or \( S_{ww}(\omega) \) = spectral density function of the vertical turbulent wind

\( S_M \) = power spectral density function of white noise excitation

\( U \) = mean wind speed

\( U_m \) = mean wind speed

\( V_0 \) = volume of air inside chamber at static position of liquid

\( W \) = relative motion of liquid inside liquid column damper to container

\( \dot{W} \) = the first derivative of \( W \) with respect to time
\( \dot{W} \) = the second derivative of \( W \) with respect to time

\( W_a \) = tolerable liquid displacement of SATLCD

\( W_k \) = relative displacement of liquid inside the \( k^{th} \) liquid column damper to container

\( \dot{W}_k \) = the first derivative of \( W_k \) with respect to time

\( \ddot{W}_k \) = the second derivative of \( W_k \) with respect to time

\( \bar{W} \) = amplitude of liquid displacement in liquid column damper

\( \bar{W}_k \) = amplitude of liquid displacement in the \( k^{th} \) liquid column damper

\( \alpha \) = torsional displacement of the bridge deck at a given position

\( \alpha_c \) = angle of attack of normal incident wind referring to the horizontal plane of bridge deck

\( \lambda \) = liquid length ratio of liquid column damper

\( \alpha_c \) = the ratio of horizontal width to total length of liquid column in the central unit of MTLCD

\( \alpha_k \) = liquid length ratio of the \( k^{th} \) liquid column damper

\( \beta_k \) = frequency spacing

\( \delta \) = head loss coefficient

\( \delta_k \) = head loss coefficient of the \( k^{th} \) liquid column damper

\( \lambda_u \) = phase angle between excitation force and excitation moment

\( \lambda_w \) = decay factor of the coherence of turbulent wind in alongwind direction

\( \lambda_w \) = decay factor of the coherence of turbulent wind in vertical direction

\( \mu \) = mass moment of inertia ratio of MTLCD to structure.

\( \mu \) = mass ratio of MTLCD to bridge

\( \mu_a \) = mass ratio of SATLCD to structure

\( \mu_{Lj} \) = mass ratio of liquid column dampers tuned to the \( j^{th} \) mode of lateral vibration of bridge

\( \mu_{Tj} \) = mass ratio of liquid column dampers tuned to the \( j^{th} \) mode of torsional vibration of bridge

\( \theta \) = torsional displacement of structure
\( \dot{\theta} \) = torsional velocity of structure

\( \ddot{\theta} \) = torsional acceleration of structure

\( \rho \) = liquid density inside liquid column damper

\( \rho_a \) = air density

\( \sigma_w \) = standard deviation of liquid velocity in liquid column damper

\( \sigma_{w_k} \) = standard deviation of liquid velocity in the kth liquid column damper

\( \xi_k \) = equivalent damping ratio of the kth liquid column damper

\( \xi_s \) = structural damping ratio

\( \xi_x \) = damping ratio in the first lateral mode of bridge

\( \xi_\theta \) = damping ratio in the first torsional mode of bridge

\( \omega \) = circular frequency of wind turbulence.

\( \bar{\omega} \) = circular natural frequency of harmonic excitation

\( \omega_d \) = circular natural frequency of liquid column damper

\( \omega_k \) = circular natural frequency of the kth liquid column damper

\( \omega_x \) = circular lateral frequency of structure without control

\( \omega_\theta \) = circular torsional frequency of structure without control

\( \omega_{up} \) = upper cutoff frequency

\( \Delta_{jm} \) = horizontal distance between points j and m

\( \Delta P_k \) = pressure tuning ratio

\( \Delta \gamma \) = frequency tuning ratio

\( \Delta \omega \) = frequency interval between the spectral lines

\( \Delta X \) = frequency bandwidth

\( \Phi_{ml} \) = random variable uniformly distributed between 0 and 2\( \pi \)

\( \psi_L(y) \) = the first lateral mode shape of bridge deck

\( \psi_T(y) \) = the first torsional mode shape of bridge deck

\( \chi_{Lw} \) = frequency-dependent aerodynamic transfer function between velocity fluctuation and buffeting force

\( \chi_{Lw} \) = frequency-dependent aerodynamic transfer function between velocity fluctuation and buffeting force
\( \chi_{D_\omega} \) = frequency-dependent aerodynamic transfer function between velocity fluctuation and buffeting force

\( \chi_{D_\nu} \) = frequency-dependent aerodynamic transfer function between velocity fluctuation and buffeting force

\( \chi_{M_\omega} \) = frequency-dependent aerodynamic transfer function between velocity fluctuation and buffeting force

\( \chi_{M_\nu} \) = frequency-dependent aerodynamic transfer function between velocity fluctuation and buffeting force
CHAPTER 1
INTRODUCTION

1.1 RESEARCH MOTIVATIONS

The rapid growth of economy in China tremendously increases commercial cooperation with Hong Kong, which burdens the busy traffic flow to a further extension. In order to cope with the sustained transport growth, many long span cable-supported bridges have been or to be built. Advances in construction technology have resulted in a trend to design and construct cable-supported bridges with a much longer span than ever. The Stonecutters cable-stayed bridge in Hong Kong with a main span of 1018m is a typical example, which crosses the entrance to the Kwai Chung container port between Tsing Yi Island and Cheung Sha Wan. Large span length to weight ratio of the deck increases its sensitivity to wind-induced vibration. In particular for the bridge under construction, its bending and torsional rigidities are much lower than those of the complete bridge due to the lacking of bridge deck continuity from pylon to pylon. Suppressing wind-induced vibration of long span cable-supported bridges in either construction stage or completed stage is thus an important issue for minimizing the loss in both economy and human life.

There are various possible ways to reduce wind-induced vibration of structures. They can be classified into three major categories, aerodynamic modification, structural modification and mechanical control devices. Aerodynamic modification changes the flow pattern around a structure so as to directly reduce aerodynamic force or moment acting on it by selecting better cross sectional shape of the structure or installing some aerodynamic devices such as spoilers, vanes and openings. However, it becomes difficult to reduce wind-induced vibration of long span bridges during construction solely by aerodynamic modification. The buffeting force is inevitably to happen to any bridge exposing to natural turbulent wind, and this phenomenon is caused not only by the way the bridge is designed but also by the atmospheric turbulence existing inherently. The increase of span length of modern cable-supported bridges will
significantly increase their buffeting response. Therefore, it may also be required to modify the structural dynamic parameters by changing mass, damping and stiffness through the change of structural materials or structural system. The increase in mass, damping and stiffness can reduce wind-induced vibration of long span bridges. However, an increase in mass leads to a decrease in natural frequency and an increase in dead load. This may not be an economical way to reduce structural vibration. An increase in natural frequency of structure due to an increase in stiffness usually results in a decrease of wind-induced vibration. In order to increase stiffness without much increase in mass, temporary tie-down cables are usually installed to cable-stayed bridge during construction. However, in some circumstances, use of temporary tie-down may probably be subjected to a high construction cost of anchor block and risk of ship collision during storm (Virlogeux 1992). Therefore, utilizing mechanical control devices to supplement damping capacity of the structure can be an alternative solution to reduce wind-induced vibration.

To ease erection and welding operation as well as avoid interaction with ship navigation, tuned mass damper was installed on the Normandy Bridge for suppressing vibrations of the two long cantilevering bridge decks. Tuned mass damper is only one of passive mechanical control devices and many mechanical control devices were actually developed in the past. They can be classified as passive, semi-active, and active. Passive control devices absorb energy from excessive vibration of the structure, which increase an overall damping of the structure, and hence reduce wind-induced vibration of the structure. Active control devices require external energy supply to apply control forces to the structure in a prescribed manner. Semi-active control devices require much less input energy in comparison with the active one and the input energy is used to modify the damper properties leading to the optimal control of structural vibration.

Extensive research studies have shown that the aerodynamic performance of building can be improved by the installation of mechanical control devices (Soong and Dargush 1997). Passive control devices such as tuned mass damper (TMD), tuned liquid damper (TLD), and tuned liquid column damper (TLCD) have been applied successfully to the building structure for reducing wind-induced horizontal vibration (Kareem 1983; Tamura 1990; Samali and Kwok 1994; Soong and Dargush 1997).
However, relatively less research works have been conducted on the control of wind-induced vibration of long span cable-supported bridges by utilizing mechanical control devices. As such, tuned liquid column dampers has been proposed and studied for reducing torsional vibrations of bridge deck (Xue et al. 2000). Both theoretical and experimental studies demonstrated that TLCD could effectively reduce torsional vibration if its parameters are properly selected. However, in determining the parameters of TLCD for the torsional vibration, it is always assumed that the natural frequency of long span bridge is precisely known. But the estimation of natural frequencies of long span bridge is usually based on a simple finite element model and often deviates from the actual values. Besides, changes in the natural frequency of the bridge may arise from the effect of structural aging or the effect of wind-bridge interaction. Possible uncertainties of structural natural frequencies may cause an offset between the natural frequency of the liquid motion and the natural frequency of the bridge. Consequently, the performance of TLCD is deteriorated significantly due to off-tuning. This is definitely a disadvantage of using single TLCD for the torsional vibration reduction of long span bridge. Therefore, it is important to investigate the performance of multiple tuned liquid column dampers (MTLCD) in reducing the torsional vibration of structures in comparison with single TLCD.

The frequency of STLCD or MTLCD depends solely on the length of liquid column, which imposes certain restrictions on its application to long span bridges. The short period of torsional vibration of long span bridge may require short liquid column length to have a proper frequency tuning. Consequently, a large number of such small TLCD containers are required, which leads to a higher cost of installation and maintenance. Therefore, new type of tune liquid column dampers shall be sought to handle this problem. To this end, multiple pressurized tuned liquid column dampers (MPTLCD) are proposed in this study to facilitate torsional vibration reduction of a structure and to improve the sensitivity under mistuning. The MPTLCD container is sealed with an air chamber at its two ends. The frequency tuning can be adjusted by manipulating static pressure inside the air chamber while the length of liquid column is fixed.
To control vibration of a structure with high natural frequency, the MPTLCD with a longer liquid column length can be used to replace the STLCD or MTLCD with a shorter liquid column length. However, for a long span cable-supported bridge during construction, its very low natural frequency may require a STLCD or a MTLCD with very long liquid column length, which may not be possible in practice. Moreover, natural frequencies of a long span bridge vary during its construction stage. Hence, novel tuned liquid column dampers with adaptive frequency tuning capacity are needed to suppress lateral or torsional vibration of long span cable-supported bridge during its construction stage. In this connection, semi-active tuned liquid column dampers (SATLCD) are proposed in this study with a semi-active control algorithm. The natural frequency of the proposed SATLCD can be adjusted by active control of air pressures at the two chambers of a PTLCD.

Wind-induced vibration of long span cable-supported bridges involves many modes of vibration. Large vibration may result from coupling of different modes of vibration. Most of previous studies pertaining to the suppression of wind-induced vibration of a bridge focused on the coupling of vertical and torsional vibrations only. With an increase in the span length and complexity of a bridge deck, significant mechanical and aerodynamic coupling may exist between the first lateral and torsional vibration under turbulent winds. However, little information is available on this topic. How to use MTLCD to reduce the coupled lateral and torsional vibration of a bridge deck should thus be explored.

The flexibility of MPTLCD in frequency tuning offers wider choice of container configurations, which makes it easier to be installed in a real long span bridge in its completion stage. The use of MPTLCD for reducing the coupled lateral and torsional vibration might be an alternative solution for some long span cable-supported bridges with relatively high lateral or torsional frequency. The 3-D finite element based buffeting analysis is often used to predict buffeting response of a long span cable-supported bridge. How to incorporate MPTLCD into the finite element model of the bridge and how to apply MPTLCD to reduce the coupled lateral and torsional vibration of a real long span bridge deck at the completion stage need to be investigated.
The configuration of a long span cable-stayed bridge varies from different construction stages and so do its natural frequencies. It is thus difficult to apply TLCD with a fixed configuration to the bridge during construction or it is not economical to design a series of TLCD with different liquid column length to suit for various construction stages. Semi-active tuned liquid column dampers with adaptive frequency tuning capacity may be a good alternative. Then, how to incorporate SATLCD into the finite element model of the bridge and how to use SATLCD with a fixed container configuration to suppress the coupled lateral and torsional vibration of a long span cable-stayed bridge during construction stage require a detail investigation.

1.2 OBJECTIVES

This thesis thus focuses on the development and application of novel tuned liquid column dampers for suppressing lateral and torsional vibration of long span cable-supported bridge during construction and at completion stage with the following major objectives:

(1) To carry out experimental investigations to access the performance of MTLCD in reducing torsional vibration of a structure with particular focus on the sensitivity of their performance to frequency tuning ratio in comparison with STLCD. An experiment method for estimating head-loss coefficient of STLCD using free vibration test will also be proposed. An analytical model for the torsional vibration of the structure with an MTLCD under either harmonic excitation or white noise excitation will then be developed and verified using the obtained experimental results together with the identified head-loss coefficient. The performance of MTLCD in reducing torsional displacement will be investigated through extensive parametric studies using the verified theoretical model.

(2) Multiple pressurized tuned liquid column dampers (MPTLCD) will be proposed to facilitate torsional vibration reduction of a structure and to improve the sensitivity under mistuning. An analytical model will be developed for torsional vibration of a structure with a MPTLCD under either harmonic or white noise
excitation. The nonlinear damping resulting from orifice and the nonlinear restoring force due to pressure inside air chambers will be linearized in the frequency domain. A comparison between nonlinear analysis in time domain and linear analysis in frequency domain will be conducted to check the validity of linearized analytical model of the MPTLCD-structure system. Extensive parametric studies will then be carried out to access the performance of MPTLCD in comparison with SPTLCD, with particular focus on the sensitivity of their performance to pressure tuning ratio.

(3) Semi-active tuned liquid column dampers (SATLCD) of adaptive frequency tuning capacity will be proposed in this study together with a semi-active control algorithm. Analytical models will be developed for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD, respectively, under either harmonic or white noise excitation. The nonlinear damping property of SATLCD will be linearized using an equivalent linearization technique. Extensive parametric studies will be carried out in the frequency domain to find the beneficial parameters by which the maximum vibration reduction can be achieved.

(4) The equations of motion for coupled lateral and torsional vibration of the bridge deck with MTLCD will be formulated using mode-by-mode spectral approach. The efficiency of MTLCD in reducing the coupled lateral and torsional vibration will be investigated through extensive parametric studies.

(5) Finite element model of MPTLCD will be developed for the sake of carrying out buffeting analysis of MPTLCD-bridge system. A real long span cable-stayed bridge will be modeled using the conventional finite element method. The developed finite element model of MPTLCD will be incorporated into the finite element model of the bridge for predicting the buffeting response of the MPTLCD-bridge system. The application of MPTLCD for the suppression of lateral and torsional vibration of a real long span cable-stayed bridge will be investigated extensively using the developed model.
(6) Finite element model of SATLCD will be developed for the sake of carrying out buffeting analysis of SATLCD-bridge system. Five different construction stages of a real long span cable-stayed bridge during construction will be selected for the study of the SATLCD performance and adaptability. The developed finite element model of SATLCD will be incorporated into the finite element model of the bridge for predicting the buffeting response of the SATLCD-bridge system. The use of SATLCD for the suppression of lateral and torsional vibration of the real long span cable-stayed bridge during construction will be investigated extensively using the developed model.

1.3 ASSUMPTIONS AND LIMITATIONS

The novel tuned liquid column dampers for suppressing wind-induced lateral and torsional vibration of long span cable-supported bridges during construction and at completion stage are developed and applied in this thesis with the following assumptions and limitations.

(1) The aerodynamic coefficients and the flutter derivatives of the bridge deck obtained from wind tunnel test are assumed to be unchanged for all stages of construction including completed stage. The variation of the aerodynamic coefficients and flutter derivatives of the bridge deck due to installation of liquid column dampers is assumed to be insignificant.

(2) Wind-induced dynamic response is assumed small and linear around the static equilibrium position of the bridge so that the nonlinear effects of the bridge motion on the aerodynamic coefficients and flutter derivatives can be ignored.

(3) The quasi-steady model is employed to obtain the self-excited force of the bridge deck in alongwind direction. The mean wind velocity is assumed to be perpendicular to the longitudinal axis of the bridge deck.
(4) The lateral and torsional vibration control by using liquid column dampers is of prime interest in this study. The vertical vibration control is not considered at the present study due to the inherent feature of liquid column dampers.

(5) The semi-active control technology is applied to control the frequency tuning of TLCD rather than to control the optimal damping of TLCD.

(6) The effect of time delay associated with build up pressure inside liquid column of SATLCD is assumed to be negligible.

(7) In the derivation of the finite element model of liquid column dampers, the liquid column damper is assumed to be installed below the torsional centre of the deck and is connected to the bridge deck by a roller support and a simply support. The axial deformation of the bridge deck between the two supports is assumed to be negligible.

1.4 THESIS LAYOUT

This thesis contains a variety of research topics to achieve the aforementioned objectives. It is divided into ten chapters and is organized in the following way.

- Chapter 1 gives a brief introduction of the research motivations, the objectives, the assumptions and limitations, and the layout of the thesis.

- Chapter 2 presents an extensive literature review on three relevant topics. The first section presents an introduction to aerodynamics of long span cable-supported bridges. The second section gives a review on various approaches for mitigating wind-induced vibration of long span cable-supported bridges by means of structural control. The third section presents a review on the research development and application of tuned liquid column dampers.

- Chapter 3 presents an experimental investigation on the performance of multiple-tuned liquid column dampers (MTLCD) for reducing torsional
vibration of structures in comparison with single-tuned liquid column dampers (STLCD). A large structure model simulating the torsional vibration of bridge deck and several STLCDs and MTLCDs of different configurations are designed and constructed. A series of harmonically forced vibration tests are conducted to evaluate the effectiveness of MTLCDs in reducing torsional vibration of the structure and to assess the performance effects of various design parameters, which include the number of TLCD units in a MTLCD, the bandwidth of a MTLCD, the frequency tuning ratio, and the moment excitation amplitude. An averaging method is also used to identify the head loss coefficients of STLCDs and MTLCDs in conjunction with the free vibration test technique.

- Chapter 4 contains a detailed theoretical study on the performance of multiple-tuned liquid column dampers (MTLCD) for reducing torsional vibration of structures in comparison with single-tuned liquid column dampers (STLCD). The analytical model is first developed for torsional vibration of a structure with an MTLCD under either harmonic excitation or white noise excitation. The experimental results are then used to verify the analytical model for coupled MTLCD-structure systems under harmonic excitation. The performance of an MTLCD and its beneficial parameters for achieving the maximum torsional response reduction to white noise excitation are finally investigated through an extensive parametric study in terms of the distance from the center line of the MTLCD to the rotational axis of the structure, the ratio of the horizontal length to the total length of liquid column, frequency bandwidth, head loss coefficient, the number of TLCD units in MTLCD, frequency tuning ratio, and the spectral level of excitation moment.

- Chapter 5 proposes multiple pressurized tuned liquid column dampers (MPTLCD) for the torsional vibration reduction of structures. The frequency tuning can be adjusted by manipulating static pressure inside the air chamber while the length of liquid column is fixed. An analytical model is developed for torsional vibration of a structure with a MPTLCD under either harmonic or white noise excitation. The nonlinear damping due to orifice and the nonlinear restoring force due to air pressure in the MPTLCD are linearized in the
frequency domain. After such linearization is proved to be satisfactory through a comparison with a nonlinear analysis in the time domain, extensive parametric studies are finally carried out in the frequency domain to find the beneficial parameters by which the maximum torsional vibration reduction can be achieved. The key parameters investigated include distance from centre line of the MPTLCD to rotational axis of the structure, ratio of horizontal length to total length of liquid column, frequency bandwidth, head loss coefficient, number of TLCD units, and pressure tuning ratio.

- Chapter 6 proposes a novel semi-active tuned liquid column damper (SATLCD) for the suppression of either lateral or torsional vibration of structure. The natural frequency of the proposed SATLCD can be adjusted by active control of air pressures at the two chambers of a PTLCD. Analytical models are developed for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD, respectively, under either harmonic or white noise excitation. The nonlinear damping property of SATLCD is linearized using an equivalent linearization technique. Extensive parametric studies are carried out in the frequency domain to find the beneficial parameters by which the maximum vibration reduction can be achieved. The parameters investigated include distance from centre line of SATLCD to rotational axis of structure, ratio of horizontal length to total length of liquid column, and head loss coefficient.

- Chapter 7 presents a theoretical investigation on the performance of multiple tuned liquid column dampers (MTLCD) for mitigating the coupled lateral and torsional vibration of a long span bridge. The cross section of the bridge has a vertical axis of symmetry but with the vertical offset between the elastic centre and the mass centre of the bridge. The external dynamic force and moment are applied at the elastic centre of the bridge. The proposed MTLCD consists of two sets of liquid column dampers with one tuned to the lateral frequency of the bridge and the other tuned to the torsional frequency of the bridge. The equations of motion for the coupled lateral and torsional vibration of the bridge with MTLCD are developed. The nonlinear damping property of MTLCD is
linearized using the equivalent linearisation technique. Extensive parametric studies are then carried out in the frequency domain to find beneficial parameters of the MTLCD for achieving maximum reduction of coupled lateral and torsional vibration of the bridge. The parameters investigated include water mass distribution between the two dampers, distance from the centre line of the MTLCD to the rotational axis of the bridge, ratio of horizontal length to total length of liquid column, head loss coefficient, and frequency tuning ratio. The performance of the MTLCD in reducing buffeting response of the bridge due to turbulent wind is finally investigated.

- Chapter 8 presents an application of multiple pressurized tuned liquid column dampers for the suppression of lateral and torsional vibration of a real long span cable-stayed bridge. Finite element model of MPTLCD is developed for the sake of carrying out buffeting analysis of MPTLCD-bridge system. The real long span cable-stayed bridge is modeled using the conventional finite element method. The developed finite element model of MPTLCD is incorporated into the finite element model of the bridge for predicting the buffeting response of the MPTLCD-bridge system. Buffeting forces and self-excited forces are simulated in time domain using a fast spectral representation method, the aerodynamic coefficient, and the flutter derivatives along with a rational function approximation approach. The performance of MPTLCD for the suppression of lateral and torsional vibration of a real long span cable-stayed bridge is investigated using the developed model through extensive parametric studies. The key parameters investigated include mass ratio, head loss coefficient, mean wind speed and frequency tuning ratio.

- Chapter 9 presents an application of semi-active tuned liquid column dampers for the suppression of lateral and torsional vibration of a real long span cable-stayed bridge during construction. Finite element model of SATLCD is developed for the sake of carrying out buffeting analysis of SATLCD-bridge system. Five different construction stages of a real long span cable-stayed bridge during construction are selected for the study of the SATLCD performance and adaptability. The developed finite element model of SATLCD is incorporated
into the finite element model of the bridge for predicting the buffeting response of the SATLCD-bridge system. The performance of SATLCD for the suppression of lateral and torsional vibration of the real long span cable-stayed bridge during construction is investigated extensively through parametric studies. The key parameters investigated include mass ratio, head loss coefficient, mean wind speed and cantilever length.

- Chapter 10 concludes all the main findings obtained in this study and provides some recommendations for further study on this topic.
CHAPTER 2
LITERATURE REVIEW

2.1 BUFFETING OF LONG SPAN CABLE-SUPPORTED BRIDGES

To cope with the needs of economic growth and recreation of the community, many long span cable-supported bridges have been built over the world in the past two decades because of their aesthetic appearance, structural efficiency and ease of construction. The world’s longest suspension bridge and the world’s longest cable-stayed bridge, namely Akashi Kaikyo suspension bridge (central span: 1991 m, Figures 2-1 and 2-2) and Tatara cable-stayed bridge (central span: 890m, Figures 2-3 and 2-4) were completed at the end of last century (Fujino, 2002). Super long span suspension bridges with main span length beyond 3000m, such as Messina Narrow Bridge in Italy (central span: 3300m) and super long span cable-stayed bridges with main span length beyond 1000m, such as Sutong Bridge in Jiangsu, China (central span: 1088m) and Stonecutter Bridge in Hong Kong, China (central span: 1018m) are to be built. The trend of longer span length raises one of the major challenges: how to find new and better means to minimize the damaging effect arising from the destructive environmental loads. Among various types of environmental load, wind load may be the most critical one to long span bridges.

The aerodynamic effects on long span cable-supported bridges can be classified into self-excited vibration and buffeting vibration. Self-excited vibration is due to structural motion under wind flow and buffeting vibration arises from wind turbulent. Flutter, vortex-induced vibration and galloping are three main kinds of self-excited vibration. Flutter and galloping are considered to be unstable vibration while vortex-induced vibration is amplitude limited vibration. Vortex-induced vibration is a resonance of structure arising from the formation of vortices alternately on the upper and lower surface of bridge deck. When the oscillatory character of the wake approximately coincides with a natural frequency of the bridge, it is possible for vibration to develop. The vibration amplitude depends on the structural damping and the
shape of the deck cross-section. The shallower and more streamlined the deck the smaller the amplitude will be. The aerodynamic effects of self-excited vibration can be reduced by means of aerodynamics measures such as optimizing deck cross-sections and installing aerodynamic devices. However, buffeting is inevitably to happen to any bridge exposing to natural wind because it is caused by the wind turbulent existing inherently in atmosphere.

With the trend of constructing bridges with longer span length, modern long span cable-supported bridges become more sensitive to strong wind than ever. This may lead to a significant increase of buffeting response of the bridge, which may result in exceeding of ultimate stress, serious fatigue damage to structural components as well as instability of the vehicles traveling on the deck and discomfort to the passengers inside the moving vehicles on the deck. The dramatic failure of the Tacoma Narrows Bridge in 1940 sparked considerable investigations into the aerodynamic behaviors of bridges by using wind tunnel tests or analytical methods. A brief review on some remarkable investigations of the aerodynamic behaviors of bridges is given as follows.

2.1.1 Wind tunnel tests and field measurements

Aerodynamic investigation on long span bridges by means of wind tunnel tests can be classified as section model test and full aeroelastic model test. Section model test measures the aerodynamic force coefficients or flutter derivatives of bridge deck (e.g. Choi and Brownjohn 1998) while full aeroelastic model test of a long span bridge can predict the buffeting responses and internal forces of bridge component (Irwin 1992 and King 1999). Apart from wind tunnel tests, field measurements can also be used to study the aerodynamic behavior of long span bridge as well as characteristics of wind speed field. A large number of field experiments have been conducted by some researchers (Larose et al., 1998). Xu et al. (2000) analyzed the mean wind speed, mean wind direction, mean wind inclination, turbulent intensity, integral scale, gust factor, wind spectrum, the acceleration response, and the natural frequencies of the Tsing Ma suspension bridge based on the measured wind and structure responses when typhoon just crossed over the bridge.
2.1.2 Analytical methods in the frequency domain

Analytical method for predicting buffeting response is an alternative way to study the aerodynamic behavior of long span cable-supported bridges. Most of the existing analytical methods for buffeting analysis of long span cable-supported bridge in the frequency domain are developed on the basis of the method proposed by Davenport or Scanlan. Davenport (1961a, 1961b, 1962a, 1962b) first applied statistical concepts of the stationary time series and random vibration theory to the buffeting analysis of long span bridges. The concept of joint acceptance function was introduced to consider the effect of the temporal and spanwise cross correlation of buffeting loading, which were assumed to be same as that of wind turbulence. Besides, the concept of aerodynamic admittance was adopted to take account of the effects of unsteadiness and spatial variation of wind turbulence surrounding the cross section. In the next few years, Davenport (1963, 1964, 1966, 1969, Davenport et al., 1971) further considered the aerodynamic stiffness and aerodynamic coupling in his theoretical model for buffeting analysis. Following his works, significant achievements were accomplished by various investigators, both in buffeting analysis and in wind tunnel experiments of real bridges (Miyata and Ito 1972; Davenport & Tanaka, 1974; Holmes, 1975; Melbourne, 1979; Irwin, 1977a, 1977b, 1978).

Extensive researches carried out by Scanlan and his co-workers in the 1970s (Scanlan and Gade, 1977; Scanlan, 1978b) indicated that self-excited aeroelastic forces would exert an important influence on the buffeting response of bridges. As such, Scanlan and his co-workers (Scanlan and Tomko, 1971; Scanlan, 1978a) later suggested that the wind excitation should include not only the aerodynamic forces arising from wind turbulence but also the aeroelastic forces due to the motion of the deck. In the aeroelastic forces they suggested, aeroelastic stiffness, damping effects and aeroelastic coupling between flexural and torsional vibrations were included in terms of sets of flutter derivatives. The aeroelastic forces due to lateral motion of the deck and its associated aeroelastic coupling were further included in terms of the eighteen frequency-dependent flutter derivatives. The buffeting forces are linear to the fluctuations of wind speed, i.e., the aerodynamic coefficients are independent of wind turbulence. The analysis results with this theory (Beliveau et al. 1977) showed the notable effects of self-excited forces on buffeting response.
The effect of the unsteadiness of aerodynamic forces is not included in the earlier Scanlan’s theory. Hence, to overcome this weakness, Gu and his colleagues, and also Scanlan himself, introduced the admittance functions, as in the Davenport’s theory, into mode-by-mode buffeting analysis with the Scanlan’s earlier theory (Gu and Xiang, 1992; Chen, 1993; Scanlan, 1993). Nowadays, the buffeting forces of lift ($L_b$), drag ($D_b$) and pitching moment ($M_b$) per unit deck length are commonly expressed as follows (Chen et al., 2000a, 2000b).

\[
L_b(t) = \frac{1}{2} \rho_s U_m^2 b \left[ C_L \chi_{Lw} (k) \frac{2u(t)}{U_m} + \left( C'_L + C_D \chi_{Lw} (k) \right) \frac{2w(t)}{U_m} \right]
\]  

(2-1a)

\[
D_b(t) = \frac{1}{2} \rho_s U_m^2 b \left[ C_D \chi_{Dw} (k) \frac{2u(t)}{U_m} + C'_D \chi_{Dw} (k) \frac{2w(t)}{U_m} \right]
\]  

(2-1b)

\[
M_b(t) = \frac{1}{2} \rho_s U_m^2 b \left[ C_M \chi_{Mw} (k) \frac{2u(t)}{U_m} + \left( C'_L + C_D \chi_{Mw} (k) \right) \frac{2w(t)}{U_m} \right]
\]  

(2-1c)

where $L_b(t)$, $D_b(t)$, and $M_b(t)$ are the buffeting lift, drag and moment respectively, on the bridge deck of unit span length at a given position; $\rho_s$ is the air density; b is the deck width; $C_L$, $C_D$ and $C_M$ are the lift, drag and moment coefficients obtained from wind tunnel tests of bridge deck section model; $C'_L$, $C'_D$ and $C'_M$ are the slopes of $C_L$, $C_D$ and $C_M$ at the angle $\alpha$, respectively; $U_m$ is the mean wind speed; $\alpha$ is the angle of attack of normal incident wind referring to the horizontal plane of the bridge deck; $u(t)$ and $w(t)$ are the wind speed components in the horizontal and vertical direction, respectively; $\chi_{Lw}$, $\chi_{Dw}$, $\chi_{Dw}$, $\chi_{Dw}$ and $\chi_{Mw}$ are the frequency-dependent aerodynamic functions or transfer function between velocity fluctuations and buffeting forces; $k = b \omega / U_m$ is the reduced frequency; and $\omega$ is the circular frequency of wind turbulence.

The self-excited forces per unit deck length are commonly described utilizing flutter derivatives as follows:

\[
D_{st}(t) = \rho_s U_m^2 b \left\{ k P'_y(v) \frac{\dot{v}}{U_m} + k P'_z(v) \frac{\dot{\alpha}}{U_m} + k^2 P'_s(v) \alpha + k^2 P'_s(v) \frac{\dot{P}}{b} \right\}
\]  

\[
+ k P'_z(v) \frac{\dot{h}}{U_m} + k^2 P'_s(v) \frac{h}{b}
\]  

(2-2a)

\[
2-4
\]
\[
L_{\infty}(t) = \rho_a U_m^2 b \left( kH_1'(v)\frac{\dot{h}}{U_m} + kH_2'(v)\frac{\dot{b}}{U_m} + k^2 H_3'(v)\alpha + k^2 H_4'(v)\frac{\dot{h}}{b} \right) 
+ kH_5'(v)\frac{\dot{p}}{U_m} + k^2 H_6'(v)\frac{p}{b}
\]

(2-2b)

\[
M_{\infty}(t) = \rho_a U_m^2 b^2 \left( kA_1'(v)\frac{\dot{h}}{U_m} + kA_2'(v)\frac{\dot{b}}{U_m} + k^2 A_3'(v)\alpha + k^2 A_4'(v)\frac{\dot{h}}{b} \right) 
+ kA_5'(v)\frac{\dot{p}}{U_m} + k^2 A_6'(v)\frac{p}{b}
\]

(2-2c)

where \( k = \omega b/U_m \) is the reduced frequency; \( \nu = 2\pi/k \); \( \omega \) is the circular frequency of vibration; \( h, p, \) and \( \alpha \) are the vertical, lateral and torsional displacement of the bridge deck at a given position, respectively; and \( H_i', P_i', A_i'(i=1-6) \) are non-dimensional flutter derivatives, which are functions of the reduced frequency and depend on the geometrical configuration of the bridge deck section.

Although some frameworks of multi-mode buffeting analysis methods had already been proposed in the eighties of last century (Lin and Yang, 1983; Scanlan, 1988), the computation work of performing the multi-mode analysis of buffeting response was enormous and time-consuming. Analysis of buffeting response was always done by mode-by-mode method before significance evolution of computation capacity in the middle of the nineties. Nowadays, the multi-mode effects and aerodynamic inter-mode coupling effects are often considered in the frequency domain (Jain et al., 1996; Xu et al., 1998; Chen et al., 1999; Katsuchi et al., 1999; Chen et al., 2000a; and Chen et al., 2001). A fully-coupled 3D buffeting analysis method in the frequency domain has been also proposed based on FEM (finite element method) and PEM (pseudo excitation method), which make the computing efficiency be conspicuously promoted. With this method, not only the effects of multi-modes and the aerodynamic inter-mode coupling, but also the varying wind properties along bridge deck and the interaction among the bridge deck, cables and towers can be automatically taken into consideration (Xu et al., 1998; Xu, 1999; Sun et al., 1999; Xu et al., 2000). However, the frequency domain analysis is restricted to linear structure excited by stationary wind loads without aerodynamic nonlinearities and only the standard deviation buffeting responses can be obtained.
2.1.3 Analytical methods in the time domain

With the rapid rising of computation capacity of the computer, it is now possible to simulate wind turbulent and analytically investigate the buffeting response in the time domain. Buffeting analysis in the time domain can take account of aerodynamic and structural nonlinearities. Lin and his co-workers presented the time-domain method for predicting bridge response to turbulent winds by using Ito’s stochastic differential equations (Lin and Arianratnam, 1978; Lin, 1979a; Lin, 1979b; Lin and Arianratnam, 1980). To consider the multi-mode coupling effect on buffeting response, Lin and Yang (1983) proposed a general linear theory for the computation of cross spectra of the deck response to turbulent wind by using the time domain method. Scanlan has made great efforts in investigating the self-excited forces. By expressing the self-excited forces with time-dependent aerodynamic indicial functions, his theory can be used in the time domain (Scanlan et al., 1974; Scanlan, 1984). The indicial functions can be either measured directly through wind tunnel test or derived from the flutter derivatives obtained often by section model wind tunnel test (Scanlan and Budlong, 1972; Scanlan et al., 1974; Scanlan, 1993; Scanlan 1996).

Most of the previous studies concerning the buffeting response have used the quasi-steady theory for modeling the aerodynamic forces, thus ignoring the frequency dependent characteristics of unsteady aerodynamic forces in the numerical scheme. Chen et al. (2000) presented a time domain approach for predicting the flutter and buffeting response utilizing frequency dependent unsteady aerodynamic forces which are expressed in terms of convolution integrals involving the aerodynamic impulse function and structural motions or fluctuating wind velocities. The aerodynamic impulse functions are obtained from the measured flutter derivatives, aerodynamic admittance function, and spanwise coherence of aerodynamic forces using rational approximations, in which all the coefficients can be determined by the linear and nonlinear least-squares methods.

2.1.4 Buffeting of long span cable-stayed bridge during construction
The concept of cable-stayed bridges is to use inclined cable to support the bridge girder so that the span length between pylons can be increased. The early cable-stayed bridges were constructed by the use of stiff girder to take care of the local bending moments between cables as well as provide enough strength for construction. By decreasing the spacing of the cable supports, the local bending moments can be reduced significantly. The modern cable-stayed bridges thus utilize closely-spaced cables with flexible deck girders. Consequently, the girder can be more flexible which reduces the global bending moment of the deck girders. The trend of constructing long span cable-supported bridges further increases the flexibility of the deck girders and hence their sensitivity to wind as well. In most cases, the design of long span cable-stayed bridges is governed by the dynamic response of the deck to turbulent wind. Vertical dynamic response is mainly controlled by the rigidity of cables which is very similar for the bridge in operation and during construction. Torsional dynamic response is mainly governed by the torsional rigidity of the box-girder and the lateral planes of stay-cables. Lateral dynamic response is controlled by the deck inertia and also by the connection conditions between deck and pylons, and deck and intermediate piers in the side spans. However, cable-stayed bridges are usually constructed by free-cantilever method. The lateral bending stiffness of the girder may be insufficient to ensure the stability of the cantilever arm. Construction situations are much more severe for lateral vibrations than in the completed bridge. The vertical and torsional vibrations of the cantilever deck girder could also be prone to vibrate under wind attack. This is one major drawback of cable-stayed bridges as compared with cable-suspension bridges.

To ensure working ability at low wind speed and structural safety at high wind speed, aerodynamics effect on long span cable stayed bridges under construction are very important. The erection of bridge girders starting from the pylon ensures that the torsional bridge deck rigidity may become active in increasing bridge flutter velocity at the beginning of the construction stage. However, the positive influence if bridge deck rigidity decreases during the construction stage and critical flutter wind velocities may become lower as the cantilever is increased. The most widely used method to strengthen the deck during construction is by the installation of temporary cable to the bridge girders. However, it has been reported that in the case of Normandie Bridge, use of temporary tie-down may probably be subjected to a high construction cost of anchor
block and risk of ship collision during storm (Virlogeux 1992). The free cantilever of the bridge had to be extended for a length of 428 m which is almost the half of the main span length out from the pylons to mid-span. The corresponding cantilever to width ratio is approximately 20 which is the longest cantilever yet constructed. Hence, the erection stages of Normandie Bridge had to be carefully investigated both analytically and experimentally. The investigations revealed that the girder could be prone to oscillate laterally and it was therefore decided to install a large tuned mass damper on the bridge deck to supplement damping capacity of the bridge (Conti et al. 1996 and Larose et al. 1996). The stability problems arising when cantilevering a very slender girder were also to be overcome by the use of tuned mass dampers for the construction of the Karnali River Bridge in Nepal (Arzoumanidis and Kunihiro 1994).

2.1.5 Simulation of wind velocity

A series of time histories of fluctuating wind velocity in vertical and alongwind directions at various points along the deck of a long span bridge is essential for performing buffeting analysis of the long span bridge in time domain. Unfortunately, the collected wind velocity time histories from field measurement are usually not enough for carrying out a buffeting analysis of long span bridges in the time domain. Several numerical methods of simulating wind velocity fields such as the spectral representation method (Shinozuka, 1971; Shinozuka and Jan, 1972; Yang, 1972, 1973; Shinozuka, 1990; Deodatis and Shinozuka, 1989) and the digital filtering method using ARMA model (Samaras et al., 1985; Li and Kareem, 1990; Minolet and Spanos, 1990) were developed for the sake of carrying buffeting analysis of structures in the time domain. The spectral representation method can provide unconditionally stable results but it is computationally expensive due to the repetitive decomposition of the spectral matrix at every frequency step in the simulation when a series of wind velocities at various locations on a long span bridge are generated. On the other hand, the digital filtering method is computationally efficient and it does not require large computer storage, but the algorithm requires extra attention to ensure the numerical stability of a discrete system (Yang et al., 1997).
A fast spectral representation approach (Cao et al. 2000), which is developed on the basis of the method proposed by Yang (1972, 1973), Deodatis and Shinozuka (1989), is particularly suitable for the simulation of stochastic wind velocity field on long span bridges. The fast spectral decomposition method involves the assumptions that the bridge deck is horizontal at the same elevation, the mean wind speed and wind spectra do not vary along the bridge deck, and the distance between any two successive points where wind speeds are simulated are the same. The time histories of the alongwind component \( u(t) \) and the vertical wind component \( w(t) \) at the \( j \)th point can be generated by the following equations

\[
\begin{align*}
u_j(t) &= \sqrt{2(\Delta \omega)} \sum_{m=1}^{n} \sum_{l=1}^{N} \sqrt{S_u(\omega_{ml})} G_{jm}(\omega_{ml}) \cos(\omega_{ml}t + \Phi_{ml}) \quad (2-3a) \\
w_j(t) &= \sqrt{2(\Delta \omega)} \sum_{m=1}^{n} \sum_{l=1}^{N} \sqrt{S_w(\omega_{ml})} G_{jm}(\omega_{ml}) \cos(\omega_{ml}t + \Phi_{ml}) \quad (2-3b)
\end{align*}
\]

where \( \Delta \omega \) is the frequency interval between the spectral lines; \( N \) is the total number of frequency interval; \( j=1,2,\ldots,n \); \( n \) is the total number of points where wind speeds are simulated; \( S_u(\omega) \) and \( S_w(\omega) \) are the PSD functions of the along wind and vertical wind respectively; \( \Phi_{ml} \) is a random variable uniformly distributed between 0 and 2\( \pi \); and

\[
G_{jm}(\omega) = \begin{cases} 0, & \text{when } 1 \leq j < m \leq n \\ \frac{c_{\lceil j-m \rceil}}{\sqrt{1 - c^2}}, & \text{when } m = 1, m \leq j \leq n \\ \frac{c_{\lfloor j-m \rfloor}}{\sqrt{1 - c^2}}, & \text{when } 2 \leq m \leq j \leq n 
\end{cases} \quad (2-4)
\]

\[
C = \exp\left(-\frac{\eta_r}{2\pi}\right), \quad \eta_r = \frac{0.7477\Delta}{L_r} \sqrt{1 + 70.78 \left(\frac{\omega L_r^2}{2\pi U_m}\right)^2} \quad (2-5)
\]

\[
\omega_{ml} = (l-1)\Delta \omega + \frac{m}{n} \Delta \omega \quad (l=1,2,\ldots,N); \quad \text{and} \quad \Delta \omega = \omega_{up}/N \quad (2-6)
\]

where \( \Delta_{jm}=\Delta[j-m] \), the horizontal distance between points \( j \) and \( m \); \( C \) is the coherence function between points \( j \) and \( m \); and \( \omega_{up} \) is the upper cutoff frequency.

Yang (1972, 1973) showed that the efficiency of simulation can be enhanced by utilizing the FFT technique. To implement the simulation with the FFT techniques, Equations (2-3a) and (2-3b) can be rewritten in the following form.

\[
u_j(p \Delta t) = \text{Re} \left[ \sum_{m=1}^{n} h_{jm}(q \Delta t) \exp\left[i \left(\frac{m \Delta t}{n}\right)(p \Delta t)\right] \right] \quad (2-7)
\]
\[ w_j(p \Delta t) = \text{Re} \left\{ \sum_{m=1}^{2N} h_{jm}(q \Delta t) \exp \left[ i \left( \frac{m \Delta t}{n} \right) (p \Delta t) \right] \right\} \]  

(2-8)

where \( p=0,1,\ldots,2N-1 \); \( j=1,2,\ldots,n \); \( q \) is the remainder of \( p/2N \); \( q=0,1,2,\ldots,n-1 \) and \( h_{jm}(q \Delta t) \) is given by

\[ h_{jm}(q \Delta t) = \sum_{l=0}^{2N-1} B_{jm}(1 \Delta \omega) \exp \left( i \frac{2\pi l q}{N} \right) \]  

(2-9)

\[ B_{jm}(1 \Delta \omega) = \begin{cases} \sqrt{2(\Delta \omega) S(\omega)} G_{jm} \left( 1 \Delta \omega + \frac{m \Delta \omega}{n} \right) \exp(i \Phi_m), & \text{when } 0 \leq l < N \\ 0, & \text{when } N \leq l < 2N \end{cases} \]  

(2-10)

where \( S(\omega) = S_u(\omega) \) or \( S_w(\omega) \).

It can be seen from Equations (2-9) and (2-10) that \( h_{jm}(q \Delta t) \) is the Fourier transformation of \( B_{jm}(1 \Delta \omega) \) and therefore the simulation can be performed with much higher efficiency by the use of the FFT technique.

2.2 VIBRATION CONTROL OF LONG SPAN BRIDGES

In the past two decades, construction of long-span cable-supported bridges has been very active in the world. However, long span bridges are often flexible and low damped and hence they are prone to vibrate under dynamic loading. The most commonly used technique for the vibration control of bridge is the vibration isolation systems. These systems are placed at the foundations, supports or bearings of the bridge structure. They are effective for vibration protection of earthquake and vehicle loadings, but less effective for wind excitations. A detailed discussion and review of the isolation technique can refer to the work by Skinner et al (1993).

The wind-induced vibration of bridges can be reduced by means of aerodynamic modification, and mechanical control devices. Many of these applications were summarized by Ito (1987, 1989) and Fujino (2002). Aerodynamics modification change the flow pattern around a structure directly to reduce aerodynamic force or moment acting on it by selecting better cross-sectional shape of suspended structure or installing some aerodynamic devices such as the use of wind-noise for decks, corner cut for pylons, center barrier, fairing, and air gap for decks (Ito 1987, 1992). In contemporary
design of long span bridges, a good aerodynamic shape of the bridge is always obtained through a series of wind tunnel tests or computational fluid mechanics. When the aerodynamic countermeasures are not able to reduce wind-induced vibrations sufficiently, mechanical control devices may be considered to be employed (Ito and Miyata 1991; Fujino 2002). These measures may apply to the bridge decks, cables and towers. Structural control technologies can generally be classified as passive, active, semi-active and hybrid control methods. Passive control systems operate without requiring an external power source and utilize the motion of structure to develop the control forces. Passive control devices such as metallic dampers, friction dampers, viscoelastic dampers, tuned mass dampers, and tuned liquid column dampers, do not increase the vibration energy in a passively controlled structural system and are thus inherently stable. As compared with passive control systems, active control is a relatively new area of research and technological development. Active control systems require external energy supply to apply forces to the structure in a prescribed manner. Control forces are developed based on feedback from sensors which measure the excitation and/or the response of the structure. These forces can be used to add and dissipate energy in structures. Active mass dampers and active tendon systems are some of the devices being developed and tested in the laboratory and in some cases, in actual structural applications. Semi-active control is a compromise between passive and active control systems. Semi-active control systems require much less input energy in comparison with the active one and the input energy is used to modify the damper properties leading to the optimal control of structural vibration. Control forces are developed based on feedback from sensors which measure the excitation and/or the response of the structure. Such examples are the variable orifice dampers, variable friction damper system, and magnetorheological (MR) dampers. A hybrid system consists of an active control system and a passive control system and thus increases the performance and robustness of the control system. Hybrid mass damper (HMD) is a combination of a passive tuned mass damper and an active control actuator and is the most common control device employed in full-scale civil engineering applications.

Many passive mechanical dampers have been successfully applied to bridge towers to mitigate wind-induced vibrations, such as the tuned mass dampers installed on the Meiko-Nishi bridge pylons and on the Akashi bridge pylons (Fujino 1993). The
tuned liquid damper which utilizes the liquid motion as an energy dissipater is installed on the Higashi Kobe Bridge pylons and on the Toda Bridge pylons (Fujino 2002). Recently, active control means of using hybrid TMD are applied to some bridge towers during erection phases, such as the active mass damper installed on the pylon of the Tokyo Port Bridge and the hybrid active TMD installed on the tower of the Hakcho Bridge (Fujino 1993). The vibration problem for long span bridges may occur in either construction or completion stages. Large wind-induced vibrations were found for some free-standing bridge towers, damping devices were installed on the towers during their construction to mitigate the tower vibration (Fujiwara et al 1991; Ueda et al 1992; Ogawa et al 1995). It was also reported that aeroelastic instability of a long suspension bridge deck may occur during the early phases of deck erection (Tate et al 1971; Yamaguchi et al 1971; NMI 1977). Therefore, some temporary measures for preventing aeroelastic instability of the bridge were applied to the bridge decks during the erection stage (NMI 1977, Brancaleoni and Borton 1981; Larsen 1997). Some vibration control measures were also applied to the completed bridges to sustain good service condition, such as the tuned mass damper installed on the bridge deck of Kessock Bridge (Wallace 1985), the tuned mass damper on the deck of Great Belt East Bridge (Larsen et al 1995), the tuned mass damper on the tower of the Aratsu-Ohashi Bridge (Yoshimura et al 1989), and the tuned mass damper on the tower of the Yokoama Bay Bridge (Saito et al 1988).

In recent years, extensive researches have been performed on the active control measures for the mitigation of bridge vibration. Leipholz and Abdel-Rohman (1986) discussed some active control mechanisms for simple span bridges, such as the use of auxiliary mass, tendons, dampers, and aerodynamic appendages. Yang and Giannopoulos (1978, 1979a, 1979b) presented dynamic analysis of a two-cable-stayed bridge subjected to active feedback control. The control forces from each cable acting on the bridge are regulated by the movement of the hydraulic ram which in turn is actuated by the control system, based on the feedback measurements of the bridge motion from the sensor. They showed that the bridge vibration can be reduced appreciably and the flutter speed can be raised significantly by a suitable choice of the design values for the active control parameters. Abdel-Rohman and Leipholz (1978a) proposed to control the lowest three modes of a simply-supported beam, simulating a
bridge, by means of a collocated actuator/sensor pair. They also presented an active tendon control mechanism for the control of bridges (Abdel-Rohman and Leipholz 1978b). Carotti et al (1987) suggested an active control system to protect a pipeline suspension bridge from excessive wind-induce vibration. Meirovitch and Ghosh (1987) discussed the use of the IMSC method to suppress the unstable flutter mode in a suspension bridge. Abdel-Rohman and Nayfeh (1987a, 1987b) presented both active and passive control mechanism for suppressing the nonlinear oscillations in bridges. Kobayashi and Nagaoka (1992) investigated the flutter control of a suspension bridge by using active control wings set above the both edges of the bridge deck. The use of the attached controlled surfaces either actively or passively beneath both edges of the bridge deck was proposed and its effectiveness was confirmed numerically as well as experimentally (Ostenfeld and Larsen, 1992; Wilde and Fujino, 1998; Wilde et al., 1999). The analysis results show that the onset of flutter wind speed can be increased by more than 50% (Wilde et al., 1999). A semi-active hydraulic bridge vibration absorber that can be retrofitted to an existing bridge was described by Patten et al (1996). An active tendon control method for cable-stayed bridges was investigated by Achkire and Preumont (1996). Preidikman and Mook (1997) developed a method for actively suppressing flutter of suspension bridges. Their approach is to attach a light-weight wing below the deck and to use a feedback control system to regulate the angle of wind incidence. They showed that this system can substantially increase the critical wind speed.

The vibration control of bridges can be implemented by passive means. In order to improve the flutter stability problem in the construction stage of suspension bridges, Brancaleoni and Brotton (1981) and Larsen (1997) investigated the use of unsymmetrical water masses ever used on the Humber bridge deck as suggested by NMI (1977). This method, placing an unsymmetrical water mass on the windward side of the deck to decrease the aerodynamic torque, was proved to be effective in increasing the critical wind speed. Brancaleoni and Brotton (1981) also investigated the use of damping plates for improving the flutter stability. As unacceptable levels of oscillations were predicted for the Dao Kanong cable-stayed bridge by the wind tunnel tests, tuned mass dampers were installed on the bridge decks and pylons (Freeman et al 1987). The effectiveness of the tuned mass dampers on the decks was investigated by Malhortra
and Wieland (1987). They showed that two separate TMDs, tuned to flexural and torsional modes respectively, could eliminate completely the response due to vortex-shedding and reduce the response under turbulent wind by nearly 35%. The flutter wind speed was also raised by 15%. The effectiveness of tuned mass dampers for controlling the bridge flutter were studied both theoretically and experimentally by Nobuto et al. in 1988 and Gu et al. in 1998. With the concern of serious vibrations due to vortex-excitation at low wind speed, Chen et al. (1993) and Larsen (1993) studied the effectiveness of tuned mass dampers in reducing the harmonic responses of bridge. In Lin et al. (2000), a combination of the bending and torsional dampers was studied for suppressing the coupled vertical and torsional buffeting responses and flutter instability of long span bridges. Pourzeynali and Datta (2002) investigated the passive control of the flutter condition of suspension bridge using a combined vertical and torsional tuned mass damper (TMD) system. The bridge-TMD system is analyzed to determine the onset wind speed of flutter instability by using a finite element approach. Apart from theoretical studies, wind tunnel studies on the performance of tuned mass dampers in reducing buffeting response of the Normandy Bridge during construction were carried out by Conti et al. (1996) and Livesey et al. (1996). To reduce the effects of wind on the incomplete bridge deck during the construction phase, a deck sway stabilizer comprising a tuned mass damper (TMD) was installed on the deck. The test results indicated that the TMD can reduce the response by 35%.

2.3 TUNED LIQUID COLUMN DAMPER AND ITS ENGINEERING APPLICATIONS

2.3.1 Development of tuned liquid column dampers

The new high strength materials incorporated with the advanced construction technologies result in very flexible and low damped structures. Serious vibrations may occur under strong winds. Even though most buildings and structures may not have safety problems under strong winds, wind-induced vibration could cause discomfort to occupants, damage to curtain wall, or malfunction of equipment inside the building. As such, it is of practical interest to develop effective control techniques for the suppression of structural vibrations due to fluctuating wind loads.
In order to reduce structural vibrations excited by wind or earthquake, various control devices and systems have been developed in the past, such as viscoelastic dampers (Caldwell 1986; Mahmoodi and Keel 1986; Tsai and Lee 1993; Nielsen et al. 1994; Lai et al. 1995; Shen and Soong 1995; Iwata et al. 1998; Nakamura and Kaneko 1998), hysteretic dampers (Skinner et al. 1975; Skinner et al. 1980), friction damper (Pall et al. 1980; Pall and Marsh 1982; Pall and Pall 1993), viscous fluid dampers (Arima et al. 1988; Constantinou et al. 1993; Makris et al. 1993), active tendons (Roorda 1975; Yang and Giannopoulos 1978), active bracing system (Soong et al. 1991; Reinhorn et al. 1993), aerodynamic appendages (Klein et al. 1972; Chang and Soong 1980), tuned mass dampers (McNamara 1977; Wiesner 1979), tuned liquid dampers (Bauer 1984; Welt and Modi 1989a,b; Fujino et al. 1992; Tamura 1995) and tuned liquid column dampers (Sakai et al. 1989; 1991a,b), etc. Many of these systems have been successfully applied to high-rise buildings, towers, and slender structures (Soong et al. 1994; Kareem and Tamura 1996; Soong and Dargush 1997; Tamura 1995, 1997).

Tuned liquid column damper (TLCD) was introduced by Sakai et al. (1989) to suppress wind or earthquake induced vibrations of tower-like structures such as tall building and pylons of cable-supported bridges. The energy of structural vibration is dissipated by the passage of liquid through an orifice in a U-shaped tube. Liquid contained in the tube can be utilized as a secondary liquid source for fire emergency. In their paper, TLCD units were installed in the structure model, which subjected to horizontal motions and the effectiveness of the TLCDs were investigated by experiments. Further to his works, Xu et al. (1992) examined the effectiveness of the TLCD to control the along-wind response of tall building structures theoretically. A structure modeled as a lumped mass multi-degree of freedom system was analyzed under the alongwind turbulence and the across wake excitation. Balendra et al. (1995) studied the effectiveness of a TLCD to control the wind-induced vibration of a tower. Zhang and Zhang (1993), and Zhang et al. (1993) proposed the use of crossed tuned liquid damper with tube-like containers for the suppression of alongwind and across wind vibrations of structures. The above studies concentrated on the horizontal vibration control under wind loading. Won et al. (1997) used random vibration theory to evaluate the performance of a flexible building with a TLCD under the random seismic load. The earthquake motion was simulated as a non-stationary stochastic process with both
frequency and amplitude modulation. Their investigations demonstrated the
effectiveness of the TLCD in suppressing earthquake responses of structures.

Aiming at facilitating frequency tuning for suppressing lateral oscillation of a
ship, Kagawa et al. (1989) and Shyu et al. (1996) proposed a pressurized tuned liquid
column damper (PTLCD) which is a U-shaped container with uniform cross-section.
Liquid is filled into the container and two chambers are formed for compressed air. The
frequency of liquid can be adjusted by air pressure inside the air chambers of the
PTLCD. The theoretical studies carried out by Shyu et al. (1996) showed, however, that
the efficiency of the PTLCD is sensitive to pressure inside the air chambers. In addition,
a newly developed liquid column damper system with a frequency adjustable device
was proposed by Shimizu and Teramura (1994) for the vibration suppression of
horizontal motion of structures. This system can adjust the frequency of the liquid
sloshing according to the control requirements. Besides, a variation of the TLCD,
termed as liquid column vibration absorber (LCVA), was proposed and investigated by
Watkins (1991), Watkins and Hitchcock (1992), and Hitchcock et al. (1997a, b). The
nominally different cross-sectional areas of the vertical and horizontal columns allow
the natural frequency of a LCVA to be a function of the container geometry, rather than
the length of liquid column alone. The flexibility can thus be obtained by selecting a
difference in the cross-sectional area of the horizontal and vertical part of the tube. A bi-
directional configuration of the LCVA was also reported by Hitchcock et al. (1997b) to
suppress the vibrations in different directions. However, the variation in the cross
sectional area of the horizontal and vertical part of the tube is very limited and hence the
flexibility of frequency tuning may not be large enough for the application in some
circumstances.

In order to investigate the influence of TLCD parameters on the control
performance, some optimal parametric studies were performed. Gao et al. (1997)
presented numerical studies for seeking the optimum parameters of the TLCD in
reducing the structural responses excited by harmonic loading. They indicated that the
optimal tuning ratio would be independent of the excitation. They also performed a
parametric analysis to the optimal parameters of the LCVA, and proposed a new type
V-shaped tuned liquid column damper to increase the capacity of resisting strong
vibrations. Chang et al. (1998) studied the optimal design parameters of single and
multiple tuned liquid column dampers under the assumption that the building vibrated in a dominant mode and was subjected to a Gaussian white noise excitation. Some useful design formulas and procedures for the single and multiple TLCDs were proposed. More recently, Yalla and Kareem (2000) presented a new approach to determine the optimum head loss coefficient for a given level of wind or seismic excitation in a single step without resorting to iterations. However, passive damper system cannot perform optimally at different levels of excitation. In order to overcome this shortcoming, semi-active TLCD with the optimal control of head loss coefficient have therefore been proposed and studied by Kareem (1994), Haroun and Pires (1994), and Yalla et al. (2002). Haroun and Pires (1994) further proposed a hybrid liquid column damper to control lateral vibration of structure under earthquake excitation by delivering a desired optimal control force to the system via controlling the orifice together with the liquid column pressure actively. More recently, Yalla and Kareem (2003) carried out experimental studies on the application of a semi-active TLCD in suppressing the lateral motion of structures. Different experiments using scale models of structures together with a semi-active TLCD were conducted. In their studies, a gainscheduled control law was experimentally verified for achieving optimal damping.

Optimization studies discussed above showed that there exists optimal tuning ratio which leads to maximum vibration reduction. However, the current design method used in TLCD assumes the natural frequency of the structure is precisely known. Error in the estimation of the natural frequencies of a structure is always presented. Aiming at improving the sensitivity of the TLCD, use of multiple tuned liquid column dampers which consists of a number of tuned liquid column dampers whose natural frequencies are distributed over a certain range around a particular fundamental frequency of the structure has been studied by Chang et al. (1998), Sadek et al. (1998) and Gao et al. (1999). These studies reported that an optimized MTLCD can be more efficient than a single optimized TLCD and provide a more robust performance in a mistuning condition when compared with the performance of a single TLCD. Due to the nonlinearity damping characteristics of TLCD, the optimal head loss coefficient is dependent on the excitation level. However, for some long span bridges with closely spaced natural frequencies, the main contributions of the response may come from several modes. In this case, it is important to develop control measures for suppressing
multimode responses of long span bridge. This may be implemented by using several dampers together, each damper being tuned to a particular mode.

All the above investigations for the tuned liquid column damper are mainly concerned with the vibration control of horizontal motions for tall building structures. Xue et al. (2000) first explored the possibility of applying the TLCD for the suppression of the torsional vibrations of structures. Both experimental and theoretical results obtained by them indicated that the TLCD could be an effective device in suppressing the pitching motion of structures if the parameters of the TLCD are properly selected. Later, Xue et al. (2002) also investigated the effectiveness of TLCD to reduce the torsional buffeting response of long span cable suspension bridge. They demonstrated that the TLCD is an effective device for either reducing buffeting response or increasing critical flutter wind speed of the bridge.

2.3.2 Application of tuned liquid column dampers

Because of new materials and advanced technologies used in construction, building structures and cable-supported bridges have now been constructed with increasing height or central span. The structures usually possess low inherent damping and are therefore prone to wind excitations. There have been several applications of TLCD in the world, one of which can be found in Japan. The TLCD has been installed at the top of the Hotel Cosima in Tokyo (see Figure 2-5). The building is a 26 story steel building with a height of 106.2 meters. This high rise building has a high height to width ratio and is prone to wind excitation. The foundation of the building is firmly connected to the ground by high strength steel pre-tensioned grout anchors. In addition, a super structure is adopted as the frame of the building in order to resist earthquakes and wind loads. The 51 tons TLCD with pressure adjustment was installed at the top floor. Field measurement results indicate that the TLCD can reduce the maximum acceleration to about 60% and the RMS acceleration to about 40%, comparing with that without control (Shimizu and Termura, 1994; Teramura and Yoshida 1996).

Twenty individual bi-directional LCVA (a variation of TLCD) have been installed in the Prospect Communications Tower in Sydney to mitigate the effects of wind
excitation (Hitchcock et al., 1999). The tower is a steel frame structure with a height of 67m located at Prospect in the western suburbs of Sydney, Australia. A LCVA system containing approximately 280 kg fresh water was installed at the 57 meter level of the tower to control structural vibrations corresponding to the first mode natural frequency of the tower. Full-scale measurement has been conducted after its construction. Comparative study on the full scale measurement results before and after the installation of LCVA indicated that the total structural damping ratio of the tower could be increased from approximately 0.55% to approximately 2.5% of critical damping. In addition, it was found that the wind-induced acceleration response of the tower was almost halved at mean wind speeds of approximately 20 m/s.

More recently, a TLCD system has been installed in the 48-story hotel and residential tower, One Wall Centre, the tallest building in Vancouver, British Columbia (see Figure 2-6). The One Wall Centre includes two 50,000 gallon (189 tons) specially-designed U-shaped tanks of water which are installed in the tower’s mechanical penthouse in order to lessen the lateral movement of building against both earthquakes and strong winds. Each TLCD has a broad horizontal chamber at the bottom with a column of water at each end, thus resembling a cup within a cup. The TLCD system not only solved the structural challenges presented but also saved an estimated 2 million dollars in construction costs compared to other conventional damping systems like tuned mass dampers. This is because the TLCD system eliminates the installation of a pump station and a backup generator at the base of the building for the fire suppression. Besides, the water tanks act as heat sinks for building’s heat pumps.

At present, no other researches discuss the application of tuned liquid column dampers for the suppression of lateral and torsional vibration of bridge deck. This thesis thus aims to develop and apply novel tuned liquid column dampers for the suppression of lateral and torsional vibration of bridge deck with particular focus on the bridge during construction and at completion.
Figure 2-1 The World’s Longest Suspension Bridge — Akashi Straits Bridge in Japan

(Courtesy of Honshu - Shikoku Bridge Authority)
Figure 2-2 Elevation of the Akashi Straits Bridge in Japan

(Courtesy of Honshu - Shikoku Bridge Authority)
Figure 2-3 The World’s Longest Cable-Stayed Bridge — Tatara Bridge in Japan

(Courtesy of Honshu - Shikoku Bridge Authority)
Figure 2-4 Elevation of the Tatara Bridge

(Courtesy of Honshu - Shikoku Bridge Authority)
(a) Liquid damper with pressure adjustment

(b) Side view of the Hotel Cosima

Figure 2-5 Liquid dampers in the Hotel Cosima (from Teramura and Yoshida, 1996)
Figure 2-6 The Tallest Building in Vancouver with a Pair of Tuned Liquid Column Dampers on its Roof

(Courtesy of Glotman-Simpson group)
CHAPTER 3

MULTIPLE TUNED LIQUID COLUMN DAMPERS FOR TORSIONAL VIBRATION CONTROL OF STRUCTURES: EXPERIMENTAL INVESTIGATION

3.1 INTRODUCTION

As mentioned in Chapter 2, modern long span cable-supported bridges are very flexible and lightly damped and thus vulnerable to wind-induced vibration. In particular, during the erection of long span bridges, the bridge deck lacks torsional continuity from pylon to pylon and its torsional stiffness is lower than that after the bridge is completed (Larsen 1995). Large torsional motion of bridge deck units may thus occur during erection due to gust winds. The permanent or temporary installation of control devices in the bridge deck units during erection may be an effective way for reducing the torsional motion of bridge deck.

Sun et al. (1995) investigated the performance of tuned liquid damper (TLD) for reducing pitching vibration of a simple structure. They developed a nonlinear analytical model for the TLD using a nonlinear shallow water wave theory and verified the model by the experimental results. The investigation demonstrated that the TLD can suppress efficiently the torsional vibration of the structure. Xue et al. (2000) then explored the possibility of applying tuned liquid column damper (TLCD) to reduce the torsional vibration of a simple structure under harmonic excitation. Both theoretical and experimental studies they carried out showed that TLCD could also reduce effectively the torsional vibration if the parameters of STLCD were properly selected. However, in determining the parameters of TLCD for the torsional vibration, it is always assumed that the natural frequency of long span bridge is precisely known. But the estimation of natural frequencies of long span bridge often deviates from the actual values. The effectiveness of STLCD could deteriorate considerably if the frequency of liquid column motion was not properly tuned to the natural frequency of structure.
This Chapter thus aims to experimentally investigate the performance of MTLCDs for reducing torsional vibration of structures in comparison with STLCDs. A large structure model simulating the torsional vibration of bridge deck and several STLCDs and MTLCDs of different configurations are designed and constructed. A series of harmonically forced vibration tests are conducted to evaluate the effectiveness of MTLCDs in terms of the number of TLCD units in a MTLCD, the bandwidth of a MTLCD, the frequency tuning ratio, and the moment excitation amplitude. An averaging method is also used to identify the head loss coefficients of STLCDs and MTLCDs in conjunction with the free vibration test technique. It should be pointed out that the harmonic excitation used in this experiment may represent, to some extent, the excitation acting on the bridge deck caused by wind-induced vortex-shedding phenomenon. In Chapter 4, a theoretical model will be developed for the torsional vibration of the structure with MTLCD under harmonic excitation or white noise excitation and verified by using the experimental data obtained from this chapter. Further investigation on the effectiveness of STLCD and MTLCD for torsional vibration reduction of structures under random excitation will be carried out in Chapter 4.

3.2 EXPERIMENTAL SETUP AND CALIBRATION

A large steel structure was designed and constructed to model the torsional vibration of a bridge deck unit (see Figure 3-1). The steel structure was supported by a pivot at the middle point of its horizontal frame so that the structure could rotate around the pivot. Two springs were installed at the two ends of the horizontal frame to provide the structure with its torsional stiffness. The test platform was rigidly connected to the horizontal frame using four vertical steel members. The whole structure could be thus seen as a single degree of freedom system rotating about the pivot. The container of a STLCD or a MTLCD was fixed on the test platform and the weight of the container without water was regarded as part of the structural weight.

Three STLCDs and two MTLCDs of different configurations were designed and constructed in the experiments with their basic dimensions listed in Table 3-1 and shown in Figure 3-2. The overall water column dimensions of the STLCD of configuration No.1 were kept the same as those of the MTLCD with 3 TLCD units
(configuration No.2) or the MTLCD with 5 TLCD units (configuration No.3) so that a fair comparison of vibration reduction performance between the STLCD and the MTLCDs could be conducted. The STLCDs of configuration No.4 and No.5 were designed mainly for the determination of head loss coefficients of the MTLCDs of configuration No.2 and No.3, respectively. The opening ratio of the orifice of all the five configurations was kept at 50% throughout the experimental work. The STLCDs and MTLCDs were made of Perspex materials. The distance between the pivot and the centre line of the bottom tube of the STLCD or the MTLCD (H) was 0.98m.

<table>
<thead>
<tr>
<th>Table 3-1 Dimensions of TLCDs Used in Experiment</th>
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<td>Configuration No.</td>
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In forced vibration tests, an electrodynamics LDS exciter (Link Dynamic System) was used to apply a moment excitation to the horizontal frame. A B&K signal generator (Type 1027) was used to command the exciter to generate harmonic moment excitation and perform frequency sweep tests through a PA500 power amplifier. A B&K load cell was placed between the exciter and the structure to measure the amplitude of moment excitation. The connection between the exciter and the structure should be allowed to have some flexibility so that the small twist angle of the horizontal frame would not affect the applied load. The structural response was measured from a Kyowa accelerometer (type ASQ-1BL) located at one side of the horizontal frame of the structure at a distance of 1.42 m to the pivot. The signal from the accelerometer was then transferred into a Kyowa signal conditioner (Type VAQ-500A). An auto compensating wave gauge was placed into the central water column of the MTLCD to measure water motion. The recorded signals of the structural response and the water motion in the central column of the MTLCD were analyzed using the Global Lab data acquisition and processing system. To determine the head loss coefficients of STLCD of configuration No.1, No.4 or No.5, free vibration tests were carried out on the SLCD to
record the time-histories of water motion for a given initial water displacement. An averaging method was then used to identify the head loss coefficient.

To have a proper design of a STLCD and a MTLCD, the torsional stiffness, the natural frequency, the structural damping ratio, and the mass moment of inertia of the steel structure should be identified. First, a series of static moments were applied to the structure and the corresponding torsional displacement of the structure were recorded, from which the torsional stiffness of the structure was identified as 13,873 Nm/rad. Secondly, a series of free vibration tests of the structure without control were performed. From the recorded free vibration decay curves, the torsional frequency of the structure was found as 0.565 Hz and the averaged damping ratio of structure was around 1%. Based on the measured natural frequency and torsional stiffness of the structure, the mass moment of inertia of the structure was estimated as 1,100 N.m.s²/rad.

3.3 DETERMINATION OF HEAD LOSS COEFFICIENT

3.3.1 Mathematical model

The coupled equations of motion of the structure equipped with a STLCD for torsional vibration control have been derived by Xue et al (2000).

\[
(I_s + I_d)\ddot{\theta} + \rho AB(H + \frac{L - B}{2})\ddot{W} + C_s \dot{\theta} + (K_s + \rho A L g H)\theta + \rho A g BW = M_s \tag{3-1}
\]

\[
\rho A L \dddot{W} + \frac{\partial A}{2} \ddot{\delta} + 2 \rho A g W + \rho A B \left( H + \frac{L - B}{2} \right) \dddot{\theta} + \rho A g B \theta = 0 \tag{3-2}
\]

with the condition

\[
W \leq \frac{L - B}{2} - \frac{d}{2} \tag{3-3}
\]

where \(I_s\) is the mass moment of inertia of the structure with respect to the pivot; \(I_d\) is the mass moment of inertia of water column of the STLCD with respect to the pivot; \(\rho\) is the density of water; \(A\) is the cross-sectional area of water column of the STLCD; \(B\) is the horizontal width of the water column of the STLCD; \(H\) is the distance from the centre line of the bottom tube of the STLCD to the pivot; \(L\) is the total length of water column; \(C_s\) is the damping coefficient of the structure; \(K_s\) is the torsional stiffness of the structure; \(g\) is the acceleration of gravity; \(\delta\) is the head loss coefficient of the STLCD.
governed by the opening ratio of orifice; \( \theta \) is the torsional displacement of the structure; \( W \) is the relative motion of water to the container; \( d \) is the thickness of water column; and \( M_s \) is the external moment excitation.

Let us consider the free vibration of a STLCD only. The corresponding equation of motion of the water column is then

\[
\ddot{W} + \omega_n^2 W = -\frac{\delta}{2L} \dot{W} \left| \frac{\dot{W}}{\dot{W}} \right| \tag{3-4}
\]

where \( \omega_n = \sqrt{2g/L} \), the natural frequency of water motion. Clearly, the energy dissipation capacity of the STLCD depends on the head loss coefficient \( \delta \). Therefore, how to determine the head loss coefficient becomes essential to the torsional vibration control of the structure using a STLCD or a MTLCD. Belenda et al (1995) applied the harmonically forced vibration test technique to the STLCD to obtain the frequency response curve of water column motion and then used the half-power bandwidth method to determine the damping ratio \( \xi \). The head loss coefficient \( \delta \) was then calculated from \( \delta = 2L\pi(\xi/W_r) \), where \( W_r \) is the resonant water displacement. This approach is based on the equivalent linear equation of water column motion in a STLCD under harmonic excitation.

In this study, an averaging method is used to determine the head loss coefficient in conjunction with the free vibration test technique. Equation (3-4) is actually a nonlinear equation with quadratic damping. If the amplitude and phase angle of water column motion vary slowly with time \( t \) during the free vibration, then the method of averaging (Nayfeh and Mook 1995) gives the first order solutions of Equation (3-4) for the water displacement and velocity responses as

\[
W = \frac{a_0}{1 + \frac{2\delta \omega_n a_0}{3\pi L} t} \cos(\omega_n t + \beta_0) \tag{3-5}
\]

\[
\dot{W} = -\frac{\omega_n a_0}{1 + \frac{2\delta \omega_n a_0}{3\pi L} t} \sin(\omega_n t + \beta_0) \tag{3-6}
\]
where \( a_o \) and \( \beta_o \) can be determined by the initial conditions. In particular, let us assume that the initial water displacement is \( W_o \) and the initial water velocity is zero. Then, from Equations (3-5) and (3-6), one may have

\[
\begin{align*}
W_o &= W(0) = a_o \cos \beta_o \\
\dot{W}_o &= \dot{W}(0) = -\omega_o a_o \sin \beta_o = 0
\end{align*}
\]  

(3-7)

Clearly, Equation (3-7) leads to

\[
\begin{align*}
a_o &= W_o \\
\beta_o &= 0
\end{align*}
\]  

(3-8)

\[
W = \frac{W_o}{1 + \frac{2\delta \omega_o}{3\pi L}} W_o \cos \omega_o t
\]  

(3-9)

The positive peak in the free vibration response curve of the water, \( W_k \), which occurs at time \( t = k(2\pi/\omega_o) \), can be then expressed as

\[
W_k = \frac{W_o}{1 + \frac{4k\delta W_o}{3L}}
\]  

(3-10)

Equation (3-10) provides a way of determining the head loss coefficient through the free vibration test of a STLCD.

### 3.3.2 Experimental results and verification

Free vibration tests with a nonzero initial displacement and a zero initial velocity of water column motion were performed on each STLCD of configuration No.1, No.4 or No.5. The opening ratio of orifice of each STLCD was 50%. The natural frequency of the water column was tuned to 0.565Hz. Figures 3-3a to 3-3c show the recorded free decay curves for the configuration No.1, No.4 and No.5 STLCD, respectively. It is seen from Figures 3-3a to 3-3c that the decay of the water motion in the No.5 STLCD is the fastest with the time while the No.1 STLCD had the slowest decay of the water column motion among the three STLCDs. By applying Equation (10) to the recorded free decay curves, one may estimate the head loss coefficient of the STLCD. The estimated values for the concerned three STLCDs are listed in Table 3-2 for different cycles \( k \) counted. It is noted that the values of head loss coefficient vary with the number of cycles counted.
to some extent. The No.1 has the lowest variation while the No.5 STLCD has the highest variation.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>k=5</th>
<th>k=10</th>
<th>k=15</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>16.20</td>
<td>17.71</td>
<td>19.12</td>
<td>17.68</td>
</tr>
<tr>
<td>No.4</td>
<td>17.28</td>
<td>19.97</td>
<td>22.29</td>
<td>19.85</td>
</tr>
<tr>
<td>No.5</td>
<td>21.27</td>
<td>25.41</td>
<td>30.04</td>
<td>25.57</td>
</tr>
</tbody>
</table>

To have a further understanding of the head loss coefficient and the proposed method, the exact solution of Equation (3-4) is obtained by numerical integration using the initial conditions and the average head loss coefficients measured in the experiments. The numerical results are plotted in Figure 3-3a to 3-3c to compare with the measured results. It is seen that the numerical free decay curve is very close to the measured free decay curve for the STLCD of configuration No.1. For the STLCD of configuration No. 5, the amplitudes of numerical free decay curve are slightly smaller than the measured amplitudes before the first 15 seconds while they are slightly larger than the measured amplitudes after the first 15 seconds. These comparative results are consistent with the effects of number of cycles found in the experiments. The comparative results also indicate that the quadratic damping model is a good model for describing the water column motion in a STLCD when the width of the STLCD is relatively large (No.1 STLCD) and it will have a small deviation when the width of the STLCD is relatively small (No.5 STLCD).

In some circumstances, the container size of an MTLCD may be so large that free vibration test of the container with water is not easy to be performed to determine the head loss coefficient. One of the possible ways to overcome this difficulty is to scale down the MTLCD container and then calibrate a smaller MTLCD container based on similitude law.

3.4 PERFORMANCE OF MTLCD

3.4.1 Definition of parameters
To assess the performance of MTLCD and to help the understanding of the fundamental characteristics of the MTLCD, several parameters are defined. They are the central frequency $f_o$, the frequency bandwidth $\Delta X$, and the constant frequency spacing $\beta_i$.

\[
f_o = \frac{f_n + f_l}{2}, \quad \Delta X = \frac{f_n - f_l}{f_s}, \quad \beta_i = f_{i+1} - f_i \quad (i = 1, 2, \ldots n) \tag{3-11}
\]

where $f_i$ is the natural frequency of water in the $i^{th}$ TLCD; $f_o$ is the natural frequency of the structure; $f_l$ and $f_n$ are the lowest and highest natural frequencies, respectively, among all the TLCD units in a MTLCD; and $n$ is assumed to be an odd number. For the MTLCD with 3 TLCD units or with 5 TLCD units investigated in this study, the lowest and highest natural frequencies of water are designated to the side water columns while the water having the central frequency is arranged in the central water column. For studying the sensitivity of the MTLCD to off-tuning situation, the frequency tuning ratio parameter $\Delta \gamma$ is defined as

\[
\Delta \gamma = \frac{f_o}{f_s} \tag{3-12}
\]

### 3.4.2 Structural response without control

The structure was first tested without control to confirm the dynamic characteristics of the structure determined from the static and free vibration tests and to obtain the structural torsional response without control for the comparison with the controlled structural response. The measured typical dynamic magnification factor (DMF) of structural torsional displacement response without control is plotted in Figure 3-4 against the frequency ratio of the applied harmonic moment to the natural frequency of the structure. The amplitude of applied harmonic moment corresponding to the measured results in Figure 3-4 is 1.375 Nm. The DMF is defined as the ratio of the harmonic torsional displacement response amplitude to the static torsional displacement, which would be produced by the static moment of the amplitude of applied harmonic moment. The measured response curves confirm that the structure is a single degree of freedom system, the natural frequency of the structure is 0.565Hz, and the structural damping ratio slightly depends on the vibration amplitude of the structure. The maximum dynamic magnification factor, $DMF_{\text{max}}$, for the structure without control is 63.71.
3.4.3 Non-linearity of structure with MTLCD

As indicated by Equation (3-2), the water motion in a STLCD or a MTLCD is non-linear. The structure after installed with a STLCD or a MTLCD thus becomes a non-linear system. Many previous studies indicate that the non-linearity of water motion is quite weak and the non-linear damping force of water column can be replaced by an equivalent linear viscous damping force (Xu 1992 and Balendra 1995). To have a better understanding of the non-linearity of the system, some forced vibration tests were performed on the structure with the STLCD (Configuration No.1) and then with the MTLCD with 5 units (Configuration No.3). The opening ratio of orifice in each TLCID was kept the same at 50%. The frequency tuning ratio $\Delta \gamma$ was selected as one. The frequency bandwidth $\Delta X$ of the MTLCD was fixed at 0.1. The excitation frequency applied to the structure is 0.565Hz. For each given excitation frequency, three different excitation amplitudes ($M_0=1.375Nm, 2.063Nm$ and $2.750Nm$) were used. The steady state response time histories and amplitudes of the structure were recorded. Figures 3-5a and 3-5b show the response time histories of the structure with the STLCD and with the MTLCD, respectively, under the harmonic excitation of 2.063Nm amplitude. It is seen that the steady state angular displacement responses of the structure are almost harmonic, which indicate that the non-linear effect of the damper damping force on the structural motion is very small. This observation is further supported by the almost linear relationship between the response amplitude and the excitation amplitude (see section 3.4.7). Therefore, one may conclude that the non-linearity of damper damping force is indeed quite weak and the damper-structure system can be regarded as an equivalent linear system.

3.4.4 Effect of number of TLCID units

The effects of the number of TLCID units in a MTLCD were investigated using the MTLCD with 3 TLCID units (Configuration No.2) and the MTLCD with 5 TLCID units together with the STLCD with 1 TLCID unit. The opening ratio of orifice in each TLCID was kept the same at 50%. The frequency tuning ratio $\Delta \gamma$ was selected as one. The frequency bandwidth $\Delta X$ of the MTLCDs was fixed at 0.1. The frequency sweep tests were performed with the excitation moment amplitude of 1.375Nm. The dynamic magnification factors of torsional displacement response of the structure and the
dynamic magnification factors of displacement response of the central water column were plotted in Figure 3-6a and Figure 3-6b, respectively, against the frequency ratio of the excitation frequency to the structural frequency. The DMF of water displacement is defined as the ratio of dynamic water displacement to the static water displacement, which would be produced when the static moment of the amplitude of harmonic moment is applied to the structure. It is seen that the DMF curve of the structure with the STLCD has two peaks around the frequency ratios of 0.965 and 1.035. The two peak values of DMF are 25.72 and 18.87, respectively. Clearly, the STLCD reduces the peak torsional displacement response of the structure significantly if compared with the DMF\text{max} of the uncontrolled structure in Figure 3-4. With the installation of the MTLCD with 3 TLCD units (configuration No.2), the DMF curve has 4 local peaks and the DMF\text{max} is only 17.15. When the TLCD units are increased to 5 (configuration No.3), the DMF curve becomes smoother with one dominant peak only. This is probably because the local peak values are reduced as the number of TLCD units is increased and the frequency spacing becomes smaller. Similar findings were reported by Fujino et al. (1993) for multiple-tuned liquid dampers in reducing the horizontal motion of structure. The similarity of both the structural response curves and the water response curves between the MTLCD of 3 TLCD units and the MTLCD of 5 TLCD units indicates that further increase of the number of TLCD units may not be necessary. Since the maximum structural peak torsional displacement response using the MTLCD is less than that using the STLCD by 30%, one may say that the effectiveness of the MTLCD is better than the STLCD in this study. The maximum dynamic magnification factor of water displacement of the MTLCD is, however, slightly larger than that of the STLCD, as shown in Figure 3-6b.

3.4.5 Effect of frequency bandwidth

The effects of frequency bandwidth of a MTLCD were investigated using the MTLCDs of configuration No.2 and No.3. The frequency sweep tests were performed with the excitation moment amplitude of 1.375 Nm. The opening ratio of orifice was taken as 50% and the frequency tuning ratio \( \Delta \gamma \) was kept at one. The frequency bandwidth of the MTLCDs varies from 0.08 to 0.14 at an interval of 0.02. The dynamic magnification factors of torsional displacement response of the structure were plotted in Figure 3-7a for the MTLCD of 3 TLCD units and Figure 3-8a for the MTLCD of 5
TLCD units. The dynamic magnification factors of displacement response of the central water column were plotted in Figure 3-7b for the MTLCD of 3 TLCD units and Figure 3-8b for the MTLCD of 5 TLCD units. Both Figure 3-7a and Figure 3-8a depict that the maximum peak torsional response of structure increases as the bandwidth of the MTLCD increases. The bandwidth of the structural frequency response curves, however, decreases with the increase of the bandwidth of the MTLCD. This is probably because increasing the bandwidth of MTLCD enlarges the frequency spacing. The effectiveness of the side water columns is reduced when the excitation frequency ratio approaches one while it is increased when the excitation frequency ratio is away from unit. As a result, the two factors, that are the maximum peak torsional response and the bandwidth of torsional frequency response curve, should be taken into consideration when one wants to determine the optimum bandwidth of the MTLCD. It is seen from Figure 3-7b and Figure 3-8b that the displacement response of the central water column is increased as the bandwidth of the MTLCD increases. This is probably because the frequency spacing is increased and the side water columns become less effective at the excitation frequency ratio of one. The motion of the central water column thus becomes large.

3.4.6 Effect of frequency tuning ratio

A STLCD is often designed to have its natural frequency equal to the natural frequency of structure. However, it is difficult to avoid the frequency offset between the structure and the STLCD in real engineering application. Hence, the effects of frequency tuning ratio on the performance of STLCDs and MTLCDs should be investigated. The effects of frequency tuning ratio were investigated using the STLCD of configuration No. 1 and the MTLCD of configuration No. 3 with the frequency bandwidth of 0.1. The frequency sweep tests were performed under the excitation moment of amplitude 1.375 Nm. The opening ratio of orifice was taken as 50%. The frequency tuning ratio varies from 0.95 to 1.05. The dynamic magnification factors of torsional displacement responses of the structure are plotted in Figure 3-9a and Figure 3-9b for the STLCD and the MTLCD, respectively. The variations of the maximum dynamic magnification factor of the structure with frequency tuning ratio are plotted in Figure 3-10 for both the STLCD and the MTLCD.
It can be seen from Figures 3-9a and 3-9b that the offset of frequency tuning does affect the structural responses for either the STLCD or the MTLCD. For the STLCD, the smaller tuning ratio of 0.95 leads to the first peak DMF much smaller than the second peak DMF while the larger tuning ratio of 1.05 results in the first peak DMF much larger than the second peak DMF. For the MTLCD, the offset of frequency tuning leads to the larger peak structural response. From Figure 3-10, one may see that the increasing rate in the maximum peak structural response against the frequency tuning ratio is generally smaller in the case of the MTLCD than that of the STLCD within the frequency tuning range from 0.95 to 1.03. This indicates that the MTLCD is less sensitive to the frequency tuning ratio than the STLCD within this range. The optimal frequency tuning ratio of the STLCD and the MTLCD is almost the same around 0.983 but the MTLCD is more effective in reducing the peak torsional response of the structure.

3.4.7 Effect of excitation moment amplitude

The damping term in Equation (3-2) for water motion is non-linear. The excitation moment amplitude may affect the performance of the MTLCD due to the nonlinearity of the damping. The torsional displacement responses of the structure under various excitation amplitudes were studied using the STLCD of configuration No. 1 and the MTLCD of configuration No. 3. The frequency bandwidth of the MTLCD was 0.1. The opening ratio of orifice was taken as 50% and the frequency tuning ratio varied from 0.95 to 1.05. The excitation moment amplitudes were taken as 1.375 Nm, 2.063 Nm and 2.750 Nm. The frequency sweep tests were performed for a combination of designated tuning ratio and excitation amplitude. The maximum peak structural response was selected from each test. The variations of the maximum dynamic magnification factor with frequency tuning ratio for different excitation amplitudes are plotted in Figure 3-11a for the STLCD and Figure 3-11b for the MTLCD.

Figure 3-11a depicts that the STLCD becomes more sensitive to the frequency tuning ratio as the excitation moment amplitude increases. However, the low sensitivity of the MTLCD to the frequency tuning ratio can be maintained for different excitation moment amplitudes, as shown in Figure 3-11b. Clearly, the experimental data show that
the MTLCD is less sensitive to the excitation moment amplitude compared with the
STLCD.

3.5 SUMMARY

An experimental investigation on the performance of multiple-tuned liquid
column dampers (MTLCD) for reducing the torsional vibration of structures was carried
out. The experimental results revealed that the MTLCD could be more effective than the
STLCD in reducing the torsional vibration of structures when the overall water column
dimensions of the STLCD and the MTLCD were kept the same. Decreasing the
frequency bandwidth of MTLCD reduced the peak torsional displacement response of
the structure but increased the bandwidth of the frequency response curve of the
structure. The offset of frequency tuning did affect the performance of both the STLCD
and the MTLCD. Compared with the STLCD, the MTLCD was less sensitive to the
frequency tuning ratio and excitation moment amplitude. The MTLCD seems to be
more robust than the STLCD in controlling the torsional vibration of structures. An
efficient method was also proposed in this chapter to determine the head loss coefficient
of a TLCD using the free vibration test technique. The method was also verified through
the comparison with the exact solution obtained by numerical integration. The
experimental results and the head loss coefficients identified in this study will be used
to verify the theoretical model in Chapter 4.
(a) Test model simulating torsional vibration of bridge deck

(b) Instrumentation set-up
(c) Schematic diagram of experimental set-up and instrumentation

**Figure 3-1 Experimental Set-up**

**Figure 3-2 MTLCD Configurations in Experiment**
(a) Configuration No.1

(b) Configuration No.4
Figure 3-3 Free Decay Curves of Water Motion
Figure 3-4 Variation of DMF with Frequency Ratio for Uncontrolled Structure
(a) STLCD (Configuration No.1)

(b) MTLCD (Configuration No.3)

Figure 3-5 Steady State Response Time Histories of Structure
(a) Torsional response of controlled structure

(b) Water displacement response in central unit

Figure 3-6 Effect of Number of TLCD Units
Figure 3-7 Effect of Bandwidth of MTLCD (Configuration No.2)
(a) Torsional response of controlled structure

(b) Water displacement response in central unit

Figure 3-8 Effect of Bandwidth of MTLCD (Configuration No.3)
Figure 3-9 Effect of Frequency Tuning Ratio
Figure 3-10 Effect of Frequency Tuning Ratio on $\text{DMF}_{\text{max}}$
(a) STLCD (Configuration No.1)

(b) MTLCD (Configuration No.3, ΔX=0.1)

Figure 3-11 Effect of Excitation Moment Amplitude
CHAPTER 4
MULTIPLE TUNED LIQUID COLUMN DAMPERS FOR TORSIONAL VIBRATION CONTROL OF STRUCTURES: THEORETICAL INVESTIGATION

4.1 INTRODUCTION

The experimental investigations of multiple tuned liquid column dampers (MTLCD) were presented in Chapter 3. The experimental results revealed that the MTLCD could be more effective and robust than the STLCD when the overall dimensions of the STLCD and the MTLCD were kept the same. However, the experimental investigation was limited to a particular structure and accordingly extensive parametric studies could not be performed to facilitate the practical design of MTLCD. Furthermore, the external excitation in the experiment was limited to a harmonic moment, which might not well represent real excitations encountered by bridge deck units during construction. A theoretical model should therefore be developed for further investigation on the effectiveness of STLCD and MTLCD for torsional vibration reduction of structures under random excitation.

Previous work on the multiple tuned mass dampers (Yamaguchi and Harnpornchai 1993; Abe and Fujino 1994; Igusa and Xu 1994), multiple tuned liquid dampers (Fujino and Sun 1993), and multiple tuned liquid column dampers (Chang et al. 1998 and Gao et al. 1999) were performed for suppressing the horizontal motion of a structure. These studies showed that multiple tuned dampers are more effective and robust than the single tuned damper of the same volume of mass or liquid. However, there is little information about the performance and robustness of MTLCD for suppressing the torsional vibration of structures under random excitation.

In this chapter, the main objectives are thus to develop an analytical model of coupled MTLCD-structure systems under either harmonic excitation or white noise excitation, to verify the analytical model using the experimental data for the case of
harmonic excitation from chapter 3, and to investigate the performance of MTLCD for reducing the torsional vibration of structures under white noise excitation. Extensive numerical parametric studies aiming to find beneficial MTLCD parameters are performed in terms of the distance from the center line of the MTLCD to the rotational axis of the structure, the ratio of the horizontal length to the total length of liquid column, frequency bandwidth, head loss coefficient, the number of TLCD units, and frequency tuning ratio. The effects of the spectral intensity level of excitation moment on the MTLCD performance are also investigated. A procedure for selecting design parameters of a MTLCD is finally suggested.

4.2 ANALYTICAL MODEL

4.2.1 Equation of motion of liquid in an MTLCD

Let us consider an MTLCD consisting of N small TLCD units (Figure 4-1) and installed in a structure subjected to torsional vibration (Figure 4-2). Each small TLCD unit is a U-shape container of uniform rectangular cross-section filled with liquid where structural vibration energy is dissipated as the liquid passes through an orifice with inherent head-loss characteristics. In consideration of dynamic equilibrium conditions and the interaction between the structure and the liquid columns in the MTLCD, the equation of motion of liquid in the kth TLCD unit can be expressed as:

\[
\rho A_k L_k \ddot{W}_k + \frac{\rho A_k}{2} \delta_k |\dot{W}_k| \dot{W}_k + 2 \rho A_k g W_k = -\rho A_k B_k \left( H_k + \frac{L_k - B_k}{2} \right) \ddot{\theta} - \rho A_k g B_k \theta \quad (4-1)
\]

Equation (4-1) is subjected to the condition that the liquid should fully retain in the horizontal part of the kth TLCD unit and thus the following equation should be satisfied at any time.

\[
W_k \leq \frac{L_k - B_k}{2} - \frac{d_k}{2} \quad (k=1,2,\ldots,N) \quad (4-2)
\]

where \( \rho \) is the density of liquid assumed to be the same for all TLCD units; \( A_k \) is the cross-sectional area of liquid column of the kth TLCD; \( L_k \) is the total length of liquid column of the kth TLCD; \( W_k \) is the relative motion of liquid column in the kth TLCD to the container; \( d_k \) is the thickness of liquid column of the kth TLCD; \( \delta_k \) is the head loss coefficient of the kth TLCD governed mainly by the opening ratio of orifice and the width of liquid column; \( g \) is the acceleration of gravity; \( B_k \) is the horizontal length of
liquid column of the kth TLCD; $H_k$ is the distance between the center line of the horizontal part of the kth TLCD and the rotational axis of the structure; $\dot{W}_k$ and $\ddot{W}_k$ represent, respectively, the first and second derivatives of $W_k$ with respect to time; and $\theta$ and $\ddot{\theta}$ are, respectively, the torsional displacement and acceleration of the structure.

4.2.2 Equation of motion of structure

Let $I_d$ denote the total mass moment of inertia of liquid columns in the MTLCD about the rotational axis of the structure. $I_s$ is the mass moment of inertia of the structure including the mass moment of inertia of the containers about the rotational axis of the structure. $K_s$ and $C_s$ are the torsional stiffness and damping coefficient of the structure, respectively. The equation of motion of the structure with the MTLCD can be expressed as:

$$\left( I_s + I_d \right) \ddot{\theta} + C_s \dot{\theta} + \left[ K_s + \sum_{k=1}^{N} \rho A_k L_k g \bar{H}_k \right] \theta = \sum_{k=1}^{N} \rho A_k B_k \left( H_k + \frac{L_k - B_k}{2} \right) \dot{W}_k + \rho A_k B_k g W_k + M_e(t)$$ \hspace{1cm} (4-3)

in which

$$I_d = \left( \sum_{k=1}^{N} I_{dk} \right)$$ \hspace{1cm} (4-4)

$$I_{dk} = \rho A_k B_k \left( \frac{H_k^2 + B_k^2}{12} \right) + \rho A_k \left( L_k - B_k \right) \left[ \frac{H_k^2 + B_k^2}{4} - \frac{H_k \left( L_k - B_k \right)}{2} + \frac{(L_k - B_k)^2}{12} \right]$$ \hspace{1cm} (k=1,2...,N) (4-5)

$$\bar{H}_k = H_k - \frac{(L_k - B_k)^2}{4L_k}, \hspace{1cm} (k=1,2,..N)$$ \hspace{1cm} (4-6)

and $M_e(t)$ is the external moment acting on the structure.

4.2.3 Equation of motion of MTLCD-structure system in matrix form

By combining Equation (4-1) with Equation (4-3), the equation of motion of the coupled MTLCD-structure system can be written in a matrix form as:

$$M \ddot{x} + C \dot{x} + K x = P$$ \hspace{1cm} (4-7)
where the vector $x$ is the displacement vector.

$$x = [0 \ W_1 \ W_2 \ldots \ W_N]^T \quad (4-8)$$

$P$ is the external moment excitation vector. $M$, $C$, and $K$ are the mass, damping and stiffness matrices of the coupled MTLCD-structure system, respectively, which can be expressed as

$$M = \begin{pmatrix}
I_s(l + \mu) & G_1 & G_2 & \ldots & G_N \\
G_1 & m_1 & 0 & \ldots & 0 \\
G_2 & 0 & m_2 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
G_N & 0 & 0 & \ldots & m_N
\end{pmatrix} \quad (4-9)$$

$$K = \begin{pmatrix}
K_s + mg & m_1 g \alpha_1 & m_2 g \alpha_2 & \ldots & m_N g \alpha_N \\
m_1 g \alpha_1 & m_1 \omega_1^2 & 0 & \ldots & 0 \\
m_2 g \alpha_2 & 0 & m_2 \omega_2^2 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
m_N g \alpha_N & 0 & 0 & \ldots & m_N \omega_N^2
\end{pmatrix} \quad (4-10)$$

$$C = \begin{pmatrix}
C_s & 0 & 0 & \ldots & 0 \\
0 & \frac{\rho A_1}{2} \delta_t |\dot{W}_1| & 0 & \ldots & 0 \\
0 & 0 & \frac{\rho A_2}{2} \delta_t |\dot{W}_2| & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & \frac{\rho A_N}{2} \delta_t |\dot{W}_N|
\end{pmatrix} \quad (4-11)$$

in which

$$\alpha_k = \frac{B_k}{L_k} ; G_k = \rho A_k B_k \left( H_k + \frac{L_k - B_k}{2} \right) ; m_k = \rho A_k L_k ; \omega_k = \sqrt{\frac{2g}{L_k}} ; m = \sum_{k=1}^{N} m_k \overline{H}_k ;$$

$$\mu = \frac{I_d}{I_s} \quad (4-12)$$
where $\alpha_k$ is the liquid length ratio of the kth TLCD; $G_k$ and $m_k$ are the liquid mass moment and mass in the kth TLCD, respectively; $\omega_k$ is the circular natural frequency of the kth TLCD; $\overline{H}_k$ is the distance of the mass center of the kth TLCD to the rotation axis; $\mu$ is the mass moment of inertia ratio of the MTLCD to the structure.

### 4.2.4 Equivalent linearization technique

It is noted that the damping terms in Equation (4-11) for liquid motion are nonlinear. Therefore, the coupled MTLCD-structure system described by Equation (4-7) is a nonlinear system. However, the experimental results for the coupled MTLCD-structure system presented in Chapter 3 indicate that the nonlinearity of the coupled system is not significant. The equivalent linearization technique can thus be applied to the system so that closed-form solutions can be obtained. As a result, in terms of the principle of equivalent energy dissipation the damping matrix in Equation (4-11) can be replaced by

$$
C = \begin{pmatrix}
2I_0\omega_z \xi_1 & 0 & 0 & \cdots & 0 \\
0 & 2m_1\omega_1 \xi_1 & 0 & \cdots & 0 \\
0 & 0 & 2m_2\omega_2 \xi_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 2m_N\omega_N \xi_N
\end{pmatrix}
$$

(4-13)

where $\omega_z$ is the circular frequency of the structure equal to $\sqrt{K_s / I_s}$; $\xi_z$ is the structural damping ratio; and $\xi_k$ is the equivalent damping ratio of the kth TLCD. If the external moment excitation is harmonic moment excitation, the equivalent damping ratio of the kth TLCD can be calculated by

$$
\xi_k = \frac{\sqrt{2} \delta_k}{3\pi \sqrt{g L_k}} \overline{W}_k \bar{\omega}
$$

(4-14)

where $\overline{W}_k$ is the amplitude of liquid motion in the kth TLCD; and $\bar{\omega}$ is the circular frequency of the applied harmonic moment. If the external excitation is a zero-mean stationary white noise excitation, the equivalent damping ratio of the kth TLCD should be calculated by

$$
\xi_k = \frac{\delta_k}{2\sqrt{\pi \sigma_{w_k} L_k}}
$$

(4-15)
where $\sigma_{e_k}$ is the standard deviation of the liquid velocity in the kth TLCD. Since the equivalent damping ratio described by either Equation (4-14) or Equation (4-15) depends on the liquid motion, iterations are generally required.

### 4.2.5 Solution for the case of harmonic excitation

When the excitation applied to the structure is a harmonic moment, the force vector $P$ in Equation (4-7) can be given by

$$P = M^T e^{i\theta} = [M_0 \ 0 \ 0 \ \cdots \ 0]^T e^{i\theta}$$  \hspace{1cm} (4-16)

where $M_0$ is the amplitude of harmonic moment excitation; and $i$ is the imaginary unit. Consider the steady state responses of both the structure and the liquid columns in the MTLCD under harmonic moment excitation.

$$x = X^T e^{i\theta t} = [\bar{\theta} \ \bar{W}_1 \ \bar{W}_2 \ \cdots \ \bar{W}_N]^T e^{i\theta t}$$  \hspace{1cm} (4-17)

Then, substituting Equations (4-16) and (4-17) into Equation (4-7) with some manipulation yields the steady state response amplitudes of both the structure and liquid columns.

$$\bar{\theta} = \frac{(-M_0) \prod_{k=1}^{N} Z_k}{Z}$$  \hspace{1cm} (4-18)

$$\bar{W}_k = \frac{M_0 \left( -\bar{\omega}_k^2 G_k + m_k g \alpha_k \right) \cdot Z_1 \cdot Z_2 \cdot \cdots \cdot \hat{Z}_k \cdots \cdot Z_N}{m_k Z}, \quad (k=1,2,\ldots,N)$$  \hspace{1cm} (4-19)

$Z_k$ and $Z$ in equations (4-18) and (4-19) are obtained as

$$Z_k = -\bar{\omega}_k^2 + \omega_k^2 + 2\pi \bar{\omega}_k \xi_k, \quad (k=1,2,\ldots,N)$$  \hspace{1cm} (4-20)

$$Z = \left[ \prod_{k=1}^{N} \left( \frac{\bar{\omega}_k^2 G_k + m_k g \alpha_k}{m_k} \right) \left( -\bar{\omega}_k^2 G_k + m_k g \alpha_k \right) \cdot Z_1 \cdot Z_2 \cdot \cdots \cdot \hat{Z}_k \cdots \cdot Z_N \right] -$$

$$\left[ -\bar{\omega}_k^2 (1 + \mu) I_x + mg + I_y \omega_z^2 + 2\pi \bar{\omega}_k I_y \xi_z \right] \prod_{k=1}^{N} Z_k$$  \hspace{1cm} (4-21)

where $Z_k$ under the sign $\hat{\cdot}$ should be deleted; and $\prod$ is a chain product sign.

### 4.2.6 Solution for the case of white noise excitation

The transfer functions of both the structural response and the liquid motion in the kth TLCD can be easily found from Equations (4-18) and (4-19).
\[ H_\theta(i\omega) = \frac{-\sum_{k=1}^{N} Z_k}{Z}, \quad H_{\omega_k}(i\omega) = \frac{-\tilde{\omega}^2 G_k + m_k g\alpha_k}{m_k Z} \cdot Z_1 \cdot Z_2 \cdots \hat{Z}_k \cdots Z_N, \quad (k=1,2 \ldots N) \] (4-22)

Assume that a zero-mean stationary white noise moment excitation of spectral intensity level \( S_\theta \) acts on the structure. The standard deviation torsional displacement and acceleration responses of the structure and the standard deviation displacement and velocity responses of the kth TLCD are given, respectively, by

\[
\sigma_\theta = \sqrt{\int_{-\infty}^{\infty} |H_\theta(i\omega)|^2 S_\theta \, d\omega}; \quad \sigma_\ddot{\theta} = \sqrt{\int_{-\infty}^{\infty} |\dot{\omega}^2| H_\theta(i\omega)|^2 S_\theta \, d\omega}; \quad (4-23)
\]

\[
\sigma_\omega = \sqrt{\int_{-\infty}^{\infty} |H_{\omega_k}(i\omega)|^2 S_\omega \, d\omega}; \quad \sigma_\omega = \sqrt{\int_{-\infty}^{\infty} |\dot{\omega}^3| H_{\omega_k}(i\omega)|^2 S_\omega \, d\omega}; \quad (4-24)
\]

It is noted that the standard deviation responses of both the structure and liquid columns are dependent of the equivalent damping ratios while Equation (4-15) shows that the equivalent damping ratios of the MTLCD is dependent of the standard deviation responses of liquid column velocities. Thus, to obtain the standard deviation responses of the structure and liquid columns, iterations are required to obtain converged results.

4.3 VERIFICATION

An experimental investigation on the relative performance of STLCD and MTLCD for reducing torsional vibration of structures was presented in Chapter 3. A large structure model simulating the torsional vibration of bridge deck and several STLCDs and MTLCDs of different configurations were designed and constructed. A series of harmonically forced vibration tests were performed to evaluate the effectiveness of MTLCD in terms of the number of TLCD units in an MTLCD, the bandwidth of an MTLCD, the frequency tuning ratio, and the moment excitation amplitude. Thus, a comparison between the analytical and experimental results can be performed to verify the analytical model developed in this chapter for the case of harmonic excitation before a parametric study is carried out for the case of white noise excitation.
To help the understanding of experimental results, several parameters that affect the performance of the MTLCD are introduced here. They are the central frequency $f_o$, the frequency bandwidth $\Delta X$, and the constant frequency spacing $\beta_k$.

\[
f_o = \frac{f_N + f_i}{2}; \quad \Delta X = \frac{f_N - f_i}{f_s}; \quad \beta_k = f_{k+1} - f_k.
\] (4-25)

where $f_k$ is the natural frequency of liquid column in the kth TLCD; $f_s$ is the natural frequency of the structure; and $f_i$ and $f_N$ are the lowest and highest natural frequencies, respectively, among all the TLCD units in an MTLCD. The sensitivity of the MTLCD to off-tuning situation is evaluated in terms of the tuning ratio parameter $\Delta g$ defined as

\[
\Delta g = \frac{f_o}{f_s}
\] (4-26)

In the experiment, a single-degree-of-freedom structure was designed and constructed to model the torsional vibration of a bridge deck unit (see Figure 4-2). The dynamic parameters of the model were identified as $f_s=0.565\,\text{Hz}$, $\xi=1\%$, and $K_s=13873\,\text{Nm/rad}$. The basic dimensions of the STLCD and the MTLCD with 5 TLCD units used in the experiment and selected for the comparison are listed in Table 4-1. The opening ratio of the orifice of either the STLCD or the MTLCD was kept at 50\% throughout the experiment and they were filled with water. The main parameters of the STLCD and the MTLCD are $H=0.98\,\text{m}$, $\rho=1000\,\text{kg/m}^3$, $\Delta g=1$, $\delta=17.68$ for the STLCD, and $\delta=25.57$ and $\Delta X=0.1$ for the MTLCD. The amplitude of the applied harmonic moment was 1.375 Nm.

<table>
<thead>
<tr>
<th>Table 4-1 Dimensions of TLCDs Used in Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>STLCD</td>
</tr>
<tr>
<td>MTLCD</td>
</tr>
</tbody>
</table>

Figures 4-3a and 4-3b show the comparisons of the frequency response curves of both structural torsional displacement and water column displacement for the STLCD and the MTLCD, respectively, in terms of the dynamic magnification factors (DMF). The DMF of either structural displacement or water displacement is defined as the ratio

4-8
of dynamic displacement response amplitude to the static displacement, which would be produced by a static moment with amplitude equal to that of the applied harmonic moment. It is seen from Figures 4-3a and 4-3b that the frequency response curves of the structure with either the STLCD or the MTLCD predicted by the developed analytical model are in good agreement with those from the experiment. Both the analytical and experimental results show that the frequency response curves of the structure with the STLCD have two peaks around the frequency ratios of 0.965 and 1.035. However, the frequency response curves of the structure with the MTLCD have one dominant peak only. The maximum peak torsional displacement response of the structure with the MTLCD is less than that with the STLCD by approximately 30%. The better performance of the MTLCD than the STLCD is mainly attributed to two reasons: one is a higher head loss coefficient in the MTLCD; and the other is a wider covering frequency range of the MTLCD so that only one peak response appears. Figures 4-3a and 4-3b also show that the analytical model can predict the main features of frequency response curves of the water column in either the STLCD or the central unit of the MTLCD. The amplitude of the water column motion predicted by the analytical model, however, is larger than that from the experiment within a certain frequency range. This discrepancy is attributed to many factors, in which the head loss coefficient measured in the experiment and the equivalent damping ratio calculated in the analysis are most influencing factors.

Figures 4-4a and 4-4b display the comparisons of the maximum DMFs of the structure with the STLCD and the MTLCD, respectively, against the frequency tuning ratio $\Delta\gamma$. Both analytical and experimental results indicate that the MTLCD is less sensitive to the frequency tuning ratio. The analytical results match the experimental results quite well, in particular for the case of the MTLCD. Thus, based on the above comparable results, the next section of this chapter investigates the performance of MTLCD in reducing torsional vibration of the structure subject to white noise excitation and the optimal parameters of the MTLCD for achieving the maximum response reduction using the developed and verified analytical model.

4.4 PARAMETRIC STUDIES
To evaluate the performance of a STLCD or an MTLCD for the case of white noise excitation, the following structural response ratio is introduced.

\[ R = \frac{\text{Standard deviation response of the structure with control}}{\text{Standard deviation response of the structure without control}} \] (4-27)

The structure and its parameters are kept the same as those used in the experiment. Some of the parameters of a MTLCD are regarded as variables in order to find out their optimal values through parametric studies.

4.4.1 Effect of distance from MTLCD to rotational axis

The effect from distance of the centerline of a MTLCD to the rotational axis of the structure, \( H \), on the torsional response reduction of the structure is depicted in Figures 4-5a and 4-5b for different mass moment of inertia ratios, \( \mu \). The parameters of the MTLCD used in the computation are \( N=5, \Delta \gamma=1, \delta=5 \) for all 5 TLCD units, and \( \Delta X=0.1 \). The spectral intensity level of white noise moment excitation is \( S_\alpha=0.7(Nm)^2/s/\text{rad} \). The total length of liquid column in the central unit, \( L_c \), and the total length of liquid column in other TLCD units are determined based on the value of frequency-tuning \( \Delta \gamma \) and frequency bandwidth \( \Delta X \). In consideration of the condition set by Equation (4-2), the ratio of the horizontal length to the total length of liquid column in the central unit of the MTLCD, \( \alpha_c \), is selected as 0.7. The horizontal length of liquid column in the central unit, \( B_c \), is then determined by \( B_c = \alpha_c L_c \). The horizontal length of liquid column in other TLCD units, \( B \), takes the same value as \( B_c \) from a viewpoint of practical use. The rest parameters of the MTLCD can then be determined once a mass moment of inertia ratio \( \mu \) is selected.

Figures 4-5a and 4-5b demonstrate that the structural torsional response reduction depends on the height \( H \) significantly. In particular, when the ratio \( H/L_c \) equals \( \alpha_c/2 \) (i.e., 0.35), the torsional displacement response of structure cannot be reduced by the MTLCD and the reduction of torsional acceleration response of structure is very small for all three mass moment of inertia ratios. The ratio \( H/L_c \) equal to \( \alpha_c/2 \) is actually the same as the ratio \( H/B \) equal to 1/2. Similar finding has been reported and explained by Xue et al. (2000) for the case of STLCD under harmonic loading. For the case of
STLCD under harmonic loading, the ratio $H/B$ equal to 1/2 and the excitation frequency equal to the structural frequency (i.e., $\tilde{\omega}/\omega_s = 1$) will lead the term $\left( -\tilde{\omega}^2 G_k + m_k g \alpha_k \right)$ in Equation (4-19) and the liquid motion to be zero (Xue et al. 2000). Consequently, the STLCD loses its function. For the case of MTLCD under white noise excitation, because of the same reason the transfer function of the liquid motion in the central unit of the MTLCD (see Equations (4-22) and (4-24)) is zero at the frequency ratio $\tilde{\omega}/\omega_s = 1$ and of very small values at other frequency ratios. Thus, the standard deviation displacement response of liquid column is very small. This leads to almost zero reduction of the structural displacement response and very small reduction of the structural acceleration response. The term $\left( -\tilde{\omega}^2 G_k + m_k g \alpha_k \right)$ is physically related to the external force caused by the torsional vibration of structure on the liquid as shown in Equation (4-1). It can also be regarded as the additional external moment caused by the liquid motion on the structure as shown in Equation (4-3). Thus, when the term $\left( -\tilde{\omega}^2 G_k + m_k g \alpha_k \right)$ tends to almost zero, the interaction between the structure and the MTLCD is very small and the MTLCD loses its function. As a result, it is very important to select a proper ratio of $H/L_c$ or $H/B$ when using the MTLCD or the STLCD for reducing the torsional vibration of structures. Nevertheless, it is clear that for the concerned structure, the further increase of the ratio $H/L_c$ beyond a value of unity will not improve the performance of the MTLCD and the ratio $H/L_c = 1$ can thus be regarded as a beneficial value. Figures 4-5a and 4-5b also show that a larger mass moment of inertia ratio results in a larger reduction of torsional vibration of the structure if the ratio of $H/L_c$ is larger than 0.35.

4.4.2 Effect of liquid length ratio

The effect of the liquid length ratio, $\alpha_c$, of the horizontal length to the total length of liquid column in the central unit of the MTLCD on the performance of the MTLCD is displayed in Figures 4-6a and 4-6b for three different distance ratios $H/L_c$. The parameters of the MTLCD and the external moment excitation used in the computation are $N=5$, $\Delta \gamma=1$, $\delta=5$ for all 5 TLCD units, $\Delta X=0.1$, $\mu=0.02$, and $S_o=0.7(Nm)^2/s/\text{rad}$. Taking the constraint Equation (4-2) into consideration and letting the horizontal lengths of all the individual TLCD be the same as the horizontal length of the central unit $B_c$, one may obtain the following constraint equation on the liquid length ratio $\alpha_c$.  

4-11
\[ \alpha_c \leq \min_{k \in N} \left[ \frac{1}{L_c} (L_k - 2W_k - d_k) \right] \] (4-28)

Clearly, the selection of the liquid length ratio should consider the total length of liquid column of the kth TLCD \( L_k \), the thickness of liquid column of the kth TLCD \( d_k \), and the relative motion of liquid column in the kth TLCD to the container \( W_k \). The relative motion of liquid column in the MTLCD is increased with increasing external excitation level. To design an MTLCD working within a wide range of external excitation, one would select a value of liquid length ratio \( \alpha_c \) slightly smaller than its upper bound value determined from Equation (4-28) in order to ensure the liquid fully retains in the horizontal part of the MTLCD all the time. Based on all these considerations, the largest value of the liquid length ratio is limited to 0.8 and a value of 0.7 is recommended in this investigation. It is seen from Figures 4-6a and 4-6b that for \( H/L_c=1 \), both the structural displacement and acceleration response ratios decrease monotonically with increasing liquid length ratio. Particularly, if the length ratio \( \alpha_c \) is selected as 0.7, both the standard deviation torsional displacement and acceleration responses of the structure are reduced by more than 35%. However, when the ratio \( H/L_c \) is reduced from 0.6 to 0.4, the performance of the MTLCD deteriorates significantly as the ratio \( H/L_c \) approaches 0.35 for which the MTLCD is not effective as explained earlier. For \( H/L_c=0.2 \), the MTLCD is still not effective when the liquid length ratio \( \alpha_c \) is small, but when the liquid length ratio \( \alpha_c \) approaches 0.8, the maximum response reduction can also reach 35% approximately.

4.4.3 Effect of frequency bandwidth and head loss coefficient

To investigate the effect of bandwidth and head loss coefficient on the performance of MTLCD, the bandwidth and head loss coefficient are taken as variables and the other parameters of the MTLCD are \( N=5, \Delta \gamma=1, \mu=0.02, \alpha_c=0.7, \) and \( H/L_c=1 \). The spectral intensity level of external moment excitation \( S_n \) is 0.7 \((\text{Nm})^2\text{s/rad}\). Figure 4-7a depicts the variation of torsional displacement response ratio with the bandwidth \( \Delta X \) for a number of head loss coefficient. Figure 4-7b displays the variation of the torsional displacement response ratio with the head loss coefficient \( \delta \) for a series of frequency bandwidth. It is seen that the effectiveness of the MTLCD is affected by
frequency bandwidth. For a given head loss coefficient, there is an optimal frequency bandwidth by which the maximum response reduction can be achieved. For instance, if the head loss coefficient is less than 10, the optimal bandwidth is around 0.08. Figure 4-7b shows that the effectiveness of the MTLCD is much affected by head loss coefficient. There is an optimal head loss coefficient, $\delta=3.5$, for the MTLCD of 5 TLCD units with the bandwidth of 0.08. If the frequency bandwidth becomes smaller, the optimal head loss coefficient becomes larger and eventually it is close to the optimal head loss coefficient of STLCD ($\delta=18$). The optimal head loss coefficient of the MTLCD is clearly much less than that of the STLCD. This is because the individual TLCD in the MTLCD is much smaller than the STLCD and the optimal head loss coefficient of a smaller STLCD is much less than that of a larger STLCD, as found by Gao et al. (1997).

### 4.4.4 Effect of number of TLCD units

To investigate the effect of the number of TLCD units on the performance of the MTLCD, the optimal head loss coefficient and frequency bandwidth to achieve the maximum reduction of structural responses should be first found for a given number of TLCD units. Therefore, not only the number of TLCD units, $N$, but also the head loss coefficient $\delta$ and frequency bandwidth $\Delta X$ are taken as variables in this investigation. The total liquid volume of the MTLCD however should remain the same no matter how many TLCD units are selected. The other fixed parameters of the MTLCD are $\alpha_c=0.7$, $H/L_c=1$, $\Delta T=1$, and $\mu=0.02$ or $\mu=0.04$. The spectral intensity level of external moment excitation $S_o$ is still $0.7\text{(Nm)}^2\text{s/rad}$. The intervals of the parameters $N$, $\delta$ and $\Delta X$ considered in the computation are 2, 0.5 and 0.01, respectively.

Table 4-2 lists the optimal values of frequency bandwidth, head loss coefficient, and equivalent damping ratio $\xi_c$ in the central unit of MTLCD for the maximum reduction of torsional displacement response of the structure against the number of TLCD units and the mass moment of inertia ratio. Table 4-3 list the same quantities but for the maximum reduction of torsional acceleration response of the structure. It is seen that with the increasing number of TLCD units, the optimal value of frequency bandwidth increases while the optimal value of head loss coefficient decreases. Similar trends have been reported by Gao et al. (1999) for the MTLCD in reducing the
horizontal vibration of a structure. For the same number of TLCD units, the optimal values of frequency bandwidth and head loss coefficient are larger for larger mass moment inertia ratio $\mu$. It is also seen that the optimal values of frequency bandwidth and head loss coefficient for the maximum reduction of torsional acceleration response of the structure are close to those for the maximum reduction of torsional displacement response of the structure.

Table 4-2 Optimal Values of Frequency Bandwidth, Head Loss Coefficient and Damping Ratio of MTLCD for Torsional Displacement Reduction

<table>
<thead>
<tr>
<th>Number of TLCD units</th>
<th>$\mu=0.02$</th>
<th></th>
<th></th>
<th>$\mu=0.04$</th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta X$</td>
<td>$\delta$</td>
<td>$\xi_c$</td>
<td>$\Delta X$</td>
<td>$\delta$</td>
<td>$\xi_c$</td>
</tr>
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<td>0</td>
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</tr>
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<td>0.09</td>
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<td>5</td>
<td>0.08</td>
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<tr>
<td>7</td>
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<td>0.13</td>
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<td>0.13</td>
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Table 4-3 Optimal Values of Frequency Bandwidth, Head Loss Coefficient and Damping Ratio of MTLCD for Torsional Acceleration Reduction

<table>
<thead>
<tr>
<th>Number of TLCD units</th>
<th>$\mu=0.02$</th>
<th></th>
<th></th>
<th>$\mu=0.04$</th>
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<tbody>
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<td>0.008434</td>
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The corresponding maximum torsional displacement and acceleration responses of the structure are plotted in Figures 4-8a and 4-8b, respectively, against the number of TLCD units. Figures 4-8a and 4-8b indicate that the performance of the optimized MTLCD ($N=1$) is better than that of the optimized STLCD ($N=1$). However, there is no significant improvement if the number of TLCD units in MTLCD exceeds five. Both
figures also show that larger mass moment inertia ratio leads to larger reduction in torsional response of the structure.

4.4.5 Effect of excitation amplitude

To investigate the effect of excitation amplitude on the performance of STLCD/MTLCD and the sensitivity of head loss coefficient to excitation amplitude, the torsional displacement response ratios of the structure are computed against head loss coefficient for four different levels of excitation amplitude. The fixed parameters of the STLCD and the MTLCD are $\Delta \gamma = 1$, $H/L_c = 1$, $\mu = 0.02$, $\alpha_c = 0.7$, $N = 5$, and $\Delta X = 0.08$. The computed results are shown in Figure 4-9a for the STLCD and Figure 4-9b for the MTLCD. The unit of the excitation amplitude in both figures is in $(Nm)^2/s/rad$. It is seen that the optimal head loss coefficient is sensitive to excitation amplitude. The optimal head loss coefficient decreases as the excitation amplitude increases for either the STLCD or the MTLCD. This is because the velocity of liquid column is increased as the excitation amplitude increases while the optimal damping ratio of liquid motion remains almost unchanged. As a result, an optimized STLCD or MTLCD with a higher excitation level will require a smaller head loss coefficient. However, the range of variation of optimal head loss coefficient of the MTLCD with the concerned excitation amplitudes is much smaller than that of STLCD. It is also seen that the achievable maximum response reduction using the MTLCD is larger than using the STLCD.

4.4.6 Effect of frequency tuning ratio

A TLCD is often designed to have its natural frequency equal to the natural frequency of structure. However, the off-tuning may occur in the engineering application. Hence, the effect of frequency tuning ratio on the performance of a STLCD or an MTLCD should be investigated. Figure 4-10a shows that the variation of torsional displacement response ratio of the controlled structure with frequency tuning ratio for a series of frequency bandwidth. The fixed parameters of the STLCD ($\Delta X = 0$) and the MTLCD ($\Delta X \neq 0$) are $\alpha_c = 0.7$, $\mu = 0.02$, $\delta = 18$ for $N = 1$, and $\delta = 3.5$ for $N = 5$. The parameter $H$ is taken the same value as the one in the preceding section. The spectral intensity level of moment excitation $S_o$ is $0.7(Nm)^2/s/rad$. It is seen that the sensitivity of the structural response with an optimized MTLCD ($\Delta X = 0.08$) to the frequency tuning
ratio is better than that with an optimized STLCD ($\Delta X=0$) in the frequency tuning range from 0.96 to 1.04. It is also seen that the sensitivity of the optimized MTLCD to the frequency tuning ratio can be further improved by increasing the bandwidth but at the cost of the less torsional vibration reduction. Hence, one can design an MTLCD, which has almost the same effectiveness as the optimized STLCD but is more robust to the frequency tuning ratio. Figure 4-10b displays the variation of standard deviation liquid displacement response in the central unit with frequency tuning ratio for a series of frequency bandwidth. Clearly, with increasing frequency bandwidth the liquid displacement in the central unit of the MTLCD is increased. This is because the frequency spacing among the TLCD units in the MTLCD is increased with increasing frequency bandwidth. Thus, for the frequency tuning ratio equal to one, the proportion of structural vibration energy absorbed by the central TLCD unit is increased, resulting in the larger liquid displacement in the central unit.

4.4.7 Procedure for selecting design parameters of a MTLCD

From a practical point of view, the design parameters for a MTLCD system may be selected in an order as follows: (1) decide the mass moment of inertia ratio of a MTLCD system, normally from 2% to 5% with a higher ratio for a larger reduction; (2) decide the number of TLCD units in the MTLCD, preferably not smaller than five; (3) estimate the liquid length ratio $\alpha_c$ in consideration Equation (4-28); (4) determine the frequency tuning ratio $\Delta \gamma$ which may be set as unity; (5) determine the value $L_c$ according to $\Delta \gamma$; (6) determine the value $B_c$ using the relationship $B_c = \alpha_c L_c$; (7) decide the value $H/L_c$ according to real structure configuration; and (8) estimate the head loss coefficient and the frequency bandwidth from Figure 4-7 or through computer simulation.

4.5 SUMMARY

An analytical model for torsional vibration of a structure with an MTLCD under either harmonic excitation or white noise excitation was developed. The computed results using the developed analytical model for the torsional vibration of a structure with either STLCD or MTLCD under harmonic excitation are in good agreement with
those obtained from the experiments. Both analytical and experimental results showed that the MTLCD is more effective in reducing structural torsional response to harmonic excitation and less sensitive to the frequency tuning ratio than the STLCD.

Extensive parametric studies on STLCDs and MTLCDs for reducing torsional vibration of a structure under white noise excitation were also carried out using the verified analytical model. The results revealed that the ratio $H/L_e$ is an important factor and should be larger than the value of $\alpha_c/2$ in order to achieve significant torsional vibration reduction. There are an optimal head loss coefficient and an optimal frequency bandwidth for an MTLCD with a given number of TLCD units. The optimal head loss coefficient is decreased but the optimal frequency bandwidth is increased with increasing number of TLCD units. The effectiveness of an optimized MTLCD increases with increasing number of TLCD units but there is no significant improvement if the number of TLCD units in the MTLCD exceeds 5. Moreover, the optimal head loss coefficient of either the STLCD or the MTLCD depends on external excitation amplitude. The sensitivity of structural response reduction with an optimized MTLCD to the frequency tuning ratio is less than that with an optimized STLCD and it can be further improved by increasing the frequency bandwidth but at the cost of less torsional vibration reduction. Thus, one may design an MTLCD, which has almost the same effectiveness as the optimized STLCD and is more robust to the frequency-tuning ratio.
Figure 4-1 Multiple Tuned Liquid Column Dampers (MTLCD)

Figure 4-2 MTLCD-Structure System
Figure 4-3 Comparison of Frequency Response Curves
Figure 4-4 Comparison of Maximum Dynamic Magnification Factors
(a) Displacement response of structure

(b) Acceleration response of structure

Figure 4-5 Effect of Distance from MTLCD to Rotational Axis
Figure 4-6 Effect of Liquid Length Ratio $\alpha_c$
Figure 4-7 Effects of Frequency Bandwidth and Head Loss Coefficient
(a) Displacement response of structure

(b) Acceleration response of structure

Figure 4-8 Effect of Number of TLCD units
Figure 4-9 Effect of Excitation Amplitude
Figure 4-10 Effect of Frequency Tuning Ratio
CHAPTER 5
MULTIPLE PRESSURIZED TUNED LIQUID COLUMN DAMPERS FOR TORSIONAL VIBRATION CONTROL OF STRUCTURES

5.1 INTRODUCTION

The effectiveness of multiple tuned liquid column dampers (MTLCD) for suppressing torsional vibration of structures was demonstrated in Chapters 3 & 4. However, the torsional frequency of some long span bridges is much higher than that in the vertical and lateral direction. The short period of torsional vibration of such long span bridges may require short liquid column length to have a proper frequency tuning. Consequently, a large number of such small TLCD containers are required, which leads to a higher cost of installation and maintenance but a low control performance. As such, multiple pressurized tuned liquid column dampers (MPTLCD) are studied in this chapter to facilitate torsional vibration control of long span bridges with relatively higher torsional frequency.

Actually, in order to facilitate the frequency tuning a variation of the TLCD, termed as liquid column vibration absorber (LCVA), was proposed and investigated by Watkins and his co-workers (1991, 1992 and 1997a, b). The nominally different cross-sectional areas of the vertical and horizontal columns allow the natural frequency of a LCVA to be a function of the container geometry, rather than the length of liquid column alone. The flexibility in the frequency tuning can thus be obtained by selecting a difference in the cross sectional area of the horizontal and vertical part of the tube. However, the variation in the cross sectional area of the horizontal and vertical part of the tube is very limited and the flexibility of frequency tuning may not be large enough for application in some circumstances. Kagawa et al. (1989) proposed a pressurized tuned liquid column damper (PTLCD) which has air and water in a sealed tank of U-shape for the suppression of ship oscillation in lateral direction. The experimental studies they carried out showed that the short period vibration of ship can be reduced
effectively by PTLCD and the natural frequency of PTLCD can be controlled by
adjusting the air pressure in the tank. A board band of PTLCD frequency can be tuned
by the manipulation of the air pressure in the tank. It is thus believed that the PTLCD is
a potential device for the suppression of structure with high frequency. Shyu et al.
(1996) further studied the performance of PTLCD with focus on the sensitivity of
PTLCD to the tuning of air pressure. The investigation showed that the reduction of
structural vibration appears to be sensitive to pressure deviation in the air chambers
Pressure drift around a predetermined pressure in the air chambers should be small to
retain the effectiveness of PTLCD.

In this chapter, multiple pressurized tuned liquid column dampers (MPTLCD) are
thus studied for the torsional vibration reduction of a structure and to improve the
sensitivity of PTLCD under pressure mistuning. The MPTLCD container is sealed with
air chamber at its two ends. The frequency tuning can be adjusted by manipulating static
pressure inside the air chamber while the length of liquid column is fixed. An analytical
model is developed for torsional vibration of a structure with a MPTLCD under either
harmonic or white noise excitation. The non-linear damping due to orifice and the non-
linear restoring force due to air pressure in the MPTLCD are linearized in the frequency
domain. After such linearization is proved to be satisfactory through a comparison with
a nonlinear analysis in the time domain, parametric studies are then carried out in the
frequency domain in terms of the distance from the centre line of the PTLCD to the
rotational axis of the structure, the ratio of the horizontal length to the total length of
liquid column, the frequency bandwidth, the head-loss coefficient, the number of
PTLCD units in the MPTLCD, and the pressure tuning ratio.

In this chapter, only the torsional vibration of a simple structure is investigated.
The application of MPTLCD for the suppression of coupled lateral and torsional
vibrations of a real long cable-stayed bridge will be further investigated in Chapter 8
based on the understanding of MPTLCD obtained from this chapter.

5.2 ANALYTICAL MODEL
5.2.1 Equation of motion of liquid in PTLCD

Let us consider a PTLCD installed in a structure subjected to torsional vibration (Figure 5-1). The PTLCD is a U-shaped container of uniform rectangular cross-section filled with liquid and two chambers filled with compressed air of static pressure \( P_o \). In consideration of dynamic equilibrium conditions and the interactions between the structure and the liquid column in the PTLCD, the equation of motion of liquid in the PTLCD can be expressed as:

\[
pAL\ddot{W} + \frac{PA}{2} \delta \dot{W} + 2pAgW = -pAB\left( H + \frac{L - B}{2} \right) - pAgB\theta - PA
\]  

Equation (5-1) is subjected to the condition that the liquid should fully retain in the horizontal part of the PTLCD. The following equation should thus be satisfied at any time.

\[
W \leq \min\{h, (L - B - d)/2\}
\]  

where \( h \) is the air chamber height of the PTLCD; \( \rho \) is the density of liquid; \( A \) is the cross-sectional area of liquid column in the PTLCD; \( L \) is the total length of liquid column of the PTLCD; \( W \) is the relative motion of liquid column in the PTLCD to the container; \( \dot{W} \) and \( \ddot{W} \) represent, respectively, the first and second derivatives of \( W \) with respect to time; and \( \theta \) and \( \ddot{\theta} \) are, respectively, the torsional displacement and acceleration of the structure; \( d \) is the thickness of liquid column of the PTLCD; \( \delta \) is the head loss coefficient of the PTLCD governed mainly by the opening ratio of orifice; \( g \) is the acceleration due to gravity; \( B \) is the horizontal length of liquid column of the PTLCD; \( H \) is the distance between the center line of the horizontal part of the PTLCD and the rotational axis of the structure; and \( P \) is the net pressure acting on the water at time \( t \).

The volume of the two chambers varies due to the water motion inside the PTLCD, so does the pressure inside the two chambers. Supposing the change in pressure and volume is an isothermal process, the variation of pressure and volume inside the two chambers of the PTLCD can be described by Boyle’s law, that is

\[
P_o V_o = P_e (V_o - AW) = P_e (V_o + AW)
\]  

5-3
where $P_c$ is the pressure in the air chamber in further compression and $P_r$ is the pressure in the air chamber in relaxation with reference to the static pressure $P_o$; $V_o$ is the volume of air inside the chamber at the static position of liquid. The restoring force $PA$ acting on the water at time $t$ is then determined by

$$PA = (P_c - P_r)A = P_o A \left[ (1 - \frac{AW}{V_o})^{-1} - \left(1 + \frac{AW}{V_o}\right)^{-1} \right] = P_o A \left[ \left(1 - \frac{W}{h}\right)^{-1} - \left(1 + \frac{W}{h}\right)^{-1} \right]$$

(5-4)

The equation of motion of liquid inside the PTLCD becomes:

$$\rho A \ddot{W} + \frac{\rho A}{2} \dot{W} \dot{W} + 2\rho A g W = \left\{ -\rho AB \left( H + \frac{L - B}{2} \right) \ddot{\theta} - \rho Ag \theta \right\} - P_o A \left[ \left(1 - \frac{W}{h}\right)^{-1} - \left(1 + \frac{W}{h}\right)^{-1} \right]$$

(5-5)

Equation (5-4) shows that the restoring force due to the pressure acting on the water inside a PTLCD is a nonlinear function of water displacement inside the PTLCD. Iterations are generally required to solve Equation (5-5) in the time domain. However, if the ratio $W/h$ is small, the restoring force in Equation (5-4) can be approximately expressed as a linear function of water displacement, that is

$$PA \approx P_o A \left[ \left(1 + \frac{W}{h}\right) - \left(1 - \frac{W}{h}\right) \right] = \frac{2P_o AW}{h}$$

(5-6)

Substitute Equation (5-6) into Equation (5-1), the nonlinear equation of liquid motion inside the PTLCD becomes:

$$\rho A \ddot{W} + \frac{\rho A}{2} \dot{W} \dot{W} + \left(2\rho A + \frac{2P_o A}{h} \right) W = -\rho AB \left( H + \frac{L - B}{2} \right) \ddot{\theta} - \rho Ag \theta$$

(5-7)

The circular natural frequency of liquid motion in the PTLCD, $\omega_d$, is then determined by

$$\omega_d^2 \approx \frac{2g}{L} \left(1 + \frac{P_o}{\rho gh}\right)$$

(5-8)

By re-arranging Equation (5-8), the static pressure $P_o$ in the PTLCD is given by

$$P_o \approx \frac{\rho LH}{2} \left[ \omega_d^2 - \frac{2g}{L} \right]$$

(5-9)

Equation (5-8) shows that the natural frequency of liquid motion inside the PTLCD is determined not only by the length of liquid column but also the static pressure $P_o$. For a
given liquid column length, the natural frequency of the liquid motion inside the PTLCD can be increased by the factor \((1+\rho_o/\rho gh)\) comparing with the traditional TLCD, which greatly facilitates frequency tuning requirement. Equation (5-9) provides a way of determining the required static pressure \(P_e\) after the frequency of the PTLCD and its water length \(L\) are selected.

### 5.2.2 Equation of motion of structure with PTLCD

Let \(I_d\) denote the total mass moment of inertia of liquid columns in a PTLCD about the rotational axis of a structure. \(I_s\) is the mass moment of inertia of the structure including the mass moment of inertia of containers about the rotational axis of structure. \(C_s\) and \(K_s\) are the torsional damping coefficient and stiffness of the structure. The equation of motion of the structure with a PTLCD can be expressed as:

\[
(I_s + I_d)\ddot{\theta} + C_s\dot{\theta} + (K_s + \rho A L g \overline{H})\dot{\theta} = -\rho A B \left(\frac{H + \frac{L-B}{2}}{2}\right)\ddot{W} + \rho A B g W + M_s(t)
\] (5-10)

in which

\[
I_d = \rho A B \left(\frac{H^2 + \frac{B^3}{12}}{12}\right) + \rho A (L - B) \left[\frac{H^2}{4} + \frac{B^2}{2} - \frac{H(L-B)}{2} + \frac{(L-B)^2}{12}\right]
\] (5-11)

\[
\overline{H} = H - \frac{(L-B)^2}{4L}
\] (5-12)

and \(M_s(t)\) is the external moment acting on the structure.

### 5.2.3 Equation of motion of liquid in MPTLCD

Let us consider a MPTLCD which consists of \(N\) small PTLCD units and is installed in a structure subjected to torsional vibration. Each small PTLCD unit is a U-shape container of same uniform rectangular cross-section. The equation of motion of liquid in the \(k\)th PTLCD unit in the MPTLCD with linearized pressure can be expressed as:
\[ \rho A_k L_k \ddot{W}_k + \frac{\rho A_k}{2} \delta_k \dot{W}_k + \left(2 \rho A_k g + \frac{2P_{o,k} A_k}{h_k}\right) W_k = \begin{cases} -\rho A_k B_k \left(H_k + \frac{L_k - B_k}{2}\right) \dot{\theta} \\ -\rho A_k B_k \theta \end{cases} \] (5-13)

Equation (5-13) is subjected to the same condition as that of Equation (5-1) and thus the following equation should be satisfied at any time.

\[ W_k \leq \min\{h_k, (L_k - B_k - d_k)/2\} \quad (k=1,2,\ldots,N) \] (5-14)

The definitions of all parameters appearing in the above equation are the same as those in section 5.2.1 and the parameter with the subscript k corresponds to the kth PTLCD unit. Equation (5-13) indicates that the natural frequency of the kth PTLCD unit \( (\omega_k) \) can be altered by the static pressure \( P_{o,k} \), determined by the following equation:

\[ \omega_k^2 = \frac{2g}{L_k} \left(1 + \frac{P_{o,k}}{\rho g h_k}\right) \] (5-15)

Re-arranging Equation (5-15) gives the static pressure \( P_{o,k} \) of the kth PTLCD unit as:

\[ P_{o,k} \approx \frac{\rho L_k h_k}{2} \left[ \omega_k^2 - \frac{2g}{L_k} \right] \] (5-16)

5.2.4 Equation of motion of structure with MPTLCD

Let \( I_d \) denote the total mass moment of inertia of liquid columns in a MPTLCD about the rotational axis of the structure. The equation of motion of the structure with the MPTLCD can be expressed as:

\[ (I_z + I_d) \ddot{\theta} + C_s \dot{\theta} + \left( K_z + \sum_{k=1}^{N} \rho A_k L_k g h_k \right) \theta = -\sum_{k=1}^{N} \rho A_k B_k \left[H_k + \frac{L_k - B_k}{2}\right] \dot{W}_k + \rho A_k B_k g W_k + M_z(t) \] \hspace{1cm} (5-17)

in which

\[ I_d = \sum_{k=1}^{N} I_{dk} \] (5-18)

\[ I_{dk} = \rho A_k B_k \left[H_k^2 + \frac{B_k^2}{12}\right] + \rho A_k \left(L_k - B_k\right) \left[H_k^2 + \frac{B_k^2}{4} - \frac{H_k(L_k - B_k)}{2} + \frac{(L_k - B_k)^2}{12}\right], \quad (k=1,2,\ldots,N) \] (5-19)
5.2.5 Equation of motion of MPTLCD-structure system in matrix form

By combining Equation (5-13) with Equation (5-17), the equation of motion of a coupled MPTLCD-structure system can be written in a matrix form as:

\[ \mathbf{M} \ddot{\mathbf{x}} + \mathbf{C} \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{P} \]  \hspace{1cm} (5-20)

where the vector \( \mathbf{x} \) is the displacement vector and \( \mathbf{P} \) is the external moment excitation vector.

\[ \mathbf{x} = [\theta \ W_1 \ W_2 \ldots \ W_N]^T \]  \hspace{1cm} (5-21)

\[ \mathbf{P} = [\mathbf{M}(t) \ 0 \ 0 \ldots \ 0]^T \]  \hspace{1cm} (5-22)

\( \mathbf{M} \), \( \mathbf{C} \), and \( \mathbf{K} \) are the mass, damping and stiffness matrices of the coupled MPTLCD-structure system, respectively, which can be expressed as

\[
\mathbf{M} = \begin{pmatrix}
I_x + I_d & G_1 & G_2 & \cdots & G_N \\
G_1 & m_1 & 0 & \cdots & 0 \\
G_2 & 0 & m_2 & \cdots & 0 \\
& \vdots & \ddots & \ddots & \vdots \\
G_N & 0 & 0 & \cdots & m_N
\end{pmatrix}
\]  \hspace{1cm} (5-23)

\[
\mathbf{C} = \begin{pmatrix}
C_s & 0 & 0 & \cdots & 0 \\
0 & \frac{\rho A_1}{2} \delta_1 & \frac{\dot{\omega}_1}{2} & \cdots & 0 \\
0 & 0 & \frac{\rho A_2}{2} \delta_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \frac{\rho A_N}{2} \delta_N & \frac{\dot{\omega}_N}{2}
\end{pmatrix}
\]  \hspace{1cm} (5-24)

\[
\mathbf{K} = \begin{pmatrix}
K_s + mg & m_1g \alpha_1 & m_2g \alpha_2 & \cdots & m_Ng \alpha_N \\
m_1g \alpha_1 & m_1 \omega_1^2 & 0 & \cdots & 0 \\
m_2g \alpha_2 & 0 & m_2 \omega_2^2 & \cdots & 0 \\
& \ddots & \ddots & \ddots & \vdots \\
m_Ng \alpha_N & 0 & 0 & \cdots & m_N \omega_N^2
\end{pmatrix}
\]  \hspace{1cm} (5-25)

in which

\[ \alpha_k = \frac{B_k}{L_k}; \ \gamma_k = \rho A_k B_k \left( H_k + \frac{L_k - B_k}{2} \right); \ \ \ m_k = \rho A_k L_k; \ \ \ m = \sum_{k=1}^{N} m_k \bar{H}_k; \ \ \ m_d = \sum_{k=1}^{N} m_k \]  \hspace{1cm} (5-26)
where $\alpha_k$ is the liquid length ratio of the kth PTLCD; $G_k$ and $m_k$ are the liquid mass moment and mass in the kth PTLCD, respectively; $\omega_k$ is the circular natural frequency of the kth PTLCD; $H_k$ is the distance of the mass center of the kth PTLCD to the rotation axis.

5.2.6 Equivalent linearization technique

It is noted that the damping terms in Equation (5-13) for liquid motion are nonlinear. Therefore, the coupled MPTLCD-structure system described by Equation (5-20) is a nonlinear system. To facilitate practical use and to carry out parametric studies in the frequency domain, the equivalent linearization technique is applied to Equation (5-13) in terms of the principle of equivalent energy dissipation. As a result, the damping matrix in Equation (5-24) can be replaced by

$$
C = \begin{pmatrix}
2I_2 \omega_s \xi_s & 0 & 0 & \cdots & 0 \\
0 & 2m_1 \omega_1 \xi_1 & 0 & \cdots & 0 \\
0 & 0 & 2m_2 \omega_2 \xi_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 2m_N \omega_N \xi_N
\end{pmatrix}
$$

(5-27)

where $\xi_s$ is the structural damping ratio; and $\xi_k$ is the equivalent damping ratio of the kth PTLCD. If the external moment excitation is harmonic moment excitation, the equivalent damping ratio of the kth PTLCD can be calculated (Gao et al. 1997) by

$$
\xi_k = \frac{2\delta_k}{L_k \omega_k} \bar{W}_k \bar{\omega}
$$

(5-28)

where $\bar{W}_k$ is the amplitude of liquid motion in the kth PTLCD; and $\bar{\omega}$ is the circular frequency of the applied harmonic moment. If the external excitation is a zero-mean stationary white noise excitation, the equivalent damping ratio of the kth PTLCD should be calculated (Xu et al. 1992) by

$$
\xi_k = \frac{\delta_k}{L_k \omega_k \sqrt{2\pi} \sigma_{\omega_k}}
$$

(5-29)

where $\sigma_{\omega_k}$ is the standard deviation of the liquid velocity in the kth PTLCD. Since the equivalent damping ratio described by either Equation (5-28) or Equation (5-29) depends on the liquid motion, iterations are generally required.
5.3 PARAMETRIC STUDIES ON THE PERFORMANCE OF PTLCD

A single degree of freedom structure with a PTLCD is considered first. The dynamic parameters of structure studied herein are $m_s=1.5012\times10^7\text{kg}$, $I_s=6.4039\times10^8\text{kglm}^2$, $\xi_s=0.01$, $K_s=6.8640\times10^9\text{Nm/ rad}$. These values are selected with reference to the first torsional mode of vibration of a real cable-stayed bridge. The torsional circular natural frequency of the structure is 3.2792rad/s. To evaluate the PTLCD/MPTLCD performance in this chapter, the mass ratio $\mu$ is defined as the ratio of total mass of liquid inside the PTLCD/MPTLCD to the mass of structure. The frequency tuning ratio $\Delta\gamma$ is introduced here.

$$\Delta\gamma = \frac{\omega_d}{\omega_s}$$ \hspace{1cm} (5-30)

where $\omega_d$ is the natural frequency of the PTLCD; $\omega_s$ is the natural frequency of the structure. For the sake of comparison, the structural response ratio $R$ is introduced as:

$$R = \frac{\text{Maximum structural response with control}}{\text{Maximum structural response without control}} \quad \text{for harmonic excitation} \hspace{1cm} (5-31)$$

$$R = \frac{\text{Standard deviation of structural response with control}}{\text{Standard deviation of structural response without control}} \quad \text{for white noise excitation} \hspace{1cm} (5-32)$$

The smallest value of the structural response ratio is often used as the indication of the effectiveness of MPTLCD in the parametric study. The harmonic excitation amplitude and the power spectral density function of the white noise excitation used in the study are: $M_o=1.0\times10^6\text{Nm}$ and $S_M=7.5\times10^{11}\text{N}^2\text{m}^2/\text{s/ rad}$, respectively.

5.3.1 Validation of linear water motion assumption

The linear equation of motion of liquid inside a PTLCD is obtained by the linearization of its nonlinear damping and nonlinear restoring force. To check the validity of the linear equation of motion of a coupled PTLCD-structure system, the Wilson-0 method is used for the numerical solution of the nonlinear equations of the PTLCD-structure system which are described by Equations (5-5) and (5-10). The obtained steady state responses of both structure and water under the harmonic excitation and the standard deviation responses of both structure and water under the
white noise excitation are compared with those obtained by the corresponding equations linearized in the frequency domain. The time history of white noise moment excitation is generated using an algorithm proposed by Shinozuka (1972). It is modelled as a band-limited Gaussian white noise with a bandwidth of 25Hz. The duration of the simulated time history of moment excitation is 1310.7s with an interval of 0.02s. The parameters of the PTLCD are N=1, Δγ=1, δ=30, α=0.7, H=3.66m, L=8m, and μ=0.02. The validity of linear equation is verified by examining the responses of the PTLCD-structure system within a range of the W/h ratio. The results from both the time domain analysis and the frequency domain analysis are plotted in Figure 5-2. It can be seen that the relative difference of the structure response ratio between the results from nonlinear equation and the results from linear equation is decreased with an increasing air chamber height. The structural response ratios and water displacements computed from the linear equation are almost the same as those from the nonlinear equation when the air chamber height is larger than 0.7m for either harmonic or white noise excitation. However, the PTLCD with a larger value of air chamber height would require a larger value of static pressure as indicated by Equation (5.9). To keep the assumption of the linearized restoring force valid, the air chamber height should be designed as large as possible but at the same time it would not lead to large static pressure requirement. A value of 1.5m is recommended in this study, which leads to the ratio \( \overline{W}/h \) and \( \sigma_w/h \) being 0.047 for the harmonic excitation case and 0.027 for the white noise excitation case.

With the selected air chamber height, the validity of the linear equation of liquid is further examined for linearized damping force by looking at the steady-state responses of the structure and water under harmonic excitation with two different amplitudes. The steady-state responses obtained are plotted in Figure 5-3. It can be seen that the results from the linear equation are almost the same as those from the nonlinear equation for a wide range of excitation amplitude. This implies the nonlinearity of water motion resulting from damping and pressure is relatively weak. Hence, the linear equation is an adequate model to describe the motion of the PTLCD-structure system if the value of air chamber height is properly selected.
5.3.2 Effect of distance from PTLCD to rotational axis

The effect of the distance from the centreline of the horizontal part of the PTLCD to the rotational axis of the structure on torsional displacement response ratio is depicted in Figure 5-4. The parameters of the PTLCD are \( N=1, \Delta y=1, \delta=30, \alpha=0.7, \)\( h=1.5 \text{m} \) and \( \mu=0.02 \). Three different PTLCDs, with water lengths \( L \) of 4m, 6m and 8m, are studied, and the corresponding static pressure \( P_o \) is 17.441kPa, 33.518kPa, 49.596kPa according to the frequency tuning requirement. The thickness of the corresponding liquid column \( d \) is selected as 0.2m, 0.3m and 0.4m, respectively, to retain a reasonable geometric size of the damper. To keep the same mass ratio of 0.02 for each case, the width (or the number) of the liquid column, which is in the direction perpendicular to the elevation of the damper, is then determined accordingly. The performance of passive TLCD is also included for the comparison with the PTLCDs. The parameters of the passive TLCD are \( N=1, \Delta y=1, \delta=30, \alpha=0.7, \) and \( \mu=0.02 \). The corresponding water length \( L_p \) is 1.83m. The thickness of the liquid column is selected as 0.1m. The width (or the number) of the liquid column is then determined based on the mass ratio of 0.02. Clearly, PLTCD provides a great flexibility for selecting different liquid lengths for a given structure. The longer the liquid column length, the larger is the air pressure, but the smaller is the depth of the liquid column which indicates the smaller number of containers required in practice. It is seen from Figure 5-4 that the reduction of torsional displacement response is significantly affected by the value of \( H \). The reduction in torsional displacement response is hardly achieved if the ratio \( H/L_p \) is close to a value of 0.35 for the case of TLCD and a value of 0.15 for the case of PTLCD with a water length of 4m. Similar finding has been reported and explained by Xue et al. (2000) for the case of TLCD in reducing the torsional vibration of a structure. As shown in Chapter 4, this phenomenon is attributed to the zero interaction between the structure and the TLCD when the ratio \( H/L_p \) approaches to \( \alpha/2 \). For the case of PTLCD under harmonic or white noise excitation, when the excitation frequency equals to the structural frequency the condition for the zero interaction between the structure and the PTLCD can be derived as follows:

\[
\frac{H}{L_p} = \frac{\alpha}{2} - T(1 - \alpha); \quad T = \frac{P_o}{2 \rho gh}
\]  

(5-33)
It is noted that Equation (5-33) can be deduced to the condition for the case of the TLCD by setting the pressure inside the air chambers of PTLCD to be zero. Due to the fact that the static pressure in the PTLCD is positive, it is easily seen from Equation (5-33) that the $H/L_p$ ratio for the zero interaction between the structure and the PTLCD is smaller than that for the TLCD. Furthermore, Equation (5-9) shows that the pressure $P_o$ increases with the water length of a PTLCD, so does the value of $T$. Hence, the ratio $H/L_p$ for the zero interaction between the structure and the PTLCD decreases with the increasing water length of the PTLCD. For the PTLCD with the water length $L$ of 6m and 8m, the corresponding value of the $H/L_p$ ratio is thus smaller than that of 4m. This feature further facilitates the application of PTLCD in practice. It is also seen from Figure 5-4 that the effectiveness of PTLCD in reducing the torsional displacement is increased with the increasing ratio of $H/L_p$ when the ratio is above 0.35. For a given $H/L_p$, PTLCD with a larger value of $L$ can achieve larger reduction in displacement response. Figure 5-4 also depicts that for the $H/L_p$ ratio beyond a value of 0.35, the performance of the PTLCD in reducing the torsional displacement is higher than that of the TLCD and it is further enhanced with the increasing ratio of $H/L_p$. The performance of PTLCD is less affected by the value of $H$ than that of TLCD, but it is still important to select a proper ratio of $H/L_p$ for the PTLCD so as to avoid the zero interaction between the structure and PTLCD.

5.3.3 Effect of liquid length ratio

The effect of liquid length ratio, $\alpha$, of the horizontal length to the total length of liquid column of the PTLCD on torsional displacement response ratio is depicted in Figure 5-5 for four different distance ratios $H/L_p$. The parameters of the PTLCD are $N=1$, $\Delta\gamma=1$, $\delta=30$, $L=8m$, $h=1.5m$, $d=0.4m$, $P_o=49.596$ kPa and $\mu=0.02$. The liquid length ratio $\alpha$ varies within the range from 0.1 to 0.8. It is seen from Figure 5-5 that for the ratio $H/L_p$ equal to 0.5 or 1, the reduction of torsional displacement is dependent on the ratio of horizontal length to total length of liquid column. An optimal value of liquid length ratio $\alpha$ exists for a maximum reduction of torsional displacement. It is also seen that increase in the ratio $H/L_p$ leads to more reduction in torsional displacement response. When the ratio $H/L_p$ increases to 2 or above, the torsional displacement
response reduces monotonically with increasing liquid length ratio $\alpha$: the larger the liquid length ratio, the greater is the torsional displacement response reduction.

5.4 PARAMETRIC STUDIES ON THE PERFORMANCE OF MPTLCD

The structure considered before is now considered in this section with a MPTLCD. To help the understanding of MPTLCD performance, several parameters are introduced first. They are the central frequency $\omega_c$, the frequency bandwidth $\Delta X$, the frequency tuning ratio $\Delta \gamma$, and the constant frequency spacing $\beta_k$.

$$\omega_c = \frac{\omega_N + \omega_L}{2}; \quad \Delta X = \frac{\omega_N - \omega_L}{\omega_s}; \quad \Delta \gamma = \frac{\omega_c}{\omega_s}; \quad \beta_k = \omega_{k+1} - \omega_k. \quad (5-34)$$

where $\omega_k$ is the natural frequency of liquid column in the $k$th PTLCD; and $\omega_L$ and $\omega_N$ are the lowest and the highest natural frequencies, respectively, among all the PTLCD units in the MPTLCD. From a practical point of view, the water length and the air chamber height of all the PTLCD units are kept the same. The frequency tuning depends on the pressure inside the air chambers of each PTLCD only. The sensitivity of the $k$th PTLCD to off-tuning situation is thus evaluated in terms of the pressure tuning ratio parameter $\Delta P_k$ defined as

$$\Delta P_k = \frac{P_{\omega_k}}{P_{L, h}} \left[ \frac{2g}{\omega_k^2 - \frac{2g}{L_k}} \right] \quad (5-35)$$

According to Equation (5-16), the pressure tuning ratio with a value of unity implies that the $k$th PTLCD in the MPTLCD is tuned to the designated frequency $\omega_k$.

5.4.1 Effect of frequency bandwidth and head-loss coefficient

To investigate the effect of bandwidth and head-loss coefficient on the performance of a MPTLCD, the bandwidth and head-loss coefficient are taken as variables and the other parameters of the MPTLCD are $N=5$, $\Delta \gamma=1$, $\Delta P=1$, $H=3.66m$, $\alpha=0.7$, $L=8m$, $h=1.5m$, $d=0.4m$, and $\mu=0.02$. The dropping of the subscript $k$ in damper parameters implies that these parameters are the same for all the PTLCD units in the MTPCD. The maximum pressure among all the PTLCD units in the MPTLCD, $P_{\text{max}}$, is 70.34kPa. Figure 5-6 depicts the variation of torsional displacement response ratio with
the bandwidth $\Delta X$ for a series of head-loss coefficients. It is seen that the effectiveness of the MPTLCD is affected by frequency bandwidth significantly. There is an optimal frequency bandwidth by which the maximum response reduction can be achieved for a given head loss coefficient. For instance, if the head loss coefficient is less than 10, the optimal bandwidth is around 0.08. Figure 5-7 displays the variation of torsional displacement response ratio with the head-loss coefficient $\delta$ for a series of frequency bandwidth. The optimal head loss coefficient for the case of harmonic excitation is larger than that for the case of white noise excitation. If the frequency bandwidth becomes smaller, the optimal head loss coefficient becomes larger and eventually it is close to the optimal head loss coefficient of SPTLCD. The optimal head loss coefficient of the MPTLCD is clearly much less than that of the SPTLCD.

5.4.2 Effect of number of PTLCD units

To investigate the effect of the number of PTLCD units on the performance of the MPTLCD, the optimal head loss coefficient and frequency bandwidth to achieve the maximum reduction of structural responses are found for a given number of PTLCD units. Therefore, not only the number of PTLCD units, $N$, but also the head loss coefficient, $\delta$, and the frequency bandwidth, $\Delta X$, are taken as variables in this investigation. The total liquid mass of the MPTLCD however remains the same regardless of the number of PTLCD units. The other fixed parameters of the MPTLCD are $\alpha=0.7$, $H=3.66m$, $\Delta \gamma=1$, $L=8m$, $h=1.5m$, $d=0.4m$, and $\mu=0.02$. The intervals of the parameters $N$, $\delta$ and $\Delta X$ considered in the computation are 2, 0.5, and 0.01, respectively. Figure 5-8 portrays the minimum torsional displacement responses of the structure against the number of PTLCD units. The values inside the parenthesis in Figure 5-8 represent the corresponding optimum head loss coefficient and frequency bandwidth. It is seen that with increasing number of PTLCD units, the optimum value of frequency bandwidth increases while the optimum value of head loss coefficient decreases. Similar trends have been reported in Chapter 4 for the MTLCD in reducing the torsional vibration of a structure. Figures 5-8 also indicates that the performance of the optimized MPTLCD ($N=1$) is better than that of the optimized STLCD ($N=1$). For the case of harmonic excitation, there is some variation in the performance of the optimized MPTLCD when the number of PTLCD units in the
MPTLCD smaller than 9, while for the case of white noise excitation, there is no significant improvement if the number of PTLCD units in the MPTLCD exceeds or equal to 5.

5.4.3 Effect of pressure tuning ratio

Some studies (Fujino et al. 1993; Gao et al. 1999) indicated that the use of multiple dampers could increase its robustness against the offset in frequency tuning due to error in the estimation of structural frequency. The natural frequency of liquid in a PTLCD is now tuned by adjusting its pressure. In a real situation it may be difficult to let the static pressure to be tuned precisely to the target frequency. To see the robustness of a MPTLCD against the pressure mistuning, the effect of the pressure tuning ratio ΔP on the MPTLCD's performance in reducing torsional displacement response ratio is thus investigated. Figure 5-9 depicts the variation of torsional displacement response ratio with the pressure tuning ratio for a series of bandwidth under either harmonic excitation or white noise excitation. The fixed parameters of the PTLCD (ΔX=0) and the MPTLCD (ΔX≠0) are Δγ=1, α=0.7, H=3.66m, L=8m, h=1.5m, d=0.4m, μ=0.02, and δ=20 for N=1, and δ=5 for N=5. The maximum pressure among all the PTLCD units in the MPTLCD, \( P_{\text{max}} \), is 66.32kPa. In the case of harmonic excitation, the performance of the MPTLCD with a bandwidth of 0.07 is higher than that of the PTLCD in reducing torsional displacement response within the pressure-turning ratio ranging from 0.95 to 1.05. The optimum pressure tuning ratio of the MPTLCD is close to that of the PTLCD. By increasing the frequency bandwidth to 0.10 and above, the sensitivity of the MPTLCD to pressure tuning ratio is improved significantly, but the effectiveness of vibration control is slightly decreased. In the case of white noise excitation, the MPTLCD with the bandwidth less than 0.16 can reduce not only the torsional displacement response but also the sensitivity to the pressure turning ratio, compared with the PTLCD. Hence, one can design a MPTLCD, which has almost the same effectiveness as the optimized PTLCD but is more robust to pressure tuning ratio.

5.5 DESIGN CONSIDERATION FOR MPTLCD
In designing a MPTLCD for suppressing torsional vibration of a structure, the value of \( H \) should be decided first according to real structure configuration. The mass moment ratio of the MPTLCD to the structure is then selected and it is usually not greater than 5\%. Once the damper mass \( m_d \) is decided, the water length \( L \) should be determined based on the available space inside the structure and the air pressure required. To keep the validity of linearized restoring force due to air pressure inside the air chambers, it is recommended that the air chamber height \( h \) be larger than one-tenth of the water length. To increase the damper robustness to the pressure or frequency mistuning, the use of multiple dampers is recommended. From a practical point of view, the water length and the air chamber height of all the PTLCD units are kept the same. The number of PTLCD units should be selected to be not less than a certain number such as five in this study. Then, the frequency tuning ratio as well as the pressure tuning ratio of the MPTLCD may be set as unity for simplicity. The selection of the liquid length ratio should be as large as possible provided that the liquid in the horizontal part is always retained under the practical excitation level. The condition specified by Equation (5-33) should also be checked at this stage with the value of \( H/L \), targeted to be greater than 2. The optimal head loss coefficient and the frequency bandwidth of the MPTLCD can be estimated through parametric studies such as Figures 5-6 & 5-7 in this study. The designation of static pressure \( P_{0,k} \) inside air chambers of the kth PTLCD is finally determined by Equation (5-16).

5.6 SUMMARY

An analytical model for torsional vibration of a structure with a MPTLCD under either harmonic excitation or white noise excitation was developed in this chapter. The interaction between the structure and the MPTLCD was fully taken into consideration. The nonlinear restoring force due to the pressure inside the air chambers was linearized as a linear function of water displacement and the nonlinear damping property of the MPTLCD was linearized using the equivalent linearization technique. The MPTLCD can provide a great flexibility for selecting a liquid length to have a proper frequency tuning through the change of air pressure. In general, the longer the liquid column length, the higher is the air pressure. For a given mass ratio, the number of containers required in a PTLCD with longer liquid length is smaller. The results obtained from the
linear equation of a PTLCD-structure system were compared with those obtained from
the nonlinear equation of the PTLCD-structure system to examine the validity of linear
model of the PTLCD-structure system. The results from the linear model are almost the
same as those from the nonlinear model if the air chamber height is selected properly.
This implied that the nonlinearity of the MPTLCD-structure system is relatively weak.
Hence, the linear equation is an adequate model to describe the MPTLCD-structure
system. Extensive parametric studies were then carried out in the frequency domain.
The investigations demonstrated that $H/L_p$ ratio is an important parameter affecting
the performance of PTLCD in reducing torsional vibration. For the same value of the $H/L_p$,
the PTLCD with a larger value of liquid length can achieve larger reduction in
displacement response. It was also found that for $H/L_p$ ratio beyond a value of 0.35, the
performance of PTLCD in reducing torsional displacement is higher than that of TLCD
and it could be further improved with the increasing of $H/L_p$ ratio. For the ratio $H/L_p$
equal to 0.5 or 1, an optimal value of $\alpha$ exists for the maximum torsional displacement
reduction. For the ratio $H/L_p$ increasing to 2 and above, the performance of MPTLCD is
improved and the torsional response reduction is increased with the increasing liquid
length ratio. There are an optimal head loss coefficient and an optimal frequency
bandwidth for a MPTLCD with a given number of PTLCD units. The optimal head loss
coefficient is decreased but the optimal frequency bandwidth is increased with the
number of PTLCD units. The effectiveness of optimised MPTLCD increases with the
increasing number of PTLCD units but there is no significant improvement if the
number of PTLCD units in the MPTLCD exceeds or equal to five. The sensitivity of
MPTLCD to pressure tuning ratio can be improved by increasing the bandwidth but at
the expense of less torsional vibration reduction. One may design a MPTLCD with
almost the same effectiveness as the optimised PTLCD but it is more robust to the
pressure-tuning ratio. The application of MPTLCD for the suppression of coupled
lateral and torsional vibration of a real long cable-stayed bridge will be investigated in
chapter 8 based on the understanding of MPTLCD accomplished from this chapter.
Figure 5-1 PTLCD-structure system
(a) Harmonic excitation

(b) White noise excitation

Figure 5-2 Nonlinear Effect of Air Chamber Height
Figure 5-3 Comparisons of Linear and Nonlinear Responses of PTLCD-Structure System
Figure 5-4 Effect of Distance from PTLCD to Rotational Axis
(a) Harmonic excitation

(b) White noise excitation

Figure 5-5 Effect of Liquid Length Ratio
Figure 5-6 Effect of Frequency Bandwidth
(a) Harmonic excitation

(b) White noise excitation

Figure 5-7 Effect of Head Loss Coefficient
Figure 5-8 Effect of Number of PTLCD units
Figure 5-9 Effect of Pressure Tuning Ratio
CHAPTER 6

SEMI-ACTIVE TUNED LIQUID COLUMN DAMPERS
FOR STRUCTURAL VIBRATION CONTROL

6.1 INTRODUCTION

As shown in Chapter 5, to control vibration of a structure with high natural frequency, the MPTLCD with a longer liquid column length can be used to replace the STLCD or MTLCD with a shorter liquid column length. However, for a long span cable-supported bridge during construction, its very low natural frequency in the lateral direction may require a STLCD or a MTLCD with a very long liquid column length, which may be impossible in practice. Moreover, the natural frequencies of a long span bridge vary during its construction stage and it is thus difficult to apply TLCD with a fixed configuration to the bridge during its construction or it is not economical to design a series of TLCDs with different configurations. Hence, tuned liquid column dampers with adaptive frequency tuning capacity should be sought for suppressing lateral or torsional vibration of a long span cable-supported bridge during its construction. The semi-active tuned liquid column dampers (SATLCD), of which the frequency can be adjusted by active control of air pressures at the two chambers of a PTLCD based on feedback of the liquid response, are investigated in this chapter.

Previous works on semi-active tuned liquid column dampers focused mainly on the optimal control of head loss coefficient, such as those performed by Kareem (1994), Haroun and Pires (1994), and Yalla et al. (2002). Haroun and Pires (1994) further proposed a hybrid liquid column damper to control lateral vibration of structure under earthquake excitation by delivering a desired optimal control force to the system via controlling the orifice together with the liquid column pressure actively. However, there is little work relevant to the improvement of TLCD frequency-tuning adaptability by the use of semi-active control technology.

The objective of this chapter is thus to develop a new semi-active tuned liquid column damper (SATLCD) with adaptive tuning capacity for reducing either lateral or
torsional vibration of a structure with very low or high frequency. The principle of SATLCD with adaptive tuning capacity is first introduced. The analytical models are then developed for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD, respectively, under either harmonic or white noise excitation. The non-linear damping property of SATLCD is linearized by an equivalent linearization technique. Extensive parametric studies are finally carried out in the frequency domain to find the beneficial parameters by which the maximum vibration reduction can be achieved. It should be pointed out that the investigation in this chapter is limited to a simple structure in order to pursue a deep understanding of the characteristic and performance of SATLCD. The application of SATLCD for the suppression of coupled lateral and torsional vibrations of a real long cable-stayed bridge during construction will be investigated in Chapter 9 based on the understanding of SATLCD obtained from this chapter.

6.2 MATHEMATICAL MODELS

To facilitate the frequency tuning of liquid column inside a TLCD to the natural frequency of a structure, two air chambers are formed at the two ends of the TLCD (see Figure 6-1 and Figure 6-2). Net external pressure in the two air chamber acts on the liquid column to change the restoring force of liquid column. The pressure is regulated according to the displacement of the liquid column in a prescribed way. As a result, the natural frequency of liquid motion inside the TLCD can be increased or decreased accordingly so as to tune it to the natural frequency of the structure. The proposed pressure system with a closed loop control installed on a TLCD is shown in Figure 6-1 for lateral vibration control and Figure 6-2 for torsional vibration control. The net pressure inside the two air chambers, sensed by the pressure transducer, is to be forced to follow or track the desired pressure determined by a computer. Any deviation from the desired pressure is fed back to the computer to take corrective action to adjust the control valve. Thus, the control system is continually monitoring and correcting pressure deviation to maintain the desired pressure acting on the liquid column.

6.2.1 Lateral vibration of SATLCD-structure system
Let us consider a SATLCD installed in a structure subjected to lateral vibration (Figure 6-1). The SATLCD is a U-shaped container of uniform rectangular cross-section filled with liquid and two air chambers filled with compressed air of static pressure $P_0$. In consideration of dynamic equilibrium condition and the interaction between the structure and the liquid column in the SATLCD, the equation of motion of a structure equipped with the SATLCD for lateral vibration control is

$$\rho A L \ddot{W} + \frac{1}{2} \rho A \delta |\dot{W}| \dot{W} + 2 \rho Ag W = -\rho AB \ddot{x} - u$$ \hspace{1cm} (6-1)

$$m_s (1 + \mu) \ddot{x} + C_x \dot{x} + K_x x = -\rho AB \dot{W} + F_s$$ \hspace{1cm} (6-2)

under the condition

$$W \leq \frac{L - B - d}{2}$$ \hspace{1cm} (6-3)

where $m_s$ is the mass of structure; $C_x$ and $K_x$ are the lateral damping coefficient and the lateral stiffness of structure, respectively; $x$ is the lateral displacement of structure; $W$ is the relative motion of liquid column inside the SATLCD to the container; $g$ is the acceleration due to gravity; $d$ is the thickness of liquid column; $\delta$ is the head loss coefficient of the SATLCD governed by the opening ratio of orifice; $\rho$ is the liquid density in the SATLCD; $A$ is the cross-sectional area of liquid column in the SATLCD; $B$ is the horizontal length of liquid column; $L$ is the total length of liquid column; $\mu$ is the mass ratio of the liquid column to the structure; $F_s$ is the external force acting on the structure. The control force $u(t)$ in the SATLCD is given by

$$u(t) = S \cdot W(t)$$ \hspace{1cm} (6-4)

$S$ is the constant displacement feedback gain of the SATLCD. The direction of the control force $u(t)$ is in the same (opposite) direction as the liquid motion $W$ when the constant displacement feedback gain is positive (negative). The circular natural frequency of liquid motion in the SATLCD, $\omega_d$, is then determined by

$$\omega_d^2 = \frac{2 \rho Ag + S}{\rho A L}$$ \hspace{1cm} (6-5)

For a targeted frequency of liquid damper, it is easily seen from Equation (6-5) that the liquid column length of the SATLCD is given by

$$L = \frac{2g}{\omega_d^2} + \frac{S}{\rho A \omega_d^2}$$ \hspace{1cm} (6-6)
Clearly, the liquid column length can be increased or decreased by adjusting the constant displacement feedback gain while keeping its frequency unchanged. The SATLCD is therefore more flexible than the traditional TLCD in which $S$ is equal to zero and there is no way for changing the liquid column length $L$. The frequency-tuning ratio, which could affect the performance of the SATLCD, is defined as

$$
\Delta \gamma = \frac{\omega_d}{\omega_x}
$$

(6-7)

where $\omega_x$ is the circular lateral frequency of the structure without control. Once the frequency and length of the liquid column are decided, the required constant displacement feedback gain of the SATLCD can be determined by

$$
S = m_d \left[ (\Delta \gamma \omega_x)^2 - \frac{2g}{L} \right]
$$

(6-8)

where $m_d$ is the mass of the liquid column. The desired control force acting on the liquid column can be provided by regulating the air pressure in the right chamber with respect to the air pressure in the left chamber to obtain a net pressure $P(t)$ as

$$
P(t) \cdot A = S \cdot W(t)
$$

(6-9)

The air pressure in the left chamber $P_L$ and in the right chamber $P_R$ is determined, respectively, by

$$
P_L = P_0 - \frac{P(t)}{2} \quad P_R = P_0 + \frac{P(t)}{2}
$$

(6-10)

Equation (6-9) provides a way of determining the required net pressure $P$ at time $t$ if the displacement feedback gain $S$ and the liquid displacement inside the SATLCD at time $t$ are known. Equation (6-5) shows that the natural frequency of liquid motion inside the SATLCD is determined not only by the length of liquid column but also the feedback gain $S$. It can also be seen from Equation (6-8) that the constant displacement feedback gain can be obtained if the structural natural frequency, the length of liquid column, the frequency-tuning ratio and the mass of liquid column are known. Equation (6-9) also shows that the net pressure $P$ required for the feedback force is linearly proportional to the liquid displacement inside the SATLCD.

6.2.2 Torsional vibration of SATLCD-structure system
In consideration of the dynamic interaction between the structure and the liquid column of the SATLCD under torsional vibration (Figure 6-2), the equation of motion of a structure equipped with the SATLCD can be derived as

\[ \rho A L \ddot{W} + \frac{1}{2} \rho A d \ddot{W} \dot{W} + 2 \rho A g W = -\rho A B \left( H + \frac{L - B}{2} \right) \dot{\theta} - \rho A B g \dot{\theta} - u \]  

(6-11)

\[ (I_s + I_d) \dddot{\theta} + C_s \dot{\theta} + (K_s + \rho A L g \bar{H}) \dot{\theta} = -\rho A B \left( H + \frac{L - B}{2} \right) \ddot{W} - \rho A B g W + M \]  

(6-12)

with the condition

\[ W \leq \frac{L - B}{2} - \frac{d}{2} \]  

(6-13)

where \( I_s \) is the mass moment of inertia of the structure; \( C_s \) and \( K_s \) are the torsional damping coefficient and the torsional stiffness of the structure, respectively; \( I_d \) denotes the total mass moment of inertia of liquid column in the SATLCD about the elastic center of the structure; \( \theta \) is the torsional displacement of the structure; \( H \) is the vertical distance between the center line of the horizontal part of SATLCD and the elastic center of the structure; \( \bar{H} \) is the distance between the mass center of liquid inside the SATLCD and the elastic center of the structure; \( M \) is the external moment acting on the elastic center of the structure; the control force \( u \) is given by Equation (6-4); and the circular natural frequency of liquid motion in the SATLCD is determined by Equation (6-5).

The frequency-tuning ratio, which could affect the performance of the SATLCD for torsional vibration reduction, is defined as

\[ \Delta \gamma = \frac{\omega_d}{\omega_0} \]  

(6-14)

where \( \omega_0 \) is the circular torsional frequency of the structure without control. By substituting Equation (6-14) into Equation (6-5) and after some manipulations, the constant displacement feedback gain of the SATLCD is given by

\[ S = m_d \left[ (\Delta \gamma \omega_0)^2 - \frac{2 g}{L} \right] \]  

(6-15)

The required external pressure acting on the liquid inside the SATLCD is determined by Equation (6-9) and Equation (6-10).
6.2.3 Equation of motion of SATLCD-structure system in matrix form

The equation of motion of the coupled SATLCD-structure system subjected to either lateral or torsional excitation can be written in a matrix form as

\[ M\ddot{y} + C\dot{y} + Ky = P \]  \hspace{1cm} (6-16)

where the vector \( y \) is the displacement vector, \( P \) is the external excitation vector, \( M \), \( C \), and \( K \) are the mass, damping, and stiffness matrices of the coupled SATLCD-structure system, respectively. For the case of lateral vibration of the system,

\[ y = [x \quad W]^T \quad P = [F \quad 0]^T \]  \hspace{1cm} (6-17)

\[ M = \begin{bmatrix} m_s + m_d & am_d \\ am_d & m_d \end{bmatrix}; \quad C = \begin{bmatrix} 2m_s\omega_x\xi_x & 0 \\ 0 & \frac{1}{2}\rho A\delta|\dot{W}| \end{bmatrix}; \quad K = \begin{bmatrix} K_s & 0 \\ 0 & 2\rho A g + S \end{bmatrix} \]  \hspace{1cm} (6-18)

For the case of torsional vibration of the system,

\[ y = [\theta \quad W]^T \quad P = [M \quad 0]^T \]  \hspace{1cm} (6-19)

\[ M = \begin{bmatrix} I_s + I_d & G \\ G & m_d \end{bmatrix}; \quad C = \begin{bmatrix} 2I_s\omega_\theta \xi_\theta & 0 \\ 0 & \frac{1}{2}\rho A\delta|\dot{W}| \end{bmatrix}; \quad K = \begin{bmatrix} K_\theta + mg & m_d ga \\ m_d ga & 2\rho A g + S \end{bmatrix} \]  \hspace{1cm} (6-20)

where

\[ m_d = \rho A L; \quad \alpha = \frac{B}{L}; \quad G = m_d a \left( H + \frac{L-B}{2} \right); \quad m = m_d \bar{H}; \quad \bar{H} = H - \frac{(L-B)^2}{4L} \]  \hspace{1cm} (6-21)

6.2.4 Equivalent linearization technique

It is noted that damping terms in Equation (6-1) and Equation (6-11) for liquid motion are non-linear and therefore the equation of the SATLCD-structure system is also nonlinear. To carry out an extensive parametric study in the frequency domain, the equivalent linearization technique is applied to nonlinear damping force of liquid motion in Equation (6-1) and Equation (6-11) in terms of the principle of equivalent energy dissipation. As a result, the nonlinear damping terms in Equation (6-1) and Equation (6-11) can be replaced by an equivalent damping coefficient. If external excitation is a harmonic excitation, the equivalent damping ratio can be calculated (Gao et al. 1997) by
\[ \zeta = \frac{2\delta}{\tilde{\omega} L} \]  
(6-22)

where \( \tilde{W} \) is the amplitude of liquid motion in the SATLCD; and \( \tilde{\omega} \) is the circular frequency of the applied harmonic excitation. If external excitation is a stationary random excitation, the equivalent damping coefficient can be calculated (Xu et al. 1992) by

\[ \zeta = \frac{1}{\sqrt{2\pi}} \frac{\delta}{\omega_d L} \sigma_w \]  
(6-23)

where \( \sigma_w \) is the standard deviation of the liquid velocity in the SATLCD. Since the equivalent damping ratio described by either Equation (6-22) or Equation (6-23) depends on the liquid motion, iterations are generally required.

6.3 PARAMETRIC STUDIES ON LATERAL VIBRATION REDUCTION

Because of lacking structural continuity from pylon to pylon, the lateral stiffness of a long span cable-stayed bridge during construction when using a double cantilever technique is much lower than that after the bridge is completed. If the traditional TLCD is used in such a case, it will require a very long liquid column length that may not be practical. As such, the semi-active tuned liquid column damper with adaptive tuning capacity is studied in this chapter to deal with the problem. The performance of the SATLCD is investigated for the lateral vibration reduction of a structure under two different kinds of loading: harmonic and white noise excitations. The dynamic parameters of a structure studied herein are \( m_x = 4.6905 \times 10^6 \text{kg} \), \( K_x = 3.6504 \times 10^6 \text{N/m} \), \( \xi_x = 0.01 \), and these values are selected from the first lateral vibration mode of a cable-stayed bridge under construction. The lateral circular natural frequency of the structure \( (\omega_x) \) is 0.2790 rad/s. It should be noted that the use of traditional TLCD for the lateral vibration suppression of the concerned structure would require a liquid column length of 252m, which is impossible in practice. To evaluate the SATLCD performance, the mass ratio \( \mu \) is defined as the ratio of the total mass of liquid inside the SATLCD to the mass of the structure. For the sake of comparison, the structural response ratio \( R \) is introduced as:

\[ R = \frac{\text{Maximum structural response with control}}{\text{Maximum structural response without control}} \quad \text{for harmonic excitation} \]  
(6-24)
\[ R = \frac{\text{Standard deviation of structural response with control}}{\text{Standard deviation of structural response without control}} \quad \text{for white noise excitation} \] (6-25)

The smallest value of structural response ratio is often used as the indication of the effectiveness of the SATLCD. The harmonic force amplitude is 6000 N and the power spectral density function of the force is \(2.5 \times 10^8 \text{N}^2/\text{s/rad}\).

### 6.3.1 Effect of liquid length ratio

The effect of liquid length ratio, \(\alpha\), which is the ratio of the horizontal length to the total length of liquid column of the SATLCD, on the lateral displacement response ratio is depicted in Figure 6-3. The other parameters of the SATLCD used in computation are \(\Delta y = 1\), \(\delta = 15\), \(L = 22\text{m}\) and \(\mu = 0.02, 0.03\) and 0.04. Note that the liquid column length selected is 22 m only, which is much shorter than 252m required by the traditional TLCD. Figure 6-3 demonstrates that SATLCD can reduce the displacement response of the structure significantly if its parameters are selected properly. When the liquid length ratio equals to 0.7, the lateral displacement reduction achieved by the SATLCD (\(\mu = 0.02\)) reaches the level of 66\% for the structure under harmonic excitation and 38\% under white noise excitation. The SATLCD with a larger value of liquid length ratio \(\alpha\) can achieve larger reduction in displacement response but at the cost of relatively larger value of pressure required for the feedback control force. For a given value of liquid length ratio \(\alpha\), SATLCD with larger liquid mass can achieve larger reduction in displacement response but the required pressure for the feedback control force decreases as the mass of liquid inside the container increases.

### 6.3.2 Effect of head-loss coefficient

To investigate the effect of head-loss coefficient on the performance of SATLCD in reducing lateral displacement response of the structure, head-loss coefficient is taken as a variable. The other parameters of SATLCD are \(\Delta y = 1\), \(\alpha = 0.7\), \(\mu = 0.03\). Figure 6-4 shows the variation of the lateral displacement response ratio with the head-loss coefficient for four different water lengths \(L = 20\text{m}, 22\text{m}, 24\text{m},\) and 26m. The
corresponding thickness of liquid column (d) is selected as 1.0m, 1.2m, 1.4m, and 1.6m, respectively, to retain a reasonable geometric size of the damper. To keep the same mass ratio of 0.03 for each case, the width of the liquid column, which is in the direction perpendicular to the elevation of the damper, is then determined accordingly. Clearly, SATLCD provides a great flexibility for selecting different liquid lengths for a given structure. It can be seen from Figure 6-4 that the effectiveness of SATLCD is affected by head loss coefficient. There exists an optimal head loss coefficient to achieve the maximum lateral displacement reduction. The effectiveness of the optimized SATLCD is almost the same for different liquid lengths, and the corresponding optimal head loss coefficient increases with the increasing length of liquid. Figure 6-4 also indicates that the pressure required for feedback control force increases as the head loss coefficient decreases. It is seen that the maximum lateral displacement reduction achieved by the optimized SATLCD reaches the level of 75% for the structure under harmonic excitation and 48% under white noise excitation.

6.3.3 Effect of frequency offset ratio

The natural frequency of liquid column damper is usually designed to be the same as that of the structure in order to have maximum vibration energy dissipation. However, the natural frequency of the structure may vary from its original value due to uncertainties in structural parameters. This uncertainty could lead to an offset in frequency between damper and structure which deteriorates the damper's control performance significantly. Gao et al. (1997) indicated that the use of multiple tuned liquid column dampers could increase its robustness against the offset in frequency tuning. Due to the nature of passive dampers, the adaptability is still limited. The feature of the SATLCD studied herein can have greater adaptability against the change in structural frequency. If the online information of structural natural frequency is available, the damper can then be adjusted to the updated structural natural frequency simply by changing the feedback gain of the damper. The sensitivity of SATLCD to offset in frequency tuning between damper and structure is therefore investigated. The change in structural frequency is achieved by altering the structural stiffness. The frequency offset ratio ($\Delta \gamma_4$) is defined as the ratio of the updated structural natural frequency to the original structural natural frequency. The performance of SATLCD
without and with self-updating of its natural frequency from the measured structural natural frequency is represented by the curves A and B, respectively, shown in Figure 6-5. The parameters of the SATLCD are $\alpha=0.7$, $\delta=15$, $L=22\text{m}$ and $\mu=0.03$. It is seen from the figure that for SATLCD without updating of its natural frequency (curve A), the displacement response ratio increases and the control performance deteriorates significantly as the frequency offset ratio deviates from a value of 1.02. On the contrary, the performance of SATLCD with updating of its natural frequency (curve B) is almost insensitive to the change in frequency offset ratio. Hence, the adaptability of SATLCD can be much enhanced by self-adjusting of its natural frequency precisely to the measured structural natural frequency.

6.4 PARAMETRIC STUDIES ON TORSIONAL VIBRATION REDUCTION

The performance of SATLCD in reducing the torsional vibration of a structure is investigated in this section. The structural dynamic parameters are $I_s=2.2099\times10^8$ kgm$^2$, $C_0=7.7036\times10^6$ Nms/rad, and $K_0=6.7135\times10^8$ Nm/rad, which are selected with reference to the first torsional vibration mode of the cable-stayed bridge during construction. The mass of structure is $4.6905\times10^6$ kg. The torsional circular natural frequency of the structure, $\omega_0$, is $1.7430\text{rad/s}$. It is noted that the use of traditional TLCs for the torsional vibration suppression of the concerned structure would require a short liquid (water in most cases) length of $6.458\text{m}$ only. This would in turn require a large number of TLC containers, leading to a higher cost of installation. The performance of SATLCD with adaptive tuning capacity is thus studied for the reduction of torsional vibration of the structure under two different kinds of loading: harmonic excitation and white noise excitation. The harmonic moment amplitude is $2.0\times10^5$ Nm and the power spectral density function of the moment is $10\times10^4$ N$^2$m$^2$/s/rad.

6.4.1 Effect of distance from SATLCD to rotational axis

The effect of the distance from the SATLCD to the rotational axis of the structure on the torsional displacement response ratio is depicted in Figure 6-6. The parameters of the SATLCD are $\Delta\gamma=1$, $\delta=50$, $\alpha=0.7$, and $\mu=0.015$. Four different SATLCD, with water lengths of $15\text{m}$, $20\text{m}$, $22\text{m}$ and $24\text{m}$, are studied. The thickness of the corresponding
liquid column \( d \) is selected as 0.7m, 1.0m, 1.2m and 1.4m, respectively, to retain a reasonable geometric size of the damper. To keep the same mass ratio of 0.015 for each case, the width of the liquid column, which is in the direction perpendicular to the elevation of the damper, is then determined accordingly. The performance of the traditional passive TLCD is also included for the comparison with the SATLCD. The parameters of the passive TLCD are \( \Delta \gamma = 1, \delta = 50, \alpha = 0.7, \) and \( \mu = 0.015. \) The water length of the passive TLCD is 6.458 m, denoted \( L_p. \) The thickness of the liquid column is selected as 0.35m. The width of the liquid column is then determined based on the mass ratio of 0.015. Clearly, SATLCD provides a great flexibility for selecting different liquid lengths for a given structure. It can be seen that the reduction of torsional displacement response is significantly affected by the value of \( H. \) The reduction in torsional displacement response is hardly achieved if the ratio \( H/L_p \) is near to a value of 0.35 for the case of the passive TLCD and a value of 0.15 for the case of the SATLCD with water length of 15m. Similar finding has been reported and explained by Xue et al. (2000) for the case of TLCD in reducing the torsional vibration of a structure. As shown in Chapter 4, this phenomenon is attributed to the zero interaction between the structure and the TLCD when the ratio \( H/L_p \) approaches to \( \omega/2. \) For the case of SATLCD under harmonic or white noise excitation, when the excitation frequency equals to the structural frequency, the condition for the zero interaction between the structure and the SATLCD can be derived as follows:

\[
\frac{H}{L_p} = \frac{a}{2} - T(1 - \alpha); \quad T = \frac{S}{4 \rho A g} \tag{6-26}
\]

It is noted that Equation (6-26) can be deduced to the condition for the case of the passive TLCD by setting the feedback displacement gain \( S \) to be zero. Due to the fact that all the concerned SATLCDs for torsional vibration control in this study have a positive constant displacement feedback gain \( S, \) it is easily seen from Equation (6-26) that the \( H/L_p \) ratio for the zero interaction between the structure and the SATLCD is smaller than that for the TLCD. Furthermore, Equation (6-15) indicates that \( T \) increases with the increasing water length of a SATLCD, and hence the ratio \( H/L_p \) for the zero interaction between the structure and the SATLCD decreases with the increasing water length of the SATLCD. For the SATLCD with water length \( L \) of 20m, 22m and 24m, the corresponding value of the \( H/L_p \) ratios is smaller than that with water length of 15m. This feature further facilitates the application of SATLCD in practice. It is also seen
from the figure that the effectiveness of SATLCD in reducing the torsional displacement increases with the increasing ratio of $H/L_p$ but the required pressure for the feedback control force also increases. For a given $H/L_p$, the SATLCD with a larger value of $L$ can achieve larger reduction in displacement response but at the expense of larger value of pressure required for the feedback control force. Figure 6-6 also depicts that for $H/L_p$ ratio beyond a value of 0.35, the performance of the SATLCD in reducing the torsional displacement is more effective than that of the passive TLCD and it is further enhanced with the increasing ratio of $H/L_p$. The performance of SATLCD is less affected by the value of $H$ than that of the passive TLCD, but it is still important to select a proper ratio of $H/L_p$ for the SATLCD so as to avoid the zero interaction between the structure and SATLCD.

6.4.2 Effect of liquid length ratio

The effect of the liquid length ratio, $\alpha$, of the horizontal length to the total length of liquid column of the SATLCD on torsional displacement response ratio is depicted in Figure 6-7 for five different distance ratios $H/L_p$. The parameters of the SATLCD are $\Delta \gamma=1$, $\delta=50$, $L=22m$ and $\mu=0.015$. The liquid length ratio $\alpha$ varies within the range from 0.1 to 0.7. It is seen from Figure 6-7 that the reduction of torsional displacement is dependent on the ratio of horizontal length to total length of liquid column. For the ratio $H/L_p$ equal to 0.25, 0.5 or 0.75, an optimal value of liquid length ratio $\alpha$ exists for a maximum reduction of torsional displacement. It is also seen that increase in the ratio $H/L_p$ leads to more reduction in torsional displacement response. When the ratio $H/L_p$ increases to 1 or above, the torsional displacement response reduces monotonically with increasing liquid length ratio $\alpha$: the larger the length ratio, the greater is the torsional displacement response reduction.

6.4.3 Effect of head-loss coefficient

To investigate the effect of head-loss coefficient on the performance of SATLCD in reducing torsional displacement response of the structure, the head-loss coefficient is taken as a variable. The other parameters of the SATLCD are $\Delta \gamma=1$, $\alpha=0.7$, $H/L_p=1$, and
μ=0.015. Figure 6-8 shows the variation of the torsional displacement response ratio with the head-loss coefficient for five different water lengths. It can be seen from Figure 6-8 that the effectiveness of the SATLCD is affected by the head loss coefficient, and there is an optimal head-loss coefficient for a given water length. The pressure required for the feedback control force of SATLCD increases with the decreasing head loss coefficient. Figure 6-8 also shows that the performance of the optimized SATLCD increases with the increasing water length L. The corresponding optimal head-loss coefficient and the pressure required for the feedback control force also increase with the increasing water length. However, the SATLCD with a larger value of L would require more space for installation. The selection of water length should consider the balance between the control effectiveness and the available space inside the structure.

6.4.4 Effect of frequency offset ratio

In real situation, it may be difficult to tune the damper frequency precisely to the structural natural frequency because of uncertainties in structural parameters. This could lead to the deterioration of the damper control performance. As already demonstrated in Section 6.3.3, the feature of the SATLCD studied herein can have greater adaptability to the change in structural frequency. The sensitivity of SATLCD to the offset in frequency between damper and structure is investigated herein for the torsional vibration. The change in structural frequency is achieved by altering the structural stiffness. The performance of SATLCD without and with updating of its natural frequency from the measured structural natural frequency is represented by the curves A and B, respectively, shown in Figure 6-9. The parameters of the SATLCD are α=0.7, δ=50, L=22m and μ=0.015. The value of H is taken the same value as one in Section 6.4.3. It is seen from the figure that for SATLCD without updating of its natural frequency (curve A), the displacement response ratio increases and the control performance deteriorates significantly as the frequency offset ratio deviates from a value of 1.02. On the contrary, the performance of SATLCD with updating of its natural frequency (curve B) is less sensitive to the change in frequency offset ratio. The performance of SATLCD in reducing torsional displacement deteriorates slightly when the frequency offset ratio decreases. The adaptability of SATLCD to the offset in
frequency between the damper and the structure is enhanced by actively adjusting its natural frequency to the measured structural natural frequency.

6.5 SUMMARY

A semi-active tuned liquid column damper (SATLCD), whose natural frequency can be altered by active control of liquid column pressure, was developed in this chapter. The analytical models for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD were developed accordingly. The SATLCD can provide a great flexibility for selecting a liquid length while keeping a proper frequency tuning through the change of air pressure acting on liquid. Another feature of the SATLCD studied herein is its adaptability to the change in structural frequency. If the online information of varying structural natural frequency is available, the frequency of the damper can be actively adjusted to the varying structural natural frequency to maintain high vibration control performance. The numerical examples carried out in this chapter demonstrated that SATLCD can effectively reduce either lateral or torsional vibration of a structure if its parameters are properly selected. There exists an optimal head loss coefficient for maximum reduction in either lateral or torsional vibration. The optimal head loss coefficient increases with the increasing liquid length of SATLCD. For the case of lateral vibration of SATLCD-structure system, the results revealed that SATLCD with a larger value of liquid length ratio can achieve larger reduction in displacement response but at the expense of larger value of pressure required for the feedback control force. For the case of torsional vibration of SATLCD-structure system, it was found that H/L_p ratio is an important parameter affecting the performance of SATLCD in reducing torsional vibration. For the same value of the H/L_p, the SATLCD with a larger value of liquid length can achieve larger reduction in displacement response. When the ratio H/L_p is increased to 1 and above, the torsional response reduction is increased with the increasing liquid length ratio. The application of SATLCD for the suppression of coupled lateral and torsional vibrations of a real long cable-stayed bridge during construction will be investigated in Chapter 9 based on the understanding of SATLCD obtained from this chapter.
Figure 6-1 Lateral Vibration of SATLCD-Structure system

Figure 6-2 Torsional Vibration of SATLCD-Structure system
Figure 6-3 Effect of Liquid Length Ratio on Lateral Vibration Reduction
Figure 6-4 Effect of Head Loss Coefficient on Lateral Vibration Reduction
Figure 6-5 Effect of Frequency Offset Ratio on Lateral Vibration Reduction
Figure 6-6 Effect of Distance from SATLCD to Rotational Axis on Torsional Vibration Reduction
Figure 6-7 Effect of Liquid Length Ratio on Torsional Vibration Reduction
Figure 6-8 Effect of Head Loss Coefficient on Torsional Vibration Reduction
(a) Harmonic excitation

(b) White noise excitation

Figure 6-9 Effect of Frequency Offset Ratio on Torsional Vibration Reduction
CHAPTER 7
MULTIPLE TUNED LIQUID COLUMN DAMPERS FOR REDUCING COUPLED LATERAL AND TORSIONAL VIBRATION OF STRUCTURES

7.1 INTRODUCTION

Previous work on the tuned liquid column dampers demonstrated that it is effective for suppressing the horizontal motion of a structure (Xu et al. 1992; Balendra et al. 1995; Gao et al. 1997) and also for suppressing the torsional motion of a structure (Xue et al. 2000). However, wind-induced vibration of long span cable-supported bridges involves many modes of vibration. Large vibration may result from coupling of different modes of vibration. Most of previous studies pertaining to the suppression of wind-induced vibration of a bridge focused on the coupling of vertical and torsional vibrations only (Lin et al. 2000). With an increase in the span length and complexity of a bridge deck, significant mechanical and aerodynamic coupling may exist between the first lateral and torsional vibration under turbulent winds. However, little information is available on this topic. How to use MTLCD to reduce the coupled lateral and torsional vibration of a bridge deck should thus be explored.

The objective of this chapter is thus to carry out a theoretical investigation on the performance of multiple tuned liquid column dampers (MTLCD) for mitigating the coupled lateral and torsional vibration of a long span bridge. The equation of motion for the coupled lateral and torsional vibration of a long span bridge with MTLCD is first presented. The bridge is modelled as a two-degree-freedom structure and its cross section has a vertical axis of symmetry but with the vertical offset between the elastic centre and the mass centre of the bridge deck. The external dynamic force and moment are applied at the elastic centre of the bridge. The proposed MTLCD consists of two sets of liquid column dampers with one tuned to the lateral frequency of the bridge and the other tuned to the torsional frequency of the bridge. The nonlinear damping property of MTLCD is linearized using the equivalent linearisation technique. Extensive
parametric studies are then carried out in the frequency domain to find beneficial parameters of the MTLCD for achieving maximum reduction of coupled lateral and torsional vibration of the bridge. The parameters investigated include water mass distribution between the two dampers, distance from the centre line of the MTLCD to the rotational axis of the bridge, ratio of horizontal length to total length of liquid column, head loss coefficient, and frequency tuning ratio. The performance of the MTLCD in reducing buffeting response of the bridge due to turbulent wind is finally investigated.

It is worth to point out that only the control of the coupled first lateral and torsional vibration of a long span bridge is considered in this chapter by using mode-by-mode spectral approach in the frequency domain. The control of wind-induced coupled lateral and torsional vibration of a real long span cable-stayed bridge involving more modes of vibration will be investigated in Chapters 8 and 9 by using the finite element approach in the time domain.

7.2 ANALYTICAL MODEL

7.2.1 Equation of motion of liquid in MTLCD

Suppose that MTLCD is installed at the middle span of a long span bridge where the amplitudes of the first lateral mode shape and the first torsional mode shape are the largest. Each mode shape is such normalised that its amplitude at the location of MTLCD is of unity. In consideration of the dynamic equilibrium condition, the interactions between the liquid in the kth TLC and the bridge can be described by the following equation (see Figure 7-1).

\[
\rho A_k L_k \ddot{W}_k + \frac{\rho A_k}{2} \delta_k |\dot{W}_k|^2 + 2 \rho A_k g \dot{W}_k = -\left[\rho A_k B_k \left(H_k + \frac{L_k - B_k}{2}\right) \dot{\theta}\right] + \rho A_k g B_k \dot{\theta} + \rho A_k B_k \ddot{x}
\]

(7-1)

where \( \rho \) is the density of liquid inside MTLCD; \( A_k \) is the cross-sectional area of liquid column in the kth TLC (k represents 1 or 2); \( \delta_k \) is the head-loss coefficient of the kth TLC governed by the opening ratio of orifice; \( B_k \) is the horizontal length of liquid column; \( L_k \) is the total length of liquid column; \( W_k \) is the relative motion of liquid column in the kth TLC to the container; \( \dot{W}_k \) and \( \ddot{W}_k \) represent the first and second
derivatives of $W_k$ with respect to time; $g$ is the acceleration of gravity; $H_k$ is the vertical distance between the centre line of the horizontal part of the container and the elastic centre $E$ of the bridge; $x$ and $\theta$ are the lateral displacement and the torsional displacement of the elastic centre of the bridge at its middle span where the normalised mode shape amplitude is of unity; $\dot{x}$ and $\ddot{x}$ represent the first and second derivatives of $x$ with respect to time; $\dot{\theta}$ and $\ddot{\theta}$ represent the first and second derivatives of $\theta$ with respect to time. The parameters of the TLCD tuned to the first lateral and torsional frequency of the bridge are denoted by subscript 1 and 2 respectively. It should be noted that Equation (7-1) is valid as long as the liquid is fully retained in the horizontal part of the kth TLCD. Hence, the following equation must be satisfied at any time.

$$W_k \leq \frac{L_k - B_k - \frac{d_k}{2}}{2} \quad (k=1,2) \quad (7-2)$$

where $d_k$ is the thickness of liquid column of the kth TLCD.

7.2.2 Equation of motion of bridge

A long span bridge can be reduced to a two-degree-of-freedom structure by considering its first lateral and first torsional modes of vibration only. The cross section of the bridge is assumed to have a vertical axis of symmetry but with an eccentricity designated by the vertical offset distance $z$ between the elastic centre $E$ and the mass centre $M$ of the bridge (see Figure 7-1). Denote $m_\xi$ the generalized mass of the bridge deck for the first lateral mode of vibration. $I_\xi$ is the generalized mass moment of inertia of the bridge deck for the first torsional mode of vibration. $K_\xi$ and $K_\theta$ are respectively the corresponding generalized modal lateral stiffness and generalized modal torsional stiffness of the bridge. $C_\xi$ and $C_\theta$ are respectively the corresponding generalized modal lateral damping coefficient and generalized modal torsional damping coefficient of the bridge. Let $y$ represent the axis of the bridge in the longitudinal direction of the deck. $\bar{m}(y)$ and $\bar{I}(y)$ are the mass and mass moment of inertia of the bridge deck per unit length. $\psi_L(y)$ and $\psi_T(y)$ are respectively the first normalised lateral and torsional mode shape of the bridge deck. $\omega_\xi$ and $\omega_\theta$ are the first circular lateral frequency and circular torsional frequency of the bridge without control. Furthermore, denote $m_0$ and $I_0$ the total mass and the total mass moment of inertia of liquid in the MTLCD about the elastic centre of the bridge. $M_0(t)$ and $F_0(t)$ are respectively the first generalised external
dynamic moment around and the first generalized external dynamic force at the elastic centre of the bridge. Then, considering the interaction between the bridge and the MTLCD yields the following equations of motion for the bridge which can be modelled as a 2DOF structure.

\[
\begin{align*}
(m_s + m_d)\ddot{x} + C_x\dot{x} + K_x x &= -(m_z + m)\ddot{\theta} - \sum_{k=1}^{2} m_k \alpha_k \ddot{W}_k + F_z(t) \\
(I_s + I_d)\ddot{\theta} + C_\theta \dot{\theta} + \left(K_\theta + \sum_{k=1}^{2} m_k g \overline{H}_k \right) \theta &= - \left[ \sum_{k=1}^{2} \left( G_k \ddot{W}_k + m_k \alpha_k g W_k \right) \right] + M_z(t)
\end{align*}
\]  

(7-3) (7-4)

in which

\[
m_s = \int_{\text{deck}} \tilde{m}(y) \psi^2_k(y) \, dy; \quad K_x = m_z \omega^2_z; \quad I_s = \int_{\text{deck}} \tilde{I}(y) \psi^2_k(y) \, dy; \quad K_\theta = I_s \omega^2_\theta
\]

(7-5)

\[
\alpha_k = \frac{B_k}{L_k}; \quad G_k = \rho A_k B_k \left( H_k + \frac{L_k - B_k}{2} \right); \quad \overline{H}_k = H_k - \frac{(L_k - B_k)^2}{4L_k}; \quad (k=1,2)
\]

(7-6)

\[
I_{dk} = m_k \left[ \alpha_k \left( H_k^2 + \frac{B_k^2}{12} \right) \right] + (1 - \alpha_k) \left[ H_k^2 + \frac{B_k^2}{4} - \frac{H_k (L_k - B_k)}{2} + \frac{(L_k - B_k)^2}{12} \right]
\]

\[
I_d = \sum_{k=1}^{2} I_{dk}
\]

(7-7)

\[
m_k = \rho A_k L_k; \quad m_d = \sum_{k=1}^{2} m_k; \quad m = \sum_{k=1}^{2} m_k \overline{H}_k
\]

(7-8)

The first term on the right hand side of Equation (7-3) and Equation (7-4) shows that the lateral and torsional motions of the bridge deck are coupled due to the offset \( z \) and the inertia effect of the liquid. The second term on the right hand side of Equation (7-1) and Equation (7-3) indicates that the lateral motion of the bridge deck and the liquid motion are coupled due to the inertia effect of the liquid. The first term on the right hand side of Equation (7-1) and the second term on the right hand side of Equation (7-4) show that the torsional motion of the bridge deck and the liquid motion are coupled due to the inertia effect as well as the gravitation effect of the liquid. Hence, the installation of the MTLCD on the bridge deck induces the coupled effect on the lateral and torsional motions of the bridge deck as well as the liquid motion inside the MTLCD in addition to the original structural coupling.
7.2.3 Equation of motion of coupled MTLCD-bridge system

By combining Equation (7-1) with Equation (7-3) and Equation (7-4), the equation of motion of the MTLCD-bridge system can be written as:

$$\mathbf{M} \ddot{\mathbf{x}} + \mathbf{C} \dot{\mathbf{x}} + \mathbf{K} \mathbf{x} = \mathbf{p} \quad (7-9)$$

where the vector $\mathbf{X}$ is the displacement vector, which consists of the lateral displacement $x$ and the torsional displacement $\theta$ of the elastic centre of the bridge at its middle span and the relative displacement of liquid to the container of the $k$th TLCD, $W_k$ ($k=1,2$).

$$\mathbf{X} = [x, W_1, W_2, \theta]^T \quad \text{and} \quad \mathbf{P} = \begin{bmatrix} F_1 & 0 & 0 & M_3 \end{bmatrix}^T \quad (7-10)$$

$\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the mass, damping and stiffness matrices of the MTLCD-bridge system, respectively. The mass matrix, damping matrix and the stiffness matrix are expressed as

$$\mathbf{M} = \begin{bmatrix}
    m_y(1+\mu) & m_1a_1 & m_2a_2 & m+m_z \\
    m_1a_1 & m_1 & 0 & G_1 \\
    m_2a_2 & 0 & m_2 & G_2 \\
    m+m_z & G_1 & G_2 & I + I_d \\
\end{bmatrix} \quad (7-11)$$

$$\mathbf{C} = \begin{bmatrix}
    C_x & 0 & 0 & 0 \\
    0 & \frac{\rho A_1}{2} \delta & W_1 & 0 \\
    0 & 0 & \frac{\rho A_2}{2} \delta & W_2 \\
    0 & 0 & 0 & C_0 \\
\end{bmatrix} \quad (7-12)$$

$$\mathbf{K} = \begin{bmatrix}
    k_x & 0 & 0 & 0 \\
    0 & m_1\omega_1^2 & 0 & m_1a_1g \\
    0 & 0 & m_2\omega_2^2 & m_2a_2g \\
    0 & m_1a_1g & m_2a_2g & K_0 + mg \\
\end{bmatrix} \quad (7-13)$$

where $\omega_k^2 = \frac{2g}{L_k}$; $\mu = \frac{m_d}{m_s}$ \quad (7-14)

It should be noted that $\omega_k$ is the circular frequency of liquid in the $k$th column and $\mu$ is the mass ratio of the liquid to the first generalised lateral modal mass of the bridge deck.
7.2.4 Equivalent linearisation technique

It is noted that the damping terms in Equation (7-1) for liquid motion are nonlinear. Therefore, the coupled MTLCD-bridge system described by Equation (7-9) is a nonlinear system. To carry out an extensive parametric study in the frequency domain, the equivalent linearisation technique is applied to the nonlinear damping force of liquid motion in Equation (7-1) in terms of the principle of equivalent energy dissipation. As a result, the damping matrix in Equation (7-12) can be replaced by

\[
C = \begin{pmatrix}
2m_1 \omega_x & \xi_x & 0 & 0 & 0 \\
0 & 2m_1 \omega_1 \xi_1 & 0 & 0 \\
0 & 0 & 2m_2 \omega_2 & \xi_2 & 0 \\
0 & 0 & 0 & 21_2 \omega_2 \xi_2 \\
\end{pmatrix}
\]  

(7-15)

where \( \xi_x \) and \( \xi_2 \) are the structural damping ratio in the first lateral and torsional mode of vibration, respectively; \( \xi_k \) is the equivalent damping ratio for the kth TLCD. The procedures of calculating the equivalent damping ratio of TLCD are provided in Chapter 4.

7.2.5 Coupled lateral and torsional response of structure with MTLCD

7.2.5.1 Harmonic excitation

A long span bridge located in a wind prone area may suffer from vortex shedding induced vibration. The excitation force and moment acting on the bridge deck due to vortex shedding may be assumed to be harmonic and their frequencies may be assumed to be the same value but with a phase angle \( \lambda \) between them. The force vector \( \mathbf{P} \) in Equation (7-9) can thus be written as:

\[
\mathbf{P} = \mathbf{F} e^{i \bar{\omega} t} = \begin{bmatrix} F_o & 0 & 0 \end{bmatrix} e^{i \lambda t}
\]  

(7-16)

where \( F_o \) and \( M_o \) are the generalised force amplitude and the generalised moment amplitude; \( \bar{\omega} \) is the circular frequency of the applied force and the applied moment; \( \lambda \) is the phase angle between the external force and the external moment; and \( i \) is the imaginary unit. The steady state responses of both the bridge and the liquid columns in the MTLCD under the harmonic excitations can be obtained as

\[
\mathbf{X} = \begin{bmatrix} \bar{x} & \bar{W}_1 & \bar{W}_2 & \bar{\theta} \end{bmatrix} e^{i \bar{\omega} t}
\]  

(7-17)
The amplitude vector of the steady-state response, \( |X(i\omega)| \), is given by

\[
|X(i\omega)| = |h(i\omega)F| = \left| \left( -\omega^2 M + i\omega C + K \right)^{-1} F \right|
\]  

(7-18)

where \( h(i\omega) \) is the matrix of the frequency response function.

### 7.2.5.2 Stationary random excitation

When the excitation applied to the bridge is stationary random excitation, the spectral matrix of the external excitation, \( S_{pp}(\omega) \), on the MTLCD-bridge system is given by

\[
S_{pp}(\omega) = \begin{bmatrix}
S_{11}(\omega) & 0 & 0 & S_{14}(\omega) \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
S_{41}(\omega) & 0 & 0 & S_{44}(\omega)
\end{bmatrix}
\]  

(7-19)

where \( S_{11} \) is the power spectral density (PSD) function of lateral force; \( S_{44} \) is the power spectral density function of moment; \( S_{14} \) and \( S_{41} \) are the cross spectral density functions between the force and the moment, and they are equal to each other.

#### 7.2.5.2.1 PSD of external excitations

If the external excitations acting on the bridge deck can be modelled as white noise random processes, the power spectral density functions of the generalised force and moment as well as their cross-spectral density functions are of constant values. For the case where real wind excitation is considered, the spectral density functions of the generalised lateral buffeting force and moment, \( S_{11}(\omega) \) and \( S_{44}(\omega) \), together with their cross-spectra density function \( S_{14}(\omega) \) can be expressed as (Dyrbeye and Hansen 1997):

\[
\begin{bmatrix}
S_{11}(\omega) \\
S_{44}(\omega) \\
S_{14}(\omega)
\end{bmatrix} = \left( \frac{1}{2 \rho_a U b} \right)^2 \begin{bmatrix}
(2C_{D}\chi_{D_a})^2|J_{DD}(\omega)|^2 & (C_{D}\chi_{D_a})^2|J_{DD}(\omega)|^2 & (C_{M}\chi_{M_a})^2|J_{MM}(\omega)|^2 & (C_{M}\chi_{M_a})^2|J_{MM}(\omega)|^2 \\
(2C_{M}\chi_{M_a} b)^2|J_{MM}(\omega)|^2 & (C_{M}\chi_{M_a} b)^2|J_{MM}(\omega)|^2 & -4C_{D}C_{M}\chi_{D_a}\chi_{M_a} b|J_{DM}(\omega)|^2 & -4C_{D}C_{M}\chi_{D_a}\chi_{M_a} b|J_{DM}(\omega)|^2 \\
-4C_{D}C_{M}\chi_{D_a}\chi_{M_a} b|J_{DM}(\omega)|^2 & -4C_{D}C_{M}\chi_{D_a}\chi_{M_a} b|J_{DM}(\omega)|^2 & (2C_{D}\chi_{D_a})^2|J_{DD}(\omega)|^2 & (2C_{D}\chi_{D_a})^2|J_{DD}(\omega)|^2
\end{bmatrix}
\]  

(7-20)

where \( \rho_a \) is the air density; \( b \) is the bridge deck width; \( U \) is the mean wind speed at the deck level; \( C_D \) and \( C_M \) are the static drag force and moment coefficients, respectively; \( \chi_{D_a}, \chi_{D_a}, \chi_{M_a}, \) and \( \chi_{M_a} \) are the aerodynamic transfer functions between fluctuating wind...
velocities and buffeting forces, which are described as functions of frequency and are dependent on the deck configuration; \( C'_b = \frac{dC_b}{d\theta} \) and \( C'_m = \frac{dC_m}{d\theta} \). The joint acceptance functions for modal loading components are \((X, Y = D, M; j = u, w)\):

\[
|J_{XY}^j(\omega)|^2 = \int_0^{L_j} \int_0^{L_j} \rho_X(y_1) \rho_Y(y_2) \varphi_j(y_1, y_2, \omega) dy_1 dy_2
\]

\[
\varphi_j(y_1, y_2, \omega) = \exp \left( -\frac{\lambda_j 0.747|y_1 - y_2|}{2\pi L_j} \sqrt{1 + 70.78 \left( \frac{\omega L_j^j}{2\pi U} \right)^2} \right)
\]

where \( g_0(y) = \psi_L(y); g_m(y) = \psi_T(y); \lambda_j \) is the decay factor of the coherence of the \( j \) component; \( L_0 \) is the bridge deck length; \( L_j^u \) and \( L_j^w \) are the integral scales of the \( j \) component in the along-wind and across-wind directions, respectively. The PSD function of the horizontal gust component \( u \) and vertical gust component \( w \) are given by the von Kármán spectra as follows.

\[
\frac{\omega S_{uu}(\omega)}{\sigma_u^2} = \frac{4Z_u}{(1 + 70.78 Z_u^2)^{3/2}}, \quad \frac{\omega S_{ww}(\omega)}{\sigma_w^2} = \frac{2(1 + 188.8 Z_w^2)Z_w}{(1 + 70.78 Z_w^2)^{1/2}}, \quad Z_j = \frac{1}{2\pi} \frac{\omega L_j^u}{U}
\]

### 7.2.5.2.2 Motion-induced aeroelastic excitations

When real wind excitation is considered for a long span bridge, the interaction between fluctuating wind and bridge motion will generate motion-induced aeroelastic excitations on the bridge deck. The generalised aeroelastic lateral force and moment can be commonly expressed as

\[
D_{\beta}(t) = \rho_u U^2 b \left( kD_{LL} P_2^* \frac{\ddot{X}}{U} + kD_{LT} P_3^* \frac{\theta}{U} + k^2 D_{LT} P_3^* \theta + k^2 D_{LL} P_4^* \frac{\dot{X}}{b} \right)
\]

\[
M_{\beta}(t) = \rho_u U^2 b^2 \left( kD_{TT} A_1^* \frac{\dot{\theta}}{U} + k^2 D_{TT} A_1^* \theta + kD_{TL} A_5^* \frac{\dot{X}}{U} + k^2 D_{TL} A_6^* \frac{\dot{X}}{b} \right)
\]

where the reduced frequency \( k = \omega b/U \); and the non-dimensional coefficients \( D_{LL}, D_{LT}, D_{TT} \) and \( D_{TL} \) are given by

\[
D_{LL} = \int_{\text{deck}} \psi_L^2(y) dy; \quad D_{LT} = \int_{\text{deck}} \psi_L(y) \psi_T(y) dy; \quad D_{TT} = \int_{\text{deck}} \psi_T^2(y) dy; \quad D_{TL} = D_{LT}
\]
The flutter derivatives $A_1^\ast$, and $A_3^\ast$ are commonly obtained from wind tunnel tests. The following flutter derivatives are often not available from wind tunnel tests, and they are given in this study based on the quasi-steady theory as follows:

$$
P_1^\ast = \frac{-C_D}{k}; \quad P_2^\ast = \frac{-C_D - C_L}{2k}; \quad P_3^\ast = \frac{-C_D}{2k^2}; \quad A_4^\ast = \frac{C_M}{k}; \quad P_4^\ast = A_6^\ast = 0 \quad (7-27)
$$

For the buffetting analysis in the frequency domain, the motion induced aeroelastic force and moment given in Equations (7-24) and (7-25) are often transferred to the left hand side of Equation (7-9) in terms of the so-called aeroelastic damping matrix $C_{ae}$ and aeroelastic stiffness matrix $K_{ae}$.

$$
C_{ae} = -\rho_s U k b \left[
\begin{array}{cccc}
D_{LL} P_1^\ast & 0 & 0 & D_{LT} P_2^\ast b \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
D_{TL} A_4^\ast b & 0 & 0 & D_{TT} A_4^\ast b^2
\end{array}
\right] \quad (7-28)
$$

$$
K_{ae} = -\rho_s U^2 k^2 \left[
\begin{array}{cccc}
D_{LL} P_4^\ast & 0 & 0 & D_{LT} P_3^\ast b \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
D_{TL} A_6^\ast b & 0 & 0 & D_{TT} A_3^\ast b^2
\end{array}
\right] \quad (7-29)
$$

The aeroelastic stiffness matrix and the aeroelastic damping matrix are then combined with the structural stiffness matrix and the structural damping matrix respectively.

**7.2.5.2.3 Random system responses**

The spectral matrix of response of the MTLCD-bridge system, $S_{yy}(\omega)$, to either the white noise excitation or the wind excitation can be determined by

$$
S_{xx}(\omega) = h(i\omega) \cdot S_{pp}(\omega) \cdot h^\top(i\omega) \quad (7-30)
$$

The standard deviations of the lateral and torsional displacement responses of the bridge at the middle span and the standard deviation of the velocity response of the $k$th TLCD are given, respectively, by
\[ \sigma_x = \sqrt{\int_{-\infty}^{\infty} S_{xx}(1,1)d\omega = \sqrt{\int_{-\infty}^{\infty} \left( h_{11}^2 S_{11} + h_{11} h_{14} S_{14} + h_{14}^2 S_{14} \right) d\omega} \]  

(7-31)

\[ \sigma_y = \sqrt{\int_{-\infty}^{\infty} S_{yy}(4,4)d\omega = \sqrt{\int_{-\infty}^{\infty} \left( h_{41}^2 S_{11} + h_{41} h_{44} S_{44} + h_{44}^2 S_{44} \right) d\omega} \]  

(7-32)

\[ \sigma_z = \sqrt{\int_{-\infty}^{\infty} S_{zz}(k+1,k+1)d\omega \quad (k=1,2)} \]

(7-33)

\[ = \sqrt{\int_{-\infty}^{\infty} \left( h_{(k+1)1}^2 S_{11} + h_{(k+1)4} h_{(k+1)4} S_{44} + h_{(k+1)4} h_{(k+1)4} S_{44} + h_{(k+1)4}^2 S_{44} \right) d\omega} \]

\[ \]  

where \( h_y(i\omega) = h_y(-i\omega) \) is the \( ij^{th} \) element of the frequency response function matrix \( h(i\omega) \). It is noted that the standard deviation responses of both the bridge and liquid columns are dependent on the equivalent damping ratios while the equivalent damping ratios of the MTLCD are dependent on the standard deviation responses of liquid velocities. Thus, to obtain the standard deviation responses of the bridge and liquid columns, iterations are required to obtain converged results.

### 7.3 PARAMETRIC STUDIES ON REDUCTION OF COUPLED VIBRATIONS

A triple tower cable-stayed bridge with an overall length of 1177m and the two main spans measured at 475m and 448m and the two side spans of 127m each is taken as an example bridge in this study. From the eigenvalue analysis of the finite element model of the cable-stayed bridge, the first lateral frequency and the first torsional frequency of the bridge deck are 1.5648 rad/s and 3.2792 rad/s, respectively. The first lateral mode shape and the first torsional mode shape both reach the maximum at the middle span of one main span and they are normalized as unity at the middle span. The first lateral modal mass and the first torsional modal mass moment of inertia of the bridge deck are 1.5012 \times 10^7 \text{ kg} and 6.4039 \times 10^8 \text{ kgm}^2, respectively. Harmonic excitation and white noise excitation are considered to be the external excitations acting on the bridge in this section. Wind excitation and buffeting response will be considered in the next section. The harmonic excitation amplitude and the power spectral density function of the white noise excitation used are: \( F_0 = 3 \times 10^4 \text{ N}, M_0 = 10 \times 10^5 \text{ Nm, S}_{11} = 4 \times 10^6 \text{ Ns/rad} \) and \( S_{44} = 7.5 \times 10^{11} \text{ Nm}^2/\text{ s/rad} \). The first lateral modal damping ratio and the first torsional modal damping ratio of the bridge are taken as a value of 0.01 in the parametric studies. The eccentricity of the bridge \( (e_b) \), which is defined as the ratio of the vertical offset
distance between its elastic centre and its mass centre (z) to its radius of gyration (r), is taken as 0.05. Most of the above structural parameters will remain unchanged in the parametric study unless otherwise specified. To help understanding of the sensitivity of the MTLCD under off-tuning situation, the frequency-tuning ratio which would affect the performance of MTLCD under off-tuning is defined as:

\[ \Delta \gamma_1 = \frac{\omega_1}{\omega_s}; \quad \Delta \gamma_2 = \frac{\omega_2}{\omega_s} \]  

(7-34)

For the sake of comparison, the structural response ratio \( R \), are introduced as:

\[ R = \max \left\{ \frac{\text{Maximum structural response with control}}{\text{Maximum structural response without control}} \right\} \text{ for harmonic excitation} \]  

(7-35)

\[ R = \frac{\text{Standard deviation of structural response with control}}{\text{Standard deviation of structural response without control}} \text{ for white noise excitation} \]  

(7-36)

It should be noted that the structural response ratio for harmonic excitation is defined as the maximum value of the ratio inside the bracket with the phase angle (\( \lambda \)) ranging from 0 to 2\( \pi \). The smallest value of the structural response ratio is often used as the indication of the effectiveness of TLCD in the parametric study. The thickness of the liquid columns, \( d_1 \) and \( d_2 \) are taken as 0.4m and 0.1m, respectively.

7.3.1 Effect of mass distribution between two dampers

The first step in designing a TLCD for suppressing the response of a bridge either in pure lateral vibration or pure torsional vibration is usually the designation of the mass ratio \( \mu \) of the damper to the bridge. In designing the MTLCD for suppressing the coupled vibration of bridge, the mass ratio should refer to the total mass ratio, that is, the ratio of the total mass of the two dampers to the mass of the bridge. Therefore, it involves the issue how to distribute the total water mass between the two dampers. The distribution of the total water mass between the two dampers can be realized through the change of the width of the dampers only in the parametric study so as to keep all the other parameters unchanged. The effects of the water mass ratio \( m_1/m_2 \) on the performance of the MTLCD for the concerned bridge are depicted in Figure 7-2. The
parameters of the MTLCD used herein are $\Delta \gamma = 1$, $\delta = 20$, $\alpha = 0.7$, $H/L_1 = 1.57$, $H/L_2 = 2.8$, and $\mu = 0.04$. The dropping of the subscript in some damper parameters implies that these parameters are the same for the two dampers. It can be seen from Figure 7-2 that the torsional displacement response ratio increases with increasing water mass ratio $m_1/m_2$ but the lateral displacement response ratio decreases with increasing water mass ratio. However, the lateral displacement response ratio becomes insensitive to the water mass ratio $m_1/m_2$ after its value beyond a value of 1.2 for harmonic excitation and a value of 0.6 for white noise excitation. In other words, no significant improvement in reducing the lateral displacement response can be achieved even though more water mass is assigned to the TLCD tuned to the lateral frequency of bridge. To gain the maximum benefit for the reduction of the coupled lateral and torsional vibration of the bridge, a response index ($R_t$) is defined as

$$R_t = (1 - a)R_l + aR_t$$ (7-37)

where $a$ is the weighting factor ranging from zero to unit; and $R_l$ and $R_t$ are the response ratios of lateral displacement and torsional displacement, respectively. Since $R_l$ and $R_t$ are the function of the water mass ratio $m_1/m_2$, the response index is also the function of the water mass ratio. Figures 7-3a and 7-3b display the variation of the response index with the water mass ratio for harmonic excitation and white noise excitation, respectively. It is seen that there exists an optimal water mass ratio for achieving the minimum response index for a given weighting factor and a given excitation type. Clearly, the optimal water mass ratio obtained for a smaller weighting factor approaches to that for the maximum torsional response reduction. On the other hand, the optimal water mass ratio achieved for a larger weighting factor comes near that for the maximum lateral response reduction. Table 7-1 lists the minimum response index and the corresponding optimal water mass ratio against the weighting factor for both harmonic excitation and white noise excitation. It is seen that for the case of the weighting factor being less than 0.3 (i.e. the torsional response reduction is more important), the water mass ratio $m_1/m_2$ should be smaller than 0.25 for the bridge under either harmonic or white noise excitation. For the case of the weighting factor equal to 0.5 (i.e. the lateral and torsional response reductions are equally important), the water mass ratio $m_1/m_2$ should be around 0.4 for the bridge under harmonic excitation and 0.95 for the bridge under white noise excitation. This implies that more water mass should be distributed to the TLCD tuned to the torsional frequency of the bridge even
for the case of lateral and torsional response reductions being equally important. For the
parametric studies in the remaining sections, the water mass ratio $m_1/m_2$ is taken as 0.33
for the concerned bridge.

Table 7-1 The Minimum Response Index and Optimal Water Mass Ratio Against
The Weighting Factor

<table>
<thead>
<tr>
<th>Harmonic excitation</th>
<th>White noise excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$m_1/m_2$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.25</td>
</tr>
<tr>
<td>0.3</td>
<td>0.30</td>
</tr>
<tr>
<td>0.4</td>
<td>0.35</td>
</tr>
<tr>
<td>0.5</td>
<td>0.40</td>
</tr>
<tr>
<td>0.6</td>
<td>0.45</td>
</tr>
<tr>
<td>0.7</td>
<td>0.55</td>
</tr>
<tr>
<td>0.8</td>
<td>0.80</td>
</tr>
</tbody>
</table>

7.3.2 Effect of distance from MTLCD to rotational axis

The effects of the distance from the MTLCD to the elastic centre on the lateral
and torsional displacement response ratios are depicted in Figure 7-4. The parameters of
the MTLCD are $\Delta \gamma = 1$, $\delta = 20$, $\alpha = 0.7$, and $\mu = 0.03$ and 0.04. It can be seen that the
reduction of torsional displacement response is significantly affected by the value of $H$
but it does not affect the lateral displacement response reduction. The reduction in
torsional displacement response is hardly achieved if the ratio $H/L_2$ is near to a value of
0.35 for the case of zero eccentricity, a value of 0.6 for the case of eccentricity equal to
0.05 and a value of 1.3 for the case of eccentricity equal to 0.2. Similar finding has been
reported and explained by Xue et al. (2000) for the case of STLCD and in Chapter 4 for
the case of MTLCD in reducing the pure torsional vibration of a structure. They found
that when the ratio $H/L_2$ equal to $\alpha/2$, the torsional displacement is hardly reduced by a
TLCD. The physical explanation of this phenomenon in the case of pure torsional
vibration of a structure with a TLCD is that when the ratio $H/L_2$ approaches to $\alpha/2$, the
interaction between the structure and the TLCD, which is $\left(-\ddot{\omega}^2 G_k + m_k g \alpha_k \right)$ for the
pure torsional vibration, tends to zero. In the case of the coupled lateral and torsional
vibration of the bridge with MTLCD, the torsional motion of the bridge and the motion
of the liquid columns are coupled in a similar way as in the case of pure torsional

7-13
vibration, as shown by the first term on the right hand side of Equation (7-1) and the second term on the right hand side of Equation (7-4). However, the coupling between the lateral motion of the bridge and the torsional motion of the bridge (the first term on the right hand side of Equation (7-4)) and the coupling between the lateral motion of the bridge and the water motion (the second term on the right hand side of Equation (7-1)) make the explicit expression for the condition where the interaction approaching to minimum becomes difficult to be obtained. Nevertheless, it can be obtained through numerical studies as shown in Figure 7-4. It can be concluded from the results that the H/L₂ ratio for the interaction tending to minimum in the coupled lateral and torsional vibration of the bridge with eccentricity is larger than that without eccentricity. It is thus very important to provide a proper ratio of H/L₂ to the MTLCD so that the weakest interaction phenomenon will not occur in the coupled vibration.

7.3.3 Effect of liquid length ratio

The effect of the liquid length ratio, αₖ, of the horizontal length to the total length of liquid column of the MTLCD on the performance of the MTLCD is displayed in Figure 7-5 for four different H/L₂ ratios. The parameters of the MTLCD are Δγ=1, δ=20 and μ=0.04. Taking the constraint Equation (7-2) into consideration, one may obtain the constraint equation on the length ratio αₖ as follows.

\[ αₖ ≤ 1 - \frac{2W_k + d_k}{L_k} \]  

(7-38)

It is seen that the selection of the liquid length ratio should consider the total length of liquid column of the kth TLCD (Lₖ), the thickness of the kth TLCD (dₖ) and the relative motion of liquid column in the kth TLCD to the container. The relative motion of liquid column in the MTLCD increases with increasing level of external excitation. Thus, to design MTLCD workable within a wide range of external excitation, one should select a value of liquid length ratio αₖ slightly smaller than its upper bound value determined from Equation (7-40) in order to ensure the liquid be fully retained in the horizontal part of the MTLCD all the time. Based on all these considerations, the largest value of the liquid length ratio is limited to 0.8 and a value of 0.7 is recommended in this study for the concerned system. It is seen from Figure 7-5 that the reduction in lateral displacement response does not depend on the value of the H/L₂ but
depends on the value of $\alpha$ significantly. Larger value of $\alpha$ can generate larger reduction in lateral displacement response. It is seen again that the MTLCD is more effective in suppressing harmonic excitation-induced lateral vibration than white noise excitation-induced lateral vibration. For the torsional vibration of the bridge, increase in the $H/L_2$ ratio leads to more reduction in torsional displacement response. For the ratio $H/L_2$ equal to one, the torsional displacement response ratio is less affected by the value of $\alpha$. When the ratio $H/L_2$ is increased to 2 and above, the performance of the MTLCD is improved and the torsional response reduction is increased with increasing value of $\alpha$.

7.3.4 Effect of head-loss coefficient

To investigate the effect of head-loss coefficient on the performance of MTLCD, the head-loss coefficient is taken as a variable and the rest parameters of the MTLCD are $\Delta y=1$, $\alpha=0.7$, $H/L_2=2.8$, and $\mu=0.03$, 0.04, and 0.05. Figure 7-6 depicts the variation of the lateral displacement and torsional displacement response ratios with the head-loss coefficient. It is seen that the effectiveness of the MTLCD is affected by the head-loss coefficient, and there is an optimal head-loss coefficient for a given mass ratio and a given excitation type. As the water mass inside the container increases (the mass ratio increases), the optimal head-loss coefficient also increases and more reduction in either lateral or torsional displacement response is achieved. It can also be seen in Figure 7-6 that the head-loss coefficient for achieving the maximum reduction in lateral displacement response is larger than that for obtaining the maximum reduction in torsional displacement response. Hence, one may use two different values of head-loss coefficient to get the maximum reduction in both lateral and torsional vibrations. Figure 7-6 also depicts that larger reduction in either lateral displacement or torsional displacement response can be achieved under the harmonic excitation and the corresponding optimal head-loss coefficient is larger than that under the white noise excitation.

7.3.5 Effect of frequency-tuning ratio

A TLCD is often designed to have its natural frequency equal to the natural frequency of bridge. However, the off-tuning may occur in the engineering application due to the error in the estimation of natural frequency of the bridge. The effects of
frequency tuning ratio on the performance of MTLCD are thus investigated. The parameters of the MTLCD used herein are \( \delta = 20 \), \( \alpha = 0.7 \), and \( \mu = 0.03, 0.04 \) and 0.05. The parameter \( H \) is taken the same value as the one in the preceding section 7.3.4. Figure 7-7 shows that the optimal tuning ratio exists for maximum reduction in lateral and torsional displacement responses. Both displacement response reductions are more sensitive to tuning ratio for harmonic excitation than white noise excitation. The optimal tuning ratio for either excitation condition is slightly decreased as the mass ratio is increased. The optimal tuning ratio of the MTLCD for the maximum reduction in lateral displacement response is also slightly different from that in torsional displacement response. Once again, Figure 7-7 depicts that larger reduction in either lateral or torsional displacement response can be achieved under harmonic excitation condition.

7.4 PERFORMANCE OF MTLCD IN REDUCING BUFFETING RESPONSE

The trend of constructing bridges with longer span leads to the increasing concern of aerodynamic performance of the bridge under wind excitation. For buffeting response of a long span bridge, wind excitation can be resolved into buffeting and self-excited force components. The buffeting forces are due to incident wind turbulences acting on the bridge and the self-excited forces are due to the interaction between the motions of the bridge and the wind. The self-excited forces will generate aeroelastic stiffness and damping which may lead to the instability of the bridge. It is thus important to investigate the performance of the MTLCD in reducing the buffeting response with the consideration of aeroelastic effects. The aerodynamic parameters are assumed to be uniform along the whole bridge deck. The aerodynamic coefficients of the deck section are \( C_D = 0.103 \); \( C_D^* = 0 \); \( C_L = 0.134 \); \( C_M = -0.011 \) and \( C_M^* = 1.06 \). The mean wind speed is normal to the bridge deck. The turbulence intensity is 0.10 for horizontal wind (\( I_u \)) and 0.05 for vertical wind (\( I_v \)). The integral length scales for horizontal wind, \( L_u^* \) and \( L_v^* \), are equal of 80m and for vertical wind, \( L_w^* \) and \( L_w^* \) are equal of 40m. The decay factors (\( \lambda_u \) and \( \lambda_w \)) are assumed to be 8. \( \chi_{D_v} \) and \( \chi_{D_w} \) are given by the Davenport's admittance function. \( \chi_{M_v} \) and \( \chi_{M_w} \) are given by the Sears admittance function. The structural lateral damping ratio and torsional damping ratio are taken as a value of 0.005. In considerations of the self-excited forces, only the flutter derivatives
$A_\tau^*$ and $A_\gamma^*$ are available from the wind tunnel test measurement (Brownjohn and Choi 2001) and they are fitted by the following functions.

$$A_\tau^* = -0.00299t^3 - 0.02121t; \quad A_\gamma^* = 0.0061t^2 - 0.0025t; \quad \text{for } 0 \leq t \leq 8 \quad \text{and} \quad t = \frac{2\pi}{k} \quad (7-39)$$

The other flutter derivatives are calculated based on the quasi-steady theory according to Equation (7-27). To have a reasonable assessment of the performance of MTLCD in reducing buffeting response of the bridge, head loss coefficient is taken as a variable to find its optimal value for a given mean wind speed. The other parameters of the MTLCD are selected as: $H/L_2=2.8$, $\alpha=0.7$, $\Delta\gamma=1$, $\mu=0.04$ and $m_1/m_2=0.33$ based on the results obtained in the parametric studies. Figure 7-8 shows the displacement response ratios of the MTLCD-bridge system under different mean wind speed without and with aeroelastic effects. The corresponding optimal head loss coefficients are also plotted in Figure 7-8. It can be seen in Figure 7-8(a) that the MTLCD can reduce both the lateral and torsional displacement responses of the bridge and the response reduction becomes more and more as mean wind speed increases. At a mean wind speed of 35m/s, the lateral and torsional response reductions reach the same level of 46%. It is also seen that the optimal head loss coefficient for the maximum bridge response reduction decreases with increasing mean wind speed. Figure 7-8(b) shows that when aeroelastic effects are taken into consideration, the performance of MTLCD is not as good as that for the case without considering aeroelastic effects. This is particularly true for the torsional vibration of the bridge. At a mean wind speed of 20m/s, the lateral and torsional displacement responses of the bridge are reduced by 40% and 24%, respectively, compared with 43% and 38% for the case without considering aeroelastic effects. Furthermore, with the aeroelastic effects included both the lateral and torsional response reductions remain almost unchanged as mean wind speed increases. This is because for the bridge and mean wind speed range concerned in this study, the aeroelastic damping ratios are positive and increase as mean wind speed increases. It should be pointed out that the phenomenon associated with aeroelastic effects and observed in this study may not be applicable to other bridges.

7.5 SUMMARY
The use of MTLCD for reducing the coupled lateral and torsional vibration was explored in this chapter for long span bridges under harmonic excitation, white noise excitation, and wind excitation. The equations of motion of the coupled MTLCD and bridge systems under different excitations were formulated. Extensive parametric studies on the MTLCD for a long span bridge under harmonic excitation and white noise excitation were carried out based on the formulation developed. The results revealed that there exists an optimal water mass distribution between the two dampers, which depends on the relative importance of torsional response reduction to lateral response reduction. It is important to provide a proper ratio of \( H/L_2 \) to the MTLCD so that the interaction between the MTLCD and bridge will not tend to be zero in the coupled lateral and torsional vibration. To achieve the maximum reduction of displacement response, the value of \( \alpha \) should be as large as possible provided that the water is retained in the horizontal part of the container. The performance of MTLCD in reducing lateral and torsional displacement responses depends on the head-loss coefficient and the tuning ratio. The optimal head-loss coefficient for achieving the maximum reduction in lateral vibration is different from that for achieving the maximum reduction in torsional vibration. However, the optimal tuning ratio for achieving the maximum reduction in lateral vibration is slightly different from that for achieving the maximum reduction in torsional vibration. The investigation on the buffeting response of the coupled MTLCD-bridge system demonstrated that the MTLCD can reduce both the lateral and torsional vibrations of the bridge, and the bridge response reduction increases with increasing mean wind speed if aeroelastic effects are not considered. However, for the bridge and mean wind speed range concerned, the aeroelastic damping ratio is positive and increases as mean wind speed increases. The bridge response reduction by the MTLCD remains almost unchanged as mean wind speed increases. The control of wind induced coupled lateral and torsional vibration of a real long span cable-stayed bridge involving more modes of vibration will be investigated in Chapters 8 and 9 by using the finite element approach in the time domain.
Figure 7-1 MTLCD-Bridge System

Figure 7-2 Effect of Water Mass Distribution between Two Dampers
Figure 7-3 Variation of Response Index with Water Mass Ratio
Figure 7-4 Effect of Distance from MTLCD to Rotational Axis
Figure 7-5 Effect of Liquid Length Ratio
(a) Lateral displacement

(b) Torsional displacement

Figure 7-6 Effect of Head-Loss Coefficient
(a) Lateral displacement

(b) Torsional displacement

Figure 7-7 Effect of Frequency Tuning Ratio
Figure 7-8 Performance of MTLCD in reducing buffeting response of the bridge
CHAPTER 8
APPLICATION OF MULTIPLE PRESSURIZED TUNED LIQUID COLUMN DAMPERS TO A LONG SPAN BRIDGE AT ITS COMPLETION STAGE

8.1 INTRODUCTION

The theoretical investigation of multiple pressurized tuned liquid column dampers (MPTLCD) was presented in Chapter 5. The results revealed that the MPTLCD can reduce effectively the torsional vibration of a structure and provide greater flexibility of selecting a liquid length for a proper frequency tuning through the change of air pressure. However, the investigation was limited to the torsional vibration of a simple structure and aimed at the basic understanding of the MPTLCD. The application of MPTLCD for reducing the coupled lateral and torsional vibration of a real long span cable-stayed bridge under fluctuating wind is thus investigated in this chapter based on the understanding accomplished in Chapter 5.

For mitigating wind-induced vibrations of a long span bridge, numerous investigations on the efficiency of tuned mass dampers and tuned liquid column dampers were undertaken (Gu et al. 1988; Larsen 1993; Lin et al. 2000; Gu et al. 2002; Xue et al. 2002). These studies demonstrated that both tuned mass dampers and tuned liquid column dampers (TLCD) are effective devices for either reducing buffeting response or increasing critical flutter wind speed of the bridge. However, most of these studies were carried out by mode-by-mode spectral approach in the frequency domain. Only the first mode of vibration in vertical or torsional direction was taken into consideration. Coupling between higher modes of vibration could not be handled properly by the use of the mode-by-mode spectral approach in the frequency domain. Besides, for some long span bridges with closely spaced natural frequencies, the main contributions of the response may come from several modes of vibration. In this case, it is important to consider the control of multimode responses of a long span bridge. Furthermore, as mentioned in Chapter 5, the fundamental torsional frequency of a long
span cable-stayed bridge is often much higher than that in the lateral or vertical direction. The MPTLCD other than the TLCD is more suitable for such an application.

In this chapter, the performance of MPTLCD in reducing the lateral and torsional vibration of a real long span cable-stayed bridge at the completion stage is investigated using a finite element based approach. Several MPTLCDs are implemented to the bridge at the proper locations for the suppression of the multi-mode lateral vibrations, with each MPTLCD being tuned to a particular lateral mode, and the fundamental torsional vibration as well. The finite element model of the MPTLCD is first developed for the sake of carrying out buffeting analysis of the MPTLCD-bridge system. A real long span cable-stayed bridge is modeled using the conventional finite element method. The developed finite element model of MPTLCD is then incorporated into the finite element model of the bridge for predicting the buffeting response of the MPTLCD-bridge system. Wind forces acting on the bridge, including both buffeting and self-excited forces, are generated in time domain using computer simulation techniques in addition to the measured aerodynamic coefficients and flutter derivatives. A direct integration method is finally employed to find the solution of the stochastic buffeting response of the bridge. The performance of MPTLCD on the buffeting response of the complete bridge is investigated through a parametric study in terms of mass ratio, head loss coefficient, mean wind speed, and frequency tuning ratio.

8.2 EQUATIONS OF MOTION OF MPTLCD-BRIDGE SYSTEM UNDER TURBULENT WIND

8.2.1 Modeling of bridge

A long span cable-stayed bridge is represented by a three-dimensional finite element model using different types of finite elements such as beam element, cable element, plate element and solid element. The inclined stay cables are modeled as cable element whose elastic modulus is modified by the Ernst's formula in order to include the sag effect of cable due to its self-weight. The geometric nonlinear stiffness of the cable is also taken into consideration. Three-dimensional Timoshenko beam elements are used to model the bridge tower and deck. The mass matrix, the stiffness matrix and
the force vector of the bridge are obtained by the use of traditional finite element method (Bathe, 1996). The damping matrix of the bridge, which is assumed to be the Rayleigh damping, is expressed as a combination of the mass and the stiffness matrices.

\[
[C_b] = \alpha_b [M_b] + \beta_b [K_b]
\]

(8-1)

where \([M_b]\), \([C_b]\) and \([K_b]\) are the mass, damping and stiffness matrices of the bridge, respectively; \(\alpha_b\) and \(\beta_b\) are the Rayleigh damping factors, which can be evaluated if the first two modal damping ratios and natural frequencies are known. The equation of motion of the bridge under turbulent wind can then be expressed as:

\[
[M_b]\dddot{\{v_b\}} + [C_b] \ddot{\{v_b\}} + [K_b] \{v_b\} = \{P_{\text{buff}}\} + \{P_{\text{se}}\}
\]

(8-2)

where the vectors \(\{v_b\}\), \(\dot{\{v_b\}}\) and \(\ddot{\{v_b\}}\) are the nodal dynamic displacement, velocity, and acceleration of the bridge, respectively. \(\{P_{\text{buff}}\}\) and \(\{P_{\text{se}}\}\) are the buffeting force vector and self-excited force vector of the bridge, respectively, which are assembled from the buffeting forces and self-excited forces acting on all the nodes of the bridge, such as those acting on the ith node of the bridge deck \(\{P_{\text{buff}}^i\}\) and \(\{P_{\text{se}}^i\}\).

\[
\begin{bmatrix}
D_b \\
L_b \\
M_b
\end{bmatrix}; \quad \begin{bmatrix}
P_{\text{buff}}^i \\
P_{\text{se}}^i
\end{bmatrix} = \begin{bmatrix}
D_{\text{se}} \\
L_{\text{se}} \\
M_{\text{se}}
\end{bmatrix}
\]

(8-3)

It is noted that the self-excited forces at a particular time instant are dependent on motion of the bridge at that time instant. Iterations are generally required at each time step to determine the self-excited forces until the prescribed convergence is satisfied.

### 8.2.2 Modeling of liquid damper

As described in Chapter 5, the PTLCD is a U-shaped container with uniform cross-sectional area. Liquid is filled into the container and the two chambers are filled with compressed air. The frequency of the liquid inside the PTLCD can be adjusted by manipulating air pressure inside the air chamber of the PTLCD. To consider the interaction between the liquid dampers and long span cable-supported bridge under wind excitation, it is expedient to derive the finite element model of the MPTLCD. Let us consider the MPTLCD which consists of \(N\) small PTLCD unit (see Figure 8-1). Each small PTLCD unit is installed below the torsional centre of the bridge deck and
connected to the transverse beam of the bridge deck by a roller support and a simply support as shown in Figure 8-2. Two additional nodes, named node 1 and node 2, are generated at the position where the MPTLCD is connected to the bridge. These two additional nodes reflect the motion of the bridge deck associated with the damper. From a viewpoint of practical use, the liquid column length (L_k) and the distance between the two vertical columns (B_k) of each PTLCD unit are assumed to be the same. The axial deformation of the transverse beam between the two supports is assumed to be negligible. Hence, the lateral displacement of the liquid damper is taken as x and the torsional displacement of the liquid damper is determined by

\[ \theta = \frac{y_2 - y_1}{B} \]  

(8-4)

where x is the lateral displacement of the node 2; y_1 and y_2 are the vertical displacements of the nodes 1 and 2 respectively; and B is the distance between the two additional nodes. In Appendix A, the Lagrangian of the MPTLCD has been shown to be

\[ L_d = \sum_{k=1}^{n} \left[ \frac{1}{2} m_k \ddot{W}_k^2 + \frac{1}{2} m_k \dot{x}^2 + \frac{1}{2} I_k \dot{\theta}^2 + m_k \dot{\theta} \dot{x} + G_k \ddot{W}_k + \dot{W}_k \dot{\theta} + m_k \ddot{H}_k \dot{x} \dot{\theta} \right] \]  

(8-5)

where m_k is the liquid mass inside the kth PTLCD; I_k is the mass moment inertia of the liquid mass inside the kth PTLCD; \( \alpha_k \) is the liquid length ratio; L_k is the length of the kth PTLCD; \( \ddot{H}_k \) is the distance from the mass center of liquid inside the kth PTLCD to the torsional center of the structure; G_k is liquid mass moment in the kth PTLCD; g is the acceleration due to gravity; P_{o_k} is the static pressure inside the two chambers of the kth PTLCD; A_k is the cross sectional area of liquid column in the kth PTLCD; h_k is the height of the air chambers of the kth PTLCD; and \( \dot{W}_k \) and \( \ddot{W}_k \) are the liquid displacement and velocity inside the kth PTLCD. The last term in Equation (8-5) is the potential energy of liquid due to the pressure inside air chamber. Equation (8-5) is subjected to the following equation at any time.

\[ W_k \leq \{h_k, (L_k-B_k-d_k)/2\} \]  

(8-6)

where d_k is the thickness of the liquid damper. The interaction between the PTLCD and the bridge in vertical direction is considered by modeling the mass of water as rest mass in the finite element model of the bridge. The entries of mass matrix m_{ij} and stiffness matrix k_{ij} of the damper element can be determined by
\[
m_{ij} = \frac{\partial}{\partial q_{i}} \left( \frac{\partial L_d}{\partial q_{j}} \right) \quad k_{ij} = -\frac{\partial}{\partial q_{j}} \left( \frac{\partial L_d}{\partial q_{i}} \right)
\]
(8-7)

\[
[q]^T = [x \quad y_1 \quad y_2 \quad W_1 \quad \ldots \quad W_N]
\]
(8-8)

After some manipulations, the mass matrix of the damper element, \([M]\) and the stiffness matrix of the damper element, \([K]\) can be written as:

\[
[M] = \begin{bmatrix}
M_{11} & M_{12} \\
M_{12} & M_{22}
\end{bmatrix} \quad \text{and} \quad [K] = \begin{bmatrix}
K_{11} & K_{12} \\
K_{12} & K_{22}
\end{bmatrix}
\]
(8-9)

where

\[
M_{11} = \frac{1}{B^2} \begin{bmatrix}
m_d B^2 & -mB & -mB \\
-mB & I_d & -I_d \\
-mB & -I_d & I_d
\end{bmatrix}
\]
(8-10)

\[
M_{12} = \begin{bmatrix}
m_1 a_1 & m_2 a_2 & \ldots & m_N a_N \\
\frac{G_1}{B} & \frac{G_2}{B} & \ldots & \frac{G_N}{B} \\
\frac{G_1}{B} & \frac{G_2}{B} & \ldots & \frac{G_N}{B}
\end{bmatrix} = M_{21}
\]
(8-11)

\[
M_{22} = \text{diag}(m_1, m_2, \ldots, m_N)
\]
(8-12)

\[
K_{11} = \frac{mg}{B^2} \begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & -1 \\
0 & -1 & 1
\end{bmatrix}
\]
(8-13)

\[
K_{12} = \frac{g}{B} \begin{bmatrix}
0 & 0 & \ldots & 0 \\
-m_1 a_1 & -m_2 a_2 & \ldots & -m_N a_N \\
m_1 a_1 & m_2 a_2 & \ldots & m_N a_N
\end{bmatrix} = K_{21}
\]
(8-14)

\[
K_{22} = \text{diag}(m_1 \omega_1^2, m_2 \omega_2^2, \ldots, m_N \omega_N^2)
\]
(8-15)

where \(\omega_k\) is the circular natural frequency of the \(k\)th PTLCD; \(m_d\) is the total liquid mass of all the PTLCD units; \(I_d\) is the total liquid mass moment of inertia of all the PTLCD units; \(m = \sum_{k=1}^{N} m_k \bar{H}_k\); and \(G_k = m_k a_k \left( H_k + \frac{L_k - B_k}{2} \right)\). The circular natural frequency of liquid motion in the \(k\)th PTLCD, \(\omega_k\), is determined by.
\[ \omega_k^2 = \frac{2g}{L_k} \left(1 + \frac{P_{o,k}}{\rho gh_k}\right) \]  

(8-16)

if the ratio \( W_k/h_k \) is small. By re-arranging Equation (8-16), the static pressure \( P_{o,k} \) inside the \( k \)th PTLCD is given by

\[ P_{o,k} = \frac{pL_k h_k}{2} \left[ \omega_k^2 - \frac{2g}{L_k} \right] \]  

(8-17)

Equation (8-16) shows that the natural frequency of liquid motion inside the PTLCD is determined not only by the length of liquid column but also the static pressure \( P_{o,k} \). For a given liquid column length, the natural frequency of the liquid motion inside the PTLCD can be increased by the factor \( 1 + P_{o,k}/\rho gh \) comparing with the traditional TLCD, which greatly facilitates the frequency tuning requirement. Equation (8-17) provides a way of determining the required static pressure \( P_{o,k} \) after the frequency of the PTLCD and its liquid column length \( L \) are selected. To assess the performance of MPTLCD, several parameters are defined. They are the central frequency \( (\omega_o^j) \), the frequency bandwidth \( (\Delta \omega^j) \), and the constant frequency spacing \( (\beta_i^j) \).

\[ \omega_o^j = \frac{\omega_o^j + \omega_n^j}{2}; \quad \Delta \omega^j = \frac{\omega_n^j - \omega_o^j}{\omega_o^j}; \quad \beta_i^j = \omega_{i+1}^j - \omega_i^j \]  

(8-18)

where \( \omega_i^j \) is the natural frequency of liquid in the \( i \)th PTLCD of the MPTLCD tuned to the \( j \)th mode of vibration of the bridge; \( \omega_s^j \) is the natural frequency of the \( j \)th mode of vibration; \( \omega_o^j \) and \( \omega_n^j \) are the lowest and highest natural frequencies, respectively, among all the PTLCD units in the MPTLCD tuned to the \( j \)th mode of structural vibration; and \( n \) is assumed to be an odd number. For studying the sensitivity of the MPTLCD to off-tuning in frequency, the frequency tuning ratio parameter \( (\Delta \gamma_j) \) of the MPTLCD tuned to the \( j \)th mode of structural vibration is defined as

\[ \Delta \gamma_j = \frac{\omega_o^j}{\omega_s^j} \]  

(8-19)

8.2.3 Modeling of MPTLCD-Bridge system
Let a MPTLCD be installed at the middle span of a long span bridge where the amplitudes of the first lateral mode shape and the first torsional mode shape are the largest. The finite element model of the MPTLCD-Bridge system can be constituted by beam elements and damper elements. The equation of motion of MPTLCD-Bridge system is easily obtained by using the conventional finite element method as:

\[
\begin{bmatrix}
M_b + M_{bd} & M_{bd} \\
M_{bd}^T & M_d
\end{bmatrix}\begin{bmatrix}
\ddot{v}_b \\
\ddot{v}_w
\end{bmatrix} + \begin{bmatrix}
C_b & 0 \\
0 & C_d
\end{bmatrix}\begin{bmatrix}
\dot{v}_b \\
\dot{v}_w
\end{bmatrix} + \begin{bmatrix}
K_b + K_{bd} & K_{bd} \\
K_{bd}^T & K_d
\end{bmatrix}\begin{bmatrix}
v_b \\
v_w
\end{bmatrix} = \begin{bmatrix}
P_{bf} + P_{sf} \\
0
\end{bmatrix}
\]

(8-20)

In Equation (8-20), \([M_b]\), \([C_b]\) and \([K_b]\) are the mass matrix, damping matrix, and stiffness matrix of the bridge alone, respectively. The matrix \([v_w]_{nx1}\) represents the liquid displacements of the \(n\) PTLCD units. The matrix \([M_{bd}]\) is associated with the inertial forces due to the global motion of liquid damper acting on the bridge and it is obtained by assembling the liquid damper matrix \([M_{11}]\) properly. The matrix \([M_d]\) corresponds to the inertia forces of the liquid inside the dampers and it is obtained by assembling the liquid damper matrix \([M_{22}]\). The matrix \([M_{bd}]\) indicates that the structural motion and liquid motion are coupled by the inertia effect of liquid and it is obtained by assembling the liquid damper matrix \([M_{12}]\). The matrices \([K_{bd}]\) and \([K_{bd}]\) are associated with the restoring forces resulting from the gravitational effect of liquid and they are obtained by assembling the liquid damper matrices \([K_{11}]\) and \([K_{12}]\). The matrix \([K_d]\) corresponds to the liquid restoring forces due to the liquid elevation difference between the two vertical columns and it is obtained by assembling the liquid damper matrix \([K_{22}]\). The matrix \([C_d]\) is associated with the nonlinear damping forces resulting from the damper orifice and it is given by

\[
[C_d] = \text{diag}\left(\frac{1}{2}\rho_w A_1 \delta_1 |\dot{W}_1|, \frac{1}{2}\rho_w A_2 \delta_2 |\dot{W}_2|, \ldots, \frac{1}{2}\rho_w A_{nT} \delta_{nT} |\dot{W}_{nT}|\right)
\]

(8-21)

where \(\rho_w\) is the density of liquid inside the PTLCD units; and \(\delta_k\) is the head loss coefficient of the \(k\)th PTLCD unit.

### 8.3 SIMULATION ALGORITHM OF WIND VELOCITY ALONG A LONG SPAN BRIDGE

A series of time histories of fluctuating wind velocity in vertical and longitudinal directions at various points along the deck of a long span bridge is essential for
performing buffeting analysis of the bridge in the time domain. Several numerical methods of simulating wind velocity fields such as the spectral representation method (Shinozuka, 1971; Shinozuka and Jan, 1972; Yang, 1972, 1973; Shinozuka, 1990; Deodatis and Shinozuka, 1989) and the digital filtering method using ARMA model (Samaras et al., 1985; Li and Kareem, 1990; Mignolet and Spanos, 1990) were developed for the sake of carrying buffeting analysis of structures in the time domain. The spectral representation method can provide unconditionally stable results but it is computationally expensive due to the repetitive decomposition of the spectral matrix at every frequency step in the simulation when a series of wind velocities at various locations on a long span bridge is generated. On the other hand, the digital filtering method is computationally efficient and it does not require large computer storage, but the algorithm requires extra attention to ensure the numerical stability of a discrete system (Yang et al., 1997)

A fast spectral representation approach (Cao et al. 2000), which is developed on the basis of the method proposed by Deodatis and Shinozuka (1989), is adopted herein for the simulation of stochastic wind velocity field on a long span bridge. The fast spectral decomposition method involves the assumptions that the bridge deck is horizontal at the same elevation, the mean wind speed and wind spectra do not vary along the bridge deck, and the distance between any two successive points where wind speeds are simulated are the same. The time histories of the alongwind component \( u(t) \) and the vertical wind component \( w(t) \) at the \( j \)th point can be generated by the following equations

\[
\begin{align*}
  u_j(t) &= \sqrt{2(\Delta \omega)} \sum_{m=1}^{n} \sum_{l=1}^{N} \sqrt{S_u(\omega_{ml})} G_{jm}(\omega_{ml}) \cos(\omega_{ml} t + \phi_{ml}) \\
  w_j(t) &= \sqrt{2(\Delta \omega)} \sum_{m=1}^{n} \sum_{l=1}^{N} \sqrt{S_w(\omega_{ml})} G_{jm}(\omega_{ml}) \cos(\omega_{ml} t + \phi_{ml})
\end{align*}
\]  

(8-22)  

(8-23)

where \( \Delta \omega \) is the frequency interval between the spectral lines; \( N \) is the total number of frequency interval; \( j=1,2, \ldots, n \); \( n \) is the total number of points where wind speeds are simulated; \( S_u(\omega) \) and \( S_w(\omega) \) are the auto PSD functions of the alongwind and vertical wind respectively. \( \phi_{ml} \) is a random variable uniformly distributed between 0 and \( 2\pi \); and
\[ G_{jm}(\omega) = \begin{cases} 0, & \text{when } 1 \leq j < m \leq n \\ C^{[j-m]}, & \text{when } m = 1, m \leq j \leq n \\ C^{-[j-m]} \sqrt{1-C^2}, & \text{when } 2 \leq m \leq j \leq n \end{cases} \]  

(8-24)

\[ C = \exp \left( -\lambda \frac{\eta_r}{2\pi} \right); \quad \eta_r = \frac{0.747\Delta_{jm}}{L_r} \sqrt{1 + 70.78 \left( \frac{\omega L_r}{2\pi U_m} \right)^2} \]  

(8-25)

\[ \omega_m = (l-1)\Delta \omega + \frac{m}{n} \Delta \omega \quad (l=1,2,\ldots,N); \quad \text{and} \quad \Delta \omega = \omega_{up}/N \]  

(8-26)

where \( \Delta_{jm} = \Delta[j-m] \), the horizontal distance between points \( j \) and \( m \); \( C \) is the coherence function between points \( j \) and \( m \); \( L_r \) is the integral scales of the \( r \) component in the across wind direction; \( \omega_{up} \) is the upper cutoff frequency; and \( U_m \) is the mean wind speed.

Yang (1972, 1973) showed that the efficiency of simulation can be further enhanced by utilizing the FFT technique. To implement the simulation with the FFT technique, Equations (8-22) and (8-23) can be rewritten in the following form.

\[ u_j(p \Delta t) = \text{Re} \left\{ \sum_{m=1}^{N} h_{jm}(q \Delta t) \exp \left( i \left( \frac{m \Delta t}{n} \right) (p \Delta t) \right) \right\} \]  

(8-27)

\[ w_j(p \Delta t) = \text{Re} \left\{ \sum_{m=1}^{N} h_{jm}(q \Delta t) \exp \left( i \left( \frac{m \Delta t}{n} \right) (p \Delta t) \right) \right\} \]  

(8-28)

where \( p=0,1,\ldots,2N\times n-1; \ j=1,2,\ldots,n; \ q \) is the remainder of \( p/2N; \ q=0,1,2,\ldots,n-1 \) and \( h_{jm}(q \Delta t) \) is given by

\[ h_{jm}(q \Delta t) = \sum_{l=0}^{2N-1} B_{jm}(l \Delta \omega) \exp \left( i \frac{\pi l q}{N} \right) \]  

(8-29)

\[ B_{jm}(l \Delta \omega) = \begin{cases} \sqrt{2(\Delta \omega)S(\omega)} G_{jm}(l \Delta \omega + \frac{m \Delta \omega}{n}) \exp (i \Phi_{nl}), & \text{when } 0 \leq l < N \\ 0, & \text{when } N \leq l < 2N \end{cases} \]  

(8-30)

where \( S(\omega) = S_\omega(\omega) \) or \( S_\omega(\omega) \).

It can be seen from Equations (8-29) and (8-30) that \( h_{jm}(q \Delta t) \) is the Fourier transformation of \( B_{jm}(l \Delta \omega) \) and therefore the simulation can be performed with much higher efficiency by the use of the FFT technique.
8.4 WIND FORCES ON A LONG SPAN CABLE-STAYED BRIDGE

In time domain buffeting analysis of a long span bridge, the aerodynamic forces acting on the bridge can be resolved into buffeting and self-excited force components. The buffeting forces are due to incident wind turbulences acting on the bridge and the self-excited forces are due to the interaction between the motions of the bridge and the wind. The formulations of the above forces acting on the bridge deck are introduced in the following sections.

8.4.1 Buffeting forces

The buffeting forces are caused by wind turbulences \( u \) and \( w \) in the longitudinal (alongwind) and vertical (upward) directions. The buffeting forces as shown in Figure 8-3 can be determined by the following expressions:

\[
D_b(t) = \frac{1}{2} \rho_a U_m^2 b \left[ C_D \frac{2u(t)}{U_m} + C_D' \frac{w(t)}{U_m} \right] \tag{8-31a}
\]

\[
L_b(t) = \frac{1}{2} \rho_a U_m^2 b \left[ C_L \frac{2u(t)}{U_m} + (C_L' + C_D) \frac{w(t)}{U_m} \right] \tag{8-31b}
\]

\[
M_b(t) = -\frac{1}{2} \rho_a U_m^2 b^2 \left[ C_M \frac{2u(t)}{U_m} + C_M' \frac{w(t)}{U_m} \right] \tag{8-31c}
\]

where \( D_b(t) \), \( L_b(t) \) and \( M_b(t) \) are the buffeting drag, lift and moment respectively, on the bridge deck unit span length at a given position; \( \rho_a \) is the air density; \( b \) is the deck width; \( C_L, C_D \) and \( C_M \) are the lift, drag and moment coefficients obtained from wind tunnel tests of the bridge deck section model; \( C_L', C_D' \) and \( C_M' \) are the slopes of \( C_L, C_D \) and \( C_M \) at the angle \( \alpha \), respectively; \( \alpha \) is the angle of attack of normal incident wind referring to the horizontal plane of the bridge deck; and \( u(t) \) and \( w(t) \) are the wind speed components in the Y-direction and Z-direction, respectively.

8.4.2 Self-excited forces

The self-excited forces on the bridge deck are caused by interaction between the wind and the bridge motion. The bridge system taps off energy from wind flow by
means of its deflection and their time derivatives. The vibration amplitude may be increased to catastrophic level if the energy of motion extracted from the flow exceeds the energy dissipated by the system through mechanical damping. The self-excited forces per unit span length are commonly expressed by the following Scanlan’s extended formula (Scanlan, 1978a, 1993).

\[
D_{se}(t) = \rho_s U_m^2 b \left\{ \frac{kP_1^*(v) \frac{\partial}{\partial t} + kP_2^*(v) \frac{b \dot{u}}{U_m} + k^2 P_3^*(v) \alpha + k^2 P_4^*(v) \frac{\partial^2}{\partial t^2}}{b} \right\}
\]  
\[ \quad + \frac{kP_5^*(v) \frac{\partial h}{\partial t} + k^2 P_6^*(v) \frac{h}{b}}{b} \]  

\[ L_{se}(t) = \rho_s U_m^2 b \left\{ \frac{kH_1^*(v) \frac{\partial h}{\partial t} + kH_2^*(v) \frac{b \dot{u}}{U_m} + k^2 H_3^*(v) \alpha + k^2 H_4^*(v) \frac{\partial h}{\partial t}}{b} \right\}
\]  
\[ \quad + \frac{kH_5^*(v) \frac{\partial h}{\partial t} + k^2 H_6^*(v) \frac{h}{b}}{b} \]  

\[ M_{se}(t) = \rho_s U_m^2 b^2 \left\{ \frac{kA_1^*(v) \frac{\partial h}{\partial t} + kA_2^*(v) \frac{b \dot{u}}{U_m} + k^2 A_3^*(v) \alpha + k^2 A_4^*(v) \frac{\partial h}{\partial t}}{b} \right\}
\]  
\[ \quad + \frac{kA_5^*(v) \frac{\partial h}{\partial t} + k^2 A_6^*(v) \frac{h}{b}}{b} \]  

where \( k = \omega b / U \) is the reduced frequency; \( \nu = 2\pi / \omega \) is the circular frequency of vibration; \( h, \dot{u}, \) and \( \alpha \) are the vertical, lateral and torsional displacement of the bridge deck at a given position, respectively; and \( H_i^*, P_i^*, A_i^*(i=1-6) \) are non-dimensional flutter derivatives, which are functions of the reduced frequency and depend on the geometrical configuration of the bridge deck section.

Based on a linear strip theory, the self-excited forces per unit span length can be expressed in terms of convolution integrals as follows (Lin and Yang 1983).

\[ D_{se}(t) = \rho_s U_m^2 \int_0^t \left[ I_{Dh}(t-\tau) h(\tau) + I_{Dp}(t-\tau) p(\tau) + I_{De}(t-\tau) \alpha(\tau) \right] d\tau \]  
\[ L_{se}(t) = \rho_s U_m^2 \int_0^t \left[ I_{Lh}(t-\tau) h(\tau) + I_{Lp}(t-\tau) p(\tau) + I_{Le}(t-\tau) \alpha(\tau) \right] d\tau \]  
\[ M_{se}(t) = \rho_s U_m^2 \int_0^t \left[ I_{Mh}(t-\tau) h(\tau) + I_{Mp}(t-\tau) p(\tau) + I_{Me}(t-\tau) \alpha(\tau) \right] d\tau \]  

where \( I_0 \) represents the impulse response function of the self-excited forces, in which the subscripts indicate the corresponding force components. The impulse response
function can be obtained from the wind tunnel test results together with the nonlinear parameter identification techniques.

The relationship between the aerodynamic impulse functions and flutter derivatives can be obtained by taking the Fourier transform of Equations (8-32) and (8-33) and comparing the corresponding terms between these equations.

\[
\tilde{I}_{14}(\omega) = k^2[H_4^* (\nu) + iH_4^* (\nu)] \\
\tilde{I}_{1p}(\omega) = k^2[H_5^* (\nu) + iH_5^* (\nu)] \\
\tilde{I}_{1a}(\omega) = k^2b[H_6^* (\nu) + iH_6^* (\nu)] \\
\tilde{I}_{1b}(\omega) = k^2[P_5^* (\nu) + iP_5^* (\nu)] \\
\tilde{I}_{1p}(\omega) = k^2[P_4^* (\nu) + iP_4^* (\nu)] \\
\tilde{I}_{1a}(\omega) = k^2b[P_4^* (\nu) + iP_4^* (\nu)] \\
\tilde{I}_{1b}(\omega) = k^2b[A_4^* (\nu) + iA_4^* (\nu)] \\
\tilde{I}_{1a}(\omega) = k^2b[A_4^* (\nu) + iA_4^* (\nu)] \\
\tilde{I}_{1b}(\omega) = k^2b[A_4^* (\nu) + iA_4^* (\nu)] \\
\tilde{I}_{1a}(\omega) = k^2b[A_4^* (\nu) + iA_4^* (\nu)] \\
\tilde{I}_{1b}(\omega) = k^2b[A_4^* (\nu) + iA_4^* (\nu)]
\]

where the over-bar denotes the Fourier transform operation; and terms containing \( i \) represent imaginary parts. Equation (8-34) shows that the impulse function can easily be obtained by the flutter derivatives of the bridge deck. However, the measurement results of flutter derivatives from wind tunnel tests are usually a series of discrete data in the frequency domain, and approximate functions should be used to develop the impulse functions as continuous functions for a time domain analysis. Lin et al. (1978) proposed to use the following rational functions to approximate the impulse functions in the frequency domain.

\[
\tilde{I}(\omega) = \left[ C_1 + iC_2 \frac{b\omega}{U_m} + \sum_{k=3}^{n} C_k \frac{i\omega}{d_k \frac{U_m}{b} + i\omega} \right] \tag{8-35}
\]

With regard to the terms corresponding to the moment induced by the torsional motion \( M_{\text{seo}}(t) \), the aerodynamic transfer function can be expressed as follows.
\[ \tilde{I}_{mb}(\omega) = k^2 b^2 [A_j^* (\nu) + i A_j^* (\nu)] = b^2 \left[ C_1 + i C_2 \frac{b \omega}{U_m} + \sum_{k=3}^{n} C_k \frac{i \omega}{d_k \frac{U_m}{b} + i \omega} \right] \]  

(8-36)

where \( C_1, \ C_2, \ C_3, \ C_k \) and \( d_k \) (\( d_k \geq 0; \ k=3,4,\ldots,n \)) are the frequency independent coefficients of the rational functions. The first and second terms in Equation (8-35) represent the aerodynamic stiffness and the aerodynamic damping respectively and the rational terms represent the unsteady components that lag the velocity term and permit an approximation of the time delays through positive values of parameter \( d_k \). For other components of self-excited force, they can be approximated using the same rational function as Equation (8-35) with the following relationships:

\[ \tilde{I}_{lk}(\omega) = \tilde{I}(\omega) \quad \tilde{I}_{lp}(\omega) = \tilde{I}(\omega) \quad \tilde{I}_{le}(\omega) = b \tilde{I}(\omega) \]  

(8-37a)

\[ \tilde{I}_{dh}(\omega) = \tilde{I}(\omega) \quad \tilde{I}_{dp}(\omega) = \tilde{I}(\omega) \quad \tilde{I}_{de}(\omega) = b \tilde{I}(\omega) \]  

(8-37b)

\[ \tilde{I}_{mb}(\omega) = b \tilde{I}(\omega) \quad \tilde{I}_{mp}(\omega) = b \tilde{I}(\omega) \quad \tilde{I}_{ma}(\omega) = b^2 \tilde{I}(\omega) \]  

(8-37c)

Equation (8-36) can be transformed in the following form.

\[ 4 \pi^2 [A_j^* (\nu) + i A_j^* (\nu)] = v^2 \left[ C_1 + i C_2 \frac{2 \pi}{v} + \sum_{k=3}^{n} C_k \frac{i 2 \pi d_k v + 4 \pi^2}{d_k^2 v^2 + 4 \pi^2} \right] \]  

(8-38)

To determine the coefficients \( C_1, \ldots, C_n, \ d_3, \ldots, d_n \), let us take \( \tilde{I}_{ma} \) as an example. Equating the real part and imaginary part of Equation (8-38), one would obtain the following relation between the flutter derivatives and the coefficients \( C_k \) and \( d_k \).

\[ \frac{C_1 v^2}{4 \pi^2} + \sum_{k=3}^{n} \frac{C_k v^2}{d_k^2 v^2 + 4 \pi^2} = \tilde{A}_j^* (v) \quad \frac{C_2 v}{2 \pi} + \sum_{k=3}^{n} \frac{C_k d_k v^3}{2 \pi d_k^2 v^2 + 8 \pi^3} = \tilde{A}_j^* (v) \]  

(8-39)

From Equation (8-39), one could obtain the coefficients \( C_1, \ldots, C_n, \ d_3, \ldots, d_n \) by using the least squares method to fit the measured flutter derivatives at different reduced frequencies. The inverse Laplace Transform of \( \tilde{I}_{ma} \) yields the aerodynamic impulse function

\[ I_{ma} (t) = b^2 \left[ C_1 \delta(t) + \frac{b}{U_m} \delta(t) + \delta(t) \sum_{k=3}^{n} C_k - \sum_{k=3}^{n} C_k d_k \frac{U_m}{b} \exp \left( - \frac{d_k U_m}{b} t \right) \right] \]  

(8-40)

where \( \delta(t) \) is the Dirac delta function. Substituting Equation (8-40) into Equation (8-33) and after some manipulations, one would obtain the self-excited moment induced by torsional motion of the bridge deck.
\[ M_a(t) = \rho_s U_m^2 b^2 \left[ C_0 a(t) + C_2 \frac{b}{U_m} \dot{a}(t) + \sum_{k=3}^{n} \int C_k \exp \left( -\frac{d_k U_m}{b} (t - \tau) \right) \dot{a}(\tau) d\tau \right] \]  

(8.41)

The derivation procedure can be applied to other self-excited force components to obtain similar formulations, which are omitted here for the sake of brevity.

### 8.5 COMPUTER PROGRAM

The equations of motion of coupled liquid column damper and long span cable-stayed bridge systems subjected to turbulent wind have been derived in the previous sections. A set of user-friendly comprehensive computer programs is then developed accordingly. These computer programs execute five major functions as follows:

- **The computer program DSAPB** is developed using the Fortran language to assemble the mass matrix, damping matrix, and stiffness matrix of the cable-stayed bridge alone based on the conventional finite element method. This program also computes the natural frequencies and mode shapes of the bridge based on the subspace iteration method. To use this computer program, only one input data file is required. The input data file should include four types of information: (1) the general information such as the number of node, the number of element type, and the number of natural frequency required; (2) the node coordinate; (3) the node restraint information; and (4) the geometric and physical property information of each element.

- **The computer program SIMURW** is developed using the Fortran language to perform the simulation of time histories of wind turbulent velocities based on the fast spectral representation approach proposed by Yang et al., (1997) and Cao et al., (2000).

- **The major computer program DSAPBDW** is developed using the Fortran language for the prediction of dynamic response of the coupled liquid column damper and cable-stayed bridge system subjected to turbulent wind. The flowchart of the main structure of this computer program is displayed in Figure 8-4. To execute this computer program, one project control data file is required,
which contains all the necessary input and output data information. The input data information consists of the general control input data information and 6 specific input data files. The 6 specific input data files include: (1) the data file for bridge information such as the mass matrix, damping matrix, and stiffness matrix of the bridge determined by the computer program DSAPB; (2) the data file for multiple pressurized liquid column damper information; (3) the data file for wind information such as the frequency independent coefficients used to determine the buffeting forces and self-excited forces in the time domain; (4) the data file for the initial displacements and velocities of all the independent degrees of freedom of both the cable-stayed bridge and multiple pressurized liquid column dampers; (5) the data file for the time histories of wind turbulent velocities in the longitudinal direction; (6) the data file for the time histories of wind turbulent velocities in the vertical direction. The flowchart of the computer program for executing the dynamic analysis of coupled multiple pressurized liquid column damper and bridge systems under turbulent wind is shown in Figure 8-5.

8.6 DYNAMIC RESPONSE OF THE BRIDGE AT ITS COMPLETION STAGE WITHOUT CONTROL

8.6.1 A long span cable-stayed bridge

A triple tower cable-stayed bridge with an overall length of 1177m and the two main spans of 448m and 475m and the two side spans of 127m each is taken for this case study (see Figure 8-6). The bridge deck consists of two carriage-way structures and the overall width of the bridge deck is 42.8m. The central pylon reaches a height of 200m above sea level with the end towers having heights of 163m and 172m above sea level respectively. The three bridge pylons are all single leg concrete pylons. The deck to tower connections offer longitudinal and lateral displacement restraint with completely free rotation about all three axes in addition to free vertical displacement. The side towers are similar but without longitudinal restraint. Two large steel tower heads, each weighing some 170 tons, are installed at the top of each tower from which 384 stay cables in four planes radiate downwards to support the bridge deck at 13.5m
intervals. The tower are further stabilized by transverse cables running from the tower head to the cross struts and the section of tower below deck level. The central towers are further stabilized longitudinally with eight longitudinal stay cables. The bridge deck is separated into two carriageway structures and each carriageway structure is formed by two longitudinal steel plate girders with steel beams spanning transversely between them at 4.5m centers. The bridge is represented by a three dimensional dynamic finite element model. Three-dimensional Timoshenko beam elements are used to model the three bridge towers. The stay cables and stabilizing cables are modeled by cable elements accounting for geometric nonlinearity due to cable tension. Each carriageway structure is represented by a three-girder model consisting of one central girder and two side girders connected by transverse links. All the girders are modeled by the three-dimensional Timoshenko beam elements with axial force effect included. The first two modes of structural damping ratio of the bridge is taken as 0.8%.

8.6.2 Simulation of wind velocity

The wind velocity field in either alongwind or vertical wind directions along the bridge girder is simulated by ninety-six (n) fluctuating wind velocity time histories at ninety-six different points evenly distributed with an interval distance of 13.5m (Δ) along the bridge deck. The PSD function of the along wind u and vertical wind w are given by the von Kármán spectra as follows.

\[
\frac{\omega S_{uu}(\omega)}{\sigma_u^2} = \frac{42Z_u}{(1 + 70.78 Z_u^2)^{5/6}}; \quad \frac{\omega S_{ww}(\omega)}{\sigma_w^2} = \frac{2(l + 188.8 Z_w^2 Z_u)}{(1 + 70.78 Z_w^2)^{5/6}}; \quad Z_j = \frac{1}{2\pi} \frac{\omega L_j^y}{U}
\]

where \(L_j^y\) is the integral scales of the j component in the along-wind direction; \(\omega\) is the frequency in rads\(^{-1}\); \(\sigma_u\) and \(\sigma_w\) are the standard deviations of wind turbulent in the along wind and vertical wind direction.

In this study, the turbulent intensity at the bridge deck level in alongwind direction (\(I_u\)) and in vertical direction (\(I_w\)) are 0.10 and 0.05, respectively. The integral scale of alongwind turbulence in the alongwind direction (\(L_u^i\)) is of 80m and that of vertical wind in the along wind direction (\(L_w^i\)) is of 40m. The parameter \(\lambda\) in Equation (8-25) is taken as 8 as a constant value. The upper cutoff frequency (\(\omega_{up}\)) is taken as
20π rad/s and the dividing number of frequency (N) is 2^{14}. The corresponding frequency interval (Δω) and the time interval of wind velocity histories (dt) are 0.0019175 rad/s and 0.05 s respectively. Figures 8-7 and 8-8 illustrate turbulent wind velocity time-histories for different mean wind speeds: 10, 20, 30, 40 and 50 m/s, in the alongwind and vertical directions at the middle section of the left main span (point A in Figure 8-5).

8.6.3 Simulation of wind forces

The drag, lift and moment coefficients (C_D, C_L, C_M) of the bridge deck measured from the wind tunnel tests are 0.103, 0.134, and 0.011 respectively, at the zero wind angle of attack with respect to the deck width of 42.8 m (Tan, 1999). The first derivatives of the drag, lift, and moment coefficients (C_D', C_L', C_M') with respect to wind angle at the zero wind angle of attack are 0.00, 5.25, and 1.06, respectively.

In the simulation of the self-excited forces, only the flutter derivatives (H_1^*, H_4^*, A_2^*, and A_3^*) of the concerned cable-stayed bridge are available from wind tunnel tests (Choi and Brownjohn 1998; Tan, 1999) and they are displayed in Figure 8-9. For the flutter derivatives in lateral direction, they are considered by the quasi-steady theory as follows:

\[ P_1^* = -\frac{C_D}{k}; \quad P_4^* = 0 \]  \hspace{1cm} (8-43)

The frequency independent coefficients determined by using the measured flutter derivatives (H_1^* and H_4^*) are listed in Table 8-1 and those determined by the measured flutter derivatives (A_2^* and A_3^*) are listed in Table 8-2 while the frequency independent coefficients determined by the flutter derivatives (P_1^* and P_4^*) are listed in Table 8-3.

<p>| Table 8-1 Frequency Independent Coefficient Based on H_1^* and H_4^* (Tan, 1999) |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|</p>
<table>
<thead>
<tr>
<th>C_{1Lh}</th>
<th>C_{2Lh}</th>
<th>C_{3Lh}</th>
<th>C_{4Lh}</th>
<th>D_{3Lh}</th>
<th>d_{4Lh}</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5463</td>
<td>-2.1835</td>
<td>10.6568</td>
<td>2.2127</td>
<td>13157000</td>
<td>28.9613</td>
</tr>
</tbody>
</table>
Table 8-2 Frequency Independent Coefficient Based on $A'_i$ and $A'_j$ (Tan, 1999)

<table>
<thead>
<tr>
<th>$C_{1Mx}$</th>
<th>$C_{2Mx}$</th>
<th>$C_{3Mx}$</th>
<th>$C_{4Mx}$</th>
<th>$D_{1Ma}$</th>
<th>$D_{4Ma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40.661</td>
<td>-0.1618</td>
<td>-40.4288</td>
<td>-0.0794</td>
<td>0.001438</td>
<td>2.3229</td>
</tr>
</tbody>
</table>

Table 8-3 Frequency Independent Coefficient Based on $P'_1$ and $P'_4$

<table>
<thead>
<tr>
<th>$C_{1Dx}$</th>
<th>$C_{2Dx}$</th>
<th>$C_{3Dx}$</th>
<th>$C_{4Dx}$</th>
<th>$D_{1Dx}$</th>
<th>$d_{4Dx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>-0.103</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

8.6.4 Dynamic characteristics of bridge

The buffeting response of a long span bridge significantly depends on its natural frequencies, mode shapes, and modal damping ratios. The dynamic characteristics of the bridge are first studied in terms of its natural frequency and mode shapes. The computed natural frequencies and mode shapes are shown in Appendix B.1, and the natural frequencies and the generalized masses and mass moments of inertia are summarized in Tables 8-4 to 8-6. The generalized mass and mass moment of inertia given in these tables are determined by

$$M'_{ij} = \int \overline{m}(y)\psi_{ij}^2(y) \, dy; \quad M''_{ij} = \int \overline{m}(y)\psi_{ij}'(y) \, dy; \quad I''_{ij} = \int \overline{l}(y)\psi_{ij}^2(y) \, dy \quad (8.44)$$

where $y$ represents the coordinate along the bridge deck; $\overline{m}(y)$ and $\overline{l}(y)$ are the mass and mass moment of inertia of the bridge deck per unit length; $\psi_{ij}(y)$, $\psi_{ij}'(y)$ and $\psi_{ij}''(y)$ are respectively the $j$th normalised lateral, vertical and torsional mode shape of the bridge deck.

It is seen from Appendix B.1 that the maximum amplitudes of both the first vertical and torsional mode shape occur at point A in the left main span and point B in the right main span. The first lateral mode of vibration is dominated by the lateral bending of the towers. For the second lateral mode of vibration, the maximum vibration amplitudes also occur at point A in the left main span and point B in the right main span. It is noted from Tables 8-4 to 8-6 that the first five lateral natural frequencies of the bridge are closely spaced between 0.23097Hz and 0.35661Hz. The MPTLCD should
thus be installed at points A and B for abating the potential largest vibration along the bridge deck.

Table 8-4 Lateral Natural Frequency and Generalized Mass of Bridge

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Generalized mass ($M_{ij}$) / kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>0.23097</td>
<td>4.89925×10^6</td>
</tr>
<tr>
<td>L2</td>
<td>0.24983</td>
<td>1.50120×10^7</td>
</tr>
<tr>
<td>L3</td>
<td>0.28891</td>
<td>7.69831×10^6</td>
</tr>
<tr>
<td>L4</td>
<td>0.32322</td>
<td>4.21742×10^6</td>
</tr>
<tr>
<td>L5</td>
<td>0.35661</td>
<td>9.01821×10^6</td>
</tr>
</tbody>
</table>

Table 8-5 Vertical Natural Frequency and Generalized Mass of Bridge

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Generalized mass ($M_{ij}$) / kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>0.17611</td>
<td>1.21292×10^7</td>
</tr>
<tr>
<td>V2</td>
<td>0.33486</td>
<td>2.37125×10^7</td>
</tr>
<tr>
<td>V3</td>
<td>0.35524</td>
<td>9.20961×10^6</td>
</tr>
<tr>
<td>V4</td>
<td>0.39146</td>
<td>1.01376×10^7</td>
</tr>
<tr>
<td>V5</td>
<td>0.47849</td>
<td>6.62296×10^6</td>
</tr>
<tr>
<td>V6</td>
<td>0.56916</td>
<td>7.46246×10^6</td>
</tr>
<tr>
<td>V7</td>
<td>0.59718</td>
<td>5.75580×10^6</td>
</tr>
</tbody>
</table>

Table 8-6 Torsional Natural Frequency and Generalized Mass Moment of Inertia of Bridge

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Generalized mass moment of inertia ($I_{ij}$) / kgm^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.52845</td>
<td>6.38805×10^8</td>
</tr>
<tr>
<td>T2</td>
<td>0.54436</td>
<td>5.99252×10^8</td>
</tr>
<tr>
<td>T3</td>
<td>0.61027</td>
<td>9.37579×10^8</td>
</tr>
</tbody>
</table>

8.6.5 Buffeting response of bridge under different mean wind speeds

To have a better understanding of buffeting response of the bridge deck, the standard deviations of the displacement responses of the bridge deck along the bridge axis are plotted in Figure 8-10 for different mean wind speeds. It is clear that the dynamic responses of the bridge increase rapidly with increasing mean wind speed. The maximum amplitudes of vibration in lateral and torsional directions always occur at point A in the left main span and point B in the right main span, which is consistent
with the modal analysis results. The results confirm that the MPTLCD should be installed at points A and B for abating the largest vibration along the bridge deck. The standard deviation and peak displacements of point A and point B at different mean wind speeds are tabulated in Tables 8-7 and 8-8 respectively. The time histories of the deck displacements in lateral, vertical and torsional directions at points A and B under the mean wind speed of 50 m/s are displayed in Figures 8-11 and 8-12. To further understand the vibration nature of the bridge deck, the spectrum analysis is performed to the time histories of deck displacement, and the resulting PSD functions are shown in Figures 8-13 and 8-14. The PSD functions of vertical and torsional displacement of the bridge deck at points A and B show that both vertical and torsional displacements of the bridge deck are dominant by one single peak at the frequency around 0.18 Hz and 0.53 Hz respectively. For the lateral displacement, the PSD function at points A and B has five different peaks at the frequency around 0.23, 0.25, 0.29, 0.32 and 0.35 Hz. These peak frequencies appearing in the PDS functions match quite well with the computed natural frequencies (see Tables 8-3 to 8-5).

### Table 8-7 Standard Deviation Displacement of the Bridge under Different Mean Wind Speeds

<table>
<thead>
<tr>
<th>$U_m$ (m/s)</th>
<th>Lateral (m)</th>
<th>Vertical (m)</th>
<th>Torsional (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>20</td>
<td>0.00282</td>
<td>0.00318</td>
<td>0.08058</td>
</tr>
<tr>
<td>30</td>
<td>0.00795</td>
<td>0.00900</td>
<td>0.21222</td>
</tr>
<tr>
<td>40</td>
<td>0.01650</td>
<td>0.01863</td>
<td>0.39909</td>
</tr>
<tr>
<td>50</td>
<td>0.02865</td>
<td>0.03225</td>
<td>0.62403</td>
</tr>
</tbody>
</table>

### Table 8-8 Peak Displacement of the Bridge under Different Mean Wind Speeds

<table>
<thead>
<tr>
<th>$U_m$ (m/s)</th>
<th>Lateral (m)</th>
<th>Vertical (m)</th>
<th>Torsional (rad)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>20</td>
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<td>0.01091</td>
<td>0.29549</td>
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<td>0.03052</td>
<td>0.78479</td>
</tr>
<tr>
<td>40</td>
<td>0.06826</td>
<td>0.06357</td>
<td>1.50056</td>
</tr>
<tr>
<td>50</td>
<td>0.12079</td>
<td>0.11110</td>
<td>2.39020</td>
</tr>
</tbody>
</table>

### 8.7 PERFORMANCE OF MPTLCD
As described in the previous section, the lateral vibration of the concerned long span bridge consists of five major frequency components and the torsional vibration consists of only one major frequency component. Therefore, it is decided that for each main span, five MPTLCDs will be tuned to the first five lateral frequencies of the bridge and one MPTLCD will be tuned to the first torsional frequency of the bridge. To investigate the robustness of MPTLCD against the offset in frequency tuning, each MPTLCD has five PTLCD units tuned at the target central frequency \( f_c^1 \) with a frequency bandwidth \( \Delta X^1 \). The mass ratio \( \mu_{lj} \) for the MPTLCD tuned to the jth mode of lateral vibration of the bridge is defined as

\[
\mu_{lj} = \frac{m_{lj}}{M_{lj}}
\]  
(8-45)

The mass ratio \( \mu_{\eta j} \) for the MPTLCD tuned to the jth mode of torsional vibration of the bridge is defined as

\[
\mu_{\eta j} = \frac{m_{\eta j}I_j}{I_{\eta j}}
\]  
(8-46)

where \( m_{lj} \) is the mass of MPTLCD tuned to the jth lateral vibration mode of the bridge; \( m_{\eta j} \) is the mass of MPTLCD tuned to the jth torsional vibration mode of the bridge; and \( I_j \) is the moment of inertia of MPTLCD per unit mass tuned to the jth torsional mode of vibration of the bridge. The total liquid mass of all the MPTLCDs (\( m_d \)) is determined by

\[
m_d = \sum_{j=1}^{n_L} m_{lj} + \sum_{j=1}^{n_T} m_{\eta j} = \mu m_s
\]  
(8-47)

where \( n_L \) is the number of lateral modes of vibration to be controlled; \( n_T \) is the number of torsional mode of vibration to be controlled; \( \mu \) is the ratio of total liquid mass to the mass of the bridge deck (\( m_s \)). For simplicity, the mass ratios \( (\mu_{L1}, \ldots, \mu_{LS}) \) of the five MPTLCDs tuned to the first five lateral frequencies are taken as the same value of \( \mu_L \). The performance of the MPTLCDs is assessed in terms of the response ratio \( R \), which is defined as the ratio of the structural response with control to the structure response without control. The mean wind speed considered is 50m/s unless it is otherwise specified. The mass of the concerned bridge deck is 4.4850\times10^7 kg. From a view point of practical use, the geometric configurations of all the PTLCD units are taken to be the same. In this study, the liquid column length is selected to be 16m with the air chamber height (h) of 2m and the thickness of liquid column of 1.1m.

8-21
8.7.1 Effect of liquid mass distribution

The application of MPTLCDs for the suppression of lateral and torsional vibration involves the issue on how to distribute the total water mass to the MPTLCD tuned to lateral vibration and the MPTLCD tuned to torsional vibration. It is therefore important to investigate the effect of the water mass ratio $\mu_L/\mu_T$ on the displacement response reduction so as to have the maximum benefit for the reduction of the coupled lateral and torsional vibration of the bridge. The effects of the water mass ratio $\mu_L/\mu_T$ on the performance of MPTLCDs for the concerned bridge are depicted in Figure 8-15. The parameters of the MPTLCDs used are $\Delta y=1$, $H=5m$, $L=16m$, $\delta_L=30$, $\delta_T=10$, $\Delta X_L=\Delta X_T=0$, $\alpha=0.8$ and $\mu=0.027$. The parameters with subscript L (T) represent the parameters of the MPTLCD tuned to the lateral (torsional) vibration of the bridge. The dropping of the subscript in some damper parameters implies that these parameters are the same for all the MPTLCDs. The maximum air pressure in the damper among all the PTLCD units is found to be 156.8 kPa. It can be seen from Figure 8-15 that the torsional displacement response ratio increases with increasing water mass ratio $\mu_L/\mu_T$ but the lateral displacement response ratio decreases with increasing water mass ratio $\mu_L/\mu_T$. However, the lateral displacement response ratio becomes relatively less sensitive to the water mass ratio $\mu_L/\mu_T$ after its value is beyond 0.75. To determine the optimal value of water mass ratio $\mu_L/\mu_T$, the displacement response index is introduced as

$$R_1 = (1-a)R_1 + aR_t$$

(8-48)

where $a$ is the weighing factor ranging from zero to unit; and $R_1$ and $R_t$ are the response ratios of lateral displacement and torsional displacement respectively. Since $R_1$ and $R_t$ are the function of the water mass ratio $\mu_L/\mu_T$, the response index is also the function of the water mass ratio. Figure 8-15 portrays that the variation of the response index with the water mass ratio for the case of the lateral and torsional displacement reduction being equally important ($a=0.5$). The figure shows that there exists an optimal water mass ratio at points A and B and they are around 0.65. To achieve better performance of MPTLCD for torsional vibration reduction, a smaller water mass ratio is selected in this study. For the parametric studies in the remaining sections, the mass ratio $\mu_L$ and $\mu_T$ are selected to be 0.85% and 1.5% with $\mu=0.027$. For this water mass ratio $\mu_L/\mu_T=0.567$, the reduction in lateral displacement and torsional displacement can achieve 29% and 25% respectively.
8.7.2 Effect of head loss coefficient

The effects of head loss coefficient on the performance of MPTLCD in reducing lateral and torsional displacements are depicted in Figure 8-16 for $\Delta X=0$ and Figure 8-17 for $\Delta X \neq 0$. The parameters of the MPTLCDs are $\Delta \gamma = 1$, $L = 16 m$, $H = 5 m$, $\alpha = 0.8$. The maximum air pressure among all the PTLCD units is 156.8 kPa. Figure 8-16 shows that the effectiveness of MPTLCD with $\Delta X=0$ is affected by head loss coefficient and the head loss coefficient for achieving maximum reduction in lateral displacement response is larger than that for achieving the maximum reduction in torsional displacement response. The optimal head loss coefficients for achieving maximum reduction in lateral and torsional displacement at point A are almost the same as those at point B. They are found to be 30 for lateral vibration and 10 for torsional vibration. The reduction of the lateral displacement is less sensitive to the head loss coefficient when it is larger than a value of thirty. It should be noted that the zero bandwidth of MPTLCD has all of its PTLCD units tuned to its central frequency. To improve the robustness of MPTLCD against the offset in frequency tuning, each MPTLCD has five PTLCD units tuned to its central frequency with a nonzero frequency bandwidth. The selection of frequency bandwidth together with head loss coefficient is a key step in designing the MPTLCD. The effect of head loss coefficient on the performance of MPTLCD with a series of nonzero bandwidth is shown in Figure 8-17. If the bandwidth becomes larger, the optimal head loss coefficient of MPTLCD is increased for achieving maximum reduction of lateral displacement but it is decreased for achieving maximum reduction of torsional displacement. This is because the lateral displacement response is composed of five frequency components and the torsional displacement response is dominated by one single frequency component. Increasing the bandwidth of MPTLCD tuned to lateral frequency would lead to the overlapping of frequency coverings of the MPTLCDs. Hence, the side PTLCD units of MPTLCD could also be beneficial to the suppression of the two nearby frequency components of its central frequency. More vibration energy can be dissipated by the MPTLCDs if it has larger head loss coefficient. As for MPTLCD tuned to torsional frequency of the bridge, larger bandwidth would lead to the frequency of the side PTLCD farther from the central frequency. The side PTLCD with smaller head loss coefficient can have larger liquid motion to dissipate more vibration energy. The optimal head loss coefficient of MPTLCD for reducing lateral displacement
and torsional displacement at point A are approximately 12 and 4 with frequency bandwidth equal to 0.17 and 0.09 respectively. For the performance of MPTLCD with nonzero bandwidth at point B, similar conclusion can be drawn for the effect of head loss coefficient under different frequency bandwidth. The optimal head loss coefficient of MPTLCD for reducing lateral displacement and torsional displacement at point B are approximately 5 and 4 with frequency bandwidth equal to 0.19 and 0.06 respectively.

8.7.3 Effect of mean wind speed

The aeroelastic effect which is directly related to wind speeds is one of the major concerns in wind-induced vibration of long span bridges. The aeroelastic effect would alter the stiffness and the damping of the bridge which depends on the mean wind speed. It is thus important to investigate the performance of the MPTLCD in reducing the buffeting response under different mean wind speeds. To have a reasonable assessment of the performance of MPTLCD, head loss coefficient is taken as a variable to find its optimal value for achieving maximum reduction of standard deviation displacement response ratio at points A and B. The maximum head loss coefficient is limited to 200. The other parameters of the MPTLCD are $L=16m$, $H=5m$, $\alpha=0.8$, and $\Delta X=0$. Figure 8-18 shows the performance of the MPTLCD under different mean wind speeds and the values inside the parenthesis in the figure represent the corresponding optimal head loss coefficient. It shows that both the standard deviation displacement and acceleration response are reduced effectively by MPTLCD. At a mean wind speed of 20m/s, the lateral acceleration reduction reaches the level of 11% and torsional acceleration reduction reaches the level of 20% by MPTLCD with zero bandwidth. However, more reduction can be achieved for both displacement and acceleration response at higher mean wind speed and the corresponding optimal head loss coefficient is decreased with the increasing mean wind speed.

The performance of MPTLCD is further examined by studying the deck displacement and acceleration at points A and B. The results for the mean wind speeds at 20m/s and 50m/s are tabulated in Tables 8-9 to 8-12. It is seen from Tables 8-9 and 8-10 that both the standard deviation displacement and acceleration responses in either lateral or torsional directions are reduced by the MPTLCD effectively. The standard
deviation displacement reduction in lateral direction reaches the level of 25% at a mean wind speed of 20m/s and the level of 29% at a mean wind speed of 50m/s. As for the reduction of standard deviation torsional displacement, it can reach the level of 21% at a mean wind speed of 20m/s and the level of 25% at a mean wind speed of 50m/s. Tables 8-11 and 8-12 show that the peak lateral displacement response reduction reaches the level of 23% and the peak lateral acceleration response reduction reaches the level of 12%. Tables 8-11 and 8-12 show that the peak torsional displacement reduction reaches the level of 20% and the peak torsional acceleration reduction reaches the level of 16%

The standard deviations of lateral, vertical and torsional displacement responses of the bridge deck along the bridge axis at the mean wind speed of 50m/s are plotted in Figure 8-19. It shows that the maximum standard deviation of the displacement response of the bridge occurs near the midpoints of the two main spans and they are reduced significantly by the MPTLCD. However, the displacement responses of the deck near the tower are hardly reduced by the MPTLCD.

<table>
<thead>
<tr>
<th>Location</th>
<th>Lateral (m)</th>
<th>Vertical (m)</th>
<th>Torsional (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>w/o control</td>
<td>0.00282</td>
<td>0.00318</td>
<td>-0.08058</td>
</tr>
<tr>
<td>(U_m=20m/s)</td>
<td>(-25.5%)</td>
<td>(-26.7%)</td>
<td>(0.29%)</td>
</tr>
<tr>
<td>With control</td>
<td>0.00210</td>
<td>0.00233</td>
<td>0.08081</td>
</tr>
<tr>
<td>(U_m=20m/s)</td>
<td>(-25.5%)</td>
<td>(-26.7%)</td>
<td>(0.29%)</td>
</tr>
<tr>
<td>w/o control</td>
<td>0.02865</td>
<td>0.03225</td>
<td>0.62403</td>
</tr>
<tr>
<td>(U_m=50m/s)</td>
<td>(-28.9%)</td>
<td>(-29.4%)</td>
<td>(0.22%)</td>
</tr>
<tr>
<td>With control</td>
<td>0.02036</td>
<td>0.02278</td>
<td>0.62539</td>
</tr>
<tr>
<td>(U_m=50m/s)</td>
<td>(-28.9%)</td>
<td>(-29.4%)</td>
<td>(0.22%)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Location</th>
<th>Lateral (m/s²)</th>
<th>Vertical (m/s²)</th>
<th>Torsional (rad/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>w/o control</td>
<td>0.01367</td>
<td>0.01340</td>
<td>0.15301</td>
</tr>
<tr>
<td>(U_m=20m/s)</td>
<td>(-22.1%)</td>
<td>(-19.3%)</td>
<td>(-3.24%)</td>
</tr>
<tr>
<td>With control</td>
<td>0.01065</td>
<td>0.01081</td>
<td>0.14805</td>
</tr>
<tr>
<td>(U_m=20m/s)</td>
<td>(-22.1%)</td>
<td>(-19.3%)</td>
<td>(-3.24%)</td>
</tr>
<tr>
<td>w/o control</td>
<td>0.13754</td>
<td>0.13407</td>
<td>1.38818</td>
</tr>
<tr>
<td>(U_m=50m/s)</td>
<td>(-23.1%)</td>
<td>(-19.8%)</td>
<td>(-4.1%)</td>
</tr>
<tr>
<td>With control</td>
<td>0.10577</td>
<td>0.10750</td>
<td>1.33098</td>
</tr>
<tr>
<td>(U_m=50m/s)</td>
<td>(-23.1%)</td>
<td>(-19.8%)</td>
<td>(-4.1%)</td>
</tr>
</tbody>
</table>

Table 8-10 Standard Deviation of Deck Acceleration with and without Control
Table 8-11 Peak Deck Displacement with and without Control

<table>
<thead>
<tr>
<th>Location</th>
<th>Lateral (m)</th>
<th>Vertical (m)</th>
<th>Torsional (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>w/o control</td>
<td>0.01094</td>
<td>0.01091</td>
<td>0.29549</td>
</tr>
<tr>
<td>(U_m=20m/s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With control</td>
<td>0.00788</td>
<td>-0.00834</td>
<td>0.29381</td>
</tr>
<tr>
<td>(U_m=20m/s)</td>
<td>(-27.9%)</td>
<td>(-23.5%)</td>
<td>(-0.57%)</td>
</tr>
<tr>
<td>w/o control</td>
<td>0.12079</td>
<td>0.11110</td>
<td>2.39020</td>
</tr>
<tr>
<td>(U_m=50m/s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With control</td>
<td>0.07748</td>
<td>0.07879</td>
<td>2.37379</td>
</tr>
<tr>
<td>(U_m=50m/s)</td>
<td>(-35.9%)</td>
<td>(-29.1%)</td>
<td>(-0.7%)</td>
</tr>
</tbody>
</table>

Table 8-12 Peak Deck Acceleration with and without Control

<table>
<thead>
<tr>
<th>Location</th>
<th>Lateral (m/s²)</th>
<th>Vertical (m/s²)</th>
<th>Torsional (rad/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>w/o control</td>
<td>0.05047</td>
<td>0.04980</td>
<td>0.71893</td>
</tr>
<tr>
<td>(U_m=20m/s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With control</td>
<td>0.04148</td>
<td>0.04386</td>
<td>0.68882</td>
</tr>
<tr>
<td>(U_m=20m/s)</td>
<td>(-17.8%)</td>
<td>(-11.9%)</td>
<td>(-4.19%)</td>
</tr>
<tr>
<td>w/o control</td>
<td>0.53980</td>
<td>0.49019</td>
<td>6.43277</td>
</tr>
<tr>
<td>(U_m=50m/s)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>With control</td>
<td>0.41774</td>
<td>0.41156</td>
<td>6.14749</td>
</tr>
<tr>
<td>(U_m=50m/s)</td>
<td>(-22.6%)</td>
<td>(-16.0%)</td>
<td>(-4.4%)</td>
</tr>
</tbody>
</table>

8.7.4 Effect of frequency tuning ratio

PTLCD is often designed to have its natural frequency equal to the natural frequency of the structure. However, due to the error in the estimation of natural frequency of the structure as well as the mistuning of the pressure inside air chambers of a MPTLCD, off-tuning in damper frequency may occur in real application. The effects of frequency tuning ratio on the performance of MPTLCD are thus investigated. The parameters of the MPTLCD are L=16m, H=5m, and α=0.8. There are four different combinations of the head loss coefficient and frequency bandwidth for the sensitivity study of the MPTLCD and they are listed in Table 8-13. The case M1 involves the parameters of an optimized MPTLCD with ΔX=0 and the case M2 contains the parameters of an optimized MPTLCD with ΔX≠0. The effect of frequency tuning ratio on the reduction of lateral displacement and torsional displacement is shown in Figures 8-20 and 8-21 respectively. It shows that the sensitivity of the optimized MPTLCD with
ΔX≠0 in reducing lateral displacement is better than that with an optimized MPTLCD with ΔX=0 in the frequency tuning range from 0.9 to 1.02 but the reduction achieved by MPTLCD with ΔX≠0 in this range is smaller. As for the reduction of torsional displacement, the sensitivity of an optimized MPTLCD with ΔX≠0 is almost the same as that of an optimized MPTLCD with ΔX=0. The figures also show that the sensitivity of the optimized MPTLCD to the frequency tuning ratio can be further improved by increasing the bandwidth but at the cost of smaller displacement reduction.

Table 8-13 Head Loss Coefficient and Frequency Bandwidth of MPTLCD in Three Case Studies

<table>
<thead>
<tr>
<th>Location of MPTLCD</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>δL ΔX_L δ_T ΔX_T</td>
<td>δL ΔX_L δ_T ΔX_T</td>
</tr>
<tr>
<td>M1</td>
<td>30 0 10 0</td>
<td>30 0 10 0</td>
</tr>
<tr>
<td>M2</td>
<td>12 0.170 4 0.09 5</td>
<td>0.190 4 0.06</td>
</tr>
<tr>
<td>M3</td>
<td>12 0.187 4 0.16 5</td>
<td>0.209 4 0.16</td>
</tr>
</tbody>
</table>

8.8 SUMMARY

The application of MPTLCD to reduce wind-induced coupled lateral and torsional vibration of a real long span cable-stayed bridge at its completion stage was investigated in Chapter 8. The flexibility of MPTLCD in frequency tuning offers wider choice of container configurations which make it easier to be installed in the real long span bridge at its completion stage. A finite element model of MPTLCD was developed in this chapter and incorporated into the finite element model of the long span bridge for predicting the buffeting response in the time domain. The detailed buffeting analysis of the real long span cable-stayed bridge with MPTLCDs in the time domain was presented in this chapter. Extensive parametric studies on MPTLCDs for reducing wind-induced coupled lateral and torsional vibration of the real long span cable-stayed bridge were carried out using the established finite element model of the MPTLCD-bridge system. The results demonstrated that MPTLCD can effectively reduce the lateral and torsional displacement of the long span cable-stayed bridge if the parameters are selected properly. There exists an optimal water mass distribution between the MPTLCD tuned to lateral frequency of the bridge and the MPTLCD tuned to torsional frequency of the bridge. The performance of MPTLCD in reducing lateral and torsional...
displacement responses depends on head loss coefficient. There exists an optimal head loss coefficient of MPTLCD for a given frequency bandwidth. The optimal head loss efficient of MPTLCD with zero bandwidth for achieving maximum reduction of torsional displacement is smaller than that for achieving maximum reduction of lateral displacement. The effectiveness of an optimized MPTLCD with zero bandwidth is increased with the increasing mean wind speed and the corresponding head loss coefficient is decreased. Further investigations demonstrated that the lateral and torsional displacement responses in the vicinity of the middle span of the bridge deck are reduced significantly by the MPTLCD and the displacement responses of the bridge deck near the tower are hardly reduced by the MPTLCD. The frequency turning sensitivity of the optimized MPTLCD in reducing displacement response can be improved by increasing the frequency bandwidth but at the cost of less displacement reduction.

Figure 8-1 Schematic Diagram of Multiple Pressurized Tuned Liquid Column Dampers

Figure 8-2 Connections between Bridge Deck and Liquid Damper
Figure 8-3 Aerodynamic Forces of Bridge Deck
Figure 8-4 Flowchart of Main Structure of Program DSAPBDW
Start

Input the data control file name

Select the type of dynamic analysis (Idtype) from the data control file

Idtype=2: MPTLCD-bridge system under turbulent wind

Read the following 6 data files required in this analysis from the data control file

Read the dynamic characteristic matrices $[M_b]$, $[C_b]$, and $[K_b]$ of the bridge, the coordinates of all nodes of the bridge, the degrees of freedom of all the nodes.

Read the number of liquid column damper and the detailed information of each of the damper, the node numbers of the bridge deck on which the MPTLCD to be installed from data file 2.

Determine the dynamic characteristic matrices $[M_{bd}]$, $[M_d]$, $[K_{bd}]$, $[K_d]$, $[K_d]$ of each damper. These matrices remain unchanged during the computation.

Read the mean wind velocity, the aerodynamic coefficients $C_D$, $C_L$, $C_M$, and the frequency independent coefficients used to determine the self-excited forces in the time domain. Also read the position of fluctuating wind velocity from data file 3.

Determine the total degrees of freedom of the liquid column dampers and bridge system

Assemble the matrices $[M_b]$, $[C_b]$, $[K_b]$ of the bridge along with the matrices $[M_{bd}]$, $[M_d]$, $[K_{bd}]$, $[K_d]$ of each damper and put them into the proper position of dynamic characteristic matrices of the MPTLCD-bridge system.
Read the initial displacement and velocity of all the independent degrees of freedom of the MPTLCD-bridge system from data file 4.

Read the time interval (Δt) and the time step (Nstep) from the data control file.

Istep = 0

Time step loop: Istep = Istep + 1

Load the time independent dynamic characteristic matrices of the MPTLCD-bridge system such as [M_b], [C_b], [K_b], [M_bb1], [M_bb2], [M_bb3], [M_d], [K_bb1], [K_bb2], [K_bb3], [K_d].

Compute the time dependent dynamic characteristic matrices [C_d] and put it into the proper position of dynamic characteristic matrices of the MPTLCD-bridge system.

Read the values of turbulent wind velocity in the horizontal and vertical directions along with the points of the bridge deck at given time t from data files 5 and 6.

Compute the buffeting force vector {P_{buf}} and put it into the proper position of external force vectors of the MPTLCD-bridge system.

Compute the self-excited force vector {P_{se}} based on the dynamic responses of the bridge at the time t-Δt and put it into the proper position of external force vectors of the liquid column dampers and bridge system.

Calculate dynamic responses of the liquid column dampers and bridge system using the Wilson-0 method.
Figure 8-5 Flowchart of Executing Structure of Main Program DSAPBDW
(a) $10.0 \text{ m/s}$

(b) $20.0 \text{ m/s}$

(c) $30.0 \text{ m/s}$
Figure 8-7 Time Histories of Wind Turbulent Velocity in Alongwind Direction at Point A
(a) 10.0 m/s

(b) 20.0 m/s

(c) 30.0 m/s
Figure 8-8 Time Histories of Wind Turbulent Velocity in Vertical Direction at Point A

(d) 40.0 m/s

(e) 50.0 m/s
Figure 8-9 Flutter Derivatives for the Concerned Bridge Deck Section (Tan, 1999)
Figure 8-10 Standard Deviation Displacement Response of the Bridge Deck under Different Mean Wind Speeds
Figure 8-11 Time histories of displacements of the bridge at point A subjected to turbulent wind ($U_m=50.0 \text{m/s}$)
Figure 8-12 Time histories of displacements of the bridge at point B subjected to turbulent wind ($U_m=50.0 \text{m/s}$)
(a) PSD function of lateral displacement

(b) PSD function of vertical displacement

(c) PSD function of torsional displacement

Figure 8-13 PSD functions of displacements of the bridge at point A subjected to turbulent wind ($U_m=50.0\text{m/s}$)
(a) PSD function of lateral displacement

(b) PSD function of vertical displacement

(c) PSD function of torsional displacement

Figure 8-14 PSD Functions of Displacements of the Bridge at Point B Subjected to Turbulent Wind ($U_m=50.0\text{m/s}$)
Figure 8-15 Effect of Water Mass Ratio on the Performance of MPTLCD
Figure 8-16 Effect of Head Loss Coefficient on the Performance of MPTLCD with Zero Bandwidth
(a) Lateral displacement

(b) Torsional displacement

Figure 8-17 Effect of Head Loss Coefficient on the Performance of MPTLCD with Nonzero Frequency Bandwidth at Point A
(a) Standard deviation of displacement

(b) Standard deviation of acceleration

Figure 8-18 Effect of Mean Wind Speed
Figure 8-19 Comparison of Standard Deviations of Bridge Deck Displacement
Figure 8-20 Effect of Frequency Tuning Ratio on Lateral Displacement
Figure 8-21 Effect of Frequency Tuning Ratio on Torsional Displacement
CHAPTER 9
APPLICATION OF SEMI-ACTIVE TUNED LIQUID COLUMN DAMPERS TO A LONG SPAN BRIDGE DURING CONSTRUCTION

9.1 INTRODUCTION

It is efficient and economic to construct long span cable-stayed bridges by using one-sided free-cantilevering approach and/or double free-cantilevering approach. Although long span cable-stayed bridges are more stable in its final condition, they are often vulnerable during construction stages. If it is located at a wind-prone region, serious attention is required for controlling the buffeting response of the incomplete bridge, as the bending stiffness of the bridge deck during the construction is much smaller than that of the complete bridge in lateral, vertical and torsional directions. Besides, the buffeting forces on the incomplete bridge also differ from those on the complete bridge because of their different configurations. It thus becomes important to explore the way of minimizing the probable disturbances to erection work which arises from the vibration of deck at low wind speed and ensuring the safety of the bridge at high wind speed.

There are various possible ways to reduce wind-induced vibration on long span cable-stayed bridges during construction. To install the temporary cable on the bridge girders is the most widely used method to strengthen the deck during construction. However, it has been reported in some circumstances that the use of temporary tie-down may probably be subjected to a high construction cost of anchor block and a risk of ship collision during storm (Virlogeux 1992). Therefore, utilizing mechanical control devices to supplement damping capacity of structure can be an alternative solution for reducing wind-induced vibration. To ease erection work and avoid interaction with ship navigation, tuned mass damper (TMD) was installed on the Normandy Bridge for suppressing everyday vibrations of the two long cantilevering bridge decks. Conti et al. (1996) demonstrated that the lateral buffeting response of the deck could be reduced by
at least 35% after the installation of the TMD on the Normandy Bridge. Takeda et al. (1998) then studied an active mass damper (AMD) for controlling the vertical buffeting response of concrete cable-stayed bridges during cantilever construction. They showed that the structural damping of the bridge was enhanced significantly by the implementation of AMD and thereby reducing the vibration of bridge. In consideration of the features of long span cable-stayed bridge during construction, semi-active tuned liquid column damper (SATLCD) with frequency adaptability has been developed in chapter 6 for the suppression of lateral and torsional vibration of the bridge deck. The results showed that the SATLCD can effectively reduce either lateral or torsional vibration of a structure and provide a great flexibility of selecting a liquid length while keeping a proper frequency tuning through the change of air pressure acting on liquid. However, the investigation presented in Chapter 6 is limited to a simple structure and aimed at achieving a basic understanding of characteristic and performance of the SATLCD for reducing either lateral or torsional vibration.

This chapter investigates the performance of the SATLCD in reducing wind-induced coupled lateral and torsional response of a real long span cable-stayed bridge under different stages of cantilever construction based on the understanding accomplished in Chapter 6. Semi-active tuned liquid column damper (SATLCD) with two control targets, the control of its natural frequency and the control of liquid motion within the tolerable limit, is developed. The desired control force acting on the liquid column is provided by regulating the pressure difference inside the air chambers at the two ends of container. Finite element model of SATLCD is developed for the sake of carrying out buffeting analysis of the SATLCD-bridge system. Five different construction stages of the real long span cable-stayed bridge during construction are selected for the study of the SATLCD performance and adaptability. The developed finite element model of SATLCD is incorporated into the finite element model of the bridge for predicting the buffeting response of the SATLCD-bridge system, and computer programs are correspondingly developed. The performance of SATLCD for the suppression of lateral and torsional vibration of the real long span cable-stayed bridge during construction is investigated extensively through parametric studies. The key parameters investigated include mass ratio, head loss coefficient, mean wind speed and cantilever length.
9.2 EQUATIONS OF MOTION OF SATLCD-BRIDGE SYSTEM UNDER TURBULENT WIND

9.2.1 Modeling of cable-supported bridge

The modeling of a long span cable-supported bridge has been introduced in Chapter 8 for its completion stage. The modeling principle for the bridge during construction is almost the same as for the completion stage. A long span cable-supported bridge under construction is represented by a three-dimensional finite element model using different types of finite elements such as beam element, cable element, plate element and solid element. The inclined stay cables are modeled as cable element whose elastic modulus is modified by the Ernst's formula in order to include the sag effect of cable due to its self-weight. The geometric nonlinear stiffness of the cable is also taken into consideration. Three-dimensional Timoshenko beam elements are used to model the bridge tower and deck. The mass matrix, the stiffness matrix and the force vector of the bridge are obtained by the use of traditional finite element method (Bathe, 1996). The damping matrix of bridge, which is assumed to be the Rayleigh damping, can be expressed as a combination of the mass and the stiffness matrices.

\[
[C_b] = \alpha_b [M_b] + \beta_b [K_b]
\]  

(9-1)

where \([M_b]\), \([C_b]\) and \([K_b]\) are the mass, damping and stiffness matrices of the bridge, respectively; \(\alpha_b\) and \(\beta_b\) are the Rayleigh damping factors, which can be evaluated if the first two modal damping ratios and natural frequencies are known. The equation of motion of bridge under turbulent wind can be expressed as:

\[
[M_b] \ddot{\mathbf{v}}_b + [C_b] \dot{\mathbf{v}}_b + [K_b] \mathbf{v}_b = \{P_{\text{buff}}\} + \{P_{\text{se}}\}
\]  

(9-2)

where the vectors \(\{\mathbf{v}_b\}, \{\dot{\mathbf{v}}_b\}\) and \(\{\ddot{\mathbf{v}}_b\}\) are the nodal dynamic displacement, velocity, and acceleration of the bridge, respectively. The buffeting wind force \(\{P_{\text{buff}}\}\) and the self-excited wind force \(\{P_{\text{se}}\}\) are given in Chapter 8.

9.2.2 Modeling of liquid damper
SATLCD with frequency adaptability capacity has been studied in Chapter 6 for the suppression of either lateral or torsional vibration of a SDOF structure. It is a U-shaped container with uniform cross-sectional area. Liquid is filled into its container and the two chambers are filled with compressed air of static pressure $P_o$. The key concept of the SATLCD studied herein is to act the control force on the liquid column by controlling a net external pressure between the two air chambers (see Figure 9-1). The net pressure is regulated by the displacement and the velocity of the liquid column in a prescribed way. As a result, the natural frequency and the damping of liquid motion inside the TLCD can be changed accordingly to the ever changing environment. The net pressure between the two air chambers, sensed by the pressure transducer, is to be forced to follow or track the desired pressure determined by a computer in accordance with the given control algorithms. Any deviation from the desired pressure is fed back to the computer to take corrective action to adjust the control valve. Thus, the control system is continually monitoring and correcting pressure deviation to maintain the desired pressure acting on the liquid column.

To consider the interaction between semi-active liquid column dampers and long span cable-supported bridge under wind excitation, it is expedient to derive the finite element model of the liquid dampers. Let us consider $N$ SATLCD units installed below the torsional centre of the bridge deck and at the locations where the vibration amplitudes of the bridge in the lateral and torsional directions are the largest. The SATLCD units are connected to the transverse beams of the bridge deck by roller supports and simply supports as shown in Figure 9-2. Two additional nodes namely node 1 and node 2 are generated at the position where a damper is connected to the bridge. These two additional nodes reflect the motion of the damper interacted with the motion of the bridge. From a view point of practical use, the liquid column length and the distance between the two vertical columns of the $k$th SATLCD unit are assumed to take the same value as all other SATLCD units. The axial deformation of the transverse beam between the two supports is assumed to be negligible and hence the lateral displacement of the liquid dampers is taken as $x$ and the torsional displacement of the liquid dampers is determined by

$$\theta = \frac{y_2 - y_1}{B} \quad (9-3)$$
where \( x \) is the lateral displacement of the node 2; \( y_1 \) and \( y_2 \) are the vertical displacements of the node 1 and 2 respectively; \( B \) is the distance between the two additional nodes. In Appendix A, the Lagrangian of \( N \) SATLCD units has been shown to be

\[
L_d = \sum_{k=1}^{n} \left[ \frac{1}{2} m_k \tilde{W}_k^2 + \frac{1}{2} m_k \hat{x}^2 + \frac{1}{2} I_k \hat{\theta}^2 + m_k \alpha_k W_k \hat{x} + G_k W_k \hat{\theta} + m_k \tilde{H}_k \hat{x} \hat{\theta} \right. \\
+ m_k g \tilde{H}_k \cos \theta - m_k g \alpha_k W_k \sin \theta - \frac{m_k g}{L_k} W_k^2 \cos \theta - \frac{S_k}{2} W_k^2 \left. \right] 
\]

(9-4)

where \( m_k \) is the liquid mass inside the \( k \)th SATLCD; \( I_k \) is the mass moment inertia of the liquid mass inside the \( k \)th SATLCD; \( \alpha_k \) is the liquid length ratio; \( L_k \) is the length of the \( k \)th SATLCD; \( \tilde{H}_k \) is the distance from the mass center of liquid inside the \( k \)th SATLCD to the torsional center of the bridge deck; \( G_k \) is the liquid mass moment in the \( k \)th SATLCD; \( g \) is the acceleration due to gravity; \( S_k \) is the constant displacement feedback gain for the SATLCD; \( W_k \) and \( \dot{W}_k \) are the liquid displacement and velocity inside the \( k \)th PTLCD. The last term in Equation (9-4) is the potential energy of liquid due to the control force \( u_k \) in the \( k \)th SATLCD. Equation (9-4) is subjected to the condition that the liquid should be fully retained in the horizontal part of the SATLCD and thus the following equation should always be satisfied.

\[
W_k \leq \frac{L_k - B_k - d_k}{2} = W_s
\]

(9-5)

where \( d_k \) is the thickness of the liquid damper; \( W_s \) is the tolerable liquid displacement of the SATLCD. The interaction between the SATLCD and the bridge in vertical direction is considered by modeling the mass of water as rest mass in the finite element model of the bridge. The entries of mass matrix \( m_{ij} \) and stiffness matrix \( k_{ij} \) of the damper element can be determined by

\[
m_{ij} = \frac{\partial}{\partial \tilde{q}_j} \left( \frac{\partial L_d}{\partial \tilde{q}_i} \right) \quad k_{ij} = -\frac{\partial}{\partial \tilde{q}_j} \left( \frac{\partial L_d}{\partial \tilde{q}_i} \right)
\]

(9-6)

\[
[q]^T = [x \ y_1 \ y_2 \ W_1 \ \ldots \ \ldots \ \ W_N]
\]

(9-7)

After some manipulations, the mass matrix of the damper element, \([M]\) and the stiffness matrix of the damper element, \([K]\) can be written as:

\[
[M] = \begin{bmatrix}
M_{11} & M_{12} \\
M_{12}^T & M_{22}
\end{bmatrix} \quad \text{and} \quad [K] = \begin{bmatrix}
K_{11} & K_{12} \\
K_{12}^T & K_{22}
\end{bmatrix}
\]

(9-8)

where
\[ M_{11} = \frac{1}{B^2} \begin{bmatrix} m_1 B^2 & -mB & -mB \\ -mB & I_d & -I_d \\ -mB & -I_d & I_d \end{bmatrix} \]  
(9-9)

\[ M_{12} = \begin{bmatrix} m_1 \alpha_1 & m_2 \alpha_2 & \ldots & m_N \alpha_N \\ -G_1 & -G_2 & \ldots & G_N \\ G_1 & G_2 & \ldots & G_N \\ G_1 & G_2 & \ldots & G_N \end{bmatrix} \]  
(9-10)

\[ M_{22} = \text{diag}(m_1, m_2, \ldots, m_N) \]  
(9-11)

\[ K_{11} = \frac{mg}{B^2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \]  
(9-12)

\[ K_{12} = \frac{g}{B} \begin{bmatrix} 0 & 0 & \ldots & 0 \\ -m_1 \alpha_1 & -m_2 \alpha_2 & \ldots & -m_N \alpha_N \\ m_1 \alpha_1 & m_2 \alpha_2 & \ldots & m_N \alpha_N \end{bmatrix} \]  
(9-13)

\[ K_{22} = \text{diag}(m_1 \omega_1^2, m_2 \omega_2^2, \ldots, m_N \omega_N^2) \]  
(9-14)

where \( \omega_k \) is the circular natural frequency of the \( k \)th SATLCD; \( m_0 \) is the total liquid mass of all SATLCDs; \( I_d \) is the total liquid mass moment of inertia of all the SATLCDs;

\[ m = \sum_{k=1}^{N} m_k \bar{H}_k ; \text{ and } G_k = m_k \alpha_k \left( H_k + \frac{L_k - B_k}{2} \right) \].

The circular natural frequency of liquid motion in the \( k \)th SATLCD, \( \omega_k \), is determined by

\[ m_k \omega_k^2 = \frac{2m_k g}{L_k} + S_k \]  
(9-15)

For a target frequency of liquid damper, it is easily seen from Equation (9-16) that the liquid column length of the SATLCD is given by

\[ L_k = \frac{2g}{\omega_k^2} + \frac{S_k}{\rho \omega_k A_k \omega_k^2} \]  
(9-16)

where \( \rho \omega_k \) is the density of liquid inside SATLCD; \( A_k \) is the cross-sectional area of the \( k \)th SATLCD. Clearly, the liquid column length can be increased or decreased by adjusting the constant displacement feedback gain while keeping its frequency unchanged. The SATLCD is therefore more flexible than the traditional TLC in which
S is equal to zero and there is no way in changing the liquid column length $L$. Once the frequency and the length of the liquid column are decided, the required constant displacement feedback gain from the SATLCD can be determined by

$$S_k = m_k \left[ \omega_n^2 - \frac{2g}{L_k} \right]$$  \hspace{1cm} (9-17)

The control force for adjusting the natural frequency of the $k$th SATLCD is given as

$$u_{nk} = S_k W_k$$  \hspace{1cm} (9-18)

The malfunction of SATLCD may be resulted from its excessive liquid motion under high mean wind speed. An on-off control algorithm is therefore employed to govern the liquid motion inside the SATLCD to be within the tolerable limit defined by Equation (9-5). This control force is regulated by varying the pressure in accordance with the on-off control strategy as follows:

$$u_{2k} = \begin{cases} \frac{1}{2} \rho_w A_k \delta_m |W_k| \dot{W}_k & \text{if } (W_k > K_p \cdot W_s) \text{ and } (\dot{W}_k \cdot W_k > 0) \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (9-19)

where $K_p$ is a value between 0 and 1; $\delta_m$ is the head loss coefficient for providing sufficient damping to the SATLCD. The choice of $\delta_m$ is affected by the value of $K_p$. The control force is provided to the liquid when the water displacement is increased to a value beyond $K_p W_k$ and the velocity is in the direction of the water displacement. The control force acting on the $k$th SATLCD is the sum of the control force based on the feedback of liquid displacement $u_{nk}$ and the control force based on the feedback of liquid velocity $u_{2k}$, which is given as

$$u_k = \begin{cases} S_k W_k + \frac{1}{2} \rho_w A_k \delta_m |W_k| \dot{W}_k & \text{if } (W_k > K_p \cdot W_s) \text{ and } (\dot{W}_k \cdot W_k > 0) \\ S_k W_k & \text{otherwise} \end{cases}$$  \hspace{1cm} (9-20)

The desired control force acting on the liquid column can be provided by regulating the air pressure in the right chamber with respect to the air pressure in the left chamber to obtain a net pressure $P_k(t)$ as

$$u_k = P_k(t) \cdot A_k$$  \hspace{1cm} (9-21)

The corresponding net pressure $P_k(t)$ in the $k$th SATLCD becomes

$$P_k(t) = \begin{cases} S_k W_k + \frac{1}{2} \rho_w A_k \delta_m |W_k| \dot{W}_k & \text{if } (W_k > K_p \cdot W_s) \text{ and } (\dot{W}_k \cdot W_k > 0) \\ \frac{S_k W_k}{A_k} & \text{otherwise} \end{cases}$$  \hspace{1cm} (9-22)
Inside the kth SATLCD, the air pressure in the left chamber \( P_L \) and in the right chamber \( P_R \) is determined, respectively, by

\[
P_L = P_0 - \frac{P_k(t)}{2} \quad P_R = P_0 + \frac{P_k(t)}{2}
\]

(9-23)

### 9.2.3 Modeling of SATLCD-Bridge system

Let SATLCDs be installed at the tip of the two long cantilever decks of a long span bridge where the vibration amplitudes in the lateral and torsional directions are the largest. The finite element model of the SATLCD-bridge system can then be constituted using beam elements and damper elements. The equation of motion of a SATLCD-Bridge system is easily obtained by using the conventional finite element method as:

\[
\begin{bmatrix}
M_b + M_{bd} & M_{bd} \\
M_{bd} & M_d
\end{bmatrix}
\begin{bmatrix}
\ddot{v}_b \\
\ddot{v}_w
\end{bmatrix}
+ \begin{bmatrix}
C_b & 0 \\
0 & C_d
\end{bmatrix}
\begin{bmatrix}
\dot{v}_b \\
\dot{v}_w
\end{bmatrix}
+ \begin{bmatrix}
K_b + K_{bd} & K_{bd} \\
K_{bd} & K_d
\end{bmatrix}
\begin{bmatrix}
v_b \\
v_w
\end{bmatrix}
= \begin{bmatrix}
P_{\text{buff}} + P_{\text{se}} \\
0
\end{bmatrix}
\]

(9-24)

In Equation (9-24), \([M_b]\), \([C_b]\) and \([K_b]\) are the mass matrix, damping matrix, and stiffness matrix of the bridge alone, respectively. The matrix \([v_w]_{nT} \times 1\) represents the liquid displacements of the \(n_T\) SATLCD units. The matrix \([M_{bd}]\) is associated with the inertial forces due to the global motion of liquid inside the damper and it is obtained by assembling the liquid damper matrix \([M_{t1}]\). The matrix \([M_d]\) corresponds to the inertial forces of the liquid inside the damper and it is obtained by assembling the liquid damper matrix \([M_{t2}]\). The matrix \([M_{bd}]\) indicates that the structural motion and liquid motion are coupled by the inertia effect of liquid and it is obtained by assembling the liquid damper matrix \([M_{t2}]\). The matrices \([K_{bd}]\) and \([K_{bd}]\) are related to the restoring forces resulting from the gravitational effect of liquid and they are obtained by assembling the liquid damper matrix \([K_{t1}]\) and \([K_{t2}]\). The matrix \([C_d]\) corresponds to the liquid restoring forces due to the liquid elevation difference between the two vertical columns and it is obtained by assembling the liquid damper matrix \([K_{t2}]\). The matrix \([C_d]\) represents the nonlinear damping forces resulting from the damper orifice and it is given by

\[
[C_d] = \text{diag}(\frac{1}{2}\rho_w A_1 \delta_1 |\hat{W}_1|, \frac{1}{2}\rho_w A_2 \delta_2 |\hat{W}_2|, \ldots, \frac{1}{2}\rho_w A_{nT} \delta_{nT} |\hat{W}_{nT}|)
\]

(9-25)

where \(\delta_k\) is the head loss coefficient of the kth SATLCD unit.
9.3 COMPUTER PROGRAM

The computer program DSAPBDW is developed using the Fortran language for the prediction of dynamic response of the coupled SATLCD and cable-stayed bridge system subjected to turbulent wind. The flowchart of the main structure of this computer program is displayed in Figure 8-3. To execute this computer program, one project control data file is required, which contains all the necessary input and output data information. The input data information consists of the general control input data information and 6 specific input data files. The 6 specific input data files include: (1) the data file for bridge information such as the mass matrix, damping matrix, and stiffness matrix of the bridge determined by the computer program DSAPB; (2) the data file for SATLCD information; (3) the data file for wind information such as the frequency independent coefficients used to determine the buffeting forces and self-excited forces in the time domain; (4) the data file for the initial displacements and velocities of all the independent degrees of freedom of both the cable-stayed bridge and SATLCDs; (5) the data file for the time histories of wind turbulent velocity in the horizontal direction; (6) the data file for the time histories of wind turbulent velocity in the vertical direction. The flowchart of the computer program for executing the dynamic analysis of coupled semi-active liquid column damper and cable-stayed bridge systems under turbulent wind is shown in Figure 9-3.

9.4 DYNAMIC RESPONSE OF BRIDGE DURING CONSTRUCTION STAGE WITHOUT CONTROL

9.4.1 A long span cable-stayed bridge

The same triple tower cable-stayed bridge considered in Chapter 8 is used in this chapter for wind-induced vibration mitigation of the bridge during its construction stage using SATLCD. Five different construction stages of the concerned bridge are selected for this study (See Figures 9-4 to 9-8). Figure 9-4 shows that the bridge under construction stage 1 is divided into three parts and they are erected simultaneously. Each part of the bridge deck is free at its two ends and its transverse restraint is
provided by the tower. The deck to tower connections offer longitudinal and lateral displacement restraints with essentially completely free rotation about all three axes together with free vertical displacement. Figure 9-5 shows that the span length of all incomplete bridge decks becomes longer at stage 2 and the central pylon is stabilized by the longitudinal stabilizing cables. The incomplete bridge deck at two side spans is fixed transversely and vertically at one end. Figures 9-6 to 9-8 depict that the span of the incomplete bridge deck at the central tower is increased gradually from stage 3 to stage 5 and the bridge is almost completed at the construction stage 5. The bridge is represented by a three dimensional dynamic finite element model, which takes into account the geometric nonlinear effect of axial forces on the bending of the bridge deck and the cable tension. The first two modal damping ratios of the bridge are taken as 0.8%.

9.4.2 Dynamic characteristics of bridge

The dynamic characteristics of the bridge are first determined for its natural frequencies and mode shapes. The computed natural frequencies and mode shapes are shown in Appendix B.2 to B.6 for the five construction stages respectively. The generalized mass and mass moment of inertia of the bridge deck are determined by

\[
\begin{align*}
M_{ij}^* &= \int \bar{m}(y)\psi_{ij}^2(y) \, dy; \\
M_{ij}^* &= \int \bar{m}(y)\psi_{ij}^2(y) \, dy; \\
I_{ij}^* &= \int \bar{I}(y)\psi_{ij}^2(y) \, dy. 
\end{align*}
\tag{9-26}
\]

where \( y \) is the coordinate along the bridge longitudinal axis; \( \bar{m}(y) \) and \( \bar{I}(y) \) are the mass and mass moment of inertia of the bridge deck per unit length; \( \psi_{ij}(y) \), \( \psi_{ij}(y) \) and \( \psi_{ij}(y) \) are respectively the \( j \)th normalised lateral, vertical and torsional mode shape of the bridge deck.

As shown in Appendix B.2 to B.6, the largest first mode shape amplitude of the bridge deck during construction occurs at the tip of the cantilever in the lateral, vertical and torsional directions. The first lateral, vertical and torsional frequencies of the bridge at the side span increase significantly when one of its ends is fixed. The natural frequencies of the bridge at the central tower part are summarized in Tables 9-1 to 9-3 from construction stage one to five. The lateral frequency of the bridge at the central tower part does not vary as much as that in vertical and torsional direction but it is much
lower than that in the other two directions. The vertical frequency of the bridge at the central tower part is increased from construction stage 1 to stage 2 and is decreased gradually from construction stage 2 to stage 5. The torsional frequency of the bridge deck is decreased from construction stage 1 to stage 5. It is seen from Appendix B.2 to B.6 that the bridge deck at the central tower part sways in an opposite direction in the first lateral mode of vibration. The geometric stiffness of cables thus provides the lateral stiffness to the bridge deck. As the deck length increases, the cable stiffness increases almost proportionally to the increase in mass, leading to small variation in the first lateral frequency of the bridge at the central tower part. The increase in vertical frequency of the bridge from construction stage 1 to stage 2 can be attributed to the effect of longitudinal stabilizing cables. The first vertical mode shape of the bridge deck at the central tower part involves the bending of the central tower. The longitudinal stabilizing cables indeed strengthen the central tower and hence the stiffness of the tower is increased after the installation of the longitudinal stabilizing cables. As the deck length is increased, the decrease in vertical or torsional frequency of the bridge deck is mainly due to the decrease of the bending stiffness of bridge deck with longer cantilever length.

<p>| Table 9-1 First Lateral Frequency and Generalized Mass of the Double Cantilever Deck at the Central Tower |
|----------------|----------------|</p>
<table>
<thead>
<tr>
<th>Stage</th>
<th>Frequency (Hz)</th>
<th>Generalized mass ( M_1^* )/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.04677</td>
<td>2.48160\times10^6</td>
</tr>
<tr>
<td>2</td>
<td>0.04606</td>
<td>3.12910\times10^6</td>
</tr>
<tr>
<td>3</td>
<td>0.04555</td>
<td>4.11101\times10^6</td>
</tr>
<tr>
<td>4</td>
<td>0.04492</td>
<td>4.24095\times10^6</td>
</tr>
<tr>
<td>5</td>
<td>0.04505</td>
<td>4.68761\times10^6</td>
</tr>
</tbody>
</table>

<p>| Table 9-2 First Vertical Frequency and Generalized Mass of the Double Cantilever Deck at the Central Tower |
|----------------|----------------|</p>
<table>
<thead>
<tr>
<th>Stage</th>
<th>Frequency (Hz)</th>
<th>Generalized mass ( M_2^* )/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.17602</td>
<td>2.00085\times10^6</td>
</tr>
<tr>
<td>2</td>
<td>0.22302</td>
<td>2.90739\times10^6</td>
</tr>
<tr>
<td>3</td>
<td>0.18148</td>
<td>2.40548\times10^6</td>
</tr>
<tr>
<td>4</td>
<td>0.16140</td>
<td>2.65769\times10^6</td>
</tr>
<tr>
<td>5</td>
<td>0.13683</td>
<td>2.30176\times10^6</td>
</tr>
</tbody>
</table>
Table 9-3 First Torsional Frequency and Generalized Mass Moment of the Double Cantilever Deck at the Central Tower

<table>
<thead>
<tr>
<th>Stage</th>
<th>Frequency (Hz)</th>
<th>Generalized mass moment of inertia (I_{y}^*) / kgm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.93165</td>
<td>1.54822×10⁸</td>
</tr>
<tr>
<td>2</td>
<td>0.77281</td>
<td>1.44241×10⁸</td>
</tr>
<tr>
<td>3</td>
<td>0.38114</td>
<td>1.28881×10⁸</td>
</tr>
<tr>
<td>4</td>
<td>0.37469</td>
<td>1.52893×10⁸</td>
</tr>
<tr>
<td>5</td>
<td>0.27757</td>
<td>2.21612×10⁸</td>
</tr>
</tbody>
</table>

9.4.3 Buffeting response of bridge at different construction stages

To have a better understanding of wind-induced vibration of the bridge, the buffeting responses of the concerned bridge at five different construction stages under a mean wind speed of 20m/s are computed, and their corresponding standard deviation displacement responses are displayed in Figure 9-9. The points A and D represent the locations at the tips of the cantilevers of the two side bridge decks. The points B and C represent the locations at the tips of the double cantilever of the bridge deck at the central tower part. For the cantilevers of the two side bridge decks under construction stage 1, serious vibration is observed in the lateral direction. However, they are decreased significantly when the side deck is fixed on the ground at one of its ends. For the double cantilever deck at the central tower part, serious lateral vibration of the double cantilever deck is observed for the five construction stages. For the vertical and torsional displacements of the double cantilever, they increase significantly as the cantilever deck length is increased. This is because the bending stiffness of the double cantilever deck in vertical and torsional directions is decreased with the increasing cantilever length. It is found that the vibration of double cantilever deck is more serious than that of the two side bridge decks for the concerned five construction stages. To control the buffeting response of the bridge deck with such a large variation in natural frequency, the SATLCD investigated in Chapter 6 seems to be a good potential device. Hence, the application of SATLCD in reducing the coupled lateral and torsional vibration of the double cantilever deck at the central tower part will be studied in this chapter with focus on its performance and adaptability during various construction stages of the bridge.
Displayed in Figures 9-10 and 9-11 are the time histories of the deck displacement responses in lateral, vertical and torsional directions at points B and C of the double cantilever deck at construction stage 5 under the mean wind speed of 20m/s. To further understand the vibration nature of the bridge deck, the spectrum analysis is also performed using the given time histories of deck displacements. The resulting PSD functions are shown in Figures 9-12 and 9-13. The PSD functions of lateral and torsional displacement of the bridge deck at points B and C show that both lateral and torsional displacements of the bridge deck are dominated by one single peak at the frequency around 0.045Hz and 0.28Hz respectively. For the vertical displacement, the PSD function at points B and C has two peaks at the frequency around 0.135 and 0.195Hz. The peak frequencies appearing in the PSD functions match quite well with the computed natural frequencies obtained from the dynamic characteristics analysis (see Tables 9-1 to 9-3).

9.5 PERFORMANCE OF SATLCD

As described in the previous section, the lateral and torsional vibration of the concerned long span bridge consists of only one major frequency component. Two SATLCDs with one tuned to the first lateral frequency of the bridge and the other tuned to the torsional frequency of the bridge are installed at the two ends of the double cantilever deck at the central tower part. The mass ratio \( \mu_{lj} \) for the SATLCD tuned to the jth mode of lateral vibration of the bridge is defined as

\[
\mu_{lj} = \frac{m_{lj}}{M_{lj}} \quad (9-27)
\]

The mass ratio \( \mu_{\tau j} \) for the SATLCD tuned to the jth mode of torsional vibration of the bridge is defined as

\[
\mu_{\tau j} = \frac{m_{\tau j}I_j}{I_{\tau j}} \quad (9-28)
\]

where \( m_{lj} \) is the mass of SATLCD tuned to the jth lateral vibration mode of the bridge; \( m_{\tau j} \) is the mass of SATLCD tuned to the jth torsional vibration mode of the bridge; \( I_j \) is the moment of inertia of SATLCD per unit mass tuned to the jth torsional mode of vibration. The total liquid mass of all the SATLCDs \( (m_d) \) is determined by

\[
m_d = \sum_{j=1}^{a_k} m_{lj} + \sum_{j=1}^{a_t} m_{\tau j} = \mu m_s \quad (9-29)
\]
where \( n_L \) is the number of lateral modes of vibration to be controlled; \( n_T \) is the number of torsional modes of vibration to be controlled; \( \mu \) is the mass ratio of the total liquid mass to the mass of the bridge deck \( (m_e) \). The performance of SATLCD is assessed in terms of the response ratio \( R \), which is defined as the ratio of the structural response with control to the structure response without control. The mean wind speed considered in this study is 20m/s unless it is otherwise specified. From a viewpoint of practical use, the geometric configurations of all the SATLCDs are taken to be the same. In this study, the liquid column length is selected to be 21.5m and the thickness of liquid column of 1.1m.

9.5.1 Effect of mass ratio

The effects of the mass ratio \( \mu \) on the performance of SATLCD in reducing lateral and torsional displacement of the double cantilever deck at the central tower part under construction stage 5 are depicted in Figure 9-14. The mass of the double cantilever deck under construction stage 5 is \( 1.34215 \times 10^7 \)kg. The parameters of the SATLCD used herein are \( H=6.5m, \alpha=0.6, \delta_1=5, \delta_2=105 \), and \( \mu_L=\mu_T \). The parameters of SATLCD tuned to the first lateral and torsional frequency of the double cantilever bridge deck are denoted by the subscripts 1 and 2 respectively. The dropping of the subscript in some damper parameters implies that these parameters are the same for the two dampers. It is seen from Figure 9-14 that the lateral displacement at points B and C are reduced significantly with the increase in mass ratio but the reduction of the torsional displacement at points B and C is less sensitive to the mass ratio. There is almost no change in torsional displacement response ratio at point C when the mass ratio is beyond a value of 2.7%. The SATLCD with a mass ratio of 2.7% is thus selected for further parametric studies on the control of buffeting response of the double cantilever deck. For this mass ratio, the decrease in torsional displacement can achieve about 15% and the decrease in the lateral displacement is about 40%. The performance of SATLCD in reducing lateral displacement at points B and C are almost the same and the reduction of torsional displacement at point B is slightly better than that at point C.

9.5.2 Effect of head loss coefficient
The effects of head loss coefficient on the performance of SATLCD in reducing lateral and torsional displacement responses of the double cantilever deck under construction stage 5 are depicted in Figure 9-15. The corresponding pressure inside the air chamber is also plotted in the figure. The parameters of the SATLCD used herein are \( H = 6.5 \text{m}, \alpha = 0.6, \mu = 0.027, \) and \( \mu_L = \mu_T. \) The two SATLCDs at each side of the double cantilever are assumed to take the same value of head loss coefficient. It is seen from the figure that the effectiveness of SATLCD is affected by head loss coefficient and the optimal head-loss coefficients exist for the maximum reduction of lateral and torsional displacement responses. The head loss coefficient for achieving maximum reduction in lateral displacement response is much smaller than that for achieving the maximum reduction in torsional displacement response. The optimal head loss coefficients for achieving maximum reduction in lateral or torsional displacement at point B are almost same as that at point C. It also shows that for torsional displacement reduction, the performance of SATLCD is less sensitive to the head loss coefficient. The performance would not be deteriorated much even the head loss coefficient slightly offsets from the optimal value. Figure 9-15 also indicates that the pressure required for feedback control force is decreased with the increasing head loss coefficient.

9.5.3 Effect of mean wind speed

The application of liquid column dampers for cable-stayed bridge during construction aims at minimizing the probable disturbances to erection work due to the deck vibration at low wind speed and ensuring the safety of the bridge deck under high wind speed. It is thus important to investigate the performance of SATLCD in reducing buffeting response of the bridge deck under different mean wind speeds. To have a reasonable assessment of the performance of SATLCD in reducing buffeting response of the bridge, head loss coefficient is taken as a variable to find its optimal value for achieving maximum reduction of standard deviation displacement response ratio at a given construction stage. The other parameters of the SATLCD used herein are \( H = 6.5 \text{m}, \alpha = 0.6, \mu = 0.027, \) and \( \mu_L = \mu_T. \) To avoid the malfunction of SATLCD due to its excessive liquid motion under high mean wind speed, additional control force is provided to control the liquid displacement within the tolerable limit when the liquid is continuously increased and beyond 75% (i.e. \( K_p = 0.75 \)) of allowable liquid
displacement. With the choice of $K_p$ equal to 0.75, head loss coefficient $\delta_m$ is selected to be 200 in order to provide sufficient damping force to the liquid inside SATLCD. Figure 9-16 shows the displacement and acceleration response ratios of the SATLCD-bridge system under different mean wind speeds. The corresponding pressure inside the air chamber is also plotted in the figure. The values inside the parenthesis in Figure 9-16 represent the corresponding optimum head loss coefficient. It shows that both the standard deviation displacement and acceleration responses are reduced effectively by the SATLCD. The reduction of lateral displacement by SATLCD is almost at the same level for the mean wind speed of 20m/s and 30m/s but it is decreased at the mean wind speed of 40m/s and 50m/s. Since the liquid displacement is controlled within the tolerable limit at high mean wind speed, the energy dissipation which mainly depends on the liquid displacement is thus smaller and results in the smaller reduction of displacement response. For the reduction of torsional displacement response, the performance of SATLCD is slightly better as the mean wind speed is increased. The reduction of lateral displacement at points B and C are almost same while the reduction of torsional displacement at point B is better than that at point C. With the increasing mean wind speed, the optimum head loss coefficient of SATLCD tuned to torsional frequency is decreased but under the influence of the addition damping force, the optimum head loss coefficient of SATLCD tuned to lateral frequency is increased with the increasing mean wind speed. The liquid displacement is controlled within the tolerable limit and therefore the standard deviation of pressure inside the SATLCD tuned to lateral frequency is almost the same at the mean wind speed of 40m/s and 50m/s. Figures 9-17 and 9-18 show the time histories of deck displacement at point B together with the pressure inside the SATLCD at point B under the mean wind speed of 20m/s. Clearly, the pressure inside the SATLCD tuned to the lateral frequency of the bridge is varied at a lower frequency and with a larger amplitude as compared with that tuned to the torsional frequency of the bridge.

The performance of SATLCD is further examined by studying the double deck displacement and acceleration at points B and C. The results for the mean wind speeds at 20m/s and 50m/s are tabulated in Tables 9-4 to 9-7. It is seen from Tables 9-4 and 9-5 that both the standard deviation displacement and acceleration responses in either lateral or torsional direction are reduced by the SATLCD effectively. The standard deviation
displacement reduction in the lateral direction reaches the level of 43% at mean wind speed of 20 m/s and the level of 29% at mean wind speed of 50 m/s. As for the reduction of standard deviation torsional displacement, it can reach the level of 15% at mean wind speed of 20 m/s and the level of 18% at the mean wind speed of 50 m/s. Tables 9-6 and 9-7 show that the peak lateral displacement response reduction reaches the level of 30% and the peak lateral acceleration response reduction reaches the level of 27%. Tables 9-6 and 9-7 show that the peak torsional displacement reduction reaches the level of 8% and the peak torsional acceleration reduction reaches the level of 16%. The standard deviations of lateral, vertical and torsional displacement responses of the bridge deck along the bridge longitudinal axis at the mean wind speed of 20 m/s are plotted in Figure 9-19. It is seen that the maximum standard deviation of the displacement response of the bridge deck occurs at the tip of the two cantilevers. The lateral displacement response of the bridge deck in the uncontrolled case is increased linearly with the increasing distance from the central tower but the torsional displacement in the uncontrolled case is fairly small and is increased suddenly near the tip of the cantilever. This may imply that the torsional stiffness of the deck near the tip of the cantilever is smaller. The lateral displacement response of the whole bridge deck is reduced effectively by the SATLCD. However, for the reduction of torsional displacement, only the part with significant torsional vibration can be reduced.

<table>
<thead>
<tr>
<th>Location</th>
<th>Lateral (m)</th>
<th>Vertical (m)</th>
<th>Torsional (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>w/o control</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(U_m=20 m/s)</td>
<td>0.29746</td>
<td>0.29755</td>
<td>0.37318</td>
</tr>
<tr>
<td>With control</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(U_m=20 m/s)</td>
<td>0.16782</td>
<td>0.16739</td>
<td>0.37243</td>
</tr>
<tr>
<td></td>
<td>(-43.6%)</td>
<td>(-43.7%)</td>
<td>(-0.2%)</td>
</tr>
<tr>
<td>w/o control</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(U_m=50 m/s)</td>
<td>1.87074</td>
<td>1.87170</td>
<td>2.45735</td>
</tr>
<tr>
<td>With control</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(U_m=50 m/s)</td>
<td>1.32582</td>
<td>1.32442</td>
<td>2.44805</td>
</tr>
<tr>
<td></td>
<td>(-29.1%)</td>
<td>(-29.2%)</td>
<td>(-0.38%)</td>
</tr>
</tbody>
</table>
Table 9-5 Standard Deviation of Deck Acceleration with and without Control

<table>
<thead>
<tr>
<th>Location</th>
<th>Lateral (m/s²)</th>
<th>Vertical (m/s²)</th>
<th>Torsional (rad/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>w/o control (U_m=20m/s)</td>
<td>0.05392</td>
<td>0.05410</td>
<td>0.42710</td>
</tr>
<tr>
<td>With control (U_m=20m/s)</td>
<td>0.04027</td>
<td>0.04049</td>
<td>0.41139</td>
</tr>
<tr>
<td>(25.3%)</td>
<td>(-25.2%)</td>
<td>(-3.68%)</td>
<td>(-23.5%)</td>
</tr>
<tr>
<td>w/o control (U_m=50m/s)</td>
<td>0.42718</td>
<td>0.42861</td>
<td>3.12393</td>
</tr>
<tr>
<td>With control (U_m=50m/s)</td>
<td>0.34159</td>
<td>0.34373</td>
<td>3.00655</td>
</tr>
<tr>
<td>(20.0%)</td>
<td>(-19.8%)</td>
<td>(-3.76%)</td>
<td>(-3.05%)</td>
</tr>
</tbody>
</table>

Table 9-6 Peak Deck Displacement with and without Control

<table>
<thead>
<tr>
<th>Location</th>
<th>Lateral (m)</th>
<th>Vertical (m)</th>
<th>Torsional (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>w/o control (U_m=20m/s)</td>
<td>0.75540</td>
<td>0.78299</td>
<td>1.37033</td>
</tr>
<tr>
<td>With control (U_m=20m/s)</td>
<td>0.45393</td>
<td>0.44787</td>
<td>1.36958</td>
</tr>
<tr>
<td>(39.9%)</td>
<td>(-42.8%)</td>
<td>(-0.05%)</td>
<td>(-2.72%)</td>
</tr>
<tr>
<td>w/o control (U_m=50m/s)</td>
<td>5.32071</td>
<td>5.29925</td>
<td>9.39659</td>
</tr>
<tr>
<td>With control (U_m=50m/s)</td>
<td>3.69404</td>
<td>3.47452</td>
<td>9.33599</td>
</tr>
<tr>
<td>(30.6%)</td>
<td>(-34.4%)</td>
<td>(-0.65%)</td>
<td>(-2.11%)</td>
</tr>
</tbody>
</table>

Table 9-7 Peak Deck Acceleration with and without Control

<table>
<thead>
<tr>
<th>Location</th>
<th>Lateral (m/s²)</th>
<th>Vertical (m/s²)</th>
<th>Torsional (rad/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>w/o control (U_m=20m/s)</td>
<td>0.21676</td>
<td>0.22076</td>
<td>1.67801</td>
</tr>
<tr>
<td>With control (U_m=20m/s)</td>
<td>0.14329</td>
<td>0.14863</td>
<td>1.49870</td>
</tr>
<tr>
<td>(33.9%)</td>
<td>(-32.7%)</td>
<td>(-10.7%)</td>
<td>(-7.63%)</td>
</tr>
<tr>
<td>w/o control (U_m=50m/s)</td>
<td>1.92871</td>
<td>1.95003</td>
<td>12.3014</td>
</tr>
<tr>
<td>With control (U_m=50m/s)</td>
<td>1.40071</td>
<td>1.39991</td>
<td>10.7848</td>
</tr>
<tr>
<td>(27.4%)</td>
<td>(-28.2%)</td>
<td>(-12.3%)</td>
<td>(-8.99%)</td>
</tr>
</tbody>
</table>

9.5.4 Effect of cantilever length

One of the challenges to apply passive liquid column dampers for the suppression of bridge deck vibration during construction probably arises from the change of bridge dynamic properties from stage to stage. It has been shown in Tables 9-1 to 9-3 that the
natural frequencies of the double cantilever are varied with the stage of construction in particular for torsional vibration of the concerned bridge. It is thus difficult to apply passive damper with fixed parameters to mitigate the bridge vibrations. However, the natural frequency of the SATLCD can be easily adjusted to match with the updated structural frequency simply by changing the feedback gain of the liquid column damper. This special feature enables it to become a potential device for the suppression of bridge deck vibration with varying frequency during construction. It is seen from Figure 9.9 that for the cantilever length smaller than 170m, the torsional vibration of the double cantilever deck is small because of the high torsional deck stiffness. At the initial stage of construction, control of lateral vibration is more important and all the SATLCDs are thus tuned to the lateral frequency of the deck at the first two construction stages.

To have a reasonable assessment of the performance of SATLCD in reducing the buffeting response of the bridge, head loss coefficient is taken as a variable to find its optimal value for achieving the maximum reduction of standard deviation displacement response ratio at a given construction stage. The other parameters of the SATLCD used herein are $H=6.5m$, $a=0.6$, $\mu=0.027$, and $\mu_L=\mu_T$. It is noted that the use of SATLCD with a fixed liquid column length ($L$) might reduce the vibration of the concerned bridge during different construction stages. If the updated structural natural frequency of the bridge deck at each stage is available, the frequency of the damper can be actively adjusted to the updated structural natural frequency to maintain high vibration control performance. The reduction of displacement response ratio of the double cantilever deck at the central tower part achieved by the SATLCDs during five different construction stages at a mean wind speed of 20m/s are plotted in Figure 9.20. It is seen that the performance of SATLCD is varied from stage to stage. For the lateral vibration reduction, the performance of SATLCD is deteriorated as the cantilever length is increased. For the torsional vibration reduction, the performance of SATLCD is less affected by the increase of the cantilever length when compared with that for lateral vibration reduction. The reduction of lateral vibration obtained by the SATLCDs reaches the level of 40% for standard deviation displacement response and the level of 15% for standard deviation acceleration response ratio. For the torsional vibration, the reduction obtained by the SATLCDs reaches the level of 15% for standard deviation
displacement response and the level of 20% for standard deviation acceleration response ratio.

9.6 SUMMARY

Semi-active tuned liquid column damper (SATLCD) with two control targets, the control of its natural frequency and the control of liquid motion within the tolerable limit, was developed and studied for the mitigation of wind-induced coupled lateral and torsional vibration of a real long span bridge during five construction stages. The SATLCD can provide a great flexibility for selecting liquid column length while keeping a proper frequency tuning through the change of air pressure acting on liquid. Another feature of the SATLCD is its adaptability to the variation of the bridge frequency under various construction stages. If the updated structural natural frequency of the bridge deck at each stage is available, the frequency of the damper can be actively adjusted to the updated structural natural frequency to maintain high vibration control performance. A finite element model of SATLCD was developed in this chapter and incorporated into the finite element model of the long span bridge for predicting the buffeting response in the time domain. Extensive parametric studies on SATLCD for the mitigation of wind-induced coupled lateral and torsional responses of the real long span cable-stayed bridge during construction were carried out in terms of the mass ratio, head loss coefficient, mean wind speed, and cantilever length. The results demonstrated that SATLCD can effectively reduce either lateral or torsional buffeting response of the cable-stayed bridge if its parameters are properly selected. There exists an optimal head loss coefficient for the maximum reduction in either lateral or torsional displacement response. The optimal head loss coefficient for achieving the maximum reduction in lateral displacement is much smaller than that for achieving the maximum reduction in torsional displacement. Both the standard deviation displacement and acceleration responses are reduced effectively by the SATLCD under the mean wind speeds at 20m/s and 50m/s. The reduction of lateral displacement response by the SATLCD is almost at the same level with the mean wind speed at 20m/s and 30m/s but it is decreased at the mean wind speed of 40m/s and 50m/s. For the reduction of torsional displacement response, the performance of the SATLCD is slightly better as the mean wind speed is increased. It was also shown that the SATLCD can effectively reduce the lateral displacement response of the entire deck but not for the torsional displacement
response. For the torsional displacement reduction of the deck, only the part with significant torsional vibration can be reduced by the SATLCD. It was also found that with a fixed value of liquid column length, the SATLCD with frequency adaptability can effectively reduce the buffeting response of the bridge deck under the five different construction stages.
Figure 9-1 Schematic Diagram of Semi-Active Tuned Liquid Column Dampers

Figure 9-2 Connections between Bridge Deck and Liquid Damper
Start

Input the data control file name

Select the type of dynamic analysis (Idtype) from the data control file

Idtype=3: SATLCD-bridge system under turbulent wind

Read the following 6 data files required in this analysis from the data control file

Read the dynamic characteristic matrices \([M_b], [C_b], \text{ and } [K_b]\) of the bridge, the coordinates of all nodes of the bridge, the degrees of freedom of all the nodes.

Read the number of liquid column damper and the detailed information of each of the damper, the node numbers of the bridge deck on which the liquid column dampers to be installed from data file 2.

Determine the dynamic characteristic matrices \([M_{bd}], [M_d], [M_{bd}], [K_{bd}], [K_d]\) of each damper. These matrices remain unchanged during the computation.

Read the mean wind velocity, the aerodynamic coefficients \(C_D, C_L, C_M\) and the frequency independent coefficients used to determine the self-excited forces in the time domain. Also read the position of fluctuating wind velocity from data file 3.

Determine the total degrees of freedom of the liquid column dampers and bridge system

Assemble the matrices \([M_b], [C_b], [K_b]\) of the bridge along with the matrices \([M_{bd}], [M_d], [M_{bd}], [K_{bd}], [K_d]\) of each damper and put them into the proper position of dynamic characteristic matrices of the liquid column dampers-bridge system.
Read the initial displacement and velocity of all the independent degrees of freedom of the liquid column dampers and bridge system from data file 4.

Read the time interval (Δt) and the time step (Nstep) from the data control file.

Istep=0

Time step loop: Istep=Istep+1

Load the time independent dynamic characteristic matrices of the liquid column dampers and bridge system such as \([M_b], [C_b], [K_b], [M_{bbd}], [M_{bd}], [M_d], [K_{bbd}], [K_{bd}], [K_d]\).

Compute the time dependent dynamic characteristic matrices \([C_d]\) and put it into the proper position of dynamic characteristic matrices of the liquid column dampers and bridge system.

Read the values of turbulent wind velocity in the horizontal and vertical directions along with the points of the bridge deck at given time \(t\) from data files 5 and 6.

Compute the buffeting force vector \([P_{buff}]\) and put it into the proper position of external force vectors of the liquid column dampers and bridge system.

Compute the self-excited force vector \([P_{se}]\) based on the dynamic responses of the bridge at the time \(t-Δt\) and put it into the proper position of external force vectors of the liquid column dampers and bridge system.

Calculate dynamic responses of the liquid column dampers and bridge system using the Wilson-θ method.
Compute the new self-excited force vector \(\{P_{se}\}\) based on the dynamic responses of the bridge obtained in the current step.

\[
(P_{se\text{-}nt} - P_{se})/P_{se} \leq \text{Tol}
\]

Yes

Store the dynamic responses of both the liquid and the bridge

\[\text{I}_{\text{step}} = \text{N}_{\text{step}}?\]

No

Yes

\[\delta_{kn} = \delta_k + \delta_m\]

\[W_k > K_p W_a \text{ and } W_k W_k > 0\]

No

\[\delta_{kn} = \delta_k\]

Output the response at the specified degrees of freedom of the bridge and the liquid response inside the column dampers

End

Figure 9-3 Flowchart of Executing Structure of Main Program DSAPBDW
Figure 9-4 Configuration of Long Span Cable-Stayed Bridge under Construction Stage 1
Figure 9-5  Configuration of Long Span Cable-Stayed Bridge under Construction Stage 2
Figure 9-6 Configuration of Long Span Cable-Stayed Bridge under Construction Stage 3
Figure 9-7 Configuration of Long Span Cable-Stayed Bridge under Construction Stage 4
Figure 9-8 Configuration of Long Span Cable-Stayed Bridge under Construction Stage 5
(a) Lateral displacement

(b) Vertical displacement
(c) Torsional displacement

Figure 9-9 Buffeting Response of Bridge Deck during Various Construction Stages at Mean Wind Speed of 20m/s
(a) Lateral displacement

(b) Vertical displacement

(c) Torsional displacement

Figure 9-10 Time Histories of Displacements of the Bridge at Point B Subjected to Turbulent Wind ($U_m=20.0\,\text{m/s}$)

9-33
Figure 9-11 Time Histories of Displacements of the Bridge at Point C Subjected to Turbulent Wind ($U_m=20.0\text{m/s}$)
(a) PSD function of lateral displacement

(b) PSD function of vertical displacement

(c) PSD function of torsional displacement

Figure 9-12 PSD Functions of Displacements of the Bridge at Point B Subjected to Turbulent Wind ($U_m=20.0\text{m/s}$)
Figure 9-13 PSD Functions of Displacements of the Bridge at Point C Subjected to Turbulent Wind ($U_m=20.0\text{m/s}$)
Figure 9-14 Effect of Mass Ratio on the Performance of SATLCD
(a) Lateral displacement

(b) Torsional displacement

Figure 9-15 Effect of Head Loss Coefficient
Figure 9-16 Effect of Mean Wind Speed
Figure 9-17 Time Histories of Displacements of the Bridge with Control at Point B under Construction Stage 5 ($U_m=20.0\text{m/s}$)
(a) SATLCD tuned to lateral frequency of the bridge

(b) SATLCD tuned to torsional frequency of the bridge

Figure 9-18 Time Histories of Pressure inside SATLCD at Point B under Construction Stage 5 (\(U_m=20.0\text{m/s}\))
(a) Lateral displacement

(b) Vertical displacement

(c) Torsional displacement

Figure 9.19 Standard Deviation of Deck Displacement of the Bridge under Construction Stage 5
(a) Standard deviation displacement response ratio

(b) Standard deviation acceleration response ratio

Figure 9-20 Effect of Cantilever Length
CHAPTER 10

CONCLUSIONS AND RECOMMENDATIONS

10.1 CONCLUSIONS

This thesis focuses on the development and application of novel tuned liquid column dampers for suppressing lateral and torsional vibration of long span cable-supported bridges during construction and at completion stage. The performance of multiple tuned liquid column dampers (MTLCD) for reducing torsional vibration of structures has been studied through a combined experimental and theoretical investigation. The use of MTLCD for reducing the coupled lateral and torsional vibration of a bridge deck has been explored using the mode-by-mode spectral approach. Two types of new liquid column dampers with greater flexibility in frequency tuning, namely the multiple pressurized tuned liquid column dampers (MPTLCD) and the semi-active tuned liquid column dampers (SATLCD), have been developed for the mitigation of wind-induced vibration of long span bridges. The buffeting analysis of a real long span cable-stayed bridge has been performed in the time domain. Finite element models of MPTLCD and SATLCD have been developed and incorporated into the finite element model of the bridge for the prediction of buffeting responses of the bridge with the control devices. Comprehensive computer programs have been written accordingly, which can be used to predict the dynamic responses of coupled liquid column damper and bridge systems under turbulent wind. The performances of MPTLCD and SATLCD in reducing wind-induced vibration of the real long span cable-stayed bridge have been assessed using the developed computer programs. The main contributions and conclusions made in this thesis can be summarized as follows:

1. An experimental investigation on the performance of MTLCD for reducing torsional vibration of structures in comparison with single-tuned liquid column damper (STLCD) was first carried out. A large structure model simulating the torsional vibration of bridge deck and several STLCDs and MTLCDs of different configurations were designed and constructed. A series of harmonically forced
vibration tests were conducted to evaluate the effectiveness of MTLCD in reducing torsional vibration of the structure. The experimental results revealed that the MTLCD could be more effective than the STLCD in reducing the torsional vibration of structures when the overall water column dimensions of the STLCD and the MTLCD were kept the same. Decreasing the frequency bandwidth of MTLCD reduced the peak torsional displacement response of the structure but increased the bandwidth of the frequency response curve of the structure. The offset of frequency tuning did affect the performance of both the STLCD and the MTLCD. Compared with the STLCD, the MTLCD was less sensitive to the frequency tuning ratio and excitation moment amplitude. The MTLCD seemed to be more robust than the STLCD in controlling the torsional vibration of structures. An efficient method was also developed to determine the head loss coefficient of a TLCD using the free vibration test technique. The method was also verified through the comparison with the exact solution obtained by numerical integration.

2. An analytical model for the torsional vibration of the structure with an MTLCD under either harmonic excitation or white noise excitation was developed and verified using the obtained experimental results together with the identified head loss coefficient. It was found that the computed results using the developed analytical model for torsional vibration of the structure with either STLCD or MTLCD under harmonic excitation were in good agreement with those obtained from the experiments. Extensive parametric studies on STLCDs and MTLCDs for reducing torsional vibration of the structure under white noise excitation were also carried out using the verified analytical model. The results revealed that the ratio $H/L_c$ is an important factor and should be larger than the value of $\alpha_c/2$ to achieve significant torsional vibration reduction. There are an optimal head loss coefficient and an optimal frequency bandwidth for a MTLCD with a given number of TLCD units. The optimal head loss coefficient is decreased but the optimal frequency bandwidth is increased with increasing number of TLCD units. The effectiveness of an optimized MTLCD increases with increasing number of TLCD units but there is no significant improvement if the number of TLCD units in the MTLCD exceeds 5. Moreover, the optimal head loss coefficient of either the STLCD or the MTLCD depends on external excitation amplitude. The sensitivity of structural response
reduction with an optimized MTLCD to the frequency tuning ratio is less than that with an optimized STLCD and it can be further improved by increasing the frequency bandwidth but at the cost of less torsional vibration reduction.

3. Multiple pressurized tuned liquid column dampers (MPTLCD) were studied for torsional vibration reduction of a structure. The MPTLCD container was sealed with air chamber at its two ends. The frequency tuning can be adjusted by manipulating the static pressure inside the air chamber while the length of liquid column is fixed. An analytical model for torsional vibration of a structure with a MPTLCD under either harmonic excitation or white noise excitation was developed. The nonlinear damping force due to orifice and the nonlinear restoring force due to the pressure inside the air chambers were linearized. The nonlinearity of the PTLCD-structure system was studied by comparing the results obtained from the linear equation of a PTLCD-structure system with those obtained from the nonlinear equation of the PTLCD-structure system. It was found that the nonlinearity of the MPTLCD-structure system is relatively weak and the linear equation is an adequate model to describe the MPTLCD-structure system. Extensive parametric studies were then carried out using the linear model in the frequency domain. The investigations demonstrated that \( H/L_p \) ratio is an important parameter affecting the performance of PTLCD in reducing torsional vibration. For the same value of the \( H/L_p \), the PTLCD with a larger value of liquid length can achieve larger reduction in displacement response. It was also found that for \( H/L_p \) ratio beyond a value of 0.35, the performance of PTLCD in reducing torsional displacement is better than that of TLCD and it can be further improved with the increasing of \( H/L_p \) ratio. For the ratio \( H/L_p \) increasing to 2 and above, the performance of MPTLCD is improved and the torsional response reduction is increased with the increasing liquid length ratio. There are an optimal head loss coefficient and an optimal frequency bandwidth for a MPTLCD with a given number of PTLCD units. The sensitivity of MPTLCD to pressure tuning ratio can be improved by increasing the bandwidth but at the expense of less torsional vibration reduction.

4. A semi-active tuned liquid column damper (SATLCD), whose natural frequency can be altered by active control of liquid column pressure, was studied for either lateral
or torsional vibration reduction. The analytical models for lateral vibration of a structure with SATLCD and torsional vibration of a structure with SATLCD were developed accordingly. The SATLCD can provide a great flexibility for selecting a liquid length while keeping a proper frequency tuning through the change of air pressure acting on liquid. Another feature of the SATLCD studied herein is its adaptability to the change in structural frequency. If the online information of varying structural natural frequency is available, the frequency of the damper can be actively adjusted to the varying structural natural frequency to maintain high vibration control performance. The numerical examples carried out in this study demonstrated that SATLCD can effectively reduce either lateral or torsional vibration of a structure if its parameters are properly selected. There exists an optimal head loss coefficient for the maximum reduction in either lateral or torsional vibration. The optimal head loss coefficient increases with the increasing liquid length of SATLCD. For the case of lateral vibration of SATLCD-structure system, the results revealed that SATLCD with a larger value of liquid length ratio can achieve larger reduction in displacement response but at the expense of larger value of pressure required for the feedback control force. For the case of torsional vibration of SATLCD-structure system, it was found that H/L_p ratio is an important parameter which affects the performance of SATLCD in reducing torsional vibration. For the same value of the H/L_p, the SATLCD with a larger value of liquid length can achieve larger reduction in displacement response. When the ratio H/L_p is increased to 1 and above, the torsional response reduction is increased with the increasing liquid length ratio.

5. The use of MTLCD for reducing the coupled lateral and torsional vibration was investigated for a bridge deck under harmonic excitation, white noise excitation, and wind excitation. An analytical model for the coupled lateral and torsional vibration of the bridge deck with a MTLCD under different excitations was developed. Extensive parametric studies on the MTLCD under both harmonic excitation and white noise excitation were carried out using the developed analytical model. The results revealed that there exists an optimal water mass distribution between the two dampers, which depends on the relative importance of torsional response reduction to lateral response reduction. It is important to provide a proper ratio of H/L_2 to the
MTLCD so that the interaction between the MTLCD and bridge deck will not tend to be zero in the coupled lateral and torsional vibration. To achieve the maximum reduction of displacement response, the value of $\alpha$ should be as large as possible provided that the water is retained in the horizontal part of the container. The performance of MTLCD in reducing lateral and torsional displacement responses depends on the head-loss coefficient and the tuning ratio. The optimal head-loss coefficient for achieving the maximum reduction in lateral vibration is different from that for achieving the maximum reduction in torsional vibration. However, the optimal tuning ratio for achieving the maximum reduction in lateral vibration is slightly different from that for achieving the maximum reduction in torsional vibration. The investigation on the buffeting response of the coupled MTLCD-bridge deck system demonstrated that the MTLCD can reduce both the lateral and torsional vibrations of the bridge deck, and the bridge deck response reduction increases with increasing mean wind speed if aeroelastic effects are not considered. However, for the bridge and mean wind speed range concerned, the aeroelastic damping ratio is positive and increases as mean wind speed increases. The bridge response reduction by the MTLCD remains almost unchanged as mean wind speed increases.

6. The application of MPTLCD on a real long span cable-stayed bridge at the completion stage was investigated. A finite element model of MPTLCD was developed and incorporated into the finite element model of long span bridge for the prediction of the buffeting response of the bridge in the time domain. The detailed buffeting analysis of the real long span cable-stayed bridge with MPTLCD in the time domain was presented. Extensive parametric studies on MPTLCD for reducing wind-induced coupled lateral and torsional vibration of the real long span cable-stayed bridge were carried out using the established finite element model for the coupled MPTLCD-bridge system. The results demonstrated that MPTLCD could effectively reduce the lateral and torsional displacement of the long span cable-stayed bridge if the parameters were selected properly. There existed an optimal water mass distribution between the MPTLCD tuned to the lateral frequencies of the bridge and the MPTLCD tuned to the torsional frequency of the bridge. The performance of MPTLCD in reducing lateral and torsional displacement responses
depended on head loss coefficient. There existed an optimal head loss coefficient of MPTLCD for a given frequency bandwidth. The effectiveness of an optimized MPTLCD with zero bandwidth increased with increasing mean wind speed and the corresponding head loss coefficient decreased. Further investigations demonstrated that the lateral and torsional displacement responses in the vicinity of the middle span of the deck could be reduced significantly by the MPTLCD. The sensitivity of the optimized MPTLCD in reducing displacement response could be improved by increasing the frequency bandwidth but at the cost of less displacement reduction.

7. Semi-active tuned liquid column damper (SATLCD) with two control targets, the control of its natural frequency and the control of liquid motion within the tolerable limit, was developed and applied for the mitigation of wind-induced coupled lateral and torsional vibration of the real long span cable-stayed bridge during construction. The SATLCD can provide a great flexibility for selecting liquid column length while keeping a proper frequency tuning through the change of air pressure acting on liquid. Another feature of the SATLCD is its adaptability to the variation of the bridge frequency under various construction stages. If the updated structural natural frequency of the bridge deck at each stage is available, the frequency of the damper can be actively adjusted to the updated structural natural frequency to maintain high vibration control performance. The finite element model of SATLCD was developed and incorporated into the finite element model of the long span bridge for predicting the buffeting response in the time domain. Extensive parametric studies on the SATLCD for the mitigation of wind-induced coupled lateral and torsional vibration of the real long span cable-stayed bridge under construction were carried out using the established finite element model of the SATLCD-bridge system. The result demonstrated that SATLCD could effectively reduce either lateral or torsional buffeting response of the cable-stayed bridge if its parameters were properly selected. There existed an optimal head loss coefficient for the maximum reduction in either lateral or torsional displacement response. The optimal head loss coefficient for achieving the maximum reduction in lateral displacement was much smaller than that for achieving the maximum reduction in torsional displacement. Both the standard deviation displacement and acceleration responses were reduced effectively by the SATLCD under the mean wind speeds at 20m/s and 50m/s. The
reduction of lateral displacement response by the SATLCD was almost at the same level with the mean wind speed at 20m/s and 30m/s but it decreased at the mean wind speed of 40m/s and 50m/s. For the reduction of torsional displacement response, the performance of SATLCD was slightly better as the mean wind speed increased. It was also shown that the SATLCD could effectively reduce the lateral displacement response of the entire bridge deck but not for the torsional displacement response of the entire bridge deck. For the torsional displacement reduction of the deck, only the part with significant torsional vibration could be reduced by the SATLCD. It was also found that with a fixed value of liquid column length, the SATLCD with frequency adaptability could effectively reduce the buffeting response of the bridge deck throughout the five construction stages.

It should be noted that the above conclusions were drawn from the concerned structure and bridge together with the parameters selected for liquid column dampers. The conclusions thus should be used with caution for other structures and bridges, and the computation with proper input data using a sophisticate program for new structures and bridges may be necessary.

10.2 RECOMMENDATIONS

Although a significant progress has been made in this thesis to the development and application of novel tuned liquid column dampers for suppressing lateral and torsional vibration of long span cable-supported bridges during construction and at completion stage, there remain some important issues deserving further study to enhance our understanding of the novel tuned liquid column dampers and their applications.

1. In the present study, semi-active tuned liquid column damper (SATLCD) has been studied for improving the frequency tuning adaptability only. Yalla et al. (2001) showed that a semi-active tuned liquid column with its orifice opening changing adaptively can perform optimally in accordance with external excitation and structure responses. Therefore, the semi-active tuned liquid column with the two control targets combined needs to be further investigated.
2. The effectiveness of the SATLCD was assessed through computation simulation based on the developed analytical model. A detailed experiment on the realization of the pressure control system on tuned liquid column damper is needed. The experimental investigation on the performance of SATLCD for vibration reduction of a simple structure is also needed to verify the developed analytical model and confirm the effectiveness of SATLCD achieved in the computation simulation.

3. The vertical vibration control of the bridge deck is not considered at the present study due to the inherent feature of liquid column dampers. However, large vibration may result from the coupling between vertical and torsional modes of vibration and the coupling between lateral and torsional modes of vibration for long span cable supported bridges. Besides, coupled flutter is a common stability problem encountered by long span bridges (Dyrbye and Hansen 1997). Therefore, it is important to develop new or hybrid control devices for suppressing coupled vibration of long span bridges in lateral, vertical and torsional directions.

4. Theoretical studies on wind-induced vibration control have shown that the control performance of MTLCD, MPTLCD or SATLCD depend on the aerodynamic characteristics of the bridge deck and is influenced by the mean wind speed. A detailed wind tunnel experiment is needed to verify wind-induced vibration control performance in either buffeting response or flutter stability before installing these dampers on the real bridge. Serious attention must be paid on the modelling issues of the bridge and dampers.

5. The effect of vehicles running on a long span bridge was neglected in the study of the performance of MPTLCD in mitigating lateral and torsional vibrations. However, it is believed that the natural frequency of the bridge would alter in accordance with the distribution, the type as well as the speed of vehicles running on the bridge. Therefore, it is important to consider the effect of vehicles on the performance and robustness of MPTLCD.

6. In the evaluation of the buffeting response of the long span bridge, the mean wind at a right angle to the longitudinal axis of the bridge deck was considered to be the
worst case. However, Zhu (2002) showed that the most unfavourable buffeting response of the bridge may not occur in the normal wind direction. It is therefore important to investigate the performance of the liquid column dampers in reducing the buffeting responses of long span cable-supported bridge under the skew wind.
APPENDIX A
MATHEMATICAL MODEL

This study focuses on the development and application of novel tuned liquid column dampers for the suppression of wind-induced lateral and torsional vibration of a long span cable-supported bridge during construction and at completion stage. Various analytical models of different liquid column dampers have been presented in this thesis. This appendix thus aims to provide detailed mathematical derivations of the models for the MTLCD-structure system, the MPTLCD-structure system, and the SAMTLCD-structure system.

A.1 MTLCD-STRUCTURE SYSTEM

Let us consider a MTLCD consisting of n small TLCD units and installed in a two-degree-of-freedom structure (see Figure A-1). Each small TLCD unit is a U-shape container of uniform cross-section filled with liquid. The equation of motion of MTLCD-structure system can be formulated by many methods, such as using the Newton’s law of motion, the D’Alembert’s principle, and the Lagrange’s equation (Goldstein et al. 2002). The Lagrange’s equation is adopted here for the derivation of the equation of motion of a MTLCD-structure system. The energy of the system is first determined in terms of the chosen coordinate frame. The MTLCD-structure system has n+2 degrees of freedom, which include the lateral motion of the structure, the torsional motion of the structure, and the water motions inside n TLCD units in the MTLCD. As shown in Figure A-1, let the two coordinates x and θ represent the lateral displacement and torsional displacement of the structure with respect to the elastic centre E. Denote the coordinate $W_k$ (k=1,......,n) as the liquid displacement relative to the container of the kth TLCD in the MTLCD.

A.1.1 Kinetic energy of MTLCD-structure system
Take the positive vertical direction upward and the positive horizontal direction on the right side. In consideration of a small volume of liquid inside the TLCD, its kinetic energy $dT$ with respect to the chosen coordinate system is given by

$$dT = \frac{1}{2} dm \left( u_i^2 + u_j^2 \right)$$  \hspace{1cm} (A-1)$$

where $u_i$ and $u_j$ are the horizontal and vertical component of the liquid velocity with respect to the chosen coordinate system; $dm$ is the mass of the small volume of liquid. Denote $s$ and $t$ as the horizontal distance and vertical distance from the elastic centre to the centroid of the small volume liquid $dm$ in the bottom tube and vertical column of container, respectively. Then, for the liquid inside the $k$th TLCD, $u_i$ and $u_j$ are given by

$$u_i = H_k \dot{\theta} + \dot{x} + \dot{W}_k$$

for liquid inside the bottom tube of container \hspace{1cm} (A-2)

$$u_j = s \dot{\theta}$$

$$u_i = t \dot{\theta} + \dot{x}$$

$$u_j = (-1)^z \left( \frac{1}{2} B_k \dot{\theta} + \dot{W}_k \right)$$

for liquid inside the vertical column of container \hspace{1cm} (A-3)

where $z$ is the odd natural number for the left vertical column and the even natural number for the right vertical column; $H_k$ is the center line of the horizontal part of the container and the elastic center; $B_k$ is the horizontal length of liquid column. By summing up the kinetic energy of all small volumes of liquid inside the container, the total kinetic energy of liquid inside the $k$th TLCD, $T_k$, is determined by

$$T_k = \int_{H_k - \frac{1}{2} B_k - W_k}^{H_k} \frac{1}{2} \rho A_k \left[ (\dot{\theta} + \dot{x})^2 + (\frac{1}{2} B_k \dot{\theta} + \dot{W}_k)^2 \right] dt$$

$$+ 2 \int_0^{\frac{1}{2} B_k} \frac{1}{2} \rho A_k \left[ (H_k \dot{\theta} + \dot{x} + \dot{W}_k)^2 + (s \dot{\theta})^2 \right] ds + \int_{H_k - \frac{1}{2} B_k - W_k}^{H_k} \frac{1}{2} \rho A_k \left[ (\dot{\theta} + \dot{x})^2 + (\frac{1}{2} B_k \dot{\theta} + \dot{W}_k)^2 \right] dt$$

$$= \frac{1}{2} m_k \ddot{W}_k^2 + \frac{1}{2} m_k \ddot{x}^2 + \frac{1}{2} l_{ik} \dot{\theta}^2 + m_k a_k \ddot{W}_k \dot{x} + G_k \ddot{W}_k \dot{\theta} + m_k \ddot{H}_k \dot{x} \dot{\theta}$$  \hspace{1cm} (A-4)$$
where $\rho$ is the density of the liquid inside the MTLCD; $A_k$ is the cross-sectional area of the $k$th TLCD; $m_k = \rho A_k L_k$; $a_k = B_k / L_k$; $G_k = \rho A_k B_k \left( H_k + \frac{L_k - B_k}{2} \right)$;

$$H_k = H_k - \frac{(L_k - B_k)^2}{4L_k};$$

$$I_k = m_k \left( a_k \left[ H_k^2 + \frac{B_k^2}{12} \right] + (1 - a_k) \left[ H_k^2 + \frac{B_k^2}{4} - \frac{H_k (L_k - B_k)}{2} + \frac{(L_k - B_k)^2}{12} \right] \right);$$

$(k=1,2,\ldots,n)$. The kinetic energy of the structure, $T_s$, is given by $T_s = \frac{1}{2} m_s \dot{x}^2 + \frac{1}{2} I_s \dot{\theta}^2$ where $m_s$ and $I_s$ are the mass and the mass moment of inertia of the structure. The kinetic energy of the MTLCD-structure system, $T$, which is composed of the kinetic energy of the structure and the kinetic energy of liquid inside MTLCD, is given by

$$T = T_s + \sum_{k=1}^{n} T_k \quad \text{(A-5)}$$

### A.1.2 Potential energy of MTLCD-structure system

At the equilibrium position of the $k$th TLCD, the potential energy of the liquid inside the $k$th TLCD, $V_k^l$, with reference to the elastic centre $E$ is given by

$$V_k^l = -\left[ \rho A_k B_k g H_k + \rho A_k (L_k - B_k) g \left( H_k - \frac{L_k - B_k}{4} \right) \right] \quad \text{(A-6)}$$

For the structure with a lateral displacement $x$ and a rotational displacement $\theta$, the potential energy of the liquid inside the $k$th TLCD, $V_k^f$, with reference to the elastic centre $E$ is determined by

$$V_k^f = -\rho A_k g B_k H_k \cos \theta - \rho A_k g \left[ \frac{L_k - B_k}{2} + W_k \right] \left[ H_k - \frac{B_k}{2} \tan \theta - \frac{1}{2} \left( \frac{L_k - B_k}{2} + W_k \right) \right] \cos \theta$$

$$- \rho A_k g \left[ \frac{L_k - B_k}{2} - W_k \right] \left[ H_k + \frac{B_k}{2} \tan \theta - \frac{1}{2} \left( \frac{L_k - B_k}{2} - W_k \right) \right] \cos \theta \quad \text{(A-7)}$$

By omitting the higher order terms, the change in potential energy of the liquid inside the $k$th TLCD, $V_k$, is then determined by
\[ V_k = V_k^r - V_k^l = -m_k g \bar{H}_k \cos \theta + m_k g \alpha_k W_k \sin \theta + \frac{1}{2} m_k \omega_k^2 W_k^2 \cos \theta \]  

(A-8)

and \( \omega_k = \sqrt{\frac{2g}{L_k}} \).

Denote \( K_x \) and \( K_\theta \) as the lateral stiffness and torsional stiffness of the structure respectively. The potential energy of the structure, \( V_s \), due to its lateral displacement and torsional displacement is

\[ V_s = \frac{1}{2} K_x \dot{x}^2 + \frac{1}{2} K_\theta \dot{\theta}^2 \]

The potential energy of the MTLCD-structure system is determined by

\[ V = V_s + \sum_{k=1}^{n} V_k \]

A.1.3 Equation of motion of MTLCD-structure system

The Lagrangian of the system is

\[ L = \left( \frac{1}{2} m \ddot{x}^2 + \frac{1}{2} I \dot{\theta}^2 \right) + \sum_{k=1}^{n} \left[ \frac{1}{2} m_k W_k^2 + \frac{1}{2} m_k \ddot{x}^2 + \frac{1}{2} I_k \dot{\theta}^2 + m_k \alpha_k W_k \ddot{x} \right] + G_k \dot{W}_k \dot{\theta} + m_k g \bar{H}_k \dot{x} \dot{\theta} + m_k g \bar{H}_k \cos \theta \]

\[ -m_k g \alpha_k W_k \sin \theta - \frac{1}{2} m_k \omega_k^2 W_k^2 \cos \theta \]  

(A-9)

The general form of the Lagrange's equation can be written as

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (i=1,2,\ldots,n+2) \]  

(A-10)

where \( q_i \) and \( \dot{q}_i \) are the ith generalized coordinate and the ith generalized velocity; \( Q_i \) is the ith generalized force. Let \( q_1 = x \), \( q_i = W_{i-1} \) \( (i=2,3,\ldots,n+1) \) and \( q_{n+2} = \theta \). The generalized forces are

\[ Q_i = F_s - C_x \dot{x} - m_z \dot{\theta} \]

\[ Q_i = -\frac{1}{2} \rho A_{i-1} \delta_{i-1} \dot{W}_{i-1} \quad (i=2,3,\ldots,n+1) \]

(A-11)

\[ Q_{n+2} = M_s - C_{\theta} \dot{\theta} - m_z \dot{x} \]

where \( F_s \) is the external dynamic force acting at the elastic centre of the structure and \( M_s \) is the external dynamic moment acting on the structure; \( \delta_i \) is the head loss coefficient of the jth TLCD governed by the opening ratio of orifice; \( z \) is the vertical distance between
the elastic center E and the mass center M of the bridge; \( C_x \) and \( C_\theta \) are the lateral damping coefficient and torsional damping coefficient of the structure.

Applying the Lagrange's equation, the equation of motion of the MTLCD-structure system can be derived.

For the lateral motion, the equation of motion of the structure,

\[
\left( m_x + \sum_{k=1}^{n} m_k \right) \ddot{x} + C_x \dot{x} + K_x x = - \left( m_z z + \sum_{k=1}^{n} m_k H_k \right) \ddot{\theta} - \sum_{k=1}^{n} m_k a_k \ddot{W}_k + F_x(t) \tag{A-12}
\]

For the liquid motion, the equation of motion of the kth TLCD,

\[
m_k \ddot{W}_k + \frac{1}{2} \rho A_k \delta_k \left| \dot{W}_k \right| \dot{W}_k + m_k \rho \omega_k^2 W_k = - \left[ G_k \ddot{\theta} + m_k g \alpha_k \theta \right] - m_k a_k \ddot{x} \tag{A-13}
\]

For the torsion motion, the equation of motion of the structure,

\[
\left( I_s + \sum_{k=1}^{n} I_k \right) \ddot{\theta} + C_\theta \dot{\theta} + \left( K_\theta + \sum_{k=1}^{n} m_k g H_k \right) \theta = - \left( \sum_{k=1}^{n} m_k H_k \right) \ddot{x} - \sum_{k=1}^{n} \left[ m_k a_k g W_k \right] + M_x(t) \tag{A-14}
\]

A.2 MPTLCD-STRUCTURE SYSTEM

The equation of motion of a MPTLCD-structure system can also be derived by using the Lagrange's equation. The MPTLCD-structure system has \( n+2 \) degrees of freedom, which are the lateral motion of the structure, the torsional motion of the structure, and the water motions inside \( n \) small PTLCD units in the MPTLCD. As shown in Figure A-2, let the two coordinates \( x \) and \( \theta \) represent the lateral displacement and torsional displacement of the structure from the elastic centre E, respectively. Denote the coordinate \( W_k \) (\( k=1, \ldots, n \)) as the liquid displacement relative to the container of the kth PTLCD in the MPTLCD.

A.2.1 Kinetic energy of MPTLCD-structure system

Take the positive vertical direction upward and the positive horizontal direction on the right side. The total kinetic energy of liquid inside the kth PTLCD, \( T_k \), is
\[ T_k = \frac{1}{2} m_k \dot{W}_k^2 + \frac{1}{2} m_k \dot{x}^2 + \frac{1}{2} I_{ek} \dot{\theta}^2 + m_k \alpha_k W_k \dot{x} + G_k W_k \dot{\theta} + m_k \bar{H}_k \ddot{x} \dot{\theta} \]  

(A-15)

The kinetic energy of the structure, \( T_s \), is given by \( T_s = \frac{1}{2} m_s \dot{x}^2 + \frac{1}{2} I_s \dot{\theta}^2 \). The kinetic energy of the MPTLCD-structure system, \( T \), which is composed of the kinetic energy of the structure and the kinetic energy of liquid inside MPTLCD, is given by

\[ T = T_s + \sum_{k=1}^{n} T_k \]  

(A-16)

### A.2.2 Potential energy of MPTLCD-structure system

The potential energy of the liquid inside the PTLCD is the sum of the potential energy of the liquid inside the PTLCD with zero pressure inside the chamber (which is given by Equation A-8) and the strain energy due to the pressure inside the chamber. The strain energy of the liquid inside the kth PTLCD due to the pressure inside the air chamber is given by

\[ V^p_k = \int_0^{w_k} p_k A_k dW_k \]  

(A-17)

As shown in Chapter 5, the restoring force \( P_k A_k \) acting on the liquid satisfies the following equation

\[ P_k A_k = \frac{2P_{ok} A_k}{h_k} W_k \]  

(A-18)

where \( P_{ok} \) is the static pressure inside the two chambers of the kth PTLCD; \( h_k \) is the height of the air chambers of the kth PTLCD. The potential energy of the liquid inside the kth PTLCD, \( V_k \), is thus determined by

\[ V_k = -m_k g \bar{H}_k \cos \theta + m_k g \alpha_k W_k \sin \theta + \frac{m_k g}{L_k} W_k^2 \cos \theta + \frac{P_{ok} A_k}{h_k} W_k^2 \]  

(A-19)

Denote \( K_x \) and \( K_\theta \) as the lateral stiffness and torsional stiffness of the structure respectively. The potential energy of the structure, \( V_s \), due to its lateral displacement and torsional displacement is

\[ V_s = \frac{1}{2} K_x x^2 + \frac{1}{2} K_\theta \theta^2 \]
The potential energy of the MPTLCD-structure system is determined by 
\[ V = V_s + \sum_{k=1}^{n} V_k. \]

A.2.3 Equation of motion of MPTLCD-structure system

The Lagrangian of the system is 
\[ L = T - V. \]

\[
L = \left( \frac{1}{2} m_x \ddot{x}^2 + \frac{1}{2} l_x \dot{\theta}^2 \right) + \sum_{k=1}^{n_k} \left[ \frac{1}{2} m_k \ddot{W}_k^2 + \frac{1}{2} l_k \dot{\theta}_k^2 + m_k a_k \ddot{W}_k \dot{x} + G_k \ddot{W}_k \dot{\theta} + m_k g \ddot{H}_k \cos \theta - m_k g \ddot{W}_k \sin \theta - \frac{m_k g}{L_k} W_k^2 \cos \theta - \frac{p_{ak} A_k}{h_k} W_k^2 \right] \tag{A-20}
\]

The general form of the Lagrange's equation can be written as

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (i=1,2,\ldots,n+2) \tag{A-21}
\]

where \( q_i \) and \( \dot{q}_i \) are the \( i \)th generalized coordinate and the \( i \)th generalized velocity; \( Q_i \) is the \( i \)th generalized force. Let \( q_1 = x, q_i = W_{i-1} \) (\( i=2,3,\ldots,n+1 \)) and \( q_{n+2} = \theta \). The generalized forces are

\[
Q_1 = F_s - C_x \ddot{x} - m_x \ddot{\theta} \\
Q_i = -\frac{1}{2} \rho A_{l-i} \delta_{l-i} |\ddot{W}_{i-1}| \quad (i=2,3,\ldots,n+1) \tag{A-22} \\
Q_{n+2} = M_s - C_\theta \ddot{\theta} - m_x \ddot{x}
\]

where \( F_s \) is the external dynamic force acting at the elastic centre of the structure and \( M_s \) is the external dynamic moment acting on the structure; \( \delta_j \) is the head loss coefficient of the \( j \)th PTLCD governed by the opening ratio of orifice; \( z \) is the vertical distance between the elastic center \( E \) and the mass center \( M \) of the bridge; \( C_x \) and \( C_\theta \) are the lateral damping coefficient and torsional damping coefficient of the structure.

Applying the Lagrange's equation, the equation of motion of the MPTLCD-structure system can be derived.

For the lateral motion, the equation of motion of the structure,

\[
\left( m_x + \sum_{k=1}^{n} m_k \right) \ddot{x} + C_x \dot{x} + K_x x = -\left( m_x z + \sum_{k=1}^{n} m_k \ddot{H}_k \right) \ddot{\theta} - \sum_{k=1}^{n} m_k a_k \ddot{W}_k + F_s(t) \tag{A-23}
\]

For the liquid motion, the equation of motion of the \( k \)th PTLCD,
\[ m_k \ddot{W}_k + \frac{1}{2} \rho A_k \delta_k \left| \dot{W}_k \right| \dot{W}_k + m_k \alpha_k^2 W_k = - \left[ G_k \ddot{\theta} + m_k g \alpha_k \theta \right] - m_k \alpha_k \ddot{x} \quad (A-24) \]

where \( \alpha_k^2 = \frac{2g}{L_k} \left( 1 + \frac{P_{ek}}{\rho gh_k} \right) \).

For the torsion motion, the equation of motion of the structure,

\[
\left( I_z + \sum_{k=1}^{n} I_k \right) \ddot{\theta} + C_\theta \dot{\theta} + \left( K_\theta + \sum_{k=1}^{n} m_k g \bar{H}_k \right) \theta = - \left( \sum_{k=1}^{n} m_k \bar{H}_k \right) \ddot{x} - \sum_{k=1}^{n} \left[ m_k \alpha_k g \dot{W}_k \right] + M_s(t) \quad (A-25)
\]

### A.3 SAMTLCD-STRUCTURE SYSTEM

The equation of motion of a SAMTLCD can also be derived by using the Lagrange's equation. The SAMTLCD-structure system has \( n + 2 \) degrees of freedom, which are the lateral motion of the structure, the torsional motion of the structure, and the water motions inside \( n \) small SATLCD units in the SAMTLCD. As shown in Figure A-3, let the two coordinates \( x \) and \( \theta \) represent the lateral displacement and torsional displacement of the structure from the elastic centre \( E \), respectively. Denote the coordinate \( W_k \) \((k=1, \ldots, n)\) as the liquid displacement relative to the container of the \( k \)th SATLCD in the SAMTLCD.

#### A.3.1 Kinetic energy of SAMTLCD-structure system

Take the positive vertical direction upward and the positive horizontal direction on the right side. The total kinetic energy of the liquid inside the \( k \)th SATLCD, \( T_k \), is determined by

\[ T_k = \frac{1}{2} m_k \dot{W}_k^2 + \frac{1}{2} m_k \dddot{x}^2 + \frac{1}{2} I_{ak} \dot{\theta}^2 + m_k \alpha_k \dot{W}_k \dddot{x} + G_k \dot{W}_k \dddot{\theta} + m_k \bar{H}_k \dddot{x} \dddot{\theta} \quad (A-26) \]

The kinetic energy of the structure, \( T_s \), is given by \( T_s = \frac{1}{2} m_s \dddot{x}^2 + \frac{1}{2} I_s \dddot{\theta}^2 \). The kinetic energy of the SAMTLCD-structure system, \( T \), which is composed of the kinetic energy of the structure and the kinetic energy of the liquid inside the SAMTLCD, is given by
T = T_s + \sum_{k=1}^{n} T_k \tag{A-27}

A.3.2 Potential energy of SAMTLC structure system

The potential energy of the liquid inside the SATLCD is the sum of the potential energy of the liquid inside the SATLCD with zero pressure inside the chamber (which is given by Equation A-8) and the strain energy due to the pressure inside the chamber. The strain energy of the liquid inside the kth SATLCD due to the pressure inside the air chamber is given by

\[ V_k^p = \int_0^{W_k} P_k A_k \, dW_k \tag{A-28} \]

As shown in Chapter 6, the restoring force \( P_k A_k \) acting on the liquid satisfies the following equation

\[ P_k A_k = S_k W_k \tag{A-29} \]

\[ S_k = m_k \left[ \omega_k^2 - \frac{2g}{L_k} \right] \tag{A-30} \]

where \( \omega_k \) is the circular natural frequency of the kth SATLCD. The potential energy of the liquid inside the kth SATLCD, \( V_k \), is then determined by

\[ V_k = -m_k g \bar{H}_k \cos \theta + m_k g a_k W_k \sin \theta + \frac{m_k g}{L_k} W_k^2 \cos \theta + \frac{S_k}{2} W_k^2 \tag{A-31} \]

Denote \( K_x \) and \( K_\theta \) as the lateral and torsional stiffness of the structure respectively. The potential energy of the structure, \( V_s \), due to its lateral displacement and torsional displacement is

\[ V_s = \frac{1}{2} K_x x^2 + \frac{1}{2} K_\theta \theta^2 \]

The potential energy of the SAMTLC structure system is determined by

\[ V = V_s + \sum_{k=1}^{n} V_k \]

A.3.3 Equation of motion of SAMTLC structure system
The Lagrangian of the system is \( L = T - V \).

\[
L = \left( \frac{1}{2} m_x \ddot{x}^2 + \frac{1}{2} I_1 \dot{\theta}^2 \right) + \sum_{k=1}^{n} \left[ \frac{1}{2} m_k \ddot{W}_k^2 + \frac{1}{2} m_k \dddot{\theta}^2 + \frac{1}{2} I_k \dot{\theta}^2 + m_k a_k \dddot{W}_k \dddot{x} \right]
+ G_k \dddot{W}_k \dot{\theta} + m_k H_k \dddot{W}_k \dddot{\theta} + m_k a_k \dddot{W}_k \dddot{\theta} + m_k g H_k \cos \theta
- m_k g a_k W_k \sin \theta - \frac{m_k g}{L_k} W_k^2 \cos \theta - \frac{S_k}{2} W_k^2
\]  

(A-32)

The general form of the Lagrange's equation can be written as

\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (i=1,2,\ldots,n+2)
\]  

(A-33)

where \( q_i \) and \( \dot{q}_i \) are the \( i \)th generalized coordinate and the \( i \)th generalized velocity; \( Q_i \) is the \( i \)th generalized force. Let \( q_1 = x, \ q_i = W_{i-1} \ (i=2,3,\ldots,n+1) \) and \( q_{n+2} = \theta \). The generalized forces are

\[
Q_1 = F_s - C_x \ddot{x} - m_x \theta \dddot{\theta}
Q_i = -\frac{1}{2} \rho A_{i-1} \delta_{i-1} |\dddot{W}_{i-1}| \quad (i=2,3,\ldots,n+1)
Q_{n+2} = M_s - C_0 \dddot{\theta} - m_x \dddot{x}
\]  

(A-34)

where \( F_s \) is the external dynamic force acting at the elastic center of the structure and \( M_s \) is the external dynamic moment acting on the structure; \( \delta_j \) is the head loss coefficient of the \( j \)th PTLCD governed by the opening ratio of orifice; \( z \) is the vertical distance between the elastic center \( E \) and the mass center \( M \) of the bridge; \( C_x \) and \( C_0 \) are the lateral damping coefficient and torsional damping coefficient of the structure.

Applying the Lagrange's equation, the equation of motion of the MPTLCD-structure system can be derived.

For the lateral motion, the equation of motion of the structure,

\[
\left( m_x + \sum_{k=1}^{n} m_k \right) \ddot{x} + C_x \dddot{x} + K_x x = -\left( m_x z + \sum_{k=1}^{n} m_k H_k \right) \dot{\theta} - m_x \sum_{k=1}^{n} m_k a_k \dddot{W}_k + F_s(t)
\]  

(A-35)

For the liquid motion, the equation of motion of the \( k \)th PTLCD,

\[
m_k \dddot{W}_k + \frac{1}{2} \rho A_k \delta_k \dddot{W}_k = -G_k \dot{\theta} + m_k g a_k \theta - m_k a_k \dddot{W}_k + m_k \omega_k^2 W_k
\]  

where \( \omega_k^2 = \frac{2 \rho A_k g + S_k}{\rho A_k L_k} \).

(A-36)

For the torsion motion, the equation of motion of the structure,
\[
\left( I_s + \sum_{k=1}^{n} I_k \right) \ddot{\theta} + C_{\theta} \dot{\theta} + \left( K_{\theta} + \sum_{k=1}^{n} m_k g H_k \right) \dot{\theta} = - \left( \sum_{k=1}^{n} m_k \dddot{H}_k \right) \dddot{x} - \sum_{k=1}^{n} \left[ m_k \alpha_k g W_k \right] + M_z(t)
\]

(A-37)
Figure A-1 MTLCD-Structure System

Figure A-2 MPTLCD-Structure system
Figure A-3 SAMTLC-Structure System
APPENDIX B

COMPUTED NATURAL MODES OF
THE CONCERNED CABLE-STAYED BRIDGE
B.1 THE CONCERNED CABLE-STAYED BRIDGE AT THE COMPLETION STAGE
B.2 THE CONCERNED CABLE-STAYED BRIDGE
AT THE CONSTRUCTION STAGE ONE
Mode No.21  \( f = 0.98426 \text{ Hz} \)

Mode No.22  \( f = 0.99083 \text{ Hz} \)

Mode No.23  \( f = 1.01362 \text{ Hz} \)

Mode No.24  \( f = 1.01773 \text{ Hz} \)
B.3 THE CONCERNED CABLE-STAYED BRIDGE
AT THE CONSTRUCTION STAGE TWO
Mode No.1

\[ f = 0.04606 \text{ Hz} \]

Mode No.2

\[ f = 0.22302 \text{ Hz} \]

Mode No.3

\[ f = 0.22972 \text{ Hz} \]

Mode No.4

\[ f = 0.26106 \text{ Hz} \]
Mode No.23
$f = 0.91611 \text{ Hz}$

Mode No.24
$f = 0.95054 \text{ Hz}$

Mode No.21
$f = 0.82427 \text{ Hz}$

Mode No.22
$f = 0.90646 \text{ Hz}$
B.4 THE CONCERNED CABLE-STAYED BRIDGE AT THE CONSTRUCTION STAGE THREE
Mode No. 11
\[ f = 0.38417 \text{ Hz} \]

Mode No. 12
\[ f = 0.38515 \text{ Hz} \]

Mode No. 9
\[ f = 0.35823 \text{ Hz} \]

Mode No. 10
\[ f = 0.38114 \text{ Hz} \]
B.5 THE CONCERNED CABLE-STAYED BRIDGE
AT THE CONSTRUCTION STAGE FOUR
B.6 THE CONCERNED CABLE-STAYED BRIDGE
AT THE CONSTRUCTION STAGE FIVE
APPENDIX C

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