The Contribution of Working Memory, Conceptual Knowledge and Calculation Principles to the Individual and Group Differences in Arithmetic Competency of Elementary School Children

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CERTIFICATE OF ORIGINALITY

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Abstract of the Study

Research findings from previous studies have suggested the importance of working memory and conceptual and procedural knowledge to the development of children's arithmetic competency. However, their influences on children's arithmetic competency have rarely been studied together and most of the findings in these areas are from overseas studies. Given the national and cultural differences and the variations in curriculum design and classroom instructions, the applicability of international studies to the local population has yet to be confirmed. This current study is a cross-sectional study to assess working memory capacity, understanding of fundamental arithmetic concepts and use of calculation principles simultaneously, as they contribute to individual and group differences in competency in early elementary arithmetical learning.

In the current study, a variety of measures were administered to 160 primary 1 to 3 children from a local mainstream school, in order to assess the functioning of different components as well as general resource of children's working memory, conceptual understanding of place-value and commutativity and the application of calculation principles. Results from analyses of variance ANOVA indicated that children of different grades and levels of arithmetic competency showed differences in performances in most of the measures of working memory with a few exceptions. Although the higher ability group of all grades outperformed their lower ability peers in the understanding of place-value and commutativity concepts, and the uses of calculation principles, the differences between the

performances of the two arithmetic ability groups tended to decrease as grade levels increased.

Hierarchical regression analyses revealed that the relationship between working memory functioning and children's arithmetical competency was not mediated by children's conceptual or procedural knowledge and vice versa. Hierarchical analyses also revealed the pattern of interaction of different components of working memory, as they contribute in explaining variance in children's arithmetic competency. All the three cognitive aspects in question were shown to have significant unique contribution and interaction in their influences on the arithmetic competency in each grade. However, it was found that these influences tended to decrease with schooling.

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Chapter One

Introduction

1. Background of the Study

Competency in elementary arithmetic (i.e., addition, multiplication, subtraction, and division) is a principal foundation of everyday modern life. It provides the essential means for dealing with a wide variety of numerical-problem-solving situations. Basic arithmetic also provides the foundation for the more advanced mathematical skills that are central to all modern scientific disciplines. Consequently, the factors underlying the individual differences in this fundamental intellectual skill are crucial for children's cognitive development (Ashcraft, 1995; Geary, 1994).

Individual differences inevitably exist in almost every domain of learning. Although Hong Kong students' generally strong performance in mathematics has long been recognized, not all local children excel at mathematics. The divergences in this intellectual domain could stem from different early learning experiences of elementary arithmetic, an irreplaceable foundation for higher-level mathematical competence.

Reading ability and mathematical ability are currently the two most highly focused domains in local education. Over the past few decades, enormous progress has been made in understanding the individual differences in reading abilities. A wide variety of assessment and intervention tools have been established as a result of the increasing understanding of the topic. Growing knowledge on phonological processing and awareness, and their importance to the acquisition of strong reading skills has led to the development of both preventive measures and intervention strategies for children with or at risk of reading difficulties (Gersten & Chard, 1999)

In contrast, the individual differences in arithmetic remain comparatively less well understood (Ginsburg, 1997). Measures that are specifically designed to diagnose children at risk of mathematical difficulties are not available; despite an estimate in previous studies, which used different behavioral criteria, that roughly 4 to 10% of children have suffered from some forms of difficulty in the learning of mathematics. (Gross-Tsur, Manor, & Shalev, 1996; Ostad, 1997; Geary, 2003). Hence it is of great importance to identify the factors underlying the individual differences in arithmetic competency.

Educators and psychologists have begun to channel efforts towards understanding the development of mathematical cognition or skills in young children. Models of mathematical cognition in normally developing children have emerged in the areas of word problem solving (Riley & Greeno, 1988), arithmetic operations (Jordan, Huttenlocher & Levine, 1992), development of counting knowledge (Geary, Bow-Thomas, & Yao, 1992; Gelman & Gallistel, 1978), and strategy use (Siegler & Jenkins, 1989). Among these areas, observable behavioral components and involved cognitive processes have received a greater amount of attention, since they are located at the first two levels of explanation in a complete theory of learning differences (Torgesen, 1999).

A growing body of research is devoted to investigating how the functioning in working memory relates to or affects children's arithmetic abilities. The vast majority of these empirical studies on working memory and arithmetic have been conducted with the multi-component model proposed originally by Baddeley and Hitch (1974), which explains a wide range of experimental and neuropsychological findings. In addition, more tasks have been developed for assessing the functioning of components in this model. In sum, this model proposes working memory that comprises a central executive and two subsidiary or "slave" systems, called the phonological loop and the visuospatial sketchpad. Though differences in sample selection and choice of measure lead to inconsistent findings, available evidences converge on the view that different components of working memory might have specialized roles in arithmetic (Ashcraft, 1995). However, the fractionation of and the interactions between different components have yet to be confirmed.

Another stream in the research on children's arithmetic focuses on their knowledge of arithmetic concepts and principles directly attributable to their connections with arithmetic learning. Among the concepts of arithmetic, place value is the most important found to be relevant to competency in elementary arithmetic (Ho & Cheng, 1997). Moreover, the understanding of and the ability to use arithmetic principles such as commutativity, associativity, inverse, and reversal principles to derive and predict unknown arithmetic facts from known facts were found to be crucial to the children's development of arithmetic reasoning and competency (Baroody, 2004).

Current available findings from the two aforementioned streams have confirmed their importance to children's arithmetic competency. However, their influences on children's arithmetic competency have rarely been studied together. Furthermore, most of the findings in these areas are from overseas studies. Given that there are national and cultural differences in mathematics performance, explained from factors such as numerical language features and variations in curriculum classroom instructions, the applicability of these findings to the local population has yet to be quantified.

2. Purposes of the Study

This current study is the first local cross-sectional study to assess working memory capacity, understanding of fundamental arithmetic concepts and use of calculation principles simultaneously, as they contribute to individual and group differences in competency in early elementary arithmetic learning. The current study aims to address several issues in the understanding of the development of elementary arithmetic.

First, it explores the differences, if any, in these three areas between children with different levels of arithmetic competency in each grade. Overseas findings indicate the presence of differences in working memory. Differences in conceptual understanding and use of calculation principles are also expected with reference to the limited findings from previous research.

Second, the study examines the pattern of contributions and interaction within and between the three cognitive domains in question. Specifically, with reference to the effect of formal instruction on the children's learning, it is expected that the relative importance of the differences in these cognitive domains to the arithmetic competency will change with the increasing level of schooling. Moreover, as conceptual understanding and use of calculation principles could be regarded as declarative and procedural knowledge resides in long-term memory, interactions between these aspects and working memory capacity are expected.

Last but not least, this study focuses, in particular, on the functions of children's working memory in elementary arithmetic. In view of inconsistency of previous findings regarding the role of the phonological loop, visuospatial sketchpad and central executive in arithmetic competency, this study will also explore in depth, the interactions between these three components in the working memory model proposed by Baddeley and Hitch (1974, 1986). Of particular interest is whether the central executive mediates the influences of the phonological loop or visuospatial sketchpad on children's arithmetic ability. This study also attempts to explore the mechanism underlying the weaker performance in working memory span tasks.

3. Significance of the Study

The present study compares the functioning of working memory, understanding of place value and commutativity, and application calculation principles children who have received different level of schooling and have manifested varying competency in performing elementary arithmetic operations. If the results reveal connections between arithmetic competency and the differences in the functioning in these cognitive areas, the information will further substantiate previous overseas findings about the role of the above cognitive aspects in the competency of elementary arithmetic, and will be useful for educators, and especially those who are in the position of helping children with difficulties in mathematical learning.

Chapter Two

Working Memory and Arithmetic Competency—Literature Review

1. Overview of Working Memory

In the exploration of memory and human cognition, there is a general agreement that working memory is a limited capacity system responsible for storing and transforming temporary information. The methods adopted by most of the studies for measuring working memory capacity focus on assessing one's working memory span. Daneman and Carpenter (1980) suggested that working memory span was a better predictor of scholastic attainment than measures, such as digit span, that tax only on temporary storage capacity in the absence of concurrent processing operations. Storing information temporarily is insufficient in reflecting the real nature of academic learning activities.

In the basic structure of a memory span task, the performance indicator is usually the amount of information that can be held in temporary storage when performing mental operations at the same time (Daneman & Carpenter, 1980). There are many different types of working memory span task with variations in the nature of the concurrent processing component. For example, in a reading span task, participants read a series of sentences for comprehension while being instructed to concurrently remember the last word in each sentence. In another task of this kind by Daneman and Carpenter (1980), termed, "listening span", the participants listen to a series of incomplete sentences and are asked to supply the missing word from each. At the end of the series, the participants attempt to recall all the missing words in the correct order. Listening span is taken as the maximum number of sentences that can be handled without error in recalling missing words. Case, Kurland and Goldberg (1982) developed a counting span task in which the participants are presented with a series of cards showing different numbers of colored spots and are asked to count the number of spots on each card. At the end of the series, the participants attempt to recall the results of all the counts. The task continues with increasing length of counting series; span is determined to be the maximum number of counts recalled before performance breaks down.

Working memory capacity has been related to the individual differences in a number of areas in human development and abilities such as spoken language comprehension (Adams, Bourke, and Willis, 1999), cognitive skills (Alloway, Gathercole, Willis & Adams, 2004), cognitive deficits in children with atypical development (Phillips, Jarrold, Baddeley, Grant & Karmiloff-Smith, 2004), and reading abilities (Chiappe, Hasher & Siegel, 2000; Swanson & Howell, 2001). Research has also confirmed that measures of working memory span are highly correlated with measures of general attainment and ability in the learning of mathematics.

A growing body of research has been devoted to the study of how the profile in working memory relates to or affects arithmetic competency in children who are developing normally, as well as in those with learning difficulties. For example, Siegel and Ryan (1989) investigated the differences between subtypes of learning-disabled children. The subtypes included reading disability, arithmetic disability, and attentional deficit disorder. The arithmetic disability was conceptualized as normal reading and language skills, but difficulty with computational arithmetic, fine motor coordination and efficient written work. Siegel and Ryan (1989) found that children were impaired in both listening span and counting span when their learning difficulties in arithmetics coexisted with problems in reading. On the other hand, children with learning difficulties in arithmetic alone were impaired only on counting span. Siegel and Ryan (1989) suggested a low-capacity domain-general working memory responsible for the pervasive learning difficulties in both reading and arithmetic learning, while an low-capacity domain-specific working memory responsible for the learning difficulty specific to arithmetic operations.

Hitch and McAuley (1991) studied ALD children aged eight to nine with their normally achieving controls. In one experiment, Hitch and McAuley examined the performance of their subjects on several working memory span tasks, for information storage during concurrent operations. In another experiment, they sought to determine whether the children with ALD would be impaired at counting or retaining temporary information when each was assessed separately. While the children with ALD were impaired specifically in the counting span task in one study, they also manifested slower counting speed and lower digit span when these two tasks were assessed separately. Hitch and McAuley (1991) argued that cognitive deficits of ALD children were not only restricted to concurrent span tasks involving counting, but also to counting speed and auditory digit span.

These studies of children with learning deficits in mathematics underscore the need for a clearer understanding of the structure and functions of working memory and its role in children's arithmetic learning.

Miyake and Shah (1999) posited that any model of working memory must address several issues. A model should specify how external information enters and remains active in the working memory system. It must also explain how information in the working memory is controlled and regulated. In addition, it should highlight the mechanism that limits the performance or capacity of working memory. Moreover, a model should address the role of working memory in performance of complex cognitive tasks and the relationship between working memory and knowledge stored in long-term memory. While there are several prominent models of working memory with a varying number of functional dimensions, the most pertinent to this study is a multi-component system originally proposed by Baddeley and Hitch (1974;

Baddeley, 1986), composed of subsystems that are able to perform the functions in Miyake and Shah's postulation. The vast majority of the empirical work on working memory and arithmetic has been conducted, using Baddeley and Hitch's multi-component model, as the model has considerable support from a significant body of converging evidence gathered from normal adults, neuropsychological patients with selective lesions and both normally and abnormally developing children (Baddeley, 1986).

Briefly, there are three components in Baddeley and Hitch's (1974) multi-component model. These are: the central executive, the phonological loop and the visuospatial sketchpad. The central executive acts as a limited capacity attentional system that initiates and controls mental operations. Its primary role is to resource the competing demands for information processing and temporary information storage necessary for performing complex cognitive tasks such as reasoning or language comprehension. It fulfils a number of separate but overlapping executive control functions, such as coordinating concurrent activities, switching retrieval plans, attending to information source, and maintaining and manipulating information in long-term memory.

The central executive is also thought to take part in coordinating the activities of the other components of the model, which act as limited capacity, modality-specific subsidiary systems (Baddeley, 1996). One of the subsidiary systems is the visuospatial sketchpad, which is dedicated to storing and manipulating spatial or visual information. Another system is the phonological loop, which is concerned with maintenance and processing verbal material. The idea that the system has limited capacity leads to the proposal that when a complex cognitive task overloads one or more subsystems, performance of the working memory as a whole is undermined.

Further evidence from studies of the normal population suggests that different components of working memory might have specialized roles in arithmetic (Ashcraft, 1995).

2. Phonological Loop

The phonological loop is a speech-based system for storing short-term verbal information and is thought to have evolved from language perception and production systems It has been fractionated into a passive phonological store and an active rehearsal process (Baddeley, 1986; 1998). Short-term verbal memory, proposed to be the primary function of phonological loop, is thought to have a central role in the acquisition and execution of basic educational skills and has often been associated with the development of children's reading, comprehension, and arithmetic skills (Geary, 1990; Swanson, 1993). Evidence linking poor arithmetic skills with a short-term verbal memory deficit comes from a number of studies, with most of these studies using verbal recall tasks that tap on the storage capacity of the phonological loop.

Logie, Gilhooly, and Wynn (1994) suggested that the role of the phonological loop was to keep a record of running verbal information, which is important to maintain accuracy in calculation. Logie et al. (1994) used articulatory suppression as an active phonological secondary task. Loading the phonological loop in this way was assumed to prevent subvocal rehearsal, and may lead to decay of verbal information stored in phonological loop. Logie et al. found that performance of the concurrent, secondary, verbal task had a significant effect on arithmetic performance. Performance of the secondary task was also disrupted by the concurrent arithmetic task.

Children with arithmetic learning difficulties in Siegel and Ryan’s (1989) study significantly under-performed normal achievers on the working memory task that involved counting and were believed to have specific difficulties in remembering arithmetic information critical to problem solution, a function that is assumed to be covered by the phonological loop in the working memory model of Baddeley and Hitch (1974, 1986).

Lee and Kang (2002) studied whether and how working memory related to arithmetic functions. Ten university students, aged twenty-four to thirty years, were administered arithmetic tasks in a session where the administration of the arithmetic task was accompanied by a phonological suppression task. The subjects were asked to whisper a non-word string while solving arithmetic problems. Results indicated that the reaction times for the multiplication task were significantly delayed by the concurrent task, causing suppression of the phonological rehearsal function.

Given that many previous studies found some form of short-term memory deficit relative to lower arithmetic competency, findings in some recent studies suggest that poor arithmetic skills might not be closely connected to capacity in short-term verbal memory span. Butterworth, Cipolotti, and Warrington (1996) reported the case of a patient, MRF, who showed poor performance on short-term verbal memory tests and abnormally fast forgetting as a result of neurological disorder, but whose arithmetic skills were intact. To assess the relationship between short-term memory capacity and performance in arithmetic calculation, MRF was administered in the Paced Auditory Serial Addition Task, in which the patient was required to add a new digit to the previously presented digit, ignoring the intervening response. Despite his rapid decrement in performance of single digits and letters with both auditory and visual presentation in a forgetting task, MRF performed well on this calculation task. MRF's profile suggests the capacity of short-term memory might not have a strong connection with arithmetic skills. However, we should be cautious in extrapolating these results to other age groups. Butterworth, Cipolotti, and Warrington (1996) suggested that adults and children may be using different procedures in solving arithmetic combination. With a well-established system of arithmetic facts, adults would be more likely to use retrieval on the task, while children would depend more on backup counting strategies which may place a heavier load on short-term memory.

However, inconsistent findings were also found in studies with children. Bull and Johnston (1997) investigated the cognitive factors responsible for children's arithmetic learning difficulties (ALD). Sixty-eight 7-year-old children were tested with measures of short-term memory, processing speed, sequencing ability, and arithmetic fact retrieval. Bull and Johnston (1997) found no significant group differences between the arithmetic difficulties group and the control group in measures of short-term memory, when reading ability had been controlled. While speed of item identification and information processing were found to be the best predictors to arithmetic attainment, short-term memory was shown to account for the least amount of variance in regression model with other variables.

The presence of these inconsistent findings not only highlights the need to explore in more detail the role of short-term memory in children's arithmetic skills, but also the potential influence from other systems such as the central executive and visuospatial sketchpad.

3. Visuospatial Sketchpad

The structure of the visuospatial sketchpad has consistently been treated as a single component for processing and storage of visual and spatial information (Logie, 1991). The involvement of functions related to the visuospatial sketchpad in arithmetic performance was considered in a number of studies.

Dehaene (1992), in a review of essential findings and current points of numerical cognition, proposed the triple-code model for human numerical cognition. In this model, there are three types of number codes in human numerical cognition, namely, visual-arabic, auditory-verbal, and analog magnitude code, which are differentially involved in various kinds of arithmetic operation and number processing. Dehaene suggested that an individual would activate an “approximate mode” through which he or she could access and manipulate a mental model of approximate quantities similar to a “mental number line” when involving comparison of quantities, or approximate calculations.

In a study of working memory impairments in children with specific arithmetic learning difficulties (ALD), McLean and Hitch (1999) administered a working memory battery of 10 tasks to 4th-graders. The battery included a Corsi Block task and a visual matrix task that were used to assess spatial and visual functions of the visuospatial sketchpad, respectively. The results showed that children with ALD under-performed age-matched controls only on spatial function but not on visual function. When chronological age was controlled, spatial function correlated with arithmetic ability but not with reading ability. However, in a study with 3rd graders and with different grouping criteria (Bull, Johnston, and Roy, 1999), spatial function, as manifested by performance in the Corsi block task, as in McLean and Hitch’s (1999) study, was not related to children’s mathematical ability.

Heathcote (1994) examined the role of visuospatial working memory in the adults’ mental addition of multidigit addends. A standard task, devised to assess ability of mental calculation of 2 3-digit addends was administered, either visually or auditorily, to 12 adults, in conditions with either spatial interference or articulatory suppression. Disruption in performance in either of these conditions suggested a slave system was involved in mental arithmetic. The results demonstrated that both phonological and visuospatial subsystems were involved in mental arithmetic, and visuospatial interference led to the greatest disruption in complex problems involving carrying. Heathcote proposed that the phonological loop acts as a storage device, retaining both the initial problem information and running total, visuospatial responsible functions such as keeping number place, and carrying in multidigit additions.

The visuospatial sketchpad was also found to be related to arithmetic function in a study by Lee and Kang (2002). Ten university subjects were given an arithmetic test in another session with a dual task for suppressing visuospatial activities. They were asked to remember the shape and location of a series of decorative figures while performing arithmetic tasks. Results indicated that the concurrent visuospatial suppression task had significantly hampered the performance of the subtraction task. Based on the results of their pilot study, Lee and Kang (2002) proposed that arithmetic function and working memory subsystems are related in an operation “type-specific” manner.

Geary and Burlingham-Dubree (1989), in a study to investigate the validity of Siegler and Shrager's (1984) strategy choice model for addition, administered the Wechsler Preschool and Primary Scale of Intelligence (WPPSI) and the Arithmetic subtest of the Wide Range Achievement Test (WRAT) and simple addition tasks to 42 pre-school children. Results indicated that differences in strategy-choice variables were significantly related to two measures that require reproduction of simple and complex geometric designs and spatial scanning. Geary and Burlingham-Dubree (1989) concluded that was a close relation between pre-school children’s visuospatial ability and the level of accuracy in their usage of strategies for addition.

Evidence from a recent study by McKenzie, Bull and Gray (2003) suggested an age-related change in the importance of phonological and visuospatial functions. In an attempt to determine the effects of the two subsidiary slave systems of working memory and strategies being used in arithmetic at various developmental points, children aged 6 to 9 years were divided into two age-groups and administered simple mental arithmetic tasks, as well as measures for working memory, in three testing conditions. The first was the baseline condition, in which arithmetic tasks were administered without any disruptions. Concurrent phonological and visuospatial interference were introduced in the other two conditions. Although previous studies with older subjects suggested the greater importance of the phonological system over than visuospatial system, in this study, the performance of younger children was more affected by the concurrent visuospatial interference, while the older children were more affected by concurrent phonological disruption. McKenzie, Bull and Gray (2003) suggested that the results indicated that the younger, primary two children were relying on strategies, which load on visuospatial sketchpad. Older children, as those primary 4 children in the study and those with a better-developed sub-vocal rehearsal function, would adopt a mixture of strategies that primarily rely on verbal function, with visuospatial function as auxiliary support. These results support the hypothesis that children use different strategies at different ages; younger children almost exclusively utilise visuospatial strategies in mental arithmetic, whereas older children use a mixture of phonological and visuospatial strategies.

4. Central Executive

The central executive of working memory has been well-known for its connection with the phonological loop and visuospatial sketchpad. However, many other functions have also been proposed as being under the control of executive functioning. Diamond (1989) suggested an inhibitory function where dominant action tendencies are suppressed in favor of more goal-appropriate behavior. Baddeley, Bressi, Della-Sala, Logie, & Spinnler (1991) studied dementia patients' performance on dual-tasks. The progressive deterioration in performance shown by these patients was suggested to be due to impairment in the functioning of the central executive. Based on a number of psychometric studies and the influence of the supervisory activating system (SAS) model of attentional control by Norman and Shallice (1980), Baddeley (1996) also suggested that the central executive is involved in attentional control, generation of random numbers, process control and switching strategies. In addition to the functions of process control and attention allocation, the central executive is thought to be able to access information from long-term memory, although it is not confirmed whether the central executive itself has storage capacities (Ericsson & Kintsch, 1995).

Cognitive difficulties associated with poor executive functioning include disrupted organizational and planning skills, generalized memory deficits, difficulties with mental flexibility, and poor task initiation, as well as behavioral difficulties such as distractibility and problems with sustained attention. In the light of this, studies in developmental neuropsychology have also suggested that the frontal lobes could be the neurological base of central executive with reference to the complex functions thought to be under the coverage of this working memory component (Welsh & Pennington, 1988).

Case (1992) summarized and reviewed the fundamental changes of attentional, executive, and self-reflexive processes between the ages of 5 and 10 years. By examining the developmental trend of the frontal lobe, he gave further suggestion to the potential role of the central executive in individual differences in cognitive skills, including arithmetic. He postulated that frontal lobe functioning in children shows its main developmental increase from the ages of 7 to 10, and children reach adult performance levels on measures of executive functioning at around 10 years of age. He presumed that different rates in developmental advancement before the age of 10 could be the source of individual differences in most of the cognitive skills requiring executive function.

In the study to examine the functional aspects of working memory and their relative contribution to adults' mental arithmetic performance, Logie et al. (1994) used a dual-task procedure to examine the extent to which disrupting each component of working memory would affect mental arithmetic performance. Random letter generation was used to disrupt functioning of the central executive. Results indicated that mental addition performance was greatly hampered by the concurrent generation of random letters. Logie et al. (1994) suggested that mental arithmetic could be loaded on central-executive resources.

The central executive has also been reported as being related to arithmetic fact retrieval. In a single case study, Kaufmann (2002) reported the performance of an adolescent, M. O. who was diagnosed with severe developmental dyscalculia, and literacy problems. Despite outstanding difficulties in retrieving arithmetic facts, M.O. demonstrated a rather well-preserved procedural skill, which was manifested by his intact ability to solve multidigit written calculations. M.O.'s performance on the Brown-Peterson Interference Task indicated a deficient central executive of working memory. Give that M.O. had intact functioning of the phonological loop, Kaufmann (2002) suggested that M.O.'s problem in retrieving arithmetic facts arose from his application of time-consuming backup strategies that relied heavily on the resources of the central executive.

However, a limited number of studies focused specifically on the relationship between the central executive and arithmetic performance in normally developing elementary children. Many of these studies were conducted with adult or neuropsychological populations. Therefore, generalization of these results to developing children may not be appropriate. Nevertheless, with the finer picture of the central executive's functional structure, gained over the past decade, a growing body of research has been devoted to learning how this component in working memory relates to or affects children academic competency.

Lehto (1995) studied the relationship between working memory, specifically the functioning of the central executive, and academic attainment of Finnish teenagers. In addition to the measures for the phonological loop, several working memory span tasks were administered to measure the capacity of working memory. A memory-updating task was also administered to specifically measure the central executive. Data about the teenagers' attainment in four academic subjects were collected for evaluating the relationship with the performance in working memory tasks. Results showed that the memory-updating task, the measure that was prescribed for assessing the capacity of central executive, correlated strongly with all academic subjects, especially with attainment in mathematics, and was not mediated by the contribution of simple memory tasks.

Passolunghi and Siegel (2001) conducted a study to investigate the relationship between impaired performance in arithmetic problem solving, working memory, short-term memory and inhibitory control of poor arithmetic problem solvers, using four working memory span tasks, as well as four a short-term memory tasks. The results showed that poor problem solvers had lower scores and made more intrusive errors, which reflected an inability to control and to ignore irrelevant or no longer relevant information. This finding gives partial and indirect support to the role of the central executive, which is believed to be responsible for attentional control in arithmetic problem solving.

In sum, the studies discussed in the above review converge on a perspective that each, of the three components of the working memory system proposed by Baddeley (central executive, phonological loop, and visuospatial sketchpad) plays a role in arithmetic under different conditions. For example, DeStefano and LeFevre (2004), in their review, suggested that using problems in mental arithmetic that involved multiple digits would be more likely to reveal interactions between all the components of the working memory system.

However, not all investigators have found a relation between working memory and performance in arithmetic tasks. Kail and Hall (1999) investigated the roles of arithmetic knowledge, processing time, memory and reading skill on the performance of arithmetic word problems. Results indicated that word-problem performance was predicted by arithmetic knowledge. Of the three general information-processing skills investigated in this research, while processing time had demonstrated the strongest and most consistent relationship with arithmetic word-problem performance, the effects of working memory on word-problem performance were the smallest and least consistent. Given the limited number of studies and inconsistent findings, more research needs to be conducted with a larger cohort of normally developed school children before a clearer picture of the influence of working memory on elementary arithmetic competency can be sketched.

Chapter Three

Understanding of Fundamental Arithmetic Concepts and Use of Calculation Principles—Literature Review

1. Overview

The development and acquisition of computational skills and conceptual understanding has been a major concern in the field of mathematical psychology (Resnick & Omanson, 1987). Over the last two decades, researchers have argued over which is more important, procedural knowledge or concepts, as well as the order of their development. Hiebert and Lefevre (1986) stated that the developmental relationship between conceptual knowledge and procedural knowledge is not as straightforward and cannot be readily conceptualized as either “skill-first” or “concept-first”. Baroody and Ginsburg (1986) also suggested an iterative relationship between the two domains, in which conceptual knowledge can lead to invention in procedural knowledge, while the application of procedural knowledge can also contribute to advances in conceptual knowledge.

Nevertheless, conceptual understanding of fundamental arithmetic concepts and procedural knowledge are two critical areas to the understanding of children’s arithmetic development. Hiebert and Lefevre (1986) defined procedural knowledge as consisting of knowing the forms of arithmetic, as well as knowing step-by-step linear or hierarchical implementation, while conceptual knowledge, on the other hand, involves linking new information to an existing knowledge structure, as well as linking existing, but isolated aspects of knowledge. They also concluded that linking conceptual and procedural knowledge could greatly and mutually benefit both domains. For example, conceptual understanding involves understanding of the properties of and relationships between arithmetic operations, which include single or multidigit addition, subtraction, multiplication and division (Hiebert & Lefevre, 1986). A thorough understanding in the concepts may predispose the ability to derive of calculation principles such as inverse, reversal, other associative-based principles, especially where standard calculation procedures are cumbersome and time-consuming.

2. Fundamental Concepts of Elementary Arithmetic

Among studies of multidigit addition, subtraction and simple multiplication, knowledge of place value and commutativity has been consistently suggested as critical to the mastery and competency of arithmetic.

Place value principle refers to the numerical notation system in which the position of each digit determines the value it represents. In integers, the digit at the far-right of the number represents units; moving to the left the next digit represents tens, the next, hundreds, and so on. Commutativity refers to the principle that changing the order of the addends, or multiplicand and multiplier does not change the sum or product. For example, both the products of 8 x 3 and 3 x 8 are equal to 24, and 12 + 33 is equal to 33 + 12.

While calculation principles are derived knowledge about arithmetic operations that can allow a wide range of variations, the emergence of several principles can be well observed in the elementary levels of arithmetic learning. Included in the current study are inverse, reversal, and associative-based principles such as operand-plus-or-minus-one and operand-plus-or-minus-ten principle. These principles vary in a number of dimensions. For example, while calculation principles are assumed to provide shortcuts in solving some originally time-consuming calculations, wherever theoretically appropriate and applicable, answers that can be provided by commutativity, inverse, and reversal principles are comparatively more apparent, whereas those by the associative-based rules are less transparent in such a way that a certain level of insight and judgment are demanded.

It should be noted that the components addressed above and below are not to be regarded as a complete list of components critical for elementary arithmetic calculations, either from a mathematical or educational point of view.

Place Value

The concept of place value is a central concept underlying both our system of arithmetic notation and the operational algorithms that form the basis of arithmetic (Geary, 1993). Our number system, taught in elementary mathematical education, is based on place value in a base-10 system; that is, the value of each digit in a number is determined by its relative position in the number. For integers, the digit at the far-right of the number represents units; moving to the left, the next digit represents an increasing power of ten, as the value of the digit increases by a factor of 10 with each move to the left.

Understanding the place value in early elementary arithmetic operations relating to the ability to process arithmetic operations ranged from adding or subtracting 10s and units (10 + 1 = 11); tens and tens (10 + 20 = 30); and to arithmetic combinations involving several multi-digit items (78 + 43 - 59 = 62). In research, tasks for assessing place-value usually involve counting and number identification, positional knowledge, and digit correspondence, in which a subject is asked to indicate the value represented by a digit in a number. In classroom practice, educators and researchers often use concrete referents to represent the multi-digit numbers to assess and teach the concept of place value. For example, Base-10 cubes or clips are the most commonly used concrete representations.

The mastery of arithmetic combinations and conceptual understanding of the place-value in a base-ten number system facilitates multi-digit calculation. To be successful in performing multi-digit addition and subtraction, children must learn the carrying procedure for addition and the borrowing procedure for subtraction. For example, to perform a multi-digit subtraction problem in a written columnar format, a child could process single columns from right to left. The child has to “borrow” when the bottom number is greater than the top number and if top number is zero. Understanding these procedures requires understanding of the concept of place value. It has been suggested that instruction emphasizing concepts of place value and how they relate to steps in a procedure usually leads to increases in both conceptual and procedure knowledge (Rittle-Johnson & Alibali, 1999). Through formal instruction, children gradually learn conventional procedures for performing multi-digit addition and subtraction problems. Skilled performance in multi-digit addition and subtraction requires knowledge of several complex procedures. Although many primary-school children are competent in following rules and procedures, they may follow the wrong ones. Memorizing a procedure without understanding its underlying principles can lead children to make consistent mistakes in questions with a more complex structure.

In a longitudinal study of first through third graders, Hiebert and Wearne (1996) found a close relationship between children’s understanding of multi-digit numbers and their level of computational skill. For example, it was shown that children who developed the earliest understanding of place-value concept in base-ten system performed at the highest level on computational tasks at the end of third grade. The finding also suggests that an early understanding of our number system leads to greater appreciation and participation in learning mathematics throughout primary school.

Ginsburg (1997) suggested that the use of a “cognitive clinical interview" method could offer a richer account of children's mathematical thinking. One of the five key areas of focus highlighted was “bugs”, which refers to systemic erroneous strategies in computations. For example, second-graders often do not correctly carry in multidigit addition (Fuson & Briars, 1990). The children either write the two-digit sums beneath each column of single-digit addends or ignore the carried values. An example of an error in addition would be that the sum for 46 + 85 would become either 1211 or 121.

Erroneous procedures related to place-value can also be easily observed in subtraction. For example, children might subtract the smaller number from the large number, regardless of whether the large number is in the minuend or subtrahend; in this case, the answer for 312 – 145 would become 233. Other areas of erroneous procedure include ordering numbers, setting out horizontally presented computations, and addition and subtraction involving carrying or trading. The occurrences of these consistent and systematic errors in multi-digit addition and subtraction procedures may indicate the presence of misunderstanding or immature understanding in mathematical knowledge, specifically in place value.

Additive & Multiplicative Commutativity

Commutativity is a case-sensitive concept involving the irrelevance of operands where changing the order of the operands in an arithmetic problem does not change the result or solution. Both addition and multiplication are commutative in nature. From a mathematical point of view, the commutative property of multiplication can be expressed as (a)(b) =(b)(a) while that of addition can be expressed as (a)+(b) =(b)+(a). For example 2 + 3 equals to 3 + 2, just as 3 x 2 equals to 2 x 3.

Baroody and Ginsburg (1986) acknowledged that the knowledge of a commutative relation may be represented as schema-based view. According to this view, relational learning and transfer are an integral part of mastering basic number combination. Children notice arithmetic relations, which allow them to utilize existing knowledge. Therefore, by noticing that addition is commutative, they can reduce by almost half their effort to memorize the basic additive combinations. In the light of this, it is not necessary for those who understand the principle to store both forms of the commuted pairs in memory (Baroody & Dowker, 2004). Understanding these relations serves to convert small-factor-first expressions such as 2 x 3 into larger-factor-first expressions such as 3 x 2. Therefore both 2 x 3 and 3 x 2 to be stored in the retrieval network as a single 2 x 3 = 6 association. This explains why children are able to retrieve big-factor-first combinations of commutated operands as fast as the smaller-factor-first combinations, while the former is usually taught long after the latter.

A similar view of commutativity has also been conceptualized in a model called the "Identical element model", proposed by Rickard, Healy, and Bourne (1994). The identical elements model incorporates three basic assumptions. First, for simplicity, the model assumes distinct and sequential perceptual, cognitive, and motor stages of performance. Second, it assumes that answer retrieval occurs exclusively within the cognitive stage. The third assumption is that representation of each arithmetic fact within the cognitive stage can be fully characterized in terms of its three essential constituent elements, which include the two corresponding presented numbers in a fact and the operation formally required.

Stated in this model, the order of the numbers in commutative operations is not represented. Thus the two orders of operand in a commuted multiplication problem map on to the same single representation within the cognitive stage. For non-commutative operations, the order is preserved (thus, 20 ÷ 4 and 4 ÷ 20 would be represented uniquely).

In addition, a problem presented in an Arabic format, such as 4 × 7, and the same problem presented in a written verbal format, such as *four times seven*, will access the exact same semantic memory "chunk" within the cognitive stage. In summary, any problems that differ only with respect to the format of presentation or the relational characteristics among the elements will access the same chunk. In contrast, problems that differ with respect to one or more elements will access completely different memory chunks. Thus, 4 × 7 and 7 × 6 will access completely different memory chunks. A later study by Rickard & Bourne (1996) provides further support to the "identical elements model" in the organization of multiplicative arithmetic skill in memory.

In light of this model, a single representation for commuted combination would also account for the transfer for practice effects to unpracticed but commuted combinations. The findings of a study by Baroody (1999) support this view. A training experiment focused on third graders with negligible mastery of multiplicative combinations involving factors from 3 to 9. After being screened for baseline information, subjects given structured training on different subtests of combinations, followed by a post-test, specifically designed to identify any transfer of learning. Results indicated that practice on a subtest of combinations could facilitate the learning of unpracticed but commuted combinations. Baroody (1999) suggested that children could devise more flexible and accurate strategies and use commutativity principles to master combinations.

Given the close comparison in the format of presentation of combinations of this kind, one would assume that it is easy for even a primary school child to recognize the commutative nature of two commuted operands and to state the answer of a combination based on their knowledge about the solution of the commuted pairs. However, previous studies have presented a less optimistic view.

Vergnaud (1988) has pointed out that the property of multiplicative commutativity of multiplication may not be readily accepted by school children. Certain instructional and curricula pre-requisites should be given before children can assimilate the concept. In an empirical test of Vergnaud's hypothesis among schooled subjects, Nunes & Bryant (1996) showed that even 9- to 10-year-old school children did not easily accept multiplicative commutativity when they tried to solve multiplicative problems in which any number could be taken as either multiplier or multiplicand.

Regarding the development of the concept of commutativity, evidences from previous studies state that children would be able to discover additive commutativity at the numerical level via computational experiences. Baroody and Gannon (1984) assessed kindergartners' understanding of the commutativity principle and their use of the *min* procedure, a more advanced mode of counting. Children were administered tasks to measure their procedural knowledge of min procedure, accompanied with judgmental tasks in which commuted pairs on addition combinations were presented. Children were considered successful on the judgmental tasks if they responded quickly and accurately to the question without overtly computing the answer. It was indicated that children who succeeded in the commutativity tasks were more likely and ready to use the min method in problem solving. Baroody and Gannon (1984) concluded that computational experience is associated with commutativity performance at this level. This perspective is consistent with Resnick's (1992) account that computational experience is sufficient to play a key role in reconstructing an understanding of commutativity at higher levels of mathematical thinking.

Contrary to the situation in the acquisition of commutativity for addition (Petitto & Ginsburg, 1982), instructions that specifically emphasize the property were suggested to have played a critical role in the development of the knowledge of multiplicative commutativity and its consequential application in solving multiplication problems.

To solve a simple addition combination, children can either access the answer by direct retrieval or use backup strategies such as finger counting and verbal counting. The situation here is similar for multiplication. For example, 6 times 7 is equal to adding 6 repeatedly for 7 times or adding 7 repeatedly for 6 times; therefore, if children fail to retrieve the answer to a multiplicative combination of 6 and 7, they can use a repeated addition method to solve it. Although frequent and successful retrieval will increase the chance of achieving understanding of this property of multiplicative commutativity, one's fixation or dependence on the use of repeated addition in solving multiplication combinations would postpone the recognition of the relations.

Data from a previous study by Schliemann et al. (1998) supports the suggestion that school instruction focusing on multiplicative commutativity is critical for the use of the commutative property for solving multiplication problems. Children approach multiplication in schools by learning about multiplication tables. They have many opportunities to realize that the same result is obtained if one multiplies 2 times 3 or 3 times 2. Moreover, they are explicitly taught that commutativity is one of the properties of multiplication, and they receive intensive training on algorithms entailing multiplicative relationships with emphasis numerical computations in which numbers are frequently used without reference to physical quantities. This type of training may accelerate use of multiplicative commutativity among school subjects.

3. Use of Calculation Principles

According to the model of mathematical development proposed by Resnick (1992), lack of understanding and erroneous usage of any derived calculation principle may suggest the areas of weakness in the children's development of arithmetic reasoning. Resnick's suggestion echoed a point suggested by Piaget that without understanding the inverse relations between addition and subtraction, no one could grasp the nature of these two operations. He added further that the understanding of inverse relations is an essential part of the groupings that underlie concrete operation in his theory. Despite its importance in revealing children's development of arithmetic reasoning, it is surprising that the topic has received less attention than other areas in mathematical cognition, in the past decades (Geary 1993).

One crucial aspect of arithmetic reasoning is the ability to derive and predict unknown arithmetic facts from known facts. The particular derived fact strategies that are the main focus of this study are commutativity (e.g. if 8 + 6 = 14, then 6 + 8 = 14), inverse (e.g. if 46 + 27 = 73, then 73 – 27 = 46), reversal (e.g. 11 – 8 = 3, then 11 – 3 = 8), associative-based principles such as operand-plus-or-minus-one and operand-plus-or-minus-ten principles (e.g. if 9 + 4 = 14, then 9 + 5 = 14 + 1 = 15; and if 19 + 14 = 33, then 19 + 24 = 33 + 10 = 43).

Dowker (1998) studied the relationship between children's calculation performance and their use of derived facts in addition and subtraction. The children were administered an arithmetic reasoning test involving use of arithmetic principles in derived fact strategies. Two numerically related problems were presented to the children in each trial. One problem was assigned as a sample and the answer to it was given. Children were asked to solve another problem, which could possibly be solved by making use of the answer of the sample problem. Trials with numerically unrelated problems were given as foils. The test consists of items that tap into children's use of a variety of principles for addition and subtraction, such as inverse principle, reversal principle, and associative-based principles. Children were regarded to be able to use a principle if they could derive answer to target problems using the answer of the corresponding sample problems. Results indicated strong associations between calculation competency and use of derived calculation principles. Also, although effect was not significant, older children in the study tended to use more derived principles in a subtraction subtest. Dowker suggested children with flexible understanding in numerical facts and procedures might have higher tendency to access arithmetic principles of various types or, conversely, that the ability to use derived principles leads to calculation competence.

Bryant, Christie and Rendu (1999) studied the understanding of the inversion principles of 5- and 6-year-old children. The children were divided into younger and older groups. Thirty-six problems, structured to assess the inversion principle, were further categorized into six testing conditions to examine whether children would understand the principle greater in some contexts than in others. Based on the results the study, Bryant and his colleagues concluded that children as young as 5 years understand and frequently used the inversion principle and that they did so in a “genuinely quantitative” way. Some children were even able to use the principle flexibly in way that they demonstrate usage of the principle even in a testing condition that required decomposition.

Rasmussen, Ho and Bisanz (2003) examined whether children in pre-school and grade one were able to use the principle of inversion. Children were present with three-term inversion problem and standard problems of similar magnitude. Similar to the study by Bryant and his colleagues (1999), three testing conditions were included to examine the children’s decisions with respect to quantitative and non-quantitative features of the problems. Results showed that both preschool and grade one children were able to use inversion quantitatively. Rasmussen and her colleagues suggested that this principle is available in some form prior to extensive formal instruction in arithmetic. Though they suggested that using inversion to solve three-term arithmetic problems could be the effect of formal schooling, they did not support the view of the role of schooling as a pre-requisite for the majority of children to use inversion.

Hanich, Jordan, Kaplan, and Dick (2001), in their multi-year longitudinal project, compared performance of grade two children with difficulties in mathematics but not in reading and second graders with difficulties in mathematics as well as in reading. Also included in the study were a group of children with difficulties in reading but not in mathematics and a group with normal achievement in reading and mathematics. They assessed areas of mathematical cognition that included basic calculation principles, with respect to a previous observation that a majority of children in grade three had demonstrated ability to derive answers from known arithmetic facts. Results indicated that both groups of children with mathematical difficulty performed worse than children in control, and children with both mathematical and reading difficulties performed worse than children with only reading difficulties. Hanich and her colleagues suggested that children with mathematical difficulties might have a weak and unstable understanding of the relationships between and within arithmetic operations. Among those principles assessed in the study, performance on commutativity was the best while performance on inversion principle was generally weak.

Yet, currently available findings about the development of mastery in these two areas are far from consistent. For example, although some of the aforementioned studies present evidence that even pre-schoolers could be capable of using inversion principles, at least, a study by Hanich and her colleagues (2001) revealed that elementary school children could demonstrate impoverished understanding and usage of calculation principles.

In summary, there is a general belief that children’s numerical understanding of concepts and usage of calculation principles will increase during the early primary school years. The relationship between arithmetic competency and the mastery of a number of underlying concepts and principles has yet to be shown.

Baroody (2004) stated that achieving mastery and competency in arithmetic skills is a complex task that requires both declarative and procedural knowledge, as well as flexibility in conceptual understanding. However, research in these areas of children’s elementary arithmetic development has received comparatively little attention and many of the currently available findings are narrowly focused on one or two areas of concepts or calculation principles. In the light of this, further studies in this area of elementary mathematical cognition nevertheless provide an informative vision of children's development of mathematical cognition, as well as important insights for instructional design (Rasmussen et al., 2003).

Chapter Four

Purposes and Hypotheses

This is the first local study to investigate the influence of working memory capacity, understanding of fundamental arithmetic concepts and use of calculation principles, on individual and group differences in early elementary arithmetic learning competency. Theoretically, there should be a cognitive explanation for the differences in arithmetic problem solving behaviors of individuals with either higher or lower arithmetic competency.

The inconsistency in findings, uses of different screening criteria, and national or culture differences in a number of factors such as curriculum design, primary mathematical language, and early childhood experiences, diminish the applicability of overseas findings to the local situation.

The first purpose of the current study is to explore the differences, if any, in the functioning of the three components of working memory with respect to the multi-component model proposed by Baddeley and Hitch (1974), and a numerical-related working memory as a whole, between children in each grade who demonstrate different levels of arithmetic competency. It is hypothesized that children with higher competency in performing elementary arithmetic operations would outperform their same grade less competent peers in the tasks for assessing functions of working memory.

With reference to previous findings that the phonological loop is involved in arithmetic functions, differences between group and grade in the performance of this component will be observed. However, of particular interest is the question of whether the difference in function is domain-specific or domain-general. Passolunghi and Siegel (2001) found that poorer problem solvers were impaired on passive storage of numerical information but not on material that included words. With reference to this recent finding, it is hypothesized that differences in performance may be seen only in digit span tasks that relate to numerical information, rather than to both phonological memory span tasks.

From the account of mathematical cognition in Dehaene's (1992) triple code model, where visual representations of magnitude could be involved in mathematical tasks, it is hypothesized that group differences in the functioning of visuospatial sketchpad will be identified. Moreover, of greater interest of is the identification of the process component involved in differences in arithmetic competency. Measures of visuospatial sketchpad used in the current study tap on two functions, a simple visual recognition function and a visual serial recall function. Previous findings do not adequately explain this issue. Bull and Johnston (1997) suggested that mathematical ability was not found to be associated with the performance in a serial recall measure of the visuospatial sketchpad. However, children's use of written columnar calculation to solve multi-digit computation has been well observed and documented. The computation method involves recall and recognition of both numbers and the positional information of how the numbers should be arranged. From this information, it is possible that both visual serial recall function and visual recognition would relate to differences in arithmetic competency.

The functions of the central executive have rarely been studied separately. Previous findings indicated a positive result for group differences in the performance in these functions of the central executive. Given the role of the central executive proposed in the multi-component model (Baddeley & Hitch, 1974), of particular interest in the study of this component is whether it mediates the influence of the phonological loop or visuospatial sketchpad on differences in children's arithmetic competency.

Theoretically, differences in performance of general working memory span task may also be observed if group performances differ in the measure for the three working memory components. Special focus on the two versions of general working memory span task is placed on whether the results of these two measures support the hypothesis that a weaker performance manifested in span task can be explained in terms of task-switching hypotheses or by resource sharing account. Task-switching account suggests that there is no active maintenance of stored information, and that competes with the execution of concurrent operations in a span task. It hypothesizes that increased processing time results in reduced span. On the other hand, resource-sharing account suggests that limited resources of working memory are shared by both concurrent tasks of processing and storage. This latter account hypothesized that a trade-off between limited resources for processing and storage could be observed (Towse, Hitch & Hutton, 1998).

The second purpose of the current study is to explore the differences, if any, in understanding of fundamental arithmetic concepts and use of calculation principles, between children with different levels of arithmetic competency in each grade. It is hypothesized that children with higher competency in performing elementary arithmetic operations would outperform their same grade less competent peers in the tasks for assessing understanding of place value and commutativity, the application of calculation principles. The current literature on this topic provides no consistent and reliable answer. While some reports suggest that pre-schoolers can manifest sophisticated conceptual understanding that leads to their learning of arithmetic procedures, some other studies have reported children in elementary school displaying a discrepancy between conceptual understanding and procedural skills, and between these areas of knowledge and measured arithmetic competency.

Among the concepts and calculation principles covered in the current study, the concept of place value has received most attention and the findings regarding its relationship with arithmetic competency are by far the most consistent. Given documented the importance of multi-digit arithmetic computation, it could be hypothesized that differences in the task of assessing the understanding of this concept exist between groups and grades.

From the similarity in structure of the problem pairs, it is assumed in this study that the answers provided by commutativity, inverse, and reversal problem-pairs are more transparent than those of problem pairs in the associative-based principles. Thus, more prominent differences in the ability to use associative-based calculation principle will be observed.

The third purpose of the current study is to explore the pattern of contributions and interaction among working memory capacity, acquired understanding of arithmetic concepts and use of calculation principles, on children differences in arithmetic competency in each grade. Of particular interest is whether understanding of arithmetic concepts and use of calculation principles mediates the influences of the working memory capacity on children's arithmetic competency, or vice versa.

Moderation refers to the investigation of the statistical interaction between two independent variables in predicting a dependent variable. Mediation refers to the covariance relationships among an independent variable, a potential mediating variable, and a dependent variable. Since correlational analysis including all the variables suggested that the inter-correlations between the measures on arithmetic competency, working memory, and the understanding of concepts and use of calculation principles were found to be significant. It is not appropriate to describe the relationship between these three domains in terms of mediation. Moreover, from a theoretical point of view, both the temporary memory storage and the coordination activities of working memory are influenced by assessing contents from long-term memory (Baddeley, 2000; Mayer and Hegarty, 1996). Therefore, the presence, as well as the retrievability of information in the long-term memory, could mediate the relationship between children's working memory functioning and their performance on timed arithmetic operations. Hence, the lower performance in working memory found among individuals with difficulties in arithmetic could also reflect the differences in acquired strategies, mental heuristics, and prior knowledge stored in the long-term memory. It is hypothesized that understanding of place value and commutativity, and knowledge in the application of calculation principles would mediate the influences of the working memory capacity on children's arithmetic competency.

Only a few studies have attempted to explore in the relationship between acquired arithmetic knowledge and the functioning of working memory and the way they relate to competency in arithmetic. For example, Keeler and Swanson (2001) investigated the relationship between working memory, declarative strategy knowledge, and mathematic achievement in children with and without mathematical disabilities. The results suggested that working memory and math achievement are related to strategy knowledge, which is an example of acquired knowledge.

Furthermore, with reference to the effect of formal instruction on children's learning, it is expected that the relative importance of the influence of these three cognitive domains on arithmetic competency change with increasing level of schooling.

Chapter Five

Methodology

1. Participants

Three hundred and seven primary one to primary three students from a normal local mainstream primary school in Kowloon district participated in the initial screening. A measure of nonverbal intelligence, the Raven's Standard Progressive Matrices, one standardized Chinese word reading test, and one arithmetic screening tests were administered in the initial screening. Given that different schools differ on the teaching schedule, as well as instructional design, only one school was used as the source of subject selection to eliminate the potential effect of instructional differences on the result. Subjects were selected for the study according to their performance in the screening tasks.

Since the study aimed to investigate the difference between students with higher and lower arithmetic abilities, students in each grade were allocated to two groups according to their percentile rank in the arithmetic test. In each grade, students whose percentile rank fell between 10 and 40 were assigned to the lower arithmetic ability group, while those whose percentile rank fell between 60 and 90 were assigned to the higher arithmetic ability group. An arithmetic test was constructed based on local curricula in mathematics and consisted of three subtests, each designed to assess children's arithmetic abilities in each grade. The test for primary-one has thirty arithmetic problems presented in horizontal format. It was composed of problems that ranged from 1- to 1-digit (e.g. 3 + 6) to 2-digit 3-item problems (e.g. 53 – 16 + 29) addition and subtraction computations. Half of the problems involve either borrowing or carrying. The test for primary-two students also has thirty arithmetic problems and is composed of problems ranged from 2- to 2-digit (e.g. 24 + 18) to 3- to 3-digit 3-item (e.g. 157 – 148 + 116) addition and subtraction computations, half of which involve either borrowing or carrying. Four simple multiplication and four division problems were included in the test for primary two students. The test for primary three students is composed of problems ranged from 3-digit 2-item (e.g. 136 – 128) to 3-digit 3-item problems (e.g. 238 + 323 – 452) addition and subtraction computations. The test for primary three has also multi-digit multiplication and division problems with residuals (e.g. 146 x 6, 206 divided by 6). Children were given 20 minutes for finishing the test.

In the current study, about 30 percents of children from each grade were required for lower ability group, so as to maintain adequate sample size for analyses. However, those at lowest and highest 10 percents of students in each grade were deliberately excluded from the study. The reasoning behind this is that students with profound learning difficulties or significant giftedness are usually observed at the two extremes of a student sample in each grade.

Subjects' reading abilities and nonverbal intellectual abilities were assessed with the standardized Chinese Reading Test and Raven’s Standard Progressive Matrices respectively. Raven's Standard Progressive Matrices were adopted to measure the children's non-verbal reasoning ability and hence to estimate their intelligence. The children were required to select, from six or eight alternatives, the one that best completed the matrix. The test was administered in a group setting with no time limit and most of the Primary 1 children were able to finish it in 30 minutes. The Chinese Word Reading Test (developed and standardized by the Hong Kong Education Department in 1988) is a 1-to-1 individual test for assessing children’s Chinese word reading skills. Material consists of 65 two-character Chinese words of primary school level arranged in ascending order of difficulty. The children were asked to read the words aloud one by one. The task ended when the child had failed to read 10 consecutive words.

Table 1. Descriptive Statistics for the Grouping Information

		Primary One		Primary Two		Primary Three	
		Lower ability	Higher ability	Lower ability	Higher ability	Lower ability	Higher ability
Total Number (N)		26	25	28	26	27	28
Age (in months)	Mean	80.65	81.68	96.00	95.84	107.70	107.82
	SD	4.62	4.28	4.35	4.54	4.19	4.34
Gender	Male	12	14	15	14	13	16
	Female	14	11	13	12	14	12
Arithmetic Test	Mean	12.84	24.76	14.07	23.88	14.88	24.89
	SD	3.756	3.68	3.90	3.81	4.32	4.07
Chinese Reading	Mean	25.80	25.68	36.85	36.8	46.8	47.10
	SD	3.28	3.13	3.27	3.22	2.77	3.217
Raven (Raw Score)	Mean	19.38	20.08	24.61	25.04	36.00	36.00
	SD	4.61	4.39	4.95	4.85	3.99	4.50

Those who were found to have a reading age not compatible with schooling or a nonverbal intelligence score in the bottom 20% of the norm for their age were excluded from the study. Children were matched on reading so as to ensure that any differences identified could be attributable to their individual difference in arithmetic cognition. T tests were performed in each grade to ensure that the two ability groups were adequately matched on reading abilities and non-verbal intelligence. Results showed that the 2 ability groups in primary one did not differ in Chinese reading ability, $t(49)= 0.206$, $p > 0.5$, and nonverbal intelligence, $t(49)= 0.552$, $p > 0.5$, but differed in arithmetic, $t(49)= 14.49$, $p < 0.01$. For primary two, the 2 ability groups did not differ in Chinese reading ability, $t(52)= 0.081$, $p > 0.5$, and nonverbal intelligence, $t(52)= 0.323$, $p > 0.5$, but did in arithmetic, $t(52)= 9.81$, $p < 0.01$. Finally, the 2 ability groups in primary three showed similar results, as they did not differ in Chinese reading ability, $t(53)= 0.314$, $p > 0.5$, and nonverbal intelligence, $t(53)= 0.432$, $p > 0.5$, but did in arithmetic, $t(53)= 13.58$, $p < 0.01$.

In addition, children with a history of repeating their grade, with profound behavioral, emotional or serious attentional problems were excluded from the pool as well. Descriptive statistics for grouping information are shown in Table 1.

2. Materials

After the initial screening, one hundred and sixty children selected for the study were administered a battery of working memory tests, which consist of tasks that were designed to assess the functioning of each of the components suggested in Baddeley and Hitch’s (1974) multi-component model. The structure of some aspects of the working memory battery was designed with reference to a similar battery used in a previous study by McLean and Hitch (1999). Children were also administered tasks designed to test their understanding of place value and commutativity, and their ability to use calculation principles.

Working memory—Phonological Loop

Forward Digit Span. The Forward Digit Span task from the WISC-R subtest (Wechsler, 1974) was used as a measure of phonological short-term memory with numerical contents. The test is composed of span items with an increasing number of digits, from two to eight digit per span length, with two trials for each span length. The researcher started from a span length of two. Digits were read to each child at a rate of 1 second per digit. A span was considered to be correct if all digits were recalled in correct order and 1 point was given. When the child succeeded on both trials of a span length, the researcher moved to a higher level with span length increased by 1. If the child failed on both trials of the same span length, testing was discontinued. Children were tested once only.

Chinese Character Span. Chinese Character Span was used as a second index of phonological short-term memory. The design and the operation of the task follows those of the auditory digit span, except that the digits are replaced by Chinese characters which have pronunciations familiar to Cantonese speaking children. Similarity in structure allows direct comparison between the performance of the two phonological span tasks.

<u>Working memory— Visuospatial Sketchpad</u>

Squares Span. This is a self-developed task with reference to the design of the Corsi blocks task (Milner, 1971; Corsi, 1972). It tests the function of serial recall of visual information. Using MS PowerPoint, a total of 9 white identical squares is presented on a laptop screen with background set at black. The squares are arranged in random positions. In each trial the child observed a sequence of white squares highlighted into green, one at a time and at a rate of 1 square per second, and no square was highlighted more than once. After having observed a sequence of white squares highlighted into green on the screen, the child was asked to respond by using a pen to point out those previously highlighted squares in correct order on the screen. A span was considered to be correct if the sequence of squares was reproduced in correct order; then 1 point was given. Each child was given 2 trials with span length of 2 squares. The first trial started with a span length of two squares and increased from two to eight squares per span length. There were 2 trials for each span length. When the child succeeded on both trials of a span length, the investigator moved to a higher level with span length increased by 1. If the child failed on the both trials of the same span length, testing was discontinued.

*Visual Memory Test***.** The measure used in the study was adapted from the Test of Visual-Perceptual Skills (non-motor)—Revised (TVPS-R) visual memory subtest. The task assesses children's ability to remember, for immediate recall, all of the characteristics of a given form, and being able to recognize this form from an array of visually similar forms. The test consists of a total of 16 stimuli. These stimuli are in general different forms ranged from a simple geometrical shape to irregular pattern of lines and shapes. Each response array from which children were asked to pick out the previously presented stimuli had five different but similar forms, with only one of them representing the correct answer. Each stimulus was displayed for 2 seconds, followed by a 2-second delay, prior to the presentation of the response array. One point was given for each correct response.

Working memory—Central Executive

*Verbal Trail***.** The ability to switch retrieval strategies was assessed in the Verbal Trail task (Reitan, 1958). The task involves generating a stream of responses by alternating between two different sequences. The child was asked to read out loud from 1 to 12 and A to L following the order (1-A-2-B-3-C…11-K-12-L). Before commencing, each child was asked to recite the alphabet in a correct order from A to L and count from 1 to 12. Each child was also asked to practice with a sequence up to "4-D". The task did not involve reading. The children were asked to recall the order of number and alphabets from their memory. The dependent variable was the time taken to complete the task all over again.

*Crossing Out Task***.** A Crossing Out task was developed with reference to a similar task used by McLean and Hitch (1999) for measuring selective attention. It is similar to a standard cancellation task except that the target is repeatedly shifted. The task involves inhibiting responses to items previously designated as targets. Each child was given a sheet of paper with

10 rows of digits. The first digit in each line was colored in red, and identified as the target while the rest were in standard black typeface. The child was asked to cross out all the digits that matched the target digit in the same row. Each line of ten digits contained 3 to 4 randomly positioned targets. Two trial rows were given for practice. The dependent variable was the time taken to complete the remaining 10 rows.

Missing Item Task. The task was adapted from the design by McLean and Hitch (1999) in an attempt to measure the capacity to hold and manipulate information accessed in long-term memory. The task was a paper and pencil task with 12 problems, each consisted of a equation on the left with both addends but no answer (i.e. 3 + 4 = A), and a second incomplete equation on the right (i.e. A = 2 + b). Each question was presented in a form similar to the following example: 3 + 4 = A = 2 + B. The child was asked to answer orally the sum of the first addition (A) and then to complete the equivalence, that is, to write down the value of the missing addend in the second addition (B) so that they have correct sum. Two practice trials were given in which the experimenter explained the task with demonstration. The logic behind this task is that the child had to access long-term memory to complete the first addition and maintain the resulting sum, and then complete the second addition with this active information and information in long-term memory. Time taken to complete the task was the dependent variable. Athough this was a timed test that children were asked to finish it as soon as possible, they were told that accuracy of answers was of greater importance.

Working memory—General Working Memory Resource

Operation Span. The Operation Span task (Turner & Engle, 1989) was used as a measure of general working memory resources where the processing element involved arithmetic calculation. In each trial of this task, a short series of simple arithmetic problems (e.g. 3 + 2 followed by 4 + 9 in a series of two problems) was presented on the screen of a laptop computer. Each child was asked to verbalize the answer to each problem and recall all the answers in the correct order. The investigator recorded the responses of each child using a number keypad. Two practice trials were given for each child. The task began at each level with a series of 2 problems, with 2 trials at each level. After the child succeeded in both trials at each level, the experiment moved to the next level with the number of problems increased by 1. Span scores were obtained by noting the maximum number of answers that could be recalled entirely correctly in the trial. Accuracy of recall was assessed without taking into account whether the initial answers to the problems were calculated correctly.

As one of the aims of the current study was to explore the mechanism underlying the performance in working memory span task, the task was repeated in two conditions: the normal and extended version. In the normal condition, calculation problems were in a format of **a** + or - **b** = **c**, where **a** and **b** are integers less than 10, with **c** as the answer to be recalled (e.g. 2 + 3 =). In the extended condition, problems were in a format of **a** + or- **b** + or- **c** + or- **d**= **e**, where a, b are integers less than 10, but c and d are either 1 or 0 (e.g. 2 + 3 – 0 + 1 =).

<u>Understanding of Place Value and Commutativity</u>

In designing the tasks for place value, past research in place-value understanding and local school curricula in mathematics were taken into consideration. There are two measures for the concept of place-value (Ho & Cheng, 1997).

Digit Representation. In this task, there were a total of six items printed on a piece of paper. In each item, a multi-digit number was printed in black with one of the digits in the number circled in red. The number of digits ranged from 2 to 5. Each child was asked how much the value of number would decrease if the circled digit were to be changed to zero. Each child was required to give both oral and written responses to each item. For example, correct responses to the number 17**9**8 with “**9**” circled in red would be an oral answer of “ninety” and a written response of 90. One practice trial with feedback of whether the answer was right or wrong only was given. One point was given for each correct response.

Columnar Calculation. In each of the six test items in this task, the child was presented with a multi-digit arithmetic problem printed horizontally. These problems ranged from 2-to 2-digit, to 3-to 3-digit, and all the solutions involved either carrying or borrowing. Each child was asked to solve the problem using written columnar calculations. This required the child to rewrite the horizontally printed problem into a vertically aligned format, with one-place of one operand matching the position of another operand. The number of place-value-related errors among the six items was the dependent variable, rather than the accuracy of the solutions. While misalignment of value place and consistently subtracting the smaller number from the larger number were examples of concept-related errors, inaccurate retrieval of arithmetic facts, and miscalculation were not the case.

Problem-pair Comparison. To assess students’ understanding of additive and multiplicative commutativity, a comparison test was developed and used. The tasks consisted of 9 items among which 4 were designed for assessing additive commutativity. An additional 2 items were for multiplicative commutativity, and 3 items were foils. In each item, two 2 to 3-item problems were presented in written form. Each child was asked to state within 10 seconds if the answers of the 2 paired problems were equivalent and explain the method he or she used to find out the answer. The child was credited with one point if he or she could state response without any overt or covert calculation, and could explain the result solely in terms of the concept of commutativity. Foils items were mixed with the commuted test items to prevent children from simply repeating the response to previous items. Foils were also used to detect any misconception of commutativity. For example, a response, which stated that 27 + 45 –19 was equal to 27 + 19 – 45 and suggested that the problems were commuted, was an incorrect response.

Use of Calculation Principles

*Hinted Calculation***.** The ability to use calculation principles was assessed with a method similar to that of Dowker (1998). Each child was asked to solve twenty-five pairs of problems in which the given answer to the first problem in each pair could be used to solve the second. Four items were given to assess the use of the commutativity principle, the inversion principle, the reversal principle, the associative-based principles of operand plus-or minus-1, and operand plus-or minus-10. The problems were presented to children both orally and visually in a horizontal format Children were told to give an oral response as soon as they knew the answer to the problem, and to respond quickly. To prevent children from calculating, a 5-second time limit was implemented. All the correct answers was scored as “1” while those incorrect or “nil” responses was scored “0”. Two-digit two-item problems were used to prevent children from simply by retrieving facts easily. Five problems preceded by answers to numerically unrelated problems were given as foils. The six types of problems were complied and mixed together into a single list of problem-pairs.

3. Procedure

All students were tested in their school by a graduate student and eleven trained undergraduate students. For initial screening, one session of approximately 60 minutes was required for group administration of Raven’s Standard Progressive Matrices and the arithmetic test for each of the three grades. Three 60-minute sessions for individual administration of Chinese reading test were conducted 1 week after the group test.

All the other measures were administered three weeks after the initial screening and by the end of school year. Each child was tested individually in a quiet area and completed the tasks in one 50-minute session. The tasks were administered to each subject in an order such that no two tasks from the same domain were administered consecutively, so as to prevent fatigue in any sensory as well as cognitive modalities.

4. Data Analyses

Differences between groups and grades were analyzed using a two-way analysis of variance for each variable of working memory and the conceptual knowledge of place value and arithmetic commutativity. Planned comparisons were launched to identify significant differences in the children's cognitive functioning. With reference to the method used by Swanson and Sachse-Lee (2001), hierarchical regression, a series of stepwise multiple regression analyses, were conducted to determine if the influence of one factor of arithmetic competency could be substantially mediated by the presence of another factor. The analyses were conducted with the variables of working memory, understanding of conceptual knowledge, and use of calculation principles as predictors, and arithmetic competency as dependent variables.

Chapter Six

Results

An overall 2 ability groups x 3 grades Two-way analysis of variance (ANOVA) was performed to assess the main effects of, and the interaction between grade and ability on the performance of each task. Partial Correlations were performed to examine the interrelations among the measures. Planned comparisons were performed to compare the performance of the six groups in the study where necessary. Hierarchical regression analyses were conducted to explore the contribution of the cognitive measures in explaining the variance in arithmetic competency.

1. Measures of Working Memory

Measures of Phonological Loop

Forward Digit Span

The means and standard deviations for performance in forward digit span and forward character span are shown in table 2. Two-way ANOVA showed a significant main effect of ability, $F (1, 154)= 54.144$, $p < 0.005$, as well as a main effect of grade $F (2, 154)= 170.47$, $p < 0.005$. The interaction between ability and grade was found to be not significant, $F (2, 154)= 0.942$, $p > 0.1$. Despite the significant main effect of ability and grade, planned comparison revealed that there was no significant difference between the Primary 1 higher-ability group and Primary 2 lower-ability group, $t(51)=0.469$, $p > 0.5$.

Forward Character Span

Two-way ANOVA showed a significant main effect of ability, $F (1, 154)= 29.69$, $p < 0.005$, as well as a main effect of grade $F (2, 154)= 123.41$, $p < 0.005$. The interaction between ability and grade was again found to be not significant, $F (2, 154)= 1.498$, $p > 0.2$.

Despite the significant main effect of ability and grade, the difference between the 2 ability groups in Primary 3 was not significant, t(53)=1.845, $p > 0.5$.

Table 2. Performance Statistics of the Selected Groups on Measures of Phonological Loop

		Primary One		Primary Two		Primary Three	
		Lower ability	Higher ability	Lower ability	Higher ability	Lower ability	Higher ability
Total Number (N)		26	25	28	26	27	28
Forward Digit Span	Mean	4.65	5.92	5.89	6.96	8.00	8.71
	SD	.74	.70	.78	.77	.73	1.08
Chinese Character Span	Mean	7.846	8.440	8.536	9.308	10.000	10.643
	SD	.78	.58	1.07	.67	.73	.48
Span Difference	Mean	3.19	2.52	2.64	2.34	2.00	1.92
	SD	.49	.58	.55	.48	.39	.71

Discrepancy between performance in Digit and Character Span

All subjects had shown a discrepancy in performance of forward character span and digit span, with performance in forward character span task higher than that in forward digit span. The discrepancy between character span and digit span was calculated for all subjects. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 15.924, $p < 0.005$, as well as a main effect of grade F (2, 154)= 35.487, $p < 0.001$. A significant interaction between ability and grade was also found, F (2, 154)= 4.019, $p < 0.05$. The significant interaction effect in difference of the discrepancy between performance in digit and character span was due to the fact that the difference in the discrepancy manifested by the 2 ability groups decreased from primary 1 with mean difference of 0.6723 [$t(49)=4.446$, $p < 0.001$, 2-tailed], to primary 3 with mean difference of 0.0714 [$t(53)=0.456$, $p < 0.65$, 2-tailed].

<u>Measures of Visual-Spatial Sketchpad</u>

TVPS-Revised Visual Memory Test

The means and standard deviations for performance in visual memory test and Square span are shown in table 3. While two-way ANOVA indicated a significant main effect of grade, F (2, 154)= 101.98, $p < 0.001$, no significant main effect of ability, F (1, 154)= 0.024, $p > 0.5$ and interaction between grade and ability, F (2, 154)= 0.070, $p > 0.5$, emerged for visual memory. A Tukey test showed that the subjects in primary 3 performed significantly better than those in primary 2, and subjects in primary 2 performed significantly better than those in primary 1 in the visual memory subtest of TVPS-Revised.

Square Span

ANOVA showed a significant main effect of ability, F (1, 154)= 69.53, $p < 0.001$, as well as a main effect of grade F (2, 154)= 86.24, $p < 0.001$, for square span task. However, the interaction between ability and grade was found to be not significant, F (2, 154)= 0.42, $p > 0.5$.

Table 3. Performance Statistics of the Selected Groups on Measures of Visuospatial Sketchpad

		Primary One		Primary Two		Primary Three	
		Lower ability	Higher ability	Lower ability	Higher ability	Lower ability	Higher ability
Total Number (N)		26	25	28	26	27	28
Visual Memory	Mean	9.46	9.48	10.46	10.46	11.48	11.42
	SD	.50	.52	.50	.50	.51	.53
Visuospatial Memory	Mean	4.02	5.04	5.03	6.10	5.96	7.00
	SD	.80	.78	.74	.74	.75	.76

Measures of Central Executive

Missing Item

The means and standard deviations for performance in measures for the central executive are displayed in Table 4. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 139.82, $p < 0.001$, as well as a main effect of grade F (2, 154)= 151.19, $p < 0.001$. A significant interaction between ability and grade was also found, F (2, 154)= 149.16, $p < 0.001$. The significant interaction effect reflected the fact that difference in the performance of the 2 ability groups decreased from primary 1 with mean difference of 70.19 [$t(49)=25.27$, $p < 0.001$, 2-tailed], to primary 3 with mean difference of 21.76 [$t(53)=21.92$, $p < 0.001$, 2-tailed].

The accuracy rate for the Missing Item was recorded. The overall mean percentages of accuracy for Primary one, two and three were 94.33%, 96.07%, and 99.45% respectively.

Verbal Trail

Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 81.92, $p < 0.001$, as well as a main effect of grade F (2, 154)= 129.60, $p < 0.001$, for Verbal Trail task. However, the interaction between ability and grade was found to be not significant, F (2, 154)= 0.039, $p > 0.5$. Tukey Multiple comparison revealed that differences between all the experiment groups were statistically significant.

Crossing Out

Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 172.63, $p < 0.001$, as well as a main effect of grade F (2, 154)= 168.56, $p < 0.001$, for Crossing out. However, the interaction between ability and grade was found to be not significant, F (2, 154)= 1.546, $p > 0.1$. Similar to the result of Verbal Trail tasks, Tukey multiple comparisons revealed that differences between all the experiment groups were statistically significant. The accuracy rate for the Crossing Out was recorded. The overall mean percentages of accuracy for primary one, two and three were 98.96%, 99.11%, and 99.57% respectively.

Table 4. Performance Statistics of the Selected Groups on Measures of Central Executive

		Primary One		Primary Two		Primary Three	
		Lower ability	Higher ability	Lower ability	Higher ability	Lower ability	Higher ability
Total Number (N)		26	25	28	26	27	28
Missing Item (Time in sec.)	Mean	388.36	318.17	274.10	236.38	208.87	187.11
	SD	11.05	8.57	7.42	6.99	4.29	3.00
Verbal Trail (Time in sec.)	Mean	52.71	46.96	41.00	35.06	29.00	23.02
	SD	2.21	2.20	2.22	2.35	2.44	1.82
Crossing Out (Time in sec.)	Mean	50.66	46.33	40.80	37.64	33.76	30.29
	SD	1.98	1.79	1.71	1.66	1.82	1.51

General Measures of Working Memory

Operation Span (Normal Version)

The statistics of performances in the two versions of Operation Span task are displayed in table 5. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 40.41, p < 0.001, as well as a main effect of grade F (2, 154)= 84.48, p < 0.001, for the normal version of operation span task. However, the interaction between ability and grade was found to be not significant, F (2, 154)= 1.488, p > 0.1.

In each grade, the high-ability group performed significantly better than the low-ability group. Planned comparisons revealed no significant difference between primary 1 higher-ability group and primary 2 lower-ability group t(51)=1.714, p > 0.05, 2-tailed, between primary 2 higher-ability group and primary 3 lower-ability group t(51)=0.159, p > 0.5, 2-tailed.

Analysis of accuracy in the mental arithmetic operation in the normal version of Operation Span task showed no significant main effects of ability and grade, as well as, interaction between the two independent variables.

Operation Span (Extended Version)

Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 73.73, $p <$ 0.001, as well as a main effect of grade F (2, 154)= 145.63, $p < 0.001$, for the extended version of operation span task. Different from the normal version was a significant interaction between ability and grade F (2, 154)= 4.12, $p < 0.05$, with difference between the two ability groups increased through primary 1 to primary 3.

Tukey test showed a similar pattern of group differences to the normal operation span. Performances of low-ability groups were significantly lower than those of the high-ability group of the same grade, but were statistically comparable to those of the high-ability group at 1 grade below.

Analysis of the accuracy in the mental arithmetic operations in the extended version of Operation Span task showed significant main effects of both ability F (1, 154)= 18.03, $p <$ 0.001, and grade F (2, 154)= 5.043, $p < 0.005$, but no significant interaction between the two independent variables F (2, 154)= 0.716, $p > 0.1$.

Discrepancy between span in Normal and Extended Version of Operation Span task

Performances of all subjects in the extended version of operation span task were either lower than or equal to those in normal operation span. Discrepancy in span between the two versions was calculated for all subjects. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 6.171, $p < 0.05$, as well as a main effect of grade F (2, 154)= 9.480, $p < 0.001$. No significant interaction effect was indicated.

Tukey test showed discrepancies in performances of low-ability groups were significantly higher than those of the high-ability group of the same grade, but were statistically comparable to those of the high-ability group at one grade below.

Discrepancy in accuracy between the two versions was also calculated and analyzed for all subjects in a similar manner to the analysis of discrepancy in span. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 16.51, $p < 0.001$, as well as a main effect of grade F (2, 154)= 5.568, $p < 0.005$. Again, no significant interaction effect was indicated.

Table 5. Performance Statistics of the Selected Groups on Operation Span Tasks

		Primary One		Primary Two		Primary Three	
		Lower ability	Higher ability	Lower ability	Higher ability	Lower ability	Higher ability
Total Number (N)		26	25	28	26	27	28
Normal Operation Span (Span)	Mean	3.07	3.60	4.00	5.00	4.96	6.00
	SD	.93	.70	.86	.84	.85	.86
Normal Operation Span (Accuracy)	Mean	97.19	98.34	97.39	98.32	97.79	98.72
	SD	5.32	3.44	3.92	3.25	3.90	2.55
Extended Operation Span (Span)	Mean	2.00	2.56	3.00	4.15	4.11	5.50
	SD	.80	.71	.60	.88	.80	.74
Extended Operation Span (Accuracy)	Mean	86.53	94.57	90.66	95.39	93.59	98.02
	SD	13.60	6.72	10.86	5.80	6.70	3.29
Span Differences	Mean	1.07	1.04	1.00	.84	.85	.50
	SD	.27	.20	.60	.36	.60	.50
Accuracy Differences	Mean	10.65	3.77	6.73	2.93	4.19	.69
	SD	12.66	5.23	8.78	5.30	5.44	1.96

Partial Correlations Between Measures of Working Memory

Partial Correlations, with the effect of chronological age under control, were obtained for all measures of working memory and are shown in table 6. The focus of these correlations is to explore the relationship between the measure for general working memory and the measures for the three components, and between the three components. Several observations are highlighted.

Strong and significant correlations were shown between the spans, and between the accuracy rates of the two versions of Operation Span task. Generally, operation Span correlated significantly with most of the other measures, except those visuospatial measures.

Strong and significant correlations among the three measures of central executive were indicated. The measure of central executive correlated significantly with most of the measures of phonological loop and visuospatial sketchpad.

No significant correlations between measures of phonological loop and visuospatial sketchpad were observed, suggesting the separate nature of the visuospatial sketchpad and phonological loop.

Table 6. Correlation Coefficients Between Measures of Working Memory, Controlling for Chronological Age

		2	3	4	5	6	7	8	9	10	11
1.	Forward Digit Span	.78**	.21	.30	-.21*	-.25*	-.14	.58*	.04	.60*	.09
2.	Forward Word Span		.60	.78	-.04*	-.31*	.08	.57*	.07	.54*	.04
3.	Visual Memory			.56*	.33*	.43*	.48*	.21	-.06	.18	-.07
4.	Square Span				-.22*	-.09	.33*	.29	-.04	.23	.11
5.	Missing Item (Time)					.81**	.84**	-.21*	-.06	-.23*	-.26*
6.	Verbal Trail (Time)						.89**	-.02	-.03	-.14	-.22
7.	Crossing Out (Time)							-.31*	-.02	-.35*	-.21
8.	Normal Operation Span (Span)								.04	.85**	.09
9.	Normal Operation Span (Calculation Accuracy)									.10	.51**
10.	Extended Operation Span (Span)										.19*
11.	Extended Operation Span (Calculation Accuracy)										

** Correlation is significant at the 0.01 level (2-tailed) * Correlation is significant at the 0.05 level (2-tailed)

2. Measures on Conceptual Knowledge

Measures of Understanding of Place Value

Digit Representation

The means and standard deviations for performance in measures for assessing conceptual understanding are shown in Table 7. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 130.78, p < 0.001, as well as a main effect of grade F (2, 154)= 240.51, p < 0.001. A significant interaction between ability and grade was also found, F (2, 154)= 3.619, p < 0.05. The significant interaction effect reflected the fact that discrepancy in the performance of the two ability groups changed from primary 1 with absolute mean difference of 1.22, to primary 2 with absolute mean difference of 0.67, and finally to primary 3 with absolute mean difference of 0.95. Planned comparison showed no significant difference between primary 1 higher-ability group and primary 2 lower-ability group, t(51)=0.591, p > 0.5, 2-tailed, while Tukey test showed that differences between other experiment groups were statistically significant.

Columnar Calculation

Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 56.17, p < 0.001, as well as a main effect of grade F (2, 154)= 61.43, p < 0.001. A significant interaction between ability and grade was also found, F (2, 154)= 7.641, p < 0.005, with difference between the two ability groups in a same grade decreased from primary 1, with significant absolute mean difference of 1.31 (t(49)=6.175, p <0.01, 2-tailed), to primary 3, with significant absolute mean difference of 0.25 (t(53)=1.457, p > 0.1, 2-tailed). Planned comparison showed no significant difference only between the 2 ability groups in primary 3, whereas differences between other experiment groups were statistically significant.

Measures of Concept of Commutativity

Problem-Pair Comparison

Since the items for primary 1 are different from those for primary 2 and 3, with two problems for assessing the concept of multiplicative commutativity removed from the item for primary 1, the number of desired responses manifested by primary 1 group was analyzed separately from that of the primary 2 and 3 groups.

The mean number of desired responses of the two ability groups in primary 1 was compared using independent-samples T-test. The results showed a significant mean difference between the two ability groups, $t(49)=2.24$, $p < 0.05$ 2-tailed. Two-way ANOVA was run for groups in primary 2 and 3. Despite a significant main effect of grade F (1, 105)= 7.61, $p < 0.01$, both main effect of ability, F (1, 105)= 1.455, $p > 0.1$, and interaction effect, F (1, 105)= 1.455, $p > 0.1$, were not significant.

Table 7. Performance Statistics of the Selected Groups on Measures of Understanding of Concepts

		Primary One		Primary Two		Primary Three	
		Lower ability	Higher ability	Lower ability	Higher ability	Lower ability	Higher ability
Total Number (N)		26	25	28	26	27	28
Place Value (Digit Representation)	Mean	2.57	3.80	3.71	4.38	4.92	5.89
	SD	.80	.40	.46	.49	.54	.31
Place Value (Columnar Calculations)	Mean	5.26	3.96	3.00	2.00	1.03	.78
	SD	.72	.78	.72	.80	.64	.62
Commutativity (Problem-pair Comparison)	Mean	2.46	2.84	5.60	5.84	6.00	6.00
	SD	.76	.37	.87	.54	.00	.00
Foils in Commutativity	Mean	.31	.12	.10	0.03	.00	.00
	SD	.62	.33	.31	.19	.00	.00

Over-generalization of the Concept of Commutativity

Foil items were introduced to check if the concept of commutativity would be over-generalized by the subjects. Two-way ANOVA revealed only significant main effect of grade for the correct response on the foil items, F (1, 154)= 6.05, $p < 0.005$, with subjects in primary 1 showing the highest tendency to over-generalize the concept of the foil items, whereas subjects in primary 3 made the least mistakes of this kind.

3. Measures on Use of Calculation Principles

Measures of the Use of Additive Commutativity

Statistics regarding the use of the five calculation principles are shown in table 8. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 17.902, $p < 0.001$, as well as main effect of grade F (2, 154)= 36.046, $p < 0.001$. A significant interaction between ability and grade was also found, F (2, 154)= 12.168, $p < 0.001$. The significant interaction effect reflected the fact that difference in usage of commutated equation in addition of the two ability groups changed from primary 1, with significant absolute mean difference of 0.946, t(49)=4.838, $p > 0.01$, 2-tailed, to no significant difference in usage between the two ability groups in primary 3.

Planned comparison showed no significant difference between primary 1 higher-ability group and primary 2 lower-ability group, t(51)=0.049, $p > 0.5$, 2-tailed. Tukey test also showed that usages of the knowledge of additive commutativity among other groups in primary 2 and primary 3 were statistically comparable. Despite the differences between groups and grades, all groups demonstrated 100% accuracy in their usage of commutated equations in solving the target items.

Measures of the Use of Reversal

A similar result was exhibited in the item for assessing the use of reversal principle. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 21.52, $p < 0.001$, as well as main effect of grade F (2, 154)= 34.54, $p < 0.001$. A significant interaction between ability and grade was also found, F (2, 154)= 5.859, $p < 0.005$, due to the lower-ability groups showing substantial increment from primary 1 (mean usage = 2.08) to primary 3 (mean usage of 3.70) in the usage of reversal principle, while higher-ability groups began with and maintained at a much higher level from primary 1.

Planned comparison showed no significant difference between primary 1 higher-ability group and primary 2 lower-ability group, t(51)=0.592, $p > 0.5$, 2-tailed, and between primary 2 higher-ability group and primary 3 lower-ability group, t(51)=0.431, $p > 0.5$, 2-tailed. Despite the differences between groups and grades, all the groups demonstrated nearly error-free usage of the hinted equations in solving reversal items.

Measures of the Use of Inversion

Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 16.79, $p < 0.001$, as well as main effect of grade F (2, 154)= 57.92, $p < 0.001$ for the usage of inversion principle. A significant interaction between ability and grade was also found, F (2, 154)= 3.46, $p < 0.05$. The lower-ability groups showed substantial increment from primary 1 (mean usage = 1.80) to primary 3 (mean usage of 3.56), while the higher-ability groups showed more steady increment from primary 1 (mean usage = 2.68) to primary 3 (mean usage of 3.85) in the usage of inversion principle.

Planned comparison showed no significant difference between primary 2 higher-ability group and primary 3 lower-ability group, t(51)=1.48, $p > 0.1$, 2-tailed, and between the 2 ability groups in primary 2, t(52)= 1.73, $p > 0.1$, 2-tailed.

Despite the differences between groups and grades, all the groups demonstrated nearly error-free usage of the hinted equations in solving inversion items.

Measures of the Use of Operand-Plus/Minus-One

A similar result was exhibited in the item for assessing the use of operand-plus/minus-one principle. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 38.99, $p < 0.001$, as well as main effect of grade F (2, 154)= 65.37, $p < 0.001$. A significant interaction between ability and grade was also found, F (2, 154)= 14.42, $p < 0.05$. The lower-ability groups showed substantial increment from primary 1 (mean usage = 0.62) to primary 3 (mean usage of 3.00), while the higher-ability groups showed more steady increment from primary 1 (mean usage = 2.28) to primary 3 (mean usage of 3.25) in the usage of addend plus-minus one principle.

Planned comparison showed no significant difference between primary 1 higher-ability group and primary 2 lower-ability group, t(51)=0.108, $p > 0.5$, 2-tailed, between primary 2 higher-ability group and primary 3 lower-ability group, t(51)= 0.276, $p > 0.5$, 2-tailed, and between the 2 ability groups in primary 2, t(52)= 0.287, $p > 0.5$, 2-tailed,

For the 144 subjects who showed usage of operand-plus/minus-one principle to solve target items, accuracy of their usage was analyzed. Two-way ANOVA showed a significant main effect of ability, F (1, 138)= 8.388, $p < 0.001$, as well as main effect of grade F (2, 138)= 8.132, $p < 0.001$. A significant interaction between ability and grade was also found, F (2, 138)= 3.405, $p < 0.05$. Though some of the primary 1 lower-ability group utilized the corresponding hinted equation to solve target items, the accuracy was very low. Substantial increments were shown from primary 1 to primary 3 in the lower-ability group. In addition to the higher usage hinted item, the higher-ability groups also demonstrated higher accuracy in their usage of the principle.

Measures of the Use of Operand-Plus/Minus-Ten

A similar result was exhibited in the item for assessing the use of operand-plus/minus-ten principle. Two-way ANOVA showed a significant main effect of ability, F (1, 154)= 26.40, p < 0.001, as well as main effect of grade F (2, 154)= 170.61, p < 0.001. However, no significant interaction between ability and grade was found, F (2, 154)=0.120, p > 0.5. Planned comparison showed only the difference between primary 1 higher-ability group and primary 2 lower-ability group was close to significant, t(51)=1.87, p = 0.061, 2-tailed.

Accuracy of usage was analyzed for the 124 subjects who showed usage of operand-plus/minus-ten principle to solve target items. Two-way ANOVA showed a significant main effect of ability, F (1, 118)= 5.137, p < 0.001, as well as main effect of grade F (2, 118)= 8.418, p < 0.05. A significant interaction between ability and grade was also found, F (2, 118)= 4.22, p < 0.05.

Partial Correlations Between Measures of Conceptual Understanding and Use of Calculation Principles

Partial Correlations, with the effect of chronological age under control, were obtained for all measures of conceptual understanding and the use of calculation principles; these are shown in table 9. The focus of these correlations is to explore whether the understanding or use of one concept or principle would be closely linked with the tendency and accuracy in the application of other principles. Several observations are highlighted.

Strong and significant negative correlation was found between Digit Representation and Columnar Calculation. This suggested a strong within-domain relationship between these two measures of place-value.

The understanding of the concept of commutativity did not correlated significantly with other measures of place-value concept and the five measures of principles, even to the additive commutativity principle. This could be due to the controlling of chronological age and the design of the measure. Only significant effect of grade was found in the difference in the performance of this measure. Due to the genuine connection between age and schooling, control of chronological age would inevitably diminish the relationship of this concept with other measures.

No significant correlations between usage and accuracy of usage were observed in the principles of additive commutativity, inversion, and reversal. This was because the accuracy of usage of additive commutativity, inversion and reversal were constantly high, ranging from 98.7% to 100%.

Regarding the two associative-based principles, significant correlation was observed between the usage of the operand-plus/minus-one and operand-plus/minus-ten principles. While usage correlated significantly with accuracy in the application of operand-plus/minus-one, there was a weak correlation between these two aspects in operand-plus/minus-ten.

Table 8. Performance Statistics of the Selected Groups on Measures of Use of Calculation Principles

		Primary One		Primary Two		Primary Three	
		Lower ability	Higher ability	Lower ability	Higher ability	Lower ability	Higher ability
Additive Commutativity (Usage)	Mean	2.65	3.60	3.60	3.73	4.00	4.00
	SD	.74	.64	.73	.45	.00	.00
Add. Commutativity (Correct Usage %)	*Mean*	*100.00*	*100.00*	*100.00*	*100.00*	*100.00*	*100.00*
	SD	*.00*	*.00*	*.00*	*.00*	*.00*	*.00*
Reversal (Usage)	Mean	2.07	3.20	3.32	3.61	3.70	3.92
	SD	.93	.95	.86	.63	.60	.26
Reversal (Correct Usage %)	Mean	99.03	99.00	100.00	100.00	100.00	100.00
	SD	4.90	5.00	.00	.00	.00	.00
Inversion (Usage)	Mean	1.87	2.68	3.07	3.26	3.55	3.85
	SD	.63	.55	.89	.82	.80	.35
Inversion (Correct Usage %)	Mean	98.71	100.00	99.10	100.00	100.00	100.00
	SD	6.53	.00	4.72	.00	.00	.00
Operand-Plus/Minus-One (Usage)	Mean	.61	2.28	2.25	2.57	3.00	3.25
	SD	.49	.73	.70	.57	.96	.92
Operand- Plus/Minus-One # (Correct Usage %)	Mean	25.00	67.33	68.45	76.92	74.07	77.67
	SD	44.72	42.62	41.41	35.92	31.71	30.68
Operand-Plus/Minus-Ten (Usage)	Mean	.15	.76	1.10	1.65	2.59	3.07
	SD	.36	.43	.78	.56	.57	1.01
Operand-Plus/Minus-Ten * (Correct Usage %)	Mean	.00	47.36	52.38	78.00	79.62	68.45
	SD	.00	51.29	46.03	38.40	35.30	34.42

Principle used by 144 subjects

* Principle used by 124 subjects

Table 9. Partial Correlation Coefficients Between Measures of Conceptual Understanding and Use of Calculation Principles, Controlling for Chronological Age

	2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1. Place Value (Digit Representation)	-.66**	-.23**	.02	.12	.01	.13	.03	.20**	-.11	.27*	-.04
2. Place Value (Columnar Calculations)		.06	-.23*	-.54*	.01	-.52**	-.02	-.55**	-.15	-.56**	-.10
3. Commutativity			.12	-.016	.20	-.01	-.03	-.18	.05	-.33	.09
4. Additive Commutativity (Usage)				.51*	-.07	.33*	-.07	.18*	.01	.20*	-.12
5. Reversal (Usage)					-.10	.75**	.01	.50**	.01	.47**	-.13
6. Reversal (Accuracy)						-.01	-.05	.04	.13	-.01	-.01
7. Inversion (Usage)							-.01	.57**	.05	.52**	-.02
8. Inversion (Accuracy)								.14	.09	.01	.02
9. Operand-Plus/Minus-One (Usage)									.27**	.75**	.17
10. Operand-Plus/Minus-One (Accuracy)										.10	.17
11. Operand-Plus/Minus-Ten (Usage)											.11
12. Operand-Plus/Minus-Ten (Accuracy)											

**Correlation is significant at the 0.01 level (2-tailed)
* Correlation is significant at the 0.05 level (2-tailed)

4. Hierarchical Regression Analyses

A series of forced-order, theoretically driven, hierarchical regression analyses was performed to identify measures that accounted for unique variances in individual differences in elementary arithmetical competency, and to determine if the contribution of some variables was mediated by the presence of other variables. Since the order of entry is known to influence the outcome of regression analyses, several matrices were constructed for each relationship to be examined. Moreover, each analysis of relationship was repeated for each grade to allow for children from different grades being administered different sets of arithmetic test.

Analysis of Variances by the Measures of Central Executive and Phonological Loop on Arithmetic Competency

To address questions about the contribution of the phonological loop and central executive on performance in arithmetic operations, their order of entry was rotated at two nested levels. The first level concerned whether the central executive measures were entered before or after the measures of phonological loop. By this means, it was possible to identify shared and unique variance associated with the two categories of measures. The second level concerned the order of entering the measure of the two phonological loop span tasks. This is to address the question of whether variance unique to the span tasks as a pair is common to them both.

Primary One

The results of analyses for primary one are summarized in Table 10. The results indicated that both measures of phonological loop and central executive accounted for 85.4% of variance. Measures of phonological loop accounted for 17.6% of variance when entered after those of central executive, whereas central executive accounted for 17% when entered after the spans. Thus, PL and CE measures mainly explained 50.8% shared variance, with both categories explaining comparable unique variance in performance in arithmetic.

While digit span or character span explained significant variance when either was entered at the penultimate step, only digit span explained further variation when entered last. Thus, the variance unique to span tasks was unique to digit span.

Table. 10 Outcome of hierarchical regression analyses for Primary 1

	PL entered before CE measures		*CE entered before PL measures*	
Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
	Digit Span entered before Character Span			
1	Forward Digit Span	.684*	CE measures	.678*
2	Forward Character Span	.000	Forward Digit Span	.174*
3	CE measures	.170*	Forward Character Span	.002
	Character Span entered before Digit Span			
1	Forward Character Span	.370*	CE measures	.678*
2	Forward Digit Span	.314*	Forward Character Span	.093*
3	CE measures	.170*	Forward Digit Span	.083*

* $p<0.05$

Primary Two

The results of analyses for primary two are summarized in Table 11. The results indicated that both measures of phonological loop and central executive accounted for 73.7% of variance. Measures of phonological loop accounted for 19.3% of variance when entered after those of central executive, whereas central executive accounted for 11.2% when entered after the spans. Thus, PL and CE measures mainly explained 43.2% shared variance, while phonological loop explained more unique variance in performance in arithmetic.

While digit span or character span explained significant variance when either was entered at the penultimate step, only digit span explained further variation when entered last. Thus, the variance unique to span tasks was unique to digit span.

Table. 11 Outcome of hierarchical regression analyses for Primary 2

Step	*PL entered before CE measures* Variable(s)	R^2 Change	*CE entered before PL measures* Variable(s)	R^2 Change
	Digit Span entered before Character Span			
1	Forward Digit Span	.623*	CE measures	.544*
2	Forward Character Span	.002	Forward Digit Span	.177*
3	CE measures#	.112*	Forward Character Span	.016
	Character Span entered before Digit Span			
1	Forward Character Span	.472*	CE measures	.544*
2	Forward Digit Span	.153*	Forward Character Span	.098*
3	CE measures	.112*	Forward Digit Span	.095*

* *p<0.05*

Primary Three

The results of analyses for primary three are summarized in Table 12. The results indicated that both measures of phonological loop and central executive accounted for 79.9% of variance. Measures of phonological loop accounted for 18.1% of variance when entered after those of central executive, whereas central executive accounted for 32.2% when entered after the spans. Thus, PL and CE measures explained 29.6% shared variance, while CE explained more unique variance in performance in arithmetic.

While digit span or character span explained significant variance when either was entered at the penultimate step, only digit span explained further variation when entered last. Thus, the variance unique to PL span tasks was unique to digit span.

Table. 12 Outcome of hierarchical regression analyses for Primary 3

Step	*PL entered before CE measures* Variable(s)	R^2 Change	*CE entered before PL measures* Variable(s)	R^2 Change
	Digit Span entered before Character Span			
1	Forward Digit Span	.469*	CE measures	.618*
2	Forward Character Span	.008	Forward Digit Span	.170*
3	CE measures	.322*	Forward Character Span	.011
	Character Span entered before Digit Span			
1	Forward Character Span	.376*	CE measures	.618*
2	Forward Digit Span	.101*	Forward Character Span	.144*
3	CE measures	.322*	Forward Digit Span	.037*

* *p<0.05*

Analysis of Variances by the Measures of Central Executive and Visuospatial Sketchpad on Arithmetic Competency

To address questions about the contribution of Visual Spatial Sketchpad (VSSP) and central executive (CE) to performance in arithmetic operations, their order of entry was rotated at two nested levels. The first level concerned whether the CE measures were entered before or after the measures of VSSP. It was possible to identify shared and unique variance associated with the two categories of measures. The second level concerned the order of entering the two measures of VSSP. This is pertinent to the question of whether variance unique to the VSSP tasks as a pair is common to them both.

Primary One

The results of analyses for primary one are summarized in Table 13. The results indicated that both measures of VSSP and CE accounted for 92.4% of variance. Measures of VSSP accounted for 33.8% of variance when entered after those of central executive, whereas central executive accounted for 35.1% when entered after the VSSP tasks. Thus, VSSP and CE measures mainly explained 23.5% shared variance; with CE measures explained more unique variance than VSSP in performance in arithmetic.

While visual memory and visuospatial memory explained significant variance when either was entered at the penultimate step, only square span explained further variation when entered last. Thus, the variance unique to span tasks was unique to square span.

Table. 13 Outcome of hierarchical regression analyses for primary 1

	VSSP entered before CE measures		*CE entered before VSSP measures*	
Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
	Square span entered before Visual memory			
1	Square Span	.456*	CE measures	.586*
2	Visual Memory	.117*	Square Span	.307*
3	CE measures	.351*	Visual Memory	.031
	Visual memory entered before Square span			
1	Visual Memory	.355*	CE measures	.586*
2	Square Span	.218*	Visual Memory	.229*
3	CE measures	.351*	Square Span	.109*

* *$p<0.05$*

Primary Two

The results of analyses for primary two are summarized in Table 14. The results indicated that both measures of VSSP and CE accounted for 81.9% of variance. Measures of VSSP accounted for 23.3% of variance when entered after those of central executive, whereas central executive accounted for 23.6% when entered after the VSSP tasks. Thus, VSSP and CE measures mainly explained 35% shared variance; with CE measures explained slightly more unique variance than VSSP in performance in arithmetic.

While visual memory and visuospatial memory explained significant variance when either was entered at the penultimate step, only square span explained further variation when entered last. Thus, the variance unique to spans tasks was unique to square span, as similar to the pattern found in Primary 1.

Table. 14 Outcome of hierarchical regression analyses for primary 2

	VSSP entered before CE measures		*CE entered before VSSP measures*	
Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
	Square span entered before Visual memory			
1	Square Span	.442*	CE measures	.481*
2	Visual Memory	.141*	Square Span	.324*
3	CE measures	.236*	Visual Memory	.014
	Visual memory entered before Square span			
1	Visual Memory	.382*	CE measures	.481*
2	Square Span	.201	Visual Memory	.212*
3	CE measures	.236*	Square Span	.126*

** $p<0.05$*

Primary Three

The results of analyses for primary three are summarized in Table 15. The results indicated that both measures of VSSP and CE accounted for 81.9% of variance. Measures of VSSP accounted for 20.2% of variance when entered after those of central executive, whereas central executive accounted for 27.5% when entered after the VSSP tasks. Thus, VSSP and CE measures mainly explained 34.2% shared variance; while CE measures explained more unique variance in performance in arithmetic.

While visual memory and visuospatial memory explained significant variance when either was entered at the penultimate step, only square span explained further variation when entered last. Thus, the variance unique to span tasks was unique to square span

Table. 15 Outcome of hierarchical regression analyses for primary 3

	VSSP entered before CE measures		*CE entered before VSSP measures*	
Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
	Square span entered before Visual memory			
1	Square Span	.341*	CE measures	.617*
2	Visual Memory	.203*	Square Span	.196*
3	CE measures	.275*	Visual Memory	.006
	Visual memory entered before Square span			
1	Visual Memory	.138*	CE measures	.617*
2	Square Span	.681*	Visual Memory	.131*
3	CE measures	.275*	Square Span	.071*

CE Measures: Crossing Out (Time), Verbal Trail (Time), Missing Item (Time)
* *p<0.05*

<u>Analysis of Variances by the Three Measures of Central Executive on Arithmetic Competency</u>

Primary One

The results of analyses for primary one are summarized in Table 16. All the three measures of CE accounted for 67.8% of variance. Measures of Missing Item accounted for 38.4% of variance when entered after the other two measures of central executive whereas Verbal Trail accounted for 5.1% when entered last. When Crossing out was entered last, variance of 6.7% of unique variance was indicated. The three measures explained 17.6% of shared variance in arithmetic performance.

Table. 16 Outcome of hierarchical regression analyses for primary 1

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
		Missing Item Entered last		
1	Crossing Out	.262*	Verbal Trail	.215*
2	Verbal Trail	.059*	Crossing Out	.079*
3	Missing Item	.384*	Missing Item	.384*
		Missing Item Entered first		
1	Missing Item	.574*	Missing Item	.574*
2	Verbal Trail	.037	Crossing Out	.053*
3	Crossing Out	.067*	Verbal Trail	.051*

* *p<0.05*

Primary Two

The results of analyses for primary two are summarized in Table 17. All the three measures of CE accounted for 64.1% of variance. Measures of Missing Item and Crossing Out accounted for 33.2% and 6.1% of variance respectively when entered after the other two measures of central executive, whereas Verbal Trail accounted for an insignificant 0.7% when entered last. The three measures explained 24.1% of shared variance in arithmetic performance.

Table. 17 Outcome of hierarchical regression analyses for primary 2

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
		Missing Item Entered last		
1	Crossing Out	.217*	Verbal Trail	.116*
2	Verbal Trail	.092*	Crossing Out	.193*
3	Missing Item	.332*	Missing Item	.332*
		Missing Item Entered first		
1	Missing Item	.412*	Missing Item	.412*
2	Verbal Trail	.168*	Crossing Out	.222*
3	Crossing Out	.061*	Verbal Trail	.007
* *p<0.05*				

Table. 18 Outcome of hierarchical regression analyses for primary 3

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
		Missing Item Entered last		
1	Crossing Out	.165*	Verbal Trail	.297*
2	Verbal Trail	.164*	Crossing Out	.033
3	Missing Item	.288*	Missing Item	.288*
		Missing Item Entered first		
1	Missing Item	.486*	Missing Item	.486*
2	Verbal Trail	.043*	Crossing Out	.132*
3	Crossing Out	.089*	Verbal Trail	.000
* *p<0.05*				

Primary Three

The results of analyses for primary three are summarized in Table 18. All the three measures of CE accounted for 61.8% of variance. Measures of Missing Item accounted for 28.8% of variance when entered after the other two measures of central executive whereas Crossing Out accounted for 8.9% when entered last. When Verbal Trail was entered last, no significant amount of unique variance was indicated. Therefore unique variance unique to Verbal Trail task is common with the other two tasks. Moreover, the three measures explained 40.5% of shared variance in arithmetic performance.

Analysis of variances by Operation Spans and measures of the Three Working Memory Components

Another 2 models of hierarchical regression were conducted to identify the unique and shared variance associated with components measures and measures of general working memory (WM).

Primary One

The results of analyses for primary one are summarized in Table 19. The components measures and general working memory together accounted for 96.9% of the variance in arithmetic skills. General WM and components measures explained 34.8% and 14.6% unique variance respectively, with 47.5 % shared variance.

Table. 19 Outcome of hierarchical regression analyses for primary 1

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
	Component Measures Entered first		*Measures of General Working Memory (WM) Entered first*	
1	Component Measures	.621*	General WM	.823*
2	General WM	.348*	Component Measures	.146*

* *p<0.05*

Primary Two

The results of analyses for primary two are summarized in Table 20. The components measures and general working memory together accounted for 90.1% of the variance. General WM and components measures explained 57.5% and 11.3% unique variance respectively, and explained 21.3 % shared variance in arithmetic skills.

Table. 20 Outcome of hierarchical regression analyses for primary 2

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
	Component Measures Entered first		*Measures of General Working Memory (WM) Entered first*	
1	Component Measures	.326*	General WM	.788*
2	General WM	.575*	Component Measures	.113*
* *p<0.05*				

Primary Three

The results of analyses for primary three are summarized in Table 21. The two categories together accounted for 82.4% of the variance in arithmetic skills. General WM and components measures explained 51.1% and 25.2% unique variance respectively, with 21.0 % shared variance.

Table.21 Outcome of hierarchical regression analyses for primary 3

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
	Component Measures Entered first		*Measures of General Working Memory (WM) Entered first*	
1	Component Measures	.313*	General WM	.572*
2	General WM	.511*	Component Measures	.252*
* *p<0.05*				

Analysis of variances by the Measures of working memory, Conceptual knowledge and Use of Principles on Arithmetic Competency

To address questions about the contribution of working memory, conceptual knowledge and uses of calculation principles in the performance in arithmetic operations, their order of entry was again rotated at two nested levels as in previous analyses. The first level concerned whether the measures of working memory (WM) were entered first or last. The second level concerned the order of entering the measures of conceptual knowledge of place value (PV) and commutativity (COMMU), and uses of calculation principles.

Primary One

The results of analyses for primary one are summarized in Table 22. The results indicated that, together, all the involved measures accounted for 94.8% of variance. Measures of conceptual knowledge of PV and COMMU, and calculations principles accounted for 29.5% and 7.5% of unique variance when being entered last, with unique variance of 43.7% explained by measures of working memory. The three domains explained 14.1% shared variance.

Table. 22 Outcome of hierarchical regression analyses for primary 1

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
	Working Memory Measures entered first			
	Uses of Calculations Principles first		*Concepts of Place Value(PV) and Commutativity (COMMU) first*	
1	WM Measures	.518*	WM Measures	.518*
2	Calculations Principles	.135*	PV and COMMU	.355*
3	PV and COMMU	.295*	Calculations Principles	.075*
	Working Memory Measures entered last			
1	Calculations Principles	.306*	PV and COMMU	.346*
2	PV and COMMU	.205*	Calculations Principles	.165*
3	WM Measures	.437*	WM Measures	.437*

* $p<0.05$

Primary Two

The results of analyses for primary two are summarized in Table 23. The results indicated that, together, all the involved measures accounted for 91.6% of variance. Measures of conceptual knowledge of Place value (PV) and Commutativity (COMMU), and calculations principles accounted for 21.5% and 10.3% of unique variance when being entered last, with unique variance of 40.7% explained by measures of working memory. The three domains explained 19.1% shared variance.

Table. 23 Outcome of hierarchical regression analyses for primary 2

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
		Working Memory Measures entered first		
	Uses of Calculations Principles first		*Concepts of Place Value(PV) and Commutativity (COMMU) first*	
1	WM Measures	.497*	WM Measures	.497*
2	Calculations Principles	.204*	PV and COMMU	.316*
3	PV and COMMU	.215*	Calculations Principles	.103*
		Working Memory Measures entered last		
1	Calculations Principles	.271*	PV and COMMU	.315*
2	PV and COMMU	.238*	Calculations Principles	.194*
3	WM Measures	.407*	WM Measures	.407*
* $p<0.05$				

Primary Three

The results of analyses for primary three are summarized in Table 24. The results indicated that, together, all the involved measures accounted for 88.3% of variance. Measures of conceptual knowledge (PV and COMMU), and calculations principles accounted for 14.3% and 15.1% of unique variance when being entered last, with unique variance of 38.7% explained by measures of working memory. The three domains explained 20.2% shared variance.

Table. 24 Outcome of hierarchical regression analyses for primary 3

Step	Variable(s)	R^2 Change	Variable(s)	R^2 Change
		Working Memory Measures entered first		
	Uses of Calculations Principles first		*Concepts of Place Value(PV) and Commutativity (COMMU) first*	
1	WM Measures	.465*	WM Measures	.465*
2	Calculations Principles	.275*	PV and COMMU	.267*
3	PV and COMMU	.143*	Calculations Principles	.151*
		Working Memory Measures entered last		
1	Calculations Principles	276*	PV and COMMU	.238*
2	PV and COMMU	.220*	Calculations Principles	.258*
3	WM Measures	.387*	WM Measures	.387*
* $p<0.05$				

Chapter Seven

Discussion

1. General Discussion

This study set out to examine the roles of the working memory and its components as portrayed in the working-memory model of Baddeley & Hitch (1974) understanding of the concepts of place value and commutativity and uses of calculation principles in arithmetic competency of children from primary one to primary three.

Group Differences in Working Memory Measures

Phonological Loop

In the current study, digit span and Chinese character span were adopted to measure the short-term memory of verbal material. The results showed that children with lower arithmetic ability under-performed those with higher ability of the same grade, and children in higher grades tend to outperform their lower grade peers, in all measures of phonological loop. The lower performance in digit span could represent a poor representation of numerical materials. However, the lower performance in Chinese character span was also exhibited by the lower-ability group in each grade. This result is surprising, given that both groups in each grade were matched on Chinese reading ability. This would not be explained by poorer access to representation of Chinese character, due to the fact that subjects in the current study are matched on reading ability. This finding also presents a different view, as could be suggested by Bull and Johnston's notion that short-term memory of the phonological loop does not account for differences in children's mathematical ability when differences in reading ability have been controlled for (Bull & Johnston, 1997). One possibility is that subjects in the lower ability groups were weak on some more general factors, such as speed of item identification and general speed of information processing, which may influence the performance of verbal memory span task. Addressing this issue is beyond the scope of the present study.

Since the designs of the two span tasks in the current study are the same, except in the nature of content materials, individual performances in the two tasks were compared. All subjects displayed a discrepancy in performance of forward character span and digit span, with performance in forward character span task higher than that in forward digit span. Analyses of the discrepancy revealed three patterns of the development of phonological loop. Firstly, the discrepancy between performance of digit span and Chinese character span became smaller as schooling increased. Secondly, discrepancies shown by higher-ability groups were consistently smaller than those of lower-ability groups. Thirdly, despite no interaction effects of grade and ability shown in both span tasks, the difference between the between-span discrepancies of the 2 ability groups converged as schooling increased. With the observation that the lower-ability group exhibited a higher increment rate in the performance of digit span task than in character span task, one plausible explanation for this feature is that formal schooling might have a stronger effect on enhancing the memory skills for numerical skills of children with lower arithmetic ability who entered school with much weaker representation of numerical information.

Visuospatial Sketchpad

TVPS-R and Square Span were used to measure the short-term visual memory and visuospatial memory respectively. It was found that the lower-ability groups under-performed their higher-ability counterparts only in square span but not in short-term visual memory. Given that the square span has no obvious arithmetic or numerical content, the finding supports the perspective that a visuospatial working memory weakness contributes to lower arithmetic performance (McLean & Hitch, 1999). The finding that square span, a measure which required serial recall, associated with individual difference in arithmetic performance supporting the prevailing perspective that mathematical ability associates with the performance in a serial recall measure of the visuospatial sketchpad (Mclean & Hitch, 1999; Dehaene, 1992; Heathcote, 1994). Since the materials used in the current study to differentiate children on their arithmetic ability were composed of multi-digit problems in which required carrying or borrowing, the findings give support to Heathcote's (1994) perspective that the influence of visual-spatial sketchpad may be more pronounced on multi-digit arithmetic problem solving. Moreover, the observation that some children in the current study used number imagery or visualizing number representation in backup counting strategies for solving simple arithmetic combinations, gives further support to the importance of the visuospatial sketchpad in the differences in arithmetic competency.

Central Executive

Three measures for assessing three functions, which were proposed to be performed by the central executive, were used in the current study; these include the Verbal Trail task for assessing the ability to switch retrieval strategies; the Crossing Out task for measuring selective attention and ability to inhibit responses to non-targets that had been designated as targets previously; and the Missing Item tasks to measure the executive function of holding and manipulating information.

While grade difference in Verbal Trail could reflect the effect of formal schooling on quicker access to the numerical and alphabetical information in long-term memory, the ability group difference could be attributed to differences in the capacity to switch strategies continuously, given that reading abilities were matched between ability groups.

The finding that children's arithmetic ability associated with performance in Crossing Out tasks is consistent with the results of Passolunghi and Siegel (2001), whereby the weaker performance in working memory of poorer arithmetic problem solvers was related to an inability to control and to ignore irrelevant or no longer relevant information. It was suggested that an inability to sustain attention on the task in hand makes the solving of arithmetic questions very difficult, particularly where slower, less efficient overt counting strategies are being used. This would result in the child's losing track of the counting procedure, which in turn leads to under- or over-counting (Geary & Burlingham-Dubree, 1989).

The results of the Missing item tasks showed that children in higher grades performed better than those in lower grades, and children with higher arithmetic competence performed better than those with lower competence. However, the difference is not necessarily attributable to executive functioning in concurrently holding and manipulating information. There were interesting observations that children, even of the same grade, might have used different strategies in solving the simple arithmetic combinations, which could have contributed to the time used to finish the task. For example, some children consumed much less time in solving simple arithmetic combinations than their same grade counterparts who used backup strategies, such as finger counting or verbalization. Descriptive statistics revealed that there was a decreasing trend in the standard deviation of time used along the 3 grades, from SD of 11.06 seconds of the primary 1 lower-ability group, down to SD of 3.01 seconds of the higher-ability group in primary 3. Thus, the variation of strategy used may contribute to the group and ability difference, in addition to that in the executive function. Therefore the interpretation of the Missing Item task difference should be tentative, bearing in mind the existence of the behavioral pattern of strategy used.

General Working Memory

Operation Span tasks were included in the current study as a general measure of working memory as a whole. Foreshadowed by the findings in phonological loop, visuospatial sketchpad and central executive, a similar pattern was exhibited in both the normal and extended version of Operation Span task, with children of higher grade outperforming children of lower grade, and in each grade, children with higher arithmetic ability outperforming those with lower ability. This current finding converges with previous findings that poor arithmetic problem solvers have weakness in working memory (Hitch & McAuley, 1991; Swanson, 1993). Moreover, it was found that children with lower arithmetic ability performed at a level close to that of the high-ability group at 1 grade below. This raises the possibility that the weaker working memory capacity of the lower arithmetic ability group may be due to a developmental delay rather than a specific deficit.

The lower performance in operation span task reflects a weaker ability to recall the serially stored data in the presence of concurrent processing operations. It was also found that both ability groups suffered a reduction in span performance in the extended version, with the low ability group exhibiting a larger degree of decrement. This finding supports the task-switching hypothesis and the notion that children with lower arithmetic ability are associated with faster decay in working memory.

However, the result from the analysis of accuracy of calculation performed during the Operation Span task also provided partial support to a resource sharing account of working memory. Given that no significant ability and grade difference exhibited in calculation accuracy in the normal version, significant ability and grade effects found on discrepancy between the calculation accuracy of normal and extended version actually reflect the decrement mainly in the accuracy of calculation in the extended version. This accuracy-span trade-off reflected that some resources would have been shared by the processing of the calculations in trials.

While association of differences in arithmetic ability and the group differences in central executive, phonological loop and visuospatial sketchpad have been confirmed by many previous studies, the nature of interactions among these three components of working memory regarding their influences on arithmetic competency has yet to be definitively shown.

One focus of the current study is to explore the interactions between the central executive and the two subsidiary storage systems, as proposed in the popular model by Baddeley and Hitch (1974), upon their influence on arithmetic competency. Using hierarchical regression analyses, this study attempted to determine whether influence of phonological loop or visuospatial sketchpad was mediated by the presence of central executive.

Relationship between Central Executive and Phonological Loop

Hierarchical regression analysis showed that phonological loop and central executive contribute unique variance to arithmetic ability in all three grades. That is, the entry of central executive first into the equation did not diminish the contribution of phonological loop to the difference in arithmetic ability or vice versa. This supports a domain-specific interpretation of phonological loop and central executive that each of them plays a specific role in arithmetic ability. However, this does not necessarily mean that the phonological loop and central executive are two distinct, independent functions, due to the presence of a high proportion of shared variance explained by them. This supports the view that the two components interact in some ways in their influence on arithmetic tasks.

Relationship between the Two Measures in Phonological Loop

While phonological loop was found to have contributed unique variance to arithmetic competency, hierarchical analyses showed that the variance was not common to both Digit Span and Character Span for phonological loop in all three grades. Character Span, when entered after Digit Span, explained no significant further variation to arithmetic competency. This may have been the result of matching reading abilities of children in each grade.

Relationship between the Central Executive and the Visuospatial Sketchpad

Hierarchical regression analysis of the model, consisting of central executive and visuospatial sketchpad, also showed that each of them contributed unique variance to arithmetic ability in all three grades. That is, the entry of central executive first into the equation did not nullify the contribution of visuospatial sketchpad to the difference in arithmetic ability or vice versa. The finding that both components under discussion elucidated unique and shared variance in arithmetic competency supported a combination of domain-specific and domain-general interpretation of their nature of interaction.

Relationship between the Two Measures in Visuospatial Sketchpad

While visuospatial sketchpad was found to have contributed unique variance to arithmetic competency, hierarchical analyses showed that the variance was not common by both visual memory and visuospatial memory in all three grades. Visual memory, when entered after Square Span, a measure of visuospatial memory, explained no significant further variation to arithmetic competency. This is in line with the results for the insignificant ability-group differences in visual memory task. Since the visual memory task used in the current study has its focus on pattern recognition with non-numerical contents, the findings suggested that visuospatial ability that requires serial recall of spatial position, is more important than the ability to recognize graphical representations to differentiate arithmetic performance. Observations of children's strategic devices also provide evidence backing this view. As observed in the screening sessions, many children were found to use written columnar calculation to solve multi-digit arithmetic problems. The use of written calculation highly reduces the importance of short-term visual memory, while keeping numbers in correct alignment places significant demands on visuospatial memory.

Relationships among the Measures for Central Executive

Hierarchical regression analysis of the model, consisting of the three measures of central executive, showed that each Missing Item and Crossing Out contributed unique variance to arithmetic ability in all three grades. However, the Verbal Trail task for assessing the function of switching retrieval plan contributed unique variance only in primary one, but not in the other two grades. Shared variance explained by the three measures also suggests interactions among them and they are three separate but interacting functions of the central executive. The finding gives support to the perspective that cognitive functions of attention selection, along with concurrent retrieval and manipulation of information in long-term memory are important to arithmetic competency (Baddeley, 1996). However, decreasing size of variance explained by the verbal trail could be the result of confounding variables such as the articulatory speed of children or the relatively lower difficulty of the tasks to older children in elementary school.

Decreasing Variance by Different Components of Working Memory from Primary One to Primary Three

Here, the important finding that the contribution by measures of working memory to the arithmetic competency tended to decrease as grade level increased, was brought out. When all the measures for the three working memory components and operation spans were included together in a testing model for hierarchical regression analyses, total variances showed a decreasing trend from 96.9% in primary 1 to 82.4 % in primary 3. With reference to separate hierarchical analyses of different components, visuospatial sketchpad showed a larger decrement in unique variance suggesting a decreasing contribution of this domain in explaining differences in elementary arithmetic competency.

Geary and Burlingham-Dubree (1989) suggested that at early stages where the child is using concrete representations to aid counting, visuospatial skills and central executive may play a greater role (Geary & Burlingham-Dubree, 1989; Luria, 1980). As children gain more experience with arithmetic procedures and facts, more of this arithmetic knowledge becomes automated in long-term memory that allows many children to use retrieval to solve arithmetic operations. The visuospatial and the central executive may play a lesser role in finding the solution at that point.

However, this finding does not contradict with those of previous studies that emphasize the role of the central executive in children's overall mathematical learning. Bull and Johnston (1997) suggested that the central executive may be most relevant in arithmetic tasks requiring a choice has to be made from many different alternative solution strategies or when older children are asked to solve more complex arithmetic problems, where some parts of the problem require a heuristic solution to be implemented. Studies investigating executive functioning in relation to different arithmetic skills at different levels of education will provide more evidence to support this claim.

General Working Memory and Other Measures of Working Memory

Also of particular interest is the consideration of whether operation span, as a general measure of working memory, adds no further influence to arithmetic ability, beyond the contribution by measures of phonological loop, visuospatial sketchpad and central executive. Hierarchical analysis showed that operation span explained unique variance in the model. This suggests that operation span covers some functions that could not be explained by the three components of working memory.

Group Differences in the Understanding of Place Value and Commutativity

Place value

Understanding of place value concepts was assessed by Digit Representation and Written Columnar Calculation. Performance in Digit Representation progressed as grade level increased and most children in primary three showed no problem in achieving the maximum score. The performance of the low-ability groups was also seen to be comparable to that of the high-ability groups at one grade below. This suggests that a significant delay in understanding of place value could have existed in those low-ability groups. Performance in Columnar Calculation was assessed in terms of number of misalignment and carrying- or borrowing-related errors. In an un-timed condition, no difference in the number of errors was found between the two ability groups in primary three.

Strong and significant negative correlation was found between Digit Representation and Columnar Calculation in the current study. This result supports the findings of Hiebert and Wearne (1996), that there is a close connection between children's understanding of multidigit numbers and their written computational skills. Children who develop the earlier understanding of place value could perform at the higher level of accuracy in written computation.

Commutativity

Understanding of commutativity concepts was assessed by problem comparison task, which involved additive and multiplicative commutativity. Children in Primary 1 were only assessed with additive commutativity. The only group difference in additive commutativity was found between the 2 ability-groups in primary 1, and most of the older children seemed to have understood this concept in addition very well. This is consistent with the notion that most children develop an understanding of commutativity with more exposure to the outcome of simple addition (Baroody & Gannon, 1984). With only a few months of formal education, those children with lower arithmetic competency in primary 1 may not benefit from the limited and erroneous exposure in simple addition, which may, in turn, lead to a delay in the understanding in additive commutativity. Given that children in Primary 2 and Primary 3 already understood additive commutativity, the grade difference that emerged in primary 2 and 3 reflected the difference in multiplicative commutativity. Since the teaching of multiplication is normally introduced in primary two, the development of the related commutativity principle usually takes place one year after the additive principle.

Foil problems were used to detect whether children over-generalized the concepts. In line with the result in Problem-pair Verification tasks is the finding that children of primary 1 would have rather immature understanding of additive commutativity and were most likely to wrongly apply the concepts to other unrelated problem types. For example a few of the children mistakenly treated the answer of 43 + 35 – 26 as equal to the answer of 43 - 35 + 26, reflecting their recognition of the "operand" part of the principle, on the one hand, but their negligence of the "operation" part of the principle, on the other hand.

Group Differences in the Usage of Calculation Principles

Additive Commutativity

On the usage of calculation principles, the results revealed that most children with higher arithmetic ability were able to apply this principle to their problem solving, even as early as primary one, given that they showed a higher tendency to use columnar calculation and backup strategies at this stage. Children with lower arithmetic ability showed delay in the application of the principle. This could be explained by the fact that children with lower arithmetic skills tend to make more errors during their learning of basic arithmetic combinations. This inconsistency in the answer of the same combination would weaken the strength of the commutative relationship between any two addends in an addition condition.

Both ability groups demonstrated high usage of the principle in primary two and three, being assisted by the notable progress of the lower-arithmetic ability group. Formal education and higher exposure to arithmetic may reduce the discrepancy in the application of this principle.

Inversion and Reversal

In comparison with usage of the additive commutativity principle, the usage of the inversion and reversal principles displayed a similar pattern of group differences, except for a lower usage of these two principles in the higher ability group in primary one. The principles of commutativity, inversion, and reversal could be regarded as highly transparent principles, whereby answers to problems can be found in the hinted equation directly without any demand on alteration. The lower usage manifested in inversion and reversal could be due to the difference in successful experiences of exposure to the pre-requisite arithmetic operations; in this case, exposure to subtraction would be the source of difference.

Similar results were found in an attempt to investigate mathematical competencies in children with different patterns of academic achievement (Hanich et al., 2001). Hanich and her colleagues (2001) suggested lags in usage if inversion was due to a weaker or immature relationship between addition and subtraction, which could be the result of instructional approaches.

Operand-plus/minus-one

While increment in usage of this principle was indicated in both higher and lower arithmetic groups, usage was generally less than that of the three calculation principles above, and fewer higher-ability children could reach maximum usage, even at primary three. Unlike the three calculation principles discussed above, the principle of operand-plus/minus-one is less easy to apply. The answers to items for testing these principles are much less apparent that users need to decompose the operands and identify the relationships between operands in the problem pair during the application of the principle. Moreover, the level of accuracy of the application reflected that some children might have encountered difficulty in applying the principle, even though they were aware of the special characteristics between operands in the test items. Moreover, to benefit from the application of this principle, a child should not only be aware of the relationship between operands, but also be able to make accurate judgment of direction of change with respect to the type of operation indicated in the problem.

Operand-Plus/minus-Ten

The usage and accuracy in the principle of operand-plus/minus-ten was even less than that of operand plus-or minus one, which could plausibly be attributed to problem-size effect. Moreover, the tendency of children to apply the operand-plus/minus-one principle associated with the tendency to apply the operand-plus/minus-ten principle. The tendency to use these principles seems to be moderated by a concept termed “confidence criterion”, which was proposed by Siegler (1988). The confidence criterion in this case would represent an internal representation against which the child gauges confidence in applying a principle correctly. Hence, children with a rigorous confidence criterion would only apply it whenever they are certain it is correct, whereas children with a more lenient criterion would use it whenever they know it is applicable.

It is also worth noting that the correlations between working memory and the two less transparent calculation principles were the highest among the five calculation principles in question. Successful application of these principles takes into account the size and judgment on the direction of change, which requires higher working memory capacity.

Patterns of contribution of working memory, understanding of Concepts, and use of calculation principles

The current study attempted to explore the contribution of working memory, understanding of fundamental concepts, and use of calculation principles to the individual differences in arithmetic competency. Hierarchical regression analysis of the model, which consisted of all measures of working memory, understanding of concepts, and uses of calculation principles, showed that each aspect contributed unique variance to arithmetic ability in all three grades. Shared variance explained by the three aspects also suggests interactions among them.

In line with the analyses of models with only working memory measures, decrement in total variance of working memory was indicated in this model, though to a lesser extent. A similar trend was indicated in the conceptual understanding. The explanation could be straightforward, as smaller differences in conceptual understanding were found between the two ability groups in the primary three children. Moreover, an increasing trend in the variances explained by the use of calculation principles was observed, through in a smaller magnitude. The size of the unique variances explained by the use of calculation principles in all three grades was smaller than that by the use of working memory, but was shown to be comparable to the unique variance explained by the understanding of concepts at the level of primary 3, though significant in all three grades. This finding suggests that application of calculation principles could be becoming more closely related to arithmetic competency throughout early elementary school years, though its role is still far less important than that of working memory.

With respect to the results of the hierarchical regression, it could be concluded that the relationship between working memory functioning and children’s arithmetic competency was not mediated by the children's understanding of place value and commutativity concepts, and the application of calculation principles.

2. Limitation of the study

The variability of strategy choice manifested by the subjects in solving the calculation problems may have affected the result of some of the tasks as measures of designated functions related to working memory. Among the tasks used in this study, Operation Span and Missing Item are the two tasks that were most susceptible to the variability of strategy choice. In particular, the structure of these two tasks is such that subjects could achieve the same score by using very different strategies (Daneman & Tardif, 1987). Such considerations suggest the need to record and analyze exactly how the tasks were performed.

As suggested in the discussion of exposure to arithmetic tasks possibly leading to the development or understanding of some fundamental concepts and relations, the factor of exposure was controlled in this study. Although children taking part in the study were assumed to have received comparable opportunities in access to arithmetic work within the school curriculum, the variation in degree and quality of extracurricular exposure should also be considered as being potentially very significant. Achieving a match in both the level of in-school exposure and extracurricular exposure may further validate the interpretation of findings in the current study.

3. Directions for future study

As indicated in the findings, the contributions of working memory and conceptual understanding, and the use of calculation principles on the differences in arithmetic competency decreased as grade levels rose. Some other factors may come into play as the level of schooling increases. For example, it was observed that some children displayed considerable level of tension and anxiety when they were asked to perform the tasks, particularly those tasks that were related to mathematics. This highlights the question of whether anxiety or other emotional factors may interfere with children's arithmetic performance. Ashcraft and Kirk (2001) found that reduced working memory capacity led to a pronounced increase in reaction time and errors when mental addition was performed concurrently with a memory load task. Generally, the results demonstrated that math anxiety, through a disturbance in working memory, affects on-line performance in math-related tasks. Future studies may need to include the role of anxiety about mathematical tasks and its affect on children's arithmetic competency.

The hypothesis that variations of strategy mediate executive functions of working memory on arithmetic competency has not been confirmed. Given its potential effects on time and accuracy of working memory span task, it is suggested that arithmetic strategies choice used by local children in solving arithmetic problems should be included in future studies in combination with other cognitive factors.

4. Conclusion

This study set out to examine the roles of working memory, understanding of fundamental concepts, and uses of calculation principles in arithmetic competency of lower primary school children. Children of different grades and levels of arithmetic ability generally showed differences in performances in most of the measures of working memory. Moreover, while understanding of concepts and uses of calculation principles were found to increase with schooling, the differences between the performances of higher and lower arithmetic ability groups were found to decrease as grade levels increase.

Although no causal relationship between the studied cognitive abilities and arithmetic competency could be concluded, the information regarding the group differences in children's performances in the cognitive tasks shed light on the potential value of launching of working memory training and direct instruction on arithmetic concepts and principles in helping elementary school children with difficulties in arithmetic.

Furthermore, all the three cognitive aspects in question were shown to be significant and unique, as well as interactive in their influences on arithmetic competency in each grade. However, the influence of them altogether on elementary arithmetic competency was found to decrease with schooling.

It is not possible to provide a complete explanation of why some children have lower arithmetic competency, as there are also many social, motivational, and educational factors involved. Consideration of these factors will help us to gain greater insight into the issue of children's arithmetic competency and mathematical development in future studies.

Bibliography

Alloway, T. P., Gathercole, S. E., Willis, C., & Adams, A. M. (2004). A structural analysis of working memory and related cognitive skills in young children. *Journal of Experimental Child Psychology, 87*(2), 85-106

Ashcraft, M. H. (1995). Cognitive arithmetic: A review of data and theory. *Cognition, 44,* 75-106.

Ashcraft, M. H. & Kirk, E. P. (2001). The relationships among working memory, math anxiety, and performance. Journal of Experimental Psychology: General, 130*, 224-237.*

Baddeley, A. D. (1996). Exploring the central executive. *The Quarterly Journal of Experimental Psychology, 49*A, 5-28.

Baddeley, A.D. (1986). *Working Memory.* Oxford: Clarendon Press.

Baddeley, A. D., & Hitch, G. J. (1974). Working memory. In G. H. Bower (Ed.), *The psychology of learning and motivation: Advances in research and theory* (Vol. 8 pp.47-90). New York: Academic Press.

Baddeley, A.D., Bressi, S., Della-Sala, S., Logic, R., & Spinnler, H. (1991). The decline of working memory in Alzheimer's disease: A longitudinal study. *Brain, 114*, 2521-2542.

Baroody, A. J. (1999). Children's relational knowledge of addition and subtraction. *Cognition and Instruction*, *17*, 137–175.

Baroody, A. J. (1994). An evaluation of the evidence supporting fact-retrieval models. *Learning and Individual Differences, 6*, 1-36

Baroody, A. J., & Gannon, K. E. (1984). The development of the commuttivity principle and economical addition strategies. *Cognition and Instruction, 1*, 321-339.

Baroody, A. J., & Ginsburg, H. P. (1986) The relationship between initial meaningful and mechanical knowledge of arithmetic. In J. Hiebert (Ed.), *Conceptual and procedural* *knowledge: The case of mathematics* (pp. 75-112). Hillside, NJ: Lawrence Erlbaum Associates.

Baroody, A. J., & Dowker, A. (Eds.). (2004). *The development of arithmetic concepts and skills: Constructing adaptive expertise.* Mahwah, NJ: Lawrence Erlbaum Associates.

Bisanz, J. (1999). The development of mathematical cognition: Arithmetic. *Journal of Experimental Child Psychology, 74*, 153-156.

Briars, D., & Siegler, R.S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology 20*, 607–618.

Bryant, P., Christie, C., & Rendu, A. (1999). *Children's understanding of the relation between addition and subtraction: Inversion, identity, and decomposition. Journal of Experimental Child Psychology 74*, 194-212.

Bull, R., & Johnston, R. S. (1997). Children's arithmetic difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology, 65*, 1-24.

Bull, R., Johnston, R.S. & Roy, J.A.(1999). Exploring the roles of the visual-spatial sketchpad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology 15,* 421–442.

Butterworth, B., Cipolotti, L., & Warrington, E. K. (1996). Short-term memory impairment and arithmetical ability. *Quarterly Journal of Experimental Psychology, 49*A, 251-262.

Campbell, J.I.D. (1994). Architectures for numerical cognition. *Cognition 53*, 1–44.

Case, R. (1992). The role of the frontal lobes in the regulation of cognitive development. *Brain and Cognition, 20*, 51-73.

Case, R. (1985). *Intellectual development: Birth to Adulthood*. New York: Academic Press.

Case, R. M., Kurland, D. M. & Goldberg, J. (1982). Operational efficiency and the growth of short-term memory span. *Journal of Experimental Child Psychology, 33*, 386-404.

Chiappe, P., Hasher, L., & Siegel, L. S. (2000). Working memory, inhibitory control, and reading disability. *Memory-and-Cognition, 28* (1), 8-17.

Corsi, P. M. (1972). Human memory and the medical temporal region of the brain. Doctoral dissertation, McGill University, Montreal, Canada.

Daneman, M. & Tardiff, T. (1987). Working memory and reading skill reexamined. In M. Coltheart (Ed.), *Attention and Performance* (Vol. 8 pp.491-508). London: Erlbaum.

Daneman, M., & Carpenter, P. A. (1980). Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behaviour, 19*, 450-466.

Davis, A., & Bamford, G. (1995). The effect of imagery on young children's ability to solve simple arithmetic. *Education Section Review, 19,* 61-68.

Dehaene, S. (1997). *The number sense*. New York: Oxford University Press.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*, 1-42.

DeStefano, D., & LeFevre, J. A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology, 16*(3), 353-386.

Diamond, A. (1989). *The development and neural bases of higher mental functions*. New York: New York Academy of Science.

Dowker, A. D. (1998). Individual differences in normal arithmetical development. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 275–302). Hove, England: Taylor & Francis.

Ericsson, K. A., & Kintsch, W. (1995). Long-term working memory. Psychological Review, 102, 211-245.

Fuson, K. C., & Briars, D. J. (1990). Using a base-ten blocks learning/ teaching approach for first- and second-grade place-value and multidigit addition and subtraction. *Journal for Research in Mathematicas Education, 21*, 180-206.

Fuson, K.C. (1988). *Children's counting and concepts of number*. New York: Springer–Verlag.

Gathercole, S. E., & Adams, A. (1994). Children's phonological working memory: Contributions of long term knowledge and rehearsal. *Journal of Memory and Language, 33*, 685-701.

Geary, D.C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology, 49*, 363-383.

Geary, D.C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*, 345-362.

Geary, D.C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association.

Geary, D. C. (2004). Learning disabilities in arithmetic: problem-solving differences and cognitive deficit. In H. L. Swanson, K. R. Harris, & S. Graham (Eds.), Handbook of learning disabilities. (pp 199-212). New York, NY: Guilford Press.

Geary, D.C., Bow-Thomas, C. C., & Yao, Y. (1992). Counting knowledge and skill in cognitive addition: A comparison of normal and mathematically disabled children. *Journal of Experimental Child Psychology, 54*, 372-391.

Geary, D.C., Bow-Thomas, C.C., Liu, F. and Siegler, R.S. (1996). Development of arithmetical competencies in Chinese and American children: Influence of age, language, and schooling. *Child Development 67*, 2022–2044.

Geary, D. C., & Brown, S. C. (1991). Cognitive addition: Strategy choice and speed of processing differences in gifted, normal, and mathematically disabled children. *Developmental Psychology, 27*, 398-406.

Geary, D.C., & Burlingham-Dubree, M. (1989). External validation of the strategy choice model for addition. *Journal of Experimental Child Psychology, 47*, 175-192.

Geary, D.C., Hamson, C.O. and Hoard, M.K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology 77*, 236–263.

Geary, D.C. and Hoard, M.K. (2001). Numerical and arithmetical deficits in learning-disabled children: Relation to dyscalculia and dyslexia. *Aphasiology 15*, 635–647.

Geary, D.C., Hoard, M.K. and Hamson, C.O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology 74*, 213–239.

Gelman, R. & Gallistel, C. R. (1978). *The Child's Understanding of Number*. Cambridge, MA: Harvard University Press.

Gersten, R., & Chard, D. (1999). Number sense: Rethinking arithmetic instruction for students with mathematical disabilities. *Journal of Special Education, 33*(1), 18-28.

Ginsburg, H. P. (1997). Mathematics learning disabilities: A review of developmental psychology. *Journal of Learning Disabilities, 30*, 20-33.

Gross-Tsur, V., Manor, O. and Shalev, R.S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology 38*, 25–33.

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. J*ournal of Educational Psychology, 93*(3), 615-626.

Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multidigit addends. *Current Psychology of Cognition, 13*, 207-245.

Hiebert , J., & Lefevre, P. (1986). Conceptual and procedural knowledge in amthematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 29-57). Hillsdale, NJ: Lawrence Erlbaum Associates.

Hiebert, J., & Wearne, D. (1996). Instruction, understanding, and skill in multidigit addition and subtraction. *Cognition and Instruction*, *14*, 251–283.

Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, *82*, 375-386.

Ho, C. S.-H., & Cheng, F. S.-F. (1997). Training in place-value concepts improves children's addition skills. *Contemporary Educational Psychology*, *22*, 495-506.

Huttenlocher, J., Jordan, N. C., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General*, *123*, 284–296.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology*, *28*, 644–653.

Jordan, N.C. & Hanich, L.B. (2000). Mathematical thinking in second-grade children with different forms of LD. *Journal of Learning Disabilities 33*, 567–578.

Jordan, N.C., Hanich, L.B. and Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development 74*, 834–850.

Kail, R., & Hall, L. K. (1999). Sources of developmental change in children's word-problem performance. *Journal of Educational Psychology, 91*, 660-668.

Kaufamn, L. (2002). More evidence for the role of the central executive in retrieving arithmetic facts—A case study of severe developmental dyscalculia. *Journal of Clinical and Experimental Neuropsychology*, *24*(3), 302-310.

Keeler, M. L., & Swanson, H. L. (2001). Does strategy knowledge influence working memory in children with mathematical disabilities? *Journal of Learning Disabilities, 34*(5), 418-434.

Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities 7*, 164–177.

Kyllonen, P. C. & Christal, R. E. (1990). Reasoning ability is (little more than) working memory capacity . *Intelligence*, *14*, 389-433.

Lee, K.-M., & Kang, S.-Y. (2002). Arithmetic operation and working memory: Differential suppression in dual tasks. *Cognition, 83*, B63-B68.

Lehto, J. (1995). Working memory and school achievement in the ninth form. *Educational Psychology, 15*, 271-281.

Lehto, J. (1996). Are executive function tests dependent on working memory capacity? *Quarterly Journal of Experimental Psychology, 49*A, 29-50.

Levine, S. C., Jordan, N. C., & Huttenlocher, J. (1992). Development of calculation abilities in young children. *Journal of Experimental Child Psychology*, *53*, 72–103.

Logie, R. H. (1991). Visuo-spatial working memory: Visual working memory or visual buffer? In C. Cornoldi & M. McDaniel (Eds.), *Imagery and cognition* (pp. 77-102). Berlin: Springer-Verlag.

Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory and Cognition, 22*, 395-410.

Luria, A. R. (1980). *Higher Cortical Functions in Man*. (2nd ed.). New York: Basic Book.

McKenzie, B., Bull, R., & Gray, C. (2003). The effect of phonological and visual-spatial interference on children's arithmetical performance. *Educational and Child Psychology, 20*(3), 93-108.

McLean, J. F. and Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology 74*, pp. 240–260.

Milner, B. (1971). Interhemisperic differences in the localization of psychological processes on man. *British Medical Bulletin, 27*, 272-277.

Miyake, A., & Shah, P. (Eds). (1999). *Models of working memory: Mechanisms of active maintenance and executive control*, Cambridge, UK: Cambridge University Press.

Nunes, T., & Bryant, P. (1996). *Children doing mathematics*. Oxford: Blackwell.

Ostad, S.A. (1997). Developmental differences in addition strategies: A comparison of mathematically disabled and mathematically normal children. *British Journal of Educational Psychology, 67*, 345–357.

Passolunghi, M. C., & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology, 80*, 44-57.

Petitto, A. L., & Ginsburg, H. P. (1982). Mental arithmetic in Africa and America: Strategies, principles and explanations. *International Journal of Psychology, 17*, 81-102

Phillips, C. E, Jarrold, C., Baddeley, A. D., Grant, J., & Karmiloff-Smith, A. (2004). Comprehension of spatial language terms in Williams syndrome: Evidence for an interaction between domains of strength and weakness. *Cortex, 40* (1), 85-101.

Piaget, J. (1997). *The child's conception of number.* London: Routedge.

Rasmussen, C., Ho, C., & Bisanz, J. (2003). Use of the mathematical principle of inversion in young children. *Journal-of-Experimental-Child-Psychology, 85*(2), 89-102.

Resnick, L. B. (1992). From protoquantities to operators: Building mathematical competence on a foundation of everydayknowledge. In G. Leinhardt, R. Putnam, & R. A. Hattrup (Eds.), analysis of arithmetic for mathematical teaching (pp. 373-425). Hillsdale, NJ: Lawrence Erlbaum Associates.

Resnick, L. B. & Omanson, S. F. (1987). Learning to understand arithmetic. In R, Glaser (Ed.), *Advances in instructional psychology: Vol. 3.* (pp.41-95). Hillsdale, NJ, England: Lawrence Erlbaum Associates.

Reitan, R. M. (1958). Validity of the trail making test as an indicator of organic brain damage. *Perceptual and Motor Skills*, *8*, 271-276.

Rickard. T. C., & Bourne, L. E. (1996). Some tests of an identical elements model of basic arithmetic skills. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 22*, 1281-1295.

Rickard, T. C., Healy, A. F., & Bourne, L. E. (1994). On the cognitive structure of basic arithmetic skills. Operation, order and symbol transfer effects. *Journal of Experimental Psychology: Leaning, memory, and cognition*, *20*, 1139-1153.

Riley, M. S., & Greeno, J. G. (1988). Developmental analysis of understanding language about quantities and of solving problems. *Cognition and Instruction*, *5*, 49–101.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology, 91*, 175-189.

Robinson, C. S., Menchetti, B. M., & Torgesen, J. K. (2002). Toward a two-factor theory of one type of mathematics disabilities. *Learning Disabilities Research & Practice, 17*(2), 81-89.

Rourke, B. P. (1993). Arithmetic disabilities, specific and otherwise: A neuropsychological perspective. *Journal of Learning Disabilities*, *26*, 214-226.

Rourke, B. P., & Conway, J. A. (1997). Disabilities of arithmetic and mathematical reasoning: Perspectives from neurology and neuropsychology. *Journal of Learning Disabilities, 30*, 34–46.

Schlieman, A. D., Araujo, C., Cassunde, M. A., Macedo, S., & Niceas, L. (1998). Use of Use of multiplicative commutativity by school children and street sellers. *Journal for Research in Mathematics Education, 29*(4), 422-435.

Seron, X., Pesenti, M., Noel, M., Deloche, G., & Comet, J. (1992). Images of numbers, or "When 98 is upper left and 6 sky blue." *Cognition, 44*, 159-196.

Shalev, R.S., Manor, O., Kerem, B., Ayali, M., Badichi, N., Friedlander, Y. and Gross-Tsur, V., 2001. Developmental dyscalculia is a familial learning disability. *Journal of Learning Disabilities*, *34*, 59–65.

Shallice, T. (1982). Specific impairments of planning. *Philosophical Transactions of the Royal Society of London, B, 298,* 199-209.

Siegel, L. S. & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development*, *60*, 973-980.

Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies*. Hillsdale, NJ: Erlbaum.

Siegler, R.S., 1988. Individual differences in strategy choices: Good students, not-so-good students, and perfectionists. *Child Development*, *59*, 833–851.

Siegler, R.S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press.

Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *Origins of cognitive skills* (pp. 229–293). Hillsdale, NJ: Erlbaum.

Swanson, H. L. (1993). Working memory in learning disability subgroups. *Journal of Experimental Child Psychology, 56,* 87-114.

Swanson, H. L., Cooney, J. B., & Brock, S. (1993). The influence of working memory and classification ability on children's word problem solution. *Journal of Experimental Child Psychology*, *55*, 374-395.

Swanson, H. L., & Howell, M. (2001). Working memory, short-term memory, and speech rate as predictors of children's reading performance at different ages. *Journal of Educational Psychology, 93* (4), 720-734.

Swanson, H. L., & Sachse-Lee, C. (2001). Mathematical problem solving and working memory in children with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Child Psychology, 79*, 294-321.

Temple, C. M. (1991). Procedural dyscalculia and number fact dyscalculia: Double dissociation in developmental dyscalculia. *Cognitive Neuropsychology, 8*, 155-176.

Torgesen, J. K. (1999). Phonological based reading disabilities: Toward a coherent theory of one kind of learning disability. In L. Spear-Swerling & R. J. Sternberg (Eds.), *Perspective on learning disabilities* (pp.231-262). New Haven, CT: Westview Press.

Towse, J. N., Hitch, G. J., & Hutton, U. (1998). A reevaluation of working memory capacity in children. *Journal of Memory and Language, 39*, 195-217.

Turner, M. L., & Engle, R. W. (1989). Is working-memory capacity task-dependent? Journal of memory and Language, 28, 127-154.

Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert & M. Behr (Eds.), Number concepts and operations in the middle grades (pp.141-161). Hillsdale, NJ: Erlbaum, and Reston, VA: National Council of Teachers of Mathematics.

Wechsler, D. (1974). *Manual for the Wechsler Intelligence Scale for Children-Revised.* New York: Psychological Corporation.

Welsh, M. C., & Pennington, B. F. (1988). Assessing fronta lobe functioning in children: Views from developmental psychology. *Developmental Neuropsychology, 4*, 199-230.

Appendix 1

Consent Letter to School and Parents

尊敬的 **校長 / 家長**：

<u>本地兒童數學運算與認知能力之研究</u>

香港理工大學護理學院「宏利兒童學習潛能發展中心」現正進行一項有關於本地初小學童數學運算能力與認知能力的研究，目的是要探討初小學童數學運算能力與其本身認知能力(如工作記憶能力、數學原則的認識與運用等)的關係。現特來函邀請 貴校參與是項研究的活動。

本研究除了探討初小學童數學運算能力及其數理的認識及應用三者之間的關係外，亦會比較數學能力稍遜的學童與一般學童的工作記憶能力及數理的認識與運用於初小(從小一至小三)的階段中的發展。有關的研究結果將有發展配合學生學習模式和差異的數學支援課程。

本研究將分爲兩個階段。首階段主要工作爲於小一至小三的級別中選擇受試學童。選擇方法包括 1)集體運算能力與智力測試，及 2)數學科老師推荐。集體測試將需要小一至小三的學童完成一項非語文智力測驗(約 30 分鐘)及於限時內完成一份基本數學運算工作紙(時限爲 30 分鐘)。研究人員將根據學童數學運算工作的表現，於小一至小三各級中選擇約 25 位數學運算能力稍遜及另外約 25 位數學運算能力中等的學童進行第二階段的研究。第一階段維期將不多於一週。

第二階段的研究主要是學童個別測試。測試工作內容將包括多項有關工作記憶的測試及有關數理意識及運用的測試。完成以上多項的個別測

試需時共約 50 分鐘。爲減低測試過程對老師及學童的影響，第二階段的測試工作可按學校及學生的情況，於短期內分多節完成。第二階段維期將不多於兩週。

基於是項研究計劃是需要了解初小各級學童於下學期完結前在運算工作上的表現，故此我們希望能獲安排於本年五月期間完成是項研究計劃中的所有測試。

本研究所得的資料，只供集体數據分析及教學研究，將不用於其他途徑，並會絕對保密。有關的研究結果將有發展配合學生學習模式和差異的數學支援課程。

敬祝　教安!

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香港理工大學
護理學院
「宏利兒童學習潛能發展中心」

<u>本地兒童數學運算與認知能力之研究</u>
<u>學校回條</u>

- 本校 願意 / 不願意 (*請刪去其一*)參與上述研究。
- 校方明白此項研究的目的，亦明白個人資料將會保密。

學校：______________________

校方負責人：______________________

聯絡電話：______________________

回覆日期: ______________________

香港理工大學
護理學院
「宏利兒童學習潛能發展中心」

<u>本地兒童數學運算與認知能力之研究</u>
<u>家長回條</u>

- 本人 *願意／不願意 子女 ____________(班別)________ (子/女姓名)於校內參與上述研究。
- 本人明白此項研究的目的，亦明白個人資料將會保密。本人亦可隨時終止子女參與是項研究。

家長簽名：______________________

家長姓名：______________________

填表日期：______________________

聯絡電話：______________________

Appendix 1

Materials of the Experiment (A Partial Listing)

1. Arithmetic Test

1.1 Problem types of the Test for Primary 1

Problem Type	Example
1 to 1-digit Addition, without carrying	3 + 6 =
1 to 1-digit Addition, with carrying	4 + 8 =
1 to 1-digit Subtraction, without borrowing	9 – 6 =
2 to 1-digit Addition, without carrying	22 + 7 =
2 to 1-digit Addition, with carrying	23 + 8 =
2 to 1-digit Subtraction, without borrowing	27 – 5 =
2 to 1-digit Subtraction, with borrowing	23 – 6 =
2 to 2-digit Addition, without carrying	23 + 16 =
2 to 2-digit Addition, with carrying	34 + 18 =
2 to 2-digit Subtraction, without borrowing	38 – 25 =
2 to 2-digit Subtraction, with borrowing	32 – 17 =
3 to 2-digit Addition, without carrying	123 + 46 =
3 to 2-digit Addition, with carrying	144 + 67 =
3 to 2-digit Subtraction, without borrowing	147 – 34 =
3 to 2-digit Subtraction, with borrowing	134 – 47=
2 to 2 to 2-digit Addition, without carrying	12 + 23 + 14 =
2 to 2 to 2-digit Addition, with carrying	17 + 24 + 16 =
2 to 2 to 2-digit Subtraction, without borrowing	58 – 12 – 15 =
2 to 2 to 2-digit Subtraction, with borrowing	53 – 18 – 16 =
2 to 2 to 2-digit Mixed (+ -), with trading	53 – 16 + 29 =
2 to 2 to 2-digit Mixed (- +), with trading	34 – 27 + 16 =

1.2 Problem types of the Test for Primary 2

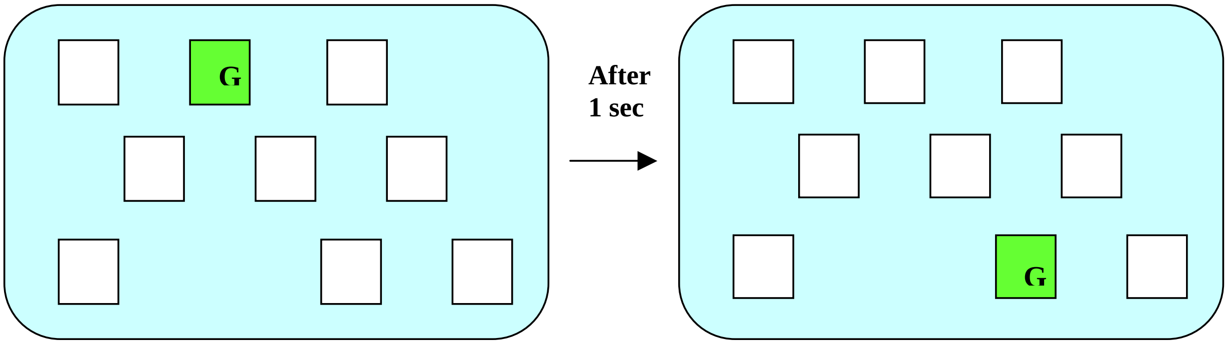
Problem Type	Example
2 to 2-digit Addition, with carrying	24 + 18 =
2 to 2-digit Subtraction, with borrowing	32 – 17 =
3 to 2-digit Addition, without carrying	123 + 46 =
3 to 2-digit Addition, with carrying	144 + 67 =
3 to 2-digit Subtraction, without borrowing	147 – 34 =
3 to 2-digit Subtraction, with borrowing	134 – 47=
3 to 3-digit Addition, without carrying	124 + 135 =
3 to 3-digit Addition, with carrying	157 + 165 =
3 to 3-digit Subtraction, without borrowing	187 – 146 =
3 to 3-digit Subtraction, with borrowing	136 – 128 =
2 to 2 to 2-digit Addition, with carrying	17 + 24 + 16 =
2 to 2 to 2-digit Subtraction, with borrowing	53 – 18 – 16 =
2 to 2 to 2-digit Mixed (+ -), with trading	48 + 23 – 32 =

2 to 2 to 2-digit Mixed (- +), with trading	34 – 27 + 16 =
3 to 3 to 3-digit addition, without carrying	132 + 125 + 102 =
3 to 3 to 3-digit addition, with carrying	147 + 135 + 152 =
3 to 3 to 3-digit subtraction, without borrowing	289 – 124 – 133 =
3 to 3 to 3-digit subtraction, with borrowing	286 – 128 – 139 =
3 to 3 to 3-digit Mixed (+ -), with trading	144 + 137 – 108 =
3 to 3 to 3-digit Mixed (- +), with trading	157 – 148 + 116 =
1 to 1 Multiplication (1)	4 x 6 =
2 by 1 Division, without residual	42 ÷ 6 =

1.3 Problem types of the Test for Primary 3

Problem Type	**Example**
3 to 3-digit Addition, with carrying	157 + 165 =
3 to 3-digit Subtraction, with borrowing	136 – 128 =
2 to 2 to 2-digit Addition, with carrying	17 + 24 + 16 =
2 to 2 to 2-digit Subtraction, with borrowing	53 – 18 – 16 =
2 to 2 to 2-digit Mixed (+ -), with trading	48 + 23 – 32 =
3 to 3 to 3-digit addition, with carrying	147 + 135 + 152 =
3 to 3 to 3-digit subtraction, with borrowing	297 – 128 – 139 =
3 to 3 to 3-digit Mixed (+ -), with trading	238 + 323 – 452 =
3 to 3 to 3-digit Mixed (- +), with trading	157 – 148 + 116 =
2 to 1 Multiplication, without carrying	12 x 4 =
2 to 1 Multiplication, with carrying	14 x 6 =
3 to 1 Multiplication, without carrying	232 x 3 =
3 to 1 Multiplication, with carrying	146 x 6 =
2 by 1 Division, without residual	42 ÷6 =
2 by 1 Division, with residual	47 ÷8 =
3 by 1 Division, without residual	182 ÷7 =
3 by 1 Division, with residual	206 ÷ 6 =
2 to 1 to 1-digit Mixed (+ x), with trading	24 + 8 x 7 =
2 to 1 to 1-digit Mixed (- x), with trading	64 – 9 x 3 =
2 to 2 to 1-digit Mixed (+ ÷), with trading	48 + 72 ÷ 8 =
2 to 2 to 1-digit Mixed (- ÷), with trading	42 – 35 ÷ 7 =

2. Squares Span.



Example of Squares Span with Span length of 2
(G = Colored in Green)

3. Crossing Out Task

目標字	*例子*											
1	~~**1**~~	**2**	**7**	**4**	**1**	**3**	**2**	**1**	**5**	**2**	**1**	**8**
9	**1**	**9**	**3**	**1**	**7**	**9**	**4**	**8**	**9**	**1**	**6**	**5**
7	**9**	**1**	**7**	**6**	**9**	**7**	**5**	**2**	**9**	**3**	**7**	**8**

4. Missing Item Task

<table>
<tr><td>3 + 4 = A = 2 + B</td><td>B=</td><td rowspan="4">Remarks
“A” is an oral response where
“B” is a written response</td></tr>
<tr><td>6 + 2 = A = 5 + B</td><td>B=</td></tr>
<tr><td>8 + 2 = A = 6 + B</td><td>B=</td></tr>
<tr><td>7 + 4 = A = 8 + B</td><td>B=</td></tr>
</table>

5. Operation Span

Normal Version	Extended Version
2 + 3 = a1 4 + 5 = a2 ***Recall a1, a2*** 4 - 1 = b1 6 + 3 = b2 7 - 5 = b3 ***Recall b1, b2, b3*** 3 + 8 = c1 9 - 3 = c2 1 + 7 = c3 6 + 5 = c4 ***Recall c1, c2, c3, c4***	2 + 3 + 1 - 0 = a1 4 + 5 + 0 - 1= a2 ***Recall a1, a2*** 4 - 1 - 1 + 0= b1 6 + 3 - 0 + 1= b2 7 - 5 - 1 + 0= b3 ***Recall b1, b2, b3*** 3 + 8 + 0 - 1= c1 9 - 3 + 0 - 1= c2 1 + 7 + 0 - 1= c3 6 + 5 - 0 + 1= c4 ***Recall c1, c2, c3, c4***

6. Digit Representation.

	Correct Response	Incorrect Response
453	50 (Written) "Five-Ten" (Representing *fifty* orally)	5 (Written) "Five" (orally)
2798	700 (Written) "Seven-hundred" (orally)	70 (Written) "Seven-Ten" (Representing *seventy* orally)

7. Problem-pair Comparison

2 x 8	**8 x 2**
45 + 26 + 17	**45 + 17 + 26**
47 - 31 + 25 (Foil)	**31 – 25 + 47**
61 + 43 (Foil)	**61 – 43**

8. Hinted Calculations.

Additive Commutativity	
23 + 65 = <u>88</u>	65 + 23 =
42 + 13 + 27 = <u>82</u>	27 + 42 + 13 =
34 + 21 + 17 = <u>72</u> (Foil)	31 + 22 + 19 =
28 + 12 – 37 = <u>3</u> (Foil)	37 + 12 – 28 =

Inversion	
41 + 23 = <u>64</u>	64 - 23 =
45 – 23 = <u>22</u>	45 – 22 =
46 – 12 = <u>34</u> (Foil)	34 – 12 =

Reversal	
54 – 37 = <u>17</u>	54 - 17 =
57 – 24 = <u>33</u>	58 – 33 =
16 + 18 = <u>34</u> (Foil)	16 + 34 =

Operand-plus/minus-one	
24 + 47 = <u>71</u>	24 + 48 =
44 – 26 = <u>18</u>	44 – 25 =

Operand-plus/minus-ten	
21 + 34 = <u>55</u>	21 + 44 =
42 + 21 = <u>63</u>	52 + 21 =